International Standard ISO 5022

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Shaped refractory products — Sampling and acceptance testing

Produits réfractaires façonnés — Échantillonnage et contrôle de réception

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Foreword

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Draft International Standards adopted by the technical committees are circulated to the member bodies for approval before their acceptance as International Standards by the ISO Council.

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It has been approved by the member bodies of the following countries :

Austria	India	Spain
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The member bodies of the following countries expressed disapproval of the document on technical grounds :

Canada
Netherlands

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Shaped refractory products — Sampling and acceptance testing

1 Scope and field of application

This International Standard gives directives for sampling shaped refractory products and for obtaining, from a sample of the smallest possible size, the most precise assessment possible, of the quality of a consignment.

The methods described below make it possible to carry out an acceptance test based on an assessment of the extent to which the specifications have been observed, but do not make it possible to determine whether the accepted consignment is suitable for a given application or to compare different qualities of parts for this same purpose.

This International Standard applies to products manufactured from refractory materials.

It may be applied when the parties concerned have agreed to do so and have therefore, by common consent, made a choice between the various possibilities put forward in this International Standard, and have specified the various parameters (see 3.2) which must be defined in order to permit the application of the methods described.

It is also possible to apply the directives forming the subject of this International Standard while modifying, by prior agreement between the parties concerned, those values which, particularly in the tables, do not follow from statistical laws (see 3.3).

2 Statistical terminology and symbols

2.1 population : The totality of items under consideration. Each of the batches formed in accordance with 3.1 represents a population.

2.2 size of the population : Number of items in the population (symbol : N).

2.3 sample : One or more items taken from a population and intended to provide information on the population and possibly to serve as a basis for a decision on the population or the process which had produced it.

2.4 size of the sample : Number of items in the sample (symbol : n).

2.5 observed value : The value of a characteristic determined as a result of an observation or test (symbol for the observed value having the number i : x_i).

2.6 extreme values :

x_{max} : largest observed value in a sample;

x_{min} : smallest observed value in a sample.

2.7 (arithmetic) mean : The arithmetic mean of the observed values in a sample is their sum divided by the size of the sample.

$$\bar{x} = \frac{1}{n}(x_1 + x_2 + \ldots + x_n) = \frac{1}{n}\sum_{i=1}^{n} x_i$$

The mean value of the population is designated by the symbol μ.

2.8 standard deviation : The standard deviation is the quantity most commonly used in statistics to characterize dispersion. It is the square root of the variance.

The standard deviation of the sample is given by the formula :

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

The standard deviation of the population is designated by the symbol σ.

In practice it is generally not convenient to compute $\bar{x}$ and s using the above formulae. Computations are made easier and their results improved using equivalent but different formulae (see [2]).

2.9 confidence interval : When it is possible to define two functions T_1 and T_2 of the values observed such that, when θ is a population parameter to be estimated, the probability

$$P[T_1 \leqslant \theta \leqslant T_2] = 1 - \alpha$$

where $1 - \alpha$ is a fixed number which is positive and less than 1, the interval between T_1 and T_2 is a confidence inverval for θ.

The limits T_1 and T_2 of the confidence interval are random variables which, as such, may have different values for each sample.

In a large series of samples, the frequency of the cases in which the interval will include θ will be approximately equal to $1 - \alpha$.

2.10 confidence level : The value $1 - \alpha$ of the probability associated with a confidence interval.

2.11 statistical tolerance interval : An interval for which it can be stated with a given level of confidence that it contains at least a specified proportion of the population.

When both limits are defined by statistics, the interval is two-sided. When one of the two limits is not finite or consists of the absolute boundary of the variable, the interval is one-sided.

2.12 inspection by attributes : A method which consists in taking note, for every item of a population or of a sample taken from this population, of the presence or absence of a certain qualitative characteristic (attribute) and in counting how many items have or do not have this characteristic.

The characteristics inspected by attribute are, for example, cracks or other defects which are visible on the outside, or else defects which are revealed on sawing or by a sonic test.

2.13 inspection by variables : A method which consists in measuring a quantitative characteristic for each item of a population or of a sample taken from this population.

The measurable characteristics are, for example, the results of dimensional measurements, of chemical analysis or of physical tests.

2.14 single sampling : A type of sampling which consists of taking only one sample per batch.

2.15 sequential sampling : A type of sampling which consists in taking successive items, or sometimes successive groups of items, but without fixing their number in advance, the decision to accept or reject the batch being taken, as soon as the results permit it according to rules laid down in advance.

2.16 acceptable quality level (AQL) : A quality level which, in a sampling plan, corresponds to a specified, but relatively high, probability of acceptance.

It is the maximum proportion of defective units in the batch, such that batches in which the percent defective does not exceed this values, are regarded as "good" and will very probably be accepted if a sampling plan is applied.

2.17 limiting quality (LQ) : A quality level which, in a sampling plan, corresponds to a specified and relatively low probability of acceptance (usually 10 %).

It is the proportion of defective units in the batch, such that batches in which the percent defective exceeds this value are regarded as "bad" and will very probably be rejected if a sampling plan is applied.

2.18 producer's risk : For a given sampling plan, the probability of rejecting a batch in which the proportion of defective items has a value fixed by the plan.

It is the probability α of rejecting a batch when the proportion of defective units in this batch equals the acceptable quality level AQL (or when its mean value is equal to the guaranteed value μ_G for the mean).

2.19 consumer's risk : For a given sampling plan, the probability of accepting a batch in which the proportion of defective items has a value fixed by the plan.

It is the probability β of accepting a batch when the proportion of defective units is equal to the limiting quality LQ (or when its mean value equals $\mu_G + \Delta\mu$ or $\mu_G - \Delta\mu$).

2.20 operating characteristic curve (OC) : A curve showing, for a given sampling plan, the probability of acceptance of a batch as a function of its actual quality.

3 General considerations and preliminary conditions for sampling

3.1 Subdivision of consignments into batches

Consignments which correspond to a large tonnage shall be subdivided into batches of 100 to 500 t made up in accordance with the objectives which are being aimed at. These batches shall be sampled and subjected to tests separately and they may be accepted separately.

It will also be necessary to subdivide into batches a consignment which comprises products belonging to different classes or in which the items have been obtained by different methods of manufacture.

Moreover, a consignment shall also be subdivided into batches according to sizes, masses and, if necessary, the shapes of the items, if the producer and consumer are agreed in thinking that these factors influence the characteristics investigated.

For the purpose of making up batches in terms of the masses of the items, it is often desirable to divide the items into the following three categories :

- category 1 : items up to 15 kg;
- category 2 : items ranging from 15 to 35 kg;
- category 3 : items in excess of 35 kg.

The making-up of batches from a consignment may be facilitated if the items are marked in such a way as to indicate the period during which they have been manufactured.

If a batch is declared to be non-complying it is possible to subdivide it into smaller batches by applying the criteria indicated above which might not have been taken into account when making it up, in order to ensure greater uniformity of each of the new batches made up, and these may be subjected to acceptance separately. This procedure may only be applied after a new agreement has come into operation between the producer and the consumer, and it is expedient to make sure that the new sampling plans which will be operated provide, for both parties, similar guarantees to those which would result from the first plan used.

3.2 Properties inspected

3.2.1 Specifications relating to the properties inspected

For each of the batches made up as indicated above, each of the properties inspected by attributes is characterised by a proportion of defective units in the batch, and each of the measurable properties is characterised by a mean value and by a standard deviation.

Statistical control of a production in respect of its quality shows that, over a period of time, the mean value (μ) of a property undergoes fluctuations which are due to inevitable variations in the raw materials, their preparation and the methods of casting and firing. The standard deviation (σ), on the other hand, generally varies less.

When the specifications are drawn up, a mean value μ_G is guaranteed by the producer : the producer guarantees, depending upon the nature of the property, that the mean value of each batch is either less than or equal to μ_G or else greater than or equal to μ_G.

The delivery contract must therefore specify, for each class of product :

- the properties on the basis of which acceptance or rejection of batches will be decided;
- for each of these properties, the specification which will be employed.

This specification may assume various forms. It may consist of fixing down :

- in the case of sampling by attributes, a maximum percentage of defective parts (which takes the form of the fixing of an acceptable quality level : AQL). The corresponding sampling plans are dealt with in clause 4.
- in the case of sampling by variables :
  - a guaranteed value (μ_G) for the mean. The corresponding sampling plans are dealt with in 5.3 and 5.5, or,
  - a limit value for the individual values (an upper limit T_s or a lower limit T_i, according to the properties).

    In this event, the delivery contract must also lay down an acceptable quality level (AQL). The corresponding sampling plans are dealt with in articles 5.4 and 5.6, or,
  - a downwardly limited and an upwardly limited value for the mean value or the individual values. The sampling plans corresponding to bilateral protection of this kind are not given in the present document.

A batch will comply with requirements if it really belongs to the class laid down in the order or specifications and if the values found for each of the properties investigated, following application of the sampling plans described below, result in a conformity decision.

A batch will not comply with requirements if it does not belong to the class laid down or if the values found for one or more than one of the properties investigated, following application of the sampling plans described below, result in a non-conformity decision.

3.2.2 Nature and number of the properties subjected to inspection — Efficiency of plans

The nature and number of the characteristic properties inspected depend upon the nature of the consignment, its intended use, all the risks which the producer and consumer agree to incur, and the expense which they agree to devote to sampling and testing.

In fact, the application of any sampling plan provides no certainty that the batch either is, or is not in conformity with the requirements : the probability of acceptance of a batch and its quality level are related through a function which is defined by the selected sampling plan.

This function is represented by the operating characteristic curve of the plan which, for convenience of use, is characterised by two points : one corresponds to the producer's risk α and the other to the consumer's risk β.

If the inspection deals with a single property, the sampling plans described below are such that :

- the producer's risk (α), which is associated with the acceptable quality level (AQL) fixed by the requirement (or which is associated with the guaranteed value μ_G for the mean) is always fixed, in the case of inspection by variables, at a value which is equal to or very close to 5 %. In the case of inspection by attributes, this risk is variable (see table 3).
- the consumer's risk (β) is associated with a batch quality level which depends directly upon the sampling plan selected for inspection. The values for this quality level (limiting quality) are given for a constant risk β = 10 % in tables 3, 9 and 10 and can be found, for different values of β, from the graphs giving the operating characteristic curves of the corresponding sampling plans (see figures 4, 5, 6 and 7).

It may be noted that the percentages of defective units associated with the consumer's risk in the various sampling plans are generally high as compared with the producer's risk. This is the result of economic considerations which induce both parties to cut the size of the inspected samples.

However when inspection is carried out on a number of quality characteristics the resulting risk increases for the producer and decreases for the consumer if it is assumed that a product subjected to inspection must meet all the requirements put on the individual properties tested to receive final acceptance. In this case, and assuming in addition that the quality characteristics involved in the various acceptance procedures are independent, table 1 gives the resulting values of α and β as a function of the number j of quality characteristics subjected to inspection.

In actual fact, the overall consumer's risk indicated in this table do not permit a full assessment of the severity of inspection. This severity is much better represented by the quality level (or percentage defective) LQ associated with a constant value of β.

This is shown in the example given in figure 1[1] (the points B_1, B_2, B_3 all correspond to β = 10 %).

In each case, only a knowledge of the complete operating characteristic curve of the plan makes it possible to know the value of one quality level corresponding to a given risk and to the number of properties inspected.

Furthermore, the properties of refractory products are not all independent of one another, and the values indicated in table 1 are therefore maximum values (for the producer) or minimum values (for the consumer).

They nevertheless lead to limitation of the number of properties inspected : the number of properties inspected by destructive tests (excluding chemical analysis) should not be greater than three.[2]

Furthermore, with the exception of methods dealt with in 5.3, the methods of sampling by variable are based on the theoretical hypothesis that the property measured is distributed in the batch in accordance with the normal law. In practice, the properties investigated will very rarely be distributed precisely in accordance with this law, but the efficiency of the methods of investigation is changed very little in cases where the distributions deviate only slightly from normal.

In cases of doubt, however, it is expedient to verify, with the aid of a statistical test (for example [2]), that the distribution of the property considered in the investigation may be regarded as normal.

Table 1 — Change in risks when the number of independant properties inspected increases

Number of properties j	Total producer's risk when the producer's risk α rises to 5 % for each property %	Total consumer's risk when the consumer's risk β rises to 10 % for each property
1	α_1 = 5,00	β_1 = 10 %
2	α_2 = 9,75	β_2 = 1 % = 10^{-2}
3	α_3 = 14,26	β_3 = 10^{-3}
4	α_4 = 18,55	β_4 = 10^{-4}
5	α_5 = 22,62	β_5 = 10^{-5}
6	α_6 = 26,49	β_6 = 10^{-6}
7	α_7 = 30,17	β_7 = 10^{-7}

1) The operating characteristic curves shown in figure 1 apply in case where, for each property, the same sampling plan used, as defined by :

- a known standard deviation (σ);
- a one sided tolerance limit for individual values;
- an AQL of 4 %;
- a sample size of 10.

These curves are graphic representations of the following function :

$$P = \Phi_j\,[(U_{1-p} - K)\sqrt{n}]$$

where

P is the probability of accepting the batch on the basis of all quality characteristics inspected;

Φ is the distribution function of the standardized normal distribution;

U_1 is the standardized deviation corresponding to a probability p;

p is the proportion of defective items in the batch subjected to inspection;

K is the constant defined by the sampling plan used;

j is the number of properties subjected to inspection.

2) It will be possible to use the items which have supplied the highest or lowest values when subjected to these three investigations, for determining certain other characteristics for information purposes.

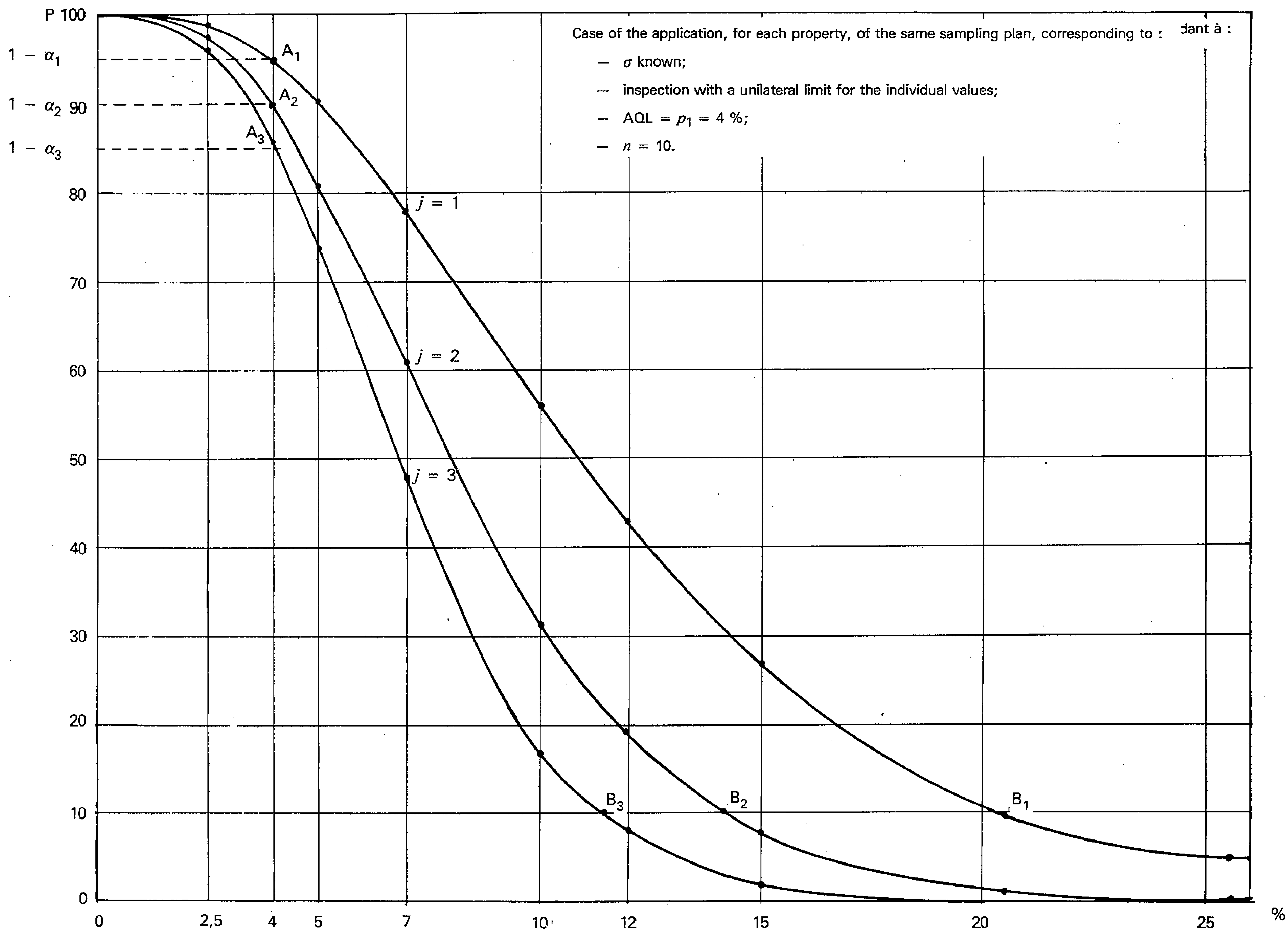

Figure 1 — Example of operating characteristic curves when several properties are investigated simultaneously

3.3 Sampling process and use of the sampling plans described in clauses 4 and 5

Every sampling plan shall be prepared, and its execution supervised, by experts who are as well acquainted with the problems of the production and use of the products as with the problems of sampling.

The taking of samples shall be carried out in such a way that all the items in the batch have the same probability of being selected and tested.

The efficiency of a sampling plan depends solely on the number of sampled units, n, whatever the size, N, of the batch, provided that n/N is less than 10 %. Tables 3, 4, 6, 9 and 10 (or the operating characteristic curves also given in this International Standard) shall be used to determine, on the basis of the required efficiency, what the sample size should be.

If experience shows that the quality of the manufacturer's production corresponds to the agreements, it is possible, when batches of the same quality are frequently subjected to acceptance procedures, either to choose a plan with lower efficiency which implies the use of a smaller sample size or to reduce the number of batches inspected while retaining a plan having the same efficiency. The same applies when statistical quality control charts are available (see 3.5).

Similarly, if it is desired to reduce the proportion of defective units associated with a given consumer's risk value, it is necessary to choose a sampling plan with higher efficiency which implies the use of a larger sample size.

The sample size, n, indicated in the tables corresponds to that number of results relating to one of the properties inspected, which must be available in order to decide whether the batch is in conformity as regards the said property.

Each method of test must define what constitutes a result. Thus according to the nature of the test, a result may be constituted either by the value obtained by applying the method of test on a single occasion, or by a value which is deduced from the values obtained by repeating the test one or more times under the conditions prescribed by the method. Each of the n "results" must be obtained from a different item.

It is therefore necessary to calculate, from the sample size corresponding to each of the properties which will be inspected, the number of items which will have to be selected while taking into account :

- the number of properties which will be inspected;
- the specifications of each of the methods of test which will be used;
- the fact that each of the items selected may, or may not, be used for investigating a number of properties;
- the possibility of problems during the handling of the items selected or during the tests;
- the way in which it is intended to settle any disagreement between the producer and the consumer : in this connection, it is recommended that the number of items selected should permit the making-up of a reserve sample for use in the event of arbitration.

3.4 Treatment of the items selected

The distribution of the items selected, their possible apportionment between the various parties concerned (producer, consumer, arbitrator) and the constitution of a reserve sample shall be indicated in the terms of the transaction as well as, if necessary, the method of sub-dividing the test pieces.

3.5 Use of statistical control charts

The tests carried out by the consumer may be considerably reduced if the manufacturer regularly plots statistical control charts of the quality of his production, and places these charts at the disposal of his customers.

It is therefore expedient that the pattern of the control chart used should be selected in such a way that it can be used with equal success for production and for the inspection of a consignment : a selection of works on this subject is given in annex D.

Control charts can be used for controlling the mean value, the standard deviation, the tolerance or the percentage of defective units.

A further advantage of regular use of control chart is that, in certain cases it supplies a good estimate of the standard deviation of the quality characteristic.

4 Sampling for non-destructive tests

4.1 Inspection of the external appearance

The specification precisely defines what should be regarded as a defective item after examination of its external appearance.

It therefore specifically states the defects, such as cracks, blemishes, deformations, firing defects, etc., which will be taken into consideration.

The acceptable proportion of defective items (AQL) is also fixed by agreement between the two parties. This proportion may often be fixed at 4 % in the case of ordinary bricks and mass-produced items, and at 1,5 % for items having complicated shapes.

The external appearance is inspected by attributes.

The sampling plans to be taken into consideration, which are defined by the sample size, n, and the acceptance number, c, may be taken from ISO 2859 [40]. Table 3 gives a selection of sampling plans for AQL's of the order of 6,5, 4,0 or 1,5. This table also gives, in column 4, the probability of acceptance P for different proportions p of defective units in the batch.

The number y of defective pieces in the sample having a size n is determined.

If $y \leqslant c$, the batch is in conformity;

If $y > c$, the batch is not in conformity.

Example :

A consignment having a total mass of 200 t comprises 20 000 pressed items, having a unit mass of 10 kg, which are divided into three formats :

- format 1 : 12 000 items
- format 2 : 500 items
- format 3 : 7 500 items

As indicated in 3.1, the consignment is subdivided into three batches corresponding to the three formats, for the purpose, in this case, of inspecting the external appearance (cracks).

The process is described in table 2.

In the case of batch 2, for example, the sampling plan used provides the following guarantees (see table 3 — AQL : 1,5 % — line 3) :

- for the producer, the risk of having a batch comprising 1,66 % of defective pieces erroneously declared not to be in conformity, is equal to 5 %;

- for the consumer, the risk of having a batch which contains 10,3 % of defective pieces erroneously declared to be in conformity, is equal to 10 %.

Table 2 — Inspection process

Batch	1	2	3
Batch size, N	12 000	500	7 500
Values drawn from table 3 for AQL = 1,5 %			
Sample size, n	315	50	200
Acceptance number, c	10	2	7
Number y of defective pieces found	8	2	8
Decision	In conformity	In conformity	Not in conformity

4.2 Inspection of dimensions

The dimensions may be inspected by attributes (i.e., in accordance with 4.1) or by variables (see [18]); the methods involving inspection by variables described in clause 5 may not be used for inspection of dimensions, because in this case a lower limit and an upper limit are generally prescribed. The sampling plans required for inspection by variables are not given in the present document but they may be published later in a second edition of this International Standard.

The single sampling plans necessary for inspection by attributes may be taken from ISO 2859 [40] or from table 3.

Table 3 — Single sampling plans for sampling by attributes in normal inspection

(1)	(2)	(3)	(4)	(5) Probability of acceptance, P						
AQL %	N	n	c	0,99	0,95	0,90	0,50	0,10	0,05	0,01
				p % defective units in the lot						
1,5	2 to 90	N or 8	0	0,13	0,64	1,3	8,30	25,0	31,2	43,8
	91 to 280	32	1	0,48	1,13	1,67	5,19	11,6	14,0	19,0
	281 to 500	50	2	0,89	1,66	2,23	5,31	10,3	12,1	15,9
	501 to 1 200	80	3	1,05	1,73	2,20	4,57	8,16	9,39	12,0
	1 201 to 3 200	125	5	1,43	2,09	2,52	4,54	7,42	8,41	10,5
	3 201 to 10 000	200	7	1,45	1,99	2,33	3,84	5,89	6,57	8,60
	10 001 to 35 000	315	10	1,51	1,96	2,23	3,39	4,89	5,38	6,40
	35 001 to 150 000	500	14	1,50	1,85	2,06	2,93	4,03	4,38	5,09
	Over 150 000	800	21	1,57	1,86	2,03	2,71	3,52	3,78	4,29
4,0	2 to 25	N or 3	0	0,33	1,70	3,45	20,6	53,6	63,2	75,4
	26 to 90	13	1	1,19	2,81	4,16	12,6	26,8	31,6	41,5
	91 to 150	20	2	2,25	4,22	5,64	13,1	24,5	28,3	35,6
	151 to 280	32	3	2,63	4,39	5,56	11,4	19,7	22,5	28,0
	281 to 500	50	5	3,66	5,34	6,42	11,3	17,8	19,9	24,3
	501 to 1 200	80	7	3,72	5,06	5,91	9,55	14,2	15,8	18,9
	1 201 to 3 200	125	10	3,82	4,94	5,62	8,53	12,3	13,6	16,1
	3 201 to 10 000	200	14	3,74	4,62	5,15	7,33	10,1	10,9	12,7
	Over 10 000	315	21	3,99	4,73	5,16	6,88	8,95	9,60	10,9
6,5	2 to 15	2	0	0,50	2,53	5,13	29,3	68,4	77,6	90,0
	16 to 50	8	1	2,00	2,64	6,88	20,1	40,6	47,1	58,9
	51 to 90	13	2	3,63	6,63	8,80	20,0	36,0	41,0	50,6
	91 to 150	20	3	4,31	7,13	9,03	18,1	30,4	34,4	42,0
	151 to 280	32	5	5,94	8,50	10,20	17,5	27,1	30,1	35,9
	281 to 500	50	7	6,06	8,20	9,53	15,2	22,4	24,7	·
	501 to 1 200	80	10	6,13	7,91	8,95	13,3	18,6	20,3	23,6
	1 201 to 3 200	125	14	5,98	7,40	8,24	11,7	16,1	17,5	20,4
	Over 3 200	200	21	6,29	7,45	8,12	10,8	14,1	15,1	17,2

Extracted from [40] for "Inspection Level II"; the sampling plans coincide with those in [17], [10] and [34].

The acceptable proportion of defective parts shall be fixed by agreement between the interested parties. It may often be possible to fix the proportion at 6,5 %.

5 Sampling for destructive tests

5.1 Introduction

The properties which are revealed by destructive tests are inspected by variables.

The statistical methods of inspection by variables described in 5.4, 5.5 and 5.6 presuppose that the property measured is distributed in the batch according to a law which is close to the normal law (see 3.2.2, last paragraph).

Sub-clauses 5.3 and 5.5 apply to the case in which the delivery contract has specified a guaranteed value (μ_G) for the mean value of the property measured.

Sub-clauses 5.4 and 5.6 apply to the case when one limit (T_s or T_i) has been set to individual values : an item is regarded as satisfactory with reference to the inspected property if the value for this item is less than T_s (or greater than T_i); if not it is regarded as defective with reference to the property measured.

The methods described in 5.3 and 5.4 may be used when the parties concerned agreed upon the assumption that the standard deviation σ of the measured property is known. This standard deviation must be estimated from larger samples than those envisaged in this clause (see annex A). The constancy of the standard deviation shall be checked at regular intervals, by means of a statistical test (see, for example, [2]).

5.2 Test sharing

By agreement between the interested parties the n units of the sample may be shared between the producer and the consumer (or, if applicable, a neutral agency) provided that it has first been verified that the laboratories do not show any significant difference in their test results (see annex B). The results will then be combined for statistical treatment; if the two interested parties so agree, then the producer may, in the case of the results which he is responsible for supplying, refer to the values in the control chart.

Agreement between the results obtained by the laboratories shall be regularly verified with the aid of statistical tests such as, for example, the t test for comparing mean values and the F test for comparing standard deviations (see [2]). If this verification reveals significant differences between the test results of the laboratories, an attempt will be made to find the causes of these differences. Until these differences have been eliminated, the test results cannot be combined for the purpose of statistical treatment.

If there are differences between the results obtained by the producer and the consumer, the results obtained by an arbitrating laboratory will decide.

5.3 Sampling plans in the case of a guaranteed value for the mean value and a known standard deviation

5.3.1 Field of application

The sampling plans given in this sub-clause shall be used when the producer and the consumer have reached an agreement on a guaranteed value for the mean and when it may be accepted that the standard deviation σ of the property is known.

5.3.2 Single sampling plans

5.3.2.1 Characteristic parameters

A single sampling plan is characterised by the sample size n and the acceptance factor K_{PRE}; these parameters will be taken from table 4, columns 1 and 2.

5.3.2.2 Treatment of the sample and decision on the batch

The tests yield n individual values, of which the mean $\bar{x}$ is calculated.

Rule governing decision, if the high values are unfavourable :

- calculate $\mu_G + K_{PRE}\,\sigma$;
- if $\bar{x} \leqslant \mu_G + K_{PRE}\,\sigma$, the batch is in conformity;
- if $\bar{x} > \mu_G + K_{PRE}\,\sigma$, the batch is not in conformity.

Rule governing decision, if the low values are unfavourable :

- calculate $\mu_G - K_{PRE}\,\sigma$;
- if $\bar{x} \geqslant \mu_G - K_{PRE}\,\sigma$, the batch is in conformity;
- if $\bar{x} < \mu_G - K_{PRE}\,\sigma$, the batch is not in conformity.

5.3.2.3 Producer's and consumer's risks

The values for K_{PRE} given in table 4 are based on a producer's risk $\alpha = 5$ % that a batch with true population mean μ equal to the guaranteed value μ_G will be rejected by chance.

The consumer's risk β is the probability that a batch with true population mean μ differing by $\Delta\mu$ from the guaranteed value μ_G will be accepted. The value of $\Delta\mu$ which corresponds to a risk $\beta = 10$ % is obtained by multiplying by σ the value of $\left(\frac{\Delta\mu}{\sigma}\right)_{\beta = 10\,\%}$ taken from column 3 of table 4.

$$\mu_{\beta = 10\,\%} = \mu_G \pm \left(\frac{\Delta\mu}{\sigma}\right)_{\beta = 10\,\%} \times \sigma$$

The + sign being used if high values of the measured characteristic are undesirable.

The operating curves of the sampling plans dealt with in table 4 are given in figure 2.

5.3.2.4 Example

Suppose the consignment mentioned in example 4.1 is to be inspected with respect to cold crushing strength. Let μ_G = 230 kgF/cm^2 be the guaranteed mean value; the standard deviation is known and is σ = 70 kgF/cm^2.

All the items in the consignment belong to mass class 1, with the result that the entire consignment constitutes one batch from the point of view of destructive tests (see 3.1).

In view of the given data, the sampling plans in 5.3 must be used. It is desired to use a single sampling plan (5.3.2). The sample size n = 14 and acceptance factor K_{PRE} = 0,44 are obtained from table 4.

The mean of the results of the 14 tests is $\bar{x}$ = 190 kgF/cm^2.

The following calculation is carried out : $\mu_G - K_{PRE}\,\sigma = 230 - 0{,}44 \times 70 = 199$.

Since $\bar{x} < \mu_G - K_{PRE}\,\sigma$ (190 < 199) a decision of non-conformity is arrived at.

The sampling plan used provides the following guarantees :

— for the producer, the risk of having a batch, the true mean of which would be 230 kgF/cm^2, declared not to be in conformity, is equal to 5 %;

— the consumer runs a risk β = 10 % of having a batch, the true mean of which would be

$$\mu_G - \left(\frac{\Delta\mu}{\sigma}\right) \times \sigma = 230 - 0{,}78 \times 70 = 175{,}4 \text{ kgF/cm}^2,$$

erroneously declared to be in conformity.

5.3.3 Sequential sampling plans

5.3.3.1 General remarks

Where as the sample size n is fixed before the start of sampling, in the case of single sampling plans, the number of items to be taken and subjected to tests depends, in sequential sampling plans, upon the results of the successive tests. After each test, three decisions are possible :

– declare conformity;

– declare non-conformity;

– continue the tests.

The number of tests to be carried out for deciding upon conformity or non-conformity is therefore dependent upon the results already obtained but is in general lower than the number of tests imposed by a single sampling plan having the same efficiency. Sequential sampling plans are therefore generally advantageous in the case of costly tests, the results of which are rapidly obtained.

Either the items will be taken from the batch one after another until a decision on the fate of the batch intervenes, or else the number of items corresponding to the value n_{max} (table 6, column 8) will be taken in one operation from the batch and these items will be successively subjected to the tests in an order which is random but is determined at the moment at which the sample is taken.

5.3.3.2 Characteristic parameters

A sequential sampling plan is characterised by the parameters b, a and r which are taken from table 6, columns 1 to 3.

Table 4 — Single sampling plans in the case of a guaranteed value for the mean[1)]

Standard deviation σ known — Sample size, n	K_{PRE} for α = 5 %	$\left(\frac{\Delta\mu}{\sigma}\right)$ β = 10 %	Standard deviation σ unknown — Sample size, n	Total mass of batch[2)] t
1	2	3	4	5
4	0,82	1,46	6	1
6	0,67	1,20	8	10
10	0,52	0,93	12	100
14	0,44	0,78	16	200
18	0,39	0,69	20	300
22	0,35	0,62	24	400
26	0,32	0,58	28	500

1) The table is suitable for unit masses limited to 35 kg. In the case of unit masses greater than 35 kg, the sample size is to be agreed upon by the contracting parties.

2) This column is given only as a guide.

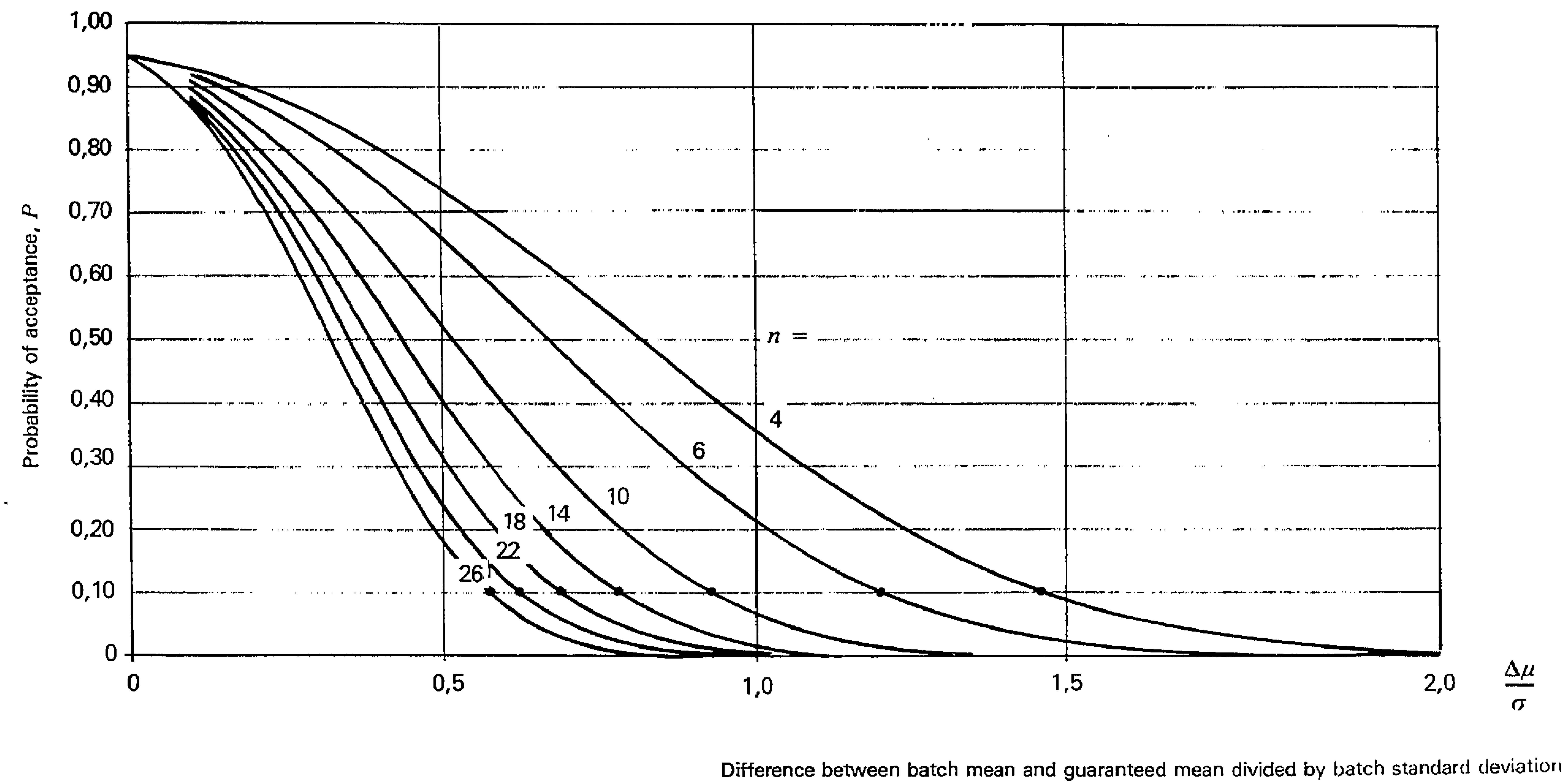

Figure 2 — Operating characteristic curves for single sampling plans defined in table 4 : testing for a guaranteed value of the population mean when the standard deviation is known[1)]

1) These curves are approximately valid for the single sampling plans in table 4 when the standard deviation is not known for the sequential plans of table 5 for the same total batch weight.

5.3.3.3 Execution of tests and decision on batch

Let n be the number of tests already carried out. After each additional test, the following summation is calculated from the n results x_i already obtained :

$$S_n = \sum_{i=1}^{n} (x_i - b)$$

The rules governing the decision are summarized in the following synoptic table :

The sampling plan may be interrupted, for $n = n_{max}$ (table 6, column 8), by applying the decisions indicated in the lower half of table 5.

Table 5 — Conformity decisions

	High values unfavourable	Low values unfavourable
Declare the batch to be in conformity if :	$S_n \leqslant a$	$S_n \geqslant a$
Declare the batch to be not in conformity if :	$S_n \geqslant r$	$S_n \leqslant r$
Continue the tests if :	$a < S_n < r$	$r < S_n < a$
Stop the tests for $n = n_{max}$ and declare the batch to be in conformity if :	$S_{n\,max} \leqslant 0$	$S_{n\,max} \geqslant 0$
Stop the tests for $n = n_{max}$ and declare the batch to be not in conformity if :	$S_{n\,max} > 0$	$S_{n\,max} < 0$

5.3.3.4 Producer's and consumer's risks

The values given in table 6 are based on a producer risk $\alpha = 5$ %; the probability of a batch, the mean μ of which is equal to the guaranteed value μ_G, being erroneously declared not to be in conformity, is therefore 5 %.

The consumer's risk β corresponds to the probability of a batch, the effective mean of which would differ by $\Delta\mu$ from the guaranteed value μ_G, being accidentally declared to be in conformity : the value of $\Delta\mu$ which corresponds to a risk $\beta = 10$ % is obtained by multiplying by σ the value $\left(\frac{\Delta\mu}{\sigma}\right)_{\beta = 10\,\%}$, obtained from column 4 in table 6,

i.e. :

$$\mu_{\beta = 10\,\%} = \mu_G \pm \left(\frac{\Delta\mu}{\sigma}\right)_{\beta = 10\,\%} \times \sigma$$

The efficiency curves of the sequential sampling plans given in this sub-clause are practically the same as those of the corresponding plans in 5.3.2 (see figure 2).

5.3.3.5 Mean sample size

The mean sample size may be found in columns 5 to 7 of table 6; it is the number of items which must be tested before the batch is declared to be in conformity or not in non-conformity; this number depends upon the true value of the mean μ :

— column 5 relates to batches, the mean μ of which is equal to the guaranteed mean μ_G;

— column 6 relates to batches, the mean value μ of which deviates by $\Delta\mu$ from the guaranteed mean μ_G (these batches therefore have only a 10 % probability of being accepted);

— column 7 relates to batches, the mean value of which deviates by $\Delta\mu/2$ from the guaranteed mean μ_G (the mean sample size reached here has almost its maximum value).

5.3.3.6 Examples

5.3.3.6.1 Example of a test for refractoriness under load

A batch of 200 t has to be inspected for acceptance from the

Table 6 — Sequential sampling plans in the case of a guaranteed value limit (μ_G) for the mean and a known standard deviation[1)]

b	a	r	$\left(\frac{\Delta\mu}{\sigma}\right)_{\beta = 10\,\%}$	Mean sample size $\bar{n}(\mu_G)$	Mean sample size $\bar{n}(\mu_G \pm \Delta\mu)$	Mean sample size $\bar{n}\left(\mu_G \pm \frac{\Delta\mu}{2}\right)$	n_{max}	Total weight of batch[2)] t
1	2	3	4	5	6	7	8	9
$\mu_G \pm 0{,}730\,\sigma$	$\mp 1{,}54\,\sigma$	$\pm 1{,}98\,\sigma$	1,46	1,9	2,2	3,1	6	1
$\mu_G \pm 0{,}600\,\sigma$	$\mp 1{,}88\,\sigma$	$\pm 2{,}41\,\sigma$	1,20	2,8	3,3	4,5	8	10
$\mu_G \pm 0{,}465\,\sigma$	$\mp 2{,}42\,\sigma$	$\pm 3{,}11\,\sigma$	0,93	4,6	5,5	7,5	13	100
$\mu_G \pm 0{,}390\,\sigma$	$\mp 2{,}89\,\sigma$	$\pm 3{,}71\,\sigma$	0,78	6,6	7,8	10,7	18	200
$\mu_G \pm 0{,}345\,\sigma$	$\mp 3{,}26\,\sigma$	$\pm 4{,}19\,\sigma$	0,69	8,4	10,0	13,7	23	300
$\mu_G \pm 0{,}310\,\sigma$	$\mp 3{,}63\,\sigma$	$\pm 4{,}66\,\sigma$	0,62	10,4	12,4	16,9	29	400
$\mu_G \pm 0{,}290\,\sigma$	$\mp 3{,}88\,\sigma$	$\pm 4{,}98\,\sigma$	0,58	11,9	14,1	19,3	33	500

1) If the high values are unfavourable, the upper signs in columns 1 to 3 must be adopted. If the low values are unfavourable, the lower signs must be adopted.

2) This column is only given as a guide.

point of view of "refractoriness under load"; a guaranteed value for the mean μ_G = 1 670 °C has been agreed upon; the standard deviation is known and its value is σ = 15 °C.

From the given data the sampling plans in chapter 5.3 must be used.

It is desired to make use of a sequential sampling plan (see 5.3.3) and the following values are therefore calculated, using table 6 :

$$b = \mu_G - 0{,}390\,\sigma = 1\,670 - (0{,}390 \times 15) = 1\,664$$

$$a = 2{,}89\,\sigma = 2{,}89 \times 15 = 43{,}4$$

$$r = -3{,}71\,\sigma = -3{,}71 \times 15 = -55{,}6$$

Furthermore, it is found that, according to table 6, colulmn 8, the sequential plan may be interrupted after the testing of n_{max} = 18 items.

The course of development of the sequential plan is indicated in table 7.

Table 7 — Development of sequential sampling plan

i	x_i	$x_i = b$ (b = 1 664)	S_n	Decision (a = 43,4; r = − 55,6)
1	1 670	6	6	Continue since $s < S_n < a$
2	1 680	16	22	
3	1 660	− 4	18	
4	1 670	6	24	
5	1 670	6	30	
6	1 660	− 4	26	
7	1 680	16	42	
8	1 660	− 4	38	
9	1 680	16	54	Declare to be in conformity since $S_n > a$

After testing n = 9 items, the decision to declare the batch to be in conformity is obtained.

The process which results in the decision may also be illustrated by the diagram in figure 3.

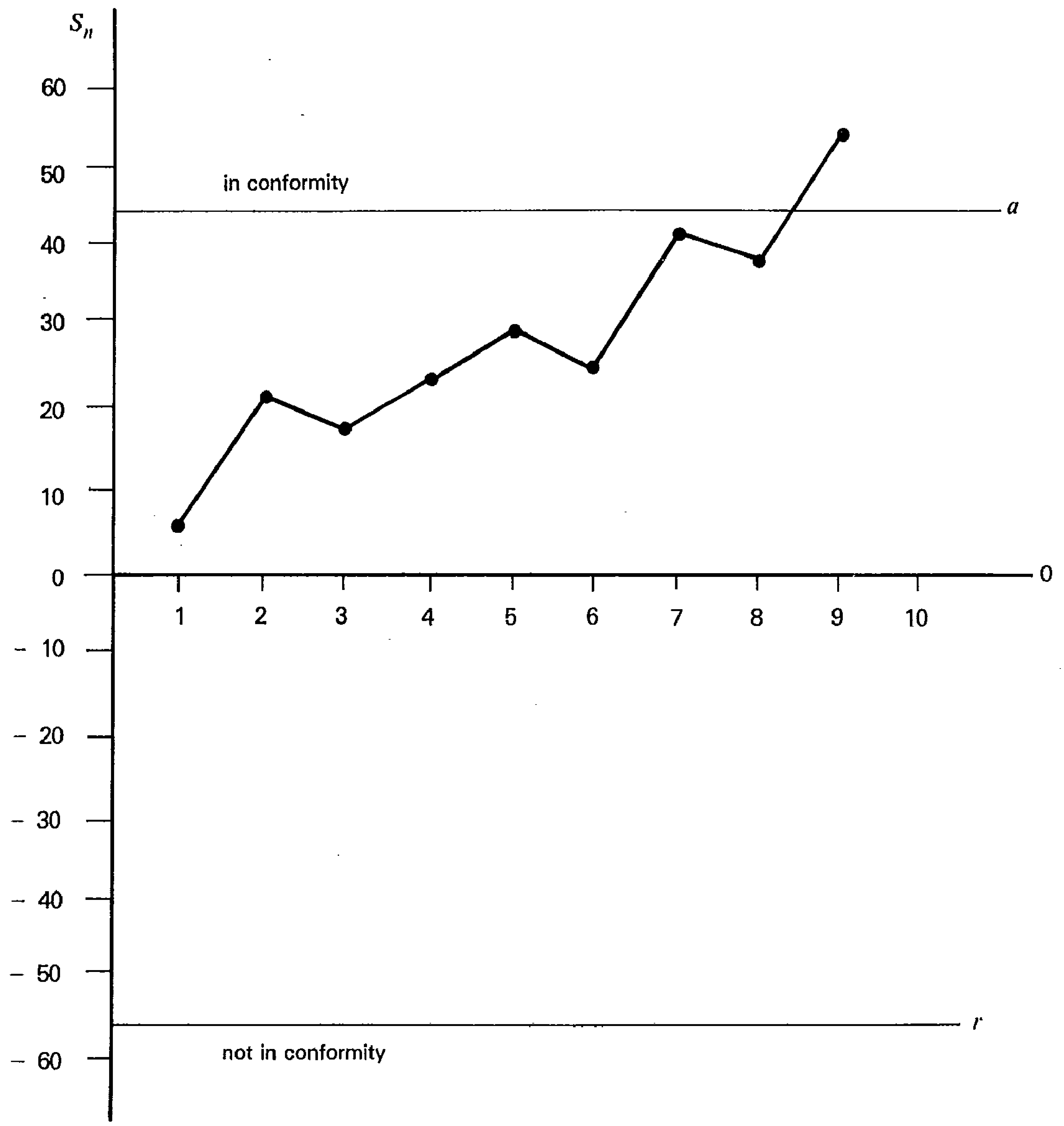

Figure 3 — Procedure for decision on conformity

5.3.3.6.2 Example of a test for thermal expansion at 1 400 °C

A 200 t batch is to be inspected for acceptance in respect of "thermal expansion at 1 400 °C"; a guaranteed mean value μ_G = 1,30 % has been agreed upon; the standard deviation is known and has a value σ = 0,05 %.

From the given data it is necessary to use the sampling plans in 5.3.

It is desired to use a sequential sampling plan (see 5.3.3) and the following values are therefore calculated, using table 6 :

$$b = \mu_G + 0{,}390\,\sigma = 1{,}30 + (0{,}390 \times 0{,}05) = 1{,}32$$

$$a = -2{,}89\,\sigma = -2{,}89 \times 0{,}05 = -0{,}145$$

$$r = 3{,}71\,\sigma = 3{,}71 \times 0{,}05 = 0{,}186$$

Furthermore, it is found that, according to table 6, column 8, the sequential plan may be interrupted after the testing of n_{max} = 18 items.

The course of development of the sequential plan is illustrated in table 8.

Table 8 — Development of sequential sampling plan

i	x_i	$x_i - b$ (b = 1,32)	S_n	Decision (a = − 0,145; r = 0,186)
1	1,29	− 0,03	− 0,03	Continue since $a < S_n < r$
2	1,30	− 0,02	− 0,05	
3	1,34	+ 0,02	− 0,03	
4	1,28	− 0,04	− 0,07	
5	1,29	− 0,03	− 0,10	
6	1,32	0	− 0,10	
7	1,31	− 0,01	− 0,11	
8	1,28	− 0,04	− 0,15	Declare to be in conformity since $S_n < a$

After testing n = 8 items the decision to declare the batch to be in conformity is obtained.

5.4 Single sampling plans with a fixed unilateral limit for individual values and a known standard deviation

5.4.1 Field of application

The single sampling plans given in this sub-clause shall be used when the producer and the consumer have agreed on a limit (an upper limit T_s or lower limit T_i), depending on the property tested, on individual values and when it can be said that the standard deviation σ is known.

5.4.2 Characteristic parameters

A single sampling plan is characterised by the sample size n and the acceptance factor K; these parameters shall be taken from table 9 according to the AQL agreed upon.

5.4.3 Treatment of the sample and decision on the batch

The tests yield n individual values. The arithmetic mean $\bar{x}$ of these values is calculated first, and then the quality index of the sample :

$$Q = \frac{T_s - \bar{x}}{\sigma}$$

or

$$Q = \frac{\bar{x} - T_i}{\sigma}$$

Rules governing the decision :

— If $Q > K$, the batch is declared to be in conformity;

— If $Q < K$, the batch is declared to be not in conformity.

Table 9 — Single sampling plans with a fixed unilateral limit for the individual values and a known standard deviation[1)]

Sample size n	AQL = 1,5 %		AQL = 2,5 %		AQL = 4,0 %		AQL = 6,5 %		Total mass of batch[2)] t
	K	LQ %	K	LQ %	K	LQ %	K	LQ %	
1	2	3	4	5	6	7	8	9	10
4	1,35	23,9	1,14	30,9	0,93	38,6	0,69	48,0	1
6	1,50	16,4	1,29	22,2	1,08	28,9	0,84	37,6	10
10	1,65	10,7	1,44	15,0	1,23	20,5	0,99	27,9	100
14	1,73	8,2	1,52	11,9	1,31	16,6	1,07	23,4	200
18	1,78	6,9	1,57	10,2	1,36	14,5	1,13	20,4	300
22	1,82	6,1	1,61	9,0	1,40	12,9	1,16	18,7	400
26	1,85	5,5	1,64	8,2	1,43	11,9	1,19	17,4	500

1) This table is suitable for unit masses which are limited to 35 kg. In the case of unit masses in excess of 35 kg, the sample size is to be agreed upon by the contracting parties.

2) This column is only given as a guide.

5.4.4 Producer's and consumer's risks

The values for K given in table 9 are based on a producer's risk of $\alpha = 5$ %; the probability of a batch, in which the proportion of defective items equals AQL, being accidentally declared not to be in conformity, is therefore 5 %.

For each AQL, table 9 contains a column giving the values of LQ for a producer's risk $\beta = 10$ %; the probability of acceptance of a batch in which the proportion of defective items is equal to the LQ value given in the table, is therefore 10 %.

The operating characteristic curves of the single plans dealt with in table 9 are given in figures 4 (AQL = 1,5 %), 5 (AQL = 2,5 %), 6 (AQL = 4 %) and 7 (AQL = 6,5 %).

5.4.5 Example

A 200 t batch is to be inspected for acceptance with respect to "apparent density".

It has been agreed to set a limit $T_i = 2{,}98$ g/cm^3 on the individual values and to choose an AQL = 4 %.

The standard deviation is known and is equal to 0,04 g/cm^3.

On the basis of these, it is necessary to use the sampling plans of section 5.4.

The sample size $n = 14$ and the acceptance factor $K = 1{,}31$ are taken from table 9.

A mean value $\bar{x} = 3{,}04$ g/cm^3 results from the tests. Hence the quality index :

$$Q = \frac{\bar{x} - T_i}{\sigma} = \frac{3{,}04 - 2{,}98}{0{,}04} = 1{,}5$$

Since $Q = 1{,}5 > K = 1{,}31$, it is decided that the batch is in conformity.

The sampling plan used gives the following guarantees :

- for the producer, the risk of having a batch in which 4,0 % of the items have an apparent density lower than 2,98 g/cm^3, being erroneously declared not to be in conformity, is equal to 5 %.
- the consumer runs a risk $\beta = 10$ % of having a batch in which 16,6 % of the items have an apparent density lower than 2,98 g/cm^3, erroneously declared to be in conformity.

5.5 Single sampling plans in the case of a guaranteed value for the mean and an unknown standard deviation[1]

5.5.1 Field of application

The single sampling plans given in this sub-clause shall be used when the producer and consumer have reached agreement on a guaranteed value for the mean and when it cannot be accepted that the standard deviation of the property is known, but when it may be estimated by the standard deviation s of the sample.

5.5.2 Characteristic parameters

A single sampling plan of this kind is characterised by the sample size n and the acceptance factor K_{PRE}; these parameters are given in table 4 in columns 4 and 2 respectively.

5.5.3 Treatment of the sample and decision on the batch

The tests yield n individual values, of which the mean $\bar{x}$ and the standard deviation s are calculated.

Rule governing decision if the high values are unfavourable :

- calculate $\mu_G + K_{PRE}\, s$;
- if $\bar{x} < \mu_G + K_{PRE}\, s$, the batch is in conformity;
- if $\bar{x} > \mu_G + K_{PRE}\, s$, the batch is not in conformity.

Rule governing decision if the low values are unfavourable :

- calculate $\mu_G - K_{PRE}\, s$;
- if $\bar{x} > \mu_G - K_{PRE}\, s$, the batch is in conformity;
- if $\bar{x} < \mu_G - K_{PRE}\, s$, the batch is not in conformity.

---

1) Theoretically, the probability of acceptance depends solely on the ratio $\Delta\mu/\sigma$ in which $\Delta\mu$ is the difference between the true mean μ of the batch and the guaranteed mean μ_G and σ is the true standard deviation of the particular batch which is subjected to sampling. In other words, two batches 1 and 2 having different standard deviations σ_1 and σ_2 and differing from μ_G by $\Delta\mu_1$ and $\Delta\mu_2$ respectively but such that :

$$\frac{\Delta\mu_1}{\sigma_1} = \frac{\Delta\mu_2}{\sigma_2}$$

will have exactly the same probability of being accepted. This is the reason why the operating characteristic curves show the probability of acceptance as a function of $(\Delta\mu/\sigma)$. **Consequently the value of $\Delta\mu$ for which the probability of acceptance is β can be calculated only if σ is known.**

The use of the sample standard deviation s in place of the true standard deviation σ will yield **approximate** results which may be interpreted by means of table 11 (column 2) showing the relationship of s to σ at the confidence level of 95 %.

The operating characteristic curves of figure 2 illustrated in section 5.3 are approximately valid for the corresponding sampling plans of this sub-clause when n is replaced by $n + 2$.

It must be remembered, however, when using these curves that, in the ratio $\Delta\mu/\sigma$, σ is the true standard deviation for the batch being inspected, which, by definition, is **unknown** (and only estimated) in the present context.

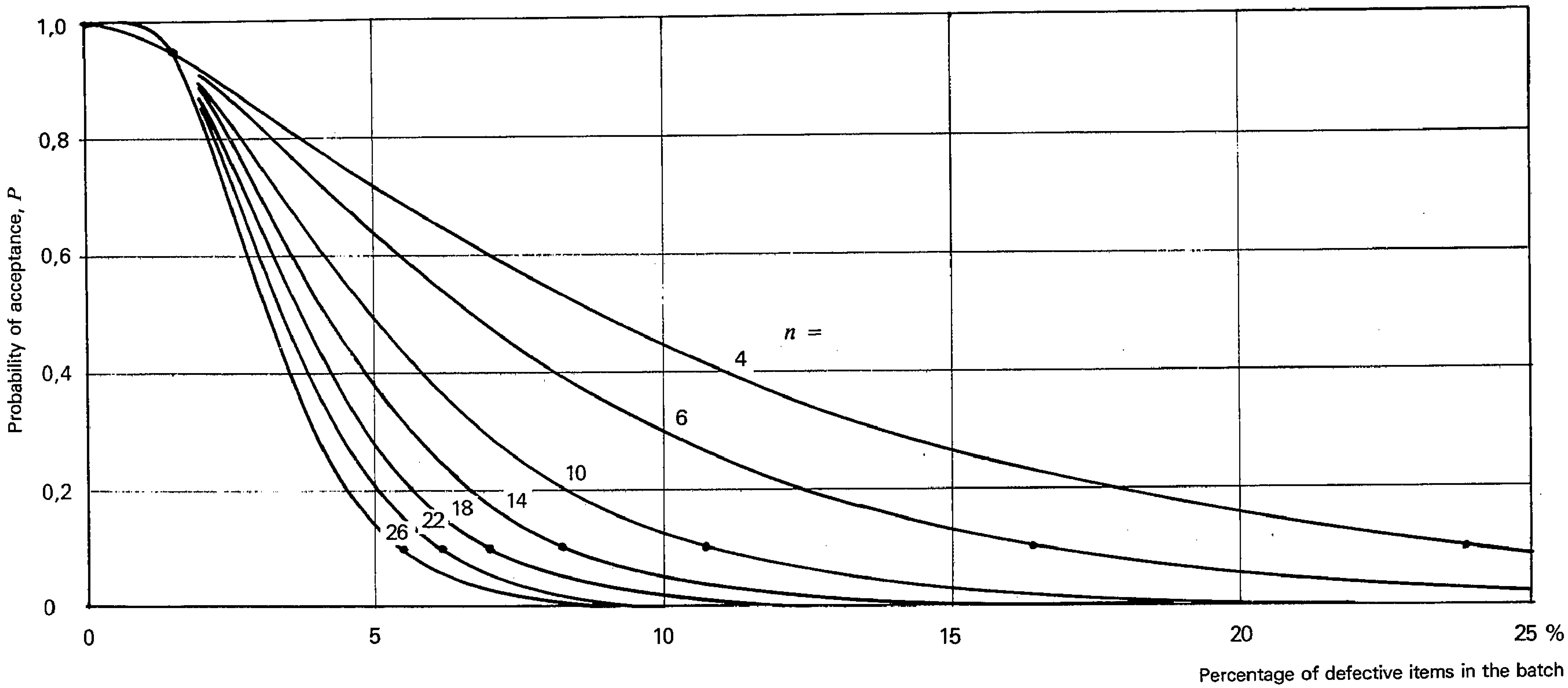

Figure 4 — Operating characteristic curves of single sampling plans of table 9, for AQL = 1,5 % : fixed unilateral limit for individual values and known standard deviation[1)]

1) These curves are approximately valid for the corresponding single sampling plans in table 10 having same AQL and same LQ (with standard deviation unknown).

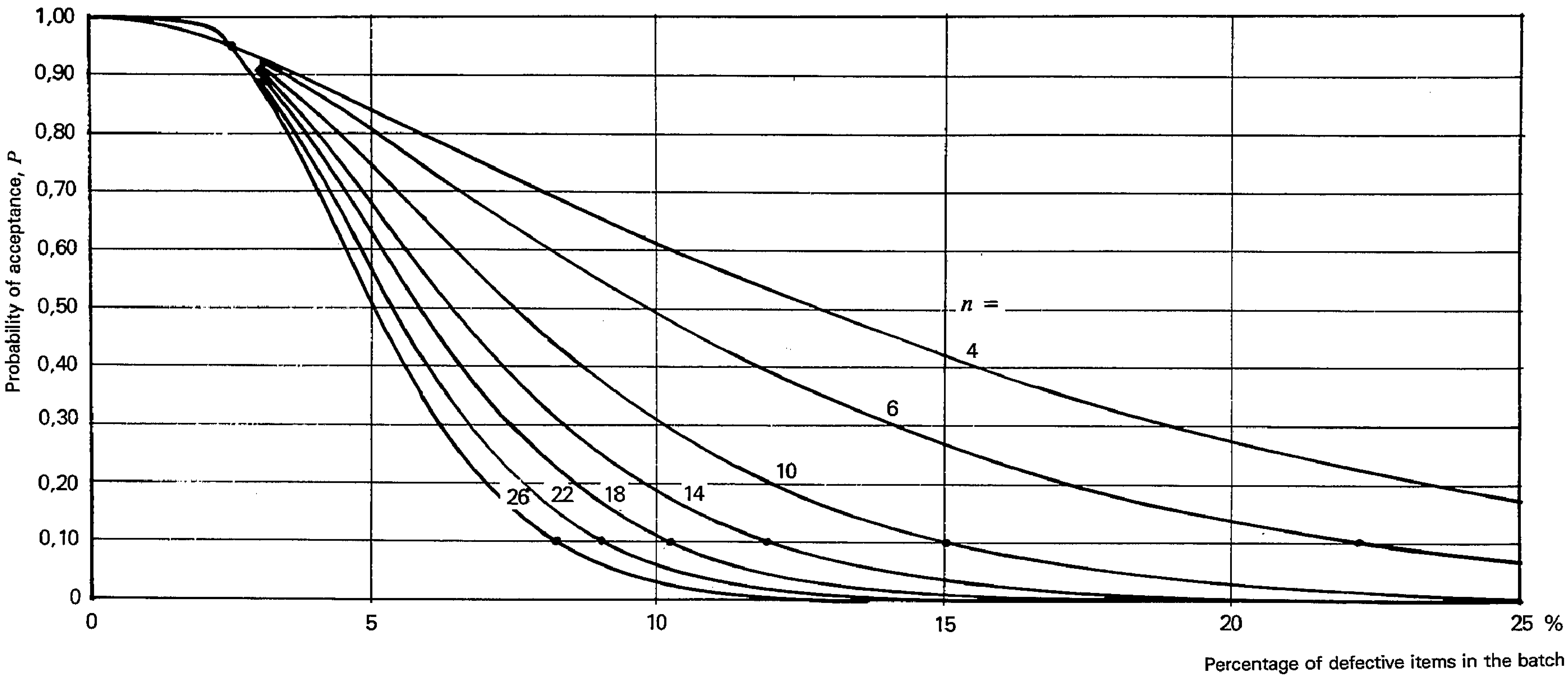

Figure 5 — Operating characteristic curves of single sampling plans of table 9, for an AQL = 2,5 % : fixed unilateral limit for individual values and known standard deviation[1)]

1) These curves are approximately valid for the corresponding single sampling plans in table 10 having same AQL and same LQ (with standard deviation unknown).

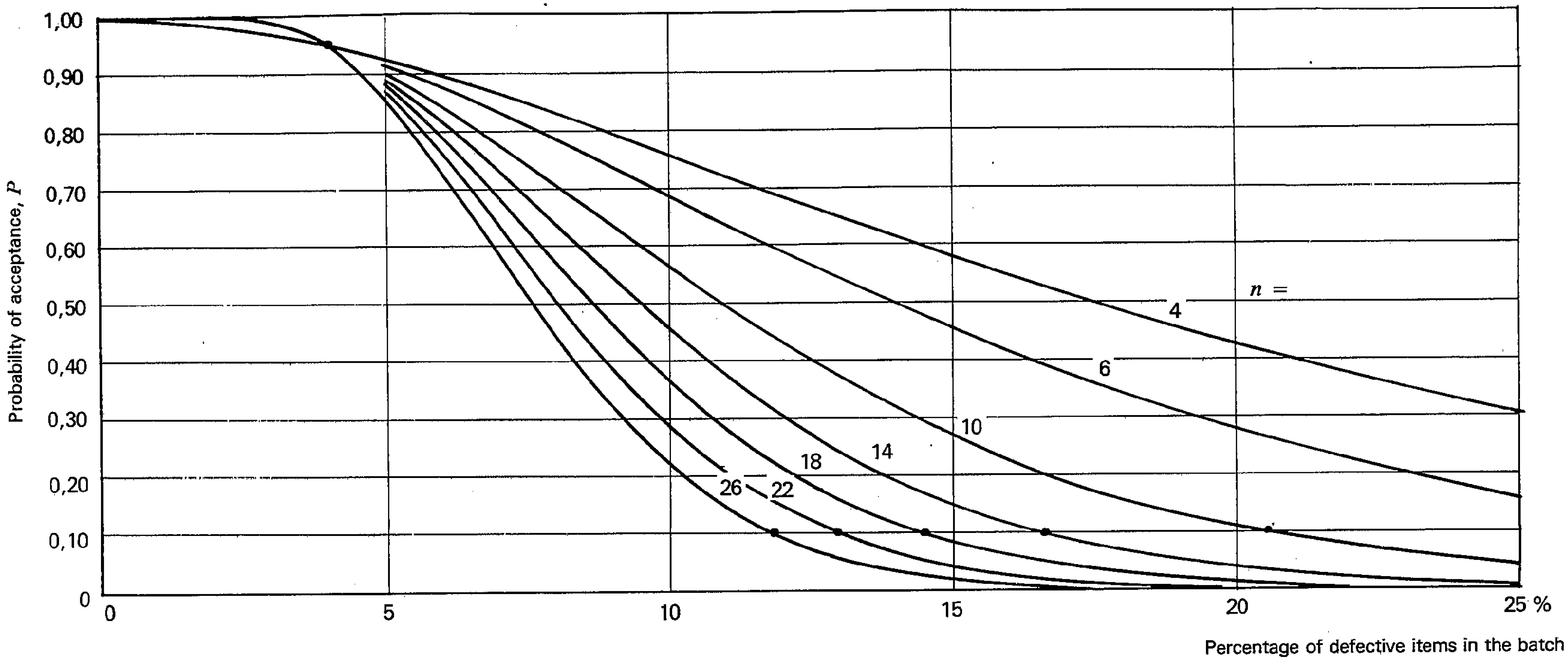

Figure 6 — Operating characteristic curves of single sampling plans of table 9, for an AQL = 4,0 % : fixed unilateral limit for individual values and known standard deviation[1)]

1) These curves are approximately valid for the corresponding single sampling plans in table 10 having same AQL and same LQ (with standard deviation unknown).

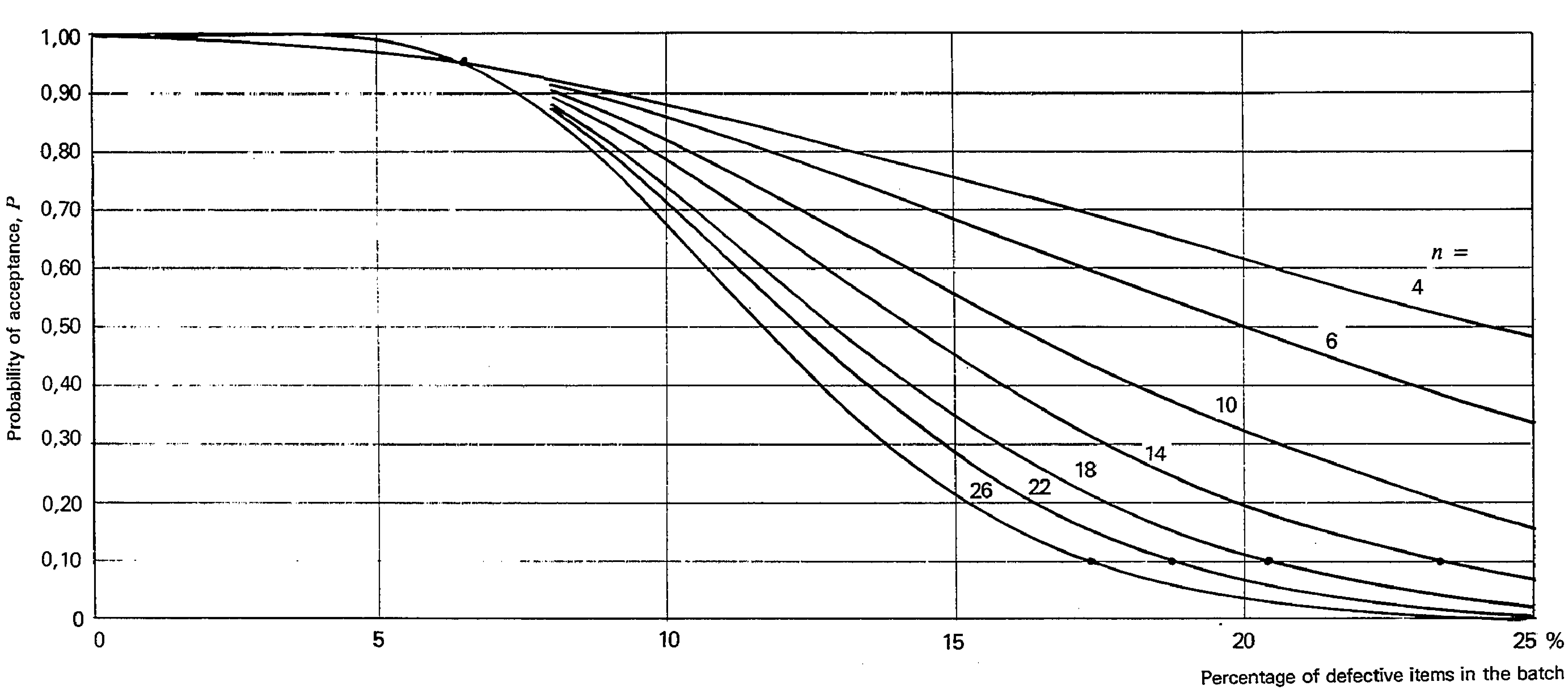

Figure 7 — Operating characteristic curves of single sampling plans of table 9, for an AQL = 6,5 % : fixed unilateral limit for individual values and known standard deviation[1)]

1) These curves are approximately valid for the corresponding single sampling plans in table 10 having same AQL and same LQ (with standard deviation unknown).

5.5.4 Producer's and consumer's risks

The values for K_{PRE} given in table 4 are based on a producer's risk $\alpha = 5$ % that a batch, with true mean value μ equal to the guaranteed mean μ_G, will be rejected by chance.

The consumer's risk β corresponds to the probability of a batch, whose true mean μ differs by $\Delta\mu$ from the guaranteed mean μ_G, will be declared to be in conformity. The value of $\Delta\mu$ which corresponds to $\beta = 10$ % is obtained by multiplying the factor $\left(\frac{\Delta\mu}{\sigma}\right)_{\beta = 10\ \%}$ of column 3 in table 4 by the true value of σ for the batch (that is unknown).

5.5.5 Example

A 200 t batch is to be inspected for acceptance with respect to "apparent density"; a value $\mu_G = 3{,}03$ g/cm^3 has been agreed upon.

The standard deviation σ being unknown, the sampling plans of section 5.5 must be used.

Table 4 indicates a sample size $n = 16$ and an acceptance factor $K_{PRE} = 0{,}44$.

The tests made on the sample give an arithmetic means $\bar{x} = 3{,}02$ g/cm^3 and a standard deviation $s = 0{,}035$ g/cm^3.

This gives $\mu_G - K_{PRE}\, s = 3{,}03 - 0{,}44 \times 0{,}035 = 3{,}015$.

Since $x = 3{,}02$ ($3{,}02 > \mu_G - K_{PRE}\, s$) the batch is declared to be in conformity.

The sampling plan used gives the following values :

- for the producer, the risk of having a batch in which the true mean would be 3,03, accidentally declared not to be in conformity, is equal to 5 %;
- the consumer runs a risk $\beta = 10$ % of having a batch in which the true mean would be approximately $3{,}03 - 0{,}78 \times 0{,}035 = 3{,}00$ g/cm^3, accidentally declared to be in conformity.

5.6 Single sampling plans with fixed unilateral limit for individual values and an unknown standard deviation

5.6.1 Field of application

The single sampling plans given in this sub-clause shall be used when the producer and consumer have agreed on a unilateral limit value (an upper limit T_s or a lower limit T_i, depending on the property being tested) for individual values and the standard deviation cannot be regarded as being known but can be estimated from the sample.

5.6.2 Characteristic parameters

A single sampling plan is characterised by the sample size n and the acceptance factor K; these parameters shall be obtained from table 10 according to the AQL agreed upon.

5.6.3 Treatment of the sample and decision on the batch

The tests yield n individual values, of which the arithmetic mean $\bar{x}$ and the standard deviation s are first calculated. The quality index of the sample is then calculated.

$$Q = \frac{T_s - \bar{x}}{s}$$

or

$$Q = \frac{\bar{x} - T_i}{s}$$

Rule governing the decision :

If $Q \geqslant K$, the batch is declared to be in conformity;

If $Q < K$, the batch is declared to be not in conformity.

5.6.4 Producer's and consumer's risks

The AQL and LQ values in table 10 are the same as those in table 9.

The producer's risk that corresponds to each AQL is approximately equal to 5 %. The consumer's risk which corresponds to each LQ is approximately equal to 10 %. This approximation is sufficient for practical purposes but it becomes less satisfactory when the sample size is less than 15. The sampling plans of this sub-clause are therefore practically equivalent to those of 5.4 with respect to their operating characteristic curves when they have the same AQL and same LQ.

5.6.5 Example

At 200 t batch is to be inspected for acceptance in respect of "porosity". An upper limit for the individual values of $T_s = 20{,}7$ %, and an AQL = 4 % have been agreed upon. Since the standard deviation σ is unknown, it is necessary to use the sampling plans of 5.6. The sample size $n = 26$ and the acceptance factor $K = 1{,}31$ are obtained from table 10.

The tests on the sample give an arithmetic mean $\bar{x} = 19{,}0$ % and a standard deviation $s = 0{,}9$ %. The quality index is obtained from these values :

$$Q = \frac{T_s - \bar{x}}{s} = \frac{20{,}7 - 19{,}0}{0{,}9} = 1{,}89$$

Since $Q = 1{,}89 > K = 1{,}31$, the result is a decision to declare the batch to be in conformity.

The sampling plan used gives the following guarantees :

— to the producer, that there exists a risk of 5 % that a batch containing 4 % of units with porosity in excess of 20,7 % will be declared not to be in conformity;

— to the consumer, that there exists a risk β = 10 % that a batch containing 16,6 % of units with porosity in excess of 20,7 % will be declared to be in conformity.

6 Report on sampling

The report on sampling shall contain the following information :

a) names of producer and consumer;

b) number and reference-marking of the batches;

c) date and place of sampling;

d) name of sampler;

e) number and identification of the samples intended for destructive tests, with an indication of the formats;

f) sampling plan;

g) values indicated by the supplier for the statistical inspection (both for non-destructive and destructive tests);

h) all the results obtained from the non-destructive tests.

Table 10 - Single sampling plans with a unilateral limit fixed for the individual values and an unknown standard deviation[1)]

AQL = 1,5 %			AQL = 2,5 %			AQL = 4,0 %			AQL = 6,5 %			Total mass of batch[2)] t
Sample size *n*	*K*	LQ en %	Sample size *n*	*K*	LQ	Sample size *n*	*K*	LQ	Sample size *n*	*K*	LQ	
1	2	3	4	5	6	7	8	9	10	11	12	13
8	1,35	23,9	7	1,14	30,9	6	0,93	38,6	5	0,69	48,0	1
13	1,50	16,4	11	1,29	22,2	9	1,08	28,9	8	0,84	37,6	10
24	1,65	10,7	20	1,44	15,0	18	1,23	20,5	14	0,99	27,9	100
35	1,73	8,2	30	1,52	11,9	26	1,31	16,6	22	1,07	23,4	200
47	1,78	6,9	40	1,57	10,2	35	1,36	14,5	29	1,13	20,4	300
58	1,82	6,1	51	1,61	9,0	44	1,40	12,9	37	1,16	18,7	400
70	1,85	5,5	61	1,64	8,2	53	1,43	11,9	44	1,19	17,4	500

1) The table is suitable for unit masses limited to 35 kg. In the case of unit masses greater than 35 kg, the sample mass is to be agreed upon by the contracting parties.

2) This column is given only as a guide.

Annex A

Determination of the arithmetic mean and standard deviation of a production

From time to time, the manufacturer shall determine the arithmetic mean μ and the standard deviation σ of his production, in respect of all the major characteristic properties.

The determinations shall be carried out with a frequency such that variations in μ and σ are known as quickly as possible. For the purpose of fixing the intervals in time between successive determinations, the following must be taken into account :

a) changes in raw materials;

b) preparation;

c) the method of casting;

d) variations in firing conditions.

It is recommended that the intervals should not exceed 1 month.

The arithmetic means $\bar{x}$ and the standard deviation s are calculated from the n measurement results obtained from the tests.

With $\bar{x}$ and s, the confidence interval of the population mean is obtained, on the level $1 - \alpha$:

$$\bar{x} - t_{(n-1; 1-\alpha/2)} \frac{s}{\sqrt{n}} \leqslant \mu \leqslant \bar{x} + t_{(n-1; 1-\alpha/2)} \frac{s}{\sqrt{n}}$$

and the confidence interval for the standard deviation σ of the population is obtained by :

$$sK_u \leqslant \sigma \leqslant sK_o$$

where

$$K_u = \sqrt{\frac{n-1}{\chi^2_{(n-1; 1-\alpha/2)}}}$$

$$K_o = \sqrt{\frac{n-1}{\chi^2_{(n-1; \alpha/2)}}}$$

The values of $t_{(n-1; 1-\alpha/2)}$, $\chi^2_{(n-1; \alpha/2)}$, $\chi^2_{(n-1; \alpha/2)}$, K_u and K_o may be found for example in ([2], [29], [36], [37], [42]).

The confidence intervals of μ and σ are given in table 11 for a confidence level of 95 % and for a few values of n.

The greater the sample size n, the narrower is the confidence interval, i.e., the more precise is the knowledge of μ and σ.

It is therefore necessary to weigh the cost of the tests against the precision desired, for the purpose of choosing the sample size n : it ought not to be lower than $n = 25$.

Determination of the confidence interval of μ and σ is also recommended in the case of quality characteristics of pieces for which no acceptance tests is carried out on grounds of cost.

Table 11 — Confidence interval of μ and σ for different sample sizes n

Sample size n	Confidence interval of the mean = 95 %	Confidence interval of the standard deviation = 95 %
2	$\bar{x} - 8{,}99\,s < \mu < \bar{x} + 8{,}99\,s$	$0{,}446\,s \leqslant \sigma \leqslant 31{,}91\,s$
5	$\bar{x} - 1{,}24\,s < \mu < \bar{x} + 1{,}24\,s$	$0{,}599\,s \leqslant \sigma \leqslant 2{,}87\,s$
10	$\bar{x} - 0{,}72\,s < \mu < \bar{x} + 0{,}72\,s$	$0{,}688\,s \leqslant \sigma \leqslant 1{,}83\,s$
15	$\bar{x} - 0{,}55\,s < \mu < \bar{x} + 0{,}55\,s$	$0{,}732\,s \leqslant \sigma \leqslant 1{,}58\,s$
20	$\bar{x} - 0{,}47\,s < \mu < \bar{x} + 0{,}47\,s$	$0{,}760\,s \leqslant \sigma \leqslant 1{,}46\,s$
25	$\bar{x} - 0{,}41\,s < \mu < \bar{x} + 0{,}41\,s$	$0{,}781\,s \leqslant \sigma \leqslant 1{,}39\,s$
30	$\bar{x} - 0{,}37\,s < \mu < \bar{x} + 0{,}37\,s$	$0{,}796\,s \leqslant \sigma \leqslant 1{,}34\,s$
40	$\bar{x} - 0{,}32\,s < \mu < \bar{x} + 0{,}32\,s$	$0{,}819\,s \leqslant \sigma \leqslant 1{,}28\,s$
50	$\bar{x} - 0{,}28\,s < \mu < \bar{x} + 0{,}28\,s$	$0{,}835\,s \leqslant \sigma \leqslant 1{,}25\,s$

Annex B

Comparison of the means of two samples

If it is necessary to compare the means of two samples with the aid of a significant test, the following method may be used :

Let $\overline{x}_1$, s_1 and $\overline{x}_2$, s_2 be, respectively, the means and the standard deviations of two samples having a size of n_1 and n_2.

Calculate :

$$s = \sqrt{\frac{(n_1 - 1)\, s_1^2 + (n_2 - 1)\, s_2^2}{n_1 + n_2 - 2}}$$

and

$$t = \frac{(\overline{x}_1 - \overline{x}_2)}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

If the value calculated for t is greater than the value given in the tables in references [2], [36], [37] for the desired confidence level and for $(n_1 + n_2 - 2)$ degrees of freedom, the means are regarded as significantly different.

It often happens that the samples are of equivalent size $(n_1 = n_2 = n)$. In this event, the test simplifies :

The quantity $\frac{\overline{x}_1 - \overline{x}_2}{\sqrt{s_2^1 + s_2^2}}$ is compared with the quantity g given in table 12 for the desired confidence level and the size of each sample n. If the value calculated is greater than the value shown in the table, the means are significantly different.

The other values of g can be calculated according to the formula $t_{2n-2}/\sqrt{n}$.

Table 12 — Values for the quantity g for the purpose of comparing the means of two samples of the same size n

Sample size n	Confidence level 95 %	Confidence level 99 %
2	3,043	7,018
3	1,603	2,658
4	1,223	1,854
5	1,031	1,500
6	0,909	1,294
7	0,824	1,155
8	0,758	1,053
9	0,707	0,974
10	0,664	0,910
15	0,529	0,713
20	0,453	0,606
25	0,402	0,536
30	0,366	0,486
40	0,315	0,417
50	0,281	0,371

Annex C

Equations used for calculating the values given in the tables

C.1 Table 4 — Single sampling plans in the case of a guaranteed value for the mean (see 5.3.2)

The values indicated in columns 2, 3 and 4 of table 4 and in figure 2 are derived from the following equations :

(1) $K_{PRE} = \frac{u_{1-\alpha}}{\sqrt{n}} = \frac{u_{95\,\%}}{\sqrt{n}} = \frac{1,645}{\sqrt{n}}$

(2) $\left(\frac{\Delta\mu}{\sigma}\right)_{\beta = 10\,\%} = \frac{u_{1-\alpha} + u_{1-\beta}}{\sqrt{n}} = \frac{u_{95\,\%} + u_{90\,\%}}{\sqrt{n}}$

$= \frac{1,645 + 1,282}{\sqrt{n}} = \frac{2,927}{\sqrt{n}}$

(3) $P\left(\frac{\Delta\mu}{\sigma}\right) = \Phi\left(1,645 - \sqrt{n}\,\frac{\Delta\mu}{\sigma}\right)$

P is the probability of acceptance;

$\Phi\mu$ is the cumulative distribution function of the standardized normal distribution.

C.2 Table 6 — Sequential sampling plans in the case of a guaranteed value limit (μ_G) for the mean and a known standard deviation (see 5.3.3)

The values indicated in table 6 are obtained, by means of the following equations, from the values of $\frac{\Delta\mu}{\sigma}$ taken from column 3 of table 4 :

(4) $l_2 = \ln\frac{1-\alpha}{\beta} = 2,25; \qquad l_1 = \ln\frac{1-\beta}{\alpha} = 2,89$

(5) $b = \mu_G \pm \frac{\sigma}{2}\left(\frac{\Delta\mu}{\sigma}\right)_{\beta = 10\,\%};$

$a = \mp\,\sigma l_1\left(\frac{\sigma}{\Delta\mu}\right)_{\beta = 10\,\%};$

$r = \pm\,\sigma l_2\left(\frac{\sigma}{\Delta\mu}\right)_{\beta = 10\,\%}$

(6) $\overline{n}(\mu_G) = [2(1-\alpha)(l_1 + l_2) - 2\,l_2]\left(\frac{\sigma}{\Delta\mu}\right)^2_{\beta = 10\,\%}$

$= 3,988\left(\frac{\sigma}{\Delta\mu}\right)^2_{\beta = 10\,\%}$

(7) $\overline{n}(\mu_G \pm \Delta\mu) = [2\,l_2 - 2\,\beta\,(l_1 + l_2)]\left(\frac{\sigma}{\Delta\mu}\right)^2_{\beta = 10\,\%}$

$= 4,752\left(\frac{\sigma}{\Delta\mu}\right)^2_{\beta = 10\,\%}$

(8) $\overline{n}\;\mu_G \pm\left(\frac{\Delta\mu}{2}\right) = l_1 l_2\left(\frac{\sigma}{\Delta\mu}\right)^2_{\beta = 10\,\%}$

$= 6,507\left(\frac{\sigma}{\Delta\mu}\right)^2_{\beta = 10\,\%}$

The value of n_{max} is obtained, for $\alpha = 5\,\%$, from the expression :

(9) $n_{max} = 10,8\left(\frac{\sigma}{\Delta\mu}\right)^2_{\beta = 10\,\%}$

The values for n_{max} in column 8 of table 6 have been rounded upwards to the nearest integer.

NOTE — If sequential sampling is curtailed at $n = n_{max}$, the sequential sampling plan is changed into a single sampling plan.

In the case of a single sampling plan, with $\alpha = 5\,\%$ according to 5.3.2, the acceptance value, which is to be compared with $\overline{x}$, has the following values (if the low values are unfavourable) :

(10) $g_E = \mu_G - K_{PRE}\,\sigma = \mu_G - \frac{1,645}{\sqrt{n}}\,\sigma$

If the sequential sampling plan is curtailed according to the rules governing the decision which are indicated in the lower half of the synoptic table, the acceptance value, which is to be compared with $\bar{x}$, becomes :

(11) $g_F = b$

Since $g_E = g_F$, the following is derived from equation (5)

(12) $\mu_G - \frac{1,645}{\sqrt{n}}\,\sigma = b = \mu_G - \frac{\Delta\mu}{2}$

and thence finally equation (9).

If it is the high values which are unfavourable, the same equation is found for n_{max}.

C.3 Table 9 — Single sampling plans with a fixed unilateral limit for the individual values and a known standard deviation (see 5.4)

The following equations make it possible to calculate K and LQ in table 9 and $P(p)$ in figures 4, 5, 6 and 7 :

$$(13)\quad K = u_{1-\text{AQL}} - \frac{u_{95\,\%}}{\sqrt{n}} = u_{1-\text{AQL}} - \frac{1{,}645}{\sqrt{n}}$$

$$(14)\quad u_{1-\text{LQ}} = K - \frac{u_{90\,\%}}{\sqrt{n}} = K - \frac{1{,}282}{\sqrt{n}}$$

$$(15)\quad P(p) = \Phi\left[\sqrt{n}\,(u_{1-p} - K)\right]$$

in which :

$$\Phi(u_{1-\text{AQL}}) = 1 - \text{AQL}$$

$$\Phi(u_{1-\text{LQ}}) = 1 - \text{LQ}$$

$$\Phi(u_{1-p}) = 1 - p$$

and $\Phi(u)$ is the integral of the standard distribution law.

The acceptance factors K et K_{PRE} (in 5.3.1) are linked (for the same sample size n) by the following relation :

$$(16)\quad K = u_{1-\text{AQL}} - K_{\text{PRE}}$$

C.4 Table 6 — Single sampling plans in the case of a guaranteed value limit (μ_G) for the mean and an unknown standard deviation (see 5.5)

If n_σ is used to designate the sample size intended for a single sampling plan with a known σ (table 4, column 1), and n_s is used to designate the sample size intended for the single sampling plan with σ unknown (table 4, column 4), the following may be written for $\alpha = 5\,\%$, and gives a satisfactory approximation :

$$(17)\quad K_{\text{PRE}} = \frac{u_{95\,\%}}{\sqrt{n_\sigma}} = \frac{1{,}645}{\sqrt{n_\sigma}} = \frac{t_{95\,\%;\,n_s-1}}{\sqrt{n_s}}$$

It is possible to calculate n_s from n_σ with the aid of this equation. The value $t_{95\,\%;\,n_s-1}$ of the t distribution may be taken, for example, from [2].

C.5 Table 10 — Single sampling plans with a unilateral limit fixed for the individual values and an unknown standard deviation (see 5.6)

The values K and LQ in table 10 correspond to those in table 9; if n_σ is used to designate the n values in table 9, and n_s is used to designate those in table 10, the following is obtained and gives a satisfactory approximation :

$$(18)\quad n_s = n_\sigma\left(1 + \frac{K^2}{2}\right)$$

Annex D

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