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## Cylindrical cavity method to measure the complex permittivity of low-loss dielectric rods

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## Cylindrical cavity method to measure the complex permittivity of low-loss dielectric rods

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# CYLINDRICAL CAVITY METHOD TO MEASURE THE COMPLEX PERMITTIVITY OF LOW-LOSS DIELECTRIC RODS

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The text of this standard is based on the following documents:

| CDV | Report on voting |
|---|---|
| 46F/242/CDV | 46F/260/RVC |

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# CYLINDRICAL CAVITY METHOD TO MEASURE THE COMPLEX PERMITTIVITY OF LOW-LOSS DIELECTRIC RODS

## 1 Scope

This International Standard relates to a measurement method for complex permittivity of a dielectric rod at microwave frequency. This method has been developed to evaluate the dielectric properties of low-loss materials in coaxial cables and electronic devices used in microwave systems. It uses the $TM_{010}$ mode in a circular cylindrical cavity and presents accurate measurement results of a dielectric rod sample, where the effect of sample insertion holes is taken into account accurately on the basis of the rigorous electromagnetic analysis.

In comparison with the conventional method described in IEC 60556 [2][1], this method has the following characteristics:

- the values of the relative permittivity $\varepsilon'$ and loss tangent $\tan\delta$ of a dielectric rod sample can be measured accurately and non-destructively;
- the measurement accuracy is within 1,0 % for $\varepsilon'$ and within 20 % for $\tan\delta$;
- the effect of sample insertion holes is corrected using correction charts presented;
- this method is applicable for the measurements on the following condition:
  - frequency: 1 GHz $\leqq f \leqq$ 10 GHz;
  - relative permittivity: $1 \leqq \varepsilon' \leqq 100$;
  - loss tangent: $10^{-4} \leqq \tan\delta \leqq 10^{-1}$.

## 2 Normative references

Void.

## 3 Measurement parameters

The measurement parameters are defined as follows:

$$\varepsilon_r = \varepsilon' - j\varepsilon'' \tag{1}$$

$$\tan\delta = \varepsilon''/\varepsilon' \tag{2}$$

where $\varepsilon'$ and $\varepsilon''$ are the real and imaginary parts of the complex relative permittivity $\varepsilon_r$.

## 4 Theory and calculation equations

A resonator structure used in these measurements is shown in Figure 1. A cavity, made with copper, with diameter $D$ and height $H$ has sample insertion holes with diameter $d_2$ and depth $g$ oriented coaxially. A dielectric rod sample of diameter $d_1$ having $\varepsilon'$ and $\tan\delta$ is inserted into the holes.

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[1] Figures in square brackets refer to the Bibliography.

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The $TM_{010}$ mode, where the electric field component in the cavity is parallel to the sample rod, is used for the measurement. Taking account of the effect of sample insertion holes calculated on the basis of the rigorous electromagnetic field analysis, $\varepsilon'$ and $\tan\delta$ are determined from the measured values of the resonant frequency $f_0$ and the unloaded $Q$-factor $Q_u$. To avoid the tedious numerical calculation and make the measurements easy, the following process is taken in this measurement:

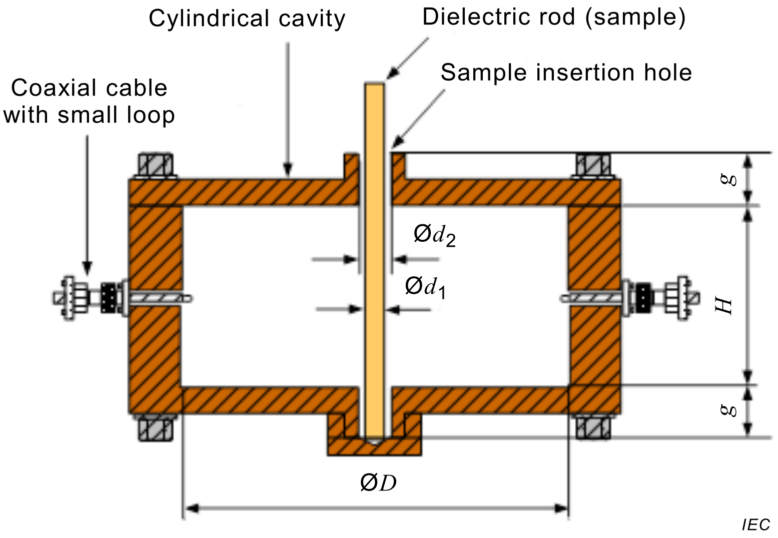


**Figure 1 – Structure of a cylindrical cavity resonator**

The following steps shall be taken:

1) At the first step, obtain approximate values $\varepsilon_p$ and $\tan\delta_p$ from the $f_0$ and $Q_u$ values by using the simple perturbation formulas, where the effect of sample insertion holes is neglected. The subscript p denotes the calculated values using the following perturbation formulas:

$$\varepsilon_p = \frac{1}{\alpha}\frac{f_0 - f_1}{f_1}\left(\frac{D}{d_1}\right)^2 + 1 \tag{3}$$

$$\tan\delta_p = \frac{1}{2\alpha\varepsilon_p}\left(\frac{D}{d_1}\right)^2\left(\frac{1}{Q_{u1}} - \frac{1}{Q_{u0}}\right) \tag{4}$$

where $\alpha = 1/J_1(x_{01})^2 = 1{,}855$.

$J_n(x)$ is the Bessel function of order n of first kind and $x_{01} = 2{,}405$ is the first root of $J_0(x) = 0$. $f_0$ and $Q_{u0}$ are the resonant frequency and unloaded $Q$-factor measured for the cavity without a sample, respectively. $f_1$ and $Q_{u1}$ are ones measured for the cavity with a sample.

2) In the second step, obtain accurate values $\varepsilon'$ and $\tan\delta$ from $\varepsilon_p$ and $\tan\delta_p$ values by using the following equations with correction factors calculated based on the rigorous analysis:

$$\varepsilon' = C_1\,\varepsilon_p \tag{5}$$

$$\tan\delta = C_2\tan\delta_p \tag{6}$$

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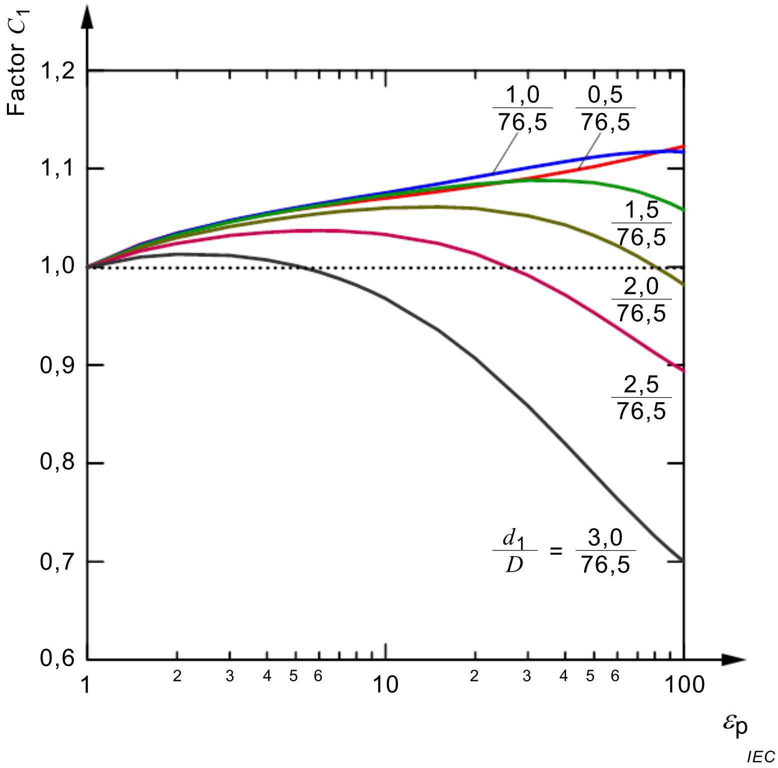
where correction factors $C_1$ and $C_2$, due to the sample insertion holes and errors included in the perturbation formulas, are calculated numerically by using the Ritz-Galerkin method [3][5], as shown in Figure 2 and Figure 3, and the corresponding data are listed in detail in Table 1, 2, and 3. The missing data of $C_1$ and $C_2$ can be obtained by interpolation or extrapolation from the tables. The correction factors shown in these figures are calculated for the cavity with $D$ = 76,5 mm, $H$ = 20,0 mm, $d_2$ = 3,0 mm, and $g$ = 10,0 mm, where the resonant frequency is about 3 GHz. $C_1$ is also used for a cavity having the same aspect ratios as $H/D$, $d_2/D$ and $g/D$.

It is found from the analysis for a cavity with insertion holes which constitute a cut-off $TM_{01}$ mode cylindrical waveguide that $f_0$ converges to a constant value for $g$>10 mm and $d_2$ = 3 mm. Therefore, the correction factors shown in Figure 2 and Figure 3 are applicable to a dielectric sample rod with $d_1$<3 mm and $\varepsilon'$ below the value calculated by the following equation for the measured value of the resonant frequency:

$$\varepsilon' \leq \left( \frac{x_{01} c}{\pi d_2 f_0} \right)^2 \tag{7}$$

where $c$ is the velocity of light in a vacuum ($c$ = 2,9 979 × 108 m/s).




**Assumptions**

| | | | |
|---|---|---|---|
| $D$ | 76,5 mm | $d_2$ | 3,0 mm |
| $H$ | 20,0 mm | $g$ | 10,0 mm |

**Figure 2 – Correction factor $C_1$ for $\varepsilon'$**

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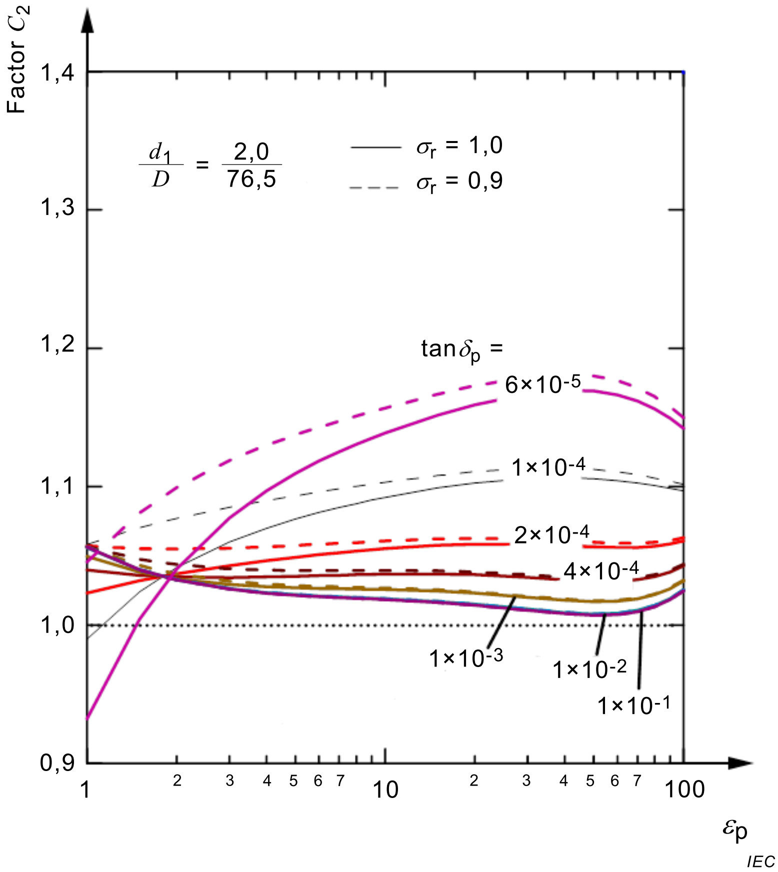
**Table 1 – Numerical values of correction factor $C_1$**

| $\varepsilon_p$ | $d_1$(mm) | | | | | |
|---|---|---|---|---|---|---|
| | 0, 5 | 1, 0 | 1, 5 | 2, 0 | 2, 5 | 3, 0 |
| 1 | 1, 000 | 1, 000 | 1, 000 | 1, 000 | 1, 000 | 1, 000 |
| 1, 5 | 1, 023 | 1, 022 | 1, 021 | 1, 019 | 1, 016 | 1, 010 |
| 2 | 1, 035 | 1, 034 | 1, 033 | 1, 030 | 1, 024 | 1, 013 |
| 3 | 1, 047 | 1, 047 | 1, 046 | 1, 041 | 1, 032 | 1, 012 |
| 4 | 1, 054 | 1, 055 | 1, 053 | 1, 047 | 1, 035 | 1, 007 |
| 5 | 1, 058 | 1, 060 | 1, 059 | 1, 051 | 1, 037 | 1, 001 |
| 6 | 1, 061 | 1, 064 | 1, 063 | 1, 054 | 1, 037 | 0, 995 |
| 7 | 1, 064 | 1, 068 | 1, 066 | 1, 056 | 1, 037 | 0, 988 |
| 8 | 1, 066 | 1, 071 | 1, 069 | 1, 058 | 1, 036 | 0, 981 |
| 9 | 1, 068 | 1, 073 | 1, 071 | 1, 059 | 1, 035 | 0, 975 |
| 10 | 1, 070 | 1, 076 | 1, 073 | 1, 060 | 1, 033 | 0, 968 |
| 15 | 1, 077 | 1, 085 | 1, 080 | 1, 061 | 1, 024 | 0, 936 |
| 20 | 1, 082 | 1, 091 | 1, 084 | 1, 060 | 1, 013 | 0, 907 |
| 30 | 1, 090 | 1, 101 | 1, 088 | 1, 052 | 0, 992 | 0, 859 |
| 40 | 1, 097 | 1, 107 | 1, 088 | 1, 043 | 0, 971 | 0, 820 |
| 50 | 1, 102 | 1, 112 | 1, 086 | 1, 032 | 0, 953 | 0, 789 |
| 60 | 1, 107 | 1, 115 | 1, 082 | 1, 021 | 0, 938 | 0, 764 |
| 70 | 1, 112 | 1, 117 | 1, 077 | 1, 011 | 0, 924 | 0, 743 |
| 80 | 1, 116 | 1, 118 | 1, 071 | 1, 001 | 0, 912 | 0, 726 |
| 90 | 1, 119 | 1, 118 | 1, 065 | 0, 991 | 0, 903 | 0, 712 |
| 100 | 1, 123 | 1, 117 | 1, 058 | 0, 982 | 0, 894 | 0, 700 |




a) Dielectric sample rod with $d_1$ = 2,0 mm

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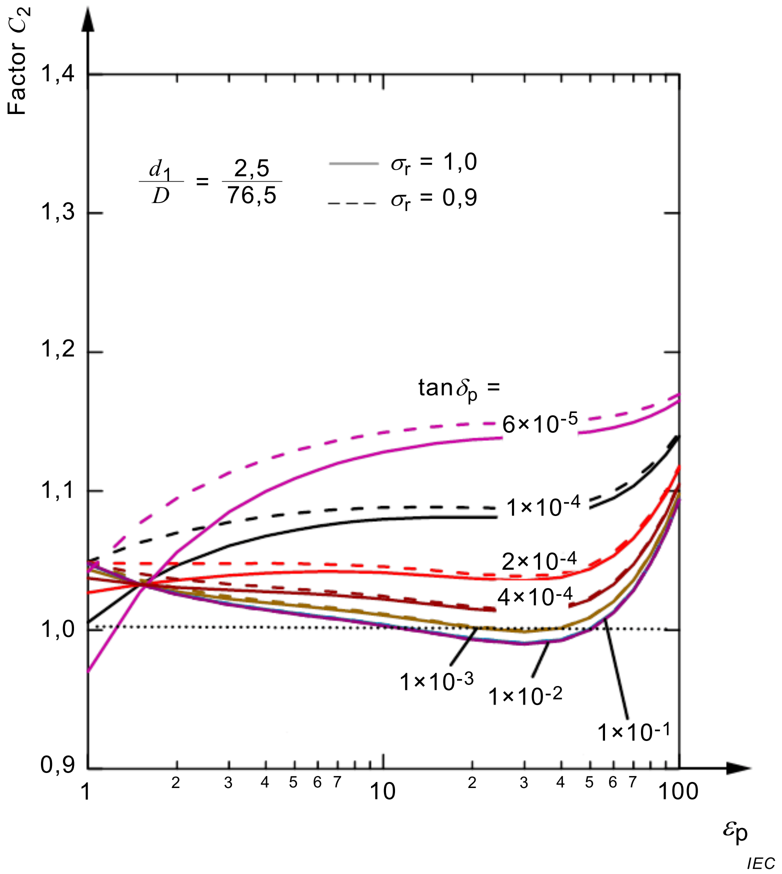


**b) Dielectric sample rod with $d_1$ = 2,5 mm**

**Assumptions**

| | | | |
|---|---|---|---|
| $D$ | 76,5 mm | $d_2$ | 3,0 mm |
| $H$ | 20,0 mm | $g$ | 10,0 mm |

**Figure 3 – Correction factor $C_2$ for tan$\delta$ with the different values of $d_1$**

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**Table 2 – Numerical values of correction factor $C_2$**

(Dielectric sample rod with $d_1$ = 2,0 mm)

$\sigma_r$=0,9

| $\varepsilon_p$ | $\tan\delta_p$ | | | | | | |
|---|---|---|---|---|---|---|---|
| | $6\times10^{-5}$ | $1\times10^{-4}$ | $2\times10^{-4}$ | $4\times10^{-4}$ | $1\times10^{-3}$ | $1\times10^{-2}$ | $1\times10^{-1}$ |
| 1 | 1, 045 | 1, 058 | 1, 057 | 1, 057 | 1, 057 | 1, 056 | 1, 056 |
| 1, 5 | 1, 081 | 1, 070 | 1, 055 | 1, 048 | 1, 043 | 1, 040 | 1, 040 |
| 2 | 1, 099 | 1, 077 | 1, 055 | 1, 044 | 1, 037 | 1, 033 | 1, 033 |
| 3 | 1, 119 | 1, 085 | 1, 055 | 1, 041 | 1, 032 | 1, 026 | 1, 026 |
| 4 | 1, 130 | 1, 090 | 1, 056 | 1, 040 | 1, 030 | 1, 024 | 1, 023 |
| 5 | 1, 137 | 1, 093 | 1, 057 | 1, 039 | 1, 029 | 1, 022 | 1, 021 |
| 6 | 1, 143 | 1, 096 | 1, 058 | 1, 039 | 1, 028 | 1, 021 | 1, 020 |
| 7 | 1, 147 | 1, 098 | 1, 059 | 1, 039 | 1, 028 | 1, 020 | 1, 020 |
| 8 | 1, 151 | 1, 100 | 1, 060 | 1, 039 | 1, 027 | 1, 020 | 1, 019 |
| 9 | 1, 154 | 1, 102 | 1, 060 | 1, 039 | 1, 027 | 1, 019 | 1, 019 |
| 10 | 1, 157 | 1, 103 | 1, 061 | 1, 039 | 1, 027 | 1, 019 | 1, 018 |
| 15 | 1, 167 | 1, 108 | 1, 062 | 1, 039 | 1, 025 | 1, 017 | 1, 016 |
| 20 | 1, 173 | 1, 111 | 1, 063 | 1, 038 | 1, 024 | 1, 015 | 1, 014 |
| 30 | 1, 179 | 1, 113 | 1, 062 | 1, 036 | 1, 021 | 1, 012 | 1, 011 |
| 40 | 1, 181 | 1, 114 | 1, 061 | 1, 034 | 1, 019 | 1, 009 | 1, 008 |
| 50 | 1, 180 | 1, 113 | 1, 060 | 1, 033 | 1, 018 | 1, 008 | 1, 007 |
| 60 | 1, 177 | 1, 111 | 1, 059 | 1, 033 | 1, 018 | 1, 009 | 1, 008 |
| 70 | 1, 172 | 1, 109 | 1, 059 | 1, 034 | 1, 019 | 1, 011 | 1, 010 |
| 80 | 1, 165 | 1, 106 | 1, 060 | 1, 036 | 1, 022 | 1, 014 | 1, 013 |
| 90 | 1, 158 | 1, 104 | 1, 061 | 1, 040 | 1, 027 | 1, 019 | 1, 018 |
| 100 | 1, 150 | 1, 102 | 1, 063 | 1, 044 | 1, 032 | 1, 025 | 1, 025 |

$\sigma_r$=1,0

| $\varepsilon_p$ | $\tan\delta_p$ | | | | | | |
|---|---|---|---|---|---|---|---|
| | $6\times10^{-5}$ | $1\times10^{-4}$ | $2\times10^{-4}$ | $4\times10^{-4}$ | $1\times10^{-3}$ | $1\times10^{-2}$ | $1\times10^{-1}$ |
| 1 | 0, 932 | 0, 990 | 1, 023 | 1, 040 | 1, 050 | 1, 056 | 1, 056 |
| 1, 5 | 1, 004 | 1, 024 | 1, 032 | 1, 036 | 1, 038 | 1, 040 | 1, 040 |
| 2 | 1, 040 | 1, 042 | 1, 037 | 1, 035 | 1, 033 | 1, 033 | 1, 032 |
| 3 | 1, 077 | 1, 060 | 1, 043 | 1, 034 | 1, 029 | 1, 026 | 1, 026 |
| 4 | 1, 097 | 1, 070 | 1, 046 | 1, 035 | 1, 028 | 1, 023 | 1, 023 |
| 5 | 1, 110 | 1, 077 | 1, 049 | 1, 035 | 1, 027 | 1, 022 | 1, 021 |
| 6 | 1, 118 | 1, 081 | 1, 051 | 1, 036 | 1, 026 | 1, 021 | 1, 020 |
| 7 | 1, 125 | 1, 085 | 1, 052 | 1, 036 | 1, 026 | 1, 020 | 1, 020 |
| 8 | 1, 131 | 1, 088 | 1, 053 | 1, 036 | 1, 026 | 1, 020 | 1, 019 |
| 9 | 1, 135 | 1, 090 | 1, 054 | 1, 037 | 1, 026 | 1, 019 | 1, 019 |
| 10 | 1, 139 | 1, 092 | 1, 055 | 1, 037 | 1, 026 | 1, 019 | 1, 018 |
| 15 | 1, 152 | 1, 099 | 1, 058 | 1, 037 | 1, 024 | 1, 017 | 1, 016 |
| 20 | 1, 159 | 1, 103 | 1, 058 | 1, 036 | 1, 023 | 1, 015 | 1, 014 |
| 30 | 1, 167 | 1, 106 | 1, 058 | 1, 034 | 1, 020 | 1, 012 | 1, 011 |
| 40 | 1, 170 | 1, 107 | 1, 057 | 1, 033 | 1, 018 | 1, 009 | 1, 008 |
| 50 | 1, 169 | 1, 106 | 1, 056 | 1, 032 | 1, 017 | 1, 008 | 1, 007 |
| 60 | 1, 166 | 1, 104 | 1, 056 | 1, 032 | 1, 017 | 1, 008 | 1, 008 |
| 70 | 1, 162 | 1, 103 | 1, 056 | 1, 033 | 1, 019 | 1, 010 | 1, 010 |
| 80 | 1, 156 | 1, 101 | 1, 057 | 1, 035 | 1, 022 | 1, 014 | 1, 013 |
| 90 | 1, 150 | 1, 099 | 1, 059 | 1, 038 | 1, 026 | 1, 019 | 1, 018 |
| 100 | 1, 142 | 1, 097 | 1, 061 | 1, 043 | 1, 032 | 1, 025 | 1, 025 |

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**Table 3 – Numerical values of correction factor $C_2$**

(Dielectric sample rod with $d_1$ = 2,5 mm)

$\sigma_r$=0,9

| $\varepsilon_p$ | $\tan\delta_p$ | | | | | | |
|---|---|---|---|---|---|---|---|
| | $6\times10^{-5}$ | $1\times10^{-4}$ | $2\times10^{-4}$ | $4\times10^{-4}$ | $1\times10^{-3}$ | $1\times10^{-2}$ | $1\times10^{-1}$ |
| 1 | 1, 042 | 1, 049 | 1, 049 | 1, 048 | 1, 048 | 1, 048 | 1, 048 |
| 1, 5 | 1, 077 | 1, 063 | 1, 048 | 1, 040 | 1, 036 | 1, 033 | 1, 033 |
| 2 | 1, 095 | 1, 070 | 1, 048 | 1, 037 | 1, 030 | 1, 026 | 1, 026 |
| 3 | 1, 113 | 1, 078 | 1, 048 | 1, 033 | 1, 024 | 1, 019 | 1, 018 |
| 4 | 1, 123 | 1, 081 | 1, 048 | 1, 031 | 1, 021 | 1, 015 | 1, 014 |
| 5 | 1, 129 | 1, 084 | 1, 048 | 1, 030 | 1, 019 | 1, 012 | 1, 012 |
| 6 | 1, 133 | 1, 086 | 1, 047 | 1, 028 | 1, 017 | 1, 010 | 1, 009 |
| 7 | 1, 136 | 1, 087 | 1, 047 | 1, 027 | 1, 015 | 1, 008 | 1, 008 |
| 8 | 1, 139 | 1, 087 | 1, 047 | 1, 026 | 1, 014 | 1, 007 | 1, 006 |
| 9 | 1, 141 | 1, 088 | 1, 046 | 1, 025 | 1, 013 | 1, 005 | 1, 004 |
| 10 | 1, 142 | 1, 088 | 1, 046 | 1, 024 | 1, 011 | 1, 004 | 1, 003 |
| 15 | 1, 146 | 1, 088 | 1, 043 | 1, 020 | 1, 006 | 0, 998 | 0, 997 |
| 20 | 1, 148 | 1, 088 | 1, 040 | 1, 017 | 1, 002 | 0, 994 | 0, 993 |
| 30 | 1, 150 | 1, 088 | 1, 039 | 1, 014 | 0, 999 | 0, 991 | 0, 990 |
| 40 | 1, 150 | 1, 089 | 1, 041 | 1, 016 | 1, 002 | 0, 993 | 0, 992 |
| 50 | 1, 152 | 1, 094 | 1, 047 | 1, 023 | 1, 009 | 1, 001 | 1, 000 |
| 60 | 1, 154 | 1, 100 | 1, 056 | 1, 034 | 1, 021 | 1, 013 | 1, 012 |
| 70 | 1, 157 | 1, 108 | 1, 068 | 1, 048 | 1, 036 | 1, 029 | 1, 028 |
| 80 | 1, 161 | 1, 118 | 1, 083 | 1, 065 | 1, 055 | 1, 048 | 1, 048 |
| 90 | 1, 165 | 1, 130 | 1, 100 | 1, 084 | 1, 075 | 1, 070 | 1, 069 |
| 100 | 1, 170 | 1, 142 | 1, 118 | 1, 106 | 1, 098 | 1, 094 | 1, 094 |

$\sigma_r$=1,0

| $\varepsilon_p$ | $\tan\delta_p$ | | | | | | |
|---|---|---|---|---|---|---|---|
| | $6\times10^{-5}$ | $1\times10^{-4}$ | $2\times10^{-4}$ | $4\times10^{-4}$ | $1\times10^{-3}$ | $1\times10^{-2}$ | $1\times10^{-1}$ |
| 1 | 0, 970 | 1, 006 | 1, 027 | 1, 037 | 1, 044 | 1, 048 | 1, 048 |
| 1, 5 | 1, 027 | 1, 033 | 1, 033 | 1, 033 | 1, 033 | 1, 033 | 1, 033 |
| 2 | 1, 056 | 1, 046 | 1, 036 | 1, 031 | 1, 028 | 1, 026 | 1, 026 |
| 3 | 1, 085 | 1, 060 | 1, 039 | 1, 029 | 1, 022 | 1, 019 | 1, 018 |
| 4 | 1, 100 | 1, 068 | 1, 041 | 1, 028 | 1, 020 | 1, 015 | 1, 014 |
| 5 | 1, 109 | 1, 072 | 1, 042 | 1, 027 | 1, 018 | 1, 012 | 1, 012 |
| 6 | 1, 115 | 1, 075 | 1, 042 | 1, 026 | 1, 016 | 1, 010 | 1, 009 |
| 7 | 1, 120 | 1, 077 | 1, 042 | 1, 025 | 1, 014 | 1, 008 | 1, 008 |
| 8 | 1, 123 | 1, 078 | 1, 042 | 1, 024 | 1, 013 | 1, 007 | 1, 006 |
| 9 | 1, 126 | 1, 079 | 1, 042 | 1, 023 | 1, 012 | 1, 005 | 1, 004 |
| 10 | 1, 128 | 1, 080 | 1, 041 | 1, 022 | 1, 011 | 1, 004 | 1, 003 |
| 15 | 1, 134 | 1, 081 | 1, 039 | 1, 018 | 1, 006 | 0, 998 | 0, 997 |
| 20 | 1, 137 | 1, 081 | 1, 037 | 1, 015 | 1, 002 | 0, 994 | 0, 993 |
| 30 | 1, 139 | 1, 081 | 1, 035 | 1, 012 | 0, 999 | 0, 990 | 0, 990 |
| 40 | 1, 141 | 1, 083 | 1, 038 | 1, 015 | 1, 001 | 0, 993 | 0, 992 |
| 50 | 1, 143 | 1, 088 | 1, 044 | 1, 022 | 1, 009 | 1, 001 | 1, 000 |
| 60 | 1, 146 | 1, 095 | 1, 054 | 1, 033 | 1, 021 | 1, 013 | 1, 012 |
| 70 | 1, 150 | 1, 104 | 1, 066 | 1, 047 | 1, 036 | 1, 029 | 1, 028 |
| 80 | 1, 154 | 1, 114 | 1, 081 | 1, 064 | 1, 054 | 1, 048 | 1, 048 |
| 90 | 1, 159 | 1, 126 | 1, 098 | 1, 084 | 1, 075 | 1, 070 | 1, 069 |
| 100 | 1, 165 | 1, 139 | 1, 116 | 1, 105 | 1, 098 | 1, 094 | 1, 094 |

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The value of relative conductivity $\sigma_r$ is determined from the measured unloaded $Q$-factor $Q_{u0}$ at $f_0$ for the $TM_{010}$ mode by the following equation:

$$\sigma_r = \left\{ Q_{u0} \frac{\delta_{s0}}{\lambda_0} \frac{2\pi\left(1+\frac{D}{2H}\right)}{x_{01}} \right\}^2 \tag{8}$$

where $\lambda_0 = c/f_0$ is the wave length, and the skin depth $\delta_{s0}$ at $f_0$ is defined as follows:

$$\delta_{s0} = \sqrt{\frac{1}{\pi f_0 \mu_0 \sigma_0}} \tag{9}$$

where $\mu_0$ is the permeability of vacuum and $\sigma_0 = 5{,}8 \times 10^7$ S/m is the conductivity of standard copper.

Measurement uncertainties of $\varepsilon'$ and $\tan\delta$, $u(\varepsilon')$ and $u(\tan\delta)$, are estimated as the mean square uncertainty and given respectively by

$$u(\varepsilon')^2 = \left(\frac{\partial \varepsilon'}{\partial f_0}\right)^2 u(f_0)^2 + \left(\frac{\partial \varepsilon'}{\partial f_1}\right)^2 u(f_1)^2 + \left(\frac{\partial \varepsilon'}{\partial d_1}\right)^2 u(d_1)^2 + \left(\frac{\partial \varepsilon'}{\partial D}\right)^2 u(D)^2 + \left(\frac{\partial \varepsilon'}{\partial C_1}\right)^2 u(C_1)^2 \tag{10}$$

$$\begin{aligned} u(\tan\delta)^2 = {} & \left(\frac{\partial \tan\delta}{\partial \varepsilon_P}\right)^2 u(\varepsilon_P)^2 + \left(\frac{\partial \tan\delta}{\partial d_1}\right)^2 u(d_1)^2 + \left(\frac{\partial \tan\delta}{\partial D}\right)^2 u(D)^2 + \\ & \left(\frac{\partial \tan\delta}{\partial Q_{u0}}\right)^2 u(Q_{u0})^2 + \left(\frac{\partial \tan\delta}{\partial Q_{u1}}\right)^2 u(Q_{u1})^2 + \left(\frac{\partial \tan\delta}{\partial C_2}\right)^2 u(C_2)^2 \end{aligned} \tag{11}$$

where $u(f_0)$, $u(f_1)$, $u(d_1)$, $u(D)$, and $u(C_1)$ are the standard uncertainties of $f_0$, $f_1$, $d_1$, $D$, and $C_1$, respectively. Also, $u(\tan\delta)$ is mainly attributed to measurement uncertainty of $\varepsilon_p$, $d_1$, $D$, $Q_{u0}$, $Q_{u1}$, and $C_2$. $u(\varepsilon_p)$, $u(d_1)$, $u(D)$, $u(Q_{u0})$, $u(Q_{u1})$, and $u(C_2)$ are the standard uncertainties of them, respectively.

## 5 Measurement system

Figure 4 shows a schematic diagram of two equipment systems required for microwave measurement. For the measurement of dielectric properties, only the information on the amplitude of transmitted power is needed, that is, the information on the phase of the transmitted power is not required. Therefore, a scalar network analyser can be used for the measurement shown in Figure 4a. However, a vector network analyser, as shown in Figure 4b, has an advantage in precision of the measurement.

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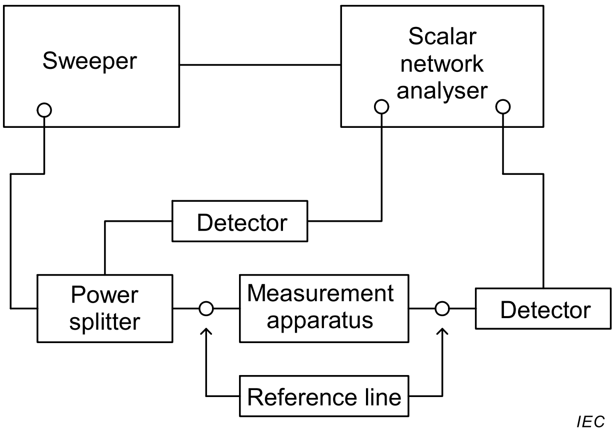
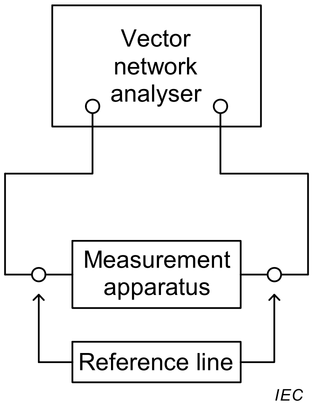



a) Scalar network analyzer system




b) Vector network analyzer system

**Figure 4 – Schematic diagram of measurement systems**

The structure of the $TM_{010}$ mode cylindrical cavity resonator used in the complex permittivity measurement is shown in Figure 1. The cavity has $D$ = 76,5 mm, $H$ = 20,0 mm, $d_2$ = 3,0 mm, and $g$ = 10,0 mm for the measurement around 3 GHz. A sample with diameter $d_1<d_2$ is coaxially inserted into the holes and excited magnetically by a pair of semi-rigid coaxial cables with a small loop at the top. The transmission-type resonator is constituted and under-coupled equally to the input and output loops with setting $S_{11} = S_{22}$.

The resonant frequency $f_0$, half-power band width $f_{BW}$, and the insertion attenuation $IA_0$ (dB) at $f_0$ are measured using a network analyser by means of the swept-frequency method, as shown in Figure 5. The value of $Q_u$ is given by

$$Q_u = \frac{Q_L}{1-10^{IA_0(\mathrm{dB})/20}}, \quad Q_L = \frac{f_0}{f_{BW}} \tag{12}$$

1) At the first step, obtain approximate values $\varepsilon_p$ and $\tan\delta_p$ from the $f_0$ and $Q_u$ values by using the simple perturbation formulas, where the effect of sample insertion holes is neglected. The subscript p denotes the calculated values using the following perturbation formulas:

$$\varepsilon_p = \frac{1}{\alpha}\frac{f_0-f_1}{f_1}\left(\frac{D}{d_1}\right)^2 + 1 \tag{3}$$

$$\tan\delta_p = \frac{1}{2\alpha\varepsilon_p}\left(\frac{D}{d_1}\right)^2\left(\frac{1}{Q_{u1}} - \frac{1}{Q_{u0}}\right) \tag{4}$$

where $\alpha = 1/J_1(x_{01})^2 = 1{,}855$. $J_n(x)$ is the Bessel function of order n of first kind and $x_{01}$ = 2,405 is the first root of $J_0(x) = 0$. $f_0$ and $Q_{u0}$ are the resonant frequency and unloaded $Q$-factor measured for the cavity without a sample, respectively. $f_1$ and $Q_{u1}$ are ones measured for the cavity with a sample.

2) In the second step, obtain accurate values $\varepsilon'$ and $\tan\delta$ from $\varepsilon_p$ and $\tan\delta_p$ values by using the following equations with correction factors calculated based on the rigorous analysis:

$$\varepsilon' = C_1\,\varepsilon_p \tag{5}$$

$$\tan\delta = C_2 \tan\delta_p \tag{6}$$

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where correction factors $C_1$ and $C_2$ due to the sample insertion holes and errors included in the perturbation formulas are calculated numerically by using the Ritz-Galerkin method [3][5], as shown in Figure 2 and Figure 3, and the corresponding data are listed in detail in Table 1, 2, and 3. The missing data of $C_1$ and $C_2$ can be obtained by interpolation or extrapolation from the tables. The correction factors shown in these figures are calculated for the cavity with $D$ = 76,5 mm, $H$ = 20,0 mm, $d_2$ = 3,0 mm, and $g$ = 10,0 mm, where the resonant frequency is about 3 GHz. $C_1$ is also used for a cavity having the same aspect ratios as $H/D$, $d_2/D$ and $g/D$.

It is found from the analysis for a cavity with insertion holes which constitute a cut-off $TM_{01}$ mode cylindrical waveguide that $f_0$ converges to a constant value for $g$>10 mm and $d_2$ = 3 mm. Therefore, the correction factors shown in Figure 2 and Figure 3 are applicable to a dielectric sample rod with $d_1$<3 mm and $\varepsilon'$ below the value calculated by the following equation for the measured value of the resonant frequency:

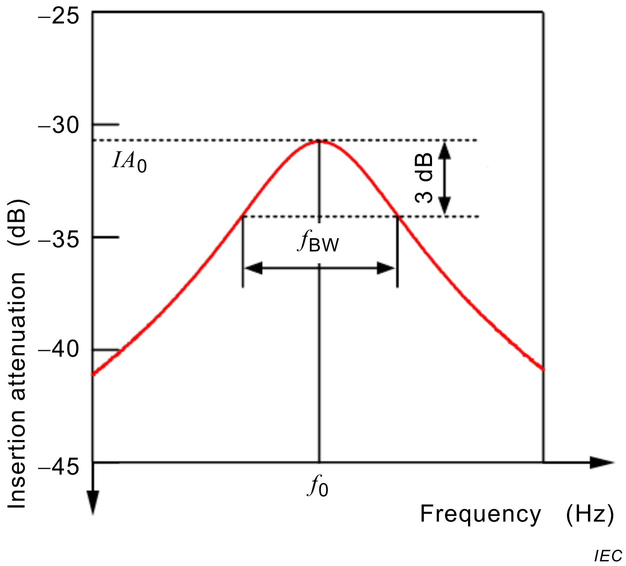


**Figure 5 – Resonance frequency $f_0$, insertion attenuation $IA_0$ and half-power band width $f_{BW}$**

## 6 Measurement procedure

### 6.1 Preparation of measurement apparatus

Set up the measurement equipment and apparatus as shown in Figure 4. The cavity resonator and dielectric samples shall be kept in a clean and dry state, as high humidity degrades unloaded $Q$. The relative humidity shall preferably be less than 60 %.

### 6.2 Measurement of reference level

The reference level, level of full transmission power, is measured first. Connect the reference line to the measurement equipment and measure the full transmission power level over the entire measurement frequency range.

### 6.3 Measurement of cavity parameters: $\sigma_r$

Set the empty cavity and adjust the insertion attenuation $IA_0$ to be around 30 dB by changing the distance between two semi-rigid cables, as shown in Figure 5.

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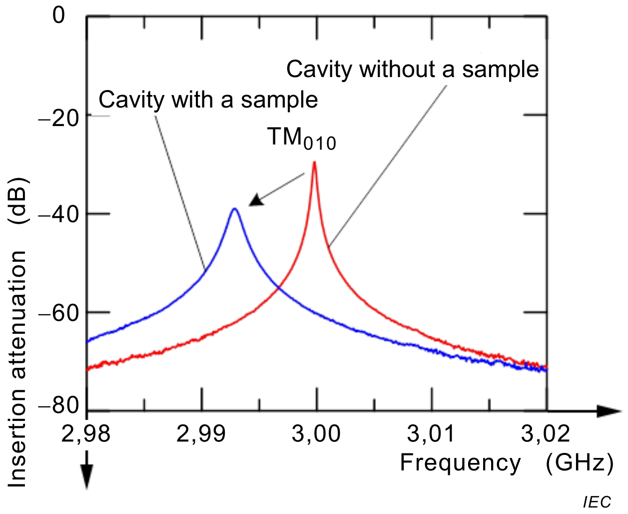
Measure $f_0$, $f_{BW}$, and $IA_0$ of the $TM_{010}$ resonant mode. Calculate $Q_{u0}$ by using Equation (12). Then, calculate $\sigma_r$ by using Equation (8). Since the value of $\sigma_r$ degrades due to oxidation of the metal surface, it shall be measured periodically. $\sigma_r$ shall preferably be more than 0,9.

### 6.4 Measurement of complex permittivity of test sample: $\varepsilon'$, tan $\delta$

Insert the test sample into the holes. Figure 6 shows the frequency responses of the $TM_{010}$ mode in the cavity with and without a sample. Measure the resonant frequency $f_1$, half-power band width $f_{BW}$ and the insertion attenuation $IA_0$. Calculate the values of $\varepsilon_p'$ and tan $\delta_p$ by using Equations (3) and (4), respectively. Then, calculate $\varepsilon'$ and tan$\delta$ values by using Equations (5) and (6).




**Assumptions**

| | | | |
|---|---|---|---|
| $D$ | 76,5 mm | $d_2$ | 3,0 mm |
| $H$ | 20,0 mm | $g$ | 10,0 mm |

**Figure 6 – Frequency responses of the $TM_{010}$ mode of cylindrical cavity**

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# Annex A
(informative)

# Example of measurement results and accuracy

## A.1 Measurement of $\varepsilon'$ and $\tan\delta$ values

The measurement results of $\varepsilon'$ and $\tan\delta$ for polyethylene rod sample are obtained as followed.

a) The parameters such as $D$, $H$ and $d_2$ of the cavity and $d_1$ of the polyethylene sample used in the measurements are shown in Table A.1.

**Table A.1 – The parameters of the cavity and the rod sample**

| $D$<br>mm | $H$<br>mm | $d_2$<br>mm | $d_1$<br>mm |
|---|---|---|---|
| 76,50<br>±0,02 | 20,00<br>±0,01 | 3,00<br>±0,01 | 2,52<br>±0,01 |

b) The resonant frequency $f_0$ and unloaded $Q$-factor $Q_{u0}$ of the $TM_{010}$ mode in the cavity without a sample and $f_1$ and $Q_{u1}$ in the cavity with a sample are measured and shown in Table A.2.

**Table A.2 – The resonant frequencies and unloaded $Q$-factors**

| $f_0$<br>(GHz) | $Q_{u0}$ | $f_1$<br>(GHz) | $Q_{u1}$ |
|---|---|---|---|
| 2,99992<br>±0,00001 | 10264<br>±5 | 2,99249<br>±0,00001 | 10073<br>±7 |

c) The approximate values $\varepsilon_p$ and $\tan\delta_p$ and the value of relative conductivity $\sigma_r$ are calculated numerically by Equations (3), (4), and (8), respectively, and the results are shown in Table A.3.

**Table A.3 – The approximate values and the relative conductivity value**

| $\varepsilon_p$ | $\tan\delta_p\,(\times 10^{-4})$ | $\sigma_r$ |
|---|---|---|
| 2,233<br>±0,010 | 2,055<br>±0,095 | 0,889<br>±0,001 |

d) The correction factors $C_1$ and $C_2$ are found from Figure 2 and Figure 3b, respectively, using the calculated values of $\varepsilon_p$, $\tan\delta_p$ and $\sigma_r$. The results are shown in Table A.4.

**Table A.4 – Correction factors and the measurement results**

| $C_1$ | $C_2$ | $\varepsilon'$ | $\tan\delta\,(\times 10^{-4})$ |
|---|---|---|---|
| 1,027<br>±0,001 | 1,047<br>±0,001 | 2,293<br>±0,010 | 2,152<br>±0,099 |

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e) The accurate values $\varepsilon'$ and $\tan\delta$ are obtained from Equations (5) and (6), and these results are also shown in Table A.4.

## A.2 Measurement uncertainty of $\varepsilon'$ and $\tan\delta$

The measurement uncertainty (see ISO/IEC Guide 98-3) of $\varepsilon'$ and $\tan\delta$ is calculated for the polyethylene sample mentioned above by Equation (10) and (11). Each sensitivity coefficients in Equations (10) and (11) are as follows:

$$\frac{\partial \varepsilon'}{\partial f_0} = \frac{1}{\alpha}\frac{1}{f_1}\left(\frac{D}{d_1}\right)^2 C_1$$

$$\frac{\partial \varepsilon'}{\partial f_1} = \frac{1}{\alpha}\left(-\frac{f_0}{f_1{}^2}\right)\left(\frac{D}{d_1}\right)^2 C_1$$

$$\frac{\partial \varepsilon'}{\partial d_1} = \frac{1}{\alpha}\frac{f_0 - f_1}{f_1}\left(-2\frac{D^2}{d_1{}^3}\right)C_1$$

$$\frac{\partial \varepsilon'}{\partial D} = \frac{1}{\alpha}\frac{f_0 - f_1}{f_1}\left(\frac{2D}{d_1{}^2}\right)C_1$$

$$\frac{\partial \varepsilon'}{\partial C_1} = \frac{1}{\alpha}\frac{f_0 - f_1}{f_1}\left(\frac{D}{d_1}\right)^2$$

$$\frac{\partial \tan\delta}{\partial \varepsilon_\mathrm{p}} = -\frac{1}{\varepsilon_\mathrm{p}{}^2}\frac{1}{2\alpha}\left(\frac{D}{d_1}\right)^2\left\{\frac{1}{Q_{u1}} - \frac{1}{Q_{u0}}\right\}C_2$$

$$\frac{\partial \tan\delta}{\partial d_1} = \frac{1}{\varepsilon_\mathrm{p}}\frac{1}{2\alpha}\left(-2\frac{D^2}{d_1{}^3}\right)\left\{\frac{1}{Q_{u1}} - \frac{1}{Q_{u0}}\right\}C_2$$

$$\frac{\partial \tan\delta}{\partial D} = \frac{1}{\varepsilon_\mathrm{p}}\frac{1}{2\alpha}\left(\frac{2D}{d_1{}^2}\right)\left\{\frac{1}{Q_{u1}} - \frac{1}{Q_{u0}}\right\}C_2$$

$$\frac{\partial \tan\delta}{\partial Q_{u0}} = \frac{1}{\varepsilon_\mathrm{p}}\frac{1}{2\alpha}\left(\frac{D}{d_1}\right)^2\left\{\frac{1}{Q_{u0}{}^2}\right\}C_2$$

$$\frac{\partial \tan\delta}{\partial Q_{u1}} = \frac{1}{\varepsilon_\mathrm{p}}\frac{1}{2\alpha}\left(\frac{D}{d_1}\right)^2\left\{-\frac{1}{Q_{u1}{}^2}\right\}C_2$$

$$\frac{\partial \tan\delta}{\partial C_2} == \frac{1}{\varepsilon_\mathrm{p}}\frac{1}{2\alpha}\left(\frac{D}{d_1}\right)^2\left\{\frac{1}{Q_{u1}} - \frac{1}{Q_{u0}}\right\}$$

The results are shown in Table A.5 and A.6.

**Table A.5 – The measurement uncertainty of $\varepsilon'$**

| | $\frac{\partial\varepsilon'}{\partial f_0}u(f_0)$ | $\frac{\partial\varepsilon'}{\partial f_1}u(f_1)$ | $\frac{\partial\varepsilon'}{\partial d_1}u(d_1)$ | $\frac{\partial\varepsilon'}{\partial D}u(D)$ | $\frac{\partial\varepsilon'}{\partial C_1}u(C_1)$ | $u(\varepsilon')$ |
|---|---|---|---|---|---|---|
| Sensitivity | $1{,}7050\times10^{-7}$ | $-1{,}7092\times10^{-7}$ | $-1{,}0054\times10^{3}$ | $3{,}3119\times10^{1}$ | $1{,}2335\times10^{0}$ | ---- |
| uncertainty | 0,0017 | 0,0017 | 0,0101 | 0,0007 | 0,0012 | 0,0104 |

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**Table A.6 – The measurement uncertainty of tan$\delta$**

| | $\frac{\partial \tan\delta}{\partial \varepsilon_P} u(\varepsilon_P)$ | $\frac{\partial \tan\delta}{\partial d_1} u(d_1)$ | $\frac{\partial \tan\delta}{\partial D} u(D)$ | $\frac{\partial \tan\delta}{\partial Q_{u0}} u(Q_{u0})$ | $\frac{\partial \tan\delta}{\partial Q_{u1}} u(Q_{u1})$ | $\frac{\partial \tan\delta}{\partial C_2} u(C_2)$ | $u(\tan\delta)$ |
|---|---|---|---|---|---|---|---|
| Sensitivity | $-9{,}6313\times10^{-5}$ | $-1{,}7073\times10^{-1}$ | $5{,}6239\times10^{-3}$ | $1{,}1053\times10^{-6}$ | $-1{,}1476\times10^{-6}$ | $2{,}0546\times10^{-4}$ | ---- |
| uncertainty | $0{,}00972\times10^{-4}$ | $0{,}01707\times10^{-4}$ | $0{,}00112\times10^{-4}$ | $0{,}05526\times10^{-4}$ | $0{,}08033\times10^{-4}$ | $0{,}00205\times10^{-4}$ | $0{,}09949\times10^{-4}$ |

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