INTERNATIONAL STANDARD

ISO 9083

First edition
2001-07-15

Calculation of load capacity of spur and helical gears — Application to marine gears

Calcul de la capacité de charge des engrenages cylindriques à dentures droite et hélicoïdale — Application aux engrenages marins

Reference number
ISO 9083:2001(E)

ISO copyright office
Case postale 56 • CH-1211 Geneva 20
Tel. + 41 22 749 01 11
Fax + 41 22 749 09 47
E-mail copyright@iso.ch
Web www.iso.ch

Printed in Switzerland

Contents

Page

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 3.

Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this International Standard may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

International Standard ISO 9083 was prepared by Technical Committee ISO/TC 60, *Gears*, Subcommittee SC 2, *Gear capacity calculation*.

Annexes A and B form a normative part of ISO 9083. Annexes C to E are for information only.

Introduction

Procedures for the calculation of the load capacity of general spur and helical gears with respect to pitting and bending strength appear in ISO 6336-1, ISO 6336-2, ISO 6336-3 and ISO 6336-5. This International Standard is derived from ISO 6336-1, ISO 6336-2 and ISO 6336-3 by the use of specific methods and assumptions considered to be applicable to marine gears. Its application requires the use of allowable stresses and material requirements that are to be found in ISO 6336-5.

Calculation of load capacity of spur and helical gears — Application to marine gears

1 Scope

The formulae specified in this International Standard are intended for the establishment of a uniformly acceptable method for calculating the pitting resistance and bending strength capacity for the endurance of the main-propulsion and auxiliary gears of ships, offshore vessels and drilling rigs, having straight or helical teeth and subject to the rules of classification societies.

The rating formulae in this International Standard are not applicable to other types of gear tooth deterioration, such as plastic yielding, micropitting, scuffing, case crushing, welding and wear, and are not applicable under vibratory conditions where there may be an unpredictable profile breakdown. The bending strength formulae are applicable to fractures at the tooth fillet, but are not applicable to fractures on the tooth working profile surfaces, failure of the gear rim, or failures of the gear blank through web and hub. This International Standard does not apply to teeth finished by forging or sintering. This standard is not applicable to gears having a poor contact pattern.

This International Standard provides a method by which different gear designs can be compared. It is not intended to assure the performance of assembled drive gear systems. It is not intended for use by the general engineering public. Instead, it is intended for use by the experienced gear designer who is capable of selecting reasonable values for the factors in these formulae based on knowledge of similar designs and awareness of the effects of the items discussed.

WARNING — The user is cautioned that the calculated results of this International Standard should be confirmed by experience.

2 Normative references

The following normative documents contain provisions which, through reference in this text, constitute provisions of this International Standard. For dated references, subsequent amendments to, or revisions of, any of these publications do not apply. However, parties to agreements based on this International Standard are encouraged to investigate the possibility of applying the most recent editions of the normative documents indicated below. For undated references, the latest edition of the normative document referred to applies. Members of ISO and IEC maintain registers of currently valid International Standards.

ISO 53:1998, *Cylindrical gears for general and heavy engineering — Standard basic rack tooth profile.*

ISO 54:1996, *Cylindrical gears for general engineering and for heavy engineering — Modules*.

ISO 701:1998, *International gear notation — Symbols for geometrical data*.

ISO 1122-1:1998, *Vocabulary of gear terms — Part 1: Definitions related to geometry*.

ISO 1328-1:1995, *Cylindrical gears — ISO system of accuracy — Part 1: Definitions and allowable values of deviations relevant to corresponding flanks of gear teeth*.

ISO 6336-1:1996, *Calculation of load capacity of spur and helical gears — Part 1: Basic principles, introduction and general influence factors*.

ISO 6336-2:1996, *Calculation of load capacity of spur and helical gears — Part 2: Calculation of surface durability (pitting)*.

ISO 6336-3:1996, *Calculation of load capacity of spur and helical gears — Part 3: Calculation of tooth bending strength*.

ISO 6336-5:1996, *Calculation of load capacity of spur and helical gears — Part 5: Strength and quality of material*.

ISO/TR 10495:1997, *Cylindrical gears — Calculation of service life under variable loads — Conditions for cylindrical gears according to ISO 6336.*

3 Terms, definitions and symbols

For the purposes of this International Standard, the terms and definitions given in ISO 1122-1 apply. For symbols, see Table 1.

Table 1 — Symbols and abbreviations used in this International Standard

Symbol	Description or term	Unit
a	centre distance [a]	mm
b	facewidth	mm
b_{B}	facewidth of an individual helix of a double helical gear	mm
c_{γ}	mean value of mesh stiffness per unit facewidth	N/(mm·µm)
c'	maximum tooth stiffness of one pair of teeth per unit facewidth (single stiffness)	N/(mm·µm)
$d_{1,2}$	reference diameter of pinion, wheel	mm
$d_{a1,2}$	tip diameter of pinion, wheel	mm
$d_{b1,2}$	base diameter of pinion, wheel	mm
$d_{f1,2}$	root diameter of pinion, wheel	mm
d_{sh}	shaft nominal diameter for bending	mm
d_{shi}	internal diameter of hollow shaft	mm
$d_{w1,2}$	working pitch diameter of pinion, wheel	mm
$d_{Na1,2}$	diameter of a circle defining the outer extremities of the usable flanks of tip chamfered/rounded gear teeth	mm
$f_{H\beta}$	tooth alignment deviation (not including helix form deviation)	µm
f_{ma}	mesh misalignment due to manufacturing deviations	µm
f_{pb}	transverse base pitch deviation (the values of f_{pt} may be used for calculation in accordance with ISO 6336-1, using tolerances complying with ISO 1328-1)	µm
f_{sh}	helix deviation due to elastic deflections	µm
g_{α}	length of path of contact	mm
h	tooth depth	mm
h_{aP}	addendum of basic rack of cylindrical gears	mm
h_{fP}	dedendum of basic rack of cylindrical gears	mm
h_{Fe}	bending moment arm for load application at the outer point of single pair tooth contact	mm
l	bearing span	mm
m^*	relative individual gear mass per unit facewidth referenced to line of action	kg/mm
m_{n}	normal module	mm
m_{red}	reduced gear pair mass per unit facewidth referenced to the line of action	kg/mm
m_{t}	transverse module	mm
$n_{1,2}$	rotation speed of pinion, of wheel	min^{-1}
n_{E}	resonance speed	min^{-1}
p_{bn}	normal base pitch	mm

Table 1 — (*Continued*)

Symbol	Description or term	Unit
p_{bt}	transverse base pitch	mm
pr	protuberance of the tool	mm
q	finishing stock allowance of tooth flank	mm
q_s	notch parameter $s_{Fn} / 2\rho_F$	—
s	tooth thickness	mm
s_{Fn}	tooth-root chord at the critical section	mm
s_R	rim thickness	mm
u	gear ratio [a] $\lvert u \rvert = \lvert z_2/z_1 \rvert \geqslant 1$	—
v	tangential speed (without subscript: at reference circle ≈ tangential speed at pitch circle)	m/s
v_p	velocity parameter	—
$x_{1,2}$	profile shift coefficient of pinion, wheel	—
y_α	running-in allowance for a gear pair	µm
y_β	running-in allowance (equivalent misalignment)	µm
z_n	virtual number of teeth of a helical gear	—
$z_{1,2}$	number of teeth of pinion, of wheel [a]	—
A	auxiliary value for the determination of f_{sh}	mm·µm/N
B	total facewidth of a double helical gear including the gap	mm
C_a	tip relief	µm
C_B	basic rack factor (same rack for pinion and wheel)	—
C_R	gear blank factor	—
E	modulus of elasticity, Young's modulus	N/mm^2
F_m	the mean transverse load at the reference cylinder ($= F_t K_A K_v$)	N
F_t	(nominal) transverse tangential load at reference cylinder	N
F_{tH}	the determinant transverse load at the reference cylinder ($= F_t K_A K_v K_{H\beta}$)	N
F_β	total helix deviation	µm
$F_{\beta x}$	initial equivalent misalignment (before running-in)	µm
$F_{\beta y}$	initial equivalent misalignment (after running-in)	µm
K_v	dynamic factor	—
K_A	application factor	—
$K_{F\alpha}$	transverse load factor (root stress)	—

Table 1 — (*Continued*)

Symbol	Description or term	Unit
$K_{F\beta}$	face load factor (root stress)	—
$K_{H\alpha}$	transverse load factor (contact stress)	—
$K_{H\beta}$	face load factor (contact stress)	—
K_{γ}	mesh load factor (takes into account the uneven distribution of the load between meshes for multiple transmission paths)	—
$M_{1,2}$	auxiliary values for the determination of $Z_{B,D}$	—
N_L	number of cycles	—
N_S	resonance ratio in the main resonance range	—
P	transmitted power	kW
Ra	arithmetic mean roughness value (as specified in ISO 4287)	µm
Rz	mean peak-to-valley roughness (as specified in ISO 4287)	µm
S_F	factor of safety from tooth breakage	—
$S_{F\ min}$	minimum safety factor (tooth breakage)	—
S_H	factor of safety from pitting	—
$S_{H\ min}$	minimum safety factor (pitting)	—
$T_{1,2}$	pinion torque, wheel torque; (nominal)	Nm
Y_F	tooth form factor	—
$Y_{R\ rel\ T}$	relative surface factor	—
Y_S	stress correction factor	—
Y_X	size factor (tooth-root)	—
Y_{β}	helix angle factor (tooth-root)	—
$Y_{\delta\ rel\ T}$	relative notch sensitivity factor	—
Z_v	speed factor	—
$Z_{B,D}$	single pair tooth contact factors for the pinion, for the wheel	—
Z_E	elasticity factor	$\sqrt{N/mm^2}$
Z_H	zone factor	—
Z_L	lubricant factor	—
Z_R	roughness factor affecting surface durability	—
Z_W	work-hardening factor	—
Z_X	size factor (pitting)	—
Z_{β}	helix angle factor (pitting)	—
Z_{ϵ}	contact ratio factor (pitting)	—

Table 1 — (*Continued*)

Symbol	Description or term	Unit
α_n	normal pressure angle	°
α_t	transverse pressure angle	°
α_{wt}	transverse pressure angle at the working pitch cylinder	°
α_{P0}	normal pressure angle of the basic rack for cylindrical gears	°
β	helix angle (without subscript — at the reference cylinder)	—
β_b	base helix angle	°
ϵ_α	transverse contact ratio	—
$\epsilon_{\alpha n}$	virtual contact ratio, transverse contact ratio of a virtual gear	—
ϵ_β	axial overlap ratio	—
ϵ_γ	total contact ratio ($\epsilon_\gamma = \epsilon_\alpha + \epsilon_\beta$)	—
κ_β	factor characterizing the equivalent misalignment after running-in	—
$\nu_{40,50}$	kinematic viscosity at 40 °C, 50 °C	—
ν_f	viscosity parameter	—
ρ_{fP}	root fillet radius of the basic rack for cylindrical gears	mm
ρ_{rel}	radius of relative curvature	mm
ρ_C	radius of relative curvature at the pitch surface	mm
ρ_F	tooth-root fillet radius at the critical section	mm
σ_B	tensile strength	N/mm^2
σ_F	tooth-root stress	N/mm^2
$\sigma_{F\,lim}$	nominal stress number (bending)	N/mm^2
σ_{FE}	allowable stress number (bending) = $\sigma_{F\,lim} Y_{ST}$	N/mm^2
σ_{FG}	tooth-root stress limit	N/mm^2
σ_{FP}	permissible tooth-root stress	N/mm^2
σ_{F0}	nominal tooth-root stress	N/mm^2
σ_H	calculated contact stress	N/mm^2
$\sigma_{H\,lim}$	allowable stress number (contact)	N/mm^2
σ_{HG}	modified allowable stress number = $\sigma_{HP} S_{H\,min}$	N/mm^2
σ_{HP}	permissible contact stress	N/mm^2
σ_{H0}	nominal contact stress	N/mm^2
$\omega_{1,2}$	angular velocity of pinion, or wheel	rad/s

[a] For external gear pairs a, u, z_1 and z_2 are positive; for internal gear pairs a, u and z_2 are negative with z_1 positive.

4 Application

4.1 Design, specific applications

4.1.1 General

Gear designers shall recognize that requirements for different applications vary considerably. Use of the procedures of this International Standard for specific applications demands a careful appraisal of all applicable considerations, in particular:

— the allowable stress of the material and the number of load repetitions;

— the consequences of any percentage of failure (failure rate);

— the appropriate factor of safety.

Design considerations to prevent fractures emanating from stress raisers in the tooth flank, tip chipping and failures of the gear blank through the web or hub should be analysed by general machine design methods.

Any variances according to the following shall be reported in the calculation statement.

a) If a more refined method of calculation is desired or if compliance with the restrictions in clause 4.1 is for any reason impractical, relevant factors may be evaluated according to the basic standard or another application standard.

b) Factors derived from reliable experience or test data may be used instead of individual factors according to this International Standard. Concerning this, the criteria for Method A in ISO 6336-1:1996, 4.1.8, are applicable.

In other respects, rating calculations shall be strictly in accordance with this International Standard if stresses, safety factors, etc. are to be classified as being in accordance with this International Standard.

This International Standard is applicable when the wheel blank, shaft/hub connections, shafts, bearings, housings, threaded connections, foundations and couplings conform to the requirements regarding accuracy, load capacity and stiffness forming the basis for the calculation of the load capacity of gears.

Although the method described in this International Standard is mainly intended for recalculation purposes, by means of iteration it can also be used to determine the load capacities of gears. The iteration is accomplished by selecting a load and calculating the corresponding safety factor against pitting, S_{H1}, for the pinion. If S_{H1} is greater than $S_{H\,min}$ the load is increased, if it is smaller than $S_{H\,min}$ the load is reduced. This is done until the load chosen corresponds to $S_{H1} = S_{H\,min}$. The same method is used for the wheel ($S_{H2} = S_{H\,min}$) and also for the safety factors against tooth breakage, $S_{F1} = S_{F2} = S_{F\,min}$.

4.1.2 Gear data

This International Standard is applicable within the following constraints.

a) Types of gear:

- external and internal, involute spur, helical and double helical gears;
- for double helical gears, it is assumed that the total tangential load is evenly distributed between the two helices; if this is not the case (e.g. due to externally applied axial forces), this shall be taken into account; the two helices are treated as two single helical gears in parallel;
- planetary and other gear trains with multiple transmission paths.

b) Range of the transverse contact ratios of actual spur and helical gear pairs:

— $1{,}2 < \epsilon_\alpha < 2{,}5$ (affects c', c_γ, K_V, $K_{H\beta}$, $K_{F\beta}$, $K_{H\alpha}$ and $K_{F\alpha}$).

c) Range of helix angles:

— β less than or equal to 30° (affects c', c_γ, K_V and $K_{H\beta}$).

d) Basic racks:

— no restriction[1)].

4.1.3 Wheel blank, wheel rim

This International Standard is applicable when s_R, the thickness of the wheel rim under the tooth roots of internal and external gears, is > 3,5 m_n.

4.1.4 Materials

These include steels (affects Z_E, $\sigma_{H\ lim}$, σ_{FE}, K_V, $K_{H\beta}$ and $K_{F\beta}$). For materials and their abbreviations used in this International Standard, see Table 2. For other materials, see ISO 6336-1, ISO 6336-2, ISO 6336-3 and ISO 6336-5

Table 2 — Materials

Material	Abbreviation
Steel (σ_B < 800 N/mm^2)	St
Through-hardening steel, alloy or carbon, through-hardened ($\sigma_B \geqslant$ 800 N/mm^2)	V
Case-hardened steel, case hardened	Eh
Steel, flame or induction hardened	IF
Nitriding steel, nitrided	NT (nitr.)
Through-hardening and case-hardening steel, nitrided	NV (nitr.)
Through-hardening and case-hardening steel, nitrocarburized	NV (nitrocarb.)

4.1.5 Lubrication

The calculation procedures are valid for gears that are spray or oil-bath lubricated using a lubricant approved by the manufacturer/designer of the gears. This validity is further subject to the condition that, at all times of operation, an adequate quantity of approved lubricant is available to the gear mesh. Provision for cooling shall ensure that temperatures assumed for purposes of calculations are not exceeded (affects lubricant film formation i.e. factors Z_L, Z_V and Z_R).

Provided that sufficient lubricant is available to the mesh, grease lubrication of slow speed auxiliaries is not excluded.

1) For all practical purposes, it may be assumed that the proportions of the basic rack of the tools are equal to those of the basic rack of the gear.

4.2 Safety factors

It is necessary to distinguish between the safety factor relative to pitting, S_H, and the safety factor relative to tooth breakage, S_F.

For a given application, adequate gear load capacity is demonstrated by the computed values of S_H and S_F being equal to or greater than the values $S_{H\,min}$ and $S_{F\,min}$, respectively.

Choice of the value of a safety factor should be based on the degree of confidence in the reliability of the available data and the consequences of possible failures.

Important factors to be considered are:

a) the allowable stress numbers used in the calculation are valid for a given probability of failure (the material values in ISO 6336-5:1996 are valid for 1 % probability of damage);

b) the specified quality and the effectiveness of quality control at all stages of manufacture;

c) the accuracy of specification of the service duty and external conditions;

d) tooth breakage is often considered to be a greater hazard than pitting.

Therefore, the chosen value for $S_{F\,min}$ should to be greater than the value chosen for $S_{H\,min}$. For calculation of actual safety factor see 6.1.5 (S_H, pitting) and 7.1.4 (S_F, tooth breakage).

It is recommended that the minimum values of the safety factors should be agreed upon between the purchaser, the manufacturer and the classification authority.

4.3 Input data

The following data shall be available for the calculations:

a) gear data:

a, z_1, z_2, m_n, d_1, d_2, d_{a1}, d_{a2}, b, x_1, x_2, α_n, β, ϵ_α, ϵ_β (see ISO 53:1998, ISO 54:1996);

b) cutter basic rack tooth profile:

h_{a0}, ρ_{a0} (see ISO 53:1998);

c) design and manufacturing data:

C_{a1}, C_{a2}, f_{pb}, $S_{H\,min}$, $S_{F\,min}$, Ra_1, Ra_2, Rz_1, Rz_2;

materials, material hardness and heat treatment details, gear accuracy grades, bearing span l, positions of gears relative to bearings, dimensions of pinion shaft d_{sh} and, when applicable, helix modification (crowning, end relief);

d) power data:

P or T or F_t, n_1, v_1, details of driving and driven machines.

Requisite geometrical data can be calculated according to national standards.

Information to be exchanged between manufacturer and purchaser should include data specifying material preferences, lubrication, safety factor and externally applied forces due to vibrations and overloads (application factor).

4.4 Numerical equations

The units listed in clause 3 shall be used in all calculations. Information that will facilitate the use of this International Standard is provided in annex C of ISO 6336-1:1996.

5 Influence factors

5.1 General

The influence factors, K_v, $K_{H\alpha}$, $K_{H\beta}$, $K_{F\alpha}$ and $K_{F\beta}$, are all dependent on the tooth load. Initially, this is the applied load (nominal tangential load multiplied by the application factor).

The factors are also interdependent and shall therefore be calculated successively as follows:

a) K_v with the applied tangential load $F_t K_A$;

b) $K_{H\beta}$ or $K_{F\beta}$ with the recalculated load $F_t K_A K_v$;

c) $K_{H\alpha}$ or $K_{F\alpha}$ (Method B) with the applied tangential load $F_t K_A K_v K_{H\beta}$.

When a gear drives two or more mating gears or is double helical, it is necessary to substitute K_A by $K_A K_\gamma$. If possible, the mesh load factor, K_γ, should preferably be determined by measurement; alternatively its value may be estimated from the available literature.

The simplification of all influence factors in this clause involves the following assumptions (also see clause 4):

a) that the pinion tooth number $z_1 < 50$;

b) that gears are of solid disc type or with heavy rims.

When details are substantially different from any of the above, refer to ISO 6336-1.

5.2 Nominal tangential load, F_t, nominal torque, T, nominal power, P

The nominal tangential load, F_t, is determined in the transverse plane at the reference cylinder. It is based on the input torque to the driven machine. This is the torque corresponding to the heaviest regular working condition. Alternatively, the nominal torque of the prime mover can be used as a basis if it corresponds to the torque requirement of the driven machine, or some other suitable basis can be chosen.

$$F_t = \frac{2\,000\ T_{1,2}}{d_{1,2}} = \frac{19\,098 \times 1\,000\ P}{d_{1,2}\ n_{1,2}} = \frac{1\,000\ P}{v} \quad (1)$$

$$T_{1,2} = \frac{F_t\, d_{1,2}}{2\,000} = \frac{1\,000\ P}{\omega_{1,2}} = \frac{9\,549\ P}{n_{1,2}} \quad (2)$$

$$P = \frac{F_t\, v}{1\,000} = \frac{T_{1,2}\,\omega_{1,2}}{1\,000} = \frac{T_{1,2}\, n_{1,2}}{9\,549} \quad (3)$$

$$v = \frac{d_{1,2}\,\omega_{1,2}}{2\,000} = \frac{d_{1,2}\,n_{1,2}}{19\,098} \quad (4)$$

$$\omega_{1,2} = \frac{\pi\,n_{1,2}}{30} = \frac{2\,000\,v}{d_{1,2}} = \frac{n_{1,2}}{9\,549} \quad (5)$$

5.3 Non-uniform load, non-uniform torque, non-uniform power

When the transmitted load is not uniform, consideration should be given not only to the peak load and its anticipated number of cycles, but also to intermediate loads and their numbers of cycles. This type of load is classed as a duty cycle and may be represented by a load spectrum. In such cases, the cumulative fatigue effect of the duty cycle shall be considered in rating the gearset. A method of calculating the effect of the loads under this condition is given in ISO TR 10495.

5.4 Maximum tangential load, $F_{t\,max}$, maximum torque, T_{max}, maximum power, P_{max}

This is the maximum tangential load, F_{tmax} (or corresponding torque, T_{max}, corresponding power, P_{max}) in the variable duty range. Its magnitude can be limited by a suitably responsive safety clutch. F_{tmax}, T_{max}, and P_{max} shall be known when safety from pitting damage and from sudden tooth breakage due to loading corresponding to the static stress limit is to be determined (see 5.5).

5.5 Application factor, K_A

5.5.1 General

The factor K_A adjusts the nominal load, F_t, in order to compensate for incremental gear loads from external sources. These additional forces are largely dependent on the characteristics of the driving and driven machines, as well as the masses and stiffness of the system, including shafts and couplings used in service.

It is recommended that the purchaser and manufacturer/designer agree on the value of the application factor with the accord of the classification authority.

5.5.2 Method A — Factor $K_{A\text{-}A}$

K_A shall be determined in this method by means of careful measurements and a comprehensive analysis of the system, or on the basis of reliable operational experience in the field of application concerned (see 5.3).

5.5.3 Method B — Factor $K_{A\text{-}B}$

If no reliable data, obtained as described in 5.5.2, are available, or even as early as the first design phase, it is possible to use the guideline values for K_A as described in annex C.

5.6 Internal dynamic factor, K_v

5.6.1 General

The dynamic factor relates the total tooth load, including internal dynamic effects of a "multi-resonance" system, to the transmitted tangential tooth load. Method B of ISO 6336-1:1996 with modifications is used in this International Standard.

In this procedure it is assumed that the gear pair consists of an elementary single mass and spring system comprising the combined masses of pinion and wheel, and the mesh stiffness of the contacting teeth. It is also assumed that each gear pair functions as a single stage pair, i.e. the influence of other stages in a multiple-stage gear system is ignored. This assumption is only tenable when the torsional stiffness (measured at the base radius of the gears), of the shaft common to a wheel and a pinion is less than the mesh stiffness. See 5.6.3 and annex B.1 for the procedure dealing with very stiff shafts.

Forces caused by torsional vibrations of the shafts and coupled masses are not covered by K_v. These forces should be included with other externally applied forces (e.g. with the application factor).

In multiple mesh gear trains there are several natural frequencies. These can be higher or lower than the natural frequency of a single gear pair with only one mesh. When such gears run in the supercritical range, analysis by Method A is recommended. See ISO 6336-1:1996, 6.3.1.

The specific load for the calculation of K_A is $(F_t K_A)/b$.

If $(F_t K_A)/b$ > 100 N/mm, then $F_m/b = (F_t K_A)/b$.

If $(F_t K_A)/b \leqslant$ 100 N/mm, then F_m/b = 100 N/mm.

When the specific loading $F_t K_A/b$ is < 50 N/mm, a particular risk of vibration exists (under some circumstances, with separation of working tooth flanks), above all for spur or helical gears of coarse quality grade running at high speed.

5.6.2 Calculation of the parameters required for evaluation of K_v

5.6.2.1 Calculation of the reduced mass, m_{red}

a) Calculation of the reduced mass, m_{red}, of a single-stage gear pair

$$m_{red} = \frac{J_1^* J_2^*}{J_1^* r_{b2}^2 + J_2^* r_{b1}^2} \tag{6}$$

where

m_{red} is the reduced mass of a gear pair, i.e. of the mass per unit facewidth of each gear, referred to its base radius or to the line of action;

$J^*_{1,2}$ are the polar moments of inertia per unit facewidth;

$r_{b1,2}$ are the base radii (= 0,5 $d_{b1,2}$)

b) Calculation of reduced mass, m_{red}, of a multi-stage gear pair

See clause B.1.

c) Calculations of reduced mass, m_{red}, of gears of less common designs

For information on the following cases, see clause B.1:

— pinion shaft with diameter at mid-tooth depth, d_{m1}, about equal to the shaft diameter;

— two rigidly connected, coaxial gears;

— one large wheel driven by two pinions;

— planetary gears;

— idler gears.

5.6.2.2 Determination of the resonance running speed (main resonance) of a gear pair

a) Resonance running speed, n_{E1}, of the pinion:

$$n_{E1} = \frac{30 \times 10^{-3}}{\pi\, z_1} \sqrt{\frac{c_\gamma}{m_{red}}} \quad \text{in min}^{-1} \tag{7}$$

with c_γ from annex A

b) Resonance ratio, N

The ratio of pinion speed to resonance speed, the resonance ratio, N, is determined as follows:

$$N = \frac{n_1}{n_{E1}} = \frac{n_1 \pi\, z_1}{30\,000} \sqrt{\frac{m_{red}}{c_\gamma}} \tag{8}$$

The resonance running speed may be above or below the running speed calculated from equation (8) because of stiffness that has not been included (e.g. the stiffness of shafts, bearings and housings) and as a result of damping. For reasons of safety, the resonance range is defined by the following:

$$N_S < N \leqslant 1{,}15 \tag{9}$$

At loads such that $(F_t K_A)/b$ is less than 100 N/mm, the lower limit of resonance ratio N_S is determined:

— if $(F_t K_A)/b < 100$ N/mm, then

$$N_S = 0{,}5 + 0{,}35 \sqrt{\frac{F_t\, K_A}{b \times 100}} \tag{10}$$

— if $(F_t K_A)/b \geqslant 100$ N/mm, then

$$N_S = 0{,}85 \tag{11}$$

5.6.2.3 Gear accuracy and running-in parameters, B_p, B_f, B_k

B_p, B_f and B_k are non-dimensional parameters used to take into account the effect of tooth deviations and profile modifications on the dynamic load.[2)]

$$B_p = \frac{c'\, f_{pb\,eff}}{K_A (F_t/b)} \tag{12}$$

$$B_f = \frac{c'\, f_{f\,eff}}{K_A (F_t/b)} \tag{13}$$

2) The amount C_a of tip relief may only be allowed for gears of accuracy grades in the range 0 to 6 as specified in ISO 1328-1:1995.

$$B_k = \left| 1 - \frac{c' C_a}{K_A (F_t/b)} \right| \tag{14}$$

with

c' as given in annex A;

C_a design amount for profile modification (tip relief at the beginning and end of tooth engagement). A value C_{ay} from running-in shall be substituted for C_a in equation (14) in the case of gears without a specified profile modification. C_{ay} can be obtained from Table 3.

The effective base pitch and profile deviations are those present after running-in. The values of $f_{pb\,eff}$ and $f_{f\,eff}$ are determined by deducting estimated running-in allowances, y_p and y_f, as follows:

$$f_{pb\,eff} = f_{pb1} - y_{p1} \quad \text{or} \quad f_{pb\,eff} = f_{pb2} - y_{p2} \tag{15}$$

whichever is the greater;

$$f_{f\,eff} = f_{f\alpha 1} - y_{f1} \quad \text{or} \quad f_{f\,eff} = f_{f\alpha 2} - y_{f2} \tag{16}$$

whichever is the greater.

5.6.2.4 Running-in allowance, y_α

a) For St, V [3]:

$$y_p = y_\alpha = \frac{160}{\sigma_{H\,lim}} f_{pb} \tag{17}$$

$$y_f = \frac{160}{\sigma_{H\,lim}} f_{f\alpha} \tag{18}$$

b) For Eh, IF, NT (nitr.), NV (nitr.), NV (nitrocarb.) [3]

$$y_p = y_\alpha = 0{,}075 \; f_{pb} \tag{19}$$

$$y_f = 0{,}075 \; f_{f\alpha} \tag{20}$$

5.6.3 Dynamic factor in the subcritical range ($N \leqslant N_S$)

In this sector, resonances may exist if the tooth mesh frequency coincides with N = 1/2 and N = 1/3. The risk of this is slight in the case of precision helical or spur gears, if the latter have suitable profile modification (gears to ISO 1328-1:1995 accuracy grade 6 or better).

When the contact ratio of straight spur gears is small or if the quality is of low grade, K_v can be just as great as in the main resonance-speed range. If this occurs, the design or operating parameters should be altered.

---

3) See Table 2 for an explanation of the abbreviations used.

Resonances at N = 1/4, 1/5, ..., are seldom troublesome because the associated vibration amplitudes are usually small.

For gear pairs where the stiffness of the driving and driven shafts is not equal, in the range $N \approx$ 0,2 ... 0,5, the tooth contact frequency can excite natural frequencies when the torsional stiffness, c, of the stiffer shaft, referred to the line of action, is of the same order of magnitude as the tooth stiffness, i.e. if $c/_{rb}^2$ is of the order of magnitude of c_γ. When this is so, dynamic load increments can exceed values calculated using equation (21).

$$K_v = (NK) + 1 \tag{21}$$

$$K = (C_{v1} B_p) + (C_{v2} \ B_f) + (C_{v3} B_k) \tag{22}$$

where

C_{v1} and C_{v2} allow for pitch and profile deviations while C_{v3} allows for the cyclic variation of mesh stiffness. See Table 3.

A value C_{ay} resulting from running-in shall be substituted for C_a in equation (14) in the case of gears without a specified profile modification. The value of C_{ay} is obtained from Table 3.

See annex A for single tooth stiffness c'.

5.6.4 Dynamic factor in the main resonance range ($N_S < N \leqslant 1{,}15$)

High quality helical gears with high total contact ratio can function satisfactorily in this sector. Spur gears of grade 5 or better as specified in ISO 1328-1:1995 shall have suitable profile modification, as specified in ISO 6336-1:1996 clause 6.4.1 item b).

Subject to above, this factor is equal to:

$$K_v = (C_{v1} B_p) + (C_{v2} B_f) + (C_{v4} B_k) + 1 \tag{23}$$

For C parameters refer to Table 3.

5.6.5 Dynamic factor in the supercritical range ($N \geqslant 1{,}5$)

Resonance peaks can occur at N = 2, 3 ... in this range. However, in the majority of cases, vibration amplitudes are small, since excitation forces with frequencies lower than meshing frequency are usually small.

For some gears in this speed range, it is also necessary to consider dynamic loads due to transverse vibration of the gear and shaft assemblies. When the critical frequency is near to the frequency of rotation, and if this condition cannot be avoided, this shall be taken into account in the evaluation of K_v.

$$K_v = (C_{v5} B_p) + (C_{v6} B_f) + C_{v7} \tag{24}$$

For C parameters, refer to Table 3.

Table 3 — Equations for the calculation of factors C_{v1} to C_{v7} and C_{ay}

	$1 < \epsilon_\gamma \leqslant 2$	$\epsilon_\gamma > 2$	
C_{v1}	0,32	0,32	
C_{v2}	0,34	$\frac{0,57}{\epsilon_\gamma - 0,3}$	
C_{v3}	0,23	$\frac{0,096}{\epsilon_\gamma - 1,56}$	
C_{v4}	0,90	$\frac{0,57 - 0,05\,\epsilon_\gamma}{\epsilon_\gamma - 1,44}$	
C_{v5}	0,47	0,47	
C_{v6}	0,47	$\frac{0,12}{\epsilon_\gamma - 1,74}$	
	$1 < \epsilon_\gamma \leqslant 1,5$	$1,5 < \epsilon_\gamma \leqslant 2,5$	$\epsilon_\gamma > 2,5$
C_{v7}	0,75	$0,125 \sin[\pi(\epsilon_\gamma - 2)] + 0,875$	1,0

$$C_{ay} = \frac{1}{18}\left(\frac{\sigma_{H\,lim}}{97} - 18,45\right)^2 + 1,5$$

NOTE When the material of the pinion (1) is different from that of the wheel (2), C_{ay1} and C_{ay2} are calculated separately; then $C_{ay} = 0,5\,(C_{ay1} + C_{ay2})$.

5.6.6 Dynamic factor in the intermediate range ($1,15 < N < 1,5$)

In this range, the dynamic factor is determined by linear interpolation between K_v at $N = 1,15$, as specified in 5.6.4 and K_v at $N = 1,5$, as specified in 5.6.5.

$$K_v = K_{v(N=1,5)} + \frac{K_{v(N=1,15)} - K_{v(N=1,5)}}{0,35}(1,5 - N) \quad (25)$$

5.7 Face load factor, $K_{H\beta}$

5.7.1 General

The face load factor adjusts gear tooth stresses to allow for the effects of uneven load distribution over the facewidth.

Methods C1 and C2 of ISO 6336-1:1996 are used with modifications in this International Standard.

5.7.2 Face load factor, $K_{H\beta\text{-C1}}$

5.7.2.1 General

The use of method C1 is appropriate for gears having the following characteristics:

a) pinion on solid or hollow shaft, d_{shi}/d_{sh} < 0,5, positioned symmetrically between bearings ($s/l \leqslant 1$; see Figure 2), (an asymmetrically positioned pinion leads to an additional bending deformation, which shall be evaluated and added to f_{ma});

b) pinion diameter about equal to shaft diameter;

c) stiff wheel and case, stiff wheel shaft, stiff bearings;

d) a contact pattern which, under load, extends over the entire facewidth;

e) no additional external loads that act on the pinion shaft (e.g. form shaft couplings);

f) running-in allowance $y_\beta \leqslant$ maximum y_β as specified in 5.7.2.3. A computed $F_{\beta x}$, may be verified using the equation:

$$F_{\beta x} = \frac{K_{H\beta} - 1}{\kappa_\beta \left(\frac{c_\gamma / 2}{F_m / b} \right)} \tag{26}$$

It is recommended that the values used for f_{ma} be verified by inspection checks, such as the tooth contact pattern in the working attitude.

Refer to annex B for application to planetary gears.

5.7.2.2 Mesh misalignment due to manufacturing tolerances, f_{ma}

a) Assembly of gears without any modification or adjustment:

$$f_{ma} = 1{,}0 \quad f_{H\beta} \tag{27}$$

b) Gear pairs with provision for adjustment (lapping or running-in under light load, adjustable bearings or appropriate helix angle modification) and gear pairs suitably crowned:

$$f_{ma} = 0{,}5 \quad f_{H\beta} \tag{28}$$

c) Gear pairs with well designed end relief:

$$f_{ma} = 0{,}7 \quad f_{H\beta} \tag{29}$$

Of a pair of gears, the larger of the values f_β of the pair shall be substituted in equations (27) to (29).

5.7.2.3 Running-in allowance, y_β, running-in factor, κ_β

The amount y_β is that by which the initial equivalent misalignment is reduced by running-in after start of operation. While κ_β is the factor characterizing the equivalent misalignment after running-in. The use of κ_β in calculations is valid only as long as κ_β is proportional to $F_{\beta x}$.

a) For St, V:

$$y_\beta = \frac{320}{\sigma_{H\,lim}} F_{\beta x}; \quad \kappa_\beta = 1 - \frac{320}{\sigma_{H\,lim}} \tag{30}$$

with $y_\beta \leqslant F_{\beta x}$ and $\kappa_\beta \geqslant 0$

when $v \leqslant 5$ m/s: no restriction;

when 5 m/s < $v \leqslant 10$ m/s: the upper limit of y_β is 25 600/$\sigma_{H\,lim}$ corresponding to $F_{\beta x}$ = 80 µm;

when v > 10 m/s: the upper limit of y_β is 12 800/$\sigma_{H\,lim}$ corresponding to $F_{\beta x}$ = 40 µm;

$\sigma_{H\,lim}$ is as specified in ISO 6336-5:1996.

b) For Eh, IF, NT (nitr.), NV (nitr.):

$$y_\beta = 0{,}15\,F_{\beta x}\ ;\ \kappa_\beta = 0{,}85 \qquad (31)$$

For all speeds, the upper limit is y_β = 6 µm, corresponding to $F_{\beta x}$ = 40 µm. When the material of the pinion differs from that of the wheel, $y_{\beta 1}$ and $\kappa_{\beta 1}$ for the pinion, and $y_{\beta 2}$ and $\kappa_{\beta 2}$ for the wheel, shall be determined separately.

The mean of the values:

$$y_\beta = \frac{y_{\beta 1} + y_{\beta 2}}{2}\ ;\ K_\beta = \frac{K_{\beta 1} + K_{\beta 1}}{2} \qquad (32)$$

is used for the calculation.

5.7.2.4 Determination of the face load factor, $K_{H\beta\text{-}C1}$

5.7.2.4.1 Gears with unmodified helices

a) Spur and single helical gears[4]:

$$K_{H\beta} = 1 + \frac{4\,000}{3\,\pi}\,\kappa_\beta\,\frac{c_\gamma}{E}\left(\frac{b}{d_1}\right)^2\left[5{,}12 + \left(\frac{b}{d_1}\right)^2\left(\frac{l}{b} - \frac{7}{12}\right)\right] + \frac{\kappa_\beta\,c_\gamma\,f_{ma}}{2F_m/b} \qquad (33)$$

b) Double helical gears[4) 5)]:

$$K_{H\beta} = 1 + \frac{4\,000}{3\,\pi}\,\kappa_\beta\,\frac{c_\gamma}{E}\left[3{,}2\left(\frac{2\,b_B}{d_1}\right)^2 + \left(\frac{B}{d_1}\right)^4\left(\frac{l}{B} - \frac{7}{12}\right)\right] + \frac{\kappa_\beta\,c_\gamma\,f_{ma}}{F_m/b_B} \qquad (34)$$

4) It is assumed that the entire torque is input at one shaft end. If the torque is input at both shaft ends or in between helices of a double helical gear, a more accurate analysis is necessary.

5) The value of $K_{H\beta}$ is for the more severely stressed helix, which is that nearer to the torqued end of the pinion; tangential load is divided equally between the two helices; i.e. a small gap width compared to the facewidth (B – 2 b_B) $\leqslant$ 0,5 b_B. As for the calculation for $K_{H\beta}$, half the tooth width (incorporating half the gap width) is used, and the obtained values are large. Thus, for double helical gears with a large gap width, method C2 of ISO 6336-1 is appropriate in this case, see 5.7.3.

5.7.2.4.2 Gears with modified helices

a) Spur and single helical gears [4)]

— with partial helix modification [6)] (with compensation for torsional deflection only):

$$K_{\mathrm{H}\beta} = 1 + \frac{4\,000}{3\,\pi}\,\kappa_\beta\,\frac{c_\gamma}{E}\left(\frac{b}{d_1}\right)^4\left(\frac{l}{b} - \frac{7}{12}\right) + \frac{\kappa_\beta\,c_\gamma\,f_{\mathrm{ma}}}{2\,F_\mathrm{m}/b} \tag{35}$$

— with full helix modification (with compensation for torsional and bending deflections):

$$K_{\mathrm{H}\beta} = 1 + \frac{\kappa_\beta\,c_\gamma\,f_{\mathrm{ma}}}{2\,F_\mathrm{m}/b} \quad \text{and} \quad K_{\mathrm{H}\beta} \geqslant 1{,}05 \tag{36}$$

b) Double helical gears[4) 5)]

— with full helix modification[7)] (with compensation for torsional and bending deflections):

$$K_{\mathrm{H}\beta} = 1 + \frac{K_\beta\,c_\gamma\,f_{\mathrm{ma}}}{\mathrm{F}_\mathrm{m}/b_\mathrm{B}} \quad \text{and} \quad K_{\mathrm{H}\beta} \geqslant 1{,}05 \tag{37}$$

The validity of equations (33) to (37) depends upon compliance with 5.7.2.1, a) to f).

5.7.3 Face load factor, $K_{\mathrm{H}\beta\text{-C2}}$

5.7.3.1 General

Taken from the basic standard, this method is arranged so that account is taken of the influences on mesh alignment, of elastic deformations of the pinion and of manufacturing inaccuracies.

$K_{\mathrm{H}\beta}$ shall be calculated from the total mesh misalignment after running in; $F_{\beta\mathrm{y}}$, which comprises the following two components.

— *Systematic error* is taken into account by f_{sh} (mesh misalignment due to shaft deflection). It is primarily caused by pinion shaft deflection, but in principle may include all mechanical deflections able to be evaluated accurately enough in both amount and direction.

— *Random error* is represented by f_{ma} (mesh misalignment due to manufacturing tolerance). The actual direction and amount of misalignment due to manufacturing cannot be evaluated; only the range is limited by manufacturing tolerance (in reference to gear accuracy grade).

---

6) Torsional deflection can be almost completely compensated for by means of a linear tooth trace or helix angle modification. In addition, crowning is necessary when compensation of bending deflection is required.

7) Full modification of both helices is necessary. Partial modification of the helix angle merely to compensate for torsional deflection is not appropriate for double helical gears which are symmetrically positioned between bearings. Torsional and bending deflections can be almost completely compensated for by means of helix angle modification. However, it is often sufficient if only the helix nearest the torque input end is modified; torsional and bending deflections of the other helix tend to compensate each other. This should be verified.

Application of helix correction and crowning:

— *helix correction* is a lead modification applied to compensate for the systematic error, and while theoretically it is possible to apply a helix correction that exactly matches the calculated deflection for a specific load and so eliminates the f_{sh} contribution to $K_{H\beta}$ for that particular load, in practice, varying loads and errors in the evaluation of f_{sh} leave a lasting influence on $K_{H\beta}$ that has to be taken into account;

— *crowning* is a lead modification comprising the best defensive strategy against the random component of misalignment. Since f_{ma} can be in either direction, crowning should be symmetric to the middle of face width.

A more exact and comprehensive analysis in accordance with ISO 6336-1 is recommended if the design does not match the requirements listed in clause 4 or if any of the following items have significant influence on mesh alignment:

— elastic deformations not caused by gear mesh forces but by external loads (e.g. belts, chains, couplings);

— elastic deformations of wheel and wheel shaft;

— elastic deformations and manufacturing inaccuracies of the gear case;

— bearing clearances and deflections;

— arrangements different from those shown in Figure 2;

— any manufacturing or other deformations that indicate the need for a more detailed analysis.

When, by this method, a value of $K_{H\beta}$ greater than 2,0 is calculated, the true value will usually be less. However if the calculated value of $K_{H\beta}$ is greater than 1,5, the design should be reconsidered (e.g. shaft stiffness increased, bearing positions changed, helix accuracy improved).

5.7.3.2 Calculation of $K_{H\beta\text{-C2}}$

The specific loading for the calculation of $K_{H\beta}$ is $(F_t K_A K_v)/b$.

If $(F_t K_A K_v)/b$ > 100 N/mm, then $F_m/b = (F_t K_A K_v)/b$.

If $(F_t K_A K_v)/b \leqslant$ 100 N/mm, then F_m/b = 100 N/mm.

$$K_{H\beta} = 1 + \frac{F_{\beta y} \, c_\gamma}{2 F_m / b} \text{ applies when } K_{H\beta} \leqslant 2 \qquad (38)$$

with the value of c_γ taken from annex A.

Note that b is the smaller of the facewidths of pinion and wheel measured at the pitch circles. Chamfers or rounding of tooth ends shall be ignored. (For double helical gears, $b = 2b_B$.)

5.7.3.3 Mesh misalignment after running-in, $F_{\beta y}$

$$F_{\beta y} = F_{\beta x} - y_\beta \qquad (39)$$

where

$F_{\beta x}$ is the mesh misalignment before running-in (see 5.7.4);

y_β is the running-in allowance (see 5.7.2.3).

5.7.4 Mesh misalignment before running-in, $F_{\beta x}$

The mesh alignment before running-in, $F_{\beta x}$, is the absolute value of the sum of manufacturing deviations, and pinion and shaft deflections, measured in the plane of action.

For gear pairs without verification of the favourable position of the contact pattern[8)]:

$$F_{\beta x} = 1{,}33\, B_1 f_{sh} + B_2\, f_{ma} \tag{40}$$

with B_1 and B_2 taken from Table 4.

For gear pairs with verification of the favourable position of the contact pattern (e.g. by adjustment of bearings):

$$F_{\beta x} = \left| 1{,}33\, B_1\, f_{sh} - f_{H\beta 5} \right| \tag{41}$$

where

$f_{H\beta 5}$ is the maximum helix slope deviation for ISO accuracy grade 5 (see ISO 1328-1:1995).

By subtracting $f_{H\beta 5}$, allowance is made for the compensatory roles of elastic deformation and manufacturing deviations.

Table 4 — Constants for use in equation (40)

No.	Helix modification		Equation constant	
	Type	Amount	B_1	B_2
1	None	—	1	1
2	Ventral crowning only	$C_\beta = 0{,}5\, f_{ma}$ [a]	1	0,5
3	Central crowning only	$C_\beta = 0{,}5\, (f_{ma} + f_{sh})$ [a]	0,5	0,5
4 [b]	Helix correction only	Corrected shape calculated to match torque being analysed	0,1 [c]	1,0
5	Helix correction plus central crowning	Case 2 plus case 4	0,1 [c]	0,5
6	End relief	Appropriate amount $C_{I(II)}$, see annex D	0,7	0,7

a For appropriate crowning, C_β, see annex D.

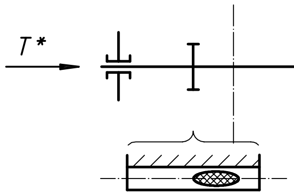
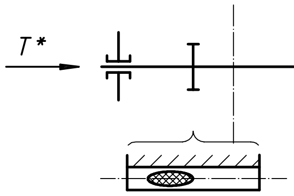
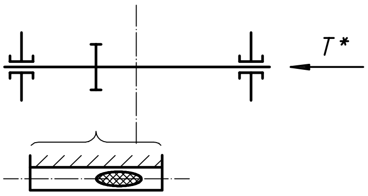
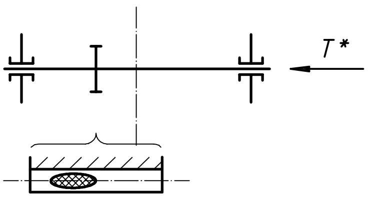
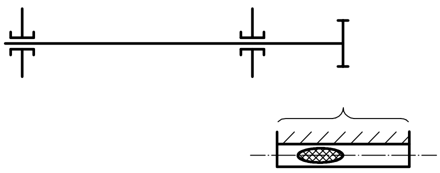
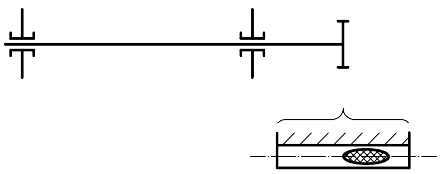
b Predominantly applied for applications with constant load conditions.

c Valid for very best practice of manufacturing, otherwise higher values appropriate.

5.7.4.1 Minimum values for $K_{H\beta\text{-}C2}$

For gear pairs without helix correction or crowning, the minimum value for $K_{H\beta}$ is 1,25; for gear pairs with appropriate helix correction and crowning, the minimum value for $K_{H\beta}$ is 1,10. A favourable contact pattern shall be verified.

8) With a favourable position of the contact pattern, the elastic deformations and the manufacturing deviations compensate each other (see Figure 1, compensative roles).

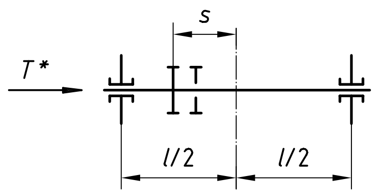
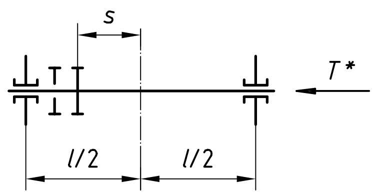
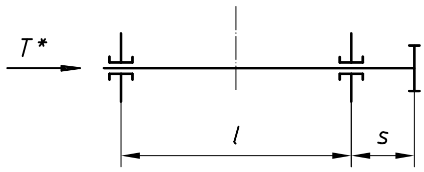
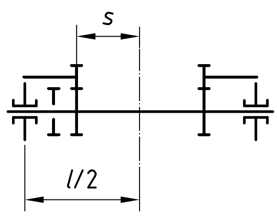
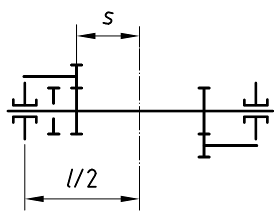
Figure	Position of contact pattern	Determination of $F_{\beta x}$
a)	Contact pattern lies towards mid bearing span 	$F_{\beta x}$ in accordance with equation (41) (compensative)
b)	Contact pattern lies away from mid bearing span 	$F_{\beta x}$ in accordance with equation (40) (augmentative)
c)	Contact pattern lies towards mid bearing span 	$F_{\beta x}$ in accordance with equation (40) $\lvert K' \rvert \times l \times s/d_1^2\ (d_1/d_{sh})^4 \leqslant B^*$ (augmentative) $F_{\beta x}$ in accordance with equation (41) $\lvert K' \rvert \times l \times s/d_1^2\ (d_1/d_{sh})^4 > B^*$ (compensative)
d)	Contact pattern lies away from mid bearing span 	$F_{\beta x}$ in accordance with equation (41) $\lvert K' \rvert \times l \times s/d_1^2\ (d_1/d_{sh})^4 \geqslant B^* - 0{,}3$ (augmentative) $F_{\beta x}$ in accordance with equation (41) $\lvert K' \rvert \times l \times s/d_1^2\ (d_1/d_{sh})^4 < B^* - 0{,}3$ (compensative)
e)	Contact pattern lies towards the bearing span 	$F_{\beta x}$ in accordance with equation (40) (augmentative)
f)	Contact pattern lies away from the bearing span 	$F_{\beta x}$ in accordance with equation (41) (compensative)

NOTE Figures a) to d) show the most common mounting arrangement with pinion between bearings. Figures e) to f) show the overhung pinion.

*T** Input or output torqued end, not dependent on direction of rotation.

*B** *B** = 1 for spur and single helical gears; *B** = 1,5 for double helical gears, the peak load intensity occurs on the helix near to the torqued end.

Figure 1 — Rules for determination of $F_{\beta x}$ with regard to contact pattern position

Factor K' with stiffening[a]	**Factor** K' without stiffening[a]	**Figure**	**Arrangement**
0,48	0,8	a)	with $s/l < 0,3$ 
– 0,48	– 0,8	b)	with $s/l < 0,3$ 
1,33	1,33	c)	with $s/l < 0,3$ 
– 0,36	– 0,6	d)	with $s/l < 0,3$ 
– 0,6	– 1,0	e)	with $s/l < 0,3$ 

[a] When $d_1/d_{sh} \geqslant 1,15$, stiffening is assumed; when $d_1/d_{sh} < 1,15$, there is no stiffening; furthermore, scarcely any or no stiffening at all is to be expected when a pinion slides on a shaft and feather key or similar fitting, nor when normally shrink fitted.

T^* Input or output torqued end, not dependent on direction of rotation.

A dashed line indicates the less deformed helix of a double helical gear.

Determine f_{sh} from the diameter in the gaps of double helical gearing mounted centrally between bearings.

Figure 2 — Constant K' to substitute in equations (42) and (43) for calculation of f_{sh}

5.7.4.2 Equivalent misalignment, f_{sh}

For spur and single helical gears:

$$f_{sh} = \frac{F_m}{b} 0{,}023 \left[\left| 1 + K' \frac{l\,s}{d_1^2} \left(\frac{d_1}{d_{sh}} \right)^4 - 0{,}3 \right| + 0{,}3 \right] \left(\frac{b}{d_1} \right)^2 \qquad (42)$$

The calculation of f_{sh} for double helical gears is relative to the helix nearest to the shaft end which is driven or which drives the load.

$$f_{sh} = \frac{F_m}{b} 0{,}046 \left[\left| 1{,}5 + K' \frac{l\,s}{d_1^2} \left(\frac{d_1}{d_{sh}} \right)^4 - 0{,}3 \right| + 0{,}3 \right] \left(\frac{b_B}{d_1} \right)^2 \qquad (43)$$

where

$b = 2b_B$;

b_B is the width of one helix.

In equations (42) and (43), K', s and l are according to Figure 2.

In Figure 2, the pinions shown in dashed lines indicate those helices of double helical gears, which have the lower value of f_{sh} and normal shrink fit (for a normal shrink fit, the supporting effect is negligible). The root diameter shall be somewhat greater than the shaft diameter.

5.7.4.3 Misalignment due to manufacturing inaccuracies, f_{ma}

The misalignment due to manufacturing inaccuracies f_{ma} equals the helix tolerance $f_{H\beta}$:

$$f_{ma} = f_{H\beta} \qquad (44)$$

The greater of the wheel and pinion value should be used.

NOTE As it is theoretically possible that manufacturing tolerances of pinion, wheel and shaft alignment may sum up to the worst case, satisfying load distribution should be verified by e.g. contact pattern control.

5.8 Face load factor, $K_{F\beta}$

$$K_{F\beta} = K_{H\beta}^{N_F} \qquad (45)$$

a) if $b/h \geqslant 3$, then

$$N_F = \frac{(b/h)^2}{1 + b/h + (b/h)^2} = \frac{1}{1 + h/b + (h/b)^2} \qquad (46)$$

b) if $b/h < 3$, then

$$N_F = 0{,}692\,3 \qquad (47)$$

where

b is the smaller of the facewidths of pinion and wheel measured at the pitch circles. Chamfers or rounding of tooth ends shall be ignored. For double helical gears the width of one helix, b_B, shall be substituted.

h is the tooth height from tip to root: $h = (d_a - d_f)/2$.

5.9 Transverse load factors, $K_{H\alpha}$, $K_{F\alpha}$

5.9.1 General

The transverse load factors account for the effect of the non-uniform distribution of transverse load between several pairs of simultaneously contacting gear teeth, as follows: $K_{H\alpha}$ for surface stress, and $K_{F\alpha}$ for tooth-root stress. Method B of ISO 6336-1:1996 is applied.

5.9.2 Determination of the transverse load factors

Equations (48) and (49) are based on the assumption that the base pitch deviations appropriate to the gear accuracy specified are distributed around the circumference of the pinion and wheel, as is consistent with normal manufacturing practice. They do not apply when the gear teeth are intentionally modified.

In the following equations use c_γ from annex A and y_α from 5.9.4.

— For gears with total contact ratio $\epsilon_\gamma \leqslant 2$:

$$K_{H\alpha} = K_{F\alpha} = \frac{\epsilon_\gamma}{2}\left(0,9 + 0,4\,\frac{c_\gamma\,(f_{pb} - y_\alpha)}{F_{tH}/b}\right) \quad (48)$$

— For gears with total contact ratio $\epsilon_\gamma > 2$:

$$K_{H\alpha} = K_{F\alpha} = 0,9 + 0,4\sqrt{\frac{2\,(\epsilon_\gamma - 1)}{\epsilon_\gamma}}\;\frac{c_\gamma\,(f_{pb} - y_\alpha)}{F_{tH}/b} \quad (49)$$

In equations (48) and (49), the larger of $(f_{pb1} - y_{\alpha 1})$ and $(f_{pb2} - y_{\alpha 2})$ is used.

5.9.3 Limiting conditions for $K_{H\alpha}$ and $K_{F\alpha}$

When, in accordance with equations (48) and (49, and

$$\text{when } K_{H\alpha} = K_{F\alpha} > \frac{\epsilon_\gamma}{\epsilon_\alpha Z_\epsilon^2}\text{, then for } K_{H\alpha} \text{ and } K_{F\alpha}\text{, substitute } \frac{\epsilon_\gamma}{\epsilon_\alpha Z_\epsilon^2} \quad (50)$$

and when $K_{H\alpha} < 1,0$, and respectively $K_{F\alpha} < 1,0$, then substitute for $K_{H\alpha}$, and respectively for $K_{F\alpha}$, the limit value 1,0.

It is recommended that the accuracy of helical gears be chosen so that $K_{H\alpha}$ and $K_{F\alpha}$ are no greater than ϵ_α. As a consequence, it may be necessary to limit the base pitch deviation tolerances of gears of coarse quality grade.

5.9.4 Running-in allowance, y_α

a) For St, V:

$$y_\alpha = \frac{160}{\sigma_{\mathrm{H\,lim}}} f_{\mathrm{pb}} \tag{51}$$

— if $v \leqslant 5$ m/s, no restriction;

— if 5 m/s < $v \leqslant 10$ m/s, the upper limit of $y_\alpha = 12\,800/\sigma_{\mathrm{H\,lim}}$, corresponding to $f_{\mathrm{pb}} < 80$ µm;

— if $v > 10$ m/s, the upper limit of $y_\alpha = 6\,400/\sigma_{\mathrm{H\,lim}}$, corresponding to $f_{\mathrm{pb}} < 40$ µm.

b) For Eh, IF, NT (nitr.) et NV (nitr.):

$$y_\alpha = 0{,}075 f_{\mathrm{pb}} \tag{52}$$

for all speeds with the restriction: the upper limit of $y_\beta = 3$ µm, corresponding to $f_{\mathrm{pb}} = 40$ µm.

6 Calculation of surface durability (pitting)

6.1 Basic formulae

6.1.1 General

The calculation of surface durability is based on the contact stress, σ_{H}, at the pitch point or at the inner (lowest) point of single pair tooth contact. The higher of the two values obtained shall be used to determine capacity. Contact stress, σ_{H}, and the permissible contact stress, σ_{Hp}, shall be calculated separately for wheel and pinion; σ_{H} shall be $\leqslant \sigma_{\mathrm{Hp}}$.

6.1.2 Determination of contact stress, σ_{H}, for the pinion

Contact stress, σ_{H}, for the pinion is calculated as follows:

$$\sigma_{\mathrm{H0}} = Z_{\mathrm{B}}\, \sigma_{\mathrm{H0}} \sqrt{K_{\mathrm{A}} K_{\mathrm{v}} K_{\mathrm{H\beta}} K_{\mathrm{H\alpha}}} \leqslant \sigma_{\mathrm{HP}} \tag{53}$$

with

$$\sigma_{\mathrm{H0}} = Z_{\mathrm{H}}\, Z_{\mathrm{E}}\, Z_{\in}\, Z_{\beta} \sqrt{\frac{F_{\mathrm{t}}}{d_1 b} \frac{u \pm 1}{u}} \quad \text{(use the negative sign for internal gears)} \tag{54}$$

where

σ_{H0} is the nominal contact stress at the pitch point; this is the stress induced in flawless (error free) gearing by application of static nominal torque;

b is the facewidth (for a double helical gear $b = 2b_{\mathrm{B}}$), and the value b of mating gears is the smaller of the facewidths at the pitch circles of pinion and wheel, ignoring any intentional transverse chamfers or tooth-end rounding; neither unhardened portions of surface-hardened gear tooth flanks nor the transition zones, shall be included;

Z_{B} is the single pair tooth contact factor for the pinion (see 6.2).

6.1.3 Determination of contact stress, σ_H, for the wheel

Contact stress, σ_H, for the pinion is calculated as

$$\sigma_H = Z_D \sigma_{H0} \sqrt{K_A K_v K_{H\beta} K_{H\alpha}} \leqslant \sigma_{HP} \tag{55}$$

where

Z_D is the single pair tooth contact factor for the wheel (see 6.2).

The total tangential load in the case of gear trains with multiple transmission paths, planetary gear systems, or split-path gear trains is not quite evenly distributed over the individual meshes (depending on design, tangential speed and manufacturing accuracy). This shall be taken into consideration by substituting $K_\gamma K_A$ for K_A in equation (53) and equation (55) to adjust the average tangential load per mesh as necessary; see clause 5.

6.1.4 Determination of permissible contact stress, σ_{HP}, for long life

In this International Standard, Method B of ISO 6336-2:1996 is used.

$$\sigma_{HP\,ref} = \frac{\sigma_{H\,lim}}{S_{H\,min}} Z_L\, Z_v\, Z_R\, Z_W\, Z_X = \frac{\sigma_{HG}}{S_{H\,min}} \tag{56}$$

The permissible contact stress (long life) shall be derived from equation (56), with the influence factors $\sigma_{H\,lim}$, $S_{H\,min}$, Z_L, Z_V, Z_R Z_W and Z_X calculated according to this International Standard. However, according to ISO 6336-2, the values of $\sigma_{H\,lim}$ are validated for $N_L = 5 \times 10^7$ load cycles (for St, V, Eh) or 2×10^6 load cycles [for If, NT (nitr.), NV (nitr.), NV (nitrocar.)]. This number is likely to be exceeded in the life of a marine gear. If this is not the case, refer to ISO 6336-2 for the limited life range. Nevertheless, values of $\sigma_{HP\,ref}$ derived from equation (56) may be substituted for σ_{Hp}, given optimum conditions, material, lubrication, manufacturing and experience; otherwise the values for σ_{Hp} are obtained for material quality MQ according to ISO 6336-5:1996 using equation (57):

For St, V, Eh:

$$\sigma_{HP} = 0{,}92\, \sigma_{HP\,ref} \left(\frac{10^{10}}{N_L} \right)^{0{,}0157} = \frac{\sigma_{HG}}{S_{H\,min}} \tag{57}$$

For If, NT (nitr.), NV (nitr.), NV (nitrocar.):

$$\sigma_{HP} = 0{,}92\, \sigma_{HP\,ref} \left(\frac{10^{10}}{N_L} \right)^{0{,}0098} = \frac{\sigma_{HG}}{S_{H\,min}}$$

6.1.5 Safety factor for surface durability, S_H

S_H shall be calculated separately for the pinion and wheel:

$$S_H = \frac{\sigma_{HG}}{\sigma_H} > S_{H\,min} \tag{58}$$

with σ_{HG} for endurance according to equation (57); σ_H shall be in accordance with equation (53) for the pinion, and with equation (55) for the wheel (see 6.1.1).

NOTE This is the calculated safety factor with regard to contact stress (Hertzian pressure). The corresponding factor relative to torque capacity is equal to the square of S_H.

For the minimum safety factor for surface durability, $S_{H\,min}$, and probability of failure, see 4.1.3 of ISO 6336-1:1996.

6.2 Single pair tooth contact factors, Z_B, Z_D

When $Z_B > 1$ or $Z_D > 1$, the factors Z_B and Z_D are used to transform the contact stress at the pitch point of spur gears to the contact stress at the inner (lowest) limit of the single pair tooth contact of the pinion or the wheel. See 6.1.1.

a) Internal gears

Z_D is always to be taken as unity.

b) Spur gears

Determine M_1 (quotient of ρ_{relC} at the pitch point $\rho_{rel\,B}$ at the inner limit (lowest point) of single tooth pair contact of the pinion) and M_2 (quotient of $\rho_{rel\,C}$ by $\rho_{rel\,D}$ of the wheel) from:

$$M_1 = \frac{\tan \alpha_{wt}}{\sqrt{\left[\sqrt{\frac{d_{a1}^2}{d_{b1}^2} - 1} - \frac{2\pi}{z_1}\right]\left[\sqrt{\frac{d_{a2}^2}{d_{b2}^2} - 1} - (\epsilon_\alpha - 1)\frac{2\pi}{z_2}\right]}} \quad (59)$$

$$M_2 = \frac{\tan \alpha_{wt}}{\sqrt{\left[\sqrt{\frac{d_{a2}^2}{d_{b2}^2} - 1} - \frac{2\pi}{z_2}\right]\left[\sqrt{\frac{d_{a1}^2}{d_{b1}^2} - 1} - (\epsilon_\alpha - 1)\frac{2\pi}{z_1}\right]}} \quad (60)$$

(See 6.5.2 for the calculation of the profile contact ratio ϵ_α.)

If $M_1 > 1$, then $Z_B = M_1$; if $M_1 \leqslant 1$, then $Z_B = 1{,}0$.

If $M_2 > 1$, then $Z_D = M_2$; if $M_2 \leqslant 1$, then $Z_D = 1{,}0$.

c) Helical gears with $\epsilon_\beta \geqslant 1$

$Z_B = Z_D = 1$

d) Helical gears with $\epsilon_\beta < 1$

Z_B and Z_D are determined by linear interpolation between the values for spur and helical gearing with $\epsilon_\beta \geqslant 1$:

$$\begin{aligned} Z_B &= M_1 - \epsilon_\beta \left(M_1 - 1\right);\ Z_B \geqslant 1 \\ Z_D &= M_2 - \epsilon_\beta \left(M_2 - 1\right);\ Z_D \geqslant 1 \end{aligned} \quad (61)$$

If Z_B or Z_D are set to unity, the contact stresses calculated using equations (53) or (55) are the values for the contact stress at the pitch cylinder.

The methods in 6.2 apply to the calculation of contact stress when the pitch point lies in the path of contact. If the pitch point is determinant and lies outside the path of contact, then Z_B and / or Z_D or both shall be determined for contact at the adjacent tip circle. For helical gears when ϵ_β is less than 1,0, Z_B and Z_D shall be determined by linear interpolation between the values (determined at the pitch point or at the adjacent tip circle as appropriate) for spur gears and those helical gears with $\epsilon_\beta \geqslant 1$.

6.3 Zone factor, Z_H

The zone factor, Z_H, accounts for the influence on Hertzian pressure of tooth flank curvature at the pitch point and transforms the tangential force at the reference cylinder to the normal force at the pitch cylinder.

$$Z_H = \sqrt{\frac{2 \cos \beta_b \cos \alpha_{wt}}{\cos^2 \alpha_t \sin \alpha_{wt}}} \quad (62)$$

6.4 Elasticity factor, Z_E

The elasticity factor, Z_E, takes into account the influences of the material properties E (modulus of elasticity) and ν (Poisson's ratio) on the contact stress. As this International Standard is only applicable to steel gears, Z_E is fixed to

$$Z_H = 189{,}8 \quad (63)$$

6.5 Contact ratio factor, Z_ϵ

6.5.1 General

The contact ratio factor, Z_ϵ, accounts for the influence of the transverse contact and overlap ratios on the surface load capacity of cylindrical gears.

a) Spur gears:

$$Z_\epsilon = \sqrt{\frac{4 - \epsilon_\alpha}{3}} \quad (64)$$

The conservative value of $Z_\epsilon = 1{,}0$ may be chosen for spur gears having a contact ratio less than 2,0.

b) Helical gears:

If $\epsilon_\beta < 1$, then

$$Z_\epsilon = \sqrt{\frac{4 - \epsilon_\alpha}{3}(1 - \epsilon_\beta) + \frac{\epsilon_\beta}{\epsilon_\alpha}} \quad (65)$$

If $\epsilon_\beta \geqslant 1$, then

$$Z_\epsilon = \sqrt{\frac{1}{\epsilon_\alpha}} \quad (66)$$

6.5.2 Transverse contact ratio, ϵ_α

$$\epsilon_\alpha = g_\alpha / p_{bt} \quad (67)$$

with length of path of contact:

$$g_{\alpha} = \frac{1}{2}\left[\sqrt{d_{a1}^2 - d_{b1}^2} \pm \sqrt{d_{a2}^2 - d_{b2}^2}\right] - a \sin\alpha_{wt} \quad (68)$$

and transverse base pitch:

$$p_{bt} = m_t \pi \cos\alpha_t \quad (69)$$

The positive sign is used for external gears, the negative sign for internal gears.

Equation (69) is only valid if the path of contact is effectively limited by the tip circle of the pinion and the wheel and not, for example, by undercut tooth profiles.

6.5.3 Overlap ratio, ϵ_β

$$\epsilon_\beta = \frac{b \sin\beta}{\pi\, m_n} \quad (70)$$

See 6.1.2 for the definition of facewidth.

6.6 Helix angle factor, Z_β

The helix angle factor, Z_β, takes account of the influence on surface stress of the helix angle.

$$Z_\beta = \sqrt{\cos\beta} \quad (71)$$

6.7 Allowable stress numbers (contact), $\sigma_{H\,lim}$

ISO 6336-5 provides information on commonly used gear materials, methods of heat treatment, and the influence of gear quality on values for allowable stress numbers, $\sigma_{H\,lim}$, derived from test results of standard reference test gears.

See, too, ISO 6336-5 for requirements concerning material and heat treatment for qualities ML, MQ, ME and MX. Material quality MQ shall be chosen for marine gears, unless otherwise agreed.

6.8 Influences on lubrication film formation, Z_L, Z_v and Z_R

6.8.1 General

As described in ISO 6336-2, Z_L accounts for the influence of nominal viscosity of the lubricant, Z_v, for the influence of tooth-flank velocities, and Z_R for the influence of surface roughness on the formation of the lubricant film in the contact zone. Method B of ISO 6336-2:1996 is used in this International Standard.

Factors shall be determined for the softer material when the hardness of meshing gears is different.

6.8.2 Lubricant factor, Z_L

Z_L can be calculated using equations (72) to (75):

$$Z_L = C_{ZL} + \frac{4\,(1{,}0 - C_{ZL})}{\left[1{,}2 + \frac{80}{v_{50}}\right]^2} = C_{ZL} + \frac{4\,(1{,}0 - C_{ZL})}{\left[1{,}2 + \frac{134}{v_{40}}\right]^2} \quad (72)$$

a) If $\sigma_{H\,lim} < 850$ /mm^2, then

$$C_{ZL} = 0{,}83 \tag{73}$$

b) If 850 N/mm^2 $\leqslant \sigma_{H\,lim} \leqslant$ 1 200 N/mm^2, then

$$C_{ZL} = \frac{\sigma_{H\,lim}}{4\,375} + 0{,}635\,7 \tag{74}$$

c) If $\sigma_{H\,lim}$ > 1 200 N/mm^2, then

$$C_{ZL} = 0{,}91 \tag{75}$$

Alternatively, Z_L can be calculated from equation (76):

$$Z_L = C_{ZL} + 4(1{,}0 - C_{ZL})\,\nu_f \tag{76}$$

where

$\nu_f = 1/(1{,}2 + 80/\nu_{50})^2$

using the viscosity parameters from Table 5.

Table 5 — Viscosity parameters

ISO viscosity class	VG 32 [a]	VG 46 [a]	VG 68 [a]	VG 100	VG 150	VG 220	VG 320
Nominal viscosity							
ν_{40} mm^2/s	32	46	68	100	150	220	320
ν_{50} mm^2/s	21	30	43	61	89	125	180
Viscosity parameter ν_f	0,040	0,067	0,107	0,158	0,227	0,295	0,370

[a] Only for high speed transmission.

6.8.3 Speed factor, Z_v

Z_v can be calculated using equations (77) and (78):

$$Z_v = C_{Zv} + \frac{2\,(1{,}0 - C_{Zv})}{\sqrt{0{,}8 + \frac{32}{v}}} \tag{77}$$

where

$$C_{Zv} = C_{ZL} + 0{,}02 \tag{78}$$

see equations (73) to (75) for the values of C_{ZL}.

Alternatively, Z_v can be calculated from equation (79):

$$Z_v = C_{Zv} + 2\,(1{,}0 - C_{Zv})\,v_p \tag{79}$$

where the velocity parameter $v_p = 1/(0{,}8 + 32/v)^{0{,}5}$

6.8.4 Roughness factor, Z_R

6.8.4.1 Calculation of Z_R

Z_R may be calculated using the following equations:

$$Z_R = \left(\frac{3}{Rz_{10}}\right)^{C_{ZR}} \tag{80}$$

where

$$Z_R = \left(\frac{1{,}29a^{1/3}}{Rz_1 + Rz_2}\right)^{C_{ZR}} \tag{81}$$

6.8.4.2 Roughness values

$$Rz = \frac{Rz_1 + Rz_2}{2} \tag{82}$$

$Rz_{1,2}$ is measured on several tooth flanks. The mean roughness Rz_1 (pinion flank) and the mean roughness Rz_2 (wheel flank) shall be determined for their surface condition after manufacture, including any running-in treatment planned as a manufacturing, commissioning or in-service process, when safe to assume that this will take place. If the stated roughness is an Ra value (= CLA value; = AA value), the following approximation may be used for the conversion:

$$Ra = \text{CLA} = \text{AA} = \frac{Rz}{6} \tag{83}$$

$$Rz_{10} = Rz \sqrt[3]{\frac{10}{\rho_{red}}} \tag{84}$$

$$\rho_{red} = \frac{\rho_1 \rho_2}{\rho_1 + \rho_2} \tag{85}$$

where

$$\rho_{1,2} = 0{,}5 d_{b1,2} \tan \alpha_t \tag{86}$$

(also applicable for internal gears, d_b then being negative sign)

6.8.4.3 Material dependent index, C_{ZR}

a) If $\sigma_{H\,lim} < 850$ N/mm^2, then

$$C_{ZR} = 0{,}15 \tag{87}$$

b) If 850 N/mm$^2 \leqslant \sigma_{H\,lim} \leqslant 1\,200$ N/mm^2, then

$$C_{ZR} = 0{,}32 - 0{,}000\,2\sigma_{H\,lim} \tag{88}$$

c) If $\sigma_{\mathrm{H\,lim}} > 1\,200$ N/mm², then

$$C_{ZR} = 0{,}08 \tag{89}$$

6.9 Work hardening factor, Z_W

As described in ISO 6336-2, the work hardening factor, Z_W, takes account of the increased surface durability due to the meshing of a steel wheel (structural steel, through-hardened steel) with a pinion significantly (≈ 200 HV or more) harder than the wheel and having smooth tooth flanks ($Rz \leqslant 6$ µm, otherwise effects of wear are not covered by this International Standard). Method B of ISO 6336-2:1996 is applied, as follows.

If HB < 130, then

$$Z_W = 1{,}2 \tag{90}$$

If 130 ⩽ HB ⩽ 470, then

$$Z_W = 1{,}2 - \frac{\mathrm{HB} - 130}{1\,700} \tag{91}$$

If HB > 470, then

$$Z_W = 1{,}0 \tag{92}$$

where HB is the Brinell hardness of the tooth flanks of the softer gear of the pair.

6.10 Size factor, Z_X

By means of Z_X, account is taken of statistical evidence indicating that the stress levels at which fatigue damage occurs decrease with an increase of component size (larger number of weak points in structure), as a consequence of the influence on subsurface defects of the smaller stress gradients that occur (theoretical stress analysis) and the influence of size on material quality (effect on forging process, variations in structure, etc.). Important influence parameters are:

a) material quality (furnace charge, cleanliness, forging);

b) heat treatment, depth of hardening, distribution of hardening;

c) radius of flank curvature;

d) module: in the case of surface hardening; depth of hardened layer relative to the size of teeth (core supporting-effect).

For through-hardened gears and for surface-hardened gears with adequate case depth relative to tooth size and radius of relative curvature, the size factor, Z_X, is taken to be 1,0.

7 Calculation of tooth bending strength

7.1 Basic formulae

7.1.1 General

As specified in ISO 6336-3, the maximum tensile stress at the tooth-root may not exceed the permissible bending stress for the material. This is the basis for rating the bending strength of gear teeth.

The actual tooth-root stress, σ_F, and the permissible bending stress, σ_{FP}, shall be calculated separately for pinion and wheel; σ_F shall be less than σ_{FP}.

7.1.2 Determination of tooth root stress σ_F

Method B of ISO 6336-3:1996 is used in this International Standard.

Tooth root stress, σ_F, is calculated as follows:

$$\sigma_F = \sigma_{F0} K_A K_v K_{F\beta} K_{F\alpha} \leqslant \sigma_{FP} \tag{93}$$

with

$$\sigma_{F0} = \frac{F_t}{b m_n} Y_F Y_S Y_\beta \tag{94}$$

In the case of gear trains with multiple transmission paths, planetary gear systems or split-path gear trains, the total tangential load is not quite evenly distributed over the individual meshes (depending on design, tangential speed and manufacturing accuracy). This shall be taken into consideration by substituting $K_\gamma K_A$ for K_A in equation (93) to adjust the average tangential load per mesh as necessary (see clause 5).

When the facewidth b (for a double helical gear $b = 2\ b_B$) is larger than that of its mating gear, the bending strength of its teeth shall be based on the smaller facewidth plus a length, not exceeding one module of any extension at each end. However, if it is foreseen that, because of crowning or end relief, contact does not extend to the end of face, then the smaller facewidth shall be used for both pinion and wheel. Facewidth b is the facewidth at the root cylinder of the gear.

7.1.3 Determination of permissible tooth root stress, σ_{FP}

$$\sigma_{FP\ ref} = \frac{\sigma_{FE}}{S_{F\ min}} Y_{\delta\ rel\ T} Y_{R\ rel\ T} Y_X = \frac{\sigma_{FG}}{S_{F\ min}} \tag{95}$$

According to ISO 6336-3, the values of $\sigma_{F\ lim}$ and σ_{FE} are validated for $N_L = 3 \times 10^6$ load cycles. This number is likely to be exceeded in the life of a marine gear. If this is not the case, refer to ISO 6336-3 for the limited life range. Nevertheless, values of $\sigma_{HP\ ref}$ derived from equation (95) may be substituted for σ_{FP}, given optimum conditions, material, manufacturing and experience; otherwise the value for σ_{FP} is obtained by equation (96).

$$\sigma_{FP} = 0{,}92\,\sigma_{FP\ ref} \left(\frac{10^{10}}{N_L}\right)^{0{,}01} = \frac{\sigma_{FG}}{S_{F\ min}} \tag{96}$$

7.1.4 Safety factor for bending strength, S_F

The factor S_F shall be calculated using the following equation:

$$S_F = \frac{\sigma_{FG}}{\sigma_F} \geqslant S_{F\ min} \tag{97}$$

S_F is calculated separately for pinion and wheel, with σ_{FG} calculated in accordance with equation (95) or (96) as appropriate, and σ_F obtained with equation (93).

More information on the safety factor and probability of failure can be found in ISO 6336-1:1996, 1.3.

7.2 Form factor, Y_F

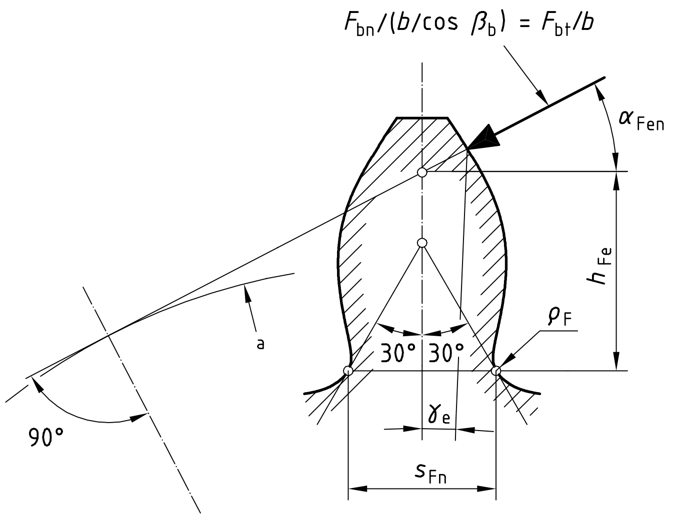
7.2.1 General

Y_F is the form factor by means of which the influence of tooth form on nominal bending stress is taken into account. Y_F is relevant to the application of load at the outer limit of single pair tooth contact. (method B of ISO 6336-3:1996).

Values of Y_F are determined for spur gears and the virtual spur gears of helical gears. Virtual spur gears have the virtual number of teeth z_n. See 7.2.4 for the calculation of z_n and other virtual gear parameters.

Y_F shall be determined separately for wheel and pinion from the following equation (see Figure 3).

$$Y_F = \frac{\frac{6h_{Fe}}{m_n}\cos\alpha_{Fen}}{\left(\frac{s_{Fn}}{m_n}\right)^2 \cos\alpha_n} \quad (98)$$



a Base circle.

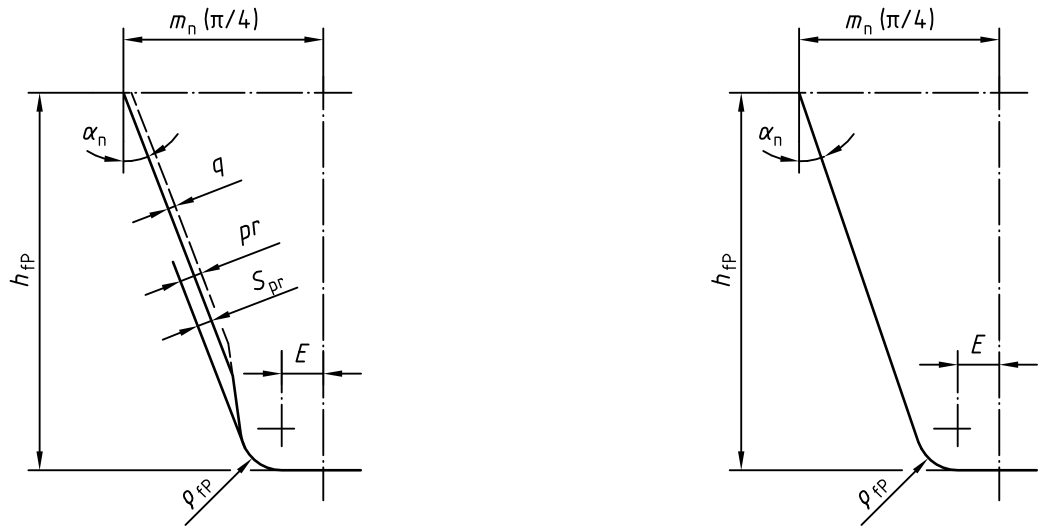
Figure 3 — Determination of dimensions of tooth-root chord at the critical section

The equations given here apply to all basic rack tooth profiles with and without undercut, but with the following restrictions:

a) the contact point of the 30° tangent lies on the tooth-root fillet;

b) the basic rack profile of the gearing has a root fillet;

c) the teeth were generated using tools such as hobs or planer-cutters having rack form teeth.



a) With undercut

b) Without undercut

Figure 4 — Dimensions and basic rack profile of teeth (finished profile)

7.2.2 Parameters required for the determination of Y_F

First, determine the auxiliary values E, G and H:

$$E = \frac{\pi}{4} m_n - h_{fP} \tan\alpha_n + \frac{s_{pr}}{\cos\alpha_n} - (1 - \sin\alpha_n) \frac{\rho_{fP}}{\cos\alpha_n} \tag{99}$$

where

$s_{pr} = pr - q$ (see Figure 4);

$s_{pr} = 0$, when gears are not undercut (see Figure 4)

$$G = \frac{\rho_{fP}}{m_n} - \frac{h_{fP}}{m_n} + x \tag{100}$$

$$H = \frac{2}{z_n}\left(\frac{\pi}{2} - \frac{E}{m_n}\right) - \frac{\pi}{3} \tag{101}$$

Next, use G and H together with $\theta = \pi/6$ as a seed value (on the right hand side) in equation (102).

$$\theta = \frac{2G}{z_n} \tan\theta - H \tag{102}$$

Use the newly calculated θ and again apply equation (102). Continue using equation (102) until there is no significant change in successive values of θ. Generally, the function converges after two or three iterations of equation (102). Use this final value of θ in equations (103), (104) and (105).

Tooth-root normal chord, s_{Fn}:

$$\frac{s_{Fn}}{m_n} = z_n \sin\left(\frac{\pi}{3} - \theta\right) + \sqrt{3}\left(\frac{G}{\cos\theta} - \frac{\rho_{fP}}{m_n}\right) \tag{103}$$

Radius of root fillet, ρ_F:

$$\frac{\rho_F}{m_n} = \frac{\rho_{fP}}{m_n} + \frac{2G^2}{\cos\theta\,(z_n \cos^2\theta - 2G)} \tag{104}$$

Bending moment arm, h_{Fe}:

$$\alpha_{en} = \arccos\left(\frac{d_{bn}}{d_{en}}\right) \tag{105}$$

$$\gamma_e = \frac{0{,}5\,\pi + 2 \times x \times \tan\alpha_n}{z_n} + \operatorname{inv}\alpha_n - \operatorname{inv}\alpha_{en} \tag{106}$$

$$\alpha_{Fen} = \alpha_{en} - \gamma_e = \tan\alpha_{en} - \operatorname{inv}\alpha_n - \frac{0{,}5\pi + 2 \times x \times \tan\alpha_n}{z_n} \tag{107}$$

$$\frac{h_{Fe}}{m_n} = 0{,}5\left[(\cos\gamma_e - \sin\gamma_e \tan\alpha_{Fen})\,\frac{d_{en}}{m_n} - z_n \cos\left(\frac{\pi}{3} - \theta\right) - \frac{G}{\cos\theta} + \frac{\rho_{fP}}{m_n}\right] \tag{108}$$

7.2.3 Internal gears

It is assumed that the value of the form factor of a special rack can be substituted as an approximate value of the form factor of an internal gear. The profile of such a rack should be a version of the basic rack profile, modified in such as way that it would generate the normal profile, including tip and root circles, of an exact counterpart gear of the internal gear. The load direction angle is α_n (see Figure 5).

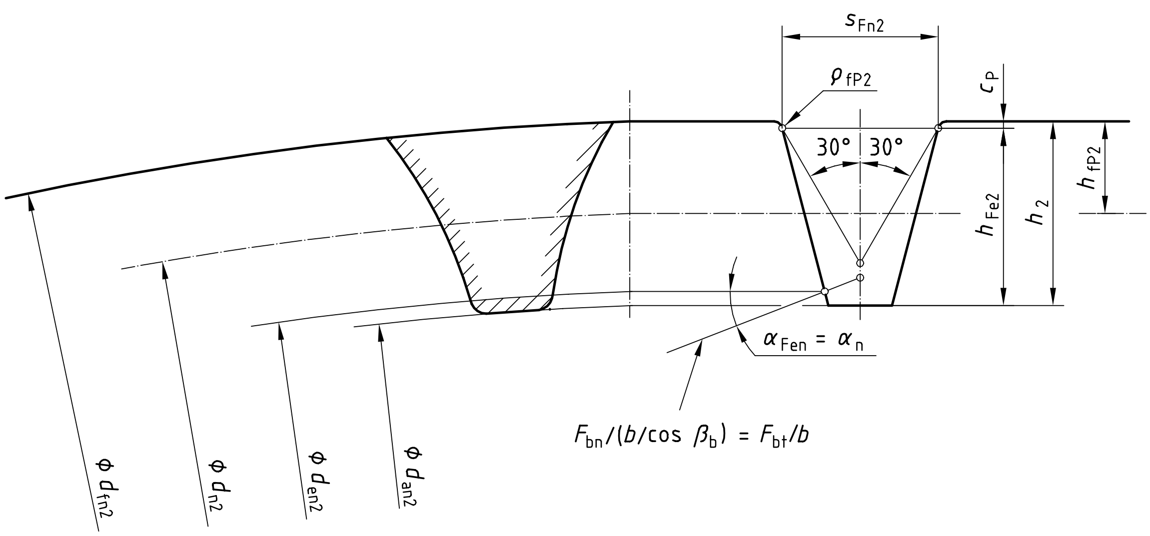


Figure 5 — Parameters for determination of form factor, Y_F, of an internal gear

The values to be used in equation (98) are determined as follows:

Tooth-root normal chord, s_{Fn2}:

$$\frac{s_{Fn2}}{m_n} = 2\left[\frac{\pi}{4} + \frac{h_{fP2} - \rho_{fP2}}{m_n}\tan\alpha_n + \frac{\rho_{fP2} - s_{pr}}{m_n\cos\alpha_n} - \frac{\rho_{fP2}}{m_n}\cos\frac{\pi}{6}\right] \tag{109}$$

where

ρ_{fP2} is tool radius (see below).

Bending moment arm, h_{Fe2}:

$$\frac{h_{Fe2}}{m_n} = \frac{d_{en2} - d_{fn2}}{2\,m_n} - \left[\frac{\pi}{4} + \left(\frac{h_{fP2}}{m_n} - \frac{d_{en2} - d_{fn2}}{2\,m_n}\right)\tan\alpha_n\right]\tan\alpha_n - \frac{\rho_{fP2}}{m_n}\left(1 - \sin\frac{\pi}{6}\right) \tag{110}$$

where

ρ_{fP2} is tool radius (see below);

d_{en2} is derived from equation (121) adding the subscript 2;

d_{fn2} is derived in the same way as d_{an} [equation (121), note that $d_{fn2} - d_{f2} = d_{n2} - d_2$].

Obtain h_{fP2} from equation (111), refer to equation (113) and related information for ρ_{fP2}.

$$h_{fP2} = \frac{d_{n2} - d_{fn2}}{2} \tag{111}$$

Root fillet radius, ρ_{F2}, tool radius, ρ_{fP2}:

When the root fillet radius, ρ_{F2}, is known, it shall be used. Otherwise:

$$\rho_{F2} = \rho_{fP2} = \frac{c_P}{1 - \sin\alpha_n} = \frac{h_{f2} - h_{Nf2}}{1 - \sin\alpha} = \frac{d_{Nf2} - d_{f2}}{2(1 - \sin\alpha_n)} \tag{112}$$

(d_{Nf2} represents the diameter of a circle near the tooth-root, containing the limits of the usable flanks of an internal gear).

If sufficient data are not available, the following approximation may be used:

$$\rho_{F2} = \rho_{fP2} = 0{,}15 m_n \tag{113}$$

Ensure that the correct sign is used; see the footnote in Table 1.

7.2.4 Parameters for the virtual gear

$$\beta_B = \arccos\sqrt{1 - (\sin\beta\cos\alpha_n)^2} \tag{114}$$

$$z_n = \frac{z}{\cos^2\beta_b\cos\beta} \tag{115}$$

Approximation:

$$z_n \approx \frac{z}{\cos^3\beta} \tag{116}$$

$$\epsilon_{\alpha n} = \frac{\epsilon_\alpha}{\cos^2\beta_b} \tag{117}$$

$$d_n = \frac{d}{\cos^2\beta_b} = m_n z_n \tag{118}$$

$$p_{bn} = \pi\, m_n \cos\alpha_n \tag{119}$$

$$d_{bn} = d_n \cos\alpha_n \tag{120}$$

$$d_{an} = d_n + d_a - d \tag{121}$$

$$d_{en} = 2\,\frac{z}{|z|}\sqrt{\left[\sqrt{\left(\frac{d_{an}}{2}\right)^2 - \left(\frac{d_{bn}}{2}\right)^2} - \frac{\pi\, d\, \cos\beta\, \cos\alpha_n}{|z|}\,(\epsilon_{\alpha n} - 1)\right]^2 + \left(\frac{d_{bn}}{2}\right)^2} \tag{122}$$

The value of z is positive for external gears and negative for internal gears (see clause 3, footnote 2).

7.3 Stress correction factor, Y_S

The stress correction factor, Y_S, is used to convert the nominal bending stress to local tooth root stress. Y_S shall be determined separately for pinion and wheel. Y_S is valid in the range $1 \leqslant q_s < 8$.

$$Y_S = (1{,}2 + 0{,}13\,L)\, q_s^{\left[1/(1{,}21 + 2{,}3/L)\right]} \tag{123}$$

where

$$L = \frac{s_{Fn}}{h_{Fe}} \tag{124}$$

$$q_s = \frac{s_{Fn}}{2\,\rho_F} \tag{125}$$

with

s_{Fn} from equation (103) for external gears, equation (109) for internal gears;

h_{Fe} from equation (108) for external gears, equation (110) for internal gears;

ρ_F from equation (104) for external gears, equation (113) for internal gears.

7.4 Helix angle factor, Y_β

The tooth-root stress of a virtual spur gear, calculated as a preliminary value, is converted by means of the helix factor, Y_β, to that of the corresponding helical gear. By this means, the oblique orientation of the lines of mesh contact is taken into account (lesser tooth-root stress).

If $\epsilon_\beta > 1$ and $\beta \leqslant 30°$, then

$$Y_\beta = 1 - \frac{\beta}{120°} \tag{126}$$

If $\epsilon_\beta > 1$ and $\beta > 30°$, then

$$Y_\beta = 0{,}75 \tag{127}$$

If $\epsilon_\beta \leqslant 1$ and $\beta \leqslant 30°$, then

$$Y_\beta = 1 - \epsilon_\beta \frac{\beta}{120°} \tag{128}$$

If $\epsilon_\beta \leqslant 1$ and $\beta > 30°$, then

$$Y_\beta = 1 - 0{,}25\,\epsilon_\beta \tag{129}$$

7.5 Tooth-root reference strength, σ_{FE}

ISO 6336-5 provides information on values of $\sigma_{F\,lim}$ and σ_{FE} for the more popular gear materials. The requirements for heat treatment processes and material quality for quality grades ML, MQ and ME are also included.

The quality MQ shall be used for marine gears unless otherwise agreed.

7.6 Relative notch sensitivity factor, $Y_{\delta\,rel\,T}$

$Y_{\delta\,rel\,T}$ approximately indicates the overstress tolerance of the material in the root fillet region. Method B of ISO 6336-3:1996 is used in this International Standard.

$$Y_{\delta\,rel\,T} = \frac{1 + \sqrt{\rho' \chi^*}}{1 + \sqrt{\rho' \chi^*_T}} \tag{130}$$

where

ρ' is the slip-layer thickness taken from Table 6 as a function of the material;

χ^*_T is the value for the standard reference test gear: $\chi^*_T = 1{,}2$;

χ^* is the relative stress gradient calculated using the following equation:[9)]

9) Applies for module m = 5 mm. The influence of size is covered by the factor Y_X (see 7.8).

$$\chi^* = 0{,}2(1 + 2q_s) \qquad (131)$$

where

q_s is the notch parameter obtained from equation (125).

Table 6 — Values for slip-layer thickness ρ'

Material [a]	ρ' mm
NT (nitr.), NV (nitr.), NV (nitrocar.)	0,100 5
V	
yield point σ_s = 500 N/mm^2	0,028 1
yield point σ_s = 600 N/mm^2	0,019 4
limit of proportionality $\sigma_{0,2}$ = 800 N/mm^2	0,006 4
limit of proportionality $\sigma_{0,2}$ = 1 000 N/mm^2	0,001 4
Eh, If	0,003 0

[a] See Table 2 for an explanation of abbreviations used.

7.7 Relative surface factor, $Y_{R\,rel\,T}$

The surface factor, $Y_{R\,rel\,T}$, accounts for the influence on tooth-root stress of the surface condition in the tooth-roots. Primarily, this is dependent on surface roughness in the tooth-root fillets.

The influence of surface condition on tooth-root bending strength does not depend solely on the surface roughness in the tooth-root fillets, but also on the size and shape (the problem of 'notches within a notch'). This subject has not been sufficiently well studied to date for it to be taken into account in this International Standard. The method applied here is only valid when scratches or similar defects deeper than $2 \times Rz$ are not present.

NOTE $2 \times Rz$ is the preliminary estimated value.

Besides surface texture, other known influences on tooth bending strength include residual compressive stresses (shot peening), grain boundary oxidation and chemical effects. When fillets are shot peened, perfectly shaped, or both, a value slightly greater than that obtained from the graph should be substituted for $Y_{R\,rel\,T}$. When grain boundary oxidation or chemical effects are present, a smaller value than that indicated by the graph should be substituted for $Y_{R\,rel\,T}$.

a) For V, Eh, IF when Rz < 1 µm

$$Y_{YR\,rel\,T} = 1{,}12 \qquad (132)$$

b) for NT (nitr.), NV (nitr.), NV (nitrocar.) when Rz < 1 µm

$$Y_{YR\,rel\,T} = 1{,}025$$

c) for V, Eh, IF if $Rz \geqslant 1$ µm

$$Y_{YR\,rel\,T} = 1{,}674 - 0{,}529\,(Rz + 1)^{0{,}1} \qquad (133)$$

d) for NT (nitr.), NV (nitr.), NV (nitrocar.) when $Rz \geqslant 1$ µm

$$Y_{\text{YR rel T}} = 4{,}299 - 3{,}259\,(Rz + 1)^{0{,}005} \qquad (134)$$

7.8 Size factor, Y_X

Y_X is used to allow for the influence of size on

— the probable distribution of weak points in the material structure,

— the stress gradients that in materials theory decrease with increasing dimensions,

— material quality, and

— as regards the quality of forging, presence of defects, etc.

Y_X is calculated in accordance with Table 7.

Table 7 — Size factor (root), Y_X

Material [a]	**Normal module** m_n	**Size factor** Y_X
V	$m_n \leqslant 5$ $5 < m_n < 30$ $30 \leqslant m_n$	$Y_X = 1{,}0$ $Y_X = 1{,}03 - 0{,}006\ m_n$ $Y_X = 0{,}85$
Eh, IF, NT (nitr.) NV (nitr.) NV (nitrocar.)	$m_n \leqslant 5$ $5 < m_n < 25$ $25 \leqslant m_n$	$Y_X = 1{,}0$ $Y_X = 1{,}05 - 0{,}01\ m_n$ $Y_X = 0{,}8$

[a] See Table 2 for an explanation of the abbreviations used.

Annex A
(normative)

Tooth stiffness parameters c' and c_γ

A.1 General

A tooth stiffness parameter represents the requisite load over 1 mm facewidth, directed along the line of action[10)] to produce in line with the load, the deformation amounting to 1 µm, of one or more pairs of deviation-free teeth in contact.

Single stiffness, c', is the maximum stiffness of a single-tooth pair of a spur gear pair. It is approximately equal to the maximum stiffness of a tooth pair in single pair contact[11)]. Single stiffness c' for helical gears is the maximum stiffness normal to the helix of one tooth pair.

Mesh stiffness, c_γ, is the mean value of stiffness of all the teeth in a mesh.

Method B from ISO 6336-1:1996, used in this International Standard, is applicable in the range $x_1 \geqslant x_2 \leqslant 2$.

A.2 Single stiffness c'

A.2.1 Calculation of c'

For specific loading, $F_t K_A / b \geqslant 100$ N/mm^2:

$$c' = 0{,}8\, c'_{th}\, C_R\, C_B \cos \beta \qquad \text{(A.1)}$$

A.2.2 Theoretical single stiffness, c'_{th}

$$c'_{th} = \frac{1}{q'} \qquad \text{(A.2)}$$

where

$$q' = C_1 + \frac{C_2}{z_{n1}} + \frac{C_3}{z_{n2}} + (C_4 x_1) + \frac{(C_5 x_1)}{z_{n1}}1 + (C_6 x_2) + \frac{(C_7 x_2)}{z_{n2}}2 + (C_8 x_1^2) + (C_9 x_2^2) \qquad \text{(A.3)}$$

---

10) The tooth deflection can be determined approximately using F_t (F_m F_{tH} ...) instead of F_{bt}. Conversion from F_t to F_{bt} (load tangent to the base cylinder) is covered by the relevant factors, or the modifications resulting from this conversion can be ignored when compared with other uncertainties (e.g. tolerances on the measured values).

11) c' at the outer limit of single pair tooth contact, can be assumed to approximate the maximum value of single stiffness when $\epsilon_\alpha > 1{,}2$.

Table A.1 — Constants for equation (A.3)

C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
0,047 23	0,155 51	0,257 91	– 0,006 35	– 0,116 54	– 0,001 93	– 0,241 88	0,005 29	0,001 82

A.2.3 Gear blank factor, C_R

C_R = 1 for gears made from solid disc blanks. For other gears:

$$C_R = 1 + \frac{\ln(b_s/b)}{5e^{s_R/(5m_n)}} \quad \text{(A.4)}$$

Boundary conditions:

when $b_s/b < 0{,}2$, substitute $b_s/b = 0{,}2$;

when $b_s/b > 1{,}2$, substitute $b_s/b = 1{,}2$.

See Figure A.1 for symbols.

A.2.4 Basic rack factor, C_B

C_B can be obtained from equation (A.5):

$$C_B = \left[1 + 0{,}5\,(1{,}2 - \frac{h_{fP}}{m_n})\right][1 - 0{,}02\,(20° - \alpha_{Pn})] \quad \text{(A.5)}$$

A.2.5 Additional information

a) Internal gears: approximate values of the theoretical single stiffness of internal gear teeth can be determined from equations (A.2), (A.3), by the substitution of infinity for z_{n2}.

b) Specific load $(F_t K_A/b) < 100$ N/mm

$$c' = 0{,}8\, c'_{th} C_R C_B \cos\beta \left[\frac{F_t K_A}{100\, b}\right]^{0{,}25} \quad \text{(A.6)}$$

c) The above is based on steel gear pairs, for other materials and material combinations, refer to ISO 6336-1:1996, clause 9.

A.2.6 Mesh stiffness, c_γ

For spur gears with $\epsilon_\alpha \geqslant 1{,}2$ and helical gears with $\beta \leqslant 30°$, the mesh stiffness:

$$c_\gamma = c'\,(0{,}75\,\epsilon_\alpha + 0{,}25) \quad \text{(A.7)}$$

with c' according to equation (A.1).

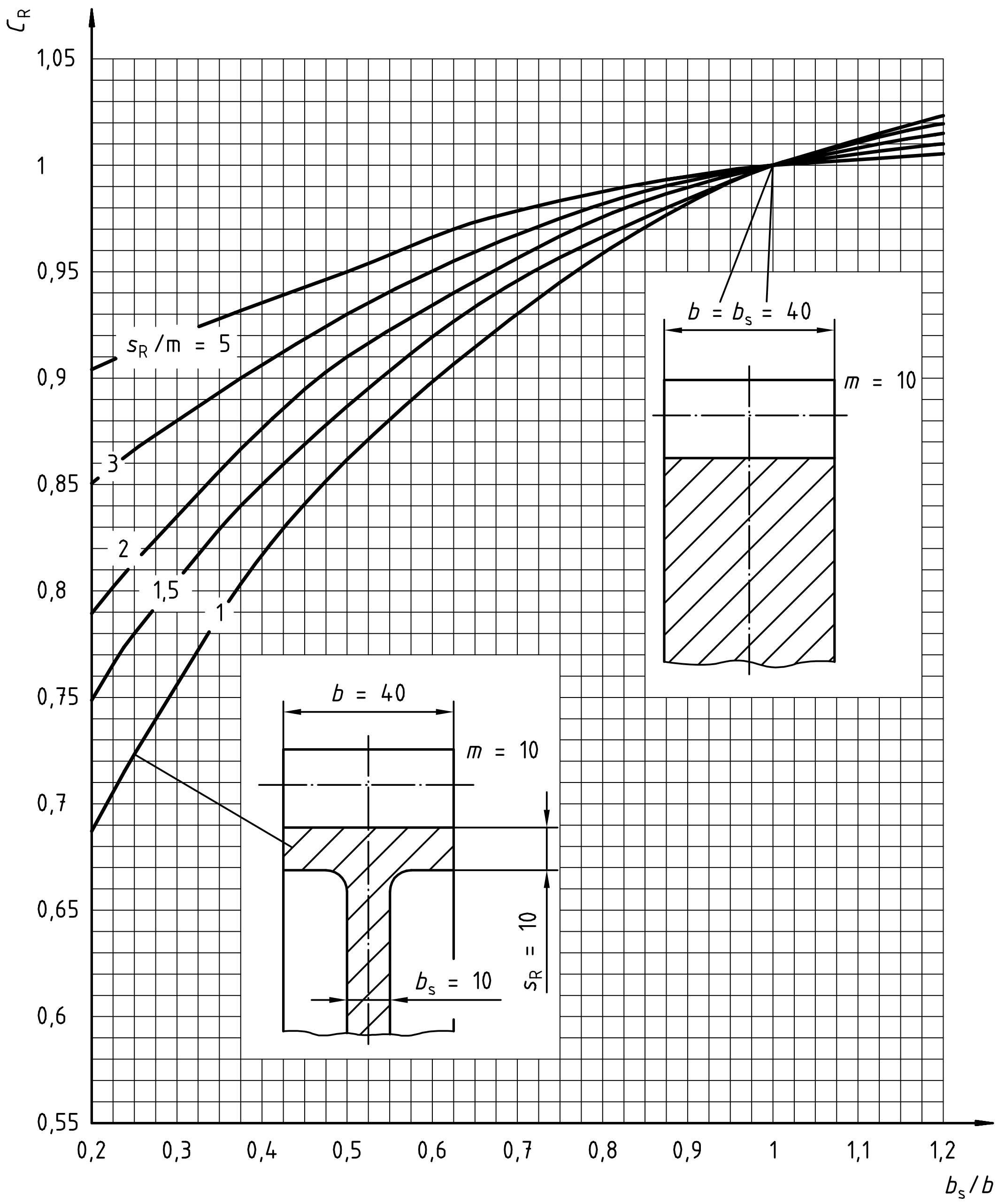


Figure A.1 — Wheel blank factor, C_R; mean values for mating gears of similar or stiffer wheel blank design

Annex B
(normative)

Special features of less common gear designs

B.1 Dynamic factor, K_V, for planetary gears

B.1.1 General

In gear trains which include multiple mesh gears such as idler gears and in epicyclic gearing, planet and sun gears, there are several natural frequencies. These can be higher or lower than the natural frequency of a single gear pair which has only one mesh.

Although values of K_v determined with the formulae in this International Standard shall be considered as unreliable, they can nevertheless be useful as preliminary assessments. It is recommended that if possible they be re-assessed using a more accurate procedure.

Method A should be preferred for the analysis of less common transmission designs. Refer to 6.1.1 of ISO 6336-1:1996 for further information.

B.1.2 Calculation of the relative mass of a gear with external teeth

Refer to 5.6.2.

B.1.3 Resonance speed determination for less common gear designs

B.1.3.1 General

The resonance speed determination for less common gear designs should be made using Method A. However, other methods may be used to approximate the effects. Some examples are the following:

a) pinion shaft with diameter at mid-tooth depth, d_{m1}, about equal to the shaft diameter;

b) two rigidly connected, coaxial gears;

c) one large wheel driven by two pinions;

d) planetary gears;

e) idler gears.

B.1.3.2 Pinion shaft diameter equal to diameter at mid-tooth depth, d_{m1}

The high torsional stiffness of the pinion shaft is to a great extent compensated by the shaft mass. Thus the resonance speed can be calculated in the normal way, using the mass of the pinion (toothed portion) and the normal mesh stiffness c_γ.

B.1.3.3 Two, rigidly connected, coaxial gears

The mass of the larger of the connected gears shall be included.

B.1.3.4 One large wheel driven by two pinions

As the mass of the wheel is normally much greater than the masses of the pinions, each mesh can be considered separately, i.e:

— as a pair comprising the first pinion and the wheel, and

— as a pair comprising the second pinion and the wheel.

B.1.3.5 Planetary gears

Because of the many transmission paths that include varieties of stiffness other than mesh stiffness, the vibratory behaviour of planetary gears is very complex. The calculation of dynamic load factors using simple formulae, such as method B, is generally quite inaccurate. Nevertheless, method B, modified as follows, can be used for a first estimate of K_v. This estimate should, if possible, be verified be means of a subsequent detailed theoretical or experimental analysis, or on the basis of operating experience. See also the introductory comments to this annex.

a) Sun gear/planet gear

The reduced mass for the determination of the resonance speed, n_{E1}, of the sun gear is given by:

$$m_{red} = \frac{J^*_{pla} \; J^*_{sun}}{\left(p \; J^*_{pla} \; r^2_{b\,sun} \right) + \left(J^*_{sun} \; r^2_{b\,pla} \right)} \tag{B.1}$$

where

J^*_{pla} and J^*_{sun} are the moments of inertia per unit facewidth of one planet gear and the sun gear respectively in kilogram square millimetres per millimetre ($kg \cdot mm^2/mm$).

$r_{b\,sun}$ $= 0{,}5\ d_{b\,sun}$;

$r_{b\,pla}$ $= 0{,}5\ d_{b\,pla}$;

p is the number of planet gears in the gear stage under consideration.

The value, m_{red}, determined from equation (B.1), shall be used in the equation for calculating N (see 5.6.2.2) where a mesh stiffness approximately equal to a single planetary gear shall be used for the mesh stiffness c_γ and the number of teeth on the sun gear shall be used for z_1.

Concerning planetary gears, it should be noted that F_t in equations (12) to (14) for B_p, B_f, B_k (see 5.6.2.3) is equal to the total tangential load applied to the sun gear, divided by the number of planet gears.

b) Planet gear/annulus gear rigidly connected to the gear case

In this case, the mass of the annulus gear can be assumed to be infinite. Thus, the relative mass becomes equal to the referred mass of the planet gear. This can be determined as follows:

$$m_{red} = \frac{J^*_{pla}}{r^2_{bpla}} \tag{B.2}$$

with the notation as above.

c) Planet gear/rotating annulus gear

In this case, the referred mass of the annulus gear can be determined as for an external wheel, and the planet gear relative mass calculated in accordance with equation (B.2). The procedure described in B.1.3.4 shall be used when the annulus gear meshes with several planet gears

B.1.3.6 Idler gears

Approximate values can be obtained from the following when the driving and driven gears are roughly of the same size, the idler gear also about the same size or a little larger:

— reduced mass

$$m_{\text{red}} = \frac{2}{\left(\dfrac{r_{\text{b1}}^2}{J_1^*} + \dfrac{2 r_{\text{b2}}^2}{J_2^*} + \dfrac{r_{\text{b3}}^2}{J_3^*} \right)} \tag{B.3}$$

— mesh stiffness

$$c_\gamma = 0{,}5 \left(c_{\gamma 1,2} + c_{\gamma 2,3} \right) \tag{B.4}$$

where

J_1^*, J_2^*, J_3^* are the moments of inertia per unit facewidth of the pinion, the idler and the wheel respectively in kilogram square millimetres per millimetre ($\text{kg} \cdot \text{mm}^2/\text{mm}$);

$c_{\gamma 1,2}$ is the mesh stiffness of the driver and idler gear pair;

$c_{\gamma 2,3}$ is the mesh stiffness of the driver and idler gear pair (see annex A for the determination of c_γ). More accurate analysis is recommenced if the reference speed is in the range 0,6 < < 1,5.

If the idler is substantially larger than the driving and driven gears or, if the driving gear or driven gear is substantially smaller than the two other gears, K_v can be calculated separately for each meshing pair, i.e.

— for the driver-idler gear combination, and

— for the idler-driven gear combination.

Values of m_{red} calculated in accordance with the above may be substituted in equation (7) of 5.6.2.2 to determine the resonance speed.

An accurate analysis is recommended for cases not mentioned here.

B.2 Face load factors, $K_{\text{H}\beta}$, $K_{\text{F}\beta}$, for simple planetary gears

The face load factor takes into account the effects of the non-uniform distribution of load over the gear facewidth on the surface stress ($K_{\text{H}\beta}$) and tooth-root stress ($K_{\text{F}\beta}$).

According to 7.2.3.1 a) and 7.6.1 of ISO 6336-1:1996, Method C1 is suitable for the gears of single planetary gear-sets in which the following features are found[12)].

Either the sun or the planet carrier and sometimes the annulus gear is free to float; otherwise a comparable division of load between the individual planet gears is achieved by greater accuracy of manufacture, flexibility or both. If necessary, refer to the above mentioned clauses for details.

Determine:

— mesh misalignment due to manufacturing deviation f_{ma} in accordance with 5.7.2.2,

— running-in factor χ_β in accordance with 5.7.2.3,

— mesh stiffness in accordance with annex A.

Any unequal division of the total tangential load between the planet gears is covered by factor $K\gamma$ (see clause 5). Thus, for these gears, $F_m = (F_t\, K_A\, K_\gamma\, K_v)$, and with F_t being the nominal tangential load transmitted per mesh, also the sum of the loads over both helices of double helical gears.

a) Spur and single helical gears (see footnote 5)

— Gear pair **without** helix modification, sun gear (Z)/planet (P), mounted on a fixed, rigid planet pin:

$$K_{H\beta} = 1 + \frac{4\,000}{3\pi}\; p\, \kappa_\beta\, \frac{c_\gamma}{E}\left(\frac{b}{d_Z}\right)^2 5{,}12 + \frac{\kappa_\beta c_\gamma f_{ma}}{2F_m/b} \qquad \text{(B.5)}$$

— For the same gear pair **with** helix modification (torsional deflection only compensated):

$K_{H\beta}$ in accordance with equation (36) and 5.7.2.4.2, and $K_{H\beta} \geqslant 1{,}05$.

— Gear pair **without** helix modification, sun gear (Z)/planet (P) with journals, mounted with bearings in the planet carrier:

$$K_{H\beta} = 1 + \frac{4\,000}{3\pi}\, \kappa_\beta\, \frac{c_\gamma}{E}\left[5{,}12\, p\left(\frac{b}{d_z}\right)^2 + 2\left(\frac{b}{d_P}\right)^4\left(\frac{l_P}{b} - \frac{7}{12}\right)\right] + \frac{\kappa_\beta c_\gamma f_{ma}}{2F_m/b} \qquad \text{(B.6)}$$

— For the same gear pair **with** full helix modification (bending and torsional deflection fully compensated):

$K_{H\beta}$ in accordance with equation (36) of 5.7.2.4.2, and $K_{H\beta} \geqslant 1{,}05$.

— Gear pair **without** helix modification, annulus gear (H)/planet (P) with journals, mounted with bearings in the planet carrier:

$$K_{H\beta} = 1 + \frac{8\,000}{3\pi}\, \kappa_\beta\, \frac{c_\gamma}{E}\left(\frac{b}{d_P}\right)^4\left(\frac{l_P}{b} - \frac{7}{12}\right) + \frac{\kappa_\beta c_\gamma f_{ma}}{2F_m/b} \qquad \text{(B.7)}$$

12) Restoring forces in toothed couplings are ignored. Restoring forces which lead to uneven distribution of load over the facewidth can occur when transmission elements are rigid and friction characteristics of flexible couplings are unsatisfactory.

— For the same gear pair **with** helix modification (bending deflection only compensated):

$K_{H\beta}$ in accordance with equation (36) of 5.7.2.4.2, and $K_{H\beta} \geqslant 1{,}05$.

— Gear pair **with** or **without** helix modification, annulus gear (H)/planet (P) mounted on a fixed, rigid planet pin:

$K_{H\beta}$ in accordance with equation (36) of 5.7.2.4.2, and $K_{H\beta} \geqslant 1{,}05$.

b) Double helical gears (see 5.7.2.4, with footnotes 4 and 5)

— Gear pair **without** helix modification, sun gear (Z)/planet (P) mounted on a fixed, rigid planet pin:

$$K_{H\beta} = 1 + \frac{4\,000}{3\pi}\; p\, \stackrel{?}{>}_{\beta}\; \frac{c_\gamma}{E} \left(\frac{2b_B}{d_Z} \right)^2 3{,}2 + \frac{\stackrel{?}{>}_{\beta} c_\gamma f_{ma}}{F_m / b_B} \qquad \text{(B.8)}$$

— For the same gear pair **with** helix modification (torsional deflection only compensated, see 5.7.2.4, footnote 4):

$K_{H\beta}$ in accordance with equation (37) of 5.7.2.4.2, and $K_{H\beta} \geqslant 1{,}05$.

— Gear pair **without** helix modification, sun gear (Z)/planet (P) with journals, mounted with bearings in a planet carrier:

$$K_{H\beta} = 1 + \frac{4\,000}{3\pi}\; \kappa_\beta\; \frac{c_\gamma}{E} \left[3{,}2\, p \left(\frac{2b_B}{d_Z} \right)^2 + 2 \left(\frac{B}{d_P} \right)^4 \left(\frac{l_P}{B} - \frac{7}{12} \right) \right] + \frac{\kappa_\beta c_\gamma f_{ma}}{F_m / b_B} \qquad \text{(B.9)}$$

— For the same gear pair **with** full helix modification (bending and torsional deflection fully compensated, see footnote 7):

$K_{H\beta}$ in accordance with equation (37) of 5.7.2.4.2, and $K_{H\beta} \geqslant 1{,}05$.

— Gear pair **without** helix modification, annulus gear (H)/planet (P) with journals, mounted with bearings in a planet carrier:

$$K_{H\beta} = 1 + \frac{8\,000}{3\pi}\; \kappa_\beta\; \frac{c_\gamma}{E} \left(\frac{B}{d_P} \right)^4 \left(\frac{l_P}{B} - \frac{7}{12} \right) + \frac{\kappa_\beta c_\gamma f_{ma}}{F_m / b_B} \qquad \text{(B.10)}$$

— For the same gear pair **with** helix modification (bending deflection only compensated):

$K_{H\beta}$ in accordance with equation (37) of 5.7.2.4.2, and $K_{H\beta} \geqslant 1{,}05$.

— Gear pair: **with** or **without** helix modification, annulus gear (H)/planet (P) mounted on a fixed, rigid planet pin:

$K_{H\beta}$ in accordance with equation (37) of 5.7.2.4.2, and $K_{H\beta} \geqslant 1{,}05$.

Annex C
(informative)

Guide values for application factor, K_A

C.1 Establishment of application factors

Application factors can best be established from a thorough analysis of service experience with a particular application (see ISO/TR 10495). For marine gears, the rules of the classification authorities shall be observed, as these are founded on extensive service experience. For the main propulsion gears of sea-going ships, a thorough analytical investigation should be made.

The factor K_A is used to modify the value F_t, to take into account loads additional to nominal loads imposed on the gears from external sources. If it is not possible to determine the equivalent tangential load (see clause 5.3) by comprehensive system analysis or from measured values using a suitable cumulative damage criterion; the empirical guidance values in C.2 can be used.

For marine gears, which are subjected to cyclic peak torques (torsional vibrations) and designed for infinite life, the application factor can be defined as the ratio between the peak cyclic torque and the nominal rated torque. The nominal rated torque is defined by the rated power and speed; it is the torque used in the load capacity calculations.

If the gear is subjected to a limited number of known loads in excess of the amount of the peak cyclic torques, this influence may be covered directly by means of a cumulative fatigue criterion, as mentioned above, or by means of an increased application factor representing the influence of the load spectrum.

It is recommended that the purchaser and manufacturer or designer agree on the value of the application factor in agreement with the applicable classification authority.

C.2 Approximate values for the application factors

K_A used in the preparation of preliminary designs can be chosen from the following values:

— for diesel driven main propulsion gears, K_A = 1,35;

— for turbine driven main propulsion gears, K_A = 1,1.

For geared transmissions of auxiliary machinery such as those listed in table C1, the following values may be used:

— for diesel driven auxiliaries, K_A = 1,5;

— for turbine and electric motor driven auxiliaries, K_A = 1,25;

— for turbine driven electricity generators, K_A = 1,1.

Table C.1 — Auxiliary machinery

Electricity generators	Cargo pumps, feed pumps
Side thrusters	Dynamic positioning thrusters
Azimuth thrusters	Platform jacking equipment
Any other equipment which is essential to the safety of a ship or other similar marine unit	

Annex D
(informative)

Guide values for crowning and end relief of teeth of cylindrical gears

D.1 General

Well designed crowning and end relief have a beneficial influence on the distribution of load over the facewidth of a gear (see 5.7). Design details should be based on a careful estimate of the deformations and manufacturing deviations of the gearing of interest. If deformations are considerable, helix angle modification might be superposed over crowning or end relief, but well designed helix modification is preferable.

D.2 Amount of crowning, C_β

The following non-mandatory rule is drawn from experience; the amount of crowning (see Figure D.1) necessary to obtain acceptable distribution of load can be determined as follows.

Subject to the limitations 10 µm $\leqslant C_\beta \leqslant$ 40 µm plus a manufacturing tolerance of 5 µm to 10 µm, and that the value b_{cal}/b would have been greater than 1 had the gears not been crowned: $C_\beta \approx 0{,}5\,F_{\beta\mathrm{x\,cv}}$.

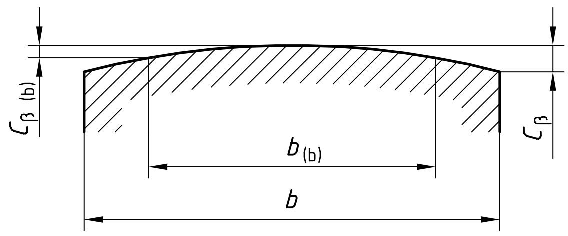


Figure D.1 — Amount of crowning, $C_{\beta\,(\mathrm{b})}$, and width, $b_{(\mathrm{b})}$

In order to avoid excessive loading of tooth ends, the crowning amount shall be calculated as:

$$C_\beta = 0{,}5\left(f_{\mathrm{sh}} + f_{\mathrm{H}\beta}\right) \quad \text{(D.1)}$$

When the gears are of such stiff construction that f_{sh} can for all practical purposes be neglected, or when the helices have been modified to compensate for deformation at mid-facewidth, the following value can be substituted:

$$C_\beta = 0{,}5\,f_{\mathrm{H}\beta} \quad \text{(D.2)}$$

Subject to the restriction 10 µm $\leqslant C_\beta \leqslant$ 25 µm plus a manufacturing tolerance of about 5 µm, 60 % to 70 % of the above values are adequate for extremely accurate and reliable high speed gears.

D.3 Amount $C_{\mathrm{I(II)}}$ and width $b_{\mathrm{I(II)}}$ of end relief

D.3.1 Method C1

This method is based on an assumed value for the equivalent misalignment of the gear pair, without end relief, and on the recommendations for the amount of gear crowning.

a) Amount of end relief (see Figure D.2)

For through hardened gears: $C_{\mathrm{I(II)}} \approx F_{\beta \mathrm{x\,cv}}$ plus a manufacturing tolerance of 5 µm to 10 µm.

Thus, by analogy with $F_{\beta \mathrm{x\,cv}}$ in clause D.1, $C_{\mathrm{I(II)}}$ should be approximately:

$$C_{\mathrm{I(II)}} = f_{\mathrm{sh}} + 1{,}5\, f_{\mathrm{H}\beta} \qquad \text{(D.3)}$$

For surface hardened and nitrided gears: $C_{\mathrm{I(II)}} \approx 0{,}5\, F_{\beta \mathrm{x\,cv}}$ plus a manufacturing tolerance of 5 µm to 10 µm.

Thus, by analogy with $F_{\beta \mathrm{x\,cv}}$ in clause D.1, $C_{\mathrm{I(II)}}$ should be approximately:

$$C_{\mathrm{I(II)}} = 0{,}5\left(f_{\mathrm{sh}} + 1{,}5\, f_{\mathrm{H}\beta}\right) \qquad \text{(D.4)}$$

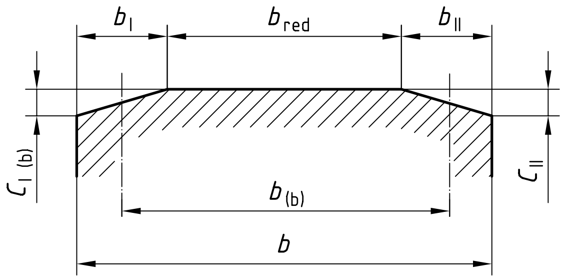


Figure D.2 — Amount $C_{\mathrm{I(II)}}$ and width $b_{\mathrm{(b)}}$ of end relief

When the gears are of such stiff construction that f_{sh} can for all practical purposes be neglected, or when the helices have been modified to compensate deformation, proceed in accordance with equation (D.2).

For very accurate and reliable gears with high tangential velocities, 60 % to 70 % of the above values is appropriate.

b) Width of end relief

For approximately constant load and higher tangential velocities: $b_{\mathrm{I(II)}}$ is the smaller of the values (0,1 b) or (1,0 m)

The following is appropriate for variable loading, low and average speeds:

$$b_{\mathrm{red}} = (0{,}5 \text{ to } 0{,}7)\, b \qquad \text{(D.5)}$$

D.3.2 Method C2

This method is based on the deflection of gear pairs, assuming uniform distribution of load over the facewidth:

$$\delta_{\mathrm{bth}} = F_{\mathrm{m}} / (b c_{\gamma}), \text{ or } F_{\mathrm{m}} = F_{\mathrm{t}} K_{\mathrm{A}} K_{\mathrm{v}} \qquad \text{(D.6)}$$

For highly accurate and reliable gears with high tangential velocities, the following are appropriate:

$$C_{I(II)} = (2 \text{ to } 3)\delta_{bth} \quad \text{(D.7)}$$

$$b_{red} = (0{,}8 \text{ to } 0{,}9)b \quad \text{(D.8)}$$

For similar gears of lesser accuracy:

$$C_{I(II)} = (3 \text{ to } 4)\delta_{bth} \quad \text{(D.9)}$$

$$b_{red} = (0{,}7 \text{ to } 0{,}8)b \quad \text{(D.10)}$$

Annex E
(informative)

Check and interpretation of tooth contact pattern

E.1 Scope and field of application

This annex describes a procedure for checking the tooth contact of marine gear units (accuracy grade 6 or better) without load or under partial load condition.

E.2 Test methods

E.2.1 General

There are two methods described for determining the tooth contact pattern:

— contact test (check of mesh without load);

— load test (contact pattern at defined load level).

E.2.2 Contact test

The contact test is an economical method for determining the sum of all manufacturing deviations. The contact test is usually performed in the completely mounted condition. If no housing is available, especially in the case of large gears, a test rig may be used. Typical applications are

— large gears for marine transmissions, and

— gears mounted on board.

The main influence factors on the tooth contact without load are given in Table E1.

Table E.1 — Main influence factors on the tooth contact without load

Tooth deviations	Housing deviations	Shaft deviations	Bearing tolerances
Pitch deviation	Shaft angle deviation	Axial runout	Bearing clearance
Profile deviation	Shaft slope deviation		Concentricity
Lead deviation			

E.2.3 Load test

The load test is applied for highly loaded gears with profile or lead modifications, or both, for comparison of the actual contact pattern with the data obtained by calculation. For the test, the load is increased in reasonable steps in order to be able to predict the load distribution at full load. At the lowest load stage the gears shall already have reached their final position. A typical sequence of load stages is: 5 %, 25 %, 50 %, 75 %, 100 % (maximum value as high as possible).

The load-dependent influence factors with influence on the tooth contact are given in Table E.2.

Table E.2 — Load-dependent influence factors influencing the tooth contact

Tooth deviations	Housing deviations	Shaft deviations	Bearing tolerances
Tooth deformation	Housing stiffness	Shaft deflection	Bearing stiffness
Hertzian deformation	Housing temperature	Shaft distortion	
Gear blank deformation			
Tooth wear			

E.2.4 Procedure

It is usual for both contact and load tests that at least three sets of teeth (for the whole plane of contact) are considered. Either the wheel or the pinion is painted with a suitable contrast colour. After several revolutions without load or at the actual load stage the transmission of colour to the mating gear (without or with moderate load), or the abrasion of the colour (high load), is used to evaluate the contact pattern.

E.2.5 Paints

E.2.5.1 Contact test

See Table E.3.

Table E.3 — Suitable paints (contact test)

Suitable paints	Manufacturer
Lukas Tuschierfarbe	Dr. Schönfeld & Co.
Diamant Tuschierfarbe	Schleifmittelwerk Kahl
Eosol Tuschierpaste	Emil Otto/Fabrik
Kruel Tuschierfarbe	Fa. C. Kreul
Norma Ölfarbe	H. Schminke & Co.
Yellow Gear Marking	Prescott & Comp. Ltd.
NOTE The above are examples of products available commercially. This information is given for the convenience of users of this International Standard and does not constitute an endorsement of ISO of these products.	

E.2.5.2 Load test

The contrast colour for load tests shall meet the following requirements:

- good contrast on metal surface;
- high temperature resistance;
- oil resistant;
- high tensile strength;
- high adhesion.

See Table E.4.

Table E.4 — Suitable paints (load test)

Suitable paints	Manufacturer
Dykem Red Layout DX-296	The Dykem Company
Eosol Anreißfarbe	Emil Otto/Fabrik
Pelikan Anreißfarbe	Pelikanwerke
Regensburger Getriebeprüflack	Regensburger Lackfabrik
Copper sulphate	
NOTE The above are examples of products available commercially. This information is given for the convenience of users of this International Standard and does not constitute an endorsement of ISO of these products.	

E.3 Evaluation of expected values for contact test and load test

E.3.1 Face contact width

The optimum contact pattern is determined on the basis of the lead modification gained by Method A, B or C of ISO 6336-1:1996. If a linear helix modification is applied the face contact, width is calculated as

$$b_{\mathrm{p}} = \frac{s_{\mathrm{c}}}{f_{\mathrm{korr}}} \times 100 \qquad \text{(E.1)}$$

where

b_{p} is the face contact width in percent;

s_{c} is the film thickness of the contrast colour, in micrometres (μm);

f_{korr} is the lead correction value, in micrometres (μm).

E.3.2 Profile contact width

The optimum contact width over the profile is determined corresponding to the actual profile and lead corrections, and the referring tolerances. Reasonably, the load value should be advised where full profile contact is expected.

E.4 Examination of contact pattern

Examination of the contact pattern is subjective and should therefore always be carried out together with the use of complete investigation records. For both the contact test and load test, the optimum contact pattern can be adjusted during the test by means of eccentric bearing rings or by adding shims at the supports to distort the housing.

Bibliography

[1] ISO 4287:1997, *Geometrical Product Specifications (GPS) — Surface texture: Profile method — Terms, definitions and surface texture parameters*.

[2] ISO 4288:1996, *Geometrical Product Specifications (GPS) — Surface texture: Profile method — Rules and procedures for the assessment of surface texture*.

[3] ISO 9084:2000, *Calculation of load capacity of spur and helical gears — Application to high speed gears and gears of similar requirements*.

[4] ISO 9085:—[13)], *Calculation of load capacity of spur and helical gears — Application for industrial gears.*

13) To be published.

ICS 21.200; 47.020.05

Price based on 58 pages