IEC TR 62461

Edition 2.0 2015-01

TECHNICAL REPORT

Radiation protection instrumentation – Determination of uncertainty in measurement

IEC TR 62461:2015-01(en)

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Radiation protection instrumentation – Determination of uncertainty in measurement

colour inside

INTERNATIONAL ELECTROTECHNICAL COMMISSION

ICS 13.280

ISBN 978-2-8322-2216-4

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RADIATION PROTECTION INSTRUMENTATION – DETERMINATION OF UNCERTAINTY IN MEASUREMENT

FOREWORD

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IEC 62461, which is a technical report, has been prepared by subcommittee 45B: Radiation protection instrumentation, of IEC technical committee 45: Nuclear instrumentation.

This second edition of IEC TR 62461 cancels and replaces the first edition, published in 2006, and constitutes a technical revision. The main changes with respect to the previous edition are as follows:

– add to the analytical method for the determination of uncertainty the Monte Carlo method for the determination of uncertainty according to supplement 1 of the Guide to the Expression of uncertainty in measurement (GUM S1), and
– add a very simple method to judge whether a measured result is significantly different from zero or not based on ISO 11929.

The text of this technical report is based on the following documents:

Enquiry draft	Report on voting
45B/783/DTR	45B/813/RVD

Full information on the voting for the approval of this technical report can be found in the report on voting indicated in the above table.

This publication has been drafted in accordance with the ISO/IEC Directives, Part 2.

The committee has decided that the contents of this publication will remain unchanged until the stability date indicated on the IEC website under "http://webstore.iec.ch" in the data related to the specific publication. At this date, the publication will be

- reconfirmed,
- withdrawn,
- replaced by a revised edition, or
- amended.

A bilingual version of this publication may be issued at a later date.

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INTRODUCTION

The ISO/IEC Guide 98-3:2008, *Uncertainty of measurement – Part 3: Guide to the expression of uncertainty in measurement (GUM:1995)* as well as its Supplement 1:2008, *Propagation of distributions using a Monte Carlo method* (GUM S1), are general guides to assess the uncertainty in measurement. This Technical Report lays emphasis on their application in the area of radiation protection and serves as a practical introduction to the GUM and its supplement 1 (GUM S1).

The process of determining the uncertainty delivers not only a numerical value of the uncertainty; in addition it produces the best estimate of the quantity to be measured which may differ from the indication of the instrument. Thus, it can also improve the result of the measurement by using information beyond the indicated value of the instrument, e.g. the energy dependence of the instrument.

RADIATION PROTECTION INSTRUMENTATION – DETERMINATION OF UNCERTAINTY IN MEASUREMENT

1 Scope

This Technical Report gives guidelines for the application of the uncertainty analysis according to ISO/IEC Guide 98-3:2008 (GUM describing an analytical method for the uncertainty determination) and its Supplement 1:2008 (GUM S1 describing a Monte Carlo method for the uncertainty determination) for measurements covered by standards of IEC Subcommittee 45B. It does not include the uncertainty associated with the concept of the measuring quantity, e. g., the difference between $H_p(10)$ on the ISO water slab phantom and on the person.

This Technical Report explains the principles of the ISO/IEC Guide 98-3:2008 (GUM),its Supplement 1:2008 (GUM S1) and the special considerations necessary for radiation protection at an example taken from individual dosimetry of external radiation. In the informative annexes, several examples are given for the application on instruments, for which SC 45B has developed standards.

This Technical Report is supposed to assist the understanding of the ISO/IEC Guide 98-3:2008 (GUM), its Supplement 1: 2008 (GUM S1), and other papers on uncertainty analysis. It cannot replace these papers nor can it provide the background and justification of the arguments leading to the concept of the ISO/IEC Guide 98-3:2008 (GUM) and its Supplement 1:2008 (GUM S1).

Finally, this Technical Report gives a very simple method to judge whether a measured result is significantly different from zero or not based on ISO 11929.

For better readability the correct terms are not always used throughout this technical report. For example, instead of “random variables of a quantity” only the “quantity” itself is stated.

2 Normative references

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

IEC 60050 (all parts): *International Electrotechnical Vocabulary* (available at http://www.electropedia.org)

ISO/IEC Guide 98-3:2008, *Uncertainty of measurement – Part 3: Guide to the expression of uncertainty in measurement (GUM:1995)*

ISO/IEC Guide 98-3, Supplement 1:2008, *Uncertainty of measurement – Part 3: Guide to the expression of uncertainty in measurement (GUM:1995) – Propagation of distributions using a Monte Carlo method*

3 Terms and definitions

For the purposes of this document, the technical terms of IEC 60050-151 [1], and IEC 60050-311 [2] as well as the following definitions taken from the ISO/IEC Guide 98-3:2008 (GUM), and its Supplement 1:2008 (GUM S1) apply[1].

3.1
calibration factor
N
quotient of the true value of a quantity and the indicated value for a specified reference radiation under specified reference conditions

3.2
conformity test
test for conformity evaluation

[SOURCE: IEC 60050-151:2001,151-16-15]

3.3
complete result of a measurement
set of values attributed to a measurand, including a value, the corresponding uncertainty and the unit of measurement

Note 1 to entry: The central value of the whole (set of values) can be selected as *measured value* and a parameter characterising the dispersion as *uncertainty*.

Note 2 to entry: The result of a measurement is related to the *indication given by the instrument* and to the values of correction obtained by calibration and by the use of a *model*.

Note 3 to entry: In this Technical Report, the "measured value", see Note 1 above, is abbreviated by M.

Note 4 to entry: In this Technical Report, the "indication given by the instrument", see Note 2 above, is abbreviated by G, and called "indicated value".

Note 5 to entry: In this Technical Report, the "model", see Note 2 above, is called "model function", see 3.10 and 5.2.

[SOURCE: IEC 60050-311:2001, 311-01-01, modified]

3.4
correction factor
K
factor to the indicated value to correct for deviation of measurement conditions from calibration conditions

3.5
coverage factor
$\boldsymbol{k_{\mathrm{cov}}}$
numerical factor used as a multiplier of the (combined) standard uncertainty in order to obtain an expanded uncertainty

Note 1 to entry: A coverage factor k_{cov} is typically in the range of 2 to 3.

[SOURCE: GUM:2008, 2.3.6]

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1 Numbers in square brackets refer to the bibliography.

3.6
decision threshold
m*
value of the estimator of the measurand, which when exceeded by the result of an actual measurement using a given measurement procedure of a measurand quantifying a physical effect, one decides that the physical effect is present

Note 1 to entry: The decision threshold is defined such that in cases where the measurement result, m, exceeds the decision threshold, m^*, the probability that the true value of the measurand is zero is less or equal to a chosen probability, α.

Note 2 to entry: If the result, m, is below the decision threshold, m^*, the result cannot be attributed to the physical effect; nevertheless it cannot be concluded that it is absent.

[SOURCE: ISO 11929:2010]

3.7
deviation
D
difference between the indicated values for the same value of the measurand of an indicating measuring instrument, or the values of a material measure, when an influence quantity assumes, successively, two different values

Note 1 to entry: This definition is applicable to all measuring instruments and influence quantities, but it should mainly be used in those cases, where this deviation is independent of the indicated value.

[SOURCE: IEC 60050-311:2001, 311-07-03, modified[2]]

3.8
distribution function
***F*(*x*)**
a function giving, for every value x, the probability that the random variable X be less than or equal to x: $F(x) = \Pr(X \leq x)$

[SOURCE: GUM:2008, C.2.4; GUM S1:2008, 3.2]

3.9
expanded uncertainty
U
quantity defining an interval about the result of a measurement that may be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to the measurand

Note 1 to entry: The expanded uncertainty is obtained by multiplying the (combined) standard uncertainty by a coverage factor.

[SOURCE: GUM:2008, 2.3.5]

3.10
indicated value
G
quantity value provided by a measuring instrument or a measuring system

Note 1 to entry: An indication is often given by the position of a pointer on the display for analogue outputs, a displayed or printed number for digital outputs, a code pattern for code outputs, or an assigned quantity value for material measures.

3.11
influence quantity
quantity that is not the measurand but that effects the result of the measurement

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2 Original term “variation (due to an influence quantity)”.

Note 1 to entry: For example, temperature of a micrometer used to measure length.

[SOURCE: GUM:2008, B.2.10]

3.12
measured value
M
value determined from the indicated value, G, by applying the model function for the measurement

Note 1 to entry: An example of a model function is given below. The calibration factor N, a deviation D, and a correction factor K are applied:

$$M = N \times K \times (G - D)$$

The calculations according to this model function are not always performed. One main purpose of this model function of the measurement is, that it is necessary for any determination of the uncertainty according to the GUM (see GUM, 3.1.6, 3.4.1 and 4.1; see also 5.2 of this Technical Report).

Note 2 to entry: In the GUM the *measured value* is called *value of the measurand*.

3.13
probability density function <for a continuous random variable>
$f(x)$
the derivative (when it exists) of the distribution function: $f(x)=\mathrm{d}F(x)/\mathrm{d}x$

Note 1 to entry: $f(x)\cdot\mathrm{d}x$ is the "probability element": $f(x)\cdot\mathrm{d}x=\Pr(x<X<x+\mathrm{d}x)$; in general: $\Pr(a < X < b) = \int_a^b f(x)\mathrm{d}x$.

[SOURCE: GUM:2008, C.2.5; GUM S1:2008, 3.3, modified by adding "in general"]

3.14
reference conditions
set of specified values and/or ranges of values of influence quantities under which the uncertainties, or limits of error, admissible for a measuring instrument are the smallest

[SOURCE: IEC 60050-311:2001, 311-06-02]

3.15
reference response
R_{ref}
response of the assembly under reference conditions to unit reference dose (rate) or activity and is expressed as:

$$R_{\mathrm{ref}} = \frac{G}{M_{\mathrm{c}}}$$

where G is the indicated value of the equipment or assembly under test and M_{c} is the true value of the reference source

3.16
relative response
R_{rel}
quotient of the response and the reference response under specified conditions

Note 1 to entry: For the specified reference conditions, the response is the reciprocal of the calibration factor.

3.17
response
R
ratio of the quantity measured under specified conditions by the equipment or assembly under test and the true value of this quantity

3.18
standard uncertainty
standard deviation associated with the measurement result or an input quantity

Note 1 to entry: See GUM:2008, 2.3.4.

Note 2 to entry: The standard uncertainty of the measurement result is sometimes called "combined standard uncertainty".

Note 3 to entry: The quotient of the standard uncertainty and the measurement result is called "relative standard uncertainty" and sometimes given as percentage.

3.19
type test
conformity test made on one or more items representative of the production

[SOURCE: IEC 60050-151:2001, 151-16-16]

3.20
uncertainty
uncertainty of measurement
parameter, associated with the result of a measurement, that characterises the dispersion of the values that could reasonably be attributed to the measurand

Note 1 to entry: The parameter may be, for example, a standard deviation (or a given multiple of it), or the half-width of an interval having a stated level of confidence (coverage probability).

[SOURCE: GUM:2008, 2.2.3]

4 List of symbols

Table 1 gives a list of the symbols (and abbreviated terms) used in the main text of this Technical Report (excluding annexes).

Table 1 – Symbols (and abbreviated terms) used in the main text (excluding annexes)

Symbol	Meaning	Unit (dose measurement)
a	Half-width of an interval for possible values of a quantity	As quantity
a_-	Lower limit of an interval for possible values of a quantity	As quantity
a_+	Upper limit of an interval for possible values of a quantity	As quantity
α	Probability to detect an effect (state a result above zero) although in reality no effect is present (the true value is zero) also called "probability of false positive decision"	–
c_k	Sensitivity coefficient for the input quantity K	Sv
c_m	Sensitivity coefficient for the input quantity M	–
c_{m0}	Sensitivity coefficient for the input quantity M_0	–
c_n	Sensitivity coefficient for the input quantity N	Sv
$F(x)$	Distribution function	–

Symbol	Meaning	Unit (dose measurement)
$f(x)$	Probability density function (for a continuous random variable) PDF	Inverse of quantity
G	Indicated value, for example, reading of the dosemeter in units of $H_p(10)$	Sv
$\hat{g}$	Best estimate of G	Sv
g	Possible value (estimate) of G	Sv
G_0	Zero reading	Sv
$\hat{g}_0$	Best estimate of G_0	Sv
g_0	Possible value (estimate) of G_0	Sv
$h(x)$	Model function, see Note 1 to 3.12	As output quantity
$H_p(10)$	Personal dose equivalent at a depth 10 mm	Sv
i	Running index (integer)	–
j	Running index (integer)	–
K	Correction factor, for example, for energy and angle of radiation incidence	–
$\hat{k}$	Best estimate of K	–
k	Possible value (estimate) of K	–
$k_{1-\alpha}$	quantile of the standardized normal distribution for a given probability α	–
k_{cov}	Coverage factor	–
L	Number of Monte Carlo trials	–
M	Measured value, for example, personal dose equivalent $H_p(10)$	Sv
M_c	True value of a reference source	Sv
$\hat{m}$	Best estimate of M	Sv
m	Possible value (estimate) of M	Sv
m^*	Decision threshold of M	Sv
N	Calibration factor	–
$\hat{n}$	Best estimate of N	–
n	Possible value (estimate) of N	–
p	Coverage probability	–
Q	Distribution function for the output quantity	–
q	Arbitrary integer	–
R_{abs}	Absolute response	–
R_{rel}	Relative response	–
$s_{\hat{g}}$	Standard deviation of the distribution of the g-values	Sv
$s_{\hat{g}0}$	Standard deviation of the distribution of the g_0-values	Sv
$s_{\hat{k}}$	Standard deviation of the distribution of the k-values	–
$s_{\hat{n}}$	Standard deviation of the distribution of the n-values	–
T	Number of input quantities	–
U	Expanded uncertainty	Sv
$u(\hat{m})$	Standard uncertainty associated with the best estimate of the measurement result, $\hat{m}$	Sv
$u_g(\hat{m})$	Uncertainty contribution to u of the input quantity G associated with the best estimate of the measurement result, $\hat{m}$	Sv
$u_{g0}(\hat{m})$	Uncertainty contribution to u of the input quantity G_0 associated with the best estimate of the measurement result, $\hat{m}$	Sv
$u_k(\hat{m})$	Uncertainty contribution to u of the input quantity K associated with the best estimate of the measurement result, $\hat{m}$	Sv

Symbol	Meaning	Unit (dose measurement)
$u_n(\hat{m})$	Uncertainty contribution to u of the input quantity N associated with the best estimate of the measurement result, $\hat{m}$	Sv
X	A non-specified quantity	As quantity
$\hat{x}$	Best estimate of X	As quantity
x	Possible value (estimate) of X	As quantity
y	Random number from the standard Gaussian distribution	–
z	Random number out of the interval 0 .. 1 (rectangular distribution)	–

5 The GUM and the GUM S1 concept

5.1 General concept of uncertainty determination

5.1.1 Overview in four steps

The GUM:2008 and its supplement 1, GUM S1:2008:

– consider available quantities influencing the measurement, e.g. the experience of the person performing the measurement,
– are partly based on the Bayes statistics (especially the GUM S1),
– are internationally accepted.

NOTE The methods of the GUM and the GUM S1 are described and explained in many papers [3] to [11].

The application of the GUM (analytical method) and GUM S1 (Monte Carlo method), not the justification or the mathematics behind it, will be described in a *simplified example* in the following subclauses. Further details can be found in the literature.

The following four steps are necessary for the propagation (determination) of uncertainty. Especially, for the first two steps, the expertise of the evaluator is essential.

– Step 1: A mathematical model function (or an algorithm) has to be stated describing the relation of the input quantities X_i and the output quantity M

$$M = h(X_1, \ldots, X_T) \tag{1}$$

where

T is the number of input quantities;

X_i is an input quantity;

M is the output quantity.

The model function should contain every quantity, including all corrections and correction factors that can contribute a significant component of uncertainty to the result of the measurement; details are given in 5.2.

– Step 2: The available information for the input quantities X_i has to be collected; details are given in 5.3.
– Step 3: The standard uncertainty $u(\hat{m})$ of the output quantity has to be calculated using either the analytical method (explained in 5.1.2) or the Monte Carlo method (explained in 5.1.3). For this step, only the application of mathematics is required. This task can, therefore, be performed completely by a computer program, for example, the software "GUM Workbench" [12] or "UncertRadio" [13]; details are given in 5.4.
– Step 4: The expanded uncertainty $U(\hat{m})$ and or the corresponding coverage interval have to be stated; details are given in 5.5.

5.1.2 Summary of the analytical method for steps 3 and 4

In this subclause, a short summary is given in the following to illustrate the analytical method:

a) Firstly, for each input quantity X_i, $i = 1..T$, the best estimate $\hat{x}_i$ and its standard uncertainty $s(\hat{x}_i)$ have to be obtained;

b) Secondly, the sensitivity coefficient, i.e. the partial derivative of the output quantity with respect to each input quantity, has to be calculated: $c_i = \partial h/\partial x_i$; this is the slope of the model function $h(x_i)$. The larger it is the stronger is the impact of the corresponding input quantity to the output quantity, thus, it is the "lever arm" or "impact" of the corresponding input quantity.

c) Thirdly, the uncertainty contribution to the output quantity due to each input quantity has to be calculated by multiplying the sensitivity coefficient and the standard uncertainty: $u_i(\hat{m}) = |c_i| \cdot s(\hat{x}_i)$.

d) Fourthly, the combined standard uncertainty for the output quantity is computed as the square root of the squared uncertainty contributions: $u_c(\hat{m}) = \sqrt{\sum_{i=1}^{n} \{u_i(\hat{m})\}^2}$; in case some (random variables expressing the state of knowledge about the according) input quantities are correlated with one another (i.e. they depend on each other), further terms need to be added to the sum under the square root sign, as detailed in 5.2 of the GUM:2008.

e) Finally, the expanded uncertainty for the output quantity has to be calculated by multiplying the standard uncertainty with the appropriated coverage factor (usually k = 2): $U_c(\hat{m}) = 2 \cdot u_c(\hat{m})$; if the probability distribution of the output quantity is not approximately Gaussian (or normal), the coverage factor may have another value, see 6.3 of the GUM:2008 .

5.1.3 Summary of the Monte Carlo method for steps 3 and 4

In this subclause, a short summary, taken from the introduction and from 5.9.6 of the GUM S1:2008, is given in the following to illustrate the Monte Carlo method:

This Supplement to the GUM is concerned with the propagation of probability distributions through the mathematical model of measurement [GUM:1995, 3.1.6] as a basis for the evaluation of uncertainty of measurement, and its implementation by a Monte Carlo method. The treatment applies to a model having any number of input quantities, and a single output quantity. The described Monte Carlo method is a practical alternative to the GUM uncertainty framework [GUM:1995, 3.4.8]. It has value when

a) linearization of the model provides an inadequate representation or

b) the probability density function (PDF) for the output quantity departs appreciably from a Gaussian distribution or a scaled and shifted t-distribution, e.g. due to marked asymmetry of dominating influence quantities (i.e. those with large uncertainties) or due to a model function with only very few influence quantities which are, in addition, non-Gaussian distributed.

The Monte Carlo method can be stated as a step-by-step procedure, see 5.9.6 of the GUM S1:2008:

a) select the number L of Monte Carlo trials to be made;

b) generate L vectors, by sampling from the assigned PDFs, as realizations of the (set of $i = 1..T$) input quantities X_i;

c) for each such vector, form the corresponding model value of $M = h(X_i)$, yielding L model values M_j with $j = 1..L$;

d) sort these L model values into increasing order, using the sorted model values to provide the distribution function for the output quantity Q;

e) calculate the average of M_1, …, M_L which is an estimate $\hat{m}$ of M, and calculate their standard deviation which is an evaluation of the standard measurement uncertainty $u(\hat{m})$ associated with $\hat{m}$, see 5.4.3 d);

f) use Q to form an appropriate coverage interval for M, for a stipulated coverage probability p, see 5.5.3.

5.1.4 Which method to use: Analytical or Monte Carlo?

The Monte Carlo method usually delivers better estimates of the result and the uncertainty if the measurement conditions are modeled properly as no approximation is applied; this is confirmed by experimental findings [11]. However, the analytical method is easier to apply for a large number of measurements as they, for example, occur in services performing daily a large number of similar measurements, and may therefore preferably be applied.

If the model function is linear and the input quantities are limited symmetrically around their centre value, then the analytical method can be used.

Otherwise, the results of both methods should be given in order to display their difference. When the 95 % coverage intervals of the Monte Carlo method and of the analytical method do not deviate by more than 10 %, then the analytical one may be used for the uncertainty determination in similar cases, i.e. a similar model function and similar or smaller values of the uncertainty of the input quantities.

5.2 Example of a model function

The basis of any measurement and the first (and probably most important) step of the uncertainty evaluation is the definition of the measurement model. This is a mathematical relationship between all the influence quantities. However, different evaluators may well have different knowledge of the process, and different understandings of how the quantities in play interact and by that state different model functions. This is an image of the scientific reality: one evaluator is aware of a specific influence quantity and thus includes it in the model function, while the other is not. As a result, different uncertainties (and maybe even different measuring results) can be calculated by different evaluators. It is, therefore, important to explain in detail which input quantities have been taken into account, even when they are regarded as negligible.

Since different measurement models typically will lead to different uncertainty evaluations, this is a source of uncertainty, too, often called “model uncertainty" [14], [15]. If different models appear comparably reasonable to the evaluator, then alternative uncertainty evaluations should be performed to assess the sensitivity of the results to the modelling assumptions, and possibly also to quantify the component uncertainty that derives from the multiplicity of such models.

The model function is in most cases an analytical function, but the GUM S1 method does not require this: it can also be an algorithm. It is important that the model gives an unambiguous value of the measurand. To explain the model, an example of a direct reading individual dosemeter will be considered. The dosemeter’s display indicates the dose directly in units of the quantity to be measured, for example, in μSv or mSv for the quantity $H_\mathrm{p}(10)$.

A proven method to set up the model function is to start from the principle of cause and effect. The cause – and the aim of the measurement – is the dose M which produces, due to the absolute response R_abs, an indication of $M \times R_\mathrm{abs}$, which is increased by the zero indication G_0. Therefore, the indication of the dosemeter is given by

$$G = M\, R_\mathrm{abs} + G_0 \tag{2}$$

where

G is the indicated value, for example, reading of the dosemeter in units of $H_\mathrm{p}(10)$;

M is the cause, for example, the personal dose equivalent $H_\mathrm{p}(10)$, which shall be measured;

R_abs is the absolute response;

G_0 is the zero reading.

The aim of the measurement is M, so the model function is

$$M = \frac{1}{R_{\text{abs}}}\left(G - G_0\right) \tag{3}$$

The inverse of the absolute response R_{abs} is given by

$$\frac{1}{R_{\text{abs}}} = \frac{N}{R_{\text{rel}}} = N\,K \tag{4}$$

where

N is the calibration factor;

R_{rel} is the response relative to the response at calibration conditions and, thus, accounts for the different influence quantities, for example, for energy and angle of radiation incidence;

K is the corresponding correction factor for deviation from calibration conditions and, thus, accounts for the different influence quantities, for example, for energy and angle of radiation incidence.

In order to have symmetrical intervals about the best estimate of the influence quantity, either R_{rel} or K is used depending which one is limited symmetrically to unity in the respective instrument specific standard, e.g. 1,0 ± 0,4. If none is limited symmetrically, the one with the interval closer to unity should be used. Exception: If the analytical method is applied K should be used in case the standard uncertainty exceeds 10 %. The reason is that a linear approximation of the model function is implicitly used for the analytical method and the approximation is not good enough for standard uncertainties exceeding 10 %, see 5.1.2 of the GUM:2008, 7.9 of GUM S1:2008, and [10].

Note 1 When the distribution of R is limited symmetrically and it is relatively wide, e.g. 1,0 ± 0,4, the relation $K = 1\,/\,R_{\text{rel}}$ is not trivial, i.e. it does not lead to a symmetrical distribution of K and it leads to another (usually not trivial) probability density function (PDF). For example, a rectangular distribution leads to a hyperbolic one. However, this is ignored in this report for two reasons: Firstly, for the sake of simplicity. Secondly, instrument specific standards only lay down limits for the response or correction factor. The transformation of these limits via $K = 1\,/\,R_{\text{rel}}$ only leads to new limits. Thus, in both cases the principle of maximum entropy (PME) implies a rectangular distribution.

NOTE 2 For a device accumulating radiation over a long period of time (for example, a personal dosemeter being worn for several hours up to months), the value of R usually is the mean of all values the input quantity took during the time of measurement.

Finally, the model function is given by

$$M = \frac{N}{R_{\text{rel}}}\left(G - G_0\right) = N\,K\left(G - G_0\right). \tag{5}$$

The model function (5) gives the relation between the measurand (measuring quantity) M, called output quantity of the evaluation (which is the measured value), and the input quantities N, R_{rel}, (or K,) G and G_0.

If one or more input quantity is in the nominator of the model function, the results of the analytical method need to be verified using Monte Carlo methods. This can be done in the following way: Determine the 95 % coverage intervals resulting from the Monte Carlo method and from the analytical method: they should not deviate by more than 10 %, see 5.1.4. A possible fallacy when performing the uncertainty analysis is to perform the analysis with formula (2) for the indicated value, but this ignores that the aim of the measurement is the cause M and its associated uncertainty and not the indicated value G.

An alternative method to define a measurement model is of interest in case some of the input quantities depend on the measurand (i.e. an implicit relation). In such cases the so called observation formula is a suitable alternative [16].

For routine measurements, often $N = R_{\mathrm{rel}} = K = 1$ and $G_0 = 0$ is assumed resulting in $M = G$, which means that no correction at all is considered. However, when the uncertainty associated with the measurement is discussed, the model function including all corrections must be considered. Thus, in any measurement, the model function is implicitly included in the measurement process.

The imperfect knowledge of the true value will be taken into account in such a way that for the evaluation both the input quantities N, R_{rel}, K, G, G_0 and the output quantity M are being replaced by random variables. Their possible values are denoted by small letters, for example, n and r, whereas all quantities are written in capital letters as in formula (5). For each quantity the possible values are characterized by a distribution, which has an expectation value (mean value) denoted by the corresponding small letter with a circumflex accent, for example, $\hat{n}$ and $\hat{r}$, and a corresponding standard deviation (standard uncertainty) of the expectation value, denoted by the letter s and the index given by the mean value, for example, $s_{\hat{n}}$ and $s_{\hat{r}}$ respectively. As seen by formula (5) the output quantity M is linked to the input quantities N, R_{rel}, (or K,) G and G_0 via the model function. Therefore, the distributions of the possible values of the input quantities lead to a distribution of the possible values of the output quantity M. This is described by the corresponding expectation value $\hat{m}$ and its standard deviation. In analogy to the symbols used for the input quantities this could be given the symbol $s_{\hat{m}}$, but in all the literature the symbol u is used, so this is followed here. The aim of the uncertainty analysis according to the GUM and the GUM S1 is the determination of $u(\hat{m})$, this should be read as "u associated with $\hat{m}$". The principle method to determine it is to vary all the input quantities within their ranges of possible values. This results in a variation of the possible values m of the output quantity, which is determined by the model function. This variation determines a distribution of the output values m whose mean value is $\hat{m}$ and whose standard deviation is $u(\hat{m})$.

$$m = \frac{n}{r_{\mathrm{rel}}}\left(g - g_0\right) = n\,k\left(g - g_0\right) \tag{6}$$

where

- n is a possible value of the calibration factor;
- r_{rel} is a possible value of the response relative to the response at calibration conditions and, thus, accounts for the different influence quantities, for example, for the energy and the angle of radiation incidence;
- k is a possible value of the correction factor for deviation from calibration conditions and, thus, accounts for the different influence quantities, for example, for the energy and the angle of radiation incidence;
- g is a possible value of the indicated value, for example, the reading of the dosemeter in units of $H_{\mathrm{p}}(10)$;
- g_0 is a possible value of the zero reading;
- m is a possible value of the measurement result, for example, of the personal dose equivalent $H_{\mathrm{p}}(10)$, and is calculated from formula (6) with the possible values for n, …, g_0;

5.3 Collection of data and existing knowledge for the example

5.3.1 General

The second step of the uncertainty analysis is the collection of data and existing knowledge. This includes both mathematical methods like statistical analysis and other methods like collecting data from data sheets, for example, calibration certificates, or using scientific and experimental experience. These other methods are the most important new item introduced by the GUM method and they are most important for realistic uncertainty calculations. This second step of the uncertainty analysis depends as well as the first step to a great extent on the experience and the knowledge of the evaluator. Different evaluators may well assign (or estimate) different values for the uncertainties of the input quantities and by that calculate different uncertainties for the output quantity. This is again an image of the scientific reality. But this should not be interpreted as an uncertainty of the uncertainty; this is due to the difference in information collected by different evaluators. If the evaluators started with the same information (and calculated correctly) the uncertainty determined by the evaluators would be the same.

In particular, the other methods mentioned above can only be reviewed if the uncertainty analysis is clearly documented. An adequate documentation method, the uncertainty budget, will be given in 5.4. In the following, these methods will be demonstrated for the mentioned example of an individual electronic dosemeter with the model function of formula (5). Therefore, the input quantities N, R_{rel}, K, G and G_0 are discussed one after the other in the following subclauses.

5.3.2 Calibration factor for the example

The individual dosemeter is calibrated at the factory under reference conditions, for example, Cs-137 radiation, 0° radiation incidence and a dose of 0,3 mSv and a dose rate of 5 mSv h^{-1}. During the calibration process, the dosemeter is adjusted so that the calibration factor is close to unity. Therefore, the calibration factor N in formula (5) should only correct for remaining imperfections in the adjustment process. Such imperfections could be due to the uncertainty of the field parameters of the calibration facility at the factory – given in the calibration certificate of that facility – and limits for the adjustment given by the factory procedure, for example, adjustment until the deviation of the reading from the reference value is less than 10 %.

NOTE The zero reading G_0 is assumed to be much smaller than the dose of 0,3 mSv used for calibration, so G_0 can be neglected when adjusting the dosemeter.

For simplicity the uncertainty of the field parameters of the calibration facility is assumed to be much less than 10 % and can, therefore, be neglected. The technicians are advised to adjust until the deviation of the reading from the reference value is less than 10 % and, furthermore, perform the adjustment as thoroughly as possible. Therefore, no possible value of n is below 0,9 or above 1,1 and most values are very close to unity. The existing knowledge about the calibration factor N is given by

$$0{,}9 \leq n \leq 1{,}1 \tag{7}$$

and by the assumption of a triangular probability density distribution of n, see Figure 1. In this example, the choice of the triangular probability density distribution is the decision of the evaluator, other conditions or other evaluators may lead to other distributions.

As shown in 5.3.6, this leads to $\hat{n} = 1{,}0$ and to the standard uncertainty of $\hat{n}$ of $s_{\hat{n}} = \dfrac{0{,}1}{\sqrt{6}} = 0{,}041$ for the analytical method. A random number from this distribution is given by $n = 0{,}9 + 0{,}1 \times (z_1 + z_2)$ with the two independent random numbers z_1 and z_2 from the uniform distribution in the interval 0..1 (needed for the Monte Carlo method).

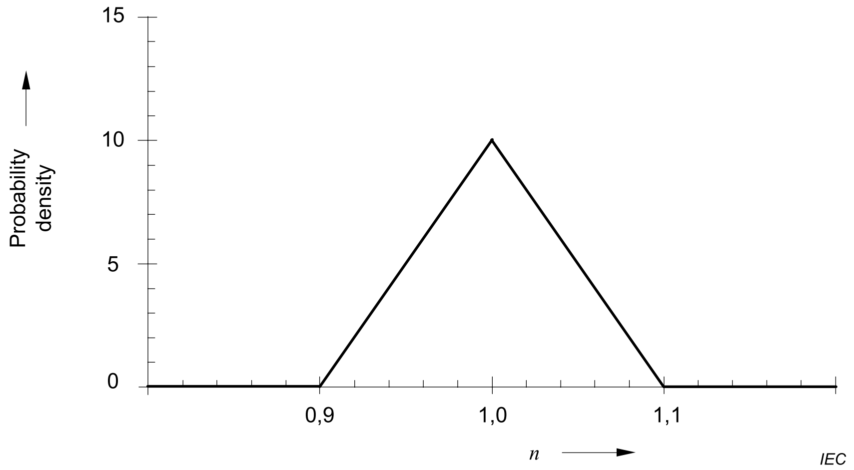


Figure 1 – Triangular probability density distribution of possible values *n* for the calibration factor *N*

5.3.3 Zero reading for the example

As mentioned above, the dosemeter indicates the dose directly in units of the quantity to be measured, thus, a digital display with a resolution of 1 μSv is assumed. During the adjustment procedure at the factory, the technicians are advised to adjust the zero reading until the dosemeter indicates 0 μSv. So the zero reading G_0 in formula (5) should only correct for remaining imperfections in the adjustment process. Due to a resolution of 1 μSv, the adjustment can only be done within ±0,5 μSv, otherwise the indication would be +1 μSv or −1 μSv. Dosemeters will normally not display negative values, but this is assumed to be possible for illustration purposes. The best estimate (mean value) of G_0 is $\hat{g}_0$ = 0 μSv. In the range of ±0,5 μSv, each possible value of g_0 has the same probability, as the indication is always 0 μSv. Consequently, the existing knowledge about the zero reading G_0 is given by

$$-0{,}5\ \mu\text{Sv} \le g_0 \le +0{,}5\ \mu\text{Sv} \tag{8}$$

and by the assumption of a rectangular probability distribution of g_0, see Figure 2. In this example, the choice of the rectangular probability density distribution is the decision of the evaluator, other conditions or other evaluators may lead to other distributions.

As shown in 5.3.6, this leads to $\hat{g}_0$ = 0 μSv and to the standard uncertainty of $\hat{g}_0$ of $s_{\hat{g}_0} = \dfrac{0{,}5\ \mu\text{Sv}}{\sqrt{3}}$ = 0,29 μSv for the analytical method. A random number from this distribution is given by $g_0 = (-0{,}5 + z_1)$ μSv with the random number z_1 from the interval 0..1 (needed for the Monte Carlo method).

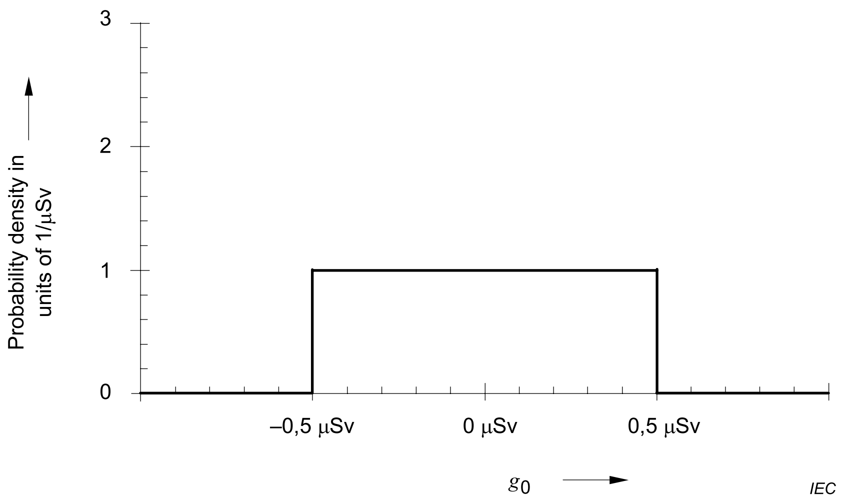


Figure 2 – Rectangular probability density distribution of possible values g_0 for the zero reading G_0

5.3.4 Reading for the example

The reading G is a statistically distributed quantity. When measuring a dose much higher than the zero reading G_0, a normal distribution of the possible reading values g is adequate and a relative standard deviation of the readings of 4 % is assumed. This is not much smaller than the requirement given in IEC 61526:2010 [17]. A best estimate of $\hat{g}$ = 500 μSv (arbitrarily chosen) leads to a distribution given by g = (500 + 20 × y) μSv with y a draw from the standard Gaussian distribution, see 5.3.6, and to a standard deviation of $s_{\hat{g}}$ = 0,04 × 500 μSv = 20 μSv. This distribution is shown in Figure 3.

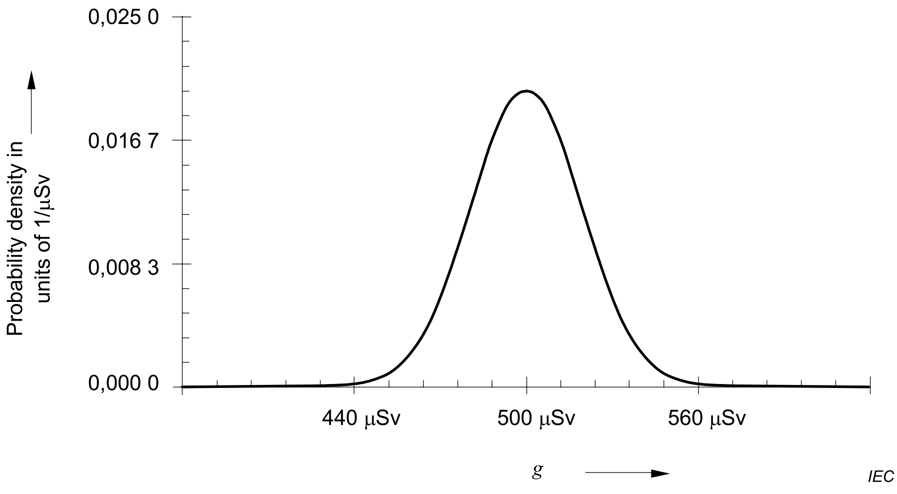


Figure 3 – Gaussian probability density distribution of possible values g for the reading G

5.3.5 Relative response or correction factor for the example

5.3.5.1 General

The relative response R_{rel} requires a more complex discussion. In general, it is composed of several separate relative responses for different influence quantities, R_{rel} being the product of all these. In case of individual monitoring, these influence quantities are determined by the workplace conditions, for example radiation energy and direction of radiation incidence, climatic conditions given by temperature and humidity, dose rate, prevailing during dose measurement. Different levels of consideration of these workplace conditions are possible.

The lowest level is the assumption that the dosemeter is adequate for the workplace. This means that the values of influence quantities prevailing at the workplace are within the rated ranges specified in the data sheet of the dosemeter. This level may be adequate for low dose values far below the dose limit.

An even worse level could be that the workplace conditions are not covered by the rated ranges, but this will not be considered here.

The highest level of consideration is given when the workplace conditions of a given dose measurement are considered in detail. The values of the influence quantities are determined by on site investigations and the corrections valid for these special conditions are applied to the dose value. The corrections can, for example, be taken from the response values determined in the course of a type test. This level of consideration may be adequate in case of an accident or when the dose value is near or above the dose limit.

In the following, two examples (low and high level of consideration of workplace conditions) are given.

5.3.5.2 Example of low level of consideration of workplace conditions

The workplace conditions are covered by the rated ranges of the influence quantities given in the relevant standard, for example, IEC 61526:2010, for the dosemeter used. In other words, the dosemeter was adequately selected for the measurement task, but the actual values of the influence quantities are not known or not considered during dose evaluation. Because the combined influence quantity “radiation energy and direction of radiation incidence” is most important, this example will focus on this influence quantity and neglect all the others. If necessary other influence quantities can be included in analogy to the method given here by introducing further relative responses or correction factors. If the dosemeter fulfils the requirements of IEC 61526:2010 the relative response to photon radiation (relative to the response to reference radiation, for example, Cs-137) is between 0,71 and 1,67 within the whole rated range. As this range is non-symmetric to unity, a transformation of variables from the relative response to the correction factor, $K = 1/R_{\mathrm{rel}}$, is done. This results in a correction factor between 1,4 and 0,6. Therefore, all possible values k of the correction factor K are within this range: $0{,}6 \leq k \leq 1{,}4$. This transformation of variables is done in this case as the centre value of the resulting variable is closer to the expected value (unity) than the centre value of the original variable, see 5.2.

The choice of the distribution of k within the range given above is guided by the following facts for a measuring period of one day:

a) The person wearing the dosemeter is changing his orientation in the radiation field during work because of the persons’ movement. Therefore, the mean value of the angle of radiation incidence is estimated to be close to the centre of the interval valid for the angle. In general, the correction factor has its extreme values for extreme values of the angle of radiation incidence. Therefore, the mean value for the correction factor is expected to be close to the centre of the interval valid for k.

b) The workplace fields, given for example, by the spectral distribution of the photons, are broader than the radiation fields used during the type test. This also causes the correction factor to be close to the centre of the interval valid for k.

c) The movement of the person also changes the radiation field he is in. This makes the range of photon energies impinging on the dosemeter even broader, enhancing the probability of a correction factor close to the centre of the interval valid for k even more.

All these statements give rise to a distribution that is even more peaked than the triangular distribution given in 5.3.2. One possible distribution is a normal (Gaussian) distribution where 99,7 % of all possible k values are within the given interval (the interval half-width is $3 \times s_{\hat{k}}$).

Concerning the normal distribution, there are 0,15 % of the possible k values below the limit of 0,6 and 0,15 % above the limit of 1,4. This is small enough to be neglected.

The existing knowledge about the correction factor K is given by

$$0{,}6 \le k \le 1{,}4 \tag{9}$$

and by a Gaussian probability distribution of k peaked at the centre of the interval. The Gaussian probability distribution was chosen as responses in workplace conditions are often quite close to 1,0 [11]. As always, the choice of the (Gaussian) probability density distribution is the decision of the evaluator, other conditions may lead to other distributions.

This leads to $\hat{k} = 1{,}0$ and to the standard uncertainty of $\hat{k}$ of $s_{\hat{k}} = \frac{0{,}4}{3} = 0{,}133$. A random number from the corresponding distribution is given by $k = (1 + 0{,}133 \times y)$ with y a draw from the standard Gaussian distribution, see 5.3.6.

5.3.5.3 Example of high level of consideration of workplace conditions

The workplace under consideration is an X-ray testing equipment for aluminium wheel rims for cars. In the respective energy range, the relative response of the dosemeter (relative to the response to reference radiation, for example, Cs-137) is low, always below unity. Therefore, it is assumed that the correction factor is between 1,0 and 1,4. Again, the indicated dose value, the reading, was 500 μSv after one working day. As this is an unexpected high value, the measured dose value should be determined considering all knowledge of the workplace.

All possible values k of the correction factor K are within the range: $1{,}0 \le k \le 1{,}4$. The arguments for the probability distribution of the k values given above are still valid for a working period of one day and will, therefore, be applied as well.

Therefore, the existing knowledge about the correction factor K for this example is given by

$$1{,}0 \le k \le 1{,}4 \tag{10}$$

and by a Gaussian probability distribution of k peaked at the centre of that interval. Again, the Gaussian probability distribution was chosen as responses in workplace conditions are often quite close to 1,0 [11].

This leads to $\hat{k} = 1{,}2$ and to the standard uncertainty of $\hat{k}$ of $s_{\hat{k}} = \frac{0{,}2}{3} = 0{,}067$. A random number from the corresponding distribution is given by $k = (1{,}2 + 0{,}067 \times y)$ with y a draw from the standard Gaussian distribution, see 5.3.6.

The corrected measured value is $\hat{m} = 1{,}2 \times 500$ μSv = 600 μSv with an associated uncertainty smaller than in case of low level consideration of workplace conditions, this is shown in 5.4.

5.3.6 Comparison of probability density distributions for input quantities

For input quantities that were determined as mean value of several measurements the standard uncertainty is given by the standard deviation of a single measurement divided by the square root of the number of the measurements – in the GUM called type A evaluation of uncertainty. To these input quantities usually a *t*-distribution can be assigned (see 6.4.9.2 and Table 1 of the GUM S1:2008).

For all the other input quantities the standard uncertainty has to be obtained by other than statistical methods, i.e. from an assumed probability density function based on the degree of belief about the value for the input quantity [often called subjective probability] – in the GUM called type B evaluation of uncertainty. In most cases one of the following probability density functions can be assumed: a rectangular, triangular or Gaussian distribution with its half-width, denoted here by the symbol a. Further distributions are given in table 1 of the GUM S1. For all these probability distributions, the most probable value, the best estimate, is the centre of the distribution, denoted here by the symbol $\hat{x}$. In practice, either the best estimate, $\hat{x}$, is given and the limits a_- and a_+ of the distribution have to be chosen symmetrical to this best estimate as $a_- = \hat{x} - a$ and $a_+ = \hat{x} + a$, or the limits a_- and a_+ are given, for example, of the correction factor discussed in 5.3.5, and the best estimate is the mean value

$$\hat{x} = \frac{a_- + a_+}{2} \tag{11}$$

For comparison purposes, the probability distributions mentioned in this Technical Report are summarized in Figure 4 and the values for the standard uncertainty and the corresponding method of computation are given in Table 2. Other distributions may also be used, if appropriate. Further examples are given in 6.4 of the GUM S1:2008.

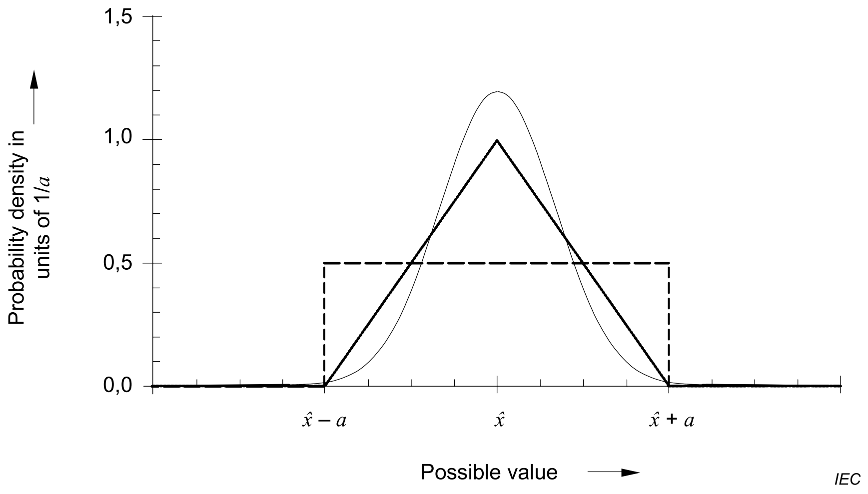


Figure 4 – Comparison of different probability density distributions of possible values: rectangular (broken line), triangular (dotted line) and Gaussian (solid line) distribution

Table 2 – Standard uncertainty and method to compute the probability density distributions shown in Figure 4

Type of distribution	Standard uncertainty	Computation method[1]	Remark
Rectangular	$\frac{a}{\sqrt{3}}$	$x = a_- + 2\,a\,z$	100 % of all possible values are within the interval from a_- to a_+ with the centre at $\hat{x}$ and a half width of a
Triangular	$\frac{a}{\sqrt{6}}$	$x = a_- + a\,(z_1 + z_2)$	100 % of all possible values are within the interval from a_- to a_+ with the centre at $\hat{x}$ and a half width of a
Gaussian	$\frac{a}{3}$	$x = \hat{x} + \frac{a}{3}\,y$	99,7 % of all possible values are within the interval from a_- to a_+ with the centre at $\hat{x}$ and a half width of a

[1] z, z_1, and z_2 denote random numbers out of the interval 0 .. 1 (rectangular distribution);

y denotes a random number from the standard Gaussian distribution.

NOTE Two values of the standard Gaussian distribution can be obtained using two independent draws z_1 and z_2 from the rectangular distribution via $y_1 = \sqrt{-2\ln(z_1)}\cos(2\pi z_2)$ and $y_2 = \sqrt{-2\ln(z_1)}\sin(2\pi z_2)$.

5.4 Calculation of the result of a measurement and its standard uncertainty (uncertainty budget)

5.4.1 General

The third step of the uncertainty analysis is the calculation of the result of a measurement and the associated standard uncertainty according to the model function. This is done using established mathematical methods and may, therefore, also be performed by software, see 5.1.1.

5.4.2 Analytical method

The standard uncertainty, $u(\hat{m})$, associated with the output quantity $\hat{m}$ depends on the standard uncertainties, s, of the input quantities. For every input quantity, the "amount" of this dependence is denoted by the symbol $u(\hat{m})$ with a subscript indicating the input quantity, for example, $u_n(\hat{m})$, $u_k(\hat{m})$, $u_g(\hat{m})$ or $u_{g0}(\hat{m})$ for the input quantities given in formula (5). This "amount" is given by the "extent" to which the output quantity is influenced by variations of the input quantity multiplied by the standard uncertainty of the input quantity. The "extent" is called "sensitivity coefficient", denoted by the symbol c with a subscript indicating the input quantity, for example, c_n, c_k, c_g or c_{g0} for the input quantities given in formula (5). In mathematical language, the "extent" is the change of the output quantity, Δm, due to a change of a particular input quantity, for example, Δn. Their quotient $\Delta m/\Delta n$ is the sensitivity coefficient. Using differential calculus, this is the partial derivative of the model function of the measurement with respect to the particular input quantity. Thus, the sensitivity coefficients according to formulas (5) and (6) are:

$$c_n = \left.\frac{\partial M}{\partial N}\right|_{N=\hat{n},\, K=\hat{k},\, G=\hat{g},\, G_0=\hat{g}_0} = \hat{k}\,(\hat{g}-\hat{g}_0) \qquad (12)$$

$$c_k = \left.\frac{\partial M}{\partial K}\right|_{N=\hat{n},\, K=\hat{k},\, G=\hat{g},\, G_0=\hat{g}_0} = \hat{n}\,(\hat{g}-\hat{g}_0)$$

$$c_g = \left.\frac{\partial M}{\partial G}\right|_{N=\hat{n},\, K=\hat{k},\, G=\hat{g},\, G_0=\hat{g}_0} = \hat{n}\,\hat{k}$$

$$c_{g_0} = \left.\frac{\partial M}{\partial G_0}\right|_{N=\hat{n},\, K=\hat{k},\, G=\hat{g},\, G_0=\hat{g}_0} = -\hat{n}\,\hat{k}$$

The contributions of the standard uncertainties of the input quantities to the standard uncertainty associated with the output quantity are then given by:

$$u_n(\hat{m}) = |c_n|\, s_{\hat{n}} \qquad (13)$$
$$u_k(\hat{m}) = |c_k|\, s_{\hat{k}}$$
$$u_g(\hat{m}) = |c_g|\, s_{\hat{g}}$$
$$u_{g0}(\hat{m}) = |c_{g0}|\, s_{\hat{g}0}$$

NOTE 1 According to the GUM, the values of $u_n(\hat{m})$, $u_k(\hat{m})$, $u_g(\hat{m})$ or $u_{g0}(\hat{m})$ are positive, so the absolute values of the sensitivity coefficients are used in formula (13).

The total standard uncertainty $u(\hat{m})$, associated with the output quantity $\hat{m}$ is given by the geometrical sum of all these contributions.

$$u(\hat{m}) = \sqrt{u_n^2(\hat{m}) + u_k^2(\hat{m}) + u_g^2(\hat{m}) + u_{g_0}^2(\hat{m})} \qquad (14)$$

NOTE 2 Formula (13) is valid for uncorrelated quantities only. Sometimes correlations can be eliminated by a proper choice of the model function. For correlated input quantities, see 5.2 of the GUM:2008.

The corresponding uncertainty budget is given in 5.4.4.

5.4.3 Monte Carlo method

The probability density function (PDF) for the output quantity M and its standard uncertainty has to be obtained from the PDFs of the input quantities via the following steps:

a) Select the number L of Monte Carlo trials to be made, at least 1 000 000 (this figure serves as an example for the following), see also 7.9 in GUM S1:2008 and [10]. The corresponding figures for this example of 1 000 000 trials are given in the following in curly brackets {};

b) Generate L vectors, by sampling from the assigned PDFs, as realizations of the (set of i = 1..4) input quantities X_i: $(N, K, G, \text{and } G_0)^{t}{}_{1..L}$;

c) For each such vector, form the corresponding model value of $M = h(X_i)$: $m = nk\left(g - g_0\right)$ which is the transformed model function, see discussion in 5.3.5, yielding L model values m_j with j = 1..L;

d) Use the L values m_j to form an estimate $m = \frac{1}{L}\sum_{j=1}^{L} m_j$ = 500 µSv of M and the standard uncertainty $u(\hat{m}) = \sqrt{\frac{1}{L-1}\sum_{j=1}^{L}(m_j - \hat{m})^2}$ = 73 µSv associated with $\hat{m}$.

NOTE $\hat{m}$ will in general not agree with the model evaluated at the best estimates of the input quantities, since, for a non-linear model $h(X)$, the expectation value of $h(X)$, $E[h(X)]$, is usually not equal to the model value of the expectation values of the input quantities, $h[E(X)]$ (see 4.1.4 in the GUM:2008). Irrespective of whether h is linear or non-linear, in the limit as L tends to infinity, $\hat{m}$ approaches $E[h(X)]$ when it exists.

5.4.4 Uncertainty budgets

The complete uncertainty analysis for a measurement – sometimes called the uncertainty budget of the measurement – should include a list of all sources of the uncertainty together with the associated probability density distributions, standard uncertainties and the methods of evaluating them. For repeated measurements, the number of observations also has to be stated. For the sake of clarity, it is recommended to present the data relevant for this analysis in the form of a table. An example of such a table for the above example of a dose measurement with an electronic dosemeter using the model function of formula (5) is given in Table 3 for low level of consideration of the workplace conditions and in Table 4 for high level of consideration of the workplace conditions. Columns 1, 2, 3, and 4 are relevant for the Monte Carlo method while columns 1, 2, 3, 5, and 6 are relevant for the analytical method.

It can be seen that in case of high level of consideration of the workplace conditions, the best estimate of the dose is enhanced from 500 µSv to 600 µSv. This is accompanied by a reduction of the standard uncertainty from 73 µSv to 48 µSv, which is equivalent to a relative standard uncertainty of 15 % and 8 %, respectively.

It can also be seen, that the results from the analytical and the Monte Carlo method are equivalent. The reason is that a linear approximation of the model function is valid in the range of the uncertainties of the input quantities. In this case, it would be sufficient to apply the analytical method for similar cases.

Table 3 – Example of an uncertainty budget for a measurement with an electronic dosemeter using the model function $M = N\,K\,(G - G_0)$ and low level of consideration of the workplace conditions, see 5.3.5.2

Quantity	Best estimate	Absolute standard uncertainty	Distribution; mean value, x; half-width, a	Sensitivity coefficient	Uncertainty contribution to output quantity
N	1,0	$\frac{0{,}1}{\sqrt{6}} = 0{,}041$	Triangular; x = 1,0; a = 0,1	500 µSv	0,041 × 500 µSv = 20,5 µSv
K	1,0	$\frac{0{,}4}{3} = 0{,}133$	Gaussian; x = 1,0; a = 0,4	500 µSv	0,133 × 500 µSv = 66,5 µSv
G	500 µSv	0,04 × 500 µSv = 20 µSv	Gaussian with one reading; x = 500 µSv; a = 60 µSv	1,0	20 µSv × 1,0 = 20 µSv
G_0	0 µSv	$\frac{0{,}5\,\mu\text{Sv}}{\sqrt{3}} = 0{,}29\ \mu\text{Sv}$	Rectangular; x = 0,0 µSv; a = 0,5 µSv	– 1,0	0,29 µSv × |– 1,0| = 0,29 µSv
M	500 µSv	73 µSv (15 %)	(Analytical method)		
M	500 µSv	73 µSv (15 %)	(Monte Carlo method)		

Table 4 – Example of an uncertainty budget for a measurement with an electronic dosemeter using the model function $M = N\,K\,(G - G_0)$ and high level of consideration of the workplace conditions, see 5.3.5.3

Quantity	Best estimate	Absolute standard uncertainty	Distribution; mean value, x; half-width, a	Sensitivity coefficient	Uncertainty contribution to output quantity
N	1,0	$\frac{0{,}1}{\sqrt{6}} = 0{,}041$	Triangular; x = 1,0; a = 0,1	600 µSv	0,041 × 600 µSv = 24,6 µSv
K	1,2	$\frac{0{,}2}{3} = 0{,}067$	Gaussian; x = 1,2; a = 0,2	500 µSv	0,067 × 500 µSv = 33,5 µSv
G	500 µSv	0,04 × 500 µSv = 20 µSv	Gaussian with one reading; x = 500 µSv; a = 60 µSv	1,2	20 µSv × 1,2 = 24 µSv
G_0	0 µSv	$\frac{0{,}5\,\mu\text{Sv}}{\sqrt{3}} = 0{,}29\ \mu\text{Sv}$	Rectangular; x = 0,0 µSv; a = 0,5 µSv	– 1,2	0,29 µSv × |– 1,2| = 0,35 µSv
M	600 µSv	48 µSv (8 %)	(Analytical method)		
M	600 µSv	48 µSv (8 %)	(Monte Carlo method)		

5.5 Statement of the measurement result and its expanded uncertainty

5.5.1 General

For Gaussian distributions, the standard uncertainty $u(\hat{m})$ defines an interval from $\hat{m} - u(\hat{m})$ to $\hat{m} + u(\hat{m})$ which covers 68 % of the possible values of the output quantity that could reasonably be attributed to the measurement. In general, a larger certainty (coverage probability or level of confidence) is asked for, therefor, typically the 95 % coverage interval is stated to represent the expanded uncertainty.

For other distributions the percentages mentioned above differ, however, the probability distribution of output quantities is often quite similar to a Gaussian, see G.2.1 of the GUM:2008.

5.5.2 Analytical method

In order to obtain the expanded uncertainty, the standard uncertainty is multiplied by a factor larger than one. The factor is called 'coverage factor', usually given the symbol k but to distinguish it from the correction factor the symbol k_{cov} is used. The expanded uncertainty is usually given the symbol U (capital letter).

For the case of low level of consideration of the workplace conditions the result is

$$M = \hat{m} \pm U(\hat{m}) = 500\ \mu\text{Sv} \pm 146\ \mu\text{Sv}\ (k_{\text{cov}} = 2) \tag{15a}$$

and in the case of high level of consideration of the workplace conditions the result is

$$M = \hat{m} \pm U(\hat{m}) = 600\ \mu\text{Sv} \pm 96\ \mu\text{Sv}\ (k_{\text{cov}} = 2) \tag{15b}$$

NOTE In the example, the increased knowledge leads to a smaller uncertainty. This is not always the case, it is also possible that an increase of knowledge leads to an enhanced uncertainty, for example, because new influence quantities were identified which were ignored previously.

To this statement an explanation should be added which in the general case will have the following content:

> The uncertainty stated is the expanded measurement uncertainty obtained by multiplying the standard uncertainty by a coverage factor $k_{\text{cov}} = 2$. It has been determined in accordance with the *Guide to the Expression of Uncertainty in Measurement*. The value of the measurand then normally lies, with a probability of approximately 95 %, within the attributed coverage interval.

As mentioned in 5.5.1 the 95 % (and accordingly $k_{\text{cov}} = 2$) are only valid for Gaussian output distributions which can, however, mostly be assumed. In case other output distributions have to be assumed, G.6.4 of the GUM:2008 should be considered.

5.5.3 Monte Carlo method

In order to obtain the expanded uncertainty, the following steps have to be applied:

a) Sort the L model values m_j (at least L = 1 000 000 values obtained according to 5.4.3) into increasing order; use these sorted model values to provide the distribution function for the output quantity Q, see Figure 5 for the distribution function of the example;

   NOTE 1 As mentioned in 5.4.3, 1 000 000 values is the minimum number of Monte Carlo trials to be used. In addition, this figure serves as an example for the following. The corresponding figures for this example of 1 000 000 are given in the following in curly brackets {}.

b) Assemble the values m_j into a histogram (with suitable cell widths) to form a frequency distribution normalized to unit area. This distribution provides an approximation to the PDF for M, see Figure 6 for the distribution of the example. Calculations are not generally carried out in terms of this histogram, the resolution of which depends on the choice of cell widths, but in terms of Q (see Figure 5). The histogram can, however, be useful as an aid to understanding the nature of the PDF, e.g. the extent of its asymmetry.

c) Use Q to form an appropriate coverage interval [m_{low}, m_{high}] for M, for a chosen coverage probability p, for example p = 0,95 = 95 % by the following: Let $q = pL$ {= 0,95 × 1 000 000 = 950 000}. If q is no integer it should be rounded to an integer. Then $(L - q)$ {= 50 000} 95 % coverage intervals [m_{low}, m_{high}] exist for M, where $m_{\text{low}} = m_j$ and $m_{\text{high}} = m_{j+q}$ for any j = 1 .. $(L - q)$ {= 1 .. 50 000}. That means $(L - q)$ {= 50 000} different coverage intervals exist. Two of them are of special interest:

1) The probabilistically symmetric p = 95 % coverage interval is given by taking $j = (L - q) / 2$ {= (1 000 000 – 950 000) / 2 = 25 000} and $j+q$ {= (25 000 + 950 000) = 975 000}. If j or q is no integer it should be rounded to an integer. This leads to $m_{\text{low}} = m_{25\,000}$ = 361 µSv and $m_{\text{high}} = m_{975\,000}$ = 648 µSv leading to $U_{\text{low}} = \hat{m} - m_{\text{low}}$ = 139 µSv and $U_{\text{high}} = m_{\text{high}} - \hat{m}$ = 148 µSv. Below and above this interval 2,5 % of the distribution are located.
2) The shortest p = 95 % coverage interval is given by determining j^* such that, for $j = 1 .. (L - q)$ = {1 .. 50 000}, the inequality $m_{j^*+q} - m_{j^*} \leq m_{j+q} - m_j$ is valid, i.e. the difference $m_{j^*+q} - m_{j^*}$ is smaller than all the others. This leads for the shortest interval to m_{low} = 356 µSv and m_{high} = 642 µSv.

In this case the shortest interval is only 0,3 % shorter than the probabilistically symmetric one as the PDF is nearly symmetric to its mean value and unimodal, i.e. it has only one maximum. In case the PDF is non-symmetric, the length of the two coverage intervals can be significantly different; a corresponding example is given in C.3.4.

For more detailed information, Clause 7 of the GUM S1:2008 may be used as a guide.

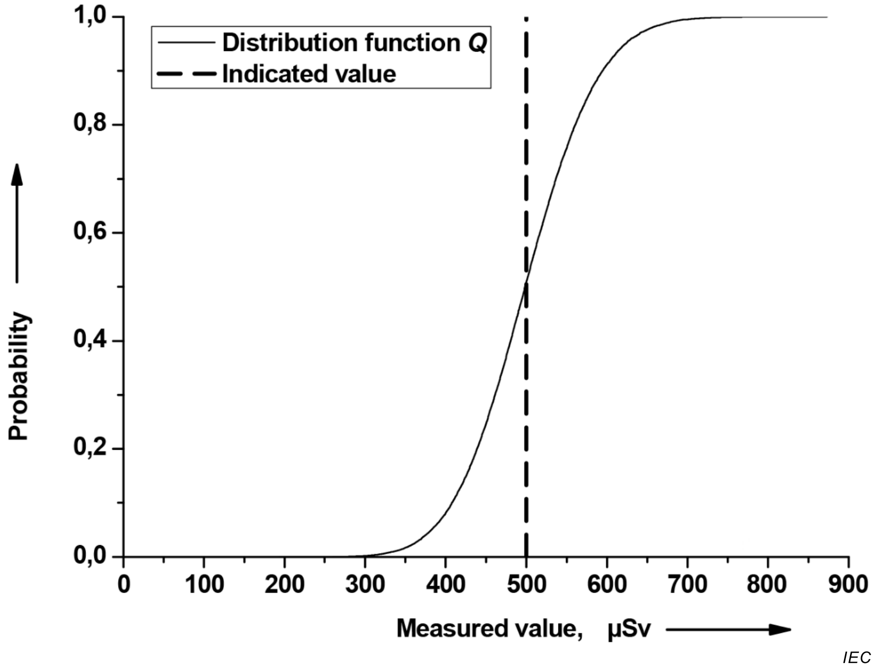


Figure 5 – Distribution function Q of the measured value

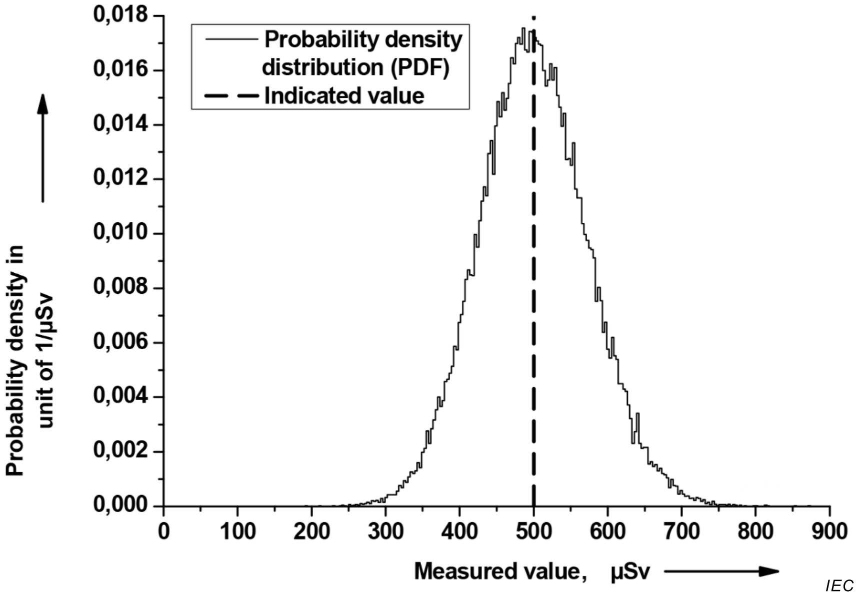


Figure 6 – Probability density distribution (PDF) of the measured value

For the above example, in the case of low level of consideration of the workplace conditions, the complete result of the measurement is given by

$$M = \hat{m}\ {}^{+U_{\text{high}}}_{-U_{\text{low}}} = (500\ {}^{+143}_{-141})\ \mu\text{Sv} \quad \text{at} \quad p = 95\ \%\ \text{(shortest interval)} \tag{16a1}$$

$$M = \hat{m}\ {}^{+U_{\text{high}}}_{-U_{\text{low}}} = (500\ {}^{+145}_{-138})\ \mu\text{Sv} \quad \text{at} \quad p = 95\ \%\ \text{(probabilistically symmetric interval)} \tag{16a2}$$

and in the case of high level of consideration of the workplace conditions, the complete result of the measurement is given by

$$M = \hat{m}\ {}^{+U_{\text{high}}}_{-U_{\text{low}}} = (600\ {}^{+93}_{-94})\ \mu\text{Sv} \quad \text{at} \quad p = 95\ \%\ \text{(shortest interval)} \tag{16b1}$$

$$M = \hat{m}\ {}^{+U_{\text{high}}}_{-U_{\text{low}}} = (600\ {}^{+97}_{-90})\ \mu\text{Sv} \quad \text{at} \quad p = 95\ \%\ \text{(probabilistically symmetric interval)} \tag{16b2}$$

In both cases, the two intervals overlap, thus, these results are consistent.

To this an explanation should be added which in the general case will have the following content:

> The uncertainty stated is the expanded measurement uncertainty with a coverage probability of p = 95 % obtained from the distribution function of the output quantity. It has been determined in accordance with Supplement 1 of the *Guide to the Expression of Uncertainty in Measurement*. The value of the measurand then normally lies, with a probability of approximately 95 %, within the attributed coverage interval (shortest or probabilistically symmetric interval).

NOTE 2 In the last line in brackets either the words “probabilistically symmetric interval” or “shortest interval” depending on which is the case should be given.

Usually, the shortest coverage interval should be stated because the corresponding range of possible values is smallest.

5.5.4 Representation of the output distribution function in a simple form (Monte Carlo method)

In case the result of an uncertainty analysis using the Monte Carlo method is used as input quantity for another uncertainty analysis using the Monte Carlo method, the arbitrarily formed distribution function should be used (an example is given in Figure 5). To represent the distribution function a simple piecewise linear interpolation as described in Annex D of the GUM S1:2008 can be used. To sample draws from this distribution function the corresponding inverse function can be used, see Clause C.2 of the GUM S1:2008.

6 Results below the decision threshold of the measuring device

This clause is applicable for measurements taking into account a gross and a background indication. According to formula (21) and 5.3.3 of ISO 11929:2010 [18], a determined primary measurement result, $\hat{m}$, for a non-negative and Gaussian distributed measurand is only significant (assumed to be larger than zero), if $\hat{m}$ is larger than the decision threshold m^*

$$m^* = k_{1-\alpha} \cdot u(0). \tag{17}$$

α is the probability to detect an effect (state a result above zero) although in reality no effect is present (the true value is zero). For a given error probability α the corresponding quantile of the standardized normal distribution $k_{1-\alpha}$ is given in Annex E of ISO 11929:2010. In this report, a value of α = 5 % is used resulting in $k_{0,95}$ = 1,65 (for α = 1 % it is $k_{0,99}$ = 2,32). $u(0)$ is the standard uncertainty of the measurand for the result zero – to be calculated according to Clause 5.

For measurands whose probability density distribution cannot assumed to be Gaussian (or similar), ISO 11929 should be considered. However, as mentioned in 5.5.1, a Gaussian (or similar) distribution can often be assumed.

NOTE 1 According to its scope ISO 11929:2010 is applicable to counting measurements. Therefore, for the purpose of this report, it is assumed that it can be applied to electronic counting dose(rate) meters, activity (rate)meters and others. In addition, it is assumed to be applicable to all kinds of measurements where a gross and a background indication are used to deduce a net indication.

NOTE 2 In the literature α is also called the “probability of the error of the first kind” or “probability of false positive decision”.

Formula (17) represents a simple approximation for the case that the probability distribution of m is Gaussian or quite similar. In case detailed calculations should be carried out ISO 11929:2010 should be used.

In case the primary measurement result $\hat{m}$ is smaller than the decision threshold m^*, then the result should be stated as follows:

> The result of the measurement cannot be stated because the measured value is below the decision threshold $m^* = k_{1-\alpha} \cdot u(0)$ determined for an error probability of α (α is usually chosen to be 5 %).
>
> The uncertainty at an indicated value of zero, $u(0)$, has been determined in accordance with Supplement 1 of the *Guide to the Expression of Uncertainty in Measurement*. $k_{1-\alpha}$ is the quantile of the standardized normal distribution.
>
> Only if the measured value exceeded the decision threshold, would the physical effect to be measured be recognized as detected. If in reality no physical effect is present, then the measured value is below m^* with a probability of 1-α (usually 95 %).

A corresponding example is given in Annex E.

7 Overview of the annexes

In Annex A and Annex B, examples of uncertainty analysis for an active photon dose rate meter according to IEC 60846-1:2009 and for a passive dosimetry system according to IEC 62387:2012 are given. For each of these instruments, two examples are given. In the first example, it is only assumed that the instrument fulfils the minimum requirements of the respective standard (low level of consideration of the workplace conditions, see 5.3.5.2). In the second example, a special measurement situation is considered, where the values of some the influence quantities are known and the appropriate corrections are applied using the results of the type test (high level of consideration of the workplace conditions, see 5.3.5.3).

Annex C contains an example of uncertainty analysis for a neutron dose rate meter according to IEC 61005:2003. This example clearly demonstrates the benefits of the Monte Carlo method in the case of a non-linear model function and standard uncertainties well beyond 10 % for those influence quantities in the nominator of the model function. In addition, the advantage of the shortest coverage interval compared to the probabilistically symmetric coverage interval is demonstrated in this example.

Annex D contains an example of uncertainty analysis for a radon activity monitor according to the IEC 61577 series. In this example, interim results are calculated and used in subsequent uncertainty analysis to obtain the final result and its corresponding uncertainty. In this example, the results of both the analytical and the Monte Carlo method are equivalent.

Annex E contains an example of uncertainty analysis for a measurement of the surface emission rate with a contamination meter according to IEC 60325:2002. In this example, the measurement result lies below the corresponding decision threshold and is, therefore, stated to be zero (according to Clause 6).

For the sake of readability, the data given in the Annexes are rounded to a reasonable number of digits. Therefore, some data seem to be inconsistent although they are not in reality.

In all Annexes the shortest coverage interval is given.

Annex A
(informative)

Example of an uncertainty analysis for a measurement with an electronic ambient dose equivalent rate meter according to IEC 60846-1:2009

A.1 General

IEC 60846-1:2009 has the title *Radiation protection instrumentation – Ambient and/or directional dose equivalent(rate) meters and/or monitors for beta, X and gamma radiation – Part 1: Portable workplace and environmental meters and monitors* [19].

For this example, a portable dose equivalent rate meter for the ambient dose equivalent rate $\dot{H}^*(10)$ for photon radiation with a logarithmic analogue display of three orders of magnitude is chosen. The lowest range covers 0,1 μSv h^{-1} to 100 μSv h^{-1}, so the measuring range starts according to IEC 60846-1:2009, 5.4 at 10 % deflection, which is equivalent to H_0 = 0,1 μSv h^{-1} × 10^{0,3} ≈ 0,2 μSv h^{-1}. This and some arbitrary assumptions lead to the following measuring range and rated ranges of use for influence quantities:

Measuring range: 0,2 μSv h^{-1} ≤ $\dot{H}^*(10)$ ≤ 1 Sv h^{-1}

Rated ranges of use:

Photon energy:	50 keV ≤ E_{ph} ≤ 1,5 MeV
Angle of incidence:	0° ≤ φ ≤ 45°
Power, pressure, geotropism:	minimum rate ranges, see IEC 60846-1:2009, Table 7.
Temperature, humidity:	minimum rate ranges for outdoor use, see IEC 60846-1:2009, Table 7.
Electromagnetic compatibility (EMC):	minimum rate ranges, see IEC 60846-1:2009, Table 8.
Mechanical disturbances:	minimum rate ranges, see IEC 60846-1:2009, Table 9.

A.2 Model function

According to 5.2, multiplicative influence quantities limited symmetrically in terms of relative response are below the line and those limited symmetrically in terms of correction factor are above the line. As the standard uncertainty of no influence quantity below the line exceeds 10 % the resulting model function can be used for both the analytical and the Monte Carlo method:

$$\dot{H}^*(10) = \frac{N_0 K_{\mathrm{n}}\, K_{E,\varphi} K_{\mathrm{temp}} K_{\mathrm{hum}} K_{\mathrm{press}} K_{\mathrm{pow}}}{R_{\mathrm{geo;rel}}} \times \left[G - D_{\mathrm{zero}} - D_{\mathrm{EMC,1}} - D_{\mathrm{EMC,2}} - D_{\mathrm{EMC,3}} - D_{\mathrm{EMC,4}} - D_{\mathrm{EMC,5}} - D_{\mathrm{micr}} - D_{\mathrm{drop}}\right] \tag{A.1}$$

where

$\dot{H}^*(10)$	is the measuring quantity ambient dose equivalent rate (measured value);
N_0	is the reference calibration factor;
K_{n}	is the correction factor for non-linearity;
$K_{E,\varphi}$	is the correction factor for photon energy and angle of incidence;
K_{temp}	is the correction factor for ambient temperature;

K_{hum} is the correction factor for relative humidity;

K_{press} is the correction factor for atmospheric pressure;

K_{pow} is the correction factor for power supplies;

$R_{geo;rel}$ is the relative response for orientation of the analogue instrument (geotropism) (includes analogue scale resolution and reading parallax);

G is the indicated value, reading of the dosemeter in units of $\dot{H}^*(10)$; (includes coefficient of variation);

D_{zero} is the deviation due to zero drift;

$D_{EMC,1}$ is the deviation due to EMC by electrostatic discharge;

$D_{EMC,2}$ is the deviation due to EMC by radiated electromagnetic fields;

$D_{EMC,3}$ is the deviation due to EMC by radiated electromagnetic fields (mobile phones and WLAN);

$D_{EMC,4}$ is the deviation due to EMC by conducted disturbances (radiofrequencies);

$D_{EMC,5}$ is the deviation due to magnetic field (50 Hz/60 Hz);

D_{micr} is the deviation due to microphonics;

D_{drop} is the deviation due to drop on surface.

NOTE In IEC 60846-1:2009, the deviation is called additional indication.

A.3 Calculation of the complete result of the measurement (measured value, probability density distribution, associated standard uncertainty, and the coverage interval)

A.3.1 General

IEC 60846-1 gives maximum permissible values for the relative response, which is the inverse of the correction factor. Almost all the influence quantities have non-symmetrical limits for the relative response leading to symmetrical limits for the correction factor.

For the combined influence quantity "radiation energy and direction of radiation incidence", these non-symmetrical limits for the relative response are 0,71 and 1,67 leading to respective limits of the correction factor of 0,6 and 1,4 (1,0 ± 40 %). In 5.3.5.2 a Gaussian distribution of the correction factor for the combined influence quantity 'radiation energy and direction of radiation incidence' is assumed. Only one of the arguments given there is valid for measurements of the ambient dose equivalent rate treated here, the argument b), saying that "The workplace fields, given for example, by the spectral distribution of the photons, are broader than the test fields used during the type test.". There is a new argument that a portable instrument is turned until the maximum indication is given. Both arguments cause the correction factor to come closer to the centre of the interval, so a triangular distribution is adequate, because only two arguments are given. According to 5.3.6, a triangular distribution with an interval half-width of 0,4 leads to the following distribution: $k = (0{,}6 + 0{,}4 \times (z_1+z_2))$ with the two independent random numbers z_1 and z_2 from the interval 0..1, see Table 2.

For the relative response due to orientation of the analogue instrument (geotropism) it is assumed that it includes the effects of analogue scale resolution and reading parallax. A limit of ± 2 % of the full scale of maximum angular deflection is given for this. The meaning is different for linear and logarithmic scales. For a linear scale of 0 μSv h^{-1} to 100 μSv h^{-1} this is equivalent to a constant deviation of ± 2 μSv h^{-1}. For a logarithmic scale, this gives a constant value for the change in relative response. If this scale covers three orders of magnitude, then a factor of ten (=10^1) is equivalent to 33 % of the maximum angular deflection and 2 % of the maximum angular deflection is equivalent to a factor of $10^{2/33} = 1{,}15$ or 15 % change in relative response. A Gaussian distribution is assumed, as three influence quantities are included, namely, the geotropism, the analogue scale resolution and the reading parallax. As the relative response is limited symmetrically and the resulting standard uncertainty is smaller than 10 % (see below) the corresponding distribution is directly used without any transformation to the correction factor. According to 5.3.6, a Gaussian distribution with an interval half-width of 0,15 leads to the following distribution: r = (1,0 + 0,05 × y) with y a draw from the standard Gaussian distribution, see Table 2.

A triangular distribution is assumed for the correction factor for intrinsic error, as it consists of two different components, the calibration factor and the non-linearity.

For all other multiplicative influence quantities, a rectangular distribution is used.

For the deviation D_{zero} it is assumed that the best estimate is 0 μSv h^{-1} and the interval half-width is 0,25 × H_0 = 0,05 μSv h^{-1}. For the deviations $D_{\text{EMC,1}}$ to $D_{\text{EMC,5}}$, D_{micr} and D_{drop} it is assumed that the best estimate is also 0 μSv h^{-1} and the interval half-width is given by the maximum permitted deviation and that the interval of possible values is symmetrical including negative deviations. For all these additive input quantities, a rectangular distribution is assumed.

A.3.2 Low level of consideration of measuring conditions

For low level of consideration of measuring conditions, it is only assumed, that the rated ranges of the instrument given above in Clause A.1 totally cover the corresponding values of the radiation field to be measured. These assumptions lead to a correction factor of the indication and to the associated uncertainty valid for these unspecified measuring conditions. Both values can be provided by the manufacturer from the results of the type test. A better specification of the measuring conditions will generally lead to a different value of the correction factor and to a smaller uncertainty.

In Table A.1, the complete uncertainty budget for an indicated value of g = 7,5 μSv h^{-1} is given. The analogue indication has a logarithmic scale from 0,1 μSv h^{-1} to 100 μSv h^{-1}, see Clause A.1. For an indication of 7,5 μSv h^{-1} the limit of the statistical fluctuation (see IEC 60846-1:2009, Table 6) is given by 5 % (for a lower limit of the measuring range of $\dot{H}_0$ = 0,2 μSv h^{-1}).

Table A.1 – Example of an uncertainty budget for a dose rate measurement according to IEC 60846-1:2009 with an instrument having a logarithmic scale and low level of consideration of the measuring conditions, see text for details

Quantity	Best estimate	Absolute standard uncertainty	Distribution; mean value, x; half-width, a	Sensitivity coefficient	Uncertainty contribution to output quantity
N_0	1,00	$0{,}05/\sqrt{6}$ = 0,020 4	Triangular; x = 1,0; a = 0,05	7,5 μSv h^{-1}	0,15 μSv h^{-1}
K_n	1,00	$0{,}18/\sqrt{6}$ = 0,073 5	Triangular; x = 1,0; a = 0,18	7,5 μSv h^{-1}	0,55 μSv h^{-1}
$K_{E,\varphi}$	1,00	$0{,}40/\sqrt{6}$ = 0,163	Triangular; x = 1,0; a = 0,4	7,5 μSv h^{-1}	1,20 μSv h^{-1}
K_{temp}	1,00	$0{,}15/\sqrt{3}$ = 0,0866	Rectangular; x = 1,0; a = 0,15	7,5 μSv h^{-1}	0,65 μSv h^{-1}
K_{hum}	1,00	$0{,}10/\sqrt{3}$ = 0,0577	Rectangular; x = 1,0; a = 0,1	7,5 μSv h^{-1}	0,43 μSv h^{-1}
K_{press}	1,00	$0{,}10/\sqrt{3}$ = 0,0577	Rectangular; x = 1,0; a = 0,1	7,5 μSv h^{-1}	0,43 μSv h^{-1}
K_{pow}	1,00	$0{,}05/\sqrt{3}$ = 0,0289	Rectangular; x = 1,0; a = 0,05	7,5 μSv h^{-1}	0,22 μSv h^{-1}
$R_{geo;rel}$	1,00	0,15/3 = 0,05	Gaussian; x = 1,0; a = 0,15	−7,5 μSv h^{-1}	0,38 μSv h^{-1}
G	7,5 μSv h^{-1}	0,05 × 7,5 μSv h^{-1} = 0,375 μSv h^{-1}	Gaussian with one reading; x = 7,5 μSv h^{-1}; a = 1,125 μSv h^{-1}	1,0	0,38 μSv h^{-1}
D_{zero}	0 μSv h^{-1}	$0{,}05\ \mu Sv\, h^{-1}/\sqrt{3}$ = 0,0289 μSv h^{-1}	Rectangular; x = 0,0 μSv h^{-1}; a = 0,05 μSv h^{-1}	−1,0	0,029 μSv h^{-1}
$D_{EMC,1}$	0 μSv h^{-1}	$0{,}7 \times 0{,}2\ \mu Sv\, h^{-1}/\sqrt{3}$ = 0,081 μSv h^{-1}	Rectangular; x = 0,0 μSv h^{-1}; a = 0,14 μSv h^{-1}	−1,0	0,081 μSv h^{-1}
$D_{EMC,2}$	0 μSv h^{-1}	$0{,}7 \times 0{,}2\ \mu Sv\, h^{-1}/\sqrt{3}$ = 0,081 μSv h^{-1}	Rectangular; x = 0,0 μSv h^{-1}; a = 0,14 μSv h^{-1}	−1,0	0,081 μSv h^{-1}
$D_{EMC,3}$	0 μSv h^{-1}	$0{,}7 \times 0{,}2\ \mu Sv\, h^{-1}/\sqrt{3}$ = 0,081 μSv h^{-1}	Rectangular; x = 0,0 μSv h^{-1}; a = 0,14 μSv h^{-1}	−1,0	0,081 μSv h^{-1}
$D_{EMC,4}$	0 μSv h^{-1}	$0{,}7 \times 0{,}2\ \mu Sv\, h^{-1}/\sqrt{3}$ = 0,081 μSv h^{-1}	Rectangular; x = 0,0 μSv h^{-1}; a = 0,14 μSv h^{-1}	−1,0	0,081 μSv h^{-1}
$D_{EMC,5}$	0 μSv h^{-1}	$0{,}7 \times 0{,}2\ \mu Sv\, h^{-1}/\sqrt{3}$ = 0,081 μSv h^{-1}	Rectangular; x = 0,0 μSv h^{-1}; a = 0,14 μSv h^{-1}	−1,0	0,081 μSv h^{-1}
D_{micr}	0 μSv h^{-1}	$0{,}7 \times 0{,}2\ \mu Sv\, h^{-1}/\sqrt{3}$ = 0,081 μSv h^{-1}	Rectangular; x = 0,0 μSv h^{-1}; a = 0,14 μSv h^{-1}	−1,0	0,081 μSv h^{-1}
D_{drop}	0 μSv h^{-1}	$0{,}7 \times 0{,}2\ \mu Sv\, h^{-1}/\sqrt{3}$ = 0,081 μSv h^{-1}	Rectangular; x = 0,0 μSv h^{-1}; a = 0,14 μSv h^{-1}	−1,0	0,081 μSv h^{-1}
$\dot{H}^*(10)$	7,50 μSv h^{-1}	1,73 μSv h^{-1} (23 %)	(Analytical method)		
$\dot{H}^*(10)$	7,52 μSv h^{-1}	1,75 μSv h^{-1} (23 %)	(Monte Carlo method)		

The complete result of the measurement of the ambient dose equivalent rate for photon radiation according to Table A.1 is:

$$\dot{H}^*(10) = (7{,}5\ ^{+3{,}5}_{-3{,}2})\ \mu\text{Sv h}^{-1}\quad \text{(Analytical method)} \tag{A.2}$$

$$\dot{H}^*(10) = (7{,}5 \pm 3{,}5)\ \mu\text{Sv h}^{-1}\quad \text{(Monte Carlo method)} \tag{A.3}$$

The two results differ by less than 10 %, therefore, the result of the analytical method can be used and the corresponding statement is:

> The uncertainty stated is the expanded measurement uncertainty obtained by multiplying the standard uncertainty by a coverage factor $k_{\text{cov}} = 2$. It has been determined in accordance with the “Guide to the Expression of Uncertainty in Measurement”. The value of the measurand normally lies, with a probability of approximately 95 %, within the attributed coverage interval.

Using the actual results of the type test and the actual measuring conditions both, the correction factor and the uncertainty can be determined for the actual measurement. In general, this will lead to a much smaller uncertainty. This is shown in the next subclause.

A.3.3 High level of consideration of measuring conditions

For high level of consideration of measuring conditions, it is assumed that the task was to measure the radiation of a Co-60 source behind a shield inside a building. So the energy was in the range from 300 keV to 1,3 MeV and the angle of incidence varies from 0° for direct ration to 45° for the stray radiation. With the results of the type test, this leads to a correction factor $K_{E,\varphi}$ of 0,92 to 1,08, again with the assumption of a triangular distribution. The temperature was 10 °C ± 1 °C leading to a correction factor of $K_{\text{temp}} = 1{,}03 \pm 0{,}01$. The relative humidity was 80 % ± 10 % leading to a correction factor of $K_{\text{hum}} = 0{,}99 \pm 0{,}005$. Power supplies were fresh batteries and the atmospheric pressure has no influence on the measurement, as the detector is a GM-tube, so both correction factors K_{pow} and K_{press} were unity and the respective standard uncertainties can be neglected. For the geotropism (and included analogue scale resolution and reading parallax) the value from the type test is 1 % of maximum angular deflection, which is equivalent to a factor of $10^{1/33} = 1{,}07$ or 7 % change in relative response, leading to the interval 1,0 ± 0,07, see above. The zero reading D_{zero} was as before, see A.3.2. In that building, electromagnetic compatibility (EMC) effects can be neglected and as the instrument is carried carefully by hand, the effects of vibration and shock can also be neglected.

In Table A.2, the complete uncertainty budget for an indicated value of $g = 7{,}5\ \mu\text{Sv h}^{-1}$ is given. For that indicated value, the type test result shows a correction factor for non-linearity of 0,96 ± 0,01 and the calibration (and adjustment) certificate gives a respective correction factor of 1,00 ± 0,04. The statistical fluctuation for that indicated value can be interpolated from the type test result to be 4,5 %.

The above considerations lead to a special correction factor of the indication of 0,98 to get the best estimate of the dose rate and give the special uncertainty for that measurement. Both values can only be determined by the user of the instrument. Required for this determination is the knowledge of the special measuring conditions and the results of the type test.

Table A.2 – Example of an uncertainty budget for a dose rate measurement according to IEC 60846-1:2009 with an instrument having a logarithmic scale and high level of consideration of the measuring conditions, see text for details

Quantity	Best estimate	Absolute standard uncertainty	Distribution; mean value, x; half-width, a	Sensitivity coefficient	Uncertainty contribution to output quantity
N_0	1,00	$0{,}04/\sqrt{6}$ = 0,016	Triangular; x = 1,0; a = 0,04	7,3 μSv h^{-1}	0,12 μSv h^{-1}
K_{n}	0,96	$0{,}01/\sqrt{6}$ = 0,004	Triangular; x = 0,96; a = 0,001	7,6 μSv h^{-1}	0,031 μSv h^{-1}
$K_{E,\varphi}$	1,00	$0{,}08/\sqrt{6}$ = 0,033	Triangular; x = 1,0; a = 0,08	7,3 μSv h^{-1}	0,24 μSv h^{-1}
K_{temp}	1,03	$0{,}01/\sqrt{3}$ = 0,006	Rectangular; x = 1,03; a = 0,01	7,1 μSv h^{-1}	0,041 μSv h^{-1}
K_{hum}	0,99	$0{,}005/\sqrt{3}$ = 0,003	Rectangular; x = 0,99; a = 0,005	7,4 μSv h^{-1}	0,21 μSv h^{-1}
K_{press}	1,00	0	Rectangular; x = 1,0; a = 0,0	7,3 μSv h^{-1}	0 μSv h^{-1}
K_{pow}	1,00	0	Rectangular; x = 1,0; a = 0,0	7,3 μSv h^{-1}	0 μSv h^{-1}
$R_{\mathrm{geo;rel}}$	1,00	0,07/3 = 0,023	Gaussian; x = 1,0; a = 0,07	−7,3 μSv h^{-1}	0,17 μSv h^{-1}
G	7,5 μSv h^{-1}	0,045 × 7,5 μSv h^{-1} = 0,3375 μSv h^{-1}	Gaussian with one reading; x = 7,5 μSv h^{-1}; a = 1,0125 μSv h^{-1}	0,98	0,33 μSv h^{-1}
D_{zero}	0 μSv h^{-1}	$0{,}05\ \mu\mathrm{Sv\,h^{-1}}/\sqrt{3}$ = 0,029 μSv h^{-1}	Rectangular; x = 0,0 μSv h^{-1}; a = 0,05 μSv h^{-1}	−0,98	0,028 μSv h^{-1}
$D_{\mathrm{EMC,1}}$	0 μSv h^{-1}	0 μSv h^{-1}	Rectangular; x = 0,0 μSv h^{-1}; a = 0,0 μSv h^{-1}	−0,98	0 μSv h^{-1}
$D_{\mathrm{EMC,2}}$	0 μSv h^{-1}	0 μSv h^{-1}	Rectangular; x = 0,0 μSv h^{-1}; a = 0,0 μSv h^{-1}	−0,98	0 μSv h^{-1}
$D_{\mathrm{EMC,3}}$	0 μSv h^{-1}	0 μSv h^{-1}	Rectangular; x = 0,0 μSv h^{-1}; a = 0,0 μSv h^{-1}	−0,98	0 μSv h^{-1}
$D_{\mathrm{EMC,4}}$	0 μSv h^{-1}	0 μSv h^{-1}	Rectangular; x = 0,0 μSv h^{-1}; a = 0,0 μSv h^{-1}	−0,98	0 μSv h^{-1}
$D_{\mathrm{EMC,5}}$	0 μSv h^{-1}	0 μSv h^{-1}	Rectangular; x = 0,0 μSv h^{-1}; a = 0,0 μSv h^{-1}	−0,98	0 μSv h^{-1}
D_{micr}	0 μSv h^{-1}	0 μSv h^{-1}	Rectangular; x = 0,0 μSv h^{-1}; a = 0,0 μSv h^{-1}	−0,98	0 μSv h^{-1}
D_{drop}	0 μSv h^{-1}	0 μSv h^{-1}	Rectangular; x = 0,0 μSv h^{-1}; a = 0,0 μSv h^{-1}	−0,98	0 μSv h^{-1}
$\dot{H}^*(10)$	7,34 μSv h^{-1}	0,46 μSv h^{-1} (6,3 %) (Analytical method)			
$\dot{H}^*(10)$	7,35 μSv h^{-1}	0,47 μSv h^{-1} (6,3 %) (Monte Carlo method)			

The complete result of the measurement of the ambient dose equivalent rate for photon radiation according to Table A.2 is:

$$\dot{H}^*(10) = (7{,}35\ ^{+0{,}91}_{-0{,}90})\ \mu\text{Sv h}^{-1}\ \ \text{(Analytical method)} \tag{A.4}$$

$$\dot{H}^*(10) = (7{,}34 \pm 0{,}92)\ \mu\text{Sv h}^{-1}\ \ \text{(Monte Carlo method)} \tag{A.5}$$

The two results differ by less than 10 %, therefore, the result of the analytical method can be used and the corresponding statement is:

> The uncertainty stated is the expanded measurement uncertainty obtained by multiplying the standard uncertainty by a coverage factor $k_{\text{cov}} = 2$. It has been determined in accordance with the *Guide to the Expression of Uncertainty in Measurement*. The value of the measurand normally lies, with a probability of approximately 95 %, within the attributed coverage interval.

The two intervals given by formulas (A.2) and (A.4) overlap, thus, the results are consistent.

Annex B
(informative)

Example of an uncertainty analysis for a measurement with a passive integrating dosimetry system according to IEC 62387:2012

B.1 General

IEC 62387:2012 has the title *Passive integrating dosimetry systems for personal and environmental monitoring of photon and beta radiation* [20].

For this example, a dosimetry system for the personal dose equivalent $H_\mathrm{p}(10)$ for photon radiation is chosen and the following measuring range and rated ranges of influence quantities:

Measuring range: 0,1 mSv < $H_\mathrm{p}(10)$ < 1 Sv

Rated ranges of use:

Photon energy: 65 keV < E_ph < 1,25 MeV

Angle of incidence: 0° < φ < 60°

The dosimetry systems uses only one detector. Therefore, the indicated value is additive.

Temperature, light, time: minimum rate ranges for outdoor use (–10 °C to +40 °C), see IEC 62387:2012, Table 13.

Electromagnetic compatibility (EMC): minimum rate ranges, see IEC 62387:2012, Table 14.

Mechanical disturbances: minimum rate ranges, see IEC 62387:2012, Table 15.

B.2 Model function

According to 5.2, multiplicative influence quantities limited symmetrically in terms of relative response are below the line and those limited symmetrically in terms of correction factor are above the line. Thus, the model function used for the example is:

$$H_\mathrm{p}(10) = N_0 \; K_\mathrm{n} \, K_{E,\varphi} \, K_\mathrm{add} \, K_\mathrm{temp} K_\mathrm{light} K_\mathrm{bup} K_\mathrm{stab} K_\mathrm{tempR} K_\mathrm{lightR} K_\mathrm{pow} \times \left[G - D_\mathrm{EMC,1} - D_\mathrm{EMC,2} - D_\mathrm{EMC,3} - D_\mathrm{EMC,4} - D_\mathrm{EMC,5} - D_\mathrm{EMC,6} - D_\mathrm{EMC,7} - D_\mathrm{drop}\right] \quad \text{(B.1)}$$

where

$H_\mathrm{p}(10)$ is the measuring quantity personal dose equivalent (measured value);

N_0 is the reference calibration factor;

K_n is the correction factor for non-linearity;

$K_{E,\varphi}$ is the correction factor for photon energy and angle of incidence;

K_add is the correction factor for additivity;

K_temp is the correction factor for ambient temperature and relative humidity of the dosemeter;

K_light is the correction factor for light exposure of the dosemeter;

K_bup is the correction factor for dose build-up, fading, self-irradiation and response to natural radiation of the dosemeter;

K_stab is the correction factor for reader instability;

K_{tempR} is the correction factor for ambient temperature of the reader;

K_{lightR} is the correction factor for light exposure of the reader;

K_{pow} is the correction factor for power supplies of the reader;

G is the indicated value, reading of the dosemeter in units of $H_{\mathrm{p}}(10)$;

$D_{\mathrm{EMC,1}}$ is the deviation due to electrostatic discharge;

$D_{\mathrm{EMC,2}}$ is the deviation due to conducted disturbances (fast transients);

$D_{\mathrm{EMC,3}}$ is the deviation due to conducted disturbances (surges);

$D_{\mathrm{EMC,4}}$ is the deviation due to conducted disturbances (radiofrequencies);

$D_{\mathrm{EMC,5}}$ is the deviation due to magnetic field (50 Hz/60 Hz);

$D_{\mathrm{EMC,6}}$ is the deviation due to conducted disturbances (voltage dips and interruptions);

$D_{\mathrm{EMC,7}}$ is the deviation due to EM by radiated electromagnetic fields;

D_{drop} is the deviation due to drop on surface.

B.3 Calculation of the complete result of the measurement (measured value, probability density distribution, associated standard uncertainty, and the coverage interval)

B.3.1 General

IEC 62387:2012 gives no type test requirements for the reference calibration factor because this cannot be tested in a type test. A dosimetry system is usually used in a dosimetry service with a high precision calibration facility. Therefore, limits of ±5 % with a triangular distribution are assumed.

IEC 62387:2012 gives maximum permissible values for the relative response, which is the inverse of the correction factor. For all influence quantities, these maximum permissible values are non-symmetrical to give symmetrical limits for the correction factor.

For the deviations $D_{\mathrm{EMC},i}$ and D_{drop} it is assumed, that the best estimate is 0 μSv and the interval of possible values is symmetrical including negative deviations. For all these input quantities, a Gaussian distribution is assumed.

B.3.2 Low level of consideration of workplace conditions

For low level of consideration of workplace conditions, it is only assumed that the rated ranges of the instrument given above in Clause B.1 totally cover the radiation field to be measured and the values of the influence quantities. These assumptions lead to a correction factor of the indication and to the associated uncertainty valid for these unspecified measuring conditions. Both values can be provided by the manufacturer from the results of the type test. A better specification of the measuring conditions will generally lead to a different value of the correction factor and to a smaller uncertainty.

In Table B.1, the complete uncertainty budget for an indicated value of g = 10 mSv is given. As the model function is linear and the input quantities are limited symmetrically around their centre value, only the result from the analytical method is given, see 5.1.4.

Table B.1 – Example of an uncertainty budget for a photon dose measurement with a passive dosimetry system according to IEC 62387-1:2007 and low level of consideration of the workplace conditions, see text for details

Quantity	Best estimate	Absolute standard uncertainty	Distribution; mean value, x; half-width, a	Sensitivity coefficient	Uncertainty contribution to output quantity
N_0	1,00	$0{,}05/\sqrt{6}$ = 0,020 4	Triangular; x = 1,0; a = 0,05	10 mSv	0,20 mSv
K_n	1,00	$0{,}10/\sqrt{3}$ = 0,057 7	Rectangular; x = 1,0; a = 0,1	10 mSv	0,58 mSv
$K_{E,\varphi}$	1,00	0,40/3 = 0,133	Gaussian; x = 1,0; a = 0,4	10 mSv	1,33 mSv
K_add	1,00	0	Rectangular; x = 1,0; a = 0,0	10 mSv	0,0 mSv
K_temp	1,00	0,20/3 = 0,066 7	Gaussian; x = 1,0; a = 0,20	10 mSv	0,67 mSv
K_light	1,00	0,1/3 = 0,033 3	Gaussian; x = 1,0; a = 0,10	10 mSv	0,33 mSv
K_bup	1,00	0,1/3 = 0,033 3	Gaussian; x = 1,0; a = 0,10	10 mSv	0,33 mSv
K_stab	1,00	0,1/3 = 0,033 3	Gaussian; x = 1,0; a = 0,10	10 mSv	0,33 mSv
K_tempR	1,00	0,1/3 = 0,033 3	Gaussian; x = 1,0; a = 0,10	10 mSv	0,33 mSv
K_lightR	1,00	0,1/3 = 0,033 3	Gaussian; x = 1,0; a = 0,10	10 mSv	0,33 mSv
K_pow	1,00	0,1/3 = 0,033 3	Gaussian; x = 1,0; a = 0,10	10 mSv	0,33 mSv
G	10 mSv	0,05 × 10 mSv = 0,50 mSv	Gaussian with one reading; x = 10,0 mSv; a = 1,50 mSv	1,00	0,50 mSv
$D_\mathrm{EMC,1}$	0 mSv	0,7 × 0,1 mSv/3 = 0,023 3 mSv	Gaussian; x = 0,0 mSv; a = 0,07 mSv	−1,00	0,023 mSv
$D_\mathrm{EMC,2}$	0 mSv	0,7 × 0,1 mSv/3 = 0,023 3 mSv	Gaussian; x = 0,0 mSv; a = 0,07 mSv	−1,00	0,023 mSv
$D_\mathrm{EMC,3}$	0 mSv	0,7 × 0,1 mSv/3 = 0,023 3 mSv	Gaussian; x = 0,0 mSv; a = 0,07 mSv	−1,00	0,023 mSv
$D_\mathrm{EMC,4}$	0 mSv	0,7 × 0,1 mSv/3 = 0,023 3 mSv	Gaussian; x = 0,0 mSv; a = 0,07 mSv	−1,00	0,023 mSv
$D_\mathrm{EMC,5}$	0 mSv	0,7 × 0,1 mSv/3 = 0,023 3 mSv	Gaussian; x = 0,0 mSv; a = 0,07 mSv	−1,00	0,023 mSv
$D_\mathrm{EMC,6}$	0 mSv	0,7 × 0,1 mSv/3 = 0,023 3 mSv	Gaussian; x = 0,0 mSv; a = 0,07 mSv	−1,00	0,023 mSv
$D_\mathrm{EMC,7}$	0 mSv	0,7 × 0,1 mSv/3 = 0,023 3 mSv	Gaussian; x = 0,0 mSv; a = 0,07 mSv	−1,00	0,023 mSv
D_drop	0 mSv	0,7 × 0,1 mSv/3 = 0,023 3 mSv	Gaussian; x = 0,0 mSv; a = 0,07 mSv	−1,00	0,023 mSv
$H_p(10)$	10,0 mSv	1,9 mSv (19 %)	(Analytical method)		

The complete result of the measurement of the personal dose equivalent for photon radiation according to Table B.1 is:

$$H_{\mathrm{p}}(10) = (10{,}0 \pm 3{,}8)\ \mathrm{mSv} \tag{B.2}$$

The uncertainty stated is the expanded measurement uncertainty obtained by multiplying the standard uncertainty by a coverage factor $k_{\mathrm{cov}} = 2$. It has been determined in accordance with the *Guide to the Expression of Uncertainty in Measurement*. The value of the measurand normally lies, with a probability of approximately 95 %, within the attributed coverage interval.

B.3.3 High level of consideration of workplace conditions

For a high level of consideration of workplace conditions, it is assumed that the workplace was in a test facility for X-ray tubes with operating voltages between 100 kV and 200 kV resulting in mean photon energies between 70 keV and 150 keV. Considering the variation of the spectrum and the different angles of radiation incidence at this real workplace, the limits of $K_{E,\varphi}$ are assumed as 1,02 and 1,14, leading to $K_{E,\varphi}$ = 1,08 ± 0,06. Again the assumption of a Gaussian distribution is justified, see above.

The temperature was 22 °C ± 6 °C leading to a correction factor of K_{temp} = 1,02 ± 0,04. All other correction factors are assumed to be 1,0 ± 0,0. As the type test showed no effect due to EMC and mechanical influences, $D_{\mathrm{EMC},i}$ and D_{mech} are assumed to be zero as well as their uncertainties.

In Table B.2, the complete uncertainty budget for an indicated value of g = 10 mSv is given. For that indicated value the type test result shows a correction factor for non-linearity of K_{n} = 0,97 ± 0,05. The measured statistical fluctuation for that indicated value can be interpolated from the type test result to be 2,5 %.

As the model function is linear and the input quantities are limited symmetrically around their centre value, only the result from the analytical method is given, see 5.1.4.

Table B.2 – Example of an uncertainty budget for a photon dose measurement with a passive dosimetry system according to IEC 62387-1:2007 and high level of consideration of the measuring conditions, see text for details

Quantity	Best estimate	Absolute standard uncertainty	Distribution; mean value, x; half-width, a	Sensitivity coefficient	Uncertainty contribution to output quantity
N_0	1,00	$0{,}05/\sqrt{6}$ = 0,020 4	Triangular; x = 1,0; a = 0,05	11 mSv	0,22 mSv
K_n	0,97	$0{,}05/\sqrt{3}$ = 0,028 9	Rectangular; x = 0,97; a = 0,05	11 mSv	0,32 mSv
$K_{E,\varphi}$	1,08	0,06/3 = 0,020	Gaussian; x = 1,08; a = 0,06	9,9 mSv	0,20 mSv
K_temp	1,02	$0{,}04/\sqrt{3}$ = 0,023 1	Rectangular; x = 1,02; a = 0,04	10 mSv	0,24 mSv
K_light	1,00	0	Gaussian; x = 1,0; a = 0,0	11 mSv	0 mSv
K_bup	1,00	0	Gaussian; x = 1,0; a = 0,0	11 mSv	0 mSv
K_stab	1,00	0	Gaussian; x = 1,0; a = 0,0	11 mSv	0 mSv
K_tempR	1,00	0	Gaussian; x = 1,0; a = 0,0	11 mSv	0 mSv
K_lightR	1,00	0	Gaussian; x = 1,0; a = 0,0	11 mSv	0 mSv
K_pow	1,00	0	Gaussian; x = 1,0; a = 0,0	11 mSv	0 mSv
G	10 mSv	0,025 × 10 mSv = 0,25 mSv	Gaussian with one reading; x = 10,0 mSv; a = 0,75 mSv	1,1	0,27 mSv
$D_\mathrm{EMC,1}$	0 mSv	0 mSv	Gaussian; x = 0,0 mSv; a = 0,0 mSv	−1,1	0 mSv
$D_\mathrm{EMC,2}$	0 mSv	0 mSv	Gaussian; x = 0,0; a = 0,0	−1,1	0 mSv
$D_\mathrm{EMC,3}$	0 mSv	0 mSv	Gaussian; x = 0,0; a = 0,0	−1,1	0 mSv
$D_\mathrm{EMC,4}$	0 mSv	0 mSv	Gaussian; x = 0,0; a = 0,0	−1,1	0 mSv
$D_\mathrm{EMC,5}$	0 mSv	0 mSv	Gaussian; x = 0,0; a = 0,0	−1,1	0 mSv
$D_\mathrm{EMC,6}$	0 mSv	0 mSv	Gaussian; x = 0,0; a = 0,0	−1,1	0 mSv
$D_\mathrm{EMC,7}$	0 mSv	0 mSv	Gaussian; x = 0,0; a = 0,0	−1,1	0 mSv
D_drop	0 mSv	0 mSv	Gaussian; x = 0,0 mSv; a = 0,0 mSv	−1,1	0 mSv
$H_\mathrm{p}(10)$	10,69 mSv	0,56 mSv (5,3 %)	(Analytical method)		

The above considerations lead to a special correction factor of the indication of 1,069 to get the best estimate of the dose and give the special uncertainty for that measurement. Both values can only be determined by the user of the instrument. Required for this determination is the knowledge of the special measuring conditions or workplace conditions and the results of the type test.

The complete result of the measurement of the personal dose equivalent for photon radiation according to Table B.2 is:

$$H_\mathrm{p}(10) = (10{,}7 \pm 1{,}1)\ \mathrm{mSv} \quad \text{(B.3)}$$

The uncertainty stated is the expanded measurement uncertainty obtained by multiplying the standard uncertainty by a coverage factor $k_{cov} = 2$. It has been determined in accordance with the *Guide to the Expression of Uncertainty in Measurement*. The value of the measurand normally lies, with a probability of approximately 95 %, within the attributed coverage interval.

The two intervals given by formulas (B.2) and (B.3) overlap, thus, the results are consistent.

Annex C
(informative)

Example of an uncertainty analysis for a measurement with an electronic direct reading neutron ambient dose equivalent meter according to IEC 61005:2003

C.1 General

IEC 61005:2003 has the title *Radiation protection instrumentation – Neutron ambient dose equivalent (rate) meters* [21].

For the example, an electronic dosemeter with digital display for the ambient dose equivalent rate $\dot{H}^*(10)$ for neutron radiation is chosen, which has the following measuring range and rated ranges of use for influence quantities:

Measuring range: $10\ \mu\text{Sv} \le \dot{H}^*(10) \le 1\ \text{Sv}$

Rated ranges of use:

Neutron energy:	$0{,}025\ \text{eV} \le E_\text{n} \le 15\ \text{MeV}$
Angle of incidence:	$0° \le \varphi \le 60°$
Power, temperature, humidity, pressure:	minimum rate ranges, see IEC 61005:2003, Table 3.
Electromagnetic compatibility (EMC):	minimum rate ranges, see IEC 61005:2003, Table 4.

C.2 Model function

According to 5.2, multiplicative influence quantities limited symmetrically in terms of relative response (which is the case for all influence quantities in IEC 61005:2003, even the ones due to electromagnetic disturbances) are below the line. Thus, the resulting model function is:

$$\dot{H}^*(10) = \frac{N_0 \times G}{R_{\text{n;rel}}\, R_{E;\text{rel}}\, R_{\varphi;\text{rel}}\, R_{\text{ph;rel}}\, R_{\text{pow;rel}}\, R_{\text{vibr;rel}}\, R_{\text{temp;rel}}\, R_{\text{tempshock;rel}}} \cdot \frac{1}{R_{\text{EMC,1;rel}}\, R_{\text{EMC,2;rel}}\, R_{\text{EMC,3;rel}}\, R_{\text{EMC,4;rel}}\, R_{\text{EMC,5;rel}}\, R_{\text{EMC,6;rel}}} \qquad \text{(C.1)}$$

where

$H^*(10)$	is the measuring quantity ambient dose equivalent rate (measured value);
N_0	is the reference calibration factor;
$R_{\text{n;rel}}$	is the relative response for non-linearity;
$R_{E;\text{rel}}$	is the relative response for neutron energy;
$R_{\varphi;\text{rel}}$	is the relative response for angle of incidence;
$R_{\text{ph;rel}}$	is the relative response for the influence of photon radiation;
$R_{\text{pow;rel}}$	is the relative response for power supplies;
$R_{\text{vibr;rel}}$	is the relative response for vibration;
$R_{\text{temp;rel}}$	is the relative response for ambient temperature;
$R_{\text{tempshock;rel}}$	is the relative response for temperature shock;
G	is the indicated value, reading of the dosemeter in units of $H^*(10)$;

$R_{\mathrm{EMC,1;rel}}$ is the relative response for EMC by electrostatic discharge;

$R_{\mathrm{EMC,2;rel}}$ is the relative response for EMC by radiated electromagnetic fields;

$R_{\mathrm{EMC,3;rel}}$ is the relative response for EMC by conducted disturbances (radiofrequencies);

$R_{\mathrm{EMC,4;rel}}$ is the relative response for EMC by conducted disturbances (surges);

$R_{\mathrm{EMC,5;rel}}$ is the relative response for EMC by conducted disturbances (fast transients/bursts);

$R_{\mathrm{EMC,6;rel}}$ is the relative response for magnetic field (50 Hz/60 Hz).

As some limits for the influence quantities have standard uncertainties larger than 10 % (see below), the model function for the analytical method is as follows (using for these influence quantities the transformed variables K):

$$\dot{H}^*(10) = \frac{K_{\mathrm{n}}\, K_E\, K_\varphi\, K_{\mathrm{temp}}\, N_0 \times G}{R_{\mathrm{ph;rel}}\, R_{\mathrm{pow;rel}}\, R_{\mathrm{vibr;rel}}\, R_{\mathrm{tempshock;rel}}} \cdot \frac{1}{R_{\mathrm{EMC,1;rel}}\, R_{\mathrm{EMC,2;rel}}\, R_{\mathrm{EMC,3;rel}}\, R_{\mathrm{EMC,4;rel}}\, R_{\mathrm{EMC,5;rel}}\, R_{\mathrm{EMC,6;rel}}} \tag{C.2}$$

where

K_{n} is the correction factor for non-linearity;

K_E is the correction factor for neutron energy;

K_φ is the correction factor for angle of incidence;

K_{temp} is the correction factor for ambient temperature.

C.3 Calculation of the complete result of the measurement (measured value, probability density distribution, associated standard uncertainty, and the coverage interval)

C.3.1 General

IEC 61005:2003 gives no type test requirements for the reference calibration factor because this cannot be tested in a type test, it can only be tested in a routine test. Therefore, limits of ±10 % with a triangular distribution are assumed.

IEC 61005:2003 gives symmetrical limits for all maximum permissible values for the relative response, see tables 2 to 4 of that standard. For the energy dependence no limits are stated, therefore, the value of ±50 % is adopted from another international standard for neutron devices [22].

For both the analytical and the Monte Carlo method, always the maximum permissible ranges of the influence quantities are assumed (low level of consideration of workplace conditions).

C.3.2 Analytical method

Formula (C.2) is used as model function. The transformation of variables leads, for the example of the relative response for neutron energy, $R_{\mathrm{E;rel}}$, from 1 ± 0,5 (which is the interval 0,5 … 1,5) to 1,33 ± 0,67 (which is the interval 0,67 … 2,0 resulting from the two limits 1/1,5 = 0,67 and 1/0,5 = 2,0) for the corresponding correction factor, K_{E}. The other response intervals are transformed to correction factor intervals accordingly in case their standard uncertainty is beyond 10 %, see below.

In Table C.1, the complete uncertainty budget for an indicated value of g = 10 mSv/h is given.

Table C.1 – Example of an uncertainty budget for a neutron dose measurement according to IEC 61005:2003 using the analytical method

Quantity	Best estimate	Absolute standard uncertainty	Distribution; mean value, x; half-width, a	Sensitivity coefficient	Uncertainty contribution to output quantity
N_0	1,00	$0{,}10/\sqrt{6}$ = 0,041	Triangular; x = 1,0; a = 0,1	15,4 mSv/h	0,63 mSv/h
G	10 mSv/h	0,20 × 10 mSv/h = 2,0 mSv/h	Gaussian with one reading; x = 10 mSv; a = 6 mSv	1,54	3,1 mSv/h
K_n	1,04	$0{,}21/\sqrt{3}$ = 0,121	Rectangular; x = 1,04; a = 0,21	14,8 mSv/h	1,8 mSv/h
K_E	1,33	$0{,}67/\sqrt{3}$ = 0,387	Rectangular; x = 1,33; a = 0,67	11,5 mSv/h	4,5 mSv/h
K_φ	1,07	$0{,}27/\sqrt{3}$ = 0,156	Rectangular; x = 1,07; a = 0,27	14,4 mSv/h	2,2 mSv/h
$R_\mathrm{ph;rel}$	1,00	$0{,}10/\sqrt{3}$ = 0,058	Rectangular; x = 1,0; a = 0,1	−15,4 mSv/h	0,89 mSv/h
$R_\mathrm{pow;rel}$	1,00	$0{,}10/\sqrt{3}$ = 0,058	Rectangular; x = 1,0; a = 0,1	−15,4 mSv/h	0,89 mSv/h
$R_\mathrm{vibr;rel}$	1,00	$0{,}15/\sqrt{3}$ = 0,087	Rectangular; x = 1,0; a = 0,15	−15,4 mSv/h	1,3 mSv/h
$K_\mathrm{temp;rel}$	1,04	$0{,}21/\sqrt{3}$ = 0,121	Rectangular; x = 1,04; a = 0,21	14,8 mSv/h	1,8 mSv/h
$R_\mathrm{tempshock;rel}$	1,00	$0{,}15/\sqrt{3}$ = 0,087	Rectangular; x = 1,0; a = 0,15	−15,4 mSv/h	1,3 mSv/h
$R_\mathrm{EMC,1;rel}$	1,00	$0{,}10/\sqrt{3}$ = 0,058	Rectangular; x = 1,0; a = 0,1	−15,4 mSv/h	0,89 mSv/h
$R_\mathrm{EMC,2;rel}$	1,00	$0{,}10/\sqrt{3}$ = 0,058	Rectangular; x = 1,0; a = 0,1	−15,4 mSv/h	0,89 mSv/h
$R_\mathrm{EMC,3;rel}$	1,00	$0{,}10/\sqrt{3}$ = 0,058	Rectangular; x = 1,0; a = 0,1	−15,4 mSv/h	0,89 mSv/h
$R_\mathrm{EMC,4;rel}$	1,00	$0{,}10/\sqrt{3}$ = 0,058	Rectangular; x = 1,0; a = 0,1	−15,4 mSv/h	0,89 mSv/h
$R_\mathrm{EMC,5;rel}$	1,00	$0{,}10/\sqrt{3}$ = 0,058	Rectangular; x = 1,0; a = 0,1	−15,4 mSv/h	0,89 mSv/h
$R_\mathrm{EMC,6;rel}$	1,00	$0{,}10/\sqrt{3}$ = 0,058	Rectangular; x = 1,0; a = 0,1	−15,4 mSv/h	0,89 mSv/h
$\dot{H}^*(10)$	15,4 mSv/h	7,2 mSv/h (47 %)	(Analytical method)		

The complete result of the measurement of the ambient dose equivalent rate for neutron radiation according to Table C.1 is:

$$\dot{H}^*(10) = (15 \pm 14)\ \mathrm{mSv/h} \tag{C.3}$$

The uncertainty stated is the expanded measurement uncertainty obtained by multiplying the standard uncertainty by a coverage factor $k_\mathrm{cov} = 2$. It has been determined in accordance with the *Guide to the Expression of Uncertainty in Measurement*. The value of the measurand normally lies, with a probability of approximately 95 %, within the attributed coverage interval.

C.3.3 Monte Carlo method

Formula (C.1) is used as model function. In Table C.2, the complete uncertainty budget for an indicated value of g = 10 mSv is given.

Table C.2 – Example of an uncertainty budget for a neutron dose rate measurement according to IEC 61005:2003 using the Monte Carlo method

Quantity	Best estimate	Absolute standard uncertainty	Distribution; mean value, x; half-width, a
N_0	1,00	$0{,}10/\sqrt{6}$ = 0,041	Triangular; x = 1,0; a = 0,1
G	10 mSv/h	0,20 × 10 mSv/h = 2,0 mSv/h	Gaussian with one reading; x = 10 mSv; a = 6 mSv
$R_{\mathrm{n;rel}}$	1,00	$0{,}20/\sqrt{3}$ = 0,115	Rectangular; x = 1,0; a = 0,2
$R_{E;\mathrm{rel}}$	1,00	$0{,}50/\sqrt{3}$ = 0,289	Rectangular; x = 1,0; a = 0,5
$R_{\varphi;\mathrm{rel}}$	1,00	$0{,}25/\sqrt{3}$ = 0,144	Rectangular; x = 1,0; a = 0,25
$R_{\mathrm{ph;rel}}$	1,00	$0{,}10/\sqrt{3}$ = 0,058	Rectangular; x = 1,0; a = 0,1
$R_{\mathrm{pow;rel}}$	1,00	$0{,}10/\sqrt{3}$ = 0,058	Rectangular; x = 1,0; a = 0,1
$R_{\mathrm{vibr;rel}}$	1,00	$0{,}15/\sqrt{3}$ = 0,087	Rectangular; x = 1,0; a = 0,15
$R_{\mathrm{temp;rel}}$	1,00	$0{,}20/\sqrt{3}$ = 0,115	Rectangular; x = 1,0; a = 0,2
$R_{\mathrm{tempshock;rel}}$	1,00	$0{,}15/\sqrt{3}$ = 0,087	Rectangular; x = 1,0; a = 0,15
$R_{\mathrm{EMC,1;rel}}$	1,00	$0{,}10/\sqrt{3}$ = 0,058	Rectangular; x = 1,0; a = 0,1
$R_{\mathrm{EMC,2;rel}}$	1,00	$0{,}10/\sqrt{3}$ = 0,058	Rectangular; x = 1,0; a = 0,1
$R_{\mathrm{EMC,3;rel}}$	1,00	$0{,}10/\sqrt{3}$ = 0,058	Rectangular; x = 1,0; a = 0,1
$R_{\mathrm{EMC,4;rel}}$	1,00	$0{,}10/\sqrt{3}$ = 0,058	Rectangular; x = 1,0; a = 0,1
$R_{\mathrm{EMC,5;rel}}$	1,00	$0{,}10/\sqrt{3}$ = 0,058	Rectangular; x = 1,0; a = 0,1
$R_{\mathrm{EMC,6;rel}}$	1,00	$0{,}10/\sqrt{3}$ = 0,058	Rectangular; x = 1,0; a = 0,1
$\dot{H}^*(10)$	12,0 mSv/h	6,1 mSv/h (51 %)	(Monte Carlo method)

The complete result of the measurement of the ambient dose equivalent rate for neutron radiation according to Table C.2 is:

$$\dot{H}^*(10) = (12\ {}^{+12}_{-9})\ \mathrm{mSv} \tag{C.4}$$

The uncertainty stated is the expanded measurement uncertainty with a coverage probability of p = 95 % obtained from the distribution function of the output quantity. It has been determined in accordance with Supplement 1 of the *Guide to the Expression of Uncertainty in Measurement*. The value of the measurand normally lies, with a probability of approximately 95 %, within the attributed coverage interval (shortest interval).

C.3.4 Comparison of the result of the analytical and the Monte Carlo method

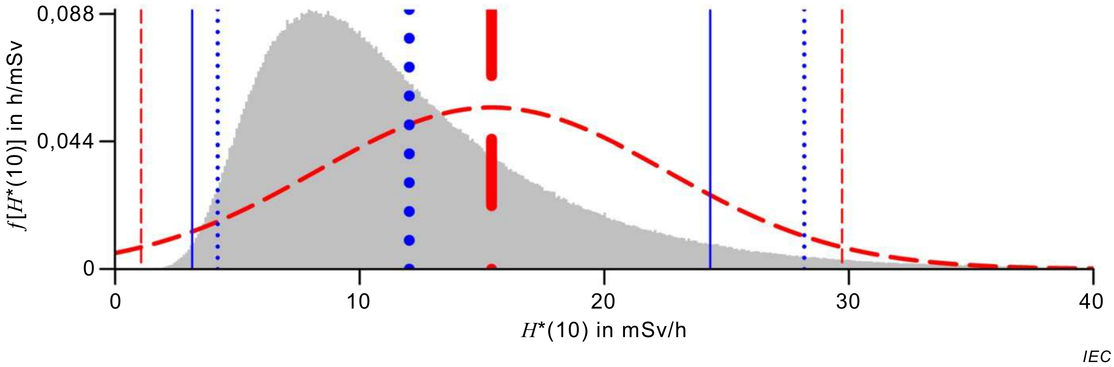
In Figure C.1 the resulting probability density function (PDF) from the Monte Carlo method and the resulting Gaussian PDF according to the analytical method are shown. It can clearly be seen that the realistic result from the Monte Carlo method is not represented by the result of the analytical method, neither the mean value nor the coverage interval. This can also be seen by comparing the data given in Table C.3 and was formerly confirmed by measurements [11].

The reason for the deviation of the best estimate from the indicated value of 10 mSv/h comes for the Monte Carlo method from the non-linear model function; the even stronger deviation for the analytical method has its reason in the artificial (but necessary) transformation of variables from response values to correction factors and the resulting ranges that are not symmetrical to unity, see C.3.2 and 5.2 (paragraph after the note). As a consequence, two different model functions are used which is the main reason for the strong deviations between the results of the analytical and the Monte Carlo method.

In conclusion, the deviation of the best estimate and the limits of the coverage interval is much larger than the criterion of 10 % introduced in 5.1.4. Therefore, the Monte Carlo method should be used in this case.

Besides the shortest coverage interval also the probabilistically symmetric coverage interval is given in Figure C.1 and Table C.2. As the probability distribution function (PDF) of the dose rate is quite non-symmetric (log-normal), quite different intervals occur although both cover 95 % of the PDF. In such cases the shortest coverage interval is clearly superior the probabilistically symmetric and should, therefore, always be stated (see 5.5.3).

This example clearly demonstrates the benefits of the Monte Carlo method, not only for the determination of uncertainty but also for the best estimate of the measured value itself. For all cases with similar non-linear model functions with large standard uncertainties of the input quantities (above about 10 %), the Monte Carlo method should to be used.



NOTE The vertical lines are the mean values (thick lines), the boundaries of the coverage interval from the analytical method (thin red dashed lines), the boundaries of the shortest coverage interval from the Monte Carlo method (blue solid lines), and the boundaries of the probabilistically symmetric coverage interval form the Monte Carlo method (blue dotted lines).

Figure C.1 – Results of the analytical (red dashed lines) and the Monte Carlo method (grey histogram and blue dotted and solid lines) for $\dot{H}^*(10)$

Table C.3 – Results of the analytical and the Monte Carlo method

	Best estimate of the measured value	95 % coverage interval
Analytical method	15 mSv/h	1 mSv/h … 29 mSv/h
Monte Carlo method (shortest coverage interval)	12 mSv/h	3 mSv/h … 24 mSv/h
Monte Carlo method (probabilistically symmetric coverage interval)	12 mSv/h	4 mSv/h … 28 mSv/h

Annex D
(informative)

Example of an uncertainty analysis for a calibration of radon activity monitor according to the IEC 61577 series

D.1 General

IEC 61577 consists of several parts with the general title *Radiation protection instrumentation – Radon and radon decay product measuring instruments* [23] to [25].

The following example shows the result of a software based method (GUM workbench [12]) to determine the uncertainty. The following text consists of the direct output of the software plus some additional text for enhancing the understanding.

The example comprises the calibration of a radon monitor by a radon reference atmosphere (realisation of the quantity activity concentration) traceable to the input quantities activity and volume.

D.2 Model function

The model function used for the example is:

$$K_T = \frac{C}{C_T - C_{T,\mathrm{bg}}}; \quad C = \frac{A}{V + V_{\mathrm{Rn}}} e^{-\Lambda T_\mathrm{c}}; \quad \Lambda = \frac{\ln 2}{T_{1/2} \times 24 \times 60}; \quad C_T = C_{T,T_\mathrm{c}} + C_{T,T_\mathrm{bg}} \qquad \text{(D.1)}$$

where Table D.1 gives the definitions and units of the quantities used.

Table D.1 – List of quantities used in formula (D.1)

Quantity	Unit	Definition
A	Bq	Activity of the radon gas standard as certified (t = 0)
V	m^3	Reference volume as certified (displaced volume considered)
V_{Rn}	m^3	Volume of the radon gas standard container
$T_{½}$	d	Half-life of Radon-222. Nuclear Data from NuDat 2002, Evaluation BNL-NCS-52142 [26]
Λ	1/min	Decay constant of Radon-222
T_c	min	Point of time of the radon gas transfer into the reference volume
C	Bq/m^3	Activity concentration of the reference atmosphere at the point of time (t = t_c)
C_T	Bq/m^3	Average of the observations (t = t_c)
$C_{T,T\mathrm{c}}$	Bq/m^3	Parameter while averaging the observations
$C_{T,\mathrm{bg}}$	Bq/m^3	Background of the radon monitor
K_T	–	Measured value (calibration factor)

D.3 Calculation of the complete result of the measurement (measured value, probability density distribution, associated standard uncertainty, and the coverage interval)

Table D.2 gives all the available data for the input quantities. These data are requested from the software.

Table D.2 – List of data available for the input quantities of formula (D.1)

Quantity	Distribution	Value	Expanded uncertainty	Coverage factor	Remark
A	Type B normal distribution	152 Bq	4 Bq	2	
V	Type B normal distribution	0,05 m^3	0,000 3 m^3	1	
V_{Rn}	Type B normal distribution	0,000 062 7 m^3	0,1 %	1	
$T_{½}$	Type B normal distribution	3,823 5 d	0,000 3 d	1	
Λ	–	–	–	–	Interim result
T_{c}	Type B rectangular distribution	499 min	–	–	Half width of limits: 10 min
C	–	–	–	–	Interim result
C_T	–	–	–	–	Interim result. The observations over a time of more than 24 h are averaged
$C_{T,T\mathrm{c}}$	Type A summarized	2 883 Bq/m^3	25 Bq/m^3	–	Degrees of freedom: 144
$C_{T,\mathrm{bg}}$	Type A summarized	12 Bq/m^3	2 Bq/m^3	–	Degrees of freedom: 144
K_T	–	–	–	–	Result

In Table D.3 the complete uncertainty budget is given. All the values are direct output of the software.

Table D.3 – Example of an uncertainty budget for the calibration of a radon monitor according to IEC 61577, see text for details

Quantity	Best estimate	Absolute standard uncertainty	Sensitivity coefficient	Uncertainty contribution to output quantity
A	152 Bq	2 Bq	0,006 5 Bq^{-1}	0,013
V	0,050 0 m^3	0,000 3 m^3	–20 m^{-3}	0,005 9
V_{Rn}	62,700 × 10^{-6} m^3	0,063 × 10^{-6} m^3	–20 m^{-3}	1,2 × 10^{-6}
$T_{½}$	3,823 5 d	0,000 3 d	0,016 d^{-1}	4,9 × 10^{-6}
Λ	125,893 × 10^{-6} 1/min	0,001 × 10^{-6} 1/min	interim result	
T_{c}	499,0 min	5,8 min	–0,000 12 min^{-1}	0,000 72
C	2 851 Bq/m^3	41 Bq/m^3	interim result	
C_T	2 895 Bq/m^3	25 Bq/m^3	interim result	
$C_{T,T\mathrm{c}}$	2 883 Bq/m^3	25 Bq/m^3	–0,00034 Bq^{-1} m^3	0,008 6
$C_{T,\mathrm{bg}}$	12 Bq/m^3	2 Bq/m^3	0 Bq^{-1} m^3	0
K_T	0,989	0,017 (1,7 %)	(Analytical method)	
K_T	0,989	0,017 (1,7 %)	(Monte Carlo method)	

The complete result of the measurement of the calibration factor of the radon monitor according to Table D.3 is:

$$K_T = 0{,}989 \pm 0{,}033 \text{ (Analytical method)} \tag{D.2}$$

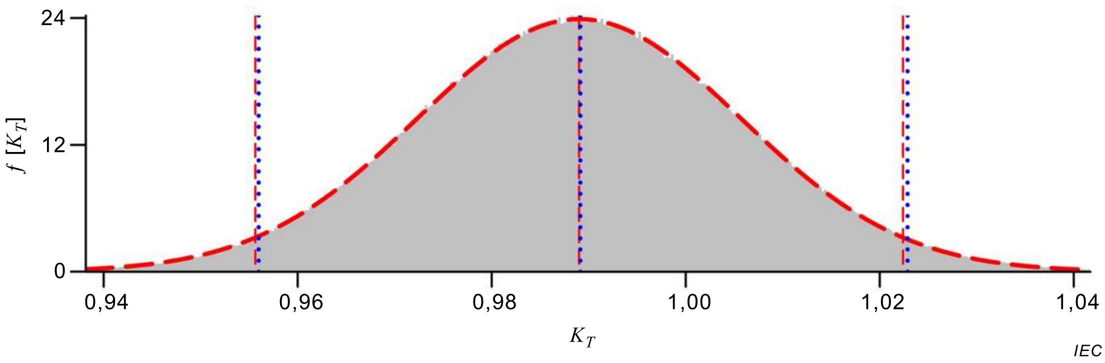
$$K_T = 0{,}989\ ^{+0{,}034}_{-0{,}033} \text{ (Monte Carlo method)} \tag{D.3}$$

The two results differ by less than 10 %, therefore, the analytical method can be used and the corresponding statement is:

The uncertainty stated is the expanded measurement uncertainty obtained by multiplying the standard uncertainty by a coverage factor $k_{\mathrm{cov}} = 2$. It has been determined in accordance with the *Guide to the Expression of Uncertainty in Measurement*. The value of the measurand normally lies, with a probability of approximately 95 %, within the attributed coverage interval.

In Figure D.1, the resulting probability density function (PDF) from the Monte Carlo method and the resulting Gaussian PDF according to the analytical method are shown. It can clearly be seen that the result are equivalent and, therefore, both methods are adequate for similar cases. For the Monte Carlo method, the shortest coverage interval and the probabilistically symmetric coverage interval are equivalent as the PDF is symmetrical, see 5.5.3.

NOTE In spite of the non-linear model function, the results are equivalent as the input quantities have rather small uncertainties, and, therefore, a linear approximation of the model function is possible.



NOTE The vertical lines are the mean values (at $k_{\mathbf{t}}$ = 0,989) and the boundaries of the coverage intervals from both methods.

Figure D.1 – Result of the analytical (red dashed lines) and the Monte Carlo method (grey histogram and blue dotted lines) for K_{T}

Annex E
(informative)

Example of an uncertainty analysis for a measurement of surface emission rate with a contamination meter according to IEC 60325:2002

E.1 General

IEC 60325:2002 has the title *Radiation protection instrumentation – Alpha, beta and alpha/beta (beta energy > 60 keV) contamination meters and monitors* [27].

For the example a contamination monitor is used to measure the surface emission rate due to beta contamination of ^{14}C with the following rated range and ranges of use for influence quantities:

Measuring range: 10 s^{-1} to 10 000 s^{-1} (in counts per second)

Area of detector: 100 cm^2

Rated ranges of use: nominal ranges

E.2 Model function

The model function used for the example is:

$$A = \frac{C - B}{D} F\, K_{\mathrm{n}}\, K_{\mathrm{hv}}\, K_{\mathrm{temp}}\, K_{\mathrm{hum}}\, K_{d,\mathrm{air}}\, K_{d,\mathrm{geo}}\, K_{\mathrm{uniform}}\, K_{\mathrm{surface}} \quad \text{(E.1)}$$

where

A is the measured surface emission rate of ^{14}C in terms of $s^{-1}\ cm^{-2}$;

C is the indicated value of the activity in terms of s^{-1};

B is the indicated value of the background in terms of s^{-1};

D is the area of the detector in terms of cm^2;

F is the calibration factor for the reference beta emitter (area related surface emission rate per indicated activity);

K_{n} is the correction factor for non-linearity;

K_{hv} is the correction factor for detector supply;

K_{temp} is the correction factor for ambient temperature;

K_{hum} is the correction factor for humidity;

$K_{d,\mathrm{air}}$ is the correction factor for distance effects due to air absorption;

$K_{d,\mathrm{geo}}$ is the correction factor for distance effects due to geometric changes;

K_{uniform} is the correction factor for effects of contamination non-uniformity;

K_{surface} is the correction factor for effects of surface absorption.

E.3 Calculation of the complete result of the measurement (measured value, probability density distribution, associated standard uncertainty, and the coverage interval)

E.3.1 General

IEC 60325 provides for test requirements and methods and specifies the allowable variations in response for various influence quantities of the monitoring equipment. It does not specify the way the monitoring is to be carried out or the effects of the non-uniformity in the contamination being measured or the effect of absorption in the surface changing the spectrum of the particles being emitted (total absorption of particles is taken into account by monitoring surface emission rate and not surface activity). The indicated activity value of the example is $c = 1\ 600\ \mathrm{s}^{-1}$ over a measuring time of 1 s, the measured background is $b = 1350\ \mathrm{s}^{-1}$ over a measuring time of 1 s and the detector area is 100 cm^2 with an upper limit of 101 cm^2 and an lower limit of 99 cm^2; the standard uncertainty of the count rate is 8,5 %. The calibration factor is determined to be 40 with a standard uncertainty of 8.

For the purposes of this example, the monitoring is assumed to be between 13 mm and 17 mm distance, i.e. at (15 ± 2) mm distance from the surface, whereas the calibration distance was 5 mm. This is considered by the correction factors $K_{d,\mathrm{air}}$ and $K_{d,\mathrm{geo}}$, which, therefore, have to correct for 10 mm additional distance.

E.3.2 Effects of distance

IEC 60325 does not specify the actual distance from the source to detector during measurements, but 10 mm is implied. However, in the act of monitoring, a fixed distance will not be adhered to, in fact it may be impossible to adhere to.

There will be two effects, air absorption and geometric changes.

Air absorption will be small for the higher energy beta emitters but will be significant in the monitoring of $^{14}\mathrm{C}$. For this radionuclide, an additional distance of 8 mm, 10 mm or 12 mm (equivalent to the above given example) results in a reduction in efficiency of about 15 %, 19 % or 23 %, respectively. For the correction factor $K_{d,\mathrm{air}}$ this results in

$$\frac{1}{1-0{,}15} \le K_{d,\mathrm{air}} \le \frac{1}{1-0{,}23} \qquad \text{(E.2)}$$

or $1{,}18 \le K_{d,\mathrm{air}} \le 1{,}30$, which gives $K_{d,\mathrm{air}} = 1{,}24 \pm 0{,}06$.

Geometric changes alter the solid angle between the detector and source. The effect of this would be zero for an infinite plane of uniform contamination. The effect could go either way for non-uniform infinite contamination. The greatest effect will be for points of contamination. The inverse square law generally will not apply as the distance between the contamination and detector is small by comparison to the dimensions of the source. It will approach a linear relationship.

For a contamination nominally 10 mm from a 10 cm × 10 cm detector plane, this geometric effect alone would cause a 10 % decrease in detection from the calibration value and changes of up to ± 2 mm in this distance will change the value of this decrease of 8 % and 12 %, respectively. For the correction factor for geometric effects, $K_{d,\mathrm{geo}}$, this results in

$$\frac{1}{1-0{,}08} \le K_{d,\mathrm{geo}} \le \frac{1}{1-0{,}12} \qquad \text{(E.3)}$$

or $1{,}087 \le K_{d,\mathrm{geo}} \le 1{,}136$, which gives $K_{d,\mathrm{geo}} = 1{,}11 \pm 0{,}02$.

It is assumed that this is an upper estimate for the geometric effects of the change in distance from 5 mm for the calibration to 15 mm for the measurement.

E.3.3 Contamination non-uniformity

The standard only considers the non-uniformity of the detector not that of the contamination. The non-uniformity of the contamination can only be determined by other tests but the effect on the measurement is likely to be comparable to that due to the non-uniformity of the detector.

For the purposes of this example, the effect of the non-uniformity of the contamination will be similar to the effect of the non-uniformity of detection over the detector area and is assumed to be K_{uniform} = 1,0 ± 0,025.

E.3.4 Surface absorption

The effect of absorption in the surface again can only be determined by assessment with regard to the nature of the surface and experience. The absorption below the surface could be regarded as not of interest since it is included in the definition of the surface emission rate, however, on the surface there could be grease or dirt which could be removed later and so would be of particular interest.

For the purposes of this example, it is assumed that the surface will be covered by a layer between 0 mg cm^{-2} and 10 mg cm^{-2} giving for ^{14}C a reduction in efficiency from 0 % to 76 %. This is equivalent to

$$\frac{1}{1-0{,}0} \leq K_{\text{surface}} \leq \frac{1}{1-0{,}76} \tag{E.4}$$

or 1,0 ≤ K_{surface} ≤ 4,17, which gives K_{surface} = 2,59 ± 1,59.

E.3.5 Other influence quantities

For the purposes of the example, it is assumed for the remaining influence quantities that their associated correction factors all have a value of 1,0 with an uncertainty equivalent to the maximum permitted value. This gives K_{n} = 1,0 ± 0,1; K_{hv} = 1,00 ± 0,01; K_{temp} = 1,0 ± 0,05 and K_{hum} = 1,0 ± 0,025.

E.3.6 Uncertainty budget

In Table E.1, the complete uncertainty budget for this example is given. It can be seen, that the uncertainty is quite large, therefore, the significance of the result should be checked. For this, in Table E.2 the uncertainty for a measured value of zero is given. It yields a value of $u(a{=}0)$ = 250 s^{-1} cm^{-2}. According to clause 6, the decision threshold is then given by $a^* = k_{0{,}95} \cdot u(a{=}0)$ = 410 s^{-1} cm^{-2} with $k_{0{,}95}$ = 1,65 for an error probability of α = 5 %. The result of the uncertainty analysis (360 s^{-1} cm^{-2}, see Table E.1) is well below the decision threshold, therefore, it is assumed that no effect of the probe is present. Thus, the final statement for the result of the measurement is as follows:

> The result of the measurement cannot be stated because the measured value is below the decision threshold $a^* = k_{1-\alpha} \cdot u(0)$ = 410 s^{-1} cm^{-2} determined for an error probability of α = 5 %.
>
> The uncertainty at an indicated value of zero, $u(0)$, has been determined in accordance with Supplement 1 of the *Guide to the Expression of Uncertainty in Measurement*. $k_{1-\alpha}$ is the quantile of the standardized normal distribution.
>
> Only if the measured value exceeded the decision threshold, would the physical effect to be measured be recognized as detected. If in reality no physical effect is present, then the measured value is below a^* = 410 s^{-1} cm^{-2} with a probability of 95 %.

Alternatively, an error probability of only 1 % can be chosen, leading to $a^* = k_{0{,}99} \cdot u(a{=}0)$ = 580 s^{-1} cm^{-2} with $k_{0{,}99}$ = 2,32 for an error probability of α = 1 %. Then, the final statement for the result of the measurement is as follows:

The result of the measurement cannot be stated because the measured value is below the decision threshold $a^* = k_{1-\alpha} \cdot u(0)$ = 580 s^{-1} cm^{-2} determined for an error probability of α = 1 %.

The uncertainty at an indicated value of zero, $u(0)$, has been determined in accordance with Supplement 1 of the *Guide to the Expression of Uncertainty in Measurement*. $k_{1-\alpha}$ is the quantile of the standardized normal distribution.

Only if the measured value exceeded the decision threshold, would the physical effect to be measured be recognized as detected. If in reality no physical effect is present, then the measured value is below a^* = 580 s^{-1} cm^{-2} with a probability of 99 %.

Table E.1 – Example of an uncertainty budget for a surface emission rate measurement according to IEC 60325:2002, see text for details

Quanti-ty	Best estimate	Absolute standard uncertainty	Distribution; mean value, x; half-width, a	Sensitivity coefficient	Uncertainty contribution to output quantity
C	1 600 s^{-1}	136 s^{-1}	Gaussian with one reading; x = 1600 s^{-1}; σ = 8,5 %	1,4 cm^{-2}	190 s^{-1} cm^{-2}
B	1350 s^{-1}	115 s^{-1}	Gaussian with one reading; x = 1350 s^{-1}; σ = 8,5 %	−1,4 cm^{-2}	160 s^{-1} cm^{-2}
D	100 cm^2	$1\,\text{cm}^2/\sqrt{3}$ = 0,58 cm^2	Rectangular; x = 100 cm^2; a = 1,0 cm^2	–3,6 s^{-1} cm^{-4}	2,1 s^{-1} cm^{-2}
F	40	24/3 = 8	Gaussian; x = 40; a = 24	8,9 s^{-1} cm^{-2}	71 s^{-1} cm^{-2}
K_{n}	1,0	$0{,}1/\sqrt{3}$ = 0,058	Rectangular; x = 1,0; a = 0,1	360 s^{-1} cm^{-2}	21 s^{-1} cm^{-2}
K_{hv}	1,0	$0{,}01/\sqrt{3}$ = 0,006	Rectangular; x = 1,0; a = 0,01	360 s^{-1} cm^{-2}	2,1 s^{-1} cm^{-2}
K_{temp}	1,0	$0{,}05/\sqrt{3}$ = 0,029	Rectangular; x = 1,0; a = 0,05	360 s^{-1} cm^{-2}	10 s^{-1} cm^{-2}
K_{hum}	1,0	$0{,}025/\sqrt{3}$ = 0,014	Rectangular; x = 1,0; a = 0,025	360 s^{-1} cm^{-2}	5,1 s^{-1} cm^{-2}
$K_{d,\text{air}}$	1,24	$0{,}06/\sqrt{3}$ = 0,035	Rectangular; x = 1,24; a = 0,06	290 s^{-1} cm^{-2}	10 s^{-1} cm^{-2}
$K_{d,\text{geo}}$	1,11	$0{,}02/\sqrt{3}$ = 0,012	Rectangular; x = 1,11; a = 0,02	320 s^{-1} cm^{-2}	3,7 s^{-1} cm^{-2}
K_{uniform}	1,0	$0{,}025/\sqrt{3}$ = 0,014	Rectangular; x = 1,0; a = 0,025	360 s^{-1} cm^{-2}	5,1 s^{-1} cm^{-2}
K_{surface}	2,59	$1{,}59/\sqrt{3}$ = 0,918	Rectangular; x = 2,59; a = 1,59	140 s^{-1} cm^{-2}	130 s^{-1} cm^{-2}
A	360 s^{-1} cm^{-2}	290 s^{-1} cm^{-2} (82 %) (Analytical method)			
A	360 s^{-1} cm^{-2}	310 s^{-1} cm^{-2} (86 %) (Monte Carlo method)			

Table E.2 – Example of an uncertainty budget for a surface emission rate measurement according to IEC 60325:2002 for the determination of the uncertainty at a measured value of zero

Quantity	Best estimate	Absolute standard uncertainty	Distribution; mean value, x; half-width, a	Sensitivity coefficient	Uncertainty contribution to output quantity
C	1 350 s^{-1}	115 s^{-1}	Gaussian with one reading; x = 1350 s^{-1}; σ = 8,5 %	1,4 cm^{-2}	160 s^{-1} cm^{-2}
B	1350 s^{-1}	115 s^{-1}	Gaussian with one reading; x = 1350 s^{-1}; σ = 8,5 %	−1,4 cm^{-2}	160 s^{-1} cm^{-2}
D	100 cm^2	$1\,\mathrm{cm}^2/\sqrt{3}$ = 0,58 cm^2	Rectangular; x = 100 cm^2; a = 1,0 cm^2	0 s^{-1} cm^{-4}	0 s^{-1} cm^{-2}
F	40	24/3 = 8	Gaussian; x = 40; a = 24	0 s^{-1} cm^{-2}	0 s^{-1} cm^{-2}
K_n	1,0	$0{,}1/\sqrt{3}$ = 0,058	Rectangular; x = 1,0; a = 0,1	0 s^{-1} cm^{-2}	0 s^{-1} cm^{-2}
K_{hv}	1,0	$0{,}01/\sqrt{3}$ = 0,006	Rectangular; x = 1,0; a = 0,01	0 s^{-1} cm^{-2}	0 s^{-1} cm^{-2}
K_{temp}	1,0	$0{,}05/\sqrt{3}$ = 0,029	Rectangular; x = 1,0; a = 0,05	0 s^{-1} cm^{-2}	0 s^{-1} cm^{-2}
K_{hum}	1,0	$0{,}025/\sqrt{3}$ = 0,014	Rectangular; x = 1,0; a = 0,025	0 s^{-1} cm^{-2}	0 s^{-1} cm^{-2}
$K_{d,air}$	1,24	$0{,}06/\sqrt{3}$ = 0,035	Rectangular; x = 1,24; a = 0,06	0 s^{-1} cm^{-2}	0 s^{-1} cm^{-2}
$K_{d,geo}$	1,11	$0{,}02/\sqrt{3}$ = 0,012	Rectangular; x = 1,11; a = 0,02	0 s^{-1} cm^{-2}	0 s^{-1} cm^{-2}
$K_{uniform}$	1,0	$0{,}025/\sqrt{3}$ = 0,014	Rectangular; x = 1,0; a = 0,025	0 s^{-1} cm^{-2}	0 s^{-1} cm^{-2}
$K_{surface}$	2,59	$1{,}59/\sqrt{3}$ = 0,918	Rectangular; x = 2,59; a = 1,59	0 s^{-1} cm^{-2}	0 s^{-1} cm^{-2}
A	0 s^{-1} cm^{-2}	230 s^{-1} cm^{-2}	(Analytical method)		
A	0 s^{-1} cm^{-2}	250 s^{-1} cm^{-2}	(Monte Carlo method)		

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