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Control charts —

Part 2: **Shewhart control charts**

Cartes de contrôle —

Partie 2: Cartes de contrôle de Shewhart

Reference number
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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 7870-2 was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 4, *Applications of statistical methods in process management*.

This first edition cancels and replaces ISO 8258:1991, which has been technically revised.

ISO 7870 consists of the following parts, under the general title Control charts:

— *Part 1: General guidelines*

— *Part 2: Shewhart control charts*

— *Part 3: Acceptance control charts*

— *Part 4: Cumulative sum charts*

— *Part 5: Specialized control charts*

EWMA control charts will from the subject of a future Part 6.

Introduction

A traditional approach to manufacturing has been to depend on production to make the product and on quality control to inspect the final product and screen out items not meeting specifications. This strategy of detection is often wasteful and uneconomical because it involves after-the-event inspection when the wasteful production has already occurred. Instead, it is much more effective to institute a strategy of prevention to avoid waste by not producing unusable output in the first place. This can be accomplished by gathering process information and analysing it so that timely action can be taken on the process itself.

Dr. Walter Shewhart in 1924 proposed the control chart as a graphical means of applying the statistical principles of significance to the control of a process. Control chart theory recognizes two kinds of variability. The first kind is random variability due to “chance causes” (also known as “common/*natural/random/inherent/uncontrollable* causes”). This is due to the wide variety of causes that are consistently present and not readily identifiable, each of which constitutes a very small component of the total variability but none of which contributes any significant amount. Nevertheless, the sum of the contributions of all of these unidentifiable random causes is measurable and is assumed to be inherent to the process. The elimination or correction of common causes may well require a decision to allocate resources to fundamentally change the process and system.

The second kind of variability represents a real change in the process. Such a change can be attributed to some identifiable causes that are not an inherent part of the process and which can, at least theoretically, be eliminated. These identifiable causes are referred to as “assignable causes” *(also known as special/unnatural/systematic/controllable causes*) of variation. They may be attributable to such matters as the lack of uniformity in material, a broken tool, workmanship or procedures, the irregular performance of equipment, or environmental changes.

A process is said to be in statistical control, or simply “in control”, when the process variability results only from random causes. Once this level of variation is determined, any deviation from this level is assumed to be the result of assignable causes that should be identified and eliminated.

Statistical process control is a methodology for establishing and maintaining a process at an acceptable and stable level so as to ensure conformity of products and services to specified requirements. The major statistical tool used to do this is the control chart, which is a graphical method of presenting and comparing information based on a sequence of observations representing the current state of a process against limits established after consideration of inherent process variability called process capability. The control chart method helps first to evaluate whether or not a process has attained, or continues in, a state of statistical control. When in such a state the process is deemed to be stable and predictable and further analysis as to the ability of the process to satisfy the requirements of the customer can then be conducted. The control chart also can be used to provide a continuous record of a quality characteristic of the process output while process activity is ongoing. Control charts aid in the detection of unnatural patterns of variation in data resulting from repetitive processes and provide criteria for detecting a lack of statistical control. The use of a control chart and its careful analysis leads to a better understanding of the process and will often result in the identification of ways to make valuable improvements.

Control charts —

Part 2:
Shewhart control charts

1 Scope

This International Standard establishes a guide to the use and understanding of the Shewhart control chart approach to the methods for statistical control of a process.

This International Standard is limited to the treatment of statistical process control methods using only the Shewhart system of charts. Some supplementary material that is consistent with the Shewhart approach, such as the use of warning limits, analysis of trend patterns and process capability is briefly introduced. There are, however, several other types of control chart procedures, a general description of which can be found in ISO 7870-1.

2 Normative references

The following referenced documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 3534-2, *Statistics — Vocabulary and symbols — Part 2: Applied statistics*

ISO 16269-4, *Statistical interpretation of data — Part 4: Detection and treatment of outliers*

ISO 5479, *Statistical interpretation of data — Tests for departure from the normal distribution*

ISO 22514 (all parts), *Statistical methods in process management — Capability and performance*

3 Terms, definitions and symbols

3.1 General

For the purposes of this document, the terms and definitions given in ISO 3534-2:2006 apply.

3.2 Symbols

NOTE The ISO/IEC Directives makes it necessary to depart from common SPC usage in respect to the differentiation between abbreviated terms and symbols. In ISO standards an abbreviated term and its symbol can differ in appearance in two ways: by font and by layout. To distinguish between abbreviated terms and symbols, abbreviated terms are given in Cambria upright and symbols in Cambria or Greek italics, as applicable. Whereas abbreviated terms can contain multiple letters, symbols consist only of a single letter. For example, the conventional abbreviation of upper control limit, UCL, is valid but its symbol in equations becomes U_{CL}. The reason for this is to avoid misinterpretation of compound letters as an indication of multiplication.

In cases of long established practice where a symbol and/or abbreviated term means different things in different applications, it is necessary to use a field limiter, thus ⟨ ⟩, to distinguish between them. This avoids the alienation of practitioners by the creation of unfamiliar abbreviated terms and symbols in their particular field that are unlike all related texts, operational manuals and dedicated software programs. An example is the abbreviated term ‘R’ and symbol ‘*R*’ which means different things in metrology from that in acceptance sampling and statistical process control. The abbreviated term ‘R’ is differentiated thus:

R ⟨metrology⟩ reproducibility limit

R ⟨SPC and acceptance sampling⟩ range

For the purposes of this document, the following symbols apply.

n Subgroup size; the number of sample observations per subgroup

k Number of subgroups

L Lower specification limit

L_{CL} Lower control limit

U Upper specification limit

U_{CL} Upper control limit

X Measured quality characteristic (individual values are expressed as (X_1, X_2, X_3,...). Sometimes the symbol Y is used instead of X

$\overline{X}$ (X bar) Subgroup average

$\overline{\overline{X}}$ (X double bar) Average of the subgroup averages

μ True process mean value

σ True process standard deviation value

σ_0 A given value of σ

$\tilde{X}$ Median of a subgroup

$\overline{\tilde{X}}$ Average of the subgroup medians

R Subgroup range: difference between the largest observation and smallest observation of a subgroup

$\overline{R}$ Average of the R values for all subgroups

R_m Moving range: the absolute value of the difference between two successive values $|X_1 - X_2|$,$|X_2 - X_3|$, etc.

$\overline{R}_m$ Average of the $(n - 1)$ R_m values in a set of n observed values

s Sample standard deviation obtained from values within a subgroup:

$$s = \sqrt{\frac{\sum (X_i - \overline{X})^2}{n-1}}$$

$\overline{s}$ Average of the subgroup sample standard deviations

$\hat{\sigma}$ Estimated process standard deviation value

p Proportion or fraction of units in a subgroup with a given classification

$\overline{p}$ Average value of the proportion or fraction

np Number of units with a given classification in a subgroup

p_0 A given value of p

np_0 A given value of np (for a given p_0)

c Number of incidences in a subgroup

c_0 A given value of c

$\bar{c}$ Average value of the c values for all subgroups

u Number of incidences per unit in a subgroup

$\bar{u}$ Average value of the u values

u_0 A given value of u

4 Nature of Shewhart control charts

A Shewhart control chart is a graph that is used to display a statistical measure obtained from either variables or attribute data. The control chart requires data from rational subgroups to be taken at approximately regular intervals from the process. The intervals may be defined in terms of time (for example hourly) or quantity (every lot). Usually, the data are obtained from the process in the form of samples or subgroups consisting of the same process characteristic, product or service with the same measurable units and the same subgroup size. From each subgroup, one or more subgroup characteristics are derived, such as the subgroup average, $\bar{X}$, and the subgroup range, R, the standard deviation, s, or a countable characteristic such as the proportion of units with a given classification.

A Shewhart control chart is a plot of the values of a given subgroup characteristic versus the subgroup number. It consists of a centre line (CL) located at a reference value of the plotted characteristic. In establishing whether or not a state of statistical control exists, the reference value is usually the average of the statistical measure being considered. For process control, the reference value may be the long-term value of the characteristic as stated in the product specifications; a value of the characteristic being plotted based on past experience with the process when in a state of statistical control, or based upon implied product or service target values.

The control chart has two statistically determined limit lines, one on either side of the centre line, which are called the upper control limit (U_{CL}) and the lower control limit (L_{CL}) (see Figure 1).

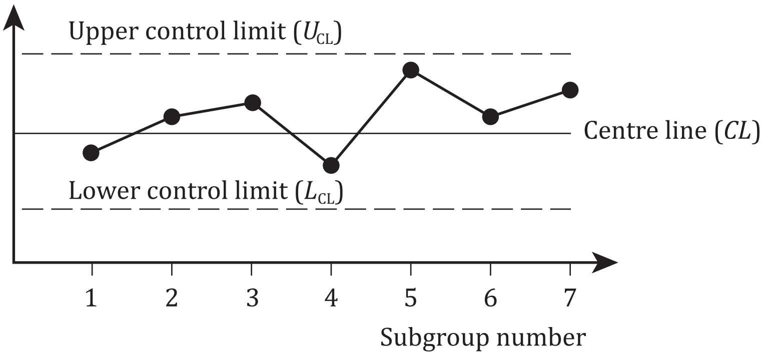


Figure 1 — Outline of a control chart

The control limits on the Shewhart charts are placed at a distance of 3 sigma on each side of the centre line, where sigma is the known or estimated standard deviation of the population. Shewhart chose to use 3 sigma limits on the basis that it made economic sense with respect to balancing the cost of looking for process problems when such problems do not exist and failing to look for problems when the process is not performing as it should. Placing the limits too close to the centre line will result in many searches for non-existing problems and yet placing the limits too far apart will increase the risk of not detecting process problems when they do exist. Under an assumption that the plotting statistic is approximately normally distributed 3 sigma limits indicate that approximately 99,7 % of the values of the statistic will be included within the control limits, provided the process is in statistical control. Interpreted another way, there is approximately a 0,3 % risk, or an average of three times in a thousand, of a plotted point being outside of either the upper or lower control limit when the process is in control. The word "approximately" is used because deviations from underlying assumptions such as the distributional form of the data will affect the probability values. In fact, the choice of k sigma limits instead of 3 sigma limits depends on costs of investigation and taking appropriate action *vis-à-vis* consequences of not taking action.

It should be noted that some practitioners prefer to use the factor 3,09 instead of 3 to provide a nominal probability value of 0,2 % or an average of one spurious observation per thousand, but Shewhart selected 3 so as not to lead to attempts to consider exact probabilities. Similarly, some practitioners use actual probability values for the charts based on non-normal distributions such as for ranges and fraction nonconforming. Again, the Shewhart control chart used ±3 sigma limits in view of the emphasis on empirical interpretation.

The possibility that a violation of the limits is really a chance event rather than a real signal is considered so small that when a point appears outside of the limits, action should be taken. Since action is required at this point, the 3 sigma control limits are sometimes called the "action limits".

Many times it is advantageous to mark 2 sigma limits on the chart also. Then, any sample value falling beyond the 2 sigma limits can serve as a warning of an impending out-of-control situation. As such, the 2 sigma limit lines are sometimes called "warning limits". While no action is required as a result of such a warning been given on the control chart, some users may wish to immediately select another subgroup of the same size to determine if corrective action is indicated.

When assessing the status of a process using control charts, two types of errors are possible. The first occurs when the process involved is actually in a state of control but a plotted point falls outside the control limits due to chance. As a result, the chart has given a signal resulting in an incorrect conclusion that the process is out of control. A cost is then incurred in an attempt to find the cause of a non-existent problem.

The second error occurs when the process involved is not in control but the plotted point falls within the control limits due to chance. In this case, the chart provides no signal and it is incorrectly concluded that the process is in statistical control. There may also be a substantial cost associated with failing to detect that a change in the process location or variability has occurred, the result of which might be the production of nonconforming output. The risk of this type of error occurring is a function of three things: the width of the control limits, the sample size, and the degree to which the process is out of control. In general, because the magnitude of the change in the process cannot be known, little can be determined about the actual size of the risk of this error.

Because it is generally impractical to make a meaningful estimate of the risk and of the cost of the second type of error in any given situation, the Shewhart control chart system is designed to control the first of these errors. When normality is assumed and 3 sigma control limits are used, the size of this first error is 0,3 %. In other words, this error will happen only about 3 times in 1 000 samples when the process is in control.

In fact the choice of k sigma limits instead of 3 sigma limits depends on costs of investigation and taking appropriate action *vis-à-vis* consequences of not taking action.

When a process is in statistical control, the control chart provides a method, which in some senses is analogous to continually testing a statistical null hypothesis that the process has not changed and remains in statistical control. Because, in Phase 1, there is often uncertainty about such matters as the probability distribution of the characteristic of interest, randomness, and the specific departures of the process characteristic from the target value that may be of concern are not usually defined in advance, the Shewhart control chart should not be considered to be a test of hypothesis in the purest sense. Walter Shewhart emphasized the empirical usefulness of the control chart for recognizing departures from an "in-control" process and de-emphasized making probabilistic interpretations.

When a plotted value falls outside of either control limit, or a series of values display an unusual pattern such as discussed in Clause 8, the state of statistical control can no longer be accepted. When this occurs, an investigation is initiated to locate the assignable cause, and the process may be stopped or adjusted. Once the assignable cause is determined and eliminated, the process is ready to continue. As discussed above, on rare occasions no assignable cause can be found and it must be concluded that the point outside the limits represents the occurrence of a very rare event, a random cause, which has resulted in a value outside of the control limits even though the process is in control.

When a process is to be studied for the first time with the objective of bringing the process into a state of statistical control, it is often found necessary to use historical data that has previously been obtained from the process or to undertake to obtain new data from a series of samples before attempting to establish the control chart. This retrospective stage during which the control chart parameters are being established is often referred to as Phase 1. Sufficient data will need to be found in order to obtain reliable estimates of the centre line and control limits for the control charts. The control limits established in Phase 1 are trial control limits as they are based upon data collected when the process may not be in control. The identification of the precise causes for signals given by the control chart at this stage may prove to be difficult because of the lack of information about the historical operating characteristics of the process. However, when special causes of variation can be identified and corrective action taken, the retrospective data from the process when under the influence of the special cause should be removed from consideration and the control chart parameters re-determined. This iterative procedure is continued until the trial control chart shows no signals and the process may then be considered to be in control and hence is stable and predictable. Because some data may have to be removed from consideration during Phase 1, the user may have to obtain additional data from the process to maintain the reliability of the parameter estimates.

Once statistical control has been established, the final trial control chart centre line and control limits identified in Phase 1 are taken as the control chart parameters for the ongoing monitoring of the process. The objective now, in what is referred to as Phase 2, is the maintenance of the process in a state of control as well as the rapid identification of special causes that may affect the process from time to time. It should be recognized that moving from Phase 1 to Phase 2 might prove to be both time consuming and difficult. It is crucial, however, since the failure to remove special causes of variation will result in the process variation being overestimated. In this case the control chart will have control limits that are set too wide apart resulting in a control chart that is not sufficiently sensitive for detecting the presence of special causes.

Details for the procedure to establish control charts for a process will be discussed below.

5 Types of control charts

Shewhart control charts are basically of two types: variables control charts and attributes control charts. For each of the control charts, there are two distinct situations:

a) when no pre-specified process parameter values are given;

b) when pre-specified process parameters values are given.

The pre-specified process values may be specified requirements or target values, or estimated values of the parameters that have been determined over the long term from data when the process is in control.

5.1 Control charts where no pre-specified values are given

The purpose here is to discover whether observed values of the plotted characteristics, such as $\overline{X}$, R or any other statistic, vary among themselves by an amount greater than that which can be attributed to chance alone. Control charts will be constructed using only the data collected from samples from the process. The control charts are used for detecting those variations caused other than by chance with the purpose being to bring the process into a state of statistical control.

5.2 Control charts with respect to given pre-specified values

The purpose here is to identify whether the observed values of $\overline{X}$, s, etc., for several subgroups of n observations each, differ from the respective given values of μ_0, σ_0, etc. by amounts greater than that expected to be due to chance causes only. The difference between charts with given parameter values and those where no pre-specified values are given is the additional requirement concerning the determination of the location of the centre and variation of the process. The specified values may be based on experience obtained by using control charts with no prior information or specified values. They may also be based on economic values established upon consideration of the need for service and cost of production or be nominal values designated by the product specifications.

Preferably, the specified values should be determined through an investigation of preliminary data that is supposed to be typical of all future data. The specified values should be compatible with the inherent process variability for effective functioning of the control charts. Control charts based on such pre-specified values are used particularly during process operation to control processes and to maintain product or service uniformity at the desired level.

5.3 Types of variables and attributes control charts

The following control charts are considered:

a) Variables control charts, used when the measurements are on a continuous scale:

   1) average ($\overline{X}$) chart and range (R) or standard deviation (s) chart;

   2) individuals (X) and moving range (R_m);

   3) median ($\tilde{X}$) chart and range (R) chart.

b) Attributes control charts, used when the measurements are countable or categorized data:

   1) p chart for number of units of a given classification per total number of units in the sample expressed as a proportion or percentage;

   2) np chart for number of units of a given classification where the sample size is constant;

   3) c chart for number of incidences where the opportunity for occurrence is fixed;

   4) u chart for the number of incidences per unit where the opportunity is variable.

Figure 2 shows a process of selecting an appropriate control chart for a given situation.

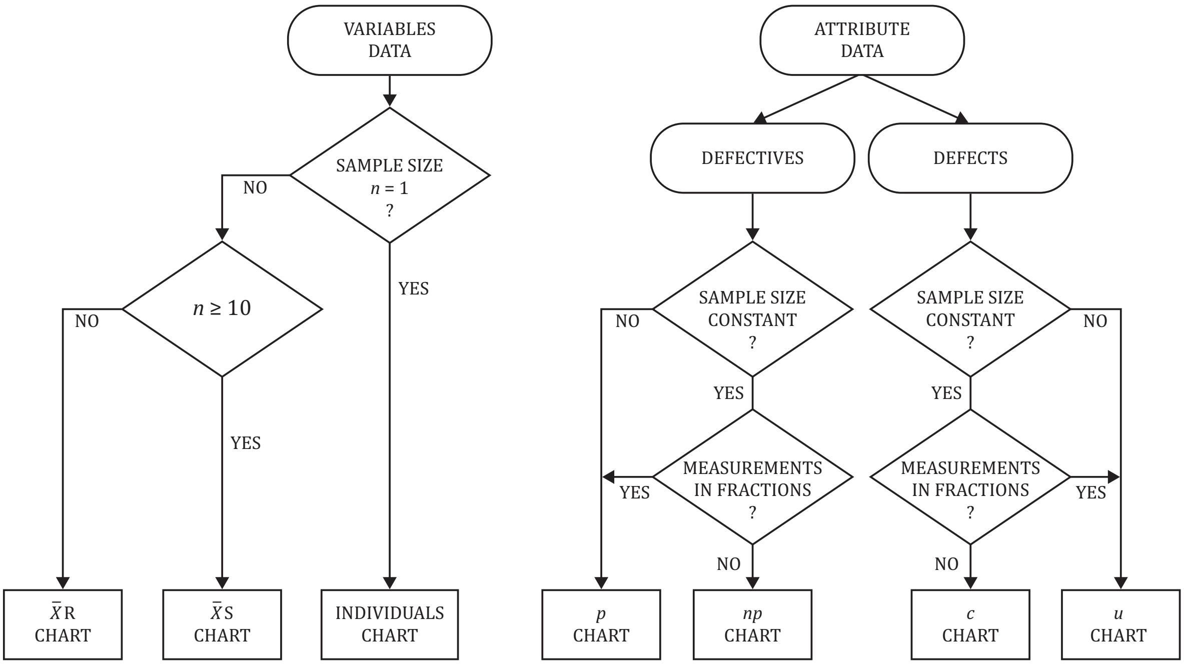


Figure 2 — Types of control charts

6 Variables control charts

Variables control charts, or charts for variables data, and especially their most customary forms, the $\bar{X}$ and R charts represent the classic application of control charting to process control.

Control charts for variables are particularly useful for several reasons:

a) Most processes, and their output, have characteristics that are measurable, hence generate variables data, so the potential applicability is broad.

b) Variables charts are more informative than attributes charts since specific information about the process mean and variance are obtained directly. Variables charts will often signal a process problem before the process has produced nonconforming items.

c) Although obtaining one item of measured data is generally more costly than obtaining one item of go/no go data, the subgroup sizes needed for variables are almost always much smaller than those for attributes, for an equivalent monitoring efficiency. This helps to reduce the total inspection cost in some cases and to shorten the time gap between the occurrence of a process problem and corrective action.

d) These charts will provide visual means to directly assess process performance regardless of the specifications. A close look at variables charts, along with review of histograms at appropriate intervals, will often lead to ideas or suggestions as to how to improve the process.

For all variables control chart applications considered in this International Standard, it is assumed that the distribution of the quality characteristic is normal (Gaussian) and departures from this assumption will affect the performance of the charts. The factors used for computing control limits were derived using the assumption of normality. Since most control limits are used as empirical guides in making decisions, reasonably small departures from normality should not cause concern. In any case, because of the central limit theorem, averages tend to be normally distributed even when individual observations are not; this makes it reasonable for evaluating control to assume normality for $\bar{X}$ charts, even for sample sizes as small as 4 or 5. When dealing with individual observations for capability study purposes, the true form of the distribution is important. Periodic checks on the continuing validity of such assumptions are advisable, particularly for ensuring that only data from a single population are being used. It should be noted that the distributions of the ranges and standard deviations are not normal. Although normality is necessarily assumed in the determination of the constants for the calculation of control limits for the range or standard deviation chart, moderate deviations from normality of the process data should not be of major concern in the use of these charts as an empirical decision procedure.

Variables charts can describe process data in terms of both spread (process variability) and location (process average). Because of this, control charts for variables are almost always prepared and analysed in pairs – one chart for location and another for spread. The chart for spread is usually analysed first, since it provides the rationale and justification for the estimation of the process standard deviation. The resulting estimate of the process standard deviation may then be used in establishing control limits for the chart for location.

Each chart can be plotted using either estimated control limits, in which case limits are based on the information contained in the sample data plotted on the chart, or pre-specified control limits based on adopted specified values applicable to the statistical measures plotted on the chart. The subscript “0” is used in Tables 1 and 3 to designate the specified values, such as μ_0 for the specified process mean or σ_0 for the specified process standard deviation.

Following are the most commonly used variables control charts.

6.1 Mean ($\bar{X}$) chart and range (R) chart or mean ($\bar{X}$) chart and standard deviation (s) chart

$\bar{X}$ and R charts can be used when subgroup sample size is small or moderately small, usually less than 10. $\bar{X}$ and s charts are preferable particularly in the case of large subgroup sample sizes ($n \geq 10$), since the range becomes increasingly less efficient at estimating the process standard deviation as the sample size gets larger. Where electronic devices are available to calculate process limits, standard deviation is preferable.

Tables 1 and 2 give the control limit formulae and the factors for each of these variables control charts.

Table 1 — Control limit formulae for Shewhart variables control charts

Statistic	Estimated control limits		Pre-specified control limits	
	Centre line	U_{CL} and L_{CL}	Centre line	U_{CL} and L_{CL}
$\bar{X}$	X	$\bar{\bar{X}} \pm A_2\bar{R}$ or $\bar{\bar{X}} \pm A_3\bar{s}$	μ_0	$\mu_0 \pm A\sigma_0$
R	$\bar{R}$	$D_4\bar{R}, D_3\bar{R}$	$d_2\sigma_0$	$D_2\sigma_0, D_1\sigma_0$
s	$\bar{s}$	$B_4\bar{s}, B_3\bar{s}$	$c_4\sigma_0$	$B_6\sigma_0, B_5\sigma_0$
NOTE μ_0 and σ_0 are pre-specified values.				

Table 2 — Factors for computing control chart lines

Observations in subgroups of size *n*	Factors for control limits											Factors for centre line	
	$\bar{X}$ Chart			*s* Chart				*R* Chart *				Using *s* *	Using *R* *
	A	A_2	A_3	B_3	B_4	B_5	B_6	D_1	D_2	D_3	D_4	C_4	d_2
2	2,121	1,880	2,659	–	3,267	–	2,606	–	3,686	–	3,267	0,7979	1,128
3	1,732	1,023	1,954	–	2,568	–	2,276	–	4,358	–	2,575	0,8862	1,693
4	1,500	0,729	1,628	–	2,266	–	2,088	–	4,698	–	2,282	0,9213	2,059
5	1,342	0,577	1,427	–	2,089	–	1,964	–	4,918	–	2,114	0,9400	2,326
6	1,225	0,483	1,287	0,030	1,970	0,029	1,874	–	5,079	–	2,004	0,9515	2,534
7	1,134	0,419	1,182	0,118	1,882	0,113	1,806	0,205	5,204	0,076	1,924	0,9594	2,704
8	1,061	0,373	1,099	0,185	1,815	0,179	1,751	0,388	5,307	0,136	1,864	0,9650	2,847
9	1,000	0,337	1,032	0,239	1,761	0,232	1,707	0,547	5,394	0,184	1,816	0,9693	2,970
10	0,949	0,308	0,975	0,284	1,716	0,276	1,669	0,686	5,469	0,223	1,777	0,9727	3,078
11	0,905	0,285	0,927	0,321	1,679	0,313	1,637	0,811	5,535	0,256	1,744	0,9754	3,173
12	0,866	0,266	0,886	0,354	1,646	0,346	1,610	0,923	5,594	0,283	1,717	0,9776	3,258
13	0,832	0,249	0,850	0,382	1,618	0,374	1,585	1,025	5,647	0,307	1,693	0,9794	3,336
14	0,802	0,235	0,817	0,406	1,594	0,399	1,563	1,118	5,696	0,328	1,672	0,9810	3,407
15	0,775	0,223	0,789	0,428	1,572	0,421	1,544	1,203	5,740	0,347	1,653	0,9823	3,472
16	0,750	0,212	0,763	0,448	1,552	0,440	1,526	1,282	5,782	0,363	1,637	0,9835	3,532
17	0,728	0,203	0,739	0,466	1,534	0,458	1,511	1,356	5,820	0,378	1,622	0,9845	3,588
18	0,707	0,194	0,718	0,482	1,518	0,475	1,496	1,424	5,856	0,391	1,609	0,9854	3,640
19	0,688	0,187	0,698	0,497	1,503	0,490	1,483	1,489	5,889	0,404	1,596	0,9862	3,689
20	0,671	0,180	0,680	0,510	1,490	0,504	1,470	1,549	5,921	0,415	1,585	0,9869	3,735
21	0,655	0,173	0,663	0,523	1,477	0,516	1,459	1,606	5,951	0,425	1,575	0,9876	3,778
22	0,640	0,167	0,647	0,534	1,466	0,528	1,448	1,660	5,979	0,435	1,567	0,9882	3,819
23	0,626	0,162	0,633	0,545	1,455	0,539	1,438	1,711	6,006	0,443	1,557	0,9887	3,858
24	0,612	0,157	0,619	0,555	1,445	0,549	1,429	1,759	6,032	0,452	1,548	0,9892	3,895
25	0,600	0,153	0,606	0,565	1,435	0,559	1,420	1,805	6,056	0,459	1,541	0,9896	3,931

* Not recommended for sample size $n > 10$.

6.2 Control chart for individuals (X) and control chart for moving ranges (R_m)

In some process control situations, it is either impossible, impractical, or it does not make sense to select rational subgroups. It is then necessary to assess process control based on individual readings using X and R_m charts.

In the case of control charts for individuals, since there are no rational subgroups to provide an estimate of variability, control limits are based on a measure of variation obtained from moving ranges of two consecutive observations. A moving range is the absolute value of the difference between successive pairs of measurements in a series; i.e. the absolute value of the difference between the first and second measurements, then between the second and third, and so on. From the moving ranges, the average moving range $\overline{R}_m$ is calculated and used for the construction of control charts. Also, from the entire collection of data, the overall average $\overline{X}$ is calculated. Table 3 gives the control limit formulae for control charts for individuals and for control charts for moving ranges.

Some caution should be exercised with respect to control charts for individuals:

a) The charts for individuals are not as sensitive to process changes as charts based on subgroups.

b) Care shall be taken in the interpretation of charts for individuals if the process distribution is not normal.

c) Charts for individuals isolate process variability from an average of consecutive differences between observations. Thus, it is implied that the data are time-ordered, and that no significant changes have occurred in the process in between the collection of any two consecutive individuals. It would be ill advised, for example, to gather data from two discontinuous campaigns of production of a batch chemical product and to calculate a moving range between the last batch of the first campaign and the first batch of the next campaign, if the production line has been stopped in between.

Table 3 — Control limit formulae for control charts for individuals

Statistic	Estimated control limits		Pre-specified control limits	
	Centre line	U_{CL} and L_{CL}	Central line	U_{CL} and L_{CL}
Individual, X	$\overline{X}$	$\overline{X} \pm 2{,}660\overline{R}_m$	μ_0	$\mu_0 \pm 3\sigma_0$
Moving Range, R_m	$\overline{R}_m$	$3{,}267\overline{R}_m$ 0	$1{,}128\sigma_0$	$3{,}686\sigma_0$ 0

NOTE 1 μ_0 and σ_0 are pre-specified values

NOTE 2 $\overline{R}_m$ denotes the average of moving ranges of 2 observations.

6.3 Control charts for medians ($\tilde{X}$)

Median charts are alternatives to $\overline{X}$ charts for the control of a process location when it is desired to reduce the influence of the extreme values in a subgroup. This might be the case for subgroups made of many automated measurements of highly variable samples such as when measuring tensile strength. Median charts are easy to use and do not require as many calculations, particularly for subgroups of small size containing an odd number of observations. This can increase shop floor acceptance of the control chart approach even more so when individual values in the subgroup are plotted together with their median on the same chart. The chart then also shows the spread of process output and gives an ongoing picture of the process variation. It should be noted that the median chart gives a marginally slower response to out-of-control conditions than the $\overline{X}$ chart.

Control limits for median charts are calculated in two ways: by using the median of the subgroup medians and the median of the ranges; or by using the average of the subgroup medians and the average of the ranges. Only the latter approach, which is easier and more convenient, is considered in this International Standard.

The control limits are calculated as follows.

6.3.1 Median chart

Centre line = $\overline{\tilde{X}}$ = average of the subgroup medians

$$U_{CL\tilde{X}} = \overline{\tilde{X}} + A_4\overline{R}$$

$$L_{\mathrm{CL}\bar{\bar{X}}} = \bar{\bar{X}} - A_4\bar{R}$$

The values of the constant A_4 are given in Table 4.

Table 4 — Values of A_4

n	2	3	4	5	6	7	8	9	10
A_4	1,880	1,187	0,796	0,691	0,548	0,508	0,433	0,412	0,362

6.3.2 Range chart

The range chart is constructed in the same way as for the case of the $\bar{X}$ and R chart in 6.1.

7 Control procedure and interpretation for variables control charts

The Shewhart system of charts stipulates that if the process location and the process variability were to remain constant at their present levels, the individual plotted statistics (e.g. $\bar{X}$, R, s) would vary by chance alone and they would seldom fall outside the control limits. Likewise, there would be no obvious trends or patterns in the data, beyond what would occur due to chance. The charts for location show where the process mean is located and indicates whether or not the process is stable with respect to the mean. The $\bar{X}$ chart, for example, reveals between-subgroups variations over time and is designed for detecting shifts in mean between the subgroups. The s or R chart reveals within-subgroup variation at a given time and is designed for detecting changes in process variation. The s or R chart must be in control before a location chart is interpreted. The following control procedure applies to the $\bar{X}$ and s (or R) charts. A similar procedure can be used for other control charts including the individual (X) chart where rational subgrouping is not appropriate.

7.1 Collect preliminary data

Gather preliminary rational subgroups (see 11.3) from a process under standard operating conditions. Compute the s (or R) of each subgroup. Compute the average ($\bar{s}$ or $\bar{R}$) of the subgroup statistics. Typically a minimum of 25 preliminary subgroups are taken to ensure reliable estimates ($\bar{s}$ or $\bar{R}$) of the process variability and consequently the control limits.

7.2 Examine the s (or R) chart

Compute and plot the trial centre line and control limits of the s (or R) chart. Examine the data points against the trial control limits for points outside the control limits or for unusual patterns or trends. For each such signal on the chart, conduct an analysis of the operation of the process to attempt to identify and remove assignable cause(s).

NOTE 1 The sampling distributions of s and R are both asymmetric about its mean value. However, for simplicity and ease in constructing the s and R chart, symmetric 3 sigma limits have been widely adopted. A lower control limit of 0 is used when the calculated lower limit is a negative value.

NOTE 2 If one fails to identify an assignable cause for a point that plots out-of-control, one should retain the point in the calculation of control limits.

7.3 Remove assignable causes and revise the chart

Exclude all subgroups affected by the identified assignable causes; then recalculate and plot the revised centre line and control limits. Examine the chart to determine if all remaining data points show statistical control when compared to the revised limits; repeat the identification/recalculation sequence if necessary.

NOTE Ensure that at least 2/3 of subgroups remain. Collect additional subgroups if necessary.

7.4 Examine the X chart

Once the standard deviations (or ranges) are in statistical control, the process variability (the within-subgroup variation) is considered to be stable. The averages can then be analysed to examine whether the process central location is changing over time. Compute and plot the centre line and control limits of the $\bar{X}$ chart. Examine the data points against the control limits for points outside of control limits or for unusual patterns or trends. Exclude any out-of-control points for which assignable causes have been identified, recalculate and plot the revised centre line and control limits. Check that all data points show statistical control when compared to the revised limits, repeat the identification/recalculation sequence if necessary.

Any subgroups that are excluded from the construction of s (or R) chart shall also be excluded from the construction of the $\bar{X}$ chart.

NOTE 1 The exclusion of subgroups representing out-of-control conditions is to ensure that the control limits are calculated reflecting only process variation due to the chance causes.

NOTE 2 Out of control situations eliminated to determine control limits must not be excluded on the plotted chart to provide vital clues to know the process behaviour and aid investigations.

7.5 Ongoing monitoring of process

When statistical control has been established so that there are no signals on the charts, these revised control limits shall be adopted for future ongoing monitoring of the process. Because the process has been demonstrated to be in a state of statistical control, there is no need to alter the control limits as additional subgroups are obtained in this monitoring phase. However, one may wish to update the control limits from time to time or whenever there is any change in the process.

In the event of a signal being given on the chart and an assignable cause identified the elimination of which required substantial changes be made to the process, it is possible or likely that the procedure of identification/recalculation outlined in 7.1 to 7.4 may be required to re-establish control of the process.

8 Pattern tests for assignable causes of variation

Systematic or non-random patterns on the control chart might indicate smaller shifts in process mean or process variability that may not be large enough to manifest themselves quickly as points outside the control limits. The analyst should be alert to any patterns of points on the chart that might indicate the influences of assignable causes in their process. A set of pattern tests can be used for interpreting patterns in Shewhart $\bar{X}$ chart and X chart is schematically presented in Figure 3.

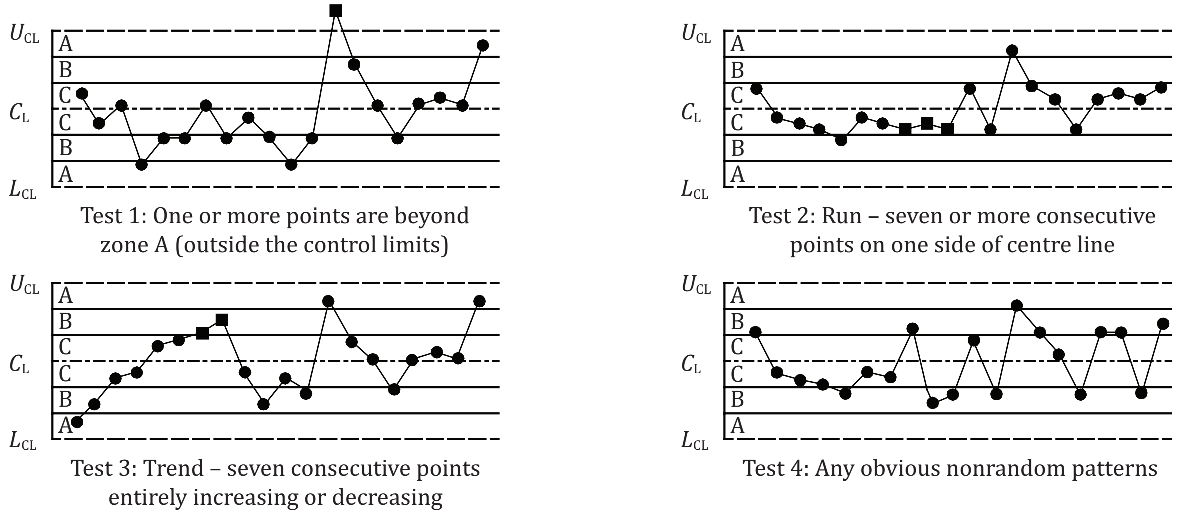


Figure 3 — Examples of pattern tests for assignable causes

NOTE 1 Some industries may use different pattern tests.

NOTE 2 With respect to the charts *p*, *np*, *c* and *u* where the lower control limit has been set to zero, it will not be possible to create the three 1-sigma zones below the centre line.

For the purpose of applying these tests, the control chart is equally divided into three zones A, B, and C on each side of the centre line, each zone being one sigma wide. This partitioning makes it easy for an investigator to detect a pattern that deviates away from a stable process. For example, the “obvious non-random patterns” of Test 4 can be more easily detected when such partitions are applied. We expect about 2/3 of the plotted points to lie in zone C in a stable process. If substantially fewer than 2/3 of the plotted points lie in zone C, as shown in the Test 4 of Figure 3, one should be concerned about such a non-random pattern in the plot. Such a pattern calls for further investigation of their process for potential assignable causes. Following are the typical signals provided by the four tests in Figure 3:

a) Test 1 signals the presence of an out-of-control condition.

b) Test 2 signals the process mean or variability has shifted from the centre line.

c) Test 3 signals a systematic linear trend in the process.

d) Test 4 signals a non-random or cyclical pattern in the process.

For a more complete discussion of these tests, see Nelson, L.S. (1984)[2] and Nelson, L.S. (1985).[3] Examples are given in Annex B.

A process with a sequence of points on the chart that violates one or more of the test rules is said to be out-of-control and its assignable causes of variation must be diagnosed and corrected. These supplementary test rules do improve the ability of the control chart to detect smaller shifts in process mean, but at the expense of higher false alarm rate. A Shewhart $\bar{X}$ or X chart with pattern tests one to three applied simultaneously has a false alarm rate of about 10 per thousand, in contrast to about 3 per thousand when only the first test is applied.

9 Process control, process capability, and process improvement

The function of a process control system is to provide statistical signals separating non-assignable from assignable causes of variation, which leaves only non-assignable variation present. The systematic elimination of assignable causes of excessive variation through continuous determined efforts of eliminating causes brings the process into a state of statistical control. Once the process is operating in statistical control, its performance is predictable and its capability to meet the specifications can be assessed. Since prediction is the essence of management, this ability to know what to expect is invaluable in terms of the process operating more consistently, more predictably and more reliably.

Process capability is determined by the total variation that comes from common causes—the minimum variation that can be achieved after all assignable causes have been eliminated. Process capability represents the performance of the process itself, as demonstrated when the process is being operated in a state of statistical control (see the ISO 22514 series). As such, the process must first be brought into statistical control before its capability can be assessed. Thus, the assessment of process capability begins after control issues in both the $\overline{X}$ and R charts have been resolved; that is, special causes have been identified, analysed, corrected and prevented from recurring and the ongoing control charts reflect a process that has remained in statistical control, preferably for at least the past 25 subgroups. In general, the distribution of the process output is compared with the engineering specifications to see whether these specifications can consistently be met.

Process capability is generally measured in terms of a process capability index C_p and C_pk. See the ISO 22514 series. A C_p value of less than 1 indicates that the process is not capable, while a C_p = 1 implies that the process is only just capable. In practice, a C_p value of 1,33 is generally taken as the minimum acceptable value fully in because there is always some sampling variation and few processes are ever in statistical control consistently.

However, it must be noted that the C_p measures only the relationship of the limits to the process spread; the location or the centring of the process is not considered. It would be possible to have any percentage of values outside the specification limits with a high C_p value. For this reason, it is important to consider the scaled distance between the process average and the closest specification limit.

In view of the above discussion, a procedure, as schematically presented in Figure 4, can be used as a guide to illustrate key steps leading towards process control, capability and improvement. Specified capability minimum requirements are the product of a negotiation between the supplier and the customer.

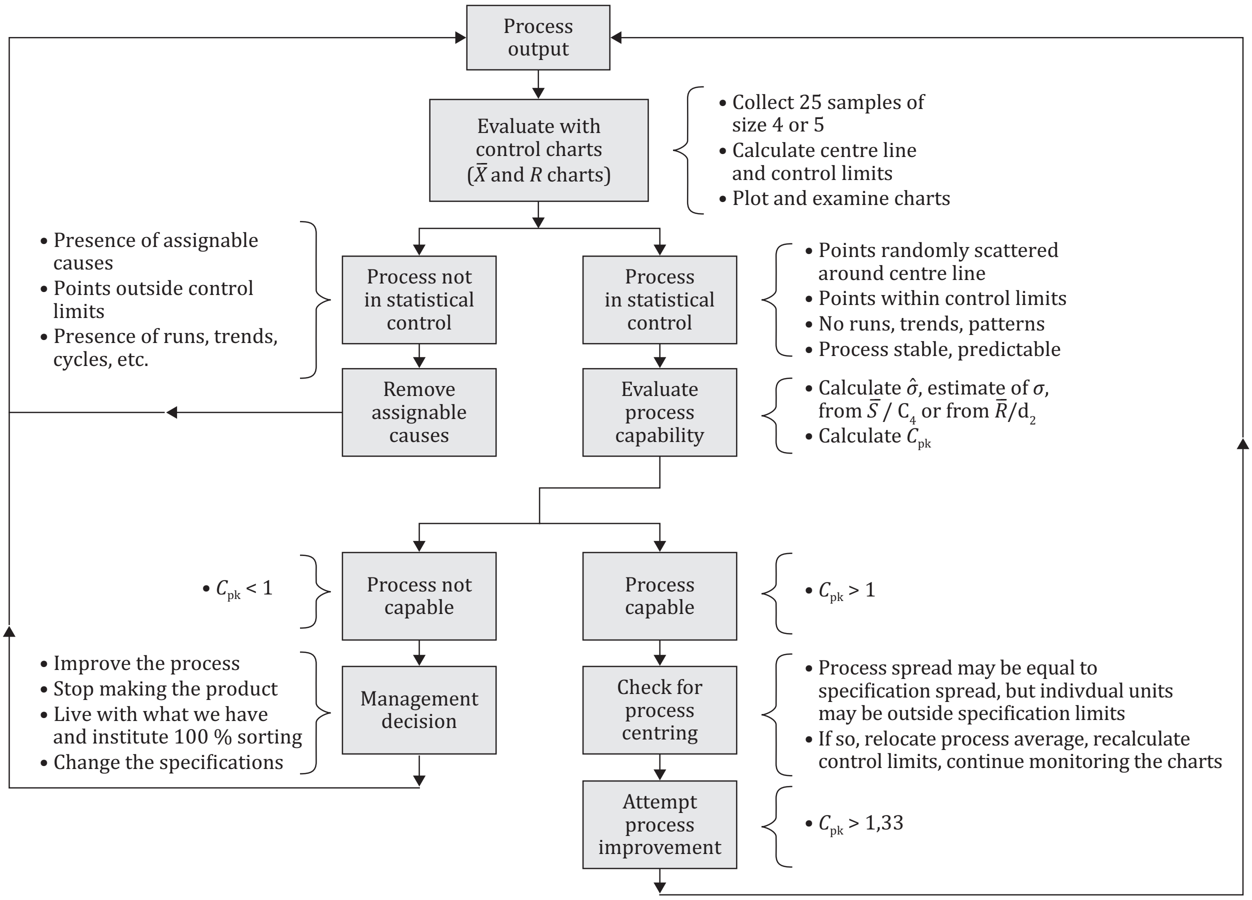


Figure 4 — Strategy for process improvement

NOTE Optimal sample size is a function of components of within and between samples variation.

10 Attributes control charts

Attribute data represent observations obtained by noting the presence or absence of some characteristic (or attribute) in each of the units in the subgroup under consideration, then counting how many units do or do not possess the attribute, or how many such events occur in the unit, group or area. Attribute data are generally rapid and inexpensive to obtain and often do not require specialized collection skills. Table 5 gives control limit formulae for attributes control charts.

There is much attention focused on the use of variable data for process improvement, but feedback data from major industries indicate that over 80 % of quality problems are attribute in nature. More emphasis, therefore, is needed on the improvement of attribute characteristics using control charts.

In the case of control charts for variables, it is common practice to maintain a pair of control charts – one for the control of the average and the other for the control of the dispersion. This is necessary because the underlying distribution in the control charts for variables is the normal distribution, which depends on these two parameters. However, in the case of control charts for attributes, a single chart will suffice since the assumed distribution has only one independent parameter, the average level. The *p* and *np* charts are based on the binomial distribution, while the *c* and *u* charts are based on the Poisson distribution.

Computations for these charts are similar except in cases where the variability in subgroup size affects the situation. When the subgroup size is constant, the same set of control limits can be used for each subgroup. However, if the number of items inspected in each subgroup varies, separate control limits have to be computed for each subgroup. *np* and *c* charts may thus be reasonably used with a constant sample size, whereas *p* and *u* charts could be used in either situation.

Where the sample size varies from sample to sample, separate control limits are calculated for each sample. The smaller the subgroup size, the wider the control bands, and vice versa. If the subgroup size does not vary appreciably, then a single set of control limits based on the average subgroup size can be used. For practical purposes, this holds well for situations in which the subgroup size is within ± 25 % of the target subgroup size.

NOTE Alternatively, control limits for smallest and largest sample size may be used. For points following in between, only the control limits may calculated.

Table 5 — Control limit formulae for Shewhart attributes control charts

Statistic	No standard values given		Standard values given	
	Centre line	3σ - control limits	Centre line	3σ - control limits
p	$\overline{p}$	$\overline{p} \pm 3\sqrt{\overline{p}(1-\overline{p})/n}$	p_0	$p_0 \pm 3\sqrt{p_0(1-p_0)/n}$
np	$n\overline{p}$	$n\overline{p} \pm 3\sqrt{n\overline{p}(1-\overline{p})}$	np_0	$np_0 \pm 3\sqrt{np_0(1-p_0)}$
c	$\overline{c}$	$\overline{c} \pm 3\sqrt{\overline{c}}$	c_0	$c_0 \pm 3\sqrt{c_0}$
u	$\overline{u}$	$\overline{u} \pm 3\sqrt{\overline{u}/n}$	u_0	$u_0 \pm 3\sqrt{u_0/n}$
NOTE 1 p_0, np_0, c_0 and u_0 are given standard values.				
NOTE 2 A lower control limit of 0 is used when the calculated lower limit is a negative value.				

An alternative procedure for situations in which the sample size varies greatly is the use of a standardized variate. For example, instead of plotting *p*, plot the standardized value

$$Z = \frac{p - p_0}{\sqrt{p_0(1-p_0)/n}}$$

or

$$Z = \frac{p - \overline{p}}{\sqrt{\overline{p}(1-\overline{p})/n}}$$

according to whether the standard value for *p* is specified or not. The centre line as well as the control limits become constant, independent of subgroup size, and are given as

centre line = 0

$U_{CL} = +3$

$L_{CL} = -3$

The *p* chart is used to determine the average percentage of nonconforming items submitted over a period of time. It brings to the attention of process personnel and management any changes in this average. The process is judged to be in statistical control in the same way as is done for the $\overline{X}$ and R charts. If all the sample points fall within the trial control limits without exhibiting any indication of an assignable cause, the process is said to be in control. In such a case, the average fraction nonconforming, $\overline{p}$, is taken as the standard value for the fraction nonconforming, p_0.

Low results on the control charts (points below the lower control limits) should be treated differently to high plots. They are indicative in changes to process by removal of common causes, but a word of caution- it might also point to lower inspection standards. When a significant break through the L_{CL} occurs, it is important to understand the causes and to institutionalize the change in the work standard.

11 Preliminary considerations before starting a control chart

11.1 Choice of critical to quality (CTQ) characteristics describing the process to control

Characteristics that critically affect the performance of the product, process, or service, and which add value to the customer should be classified at the quality planning stage. These characteristics, where variation is the significant factor of the process should be selected to have a decisive effect on product or service quality and to ensure the stability and predictability of the processes. These may be aspects directly related to evaluation of the performance of the process – for example, related to the environment, health, customer satisfaction – or a process parameter whose performance is vital in achieving the design intent. Control charts should be introduced during the early stage of process development to collect data and information about a new product and process feasibility to achieve process capability prior to production. This enables the processes to be optimized, and any design or process improvement made for a better product or service produced.

11.2 Analysis of the process

If possible, a detailed analysis of the process should be made to determine:

a) the kind and location of causes that may give rise to irregularities;

b) the effect of the imposition of specifications;

c) the method and location of inspection;

d) all other pertinent factors that may affect the production process.

Analysis should also be performed to determine the stability of processes, the accuracy of testing equipment, the quality of the outputs of the processes, and the patterns of correlation between the types and causes of nonconformities. The conditions of operations are required to make arrangements to adjust the production process and equipment, if needed, as well as to devise plans for the statistical control of processes. This will help pinpoint the most optimal place to establish controls and identify quickly any irregularities in the performance of the process to allow for prompt corrective action.

11.3 Choice of rational subgroups

At the basis of control charts is Shewhart's central idea of the division of observations into what are called "rational subgroups"; that is the classification of the observations under consideration into subgroups, within which the variations may be considered to be due to chance causes only, but between which any difference may be due to assignable causes which the control chart is intended to detect.

This depends on some technical knowledge and familiarity with the process conditions and the conditions under which the data were taken. By identifying each subgroup with a time or a source, specific causes of trouble may be more readily traced and corrected, if advantageous. Inspection and test records given in the order in which the observations were taken provide a basis for subgrouping with respect to time. This is commonly useful in manufacturing where it is important to maintain the production cause system constant with time.

In collecting data it should always be remembered that analysis will be greatly facilitated if care is taken to select the samples that can be properly treated as separate rational subgroups. If possible, the subgroup size should be kept constant to facilitate calculations and interpretation. However, it should be noted that the principles of Shewhart charts can equally be applied to situations where subgroup size varies.

11.4 Frequency and size of subgroups

No general rules may be laid down for the frequency of subgroups or the subgroup size. The frequency and size of subgroup may depend upon the cost of taking and analysing samples and allied practical considerations. For instance, large subgroups taken at less frequent intervals may detect a small shift in the process average more accurately, but small subgroups taken at more frequent intervals will detect a large shift more quickly. Often, the subgroup size is taken to be 4 or 5, while the sampling frequency is generally high in the beginning and low once a state of statistical control is reached. Normally, 25 subgroups of size 4 or 5 are considered adequate for providing preliminary estimates.

It is worth noting that sampling frequency, statistical control and process capability need to be considered together. The reasoning is as follows. The value of the average range $\overline{R}$ is often used to estimate σ. The number of sources of variation increases as the time interval between samples within a subgroup increases. Therefore, spreading out the samples within a subgroup over time will increase $\overline{R}$, increase the estimate of σ, widen the control limits and will thus appear to decrease the process capability index. Conversely, it is possible to increase process capability by consecutive piece sampling, giving a small $\overline{R}$ and σ estimate.

11.5 Preliminary data collection

After having decided upon the quality characteristic which is to be controlled and the frequency and size of the subgroup to be taken, some initial inspection data or measurements have to be collected and analysed for the purpose of providing preliminary control chart values that are needed in determining the centre line and control limits to be drawn on the chart. The preliminary data may be collected subgroup by subgroup until the recommended 25 subgroups have been obtained from a continuous run of the production process. Care shall be exercised that, during the course of this initial data collection, the process is not unduly influenced intermittently by extraneous factors such as change in the feed of raw material, operators, operations, machine settings, etc. In other words, the process should exhibit a state of stability during the period when preliminary data are being gathered.

11.6 Out of control action plan

There is an important connection between the two types of variation found and the types of action necessary to reduce them. Control charts can detect special causes of variation. Discovering the source of the special cause and taking the remedial actions is usually the responsibility of operators, supervisors or engineers directly associated with the process. Management are responsible for more than 80 % of the causes and must take action on the common causes in the system. Special causes are identified locally and can usually be actioned by the process owners. Processes are often adjusted as remedial action when management action on the system is needed on the root cause which might be different sources of raw material, machine maintenance, gauging or an unreliable method. Close teamwork is the key to long term continual improvement.

When the process is inherently non-capable or is capable but goes out of statistical control and is found to be producing nonconforming product, then 100 % inspection is normally instituted until the process is corrected.

Consistency of inspection needs to be assured. Uncertainty measurement needs to be kept in harmless tolerable limits.

12 Steps in the construction of control charts

The steps involved in the construction of the $\overline{X}$ chart and the R chart, for the case when no standard values are given, are described in 12.1 to 12.3. They are described in the form of an example in Annex A. In the construction of other control charts, the same basic steps shall be followed but the computational method for determining control limits and centre line are different. A typical format of a standard control chart form is shown in Figure 5. Modifications to this form can be made in concert with the particular requirements of a process control situation.

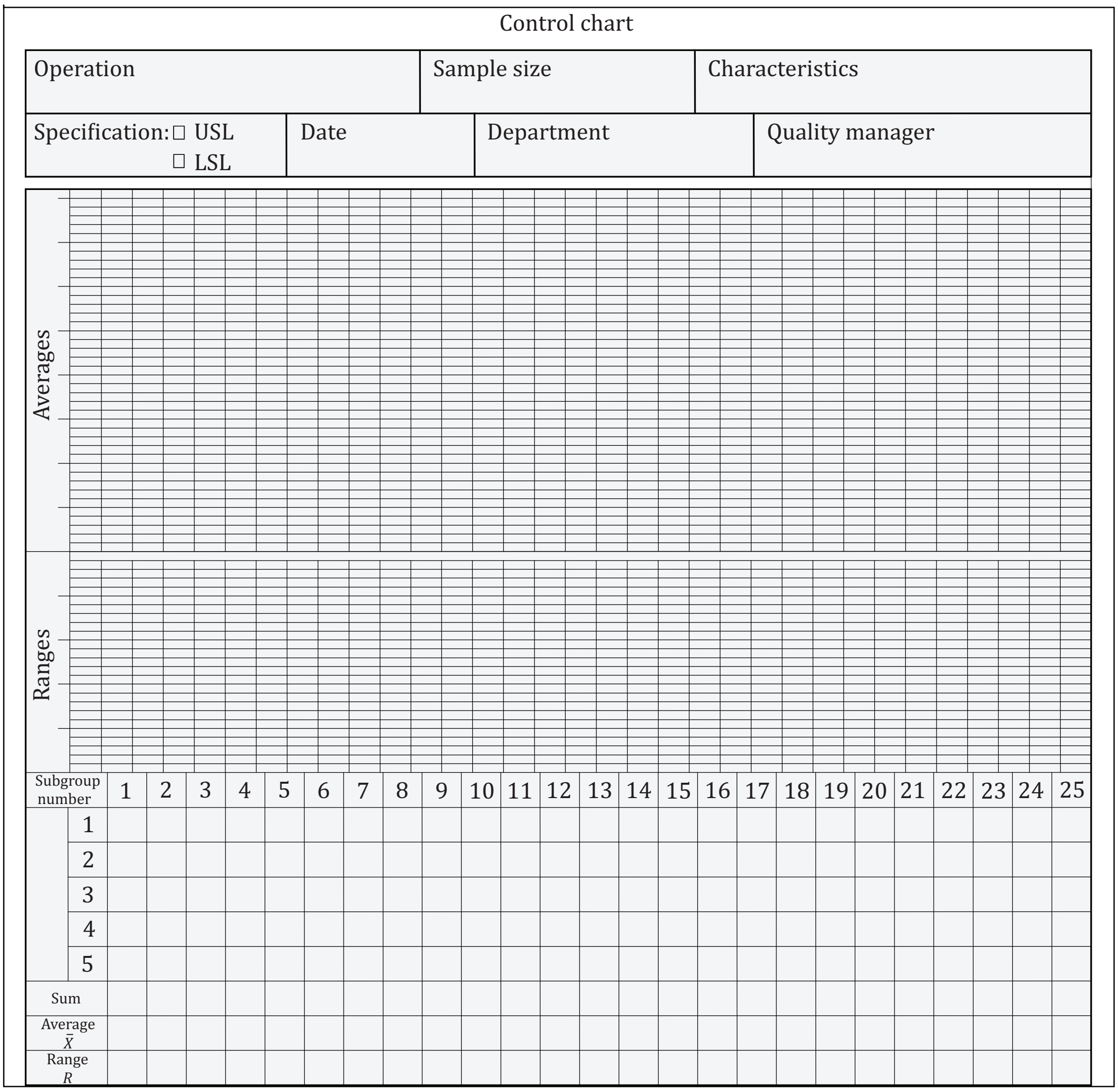


Figure 5 — General format of a variables control chart

12.1 Determine data collection strategy

If the preliminary data were not taken in subgroups according to a prescribed plan, break up the total set of observed values into sequential subgroups, according to the criteria for rational subgroups as discussed in 10.3. The subgroups must be of the same structure and size. The items of any one subgroup should have what is believed to be some important common factor, for example units produced during the same short interval of time or units coming from one of several distinct sources or locations. The different subgroups should represent possible or suspect differences in the process that produced them, for example different intervals of time or different sources or locations.

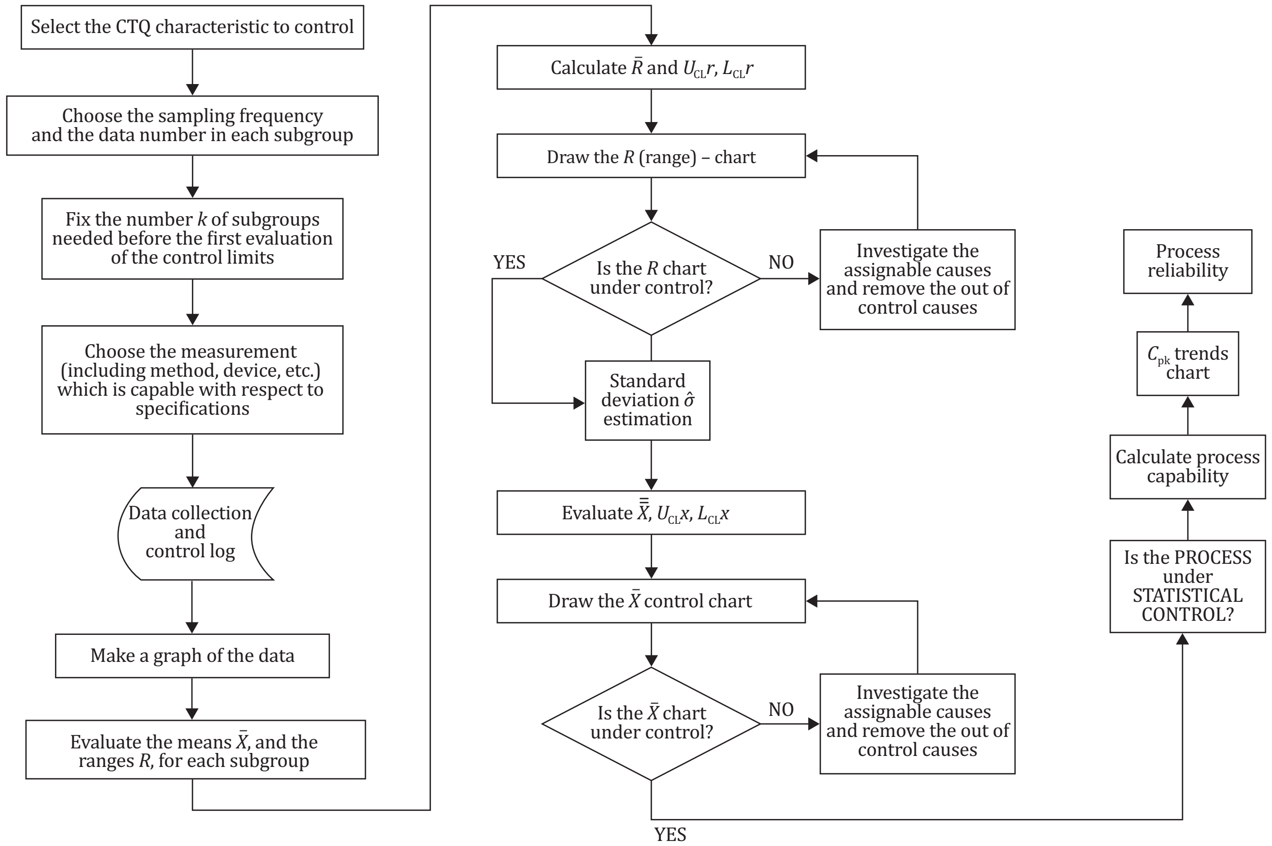


Figure 6 — Systems approach to the construction of variables control charts

NOTE Prepare a list of known-sources of chance and assignable causes of variation.

12.2 Data collection and computation

For each subgroup, calculate the average, $\bar{X}$, and the range, R. Then, compute the grand average of all the observed values, $\bar{\bar{X}}$, and the average range, $\bar{R}$.

12.3 Plotting $\bar{X}$ chart and R chart

On a suitable form or graph paper, lay out an $\bar{X}$ chart and an R chart. The vertical scale on the left is used for $\bar{X}$ and for R and the horizontal scale is used for the subgroup number. Plot the computed values for $\bar{X}$ on the chart for averages and plot the computed values for R on the chart for ranges.

On these respective charts, draw solid horizontal lines to represent $\bar{\bar{X}}$ and $\bar{R}$.

Place the control limits on these charts. On the $\bar{X}$ chart, draw two horizontal dashed lines at $\bar{\bar{X}} \pm A_2 \bar{R}$ and, on the R chart, draw two horizontal dashed lines at $D_3 \bar{R}$ and $D_4 \bar{R}$, where A_2, D_3 and D_4 are based on n, the number of observations in a subgroup, and are given in Table 2. The L_{CL} on the R chart is not needed whenever n is less than 7 since the ensuing value of D_3 is considered zero.

13 Caution with Shewhart control charts

There are some practical situations, as given below, where some caution may be needed in using Shewhart control chart.

13.1 General caution

The variation within a subgroup may not necessarily be due to chance causes alone. The subgroup is composed of a treatment lot; that is, the variability within a subgroup is the variability within a lot. The subgroup has a meaning from the viewpoints of both physical aspect and quality assurance. Therefore, it is necessary to control the variability within a treatment lot using R chart.

Figure 7 shows $\bar{X}$ and R control chart in the early-stage mass production of a heat treatment process. This is a $\bar{X}$ and R control chart where no standard values are given. R chart indicates process in state of control, but $\bar{X}$ chart shows many points and situations out-of-control.

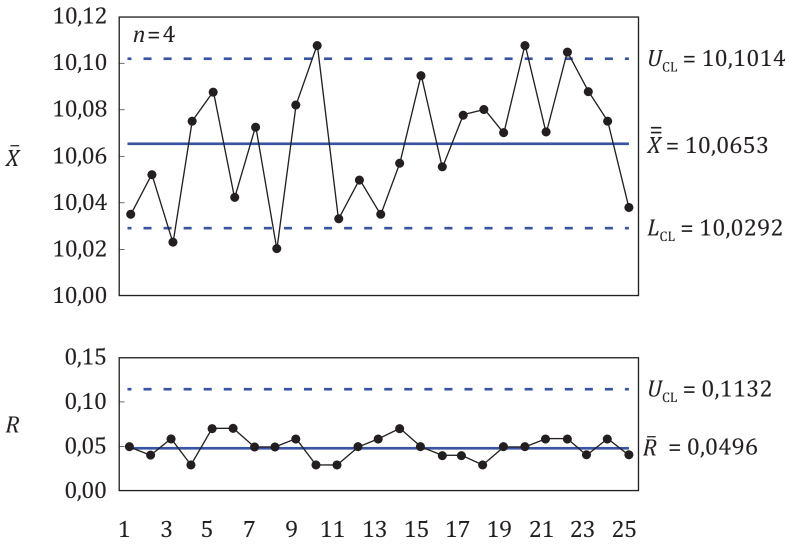


Figure 7 — Ordinary $\bar{X}$ and R chart in the early-stage mass production

On the other hand, Figure 8 shows another $\bar{X}$ and R chart for the same data as that in Figure 7, where the control limits of $\bar{X}$ chart are calculated on the basis of the overall process variability instead of the mean of ranges ($\bar{R}$).

Figure 8 indicates that process is in-control. At that time if the process performance is well satisfied, it can be decided that the process can proceed to the routine mass production stage from the early-stage mass production. Then the control limits of $\bar{X}$ and R chart in Figure 8 are used as a standard control level in the routine mass production. This means that the random variability due to some allowable causes between subgroups in the early-stage mass production is included as the variability due to chance causes.

Therefore, it should be noted that variability within a subgroup does not necessarily mean variability due to chance causes only. However, 17 to 24 points on $\bar{X}$ chart falling above the central line, and the increasing trend from 9 to 24 points, along with clustering of points about $\bar{R}$ on range chart, do indicate potential for improvement through detection and elimination of assignable causes.

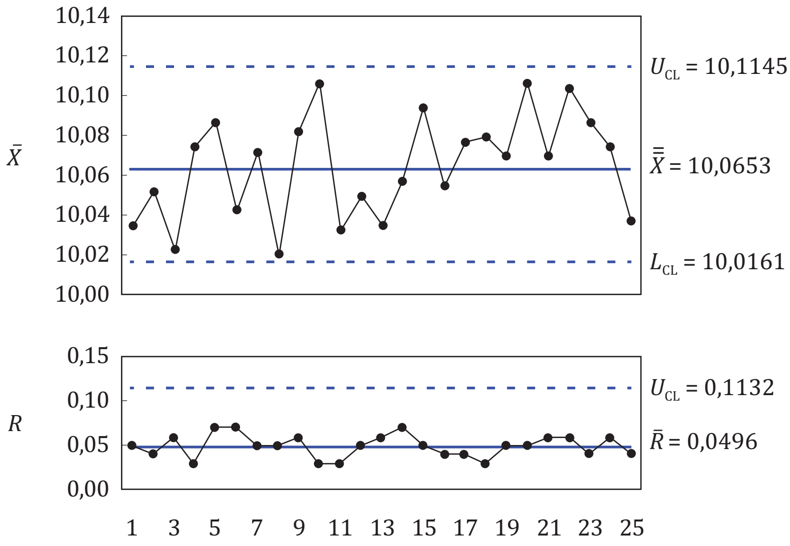


Figure 8 — $\bar{X}$ and R chart, where the control lines of $\bar{X}$ chart are given from the overall process variability instead of the mean of range $\bar{R}$

13.2 Correlated data

In the presence of data correlation, the following equation, which is a fundamental equation in conducting a $\bar{X}$ chart with the sample size n, does not hold:

$$\sigma^2(\text{individuals}) = \frac{\sigma^2(\bar{X})}{n}$$

Therefore, if the control limits are calculated in the ordinary procedure, they are misplaced.

In such case, the process model should be identified and then the residuals from the model should be regarded as the observations. An alternative way is that the control limits should be calculated from the variability of $\bar{X}$. One should consult a specialist for advice.

13.3 Use of alternative rule to the three-sigma rule

The Shewhart control chart for the average will detect a large sustained shift in the process mean level quickly. However if the shift in the mean is small, of magnitude 1,5 standard deviations or less, the Shewhart $\bar{X}$ control chart does not perform well. Therefore, in such cases, if the small shift in the process mean from a desirable level has to be detected as soon as possible, then additional pattern tests are usually employed. However, such supplemental rules may increase the false alarm rate, that is, the probability of observing a signal on the chart through the application of these rules increases substantially. On the other hand, when the control chart without standard values is used in the early-stage mass production, the supplementary rules given in Clause 8 should be considered for improving process performance. Alternative strategy is to use the control charts, such as, the Exponentially Weighted Moving Average (EWMA) chart or a Cusum chart.

Another rule is to replace the conventional out-of-control signal criterion as well as the position of the control limits on the chart. A signal will be given on the $\bar{X}$ chart if two out of three points lie beyond 2 σ limits. When using this “two of three” criterion it is recommended that the usual 3 sigma control limits on the $\bar{X}$ chart be replaced by control limit lines placed 1,78 sigma on either side of the centre line. Use of this rule and these control limits will produce a chart with false alarm rate equivalent to that of conventional Shewhart control chart having a rule of one point outside the 3 sigma control limits.

However, the probability of detecting small to moderate shifts increases substantially with the use of this modified criterion.

Annex A
(informative)

Illustrative examples

A.1 Variables control charts

A.1.1 $\overline{X}$ chart and *R* chart - μ and σ unknown

A.1.2 $\overline{X}$ chart and *s* chart - μ and σ given

A.1.3 Control charts for individuals and moving ranges - μ and σ unknown

A.1.4 Median chart and *R* chart - μ and σ unknown

A.2 Attributes control charts

A.2.1 *p* chart - no p_0 value given

A.2.2 *np* chart- no p_0 value given

A.2.3 *c* chart- no c_0 value given

A.2.4 *u* chart- no u_0 value given

A.3 Variables control charts

A.3.1 $\overline{X}$ chart and *R* chart – μ and σ unknown

A supplier of houses for water pumps wishes to control a turning process using a control chart. An important characteristic is the bearing diameter. Measurements from a new production are taken every hour for total 25 samples. The maximum and minimum values in the subgroup samples are given in Table A.1.

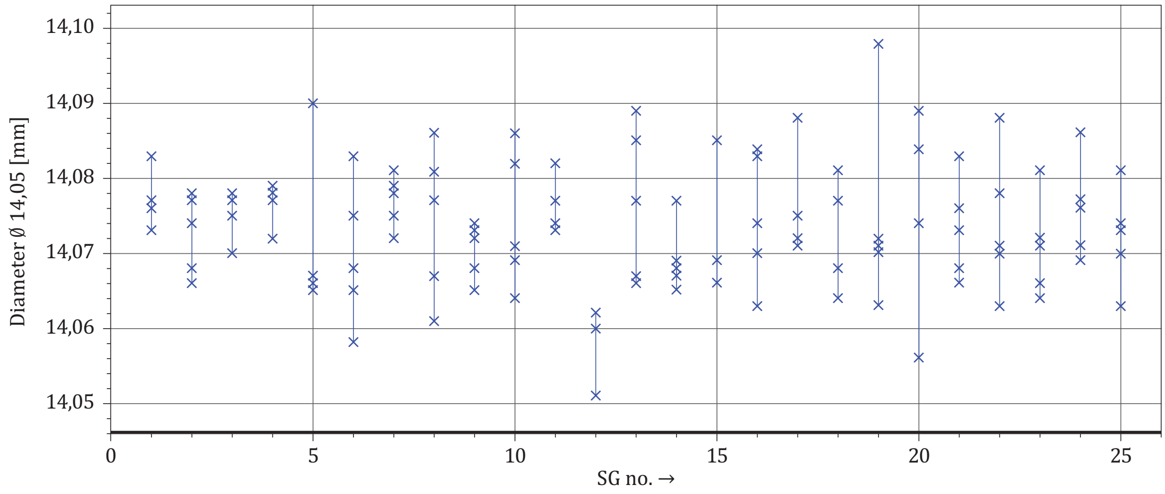


Figure A.1 — Plotted values

NOTE 1 The data consist of 125 data points, which are from 25 subgroups (SG) of sample size 5. The 125 data points are plotted in Figure A.1 and standard calculations on the subgroups are given in Table A.1.

NOTE 2 A histogram may also be plotted alongside. This chart seen together with the histogram will display the process behaviour transparently. Central tendency and freak observations become more obvious which otherwise are not easy to observe.

Table A.1 — Subgroup results from measurement of bearing diameter

j	$\overline{X}_j$	$x_{min\,j}$	$x_{max\,j}$	R_j
1	14,076 4	14,073	14,083	0,010
2	14,072 6	14,066	14,078	0,012
3	14,075 4	14,070	14,078	0,008
4	14,077 0	14,072	14,079	0,007
5	14,070 8	14,065	14,090	0,025
6	14,069 8	14,058	14,083	0,025
7	14,077 0	14,072	14,081	0,009
8	14,074 4	14,061	14,086	0,025
9	14,070 4	14,065	14,074	0,009
10	14,074 4	14,064	14,086	0,022
11	14,076 6	14,073	14,082	0,009
12	14,056 8	14,051	14,062	0,011
13	14,076 8	14,066	14,089	0,023
14	14,069 2	14,065	14,077	0,012
15	14,071 6	14,066	14,085	0,019
16	14,074 8	14,063	14,084	0,021
17	14,075 4	14,071	14,088	0,017
18	14,073 4	14,064	14,081	0,017
19	14,074 8	14,063	14,098	0,035
20	14,075 4	14,056	14,089	0,033

Table A.1 *(continued)*

j	$\overline{X}_j$	xmin j	xmax j	Rj
21	14,073 2	14,066	14,083	0,017
22	14,074 0	14,063	14,088	0,025
23	14,070 8	14,064	14,081	0,017
24	14,076 0	14,069	14,086	0,017
25	14,072 2	14,063	14,081	0,018

Since μ and σ in this example are unknown, $\overline{\overline{X}}$ and $\overline{R}$ are calculated based on the total set of values.

The averages ($\overline{X}_j$) and the ranges (R_j) are calculated for each subgroup j (see Table A.1).

Based on these calculations,

$$\overline{\overline{X}} = \frac{1}{k}\sum_{j=1}^{k}\overline{x}_j = 14{,}073\,2\ \text{mm}$$

$$\overline{R} = \frac{1}{k}\sum_{j=1}^{k}R_j = 0{,}017\,7\ \text{mm}$$

where k is the number of subgroups.

The first step is to plot an R chart and evaluate its state of control.

The values of D_3 and D_4 are taken from Table 2 where n = 5

R chart

Centre line $C_L = \overline{R}$ = 0,017 7 mm

$U_{CL} = D_4 \times \overline{R}$ = 2,114 × 0,017 7 = 0,037 5mm

$L_{CL} = D_3 \times \overline{R}$ where D_3 = 0 when the sample size is < 7

The R chart indicates a process in control.

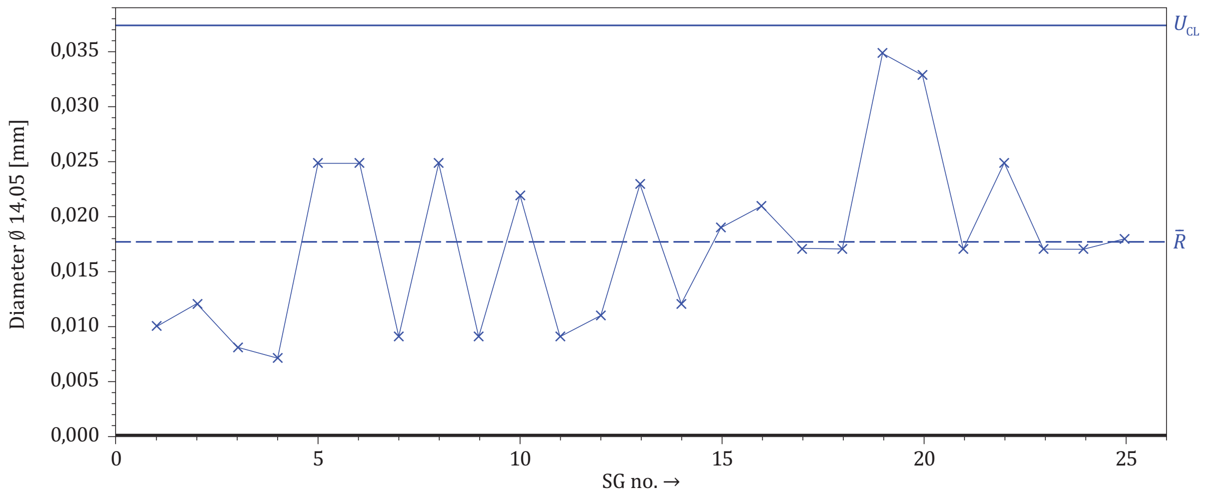


Figure A.2 — *R* chart – bearing diameter

Then an $\bar{X}$ chart can be calculated based on the $\bar{X}$ and R values

$\bar{X}$ chart:

Centre line $C_L = \bar{\bar{X}}$ = 14,073 17 mm

$$U_{CL} = \bar{\bar{X}} + A_2 \times \bar{R} = 14{,}073\ 17 + (0{,}577 \times 0{,}017\ 72) = 14{,}083\ 41\ \text{mm} \sim 14{,}083\ 4\ \text{mm}$$

$$L_{CL} = \bar{\bar{X}} - A_2 \times \bar{R} = 14{,}073\ 17 - (0{,}577 \times 0{,}017\ 72) = 14{,}062\ 93\ \text{mm} \sim 14{,}062\ 9\ \text{mm}$$

The value of the factor A_2 is taken from Table 2 where $n = 5$

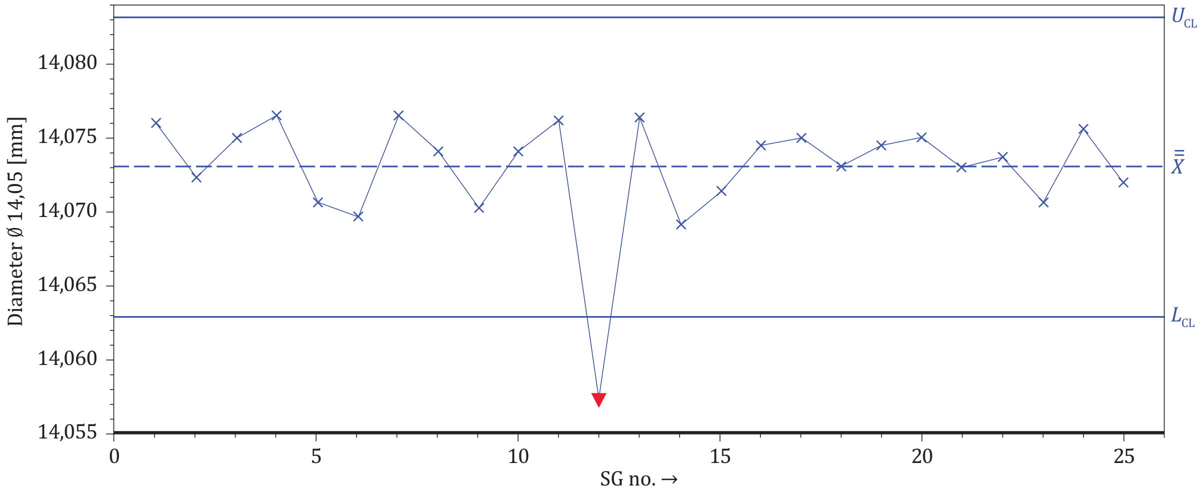


Figure A.3 — $\bar{X}$ chart – bearing diameter

The $\bar{X}$ chart indicates a process out of control.

The examination of the $\bar{X}$ chart reveals that subgroup 12 is out of control. It indicates that some assignable causes of variation may be operating.

We therefore omit subgroup 12 from the calculations and find:

$$\bar{\bar{X}} = \frac{1}{k}\sum_{j=1}^{k} \bar{x}_j = 14{,}073\,85\ \text{mm}$$

$$\bar{R} = \frac{1}{k}\sum_{j=1}^{k} R_j = 0{,}018\,00\ \text{mm}$$

The revised $\bar{X}$ chart

Centre line $C_L = \bar{\bar{X}}$ = 14,074 01 mm

$$U_{CL} = \bar{\bar{X}} + A_2 \times \bar{R} = 14{,}073\ 85 + (0{,}577 \times 0{,}018\ 00) = 14{,}084\ 23\ \text{mm} \sim 14{,}084\ 2\ \text{mm}$$

$$L_{CL} = \bar{\bar{X}} - A_2 \times \bar{R} = 14{,}073\ 85 - (0{,}577 \times 0{,}018\ 00) = 14{,}063\ 468\ \text{mm} \sim 14{,}063\ 5\ \text{mm}$$

The revised R chart:

Centre line $C_L = \overline{R} = 0{,}018\ 00$ mm

$U_{CL} = D_4 \times \overline{R} = 2{,}114 \times 0{,}01800 = 0{,}038\ 05 \text{ mm} \sim 0{,}038\ 1 \text{ mm}$

$L_{CL} = D_3 \times \overline{R}$ where $D_3 = 0$ when the sample size is < 7

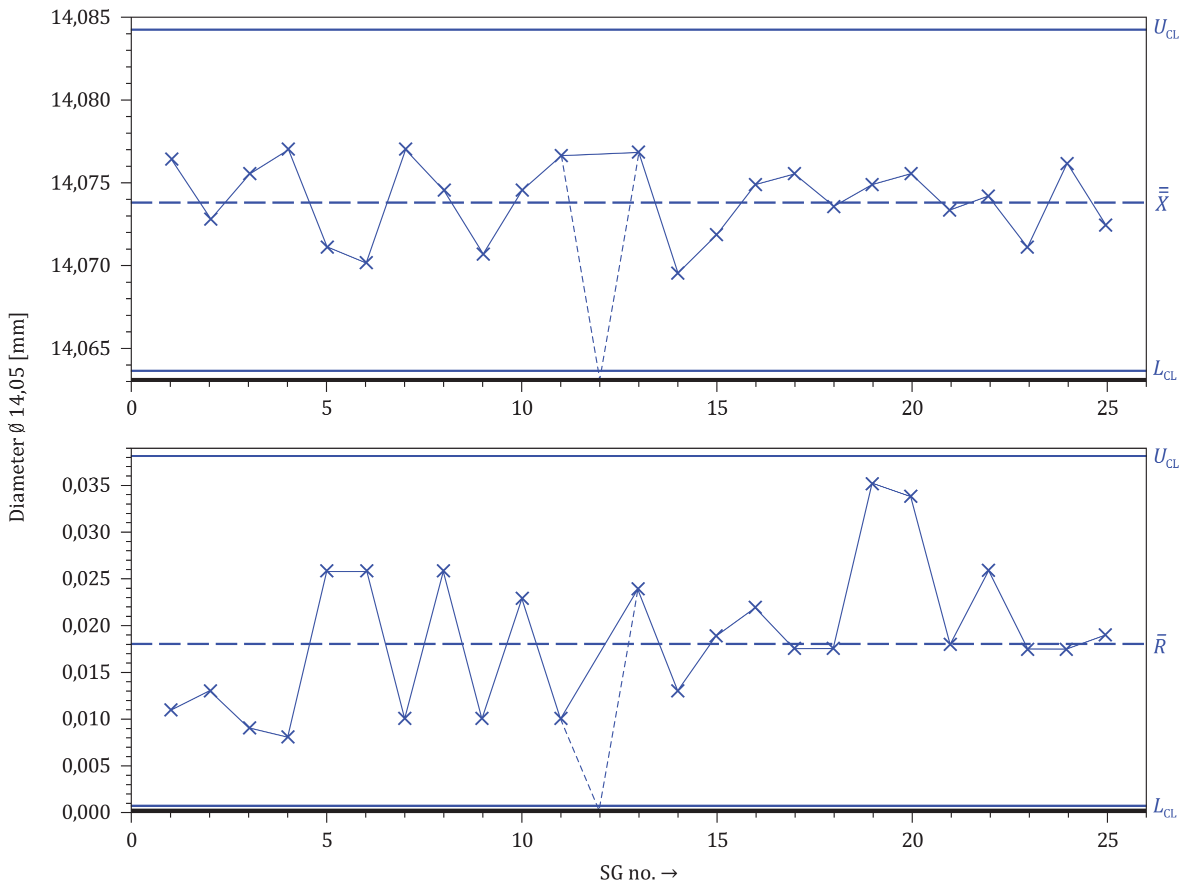


Figure A.4 — $\overline{X}$ and R chart – bearing diameter

After the deletion of subgroup 12 the process is in control and the above calculated control limits can be used to control the process in the future.

A.3.2 $\overline{X}$ chart and s chart – μ and σ given from productions in the past

A producer of batteries wishes to control the mass of his batteries such that the mean mass of the batteries is 29,87 g. A process analysis from a former production has shown that the standard deviation of the process can be assumed to 0,062 g.

Since the standard values are $\mu_0 = 29{,}87$ g and $\sigma_0 = 0{,}062$ g, the control chart can be immediately constructed using the formulae given in Table 1 and the factors A, C_4, D_2 and D_1 given in Table 2 using a subgroup size of 5.

Centre line $C_L = \mu_0 = 29{,}87$ mm

$U_{CL} = \mu_0 + A\sigma_0 = 29{,}87 + (1{,}342 \times 0{,}062) = 29{,}953\ 2 \text{ mm} \approx 29{,}953 \text{ mm}$

$L_{CL} = \mu_0 - A\sigma_0$ = 29,87 – (1,342 × 0,062) = 29,786 8 mm ≈ 29,787 mm

Centre line = $C_4\sigma_0$ = 0,94 × 0,062 = 0,058 28 mm ≈ 0,058 3 mm

$U_{CL} = B_6\sigma_0$ = 1,964 × 0,062= 0,121 768 mm ≈ 0,121 8 mm

$L_{CL} = B_5\sigma_0$ where B_5 = 0 when the sample size is < 6

Twenty-five samples of size 5 are now selected from the production process and their subgroup average and standard deviation are calculated, as shown in Table A.2.

Table A.2 — Subgroup results from production of batteries

j	$\overline{X}_j$	s_j
1	29,816	0,052
2	29,932	0,022
3	29,858	0,066
4	29,824	0,023
5	29,888	0,036
6	29,830	0,066
7	29,868	0,043
8	29,876	0,038
9	29,910	0,064
10	29,802	0,049
11	29,884	0,019
12	29,880	0,019
13	29,916	0,031
14	29,898	0,040
15	29,946	0,058
16	29,842	0,045
17	29,824	0,063
18	29,904	0,056
19	29,912	0,056
20	29,886	0,048
21	29,908	0,073
22	29,852	0,041
23	29,828	0,048
24	29,904	0,065
25	29,902	0,013

The subgroup results are plotted together with the control limits calculated above (see Figure A.5).

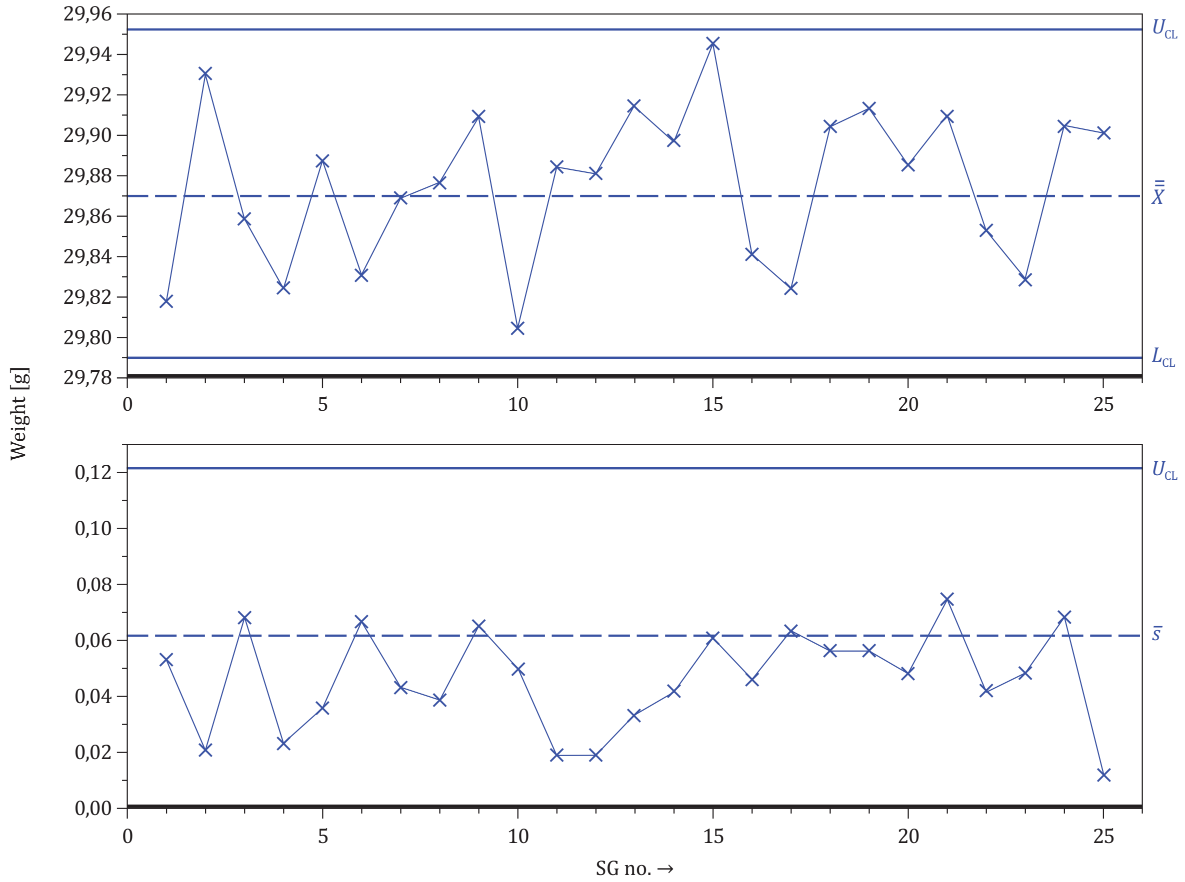


Figure A.5 — $\bar{X}$ chart and s chart – mass of batteries

The chart shown in Figure A.5 indicates that the process is in statistical control.

A.3.3 Control charts for individuals and moving ranges: μ and σ unknown

Table A.3 gives the results of laboratory analysis of "percent moisture" of samples from 25 successive lots of skim milk powder. A sample of skim milk powder, representing a lot, is analysed in the laboratory for such various characteristics as fat, moisture, acidity, solubility index, sedimentation, bacteria and whey protein. It was intended to control the percentage of moisture below 4 % for this process. The sampling variation within a single lot was found to be negligible, so it was decided to take only one observation per lot and to set control limits on the basis of the moving range of successive lots.

Table A.3 — Percent moisture for 25 successive samples of skim milk powder

Lot No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
X: % moisture	2,9	3,2	3,6	4,3	3,8	3,5	3,0	3,1	3,6	3,5	3,1	3,4	3,4
R_m		0,3	0,4	0,7	0,5	0,3	0,5	0,1	0,5	0,1	0,4	0,3	0
Lot No.	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
X: % moisture	3,6	3,3	3,9	3,5	3,6	3,3	3,0	3,4	3,8	3,5	3,2	3,5	
R_m	0,2	0,3	0,6	0,4	0,1	0,3	0,3	0,4	0,4	0,3	0,3	0,3	

Calculation of $\overline{X}$ and $\overline{R}$:

$$\overline{X} = \frac{2,9+3,2+...+3,5}{25} = \frac{86}{25} = 3,44\,\%$$

$$\overline{R} = \frac{0,3+0,4+...+0,3}{24} = \frac{8}{24} = 0,33\,\%$$

Control chart lines for moving ranges, R:

Centre line $C_L = \overline{R}$ = 0,33 %

$U_{CL} = D_4 \overline{R}$ = 3,267 × 0,33 = 1,078 ~ 1,08

$L_{CL} = D_3 \overline{R}$ = 0 × 0,33

The values of the factors D_3 and D_4 are obtained from Table 2 for n = 2. Since the range chart exhibits a state of statistical control, the plotting of the control chart for individuals can be carried out.

Control chart for individuals, X:

Centre line $C_L = \overline{X}$ = 3,44 %

$U_{CL} = \overline{X} + A_3 \overline{R}$ = 3,44 + (2,66 × 0,33) = 4,317 8 ~ 4,32

$L_{CL} = \overline{X} - A_3 \overline{R}$ = 3,44 − (2,66 × 0,33) = 2,562 2 ~ 2,5

The formulae for control limits and the value of the factor E_2 are given in Tables 2 and 3. The control charts are plotted in Figure A.6. The control charts indicate that the process is in statistical control.

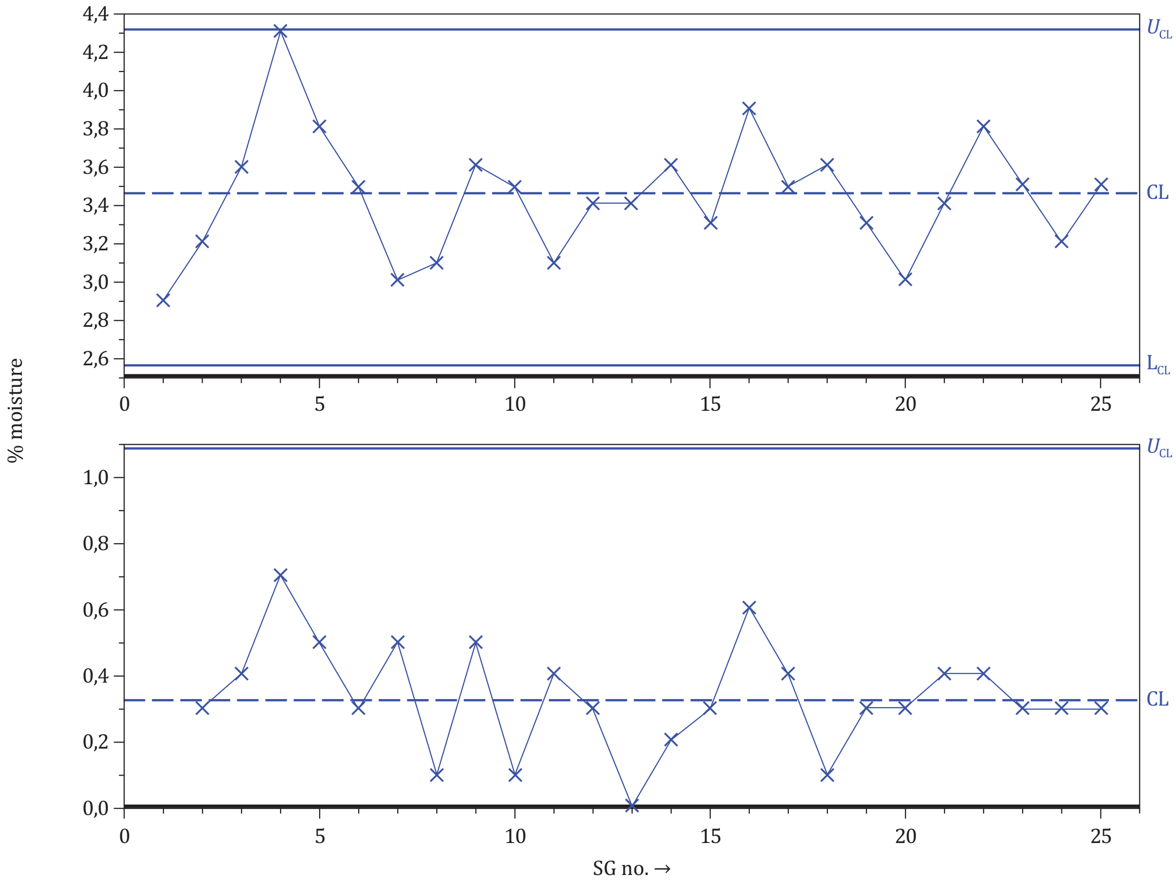


Figure A.6 — Control chart for individuals, *X* and moving range, R_m of skim milk powder

A.3.4 Median chart and *R* chart: μ and σ unknown

A machine is manufacturing DVDs with specified thickness between 1,20 mm and 1,25 mm. Samples of size 5 are drawn every half hour and their thickness in millimetres is recorded as shown in the Table A.4. It was decided to install a median chart for the purpose of controlling the quality. The values of medians and ranges are also shown in Table A.4.

Table A.4 — Control data for thickness of DVDs

Values in units of 0,001 mm							
Subgroup No.	**Thickness**					**Median $\tilde{X}$**	**Range *R***
	X_1	X_2	X_3	X_4	X_5		
1	14	8	12	12	8	12	6
2	11	10	13	8	10	10	5
3	11	12	16	14	9	12	7
4	16	12	17	15	13	15	5
5	15	12	14	10	7	12	8
6	13	8	15	15	8	13	7
7	14	12	13	10	16	13	6
8	11	10	8	16	10	10	8
9	14	10	12	9	7	10	7

Table A.4 *(continued)*

Values in units of 0,001 mm							
Subgroup No.	**Thickness**					**Median $\tilde{X}$**	**Range R**
	X_1	X_2	X_3	X_4	X_5		
10	12	10	12	14	10	12	4
11	10	12	8	10	12	10	4
12	10	10	8	8	10	10	2
13	8	12	10	8	10	10	4
14	13	8	11	14	12	12	6
15	7	8	14	13	11	11	7
16	10	12	6	9	13	10	7
17	17	13	11	10	14	13	7
18	10	17	14	14	9	14	8
19	14	13	15	16	15	15	3
20	10	15	8	11	8	10	7

Calculate the average of subgroup medians and ranges as follows:

$$\bar{\tilde{X}} = \frac{12+10+12+...10}{20} = \frac{234}{20} = 11,70$$

$$\bar{R} = \frac{6+5+7...7}{20} = \frac{118}{20} = 5,90$$

The range chart is calculated as follows:

Centre line $C_L = \bar{R} = 5,90$

$U_{CL} = D_4\bar{R} = 2,114 \times 5,90 = 12,472\,6 \sim 12,5$

$L_{CL} = D_3\bar{R} = 0 \times 5,90$ (since n is less than 7, L_{CL} is not shown)

The value of the constants D_3 and D_4 are taken from Table 2 for $n = 5$. Since the range chart exhibits a state of control, the median chart lines can be calculated.

Median control chart:

Centre line $C_L = \bar{\tilde{X}} = 11,70$

$U_{CL} = \bar{\tilde{X}} + A_4\bar{R} = 11,70 + (0,691 \times 5,9) = 15,776\,9\ \mu m \sim 15,78\ \mu m$

$L_{CL} = \bar{\tilde{X}} - A_4\bar{R} = 11,70 - (0.691 \times 5,9) = 7,623\,1\ \mu m \sim 7,62\ \mu m$

The value of A_4 is taken from Table 4 for $n = 5$. The graphs are plotted in the Figure A.7. As is evident from the chart, the process is exhibiting a state of statistical control.

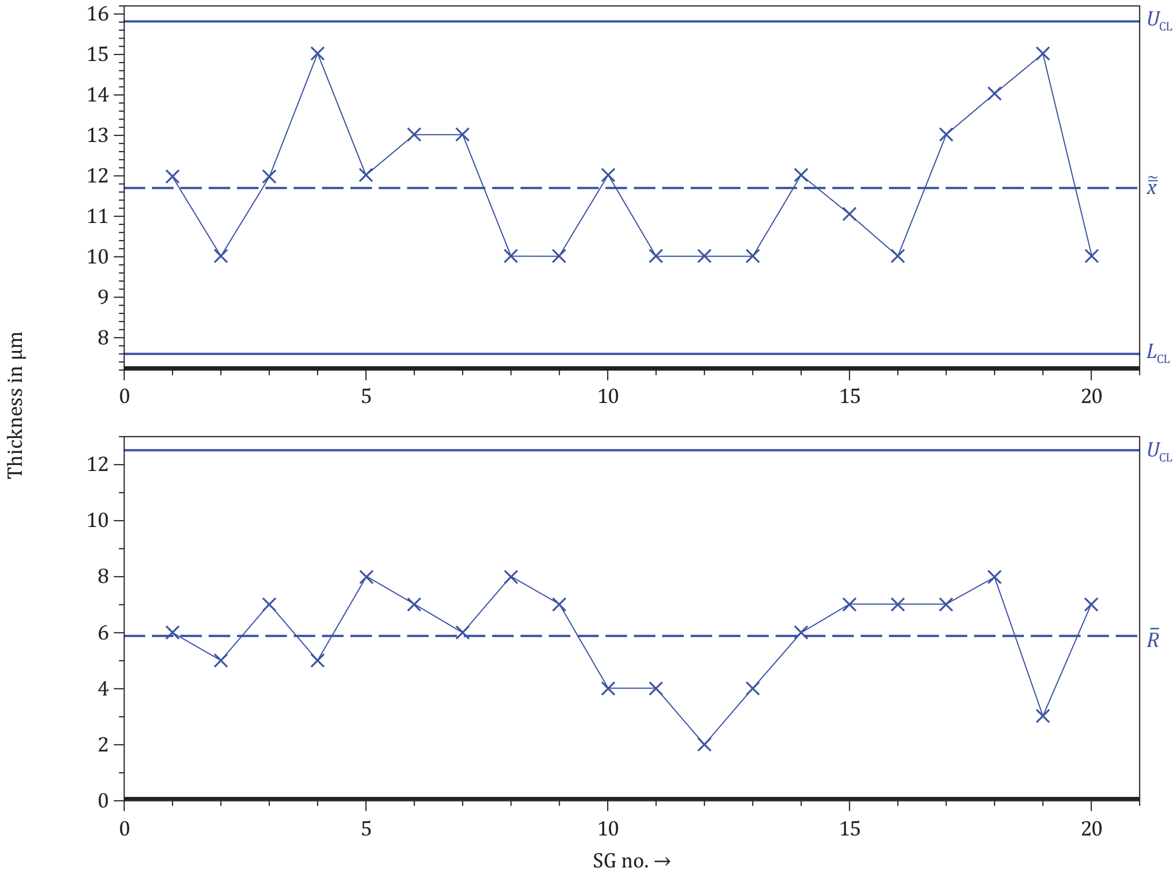


Figure A.7 — Median chart and range chart for thickness of DVDs

A.4 Attributes Control Charts

A.4.1 *p* chart: no p_0 value given

In a manufacturing company producing radio transistors, it was decided to install a fraction nonconforming *p* chart. Data were collected and analysed for a period of 1 month. From each day's production a random sample was collected at the end of the day and examined for the number of nonconforming items. The data are shown in Table A.5.

Table A.5 — Radio transistors: *p* chart (initial data)

Day	Number inspected	Number nonconforming	Fraction nonconforming
1	158	11	0,070
2	140	11	0,079
3	140	8	0,057
4	155	6	0,039
5	160	4	0,025
6	144	7	0,049
7	139	10	0,072
8	151	11	0,073
9	163	9	0,055

Table A.5 *(continued)*

Day	Number inspected	Number nonconforming	Fraction nonconforming
10	148	5	0,034
11	150	2	0,013
12	153	7	0,046
13	149	7	0,047
14	145	8	0,055
15	160	6	0,038
16	165	15	0,091
17	136	18	0,132
18	153	10	0,065
19	150	9	0,060
20	148	5	0,034
21	135	0	0,000
22	165	12	0,073
23	143	10	0,070
24	138	8	0,058
25	144	14	0,097
26	161	20	0,124
Total	3 893	233	

The values of the fraction nonconforming calculated for each subgroup are also given in Table A.5. The average fraction nonconforming for the month is calculated as follows:

$$\bar{p} = \frac{N_{\text{nc,tot}}}{N_{\text{i,tot}}} = \frac{233}{3\,893} = 0{,}06$$

where

$N_{\text{nc,tot}}$ is the total number nonconforming

$N_{\text{i,tot}}$ is the total number inspected

Since subgroup sizes are different, the U_{CL} and L_{CL} values shall be calculated for each subgroup separately from

$$U_{\text{CL}} = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$L_{\text{CL}} = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

where n is the size of the subgroup.

Table A.6 shows the calculation results for every subgroup.

Table A.6 — Radio transistors - calculation results

Subgroup number	Number inspected	Fraction nonconforming p	U_{CL}	L_{CL}
1	158	0,070	0,117	0,003
2	140	0,079	0,120	0,000
3	140	0,057	0,120	0,000
4	155	0,039	0,117	0,003
5	160	0,025	0,116	0,004
6	144	0,049	0,119	0,001
7	139	0,072	0,120	0,000
8	151	0,073	0,118	0,002
9	163	0,055	0,116	0,004
10	148	0,034	0,119	0,001
11	150	0,013	0,118	0,002
12	153	0,046	0,118	0,002
13	149	0,047	0,118	0,002
14	145	0,055	0,119	0,001
15	160	0,038	0,116	0,004
16	165	0,091	0,115	0,005
17	136	0,132	0,121	0,000
18	153	0,065	0,118	0,002
19	150	0,060	0,118	0,002
20	148	0,034	0,119	0,001
21	135	0,000	0,121	0,000
22	165	0,073	0,115	0,005
23	143	0,070	0,120	0,000
24	138	0,058	0,121	0,000
25	144	0,097	0,119	0,001
26	161	0,124	0,116	0,004
Total	3893			

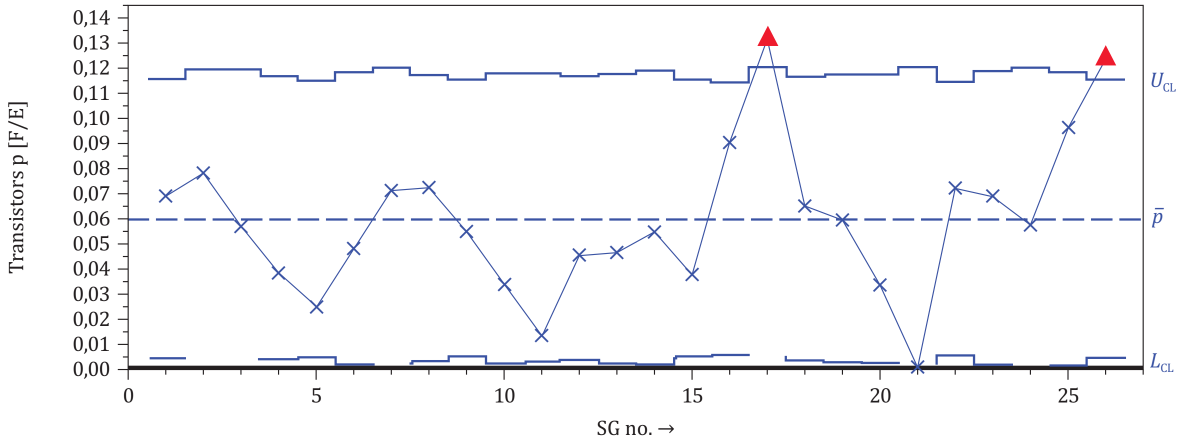


Figure A.8 — *p* chart for nonconforming radio transistors

It can be seen that plotting the U_{CL} and L_{CL} values for each subgroup is a time-consuming task. It can be observed from Figure A.8 that the fractions nonconforming for subgroup numbers 17 and 26 are falling outside their corresponding upper control limits. These two subgroups are omitted from the data for calculation of the control limits as they are shown to be subject to variations different from those affecting the other subgroups. To include them in the computations would result in an overstated process average and control limits which would not reflect the true random variations. The reasons for these high values should be sought so that corrective action may be taken to prevent future occurrences. A revised average fraction nonconforming is calculated from the remaining 24 subgroup values:

$$\bar{p} = \frac{195}{3\,596} = 0,054$$

Calculating the revised U_{CL} and L_{CL} values for each subgroup, using the revised $\bar{p}$ value, would reveal that all the fractions nonconforming are within their corresponding control limits. Hence, this revised value of $\bar{p}$ is taken as the standard fraction nonconforming for the purpose of installation of control charts. Thus, p_0 = 0,054.

As remarked above, the plotting of upper control limits for each subgroup of varying sizes is a time-consuming and tedious process. However, since the subgroup sizes do not vary widely from the average subgroup size, which comes out to be 150, the revised *p* chart (using p_0 = 0,054) can be plotted with an upper control limit using a subgroup size of n = 150, as the average subgroup size.

Thus, the revised *p* chart lines are calculated as follows.

Centre line $C_L = p_0 = 0,054$

$$U_{CL} = p_0 + 3\sqrt{\frac{p_0(1-p_0)}{n}} = 0,054 + 3\sqrt{\frac{0,054(1-0,054)}{150}} = 0,109$$

$$L_{CL} = p_0 - 3\sqrt{\frac{p_0(1-p_0)}{n}} = 0,054 - 3\sqrt{\frac{0,054(1-0,054)}{150}}$$

NOTE Since negative values are not possible, the lower limit is not shown.

The revised *p* chart is plotted below in Figure A.9. The process is exhibiting a state of statistical control.

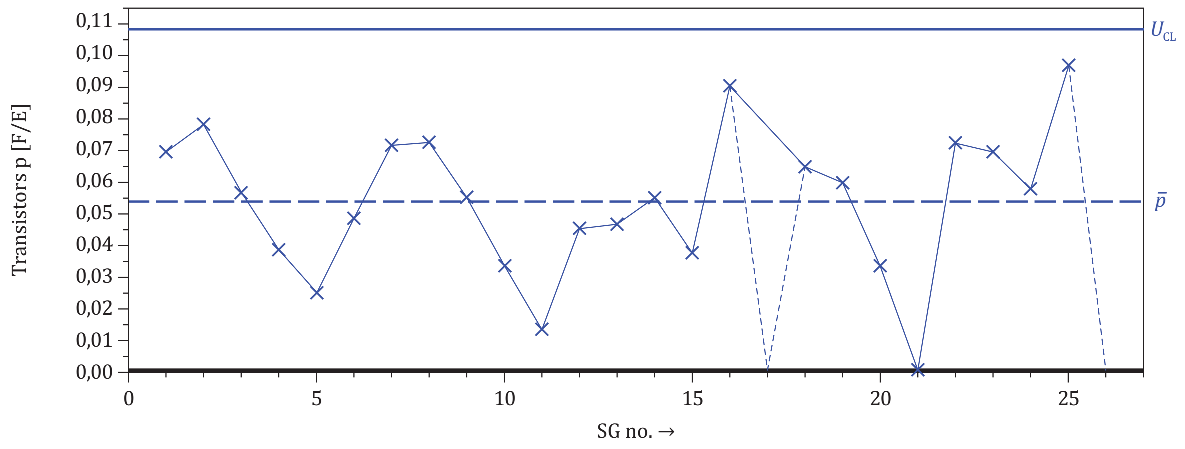


Figure A.9 — Revised *p* chart for nonconforming radio transistors

A.4.2 *np* chart: no p_0 value given

The data in Table A.7 give the number of nonconforming units per hour regarding malfunctions found by 100 % inspection on small switches with automatic inspection devices. The switches are produced in an automatic assembly line. Since the malfunction is serious, the percent nonconforming is used to identify when the assembly line is out of control. An *np* chart is prepared by gathering data of 25 groups as the preliminary data since the number inspected is constant.

Table A.7 — Preliminary data: switches

Subgroup number	Number of switches inspected	Number of nonconforming switches	Percent nonconforming
1	4000	8	0,200
2	4000	14	0,350
3	4000	10	0,250
4	4000	4	0,100
5	4000	13	0,325
6	4000	9	0,225
7	4000	7	0,175
8	4000	11	0,275
9	4000	15	0,375
10	4000	13	0,325
11	4000	5	0,125
12	4000	14	0,350
13	4000	12	0,300
14	4000	8	0,200
15	4000	15	0,375
16	4000	11	0,275
17	4000	9	0,225
18	4000	18	0,450
19	4000	6	0,150
20	4000	12	0,300

Table A.7 *(continued)*

Subgroup number	Number of switches inspected	Number of nonconforming switches	Percent nonconforming
21	4000	6	0,150
22	4000	12	0,300
23	4000	8	0,200
24	4000	15	0,375
25	4000	14	0,350
Total	100000	269	

The centre line and the control limits are calculated below and plotted.

Calculation of the *np* chart:

$$\text{Centre line } C_{\text{L}} = n\overline{p} = \frac{8+14+\ldots+14}{25} = 10{,}76$$

$$U_{\text{CL}} = n\overline{p} + 3\sqrt{n\overline{p}(1-\overline{p})} = 10{,}76 + 3\sqrt{10{,}76(1-0{,}0027)} = 20{,}59$$

$$L_{\text{CL}} = n\overline{p} - 3\sqrt{n\overline{p}(1-\overline{p})} = 10{,}76 - 3\sqrt{10{,}76(1-0{,}0027)} = 0{,}93$$

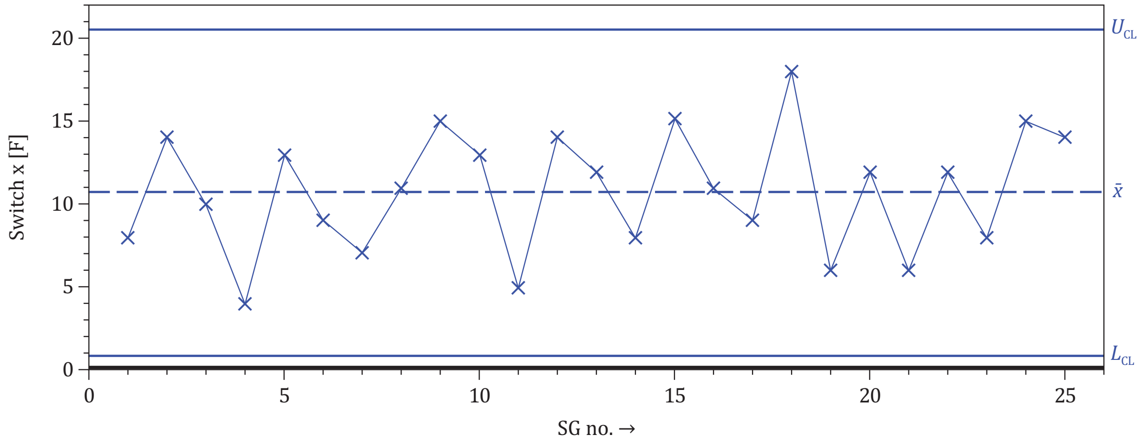


Figure A.10 — *np* chart for switches

Figure A.10 indicates that the quality of switches is in statistical control. These control limits may now be used for future subgroups until such time that the process is altered or that the process goes out of statistical control. Note that since the process is in statistical control, it is unlikely that any improvement can be made without a process change. For future control sample size 500 instead of 4 000 may be adequate.

If an improvement is made, then different control limits will have to be computed for future subgroups to reflect the altered process performance. If the process has been improved (smaller *np* value), use the new limits, but if the process has deteriorated (higher *np* value), search for additional assignable causes.

A.4.3 *c* chart: c_0 value not given

A manufacturer of videotape wishes to control the number of spot nonconformities in videotape. The following data give the number of spot nonconformities found by examining successively the surface of 20 hoops of video tape, each being 350 m long, from a certain production process in which one end of the videotape is investigated.

In order to control this process, it is intended to apply a *c* chart plotting the number of spot nonconformities. The data for 20 hoops, given in Table A.8, are taken as the preliminary data to prepare a *c* chart.

Table A.8 — Preliminary data: videotape

Hoop Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Total
Number of spot non-conformities	7	1	2	5	0	6	2	0	4	4	6	3	3	3	1	6	3	1	5	6	68

The centre line and control limits are calculated below and plotted.

Centre line $C_L = \bar{c} = \dfrac{7+1+\ldots+6}{20} = 3,4$

$$U_{CL} = \bar{c} + 3\sqrt{\bar{c}} = 3,4 + 3\sqrt{3,4} = 8,9$$

$$L_{CL} = \bar{c} - 3\sqrt{\bar{c}} = 3,4 - 3\sqrt{3,4}$$

When the lower control limit (L_{CL}) is negative, there is no lower control limit.

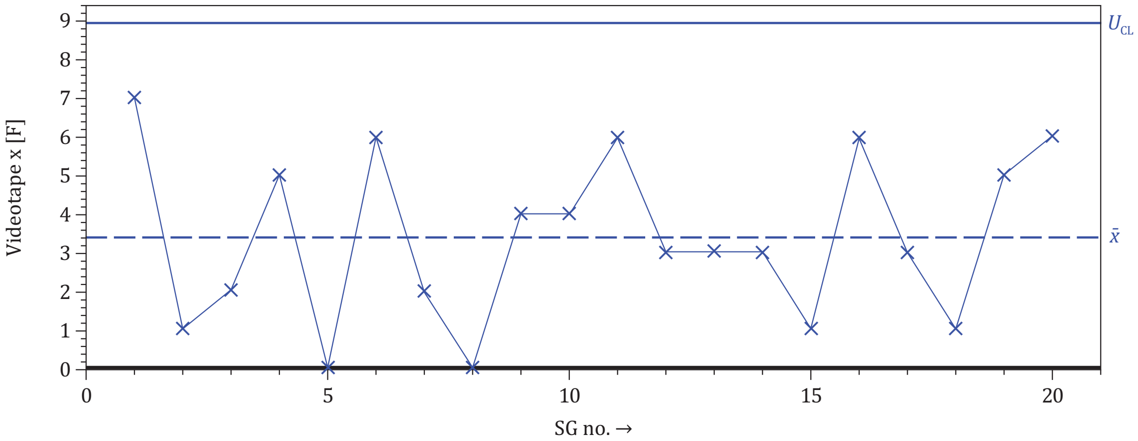


Figure A.11 — *c* chart for data from production of videotape

The preliminary data indicate that the process is in a state of statistical control.

A.4.4 *u* chart - no u_0 value given

In a tyre manufacturing plant, 50 tyres were inspected every half hour and the total number of nonconformities and number of nonconformities per unit were recorded. It was decided to install a *u* chart for the number of nonconformities per unit to study the state of control of the process. The data are shown in Table A.9.

The average of the *u* values is calculated from the table as follows.

Table A.9 — Number of nonconformities per unit (units inspected per subgroup, n = 50)

Subgroup Number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	
***c*: Number of nonconformities**	4	5	3	6	2	1	5	6	2	4	
***u*: Number of nonconformities per unit**	0,08	0,10	0,06	0,12	0,04	0,02	0,10	0,12	0,04	0,08	
Subgroup Number	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**Total**
***c*: Number of nonconformities**	7	5	2	3	5	1	2	6	3	5	77
***u*: Number of nonconformities per unit**	0,14	0,10	0,04	0,06	0,10	0,02	0,04	0,12	0,06	0,10	0,077

Divide the total number of nonconformities (from the row of c values) by the total number of units inspected.

$$\bar{u} = \frac{\sum c}{\sum n} = \frac{77}{20 \cdot 50} = 0{,}077$$

$$U_{\mathrm{CL}} = \bar{u} + 3\sqrt{\frac{\bar{u}}{n}} = 0{,}077 + 3\sqrt{\frac{0{,}077}{50}} = 0{,}194\,72 \approx 0{,}195$$

$$L_{\mathrm{CL}} = \bar{u} - 3\sqrt{\frac{\bar{u}}{n}} = 0{,}077 - 3\sqrt{\frac{0{,}077}{50}}$$

NOTE Since negative values are not possible, the lower limit is not shown.

The data and control lines are plotted in Figure A.12 below.

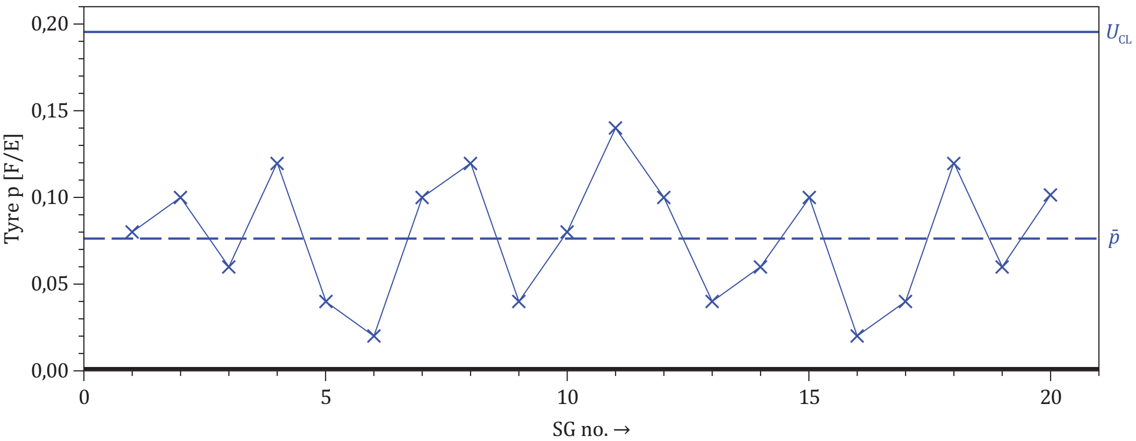


Figure A.12 — *u* chart for data from production of tyres

Figure A.12 indicates that the process is in a state of statistical control.

NOTE Since subgroup sizes are constant, a c chart could have been used instead.

Annex B
(informative)

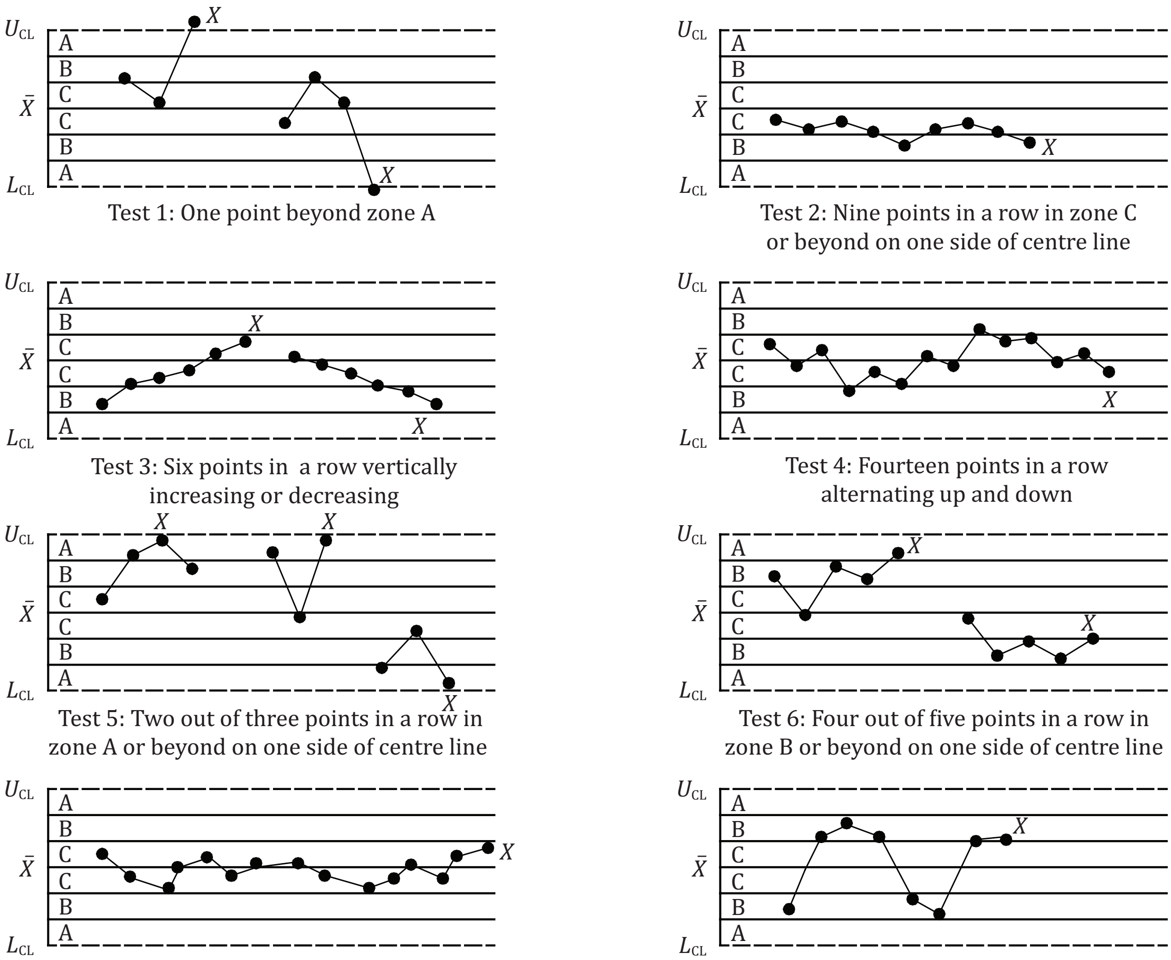
Practical notices on the pattern tests for assignable causes of variation

Practical notices on using the pattern tests in Figure 3 are given as follows.

a) There are many different pattern tests available. The pattern tests shown in Figure 3 are commonly used tests. The aim of Shewhart control charts is to verify whether the process is in a stable or unstable condition. For example, as the oxidation process in semiconductor manufacturing process tends to undergo influence of the atmospheric pressure, runs are apt to appear in the control charts. However, such a state is not considered as unusual but usual. Therefore, the battery of pattern tests in Figure 3 should not be regarded as specified rules but rather used as a kind of guideline. The pattern tests should be specified according to usual state of process.

b) As shown in Clause 8, if some tests in Figure 3 used together, then the probability of type one error may become too large. However, in early stage production, the purpose of statistical process control is to bring the process into a stable state and improve the process for better process performance. Therefore, we must positively and rapidly detect assignable causes by using some tests in Figure 3; however, the probability of type one error may become too large. It can be considered as exploratory data analysis. On the other hand, when the production stage is transferred to the routine mass production, the purpose of statistical process control is to maintain the process in a state of control. In this case a very small probability of type one error is required. Therefore, using some tests together should be avoided. Test 1 is fundamental rule of Shewhart control chart, but it is an omnibus test. If a relatively small shift and/or a trend in the process mean tends to appear, then it is helpful to use a supplementary rule. For example, Test 5 of the Western Electric Rules can be specified as a supplemental rule in addition to Test 1.

c) Western Electric Rules: There are many different criteria for identifying assignable causes. One of the commonly used rules since the 1950s is the test criteria known as the Western Electric Rules or AT&T Rules. Figure B.1 shows the eight typical test criteria given in these rules. As noted earlier, the decision as to which criteria to use depends on the process being studied.



Test 1: One point beyond zone A

Test 2: Nine points in a row in zone C or beyond on one side of centre line

Test 3: Six points in a row vertically increasing or decreasing

Test 4: Fourteen points in a row alternating up and down

Test 5: Two out of three points in a row in zone A or beyond on one side of centre line

Test 6: Four out of five points in a row in zone B or beyond on one side of centre line

Test 7: Fifteen points in a row in zone C above and below centre line

Test 8: Eight points in a row on both sides of centre line with none in zone C

Figure B.1 — Tests for assignable causes

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