TECHNICAL REPORT

ISO TR 5307

First edition
1991-12-01

Solid fertilizers — Derivation of a sampling plan for the evaluation of a large delivery

Matières fertilisantes solides — Fondements théoriques du plan d'échantillonnage destiné à l'évaluation d'une grosse livraison

Reference number
ISO/TR 5307:1991(E)

Contents

Annexes

International Organization for Standardization
Case Postale 56 • CH-1211 Genève 20 • Switzerland

Printed in Switzerland

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The main task of technical committees is to prepare International Standards, but in exceptional circumstances a technical committee may propose the publication of a Technical Report of one of the following types:

— type 1, when the required support cannot be obtained for the publication of an International Standard, despite repeated efforts;

— type 2, when the subject is still under technical development or where for any other reason there is the future but not immediate possibility of an agreement on an International Standard;

— type 3, when a technical committee has collected data of a different kind from that which is normally published as an International Standard ("state of the art", for example).

Technical Reports of types 1 and 2 are subject to review within three years of publication, to decide whether they can be transformed into International Standards. Technical Reports of type 3 do not necessarily have to be reviewed until the data they provide are considered to be no longer valid or useful.

ISO/TR 5307, which is a Technical Report of type 3, was prepared by Technical Committee ISO/TC 134, *Fertilizers and soil conditioners*.

This document is a type 3 Technical Report. It is not envisaged that it will be published as an International Standard. It gives the mathematical derivation of the sampling plan specified in ISO 8634.

Annexes A and B are for information only.

Introduction

Within the framework of its work on sampling, Technical Committee 134 "Fertilizers and Soil Conditioners" has, through its subcommittee 2, carried out statistical studies on various sampling plans which may be used to assess large deliveries of fertilizers. This work complements other standards for fertilizers, currently under preparation, and provides the theoretical background necessary to appreciate fully the requirements of those standards. This technical report (type 3), which is different from the international standards usually produced by ISO/TC 134/SC4, is intended to act as a complement to them, as a basis for the sampling of fertilizer deliveries.

Each country has its own regulations applicable to the fertilizer trade; an official department is responsible for carrying out checks regarding application of the regulations. If these regulations are violated, sanctions may be taken against those responsible for placing the fertilizer on the market in that country. In the case of an imported delivery, it is the representative of the manufacturing company in the country, or the importer who is considered by the relevant authorities to be responsible for the declared contents shown on labels or other documentation accompanying the fertilizers.

ISO 8634 concerns the case of an importer who resells, on his own responsibility, a large amount of fertilizer received from abroad. After unloading, this delivery is resold in smaller lots to traders (dealers or farmer cooperatives) who will themselves be direct suppliers to farmers. In the case in question, it is the importer whose name is associated with the fertilizer; and it is therefore he who will be considered by the retailers and users to be responsible for the declared contents.

ISO 8634 is designed for **acceptance inspection**. It determines the rules for:

a) sampling (i.e. the sampling plan);

b) acceptance (the acceptance or rejection of the delivery);

and both apply to the bulk delivery imported.

The location of the acceptance inspection, as defined in ISO 8634, in the chain of transactions can be represented by the following diagram:

Country of manufacture	Importing country

Seller → **Importer** → **Retailers** → **Farmers**

. . .   .
. . .   .
. . .   .

Acceptance inspection in accordance with ISO 8634 (no loading or unloading)	Official inspections of lots resold in accordance with national regulations

Solid fertilizers — Derivation of a sampling plan for the evaluation of a large delivery

1 Scope

This Technical Report presents the sampling theory which has resulted in the definition of the sampling plan described in ISO 8634[1].

The sampling plan is applicable to a large delivery of more than 250 t of fertilizer supplied to another party, for resale, on his own responsibility, in small lots, each of which would be subject to legislation.

By large amount is understood, for example, a full boat-load (5,000 t, 10,000 t or more) thus corresponding to a relatively long period of manufacture, but the theory applies to any delivery of 250 t or more.

2 References

ISO 8157:1984, *Fertilizers and soil conditioners - Vocabulary.*

ISO 8634:-[1], *Solid fertilizers - Sampling plan for the evaluation of a large delivery.*

3 Notation and symbols

The following symbols appear in this Technical Report and have the meanings assigned to them below.

μ, $\underline{\sigma}$	Actual mean value and standard deviation between sampling units in the delivery.
μ_a, $\underline{\sigma}_a$	Mean value and standard deviation between sampling units in a delivery of just acceptable quality.

1) To be published.

μ_r, σ_r	Mean value and standard deviation between sampling units in a delivery of just unacceptable quality.
μ_e, σ_e, μ_f, σ_f	Mean and standard deviation, respectively, of two lots which can be considered by the importer to be of the same quality.
U	Number of sampling units in the delivery.
N	Number of sampling units to be selected during the sampling of the delivery. (Increments).
N'	Number of analyses to be carried out on the N increments during the inspection of the delivery.
N_R	Number of sampling units contained in the smallest lot presented for resale.
k	Number of increments to be combined into each aggregate sample for analysis.
n	Number of sampling units which will be mandatorily selected during the official sampling of a lot of N_R sampling units.
$\bar{x}$	Mean value found by analysis after the selection of n sampling units from a lot of sampling units.
s	Estimate of $\sigma/\sqrt{k}$ with the aid of the N' analyses, where σ is the standard deviation between the sampling units in the delivery.
X_i	Analytical result obtained on the sample of rank i.
$\bar{X}$	Estimate of the mean value of the delivery with the aid of the N' analyses.
D	Declared value e.g. of a plant nutrient in the fertilizer delivery.
L	Official inspection limit value which depends on the declared value(D). It may be equal to D or less than D by a prescribed tolerance which may depend on the size of the lot sold.

r_a	Probability that the mean value of n sampling units is lower than the official limit value (L), just acceptable by the importer.
r_r	Probability that the mean value of n sampling units is lower than the official limit value (L), just unacceptable by the importer.
$\underline{\alpha}$	Probability of rejection of a delivery of just acceptable quality (seller's or producer's risk).
$\underline{\beta}$	Probability of acceptance of a delivery of just unacceptable quality (importer's or consumer's risk).
u_{1-r_a}	Value of the standardized normal variable such that $Pr[u > u_{1-r_a}]$ equals r_a .
u_{1-r_r}	Value of the standardized normal variable such that $Pr[u > u_{1-r_r}]$ equals r_r .
$u_{1-\underline{\alpha}}$	Value of the standardized normal variable such that $Pr[u > u_{1-\underline{\alpha}}]$ equals $\underline{\alpha}$.
$u_{1-\underline{\beta}}$	Value of the standardized normal variable such that $Pr[u > u_{1-\underline{\beta}}]$ equals $\underline{\beta}$.
$\underline{\delta}$	Non-centrality parameter.
K	Calculation coefficient which is dependent on n, the risk levels $\underline{\alpha}$ and $\underline{\beta}$ and the probability levels r_a and r_r.
a	Constant factor dependent on N' which represents the uncertainty associated with the estimate of the standard deviation.
t_0	Value of the non-central Student ratio corresponding to the level of probability for a non-centrality parameter equal to $\dfrac{\sqrt{N}\, u_{1-r_a}}{\sqrt{n}}$
B_0	Limit value of the estimate calculated from t_0.

A, *B*	Calculation intermediates used during the estimation of the lot after analysis.
F	Calculation intermediate used to facilitate the calculation of *k* and *N*.

4 Preliminary hypotheses

The sampling plan has been drawn up on the assumption that there is no serial correlation between the successive units of the delivery.

The *N* units inspected are selected at random from the delivery, each unit having the same chance of being selected, and the *N* groups of *k* units made up at random from the *N*. It is also understood that the lots made up by the importer represent a random sample from among the *U* bags of the delivery and that the increments taken from a lot by the authorities responsible for the inspection are taken at random from the lot.

In the subsequent theory, it is assumed that a single plant nutrient is of interest or that, if this is not the case, each plant nutrient is considered separately. It is also assumed that the fertilizer is packaged. although similar arguments will also apply to products in bulk.

The analytical error is considered to be negligible in relation to the sampling error.

Finally, it is assumed once and for all, as has been shown by the studies of data from production and dispatch inspection carried out in various countries:

a) that the mean concentration of a certain component or value in the sampling units (e.g. bag) constituting a definite lot of fertilizer shall be considered as a random quantity which obeys a normal distribution;

b) that the distribution of this random quantity does not depend, at least for sufficiently large lots, on their size.

5 Principle of the sampling plan

5.1 General

The sampling plan described in ISO 8634 defines a pair of numbers, *N* and *N'*, which depend on:

a) the legal requirements of the importing country (acceptable limit for the value and the size of the smallest lot which can be inspected);

b) the risks which the importer accepts.

NOTE - It should be remembered that it is intended for the inspection of the delivery received by the importer, and not for the lots resold by the same importer.

N is the number of increments which are to be taken from the delivery and N' the number of analyses to be carried out on these N increments.

The N increments are combined and mixed k by k (k is a whole number), thus resulting in N' aggregate samples ($N = kN'$) and an analysis is carried out on each of these N' aggregate samples.

This procedure is explained by the relatively long and costly nature for the analyses for determining the content of the various fertilizer nutrients.

The sampling plan adopted is based on the use of two non-central Student distributions.

As the standard deviation of the population is only known through N' analyses and the corresponding estimate s, the confidence intervals to be used should draw on Student's distribution and not Gaussian distribution. Moreover, in the present case, the two central values of the limit distributions which the buyer's and seller's risks should cover, will be defined on the basis of a fixed value (L) by a shift based on the standard deviation $\underline{\sigma}$ of the population. In this case, the reduced value of the interval between the value L and the confidence interval limits obeys a non-central Student distribution, which has been tabulated in particular by Neyman and Tokarska. It depends only on the shift of the central value (in relative value) in relation to the standard deviation $\underline{\sigma}$ of the population.

Given that in each non-central Student test (one linked to the seller's risk, and the other to the buyer's risk) the same standard deviation (i.e. $\underline{\sigma}_a$ or $\underline{\sigma}_r$) arose in the non-centrality parameter and in the dispersion of the mean of the N sample values, then the determination of N and N' is independent of the value of the actual standard deviation of the lot.

5.2 Information

This is of two types. The first type is derived from the national regulations of the importing country. That is:

n — The number of sampling units from which, in accordance with the regulations, partial samples are to be taken, in the case of the smallest lot that can be inspected.

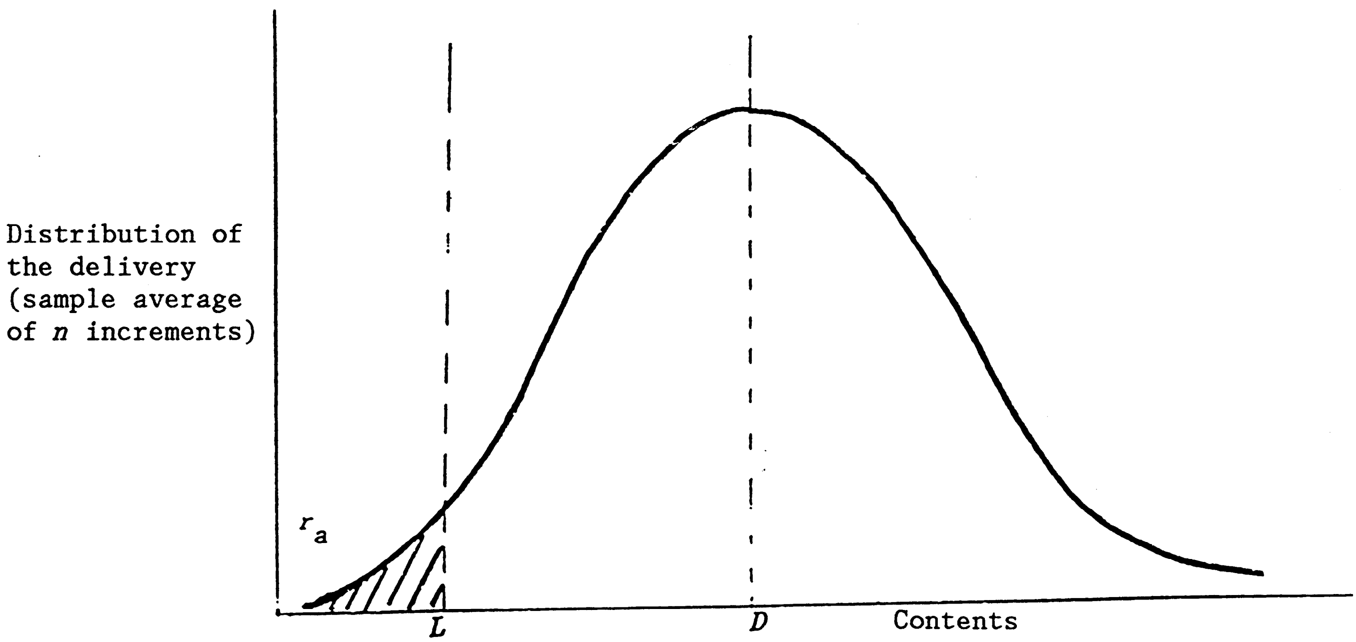
L — The official inspection limit; if the declared value is D, it can be equal to D or less than D by a permitted tolerance which may or may not be a function of the number of lots inspected. ($L = D - T$, if T = tolerance).

The second type can be fixed by mutual agreement between the two contracting parties (the supplier and the importer), taking into account the conditions of application of the regulations in the importing country (frequency and stringency of inspections, punitive sanctions, etc.):

That is:

r_a This is the fundamental parameter as **it defines the "level of quality" which shall be the minimum objective of the manufacturer in production, in order to give satisfaction to the importer** (see figure 1 and 7.1).

Production will normally be centred upon the declared value *D*; but it is not sufficient for it to fulfil this condition. What is required by the importer, and it should be noted that he is not the user, is to be able to resell small lots without being penalised by the official inspection service. He therefore wishes it to be impossible to draw from the overall delivery small lots which, after sampling, reveal average contents less than *L*, under official inspection conditions. The ideal would be for the production to contain no small lot likely to appear on inspection to have a value less than *L*; but this ideal is impossible to attain under practical manufacturing conditions and would only be verifiable by a full inspection, at a prohibitive cost. The importer therefore accepts a certain percentage of incorrect units (i.e. bags, if the bag is a sampling unit) defining the quality level of the overall delivery which he considers acceptable; this percentage is expressed by the parameter r_a which can be defined as "the probability, which is just acceptable to the importer, that the average value of *n* sampling units is less than the official limit *L*".



Minimum quality defining an acceptable delivery

Figure 1 – The relationship between *D*, *L* and r_a

NOTE - The regulations of the importing country may also require that the complete delivery should respect the declared content *D*. In this case, the tolerance *T* is only applicable to small lots resold and their acceptance is accompanied by a verification of the compensations between recorded under-contents and over-contents. This aspect of the question is not examined in this technical report.

$\underline{\alpha}$ **The seller's risk** or 'probability of rejecting a delivery of acceptable quality' (see also 7.1).

This technical report defines a statistical test. The aim of the sampling plan is to obtain sufficient information to be able to say, with certain well defined risks, whether or not the delivery is indeed of the acceptable quality level as defined above by r_a. This actual quality of the delivery cannot, in any case, be known with absolute certainty. Certain risks have to be accepted. These risks are defined in relation to a **hypothesis** of what the delivery is in fact. As far as the seller's risk, $\underline{\alpha}$, is concerned, the supposition is made that the manufacturer has in fact supplied a correct delivery, i.e. corresponding to the quality level acceptable to the importer, defined as a limit by r_a. With this hypothesis - of a correct delivery, it may happen, by the misfortune of the seller, that the random drawing of sampling units results in a sample which gives a distorted image of the delivery, making it to be declared defective whereas it is in fact correct.

The manufacturer thus sees the delivery wrongly rejected, whilst with this hypothesis his delivery is actually correct.

For the manufacturer, the sampling plan should be such that a correct delivery is only wrongly rejected in less than r_a % of cases. This risk, or the seller's risk, is the maximum which the manufacturer agrees to bear.

r_r This parameter defines the 'quality level' of the delivery, which is too low to be acceptable to the importer (see also 7.1).

It was found above, with regard to r_a, that the importer could not require an ideal which is impossible to obtain or that the total absence of sampling units of content less than *L* had to be checked. But he should require that the sampling plan guarantees him against an excessive proportion of small lots which on inspection are found to be deficient in content. This is why he requires that he should define as unacceptable a delivery which, on inspection, contains more than r_r % of small lots in which the mean value (based on *n* sampling units) is less than the official limit *L*.

$\underline{\beta}$ **The importer's risk**

This, for the importer, complements the seller's risk $\underline{\alpha}$ (see also 7.1).

In the hypothesis in which the actual quality level of the delivery is as low as r_r, the importer wishes to be sure that the sampling plan will not lead him wrongly to accept the delivery as correct, in more than $\underline{\beta}$ % of cases.

5.3 Objective of the determination

5.3.1 General

Two points should be considered.

Firstly, the sampling plan itself, i.e. the increments to be taken, their combinations (applicable) the number of analyses etc.

Secondly, the rule of acceptance, in accordance with the results obtained.

5.3.2 The sampling plan

This will define the number of sampling units (bags), N, from which the increments are to be taken.

Either each increment, obtained from a sampling unit, is analysed; or they are grouped in twos, threes or fours etc... (k) and in this case an analysis is only carried out for each of the groups of k aggregate samples (N' analyses).

Thus: $k = N/N'$

If $k = 1$, it is sufficient to determine N; in other cases, the plan determines N and k, and hence N'.

5.3.3 Rule of acceptance

The sampling plan leads to N' results being obtained for each content to be determined. With these N' results, the standard deviation of the delivery, s, can be estimated.

The standard also gives the value of a coefficient K which, as will be seen later on, depends on the preceding data: r_a, r_r, $\underline{\alpha}$, $\underline{\beta}$ and n.

Taking $\overline{X}$ as the mean of the N' results, the delivery will be accepted if:

$$\overline{X} \geq L + Ks, \text{ or } B \geq B_0$$

but if:

$$\overline{X} < L + Ks, \text{ or } B < B_0$$

then the delivery will be rejected.

6 Theory of the sampling plan

6.1 Definitions

The delivery is made up of U sampling units (bags, etc.).

The mean content of the delivery is μ, its standard deviation $\underline{\sigma}$ and its declared content D.

The importer does not know either μ or $\underline{\sigma}$. On the other hand, he knows D which is the declared composition under which the fertilizer is to be sold, and he is faced with the problem of ensuring, by suitable sampling and analysis, that the lots which he sells will conform to the specifications of local regulations.

It is assumed that local regulations generally require that an analysis, carried out after sampling any lot (which, in extreme cases, may consist of a single bag), shall not fall below a limit L. This may be the declared composition D, or it may be less by a permitted tolerance (which will generally depend on the size of the lot).

In view of the fact that the importer will usually resell the fertilizer in sub-lots of varying size, the sampling scheme should be designed to give suitable protection to the smallest lots which he intends to sell.

Assuming that the smallest lot intended for sale is N_R bags then, for inspection at this stage (resold lots) a sample of n bags will be taken (n is thus fixed by local regulations).

Using these n bags, an aggregate sample will be made up, the analysis of which will lead to the (mean) value $\bar{x}$.

$\bar{x}$ is the (mean) value of the sample of n bags; it is the **estimated** mean value for the lot of N_R bags.

As assumed above, local legislation considers the lot of N_R bags to be acceptable if $\bar{x} \geq L$.

6.2 Determination of limits

If the distribution of $\bar{x}$ observed on different groups of n bags selected from each lot of N_R bags is normal (Gaussian), which is the case when n is not too small, the limit qualities of acceptable or non-acceptable deliveries of fertilizer may be determined as follows:

A delivery will be considered to be of acceptable quality if the probability of the average, $\bar{x}$, of n bags selected at random being less than L is equal to or less than r_a.

If μ_a and $\underline{\sigma}_a$ respectively are the mean and the standard deviation of a delivery of just acceptable quality, i.e. for which the probability that $\bar{x} < L$ is exactly r_a and if u_a is such that a unit normal variable is greater than u_a with a probability of r_a, then:

$$L = \mu_a - \frac{u_a \underline{\sigma}_a}{\sqrt{n}} \qquad \text{- E.1}$$

Likewise, a delivery will be considered to be of unacceptable quality if the probability of the mean $\bar{x}$ of n bags selected at random being less than L is equal to or greater than r_r. If μ_r

and σ_r respectively are the mean value and the standard deviation of a delivery in the limit case for which the probability that $\bar{x} < L$ is exactly r_r and if u_r is such that a unit normal variable is greater than u_r with a probability of r_r, then:

$$L = \mu_r - \frac{u_r \sigma_r}{\sqrt{n}} \qquad \text{- E.2}$$

NOTE - For material to be of acceptable or unacceptable quality depends on both the mean μ and the standard deviation σ because the "quality" is considered in relation to the probability that the smallest lots do not satisfy the requirements of local regulations. This probability, in effect, depends on μ and σ as follows:

$$u = \frac{\mu - L}{\sigma} \sqrt{n}$$

controlled by the same number, n, of sampling units selected during the official local inspection.

For smaller lots of equal size, two deliveries respectively of means μ_e and μ_f and standard deviations σ_e and σ_f may be said to be of the same quality (in the above sense) if:

$$\frac{\mu_e - L}{\sigma_e} = \frac{\mu_f - L}{\sigma_f}$$

In particular, all the combinations of the values of μ_a and σ_a complying with (6.1) above, will correspond to a fertilizer of acceptable limit quality and all the combinations of values μ_r and σ_r complying with (6.2) above will correspond to a fertilizer of unacceptable limit quality. It should be noted that the material of acceptable limit quality and the material of unacceptable limit quality will not generally have the same standard deviations.

6.3 Use of two non-central t distributions

Having defined the material of acceptable limit quality (μ_a, σ_a) and of unacceptable limit quality (μ_r, σ_r), a test is now considered for the acceptance or rejection of a larger delivery. N sampling units are selected from the delivery and grouped in N' aggregate samples each containing k increments (i.e. $N = kN'$). An analysis is made of each of the aggregate samples to ascertain the quality of the delivery.

This, in terms of $\frac{\mu - L}{\sigma}$ cannot be known exactly but can be estimated by the expression:

$$\frac{\bar{X} - L}{s \sqrt{k}}$$

in which $\bar{X}$ is the analytical mean of N' aggregate samples and s is the estimate of the standard deviation between the N' analysis. The distribution of the expression:

$$\frac{\sqrt{N}\,(\bar{X} - L)}{s\,\sqrt{k}}$$

is in non-central t with $N' - 1$ degrees of freedom and the non-centrality parameter

$$\underline{\delta} = \frac{\sqrt{N}\,(\mu - L)}{\underline{\sigma}}$$

It is necessary to determine the values of N, N' and t such that for a delivery of acceptable limit quality (μ_a, $\underline{\sigma}_a$) the probability of rejection of the delivery is sufficiently small and equal to $\underline{\alpha}$; and for a delivery of unacceptable limit quality (μ_r, $\underline{\sigma}_r$), the probability of acceptance of the delivery is also small and equal to $\underline{\beta}$.

In addition for 'acceptable' material:

$$s = s_a = \sqrt{N}\,\frac{(\mu_a - L)}{\underline{\sigma}_a} = \sqrt{N}\,\frac{u_a}{\sqrt{n}}$$

and:

$$Pr\left(\frac{\bar{X} - L}{s}\sqrt{N'} < t_{1-\underline{\alpha}}/s = s_a\right) = \underline{\alpha}$$

whilst for 'unacceptable' material:

$$s = s_r = \sqrt{N}\,\frac{(\mu_r - L)}{\underline{\sigma}_r} = \sqrt{N}\,\frac{u_r}{\sqrt{n}}$$

$$Pr\left(\frac{\bar{X} - L}{s}\sqrt{N'} \geq t_{1-\underline{\beta}}/s = s_r\right) = \underline{\beta}$$

6.4 Determination of *N* and *N'*

6.4.1 ***Theory***. It is not possible to obtain mathematical expressions for N and N' based on the non-central Student distribution. The values sought may be obtained by trial and error using the Neymann and Tokarska tables, or more accurately by the use of the Lieberman and Resnikoff tables. However, they can be more conveniently determined by calculation using certain justified approximations.

The comparison of the assay $\bar{X}$ found at a limit calls for an expression of the type $\bar{X} - Ks$. As a first approximation it is assumed that all variables of the form $\bar{X} - Ks$ have a normal distribution.

This approximation is justified if N' is large as $\bar{X}$ has normal distribution and Ks therefore has a distribution which tends towards normal distribution if N' is large. It has been verified that this can be applied even for small values of N' (down to $N' = 5$) by comparison with a graphic resolution of the use of non-central Student distributions.

Allowing this approximation, then $\bar{X}$ and s are independent and $\bar{X} - Ks$ has:

a mean of: $\mu - \frac{a K \sigma}{\sqrt{k}}$

and a variance of: $\frac{\sigma^2}{k} \left[\frac{1}{N'} + (1 - a^2)K^2\right]$

with $a = \dfrac{\Gamma\left(\frac{N'}{2}\right)}{\Gamma\left(\frac{N'-1}{2}\right)} \sqrt{\dfrac{2}{N'-1}}$

where Γ is the Euler function.

NOTE. The value of a tends to 1 as N' tends to infinity.

Considering the risk of the first type, there is a probability α of rejecting (wrongly) an acceptable delivery, that is, a delivery for which:

$$\mu - \frac{u_{1-r_a} \sigma}{\sqrt{n}} = L \qquad \text{- E.3}$$

In this limit case $\bar{X} - Ks$ will be equal to a value ℓ given by ℓ = mean of $(\bar{X} - Ks) - u_{1-\alpha}$ (standard deviation of $\bar{X} - Ks$).

Thus:

$$\ell = \mu - \frac{a K \sigma}{\sqrt{k}} - \frac{u_{1-\alpha} \sigma}{\sqrt{k}} \left[\frac{1}{N'} + (1 - a^2) K^2\right]^{\frac{1}{2}} \qquad \text{- E.4}$$

where

$u_{1-\alpha}$ is the standardized normal variable corresponding to the probability α.

Eliminating μ between these two equations E.3 and E.4 gives:

$$\ell = L + \frac{u_{1-r_a} \sigma}{\sqrt{n}} - \frac{a K \sigma}{\sqrt{k}} - \frac{u_{1-\alpha} \sigma}{\sqrt{k}} \left[\frac{1}{N'} + (1 - a^2)K^2\right]^{\frac{1}{2}}$$

ℓ depends on the unknown standard deviation, unless the value of K is chosen such that $\ell = L$. This value is given by:

$$K = \frac{1}{a} \left\{u_{1-r_a} \sqrt{\frac{k}{n}} - u_{1-\alpha} \left[\frac{1}{N'} + (1 - a^2)K^2\right]^{\frac{1}{2}}\right\} \qquad \text{- E.5}$$

In the same way, for the risk of the second type, calling β the probability of accepting (wrongly) a just unacceptable delivery that is, a delivery for which:

$$\mu - \frac{u_{1-r_r}\,\underline{\sigma}}{\sqrt{n}} = L$$

$$\underline{\ell} = L\,\frac{u_{1-r_r}\,\underline{\sigma}}{\sqrt{n}} - \frac{a\,K\,\sigma}{\sqrt{k}} + \frac{u_{1-\beta}\underline{\sigma}}{\sqrt{k}}\left[\frac{1}{N'} + (1-a^2)\,K^2\right]^{\frac{1}{2}}$$

which, for $\underline{\ell}$ = L, requires the following:

$$K = \frac{1}{a}\left\{u_{1-r_r}\frac{k}{\sqrt{n}} + u_{1-\underline{\beta}}\left[\frac{1}{N'} + (1-a^2)K^2\right]^{\frac{1}{2}}\right\} \qquad \text{- E.6}$$

Equating E.5 and E.6 gives:

$$(u_{1-r_a} - u_{1-r_r})\frac{k}{\sqrt{n}} = (u_{1-\underline{\alpha}} + u_{1-\underline{\beta}})\left[\frac{1}{N'} + (1-a^2)K^2\right]^{\frac{1}{2}} \qquad \text{- E.7}$$

Deducing from E.7 the value of the expression raised to the power "1/2" and substituting back into (E.5) gives:

$$K = \frac{1}{a}\frac{k}{\sqrt{n}}\left[\frac{u_{1-\underline{\alpha}}\,u_{1-r_r} + u_{1-\underline{\beta}}\,u_{1-r_a}}{u_{1-\underline{\alpha}} + u_{1-\underline{\beta}}}\right] \qquad \text{- E.8}$$

Replacing *K* by E.8 in the equation E.7 gives:

$$(u_{1-r_a} - u_{1-r_r})^2\frac{k}{n} = \frac{(u_{1-\alpha} + u_{1-\beta})^2}{N'} + \frac{k\,(1-a^2)}{a^2\,n}\left[u_{1-\underline{\alpha}}\,u_{1-r_r} + u_{1-\underline{\beta}}\,u_{1-r_a}\right]^2$$

hence (as $kN' = N$):

$$N = (u_{1-\underline{\alpha}} + u_{1-\underline{\beta}})^2\left[\frac{(u_{1-r_a} - u_{1-r_r})^2}{n} - \frac{1-a^2}{na^2}(u_{1-\underline{\alpha}}\,u_{1-r_r} + u_{1-\underline{\beta}}\,u_{1-r_a})^2\right]^{-1}$$

Given that *N* and *N'* are to be whole numbers, this becomes in fact:

$$N \geq (u_{1-\underline{\alpha}} + u_{1-\underline{\beta}})^2\left[\frac{(u_{1-r_a} - u_{1-r_r})^2}{n} - \frac{1-a^2}{na^2}(u_{1-\underline{\alpha}}\,u_{1-r_r} + u_{1-\underline{\beta}}\,u_{1-r_a})^2\right]^{-1} \qquad \text{- E.9}$$

Moreover, by definition $\underline{a}$, and consequently $\frac{1 - a^2}{a^2}$, is only a function of N'.

This relation has been calculated and is shown in the table in annex B for values of N' between 5 and 30.

Consequently, the inequality E.9 can be expressed in the form:

$$kN' \geq g\ (N') \text{ or } k \geq f\ (N')$$

And therefore, for each whole value of k, the whole value of N' can be chosen, such that this inequality is just satisfied.

However, k must be positive, so the choice of N' is restricted to those values for which

$$\frac{1 - a^2}{a^2} \leq \left[\frac{u_{1-r_a} - u_{1-r_r}}{u_{1-\underline{\alpha}}\, u_{1-r_r} + u_{1-\underline{\beta}}\, u_{1-r_a}}\right]^2 \qquad \text{- E.10}$$

Having calculated the limit value of $\frac{1 - a^2}{a^2}$ with the aid of E.10, the table in annex B allows the determination of the smallest possible value of N', i.e. N'_0.

The smallest values of N' greater than N'_0 which will satisfy the inequality E.9 are then determined with the chosen values of k.

N is deduced from the known value of N' and k.

6.4.2 ***Simplified calculation***. The calculation can be greatly simplified as soon as N' becomes quite large (i.e. more than 30).

It is then possible to approximate $\frac{1 - a^2}{a^2}$ to $\frac{1}{2\ N'}$ and it is sufficient to use the formula:

$$N \geq n \left[\frac{u_{1-\underline{\alpha}} + u_{1-\underline{\beta}}}{u_{1-r_a} - u_{1-r_r}}\right]^2 \left(1 + \frac{K^2}{2}\right)$$

where

$$K^2 = \frac{k}{n} \left[\frac{u_{1-r_a}\, u_{1-\underline{\beta}} + u_{1-r_r}\, u_{1-\underline{\alpha}}}{u_{1-\underline{\alpha}} + u_{1-\underline{\beta}}}\right]^2$$

7 Practical procedure for the determination of *N* and *N'*

7.1 Basic information

Local legislation defines n which is the minimum number of bags for constituting the sample required for inspecting the quality of a lot of size N.

The importer, in agreement with the exporter, fixes the values for the probability levels α, β, r_a and r_r; these values, although expressed in percentage form have to be converted into fractions for calculation purposes.

7.2 Calculation

7.2.1 Table 9 give the values of u (standardized normal variable) corresponding to the probabilities r_a, r_r, $1-\alpha$ and $1-\beta$ i.e.

$$u_{1-r_a},\ u_{1-r_r},\ u_{1-\alpha} \text{ and } u_{1-\beta}.$$

7.2.2 To determine the minimum value of N', calculate:

$$\left(\frac{1-a^2}{a^2}\right)_0 = \left[\frac{u_{1-r_a} - u_{1-r_r}}{u_{1-\alpha}\, u_{1-r_r} + u_{1-\beta}\, u_{1-r_a}}\right]^2$$

Using table 10, determine the value of N' such that the ratio $\frac{1-a^2}{a^2}$ corresponding to this value of N' is just less than $\left(\frac{1-a^2}{a^2}\right)_0$ calculated above. Taking N'_0 to be that particular value of N'; this is the smallest possible value for N'.

7.2.3 Leaving $\frac{1-a^2}{a^2}$ undetermined, calculate the expression:

$$F = n\left(u_{1-\alpha} + u_{1-\beta}\right)^2 \left[\left(u_{1-r_a} - u_{1-r_r}\right)^2 - \frac{1-a^2}{a^2}\left(u_{1-\alpha}\, u_{1-r_r} + u_{1-\beta}\, u_{1-r_a}\right)^2\right]^{-1}$$

With the aid of the table in annex B, determine the values of $\frac{1-a^2}{a^2}$ for $N' = N'_0$, $N'_0 + 1 N'_0 + 2$ etc. and calculate for each of these values of N' the corresponding values of F and hence of k as k is the whole part of $\left(\frac{F}{N'} + 1\right)$ and thus N as $N = kN'$.

The calculation is concluded when a value of N' is obtained (N'm) greater than the corresponding value of F.

7.3 Simplified calculation when $N' > 30$

N is calculated as a function of k from the fixed values of n, $u_{1-\alpha}$, $u_{1-\beta}$, u_{1-r_a}, u_{1-r_r} by the formulae in 6.4.2.

Draw up the following table noting that $N' = \frac{N}{k}$ and therefore that the value of N multiple of k immediately above the calculated figure has to be chosen.

Table 1

k	N'	N
1	Z_1	Z_1
2	$Z_2/2$	Z_2
3	$Z_3/3$	Z_3
4	$Z_4/4$	Z_4
5	$Z_5/5$	Z_5

Transfer all the results to table 2:

Table 2

N'	$\frac{1-a^2}{a^2}$	F	k	N
N'_0				
$N'_0 + 1$				
$N'_0 + 2$				
....				
....				
$N'_0 +$				

With the aid of table 2, all possible pairs of N and N' may be obtained.

For practical application, these possible pairs of N and N' may be collected in a simplified table (see table 3).

Table 3

N	N'

8 Examples of calculations

8.1 Calculation by the complete process

8.1.1 ***General***. The smallest lots which the importer will resell will be lots of one bag (N = 1) for which particular content levels will be guaranteed, for example, 25 % ammonical nitrogen (L = 25 %).

Clearly local regulations stipulate, for lots of 1 bag, that a sample of 1 bag (n = 1) is taken, the content ($\bar{x}$) of which will be compared with the limit (L); and if $\bar{x} < L$, the lot of 1 bag will be considered as defective and action will be taken against the importer.

The importer decides to consider as acceptable a delivery for which the probability, r_a, of making up, from this delivery, a defective lot of 1 bag would be at most 1 %. That is to say that the importer agrees to sell one defective lot of 1 bag out of 100 lots of 1 bag (i.e. r_a = 1 % or 0,01).

In addition, he decides to consider as unacceptable a delivery for which the probability of making up, from this delivery, a defective lot of 1 bag would be equal to or greater than 10 % (i.e r_r = 10 % or 0,10).

Finally, he accepts the value 5 % as the risk of wrongly accepting a delivery which should have been considered unacceptable (i.e. β = 5 % or 0,05).

For his part, the seller agrees to the value 5 % as the risk of having a delivery wrongly rejected which should have been considered acceptable (i.e. α = 5 % or 0,05).

The problem is to determine the number, N, of bags to be selected from the delivery and the number N' of analyses to be carried out on these bags in order to achieve all these conditions. According to the definitions above, we have:

N = 1

n = 1

L = 25 % ammoniacal nitrogen

r_a = 1 %, hence u_{1-r_a} = 2,326 (see table A.1)

r_r = 10 %, hence u_{1-r_r} = 1,282 (see table A.1)

$\underline{\alpha}$ = 5 %, hence $u_{1-\underline{\alpha}}$ = 1,645 (see table A.1)

$\underline{\beta}$ = 5 %, hence $u_{1-\underline{\beta}}$ = 1,645 (see table A.1)

8.1.2 Minimum value of *N'*

The equation:

$$\left(\frac{1-a^2}{a^2}\right)_0 = \left[\frac{u_{1-r_a} - u_{1-r_r}}{u_{1-\underline{\alpha}}\, u_{1-r_r} + u_{1-\underline{\beta}}\, u_{1-r_a}}\right]^2$$

gives:

$$\left(\frac{1-a^2}{a^2}\right) = 0{,}0309$$

Table B.1 shows that this implies $N' > 17$ i.e. $N'_0 = 18$.

8.1.3 Calculation of *F*

In view of the values given to the parameters:

n, u_{1-r_a}, u_{1-r_r}, $u_{1-\underline{\alpha}}$, $u_{1-\underline{\beta}}$ and

$$F = n\left(u_{1-\underline{\alpha}} + u_{1-\underline{\beta}}\right)^2 \left[\left(u_{1-r_a} - u_{1-r_r}\right)^2 - \frac{1-a^2}{a^2}\left(u_{1-\underline{\alpha}}\, u_{1-r_r} + u_{1-\underline{\beta}}\, u_{1-r_a}\right)^2\right]^{-1}$$

gives:

$$F = 10{,}8241\left[1{,}0899 - \frac{1-a^2}{a^2}(35{,}2261)\right]^{-1}$$

8.1.4 Calculation of *k* and *N*

For N' = 18:

$$\frac{1-a^2}{a^2} = 0{,}0299$$

hence, F = 295,42

and $\frac{F}{N'} + 1$ = 17,4 the whole part of which is 17 and thus k = 17 and consequently, $N = kN'$ = 306

For N' = 19:

$$\frac{1-a^2}{a^2} = 0{,}0282$$

hence, F = 112,14

and $\frac{F}{N'} + 1$ = 6,90 the whole part of which is 6 and thus k = 6 and consequently, N = 114.

These calculations result in the values listed in table 4.

Table 4

N'	$\frac{1 - a^2}{a^2}$	F	k	N
18	0,0299	295,4	17	306
19	0,0282	112,1	6	114
20	0,0267	72,9	4	80
21	0,0253	54,5	3	63
22	0,0241	44,9	3	66
23	0,0230	38,7	2	46
24	0,0220	34,4	2	48
25	0,0210	30,9	2	50
26	0,0202	28,6	2	52
27	0,0194	26,6	1	27

The list in table 4 stops at $N' = 27$, for which $F < N'$.

A series of pairs, N and N', is thus obtained (see table 5) which satisfy the stipulated conditions.

Table 5

N	N'
306	18
114	19
80	20
63	21
46	23
27	27

The choice between the various possible pairs, N and N', will depend on practical conditions, such as the respective marginal costs of sampling and analysis.

Thus, if 27 bags are selected, each of these should be analysed individually. If, on the other hand, 46 bags are selected, they could be grouped in pairs, so that there would only be 23 analyses to perform.

At the limit, this would be beneficial if the cost of an analysis was 5 or more times as high as that of the taking of one more sample, but more extensive grouping of increments with a view to reducing the number of analyses would not, in financial terms, be worth the trouble.

8.2 Simplified calculation

With the same values for the parameters as above,

$$K = \frac{2,326 \times 1,645 + 1,282 \times 1,645}{1,645 + 1,645} \sqrt{k}$$

$$\text{and } N \geq \left(\frac{1,645 + 1,645}{2,326 - 1,282}\right)^2 \left(1 + \frac{K^2}{2}\right)$$

hence $N \geq 9,92 + 16,27\ k$

These equations lead to the following table:

Table 6

k	N'	N
1	27	27
2	22	44
3	20	60
4	19	76
6	18	108
17	17	289

9 Effect of the values of the various parameters on *N* and *K*

The following equation is obtained by the simplified calculation procedure:

$$N = n \left[\frac{u_{1-\alpha} + u_{1-\beta}}{u_{1-r_a} - u_{1-r_r}}\right]^2 \left[1 + \frac{k}{2n} \left\{ \left(u_{1-r_r} + u_{1-\beta} \frac{u_{1-r_a} - u_{1-r_r}}{u_{1-\alpha} + u_{1-\beta}} \right)^2 \right\}\right]$$

that is to say it is of the form:

$\frac{A}{X^2} + \frac{B}{X} + C$ in $(u_{1-r_a} - u_{1-r_r})$ (with A, B and $C > 0$);

or $AX^2 + BX + C$ in $(u_{1-\alpha} + u_{1-\beta})$ (with A, B and $C > 0$);

or $AX^2 + B$ (with A and $B > 0$) in $(u_{1-r_a} - u_{1-r_r})$

Similarly,

$$K = \frac{u_{1-r_a}\, u_{1-\beta} + u_{1-r_r}\, u_{1-\alpha}}{u_{1-\alpha} + u_{1-\beta}} \sqrt{\frac{k}{n}} \text{ or } \left(u_{1-r_r} + \frac{(u_{1-r_a} - u_{1-r_r})\, u_{1-\beta}}{u_{1-\alpha} + u_{1-\beta}} \right) \sqrt{\frac{k}{n}}$$

which is of the form '$\frac{A}{X} + B$' in '$(u_{1-\alpha} + u_{1-\beta})$' or' $AX + B$' in '$(u_{1-r_a} - u_{1-r_r})$'

Table 7 summarizes the effect of a changing the various parameters:

Table 7

Variation in the parameters	*N* (sampling cost)	*K* (stringency of the inspection)
Reduction in the difference between the quality levels: r_a and r_r	Increases very rapidly	Increases (more rapidly if r_r decreases than if r_a increases)
Reduction in the sum of risks α and β	Increases very rapidly and increasingly so if the risks are initially greater	Increases relatively slowly (if β decreases more quickly than α)
Reduction in the sum of defective percentages corresponding to the two quality levels $r_a + r_r$	Increases very rapidly	Increases fairly rapidly if r_r decreases more quickly than r_a
Increase in *n*	Increases linearly as a function of *n*	Decreases in proportion to the square root of *n*

The effect of the various parameters is shown in numerical terms in table 8 which gives values of N, N' and K for two values of k based on the simplified calculation. It should be noted, however, that each set of values for the parameters leads to a complete series of values for N and N' all of which will fulfil the requirements of the theory (see 7.2.4 and table 6).

Table 8 – Effect of the various parameters on the values of N, N' and K										
					For $k = 1$			For $k = 5$		
n	α 1)	β 1)	r_a 1)	r_r 1)	N	N'	K	N	N'	K
1	1	5	0,5	5	56	56	2,03	205	41	4,54
1	1	5	0,5	10	25	25	1,82	90	18	4,07
1	1	5	1	5	98	98	1,93	350	70	4,31
1	1	10	0,5	5	45	45	1,98	160	32	4,42
1	1	10	0,5	10	20	20	1,74	65	13	3,90
1	5	5	0,5	5	41	41	2,11	150	30	4,72
1	5	5	0,5	10	19	19	1,93	65	13	4,31
1	5	5	1	10	27	27	1,80	90	18	4,03
1	5	10	0,5	10	14	14	1,85	50	10	4,13
5	1	5	0,5	5	129	129	0,91	280	56	2,03
5	1	5	0,5	10	63	63	0,81	125	25	1,82
10	1	1	0,5	5	305	305	0,67	525	105	1,49
10	1	5	0,5	5	220	220	0,64	370	74	1,44
10	5	10	1	10	91	91	0,55	140	28	1,23

1) The values are expressed as percentages (see 7.1).

10 Evaluation of a delivery

For each plant nutrient guaranteed a Student test is applied, using the N' analysis results obtained, calculating successively the following intermediate values A and B:

$$A = \sum_{i=1}^{i=N'} (X_i - \bar{X})^2 \quad \text{and} \quad B = \frac{\bar{X} - L}{\sqrt{A}}$$

where

$\bar{X}$ is the mean of the analytical results calculated to 2 decimal places;

X_i is the analytical result obtained on the aggregate sample of rank i;

L is the official limit as defined in clause 3.

The delivery does not comply with the evaluation criteria chosen for a given plant nutrient if the value B corresponding to this plant nutrient is less than the value B_0 which is given, as a function of the number of analyses N', of N increments and of u_{1-r_a} and n.

$$\frac{\bar{X} - L}{s} \sqrt{N'}$$

corresponds to the limit non-central Student ratio

corresponding to a probability level α, with the non-centrality parameter equal to:

$$\sqrt{N} \frac{u_{1-r_a}}{\sqrt{n}}$$

The value B_0 is then calculated from the equation:

$$B_0 = \frac{t_0}{\sqrt{N' (N' - 1)}}$$

Annex A

Table A.1 - Values of the standardized normal variable u as a function of P

P	,000	,001	,002	,003	,004	,005	,006	,007	,008	,009	,010	
,00	∞	3,0902	2,8782	2,7478	2,6521	2,5758	2,5121	2,4573	2,4089	2,3656	2,3263	,99
,01	2,3263	2,2904	2,2571	2,2262	2,1973	2,1701	2,1444	2,1201	2,0969	2,0749	2,0537	,98
,02	2,0537	2,0335	2,0141	1,9954	1,9774	1,9600	1,9431	1,9268	1,9110	1,8957	1,8808	,97
,03	1,8808	1,8663	1,8522	1,8384	1,8250	1,8119	1,7991	1,7866	1,7744	1,7624	1,7507	,96
,04	1,7507	1,7392	1,7279	1,7169	1,7060	1,6954	1,6849	1,6747	1,6646	1,6546	1,6449	,95
,05	1,6449	1,6352	1,6258	1,6164	1,6072	1,5982	1,5893	1,5805	1,5718	1,5632	1,5548	,94
,06	1,5548	1,5464	1,5382	1,5301	1,5220	1,5141	1,5063	1,4085	1,4909	1,4833	1,4758	,93
,07	1,4758	1,4684	1,4611	1,4538	1,4466	1,4395	1,4325	1,4255	1,4187	1,4118	1,4051	,92
,08	1,4051	1,3984	1,3917	1,3852	1,3787	1,3722	1,3658	1,3595	1,3532	1,3469	1,3408	,91
,09	1,3408	1,3346	1,3285	1,3225	1,3165	1,3106	1,3047	1,2988	1,2930	1,2873	1,2816	,90
,10	1,2816	1,2759	1,2702	1,2646	1,2591	1,2536	1,2481	1,2426	1,2372	1,2319	1,2265	,89
,11	1,2265	1,2212	1,2160	1,2107	1,2055	1,2004	1,1952	1,1901	1,1850	1,1800	1,1750	,88
,12	1,1750	1,1700	1,1650	1,1601	1,1552	1,1503	1,1455	1,1407	1,1359	1,1311	1,1264	,87
,13	1,1264	1,1217	1,1170	1,1123	1,1077	1,1031	1,0985	1,0939	1,0893	1,0848	1,0803	,86
,14	2,0803	1,0758	1,0714	1,0669	1,0625	1,0581	1,0537	1,0494	1,0450	1,0407	1,0364	,85
,15	1,0364	1,0322	1,0279	1,0237	1,0194	1,0152	1,0110	1,0069	1,0027	0,9986	0,9945	,84
,16	0,9945	0,9904	0,9863	0,9822	0,9782	0,9741	0,9701	0,9661	0,9621	0,9581	0,9542	,83
,17	0,9542	0,9502	0,9463	0,9424	0,9385	0,9346	0,9307	0,9269	0,9230	0,9192	0,9154	,82
,18	0,9154	0,9116	0,9078	0,9040	0,9002	0,8965	0,8927	0,8890	0,8853	0,8816	0,8779	,81
,19	0,8779	0,8742	0,8705	0,8669	0,8633	0,8596	0,8560	0,8524	0,8488	0,8452	0,8416	,80
,20	0,8416	8,8381	0,8345	0,8310	0,8274	0,8239	0,8204	0,8169	0,8134	0,8099	0,8064	,79
,21	0,8064	0,8030	0,7995	0,7961	0,7926	0,7892	0,7858	0,7824	0,7790	0,7756	0,7722	,78
,22	0,7722	0,7688	0,7655	0,7621	0,7588	0,7554	0,7521	0,7488	0,7454	0,7421	0,7388	,77
,23	0,7388	0,7356	0,7323	0,7290	0,7257	0,7225	0,7192	0,7160	0,7128	0,7095	0,7063	,76
,24	0,7063	0,7031	0,6999	0,6967	0,6935	0,6903	0,6871	0,6840	0,6808	0,6776	0,6745	,75
,25	0,6745	0,6713	0,6682	0,6651	0,6620	0,6588	0,6557	0,6526	0,6495	0,6464	0,6433	,74
,26	0,6433	0,6403	0,6372	0,6341	0,6311	0,6280	0,6250	0,6219	0,6189	0,6158	0,6128	,73
,27	0,6128	0,6098	0,6068	0,6038	0,6008	0,5978	0,5948	0,5918	0,5888	0,5858	0,5828	,72
,28	0,5828	0,5799	0,5769	0,5740	0,5710	0,5681	0,5651	0,5622	0,5592	0,5563	0,5534	,71
,29	0,5534	0,5505	0,5476	0,5446	0,5417	0,5388	0,5359	0,5330	0,5302	0,5273	0,5244	,70
,30	0,5244	0,5215	0,5187	0,5158	0,5129	0,5101	0,5072	0,5044	0,5015	0,4987	0,4959	,69
,31	0,4959	0,4930	0,4902	0,4874	0,4845	0,4817	0,4789	0,4761	0,4733	0,4705	0,4677	,68
,32	0,4677	0,4619	0,4621	0,4593	0,4565	0,4538	0,4510	0,4482	0,4454	0,4427	0,4399	,67
,33	0,4399	0,4372	0,4344	0,4316	0,4289	0,4261	0,4234	0,4207	0,4179	0,4152	0,4125	,66
,34	0,4125	0,4097	0,4070	0,4043	0,4016	0,3989	0,3961	0,3934	0,3907	0,3880	0,3853	,65
,35	0,3853	0,3826	0,3799	0,3772	0,3745	0,3719	0,3692	0,3665	0,3638	0,3611	0,3585	,64
,36	0,3585	0,3558	0,3531	0,3505	0,3478	0,3451	0,3425	0,3398	0,3372	0,3345	0,3319	,63
,37	0,3319	0,3292	0,3266	0,3239	0,3213	0,3186	0,3160	0,3134	0,3107	0,3081	0,3055	,62
,38	0,3055	0,3029	0,3002	0,2976	0,2950	0,2924	0,2898	0,2871	0,2845	0,2819	0,2793	,61
,39	0,2793	0,2767	0,2741	0,2715	0,2689	0,2663	0,2637	0,2611	0,2585	0,2559	0,2533	,60
,40	0,2533	0,2508	0,2482	0,2456	0,2430	0,2404	0,2378	0,2353	0,2327	0,2301	0,2275	,59
,41	0,2275	0,2250	0,2224	0,2198	0,2173	0,2147	0,2121	0,2096	0,2070	0,2045	0,2019	,58
,42	0,2019	0,1993	0,1968	0,1942	0,1917	0,1891	0,1866	0,1840	0,1815	0,1789	0,1764	,57
,43	0,1764	0,1738	0,1713	0,1687	0,1662	0,1637	0,1611	0,1586	0,1560	0,1535	0,1510	,56
,44	0,1510	0,1484	0,1459	0,1434	0,1408	0,1383	0,1358	0,1332	0,1307	0,1282	0,1257	,55
,45	0,1257	0,1231	0,1206	0,1181	0,1156	0,1130	0,1105	0,1080	0,1055	0,1030	0,1004	,54
,46	0,1004	0,0979	0,0954	0,0929	0,0904	0,0878	0,0853	0,0828	0,0803	0,0778	0,0753	,53
,47	0,0753	0,0728	0,0702	0,0677	0,0652	0,0627	0,0602	0,0577	0,0552	0,0527	0,0502	,52
,48	0,0502	0,0476	0,0451	0,0126	0,0401	0,0376	0,0351	0,0326	0,0301	0,0276	0,0251	,51
,49	0,0251	0,0226	0,0201	0,0176	0,0150	0,0125	0,0100	0,0075	0,0050	0,0025	0,0000	,50

For example: for a probability of 10,5 % (P = 0,105)

u = 1,2536

Annex B

Table B.1 – Values of $(1 - a^2)/a^2$

N	a	$(1 - a^2)/a^2$
5	,9400	,1317
6	,9515	,1045
7	,9594	,0865
8	,9650	,0738
9	,9693	,0643
10	,9727	,0570
11	,9753	,0512
12	,9776	,0464
13	,9794	,0425
14	,9810	,0392
15	,9823	,0363
16	,9835	,0338
17	,9845	,0317
18	,9854	,0299
19	,9862	,0282
20	,9869	,0267
21	,9876	,0253
22	,9882	,0241
23	,9887	,0230
24	,9892	,0220
25	,9896	,0210
26	,9901	,0202
27	,9904	,0194
28	,9908	,0187
29	,9911	,0180
30	,9914	,0174

UDC 631.8:519.243

Descriptors: fertilizers, sampling, sampling tables, statistical distribution, standard deviation, rules of calculation.

Price based on 18 pages