INTERNATIONAL STANDARD

ISO 16063-43

First edition
2015-11-15

Methods for the calibration of vibration and shock transducers —

Part 43: Calibration of accelerometers by model-based parameter identification

Méthodes pour l'étalonnage des transducteurs de vibrations et de chocs —

Partie 43: Étalonnage des accéléromètres par identification des paramètres à base de modèle

Reference number
ISO 16063-43:2015(E)

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Foreword

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The committee responsible for this document is ISO/TC 108, *Mechanical vibration, shock and condition monitoring*, Subcommittee SC 3, *Use and calibration of vibration and shock measuring instruments*.

ISO 16063 consists of the following parts, under the general title *Methods for the calibration of vibration and shock transducers:*

— Part 1: *Basic concepts*

— Part 11: *Primary vibration calibration by laser interferometry*

— Part 12: *Primary vibration calibration by the reciprocity method*

— Part 13: *Primary shock calibration using laser interferometry*

— Part 15: *Primary angular vibration calibration by laser interferometry*

— Part 16: *Calibration by Earth's gravitation*

— Part 21: *Vibration calibration by comparison with a reference transducer*

— Part 22: *Shock calibration by comparison to a reference transducer*

— Part 31: *Testing of transverse vibration sensitivity*

— Part 41: *Calibration of laser vibrometers*

— Part 42: *Calibration of seismometers with high accuracy using acceleration of gravity*

— Part 43: *Calibration of accelerometers by model-based parameter identification*

The following parts are under preparation:

— Part 17: *Primary calibration by centrifuge*

— Part 32: *Resonance testing — Testing the frequency and the phase response of accelerometers by means of shock excitation*

— Part 33: *Testing of magnetic field sensitivity*

Introduction

The ISO 16063-series describes in several of its parts (ISO 16063-1, ISO 16063-11, ISO 16063-13, ISO 16063-21 and ISO 16063-22) the devices and procedures to be used for calibration of vibration sensors. The approaches taken can be divided in two classes: one for the use of stationary signals, namely sinusoidal or multi-sinus excitation; and the other for transient signals, namely shock excitation. While the first provides the lowest uncertainties due to intrinsic and periodic repeatability, the second aims at the high intensity range where periodic excitation is usually not feasible due to power constraints of the calibration systems.

The results of the first class are given in terms of a complex transfer sensitivity in the frequency domain and are, therefore, not directly applicable to transient time-domain application.

The results of the second class are given as a single value, the peak ratio, in the time domain that neglects (knowingly) the frequency-dependent dynamic response of the transducer to transient input signals with spectral components in the resonance area of the transducer's response. As a consequence of this "peak ratio characterization", the calibration result might exhibit a strong dependence on the shape of the transient input signal applied for the calibration and, therefore, from the calibration device.

This has two serious consequences:

a) The calibration with shock excitation in accordance with ISO 16063-13 or ISO 16063-22 is of limited use as far as the dissemination of units is concerned. That is, the shock sensitivities S_{sh} determined by calibrations on a device in a primary laboratory might not be applicable to the customer's device in the secondary calibration lab, simply due to a different signal shape and thus spectral constitution of the secondary device's shock excitation signal.

b) A comparison of calibration results from different calibration facilities with respect to consistency of the estimated measurement uncertainties, e.g. for validation purposes in an accreditation process, is not feasible if the facilities apply input signals of differing spectral composition.

The approach taken in this part of ISO 16063 is a mathematical model description of the accelerometer as a dynamic system with mechanical input and electrical output, where the latter is assumed to be proportional to an intrinsic mechanical quantity (e.g. deformation). The estimates of the parameters of that model and the associated uncertainties are then determined on the basis of calibration data achieved with established methods (ISO 16063-11, ISO 16063-13, ISO 16063-21 and ISO 16063-22). The complete model with quantified parameters and their respective uncertainties can subsequently be used to either calculate the time-domain response of the sensor to arbitrary transient signals (including time-dependent uncertainties) or as a starting point for a process to estimate the unknown transient input of the sensor from its measured time-dependent output signal (ISO 16063-11 or ISO 16063-13).

As a side effect, the method also usually provides an estimate of a continued frequency-domain transfer sensitivity of the model.

In short, this part of ISO 16063 prescribes methods and procedures that enable the user to

— calibrate vibration transducers for precise measurements of transient input,

— perform comparison measurements for validation using transient excitation,

— predict transient input signals and the time-dependent measurement uncertainty, and

— compensate the effects of the frequency-dependent response of vibration transducers (in real time) and thus expand the applicable bandwidth of the transducer.

Methods for the calibration of vibration and shock transducers —

Part 43:
Calibration of accelerometers by model-based parameter identification

1 Scope

This part of ISO 16063 prescribes terms and methods on the estimation of parameters used in mathematical models describing the input/output characteristics of vibration transducers, together with the respective parameter uncertainties. The described methods estimate the parameters on the basis of calibration data collected with standard calibration procedures in accordance with ISO 16063-11, ISO 16063-13, ISO 16063-21 and ISO 16063-22. The specification is provided as an extension of the existing procedures and definitions in those International Standards. The uncertainty estimation described conforms to the methods established by ISO/IEC Guide 98-3 and ISO/IEC Guide 98-3:2008/Supplement 1:2008.

The new characterization described in this document is intended to improve the quality of calibrations and measurement applications with broadband/transient input, like shock. It provides the means of a characterization of the vibration transducer's response to a transient input and, therefore, provides a basis for the accurate measurement of transient vibrational signals with the prediction of an input from an acquired output signal. The calibration data for accelerometers used in the aforementioned field of applications should additionally be evaluated and documented in accordance with the methods described below, in order to provide measurement capabilities and uncertainties beyond the limits drawn by the single value characterization given by ISO 16063-13 and ISO 16063-22.

2 Normative references

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 2041, *Mechanical vibration, shock and condition monitoring — Vocabulary*

ISO 16063-11, *Methods for the calibration of vibration and shock transducers — Part 11: Primary vibration calibration by laser interferometry*

ISO 16063-13, *Methods for the calibration of vibration and shock transducers — Part 13: Primary shock calibration using laser interferometry*

ISO 16063-21, *Methods for the calibration of vibration and shock transducers — Part 21: Vibration calibration by comparison to a reference transducer*

ISO 16063-22, *Methods for the calibration of vibration and shock transducers — Part 22: Shock calibration by comparison to a reference transducer*

ISO/IEC Guide 98-3, *Uncertainty of measurement — Part 3: Guide to the expression of uncertainty in measurement (GUM:1995)*

ISO/IEC Guide 98-3:2008/Supplement 1:2008, *Uncertainty of measurement — Part 3: Guide to the expression of uncertainty in measurement (GUM:1995) — Supplement 1: Propagation of distributions using a Monte Carlo method*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 2041 apply.

4 List of symbols

The symbols used in the formulae are listed in order of occurrence in the text.

$x, \dot{x}, \ddot{x}$	Output quantity of the respective sensor and its single and double derivative over time
δ	Damping coefficient of the model equation in the time domain
ω_0	Resonant circular frequency of the model
ρ	Electro mechanical conversion factor
i	Imaginary unit, $i = \sqrt{-1}$
H	Complex valued transfer function
S	Magnitude of the transfer function
ϕ	Phase of the transfer function
G	Reciprocal of the complex valued transfer function
μ	Parameter vector
S_m	Magnitude of the transfer function for a circular frequency, ω_m
ϕ_m	Phase of the transfer function for a circular frequency, ω_m
R	Real part of the complex valued transfer function
J	Imaginary part of the complex valued transfer function
y	Vector of real and imaginary parts of the measured transfer function
V_y	Covariance matrix of y
D	Coefficients matrix
$\hat{\mu}$	Vector of parameter estimates
$V_{\hat{\mu}}$	Covariance matrix of $\hat{\mu}$
S_0	Magnitude of the transfer function at low frequencies
A_μ	Transformation matrix for analytical uncertainty propagation
$V_{\rho,\omega_0,\delta}$	Covariance matrix of the model parameters
s	Frequency analogon in the s-domain (s-transform)
A	Acceleration in the s-domain
X	Output quantity of the respective sensor in the s-domain

z^{-1}	Back shift operator used in the bilinear transform (z-transform)
T	Sampling interval
x_k	Measurand sample at the time step k
b,c_1,c_2,Λ	Model parameters in the case of discretized time domain data
v	Substitutional parameters for the time domain parameter estimation
$\hat{v}$	Estimates of v by weighted least squares fitting
$V_{\hat{v}}$	Covariance matrix of the estimated parameters $\hat{v}$
Ω	Circular frequency normalized to the sample rate
X^2	Sum of weighted squared residuals
y_i	Calculated sensor output in the time domain based on estimated parameters

5 Consideration of typical frequency response and transient excitation

A typical acceleration transducer has a complex frequency response. This is usually given in terms of magnitude and phase with a shape, as shown in Figure 1. The magnitude is given in arbitrary units (a.u.).

This response function is subsequently sampled with lowest uncertainties by a calibration method in accordance with ISO 16063-11 or ISO 16063-21 making use of periodic excitation.

In applications with transient input signals, the sensor is then exposed to broadband excitation in terms of the frequency domain. The response in this case cannot be calculated with the help of a single (complex) value like the transfer sensitivity. Rather, the response can be considered to be a sensitivity that is weighted by those components in the frequency response that are excited by the spectral contents of the input signal.

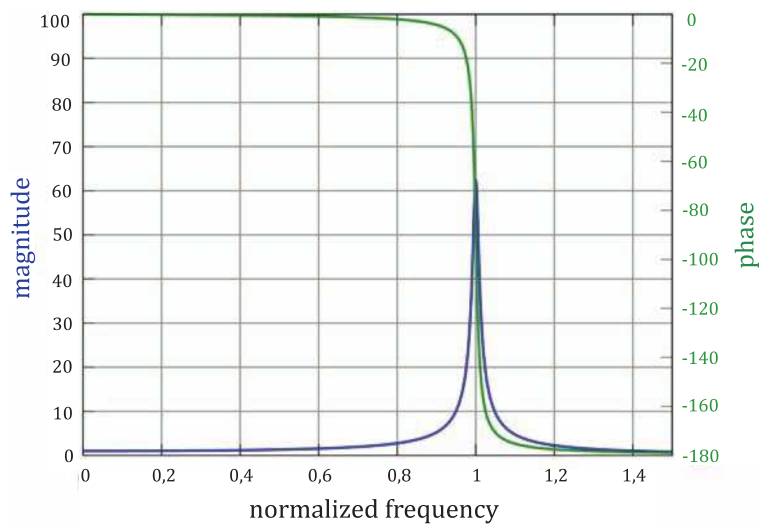
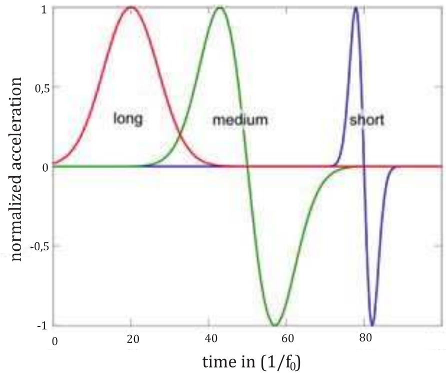
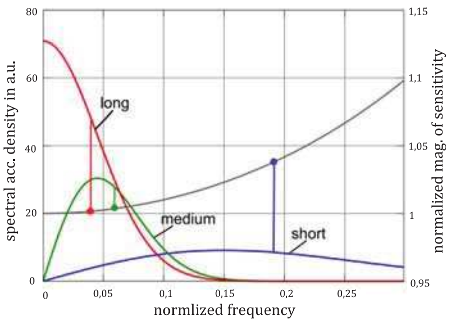


Figure 1 — Complex frequency response of a typical accelerometer in terms of magnitude of sensitivity (blue) and phase delay (green) over the normalized frequency

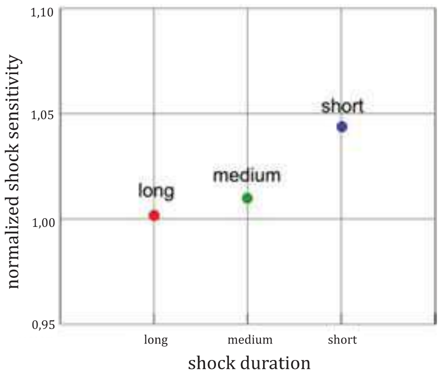
Figure 2 gives a pictorial representation of three examples of possible shock excitation signals and their respective spectra as compared to the frequency response of a typical sensor. It shows the projection of the centre of mass of the magnitude of the spectral density curve onto the sensitivity curve of a typical accelerometer. This demonstrates that a single value characterization of a transducer by shock calibration cannot sufficiently describe the dynamic behaviour.



a) Time domain representation of a long monopole (red), medium dipole (green), and short dipole (blue) shock



b) Frequency domain representation (magnitude) with the projection of the spectral centre point onto the sensitivity curve of a typical accelerometer response



c) Corresponding shock sensitivity (peak ratio) of a typical accelerometer (right)

Figure 2 — Comparison of the characteristics of three different shock signals

6 General approach

The general idea behind "model-based parameter identification" is to describe the input/output behaviour of a transducer type of certain design and construction with the help of a dynamic mathematical model. The detailed properties of an individual transducer are represented in that model by a set of parameters. Associated with the set of estimates of the model parameters is a respective set of uncertainties. The aim of the calibration is to provide measurement results that allow for the mathematical estimation of this parameter set and the evaluation of corresponding uncertainties.

NOTE 1 The parameter sets can include functions of variables to cover temperature sensitivity or mass loading effects.

This general approach is not new, and is already well-established in the fields of science and engineering under the term "identification of dynamic systems". However, in the field of transducer calibration, special emphasis has to be put on the validation of the applicability of the methods used and on the reliable calculation of uncertainties and respective coverage intervals.

NOTE 2 In this part of ISO 16063, the procedure of model-based parameter identification and further considerations is presented for a linear mass-spring-damper model of a seismic pick-up. However, this is only one example. The same approach can be used for more complicated mathematical models as long as they can be described as linear time-invariant (LTI) systems.

7 Linear mass-spring-damper model

7.1 Model

According to the investigation described in References [1], [2] and [3], some accelerometers can be described by a simple linear mass-spring-damper model in their specified working range. That means they follow the general equation of motion of the form[2] as given in Formula (1):

$$\ddot{x} + 2\delta\omega_0\dot{x} + \omega_0^2 x = \rho a\left(t\right) \tag{1}$$

where

δ is the damping coefficient;

ω_0 is the circular resonant frequency of the system;

ρ is the electro mechanical conversion factor.

This model describes the dynamic output $x\left(t\right)$ (e.g. charge or voltage) as a function of the acceleration input $a\left(t\right)$.

For such a linear system the transfer function $H\left(i\omega\right)$ in the frequency domain is independent of the acceleration amplitude and is given in Formula (2):

$$H\left(i\omega\right) = \frac{\rho}{\omega_0^2 + 2\delta\omega_0 i\omega + \left(i\omega\right)^2} = S\left(\omega\right)\cdot e^{i\phi\left(\omega\right)} \tag{2}$$

The inverse of this transfer function is given in Formula (3):

$$G\left(i\omega\right) = H\left(i\omega\right)^{-1} = \rho^{-1}\left(\omega_0^2 + 2i\omega\delta\omega_0 - \omega^2\right) = S^{-1}\left(\omega\right)\cdot \mathrm{e}^{-i\phi\left(\omega\right)} \quad (3)$$

where

$S\left(\omega\right)$ is the magnitude;

$\phi\left(\omega\right)$ is the phase of the response.

7.2 Identification by sinusoidal calibration data

7.2.1 Parameter identification

Starting from calibration measurements with sinusoidal excitation in accordance with, for example, ISO 16063-11 or ISO 16063-21, the frequency response $H\left(i\omega\right)$ can be directly determined as described by Formula (2) taking into account the well-known frequency response of any conditioning amplifier.

NOTE The model assumes that any additional response function of a measuring amplifier is eliminated prior to the identification process, which is usually the case.

Substituting a parameter vector, as given in Formula (4):

$$\mu^T = \left(\mu_1,\mu_2,\mu_3\right) = \left(\frac{\omega_0^2}{\rho},\frac{2\delta\omega_0}{\rho},\frac{1}{\rho}\right) \quad (4)$$

Formula (3) transforms into Formula (5):

$$G\left(i\omega\right) = \frac{1}{H\left(i\omega\right)} = \mu_1 + i\mu_2\omega - \mu_3\omega^2 = g^T\left(\omega\right)\cdot \mu \quad (5)$$

where

$$g^T\left(\omega\right) = \left(1,i\omega,-\omega^2\right)$$

According to this relation, the parameter vector μ can be estimated by weighted linear least squares, where the weights are chosen according to the uncertainties known from the calibration procedures in accordance with ISO 16063-11 or ISO 16063-21 as follows.

Let $S_m = S\left(\omega_m\right)$, $\phi_m = \phi\left(\omega_m\right)$ denote the magnitude and the phase of the frequency response from calibration measurements with associated standard uncertainties $u\left(S_m\right)$, $u\left(\phi_m\right)$ at the frequencies ω_m, m = 1, 2,..., L. Then the real part $R\left(S^{-1}\cdot e^{-i\phi}\right)$ and imaginary part $J\left(S^{-1}\cdot e^{-i\phi}\right)$ are given by Formula (6):

$$R\left(S,\phi\right) = R\left(S^{-1}\cdot e^{-i\phi}\right) = S^{-1}cos\left(\phi\right)\ ,\ J\left(S,\phi\right) = \mathrm{Im}\left(S^{-1}\cdot e^{-i\phi}\right) = -S^{-1}sin\left(\phi\right) \quad (6)$$

This is, in principle, a nonlinear transform which should be adequately handled for uncertainty calculations by, for example, Monte Carlo methods (see ISO/IEC Guide 98-3:2008/Supplement 1:2008 for details). However, given that the uncertainties of measurement are small enough, the direct propagation of uncertainties can be calculated in accordance with ISO/IEC Guide 98-3, as shown in Formula (7):

$$u^2(R_m) = \frac{u^2(s_m)}{s_m^4} cos^2(\phi_m) + \frac{u^2(\phi_m)}{s_m^2} sin^2(\phi_m)$$
$$u^2(J_m) = \frac{u^2(s_m)}{s_m^4} sin^2(\phi_m) + \frac{u^2(\phi_m)}{s_m^2} cos^2(\phi_m) \quad (7)$$
$$u(R_m, J_m) = \frac{-u^2(s_m)}{s_m^4} sin(\phi_m) cos(\phi_m) + \frac{u^2(\phi_m)}{s_m^2} sin(\phi_m) cos(\phi_m)$$

where $R_m = R(S_m, \phi_m)$ and $J_m = J(S_m, \phi_m)$

Then let Formula (8) be the transformed vector of the measurands:

$$y^T = \left[R(S_1, \phi_1), \ldots, R(S_L, \phi_L), J(S_1, \phi_1), \ldots, J(S_L, \phi_L)\right] \quad (8)$$

With the assumption that S and ϕ are uncorrelated measurands, the $2L \times 2L$ covariance matrix V_y becomes Formula (9):

$$V_y = \begin{pmatrix} u^2(R_1) & 0 & u(R_1 J_1) & 0 \\ & \ddots & & \ddots \\ 0 & u^2(R_L) & & u(R_L, J_L) \\ u(R_1 J_1) & & u^2(J_1) & 0 \\ & \ddots & & \ddots \\ 0 & u(R_L, J_L) & 0 & u^2(J_L) \end{pmatrix} \quad (9)$$

where D is the 2L times 3 matrix of the real and imaginary parts of $g^T(\omega)$, as given in Formula (10):

$$D = \begin{pmatrix} 1 & 0 & -\omega_1^2 \\ 1 & 0 & -\omega_2^2 \\ & \vdots & \\ 1 & 0 & -\omega_L^2 \\ 0 & \omega_1 & 0 \\ 0 & \omega_2 & 0 \\ & \vdots & \\ 0 & \omega_L & 0 \end{pmatrix} \quad (10)$$

The weighted least square estimate of the parameters can be calculated according to Formula (11):

$$\hat{\mu} = (D^T V_y^{-1} D)^{-1} D^T V_y^{-1} y \quad (11)$$

The uncertainties associated with the estimated parameter set $\hat{\mu}$, i.e. the covariance matrix, are given by Formula (12):

$$V_{\hat{\mu}} = \left(D^T V_y^{-1} D\right)^{-1} \tag{12}$$

The original model parameters can subsequently be calculated by transforming Formula (4) as Formula (13):

$$\begin{aligned} \rho &= \mu_3^{-1} \\ \omega_0 &= \sqrt{\frac{\mu_1}{\mu_3}} \\ \delta &= \frac{\mu_2}{\sqrt{\mu_1 \cdot \mu_3}} \end{aligned} \tag{13}$$

Sometimes it is more convenient to write Formula (2) in terms of $S_0 = \rho / \omega_0^2$ instead of ρ where S_0 describes the sensitivity for low frequencies. The corresponding parameter equation is given by Formula (14):

$$S_0 = \frac{1}{\mu_1} \tag{14}$$

Since the inverse transform from Formula (4) to Formula (13) is nonlinear the uncertainties associated with the model parameter set should be adequately handled for uncertainty calculations by, for example, Monte Carlo methods as described in Reference [1].

Figure 3 gives a flow chart representation of the whole analysis process.

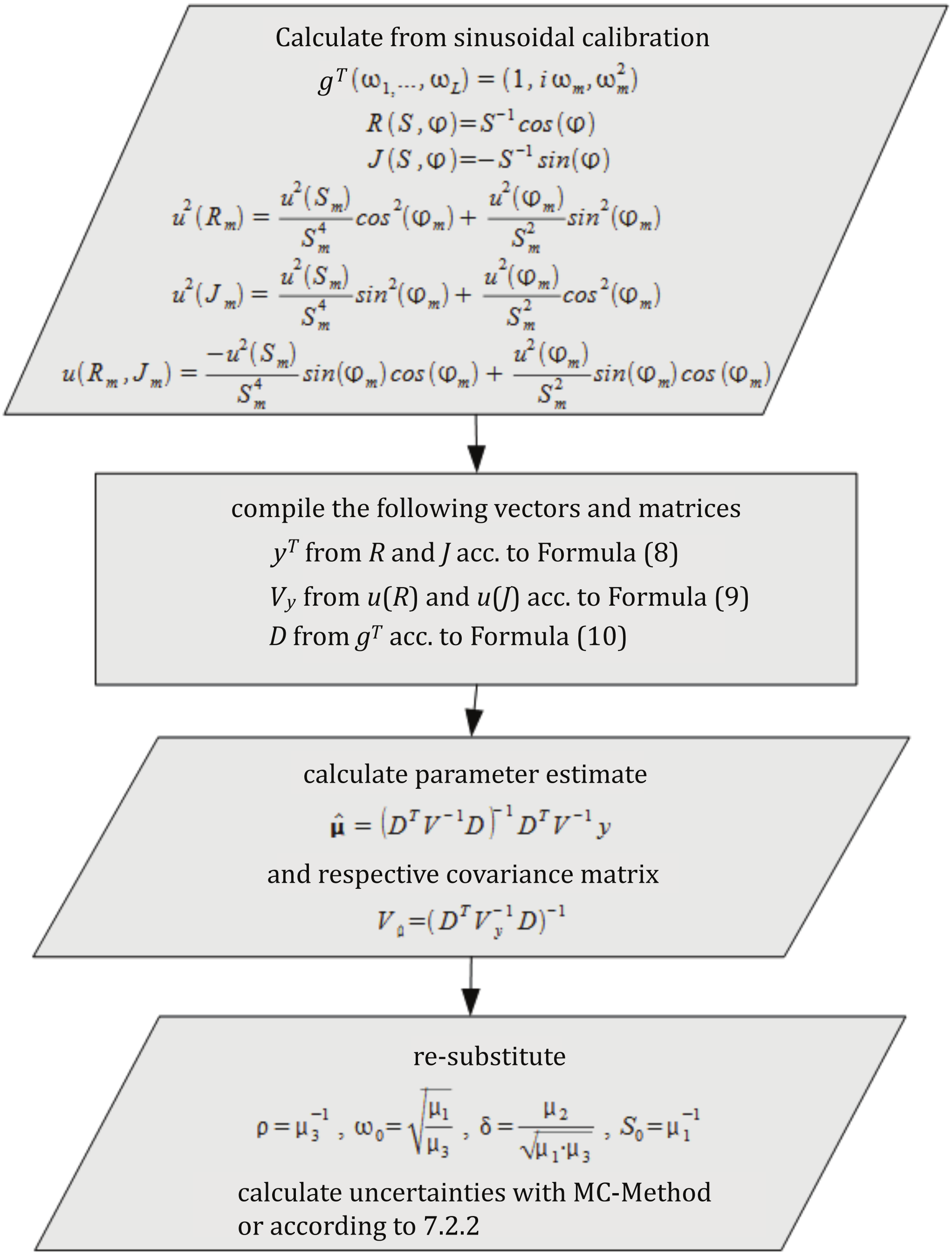


Figure 3 — Flowchart of the process of parameter identification upon sinusoidal calibration data

7.2.2 Uncertainties of model parameters by analytic propagation

In cases where the total expanded relative uncertainty of measurement of the magnitude S_m is less than 1 % and the total uncertainty of measurement of the phase ϕ_m is less than 2°, a conventional propagation of uncertainty is feasible, although Formula (13) states a strongly nonlinear relationship.

With the transformation matrix given in Formula (15):

$$A_\mu = \begin{pmatrix} \frac{\partial \rho}{\partial \mu_1} & \frac{\partial \rho}{\partial \mu_2} & \frac{\partial \rho}{\partial \mu_3} \\ \frac{\partial \omega_0}{\partial \mu_1} & \frac{\partial \omega_0}{\partial \mu_2} & \frac{\partial \omega_0}{\partial \mu_3} \\ \frac{\partial \delta}{\partial \mu_1} & \frac{\partial \delta}{\partial \mu_2} & \frac{\partial \delta}{\partial \mu_3} \end{pmatrix} \tag{15}$$

The Covariance matrix of the model parameters $V_{\rho,\omega_0,\delta}$ can be calculated from the covariance matrix $V_{\hat{\mu}}$ by Formula (16):

$$V_{\rho,\omega_0,\delta} = A_\mu V_{\hat{\mu}} A_\mu^T \tag{16}$$

where the square roots of the diagonal elements of $V_{\rho,\omega_0,\delta}$ state the uncertainties of the model parameters.

The uncertainty for S_0 can be calculated accordingly by substituting S_0 for ρ in Formula (15).

NOTE The nonlinear relationship in Formulae (13) and (14) requires an appropriate handling of the uncertainty propagation in accordance with ISO/IEC Guide 98-3:2008/Supplement 1:2008 for the general case. Only in the case of reduced input uncertainties is the linearization described in 7.2.2 applicable. For the given model, a comprehensive description of the general case is given in Reference [1].

7.3 Identification by shock calibration data in the frequency domain

7.3.1 Identification of the model parameters

Starting from calibration measurements with shock excitation in accordance with, for example, ISO 16063-13 or ISO 16063-22, it is possible to estimate the model parameters using a special pre-processing step with subsequent identification similar to the procedure described in the previous clause.

For the substitution, a discretization of the continuous time given in Formula (1) is necessary. For that purpose, the classical s-transform is used, which leads in the case of Formula (1) to the transformed equation given in Formula (17):

$$\left(s^2 + 2\delta\omega_0 s + \omega_0^2\right) X\left(s\right) = \rho A\left(s\right) \tag{17}$$

The discretization follows, for example, a bilinear mapping of the s-plane to the z-plane of the kind given in Formula (18):

$$s \to \frac{2}{T} \cdot \frac{1 - z^{-1}}{1 + z^{-1}} \tag{18}$$

where T is the sampling interval.

NOTE 1 It is reported in Reference [10] that a systematic deviation between continuous frequency representation and discretized frequencies due to the transform is reduced by applying a pre-warping.

Substituting Formula (18) into Formula (17), taking the sampled time series x_i and a_i for the respective variables and applying the back shift operator z^{-1}, ($z^{-1} \cdot x_i = x_{i-1}$) of the z-transform properly, results in the discretized version of the model equation (see Reference [3]), as given in Formula (19):

$$x_k = -c_1 x_{k-1} - c_2 x_{k-2} + b\left(a_k + 2a_{k-1} + a_{k-2}\right) \tag{19}$$

NOTE 2 The error introduced by the discretization with respect to the sample rate needs some further investigation and consideration in the uncertainty budget. At the time of writing, no publications are available on this topic.

The parameters of Formula (19) are related to the continuous model parameters according to Formula (20):

$$\begin{aligned} b &= \frac{\rho T^2}{4\Lambda} \\ c_1 &= \frac{\omega_0^2 T^2 - 4}{2\Lambda} \\ c_2 &= \frac{4 - 4\delta\omega_0 T + \omega_0^2 T^2}{4\Lambda} \end{aligned} \tag{20}$$

where $\Lambda = 1 + \delta\omega_0 T + \frac{\omega_0^2 T^2}{4}$

It is clear from this derivation that the parameters of the discretized model are dependent upon the sample rate T^{-1} and are therefore closely related to the calibration set-up. The sample rate used for the measurement should be at least five times greater than the frequency at which the first significant resonance of the sensor under calibration occurs. In order to avoid additional significant uncertainty components due to lack of resolution over frequency regions in which resonances occur, a better sampling frequency would be a factor of 10 or more greater than the frequency at which the first significant resonance of the sensor under calibration occurs.

The introduced discrete time model Formula (19) has a (periodic) frequency response of the form given in Formula (21):

$$H\left(e^{j\Omega}\right) = \frac{b\left(1 + 2e^{-j\Omega} + e^{-j2\Omega}\right)}{1 + c_1 e^{-j\Omega} + c_2 e^{-j2\Omega}} \tag{21}$$

where $\Omega = \omega / f_s = \omega T$ is the radian frequency normalized to the sample rate.

Just like Formula (5) given in 7.2, the inverse of this frequency response $G\left(e^{j\Omega}\right)$ is linear in the parameters, after some obvious substitution, given in Formula (22):

$$G\left(e^{j\Omega}\right) = H^{-1}\left(e^{j\Omega}\right) = \frac{1 + c_1 e^{-j\Omega} + c_2 e^{-j2\Omega}}{b\left(1 + 2e^{-j\Omega} + e^{-j2\Omega}\right)} = \frac{v_1 + v_2 e^{-j\Omega} + v_3 e^{-j2\Omega}}{\left(1 + 2e^{-j\Omega} + e^{-j2\Omega}\right)} \tag{22}$$

with the substitute vector, given in Formula (23):

$$v^T = \left[v_1, v_2, v_3\right] = \left[1/b, c_1/b, c_2/b\right] \tag{23}$$

With this inverse frequency response the general approach taken already in 7.2 can be followed. For the sake of completeness this will is worked out in more detail as follows.

Let $X\left(n\right)$ and $A\left(n\right)$ be the components of the discrete Fourier transform (DFT) of the sampled time series x_k and a_k respectively with n = 0, 1, ..., $N - 1$. Here, the influence of a conditioning amplifier can be eliminated by multiplying A(n) by the measured complex frequency response of the amplifier for the frequency $\frac{1}{2T}\frac{n}{N}$ in order to compensate the response later in Formula (24). In cases where AC-coupled conditioning amplifiers are used, the terms for $n = 0$ should be omitted, because they describe the DC-component of the signals, which vanishes for X_0. Formula (22) implies the relation given in Formula (24):

$$G_n = \frac{A(n)}{X(n)} = f_n^T v \tag{24}$$

with the vector f_n^T given as in Formula (25) from Formula (22):

$$f_n^T = \left[1, e^{-j(2\pi/N)n}, e^{-j2(2\pi/N)n}\right] \cdot \frac{1}{1 + 2e^{-j(2\pi/N)n} + e^{-j2(2\pi/N)n}} \tag{25}$$

Formula (24) is the analogue to Formula (5) given in 7.2. Note that the careful choice of the values for n offers the opportunity to limit the process to the relevant frequency range of the measurement.

Estimation of the parameter vector v is then performed by weighted least square fitting, i.e. by minimizing Formula (26):

$$\chi^2 = \sum_{n \in v} \left(\frac{\mathrm{Re}\left(G_n - f_n^T v\right)^2}{u^2\left[\mathrm{Re}\left(G_n\right)\right]} + \frac{\mathrm{Im}\left(G_n - f_n^T v\right)^2}{u^2\left[\mathrm{Im}\left(G_n\right)\right]} \right) \tag{26}$$

with respect to v, where the values of n are chosen such that the relevant frequency range is covered. The weights are again chosen to be the reciprocals of the squared standard uncertainties associated with the measured G_n. This uncertainty is estimated by the relation given in Formula (27):

$$u^2\left[\mathrm{Re}\left(G_n\right)\right] = u^2\left[\mathrm{Im}\left(G_n\right)\right] = \frac{u_0^2}{\left|X_n^2\right|} \tag{27}$$

The unknown uncertainty u_0 is determined by requiring that the minimum of Formula (19) equals the degree of freedom (number of data entering the fit minus number of adjusted model parameters). The components of G_n given in Formula (28) are considered to be uncorrelated:

$$R_n = R\left[G(n)\right] \text{ and } J_n = I\left[G(n)\right] n_1 \le n \le n_2 \tag{28}$$

NOTE 3 This assumption is treated in the appendix of Reference [2] in more detail.

The procedure for minimizing Formula (26) is similar to that described in 7.2.1, as given in Formulae (29), (30) and (31):

$$y^T = \left(R_{n_1}, \ldots, R_{n_2}, J_{n_1}, \ldots, J_{n_2}\right), \tag{29}$$

$$V_y = \begin{pmatrix} u^2\left(R_{n_1}\right) & & & & & \\ & \ddots & & & 0 & \\ & & u^2\left(R_{n_2}\right) & & & \\ & & & u^2\left(J_{n_1}\right) & & \\ & 0 & & & \ddots & \\ & & & & & u^2\left(J_{n_2}\right) \end{pmatrix} \tag{30}$$

$$D = \begin{pmatrix} \mathrm{Re}\left(f_{n_1}^T\right) \\ \vdots \\ \mathrm{Re}\left(f_{n_2}^T\right) \\ \mathrm{Im}\left(f_{n_1}^T\right) \\ \vdots \\ \mathrm{Im}\left(f_{n_2}^T\right) \end{pmatrix} \tag{31}$$

This means the weighted least square estimate of the parameters can be calculated according to Formula (32):

$$\hat{v} = D^T V_y^{-1} D^{-1} D^T V_y^{-1} y \tag{32}$$

The uncertainties associated with the estimated parameter set $\hat{v}$; i.e. the covariance matrix, are given by Formula (33):

$$V_{\hat{v}} = \left(D^T V_y^{-1} D\right)^{-1} \tag{33}$$

The estimate of the original (continuous time model) parameter set can subsequently be calculated via Formula (23) and Formula (32) according to Formula (34):

$$\begin{aligned} \rho &= \frac{16}{T^2\left(v_1 - v_2 + v_3\right)} \\ \omega_0^2 &= \frac{4\left(v_1 + v_2 + v_3\right)}{T^2\left(v_1 - v_2 + v_3\right)} \\ \delta &= \frac{v_1 - v_3}{\sqrt{v_1 - v_2 + v_3}\sqrt{v_1 + v_2 + v_3}} \\ S_0 &= \frac{2}{\sqrt{v_1 + v_2 + v_3}} \end{aligned} \tag{34}$$

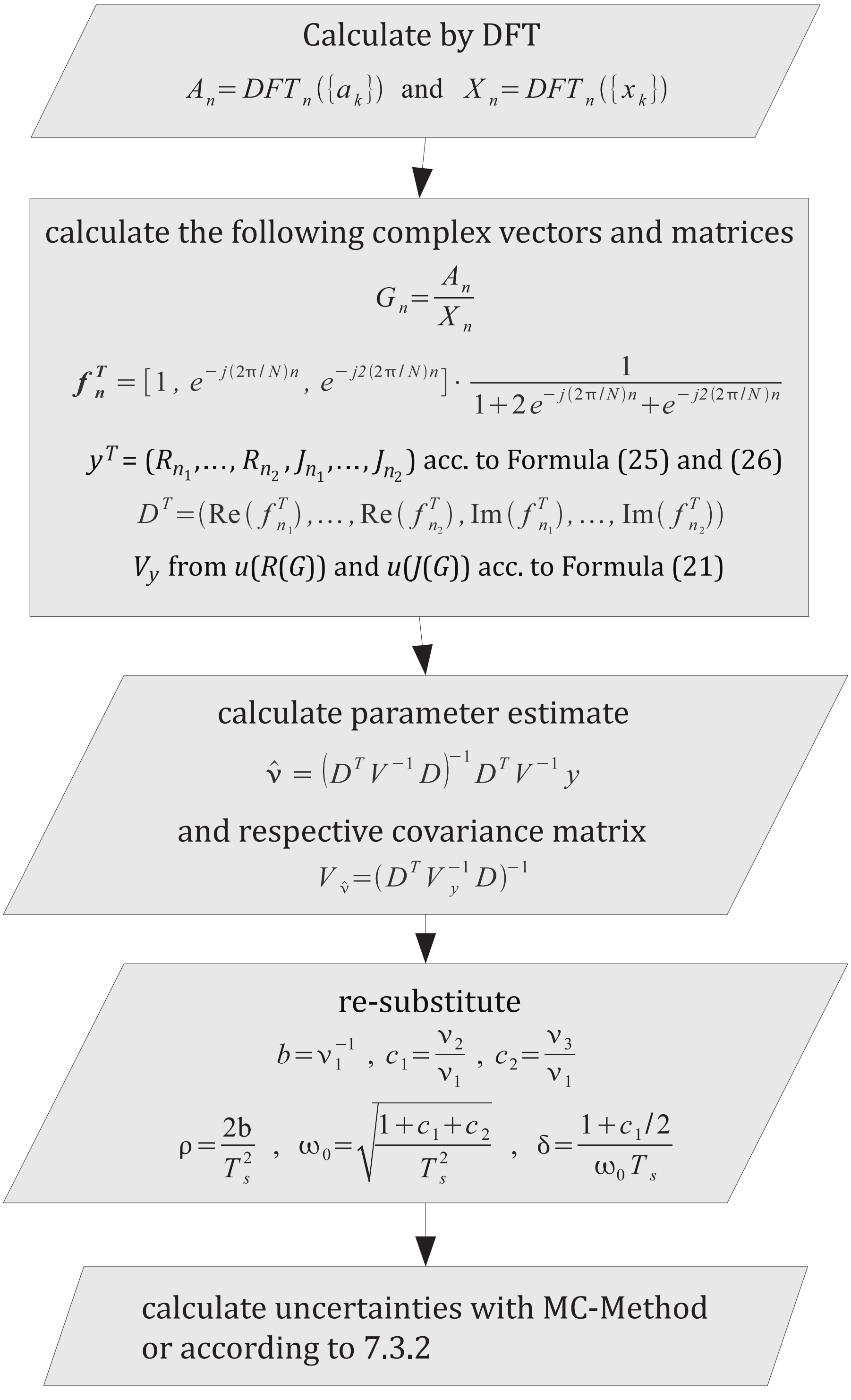


Figure 4 — Flowchart of the process of parameter identification upon shock calibration data in the frequency domain

7.3.2 Uncertainties of model parameters by analytical propagation

In cases where the total expanded relative uncertainty of measurement for the sampled time series is sufficiently small, a conventional propagation of uncertainty is feasible, although Formula (34) states a strongly nonlinear relationship.

The covariance matrix $V_{\hat{v}}$ of the identified parameters is calculated according to Formula (33).

With the transformation matrix given in Formula (35):

$$A_v = \begin{pmatrix} \frac{\partial\rho}{\partial v_1} & \frac{\partial\rho}{\partial v_2} & \frac{\partial\rho}{\partial v_3} \\ \frac{\partial\omega_0}{\partial v_1} & \frac{\partial\omega_0}{\partial v_2} & \frac{\partial\omega_0}{\partial v_3} \\ \frac{\partial\delta}{\partial v_1} & \frac{\partial\delta}{\partial v_2} & \frac{\partial\delta}{\partial v_3} \end{pmatrix} \tag{35}$$

The covariance matrix of the model parameters $V_{\rho,\omega_0,\delta}$ can be calculated from the covariance matrix $V_{\hat{v}}$, as given in Formula (36):

$$V_{\rho,\omega_0,\delta} = A_v V_{\hat{v}} A_v^T \tag{36}$$

where the square roots of the diagonal elements of $V_{\rho,\omega_0,\delta}$ state the uncertainties of the model parameters.

The uncertainty for S_0 can be calculated accordingly by substituting S_0 for ρ in Formula (35).

NOTE The nonlinear relationship in Formula (34) again requires an appropriate handling of the uncertainty propagation in accordance with ISO/IEC Guide 98-3:2008/Supplement 1:2008 for the general case. The linearization described in 7.3.2 is only applicable in the case of reduced input uncertainties. For the given model, a comprehensive description of the general case is given in Reference [3].

8 Practical considerations

8.1 The influence of the measurement chain

The physical model, which gave guidance for the model in Formula (1), is that of a simple seismic pick-up. As such, it does not allow for different frequency responses of the additional components in the measurement chain, e.g. a charge amplifier. In order to account for the conditioning elements of a measuring chain, it is necessary to either compensate the frequency response of the respective component in the measurement channel or to introduce its response in the acceleration measuring channel by appropriate filtering.

The goal is symmetry in the frequency response of the acceleration measuring channel and the conditioning part of the channel of the device under test. The compensation is necessary for magnitude as well as for the phase shift component of the compensated response.

NOTE A third approach would be an extension of the model that incorporates the conditioning amplifier, thus it would be possible to perform a parameter identification of the measuring chain. However, such an approach has not been investigated or even validated at the time of writing.

8.2 Synchronicity of the measurement channels

The physical model which is the origin of Formula (1) implies a certain phase to the complex valued transfer function given in Formula (2). As far as time domain measurements are concerned, this relates to a certain (frequency-dependent) delay between input and output of the accelerometer.

This relationship imposes a strong necessity for synchronous data acquisition. This means, on the one hand, that the sampling of the time series needs to be well synchronised. On the other hand, however, any phase-delay according to measuring or conditioning components in the measuring chains needs to be compensated by appropriate shifting, such that the delay in input and output channel generated by the measurement hardware are equal. Of course, this does not include the device under test.

8.3 Properties of the source data used for the identification

In order to achieve meaningful parameter estimates from the identification process, the spectrum, i.e. the frequency points from the input, have to be chosen carefully. Specifically, two issues should be taken into account:

a) If disturbing resonances (e.g. transverse or housing resonances), which are not covered by the chosen model of the transducer, appear in the used frequency range. The respective frequencies should be either eliminated from the identification process or weighted by a larger uncertainty of appropriate magnitude.

b) For a proper identification of resonance frequency and damping, it is essential for the input data to cover an informative part of the spectrum with respect to these parameters. This means that it is advisable to include frequencies as close to the resonance as possible, or even beyond the resonance. At a minimum, the frequency range covered should include frequencies at which the sensitivity increases by about 10 % relative to that of S_0, the sensitivity at low frequencies. The frequencies in the immediate neighbourhood of the resonance shall, however, be excluded due to the typically high measurement uncertainty.

8.4 Empirical test of model and parameter validity

8.4.1 Sinusoidal calibration data

The empirical test following the procedure given in 7.2 is as follows:

— Use the identified parameters from Formula (13) and substitute their values in Formula (2).

— Plot the resulting continuous function $H(i\omega)$ in magnitude and phase together with the original discrete input data, $S\left(\omega_m\right)$ and $\phi\left(\omega_m\right)$, respectively.

The continuous function should be a good approximation of the measurements.

8.4.2 Shock calibration data

For the procedure given in 7.3, the test compares the output y_k of a forward simulation (output prediction) with the experimentally acquired output x_k of the accelerometer.

— From the set of parameters from Formula (34) calculate the parameters (b,c_1,c_2) according to Formula (20).

— Calculate the simulated accelerometer output samples y_k according to Formula (19) based on the acquired experimental input acceleration samples a_k where

$$y_1 = y_2 = 0$$
$$y_3 = b\left(a_3 + 2a_2 + a_1\right)$$
$$y_4 = -c_1 y_3 + b\left(a_4 + 2a_3 + a_2\right)$$
$$y_k = -c_1 y_{k-1} - c_2 y_{k-2} + b\left(a_k + 2a_{k-1} + a_{k-2}\right) \text{ for } 5 \le k \le N$$

— These calculated samples y_k should give a good approximation of the originally measured output samples x_k from the shock calibration.

8.5 Statistical test of model validity

8.5.1 General

The procedures for parameter identification described in Clause 7 are all based on linear regression algorithms. Thus, in all cases the weighted sum of the squared deviations of the fitted curve from the actual measurement data χ^2 is minimized. For the minimized value $\chi^2_{\min}$ the criterion given in Formula (37) could be used to validate the consistency of the measurement data with the quantified model:

$$\chi^2_{\min} \leq \chi^2_{\nu,1-\mathrm{p}/2} \tag{37}$$

Here $\chi^2_{\nu,\mathrm{p}}$ denotes the p-th quantile of a χ^2-Distribution with ν degrees of freedom. The choice of p should be p ≤ 0,05.

For the different signal types prescribed, Formulae (38) and (39) given in 8.5.2 and 8.5.3 apply.

NOTE The statistical tests provide information on the quality of the fit of the model-equation to the measured data. They are not sufficient to infer the validity of the model. For this purpose, empirical tests based on the comparison of predicted and measured sensor input or output have proven to be an appropriate tool.[1][3]

8.5.2 Statistical test for sinosoidal data

As given in Formula (38):

$$\chi^2_{\min} = \left(y - H\hat{\mu}\right)^T V_y^{-1} \left(y - H\hat{\mu}\right) \text{ and } \nu = 2L - 3 \tag{38}$$

where L is the number of frequencies at which magnitude and phase response measurements are performed and subsequently used for identification.[1]

8.5.3 Statistical test for shock data and the frequency domain evaluation

As given in Formula (39):

$$\chi^2_{\min} = \left(y - H\hat{\nu}\right)^T V_y^{-1} \left(y - H\hat{\nu}\right) \tag{39}$$

With a degree of freedom being $2L-3$, where L is, in this case, the number of frequencies from the DFT actually used for the identification, see Formula (26). This test is only applicable if the constant uncertainty coefficient u_0 introduced in Formula (27) was estimated by means of a measurement uncertainty budget and not by applying the degree of freedom criterion given in Reference [3].

9 Reporting of results

9.1 Common considerations on the reporting

As described in the Scope, the purpose of the methods prescribed in this part of ISO 16063 is an extension of the data analysis prescribed in ISO 16063-11, ISO 16063-13, ISO 16063-21 and ISO 16063-22. Consequently, the results derived from the application of the described procedures (together with a reference to the explicit procedure) should be appended to a report in accordance with these established International Standards. Thus, the conditions of measurement and data acquisition are not explicitly covered here, but have to be provided in accordance with the appropriate International Standard.

The model-based approach taken here eliminates the need to report results in dependence of amplitude, intensity or frequency, at least for shock calibrations in accordance with ISO 16063-13 and ISO 16063-22.

If it appears from the processing that results from subsets of the measurement data give discrepant results, the validity of the employed model equation has to be reassessed.

9.2 Results and conditions to be reported

The fundamental results derived from the calculations described in Clause 7 are the parameters of the continuous time model of the vibration transducer, (ρ,ω_0,δ) or (S_0,ω_0,δ), and their associated uncertainties, $u(\rho),u(\omega_0),u(\delta)$ or $u(S_0),u(\omega_0),u(\delta)$, respectively. They have to be documented in the report. These results are directly applicable for the comparison of measurement results, e.g. for validation in the framework of accreditation.

Another set of results that could be provided together with the report are parameters related to the discrete time model (b,c_1,c_2 in Formula (20)) with their associated uncertainties, which are an intermediate result of the calculation shown in 7.3 or can be calculated from the aforementioned continuous time domain parameters and uncertainties. It is important, however, to be aware that these quantities depend on the sampling rate of the data acquisition system (T^{-1}) and the type of mapping used for the discretization (given in Formula (18)) in conjunction with the parameters and their associated uncertainties. Thus, they are only directly applicable if the sampling rates of the calibration system and the measurement application are identical or if they are transformed accordingly. This transformation includes the measurement uncertainty. Since those parameter sets are not unambiguous, it is mandatory to state the applicable sampling rate and the type of the mapping used for the discretization together with the quantities.

For example, in the form

$$\hat{c}_1 = \ldots, \hat{c}_2 = \ldots, \hat{b} = \ldots; \qquad u\left(\hat{c}_1\right) = \ldots, u\left(\hat{c}_2\right) = \ldots, u\left(\hat{b}\right) = \ldots; \qquad s \to \frac{2}{T}\cdot\frac{1-z^{-1}}{1+z^{-1}} \qquad SR = 500\ \text{kS/s}$$

Information about the goodness of fit, the maximum deviation or the root of summed squared deviations (variance) between the measured data and the resulting transfer function according to the evaluation in 8.4.1 or 8.4.2, respectively, may be reported.

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ICS 17.160

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