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2004-08

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BACKGROUND OF TERMS AND DEFINITIONS OF CASCADED TWO-PORTS

FOREWORD

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IEC 62152, which is a technical report, has been prepared by IEC technical committee 46: Cables, wires, waveguides, r.f. connectors, r.f. and microwave passive components and accessories.

The text of this technical report is based on the following documents:

Enquiry draft	Report on voting
46/129/DTR	46/133/RVC

Full information on the voting for the approval of this technical report can be found in the report on voting indicated in the above table.

This publication has been drafted in accordance with the ISO/IEC Directives, Part 2.

The committee has decided that the contents of this publication will remain unchanged until the maintenance result date indicated on the IEC web site under "http://webstore.iec.ch" in the data related to the specific publication. At this date, the publication will be

- reconfirmed;
- withdrawn;
- replaced by a revised edition, or
- amended.

A bilingual edition of this document may be issued at a later date.

BACKGROUND OF TERMS AND DEFINITIONS OF CASCADED TWO-PORTS

1 General

It is important and practical that components of a transmission chain can be separated and tested separately. This means well-defined interfaces and measuring techniques including agreed terms and definitions. It is advantageous to operate, by the square root of a reference impedance (normally application impedance of the system), with normalized voltage waves corresponding to the square root of power waves.

This technical report has two main goals. It lays the foundation for agreement on the fundamental terms and definitions to be used world wide in describing the transmission properties of a two-port or quadripole end and builds a bridge between the classical quadripole theory and the scattering matrix presentation which is based on incident and reflecting square root of power waves at the input and output of a two-port. Finally, it is shown that the two concepts are bound together through simple equations and are fundamentally identical.

The quadripole theory was originally developed for voice- and carrier-frequency technologies and transmission, and later for microwaves, but both can be used through the whole frequency range.

2 Operational, image and insertion transfer functions and complex attenuations or losses

a) Operational transfer function

T_B is defined as the square root of the power wave into the load (equal to reference impedance R_2) of a two-port $\sqrt{P_2}$ compared with an unreflected square root of power wave $\sqrt{P_0}$ from the generator with a source impedance equal to the reference impedance R_1.

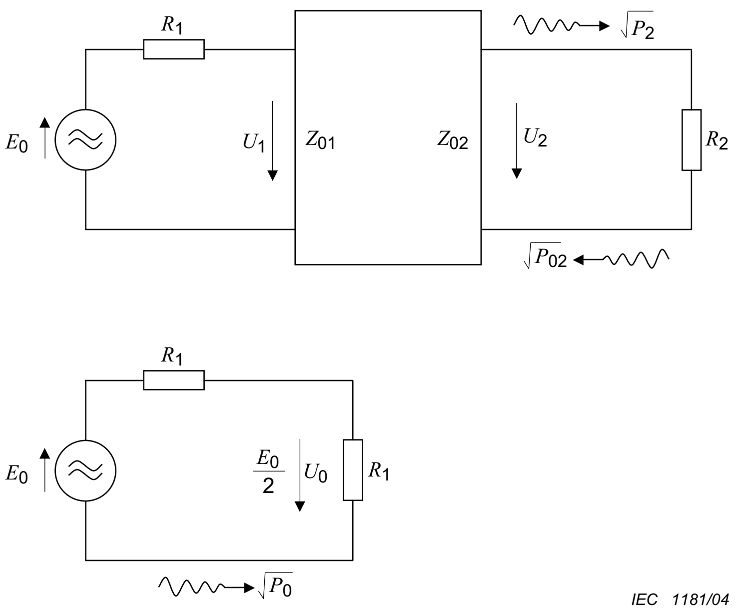


Figure 1 – Defining the transfer functions of a two-port

$$T_B = \frac{\sqrt{P_2}}{\sqrt{P_0}} = \frac{U_2 / \sqrt{R_2}}{U_0 / \sqrt{R_1}} = S_{21} = \frac{\sqrt{P_2}}{\sqrt{P_0}} \Bigg|_{\sqrt{P_{02}}=0} \tag{1}$$

which is equal to the forward transfer scattering parameter S_{21}.

The operational transfer function becomes

b) the image transfer function T when the reference impedance becomes equal to the input and output characteristic impedances Z_{01} and Z_{02} of the two-port; and

c) the insertion transfer function T'_B when $R_1 = R_2 = R$.

Correspondingly, the complex attenuations or losses are as follows.

Complex operational attenuation

$$\Gamma_B = A_B + jB_B = \ln\frac{1}{T_B} = -20\ \log|T_B|\ \text{ in } [\text{dB}] - j \cdot \arg(T_B)\ \text{ in } [\text{rad}] \tag{2}$$

Complex image attenuation

$$\Gamma = A + jB = \ln\frac{1}{T} = -20\ \log|T|\ \text{ in } [\text{dB}] - j \cdot \arg(T)\ \text{ in } [\text{rad}] \tag{3}$$

Complex insertion attenuation or loss

$$\Gamma'_B\Big|_{R_1=R_2=R} = A'_B + jB'_B = \ln\frac{1}{T'_B} = -20\ \log|T'_B|\ \text{ in } [\text{dB}] - j \cdot \arg(T'_B)\ \text{ in } [\text{rad}] \tag{4}$$

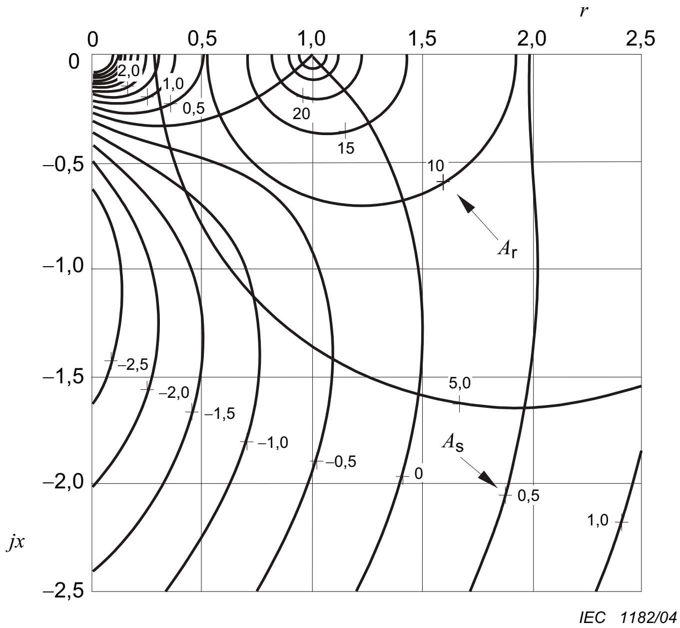
3 Terms and definitions

For the purposes of this document, the following terms and definitions apply.

3.1
operational attenuation
quotient of the unreflected square root of the power wave fed into the reference impedance of the input of the two-port and the square root of the power wave consumed by the load of the two-port expressed in dB and radians

NOTE By defining a new quantity operational insertion loss in the same way as the operational attenuation, at least when the reference impedances on both sides of the two-port are the same, the problem of insertion loss and operational attenuation is solved.

3.2
operational insertion loss
quotient of the unreflected square root of the power wave fed into the reference impedance of the input of the two-port and the square root of the power wave consumed by the load of the two-port expressed in dB and radians



Reflection loss $$A_s = 20\log\left|\frac{z_N + 1}{2\sqrt{z_N}}\right| \ [\mathrm{dB}]$$

Return loss $$A_r = 20\log\left|\frac{z_N + 1}{z_N - 1}\right| \ [\mathrm{dB}]$$

$z_N = \dfrac{Z_2}{Z_1}$ (= normalized impedance) = $r + jx$

Figure 2 – Constant value A_s and A_r curves on a complex plane $z = x + jy$

3.3
operational attenuation and insertion loss

quotient of the unreflected square root of the power wave fed into the reference impedance of the input of the two-port and the square root of the power wave consumed by the load of the two-port expressed in dB and radians

NOTE In the IEV, insertion loss is understood as the loss produced by inserting a two-port into a separated point of the transmission chain. Because of varying terminating impedances of the two-port, this leads to insertion loss or operational attenuation deviation, that is, depending on where, in the chain, the two-port is inserted.

It is obvious that the insertion of a two-port with a certain operational attenuation or operational insertion loss causes different attenuation increases (or decreases) in separate circuit points of different impedances.

This is called the Insertion Loss Deviation (ILD).

ILD has proved to be a very important subject of discussion in the standardization of a data channel.

Annex A
(normative)
Concepts of normalized voltage waves, square root of power waves and operational attenuation and losses

A.1 General

It is important and practical that components of a transmission chain can be separated and tested separately. This means well-defined interfaces and measuring techniques including agreed terms and definitions. It is advantageous to operate, by the square root of a reference impedance (normally application impedance of the system), with normalized voltage waves corresponding to the square root of power waves.

In this way, for instance, the scattering parameters are defined. For example, S_{21} is the forward operational transfer function and S_{11} is the operational reflection coefficient.

Two of the reasons for using the square root of the impedance normalized voltage waves or the square root of the power waves are

a) that the network analyser is measuring voltages; and

b) because the natural logarithm, ln, of a complex quantity $z = x + \mathrm{j}y = |z| \cdot e^{\mathrm{j} \cdot \arg z}$ is directly l, and ln|z| nepers can be expressed in decibels $20 \cdot \log_{10}|z|$ and the imaginary part still remains arg(z) in radians, as, for example,

$$\Gamma_{\mathrm{B}} = A_{\mathrm{B}} + \mathrm{j}B_{\mathrm{B}} = -20\log_{10}|S_{21}| + \mathrm{j}\arg(z)$$

(see equation (A.1)).

A.2 Complex operational attenuation or operational propagation coefficient Γ_{B}

The complex operational attenuation (complex operational loss) introduced by a two-port component, cascade of components, link, cable assembly etc. into a system is defined by using the scattering parameter S_{21} as

$$\Gamma_{\mathrm{B}} = A_{\mathrm{B}} + \mathrm{j}B_{\mathrm{B}} = \ln(1/S_{21}) = -\ln|S_{21}| - \mathrm{j}\cdot\arg(S_{21}) \qquad \text{(A.1a)}$$

$$\Gamma_{\mathrm{B}} = A_{\mathrm{B}} + \mathrm{j}B_{\mathrm{B}} = -20\log_{10}|S_{21}| - \mathrm{j}\cdot\arg(S_{21}) \qquad \text{(A.1b)}$$

where

in (A.1a) $-\ln|S_{21}| = A_{\mathrm{B}}$ $[\mathrm{Np}]$

in (A.1b) $-20\log_{10}|S_{21}| = A_{\mathrm{B}}$ $[\mathrm{dB}]$

in (A.1a) and (A.1b) $-\arg(S_{21}) = B_{\mathrm{B}}$ $[\mathrm{rad}]$

where

A_{B} is the operational attenuation = 20 $\log_{10}(1/|S_{21}|)$ (dB)

B_{B} is the operational attenuation phase constant = –arg(S_{21}) (rad)

NOTE 1 A_B is equal to the ratio of the unreflected complex power (voltage × current) sent into a two-port, to the complex power consumed by the load of the two-port, in decibels. The load is normally a resistance equal to the application impedance of the system Z_N. When the generator and load impedances are the same **operational attenuation** becomes **insertion loss.**

NOTE 2 From the theory of complex functions:

$$\ln z = \ln|z| + \mathrm{j}\cdot\arg z$$

where

$$z = x + \mathrm{j}y = |z|\cdot e^{\mathrm{j}\cdot\arg z}$$

and, by using the square root of power waves, we can write, for the natural logarithms of the ratio of two square root of complex power waves:

$$\ln\frac{\sqrt{P_1}}{\sqrt{P_2}} = \ln\left|\frac{\sqrt{P_1}}{\sqrt{P_2}}\right| + \mathrm{j}\cdot\arg\left(\frac{\sqrt{P_1}}{\sqrt{P_2}}\right) = \Gamma = A + \mathrm{j}B$$

where A is in nepers and B in radians.

When A is expressed in decibels, B will not be affected; it remains in radians.

A.3 Impedance

a) The nominal characteristic impedance Z_{CN} (of a two-port) is the resistive part of the mean characteristic impedance Z_{C} specified with tolerance at a given frequency.

b) Z_{N} is the nominal impedance of the system terminals between which the two-port is operating.

c) Z_{R} is the (nominal) reference impedance used in measurements. Normally $Z_{\mathrm{R}} = Z_{\mathrm{N}}$.

A.4 Operational reflection coefficient

The operational reflection coefficient of the two-port is equal to the scattering parameter S_{11} of a two-port. It equals the reflection coefficient r_{c} at the input when the two-port is terminated with its reference impedances Z_{R}, normally equal to the nominal impedances of the system terminals.

$$S_{11} = r_{\mathrm{B}} = \frac{Z_{\mathrm{in}} - Z_{\mathrm{R}}}{Z_{\mathrm{in}} + Z_{\mathrm{R}}} \tag{A.2}$$

A.5 Return loss

a) Complex operational return loss RL_{B}

$$\begin{aligned} RL_{\mathrm{B}} &= \ln\frac{1}{r_{\mathrm{B}}} = -\ln(r_{\mathrm{B}}) = -\ln|r_{\mathrm{B}}|\,[\mathrm{Np}] - j\cdot\arg(r_{\mathrm{B}})\,[\mathrm{rad}] \\ &= -20\cdot\log_{10}|r_{\mathrm{B}}|\,[\mathrm{dB}] - j\cdot\arg(r_{\mathrm{B}})\,[\mathrm{rad}] \end{aligned} \tag{A.3}$$

b) Structural return loss SRL

The return loss where the mismatch effects at the input and output of two-port have been eliminated (compare with the continuous wave (CW) burst measurement method).

NOTE It is important to define the structural return loss, although it is not measured direct from the cable assemblies, because it shows that there are differences between different kinds of return losses.

c) Reflection loss of a junction (see Figure 2)

$$\Gamma_{\mathrm{r}} = -\ln\sqrt{(1-S^2)} = -\ln\left|\sqrt{(1-S^2)}\right| \;[\mathrm{Np}] - \mathrm{j}\cdot\arg(\sqrt{(1-S^2)}) \;[\mathrm{rad}] \tag{A.4a}$$

or

$$\Gamma_{\mathrm{r}} = -\ln\sqrt{(1-S^2)} = -20\cdot\log_{10}\left|\sqrt{(1-S^2)}\right| \;[\mathrm{dB}] - \mathrm{j}\cdot\arg(\sqrt{(1-S^2)}) \;[\mathrm{rad}] \tag{A.4b}$$

$$\Gamma_{\mathrm{r}} = -\ln\sqrt{(1-S^2)} = -10\cdot\log_{10}\left|(1-S^2)\right| \;[\mathrm{dB}] - \mathrm{j}\cdot\frac{1}{2}\arg(1-S^2) \;[\mathrm{rad}] \tag{A.4c}$$

d) Mismatch loss of a junction (not recommended)

$$\Gamma_{\mathrm{m}} = -\ln\sqrt{(1-|S|^2)} = -\ln\left|\sqrt{(1-|S|^2)}\right| \;[\mathrm{Np}] - \mathrm{j}\cdot\arg(\sqrt{(1-|S|^2)}) \;[\mathrm{rad}] \tag{A.5a}$$

or

$$\Gamma_{\mathrm{m}} = -\ln\sqrt{(1-|S|^2)} = -20\cdot\log_{10}\left|\sqrt{(1-|S|^2)}\right| \;[\mathrm{dB}] - \mathrm{j}\cdot\arg(\sqrt{(1-|S|^2)}\,) \;[\mathrm{rad}] \tag{A.5b}$$

$$\Gamma_{\mathrm{m}} = -\ln\sqrt{(1-|S|^2)} = -10\cdot\log_{10}\left|(1-|S|^2)\right| \;[\mathrm{dB}] - \mathrm{j}\cdot\frac{1}{2}\arg(1-|S|^2) \;[\mathrm{rad}] \tag{A.5c}$$

In c) and d) S is the complex reflection coefficient of the junction

P ----->

________________|______________

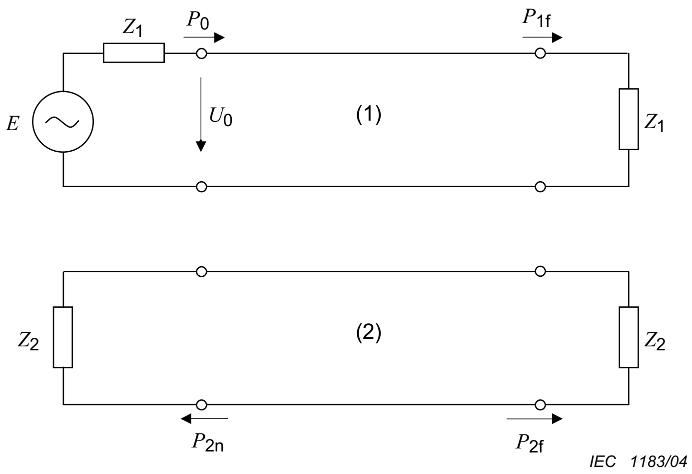
Z_1 Z_2

$$S = r = \frac{Z_2 - Z_1}{Z_2 + Z_1} \tag{A.6}$$

A.6 General coupling transfer function

This is distinguished between the near-end and far-end coupling transfer functions T_{n} and T_{f}.

$$T_{\mathrm{n,f}} = \frac{\sqrt{P_{\mathrm{2n,f}}}}{\sqrt{P_0}} = \frac{U_{\mathrm{2n,f}} / \sqrt{Z_{\mathrm{2n,f}}}}{U_0 / \sqrt{Z_1}} = \frac{\sqrt{Z_1}}{U_0}\frac{U_{\mathrm{2n,f}}}{\sqrt{Z_{\mathrm{2n,f}}}} \tag{A.7}$$



Key

P_0 is the unreflected power sent into the near end of the system (1).

System (1) is disturbing system (2).

Figure A.1 – Coupling between two systems

Coupling transfer function is a general term valid through the whole frequency range.

It may be expressed in decibels and radians

$$T_{n,f}\ [\mathrm{dB}\ \&\ \mathrm{rad}] = 20\log_{10}\left|\frac{\sqrt{P_{2n,f}}}{\sqrt{P_0}}\right| [\mathrm{dB}]\ + \mathrm{j}\cdot\arg\left(T_{n,f}\right)\ [\mathrm{rad}] \qquad \text{(A.8)}$$

and the (complex) operational transfer, coupling screening, unbalance, attenuation, etc. are

$$\Gamma_x = A_x + jB_x = -20\ \log_{10}|T| - j\ \arg(T) \qquad \text{(A.9)}$$

where

A_x is the (operational) attenuation (dB);

B_x is the (operational) attenuation phase constant (rad).

A.7 Benefits of the concept of operational quantities

Measurements are always taken between well-defined resistive terminations.

This means that the impedances at a reference plane between the cascaded units of the system are specified.

Individual units can be specified and tested separately and made by different manufacturers.

This makes open systems, networks and cabling possible.

Annex B
(normative)
Two-port transmission technique – Terms

a) Image transfer function $T = \frac{\sqrt{P_{\mathrm{OUT}}}}{\sqrt{P_{\mathrm{IN}}}} = \frac{V_{\mathrm{OUT}}}{V_{\mathrm{IN}}}$

b) Image transfer attenuation or loss$A$ = 20 log $\left|\frac{1}{T}\right|$

Z_{C1} and Z_{C2} are the image or characteristic impedances of the input or output of the two-port, equal to the input and output impedances when the opposite port is terminated with its image impedance.

$\sqrt{P_{\mathrm{IN}}} = V_{\mathrm{IN}}$ and $\sqrt{P_{\mathrm{OUT}}} = V_{\mathrm{OUT}}$ are the input and output square root of complex powers.

When defining the image, there are reflections at the input and output; in other words, the input and output are terminated with their image impedance.

c) Complex image attenuation $\Gamma = 20\log\left|\frac{1}{T}\right|[\mathrm{dB}] + \mathrm{j}\arg\frac{1}{T}[\mathrm{rad}] = A + jB$

d) Image attenuation A=20 log $\left|\frac{1}{T}\right|[\mathrm{dB}]$

e) Image phase shift B=$\arg\frac{1}{T}[\mathrm{rad}]$

f) Image phase propagation time or delay $\tau_{\mathrm{p}} = \frac{B}{\omega}$

g) Image group propagation time or delay $\tau_{\mathrm{g}} = \frac{\mathrm{d}B}{\mathrm{d}\omega}$

h) Image phase velocity $v_{\mathrm{p}} = \frac{1}{\tau_{\mathrm{p}}}$

i) Image group velocity $v_{\mathrm{g}} = \frac{1}{\tau_{\mathrm{g}}}$

j) Complex operational attenuation $\Gamma_{\mathrm{B}} = A_{\mathrm{B}} + \mathrm{j}B_{\mathrm{B}}$

k) Operational attenuation $A_{\mathrm{B}} = 20\ \log\left(\frac{V_{i1}}{V_{r2}}\Big|_{Vi2=0}\right)$

l) Operational phase shift $B_{\mathrm{B}} = \arg\left(\frac{V_{i1}}{V_{r2}}\Big|_{Vi2=0}\right)$

Annex C
(normative)
Two-port theory and fundamental concepts in transmission engineering[1]

C.1 General

This annex has two main goals. It lays the foundation for the fundamental terms and definitions to be used world wide in describing the transmission properties of a two-port or quadripole and builds a bridge between the classical quadripole theory and the scattering matrix presentation, which is based on the incident and reflecting square root of power waves at the input and output of a two-port. Finally, it is shown that the two concepts are bound together by simple equations which are fundamentally identical.

The two-port theory was originally developed for voice and carrier technologies, transmission and later for microwaves, but it can be used for the whole frequency range and for various applications.

In the following Clauses, we will use the term two-port exclusively.

C.2 Transfer equations for a passive two-port

For a passive impedance-symmetrical two-port (see Clause C.4 and Figure C.1), the following equations are valid.

$$U_1 = U_\mathrm{i} + U_\mathrm{r} \tag{C.1}$$

$$I_1 = I_\mathrm{i} - I_\mathrm{r} = \frac{U_\mathrm{i}}{Z_0} - \frac{U_\mathrm{r}}{Z_0} \tag{C.2}$$

$$U_2 = U_\mathrm{i} e^{-\Gamma} + U_\mathrm{r} e^{\Gamma} \tag{C.3}$$

$$I_2 = I_1 e^{-\Gamma} - I_\mathrm{r} e^{\Gamma} \tag{C.4}$$

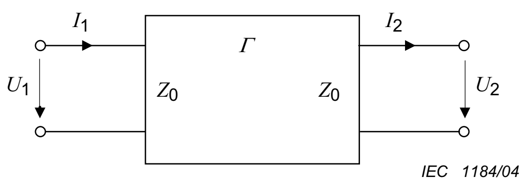


Figure C.1 – A quadripole or two-port

[1] L.HALME: CHAPTER L4, part of English version of the L. Halme´s book (Halme, L.K.: *Johtotransmissio ja sähkömagneettinen suojaus*, (Transmission on lines and electromagnetic screening, in Finnish), Parts A and B, Otakustantamo 2nd Eddition Helsinki 1989, 605 pages), corrected by J. Walling (2000-09-27).

Where Z_0 is the image impedance of the two-port, $\Gamma = A + \mathrm{j}B$ is the complex image attenuation or the image transfer constant. It equals the complex image attenuation of a two-port terminated in its image impedance (see Clause C.7). U_i and I_i represent the incident voltage and current waves fed to the input of the two-port, while U_r and I_r represent the voltage and current waves reflected back to the input from the output of the two-port. By solving equations (C.1) and (C.2) for U_i, U_r, I_i and I_r and by substituting these into equations (C.3) and (C.4), we obtain the actual voltage and current at the output terminals:

$$U_2 = U_1 \cosh\Gamma - Z_0 I_1 \sinh\Gamma \tag{C.5}$$

$$I_2 = I_1 \cosh\Gamma - \frac{U_1}{Z_0}\sinh\Gamma \tag{C.6}$$

By solving equations (C.5) and (C.6) for U_1 and I_1 we obtain

$$U_1 = U_2 \cosh\Gamma + Z_0 I_2 \sinh\Gamma \tag{C.7}$$

$$I_1 = I_2 \cosh\Gamma + \frac{U_2}{Z_0}\sinh\Gamma \tag{C.8}$$

From which we can deduce that equations (C.7) and (C.8) for input terminals can be obtained from the equations (C.5) and (C.6) for output terminals by interchanging the voltages, by interchanging the currents and by replacing Γ with $-\Gamma$.

From equations (C.5), (C.6), (C.7) and (C.8), we can also solve the currents expressed by means of the voltages, as well as the voltages expressed by means of the currents:

$$I_1 = \frac{U_1}{Z_0}\coth\Gamma - \frac{U_2}{Z_0}\frac{1}{\sinh\Gamma} \tag{C.9}$$

$$I_2 = \frac{U_1}{Z_0}\frac{1}{\sinh\Gamma} - \frac{U_2}{Z_0}\coth\Gamma \tag{C.10}$$

$$U_1 = Z_0 I_1 \coth\Gamma - Z_0 I_2 \frac{1}{\sinh\Gamma} \tag{C.11}$$

$$U_2 = Z_0 I_1 \frac{1}{\sinh\Gamma} - Z_0 I_2 \coth\Gamma. \tag{C.12}$$

C.3 Chain matrix

Equations (C.7) and (C.8) can be presented in matrix form

$$\begin{bmatrix} U_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \cosh\Gamma & Z_0 \sinh\Gamma \\ \frac{1}{Z_0}\sinh\Gamma & \cosh\Gamma \end{bmatrix} \begin{bmatrix} U_2 \\ I_2 \end{bmatrix} \tag{C.13}$$

Here, the multiplier matrix is called the chain matrix and is generally expressed as:

$$\begin{bmatrix} U_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} U_2 \\ I_2 \end{bmatrix} \quad \text{(C.14)}$$

Where the constants A, B, C and D forming the chain matrix are called the transfer parameters. They are bound to each other by the relation

$$AD - BC = 1 \quad \text{(C.15a)}$$

The transfer parameters can be calculated by alternately considering the output of the two-pole either as short-circuited or open-circuited, whereby

$$A = \left(\frac{U_1}{U_2}\right)_{I_2=0} \qquad B = \left(\frac{U_1}{I_2}\right)_{U_2=0}$$
$$C = \left(\frac{I_1}{U_2}\right)_{I_2=0} \qquad D = \left(\frac{I_1}{I_2}\right)_{U_2=0} \quad \text{(C.15b)}$$

The chain matrix is well suited for the examination of cascaded two-ports.

An impedance-unsymmetrical two-port (see Clause C.3) can be treated as a symmetrical one by cascading it (as shown by Figure C.2) with an ideal transformer with a turns ratio of

$$K = \frac{N_1}{N_2} = \frac{1}{n} = \sqrt{\frac{Z_{01}}{Z_{02}}} \quad \text{(C.16)}$$

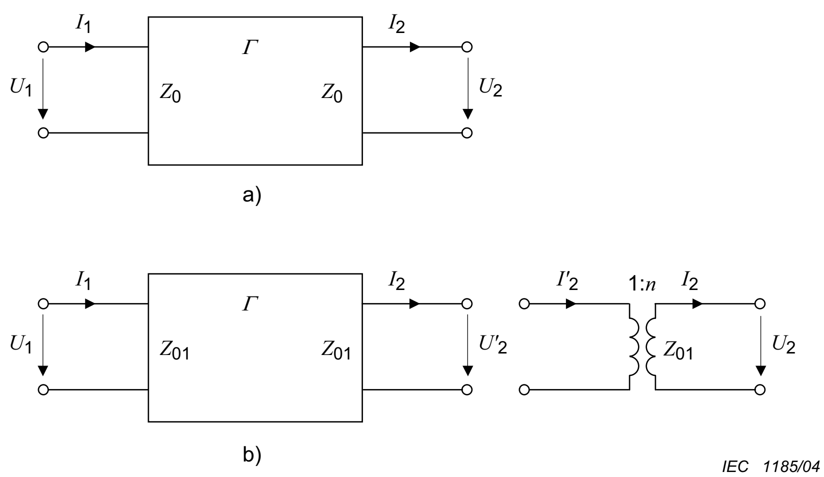


Figure C.2 – An impedance-unsymmetrical two-port (a) with its equivalent circuit (b)

We are here concerned with the cascading (or chaining) of two-ports, whereby the calculations can be appropriately carried out by means of chain matrices.

Let us suppose two two-port with the chain matrices A_1 and A_2 being interconnected as shown by Figure C.3.

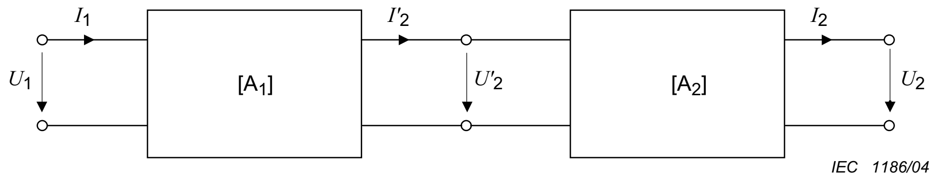


Figure C.3 – Two chained two-ports

The matrix equations, with the direction arrows as indicated in Figure C.3, are

$$\begin{bmatrix} U_1 \\ I_1 \end{bmatrix} = [A_1]\begin{bmatrix} U'_2 \\ I'_2 \end{bmatrix} \qquad \begin{bmatrix} U'_2 \\ I'_2 \end{bmatrix} = [A_2]\begin{bmatrix} U_2 \\ I_2 \end{bmatrix} \tag{C.17}$$

The combining of equations (C.17) yields

$$\begin{bmatrix} U_1 \\ I_1 \end{bmatrix} = [A_1][A_2]\begin{bmatrix} U_2 \\ I_2 \end{bmatrix} = [A]\begin{bmatrix} U_2 \\ I_2 \end{bmatrix} \tag{C.18}$$

where $[A] = [A_1][A_2]$.

The matrix A is hence obtained as a product between the chain matrices of the two-ports to be chained.

The turns ratio of the transformer in Figure C.2 can be rewritten as

$$K = \frac{1}{n} = \frac{U'_2}{U_2} = \frac{I_2}{I_2'} = \sqrt{\frac{Z_{01}}{Z_{02}}}$$

and the transfer equation of the transformer is obtained in the matrix form

$$\begin{bmatrix} U'_2 \\ I'_2 \end{bmatrix} = \begin{bmatrix} \sqrt{\dfrac{Z_{01}}{Z_{02}}} & 0 \\ 0 & \sqrt{\dfrac{Z_{02}}{Z_{01}}} \end{bmatrix}\begin{bmatrix} U_2 \\ I_2 \end{bmatrix} = [A_2]\begin{bmatrix} U_2 \\ I_2 \end{bmatrix} \tag{C.19}$$

In accordance with equation (C.13), the chain matrix A_1 of a symmetrical two-port is equal to

$$[A_1] = \begin{bmatrix} \cosh \Gamma & Z_{01} \sinh \Gamma \\ \dfrac{1}{Z_{01}} \sinh \Gamma & \cosh \Gamma \end{bmatrix} \tag{C.20}$$

The matrix A thus becomes

$$[A]=[A_1][A_2]=\begin{bmatrix}\cosh\Gamma & Z_{01}\sinh\Gamma\\ \frac{1}{Z_{01}}\sinh\Gamma & \cosh\Gamma\end{bmatrix}\begin{bmatrix}\sqrt{\frac{Z_{01}}{Z_{02}}} & 0\\ 0 & \sqrt{\frac{Z_{02}}{Z_{01}}}\end{bmatrix}=\begin{bmatrix}\sqrt{\frac{Z_{01}}{Z_{02}}}\cosh\Gamma & Z_{01}\sqrt{\frac{Z_{02}}{Z_{01}}}\sinh\Gamma\\ \frac{1}{Z_{01}}\sqrt{\frac{Z_{01}}{Z_{02}}}\sinh\Gamma & \sqrt{\frac{Z_{02}}{Z_{01}}}\cosh\Gamma\end{bmatrix} \quad \text{(C.21)}$$

The transfer equations of an impedance-unsymmetrical two-port can be written in the matrix form as follows

$$\begin{bmatrix}U_1\\ I_1\end{bmatrix}=\begin{bmatrix}\sqrt{\frac{Z_{01}}{Z_{02}}}\cosh\Gamma & Z_{01}\sqrt{\frac{Z_{02}}{Z_{01}}}\sinh\Gamma\\ \frac{1}{Z_{01}}\sqrt{\frac{Z_{01}}{Z_{02}}}\sinh\Gamma & \sqrt{\frac{Z_{02}}{Z_{01}}}\cosh\Gamma\end{bmatrix}\begin{bmatrix}U_2\\ I_2\end{bmatrix} \quad \text{(C.22)}$$

This matrix equation can also be solved for U_2 and I_2.

$$\begin{bmatrix}U_2\\ I_2\end{bmatrix}=[A]^{-1}\begin{bmatrix}U_1\\ I_1\end{bmatrix} \quad \text{(C.23)}$$

$$\begin{bmatrix}U_2\\ I_2\end{bmatrix}=\begin{bmatrix}\sqrt{\frac{Z_{02}}{Z_{01}}}\cosh\Gamma & -Z_{01}\sqrt{\frac{Z_{02}}{Z_{01}}}\sinh\Gamma\\ -\frac{1}{Z_{01}}\sqrt{\frac{Z_{01}}{Z_{02}}}\sinh\Gamma & \sqrt{\frac{Z_{01}}{Z_{02}}}\cosh\Gamma\end{bmatrix}\begin{bmatrix}U_1\\ I_1\end{bmatrix} \quad \text{(C.24)}$$

From the matrix equations (C.22) and (C.24), we can obtain the following transfer equations for an impedance-unsymmetrical two-port:

$$U_1=\sqrt{\frac{Z_{01}}{Z_{02}}}U_2\cosh\Gamma+Z_{01}\sqrt{\frac{Z_{02}}{Z_{01}}}I_2\sinh\Gamma \quad \text{(C.25)}$$

$$I_1=\sqrt{\frac{Z_{02}}{Z_{01}}}I_2\cosh\Gamma+\frac{1}{Z_{01}}\sqrt{\frac{Z_{01}}{Z_{02}}}U_2\sinh\Gamma \quad \text{(C.26)}$$

$$U_2=\sqrt{\frac{Z_{02}}{Z_{01}}}U_1\cosh\Gamma-Z_{01}\sqrt{\frac{Z_{02}}{Z_{01}}}I_1\sinh\Gamma \quad \text{(C.27)}$$

$$I_2=\sqrt{\frac{Z_{01}}{Z_{02}}}I_1\cosh\Gamma-\frac{1}{Z_{01}}\sqrt{\frac{Z_{01}}{Z_{02}}}U_1\sinh\Gamma \quad \text{(C.28)}$$

The end results obtained can also be obtained direct from the transfer equations of an impedance-symmetrical two-port on the basis of Figure C.2.

By solving equations (C.25), (C.26), (C.27) and (C.28,) currents can be expressed by means of voltages or vice-versa, resulting in the following expressions:

$$I_1 = \frac{U_1}{Z_{01}} \coth \Gamma - \frac{U_2}{Z_{01}} \sqrt{\frac{Z_{01}}{Z_{02}}} \frac{1}{\sinh \Gamma} \qquad \text{(C.29)}$$

$$I_2 = \frac{U_1}{Z_{01}} \sqrt{\frac{Z_{01}}{Z_{02}}} \frac{1}{\sinh \Gamma} - \frac{U_2}{Z_{02}} \coth \Gamma \qquad \text{(C.30)}$$

$$U_1 = Z_{01} I_1 \coth \Gamma - Z_{01} \sqrt{\frac{Z_{02}}{Z_{01}}} I_2 \frac{1}{\sinh \Gamma} \qquad \text{(C.31)}$$

$$U_2 = Z_{01} \sqrt{\frac{Z_{02}}{Z_{01}}} I_1 \frac{1}{\sinh \Gamma} - Z_{02} I_2 \coth \Gamma \qquad \text{(C.32)}$$

NOTE A short reminder on matrices:

$$M_2 = K * M_1$$
$$M_1 = K^{-1} * M_2$$

When $M_2 = K * M_1$, where M_1, M_2 and K are matrices, then $M_1 = K^{-1} * M_2$, where K^{-1} is the inverse matrix of K.

When

$$K = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

the inverse is

$$K^{-1} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} = \frac{1}{\Delta} * \begin{bmatrix} D & -B \\ -C & A \end{bmatrix}$$

where the determinant is $\Delta = AD - BC$.

C.4 The symmetries and impedances of a two-port

Let us examine the two two-ports illustrated in Figures C.4 and C.5.

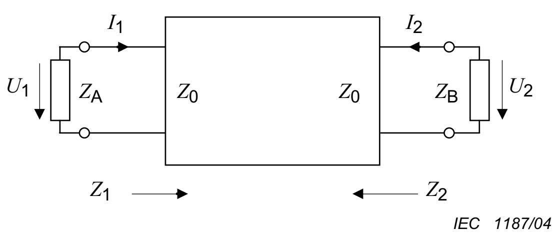


Figure C.4 – An impedance-symmetrical two-port with $Z_1 = Z_2$, when $Z_A = Z_B$

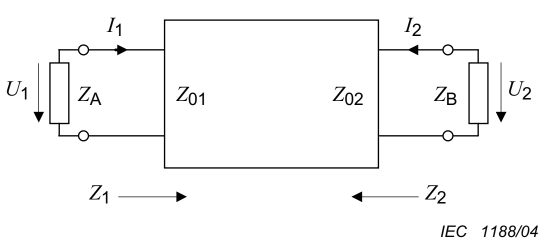


Figure C.5 – An impedance-unsymmetrical two-port for which $Z_1 \neq Z_2$ when $Z_A = Z_B$

The two-port in accordance with Figure C.4 is referred to as impedance-symmetrical or port-symmetrical, while the two-port of Figure C.5 is called an impedance-unsymmetrical or port-unsymmetrical.

If the complex composite loss (see Clause C.7) in the direction A->B is equal to that in the direction B->A for any values of generator and terminating impedance, then the two-port is referred to as transfer-symmetrical or reciprocal. Two-ports that consist of passive components (except gyrators) are always reciprocal. A two-port with none of its properties depending on the direction of transmission is both reciprocal and impedance-symmetrical. Such a two-port is referred to as longitudinally symmetrical. The input terminals of a two-port are earth-symmetrical, if the admittances measured at each input terminal relative to earth are equal. In this case we speak of transversal symmetry of the two-port [3][2].

In addition to the complex image attenuation $\Gamma = A + \mathrm{j}B$, there is another characteristic quantity for a two-port, that is, the image impedance. The image impedances Z_{01} and Z_{02} of a two-port in accordance with Figure C.5 can be determined by means of short-circuit and open-circuit measurements:

$$Z_{01} = \sqrt{Z_{1\mathrm{k}} Z_{1\mathrm{t}}} \qquad\qquad Z_{02} = \sqrt{Z_{2\mathrm{k}} Z_{2\mathrm{t}}}$$

where the subscripts k and t refer to the short-circuit and open-circuit conditions, respectively.

Let us recall the equations (C.7) and (C.8) valid for a longitudinally symmetrical two-port:

$$\begin{aligned} U_1 &= U_2 \cosh\Gamma + Z_0 I_2 \sinh\Gamma \\ I_1 &= I_2 \cosh\Gamma + \frac{U_2}{Z_0} \sinh\Gamma \end{aligned} \qquad \text{(C.33)}$$

Taking into account that $U_2 = Z_\mathrm{B} I_2$ (see Figure C.6), we obtain with equations (C.33) the input impedance of the two-port:

$$Z_1 = \frac{U_1}{I_1} = Z_0 \frac{Z_\mathrm{B} + Z_0 \tanh\Gamma}{Z_0 + Z_\mathrm{B} \tanh\Gamma} \qquad \text{(C.34)}$$

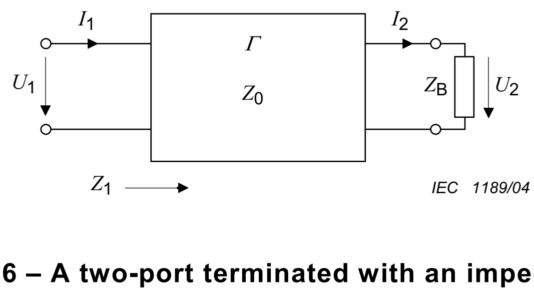


Figure C.6 – A two-port terminated with an impedance Z_B

Hence, the input impedance Z_1 depends on the properties of the two-port as well as on the terminating load impedance Z_B. It can be shown that when the attenuation A is high, Z_1 is only slightly affected by Z_B. From equation (C.34), we see that $Z_1 \approx Z_0$, when tanh $|\Gamma| \approx 1$, i.e. when $A > 2$ Np. The input impedance is then solely determined by the properties of the two-port. A two-port is called electrically short, when $A \ll 2$ Np and $B \ll \pi/2$, and correspondingly electrically long, when $A \geq 2$ Np and $B \geq \pi/2$.

2 Numbers in square brackets refer to the reference documents at the end of this Annex.

When the output is short-circuited (Z_B = 0), we have

$$Z_{1k} = Z_0 \tanh \Gamma \tag{C.35}$$

If Z_B = 0, but $A \rightarrow 0$ and $B = \pi/2$, then $Z_{1k} \rightarrow \infty$. This applies, for example, to a lossless short-circuited line with length $\lambda/4$. When the output is open ($Z_B = \infty$), we have

$$Z_{1t} = Z_0 \frac{1}{\tanh \Gamma} \tag{C.36}$$

Further, when $A \rightarrow 0$ and $B \rightarrow \pi/2$, then $Z_{1t} \rightarrow 0$. Additionally we have $\Gamma = \mathrm{j}B$. Replacing B by $\pi/2$, the equation (C.34) can then be written in the following form:

$$Z_1 = \frac{{Z_0}^2}{Z_B} \tag{C.37}$$

Using the conversion (C.37), the impedance Z_B can be transformed into an impedance Z_1. This is only feasible at the exact frequency for which the length of the lossless line is $\lambda/4$, corresponding to a so-called quarter-wavelength transformer. Equations (C.35) and (C.36) reveal that also the Z_0 and Γ of a longitudinally symmetrical two-port can be determined from the short-circuit and open-circuit impedances.

C.5 Impedance matching

If the image impedances of the two-ports to be cascaded differ from each other, reflections will be generated at the interconnection points, and those reflections then affect the uniformity of transmission. In telecommunication engineering, to avoid reflections in transmission, it is important that the impedances of the consecutive sections included in a transmission path are carefully matched to each other, i.e. the characteristic impedances of the devices to be cascaded shall very closely equal each other. A non-distorted transmission will only be possible under such conditions. However, it should be noted that one single major mismatch can be allowed within each repeater section; for example, provided that all other mismatches are small enough, because at least two mismatches are required for the generation of a propagating, signal-distorting forward-echo.

By substituting the quantities $U_2 = I_2 Z_0$, which correspond to a proper matching ($Z_B = Z_0$) into equations (C.33), we obtain

$$U_1 = U_2\, \mathrm{e}^{\Gamma} = I_2 Z_0\, \mathrm{e}^{\Gamma} \tag{C.38}$$

$$I_1 = I_2\, \mathrm{e}^{\Gamma} \tag{C.39}$$

from which it follows that the input impedance is

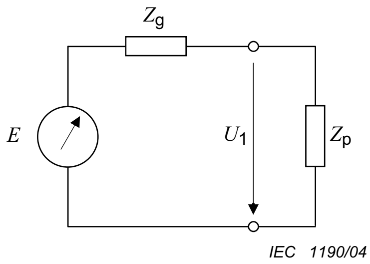
$$Z_1 = \frac{U_1}{I_1} = \frac{I_2 Z_0\, \mathrm{e}^{\Gamma}}{I_2\, \mathrm{e}^{\Gamma}} = Z_0 \tag{C.40}$$

Hence the input impedance is under these conditions independent of Γ.

Correct matching enables the greatest possible complex power to be transmitted from a generator to the load. (In the literature, the term complex power often refers to the quantity UI^*, while the quantity UI is called the apparent power. In transmission engineering, it is logical to use the term complex power to denote the product of voltage and current phasors.)

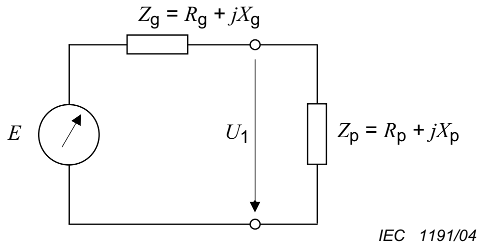
Hence,

$$P = UI \tag{C.41}$$



$Z_g = Z_p$

Figure C.7 – Reflection less matching



$Z_g = Z_p$* or $R_g = R_p$ with $X_g = X_p = 0$

Figure C.8 – Power matching for maximizing the effective power.

The complex power obtained with the load Z_B is

$$P = \frac{E^2 Z_p}{(Z_g + Z_p)^2} \tag{C.42}$$

which reaches a maximum when $Z_P = Z_g$, which yields

$$P_{max} = \frac{E^2}{4Z_g} \tag{C.43}$$

With $Z_g = R_g + jX_g$ and $Z_p = R_p + jX_p$ the greatest possible effective power is absorbed by the load when $R_g = R_p$ and $j(X_g + X_p) = 0$. The condition is met when both imaginary parts are zeros, or when the impedances are complex conjugates, i.e. $Z_g = Z^*_p$. This kind of matching is called power matching. It is commonly used when matching transmitters to antennas, but, being normally valid at a single frequency only (the tuning frequency), it has found no applications in broad-band transmission techniques. Even a two-port (its output or input, respectively) can be considered as a power source or a load.

The input or output impedance of a two-port can be built out in such a way as to be resistive while being independent of frequency, under the condition that it is represented by a series combination of R and L, or R and C. For example, if an impedance $R + 1/j\omega C$ is connected in parallel with an impedance $R + j\omega L$ by choosing $C = L/R^2$, a frequency-independent resistive impedance R will be obtained.

C.6 Level concepts

The term level is used to indicate a relative or an absolute value. If the power, voltage or current along a transmission system is concerned, one speaks of power, voltage or current levels.

When comparing the power, voltage or current at a measuring point with the respective quantity at the feeding point of the transmission system, we are concerned with a relative level, whereas, when the comparison is made to a standardized reference value, an absolute level will be obtained.

Levels are commonly expressed in decibels (dB), more seldom in nepers (Np). The use of nepers is actually restricted to some theoretical calculations. The units are related by

1 dB = 0,05/lg e = 0,1151 Np or 1 Np = 20 lg e = 8,686 dB

If P_x and V_x denote the power and the voltage at the measuring point, while P_A and V_A are the corresponding values at the feeding point (input) of the system, the relative power level is

$$N = 10\ \lg \frac{P_x}{P_A} \quad [\mathrm{dB}] = \frac{1}{2} \ln \frac{P_x}{P_A} \quad [\mathrm{Np}] \qquad \text{(C.44)}$$

and the relative voltage level is

$$N_v = 20\ \lg \frac{V_x}{V_A} \quad [\mathrm{dB}] = \ln \frac{V_x}{V_A} \quad [\mathrm{Np}] \qquad \text{(C.45)}$$

The relative level at the input of the system is always zero.

If $\underline{P_1}$ and $\underline{V_1}$ are the standardized reference values, the absolute power level is given by

$$N = 10 \lg \frac{P_x}{P_1} [\mathrm{dB}] = \frac{1}{2} \ln \frac{P_x}{P_1} [\mathrm{Np}] \qquad \text{(C.46)}$$

while the absolute voltage level is

$$N_v = 20 \lg \frac{V_x}{V_1} [\mathrm{dB}] = \ln \frac{V_x}{V_1} [\mathrm{Np}] \qquad \text{(C.47)}$$

In telecommunication engineering, the reference for absolute power levels is 1 mW and the reference for absolute voltage levels is 0,775 V, which corresponds to 1 mW in a 600 Ω load. Nowadays, voltage levels are seldom used in telecommunication engineering, to avoid confusion. There is a tendency towards an exclusive use of power levels.

In conjunction with broadcast relaying, community antennas and closed-link television systems, instead, voltage levels based on a reference of 1 μV have been adopted. A reference impedance of 75 Ω is implied; however, in the last two systems so that one is here actually concerned with power levels.

To discriminate between the above absolute levels with references 1 mW and 1 μV, respectively, the designations dBm (dB(mW)) and dB(μV) have been adopted. In transmission engineering, it proved advantageous to use a nominal level, reached when a power 1 mW is fed to the input or prevails at a fictive reference point in the system. Relative to this 1 mW point, the nominal level of the system is always 0 dB(mW) and is denoted 0 dBm0. In other words, the nominal level along the entire system can be thought to be 0 dBm0 (see Figure C.9). Hence designation –50 dBm0, for example, means a level which lies 50 dB below the nominal level of the system.

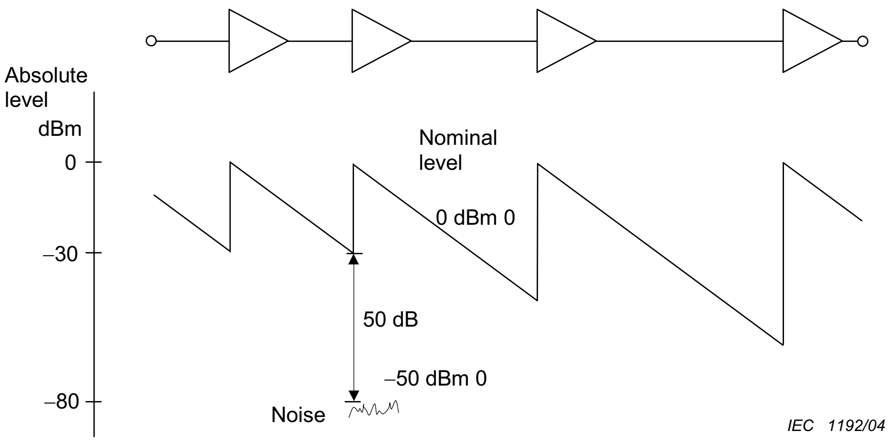


Figure C.9 – Absolute and nominal level in a system

In speech transmission, it is often appropriate to weight a disturbing noise signal in accordance with the sensitivity curve of the ear. Such a psophometrically weighted noise level, being, for example, 50 dB below the nominal level, is designated as –50 dB0p. The matter can also be expressed so that we have here a psophometrically weighted noise power reduced to the 0 dB(mW) point (1 mW point) and having a level 50 dB below 1 mW.

When it is necessary to emphasise that a level is a relative level or, respectively, a voltage level, designations dBr and dBu are employed.

C.7 Attenuation and gain concepts

The complex image attenuation or image transfer constant Γ of a two-port is defined as a logarithmic ratio between the power $P_1 = U_1 I_1$ fed to the input terminals and the power $P_2 = U_2 I_2$ obtained at the output, when the two-port is terminated in an impedance which is equal to the output image impedance of the two-port (see Figure C.10).

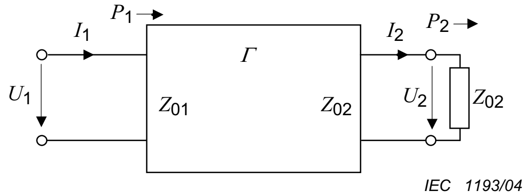


Figure C.10 – Definition of the complex image attenuation Γ of a two-port

$$\Gamma = A + jB = 10\,\lg\frac{P_1}{P_2}\,[\text{dB}] = \frac{1}{2}\ln\frac{P_1}{P_2}\,[\text{Np}]$$

$$= 10\,\lg\frac{U_1 I_1}{U_2 I_2}\,[\text{dB}] = \frac{1}{2}\ln\frac{U_1 I_1}{U_2 I_2}\,[\text{Np}] \quad \text{(C.48)}$$

$$= 20\,\lg\frac{U_1}{U_2}\sqrt{\frac{Z_{02}}{Z_{01}}}\,[\text{dB}] = \ln\frac{U_1}{U_2}\sqrt{\frac{Z_{02}}{Z_{01}}}\,[\text{Np}]$$

$$= 20\,\lg\frac{I_1}{I_2}\sqrt{\frac{Z_{01}}{Z_{02}}}\,[\text{dB}] = \ln\frac{I_1}{I_2}\sqrt{\frac{Z_{01}}{Z_{02}}}\,[\text{Np}] \quad \text{(C.49)}$$

Hence we have

$$A = 10\ \lg\left|\frac{P_1}{P_2}\right|[\text{dB}] = \frac{1}{2}\ \ln\left|\frac{P_1}{P_2}\right|[\text{Np}] \quad \text{(C.50)}$$

$$B = \frac{1}{2}\arg\frac{P_1}{P_2} = \frac{1}{2}(\angle P_1 - \angle P_2) \quad \text{(C.51)}$$

A is the image attenuation and B is the image phase constant. If the two-port is impedance-symmetrical ($Z_{01} = Z_{02}$), the equations are more simple and we obtain the expression

$$\Gamma = 20\ \lg\frac{U_1}{U_2}\,[\text{dB}] = \ln\frac{U_1}{U_2}\,[\text{Np}]$$

$$= 20\ \lg\frac{I_1}{I_2}\,[\text{dB}] = \ln\frac{I_1}{I_2}\,[\text{Np}] \quad \text{(C.52)}$$

During actual operation, a two-port often lies between terminating devices with impedances differing from the image impedances of the two-port. It is then appropriate to speak of operational attenuation. The complex operational attenuation or complex operational transfer constant Γ is defined as a logarithmic ratio between the power $P_1' = E^2/4Z_g$ fed by the generator to a load equal to its internal impedance Z_g, and the power $P_2 = U_2 I_2 = U_2^2/Z_p$ obtained to the load Z_p at the output of the two-port (see Figure C.11).

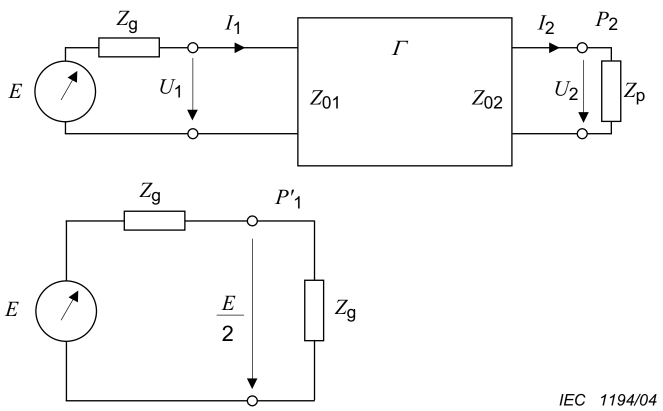


Figure C.11 – Definition of the complex operational attenuation of a two-port

$$\Gamma_{\mathrm{B}} = A_{\mathrm{B}} + \mathrm{j}B_{\mathrm{B}} = 10\,\lg\frac{P_1'}{P_2}\,[\mathrm{dB}] = \frac{1}{2}\ln\frac{P_1'}{P_2}\,[\mathrm{Np}]$$
$$= 20\,\lg\frac{E}{2U_2}\sqrt{\frac{Z_{\mathrm{p}}}{Z_{\mathrm{g}}}}\,[\mathrm{dB}] \quad = \ln\frac{E}{2U_2}\sqrt{\frac{Z_{\mathrm{p}}}{Z_{\mathrm{g}}}}\,[\mathrm{Np}] \tag{C.53}$$

$$A_{\mathrm{B}} = 10\,\lg\left|\frac{P_1'}{P_2}\right|[\mathrm{dB}] = \frac{1}{2}\;\ln\left|\frac{P_1'}{P_2}\right|\;[\mathrm{Np}] \tag{C.54}$$

$$B_{\mathrm{B}} = \frac{1}{2}\arg\frac{P_1'}{P_2} = \frac{1}{2}(\angle P_1' - \angle P_2) \tag{C.55}$$

The expression $\frac{2U_2}{E}\sqrt{\frac{Z_{\mathrm{g}}}{Z_{\mathrm{p}}}}$ is called the operational transfer constant H_{B}.

A_{B} is the operational attenuation and B_{B} the operational phase constant. If the impedances of the generator and the load are equal (Z_{g} = Z_{p}), then the equations are simplified and we obtain for the operational transfer constant

$$\Gamma_{\mathrm{B}} = 20\,\lg\frac{E}{2U_2}\,[\mathrm{dB}] = \ln\frac{E}{2U_2}\,[\mathrm{Np}] \tag{C.56}$$

The complex operational gain $-\Gamma$ is the opposite number of the complex operational attenuation:

$$-\Gamma_{\mathrm{B}} = -A_{\mathrm{B}} - \mathrm{j}B_{\mathrm{B}} = -10\lg\frac{P_1'}{P_2}[\mathrm{dB}] = 10\lg\frac{P_2}{P_1'}[\mathrm{dB}]$$
$$= 20\,\lg\frac{2U_2}{E}\sqrt{\frac{Z_{\mathrm{g}}}{Z_{\mathrm{p}}}}\,[\mathrm{dB}] \;=\; \ln\frac{2U_2}{E}\sqrt{\frac{Z_{\mathrm{g}}}{Z_{\mathrm{p}}}}\,[\mathrm{Np}] \tag{C.57}$$

-A_{B} is the operational gain and $-B_{\mathrm{B}}$ is the operational gain phase angle.

Residual attenuation: The amplifiers (repeaters) connected in a transmission line cancel a part of the attenuation caused by the lines. The remaining part is called the residual attenuation. The residual attenuation is equal to the difference of the total attenuation of all lines and components in the transmission path, A_{k} and the sum of the gain of all amplifiers S_{k} in the transmission line:

$$A = A_{\mathrm{k}} - S_{\mathrm{k}} \quad [\mathrm{dB}] \tag{C.58}$$

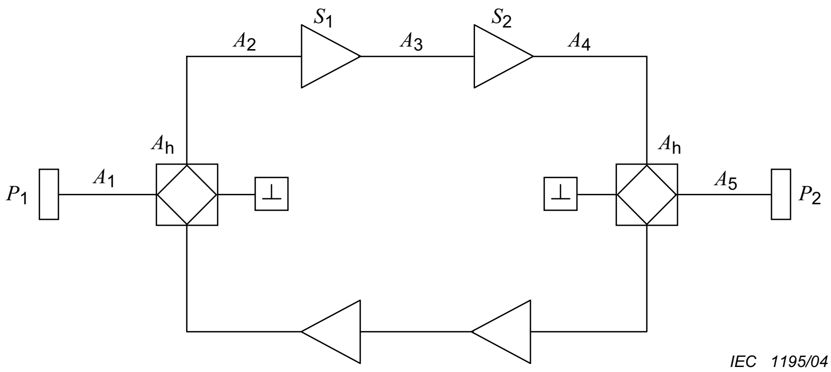


Figure C.12 – Definition of residual attenuation

The residual attenuation of the 2/4-wire line shown by Figure C.12 is equal to

$$A = (A_1 + A_2 + A_3 + A_4 + A_5 + 2A_h) - (S_1 + S_2)$$

where

$A_{1...5}$	are the attenuations of different line sections;
A_h	is the attenuation of the hybrid networks;
S_1 and S_2	are the gain of the respective repeater.

The reference equivalent is a measure for the speech-transmitting capabilities of a telephone connection. It is defined as the attenuation that must be added to the attenuation of a reference system so that the loudness of the speech through the damped reference system is equal to that through the actual system under examination. If the system under test is less sensitive than the reference system, the reference equivalent is considered to be positive.

As an international reference system, the NOSFER system (residing in the laboratories of CCITT) is employed. When measuring the sending reference equivalent (see Figure C.13), a person alternately speaks to the microphone of the system under test and to the microphone of NOSFER. There is a VU meter in the transmitting circuit of NOSFER that enables the speaker to exercise constant loudness. A listener adjusts the attenuator (A_R) in NOSFER in such a manner that equal loudness is experienced through both systems.

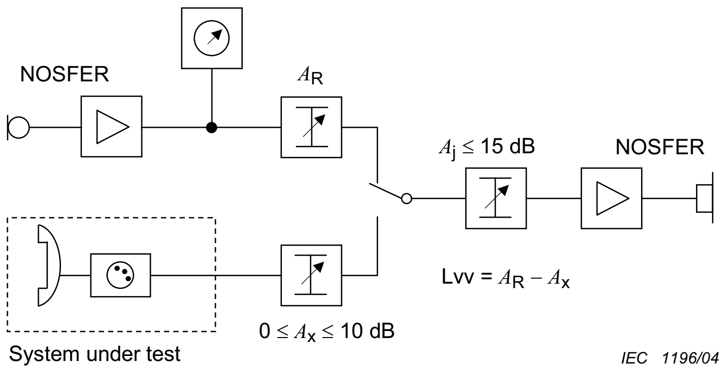


Figure C.13 – Measurement of the sending reference equivalent

When measuring the receiving reference equivalent (see Figure C.14) the speaker speaks into the NOSFER microphone, while the listener alternately listens through both systems, equalizing the loudness by means of an attenuator in similar fashion as the measurement of the sending reference equivalent.

The standard deviation of the test results in the case of a trained testing team is usually of the order (1,5 to 2,5) dB, while the 95 % margin lies within the range (±0,5 to ±4) dB.

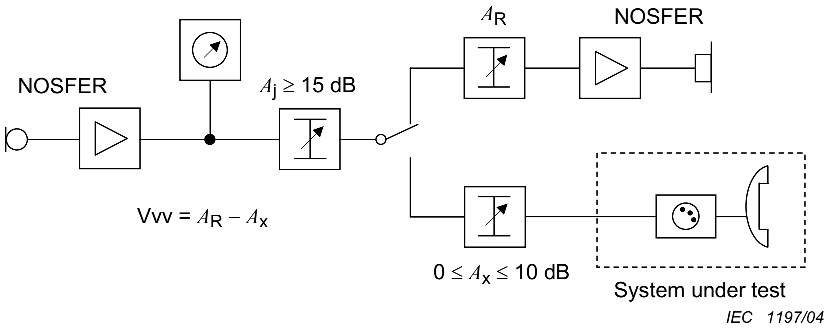


Figure C.14 – Measurement of the receiving reference equivalent

Attempts have been made to replace the subjective method of measurement by an objective one. One such method is at the moment under consideration in CCITT. However, the results obtained by objective methods have yet to coincide with adequate precision with those obtained by using the subjective method.

C.8 Concepts related to return loss and matching

Let us examine the circuit seen in Figure C.15, where U_i denotes the incident voltage wave that reaches a reflection point, while U_r is the voltage wave reflected back from the point of reflection.

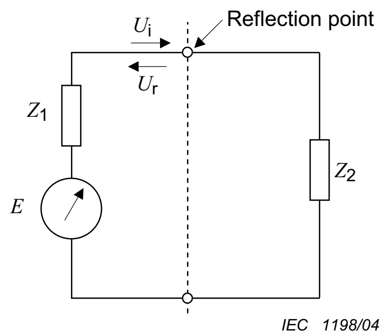


Figure C.15 – Definition of the complex return loss

The reflection coefficient ρ is the ratio between the reflected and incident waves:

$$\rho = \frac{U_r}{U_i} = \frac{Z_2 - Z_1}{Z_2 + Z_1} \qquad \text{(C.59)}$$

The complex return loss Γ_r is correspondingly defined as

$$\begin{aligned} \Gamma_r &= A_r + jB_r = 20 \lg \frac{1}{\rho} \ [\mathrm{dB}] = \ln \frac{1}{\rho} \ [\mathrm{Np}] \\ &= 20 \lg \frac{Z_2 + Z_1}{Z_2 - Z_1} \ [\mathrm{dB}] = \ln \frac{Z_2 + Z_1}{Z_2 - Z_1} \ [\mathrm{Np}]. \end{aligned} \qquad \text{(C.60), (C.61)}$$

$$A_r = 20\lg\left|\frac{Z_2+Z_1}{Z_2-Z_1}\right|[\mathrm{dB}] = \ln\left|\frac{Z_2+Z_1}{Z_2-Z_1}\right|[\mathrm{Np}] \tag{C.62}$$

$$B_r = \arg\frac{Z_2+Z_1}{Z_2-Z_1} = \angle\frac{Z_2+Z_1}{Z_2-Z_1}[\mathrm{rad}] \tag{C.63}$$

A_r is the return loss and B_r is the reflection phase constant.

The expression (C.62) for return loss can be rewritten in the form

$$A_r = 20\ \lg\left|\frac{z_N+1}{z_N-1}\right|[\mathrm{dB}] \tag{C.64}$$

where $z_N = Z_2/Z_1$ is the normalized impedance. If the reflection coefficient is constant, then also the term $|z_N + 1|/|z_N - 1|$ is constant. In accordance with Figure C.16, the numerator and the denominator can be considered as sections of lines, which indicate the distances of the end point P of the vector z_N from the points (–1,0) and (1,0), respectively. All other points, for which the ratio between the distances from the points (–1,0) and (1,0) equals the above constant, are found by drawing through P a so-called Apollonius' circle. It is formed by the points, for which the ratio of the distances from two fixed points is constant. The Apollonius' circle can be constructed by separating equal sections with length $|z_N - 1|$ on both sides of point (1,0) along a line, which is parallel to $|z_N + 1|$ and passes the point (1,0). When two lines are drawn through the point P and the external ends of the above sections, two intersection points are obtained on the real axis. The distance between these intersection points determines the diameter of the Apollonius' circle.

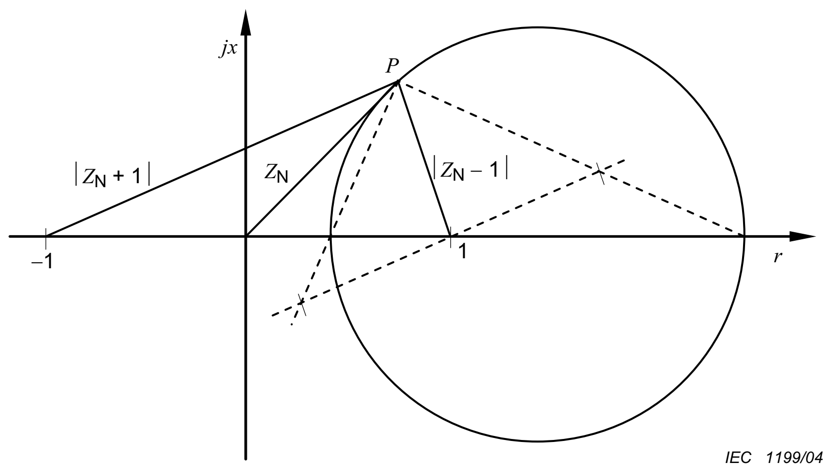
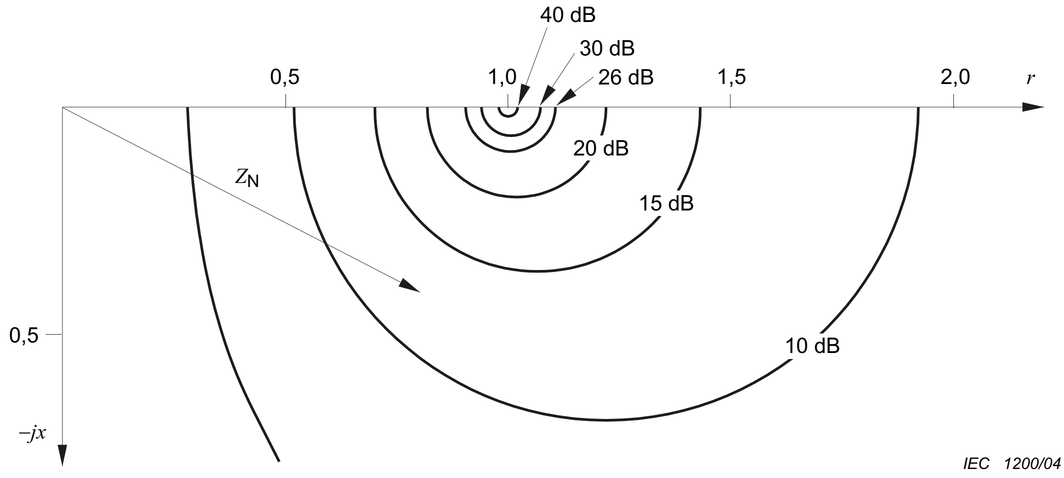


Figure C.16 – Apollonius' circle
(formed by the points for which the ratio of distances from the points (-1,0) and (1,0) is constant)

The points on the periphery of the circle in Figure C.16 represent a constant value of return loss. Inside the circle, the return loss is greater and, outside the circle, smaller than on the periphery. Because the circle is symmetrical in relation to the real axis, only one-half of the circle is usually drawn. By drawing several Apollonius' circles, each of them corresponding to a different value of return loss, a chart in accordance with Figure C.17 will be obtained. Any normalized impedance, z_N, drawn on the chart then directly gives the corresponding return loss in dB. In the example shown, $A_r \approx 12$ dB.



A_r = 20 lg $|z_N + 1|/|z_N - 1|$ where $z_N = r + jx$ is the normalized impedance

Figure C.17 – Return loss

When substituting Z_2 = 0 (short-circuit) or $Z_2 = \infty$ (open-circuit) to equation (C.62), A_r will vanish. When $Z_2 = Z_1$ (proper matching), there will be no reflections and, consequently, $A_r = \infty$.

The complex reflection loss Γ_s is

$$\Gamma_s = A_s + jB_s = 20\,\lg\frac{Z_2 + Z_1}{2\sqrt{Z_2 Z_1}}\ \left[\mathrm{dB}\right] = \ln\frac{Z_2 + Z_1}{2\sqrt{Z_2 Z_1}}\ \left[\mathrm{Np}\right] \tag{C.65}$$

$$\begin{aligned} &= 20\,\lg\frac{1}{\sqrt{1-\rho^2}}\ \left[\mathrm{dB}\right] = \ln\frac{1}{\sqrt{1-\rho^2}}\ \left[\mathrm{Np}\right] \\ &= 10\,\lg\frac{1}{1-\rho^2}\ \left[\mathrm{dB}\right] = \ln\frac{1}{1-\rho^2}\ \left[\mathrm{Np}\right]. \end{aligned} \tag{C.66}$$

$$A_s = 20\,\lg\left|\frac{Z_2 + Z_1}{2\sqrt{Z_2 Z_1}}\right|\left[\mathrm{dB}\right] = \ln\left|\frac{Z_2 + Z_1}{2\sqrt{Z_2 Z_1}}\right|\left[\mathrm{Np}\right] \tag{C.67}$$

$$B_s = \arg\frac{Z_2 + Z_1}{2\sqrt{Z_2 Z_1}} = \angle\frac{Z_2 + Z_1}{2\sqrt{Z_2 Z_1}}\left[\mathrm{rad}\right] \tag{C.68}$$

A_s is the reflection loss and B_s is the reflection loss phase angle. The quantity Γ_s indicates how much the complex power transferred through the reflection point to the actual load Z_2 has been attenuated in comparison with the unreflected complex power transmitted through the reflection point, if the load were equal to Z_1 (no reflection). Hence, equation (C.67) indicates that, with proper matching, i.e. $Z_2 = Z_1$ and A_s **=** 0, there exists also a number of other impedance pairs for which the reflection loss is zero. This is shown in Figures C.18 and C.19, which is a combination of circles with constant return loss and curves for constant values of A_s, all in a complex plane.

The right-hand side of the complex plane can be transformed into a circle with unit radius and with centre at point (1,0), whereby we obtain a so-called Smith's chart for transmission lines. There, the Apollonius' circles of constant return loss are transformed into concentric circles with central point (1,0), whereby the variation of impedance along the line, caused by the mismatch between the line and the load, can be directly read by following a circle that passes the point P for normalized load impedance. One clock-wise turn corresponds to a half-wavelength toward the generator (see, for example, the references [1] or [2]). If the line is lossy, the reflection attenuation does not remain constant when proceeding toward the generator, and the variation of impedance along the line then forms a converging spiral in the Smith chart.

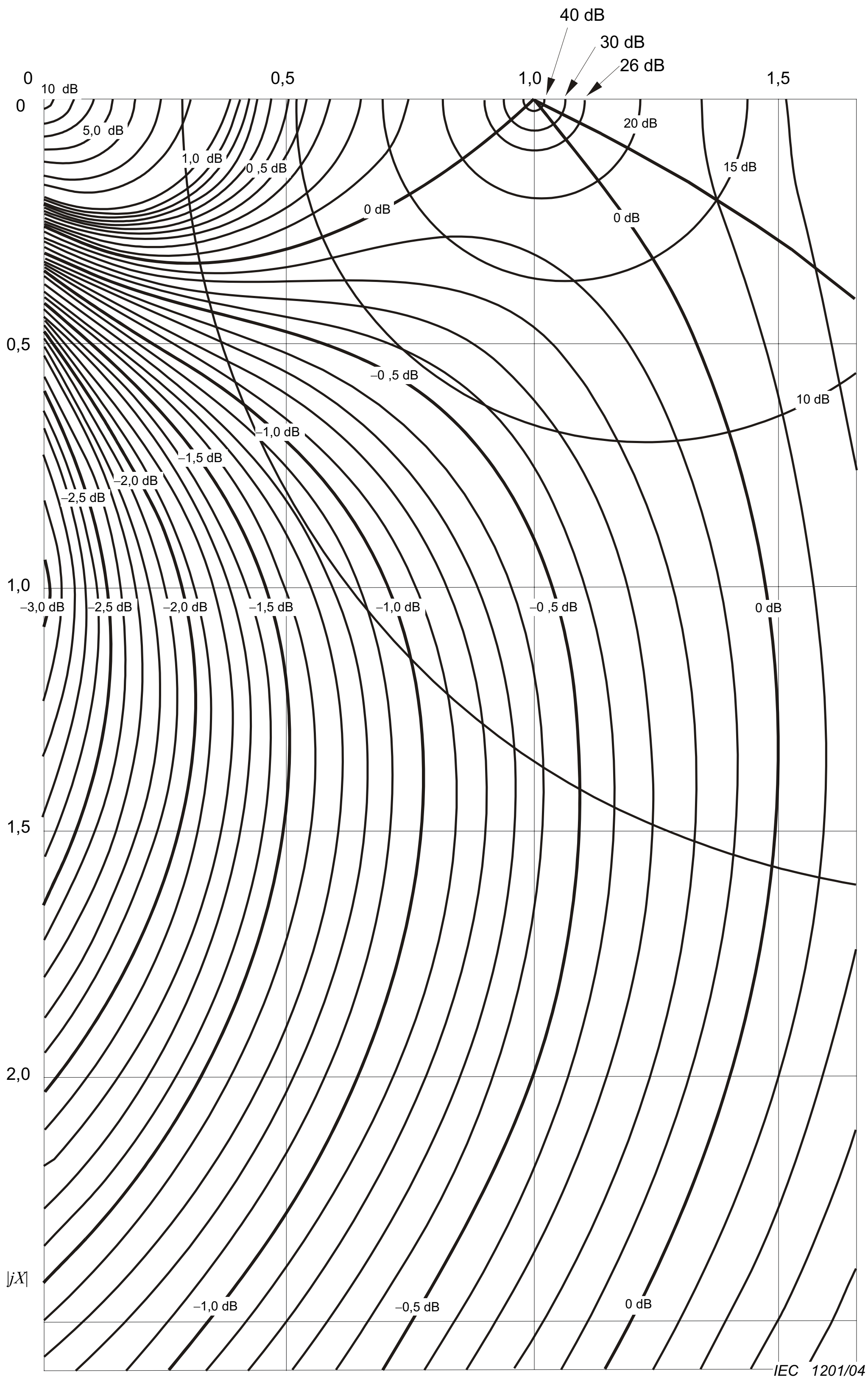
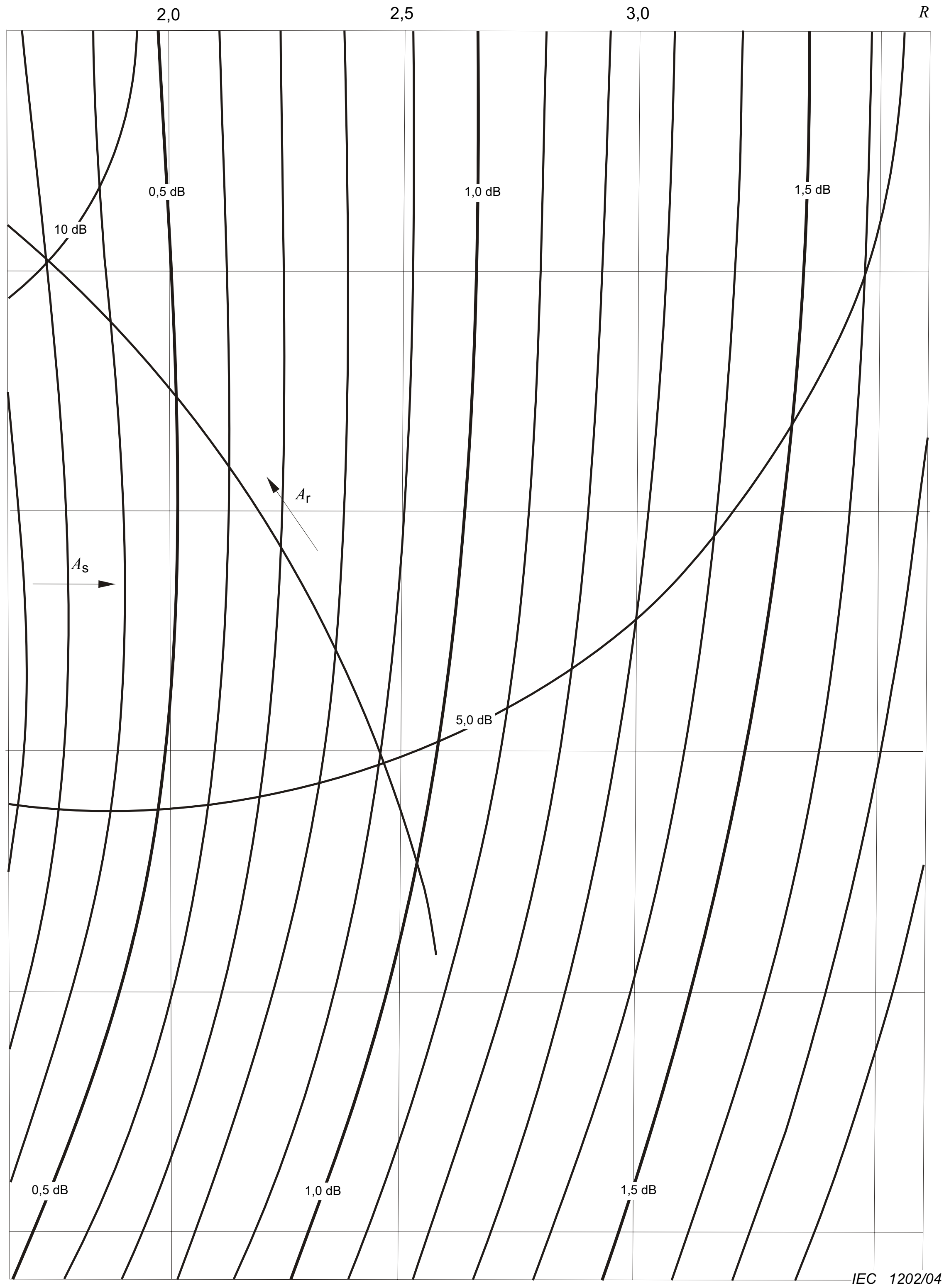


Figure C.18 – Curves for constant values of A_s or A_r in the complex plane



Reflection loss $A_s = 20\log\left|\frac{z_N + 1}{2\sqrt{z_N}}\right|$ [dB]   Return loss $A_r = 20\log\left|\frac{z_N + 1}{z_N - 1}\right|$ [dB]

$$z_\mathrm{N} = \frac{Z_2}{Z_1} \quad \text{(= normalized impedance) = } r + jx$$

Figure C.19 – Curves for constant values of A_s or $A_{r\ in}$ the complex plane

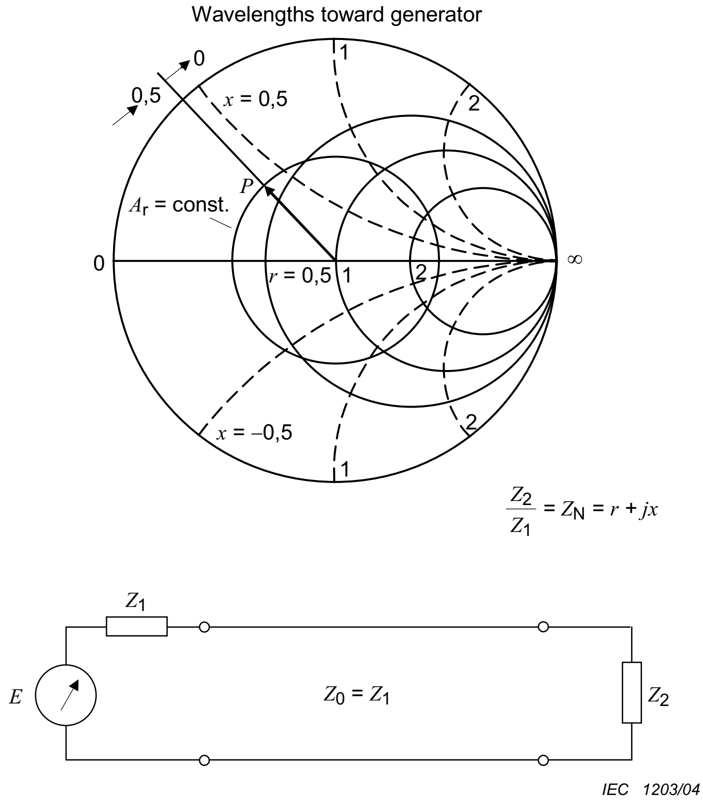


Figure C.20 – Smith chart for transmission lines

The voltage-standing-wave ratio $VSWR$ is the ratio between maximum and minimum values of the line voltages:

$$VSWR = \frac{U_{\max}}{U_{\min}} = \frac{1+|\rho|}{1-|\rho|} \tag{C.69}$$

where the reflection coefficient $\rho = \frac{Z_2 - Z_1}{Z_2 + Z_1} = \frac{z_N - 1}{z_N + 1}$.

From equation (C.69) we can calculate the absolute value of the reflection coefficient

$$|\rho| = \frac{VSWR-1}{VSWR+1} \approx \frac{VSWR-1}{2} \tag{C.70}$$

when $VSWR \approx 1$ or ρ << 1.

C.9 Scattering parameter

C.9.1 Scattering parameter of a one-port

We can characterize a port, as shown in Figure C.21, by incident (i) and reflected (r) voltage, current and square-root of power waves.

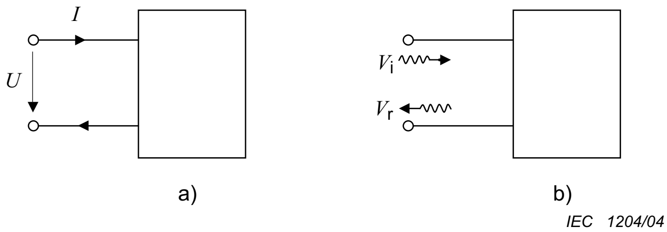


Figure C.21 – One-port

$$U = U_i + U_r \tag{C.71}$$

$$I = I_i - I_r = \frac{U_i}{R_0} - \frac{U_r}{R_0} \tag{C.72}$$

$$U_i = \frac{1}{2}(U + R_0 I) \tag{C.73}$$

$$U_r = \frac{1}{2}(U - R_0 I) \tag{C.74}$$

U and I are respectively the voltage and current at the terminals of the one-port and R_0 can be regarded as the image impedance of the one-port. Compare with the characteristic impedance of the homogenous transmission line in Figure C.22. For practical applications, it is advantageous to choose for the characteristic impedance nominal values, for example, 50 Ω, 75 Ω, 100 Ω, 120 Ω, 150 Ω.

This impedance is also used as reference impedance for measurements. This impedance does not necessarily correspond to the image impedance of the one-port, because V_i is defined as the unreflected square root of the power entering into the one-port and the square root of the fictive power, which is calculated or measured by matching the generator with this impedance.

Recalling that the square-root of power is

$$\sqrt{P} = \sqrt{UI} = \frac{U}{\sqrt{R_0}} = \sqrt{R_0} I = V \tag{C.75}$$

$$\sqrt{P_i} = V_i = \frac{U_i}{\sqrt{R_0}} = \frac{1}{2}\left(\frac{U}{\sqrt{R_0}} + \sqrt{R_0} I\right) \tag{C.76}$$

$$\sqrt{P_{\mathrm{r}}} = V_{\mathrm{r}} = \frac{U_{\mathrm{r}}}{\sqrt{R_0}} = \frac{1}{2}\left(\frac{U}{\sqrt{R_0}} - \sqrt{R_0}\, I\right) \tag{C.77}$$

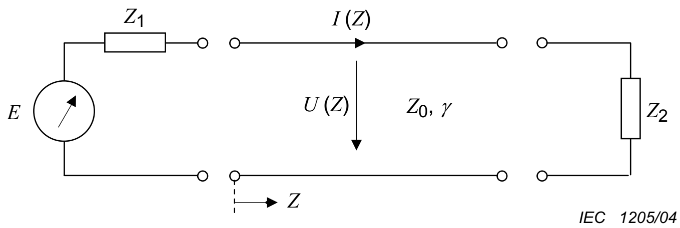


Figure C.22 – Homogenous transmission line

The relation between the incident and the reflected wave can be expressed by means of the scattering parameter S:

$$V_{\mathrm{r}} = S\, V_{\mathrm{i}} \tag{C.78}$$

The parameter S is here identical to the reflection coefficient ρ, which equally represents the ratio of the reflected voltage to the incident voltage at the reflection plane (see Clause C.8). From the definition for V_{i} and V_{r}, it follows that

$$\frac{U}{\sqrt{R_0}} - \sqrt{R_0}\, I = S\left(\frac{U}{\sqrt{R_0}} + \sqrt{R_0}\, I\right) \tag{C.79}$$

the solution of which gives

$$S = \frac{Z - R_0}{Z + R_0} \tag{C.80}$$

where $Z = U\, I$ is the input impedance of the one-port.

If $Z = R_0$, the voltage of the reflected wave is $V_{\mathrm{r}} = 0$. The inverse value of S, when expressed in dB or Np and radians, is called the complex return loss Γ_{r} (compare with equation C.61):

$$\Gamma_{\mathrm{r}} = 20\ \lg\left|\frac{1}{S}\right|\ [\mathrm{dB}]\quad + \mathrm{j}\arg\frac{1}{S}\ [\mathrm{rad}]$$

or (C.81)

$$= \ln\left|\frac{1}{S}\right|\ [\mathrm{Np}]\quad + \mathrm{j}\arg\frac{1}{S}\ [\mathrm{rad}]$$

V_{i} at a one-port, which is fed from a generator with an internal impedance Z_{g} equal to the image impedance of the one-port Z_0, is:

$$V_{\mathrm{i}} = \frac{E}{2\sqrt{Z_{\mathrm{g}}}} \tag{C.82}$$

By definition

$$V_{\mathrm{i}} = \frac{1}{2}\left(\frac{U}{\sqrt{R_0}} + \sqrt{R_0} I\right) \tag{C.83a}$$

Figure C.23 yields

$$U = E - IZ_g \tag{C.83b}$$

and

$$I = \frac{E}{Z_g + Z} \tag{C.83c}$$

When U and I are substituted into equation (C.83), expression (C.82) is obtained:

$$V_{\mathrm{i}} = \frac{1}{2\sqrt{R_0}}\left[\left(E - \frac{E\,Z_{\mathrm{g}}}{Z_{\mathrm{g}} + Z}\right) + \frac{R_0 E}{Z_{\mathrm{g}} + Z}\right] = \frac{E}{2\sqrt{R_0}}\left(1 + \frac{R_0 - Z_{\mathrm{g}}}{Z_{\mathrm{g}} + Z}\right) = \left.\frac{E}{2\sqrt{Z_{\mathrm{g}}}}\right|_{Z_{\mathrm{g}} = R_{\mathrm{o}}} \tag{C.83d}$$

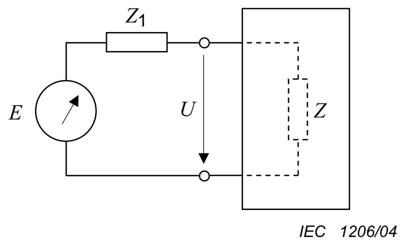


Figure C.23 – One-port fed from a generator with source impedance *Z*g

The reflected wave vanishes, if $Z = R_0$. On the other hand, there are no reflections between the generator and the load, if their impedance is equal, i.e. if $Z_{\mathrm{g}} = Z$. This condition is equivalent to a properly matched generator and the load impedance.

The maximum effective power is transmitted to the load, when $Z_{\mathrm{g}} = Z^*$ (see Clause C.5), whereas the maximum complex power is reached when $Z_{\mathrm{g}} = Z$. In accordance with equations (C.76) and (C.82), the maximum complex power is

$$P' = V_{\mathrm{i}}^2 = \frac{E^2}{4Z_{\mathrm{g}}} \tag{C.84a}$$

From the expressions (C.74) and (C.75) we obtain

$$U = (V_{\mathrm{i}} + V_{\mathrm{r}})\sqrt{R_0} \quad \text{and} \quad I = (V_{\mathrm{i}} - V_{\mathrm{r}})/\sqrt{R_0} \qquad \text{(C.84b and C.84c)}$$

This yields the expression for the complex power absorbed by the one-port, which is represented by the load:

$$P = UI = V_{\mathrm{i}}^{2} - V_{\mathrm{r}}^{2} \tag{C.85}$$

Substitution of equation (C.79) and (C.80) yields

$$P = V_{\mathrm{i}}^{2}(1 - S^{2}) = V_{\mathrm{i}}^{2}\left[1 - \left(\frac{Z - Z_{\mathrm{g}}}{Z + Z_{\mathrm{g}}}\right)^{2}\right] \tag{C.86a}$$

If the impedance Z_{g} of the generator, that feeds the one-port, is taken as reference impedance then the maximum complex power at the load is

$$P' = V_{\mathrm{i}}^{2} \tag{C.86b}$$

We obtain then the ratio between the complex power absorbed in the one-port and the actual complex power if the one-port is represented by the reference impedance Z:

$$\frac{P'}{P} = \left(\frac{(Z + Z_{\mathrm{g}})}{4ZZ_{\mathrm{g}}}\right)^{2} = \frac{1}{1 - S^{2}} \tag{C.87}$$

Compare to equations (C.65) and (C.66). Expressed in logarithmic units, this is called the complex reflection loss

$$\begin{aligned}
\Gamma_{\mathrm{s}} &= 20\ \lg\left|\frac{Z + Z_{\mathrm{g}}}{2\sqrt{Z\ Z_{\mathrm{g}}}}\right| [\mathrm{dB}]\ + \mathrm{j}\ \arg\left(\frac{Z + Z_{\mathrm{g}}}{2\ \sqrt{Z\ Z_{\mathrm{g}}}}\right) [\mathrm{rad}] \\
&= -10\lg\left|1 - S^{2}\right|\ [\mathrm{dB}] - \mathrm{j}\,\frac{1}{2}\arg(1 - S^{2})\ [\mathrm{rad}] \\
&= -\frac{1}{2}\ln\left|1 - S^{2}\right|\ [\mathrm{Np}] - \mathrm{j}\,\frac{1}{2}\arg(1 - S^{2})\ [\mathrm{rad}]
\end{aligned} \tag{C.88}$$

C.9.2 Scattering parameters and scattering matrix of a two-port

A two-port, shown in Figure C.24, can be treated as two individual one-ports, face to face.

For both one-ports, the incident and reflected waves are characterized by

$$
\begin{aligned}
V_{\mathrm{i}1} &= \frac{1}{2}\left(\frac{U_1}{\sqrt{R_{01}}} + \sqrt{R_{01}}I_1\right) \\
V_{\mathrm{r}1} &= \frac{1}{2}\left(\frac{U_1}{\sqrt{R_{01}}} - \sqrt{R_{01}}I_1\right) \\
V_{\mathrm{i}2} &= \frac{1}{2}\left(\frac{U_2}{\sqrt{R_{02}}} + \sqrt{R_{02}}I_2\right) \\
V_{\mathrm{r}2} &= \frac{1}{2}\left(\frac{U_2}{\sqrt{R_{02}}} - \sqrt{R_{02}}I_2\right)
\end{aligned} \qquad \text{(C.90)}
$$

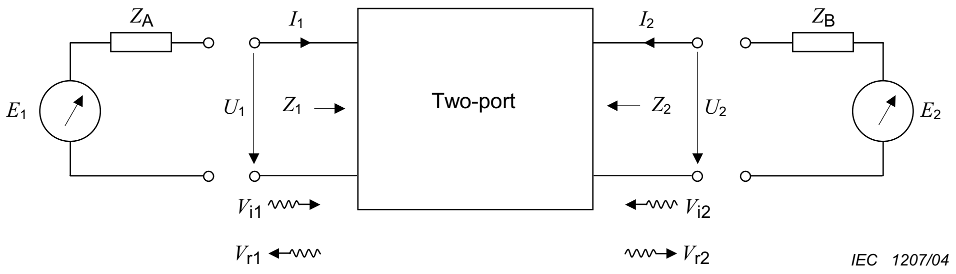


Figure C.24 – Two-port

Where R_{01} and R_{02} is the reference impedance at input and output, respectively, $V_{\mathrm{i}1}$ and $V_{\mathrm{r}1}$ are represented by the square roots of incident (unreflected) and reflected complex powers (see note) at port 1, and $V_{\mathrm{i}2}$ and $V_{\mathrm{r}2}$ are those at port 2.

NOTE Complex power is the product = UI . Apparent power is the product UI^*, which is used in electrical power technique, where the angle between the voltage and current is of interest. I^* is the complex conjugate of the current I .

The scattering parameters S_{mn} of a two-port are defined as follows:

$$
\begin{aligned}
V_{\mathrm{r}1} &= S_{11}V_{\mathrm{i}1} + S_{12}V_{\mathrm{i}2} \\
V_{\mathrm{r}2} &= S_{21}V_{\mathrm{i}1} + S_{22}V_{\mathrm{i}2}
\end{aligned} \qquad \text{(C.91)}
$$

or in a matrix form

$$
\begin{bmatrix} V_{\mathrm{r}1} \\ V_{\mathrm{r}2} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} V_{\mathrm{i}1} \\ V_{\mathrm{i}2} \end{bmatrix} \qquad \text{(C.92)}
$$

where the matrix

$$
[\mathbf{S}] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \qquad \text{(C.93)}
$$

is called the scattering matrix [4]. It has the elements:

$$S_{11} = \left.\frac{V_{r1}}{V_{i1}}\right|_{V_{i2}=0} = \frac{Z_1 - Z_A}{Z_1 + Z_A} = \rho_{B11}$$

$$S_{12} = \left.\frac{V_{r1}}{V_{i2}}\right|_{V_{i1}=0} = \frac{2U_1}{E_2}\sqrt{\frac{Z_B}{Z_A}} = H_{B12} \qquad \text{(C.94a)}$$

$$S_{21} = \left.\frac{V_{r2}}{V_{i1}}\right|_{V_{i2}=0} = \frac{2U_2}{E_1}\sqrt{\frac{Z_A}{Z_B}} = H_{B21}$$

$$S_{22} = \left.\frac{V_{r2}}{V_{i2}}\right|_{V_{i1}=0} = \frac{Z_2 - Z_B}{Z_2 + Z_B} = \rho_{B22}$$

with

$$\Gamma_B = 20 \ \lg \frac{1}{H_B} \quad [\text{dB}] \qquad \text{(C.94b)}$$

The above refers to the quantities under operational conditions, i.e. to the complex operational reflection coefficient ρ_B and to the operational transfer function H_B (see equation (C.53). They are directly derived from the condition that V_{i2} and V_{i1} are zero, a condition which is satisfied, in accordance with equation C.90) as soon as the terminal impedances Z_A and Z_B are equal to the reference impedance R_{01} and R_{02}, respectively. Hence,

S_{11} or ρ_{B11} is the complex operational reflection coefficient at the input;

S_{22} or ρ_{B22} is the complex operational reflection coefficient at the output;

S_{21} or H_{B21} is the operational transfer function in the forward direction;

S_{12} or H_{B12} is the operational transfer function in the backward direction.

The scattering matrix can thus be written in the form

$$[\mathbf{S}] = \begin{bmatrix} \rho_{B11} & H_{B12} \\ H_{B21} & \rho_{B22} \end{bmatrix} \qquad \text{(C.95a)}$$

$$S_{11} = \left.\frac{V_{r1}}{V_{i1}}\right|_{V_{i2}=0} \qquad \text{(C.95b)}$$

The connection between the scattering parameters and the above-mentioned working quantities can be derived as follows: the condition V_{i2} = 0 and its impact on the reflection factor at the input of the two-port is considered first.

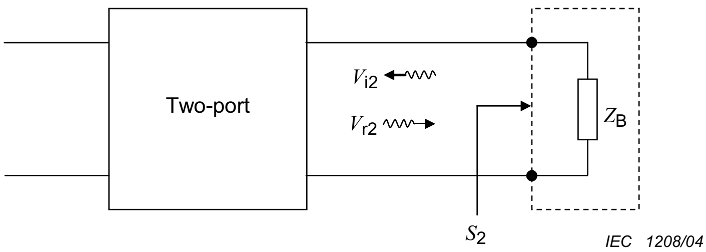


Figure C.25 – Termination Z_B by virtue of the stray parameters of the two-port

Let us consider the influence of the termination Z_B on the parameters V_{i2} and V_{r2} of the two-port. The scattering parameter of the termination is

$$S_2 = \frac{Z_B - R_{02}}{Z_B + R_{02}} \quad \text{(C.95c)}$$

where R_{02} is the reference impedance at the output of the two-port. The connection between V_{i2} and V_{r2} is obtained by the relation

$$V_{i2} = S_2 V_{r2} \quad \text{(C.95d)}$$

If the reference impedance is chosen to be equal to the impedance Z_B of the termination, then $S_2 = 0$ and $V_{i2} = 0$. Similarly, it can be shown that $V_{i1} = 0$, when $R_{01} = Z_A$.

Hence, we can conclude that, if the terminal impedances Z_A and Z_B are selected to be equal as reference impedances, the parameters V_{i1} and V_{i2} are equivalent to zero.

The input and output impedances of the two-port are

$$Z_1 = \frac{U_1}{I_1} \qquad Z_2 = \frac{U_2}{I_2} \quad \text{(C.96)}$$

When $R_{01} = Z_A$, we have

$$S_{11} = \frac{\frac{1}{2}\left[\frac{U_1}{\sqrt{Z_A}} - \sqrt{Z_A} I_1\right]}{\frac{1}{2}\left[\frac{U_1}{\sqrt{Z_A}} + \sqrt{Z_A} I_1\right]} = \frac{Z_1 - Z_A}{Z_1 + Z_A} = \rho_{B11} \quad \text{(C.97a)}$$

Similarly, when $R_{02} = Z_B$, we have

$$S_{22} = \frac{Z_2 - Z_B}{Z_2 + Z_B} = \rho_{B22} \quad \text{(C.97b)}$$

By definition, then,

$$S_{21} = \left.\frac{V_{r2}}{V_{i1}}\right|_{V_{i2}=0} \quad \text{(C.97c)}$$

As derived above, we have $V_{i2} = 0$ for $R_{02} = Z_B$. If an e.m.f. E_1 with an internal impedance equal to the reference impedance R_{01} is connected to the input terminals, then equation (C.82) yields

$$V_{i1} = \frac{E_1}{2\sqrt{Z_A}} \quad \text{(C.98)}$$

with

$$V_{i2} = 0 = \frac{1}{2}\left(\frac{U_2}{\sqrt{Z_B}} + \sqrt{Z_B} I_2\right) \tag{C.99a}$$

It follows that

$$\frac{U_2}{\sqrt{Z_B}} = -\sqrt{Z_B} I_2 \tag{C.99b}$$

By substituting this into the expression

$$V_{r2} = \frac{1}{2}\left(\frac{U_2}{\sqrt{Z_B}} - \sqrt{Z_B} I_2\right) \tag{C.99c}$$

we obtain

$$V_{r2} = \frac{U_2}{\sqrt{Z_B}} \tag{C.100}$$

and

$$S_{21} = \frac{2U_2}{E_1}\sqrt{\frac{Z_A}{Z_B}} = H_{B21} \tag{C.101}$$

Correspondingly we have:

$$S_{12} = \frac{2U_1}{E_2}\sqrt{\frac{Z_B}{Z_A}} = H_{B12} \tag{C.102}$$

We note that S_{21} and S_{12} are voltage ratios corrected to the terminal impedances and they describe under operating conditions the transfer function of the two-port. Therefore, we have the name composite transfer function. The complex composite loss of the two-port in the forward and the backward direction, respectively, is

$$\begin{aligned} \Gamma_{B21} &= 20 \lg \frac{E_1}{2U_2}\sqrt{\frac{Z_B}{Z_A}} = -20 \lg |S_{21}| \ [\mathrm{dB}] - \mathrm{j} \arg S_{21} \ [\mathrm{rad}] \\ \Gamma_{B12} &= 20 \lg \frac{E_2}{2U_1}\sqrt{\frac{Z_A}{Z_B}} = -20 \lg |S_{12}| \ [\mathrm{dB}] - \mathrm{j} \arg S_{12} \ [\mathrm{rad}] \end{aligned} \tag{C.103}$$

In the passive transfer-balanced case i.e. if the two-port is reciprocal, S_{21} is equal to S_{12}.

The usefulness of the scattering matrix in determining the transmission properties of the two-port is based on the direct measurements of composite reflection and composite loss coefficients at the input and output of the two-port, when it is terminated with a reference impedance.

It follows from the above that, in the matched case, V_i and V_r are square roots of the complex power at the input and output at the two-port, when the reference impedance is chosen to be Z_A . E_1' is the internal electromotive-force at port 1 of the two-port. V_i2 and V_r2 are obtained by changing the sub-index 1 → 2 and impedance $Z_\mathrm{A} \rightarrow Z_\mathrm{B}$.

$$V_{\mathrm{i}1} = \frac{E_1}{2\sqrt{Z_\mathrm{A}}} = \frac{U_1 + Z_\mathrm{A} I_1}{2\sqrt{Z_\mathrm{A}}}$$

$$V_{\mathrm{r}1} = \frac{E_1'}{2\sqrt{Z_A}} = \frac{U_1 - Z_\mathrm{A} I_1}{2\sqrt{Z_\mathrm{A}}} \qquad \text{(C.104)}$$

In the literature [5], parameters V_i and V_r are defined utilizing the so-called available power. The results are identical to those previously mentioned, but only when the reference impedances are purely resistive.

In this case, we have

$$S_{11} = \frac{Z_1 - Z_\mathrm{A}^{\ *}}{Z_1 + Z_\mathrm{A}} \qquad \text{(C.105a)}$$

$$S_{21} = \frac{2U_2}{E_1} \frac{\sqrt{\mathrm{Re}\, Z_\mathrm{A}\, \mathrm{Re}\, Z_\mathrm{B}}}{Z_\mathrm{B}} \qquad \text{(C.105b)}$$

These expressions are identical to (C.97) and (C.101) only when Z_A and Z_B are resistive. In line transmission, it is generally not wise to use the so-called available power matching, because there will always remain reflections (see Clause C.5). It is worthwhile to mention the available power matching, which is deployed in lump loaded line, but, also in this case, consideration is given to reflectionless matching at the ends of the line. Another example of available power matching is required for matching a transmitter to an antenna.

C.10 Examples

C.10.1 Example 1

Consider an ideal transformer with a turns ratio n:1 (Figure C.26). Then,

$$U_1 = nU_2$$

$$I_1 = -\frac{1}{n} I_2 \qquad \text{(C.106)}$$

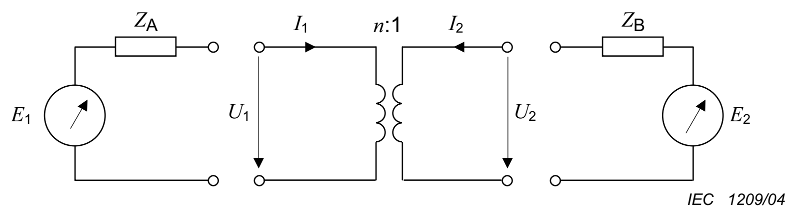


Figure C.26 – Ideal transformer

Let us choose terminal impedances as reference impedance.

Then

$$S_{11} = \frac{n^2 Z_B - Z_A}{n^2 Z_B + Z_A} \tag{C.107a}$$

$$S_{22} = \frac{\frac{Z_A}{n^2} - Z_B}{\frac{Z_A}{n^2} + Z_B} = \frac{Z_A - n^2 Z_B}{Z_A + n^2 Z_B} \tag{C.107b}$$

$$\begin{aligned} S_{21} &= \frac{2U_2}{E_1}\sqrt{\frac{Z_A}{Z_B}} = \frac{2E_1 n^2 Z_B}{(Z_A + n^2 Z_B) n E_1}\sqrt{\frac{Z_A}{Z_B}} \\ &= \frac{2nZ_B}{Z_A + n^2 Z_B}\sqrt{\frac{Z_A}{Z_B}} \end{aligned} \tag{C.108a}$$

$$S_{12} = \frac{2U_1}{E_1}\sqrt{\frac{Z_B}{Z_A}} = \frac{2nZ_A}{Z_A + n^2 Z_B}\sqrt{\frac{Z_B}{Z_A}} \tag{C.108b}$$

Therefore, the scattering matrix of the ideal transformer is

$$[\mathbf{S}] = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \tag{C.108c}$$

where

$$\begin{aligned} A &= \frac{n^2 Z_B - Z_A}{n^2 Z_B + Z_A} \\ B &= \frac{2nZ_A}{Z_A + n^2 Z_B}\sqrt{\frac{Z_B}{Z_A}} \\ C &= \frac{2nZ_B}{Z_A + n^2 Z_B}\sqrt{\frac{Z_A}{Z_B}} \\ D &= \frac{Z_A - n^2 Z_B}{Z_A + n^2 Z_B} \end{aligned} \tag{C.108d}$$

If $Z_A = n^2 Z_B$, then

$$[\mathbf{S}] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{C.108e}$$

Hence, both ports are reflectionless, and attenuation is = 0 dB, i.e. the transfer function is equal to one.

C.10.2 Example 2

Let us determine the scattering matrix of a passive, reciprocal two-port terminated by Z_A and Z_B. The image impedances of the two-port are Z_{01} and Z_{02} , the complex image attenuation is Γ and input impedances are Z_1 and Z_2 .

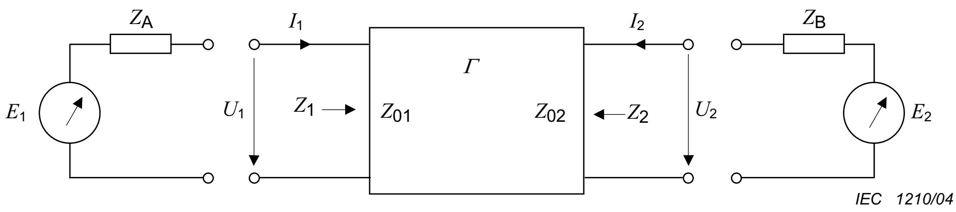


Figure C.27 – Determination of a scattering matrix of a passive reciprocal two-port

From expression (C.94) we obtain S_{11} and S_{22}

$$S_{11} = \frac{Z_1 - Z_A}{Z_1 + Z_A} \tag{C.109a}$$

$$S_{22} = \frac{Z_2 - Z_B}{Z_2 + Z_B} \tag{C.109b}$$

We find the voltage U_2 to be

$$\begin{aligned} U_2 &= \frac{Z_{01}E_1}{Z_A + Z_{01}} \sqrt{\frac{Z_{02}}{Z_{01}}} e^{-\Gamma} \frac{2Z_B}{Z_B + Z_{02}} \\ &+ \frac{Z_{01}E_1}{Z_A + Z_{01}} \sqrt{\frac{Z_{02}}{Z_{01}}} e^{-\Gamma} \frac{Z_B - Z_{02}}{Z_B + Z_{02}} \sqrt{\frac{Z_{01}}{Z_{02}}} e^{-\Gamma} \frac{Z_A - Z_{01}}{Z_A + Z_{01}} \sqrt{\frac{Z_{02}}{Z_{01}}} e^{-\Gamma} \frac{2Z_B}{Z_B + Z_{02}} \\ &+ \frac{Z_{01}E_1}{Z_A + Z_{01}} \sqrt{\frac{Z_{02}}{Z_{01}}} e^{-\Gamma} \frac{Z_B - Z_{02}}{Z_B + Z_{02}} \sqrt{\frac{Z_{01}}{Z_{02}}} e^{-\Gamma} \frac{Z_A - Z_{01}}{Z_A + Z_{01}} \sqrt{\frac{Z_{02}}{Z_{01}}} e^{-\Gamma} \\ &\cdot \frac{Z_B - Z_{02}}{Z_B + Z_{02}} \sqrt{\frac{Z_{01}}{Z_{02}}} e^{-\Gamma} \frac{Z_A - Z_{01}}{Z_A + Z_{01}} \sqrt{\frac{Z_{02}}{Z_{01}}} e^{-\Gamma} \frac{2Z_B}{Z_B + Z_{01}} + \ldots \end{aligned} \tag{C.110a}$$

$$\begin{aligned} U_2 &= \frac{Z_{01}E_1}{Z_A + Z_{01}} \sqrt{\frac{Z_{02}}{Z_{01}}} \frac{2Z_B}{Z_B + Z_{02}} e^{-\Gamma} \\ &\cdot \left[1 + \frac{Z_B - Z_{02}}{Z_A + Z_{02}} \frac{Z_A - Z_{01}}{Z_A + Z_{01}} e^{-2\Gamma} + \left(\frac{Z_B - Z_{02}}{Z_B + Z_{02}} \frac{Z_A - Z_{01}}{Z_A + Z_{01}} e^{-2\Gamma} \right)^2 + \ldots \right] \end{aligned} \tag{C.110b}$$

$$U_2 = \frac{Z_{01}E_1}{Z_A + Z_{01}} \frac{2Z_B}{Z_B + Z_{02}} \sqrt{\frac{Z_{02}}{Z_{01}}}\, e^{-\Gamma} \frac{1}{1 - \frac{Z_A - Z_{01}}{Z_A + Z_{01}} \frac{Z_B - Z_{02}}{Z_B + Z_{02}} e^{-2\Gamma}} \tag{C.110c}$$

By substituting this into expression (C.94), we get

$$S_{21} = \frac{2U_2}{E_1}\sqrt{\frac{Z_A}{Z_B}} = \frac{2\sqrt{Z_A Z_{01}}}{Z_A + Z_{01}} \frac{2\sqrt{Z_B Z_{02}}}{Z_B + Z_{02}}\, e^{-\Gamma} \frac{1}{1 - \frac{Z_A - Z_{01}}{Z_A + Z_{01}} \frac{Z_B - Z_{02}}{Z_B + Z_{02}} e^{-2\Gamma}} \tag{C.111}$$

Correspondingly S_{12}:

$$S_{12} = \frac{2U_1}{E_1}\sqrt{\frac{Z_B}{Z_A}} = \frac{2\sqrt{Z_B Z_{02}}}{Z_B + Z_{02}} \frac{2\sqrt{Z_A Z_{01}}}{Z_A + Z_{01}}\, e^{-\Gamma} \frac{1}{1 - \frac{Z_B - Z_{02}}{Z_B + Z_{02}} \frac{Z_A - Z_{01}}{Z_A + Z_{01}} e^{-2\Gamma}} \tag{C.112a}$$

By substituting expressions (C.109), (C.111) and (C.112) into the matrix (C.93), we get

$$[\mathbf{S}] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \tag{C.112b}$$

we obtain the scattering matrix of the two-port shown in Figure C.27.

C.11 Reference documents

[1] Goubau, G., *Design of Surface-Wave Transmission Lines*, Electronics, vol. 27, April 1954

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