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IEC 61691-3-2

First edition
2001-06

Behavioural languages –

Part 3-2:
Mathematical operation in VHDL

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INTERNATIONAL ELECTROTECHNICAL COMMISSION

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BEHAVIOURAL LANGUAGES –

Part 3-2: Mathematical operation in VHDL

FOREWORD

1) The IEC (International Electrotechnical Commission) is a worldwide organization for standardization comprising all national electrotechnical committees (IEC National Committees). The object of the IEC is to promote international co-operation on all questions concerning standardization in the electrical and electronic fields. To this end and in addition to other activities, the IEC publishes International Standards. Their preparation is entrusted to technical committees; any IEC National Committee interested in the subject dealt with may participate in this preparatory work. International, governmental and non-governmental organizations liaising with the IEC also participate in this preparation. The IEC collaborates closely with the International Organization for Standardization (ISO) in accordance with conditions determined by agreement between the two organizations.
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6) Attention is drawn to the possibility that some of the elements of this International Standard may be the subject of patent rights. The IEC shall not be held responsible for identifying any or all such patent rights.

International Standard IEC 61691-3-2 has been prepared by IEC technical committee 93: Design automation.

This standard is based on IEEE Std 1076-2 (1996): *IEEE Standard VHDL mathematical packages.*

The text of this standard is based on the following documents:

FDIS	Report on voting
93/131/FDIS	93/141/RVD

Full information on the voting for the approval of this standard can be found in the report on voting indicated in the above table.

This standard does not follow the rules for the structure of international standards given in Part 3 of the ISO/IEC Directives.

IEC 61691 consists of the following parts, under the general title: *Behavioural languages:*

IEC 61691-1:1997, VHDL language reference manual [1)]

IEC 61691-2:2001, Part 2: VHDL multilogic system for model interoperability

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1) The edition 2 with the title: VHSIC hardware description language VHDL (1076a) (under consideration) will replace it.

IEC 61691-3-1, Part 3-1: Analog description in VHDL (under consideration)

IEC 61691-3-2:2001, Part 3-2: Mathematical operation in VHDL

IEC 61691-3-3:2001, Part 3-3: Synthesis in VHDL

IEC 61691-3-4, Part 3-4: Timing expressions in VHDL (under consideration)

IEC 61691-3-5, Part 3-5: Library utilities in VHDL (under consideration)

The committee has decided that the contents of this publication will remain unchanged until 2004. At this date, the publication will be

- reconfirmed;
- withdrawn;
- replaced by a revised edition, or
- amended.

INTRODUCTION

This set of packages provides a standard for the declaration of most frequently used real and complex elementary functions required for numerically oriented modeling applications. Use of these packages with their defined data types, constants, and functions is intended to provide a mechanism for writing VHDL models (compliant with IEEE Std 1076-1993) that are portable and interoperable with other VHDL models adhering to this standard. The standard serves a broad class of applications with reasonable ease of use and requires implementations that are of high quality.

This standard includes package bodies, as described in annex B, which are available in electronic format either on a diskette affixed to the back cover, or as a downloadable file from the IEC Web Store.

BEHAVIOURAL LANGUAGES - Part 3-2: Mathematical operation in VHDL

1. Overview

1.1 Scope

This standard is embodied in the MATH_REAL and MATH_COMPLEX package declarations, and in the semantics of the standard mathematical definition and the conventional meaning of the functions that are part of this standard, along with 1.3. The information in annex A is a guide to users and implementors and is not a normative part of this standard, but suggests ways in which one might use this set of packages. The information in annex B is provided as a guideline for implementors and is not a normative part of this standard, but suggests ways in which implementors may implement this standard. The functions in this set of packages were chosen because of their widespread utility, as well as because they are needed to support general floating-point usage and to build other generic packages.

1.2 Constants, types, and functions provided

The following constants of type REAL are provided:

MATH_E	MATH_LOG_OF_2	MATH_DEG_TO_RAD
MATH_1_OVER_E	MATH_LOG_OF_10	MATH_RAD_TO_DEG
	MATH_LOG2_OF_E	
MATH_PI	MATH_LOG10_OF_E	
MATH_2_PI		
MATH_1_OVER_PI	MATH_SQRT_2	
MATH_PI_OVER_2	MATH_1_OVER_SQRT_2	
MATH_PI_OVER_3	MATH_SQRT_PI	
MATH_PI_OVER_4		
MATH_3_PI_OVER_2		

The following functions/procedures of type REAL are provided:

SIGN(X)	EXP(X)	SINH(X)
CEIL(X)	LOG(X)	COSH(X)
FLOOR(X)	LOG2(X)	TANH(X)
ROUND(X)	LOG10(X)	
TRUNC(X)	LOG(X, BASE)	ARCSINH(X)
"MOD"(X, Y)		ARCCOSH(X)
	SIN(X)	ARCTANH(X)
REALMAX(X, Y)	COS(X)	
REALMIN(X, Y)	TAN(X)	
UNIFORM(SEED1, SEED2, X)		
SQRT(X)	ARCSIN(X)	
CBRT(X)	ARCCOS(X)	
"**"(X, Y)	ARCTAN(Y)	
	ARCTAN(Y, X)	

The following types and subtypes are provided:

COMPLEX	POSITIVE_REAL
COMPLEX_POLAR	PRINCIPAL_VALUE

The following constants of type COMPLEX are provided:

MATH_CBASE_1 MATH_CBASE_J MATH_CZERO

The following type conversion functions for COMPLEX and COMPLEX_POLAR are provided:

CMPLX(X, Y)	POLAR_TO_COMPLEX(Z)
COMPLEX_TO_POLAR(Z)	GET_PRINCIPAL_VALUE(X)

The following overloaded relational functions for type COMPLEX_POLAR are provided:

"="(L, R) "/="(L, R)

The following functions for type COMPLEX and COMPLEX_POLAR are provided:

"ABS"(Z)	EXP(Z)	SIN(Z)
ARG(Z)	LOG(Z)	COS(Z)
	LOG2(Z)	
"-"(Z)	LOG10(Z)	SINH(Z)
CONJ(Z)	LOG(Z, BASE)	COSH(Z)
SQRT(Z)		

The following arithmetic functions for type COMPLEX and COMPLEX_POLAR are provided:

"+"	"*"
"-"	"/"

1.3 Conformance with this standard

The following conformance rules shall apply as they pertain to the use and implementation of this standard:

a) The package declarations may be modified to include additional data required by tools, but modifications shall in no way change the external interfaces or simulation behavior of the description. It is permissible to add comments and/or attributes to the package declarations, but not to change or delete any original lines of the approved package declaration.

b) The standard mathematical definition and conventional meaning of the mathematical functions that are part of this standard, together with the MATH_REAL and MATH_COMPLEX package declarations, represent the formal semantics of the implementation of the MATH_REAL and MATH_COMPLEX packages. An implementation is provided as a guideline in annex B. Implementors of these packages may choose to simply compile the package bodies provided in annex B, or they may choose to implement the package bodies in the most efficient form available to them. Implementations should conform to the semantics and minimum precision required by this standard.

c) The MATH_REAL package shall be built on top of the standard data type and precision requirements for floating point operations defined in IEEE Std 1076-1993 (STD.STANDARD).

d) The minimum precision required is that of IEEE Std 1076-1993. Because of this reason and the fact that the functions are implemented on digital computers with only finite precision, the functions and constants in this set of packages can, at best, only approximate the corresponding mathematically defined functions and constants. An implementation is allowed to provide a higher precision than the minimum required.

e) For some functions, the implementation shall deliver “prescribed results” for certain special arguments, as defined in the comments for the functions in the function declaration. The purpose is to strengthen the accuracy requirements at special argument values. Prescribed results take precedence over maximum relative error requirements.

f) The semantics of the standard require that all the functions in the packages detect and report invalid parameters (out of valid domain) through an assert statement. The domain of valid values is indicated in the MATH_REAL and MATH_COMPLEX package declarations. The default value of the severity level shall be Error.

g) The semantics of the standard do not require detection of overflow or underflow. Therefore, detection of underflow/overflow is optional and implementation dependent.

h) If an implementation chooses to provide any extensions beyond the minimum requirements of this standard (e.g., precision, overflow handling), then it shall document its behavior accordingly.

i) The MATH_REAL and MATH_COMPLEX packages shall be compiled into a library symbolically named IEEE.

2. References

This standard shall be used in conjunction with the following publications:

IEEE Std 754-1985 (Reaff 1990), IEEE Standard for Binary Floating-Point Arithmetic (ANSI).[1]

IEEE Std 1076-1993, IEEE Standard VHDL Language Reference Manual (ANSI).

[1]IEEE publications are available from the Institute of Electrical and Electronics Engineers, 445 Hoes Lane, P. O. Box 1331, Piscataway, NJ 08855-1331, USA.

3. Package declarations

The declaration of each function includes the following information: description of the mathematical definition of the function; values to be returned by the function for special arguments; valid domain of values for the input arguments; error conditions; range of values into which the function maps the values in its domain; and notes on special accuracy situations, reachable values, usable domains, or algorithms to be used by an implementation.

3.1 MATH_REAL

```
------------------------------------------------------------------------
--
-- Copyright 1996 by IEEE. All rights reserved.
--
-- This source file is an essential part of IEEE Std 1076.2-1996, IEEE Standard
-- VHDL Mathematical Packages. This source file may not be copied, sold, or
-- included with software that is sold without written permission from the IEEE
-- Standards Department. This source file may be used to implement this standard
-- and may be distributed in compiled form in any manner so long as the
-- compiled form does not allow direct decompilation of the original source file.
-- This source file may be copied for individual use between licensed users.
-- This source file is provided on an AS IS basis. The IEEE disclaims ANY
-- WARRANTY EXPRESS OR IMPLIED INCLUDING ANY WARRANTY OF MERCHANTABILITY
-- AND FITNESS FOR USE FOR A PARTICULAR PURPOSE. The user of the source
-- file shall indemnify and hold IEEE harmless from any damages or liability
-- arising out of the use thereof.
--
-- Title:       Standard VHDL Mathematical Packages (IEEE Std 1076.2-1996,
--              MATH_REAL)
--
-- Library:     This package shall be compiled into a library
--              symbolically named IEEE.
--
-- Developers:  IEEE DASC VHDL Mathematical Packages Working Group
--
-- Purpose:     This package defines a standard for designers to use in
--              describing VHDL models that make use of common REAL constants
--              and common REAL elementary mathematical functions.
--
-- Limitation:  The values generated by the functions in this package may
--              vary from platform to platform, and the precision of results
--              is only guaranteed to be the minimum required by IEEE Std 1076-
--              1993.
--
-- Notes:
--              No declarations or definitions shall be included in, or
--              excluded from, this package.
--              The "package declaration" defines the types, subtypes, and
--              declarations of MATH_REAL.
--              The standard mathematical definition and conventional meaning
--              of the mathematical functions that are part of this standard
--              represent the formal semantics of the implementation of the
--              MATH_REAL package declaration.  The purpose of the MATH_REAL
--              package body is to provide a guideline for implementations to
--              verify their implementation of MATH_REAL.  Tool developers may
--              choose to implement the package body in the most efficient
--              manner available to them.
```

```
--
-- ------------------------------------------------------------------------------
-- Version    : 1.5
-- Date       : 24 July 1996
-- ------------------------------------------------------------------------------

package MATH_REAL is
    constant CopyRightNotice: STRING
      := "Copyright 1996 IEEE. All rights reserved.";

    --
    -- Constant Definitions
    --
    constant  MATH_E : REAL := 2.71828_18284_59045_23536;
                                                        -- Value of e
    constant  MATH_1_OVER_E : REAL := 0.36787_94411_71442_32160;
                                                        -- Value of 1/e
    constant  MATH_PI : REAL := 3.14159_26535_89793_23846;
                                                        -- Value of pi
    constant  MATH_2_PI : REAL := 6.28318_53071_79586_47693;
                                                        -- Value of 2*pi
    constant  MATH_1_OVER_PI : REAL := 0.31830_98861_83790_67154;
                                                        -- Value of 1/pi
    constant  MATH_PI_OVER_2 : REAL := 1.57079_63267_94896_61923;
                                                        -- Value of pi/2
    constant  MATH_PI_OVER_3 : REAL := 1.04719_75511_96597_74615;
                                                        -- Value of pi/3
    constant  MATH_PI_OVER_4 : REAL := 0.78539_81633_97448_30962;
                                                        -- Value of pi/4
    constant  MATH_3_PI_OVER_2 : REAL := 4.71238_89803_84689_85769;
                                                        -- Value 3*pi/2
    constant  MATH_LOG_OF_2 : REAL := 0.69314_71805_59945_30942;
                                                        -- Natural log of 2
    constant  MATH_LOG_OF_10 : REAL := 2.30258_50929_94045_68402;
                                                        -- Natural log of 10
    constant  MATH_LOG2_OF_E : REAL := 1.44269_50408_88963_4074;
                                                        -- Log base 2 of e
    constant  MATH_LOG10_OF_E: REAL := 0.43429_44819_03251_82765;
                                                        -- Log base 10 of e
    constant  MATH_SQRT_2: REAL := 1.41421_35623_73095_04880;
                                                        -- square root of 2
    constant  MATH_1_OVER_SQRT_2: REAL := 0.70710_67811_86547_52440;
                                                        -- square root of 1/2
    constant  MATH_SQRT_PI: REAL := 1.77245_38509_05516_02730;
                                                        -- square root of pi
    constant  MATH_DEG_TO_RAD: REAL := 0.01745_32925_19943_29577;
                                     -- Conversion factor from degree to radian
    constant  MATH_RAD_TO_DEG: REAL := 57.29577_95130_82320_87680;
                                     -- Conversion factor from radian to degree

    --
    -- Function Declarations
    --
    function SIGN (X: in REAL ) return REAL;
        -- Purpose:
        --         Returns 1.0 if X > 0.0; 0.0 if X = 0.0; -1.0 if X < 0.0
        -- Special values:
        --         None
        -- Domain:
```

```
    --          X in REAL
    -- Error conditions:
    --          None
    -- Range:
    --          ABS(SIGN(X)) <= 1.0
    -- Notes:
    --          None

function CEIL (X : in REAL ) return REAL;
    -- Purpose:
    --          Returns smallest INTEGER value (as REAL) not less than X
    -- Special values:
    --          None
    -- Domain:
    --          X in REAL
    -- Error conditions:
    --          None
    -- Range:
    --          CEIL(X) is mathematically unbounded
    -- Notes:
    --          a) Implementations have to support at least the domain
    --                 ABS(X) < REAL(INTEGER'HIGH)

function FLOOR (X : in REAL ) return REAL;
    -- Purpose:
    --          Returns largest INTEGER value (as REAL) not greater than X
    -- Special values:
    --          FLOOR(0.0) = 0.0
    -- Domain:
    --          X in REAL
    -- Error conditions:
    --          None
    -- Range:
    --          FLOOR(X) is mathematically unbounded
    -- Notes:
    --          a) Implementations have to support at least the domain
    --                 ABS(X) < REAL(INTEGER'HIGH)

function ROUND (X : in REAL ) return REAL;
    -- Purpose:
    --          Rounds X to the nearest integer value (as real). If X is
    --          halfway between two integers, rounding is away from 0.0
    -- Special values:
    --          ROUND(0.0) = 0.0
    -- Domain:
    --          X in REAL
    -- Error conditions:
    --          None
    -- Range:
    --          ROUND(X) is mathematically unbounded
    -- Notes:
    --          a) Implementations have to support at least the domain
    --                 ABS(X) < REAL(INTEGER'HIGH)

function TRUNC (X : in REAL ) return REAL;
    -- Purpose:
    --          Truncates X towards 0.0 and returns truncated value
    -- Special values:
    --          TRUNC(0.0) = 0.0
```

```
        -- Domain:
        --         X in REAL
        -- Error conditions:
        --         None
        -- Range:
        --         TRUNC(X) is mathematically unbounded
        -- Notes:
        --         a) Implementations have to support at least the domain
        --                ABS(X) < REAL(INTEGER'HIGH)

    function "MOD" (X, Y: in REAL ) return REAL;
        -- Purpose:
        --         Returns floating point modulus of X/Y, with the same sign as
        --         Y, and absolute value less than the absolute value of Y, and
        --         for some INTEGER value N the result satisfies the relation
        --         X = Y*N + MOD(X,Y)
        -- Special values:
        --         None
        -- Domain:
        --         X in REAL; Y in REAL and Y /= 0.0
        -- Error conditions:
        --         Error if Y = 0.0
        -- Range:
        --         ABS(MOD(X,Y)) < ABS(Y)
        -- Notes:
        --         None

    function REALMAX (X, Y : in REAL ) return REAL;
        -- Purpose:
        --         Returns the algebraically larger of X and Y
        -- Special values:
        --         REALMAX(X,Y) = X when X = Y
        -- Domain:
        --         X in REAL; Y in REAL
        -- Error conditions:
        --         None
        -- Range:
        --         REALMAX(X,Y) is mathematically unbounded
        -- Notes:
        --         None

    function REALMIN (X, Y : in REAL ) return REAL;
        -- Purpose:
        --         Returns the algebraically smaller of X and Y
        -- Special values:
        --         REALMIN(X,Y) = X when X = Y
        -- Domain:
        --         X in REAL; Y in REAL
        -- Error conditions:
        --         None
        -- Range:
        --         REALMIN(X,Y) is mathematically unbounded
        -- Notes:
        --         None

    procedure UNIFORM(variable SEED1,SEED2:inout POSITIVE; variable X:out REAL);
        -- Purpose:
        --         Returns, in X, a pseudo-random number with uniform
        --         distribution in the open interval (0.0, 1.0).
```

```
    -- Special values:
    --         None
    -- Domain:
    --         1 <= SEED1 <= 2147483562; 1 <= SEED2 <= 2147483398
    -- Error conditions:
    --         Error if SEED1 or SEED2 outside of valid domain
    -- Range:
    --         0.0 < X < 1.0
    -- Notes:
    --         a) The semantics for this function are described by the
    --            algorithm published by Pierre L'Ecuyer in "Communications
    --            of the ACM," vol. 31, no. 6, June 1988, pp. 742-774.
    --            The algorithm is based on the combination of two
    --            multiplicative linear congruential generators for 32-bit
    --            platforms.
    --
    --         b) Before the first call to UNIFORM, the seed values
    --           (SEED1, SEED2) have to be initialized to values in the range
    --            [1, 2147483562] and [1, 2147483398] respectively.  The
    --            seed values are modified after each call to UNIFORM.
    --
    --         c) This random number generator is portable for 32-bit
    --           computers, and it has a period of ~2.30584*(10**18) for each
    --            set of seed values.
    --
    --         d) For information on spectral tests for the algorithm, refer
    --            to the L'Ecuyer article.

function SQRT (X : in REAL ) return REAL;
    -- Purpose:
    --         Returns square root of X
    -- Special values:
    --         SQRT(0.0) = 0.0
    --         SQRT(1.0) = 1.0
    -- Domain:
    --         X >= 0.0
    -- Error conditions:
    --         Error if X < 0.0
    -- Range:
    --         SQRT(X) >= 0.0
    -- Notes:
    --         a) The upper bound of the reachable range of SQRT is
    --            approximately given by:
    --                SQRT(X) <= SQRT(REAL'HIGH)

function CBRT (X : in REAL ) return REAL;
    -- Purpose:
    --         Returns cube root of X
    -- Special values:
    --         CBRT(0.0) = 0.0
    --         CBRT(1.0) = 1.0
    --         CBRT(-1.0) = -1.0
    -- Domain:
    --         X in REAL
    -- Error conditions:
    --         None
    -- Range:
    --         CBRT(X) is mathematically unbounded
    -- Notes:
```

```
        --         a) The reachable range of CBRT is approximately given by:
        --                ABS(CBRT(X)) <= CBRT(REAL'HIGH)

    function "**" (X : in INTEGER; Y : in REAL) return REAL;
        -- Purpose:
        --         Returns Y power of X ==>  X**Y
        -- Special values:
        --         X**0.0 = 1.0; X /= 0
        --         0**Y = 0.0; Y > 0.0
        --         X**1.0 = REAL(X); X >= 0
        --         1**Y = 1.0
        -- Domain:
        --         X > 0
        --         X = 0 for Y > 0.0
        --         X < 0 for Y = 0.0
        -- Error conditions:
        --         Error if X < 0 and Y /= 0.0
        --         Error if X = 0 and Y <= 0.0
        -- Range:
        --         X**Y >= 0.0
        -- Notes:
        --         a) The upper bound of the reachable range for "**" is
        --            approximately given by:
        --                X**Y <= REAL'HIGH

    function "**" (X : in REAL; Y : in REAL) return REAL;
        -- Purpose:
        --         Returns Y power of X ==>  X**Y
        -- Special values:
        --         X**0.0 = 1.0; X /= 0.0
        --         0.0**Y = 0.0; Y > 0.0
        --         X**1.0 = X; X >= 0.0
        --         1.0**Y = 1.0
        -- Domain:
        --         X > 0.0
        --         X = 0.0 for Y > 0.0
        --         X < 0.0 for Y = 0.0
        -- Error conditions:
        --         Error if X < 0.0 and Y /= 0.0
        --         Error if X = 0.0 and Y <= 0.0
        -- Range:
        --         X**Y >= 0.0


        -- Notes:
        --         a) The upper bound of the reachable range for "**" is
        --            approximately given by:
        --                X**Y <= REAL'HIGH

    function EXP (X : in REAL ) return REAL;
        -- Purpose:
        --         Returns e**X; where e = MATH_E
        -- Special values:
        --         EXP(0.0) = 1.0
        --         EXP(1.0) = MATH_E
        --         EXP(-1.0) = MATH_1_OVER_E
        --         EXP(X) = 0.0 for X <= -LOG(REAL'HIGH)
        -- Domain:
        --         X in REAL such that EXP(X) <= REAL'HIGH
```

```
    -- Error conditions:
    --         Error if X > LOG(REAL'HIGH)
    -- Range:
    --         EXP(X) >= 0.0
    -- Notes:
    --         a) The usable domain of EXP is approximately given by:
    --                X <= LOG(REAL'HIGH)

function LOG (X : in REAL ) return REAL;
    -- Purpose:
    --         Returns natural logarithm of X
    -- Special values:
    --         LOG(1.0) = 0.0
    --         LOG(MATH_E) = 1.0
    -- Domain:
    --         X > 0.0
    -- Error conditions:
    --         Error if X <= 0.0
    -- Range:
    --         LOG(X) is mathematically unbounded
    -- Notes:
    --         a) The reachable range of LOG is approximately given by:
    --                LOG(0+) <= LOG(X) <= LOG(REAL'HIGH)

function LOG2 (X : in REAL ) return REAL;
    -- Purpose:
    --         Returns logarithm base 2 of X
    -- Special values:
    --         LOG2(1.0) = 0.0
    --         LOG2(2.0) = 1.0
    -- Domain:
    --         X > 0.0
    -- Error conditions:
    --         Error if X <= 0.0
    -- Range:
    --         LOG2(X) is mathematically unbounded
    -- Notes:
    --         a) The reachable range of LOG2 is approximately given by:
    --                LOG2(0+) <= LOG2(X) <= LOG2(REAL'HIGH)

function LOG10 (X : in REAL ) return REAL;
    -- Purpose:
    --         Returns logarithm base 10 of X
    -- Special values:
    --         LOG10(1.0) = 0.0
    --         LOG10(10.0) = 1.0
    -- Domain:
    --         X > 0.0
    -- Error conditions:
    --         Error if X <= 0.0
    -- Range:
    --         LOG10(X) is mathematically unbounded
    -- Notes:
    --         a) The reachable range of LOG10 is approximately given by:
    --                LOG10(0+) <= LOG10(X) <= LOG10(REAL'HIGH)

function LOG (X: in REAL; BASE: in REAL) return REAL;
    -- Purpose:
    --         Returns logarithm base BASE of X
```

```
        -- Special values:
        --         LOG(1.0, BASE) = 0.0
        --         LOG(BASE, BASE) = 1.0
        -- Domain:
        --         X > 0.0
        --         BASE > 0.0
        --         BASE /= 1.0
        -- Error conditions:
        --         Error if X <= 0.0
        --         Error if BASE <= 0.0
        --         Error if BASE = 1.0
        -- Range:
        --         LOG(X, BASE) is mathematically unbounded
        -- Notes:
        --         a) When BASE > 1.0, the reachable range of LOG is
        --            approximately given by:
        --                LOG(0+, BASE) <= LOG(X, BASE) <= LOG(REAL'HIGH, BASE)
        --         b) When 0.0 < BASE < 1.0, the reachable range of LOG is
        --            approximately given by:
        --                LOG(REAL'HIGH, BASE) <= LOG(X, BASE) <= LOG(0+, BASE)

    function  SIN (X : in REAL ) return REAL;
        -- Purpose:
        --         Returns sine of X; X in radians
        -- Special values:
        --         SIN(X) = 0.0 for X = k*MATH_PI, where k is an INTEGER
        --         SIN(X) = 1.0 for X = (4*k+1)*MATH_PI_OVER_2, where k is an
        --                                                              INTEGER
        --         SIN(X) = -1.0 for X = (4*k+3)*MATH_PI_OVER_2, where k is an
        --                                                              INTEGER
        -- Domain:
        --         X in REAL
        -- Error conditions:
        --         None
        -- Range:
        --         ABS(SIN(X)) <= 1.0
        -- Notes:
        --         a) For larger values of ABS(X), degraded accuracy is allowed.

    function  COS ( X : in REAL ) return REAL;
        -- Purpose:
        --         Returns cosine of X; X in radians
        -- Special values:
        --         COS(X) = 0.0 for X = (2*k+1)*MATH_PI_OVER_2, where k is an
        --                                                               INTEGER
        --         COS(X) = 1.0 for X = (2*k)*MATH_PI, where k is an INTEGER
        --         COS(X) = -1.0 for X = (2*k+1)*MATH_PI, where k is an INTEGER
        -- Domain:
        --         X in REAL
        -- Error conditions:
        --         None
        -- Range:
        --         ABS(COS(X)) <= 1.0
        -- Notes:
        --         a) For larger values of ABS(X), degraded accuracy is allowed.

    function  TAN (X : in REAL ) return REAL;
        -- Purpose:
        --         Returns tangent of X; X in radians
```

```
    -- Special values:
    --         TAN(X) = 0.0 for X = k*MATH_PI, where k is an INTEGER
    -- Domain:
    --         X in REAL and
    --         X /= (2*k+1)*MATH_PI_OVER_2, where k is an INTEGER
    -- Error conditions:
    --         Error if X = ((2*k+1) * MATH_PI_OVER_2), where k is an
    --                                                          INTEGER
    -- Range:
    --         TAN(X) is mathematically unbounded
    -- Notes:
    --         a) For larger values of ABS(X), degraded accuracy is allowed.

function  ARCSIN (X : in REAL ) return REAL;
    -- Purpose:
    --         Returns inverse sine of X
    -- Special values:
    --         ARCSIN(0.0) = 0.0
    --         ARCSIN(1.0) = MATH_PI_OVER_2
    --         ARCSIN(-1.0) = -MATH_PI_OVER_2
    -- Domain:
    --         ABS(X) <= 1.0
    -- Error conditions:
    --         Error if ABS(X) > 1.0
    -- Range:
    --         ABS(ARCSIN(X) <= MATH_PI_OVER_2
    -- Notes:
    --         None

function  ARCCOS (X : in REAL ) return REAL;
    -- Purpose:
    --         Returns inverse cosine of X
    -- Special values:
    --         ARCCOS(1.0) = 0.0
    --         ARCCOS(0.0) = MATH_PI_OVER_2
    --         ARCCOS(-1.0) = MATH_PI
    -- Domain:
    --         ABS(X) <= 1.0
    -- Error conditions:
    --         Error if ABS(X) > 1.0

    -- Range:
    --         0.0 <= ARCCOS(X) <= MATH_PI
    -- Notes:
    --         None

function  ARCTAN (Y : in REAL) return REAL;
    -- Purpose:
    --         Returns the value of the angle in radians of the point
    --        (1.0, Y), which is in rectangular coordinates
    -- Special values:
    --         ARCTAN(0.0) = 0.0
    -- Domain:
    --         Y in REAL
    -- Error conditions:
    --         None
    -- Range:
    --         ABS(ARCTAN(Y)) <= MATH_PI_OVER_2
    -- Notes:
```

```
        --          None

    function  ARCTAN (Y : in REAL; X : in REAL) return REAL;
        -- Purpose:
        --          Returns the principal value of the angle in radians of
        --          the point (X, Y), which is in rectangular coordinates
        -- Special values:
        --          ARCTAN(0.0, X) = 0.0 if X > 0.0
        --          ARCTAN(0.0, X) = MATH_PI if X < 0.0
        --          ARCTAN(Y, 0.0) = MATH_PI_OVER_2 if Y > 0.0
        --          ARCTAN(Y, 0.0) = -MATH_PI_OVER_2 if Y < 0.0
        -- Domain:
        --          Y in REAL
        --          X in REAL, X /= 0.0 when Y = 0.0
        -- Error conditions:
        --          Error if X = 0.0 and Y = 0.0
        -- Range:
        --          -MATH_PI < ARCTAN(Y,X) <= MATH_PI
        -- Notes:
        --          None

    function SINH (X : in REAL) return REAL;
        -- Purpose:
        --          Returns hyperbolic sine of X
        -- Special values:
        --          SINH(0.0) = 0.0
        -- Domain:
        --          X in REAL
        -- Error conditions:
        --          None
        -- Range:
        --          SINH(X) is mathematically unbounded
        -- Notes:
        --          a) The usable domain of SINH is approximately given by:
        --                 ABS(X) <= LOG(REAL'HIGH)


    function COSH (X : in REAL) return REAL;
        -- Purpose:
        --          Returns hyperbolic cosine of X
        -- Special values:
        --          COSH(0.0) = 1.0
        -- Domain:
        --          X in REAL
        -- Error conditions:
        --          None
        -- Range:
        --          COSH(X) >= 1.0
        -- Notes:
        --          a) The usable domain of COSH is approximately given by:
        --                 ABS(X) <= LOG(REAL'HIGH)

    function TANH (X : in REAL) return REAL;
        -- Purpose:
        --          Returns hyperbolic tangent of X
        -- Special values:
        --          TANH(0.0) = 0.0
        -- Domain:
        --          X in REAL
```

```
        -- Error conditions:
        --         None
        -- Range:
        --         ABS(TANH(X)) <= 1.0
        -- Notes:
        --         None

    function ARCSINH (X : in REAL) return REAL;
        -- Purpose:
        --         Returns inverse hyperbolic sine of X
        -- Special values:
        --         ARCSINH(0.0) = 0.0
        -- Domain:
        --         X in REAL
        -- Error conditions:
        --         None
        -- Range:
        --         ARCSINH(X) is mathematically unbounded
        -- Notes:
        --         a) The reachable range of ARCSINH is approximately given by:
        --                ABS(ARCSINH(X)) <= LOG(REAL'HIGH)

    function ARCCOSH (X : in REAL) return REAL;
        -- Purpose:
        --         Returns inverse hyperbolic cosine of X
        -- Special values:
        --         ARCCOSH(1.0) = 0.0
        -- Domain:
        --         X >= 1.0
        -- Error conditions:
        --         Error if X < 1.0
        -- Range:
        --         ARCCOSH(X) >= 0.0
        -- Notes:
        --         a) The upper bound of the reachable range of ARCCOSH is
        --            approximately given by:   ARCCOSH(X) <= LOG(REAL'HIGH)

    function ARCTANH (X : in REAL) return REAL;
        -- Purpose:
        --         Returns inverse hyperbolic tangent of X
        -- Special values:
        --         ARCTANH(0.0) = 0.0
        -- Domain:
        --         ABS(X) < 1.0
        -- Error conditions:
        --         Error if ABS(X) >= 1.0
        -- Range:
        --         ARCTANH(X) is mathematically unbounded
        -- Notes:
        --         a) The reachable range of ARCTANH is approximately given by:
        --                ABS(ARCTANH(X)) < LOG(REAL'HIGH)

end  MATH_REAL;
```

3.2 MATH_COMPLEX

```
------------------------------------------------------------------------
--
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--
-- This source file is an essential part of IEEE Std 1076.2-1996, IEEE Standard
-- VHDL Mathematical Packages. This source file may not be copied, sold, or
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-- WARRANTY EXPRESS OR IMPLIED INCLUDING ANY WARRANTY OF MERCHANTABILITY
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-- arising out of the use thereof.
--
-- Title:       Standard VHDL Mathematical Packages (IEEE Std 1076.2-1996,
--              MATH_COMPLEX)
--
-- Library:     This package shall be compiled into a library
--              symbolically named IEEE.
--
-- Developers:  IEEE DASC VHDL Mathematical Packages Working Group
--
-- Purpose:     This package defines a standard for designers to use in
--              describing VHDL models that make use of common COMPLEX
--              constants and common COMPLEX mathematical functions and
--              operators.
--
-- Limitation:  The values generated by the functions in this package may
--              vary from platform to platform, and the precision of results
--              is only guaranteed to be the minimum required by IEEE Std 1076-
--              1993.
--
-- Notes:
--              No declarations or definitions shall be included in, or
--              excluded from, this package.
--              The "package declaration" defines the types, subtypes, and
--              declarations of MATH_COMPLEX.
--              The standard mathematical definition and conventional meaning
--              of the mathematical functions that are part of this standard
--              represent the formal semantics of the implementation of the
--              MATH_COMPLEX package declaration.  The purpose of the
--              MATH_COMPLEX package body is to provide a guideline for
--              implementations to verify their implementation of MATH_COMPLEX.
--              Tool developers may choose to implement the package body in
--              the most efficient manner available to them.
--
-- ------------------------------------------------------------------------
-- Version    : 1.5
-- Date       : 24 July 1996
-- ------------------------------------------------------------------------

use WORK.MATH_REAL.all;
package MATH_COMPLEX is
```

```
constant CopyRightNotice: STRING
  := "Copyright 1996 IEEE. All rights reserved.";

--
-- Type Definitions
--
type COMPLEX is
    record
            RE: REAL;        -- Real part
            IM: REAL;        -- Imaginary part
    end record;

subtype POSITIVE_REAL is REAL range 0.0 to REAL'HIGH;

subtype PRINCIPAL_VALUE is REAL range -MATH_PI to MATH_PI;

type COMPLEX_POLAR is
    record
            MAG: POSITIVE_REAL;    -- Magnitude
            ARG: PRINCIPAL_VALUE;  -- Angle in radians; -MATH_PI is illegal
    end record;

--
-- Constant Definitions
--
constant  MATH_CBASE_1: COMPLEX := COMPLEX'(1.0, 0.0);
constant  MATH_CBASE_J: COMPLEX := COMPLEX'(0.0, 1.0);
constant  MATH_CZERO: COMPLEX := COMPLEX'(0.0, 0.0);


--
-- Overloaded equality and inequality operators for COMPLEX_POLAR
-- (equality and inequality operators for COMPLEX are predefined)
--

function "=" ( L: in COMPLEX_POLAR;  R: in COMPLEX_POLAR ) return BOOLEAN;
    -- Purpose:
    --         Returns TRUE if L is equal to R and returns FALSE otherwise
    -- Special values:
    --         COMPLEX_POLAR'(0.0, X) = COMPLEX_POLAR'(0.0, Y) returns TRUE
    --         regardless of the value of X and Y.
    -- Domain:
    --         L in COMPLEX_POLAR and L.ARG /= -MATH_PI
    --         R in COMPLEX_POLAR and R.ARG /= -MATH_PI
    -- Error conditions:
    --         Error if L.ARG = -MATH_PI
    --         Error if R.ARG = -MATH_PI
    -- Range:
    --         "="(L,R) is either TRUE or FALSE
    -- Notes:
    --         None

function "/=" ( L: in COMPLEX_POLAR;  R: in COMPLEX_POLAR ) return BOOLEAN;
    -- Purpose:
    --         Returns TRUE if L is not equal to R and returns FALSE
    --         otherwise
    -- Special values:
    --         COMPLEX_POLAR'(0.0, X) /= COMPLEX_POLAR'(0.0, Y) returns
    --         FALSE regardless of the value of X and Y.
```

```
        -- Domain:
        --         L in COMPLEX_POLAR and L.ARG /= -MATH_PI
        --         R in COMPLEX_POLAR and R.ARG /= -MATH_PI
        -- Error conditions:
        --         Error if L.ARG = -MATH_PI
        --         Error if R.ARG = -MATH_PI
        -- Range:
        --         "/="(L,R) is either TRUE or FALSE
        -- Notes:
        --         None

    --
    -- Function Declarations
    --
    function CMPLX(X: in REAL;  Y: in REAL:= 0.0 ) return COMPLEX;
        -- Purpose:
        --         Returns COMPLEX number X + iY
        -- Special values:
        --         None
        -- Domain:
        --         X in REAL
        --         Y in REAL
        -- Error conditions:
        --         None
        -- Range:
        --         CMPLX(X,Y) is mathematically unbounded
        -- Notes:
        --         None

    function GET_PRINCIPAL_VALUE(X: in REAL ) return PRINCIPAL_VALUE;
        -- Purpose:
        --         Returns principal value of angle X; X in radians
        -- Special values:
        --         None
        -- Domain:
        --         X in REAL
        -- Error conditions:
        --         None
        -- Range:
        --         -MATH_PI < GET_PRINCIPAL_VALUE(X) <= MATH_PI
        -- Notes:
        --         None

    function COMPLEX_TO_POLAR(Z: in COMPLEX ) return COMPLEX_POLAR;
        -- Purpose:
        --         Returns principal value COMPLEX_POLAR of Z
        -- Special values:
        --         COMPLEX_TO_POLAR(MATH_CZERO) = COMPLEX_POLAR'(0.0, 0.0)
        --         COMPLEX_TO_POLAR(Z) = COMPLEX_POLAR'(ABS(Z.IM),
        --                               SIGN(Z.IM)*MATH_PI_OVER_2) if Z.RE = 0.0
        -- Domain:
        --         Z in COMPLEX
        -- Error conditions:
        --         None
        -- Range:
        --         result.MAG >= 0.0
        --         -MATH_PI < result.ARG <= MATH_PI
        -- Notes:
        --         None
```

```
function POLAR_TO_COMPLEX(Z: in COMPLEX_POLAR ) return COMPLEX;
    -- Purpose:
    --         Returns COMPLEX value of Z
    -- Special values:
    --         None
    -- Domain:
    --         Z in COMPLEX_POLAR and Z.ARG /= -MATH_PI
    -- Error conditions:
    --         Error if Z.ARG = -MATH_PI
    -- Range:
    --         POLAR_TO_COMPLEX(Z) is mathematically unbounded
    -- Notes:
    --         None

function "ABS"(Z: in COMPLEX ) return POSITIVE_REAL;
    -- Purpose:
    --         Returns absolute value (magnitude) of Z
    -- Special values:
    --         None
    -- Domain:
    --         Z in COMPLEX
    -- Error conditions:
    --         None
    -- Range:
    --         ABS(Z) is mathematically unbounded
    -- Notes:
    --         ABS(Z) = SQRT(Z.RE*Z.RE + Z.IM*Z.IM)

function "ABS"(Z: in COMPLEX_POLAR ) return POSITIVE_REAL;
    -- Purpose:
    --         Returns absolute value (magnitude) of Z
    -- Special values:
    --         None
    -- Domain:
    --         Z in COMPLEX_POLAR and Z.ARG /= -MATH_PI
    -- Error conditions:
    --         Error if Z.ARG = -MATH_PI
    -- Range:
    --         ABS(Z) >= 0.0
    -- Notes:
    --         ABS(Z) = Z.MAG

function ARG(Z: in COMPLEX ) return PRINCIPAL_VALUE;
    -- Purpose:
    --         Returns argument (angle) in radians of the principal
    --         value of Z
    -- Special values:
    --         ARG(Z) = 0.0 if Z.RE >= 0.0 and Z.IM = 0.0
    --         ARG(Z) = SIGN(Z.IM)*MATH_PI_OVER_2 if Z.RE = 0.0
    --         ARG(Z) = MATH_PI if Z.RE < 0.0        and Z.IM = 0.0
    -- Domain:
    --         Z in COMPLEX
    -- Error conditions:
    --         None
    -- Range:
    --         -MATH_PI < ARG(Z) <= MATH_PI
    -- Notes:
    --         ARG(Z) = ARCTAN(Z.IM, Z.RE)
```

```
    function ARG(Z: in COMPLEX_POLAR ) return PRINCIPAL_VALUE;
        -- Purpose:
        --         Returns argument (angle) in radians of the principal
        --         value of Z
        -- Special values:
        --         None
        -- Domain:
        --         Z in COMPLEX_POLAR and Z.ARG /= -MATH_PI
        -- Error conditions:
        --         Error if Z.ARG = -MATH_PI
        -- Range:
        --         -MATH_PI < ARG(Z) <= MATH_PI
        -- Notes:
        --         ARG(Z) = Z.ARG


    function "-" (Z: in COMPLEX ) return COMPLEX;
        -- Purpose:
        --         Returns unary minus of Z
        -- Special values:
        --         None
        -- Domain:
        --         Z in COMPLEX
        -- Error conditions:
        --         None
        -- Range:
        --         "-"(Z) is mathematically unbounded
        -- Notes:
        --         Returns -x -jy for Z= x + jy

    function "-" (Z: in COMPLEX_POLAR ) return COMPLEX_POLAR;
        -- Purpose:
        --         Returns principal value of unary minus of Z
        -- Special values:
        --         "-"(Z) = COMPLEX_POLAR'(Z.MAG, MATH_PI) if Z.ARG = 0.0
        -- Domain:
        --         Z in COMPLEX_POLAR and Z.ARG /= -MATH_PI
        -- Error conditions:
        --         Error if Z.ARG = -MATH_PI
        -- Range:
        --         result.MAG >= 0.0
        --         -MATH_PI < result.ARG <= MATH_PI
        -- Notes:
        --         Returns COMPLEX_POLAR'(Z.MAG, Z.ARG - SIGN(Z.ARG)*MATH_PI) if
        --                Z.ARG /= 0.0

    function CONJ (Z: in COMPLEX) return COMPLEX;
        -- Purpose:
        --         Returns complex conjugate of Z
        -- Special values:
        --         None
        -- Domain:
        --         Z in COMPLEX
        -- Error conditions:
        --         None
        -- Range:
        --         CONJ(Z) is mathematically unbounded
        -- Notes:
```

```
    --          Returns x -jy for Z= x + jy

function CONJ (Z: in COMPLEX_POLAR) return COMPLEX_POLAR;
    -- Purpose:
    --          Returns principal value of complex conjugate of Z
    -- Special values:
    --          CONJ(Z) = COMPLEX_POLAR'(Z.MAG, MATH_PI) if Z.ARG = MATH_PI
    -- Domain:
    --          Z in COMPLEX_POLAR and Z.ARG /= -MATH_PI
    -- Error conditions:
    --          Error if Z.ARG = -MATH_PI
    -- Range:
    --          result.MAG >= 0.0
    --          -MATH_PI < result.ARG <= MATH_PI
    -- Notes:
    --          Returns COMPLEX_POLAR'(Z.MAG, -Z.ARG) if Z.ARG /= MATH_PI

function SQRT(Z: in COMPLEX ) return COMPLEX;
    -- Purpose:
    --          Returns square root of Z with positive real part
    --          or, if the real part is zero, the one with nonnegative
    --          imaginary part
    -- Special values:
    --          SQRT(MATH_CZERO) = MATH_CZERO
    -- Domain:
    --          Z in COMPLEX
    -- Error conditions:
    --          None
    -- Range:
    --          SQRT(Z) is mathematically unbounded
    -- Notes:
    --          None

function SQRT(Z: in COMPLEX_POLAR ) return COMPLEX_POLAR;
    -- Purpose:
    --          Returns square root of Z with positive real part
    --          or, if the real part is zero, the one with nonnegative
    --          imaginary part
    -- Special values:
    --          SQRT(Z) = COMPLEX_POLAR'(0.0, 0.0) if Z.MAG = 0.0
    -- Domain:
    --          Z in COMPLEX_POLAR and Z.ARG /= -MATH_PI
    -- Error conditions:
    --          Error if Z.ARG = -MATH_PI
    -- Range:
    --          result.MAG >= 0.0
    --          -MATH_PI < result.ARG <= MATH_PI
    -- Notes:
    --          None

function EXP(Z: in COMPLEX ) return COMPLEX;
    -- Purpose:
    --          Returns exponential of Z
    -- Special values:
    --          EXP(MATH_CZERO) = MATH_CBASE_1
    --          EXP(Z) = -MATH_CBASE_1 if Z.RE = 0.0 and ABS(Z.IM) = MATH_PI
    --          EXP(Z) = SIGN(Z.IM)*MATH_CBASE_J if Z.RE = 0.0 and
    --                                              ABS(Z.IM) =  MATH_PI_OVER_2
    -- Domain:
```

```
        --         Z in COMPLEX
        -- Error conditions:
        --         None
        -- Range:
        --         EXP(Z) is mathematically unbounded
        -- Notes:
        --         None


    function EXP(Z: in COMPLEX_POLAR ) return COMPLEX_POLAR;
        -- Purpose:
        --         Returns principal value of exponential of Z
        -- Special values:
        --         EXP(Z) = COMPLEX_POLAR'(1.0, 0.0) if Z.MAG =0.0 and
        --                                                        Z.ARG = 0.0
        --         EXP(Z) = COMPLEX_POLAR'(1.0, MATH_PI) if Z.MAG = MATH_PI and
        --                                          ABS(Z.ARG) = MATH_PI_OVER_2
        --         EXP(Z) = COMPLEX_POLAR'(1.0, MATH_PI_OVER_2) if
        --                                          Z.MAG = MATH_PI_OVER_2 and
        --                                          Z.ARG = MATH_PI_OVER_2
        --         EXP(Z) = COMPLEX_POLAR'(1.0, -MATH_PI_OVER_2) if
        --                                          Z.MAG = MATH_PI_OVER_2 and
        --                                          Z.ARG = -MATH_PI_OVER_2
        -- Domain:
        --         Z in COMPLEX_POLAR and Z.ARG /= -MATH_PI
        -- Error conditions:
        --         Error if Z.ARG = -MATH_PI
        -- Range:
        --         result.MAG >= 0.0
        --         -MATH_PI < result.ARG <= MATH_PI
        -- Notes:
        --         None

    function LOG(Z: in COMPLEX ) return COMPLEX;
        -- Purpose:
        --         Returns natural logarithm of Z
        -- Special values:
        --         LOG(MATH_CBASE_1) = MATH_CZERO
        --         LOG(-MATH_CBASE_1) = COMPLEX'(0.0, MATH_PI)
        --         LOG(MATH_CBASE_J) = COMPLEX'(0.0, MATH_PI_OVER_2)
        --         LOG(-MATH_CBASE_J) = COMPLEX'(0.0, -MATH_PI_OVER_2)
        --         LOG(Z) = MATH_CBASE_1 if Z = COMPLEX'(MATH_E, 0.0)
        -- Domain:
        --         Z in COMPLEX and ABS(Z) /= 0.0
        -- Error conditions:
        --         Error if ABS(Z) = 0.0
        -- Range:
        --         LOG(Z) is mathematically unbounded
        -- Notes:
        --         None

    function LOG2(Z: in COMPLEX ) return COMPLEX;
        -- Purpose:
        --         Returns logarithm base 2 of Z
        -- Special values:
        --         LOG2(MATH_CBASE_1) = MATH_CZERO
        --         LOG2(Z) = MATH_CBASE_1 if Z = COMPLEX'(2.0, 0.0)
        -- Domain:
```

```
    --         Z in COMPLEX and ABS(Z) /= 0.0
    -- Error conditions:
    --         Error if ABS(Z) = 0.0
    -- Range:
    --         LOG2(Z) is mathematically unbounded
    -- Notes:
    --         None

function LOG10(Z: in COMPLEX ) return COMPLEX;
    -- Purpose:
    --         Returns logarithm base 10 of Z
    -- Special values:
    --         LOG10(MATH_CBASE_1) = MATH_CZERO
    --         LOG10(Z) = MATH_CBASE_1 if Z = COMPLEX'(10.0, 0.0)
    -- Domain:
    --         Z in COMPLEX and ABS(Z) /= 0.0
    -- Error conditions:
    --         Error if ABS(Z) = 0.0
    -- Range:
    --         LOG10(Z) is mathematically unbounded
    -- Notes:
    --         None

function LOG(Z: in COMPLEX_POLAR ) return COMPLEX_POLAR;
    -- Purpose:
    --         Returns principal value of natural logarithm of Z
    -- Special values:
    --         LOG(Z) = COMPLEX_POLAR'(0.0, 0.0) if Z.MAG = 1.0 and
    --                                              Z.ARG = 0.0
    --         LOG(Z) = COMPLEX_POLAR'(MATH_PI, MATH_PI_OVER_2) if
    --                                 Z.MAG = 1.0 and Z.ARG = MATH_PI
    --         LOG(Z) = COMPLEX_POLAR'(MATH_PI_OVER_2, MATH_PI_OVER_2) if
    --                                 Z.MAG = 1.0 and  Z.ARG = MATH_PI_OVER_2
    --         LOG(Z) = COMPLEX_POLAR'(MATH_PI_OVER_2, -MATH_PI_OVER_2) if
    --                                 Z.MAG = 1.0 and  Z.ARG = -MATH_PI_OVER_2
    --         LOG(Z) = COMPLEX_POLAR'(1.0, 0.0) if Z.MAG = MATH_E and
    --                                              Z.ARG = 0.0
    -- Domain:
    --         Z in COMPLEX_POLAR and Z.ARG /= -MATH_PI
    --         Z.MAG /= 0.0
    -- Error conditions:
    --         Error if Z.ARG = -MATH_PI
    --         Error if Z.MAG = 0.0
    -- Range:
    --         result.MAG >= 0.0
    --         -MATH_PI < result.ARG <= MATH_PI
    -- Notes:
    --         None

function LOG2(Z: in COMPLEX_POLAR ) return COMPLEX_POLAR;
    -- Purpose:
    --         Returns principal value of logarithm base 2 of Z
    -- Special values:
    --         LOG2(Z) = COMPLEX_POLAR'(0.0, 0.0) if Z.MAG = 1.0 and
    --                                               Z.ARG = 0.0
    --         LOG2(Z) = COMPLEX_POLAR'(1.0, 0.0) if Z.MAG = 2.0 and
    --                                              Z.ARG = 0.0
    -- Domain:
    --         Z in COMPLEX_POLAR and Z.ARG /= -MATH_PI
```

```
        --         Z.MAG /= 0.0
        -- Error conditions:
        --         Error if Z.ARG = -MATH_PI
        --         Error if Z.MAG = 0.0
        -- Range:
        --         result.MAG >= 0.0
        --         -MATH_PI < result.ARG <= MATH_PI
        -- Notes:
        --        None

    function LOG10(Z: in COMPLEX_POLAR ) return COMPLEX_POLAR;
        -- Purpose:
        --         Returns principal value of logarithm base 10 of Z
        -- Special values:
        --         LOG10(Z) = COMPLEX_POLAR'(0.0, 0.0) if Z.MAG = 1.0 and
        --                                                 Z.ARG = 0.0
        --         LOG10(Z) = COMPLEX_POLAR'(1.0, 0.0) if Z.MAG = 10.0 and
        --                                                 Z.ARG = 0.0
        -- Domain:
        --         Z in COMPLEX_POLAR and Z.ARG /= -MATH_PI
        --         Z.MAG /= 0.0
        -- Error conditions:
        --         Error if Z.ARG = -MATH_PI
        --         Error if Z.MAG = 0.0
        -- Range:
        --         result.MAG >= 0.0
        --         -MATH_PI < result.ARG <= MATH_PI
        -- Notes:
        --         None

    function LOG(Z: in COMPLEX; BASE: in REAL) return COMPLEX;
        -- Purpose:
        --         Returns logarithm base BASE of Z
        -- Special values:
        --         LOG(MATH_CBASE_1, BASE) = MATH_CZERO
        --         LOG(Z,BASE) = MATH_CBASE_1 if Z = COMPLEX'(BASE, 0.0)
        -- Domain:
        --         Z in COMPLEX and ABS(Z) /= 0.0
        --         BASE > 0.0
        --         BASE /= 1.0
        -- Error conditions:
        --         Error if ABS(Z) = 0.0
        --         Error if BASE <= 0.0
        --         Error if BASE = 1.0
        -- Range:
        --         LOG(Z,BASE) is mathematically unbounded
        -- Notes:
        --         None

    function LOG(Z: in COMPLEX_POLAR; BASE: in REAL ) return COMPLEX_POLAR;
        -- Purpose:
        --         Returns principal value of logarithm base BASE of Z
        -- Special values:
        --         LOG(Z, BASE) = COMPLEX_POLAR'(0.0, 0.0) if Z.MAG = 1.0 and
        --                                                    Z.ARG = 0.0
        --         LOG(Z, BASE) = COMPLEX_POLAR'(1.0, 0.0) if Z.MAG = BASE and
        --                                                    Z.ARG = 0.0
        -- Domain:
        --         Z in COMPLEX_POLAR and Z.ARG /= -MATH_PI
```

```
    --          Z.MAG /= 0.0
    --          BASE > 0.0
    --          BASE /= 1.0
    -- Error conditions:
    --          Error if Z.ARG = -MATH_PI
    --          Error if Z.MAG = 0.0
    --          Error if BASE <= 0.0
    --          Error if BASE = 1.0
    -- Range:
    --          result.MAG >= 0.0
    --          -MATH_PI < result.ARG <= MATH_PI
    -- Notes:
    --          None

function SIN (Z : in COMPLEX ) return COMPLEX;
    -- Purpose:
    --          Returns sine of Z
    -- Special values:
    --          SIN(MATH_CZERO) = MATH_CZERO
    --          SIN(Z) = MATH_CZERO if Z = COMPLEX'(MATH_PI, 0.0)
    -- Domain:
    --          Z in COMPLEX
    -- Error conditions:
    --          None
    -- Range:
    --          ABS(SIN(Z)) <= SQRT(SIN(Z.RE)*SIN(Z.RE) +
    --                                          SINH(Z.IM)*SINH(Z.IM))
    -- Notes:
    --          None

function SIN (Z : in COMPLEX_POLAR ) return COMPLEX_POLAR;
    -- Purpose:
    --          Returns principal value of sine of Z
    -- Special values:
    --          SIN(Z) = COMPLEX_POLAR'(0.0, 0.0) if Z.MAG = 0.0 and
    --                                              Z.ARG = 0.0
    --          SIN(Z) = COMPLEX_POLAR'(0.0, 0.0) if Z.MAG = MATH_PI and
    --                                              Z.ARG = 0.0
    -- Domain:
    --          Z in COMPLEX_POLAR and Z.ARG /= -MATH_PI
    -- Error conditions:
    --          Error if Z.ARG = -MATH_PI
    -- Range:
    --          result.MAG >= 0.0
    --          -MATH_PI < result.ARG <= MATH_PI
    -- Notes:
    --          None

function  COS (Z : in COMPLEX ) return COMPLEX;
    -- Purpose:
    --          Returns cosine of Z
    -- Special values:
    --          COS(Z) = MATH_CZERO if Z = COMPLEX'(MATH_PI_OVER_2, 0.0)
    --          COS(Z) = MATH_CZERO if Z = COMPLEX'(-MATH_PI_OVER_2, 0.0)
    -- Domain:
    --          Z in COMPLEX
    -- Error conditions:
    --          None
    -- Range:
```

```
        --         ABS(COS(Z)) <= SQRT(COS(Z.RE)*COS(Z.RE) +
        --                                          SINH(Z.IM)*SINH(Z.IM))
        -- Notes:
        --         None


    function  COS (Z : in COMPLEX_POLAR ) return COMPLEX_POLAR;
        -- Purpose:
        --         Returns principal value of cosine of Z
        -- Special values:
        --         COS(Z) = COMPLEX_POLAR'(0.0, 0.0) if Z.MAG = MATH_PI_OVER_2
        --                                                 and Z.ARG = 0.0
        --         COS(Z) = COMPLEX_POLAR'(0.0, 0.0) if Z.MAG = MATH_PI_OVER_2
        --                                                 and Z.ARG = MATH_PI
        -- Domain:
        --         Z in COMPLEX_POLAR and Z.ARG /= -MATH_PI
        -- Error conditions:
        --         Error if Z.ARG = -MATH_PI
        -- Range:
        --         result.MAG >= 0.0
        --         -MATH_PI < result.ARG <= MATH_PI
        -- Notes:
        --         None

    function SINH (Z : in COMPLEX ) return COMPLEX;
        -- Purpose:
        --         Returns hyperbolic sine of Z
        -- Special values:
        --         SINH(MATH_CZERO) = MATH_CZERO
        --         SINH(Z) = MATH_CZERO if Z.RE = 0.0 and Z.IM = MATH_PI
        --         SINH(Z) = MATH_CBASE_J if Z.RE = 0.0 and
        --                                                 Z.IM = MATH_PI_OVER_2
        --         SINH(Z) = -MATH_CBASE_J if Z.RE = 0.0 and
        --                                                 Z.IM = -MATH_PI_OVER_2
        -- Domain:
        --         Z in COMPLEX
        -- Error conditions:
        --         None
        -- Range:
        --         ABS(SINH(Z)) <= SQRT(SINH(Z.RE)*SINH(Z.RE) +
        --                                          SIN(Z.IM)*SIN(Z.IM))
        -- Notes:
        --         None

    function SINH (Z : in COMPLEX_POLAR ) return COMPLEX_POLAR;
        -- Purpose:
        --         Returns principal value of hyperbolic sine of Z
        -- Special values:
        --         SINH(Z) = COMPLEX_POLAR'(0.0, 0.0) if Z.MAG = 0.0 and
        --                                                 Z.ARG = 0.0
        --         SINH(Z) = COMPLEX_POLAR'(0.0, 0.0) if Z.MAG = MATH_PI and
        --                                                 Z.ARG = MATH_PI_OVER_2
        --         SINH(Z) = COMPLEX_POLAR'(1.0, MATH_PI_OVER_2) if Z.MAG =
        --                         MATH_PI_OVER_2 and Z.ARG = MATH_PI_OVER_2
        --         SINH(Z) = COMPLEX_POLAR'(1.0, -MATH_PI_OVER_2) if Z.MAG =
        --                         MATH_PI_OVER_2 and Z.ARG = -MATH_PI_OVER_2
        -- Domain:
        --         Z in COMPLEX_POLAR and Z.ARG /= -MATH_PI
        -- Error conditions:
```

```
    --         Error if Z.ARG = -MATH_PI
    -- Range:
    --         result.MAG >= 0.0
    --         -MATH_PI < result.ARG <= MATH_PI
    -- Notes:
    --         None

function COSH (Z : in COMPLEX ) return COMPLEX;
    -- Purpose:
    --         Returns hyperbolic cosine of Z
    -- Special values:
    --         COSH(MATH_CZERO) = MATH_CBASE_1
    --         COSH(Z) = -MATH_CBASE_1 if Z.RE = 0.0 and Z.IM = MATH_PI
    --         COSH(Z) = MATH_CZERO if Z.RE = 0.0 and Z.IM = MATH_PI_OVER_2
    --         COSH(Z) = MATH_CZERO if Z.RE = 0.0 and Z.IM = -MATH_PI_OVER_2
    -- Domain:
    --         Z in COMPLEX
    -- Error conditions:
    --         None
    -- Range:
    --         ABS(COSH(Z)) <= SQRT(SINH(Z.RE)*SINH(Z.RE) +
    --                                        COS(Z.IM)*COS(Z.IM))
    -- Notes:
    --         None


function COSH (Z : in COMPLEX_POLAR ) return COMPLEX_POLAR;
    -- Purpose:
    --         Returns principal value of hyperbolic cosine of Z
    -- Special values:
    --         COSH(Z) = COMPLEX_POLAR'(1.0, 0.0) if Z.MAG = 0.0 and
    --                                               Z.ARG = 0.0
    --         COSH(Z) = COMPLEX_POLAR'(1.0, MATH_PI) if Z.MAG = MATH_PI and
    --                                               Z.ARG = MATH_PI_OVER_2
    --         COSH(Z) = COMPLEX_POLAR'(0.0, 0.0) if Z.MAG =
    --                         MATH_PI_OVER_2 and Z.ARG = MATH_PI_OVER_2
    --         COSH(Z) = COMPLEX_POLAR'(0.0, 0.0) if Z.MAG =
    --                         MATH_PI_OVER_2 and Z.ARG = -MATH_PI_OVER_2
    -- Domain:
    --         Z in COMPLEX_POLAR and Z.ARG /= -MATH_PI
    -- Error conditions:
    --         Error if Z.ARG = -MATH_PI
    -- Range:
    --         result.MAG >= 0.0
    --         -MATH_PI < result.ARG <= MATH_PI
    -- Notes:
    --         None

--
-- Arithmetic Operators
--

function "+" ( L: in COMPLEX;  R: in COMPLEX ) return COMPLEX;
    -- Purpose:
    --         Returns arithmetic addition of L and R
    -- Special values:
    --         None
    -- Domain:
    --         L in COMPLEX
```

```
        --         R in COMPLEX
        -- Error conditions:
        --         None
        -- Range:
        --         "+"(Z) is mathematically unbounded
        -- Notes:
        --         None

    function "+" ( L: in REAL;     R: in COMPLEX ) return COMPLEX;
        -- Purpose:
        --         Returns arithmetic addition of L and R
        -- Special values:
        --         None
        -- Domain:
        --         L in REAL
        --         R in COMPLEX
        -- Error conditions:
        --         None
        -- Range:
        --         "+"(Z) is mathematically unbounded
        -- Notes:
        --         None

    function "+" ( L: in COMPLEX;  R: in REAL )    return COMPLEX;
        -- Purpose:
        --         Returns arithmetic addition of L and R
        -- Special values:
        --         None
        -- Domain:
        --         L in COMPLEX
        --         R in REAL
        -- Error conditions:
        --         None
        -- Range:
        --         "+"(Z) is mathematically unbounded
        -- Notes:
        --         None

    function "+" ( L: in COMPLEX_POLAR; R: in COMPLEX_POLAR)
                                                    return COMPLEX_POLAR;
        -- Purpose:
        --         Returns arithmetic addition of L and R
        -- Special values:
        --         None
        -- Domain:
        --         L in COMPLEX_POLAR and L.ARG /= -MATH_PI
        --         R in COMPLEX_POLAR and R.ARG /= -MATH_PI
        -- Error conditions:
        --         Error if L.ARG = -MATH_PI
        --         Error if R.ARG = -MATH_PI
        -- Range:
        --         result.MAG >= 0.0
        --         -MATH_PI < result.ARG <= MATH_PI
        -- Notes:
        --         None


    function "+" ( L: in REAL;  R: in COMPLEX_POLAR) return COMPLEX_POLAR;
        -- Purpose:
```

```
    --         Returns arithmetic addition of L and R
    -- Special values:
    --         None
    -- Domain:
    --         L in REAL
    --         R in COMPLEX_POLAR and R.ARG /= -MATH_PI
    -- Error conditions:
    --         Error if R.ARG = -MATH_PI
    -- Range:
    --         result.MAG >= 0.0
    --         -MATH_PI < result.ARG <= MATH_PI
    -- Notes:
    --         None

function "+" ( L: in COMPLEX_POLAR;  R: in REAL) return COMPLEX_POLAR;
    -- Purpose:
    --         Returns arithmetic addition of L and R
    -- Special values:
    --         None
    -- Domain:
    --         L in COMPLEX_POLAR and L.ARG /= -MATH_PI
    --         R in REAL
    -- Error conditions:
    --         Error if L.ARG = -MATH_PI
    -- Range:
    --         result.MAG >= 0.0
    --         -MATH_PI < result.ARG <= MATH_PI
    -- Notes:
    --         None

function "-" ( L: in COMPLEX;  R: in COMPLEX ) return COMPLEX;
    -- Purpose:
    --         Returns arithmetic subtraction of L minus R
    -- Special values:
    --         None
    -- Domain:
    --         L in COMPLEX
    --         R in COMPLEX
    -- Error conditions:
    --         None
    -- Range:
    --         "-"(Z) is mathematically unbounded
    -- Notes:
    --         None

function "-" ( L: in REAL;     R: in COMPLEX ) return COMPLEX;
    -- Purpose:
    --         Returns arithmetic subtraction of L minus R
    -- Special values:
    --         None
    -- Domain:
    --         L in REAL
    --         R in COMPLEX
    -- Error conditions:
    --         None
    -- Range:
    --         "-"(Z) is mathematically unbounded
    -- Notes:
    --         None
```

```
    function "-" ( L: in COMPLEX;  R: in REAL )    return COMPLEX;
        -- Purpose:
        --         Returns arithmetic subtraction of L minus R
        -- Special values:
        --         None
        -- Domain:
        --         L in COMPLEX
        --         R in REAL
        -- Error conditions:
        --         None
        -- Range:
        --         "-"(Z) is mathematically unbounded
        -- Notes:
        --         None

    function "-" ( L: in COMPLEX_POLAR; R: in COMPLEX_POLAR)
                                                    return COMPLEX_POLAR;
        -- Purpose:
        --         Returns arithmetic subtraction of L minus R
        -- Special values:
        --         None
        -- Domain:
        --         L in COMPLEX_POLAR and L.ARG /= -MATH_PI
        --         R in COMPLEX_POLAR and R.ARG /= -MATH_PI
        -- Error conditions:
        --         Error if L.ARG = -MATH_PI
        --         Error if R.ARG = -MATH_PI
        -- Range:
        --         result.MAG >= 0.0
        --         -MATH_PI < result.ARG <= MATH_PI
        -- Notes:
        --         None

    function "-" ( L: in REAL;  R: in COMPLEX_POLAR) return COMPLEX_POLAR;
        -- Purpose:
        --         Returns arithmetic subtraction of L minus R
        -- Special values:
        --         None
        -- Domain:
        --         L in REAL
        --         R in COMPLEX_POLAR and R.ARG /= -MATH_PI
        -- Error conditions:
        --         Error if R.ARG = -MATH_PI
        -- Range:
        --         result.MAG >= 0.0
        --         -MATH_PI < result.ARG <= MATH_PI
        -- Notes:
        --         None


    function "-" ( L: in COMPLEX_POLAR;  R: in REAL) return COMPLEX_POLAR;
        -- Purpose:
        --         Returns arithmetic subtraction of L minus R
        -- Special values:
        --         None
        -- Domain:
        --         L in COMPLEX_POLAR and L.ARG /= -MATH_PI
        --         R in REAL
```

```
    -- Error conditions:
    --         Error if L.ARG = -MATH_PI
    -- Range:
    --         result.MAG >= 0.0
    --         -MATH_PI < result.ARG <= MATH_PI
    -- Notes:
    --         None

function "*" ( L: in COMPLEX;  R: in COMPLEX ) return COMPLEX;
    -- Purpose:
    --         Returns arithmetic multiplication of L and R
    -- Special values:
    --         None
    -- Domain:
    --         L in COMPLEX
    --         R in COMPLEX
    -- Error conditions:
    --         None
    -- Range:
    --         "*"(Z) is mathematically unbounded
    -- Notes:
    --         None

function "*" ( L: in REAL;  R: in COMPLEX ) return COMPLEX;
    -- Purpose:
    --         Returns arithmetic multiplication of L and R
    -- Special values:
    --         None
    -- Domain:
    --         L in REAL
    --         R in COMPLEX
    -- Error conditions:
    --         None
    -- Range:
    --         "*"(Z) is mathematically unbounded
    -- Notes:
    --         None

function "*" ( L: in COMPLEX;  R: in REAL )  return COMPLEX;
    -- Purpose:
    --         Returns arithmetic multiplication of L and R
    -- Special values:
    --         None
    -- Domain:
    --         L in COMPLEX
    --         R in REAL
    -- Error conditions:
    --         None
    -- Range:
    --         "*"(Z) is mathematically unbounded
    -- Notes:
    --         None

function "*" ( L: in COMPLEX_POLAR; R: in COMPLEX_POLAR)
                                                    return COMPLEX_POLAR;
    -- Purpose:
    --         Returns arithmetic multiplication of L and R
    -- Special values:
    --         None
```

```
        -- Domain:
        --         L in COMPLEX_POLAR and L.ARG /= -MATH_PI
        --         R in COMPLEX_POLAR and R.ARG /= -MATH_PI
        -- Error conditions:
        --         Error if L.ARG = -MATH_PI
        --         Error if R.ARG = -MATH_PI
        -- Range:
        --         result.MAG >= 0.0
        --         -MATH_PI < result.ARG <= MATH_PI
        -- Notes:
        --         None

    function "*" ( L: in REAL;  R: in COMPLEX_POLAR) return COMPLEX_POLAR;
        -- Purpose:
        --         Returns arithmetic multiplication of L and R
        -- Special values:
        --         None
        -- Domain:
        --         L in REAL
        --         R in COMPLEX_POLAR and R.ARG /= -MATH_PI
        -- Error conditions:
        --         Error if R.ARG = -MATH_PI
        -- Range:
        --         result.MAG >= 0.0
        --         -MATH_PI < result.ARG <= MATH_PI
        -- Notes:
        --         None

    function "*" ( L: in COMPLEX_POLAR;  R: in REAL) return COMPLEX_POLAR;
        -- Purpose:
        --         Returns arithmetic multiplication of L and R
        -- Special values:
        --         None
        -- Domain:
        --         L in COMPLEX_POLAR and L.ARG /= -MATH_PI
        --         R in REAL
        -- Error conditions:
        --         Error if L.ARG = -MATH_PI
        -- Range:
        --         result.MAG >= 0.0
        --         -MATH_PI < result.ARG <= MATH_PI
        -- Notes:
        --         None


    function "/" ( L: in COMPLEX;  R: in COMPLEX ) return COMPLEX;
        -- Purpose:
        --         Returns arithmetic division of L by R
        -- Special values:
        --         None
        -- Domain:
        --         L in COMPLEX
        --         R in COMPLEX and R /= MATH_CZERO
        -- Error conditions:
        --         Error if R = MATH_CZERO
        -- Range:
        --         "/"(Z) is mathematically unbounded
        -- Notes:
        --         None
```

```
function "/" ( L: in REAL;  R: in COMPLEX ) return COMPLEX;
    -- Purpose:
    --         Returns arithmetic division of L by R
    -- Special values:
    --         None
    -- Domain:
    --         L in REAL
    --         R in COMPLEX and R /= MATH_CZERO
    -- Error conditions:
    --         Error if R = MATH_CZERO
    -- Range:
    --         "/"(Z) is mathematically unbounded
    -- Notes:
    --         None

function "/" ( L: in COMPLEX;  R: in REAL )    return COMPLEX;
    -- Purpose:
    --         Returns arithmetic division of L by R
    -- Special values:
    --         None
    -- Domain:
    --         L in COMPLEX
    --         R in REAL and R /= 0.0
    -- Error conditions:
    --         Error if R = 0.0
    -- Range:
    --         "/"(Z) is mathematically unbounded
    -- Notes:
    --         None

function "/" ( L: in COMPLEX_POLAR; R: in COMPLEX_POLAR)
                                                    return COMPLEX_POLAR;
    -- Purpose:
    --         Returns arithmetic division of L by R
    -- Special values:
    --         None
    -- Domain:
    --         L in COMPLEX_POLAR and L.ARG /= -MATH_PI
    --         R in COMPLEX_POLAR and R.ARG /= -MATH_PI
    --         R.MAG > 0.0
    -- Error conditions:
    --         Error if R.MAG <= 0.0
    --         Error if L.ARG = -MATH_PI
    --         Error if R.ARG = -MATH_PI
    -- Range:
    --         result.MAG >= 0.0
    --         -MATH_PI < result.ARG <= MATH_PI
    -- Notes:
    --         None

function "/" ( L: in REAL;  R: in COMPLEX_POLAR) return COMPLEX_POLAR;
    -- Purpose:
    --         Returns arithmetic division of L by R
    -- Special values:
    --         None
    -- Domain:
    --         L in REAL
    --         R in COMPLEX_POLAR and R.ARG /= -MATH_PI
```

```
        --          R.MAG > 0.0
        -- Error conditions:
        --          Error if R.MAG <= 0.0
        --          Error if R.ARG = -MATH_PI
        -- Range:
        --          result.MAG >= 0.0
        --          -MATH_PI < result.ARG <= MATH_PI
        -- Notes:
        --          None

    function "/" ( L: in COMPLEX_POLAR;  R: in REAL) return COMPLEX_POLAR;
        -- Purpose:
        --          Returns arithmetic division of L by R
        -- Special values:
        --          None
        -- Domain:
        --          L in COMPLEX_POLAR and L.ARG /= -MATH_PI
        --          R /= 0.0
        -- Error conditions:
        --          Error if L.ARG = -MATH_PI
        --          Error if R = 0.0
        -- Range:
        --          result.MAG >= 0.0
        --          -MATH_PI < result.ARG <= MATH_PI
        -- Notes:
        --          None
end  MATH_COMPLEX;
```

Annex A

(informative)

Using the MATH_REAL and MATH_COMPLEX packages

The information in this annex is intended to be a brief guide to using the MATH_REAL and MATH_COMPLEX packages, but it is not a normative part of the standard. As a standard, this set of packages provides a means of building models that interoperate and port to different tools, provided that the user adheres to a set of guidelines required by the standard and the strict typing imposed by the VHDL language (IEEE Std 1076-1993).

A.1 Predefined data types, operators, and precision for MATH_REAL

The MATH_REAL package is built on top of the standard data type (REAL), operators, and precision requirements for floating-point operations defined in IEEE Std 1076-1993 (STD.STANDARD).

A.2 Use of constants in MATH_REAL

The precision of the constants declared in the package covers the IEEE Std 754-1985 double-precision accuracy (i.e., 16 digits) in order to allow implementations to support a greater accuracy than the one required for compliance (6 digits). However, models written based on the extra accuracy may not be supported by some tools, since only 6 digits of accuracy are guaranteed by this standard.

A.3 Use and constraints of pseudo-random number generator in MATH_REAL

The pseudo-random number generator provided with the package is platform independent. In order to generate a chain of pseudo-random numbers, the seed values shall be set only in the first call to the function. A different chain of numbers is started every time the seed values are set. If multiple chains of pseudo-random numbers are required, then different sets of seed values have to be used for every chain.

A.4 Precision across different platforms

It is important to note that the math package results may be slightly different on different platforms because of variations in hardware support for floating-point arithmetic. These differences might not be immediately apparent to the average VHDL user. However, since most workstations use the IEEE Std 754-1985 floating-point format, the variations are likely to be limited in practice.

A.5 Handling of overflow/underflow conditions

The detection of underflow/overflow is optional and implementation dependent.

A.6 Testbench for the packages

A non-exhaustive testbench for the packages MATH_REAL and MATH_COMPLEX can be found in the following Web location: *http://vhdl.org/vi/math/testbench/*.

A.7 Compatibility with VHDL analog standard

At the current time, the working group for the analog VHDL standard is contemplating the modification of the precision of the real data type. This version of the MATH_REAL and MATH_COMPLEX packages is based on the data types defined in IEEE Std 1076-1993. In the future, when IEEE Std 1076 changes, the plan is to review this standard and modify as necessary for compliance with IEEE Std 1076.

A.8 Overloading side effect

Note that there is a side effect of adding functions for COMPLEX_POLAR when numerical expressions are used. Numerical arguments for these functions are ambiguous, unless a qualifier is used to disambiguate them. For example, SIN((0.0, 0.0)) is ambiguous. One has to say either SIN(COMPLEX'(0.0, 0.0)) or SIN(COMPLEX_POLAR'(0.0, 0.0)).

A.9 Synthesizability of functions

Synthesizability of the functions part of this standard is beyond the scope of this standard.

Annex B

(informative)

Package bodies

The information in this annex is intended to provide a guideline for implementors and is not a normative part of this standard, but suggests ways in which implementors may implement the MATH_REAL and MATH_COMPLEX packages. Implementors may also use this information as a guideline to verify their implementation of this standard.

The standard package bodies (subclauses B.1 and B.2) are on the diskette that is included with this standard. These standard package bodies are an official part of this standard. Please consult this diskette for the contents of these standard package bodies and for the standard package declarations as well.

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