# INTERNATIONAL STANDARD

ISO 17123-6

Second edition 2012-06-01

# Optics and optical instruments — Field procedures for testing geodetic and surveying instruments —

# Part 6: **Rotating lasers**

Optique et instruments d'optique — Méthodes d'essai sur site des instruments géodésiques et d'observation —

Partie 6: Lasers rotatifs



Reference number ISO 17123-6:2012(E)



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#### **Foreword**

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 17123-6 was prepared by Technical Committee ISO/TC 172, *Optics and photonics*, Subcommittee SC 6, *Geodetic and surveying instruments*.

This second edition cancels and replaces the first edition (ISO 17123-6:2003), which has been technically revised.

ISO 17123 consists of the following parts, under the general title *Optics and optical instruments* — *Field procedures for testing geodetic and surveying instruments*:

- Part 1: Theory
- Part 2: Levels
- Part 3: Theodolites
- Part 4: Electro-optical distance meters (EDM measurements to reflectors)
- Part 5: Total stations
- Part 6: Rotating lasers
- Part 7: Optical plumbing instruments
- Part 8: GNSS field measurement systems in real-time kinematic (RTK)

Annexes A, B and C of this part of ISO 17123 are for information only.

#### Introduction

This part of ISO 17123 specifies field procedures for adoption when determining and evaluating the uncertainty of measurement results obtained by geodetic instruments and their ancillary equipment, when used in building and surveying measuring tasks. Primarily, these tests are intended to be field verifications of suitability of a particular instrument for the immediate task. They are not proposed as tests for acceptance or performance evaluations that are more comprehensive in nature.

The definition and concept of uncertainty as a quantitative attribute to the final result of measurement was developed mainly in the last two decades, even though error analysis has already long been a part of all measurement sciences. After several stages, the CIPM (Comité Internationale des Poids et Mesures) referred the task of developing a detailed guide to ISO. Under the responsibility of the ISO Technical Advisory Group on Metrology (TAG 4), and in conjunction with six worldwide metrology organizations, a guidance document on the expression of measurement uncertainty was compiled with the objective of providing rules for use within standardization, calibration, laboratory, accreditation and metrology services. ISO/IEC Guide 98-3 was first published as the *Guide to the Expression of Uncertainty in Measurement* (GUM) in 1995.

With the introduction of uncertainty in measurement in ISO 17123 (all parts), it is intended to finally provide a uniform, quantitative expression of measurement uncertainty in geodetic metrology with the aim of meeting the requirements of customers.

ISO 17123 (all parts) provides not only a means of evaluating the precision (experimental standard deviation) of an instrument, but also a tool for defining an uncertainty budget, which allows for the summation of all uncertainty components, whether they are random or systematic, to a representative measure of accuracy, i.e. the combined standard uncertainty.

ISO 17123 (all parts) therefore provides, for defining for each instrument investigated by the procedures, a proposal for additional, typical influence quantities, which can be expected during practical use. The customer can estimate, for a specific application, the relevant standard uncertainty components in order to derive and state the uncertainty of the measuring result.



# Optics and optical instruments — Field procedures for testing geodetic and surveying instruments —

## Part 6:

### **Rotating lasers**

#### 1 Scope

This part of ISO 17123 specifies field procedures to be adopted when determining and evaluating the precision (repeatability) of rotating lasers and their ancillary equipment when used in building and surveying measurements for levelling tasks. Primarily, these tests are intended to be field verifications of the suitability of a particular instrument for the immediate task at hand and to satisfy the requirements of other standards. They are not proposed as tests for acceptance or performance evaluations that are more comprehensive in nature.

This part of ISO 17123 can be thought of as one of the first steps in the process of evaluating the uncertainty of a measurement (more specifically a measurand). The uncertainty of a result of a measurement is dependent on a number of parameters. Therefore this part of ISO 17123 differentiates between different measures of accuracy and objectives in testing, like repeatability and reproducibility (between-day repeatability), and of course gives a thorough assessment of all possible error sources, as prescribed by ISO/IEC Guide 98-3 and ISO 17123-1.

These field procedures have been developed specifically for *in situ* applications without the need for special ancillary equipment and are purposefully designed to minimize atmospheric influences.

#### 2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 3534-1, Statistics — Vocabulary and symbols — Part 1: General statistical terms and terms used in probability

ISO 4463-1, Measurement methods for building — Setting-out and measurement — Part 1: Planning and organization, measuring procedures, acceptance criteria

ISO 7077, Measuring methods for building — General principles and procedures for the verification of dimensional compliance

ISO 7078, Building construction — Procedures for setting out, measurement and surveying — Vocabulary and guidance notes

ISO 9849, Optics and optical instruments — Geodetic and surveying instruments — Vocabulary

ISO 17123-1, Optics and optical instruments — Field procedures for testing geodetic and surveying instruments — Part 1: Theory

ISO 17123-2, Optics and optical instruments — Field procedures for testing geodetic and surveying instruments — Part 2: Levels

ISO/IEC Guide 98-3, Uncertainty of measurement — Part 3: Guide to the expression of uncertainty in measurement (GUM:1995)

ISO/IEC Guide 99, International vocabulary of metrology — Basic and general concepts and associated terms (VIM)

#### 3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 3534-1, ISO 4463-1, ISO 7077, ISO 7078, ISO 9849, ISO 17123-1, ISO 17123-2, ISO/IEC Guide 98-3 and ISO/IEC Guide 99 apply.

#### 4 General

#### 4.1 Requirements

Before commencing surveying, it is important that the operator investigates that the precision in use of the measuring equipment is appropriate to the intended measuring task.

The rotating laser and its ancillary equipment shall be in known and acceptable states of permanent adjustment according to the methods specified in the manufacturer's handbook, and used with tripods and levelling staffs as recommended by the manufacturer.

The results of these tests are influenced by meteorological conditions, especially by the temperature gradient. An overcast sky and low wind speed guarantee the most favourable weather conditions. The particular conditions to be taken into account may vary depending on the location where the tasks are to be undertaken. Note should also be taken of the actual weather conditions at the time of measurements and the type of surface above which the measurements are performed. The conditions chosen for the tests should match those expected when the intended measuring task is actually carried out (see ISO 7077 and ISO 7078).

This part of ISO 17123 describes two different field procedures as given in Clauses 5 and 6. The operator shall choose the procedure which is most relevant to the project's particular requirements.

#### 4.2 Procedure 1: Simplified test procedure

The simplified test procedure provides an estimate as to whether the precision of a given item of rotating-laser equipment is within the specified permitted deviation, according to ISO 4463-1.

This test procedure is normally intended for checking the precision (see ISO/IEC Guide 99:2007, 2.15) of a rotating laser to be used for area levelling applications, for tasks where measurements with unequal site lengths are common practice, e.g. building construction sites.

The simplified test procedure is based on a limited number of measurements. Therefore, a significant standard deviation and the standard uncertainty (Type A), respectively, cannot be obtained. If a more precise assessment of the rotating laser under field conditions is required, it is recommended to adopt the more rigorous full test procedure as given in Clause 6.

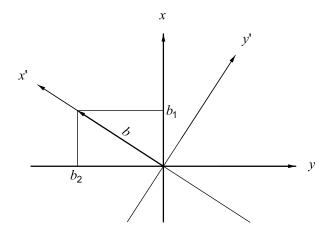
This test procedure relies on having a test field with height differences which are accepted as true values. If such a test field is not available, it is necessary to determine the unknown height differences (see Figures 1 and 2), using an optical level of accuracy (see ISO 17123-2) higher than the rotating laser required for the measuring task. If, however, a test field with known height differences cannot be established, it will be necessary to apply the full test procedure as given in Clause 6.

If no levelling instrument is available, the rotating laser to be tested can be used to determine the true values by measuring height differences between all points with central setups. At each setup, two height differences have to be observed by rotating the laser plane by 180°. The mean value of repeated readings in both positions will provide the height differences which are accepted as true.

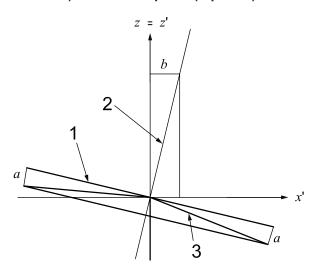
#### 4.3 Procedure 2: Full test procedure

The full test procedure shall be adopted to determine the best achievable measure of precision of a particular rotating laser and its ancillary equipment under field conditions, by a single survey team.

Further, this test procedure serves to determine the deflective deviation, a, and both components,  $b_1$  and  $b_2$ , of the deviation of the rotating axis from the true vertical,  $b = \sqrt{b_1^2 + b_2^2}$ , of the rotating laser (see Figure 1).



#### a) Horizontal plane (top view)



b) Vertical plane through x' (side view)

#### Key

- 1 inclined plane
- 2 cone axis
- 3 inclined cone

Figure 1 — Deflective deviations a and b (see Figure 5)

The recommended measuring distances within the test field (see Figure 3) are 40 m. Sight lengths greater than 40 m may be adopted for this precision-in-use test only, where the project specification may dictate, or where it is determining the range of the measure of precision of a rotating laser at respective distances.

The test procedure given in Clause 6 of this part of ISO 17123 is intended for determining the measure of precision in use of a particular rotating laser. This measure of precision in use is expressed in terms of the experimental standard deviation, s, of a height difference between the instrument level and a levelling staff (reading at the staff) at a distance of 40 m. This experimental standard deviation corresponds to the standard uncertainty of Type A:

 $s_{\text{ISO-ROLAS}} = u_{\text{ISO-ROLAS}}$ 

Further, this procedure may be used to determine:

 the standard uncertainty as a measure of precision in use of rotating lasers by a single survey team with a single instrument and its ancillary equipment at a given time;

- the standard uncertainty as a measure of precision in use of a single instrument over time and differing environmental conditions;
- the standard uncertainties as a measure of precision in use of several rotating lasers in order to enable a comparison of their respective achievable precisions to be obtained under similar field conditions.

Statistical tests should be applied to determine whether the experimental standard deviation, s, obtained belongs to the population of the instrumentation's theoretical standard deviation,  $\sigma$ , whether two tested samples belong to the same population, whether the deflective deviation, a, is equal to zero, and whether the deviation, b, of the rotating axis from the true vertical of the rotating laser is equal to zero.

#### Simplified test procedure

#### Configuration of the test field 5.1

To keep the influence of refraction as small as possible, a reasonably horizontal test area shall be chosen. Six fixed target points, 1, 2, 3, 4, 5 and 6, shall be set up in approximately the same horizontal plane at different distances, between 10 m and 60 m apart from the instrument station S. The directions from the instrument to the six fixed points shall be spread over the horizon as equally as possible (see Figure 2).

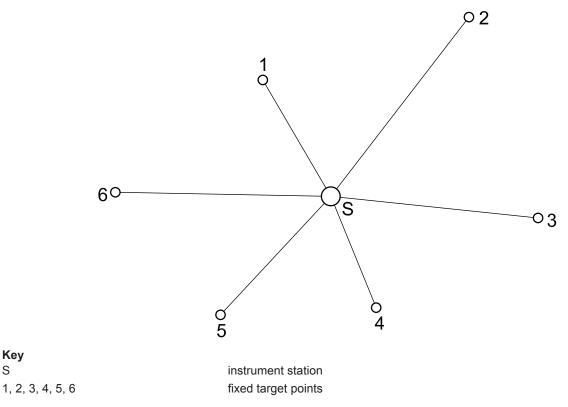


Figure 2 — Configuration of the test field for the simplified test procedure

To ensure reliable results, the target points shall be marked in a stable manner and reliably fixed during the test measurements, including repeat measurements.

The height differences between the six fixed points, 1, 2, 3, 4, 5 and 6, shall be determined using an optical level of known high accuracy as described in Clause 4.

The following five height differences between the t = 6 target points are known:

$$\overline{d} = \begin{pmatrix} \overline{d}_{2,1} \\ \vdots \\ \overline{d}_{t,t-1} \end{pmatrix} \quad t = 2, \dots 6 \tag{1}$$

Key

#### 5.2 Measurements

The instrument shall be set up in a stable manner above point S. Before commencing the measurements, the laser beam shall become steady. To ensure that the laser plane of the instrument remains unchanged during the whole measuring cycle, a fixed target shall be observed before and after each set, j, of measurements, (j = 1, ..., 5).

Six separate readings,  $x_{j,1}$  to  $x_{j,6}$ , on the scale of the levelling staff shall be carried out to each fixed target point, 1, 2, 3, 4, 5 and 6. Between two sets of readings the instrument shall be lifted, turned clockwise approximately 70°, placed in a slightly different position and relevelled. The time between any two sets of readings shall be at least 10 minutes.

Each reading shall be taken in a precise mode according to the recommendations of the manufacturer.

#### 5.3 Calculation

The evaluation of the readings  $x_i$  for each set j is based on the following differences:

$$\begin{pmatrix} d_{j,2,1} \\ \vdots \\ d_{j,t,t-1} \end{pmatrix} = \begin{pmatrix} x_{j,2} - x_{j,1} \\ \vdots & -\vdots \\ x_{j,t} - x_{j,t-1} \end{pmatrix} \quad j = 1, ..., 5 \quad and \quad t = 2, ... 6$$
 (2)

respectively

$$d_j = x_{j,t} - x_{j,t-1} \tag{3}$$

where

t is the number of the target point.

Calculating  $\bar{d}$ , the mean of the differences  $d_j$ , the residual vector of the height differences in set j is obtained by

$$r_j = \overline{d} - d_j$$
  $j = 1, ..., 5$  (4)

Finally the sum of the residual squares of all five sets yields

$$\sum r^2 = \sum_{j=1}^{5} r_j^T r_j \ . \tag{5}$$

 $v = 5 \times (6 - 1) = 25$ 

is the corresponding number of degrees of freedom.

$$s = \sqrt{\frac{\sum r^2}{v}} = u_{\rm ISO} \tag{6}$$

where s is the experimental standard deviation and  $u_{ISO}$  the standard uncertainty (Type A) of a single measured height difference,  $d_{j,t,t-1}$ , between two points of the test field. This represents in this part of ISO 17123 a measure of precision relative to the standard uncertainty of a Type A evaluation. This value includes systematic and random errors.

#### 6 Full test procedure

#### 6.1 Configuration of the test line

To keep the influence of refraction as small as possible, a reasonably horizontal test area shall be chosen. The ground shall be compact and the surface shall be uniform; roads covered with asphalt or concrete shall be avoided. If there is direct sunlight, the instrument and the levelling staffs shall be shaded, for example by an umbrella.

Two levelling points, A and B, shall be set up approximately 40 m apart. To ensure reliable results, the levelling staffs shall be set up in stable positions, reliably fixed during the test measurements, including any repeat measurements. The instrument shall be placed at the positions S1, S2 and S3. The distances from the instrument's positions to the levelling points shall be in accordance with Figure 3. The position S1 shall be chosen equidistant between the levelling points, A and B (40/2 = 20 m).

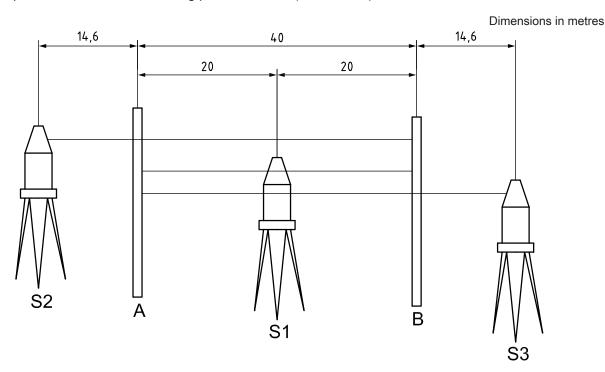


Figure 3 — Configuration of the test line for the full test procedure

#### 6.2 Measurements

Before commencing the measurements, the instrument shall be adjusted as specified by the manufacturer.

For the full test procedure, i=4 series of measurements should be performed. In each series, three instrument setups S1, S2 and S3 are chosen, according to the configuration given in Figure 3. At any setup n=4 sets of readings are taken. Each set consists of two readings  $x_{Aj}$ , and  $x_{Bj}$ , namely to rod A and to rod B. After each set, the orientation of the instrument has to be changed clockwise about 90° (see Figure 4). Hence one series consists of  $j=3\times 4=12$  readings for each rod. In order to ensure that the instrument deviation b is aligned properly during the measurements, the instrument has to be oriented at the three positions S1, S2 and S3 in the same direction and the sense of rotation has to be maintained.

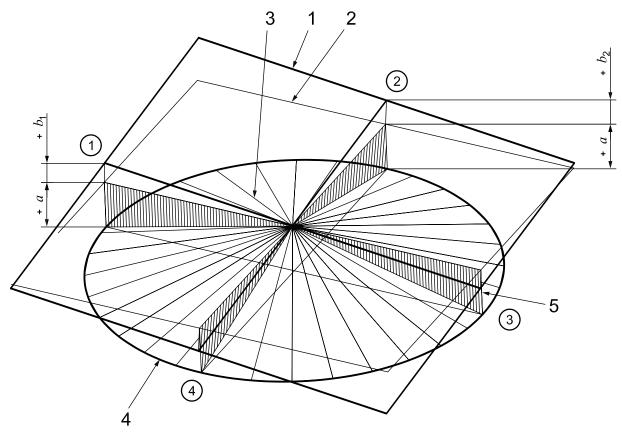
With each new setup of the chosen reference direction (reference marks on the tripod head), the instrument shall be relevelled carefully. If the instrument is provided with a compensator, care shall be taken that it functions properly. It is recommended to assign the four orientations of the instrument on the ground plate. The numbering of the 12 measurements can be represented for each measuring set as shown in Figure 4. All readings shall be taken in a precise mode according to the recommendations of the manufacturer.

Instrument so for each the si = 1,,	series	<b>I-</b> A	61 <b>I</b> B	S2 A	<b>I</b> B	<b>I</b>	S3 <b>I</b> → B
Set $n$ orienta $n = 1, \dots,$		Reading j = 1	s x <sub>Aj</sub> , x <sub>Bj</sub> ,, 4	Readings j = 5,	<i>x</i> <sub>A<i>j</i></sub> , <i>x</i> <sub>B<i>j</i></sub> , 8	Readi <i>j</i> =	ings $x_{A_j}, x_{B_j}$ 9, , 12
Set 1	$\sim$	<i>x</i> <sub>A1</sub>	<i>x</i> <sub>B1</sub>	<i>x</i> <sub>A5</sub>	<i>x</i> <sub>B5</sub>	<i>x</i> <sub>A9</sub>	<i>x</i> <sub>B9</sub>
Set 2	<del>)</del>	<i>x</i> <sub>A2</sub>	$x_{B2}$	<i>x</i> <sub>A6</sub>	$x_{B6}$	<i>x</i> <sub>A10</sub>	$x_{B10}$
Set 3	7	$x_{A3}$	$x_{B3}$	<i>x</i> <sub>A7</sub>	<i>x</i> <sub>B7</sub>	<i>x</i> <sub>A11</sub>	<i>x</i> <sub>B11</sub>
Set 4	<del>\</del>	$x_{A4}$	$x_{B4}$	<i>x</i> <sub>A8</sub>	x <sub>B8</sub>	<i>х</i> <sub>А12</sub>	<i>x</i> <sub>B12</sub>

Figure 4 — Arrangement of measurements

#### 6.3 Calculation

The possible deviations of a rotating laser may be modelled as shown in Figure 5.



#### Key

- 1 horizontal plane
- 2 inclined plane
- 3 inclined cone
- 4 radius of cone = 40 m
- 5 height of cone, a
- O direction

Figure 5 — Model of instrument deviations

In order to create a horizontal sighting in the described measuring configuration, the readings at the levelling staffs for selected sighting distances can be corrected in respect of the deviations a and b (see Table 1).

Table 1 — Corrections of the readings

Direction		Distance	
Direction	14,6 m	20,0 m	54,6 m
1	$0,365(a+b_1)$	$0,500(a+b_1)$	$1,365(a+b_1)$
2	$0,365(a+b_2)$	$0,500(a+b_2)$	$1,365(a+b_2)$
3	$0,365(a-b_1)$	$0,500(a-b_1)$	1,365( <i>a</i> – <i>b</i> <sub>1</sub> )
4	$0,365(a-b_2)$	$0,500(a-b_2)$	1,365( <i>a</i> – <i>b</i> <sub>2</sub> )

From the observation formulae for the  $i^{th}$  series, the residuals,  $r_1$  to  $r_{12}$ , are obtained (see Table 2).

Table 2 — Observation formulae for the ith series

p = 2,0	p = 0.5	p = 0.5
$r_1 = h - b_1 - (x_{B,1} - x_{A,1})$	$r_5 = h + a - b_1 - (x_{B,5} - x_{A,5})$	$r_9 = h - a - b_1 - (x_{B,9} - x_{A,9})$
$r_2 = h + b_2 - (x_{B,2} - x_{A,2})$	$r_6 = h + a + b_2 - (x_{B,6} - x_{A,6})$	$r_{10} = h - a + b_2 - (x_{B,10} - x_{A,10})$
$r_3 = h + b_1 - (x_{B,3} - x_{A,3})$	$r_7 = h + a + b_1 - (x_{B,7} - x_{A,7})$	$r_{11} = h - a + b_1 - (x_{B,11} - x_{A,11})$
$r_4 = h - b_2 - (x_{B,4} - x_{A,4})$	$r_8 = h + a - b_2 - (x_{B,8} - x_{A,8})$	$r_{12} = h - a - b_2 - (x_{B,12} - x_{A,12})$

#### where

With 12 observations and four unknown parameters, h, a,  $b_1$ ,  $b_2$ , we have an over-determined system, which leads to a parametric adjustment. As the observation formulae are already linear, Table 2 can easily be transferred in matrix notation:

$$r = A\tilde{y} - x \tag{7}$$

#### where

r is the (12 × 1) residual vector of the  $r_j$ , j = 1, ..., 12;

 $x = x_B - x_A$  is the (12 × 1) quasi-observation vector of the height differences, with

 $x_A$  (12 × 1), reading vector  $x_{Aj}$ , j = 1, ..., 12 of the levelling staff A and

 $x_{\rm B}$  (12 × 1), reading vector  $x_{\rm B}$ , j=1,...,12 of the levelling staff B;

 $\tilde{y}$  (4 × 1) is the vector of the unknown parameters.

#### With the design matrix

p is the weighting factor for one reading at the levelling staff (p = 1 for a sighting distance of 40 m);

h is the height difference between the levelling staffs B and A.

the solution vector of the unknown parameters is

$$\tilde{y} = \begin{pmatrix} h \\ a \\ b_1 \\ b_2 \end{pmatrix} = (A^T P A)^{-1} A^T P x = N^{-1} A^T P x \tag{9}$$

The weight matrix

$$p = \begin{pmatrix} p_1 & 0 \\ & \ddots & \\ 0 & p_{12} \end{pmatrix} \text{ is given by } diag (pj) = (2 \ 2 \ 2 \ 2 \ 0.5 \ 0.5 \ 0.5 \ 0.5 \ 0.5 \ 0.5 \ 0.5 \ 0.5)$$
 (10)

Inserting Formulae (8) and (10) into Formula (9) the solution vector will be obtained finally by

Regarding Formula (7) the experimental standard deviation for a sighting distance of 40 m is given by

$$s = \sqrt{\frac{r^T P r}{v}}$$
 with  $v = 12 - 4 = 8$  (12)

From all series i = 1, ..., 4 of observations we can derive the mean values of the parameters

$$\tilde{y} = \frac{1}{4} \sum_{i=1}^{4} \tilde{y}_i = \frac{1}{4} \sum_{i=1}^{4} \begin{pmatrix} h \\ a \\ b_1 \\ b_2 \end{pmatrix}$$
 (13)

Finally we get the total deviation of the rotating axis from the true vertical of the rotating laser, referenced to a sighting distance of 40 m:

$$b = \sqrt{b_1^2 + b_2^2} \tag{14}$$

With Formula (12) the overall experimental standard deviation of all series i = 1, ..., 4 yields

$$s = \sqrt{\frac{\sum_{i=1}^{4} s_i^2}{4}} = \sqrt{\frac{\sum_{i=1}^{4} r_i^T P r_i}{4}}$$
 (15)

Herewith we can state the standard uncertainty (Type A) of a height difference, h, between the instrument level and a levelling staff (reading at the levelling staff) referenced to a sighting distance of 40 m:

$$u_{\mathsf{ISO-ROLAS}} = s$$
 (16)

The experimental standard deviation for the parameters of all series can be calculated by

$$s(\tilde{y}) = s\sqrt{\frac{1}{4}\operatorname{diag} Q} \tag{17}$$

where

$$Q = \begin{pmatrix} 1/12 & 0 & 0 & 0 \\ 0 & 1/4 & 0 & 0 \\ 0 & 0 & 1/6 & 0 \\ 0 & 0 & 0 & 1/6 \end{pmatrix}$$
 (18)

Thus the standard deviations and the standard uncertainties (Type A), respectively, of the parameters are given by

$$s_h = u(h) = 0.14 s$$
 (19)

$$s_a = u(a) = 0.25 s$$
 (20)

$$s_{b_1} = s_{b_2} = s_{b_{12}} = 0.20 s (21)$$

Applying the law of variance covariance propagation on Formula (14), the experimental standard deviation of the parameter b can be written as

$$s_b = \frac{1}{b} \sqrt{b_1^2 s_{b_1}^2 + b_2^2 s_{b_2}^2} \tag{22}$$

Using Formula (21) leads to

$$s_b = \frac{1}{b} \sqrt{(b_1^2 + b_2^2) s_{b_{12}}^2} = s_{b_{12}}$$
 (23)

and

$$s_b = u(b) = 0,20 s$$
 (24)

#### 6.4 Statistical tests

#### 6.4.1 General

Statistical tests are recommended for the full test procedure only.

For the interpretation of the results, statistical tests shall be carried out using

- the experimental standard deviation, s, of a height difference, h, between the instrument level and a levelling staff (reading at the levelling staff) referenced to a sighting distance of 40 m,
- the deflective deviation, a, referenced to a sighting distance of 40 m and its standard deviation,  $s_a$ , and
- the total deviation, b, of the rotating axis from the true vertical of the rotating laser referenced to a sighting distance of 40 m and its standard deviation,  $s_b$ ,

in order to answer the following questions (see Table 3).

a) Is the calculated experimental standard deviation, s, for one reading at a levelling staff referenced to a sighting distance of 40 m, smaller than the value,  $\sigma$ , stated by the manufacturer or smaller than another predetermined value,  $\sigma$ ?

Usually the manufacturers state the precision by the deflective angle from the horizontal, which should be interpreted to the corresponding standard deviation,  $\sigma$ , at the distance of 40 m.

b) Do two experimental standard deviations, s and  $\tilde{s}$ , as determined from two different samples of measurements, belong to the same population, assuming that both samples have the same number of degrees of freedom, v?

The experimental standard deviations, s and  $\tilde{s}$ , may be obtained from

- two samples of measurements by the same instrument at different times, or
- two samples of measurements by different instruments.
- c) Is the deflective deviation, *a*, equal to zero?
- d) Is the total deviation, b, of the rotating axis from the true vertical equal to zero?

For the following tests, a confidence level of  $1-\alpha=0.95$  and, according to the design of the measurements, a number of degrees of freedom of v=32 are assumed.

Table 3 — Statistical tests

Question	Null hypothesis	Alternative hypothesis
a)	$s = \sigma$	$s > \sigma$
b)	$\sigma = \tilde{\sigma}$	$\sigma \neq \tilde{\sigma}$
c)	a = 0	<i>a</i> ≠ 0
d)		<i>b</i> ≠ <b>0</b>

#### 6.4.2 Question a)

The null hypothesis stating that the experimental standard deviation, s, is smaller than or equal to a theoretical or a predetermined value,  $\sigma$ , is not rejected if the following condition is fulfilled:

$$s \le \sigma \cdot \sqrt{\frac{\chi_{1-\alpha}^2(v)}{v}} \tag{25}$$

$$s \le \sigma \cdot \sqrt{\frac{\chi_{0,95}^2(32)}{32}} \tag{26}$$

$$\chi_{0.95}^2(32) = 46,19 \tag{27}$$

$$s \le \sigma \cdot \sqrt{\frac{46,19}{32}} \tag{28}$$

$$s \le \sigma \cdot 1{,}20 \tag{29}$$

Otherwise, the null hypothesis is rejected.

#### 6.4.3 Question b)

In the case of two different samples, a test indicates whether the experimental standard deviations, s and  $\tilde{s}$ , belong to the same population. The corresponding null hypothesis,  $\sigma = \tilde{\sigma}$ , is not rejected if the following condition is fulfilled:

$$\frac{1}{F_{1-\alpha/2}(v,v)} \le \frac{s^2}{\tilde{s}^2} \le F_{1-\alpha/2}(v,v) \tag{30}$$

$$\frac{1}{F_{0.975}(32,32)} \le \frac{s^2}{\tilde{s}^2} \le F_{0,975}(32,32) \tag{31}$$

$$F_{0,975}(32,32) = 2,02 (32)$$

$$0.50 \le \frac{s^2}{\tilde{s}^2} \le 2.02 \tag{33}$$

Otherwise, the null hypothesis is rejected.

#### 6.4.4 Question c)

The null hypothesis, stating that the deflective deviation, a, of the rotating laser is equal to zero, is not rejected if the following condition is fulfilled:

$$|a| \le s_a \cdot t_{1-\alpha/2}(v) \tag{34}$$

$$|a| \le s_a \cdot t_{0,975}(32) \tag{35}$$

$$s_a = 0.25 \cdot s \tag{36}$$

$$t_{0.975}(32) = 2,04 \tag{37}$$

$$|a| \le 0.51s \tag{38}$$

Otherwise, the null hypothesis is rejected.

#### 6.4.5 Question d)

The null hypothesis stating that the total deviation, b, of the rotating axis from the true vertical of the rotating laser is equal to zero is not rejected if the following condition is fulfilled:

$$b \le s_b \cdot t_{1-\alpha/2}(v) \tag{39}$$

$$b \le s_b \cdot t_{0.975}(32) \tag{40}$$

$$s_b = 0.20 \cdot s \tag{41}$$

$$t_{0.975}(32) = 2{,}04\tag{42}$$

$$b \le 0.42 \cdot s \tag{43}$$

Otherwise, the null hypothesis is rejected.

NOTE In practice, the parameters *a* and *b* may significantly influence the height readings.

## 7 Influence quantities and combined standard uncertainty evaluation (Type A and Type B)

The standard uncertainty  $u_{\rm ISO-ROLAS}$ , calculated in 6.3, represents a measure of precision for the described test configuration under the prevailing test conditions. To achieve in practice a realistic estimation of a quantitative measure of accuracy for a special application, we have to assemble all possible influence quantities that can affect the measurement result. Through this we can derive uncertainty components either by a Type A or Type B evaluation. Rules for evaluating the different standard uncertainties and their summation to the combined standard uncertainty are given in ISO 17123-1. Subsequent possible influence quantities using the rotating laser are compiled.

Table 4 — Typical influence quantities of the rotating laser

Sources of uncertainty (influence quantity)	Evaluation	Distribution
I. Relevant sources of the rotating laser		
Typical measure of precision for measuring distances up to 40 m	Type A <sup>u</sup> ISO-ROLAS	normal
Expanded measurement range	Type A	normal
Non-orthogonality between rotating axis and laser path, parameter <i>a</i>	Type B	rectangular
Inclination of the rotating laser plane or standing (rotating) axis, parameter $b$	Type B	triangular
Instability of laser beam, variation of the parameters $\boldsymbol{a}$ and $\boldsymbol{b}$ by temperature	Type B	rectangular
II. Error sources of the staff		
Tilts of the staff, maladjustment of spot bubble	Туре В	rectangular
Zero error	Type B	rectangular
Staff scale	Type B	rectangular
Junction error	Type B	rectangular
III. Error patterns of the electronic reading		
Round-off error	Type B	rectangular
Vibration effects	Type A	normal
Dependence on light intensity	Type B	rectangular

#### Table 4 (continued)

Sources of uncertainty (influence quantity)	Evaluation	Distribution
IV. General influence quantities		
Dependence on atmospheric conditions	Type B	rectangular
Error due to subsidence of instrument	Type B	rectangular
Curvature of the earth	Type B	rectangular

To calculate the standard uncertainties of the individual influence quantities in consideration of their distributions, in many cases it is advised to determine or estimate upper and lower limits; additionally it is necessary to state the probability that the value considerably lies in this interval. Elaborated advices are additionally given in ISO/IEC Guide 98-3:2008, 4.3.

Sometimes it is useful, after the calculation of the combined standard uncertainty, to indicate the expanded uncertainty due to the higher level of confidence that will better meet the accuracy indication of a measuring tolerance.

An example for the calculation of a typical uncertainty budget of the rotating laser is given in Annex C.

#### **Annex A**

(informative)

### **Example of the simplified test procedure**

#### A.1 Configuration of the test field

A level of known sufficient accuracy is used to determine the reference heights (relative heights) of the six target points of the test field.

The experimental standard deviation and the standard uncertainty, respectively, of a single height difference is determined according to the full test procedure as given in ISO 17123-2:2001, Clause 6.

$$s_{\overline{x}} = u_{\overline{x}} = 0.2 \text{ mm}$$

The relative heights of the six target points and the height differences were obtained as

 $\bar{x}_1 = 1,702 2 \text{ m}$ 

$\bar{x}_2 = 1,5214 \text{ m}$	$\bar{d}_{2,1} = -0.180 \text{ s m}$
2 '	471 0,100 0 111

$$\bar{x}_3 = 1,637.6 \text{ m}$$
  $\bar{d}_{3,2} = +0,116.2 \text{ m}$ 

$$\bar{x}_4 = 1,712 \text{ 4 m}$$
  $\bar{d}_{4,3} = +0,074 \text{ 8 m}$ 

$$\overline{x}_5 = 1,5610 \text{ m}$$
  $\overline{d}_{5,4} = -0,1514 \text{ m}$ 

$$\overline{x}_{6} = 1,608 \text{ 8 m}$$
  $\overline{d}_{6,5} = +0,047 \text{ 8 m}$ 

$$\Sigma = -0.093 \text{ 4 m} = \overline{x}_6 - \overline{x}_1$$

#### A.2 Measurements

Table A.1 contains the measured values,  $x_{j,t}$ , in columns 1 to 3, and the height differences,  $\overline{d}_{t,t-1}$ , in column 5, as given in A.1.

Observer: S. Miller

Weather: cloudy, + 11 °C

Instrument type and number: NN xxx 630401

Date: 1999-04-15

Table A.1 — Measurements

1	2	3	4	5	6	7
j	t	$x_t$	$d_{t,t-1}$	$\bar{d}_{t,t-1}$	$r_{t,t-1}$	$r_{t,t-1}^2$
		m	m	m	mm	mm <sup>2</sup>
1	1	2,215				
	2	2,033	- 0,182	- 0,180 8	+ 1,2	1,44
	3	2,150	+0,117	+ 0,116 2	- 0,8	0,64
	4	2,225	+ 0,075	+ 0,074 8	- 0,2	0,04
	5	2,073	- 0,152	- 0,151 4	+ 0,6	0,36
	6	2,122	+ 0,049	+ 0,047 8	- 1,2	1,44
2	1	1,915				
	2	1,736	- 0,179		- 1,8	3,24
	3	1,851	+ 0,115		+ 1,2	1,44
	4	1,926	+ 0,075		- 0,2	0,04
	5	1,776	- 0,150		- 1,4	1,96
	6	1,824	+ 0,048		- 0,2	0,04
3	1	1,224				
	2	1,042	- 0,182		+ 1,2	1,44
	3	1,158	+ 0,116		+ 0,2	0,04
	4	1,232	+ 0,074		+ 0,8	0,64
	5	1,081	- 0,151		- 0,4	0,16
	6	1,128	+ 0,047		+ 0,8	0,64
4	1	1,585				
	2	1,404	- 0,181		+ 0,2	0,04
	3	1,521	+ 0,117		- 0,8	0,64
	4	1,595	+ 0,074		+ 0,8	0,64
	5	1,443	- 0,152		+ 0,6	0,36
	6	1,489	+ 0,046		+ 1,8	3,24
5	1	1,777				
	2	1,596	- 0,181		+ 0,2	0,04
	3	1,712	+ 0,116		+ 0,2	0,04
	4	1,788	+ 0,076		- 1,2	1,44
	5	1,637	- 0,151		- 0,4	0,16
	6	1,684	+ 0,047		+ 0,8	0,64
Σ			- 0,469	- 0,093 4	+2,0	20,80
$\sum_{j=1}^{5} (x_{j,6}$	$(x_{j,1})$	- 0,469				

#### A.3 Calculation

First, the height differences,  $d_{t,t-1}$ , are calculated according to Formula (2) (see column 4 in Table A.1). Then, the residuals,  $r_{t,t-1}$ , are obtained according to Formula (4) (see column 6 in Table A.1). The sum of squares of the residuals,  $\Sigma r^2$ , is equal to 20,80 mm<sup>2</sup> (see the last line of column 7 in Table A.1). Since the number of degrees of freedom, v, is equal to 25, the standard deviation and the standard uncertainty, respectively, of a height difference,  $d_{t,t-1}$ , are calculated according to Formula (6):

$$s = \sqrt{\frac{20,80 \text{ mm}^2}{25}} = u_{\text{ISO}} = 0,9 \text{ mm}$$

There are two arithmetic checks in Table A.1:

- the value in the last line of column 3 shall be equal to the sum of column 4: -0.469 = -0.469;
- five times the sum of column 5, minus the sum of column 4 shall be the sum of column 6:  $5 \times (-0.093 \text{ 4}) (-0.469) = 0.002$ .

# Annex B (informative)

### **Example of the full test procedure**

#### **B.1** Measurements

Table B.1 contains the measured values,  $x_{Aj}$  and  $x_{Bj}$ , of the  $i^{th}$  series of measurements (the series of measurements numbers 2, 3 and 4 were not printed).

Observer: S. Miller

Weather: sunny, + 10 °C

Instrument type and number: NN xxx 630401

Date: 1999-04-15

Table B.1 — Measurements and residuals of series No. 1

1	2	3	4	5
Instrument point	j	$x_{Aj}$	$x_{Bj}$	$x_j$
		m	m	m
1	1	1,779	1,537	- 0,242
	2	1,780	1,536	- 0,244
	3	1,783	1,535	- 0,248
	4	1,783	1,536	- 0,247
2	5	1,596	1,352	- 0,244
	6	1,600	1,352	- 0,248
	7	1,604	1,353	- 0,251
	8	1,601	1,353	- 0,248
3	9	1,633	1,393	- 0,240
	10	1,634	1,390	- 0,244
	11	1,630	1,387	- 0,243
	12	1,631	1,390	- 0,241
Σ		20,054	17,114	- 2,940

#### **B.2** Calculation

The four unknown parameters, h, a,  $b_1$  and  $b_2$  are calculated according to Formula (11):

$$\tilde{y}_1 = \begin{pmatrix} h \\ a \\ b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} 1/6 & 1/6 & 1/6 & 1/6 & 1/24 & 1/24 & 1/24 & 1/24 & 1/24 & 1/24 & 1/24 & 1/24 \\ 0 & 0 & 0 & 0 & 1/8 & 1/8 & 1/8 & 1/8 & 1/8 & 1/8 & 1/8 & 1/8 \\ 0 & 1/3 & 0 & -1/12 & 0 & 1/12 & 0 & -1/12 & 0 & 1/12 & 0 & -1/12 \end{pmatrix} \begin{bmatrix} -0,244 \\ -0,248 \\ -0,247 \\ -0,244 \\ -0,248 \\ -0,240 \\ -0,244 \\ -0,243 \\ -0,241 \end{bmatrix}$$

$$= \begin{pmatrix} -0.2451 \\ -0.0029 \\ -0.0028 \\ -0.0008 \end{pmatrix} [m]$$

Following Formula (6), the residuals can be calculated to

$$r = \begin{pmatrix} 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 1 & -1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & -1 & -1 & 0 \\ 1 & -1 & 0 & 1 \\ 1 & -1 & 1 & 0 \\ 1 & -1 & 0 & 1 \\ 1 & -1 & 0 & 1 \\ 1 & -1 & 0 & 1 \\ 1 & -1 & 0 & 1 \\ 1 & -1 & 0 & -1 \end{pmatrix} \begin{bmatrix} -0,242 \\ -0,248 \\ -0,0249 \\ -0,0028 \\ -0,0008 \end{bmatrix} \begin{bmatrix} m \end{bmatrix} - \begin{pmatrix} -0,3 \\ -0,3 \\ 0,247 \\ -0,244 \\ -0,248 \\ -0,241 \\ -0,248 \\ -0,240 \\ -0,244 \\ -0,243 \\ -0,241 \end{pmatrix} \begin{bmatrix} m \end{bmatrix} = \begin{pmatrix} -0,3 \\ -0,3 \\ 0,1 \\ 1,1 \\ -1,2 \\ 0,8 \\ 0,2 \\ -0,8 \\ 0,6 \\ 2,6 \\ -2,0 \\ -2,0 \\ -2,0 \end{pmatrix}$$

According to Formula (12) the standard deviation of the first series over a distance of 40 m (standard deviation of the weighting unit) yields

$$s_1 = \sqrt{\frac{11,74 \text{ mm}^2}{8}} = 1,2 \text{ mm}$$
.

The corresponding results of the second, third and fourth series are for the parameters

$$\tilde{y}_2 = \begin{pmatrix} -0,2448 \\ -0,0033 \\ -0,0025 \\ -0,0013 \end{pmatrix} \llbracket \mathbf{m} \rrbracket, \quad \tilde{y}_3 = \begin{pmatrix} -0,2454 \\ -0,0036 \\ -0,0021 \\ -0,0008 \end{pmatrix} \llbracket \mathbf{m} \rrbracket, \quad \tilde{y}_4 = \begin{pmatrix} -0,2454 \\ -0,0036 \\ -0,0027 \\ -0,0007 \end{pmatrix} \llbracket \mathbf{m} \rrbracket$$

and for the standard deviations

$$s_2 = 0.9 \text{ mm}$$
  $s_3 = 1.1 \text{ mm}$   $s_4 = 0.9 \text{ mm}$ .

-0,242

For every series, the following holds:

$$v = v_1 = v_2 = v_3 = v_4 = 8$$

The standard uncertainty (Type A) and the parameters over all series are calculated according to Formulae (13), (14) and (15)

$$v = 4 \cdot v_i = 32$$

$$s = \sqrt{\frac{(1,2 \text{ mm})^2 + (0,9 \text{ mm})^2 + (1,1 \text{ mm})^2 + (0,9 \text{ mm})^2}{4}} = 1,0 \text{ mm}$$

 $u_{\text{ISO-ROLAS}} = 1.0 \text{ mm}$ 

$$h = \frac{-0,2451 - 0,2448 - 0,2454 - 0,2454}{4} = -0,2452 \text{ m}$$

$$a = \frac{-0,0029 - 0,0033 - 0,0036 - 0,0036}{4} = -0,0034 \text{ m}$$

$$b_1 = \frac{-0,0028 - 0,0025 - 0,0021 - 0,0027}{4} = -0,0025 \text{ m}$$

$$b_2 = \frac{0,0008 + 0,0013 + 0,0008 + 0,0007}{4} = +0,0009 \,\mathrm{m}$$

$$b = \sqrt{(-0,0025 \text{ m})^2 + (0,0009 \text{ m})^2} = -0,0027 \text{ m}$$

With Formulae (19), (20) and (24), the experimental standard deviations and the standard uncertainties, respectively, of the parameters are

$$s_h = u(h) = 0.14 \cdot 1.0 = 0.14 \text{ mm}$$

$$s_a = u(a) = 0.25 \cdot 1.0 = 0.25 \text{ mm}$$

$$s_b = u(b) = 0.20 \cdot 1.0 = 0.20 \text{ mm}$$

#### **B.3** Statistical tests

#### B.3.1 Statistical test according to question a) (see 6.4.1)

If the manufacturer has stated for

 $\sigma$  = 2,0 mm s = 1,0 mm

and it was obtained for

with

, – 1,0 11

v = 32

we get according to Formula (29)

 $1,0 \text{ mm} \le 2,0 \text{ mm} \cdot 1,20 \le 2,4 \text{ mm}$ 

Since the above condition is fulfilled, the null hypothesis stating that the empirically determined standard deviation s = 1.0 mm is smaller than or equal to the manufacturer's value  $\sigma = 2.0$  mm is not rejected at the confidence level of 95 %.

#### B.3.2 Statistical test according to question b) (see 6.4.1)

s = 1.0 mm

 $\tilde{s} = 1.9 \text{ mm}$ 

v = 32

$$0,50 \le \frac{1,00}{3,61} \le 2,02$$

 $0,50 \le 0,28 \le 2,02$ 

Since the above condition is not fulfilled, the null hypothesis stating that the experimental standard deviations, s = 1,0 mm and  $\tilde{s} = 1,9$  mm, belong to the same population is rejected at the confidence level of 95 %.

#### B.3.3 Statistical test according to question c) (see 6.4.1)

s = 1,0 mm

v = 32

a = -3,4 mm

 $s_a = 0.25 \text{ mm}$ 

 $|-3,4| \text{ mm} \le 0,25 \text{ mm} \cdot 2,04 \le 0,5 \text{ mm}$ 

Since the above condition is not fulfilled, the null hypothesis stating that the deflective deviation, a, is equal to zero is rejected at the confidence level of 95 %.

#### B.3.4 Statistical test according to question d) (see 6.4.1)

s = 1,0 mm

v = 32

b = -2,7 mm

 $s_b = 0.20 \text{ mm}$ 

 $2,7 \text{ mm} \le 0,20 \text{ mm} \cdot 2,04 \le 0,4 \text{ mm}$ 

Since the above condition is not fulfilled, the null hypothesis stating that the total deviation, b, of the rotating axis from the true vertical is equal to zero is rejected at the confidence level of 95 %.

The rotating laser should therefore be adjusted.

# Annex C (informative)

### Example for the calculation of an uncertainty budget

#### C.1 The measuring task and measuring conditions

The rotating laser used for this measuring purpose was checked by the full test procedure according to this part of ISO 17123. The results obtained are reported in Annex B. The rotating laser was used during airfield construction up to distances of 120 m. As the test procedure approved the manufacturer's stated standard uncertainty for application up to 40 m, a special test for distances up to 120 m was performed. Repeated measurements of this height difference  $h_{120}$  yielded under good meteorological conditions a standard uncertainty (Type A)  $u_{120} = 9.0$  mm. During construction and use of the rotating laser it was sunny with a high temperature (32°C) and strong scintillation effects. Regarding the results of the full test procedure, the supervising surveyor stated and quantified for the described application (operation up to 120 m) the following sources of uncertainty (see Table 4):

- Expanded measurement range (120 m) Type A  $u_{120} = 9 \text{ mm}$
- Parameter  $a = -1.5 \pm 1.0 \, \text{mm}/40 \, \text{m}$ , estimated after adjustment, for 120 m:  $a_{120} = -4.5 \pm 0.0 \, \text{mm}$
- Parameter b = -1.5 ( $\pm 1$ ) mm/40 m, estimated after adjustment, for 120 m:  $b_{120} = -4.5$  ( $\pm 3$ ) mm
- Tilts of staff, impact on the height difference:  $dh_1 = (+)0,4$  mm
- Zero error,  $dh_2 = (+)0.5 \text{ mm}$
- Junction error  $dh_3 = (-)2,0 \text{ mm}$
- Dependence on atmospheric conditions  $dh_4 = \pm 6$  mm
- Subsidence of instrument,  $dh_5 = (-)2 \text{ mm}$
- Curvature of the earth  $dh_6 = (+)1,1$  mm

#### C.2 Calculation of the uncertainty budget

Table C.1 — Uncertainty budget

Input quantity $X_l$	Input estimates $a$ $x_i$ [dim]	Standard uncertainty $u(x_i)$ [dim]	Distribution	Sensitivity coefficients $c_{\rm i} \equiv \partial f / \partial x_{\rm i}$ [dim]	$u\left(x_{t}\right) \equiv c_{t} \cdot u\left(x_{t}\right)$ [mm]	Type of evaluation, source of uncertainty Equation reference in ISO 17123-1 Probability $p$
Ч	<i>h</i> 120	9,0 mm	normal	1	0,6	A, expanded range test series
		7 7	-	•	2	B, deflective deviation $u(a)$ according to equ. (57)
σ	-4,5 MM	1,74 MM	rectangular	<del></del>	7,61	$a+=-1,5$ $a_{-}=-7,5$ $a_{-}=-4,5$
						p = 100 %
						B, deflective deviation
.4	7 1	000		*	c	u(b) according to equ. (58)
0	0, 1	0,92	llallgulal	_	0,92	$a_+ = 0,0$ $a = -4,5$ $a = 2,25$
						p = 100 %
						B, tilt of staff
186	2 7	77	2011000	٠	2,5	$u(dh_1)$ according to equ. (57)
lun	, ,	, 5,	בפרמומח	-	, 7	$a_+ = 0, 4$ $a = 0, 0$ $a = 0, 2$
						p = 100 %
						B, zero error, general knowledge
dho	7 2 20	0	rectoor	•	0 47	$u(dh_2)$ according to equ. (57)
Zun	) )	<u>†</u>	פכומות	-	<u>,</u>	$a_+ = 0.5$ $a = 0.0$ $a = 0.25$
						<i>p</i> = 100 %
						B, junction error, general knowledge
Jh.	- 2 O mm	α α	relinguetoer	•	α <sub>3</sub> C	$u(dh_3)$ according to equ. (57)
5117	5,	)		-	o o	$a_+ = 0,0$ $a = -2,0$ $a = 1,0$
						p = 100 %
a Maximum estim	Maximum estimated value, no correction for the measured input quantity $\hbar$ .	on for the measured i	nput quantity h.			

Table C.1 (continued)

m rectangular rectangular rectangular	$X_i$	$x_i$ [dim]	uncertainty $u(x_i)$ [dim]		coefficients $c_{\rm i} \equiv \partial f / \partial x_{\rm i}$	[mm]	Equation reference in ISO 17123-1 Probability $p$	
0,0 mm         3,48 mm         rectangular         1         3,48         u(dh <sub>4</sub> ) according a <sub>+</sub> = 6,0 a <sub>-</sub> = -6, a <sub>-</sub> = -6, a <sub>-</sub> = -6, a <sub>-</sub> -2,0 mm         0,58         rectangular         1         0,58         u(dh <sub>5</sub> ) according a <sub>+</sub> = 0,0 a <sub>-</sub> = -2, p <sub>-</sub> 1,1 mm         0,64         rectangular         1         B, curvature of th B, curvature of th a <sub>+</sub> = 1,1 a <sub>-</sub> = 0,0           h120         h120         10,1 mm					[dim]		B, atmospheric condition	
-2,0 mm 0,58 rectangular 1 0,58 $\frac{a_{+}=6,0 \ a_{-}=-6,}{p=100 \%}$ B, subsidence of $\frac{n(dh_{5}) \ a_{+}=0,0 \ a_{-}=-2,}{p=100 \%}$ B, curvature of th $\frac{n(dh_{6}) \ a_{-}=-2,}{p=100 \%}$ B, curvature of th $\frac{n(dh_{6}) \ a_{-}=-2,}{p=100 \%}$ $\frac{n(dh_{6}) \ a_{-}=-2,}{p=100 \%}$ $\frac{n(dh_{6}) \ a_{-}=-2,}{p=100 \%}$			07	9	7	0	$u(dh_4)$ according to equ. (57)	
-2,0 mm 0,58 rectangular 1 0,58 8, subsidence of B, subsi		0,0	0,40	rectangular	_	0,40	$a_+ = 6, 0$ $a = -6, 0$ $a = 6, 0$	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$							p = 100 %	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$							B, subsidence of instrument	
1,1 mm 0,64 rectangular 1 0,64 $\frac{a_{+}=0,0 \ a_{-}=-2,}{p=100 \%}$ B, curvature of th $\frac{u(dh_{6})}{a_{+}=1,1 \ a_{-}=0,0}$ $\frac{h_{120}}{h_{120}}$ 10,1 mm		20 0 20	α C	relinguetoer	۲	α Υ	$u(dh_5)$ according to equ. (57)	
1,1 mm         0,64         rectangular         1         0,64 $a_+ = 1,1$ $a = 0,0$ $h_{120}$ $h_{120}$ 10,1 mm         10,1 mm		) ) 	o o		-	o o	$a_+ = 0.0$ $a = -2.0$ $a = 1.0$	
1,1 mm 0,64 rectangular 1 0,64 $\frac{u(dh_6)}{a_+ = 1,1}$ a. $\frac{u(dh_6)}{a_+ $							p = 100 %	
1,1 mm         0,64         rectangular         1         0,64 $u(dh_6)$ according $h_{120}$ $h_{120}$ $h_{120}$ $h_{120}$ $h_{120}$							B, curvature of the earth	
$h_{120}$		7	790	1011001	۲	79	$u(dh_6)$ according to equ. (57)	
h <sub>120</sub> p = 100 %		-,-	Ď Ď	iectaligual	_	, O	$a_{+} = 1,1$ $a_{-} = 0,0$ $a = 0,55$	
h <sub>120</sub> 10,1 mm							p = 100 %	
	ë <del>ن</del>	0079				10.1 mm	20,000	
	ult	021				- - - - -		

The combined standard uncertainty  $u_{c \; \text{ROTLAS 120}}$  is obtained by the law of propagation of uncertainty and will indicate the expected uncertainty of a height difference measured by the rotating laser under the given conditions up to a range of 120 m:

$$u_{c\,\text{ROTLAS}\,120} = \sqrt{u_{120}^2 + u_{a_{120}}^2 + u_{b_{120}}^2 + u_{dh_1}^2 + u_{dh_2}^2 + u_{dh_3}^2 + u_{dh_4}^2 + u_{dh_5}^2 + u_{dh_6}^2} = 10,1 \text{ mm}.$$

Though some of the uncertainty components do not influence the total uncertainty budget significantly, it is important to prove this fact.

#### C.3 Expanded uncertainty

In industrial interdisciplinary application, it is often useful to state a measure of uncertainty that defines an interval about the measurement result. The measure of uncertainty that meets the requirements of providing this interval is the expanded uncertainty.

$$U_{\text{ROTLAS 120}} = k \cdot u_c$$
 ROTLAS 120

with k = 2, which corresponds to a particular level of confidence of p = 95 %,

$$U_{\mbox{ROTLAS 120}}$$
 = 2  $\cdot$  10,1 =  $\pm$  20 mm .

## **Bibliography**

[1] ISO/IEC Guide 98-1:2009, Uncertainty of measurement — Part 1: Introduction to the expression of uncertainty in measurement

ICS 17.180.30

Price based on 27 pages