INTERNATIONAL STANDARD

ISO 7507-3

Second edition
2006-07-15

Petroleum and liquid petroleum products — Calibration of vertical cylindrical tanks —

Part 3: **Optical-triangulation method**

Pétrole et produits pétroliers liquides — Jaugeage des réservoirs cylindriques verticaux —

Partie 3: Méthode par triangulation optique

Reference number
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Foreword

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International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 7507-3 was prepared by Technical Committee ISO/TC 28, *Petroleum products and lubricants*, Subcommittee SC 3, *Static petroleum measurement*.

This second edition cancels and replaces the first edition (ISO 7507-3:1993), which has been technically revised.

ISO 7507 consists of the following parts, under the general title *Petroleum and liquid petroleum products — Calibration of vertical cylindrical tanks*:

— *Part 1: Strapping method*

— *Part 2: Optical-reference-line method*

— *Part 3: Optical-triangulation method*

— *Part 4: Internal electro-optical distance-ranging method*

— *Part 5: External electro-optical distance-ranging method*

Introduction

This part of ISO 7507 describes the calibration of vertical cylindrical tanks by means of optical triangulation using theodolites. The circumference of the tank is determined at different levels by reference to a base line, which can be either a reference circumference measured by strapping or a base line between two stations of a theodolite measured by means of a tape or by an optical method. External circumferences are corrected to give true internal circumferences.

The method is an alternative to other methods such as strapping (ISO 7507-1) and the optical-reference-line method (ISO 7507-2).

Petroleum and liquid petroleum products — Calibration of vertical cylindrical tanks —

Part 3: Optical-triangulation method

1 Scope

This part of ISO 7507 specifies a calibration procedure for application to tanks above 8 m in diameter with cylindrical courses that are substantially vertical. It provides a method for determining the volumetric quantity contained within a tank at gauged liquid levels. The measurements required to determine the radius are made either internally (Clause 10) or externally (Clause 11). The external method is applicable only to tanks that are free of insulation.

This method is suitable for tanks tilted up to a 3 % deviation from the vertical provided that a correction is applied for the measured tilt as described in ISO 7507-1.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 7507-1:2003, *Petroleum and liquid petroleum products — Calibration of vertical cylindrical tanks — Part 1: Strapping method*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 7507-1 and the following apply.

3.1
total station
theodolite with built-in distance meter that coincides with the optical axis of the instrument

4 Precautions

The general precautions and safety precautions specified in ISO 7507-1 shall apply to this part of ISO 7507.

5 Equipment

5.1 Equipment for measurement of angles

5.1.1 Theodolite, with angular resolution equal to or better than 0,2 mgon (1 mgon = 0,25 s).

Each theodolite shall be mounted on a tripod that is firm and stable. The legs of the tripod shall be steadied by means of magnetic bearers (or any equivalent system) when being used for the internal method. The theodolites shall be checked either periodically or prior to the tank measurements as described in Annex F.

Alternatively, a total station can be used along with a prism mounted on the other station. The total station shall meet the same requirements for the angular measurements as the theodolites. The distance measurement shall have a resolution equal to or better than 0,1 mm. The distance meter shall be calibrated together with the used prism with an extended calibration uncertainty on the order of 1 mm or better. It shall be possible to mount the prism on the tripod in the same position as the theodolite/total station.

5.1.2 Laser-beam emitter, low-power, equipped with a device, such as a fibre-optic light-transfer system and a theodolite-telescope eye-piece connection, by which the laser beam can be transmitted through a theodolite. The laser beam shall be coincident with the optical axis of the telescope.

5.1.3 Weights, heavy, to set round the theodolite stations to prevent movement of the tank bottom plate.

5.1.4 Lighting, for use inside the tank to allow measurements to be read accurately.

5.2 Stadia

Stadia, at least 2 m long, of a material whose thermal expansion is known.

The graduated length between two marks shall be calibrated. Extended calibration uncertainty should be on the order of 0,05 mm. It shall be possible to mount the stadia on the tripod in the same position as the theodolite.

NOTE The stadia is not used when the calibration is carried out using a total station.

6 Equipment set-up and procedure

6.1 Preparation of tank

For new tanks or for tanks after repair, fill the tank to its normal working capacity at least once and allow it to stand for at least 24 h prior to calibration.

6.2 Establishment of calibration conditions

If the tank is calibrated with liquid in it, record the depth, temperature and density of the liquid at the time of calibration. Do not make transfers of liquid during the calibration.

Measure or estimate the worst-case gradient of tank-shell temperatures at the time of calibration.

NOTE 1 The temperature gradient is used to estimate the uncertainties of the measured tank radii (see 13.2 and E.3.5.3).

NOTE 2 The highest temperature is usually found on the sunny side at the top of the tank, the lowest temperature on the shady side at the bottom of the tank.

6.3 Set-up of theodolites and/or total stations

6.3.1 Set up each theodolite or total station with care, according to the procedure and instructions given by the manufacturer. In addition, follow the procedures described in 6.3.2 and 6.3.3.

6.3.2 Set up the instrument so as to be stable.

For the internal method, steady the bottom of the tank near the theodolite or total station by installing weights or other heavy objects around the station if there is a risk of the station moving during the calibration. Mount the legs of the tripod on magnetic bearers (or any equivalent system) to prevent the legs from sliding on the tank bottom.

For the external method, drive the legs of the tripod fully home into the ground.

6.3.3 Set the bed plate of the instrument as near as possible to the horizontal.

NOTE This ensures verticality of the swivel axis of the theodolite or total station.

6.3.4 The calibration equipment shall be placed at the site for typically 1 h in order to reach ambient temperature before commencement of the actual calibration procedure.

7 Stadia set-up and procedure

7.1 Mount the stadia on the tripod with care according to the procedure and instructions given by the manufacturer. In addition, follow the procedures described in 7.2 and 7.3.

7.2 Mount the stadia horizontally and perpendicular to the aiming axis by adjusting the device on the stadia.

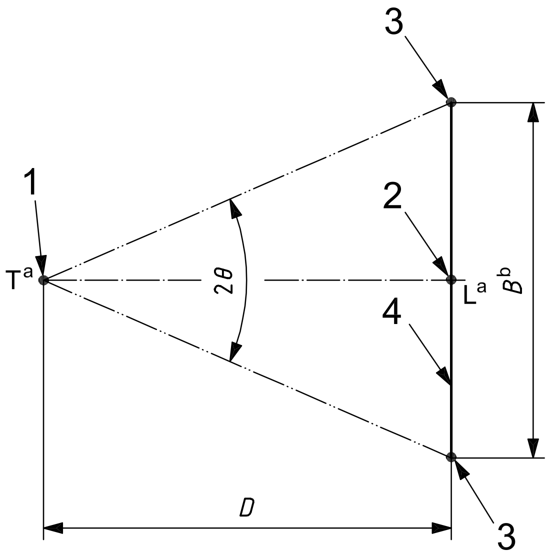
7.3 Once setting-up is complete, lock the stadia in position and verify that it is horizontal and perpendicular.

8 Measurement of horizontal distance between two theodolite stations using a stadia

8.1 This procedure for determining the distance using a stadia is not recommended if the distance between the stations is above 25 m.

8.2 Take the measurement prior to the commencement of the optical readings. Set up the stadia as described in Clause 7.

Measure the horizontal angle, 2θ, subtended at the theodolite (see Figure 1) by the two marks on the stadia, using the theodolite.



Key

1 theodolite 1
2 theodolite 2 (laser)
3 stadia mark
4 stadia

[a] Points T and L are interchangeable.
[b] B, the distance between the two reference marks on the stadia, equals 2 m.

Figure 1 — Measurement of distance between two theodolites

8.3 Compute the horizontal distance, D, between the two theodolite stations from Equation (1):

$$D = \frac{B}{2 \times \tan\theta} \quad (1)$$

where

B is the distance between the two reference marks on the stadia (corrected for thermal expansion, if necessary);

θ is half the angle subtended at theodolite, T, by the two reference marks.

8.4 Carry out the measurement of the angle 2θ and the computation of the distance, D, a minimum of five times while turning and re-pointing the theodolite in between, and calculate and record the average value. Two standard deviations of the mean of the distance, D, shall be less than half of the tolerance given in Table 3 or the entire procedure shall be repeated.

8.5 Re-determine the distance, D, after completion of all the optical measurements described in 10.13.

The average distances computed before and after the optical measurements shall agree within the tolerances given in Table 3. If they do not, repeat the calibration procedure until a set of measurements is obtained with the average values for D at the beginning and end within the tolerances.

8.6 The average of all measurements of distance, D, shall be used in further calculations.

9 Measurement of horizontal distance between two theodolite stations using a total station

9.1 This procedure for determining the distance between theodolite stations is not recommended if the distance between the stations is less than 10 m.

9.2 Set up the prism at the second tripod.

9.3 Carry out the measurement of the distance, D, a minimum of five times while turning and re-pointing the total station in between, and calculate and record the average value. Two standard deviations of the mean of the distance, D, shall be less than half of the tolerance given in Table 3 or the entire procedure shall be repeated.

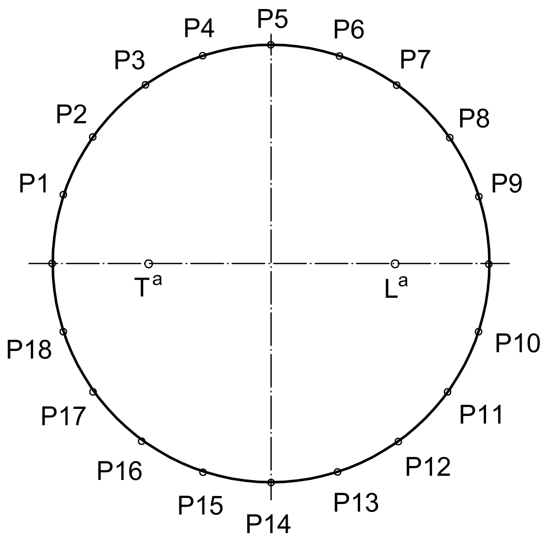
9.4 Re-determine the distance, D, after completion of all the optical measurements described in 10.13.

The average distances computed before and after the optical measurements shall agree within the tolerances given in Table 3. If they do not, repeat the calibration procedure until a set of measurements is obtained with the average values for D at the beginning and end within the tolerances.

9.5 The average of all measurements of distance, D, shall be used in further calculations.

10 Procedure for internal optical tank wall measurements

10.1 Set up two theodolite stations inside the tank as illustrated in Figure 2 and as described in 6.3.



[a] T and L are interchangeable theodolite and laser theodolite stations.

Figure 2 — Example of locations of theodolite stations and wall points for internal procedure

10.2 Locate the two stations approximately on a diametrical plane and at least one quarter diameter apart. Adjust the theodolites and measure the distance, D, between T and L as described in Clause 8 or Clause 9.

10.3 Set the reference axis, TL, optically on the horizontal planes (circles) of both instruments by sighting from each instrument the vertical graticule wires of the other instrument as described in 10.4 to 10.7.

10.4 Ensure that the laser is shut off in order to avoid exposure.

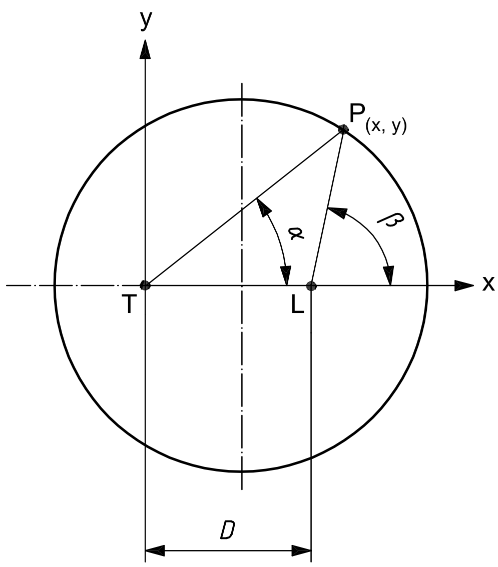
10.5 Adjust the theodolite, T, to set the telescope to infinity and illuminate the eyepiece of this telescope with a light source.

10.6 Sight the object lens of the theodolite, T, from the telescope of the laser theodolite, L, and continue focussing until the graticules become visible. Make the vertical graticule wires coincide by using the adjusting device on the laser theodolite, L.

10.7 Repeat the operation from the theodolite. Repeat the operation as many times as is necessary until the vertical graticule wires coincide perfectly.

10.8 The TL axis is now set. Record the relative locations of the two theodolites by taking readings on both horizontal scales as the horizontal reference angles.

10.9 Switch on the laser beam. This beam is then used to provide a series of points on the tank shell wall. Sight these points in turn using the other theodolite and take and record the horizontal-scale readings on both instruments. Do not locate measurement points closer to the reference angle (the line through T and L) than 10 gon (Figure 3, angle α or β, whichever is smaller).



Key

T theodolite station

L laser theodolite station

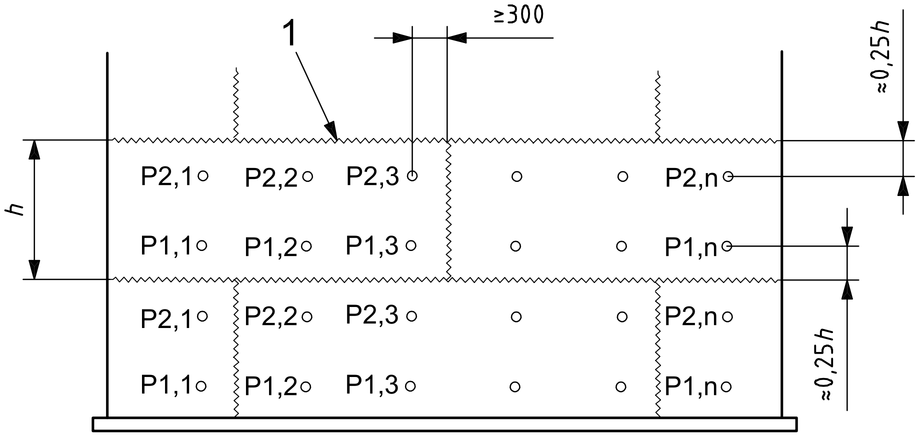
P arbitrary point on the tank

Figure 3 — Horizontal angles between sightings on points on the tank wall and the reference axis TL

10.10 The minimum number of points on the tank shell wall per circumference shall be as given in Table 1. These points shall not be closer than 300 mm from the vertical weld seam.

For each course, there shall be two sets of points: one set on a circumference at about 1/4 of the course height above the lower horizontal seam, and the other at about 1/4 of the course height below the upper horizontal seam as shown in Figure 4.

Dimensions in millimetres



Key

1 seam

Figure 4 — Location of sets of points on tank wall

10.11 Determine the horizontal angles, α and β, of all the points along a horizontal set, as shown in Figure 3, by the theodolite and the laser beam. Then move to the next level.

NOTE This ensures that each set of points on the tank wall is at the same level for a given circumference.

Table 1 — Minimum number of points per circumference for internal procedure

Circumference m	Minimum number of points
$\leqslant$ 50	10
> 50, $\leqslant$ 100	12
> 100, $\leqslant$ 150	16
> 150, $\leqslant$ 200	20
> 200, $\leqslant$ 250	24
> 250, $\leqslant$ 300	30
> 300	36

To avoid systematic errors, the number of points divided by the number of plates in the tank segments should not be equal to an integer (e.g. 1, 2, 3, etc.).

For riveted tanks, it is recommended that at least three points are sighted in every tank plate at every height, one in the centre and two at the extremes of the plate width (near the vertical seams).

10.12 After completion of the optical measurement of all the points, re-determine the horizontal distance, D, between T and L (see 8.5 and 9.4) and repeat the calibration if necessary.

10.13 Check the axis, TL, by switching off the laser and repeating the operations described in 10.3 to 10.8. The original and final horizontal reference angles shall be within the tolerance specified in 12.2. If not, repeat the calibration procedures until a set of readings ending in such agreement is obtained. Record the average values of the horizontal reference angles.

11 Procedures for external measurements

11.1 General

The measurements shall be related either to a reference circumference using the procedure described in 11.2 or to reference distances measured between pairs of theodolite stations as described in 11.3.

11.2 Reference circumference measured by strapping

11.2.1 Reference circumference

The reference circumference has a direct impact on the calibrated volume of the entire tank. It is necessary, therefore, that it be measured as accurately as possible.

Determine the reference circumference using the reference method described in ISO 7507-1 and the following items a) and b).

a) Take multiple measurements of the reference circumference either prior to the commencement or after the completion of the optical readings. If the first three consecutive measurements agree within the tolerances specified in Table 4, take their mean average as the reference circumference and their standard deviation as the standard uncertainty. If they do not agree within the tolerances specified in Table 4, repeat the measurements until two standard deviations of the mean of all measurements is less than the half of the tolerances specified in Table 4. Use the mean as the measured reference circumference and the standard deviation as the standard uncertainty. Use standard procedures to eliminate obvious outliers.

b) Take the measurement of the reference circumference at a position where work conditions allow reliable measurements and that is within the focal range of the optical instrument. Strap the tank, aiming at one of the following levels:

   1) about 1/4 of the course height above the lower horizontal seam,

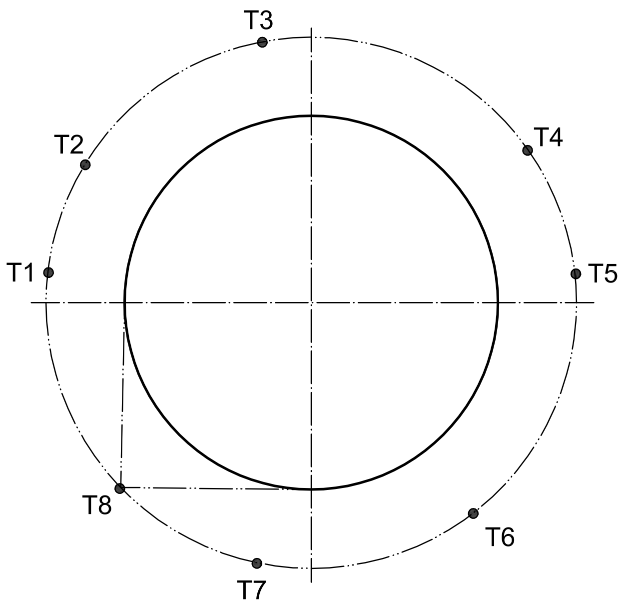
   2) about 1/4 of the course height below the upper horizontal seam,

   and repeat the measurement to achieve agreement within the tolerances specified in Table 4.

11.2.2 Theodolite readings

11.2.2.1 Set up the theodolite outside the tank, as shown in Figure 5 for eight theodolite stations and as described in 6.3.

The minimum number of stations (T1, T2, etc.) per circumference shall be as given in Table 2.



Key

T1 …… T8 theodolite stations

Figure 5 — Example of theodolite station locations for external procedures based on a reference circumference

Table 2 — Minimum number of theodolite stations for external procedures

Tank circumference m	Minimum number of stations
⩽ 50	5
> 50, ⩽ 100	6
> 100, ⩽ 150	8
> 150, ⩽ 200	10
> 200, ⩽ 250	12
> 250, ⩽ 300	15
> 300	18

To avoid systematic errors, the number of stations divided by the number of plates in the tank segments should not be equal to an integer (e.g. 1, 2, 3, etc.).

The theodolite positions should be such that the targets avoid the vertical welds between tank segments.

Care should be taken, especially for smaller tanks, that the target points are evenly distributed around the tank.

11.2.2.2 From each station and for each level (see 11.2.2.3 and 11.2.2.4), make a sighting tangentially to the tank on either side of the theodolite as shown in Figure 5. Maintain the same vertical angle of the theodolite in both sightings.

NOTE This ensures that the intended targets on the tank are at the same level for a given circumference.

Record the horizontal angles subtended by the tangents at the theodolite.

11.2.2.3 Make sightings at the height at which the reference circumference was measured (see 11.2.1). The angles at the strapped height shall be measured twice (before and after measurements of the angles at other heights; see 11.2.2.4). The subtended angles shall agree with each other to within 0,01 gon. If they do not, repeat the measurements until two standard deviations from the mean fit within 1/2 of this tolerance. In further calculations, use the average and standard deviation of the measurements.

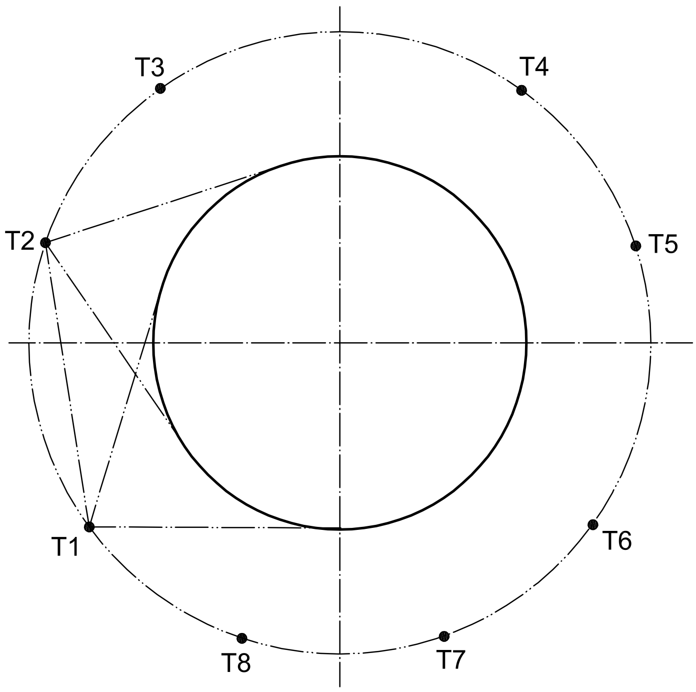
If agreement is still not obtained, repeat the measurements at this station.

11.2.2.4 For each theodolite station (e.g. T1), sight each of the courses at two levels, one at about 1/4 of the course height above the lower horizontal seam, the other at about 1/4 of the course height below the upper horizontal seam.

11.2.2.5 Move the theodolite from station T1 to T2 to T3, etc., until the whole circumference is covered. Repeat all the above steps at each station (i.e. T1, T2, etc.), for each level. Record the horizontal angles for each of the points sighted.

11.3 Reference distances measured between pairs of theodolite stations

11.3.1 Set up the two theodolite stations outside the tank, as shown in Figure 6 for eight stations, and as described in 6.3, using a theodolite (5.1.1) and a second tripod. The minimum number of stations (T1, T2, etc.) per circumference shall be as given in Table 2.



Key

T1 …… T8 theodolite stations

Figure 6 — Example of theodolite station locations for external procedure based on reference distances between pairs of theodolites

11.3.2 Determine the horizontal distance, T1 to T2, between the two theodolite stations by using the stadia as described in Clause 8 (T1 to T2 = D) with the stadia mounted at T2 as described in 6.3, or by using a total station as described in Clause 9 (T1 to T2 = D) with the prism mounted at T2.

11.3.3 From station T1, sight the tank wall tangentially on either side, maintaining the same vertical angle of the theodolite for the two observations, and record the horizontal angle subtended at the theodolite.

11.3.4 Leaving the tripod supports in the same position, interchange the stadia (or prism) and the theodolite, so that the stadia (or prism) is at location T1 and the theodolite is at location T2.

Repeat the determinations described in 11.3.2 and 11.3.3.

11.3.5 The value for D obtained in 11.3.2 shall agree with that obtained in 11.3.4 within the tolerances given in 12.1. If agreement is not obtained, repeat the measurements, starting at station T1, until two consecutive values do so agree. Record the arithmetic mean of the two values as the horizontal distance T1 to T2.

11.3.6 Transfer the tripod set up at T1 to T3, leaving the tripod set up at T2 in place. Apply the procedure in 11.3.2 to 11.3.4 for locations T1 and T2 to locations T2 and T3.

11.3.7 Continue the procedures for all subsequent stations around the circumference until station T1 is reached again.

11.3.8 For each course, repeat the procedure described in 11.3.2 to 11.3.7 at two levels, one at about 1/4 of the course height above the lower horizontal seam and the other at about 1/4 of the course height below the upper horizontal seam.

12 Tolerances

12.1 Distances between theodolites

The measurements of the distance, D, between the two theodolite stations taken before and after other optical readings shall not differ by more than the tolerances given in Table 3.

Table 3 — Tolerance on distance between theodolites

Distance m	Tolerance mm
$\leqslant 25$	2
$> 25, \leqslant 50$	4
$> 50, \leqslant 100$	6

12.2 Horizontal angles

The repeated values for the measurement of horizontal angles using the theodolites shall not differ by more than 0,01 gon.

12.3 Reference circumference

The reference circumference measurements taken before and after the optical readings (see 11.2.1) shall not differ by more than the tolerances given in Table 4.

Table 4 — Tolerance on reference circumference

Circumferential measurement m	Tolerance mm
$\leqslant 25$	2
$> 25, \leqslant 50$	3
$> 50, \leqslant 100$	5
$> 100, \leqslant 200$	6
> 200	8

13 Other measurements for tank calibrations

13.1 Tank-bottom calibrations

Refer to ISO 7507-1.

13.2 Other measurements and data

13.2.1 Determine, using calibrated equipment, and process the following data as described in ISO 7507-1:

a) plate and paint thickness;

b) height of the courses;

c) density and working temperature of the liquid to be stored in the tank;

d) ambient temperature and the temperature of the liquid at the time of measurement;

e) maximum filling height;

f) deadwood;

g) number, width and thickness of any vertical welds or overlaps;

h) tilt of the tank as shown by the plumbline deviations;

i) shape, landing height and apparent mass in air of any floating roof or cover.

The velocity of sound in paint can be substantially different from its velocity in metal. When measuring thickness of paint, care should be taken to use proper instrumentation, e.g. echo-to-echo ultrasonic gauge, while following a suitable procedure. Alternatively, if it is necessary to scrape the paint, the thickness can then be measured by a single-echo ultrasonic gauge.

NOTE The average value and the range of tank shell temperatures are required for uncertainty analysis (see Annex A.).

13.2.2 It is necessary to refer each tank dip to the dip-point, which might be in a position different from the datum-point used for the purpose of tank calibration (e.g. a point on the bottom angle). Determine any difference in level between the datum-point and dip-point and record it.

13.2.3 Measure the overall height of the reference point on each dip-hatch (upper reference point) above the dip-point using the dip-tape and dip-weight as specified in ISO 7507-1. Record this overall height, to the nearest millimetre, and permanently mark it on the tank adjacent to that dip-hatch.

13.2.4 If possible, compare measurements with the corresponding dimensions shown in the drawings and verify any measurement that shows a significant discrepancy.

13.2.5 Measurement of the temperature of the tank shell is important for correcting the measured radii at the time of tank calibration. Temperatures should be measured around the tank (at least at four points), near the bottom of the shell and again near the top of the shell. Average temperatures should be used to correct the measured radii for

a) differential expansion of the tank shell and the stadia in the internal method;

b) differential expansion of the tank shell between the strapped circumference and the rest of the tank in the reference circumference method;

c) general expansion of tanks at all heights in the reference distance method.

14 Calculations and development of tank capacity tables

14.1 From the internal procedure

NOTE See also Clause 10.

Compute the internal radius of the tank by the procedures described in Annex A and Annex B for each level, i.e. two levels per course.

14.2 From the reference circumference procedure

NOTE See also 11.2.

Compute the internal radius of the tank by the procedure described in Annex C for each level, i.e. two levels per course.

14.3 From the reference distances between pairs of theodolites

NOTE See also 11.3.

Compute the internal radius of the tank by the procedure described in Annex D for each level, i.e. two levels per course.

14.4 Corrections

Assuming that the capacity table has been calculated from internal radii (circumferences), the following corrections described in ISO 7507-1 shall be applied to it:

a) strapping over vertical seams (if lap-welded or riveted) or obstructions;

b) hydrostatic-head effect;

c) correction to the reference temperature of the tank stated in the certificate (expansion or contraction of the tank shell due to temperature effects). Note that for the external procedure based on reference circumference, a correction is only applied when the tank shell thermal expansion coefficient differs from that of the strapping tape (a tank of stainless steel, for example);

d) tilt of the tank;

e) mass of any floating roof or cover;

f) deadwood.

14.5 Tank capacity table

Calculate and prepare the tank capacity table as described in ISO 7507-1. Calculations may be undertaken in radii (in ISO 7507-1 the calculations are based on circumferences).

Annex A
(normative)

Computation of internal radii from internal measurements

A.1 The coordinates (x,y) of a point, P, on the tank shell wall relative to a system of rectangular axes with their centre at T, as shown in Figure 3, shall be determined from Equations (1) and (2):

$$y = x \tan \alpha \tag{A.1}$$

$$y = (x - D) \tan \beta \tag{A.2}$$

where

D is the distance, expressed in metres, between the theodolite stations (see Clause 8 or Clause 9 and Figure 3);

α is the horizontal angle between the point (i.e. P) on the shell wall and the x-axis at the theodolite station (see Figure 3);

β is the horizontal angle between the point (i.e. P) on the shell wall and the x-axis at the laser theodolite station (see Figure 3).

From Equations (A.1) and (A.2),

$$x = \frac{D \tan \beta}{\tan \beta - \tan \alpha} \tag{A.3}$$

A.2 Using Equations (A.3) and (A.1), compute the coordinates (x,y) for all points under consideration. Report the following data for each height at which horizontal sets of points were selected (see 10.10):

Course 1:

height 1: P1,1 (x,y), P1,2 (x,y), ... P1,n (x,y)

height 2: P2,1 (x,y), P2,2 (x,y), ... P2,n (x,y)

Course 2:

height 1: P1,1 (x,y), P1,2 (x,y), ... P1,n (x,y)

height 2: P2,1 (x,y), P2,2 (x,y), ... P2,n (x,y)

Course N:

height 1: P1,1 (x,y), P1,2 (x,y), ... P1,n (x,y)

height 2: P2,1 (x,y), P2,2 (x,y), ... P2,n (x,y)

A.3 Compute the radius for each height within each course using the method described in Annex B.

Annex B
(normative)

Determination of the radius of the circle by the least-squares method

B.1 Problem

To determine the radius of the circle that best fits the n points (x_i, y_i), where $i = 1, 2, \ldots n$, obtained from the calculation given in Annex A.

B.2 Principle

The selected criterion for what is the best fit is that the sum of the squares of the distances from the points (x_i, y_i) to the circumference of the circle should be a minimum.

B.3 Theoretical solution

Distance of point (x_i, y_i) from the circumference of the circle is calculated from Equation (B.1):

$$P_{(xi,\, yi)} = \left(\sqrt{(x_i - a)^2 + (y_i - b)^2} \right) - r \quad \text{(B.1)}$$

where (a, b) are the coordinates of the centre point of the circle shown in Figure B.1.

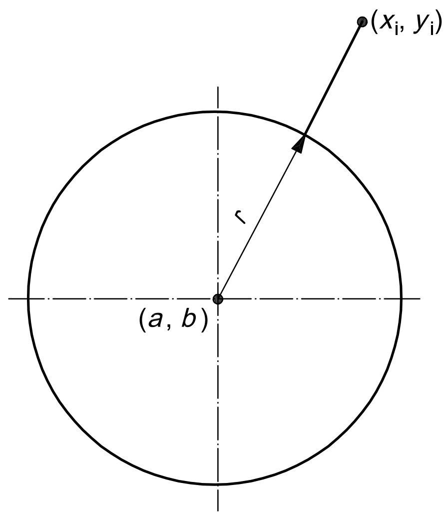


Figure B.1 — Circle and coordinates

The sum of the squares of the distances from the n points to the circle is, therefore, given by Equation (B.2):

$$\sum P_{(xi,\, yi)} = \sum \left\{ \left[\sqrt{(x_i - a)^2 + (y_i - b)^2} \right] - r \right\}^2 \quad \text{(B.2)}$$

The condition that this is a minimum leads to Equations (B.3), (B.4) and (B.5) for the three unknown values a, b and r:

$$na = \left[\sum x_i\right] - \left[r\sum \frac{(x_i - a)}{r_i}\right] \tag{B.3}$$

$$nb = \left[\sum y_i\right] - \left[r\sum \frac{(y_i - b)}{r_i}\right] \tag{B.4}$$

$$nr = \sum r_i \tag{B.5}$$

where

$$r_i = \sqrt{(x_i - a)^2 + (y_i - b)^2} \tag{B.6}$$

n is the number of measured points.

B.4 Calculations

The three Equations (B.3), (B.4) and (B.5) may be solved by any iterative method. A suggested method for solving these three equations is as follows:

Step 1: Set a, b and r to zero.

Step 2: Calculate the n values r_i from Equation (B.6).

Step 3: If any of these are zero, replace them by a value of 1 mm (this is to avoid division by zero in the next step).

Step 4: Calculate the new values of a, b and r from Equations (B.7), (B.8) and (B.9):

New value of $a = \left[\sum x_i - r\sum (x_i - a)/r_i\right] \times 1/n$ (B.7)

New value of $b = \left[\sum y_i - r\sum (y_i - b)/r_i\right] \times 1/n$ (B.8)

New value of $r = \left(\sum r_i\right) \times 1/n$ (B.9)

Step 5: If the new value of r differs from the old value by more than 0,01 mm, replace the old values of a, b and r by the new values and go back to step 2, otherwise go to step 6.

Step 6: Round the new value of r to the nearest millimetre as the internal radius for the set of points.

If any other iterative method is used, the intention specified in step 5, that two successive estimates of r shall differ by no more than 0,01 mm, shall apply.

B.5 Example

B.5.1 Data

Suppose that the distance $D = 22\,612{,}0$ mm and that, at one level, the angles α and β for 16 points on the tank wall for the internal method (see Clause 9) are as shown in Table B.1.

Table B.1 — Data for example

Point	α gon	β gon
1	32,985 0	72,455 9
2	23,554 7	56,077 1
3	12,419 3	32,619 7
4	384,814 9	355,411 8
5	369,505 4	317,683 6
6	349,318 3	283,517 2
7	331,067 6	262,377 7
8	306,710 2	242,499 2
9	284,301 9	229,719 5
10	178,136 3	192,604 0
11	148,702 9	180,271 3
12	127,961 6	168,833 6
13	106,487 5	153,790 0
14	84,816 9	135,095 2
15	61,113 4	110,345 1
16	49,932 8	96,709 6

B.5.2 Solution

Calculate coordinates (x, y) for each point as described in Annex A. The coordinates are shown in Table B.2.

Table B.2 — Calculated coordinates

Point	x mm	y mm
1	30 693,2	17 497,5
2	33 256,6	12 898,9
3	34 856,3	6 887,4
4	31 778,8	− 7 727,2
5	26 542,7	− 13 785,0
6	17 796,5	− 18 181,7
7	9 987,7	− 18 815,2
8	1 740,7	− 16 453,2
9	− 3 285,3	− 1 3051,9
10	− 10 954,0	3 917,2
11	− 10 037,2	10 454,7
12	− 7 550,8	16 071,5
13	− 2 257,0	22 071,2
14	6 407,5	26 354,9
15	18 321,8	26 168,3
16	23 842,8	23 792,5

Determine the radius of the best circle, using the least-squares method described in B.3. For the specific example, the radius of the best circle, obtained by iteration as shown in Table B.3, is 22 983 mm.

Table B.3 — Solution by iteration

a mm	*b* mm	*r* mm
12 571,271 94	4 881,238 76	25 294,534 95
12 284,413 59	4 422,656 68	22 952,768 46
12 166,734 76	4 241,896 69	22 967,467 06
12 106,852 85	4 153,733 83	22 975,074 20
12 076,253 64	4 110,737 30	22 979,199 36
12 060,577 88	4 089,758 82	22 981,330 47
12 052,536 73	4 079,520 82	22 982,408 79
12 048,408 99	4 074,523 95	22 982,949 28
12 046,289 29	4 072,085 08	22 983,219 03
12 045,200 50	4 070,894 76	22 983,353 43
12 044,641 15	4 070,313 85	22 983,420 35
12 044,353 76	4 070,030 36	22 983,453 67
12 044,206 08	4 069,892 03	22 983,470 26
12 044,130 19	4 069,824 54	22 983,478 53
12 044,091 19	4 069,791 62	22 983,482 66
12 044,071 14	4 069,775 55	22 983,484 72
12 044,060 83	4 069,767 72	22 983,485 74
12 044,055 54	4 069,763 90	22 983,486 25
12 044,052 81	4 069,762 04	22 983,486 51
12 044,051 41	4 069,761 13	22 983,486 64
12 044,050 70	4 069,760 69	22 983,486 70
12 044,050 33	4 069,760 47	22 983,486 74
12 044,050 13	4 069,760 37	22 983,486 75
12 044,050 04	4 069,760 31	22 983,486 76
12 044,049 99	4 069,760 29	22 983,486 76
12 044,049 96	4 069,760 28	22 983,486 77
12 044,049 95	4 069,760 27	22 983,486 77
12 044,049 94	4 069,760 27	22 983,486 77
12 044,049 94	4 069,760 27	22 983,486 77
12 044,049 94	4 069,760 27	22 983,486 77

Annex C
(normative)

Computation of internal radii from reference circumference and external measurements

C.1 The horizontal distance, D_{TO}, between points T and O in Figure C.1 is constant for all levels at which measurements were taken on the tank. Compute its value from the reference circumference using Equation (C.1):

$$D_{TO} = \frac{C_{em}}{2\pi} \times \frac{1}{\sin\theta_1} \tag{C.1}$$

where

C_{em} is the reference circumference, as determined in 11.2.1;

$2\theta_1$ is the horizontal angle subtended at the theodolite for the reference level, as determined in 11.2.2.3.

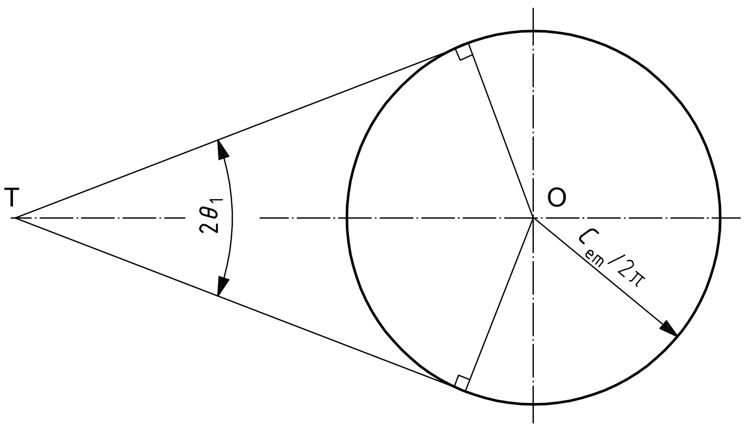


Figure C.1 — θ_1 at reference level

C.2 If r is the external radius at any other level and the corresponding horizontal angle at the theodolite station T is $2\theta_2$ (see 11.2.2.2), then, since the distance D_{TO} is constant, r can be calculated using Equations (C.2) and (C.3)

$$\frac{C}{2\pi} \times \frac{1}{\sin\theta_1} = \frac{r}{\sin\theta_2} \tag{C.2}$$

$$r = \frac{C}{2\pi} \times \frac{\sin\theta_2}{\sin\theta_1} \tag{C.3}$$

Calculate the external radii at all heights for each of the other theodolite positions in the same way.

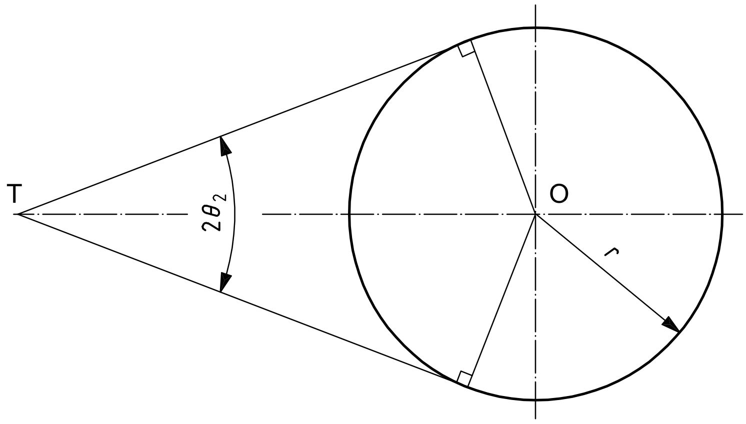


Figure C.2 — θ_2 at another level

C.3 Take the external radius at each level to be the average of the individual radii calculated for that level. Subtract the thickness of the plate and paint to give the corresponding internal radius.

Annex D
(normative)

Computation of internal radii from reference distances between pairs of theodolite stations

D.1 The following field measurements, shown in Figure D.1, are recorded for each set of readings for adjacent theodolite stations (see 11.3);

— D is the distance, in metres, between the theodolite stations T1 and T2 (11.3.2);

— D_1 is the distance, in metres, between the theodolite stations T1 and point O;

— D_2 is the distance, in metres, between the theodolite stations T2 and point O;

— $2\theta_1$ is the horizontal angle, in radians, subtended by the tangents T1-A1 and T1-A4 at theodolite station T1 (11.3.3);

— $2\theta_2$ is the horizontal angle, in radians, between the tangents T2-A2 and T2-A3;

— α is the horizontal angle, in radians, between the tangent T1-A1 and the line T1-T2;

— β is the horizontal angle, in radians, between the tangent T2-A3 and the line T1-T2.

D.2 To calculate the external radii at points A1, A2, A3 and A4, it is assumed that the distance between points O and A1 equals the distance between points O and A4, which is equal to r_1, and the distance between points O and A2 equals the distance between points O and A3, which is equal to r_2. Calculate the values of r_1 and r_2, expressed in metres, from Equations D.1 and D.2, which are derived as follows:

$$\angle \text{ O-T1-T2} = \alpha + \theta_1 \tag{D.1}$$

$$\angle \text{ O-T2-T1} = \beta + \theta_2 \tag{D.2}$$

$$\angle \text{ T2-O-T1} = \pi - \angle \text{ O-T1-T2} - \angle \text{ O-T2-T1} = \pi - (\alpha + \beta + \theta_1 + \theta_2) = \varPhi \tag{D.3}$$

The values of D_1 and D_2 can be obtained from triangle O-T1-T2 using the sine rule, as given in Equations (D.4) to (D.6):

$$\frac{D_{12}}{\sin(\text{T2-O-T1})} = \frac{D_2}{\sin(\text{O-T1-T2})} = \frac{D_1}{\sin(\text{O-T2-T1})} \tag{D.4}$$

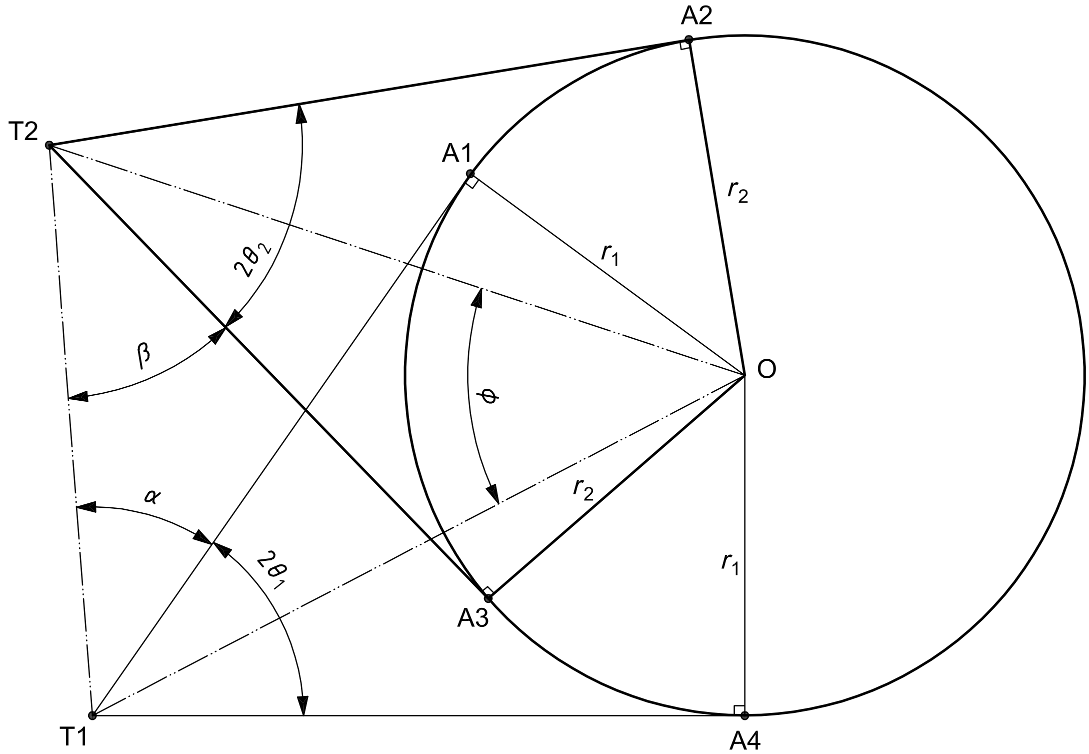


Figure D.1 — External radii from reference distances between pairs of theodolite stations

$$D_2 = \frac{\sin(\alpha + \theta_1)}{\sin\phi} \times D_{12} \tag{D.5}$$

$$D_1 = \frac{\sin(\beta + \theta_2)}{\sin\phi} \times D_{12} \tag{D.6}$$

External radius, r_2, the distance between points O and A2, which equals the distance between points O and A3, can be calculated from Equations (D.7) and (D.8):

$$r_2 = D_2 \sin\theta_2 \tag{D.7}$$

$$r_2 = D_{12} \sin\theta_2 \left[\frac{\sin(\alpha + \theta_1)}{\sin\phi} \right] \tag{D.8}$$

External radius, r_1, the distance between points O and A1, which equals the distance between points O and A4, can be calculated from Equations (D.9) and (D.10):

$$r_1 = D_1 \sin\theta_1 \tag{D.9}$$

$$r_1 = D_{12} \sin\theta_1 \left[\frac{\sin(\beta + \theta_2)}{\sin\phi} \right] \tag{D.10}$$

D.3 Calculate r_1 and r_2 for each of the other pairs of theodolite stations around the tank and at each level. Subtract the thickness of the plate and the paint (see 13.2.1) to give the corresponding internal radii.

Take the internal radius for each tank level to be the average of the individual radii calculated for that level.

Annex E
(informative)

Calibration uncertainties

E.1 Introduction

This annex describes calculations used in estimation of measurement uncertainties using the optical triangulation method for calibrating vertical cylindrical tanks.

The calculations follow the guidelines set out in the GUM[1].

As this part of ISO 7507 describes three different measurement principles, the measurement uncertainties for the different principles are discussed separately in Clauses E.3 and E.4. These clauses describe uncertainty components that derive from the measurement and calculation of the tank radius. The final tank table is also based upon other measurement and calculations. The uncertainty components of these measurements and calculations are described in ISO 7507-1.

The uncertainty components described can vary depending on the equipment and measurement conditions. Each measurement and its object are unique and it is necessary to evaluate the uncertainty components for each measurement separately.

The part of ISO 7507 describes three different 2-dimensional measurements. The procedures do not give full 3-dimensional information. Uncertainties due to the fact that the measurements are not performed in a perfect plane (differences in height when sighting the points) are not considered in this evaluation.

Any possible co-variances are disregarded.

E.2 Symbols

The following terms and their abbreviations have been used in this annex.

k	Coverage factor, used for conversions between standard and expanded uncertainties	—
$U(B_{\text{stadia}})$	Expanded uncertainty of the stadia received from calibration certificate	metre
$u(B_{\text{stadia}})$	Standard uncertainty of the length of the stadia	metre
$u(B_{\text{Tstadia}})$	Standard uncertainty due to temperature of the stadia	metre
$u(B_{\alpha\text{stadia}})$	Standard uncertainty due to thermal expansion coefficient of the stadia	metre
$U(B_{\text{dm}})$	Expanded uncertainty of the distance meter received from calibration certificate	metre
$u(B_{\text{dm}})$	Standard uncertainty of the distance meter	metre
$e(B_{\text{mis}})$	The maximum estimated misalignment error of the stadia setup	metre
$u(B_{\text{mis}})$	Standard uncertainty of the misalignment of the stadia setup	metre
s	Estimated standard deviation	metre

$u(D_{rep})$	Standard uncertainty corresponding to the variation in repeated measurements	metre
$U(D_{rep})$	Expanded uncertainty due to repeatability of the measurements	metre
T_{stadia}	Temperature of the stadia	degree Celsius
$e(T_{stadia})$	Maximum estimated error of the temperature of the stadia	degree Celsius
$u(T_{stadia})$	Standard uncertainty of the temperature of the stadia	degree Celsius
α_{stadia}	Thermal linear expansion coefficient of the stadia	per degree Celsius
$e(\alpha_{stadia})$	Maximum estimated error of coefficient of linear expansion of the stadia	per degree Celsius
$e(\alpha_{tk})$	Maximum estimated error of coefficient of linear expansion of the tank shell	per degree Celsius
$u(\alpha_{tk})$	Standard uncertainty of coefficient of linear expansion of the tank shell	per degree Celsius
$U(\alpha_{stadia})$	Standard uncertainty of coefficient of linear expansion of the stadia	per degree Celsius
u_{th}	Standard uncertainty of the theodolite	gon
θ	Half of the angle subtended at varying heights of the tank	gon
θ_{tr}	Resolution of the theodolite	gon
$u(\theta_{tr})$	Standard uncertainty corresponding to the resolution of the instrument	gon
$e(\theta_{ra})$	Maximum estimated error of determination of reference axis	gon
$u(\theta_{ra})$	Standard uncertainty of reference axis	gon
$e(\theta_{pi})$	Estimated error of misalignment of laser to theodolite	gon
$u(\theta_{pi})$	Standard uncertainty due to misalignment of laser to theodolite	gon
$u(\theta_{th})$	Standard uncertainty of measured angles due to non-linearity of the theodolite	gon
$u(\theta_{r})$	Standard uncertainty of measured angles due to limited resolution of the theodolite	gon
$u(\theta_{tot})$	Total standard uncertainty of horizontal angles	gon
C_{em}	Circumference measured at reference height	metre
$u(C_{em})$	Standard uncertainty of circumference measured at reference height	metre
D	Distance between theodolite stations	metre
$u(D)$	Standard uncertainty of distance between theodolite stations	metre
$u(X)$	Standard uncertainty of the coordinate x	metre
$u(Y)$	Standard uncertainty of the coordinate y	metre
$u(I_s)$	Standard uncertainty of the radius due to deformation	metre
$e(\alpha_{tank})$	Maximum estimated error of coefficient of linear expansion of the tank	per degree Celsius
$u(\alpha_{tank})$	Standard uncertainty of coefficient of linear expansion of the tank	per degree Celsius
T_{tank}	Temperature of the tank	degree Celsius
ΔT_{tank}	Maximum temperature deviation of the tank from the mean temperature	degree Celsius
$e(T_{tk})$	Estimated range of temperatures of the tank shell	degree Celsius

$u(T_{tk})$	Standard uncertainty of the temperature of the tank shell	degree Celsius
φ_{Tilt}	Maximum tilt of the tank	percent
$u(\varphi_{Tilt})$	Standard uncertainty of the tilt of the tank	percent
$u(R_{meas})$	Standard uncertainty of the measured radius	metre
$u(R_{LS})$	Standard uncertainty of radius of fitted circles due to tank deformation	metre
$u(R_{ang})$	Standard uncertainty of fitting circles to measured co-ordinates of the tank shell	metre
$e(R_{mis})$	Estimated errors of the radius due mis-sighting on deformed tanks	metre
$u(R_{mis})$	Standard uncertainty of the radius due mis-sighting on deformed tanks	metre
$u(R_{total})$	Total standard uncertainty of the radius	metre
r	Radius calculated from measurement	metre
R_{nom}	Nominal (average) radius of the tank	metre
$u(A)$	Standard uncertainty of the area	square metre
$u(R_{def})$	Standard uncertainty of the radius due to deformation	metre
T_{ref}	Temperature at which the stadia was calibrated	degree Celsius
$u(t_{mp})$	Standard uncertainty of thickness of metal and paint of the tank shell	metre

E.3 Measurement uncertainties, internal method

E.3.1 Uncertainties of distance between theodolites using stadia

E.3.1.1 Stadia length

The stadia is calibrated traceable to a standard. The expanded uncertainty, $U(B_{stadia})$, given by the calibration certificate (typically 0,05 mm = 5×10^{-5} m), with a coverage factor k (usually, $k = 2$, corresponding to 95 % confidence level), yields the standard uncertainty, expressed in metres, given in Equation (E.1):

$$u\left(B_{stadia}\right) = \frac{U\left(B_{stadia}\right)}{k} \quad \text{(E.1)}$$

E.3.1.2 Stadia set-up

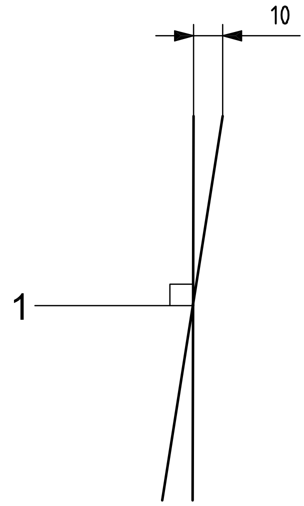
The misalignment of the set-up of the stadia may be estimated to be within 10 mm (using a 2 m stadia), covering misalignment in all axes, contributing to the error of the length of the stadia $e(B_{mis})$ = 0,1 mm = 10^{-4} m. See Figure E.1.

Assuming rectangular distribution, the corresponding standard uncertainty, expressed in metres, is given by Equation (E.2):

$$u\left(B_{mis}\right) = \frac{e\left(B_{mis}\right)}{\sqrt{3}} \quad \text{(E.2)}$$

NOTE The factor $\sqrt{3}$ corresponds to a rectangular distribution.

Dimensions in millimetres



Key

1 aiming axis

Figure E.1 — Stadia misalignment

E.3.1.3 Measured distance between theodolites

The distance, D, is determined at least 10 times and the average value is used. The distance, D, is then checked after completion of the tank wall measurements. Short-term variations of factors influencing the measurement show in the repeatability of the readings.

The distance, D, used in calculations is based on the average of n readings. The estimated standard deviation, $s(D)$, is used to achieve the contribution of the repeatability, $u(D_{\text{rep}})$, to the uncertainty of the distance D between the theodolite stations, where both $s(D)$ and $u(D_{\text{rep}})$ are expressed in metres, as given in Equation (E.3):

$$u\left(D_{\text{rep}}\right) = \frac{s\left(D\right)}{\sqrt{n}} \tag{E.3}$$

NOTE The factor $\sqrt{n}$ is due to the averaging of the measurements. Typically, $n = 10$ (5 initial + 5 final) readings are used to estimate the distance.

E.3.1.4 Temperature of the stadia

The stadia expands with temperature no matter what material it is made of. If the temperature of the stadia is measured, a maximum error of 2 °C, $e(T_{\text{stadia}}) = 2$ °C, may be assumed. If the temperature is not measured, $e(T_{\text{stadia}})$ is the difference between ambient temperature and the temperature at which the stadia was calibrated. This can be as much as 20 °C, even more in extreme ambient temperatures.

This gives the corresponding standard uncertainty, expressed in metres, as given in Equation (E.4):

$$u\left(B_{\text{T stadia}}\right) = \frac{B \times \alpha_{\text{stadia}} \times e\left(T_{\text{stadia}}\right)}{\sqrt{3}} \tag{E.4}$$

where

B is the length of the stadia, expressed in metres;

α_{stadia} is the thermal expansion coefficient of the stadia, expressed in °C^{-1}, typically 1×10^{-6} °C^{-1} for invar and 11×10^{-6} °C^{-1} for steel.

NOTE The factor $\sqrt{3}$ corresponds to a rectangular distribution.

E.3.1.5 Thermal expansion of the stadia

The uncertainty of the length of the stadia is due to the uncertainty of its thermal expansion coefficient. The error of the thermal expansion coefficient, $e(\alpha_{\text{stadia}})$, can be estimated as typically being equal to 1×10^{-6} °C^{-1} for invar, and approximately the same for steel. The corresponding standard uncertainty, expressed in metres, is given by Equation (E.5):

$$u\left(B_{\alpha\,\text{stadia}}\right) = \frac{B \times \left(T_{\text{ref}} - T_{\text{stadia}}\right) \times e\left(\alpha_{\text{stadia}}\right)}{\sqrt{3}} \tag{E.5}$$

NOTE The factor $\sqrt{3}$ corresponds to a rectangular distribution.

E.3.1.6 Linearity of angular measurements by theodolite

The estimated standard uncertainty, $u(\theta_{\text{th}})$, due to non-linearity of the angular part of the theodolite is given by the procedure in Annex F.

E.3.1.7 Angular resolution of theodolite

If the resolution of the instrument is θ_{tr}, expressed in mgon, (typically $\theta_{\text{tr}} = 0{,}2$ mgon), then the corresponding standard uncertainty, expressed in mgon, is given by Equation (E.6):

$$u\left(\theta_{\text{tr}}\right) = \frac{\theta_{\text{tr}}}{2\sqrt{3}} \tag{E.6}$$

NOTE The factor $\sqrt{3}$ corresponds to a rectangular distribution.

E.3.1.8 Total uncertainty of the distance between theodolites

The total uncertainty of the distance, $u(D)$, is calculated through partial derivatives of the different components

$$\frac{\partial D}{\partial B} = \frac{\cot\theta}{2} = \frac{D}{B}$$

$$\frac{\partial D}{\partial \theta} = \frac{B}{2} \times \left(-\frac{1}{\sin^2\theta}\right) = -\frac{B}{2 \times \sin^2\theta}$$

The result is given as Equation (E.7):

$$u(D)=\left\{\left(\frac{D}{B}\right)^2\times\left[u\left(B_{\text{stadia}}\right)^2+u\left(B_{\text{mis}}\right)^2+u\left(B_{\text{Tstadia}}\right)^2+u\left(B_{\alpha\text{stadia}}\right)^2\right]+\ldots\right.$$
$$\left.\ldots+\left[\frac{B}{2\sin^2\left(\frac{\theta\times\pi}{200}\right)}\right]^2\times\left[u\left(\theta_{\text{th}}\right)^2+u\left(\theta_{\text{tr}}\right)^2\right]\times\left(\frac{\pi}{200\,000}\right)^2+u\left(D_{\text{rep}}\right)^2\right\}^{1/2} \quad \text{(E.7)}$$

where all dimensions and their uncertainties are expressed in metres, angles are expressed in gon and their uncertainties are expressed in mgon.

E.3.2 Uncertainties of distance between theodolites using total station

E.3.2.1 Distance meter

The distance meter is calibrated traceable to a standard along with a specific prism. The expanded uncertainty, $U(B_{\text{dm}})$, given by the calibration certificate [typically $U(B_{\text{dm}})$, expressed in metres, which equals $(5\times10^{-4}+2\times10^{-5}\times D_{\text{m}})$, where D_{m} is the measured distance, in metres, with a coverage factor k (usually, $k=2$, corresponding to 95 % confidence level], yields the standard uncertainty, expressed in metres, as given in Equation (E.8):

$$u\left(B_{\text{dm}}\right)=\frac{U\left(B_{\text{dm}}\right)}{k} \quad \text{(E.8)}$$

E.3.2.2 Measured distance between theodolites

The distance, D, is determined at least 10 times and the average value is used. The distance, D, is then checked after completion of the tank wall measurements. Short-term variations of factors influencing the measurement show in the repeatability of the readings.

The distance, D, used in calculations is based on the average of n readings. The estimated standard deviation, $s(D)$, is used to achieve the contribution of the repeatability $u(D_{\text{rep}})$ to the uncertainty of the distance, D, between the theodolite stations, expressed in metres, as given in Equation (E.9).

$$u\left(L_{\text{rep}}\right)=\frac{s\left(D\right)}{\sqrt{n}} \quad \text{(E.9)}$$

NOTE The factor $\sqrt{n}$ is due to the averaging of the measurements. Typically, $n=10$ (5 initial + 5 final) readings are used to estimate the distance.

E.3.3 Uncertainty of horizontal angles

E.3.3.1 Angular linearity of theodolite

See E.3.1.6.

E.3.3.2 Angular resolution of theodolite

See E.3.1.7.

E.3.3.3 Reference axis

The co-ordinates are based on angles relative to the reference axis. The alignment of the reference axis, however, is not perfect. The estimated error of $e(\theta_{\text{ra}}) = 3$ mgon. The standard deviation, expressed in mgon, is given by Equation (E.10):

$$u\left(\theta_{\text{ra}}\right) = \frac{e\left(\theta_{\text{ra}}\right)}{\sqrt{3}} \tag{E.10}$$

NOTE The factor $\sqrt{3}$ corresponds to a rectangular distribution.

E.3.3.4 Misalignment of laser with theodolite

Misalignment results in systematic error. The estimated value of this error, $e(\theta_{\text{pi}}) = 5$ mgon, may be used, resulting in an uncertainty, expressed in mgon, as given in Equation (E.11):

$$u\left(\theta_{\text{pi}}\right) = \frac{e\left(\theta_{\text{pi}}\right)}{3^{1/2}} \tag{E.11}$$

NOTE The Factor $3^{1/2}$ corresponds to a rectangular distribution.

E.3.3.5 Total uncertainty of horizontal angles

The total uncertainty of the horizontal angles, α and β, is obtained as RMS of the different components. As the components are independent of measured angles, absolute uncertainties of α and β are identical, expressed in radians, as given in Equation (E.12):

$$u(\alpha) = u(\beta) = \left(\frac{\pi}{200\ 000}\right) \times \left[u\left(\theta_{\text{pi}}\right)^2 + u\left(\theta_{\text{ra}}\right)^2 + u\left(\theta_{\text{tr}}\right)^2 + u\left(\theta_{\text{th}}\right)^2\right]^{1/2} \tag{E.12}$$

where all uncertainties $u(\theta)$ are expressed in mgon.

E.3.4 Uncertainty of co-ordinates

To estimate the total uncertainties of the planar co-ordinates the worst actual apex angle is used. The total uncertainties of the co-ordinates, $u(X)$ and $u(Y)$, are calculated using partial derivatives of the different components, as given in Equations (E.13) to (E.20).

$$u(X) = \sqrt{u\left(X_{\text{D}}\right)^2 + u\left(X_{\alpha}\right)^2 + u\left(X_{\beta}\right)^2} \tag{E.13}$$

where

$$u\left(X_{\text{D}}\right) = u(D) \times \frac{\tan\beta}{(\tan\beta - \tan\alpha)} = u(D) \times \frac{X}{D} \tag{E.14}$$

$$u\left(X_{\alpha}\right) = u(\alpha) \times D \times \frac{\tan\beta}{\cos^2\alpha \times (\tan\beta - \tan\alpha)^2} = u(\alpha) \times \frac{X}{\cos^2\alpha \times (\tan\beta - \tan\alpha)} \tag{E.15}$$

$$u\left(X_{\beta}\right) = u(\beta) \times D \times \frac{\dfrac{\tan\beta - \tan\alpha}{\cos^2\beta} - \dfrac{\tan\beta}{\cos^2\beta}}{(\tan\beta - \tan\alpha)^2} = u(\beta) \times \frac{-2 \times Y}{\sin 2\beta \times (\tan\beta - \tan\alpha)} \tag{E.16}$$

$$u\left(Y\right)=\sqrt{u\left(Y_{\mathrm{D}}\right)^{2}+u\left(Y_{\alpha}\right)^{2}+u\left(Y_{\beta}\right)^{2}} \tag{E.17}$$

where

$$u\left(Y_{\mathrm{D}}\right)=u\left(D\right)\times\frac{\tan\alpha\times\tan\beta}{(\tan\beta-\tan\alpha)}=u\left(D\right)\times\frac{Y}{D} \tag{E.18}$$

$$u\left(Y_{\alpha}\right)=u\left(\alpha\right)\times D\times\tan\beta\times\frac{\dfrac{\tan\beta-\tan\alpha}{\cos^{2}\alpha}+\dfrac{\tan\alpha}{\cos^{2}\alpha}}{(\tan\beta-\tan\alpha)^{2}}=u\left(\alpha\right)\times\frac{X\times\tan\beta}{\cos^{2}\alpha\times(\tan\beta-\tan\alpha)} \tag{E.19}$$

$$u\left(Y_{\beta}\right)=u\left(\beta\right)\times D\times\tan\alpha\times\frac{\dfrac{\tan\beta-\tan\alpha}{\cos^{2}\beta}-\dfrac{\tan\beta}{\cos^{2}\beta}}{(\tan\beta-\tan\alpha)^{2}}=u\left(\beta\right)\times\frac{-2\times Y\times\tan\alpha}{\sin2\beta\times(\tan\beta-\tan\alpha)} \tag{E.20}$$

NOTE 1 Equations (E.13) to (E.20) neglect covariances between X_{D}, X_{α} and X_{β} and also those between Y_{D}, Y_{α} and Y_{β}.

NOTE 2 All uncertainties of X and Y are expressed in metres while those of α and β are expressed in radians.

E.3.5 Uncertainty of radius

E.3.5.1 Fitted radius

A circle with radius, R, is fitted to the measured coordinates of the tank shell using Equation (E.21):

$$R=\frac{\sum_{i=1}^{n}R_{i}}{n}=\frac{\sum_{i=1}^{n}\sqrt{\left(x_{i}-A_{x}\right)^{2}+\left(y_{i}-B_{y}\right)^{2}}}{n} \tag{E.21}$$

where

n is the total number of target points at one height;

A_x and B_y are co-ordinates of the centre of the fitted circle at this height.

The uncertainty, $u(R_{\mathrm{ang}})$, of fitting circles to measured coordinates of the tank shell is calculated using Equation (E.22):

$$u\left(R_{\mathrm{ang}}\right)=\frac{\sqrt{\sum\left[u\left(X_{i}\right)\times\frac{x_{i}-A_{x}}{R_{i}}\right]^{2}+\sum\left(uY_{i}\times\frac{Y_{i}-B_{y}}{R_{i}}\right)^{2}+\left[u\left(A_{x}\right)\times\sum\left(\frac{x_{i}-A_{x}}{R_{i}}\right)\right]^{2}+\left[u\left(B_{y}\right)\times\sum\left(\frac{y_{i}-B_{y}}{R_{i}}\right)\right]^{2}}}{n} \tag{E.22}$$

where

X_i and Y_i are the co-ordinates for each target point as above, expressed in metres;

$u(X_i)$ and $u(Y_i)$ are uncertainties of the co-ordinates for each target point as above, expressed in metres

$u(A_x)$ and $u(B_y)$ are standard uncertainties of the centre of the fitted circle, expressed in metres

NOTE Equations (E.21) and (E.22) neglect covariances.

E.3.5.2 Deformation resulting from fitting

The tank is more or less deformed, i.e. the tank is not a perfect cylinder. This has often a great impact on the uncertainty of the radius. The procedure for fitting circles to sets of measured points is described in Annex B. To estimate the standard uncertainty of the fitting, $u(R_{LS})$, the standard deviation of the residuals, expressed in metres, is used as given in Equation (E.23):

$$u\left(R_{\text{LS}}\right)=\frac{s}{\sqrt{n}} \tag{E.23}$$

NOTE The factor $\sqrt{n}$ is due to an average value of n readings and is used to calculate the radius.

E.3.5.3 Uncertainties due to temperature

The standard uncertainty of tank radii corrected for differential thermal expansion of the tank shell includes the following :

- standard uncertainty of the coefficients of expansion of the tank;
- standard uncertainty of the tank shell temperature.

Using the variables

α_{tk} coefficient of linear expansion of the tank shell material, expressed in reciprocal degrees Celsius;

T_{ref} reference temperature of the tank (zero uncertainty), expressed in degrees Celsius;

T_{tk} tank shell temperature at calibration, expressed in degrees Celsius,

the maximum range of the tank shell temperatures is $e(T_{\text{tk}})$. This should be estimated from temperature measurements (hottest to coldest points on the tank shell) or estimates of thermal gradients at the time of calibration; assuming rectangular distribution, the standard uncertainty, expressed in degrees Celsius, is given by Equation (E.24):

$$u\left(T_{\text{tk}}\right)=\frac{e\left(T_{\text{tk}}\right)}{2\times 3^{1/2}} \tag{E.24}$$

Maximum error of estimate of the linear expansion coefficient, $e(\alpha_{\text{tk}})$, are typically equal to 2×10^{-6} °C^{-1}; assuming rectangular distribution, the standard uncertainty, expressed in °C^{-1}, is given by Equation (E.25):

$$u\left(\alpha_{\text{tk}}\right)=\frac{e\left(\alpha_{\text{tk}}\right)}{2\times 3^{1/2}} \tag{E.25}$$

The standard uncertainty of the corrected tank radius, expressed in metres, is given by Equation (E.26):

$$u\left(R_{\text{th}}\right)=R\times\left\{\left[\alpha_{\text{tk}}\times u\left(T_{\text{tk}}\right)\right]^2+\left[u\left(\alpha_{\text{tk}}\right)\times\left(T_{\text{tk}}-T_{\text{ref}}\right)^2\right]\right\}^{1/2} \tag{E.26}$$

E.3.5.4 Total uncertainty of the radius

The total standard uncertainty of the radius. $u(R_{\text{total}})$, expressed in metres, is calculated as given in Equation (E.27):

$$u\left(R_{\text{total}}\right)=\sqrt{u\left(R_{\text{ang}}\right)^2+u\left(R_{\text{LS}}\right)^2+u\left(R_{\text{th}}\right)^2} \tag{E.27}$$

E.3.6 Uncertainty of cross-sectional area

As the cross-sectional area, A, is derived from the formula $\pi \times R^2$, its uncertainty, expressed in square metres, is given by Equation (E.28):

$$u(A) = 2 \times \pi \times R \times u(R_{\text{total}}) \tag{E.28}$$

E.4 Measurement uncertainties, external measurements with reference circumference measured by strapping

E.4.1 Uncertainty reference circumference

For the standard uncertainty of the measured external circumference, $u(C_{\text{em}})$, see ISO 7507-1:2003, D.5.1.

E.4.2 Uncertainty of horizontal angles

E.4.2.1 Angular linearity of theodolite

See E.3.1.6.

E.4.2.2 Angular resolution of theodolite

See E.3.1.7.

E.4.2.3 Total uncertainty of horizontal angles

The total uncertainty of the horizontal angle, θ, expressed in radians, is obtained as RMS of the different components, as given in Equation (E.29):

$$u(\theta_{\text{tot}}) = \left(\frac{\pi}{200\,000}\right) \times \left[u(\theta_{\text{th}})^2 + u(\theta_{\text{r}})^2\right]^{1/2} \tag{E.29}$$

where $u(\theta_{\text{th}})$ and $u(\theta_{\text{r}})$ are expressed in mgon.

E.4.3 Uncertainty of radius

E.4.3.1 General

To estimate the uncertainty of the calculated radius, the uncertainty of the reference circumference, $u(C_{\text{em}})$, is used together with the uncertainty of the horizontal angles, $u(\Phi_1)$. The uncertainty of the radius is calculated using the components given in E.4.3.2 to E.4.3.7.

E.4.3.2 Measured radius

The uncertainty of the measured radius, expressed in metres, can be calculated as given in Equation (E.30):

$$u(R_{\text{meas}}) = R_{\text{nom}} \times \sqrt{\left[\frac{u(C_{\text{em}})}{C_{\text{em}}}\right]^2 + 2 \times \left[\frac{u(\theta_{\text{tot}})}{\tan\theta}\right]^2} \tag{E.30}$$

where

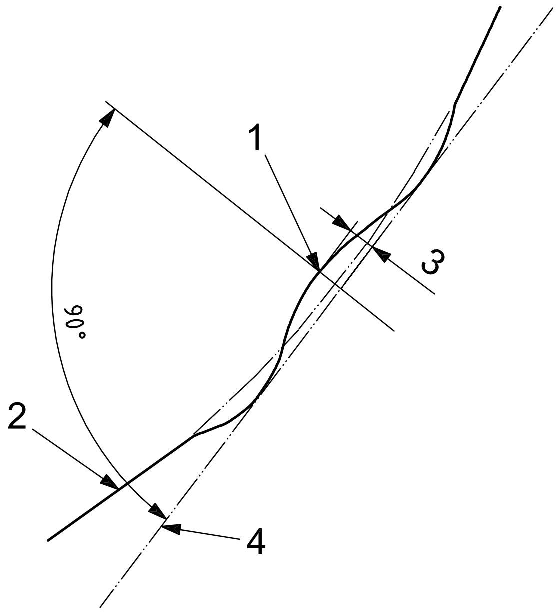
$u(C_{em})$ is the uncertainty, in metres, of the strapped circumference (at the reference height);

θ is one half of the smallest angles, in radians, subtended at all heights at all stations;

R_{nom} is the nominal radius of the tank, in metres.

E.4.3.3 Tangential mis-sighting due to tank shell deformation

Due to deformation, the tangential sighting might be at a location in front of or behind the theoretical tangent of a perfect cylinder (see Figure E.2).



Key

1 actual radius
2 tank shell
3 additional measured radius
4 line of sight

Figure E.2 — Tangential mis-sighting

The error is single-sided and, based on experience, the error in the tank radius, $e(R_{mis})$, can be of the order of 5 mm (5×10^{-3} m).

The resulting component of standard uncertainty, $u(R_{mis})$, of the tank radius, expressed in metres, can be estimated as given in Equation (E.31):

$$u(R_{mis}) = \frac{e(R_{mis})}{\sqrt{3}} \tag{E.31}$$

NOTE The factor $\sqrt{3}$ corresponds to a rectangular distribution.

E.4.3.4 Tank deformation

The tank is more or less deformed, i.e. the tank is not a perfect cylinder. This has often a great impact on the uncertainty of the radius. To estimate the uncertainty, $u(R_{def})$, of the radius, the standard deviation of the residuals (corrected for tilt), expressed in metres, is used as given in Equation (E.32):

$$u\left(R_{def}\right) = \frac{s}{\sqrt{n}} \quad \text{(E.32)}$$

NOTE 1 Factor $\sqrt{n}$ is due to the fact that n residuals were used to calculate the average radius.

NOTE 2 One angular difference is normally taken from each of n stations, resulting in n being the number of stations.

NOTE 3 The effect of tank tilt can be the major contribution to the uncertainty of the radius. There are several methods for correcting for it that can remove more or less of the uncertainty of the radius.

E.4.3.5 Uncertainties due to temperature

See E.3.5.3.

E.4.3.6 Uncertainties due to thickness of tank shell material

See ISO 7507-1.

NOTE The uncertainty, $u(t_{mp})$, of tank shell metal and paint adds directly to that of the radius.

E.4.3.7 Total uncertainty of the internal radius

The total standard uncertainty, $u(R_{total})$, of the radius, expressed in metres, is calculated as given in Equation (E.33):

$$u\left(R_{total}\right) = \sqrt{u\left(R_{meas}\right)^2 + u\left(R_{mis}\right)^2 + u\left(R_{def}\right)^2 + u\left(R_{th}\right)^2 + u\left(t_{mp}\right)^2} \quad \text{(E.33)}$$

E.4.4 Uncertainty of cross-sectional area

As the cross-sectional area is derived from the formula $\pi \times R^2$, its uncertainty, expressed in square metres, is given in Equation (E.34):

$$u\left(A\right) = 2 \times \pi \times R \times u\left(R_{total}\right) \quad \text{(E.34)}$$

Annex F
(normative)

Procedure for checking the theodolite(s)

F.1 General

The theodolites used for measurement according to this part of ISO 7507 shall be checked according to the procedure in this annex.

This procedure shall be carried out to check the measurement of horizontal angles by theodolites. The check should be carried out under as stable environmental conditions as possible. The check shall be carried out on a regular basis, or in case of one-time use of the instrument, just prior to the calibration of the tank.

NOTE Unstable set-up, bad lighting and varying ambient conditions during the actual calibration of the tank influence the result and increase the measurement uncertainty.

F.2 Procedure

F.2.1 Set up each theodolite with care according to the procedure and instructions given by the manufacturer.

NOTE It is not necessary to carry out the checking on a tank.

F.2.2 Point the telescope at a clearly defined point that lies as close to the horizontal plane of the theodolite as possible. Measure and record the horizontal angle.

F.2.3 Make a two-face measurement by turning the telescope 200 gon in, e.g. first vertical and then horizontal direction, and reposition it at the same point as in F.2.2. Measure and record the horizontal angle.

F.2.4 Repeat F.2.2 and F.2.3 four times to obtain a set of five collimation checks.

F.2.5 Repeat F.2.2 to F.2.4 at two other clearly defined points, with approximately 66 gon horizontally apart to obtain three separate sets of collimation checks.

NOTE This covers the entire angular range of the device (400 gon).

F.3 Handling the results

F.3.1 Calculate the worst-case difference between any two measurements at each measured point and standard deviation of each set. Since the theodolite can be used around its full axis, the set that resulted in the worst average deviation shall be used.

F.3.2 The standard uncertainty, $u(\theta_{\text{th}})$, of the measurement of horizontal angles by the theodolite, expressed in mgon, can then be estimated as given in Equation (F.1):

$$u(\theta_{\text{th}}) = \sqrt{\left(\frac{\overline{\Delta\varepsilon}}{\sqrt{3}}\right)^2 + \left(\frac{s}{\sqrt{5}}\right)^2} \quad \text{(F.1)}$$

where

$\overline{\Delta\varepsilon}$ is the worst-case difference in two-face measurements at all measured points, in mgon;

s is the worst-case of the standard deviations of the measurements at each measured set, in mgon;

$\sqrt{3}$ corresponds to the factor for a rectangular distribution;

$\sqrt{5}$ is due to the five measurements in each set.

F.3.3 The result of the theodolite checking procedure is used to

a) estimate the uncertainty of angular measurements used in Annex E;

b) reject the theodolite if the angular uncertainty calculated in F.2.2 is greater than 5 mgon.

Bibliography

[1] *Guide to the expression of uncertainty in measurement (GUM)*, BIPM/IEC/IFCC/ISO/IUPAC/IUPAP/OIML

[2] ISO 7507-2, *Petroleum and liquid petroleum products — Calibration of vertical cylindrical tanks — Part 2: Optical-reference-line method*

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