INTERNATIONAL STANDARD

ISO 16269-8

First edition
2004-09-15

Statistical interpretation of data —

Part 8:
Determination of prediction intervals

Interprétation statistique des données —

Partie 8: Détermination des intervalles de prédiction

Reference number
ISO 16269-8:2004(E)

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Published in Switzerland

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 16269-8 was prepared by Technical Committee ISO/TC 69, *Application of statistical methods*.

ISO 16269 consists of the following parts, under the general title *Statistical interpretation of data*:

— *Part 6: Determination of statistical tolerance intervals*

— *Part 7: Median — Estimation and confidence intervals*

— *Part 8: Determination of prediction intervals*

Introduction

Prediction intervals are of value wherever it is desired or required to predict the results of a future sample of a given number of discrete items from the results of an earlier sample of items produced under identical conditions. They are of particular use to engineers who need to be able to set limits on the performance of a relatively small number of manufactured items. This is of increasing importance with the recent shift towards small-scale production in some industries.

Despite the first review article on prediction intervals and their applications being published as long ago as 1973, there is still a surprising lack of awareness of their value, perhaps due in part to the inaccessibility of the research work for the potential user, and also partly due to confusion with confidence intervals and statistical tolerance intervals. The purpose of this part of ISO 16269 is therefore twofold:

— to clarify the differences between prediction intervals, confidence intervals and statistical tolerance intervals;

— to provide procedures for some of the more useful types of prediction interval, supported by extensive, newly-computed tables.

For information on prediction intervals that are outside the scope of this part of ISO 16269, the reader is referred to the Bibliography.

Statistical interpretation of data —

Part 8: Determination of prediction intervals

1 Scope

This part of ISO 16269 specifies methods of determining prediction intervals for a single continuously distributed variable. These are ranges of values of the variable, derived from a random sample of size n, for which a prediction relating to a further randomly selected sample of size m from the same population may be made with a specified confidence.

Three different types of population are considered, namely:

a) normally distributed with unknown standard deviation;

b) normally distributed with known standard deviation;

c) continuous but of unknown form.

For each of these three types of population, two methods are presented, one for one-sided prediction intervals and one for symmetric two-sided prediction intervals. In all cases, there is a choice from among six confidence levels.

The methods presented for cases a) and b) may also be used for non-normally distributed populations that can be transformed to normality.

For cases a) and b) the tables presented in this part of ISO 16269 are restricted to prediction intervals containing *all* the further m sampled values of the variable. For case c) the tables relate to prediction intervals that contain at least $m - r$ of the next m values, where r takes values from 0 to 10 or 0 to $m - 1$, whichever range is smaller.

For normally distributed populations a procedure is also provided for calculating prediction intervals for the mean of m further observations.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: Probability and general statistical terms*

ISO 3534-2, *Statistics — Vocabulary and symbols — Part 2: Statistical quality control*

3 Terms, definitions and symbols

3.1 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 3534-1 and ISO 3534-2 and the following apply.

3.1.1
prediction interval
interval determined from a random sample from a population in such a way that one may have a specified level of confidence that no fewer than a given number of values in a further random sample of a given size from the same population will fall

NOTE In this context, the confidence level is the long-run proportion of intervals constructed in this manner that will have this property.

3.1.2
order statistics
sample values identified by their position after ranking in non-decreasing order of magnitude

NOTE The sample values in order of selection are denoted in this part of ISO 16269 by $x_1, x_2, \ldots, x_n$. After arranging in non-decreasing order, they are denoted by $x_{[1]}, x_{[2]}, \ldots, x_{[n]}$, where $x_{[1]} \leqslant x_{[2]} \leqslant \ldots \leqslant x_{[n]}$. The word "non-decreasing" is used in preference to "increasing" to include the case where two or more values are equal, at least to within measurement error. Sample values that are equal to one another are assigned distinct, contiguous integer subscripts in square brackets when represented as order statistics.

3.2 Symbols

a lower limit to the values of the variable in the population

α nominal maximum probability that more than r observations from the further random sample of size m will lie outside the prediction interval

b upper limit to the values of the variable in the population

C confidence level expressed as a percentage: $C = 100\ (1 - \alpha)$

k prediction interval factor

m size of further random sample to which the prediction applies

n size of random sample from which the prediction interval is derived

s sample standard deviation: $s = \sqrt{\sum_{i=1}^{n} \left(x_i - \bar{x}\right)^2 \Big/ \left(n-1\right)}$

r specified maximum number of observations from the further random sample of size m that will not lie in the prediction interval

T_1 lower prediction limit

T_2 upper prediction limit

x_i ith observation in a random sample

$x_{[i]}$ ith order statistic

$\overline{x}$ sample mean: $\overline{x} = \sum_{i=1}^{n} x_i / n$

4 Prediction intervals

4.1 General

A *two-sided prediction interval* is an interval of the form (T_1, T_2), where $T_1 < T_2$; T_1 and T_2 are derived from a random sample of size n and are called the *lower* and *upper prediction limits*, respectively.

If a and b are respectively the lower and upper limits of the variable in the population, a *one-sided prediction interval* will be of the form (T_1, b) or (a, T_2).

NOTE 1 For practical purposes a is often taken to be zero for variables that cannot be negative, and b is often taken to be infinity for variables with no natural upper limit.

NOTE 2 Sometimes a population is treated as normal for the purpose of determining a prediction interval, even when it has a finite limit. This may seem incongruous, as the normal distribution ranges from minus infinity to plus infinity. However, in practice, many populations with a finite limit are closely approximated by a normal distribution.

The practical meaning of a prediction interval relating to individual sample values is that the experimenter claims that a further random sample of m values from the same population will have at most r values not lying in the interval, while admitting a small nominal probability that this assertion may be wrong. The nominal probability that an interval constructed in such a way satisfies the claim is called the confidence level.

The practical meaning of a prediction interval relating to a sample mean is that the experimenter claims that the *mean* of a further random sample of m values from the same population will lie in the interval, while admitting a small nominal probability that this assertion may be wrong. Again, the nominal probability that an interval constructed in such a way satisfies the claim is called the confidence level.

This part of ISO 16269 presents procedures applicable to a normally distributed population for $r = 0$ and procedures applicable to the mean of a further sample from a normally distributed population. It also provides procedures applicable to populations of unknown distributional form for $r = 0, 1, \ldots, 10$ or 0 to $m - 1$, whichever range is smaller. In all cases, the tables present prediction interval factors or sample sizes that provide *at least* the stated level of confidence. In general, the actual confidence level is marginally greater than the stated level.

The limits of the prediction intervals for normally distributed populations are at a distance of k times the sample standard deviation (or, where known, the population standard deviation) from the sample mean, where k is the prediction interval factor. In the case of unknown population standard deviation, the value of k becomes very large for small values of n in combination with large values of m and high levels of confidence. Use of large values of k, for example in excess of 10 or 15, should be avoided whenever possible, as the resulting prediction intervals are likely to be too wide to be of any practical use, other than to indicate that the initial sample was too small to yield any useful information about future values. Moreover, for large values of k the integrity of the resulting prediction intervals could be badly compromised by even small departures from normality. Values of k up to 250 are included in the tables primarily to show how rapidly k decreases as the initial sample size n increases.

For prediction intervals relating to the individual values in a further sample, Form A may be used to organize the calculations for a normally distributed population and Form C when the population is of unknown distributional form. Form B is provided to assist with the calculation of a prediction interval for the mean of a further sample from a normally distributed population.

Annexes A to D provide tables of prediction interval factors. Annexes E and F provide tables of sample sizes required when the population is of unknown distributional form. Annex G gives the procedure for interpolating in the tables when the required combination of n, m and confidence level is not tabulated. Annex H presents the statistical theory underlying the tables.

4.2 Comparison with other types of statistical interval

4.2.1 Choice of type of interval

In practice, it is often the case that predictions are required for a *finite* number of observations based on the results of an initial random sample. These are the circumstances under which this part of ISO 16269 is appropriate. There is sometimes confusion with other types of statistical interval. Subclauses 4.2.2 and 4.2.3 are presented in order to clarify the distinctions.

4.2.2 Comparison with a statistical tolerance interval

A prediction interval for individual sample values is an interval, derived from a random sample from a population, about which a confidence statement may be made concerning the maximum *number* of values in a further random *sample* from the population that will lie outside the interval. A statistical tolerance interval (such as that defined in ISO 16269-6) is also an interval derived from a random sample from a population for which a confidence statement may be made; however, the statement in this case relates to the maximum *proportion* of values in the *population* lying outside the interval (or, equivalently, to the *minimum* proportion of values in the population lying *inside* the interval).

NOTE 1 A statistical tolerance interval constant is the limit of a prediction interval constant as the future sample size, m, tends to infinity while the number, r, of items in the future sample falling outside the interval remains a constant fraction of m, provided $r > 0$. This is illustrated in Table 1 for a 95 % confidence level for one-sided and two-sided intervals when r/m = 0,1.

However, there is no such analogy between statistical tolerance interval constants and prediction interval constants for r = 0, the case on which this part of ISO 16269 is primarily focussed.

Table 1 — Example of prediction interval constants

r	1	2	5	10	20	50	100	1 000	Statistical tolerance interval constants for a minimum proportion of 0,9 of the population covered
m	10	20	50	100	200	500	1 000	10 000	
	Prediction interval constants								
One-sided intervals	1,887	1,846	1,767	1,718	1,686	1,663	1,655	1,647	1,646
Two-sided intervals	2,208	2,172	2,103	2,061	2,034	2,014	2,007	2,000	2,000

NOTE 2 The case r = 0 is particularly important in applications related to safety.

4.2.3 Comparison with a confidence interval for the mean

A prediction interval for a mean is an interval, derived from a random sample from a population, for which it may be asserted with a given level of confidence that the mean of a further random *sample* of specified size will lie. A confidence interval for a mean (such as that defined in ISO 2602) is also an interval derived from a random sample from a population for which a confidence statement may be made; however, the statement in this case relates to the mean of the *population*.

5 Prediction intervals for all observations in a further sample from a normally distributed population with unknown population standard deviation

5.1 One-sided intervals

A one-sided prediction interval relating to a normally distributed population with unknown population standard deviation is of the form $(\bar{x} - ks, b)$ or $(a, \bar{x} + ks)$ where the values of the sample mean $\bar{x}$ and the sample standard deviation s are determined from a random sample of size n from the population. The prediction interval factor k depends on n, on the further sample size m and on the confidence level C; values of k are presented in Annex A.

EXAMPLE The pressures in gun barrels caused by firing artillery shells of a given type are known from past experience to be closely approximated by a normal distribution. A sample of 20 rounds has a mean pressure of 562,3 MPa and a standard deviation of pressure of 8,65 MPa. A batch of 5 000 further rounds in total is to be produced under identical manufacturing conditions. What barrel pressure can one be 95 % confident will not be exceeded by any of the 5 000 shells fired under identical conditions ?

Table A.2 provides prediction interval factors at the 95 % confidence level. From Table A.2 it is found that the appropriate prediction interval factor is k = 5,251. The upper limit to a one-sided prediction interval at 95 % confidence is therefore

$$\bar{x} + ks = 562{,}3 + 5{,}251 \times 8{,}65 = 607{,}7 \text{ MPa}$$

Hence one may be 95 % confident that none of the batch of 5 000 rounds will produce a barrel pressure in excess of 607,7 MPa.

This example is also used to illustrate the use of Form A.

5.2 Symmetric two-sided intervals

A symmetric two-sided prediction interval for a normally distributed population with unknown population standard deviation is of the form $(\bar{x} - ks, \bar{x} + ks)$. The prediction interval factor k depends on n, on the further sample size m and on the confidence level C; values of k are presented in Annex B.

EXAMPLE The time to detonation of a particular type of hand grenade after the pin has been removed is known to have an approximate normal distribution. A random sample of size 30 is drawn and tested, and the times to detonation are recorded. The sample mean time is 5,140 s and the sample standard deviation is 0,241 s. A symmetric two-sided prediction interval is required for all of the next lot of 10 000 grenades at 99 % confidence.

Table B.4 provides prediction interval factors at the 99 % confidence level. Entering Table B.4 with n = 30 and m = 10 000 yields the value k = 6,059. The symmetric prediction interval is

$$(\bar{x} - ks,\ \bar{x} + ks) = (5{,}140 - 6{,}059 \times 0{,}241,\ 5{,}140 + 6{,}059 \times 0{,}241) = (3{,}68,\ 6{,}60)$$

One may therefore be 99 % confident that none of the next lot of 10 000 grenades will have a time to detonation outside the range 3,68 s to 6,60 s.

5.3 Prediction intervals for non-normally distributed populations that can be transformed to normality

For non-normally distributed populations that can be transformed to normality, first the procedures for normally distributed populations are applied to the transformed data; the prediction interval is then found by applying the inverse transformation to the resulting prediction limits.

EXAMPLE Suppose that for the data of the example in 5.2 it is known instead that times to detonation are approximately log-normally distributed, i.e. the logarithm of the time to detonation is approximately normally distributed. The sample times $x_1, x_2, \ldots, x_n$ are accordingly transformed to normality by taking their natural logarithms, namely $y_i = \ln x_i$ for i = 1, 2, …, 30. Suppose that the sample mean of the transformed data is $\bar{y}$ = 1,60 and the sample standard deviation is s_y = 0,05. The prediction interval factor for 99 % confidence that none of the next 10 000 times falls outside a two-sided interval is, of course, unchanged at k = 6,059. The symmetric prediction interval for the transformed data is

$$(\bar{y} - ks_y,\ \bar{y} + ks_y) = (1{,}60 - 6{,}059 \times 0{,}05,\ 1{,}60 + 6{,}059 \times 0{,}05) = (1{,}297,\ 1{,}903)$$

The units of measurement of y are log-seconds. The inverse transformation to convert the units back to seconds is exponentiation. The prediction interval at 99 % confidence for the time to detonation of all of the next ten thousand grenades is therefore

$$\left(\mathrm{e}^{1{,}297},\ \mathrm{e}^{1{,}903}\right) = (3{,}66,\ 6{,}71) \text{ s}$$

NOTE 1 The same result would have been obtained using logarithms to any other base, provided that the antilogarithm to the same base is used when converting back to the original units.

NOTE 2 When a two-sided prediction interval is determined in accordance with 5.2 or 6.2, its limits for normally distributed populations are symmetric about (i.e. equidistant from) the estimated median of the population. This symmetry is lost for non-normally distributed populations that are transformed to normality in accordance with 5.3 or 6.3.

5.4 Determination of a suitable initial sample size, n, for a given maximum value of the prediction interval factor, k

Sometimes the confidence level, future sample size m and approximate desired value of the prediction interval factor are given and it is required to determine the initial sample size n. Locate the table for the given confidence level and the sidedness of the prediction interval (i.e. one of the tables in Annex A for a one-sided interval or one of the tables in Annex B for two-sided intervals) and find the column for the given value of m. Look down this column until the first value of k no greater than the given maximum is found. The value of n in the leftmost column of this row of the table gives the required initial sample size.

NOTE If the entry at the bottom of this column exceeds the maximum acceptable value of k then there is no initial sample size large enough to satisfy the requirement. A reduction in the confidence level should be considered.

EXAMPLE Consider a situation in acceptance sampling in which, prior to the use of this part of ISO 16269, it has been the practice to accept lots of size 5 000 whenever $\bar{x} + 4{,}75\, s \leqslant 0{,}1$, where x is the normally distributed porosity of a sintered component and $\bar{x}$ and s are the sample mean and sample standard deviation based on a random sample of size 30 from a normally distributed population. Suppose that it has been decided to replace this acceptance criterion with one that will provide 95 % confidence that none of the items in the lot has $x > 0{,}1$. The producer says that he will be satisfied with the acceptance criterion provided the prediction interval factor is no larger than the 4,75 that he is used to, subject to the sample size requirement not being excessive.

Look down the column for m equal to 5 000 on the third page of Table A.2. It is found that k = 4,771 for sample size 40, but falls below 4,75 to k = 4,717 for a sample of size 45. The producer agrees to increase the sample size to 45 with k = 4,717 for future lots.

5.5 Determination of the confidence level corresponding to a given prediction interval

Rather than determining the prediction interval corresponding to a given confidence level, it may sometimes be required to determine, from the initial sample, the confidence level corresponding to a specified interval. This may be a one-sided interval $(\bar{x} - ks, b)$ or $(a, \bar{x} + ks)$, or a two-sided interval $(\bar{x} - ks, \bar{x} + ks)$ that is symmetrical about the sample mean.

First calculate the value of k corresponding to the desired prediction interval. The confidence level for this interval can then be found by interpolation between tabulated values, as specified in G.1.4.

6 Prediction intervals for all observations in a further sample from a normally distributed population with known population standard deviation

6.1 One-sided intervals

A one-sided prediction interval for a normally distributed population with known population standard deviation σ is of the form $(\bar{x} - k\sigma, b)$ or $(a, \bar{x} + k\sigma)$. The prediction interval factor k depends on n, on the further sample size m and on the confidence level C; values of k are presented in Annex C.

EXAMPLE 150 mm diameter vitrified clay pipes produced by a given process are known to have lengths that are normally distributed with a standard deviation of 4,49 mm. A sample of 50 pipes is found to have a mean of 1 760,60 mm. What length can one be 99 % confident that all of the next 1 000 pipes will exceed ?

Entering Table C.4 with n = 50 and m = 1 000, the appropriate prediction interval factor is found to be k = 4,306. The lower prediction limit for all of the next 1 000 lengths is therefore

$$\bar{x} - k\sigma = 1760{,}60 - 4{,}306 \times 4{,}49 = 1741$$

Hence one may be 99 % confident that none of the lengths of the next 1 000 pipes will be less than 1 741 mm.

This kind of information could be useful if the manufacturer were thinking of providing a warranty for his production. For example, here the manufacturer would be on fairly safe ground in guaranteeing a length of at least 1 740 mm.

6.2 Symmetric two-sided intervals

A symmetric two-sided prediction interval for a normally distributed population with known population standard deviation σ is of the form $(\bar{x} - k\sigma, \bar{x} + k\sigma)$. The prediction interval factor k depends on n, on the further sample size m and on the confidence level C; values of k are presented in Annex D.

EXAMPLE Suppose that for the data of the example in 6.1 it is required to calculate a two-sided prediction interval for all of the next 10 000 pipe lengths at 95 % confidence. The appropriate table for a confidence level of 95 % is Table D.2. Entering Table D.2 with n = 50 and m = 10 000, it is found that the two-sided prediction interval factor is 4,605. The prediction interval is

$$(\bar{x} - k\sigma,\ \bar{x} + k\sigma) = (1\,760{,}60 - 4{,}605 \times 4{,}49,\ 1\,760{,}60 + 4{,}605 \times 4{,}49) = (1\,739{,}9,\ 1\,781{,}3)$$

One may therefore be 95 % confident that all of the further 10 000 pipes have lengths that lie between 1 739,9 mm and 1 781,3 mm.

6.3 Prediction intervals for non-normally distributed populations that can be transformed to normality

For non-normally distributed populations that can be transformed to normality, the procedures for determining a prediction interval for known population standard deviation are similar to those for unknown population standard deviation, described in 5.3. First the procedures for normally distributed populations are applied to the transformed data; then the prediction interval is found by applying the inverse transformation to the resulting prediction limits.

EXAMPLE The fatigue life of a structural component for an aircraft is known to have an approximately lognormal distribution, i.e. the logarithm of the failure time can be assumed to be normally distributed. The standard deviation of $\log_{10}$(life) is also known from previous experience to be approximately equal to 0,11. Six samples of the component are subjected to fatigue testing and the number of loading cycles to failure recorded as follows:

— 229 200;

— 277 900;

— 332 400;

— 369 700;

— 380 800;

— 406 300.

Two further components are to be manufactured to the same specification. How many loading cycles can be applied whilst having 99,9 % confidence that none of these two components will fail?

Taking logarithms to base 10 and then averaging, the mean of $x = \log_{10}$(life) is found to be $\bar{x}$ = 5,513 86. Entering Table C.6 with n = 6 and m = 2, the appropriate one-sided prediction interval factor is found to be k = 3,554. The lower prediction limit for all of the next two components is therefore

$$\bar{x} - k\sigma = 5{,}513\,86 - 3{,}554 \times 0{,}11 = 5{,}122\,92$$

and, taking antilogarithms, $10^{5,122\,92}$ = 132 715.

Hence, taking into account that the standard deviation of $\log_{10}$(life) is only known to two significant figures, one may be 99,9 % confident that the further two components will survive for at least 130 000 loading cycles.

6.4 Determination of a suitable initial sample size, n, for a given value of k

The procedure is the same as described in 5.4 except that Annex C or D is used instead of Annex A or B.

6.5 Determination of the confidence level corresponding to a given prediction interval

The confidence level corresponding to a one-sided interval $(\bar{x} - k\sigma, b)$ or $(a, \bar{x} + k\sigma)$, or a two-sided interval $(\bar{x} - k\sigma, \bar{x} + k\sigma)$ that is symmetrical about the sample mean, may be calculated from Annexes C and D.

First calculate the value of k corresponding to the desired prediction interval. The confidence level for this interval can then be found by interpolation between tabulated values, as described in G.1.4.

7 Prediction intervals for the mean of a further sample from a normally distributed population

A simple two-stage process may be used to obtain the prediction interval factor for the mean of a further sample of m observations from the same normally distributed population, using the same tables. First find the prediction interval factor corresponding to a single future observation. Then multiply this prediction interval factor by $\sqrt{(n+m)/[m(n+1)]}$. This procedure applies to one-sided and two-sided intervals and to the cases of both known and unknown population standard deviation.

EXAMPLE Suppose that for the data of the example in 6.1 it is required to provide a lower prediction limit at 99 % confidence on the mean of the lengths of the next 1 000 pipes. From Table C.4 it is found that the prediction interval factor for an initial sample size of 50 and a single future observation is 2,350. The required prediction interval factor is therefore

$$k = 2{,}350 \times \sqrt{(n+m)/[m(n+1)]} = 2{,}350 \times \sqrt{1\,050/51\,000} = 0{,}337\,2$$

It follows that the lower prediction limit for the mean length of the next 1 000 pipes is

$$\bar{x} - k\sigma = 1\,760{,}60 - 0{,}337\,2 \times 4{,}49 = 1\,759 \text{ mm}$$

This example is used to illustrate the use of Form B.

8 Distribution-free prediction intervals

8.1 General

When the variable is continuous but the distributional form of the population is unknown, distribution-free methods should be used to produce prediction intervals. These are based on the order statistics $x_{[1]}, x_{[2]}, \ldots, x_{[n]}$. In general, one-sided distribution-free prediction intervals are of the form $(x_{[i]}, b)$ or $(a, x_{[i]})$ where $1 \leqslant i \leqslant n$, while two-sided distribution-free prediction intervals are of the form $(x_{[i]}, x_{[j]})$ where $1 \leqslant i \leqslant j \leqslant n$. This part of ISO 16269 provides procedures for one-sided intervals of the form $(x_{[1]}, b)$ or $(a, x_{[n]})$ and two-sided intervals of the form $(x_{[1]}, x_{[n]})$.

The problem with such intervals is in determining how large the initial sample size needs to be in order that one may have the required confidence that the prediction interval will contain at least $m - r$ values from the next m. Annexes E and F are provided for this purpose.

8.2 One-sided intervals

Tables E.1 to E.6 provide initial sample sizes n from which one may have confidence C that the one-sided distribution-free prediction interval $(x_{[1]}, b)$ [or alternatively $(a, x_{[n]})$] will include at least $m - r$ of a further sample of m values from the same population, for a range of values of C, m and r.

EXAMPLE A distribution-free lower prediction limit is required for the strength in bending of vitrified clay pipes, such that one may be 90 % confident that no more than 10 pipes in each further batch of 200 will have a lower strength. What initial sample size is required?

Table E.1 provides the initial sample sizes for a confidence level of 90 %. Entering this table with m = 200 and r = 10 it is found that the appropriate sample size is n = 46. A random sample of 46 pipes is drawn and tested for strength in bending. The lowest strength is found to be 6,4 kN·m. Thus one may be 90 % confident that, for pipes manufactured under identical conditions to the initial sample, no more than 10 pipes in each batch of 200 will have strength in bending below 6,4 kN·m.

8.3 Two-sided intervals

Tables F.1 to F.6 provide initial sample sizes n from which one may have confidence C that the two-sided distribution-free prediction interval ($x_{[1]}$, $x_{[n]}$) will include at least $m - r$ of a further sample of m values from the same population, for a range of values of C, m and r.

EXAMPLE A supplier supplies car batteries in batches of 100, and wishes to provide some kind of guarantee to his retail customers on the range of values of the voltage, x, in each batch. As he is uncertain about the voltage distribution, he decides to use a distribution-free approach. What initial sample size would he need in order to be 90 % confident that no more than one battery in each batch has a voltage outside the range of voltages in his sample?

Table F.1 provides the initial sample sizes for a confidence level of 90 %. Entering this table with m = 100 and r = 1 yields an initial sample size of n = 410. The supplier tests 410 batteries and finds the lowest voltage to be $x_{[1]}$ = 11,81 V and the highest to be $x_{[410]}$ = 12,33 V. He therefore provides a guarantee that no more than one battery per batch has a voltage outside the range 11,81 V to 12,33 V.

This example is also used to illustrate the use of Form C. Note that if the supplier wished to have 90 % confidence that *no* batteries in batches of 100 have voltages outside the limits, then the limits would have to be based on a sample of 1 850 batteries, i.e. more than four times as many.

Form A — Calculation of a prediction interval for all items in a further sample of observations from a normally distributed population

Blank form	Completed form
Data identification	**Data identification**
Data and observation procedure:	Data and observation procedure: Barrel pressures resulting from 20 artillery rounds of a given specification fired at a temperature of 55 °C. Require the upper limit to a one-sided prediction interval at 95 % confidence for all of the next 5 000 rounds.
Units:	Units: megapascals (MPa)
Remarks:	Remarks: Population mean pressure and standard deviation of pressure unknown.
Information required	**Information required**
Initial sample size: n =	Initial sample size: n = 20
Further sample size: m =	Further sample size: m = 5 000
Confidence level (%) C =	Confidence level (%) C = 95 %
a) One-sided interval for unknown σ ☐	a) One-sided interval for unknown σ ☒
b) Two-sided interval for unknown σ ☐	b) Two-sided interval for unknown σ ☐
c) One-sided interval for known σ ☐	c) One-sided interval for known σ ☐
d) Two-sided interval for known σ ☐	d) Two-sided interval for known σ ☐
For c) or d), the population standard deviation is σ =	For c) or d), the population standard deviation is σ =
For a) or c) with an upper prediction limit, the lower limit to x in the population is required: $T_1 = a$ =	For a) or c) with an upper prediction limit, the lower limit to x in the population is required: $T_1 = a$ = 0
For a) or c) with a lower prediction limit, the upper limit to x in the population is required: $T_2 = b$ =	For a) or c) with a lower prediction limit, the upper limit to x in the population is required: $T_2 = b$ =
Initial calculations required	**Initial calculations required**
Sample mean: $\bar{x}$ =	Sample mean: $\bar{x}$ = 562,3 MPa
For a) and b),	For a) and b),
sample standard deviation: s =	sample standard deviation: s = 8,65 MPa
Determination of prediction interval factor	**Determination of prediction interval factor**
a) Look up $k = k_{n,m}$ in Annex A: k =	a) Look up $k = k_{n,m}$ in Annex A: k = 5,251
b) Look up $k = k_{n,m}$ in Annex B: k =	b) Look up $k = k_{n,m}$ in Annex B: k =
c) Look up $k = k_{n,m}$ in Annex C: k =	c) Look up $k = k_{n,m}$ in Annex C: k =
d) Look up $k = k_{n,m}$ in Annex D: k =	d) Look up $k = k_{n,m}$ in Annex D: k =
Determination of the prediction limits	**Determination of the prediction limits**
For a) with a lower prediction limit, or for b),	For a) with a lower prediction limit, or for b),
$T_1 = \bar{x} - ks =$	$T_1 = \bar{x} - ks =$
For c) with a lower prediction limit, or for d),	For c) with a lower prediction limit, or for d),
$T_1 = \bar{x} - k\sigma =$	$T_1 = \bar{x} - k\sigma =$
For a) with an upper prediction limit, or for b),	For a) with an upper prediction limit, or for b),
$T_2 = \bar{x} + ks =$	$T_2 = \bar{x} + ks =$ 607,7 MPa
For c) with a upper prediction limit, or for d),	For c) with a upper prediction limit, or for d),
$T_2 = \bar{x} + k\sigma =$	$T_2 = \bar{x} + k\sigma =$
Result	**Result**
The prediction interval for all of the next m =	The prediction interval for all of the next m = 5 000
observations at confidence level C = % is	observations at confidence level C = 95 % is
(T_1, T_2) = (,).	(T_1, T_2) = (0, 607,7).

Form B — Calculation of a prediction interval for the mean of a further sample of observations from a normally distributed population

Blank form	Completed form
Data identification	**Data identification**
Data and observation procedure:	Data and observation procedure: Lengths of fifty 150 mm diameter vitrified clay pipes. Require the lower limit to a one-sided prediction interval at 99 % confidence for the mean length of all of the next 1 000 pipes.
Units:	Units: millimetres
Remarks:	Remarks: Population mean unknown but population standard deviation known to be 4,49 mm.
Information required	**Information required**
Initial sample size: n =	Initial sample size: n = 50
Further sample size: m =	Further sample size: m = 1 000
Confidence level (%) C =	Confidence level (%) C = 99 %
a) One-sided interval for unknown σ ☐	a) One-sided interval for unknown σ ☐
b) Two-sided interval for unknown σ ☐	b) Two-sided interval for unknown σ ☐
c) One-sided interval for known σ ☐	c) One-sided interval for known σ ☒
d) Two-sided interval for known σ ☐	d) Two-sided interval for known σ ☐
For c) or d), the population standard deviation is σ =	For c) or d), the population standard deviation is σ = 4,49 mm
For a) or c) with an upper prediction limit, the lower limit to x in the population is required: $T_1 = a$ =	For a) or c) with an upper prediction limit, the lower limit to x in the population is required: $T_1 = a$ =
For a) or c) with a lower prediction limit, the upper limit to x in the population is required: $T_2 = b$ =	For a) or c) with a lower prediction limit, the upper limit to x in the population is required: $T_2 = b$ = 1 800 mm
Initial calculations required	**Initial calculations required**
Sample mean: $\bar{x}$ =	Sample mean: $\bar{x}$ = 1 760,60 mm
For a) and b),	For a) and b),
sample standard deviation: s =	sample standard deviation: s =
Determination of prediction interval factor	**Determination of prediction interval factor**
a) Look up $k_{n,1}$ in Annex A: $k_{n,1}$ =	a) Look up $k_{n,1}$ in Annex A: $k_{n,1}$ =
b) Look up $k_{n,1}$ in Annex B: $k_{n,1}$ =	b) Look up $k_{n,1}$ in Annex B: $k_{n,1}$ =
c) Look up $k_{n,1}$ in Annex C: $k_{n,1}$ =	c) Look up $k_{n,1}$ in Annex C: $k_{n,1}$ = 2,350
d) Look up $k_{n,1}$ in Annex D: $k_{n,1}$ =	d) Look up $k_{n,1}$ in Annex D: $k_{n,1}$ =
Then calculate $k = k_{n,1} \times \sqrt{(n+m)/[m(n+1)]} =$	Then calculate $k = k_{n,1} \times \sqrt{(n+m)/[m(n+1)]} =$ 0,337 2
Determination of the prediction limits	**Determination of the prediction limits**
For a) with a lower prediction limit, or for b),	For a) with a lower prediction limit, or for b),
$T_1 = \bar{x} - ks =$	$T_1 = \bar{x} - ks =$
For c) with a lower prediction limit, or for d),	For c) with a lower prediction limit, or for d),
$T_1 = \bar{x} - k\sigma =$	$T_1 = \bar{x} - k\sigma =$ 1759
For a) with an upper prediction limit, or for b),	For a) with an upper prediction limit, or for b),
$T_2 = \bar{x} + ks =$	$T_2 = \bar{x} + ks =$
For c) with a upper prediction limit, or for d),	For c) with a upper prediction limit, or for d),
$T_2 = \bar{x} + k\sigma =$	$T_2 = \bar{x} + k\sigma =$
Result	**Result**
The prediction interval for the mean of the next m =	The prediction interval for the mean of the next m = 1 000
observations at confidence level C = % is	observations at confidence level C = 99 % is
(T_1, T_2) = (,).	(T_1, T_2) = (1 759, 1 800).

Form C — Calculation of a distribution-free prediction interval for ($m - r$) of a further m observations from the same population

Blank form	Completed form
Data identification	**Data identification**
Data and observation procedure:	Data and observation procedure: Car batteries supplied in batches of 100. Require the sample size n such that one may have 90 % confidence that the two-sided prediction interval derived from this sample will contain at least 99 of each batch's voltages.
Units:	Units: volts
Remarks:	Remarks: The type of distribution of voltages is unknown, so a distribution-free prediction interval is required.
Information required	**Information required**
Further sample size: m =	Further sample size: m = 100
Maximum number of further observations allowed to be outside interval: r =	Maximum number of further observations allowed to be outside interval: r = 1
Confidence level (%) C =	Confidence level (%) C = 90 %
a) One-sided interval ☐	a) One-sided interval ☐
b) Two-sided interval ☐	b) Two-sided interval ☒
For a) with an upper prediction limit, the lower limit to x in the population is required: $T_1 = a$ =	For a) with an upper prediction limit, the lower limit to x in the population is required: $T_1 = a$ =
For a) with a lower prediction limit, the upper limit to x in the population is required: $T_2 = b$ =	For a) with a lower prediction limit, the upper limit to x in the population is required: $T_2 = b$ =
Determination of the initial sample size	**Determination of the initial sample size**
For a), enter Annex E with C, m and r to find the initial sample size: n =	For a), enter Annex E with C, m and r to find the initial sample size: n =
For b), enter Annex F with C, m and r to find the initial sample size: n =	For b), enter Annex F with C, m and r to find the initial sample size: n = 410
Determination of the prediction limits	**Determination of the prediction limits**
For a) with a lower prediction limit, or for b),	For a) with a lower prediction limit, or for b),
$T_1 = x_{[1]}$ =	$T_1 = x_{[1]}$ = 11,81
For a) with an upper prediction limit, or for b),	For a) with an upper prediction limit, or for b),
$T_2 = x_{[n]}$ =	$T_2 = x_{[n]}$ = 12,33
Result	**Result**
The distribution-free prediction interval for all but at most r = of the next m = observations at confidence level C = % is	The distribution-free prediction interval for all but at most r = 1 of the next m = 100 observations at confidence level C = 90 % is
(T_1, T_2) = (,).	(T_1, T_2) = (11,81, 12,33).

Annex A
(normative)

Tables of one-sided prediction interval factors, k, for unknown population standard deviation

Table A.1 — One-sided prediction interval factors, k, at confidence level 90 % for unknown population standard deviation

n	m										
	1	2	3	4	5	6	7	8	9	10	15
2	3,770	6,058	7,595	8,730	9,620	10,345	10,954	11,476	11,932	12,335	13,844
3	2,178	3,066	3,615	4,010	4,316	4,566	4,775	4,955	5,112	5,252	5,778
4	1,832	2,484	2,873	3,150	3,364	3,538	3,684	3,810	3,919	4,017	4,386
5	1,680	2,240	2,567	2,798	2,976	3,120	3,241	3,345	3,436	3,517	3,824
6	1,595	2,106	2,400	2,606	2,765	2,893	3,001	3,094	3,175	3,247	3,520
7	1,540	2,020	2,294	2,485	2,632	2,751	2,850	2,935	3,010	3,076	3,328
8	1,501	1,961	2,221	2,402	2,540	2,652	2,746	2,826	2,897	2,959	3,196
9	1,473	1,918	2,168	2,341	2,474	2,580	2,670	2,747	2,814	2,873	3,099
10	1,451	1,885	2,127	2,295	2,422	2,525	2,612	2,686	2,750	2,808	3,025
11	1,434	1,858	2,095	2,258	2,382	2,482	2,566	2,637	2,700	2,756	2,967
12	1,420	1,837	2,069	2,228	2,349	2,447	2,529	2,599	2,660	2,714	2,919
13	1,408	1,820	2,047	2,204	2,322	2,418	2,498	2,566	2,626	2,679	2,880
14	1,398	1,805	2,029	2,183	2,300	2,394	2,472	2,539	2,598	2,650	2,847
15	1,390	1,792	2,013	2,165	2,280	2,373	2,450	2,516	2,574	2,625	2,818
16	1,382	1,781	2,000	2,150	2,264	2,355	2,431	2,496	2,553	2,604	2,794
17	1,376	1,772	1,989	2,137	2,249	2,339	2,415	2,479	2,535	2,585	2,773
18	1,370	1,763	1,978	2,125	2,236	2,326	2,400	2,464	2,519	2,568	2,754
19	1,365	1,756	1,969	2,115	2,225	2,313	2,387	2,450	2,505	2,554	2,737
20	1,361	1,749	1,961	2,106	2,215	2,303	2,376	2,438	2,492	2,541	2,723
25	1,344	1,725	1,931	2,071	2,177	2,262	2,333	2,393	2,445	2,492	2,667
30	1,334	1,709	1,911	2,049	2,153	2,236	2,305	2,363	2,415	2,460	2,631
35	1,326	1,697	1,898	2,033	2,136	2,217	2,285	2,343	2,393	2,438	2,605
40	1,320	1,689	1,887	2,022	2,123	2,204	2,270	2,328	2,377	2,421	2,586
45	1,316	1,683	1,880	2,013	2,113	2,193	2,259	2,316	2,365	2,408	2,572
50	1,312	1,678	1,873	2,006	2,105	2,185	2,250	2,306	2,355	2,398	2,560
60	1,307	1,670	1,864	1,995	2,094	2,172	2,237	2,292	2,341	2,383	2,543
70	1,304	1,664	1,857	1,988	2,085	2,163	2,228	2,282	2,330	2,372	2,530
80	1,301	1,660	1,853	1,982	2,079	2,156	2,221	2,275	2,322	2,364	2,521
90	1,299	1,657	1,849	1,978	2,074	2,151	2,215	2,269	2,316	2,358	2,514
100	1,297	1,655	1,846	1,974	2,071	2,147	2,211	2,265	2,312	2,353	2,508
150	1,292	1,647	1,837	1,964	2,059	2,135	2,198	2,251	2,297	2,338	2,491
200	1,290	1,644	1,832	1,959	2,054	2,129	2,191	2,244	2,290	2,331	2,483
250	1,288	1,641	1,829	1,956	2,050	2,125	2,188	2,240	2,286	2,327	2,478
300	1,287	1,640	1,828	1,954	2,048	2,123	2,185	2,238	2,283	2,324	2,475
350	1,286	1,639	1,826	1,952	2,047	2,121	2,183	2,236	2,281	2,322	2,472
400	1,286	1,638	1,825	1,951	2,045	2,120	2,182	2,234	2,280	2,320	2,470
450	1,285	1,638	1,825	1,950	2,044	2,119	2,181	2,233	2,279	2,319	2,469
500	1,285	1,637	1,824	1,950	2,044	2,118	2,180	2,232	2,278	2,318	2,468
600	1,285	1,636	1,823	1,949	2,043	2,117	2,179	2,231	2,276	2,316	2,466
700	1,284	1,636	1,823	1,948	2,042	2,116	2,178	2,230	2,275	2,315	2,465
800	1,284	1,635	1,822	1,947	2,041	2,116	2,177	2,229	2,275	2,315	2,464
900	1,284	1,635	1,822	1,947	2,041	2,115	2,176	2,229	2,274	2,314	2,463
1 000	1,284	1,635	1,821	1,947	2,040	2,115	2,176	2,228	2,274	2,314	2,463
∞	1,282	1,633	1,819	1,944	2,037	2,111	2,172	2,224	2,269	2,309	2,458

Table A.1 — One-sided prediction interval factors, k, at confidence level 90 % for unknown population standard deviation *(continued)*

n	m										
	20	30	40	50	60	80	100	150	200	250	500
2	14,870	16,249	17,183	17,884	18,442	19,297	19,940	21,068	21,838	22,421	24,153
3	6,139	6,631	6,967	7,221	7,424	7,737	7,973	8,390	8,676	8,894	9,542
4	4,640	4,988	5,227	5,408	5,553	5,777	5,946	6,246	6,453	6,610	7,079
5	4,036	4,326	4,526	4,678	4,800	4,988	5,131	5,384	5,559	5,691	6,089
6	3,708	3,968	4,146	4,282	4,391	4,560	4,689	4,916	5,073	5,192	5,551
7	3,502	3,741	3,907	4,032	4,134	4,290	4,409	4,620	4,766	4,877	5,210
8	3,360	3,585	3,741	3,860	3,955	4,103	4,215	4,415	4,553	4,658	4,974
9	3,256	3,471	3,620	3,733	3,824	3,965	4,073	4,264	4,396	4,497	4,799
10	3,176	3,383	3,526	3,635	3,723	3,859	3,963	4,147	4,275	4,372	4,665
11	3,112	3,313	3,452	3,558	3,643	3,775	3,876	4,055	4,179	4,273	4,557
12	3,061	3,257	3,392	3,495	3,578	3,707	3,805	3,979	4,100	4,192	4,470
13	3,019	3,210	3,342	3,443	3,524	3,650	3,746	3,916	4,035	4,125	4,397
14	2,983	3,170	3,300	3,399	3,478	3,602	3,696	3,863	3,979	4,068	4,334
15	2,952	3,136	3,264	3,361	3,439	3,560	3,653	3,817	3,932	4,019	4,281
16	2,926	3,107	3,232	3,328	3,405	3,525	3,616	3,778	3,890	3,976	4,235
17	2,903	3,081	3,205	3,299	3,375	3,493	3,583	3,743	3,854	3,938	4,194
18	2,882	3,059	3,181	3,274	3,349	3,465	3,554	3,712	3,822	3,905	4,158
19	2,864	3,039	3,160	3,252	3,326	3,441	3,529	3,685	3,793	3,876	4,125
20	2,848	3,021	3,140	3,231	3,305	3,419	3,506	3,660	3,767	3,849	4,096
25	2,788	2,954	3,068	3,156	3,226	3,335	3,419	3,567	3,670	3,749	3,986
30	2,748	2,910	3,021	3,106	3,174	3,280	3,361	3,505	3,605	3,682	3,912
35	2,721	2,878	2,988	3,071	3,137	3,241	3,320	3,461	3,559	3,634	3,859
40	2,700	2,855	2,963	3,044	3,110	3,212	3,290	3,428	3,524	3,597	3,819
45	2,684	2,837	2,943	3,024	3,089	3,189	3,266	3,402	3,497	3,569	3,788
50	2,671	2,823	2,928	3,008	3,072	3,171	3,247	3,382	3,475	3,547	3,762
60	2,652	2,802	2,905	2,983	3,046	3,144	3,218	3,351	3,442	3,513	3,724
70	2,639	2,787	2,888	2,966	3,028	3,124	3,198	3,328	3,419	3,488	3,697
80	2,629	2,775	2,876	2,953	3,014	3,110	3,183	3,312	3,401	3,470	3,676
90	2,621	2,766	2,867	2,943	3,004	3,099	3,171	3,299	3,388	3,455	3,660
100	2,615	2,759	2,859	2,935	2,995	3,09	3,161	3,288	3,376	3,444	3,647
150	2,596	2,738	2,836	2,911	2,970	3,062	3,133	3,257	3,343	3,409	3,607
200	2,587	2,728	2,825	2,898	2,957	3,049	3,118	3,241	3,327	3,392	3,587
250	2,581	2,722	2,818	2,891	2,950	3,041	3,110	3,232	3,317	3,381	3,575
300	2,577	2,718	2,814	2,886	2,945	3,035	3,104	3,226	3,310	3,374	3,567
350	2,575	2,715	2,810	2,883	2,941	3,031	3,100	3,221	3,305	3,369	3,561
400	2,573	2,712	2,808	2,880	2,939	3,029	3,097	3,218	3,302	3,365	3,557
450	2,571	2,711	2,806	2,878	2,936	3,026	3,094	3,215	3,299	3,362	3,554
500	2,570	2,709	2,805	2,877	2,935	3,024	3,092	3,213	3,296	3,360	3,551
600	2,568	2,707	2,802	2,874	2,932	3,022	3,090	3,210	3,293	3,356	3,547
700	2,567	2,706	2,801	2,873	2,931	3,020	3,088	3,208	3,291	3,354	3,544
800	2,566	2,705	2,800	2,871	2,929	3,018	3,086	3,206	3,289	3,352	3,542
900	2,565	2,704	2,799	2,870	2,928	3,017	3,085	3,205	3,288	3,351	3,540
1 000	2,565	2,703	2,798	2,870	2,927	3,016	3,084	3,204	3,286	3,349	3,539
∞	2,559	2,697	2,791	2,862	2,920	3,008	3,075	3,194	3,276	3,339	3,527

Table A.1 — One-sided prediction interval factors, k, at confidence level 90 % for unknown population standard deviation *(continued)*

n	m									
	1 000	2 000	5 000	10 000	20 000	50 000	100 000	200 000	500 000	1 000 000
2	25,783	27,327	29,256	30,642	31,972	33,657	34,882	36,068	37,583	38,692
3	10,157	10,741	11,473	12,000	12,508	13,152	13,620	14,074	14,655	15,080
4	7,525	7,949	8,483	8,867	9,238	9,709	10,051	10,384	10,809	11,120
5	6,467	6,828	7,281	7,609	7,925	8,326	8,618	8,902	9,264	9,530
6	5,892	6,218	6,628	6,925	7,211	7,574	7,839	8,096	8,425	8,667
7	5,528	5,832	6,215	6,491	6,758	7,098	7,345	7,586	7,893	8,119
8	5,275	5,564	5,927	6,190	6,444	6,767	7,002	7,231	7,523	7,738
9	5,088	5,365	5,714	5,967	6,211	6,521	6,748	6,968	7,249	7,456
10	4,944	5,212	5,550	5,795	6,031	6,332	6,551	6,764	7,037	7,238
11	4,829	5,090	5,419	5,657	5,887	6,180	6,394	6,602	6,868	7,063
12	4,735	4,990	5,311	5,544	5,769	6,056	6,265	6,468	6,729	6,920
13	4,657	4,906	5,221	5,450	5,671	5,952	6,157	6,357	6,612	6,800
14	4,590	4,835	5,145	5,370	5,587	5,863	6,065	6,261	6,513	6,697
15	4,532	4,774	5,079	5,300	5,514	5,787	5,986	6,179	6,427	6,609
16	4,482	4,721	5,022	5,240	5,451	5,720	5,917	6,108	6,352	6,532
17	4,439	4,674	4,971	5,187	5,395	5,661	5,856	6,044	6,286	6,464
18	4,400	4,632	4,926	5,139	5,346	5,609	5,801	5,988	6,228	6,404
19	4,365	4,595	4,886	5,097	5,302	5,562	5,753	5,938	6,175	6,349
20	4,333	4,561	4,850	5,059	5,262	5,520	5,709	5,892	6,128	6,300
25	4,214	4,433	4,711	4,913	5,109	5,358	5,540	5,718	5,945	6,112
30	4,134	4,347	4,618	4,814	5,005	5,248	5,426	5,599	5,821	5,984
35	4,076	4,285	4,550	4,742	4,929	5,167	5,342	5,512	5,729	5,889
40	4,032	4,237	4,498	4,687	4,871	5,106	5,278	5,445	5,659	5,817
45	3,998	4,200	4,457	4,644	4,825	5,057	5,227	5,392	5,604	5,759
50	3,970	4,170	4,424	4,609	4,788	5,017	5,185	5,348	5,558	5,712
60	3,928	4,124	4,373	4,555	4,731	4,956	5,121	5,281	5,487	5,639
70	3,897	4,091	4,336	4,515	4,689	4,911	5,073	5,232	5,435	5,585
80	3,874	4,065	4,308	4,485	4,657	4,876	5,037	5,194	5,395	5,543
90	3,856	4,046	4,286	4,461	4,631	4,849	5,008	5,163	5,363	5,510
100	3,842	4,029	4,268	4,442	4,610	4,826	4,984	5,138	5,336	5,482
150	3,797	3,980	4,213	4,382	4,546	4,756	4,910	5,060	5,253	5,395
200	3,775	3,955	4,184	4,351	4,513	4,719	4,871	5,019	5,209	5,349
250	3,761	3,940	4,167	4,332	4,492	4,697	4,847	4,993	5,181	5,319
300	3,752	3,930	4,155	4,319	4,478	4,681	4,830	4,975	5,162	5,299
350	3,746	3,923	4,147	4,310	4,468	4,670	4,818	4,962	5,148	5,284
400	3,741	3,917	4,141	4,303	4,461	4,662	4,809	4,953	5,137	5,273
450	3,737	3,913	4,136	4,298	4,455	4,655	4,802	4,945	5,129	5,264
500	3,734	3,910	4,132	4,293	4,450	4,650	4,796	4,939	5,122	5,257
600	3,729	3,904	4,126	4,287	4,443	4,642	4,788	4,930	5,112	5,246
700	3,726	3,901	4,122	4,282	4,438	4,636	4,781	4,923	5,105	5,239
800	3,724	3,898	4,118	4,279	4,434	4,632	4,777	4,918	5,099	5,233
900	3,722	3,896	4,116	4,276	4,431	4,628	4,773	4,914	5,095	5,228
1 000	3,720	3,894	4,114	4,274	4,428	4,626	4,770	4,911	5,091	5,224
∞	3,706	3,878	4,096	4,254	4,406	4,601	4,743	4,882	5,060	5,190

NOTE This table provides factors k such that one may be at least 90 % confident that none of the next m observations from a normally distributed population will lie outside the range $(-\infty, \bar{x} + ks)$, where $\bar{x}$ and s are derived from a random sample of size n from the same population. Similarly for the range $(\bar{x} - ks, \infty)$.

Table A.2 — One-sided prediction interval factors, k, at confidence level 95 % for unknown population standard deviation

n	m										
	1	2	3	4	5	6	7	8	9	10	15
2	7,733	12,253	15,309	17,572	19,347	20,794	22,01	23,053	23,964	24,770	27,786
3	3,372	4,572	5,328	5,876	6,303	6,652	6,946	7,198	7,420	7,616	8,359
4	2,632	3,402	3,871	4,209	4,472	4,687	4,868	5,024	5,161	5,282	5,744
5	2,336	2,952	3,321	3,584	3,788	3,955	4,096	4,217	4,323	4,418	4,779
6	2,177	2,716	3,033	3,259	3,434	3,576	3,696	3,800	3,891	3,972	4,280
7	2,078	2,570	2,857	3,061	3,218	3,345	3,453	3,546	3,627	3,700	3,976
8	2,010	2,472	2,738	2,927	3,072	3,190	3,289	3,374	3,449	3,516	3,771
9	1,961	2,400	2,653	2,830	2,967	3,077	3,171	3,251	3,321	3,384	3,623
10	1,923	2,346	2,588	2,757	2,887	2,993	3,081	3,158	3,225	3,285	3,512
11	1,894	2,304	2,537	2,700	2,825	2,927	3,012	3,085	3,149	3,207	3,424
12	1,870	2,270	2,497	2,655	2,776	2,874	2,956	3,027	3,089	3,144	3,354
13	1,850	2,242	2,463	2,617	2,735	2,830	2,910	2,979	3,039	3,093	3,297
14	1,834	2,219	2,435	2,586	2,701	2,794	2,872	2,939	2,997	3,050	3,248
15	1,820	2,199	2,411	2,559	2,672	2,763	2,839	2,905	2,962	3,013	3,207
16	1,808	2,182	2,391	2,536	2,647	2,736	2,811	2,875	2,932	2,982	3,172
17	1,797	2,167	2,373	2,516	2,625	2,713	2,787	2,850	2,906	2,955	3,142
18	1,788	2,154	2,358	2,499	2,606	2,693	2,766	2,828	2,882	2,931	3,115
19	1,780	2,142	2,344	2,484	2,590	2,675	2,747	2,808	2,862	2,910	3,091
20	1,772	2,132	2,332	2,470	2,575	2,659	2,730	2,791	2,844	2,891	3,070
25	1,745	2,094	2,287	2,419	2,520	2,601	2,668	2,726	2,777	2,822	2,992
30	1,728	2,070	2,258	2,386	2,484	2,563	2,628	2,684	2,733	2,777	2,941
35	1,715	2,052	2,237	2,364	2,459	2,536	2,600	2,655	2,703	2,745	2,906
40	1,706	2,040	2,222	2,347	2,441	2,517	2,580	2,633	2,680	2,722	2,880
45	1,699	2,030	2,210	2,334	2,427	2,502	2,564	2,617	2,663	2,704	2,859
50	1,694	2,022	2,201	2,323	2,416	2,490	2,551	2,604	2,650	2,690	2,843
60	1,685	2,011	2,188	2,308	2,399	2,472	2,532	2,584	2,629	2,669	2,820
70	1,680	2,002	2,178	2,297	2,387	2,459	2,519	2,570	2,615	2,655	2,803
80	1,675	1,996	2,171	2,289	2,379	2,450	2,509	2,560	2,604	2,643	2,791
90	1,672	1,992	2,165	2,283	2,372	2,443	2,502	2,552	2,596	2,635	2,781
100	1,669	1,988	2,161	2,278	2,367	2,437	2,496	2,546	2,590	2,628	2,773
150	1,661	1,977	2,148	2,263	2,351	2,420	2,478	2,527	2,570	2,608	2,750
200	1,657	1,971	2,141	2,256	2,343	2,412	2,469	2,518	2,560	2,598	2,739
250	1,655	1,968	2,137	2,252	2,338	2,407	2,464	2,512	2,555	2,592	2,732
300	1,653	1,966	2,135	2,249	2,335	2,403	2,460	2,509	2,551	2,588	2,728
350	1,652	1,964	2,133	2,247	2,333	2,401	2,458	2,506	2,548	2,585	2,725
400	1,651	1,963	2,131	2,245	2,331	2,399	2,456	2,504	2,546	2,583	2,722
450	1,651	1,962	2,130	2,244	2,330	2,398	2,454	2,502	2,544	2,581	2,720
500	1,650	1,962	2,129	2,243	2,329	2,397	2,453	2,501	2,543	2,580	2,719
600	1,649	1,960	2,128	2,242	2,327	2,395	2,451	2,499	2,541	2,578	2,717
700	1,649	1,960	2,127	2,241	2,326	2,394	2,450	2,498	2,540	2,577	2,715
800	1,648	1,959	2,127	2,240	2,325	2,393	2,449	2,497	2,539	2,576	2,714
900	1,648	1,959	2,126	2,239	2,324	2,392	2,448	2,496	2,538	2,575	2,713
1 000	1,648	1,958	2,126	2,239	2,324	2,392	2,448	2,496	2,537	2,574	2,712
∞	1,645	1,955	2,122	2,235	2,319	2,387	2,443	2,490	2,532	2,568	2,706

Table A.2 — One-sided prediction interval factors, k, at confidence level 95 % for unknown population standard deviation *(continued)*

n	*m*										
	20	30	40	50	60	80	100	150	200	250	500
2	29,837	32,597	34,466	35,868	36,985	38,696	39,984	42,242	43,785	44,952	48,421
3	8,869	9,566	10,043	10,404	10,692	11,138	11,474	12,068	12,477	12,786	13,712
4	6,064	6,503	6,805	7,034	7,219	7,503	7,719	8,101	8,365	8,565	9,165
5	5,029	5,374	5,613	5,794	5,940	6,166	6,338	6,643	6,853	7,013	7,494
6	4,495	4,791	4,997	5,154	5,280	5,475	5,624	5,888	6,071	6,210	6,629
7	4,169	4,435	4,620	4,762	4,875	5,052	5,187	5,426	5,592	5,718	6,098
8	3,949	4,195	4,366	4,497	4,602	4,766	4,890	5,112	5,266	5,384	5,738
9	3,790	4,021	4,182	4,305	4,404	4,558	4,676	4,885	5,031	5,142	5,476
10	3,670	3,890	4,043	4,160	4,254	4,401	4,513	4,713	4,851	4,957	5,277
11	3,576	3,787	3,934	4,046	4,136	4,277	4,385	4,577	4,710	4,812	5,120
12	3,501	3,704	3,846	3,954	4,041	4,177	4,282	4,467	4,596	4,695	4,993
13	3,439	3,636	3,773	3,878	3,963	4,095	4,196	4,376	4,502	4,598	4,888
14	3,387	3,579	3,712	3,815	3,897	4,026	4,124	4,300	4,422	4,516	4,799
15	3,343	3,530	3,661	3,761	3,841	3,967	4,063	4,235	4,355	4,446	4,723
16	3,305	3,488	3,616	3,714	3,793	3,916	4,011	4,179	4,296	4,386	4,657
17	3,272	3,452	3,577	3,673	3,751	3,872	3,965	4,130	4,245	4,333	4,600
18	3,243	3,420	3,543	3,638	3,714	3,833	3,924	4,087	4,200	4,287	4,549
19	3,217	3,392	3,513	3,606	3,681	3,799	3,888	4,048	4,160	4,245	4,504
20	3,194	3,367	3,486	3,578	3,652	3,768	3,856	4,014	4,124	4,208	4,464
25	3,110	3,273	3,386	3,473	3,543	3,653	3,736	3,886	3,990	4,070	4,312
30	3,055	3,212	3,321	3,405	3,472	3,578	3,658	3,802	3,902	3,979	4,211
35	3,017	3,170	3,276	3,357	3,423	3,525	3,603	3,742	3,839	3,914	4,140
40	2,988	3,138	3,242	3,322	3,386	3,485	3,562	3,698	3,793	3,866	4,086
45	2,967	3,114	3,216	3,294	3,357	3,455	3,530	3,664	3,757	3,828	4,045
50	2,949	3,095	3,196	3,272	3,334	3,431	3,505	3,637	3,728	3,799	4,012
60	2,924	3,066	3,165	3,240	3,301	3,395	3,467	3,596	3,685	3,754	3,962
70	2,905	3,046	3,143	3,217	3,277	3,370	3,441	3,567	3,655	3,722	3,926
80	2,892	3,031	3,127	3,200	3,259	3,351	3,421	3,545	3,632	3,698	3,899
90	2,881	3,019	3,114	3,187	3,245	3,336	3,405	3,529	3,614	3,680	3,879
100	2,873	3,010	3,104	3,176	3,234	3,324	3,393	3,515	3,600	3,665	3,862
150	2,848	2,982	3,075	3,145	3,202	3,289	3,356	3,475	3,558	3,621	3,812
200	2,836	2,969	3,060	3,129	3,185	3,272	3,338	3,455	3,537	3,599	3,787
250	2,829	2,960	3,051	3,120	3,176	3,262	3,327	3,444	3,524	3,586	3,772
300	2,824	2,955	3,045	3,114	3,169	3,255	3,320	3,436	3,516	3,577	3,763
350	2,820	2,951	3,041	3,110	3,165	3,250	3,315	3,430	3,510	3,571	3,755
400	2,818	2,948	3,038	3,106	3,161	3,246	3,311	3,426	3,506	3,566	3,750
450	2,816	2,946	3,036	3,104	3,158	3,243	3,308	3,423	3,502	3,563	3,746
500	2,814	2,944	3,034	3,102	3,156	3,241	3,305	3,420	3,499	3,560	3,743
600	2,812	2,941	3,031	3,099	3,153	3,238	3,302	3,416	3,495	3,556	3,738
700	2,810	2,940	3,029	3,096	3,151	3,235	3,299	3,413	3,492	3,552	3,734
800	2,809	2,938	3,027	3,095	3,149	3,233	3,297	3,411	3,490	3,550	3,732
900	2,808	2,937	3,026	3,093	3,148	3,232	3,296	3,410	3,488	3,548	3,730
1 000	2,807	2,936	3,025	3,092	3,147	3,231	3,295	3,408	3,487	3,547	3,728
∞	2,800	2,928	3,016	3,083	3,137	3,220	3,284	3,396	3,474	3,534	3,713

Table A.2 — One-sided prediction interval factors, k, at confidence level 95 % for unknown population standard deviation *(continued)*

n	m									
	1 000	2 000	5 000	10 000	20 000	50 000	100 000	200 000	500 000	1 000 000
2	51,686	54,779	58,642	61,418	64,085	67,460	69,914	72,291	75,325	77,547
3	14,589	15,424	16,470	17,225	17,951	18,872	19,542	20,192	21,023	21,632
4	9,736	10,280	10,964	11,458	11,933	12,538	12,978	13,406	13,952	14,353
5	7,952	8,390	8,941	9,339	9,724	10,212	10,569	10,914	11,357	11,681
6	7,029	7,411	7,894	8,243	8,580	9,008	9,321	9,625	10,013	10,298
7	6,462	6,811	7,251	7,569	7,877	8,269	8,554	8,832	9,187	9,448
8	6,077	6,402	6,813	7,111	7,399	7,765	8,033	8,293	8,625	8,870
9	5,797	6,106	6,495	6,778	7,051	7,399	7,653	7,900	8,216	8,448
10	5,584	5,879	6,252	6,523	6,785	7,119	7,363	7,600	7,904	8,127
11	5,416	5,700	6,061	6,322	6,575	6,898	7,133	7,362	7,656	7,872
12	5,280	5,555	5,905	6,159	6,404	6,718	6,947	7,169	7,455	7,664
13	5,167	5,435	5,776	6,023	6,263	6,568	6,791	7,009	7,287	7,492
14	5,071	5,334	5,666	5,908	6,143	6,442	6,660	6,873	7,146	7,346
15	4,990	5,247	5,573	5,810	6,040	6,333	6,548	6,756	7,024	7,221
16	4,919	5,171	5,491	5,725	5,950	6,239	6,450	6,655	6,918	7,112
17	4,857	5,105	5,420	5,650	5,872	6,156	6,364	6,566	6,825	7,016
18	4,802	5,047	5,357	5,583	5,803	6,083	6,287	6,487	6,743	6,931
19	4,754	4,995	5,301	5,524	5,741	6,017	6,219	6,416	6,669	6,855
20	4,710	4,948	5,251	5,471	5,685	5,958	6,158	6,353	6,603	6,787
25	4,546	4,772	5,060	5,270	5,474	5,734	5,926	6,112	6,351	6,527
30	4,436	4,654	4,932	5,135	5,332	5,584	5,769	5,949	6,180	6,351
35	4,359	4,570	4,840	5,038	5,229	5,475	5,655	5,831	6,057	6,223
40	4,300	4,507	4,771	4,964	5,152	5,392	5,569	5,741	5,962	6,125
45	4,254	4,458	4,717	4,906	5,091	5,327	5,500	5,670	5,887	6,048
50	4,218	4,418	4,673	4,859	5,041	5,274	5,445	5,612	5,826	5,984
60	4,163	4,358	4,606	4,789	4,966	5,193	5,360	5,523	5,733	5,888
70	4,123	4,315	4,558	4,737	4,911	5,134	5,298	5,458	5,664	5,816
80	4,094	4,282	4,522	4,698	4,869	5,089	5,250	5,408	5,611	5,761
90	4,071	4,256	4,494	4,667	4,836	5,053	5,213	5,369	5,570	5,718
100	4,052	4,236	4,471	4,643	4,810	5,025	5,182	5,337	5,535	5,682
150	3,996	4,174	4,401	4,567	4,729	4,936	5,088	5,237	5,429	5,571
200	3,968	4,143	4,366	4,529	4,687	4,890	5,040	5,185	5,373	5,512
250	3,952	4,125	4,345	4,506	4,662	4,863	5,010	5,154	5,339	5,476
300	3,941	4,112	4,331	4,490	4,645	4,844	4,990	5,132	5,316	5,451
350	3,933	4,103	4,321	4,479	4,633	4,830	4,975	5,117	5,299	5,433
400	3,927	4,097	4,313	4,471	4,624	4,820	4,964	5,105	5,286	5,420
450	3,922	4,092	4,307	4,464	4,617	4,812	4,956	5,096	5,276	5,409
500	3,918	4,087	4,302	4,459	4,611	4,806	4,949	5,089	5,268	5,401
600	3,913	4,081	4,295	4,451	4,603	4,797	4,939	5,078	5,256	5,388
700	3,909	4,077	4,290	4,446	4,597	4,790	4,931	5,070	5,248	5,379
800	3,906	4,073	4,286	4,442	4,592	4,785	4,926	5,064	5,241	5,372
900	3,903	4,071	4,283	4,438	4,588	4,781	4,922	5,059	5,236	5,366
1 000	3,902	4,069	4,281	4,436	4,586	4,777	4,918	5,055	5,232	5,362
∞	3,885	4,050	4,260	4,412	4,560	4,749	4,887	5,022	5,195	5,323

NOTE This table provides factors k such that one may be at least 95 % confident that none of the next m observations from a normally distributed population will lie outside the range $(-\infty, \bar{x} + ks)$, where $\bar{x}$ and s are derived from a random sample of size n from the same population. Similarly for the range $(\bar{x} - ks, \infty)$.

Table A.3 — One-sided prediction interval factors, k, at confidence level 97,5 % for unknown population standard deviation

n	m										
	1	2	3	4	5	6	7	8	9	10	15
2	15,562	24,575	30,678	35,199	38,746	41,640	44,070	46,156	47,977	49,589	55,621
3	4,969	6,629	7,683	8,451	9,052	9,543	9,956	10,312	10,624	10,902	11,950
4	3,559	4,491	5,068	5,486	5,812	6,079	6,305	6,500	6,671	6,824	7,403
5	3,042	3,738	4,161	4,466	4,703	4,898	5,062	5,205	5,330	5,441	5,867
6	2,777	3,360	3,709	3,960	4,155	4,315	4,449	4,566	4,669	4,761	5,111
7	2,616	3,134	3,440	3,659	3,830	3,969	4,086	4,188	4,278	4,357	4,663
8	2,509	2,983	3,262	3,461	3,615	3,741	3,847	3,939	4,019	4,092	4,367
9	2,431	2,876	3,136	3,320	3,463	3,579	3,677	3,762	3,837	3,903	4,158
10	2,373	2,796	3,042	3,215	3,349	3,458	3,551	3,630	3,700	3,763	4,002
11	2,328	2,734	2,968	3,134	3,261	3,365	3,453	3,528	3,595	3,654	3,881
12	2,291	2,684	2,910	3,069	3,191	3,291	3,375	3,447	3,511	3,567	3,784
13	2,262	2,644	2,862	3,016	3,134	3,230	3,311	3,381	3,442	3,497	3,705
14	2,237	2,610	2,823	2,972	3,087	3,180	3,258	3,326	3,385	3,438	3,640
15	2,216	2,581	2,789	2,935	3,046	3,137	3,213	3,279	3,337	3,388	3,585
16	2,198	2,557	2,760	2,903	3,012	3,101	3,175	3,239	3,296	3,346	3,537
17	2,182	2,535	2,736	2,875	2,983	3,069	3,142	3,205	3,26	3,309	3,496
18	2,168	2,517	2,714	2,851	2,957	3,042	3,113	3,175	3,229	3,277	3,461
19	2,156	2,500	2,695	2,830	2,934	3,018	3,088	3,149	3,202	3,249	3,429
20	2,145	2,486	2,678	2,811	2,914	2,996	3,065	3,125	3,177	3,224	3,401
25	2,105	2,432	2,615	2,742	2,839	2,917	2,982	3,039	3,088	3,132	3,298
30	2,080	2,398	2,575	2,698	2,791	2,866	2,929	2,983	3,031	3,073	3,232
35	2,062	2,374	2,547	2,667	2,758	2,831	2,892	2,945	2,991	3,032	3,186
40	2,048	2,356	2,527	2,644	2,733	2,805	2,865	2,916	2,961	3,001	3,153
45	2,038	2,343	2,511	2,627	2,714	2,785	2,844	2,895	2,939	2,978	3,127
50	2,030	2,332	2,498	2,613	2,700	2,769	2,828	2,878	2,921	2,960	3,106
60	2,018	2,316	2,480	2,592	2,678	2,746	2,803	2,852	2,895	2,933	3,076
70	2,010	2,305	2,467	2,578	2,662	2,730	2,786	2,834	2,876	2,914	3,055
80	2,003	2,296	2,457	2,567	2,651	2,718	2,773	2,821	2,863	2,900	3,039
90	1,998	2,290	2,450	2,559	2,642	2,708	2,764	2,811	2,852	2,889	3,027
100	1,995	2,285	2,444	2,553	2,635	2,701	2,756	2,803	2,844	2,880	3,017
150	1,983	2,269	2,426	2,533	2,614	2,679	2,732	2,778	2,819	2,854	2,988
200	1,977	2,262	2,417	2,523	2,603	2,668	2,721	2,767	2,806	2,841	2,974
250	1,974	2,257	2,412	2,518	2,597	2,661	2,714	2,759	2,799	2,834	2,966
300	1,972	2,254	2,409	2,514	2,593	2,657	2,710	2,755	2,794	2,829	2,960
350	1,970	2,252	2,406	2,511	2,590	2,654	2,706	2,751	2,791	2,825	2,956
400	1,969	2,251	2,404	2,509	2,588	2,651	2,704	2,749	2,788	2,823	2,953
450	1,968	2,249	2,403	2,507	2,586	2,650	2,702	2,747	2,786	2,820	2,951
500	1,967	2,248	2,402	2,506	2,585	2,648	2,700	2,745	2,784	2,819	2,949
600	1,966	2,247	2,400	2,504	2,583	2,646	2,698	2,743	2,782	2,816	2,946
700	1,965	2,246	2,399	2,503	2,582	2,644	2,697	2,741	2,780	2,815	2,944
800	1,965	2,245	2,398	2,502	2,580	2,643	2,695	2,740	2,779	2,813	2,942
900	1,964	2,244	2,397	2,501	2,580	2,642	2,694	2,739	2,778	2,812	2,941
1 000	1,964	2,244	2,397	2,500	2,579	2,642	2,694	2,738	2,777	2,811	2,940
∞	1,960	2,239	2,391	2,495	2,573	2,635	2,687	2,731	2,770	2,804	2,932

Table A.3 — One-sided prediction interval factors, k, at confidence level 97,5 % for unknown population standard deviation *(continued)*

n	m										
	20	30	40	50	60	80	100	150	200	250	500
2	59,722	65,242	68,982	71,787	74,020	77,444	80,020	84,537	87,625	89,958	96,899
3	12,673	13,659	14,335	14,846	15,255	15,887	16,364	17,208	17,787	18,227	19,542
4	7,806	8,359	8,741	9,031	9,264	9,625	9,898	10,383	10,718	10,972	11,734
5	6,164	6,574	6,858	7,074	7,249	7,519	7,724	8,090	8,342	8,534	9,112
6	5,355	5,695	5,931	6,111	6,256	6,482	6,654	6,960	7,172	7,333	7,820
7	4,877	5,174	5,381	5,540	5,667	5,866	6,018	6,288	6,476	6,619	7,051
8	4,561	4,830	5,018	5,161	5,278	5,458	5,597	5,843	6,015	6,145	6,541
9	4,337	4,585	4,759	4,893	5,000	5,168	5,297	5,526	5,685	5,807	6,176
10	4,169	4,403	4,566	4,691	4,793	4,951	5,072	5,288	5,438	5,553	5,902
11	4,040	4,261	4,416	4,535	4,632	4,782	4,897	5,103	5,246	5,356	5,688
12	3,936	4,148	4,297	4,410	4,503	4,646	4,757	4,954	5,092	5,197	5,516
13	3,852	4,056	4,199	4,308	4,397	4,536	4,642	4,832	4,965	5,067	5,375
14	3,782	3,979	4,117	4,223	4,309	4,443	4,546	4,730	4,859	4,958	5,257
15	3,722	3,914	4,048	4,151	4,235	4,365	4,465	4,644	4,769	4,865	5,156
16	3,672	3,858	3,989	4,089	4,171	4,298	4,395	4,570	4,692	4,786	5,070
17	3,628	3,810	3,938	4,036	4,115	4,239	4,335	4,506	4,625	4,716	4,995
18	3,589	3,768	3,893	3,989	4,067	4,188	4,282	4,449	4,566	4,656	4,928
19	3,555	3,731	3,853	3,948	4,024	4,143	4,235	4,399	4,514	4,602	4,870
20	3,525	3,698	3,818	3,911	3,986	4,103	4,193	4,355	4,467	4,554	4,817
25	3,414	3,576	3,688	3,775	3,845	3,955	4,039	4,189	4,295	4,376	4,622
30	3,343	3,497	3,605	3,688	3,754	3,859	3,939	4,082	4,183	4,260	4,495
35	3,294	3,443	3,547	3,627	3,691	3,792	3,869	4,007	4,104	4,178	4,405
40	3,258	3,403	3,504	3,582	3,645	3,742	3,818	3,952	4,046	4,118	4,338
45	3,230	3,372	3,471	3,547	3,609	3,705	3,778	3,909	4,001	4,072	4,287
50	3,208	3,348	3,446	3,520	3,581	3,675	3,747	3,876	3,966	4,035	4,246
60	3,175	3,312	3,407	3,480	3,539	3,630	3,700	3,826	3,913	3,980	4,184
70	3,152	3,287	3,380	3,451	3,509	3,599	3,667	3,790	3,876	3,941	4,141
80	3,135	3,268	3,360	3,430	3,487	3,575	3,643	3,764	3,848	3,913	4,109
90	3,122	3,253	3,345	3,414	3,470	3,557	3,624	3,743	3,827	3,890	4,084
100	3,112	3,242	3,332	3,401	3,457	3,543	3,609	3,727	3,809	3,872	4,064
150	3,081	3,208	3,296	3,363	3,417	3,501	3,565	3,679	3,759	3,820	4,004
200	3,065	3,191	3,278	3,344	3,397	3,480	3,543	3,655	3,734	3,793	3,975
250	3,056	3,181	3,267	3,332	3,385	3,467	3,530	3,641	3,719	3,778	3,957
300	3,050	3,174	3,260	3,325	3,377	3,459	3,521	3,632	3,709	3,768	3,946
350	3,046	3,169	3,254	3,319	3,372	3,453	3,515	3,625	3,702	3,760	3,937
400	3,043	3,166	3,251	3,315	3,367	3,448	3,510	3,620	3,696	3,755	3,931
450	3,040	3,163	3,248	3,312	3,364	3,445	3,507	3,616	3,692	3,750	3,926
500	3,038	3,161	3,245	3,310	3,362	3,442	3,504	3,613	3,689	3,747	3,923
600	3,035	3,157	3,242	3,306	3,358	3,438	3,499	3,608	3,684	3,742	3,917
700	3,033	3,155	3,239	3,303	3,355	3,435	3,496	3,605	3,680	3,738	3,913
800	3,031	3,153	3,237	3,301	3,353	3,433	3,494	3,603	3,678	3,735	3,910
900	3,030	3,152	3,236	3,300	3,351	3,431	3,492	3,601	3,676	3,733	3,907
1 000	3,029	3,151	3,235	3,299	3,350	3,430	3,491	3,599	3,674	3,732	3,905
∞	3,020	3,141	3,224	3,288	3,339	3,418	3,478	3,585	3,660	3,716	3,888

Table A.3 — One-sided prediction interval factors, k, at confidence level 97,5 % for unknown population standard deviation *(continued)*

n	m									
	1 000	2 000	5 000	10 000	20 000	50 000	100 000	200 000	500 000	1 000 000
2	103,432	109,620	117,349	122,904	128,238	134,993	139,903	144,658	150,729	155,175
3	20,788	21,974	23,462	24,534	25,567	26,877	27,831	28,755	29,937	30,804
4	12,460	13,152	14,023	14,652	15,259	16,029	16,590	17,135	17,832	18,343
5	9,664	10,191	10,856	11,337	11,801	12,391	12,821	13,239	13,774	14,166
6	8,285	8,731	9,295	9,702	10,096	10,597	10,963	11,318	11,773	12,107
7	7,465	7,863	8,365	8,729	9,081	9,529	9,856	10,174	10,582	10,881
8	6,920	7,285	7,746	8,081	8,405	8,818	9,119	9,412	9,788	10,064
9	6,530	6,872	7,304	7,618	7,921	8,309	8,592	8,867	9,219	9,479
10	6,237	6,561	6,970	7,268	7,557	7,925	8,194	8,455	8,791	9,037
11	6,008	6,317	6,710	6,995	7,271	7,624	7,882	8,133	8,455	8,691
12	5,824	6,122	6,500	6,775	7,041	7,381	7,630	7,873	8,184	8,412
13	5,673	5,961	6,327	6,593	6,851	7,181	7,423	7,658	7,960	8,182
14	5,546	5,826	6,181	6,441	6,692	7,013	7,249	7,478	7,772	7,988
15	5,438	5,711	6,058	6,311	6,556	6,870	7,100	7,324	7,611	7,822
16	5,345	5,611	5,951	6,198	6,439	6,746	6,971	7,190	7,472	7,679
17	5,264	5,525	5,857	6,100	6,336	6,637	6,858	7,074	7,350	7,554
18	5,193	5,449	5,775	6,014	6,245	6,542	6,759	6,971	7,243	7,443
19	5,129	5,381	5,702	5,937	6,165	6,457	6,671	6,879	7,147	7,344
20	5,073	5,321	5,637	5,868	6,093	6,380	6,591	6,797	7,061	7,256
25	4,862	5,094	5,391	5,609	5,821	6,093	6,292	6,486	6,737	6,921
30	4,723	4,945	5,229	5,438	5,641	5,901	6,092	6,279	6,520	6,697
35	4,625	4,839	5,114	5,315	5,512	5,764	5,949	6,130	6,363	6,535
40	4,552	4,760	5,027	5,223	5,414	5,660	5,840	6,017	6,244	6,412
45	4,495	4,699	4,960	5,151	5,338	5,578	5,755	5,928	6,151	6,315
50	4,450	4,650	4,906	5,093	5,277	5,513	5,686	5,856	6,075	6,237
60	4,383	4,576	4,824	5,006	5,184	5,413	5,582	5,747	5,960	6,117
70	4,335	4,524	4,766	4,944	5,117	5,341	5,506	5,667	5,876	6,029
80	4,299	4,484	4,722	4,896	5,067	5,286	5,448	5,607	5,811	5,962
90	4,271	4,454	4,688	4,860	5,027	5,244	5,403	5,559	5,760	5,909
100	4,249	4,429	4,660	4,830	4,996	5,209	5,366	5,520	5,719	5,866
150	4,183	4,356	4,578	4,741	4,900	5,104	5,254	5,402	5,592	5,733
200	4,150	4,320	4,537	4,696	4,851	5,051	5,197	5,341	5,527	5,664
250	4,131	4,298	4,513	4,669	4,822	5,018	5,163	5,305	5,487	5,622
300	4,118	4,284	4,496	4,651	4,803	4,997	5,140	5,280	5,460	5,594
350	4,108	4,274	4,485	4,639	4,789	4,982	5,123	5,262	5,441	5,573
400	4,101	4,266	4,476	4,629	4,778	4,970	5,111	5,249	5,427	5,558
450	4,096	4,260	4,469	4,622	4,770	4,961	5,101	5,239	5,415	5,546
500	4,092	4,255	4,464	4,616	4,764	4,954	5,094	5,230	5,406	5,536
600	4,085	4,248	4,455	4,607	4,754	4,943	5,082	5,218	5,393	5,522
700	4,081	4,243	4,450	4,601	4,747	4,936	5,074	5,209	5,383	5,512
800	4,077	4,239	4,445	4,596	4,742	4,930	5,068	5,202	5,376	5,504
900	4,075	4,236	4,442	4,592	4,738	4,925	5,063	5,197	5,370	5,498
1 000	4,072	4,234	4,439	4,589	4,735	4,922	5,059	5,193	5,366	5,493
∞	4,053	4,212	4,415	4,563	4,706	4,890	5,024	5,156	5,325	5,450

NOTE This table provides factors k such that one may be at least 97,5 % confident that none of the next m observations from a normally distributed population will lie outside the range $(-\infty, \bar{x} + ks)$, where $\bar{x}$ and s are derived from a random sample of size n from the same population. Similarly for the range $(\bar{x} - ks, \infty)$.

Table A.4 — One-sided prediction interval factors, k, at confidence level 99 % for unknown population standard deviation

n	m										
	1	2	3	4	5	6	7	8	9	10	15
2	38,973	61,484	76,735	88,036	96,901	104,135	110,209	115,423	119,976	124,006	139,087
3	8,042	10,632	12,287	13,495	14,441	15,214	15,866	16,428	16,921	17,359	19,016
4	5,077	6,306	7,074	7,633	8,071	8,430	8,734	8,997	9,228	9,434	10,219
5	4,105	4,943	5,459	5,833	6,127	6,368	6,572	6,748	6,904	7,043	7,574
6	3,635	4,298	4,702	4,993	5,222	5,409	5,568	5,706	5,827	5,936	6,351
7	3,360	3,927	4,268	4,513	4,705	4,863	4,996	5,112	5,214	5,305	5,655
8	3,180	3,686	3,988	4,204	4,373	4,511	4,629	4,730	4,820	4,900	5,208
9	3,054	3,517	3,792	3,989	4,142	4,267	4,373	4,465	4,546	4,619	4,898
10	2,960	3,393	3,649	3,831	3,972	4,088	4,186	4,271	4,346	4,413	4,670
11	2,887	3,298	3,538	3,709	3,842	3,951	4,043	4,122	4,192	4,255	4,495
12	2,830	3,222	3,451	3,614	3,740	3,843	3,930	4,005	4,071	4,130	4,358
13	2,783	3,161	3,381	3,536	3,657	3,755	3,838	3,910	3,973	4,030	4,247
14	2,744	3,110	3,323	3,472	3,588	3,683	3,763	3,832	3,893	3,947	4,155
15	2,711	3,068	3,274	3,419	3,531	3,622	3,699	3,766	3,825	3,877	4,078
16	2,683	3,031	3,232	3,373	3,482	3,571	3,646	3,710	3,767	3,818	4,013
17	2,659	3,000	3,196	3,334	3,440	3,526	3,599	3,662	3,718	3,767	3,956
18	2,638	2,973	3,165	3,300	3,403	3,488	3,559	3,620	3,674	3,723	3,907
19	2,619	2,949	3,137	3,269	3,371	3,454	3,524	3,584	3,637	3,684	3,864
20	2,603	2,927	3,113	3,243	3,343	3,424	3,492	3,551	3,603	3,649	3,826
25	2,542	2,849	3,024	3,145	3,239	3,314	3,378	3,432	3,480	3,523	3,687
30	2,503	2,800	2,967	3,083	3,172	3,245	3,305	3,357	3,403	3,444	3,599
35	2,476	2,765	2,928	3,041	3,127	3,196	3,255	3,305	3,349	3,388	3,538
40	2,456	2,740	2,899	3,009	3,093	3,161	3,218	3,267	3,310	3,348	3,493
45	2,441	2,721	2,877	2,985	3,068	3,134	3,190	3,238	3,280	3,318	3,459
50	2,429	2,705	2,860	2,966	3,048	3,113	3,168	3,215	3,257	3,293	3,433
60	2,412	2,683	2,834	2,938	3,018	3,082	3,136	3,182	3,222	3,258	3,393
70	2,399	2,667	2,816	2,919	2,997	3,060	3,113	3,158	3,197	3,233	3,366
80	2,390	2,655	2,803	2,904	2,982	3,044	3,096	3,140	3,179	3,214	3,345
90	2,383	2,646	2,792	2,893	2,970	3,031	3,083	3,127	3,165	3,200	3,329
100	2,377	2,639	2,784	2,884	2,960	3,021	3,072	3,116	3,154	3,188	3,317
150	2,360	2,617	2,760	2,858	2,932	2,992	3,042	3,084	3,122	3,155	3,280
200	2,352	2,607	2,748	2,845	2,918	2,977	3,026	3,069	3,105	3,138	3,261
250	2,347	2,600	2,741	2,837	2,910	2,969	3,017	3,059	3,096	3,128	3,251
300	2,343	2,596	2,736	2,832	2,904	2,963	3,011	3,053	3,089	3,122	3,243
350	2,341	2,593	2,733	2,828	2,901	2,959	3,007	3,049	3,085	3,117	3,238
400	2,339	2,591	2,730	2,825	2,898	2,956	3,004	3,045	3,082	3,114	3,234
450	2,338	2,589	2,728	2,823	2,895	2,953	3,002	3,043	3,079	3,111	3,231
500	2,337	2,588	2,726	2,822	2,894	2,951	3,000	3,041	3,077	3,109	3,229
600	2,335	2,586	2,724	2,819	2,891	2,949	2,997	3,038	3,074	3,105	3,226
700	2,334	2,584	2,722	2,817	2,889	2,947	2,994	3,036	3,071	3,103	3,223
800	2,333	2,583	2,721	2,816	2,887	2,945	2,993	3,034	3,070	3,101	3,221
900	2,332	2,582	2,720	2,815	2,886	2,944	2,992	3,033	3,068	3,100	3,220
1 000	2,332	2,582	2,719	2,814	2,885	2,943	2,991	3,032	3,067	3,099	3,218
∞	2,327	2,575	2,712	2,806	2,877	2,934	2,982	3,023	3,058	3,089	3,208

Table A.4 — One-sided prediction interval factors, k, at confidence level 99 % for unknown population standard deviation *(continued)*

n	m										
	20	30	40	50	60	80	100	150	200	250	500
2	149,338	163,139	172,488	179,502	185,086	193,645	200,085	211,379	219,099	224,934	242,287
3	20,158	21,718	22,788	23,598	24,246	25,246	26,003	27,339	28,258	28,955	31,039
4	10,764	11,516	12,035	12,430	12,747	13,239	13,612	14,273	14,729	15,076	16,118
5	7,946	8,461	8,819	9,092	9,312	9,654	9,914	10,376	10,696	10,939	11,673
6	6,644	7,050	7,333	7,550	7,724	7,997	8,204	8,574	8,831	9,027	9,617
7	5,902	6,246	6,486	6,671	6,820	7,052	7,229	7,546	7,767	7,935	8,444
8	5,426	5,729	5,942	6,105	6,237	6,444	6,601	6,884	7,080	7,231	7,686
9	5,095	5,370	5,563	5,711	5,832	6,019	6,164	6,421	6,601	6,739	7,156
10	4,851	5,106	5,284	5,421	5,533	5,707	5,841	6,080	6,247	6,375	6,764
11	4,665	4,903	5,071	5,199	5,304	5,467	5,593	5,818	5,975	6,096	6,463
12	4,519	4,744	4,902	5,024	5,123	5,278	5,397	5,610	5,760	5,874	6,223
13	4,400	4,614	4,765	4,882	4,976	5,124	5,237	5,442	5,584	5,694	6,028
14	4,302	4,508	4,652	4,764	4,854	4,996	5,106	5,302	5,439	5,545	5,866
15	4,220	4,418	4,557	4,665	4,752	4,889	4,995	5,184	5,317	5,419	5,730
16	4,150	4,342	4,477	4,581	4,665	4,798	4,900	5,084	5,212	5,311	5,613
17	4,089	4,276	4,407	4,508	4,590	4,719	4,818	4,997	5,122	5,218	5,511
18	4,037	4,219	4,347	4,445	4,525	4,651	4,747	4,921	5,043	5,137	5,423
19	3,991	4,168	4,293	4,389	4,468	4,590	4,685	4,854	4,973	5,065	5,345
20	3,950	4,124	4,246	4,340	4,417	4,537	4,629	4,795	4,912	5,001	5,276
25	3,801	3,961	4,074	4,160	4,230	4,340	4,425	4,578	4,685	4,767	5,020
30	3,707	3,858	3,964	4,046	4,112	4,216	4,295	4,439	4,540	4,617	4,855
35	3,642	3,787	3,889	3,967	4,030	4,129	4,206	4,343	4,439	4,513	4,740
40	3,594	3,735	3,833	3,909	3,970	4,066	4,140	4,272	4,365	4,437	4,656
45	3,558	3,695	3,791	3,865	3,925	4,018	4,090	4,219	4,309	4,378	4,591
50	3,530	3,664	3,758	3,830	3,889	3,980	4,050	4,176	4,264	4,332	4,540
60	3,488	3,618	3,709	3,779	3,836	3,924	3,992	4,113	4,199	4,264	4,464
70	3,458	3,586	3,675	3,743	3,798	3,885	3,951	4,069	4,152	4,216	4,411
80	3,436	3,562	3,650	3,717	3,771	3,856	3,921	4,037	4,118	4,181	4,371
90	3,419	3,544	3,630	3,696	3,750	3,833	3,897	4,012	4,092	4,153	4,341
100	3,406	3,529	3,615	3,680	3,733	3,815	3,879	3,992	4,071	4,132	4,316
150	3,366	3,486	3,569	3,632	3,683	3,763	3,824	3,933	4,009	4,067	4,245
200	3,347	3,464	3,546	3,608	3,659	3,737	3,797	3,904	3,979	4,036	4,210
250	3,335	3,452	3,533	3,594	3,644	3,722	3,781	3,887	3,961	4,017	4,189
300	3,328	3,443	3,524	3,585	3,635	3,712	3,770	3,876	3,949	4,005	4,175
350	3,322	3,437	3,517	3,579	3,628	3,704	3,763	3,867	3,940	3,996	4,165
400	3,318	3,433	3,513	3,574	3,623	3,699	3,757	3,861	3,934	3,989	4,158
450	3,315	3,430	3,509	3,570	3,619	3,695	3,753	3,857	3,929	3,984	4,152
500	3,312	3,427	3,506	3,567	3,615	3,691	3,749	3,853	3,925	3,980	4,148
600	3,309	3,423	3,502	3,562	3,611	3,686	3,744	3,847	3,919	3,974	4,141
700	3,306	3,420	3,499	3,559	3,607	3,683	3,740	3,843	3,915	3,970	4,136
800	3,304	3,418	3,496	3,556	3,605	3,680	3,738	3,840	3,912	3,966	4,132
900	3,302	3,416	3,494	3,554	3,603	3,678	3,736	3,838	3,909	3,964	4,129
1 000	3,301	3,414	3,493	3,553	3,601	3,676	3,734	3,836	3,907	3,962	4,127
∞	3,290	3,402	3,480	3,539	3,587	3,661	3,718	3,819	3,890	3,944	4,107

Table A.4 — One-sided prediction interval factors, k, at confidence level 99 % for unknown population standard deviation *(continued)*

n	m									
	1 000	2 000	5 000	10 000	20 000	50 000	100 000	200 000	500 000	1 000 000
2	> 250	> 250	> 250	> 250	> 250	> 250	> 250	> 250	> 250	> 250
3	33,015	34,896	37,255	38,957	40,594	42,672	44,185	45,653	47,528	48,903
4	17,110	18,056	19,248	20,109	20,938	21,993	22,762	23,507	24,462	25,162
5	12,373	13,044	13,890	14,502	15,093	15,844	16,393	16,925	17,607	18,107
6	10,183	10,726	11,412	11,909	12,390	13,001	13,448	13,882	14,438	14,846
7	8,932	9,402	9,996	10,427	10,844	11,376	11,764	12,142	12,625	12,981
8	8,124	8,545	9,080	9,469	9,845	10,324	10,675	11,016	11,453	11,774
9	7,558	7,946	8,439	8,797	9,144	9,587	9,911	10,226	10,630	10,927
10	7,140	7,502	7,963	8,299	8,625	9,040	9,344	9,640	10,020	10,299
11	6,817	7,160	7,597	7,915	8,224	8,618	8,907	9,188	9,548	9,814
12	6,561	6,888	7,305	7,609	7,904	8,281	8,557	8,827	9,172	9,426
13	6,352	6,666	7,066	7,359	7,643	8,006	8,272	8,531	8,864	9,109
14	6,178	6,481	6,868	7,150	7,425	7,776	8,034	8,285	8,608	8,845
15	6,032	6,325	6,700	6,974	7,241	7,582	7,832	8,076	8,390	8,620
16	5,906	6,191	6,556	6,823	7,082	7,415	7,659	7,897	8,202	8,427
17	5,797	6,075	6,431	6,691	6,945	7,269	7,508	7,740	8,039	8,259
18	5,702	5,973	6,321	6,576	6,824	7,142	7,375	7,603	7,896	8,112
19	5,618	5,884	6,224	6,474	6,717	7,029	7,258	7,481	7,769	7,981
20	5,543	5,804	6,138	6,383	6,621	6,928	7,153	7,373	7,656	7,864
25	5,266	5,507	5,816	6,043	6,265	6,551	6,761	6,966	7,230	7,425
30	5,087	5,314	5,607	5,822	6,032	6,303	6,503	6,698	6,950	7,135
35	4,962	5,179	5,459	5,666	5,867	6,127	6,319	6,507	6,749	6,928
40	4,870	5,079	5,350	5,549	5,744	5,996	6,182	6,363	6,598	6,772
45	4,799	5,002	5,265	5,459	5,648	5,893	6,074	6,251	6,480	6,650
50	4,743	4,941	5,197	5,387	5,572	5,811	5,988	6,161	6,385	6,551
60	4,659	4,850	5,097	5,279	5,457	5,688	5,858	6,025	6,242	6,402
70	4,601	4,786	5,025	5,202	5,375	5,599	5,765	5,927	6,138	6,293
80	4,557	4,738	4,972	5,145	5,314	5,532	5,694	5,853	6,059	6,211
90	4,523	4,701	4,931	5,100	5,266	5,481	5,639	5,795	5,997	6,147
100	4,496	4,672	4,898	5,065	5,228	5,439	5,595	5,749	5,947	6,094
150	4,417	4,585	4,800	4,959	5,114	5,314	5,462	5,608	5,796	5,935
200	4,378	4,542	4,752	4,907	5,057	5,252	5,396	5,537	5,719	5,855
250	4,355	4,517	4,723	4,875	5,024	5,215	5,356	5,494	5,673	5,806
300	4,340	4,500	4,704	4,855	5,001	5,190	5,330	5,466	5,643	5,773
350	4,329	4,488	4,691	4,840	4,985	5,173	5,311	5,446	5,621	5,750
400	4,321	4,479	4,681	4,829	4,974	5,159	5,297	5,431	5,604	5,733
450	4,314	4,472	4,673	4,820	4,964	5,149	5,286	5,419	5,592	5,719
500	4,309	4,466	4,667	4,814	4,957	5,141	5,277	5,410	5,581	5,708
600	4,302	4,458	4,657	4,803	4,946	5,129	5,264	5,396	5,566	5,692
700	4,296	4,452	4,651	4,796	4,938	5,120	5,254	5,386	5,555	5,681
800	4,292	4,447	4,646	4,791	4,932	5,114	5,247	5,378	5,547	5,672
900	4,289	4,444	4,642	4,787	4,928	5,109	5,242	5,372	5,541	5,665
1 000	4,287	4,441	4,639	4,783	4,924	5,105	5,238	5,368	5,536	5,660
∞	4,264	4,417	4,611	4,753	4,891	5,069	5,199	5,326	5,490	5,612

NOTE This table provides factors k such that one may be at least 99 % confident that none of the next m observations from a normally distributed population will lie outside the range $(-\infty, \bar{x} + ks)$, where $\bar{x}$ and s are derived from a random sample of size n from the same population. Similarly for the range $(\bar{x} - ks, \infty)$.

Table A.5 — One-sided prediction interval factors, k, at confidence level 99,5 % for unknown population standard deviation

n	m										
	1	2	3	4	5	6	7	8	9	10	15
2	77,964	122,981	153,482	176,082	193,812	208,279	220,429	230,86	239,97	248,03	>250
3	11,461	15,108	17,441	19,147	20,483	21,575	22,497	23,291	23,988	24,607	26,949
4	6,531	8,059	9,019	9,719	10,268	10,719	11,101	11,431	11,721	11,981	12,968
5	5,044	6,020	6,625	7,065	7,410	7,694	7,935	8,144	8,328	8,493	9,123
6	4,356	5,097	5,551	5,880	6,139	6,351	6,532	6,688	6,827	6,950	7,425
7	3,964	4,579	4,952	5,222	5,433	5,607	5,755	5,883	5,996	6,097	6,487
8	3,712	4,249	4,573	4,806	4,988	5,138	5,265	5,375	5,473	5,560	5,897
9	3,537	4,022	4,312	4,520	4,683	4,816	4,930	5,028	5,115	5,193	5,492
10	3,409	3,856	4,122	4,313	4,461	4,583	4,686	4,776	4,855	4,926	5,199
11	3,311	3,730	3,978	4,155	4,293	4,406	4,502	4,585	4,659	4,724	4,977
12	3,233	3,631	3,865	4,032	4,162	4,268	4,358	4,436	4,505	4,566	4,803
13	3,170	3,551	3,774	3,932	4,056	4,156	4,242	4,316	4,381	4,439	4,663
14	3,119	3,485	3,699	3,851	3,969	4,065	4,146	4,217	4,279	4,335	4,549
15	3,075	3,430	3,636	3,783	3,896	3,989	4,067	4,135	4,194	4,248	4,453
16	3,038	3,383	3,583	3,725	3,834	3,924	3,999	4,065	4,122	4,174	4,372
17	3,006	3,342	3,537	3,675	3,781	3,868	3,941	4,005	4,061	4,111	4,302
18	2,978	3,307	3,498	3,632	3,735	3,820	3,891	3,953	4,007	4,056	4,242
19	2,954	3,277	3,463	3,594	3,695	3,778	3,847	3,907	3,960	4,008	4,189
20	2,932	3,249	3,432	3,560	3,660	3,740	3,808	3,867	3,919	3,965	4,142
25	2,853	3,150	3,320	3,439	3,530	3,605	3,667	3,721	3,769	3,811	3,973
30	2,802	3,087	3,249	3,362	3,449	3,519	3,578	3,629	3,674	3,714	3,866
35	2,768	3,044	3,200	3,309	3,393	3,460	3,517	3,566	3,609	3,647	3,793
40	2,742	3,012	3,164	3,270	3,352	3,417	3,473	3,520	3,562	3,599	3,740
45	2,723	2,988	3,137	3,241	3,320	3,385	3,439	3,485	3,526	3,562	3,700
50	2,707	2,968	3,116	3,218	3,296	3,359	3,412	3,457	3,497	3,533	3,668
60	2,684	2,940	3,084	3,184	3,260	3,321	3,373	3,417	3,456	3,490	3,621
70	2,668	2,920	3,062	3,160	3,234	3,295	3,345	3,388	3,426	3,460	3,588
80	2,656	2,906	3,045	3,142	3,216	3,275	3,325	3,367	3,405	3,438	3,564
90	2,647	2,894	3,033	3,128	3,201	3,260	3,309	3,351	3,388	3,421	3,545
100	2,640	2,885	3,023	3,118	3,190	3,248	3,297	3,338	3,375	3,407	3,531
150	2,618	2,859	2,993	3,085	3,156	3,213	3,260	3,301	3,336	3,368	3,487
200	2,608	2,846	2,978	3,070	3,139	3,195	3,242	3,282	3,317	3,348	3,466
250	2,601	2,838	2,970	3,060	3,129	3,185	3,231	3,271	3,305	3,336	3,453
300	2,597	2,833	2,964	3,054	3,123	3,178	3,224	3,263	3,298	3,329	3,444
350	2,594	2,829	2,960	3,050	3,118	3,173	3,219	3,258	3,293	3,323	3,438
400	2,592	2,826	2,957	3,046	3,114	3,169	3,215	3,254	3,289	3,319	3,434
450	2,590	2,824	2,954	3,044	3,112	3,166	3,212	3,251	3,285	3,316	3,430
500	2,589	2,822	2,952	3,042	3,110	3,164	3,210	3,249	3,283	3,313	3,428
600	2,587	2,820	2,949	3,039	3,106	3,161	3,206	3,245	3,279	3,309	3,423
700	2,585	2,818	2,947	3,036	3,104	3,158	3,204	3,243	3,277	3,307	3,421
800	2,584	2,817	2,946	3,035	3,102	3,157	3,202	3,241	3,275	3,305	3,418
900	2,583	2,816	2,945	3,033	3,101	3,155	3,200	3,239	3,273	3,303	3,417
1 000	2,583	2,815	2,944	3,032	3,100	3,154	3,199	3,238	3,272	3,302	3,415
∞	2,576	2,807	2,935	3,023	3,090	3,144	3,189	3,227	3,261	3,290	3,403

Table A.5 — One-sided prediction interval factors, k, at confidence level 99,5 % for unknown population standard deviation *(continued)*

n	m										
	20	30	40	50	60	80	100	150	200	250	500
2	> 250	> 250	> 250	> 250	> 250	> 250	> 250	> 250	> 250	> 250	> 250
3	28,565	30,771	32,285	33,431	34,348	35,763	36,834	38,725	40,025	41,011	43,962
4	13,655	14,602	15,257	15,755	16,155	16,775	17,246	18,081	18,658	19,096	20,412
5	9,564	10,177	10,602	10,927	11,189	11,596	11,906	12,458	12,839	13,130	14,006
6	7,760	8,225	8,550	8,799	9,001	9,314	9,553	9,979	10,275	10,501	11,184
7	6,762	7,146	7,416	7,623	7,790	8,051	8,251	8,607	8,856	9,045	9,620
8	6,134	6,467	6,701	6,881	7,027	7,255	7,429	7,741	7,959	8,126	8,631
9	5,705	6,002	6,211	6,373	6,503	6,708	6,865	7,146	7,343	7,494	7,951
10	5,393	5,664	5,855	6,003	6,123	6,310	6,454	6,713	6,894	7,032	7,454
11	5,156	5,408	5,585	5,722	5,834	6,008	6,142	6,383	6,552	6,681	7,075
12	4,971	5,207	5,374	5,502	5,607	5,771	5,897	6,124	6,283	6,405	6,777
13	4,822	5,046	5,203	5,325	5,424	5,579	5,699	5,914	6,065	6,181	6,536
14	4,700	4,913	5,063	5,180	5,274	5,422	5,536	5,742	5,886	5,997	6,337
15	4,598	4,802	4,947	5,058	5,148	5,291	5,400	5,598	5,736	5,843	6,169
16	4,512	4,708	4,847	4,955	5,042	5,179	5,285	5,475	5,609	5,712	6,027
17	4,438	4,628	4,762	4,866	4,950	5,083	5,185	5,370	5,499	5,599	5,904
18	4,373	4,558	4,688	4,789	4,871	5,000	5,099	5,278	5,404	5,501	5,797
19	4,317	4,497	4,624	4,722	4,801	4,926	5,023	5,197	5,320	5,414	5,703
20	4,267	4,443	4,567	4,662	4,740	4,862	4,956	5,126	5,246	5,338	5,620
25	4,087	4,247	4,359	4,446	4,516	4,627	4,712	4,866	4,975	5,058	5,315
30	3,974	4,123	4,228	4,309	4,375	4,479	4,558	4,702	4,803	4,881	5,121
35	3,896	4,038	4,139	4,216	4,279	4,377	4,453	4,589	4,685	4,759	4,987
40	3,839	3,977	4,074	4,148	4,208	4,303	4,376	4,507	4,599	4,670	4,889
45	3,796	3,930	4,024	4,096	4,155	4,247	4,317	4,444	4,534	4,603	4,814
50	3,762	3,893	3,985	4,056	4,113	4,202	4,271	4,395	4,482	4,549	4,755
60	3,712	3,839	3,928	3,996	4,051	4,137	4,204	4,323	4,406	4,471	4,668
70	3,678	3,801	3,888	3,954	4,008	4,092	4,156	4,272	4,353	4,416	4,607
80	3,652	3,773	3,858	3,923	3,976	4,058	4,121	4,235	4,314	4,375	4,562
90	3,632	3,752	3,835	3,900	3,951	4,032	4,095	4,206	4,284	4,344	4,527
100	3,616	3,735	3,817	3,881	3,932	4,012	4,073	4,183	4,260	4,319	4,500
150	3,570	3,685	3,764	3,825	3,875	3,952	4,011	4,116	4,190	4,247	4,419
200	3,547	3,660	3,738	3,798	3,847	3,922	3,980	4,084	4,156	4,211	4,380
250	3,534	3,645	3,723	3,782	3,830	3,905	3,962	4,064	4,135	4,190	4,356
300	3,525	3,636	3,713	3,771	3,819	3,893	3,950	4,051	4,122	4,176	4,341
350	3,518	3,629	3,705	3,764	3,811	3,885	3,941	4,042	4,112	4,166	4,330
400	3,514	3,624	3,700	3,758	3,805	3,879	3,935	4,035	4,105	4,159	4,322
450	3,510	3,620	3,696	3,754	3,801	3,874	3,930	4,030	4,100	4,153	4,315
500	3,507	3,616	3,692	3,750	3,797	3,870	3,926	4,026	4,095	4,148	4,310
600	3,503	3,612	3,687	3,745	3,792	3,864	3,920	4,019	4,088	4,141	4,303
700	3,499	3,608	3,684	3,741	3,788	3,860	3,916	4,015	4,084	4,137	4,297
800	3,497	3,606	3,681	3,738	3,785	3,857	3,913	4,011	4,080	4,133	4,293
900	3,495	3,604	3,679	3,736	3,783	3,855	3,910	4,009	4,077	4,130	4,290
1 000	3,494	3,602	3,677	3,735	3,781	3,853	3,908	4,007	4,075	4,128	4,287
∞	3,481	3,588	3,662	3,719	3,765	3,836	3,890	3,988	4,056	4,107	4,265

Table A.5 — One-sided prediction interval factors, k, at confidence level 99,5 % for unknown population standard deviation *(continued)*

n	m									
	1 000	2 000	5 000	10 000	20 000	50 000	100 000	200 000	500 000	1 000 000
2	> 250	> 250	> 250	> 250	> 250	> 250	> 250	> 250	> 250	> 250
3	46,759	49,421	52,761	55,170	57,488	60,430	62,573	64,650	67,305	69,252
4	21,665	22,862	24,368	25,456	26,505	27,839	28,811	29,754	30,961	31,847
5	14,843	15,645	16,657	17,389	18,096	18,995	19,652	20,289	21,105	21,704
6	11,837	12,465	13,259	13,835	14,391	15,099	15,617	16,120	16,764	17,236
7	10,172	10,703	11,375	11,864	12,336	12,939	13,379	13,807	14,356	14,759
8	9,118	9,587	10,183	10,616	11,036	11,571	11,962	12,343	12,831	13,190
9	8,392	8,819	9,361	9,756	10,138	10,627	10,984	11,332	11,778	12,106
10	7,862	8,257	8,759	9,126	9,481	9,935	10,268	10,591	11,007	11,312
11	7,458	7,828	8,300	8,644	8,979	9,406	9,720	10,025	10,416	10,705
12	7,138	7,489	7,937	8,264	8,581	8,988	9,286	9,576	9,949	10,224
13	6,880	7,215	7,642	7,955	8,259	8,648	8,934	9,212	9,570	9,833
14	6,667	6,988	7,399	7,700	7,992	8,367	8,642	8,910	9,255	9,509
15	6,488	6,797	7,194	7,484	7,767	8,130	8,396	8,655	8,989	9,235
16	6,335	6,634	7,019	7,300	7,575	7,926	8,185	8,437	8,762	9,001
17	6,203	6,494	6,867	7,141	7,408	7,751	8,002	8,248	8,564	8,797
18	6,088	6,371	6,735	7,002	7,262	7,597	7,843	8,083	8,392	8,620
19	5,987	6,263	6,618	6,879	7,134	7,461	7,702	7,937	8,239	8,462
20	5,897	6,167	6,515	6,770	7,020	7,340	7,576	7,807	8,103	8,322
25	5,567	5,814	6,132	6,367	6,596	6,891	7,109	7,322	7,597	7,800
30	5,356	5,587	5,885	6,106	6,321	6,600	6,805	7,007	7,266	7,458
35	5,210	5,429	5,713	5,923	6,129	6,394	6,591	6,783	7,032	7,216
40	5,103	5,313	5,586	5,787	5,985	6,241	6,431	6,616	6,857	7,034
45	5,021	5,224	5,488	5,683	5,875	6,123	6,306	6,487	6,720	6,893
50	4,956	5,154	5,410	5,600	5,787	6,028	6,207	6,383	6,610	6,779
60	4,861	5,050	5,295	5,477	5,656	5,887	6,058	6,227	6,445	6,607
70	4,794	4,977	5,214	5,390	5,563	5,786	5,952	6,115	6,327	6,484
80	4,744	4,923	5,154	5,325	5,493	5,711	5,872	6,031	6,238	6,391
90	4,706	4,882	5,108	5,275	5,440	5,653	5,811	5,966	6,168	6,318
100	4,676	4,849	5,071	5,235	5,397	5,606	5,761	5,914	6,112	6,259
150	4,587	4,751	4,962	5,118	5,270	5,467	5,613	5,757	5,943	6,082
200	4,544	4,703	4,909	5,060	5,208	5,399	5,540	5,679	5,859	5,993
250	4,518	4,675	4,877	5,026	5,171	5,358	5,497	5,633	5,809	5,940
300	4,501	4,656	4,856	5,003	5,146	5,331	5,468	5,602	5,776	5,904
350	4,489	4,643	4,841	4,987	5,129	5,312	5,448	5,580	5,752	5,879
400	4,480	4,633	4,830	4,975	5,116	5,298	5,432	5,564	5,734	5,860
450	4,473	4,626	4,821	4,965	5,106	5,287	5,420	5,551	5,721	5,846
500	4,467	4,619	4,815	4,958	5,098	5,278	5,411	5,541	5,710	5,834
600	4,459	4,610	4,804	4,947	5,086	5,265	5,397	5,526	5,693	5,817
700	4,453	4,604	4,797	4,939	5,077	5,255	5,387	5,515	5,681	5,804
800	4,448	4,599	4,791	4,933	5,071	5,248	5,379	5,507	5,673	5,795
900	4,445	4,595	4,787	4,928	5,066	5,243	5,373	5,501	5,666	5,788
1 000	4,442	4,592	4,784	4,925	5,062	5,238	5,369	5,496	5,661	5,782
∞	4,417	4,565	4,753	4,892	5,026	5,199	5,327	5,451	5,612	5,731

NOTE This table provides factors k such that one may be at least 99,5 % confident that none of the next m observations from a normally distributed population will lie outside the range $(-\infty, \bar{x} + ks)$, where $\bar{x}$ and s are derived from a random sample of size n from the same population. Similarly for the range $(\bar{x} - ks, \infty)$.

Table A.6 — One-sided prediction interval factors, k, at confidence level 99,9 % for unknown population standard deviation

n	m										
	1	2	3	4	5	6	7	8	9	10	15
2	> 250	> 250	> 250	> 250	> 250	> 250	> 250	> 250	> 250	> 250	> 250
3	25,782	33,908	39,116	42,924	45,908	48,349	50,408	52,183	53,740	55,124	60,361
4	11,421	13,997	15,623	16,812	17,746	18,514	19,164	19,727	20,223	20,665	22,351
5	7,858	9,278	10,165	10,813	11,324	11,744	12,101	12,411	12,685	12,930	13,867
6	6,366	7,348	7,956	8,400	8,750	9,038	9,283	9,496	9,685	9,853	10,503
7	5,568	6,331	6,801	7,142	7,411	7,632	7,821	7,985	8,131	8,261	8,764
8	5,076	5,712	6,100	6,381	6,603	6,785	6,941	7,076	7,196	7,303	7,718
9	4,745	5,298	5,634	5,876	6,067	6,224	6,358	6,474	6,577	6,669	7,027
10	4,507	5,003	5,302	5,518	5,687	5,826	5,945	6,048	6,139	6,221	6,538
11	4,328	4,783	5,055	5,251	5,405	5,531	5,638	5,731	5,814	5,888	6,175
12	4,190	4,612	4,864	5,045	5,187	5,303	5,402	5,488	5,564	5,632	5,896
13	4,078	4,476	4,712	4,882	5,014	5,122	5,214	5,295	5,365	5,429	5,674
14	3,988	4,365	4,589	4,749	4,873	4,976	5,062	5,138	5,204	5,264	5,495
15	3,912	4,273	4,486	4,638	4,757	4,854	4,937	5,008	5,071	5,128	5,347
16	3,848	4,196	4,400	4,546	4,659	4,752	4,831	4,899	4,959	5,014	5,222
17	3,794	4,129	4,326	4,467	4,576	4,665	4,741	4,806	4,864	4,916	5,116
18	3,746	4,072	4,263	4,399	4,504	4,590	4,663	4,726	4,782	4,832	5,025
19	3,705	4,022	4,208	4,339	4,441	4,525	4,595	4,657	4,711	4,759	4,945
20	3,668	3,978	4,159	4,287	4,386	4,467	4,536	4,596	4,648	4,695	4,876
25	3,536	3,819	3,983	4,099	4,188	4,261	4,322	4,376	4,423	4,464	4,625
30	3,453	3,720	3,873	3,981	4,065	4,133	4,190	4,239	4,283	4,322	4,471
35	3,396	3,652	3,799	3,902	3,981	4,045	4,100	4,147	4,188	4,225	4,366
40	3,354	3,602	3,744	3,844	3,920	3,982	4,035	4,080	4,119	4,155	4,290
45	3,323	3,565	3,703	3,800	3,874	3,934	3,985	4,029	4,067	4,102	4,232
50	3,298	3,535	3,671	3,765	3,838	3,897	3,946	3,989	4,027	4,060	4,187
60	3,262	3,492	3,623	3,715	3,785	3,842	3,890	3,931	3,967	3,999	4,122
70	3,236	3,462	3,590	3,680	3,748	3,804	3,850	3,890	3,925	3,957	4,076
80	3,217	3,440	3,566	3,654	3,721	3,775	3,821	3,860	3,895	3,925	4,042
90	3,202	3,422	3,547	3,634	3,700	3,754	3,799	3,837	3,871	3,901	4,016
100	3,191	3,409	3,532	3,618	3,683	3,736	3,781	3,819	3,853	3,882	3,996
150	3,157	3,369	3,488	3,571	3,634	3,686	3,729	3,766	3,798	3,827	3,936
200	3,140	3,349	3,466	3,548	3,611	3,661	3,703	3,739	3,771	3,799	3,906
250	3,130	3,337	3,454	3,535	3,596	3,646	3,688	3,724	3,755	3,783	3,889
300	3,123	3,329	3,445	3,526	3,587	3,636	3,678	3,713	3,745	3,772	3,877
350	3,119	3,324	3,439	3,519	3,580	3,629	3,671	3,706	3,737	3,765	3,869
400	3,115	3,320	3,435	3,514	3,575	3,624	3,665	3,701	3,731	3,759	3,863
450	3,112	3,316	3,431	3,511	3,571	3,620	3,661	3,696	3,727	3,754	3,858
500	3,110	3,314	3,428	3,508	3,568	3,617	3,658	3,693	3,724	3,751	3,854
600	3,107	3,310	3,424	3,503	3,564	3,612	3,653	3,688	3,718	3,746	3,849
700	3,105	3,307	3,421	3,500	3,560	3,609	3,649	3,684	3,715	3,742	3,845
800	3,103	3,305	3,419	3,498	3,558	3,606	3,647	3,682	3,712	3,739	3,842
900	3,102	3,304	3,417	3,496	3,556	3,604	3,645	3,679	3,710	3,737	3,839
1 000	3,100	3,302	3,416	3,494	3,554	3,603	3,643	3,678	3,708	3,735	3,837
∞	3,091	3,291	3,403	3,481	3,540	3,588	3,628	3,663	3,693	3,719	3,821

Table A.6 — One-sided prediction interval factors, k, at confidence level 99,9 % for unknown population standard deviation *(continued)*

n	m										
	20	30	40	50	60	80	100	150	200	250	500
2	> 250	> 250	> 250	> 250	> 250	> 250	> 250	> 250	> 250	> 250	> 250
3	63,974	68,909	72,295	74,857	76,909	80,074	82,471	86,700	89,610	91,817	98,419
4	23,526	25,145	26,266	27,119	27,805	28,867	29,674	31,105	32,093	32,845	35,102
5	14,526	15,441	16,078	16,565	16,958	17,568	18,033	18,861	19,434	19,872	21,189
6	10,962	11,602	12,050	12,394	12,672	13,105	13,437	14,028	14,439	14,753	15,701
7	9,120	9,619	9,970	10,240	10,459	10,801	11,063	11,531	11,858	12,108	12,865
8	8,013	8,428	8,720	8,945	9,128	9,415	9,635	10,029	10,304	10,516	11,157
9	7,281	7,639	7,893	8,088	8,247	8,496	8,688	9,033	9,274	9,459	10,022
10	6,763	7,082	7,307	7,481	7,623	7,846	8,018	8,327	8,543	8,709	9,217
11	6,379	6,668	6,872	7,031	7,160	7,363	7,519	7,801	7,999	8,151	8,616
12	6,084	6,349	6,538	6,684	6,803	6,990	7,134	7,395	7,578	7,720	8,152
13	5,849	6,097	6,272	6,408	6,519	6,694	6,829	7,073	7,244	7,377	7,782
14	5,659	5,892	6,057	6,185	6,289	6,454	6,581	6,811	6,972	7,097	7,480
15	5,502	5,723	5,879	6,000	6,099	6,255	6,375	6,593	6,747	6,866	7,230
16	5,371	5,580	5,729	5,845	5,939	6,088	6,203	6,411	6,558	6,671	7,020
17	5,258	5,459	5,602	5,713	5,803	5,945	6,055	6,255	6,396	6,505	6,839
18	5,162	5,355	5,492	5,599	5,686	5,823	5,929	6,121	6,256	6,361	6,684
19	5,078	5,265	5,397	5,500	5,584	5,716	5,818	6,004	6,135	6,236	6,548
20	5,004	5,185	5,313	5,413	5,494	5,622	5,721	5,901	6,028	6,126	6,429
25	4,739	4,900	5,013	5,101	5,173	5,286	5,373	5,531	5,643	5,730	5,997
30	4,576	4,723	4,828	4,908	4,974	5,078	5,158	5,303	5,405	5,484	5,729
35	4,465	4,604	4,702	4,778	4,840	4,937	5,012	5,148	5,244	5,318	5,547
40	4,385	4,518	4,612	4,684	4,743	4,835	4,907	5,036	5,127	5,197	5,415
45	4,324	4,453	4,543	4,613	4,670	4,759	4,827	4,952	5,039	5,107	5,315
50	4,277	4,402	4,490	4,557	4,612	4,699	4,765	4,886	4,970	5,036	5,237
60	4,208	4,328	4,412	4,476	4,529	4,611	4,675	4,790	4,870	4,932	5,124
70	4,160	4,276	4,357	4,420	4,471	4,551	4,612	4,723	4,801	4,861	5,045
80	4,124	4,238	4,317	4,379	4,428	4,506	4,566	4,674	4,750	4,808	4,987
90	4,097	4,209	4,287	4,347	4,396	4,472	4,531	4,636	4,710	4,768	4,943
100	4,075	4,185	4,263	4,322	4,370	4,445	4,503	4,607	4,680	4,736	4,908
150	4,012	4,118	4,192	4,248	4,294	4,366	4,421	4,520	4,589	4,642	4,805
200	3,981	4,085	4,157	4,212	4,257	4,327	4,381	4,478	4,545	4,597	4,756
250	3,963	4,065	4,136	4,191	4,235	4,304	4,357	4,453	4,519	4,570	4,726
300	3,951	4,052	4,123	4,177	4,221	4,289	4,342	4,436	4,502	4,552	4,707
350	3,942	4,043	4,113	4,167	4,210	4,279	4,331	4,424	4,490	4,540	4,693
400	3,936	4,036	4,106	4,159	4,203	4,271	4,322	4,415	4,480	4,530	4,683
450	3,931	4,031	4,100	4,154	4,197	4,264	4,316	4,409	4,473	4,523	4,675
500	3,927	4,026	4,096	4,149	4,192	4,259	4,311	4,403	4,468	4,517	4,668
600	3,921	4,020	4,089	4,142	4,185	4,252	4,303	4,395	4,459	4,509	4,659
700	3,916	4,015	4,084	4,137	4,180	4,247	4,298	4,389	4,453	4,502	4,652
800	3,913	4,012	4,081	4,133	4,176	4,243	4,294	4,385	4,449	4,498	4,647
900	3,911	4,009	4,078	4,131	4,173	4,240	4,290	4,382	4,445	4,494	4,643
1 000	3,909	4,007	4,076	4,128	4,171	4,237	4,288	4,379	4,443	4,491	4,640
∞	3,891	3,988	4,056	4,108	4,150	4,215	4,265	4,355	4,418	4,466	4,612

Table A.6 — One-sided prediction interval factors, *k*, at confidence level 99,9 % for unknown population standard deviation *(continued)*

n	*m*									
	1 000	2 000	5 000	10 000	20 000	50 000	100 000	200 000	500 000	1 000 000
2	> 250	> 250	> 250	> 250	> 250	> 250	> 250	> 250	> 250	> 250
3	104,678	110,635	118,111	123,501	128,690	135,274	140,068	144,717	150,660	155,016
4	37,251	39,305	41,890	43,758	45,560	47,849	49,519	51,139	53,211	54,732
5	22,449	23,656	25,179	26,283	27,349	28,705	29,694	30,655	31,886	32,789
6	16,611	17,486	18,592	19,395	20,171	21,161	21,883	22,585	23,485	24,146
7	13,594	14,296	15,187	15,835	16,461	17,261	17,845	18,414	19,143	19,678
8	11,776	12,374	13,134	13,688	14,224	14,909	15,410	15,898	16,523	16,983
9	10,568	11,096	11,769	12,260	12,736	13,344	13,789	14,223	14,780	15,189
10	9,710	10,188	10,798	11,243	11,676	12,229	12,635	13,030	13,538	13,911
11	9,069	9,509	10,072	10,483	10,883	11,395	11,771	12,137	12,608	12,954
12	8,573	8,983	9,509	9,894	10,268	10,748	11,100	11,444	11,886	12,211
13	8,178	8,564	9,060	9,423	9,777	10,231	10,564	10,890	11,308	11,616
14	7,856	8,222	8,693	9,038	9,375	9,808	10,125	10,436	10,835	11,129
15	7,588	7,937	8,387	8,718	9,040	9,455	9,760	10,057	10,441	10,723
16	7,362	7,697	8,129	8,447	8,757	9,156	9,450	9,736	10,106	10,379
17	7,169	7,491	7,908	8,214	8,514	8,899	9,183	9,461	9,819	10,083
18	7,001	7,313	7,716	8,013	8,303	8,677	8,952	9,222	9,569	9,825
19	6,855	7,158	7,548	7,836	8,118	8,482	8,750	9,012	9,350	9,600
20	6,727	7,020	7,400	7,680	7,955	8,309	8,570	8,826	9,156	9,400
25	6,262	6,523	6,861	7,113	7,359	7,678	7,914	8,146	8,445	8,666
30	5,971	6,211	6,522	6,754	6,982	7,277	7,496	7,711	7,990	8,196
35	5,773	5,997	6,289	6,507	6,721	6,999	7,206	7,409	7,672	7,868
40	5,630	5,842	6,119	6,326	6,530	6,795	6,992	7,186	7,437	7,624
45	5,521	5,725	5,990	6,188	6,384	6,638	6,828	7,014	7,257	7,437
50	5,436	5,633	5,889	6,080	6,269	6,515	6,698	6,878	7,113	7,288
60	5,312	5,498	5,740	5,921	6,100	6,332	6,505	6,676	6,899	7,065
70	5,226	5,405	5,637	5,810	5,981	6,204	6,370	6,534	6,747	6,907
80	5,163	5,336	5,561	5,728	5,894	6,109	6,269	6,428	6,634	6,788
90	5,114	5,283	5,503	5,666	5,827	6,036	6,192	6,346	6,547	6,697
100	5,076	5,242	5,457	5,616	5,774	5,978	6,131	6,282	6,478	6,624
150	4,965	5,121	5,322	5,472	5,619	5,809	5,951	6,091	6,273	6,408
200	4,910	5,062	5,257	5,401	5,543	5,727	5,863	5,998	6,172	6,302
250	4,878	5,027	5,219	5,360	5,499	5,679	5,812	5,943	6,113	6,240
300	4,857	5,004	5,193	5,333	5,470	5,647	5,778	5,907	6,074	6,198
350	4,842	4,988	5,175	5,313	5,449	5,624	5,754	5,881	6,046	6,169
400	4,831	4,976	5,162	5,299	5,433	5,607	5,736	5,862	6,026	6,147
450	4,822	4,966	5,151	5,288	5,421	5,594	5,722	5,847	6,010	6,130
500	4,815	4,959	5,143	5,279	5,412	5,584	5,711	5,835	5,997	6,117
600	4,805	4,947	5,131	5,265	5,398	5,568	5,694	5,818	5,978	6,097
700	4,798	4,939	5,122	5,256	5,387	5,557	5,682	5,805	5,965	6,083
800	4,792	4,933	5,115	5,249	5,380	5,549	5,673	5,796	5,954	6,072
900	4,788	4,929	5,110	5,243	5,374	5,542	5,667	5,789	5,947	6,064
1 000	4,784	4,925	5,106	5,239	5,369	5,537	5,661	5,783	5,940	6,057
∞	4,754	4,892	5,069	5,200	5,327	5,491	5,612	5,731	5,885	5,998

NOTE This table provides factors k such that one may be at least 99,9 % confident that none of the next m observations from a normally distributed population will lie outside the range $(-\infty, \bar{x} + ks)$, where $\bar{x}$ and s are derived from a random sample of size n from the same population. Similarly for the range $(\bar{x} - ks, \infty)$.

Annex B
(normative)

Tables of two-sided prediction interval factors, k, for unknown population standard deviation

Table B.1 — Two-sided prediction interval factors, k, at confidence level 90 % for unknown population standard deviation

n	m										
	1	2	3	4	5	6	7	8	9	10	15
2	7,733	10,811	12,608	13,845	14,775	15,515	16,126	16,644	17,093	17,488	18,952
3	3,372	4,394	5,000	5,425	5,749	6,009	6,225	6,410	6,571	6,713	7,244
4	2,632	3,330	3,742	4,033	4,256	4,435	4,585	4,714	4,826	4,926	5,299
5	2,336	2,910	3,246	3,484	3,666	3,813	3,936	4,042	4,134	4,216	4,526
6	2,177	2,686	2,982	3,191	3,351	3,481	3,589	3,682	3,764	3,837	4,110
7	2,078	2,547	2,818	3,009	3,155	3,274	3,373	3,458	3,533	3,599	3,850
8	2,010	2,452	2,707	2,885	3,022	3,133	3,225	3,305	3,375	3,437	3,672
9	1,961	2,383	2,626	2,795	2,925	3,030	3,118	3,194	3,260	3,319	3,542
10	1,923	2,331	2,564	2,727	2,851	2,952	3,036	3,109	3,172	3,229	3,443
11	1,894	2,290	2,516	2,673	2,793	2,891	2,972	3,042	3,104	3,158	3,365
12	1,870	2,257	2,477	2,630	2,747	2,841	2,92	2,988	3,048	3,101	3,301
13	1,850	2,230	2,445	2,594	2,708	2,801	2,878	2,944	3,002	3,054	3,249
14	1,834	2,207	2,418	2,565	2,676	2,767	2,842	2,907	2,964	3,014	3,205
15	1,820	2,188	2,395	2,539	2,649	2,738	2,812	2,875	2,931	2,98	3,168
16	1,808	2,171	2,376	2,517	2,625	2,713	2,785	2,848	2,903	2,951	3,135
17	1,797	2,157	2,359	2,498	2,605	2,691	2,763	2,824	2,878	2,926	3,107
18	1,788	2,144	2,344	2,482	2,587	2,672	2,743	2,803	2,857	2,904	3,083
19	1,780	2,133	2,331	2,467	2,571	2,655	2,725	2,785	2,838	2,884	3,061
20	1,772	2,123	2,319	2,454	2,557	2,640	2,709	2,769	2,820	2,867	3,041
25	1,745	2,086	2,275	2,405	2,504	2,584	2,650	2,707	2,757	2,801	2,968
30	1,728	2,062	2,247	2,374	2,470	2,548	2,612	2,668	2,716	2,759	2,921
35	1,715	2,045	2,227	2,352	2,446	2,522	2,586	2,640	2,687	2,729	2,888
40	1,706	2,032	2,212	2,336	2,429	2,504	2,566	2,619	2,666	2,707	2,863
45	1,699	2,023	2,201	2,323	2,415	2,489	2,551	2,604	2,650	2,690	2,844
50	1,694	2,015	2,192	2,313	2,405	2,478	2,539	2,591	2,637	2,677	2,829
60	1,685	2,004	2,179	2,298	2,389	2,461	2,521	2,572	2,617	2,657	2,806
70	1,680	1,996	2,169	2,288	2,377	2,449	2,508	2,559	2,604	2,643	2,791
80	1,675	1,990	2,162	2,280	2,369	2,440	2,499	2,549	2,593	2,632	2,779
90	1,672	1,985	2,157	2,274	2,362	2,433	2,492	2,542	2,585	2,624	2,770
100	1,669	1,982	2,153	2,269	2,357	2,427	2,486	2,536	2,579	2,618	2,762
150	1,661	1,971	2,140	2,255	2,342	2,411	2,468	2,518	2,560	2,598	2,740
200	1,657	1,965	2,133	2,248	2,334	2,403	2,460	2,509	2,551	2,589	2,730
250	1,655	1,962	2,130	2,244	2,329	2,398	2,455	2,503	2,545	2,583	2,723
300	1,653	1,960	2,127	2,241	2,326	2,395	2,451	2,500	2,542	2,579	2,719
350	1,652	1,958	2,125	2,239	2,324	2,392	2,449	2,497	2,539	2,576	2,716
400	1,651	1,957	2,124	2,237	2,322	2,391	2,447	2,495	2,537	2,574	2,713
450	1,651	1,956	2,123	2,236	2,321	2,389	2,446	2,494	2,536	2,573	2,712
500	1,650	1,956	2,122	2,235	2,320	2,388	2,445	2,493	2,534	2,571	2,710
600	1,649	1,955	2,121	2,234	2,319	2,387	2,443	2,491	2,533	2,570	2,708
700	1,649	1,954	2,120	2,233	2,318	2,385	2,442	2,490	2,531	2,568	2,707
800	1,648	1,953	2,119	2,232	2,317	2,385	2,441	2,489	2,530	2,567	2,705
900	1,648	1,953	2,119	2,231	2,316	2,384	2,440	2,488	2,530	2,566	2,704
1 000	1,648	1,953	2,118	2,231	2,316	2,383	2,439	2,487	2,529	2,566	2,704
∞	1,645	1,949	2,115	2,227	2,311	2,379	2,434	2,482	2,523	2,560	2,697

Table B.1 — Two-sided prediction interval factors, k, at confidence level 90 % for unknown population standard deviation *(continued)*

n	m										
	20	30	40	50	60	80	100	150	200	250	500
2	19,941	21,269	22,169	22,845	23,384	24,212	24,836	25,933	26,684	27,253	28,948
3	7,607	8,097	8,432	8,684	8,886	9,197	9,431	9,845	10,130	10,345	10,989
4	5,555	5,904	6,143	6,324	6,469	6,692	6,861	7,160	7,366	7,522	7,989
5	4,739	5,030	5,230	5,381	5,503	5,691	5,833	6,085	6,259	6,391	6,786
6	4,299	4,558	4,737	4,872	4,981	5,149	5,277	5,503	5,659	5,778	6,134
7	4,024	4,262	4,427	4,552	4,652	4,808	4,926	5,135	5,280	5,390	5,722
8	3,835	4,059	4,213	4,331	4,425	4,572	4,683	4,881	5,018	5,122	5,435
9	3,697	3,909	4,057	4,169	4,259	4,399	4,505	4,694	4,825	4,925	5,225
10	3,591	3,795	3,937	4,045	4,131	4,266	4,368	4,550	4,677	4,773	5,062
11	3,508	3,705	3,842	3,946	4,030	4,160	4,260	4,436	4,558	4,652	4,933
12	3,440	3,632	3,765	3,866	3,948	4,075	4,171	4,343	4,462	4,553	4,827
13	3,385	3,572	3,701	3,800	3,880	4,003	4,098	4,266	4,382	4,471	4,739
14	3,338	3,521	3,647	3,744	3,822	3,943	4,036	4,200	4,314	4,401	4,664
15	3,298	3,477	3,602	3,696	3,773	3,892	3,983	4,144	4,256	4,342	4,600
16	3,263	3,439	3,562	3,655	3,730	3,847	3,936	4,095	4,206	4,290	4,544
17	3,233	3,407	3,527	3,619	3,693	3,808	3,896	4,053	4,161	4,244	4,495
18	3,207	3,378	3,497	3,587	3,66	3,774	3,861	4,015	4,122	4,204	4,452
19	3,183	3,352	3,469	3,559	3,631	3,743	3,829	3,981	4,087	4,168	4,413
20	3,162	3,329	3,445	3,533	3,605	3,716	3,800	3,951	4,056	4,136	4,378
25	3,084	3,243	3,354	3,439	3,507	3,613	3,694	3,838	3,938	4,015	4,247
30	3,033	3,187	3,295	3,376	3,442	3,545	3,623	3,763	3,860	3,934	4,159
35	2,997	3,148	3,253	3,332	3,397	3,497	3,573	3,709	3,804	3,877	4,096
40	2,971	3,119	3,221	3,300	3,363	3,461	3,536	3,669	3,762	3,833	4,049
45	2,950	3,096	3,197	3,274	3,336	3,433	3,507	3,638	3,730	3,800	4,011
50	2,934	3,078	3,178	3,254	3,315	3,411	3,483	3,613	3,704	3,773	3,982
60	2,910	3,051	3,149	3,224	3,284	3,377	3,449	3,576	3,664	3,732	3,937
70	2,892	3,032	3,129	3,202	3,262	3,354	3,424	3,549	3,636	3,703	3,904
80	2,880	3,018	3,114	3,186	3,245	3,336	3,405	3,529	3,615	3,681	3,880
90	2,870	3,007	3,102	3,174	3,232	3,322	3,391	3,514	3,599	3,664	3,861
100	2,862	2,998	3,092	3,164	3,222	3,311	3,380	3,501	3,586	3,650	3,846
150	2,838	2,972	3,064	3,134	3,191	3,278	3,345	3,464	3,546	3,609	3,800
200	2,826	2,959	3,050	3,120	3,175	3,262	3,328	3,445	3,527	3,589	3,776
250	2,819	2,951	3,042	3,111	3,166	3,252	3,318	3,434	3,515	3,576	3,762
300	2,815	2,946	3,036	3,105	3,160	3,246	3,311	3,427	3,507	3,568	3,753
350	2,811	2,942	3,032	3,101	3,156	3,241	3,306	3,421	3,501	3,562	3,746
400	2,809	2,939	3,029	3,097	3,152	3,237	3,302	3,417	3,497	3,558	3,741
450	2,807	2,937	3,027	3,095	3,150	3,235	3,299	3,414	3,494	3,554	3,738
500	2,805	2,935	3,025	3,093	3,148	3,232	3,297	3,412	3,491	3,552	3,734
600	2,803	2,933	3,022	3,090	3,145	3,229	3,294	3,408	3,487	3,547	3,730
700	2,801	2,931	3,020	3,088	3,143	3,227	3,291	3,405	3,484	3,545	3,726
800	2,800	2,930	3,019	3,086	3,141	3,225	3,289	3,403	3,482	3,542	3,724
900	2,799	2,929	3,018	3,085	3,140	3,224	3,288	3,402	3,480	3,541	3,722
1 000	2,798	2,928	3,017	3,084	3,139	3,223	3,287	3,400	3,479	3,539	3,720
∞	2,792	2,920	3,008	3,076	3,129	3,213	3,276	3,389	3,467	3,527	3,706

Table B.1 — Two-sided prediction interval factors, k, at confidence level 90 % for unknown population standard deviation *(continued)*

n	m									
	1 000	2 000	5 000	10 000	20 000	50 000	100 000	200 000	500 000	1 000 000
2	30,549	32,068	33,970	35,340	36,656	38,325	39,539	40,716	42,220	43,322
3	11,599	12,180	12,908	13,433	13,938	14,580	15,047	15,499	16,078	16,502
4	8,433	8,856	9,387	9,771	10,140	10,609	10,951	11,283	11,707	12,018
5	7,163	7,522	7,974	8,300	8,615	9,015	9,306	9,589	9,951	10,217
6	6,473	6,798	7,206	7,501	7,786	8,148	8,412	8,669	8,997	9,237
7	6,037	6,339	6,720	6,995	7,261	7,599	7,845	8,085	8,391	8,616
8	5,735	6,021	6,382	6,643	6,896	7,217	7,451	7,679	7,970	8,184
9	5,511	5,786	6,132	6,383	6,625	6,934	7,159	7,378	7,658	7,864
10	5,339	5,604	5,939	6,182	6,417	6,715	6,933	7,145	7,417	7,616
11	5,201	5,459	5,785	6,021	6,250	6,541	6,753	6,959	7,224	7,418
12	5,089	5,341	5,659	5,890	6,113	6,397	6,605	6,807	7,066	7,256
13	4,995	5,242	5,554	5,780	5,999	6,278	6,481	6,679	6,933	7,120
14	4,916	5,158	5,464	5,687	5,902	6,176	6,376	6,571	6,820	7,004
15	4,848	5,086	5,387	5,606	5,818	6,088	6,285	6,477	6,723	6,904
16	4,788	5,023	5,320	5,536	5,745	6,011	6,206	6,395	6,638	6,816
17	4,736	4,968	5,261	5,474	5,680	5,944	6,136	6,323	6,563	6,739
18	4,690	4,918	5,208	5,419	5,623	5,883	6,074	6,259	6,496	6,671
19	4,648	4,875	5,162	5,370	5,572	5,830	6,018	6,201	6,437	6,610
20	4,611	4,835	5,119	5,326	5,526	5,781	5,968	6,150	6,383	6,554
25	4,470	4,685	4,958	5,157	5,350	5,596	5,776	5,951	6,176	6,342
30	4,375	4,584	4,850	5,043	5,230	5,470	5,645	5,816	6,035	6,197
35	4,307	4,512	4,771	4,960	5,144	5,378	5,550	5,718	5,933	6,091
40	4,256	4,456	4,711	4,897	5,078	5,308	5,477	5,642	5,854	6,009
45	4,216	4,413	4,664	4,847	5,025	5,253	5,420	5,582	5,791	5,945
50	4,183	4,378	4,626	4,807	4,983	5,208	5,373	5,533	5,740	5,892
60	4,134	4,325	4,568	4,746	4,918	5,139	5,300	5,458	5,661	5,811
70	4,099	4,287	4,526	4,701	4,870	5,088	5,247	5,403	5,603	5,750
80	4,072	4,258	4,494	4,666	4,834	5,049	5,207	5,360	5,558	5,704
90	4,051	4,235	4,468	4,639	4,805	5,018	5,174	5,326	5,522	5,667
100	4,034	4,216	4,448	4,617	4,782	4,993	5,148	5,299	5,493	5,636
150	3,983	4,160	4,385	4,550	4,710	4,915	5,065	5,212	5,401	5,541
200	3,957	4,131	4,353	4,515	4,672	4,874	5,022	5,167	5,353	5,490
250	3,941	4,114	4,334	4,494	4,649	4,849	4,995	5,138	5,322	5,458
300	3,931	4,103	4,321	4,480	4,634	4,832	4,977	5,119	5,301	5,436
350	3,924	4,094	4,311	4,469	4,623	4,820	4,964	5,105	5,286	5,420
400	3,918	4,088	4,304	4,462	4,614	4,810	4,954	5,094	5,275	5,408
450	3,914	4,083	4,298	4,455	4,608	4,803	4,946	5,086	5,266	5,398
500	3,910	4,079	4,294	4,451	4,603	4,797	4,940	5,079	5,258	5,390
600	3,905	4,073	4,287	4,443	4,595	4,788	4,930	5,069	5,247	5,379
700	3,901	4,069	4,283	4,438	4,589	4,782	4,924	5,062	5,239	5,370
800	3,898	4,066	4,279	4,434	4,585	4,777	4,918	5,056	5,233	5,364
900	3,896	4,064	4,276	4,431	4,581	4,773	4,914	5,052	5,228	5,359
1 000	3,894	4,062	4,274	4,429	4,579	4,770	4,911	5,048	5,225	5,355
∞	3,878	4,044	4,254	4,406	4,554	4,743	4,882	5,017	5,190	5,318

NOTE This table provides factors k such that one may be at least 90 % confident that none of the next m observations from a normally distributed population will lie outside the range $(\bar{x} - ks, \bar{x} + ks)$, where $\bar{x}$ and s are derived from a random sample of size n from the same population.

Table B.2 — Two-sided prediction interval factors, *k*, at confidence level 95 % for unknown population standard deviation

n	*m*										
	1	2	3	4	5	6	7	8	9	10	15
2	15,562	21,708	25,299	27,773	29,635	31,115	32,338	33,375	34,274	35,064	37,996
3	4,969	6,392	7,243	7,842	8,299	8,667	8,974	9,236	9,464	9,666	10,421
4	3,559	4,412	4,923	5,286	5,564	5,790	5,979	6,141	6,282	6,408	6,881
5	3,042	3,697	4,087	4,364	4,578	4,751	4,897	5,022	5,132	5,229	5,597
6	2,777	3,334	3,663	3,896	4,077	4,223	4,347	4,453	4,546	4,628	4,942
7	2,616	3,115	3,407	3,615	3,775	3,905	4,015	4,109	4,192	4,265	4,545
8	2,509	2,968	3,237	3,427	3,573	3,692	3,793	3,879	3,955	4,022	4,279
9	2,431	2,864	3,115	3,293	3,429	3,541	3,634	3,715	3,785	3,848	4,088
10	2,373	2,786	3,024	3,192	3,321	3,427	3,515	3,591	3,658	3,718	3,945
11	2,328	2,725	2,953	3,114	3,238	3,338	3,422	3,495	3,559	3,616	3,833
12	2,291	2,676	2,897	3,051	3,170	3,267	3,348	3,418	3,480	3,535	3,743
13	2,262	2,636	2,850	3,000	3,116	3,209	3,288	3,355	3,415	3,468	3,669
14	2,237	2,603	2,812	2,958	3,070	3,161	3,237	3,303	3,361	3,412	3,608
15	2,216	2,574	2,779	2,922	3,031	3,120	3,195	3,259	3,315	3,365	3,556
16	2,198	2,550	2,751	2,891	2,998	3,085	3,158	3,221	3,276	3,325	3,511
17	2,182	2,529	2,727	2,864	2,969	3,055	3,126	3,188	3,242	3,290	3,472
18	2,168	2,511	2,705	2,841	2,944	3,028	3,098	3,159	3,212	3,259	3,439
19	2,156	2,495	2,687	2,820	2,922	3,005	3,074	3,133	3,186	3,232	3,409
20	2,145	2,481	2,670	2,802	2,903	2,984	3,052	3,111	3,162	3,208	3,382
25	2,105	2,428	2,609	2,734	2,830	2,907	2,972	3,027	3,076	3,119	3,283
30	2,080	2,394	2,569	2,691	2,783	2,858	2,920	2,974	3,020	3,062	3,220
35	2,062	2,370	2,542	2,660	2,751	2,823	2,884	2,936	2,982	3,022	3,176
40	2,048	2,352	2,522	2,638	2,727	2,798	2,858	2,909	2,953	2,993	3,143
45	2,038	2,339	2,506	2,621	2,708	2,779	2,837	2,888	2,932	2,971	3,118
50	2,030	2,328	2,494	2,608	2,694	2,763	2,821	2,871	2,914	2,953	3,098
60	2,018	2,313	2,476	2,587	2,672	2,741	2,797	2,846	2,889	2,926	3,069
70	2,010	2,302	2,463	2,573	2,657	2,724	2,781	2,829	2,871	2,908	3,048
80	2,003	2,293	2,453	2,563	2,646	2,713	2,768	2,816	2,857	2,894	3,033
90	1,998	2,287	2,446	2,555	2,637	2,703	2,758	2,806	2,847	2,883	3,021
100	1,995	2,282	2,440	2,548	2,630	2,696	2,751	2,798	2,839	2,875	3,012
150	1,983	2,267	2,422	2,529	2,610	2,674	2,728	2,774	2,814	2,850	2,983
200	1,977	2,259	2,414	2,520	2,599	2,663	2,717	2,762	2,802	2,837	2,969
250	1,974	2,255	2,409	2,514	2,593	2,657	2,710	2,755	2,795	2,830	2,961
300	1,972	2,252	2,405	2,510	2,589	2,653	2,705	2,751	2,790	2,825	2,956
350	1,970	2,250	2,403	2,507	2,586	2,650	2,702	2,747	2,786	2,821	2,952
400	1,969	2,248	2,401	2,505	2,584	2,647	2,700	2,745	2,784	2,818	2,949
450	1,968	2,247	2,400	2,504	2,583	2,646	2,698	2,743	2,782	2,816	2,946
500	1,967	2,246	2,398	2,503	2,581	2,644	2,697	2,741	2,780	2,815	2,945
600	1,966	2,244	2,397	2,501	2,579	2,642	2,694	2,739	2,778	2,812	2,942
700	1,965	2,243	2,395	2,499	2,578	2,641	2,693	2,737	2,776	2,811	2,940
800	1,965	2,242	2,395	2,498	2,577	2,639	2,692	2,736	2,775	2,809	2,939
900	1,964	2,242	2,394	2,498	2,576	2,639	2,691	2,735	2,774	2,808	2,937
1 000	1,964	2,241	2,393	2,497	2,575	2,638	2,69	2,734	2,773	2,807	2,936
∞	1,960	2,237	2,388	2,491	2,569	2,632	2,683	2,728	2,766	2,800	2,928

Table B.2 — Two-sided prediction interval factors, k, at confidence level 95 % for unknown population standard deviation *(continued)*

n	m										
	20	30	40	50	60	80	100	150	200	250	500
2	39,975	42,635	44,438	45,792	46,872	48,530	49,780	51,977	53,482	54,621	58,017
3	10,936	11,635	12,112	12,473	12,760	13,204	13,539	14,131	14,537	14,845	15,766
4	7,206	7,650	7,955	8,185	8,370	8,656	8,872	9,254	9,518	9,718	10,317
5	5,852	6,201	6,442	6,624	6,771	6,998	7,170	7,475	7,686	7,846	8,327
6	5,160	5,459	5,666	5,823	5,950	6,146	6,295	6,560	6,742	6,882	7,300
7	4,740	5,008	5,194	5,336	5,450	5,627	5,761	6,000	6,166	6,292	6,672
8	4,458	4,705	4,876	5,007	5,112	5,276	5,401	5,622	5,776	5,893	6,246
9	4,255	4,487	4,647	4,770	4,869	5,023	5,140	5,349	5,494	5,604	5,938
10	4,103	4,322	4,475	4,591	4,685	4,831	4,943	5,142	5,280	5,385	5,703
11	3,984	4,193	4,339	4,451	4,541	4,681	4,788	4,979	5,111	5,213	5,519
12	3,888	4,090	4,230	4,338	4,425	4,560	4,663	4,847	4,975	5,073	5,369
13	3,810	4,005	4,141	4,245	4,329	4,460	4,560	4,739	4,863	4,958	5,246
14	3,745	3,934	4,066	4,167	4,249	4,376	4,474	4,648	4,769	4,861	5,142
15	3,689	3,874	4,003	4,101	4,181	4,305	4,400	4,570	4,688	4,779	5,053
16	3,641	3,822	3,948	4,044	4,122	4,244	4,337	4,503	4,619	4,708	4,976
17	3,600	3,777	3,900	3,995	4,071	4,191	4,282	4,445	4,558	4,645	4,909
18	3,564	3,738	3,859	3,952	4,027	4,144	4,233	4,393	4,505	4,591	4,850
19	3,532	3,703	3,822	3,913	3,987	4,102	4,190	4,348	4,458	4,542	4,797
20	3,503	3,672	3,789	3,879	3,952	4,065	4,152	4,307	4,416	4,498	4,750
25	3,398	3,556	3,667	3,752	3,820	3,927	4,009	4,155	4,257	4,336	4,573
30	3,330	3,482	3,588	3,669	3,735	3,837	3,915	4,056	4,154	4,229	4,457
35	3,282	3,430	3,533	3,611	3,675	3,774	3,850	3,986	4,081	4,153	4,374
40	3,247	3,392	3,492	3,568	3,630	3,727	3,801	3,934	4,026	4,097	4,313
45	3,221	3,362	3,460	3,536	3,596	3,691	3,764	3,894	3,984	4,054	4,265
50	3,199	3,339	3,435	3,509	3,569	3,663	3,734	3,862	3,951	4,019	4,226
60	3,168	3,304	3,398	3,471	3,529	3,620	3,690	3,814	3,901	3,967	4,169
70	3,146	3,279	3,372	3,443	3,501	3,590	3,658	3,780	3,865	3,930	4,128
80	3,129	3,261	3,353	3,423	3,479	3,567	3,635	3,755	3,839	3,903	4,098
90	3,116	3,247	3,338	3,407	3,463	3,550	3,617	3,735	3,818	3,881	4,074
100	3,106	3,236	3,326	3,395	3,450	3,536	3,602	3,720	3,802	3,864	4,055
150	3,076	3,203	3,290	3,357	3,411	3,495	3,559	3,673	3,753	3,813	3,998
200	3,061	3,186	3,273	3,339	3,392	3,475	3,538	3,650	3,728	3,788	3,969
250	3,052	3,176	3,262	3,328	3,380	3,462	3,525	3,636	3,714	3,773	3,952
300	3,046	3,170	3,255	3,320	3,373	3,454	3,516	3,627	3,704	3,763	3,941
350	3,042	3,165	3,250	3,315	3,367	3,449	3,510	3,621	3,697	3,756	3,933
400	3,039	3,162	3,247	3,311	3,363	3,444	3,506	3,616	3,692	3,750	3,927
450	3,036	3,159	3,244	3,308	3,360	3,441	3,502	3,612	3,688	3,746	3,922
500	3,034	3,157	3,241	3,306	3,358	3,438	3,500	3,609	3,685	3,743	3,919
600	3,031	3,153	3,238	3,302	3,354	3,434	3,495	3,604	3,680	3,738	3,913
700	3,029	3,151	3,235	3,300	3,351	3,431	3,492	3,601	3,677	3,734	3,909
800	3,027	3,149	3,234	3,298	3,349	3,429	3,490	3,599	3,674	3,732	3,906
900	3,026	3,148	3,232	3,296	3,348	3,427	3,488	3,597	3,672	3,730	3,904
1 000	3,025	3,147	3,231	3,295	3,346	3,426	3,487	3,595	3,671	3,728	3,902
∞	3,016	3,137	3,221	3,284	3,335	3,414	3,474	3,582	3,656	3,713	3,885

Table B.2 — Two-sided prediction interval factors, k, at confidence level 95 % for unknown population standard deviation *(continued)*

n	m									
	1 000	2 000	5 000	10 000	20 000	50 000	100 000	200 000	500 000	1 000 000
2	61,224	64,269	68,080	70,824	73,462	76,806	79,239	81,597	84,610	86,818
3	16,639	17,470	18,512	19,263	19,987	20,905	21,574	22,222	23,051	23,659
4	10,887	11,430	12,113	12,606	13,081	13,684	14,124	14,551	15,097	15,497
5	8,784	9,221	9,772	10,170	10,553	11,041	11,397	11,742	12,184	12,509
6	7,699	8,081	8,563	8,911	9,248	9,675	9,987	10,291	10,679	10,963
7	7,035	7,383	7,821	8,139	8,446	8,837	9,122	9,399	9,754	10,014
8	6,584	6,908	7,318	7,615	7,902	8,267	8,534	8,793	9,125	9,369
9	6,257	6,564	6,952	7,234	7,506	7,852	8,106	8,352	8,667	8,899
10	6,009	6,302	6,673	6,943	7,204	7,536	7,779	8,015	8,318	8,540
11	5,813	6,095	6,453	6,714	6,965	7,286	7,521	7,749	8,042	8,256
12	5,654	5,928	6,275	6,527	6,771	7,083	7,311	7,533	7,817	8,026
13	5,522	5,789	6,127	6,373	6,611	6,914	7,137	7,353	7,630	7,834
14	5,412	5,672	6,002	6,242	6,475	6,772	6,989	7,201	7,472	7,671
15	5,317	5,572	5,895	6,130	6,358	6,650	6,863	7,070	7,336	7,532
16	5,235	5,485	5,802	6,033	6,257	6,543	6,753	6,957	7,219	7,411
17	5,163	5,409	5,721	5,948	6,169	6,450	6,657	6,857	7,115	7,305
18	5,100	5,342	5,649	5,873	6,090	6,368	6,571	6,769	7,024	7,211
19	5,044	5,282	5,585	5,806	6,020	6,294	6,495	6,691	6,942	7,126
20	4,993	5,228	5,528	5,746	5,958	6,228	6,427	6,620	6,868	7,051
25	4,804	5,026	5,310	5,518	5,719	5,977	6,167	6,351	6,588	6,762
30	4,678	4,892	5,166	5,366	5,560	5,809	5,992	6,170	6,399	6,568
35	4,589	4,797	5,062	5,256	5,445	5,688	5,866	6,039	6,262	6,427
40	4,522	4,725	4,984	5,174	5,358	5,595	5,770	5,939	6,158	6,319
45	4,470	4,668	4,923	5,109	5,290	5,523	5,694	5,861	6,076	6,234
50	4,428	4,623	4,873	5,056	5,235	5,464	5,632	5,797	6,009	6,165
60	4,365	4,555	4,799	4,977	5,151	5,374	5,539	5,699	5,906	6,059
70	4,320	4,506	4,745	4,920	5,090	5,309	5,470	5,628	5,831	5,981
80	4,286	4,470	4,704	4,876	5,044	5,260	5,418	5,573	5,773	5,921
90	4,260	4,441	4,672	4,842	5,008	5,220	5,377	5,530	5,728	5,874
100	4,239	4,418	4,647	4,815	4,978	5,189	5,344	5,495	5,691	5,835
150	4,176	4,349	4,569	4,731	4,889	5,091	5,241	5,387	5,575	5,714
200	4,144	4,314	4,530	4,689	4,843	5,041	5,188	5,330	5,515	5,651
250	4,125	4,293	4,507	4,663	4,815	5,011	5,155	5,296	5,478	5,612
300	4,113	4,279	4,491	4,646	4,797	4,991	5,133	5,273	5,452	5,585
350	4,104	4,269	4,480	4,634	4,784	4,976	5,118	5,256	5,434	5,566
400	4,097	4,262	4,471	4,625	4,774	4,965	5,106	5,243	5,421	5,552
450	4,092	4,256	4,465	4,617	4,766	4,956	5,096	5,233	5,410	5,540
500	4,088	4,251	4,459	4,612	4,760	4,949	5,089	5,225	5,401	5,531
600	4,081	4,244	4,452	4,603	4,750	4,939	5,078	5,214	5,388	5,517
700	4,077	4,239	4,446	4,597	4,744	4,932	5,070	5,205	5,379	5,507
800	4,074	4,236	4,442	4,592	4,739	4,926	5,064	5,199	5,372	5,500
900	4,071	4,233	4,438	4,589	4,735	4,922	5,059	5,194	5,366	5,494
1 000	4,069	4,230	4,436	4,586	4,732	4,918	5,056	5,190	5,362	5,489
∞	4,050	4,210	4,412	4,560	4,703	4,887	5,022	5,153	5,323	5,447

NOTE This table provides factors k such that one may be at least 95 % confident that none of the next m observations from a normally distributed population will lie outside the range $(\bar{x} - ks, \bar{x} + ks)$, where $\bar{x}$ and s are derived from a random sample of size n from the same population.

Table B.3 — Two-sided prediction interval factors, k, at confidence level 97,5 % for unknown population standard deviation

n	m										
	1	2	3	4	5	6	7	8	9	10	15
2	31,172	43,457	50,640	55,588	59,311	62,273	64,718	66,793	68,590	70,172	76,037
3	7,166	9,163	10,362	11,207	11,854	12,375	12,808	13,179	13,503	13,788	14,859
4	4,670	5,726	6,363	6,816	7,165	7,449	7,687	7,891	8,069	8,228	8,825
5	3,830	4,587	5,042	5,367	5,619	5,824	5,996	6,144	6,274	6,390	6,828
6	3,417	4,034	4,403	4,666	4,870	5,036	5,176	5,297	5,403	5,498	5,858
7	3,174	3,710	4,029	4,256	4,432	4,576	4,697	4,801	4,893	4,975	5,288
8	3,014	3,498	3,784	3,988	4,146	4,275	4,383	4,477	4,560	4,633	4,914
9	2,901	3,349	3,613	3,800	3,945	4,063	4,163	4,249	4,324	4,392	4,650
10	2,817	3,238	3,485	3,660	3,795	3,906	3,999	4,079	4,150	4,213	4,454
11	2,751	3,153	3,387	3,553	3,681	3,785	3,873	3,949	4,016	4,075	4,303
12	2,699	3,085	3,309	3,467	3,590	3,689	3,773	3,845	3,909	3,966	4,183
13	2,657	3,030	3,246	3,398	3,516	3,611	3,692	3,761	3,822	3,877	4,085
14	2,622	2,984	3,194	3,341	3,454	3,546	3,624	3,691	3,750	3,803	4,004
15	2,592	2,946	3,149	3,292	3,403	3,492	3,568	3,633	3,690	3,741	3,936
16	2,567	2,913	3,112	3,251	3,359	3,446	3,519	3,582	3,638	3,688	3,877
17	2,545	2,885	3,079	3,216	3,32	3,406	3,477	3,539	3,593	3,642	3,827
18	2,526	2,860	3,051	3,184	3,287	3,371	3,441	3,501	3,555	3,602	3,783
19	2,509	2,838	3,026	3,157	3,258	3,340	3,409	3,468	3,520	3,567	3,744
20	2,494	2,819	3,004	3,133	3,232	3,313	3,380	3,439	3,490	3,536	3,710
25	2,439	2,748	2,922	3,044	3,137	3,213	3,276	3,331	3,379	3,421	3,584
30	2,403	2,702	2,870	2,987	3,077	3,149	3,210	3,262	3,308	3,348	3,503
35	2,379	2,671	2,834	2,948	3,035	3,105	3,164	3,214	3,258	3,298	3,447
40	2,361	2,647	2,808	2,919	3,004	3,073	3,130	3,179	3,222	3,261	3,406
45	2,347	2,630	2,788	2,897	2,981	3,048	3,104	3,153	3,195	3,232	3,375
50	2,336	2,616	2,772	2,880	2,962	3,028	3,084	3,131	3,173	3,210	3,350
60	2,320	2,595	2,748	2,854	2,935	2,999	3,054	3,100	3,141	3,177	3,314
70	2,308	2,580	2,732	2,836	2,915	2,979	3,033	3,078	3,118	3,154	3,288
80	2,300	2,569	2,719	2,823	2,901	2,964	3,017	3,062	3,101	3,137	3,269
90	2,293	2,561	2,710	2,812	2,890	2,952	3,005	3,049	3,089	3,123	3,254
100	2,288	2,554	2,702	2,804	2,881	2,943	2,995	3,039	3,078	3,113	3,243
150	2,272	2,535	2,680	2,779	2,855	2,916	2,966	3,010	3,048	3,081	3,208
200	2,265	2,525	2,669	2,767	2,842	2,902	2,952	2,995	3,033	3,066	3,191
250	2,260	2,519	2,662	2,760	2,835	2,894	2,944	2,987	3,024	3,057	3,181
300	2,257	2,515	2,658	2,755	2,829	2,889	2,938	2,981	3,018	3,051	3,174
350	2,255	2,512	2,655	2,752	2,826	2,885	2,934	2,977	3,014	3,046	3,170
400	2,253	2,510	2,652	2,749	2,823	2,882	2,931	2,974	3,010	3,043	3,166
450	2,252	2,509	2,650	2,747	2,821	2,880	2,929	2,971	3,008	3,040	3,163
500	2,251	2,507	2,649	2,746	2,819	2,878	2,927	2,969	3,006	3,038	3,161
600	2,249	2,506	2,647	2,744	2,817	2,876	2,925	2,966	3,003	3,035	3,158
700	2,248	2,504	2,645	2,742	2,815	2,874	2,923	2,964	3,001	3,033	3,155
800	2,248	2,503	2,644	2,741	2,814	2,872	2,921	2,963	2,999	3,032	3,153
900	2,247	2,502	2,643	2,740	2,813	2,871	2,920	2,962	2,998	3,030	3,152
1 000	2,246	2,502	2,642	2,739	2,812	2,870	2,919	2,961	2,997	3,029	3,151
∞	2,242	2,496	2,636	2,732	2,804	2,862	2,911	2,952	2,988	3,020	3,141

Table B.3 — Two-sided prediction interval factors, k, at confidence level 97,5 % for unknown population standard deviation *(continued)*

n	m										
	20	30	40	50	60	80	100	150	200	250	500
2	79,996	85,317	88,924	91,634	93,795	97,112	99,613	104,008	107,020	109,299	116,095
3	15,590	16,582	17,260	17,771	18,180	18,810	19,287	20,127	20,704	21,142	22,452
4	9,237	9,799	10,185	10,478	10,713	11,076	11,351	11,837	12,172	12,427	13,190
5	7,132	7,550	7,838	8,057	8,233	8,505	8,712	9,079	9,333	9,526	10,105
6	6,108	6,453	6,692	6,874	7,021	7,248	7,421	7,729	7,942	8,104	8,592
7	5,506	5,807	6,017	6,177	6,306	6,506	6,659	6,930	7,119	7,262	7,695
8	5,110	5,383	5,572	5,717	5,834	6,016	6,155	6,402	6,574	6,705	7,101
9	4,831	5,082	5,257	5,391	5,500	5,668	5,797	6,027	6,186	6,308	6,677
10	4,623	4,858	5,023	5,148	5,250	5,408	5,530	5,746	5,896	6,011	6,360
11	4,463	4,685	4,841	4,960	5,056	5,207	5,322	5,527	5,671	5,780	6,112
12	4,335	4,547	4,696	4,810	4,902	5,046	5,156	5,353	5,490	5,595	5,914
13	4,231	4,435	4,578	4,687	4,776	4,914	5,020	5,210	5,342	5,443	5,751
14	4,145	4,342	4,479	4,585	4,671	4,804	4,907	5,090	5,218	5,316	5,614
15	4,072	4,263	4,396	4,499	4,582	4,712	4,811	4,989	5,114	5,209	5,499
16	4,010	4,196	4,325	4,425	4,506	4,632	4,729	4,902	5,024	5,117	5,399
17	3,957	4,137	4,264	4,361	4,440	4,563	4,658	4,827	4,946	5,036	5,313
18	3,910	4,086	4,210	4,305	4,382	4,503	4,595	4,761	4,877	4,966	5,237
19	3,868	4,041	4,163	4,256	4,331	4,450	4,540	4,703	4,816	4,904	5,169
20	3,832	4,002	4,121	4,212	4,286	4,402	4,491	4,651	4,763	4,848	5,109
25	3,697	3,855	3,966	4,051	4,120	4,227	4,310	4,459	4,562	4,642	4,886
30	3,611	3,761	3,866	3,947	4,013	4,115	4,193	4,334	4,433	4,509	4,740
35	3,552	3,696	3,798	3,875	3,938	4,036	4,112	4,248	4,342	4,415	4,638
40	3,508	3,649	3,747	3,822	3,883	3,979	4,052	4,183	4,275	4,346	4,562
45	3,474	3,612	3,708	3,782	3,841	3,935	4,006	4,134	4,224	4,293	4,503
50	3,448	3,583	3,678	3,750	3,808	3,900	3,970	4,095	4,183	4,251	4,457
60	3,409	3,541	3,632	3,702	3,759	3,848	3,916	4,038	4,123	4,188	4,387
70	3,382	3,511	3,600	3,669	3,725	3,811	3,878	3,997	4,080	4,144	4,338
80	3,361	3,488	3,577	3,644	3,699	3,784	3,850	3,967	4,048	4,111	4,302
90	3,346	3,471	3,559	3,625	3,679	3,763	3,828	3,943	4,024	4,085	4,273
100	3,333	3,458	3,544	3,610	3,664	3,747	3,811	3,925	4,004	4,065	4,251
150	3,296	3,417	3,501	3,565	3,617	3,698	3,759	3,869	3,946	4,005	4,184
200	3,278	3,397	3,480	3,543	3,594	3,673	3,734	3,842	3,918	3,975	4,151
250	3,267	3,385	3,467	3,530	3,580	3,659	3,719	3,826	3,901	3,958	4,131
300	3,260	3,377	3,459	3,521	3,571	3,649	3,709	3,815	3,889	3,946	4,118
350	3,255	3,372	3,453	3,515	3,565	3,643	3,702	3,808	3,881	3,938	4,109
400	3,251	3,368	3,449	3,510	3,560	3,637	3,696	3,802	3,875	3,931	4,102
450	3,248	3,364	3,445	3,507	3,556	3,633	3,692	3,797	3,871	3,927	4,096
500	3,246	3,362	3,442	3,504	3,553	3,630	3,689	3,794	3,867	3,923	4,092
600	3,242	3,358	3,438	3,499	3,549	3,626	3,684	3,789	3,861	3,917	4,085
700	3,239	3,355	3,435	3,496	3,546	3,622	3,681	3,785	3,857	3,913	4,081
800	3,238	3,353	3,433	3,494	3,543	3,620	3,678	3,782	3,854	3,910	4,077
900	3,236	3,351	3,431	3,492	3,541	3,618	3,676	3,780	3,852	3,907	4,075
1 000	3,235	3,350	3,430	3,491	3,540	3,616	3,674	3,778	3,850	3,905	4,072
∞	3,224	3,339	3,418	3,478	3,527	3,602	3,660	3,762	3,834	3,888	4,053

Table B.3 — Two-sided prediction interval factors, k, at confidence level 97,5 % for unknown population standard deviation *(continued)*

n	m									
	1 000	2 000	5 000	10 000	20 000	50 000	100 000	200 000	500 000	1 000 000
2	122,512	128,604	136,230	141,719	146,998	153,69	158,558	163,277	169,306	173,722
3	23,693	24,875	26,357	27,426	28,456	29,762	30,713	31,636	32,816	33,681
4	13,915	14,608	15,478	16,107	16,713	17,482	18,043	18,588	19,284	19,795
5	10,657	11,185	11,850	12,331	12,794	13,384	13,815	14,232	14,767	15,159
6	9,058	9,504	10,067	10,475	10,869	11,370	11,735	12,090	12,545	12,879
7	8,109	8,506	9,008	9,372	9,724	10,172	10,499	10,817	11,224	11,523
8	7,480	7,844	8,306	8,640	8,964	9,376	9,677	9,970	10,345	10,620
9	7,031	7,372	7,803	8,117	8,420	8,806	9,089	9,364	9,716	9,975
10	6,694	7,017	7,426	7,723	8,011	8,378	8,646	8,907	9,242	9,488
11	6,432	6,740	7,131	7,415	7,691	8,042	8,299	8,550	8,871	9,107
12	6,221	6,517	6,893	7,167	7,433	7,772	8,020	8,262	8,572	8,800
13	6,047	6,334	6,698	6,963	7,221	7,549	7,790	8,024	8,325	8,546
14	5,902	6,180	6,534	6,792	7,042	7,362	7,597	7,825	8,118	8,333
15	5,779	6,050	6,395	6,646	6,891	7,203	7,432	7,654	7,941	8,151
16	5,673	5,937	6,274	6,520	6,759	7,065	7,289	7,507	7,788	7,994
17	5,580	5,839	6,169	6,411	6,645	6,945	7,164	7,379	7,654	7,856
18	5,499	5,753	6,077	6,314	6,544	6,839	7,055	7,265	7,536	7,735
19	5,427	5,676	5,995	6,228	6,454	6,744	6,957	7,164	7,431	7,627
20	5,362	5,608	5,922	6,151	6,374	6,660	6,869	7,074	7,336	7,530
25	5,122	5,352	5,646	5,862	6,072	6,341	6,539	6,732	6,980	7,163
30	4,965	5,184	5,465	5,671	5,872	6,130	6,319	6,504	6,743	6,919
35	4,854	5,065	5,336	5,535	5,729	5,978	6,162	6,341	6,572	6,742
40	4,772	4,977	5,240	5,433	5,622	5,864	6,043	6,217	6,443	6,609
45	4,708	4,908	5,165	5,353	5,538	5,775	5,950	6,120	6,341	6,504
50	4,657	4,853	5,104	5,289	5,470	5,703	5,874	6,042	6,259	6,418
60	4,582	4,771	5,014	5,193	5,368	5,594	5,760	5,923	6,133	6,289
70	4,528	4,712	4,949	5,124	5,295	5,515	5,677	5,836	6,042	6,194
80	4,487	4,668	4,901	5,072	5,240	5,455	5,615	5,771	5,972	6,121
90	4,456	4,634	4,863	5,032	5,196	5,409	5,565	5,719	5,917	6,064
100	4,431	4,607	4,833	4,999	5,161	5,371	5,525	5,677	5,873	6,018
150	4,357	4,526	4,742	4,901	5,056	5,256	5,404	5,549	5,736	5,875
200	4,321	4,486	4,697	4,852	5,004	5,199	5,343	5,484	5,666	5,801
250	4,299	4,462	4,670	4,823	4,972	5,164	5,305	5,444	5,623	5,756
300	4,284	4,446	4,652	4,803	4,951	5,141	5,280	5,418	5,595	5,726
350	4,274	4,434	4,639	4,789	4,936	5,124	5,263	5,399	5,574	5,704
400	4,266	4,426	4,629	4,779	4,924	5,111	5,249	5,384	5,558	5,687
450	4,260	4,419	4,622	4,771	4,916	5,102	5,239	5,373	5,546	5,674
500	4,255	4,414	4,616	4,764	4,908	5,094	5,231	5,364	5,537	5,664
600	4,248	4,406	4,607	4,754	4,898	5,082	5,218	5,351	5,522	5,649
700	4,243	4,400	4,601	4,747	4,890	5,074	5,209	5,341	5,512	5,638
800	4,239	4,396	4,596	4,742	4,885	5,068	5,202	5,334	5,504	5,629
900	4,236	4,393	4,592	4,738	4,880	5,063	5,197	5,329	5,498	5,623
1 000	4,234	4,390	4,589	4,735	4,877	5,059	5,193	5,324	5,493	5,618
∞	4,212	4,366	4,563	4,706	4,846	5,024	5,156	5,284	5,450	5,572

NOTE This table provides factors k such that one may be at least 97,5 % confident that none of the next m observations from a normally distributed population will lie outside the range $(\bar{x} - ks, \bar{x} + ks)$, where $\bar{x}$ and s are derived from a random sample of size n from the same population.

Table B.4 — Two-sided prediction interval factors, k, at confidence level 99 % for unknown population standard deviation

n	m										
	1	2	3	4	5	6	7	8	9	10	15
2	77,964	108,673	126,629	138,998	148,307	155,711	161,825	167,013	171,506	175,460	190,123
3	11,461	14,604	16,496	17,831	18,853	19,676	20,362	20,949	21,461	21,913	23,608
4	6,531	7,943	8,800	9,412	9,885	10,269	10,591	10,868	11,110	11,325	12,138
5	5,044	5,973	6,536	6,940	7,254	7,510	7,725	7,911	8,074	8,220	8,771
6	4,356	5,072	5,504	5,814	6,056	6,253	6,420	6,564	6,691	6,804	7,235
7	3,964	4,563	4,923	5,181	5,382	5,547	5,686	5,806	5,912	6,007	6,368
8	3,712	4,238	4,553	4,778	4,954	5,097	5,219	5,324	5,416	5,499	5,816
9	3,537	4,014	4,298	4,500	4,658	4,787	4,896	4,991	5,074	5,149	5,434
10	3,409	3,850	4,111	4,297	4,442	4,561	4,661	4,748	4,824	4,893	5,155
11	3,311	3,725	3,969	4,143	4,278	4,389	4,482	4,563	4,634	4,698	4,942
12	3,233	3,627	3,858	4,022	4,149	4,253	4,341	4,417	4,485	4,545	4,775
13	3,170	3,547	3,768	3,924	4,045	4,144	4,228	4,300	4,364	4,421	4,640
14	3,119	3,482	3,693	3,844	3,960	4,055	4,135	4,204	4,265	4,320	4,529
15	3,075	3,427	3,631	3,776	3,888	3,980	4,057	4,123	4,182	4,235	4,436
16	3,038	3,380	3,579	3,719	3,827	3,916	3,990	4,055	4,112	4,163	4,357
17	3,006	3,340	3,533	3,670	3,775	3,861	3,934	3,996	4,051	4,101	4,289
18	2,978	3,305	3,494	3,627	3,730	3,814	3,884	3,945	3,999	4,047	4,230
19	2,954	3,275	3,459	3,590	3,690	3,772	3,841	3,900	3,953	4,000	4,179
20	2,932	3,247	3,429	3,557	3,655	3,735	3,803	3,861	3,912	3,958	4,133
25	2,853	3,148	3,317	3,436	3,527	3,601	3,663	3,717	3,764	3,806	3,967
30	2,802	3,086	3,247	3,360	3,446	3,516	3,575	3,626	3,671	3,710	3,862
35	2,768	3,043	3,198	3,307	3,390	3,458	3,515	3,563	3,606	3,644	3,790
40	2,742	3,011	3,163	3,269	3,350	3,415	3,470	3,518	3,559	3,596	3,737
45	2,723	2,987	3,136	3,240	3,319	3,383	3,437	3,483	3,524	3,560	3,697
50	2,707	2,968	3,114	3,216	3,294	3,357	3,410	3,456	3,495	3,531	3,666
60	2,684	2,940	3,083	3,183	3,258	3,320	3,371	3,415	3,454	3,488	3,619
70	2,668	2,920	3,061	3,159	3,233	3,293	3,344	3,387	3,425	3,459	3,587
80	2,656	2,905	3,045	3,141	3,215	3,274	3,324	3,366	3,404	3,437	3,563
90	2,647	2,894	3,032	3,127	3,200	3,259	3,308	3,350	3,387	3,420	3,544
100	2,640	2,885	3,022	3,117	3,189	3,247	3,296	3,337	3,374	3,406	3,529
150	2,618	2,858	2,992	3,085	3,155	3,212	3,259	3,300	3,335	3,367	3,486
200	2,608	2,845	2,978	3,069	3,138	3,194	3,241	3,281	3,316	3,347	3,465
250	2,601	2,837	2,969	3,060	3,129	3,184	3,230	3,270	3,305	3,335	3,452
300	2,597	2,832	2,963	3,053	3,122	3,177	3,223	3,263	3,297	3,328	3,444
350	2,594	2,829	2,959	3,049	3,117	3,172	3,218	3,257	3,292	3,322	3,438
400	2,592	2,826	2,956	3,046	3,114	3,169	3,214	3,254	3,288	3,318	3,433
450	2,590	2,824	2,954	3,043	3,111	3,166	3,211	3,250	3,285	3,315	3,430
500	2,589	2,822	2,952	3,041	3,109	3,163	3,209	3,248	3,282	3,312	3,427
600	2,587	2,819	2,949	3,038	3,106	3,160	3,206	3,244	3,278	3,309	3,423
700	2,585	2,818	2,947	3,036	3,103	3,158	3,203	3,242	3,276	3,306	3,420
800	2,584	2,816	2,945	3,034	3,102	3,156	3,201	3,240	3,274	3,304	3,418
900	2,583	2,815	2,944	3,033	3,100	3,154	3,200	3,238	3,272	3,302	3,416
1 000	2,583	2,814	2,943	3,032	3,099	3,153	3,199	3,237	3,271	3,301	3,414
∞	2,576	2,807	2,935	3,023	3,090	3,143	3,188	3,226	3,260	3,290	3,402

Table B.4 — Two-sided prediction interval factors, k, at confidence level 99 % for unknown population standard deviation *(continued)*

n	m										
	20	30	40	50	60	80	100	150	200	250	500
2	200,023	213,327	222,345	229,120	234,522	242,817	249,069	> 250	> 250	> 250	> 250
3	24,767	26,338	27,412	28,222	28,871	29,870	30,625	31,958	32,874	33,568	35,646
4	12,698	13,465	13,992	14,391	14,712	15,207	15,583	16,247	16,706	17,054	18,098
5	9,154	9,681	10,046	10,323	10,546	10,891	11,154	11,620	11,942	12,187	12,923
6	7,535	7,951	8,240	8,460	8,637	8,913	9,123	9,496	9,755	9,952	10,545
7	6,622	6,973	7,218	7,405	7,556	7,791	7,971	8,291	8,513	8,682	9,193
8	6,038	6,348	6,564	6,730	6,864	7,072	7,232	7,516	7,714	7,865	8,323
9	5,635	5,915	6,111	6,261	6,383	6,573	6,718	6,978	7,158	7,297	7,716
10	5,340	5,597	5,778	5,917	6,030	6,206	6,340	6,581	6,749	6,878	7,268
11	5,115	5,355	5,524	5,654	5,760	5,925	6,051	6,277	6,435	6,556	6,924
12	4,937	5,165	5,324	5,447	5,547	5,703	5,822	6,037	6,187	6,301	6,651
13	4,794	5,010	5,162	5,279	5,374	5,523	5,637	5,842	5,985	6,095	6,429
14	4,677	4,883	5,029	5,141	5,232	5,374	5,484	5,680	5,818	5,923	6,245
15	4,578	4,777	4,917	5,025	5,112	5,249	5,355	5,544	5,677	5,779	6,090
16	4,494	4,687	4,822	4,926	5,011	5,143	5,245	5,429	5,557	5,656	5,957
17	4,422	4,609	4,740	4,841	4,923	5,052	5,151	5,329	5,454	5,550	5,842
18	4,360	4,541	4,669	4,767	4,847	4,972	5,068	5,242	5,363	5,457	5,742
19	4,305	4,482	4,606	4,702	4,780	4,902	4,996	5,165	5,284	5,375	5,654
20	4,257	4,429	4,551	4,644	4,720	4,840	4,932	5,097	5,213	5,303	5,575
25	4,080	4,238	4,349	4,434	4,504	4,613	4,697	4,848	4,954	5,036	5,286
30	3,968	4,117	4,221	4,301	4,367	4,469	4,548	4,689	4,789	4,866	5,101
35	3,892	4,034	4,133	4,210	4,272	4,370	4,445	4,580	4,675	4,748	4,972
40	3,836	3,973	4,069	4,143	4,203	4,297	4,369	4,500	4,591	4,661	4,877
45	3,793	3,927	4,020	4,092	4,150	4,242	4,312	4,438	4,527	4,595	4,804
50	3,760	3,890	3,982	4,052	4,109	4,198	4,267	4,390	4,477	4,543	4,747
60	3,710	3,837	3,925	3,993	4,048	4,134	4,200	4,319	4,402	4,466	4,662
70	3,676	3,799	3,886	3,952	4,005	4,089	4,154	4,269	4,350	4,412	4,603
80	3,650	3,772	3,856	3,921	3,974	4,056	4,119	4,232	4,311	4,372	4,558
90	3,631	3,750	3,834	3,898	3,950	4,030	4,093	4,204	4,282	4,342	4,524
100	3,615	3,733	3,816	3,879	3,930	4,010	4,072	4,181	4,258	4,317	4,497
150	3,569	3,683	3,763	3,824	3,874	3,951	4,009	4,115	4,189	4,245	4,418
200	3,546	3,659	3,737	3,797	3,846	3,921	3,979	4,082	4,155	4,210	4,379
250	3,533	3,644	3,722	3,781	3,829	3,904	3,961	4,063	4,134	4,189	4,355
300	3,524	3,635	3,712	3,771	3,818	3,892	3,949	4,050	4,121	4,175	4,340
350	3,518	3,628	3,704	3,763	3,810	3,884	3,940	4,041	4,111	4,165	4,329
400	3,513	3,623	3,699	3,757	3,804	3,878	3,934	4,034	4,104	4,158	4,321
450	3,509	3,619	3,695	3,753	3,800	3,873	3,929	4,029	4,099	4,152	4,314
500	3,506	3,616	3,691	3,750	3,796	3,869	3,925	4,025	4,094	4,148	4,309
600	3,502	3,611	3,686	3,744	3,791	3,864	3,919	4,019	4,088	4,141	4,302
700	3,499	3,607	3,683	3,741	3,787	3,860	3,915	4,014	4,083	4,136	4,296
800	3,496	3,605	3,680	3,738	3,784	3,857	3,912	4,011	4,079	4,132	4,292
900	3,495	3,603	3,678	3,736	3,782	3,854	3,909	4,008	4,077	4,129	4,289
1 000	3,493	3,601	3,676	3,734	3,780	3,852	3,907	4,006	4,075	4,127	4,287
∞	3,480	3,587	3,662	3,718	3,764	3,835	3,890	3,987	4,055	4,107	4,264

Table B.4 — Two-sided prediction interval factors, k, at confidence level 99 % for unknown population standard deviation *(continued)*

n	m									
	1 000	2 000	5 000	10 000	20 000	50 000	100 000	200 000	500 000	1 000 000
2	> 250	> 250	> 250	> 250	> 250	> 250	> 250	> 250	> 250	> 250
3	37,614	39,489	41,841	43,537	45,170	47,243	48,753	50,217	52,089	53,462
4	19,090	20,038	21,230	22,090	22,920	23,975	24,743	25,489	26,444	27,143
5	13,626	14,298	15,145	15,758	16,349	17,101	17,650	18,183	18,865	19,365
6	11,113	11,657	12,344	12,842	13,323	13,935	14,382	14,816	15,372	15,781
7	9,683	10,154	10,749	11,181	11,598	12,130	12,519	12,896	13,380	13,735
8	8,762	9,184	9,720	10,109	10,485	10,964	11,315	11,656	12,093	12,414
9	8,119	8,507	9,000	9,359	9,706	10,149	10,472	10,787	11,192	11,488
10	7,644	8,007	8,468	8,804	9,129	9,544	9,848	10,144	10,524	10,803
11	7,279	7,622	8,058	8,376	8,684	9,078	9,367	9,647	10,008	10,273
12	6,989	7,316	7,732	8,036	8,330	8,707	8,983	9,252	9,597	9,850
13	6,753	7,066	7,466	7,758	8,041	8,404	8,669	8,928	9,261	9,505
14	6,557	6,859	7,245	7,527	7,801	8,151	8,408	8,659	8,981	9,217
15	6,391	6,684	7,058	7,331	7,597	7,937	8,187	8,430	8,743	8,973
16	6,250	6,534	6,898	7,164	7,422	7,754	7,997	8,234	8,539	8,764
17	6,127	6,404	6,759	7,018	7,271	7,594	7,832	8,064	8,362	8,581
18	6,020	6,291	6,637	6,891	7,137	7,454	7,687	7,914	8,206	8,421
19	5,926	6,190	6,529	6,778	7,020	7,330	7,558	7,781	8,068	8,279
20	5,842	6,101	6,433	6,677	6,915	7,220	7,444	7,663	7,944	8,152
25	5,531	5,770	6,077	6,303	6,524	6,807	7,016	7,220	7,483	7,676
30	5,331	5,556	5,846	6,059	6,268	6,537	6,735	6,929	7,179	7,363
35	5,191	5,406	5,683	5,887	6,087	6,345	6,535	6,722	6,962	7,140
40	5,088	5,295	5,562	5,760	5,953	6,202	6,386	6,566	6,799	6,972
45	5,009	5,210	5,469	5,661	5,848	6,091	6,270	6,445	6,672	6,840
50	4,947	5,142	5,395	5,582	5,765	6,001	6,176	6,348	6,570	6,734
60	4,854	5,042	5,284	5,464	5,640	5,867	6,036	6,201	6,415	6,574
70	4,789	4,971	5,206	5,380	5,551	5,771	5,935	6,095	6,303	6,458
80	4,740	4,918	5,148	5,317	5,484	5,699	5,859	6,016	6,219	6,369
90	4,703	4,877	5,102	5,269	5,432	5,643	5,800	5,953	6,153	6,300
100	4,673	4,845	5,066	5,230	5,391	5,598	5,752	5,903	6,099	6,245
150	4,585	4,749	4,960	5,115	5,267	5,463	5,609	5,752	5,937	6,075
200	4,542	4,702	4,907	5,058	5,206	5,396	5,538	5,676	5,856	5,989
250	4,517	4,674	4,876	5,024	5,169	5,356	5,495	5,631	5,806	5,937
300	4,500	4,655	4,855	5,002	5,145	5,330	5,466	5,600	5,774	5,902
350	4,488	4,642	4,840	4,986	5,128	5,311	5,446	5,579	5,750	5,877
400	4,479	4,632	4,829	4,974	5,115	5,297	5,431	5,563	5,733	5,859
450	4,472	4,625	4,821	4,964	5,105	5,286	5,419	5,550	5,719	5,845
500	4,466	4,619	4,814	4,957	5,097	5,277	5,410	5,540	5,708	5,833
600	4,458	4,609	4,804	4,946	5,085	5,264	5,396	5,525	5,692	5,816
700	4,452	4,603	4,796	4,938	5,076	5,255	5,386	5,514	5,681	5,804
800	4,448	4,598	4,791	4,932	5,070	5,248	5,378	5,506	5,672	5,794
900	4,444	4,594	4,787	4,928	5,065	5,242	5,373	5,500	5,665	5,787
1 000	4,441	4,591	4,783	4,924	5,061	5,238	5,368	5,495	5,660	5,781
∞	4,417	4,564	4,753	4,891	5,026	5,199	5,326	5,451	5,612	5,730

NOTE This table provides factors k such that one may be at least 99 % confident that none of the next m observations from a normally distributed population will lie outside the range $(\bar{x} - ks, \bar{x} + ks)$, where $\bar{x}$ and s are derived from a random sample of size n from the same population.

Table B.5 — Two-sided prediction interval factors, k, at confidence level 99,5 % for unknown population standard deviation

n	m										
	1	2	3	4	5	6	7	8	9	10	15
2	155,937	217,353	> 250	> 250	> 250	> 250	> 250	> 250	> 250	> 250	> 250
3	16,269	20,708	23,381	25,268	26,713	27,878	28,848	29,679	30,402	31,042	33,440
4	8,334	10,101	11,177	11,946	12,541	13,025	13,430	13,779	14,084	14,356	15,380
5	6,132	7,223	7,888	8,365	8,737	9,040	9,296	9,516	9,710	9,883	10,539
6	5,156	5,964	6,454	6,807	7,082	7,308	7,498	7,663	7,808	7,938	8,431
7	4,615	5,272	5,669	5,955	6,178	6,361	6,516	6,650	6,768	6,874	7,278
8	4,274	4,839	5,179	5,424	5,614	5,771	5,903	6,018	6,120	6,210	6,558
9	4,040	4,544	4,846	5,062	5,231	5,370	5,487	5,589	5,678	5,759	6,067
10	3,870	4,331	4,605	4,801	4,954	5,080	5,186	5,279	5,360	5,433	5,713
11	3,741	4,169	4,423	4,604	4,746	4,862	4,960	5,045	5,120	5,187	5,445
12	3,640	4,043	4,281	4,451	4,583	4,691	4,783	4,863	4,933	4,995	5,237
13	3,558	3,942	4,167	4,328	4,453	4,555	4,642	4,717	4,783	4,842	5,070
14	3,491	3,858	4,074	4,227	4,346	4,443	4,526	4,597	4,660	4,716	4,933
15	3,435	3,789	3,996	4,143	4,257	4,350	4,429	4,497	4,558	4,612	4,819
16	3,388	3,730	3,930	4,072	4,182	4,272	4,348	4,413	4,471	4,523	4,722
17	3,347	3,680	3,874	4,011	4,117	4,204	4,278	4,341	4,397	4,447	4,640
18	3,311	3,636	3,825	3,958	4,062	4,146	4,217	4,279	4,333	4,382	4,568
19	3,280	3,598	3,782	3,912	4,013	4,095	4,164	4,224	4,277	4,324	4,506
20	3,253	3,564	3,744	3,871	3,970	4,050	4,118	4,176	4,228	4,274	4,450
25	3,152	3,441	3,607	3,724	3,814	3,887	3,949	4,002	4,049	4,091	4,251
30	3,089	3,364	3,521	3,631	3,716	3,785	3,843	3,893	3,937	3,976	4,126
35	3,045	3,310	3,462	3,567	3,649	3,715	3,770	3,818	3,860	3,897	4,040
40	3,013	3,271	3,418	3,521	3,600	3,664	3,717	3,764	3,804	3,840	3,978
45	2,989	3,242	3,386	3,486	3,563	3,625	3,677	3,722	3,762	3,797	3,931
50	2,969	3,219	3,360	3,458	3,534	3,595	3,646	3,690	3,729	3,763	3,894
60	2,941	3,184	3,322	3,417	3,491	3,550	3,599	3,642	3,680	3,713	3,840
70	2,921	3,160	3,295	3,389	3,461	3,519	3,567	3,609	3,645	3,678	3,802
80	2,907	3,143	3,276	3,368	3,438	3,495	3,543	3,584	3,620	3,652	3,774
90	2,895	3,129	3,260	3,352	3,421	3,478	3,525	3,565	3,601	3,632	3,752
100	2,886	3,118	3,248	3,339	3,408	3,464	3,510	3,550	3,585	3,617	3,735
150	2,859	3,086	3,213	3,301	3,368	3,422	3,467	3,506	3,540	3,570	3,685
200	2,846	3,070	3,195	3,282	3,348	3,401	3,446	3,484	3,518	3,547	3,660
250	2,838	3,061	3,185	3,271	3,336	3,389	3,433	3,471	3,504	3,534	3,645
300	2,833	3,054	3,178	3,264	3,329	3,381	3,425	3,463	3,496	3,525	3,636
350	2,830	3,050	3,173	3,258	3,323	3,375	3,419	3,457	3,489	3,518	3,629
400	2,827	3,047	3,169	3,254	3,319	3,371	3,415	3,452	3,485	3,514	3,624
450	2,825	3,044	3,167	3,251	3,316	3,368	3,411	3,448	3,481	3,510	3,620
500	2,823	3,042	3,164	3,249	3,313	3,365	3,408	3,446	3,478	3,507	3,616
600	2,820	3,039	3,161	3,245	3,309	3,361	3,404	3,441	3,474	3,503	3,612
700	2,818	3,037	3,158	3,243	3,307	3,358	3,401	3,438	3,471	3,499	3,608
800	2,817	3,035	3,157	3,241	3,305	3,356	3,399	3,436	3,468	3,497	3,606
900	2,816	3,034	3,155	3,239	3,303	3,355	3,398	3,434	3,467	3,495	3,604
1 000	2,815	3,033	3,154	3,238	3,302	3,353	3,396	3,433	3,465	3,494	3,602
∞	2,808	3,023	3,144	3,227	3,290	3,341	3,384	3,420	3,452	3,481	3,588

Table B.5 — Two-sided prediction interval factors, k, at confidence level 99,5 % for unknown population standard deviation *(continued)*

n	m										
	20	30	40	50	60	80	100	150	200	250	500
2	> 250	> 250	> 250	> 250	> 250	> 250	> 250	> 250	> 250	> 250	> 250
3	35,080	37,304	38,823	39,970	40,888	42,302	43,372	45,258	46,554	47,538	50,479
4	16,087	17,054	17,720	18,224	18,629	19,255	19,730	20,570	21,149	21,589	22,908
5	10,995	11,623	12,057	12,388	12,654	13,067	13,380	13,937	14,321	14,614	15,495
6	8,777	9,255	9,587	9,841	10,046	10,364	10,606	11,037	11,336	11,563	12,250
7	7,562	7,956	8,232	8,442	8,613	8,878	9,080	9,441	9,691	9,883	10,461
8	6,802	7,143	7,382	7,565	7,713	7,944	8,121	8,437	8,657	8,825	9,333
9	6,285	6,589	6,802	6,966	7,099	7,306	7,465	7,750	7,948	8,100	8,560
10	5,911	6,188	6,382	6,532	6,653	6,843	6,989	7,250	7,432	7,572	7,996
11	5,628	5,884	6,065	6,203	6,316	6,492	6,628	6,871	7,041	7,171	7,567
12	5,408	5,647	5,816	5,946	6,052	6,217	6,345	6,573	6,733	6,856	7,230
13	5,231	5,457	5,616	5,739	5,839	5,996	6,117	6,333	6,485	6,602	6,957
14	5,086	5,301	5,453	5,570	5,665	5,814	5,929	6,136	6,281	6,393	6,733
15	4,965	5,171	5,316	5,428	5,519	5,663	5,773	5,971	6,110	6,217	6,544
16	4,863	5,061	5,201	5,308	5,396	5,534	5,640	5,831	5,965	6,068	6,384
17	4,776	4,967	5,101	5,206	5,290	5,423	5,526	5,711	5,840	5,940	6,246
18	4,700	4,885	5,015	5,116	5,198	5,327	5,427	5,606	5,732	5,829	6,126
19	4,634	4,813	4,940	5,038	5,118	5,243	5,340	5,514	5,637	5,731	6,020
20	4,575	4,750	4,874	4,970	5,047	5,169	5,264	5,433	5,553	5,645	5,927
25	4,364	4,522	4,634	4,720	4,790	4,900	4,985	5,138	5,246	5,329	5,585
30	4,231	4,379	4,483	4,563	4,628	4,731	4,810	4,952	5,053	5,130	5,368
35	4,141	4,281	4,380	4,456	4,518	4,615	4,690	4,825	4,920	4,993	5,218
40	4,075	4,210	4,305	4,378	4,437	4,531	4,602	4,732	4,823	4,893	5,109
45	4,025	4,156	4,248	4,319	4,376	4,467	4,536	4,661	4,749	4,817	5,025
50	3,986	4,114	4,204	4,273	4,329	4,416	4,484	4,606	4,691	4,757	4,959
60	3,928	4,052	4,138	4,204	4,258	4,343	4,407	4,524	4,606	4,669	4,863
70	3,888	4,008	4,092	4,157	4,209	4,291	4,354	4,467	4,547	4,608	4,795
80	3,859	3,976	4,059	4,122	4,173	4,253	4,315	4,425	4,503	4,563	4,745
90	3,836	3,952	4,033	4,095	4,145	4,224	4,285	4,393	4,469	4,528	4,707
100	3,818	3,932	4,012	4,074	4,123	4,201	4,261	4,368	4,443	4,500	4,677
150	3,765	3,875	3,952	4,011	4,059	4,133	4,190	4,293	4,364	4,420	4,588
200	3,738	3,847	3,922	3,980	4,027	4,100	4,156	4,256	4,326	4,380	4,544
250	3,723	3,830	3,905	3,962	4,008	4,080	4,135	4,234	4,303	4,356	4,518
300	3,713	3,819	3,893	3,950	3,996	4,067	4,122	4,220	4,288	4,341	4,501
350	3,705	3,811	3,885	3,941	3,987	4,058	4,112	4,210	4,278	4,330	4,489
400	3,700	3,805	3,879	3,935	3,980	4,051	4,105	4,202	4,270	4,322	4,480
450	3,696	3,801	3,874	3,930	3,975	4,046	4,100	4,196	4,264	4,315	4,473
500	3,692	3,797	3,870	3,926	3,971	4,041	4,095	4,191	4,259	4,310	4,467
600	3,687	3,792	3,864	3,920	3,965	4,035	4,088	4,184	4,251	4,303	4,459
700	3,684	3,788	3,860	3,916	3,961	4,030	4,084	4,179	4,246	4,297	4,453
800	3,681	3,785	3,857	3,913	3,957	4,027	4,080	4,176	4,242	4,293	4,448
900	3,679	3,783	3,855	3,910	3,955	4,024	4,077	4,173	4,239	4,290	4,445
1 000	3,677	3,781	3,853	3,908	3,953	4,022	4,075	4,170	4,237	4,287	4,442
∞	3,662	3,765	3,836	3,890	3,935	4,003	4,056	4,149	4,215	4,265	4,417

Table B.5 — Two-sided prediction interval factors, k, at confidence level 99,5 % for unknown population standard deviation *(continued)*

n	m									
	1 000	2 000	5 000	10 000	20 000	50 000	100 000	200 000	500 000	1 000 000
2	> 250	> 250	> 250	> 250	> 250	> 250	> 250	> 250	> 250	> 250
3	53,266	55,919	59,250	61,651	63,964	66,899	69,037	71,110	73,761	75,704
4	24,164	25,362	26,869	27,958	29,007	30,341	31,313	32,257	33,464	34,349
5	16,335	17,139	18,153	18,887	19,595	20,495	21,152	21,790	22,607	23,206
6	12,907	13,537	14,333	14,910	15,467	16,176	16,694	17,198	17,842	18,316
7	11,015	11,548	12,223	12,712	13,185	13,789	14,230	14,658	15,207	15,610
8	9,823	10,294	10,891	11,324	11,744	12,280	12,672	13,053	13,541	13,900
9	9,003	9,431	9,974	10,369	10,752	11,241	11,599	11,947	12,393	12,721
10	8,406	8,802	9,305	9,672	10,027	10,481	10,814	11,138	11,553	11,859
11	7,951	8,322	8,794	9,139	9,473	9,901	10,214	10,519	10,911	11,199
12	7,592	7,943	8,391	8,718	9,036	9,442	9,740	10,030	10,403	10,677
13	7,303	7,638	8,065	8,378	8,682	9,071	9,356	9,634	9,991	10,254
14	7,063	7,385	7,796	8,096	8,388	8,763	9,037	9,305	9,649	9,903
15	6,863	7,172	7,569	7,859	8,141	8,503	8,769	9,028	9,361	9,607
16	6,692	6,991	7,375	7,656	7,930	8,281	8,539	8,791	9,115	9,353
17	6,544	6,835	7,208	7,481	7,748	8,090	8,341	8,586	8,901	9,134
18	6,416	6,699	7,062	7,328	7,588	7,922	8,167	8,406	8,715	8,942
19	6,303	6,579	6,933	7,194	7,448	7,774	8,014	8,248	8,550	8,772
20	6,203	6,473	6,819	7,074	7,323	7,642	7,877	8,107	8,403	8,621
25	5,836	6,081	6,398	6,631	6,860	7,154	7,371	7,583	7,856	8,058
30	5,602	5,831	6,127	6,346	6,561	6,837	7,042	7,242	7,500	7,691
35	5,439	5,657	5,938	6,146	6,351	6,615	6,810	7,001	7,248	7,431
40	5,321	5,529	5,799	5,999	6,195	6,449	6,637	6,821	7,060	7,236
45	5,230	5,431	5,692	5,885	6,075	6,321	6,503	6,682	6,913	7,085
50	5,158	5,354	5,607	5,795	5,979	6,219	6,396	6,570	6,796	6,963
60	5,053	5,240	5,481	5,661	5,837	6,066	6,236	6,403	6,619	6,780
70	4,979	5,159	5,393	5,566	5,737	5,957	6,122	6,283	6,492	6,648
80	4,924	5,100	5,327	5,496	5,661	5,876	6,036	6,193	6,397	6,548
90	4,882	5,054	5,277	5,441	5,603	5,813	5,969	6,123	6,322	6,471
100	4,849	5,018	5,237	5,398	5,557	5,763	5,916	6,067	6,263	6,408
150	4,751	4,912	5,118	5,271	5,420	5,614	5,758	5,899	6,083	6,220
200	4,704	4,859	5,060	5,208	5,353	5,541	5,680	5,816	5,994	6,125
250	4,675	4,829	5,026	5,171	5,313	5,497	5,633	5,767	5,940	6,069
300	4,657	4,808	5,003	5,146	5,287	5,468	5,602	5,734	5,905	6,031
350	4,643	4,794	4,987	5,129	5,268	5,448	5,580	5,711	5,879	6,005
400	4,633	4,783	4,975	5,116	5,254	5,432	5,564	5,693	5,861	5,985
450	4,626	4,774	4,965	5,106	5,243	5,420	5,551	5,680	5,846	5,969
500	4,619	4,768	4,958	5,098	5,235	5,411	5,541	5,669	5,834	5,957
600	4,610	4,758	4,947	5,086	5,222	5,397	5,526	5,653	5,817	5,938
700	4,604	4,751	4,939	5,077	5,212	5,387	5,515	5,641	5,804	5,925
800	4,599	4,745	4,933	5,071	5,206	5,379	5,507	5,633	5,795	5,915
900	4,595	4,741	4,928	5,066	5,200	5,373	5,501	5,626	5,788	5,908
1 000	4,592	4,738	4,925	5,062	5,196	5,369	5,496	5,621	5,782	5,902
∞	4,565	4,708	4,892	5,026	5,158	5,327	5,451	5,573	5,731	5,847

NOTE This table provides factors k such that one may be at least 99,5 % confident that none of the next m observations from a normally distributed population will lie outside the range $(\bar{x} - ks, \bar{x} + ks)$, where $\bar{x}$ and s are derived from a random sample of size n from the same population.

Table B.6 — Two-sided prediction interval factors, k, at confidence level 99,9 % for unknown population standard deviation

n	m										
	1	2	3	4	5	6	7	8	9	10	15
2	> 250	> 250	> 250	> 250	> 250	> 250	> 250	> 250	> 250	> 250	> 250
3	36,488	46,400	52,375	56,594	59,825	62,428	64,599	66,456	68,074	69,505	74,870
4	14,450	17,451	19,284	20,595	21,611	22,436	23,129	23,726	24,248	24,712	26,465
5	9,433	11,037	12,020	12,729	13,281	13,732	14,114	14,443	14,732	14,990	15,970
6	7,420	8,504	9,168	9,648	10,024	10,332	10,593	10,819	11,018	11,195	11,875
7	6,371	7,199	7,704	8,070	8,357	8,592	8,792	8,965	9,118	9,255	9,780
8	5,736	6,416	6,830	7,128	7,362	7,555	7,719	7,861	7,986	8,099	8,531
9	5,315	5,899	6,253	6,508	6,708	6,873	7,013	7,135	7,242	7,338	7,710
10	5,015	5,534	5,846	6,072	6,248	6,393	6,517	6,624	6,719	6,804	7,132
11	4,791	5,263	5,545	5,749	5,908	6,039	6,150	6,247	6,332	6,409	6,705
12	4,619	5,054	5,314	5,501	5,647	5,767	5,869	5,957	6,035	6,106	6,377
13	4,481	4,888	5,131	5,304	5,440	5,552	5,646	5,728	5,801	5,866	6,118
14	4,369	4,754	4,982	5,146	5,273	5,378	5,466	5,544	5,612	5,673	5,909
15	4,277	4,643	4,860	5,014	5,135	5,234	5,318	5,391	5,455	5,513	5,736
16	4,199	4,549	4,756	4,904	5,019	5,114	5,194	5,263	5,324	5,379	5,591
17	4,132	4,470	4,669	4,811	4,921	5,011	5,088	5,154	5,213	5,266	5,468
18	4,074	4,401	4,593	4,730	4,836	4,923	4,997	5,061	5,117	5,168	5,363
19	4,024	4,341	4,527	4,660	4,762	4,847	4,918	4,979	5,034	5,083	5,271
20	3,980	4,289	4,470	4,598	4,698	4,779	4,848	4,908	4,961	5,008	5,190
25	3,820	4,100	4,262	4,377	4,466	4,539	4,600	4,653	4,700	4,742	4,902
30	3,720	3,982	4,134	4,240	4,323	4,390	4,446	4,495	4,539	4,577	4,725
35	3,652	3,902	4,046	4,147	4,225	4,289	4,342	4,389	4,429	4,466	4,605
40	3,603	3,844	3,983	4,080	4,155	4,216	4,267	4,312	4,351	4,385	4,519
45	3,565	3,800	3,935	4,029	4,102	4,161	4,211	4,253	4,291	4,325	4,453
50	3,536	3,766	3,897	3,989	4,060	4,118	4,166	4,208	4,245	4,277	4,402
60	3,492	3,715	3,842	3,931	3,999	4,055	4,101	4,141	4,177	4,208	4,328
70	3,462	3,680	3,804	3,890	3,957	4,011	4,056	4,095	4,129	4,160	4,276
80	3,440	3,654	3,775	3,860	3,926	3,978	4,023	4,061	4,094	4,124	4,238
90	3,423	3,634	3,754	3,837	3,902	3,954	3,997	4,035	4,068	4,097	4,209
100	3,409	3,618	3,736	3,819	3,883	3,934	3,977	4,014	4,046	4,075	4,186
150	3,369	3,571	3,686	3,766	3,827	3,876	3,917	3,953	3,984	4,012	4,118
200	3,349	3,548	3,661	3,739	3,799	3,848	3,888	3,923	3,954	3,981	4,085
250	3,337	3,535	3,646	3,724	3,783	3,831	3,871	3,906	3,936	3,963	4,065
300	3,329	3,526	3,636	3,713	3,772	3,820	3,860	3,894	3,924	3,951	4,052
350	3,324	3,519	3,629	3,706	3,765	3,812	3,852	3,886	3,915	3,942	4,043
400	3,320	3,514	3,624	3,701	3,759	3,806	3,845	3,879	3,909	3,936	4,036
450	3,317	3,511	3,620	3,696	3,754	3,801	3,841	3,875	3,904	3,931	4,031
500	3,314	3,508	3,617	3,693	3,751	3,798	3,837	3,871	3,900	3,927	4,026
600	3,310	3,503	3,612	3,688	3,746	3,792	3,831	3,865	3,894	3,921	4,020
700	3,307	3,500	3,609	3,684	3,742	3,788	3,827	3,861	3,890	3,916	4,015
800	3,305	3,498	3,606	3,682	3,739	3,786	3,824	3,858	3,887	3,913	4,012
900	3,304	3,496	3,604	3,679	3,737	3,783	3,822	3,855	3,885	3,911	4,009
1 000	3,302	3,494	3,603	3,678	3,735	3,781	3,820	3,854	3,883	3,909	4,007
∞	3,291	3,481	3,588	3,663	3,719	3,765	3,804	3,836	3,865	3,891	3,988

Table B.6 — Two-sided prediction interval factors, k, at confidence level 99,9 % for unknown population standard deviation *(continued)*

n	m										
	20	30	40	50	60	80	100	150	200	250	500
2	> 250	> 250	> 250	> 250	> 250	> 250	> 250	> 250	> 250	> 250	> 250
3	78,537	83,513	86,913	89,480	91,533	94,697	97,091	101,311	104,213	106,414	112,995
4	27,676	29,333	30,474	31,339	32,033	33,106	33,921	35,361	36,355	37,110	39,375
5	16,653	17,595	18,247	18,744	19,143	19,763	20,235	21,072	21,651	22,092	23,418
6	12,351	13,012	13,472	13,824	14,108	14,549	14,885	15,485	15,900	16,217	17,173
7	10,150	10,666	11,026	11,303	11,526	11,875	12,141	12,617	12,947	13,200	13,964
8	8,837	9,264	9,565	9,796	9,983	10,275	10,498	10,899	11,178	11,391	12,039
9	7,973	8,342	8,602	8,802	8,964	9,219	9,414	9,764	10,008	10,195	10,764
10	7,364	7,692	7,923	8,101	8,246	8,473	8,648	8,961	9,180	9,349	9,861
11	6,915	7,211	7,420	7,582	7,714	7,920	8,079	8,365	8,565	8,719	9,189
12	6,570	6,842	7,034	7,183	7,304	7,495	7,641	7,906	8,091	8,234	8,670
13	6,297	6,550	6,729	6,867	6,980	7,158	7,295	7,542	7,715	7,849	8,258
14	6,077	6,313	6,481	6,612	6,718	6,884	7,013	7,246	7,410	7,536	7,922
15	5,895	6,118	6,277	6,400	6,501	6,659	6,781	7,001	7,157	7,276	7,644
16	5,742	5,955	6,106	6,223	6,319	6,469	6,585	6,796	6,944	7,058	7,409
17	5,613	5,816	5,961	6,073	6,164	6,308	6,419	6,621	6,763	6,872	7,209
18	5,501	5,697	5,835	5,943	6,031	6,169	6,276	6,470	6,606	6,712	7,036
19	5,404	5,593	5,727	5,830	5,915	6,048	6,151	6,338	6,470	6,572	6,886
20	5,320	5,502	5,631	5,732	5,813	5,942	6,042	6,223	6,351	6,449	6,753
25	5,016	5,176	5,290	5,378	5,450	5,563	5,651	5,809	5,921	6,008	6,276
30	4,829	4,976	5,080	5,161	5,226	5,329	5,409	5,554	5,656	5,735	5,980
35	4,703	4,841	4,938	5,014	5,075	5,171	5,246	5,381	5,477	5,550	5,778
40	4,612	4,744	4,836	4,908	4,966	5,058	5,129	5,257	5,347	5,417	5,633
45	4,544	4,670	4,759	4,828	4,884	4,972	5,040	5,163	5,250	5,317	5,523
50	4,490	4,613	4,699	4,766	4,820	4,905	4,971	5,090	5,174	5,238	5,438
60	4,412	4,529	4,612	4,675	4,727	4,808	4,871	4,984	5,063	5,124	5,313
70	4,357	4,471	4,551	4,612	4,662	4,741	4,801	4,910	4,986	5,045	5,227
80	4,318	4,428	4,506	4,566	4,615	4,691	4,750	4,855	4,930	4,987	5,163
90	4,287	4,396	4,472	4,531	4,579	4,653	4,711	4,814	4,887	4,943	5,115
100	4,263	4,370	4,445	4,503	4,550	4,623	4,680	4,781	4,853	4,908	5,076
150	4,192	4,294	4,366	4,421	4,466	4,536	4,589	4,686	4,753	4,805	4,965
200	4,157	4,257	4,327	4,381	4,425	4,493	4,545	4,639	4,705	4,756	4,910
250	4,136	4,235	4,304	4,357	4,400	4,468	4,519	4,612	4,676	4,726	4,878
300	4,123	4,221	4,289	4,342	4,384	4,451	4,502	4,593	4,657	4,707	4,857
350	4,113	4,211	4,279	4,331	4,373	4,439	4,490	4,580	4,644	4,693	4,842
400	4,106	4,203	4,271	4,322	4,364	4,430	4,480	4,571	4,634	4,683	4,831
450	4,100	4,197	4,264	4,316	4,358	4,423	4,473	4,563	4,626	4,675	4,822
500	4,096	4,192	4,259	4,311	4,353	4,418	4,468	4,557	4,620	4,668	4,815
600	4,089	4,185	4,252	4,303	4,345	4,410	4,459	4,549	4,611	4,659	4,805
700	4,084	4,180	4,247	4,298	4,339	4,404	4,453	4,542	4,604	4,652	4,798
800	4,081	4,176	4,243	4,294	4,335	4,399	4,449	4,537	4,599	4,647	4,792
900	4,078	4,173	4,240	4,291	4,332	4,396	4,445	4,534	4,596	4,643	4,788
1 000	4,076	4,171	4,237	4,288	4,329	4,393	4,443	4,531	4,593	4,640	4,784
∞	4,056	4,150	4,215	4,265	4,306	4,369	4,418	4,504	4,565	4,612	4,754

Table B.6 — Two-sided prediction interval factors, k, at confidence level 99,9 % for unknown population standard deviation *(continued)*

n	m									
	1 000	2 000	5 000	10 000	20 000	50 000	100 000	200 000	500 000	1 000 000
2	> 250	> 250	> 250	> 250	> 250	> 250	> 250	> 250	> 250	> 250
3	119,233	125,172	132,625	138,000	143,176	149,745	154,530	159,170	165,110	169,460
4	41,530	43,587	46,174	48,044	49,847	52,137	53,807	55,428	57,501	59,022
5	24,684	25,896	27,424	28,530	29,598	30,956	31,947	32,909	34,142	35,046
6	18,089	18,968	20,079	20,885	21,663	22,654	23,378	24,082	24,983	25,645
7	14,698	15,404	16,299	16,949	17,577	18,379	18,964	19,534	20,264	20,800
8	12,663	13,264	14,027	14,583	15,121	15,807	16,309	16,798	17,425	17,885
9	11,314	11,845	12,520	13,013	13,490	14,100	14,546	14,980	15,538	15,948
10	10,357	10,838	11,450	11,897	12,331	12,886	13,292	13,688	14,196	14,570
11	9,645	10,087	10,652	11,065	11,466	11,979	12,355	12,722	13,193	13,539
12	9,094	9,507	10,034	10,420	10,795	11,276	11,629	11,973	12,415	12,740
13	8,656	9,044	9,541	9,906	10,260	10,715	11,048	11,374	11,793	12,101
14	8,299	8,667	9,139	9,486	9,823	10,256	10,574	10,885	11,284	11,578
15	8,003	8,354	8,805	9,136	9,459	9,874	10,179	10,477	10,860	11,142
16	7,753	8,090	8,523	8,841	9,151	9,551	9,844	10,131	10,501	10,773
17	7,540	7,864	8,281	8,588	8,888	9,273	9,557	9,835	10,192	10,456
18	7,355	7,668	8,071	8,369	8,659	9,033	9,308	9,577	9,924	10,181
19	7,194	7,497	7,888	8,177	8,459	8,822	9,090	9,352	9,690	9,939
20	7,053	7,347	7,727	8,007	8,282	8,636	8,897	9,152	9,482	9,725
25	6,541	6,802	7,140	7,391	7,638	7,956	8,192	8,422	8,721	8,942
30	6,221	6,460	6,771	7,002	7,230	7,524	7,743	7,957	8,235	8,441
35	6,004	6,227	6,518	6,735	6,948	7,226	7,431	7,634	7,896	8,091
40	5,847	6,058	6,334	6,540	6,742	7,006	7,202	7,395	7,646	7,832
45	5,728	5,930	6,194	6,391	6,585	6,838	7,027	7,212	7,454	7,633
50	5,635	5,830	6,084	6,274	6,462	6,706	6,888	7,067	7,301	7,474
60	5,499	5,683	5,924	6,103	6,280	6,510	6,682	6,852	7,073	7,238
70	5,405	5,582	5,812	5,983	6,152	6,373	6,538	6,700	6,912	7,070
80	5,336	5,507	5,729	5,895	6,058	6,272	6,431	6,588	6,792	6,945
90	5,284	5,450	5,666	5,828	5,987	6,194	6,348	6,501	6,700	6,848
100	5,242	5,405	5,617	5,774	5,930	6,132	6,283	6,432	6,626	6,771
150	5,121	5,274	5,472	5,619	5,763	5,951	6,091	6,229	6,409	6,543
200	5,062	5,210	5,402	5,543	5,683	5,864	5,998	6,130	6,302	6,431
250	5,027	5,172	5,360	5,499	5,635	5,812	5,943	6,072	6,240	6,364
300	5,004	5,148	5,333	5,470	5,604	5,778	5,907	6,034	6,198	6,321
350	4,988	5,130	5,313	5,449	5,582	5,754	5,881	6,006	6,169	6,290
400	4,976	5,117	5,299	5,433	5,565	5,736	5,862	5,986	6,147	6,267
450	4,966	5,107	5,288	5,421	5,552	5,722	5,847	5,970	6,130	6,249
500	4,959	5,099	5,279	5,412	5,542	5,711	5,835	5,958	6,117	6,235
600	4,947	5,086	5,265	5,398	5,527	5,694	5,818	5,939	6,097	6,214
700	4,939	5,078	5,256	5,387	5,516	5,682	5,805	5,926	6,083	6,199
800	4,933	5,071	5,249	5,380	5,508	5,673	5,796	5,916	6,072	6,188
900	4,929	5,066	5,243	5,374	5,502	5,667	5,789	5,908	6,064	6,179
1 000	4,925	5,062	5,239	5,369	5,497	5,661	5,783	5,902	6,057	6,172
∞	4,892	5,027	5,200	5,327	5,452	5,612	5,731	5,848	5,998	6,110

NOTE This table provides factors k such that one may be at least 99,9 % confident that none of the next m observations from a normally distributed population will lie outside the range $(\bar{x} - ks, \bar{x} + ks)$, where $\bar{x}$ and s are derived from a random sample of size n from the same population.

Annex C
(normative)

Tables of one-sided prediction interval factors, k, for known population standard deviation

Table C.1 — One-sided prediction interval factors, k, at confidence level 90 % for known population standard deviation

n	m										
	1	2	3	4	5	6	7	8	9	10	15
2	1,570	1,964	2,173	2,314	2,418	2,502	2,570	2,629	2,679	2,724	2,890
3	1,480	1,862	2,066	2,203	2,305	2,386	2,453	2,510	2,559	2,603	2,765
4	1,433	1,809	2,009	2,143	2,244	2,324	2,390	2,446	2,494	2,537	2,697
5	1,404	1,776	1,973	2,106	2,206	2,284	2,350	2,405	2,453	2,496	2,654
6	1,385	1,753	1,949	2,081	2,179	2,257	2,322	2,377	2,425	2,467	2,624
7	1,371	1,737	1,931	2,062	2,160	2,238	2,302	2,357	2,404	2,446	2,602
8	1,360	1,724	1,918	2,048	2,145	2,223	2,287	2,341	2,388	2,430	2,585
9	1,351	1,714	1,907	2,037	2,134	2,211	2,275	2,329	2,376	2,417	2,572
10	1,345	1,706	1,899	2,028	2,125	2,201	2,265	2,319	2,366	2,407	2,561
11	1,339	1,700	1,892	2,021	2,117	2,194	2,257	2,311	2,357	2,398	2,552
12	1,334	1,694	1,886	2,014	2,111	2,187	2,250	2,304	2,350	2,391	2,545
13	1,330	1,690	1,881	2,009	2,105	2,181	2,244	2,298	2,344	2,385	2,538
14	1,327	1,686	1,876	2,005	2,100	2,176	2,239	2,293	2,339	2,380	2,533
15	1,324	1,682	1,873	2,001	2,096	2,172	2,235	2,288	2,335	2,375	2,528
16	1,321	1,679	1,869	1,997	2,093	2,168	2,231	2,284	2,331	2,371	2,524
17	1,319	1,677	1,867	1,994	2,089	2,165	2,228	2,281	2,327	2,368	2,520
18	1,317	1,674	1,864	1,991	2,087	2,162	2,225	2,278	2,324	2,365	2,517
19	1,315	1,672	1,862	1,989	2,084	2,160	2,222	2,275	2,321	2,362	2,514
20	1,314	1,670	1,859	1,987	2,082	2,157	2,220	2,273	2,319	2,359	2,511
25	1,307	1,663	1,851	1,978	2,073	2,148	2,210	2,263	2,309	2,349	2,500
30	1,303	1,658	1,846	1,973	2,067	2,142	2,204	2,257	2,302	2,343	2,493
35	1,300	1,654	1,842	1,968	2,063	2,138	2,200	2,252	2,298	2,338	2,488
40	1,298	1,652	1,839	1,965	2,060	2,134	2,196	2,249	2,294	2,335	2,485
45	1,296	1,649	1,837	1,963	2,057	2,132	2,194	2,246	2,292	2,332	2,482
50	1,295	1,648	1,835	1,961	2,055	2,130	2,191	2,244	2,289	2,330	2,479
60	1,293	1,645	1,833	1,958	2,052	2,127	2,188	2,241	2,286	2,326	2,476
70	1,291	1,644	1,831	1,956	2,050	2,124	2,186	2,238	2,284	2,324	2,473
80	1,290	1,642	1,829	1,955	2,048	2,123	2,184	2,237	2,282	2,322	2,471
90	1,289	1,641	1,828	1,953	2,047	2,121	2,183	2,235	2,281	2,321	2,470
100	1,288	1,640	1,827	1,952	2,046	2,120	2,182	2,234	2,279	2,319	2,469
150	1,286	1,638	1,824	1,950	2,043	2,117	2,179	2,231	2,276	2,316	2,465
200	1,285	1,637	1,823	1,948	2,042	2,116	2,177	2,229	2,274	2,314	2,463
250	1,285	1,636	1,822	1,947	2,041	2,115	2,176	2,228	2,273	2,313	2,462
300	1,284	1,635	1,822	1,947	2,040	2,114	2,175	2,228	2,273	2,313	2,461
350	1,284	1,635	1,821	1,946	2,040	2,114	2,175	2,227	2,272	2,312	2,461
400	1,284	1,635	1,821	1,946	2,039	2,113	2,175	2,227	2,272	2,312	2,461
450	1,283	1,634	1,821	1,946	2,039	2,113	2,174	2,226	2,272	2,311	2,460
500	1,283	1,634	1,820	1,945	2,039	2,113	2,174	2,226	2,271	2,311	2,460
600	1,283	1,634	1,820	1,945	2,038	2,113	2,174	2,226	2,271	2,311	2,460
700	1,283	1,634	1,820	1,945	2,038	2,112	2,174	2,226	2,271	2,311	2,459
800	1,283	1,634	1,820	1,945	2,038	2,112	2,173	2,225	2,271	2,310	2,459
900	1,283	1,634	1,820	1,945	2,038	2,112	2,173	2,225	2,270	2,310	2,459
1 000	1,283	1,633	1,820	1,945	2,038	2,112	2,173	2,225	2,270	2,310	2,459
∞	1,282	1,633	1,819	1,944	2,037	2,111	2,172	2,224	2,269	2,309	2,458

Table C.1 — One-sided prediction interval factors, k, at confidence level 90 % for known population standard deviation *(continued)*

n	m										
	20	30	40	50	60	80	100	150	200	250	500
2	3,003	3,156	3,260	3,339	3,402	3,499	3,573	3,703	3,792	3,860	4,064
3	2,876	3,026	3,128	3,206	3,268	3,363	3,436	3,564	3,652	3,719	3,921
4	2,806	2,954	3,055	3,132	3,193	3,288	3,359	3,486	3,574	3,640	3,839
5	2,762	2,908	3,008	3,084	3,145	3,239	3,310	3,436	3,523	3,589	3,787
6	2,731	2,876	2,976	3,051	3,111	3,205	3,275	3,400	3,487	3,552	3,749
7	2,708	2,853	2,952	3,027	3,087	3,179	3,250	3,374	3,460	3,525	3,722
8	2,691	2,835	2,933	3,008	3,068	3,160	3,230	3,354	3,439	3,504	3,700
9	2,677	2,820	2,919	2,993	3,052	3,144	3,214	3,338	3,423	3,488	3,683
10	2,666	2,809	2,907	2,981	3,040	3,132	3,201	3,324	3,410	3,474	3,669
11	2,657	2,799	2,897	2,971	3,030	3,121	3,191	3,314	3,398	3,463	3,657
12	2,649	2,791	2,888	2,962	3,021	3,112	3,182	3,304	3,389	3,453	3,647
13	2,642	2,784	2,881	2,955	3,014	3,105	3,174	3,296	3,381	3,445	3,638
14	2,637	2,778	2,875	2,949	3,007	3,098	3,167	3,289	3,374	3,438	3,631
15	2,632	2,773	2,870	2,943	3,002	3,093	3,161	3,283	3,368	3,432	3,624
16	2,627	2,768	2,865	2,938	2,997	3,088	3,156	3,278	3,362	3,426	3,619
17	2,624	2,764	2,861	2,934	2,993	3,083	3,152	3,274	3,358	3,421	3,614
18	2,620	2,761	2,857	2,930	2,989	3,079	3,148	3,269	3,353	3,417	3,609
19	2,617	2,758	2,854	2,927	2,985	3,076	3,144	3,266	3,349	3,413	3,605
20	2,614	2,755	2,851	2,924	2,982	3,072	3,141	3,262	3,346	3,410	3,601
25	2,603	2,743	2,839	2,912	2,970	3,060	3,128	3,249	3,332	3,396	3,587
30	2,596	2,736	2,831	2,904	2,962	3,051	3,120	3,240	3,323	3,387	3,577
35	2,591	2,730	2,826	2,898	2,956	3,045	3,113	3,234	3,317	3,380	3,570
40	2,587	2,726	2,821	2,894	2,951	3,041	3,109	3,229	3,312	3,375	3,565
45	2,584	2,723	2,818	2,890	2,948	3,037	3,105	3,225	3,308	3,371	3,561
50	2,582	2,720	2,815	2,887	2,945	3,034	3,102	3,222	3,305	3,368	3,557
60	2,578	2,716	2,811	2,883	2,941	3,030	3,098	3,218	3,300	3,363	3,552
70	2,575	2,714	2,809	2,880	2,938	3,027	3,095	3,214	3,297	3,360	3,549
80	2,573	2,712	2,806	2,878	2,936	3,025	3,092	3,212	3,294	3,357	3,546
90	2,572	2,710	2,805	2,876	2,934	3,023	3,090	3,210	3,292	3,355	3,544
100	2,570	2,709	2,803	2,875	2,933	3,021	3,089	3,208	3,291	3,354	3,542
150	2,567	2,705	2,799	2,871	2,928	3,017	3,084	3,204	3,286	3,349	3,537
200	2,565	2,703	2,797	2,869	2,926	3,015	3,082	3,201	3,284	3,346	3,535
250	2,564	2,702	2,796	2,868	2,925	3,014	3,081	3,200	3,282	3,345	3,533
300	2,563	2,701	2,795	2,867	2,924	3,013	3,080	3,199	3,281	3,344	3,532
350	2,562	2,700	2,795	2,866	2,923	3,012	3,079	3,198	3,281	3,343	3,531
400	2,562	2,700	2,794	2,866	2,923	3,012	3,079	3,198	3,280	3,343	3,531
450	2,562	2,699	2,794	2,865	2,923	3,011	3,078	3,197	3,280	3,342	3,530
500	2,561	2,699	2,794	2,865	2,922	3,011	3,078	3,197	3,279	3,342	3,530
600	2,561	2,699	2,793	2,865	2,922	3,010	3,078	3,197	3,279	3,341	3,529
700	2,561	2,698	2,793	2,864	2,922	3,010	3,077	3,196	3,278	3,341	3,529
800	2,561	2,698	2,793	2,864	2,921	3,010	3,077	3,196	3,278	3,341	3,529
900	2,560	2,698	2,792	2,864	2,921	3,010	3,077	3,196	3,278	3,341	3,529
1 000	2,560	2,698	2,792	2,864	2,921	3,010	3,077	3,196	3,278	3,340	3,528
∞	2,559	2,697	2,791	2,862	2,920	3,008	3,075	3,194	3,276	3,339	3,527

Table C.1 — One-sided prediction interval factors, k, at confidence level 90 % for known population standard deviation *(continued)*

n	m									
	1 000	2 000	5 000	10 000	20 000	50 000	100 000	200 000	500 000	1 000 000
2	4,257	4,441	4,673	4,841	5,002	5,208	5,358	5,503	5,689	5,825
3	4,112	4,295	4,525	4,691	4,852	5,056	5,205	5,350	5,535	5,671
4	4,029	4,211	4,440	4,605	4,765	4,968	5,117	5,261	5,445	5,581
5	3,976	4,156	4,384	4,549	4,708	4,910	5,058	5,202	5,386	5,521
6	3,937	4,117	4,344	4,508	4,667	4,869	5,016	5,160	5,343	5,478
7	3,909	4,088	4,314	4,478	4,636	4,837	4,984	5,127	5,311	5,445
8	3,887	4,065	4,291	4,454	4,612	4,813	4,960	5,102	5,285	5,419
9	3,869	4,047	4,272	4,435	4,592	4,793	4,939	5,082	5,264	5,398
10	3,854	4,032	4,256	4,419	4,576	4,776	4,923	5,065	5,247	5,381
11	3,842	4,019	4,243	4,406	4,563	4,763	4,909	5,051	5,233	5,367
12	3,832	4,009	4,232	4,394	4,551	4,751	4,897	5,039	5,220	5,354
13	3,823	4,000	4,223	4,385	4,541	4,741	4,886	5,028	5,210	5,343
14	3,815	3,992	4,215	4,376	4,533	4,732	4,877	5,019	5,200	5,334
15	3,808	3,985	4,207	4,369	4,525	4,724	4,869	5,011	5,192	5,325
16	3,802	3,979	4,201	4,362	4,518	4,717	4,862	5,004	5,185	5,318
17	3,797	3,973	4,195	4,356	4,512	4,711	4,856	4,997	5,178	5,311
18	3,792	3,968	4,190	4,351	4,507	4,705	4,850	4,991	5,172	5,305
19	3,788	3,964	4,185	4,346	4,502	4,700	4,845	4,986	5,167	5,300
20	3,784	3,960	4,181	4,342	4,497	4,696	4,841	4,981	5,162	5,295
25	3,769	3,944	4,165	4,325	4,480	4,678	4,822	4,963	5,143	5,276
30	3,759	3,934	4,154	4,314	4,469	4,666	4,810	4,950	5,130	5,262
35	3,752	3,926	4,146	4,306	4,460	4,657	4,801	4,941	5,121	5,253
40	3,746	3,920	4,140	4,299	4,454	4,650	4,794	4,934	5,113	5,245
45	3,742	3,916	4,135	4,295	4,449	4,645	4,789	4,928	5,108	5,239
50	3,739	3,912	4,131	4,291	4,444	4,641	4,784	4,924	5,103	5,235
60	3,733	3,907	4,126	4,285	4,438	4,634	4,778	4,917	5,096	5,227
70	3,729	3,903	4,121	4,280	4,434	4,630	4,773	4,912	5,091	5,222
80	3,727	3,900	4,118	4,277	4,430	4,626	4,769	4,908	5,087	5,218
90	3,724	3,897	4,116	4,274	4,428	4,623	4,766	4,906	5,084	5,215
100	3,723	3,895	4,114	4,272	4,426	4,621	4,764	4,903	5,082	5,213
150	3,717	3,890	4,108	4,266	4,419	4,615	4,757	4,896	5,074	5,205
200	3,715	3,887	4,105	4,263	4,416	4,611	4,754	4,893	5,071	5,202
250	3,713	3,885	4,103	4,261	4,414	4,609	4,752	4,890	5,068	5,199
300	3,712	3,884	4,102	4,260	4,413	4,608	4,750	4,889	5,067	5,198
350	3,711	3,883	4,101	4,259	4,412	4,607	4,749	4,888	5,066	5,197
400	3,710	3,883	4,100	4,258	4,411	4,606	4,749	4,887	5,065	5,196
450	3,710	3,882	4,100	4,258	4,411	4,606	4,748	4,887	5,064	5,195
500	3,710	3,882	4,100	4,257	4,410	4,605	4,748	4,886	5,064	5,195
600	3,709	3,881	4,099	4,257	4,410	4,604	4,747	4,885	5,063	5,194
700	3,709	3,881	4,098	4,256	4,409	4,604	4,746	4,885	5,063	5,193
800	3,708	3,881	4,098	4,256	4,409	4,604	4,746	4,885	5,062	5,193
900	3,708	3,880	4,098	4,256	4,409	4,603	4,746	4,884	5,062	5,193
1 000	3,708	3,880	4,098	4,256	4,408	4,603	4,745	4,884	5,062	5,192
∞	3,706	3,878	4,096	4,254	4,406	4,601	4,743	4,882	5,060	5,190

NOTE This table provides factors k such that one may be at least 90 % confident that none of the next m observations from a normally distributed population will lie outside the range $(-\infty, \bar{x} + k\sigma)$, where $\bar{x}$ is derived from a random sample of size n from the same population. Similarly for the range $(\bar{x} - k\sigma, \infty)$.

Table C.2 — One-sided prediction interval factors, k, at confidence level 95 % for known population standard deviation

n	m										
	1	2	3	4	5	6	7	8	9	10	15
2	2,015	2,371	2,563	2,693	2,791	2,868	2,933	2,987	3,035	3,077	3,234
3	1,900	2,243	2,429	2,554	2,649	2,724	2,786	2,839	2,885	2,926	3,078
4	1,840	2,176	2,357	2,480	2,573	2,646	2,707	2,759	2,804	2,844	2,994
5	1,802	2,134	2,313	2,434	2,525	2,598	2,658	2,709	2,754	2,793	2,941
6	1,777	2,105	2,283	2,403	2,493	2,565	2,624	2,675	2,719	2,758	2,904
7	1,759	2,085	2,261	2,380	2,469	2,540	2,600	2,650	2,694	2,733	2,878
8	1,745	2,069	2,244	2,362	2,451	2,522	2,581	2,631	2,675	2,713	2,857
9	1,734	2,057	2,231	2,349	2,437	2,508	2,566	2,616	2,659	2,698	2,841
10	1,726	2,047	2,220	2,338	2,426	2,496	2,554	2,604	2,647	2,685	2,828
11	1,718	2,039	2,212	2,329	2,416	2,486	2,544	2,594	2,637	2,675	2,818
12	1,713	2,032	2,204	2,321	2,408	2,478	2,536	2,586	2,628	2,666	2,809
13	1,707	2,026	2,198	2,314	2,402	2,471	2,529	2,578	2,621	2,659	2,801
14	1,703	2,021	2,193	2,309	2,396	2,466	2,523	2,572	2,615	2,653	2,794
15	1,699	2,017	2,188	2,304	2,391	2,460	2,518	2,567	2,610	2,647	2,789
16	1,696	2,013	2,184	2,300	2,387	2,456	2,513	2,562	2,605	2,643	2,784
17	1,693	2,010	2,180	2,296	2,383	2,452	2,509	2,558	2,601	2,638	2,779
18	1,690	2,007	2,177	2,293	2,379	2,448	2,506	2,554	2,597	2,634	2,775
19	1,688	2,004	2,174	2,290	2,376	2,445	2,502	2,551	2,594	2,631	2,772
20	1,686	2,002	2,172	2,287	2,373	2,442	2,499	2,548	2,591	2,628	2,768
25	1,678	1,992	2,162	2,277	2,363	2,431	2,488	2,537	2,579	2,616	2,756
30	1,673	1,986	2,155	2,270	2,356	2,424	2,481	2,529	2,571	2,608	2,748
35	1,669	1,982	2,151	2,265	2,350	2,419	2,475	2,524	2,566	2,603	2,742
40	1,666	1,979	2,147	2,261	2,347	2,415	2,471	2,520	2,561	2,598	2,737
45	1,664	1,976	2,144	2,258	2,344	2,412	2,468	2,516	2,558	2,595	2,734
50	1,662	1,974	2,142	2,256	2,341	2,409	2,466	2,514	2,556	2,593	2,731
60	1,659	1,971	2,139	2,252	2,337	2,405	2,462	2,510	2,552	2,589	2,727
70	1,657	1,968	2,136	2,250	2,335	2,403	2,459	2,507	2,549	2,586	2,724
80	1,656	1,967	2,134	2,248	2,333	2,401	2,457	2,505	2,547	2,584	2,722
90	1,654	1,965	2,133	2,246	2,331	2,399	2,455	2,503	2,545	2,582	2,720
100	1,654	1,964	2,132	2,245	2,330	2,398	2,454	2,502	2,544	2,580	2,718
150	1,651	1,961	2,128	2,242	2,327	2,394	2,450	2,498	2,540	2,576	2,714
200	1,649	1,960	2,127	2,240	2,325	2,392	2,448	2,496	2,538	2,574	2,712
250	1,649	1,959	2,126	2,239	2,324	2,391	2,447	2,495	2,537	2,573	2,711
300	1,648	1,958	2,125	2,238	2,323	2,390	2,446	2,494	2,536	2,572	2,710
350	1,648	1,958	2,125	2,238	2,322	2,390	2,446	2,494	2,535	2,572	2,709
400	1,647	1,957	2,124	2,237	2,322	2,390	2,446	2,493	2,535	2,571	2,709
450	1,647	1,957	2,124	2,237	2,322	2,389	2,445	2,493	2,534	2,571	2,708
500	1,647	1,957	2,124	2,237	2,321	2,389	2,445	2,493	2,534	2,571	2,708
600	1,647	1,957	2,123	2,236	2,321	2,389	2,445	2,492	2,534	2,570	2,708
700	1,647	1,956	2,123	2,236	2,321	2,388	2,444	2,492	2,533	2,570	2,707
800	1,646	1,956	2,123	2,236	2,321	2,388	2,444	2,492	2,533	2,570	2,707
900	1,646	1,956	2,123	2,236	2,320	2,388	2,444	2,492	2,533	2,570	2,707
1 000	1,646	1,956	2,123	2,236	2,320	2,388	2,444	2,491	2,533	2,570	2,707
∞	1,645	1,955	2,122	2,235	2,319	2,387	2,443	2,490	2,532	2,568	2,706

Table C.2 — One-sided prediction interval factors, k, at confidence level 95 % for known population standard deviation *(continued)*

n	m										
	20	30	40	50	60	80	100	150	200	250	500
2	3,341	3,487	3,587	3,663	3,723	3,817	3,888	4,014	4,101	4,167	4,365
3	3,183	3,325	3,422	3,496	3,556	3,647	3,717	3,840	3,926	3,991	4,186
4	3,096	3,236	3,332	3,405	3,463	3,554	3,622	3,744	3,828	3,892	4,085
5	3,042	3,180	3,275	3,347	3,404	3,494	3,562	3,682	3,765	3,829	4,020
6	3,005	3,141	3,235	3,306	3,364	3,452	3,520	3,639	3,722	3,785	3,975
7	2,977	3,113	3,206	3,277	3,334	3,422	3,489	3,607	3,689	3,752	3,941
8	2,956	3,091	3,184	3,254	3,311	3,398	3,465	3,583	3,665	3,727	3,915
9	2,940	3,074	3,166	3,236	3,293	3,380	3,446	3,564	3,645	3,707	3,894
10	2,926	3,060	3,152	3,222	3,278	3,365	3,431	3,548	3,629	3,691	3,877
11	2,915	3,048	3,140	3,210	3,266	3,352	3,418	3,535	3,616	3,677	3,863
12	2,906	3,039	3,130	3,200	3,255	3,342	3,407	3,524	3,605	3,666	3,851
13	2,898	3,031	3,122	3,191	3,247	3,333	3,398	3,515	3,595	3,656	3,841
14	2,891	3,024	3,115	3,184	3,239	3,325	3,390	3,506	3,587	3,648	3,833
15	2,885	3,017	3,108	3,177	3,233	3,318	3,384	3,499	3,580	3,641	3,825
16	2,880	3,012	3,103	3,171	3,227	3,312	3,378	3,493	3,573	3,634	3,818
17	2,876	3,007	3,098	3,166	3,222	3,307	3,372	3,488	3,568	3,629	3,812
18	2,871	3,003	3,093	3,162	3,217	3,303	3,367	3,483	3,563	3,624	3,807
19	2,868	2,999	3,089	3,158	3,213	3,298	3,363	3,478	3,558	3,619	3,802
20	2,864	2,996	3,086	3,154	3,209	3,295	3,359	3,474	3,554	3,615	3,798
25	2,852	2,982	3,072	3,140	3,195	3,280	3,345	3,459	3,539	3,599	3,781
30	2,843	2,973	3,063	3,131	3,186	3,270	3,335	3,449	3,528	3,588	3,770
35	2,837	2,967	3,056	3,124	3,179	3,263	3,328	3,442	3,521	3,581	3,762
40	2,832	2,962	3,051	3,119	3,174	3,258	3,322	3,436	3,515	3,575	3,756
45	2,829	2,958	3,048	3,115	3,170	3,254	3,318	3,432	3,510	3,570	3,752
50	2,826	2,955	3,045	3,112	3,166	3,251	3,315	3,428	3,507	3,567	3,748
60	2,822	2,951	3,040	3,107	3,162	3,246	3,310	3,423	3,502	3,561	3,742
70	2,819	2,948	3,037	3,104	3,158	3,242	3,306	3,419	3,498	3,558	3,738
80	2,816	2,945	3,034	3,101	3,156	3,239	3,303	3,416	3,495	3,555	3,735
90	2,814	2,943	3,032	3,099	3,154	3,237	3,301	3,414	3,493	3,552	3,732
100	2,813	2,942	3,030	3,098	3,152	3,236	3,299	3,412	3,491	3,550	3,731
150	2,809	2,937	3,026	3,093	3,147	3,231	3,294	3,407	3,485	3,545	3,725
200	2,806	2,935	3,023	3,091	3,145	3,228	3,292	3,404	3,483	3,542	3,722
250	2,805	2,934	3,022	3,089	3,143	3,227	3,290	3,403	3,481	3,541	3,720
300	2,804	2,933	3,021	3,088	3,142	3,226	3,289	3,402	3,480	3,539	3,719
350	2,803	2,932	3,020	3,087	3,141	3,225	3,288	3,401	3,479	3,539	3,718
400	2,803	2,931	3,020	3,087	3,141	3,224	3,288	3,401	3,479	3,538	3,717
450	2,803	2,931	3,019	3,087	3,140	3,224	3,287	3,400	3,478	3,538	3,717
500	2,802	2,931	3,019	3,086	3,140	3,224	3,287	3,400	3,478	3,537	3,717
600	2,802	2,930	3,019	3,086	3,140	3,223	3,286	3,399	3,477	3,537	3,716
700	2,802	2,930	3,018	3,085	3,139	3,223	3,286	3,399	3,477	3,536	3,716
800	2,801	2,930	3,018	3,085	3,139	3,222	3,286	3,399	3,476	3,536	3,715
900	2,801	2,930	3,018	3,085	3,139	3,222	3,286	3,398	3,476	3,536	3,715
1 000	2,801	2,929	3,018	3,085	3,139	3,222	3,285	3,398	3,476	3,536	3,715
∞	2,800	2,928	3,016	3,083	3,137	3,220	3,284	3,396	3,474	3,534	3,713

Table C.2 — One-sided prediction interval factors, k, at confidence level 95 % for known population standard deviation *(continued)*

n	m									
	1 000	2 000	5 000	10 000	20 000	50 000	100 000	200 000	500 000	1 000 000
2	4,555	4,735	4,963	5,129	5,288	5,491	5,639	5,783	5,967	6,102
3	4,372	4,551	4,776	4,939	5,097	5,299	5,446	5,588	5,771	5,906
4	4,269	4,446	4,670	4,832	4,989	5,188	5,335	5,477	5,659	5,793
5	4,203	4,378	4,600	4,761	4,917	5,116	5,261	5,403	5,584	5,718
6	4,156	4,331	4,551	4,712	4,867	5,065	5,209	5,350	5,531	5,664
7	4,121	4,295	4,515	4,674	4,829	5,026	5,170	5,311	5,491	5,623
8	4,095	4,267	4,486	4,645	4,799	4,996	5,140	5,280	5,459	5,591
9	4,073	4,245	4,463	4,622	4,776	4,971	5,115	5,255	5,434	5,566
10	4,056	4,227	4,445	4,603	4,756	4,952	5,095	5,234	5,413	5,545
11	4,041	4,213	4,429	4,587	4,740	4,935	5,078	5,217	5,395	5,527
12	4,029	4,200	4,416	4,574	4,726	4,921	5,063	5,202	5,380	5,512
13	4,019	4,189	4,405	4,562	4,714	4,909	5,051	5,190	5,368	5,499
14	4,010	4,180	4,395	4,552	4,704	4,898	5,040	5,179	5,356	5,487
15	4,002	4,172	4,387	4,543	4,695	4,889	5,031	5,169	5,347	5,477
16	3,995	4,164	4,379	4,536	4,687	4,881	5,023	5,161	5,338	5,468
17	3,989	4,158	4,373	4,529	4,680	4,873	5,015	5,153	5,330	5,461
18	3,983	4,152	4,367	4,523	4,674	4,867	5,008	5,146	5,323	5,453
19	3,978	4,147	4,361	4,517	4,668	4,861	5,002	5,140	5,317	5,447
20	3,974	4,142	4,356	4,512	4,663	4,856	4,997	5,135	5,311	5,441
25	3,956	4,125	4,338	4,493	4,643	4,835	4,976	5,113	5,289	5,419
30	3,945	4,112	4,325	4,480	4,630	4,821	4,962	5,098	5,274	5,403
35	3,936	4,104	4,316	4,470	4,620	4,811	4,951	5,088	5,263	5,392
40	3,930	4,097	4,309	4,463	4,613	4,804	4,944	5,080	5,255	5,384
45	3,925	4,092	4,304	4,458	4,607	4,798	4,937	5,074	5,248	5,377
50	3,921	4,088	4,299	4,453	4,602	4,793	4,932	5,069	5,243	5,372
60	3,915	4,082	4,293	4,446	4,595	4,786	4,925	5,061	5,235	5,364
70	3,911	4,077	4,288	4,442	4,590	4,780	4,920	5,055	5,230	5,358
80	3,908	4,074	4,285	4,438	4,587	4,777	4,916	5,051	5,225	5,354
90	3,905	4,071	4,282	4,435	4,584	4,773	4,913	5,048	5,222	5,350
100	3,903	4,069	4,280	4,433	4,581	4,771	4,910	5,045	5,219	5,348
150	3,897	4,063	4,273	4,426	4,574	4,764	4,902	5,038	5,211	5,339
200	3,894	4,060	4,270	4,423	4,571	4,760	4,899	5,034	5,207	5,335
250	3,892	4,058	4,268	4,420	4,569	4,758	4,896	5,031	5,205	5,333
300	3,891	4,057	4,266	4,419	4,567	4,756	4,895	5,030	5,203	5,331
350	3,890	4,056	4,265	4,418	4,566	4,755	4,894	5,029	5,202	5,330
400	3,889	4,055	4,265	4,417	4,565	4,754	4,893	5,028	5,201	5,329
450	3,889	4,054	4,264	4,417	4,565	4,754	4,892	5,027	5,201	5,328
500	3,889	4,054	4,264	4,416	4,564	4,753	4,892	5,027	5,200	5,328
600	3,888	4,053	4,263	4,416	4,564	4,753	4,891	5,026	5,199	5,327
700	3,888	4,053	4,263	4,415	4,563	4,752	4,890	5,025	5,199	5,326
800	3,887	4,053	4,262	4,415	4,563	4,752	4,890	5,025	5,198	5,326
900	3,887	4,052	4,262	4,414	4,562	4,751	4,890	5,025	5,198	5,325
1 000	3,887	4,052	4,262	4,414	4,562	4,751	4,889	5,024	5,198	5,325
∞	3,885	4,050	4,260	4,412	4,560	4,749	4,887	5,022	5,195	5,323

NOTE This table provides factors k such that one may be at least 95 % confident that none of the next m observations from a normally distributed population will lie outside the range $(-\infty, \overline{x} + k\sigma)$, where $\overline{x}$ is derived from a random sample of size n from the same population. Similarly for the range $(\overline{x} - k\sigma, \infty)$.

Table C.3 — One-sided prediction interval factors, k, at confidence level 97,5 % for known population standard deviation

n	m										
	1	2	3	4	5	6	7	8	9	10	15
2	2,401	2,728	2,906	3,027	3,118	3,191	3,252	3,303	3,348	3,388	3,537
3	2,264	2,577	2,748	2,865	2,952	3,023	3,081	3,131	3,174	3,212	3,357
4	2,192	2,498	2,665	2,778	2,864	2,933	2,990	3,038	3,081	3,118	3,259
5	2,148	2,449	2,613	2,725	2,809	2,877	2,933	2,981	3,022	3,059	3,198
6	2,118	2,415	2,578	2,688	2,772	2,839	2,894	2,941	2,983	3,019	3,156
7	2,096	2,391	2,552	2,662	2,745	2,811	2,866	2,913	2,954	2,990	3,126
8	2,079	2,373	2,533	2,642	2,724	2,790	2,844	2,891	2,932	2,968	3,103
9	2,066	2,358	2,517	2,626	2,708	2,773	2,827	2,874	2,914	2,950	3,085
10	2,056	2,347	2,505	2,613	2,695	2,760	2,814	2,860	2,900	2,936	3,070
11	2,048	2,337	2,495	2,603	2,684	2,749	2,803	2,849	2,889	2,924	3,058
12	2,040	2,329	2,487	2,594	2,675	2,740	2,793	2,839	2,879	2,915	3,048
13	2,034	2,323	2,480	2,587	2,667	2,732	2,785	2,831	2,871	2,906	3,039
14	2,029	2,317	2,473	2,580	2,661	2,725	2,778	2,824	2,864	2,899	3,031
15	2,025	2,312	2,468	2,575	2,655	2,719	2,772	2,818	2,858	2,893	3,025
16	2,021	2,307	2,463	2,570	2,650	2,714	2,767	2,813	2,852	2,887	3,019
17	2,017	2,303	2,459	2,565	2,645	2,709	2,763	2,808	2,848	2,883	3,014
18	2,014	2,300	2,455	2,561	2,641	2,705	2,758	2,804	2,843	2,878	3,010
19	2,011	2,297	2,452	2,558	2,638	2,702	2,755	2,800	2,840	2,874	3,006
20	2,009	2,294	2,449	2,555	2,635	2,698	2,751	2,797	2,836	2,871	3,002
25	1,999	2,283	2,438	2,543	2,622	2,686	2,739	2,784	2,823	2,858	2,988
30	1,993	2,276	2,430	2,535	2,614	2,678	2,730	2,775	2,814	2,849	2,979
35	1,988	2,271	2,425	2,529	2,608	2,672	2,724	2,769	2,808	2,843	2,972
40	1,985	2,267	2,420	2,525	2,604	2,667	2,719	2,764	2,803	2,838	2,967
45	1,982	2,264	2,417	2,522	2,601	2,664	2,716	2,761	2,800	2,834	2,964
50	1,980	2,261	2,415	2,519	2,598	2,661	2,713	2,758	2,797	2,831	2,960
60	1,977	2,258	2,411	2,515	2,594	2,657	2,709	2,753	2,792	2,827	2,956
70	1,974	2,255	2,408	2,512	2,591	2,653	2,706	2,750	2,789	2,823	2,952
80	1,973	2,253	2,406	2,510	2,588	2,651	2,703	2,748	2,787	2,821	2,950
90	1,971	2,252	2,404	2,508	2,587	2,649	2,702	2,746	2,785	2,819	2,948
100	1,970	2,250	2,403	2,507	2,585	2,648	2,700	2,745	2,783	2,817	2,946
150	1,967	2,247	2,399	2,503	2,581	2,644	2,696	2,740	2,779	2,813	2,942
200	1,965	2,245	2,397	2,501	2,579	2,642	2,694	2,738	2,777	2,811	2,939
250	1,964	2,244	2,396	2,500	2,578	2,640	2,692	2,737	2,775	2,809	2,938
300	1,964	2,243	2,395	2,499	2,577	2,639	2,691	2,736	2,774	2,808	2,937
350	1,963	2,243	2,395	2,498	2,576	2,639	2,691	2,735	2,774	2,808	2,936
400	1,963	2,242	2,394	2,498	2,576	2,638	2,690	2,735	2,773	2,807	2,936
450	1,963	2,242	2,394	2,498	2,576	2,638	2,690	2,734	2,773	2,807	2,935
500	1,962	2,242	2,394	2,497	2,575	2,638	2,690	2,734	2,772	2,807	2,935
600	1,962	2,241	2,393	2,497	2,575	2,637	2,689	2,733	2,772	2,806	2,934
700	1,962	2,241	2,393	2,497	2,575	2,637	2,689	2,733	2,772	2,806	2,934
800	1,962	2,241	2,393	2,496	2,574	2,637	2,689	2,733	2,771	2,806	2,934
900	1,962	2,241	2,393	2,496	2,574	2,637	2,688	2,733	2,771	2,805	2,934
1 000	1,961	2,241	2,393	2,496	2,574	2,636	2,688	2,733	2,771	2,805	2,933
∞	1,960	2,239	2,391	2,495	2,573	2,635	2,687	2,731	2,770	2,804	2,932

Table C.3 — One-sided prediction interval factors, k, at confidence level 97,5 % for known population standard deviation *(continued)*

n	m										
	20	30	40	50	60	80	100	150	200	250	500
2	3,640	3,779	3,876	3,948	4,007	4,098	4,166	4,289	4,373	4,437	4,631
3	3,455	3,591	3,684	3,755	3,812	3,900	3,967	4,086	4,169	4,231	4,421
4	3,356	3,489	3,580	3,649	3,705	3,792	3,858	3,975	4,056	4,118	4,304
5	3,294	3,424	3,514	3,583	3,638	3,723	3,788	3,904	3,984	4,045	4,230
6	3,251	3,380	3,469	3,537	3,591	3,676	3,740	3,855	3,934	3,995	4,178
7	3,220	3,348	3,436	3,503	3,557	3,641	3,705	3,819	3,898	3,958	4,140
8	3,196	3,323	3,411	3,478	3,531	3,615	3,678	3,791	3,870	3,930	4,110
9	3,177	3,304	3,391	3,458	3,511	3,594	3,657	3,770	3,848	3,907	4,087
10	3,162	3,288	3,375	3,441	3,494	3,577	3,640	3,752	3,830	3,889	4,068
11	3,149	3,275	3,362	3,428	3,481	3,563	3,626	3,737	3,815	3,874	4,053
12	3,139	3,264	3,351	3,416	3,469	3,551	3,614	3,725	3,802	3,861	4,039
13	3,130	3,255	3,341	3,407	3,460	3,542	3,604	3,715	3,792	3,851	4,028
14	3,123	3,247	3,333	3,399	3,451	3,533	3,595	3,706	3,783	3,841	4,019
15	3,116	3,240	3,326	3,391	3,444	3,525	3,588	3,698	3,775	3,833	4,010
16	3,110	3,234	3,320	3,385	3,438	3,519	3,581	3,691	3,768	3,826	4,003
17	3,105	3,229	3,314	3,379	3,432	3,513	3,575	3,685	3,762	3,820	3,996
18	3,100	3,224	3,309	3,374	3,427	3,508	3,570	3,680	3,756	3,814	3,990
19	3,096	3,220	3,305	3,370	3,422	3,503	3,565	3,675	3,751	3,809	3,985
20	3,092	3,216	3,301	3,366	3,418	3,499	3,561	3,670	3,747	3,805	3,980
25	3,078	3,201	3,286	3,351	3,402	3,483	3,544	3,654	3,730	3,787	3,962
30	3,069	3,191	3,276	3,340	3,392	3,472	3,534	3,642	3,718	3,776	3,950
35	3,062	3,184	3,269	3,333	3,384	3,465	3,526	3,634	3,710	3,767	3,941
40	3,057	3,179	3,263	3,327	3,379	3,459	3,520	3,628	3,704	3,761	3,935
45	3,053	3,175	3,259	3,323	3,374	3,454	3,515	3,624	3,699	3,756	3,930
50	3,049	3,171	3,255	3,319	3,371	3,451	3,511	3,620	3,695	3,752	3,926
60	3,045	3,166	3,250	3,314	3,365	3,445	3,506	3,614	3,689	3,746	3,919
70	3,041	3,163	3,246	3,310	3,362	3,441	3,502	3,610	3,685	3,742	3,915
80	3,039	3,160	3,244	3,307	3,359	3,438	3,499	3,607	3,682	3,739	3,912
90	3,036	3,158	3,242	3,305	3,356	3,436	3,497	3,604	3,679	3,736	3,909
100	3,035	3,156	3,240	3,303	3,355	3,434	3,495	3,603	3,677	3,734	3,907
150	3,030	3,151	3,235	3,298	3,349	3,429	3,489	3,597	3,671	3,728	3,901
200	3,028	3,149	3,232	3,296	3,347	3,426	3,486	3,594	3,668	3,725	3,897
250	3,026	3,147	3,230	3,294	3,345	3,424	3,485	3,592	3,667	3,724	3,896
300	3,025	3,146	3,229	3,293	3,344	3,423	3,484	3,591	3,665	3,722	3,894
350	3,024	3,145	3,229	3,292	3,343	3,422	3,483	3,590	3,665	3,721	3,893
400	3,024	3,145	3,228	3,292	3,343	3,422	3,482	3,589	3,664	3,721	3,893
450	3,023	3,144	3,228	3,291	3,342	3,421	3,482	3,589	3,663	3,720	3,892
500	3,023	3,144	3,227	3,291	3,342	3,421	3,481	3,589	3,663	3,720	3,892
600	3,023	3,143	3,227	3,290	3,341	3,420	3,481	3,588	3,663	3,719	3,891
700	3,022	3,143	3,226	3,290	3,341	3,420	3,480	3,588	3,662	3,719	3,891
800	3,022	3,143	3,226	3,290	3,341	3,420	3,480	3,587	3,662	3,719	3,890
900	3,022	3,143	3,226	3,289	3,340	3,419	3,480	3,587	3,662	3,718	3,890
1 000	3,022	3,142	3,226	3,289	3,340	3,419	3,480	3,587	3,661	3,718	3,890
∞	3,020	3,141	3,224	3,288	3,339	3,418	3,478	3,585	3,660	3,716	3,888

Table C.3 — One-sided prediction interval factors, *k*, at confidence level 97,5 % for known population standard deviation *(continued)*

n	*m*									
	1 000	2 000	5 000	10 000	20 000	50 000	100 000	200 000	500 000	1 000 000
2	4,816	4,994	5,218	5,381	5,538	5,739	5,885	6,028	6,210	6,345
3	4,603	4,777	4,998	5,159	5,314	5,512	5,657	5,798	5,979	6,113
4	4,484	4,656	4,874	5,033	5,187	5,383	5,527	5,667	5,847	5,979
5	4,407	4,578	4,794	4,952	5,104	5,299	5,442	5,582	5,760	5,892
6	4,354	4,523	4,738	4,894	5,046	5,240	5,382	5,521	5,699	5,830
7	4,314	4,482	4,696	4,852	5,003	5,196	5,337	5,475	5,652	5,783
8	4,284	4,451	4,664	4,819	4,969	5,161	5,302	5,440	5,616	5,746
9	4,260	4,427	4,638	4,793	4,942	5,134	5,274	5,411	5,587	5,717
10	4,240	4,406	4,617	4,771	4,920	5,111	5,251	5,388	5,563	5,693
11	4,224	4,390	4,600	4,753	4,902	5,093	5,232	5,369	5,544	5,673
12	4,211	4,376	4,586	4,738	4,887	5,077	5,216	5,352	5,527	5,656
13	4,199	4,364	4,573	4,726	4,874	5,063	5,202	5,338	5,512	5,641
14	4,189	4,353	4,562	4,715	4,862	5,052	5,190	5,326	5,500	5,628
15	4,180	4,344	4,553	4,705	4,852	5,041	5,180	5,315	5,489	5,617
16	4,173	4,336	4,545	4,696	4,844	5,032	5,171	5,306	5,479	5,607
17	4,166	4,329	4,537	4,689	4,836	5,024	5,163	5,297	5,471	5,598
18	4,160	4,323	4,531	4,682	4,829	5,017	5,155	5,290	5,463	5,591
19	4,154	4,317	4,525	4,676	4,823	5,011	5,149	5,283	5,456	5,584
20	4,149	4,312	4,519	4,670	4,817	5,005	5,143	5,277	5,450	5,577
25	4,131	4,293	4,499	4,649	4,796	4,982	5,120	5,254	5,426	5,553
30	4,118	4,280	4,485	4,635	4,781	4,967	5,104	5,238	5,409	5,536
35	4,109	4,270	4,475	4,625	4,770	4,956	5,093	5,226	5,398	5,524
40	4,102	4,263	4,468	4,617	4,763	4,948	5,085	5,218	5,389	5,515
45	4,097	4,258	4,462	4,611	4,756	4,942	5,078	5,211	5,382	5,508
50	4,092	4,253	4,457	4,607	4,751	4,937	5,073	5,205	5,376	5,502
60	4,086	4,246	4,450	4,599	4,744	4,929	5,065	5,197	5,368	5,493
70	4,081	4,242	4,445	4,594	4,739	4,923	5,059	5,191	5,362	5,487
80	4,078	4,238	4,442	4,590	4,734	4,919	5,055	5,187	5,357	5,482
90	4,075	4,235	4,439	4,587	4,731	4,916	5,051	5,184	5,354	5,479
100	4,073	4,233	4,436	4,585	4,729	4,913	5,049	5,181	5,351	5,476
150	4,066	4,226	4,429	4,577	4,721	4,905	5,041	5,173	5,342	5,467
200	4,063	4,223	4,426	4,574	4,717	4,902	5,037	5,168	5,338	5,463
250	4,061	4,221	4,424	4,572	4,715	4,899	5,034	5,166	5,335	5,460
300	4,060	4,219	4,422	4,570	4,714	4,898	5,033	5,164	5,334	5,458
350	4,059	4,218	4,421	4,569	4,713	4,896	5,031	5,163	5,332	5,457
400	4,058	4,218	4,420	4,568	4,712	4,896	5,030	5,162	5,331	5,456
450	4,058	4,217	4,420	4,568	4,711	4,895	5,030	5,161	5,331	5,455
500	4,057	4,217	4,419	4,567	4,711	4,894	5,029	5,161	5,330	5,455
600	4,056	4,216	4,419	4,566	4,710	4,894	5,028	5,160	5,329	5,454
700	4,056	4,215	4,418	4,566	4,709	4,893	5,028	5,159	5,329	5,453
800	4,056	4,215	4,418	4,565	4,709	4,893	5,027	5,159	5,328	5,453
900	4,055	4,215	4,417	4,565	4,709	4,892	5,027	5,159	5,328	5,453
1 000	4,055	4,214	4,417	4,565	4,708	4,892	5,027	5,158	5,328	5,452
∞	4,053	4,212	4,415	4,563	4,706	4,890	5,024	5,156	5,325	5,450

NOTE This table provides factors k such that one may be at least 97,5 % confident that none of the next m observations from a normally distributed population will lie outside the range $(-\infty, \bar{x} + k\sigma)$, where $\bar{x}$ is derived from a random sample of size n from the same population. Similarly for the range $(\bar{x} - k\sigma, \infty)$.

Table C.4 — One-sided prediction interval factors, k, at confidence level 99 % for known population standard deviation

n	m										
	1	2	3	4	5	6	7	8	9	10	15
2	2,850	3,146	3,309	3,420	3,505	3,573	3,629	3,677	3,719	3,757	3,897
3	2,687	2,969	3,125	3,232	3,312	3,377	3,431	3,478	3,518	3,554	3,688
4	2,601	2,876	3,028	3,132	3,211	3,274	3,326	3,371	3,411	3,445	3,577
5	2,549	2,819	2,968	3,070	3,147	3,209	3,261	3,305	3,344	3,378	3,507
6	2,513	2,780	2,927	3,028	3,104	3,166	3,217	3,260	3,298	3,332	3,460
7	2,487	2,752	2,898	2,998	3,073	3,134	3,184	3,228	3,266	3,299	3,425
8	2,468	2,730	2,875	2,974	3,050	3,110	3,160	3,203	3,240	3,274	3,399
9	2,453	2,714	2,858	2,956	3,031	3,091	3,141	3,184	3,221	3,254	3,378
10	2,440	2,700	2,843	2,942	3,016	3,076	3,125	3,168	3,205	3,238	3,362
11	2,430	2,689	2,832	2,930	3,004	3,063	3,113	3,155	3,192	3,225	3,348
12	2,422	2,680	2,822	2,920	2,993	3,053	3,102	3,144	3,181	3,214	3,337
13	2,415	2,672	2,814	2,911	2,985	3,044	3,093	3,135	3,172	3,204	3,327
14	2,408	2,665	2,807	2,904	2,977	3,036	3,085	3,127	3,164	3,196	3,319
15	2,403	2,659	2,801	2,897	2,971	3,029	3,078	3,120	3,157	3,189	3,312
16	2,398	2,654	2,795	2,892	2,965	3,024	3,073	3,114	3,151	3,183	3,305
17	2,394	2,650	2,790	2,887	2,960	3,018	3,067	3,109	3,146	3,178	3,300
18	2,391	2,646	2,786	2,882	2,955	3,014	3,063	3,104	3,141	3,173	3,295
19	2,387	2,642	2,782	2,878	2,951	3,010	3,058	3,100	3,136	3,169	3,290
20	2,384	2,639	2,779	2,875	2,948	3,006	3,055	3,096	3,133	3,165	3,286
25	2,373	2,626	2,766	2,861	2,934	2,992	3,040	3,082	3,118	3,150	3,271
30	2,365	2,618	2,757	2,852	2,924	2,982	3,031	3,072	3,108	3,140	3,260
35	2,360	2,612	2,751	2,846	2,918	2,976	3,024	3,065	3,101	3,133	3,253
40	2,356	2,607	2,746	2,841	2,913	2,970	3,019	3,060	3,096	3,127	3,247
45	2,353	2,604	2,742	2,837	2,909	2,966	3,014	3,056	3,091	3,123	3,243
50	2,350	2,601	2,739	2,834	2,906	2,963	3,011	3,052	3,088	3,120	3,239
60	2,346	2,597	2,735	2,829	2,901	2,959	3,006	3,047	3,083	3,115	3,234
70	2,343	2,594	2,732	2,826	2,898	2,955	3,003	3,044	3,080	3,111	3,230
80	2,341	2,591	2,729	2,824	2,895	2,953	3,000	3,041	3,077	3,108	3,228
90	2,340	2,590	2,727	2,822	2,893	2,950	2,998	3,039	3,075	3,106	3,225
100	2,338	2,588	2,726	2,820	2,892	2,949	2,997	3,037	3,073	3,105	3,224
150	2,335	2,584	2,721	2,816	2,887	2,944	2,992	3,032	3,068	3,100	3,218
200	2,333	2,582	2,719	2,813	2,885	2,942	2,989	3,030	3,066	3,097	3,216
250	2,331	2,581	2,718	2,812	2,883	2,940	2,988	3,028	3,064	3,095	3,214
300	2,331	2,580	2,717	2,811	2,882	2,939	2,987	3,027	3,063	3,094	3,213
350	2,330	2,579	2,716	2,810	2,881	2,939	2,986	3,027	3,062	3,094	3,212
400	2,330	2,579	2,716	2,810	2,881	2,938	2,986	3,026	3,062	3,093	3,212
450	2,329	2,578	2,715	2,809	2,881	2,938	2,985	3,026	3,061	3,093	3,211
500	2,329	2,578	2,715	2,809	2,880	2,937	2,985	3,026	3,061	3,092	3,211
600	2,329	2,578	2,715	2,809	2,880	2,937	2,984	3,025	3,060	3,092	3,211
700	2,329	2,577	2,714	2,808	2,879	2,936	2,984	3,025	3,060	3,092	3,210
800	2,328	2,577	2,714	2,808	2,879	2,936	2,984	3,024	3,060	3,091	3,210
900	2,328	2,577	2,714	2,808	2,879	2,936	2,984	3,024	3,060	3,091	3,210
1 000	2,328	2,577	2,714	2,808	2,879	2,936	2,983	3,024	3,059	3,091	3,209
∞	2,327	2,575	2,712	2,806	2,877	2,934	2,982	3,023	3,058	3,089	3,208

Table C.4 — One-sided prediction interval factors, k, at confidence level 99 % for known population standard deviation *(continued)*

n	m										
	20	30	40	50	60	80	100	150	200	250	500
2	3,994	4,126	4,217	4,287	4,343	4,43	4,496	4,613	4,695	4,757	4,945
3	3,781	3,908	3,996	4,064	4,118	4,201	4,265	4,379	4,459	4,519	4,702
4	3,667	3,791	3,878	3,943	3,996	4,078	4,141	4,252	4,330	4,389	4,568
5	3,596	3,718	3,803	3,868	3,920	4,000	4,062	4,172	4,248	4,307	4,483
6	3,548	3,668	3,752	3,816	3,867	3,947	4,008	4,117	4,192	4,250	4,425
7	3,512	3,632	3,715	3,778	3,829	3,908	3,969	4,076	4,151	4,209	4,382
8	3,486	3,604	3,687	3,750	3,800	3,879	3,939	4,046	4,120	4,177	4,349
9	3,465	3,583	3,665	3,727	3,777	3,856	3,915	4,022	4,096	4,152	4,324
10	3,448	3,565	3,647	3,709	3,759	3,837	3,896	4,002	4,076	4,132	4,303
11	3,434	3,551	3,632	3,694	3,744	3,821	3,881	3,986	4,059	4,116	4,286
12	3,422	3,539	3,620	3,681	3,731	3,808	3,867	3,973	4,046	4,102	4,271
13	3,412	3,529	3,609	3,671	3,720	3,797	3,856	3,961	4,034	4,090	4,259
14	3,403	3,520	3,600	3,662	3,711	3,788	3,847	3,951	4,024	4,080	4,248
15	3,396	3,512	3,592	3,654	3,703	3,780	3,838	3,943	4,015	4,071	4,239
16	3,389	3,505	3,585	3,647	3,696	3,772	3,831	3,935	4,008	4,063	4,231
17	3,384	3,499	3,579	3,640	3,690	3,766	3,824	3,928	4,001	4,056	4,224
18	3,379	3,494	3,574	3,635	3,684	3,760	3,819	3,923	3,995	4,050	4,218
19	3,374	3,489	3,569	3,630	3,679	3,755	3,813	3,917	3,989	4,045	4,212
20	3,370	3,485	3,565	3,625	3,674	3,751	3,809	3,912	3,984	4,040	4,207
25	3,354	3,469	3,548	3,608	3,657	3,733	3,791	3,894	3,966	4,021	4,187
30	3,343	3,458	3,537	3,597	3,646	3,721	3,779	3,882	3,953	4,008	4,174
35	3,336	3,450	3,529	3,589	3,637	3,713	3,770	3,873	3,944	3,999	4,164
40	3,330	3,444	3,523	3,583	3,631	3,706	3,764	3,866	3,938	3,992	4,157
45	3,326	3,439	3,518	3,578	3,626	3,701	3,759	3,861	3,932	3,987	4,152
50	3,322	3,436	3,514	3,574	3,622	3,697	3,755	3,857	3,928	3,982	4,147
60	3,317	3,430	3,508	3,568	3,617	3,692	3,749	3,851	3,922	3,976	4,140
70	3,313	3,426	3,504	3,564	3,612	3,687	3,744	3,846	3,917	3,971	4,136
80	3,310	3,423	3,501	3,561	3,609	3,684	3,741	3,843	3,914	3,968	4,132
90	3,308	3,421	3,499	3,559	3,607	3,682	3,739	3,840	3,911	3,965	4,129
100	3,306	3,419	3,497	3,557	3,605	3,680	3,737	3,838	3,909	3,963	4,127
150	3,301	3,413	3,491	3,551	3,599	3,674	3,730	3,832	3,903	3,957	4,120
200	3,298	3,411	3,489	3,548	3,596	3,671	3,727	3,829	3,899	3,953	4,117
250	3,296	3,409	3,487	3,546	3,594	3,669	3,726	3,827	3,898	3,951	4,115
300	3,295	3,408	3,486	3,545	3,593	3,668	3,724	3,826	3,896	3,950	4,114
350	3,294	3,407	3,485	3,544	3,592	3,667	3,724	3,825	3,895	3,949	4,113
400	3,294	3,406	3,484	3,544	3,592	3,666	3,723	3,824	3,895	3,949	4,112
450	3,293	3,406	3,484	3,543	3,591	3,666	3,722	3,824	3,894	3,948	4,111
500	3,293	3,405	3,483	3,543	3,591	3,665	3,722	3,823	3,894	3,948	4,111
600	3,292	3,405	3,483	3,542	3,590	3,665	3,721	3,823	3,893	3,947	4,110
700	3,292	3,405	3,482	3,542	3,590	3,664	3,721	3,822	3,893	3,946	4,110
800	3,292	3,404	3,482	3,541	3,589	3,664	3,721	3,822	3,892	3,946	4,109
900	3,291	3,404	3,482	3,541	3,589	3,664	3,720	3,822	3,892	3,946	4,109
1 000	3,291	3,404	3,482	3,541	3,589	3,663	3,720	3,821	3,892	3,946	4,109
∞	3,290	3,402	3,480	3,539	3,587	3,661	3,718	3,819	3,890	3,944	4,107

Table C.4 — One-sided prediction interval factors, k, at confidence level 99 % for known population standard deviation *(continued)*

n	m									
	1 000	2 000	5 000	10 000	20 000	50 000	100 000	200 000	500 000	1 000 000
2	5,125	5,298	5,518	5,677	5,832	6,029	6,174	6,315	6,495	6,628
3	4,877	5,047	5,262	5,418	5,571	5,765	5,908	6,046	6,225	6,356
4	4,741	4,907	5,119	5,274	5,424	5,616	5,757	5,894	6,071	6,201
5	4,654	4,818	5,028	5,181	5,329	5,520	5,659	5,796	5,971	6,100
6	4,594	4,756	4,964	5,116	5,263	5,452	5,591	5,726	5,900	6,029
7	4,549	4,711	4,917	5,068	5,214	5,402	5,540	5,674	5,848	5,975
8	4,516	4,676	4,881	5,031	5,177	5,363	5,500	5,634	5,807	5,934
9	4,489	4,649	4,853	5,002	5,147	5,333	5,469	5,603	5,774	5,901
10	4,467	4,627	4,830	4,978	5,123	5,308	5,444	5,577	5,748	5,874
11	4,450	4,608	4,811	4,959	5,102	5,287	5,423	5,555	5,726	5,852
12	4,435	4,593	4,795	4,942	5,086	5,270	5,405	5,537	5,707	5,833
13	4,422	4,580	4,781	4,928	5,071	5,255	5,390	5,522	5,691	5,817
14	4,411	4,568	4,769	4,916	5,059	5,242	5,377	5,508	5,678	5,803
15	4,402	4,559	4,759	4,905	5,048	5,231	5,365	5,497	5,666	5,790
16	4,393	4,550	4,750	4,896	5,038	5,221	5,355	5,486	5,655	5,780
17	4,386	4,542	4,742	4,888	5,030	5,212	5,346	5,477	5,646	5,770
18	4,379	4,535	4,735	4,881	5,023	5,205	5,338	5,469	5,637	5,762
19	4,373	4,529	4,728	4,874	5,016	5,198	5,331	5,462	5,630	5,754
20	4,368	4,524	4,723	4,868	5,010	5,191	5,325	5,455	5,623	5,747
25	4,347	4,503	4,701	4,846	4,986	5,167	5,300	5,430	5,597	5,721
30	4,334	4,489	4,686	4,830	4,971	5,151	5,283	5,413	5,580	5,703
35	4,324	4,478	4,675	4,819	4,960	5,139	5,272	5,401	5,567	5,690
40	4,317	4,471	4,667	4,811	4,951	5,131	5,263	5,392	5,558	5,680
45	4,311	4,465	4,661	4,805	4,944	5,124	5,256	5,384	5,550	5,673
50	4,306	4,460	4,656	4,800	4,939	5,118	5,250	5,379	5,544	5,667
60	4,299	4,453	4,649	4,792	4,931	5,110	5,241	5,370	5,535	5,658
70	4,294	4,448	4,643	4,786	4,926	5,104	5,235	5,364	5,529	5,651
80	4,291	4,444	4,639	4,782	4,921	5,100	5,231	5,359	5,524	5,646
90	4,288	4,441	4,636	4,779	4,918	5,096	5,227	5,355	5,521	5,642
100	4,285	4,438	4,634	4,776	4,915	5,093	5,225	5,353	5,518	5,639
150	4,278	4,431	4,626	4,769	4,907	5,085	5,216	5,344	5,509	5,630
200	4,275	4,427	4,622	4,765	4,903	5,081	5,212	5,339	5,504	5,625
250	4,273	4,425	4,620	4,762	4,901	5,079	5,209	5,337	5,501	5,623
300	4,271	4,424	4,618	4,761	4,899	5,077	5,207	5,335	5,500	5,621
350	4,270	4,423	4,617	4,760	4,898	5,076	5,206	5,334	5,498	5,620
400	4,270	4,422	4,617	4,759	4,897	5,075	5,205	5,333	5,497	5,619
450	4,269	4,421	4,616	4,758	4,897	5,074	5,205	5,332	5,496	5,618
500	4,268	4,421	4,615	4,758	4,896	5,074	5,204	5,332	5,496	5,617
600	4,268	4,420	4,615	4,757	4,895	5,073	5,203	5,331	5,495	5,616
700	4,267	4,420	4,614	4,756	4,895	5,072	5,203	5,330	5,494	5,616
800	4,267	4,419	4,614	4,756	4,894	5,072	5,202	5,330	5,494	5,615
900	4,267	4,419	4,613	4,756	4,894	5,071	5,202	5,329	5,493	5,615
1 000	4,266	4,419	4,613	4,755	4,894	5,071	5,201	5,329	5,493	5,614
∞	4,264	4,417	4,611	4,753	4,891	5,069	5,199	5,326	5,490	5,612

NOTE This table provides factors k such that one may be at least 99 % confident that none of the next m observations from a normally distributed population will lie outside the range $(-\infty, \bar{x} + k\sigma)$, where $\bar{x}$ is derived from a random sample of size n from the same population. Similarly for the range $(\bar{x} - k\sigma, \infty)$.

Table C.5 — One-sided prediction interval factors, k, at confidence level 99,5 % for known population standard deviation

n	m										
	1	2	3	4	5	6	7	8	9	10	15
2	3,155	3,432	3,586	3,692	3,772	3,836	3,890	3,936	3,976	4,012	4,146
3	2,975	3,238	3,385	3,485	3,562	3,623	3,674	3,718	3,757	3,791	3,919
4	2,880	3,137	3,279	3,377	3,451	3,511	3,560	3,603	3,640	3,673	3,798
5	2,822	3,074	3,213	3,309	3,382	3,441	3,490	3,532	3,568	3,601	3,723
6	2,783	3,031	3,169	3,264	3,336	3,393	3,442	3,483	3,519	3,551	3,672
7	2,754	3,000	3,137	3,231	3,302	3,359	3,407	3,448	3,484	3,515	3,635
8	2,733	2,977	3,112	3,205	3,276	3,333	3,381	3,421	3,457	3,488	3,607
9	2,716	2,958	3,093	3,186	3,256	3,313	3,360	3,400	3,436	3,467	3,585
10	2,702	2,944	3,078	3,170	3,240	3,296	3,343	3,383	3,419	3,450	3,567
11	2,691	2,931	3,065	3,157	3,227	3,283	3,329	3,370	3,405	3,436	3,553
12	2,682	2,921	3,054	3,146	3,216	3,271	3,318	3,358	3,393	3,424	3,541
13	2,674	2,913	3,045	3,137	3,206	3,262	3,308	3,348	3,383	3,414	3,530
14	2,667	2,905	3,038	3,129	3,198	3,254	3,300	3,340	3,374	3,405	3,521
15	2,661	2,899	3,031	3,122	3,191	3,246	3,293	3,332	3,367	3,398	3,514
16	2,656	2,893	3,025	3,116	3,185	3,240	3,286	3,326	3,360	3,391	3,507
17	2,651	2,888	3,020	3,110	3,179	3,234	3,281	3,320	3,355	3,385	3,501
18	2,647	2,884	3,015	3,106	3,174	3,230	3,276	3,315	3,349	3,380	3,495
19	2,643	2,880	3,011	3,101	3,170	3,225	3,271	3,310	3,345	3,375	3,491
20	2,640	2,876	3,007	3,098	3,166	3,221	3,267	3,306	3,341	3,371	3,486
25	2,627	2,863	2,993	3,083	3,151	3,206	3,251	3,291	3,325	3,355	3,470
30	2,619	2,853	2,983	3,073	3,141	3,196	3,241	3,280	3,314	3,344	3,459
35	2,613	2,847	2,977	3,066	3,134	3,188	3,234	3,273	3,307	3,337	3,451
40	2,608	2,842	2,971	3,061	3,128	3,183	3,228	3,267	3,301	3,331	3,445
45	2,605	2,838	2,967	3,057	3,124	3,178	3,224	3,263	3,296	3,327	3,440
50	2,602	2,835	2,964	3,053	3,121	3,175	3,220	3,259	3,293	3,323	3,436
60	2,598	2,830	2,959	3,048	3,116	3,170	3,215	3,254	3,288	3,318	3,431
70	2,595	2,827	2,956	3,045	3,112	3,166	3,211	3,250	3,284	3,314	3,427
80	2,592	2,825	2,953	3,042	3,109	3,163	3,208	3,247	3,281	3,311	3,424
90	2,591	2,823	2,951	3,040	3,107	3,161	3,206	3,245	3,279	3,309	3,422
100	2,589	2,821	2,950	3,038	3,105	3,159	3,205	3,243	3,277	3,307	3,420
150	2,585	2,816	2,945	3,033	3,100	3,154	3,199	3,238	3,271	3,301	3,414
200	2,583	2,814	2,942	3,031	3,098	3,152	3,197	3,235	3,269	3,299	3,411
250	2,581	2,813	2,941	3,029	3,096	3,150	3,195	3,234	3,267	3,297	3,410
300	2,581	2,812	2,940	3,028	3,095	3,149	3,194	3,232	3,266	3,296	3,408
350	2,580	2,811	2,939	3,028	3,095	3,148	3,193	3,232	3,265	3,295	3,408
400	2,580	2,811	2,939	3,027	3,094	3,148	3,193	3,231	3,265	3,294	3,407
450	2,579	2,810	2,938	3,027	3,094	3,147	3,192	3,231	3,264	3,294	3,407
500	2,579	2,810	2,938	3,026	3,093	3,147	3,192	3,230	3,264	3,294	3,406
600	2,578	2,809	2,938	3,026	3,093	3,146	3,191	3,230	3,263	3,293	3,406
700	2,578	2,809	2,937	3,025	3,092	3,146	3,191	3,229	3,263	3,293	3,405
800	2,578	2,809	2,937	3,025	3,092	3,146	3,191	3,229	3,263	3,292	3,405
900	2,578	2,809	2,937	3,025	3,092	3,146	3,190	3,229	3,262	3,292	3,405
1 000	2,578	2,809	2,937	3,025	3,092	3,145	3,190	3,229	3,262	3,292	3,404
∞	2,576	2,807	2,935	3,023	3,090	3,144	3,189	3,227	3,261	3,290	3,403

Table C.5 — One-sided prediction interval factors, k, at confidence level 99,5 % for known population standard deviation *(continued)*

n	m										
	20	30	40	50	60	80	100	150	200	250	500
2	4,239	4,366	4,454	4,522	4,576	4,660	4,724	4,838	4,918	4,978	5,162
3	4,008	4,130	4,215	4,279	4,331	4,412	4,474	4,584	4,661	4,720	4,897
4	3,885	4,004	4,086	4,149	4,200	4,279	4,339	4,447	4,522	4,579	4,753
5	3,808	3,925	4,006	4,068	4,118	4,196	4,255	4,361	4,435	4,491	4,662
6	3,756	3,872	3,952	4,013	4,062	4,139	4,197	4,302	4,375	4,431	4,600
7	3,718	3,833	3,912	3,973	4,021	4,097	4,155	4,259	4,331	4,387	4,554
8	3,690	3,803	3,882	3,942	3,990	4,066	4,124	4,227	4,298	4,353	4,520
9	3,667	3,780	3,858	3,918	3,966	4,041	4,099	4,201	4,272	4,327	4,492
10	3,649	3,761	3,839	3,899	3,947	4,021	4,078	4,180	4,251	4,306	4,470
11	3,634	3,746	3,823	3,883	3,931	4,005	4,062	4,163	4,234	4,288	4,452
12	3,622	3,733	3,810	3,869	3,917	3,991	4,048	4,149	4,219	4,273	4,437
13	3,611	3,722	3,799	3,858	3,906	3,980	4,036	4,137	4,207	4,261	4,424
14	3,602	3,713	3,790	3,848	3,896	3,969	4,026	4,126	4,197	4,250	4,413
15	3,594	3,705	3,781	3,840	3,887	3,961	4,017	4,117	4,187	4,241	4,403
16	3,587	3,697	3,774	3,832	3,880	3,953	4,009	4,109	4,179	4,233	4,395
17	3,581	3,691	3,767	3,826	3,873	3,946	4,002	4,102	4,172	4,225	4,387
18	3,575	3,685	3,762	3,820	3,867	3,940	3,996	4,096	4,166	4,219	4,381
19	3,570	3,680	3,757	3,815	3,862	3,935	3,991	4,091	4,160	4,213	4,375
20	3,566	3,676	3,752	3,810	3,857	3,930	3,986	4,086	4,155	4,208	4,369
25	3,549	3,658	3,734	3,792	3,839	3,911	3,967	4,066	4,135	4,188	4,349
30	3,538	3,647	3,722	3,780	3,827	3,899	3,954	4,053	4,122	4,175	4,335
35	3,530	3,638	3,714	3,771	3,818	3,890	3,945	4,044	4,113	4,165	4,325
40	3,524	3,632	3,707	3,765	3,811	3,883	3,939	4,037	4,106	4,158	4,317
45	3,519	3,627	3,702	3,760	3,806	3,878	3,933	4,032	4,100	4,152	4,312
50	3,515	3,623	3,698	3,756	3,802	3,874	3,929	4,027	4,096	4,148	4,307
60	3,509	3,617	3,692	3,750	3,796	3,868	3,923	4,021	4,089	4,141	4,300
70	3,505	3,613	3,688	3,745	3,791	3,863	3,918	4,016	4,084	4,136	4,295
80	3,502	3,610	3,685	3,742	3,788	3,860	3,915	4,012	4,081	4,133	4,291
90	3,500	3,608	3,682	3,739	3,785	3,857	3,912	4,010	4,078	4,130	4,288
100	3,498	3,606	3,680	3,737	3,783	3,855	3,910	4,008	4,076	4,128	4,286
150	3,492	3,600	3,674	3,731	3,777	3,849	3,903	4,001	4,069	4,121	4,279
200	3,489	3,597	3,671	3,728	3,774	3,846	3,900	3,998	4,066	4,118	4,275
250	3,488	3,595	3,669	3,726	3,772	3,844	3,898	3,996	4,064	4,116	4,273
300	3,486	3,594	3,668	3,725	3,771	3,842	3,897	3,994	4,062	4,114	4,272
350	3,486	3,593	3,667	3,724	3,770	3,841	3,896	3,993	4,061	4,113	4,271
400	3,485	3,592	3,667	3,724	3,769	3,841	3,895	3,993	4,061	4,113	4,270
450	3,484	3,592	3,666	3,723	3,769	3,840	3,895	3,992	4,06	4,112	4,270
500	3,484	3,591	3,666	3,723	3,768	3,840	3,894	3,992	4,060	4,111	4,269
600	3,484	3,591	3,665	3,722	3,768	3,839	3,894	3,991	4,059	4,111	4,268
700	3,483	3,590	3,665	3,722	3,767	3,839	3,893	3,991	4,058	4,110	4,268
800	3,483	3,590	3,664	3,721	3,767	3,838	3,893	3,990	4,058	4,110	4,267
900	3,483	3,590	3,664	3,721	3,767	3,838	3,893	3,990	4,058	4,110	4,267
1 000	3,482	3,590	3,664	3,721	3,767	3,838	3,892	3,990	4,058	4,109	4,267
∞	3,481	3,588	3,662	3,719	3,765	3,836	3,890	3,988	4,056	4,107	4,265

Table C.5 — One-sided prediction interval factors, k, at confidence level 99,5 % for known population standard deviation *(continued)*

n	m									
	1 000	2 000	5 000	10 000	20 000	50 000	100 000	200 000	500 000	1 000 000
2	5,338	5,508	5,724	5,882	6,034	6,230	6,373	6,512	6,691	6,823
3	5,069	5,234	5,445	5,599	5,749	5,940	6,081	6,218	6,394	6,524
4	4,921	5,084	5,291	5,442	5,590	5,778	5,917	6,053	6,227	6,355
5	4,828	4,988	5,193	5,342	5,488	5,674	5,812	5,946	6,118	6,246
6	4,764	4,922	5,124	5,273	5,417	5,602	5,738	5,871	6,042	6,169
7	4,717	4,874	5,075	5,221	5,365	5,548	5,683	5,816	5,986	6,112
8	4,681	4,837	5,036	5,182	5,325	5,507	5,642	5,773	5,942	6,067
9	4,653	4,808	5,006	5,152	5,293	5,475	5,609	5,739	5,908	6,032
10	4,630	4,784	4,982	5,127	5,267	5,448	5,582	5,712	5,880	6,004
11	4,611	4,765	4,962	5,106	5,246	5,427	5,559	5,689	5,857	5,980
12	4,595	4,749	4,945	5,089	5,229	5,408	5,541	5,670	5,837	5,960
13	4,582	4,735	4,931	5,074	5,214	5,393	5,525	5,654	5,820	5,943
14	4,571	4,723	4,918	5,061	5,201	5,379	5,511	5,640	5,806	5,929
15	4,561	4,713	4,908	5,050	5,189	5,368	5,499	5,628	5,793	5,916
16	4,552	4,704	4,898	5,041	5,179	5,357	5,489	5,617	5,782	5,905
17	4,544	4,696	4,890	5,032	5,170	5,348	5,479	5,607	5,773	5,895
18	4,537	4,689	4,882	5,024	5,163	5,340	5,471	5,599	5,764	5,886
19	4,531	4,682	4,876	5,017	5,156	5,333	5,464	5,591	5,756	5,878
20	4,525	4,677	4,870	5,011	5,149	5,326	5,457	5,585	5,749	5,871
25	4,504	4,654	4,847	4,988	5,125	5,301	5,431	5,558	5,722	5,843
30	4,490	4,640	4,831	4,972	5,109	5,285	5,414	5,541	5,704	5,825
35	4,479	4,629	4,820	4,961	5,097	5,273	5,402	5,528	5,691	5,811
40	4,472	4,621	4,812	4,952	5,088	5,263	5,392	5,519	5,681	5,801
45	4,466	4,615	4,806	4,945	5,081	5,256	5,385	5,511	5,674	5,794
50	4,461	4,610	4,800	4,940	5,076	5,251	5,379	5,505	5,668	5,787
60	4,454	4,602	4,793	4,932	5,068	5,242	5,371	5,496	5,658	5,778
70	4,448	4,597	4,787	4,926	5,062	5,236	5,364	5,490	5,652	5,771
80	4,444	4,593	4,783	4,922	5,057	5,232	5,360	5,485	5,647	5,766
90	4,441	4,590	4,780	4,919	5,054	5,228	5,356	5,481	5,643	5,762
100	4,439	4,587	4,777	4,916	5,051	5,225	5,353	5,478	5,640	5,759
150	4,432	4,580	4,769	4,908	5,043	5,217	5,344	5,469	5,631	5,750
200	4,428	4,576	4,765	4,904	5,039	5,212	5,340	5,465	5,626	5,745
250	4,426	4,574	4,763	4,901	5,036	5,210	5,337	5,462	5,623	5,742
300	4,424	4,572	4,761	4,900	5,035	5,208	5,336	5,460	5,621	5,740
350	4,423	4,571	4,760	4,899	5,033	5,207	5,334	5,459	5,620	5,739
400	4,423	4,570	4,759	4,898	5,033	5,206	5,333	5,458	5,619	5,738
450	4,422	4,570	4,759	4,897	5,032	5,205	5,333	5,457	5,618	5,737
500	4,422	4,569	4,758	4,897	5,031	5,205	5,332	5,457	5,618	5,737
600	4,421	4,569	4,757	4,896	5,030	5,204	5,331	5,456	5,617	5,736
700	4,420	4,568	4,757	4,895	5,030	5,203	5,331	5,455	5,616	5,735
800	4,420	4,568	4,756	4,895	5,029	5,203	5,330	5,455	5,616	5,734
900	4,420	4,567	4,756	4,894	5,029	5,202	5,330	5,454	5,615	5,734
1 000	4,419	4,567	4,756	4,894	5,029	5,202	5,329	5,454	5,615	5,734
∞	4,417	4,565	4,753	4,892	5,026	5,199	5,327	5,451	5,612	5,731

NOTE This table provides factors k such that one may be at least 99,5 % confident that none of the next m observations from a normally distributed population will lie outside the range $(-\infty, \bar{x} + k\sigma)$, where $\bar{x}$ is derived from a random sample of size n from the same population. Similarly for the range $(\bar{x} - k\sigma, \infty)$.

Table C.6 — One-sided prediction interval factors, k, at confidence level 99,9 % for known population standard deviation

n	m										
	1	2	3	4	5	6	7	8	9	10	15
2	3,785	4,028	4,165	4,259	4,331	4,389	4,438	4,479	4,516	4,548	4,670
3	3,569	3,799	3,928	4,018	4,086	4,141	4,187	4,226	4,261	4,292	4,408
4	3,455	3,679	3,804	3,891	3,957	4,010	4,055	4,093	4,127	4,157	4,270
5	3,386	3,605	3,728	3,813	3,878	3,930	3,974	4,011	4,044	4,073	4,184
6	3,338	3,554	3,676	3,760	3,824	3,875	3,918	3,955	3,988	4,017	4,126
7	3,304	3,518	3,638	3,721	3,784	3,836	3,878	3,915	3,947	3,976	4,084
8	3,278	3,490	3,609	3,692	3,755	3,806	3,848	3,884	3,916	3,945	4,052
9	3,258	3,469	3,587	3,669	3,732	3,782	3,824	3,860	3,892	3,920	4,027
10	3,242	3,451	3,569	3,651	3,713	3,763	3,805	3,841	3,873	3,901	4,007
11	3,228	3,437	3,554	3,636	3,698	3,748	3,789	3,825	3,857	3,884	3,990
12	3,217	3,425	3,542	3,623	3,685	3,735	3,776	3,812	3,843	3,871	3,976
13	3,207	3,415	3,532	3,612	3,674	3,724	3,765	3,801	3,832	3,860	3,965
14	3,199	3,406	3,523	3,603	3,665	3,714	3,755	3,791	3,822	3,850	3,954
15	3,192	3,399	3,515	3,595	3,656	3,706	3,747	3,783	3,814	3,841	3,946
16	3,186	3,392	3,508	3,588	3,649	3,699	3,740	3,775	3,806	3,834	3,938
17	3,180	3,386	3,502	3,582	3,643	3,692	3,733	3,769	3,800	3,827	3,931
18	3,175	3,381	3,496	3,576	3,637	3,686	3,728	3,763	3,794	3,821	3,925
19	3,171	3,376	3,492	3,571	3,632	3,681	3,722	3,758	3,788	3,816	3,920
20	3,167	3,372	3,487	3,567	3,628	3,677	3,718	3,753	3,784	3,811	3,915
25	3,152	3,356	3,471	3,550	3,611	3,659	3,700	3,735	3,766	3,793	3,896
30	3,142	3,345	3,460	3,539	3,599	3,648	3,688	3,723	3,754	3,781	3,884
35	3,135	3,338	3,452	3,530	3,591	3,639	3,680	3,715	3,745	3,772	3,875
40	3,129	3,332	3,446	3,524	3,584	3,633	3,673	3,708	3,739	3,766	3,868
45	3,125	3,327	3,441	3,520	3,580	3,628	3,668	3,703	3,733	3,760	3,863
50	3,121	3,324	3,437	3,516	3,576	3,624	3,664	3,699	3,729	3,756	3,859
60	3,116	3,318	3,432	3,510	3,570	3,618	3,658	3,693	3,723	3,750	3,852
70	3,113	3,314	3,428	3,506	3,566	3,614	3,654	3,689	3,719	3,746	3,848
80	3,110	3,311	3,425	3,503	3,563	3,611	3,651	3,685	3,716	3,743	3,844
90	3,108	3,309	3,422	3,500	3,560	3,608	3,648	3,683	3,713	3,740	3,842
100	3,106	3,307	3,420	3,499	3,558	3,606	3,646	3,681	3,711	3,738	3,840
150	3,101	3,302	3,415	3,493	3,552	3,600	3,640	3,675	3,705	3,732	3,833
200	3,098	3,299	3,412	3,490	3,549	3,597	3,637	3,672	3,702	3,729	3,830
250	3,097	3,298	3,410	3,488	3,548	3,595	3,636	3,670	3,700	3,727	3,828
300	3,096	3,296	3,409	3,487	3,546	3,594	3,634	3,669	3,699	3,726	3,827
350	3,095	3,296	3,408	3,486	3,546	3,593	3,633	3,668	3,698	3,725	3,826
400	3,095	3,295	3,408	3,486	3,545	3,593	3,633	3,667	3,697	3,724	3,825
450	3,094	3,295	3,407	3,485	3,544	3,592	3,632	3,667	3,697	3,724	3,825
500	3,094	3,294	3,407	3,485	3,544	3,592	3,632	3,666	3,696	3,723	3,824
600	3,093	3,294	3,406	3,484	3,543	3,591	3,631	3,666	3,696	3,722	3,824
700	3,093	3,293	3,406	3,484	3,543	3,591	3,631	3,665	3,695	3,722	3,823
800	3,093	3,293	3,405	3,483	3,543	3,591	3,631	3,665	3,695	3,722	3,823
900	3,092	3,293	3,405	3,483	3,542	3,590	3,630	3,665	3,695	3,721	3,823
1 000	3,092	3,293	3,405	3,483	3,542	3,590	3,630	3,664	3,695	3,721	3,823
∞	3,091	3,291	3,403	3,481	3,540	3,588	3,628	3,663	3,693	3,719	3,821

Table C.6 — One-sided prediction interval factors, k, at confidence level 99,9 % for known population standard deviation *(continued)*

n	*m*										
	20	30	40	50	60	80	100	150	200	250	500
2	4,755	4,873	4,954	5,017	5,067	5,146	5,206	5,313	5,388	5,445	5,619
3	4,489	4,601	4,678	4,738	4,786	4,861	4,918	5,021	5,093	5,148	5,315
4	4,348	4,457	4,532	4,590	4,637	4,709	4,765	4,865	4,935	4,988	5,151
5	4,261	4,367	4,441	4,498	4,544	4,616	4,670	4,768	4,837	4,889	5,049
6	4,202	4,307	4,380	4,436	4,481	4,552	4,606	4,702	4,770	4,822	4,979
7	4,159	4,263	4,335	4,391	4,435	4,505	4,559	4,655	4,721	4,773	4,929
8	4,126	4,230	4,301	4,356	4,401	4,470	4,523	4,618	4,685	4,736	4,890
9	4,101	4,203	4,275	4,330	4,374	4,443	4,495	4,590	4,656	4,706	4,860
10	4,080	4,182	4,254	4,308	4,352	4,420	4,473	4,567	4,633	4,683	4,836
11	4,064	4,165	4,236	4,290	4,334	4,402	4,454	4,548	4,614	4,664	4,816
12	4,050	4,151	4,221	4,275	4,319	4,387	4,439	4,532	4,598	4,647	4,800
13	4,038	4,139	4,209	4,263	4,306	4,374	4,426	4,519	4,584	4,634	4,785
14	4,027	4,128	4,198	4,252	4,295	4,363	4,415	4,507	4,572	4,622	4,773
15	4,018	4,119	4,189	4,242	4,286	4,353	4,405	4,497	4,562	4,612	4,763
16	4,011	4,111	4,181	4,234	4,277	4,345	4,396	4,489	4,553	4,603	4,753
17	4,004	4,104	4,173	4,227	4,270	4,337	4,389	4,481	4,545	4,595	4,745
18	3,997	4,097	4,167	4,220	4,263	4,330	4,382	4,474	4,538	4,588	4,738
19	3,992	4,092	4,161	4,214	4,257	4,325	4,376	4,468	4,532	4,581	4,731
20	3,987	4,087	4,156	4,209	4,252	4,319	4,370	4,462	4,526	4,576	4,725
25	3,968	4,067	4,136	4,189	4,232	4,299	4,350	4,441	4,505	4,554	4,703
30	3,955	4,054	4,123	4,176	4,218	4,285	4,336	4,427	4,490	4,539	4,688
35	3,946	4,045	4,113	4,166	4,209	4,275	4,326	4,417	4,480	4,529	4,677
40	3,939	4,038	4,106	4,159	4,201	4,267	4,318	4,409	4,472	4,521	4,669
45	3,934	4,032	4,101	4,153	4,196	4,262	4,312	4,403	4,466	4,515	4,663
50	3,930	4,028	4,096	4,149	4,191	4,257	4,308	4,398	4,461	4,510	4,658
60	3,923	4,021	4,090	4,142	4,184	4,250	4,301	4,391	4,454	4,503	4,650
70	3,919	4,017	4,085	4,137	4,179	4,245	4,296	4,386	4,449	4,497	4,645
80	3,915	4,013	4,081	4,133	4,176	4,241	4,292	4,382	4,445	4,493	4,640
90	3,913	4,010	4,078	4,131	4,173	4,239	4,289	4,379	4,442	4,490	4,637
100	3,910	4,008	4,076	4,128	4,170	4,236	4,287	4,377	4,440	4,488	4,635
150	3,904	4,002	4,069	4,122	4,164	4,229	4,279	4,369	4,432	4,480	4,627
200	3,901	3,998	4,066	4,118	4,160	4,226	4,276	4,366	4,429	4,477	4,623
250	3,899	3,996	4,064	4,116	4,158	4,224	4,274	4,364	4,426	4,474	4,621
300	3,897	3,995	4,063	4,115	4,157	4,222	4,272	4,362	4,425	4,473	4,619
350	3,897	3,994	4,062	4,114	4,156	4,221	4,271	4,361	4,424	4,472	4,618
400	3,896	3,993	4,061	4,113	4,155	4,220	4,271	4,360	4,423	4,471	4,618
450	3,895	3,993	4,061	4,112	4,154	4,220	4,270	4,360	4,422	4,471	4,617
500	3,895	3,992	4,060	4,112	4,154	4,219	4,270	4,359	4,422	4,470	4,616
600	3,894	3,992	4,059	4,111	4,153	4,219	4,269	4,359	4,421	4,469	4,616
700	3,894	3,991	4,059	4,111	4,153	4,218	4,268	4,358	4,421	4,469	4,615
800	3,893	3,991	4,059	4,110	4,152	4,218	4,268	4,358	4,420	4,468	4,615
900	3,893	3,990	4,058	4,110	4,152	4,218	4,268	4,357	4,420	4,468	4,614
1 000	3,893	3,990	4,058	4,110	4,152	4,217	4,267	4,357	4,420	4,468	4,614
∞	3,891	3,988	4,056	4,108	4,150	4,215	4,265	4,355	4,418	4,466	4,612

Table C.6 — One-sided prediction interval factors, k, at confidence level 99,9 % for known population standard deviation *(continued)*

n	m									
	1 000	2 000	5 000	10 000	20 000	50 000	100 000	200 000	500 000	1 000 000
2	5,787	5,950	6,158	6,311	6,459	6,649	6,788	6,924	7,100	7,229
3	5,476	5,634	5,835	5,983	6,126	6,311	6,447	6,581	6,752	6,879
4	5,309	5,462	5,659	5,803	5,944	6,126	6,259	6,390	6,559	6,683
5	5,204	5,355	5,548	5,690	5,829	6,008	6,140	6,269	6,435	6,558
6	5,132	5,281	5,473	5,613	5,750	5,927	6,057	6,185	6,349	6,471
7	5,080	5,228	5,417	5,556	5,692	5,867	5,997	6,123	6,287	6,408
8	5,041	5,187	5,375	5,513	5,648	5,822	5,950	6,076	6,239	6,359
9	5,010	5,156	5,342	5,480	5,614	5,787	5,914	6,039	6,201	6,320
10	4,985	5,130	5,316	5,453	5,586	5,758	5,885	6,009	6,170	6,289
11	4,965	5,109	5,294	5,430	5,563	5,734	5,861	5,985	6,145	6,263
12	4,947	5,091	5,276	5,411	5,544	5,715	5,841	5,964	6,124	6,242
13	4,933	5,076	5,260	5,395	5,528	5,698	5,824	5,947	6,106	6,224
14	4,920	5,063	5,247	5,382	5,514	5,683	5,809	5,932	6,090	6,208
15	4,909	5,052	5,235	5,370	5,501	5,671	5,796	5,918	6,077	6,194
16	4,900	5,042	5,225	5,359	5,491	5,660	5,785	5,907	6,065	6,182
17	4,891	5,034	5,216	5,350	5,481	5,650	5,775	5,897	6,055	6,172
18	4,884	5,026	5,208	5,342	5,473	5,641	5,766	5,888	6,045	6,162
19	4,877	5,019	5,201	5,335	5,465	5,634	5,758	5,880	6,037	6,154
20	4,871	5,013	5,194	5,328	5,458	5,627	5,751	5,872	6,030	6,146
25	4,848	4,989	5,170	5,303	5,432	5,600	5,723	5,844	6,001	6,117
30	4,832	4,973	5,153	5,286	5,415	5,582	5,705	5,826	5,982	6,097
35	4,821	4,961	5,141	5,273	5,403	5,569	5,692	5,812	5,968	6,083
40	4,813	4,953	5,132	5,264	5,393	5,559	5,682	5,802	5,958	6,073
45	4,806	4,946	5,125	5,257	5,386	5,552	5,674	5,794	5,950	6,064
50	4,801	4,941	5,120	5,251	5,380	5,546	5,668	5,788	5,943	6,058
60	4,793	4,933	5,111	5,243	5,371	5,537	5,659	5,779	5,933	6,048
70	4,788	4,927	5,105	5,237	5,365	5,530	5,652	5,772	5,926	6,041
80	4,783	4,922	5,101	5,232	5,360	5,525	5,647	5,767	5,921	6,036
90	4,780	4,919	5,097	5,229	5,357	5,522	5,643	5,763	5,917	6,031
100	4,778	4,916	5,095	5,226	5,354	5,519	5,640	5,760	5,914	6,028
150	4,770	4,908	5,086	5,217	5,345	5,510	5,631	5,750	5,904	6,018
200	4,766	4,904	5,082	5,213	5,340	5,505	5,626	5,745	5,899	6,013
250	4,763	4,902	5,079	5,210	5,338	5,502	5,624	5,743	5,896	6,010
300	4,762	4,900	5,078	5,208	5,336	5,500	5,622	5,741	5,894	6,008
350	4,761	4,899	5,077	5,207	5,335	5,499	5,620	5,739	5,893	6,007
400	4,760	4,898	5,076	5,206	5,334	5,498	5,619	5,738	5,892	6,006
450	4,759	4,897	5,075	5,206	5,333	5,497	5,619	5,738	5,891	6,005
500	4,759	4,897	5,074	5,205	5,332	5,497	5,618	5,737	5,890	6,004
600	4,758	4,896	5,074	5,204	5,332	5,496	5,617	5,736	5,890	6,003
700	4,757	4,896	5,073	5,203	5,331	5,495	5,616	5,735	5,889	6,003
800	4,757	4,895	5,073	5,203	5,330	5,495	5,616	5,735	5,888	6,002
900	4,756	4,895	5,072	5,203	5,330	5,494	5,616	5,734	5,888	6,002
1 000	4,756	4,894	5,072	5,202	5,330	5,494	5,615	5,734	5,888	6,001
∞	4,754	4,892	5,069	5,200	5,327	5,491	5,612	5,731	5,885	5,998

NOTE This table provides factors k such that one may be at least 99,9 % confident that none of the next m observations from a normally distributed population will lie outside the range $(-\infty, \bar{x} + k\sigma)$, where $\bar{x}$ is derived from a random sample of size n from the same population. Similarly for the range $(\bar{x} - k\sigma, \infty)$.

Annex D
(normative)

Tables of two-sided prediction interval factors, k, for known population standard deviation

Table D.1 — Two-sided prediction interval factors, k, at confidence level 90 % for known population standard deviation

n	m										
	1	2	3	4	5	6	7	8	9	10	15
2	2,015	2,370	2,563	2,693	2,790	2,868	2,932	2,987	3,035	3,077	3,234
3	1,900	2,242	2,427	2,553	2,647	2,723	2,785	2,838	2,884	2,925	3,078
4	1,840	2,174	2,355	2,478	2,571	2,644	2,706	2,758	2,803	2,843	2,993
5	1,802	2,132	2,310	2,432	2,523	2,595	2,656	2,707	2,752	2,791	2,939
6	1,777	2,103	2,280	2,400	2,490	2,562	2,621	2,672	2,716	2,756	2,902
7	1,759	2,082	2,257	2,376	2,466	2,537	2,596	2,647	2,691	2,730	2,875
8	1,745	2,066	2,240	2,359	2,447	2,518	2,577	2,627	2,671	2,710	2,854
9	1,734	2,053	2,227	2,345	2,433	2,504	2,562	2,612	2,656	2,694	2,838
10	1,726	2,043	2,216	2,333	2,421	2,492	2,550	2,600	2,643	2,681	2,824
11	1,718	2,035	2,207	2,324	2,412	2,482	2,540	2,589	2,633	2,671	2,813
12	1,713	2,028	2,200	2,316	2,404	2,474	2,531	2,581	2,624	2,662	2,804
13	1,707	2,022	2,193	2,309	2,397	2,467	2,524	2,574	2,616	2,654	2,796
14	1,703	2,017	2,188	2,304	2,391	2,460	2,518	2,567	2,610	2,648	2,790
15	1,699	2,013	2,183	2,299	2,386	2,455	2,513	2,562	2,604	2,642	2,784
16	1,696	2,009	2,179	2,294	2,381	2,450	2,508	2,557	2,600	2,637	2,778
17	1,693	2,005	2,175	2,291	2,377	2,446	2,504	2,553	2,595	2,633	2,774
18	1,690	2,002	2,172	2,287	2,374	2,443	2,500	2,549	2,591	2,629	2,770
19	1,688	2,000	2,169	2,284	2,370	2,439	2,497	2,545	2,588	2,625	2,766
20	1,686	1,997	2,166	2,281	2,368	2,436	2,494	2,542	2,585	2,622	2,763
25	1,678	1,988	2,156	2,270	2,356	2,425	2,482	2,531	2,573	2,610	2,750
30	1,673	1,981	2,149	2,263	2,349	2,417	2,474	2,523	2,565	2,602	2,741
35	1,669	1,977	2,144	2,258	2,344	2,412	2,469	2,517	2,559	2,596	2,735
40	1,666	1,973	2,141	2,254	2,340	2,408	2,464	2,513	2,555	2,592	2,731
45	1,664	1,971	2,138	2,251	2,337	2,405	2,461	2,509	2,551	2,588	2,727
50	1,662	1,969	2,136	2,249	2,334	2,402	2,458	2,507	2,548	2,585	2,724
60	1,659	1,965	2,132	2,245	2,330	2,398	2,454	2,502	2,544	2,581	2,720
70	1,657	1,963	2,130	2,243	2,328	2,395	2,452	2,500	2,541	2,578	2,716
80	1,656	1,961	2,128	2,241	2,326	2,393	2,449	2,497	2,539	2,576	2,714
90	1,654	1,960	2,126	2,239	2,324	2,392	2,448	2,496	2,537	2,574	2,712
100	1,654	1,959	2,125	2,238	2,323	2,390	2,446	2,494	2,536	2,573	2,711
150	1,651	1,956	2,122	2,234	2,319	2,386	2,442	2,490	2,532	2,569	2,706
200	1,649	1,954	2,120	2,232	2,317	2,384	2,440	2,488	2,530	2,566	2,704
250	1,649	1,953	2,119	2,231	2,316	2,383	2,439	2,487	2,528	2,565	2,703
300	1,648	1,953	2,118	2,230	2,315	2,382	2,438	2,486	2,528	2,564	2,702
350	1,648	1,952	2,118	2,230	2,314	2,382	2,438	2,486	2,527	2,564	2,701
400	1,647	1,952	2,117	2,230	2,314	2,381	2,437	2,485	2,527	2,563	2,701
450	1,647	1,951	2,117	2,229	2,314	2,381	2,437	2,485	2,526	2,563	2,700
500	1,647	1,951	2,117	2,229	2,313	2,381	2,437	2,484	2,526	2,563	2,700
600	1,647	1,951	2,116	2,229	2,313	2,380	2,436	2,484	2,526	2,562	2,700
700	1,647	1,951	2,116	2,228	2,313	2,380	2,436	2,484	2,525	2,562	2,699
800	1,646	1,951	2,116	2,228	2,313	2,380	2,436	2,484	2,525	2,562	2,699
900	1,646	1,950	2,116	2,228	2,312	2,380	2,436	2,483	2,525	2,561	2,699
1 000	1,646	1,950	2,116	2,228	2,312	2,380	2,436	2,483	2,525	2,561	2,699
∞	1,645	1,949	2,115	2,227	2,311	2,379	2,434	2,482	2,523	2,560	2,697

Table D.1 — Two-sided prediction interval factors, k, at confidence level 90 % for known population standard deviation *(continued)*

n	m										
	20	30	40	50	60	80	100	150	200	250	500
2	3,341	3,487	3,587	3,663	3,723	3,817	3,888	4,014	4,101	4,167	4,365
3	3,182	3,324	3,422	3,496	3,555	3,647	3,717	3,840	3,925	3,990	4,186
4	3,095	3,235	3,331	3,404	3,462	3,553	3,622	3,743	3,828	3,892	4,085
5	3,040	3,179	3,273	3,345	3,403	3,493	3,561	3,681	3,765	3,828	4,020
6	3,002	3,139	3,233	3,305	3,362	3,451	3,518	3,638	3,721	3,784	3,974
7	2,975	3,110	3,204	3,275	3,332	3,420	3,487	3,606	3,688	3,751	3,940
8	2,953	3,088	3,181	3,252	3,308	3,396	3,463	3,581	3,663	3,725	3,913
9	2,936	3,071	3,163	3,234	3,290	3,377	3,444	3,562	3,643	3,705	3,893
10	2,923	3,057	3,149	3,219	3,275	3,362	3,428	3,546	3,627	3,689	3,875
11	2,911	3,045	3,137	3,207	3,263	3,349	3,415	3,532	3,613	3,675	3,861
12	2,902	3,035	3,127	3,196	3,252	3,339	3,404	3,521	3,602	3,664	3,849
13	2,894	3,027	3,118	3,187	3,243	3,329	3,395	3,512	3,592	3,654	3,839
14	2,887	3,019	3,111	3,180	3,235	3,322	3,387	3,503	3,584	3,645	3,830
15	2,881	3,013	3,104	3,173	3,229	3,315	3,380	3,496	3,576	3,638	3,822
16	2,875	3,007	3,098	3,167	3,223	3,309	3,374	3,490	3,570	3,631	3,815
17	2,871	3,002	3,093	3,162	3,217	3,303	3,368	3,484	3,564	3,625	3,809
18	2,866	2,998	3,089	3,158	3,213	3,298	3,363	3,479	3,559	3,620	3,804
19	2,863	2,994	3,085	3,153	3,209	3,294	3,359	3,475	3,554	3,615	3,799
20	2,859	2,991	3,081	3,150	3,205	3,290	3,355	3,470	3,550	3,611	3,795
25	2,846	2,977	3,067	3,135	3,190	3,275	3,340	3,455	3,534	3,595	3,778
30	2,837	2,968	3,057	3,125	3,180	3,265	3,330	3,444	3,523	3,584	3,766
35	2,831	2,961	3,050	3,118	3,173	3,258	3,322	3,436	3,516	3,576	3,758
40	2,826	2,956	3,045	3,113	3,168	3,252	3,317	3,431	3,510	3,570	3,752
45	2,822	2,952	3,041	3,109	3,164	3,248	3,312	3,426	3,505	3,565	3,747
50	2,819	2,949	3,038	3,106	3,160	3,245	3,309	3,423	3,501	3,561	3,743
60	2,815	2,944	3,033	3,101	3,155	3,239	3,303	3,417	3,496	3,556	3,737
70	2,811	2,941	3,030	3,097	3,152	3,236	3,300	3,413	3,492	3,552	3,732
80	2,809	2,938	3,027	3,095	3,149	3,233	3,297	3,410	3,489	3,549	3,729
90	2,807	2,936	3,025	3,092	3,147	3,231	3,294	3,408	3,486	3,546	3,727
100	2,805	2,934	3,023	3,091	3,145	3,229	3,293	3,406	3,484	3,544	3,725
150	2,801	2,930	3,018	3,086	3,140	3,224	3,287	3,400	3,479	3,539	3,719
200	2,798	2,927	3,016	3,083	3,137	3,221	3,285	3,398	3,476	3,536	3,716
250	2,797	2,926	3,014	3,082	3,136	3,219	3,283	3,396	3,474	3,534	3,714
300	2,796	2,925	3,013	3,081	3,135	3,218	3,282	3,395	3,473	3,533	3,712
350	2,796	2,924	3,013	3,080	3,134	3,218	3,281	3,394	3,472	3,532	3,712
400	2,795	2,924	3,012	3,079	3,133	3,217	3,281	3,393	3,472	3,531	3,711
450	2,795	2,923	3,012	3,079	3,133	3,216	3,280	3,393	3,471	3,531	3,710
500	2,794	2,923	3,011	3,079	3,133	3,216	3,280	3,393	3,471	3,530	3,710
600	2,794	2,922	3,011	3,078	3,132	3,216	3,279	3,392	3,470	3,530	3,709
700	2,794	2,922	3,011	3,078	3,132	3,215	3,279	3,392	3,470	3,529	3,709
800	2,793	2,922	3,010	3,077	3,131	3,215	3,279	3,391	3,469	3,529	3,709
900	2,793	2,922	3,010	3,077	3,131	3,215	3,278	3,391	3,469	3,529	3,708
1 000	2,793	2,921	3,010	3,077	3,131	3,215	3,278	3,391	3,469	3,529	3,708
∞	2,792	2,920	3,008	3,076	3,129	3,213	3,276	3,389	3,467	3,527	3,706

Table D.1 — Two-sided prediction interval factors, k, at confidence level 90 % for known population standard deviation *(continued)*

n	m									
	1 000	2 000	5 000	10 000	20 000	50 000	100 000	200 000	500 000	1 000 000
2	4,555	4,735	4,963	5,129	5,288	5,491	5,639	5,783	5,967	6,102
3	4,372	4,551	4,776	4,939	5,097	5,299	5,446	5,588	5,771	5,906
4	4,269	4,446	4,670	4,832	4,989	5,188	5,334	5,477	5,659	5,793
5	4,202	4,378	4,600	4,761	4,917	5,116	5,261	5,403	5,584	5,717
6	4,155	4,330	4,551	4,711	4,866	5,064	5,209	5,350	5,531	5,664
7	4,121	4,294	4,514	4,674	4,828	5,026	5,170	5,310	5,491	5,623
8	4,093	4,266	4,485	4,645	4,799	4,995	5,139	5,279	5,459	5,591
9	4,072	4,244	4,463	4,621	4,775	4,971	5,114	5,254	5,433	5,565
10	4,054	4,226	4,444	4,602	4,755	4,951	5,094	5,234	5,412	5,544
11	4,040	4,211	4,428	4,586	4,739	4,934	5,077	5,216	5,395	5,526
12	4,027	4,198	4,415	4,572	4,725	4,920	5,063	5,201	5,380	5,511
13	4,017	4,187	4,403	4,561	4,713	4,908	5,050	5,189	5,367	5,498
14	4,007	4,178	4,394	4,551	4,703	4,897	5,039	5,178	5,356	5,486
15	3,999	4,169	4,385	4,542	4,694	4,888	5,030	5,168	5,346	5,476
16	3,992	4,162	4,377	4,534	4,686	4,879	5,021	5,159	5,337	5,467
17	3,986	4,156	4,371	4,527	4,678	4,872	5,014	5,152	5,329	5,459
18	3,980	4,150	4,364	4,521	4,672	4,865	5,007	5,145	5,322	5,452
19	3,975	4,144	4,359	4,515	4,666	4,859	5,001	5,139	5,316	5,446
20	3,971	4,140	4,354	4,510	4,661	4,854	4,995	5,133	5,310	5,440
25	3,953	4,121	4,335	4,490	4,641	4,833	4,974	5,111	5,287	5,417
30	3,941	4,109	4,322	4,477	4,627	4,819	4,959	5,096	5,272	5,401
35	3,932	4,100	4,312	4,467	4,617	4,808	4,949	5,085	5,261	5,390
40	3,926	4,093	4,305	4,460	4,609	4,801	4,941	5,077	5,252	5,381
45	3,921	4,088	4,300	4,454	4,603	4,794	4,934	5,071	5,246	5,375
50	3,916	4,083	4,295	4,449	4,599	4,789	4,929	5,065	5,240	5,369
60	3,910	4,077	4,288	4,442	4,591	4,782	4,922	5,057	5,232	5,361
70	3,906	4,072	4,284	4,437	4,586	4,777	4,916	5,052	5,226	5,355
80	3,902	4,069	4,280	4,433	4,582	4,772	4,912	5,048	5,222	5,350
90	3,900	4,066	4,277	4,430	4,579	4,769	4,909	5,044	5,218	5,347
100	3,898	4,064	4,275	4,428	4,577	4,767	4,906	5,041	5,216	5,344
150	3,891	4,057	4,268	4,421	4,569	4,759	4,898	5,033	5,207	5,335
200	3,888	4,054	4,264	4,417	4,566	4,755	4,894	5,029	5,203	5,331
250	3,886	4,052	4,262	4,415	4,563	4,753	4,892	5,027	5,200	5,328
300	3,885	4,051	4,261	4,414	4,562	4,751	4,890	5,025	5,199	5,327
350	3,884	4,050	4,260	4,413	4,561	4,750	4,889	5,024	5,198	5,325
400	3,883	4,049	4,259	4,412	4,560	4,749	4,888	5,023	5,197	5,324
450	3,883	4,048	4,258	4,411	4,559	4,749	4,887	5,022	5,196	5,324
500	3,882	4,048	4,258	4,411	4,559	4,748	4,887	5,022	5,195	5,323
600	3,882	4,047	4,257	4,410	4,558	4,747	4,886	5,021	5,194	5,322
700	3,881	4,047	4,257	4,410	4,558	4,747	4,885	5,020	5,194	5,322
800	3,881	4,046	4,256	4,409	4,557	4,746	4,885	5,020	5,193	5,321
900	3,881	4,046	4,256	4,409	4,557	4,746	4,885	5,020	5,193	5,321
1 000	3,880	4,046	4,256	4,409	4,557	4,746	4,884	5,019	5,193	5,320
∞	3,878	4,044	4,254	4,406	4,554	4,743	4,882	5,017	5,190	5,318

NOTE This table provides factors k such that one may be at least 90 % confident that none of the next m observations from a normally distributed population will lie outside the range $(\bar{x} - k\sigma, \bar{x} + k\sigma)$, where $\bar{x}$ is derived from a random sample of size n from the same population.

Table D.2 — Two-sided prediction interval factors, k, at confidence level 95 % for known population standard deviation

n	m										
	1	2	3	4	5	6	7	8	9	10	15
2	2,401	2,727	2,906	3,027	3,118	3,191	3,252	3,303	3,348	3,388	3,537
3	2,264	2,577	2,748	2,864	2,952	3,022	3,081	3,130	3,174	3,212	3,356
4	2,192	2,497	2,664	2,778	2,864	2,932	2,989	3,038	3,080	3,118	3,259
5	2,148	2,448	2,612	2,724	2,808	2,876	2,932	2,980	3,022	3,059	3,198
6	2,118	2,414	2,577	2,687	2,771	2,838	2,893	2,940	2,982	3,018	3,156
7	2,096	2,390	2,551	2,661	2,743	2,810	2,865	2,912	2,953	2,989	3,125
8	2,079	2,372	2,531	2,640	2,723	2,788	2,843	2,890	2,930	2,966	3,102
9	2,066	2,357	2,516	2,624	2,706	2,772	2,826	2,872	2,913	2,949	3,083
10	2,056	2,345	2,504	2,612	2,693	2,758	2,812	2,858	2,899	2,934	3,068
11	2,048	2,336	2,493	2,601	2,682	2,747	2,801	2,847	2,887	2,923	3,056
12	2,040	2,328	2,485	2,592	2,673	2,738	2,791	2,837	2,877	2,913	3,046
13	2,034	2,321	2,478	2,585	2,665	2,730	2,783	2,829	2,869	2,904	3,037
14	2,029	2,315	2,471	2,578	2,659	2,723	2,776	2,822	2,862	2,897	3,030
15	2,025	2,310	2,466	2,572	2,653	2,717	2,770	2,816	2,856	2,891	3,023
16	2,021	2,306	2,461	2,567	2,648	2,712	2,765	2,811	2,850	2,885	3,017
17	2,017	2,302	2,457	2,563	2,643	2,707	2,760	2,806	2,845	2,880	3,012
18	2,014	2,298	2,453	2,559	2,639	2,703	2,756	2,802	2,841	2,876	3,008
19	2,011	2,295	2,450	2,556	2,635	2,699	2,752	2,798	2,837	2,872	3,004
20	2,009	2,292	2,447	2,553	2,632	2,696	2,749	2,794	2,834	2,869	3,000
25	1,999	2,281	2,435	2,540	2,620	2,683	2,736	2,781	2,820	2,855	2,986
30	1,993	2,274	2,428	2,532	2,612	2,675	2,727	2,772	2,811	2,846	2,976
35	1,988	2,269	2,422	2,527	2,606	2,669	2,721	2,766	2,805	2,840	2,970
40	1,985	2,265	2,418	2,522	2,601	2,664	2,716	2,761	2,800	2,835	2,964
45	1,982	2,262	2,415	2,519	2,598	2,661	2,713	2,758	2,796	2,831	2,960
50	1,980	2,259	2,412	2,516	2,595	2,658	2,710	2,755	2,793	2,828	2,957
60	1,977	2,256	2,408	2,512	2,591	2,653	2,706	2,750	2,789	2,823	2,953
70	1,974	2,253	2,405	2,509	2,588	2,650	2,702	2,747	2,786	2,820	2,949
80	1,973	2,251	2,403	2,507	2,585	2,648	2,700	2,744	2,783	2,818	2,946
90	1,971	2,249	2,401	2,505	2,583	2,646	2,698	2,743	2,781	2,816	2,944
100	1,970	2,248	2,400	2,504	2,582	2,645	2,697	2,741	2,780	2,814	2,943
150	1,967	2,244	2,396	2,500	2,578	2,640	2,692	2,737	2,775	2,809	2,938
200	1,965	2,243	2,394	2,498	2,576	2,638	2,690	2,734	2,773	2,807	2,936
250	1,964	2,241	2,393	2,496	2,574	2,637	2,689	2,733	2,772	2,806	2,934
300	1,964	2,241	2,392	2,496	2,574	2,636	2,688	2,732	2,771	2,805	2,933
350	1,963	2,240	2,392	2,495	2,573	2,635	2,687	2,731	2,770	2,804	2,932
400	1,963	2,240	2,391	2,495	2,572	2,635	2,687	2,731	2,769	2,804	2,932
450	1,963	2,239	2,391	2,494	2,572	2,634	2,686	2,731	2,769	2,803	2,932
500	1,962	2,239	2,391	2,494	2,572	2,634	2,686	2,730	2,769	2,803	2,931
600	1,962	2,239	2,390	2,493	2,571	2,634	2,686	2,730	2,768	2,802	2,931
700	1,962	2,239	2,390	2,493	2,571	2,633	2,685	2,729	2,768	2,802	2,930
800	1,962	2,238	2,390	2,493	2,571	2,633	2,685	2,729	2,768	2,802	2,930
900	1,962	2,238	2,390	2,493	2,571	2,633	2,685	2,729	2,768	2,802	2,930
1 000	1,961	2,238	2,389	2,493	2,571	2,633	2,685	2,729	2,767	2,802	2,930
∞	1,960	2,237	2,388	2,491	2,569	2,632	2,683	2,728	2,766	2,800	2,928

Table D.2 — Two-sided prediction interval factors, k, at confidence level 95 % for known population standard deviation *(continued)*

n	m										
	20	30	40	50	60	80	100	150	200	250	500
2	3,640	3,779	3,876	3,948	4,007	4,098	4,166	4,289	4,373	4,437	4,631
3	3,455	3,591	3,684	3,755	3,812	3,900	3,967	4,086	4,169	4,231	4,421
4	3,356	3,488	3,580	3,649	3,705	3,791	3,857	3,975	4,056	4,118	4,304
5	3,293	3,424	3,514	3,582	3,637	3,723	3,788	3,904	3,984	4,045	4,230
6	3,250	3,379	3,468	3,536	3,591	3,675	3,740	3,854	3,934	3,995	4,178
7	3,219	3,347	3,435	3,503	3,557	3,641	3,705	3,818	3,897	3,957	4,139
8	3,195	3,322	3,410	3,477	3,531	3,614	3,678	3,791	3,869	3,929	4,110
9	3,176	3,303	3,390	3,457	3,510	3,593	3,656	3,769	3,847	3,906	4,086
10	3,161	3,287	3,374	3,440	3,493	3,576	3,639	3,751	3,829	3,888	4,067
11	3,148	3,274	3,361	3,427	3,480	3,562	3,625	3,736	3,814	3,873	4,052
12	3,137	3,263	3,349	3,415	3,468	3,550	3,613	3,724	3,801	3,860	4,039
13	3,128	3,253	3,340	3,405	3,458	3,540	3,603	3,714	3,791	3,850	4,027
14	3,121	3,245	3,332	3,397	3,450	3,531	3,594	3,705	3,782	3,840	4,018
15	3,114	3,238	3,324	3,390	3,442	3,524	3,586	3,697	3,773	3,832	4,009
16	3,108	3,232	3,318	3,383	3,436	3,517	3,579	3,690	3,766	3,825	4,002
17	3,103	3,227	3,313	3,378	3,430	3,511	3,573	3,684	3,760	3,819	3,995
18	3,098	3,222	3,308	3,373	3,425	3,506	3,568	3,678	3,755	3,813	3,989
19	3,094	3,218	3,303	3,368	3,420	3,501	3,563	3,673	3,750	3,808	3,984
20	3,090	3,214	3,299	3,364	3,416	3,497	3,559	3,669	3,745	3,803	3,979
25	3,076	3,199	3,284	3,348	3,400	3,481	3,542	3,652	3,728	3,786	3,961
30	3,066	3,189	3,273	3,338	3,390	3,470	3,531	3,640	3,716	3,774	3,948
35	3,059	3,181	3,266	3,330	3,382	3,462	3,523	3,632	3,708	3,765	3,939
40	3,054	3,176	3,260	3,324	3,376	3,456	3,517	3,626	3,701	3,759	3,933
45	3,050	3,172	3,256	3,320	3,372	3,452	3,513	3,621	3,696	3,754	3,927
50	3,046	3,168	3,252	3,316	3,368	3,448	3,509	3,617	3,692	3,750	3,923
60	3,041	3,163	3,247	3,311	3,363	3,442	3,503	3,611	3,686	3,744	3,917
70	3,038	3,159	3,243	3,307	3,359	3,438	3,499	3,607	3,682	3,739	3,912
80	3,035	3,157	3,241	3,304	3,356	3,435	3,496	3,604	3,679	3,736	3,909
90	3,033	3,155	3,238	3,302	3,353	3,433	3,494	3,602	3,676	3,734	3,906
100	3,032	3,153	3,237	3,300	3,352	3,431	3,492	3,600	3,674	3,732	3,904
150	3,027	3,148	3,231	3,295	3,346	3,425	3,486	3,594	3,668	3,725	3,898
200	3,024	3,145	3,229	3,292	3,343	3,423	3,483	3,591	3,665	3,722	3,895
250	3,023	3,144	3,227	3,291	3,342	3,421	3,481	3,589	3,664	3,720	3,893
300	3,022	3,142	3,226	3,289	3,341	3,420	3,480	3,588	3,662	3,719	3,891
350	3,021	3,142	3,225	3,289	3,340	3,419	3,479	3,587	3,661	3,718	3,890
400	3,020	3,141	3,225	3,288	3,339	3,418	3,479	3,586	3,661	3,718	3,890
450	3,020	3,141	3,224	3,288	3,339	3,418	3,478	3,586	3,660	3,717	3,889
500	3,020	3,140	3,224	3,287	3,338	3,418	3,478	3,585	3,660	3,717	3,889
600	3,019	3,140	3,223	3,287	3,338	3,417	3,477	3,585	3,659	3,716	3,888
700	3,019	3,139	3,223	3,286	3,337	3,417	3,477	3,584	3,659	3,716	3,888
800	3,018	3,139	3,223	3,286	3,337	3,416	3,477	3,584	3,659	3,715	3,887
900	3,018	3,139	3,222	3,286	3,337	3,416	3,476	3,584	3,658	3,715	3,887
1 000	3,018	3,139	3,222	3,286	3,337	3,416	3,476	3,584	3,658	3,715	3,887
∞	3,016	3,137	3,221	3,284	3,335	3,414	3,474	3,582	3,656	3,713	3,885

Table D.2 — Two-sided prediction interval factors, k, at confidence level 95 % for known population standard deviation *(continued)*

n	m									
	1 000	2 000	5 000	10 000	20 000	50 000	100 000	200 000	500 000	1 000 000
2	4,816	4,994	5,218	5,381	5,538	5,739	5,885	6,028	6,210	6,345
3	4,603	4,777	4,998	5,159	5,314	5,512	5,657	5,798	5,979	6,113
4	4,483	4,656	4,874	5,033	5,187	5,383	5,527	5,667	5,847	5,979
5	4,407	4,578	4,794	4,952	5,104	5,299	5,442	5,582	5,760	5,892
6	4,353	4,523	4,738	4,894	5,046	5,240	5,382	5,521	5,699	5,830
7	4,314	4,482	4,696	4,852	5,003	5,196	5,337	5,475	5,652	5,783
8	4,284	4,451	4,664	4,819	4,969	5,161	5,302	5,440	5,616	5,746
9	4,259	4,426	4,638	4,792	4,942	5,134	5,274	5,411	5,587	5,717
10	4,240	4,406	4,617	4,771	4,920	5,111	5,251	5,388	5,563	5,693
11	4,224	4,389	4,600	4,753	4,902	5,092	5,232	5,368	5,543	5,672
12	4,210	4,375	4,585	4,738	4,886	5,077	5,216	5,352	5,527	5,655
13	4,198	4,363	4,572	4,725	4,873	5,063	5,202	5,338	5,512	5,641
14	4,188	4,353	4,562	4,714	4,862	5,051	5,190	5,325	5,500	5,628
15	4,179	4,343	4,552	4,704	4,852	5,041	5,180	5,315	5,489	5,617
16	4,172	4,335	4,544	4,696	4,843	5,032	5,170	5,305	5,479	5,607
17	4,165	4,328	4,536	4,688	4,835	5,024	5,162	5,297	5,470	5,598
18	4,159	4,322	4,530	4,681	4,828	5,016	5,155	5,289	5,463	5,590
19	4,153	4,316	4,524	4,675	4,822	5,010	5,148	5,282	5,456	5,583
20	4,148	4,311	4,518	4,670	4,816	5,004	5,142	5,276	5,449	5,577
25	4,129	4,291	4,498	4,648	4,794	4,982	5,119	5,253	5,425	5,552
30	4,116	4,278	4,484	4,634	4,780	4,966	5,103	5,237	5,408	5,535
35	4,107	4,268	4,474	4,624	4,769	4,955	5,092	5,225	5,397	5,523
40	4,100	4,261	4,466	4,616	4,761	4,947	5,083	5,216	5,387	5,514
45	4,094	4,256	4,460	4,610	4,755	4,940	5,077	5,209	5,380	5,506
50	4,090	4,251	4,456	4,605	4,750	4,935	5,071	5,204	5,375	5,501
60	4,084	4,244	4,448	4,597	4,742	4,927	5,063	5,196	5,366	5,492
70	4,079	4,239	4,443	4,592	4,737	4,922	5,057	5,190	5,360	5,486
80	4,075	4,236	4,439	4,588	4,733	4,917	5,053	5,185	5,355	5,481
90	4,072	4,233	4,436	4,585	4,729	4,914	5,050	5,182	5,352	5,477
100	4,070	4,230	4,434	4,583	4,727	4,911	5,047	5,179	5,349	5,474
150	4,064	4,223	4,427	4,575	4,719	4,903	5,039	5,170	5,340	5,465
200	4,060	4,220	4,423	4,571	4,715	4,899	5,034	5,166	5,336	5,461
250	4,058	4,218	4,421	4,569	4,713	4,897	5,032	5,164	5,333	5,458
300	4,057	4,217	4,419	4,568	4,711	4,895	5,030	5,162	5,331	5,456
350	4,056	4,216	4,418	4,566	4,710	4,894	5,029	5,161	5,330	5,455
400	4,055	4,215	4,418	4,566	4,709	4,893	5,028	5,160	5,329	5,454
450	4,055	4,214	4,417	4,565	4,709	4,893	5,027	5,159	5,328	5,453
500	4,054	4,214	4,417	4,564	4,708	4,892	5,027	5,159	5,328	5,453
600	4,054	4,213	4,416	4,564	4,707	4,891	5,026	5,158	5,327	5,452
700	4,053	4,213	4,415	4,563	4,707	4,891	5,025	5,157	5,326	5,451
800	4,053	4,212	4,415	4,563	4,706	4,890	5,025	5,157	5,326	5,451
900	4,052	4,212	4,415	4,562	4,706	4,890	5,025	5,156	5,326	5,450
1 000	4,052	4,212	4,414	4,562	4,706	4,890	5,024	5,156	5,325	5,450
∞	4,050	4,210	4,412	4,560	4,703	4,887	5,022	5,153	5,323	5,447

NOTE This table provides factors k such that one may be at least 95 % confident that none of the next m observations from a normally distributed population will lie outside the range $(\bar{x} - k\sigma, \bar{x} + k\sigma)$, where $\bar{x}$ is derived from a random sample of size n from the same population.

Table D.3 — Two-sided prediction interval factors, k, at confidence level 97,5 % for known population standard deviation

n	m										
	1	2	3	4	5	6	7	8	9	10	15
2	2,746	3,048	3,215	3,329	3,415	3,484	3,541	3,590	3,633	3,671	3,813
3	2,589	2,878	3,037	3,146	3,228	3,294	3,349	3,396	3,437	3,474	3,610
4	2,506	2,788	2,943	3,049	3,129	3,194	3,247	3,293	3,333	3,369	3,502
5	2,456	2,732	2,885	2,989	3,068	3,131	3,184	3,229	3,268	3,303	3,434
6	2,421	2,695	2,845	2,948	3,026	3,089	3,141	3,185	3,224	3,258	3,388
7	2,397	2,667	2,816	2,918	2,996	3,058	3,109	3,153	3,192	3,226	3,354
8	2,378	2,647	2,795	2,896	2,973	3,034	3,085	3,129	3,167	3,201	3,329
9	2,363	2,630	2,778	2,878	2,955	3,016	3,067	3,110	3,148	3,182	3,309
10	2,351	2,617	2,764	2,864	2,940	3,001	3,052	3,095	3,133	3,166	3,293
11	2,342	2,607	2,752	2,852	2,928	2,989	3,039	3,082	3,120	3,153	3,279
12	2,333	2,598	2,743	2,843	2,918	2,978	3,029	3,072	3,109	3,143	3,268
13	2,327	2,590	2,735	2,834	2,909	2,970	3,020	3,063	3,100	3,133	3,259
14	2,321	2,583	2,728	2,827	2,902	2,962	3,012	3,055	3,092	3,126	3,250
15	2,315	2,578	2,722	2,821	2,896	2,956	3,006	3,048	3,086	3,119	3,243
16	2,311	2,573	2,717	2,815	2,890	2,950	3,000	3,043	3,080	3,113	3,237
17	2,307	2,568	2,712	2,811	2,885	2,945	2,995	3,037	3,074	3,107	3,231
18	2,303	2,564	2,708	2,806	2,881	2,940	2,990	3,033	3,070	3,103	3,227
19	2,300	2,561	2,704	2,802	2,877	2,936	2,986	3,029	3,066	3,098	3,222
20	2,297	2,557	2,701	2,799	2,873	2,933	2,982	3,025	3,062	3,095	3,218
25	2,286	2,545	2,688	2,786	2,860	2,919	2,968	3,010	3,047	3,080	3,203
30	2,279	2,537	2,679	2,777	2,850	2,910	2,959	3,001	3,038	3,070	3,193
35	2,274	2,531	2,673	2,770	2,844	2,903	2,952	2,994	3,031	3,063	3,185
40	2,270	2,527	2,669	2,766	2,839	2,898	2,947	2,989	3,025	3,058	3,180
45	2,267	2,524	2,665	2,762	2,835	2,894	2,943	2,985	3,021	3,054	3,176
50	2,264	2,521	2,662	2,759	2,832	2,891	2,940	2,982	3,018	3,050	3,172
60	2,261	2,517	2,658	2,754	2,828	2,886	2,935	2,977	3,013	3,045	3,167
70	2,258	2,514	2,655	2,751	2,824	2,883	2,932	2,973	3,010	3,042	3,163
80	2,256	2,512	2,652	2,749	2,822	2,880	2,929	2,971	3,007	3,039	3,161
90	2,254	2,510	2,650	2,747	2,820	2,878	2,927	2,969	3,005	3,037	3,158
100	2,253	2,508	2,649	2,745	2,818	2,877	2,925	2,967	3,003	3,035	3,157
150	2,249	2,504	2,645	2,741	2,814	2,872	2,921	2,962	2,998	3,030	3,151
200	2,247	2,502	2,642	2,739	2,811	2,870	2,918	2,960	2,996	3,028	3,149
250	2,246	2,501	2,641	2,737	2,810	2,868	2,917	2,958	2,994	3,026	3,147
300	2,246	2,500	2,640	2,736	2,809	2,867	2,916	2,957	2,993	3,025	3,146
350	2,245	2,500	2,640	2,736	2,808	2,867	2,915	2,956	2,993	3,025	3,146
400	2,245	2,499	2,639	2,735	2,808	2,866	2,914	2,956	2,992	3,024	3,145
450	2,244	2,499	2,639	2,735	2,807	2,866	2,914	2,956	2,992	3,024	3,145
500	2,244	2,498	2,639	2,734	2,807	2,865	2,914	2,955	2,991	3,023	3,144
600	2,244	2,498	2,638	2,734	2,807	2,865	2,913	2,955	2,991	3,023	3,144
700	2,244	2,498	2,638	2,734	2,806	2,864	2,913	2,954	2,991	3,023	3,143
800	2,243	2,498	2,638	2,733	2,806	2,864	2,913	2,954	2,990	3,022	3,143
900	2,243	2,497	2,637	2,733	2,806	2,864	2,912	2,954	2,990	3,022	3,143
1 000	2,243	2,497	2,637	2,733	2,806	2,864	2,912	2,954	2,990	3,022	3,143
∞	2,242	2,496	2,636	2,732	2,804	2,862	2,911	2,952	2,988	3,020	3,141

Table D.3 — Two-sided prediction interval factors, k, at confidence level 97,5 % for known population standard deviation *(continued)*

n	m										
	20	30	40	50	60	80	100	150	200	250	500
2	3,911	4,045	4,137	4,208	4,264	4,352	4,419	4,537	4,620	4,682	4,871
3	3,705	3,834	3,923	3,991	4,046	4,131	4,195	4,311	4,391	4,451	4,636
4	3,594	3,720	3,807	3,874	3,927	4,011	4,074	4,187	4,265	4,325	4,506
5	3,525	3,649	3,735	3,800	3,853	3,935	3,997	4,109	4,186	4,245	4,424
6	3,478	3,600	3,685	3,750	3,802	3,883	3,945	4,055	4,131	4,190	4,366
7	3,443	3,565	3,649	3,713	3,765	3,845	3,906	4,015	4,091	4,149	4,325
8	3,417	3,538	3,621	3,685	3,736	3,816	3,877	3,985	4,061	4,118	4,293
9	3,396	3,516	3,600	3,663	3,714	3,793	3,854	3,962	4,037	4,094	4,267
10	3,380	3,499	3,582	3,645	3,696	3,775	3,835	3,943	4,017	4,074	4,247
11	3,366	3,485	3,568	3,630	3,681	3,760	3,820	3,927	4,001	4,058	4,230
12	3,355	3,473	3,556	3,618	3,669	3,747	3,807	3,914	3,988	4,044	4,216
13	3,345	3,463	3,545	3,608	3,658	3,736	3,796	3,902	3,976	4,033	4,204
14	3,336	3,455	3,536	3,599	3,649	3,727	3,786	3,893	3,966	4,023	4,193
15	3,329	3,447	3,529	3,591	3,641	3,719	3,778	3,884	3,958	4,014	4,184
16	3,323	3,440	3,522	3,584	3,634	3,712	3,771	3,877	3,950	4,006	4,176
17	3,317	3,435	3,516	3,578	3,628	3,705	3,765	3,870	3,943	3,999	4,169
18	3,312	3,429	3,511	3,572	3,622	3,700	3,759	3,864	3,937	3,993	4,163
19	3,307	3,425	3,506	3,568	3,617	3,695	3,754	3,859	3,932	3,988	4,157
20	3,303	3,420	3,501	3,563	3,613	3,690	3,749	3,854	3,927	3,983	4,152
25	3,288	3,404	3,485	3,546	3,596	3,673	3,732	3,836	3,909	3,964	4,133
30	3,277	3,394	3,474	3,535	3,584	3,661	3,720	3,824	3,896	3,952	4,120
35	3,270	3,386	3,466	3,527	3,576	3,653	3,711	3,815	3,888	3,943	4,110
40	3,264	3,380	3,460	3,521	3,570	3,647	3,705	3,809	3,881	3,936	4,103
45	3,260	3,375	3,455	3,516	3,565	3,642	3,700	3,804	3,876	3,931	4,098
50	3,256	3,372	3,452	3,512	3,561	3,638	3,696	3,799	3,872	3,927	4,093
60	3,251	3,366	3,446	3,507	3,556	3,632	3,690	3,793	3,865	3,920	4,087
70	3,247	3,362	3,442	3,503	3,552	3,628	3,686	3,789	3,861	3,916	4,082
80	3,244	3,359	3,439	3,500	3,548	3,624	3,682	3,786	3,857	3,912	4,078
90	3,242	3,357	3,437	3,497	3,546	3,622	3,680	3,783	3,855	3,910	4,076
100	3,240	3,355	3,435	3,495	3,544	3,620	3,678	3,781	3,853	3,907	4,073
150	3,235	3,350	3,429	3,489	3,538	3,614	3,672	3,775	3,846	3,901	4,067
200	3,232	3,347	3,426	3,487	3,535	3,611	3,669	3,772	3,843	3,898	4,063
250	3,231	3,345	3,424	3,485	3,534	3,609	3,667	3,770	3,841	3,896	4,061
300	3,230	3,344	3,423	3,484	3,532	3,608	3,666	3,768	3,840	3,895	4,060
350	3,229	3,343	3,423	3,483	3,532	3,607	3,665	3,768	3,839	3,894	4,059
400	3,228	3,343	3,422	3,482	3,531	3,606	3,664	3,767	3,838	3,893	4,058
450	3,228	3,342	3,421	3,482	3,530	3,606	3,664	3,766	3,838	3,892	4,058
500	3,228	3,342	3,421	3,481	3,530	3,606	3,663	3,766	3,837	3,892	4,057
600	3,227	3,341	3,421	3,481	3,529	3,605	3,663	3,765	3,837	3,891	4,057
700	3,227	3,341	3,420	3,480	3,529	3,605	3,662	3,765	3,836	3,891	4,056
800	3,226	3,341	3,420	3,480	3,529	3,604	3,662	3,765	3,836	3,890	4,056
900	3,226	3,340	3,420	3,480	3,528	3,604	3,662	3,764	3,836	3,890	4,055
1 000	3,226	3,340	3,419	3,480	3,528	3,604	3,661	3,764	3,835	3,890	4,055
∞	3,224	3,339	3,418	3,478	3,527	3,602	3,660	3,762	3,834	3,888	4,053

Table D.3 — Two-sided prediction interval factors, k, at confidence level 97,5 % for known population standard deviation *(continued)*

n	m									
	1 000	2 000	5 000	10 000	20 000	50 000	100 000	200 000	500 000	1 000 000
2	5,053	5,227	5,448	5,608	5,763	5,962	6,107	6,248	6,429	6,562
3	4,813	4,983	5,200	5,358	5,511	5,706	5,849	5,988	6,168	6,299
4	4,680	4,848	5,062	5,217	5,368	5,561	5,703	5,841	6,019	6,149
5	4,596	4,762	4,973	5,127	5,276	5,468	5,608	5,745	5,921	6,051
6	4,537	4,701	4,911	5,063	5,212	5,402	5,542	5,678	5,853	5,982
7	4,494	4,657	4,865	5,017	5,164	5,353	5,492	5,627	5,801	5,930
8	4,461	4,623	4,830	4,981	5,127	5,315	5,454	5,588	5,762	5,890
9	4,434	4,596	4,802	4,952	5,098	5,285	5,423	5,557	5,730	5,857
10	4,413	4,574	4,779	4,929	5,074	5,261	5,398	5,532	5,704	5,831
11	4,396	4,556	4,760	4,910	5,055	5,241	5,377	5,511	5,682	5,809
12	4,381	4,541	4,745	4,894	5,038	5,224	5,360	5,493	5,664	5,791
13	4,369	4,528	4,731	4,880	5,024	5,209	5,345	5,478	5,649	5,775
14	4,358	4,517	4,720	4,868	5,012	5,196	5,332	5,465	5,635	5,761
15	4,348	4,507	4,710	4,857	5,001	5,185	5,321	5,453	5,623	5,749
16	4,340	4,499	4,701	4,848	4,992	5,176	5,311	5,443	5,613	5,738
17	4,333	4,491	4,693	4,840	4,983	5,167	5,302	5,434	5,604	5,729
18	4,326	4,484	4,686	4,833	4,976	5,160	5,294	5,426	5,596	5,721
19	4,321	4,478	4,679	4,826	4,969	5,153	5,287	5,419	5,588	5,713
20	4,315	4,473	4,674	4,821	4,963	5,146	5,281	5,412	5,581	5,706
25	4,295	4,452	4,652	4,798	4,940	5,123	5,257	5,387	5,556	5,680
30	4,282	4,438	4,637	4,783	4,925	5,107	5,240	5,371	5,538	5,662
35	4,272	4,428	4,627	4,772	4,914	5,095	5,228	5,358	5,526	5,650
40	4,264	4,420	4,619	4,764	4,905	5,086	5,219	5,349	5,517	5,640
45	4,259	4,414	4,613	4,758	4,899	5,080	5,212	5,342	5,509	5,633
50	4,254	4,410	4,608	4,753	4,893	5,074	5,207	5,336	5,503	5,627
60	4,247	4,403	4,600	4,745	4,886	5,066	5,198	5,328	5,494	5,617
70	4,242	4,397	4,595	4,739	4,880	5,060	5,192	5,322	5,488	5,611
80	4,239	4,394	4,591	4,735	4,876	5,056	5,188	5,317	5,483	5,606
90	4,236	4,391	4,588	4,732	4,872	5,052	5,184	5,313	5,480	5,602
100	4,233	4,388	4,585	4,729	4,870	5,049	5,181	5,310	5,477	5,599
150	4,226	4,381	4,578	4,722	4,862	5,041	5,173	5,302	5,468	5,590
200	4,223	4,377	4,574	4,718	4,858	5,037	5,169	5,297	5,463	5,585
250	4,221	4,375	4,572	4,715	4,855	5,034	5,166	5,295	5,460	5,583
300	4,219	4,374	4,570	4,714	4,854	5,033	5,164	5,293	5,459	5,581
350	4,218	4,373	4,569	4,713	4,852	5,032	5,163	5,292	5,457	5,580
400	4,218	4,372	4,568	4,712	4,852	5,031	5,162	5,291	5,456	5,579
450	4,217	4,371	4,568	4,711	4,851	5,030	5,162	5,290	5,456	5,578
500	4,217	4,371	4,567	4,711	4,850	5,029	5,161	5,289	5,455	5,577
600	4,216	4,370	4,566	4,710	4,850	5,029	5,160	5,289	5,454	5,576
700	4,215	4,370	4,566	4,709	4,849	5,028	5,160	5,288	5,453	5,576
800	4,215	4,369	4,565	4,709	4,849	5,028	5,159	5,288	5,453	5,575
900	4,215	4,369	4,565	4,709	4,848	5,027	5,159	5,287	5,453	5,575
1 000	4,215	4,369	4,565	4,708	4,848	5,027	5,158	5,287	5,452	5,574
∞	4,212	4,366	4,563	4,706	4,846	5,024	5,156	5,284	5,450	5,572

NOTE This table provides factors k such that one may be at least 97,5 % confident that none of the next m observations from a normally distributed population will lie outside the range $(\bar{x} - k\sigma, \bar{x} + k\sigma)$, where $\bar{x}$ is derived from a random sample of size n from the same population.

Table D.4 — Two-sided prediction interval factors, k, at confidence level 99 % for known population standard deviation

n	m										
	1	2	3	4	5	6	7	8	9	10	15
2	3,155	3,432	3,586	3,692	3,772	3,836	3,890	3,936	3,976	4,012	4,146
3	2,975	3,238	3,385	3,485	3,562	3,623	3,674	3,718	3,757	3,791	3,919
4	2,880	3,136	3,279	3,377	3,451	3,511	3,560	3,603	3,640	3,673	3,798
5	2,822	3,074	3,213	3,309	3,382	3,441	3,490	3,532	3,568	3,601	3,723
6	2,783	3,031	3,169	3,264	3,336	3,393	3,442	3,483	3,519	3,551	3,672
7	2,754	3,000	3,136	3,230	3,302	3,359	3,407	3,448	3,484	3,515	3,635
8	2,733	2,977	3,112	3,205	3,276	3,333	3,380	3,421	3,457	3,488	3,607
9	2,716	2,958	3,093	3,186	3,256	3,312	3,360	3,400	3,435	3,467	3,585
10	2,702	2,943	3,077	3,170	3,240	3,296	3,343	3,383	3,418	3,450	3,567
11	2,691	2,931	3,065	3,157	3,226	3,282	3,329	3,369	3,404	3,435	3,553
12	2,682	2,921	3,054	3,146	3,215	3,271	3,318	3,358	3,393	3,424	3,540
13	2,674	2,912	3,045	3,136	3,206	3,262	3,308	3,348	3,383	3,413	3,530
14	2,667	2,905	3,037	3,129	3,198	3,253	3,300	3,339	3,374	3,405	3,521
15	2,661	2,899	3,031	3,122	3,191	3,246	3,292	3,332	3,367	3,397	3,513
16	2,656	2,893	3,025	3,116	3,184	3,240	3,286	3,325	3,360	3,391	3,506
17	2,651	2,888	3,020	3,110	3,179	3,234	3,280	3,320	3,354	3,385	3,500
18	2,647	2,884	3,015	3,105	3,174	3,229	3,275	3,315	3,349	3,380	3,495
19	2,643	2,880	3,011	3,101	3,170	3,225	3,271	3,310	3,344	3,375	3,490
20	2,640	2,876	3,007	3,097	3,166	3,221	3,267	3,306	3,340	3,371	3,486
25	2,627	2,862	2,993	3,082	3,151	3,205	3,251	3,290	3,324	3,355	3,469
30	2,619	2,853	2,983	3,073	3,141	3,195	3,241	3,280	3,314	3,344	3,458
35	2,613	2,847	2,976	3,066	3,133	3,188	3,233	3,272	3,306	3,336	3,450
40	2,608	2,842	2,971	3,060	3,128	3,182	3,228	3,266	3,300	3,331	3,444
45	2,605	2,838	2,967	3,056	3,124	3,178	3,223	3,262	3,296	3,326	3,440
50	2,602	2,835	2,964	3,053	3,120	3,175	3,220	3,259	3,292	3,322	3,436
60	2,598	2,830	2,959	3,048	3,115	3,169	3,215	3,253	3,287	3,317	3,430
70	2,595	2,827	2,956	3,044	3,112	3,166	3,211	3,249	3,283	3,313	3,426
80	2,592	2,824	2,953	3,042	3,109	3,163	3,208	3,247	3,280	3,310	3,423
90	2,591	2,822	2,951	3,039	3,107	3,161	3,206	3,244	3,278	3,308	3,421
100	2,589	2,821	2,949	3,038	3,105	3,159	3,204	3,243	3,276	3,306	3,419
150	2,585	2,816	2,944	3,033	3,100	3,154	3,199	3,237	3,271	3,301	3,413
200	2,583	2,814	2,942	3,030	3,097	3,151	3,196	3,235	3,268	3,298	3,411
250	2,581	2,812	2,941	3,029	3,096	3,150	3,194	3,233	3,267	3,296	3,409
300	2,581	2,811	2,940	3,028	3,095	3,148	3,193	3,232	3,265	3,295	3,408
350	2,580	2,811	2,939	3,027	3,094	3,148	3,193	3,231	3,265	3,294	3,407
400	2,580	2,810	2,938	3,026	3,093	3,147	3,192	3,230	3,264	3,294	3,406
450	2,579	2,810	2,938	3,026	3,093	3,147	3,192	3,230	3,264	3,293	3,406
500	2,579	2,810	2,938	3,026	3,093	3,146	3,191	3,230	3,263	3,293	3,406
600	2,578	2,809	2,937	3,025	3,092	3,146	3,191	3,229	3,263	3,292	3,405
700	2,578	2,809	2,937	3,025	3,092	3,145	3,190	3,229	3,262	3,292	3,405
800	2,578	2,808	2,936	3,025	3,091	3,145	3,190	3,228	3,262	3,292	3,404
900	2,578	2,808	2,936	3,024	3,091	3,145	3,190	3,228	3,262	3,292	3,404
1 000	2,578	2,808	2,936	3,024	3,091	3,145	3,190	3,228	3,262	3,291	3,404
∞	2,576	2,807	2,935	3,023	3,090	3,143	3,188	3,226	3,260	3,290	3,402

Table D.4 — Two-sided prediction interval factors, k, at confidence level 99 % for known population standard deviation *(continued)*

n	m										
	20	30	40	50	60	80	100	150	200	250	500
2	4,239	4,366	4,454	4,522	4,576	4,660	4,724	4,838	4,918	4,978	5,162
3	4,008	4,130	4,215	4,279	4,331	4,412	4,474	4,584	4,661	4,720	4,897
4	3,885	4,004	4,086	4,149	4,200	4,279	4,339	4,447	4,522	4,579	4,753
5	3,808	3,925	4,006	4,068	4,118	4,196	4,255	4,361	4,435	4,491	4,662
6	3,756	3,872	3,952	4,013	4,062	4,139	4,197	4,302	4,375	4,431	4,600
7	3,718	3,833	3,912	3,972	4,021	4,097	4,155	4,259	4,331	4,387	4,554
8	3,690	3,803	3,882	3,942	3,990	4,066	4,124	4,227	4,298	4,353	4,520
9	3,667	3,780	3,858	3,918	3,966	4,041	4,098	4,201	4,272	4,327	4,492
10	3,649	3,761	3,839	3,898	3,946	4,021	4,078	4,180	4,251	4,305	4,470
11	3,634	3,746	3,823	3,883	3,930	4,005	4,062	4,163	4,234	4,288	4,452
12	3,621	3,733	3,810	3,869	3,917	3,991	4,048	4,149	4,219	4,273	4,437
13	3,611	3,722	3,799	3,858	3,905	3,979	4,036	4,137	4,207	4,261	4,424
14	3,602	3,712	3,789	3,848	3,896	3,969	4,026	4,126	4,196	4,250	4,413
15	3,594	3,704	3,781	3,840	3,887	3,961	4,017	4,117	4,187	4,241	4,403
16	3,587	3,697	3,774	3,832	3,879	3,953	4,009	4,109	4,179	4,233	4,395
17	3,581	3,691	3,767	3,826	3,873	3,946	4,002	4,102	4,172	4,225	4,387
18	3,575	3,685	3,761	3,820	3,867	3,940	3,996	4,096	4,166	4,219	4,381
19	3,570	3,680	3,756	3,815	3,861	3,935	3,991	4,090	4,160	4,213	4,374
20	3,566	3,675	3,752	3,810	3,857	3,930	3,986	4,085	4,155	4,208	4,369
25	3,549	3,658	3,734	3,792	3,838	3,911	3,967	4,066	4,135	4,188	4,348
30	3,537	3,646	3,722	3,780	3,826	3,899	3,954	4,053	4,122	4,175	4,335
35	3,529	3,638	3,713	3,771	3,817	3,890	3,945	4,044	4,112	4,165	4,325
40	3,523	3,632	3,707	3,764	3,811	3,883	3,938	4,037	4,105	4,158	4,317
45	3,518	3,627	3,702	3,759	3,806	3,878	3,933	4,031	4,100	4,152	4,311
50	3,515	3,623	3,698	3,755	3,801	3,873	3,929	4,027	4,095	4,148	4,307
60	3,509	3,617	3,692	3,749	3,795	3,867	3,922	4,020	4,089	4,141	4,300
70	3,505	3,613	3,688	3,745	3,791	3,863	3,918	4,016	4,084	4,136	4,295
80	3,502	3,609	3,684	3,741	3,788	3,859	3,914	4,012	4,080	4,132	4,291
90	3,499	3,607	3,682	3,739	3,785	3,857	3,911	4,009	4,077	4,130	4,288
100	3,497	3,605	3,680	3,737	3,783	3,855	3,909	4,007	4,075	4,127	4,286
150	3,492	3,599	3,674	3,731	3,777	3,848	3,903	4,000	4,068	4,120	4,278
200	3,489	3,596	3,671	3,728	3,773	3,845	3,900	3,997	4,065	4,117	4,275
250	3,487	3,594	3,669	3,726	3,772	3,843	3,898	3,995	4,063	4,115	4,273
300	3,486	3,593	3,668	3,724	3,770	3,842	3,896	3,994	4,062	4,114	4,271
350	3,485	3,592	3,667	3,724	3,769	3,841	3,895	3,993	4,061	4,113	4,270
400	3,484	3,592	3,666	3,723	3,769	3,840	3,895	3,992	4,060	4,112	4,270
450	3,484	3,591	3,666	3,722	3,768	3,840	3,894	3,992	4,059	4,111	4,269
500	3,483	3,591	3,665	3,722	3,768	3,839	3,894	3,991	4,059	4,111	4,269
600	3,483	3,590	3,665	3,721	3,767	3,839	3,893	3,991	4,058	4,110	4,268
700	3,482	3,590	3,664	3,721	3,767	3,838	3,893	3,990	4,058	4,110	4,267
800	3,482	3,589	3,664	3,721	3,766	3,838	3,892	3,990	4,057	4,109	4,267
900	3,482	3,589	3,664	3,720	3,766	3,838	3,892	3,989	4,057	4,109	4,267
1 000	3,482	3,589	3,663	3,720	3,766	3,837	3,892	3,989	4,057	4,109	4,266
∞	3,480	3,587	3,662	3,718	3,764	3,835	3,890	3,987	4,055	4,107	4,264

Table D.4 — Two-sided prediction interval factors, k, at confidence level 99 % for known population standard deviation *(continued)*

n	m									
	1 000	2 000	5 000	10 000	20 000	50 000	100 000	200 000	500 000	1 000 000
2	5,338	5,508	5,724	5,882	6,034	6,230	6,373	6,512	6,691	6,823
3	5,069	5,234	5,445	5,599	5,749	5,940	6,081	6,218	6,394	6,524
4	4,921	5,084	5,291	5,442	5,590	5,778	5,917	6,053	6,227	6,355
5	4,828	4,988	5,193	5,342	5,488	5,674	5,812	5,946	6,118	6,246
6	4,764	4,922	5,124	5,273	5,417	5,602	5,738	5,871	6,042	6,169
7	4,717	4,874	5,074	5,221	5,365	5,548	5,683	5,816	5,986	6,112
8	4,681	4,837	5,036	5,182	5,325	5,507	5,642	5,773	5,942	6,067
9	4,653	4,808	5,006	5,152	5,293	5,475	5,609	5,739	5,908	6,032
10	4,630	4,784	4,982	5,127	5,267	5,448	5,582	5,712	5,880	6,004
11	4,611	4,765	4,962	5,106	5,246	5,427	5,559	5,689	5,857	5,980
12	4,595	4,749	4,945	5,089	5,229	5,408	5,541	5,670	5,837	5,960
13	4,582	4,735	4,931	5,074	5,213	5,393	5,525	5,654	5,820	5,943
14	4,570	4,723	4,918	5,061	5,200	5,379	5,511	5,640	5,806	5,929
15	4,560	4,713	4,908	5,050	5,189	5,368	5,499	5,628	5,793	5,916
16	4,552	4,704	4,898	5,040	5,179	5,357	5,489	5,617	5,782	5,905
17	4,544	4,696	4,890	5,032	5,170	5,348	5,479	5,607	5,773	5,895
18	4,537	4,689	4,882	5,024	5,163	5,340	5,471	5,599	5,764	5,886
19	4,531	4,682	4,876	5,017	5,155	5,333	5,464	5,591	5,756	5,878
20	4,525	4,676	4,870	5,011	5,149	5,326	5,457	5,584	5,749	5,871
25	4,504	4,654	4,847	4,988	5,125	5,301	5,431	5,558	5,722	5,843
30	4,489	4,639	4,831	4,972	5,109	5,284	5,414	5,541	5,704	5,825
35	4,479	4,629	4,820	4,960	5,097	5,272	5,402	5,528	5,691	5,811
40	4,471	4,621	4,812	4,952	5,088	5,263	5,392	5,518	5,681	5,801
45	4,465	4,615	4,805	4,945	5,081	5,256	5,385	5,511	5,673	5,793
50	4,460	4,610	4,800	4,940	5,076	5,250	5,379	5,505	5,667	5,787
60	4,453	4,602	4,792	4,932	5,067	5,242	5,370	5,496	5,658	5,778
70	4,448	4,597	4,787	4,926	5,062	5,236	5,364	5,490	5,651	5,771
80	4,444	4,593	4,782	4,922	5,057	5,231	5,359	5,485	5,647	5,766
90	4,441	4,589	4,779	4,918	5,054	5,228	5,356	5,481	5,643	5,762
100	4,439	4,587	4,777	4,916	5,051	5,225	5,353	5,478	5,640	5,759
150	4,431	4,579	4,769	4,907	5,043	5,216	5,344	5,469	5,630	5,749
200	4,428	4,576	4,765	4,903	5,038	5,212	5,340	5,465	5,626	5,745
250	4,425	4,573	4,762	4,901	5,036	5,209	5,337	5,462	5,623	5,742
300	4,424	4,572	4,761	4,899	5,034	5,208	5,335	5,460	5,621	5,740
350	4,423	4,571	4,760	4,898	5,033	5,206	5,334	5,459	5,620	5,739
400	4,422	4,570	4,759	4,897	5,032	5,205	5,333	5,458	5,619	5,738
450	4,421	4,569	4,758	4,897	5,031	5,205	5,332	5,457	5,618	5,737
500	4,421	4,569	4,758	4,896	5,031	5,204	5,332	5,456	5,617	5,736
600	4,420	4,568	4,757	4,895	5,030	5,203	5,331	5,455	5,616	5,735
700	4,420	4,567	4,756	4,895	5,029	5,203	5,330	5,455	5,616	5,734
800	4,419	4,567	4,756	4,894	5,029	5,202	5,330	5,454	5,615	5,734
900	4,419	4,567	4,756	4,894	5,029	5,202	5,329	5,454	5,615	5,734
1 000	4,419	4,567	4,755	4,894	5,028	5,202	5,329	5,454	5,614	5,733
∞	4,417	4,564	4,753	4,891	5,026	5,199	5,326	5,451	5,612	5,730

NOTE This table provides factors k such that one may be at least 99 % confident that none of the next m observations from a normally distributed population will lie outside the range $(\bar{x} - k\sigma, \bar{x} + k\sigma)$, where $\bar{x}$ is derived from a random sample of size n from the same population.

Table D.5 — Two-sided prediction interval factors, k, at confidence level 99,5 % for known population standard deviation

n	m										
	1	2	3	4	5	6	7	8	9	10	15
2	3,438	3,699	3,845	3,945	4,022	4,083	4,134	4,178	4,217	4,251	4,380
3	3,242	3,489	3,628	3,723	3,796	3,854	3,903	3,945	3,981	4,014	4,137
4	3,139	3,379	3,514	3,606	3,677	3,733	3,781	3,821	3,857	3,889	4,008
5	3,075	3,311	3,443	3,534	3,603	3,659	3,705	3,745	3,780	3,811	3,928
6	3,032	3,265	3,395	3,485	3,553	3,608	3,654	3,693	3,728	3,758	3,874
7	3,001	3,232	3,361	3,449	3,517	3,571	3,617	3,656	3,690	3,720	3,834
8	2,978	3,207	3,334	3,422	3,489	3,543	3,589	3,627	3,661	3,691	3,805
9	2,959	3,187	3,314	3,401	3,468	3,522	3,566	3,605	3,638	3,668	3,781
10	2,945	3,171	3,297	3,384	3,451	3,504	3,549	3,587	3,620	3,650	3,762
11	2,932	3,158	3,284	3,370	3,436	3,490	3,534	3,572	3,605	3,635	3,747
12	2,922	3,147	3,272	3,359	3,425	3,478	3,522	3,560	3,593	3,622	3,734
13	2,913	3,137	3,263	3,349	3,414	3,467	3,511	3,549	3,582	3,612	3,723
14	2,906	3,129	3,254	3,340	3,406	3,458	3,502	3,540	3,573	3,603	3,713
15	2,900	3,123	3,247	3,333	3,398	3,451	3,495	3,532	3,565	3,595	3,705
16	2,894	3,116	3,241	3,326	3,392	3,444	3,488	3,526	3,558	3,588	3,698
17	2,889	3,111	3,235	3,321	3,386	3,438	3,482	3,519	3,552	3,581	3,692
18	2,884	3,106	3,230	3,316	3,381	3,433	3,477	3,514	3,547	3,576	3,686
19	2,880	3,102	3,226	3,311	3,376	3,428	3,472	3,509	3,542	3,571	3,681
20	2,877	3,098	3,222	3,307	3,372	3,424	3,467	3,505	3,537	3,566	3,676
25	2,863	3,083	3,206	3,291	3,356	3,407	3,451	3,488	3,521	3,549	3,659
30	2,854	3,073	3,196	3,280	3,345	3,397	3,440	3,477	3,509	3,538	3,647
35	2,847	3,066	3,189	3,273	3,337	3,389	3,432	3,469	3,501	3,530	3,639
40	2,842	3,061	3,183	3,267	3,331	3,383	3,426	3,463	3,495	3,524	3,632
45	2,839	3,057	3,179	3,263	3,327	3,378	3,421	3,458	3,490	3,519	3,627
50	2,835	3,054	3,175	3,259	3,323	3,375	3,418	3,454	3,487	3,515	3,623
60	2,831	3,049	3,170	3,254	3,318	3,369	3,412	3,449	3,481	3,510	3,618
70	2,828	3,045	3,166	3,250	3,314	3,365	3,408	3,445	3,477	3,505	3,613
80	2,825	3,042	3,164	3,247	3,311	3,362	3,405	3,442	3,474	3,502	3,610
90	2,823	3,040	3,161	3,245	3,309	3,360	3,403	3,439	3,471	3,500	3,608
100	2,822	3,039	3,160	3,243	3,307	3,358	3,401	3,437	3,470	3,498	3,606
150	2,817	3,034	3,154	3,238	3,301	3,353	3,395	3,432	3,464	3,492	3,600
200	2,815	3,031	3,152	3,235	3,299	3,350	3,392	3,429	3,461	3,489	3,597
250	2,813	3,030	3,150	3,234	3,297	3,348	3,391	3,427	3,459	3,488	3,595
300	2,812	3,028	3,149	3,233	3,296	3,347	3,390	3,426	3,458	3,486	3,594
350	2,812	3,028	3,148	3,232	3,295	3,346	3,389	3,425	3,457	3,486	3,593
400	2,811	3,027	3,148	3,231	3,295	3,346	3,388	3,425	3,457	3,485	3,592
450	2,811	3,027	3,147	3,231	3,294	3,345	3,388	3,424	3,456	3,485	3,592
500	2,810	3,026	3,147	3,230	3,294	3,345	3,387	3,424	3,456	3,484	3,591
600	2,810	3,026	3,147	3,230	3,293	3,344	3,387	3,423	3,455	3,484	3,591
700	2,810	3,026	3,146	3,229	3,293	3,344	3,386	3,423	3,455	3,483	3,590
800	2,809	3,025	3,146	3,229	3,293	3,343	3,386	3,423	3,454	3,483	3,590
900	2,809	3,025	3,146	3,229	3,292	3,343	3,386	3,422	3,454	3,483	3,590
1 000	2,809	3,025	3,146	3,229	3,292	3,343	3,386	3,422	3,454	3,482	3,590
∞	2,808	3,023	3,144	3,227	3,290	3,341	3,384	3,420	3,452	3,481	3,588

Table D.5 — Two-sided prediction interval factors, k, at confidence level 99,5 % for known population standard deviation *(continued)*

n	m										
	20	30	40	50	60	80	100	150	200	250	500
2	4,469	4,592	4,677	4,742	4,795	4,876	4,939	5,050	5,127	5,186	5,366
3	4,222	4,339	4,421	4,483	4,533	4,611	4,671	4,778	4,853	4,910	5,083
4	4,091	4,205	4,284	4,345	4,394	4,470	4,528	4,632	4,705	4,760	4,929
5	4,009	4,121	4,199	4,259	4,307	4,382	4,439	4,541	4,613	4,667	4,833
6	3,954	4,065	4,141	4,200	4,248	4,321	4,378	4,479	4,549	4,603	4,767
7	3,914	4,023	4,099	4,158	4,205	4,278	4,334	4,434	4,504	4,557	4,720
8	3,883	3,992	4,068	4,125	4,172	4,245	4,300	4,400	4,469	4,522	4,683
9	3,860	3,968	4,043	4,100	4,146	4,219	4,274	4,373	4,441	4,494	4,655
10	3,840	3,948	4,023	4,080	4,126	4,198	4,253	4,351	4,419	4,472	4,632
11	3,824	3,932	4,006	4,063	4,109	4,180	4,235	4,333	4,401	4,454	4,613
12	3,811	3,918	3,992	4,049	4,095	4,166	4,221	4,318	4,386	4,438	4,597
13	3,800	3,906	3,980	4,037	4,083	4,154	4,208	4,305	4,373	4,425	4,583
14	3,790	3,897	3,970	4,027	4,072	4,143	4,197	4,294	4,362	4,414	4,572
15	3,782	3,888	3,961	4,018	4,063	4,134	4,188	4,285	4,353	4,404	4,561
16	3,775	3,880	3,954	4,010	4,055	4,126	4,180	4,277	4,344	4,396	4,553
17	3,768	3,874	3,947	4,003	4,048	4,119	4,173	4,269	4,337	4,388	4,545
18	3,762	3,868	3,941	3,997	4,042	4,113	4,166	4,263	4,330	4,381	4,538
19	3,757	3,862	3,935	3,991	4,037	4,107	4,161	4,257	4,324	4,375	4,532
20	3,752	3,858	3,931	3,986	4,031	4,102	4,155	4,252	4,319	4,370	4,526
25	3,735	3,839	3,912	3,967	4,012	4,082	4,136	4,231	4,298	4,349	4,504
30	3,723	3,827	3,899	3,955	3,999	4,069	4,122	4,218	4,284	4,335	4,490
35	3,714	3,818	3,890	3,946	3,990	4,060	4,113	4,208	4,274	4,325	4,480
40	3,708	3,811	3,884	3,939	3,983	4,053	4,106	4,201	4,267	4,318	4,472
45	3,703	3,806	3,878	3,933	3,978	4,047	4,100	4,195	4,261	4,312	4,466
50	3,699	3,802	3,874	3,929	3,974	4,043	4,096	4,191	4,257	4,307	4,461
60	3,693	3,796	3,868	3,923	3,967	4,036	4,089	4,184	4,250	4,300	4,454
70	3,688	3,791	3,863	3,918	3,962	4,032	4,084	4,179	4,245	4,295	4,449
80	3,685	3,788	3,860	3,915	3,959	4,028	4,081	4,175	4,241	4,291	4,445
90	3,682	3,786	3,857	3,912	3,956	4,025	4,078	4,172	4,238	4,288	4,442
100	3,680	3,783	3,855	3,910	3,954	4,023	4,076	4,170	4,236	4,286	4,439
150	3,674	3,777	3,849	3,903	3,948	4,016	4,069	4,163	4,229	4,279	4,432
200	3,671	3,774	3,846	3,900	3,944	4,013	4,066	4,160	4,225	4,275	4,428
250	3,669	3,772	3,844	3,898	3,942	4,011	4,064	4,158	4,223	4,273	4,426
300	3,668	3,771	3,842	3,897	3,941	4,010	4,062	4,156	4,222	4,272	4,424
350	3,667	3,770	3,841	3,896	3,940	4,009	4,061	4,155	4,221	4,271	4,423
400	3,667	3,769	3,841	3,895	3,939	4,008	4,061	4,155	4,220	4,270	4,423
450	3,666	3,769	3,840	3,895	3,939	4,008	4,060	4,154	4,219	4,270	4,422
500	3,666	3,768	3,840	3,894	3,938	4,007	4,060	4,153	4,219	4,269	4,422
600	3,665	3,768	3,839	3,894	3,938	4,006	4,059	4,153	4,218	4,268	4,421
700	3,665	3,767	3,839	3,893	3,937	4,006	4,058	4,152	4,218	4,268	4,420
800	3,664	3,767	3,838	3,893	3,937	4,006	4,058	4,152	4,217	4,267	4,420
900	3,664	3,767	3,838	3,893	3,937	4,005	4,058	4,152	4,217	4,267	4,420
1 000	3,664	3,767	3,838	3,892	3,936	4,005	4,058	4,151	4,217	4,267	4,419
∞	3,662	3,765	3,836	3,890	3,935	4,003	4,056	4,149	4,215	4,265	4,417

Table D.5 — Two-sided prediction interval factors, k, at confidence level 99,5 % for known population standard deviation *(continued)*

n	m									
	1 000	2 000	5 000	10 000	20 000	50 000	100 000	200 000	500 000	1 000 000
2	5,538	5,705	5,918	6,073	6,224	6,417	6,558	6,696	6,874	7,005
3	5,250	5,412	5,618	5,770	5,917	6,105	6,244	6,379	6,554	6,682
4	5,093	5,251	5,454	5,602	5,747	5,932	6,069	6,202	6,374	6,501
5	4,994	5,150	5,350	5,496	5,639	5,822	5,957	6,088	6,259	6,384
6	4,926	5,081	5,278	5,423	5,564	5,745	5,879	6,009	6,178	6,302
7	4,877	5,030	5,225	5,369	5,509	5,689	5,821	5,951	6,118	6,242
8	4,840	4,991	5,186	5,328	5,467	5,646	5,777	5,906	6,073	6,196
9	4,810	4,961	5,154	5,296	5,434	5,612	5,743	5,871	6,036	6,159
10	4,786	4,936	5,129	5,270	5,407	5,584	5,715	5,842	6,007	6,129
11	4,767	4,916	5,108	5,248	5,385	5,562	5,692	5,819	5,983	6,104
12	4,750	4,899	5,090	5,230	5,367	5,543	5,672	5,799	5,963	6,084
13	4,736	4,885	5,075	5,215	5,351	5,527	5,656	5,782	5,945	6,066
14	4,724	4,873	5,063	5,202	5,338	5,513	5,641	5,768	5,930	6,051
15	4,714	4,862	5,051	5,190	5,326	5,501	5,629	5,755	5,917	6,038
16	4,705	4,852	5,042	5,180	5,316	5,490	5,618	5,744	5,906	6,026
17	4,697	4,844	5,033	5,171	5,307	5,480	5,609	5,734	5,896	6,016
18	4,689	4,837	5,025	5,164	5,298	5,472	5,600	5,725	5,887	6,007
19	4,683	4,830	5,018	5,156	5,291	5,465	5,592	5,717	5,879	5,998
20	4,677	4,824	5,012	5,150	5,285	5,458	5,585	5,710	5,872	5,991
25	4,655	4,801	4,988	5,126	5,260	5,432	5,559	5,683	5,844	5,963
30	4,640	4,786	4,972	5,109	5,243	5,415	5,541	5,665	5,825	5,944
35	4,629	4,775	4,961	5,097	5,231	5,402	5,529	5,652	5,812	5,930
40	4,621	4,767	4,952	5,089	5,222	5,393	5,519	5,642	5,802	5,920
45	4,615	4,760	4,946	5,082	5,215	5,386	5,511	5,635	5,794	5,912
50	4,610	4,755	4,940	5,076	5,209	5,380	5,506	5,629	5,788	5,905
60	4,603	4,747	4,932	5,068	5,200	5,371	5,497	5,620	5,778	5,896
70	4,597	4,742	4,926	5,062	5,194	5,365	5,490	5,613	5,772	5,889
80	4,593	4,737	4,922	5,058	5,190	5,360	5,485	5,608	5,766	5,884
90	4,590	4,734	4,919	5,054	5,186	5,356	5,482	5,604	5,763	5,880
100	4,588	4,732	4,916	5,051	5,183	5,353	5,479	5,601	5,759	5,876
150	4,580	4,724	4,908	5,043	5,175	5,344	5,469	5,592	5,750	5,867
200	4,576	4,720	4,904	5,039	5,171	5,340	5,465	5,587	5,745	5,862
250	4,574	4,718	4,901	5,036	5,168	5,337	5,462	5,584	5,742	5,859
300	4,572	4,716	4,900	5,035	5,166	5,336	5,460	5,583	5,740	5,857
350	4,571	4,715	4,899	5,034	5,165	5,334	5,459	5,581	5,739	5,856
400	4,570	4,714	4,898	5,033	5,164	5,333	5,458	5,580	5,738	5,855
450	4,570	4,713	4,897	5,032	5,163	5,333	5,457	5,580	5,737	5,854
500	4,569	4,713	4,897	5,031	5,163	5,332	5,457	5,579	5,737	5,853
600	4,569	4,712	4,896	5,031	5,162	5,331	5,456	5,578	5,736	5,852
700	4,568	4,711	4,895	5,030	5,161	5,331	5,455	5,577	5,735	5,851
800	4,568	4,711	4,895	5,029	5,161	5,330	5,455	5,577	5,734	5,851
900	4,567	4,711	4,894	5,029	5,161	5,330	5,454	5,576	5,734	5,851
1 000	4,567	4,710	4,894	5,029	5,160	5,329	5,454	5,576	5,734	5,850
∞	4,565	4,708	4,892	5,026	5,158	5,327	5,451	5,573	5,731	5,847

NOTE This table provides factors k such that one may be at least 99,5 % confident that none of the next m observations from a normally distributed population will lie outside the range $(\bar{x} - k\sigma, \bar{x} + k\sigma)$, where $\bar{x}$ is derived from a random sample of size n from the same population.

Table D.6 — Two-sided prediction interval factors, k, at confidence level 99,9 % for known population standard deviation

n	*m*										
	1	2	3	4	5	6	7	8	9	10	15
2	4,031	4,262	4,392	4,483	4,552	4,608	4,654	4,694	4,729	4,760	4,879
3	3,800	4,019	4,142	4,228	4,293	4,346	4,390	4,428	4,461	4,491	4,603
4	3,679	3,892	4,011	4,094	4,158	4,209	4,252	4,288	4,321	4,349	4,458
5	3,605	3,813	3,930	4,012	4,074	4,124	4,166	4,202	4,234	4,262	4,368
6	3,555	3,760	3,876	3,956	4,017	4,067	4,108	4,143	4,175	4,202	4,307
7	3,518	3,721	3,836	3,915	3,976	4,025	4,066	4,101	4,132	4,159	4,263
8	3,491	3,692	3,806	3,885	3,945	3,993	4,034	4,069	4,100	4,127	4,230
9	3,469	3,669	3,782	3,861	3,920	3,969	4,009	4,044	4,074	4,101	4,204
10	3,452	3,651	3,763	3,841	3,901	3,949	3,989	4,024	4,054	4,081	4,183
11	3,437	3,636	3,748	3,825	3,885	3,933	3,973	4,007	4,037	4,064	4,165
12	3,425	3,623	3,735	3,812	3,871	3,919	3,959	3,993	4,023	4,050	4,151
13	3,415	3,613	3,724	3,801	3,860	3,907	3,947	3,981	4,011	4,038	4,139
14	3,407	3,603	3,714	3,791	3,850	3,897	3,937	3,971	4,001	4,027	4,128
15	3,399	3,595	3,706	3,783	3,841	3,889	3,928	3,962	3,992	4,019	4,119
16	3,392	3,588	3,699	3,775	3,834	3,881	3,921	3,955	3,984	4,011	4,111
17	3,386	3,582	3,692	3,769	3,827	3,874	3,914	3,948	3,977	4,004	4,104
18	3,381	3,577	3,687	3,763	3,821	3,868	3,908	3,942	3,971	3,998	4,097
19	3,377	3,572	3,682	3,758	3,816	3,863	3,902	3,936	3,966	3,992	4,092
20	3,372	3,567	3,677	3,753	3,811	3,858	3,897	3,931	3,961	3,987	4,087
25	3,356	3,550	3,659	3,735	3,793	3,840	3,879	3,912	3,942	3,968	4,067
30	3,345	3,539	3,648	3,723	3,781	3,827	3,866	3,900	3,929	3,955	4,054
35	3,338	3,531	3,639	3,715	3,772	3,819	3,857	3,891	3,920	3,946	4,045
40	3,332	3,524	3,633	3,708	3,766	3,812	3,851	3,884	3,913	3,939	4,038
45	3,327	3,520	3,628	3,703	3,761	3,807	3,846	3,879	3,908	3,934	4,032
50	3,324	3,516	3,624	3,699	3,756	3,803	3,841	3,875	3,904	3,930	4,028
60	3,318	3,510	3,618	3,693	3,750	3,796	3,835	3,868	3,897	3,923	4,021
70	3,314	3,506	3,614	3,689	3,746	3,792	3,831	3,864	3,893	3,919	4,017
80	3,312	3,503	3,611	3,685	3,743	3,789	3,827	3,860	3,889	3,915	4,013
90	3,309	3,500	3,608	3,683	3,740	3,786	3,825	3,858	3,887	3,913	4,010
100	3,307	3,499	3,606	3,681	3,738	3,784	3,823	3,856	3,885	3,910	4,008
150	3,302	3,493	3,600	3,675	3,732	3,778	3,816	3,849	3,878	3,904	4,002
200	3,299	3,490	3,597	3,672	3,729	3,775	3,813	3,846	3,875	3,901	3,998
250	3,298	3,488	3,595	3,670	3,727	3,773	3,811	3,844	3,873	3,899	3,996
300	3,297	3,487	3,594	3,669	3,726	3,771	3,810	3,843	3,872	3,897	3,995
350	3,296	3,486	3,593	3,668	3,725	3,771	3,809	3,842	3,871	3,897	3,994
400	3,295	3,486	3,593	3,667	3,724	3,770	3,808	3,841	3,870	3,896	3,993
450	3,295	3,485	3,592	3,667	3,724	3,769	3,808	3,841	3,870	3,895	3,993
500	3,294	3,485	3,592	3,666	3,723	3,769	3,807	3,840	3,869	3,895	3,992
600	3,294	3,484	3,591	3,666	3,723	3,768	3,807	3,840	3,869	3,894	3,992
700	3,293	3,484	3,591	3,665	3,722	3,768	3,806	3,839	3,868	3,894	3,991
800	3,293	3,483	3,591	3,665	3,722	3,768	3,806	3,839	3,868	3,893	3,991
900	3,293	3,483	3,590	3,665	3,721	3,767	3,806	3,839	3,867	3,893	3,990
1 000	3,293	3,483	3,590	3,664	3,721	3,767	3,805	3,838	3,867	3,893	3,990
∞	3,291	3,481	3,588	3,663	3,719	3,765	3,804	3,836	3,865	3,891	3,988

Table D.6 — Two-sided prediction interval factors, k, at confidence level 99,9 % for known population standard deviation *(continued)*

n	m										
	20	30	40	50	60	80	100	150	200	250	500
2	4,961	5,074	5,154	5,214	5,263	5,339	5,398	5,502	5,575	5,631	5,801
3	4,681	4,789	4,864	4,922	4,968	5,041	5,097	5,196	5,266	5,319	5,482
4	4,533	4,638	4,711	4,767	4,812	4,883	4,937	5,034	5,102	5,153	5,312
5	4,442	4,545	4,617	4,671	4,716	4,785	4,838	4,933	4,999	5,050	5,206
6	4,380	4,482	4,552	4,606	4,650	4,718	4,771	4,865	4,930	4,980	5,134
7	4,336	4,436	4,506	4,559	4,603	4,670	4,722	4,815	4,880	4,930	5,081
8	4,302	4,401	4,471	4,524	4,567	4,634	4,685	4,777	4,842	4,891	5,042
9	4,275	4,374	4,443	4,496	4,538	4,605	4,656	4,748	4,812	4,861	5,011
10	4,254	4,352	4,421	4,473	4,516	4,582	4,633	4,724	4,788	4,837	4,986
11	4,236	4,334	4,402	4,455	4,497	4,563	4,614	4,705	4,768	4,817	4,965
12	4,222	4,319	4,387	4,439	4,481	4,547	4,598	4,688	4,751	4,800	4,948
13	4,209	4,306	4,374	4,426	4,468	4,534	4,584	4,674	4,737	4,786	4,933
14	4,198	4,295	4,363	4,415	4,457	4,522	4,572	4,662	4,725	4,773	4,921
15	4,189	4,286	4,353	4,405	4,447	4,512	4,562	4,652	4,715	4,763	4,910
16	4,181	4,277	4,345	4,396	4,438	4,503	4,553	4,643	4,706	4,754	4,900
17	4,174	4,270	4,337	4,389	4,431	4,496	4,546	4,635	4,697	4,745	4,892
18	4,167	4,263	4,331	4,382	4,424	4,489	4,539	4,628	4,690	4,738	4,884
19	4,161	4,258	4,325	4,376	4,418	4,483	4,532	4,621	4,684	4,732	4,877
20	4,156	4,252	4,319	4,371	4,412	4,477	4,527	4,616	4,678	4,726	4,871
25	4,136	4,232	4,299	4,350	4,391	4,456	4,505	4,594	4,656	4,703	4,848
30	4,123	4,218	4,285	4,336	4,377	4,441	4,491	4,579	4,641	4,688	4,832
35	4,114	4,209	4,275	4,326	4,367	4,431	4,480	4,568	4,630	4,677	4,821
40	4,106	4,201	4,268	4,318	4,359	4,423	4,472	4,560	4,622	4,669	4,813
45	4,101	4,196	4,262	4,312	4,353	4,417	4,466	4,554	4,616	4,663	4,806
50	4,096	4,191	4,257	4,308	4,349	4,413	4,462	4,549	4,611	4,658	4,801
60	4,090	4,184	4,250	4,301	4,342	4,405	4,454	4,542	4,603	4,650	4,793
70	4,085	4,179	4,245	4,296	4,336	4,400	4,449	4,537	4,598	4,645	4,788
80	4,081	4,176	4,241	4,292	4,333	4,396	4,445	4,533	4,594	4,641	4,783
90	4,078	4,173	4,239	4,289	4,330	4,393	4,442	4,529	4,590	4,637	4,780
100	4,076	4,170	4,236	4,287	4,327	4,391	4,440	4,527	4,588	4,635	4,778
150	4,070	4,164	4,229	4,279	4,320	4,384	4,432	4,519	4,580	4,627	4,770
200	4,066	4,160	4,226	4,276	4,317	4,380	4,429	4,516	4,577	4,623	4,766
250	4,064	4,158	4,224	4,274	4,314	4,378	4,426	4,513	4,574	4,621	4,763
300	4,063	4,157	4,222	4,272	4,313	4,376	4,425	4,512	4,573	4,619	4,762
350	4,062	4,156	4,221	4,271	4,312	4,375	4,424	4,511	4,572	4,618	4,761
400	4,061	4,155	4,220	4,271	4,311	4,375	4,423	4,510	4,571	4,618	4,760
450	4,061	4,154	4,220	4,270	4,311	4,374	4,422	4,509	4,570	4,617	4,759
500	4,060	4,154	4,219	4,270	4,310	4,373	4,422	4,509	4,570	4,616	4,759
600	4,059	4,153	4,219	4,269	4,309	4,373	4,421	4,508	4,569	4,616	4,758
700	4,059	4,153	4,218	4,268	4,309	4,372	4,421	4,508	4,568	4,615	4,757
800	4,059	4,152	4,218	4,268	4,309	4,372	4,420	4,507	4,568	4,615	4,757
900	4,058	4,152	4,218	4,268	4,308	4,371	4,420	4,507	4,568	4,614	4,756
1 000	4,058	4,152	4,217	4,267	4,308	4,371	4,420	4,507	4,567	4,614	4,756
∞	4,056	4,150	4,215	4,265	4,306	4,369	4,418	4,504	4,565	4,612	4,754

Table D.6 — Two-sided prediction interval factors, k, at confidence level 99,9 % for known population standard deviation *(continued)*

n	m									
	1 000	2 000	5 000	10 000	20 000	50 000	100 000	200 000	500 000	1 000 000
2	5,966	6,126	6,331	6,481	6,627	6,814	6,952	7,087	7,261	7,389
3	5,640	5,794	5,991	6,136	6,278	6,460	6,594	6,725	6,895	7,020
4	5,466	5,616	5,808	5,949	6,088	6,266	6,397	6,526	6,692	6,815
5	5,357	5,504	5,693	5,832	5,968	6,143	6,273	6,400	6,564	6,685
6	5,283	5,428	5,615	5,752	5,886	6,059	6,187	6,313	6,475	6,595
7	5,229	5,373	5,558	5,694	5,827	5,998	6,125	6,249	6,410	6,529
8	5,188	5,331	5,514	5,649	5,781	5,952	6,077	6,201	6,360	6,479
9	5,156	5,298	5,480	5,615	5,746	5,915	6,040	6,163	6,321	6,439
10	5,130	5,272	5,453	5,587	5,717	5,886	6,010	6,132	6,290	6,407
11	5,109	5,250	5,431	5,564	5,694	5,861	5,985	6,107	6,264	6,381
12	5,092	5,232	5,412	5,544	5,674	5,841	5,965	6,086	6,243	6,359
13	5,076	5,216	5,396	5,528	5,657	5,824	5,947	6,068	6,224	6,340
14	5,064	5,203	5,382	5,514	5,643	5,809	5,932	6,052	6,208	6,324
15	5,052	5,191	5,370	5,502	5,630	5,796	5,919	6,039	6,195	6,310
16	5,042	5,181	5,360	5,491	5,619	5,785	5,907	6,027	6,183	6,298
17	5,034	5,172	5,350	5,481	5,610	5,775	5,897	6,017	6,172	6,287
18	5,026	5,164	5,342	5,473	5,601	5,766	5,888	6,008	6,162	6,277
19	5,019	5,157	5,335	5,465	5,593	5,758	5,880	5,999	6,154	6,268
20	5,013	5,151	5,328	5,459	5,586	5,751	5,873	5,992	6,146	6,261
25	4,989	5,126	5,303	5,433	5,560	5,723	5,845	5,963	6,117	6,231
30	4,973	5,110	5,286	5,415	5,542	5,705	5,826	5,944	6,097	6,211
35	4,961	5,098	5,273	5,403	5,529	5,692	5,812	5,930	6,083	6,196
40	4,953	5,089	5,264	5,393	5,519	5,682	5,802	5,920	6,073	6,186
45	4,946	5,082	5,257	5,386	5,512	5,674	5,794	5,912	6,064	6,177
50	4,941	5,077	5,251	5,380	5,506	5,668	5,788	5,906	6,058	6,171
60	4,933	5,068	5,243	5,371	5,497	5,659	5,779	5,896	6,048	6,161
70	4,927	5,062	5,237	5,365	5,491	5,652	5,772	5,889	6,041	6,153
80	4,923	5,058	5,232	5,360	5,486	5,647	5,767	5,884	6,036	6,148
90	4,919	5,055	5,229	5,357	5,482	5,644	5,763	5,880	6,031	6,144
100	4,916	5,052	5,226	5,354	5,479	5,640	5,760	5,877	6,028	6,140
150	4,908	5,043	5,217	5,345	5,470	5,631	5,750	5,867	6,018	6,130
200	4,904	5,039	5,213	5,340	5,465	5,626	5,745	5,862	6,013	6,125
250	4,902	5,037	5,210	5,338	5,463	5,624	5,743	5,859	6,010	6,122
300	4,900	5,035	5,208	5,336	5,461	5,622	5,741	5,857	6,008	6,120
350	4,899	5,034	5,207	5,335	5,460	5,620	5,739	5,856	6,007	6,119
400	4,898	5,033	5,206	5,334	5,459	5,619	5,738	5,855	6,006	6,117
450	4,897	5,032	5,206	5,333	5,458	5,619	5,738	5,854	6,005	6,117
500	4,897	5,032	5,205	5,332	5,457	5,618	5,737	5,853	6,004	6,116
600	4,896	5,031	5,204	5,332	5,456	5,617	5,736	5,852	6,003	6,115
700	4,896	5,030	5,203	5,331	5,456	5,616	5,735	5,852	6,003	6,114
800	4,895	5,030	5,203	5,330	5,455	5,616	5,735	5,851	6,002	6,114
900	4,895	5,030	5,203	5,330	5,455	5,616	5,734	5,851	6,002	6,113
1 000	4,894	5,029	5,202	5,330	5,454	5,615	5,734	5,851	6,001	6,113
∞	4,892	5,027	5,200	5,327	5,452	5,612	5,731	5,848	5,998	6,110

NOTE This table provides factors k such that one may be at least 99,9 % confident that none of the next m observations from a normally distributed population will lie outside the range $(\bar{x} - k\sigma, \bar{x} + k\sigma)$, where $\bar{x}$ is derived from a random sample of size n from the same population.

Annex E
(normative)

Tables of sample sizes for one-sided distribution-free prediction intervals

Table E.1 — Sample sizes, n, for one-sided distribution-free prediction intervals at confidence level 90 %

m	r										
	0	1	2	3	4	5	6	7	8	9	10
1	9										
2	18	3									
3	27	6	2								
4	36	8	4	2							
5	45	10	5	3	2						
6	54	12	6	4	3	2					
7	63	14	7	5	3	2	2				
8	72	17	8	5	4	3	2	2			
9	81	19	10	6	4	3	3	2	1		
10	90	21	11	7	5	4	3	2	2	1	
15	135	32	17	11	8	6	5	4	4	3	3
20	180	43	22	15	11	9	7	6	5	4	4
30	270	64	34	23	17	13	11	9	8	7	6
40	360	86	46	30	23	18	15	13	11	10	9
50	450	108	57	38	29	23	19	16	14	12	11
60	540	129	69	46	34	27	23	19	17	15	13
80	720	172	92	62	46	37	30	26	23	20	18
100	900	216	115	77	58	46	38	33	28	25	23
150	1 350	324	173	116	87	69	58	49	43	38	34
200	1 800	432	230	155	116	93	77	66	58	51	46
250	2 250	540	288	194	146	116	97	83	72	64	58
500	4 500	1 081	577	388	292	233	194	166	145	129	116
1 000	9 000	2 162	1 154	778	584	467	389	333	291	258	232
2 000	18 000	4 324	2 308	1 556	1 169	935	778	666	582	517	465
5 000	45 000	10 811	5 772	3 891	2 924	2 338	1 947	1 667	1 457	1 294	1 164
10 000	90 000	21 622	11 544	7 782	5 848	4 677	3 894	3 335	2 915	2 589	2 328
20 000	180 000	43 245	23 088	15 565	11 697	9 355	7 789	6 670	5 830	5 178	4 656
50 000	450 000	108 113	57 721	38 913	29 244	23 389	19 474	16 675	14 577	12 946	11 642
100 000	900 000	216 227	115 443	77 827	58 489	46 779	38 949	33 351	29 154	25 892	23 284
200 000	1 800 000	432 455	230 886	155 655	116 978	93 559	77 898	66 704	58 309	51 784	46 569
500 000	4 500 000	1 081 138	577 217	389 139	292 446	233 899	194 747	166 760	145 774	129 462	116 423
1 000 000	9 000 000	2 162 277	1 154 434	778 279	584 893	467 799	389 495	333 521	291 549	258 925	232 846

NOTE This table provides sample sizes n for which one may be at least 90 % confident that not more than r of the next m observations from the same population will lie outside the interval $(-\infty, x_{[n]})$. Similarly for the interval $(x_{[1]}, \infty)$.

Table E.2 — Sample sizes, n, for one-sided distribution-free prediction intervals at confidence level 95 %

m	r										
	0	1	2	3	4	5	6	7	8	9	10
1	19										
2	38	5									
3	57	9	3								
4	76	12	5	3							
5	95	16	7	4	2						
6	114	20	9	5	3	2					
7	133	23	11	6	4	3	2				
8	152	26	12	8	5	4	3	2			
9	171	30	14	9	6	4	3	3	2		
10	190	33	16	10	7	5	4	3	2	2	
15	285	51	24	15	11	8	7	6	5	4	3
20	380	68	33	21	15	12	9	8	7	6	5
30	570	103	50	32	23	18	15	12	11	9	8
40	760	138	67	43	32	25	20	17	15	13	11
50	950	172	84	55	40	31	26	22	19	16	15
60	1 140	207	102	66	48	38	31	26	23	20	18
80	1 520	277	136	88	64	51	42	35	30	27	24
100	1 900	346	170	110	81	64	52	44	38	34	30
150	2 850	520	256	166	122	96	79	67	58	51	46
200	3 800	693	342	222	163	128	106	90	78	69	62
250	4 750	867	427	278	204	161	132	112	98	86	77
500	9 500	1 735	856	556	409	323	266	226	196	174	155
1 000	19 000	3 471	1 713	1 114	819	646	533	453	394	348	312
2 000	38 000	6 943	3 428	2 228	1 640	1 294	1 067	907	789	697	625
5 000	95 000	17 359	8 571	5 573	4 102	3 237	2 670	2 270	1 974	1 745	1 564
10 000	190 000	34 720	17 143	11 146	8 205	6 474	5 340	4 541	3 948	3 492	3 129
20 000	380 000	69 442	34 287	22 294	16 410	12 950	10 681	9 083	7 898	6 985	6 260
50 000	950 000	173 606	85 720	55 736	41 027	32 376	26 705	22 710	19 746	17 463	15 651
100 000	1 900 000	347 212	171 440	111 473	82 055	64 754	53 412	45 420	39 494	34 927	31 302
200 000	3 800 000	694 426	342 882	222 947	164 112	129 509	106 824	90 842	78 989	69 856	62 605
500 000	9 500 000	1 736 067	857 207	557 370	410 281	323 773	267 063	227 107	197 474	174 640	156 515
1 000 000	19 000 000	3 472 134	1 714 417	1 114 741	820 563	647 548	534 126	454 214	394 950	349 282	313 031

NOTE This table provides sample sizes n for which one may be at least 95 % confident that not more than r of the next m observations from the same population will lie outside the interval $(-\infty, x_{[n]})$. Similarly for the interval $(x_{[1]}, \infty)$.

Table E.3 — Sample sizes, n, for one-sided distribution-free prediction intervals at confidence level 97,5 %

m	r										
	0	1	2	3	4	5	6	7	8	9	10
1	39										
2	78	8									
3	117	14	5								
4	156	19	7	4							
5	195	24	10	5	3						
6	234	30	12	7	4	3					
7	273	35	15	9	6	4	3				
8	312	40	17	10	7	5	3	2			
9	351	46	20	12	8	6	4	3	2		
10	390	51	22	13	9	7	5	4	3	2	
15	585	78	34	21	15	11	9	7	6	5	4
20	780	104	46	28	20	15	12	10	8	7	6
30	1 170	158	71	44	31	24	19	16	14	12	10
40	1 560	211	95	59	42	32	26	22	19	16	14
50	1 950	264	119	74	53	41	33	28	24	21	18
60	2 340	317	143	89	64	49	40	34	29	25	22
80	3 120	424	192	119	86	66	54	45	39	34	30
100	3 900	530	240	150	107	83	68	57	49	43	38
150	5 850	797	361	225	162	126	102	86	74	65	58
200	7 800	1 063	482	301	217	168	137	116	100	88	78
250	9 750	1 329	603	377	271	211	172	145	125	110	98
500	19 500	2 660	1 208	756	544	423	345	291	252	222	198
1 000	39 000	5 322	2 418	1 513	1 090	848	692	584	505	445	397
2 000	78 000	10 647	4 838	3 028	2 181	1 697	1 386	1 170	1 012	891	795
5 000	195 000	26 621	12 098	7 573	5 455	4 245	3 467	2 928	2 532	2 229	1 991
10 000	390 000	53 243	24 198	15 147	10 911	8 491	6 937	5 857	5 065	4 460	3 983
20 000	780 000	106 489	48 397	30 296	21 824	16 985	13 875	11 715	10 131	8 921	7 967
50 000	1 950 000	266 226	120 996	75 742	54 562	42 464	34 689	29 290	25 330	22 305	19 920
100 000	3 900 000	532 453	241 993	151 485	109 126	84 929	69 380	58 582	50 661	44 611	39 842
200 000	7 800 000	1 064 909	483 989	302 972	218 254	169 861	138 761	117 165	101 325	89 224	79 685
500 000	19 500 000	2 662 276	1 209 974	757 431	545 638	424 654	346 905	292 915	253 314	223 061	199 215
1 000 000	39 000 000	5 324 552	2 419 950	1 514 865	1 091 277	849 310	693 813	585 831	506 629	446 124	398 432

NOTE This table provides sample sizes n for which one may be at least 97,5 % confident that not more than r of the next m observations from the same population will lie outside the interval $(-\infty, x_{[n]})$. Similarly for the interval $(x_{[1]}, \infty)$.

Table E.4 — Sample sizes, n, for one-sided distribution-free prediction intervals at confidence level 99 %

m	r										
	0	1	2	3	4	5	6	7	8	9	10
1	99										
2	198	13									
3	297	23	7								
4	396	32	11	5							
5	495	41	15	8	4						
6	594	50	18	10	6	4					
7	693	59	22	12	8	5	3				
8	792	68	26	14	9	6	5	3			
9	891	77	29	16	11	8	6	4	3		
10	990	86	33	19	12	9	7	5	4	3	
15	1 485	131	51	30	20	15	11	9	8	6	5
20	1 980	176	70	40	28	21	16	13	11	9	8
30	2 970	266	106	62	43	32	26	21	18	15	13
40	3 960	356	142	84	58	44	35	29	24	21	19
50	4 950	446	179	105	73	55	44	37	31	27	24
60	5 940	536	215	127	88	67	54	44	38	33	29
80	7 920	716	288	170	118	90	72	60	51	45	39
100	9 900	896	361	213	149	113	91	76	65	56	50
150	14 850	1 346	543	322	224	171	137	114	98	86	76
200	19 800	1 796	725	430	300	228	184	153	131	115	102
250	24 750	2 246	907	538	375	286	230	192	165	144	128
500	49 500	4 496	1 818	1 078	753	575	463	387	332	290	258
1 000	99 000	8 996	3 638	2 160	1 509	1 152	928	776	666	583	518
2 000	198 000	17 996	7 280	4 322	3 021	2 306	1 859	1 554	1 334	1 168	1 038
5 000	495 000	44 996	18 205	10 809	7 557	5 770	4 651	3 889	3 338	2 922	2 597
10 000	990 000	89 996	36 413	21 620	15 116	11 542	9 305	7 781	6 679	5 847	5 197
20 000	1 980 000	179 996	72 829	43 243	30 235	23 086	18 612	15 563	13 360	11 696	10 396
50 000	4 950 000	449 995	182 076	108 111	75 592	57 719	46 533	38 912	33 403	29 243	25 993
100 000	9 900 000	899 996	364 156	216 225	151 186	115 441	93 067	77 826	66 808	58 487	51 989
200 000	19 800 000	1 799 993	728 314	432 453	302 375	230 885	186 137	155 654	133 618	116 977	103 980
500 000	49 500 000	4 499 992	1 820 790	1 081 135	755 939	577 215	465 347	389 137	334 048	292 445	259 953
1 000 000	99 000 000	8 999 983	3 641 583	2 162 273	1 511 881	1 154 431	930 696	778 277	668 098	584 891	519 909

NOTE This table provides sample sizes n for which one may be at least 99 % confident that not more than r of the next m observations from the same population will lie outside the interval $(-\infty, x_{[n]})$. Similarly for the interval $(x_{[1]}, \infty)$.

Table E.5 — Sample sizes, n, for one-sided distribution-free prediction intervals at confidence level 99,5 %

m	r										
	0	1	2	3	4	5	6	7	8	9	10
1	199										
2	398	19									
3	597	33	9								
4	796	46	14	6							
5	995	59	19	9	5						
6	1 194	72	24	12	7	4					
7	1 393	86	29	15	9	6	4				
8	1 592	99	34	18	11	8	5	4			
9	1 791	112	39	21	13	9	7	5	3		
10	1 990	125	44	24	15	11	8	6	4	3	
15	2 985	191	68	38	25	18	14	11	9	7	6
20	3 980	257	93	51	34	25	20	16	13	11	9
30	5 970	388	141	79	53	39	31	25	21	18	16
40	7 960	520	190	107	72	54	42	35	29	25	22
50	9 950	651	238	134	91	68	54	44	37	32	28
60	11 940	782	287	162	110	82	65	54	45	39	34
80	15 920	1 045	383	217	148	110	88	72	61	53	47
100	19 900	1 308	480	272	185	139	110	91	77	67	59
150	29 850	1 965	723	410	280	210	167	138	118	102	90
200	39 800	2 622	965	548	374	281	223	185	158	137	121
250	49 750	3 279	1 208	687	468	352	280	232	198	172	152
500	99 500	6 565	2 420	1 377	939	706	563	467	398	347	307
1 000	199 000	13 136	4 844	2 757	1 882	1 415	1 129	936	799	696	616
2 000	398 000	26 278	9 692	5 518	3 768	2 833	2 260	1 876	1 601	1 395	1 235
5 000	995 000	65 704	24 236	13 799	9 424	7 088	5 655	4 693	4 006	3 491	3 091
10 000	1 990 000	131 416	48 476	27 602	18 851	14 180	11 314	9 390	8 014	6 984	6 185
20 000	3 980 000	262 836	96 956	55 208	37 705	28 362	22 630	18 782	16 030	13 970	12 373
50 000	9 950 000	657 099	242 397	138 027	94 267	70 910	56 580	46 959	40 080	34 930	30 936
100 000	19 900 000	1 314 208	484 800	276 057	188 537	141 824	113 163	93 920	80 162	69 862	61 874
200 000	39 800 000	2 628 415	969 604	552 116	377 076	283 652	226 330	187 842	160 327	139 726	123 751
500 000	99 500 000	6 571 043	2 424 010	1 380 295	942 696	709 131	565 828	469 611	400 822	349 320	309 381
1 000 000	199 000 000	13 142 098	4 848 024	2 760 590	1 885 396	1 418 268	1 131 657	939 225	801 646	698 643	618 764

NOTE This table provides sample sizes n for which one may be at least 99,5 % confident that not more than r of the next m observations from the same population will lie outside the interval $(-\infty, x_{[n]})$. Similarly for the interval $(x_{[1]}, \infty)$.

Table E.6 — Sample sizes, n, for one-sided distribution-free prediction intervals at confidence level 99,9 %

m	r										
	0	1	2	3	4	5	6	7	8	9	10
1	999										
2	1 998	44									
3	2 997	75	17								
4	3 996	107	26	10							
5	4 995	137	36	16	8						
6	5 994	168	45	21	11	7					
7	6 993	199	54	25	15	9	6				
8	7 992	230	63	30	18	12	8	5			
9	8 991	260	72	35	21	14	10	7	5		
10	9 990	291	81	39	24	16	12	9	6	4	
15	14 985	444	126	63	39	27	20	16	13	10	9
20	19 980	597	171	86	54	38	29	23	19	16	13
30	29 970	904	261	132	84	60	46	37	30	26	22
40	39 960	1 210	351	178	114	81	63	50	42	36	31
50	49 950	1 516	441	225	144	103	79	64	53	46	40
60	59 940	1 822	531	271	173	125	96	78	65	56	48
80	79 920	2 435	711	363	233	168	130	105	88	76	66
100	99 900	3 047	891	456	293	211	164	133	111	95	83
150	149 850	4 579	1 341	687	442	319	248	201	169	145	127
200	199 800	6 110	1 791	918	591	428	332	270	227	195	171
250	249 750	7 641	2 241	1 149	740	536	416	339	284	245	215
500	499 500	15 297	4 491	2 305	1 485	1 076	837	681	573	494	433
1 000	999 000	30 608	8 992	4 617	2 976	2 157	1 678	1 367	1 150	991	870
2 000	1 998 000	61 231	17 992	9 240	5 957	4 320	3 361	2 738	2 305	1 987	1 744
5 000	4 995 000	153 098	44 992	23 111	14 900	10 806	8 409	6 853	5 768	4 972	4 365
10 000	9 990 000	306 211	89 992	46 227	29 805	21 618	16 823	13 709	11 540	9 949	8 734
20 000	19 980 000	612 435	179 990	92 462	59 616	43 240	33 650	27 423	23 084	19 901	17 472
50 000	49 950 000	1 531 108	449 990	231 164	149 048	108 109	84 129	68 565	57 717	49 759	43 687
100 000	99 900 000	3 062 216	899 981	462 332	298 101	216 222	168 265	137 134	115 439	99 523	87 377
200 000	199 800 000	6 124 456	1 799 980	924 670	596 209	432 446	336 533	274 270	230 883	199 048	174 759
500 000	499 500 000	15 311 170	4 499 944	2 311 689	1 490 522	1 081 127	841 340	685 679	577 208	497 627	436 903
1 000 000	999 000 000	30 622 339	8 999 912	4 623 361	2 981 044	2 162 257	1 682 677	1 371 363	1 154 424	995 253	873 812

NOTE This table provides sample sizes n for which one may be at least 99,9 % confident that not more than r of the next m observations from the same population will lie outside the interval $(-\infty, x_{[n]})$. Similarly for the interval $(x_{[1]}, \infty)$.

Annex F
(normative)

Tables of sample sizes for two-sided distribution-free prediction intervals

Table F.1 — Sample sizes, n, for two-sided distribution-free prediction intervals at confidence level 90 %

m	r										
	0	1	2	3	4	5	6	7	8	9	10
1	19										
2	38	7									
3	56	11	5								
4	75	15	7	4							
5	93	19	9	6	4						
6	112	23	11	7	5	3					
7	130	28	14	9	6	4	3				
8	149	32	16	10	7	5	4	3			
9	167	36	18	11	8	6	5	4	3		
10	186	40	20	13	9	7	6	4	4	3	
15	278	61	31	20	14	11	9	8	6	5	5
20	371	81	41	27	20	15	13	10	9	8	7
30	556	122	62	41	30	24	19	16	14	12	11
40	740	163	84	55	41	32	26	22	19	17	15
50	925	204	105	69	51	40	33	28	24	21	19
60	1 110	245	126	83	61	49	40	34	29	26	23
80	1 480	328	169	111	82	65	54	46	40	35	31
100	1 850	410	211	139	103	82	67	57	50	44	39
150	2 774	615	317	209	155	123	102	86	75	66	59
200	3 698	820	423	280	207	164	136	116	101	89	80
250	4 623	1 026	529	350	259	206	170	145	126	111	100
500	9 244	2 053	1 059	700	520	412	341	291	253	224	201
1 000	18 488	4 106	2 119	1 402	1 041	826	683	582	507	449	402
2 000	36 975	8 213	4 240	2 805	2 083	1 652	1 367	1 165	1 015	899	806
5 000	92 435	20 535	10 601	7 015	5 210	4 132	3 420	2 915	2 539	2 248	2 017
10 000	184 869	41 071	21 204	14 031	10 420	8 266	6 841	5 831	5 078	4 497	4 034
20 000	369 738	82 144	42 409	28 063	20 842	16 533	13 683	11 663	10 158	8 995	8 069
50 000	924 343	205 361	106 024	70 160	52 106	41 334	34 209	29 158	25 396	22 488	20 174
100 000	1 848 684	410 724	212 050	140 320	104 212	82 669	68 419	58 318	50 794	44 978	40 349
200 000	3 697 368	821 449	424 101	280 642	208 426	165 339	136 839	116 636	101 589	89 956	80 700
500 000	9 243 418	2 053 623	1 060 253	701 605	521 066	413 349	342 098	291 592	253 974	224 892	201 751
1 000 000	18 486 835	4 107 249	2 120 508	1 403 212	1 042 133	826 700	684 197	583 185	507 948	449 786	403 503

NOTE This table provides sample sizes n for which one may be at least 90 % confident that not more than r of the next m observations from the same population will lie outside the interval ($x_{[1]}$, $x_{[n]}$).

Table F.2 — Sample sizes, n, for two-sided distribution-free prediction intervals at confidence level 95 %

m	r										
	0	1	2	3	4	5	6	7	8	9	10
1	39										
2	78	10									
3	116	17	6								
4	155	23	10	5							
5	193	30	13	7	4						
6	232	36	16	9	6	4					
7	270	42	19	11	8	5	4				
8	309	49	22	13	9	7	5	4			
9	347	55	25	15	10	8	6	5	3		
10	386	62	28	17	12	9	7	5	4	3	
15	578	94	43	27	19	14	11	9	8	7	6
20	771	126	58	36	26	20	16	13	11	10	8
30	1 156	189	89	56	40	31	25	21	18	15	14
40	1 541	253	119	75	54	42	34	28	24	21	19
50	1 926	317	149	94	68	53	43	36	31	27	24
60	2 311	381	179	113	82	63	52	43	37	33	29
80	3 080	509	240	152	109	85	69	58	50	44	39
100	3 850	637	300	190	137	107	87	74	63	56	50
150	5 775	956	451	286	207	161	132	111	96	84	75
200	7 700	1 275	602	382	276	216	176	149	129	113	101
250	9 624	1 595	754	478	346	270	221	186	161	142	126
500	19 248	3 192	1 509	958	694	541	443	374	324	285	254
1 000	38 495	6 386	3 020	1 917	1 389	1 085	888	750	649	572	511
2 000	76 988	12 774	6 043	3 836	2 781	2 171	1 777	1 502	1 300	1 145	1 023
5 000	192 469	31 939	15 110	9 593	6 954	5 430	4 444	3 757	3 251	2 865	2 559
10 000	384 937	63 880	30 222	19 187	13 910	10 862	8 890	7 516	6 504	5 731	5 120
20 000	769 873	127 763	60 447	38 377	27 823	21 726	17 783	15 033	13 011	11 463	10 241
50 000	1 924 681	319 409	151 120	95 944	69 559	54 318	44 459	37 585	32 529	28 659	25 605
100 000	3 849 361	638 821	302 243	191 891	139 120	108 637	88 920	75 171	65 059	57 320	51 211
200 000	7 698 722	1 277 645	604 488	383 783	278 242	217 276	177 841	150 344	130 120	114 642	102 424
500 000	19 246 802	3 194 115	1 511 223	959 462	695 608	543 193	444 605	375 863	325 303	286 607	256 063
1 000 000	38 493 604	6 388 228	3 022 450	1 918 925	1 391 219	1 086 387	889 213	751 728	650 608	573 215	512 127

NOTE This table provides sample sizes n for which one may be at least 95 % confident that not more than r of the next m observations from the same population will lie outside the interval $(x_{[1]}, x_{[n]})$.

Table F.3 — Sample sizes, *n*, for two-sided distribution-free prediction intervals at confidence level 97,5 %

m	*r*										
	0	1	2	3	4	5	6	7	8	9	10
1	79										
2	158	14									
3	236	24	8								
4	315	34	13	6							
5	393	44	17	9	5						
6	472	54	21	12	8	5					
7	550	63	26	15	9	7	4				
8	629	73	30	17	11	8	6	4			
9	707	82	34	20	13	9	7	5	4		
10	786	92	38	22	15	11	8	6	5	4	
15	1 178	140	59	35	24	18	14	11	9	8	7
20	1 571	188	80	48	33	25	20	16	14	12	10
30	2 356	284	121	73	51	39	31	26	22	19	16
40	3 141	380	163	98	69	52	42	35	30	26	23
50	3 926	476	204	123	87	66	53	44	38	33	29
60	4 711	572	246	149	105	80	64	54	46	40	35
80	6 281	765	329	199	140	107	87	72	62	54	48
100	7 851	957	412	250	176	135	109	91	78	68	60
150	11 776	1 437	620	376	266	204	164	138	118	103	92
200	15 700	1 917	827	502	355	272	220	184	158	138	123
250	19 625	2 397	1 035	629	444	341	276	231	198	174	154
500	39 249	4 798	2 073	1 260	891	684	554	464	399	349	311
1 000	78 498	9 601	4 148	2 524	1 785	1 371	1 110	930	800	701	623
2 000	156 995	19 205	8 300	5 050	3 572	2 745	2 222	1 862	1 601	1 404	1 249
5 000	392 485	48 019	20 754	12 629	8 934	6 866	5 558	4 660	4 007	3 513	3 125
10 000	784 969	96 042	41 511	25 260	17 871	13 735	11 117	9 321	8 016	7 027	6 253
20 000	1 569 937	192 087	83 026	50 524	35 744	27 472	22 237	18 645	16 035	14 057	12 507
50 000	3 924 842	480 223	207 569	126 313	89 364	68 684	55 596	46 615	40 090	35 145	31 272
100 000	7 849 683	960 449	415 141	252 629	178 731	137 371	111 195	93 233	80 183	70 292	62 546
200 000	15 699 364	1 920 902	830 285	505 261	357 465	274 745	222 393	186 469	160 369	140 586	125 093
500 000	39 248 409	4 802 260	2 075 719	1 263 154	893 665	686 866	555 986	466 175	400 925	351 467	312 737
1 000 000	78 496 817	9 604 522	4 151 435	2 526 315	1 787 335	1 373 734	1 111 972	932 352	801 852	702 937	625 476

NOTE This table provides sample sizes n for which one may be at least 97,5 % confident that not more than r of the next m observations from the same population will lie outside the interval $(x_{[1]}, x_{[n]})$.

Table F.4 — Sample sizes, n, for two-sided distribution-free prediction intervals at confidence level 99 %

m	r										
	0	1	2	3	4	5	6	7	8	9	10
1	199										
2	398	23									
3	596	40	12								
4	795	56	18	9							
5	993	72	25	12	7						
6	1 192	88	31	16	10	6					
7	1 390	105	37	20	12	8	6				
8	1 589	121	43	23	15	10	7	5			
9	1 787	137	49	27	17	12	9	7	5		
10	1 986	153	56	30	20	14	10	8	6	5	
15	2 978	233	86	48	32	23	18	14	12	10	8
20	3 971	312	117	66	44	32	25	20	17	14	12
30	5 956	472	178	101	68	50	40	32	27	23	20
40	7 941	632	239	136	92	69	54	44	37	32	28
50	9 926	792	300	171	116	87	69	56	48	41	36
60	11 911	952	361	206	140	105	83	68	58	50	44
80	15 881	1 271	483	276	188	141	112	92	78	68	60
100	19 851	1 591	605	346	236	177	141	116	99	86	75
150	29 776	2 390	910	521	356	267	212	176	150	130	115
200	39 701	3 188	1 215	696	476	357	284	235	200	174	154
250	49 626	3 987	1 520	871	596	447	356	295	251	219	193
500	99 250	7 982	3 044	1 747	1 195	898	716	593	506	440	389
1 000	198 499	15 970	6 094	3 499	2 394	1 800	1 435	1 190	1 014	883	781
2 000	396 998	31 947	12 193	7 002	4 792	3 604	2 874	2 383	2 032	1 769	1 566
5 000	992 493	79 878	30 489	17 511	11 985	9 016	7 190	5 963	5 085	4 428	3 918
10 000	1 984 984	159 763	60 984	35 026	23 974	18 036	14 383	11 928	10 173	8 858	7 840
20 000	3 969 968	319 533	121 972	70 056	47 951	36 076	28 770	23 860	20 348	17 720	15 682
50 000	9 924 918	798 844	304 938	175 148	119 883	90 196	71 931	59 654	50 875	44 304	39 210
100 000	19 849 835	1 597 694	609 882	350 299	239 770	180 395	143 864	119 311	101 753	88 611	78 423
200 000	39 699 669	3 195 397	1 219 768	700 604	479 543	360 793	287 732	238 626	203 509	177 225	156 849
500 000	99 249 170	7 988 499	3 049 425	1 751 515	1 198 865	901 988	719 337	596 569	508 778	443 066	392 127
1 000 000	198 498 339	15 976 997	6 098 860	3 503 032	2 397 732	1 803 979	1 438 677	1 193 141	1 017 558	886 135	784 257

NOTE This table provides sample sizes n for which one may be at least 99 % confident that not more than r of the next m observations from the same population will lie outside the interval $(x_{[1]}, x_{[n]})$.

Table F.5 — Sample sizes, n, for two-sided distribution-free prediction intervals at confidence level 99,5 %

m	r										
	0	1	2	3	4	5	6	7	8	9	10
1	399										
2	798	34									
3	1 196	58	15								
4	1 595	81	24	10							
5	1 993	105	32	15	8						
6	2 392	128	40	20	12	7					
7	2 790	151	49	25	15	10	6				
8	3 189	174	57	29	18	12	9	6			
9	3 587	197	65	34	21	14	10	8	5		
10	3 986	221	73	38	24	17	12	9	7	5	
15	5 978	337	113	60	39	28	21	17	14	11	9
20	7 971	452	153	82	54	39	30	24	20	17	14
30	11 956	684	233	126	83	61	47	38	32	27	24
40	15 941	916	314	170	113	82	64	52	44	38	33
50	19 926	1 147	394	214	142	104	81	66	56	48	42
60	23 911	1 379	474	258	171	126	99	81	68	58	51
80	31 881	1 842	634	347	230	169	133	109	92	79	69
100	39 851	2 305	795	435	289	213	167	137	116	100	88
150	59 776	3 463	1 196	655	436	322	253	207	175	152	133
200	79 700	4 620	1 597	875	583	430	339	278	235	203	179
250	99 625	5 778	1 998	1 095	729	539	424	348	295	255	224
500	199 250	11 567	4 002	2 196	1 464	1 082	853	701	593	514	452
1 000	398 498	23 144	8 011	4 397	2 932	2 169	1 710	1 406	1 191	1 031	909
2 000	796 996	46 298	16 030	8 800	5 869	4 343	3 424	2 815	2 385	2 066	1 821
5 000	1 992 488	115 762	40 085	22 007	14 681	10 864	8 565	7 043	5 968	5 171	4 557
10 000	3 984 975	231 534	80 177	44 020	29 366	21 733	17 134	14 091	11 940	10 345	9 118
20 000	7 969 948	463 078	160 360	88 046	58 738	43 470	34 272	28 185	23 883	20 693	18 239
50 000	19 924 870	1 157 716	400 911	220 123	146 851	108 681	85 687	70 469	59 714	51 739	45 604
100 000	39 849 738	2 315 435	801 827	440 252	293 708	217 367	171 379	140 942	119 433	103 482	91 211
200 000	79 699 475	4 630 877	1 603 661	880 508	587 419	434 738	342 762	281 888	238 869	206 968	182 425
500 000	199 248 686	11 577 210	4 009 167	2 201 274	1 468 556	1 086 853	856 911	704 727	597 177	517 424	456 068
1 000 000	398 497 372	23 154 418	8 018 333	4 402 555	2 937 116	2 173 709	1 713 821	1 409 456	1 194 360	1 034 851	912 138

NOTE This table provides sample sizes n for which one may be at least 99,5 % confident that not more than r of the next m observations from the same population will lie outside the interval ($x_{[1]}$, $x_{[n]}$).

Table F.6 — Sample sizes, n, for two-sided distribution-free prediction intervals at confidence level 99,9 %

m	r										
	0	1	2	3	4	5	6	7	8	9	10
1	1 999										
2	3 998	76									
3	5 996	132	27								
4	7 995	186	43	17							
5	9 993	240	58	25	12						
6	11 992	294	73	32	18	10					
7	13 990	347	88	40	22	14	9				
8	15 989	401	103	47	27	18	12	8			
9	17 987	454	117	54	32	21	15	10	7		
10	19 986	508	132	62	36	24	17	13	9	7	
15	29 978	775	205	98	59	41	30	23	19	15	12
20	39 970	1 043	278	134	82	57	43	33	27	22	19
30	59 955	1 577	425	206	127	89	67	53	44	37	32
40	79 940	2 112	571	278	172	121	92	73	60	51	44
50	99 924	2 646	717	350	218	153	117	93	77	65	57
60	119 909	3 180	863	422	263	186	141	113	94	80	69
80	159 878	4 249	1 156	566	353	250	191	153	127	108	94
100	199 848	5 318	1 448	710	443	314	240	193	160	136	119
150	299 771	7 990	2 179	1 070	669	475	363	292	243	207	181
200	399 694	10 661	2 910	1 429	895	635	486	391	326	278	243
250	499 618	13 333	3 640	1 789	1 120	796	609	491	409	349	305
500	999 234	26 692	7 294	3 588	2 249	1 598	1 225	987	823	704	614
1 000	1 998 467	53 410	14 602	7 186	4 505	3 204	2 457	1 980	1 652	1 414	1 234
2 000	3 996 933	106 846	29 218	14 382	9 018	6 415	4 921	3 966	3 310	2 833	2 473
5 000	9 992 331	267 151	73 065	35 969	22 557	16 048	12 311	9 924	8 282	7 091	6 190
10 000	19 984 661	534 332	146 143	71 948	45 123	32 102	24 629	19 855	16 571	14 187	12 384
20 000	39 969 321	1 068 678	292 298	143 907	90 254	64 212	49 264	39 715	33 147	28 379	24 773
50 000	99 923 302	2 671 730	730 766	359 778	225 648	160 538	123 169	99 296	82 876	70 954	61 942
100 000	199 846 603	5 343 496	1 461 544	719 563	451 301	321 082	246 346	198 598	165 757	141 916	123 888
200 000	399 693 205	10 686 990	2 923 092	1 439 141	902 609	642 169	492 695	397 201	331 517	283 837	247 780
500 000	999 233 011	26 717 474	7 307 768	3 597 853	2 256 531	1 605 428	1 231 739	993 024	828 797	709 598	619 455
1 000 000	1 998 466 021	53 435 038	14 615 520	7 195 726	4 513 056	3 210 873	2 463 481	1 986 048	1 657 593	1 419 194	1 238 925

NOTE This table provides sample sizes n for which one may be at least 99,9 % confident that not more than r of the next m observations from the same population will lie outside the interval $(x_{[1]}, x_{[n]})$.

Annex G
(normative)

Interpolating in the tables

G.1 Interpolating in the tables of Annexes A to D

G.1.1 Interpolating to determine k for a value of n that is not tabulated

Between any adjacent pair of tabulated values of n down each column of the tables, k is approximately linear in $1/n$. Thus, for any value of n between adjacent tabulated values n_0 and n_1, $(n_0 < n_1)$ an approximation to the value of $k_{n,m}$ may be found by linear interpolation from

$$k_{n,m} \cong (1-\lambda)k_{n_0,m} + \lambda k_{n_1,m}$$

where

$$\lambda = \frac{1/n_0 - 1/n}{1/n_0 - 1/n_1}$$

EXAMPLE Suppose the value of k for $n = 120$ and $m = 2\,000$ is required for a symmetrical two-sided prediction interval at a confidence level of 99 % for a normally distributed population with an unknown standard deviation.

Reading the value from the column of Table B.4 corresponding to $m = 2\,000$, it is found that $k_{n_0,m} = k_{100,\,2\,000} = 4{,}845$ and $k_{n_1,m} = k_{150,\,2\,000} = 4{,}749$. Hence

$$\lambda = \frac{1/100 - 1/120}{1/100 - 1/150} = 0{,}5$$

The required value of $k_{120,\,2\,000}$ is therefore approximately

$$k_{120,\,2\,000} \cong (1 - 0{,}5)\, k_{100,\,2\,000} + 0{,}5\, k_{150,\,2\,000} = 0{,}5 \times 4{,}845 + 0{,}5 \times 4{,}749 = 4{,}797$$

G.1.2 Interpolating to determine k for a value of m that is not tabulated

Between any adjacent pair of tabulated values of m along each row of the tables, k is approximately linear in $\ln(m)$. Thus, for any value of m between adjacent tabulated values m_0 and m_1, $(m_0 < m_1)$, an approximation to the value of $k_{n,\,m}$ may be found by linear interpolation from

$$k_{n,m} \cong (1-\lambda)k_{n,m_0} + \lambda k_{n,m_1}$$

where

$$\lambda = \frac{\ln(m/m_0)}{\ln(m_1/m_0)}$$

EXAMPLE Suppose the value of k for $n = 100$ and $m = 2\,200$ is required for a one-sided prediction interval at a confidence level of 99,9 % for a normally distributed population with a known standard deviation.

From the row of Table C.6 corresponding to $n = 100$ it is found that $k_{n,m_0} = k_{100,\,2\,000} = 4{,}916$ and $k_{n,m_1} = k_{100,\,5\,000} = 5{,}095$. Hence

$$\lambda = \frac{\ln(2\,200/2\,000)}{\ln(5\,000/2\,000)} = \frac{\ln(1{,}1)}{\ln(2{,}5)} = \frac{0{,}095\,31}{0{,}916\,29} = 0{,}104\,02$$

The required value of $k_{100,\ 2\ 200}$ is therefore approximately

$$(1 - 0{,}104\ 02)\, k_{100,\ 2\ 000} + 0{,}104\ 02\, k_{100,\ 5\ 000} = 0{,}895\ 98 \times 4{,}916 + 0{,}104\ 02 \times 5{,}095 = 4{,}935$$

NOTE The expression ln(x) represents the natural logarithm of x, i.e. $\log_e x$. Logarithms to other bases may be used, as they will produce the same interpolated value.

G.1.3 Interpolating to determine k for values of n and m neither of which is tabulated

The procedure when neither n nor m is tabulated is a combination of the methods described in G.1.1 and G.1.2, either by applying G.1.1 twice followed by applying G.1.2 once or by applying G.1.2 twice followed by G.1.1 once.

G.1.4 Interpolating to determine the confidence level for a given value of k

The confidence level may be required after the initial random sample has been drawn and inspected and the value obtained for k has been determined for a specified limit or limits on the value of the variable. To interpolate among the tabulated confidence levels, make use of the fact that, between any two adjacent tabulated values of the confidence level, k is approximately linear in ln(α). It follows that, for any value of $100(1 - \alpha)$ % between adjacent tabulated confidence levels $100(1 - \alpha_0)$ % and $100(1 - \alpha_1)$ %, ($\alpha_0 > \alpha_1$), an approximation to the value of α may be determined from

$$\alpha \cong \alpha_0 \left(\frac{\alpha_1}{\alpha_0} \right)^{\lambda}$$

where

$$\lambda = \frac{k_{n,m,\alpha_0} - k}{k_{n,m,\alpha_1} - k_{n,m,\alpha_0}}$$

The required confidence level is then $100(1 - \alpha)$ %.

EXAMPLE Suppose a random sample of size $n = 20$ from a normally distributed population has yielded a sample mean of $\bar{x} = 20{,}5$ units and sample standard deviation of $s = 2{,}5$ units. With what confidence can it be asserted that all of the next 100 observations will lie below 30 units?

The appropriate value of k is (30 – 20,5)/2,5 = 3,8. The nearest tabulated values of k for $n = 20$ and $m = 100$ are $k = 3{,}506$ for confidence level 90 % (i.e. $\alpha_0 = 0{,}10$) in Table A.1 and $k = 3{,}856$ for confidence level 95 % (i.e. $\alpha_1 = 0{,}05$) in Table A.2. Hence

$$\lambda = \frac{k_{n,m,\alpha_0} - k}{k_{n,m,\alpha_1} - k_{n,m,\alpha_0}} = \frac{3{,}8 - 3{,}506}{3{,}856 - 3{,}506} = \frac{0{,}294}{0{,}350} = 0{,}84$$

and

$$\alpha \cong 0{,}10 \times \left(\frac{0{,}05}{0{,}10} \right)^{0{,}84} = 0{,}055\ 9$$

It follows that the required confidence level is $100(1 - \alpha)\ \% = 94{,}4\ \%$.

G.2 Interpolating in the tables of Annexes E and F

G.2.1 Interpolating to determine n for a value of m that is not tabulated, for a given value of r

Between any adjacent pair of tabulated values of m down each column of the tables, n is approximately linear in m. Thus, for any value of m between adjacent tabulated values m_0 and m_1, ($m_0 < m_1$) an approximation to the value of $n_{m,r}$ may be found by linear interpolation from

$$n_{m,r} \cong (1-\lambda) n_{m_0,r} + \lambda n_{m_1,r}$$

where

$$\lambda = \frac{m - m_0}{m_1 - m_0}$$

EXAMPLE The sample size n for a two-sided distribution-free prediction interval is required such that one may be 99 % confident that the interval includes at least 87 of the next 88 observations.

Here m = 88 and r = 1. From Table F.4 it is found that m_0 = 80, m_1 = 100, $n_{80,\,1}$ = 1 271 and $n_{100,\,1}$ = 1 591. Thus

$$\lambda = \frac{88-80}{100-80} = \frac{8}{20} = 0{,}4$$

and

$$n_{88,\,1} \cong (1-0{,}4) \times 1\,271 + 0{,}4 \times 1\,591 = 1\,399$$

G.2.2 Interpolating to determine n for a confidence level that is not tabulated, for given values of m and r

For given values of m and r the value of ln(n) between any adjacent pair of tabulated confidence levels is approximately linear in ln[(1 – α)/α]. For the appropriate values of m and r, denote the confidence level that corresponds to the nearest tabulated value of n less than the specified value by 100(1 – α_0) % and the next higher confidence level by 100(1 – α_1) %. Denote also the corresponding values of n by n_0 and n_1, respectively. Then an approximation to the required sample size is given by

$$n \cong \exp[(1 - \lambda) \ln(n_0) + \lambda \ln(n_1)]$$

where

$$\lambda = \frac{\ln\left[\dfrac{\alpha(1-\alpha_0)}{\alpha_0(1-\alpha)}\right]}{\ln\left[\dfrac{\alpha_1(1-\alpha_0)}{\alpha_0(1-\alpha_1)}\right]}$$

EXAMPLE Suppose an interval of the form ($x_{[1]}$, $x_{[n]}$) is required such that one may have 98 % confidence (i.e. α = 0,02) that not more than one of the next 100 observations falls outside the interval. As a two-sided interval is required, Annex F is applicable. The nearest tabulated confidence level below 98 % is 97,5 % in Table F.3, so α_0 = 0,025. The next higher tabulated confidence level is 99 % in Table F.4, so α_1 = 0,01. The initial sample sizes from these two tables corresponding to m = 100 and r = 1 are n_0 = 957 and n_1 = 1 591. Hence

$$\lambda = \frac{\ln\left(\dfrac{0{,}02 \times 0{,}975}{0{,}025 \times 0{,}98}\right)}{\ln\left(\dfrac{0{,}01 \times 0{,}975}{0{,}025 \times 0{,}99}\right)} = \frac{-0{,}228\,26}{-0{,}931\,56} = 0{,}245\,03$$

and

$$n \cong \exp[(1-0,245\,03)\times \ln(957)+0,245\,03\times \ln(1\,591)] = \exp(6,988\,36) = 1\,083,94$$

An initial sample size of about 1 084 is therefore necessary to provide the required level of confidence.

Annex H
(informative)

Statistical theory underlying the tables

H.1 One-sided sided prediction intervals for a normally distributed population with unknown population standard deviation (see Annex A)

H.1.1 The data

It is assumed that a random sample of n observations $x_1, x_2, \ldots, x_n$ has been drawn from a normally distributed population with unknown mean μ and unknown standard deviation σ. The sample mean is $\bar{x}$ and the sample standard deviation is s.

H.1.2 The problem

For given values of n, m and α, the smallest factor k is required such that one may have at least $100(1 - \alpha)$ % confidence that none of m further observations will exceed $\bar{x} + ks$. From symmetry considerations, this is the same as the value of k for which one may have $100(1 - \alpha)$ % confidence that none of the m further observations will lie below $\bar{x} - ks$.

H.1.3 The solution for finite n

The required prediction interval factor is the smallest value of k such that

$$\int_0^{\infty} g(s) \int_{-\infty}^{\infty} \Phi^m(\bar{x} + ks)\ f(\bar{x})\ \mathrm{d}\bar{x}\mathrm{d}s \geqslant 1 - \alpha \qquad \text{(H.1)}$$

where $f(\bar{x})$ and $g(s)$ are respectively the probability density functions of the sample mean and the sample standard deviation from the standard normal distribution and Φ is its distribution function, i.e.

$$f(\bar{x}) = \sqrt{\frac{n}{2\pi}} \exp\left(-\frac{n}{2}\bar{x}^2\right), -\infty < \bar{x} < \infty$$

$$g(s) = \frac{\nu^{\nu/2}\ s^{\nu-1}}{2^{(\nu/2)-1}\ \Gamma\left(\frac{\nu}{2}\right)} \exp(-\nu s^2/2),\ s \geqslant 0$$

$$\Phi(t) = \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}u^2\right) \mathrm{d}u$$

where

$$\Gamma\left(\frac{\nu}{2}\right) = \int_0^{\infty} x^{\frac{\nu}{2}-1} \exp(-x)\mathrm{d}x$$

$\nu = n - 1$.

For each given combination of values of n, m and α, the smallest value of k (to three decimal places of accuracy) satisfying Inequality (H.1) has been found by an iterative procedure and is presented in Annex A.

H.1.4 The solution for infinite n

As n tends to infinity, Inequality (H.1) tends to

$$\Phi^m(k) \geqslant 1 - \alpha \tag{H.2}$$

Inequality (H.2) can be solved explicitly to give

$$k \geqslant \Phi^{-1}\left[(1-\alpha)^{\frac{1}{m}}\right] \tag{H.3}$$

The smallest values of k (to three decimal places of accuracy) satisfying Inequality (H.3) are presented in the final row of each table of Annex A.

H.2 Two-sided prediction intervals for a normally distributed population with unknown population standard deviation (see Annex B)

H.2.1 The data

The data are the same as in H.1.1.

H.2.2 The problem

For given values of n, m and α, the smallest factor k is required such that one may have at least $100(1 - \alpha)$ % confidence that none of m further observations will lie outside the range $\bar{x} - ks$ to $\bar{x} + ks$.

H.2.3 The solution for finite n

The required prediction interval factor is the smallest value of k such that

$$\int_0^{\infty} g(s) \int_{-\infty}^{\infty} [\Phi(\bar{x}+ks) - \Phi(\bar{x}-ks)]^m f(\bar{x}) \mathrm{d}\bar{x} \mathrm{d}s \geqslant 1-\alpha \tag{H.4}$$

For each given combination of values of n, m and α, the smallest value of k (to three decimal places of accuracy) satisfying Inequality (H.4) has been found by an iterative procedure and is presented in Annex B.

H.2.4 The solution for infinite n

As n tends to infinity, Inequality (H.4) tends to

$$[\Phi(k) - \Phi(-k)]^m \geqslant 1 - \alpha \tag{H.5}$$

Inequality (H.5) can be solved explicitly to give

$$k \geqslant \Phi^{-1}\left\{\frac{1}{2}\left[1+(1-\alpha)^{\frac{1}{m}}\right]\right\} \tag{H.6}$$

The smallest values of k (to three decimal places of accuracy) satisfying Inequality (H.6) are presented in the final row of each table of Annex B.

H.3 One-sided prediction intervals for a normally distributed population with known population standard deviation (see Annex C)

H.3.1 The data

It is assumed that a random sample of n observations $x_1, x_2, \ldots, x_n$ has been drawn from a normally distributed population with unknown mean μ and known standard deviation σ.

H.3.2 The problem

For given values of n, m and α, the smallest factor k is required such that one may have at least $100(1 - \alpha)$ % confidence that none of m further observations will exceed $\bar{x} + k\sigma$. From symmetry considerations, this is the same as the value of k for which one may have $100(1 - \alpha)$ % confidence that none of m further observations will lie below $\bar{x} - k\sigma$.

H.3.3 The solution for finite n

The required prediction interval factor is the smallest value of k such that

$$\int_{-\infty}^{\infty} \Phi^m(\bar{x} + k)\ f(\bar{x})\,\mathrm{d}\bar{x} \geqslant 1 - \alpha \qquad \text{(H.7)}$$

For each given combination of values of n, m and α, the smallest value of k (to three decimal places of accuracy) satisfying Inequality H.7) has been found by an iterative procedure and is presented in Annex C.

H.3.4 The solution for infinite n

As n tends to infinity, Inequality (H.7) tends to Inequality (H.2), the solution to which is given by Inequality (H.3). Hence, the final row of each table of Annex C is the same as the corresponding final row of each table of Annex A.

H.4 Two-sided prediction intervals for a normally distributed population with known population standard deviation (see Annex D)

H.4.1 The data

The data are the same as in H.3.1.

H.4.2 The problem

For given values of n, m and α, the smallest factor k is required such that one may have at least $100(1 - \alpha)$ % confidence that none of m further observations will lie outside the range $\bar{x} - k\sigma$ to $\bar{x} + k\sigma$.

H.4.3 The solution for finite n

The required prediction interval factor is the smallest solution in k to

$$\int_{-\infty}^{\infty} [\Phi(\bar{x} + k) - \Phi(\bar{x} - k)]^m\ f(\bar{x})\,\mathrm{d}\bar{x} \geqslant 1 - \alpha \qquad \text{(H.8)}$$

For each given combination of values of n, m and α, the smallest value of k (to three decimal places of accuracy) satisfying Inequality (H.8) has been found by an iterative procedure and is presented in Annex D.

H.4.4 The solution for infinite n

As n tends to infinity, Inequality (H.8) tends to Inequality (H.5), the solution to which is given by Inequality (H.6). Hence the final row of each of the tables of Annex D is the same as the final row of the corresponding table of Annex B.

H.5 Prediction intervals for the mean of a further sample from a normally distributed population

H.5.1 One-sided prediction interval for unknown population standard deviation

A one-sided prediction interval of the form ($\bar{x} - ks$, ∞) or ($-\infty$, $\bar{x} + ks$) for the *mean* of a further m observations from the same normally distributed population, based on a sample of size n, has confidence level 100(1 – α) % if

$$k = t_{n-1,1-\alpha} \sqrt{\frac{1}{n} + \frac{1}{m}} \tag{H.9}$$

where $t_{n-1,\,1-\alpha}$ is the upper α-fractile of the t-distribution with $n - 1$ degrees of freedom. This can be calculated directly if suitable tables of the t-distribution are available. An alternative that does not require tables of the t-distribution is as follows. When m is equal to 1, the required value of k is the same as the prediction interval factor for a further sample of size 1, i.e.

$$k_{n,1,1-\alpha} = t_{n-1,1-\alpha} \sqrt{\frac{1}{n} + 1} \tag{H.10}$$

It can be deduced from Equations (H.9) and (H.10) that

$$k = \frac{k_{n,1,1-\alpha} \sqrt{\frac{1}{n} + \frac{1}{m}}}{\sqrt{\frac{1}{n} + 1}} = k_{n,1,1-\alpha} \sqrt{\frac{n+m}{m(n+1)}} \tag{H.11}$$

where $k_{n,\,1,\,1-\alpha}$ is given in Annex A corresponding to confidence level 100(1 – α) % for the given value of n and for m = 1.

H.5.2 Two-sided prediction interval for unknown population standard deviation

A two-sided prediction interval of the form ($\bar{x} - ks$, $\bar{x} + ks$) for the mean of a further m observations from the same normally distributed population, based on a sample of size n, has confidence level 100(1 – α) % if

$$k = t_{n-1,1-\alpha/2} \sqrt{\frac{1}{n} + \frac{1}{m}}$$

By similar reasoning to that given in H.5.1, it may be deduced that

$$k = k_{n,1,1-\alpha} \sqrt{\frac{n+m}{m(n+1)}} \tag{H.12}$$

where $k_{n,\,1,\,1-\alpha}$ is given in Annex B corresponding to a confidence level 100(1 – α) % for the given value of n and for m = 1.

H.5.3 One-sided prediction interval for known population standard deviation

A one-sided prediction interval of the form $(\bar{x} - k\sigma, \infty)$ or $(-\infty, \bar{x} + k\sigma)$ for the mean of a further m observations from the same normally distributed population, based on a sample of size n, has confidence level $100(1 - \alpha)$ % if

$$k = z_{1-\alpha}\sqrt{\frac{1}{n} + \frac{1}{m}}$$

where $z_{1-\alpha}$ is the upper α-fractile of the standard normal distribution. By similar reasoning to that given in H.5.1, it may be deduced that

$$k = k_{n,1,1-\alpha}\sqrt{\frac{n+m}{m(n+1)}} \tag{H.13}$$

where $k_{n,1,1-\alpha}$ is given in Annex C corresponding to confidence level $100(1 - \alpha)$ % for the given value of n and for $m = 1$.

H.5.4 Two-sided prediction interval for known population standard deviation

A two-sided prediction interval of the form $(\bar{x} - k\sigma, \bar{x} + k\sigma)$ for the mean of a further m observations from the same normally distributed population, based on a sample of size n, has confidence level $100(1 - \alpha)$ % if

$$k = z_{1-\alpha/2}\sqrt{\frac{1}{n} + \frac{1}{m}}$$

By similar reasoning to that given in H.5.1, it may be deduced that

$$k = k_{n,1,1-\alpha}\sqrt{\frac{n+m}{m(n+1)}} \tag{H.14}$$

where $k_{n,1,1-\alpha}$ is given in Annex D corresponding to confidence level $100(1 - \alpha)$ % for the given value of n and for $m = 1$.

H.6 One-sided distribution-free prediction intervals (see Annex E)

H.6.1 The data

It is assumed that a random sample of n observations $x_1, x_2, \ldots, x_n$ will be drawn from a population whose distribution is unknown.

H.6.2 The problem

Denote the smallest of the n observations by $x_{[1]}$ and the largest by $x_{[n]}$. The one-sided prediction intervals considered in most detail in this part of ISO 16269 are either the interval $(-\infty, x_{[n]})$ or the interval $(x_{[1]}, \infty)$. It is known that there will be m further observations, and it is required to determine the smallest value of n such that, for given m, α and r, one may have at least $100(1 - \alpha)$ % confidence that not more than r of the m further observations will lie outside the one-sided prediction interval.

H.6.3 The solution

The initial sample size n must satisfy Inequality (H.15):

$$\frac{\sum_{i=0}^{r} \binom{n-1+m-i}{n-1}}{\binom{n+m}{n}} \geqslant 1-\alpha \qquad \text{(H.15)}$$

where $\binom{a}{b} = \frac{a!}{b!\,(a-b)!}$

For each given combination of values of m, α and r, the smallest integer value of n satisfying Inequality (H.15) has been found by an iterative procedure and is presented in Annex E.

H.6.4 More general one-sided distribution-free prediction intervals

If a narrower interval is desired and a larger initial sample size can be tolerated, order statistics other than the most extreme ones may be used. Such intervals also have the advantage that they are not so likely to be influenced by outliers. Denote the tth smallest of the n observations by $x_{[t]}$ and the tth largest by $x_{[n+1-t]}$. The more general one-sided prediction intervals considered here are either the interval $(-\infty, x_{[n+1-t]})$ or the interval $(x_{[t]}, \infty)$. It is known that there will be m further observations, and it is required to determine the smallest value of n such that, for given m, t, α and r, one may have at least $100(1-\alpha)$ % confidence that not more than r of the m further observations will lie outside the one-sided interval. The solution is the smallest value of n satisfying

$$\frac{\sum_{i=0}^{r} \binom{n-t+m-i}{n-t}}{\binom{n+m}{n}} \geqslant 1-\alpha \qquad \text{(H.16)}$$

Due to space limitations, tables of the solutions in n to Inequality (H.16) are not provided in this part of ISO 16269 for values of t other than 1.

H.7 Two-sided distribution-free prediction intervals (see Annex F)

H.7.1 The data

The data are the same as in H.6.1.

H.7.2 The problem

The two-sided prediction intervals considered in most detail in this part of ISO 16269 are of the form $(x_{[1]}, x_{[n]})$, i.e. the range of the initial n observations. It is known that there will be m further observations, and it is required to determine the smallest value of n such that, for given m, α and r, one may have at least $100(1-\alpha)$ % confidence that not more than r of the m further observations will lie outside the two-sided prediction interval.

H.7.3 The solution

The initial sample size n must satisfy Inequality (H.17):

$$\frac{\sum_{i=0}^{r} (i+1)\binom{n-2+m-i}{n-2}}{\binom{n+m}{n}} \geqslant 1-\alpha \qquad \text{(H.17)}$$

For each given combination of values of m, α and r, the smallest integer value of n satisfying Inequality (H.17) has been found by an iterative procedure and is presented in Annex F.

H.7.4 More general two-sided distribution-free prediction intervals

If a narrower interval is desired and a larger initial sample size can be tolerated, order statistics other than the most extreme ones may be used. Such intervals also have the advantage that they are not as likely to be influenced by outliers. Denote the tth smallest of the n observations by $x_{[t]}$ and the tth largest by $x_{[n+1-t]}$. The more general two-sided prediction intervals considered here are of the form $(x_{[t]}, x_{[n+1-t]})$. It is known that there will be m further observations, and it is required to determine the smallest value of n such that, for given m, t, α and r, one may have at least $100(1-\alpha)$ % confidence that not more than r of the m further observations will lie outside the range $(x_{[t]}, x_{[n+1-t]})$. The solution is the smallest value of n satisfying Inequality (H.18):

$$\frac{\sum_{i=0}^{r} \binom{n-2t+m-i}{n-2t} \sum_{j=0}^{i} \binom{t-1+j}{t-1}\binom{t-1+i-j}{t-1}}{\binom{n+m}{n}} \geqslant 1-\alpha \qquad \text{(H.18)}$$

Due to space limitations, tables of the solutions in n to Inequality (H.18) are not provided in this part of ISO 16269 for values of t other than 1.

Bibliography

[1] ISO 2602, *Statistical interpretation of test results — Estimation of the mean — Confidence interval*

[2] ISO 16269-6, *Statistical interpretation of data — Part 6: Determination of statistical tolerance intervals*

[3] HAHN, G.J. Factors for calculating two-sided prediction intervals for samples from a normal distribution. *Journal of the American Statistical Association*, **64**, 1969, pp. 878-888

[4] HAHN, G.J. Additional factors for calculating prediction intervals for samples from a normal distribution. *Journal of the American Statistical Association*, **65**, 1970, pp. 1668-1676

[5] HAHN, G.J. and NELSON, W. A survey of prediction intervals and their applications. *Journal of Quality Technology*, **5**, 1973, pp. 178-188

[6] HAHN, G.J. and MEEKER, W.Q. *Statistical Intervals — A Guide for Practitioners*. New York, John Wiley and Sons Inc., 1991

[7] HALL, I.J., PRAIRIE, R.R. and MOTLAGH, C.K. Non-parametric prediction intervals. *Journal of Quality Technology*, **7**, 1975, pp. 109-114

[8] PATEL, J.K. Prediction intervals — A review. *Communications in Statistics — Theory and Methods.* **18**(7), 1989, pp. 2393-2465

ICS 03.120.30

Price based on 108 pages