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Edition 2.0 2015-04

TECHNICAL REPORT

Short-circuit currents – Calculation of effects –
Part 2: Examples of calculation

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FOREWORD

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IEC TR 60865-2, which is a technical report, has been prepared by IEC technical committee 73: Short-circuit currents.

This second edition cancels and replaces the first edition published in 1994. This edition constitutes a technical revision.

This edition includes the following significant technical changes with respect to the previous edition.

a) The determinations for auto reclosure together with rigid conductors have been revised.

b) The configurations in cases of flexible conductor arrangements have been changed.

c) The influence of mid-span droppers to the span has been included.

d) For vertical cable-connection the displacement and the tensile force onto the lower fixing point may be calculated now.

e) Additional recommendations for foundation loads due to tensile forces have been added.

f) The subclause for determination of the thermal equivalent short-circuits current has been deleted (is part of IEC 60909-0:2001 now).

g) The standard IEC 60865-1:2011 has been reorganized and some of the symbols have been changed to follow the conceptual characteristic of international standards.

The text of this technical report is based on the following documents:

Enquiry draft	Report on voting
73/168/DTR	73/173/RVC

Full information on the voting for the approval of this technical report can be found in the report on voting indicated in the above table.

This publication has been drafted in accordance with the ISO/IEC Directives, Part 2.

A list of all parts in the IEC 60865 series, published under the general title *Short-circuit currents – Calculations of effects*, can be found on the IEC website.

The committee has decided that the contents of this publication will remain unchanged until the stability date indicated on the IEC website under "http://webstore.iec.ch" in the data related to the specific publication. At this date, the publication will be

- reconfirmed,
- withdrawn,
- replaced by a revised edition, or
- amended.

A bilingual version of this publication may be issued at a later date.

SHORT-CIRCUIT CURRENTS – CALCULATION OF EFFECTS

Part 2: Examples of calculation

1 Scope

The object of this part of IEC 60865, which is a Technical Report, is to show the application of procedures for the calculation of mechanical and thermal effects due to short circuits as presented in IEC 60865-1. Thus, this technical report is an addition to IEC 60865-1. It does not, however, change the basis for standardized procedures given in that publication.

The following points should particularly be noted:

a) The examples in this Technical Report illustrate how to make the calculations according to IEC 60865-1 in a simplified and easy-to-follow manner. They are not intended as a check for computer programs.
b) The numbers in parentheses at the end of the equations refer to the equations in IEC 60865-1:2011.
c) The system voltages are referred to as nominal voltages.
d) The results are rounded to three significant digits.
e) Short-circuit effects appear as exceptional load in addition to the mechanical loads of the normal operation of a switchgear. In the following examples with rigid conductors, a possible static preloading is therefore calculated too. Depending on whether it concerns the load of the normal operation or the load during the short-circuit different safety factors come to use. The height of these factors has been chosen typically and is recommended for the use. However, other safety factors may be necessary depending on the safety concept.

2 Normative references

IEC 60865-1:2011, *Short-Circuit Currents – Calculation of Effects – Part 1: Definitions and calculation methods*

IEC 60909-0:2001, *Short-circuit currents in three-phase AC systems – Part 0: Calculation of currents*

3 Symbols and units

For symbols and units, reference is made to IEC 60865-1:2011.

In addition, the following symbols are used:

$F_{\mathrm{str,k}}$	Dead load (characteristic value)	N
$F_{\mathrm{str,d}}$	Dead load (design value)	N
$F_{\mathrm{st,r,d}}$	Force on support of rigid conductors (design value) due to dead load	N
h_{S}, h_{I}	Height of the substructure, insulator	m
H_{s}	Horizontal component of the force at the lower fixing point of one sub-conductor of a dropper	N

$J_{st,m}$	Second moment of main conductor area with respect to the direction of the dead load	m^4
I_k	Steady-state short-circuit current (r.m.s) according to IEC 60909-0	A
l_{eff}	Effective length of a span	m
l_f	Form factor of a span	m
l_h	Extend of one head armature and clamp	m
m, n	Factor for heat effect of the d.c. component and a.c. component	1
$M_{S,d}$, $M_{I,d}$	Bending moment on the bottom on the substructure, insulator (design value)	Nm
V_s	Vertical component of the force at the upper fixing point of one sub-conductor of a dropper	N
$W_{st,m}$	Section modulus of main conductor with respect to the direction of the dead load	m^3
γ_F	Partial safety factor for action	1
γ_M	Partial safety factor for material property	1
$\sigma_{st,m,d}$	Bending stress caused by the dead load (design value)	N/m^2
$\sigma_{st,m,k}$	Bending stress caused by the dead load (characteristic value)	N/m^2

4 Example 1 – Mechanical effects on a 10 kV arrangement with single rigid conductors

4.1 General

The basis for the calculation in this example is a three-phase 10 kV busbar with one conductor per phase. The conductors are continuous beams with equidistant simple supports. The conductor arrangement is shown in Figure 1. According to IEC 61936-1 [1][1], the calculation is done for the normal load case considering the dead load of the busbar and the exceptional load case considering the combination of effects of short-circuit currents and dead load.

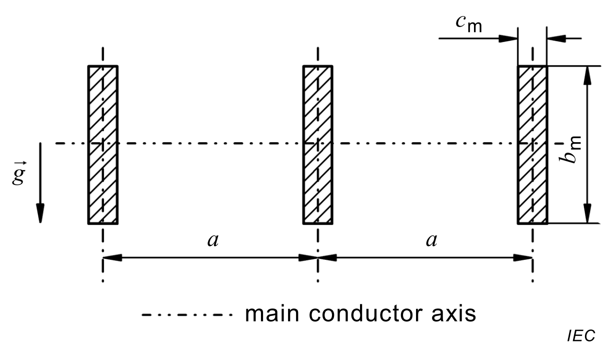


Figure 1 – Conductor arrangement

[1] The numbers in square brackets refer to the Bibliography.

4.2 Data

Initial symmetrical three-phase short-circuit current (r.m.s.)	I''_k	=	16 kA
Factor for the calculation of the peak short-circuit current	κ	=	1,35
System frequency	f	=	50 Hz
No automatic reclosing			
Number of spans		≥	3
Centre-line distance between supports	l	=	1 m
Centre-line distance between conductors	a	=	0,2 m
Rectangular conductor EN AW-6101B T7			
– Dimensions	b_m	=	60 mm
	c_m	=	10 mm
– Mass per unit length of main conductor	m'_m	=	1,62 kg/m
– Young's modulus	E	=	70 000 N/mm²
– Stress corresponding to the yield point	f_y	=	120 N/mm² to 180 N/mm²
Conventional value of acceleration of gravity	g	=	9,81 m/s²
Partial safety factors; for example according to EN 1990 [2]			
– Normal load case	γ_F	=	1,35
	γ_M	=	1,1
– Exceptional load case	$\gamma_F\ \gamma_M$	=	1,0

NOTE Safety factors differ in national standards.

4.3 Normal load case: Conductor stress and forces on the supports caused by dead load

The dead load on the conductor is:

$$F_{str,k} = m'_m\, l\, g = 1{,}62\,\frac{kg}{m}\cdot 1{,}00\ m\cdot 9{,}81\,\frac{m}{s^2} = 15{,}9\ N$$

$$F_{str,d} = \gamma_F\, F_{str,k} = 1{,}35\cdot 15{,}9\ N = 21{,}5\ N$$

The conductor bending stress is:

$$\sigma_{st,m,k} = \frac{F_{str,k}\, l}{8 W_{st,m}} = \frac{15{,}9\ N\cdot 1{,}00\ m}{8\cdot 6\cdot 10^{-6}\ m^3} = 0{,}33\cdot 10^6\ N/m^2 = 0{,}33\ N/mm^2$$

$$\sigma_{st,m,d} = \gamma_F\, \sigma_{st,m,k} = 1{,}35\cdot 0{,}33\ N/mm^2 = 0{,}45\ N/mm^2$$

with

$$J_{\text{st,m}} = \frac{c_{\text{m}} b_{\text{m}}^3}{12} = \frac{0{,}010 \cdot 0{,}060^3}{12} \text{ m}^4 = 1{,}8 \cdot 10^{-7} \text{ m}^4$$

$$W_{\text{st,m}} = \frac{J_{\text{st,m}}}{b_{\text{m}}/2} = \frac{1{,}8 \cdot 10^{-7} \text{ m}^4}{0{,}03 \text{ m}} = 6 \cdot 10^{-6} \text{ m}^3$$

NOTE The equation for the calculation of $\sigma_{\text{st,m,k}}$ gives the maximum value for two spans. The actual value for three or more spans is slightly lower.

The conductors have sufficient strength if

$$\sigma_{\text{st,m,d}} \le \frac{f_{\text{y}}}{\gamma_{\text{M}}}$$

with the lower value of f_{y}. The partial safety factors for normal load case γ_{F}, γ_{M} see 4.2. This gives:

$$\sigma_{\text{st,m,d}} = 0{,}45 \text{ N/mm}^2 \quad \text{less than} \quad \frac{f_{\text{y}}}{\gamma_{\text{M}}} = \frac{120 \text{ N/mm}^2}{1{,}1} = 109 \text{ N/mm}^2$$

The forces on the supports are in the direction of the dead load:

– for the outer supports (A) with $\alpha_{\text{A}} = 0{,}4$, see IEC 60865-1:2011, Table 3:

$$F_{\text{st,r,dA}} = \alpha_{\text{A}} F_{\text{str,d}} = 0{,}4 \cdot 21{,}5 \text{ N} = 8{,}6 \text{ N}$$

– for the inner supports (B) with $\alpha_{\text{B}} = 1{,}1$, see IEC 60865-1:2011, Table 3:

$$F_{\text{st,r,dB}} = \alpha_{\text{B}} F_{\text{str,d}} = 1{,}1 \cdot 21{,}5 \text{ N} = 23{,}7 \text{ N}$$

NOTE In some standards the safety factors for the supports can include the partial safety factor γ_{F} for action.

4.4 Exceptional load case: Effects of short-circuit currents

4.4.1 Maximum force on the central main conductor

The maximum electromagnetic force on the central main conductor is:

$$F_{\text{m3}} = \frac{\mu_0}{2\pi} \frac{\sqrt{3}}{2} i_{\text{p}}^2 \frac{l}{a_{\text{m}}} = \frac{4\pi \cdot 10^{-7}}{2\pi} \frac{\text{Vs}}{\text{Am}} \cdot \frac{\sqrt{3}}{2} \cdot \left(30{,}6 \cdot 10^3 \text{ A}\right)^2 \cdot \frac{1{,}00 \text{ m}}{0{,}202 \text{ m}} = 803 \text{ N} \tag{2}$$

where

$$i_{\text{p}} = \kappa \sqrt{2} I_{\text{k}}'' = 1{,}35 \cdot \sqrt{2} \cdot 16 \text{ kA} = 30{,}6 \text{ kA} = 30{,}6 \cdot 10^3 \text{ A}$$

and the effective distance between the main conductors

$$a_{\text{m}} = \frac{a}{k_{12}} = \frac{0{,}20 \text{ m}}{0{,}99} = 0{,}202 \text{ m} \tag{6}$$

with k_{12} according to IEC 60865-1:2011, Figure 1 with $a_{1\text{s}} = a$, $b_{\text{s}} = b_{\text{m}}$, $c_{\text{s}} = c_{\text{m}}$, for $b_{\text{m}}/c_{\text{m}}$ = 60 mm/10 mm = 6, and a/c_{m} = 200 mm/10 mm = 20.

4.4.2 Conductor stress and forces on the supports

4.4.2.1 General

The calculations can be made according to the following 4.4.2.2 or 4.4.2.3.

4.4.2.2 Simplified method

4.4.2.2.1 Conductor bending stress

The maximum bending stress is:

$$\sigma_{\text{m,d}} = V_{\sigma\text{m}} V_{\text{rm}} \beta \frac{F_{\text{m3}} l}{8 W_{\text{m}}} = 1{,}0 \cdot 0{,}73 \cdot \frac{803 \text{ N} \cdot 1{,}00 \text{ m}}{8 \cdot 1 \cdot 10^{-6} \text{ m}^3} = 73{,}3 \cdot 10^6 \text{ N/m}^2 = 73{,}3 \text{ N/mm}^2 \quad (9)$$

where

$V_{\sigma\text{m}} V_{\text{rm}}$ = 1,0 $(V_{\sigma\text{m}} V_{\text{rm}})_{\text{max}}$	according to IEC 60865-1:2011, Table 2
β = 0,73	according to IEC 60865-1:2011, Table 3

$$W_{\text{m}} = \frac{J_{\text{m}}}{c_{\text{m}}/2} = \frac{0{,}5 \cdot 10^{-8} \text{ m}^4}{0{,}005 \text{ m}} = 1 \cdot 10^{-6} \text{m}^3$$

The busbar is assumed to withstand the short-circuit force if

$$\sigma_{\text{m,d}} + \sigma_{\text{st,m,k}} \le q f_{\text{y}} \quad (11)$$

with the lower value of f_{y}. $\sigma_{\text{st,m,k}}$ see 4.3. For rectangular cross-section q = 1,5, see IEC 60865-1:2011, Table 4. This gives:

$$\sigma_{\text{m,d}} + \sigma_{\text{st,m,k}} = 73{,}3 \text{ N/mm}^2 + 0{,}33 \text{ N/mm}^2 = 73{,}6 \text{ N/mm}^2 \quad \text{less than} \quad q f_{\text{y}} = 1{,}5 \cdot 120 \text{ N/mm}^2 = 180 \text{ N/mm}^2$$

4.4.2.2.2 Forces on the supports

The equivalent static force on the supports is:

$$F_{\text{r,d}} = V_{\text{F}} V_{\text{rm}} \alpha F_{\text{m3}} \quad (15)$$

According to IEC 60865-1:2011, Table 2, with the upper value of f_{y} and $\sigma_{\text{tot,d}} = \sigma_{\text{m,d}} + \sigma_{\text{st,m,k}}$ it is:

$$\frac{\sigma_{\text{tot,d}}}{0{,}8 f_{\text{y}}} = \frac{73{,}6 \text{ N/mm}^2}{0{,}8 \cdot 180 \text{ N/mm}^2} = 0{,}511$$

Therefore, with a three-phase short-circuit we meet range 2 in IEC 60865-1:2011, Table 2,

$$0{,}370 < \frac{\sigma_{\text{tot,d}}}{0{,}8 f_{\text{y}}} < 1$$

hence

$$V_F V_{rm} = \frac{0{,}8\, f_y}{\sigma_{tot,d}} = \frac{1}{0{,}511} = 1{,}96$$

For the outer supports (A) it is with α_A = 0,4, see IEC 60865-1:2011, Table 3:

$$F_{r,dA} = V_F V_{rm} \alpha_A F_{m3} = 1{,}96 \cdot 0{,}4 \cdot 803\ \text{N} = 630\ \text{N}$$

For the inner supports (B) it is with α_B = 1,1, see IEC 60865-1:2011, Table 3:

$$F_{r,dB} = V_F V_{rm} \alpha_B F_{m3} = 1{,}96 \cdot 1{,}1 \cdot 803\ \text{N} = 1731\ \text{N}$$

4.4.2.3 Detailed method

4.4.2.3.1 Relevant natural frequency f_{cm} and factors V_F, V_{rm} and $V_{\sigma m}$

The relevant natural frequency of the main conductor is:

$$f_{cm} = \frac{\gamma}{l^2}\sqrt{\frac{E J_m}{m'_m}} = \frac{3{,}56}{(1{,}00\ \text{m})^2} \cdot \sqrt{\frac{7 \cdot 10^{10}\ \text{N/m}^2 \cdot 0{,}5 \cdot 10^{-8}\ \text{m}^4}{1{,}62\ \text{kg/m}}} = 52{,}3\ \text{Hz} \qquad (16)$$

where

γ = 3,56 according to IEC 60865-1:2011, Table 3

J_m = 0,5 · 10^{-8}m^4 see 4.4.2.2.1

The frequency ratio is:

$$\frac{f_{cm}}{f} = \frac{52{,}3\ \text{Hz}}{50\ \text{Hz}} = 1{,}05$$

From Figure 4 and 5.7.3 of IEC 60865-1:2011, the following values for the factors V_F, $V_{\sigma m}$ and V_{rm} are obtained:

$$V_F = 1{,}8$$
$$V_{\sigma m} = 1{,}0$$
$$V_{rm} = 1{,}0$$

4.4.2.3.2 Conductor bending stress

The maximum bending stress is:

$$\sigma_{m,d} = V_{\sigma m} V_{rm} \beta \frac{F_{m3}\, l}{8 W_m} = 1{,}0 \cdot 1{,}0 \cdot 0{,}73 \cdot \frac{803\ \text{N} \cdot 1{,}00\ \text{m}}{8 \cdot 1 \cdot 10^{-6}\ \text{m}^3} = 73{,}3 \cdot 10^6\ \text{N/m}^2 = 73{,}3\ \text{N/mm}^2 \qquad (9)$$

where

$V_{\sigma m} V_{rm}$ = 1,0·1,0 according to 4.4.2.3.1

β = 0,73 according to IEC 60865-1:2011, Table 3

W_m = 1·10^{-6}m^3 see 4.4.2.2.1

The busbar is assumed to withstand the short-circuit force if

$$\sigma_{m,d} + \sigma_{st,m,k} \leq q\, f_y \tag{11}$$

with the lower value of f_y. $\sigma_{st,m,k}$ see 4.3. For rectangular cross-section $q = 1{,}5$, see IEC 60865-1:2011, Table 4. This gives:

$$\sigma_{m,d} + \sigma_{st,m,k} = 73{,}3\ \text{N/mm}^2 + 0{,}33\ \text{N/mm}^2 = 73{,}6\ \text{N/mm}^2 \ \text{ less than } \ q\, f_y = 1{,}5 \cdot 120\ \text{N/mm}^2 = 180\ \text{N/mm}^2$$

4.4.2.3.3 Forces on the supports

The equivalent static force on supports becomes:

$$F_{r,d} = V_F\, V_{rm}\, \alpha\, F_{m3} \tag{15}$$

According to IEC 60865-1:2011, Table 2, with the upper value of f_y and $\sigma_{tot,d} = \sigma_{m,d} + \sigma_{st,m,k}$ it is:

$$\frac{\sigma_{tot,d}}{0{,}8\, f_y} = \frac{73{,}6\ \text{N/mm}^2}{0{,}8 \cdot 180\ \text{N/mm}^2} = 0{,}511$$

Therefore, with a three-phase short-circuit we meet range 2 in IEC 60865-1:2011, Table 2,

$$0{,}370 < \frac{\sigma_{tot,d}}{0{,}8\, f_y} < 1$$

hence

$$V_F\, V_{rm} = \frac{0{,}8\, f_y}{\sigma_{tot,d}} = \frac{1}{0{,}511} = 1{,}96$$

According to 4.4.2.3.1 above, $V_F\, V_{rm} = 1{,}8 \cdot 1{,}0 = 1{,}8$ which is less than the value 1,96 according to IEC 60865-1:2011, Table 2.

For the outer supports (A) it is with $\alpha_A = 0{,}4$, see IEC 60865-1:2011, Table 3:

$$F_{r,dA} = V_F\, V_{rm}\, \alpha_A\, F_{m3} = 1{,}8 \cdot 1{,}0 \cdot 0{,}4 \cdot 803\ \text{N} = 578\ \text{N}$$

For the inner supports (B) it is with $\alpha_B = 1{,}1$, see IEC 60865-1:2011, Table 3:

$$F_{r,dB} = V_F\, V_{rm}\, \alpha_B\, F_{m3} = 1{,}8 \cdot 1{,}0 \cdot 1{,}1 \cdot 803\ \text{N} = 1590\ \text{N}$$

4.5 Conclusions

The busbar will withstand the dead load

The calculated bending stress is	$\sigma_{st,m,d}$	1 N/mm²
The outer supports have to withstand a vertical force of	$F_{st,r,dA}$	9 N
The inner supports have to withstand a vertical force of	$F_{st,r,dB}$	24 N

		Simplified method	Detailed method
The busbar will withstand the short-circuit load			
The calculated bending stress is	$\sigma_{\text{tot,d}}$	74 N/mm^2	74 N/mm^2
The outer supports have to withstand an equivalent static force of	$F_{\text{r,dA}}$	630 N	580 N
The inner supports have to withstand an equivalent static force of	$F_{\text{r,dB}}$	1 740 N	1 590 N

The stresses and forces are rounded.

The forces calculated with the detailed method are less than calculated with the simplified method.

5 Example 2 – Mechanical effects on a 10 kV arrangement with multiple rigid conductors

5.1 General

The basis for the calculation in this example is the same three-phase 10 kV busbar as in Example 1, but now with three sub-conductors per main conductor as shown in Figure 2. The cross-sections of the sub-conductors are 60 mm × 10 mm as the conductors of Example 1. The connecting pieces are spacers. According to IEC 61936-1 [1], the calculation is done for the normal load case considering the dead load of the busbar and the exceptional load case considering the combination of effects of short-circuit currents and dead load.

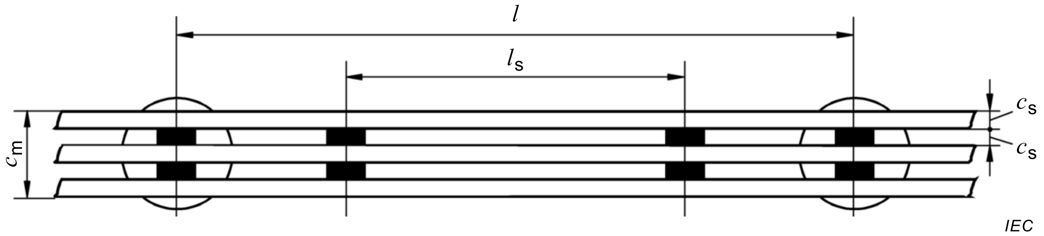


Figure 2 – Position of the sub-conductors and connecting pieces

5.2 Data (additional to the data of Example 1)

Number of sub-conductors	n =	3
Dimension of sub-conductor in the direction of the force	c_s =	10 mm
Number of sets of spacers	k =	2
Centre-line distance between connecting pieces	l_s =	0,5 m
Dimension of spacers of EN AW-6101B T7		60 mm × 60 mm × 10 mm

5.3 Normal load case: Conductor stress and forces on the supports caused by dead load

In 4.3 (Example 1), the following values are calculated for one conductor

Dead load of the conductor	$F_{\text{str,k}}$	=	15,9 N
	$F_{\text{str,d}}$	=	21,5 N
Conductor bending stress	$\sigma_{\text{st,m,k}}$	=	0,33 N/mm^2
	$\sigma_{\text{st,m,d}}$	=	0,45 N/mm^2

In this Example 2, the conductor bending stress is the same as in Example 1, 4.3. According to the number of sub-conductors, the vertical forces on the supports are n times higher

– for the outer supports (A) with α_A = 0,4, see IEC 60865-1:2011, Table 3:

$$F_{\text{st,r,dA}} = n\,\alpha_A\,F_{\text{str,d}} = 3 \cdot 0{,}4 \cdot 21{,}5\ \text{N} = 25{,}8\ \text{N}$$

– for the inner supports (B) with α_B = 1,1, see IEC 60865-1:2011, Table 3:

$$F_{\text{st,r,dB}} = n\,\alpha_B\,F_{\text{str,d}} = 3 \cdot 1{,}1 \cdot 21{,}5\ \text{N} = 71{,}0\ \text{N}$$

5.4 Exceptional load case: Effects of short-circuit currents

5.4.1 Maximum forces on the conductors

5.4.1.1 Maximum force on the central main conductor

The maximum electromagnetic force on the central main conductor is:

$$F_{\text{m3}} = \frac{\mu_0}{2\pi}\frac{\sqrt{3}}{2}\,i_p^2\,\frac{l}{a_m} = \frac{4\pi \cdot 10^{-7}}{2\pi}\,\frac{\text{Vs}}{\text{Am}} \cdot \frac{\sqrt{3}}{2} \cdot \left(30{,}6 \cdot 10^3\ \text{A}\right)^2 \cdot \frac{1{,}00\ \text{m}}{0{,}20\ \text{m}} = 811\ \text{N} \qquad (2)$$

where

$$i_p = \kappa\sqrt{2}\,I_k'' = 1{,}35 \cdot \sqrt{2} \cdot 16\ \text{kA} = 30{,}6\ \text{kA} = 30{,}6 \cdot 10^3\ \text{A}$$

and the effective distance between the main conductors

$$a_m = \frac{a}{k_{12}} = \frac{0{,}2\ \text{m}}{1{,}00} = 0{,}20\ \text{m} \qquad (6)$$

with k_{12} according to IEC 60865-1:2011, Figure 1 with $a_{1s} = a$, $b_s = b_m$, $c_s = c_m$, for b_m/c_m = 60 mm/50 mm = 1,2 and a/c_m = 200 mm/50 mm = 4. The dimensions b_m and c_m are shown in IEC 60865-1:2011, Figure 2 b).

5.4.1.2 Maximum force on the sub-conductor

The maximum electromagnetic force on the outer sub-conductor between two adjacent connecting pieces is:

$$F_s = \frac{\mu_0}{2\pi}\left(\frac{i_p}{n}\right)^2\frac{l_s}{a_s} = \frac{4\pi \cdot 10^{-7}}{2\pi}\,\frac{\text{Vs}}{\text{Am}} \cdot \left(\frac{30{,}6 \cdot 10^3\ \text{A}}{3}\right)^2 \cdot \frac{0{,}5\ \text{m}}{20{,}2 \cdot 10^{-3}\ \text{m}} = 515\ \text{N} \qquad (4)$$

where

$$\frac{1}{a_\mathrm{s}} = \frac{k_{12}}{a_{12}} + \frac{k_{13}}{a_{13}} = \frac{0{,}60}{20\ \mathrm{mm}} + \frac{0{,}78}{40\ \mathrm{mm}} = \frac{1}{20{,}2\ \mathrm{mm}} \tag{8}$$

with k_{12} and k_{13} from IEC 60865-1:2011, Figure 1:

- k_{12} = 0,60 for a_{12}/c_s = 20 mm/10 mm = 2 and $b_\mathrm{s}/c_\mathrm{s} = b_\mathrm{m}/c_\mathrm{s}$ = 60 mm/10 mm = 6
- k_{13} = 0,78 for a_{13}/c_s = 40 mm/10 mm = 4 and $b_\mathrm{s}/c_\mathrm{s} = b_\mathrm{m}/c_\mathrm{s}$ = 60 mm/10 mm = 6

or a_s from IEC 60865-1:2011, Table 1.

5.4.2 Conductor stress and forces on the supports

5.4.2.1 General

The calculations can be made according to 5.4.2.2 or 5.4.2.3.

5.4.2.2 Simplified method

5.4.2.2.1 Bending stress caused by the forces between the main conductors

The maximum bending stress caused by the forces between the main conductors is:

$$\sigma_\mathrm{m,d} = V_{\sigma\mathrm{m}} V_\mathrm{rm}\, \beta\, \frac{F_\mathrm{m3}\, l}{8 W_\mathrm{m}} = 1{,}0 \cdot 0{,}73 \cdot \frac{811\ \mathrm{N} \cdot 1{,}00\ \mathrm{m}}{8 \cdot 3 \cdot 10^{-6}\ \mathrm{m}^3} = 24{,}7 \cdot 10^6\ \mathrm{N/m}^2 = 24{,}7\ \mathrm{N/mm}^2 \tag{9}$$

where

$V_{\sigma\mathrm{m}} V_\mathrm{rm} = 1{,}0 = (V_{\sigma\mathrm{m}} V_\mathrm{rm})_\mathrm{max}$	according to IEC 60865-1, Table 2
$\beta = 0{,}73$	according to IEC 60865-1, Table 3
$J_\mathrm{s} = \frac{c_\mathrm{s}^3 b_\mathrm{s}}{12} = \frac{0{,}010^3 \cdot 0{,}060}{12}\ \mathrm{m}^4 = 0{,}5 \cdot 10^{-8}\ \mathrm{m}^4$	
$W_\mathrm{s} = \frac{J_\mathrm{s}}{c_\mathrm{s}/2} = \frac{0{,}5 \cdot 10^{-8}\ \mathrm{m}^4}{0{,}005\ \mathrm{m}} = 1 \cdot 10^{-6}\ \mathrm{m}^3$	
$W_\mathrm{m} = n\, W_\mathrm{s} = 3 \cdot 1 \cdot 10^{-6}\ \mathrm{m}^3 = 3 \cdot 10^{-6}\ \mathrm{m}^3$	according to IEC 60865-1, 5.4.2

5.4.2.2.2 Bending stress caused by the forces between the sub-conductors

The maximum bending stress caused by the forces between the sub-conductors is:

$$\sigma_\mathrm{s,d} = V_{\sigma\mathrm{s}} V_\mathrm{rs} \frac{F_\mathrm{s}\, l_\mathrm{s}}{16 W_\mathrm{s}} = 1{,}0 \cdot \frac{515\ \mathrm{N} \cdot 0{,}5\ \mathrm{m}}{16 \cdot 1 \cdot 10^{-6}\ \mathrm{m}^3} = 16{,}1 \cdot 10^6\ \mathrm{N/m}^2 = 16{,}1\ \mathrm{N/mm}^2 \tag{10}$$

where

$V_{\sigma\mathrm{s}} V_\mathrm{rs} = 1{,}0 = (V_{\sigma\mathrm{s}} V_\mathrm{rs})_\mathrm{max}$	according to IEC 60865-1:2011, Table 2
W_s = $1 \cdot 10^{-6} \mathrm{m}^3$	see 5.4.2.2.1

5.4.2.2.3 Total conductor stress

The total conductor stress is with the stresses calculated in 5.4.2.2.1, 5.4.2.2.2 and 5.3:

$$\sigma_\mathrm{tot,d} = \sigma_\mathrm{m,d} + \sigma_\mathrm{s,d} + \sigma_\mathrm{st,m,k} = 24{,}7\ \mathrm{N/mm}^2 + 16{,}1\ \mathrm{N/mm}^2 + 0{,}33\ \mathrm{N/mm}^2 = 41{,}1\ \mathrm{N/mm}^2 \tag{12}$$

The busbar is assumed to withstand the short-circuit force if

$$\sigma_{tot,d} \leq q\, f_y \tag{13}$$

with the lower value of f_y. For rectangular cross-sections q = 1,5, see IEC 60865-1:2011, Table 4 or 5.4.2. This gives:

$$\sigma_{tot,d} = 41{,}1\,\text{N/mm}^2 \quad \text{less than} \quad q\, f_y = 1{,}5 \cdot 120\,\text{N/mm}^2 = 180\,\text{N/mm}^2$$

It is recommended that the stress caused by the forces between sub-conductors holds

$$\sigma_{s,d} \leq f_y \tag{14}$$

with the lower value of f_y. This gives:

$$\sigma_{s,d} = 16{,}1\,\text{N/mm}^2 \quad \text{less than} \quad f_y = 120\,\text{N/mm}^2$$

5.4.2.2.4 Forces on the supports

The equivalent static force on supports is:

$$F_{r,d} = V_F\, V_{rm}\, \alpha\, F_{m3} \tag{15}$$

According to IEC 60865-1:2011, Table 2, with the upper value of f_y it is:

$$\frac{\sigma_{tot,d}}{0{,}8\, f_y} = \frac{41{,}1\,\text{N/mm}^2}{0{,}8 \cdot 180\,\text{N/mm}^2} = 0{,}285$$

therefore with a three-phase short-circuit we meet range 1 in IEC 60865-1:2011, Table 2,

$$\frac{\sigma_{tot,d}}{0{,}8\, f_y} < 0{,}370$$

hence

$$V_F\, V_{rm} = 2{,}7$$

For the outer supports (A) it is with α_A = 0,4, see IEC 60865-1:2011, Table 3:

$$F_{r,dA} = V_F\, V_{rm}\, \alpha_A\, F_{m3} = 2{,}7 \cdot 0{,}4 \cdot 811\,\text{N} = 876\,\text{N}$$

For the inner supports (B) it is with α_B = 1,1, see IEC 60865-1:2011, Table 3:

$$F_{r,dB} = V_F\, V_{rm}\, \alpha_B\, F_{m3} = 2{,}7 \cdot 1{,}1 \cdot 811\,\text{N} = 2409\,\text{N}$$

5.4.2.3 Detailed method

5.4.2.3.1 Relevant natural frequency f_{cm} of the main conductors, f_{cs} of the sub-conductors and factors V_F, $V_{\sigma m}$, $V_{\sigma s}$, V_{rm} and V_{rs}

The relevant natural frequency of the main conductors is:

$$f_{cm} = e\frac{\gamma}{l^2}\sqrt{\frac{E\,J_s}{m'_s}} = 0{,}97\cdot\frac{3{,}56}{(1{,}00\ \text{m})^2}\cdot\sqrt{\frac{7\cdot10^{10}\ \text{N/m}^2\cdot0{,}5\cdot10^{-8}\ \text{m}^4}{1{,}62\ \text{kg/m}}} = 50{,}8\ \text{Hz} \tag{17}$$

where

e = 0,97 according to IEC 60865-1:2011, Figure 3c), for $k = 2\left(\frac{l_s}{l} = 0{,}5\right)$ and the radio

$$\frac{m_z}{n m'_s l} = \frac{1{,}62\ \text{kg/m}\cdot0{,}06\ \text{m}\cdot2}{3\cdot1{,}62\ \text{kg/m}\cdot1{,}00\ \text{m}} = 0{,}04$$

γ = 3,56 according to IEC 60865-1:2011, Table 3

$J_s = 0{,}5\cdot10^{-8}\text{m}^4$ see 5.4.2.2.1

The relevant natural frequency of the sub-conductors is:

$$f_{cs} = \frac{3{,}56}{l_s^2}\sqrt{\frac{E\,J_s}{m'_s}} = \frac{3{,}56}{(0{,}5\ \text{m})^2}\cdot\sqrt{\frac{7\cdot10^{10}\ \text{N/m}^2\cdot0{,}5\cdot10^{-8}\text{m}^4}{1{,}62\ \text{kg/m}}} = 209\ \text{Hz} \tag{18}$$

The frequency ratios are:

$$\frac{f_{cm}}{f} = \frac{50{,}8\ \text{Hz}}{50\ \text{Hz}} = 1{,}02$$

$$\frac{f_{cs}}{f} = \frac{209\ \text{Hz}}{50\ \text{Hz}} = 4{,}18$$

This gives from IEC 60865-1:2011, Figure 4 and 5.7.3, the following values for the factors V_F, $V_{\sigma m}$, $V_{\sigma s}$, V_{rm} and V_{rs}:

$$V_F = 1{,}8$$

$$V_{\sigma m} = 1{,}0 \qquad V_{\sigma s} = 1{,}0$$

$$V_{rm} = 1{,}0 \qquad V_{rs} = 1{,}0$$

5.4.2.3.2 Bending stress caused by the forces between the main conductors

The maximum bending stress caused by the forces between the main conductors is:

$$\sigma_{m,d} = V_{\sigma m} V_{rm}\,\beta\,\frac{F_{m3}\,l}{8W_m} = 1{,}0\cdot1{,}0\cdot0{,}73\cdot\frac{811\ \text{N}\cdot1{,}00\ \text{m}}{8\cdot3\cdot10^{-6}\ \text{m}^3} = 24{,}7\cdot10^6\ \text{N/m}^2 = 24{,}7\ \text{N/mm}^2 \tag{9}$$

where

$V_{\sigma m}V_{rm}$ = 1,0·1,0 according to 5.4.2.3.1

β = 0,73 according to IEC 60865-1:2011, Table 3

W_m = 3·10^{-6}m^3 see 5.4.2.2.1

5.4.2.3.3 Bending stress caused by the forces between the sub-conductors

The maximum bending stress caused by the forces between the sub-conductors is:

$$\sigma_{s,d} = V_{\sigma s} V_{rs} \frac{F_s\, l_s}{16 W_s} = 1{,}0 \cdot 1{,}0 \cdot \frac{515\ \text{N} \cdot 0{,}5\ \text{m}}{16 \cdot 1 \cdot 10^{-6}\ \text{m}^3} = 16{,}1 \cdot 10^6\ \text{N/m}^2 = 16{,}1\ \text{N/mm}^2 \qquad (10)$$

where

$V_{\sigma s} V_{rs}$ = 1,0·1,0 according to 5.4.2.3.1

W_s = 1·10^{-6}m^3 see 5.4.2.2.1

5.4.2.3.4 Total bending stress in the busbar

The total conductor stress is with the stresses calculated in 5.4.2.3.2 and 5.4.2.3.3 and 5.3:

$$\sigma_{tot,d} = \sigma_{m,d} + \sigma_{s,d} + \sigma_{st,m,k} = 24{,}7\ \text{N/mm}^2 + 16{,}1\ \text{N/mm}^2 + 0{,}33\ \text{N/mm}^2 = 41{,}1\ \text{N/mm}^2 \qquad (12)$$

The busbar is assumed to withstand the short-circuit force if

$$\sigma_{tot,d} \le q\, f_y \qquad (13)$$

with the lower value of f_y. For rectangular cross-sections q = 1,5, see IEC 60865-1:2011, Table 4. This gives:

$$\sigma_{tot,d} = 41{,}1\ \text{N/mm}^2 \quad \text{less than} \quad q\, f_y = 1{,}5 \cdot 120\ \text{N/mm}^2 = 180\ \text{N/mm}^2$$

It is recommended a value

$$\sigma_{s,d} \le f_y \qquad (14)$$

with the lower value of f_y. This gives:

$$\sigma_{s,d} = 16{,}1\ \text{N/mm}^2 \quad \text{less than} \quad f_y = 120\ \text{N/mm}^2$$

5.4.2.3.5 Forces on the supports

The equivalent static force on supports is:

$$F_{r,d} = V_F\, V_{rm}\, \alpha\, F_{m3} \qquad (15)$$

According to IEC 60865-1:2011, Table 2, with the upper value of f_y it is:

$$\frac{\sigma_{tot,d}}{0{,}8\, f_y} = \frac{41{,}1\ \text{N/mm}^2}{0{,}8 \cdot 180\ \text{N/mm}^2} = 0{,}285$$

Therefore with a three-phase short-circuit we meet range 1 in IEC 60865-1:2011, Table 2,

$$\frac{\sigma_{\mathrm{tot,d}}}{0{,}8\, f_{\mathrm{y}}} < 0{,}370$$

hence

$$V_{\mathrm{F}}\, V_{\mathrm{rm}} = 2{,}7$$

According to 5.4.2.3.1 above, $V_{\mathrm{F}}\, V_{\mathrm{rm}}$ = 1,8 · 1,0 = 1,8, which is less than the value 2,7 obtained from IEC 60865-1:2011, Table 2.

For the outer supports (A) it is with α_{A} = 0,4, see IEC 60865-1:2011, Table 3:

$$F_{\mathrm{r,dA}} = V_{\mathrm{F}}\, V_{\mathrm{rm}}\, \alpha_{\mathrm{A}}\, F_{\mathrm{m3}} = 1{,}8 \cdot 1{,}0 \cdot 0{,}4 \cdot 811\,\mathrm{N} = 584\,\mathrm{N}$$

For the inner supports (B) it is with α_{B} = 1,1; see IEC 60865-1:2011, Table 3:

$$F_{\mathrm{r,dB}} = V_{\mathrm{F}}\, V_{\mathrm{rm}}\, \alpha_{\mathrm{B}}\, F_{\mathrm{m3}} = 1{,}8 \cdot 1{,}0 \cdot 1{,}1 \cdot 811\,\mathrm{N} = 1606\,\mathrm{N}$$

5.5 Conclusions

The busbar will withstand the dead load			
The calculated bending stress is	$\sigma_{\mathrm{st,m,d}}$	1 N/mm^2	
The outer supports have to withstand a vertical force of	$F_{\mathrm{st,r,dA}}$	26 N	
The inner supports have to withstand a vertical force of	$F_{\mathrm{st,r,dB}}$	71 N	
		Simplified method	Detailed method
The busbar will withstand the short-circuit force			
The calculated bending stresses are	$\sigma_{\mathrm{tot,d}}$	42 N/mm^2	42 N/mm^2
	$\sigma_{\mathrm{s,d}}$	17 N/mm^2	17 N/mm^2
The outer supports have to withstand an equivalent static force of	$F_{\mathrm{r,dA}}$	880 N	590 N
The inner supports have to withstand an equivalent static force of	$F_{\mathrm{r,dB}}$	2 410 N	1 610 N

The forces calculated with the detailed method are less than calculated with the simplified method.

6 Example 3. – Mechanical effects on a high-voltage arrangement with rigid conductors

6.1 General

The basis for the calculation in this example is a three-phase 380 kV busbar, with one tubular conductor per phase. The conductor arrangement is shown in Figure 3. This example includes calculations without and with automatic reclosing. Without automatic reclosing only one short-circuit current duration exists, with automatic reclosing two short-circuit current durations exist with an interval without current flow.

According to IEC 61936-1 [1], the calculation is done for the normal load case considering the dead load of the busbar and the exceptional load case considering the combination of effects of short-circuit currents and dead load.

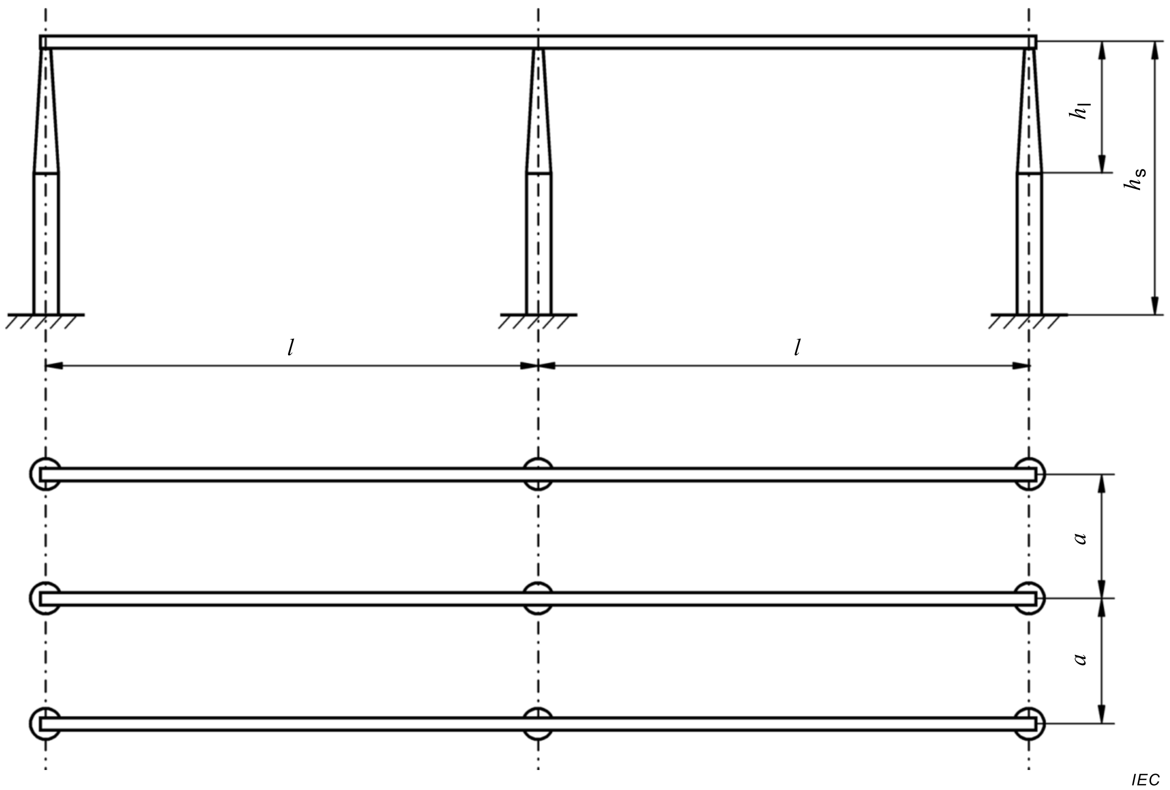


Figure 3 – Two-span arrangement with tubular conductors

6.2 Data

Initial symmetrical three-phase short-circuit current (r.m.s.)	I''_k	=	50 kA
Factor for the calculation of the peak short-circuit current	κ	=	1,81
System frequency	f	=	50 Hz
Number of spans			2
Centre-line distance between supports	l	=	18 m
Centre-line distance between conductors	a	=	5 m
Height of the insulator with clamp	h_I	=	3,7 m
Height of the support	h_S	=	7,0 m
Tubular conductor 160 mm × 6 mm EN AW-6101B T6			
– Mass per unit length	m'_m	=	7,84 kg/m
– Outer diameter	d	=	160 mm
– Wall thickness	t	=	6 mm
– Young’s modulus	E	=	70 000 N/mm^2

– Stress corresponding to the yield point	f_y	=	160 N/mm^2 to 240 N/mm^2
Conventional value of acceleration of gravity	g	=	9,81 m/s^2

Partial safety factors; for example according to EN 1990 [2]

– Normal load case	γ_F	=	1,35
	γ_M	=	1,1
– Exceptional load case	$\gamma_F\ \gamma_M$	=	1,0

NOTE Safety factors differ in national standards.

6.3 Normal load case: Conductor stress and forces on the supports caused by dead load

The dead load on the conductor is

$$F_{str.k} = m'_m\, l\, g = 7{,}84\ \frac{kg}{m} \cdot 18\ m \cdot 9{,}81\ \frac{m}{s^2} = 1384\ N$$

$$F_{str,d} = \gamma_F\, F_{str,k} = 1{,}35 \cdot 1384\ N = 1868\ N$$

The conductor bending stress is

$$\sigma_{st,m,k} = \frac{F_{str}\, l}{8 W_m} = \frac{1384\ N \cdot 18\ m}{8 \cdot 108 \cdot 10^{-6}\ m^3} = 28{,}8 \cdot 10^6\ N/m^2 = 28{,}8\ N/mm^2$$

$$\sigma_{st,m,d} = \gamma_F\, \sigma_{st,m,k} = 1{,}35 \cdot 28{,}8\ N/mm^2 = 38{,}9\ N/mm^2$$

with

$$J_m = \frac{\pi}{64}\left(d^4 - (d-2t)^4\right) = \frac{\pi}{64} \cdot \left(0{,}16^4 - (0{,}16 - 2 \cdot 0{,}006)^4\right) m^4 = 8{,}62 \cdot 10^{-6}\ m^4$$

$$W_m = \frac{J_m}{d/2} = \frac{8{,}62 \cdot 10^{-6}\ m^4}{0{,}16/2\ m} = 108 \cdot 10^{-6}\ m^3$$

The conductors have sufficient strength if

$$\sigma_{st,m,d} \le \frac{f_y}{\gamma_M}$$

with the lower value of f_y. The partial safety factors for normal load case γ_F, γ_M see 6.2. This gives:

$$\sigma_{st,m,d} = 38{,}9\ N/mm^2 \quad \text{less than} \quad \frac{f_y}{\gamma_M} = \frac{160\ N/mm^2}{1{,}1} = 145\ N/mm^2$$

The force on the supports are in the direction of the dead load:

– for the outer supports (A) with α_A = 0,375, see IEC 60865-1:2011, Table 3:

$$F_{st,r,dA} = \alpha_A\, F_{str,d} = 0{,}375 \cdot 1868\ N = 701\ N = 0{,}701\ kN$$

– for the inner supports (B) with α_B = 1,25, see IEC 60865-1:2011, Table 3:

$$F_{\text{st,r,dB}} = \alpha_{\text{B}} F_{\text{str,d}} = 1{,}25 \cdot 1868\ \text{N} = 2335\ \text{N} = 2{,}335\ \text{kN}$$

6.4 Exceptional load case: Effects of short-circuit currents

6.4.1 Maximum force on the central main conductor

The maximum electromagnetic force on the central main conductor becomes:

$$F_{\text{m3}} = \frac{\mu_0}{2\pi} \frac{\sqrt{3}}{2} i_{\text{p}}^2 \frac{l}{a_{\text{m}}} = \frac{4\pi \cdot 10^{-7}}{2\pi} \frac{\text{Vs}}{\text{Am}} \cdot \frac{\sqrt{3}}{2} \cdot \left(128 \cdot 10^3\ \text{A}\right)^2 \cdot \frac{18\ \text{m}}{5\ \text{m}} = 10200\ \text{N} = 10{,}2\ \text{kN} \qquad (2)$$

where

$$i_{\text{p}} = \kappa \sqrt{2}\, I_{\text{k}}'' = 1{,}81 \cdot \sqrt{2} \cdot 50\ \text{kA} = 128\ \text{kA} = 128 \cdot 10^3\ \text{A}$$

and $a_{\text{m}} = a = 5$ m according to IEC 60865-1:2011, 5.3.

6.4.2 Conductor stress and forces on the supports

6.4.2.1 General

The calculations can be made according to the following 6.4.2.2 or 6.4.2.3.

6.4.2.2 Simplified method

6.4.2.2.1 Calculation without three-phase automatic reclosing

6.4.2.2.1.1 Conductor bending stress

The maximum bending stress is:

$$\sigma_{\text{m,d}} = V_{\sigma\text{m}} V_{\text{rm}} \beta \frac{F_{\text{m3}}\, l}{8 W_{\text{m}}} = 1{,}0 \cdot 0{,}73 \cdot \frac{10{,}2 \cdot 10^3\ \text{N} \cdot 18\ \text{m}}{8 \cdot 108 \cdot 10^{-6}\ \text{m}^3} = 155 \cdot 10^6\ \text{N/m}^2 = 155\ \text{N/mm}^2 \qquad (9)$$

where

$V_{\sigma\text{m}} V_{\text{rm}} = 1{,}0 = \left(V_{\sigma\text{m}} V_{\text{rm}}\right)_{\max}$	according to IEC 60865-1:2011, Table 2
$\beta = 0{,}73$	according to IEC 60865-1:2011, Table 3
$W_{\text{m}} = 108 \cdot 10^{-6} \text{m}^3$	see 6.3

For tubular cross-section, the total bending stress becomes:

$$\sigma_{\text{tot,d}} = \sqrt{\sigma_{\text{m,d}}^2 + \sigma_{\text{st,m,k}}^2} = \sqrt{155^2 + 28{,}8^2}\ \text{N/mm}^2 = 158\ \text{N/mm}^2$$

The busbar is assumed to withstand the short-circuit force if

$$\sigma_{\text{tot,d}} \le q\, f_{\text{y}} \qquad (11)$$

with the lower value of f_{y}. For tubular cross-section in accordance with IEC 60865-1:2011, Table 4:

$$q = 1{,}7\frac{1-(1-2t/d)^3}{1-(1-2t/d)^4} = 1{,}7\cdot\frac{1-(1-2\cdot 0{,}006\text{ m}/(0{,}160\text{ m}))^3}{1-(1-2\cdot 0{,}006\text{ m}/(0{,}160\text{ m}))^4} = 1{,}32$$

This gives:

$$\sigma_{\text{tot,d}} = 158\text{ N/mm}^2 \quad \text{less than} \quad q\,f_{\text{y}} = 1{,}32\cdot 160\text{ N/mm}^2 = 211\text{ N/mm}^2$$

6.4.2.2.1.2 Forces on the supports and moments on the substructures

The equivalent static force on supports is:

$$F_{\text{r,d}} = V_{\text{F}}\,V_{\text{rm}}\,\alpha\,F_{\text{m3}} \tag{15}$$

According to IEC 60865-1:2011, Table 2, with the upper value of f_{y} it is:

$$\frac{\sigma_{\text{tot,d}}}{0{,}8\,f_{\text{y}}} = \frac{158\text{ N/mm}^2}{0{,}8\cdot 240\text{ N/mm}^2} = 0{,}823$$

Therefore we meet range 2 in IEC 60865-1:2011, Table 2,

$$0{,}370 < \frac{\sigma_{\text{tot,d}}}{0{,}8\,f_{\text{y}}} < 1$$

hence

$$V_{\text{F}}\,V_{\text{rm}} = \frac{0{,}8\,f_{\text{y}}}{\sigma_{\text{tot,d}}} = \frac{1}{0{,}823} = 1{,}22$$

For the outer supports (A) it is with α_{A} = 0,375, see IEC 60865-1:2011, Table 3:

$$F_{\text{r,dA}} = V_{\text{F}}\,V_{\text{rm}}\,\alpha_{\text{A}}\,F_{\text{m3}} = 1{,}22\cdot 0{,}375\cdot 10{,}2\text{ kN} = 4{,}67\text{ kN}$$

For the inner supports (B) it is with α_{B} = 1,25, see IEC 60865-1:2011, Table 3:

$$F_{\text{r,dB}} = V_{\text{F}}\,V_{\text{rm}}\,\alpha_{\text{B}}\,F_{\text{m3}} = 1{,}22\cdot 1{,}25\cdot 10{,}2\text{ kN} = 15{,}6\text{ kN}$$

The bending moments on the substructures are:

– on the bottom of the outer insulators

$$M_{\text{IA,d}} = F_{\text{r,dA}}\,h_{\text{I}} = 4{,}67\text{ kN}\cdot 3{,}7\text{ m} = 17{,}3\text{ kNm}$$

– on the bottom of the outer supports

$$M_{\text{SA,d}} = F_{\text{r,dA}}\,h_{\text{S}} = 4{,}67\text{ kN}\cdot 7{,}0\text{ m} = 32{,}7\text{ kNm}$$

– on the bottom of the inner insulators

$$M_{\text{IB,d}} = F_{\text{r,dB}}\,h_{\text{I}} = 15{,}6\text{ kN}\cdot 3{,}7\text{ m} = 57{,}7\text{ kNm}$$

– on the bottom of the inner supports

$$M_{\text{SB,d}} = F_{\text{r,dB}}\,h_{\text{S}} = 15{,}6\text{ kN}\cdot 7{,}0\text{ m} = 109\text{ kNm}$$

6.4.2.2.2 Calculation with three-phase automatic reclosing

6.4.2.2.2.1 General

In networks with three-phase automatic reclosing different mechanical stresses can occur during the first and the second short-circuit current flow duration. In 6.4.2.2.1, the stresses and forces during the first short-circuit current flow duration are calculated.

6.4.2.2.2.2 Conductor bending stress

The maximum bending stress during the second short-circuit current flow duration is:

$$\sigma_{\text{m,d}} = V_{\sigma\text{m}} V_{\text{rm}} \beta \frac{F_{\text{m3}} l}{8 W_{\text{m}}} = 1{,}8 \cdot 0{,}73 \cdot \frac{10{,}2 \cdot 10^{3}\ \text{N} \cdot 18\ \text{m}}{8 \cdot 108 \cdot 10^{-6}\ \text{m}^{3}} = 279 \cdot 10^{6}\ \text{N/m}^{2} = 279\ \text{N/mm}^{2} \qquad (9)$$

where

$V_{\sigma\text{m}} V_{\text{rm}} = 1{,}8 = (V_{\sigma\text{m}} V_{\text{rm}})_{\text{max}}$	according to IEC 60865-1:2011, Table 2
β = 0,73	according to IEC 60865-1:2011, Table 3
W_{m} = $108 \cdot 10^{-6} \text{m}^3$	see 6.3

The bending stress during the second short-circuit current flow duration is greater than during the first short-circuit current flow duration calculated in 6.4.2.2.1.1.

The total bending stress becomes:

$$\sigma_{\text{tot,d}} = \sqrt{\sigma_{\text{m,d}}^{2} + \sigma_{\text{st,m,k}}^{2}} = \sqrt{279^{2} + 28{,}8^{2}}\ \text{N/mm}^{2} = 281\ \text{N/mm}^{2}$$

The busbar is assumed to withstand the short-circuit force if

$$\sigma_{\text{tot,d}} \leq q f_{\text{y}} \qquad (11)$$

with the lower value of f_{y} and where q = 1,32, see 6.4.2.2.1.1. This gives:

$$\sigma_{\text{tot,d}} = 281\ \text{N/mm}^{2} \quad \text{greater than} \quad q f_{\text{y}} = 1{,}32 \cdot 160\ \text{N/mm}^{2} = 211\ \text{N/mm}^{2}$$

Considering only the result of the simplified method, the busbar is not assumed to withstand the short-circuit force. Therefore it is necessary to apply the detailed method to verify that the conductors are assumed to withstand the short-circuit force.

6.4.2.2.2.3 Forces on the supports

The following calculation is made only for information, as a result of the conductors not withstanding the short-circuit force according to the simplified method.

The equivalent static force on the supports is:

$$F_{\text{r,d}} = V_{\text{F}} V_{\text{rm}} \alpha F_{\text{m3}} \qquad (15)$$

During the first short-circuit current flow it is, see 6.4.2.2.1.2:

$$(V_{\text{F}} V_{\text{rm}})_{1} = 1{,}22$$

According to IEC 60865-1:2011, Table 2, with the upper value of f_y it is during the second short-circuit current flow:

$$\frac{\sigma_{tot,d}}{0{,}8\,f_y} = \frac{281\,\text{N/mm}^2}{0{,}8 \cdot 240\,\text{N/mm}^2} = 1{,}46$$

therefore we meet range 3 in IEC 60865-1:2011, Table 2,

$$1 < \frac{\sigma_{tot,d}}{0{,}8\,f_y}$$

hence during the second short-circuit current flow

$$\left(V_F\,V_{rm}\right)_2 = 1{,}0$$

According to IEC 60865-1:2011, 5.6, the greater of both values is to be inserted in Equation (15):

$$V_F\,V_{rm} = \max\left\{\left(V_F\,V_{rm}\right)_1 ; \left(V_F\,V_{rm}\right)_2\right\} = \max\left\{1{,}22 ; 1{,}00\right\} = 1{,}22$$

For the outer supports (A) it is with α_A = 0,375, see IEC 60865-1:2011, Table 3:

$$F_{r,dA} = V_F\,V_{rm}\,\alpha_A\,F_{m3} = 1{,}22 \cdot 0{,}375 \cdot 10{,}2\,\text{kN} = 4{,}67\,\text{kN}$$

For the inner supports (B) it is with α_B = 1,25, see IEC 60865-1:2011, Table 3:

$$F_{r,dB} = V_F\,V_{rm}\,\alpha_B\,F_{m3} = 1{,}22 \cdot 1{,}25 \cdot 10{,}2\,\text{kN} = 15{,}6\,\text{kN}$$

6.4.2.3 Detailed method

6.4.2.3.1 Relevant natural frequency f_{cm} and factors V_F, $V_{\sigma m}$ and V_{rm}

The relevant natural frequency of the main conductors is:

$$f_{cm} = \frac{\gamma}{l^2}\sqrt{\frac{E\,J_m}{m'_m}} = \frac{2{,}45}{(18\,\text{m})^2} \cdot \sqrt{\frac{7 \cdot 10^{10}\,\text{N/m}^2 \cdot 8{,}62 \cdot 10^{-6}\,\text{m}^4}{7{,}84\,\text{kg/m}}} = 2{,}10\,\text{Hz} \qquad (16)$$

where

γ = 2,45 according to IEC 60865-1:2011, Table 3

J_m = 8,62·10^{-6}m^4 see 6.3

The frequency ratio is:

$$\frac{f_{cm}}{f} = \frac{2{,}10\,\text{Hz}}{50\,\text{Hz}} = 0{,}042$$

For this ratio, the factors V_F, $V_{\sigma m}$ and V_{rm} are according to IEC 60865-1:2011, 5.7.3, Figure 4 and Figure 5:

$V_F = 0{,}36$
$V_{\sigma m} = 0{,}32$
$V_{rm} = 1{,}0$ without three-phase automatic reclosing
$V_{rm} = 1{,}8$ with three-phase automatic reclosing

6.4.2.3.2 Calculation without three-phase automatic reclosing

6.4.2.3.2.1 Conductor bending stress

The maximum bending stress is:

$$\sigma_{m,d} = V_{\sigma m} V_{rm} \beta \frac{F_{m3}\, l}{8 W_m} = 0{,}32 \cdot 1{,}0 \cdot 0{,}73 \cdot \frac{10{,}2 \cdot 10^3 \text{ N} \cdot 18 \text{ m}}{8 \cdot 108 \cdot 10^{-6} \text{ m}^3} = 49{,}6 \cdot 10^6 \text{ N/m}^2 = 49{,}6 \text{ N/mm}^2 \qquad (9)$$

where

$V_{\sigma m} V_{rm}$ = 0,32·1,0 = 0,32 according to 6.4.2.3.1, value which is less than 1,0 = $(V_{\sigma m} V_{rm})_{max}$ according to IEC 60865-1:2011, Table 2

β = 0,73 according to IEC 60865-1:2011, Table 3

W_m = $108 \cdot 10^{-6} \text{m}^3$ see 6.3

The total bending stress is:

$$\sigma_{tot,d} = \sqrt{\sigma_{m,d}^2 + \sigma_{st,m,k}^2} = \sqrt{49{,}6^2 + 28{,}8^2} \text{ N/mm}^2 = 57{,}4 \text{ N/mm}^2$$

The busbar is assumed to withstand the short-circuit force if

$$\sigma_{tot,d} \le q\, f_y \qquad (11)$$

with the lower value of f_y and where q = 1,32, see 6.4.2.2.1.1. This gives:

$$\sigma_{tot,d} = 57{,}4 \text{ N/mm}^2 \quad \text{less than} \quad q\, f_y = 1{,}32 \cdot 160 \text{ N/mm}^2 = 211 \text{ N/mm}^2$$

6.4.2.3.2.2 Forces on the supports and moments on the substructures

The equivalent static force on the supports is:

$$F_{r,d} = V_F V_{rm} \alpha F_{m3} \qquad (15)$$

According to 6.4.2.3.1 above, $V_F V_{rm} = 0{,}36 \cdot 1{,}0 = 0{,}36$ which is lower than the value $1{,}0 = (V_{\sigma m} V_{rm})_{max}$ according to IEC 60865-1:2011, Table 2.

For the outer supports (A) it is with α_A = 0,375, see IEC 60865-1:2011, Table 3:

$$F_{r,dA} = V_F V_{rm} \alpha_A F_{m3} = 0{,}36 \cdot 1{,}0 \cdot 0{,}375 \cdot 10{,}2 \text{ kN} = 1{,}38 \text{ kN}$$

For the inner supports (B) it is with α_B = 1,25, see IEC 60865-1:2011, Table 3:

$$F_{r,dB} = V_F V_{rm} \alpha_B F_{m3} = 0{,}36 \cdot 1{,}0 \cdot 1{,}25 \cdot 10{,}2 \text{ kN} = 4{,}59 \text{ kN}$$

The bending moments on the substructures are:

– on the bottom of the outer insulators

$$M_{\mathrm{IA,d}} = F_{\mathrm{r,dA}}\, h_{\mathrm{I}} = 1{,}38\ \mathrm{kN} \cdot 3{,}7\ \mathrm{m} = 5{,}11\,\mathrm{kNm}$$

– on the bottom of the outer supports

$$M_{\mathrm{SA,d}} = F_{\mathrm{r,dA}}\, h_{\mathrm{S}} = 1{,}38\ \mathrm{kN} \cdot 7{,}0\ \mathrm{m} = 9{,}66\ \mathrm{kNm}$$

– on the bottom of the inner insulators

$$M_{\mathrm{IB,d}} = F_{\mathrm{r,dB}}\, h_{\mathrm{I}} = 4{,}59\ \mathrm{kN} \cdot 3{,}7\ \mathrm{m} = 17{,}0\ \mathrm{kNm}$$

– on the bottom of the inner supports

$$M_{\mathrm{SB,d}} = F_{\mathrm{r,dB}}\, h_{\mathrm{S}} = 4{,}59\ \mathrm{kN} \cdot 7{,}0\ \mathrm{m} = 32{,}1\,\mathrm{kNm}$$

6.4.2.3.2.3 Calculation with three-phase automatic reclosing

6.4.2.3.2.3.1 Conductor bending stress

The maximum bending stress during the second short-circuit current flow duration is:

$$\sigma_{\mathrm{m,d}} = V_{\sigma\mathrm{m}} V_{\mathrm{rm}}\, \beta\, \frac{F_{\mathrm{m3}}\, l}{8 W_{\mathrm{m}}} = 0{,}58 \cdot 0{,}73 \cdot \frac{10{,}2 \cdot 10^{3}\ \mathrm{N} \cdot 18\ \mathrm{m}}{8 \cdot 108 \cdot 10^{-6}\ \mathrm{m}^{3}} = 90{,}0 \cdot 10^{6}\ \mathrm{N/m^{2}} = 90{,}0\ \mathrm{N/mm^{2}} \qquad (9)$$

where

$V_{\sigma\mathrm{m}} V_{\mathrm{rm}}$ = 0,32·1,8 = 0,58	according to 6.4.2.3.1 above, value which is less than 1,8 = $(V_{\sigma\mathrm{m}} V_{\mathrm{rm}})_{\mathrm{max}}$ according to IEC 60865-1:2011, Table 2
β = 0,73	according to IEC 60865-1:2011, Table 3
W_{m} = 108·10^{-6}m^{3}	see 6.3

The bending stress during the second short-circuit current flow duration is greater than during the first short-circuit current flow duration, see 6.4.2.3.2.1.

The total bending stress is:

$$\sigma_{\mathrm{tot,d}} = \sqrt{\sigma_{\mathrm{m,d}}^{2} + \sigma_{\mathrm{st,m,k}}^{2}} = \sqrt{90{,}0^{2} + 28{,}8^{2}}\ \mathrm{N/mm^{2}} = 94{,}5\ \mathrm{N/mm^{2}}$$

The busbar is assumed to withstand the short-circuit force if

$$\sigma_{\mathrm{tot,d}} \le q\, f_{\mathrm{y}} \qquad (11)$$

with the lower value of f_{y} and where q = 1,32, see 6.4.2.2.1.1. This gives:

$$\sigma_{\mathrm{tot,d}} = 94{,}5\ \mathrm{N/mm^{2}} \quad \text{less than} \quad q\, f_{\mathrm{y}} = 1{,}32 \cdot 160\ \mathrm{N/mm^{2}} = 211\,\mathrm{N/mm^{2}}$$

6.4.2.3.2.3.2 Forces on the supports and moments on the substructures

The equivalent static force on the supports is:

$$F_{r,d} = V_F V_{rm} \alpha F_{m3} \tag{15}$$

During the first short-circuit current flow it is, see 6.4.2.3.1:

$$(V_F V_{rm})_1 = 0{,}36 \cdot 1{,}0 = 0{,}36$$

During the second current flow it is according to 6.4.2.3.1

$$(V_F V_{rm})_2 = 0{,}36 \cdot 1{,}8 = 0{,}65$$

which is lower than the value 1,0 according to IEC 60865-1:2011, Table 2.

According to IEC 60865-1:2011, 5.6, the greater of both values is to be inserted in Equation (15):

$$V_F V_{rm} = \max\left\{(V_F V_{rm})_1 ; (V_F V_{rm})_2\right\} = \max\{0{,}36 ; 0{,}65\} = 0{,}65$$

For the outer supports (A) it is with α_A = 0,375, see IEC 60865-1:2011, Table 3:

$$F_{r,dA} = V_F V_{rm} \alpha_A F_{m3} = 0{,}65 \cdot 0{,}375 \cdot 10{,}2 \text{ kN} = 2{,}49 \text{ kN}$$

For the inner supports (B) it is with α_B = 1,25, see IEC 60865-1:2011, Table 3:

$$F_{r,dB} = V_F V_{rm} \alpha_B F_{m3} = 0{,}65 \cdot 1{,}25 \cdot 10{,}2 \text{ kN} = 8{,}29 \text{ kN}$$

The bending moments on the substructures are:

– on the bottom of the outer insulators

$$M_{IA,d} = F_{r,dA} h_I = 2{,}49 \text{ kN} \cdot 3{,}7 \text{ m} = 9{,}21 \text{ kNm}$$

– on the bottom of the outer supports

$$M_{SA,d} = F_{r,dA} h_S = 2{,}49 \text{ kN} \cdot 7{,}0 \text{ m} = 17{,}4 \text{ kNm}$$

– on the bottom of the inner insulators

$$M_{IB,d} = F_{r,dB} h_I = 8{,}29 \text{ kN} \cdot 3{,}7 \text{ m} = 30{,}7 \text{ kNm}$$

– on the bottom of the inner supports

$$M_{SB,d} = F_{r,dB} h_S = 8{,}29 \text{ kN} \cdot 7{,}0 \text{ m} = 58{,}0 \text{ kNm}$$

6.4.3 Conclusions

The busbar will withstand the dead load

The calculated bending stress is	$\sigma_{st,m,d}$	39 N/mm^2
The outer supports have to withstand a vertical force of	$F_{st,r,dA}$	0,701 kN
The inner supports have to withstand a vertical force of	$F_{st,r,dB}$	2,335 kN

		Simplified method	Detailed method
a) Without three-phase automatic reclosure			
The busbar will withstand the short-circuit force			
The calculated bending stress is	$\sigma_{\text{tot,d}}$	158 N/mm^2	57 N/mm^2
The outer supports have to withstand an equivalent static force of	$F_{\text{r,dA}}$	4,7 kN	1,38 kN
The inner supports have to withstand an equivalent static force of	$F_{\text{r,dB}}$	15,6 kN	4,59 kN
b) With three-phase automatic reclosure			
The calculated bending stress is	$\sigma_{\text{tot,d}}$	281 N/mm^2	95 N/mm^2
The busbar is assumed to withstand the short-circuit force when using the detailed method but not when using the simplified method			
The outer supports have to withstand an equivalent static force of	$F_{\text{r,dA}}$	4,7 kN	2,49 kN
The inner supports have to withstand an equivalent static force of	$F_{\text{r,dB}}$	15,6 kN	8,29 kN

7 Example 4. – Mechanical effects on a 110 kV arrangement with slack conductors

7.1 General

The basis for the calculations in this example is a three-phase flexible busbar connection with one all-aluminium stranded conductor per phase with varying distances between the conductors. The anchor points at each end of the span are post insulators on steel substructures as shown in Figure 4.

The effective length of the span is the distance between the axis of the supports reduced by

– the extend of the connection plate of the equipment including the clamp, and
– an additional form factor which depends on conductor stiffness and mounting form, for example 0,1 m to 0,3 m.

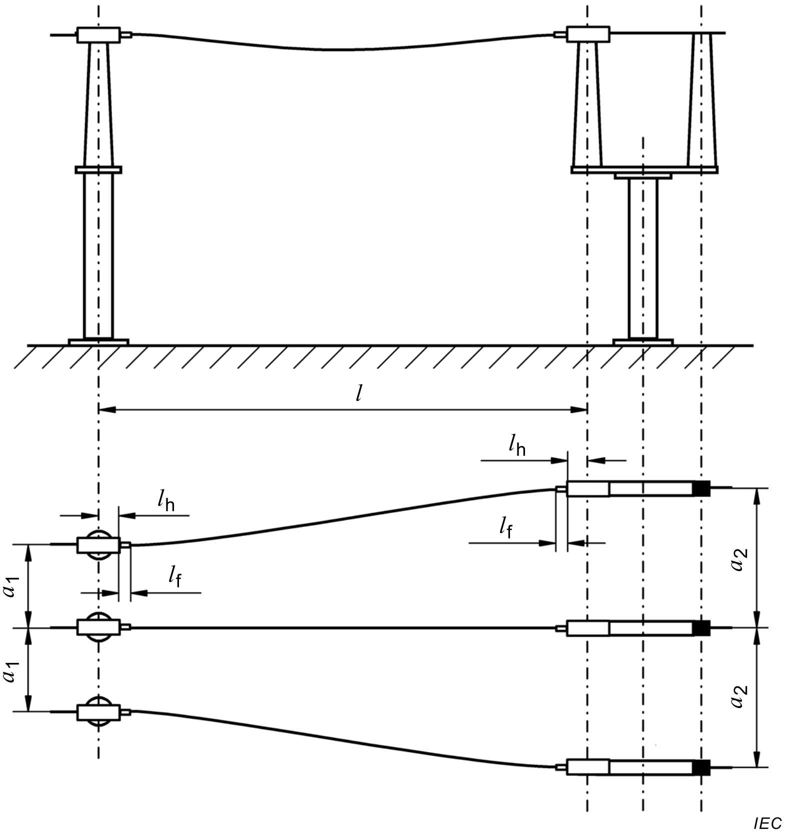


Figure 4 – Arrangement with slack conductors

7.2 Data

Initial symmetrical three-phase short-circuit current (r.m.s.)	I''_k	=	19 kA
Duration of the first short-circuit current flow	T_{k1}	=	0,3 s
Centre-line distance between supports	l	=	11,5 m
Extend of one head armature and clamp	l_h	=	0,4 m
Form factor	l_f	=	0,15 m
Centre-line distances between conductors	a_1	=	1,6 m
	a_2	=	2,4 m
Resultant spring constant of both span supports	S	=	100 N/mm
All-aluminium stranded conductor EN 243-AL1			
– Number of sub-conductors	n	=	1
– Cross section	A_s	=	243 mm^2
– Mass per unit length	m'_s	=	0,671 kg/m
– Young's modulus	E	=	55 000 N/mm^2
Static tensile force of one flexible main conductor at a temperature of −20°C (local minimum winter temperature)	$F_{st,-20}$	=	350 N
Static tensile force of one flexible main conductor at a temperature of 60°C (maximum operating temperature)	$F_{st,60}$	=	250 N
Conventional value of acceleration of gravity	g	=	9,81 m/s^2

7.3 Electromagnetic load and characteristic parameters

The characteristic electromagnetic load per unit length is:

$$F' = \frac{\mu_0}{2\pi} 0{,}75 \frac{(I_k'')^2}{a} \frac{l_c}{l_{eff}} = \frac{4\pi \cdot 10^{-7}}{2\pi} \frac{Vs}{Am} \cdot 0{,}75 \cdot \frac{\left(19 \cdot 10^3 \text{ A}\right)^2}{2 \text{ m}} \cdot 1{,}0 = 27{,}1 \text{ N/m} \quad (19a)$$

with the effective length of the span

$$l_{eff} = l - 2l_h - 2l_f = 11{,}5 \text{ m} - 2 \cdot 0{,}4 \text{ m} - 2 \cdot 0{,}15 \text{ m} = 10{,}4 \text{ m}$$

and

$$l_c = l_{eff}$$

and an equivalent distance

$$a = \frac{a_1 + a_2}{2} = \frac{1{,}6 \text{ m} + 2{,}4 \text{ m}}{2} = 2 \text{ m}.$$

The parameter r is:

$$r = \frac{F'}{n m_s' g} = \frac{27{,}1 \text{ N/m}}{1 \cdot 0{,}671 \text{ kg/m} \cdot 9{,}81 \text{ m/s}^2} = 4{,}12 \quad (20)$$

The direction of the resulting force on the conductor is:

$$\delta_1 = \arctan r = \arctan 4{,}12 = 76{,}4° \quad (21)$$

The equivalent static conductor sags at midspan are:

$$f_{es,-20} = \frac{n m_s' g l_{eff}^2}{8 F_{st,-20}} = \frac{1 \cdot 0{,}671 \text{ kg/m} \cdot 9{,}81 \text{ m/s}^2 \cdot (10{,}4 \text{ m})^2}{8 \cdot 350 \text{ N}} = 0{,}254 \text{ m}$$

$$f_{es,60} = \frac{n m_s' g l_{eff}^2}{8 F_{st,60}} = \frac{1 \cdot 0{,}671 \text{ kg/m} \cdot 9{,}81 \text{ m/s}^2 \cdot (10{,}4 \text{ m})^2}{8 \cdot 250 \text{ N}} = 0{,}356 \text{ m} \quad (22)$$

The periods of the conductor oscillation are:

$$T_{-20} = 2\pi \sqrt{0{,}8 \frac{f_{es,-20}}{g}} = 2\pi \sqrt{0{,}8 \frac{0{,}254}{9{,}81 \text{ m/s}^2}} = 0{,}904 \text{ s}$$

$$T_{60} = 2\pi \sqrt{0{,}8 \frac{f_{es,60}}{g}} = 2\pi \sqrt{0{,}8 \frac{0{,}356}{9{,}81 \text{ m/s}^2}} = 1{,}071 \text{ s} \quad (23)$$

The resulting periods of the conductor oscillation are:

$$T_{\text{res,-20}} = \frac{T_{\text{-20}}}{\sqrt[4]{1+r^2}\left[1-\frac{\pi^2}{64}\left(\frac{\delta_1}{90°}\right)^2\right]} = \frac{0{,}904}{\sqrt[4]{1+4{,}12^2}\left[1-\frac{\pi^2}{64}\left(\frac{76{,}4°}{90°}\right)^2\right]} = 0{,}494\ \text{s}$$

$$T_{\text{res,60}} = \frac{T_{60}}{\sqrt[4]{1+r^2}\left[1-\frac{\pi^2}{64}\left(\frac{\delta_1}{90°}\right)^2\right]} = \frac{1{,}071}{\sqrt[4]{1+4{,}12^2}\left[1-\frac{\pi^2}{64}\left(\frac{76{,}4°}{90°}\right)^2\right]} = 0{,}585\ \text{s} \quad (24)$$

The stiffness norms are:

$$N_{\text{-20}} = \frac{1}{S\,l_{\text{eff}}} + \frac{1}{n\,E_{\text{eff,-20}}\,A_{\text{s}}} = \frac{1}{10^5\ \text{N/m}\cdot 10{,}4\ \text{m}} + \frac{1}{1\cdot 1{,}82\cdot 10^{10}\ \text{N/m}^2\cdot 243\cdot 10^{-6}\ \text{m}^2} = 1{,}188\cdot 10^{-6}\ \text{1/N}$$

$$N_{60} = \frac{1}{S\,l_{\text{eff}}} + \frac{1}{n\,E_{\text{eff,60}}\,A_{\text{s}}} = \frac{1}{10^5\ \text{N/m}\cdot 10{,}4\ \text{m}} + \frac{1}{1\cdot 1{,}78\cdot 10^{10}\ \text{N/m}^2\cdot 243\cdot 10^{-6}\ \text{m}^2} = 1{,}193\cdot 10^{-6}\ \text{1/N} \quad (25)$$

with the actual Young's moduli

$$E_{\text{eff,-20}} = E\left[0{,}3+0{,}7\sin\left(\frac{F_{\text{st,-20}}}{n\,A_{\text{s}}\,\sigma_{\text{fin}}}90°\right)\right] = 55\cdot 10^9\ \frac{\text{N}}{\text{m}^2}\cdot\left[0{,}3+0{,}7\sin\left(\frac{1{,}44\cdot 10^6\ \text{N/m}^2}{50\cdot 10^6\ \text{N/m}^2}\cdot 90°\right)\right]$$

$$= 1{,}82\cdot 10^{10}\ \text{N/m}^2 \quad (26)$$

$$E_{\text{eff,60}} = E\left[0{,}3+0{,}7\sin\left(\frac{F_{\text{st,60}}}{n\,A_{\text{s}}\,\sigma_{\text{fin}}}90°\right)\right] = 55\cdot 10^9\ \frac{\text{N}}{\text{m}^2}\cdot\left[0{,}3+0{,}7\sin\left(\frac{1{,}03\cdot 10^6\ \text{N/m}^2}{50\cdot 10^6\ \text{N/m}^2}\cdot 90°\right)\right]$$

$$= 1{,}78\cdot 10^{10}\ \text{N/m}^2$$

because

$$\frac{F_{\text{st,-20}}}{n\,A_{\text{s}}} = \frac{350\ \text{N}}{1\cdot 243\cdot 10^{-6}\ \text{m}^2} = 1{,}44\cdot 10^6\ \text{N/m}^2 \quad \text{less than} \quad \sigma_{\text{fin}} = 50\cdot 10^6\ \text{N/m}^2$$

$$\frac{F_{\text{st,60}}}{n\,A_{\text{s}}} = \frac{250\ \text{N}}{1\cdot 243\cdot 10^{-6}\ \text{m}^2} = 1{,}03\cdot 10^6\ \text{N/m}^2 \quad \text{less than} \quad \sigma_{\text{fin}} = 50\cdot 10^6\ \text{N/m}^2$$

The stress factors are:

$$\zeta_{\text{-20}} = \frac{(n\,g\,m'_{\text{s}}\,l_{\text{eff}})^2}{24\,F_{\text{st,-20}}^3\,N_{\text{-20}}} = \frac{\left(1\cdot 9{,}81\ \text{m/s}^2\cdot 0{,}671\ \text{kg/m}\cdot 10{,}4\ \text{m}\right)^2}{24(350\ \text{N})^3\cdot 1{,}188\cdot 10^{-6}\ \text{1/N}} = 3{,}84$$

$$\zeta_{60} = \frac{(n\,g\,m'_{\text{s}}\,l_{\text{eff}})^2}{24F_{\text{st,60}}^3\,N_{60}} = \frac{\left(1\cdot 9{,}81\ \text{m/s}^2\cdot 0{,}671\ \text{kg/m}\cdot 10{,}4\ \text{m}\right)^2}{24(250\ \text{N})^3\cdot 1{,}193\cdot 10^{-6}\ \text{1/N}} = 10{,}5 \quad (28)$$

Because

$$T_{\text{k1}} = 0{,}3\ \text{s} \quad \text{less than} \quad 0{,}4\,T_{\text{-20}} = 0{,}4\cdot 0{,}904\ \text{s} = 0{,}361\ \text{s}$$

$$T_{\text{k1}} = 0{,}3\ \text{s} \quad \text{less than} \quad 0{,}4\,T_{60} = 0{,}4\cdot 1{,}071\ \text{s} = 0{,}428\ \text{s}$$

in the Equations (29), (32), and (35) it is to be inserted:

$$T_{k1} = 0,3 \text{ s}$$

The swing-out angles at the end of short-circuit current flow are:

$$\delta_{end,-20} = \delta_{end,60} = 2\,\delta_1 = 2 \cdot 76,4° = 153° \tag{29}$$

because

$$\frac{T_{k1}}{T_{res,-20}} = \frac{0,3 \text{ s}}{0,494 \text{ s}} = 0,607 \quad \text{greater than} \quad 0,5$$
$$\frac{T_{k1}}{T_{res,60}} = \frac{0,3 \text{ s}}{0,585 \text{ s}} = 0,513 \quad \text{greater than} \quad 0,5$$

The maximum swing-out angles $\delta_{max,-20}$ and $\delta_{max,60}$ depend respectively on χ_{-20} and χ_{60} which depend on $\delta_{end,-20}$ and $\delta_{end,60}$:

For $\delta_{end,-20} = \delta_{end,60} = 153°$ greater than 90° it is:

$$\chi_{-20} = \chi_{60} = 1 - r = 1 - 4,12 = -3,12 \tag{30}$$

and for $\chi_{-20} = \chi_{60} = -3,12$ less than $-0,985$ it is:

$$\delta_{max,-20} = \delta_{max,60} = 180° \tag{31}$$

7.4 Tensile force $F_{t,d}$ during short-circuit caused by swing out

The calculation is done according to IEC 60865-1:2011, 6.2.3.

The load parameters are:

$$\varphi_{60} = \varphi_{-20} = 3\left(\sqrt{1+r^2} - 1\right) = 3\left(\sqrt{1+4,12^2} - 1\right) = 9,72 \tag{32}$$

because

$$T_{k1} = 0,3 \text{ s} \quad \text{greater than} \quad \frac{T_{res,-20}}{4} = \frac{0,494 \text{ s}}{4} = 0,124 \text{ s}$$
$$T_{k1} = 0,3 \text{ s} \quad \text{greater than} \quad \frac{T_{res,60}}{4} = \frac{0,585 \text{ s}}{4} = 0,146 \text{ s}$$

According to IEC 60865-1:2011, Figure 8, the factors ψ_{-20} and ψ_{60} are:

– for $\varphi_{-20} = 9,72$ and $\zeta_{-20} = 3,84$:

$$\psi_{-20} = 0,594$$

– for $\varphi_{60} = 9,72$ and $\zeta_{60} = 10,5$:

$$\psi_{60} = 0,745$$

The tensile forces during the short-circuit are:

$$F_{\text{t,d,-20}} = F_{\text{st,-20}}\left(1+\varphi_{\text{-20}}\,\psi_{\text{-20}}\right) = 350\ \text{N}\cdot\left(1+9{,}72\cdot 0{,}594\right) = 2371\,\text{N} = 2{,}37\ \text{kN}$$
$$F_{\text{t,d,60}} = F_{\text{st,60}}\left(1+\varphi_{60}\,\psi_{60}\right) = 250\ \text{N}\cdot\left(1+9{,}72\cdot 0{,}745\right) = 2060\ \text{N} = 2{,}06\ \text{kN} \qquad (33)$$

The tensile force $F_{\text{t,d}}$ is the maximum value of $F_{\text{t,d,-20}}$ and $F_{\text{t,d,60}}$:

$$F_{\text{t,d}} = \max\left\{F_{\text{t,d,-20}}\,;F_{\text{t,d,60}}\right\} = \max\left\{2{,}37\ \text{kN}\,;2{,}06\ \text{kN}\right\} = 2{,}37\ \text{kN}$$

7.5 Dynamic conductor sag at midspan

All the following quantities are calculated at a conductor temperature of 60°C which leads to a greater conductor sag than a conductor temperature of –20°C.

The elastic expansion is:

$$\varepsilon_{\text{ela}} = N_{60}\left(F_{\text{t,d,60}} - F_{\text{st,60}}\right) = 1{,}193\cdot 10^{-6}\,\frac{1}{\text{N}}\cdot\left(2060\ \text{N} - 250\ \text{N}\right) = 2{,}16\cdot 10^{-3} \qquad (34)$$

The thermal expansion is:

$$\varepsilon_{\text{th}} = c_{\text{th}}\left(\frac{I_k''}{n\,A_\text{s}}\right)^2\frac{T_{\text{res,60}}}{4} = 0{,}27\cdot 10^{-18}\,\frac{\text{m}^4}{\text{A}^2\text{s}}\cdot\left(\frac{19\cdot 10^3\ \text{A}}{1\cdot 243\cdot 10^{-6}\ \text{m}^2}\right)^2\cdot\frac{0{,}585\ \text{s}}{4} = 2{,}41\cdot 10^{-4} \qquad (35)$$

because

$$T_{\text{k1}} = 0{,}3\ \text{s} \qquad \text{greater than} \qquad \frac{T_{\text{res,60}}}{4} = \frac{0{,}585\ \text{s}}{4} = 0{,}146\ \text{s}$$

with

$$c_{\text{th}} = 0{,}27\cdot 10^{-18}\ \text{m}^4/\left(\text{A}^2\text{s}\right) \qquad \text{for all-aluminium conductors}$$

The factor C_D is:

$$C_\text{D} = \sqrt{1+\frac{3}{8}\left(\frac{l_{\text{eff}}}{f_{\text{es,60}}}\right)^2\left(\varepsilon_{\text{ela}}+\varepsilon_{\text{th}}\right)} = \sqrt{1+\frac{3}{8}\left(\frac{10{,}4\ \text{m}}{0{,}356\ \text{m}}\right)^2\left(2{,}16\cdot 10^{-3}+2{,}41\cdot 10^{-4}\right)} = 1{,}33 \qquad (36)$$

The factor C_F is:

$$C_\text{F} = 1{,}15 \qquad (37)$$

because

$$r = 4{,}12 \qquad \text{greater than} \qquad 1{,}8$$

The dynamic conductor sag at midspan is:

$$f_{\text{ed}} = C_{\text{F}}\, C_{\text{D}}\, f_{\text{es,60}} = 1{,}15 \cdot 1{,}33 \cdot 0{,}356\ \text{m} = 0{,}55\ \text{m} \tag{38}$$

7.6 Tensile force $F_{\text{f,d}}$ after short-circuit caused by drop

Because

$$r = 4{,}12 \qquad \text{greater than} \qquad 0{,}6$$

and

$$\delta_{\text{max,-20}} = \delta_{\text{max,60}} = 180° \qquad \text{greater than} \qquad 70°$$

the drop force after short-circuit $F_{\text{f,d}}$ is significant:

$$\begin{aligned} F_{\text{f,d,-20}} &= 1{,}2\, F_{\text{st,-20}} \sqrt{1 + 8\, \zeta_{\text{-20}} \frac{\delta_{\text{max,-20}}}{180°}} = 1{,}2 \cdot 350\ \text{N} \cdot \sqrt{1 + 8 \cdot 3{,}84 \cdot \frac{180°}{180°}} = 2366\ \text{N} = 2{,}37\ \text{kN} \\ F_{\text{f,d,60}} &= 1{,}2\, F_{\text{st,60}} \sqrt{1 + 8\, \zeta_{60} \frac{\delta_{\text{max,60}}}{180°}} = 1{,}2 \cdot 250\ \text{N} \cdot \sqrt{1 + 8 \cdot 10{,}5 \cdot \frac{180°}{180°}} = 2766\ \text{N} = 2{,}77\ \text{kN} \end{aligned} \tag{43}$$

The drop force $F_{\text{f,d}}$ is the maximum of $F_{\text{f,d,-20}}$ and $F_{\text{f,d,60}}$:

$$F_{\text{f,d}} = \max\left\{F_{\text{f,-20}}\,;\, F_{\text{f,d,60}}\right\} = \max\left\{2{,}37\ \text{kN}\,;\, 2{,}77\ \text{kN}\right\} = 2{,}77\ \text{kN}$$

7.7 Horizontal span displacement b_{h} and minimum air clearance a_{min}

The maximum horizontal span displacement is:

$$b_{\text{h}} = f_{\text{ed}} = 0{,}55\ \text{m} \tag{44}$$

because

$$\delta_{\text{max,60}} = 180° \qquad \text{greater than} \qquad 90°$$

and the minimum air clearance is:

$$a_{\text{min}} = a - 2 b_{\text{h}} = 2\ \text{m} - 2 \cdot 0{,}55\ \text{m} = 0{,}90\ \text{m} \tag{48}$$

7.8 Conclusions

According to IEC 60865-1:2011, 6.5.1 and 6.5.3, the supports (post insulators and steel structures) and the foundations have to withstand a bending force

$$\max\left\{F_{\text{t,d}}\,;\, F_{\text{f,d}}\right\} = \max\left\{2{,}37\ \text{kN}\,;\, 2{,}77\ \text{kN}\right\} = 2{,}77\ \text{kN}$$

given by the tensile force $F_{\text{f,d}}$ after short-circuit caused by drop.

The clamping device for the conductor anchoring shall be specified with a rating based on the force

$$\max\{1{,}5\,F_{\text{t,d}}\,;1{,}0\,F_{\text{f,d}}\} = \max\{1{,}5\cdot 2{,}37\text{ kN}\,;1{,}0\cdot 2{,}77\text{ kN}\} = \max\{3{,}56\text{ kN}\,;2{,}77\text{ kN}\} = 3{,}56\text{ kN}$$

The horizontal displacement is 0,55 m and the minimum air clearance is 0,90 m.

8 Example 5. – Mechanical effects on strained conductors

8.1 General

The basis for the calculations in this example is a three-phase 380 kV arrangement with strained twin-bundle conductors as shown in Figure 5. In the span there are two connections of pantograph-disconnectors, which also operate as spacers, and between the connections one spacer.

The calculation is carried out for two different centre-line distances between sub-conductors showing the effect of the pinch force.

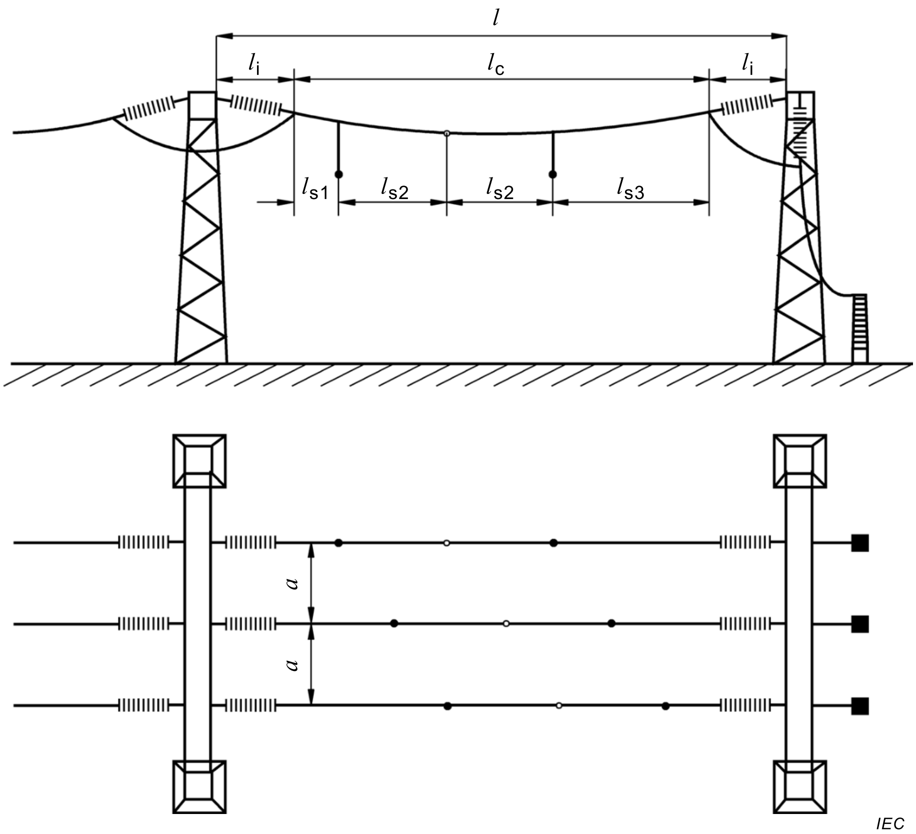


Figure 5 – Arrangement with strained conductors

8.2 Common data

Initial symmetrical three-phase short-circuit current (r.m.s.)	I_k''	=	63 kA
Factor for the calculation of the peak short-circuit current	κ	=	1,81
Duration of the short-circuit current flow	T_{k1}	=	0,5 s
System frequency	f	=	50 Hz
Centre-line distance between supports	l	=	48 m
Length of one insulator chain	l_i	=	5,3 m

Cord length $l_c = l - 2l_i$	l_c	=	37,4 m
Centre-line distance between conductors	a	=	5 m
Resultant spring constant of both span supports of one span	S	=	500 N/mm
Twin conductor 2 EN 1046-AL1/45-ST1A			
– Number of sub-conductors	n	=	2
– Outer diameter of one sub-conductor	d	=	43 mm
– Cross-section of one sub-conductor	A_s	=	1 090 mm^2
– Mass per unit length of one sub-conductor	m'_s	=	3,25 kg/m
– Young's modulus	E	=	60 000 N/mm^2
Static tensile force of one flexible main conductor at a temperature of −20°C (local minimum winter temperature)	$F_{st,-20}$	=	17,8 kN
Static tensile force of one flexible main conductor at a temperature of 60°C (maximum operating temperature)	$F_{st,60}$	=	15,4 kN
Additional concentrated masses representing the connections of panthograph disconnectors			
– Number of spacers	n_c	=	3
– Mass of one connection	m_c	=	36 kg
– Mass of one spacer	m_{cs}	=	2 kg
– Distances	l_{s1}	=	4,2 m
	l_{s2}	=	9,5 m
	l_{s3}	=	14,2 m
Conventional value of acceleration of gravity	g	=	9,81 m/s^2

8.3 Centre-line distance between sub-conductors a_s = 0,1 m

8.3.1 Electromagnetic load and characteristic parameters

The characteristic electromagnetic load per unit length is:

$$F' = \frac{\mu_0}{2\pi} 0{,}75 \frac{(I''_k)^2}{a} \frac{l_c}{l} = \frac{4\pi \cdot 10^{-7}}{2\pi} \frac{\text{Vs}}{\text{Am}} \cdot 0{,}75 \cdot \frac{\left(63 \cdot 10^3\ \text{A}\right)^2}{5\ \text{m}} \cdot \frac{37{,}4\ \text{m}}{48\ \text{m}} = 92{,}8\ \text{N/m} \qquad (19a)$$

The parameter r is:

$$r = \frac{F'}{n\, m'_{sc}\, g} = \frac{92{,}8\ \text{N/m}}{2 \cdot 4{,}24\ \text{kg/m} \cdot 9{,}81\ \text{m/s}^2} = 1{,}12 \qquad (20)$$

where m'_{sc} is the resulting mass per unit length of one sub-conductor including concentrated masses:

$$m'_{sc} = m'_s + \frac{2\, m_c + m_{cs}}{n\, l_c} = 3{,}25\ \frac{\text{kg}}{\text{m}} + \frac{2 \cdot 36\ \text{kg} + 2\ \text{kg}}{2 \cdot 37{,}4\ \text{m}} = 4{,}24\ \text{kg/m}$$

The direction of the resulting force on the conductor is:

$$\delta_1 = \arctan r = \arctan 1{,}12 = 48{,}2^\circ \qquad (21)$$

The equivalent static conductor sags at midspan are:

$$\begin{aligned} f_{\text{es,-20}} &= \frac{n\, m'_{\text{sc}}\, g\, l^2}{8\, F_{\text{st,-20}}} = \frac{2 \cdot 4{,}24\ \text{kg/m} \cdot 9{,}81\ \text{m/s}^2 \cdot (48\ \text{m})^2}{8 \cdot 17{,}8 \cdot 10^3\ \text{N}} = 1{,}35\ \text{m} \\ f_{\text{es,60}} &= \frac{n\, m'_{\text{sc}}\, g\, l^2}{8\, F_{\text{st,60}}} = \frac{2 \cdot 4{,}24\ \text{kg/m} \cdot 9{,}81\ \text{m/s}^2 \cdot (48\ \text{m})^2}{8 \cdot 15{,}4 \cdot 10^3\ \text{N}} = 1{,}56\ \text{m} \end{aligned} \qquad (22)$$

The periods of the conductor oscillation are:

$$\begin{aligned} T_{\text{-20}} &= 2\pi \sqrt{0{,}8 \frac{f_{\text{es,-20}}}{g}} = 2\pi \sqrt{0{,}8 \frac{1{,}35\ \text{m}}{9{,}81\ \text{m/s}^2}} = 2{,}09\ \text{s} \\ T_{60} &= 2\pi \sqrt{0{,}8 \frac{f_{\text{es,60}}}{g}} = 2\pi \sqrt{0{,}8 \frac{1{,}56\ \text{m}}{9{,}81\ \text{m/s}^2}} = 2{,}24\ \text{s} \end{aligned} \qquad (23)$$

The resulting periods of the conductor oscillation are:

$$\begin{aligned} T_{\text{res,-20}} &= \frac{T_{\text{-20}}}{\sqrt[4]{1+r^2}\left[1 - \frac{\pi^2}{64}\left(\frac{\delta_1}{90^\circ}\right)^2\right]} = \frac{2{,}09}{\sqrt[4]{1+1{,}12^2}\left[1 - \frac{\pi^2}{64}\left(\frac{48{,}2^\circ}{90^\circ}\right)^2\right]} = 1{,}79\ \text{s} \\ T_{\text{res,60}} &= \frac{T_{60}}{\sqrt[4]{1+r^2}\left[1 - \frac{\pi^2}{64}\left(\frac{\delta_1}{90^\circ}\right)^2\right]} = \frac{2{,}24}{\sqrt[4]{1+1{,}12^2}\left[1 - \frac{\pi^2}{64}\left(\frac{48{,}2^\circ}{90^\circ}\right)^2\right]} = 1{,}91\ \text{s} \end{aligned} \qquad (24)$$

The stiffness norms are:

$$\begin{aligned} N_{\text{-20}} &= \frac{1}{S\, l} + \frac{1}{n\, E_{\text{eff,-20}}\, A_{\text{s}}} = \frac{1}{5 \cdot 10^5\ \text{N/m} \cdot 48\ \text{m}} + \frac{1}{2 \cdot 2{,}87 \cdot 10^{10}\ \text{N/m}^2 \cdot 1090 \cdot 10^{-6}\ \text{m}^2} = 5{,}77 \cdot 10^{-8}\ \text{1/N} \\ N_{60} &= \frac{1}{S\, l} + \frac{1}{n\, E_{\text{s,60}}\, A_{\text{s}}} = \frac{1}{5 \cdot 10^5\ \text{N/m} \cdot 48\ \text{m}} + \frac{1}{2 \cdot 2{,}72 \cdot 10^{10}\ \text{N/m}^2 \cdot 1090 \cdot 10^{-6}\ \text{m}^2} = 5{,}85 \cdot 10^{-8}\ \text{1/N} \end{aligned} \qquad (25)$$

with the actual Young's moduli

$$\begin{aligned} E_{\text{eff,-20}} &= E\left[0{,}3 + 0{,}7 \sin\left(\frac{F_{\text{st,-20}}}{n\, A_{\text{s}}\, \sigma_{\text{fin}}} 90^\circ\right)\right] = 6 \cdot 10^{10}\ \frac{\text{N}}{\text{m}^2} \cdot \left[0{,}3 + 0{,}7 \sin\left(\frac{8{,}17 \cdot 10^6\ \text{N/m}^2}{50 \cdot 10^6\ \text{N/m}^2} 90^\circ\right)\right] \\ &= 2{,}87 \cdot 10^{10}\ \text{N/m}^2 \\ E_{\text{eff,60}} &= E\left[0{,}3 + 0{,}7 \sin\left(\frac{F_{\text{st,60}}}{n\, A_{\text{s}}\, \sigma_{\text{fin}}} 90^\circ\right)\right] = 6 \cdot 10^{10}\ \frac{\text{N}}{\text{m}^2} \cdot \left[0{,}3 + 0{,}7 \sin\left(\frac{7{,}06 \cdot 10^6\ \text{N/m}^2}{50 \cdot 10^6\ \text{N/m}^2} 90^\circ\right)\right] \\ &= 2{,}72 \cdot 10^{10}\ \text{N/m}^2 \end{aligned} \qquad (26)$$

because

$$\frac{F_{st,-20}}{n\,A_s} = \frac{17{,}8 \cdot 10^3\ \text{N}}{2 \cdot 1090 \cdot 10^{-6}\ \text{m}^2} = 8{,}17 \cdot 10^6\ \text{N/m}^2 \qquad \text{less than} \qquad \sigma_{fin} = 50 \cdot 10^6\ \text{N/m}^2$$

$$\frac{F_{st,60}}{n\,A_s} = \frac{15{,}4 \cdot 10^3\ \text{N}}{2 \cdot 1090 \cdot 10^{-6}\ \text{m}^2} = 7{,}06 \cdot 10^6\ \text{N/m}^2 \qquad \text{less than} \qquad \sigma_{fin} = 50 \cdot 10^6\ \text{N/m}^2$$

The stress factors are:

$$\zeta_{-20} = \frac{(n\,g\,m'_{sc}\,l)^2}{24\,F_{st,-20}^3\,N_{-20}} = \frac{\left(2 \cdot 9{,}81\ \text{m/s}^2 \cdot 4{,}24\ \text{kg/m} \cdot 48\ \text{m}\right)^2}{24\left(17{,}8 \cdot 10^3\ \text{N}\right)^3 \cdot 5{,}77 \cdot 10^{-8}\ \text{1/N}} = 2{,}04$$

$$\zeta_{60} = \frac{(n\,g\,m'_{sc}\,l)^2}{24\,F_{st,60}^3\,N_{60}} = \frac{\left(2 \cdot 9{,}81\ \text{m/s}^2 \cdot 4{,}24\ \text{kg/m} \cdot 48\ \text{m}\right)^2}{24\left(15{,}4 \cdot 10^3\ \text{N}\right)^3 \cdot 5{,}85 \cdot 10^{-8}\ \text{1/N}} = 3{,}11 \qquad (28)$$

Because

$$T_{k1} = 0{,}5\ \text{s} \qquad \text{less than} \qquad 0{,}4\,T_{-20} = 0{,}4 \cdot 2{,}09\ \text{s} = 0{,}836\ \text{s}$$

$$T_{k1} = 0{,}5\ \text{s} \qquad \text{less than} \qquad 0{,}4\,T_{60} = 0{,}4 \cdot 2{,}24\ \text{s} = 0{,}896\ \text{s}$$

in the Equations (29), (32), and (35) it is to be inserted:

$$T_{k1} = 0{,}5\ \text{s}$$

The swing-out angles at the end of short-circuit current flow are:

$$\delta_{end,-20} = \delta_1\left[1-\cos\left(360° \frac{T_{k1}}{T_{res,-20}}\right)\right] = 48{,}2° \cdot \left[1-\cos\left(360° \cdot \frac{0{,}5\ \text{s}}{1{,}79\ \text{s}}\right)\right] = 57{,}0°$$

$$\delta_{end,60} = \delta_1\left[1-\cos\left(360° \frac{T_{k1}}{T_{res,60}}\right)\right] = 48{,}2° \cdot \left[1-\cos\left(360° \cdot \frac{0{,}5\ \text{s}}{1{,}91\ \text{s}}\right)\right] = 51{,}8° \qquad (29)$$

because

$$\frac{T_{k1}}{T_{res,-20}} = \frac{0{,}5\ \text{s}}{1{,}79\ \text{s}} = 0{,}279 \qquad \text{less than} \qquad 0{,}5$$

$$\frac{T_{k1}}{T_{res,60}} = \frac{0{,}5\ \text{s}}{1{,}91\ \text{s}} = 0{,}262 \qquad \text{less than} \qquad 0{,}5$$

The maximum swing-out angles $\delta_{max,-20}$ and $\delta_{max,60}$ depend respectively on χ_{-20} and χ_{60} which depend on $\delta_{end,-20}$ and $\delta_{end,60}$:

– for 0 less than $\delta_{end,-20}$ = 57,0° less than 90° is:

$$\chi_{-20} = 1 - r\sin\delta_{end,-20} = 1 - 1{,}12 \cdot \sin 57{,}0° = 0{,}0607 \qquad (30)$$

and for −0,985 less than χ_{-20} = 0,0607 less than 0,766:

$$\delta_{max,-20} = 10° + \arccos\chi_{-20} = 10° + \arccos 0{,}0607 = 96{,}5° \qquad (31)$$

– for 0 less than $\delta_{\text{end,60}}$ = 51,8° less than 90° is:

$$\chi_{60} = 1 - r \sin \delta_{\text{end,60}} = 1 - 1{,}12 \cdot \sin 51{,}8° = 0{,}120 \tag{30}$$

and for −0,985 less than χ_{60} = 0,120 less than 0,766:

$$\delta_{\text{max,60}} = 10° + \arccos \chi_{60} = 10° + \arccos 0{,}120 = 93{,}1° \tag{31}$$

8.3.2 Tensile force $F_{\text{t,d}}$ during short-circuit caused by swing out

The calculation is done according to IEC 60865-1:2011, 6.2.3.

The load parameters are:

$$\varphi_{60} = \varphi_{-20} = 3\left(\sqrt{1+r^2} - 1\right) = 3\left(\sqrt{1+1{,}12^2} - 1\right) = 1{,}50 \tag{32}$$

because

$$T_{\text{k1}} = 0{,}5 \text{ s} \quad \text{greater than} \quad \frac{T_{\text{res,-20}}}{4} = \frac{1{,}79 \text{ s}}{4} = 0{,}448 \text{ s}$$

$$T_{\text{k1}} = 0{,}5 \text{ s} \quad \text{greater than} \quad \frac{T_{\text{res,60}}}{4} = \frac{1{,}91 \text{ s}}{4} = 0{,}478 \text{ s}$$

According to IEC 60865-1:2011, Figure 8, the factors ψ_{-20} and ψ_{60} are:

– for φ_{-20} = 1,50 and ζ_{-20} = 2,04:

$$\psi_{-20} = 0{,}691$$

– for φ_{60} = 1,50 and ζ_{60} = 3,11:

$$\psi_{60} = 0{,}759$$

The tensile forces during the short-circuit forces are:

$$F_{\text{t,d,-20}} = F_{\text{st,-20}}(1+\varphi_{-20}\psi_{-20}) = 17{,}8 \text{ kN}\cdot(1+1{,}50\cdot 0{,}691) = 36{,}3 \text{ kN}$$

$$F_{\text{t,d,60}} = F_{\text{st,60}}(1+\varphi_{60}\psi_{60}) = 15{,}4 \text{ kN}\cdot(1+1{,}50\cdot 0{,}759) = 32{,}9 \text{ kN} \tag{33}$$

The tensile force $F_{\text{t,d}}$ is the maximum value of $F_{\text{t,d,-20}}$ and $F_{\text{t,d,60}}$:

$$F_{\text{t,d}} = \max\left\{F_{\text{t,d,-20}}; F_{\text{t,d,60}}\right\} = \max\left\{36{,}3 \text{ kN}; 32{,}9 \text{ kN}\right\} = 36{,}3 \text{ kN}$$

8.3.3 Dynamic conductor sag at midspan

All the following quantities are calculated at a conductor temperature of 60 °C which leads to a greater conductor sag than at a conductor temperature of −20 °C.

The elastic expansion is:

$$\varepsilon_{\text{ela}} = N_{60}\left(F_{\text{t,d,60}} - F_{\text{st,60}}\right) = 5{,}85 \cdot 10^{-8} \frac{1}{\text{N}} \cdot \left(32{,}9 - 15{,}4\right) \cdot 10^3 \text{ N} = 1{,}02 \cdot 10^{-3} \tag{34}$$

The thermal expansion is:

$$\varepsilon_{\text{th}} = c_{\text{th}}\left(\frac{I''_{\text{k}}}{n\,A_{\text{s}}}\right)^2 \frac{T_{\text{res,60}}}{4} = 0{,}27\cdot 10^{-18}\ \frac{\text{m}^4}{\text{A}^2\,\text{s}}\cdot\left(\frac{63\cdot 10^3\ \text{A}}{2\cdot 1090\cdot 10^{-6}\ \text{m}^2}\right)^2\cdot\frac{1{,}91\ \text{s}}{4} = 1{,}08\cdot 10^{-4} \qquad (35)$$

because

$$T_{\text{k1}} = 0{,}5\ \text{s} \qquad \text{greater than} \qquad \frac{T_{\text{res,60}}}{4} = \frac{1{,}91\ \text{s}}{4} = 0{,}478\ \text{s}$$

and for ASCR conductors with $A_{\text{Al}}/A_{\text{St}} = 1046\ \text{mm}/45\ \text{mm} = 23{,}2$ greater than 6:

$$c_{\text{th}} = 0{,}27\cdot 10^{-18}\ \text{m}^4/(\text{A}^2\text{s})$$

The factor C_{D} is:

$$C_{\text{D}} = \sqrt{1+\frac{3}{8}\left(\frac{l}{f_{\text{es,60}}}\right)^2\left(\varepsilon_{\text{ela}}+\varepsilon_{\text{th}}\right)} = \sqrt{1+\frac{3}{8}\left(\frac{48\ \text{m}}{1{,}56\ \text{m}}\right)^2\left(1{,}02\cdot 10^{-3}+1{,}08\cdot 10^{-4}\right)} = 1{,}18 \qquad (36)$$

The factor C_{F} is:

$$C_{\text{F}} = 0{,}97 + 0{,}1r = 0{,}97 + 0{,}1\cdot 1{,}12 = 1{,}08 \qquad (37)$$

because

$$0{,}8 \qquad \text{less than} \qquad r = 1{,}12 \qquad \text{less than} \qquad 1{,}8$$

The dynamic conductor sag at midspan is:

$$f_{\text{ed}} = C_{\text{F}}\,C_{\text{D}}\,f_{\text{es,60}} = 1{,}08\cdot 1{,}18\cdot 1{,}56\ \text{m} = 1{,}99\ \text{m} \qquad (38)$$

8.3.4 Tensile force $F_{\text{f,d}}$ after short-circuit caused by drop

Because

$$r = 1{,}12 \qquad \text{greater than} \qquad 0{,}6$$

and

$$\delta_{\text{max,-20}} = 96{,}5^\circ \qquad \text{greater than} \qquad 70^\circ$$
$$\delta_{\text{max,60}} = 93{,}1^\circ \qquad \text{greater than} \qquad 70^\circ$$

the tensile force after short-circuit $F_{\text{f,d}}$ is significant:

$$\begin{aligned} F_{\text{f,d,-20}} &= 1{,}2\cdot F_{\text{st,-20}}\sqrt{1+8\zeta_{\text{-20}}\frac{\delta_{\text{max,-20}}}{180^\circ}} = 1{,}2\cdot 17{,}8\ \text{kN}\cdot\sqrt{1+8\cdot 2{,}04\cdot\frac{96{,}5^\circ}{180^\circ}} = 66{,}7\ \text{kN} \\ F_{\text{f,d,60}} &= 1{,}2\cdot F_{\text{st,60}}\sqrt{1+8\zeta_{60}\frac{\delta_{\text{max,60}}}{180^\circ}} = 1{,}2\cdot 15{,}4\ \text{kN}\cdot\sqrt{1+8\cdot 3{,}11\cdot\frac{93{,}1^\circ}{180^\circ}} = 68{,}8\ \text{kN} \end{aligned} \qquad (43)$$

The tensile force $F_{f,d}$ is the maximum of $F_{f,d,-20}$ and $F_{f,d,60}$:

$$F_{f,d} = \max\{F_{f,d,-20}; F_{f,d,60}\} = \max\{66,7\ \text{kN}; 68,8\ \text{kN}\} = 68,8\ \text{kN}$$

8.3.5 Horizontal span displacement b_h and minimum air clearance a_{min}

The maximum horizontal span displacement for strained conductors with $l_c = l - 2l_i$ is:

$$b_h = f_{ed} \sin\delta_1 = 1,99\ \text{m} \cdot \sin 48,2° = 1,48\ \text{m} \tag{45}$$

because

$$\delta_{max,60} = 93,1° \qquad \text{greater than} \qquad \delta_1 = 48,2°$$

and the minimum air clearance is:

$$a_{min} = a - 2b_h = 5\ \text{m} - 2 \cdot 1,48\ \text{m} = 2,04\ \text{m} \tag{48}$$

8.3.6 Pinch force $F_{pi,d}$

The sub-conductors clash effectively during short-circuit because Equation (53) is fulfilled:

$$\frac{a_s}{d} = \frac{0,1\ \text{m}}{0,043\ \text{m}} = 2,33 \qquad \text{less than} \qquad 2,5 \tag{53}$$

and

$$l_s = 9,35\ \text{m} \qquad \text{greater than} \qquad 70\,a_s = 70 \cdot 0,1\ \text{m} = 7\ \text{m} \tag{53}$$

with

$$l_s = \frac{l_{s1} + 2l_{s2} + l_{s3}}{4} = \frac{l_c}{4} = \frac{4,2 + 2 \cdot 9,5 + 14,2}{4}\ \text{m} = \frac{37,4\ \text{m}}{4} = 9,35\ \text{m}$$

The tensile forces caused by pinch are:

$$\begin{aligned} F_{pi,d,-20} &= 1,1 F_{t,d,-20} = 1,1 \cdot 36,3\ \text{kN} = 39,9\ \text{kN} \\ F_{pi,d,60} &= 1,1 F_{t,d,60} = 1,1 \cdot 32,9\ \text{kN} = 36,2\ \text{kN} \end{aligned} \tag{51}$$

$F_{t,d,-20}$ and $F_{t,d,60}$ are calculated in 8.3.2.

The pinch force $F_{pi,d}$ is the maximum of $F_{pi,d,-20}$ and $F_{pi,d,60}$:

$$F_{pi,d} = \max\{F_{pi,d,-20}; F_{pi,d,60}\} = \max\{39,9\ \text{kN}; 36,2\ \text{kN}\} = 39,9\ \text{kN}$$

8.3.7 Conclusions

According to IEC 60865-1:2011, 6.5.2, to the structure, the insulators and the connectors the maximum value of $F_{t,d}$, $F_{f,d}$ and $F_{pi,d}$ shall be applied as a static load:

$$\max\left\{F_{t,d};F_{f,d};F_{pi,d}\right\} = \max\left\{36{,}3\text{ kN};68{,}8\text{ kN};39{,}9\text{ kN}\right\} = 68{,}8\text{ kN}$$

given by the tensile force $F_{f,d}$ after short-circuit caused by drop.

The maximum horizontal displacement is 1,48 m and the minimum air clearance is 2,04 m.

8.4 Centre-line distance between sub-conductors a_s = 0,4 m

8.4.1 Preliminary remarks

In this case it is

$$\frac{a_s}{d} = \frac{0{,}400\text{ m}}{0{,}043\text{ m}} = 9{,}30$$

and neither Equation (52) nor Equation (53) of IEC 60865-1:2011 is fulfilled. Therefore the pinch force $F_{pi,d}$ is to be calculated with the Equations (54) and following of IEC 60865-1:2011, 6.4. The other results are the same as 8.3.2, 8.3.3, 8.3.4 and 8.3.5, they do not depend on the centre-line distance of the sub-conductors:

Tensile force during short-circuit	$F_{t,d}$	=	36,3 kN
Tensile force after short-circuit	$F_{f,d}$	=	68,8 kN
Maximum horizontal displacement	b_h	=	1,48 m
Minimum air clearance	a_{min}	=	2,04 m

8.4.2 Characteristic dimensions and parameters

The short-circuit current force between the sub-conductors is:

$$F_v = (n-1)\frac{\mu_0}{2\pi}\left(\frac{I_k''}{n}\right)^2 \frac{l_s}{a_s}\frac{\nu_2}{\nu_3} \tag{54}$$

The factor ν_1 for calculation of ν_2 is:

$$\nu_1 = f\frac{1}{\sin\frac{180°}{n}}\sqrt{\frac{(a_s-d)m_s'}{\frac{\mu_0}{2\pi}\left(\frac{I_k''}{n}\right)^2\frac{n-1}{a_s}}} = 50\,\frac{1}{\text{s}}\cdot\frac{1}{\sin\frac{180°}{2}}\cdot\sqrt{\frac{(0{,}400\text{ m}-0{,}043\text{ m})\cdot 3{,}25\text{ kg/m}}{\frac{4\pi\cdot 10^{-7}}{2\pi}\frac{\text{Vs}}{\text{Am}}\cdot\left(\frac{63\cdot 10^3\text{ A}}{2}\right)^2\cdot\frac{2-1}{0{,}400\text{ m}}}} = 2{,}42 \tag{55}$$

According to IEC 60865-1:2011, Figure 9, the factor ν_2 for ν_1 = 2,42 and κ = 1,81 is:

$$\nu_2 = 2{,}22$$

According to IEC 60865-1:2011, Figure 10, the factor ν_3 for a_s/d = 9,3 is:

$$\nu_3 = 0{,}250$$

With this the short-circuit current force between the sub-conductors is:

$$F_{\text{v}} = (n-1)\frac{\mu_0}{2\pi}\left(\frac{I_{\text{k}}''}{n}\right)^2 \frac{l_{\text{s}}}{a_{\text{s}}}\frac{\nu_2}{\nu_3} = (2-1)\cdot\frac{4\pi\cdot10^{-7}}{2\pi}\frac{\text{Vs}}{\text{Am}}\cdot\left(\frac{63\cdot10^3\ \text{A}}{2}\right)^2\cdot\frac{9{,}35\ \text{m}}{0{,}4\ \text{m}}\cdot\frac{2{,}22}{0{,}25} = 41{,}2\cdot10^3\ \text{N} = 41{,}2\ \text{kN} \tag{54}$$

The strain factors are:

$$\begin{aligned}
\varepsilon_{\text{st,-20}} &= 1{,}5\frac{F_{\text{st,-20}}\, l_{\text{s}}^2\, N_{\text{-20}}}{(a_{\text{s}}-d)^2}\left(\sin\frac{180°}{n}\right)^2 = 1{,}5\cdot\frac{17{,}8\cdot10^3\ \text{N}\cdot(9{,}35\ \text{m})^2\cdot5{,}77\cdot10^{-8}\ 1/\text{N}}{(0{,}400\ \text{m}-0{,}043\ \text{m})^2}\cdot\left(\sin\frac{180°}{2}\right)^2 = 1{,}06\\
\varepsilon_{\text{st,60}} &= 1{,}5\frac{F_{\text{st,60}}\, l_{\text{s}}^2\, N_{60}}{(a_{\text{s}}-d)^2}\left(\sin\frac{180°}{n}\right)^2 = 1{,}5\cdot\frac{15{,}4\cdot10^3\ \text{N}\cdot(9{,}35\ \text{m})^2\cdot5{,}85\cdot10^{-8}\ 1/\text{N}}{(0{,}400\ \text{m}-0{,}043\ \text{m})^2}\cdot\left(\sin\frac{180°}{2}\right)^2 = 0{,}927
\end{aligned} \tag{56}$$

$$\begin{aligned}
\varepsilon_{\text{pi,-20}} &= 0{,}375\,n\frac{F_{\text{v}}\, l_{\text{s}}^3\, N_{\text{-20}}}{(a_{\text{s}}-d)^3}\left(\sin\frac{180°}{n}\right)^3 = 0{,}375\cdot2\cdot\frac{41{,}2\cdot10^3\ \text{N}(9{,}35\ \text{m})^3\, 5{,}77\cdot10^{-8}\ 1/\text{N}}{(0{,}400\ \text{m}-0{,}043\ \text{m})^3}\cdot\left(\sin\frac{180°}{2}\right)^3 = 32{,}0\\
\varepsilon_{\text{pi,60}} &= 0{,}375\,n\frac{F_{\text{v}}\, l_{\text{s}}^3\, N_{60}}{(a_{\text{s}}-d)^3}\left(\sin\frac{180°}{n}\right)^3 = 0{,}375\cdot2\cdot\frac{41{,}2\cdot10^3\ \text{N}(9{,}35\ \text{m})^3\, 5{,}85\cdot10^{-8}\ 1/\text{N}}{(0{,}400\ \text{m}-0{,}043\ \text{m})^3}\cdot\left(\sin\frac{180°}{2}\right)^3 = 32{,}5
\end{aligned} \tag{57}$$

The parameters $j_{\text{-20}}$ and j_{60} are:

$$\begin{aligned}
j_{\text{-20}} &= \sqrt{\frac{\varepsilon_{\text{pi,-20}}}{1+\varepsilon_{\text{st,-20}}}} = \sqrt{\frac{32{,}0}{1+1{,}06}} = 3{,}94\\
j_{60} &= \sqrt{\frac{\varepsilon_{\text{pi,60}}}{1+\varepsilon_{\text{st,60}}}} = \sqrt{\frac{32{,}5}{1+0{,}927}} = 4{,}11
\end{aligned} \tag{58}$$

8.4.3 Pinch force $F_{\text{pi,d}}$

Because

$$\begin{aligned}
j_{\text{-20}} &= 3{,}94 \quad \text{greater than} \quad 1\\
j_{60} &= 4{,}11 \quad \text{greater than} \quad 1
\end{aligned}$$

the sub-conductors clash and the tensile forces due to contraction are calculated according to IEC 60865-1:2011, 6.4.2:

$$\begin{aligned}
F_{\text{pi,d,-20}} &= F_{\text{st,-20}}\left(1+\frac{\nu_{\text{e,-20}}}{\varepsilon_{\text{st,-20}}}\xi_{\text{-20}}\right)\\
F_{\text{pi,d,60}} &= F_{\text{st,60}}\left(1+\frac{\nu_{\text{e,60}}}{\varepsilon_{\text{st,60}}}\xi_{60}\right)
\end{aligned} \tag{59}$$

According to IEC 60865-1:2011, Figure 11, and

– with $j_{\text{-20}}$ = 3,94 and $\varepsilon_{\text{st,-20}}$ = 1,06 the factor $\xi_{\text{-20}}$ is:

$$\xi_{\text{-20}} = 2{,}86$$

– with j_{60} = 4,11 and $\varepsilon_{\text{st,60}}$ = 0,927 the factor ξ_{60} is:

$$\xi_{60} = 2{,}91$$

The factors $\nu_{e,-20}$ and $\nu_{e,60}$ are:

$$\nu_{e,-20} = \frac{1}{2} + \left[\frac{9}{8} n(n-1) \frac{\mu_0}{2\pi} \left(\frac{I''_k}{n} \right)^2 N_{-20}\, \nu_2 \left(\frac{l_s}{a_s - d} \right)^4 \frac{\left(\sin\frac{180°}{n} \right)^4}{\xi_{-20}^3} \left\{ 1 - \frac{\arctan\sqrt{\nu_4}}{\sqrt{\nu_4}} \right\} - \frac{1}{4} \right]^{1/2}$$

$$= \frac{1}{2} + \left[\frac{9}{8} \cdot 2 \cdot (2-1) \cdot \frac{4\pi \cdot 10^{-7}}{2\pi} \frac{\text{Vs}}{\text{Am}} \cdot \left(\frac{63 \cdot 10^3\ \text{A}}{2} \right)^2 \cdot 5{,}77 \cdot 10^{-8} \frac{1}{\text{N}} \cdot 2{,}22 \cdot \left(\frac{9{,}35\ \text{m}}{0{,}400\ \text{m} - 0{,}043\ \text{m}} \right)^4 \right. \quad (60)$$

$$\left. \cdot \frac{\left(\sin\frac{180°}{2} \right)^4}{2{,}87^3} \left\{ 1 - \frac{\arctan\sqrt{8{,}3}}{\sqrt{8{,}3}} \right\} - \frac{1}{4} \right]^{1/2}$$

$$= 1{,}14$$

$$\nu_{e,60} = \frac{1}{2} + \left[\frac{9}{8} n(n-1) \frac{\mu_0}{2\pi} \left(\frac{I''_k}{n} \right)^2 N_{60}\, \nu_2 \left(\frac{l_s}{a_s - d} \right)^4 \frac{\left(\sin\frac{180°}{n} \right)^4}{\xi_{60}^3} \left\{ 1 - \frac{\arctan\sqrt{\nu_4}}{\sqrt{\nu_4}} \right\} - \frac{1}{4} \right]^{1/2}$$

$$= \frac{1}{2} + \left[\frac{9}{8} \cdot 2 \cdot (2-1) \cdot \frac{4\pi \cdot 10^{-7}}{2\pi} \frac{\text{Vs}}{\text{Am}} \cdot \left(\frac{63 \cdot 10^3\ \text{A}}{2} \right)^2 \cdot 5{,}85 \cdot 10^{-8} \frac{1}{\text{N}} \cdot 2{,}22 \cdot \left(\frac{9{,}35\ \text{m}}{0{,}400\ \text{m} - 0{,}043\ \text{m}} \right)^4 \right. \quad (60)$$

$$\left. \cdot \frac{\left(\sin\frac{180°}{2} \right)^4}{2{,}91^3} \left\{ 1 - \frac{\arctan\sqrt{8{,}3}}{\sqrt{8{,}3}} \right\} - \frac{1}{4} \right]^{1/2}$$

$$= 1{,}12$$

with the factor ν_4:

$$\nu_4 = \frac{a_s - d}{d} = \frac{0{,}400\ \text{m} - 0{,}043\ \text{m}}{0{,}043\ \text{m}} = 8{,}3 \quad (61)$$

With this, the tensile forces caused by pinch are:

$$F_{pi,d,-20} = F_{st,-20} \left(1 + \frac{\nu_{e,-20}}{\varepsilon_{st,-20}} \xi_{-20} \right) = 17{,}8\ \text{kN} \cdot \left(1 + \frac{1{,}14}{1{,}06} \cdot 2{,}86 \right) = 72{,}6\ \text{kN}$$

$$F_{pi,d,60} = F_{st,60} \left(1 + \frac{\nu_{e,60}}{\varepsilon_{st,60}} \xi_{60} \right) = 15{,}4\ \text{kN} \cdot \left(1 + \frac{1{,}12}{0{,}927} \cdot 2{,}91 \right) = 69{,}5\ \text{kN} \quad (59)$$

The pinch-force $F_{pi,d}$ is the maximum value of $F_{pi,d,-20}$ and $F_{pi,d,60}$:

$$F_{pi,d} = \max\left\{ F_{pi,d,-20} ; F_{pi,d,60} \right\} = \max\{72{,}6\ \text{kN}; 69{,}5\ \text{kN}\} = 72{,}6\ \text{kN}$$

8.4.4 Conclusions

According to IEC 60865-1:2011, 6.5.2 and 6.5.3, to the structure, the insulators and the connectors and to the foundations the maximum value of $F_{\mathrm{t,d}}$, $F_{\mathrm{f,d}}$ and $F_{\mathrm{pi,d}}$ shall be applied as a static load:

$$\max\left\{F_{\mathrm{t,d}};F_{\mathrm{f,d}};F_{\mathrm{pi,d}}\right\} = \max\left\{36{,}3\ \mathrm{kN};68{,}8\ \mathrm{kN};72{,}6\ \mathrm{kN}\right\} = 72{,}6\ \mathrm{kN}$$

given by the pinch force $F_{\mathrm{pi,d}}$.

The maximum horizontal displacement is 1,48 m and the minimum air clearance is 2,04 m.

9 Example 6 – Mechanical effects on strained conductors with dropper in the middle of the span

9.1 General

The basis for the calculations in this example is a three-phase 380 kV arrangement with strained twin-bundle conductors as shown in Figure 6. In the span there is one connection of pantograph-disconnectors and one dropper in midspan, both operate as spacers.

The calculation is carried out for the arrangement of droppers with plane parallel to the main conductors and plane perpendicular to the main conductors.

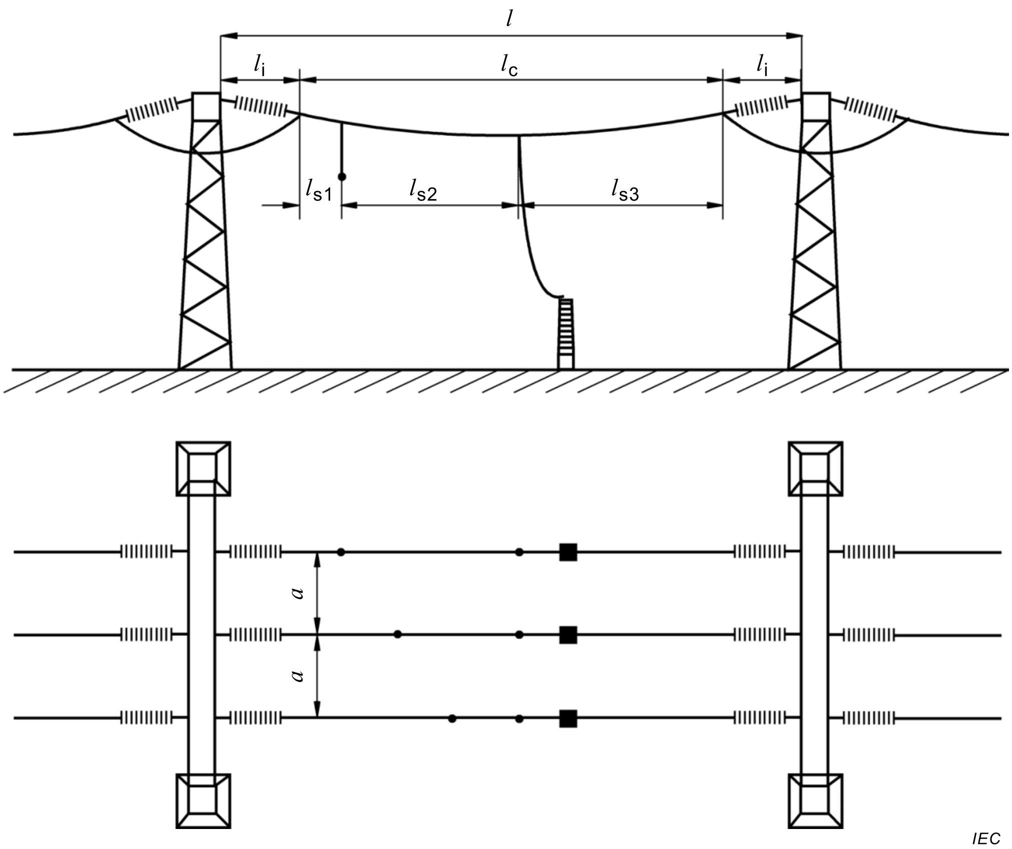


Figure 6 – Arrangement with strained conductors and droppers in midspan. Plane of the droppers parallel to the main conductors

9.2 Common data

Initial symmetrical three-phase short-circuit current (r.m.s.)	I''_k	=	63 kA
Factor for the calculation of the peak short-circuit current	κ	=	1,81
Duration of the first short-circuit current flow	T_{k1}	=	0,5 s
System frequency	f	=	50 Hz
Centre-line distance between supports	l	=	48 m
Length of one insulator chain	l_i	=	5,3 m
Cord length $l_c = l - 2l_i$	l_c	=	37,4 m
Centre-line distance between conductors	a	=	5 m
Resultant spring constant of both span supports of one span	S	=	500 N/mm
Twin conductor 2 EN 1046-AL1/45-ST1A			
– Number of sub-conductors	n	=	2
– Centre-line distance between sub-conductors	a_s	=	0,1 m
– Outer diameter of one sub-conductor	d	=	43 mm
– Cross-section of one sub-conductor	A_s	=	1 090 mm^2
– Mass per unit length of one sub-conductor	m'_s	=	3,25 kg/m
– Young's modulus	E	=	60 000 N/mm^2
Static tensile force of one flexible main conductor at a temperature of –20°C (local minimum winter temperature)	$F_{st,-20}$	=	17,4 kN
Static tensile force of one flexible main conductor at a temperature of 60°C (maximum operating temperature)	$F_{st,60}$	=	15,0 kN
Additional concentrated masses representing the connections of panthograph disconnectors			
– Number of spacers	n_c	=	2
– Mass of one connection	m_c	=	36 kg
– Distances	l_{s1}	=	2,5 m
	l_{s2}	=	18,6 m
	l_{s3}	=	16,3 m
Conventional value of acceleration of gravity	g	=	9,81 m/s^2

9.3 Plane of the dropper parallel to the main conductors

9.3.1 General

In addition to 9.2, the following data are given:

Height of the dropper at a main conductor temperature of 60°C (maximum operating temperature)	h	=	7 m
Width of the dropper at a main conductor temperature of 60°C (maximum operating temperature)	w	=	2 m
Cord length of the dropper	l_v	=	7,6 m

9.3.2 Current flow along the whole length of the main conductor span

9.3.2.1 Electromagnetic load and characteristic parameters

The characteristic electromagnetic load per unit length is:

$$F' = \frac{\mu_0}{2\pi} 0{,}75 \frac{\left(I_k''\right)^2}{a} \frac{l_c}{l} = \frac{4\pi \cdot 10^{-7}}{2\pi} \frac{\text{Vs}}{\text{Am}} \cdot 0{,}75 \cdot \frac{\left(63 \cdot 10^3 \text{ A}\right)^2}{5 \text{ m}} \cdot \frac{37{,}4 \text{ m}}{48 \text{ m}} = 92{,}8 \text{ N/m} \tag{19a}$$

The parameter r is:

$$r = \frac{F'}{n\, m'_{sc}\, g} = \frac{92{,}8 \text{ N/m}}{2 \cdot 3{,}73 \text{ kg/m} \cdot 9{,}81 \text{ m/s}^2} = 1{,}27 \tag{20}$$

where m'_{sc} is the resulting mass per unit length of one sub-conductor including concentrated mass:

$$m'_{sc} = m'_s + \frac{m_c}{n\, l_c} = 3{,}25 \frac{\text{kg}}{\text{m}} + \frac{36 \text{ kg}}{2 \cdot 37{,}4 \text{ m}} = 3{,}73 \text{ kg/m}$$

The direction of the resulting force on the conductor is:

$$\delta_1 = \arctan r = \arctan 1{,}27 = 51{,}8° \tag{21}$$

The equivalent static conductor sags at midspan are:

$$f_{es,-20} = \frac{n\, m'_{sc}\, g\, l^2}{8 F_{st,-20}} = \frac{2 \cdot 3{,}73 \text{ kg/m} \cdot 9{,}81 \text{ m/s}^2 \left(48 \text{ m}\right)^2}{8 \cdot 17{,}4 \cdot 10^3 \text{ N}} = 1{,}21 \text{ m}$$

$$f_{es,60} = \frac{n\, m'_{sc}\, g\, l^2}{8 F_{st,60}} = \frac{2 \cdot 3{,}73 \text{ kg/m} \cdot 9{,}81 \text{ m/s}^2 \left(48 \text{ m}\right)^2}{8 \cdot 15{,}0 \cdot 10^3 \text{ N}} = 1{,}41 \text{ m} \tag{22}$$

The periods of the conductor oscillation are:

$$T_{-20} = 2\pi \sqrt{0{,}8 \frac{f_{es,-20}}{g}} = 2\pi \sqrt{0{,}8 \cdot \frac{1{,}21 \text{ m}}{9{,}81 \text{ m/s}^2}} = 1{,}97 \text{ s}$$

$$T_{60} = 2\pi \sqrt{0{,}8 \frac{f_{es,60}}{g}} = 2\pi \sqrt{0{,}8 \cdot \frac{1{,}41 \text{ m}}{9{,}81 \text{ m/s}^2}} = 2{,}13 \text{ s} \tag{23}$$

The resulting periods of the conductor oscillation are:

$$T_{\text{res,-20}} = \frac{T_{\text{-20}}}{\sqrt[4]{1+r^2}\left[1-\frac{\pi^2}{64}\left(\frac{\delta_1}{90°}\right)^2\right]} = \frac{1{,}97}{\sqrt[4]{1+1{,}27^2}\left[1-\frac{\pi^2}{64}\left(\frac{51{,}8°}{90°}\right)^2\right]} = 1{,}63 \text{ s}$$

$$T_{\text{res,60}} = \frac{T_{60}}{\sqrt[4]{1+r^2}\left[1-\frac{\pi^2}{64}\left(\frac{\delta_1}{90°}\right)^2\right]} = \frac{2{,}13}{\sqrt[4]{1+1{,}27^2}\left[1-\frac{\pi^2}{64}\left(\frac{51{,}8°}{90°}\right)^2\right]} = 1{,}77 \text{ s} \quad (24)$$

The stiffness norms are:

$$N_{\text{-20}} = \frac{1}{S\,l} + \frac{1}{n\,E_{\text{eff,-20}}\,A_\text{s}} = \frac{1}{5\cdot10^5 \text{ N/m}\cdot 48 \text{ m}} + \frac{1}{2\cdot 2{,}84\cdot10^{10} \text{ N/m}^2\cdot 1090\cdot10^{-6} \text{ m}^2} = 5{,}78\cdot10^{-8} \text{ 1/N}$$

$$N_{60} = \frac{1}{S\,l} + \frac{1}{n\,E_{\text{s,60}}\,A_\text{s}} = \frac{1}{5\cdot10^5 \text{ N/m}\cdot 48 \text{ m}} + \frac{1}{2\cdot 2{,}70\cdot10^{10} \text{ N/m}^2\cdot 1090\cdot10^{-6} \text{ m}^2} = 5{,}87\cdot10^{-8} \text{ 1/N} \quad (25)$$

with the actual Young's moduli

$$E_{\text{eff,-20}} = E\left[0{,}3+0{,}7\sin\left(\frac{F_{\text{st,-20}}}{n\,A_\text{s}\,\sigma_{\text{fin}}}90°\right)\right] = 6\cdot10^{10}\ \frac{\text{N}}{\text{m}^2}\cdot\left[0{,}3+0{,}7\sin\left(\frac{7{,}98\cdot10^6 \text{ N/m}^2}{5\cdot10^7 \text{ N/m}^2}\cdot 90°\right)\right]$$
$$= 2{,}84\cdot10^{10} \text{ N/m}^2$$

$$E_{\text{eff,60}} = E\left[0{,}3+0{,}7\sin\left(\frac{F_{\text{st,60}}}{n\,A_\text{s}\,\sigma_{\text{fin}}}90°\right)\right] = 6\cdot10^{10}\ \frac{\text{N}}{\text{m}^2}\cdot\left[0{,}3+0{,}7\sin\left(\frac{6{,}88\cdot10^6 \text{ N/m}^2}{5\cdot10^7 \text{ N/m}^2}\cdot 90°\right)\right]$$
$$= 2{,}70\cdot10^{10} \text{ N/m}^2 \quad (26)$$

because

$$\frac{F_{\text{st,-20}}}{n\,A_\text{s}} = \frac{17{,}4\cdot10^3 \text{ N}}{2\cdot1090\cdot10^{-6} \text{ m}^2} = 7{,}98\cdot10^6 \text{ N/m}^2 \quad \text{less than} \quad \sigma_{\text{fin}} = 50\cdot10^6 \text{ N/m}^2$$

$$\frac{F_{\text{st,60}}}{n\,A_\text{s}} = \frac{15{,}0\cdot10^3 \text{ N}}{2\cdot1090\cdot10^{-6} \text{ m}^2} = 6{,}88\cdot10^6 \text{ N/m}^2 \quad \text{less than} \quad \sigma_{\text{fin}} = 50\cdot10^6 \text{ N/m}^2$$

The stress factors are:

$$\zeta_{\text{-20}} = \frac{(n\,g\,m'_{\text{sc}}\,l)^2}{24\,F_{\text{st,-20}}^3\,N_{\text{-20}}} = \frac{\left(2\cdot9{,}81 \text{ m/s}^2\cdot3{,}73 \text{ kg/m}\cdot48 \text{ m}\right)^2}{24\left(17{,}4\cdot10^3 \text{ N}\right)^3\cdot5{,}78\cdot10^{-8} \text{ 1/N}} = 1{,}69$$

$$\zeta_{60} = \frac{(n\,g\,m'_{\text{sc}}\,l)^2}{24\,F_{\text{st,60}}^3\,N_{60}} = \frac{\left(2\cdot9{,}81 \text{ m/s}^2\cdot3{,}73 \text{ kg/m}\cdot48 \text{ m}\right)^2}{24\left(15{,}0\cdot10^3 \text{ N}\right)^3\cdot5{,}87\cdot10^{-8} \text{ 1/N}} = 2{,}60 \quad (28)$$

Because

$$T_{\text{k1}} = 0{,}5 \text{ s} \quad \text{less than} \quad 0{,}4\,T_{\text{-20}} = 0{,}4\cdot1{,}97 \text{ s} = 0{,}788 \text{ s}$$

$$T_{\text{k1}} = 0{,}5 \text{ s} \quad \text{less than} \quad 0{,}4\,T_{60} = 0{,}4\cdot2{,}13 \text{ s} = 0{,}852 \text{ s}$$

in the Equations (29), (32), and (35) it is to be inserted:

$$T_{k1} = 0{,}5 \text{ s}$$

The swing-out angles at the end of short-circuit current flow are:

$$\delta_{end,-20} = \delta_1\left[1-\cos\left(360°\frac{T_{k1}}{T_{res,-20}}\right)\right] = 51{,}8°\cdot\left[1-\cos\left(360°\cdot\frac{0{,}5 \text{ s}}{1{,}63 \text{ s}}\right)\right] = 69{,}9°$$
$$\delta_{end,60} = \delta_1\left[1-\cos\left(360°\frac{T_{k1}}{T_{res,60}}\right)\right] = 51{,}8°\cdot\left[1-\cos\left(360°\cdot\frac{0{,}5 \text{ s}}{1{,}77 \text{ s}}\right)\right] = 62{,}3° \quad (29)$$

because

$$\frac{T_{k1}}{T_{res,-20}} = \frac{0{,}5 \text{ s}}{1{,}63 \text{ s}} = 0{,}307 \quad \text{less than} \quad 0{,}5$$
$$\frac{T_{k1}}{T_{res,60}} = \frac{0{,}5 \text{ s}}{1{,}77 \text{ s}} = 0{,}283 \quad \text{less than} \quad 0{,}5$$

The maximum swing-out angles $\delta_{max,-20}$ and $\delta_{max,60}$ depend respectively on χ_{-20} and χ_{60} which depend on $\delta_{end,-20}$ and $\delta_{end,60}$:

– for 0 less than $\delta_{end,-20}$ = 69,9° less than 90° is:

$$\chi_{-20} = 1 - r\sin\delta_{end,-20} = 1 - 1{,}27\cdot\sin 69{,}9° = -0{,}193 \quad (30)$$

and for −0,985 less than χ_{-20} = −0,193 less than 0,766:

$$\delta_{max,-20} = 10° + \arccos\chi_{-20} = 10° + \arccos(-0{,}193) = 111° \quad (31)$$

– for 0 less than $\delta_{end,60}$ = 62,3° less than 90° is:

$$\chi_{60} = 1 - r\sin\delta_{end,60} = 1 - 1{,}27\cdot\sin 62{,}3° = -0{,}124 \quad (30)$$

and for −0,985 less than χ_{60} = −0,124 less than 0,766:

$$\delta_{max,60} = 10° + \arccos\chi_{60} = 10° + \arccos(-0{,}124) = 107° \quad (31)$$

9.3.2.2 Tensile force $F_{t,d}$ during short-circuit caused by swing out without dropper in midspan

The calculation is done according to IEC 60865-1:2011, 6.2.3.

The load parameters are:

$$\varphi_{-20} = \varphi_{60} = 3\left(\sqrt{1+r^2} - 1\right) = 3\left(\sqrt{1+1{,}27^2} - 1\right) = 1{,}85 \quad (32)$$

because

$$T_{k1} = 0{,}5\ \text{s} \quad \text{greater than} \quad \frac{T_{\text{res,-20}}}{4} = \frac{1{,}63\ \text{s}}{4} = 0{,}408\ \text{s}$$

$$T_{k1} = 0{,}5\ \text{s} \quad \text{greater than} \quad \frac{T_{\text{res,60}}}{4} = \frac{1{,}77\ \text{s}}{4} = 0{,}443\ \text{s}$$

According to IEC 60865-1:2011, Figure 8, the factors $\psi_{\text{-20}}$ and ψ_{60} are:

– for $\varphi_{\text{-20}}$ = 1,85 and $\zeta_{\text{-20}}$ = 1,69:

$$\psi_{\text{-20}} = 0{,}641$$

– for φ_{60} = 1,85 and ζ_{60} = 2,60:

$$\psi_{60} = 0{,}714$$

The tensile forces during the short-circuit are:

$$\begin{aligned} F_{\text{t,d,-20}} &= F_{\text{st,-20}}\left(1+\varphi_{\text{-20}}\,\psi_{\text{-20}}\right) = 17{,}4\ \text{kN}\cdot\left(1+1{,}85\cdot 0{,}641\right) = 38{,}0\ \text{kN} \\ F_{\text{t,d,60}} &= F_{\text{st,60}}\left(1+\varphi_{60}\,\psi_{60}\right) = 15{,}0\ \text{kN}\cdot\left(1+1{,}85\cdot 0{,}714\right) = 34{,}8\ \text{kN} \end{aligned} \qquad (33)$$

The tensile force $F_{\text{t,d}}$ is the maximum value of $F_{\text{t,d,-20}}$ and $F_{\text{t,d,60}}$:

$$F_{\text{t,d}} = \max\left\{F_{\text{t,d,-20}}\,;F_{\text{t,d,60}}\right\} = \max\left\{38{,}0\ \text{kN};34{,}8\ \text{kN}\right\} = 38{,}0\ \text{kN}$$

9.3.2.3 Dynamic conductor sag at midspan

The elastic expansions are:

$$\begin{aligned} \varepsilon_{\text{ela,-20}} &= N_{\text{-20}}\left(F_{\text{t,d,-20}} - F_{\text{st,-20}}\right) = 5{,}78\cdot 10^{-8}\ \frac{1}{\text{N}}\cdot\left(38{,}0-17{,}4\right)\cdot 10^{3}\ \text{N} = 1{,}19\cdot 10^{-3} \\ \varepsilon_{\text{ela,60}} &= N_{60}\left(F_{\text{t,d,60}} - F_{\text{st,60}}\right) = 5{,}87\cdot 10^{-8}\ \frac{1}{\text{N}}\cdot\left(34{,}8-15{,}0\right)\cdot 10^{3}\ \text{N} = 1{,}16\cdot 10^{-3} \end{aligned} \qquad (34)$$

The thermal expansions are:

$$\begin{aligned} \varepsilon_{\text{th,-20}} &= c_{\text{th}}\left(\frac{I_k''}{n\,A_{\text{s}}}\right)^2 \frac{T_{\text{res,-20}}}{4} = 0{,}27\cdot 10^{-18}\ \frac{\text{m}^4}{\text{A}^2\,\text{s}}\cdot\left(\frac{63\cdot 10^3\ \text{A}}{2\cdot 1090\cdot 10^{-6}\ \text{m}^2}\right)^2\cdot\frac{1{,}63\ \text{s}}{4} = 0{,}919\cdot 10^{-4} \\ \varepsilon_{\text{th,60}} &= c_{\text{th}}\left(\frac{I_k''}{n\,A_{\text{s}}}\right)^2 \frac{T_{\text{res,60}}}{4} = 0{,}27\cdot 10^{-18}\ \frac{\text{m}^4}{\text{A}^2\,\text{s}}\cdot\left(\frac{63\cdot 10^3\ \text{A}}{2\cdot 1090\cdot 10^{-6}\ \text{m}^2}\right)^2\cdot\frac{1{,}77\ \text{s}}{4} = 0{,}998\cdot 10^{-4} \end{aligned} \qquad (35)$$

because

$$T_{k1} = 0{,}5\ \text{s} \quad \text{greater than} \quad \frac{T_{\text{res,-20}}}{4} = \frac{1{,}63\ \text{s}}{4} = 0{,}408\ \text{s}$$

$$T_{k1} = 0{,}5\ \text{s} \quad \text{greater than} \quad \frac{T_{\text{res,60}}}{4} = \frac{1{,}77\ \text{s}}{4} = 0{,}443\ \text{s}$$

and for ASCR conductors with $A_{\text{Al}}/A_{\text{St}} = 1046\ \text{mm}/45\ \text{mm} = 23{,}2$ greater than 6

$$c_{\text{th}} = 0,27 \cdot 10^{-18}\ \text{m}^4/(\text{A}^2\text{s})$$

The factors C_{D} are:

$$C_{\text{D,-20}} = \sqrt{1 + \frac{3}{8}\left(\frac{l}{f_{\text{es,-20}}}\right)^2 (\varepsilon_{\text{ela,-20}} + \varepsilon_{\text{th,-20}})} = \sqrt{1 + \frac{3}{8}\left(\frac{48\ \text{m}}{1,21\ \text{m}}\right)^2 (1,19 \cdot 10^{-3} + 0,919 \cdot 10^{-4})} = 1,33$$
$$C_{\text{D,60}} = \sqrt{1 + \frac{3}{8}\left(\frac{l}{f_{\text{es,60}}}\right)^2 (\varepsilon_{\text{ela,60}} + \varepsilon_{\text{th,60}})} = \sqrt{1 + \frac{3}{8}\left(\frac{48\ \text{m}}{1,41\ \text{m}}\right)^2 (1,16 \cdot 10^{-3} + 0,998 \cdot 10^{-4})} = 1,24 \quad (36)$$

The factor C_{F} is:

$$C_{\text{F}} = 0,97 + 0,1r = 0,97 + 0,1 \cdot 1,27 = 1,10 \quad (37)$$

because

0,8 less than $r = 1,27$ less than 1,8

The dynamic conductor sags at midspan are:

$$f_{\text{ed,-20}} = C_{\text{F}}\, C_{\text{D,-20}}\, f_{\text{es,-20}} = 1,10 \cdot 1,33 \cdot 1,21\ \text{m} = 1,77\ \text{m}$$
$$f_{\text{ed,60}} = C_{\text{F}}\, C_{\text{D,60}}\, f_{\text{es,60}} = 1,10 \cdot 1,24 \cdot 1,41\ \text{m} = 1,92\ \text{m} \quad (38)$$

9.3.2.4 Tensile force $F_{\text{t,d}}$ during short-circuit caused by swing out with dropper in midspan

The calculation is done according to IEC 60865-1:2011, 6.2.5, because

$$\sqrt{(h_{\text{-20}} + f_{\text{es,-20}} + f_{\text{ed,-20}})^2 + w^2} = \sqrt{(7,2\ \text{m} + 1,21\ \text{m} + 1,77\ \text{m})^2 + (2\ \text{m})^2} = 10,4\ \text{m} \quad \text{greater than} \quad l_{\text{v}} = 7,6\ \text{m}$$
$$\sqrt{(h_{60} + f_{\text{es,60}} + f_{\text{ed,60}})^2 + w^2} = \sqrt{(7,0\ \text{m} + 1,41\ \text{m} + 1,92\ \text{m})^2 + (2\ \text{m})^2} = 10,5\ \text{m} \quad \text{greater than} \quad l_{\text{v}} = 7,6\ \text{m}$$

with the dropper height at –20°C due to the change of sag with the temperature of the main conductor

$$h_{\text{-20}} = h_{60} + (f_{\text{es,60}} - f_{\text{es,-20}}) = 7,0\ \text{m} + (1,41\ \text{m} - 1,21\ \text{m}) = 7,2\ \text{m}$$

and $h_{60} = h = 7,0\ \text{m}$.

The actual swing-out angles are:

$$\delta_{\text{-20}} = \arccos \frac{(h_{\text{-20}} + f_{\text{es,-20}})^2 + f_{\text{ed,-20}}^2 - (l_{\text{v}}^2 - w^2)}{2 f_{\text{ed,-20}}(h_{-20} + f_{\text{es,-20}})}$$
$$= \arccos \frac{(7,2\ \text{m} + 1,21\ \text{m})^2 + (1,77\ \text{m})^2 - ((7,6\ \text{m})^2 - (2,0\ \text{m})^2)}{2 \cdot 1,77\ \text{m} \cdot (7,2\ \text{m} + 1,21\ \text{m})} = 47,5° \quad (39)$$

$$\delta_{60} = \arccos\frac{\left(h_{60} + f_{es,60}\right)^2 + f_{ed,60}^2 - \left(l_v^2 - w^2\right)}{2 f_{ed,60}\left(h_{60} + f_{es,60}\right)} = \arccos\frac{\left(7{,}0\text{ m} + 1{,}41\text{ m}\right)^2 + \left(1{,}92\text{ m}\right)^2 - \left(\left(7{,}6\text{ m}\right)^2 - \left(2{,}0\text{ m}\right)^2\right)}{2\cdot 1{,}92\text{ m}\cdot\left(7{,}0\text{ m} + 1{,}41\text{ m}\right)} = 50{,}2^\circ \quad (39)$$

The load parameters are:

$$\begin{aligned} \varphi_{-20} &= 3\left(r\sin\delta_{-20} + \cos\delta_{-20} - 1\right) = 3\left(1{,}27\sin 47{,}5^\circ + \cos 47{,}5^\circ - 1\right) = 1{,}84 \\ \varphi_{60} &= 3\left(r\sin\delta_{60} + \cos\delta_{60} - 1\right) = 3\left(1{,}27\sin 50{,}2^\circ + \cos 50{,}2^\circ - 1\right) = 1{,}85 \end{aligned} \quad (41)$$

because

$$\begin{aligned} \delta_{-20} &= 47{,}5^\circ \quad \text{less than} \quad \delta_1 = 51{,}8^\circ \\ \delta_{60} &= 50{,}2^\circ \quad \text{less than} \quad \delta_1 = 51{,}8^\circ \end{aligned}$$

and also

$$\begin{aligned} \delta_{end,-20} &= 69{,}9^\circ \quad \text{greater than} \quad \delta_{-20} = 47{,}5^\circ \\ \delta_{end,60} &= 62{,}3^\circ \quad \text{greater than} \quad \delta_{60} = 50{,}2^\circ \end{aligned}$$

According to IEC 60865-1:2011, Figure 8, the factors ψ_{-20} and ψ_{60} are:

– for φ_{-20} = 1,84 and ζ_{-20} = 1,69:

$$\psi_{-20} = 0{,}641$$

– for φ_{60} = 1,85 and ζ_{60} = 2,60:

$$\psi_{60} = 0{,}714$$

The tensile forces during the short-circuit are:

$$\begin{aligned} F_{t,d,-20} &= F_{st,-20}\left(1 + \varphi_{-20}\psi_{-20}\right) = 17{,}4\text{ kN}\cdot\left(1 + 1{,}84\cdot 0{,}641\right) = 37{,}9\text{ kN} \\ F_{t,d,60} &= F_{st,60}\left(1 + \varphi_{60}\psi_{60}\right) = 15{,}0\text{ kN}\cdot\left(1 + 1{,}85\cdot 0{,}714\right) = 34{,}8\text{ kN} \end{aligned} \quad (42)$$

The tensile force $F_{t,d}$ is the maximum value of $F_{t,d,-20}$ and $F_{t,d,60}$:

$$F_{t,d} = \max\left\{F_{t,d,-20}; F_{t,d,60}\right\} = \max\left\{37{,}9\text{ kN}; 34{,}8\text{ kN}\right\} = 37{,}9\text{ kN}$$

9.3.2.5 Tensile force $F_{f,d}$ after short-circuit caused by drop

Because

$$r = 1{,}27 \quad \text{greater than} \quad 0{,}6$$

and

$$\begin{aligned} \delta_{max,-20} &= 111^\circ \quad \text{greater than} \quad 70^\circ \\ \delta_{max,60} &= 107^\circ \quad \text{greater than} \quad 70^\circ \end{aligned}$$

however

$$\delta_{-20} = 47{,}5^\circ \qquad \text{less than} \qquad 60^\circ$$
$$\delta_{60} = 50{,}2^\circ \qquad \text{less than} \qquad 60^\circ$$

the tensile force $F_{f,d}$ after short-circuit is not significant.

When calculating according to IEC 60865-1:2011, 6.2.3, in addition the tensile force $F_{f,d}$ after short-circuit is to be calculated according to IEC 60865-1:2011, 6.2.6. Because

$$r = 1{,}27 \qquad \text{greater than} \qquad 0{,}6$$

and

$$\delta_{max,-20} = 111^\circ \qquad \text{greater than} \qquad 70^\circ$$
$$\delta_{max,60} = 107^\circ \qquad \text{greater than} \qquad 70^\circ$$

The tensile forces after short-circuit are:

$$\begin{aligned} F_{f,d,-20} &= 1{,}2 \cdot F_{st,-20} \sqrt{1 + 8\zeta_{-20}\frac{\delta_{max,-20}}{180^\circ}} = 1{,}2 \cdot 17{,}4\ \text{kN} \cdot \sqrt{1 + 8 \cdot 1{,}69 \cdot \frac{111^\circ}{180^\circ}} = 63{,}8\ \text{kN} \\ F_{f,d,60} &= 1{,}2 \cdot F_{st,60} \sqrt{1 + 8\zeta_{60}\frac{\delta_{max,60}}{180^\circ}} = 1{,}2 \cdot 15{,}0\ \text{kN} \cdot \sqrt{1 + 8 \cdot 2{,}60 \cdot \frac{107^\circ}{180^\circ}} = 65{,}8\ \text{kN} \end{aligned} \qquad (43)$$

The tensile force $F_{f,d}$ is the maximum of $F_{f,d,-20}$ and $F_{f,d,60}$:

$$F_{f,d} = \max\{F_{f,d,-20}; F_{f,d,60}\} = \max\{63{,}8\ \text{kN}; 65{,}8\ \text{kN}\} = 65{,}8\ \text{kN}$$

9.3.2.6 Horizontal span displacement b_h and minimum air clearance a_{min}

All the following quantities are calculated at a conductor temperature of 60°C which leads to a greater conductor sag than a conductor temperature of −20°C.

The maximum horizontal span displacement for stranded conductors with $l_c = l - 2l_i$ is:

$$b_h = f_{ed,60} \sin\delta_{60} = 1{,}92\ \text{m} \cdot \sin 50{,}2^\circ = 1{,}48\ \text{m} \qquad (47)$$

because

$$\delta_{60} = 50{,}2^\circ \qquad \text{less than} \qquad \delta_{max,60} = 107^\circ$$
$$\delta_{60} = 50{,}2^\circ \qquad \text{less than} \qquad \delta_1 = 51{,}8^\circ$$

The minimum air clearance is:

$$a_{min} = a - 2b_h = 5\ \text{m} - 2 \cdot 1{,}48\ \text{m} = 2{,}04\ \text{m} \qquad (48)$$

When calculating according to IEC 60865-1:2011, 6.2.3, without dropper in midspan, in addition the horizontal displacement b_h and the minimum air clearance a_{min} shall be calculated according to IEC 60865-1:2011, 6.2.7.

$$b_h = f_{ed,60} \sin \delta_1 = 1{,}92 \text{ m} \cdot \sin 51{,}8° = 1{,}51 \text{ m} \tag{45}$$

because

$$\delta_{max,60} = 107° \qquad \text{greater than} \qquad \delta_1 = 51{,}8°$$

and the minimum air clearance is:

$$a_{min} = a - 2b_h = 5 \text{ m} - 2 \cdot 1{,}51 \text{ m} = 1{,}98 \text{ m} \tag{48}$$

9.3.2.7 Pinch force $F_{pi,d}$

The sub-conductors clash effectively during short-circuit because Equation (53) is fulfilled:

$$\frac{a_s}{d} = \frac{0{,}1 \text{ m}}{0{,}043 \text{ m}} = 2{,}33 \qquad \text{less than} \qquad 2{,}5 \tag{53}$$

and

$$l_s = 12{,}5 \text{ m} \qquad \text{greater than} \qquad 70\, a_s = 70 \cdot 0{,}1 \text{ m} = 7 \text{ m} \tag{53}$$

with

$$l_s = \frac{l_{s1} + l_{s2} + l_{s3}}{3} = \frac{l_c}{3} = \frac{2{,}5 + 18{,}6 + 16{,}3}{3} \text{ m} = \frac{37{,}4 \text{ m}}{3} = 12{,}5 \text{ m}$$

The tensile forces caused by pinch are:

$$\begin{aligned} F_{pi,d,-20} &= 1{,}1 F_{t,d,-20} = 1{,}1 \cdot 38{,}0 \text{ kN} = 41{,}8 \text{ kN} \\ F_{pi,d,60} &= 1{,}1 F_{t,d,60} = 1{,}1 \cdot 34{,}8 \text{ kN} = 38{,}3 \text{ kN} \end{aligned} \tag{51}$$

$F_{t,d,-20}$ and $F_{t,d,60}$ are calculated in 9.3.2.2.

The pinch force $F_{pi,d}$ is the maximum of $F_{pi,d,-20}$ and $F_{pi,d,60}$:

$$F_{pi,d} = \max\left\{F_{pi,d,-20}; F_{pi,d,60}\right\} = \max\left\{41{,}8 \text{ kN}; 38{,}3 \text{ kN}\right\} = 41{,}8 \text{ kN}$$

9.3.2.8 Conclusions

According to IEC 60865-1:2011, 6.5.2 and 6.5.3, to the structure, the insulators and the connectors and to the foundations the maximum value of $F_{t,d}$, $F_{f,d}$ and $F_{pi,d}$ shall be applied as a static load:

$$\max\left\{F_{t,d}; F_{f,d}; F_{pi,d}\right\} = \max\left\{37{,}9 \text{ kN}; 0 \text{ kN}; 41{,}8 \text{ kN}\right\} = 41{,}8 \text{ kN}$$

given by the tensile force $F_{pi,d}$ caused by pinch.

The maximum horizontal displacement is 1,48 m and the minimum air clearance is 2,04 m.

When calculating according to IEC 60865-1:2011, 6.2.3, without dropper in midspan, to the structure, the insulators and the connectors and to the foundations the maximum value of $F_{t,d}$, $F_{f,d}$ and $F_{pi,d}$ shall be applied as a static load:

$$\max\left\{F_{t,d};F_{f,d};F_{pi,d}\right\}=\max\left\{38{,}0\text{ kN};65{,}8\text{ kN};41{,}8\text{ kN}\right\}=65{,}8\text{ kN}$$

given by the tensile force $F_{f,d}$ after short-circuit caused by drop without dropper in midspan.

The maximum horizontal displacement is 1,51 m and the minimum air clearance is 1,98 m.

9.3.3 Current flow along half of the length of the main conductor and along the dropper

9.3.3.1 Electromagnetic load and characteristic parameters

The characteristic electromagnetic load per unit length is:

$$F'=\frac{\mu_0}{2\pi}0{,}75\frac{(I_k'')^2}{a}\frac{l_c/2+l_v/2}{l}=\frac{4\pi\cdot 10^{-7}}{2\pi}\frac{\text{Vs}}{\text{Am}}\cdot 0{,}75\cdot\frac{\left(63\cdot 10^3\text{ A}\right)^2}{5\text{ m}}\cdot\frac{37{,}4\text{ m}/2+7{,}6\text{ m}/2}{48\text{ m}}=55{,}8\text{ N/m} \quad (19b)$$

The parameter r is:

$$r=\frac{F'}{n\,m'_{sc}\,g}=\frac{55{,}8\text{ N/m}}{2\cdot 3{,}73\text{ kg/m}\cdot 9{,}81\text{ m/s}^2}=0{,}763 \quad (20)$$

where

m'_{sc} = 3,73 kg/m see 9.3.2.1

The direction of the resulting force on the conductor is:

$$\delta_1=\arctan r=\arctan 0{,}763=37{,}3° \quad (21)$$

The equivalent static conductor sags at midspan are, see 9.3.1.1:

$$f_{es,-20}=1{,}21\text{ m} \qquad f_{es,60}=1{,}41\text{ m} \quad (22)$$

The periods of the conductor oscillation are, see 9.3.2.1:

$$T_{-20}=1{,}97\text{ s} \qquad T_{60}=2{,}13\text{ s} \quad (23)$$

The resulting periods of the conductor oscillation are:

$$T_{res,-20}=\frac{T_{-20}}{\sqrt[4]{1+r^2}\left[1-\frac{\pi^2}{64}\left(\frac{\delta_1}{90°}\right)^2\right]}=\frac{1{,}97}{\sqrt[4]{1+0{,}763^2}\left[1-\frac{\pi^2}{64}\left(\frac{37{,}3°}{90°}\right)^2\right]}=1{,}80\text{ s}$$

$$T_{res,60}=\frac{T_{60}}{\sqrt[4]{1+r^2}\left[1-\frac{\pi^2}{64}\left(\frac{\delta_1}{90°}\right)^2\right]}=\frac{2{,}13}{\sqrt[4]{1+0{,}763^2}\left[1-\frac{\pi^2}{64}\left(\frac{37{,}3°}{90°}\right)^2\right]}=1{,}95\text{ s} \quad (24)$$

The stiffness norms are, see 9.3.2.1:

$$N_{-20} = 5,78 \cdot 10^{-8}\ 1/\text{N} \qquad N_{60} = 5,87 \cdot 10^{-8}\ 1/\text{N} \tag{25}$$

The stress factors are, see 9.3.2.1:

$$\zeta_{-20} = 1,69 \qquad \zeta_{60} = 2,60 \tag{28}$$

In the Equations (29), (32), and (35) it is to be inserted, see 9.3.2.1:

$$T_{k1} = 0,5\ \text{s}$$

The swing-out angles at the end of short-circuit current flow are:

$$\begin{aligned} \delta_{\text{end,-20}} &= \delta_1 \left[1 - \cos\left(360° \frac{T_{k1}}{T_{\text{res,-20}}}\right)\right] = 37,3° \cdot \left[1 - \cos\left(360° \cdot \frac{0,5\ \text{s}}{1,80\ \text{s}}\right)\right] = 43,8° \\ \delta_{\text{end,60}} &= \delta_1 \left[1 - \cos\left(360° \frac{T_{k1}}{T_{\text{res,60}}}\right)\right] = 37,3° \cdot \left[1 - \cos\left(360° \cdot \frac{0,5\ \text{s}}{1,95\ \text{s}}\right)\right] = 38,8° \end{aligned} \tag{29}$$

because

$$\begin{aligned} \frac{T_{k1}}{T_{\text{res,-20}}} &= \frac{0,5\ \text{s}}{1,80\ \text{s}} = 0,278 \quad \text{less than} \quad 0,5 \\ \frac{T_{k1}}{T_{\text{res,60}}} &= \frac{0,5\ \text{s}}{1,95\ \text{s}} = 0,256 \quad \text{less than} \quad 0,5 \end{aligned}$$

The maximum swing-out angles $\delta_{\text{max,-20}}$ and $\delta_{\text{max,60}}$ depend respectively on χ_{-20} and χ_{60} which depend on $\delta_{\text{end,-20}}$ and $\delta_{\text{end,60}}$:

– for 0 less than $\delta_{\text{end,-20}}$ = 43,8° less than 90° is:

$$\chi_{-20} = 1 - r \sin\delta_{\text{end,-20}} = 1 - 0,763 \cdot \sin 43,8° = 0,472 \tag{30}$$

and for −0,985 less than χ_{-20} = 0,472 less than 0,766:

$$\delta_{\text{max,-20}} = 10° + \arccos \chi_{-20} = 10° + \arccos 0,472 = 71,8° \tag{31}$$

– for 0 less than $\delta_{\text{end,60}}$ = 38,8° less than 90° is:

$$\chi_{60} = 1 - r \sin\delta_{\text{end,60}} = 1 - 0,763 \cdot \sin 38,8° = 0,522 \tag{30}$$

and for −0,985 less than χ_{60} = 0,522 less than 0,766:

$$\delta_{\text{max,60}} = 10° + \arccos \chi_{60} = 10° + \arccos 0,522 = 68,5° \tag{31}$$

9.3.3.2 Tensile force $F_{t,d}$ during short-circuit caused by swing out without dropper in midspan

The calculation is done according to IEC 60865-1:2011, 6.2.3.

The load parameters are:

$$\varphi_{60} = \varphi_{\text{-20}} = 3\left(\sqrt{1+r^2}-1\right) = 3\left(\sqrt{1+0{,}763^2}-1\right) = 0{,}774 \quad (32)$$

because

$$T_{\text{k1}} = 0{,}5\ \text{s} \quad \text{greater than} \quad \frac{T_{\text{res,-20}}}{4} = \frac{1{,}80\ \text{s}}{4} = 0{,}450\ \text{s}$$

$$T_{\text{k1}} = 0{,}5\ \text{s} \quad \text{greater than} \quad \frac{T_{\text{res,60}}}{4} = \frac{1{,}95\ \text{s}}{4} = 0{,}488\ \text{s}$$

According to IEC 60865-1:2011, Figure 8, the factors $\psi_{\text{-20}}$ and ψ_{60} are:

– for $\varphi_{\text{-20}} = 0{,}774$ and $\zeta_{\text{-20}} = 1{,}69$:

$$\psi_{\text{-20}} = 0{,}702$$

– for $\varphi_{60} = 0{,}774$ and $\zeta_{60} = 2{,}60$:

$$\psi_{60} = 0{,}774$$

The tensile forces during the short-circuit are:

$$\begin{aligned} F_{\text{t,d,-20}} &= F_{\text{st,-20}}\left(1+\varphi_{\text{-20}}\,\psi_{\text{-20}}\right) = 17{,}4\ \text{kN}\cdot\left(1+0{,}774\cdot 0{,}702\right) = 26{,}9\ \text{kN} \\ F_{\text{t,d,60}} &= F_{\text{st,60}}\left(1+\varphi_{60}\,\psi_{60}\right) = 15{,}0\ \text{kN}\cdot\left(1+0{,}774\cdot 0{,}774\right) = 24{,}0\ \text{kN} \end{aligned} \quad (33)$$

The tensile force $F_{\text{t,d}}$ is the maximum value of $F_{\text{t,d,-20}}$ and $F_{\text{t,d,60}}$:

$$F_{\text{t,d}} = \max\left\{F_{\text{t,d,-20}}; F_{\text{t,d,60}}\right\} = \max\left\{26{,}9\ \text{kN}; 24{,}0\ \text{kN}\right\} = 26{,}9\ \text{kN}$$

9.3.3.3 Dynamic conductor sag at midspan

The elastic expansions are:

$$\begin{aligned} \varepsilon_{\text{ela,-20}} &= N_{\text{-20}}\left(F_{\text{t,d,-20}} - F_{\text{st,-20}}\right) = 5{,}78\cdot 10^{-8}\ \frac{1}{\text{N}}\cdot\left(26{,}9-17{,}4\right)\cdot 10^{3}\ \text{N} = 0{,}543\cdot 10^{-3} \\ \varepsilon_{\text{ela,60}} &= N_{60}\left(F_{\text{t,d,60}} - F_{\text{st,60}}\right) = 5{,}87\cdot 10^{-8}\ \frac{1}{\text{N}}\cdot\left(24{,}0-15{,}0\right)\cdot 10^{3}\ \text{N} = 0{,}528\cdot 10^{-3} \end{aligned} \quad (34)$$

The thermal expansions are:

$$\begin{aligned} \varepsilon_{\text{th,-20}} &= c_{\text{th}}\left(\frac{I_{\text{k}}''}{n\,A_{\text{s}}}\right)^2 \frac{T_{\text{res,-20}}}{4} = 0{,}27\cdot 10^{-18}\ \frac{\text{m}^4}{\text{A}^2\,\text{s}}\cdot\left(\frac{63\cdot 10^3\ \text{A}}{2\cdot 1090\cdot 10^{-6}\ \text{m}^2}\right)^2\cdot\frac{1{,}80\ \text{s}}{4} = 1{,}02\cdot 10^{-4} \\ \varepsilon_{\text{th,60}} &= c_{\text{th}}\left(\frac{I_{\text{k}}''}{n\,A_{\text{s}}}\right)^2 \frac{T_{\text{res,60}}}{4} = 0{,}27\cdot 10^{-18}\ \frac{\text{m}^4}{\text{A}^2\,\text{s}}\cdot\left(\frac{63\cdot 10^3\ \text{A}}{2\cdot 1090\cdot 10^{-6}\ \text{m}^2}\right)^2\cdot\frac{1{,}95\ \text{s}}{4} = 1{,}10\cdot 10^{-4} \end{aligned} \quad (35)$$

because

$$T_{k1} = 0{,}5\ \text{s} \qquad \text{greater than} \qquad \frac{T_{\text{res},-20}}{4} = \frac{1{,}80\ \text{s}}{4} = 0{,}450\ \text{s}$$
$$T_{k1} = 0{,}5\ \text{s} \qquad \text{greater than} \qquad \frac{T_{\text{res},60}}{4} = \frac{1{,}95\ \text{s}}{4} = 0{,}488\ \text{s}$$

and for ASCR conductors with $A_{\text{Al}}/A_{\text{St}} = 1046\ \text{mm}/45\ \text{mm} = 23{,}2$ greater than 6

$$c_{\text{th}} = 0{,}27 \cdot 10^{-18}\ \text{m}^4/\left(\text{A}^2\text{s}\right)$$

The factors C_D are:

$$C_{\text{D},-20} = \sqrt{1+\frac{3}{8}\left(\frac{l}{f_{\text{es},-20}}\right)^2\left(\varepsilon_{\text{ela},-20}+\varepsilon_{\text{th},-20}\right)} = \sqrt{1+\frac{3}{8}\left(\frac{48\ \text{m}}{1{,}21\ \text{m}}\right)^2\left(0{,}543\cdot 10^{-3}+1{,}02\cdot 10^{-4}\right)} = 1{,}18$$
$$C_{\text{D},60} = \sqrt{1+\frac{3}{8}\left(\frac{l}{f_{\text{es},60}}\right)^2\left(\varepsilon_{\text{ela},60}+\varepsilon_{\text{th},60}\right)} = \sqrt{1+\frac{3}{8}\left(\frac{48\ \text{m}}{1{,}41\ \text{m}}\right)^2\left(0{,}528\cdot 10^{-3}+1{,}10\cdot 10^{-4}\right)} = 1{,}13 \qquad (36)$$

The factor C_F is:

$$C_F = 1{,}05 \qquad (37)$$

because

$$r = 0{,}763 \qquad \text{less than} \qquad 0{,}8$$

The dynamic conductor sags at midspan are:

$$f_{\text{ed},-20} = C_F\, C_{\text{D},-20}\, f_{\text{es},-20} = 1{,}05\cdot 1{,}18\cdot 1{,}21\ \text{m} = 1{,}50\ \text{m}$$
$$f_{\text{ed},60} = C_F\, C_{\text{D},60}\, f_{\text{es},60} = 1{,}05\cdot 1{,}13\cdot 1{,}41\ \text{m} = 1{,}67\ \text{m} \qquad (38)$$

9.3.3.4 Tensile force $F_{t,d}$ during short-circuit caused by swing out with dropper in midspan

The calculation is done according to IEC 60865-1:2011, 6.2.5, because

$$\sqrt{\left(h_{-20}+f_{\text{es},-20}+f_{\text{ed},-20}\right)^2+w^2} = \sqrt{(7{,}2\ \text{m}+1{,}21\ \text{m}+1{,}50\ \text{m})^2+(2\ \text{m})^2} = 10{,}1\ \text{m} \quad \text{greater than} \quad l_v = 7{,}6\ \text{m}$$
$$\sqrt{\left(h_{60}+f_{\text{es},60}+f_{\text{ed},60}\right)^2+w^2} = \sqrt{(7{,}0\ \text{m}+1{,}41\ \text{m}+1{,}67\ \text{m})^2+(2\ \text{m})^2} = 10{,}3\ \text{m} \quad \text{greater than} \quad l_v = 7{,}6\ \text{m}$$

with the dropper height at –20°C due to the change of sag with the temperature of the main conductor

$$h_{-20} = h_{60} + \left(f_{\text{es},60} - f_{\text{es},-20}\right) = 7{,}0\ \text{m} + (1{,}41\ \text{m} - 1{,}21\ \text{m}) = 7{,}2\ \text{m}$$

and $h_{60} = h = 7{,}0\ \text{m}$.

The actual swing-out angles are:

$$\delta_{-20} = \arccos\frac{\left(h_{-20} + f_{\text{es,-20}}\right)^2 + f_{\text{ed,-20}}^2 - \left(l_{\text{v}}^2 - w^2\right)}{2 f_{\text{ed,-20}}\left(h_{-20} + f_{\text{es,-20}}\right)} = \arccos\frac{\left(7{,}2\ \text{m} + 1{,}21\ \text{m}\right)^2 + \left(1{,}50\ \text{m}\right)^2 - \left(\left(7{,}6\ \text{m}\right)^2 - \left(2{,}0\ \text{m}\right)^2\right)}{2\cdot 1{,}50\ \text{m}\cdot\left(7{,}2\ \text{m} + 1{,}21\ \text{m}\right)} = 40{,}4^\circ \quad (39)$$

$$\delta_{60} = \arccos\frac{\left(h_{60} + f_{\text{es,60}}\right)^2 + f_{\text{ed,60}}^2 - \left(l_{\text{v}}^2 - w^2\right)}{2 f_{\text{ed,60}}\left(h_{60} + f_{\text{es,60}}\right)} = \arccos\frac{\left(7{,}0\ \text{m} + 1{,}41\ \text{m}\right)^2 + \left(1{,}67\ \text{m}\right)^2 - \left(\left(7{,}6\ \text{m}\right)^2 - \left(2{,}0\ \text{m}\right)^2\right)}{2\cdot 1{,}67\ \text{m}\cdot\left(7{,}0\ \text{m} + 1{,}41\ \text{m}\right)} = 45{,}3^\circ \quad (39)$$

The load parameters are:

$$\varphi_{-20} = \varphi_{60} = 3\left(\sqrt{1 + r^2} - 1\right) = 3\left(\sqrt{1 + 0{,}763^2} - 1\right) = 0{,}774 \quad (40)$$

because

$$\delta_{-20} = 40{,}4^\circ \quad \text{greater than} \quad \delta_1 = 37{,}3^\circ$$
$$\delta_{60} = 45{,}3^\circ \quad \text{greater than} \quad \delta_1 = 37{,}3^\circ$$

and also

$$T_{\text{k1}} = 0{,}5\ \text{s} \quad \text{greater than} \quad \frac{T_{\text{res,-20}}}{4} = \frac{1{,}80\ \text{s}}{4} = 0{,}450\ \text{s}$$
$$T_{\text{k1}} = 0{,}5\ \text{s} \quad \text{greater than} \quad \frac{T_{\text{res,60}}}{4} = \frac{1{,}95\ \text{s}}{4} = 0{,}488\ \text{s}$$

According to IEC 60865-1:2011, Figure 8, the factors ψ_{-20} and ψ_{60} are:

– for φ_{-20} = 0,774 and ζ_{-20} = 1,69:

$$\psi_{-20} = 0{,}702$$

– for φ_{60} = 0,774 and ζ_{60} = 2,60:

$$\psi_{60} = 0{,}774$$

The tensile forces during the short-circuit are:

$$F_{\text{t,d,-20}} = F_{\text{st,-20}}\left(1 + \varphi_{-20}\psi_{-20}\right) = 17{,}4\ \text{kN}\cdot\left(1 + 0{,}774\cdot 0{,}702\right) = 26{,}9\ \text{kN}$$
$$F_{\text{t,d,60}} = F_{\text{st,60}}\left(1 + \varphi_{60}\psi_{60}\right) = 15{,}0\ \text{kN}\cdot\left(1 + 0{,}774\cdot 0{,}774\right) = 24{,}0\ \text{kN} \quad (42)$$

The tensile force $F_{\text{t,d}}$ is the maximum value of $F_{\text{t,d,-20}}$ and $F_{\text{t,d,60}}$:

$$F_{\text{t,d}} = \max\left\{F_{\text{t,d,-20}}; F_{\text{t,d,60}}\right\} = \max\left\{26{,}9\ \text{kN}; 24{,}0\ \text{kN}\right\} = 26{,}9\ \text{kN}$$

9.3.3.5 Tensile force $F_{f,d}$ after short-circuit caused by drop

Because

$$r = 0{,}763 \quad \text{greater than} \quad 0{,}6$$

and

$$\delta_{max,-20} = 71{,}8° \quad \text{greater than} \quad 70°$$
$$\delta_{max,60} = 68{,}5° \quad \text{less than} \quad 70°$$

however

$$\delta_{-20} = 40{,}4° \quad \text{less than} \quad 60°$$
$$\delta_{60} = 45{,}6° \quad \text{less than} \quad 60°$$

the tensile force $F_{f,d}$ after short-circuit is not significant.

When calculating according to IEC 60865-1:2011, 6.2.3, in addition the tensile force $F_{f,d}$ after short-circuit is to be calculated according to IEC 60865-1:2011, 6.2.6. Because

$$r = 0{,}763 \quad \text{greater than} \quad 0{,}6$$

and

$$\delta_{max,-20} = 71{,}8° \quad \text{greater than} \quad 70°$$
$$\delta_{max,60} = 68{,}5° \quad \text{less than} \quad 70°$$

the drop force becomes:

$$F_{f,d,-20} = 1{,}2 \cdot F_{st,-20} \sqrt{1 + 8\zeta_{-20} \frac{\delta_{max,-20}}{180°}} = 1{,}2 \cdot 17{,}4 \text{ kN} \cdot \sqrt{1 + 8 \cdot 1{,}69 \cdot \frac{71{,}8°}{180°}} = 52{,}8 \text{ kN} \tag{43}$$
$$F_{f,d,60} = 0 \text{ kN}$$

The tensile force $F_{f,d}$ is the maximum of $F_{f,d,-20}$ and $F_{f,d,60}$:

$$F_{f,d} = \max\left\{F_{f,d,-20}; F_{f,d,60}\right\} = \max\left\{52{,}8 \text{ kN}; 0 \text{ kN}\right\} = 52{,}8 \text{ kN}$$

9.3.3.6 Horizontal span displacement b_h and minimum air clearance a_{min}

All the following quantities are calculated at a conductor temperature of 60°C which leads to a greater conductor sag than a conductor temperature of −20°C.

The maximum horizontal span displacement for stranded conductors with $l_c = l - 2l_i$ is:

$$b_h = f_{ed,60} \sin\delta_1 = 1{,}68 \text{ m} \cdot \sin 37{,}3° = 1{,}02 \text{ m} \tag{47}$$

because

$$\delta_{60} = 45{,}3° \quad \text{less than} \quad \delta_{\max,60} = 68{,}5°$$
$$\delta_{60} = 45{,}3° \quad \text{greater than} \quad \delta_1 = 37{,}3°$$

and the minimum air clearance is:

$$a_{\min} = a - 2b_{\rm h} = 5\ {\rm m} - 2 \cdot 1{,}02\ {\rm m} = 2{,}96\ {\rm m} \qquad (48)$$

When calculating according to IEC 60865-1:2011, 6.2.3, without dropper in midspan, in addition the horizontal displacement b_h and the minimum air clearance a_{min} shall be calculated according to IEC 60865-1:2011, 6.2.7:

$$b_{\rm h} = f_{\rm ed,60} \sin \delta_1 = 1{,}68\ {\rm m} \cdot \sin 37{,}3° = 1{,}02\ {\rm m} \qquad (45)$$

because

$$\delta_{\max,60} = 68{,}5° \quad \text{greater than} \quad \delta_1 = 37{,}3°$$

and the minimum air clearance is:

$$a_{\min} = a - 2b_{\rm h} = 5\ {\rm m} - 2 \cdot 1{,}02\ {\rm m} = 2{,}96\ {\rm m} \qquad (48)$$

9.3.3.7 Pinch force $F_{pi,d}$

The sub-conductors clash effectively during short-circuit because Equation (53) is fulfilled, see 9.3.2.7.

The tensile forces caused by pinch are:

$$\begin{aligned} F_{\rm pi,d,-20} &= 1{,}1 F_{\rm t,d,-20} = 1{,}1 \cdot 26{,}9\ {\rm kN} = 29{,}6\ {\rm kN} \\ F_{\rm pi,d,60} &= 1{,}1 F_{\rm t,d,60} = 1{,}1 \cdot 24{,}0\ {\rm kN} = 26{,}4\ {\rm kN} \end{aligned} \qquad (51)$$

$F_{t,d,-20}$ and $F_{t,d,60}$ are calculated in 9.3.3.2.

The pinch force $F_{pi,d}$ is the maximum of $F_{pi,d,-20}$ and $F_{pi,d,60}$:

$$F_{\rm pi,d} = \max\left\{F_{\rm pi,d,-20}; F_{\rm pi,d,60}\right\} = \max\left\{29{,}6\ {\rm kN}; 26{,}4\ {\rm kN}\right\} = 29{,}6\ {\rm kN}$$

9.3.3.8 Conclusions

According to IEC 60865-1:2011, 6.5.2 and 6.5.3, to the structure, the insulators and the connectors and to the foundations the maximum value of $F_{t,d}$, $F_{f,d}$ and $F_{pi,d}$ shall be applied as a static load:

$$\max\left\{F_{\rm t,d}; F_{\rm f,d}; F_{\rm pi,d}\right\} = \max\left\{26{,}9\ {\rm kN}; 0\ {\rm kN}; 29{,}6\ {\rm kN}\right\} = 29{,}6\ {\rm kN}$$

given by the tensile force $F_{pi,d}$ caused by pinch.

The maximum horizontal displacement is 1,02 m and the minimum air clearance is 2,96 m.

When calculating according to IEC 60865-1:2011, 6.2.3, without dropper in midspan, to the structure, the insulators and the connectors and to the foundations the maximum value of $F_{t,d}$, $F_{f,d}$ and $F_{pi,d}$ shall be applied as a static load:

$$\max\{F_{t,d};F_{f,d};F_{pi,d}\} = \max\{26,9\text{ kN};52,8\text{ kN};29,6\text{ kN}\} = 52,8\text{ kN}$$

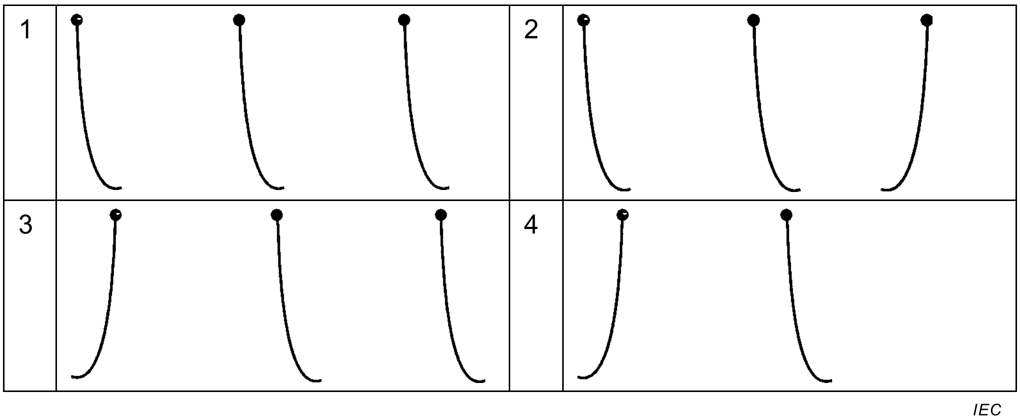
given by the tensile force $F_{f,d}$ after short-circuit caused by drop without dropper in midspan.

The maximum horizontal displacement is 1,02 m and the minimum air clearance is 2,96 m.

9.4 Plane of the dropper perpendicular to the main conductors

9.4.1 General

Droppers in three-phase systems can be arranged as shown in Figure 7. It is not possible to predict in which pair of main conductors the short-circuit currents flow. In all cases, the calculation should be done according to IEC 60865-1:2011.



In two-line-systems with outwards mounted droppers arranged according to configuration 4 in Figure 1, the maximum horizontal displacement b_h shall be calculated according to IEC 60865-1:2011, 6.2.5. The minimum air-clearance can be calculated by replacing in Equation (39) the plus-sign by a minus sign.

Figure 7 – Possible arrangement of perpendicular droppers in three-phase system and two-line system

In addition to 9.2, the following data are given:

Height of the dropper at a main conductor temperature of 60°C (maximum operating temperature)	h	=	7 m
Width of the dropper at a main conductor temperature of 60°C (maximum operating temperature)	w	=	1,5 m
Cord length of the dropper	l_v	=	7,4 m

9.4.2 Current flow along the whole length of the main conductor span

9.4.2.1 Electromagnetic load and characteristic parameters

The electromagnetic load and the characteristic parameters are calculated in 9.3.2.1, they do not depend on the dropper:

The characteristic electromagnetic load per unit length is:

$$F' = 92{,}8\ \text{N/m} \tag{19a}$$

The parameter r is:

$$r = 1{,}27 \tag{20}$$

The direction of the resulting force on the conductor is:

$$\delta_1 = 51{,}8° \tag{21}$$

The equivalent static conductor sags at midspan are:

$$f_{\text{es,-20}} = 1{,}21\ \text{m} \qquad f_{\text{es,60}} = 1{,}41\ \text{m} \tag{22}$$

The periods of the conductor oscillation are:

$$T_{\text{-20}} = 1{,}97\ \text{s} \qquad T_{60} = 2{,}13\ \text{s} \tag{23}$$

The resulting periods of the conductor oscillation are:

$$T_{\text{res,-20}} = 1{,}63\ \text{s} \qquad T_{\text{res,60}} = 1{,}77\ \text{s} \tag{24}$$

The stiffness norms are:

$$N_{\text{-20}} = 5{,}78 \cdot 10^{-8}\ \text{1/N} \qquad N_{60} = 5{,}87 \cdot 10^{-8}\ \text{1/N} \tag{25}$$

The stress factors are:

$$\zeta_{\text{-20}} = 1{,}69 \qquad \zeta_{60} = 2{,}60 \tag{28}$$

In the Equations (29), (32), and (35) it is to be inserted

$$T_{\text{k1}} = 0{,}5\ \text{s}$$

The swing-out angles at the end of short-circuit current flow are:

$$\delta_{\text{end,-20}} = 69{,}9° \qquad \delta_{\text{end,60}} = 62{,}3° \tag{29}$$

The maximum swing-out angles are:

$$\delta_{\text{max,-20}} = 111° \qquad \delta_{\text{max,60}} = 107° \tag{31}$$

9.4.2.2 Tensile force $F_{\text{t,d}}$ during short-circuit caused by swing out without dropper in midspan

The tensile force $F_{\text{t,d}}$ during short-circuit caused by swing out without dropper in midspan is the same as in 9.3.2.2, it does not depend on the dropper.

The tensile forces during the short-circuit are:

$$F_{\text{t,d,-20}} = 38{,}0\ \text{kN} \qquad F_{\text{t,d,60}} = 34{,}8\ \text{kN} \tag{33}$$

The tensile force $F_{\text{t,d}}$ is the maximum value of $F_{\text{t,d,-20}}$ and $F_{\text{t,d,60}}$:

$$F_{\text{t,d}} = \max\left\{F_{\text{t,d,-20}};F_{\text{t,d,60}}\right\} = \max\left\{38{,}0\ \text{kN};34{,}8\ \text{kN}\right\} = 38{,}0\ \text{kN}$$

9.4.2.3 Dynamic conductor sag at midspan

The dynamic conductor sags at midspan are the same as in 9.3.2.3:

$$f_{\text{ed,-20}} = 1{,}77\ \text{m} \qquad f_{\text{ed,60}} = 1{,}92\ \text{m} \tag{38}$$

9.4.2.4 Tensile force $F_{\text{t,d}}$ during short-circuit caused by swing out with dropper in midspan

The calculation is done according to IEC 60865-1:2011, 6.2.5, because

$$\sqrt{\left(h_{\text{-20}} + f_{\text{es,-20}}\right)^2 + w^2} + f_{\text{ed,-20}} = \sqrt{\left(7{,}2\ \text{m} + 1{,}21\ \text{m}\right)^2 + \left(1{,}5\ \text{m}\right)^2} + 1{,}77\ \text{m} = 10{,}3\ \text{m} \quad \text{greater than} \quad l_{\text{v}} = 7{,}4\ \text{m}$$

$$\sqrt{\left(h_{60} + f_{\text{es,60}}\right)^2 + w^2} + f_{\text{ed,60}} = \sqrt{\left(7{,}0\ \text{m} + 1{,}41\ \text{m}\right)^2 + \left(1{,}5\ \text{m}\right)^2} + 1{,}92\ \text{m} = 10{,}5\ \text{m} \quad \text{greater than} \quad l_{\text{v}} = 7{,}4\ \text{m}$$

with the dropper height at −20°C due to the change of sag with the temperature of the main conductor

$$h_{\text{-20}} = h_{60} + \left(f_{\text{es,60}} - f_{\text{es,-20}}\right) = 7{,}0\ \text{m} + \left(1{,}41\ \text{m} - 1{,}21\ \text{m}\right) = 7{,}2\ \text{m}$$

and $h_{60} = h = 7{,}0\ \text{m}$.

The actual swing-out angles are:

$$\begin{aligned}
\delta_{\text{-20}} &= \arccos\frac{\left(h_{\text{-20}} + f_{\text{es,-20}}\right)^2 + f_{\text{ed,-20}}^2 - \left(l_{\text{v}}^2 - w^2\right)}{2 f_{\text{ed,-20}}\sqrt{\left(h_{-20} + f_{\text{es,-20}}\right)^2 + w^2}} + \arccos\frac{h_{-20} + f_{\text{es,-20}}}{\sqrt{\left(h_{-20} + f_{\text{es,-20}}\right)^2 + w^2}} \\
&= \arccos\frac{\left(7{,}2\ \text{m} + 1{,}21\ \text{m}\right)^2 + \left(1{,}77\ \text{m}\right)^2 - \left(\left(7{,}4\ \text{m}\right)^2 - \left(1{,}5\ \text{m}\right)^2\right)}{2\cdot 1{,}77\ \text{m}\cdot\sqrt{\left(7{,}2\ \text{m} + 1{,}21\ \text{m}\right)^2 + \left(1{,}5\ \text{m}\right)^2}} + \arccos\frac{7{,}2\ \text{m} + 1{,}21\ \text{m}}{\sqrt{\left(7{,}2\ \text{m} + 1{,}21\ \text{m}\right)^2 + \left(1{,}5\ \text{m}\right)^2}} \\
&= 55{,}2^\circ \\
\delta_{60} &= \arccos\frac{\left(h_{60} + f_{\text{es,60}}\right)^2 + f_{\text{ed,60}}^2 - \left(l_{\text{v}}^2 - w^2\right)}{2 f_{\text{ed,60}}\sqrt{\left(h_{60} + f_{\text{es,60}}\right)^2 + w^2}} + \arccos\frac{h_{60} + f_{\text{es,60}}}{\sqrt{\left(h_{60} + f_{\text{es,60}}\right)^2 + w^2}} = \\
&= \arccos\frac{\left(7{,}0\ \text{m} + 1{,}41\ \text{m}\right)^2 + \left(1{,}92\ \text{m}\right)^2 - \left(\left(7{,}4\ \text{m}\right)^2 - \left(1{,}5\ \text{m}\right)^2\right)}{2\cdot 1{,}92\ \text{m}\cdot\sqrt{\left(7{,}0\ \text{m} + 1{,}41\ \text{m}\right)^2 + \left(1{,}5\ \text{m}\right)^2}} + \arccos\frac{7{,}0\ \text{m} + 1{,}41\ \text{m}}{\sqrt{\left(7{,}0\ \text{m} + 1{,}41\ \text{m}\right)^2 + \left(1{,}5\ \text{m}\right)^2}} \\
&= 58{,}2^\circ
\end{aligned} \tag{39}$$

The load parameters are:

$$\varphi_{60} = \varphi_{-20} = 3\left(\sqrt{1+r^2}-1\right) = 3\left(\sqrt{1+1{,}27^2}-1\right) = 1{,}85 \quad (32)$$

because

$$\delta_{-20} = 55{,}2° \quad \text{greater than} \quad \delta_1 = 51{,}8°$$
$$\delta_{60} = 58{,}2° \quad \text{greater than} \quad \delta_1 = 51{,}8°$$

and

$$T_{k1} = 0{,}5\text{ s} \quad \text{greater than} \quad \frac{T_{res,-20}}{4} = \frac{1{,}63\text{ s}}{4} = 0{,}408\text{ s}$$
$$T_{k1} = 0{,}5\text{ s} \quad \text{greater than} \quad \frac{T_{res,60}}{4} = \frac{1{,}77\text{ s}}{4} = 0{,}443\text{ s}$$

According to IEC 60865-1:2011, Figure 8, the factors ψ_{-20} and ψ_{60} are:

– for φ_{-20} = 1,85 and ζ_{-20} = 1,69:

$$\psi_{-20} = 0{,}641$$

– for φ_{60} = 1,85 and ζ_{60} = 2,60:

$$\psi_{60} = 0{,}714$$

The tensile forces during the short-circuit are:

$$F_{t,d,-20} = F_{st,-20}\left(1+\varphi_{-20}\,\psi_{-20}\right) = 17{,}4\text{ kN}\cdot\left(1+1{,}85\cdot 0{,}641\right) = 38{,}0\text{ kN}$$
$$F_{t,d,60} = F_{st,60}\left(1+\varphi_{60}\,\psi_{60}\right) = 15{,}0\text{ kN}\cdot\left(1+1{,}85\cdot 0{,}714\right) = 34{,}8\text{ kN} \quad (42)$$

The tensile force $F_{t,d}$ is the maximum value of $F_{t,d,-20}$ and $F_{t,d,60}$:

$$F_{t,d} = \max\left\{F_{t,d,-20}; F_{t,d,60}\right\} = \max\left\{38{,}0\text{ kN}; 34{,}8\text{ kN}\right\} = 38{,}0\text{ kN}$$

9.4.2.5 Tensile force $F_{f,d}$ after short-circuit caused by drop

Because

$$r = 1{,}27 \quad \text{greater than} \quad 0{,}6$$

and

$$\delta_{max,-20} = 111° \quad \text{greater than} \quad 70°$$
$$\delta_{max,60} = 107° \quad \text{greater than} \quad 70°$$

however

$$\delta_{-20} = 55{,}2° \quad \text{less than} \quad 60°$$
$$\delta_{60} = 58{,}2° \quad \text{less than} \quad 60°$$

the tensile force $F_{f,d}$ after short-circuit is not significant.

When calculating according to IEC 60865-1:2011, 6.2.3, in addition the tensile force $F_{f,d}$ after short-circuit is to be calculated according to IEC 60865-1:2011, 6.2.6. Because

$$r = 1{,}27 \quad \text{greater than} \quad 0{,}6$$

and

$$\delta_{max,-20} = 111° \quad \text{greater than} \quad 70°$$
$$\delta_{max,60} = 107° \quad \text{greater than} \quad 70°$$

the drop force becomes:

$$F_{f,d,-20} = 1{,}2 \cdot F_{st,-20} \sqrt{1 + 8\zeta_{-20} \frac{\delta_{max,-20}}{180°}} = 1{,}2 \cdot 17{,}4 \text{ kN} \cdot \sqrt{1 + 8 \cdot 1{,}69 \cdot \frac{111°}{180°}} = 63{,}8 \text{ kN}$$
$$F_{f,d,60} = 1{,}2 \cdot F_{st,60} \sqrt{1 + 8\zeta_{60} \frac{\delta_{max,60}}{180°}} = 1{,}2 \cdot 15{,}0 \text{ kN} \cdot \sqrt{1 + 8 \cdot 2{,}60 \cdot \frac{107°}{180°}} = 65{,}8 \text{ kN} \qquad (43)$$

The tensile force $F_{f,d}$ is the maximum of $F_{f,d,-20}$ and $F_{f,d,60}$:

$$F_{f,d} = \max\left\{F_{f,d,-20}; F_{f,d,60}\right\} = \max\left\{63{,}8 \text{ kN}; 65{,}8 \text{ kN}\right\} = 65{,}8 \text{ kN}$$

9.4.2.6 Horizontal span displacement b_h and minimum air clearance a_{min}

All the following quantities are calculated at a conductor temperature of 60°C which leads to a greater conductor sag than a conductor temperature of −20°C.

The maximum horizontal span displacement for stranded conductors with $l_c = l - 2l_i$ is:

$$b_h = f_{ed,60} \sin\delta_1 = 1{,}92 \text{ m} \cdot \sin 51{,}8° = 1{,}51 \text{ m} \qquad (47)$$

because

$$\delta_{60} = 58{,}2° \quad \text{less than} \quad \delta_{max,60} = 107°$$

and because

$$\delta_{60} = 58{,}2° \quad \text{greater than} \quad \delta_1 = 51{,}8°$$

and the minimum air clearance is:

$$a_{min} = a - 2b_h = 5 \text{ m} - 2 \cdot 1{,}51 \text{ m} = 1{,}98 \text{ m} \qquad (48)$$

When calculating according to IEC 60865-1:2011, 6.2.3, without dropper in midspan, in addition the horizontal displacement b_h and the minimum air clearance a_{min} shall be calculated according to IEC 60865-1:2011, 6.2.7:

$$b_h = f_{ed,60} \sin\delta_1 = 1{,}92 \text{ m} \cdot \sin 51{,}8° = 1{,}51 \text{ m} \qquad (45)$$

because

$$\delta_1 = 51{,}8^\circ \qquad \text{less than} \qquad \delta_{max,60} = 107^\circ$$

and the minimum air clearance is:

$$a_{min} = a - 2b_h = 5\,\text{m} - 2\cdot 1{,}51\,\text{m} = 1{,}98\,\text{m} \tag{48}$$

9.4.2.7 Pinch force $F_{pi,d}$

The sub-conductors clash effectively during short-circuit because Equation (53) is fulfilled, see 9.3.2.7.

The tensile forces caused by pinch are:

$$\begin{aligned} F_{pi,d,-20} &= 1{,}1 F_{t,d,-20} = 1{,}1\cdot 38{,}0\,\text{kN} = 41{,}8\,\text{kN} \\ F_{pi,d,60} &= 1{,}1 F_{t,d,60} = 1{,}1\cdot 34{,}8\,\text{kN} = 38{,}3\,\text{kN} \end{aligned} \tag{51}$$

$F_{t,d,-20}$ and $F_{t,d,60}$ are calculated in 9.3.2.2.

The pinch force $F_{pi,d}$ is the maximum of $F_{pi,d,-20}$ and $F_{pi,d,60}$:

$$F_{pi,d} = \max\left\{F_{pi,d,-20}; F_{pi,d,60}\right\} = \max\{41{,}8\,\text{kN}; 38{,}3\,\text{kN}\} = 41{,}8\,\text{kN}$$

9.4.2.8 Conclusions

According to IEC 60865-1:2011, 6.5.2 and 6.5.3, to the structure, the insulators and the connectors and to the foundations the maximum value of $F_{t,d}$, $F_{f,d}$ and $F_{pi,d}$ shall be applied as a static load:

$$\max\left\{F_{t,d}; F_{f,d}; F_{f,d}\right\} = \max\{38{,}0\,\text{kN}; 0\,\text{kN}; 41{,}8\,\text{kN}\} = 41{,}8\,\text{kN}$$

given by the tensile force $F_{pi,d}$ caused by pinch.

The maximum horizontal displacement is 1,51 m and the minimum air clearance is 1,98 m.

When calculating according to IEC 60865-1:2011, 6.2.3, without dropper in midspan, to the structure, the insulators and the connectors and to the foundations the maximum value of $F_{t,d}$, $F_{f,d}$ and $F_{pi,d}$ shall be applied as a static load:

$$\max\left\{F_{t,d}; F_{f,d}; F_{pi,d}\right\} = \max\{38{,}0\,\text{kN}; 65{,}8\,\text{kN}; 41{,}8\,\text{kN}\} = 65{,}8\,\text{kN}$$

given by the tensile force $F_{f,d}$ after short-circuit caused by drop without dropper in midspan.

The maximum horizontal displacement is 1,51 m and the minimum air clearance is 1,98 m and has the same value as with dropper.

9.4.3 Current flow along half of the length of the main conductor and along the dropper

9.4.3.1 Electromagnetic load and characteristic parameters

The characteristic electromagnetic load per unit length is:

$$F' = \frac{\mu_0}{2\pi} 0{,}75 \frac{(I_k'')^2}{a} \frac{l_c/2 + l_v/2}{l} = \frac{4\pi \cdot 10^{-7}}{2\pi} \frac{\text{Vs}}{\text{Am}} \cdot 0{,}75 \cdot \frac{\left(63 \cdot 10^3 \text{ A}\right)^2}{5 \text{ m}} \cdot \frac{37{,}4 \text{ m}/2 + 7{,}4 \text{ m}/2}{48 \text{ m}} = 55{,}6 \text{ N/m} \quad (19b)$$

The parameter r is:

$$r = \frac{F'}{n\, m'_{sc}\, g} = \frac{55{,}6 \text{ N/m}}{2 \cdot 3{,}73 \text{ kg/m} \cdot 9{,}81 \text{ m/s}^2} = 0{,}760 \quad (20)$$

where

m'_{sc} = 3,73 kg/m see 9.3.2.1

The direction of the resulting force on the conductor is:

$$\delta_1 = \arctan r = \arctan 0{,}760 = 37{,}2° \quad (21)$$

The equivalent static conductor sags at midspan are, see 9.3.2.1:

$$f_{es,-20} = 1{,}21 \text{ m} \qquad f_{es,60} = 1{,}41 \text{ m} \quad (22)$$

The periods of the conductor oscillation are, see 9.3.2.1:

$$T_{-20} = 1{,}97 \text{ s} \qquad T_{60} = 2{,}13 \text{ s} \quad (23)$$

The resulting periods of the conductor oscillation are:

$$T_{res,-20} = \frac{T_{-20}}{\sqrt[4]{1+r^2}\left[1 - \frac{\pi^2}{64}\left(\frac{\delta_1}{90°}\right)^2\right]} = \frac{1{,}97}{\sqrt[4]{1+0{,}760^2}\left[1 - \frac{\pi^2}{64}\left(\frac{37{,}2°}{90°}\right)^2\right]} = 1{,}81 \text{ s}$$

$$T_{res,60} = \frac{T_{60}}{\sqrt[4]{1+r^2}\left[1 - \frac{\pi^2}{64}\left(\frac{\delta_1}{90°}\right)^2\right]} = \frac{2{,}13}{\sqrt[4]{1+0{,}760^2}\left[1 - \frac{\pi^2}{64}\left(\frac{37{,}2°}{90°}\right)^2\right]} = 1{,}95 \text{ s} \quad (24)$$

The stiffness norms are, see 9.3.2.1:

$$N_{-20} = 5{,}78 \cdot 10^{-8} \text{ 1/N} \qquad N_{60} = 5{,}87 \cdot 10^{-8} \text{ 1/N} \quad (25)$$

The stress factors are, see 9.3.2.1:

$$\zeta_{-20} = 1{,}69 \qquad \zeta_{60} = 2{,}60 \quad (28)$$

In the Equations (29), (32), and (35) it is to be inserted, see 9.3.2.1:

$$T_{k1} = 0{,}5 \text{ s}$$

The swing-out angles at the end of short-circuit current flow are:

$$\delta_{\text{end,-20}} = \delta_1\left[1-\cos\left(360°\frac{T_{\text{k1}}}{T_{\text{res,-20}}}\right)\right] = 37{,}2°\cdot\left[1-\cos\left(360°\cdot\frac{0{,}5\ \text{s}}{1{,}81\ \text{s}}\right)\right] = 43{,}3°$$
$$\delta_{\text{end,60}} = \delta_1\left[1-\cos\left(360°\frac{T_{\text{k1}}}{T_{\text{res,60}}}\right)\right] = 37{,}2°\cdot\left[1-\cos\left(360°\cdot\frac{0{,}5\ \text{s}}{1{,}95\ \text{s}}\right)\right] = 38{,}7° \quad (29)$$

because

$$\frac{T_{\text{k1}}}{T_{\text{res,-20}}} = \frac{0{,}5\ \text{s}}{1{,}81\ \text{s}} = 0{,}276 \quad \text{less than} \quad 0{,}5$$
$$\frac{T_{\text{k1}}}{T_{\text{res,60}}} = \frac{0{,}5\ \text{s}}{1{,}95\ \text{s}} = 0{,}256 \quad \text{less than} \quad 0{,}5$$

The maximum swing-out angles $\delta_{\text{max,-20}}$ and $\delta_{\text{max,60}}$ depend respectively on $\chi_{\text{-20}}$ and χ_{60} which depend on $\delta_{\text{end,-20}}$ and $\delta_{\text{end,60}}$:

– for 0 less than $\delta_{\text{end,-20}}$ = 43,3° less than 90° is:

$$\chi_{\text{-20}} = 1 - r\sin\delta_{\text{end,-20}} = 1 - 0{,}760\cdot\sin 43{,}3° = 0{,}479 \quad (30)$$

and for −0,985 less than $\chi_{\text{-20}}$ = 0,479 less than 0,766:

$$\delta_{\text{max,-20}} = 10° + \arccos\chi_{\text{-20}} = 10° + \arccos 0{,}479 = 71{,}4° \quad (31)$$

– for 0 less than $\delta_{\text{end,60}}$ = 38,7° less than 90° is:

$$\chi_{60} = 1 - r\sin\delta_{\text{end,60}} = 1 - 0{,}760\cdot\sin 38{,}7° = 0{,}525 \quad (30)$$

and for −0,985 less than χ_{60} = 0,525 less than 0,766:

$$\delta_{\text{max,60}} = 10° + \arccos\chi_{60} = 10° + \arccos 0{,}525 = 68{,}3° \quad (31)$$

9.4.3.2 Tensile force $F_{\text{t,d}}$ during short-circuit caused by swing out without dropper in midspan

The calculation is done according to IEC 60865-1:2011, 6.2.3.

The load parameters are:

$$\varphi_{60} = \varphi_{\text{-20}} = 3\left(\sqrt{1+r^2}-1\right) = 3\left(\sqrt{1+0{,}760^2}-1\right) = 0{,}768 \quad (32)$$

because

$$T_{\text{k1}} = 0{,}5\ \text{s} \quad \text{greater than} \quad \frac{T_{\text{res,-20}}}{4} = \frac{1{,}81\ \text{s}}{4} = 0{,}453\ \text{s}$$
$$T_{\text{k1}} = 0{,}5\ \text{s} \quad \text{greater than} \quad \frac{T_{\text{res,60}}}{4} = \frac{1{,}95\ \text{s}}{4} = 0{,}488\ \text{s}$$

According to IEC 60865-1:2011, Figure 8, the factors $\psi_{\text{-20}}$ and ψ_{60} are:

– for $\varphi_{\text{-20}}$ = 0,768 and $\zeta_{\text{-20}}$ = 1,69:

$$\psi_{-20} = 0{,}702$$

– for $\varphi_{60} = 0{,}768$ and $\zeta_{60} = 2{,}60$:

$$\psi_{60} = 0{,}775$$

The tensile forces during the short-circuit are:

$$\begin{aligned} F_{\text{t,d,-20}} &= F_{\text{st,-20}}\left(1+\varphi_{-20}\,\psi_{-20}\right) = 17{,}4\ \text{kN}\cdot\left(1+0{,}768\cdot 0{,}702\right) = 26{,}8\ \text{kN} \\ F_{\text{t,d,60}} &= F_{\text{st,60}}\left(1+\varphi_{60}\,\psi_{60}\right) = 15{,}0\ \text{kN}\cdot\left(1+0{,}768\cdot 0{,}775\right) = 23{,}9\ \text{kN} \end{aligned} \tag{33}$$

The tensile force $F_{\text{t,d}}$ is the maximum value of $F_{\text{t,d,-20}}$ and $F_{\text{t,d,60}}$:

$$F_{\text{t,d}} = \max\left\{F_{\text{t,d,-20}};F_{\text{t,d,60}}\right\} = \max\left\{26{,}8\ \text{kN};23{,}9\ \text{kN}\right\} = 26{,}8\ \text{kN}$$

9.4.3.3 Dynamic conductor sag at midspan

The elastic expansions are:

$$\begin{aligned} \varepsilon_{\text{ela,-20}} &= N_{-20}\left(F_{\text{t,d,-20}} - F_{\text{st,-20}}\right) = 5{,}78\cdot 10^{-8}\ \frac{1}{\text{N}}\cdot\left(26{,}8-17{,}4\right)10^{3}\ \text{N} = 0{,}543\cdot 10^{-3} \\ \varepsilon_{\text{ela,60}} &= N_{60}\left(F_{\text{t,d,60}} - F_{\text{st,60}}\right) = 5{,}87\cdot 10^{-8}\ \frac{1}{\text{N}}\cdot\left(23{,}9-15{,}0\right)10^{3}\ \text{N} = 0{,}522\cdot 10^{-3} \end{aligned} \tag{34}$$

The thermal expansions are:

$$\begin{aligned} \varepsilon_{\text{th,-20}} &= c_{\text{th}}\left(\frac{I_{\text{k}}''}{n\,A_{\text{s}}}\right)^{2}\frac{T_{\text{res,-20}}}{4} = 0{,}27\cdot 10^{-18}\ \frac{\text{m}^4}{\text{A}^2\,\text{s}}\cdot\left(\frac{63\cdot 10^{3}\ \text{A}}{2\cdot 1090\cdot 10^{-6}\ \text{m}^2}\right)^{2}\cdot\frac{1{,}81\ \text{s}}{4} = 1{,}02\cdot 10^{-4} \\ \varepsilon_{\text{th,60}} &= c_{\text{th}}\left(\frac{I_{\text{k}}''}{n\,A_{\text{s}}}\right)^{2}\frac{T_{\text{res,60}}}{4} = 0{,}27\cdot 10^{-18}\ \frac{\text{m}^4}{\text{A}^2\,\text{s}}\cdot\left(\frac{63\cdot 10^{3}\ \text{A}}{2\cdot 1090\cdot 10^{-6}\ \text{m}^2}\right)^{2}\cdot\frac{1{,}95\ \text{s}}{4} = 1{,}10\cdot 10^{-4} \end{aligned} \tag{35}$$

because

$$T_{\text{k1}} = 0{,}5\ \text{s} \quad \text{greater than} \quad \frac{T_{\text{res,-20}}}{4} = \frac{1{,}81\ \text{s}}{4} = 0{,}453\ \text{s}$$

$$T_{\text{k1}} = 0{,}5\ \text{s} \quad \text{greater than} \quad \frac{T_{\text{res,60}}}{4} = \frac{1{,}95\ \text{s}}{4} = 0{,}488\ \text{s}$$

and for ASCR conductors with $A_{\text{Al}}/A_{\text{St}} = 1046\ \text{mm}/45\ \text{mm} = 23{,}2$ greater than 6

$$c_{\text{th}} = 0{,}27\cdot 10^{-18}\ \text{m}^4/\left(\text{A}^2\text{s}\right)$$

The factors C_{D} are:

$$C_{D,-20} = \sqrt{1+\frac{3}{8}\left(\frac{l}{f_{es,-20}}\right)^2 \left(\varepsilon_{ela,-20}+\varepsilon_{th,-20}\right)} = \sqrt{1+\frac{3}{8}\left(\frac{48\ \text{m}}{1,21\ \text{m}}\right)^2 \left(0,543\cdot 10^{-3}+1,02\cdot 10^{-4}\right)} = 1,18$$
$$C_{D,60} = \sqrt{1+\frac{3}{8}\left(\frac{l}{f_{es,60}}\right)^2 \left(\varepsilon_{ela,60}+\varepsilon_{th,60}\right)} = \sqrt{1+\frac{3}{8}\left(\frac{48\ \text{m}}{1,41\ \text{m}}\right)^2 \left(0,522\cdot 10^{-3}+1,10\cdot 10^{-4}\right)} = 1,13 \quad (36)$$

The factor C_F is:

$$C_F = 1,05 \quad (37)$$

because

$r = 0,760$ less than 0,8

The dynamic conductor sags at midspan are:

$$f_{ed,-20} = C_F\, C_{D,-20}\, f_{es,-20} = 1,05\cdot 1,18\cdot 1,21\ \text{m} = 1,50\ \text{m}$$
$$f_{ed,60} = C_F\, C_{D,60}\, f_{es,60} = 1,05\cdot 1,13\cdot 1,41\ \text{m} = 1,67\ \text{m} \quad (38)$$

9.4.3.4 Tensile force $F_{t,d}$ during short-circuit caused by swing out with dropper in midspan

The calculation is done according to IEC 60865-1:2011, 6.2.5, because

$$\sqrt{\left(h_{-20}+f_{es,-20}\right)^2+w^2}+f_{ed,-20} = \sqrt{(7,2\ \text{m}+1,21\ \text{m})^2+(1,5\ \text{m})^2}+1,77\ \text{m} = 10,3\ \text{m} \quad \text{greater than} \quad l_v = 7,4\ \text{m}$$
$$\sqrt{\left(h_{60}+f_{es,60}\right)^2+w^2}+f_{ed,60} = \sqrt{(7,0\ \text{m}+1,41\ \text{m})^2+(1,5\ \text{m})^2}+1,92\ \text{m} = 10,5\ \text{m} \quad \text{greater than} \quad l_v = 7,4\ \text{m}$$

with the dropper height at −20°C due to the change of sag with the temperature of the main conductor

$$h_{-20} = h_{60} + \left(f_{es,60} - f_{es,-20}\right) = 7,0\ \text{m} + (1,41\ \text{m} - 1,21\ \text{m}) = 7,2\ \text{m}$$

and $h_{60} = h = 7,0\ \text{m}$.

The actual swing-out angles are:

$$\begin{aligned}
\delta_{-20} &= \arccos\frac{\left(h_{-20}+f_{\text{es},-20}\right)^2+f_{\text{ed},-20}^2-\left(l_\text{v}^2-w^2\right)}{2f_{\text{ed},-20}\sqrt{\left(h_{-20}+f_{\text{es},-20}\right)^2+w^2}}+\arccos\frac{h_{-20}+f_{\text{es},-20}}{\sqrt{\left(h_{-20}+f_{\text{es},-20}\right)^2+w^2}}\\
&= \arccos\frac{(7{,}2\text{ m}+1{,}21\text{ m})^2+(1{,}50\text{ m})^2-\left((7{,}4\text{ m})^2-(1{,}5\text{ m})^2\right)}{2\cdot 1{,}50\text{ m}\cdot\sqrt{(7{,}2\text{ m}+1{,}21\text{ m})^2+(1{,}5\text{ m})^2}}+\arccos\frac{7{,}2\text{ m}+1{,}21\text{ m}}{\sqrt{(7{,}2\text{ m}+1{,}21\text{ m})^2+(1{,}5\text{ m})^2}}\\
&= 47{,}1^\circ\\
\delta_{60} &= \arccos\frac{\left(h_{60}+f_{\text{es},60}\right)^2+f_{\text{ed},60}^2-\left(l_\text{v}^2-w^2\right)}{2f_{\text{ed},60}\sqrt{\left(h_{60}+f_{\text{es},60}\right)^2+w^2}}+\arccos\frac{h_{60}+f_{\text{es},60}}{\sqrt{\left(h_{60}+f_{\text{es},60}\right)^2+w^2}}=\\
&= \arccos\frac{(7{,}0\text{ m}+1{,}41\text{ m})^2+(1{,}67\text{ m})^2-\left((7{,}4\text{ m})^2-(1{,}5\text{ m})^2\right)}{2\cdot 1{,}67\text{ m}\cdot\sqrt{(7{,}0\text{ m}+1{,}41\text{ m})^2+(1{,}5\text{ m})^2}}+\arccos\frac{7{,}0\text{ m}+1{,}41\text{ m}}{\sqrt{(7{,}0\text{ m}+1{,}41\text{ m})^2+(1{,}5\text{ m})^2}}\\
&= 52{,}7^\circ
\end{aligned} \tag{39}$$

The load parameters are:

$$\varphi_{-20}=\varphi_{60}=3\left(\sqrt{1+r^2}-1\right)=3\left(\sqrt{1+0{,}760^2}-1\right)=0{,}768 \tag{40}$$

because

$$\begin{aligned}
\delta_{-20} &= 47{,}1^\circ \quad \text{greater than} \quad \delta_1 = 37{,}2^\circ\\
\delta_{60} &= 52{,}7^\circ \quad \text{greater than} \quad \delta_1 = 37{,}2^\circ
\end{aligned}$$

and also

$$\begin{aligned}
T_{\text{k1}} &= 0{,}5\text{ s} \quad \text{greater than} \quad \frac{T_{\text{res},-20}}{4}=\frac{1{,}81\text{ s}}{4}=0{,}453\text{ s}\\
T_{\text{k1}} &= 0{,}5\text{ s} \quad \text{greater than} \quad \frac{T_{\text{res},60}}{4}=\frac{1{,}95\text{ s}}{4}=0{,}488\text{ s}
\end{aligned}$$

According to IEC 60865-1:2011, Figure 8, the factors ψ_{-20} and ψ_{60} are:

– for φ_{-20} = 0,768 and ζ_{-20} = 1,69:

$$\psi_{-20} = 0{,}702$$

– for φ_{60} = 0,768 and ζ_{60} = 2,60:

$$\psi_{60} = 0{,}775$$

The tensile forces during the short-circuit are:

$$\begin{aligned}
F_{\text{t,d},-20} &= F_{\text{st},-20}\left(1+\varphi_{-20}\,\psi_{-20}\right)=17{,}4\text{ kN}\cdot(1+0{,}768\cdot 0{,}702)=26{,}8\text{ kN}\\
F_{\text{t,d},60} &= F_{\text{st},60}\left(1+\varphi_{60}\,\psi_{60}\right)=15{,}0\text{ kN}\cdot(1+0{,}768\cdot 0{,}775)=23{,}9\text{ kN}
\end{aligned} \tag{42}$$

The tensile force $F_{\text{t,d}}$ is the maximum value of $F_{\text{t,d},-20}$ and $F_{\text{t,d},60}$:

$$F_{\text{t,d}}=\max\left\{F_{\text{t,d},-20};F_{\text{t,d},60}\right\}=\max\left\{26{,}8\text{ kN};23{,}9\text{ kN}\right\}=26{,}8\text{ kN}$$

9.4.3.5 Tensile force $F_{f,d}$ after short-circuit caused by drop

Because

$$r = 0{,}760 \quad \text{greater than} \quad 0{,}6$$

and

$$\delta_{max,-20} = 71{,}4° \quad \text{greater than} \quad 70°$$
$$\delta_{max,60} = 68{,}3° \quad \text{less than} \quad 70°$$

however

$$\delta_{-20} = 47{,}1° \quad \text{less than} \quad 60°$$
$$\delta_{60} = 52{,}7° \quad \text{less than} \quad 60°$$

the tensile force $F_{f,d}$ after short-circuit is not significant.

When calculating according to IEC 60865-1:2011, 6.2.3, in addition the tensile force $F_{f,d}$ after short-circuit is to be calculated according to IEC 60865-1:2011, 6.2.6. Because

$$r = 0{,}760 \quad \text{greater than} \quad 0{,}6$$

and

$$\delta_{max,-20} = 71{,}4° \quad \text{greater than} \quad 70°$$
$$\delta_{max,60} = 68{,}3° \quad \text{less than} \quad 70°$$

the drop forces become:

$$F_{f,d,-20} = 1{,}2 \cdot F_{st,-20} \sqrt{1 + 8\zeta_{-20} \frac{\delta_{max,-20}}{180°}} = 1{,}2 \cdot 17{,}4 \text{ kN} \cdot \sqrt{1 + 8 \cdot 1{,}69 \cdot \frac{71{,}4°}{180°}} = 52{,}7 \text{ kN} \qquad (43)$$
$$F_{f,d,60} = 0 \text{ kN}$$

The tensile force $F_{f,d}$ is the maximum of $F_{f,d,-20}$ and $F_{f,d,60}$:

$$F_{f,d} = \max\{F_{f,d,-20}; F_{f,d,60}\} = \max\{52{,}7 \text{ kN}; 0 \text{ kN}\} = 52{,}7 \text{ kN}$$

9.4.3.6 Horizontal span displacement b_h and minimum air clearance a_{min}

All the following quantities are calculated at a conductor temperature of 60°C which leads to a greater conductor sag than a conductor temperature of –20°C.

The maximum horizontal span displacement for strained conductors with $l_c = l - 2l_i$ is:

$$b_h = f_{ed,60} \sin\delta_1 = 1{,}67 \text{ m} \cdot \sin 37{,}2° = 1{,}02 \text{ m} \qquad (47)$$

because

$$\delta_{60} = 57{,}2° \quad \text{less than} \quad \delta_{max,60} = 68{,}3°$$
$$\delta_{60} = 52{,}7° \quad \text{greater than} \quad \delta_1 = 37{,}2°$$

and the minimum air clearance is:

$$a_{\text{min}} = a - 2b_{\text{h}} = 5\text{ m} - 2 \cdot 1{,}02\text{ m} = 2{,}96\text{ m} \tag{48}$$

When calculating according to IEC 60865-1:2011, 6.2.3, without dropper in midspan, in addition the horizontal displacement b_{h} and the minimum air clearance a_{min} shall be calculated according to IEC 60865-1:2011, 6.2.7:

$$b_{\text{h}} = f_{\text{ed,60}} \sin\delta_1 = 1{,}67\text{ m} \cdot \sin 37{,}2° = 1{,}02\text{ m} \tag{45}$$

because

$$\delta_{\text{max,60}} = 68{,}3° \qquad \text{greater than} \qquad \delta_1 = 37{,}2°$$

and the minimum air clearance is:

$$a_{\text{min}} = a - 2b_{\text{h}} = 5\text{ m} - 2 \cdot 1{,}02\text{ m} = 2{,}96\text{ m} \tag{48}$$

9.4.3.7 Pinch force $F_{\text{pi,d}}$

The sub-conductors clash effectively during short circuit because Equation (53) is fulfilled, see 9.3.2.6.

The tensile forces caused by pinch are:

$$\begin{aligned} F_{\text{pi,d,-20}} &= 1{,}1 F_{\text{t,d,-20}} = 1{,}1 \cdot 26{,}8\text{ kN} = 29{,}5\text{ kN} \\ F_{\text{pi,d,60}} &= 1{,}1 F_{\text{t,d,60}} = 1{,}1 \cdot 23{,}9\text{ kN} = 26{,}3\text{ kN} \end{aligned} \tag{51}$$

$F_{\text{t,d,-20}}$ and $F_{\text{t,d,60}}$ are calculated in 9.4.3.4.

The pinch force $F_{\text{pi,d}}$ is the maximum of $F_{\text{pi,d,-20}}$ and $F_{\text{pi,d,60}}$:

$$F_{\text{pi,d}} = \max\left\{F_{\text{pi,d,-20}}; F_{\text{pi,d,60}}\right\} = \max\left\{29{,}5\text{ kN}; 26{,}3\text{ kN}\right\} = 29{,}5\text{ kN}$$

9.4.3.8 Conclusions

According to IEC 60865-1:2011, 6.5.2 and 6.5.3, to the structure, the insulators and the connectors and to the foundations the maximum value of $F_{\text{t,d}}$, $F_{\text{f,d}}$ and $F_{\text{pi,d}}$ shall be applied as a static load:

$$\max\left\{F_{\text{t,d}}; F_{\text{f,d}}; F_{\text{pi,d}}\right\} = \max\left\{26{,}8\text{ kN}; 0\text{ kN}; 29{,}5\text{ kN}\right\} = 29{,}5\text{ kN}$$

given by the tensile force $F_{\text{pi,d}}$ caused by pinch.

The maximum horizontal displacement is 1,02 m and the minimum air clearance is 2,96 m.

When calculating according to IEC 60865-1:2011, 6.2.3, without dropper in midspan, to the structure, the insulators and the connectors and to the foundations the maximum value of $F_{\text{t,d}}$, $F_{\text{f,d}}$ and $F_{\text{pi,d}}$ shall be applied as a static load:

$$\max\{F_{t,d}; F_{f,d}; F_{pi,d}\} = \max\{26{,}8\ \text{kN}; 52{,}8\ \text{kN}; 29{,}5\ \text{kN}\} = 52{,}8\ \text{kN}$$

given by the tensile force $F_{f,d}$ after short-circuit caused by drop without dropper in midspan.

The maximum horizontal displacement is 1,02 m and the minimum air clearance is 2,96 m.

10 Example 7 – Mechanical effects on vertical main conductors (droppers)

10.1 General

The basis for the calculation in this example is a three-phase 380-kV-arrangement with droppers as shown in Figure 8. The droppers are fixed at the lower fixing point with horizontal connectors. At the upper end a V-shaped insulator-chain is given to prevent a swing-out of the fixing-point in the direction of the short-circuit force.

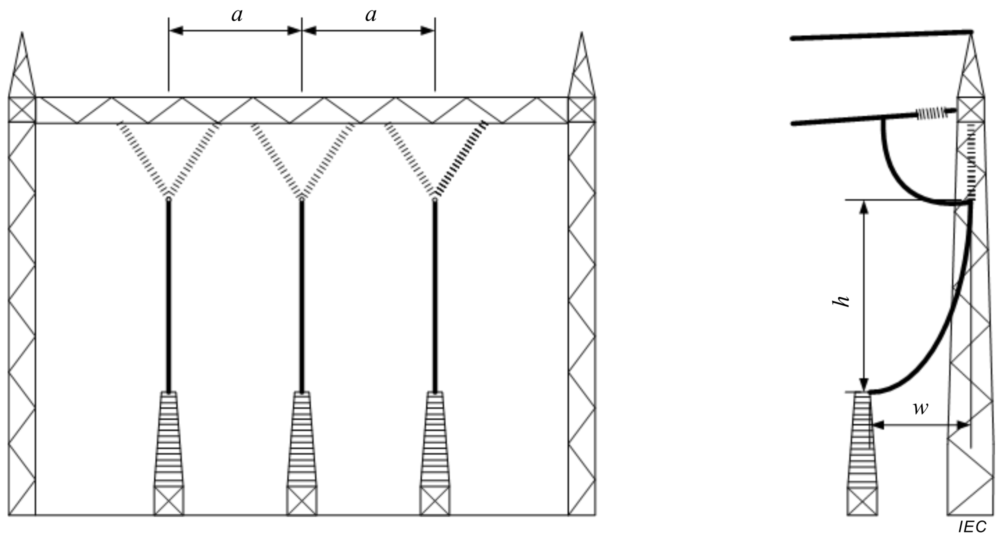


Figure 8 – Arrangement with strained conductors

10.2 Data

Initial symmetrical three-phase short-circuit current (r.m.s.)	I_k''	=	40 kA
Factor for the calculation of the peak short-circuit current	κ	=	1,81
System frequency	f	=	50 Hz
Height of dropper	h	=	12,3 m
Width of dropper	w	=	5 m
Centre-line distance between supports $l = \sqrt{h^2 + w^2} = \sqrt{(12{,}3\ \text{m})^2 + (5\ \text{m})^2}$	l	=	13,28 m
Cord length of the dropper	l_v	=	14 m
Centre-line distance between conductors	a	=	6 m
Centre-line distance between connecting pieces	l_s	=	13,28 m
Centre-line distance between sub-conductors	a_s	=	0,1 m
Resultant spring constant of both span supports of one span	S	=	100 N/mm

Twin conductor 2 EN 550-AL1/71-ST1A

– Number of sub-conductors	n	=	2
– Sub-conductor diameter	d	=	32,2 mm
– Sub-conductor cross-section	A_s	=	611,2 mm^2
– Sub-conductor mass per unit length	m'_s	=	1,95 kg/m
– Young's modulus	E	=	62 000 N/mm^2

Conventional value of acceleration of gravity g = 9,81 m/s^2

10.3 Short-circuit tensile force and maximum horizontal displacement

The short-circuit tensile force is:

$$F_{t,d} = \frac{5}{3} l_v \frac{\mu_0}{2\pi} \frac{(I''_k)^2}{a} \frac{l_v}{w} = \frac{5}{3} \cdot 14\ \text{m} \cdot \frac{4\pi \cdot 10^{-7}}{2\pi} \frac{\text{Vs}}{\text{Am}} \cdot \frac{\left(40 \cdot 10^3\ \text{A}\right)^2}{6\ \text{m}} \cdot \frac{14\ \text{m}}{5\ \text{m}} = 3484\ \text{N} = 3{,}48\ \text{kN} \quad (49)$$

for cable length

$$1{,}4\, w = 1{,}4 \cdot 5\ \text{m} = 7\ \text{m} \quad \text{less than} \quad l_v = 14\ \text{m} \quad \text{less than} \quad 3{,}3\, w = 3{,}3 \cdot 5\ \text{m} = 16{,}5\ \text{m}$$

The maximum horizontal displacement is:

$$\begin{aligned} b_h &= \left[0{,}6\sqrt{\frac{l_v}{l} - 1} + 0{,}44\left(\frac{l_v}{l} - 1\right) - 0{,}32\ln\frac{l_v}{l}\right]\frac{l^2}{l_v} \\ &= \left[0{,}6\sqrt{\frac{14\ \text{m}}{13{,}28\ \text{m}} - 1} + 0{,}44\left(\frac{14\ \text{m}}{13{,}28\ \text{m}} - 1\right) - 0{,}32\ln\frac{14\ \text{m}}{13{,}28\ \text{m}}\right] \cdot \frac{(13{,}28\ \text{m})^2}{14\ \text{m}} \\ &= 1{,}85\ \text{m} \end{aligned} \quad (50)$$

for cable length

$$l_v = 14\ \text{m} \quad \text{less than} \quad 2l = 2 \cdot 13{,}28\ \text{m} = 26{,}6\ \text{m}$$

The minimum air clearance is:

$$a_{min} = a - 2b_h = 6\ \text{m} - 2 \cdot 1{,}85\ \text{m} = 2{,}3\ \text{m} \quad (48)$$

10.4 Pinch force

10.4.1 Static tensile force regarding droppers

The horizontal component of the force caused by one sub-conductor at the lower fixing point may be calculated with the conductor sagging curve as a parabola, for example as in [3]:

$$H_s = \sqrt{\frac{1}{24} \frac{(m'_s g)^2 w^2}{\frac{\sqrt{l_v^2 - h^2}}{w} - 1}} = \sqrt{\frac{1}{24} \cdot \frac{\left(1{,}95\ \text{kg/m} \cdot 9{,}81\ \text{m/s}^2\right)^2 \cdot (5\ \text{m})^2}{\frac{\sqrt{(14\ \text{m})^2 - (12{,}3\ \text{m})^2}}{5\ \text{m}} - 1}} = 33{,}6\ \text{N} = 34\ \text{N}$$

The vertical component of the force caused by one sub-conductor at the upper fixing point is:

$$V_\mathrm{s} = m'_\mathrm{s}\, l_\mathrm{v}\, g = 1{,}95\,\frac{\mathrm{kg}}{\mathrm{m}}\cdot 14\ \mathrm{m}\cdot 9{,}81\,\frac{\mathrm{m}}{\mathrm{s}^2} = 268\ \mathrm{N}$$

The average tensile force in the dropper is:

$$F_\mathrm{st} = n\,\frac{\left(H_\mathrm{s} + V_\mathrm{s}\right)}{2} = 2\cdot\frac{\left(268\ \mathrm{N} + 34\ \mathrm{N}\right)}{2} = 302\ \mathrm{N}$$

10.4.2 Characteristic dimensions and parameters

Because

$$\frac{a_\mathrm{s}}{d} = \frac{0{,}100\ \mathrm{m}}{0{,}0322\ \mathrm{m}} = 3{,}11 \qquad \text{and} \qquad \frac{l_\mathrm{s}}{a_\mathrm{s}} = \frac{13{,}28\ \mathrm{m}}{0{,}100\ \mathrm{m}} = 133$$

neither Equation (52) nor Equation (53) of IEC 60865-1:2011 is fulfilled, the pinch force $F_\mathrm{pi,d}$ is to be calculated with the Equations (54) and following of IEC 60865-1:2011, 6.4.

The short-circuit current force between the sub-conductors is:

$$F_\mathrm{v} = \left(n-1\right)\frac{\mu_0}{2\pi}\left(\frac{I''_\mathrm{k}}{n}\right)^2 \frac{l_\mathrm{s}}{a_\mathrm{s}}\frac{\nu_2}{\nu_3} \qquad (54)$$

The factor ν_1 for calculation of ν_2 is:

$$\nu_1 = f\,\frac{1}{\sin\frac{180^\circ}{n}}\sqrt{\frac{\left(a_\mathrm{s}-d\right)m'_\mathrm{s}}{\frac{\mu_0}{2\pi}\left(\frac{I''_\mathrm{k}}{n}\right)^2\frac{n-1}{a_\mathrm{s}}}} = 50\,\frac{1}{\mathrm{s}}\cdot\frac{1}{\sin\frac{180^\circ}{2}}\cdot\sqrt{\frac{\left(0{,}100\ \mathrm{m} - 0{,}0322\ \mathrm{m}\right)\cdot 1{,}95\ \mathrm{kg/m}}{\frac{4\pi\cdot 10^{-7}}{2\pi}\,\frac{\mathrm{Vs}}{\mathrm{Am}}\cdot\left(\frac{40\cdot 10^3\ \mathrm{A}}{2}\right)^2\cdot\frac{2-1}{0{,}100\ \mathrm{m}}}} = 0{,}643 \qquad (55)$$

According to IEC 60865-1:2011, Figure 9, the factor ν_2 for ν_1 = 0,643 and κ = 1,81 is:

$$\nu_2 = 2{,}11$$

According to IEC 60865-1:2011, Figure 10, the factor ν_3 for a_s/d = 3,11 is:

$$\nu_3 = 0{,}483$$

With this the short-circuit current force between the sub-conductors is:

$$F_\mathrm{v} = \left(n-1\right)\frac{\mu_0}{2\pi}\left(\frac{I''_\mathrm{k}}{n}\right)^2\frac{l_\mathrm{s}}{a_\mathrm{s}}\frac{\nu_2}{\nu_3} = \left(2-1\right)\cdot\frac{4\pi\cdot 10^{-7}}{2\pi}\,\frac{\mathrm{Vs}}{\mathrm{Am}}\cdot\left(\frac{40\cdot 10^3\ \mathrm{A}}{2}\right)^2\cdot\frac{13{,}28\ \mathrm{m}}{0{,}1\ \mathrm{m}}\cdot\frac{2{,}11}{0{,}483} = 46411\cdot 10^3\ \mathrm{N} = 46{,}4\ \mathrm{kN} \qquad (54)$$

The stiffness norm is:

$$N = \frac{1}{S\,l} + \frac{1}{n\,E_\mathrm{eff}\,A_\mathrm{s}} = \frac{1}{1\cdot 10^5\ \mathrm{N/m}\cdot 13{,}28\ \mathrm{m}} + \frac{1}{2\cdot 1{,}89\cdot 10^{10}\ \mathrm{N/m^2}\cdot 611{,}2\cdot 10^{-6}\ \mathrm{m^2}} = 79{,}6\cdot 10^{-8}\ 1/\mathrm{N} \qquad (25)$$

with the actual Young's modulus

$$E_{\text{eff}} = E\left[0{,}3 + 0{,}7\sin\left(\frac{F_{\text{st}}}{n\,A_{\text{s}}\,\sigma_{\text{fin}}}90^\circ\right)\right] = 6{,}2\cdot 10^{10}\ \frac{\text{N}}{\text{m}^2}\cdot\left[0{,}3 + 0{,}7\sin\left(\frac{0{,}247\cdot 10^6\ \text{N/m}^2}{50\cdot 10^6\ \text{N/m}^2}\cdot 90^\circ\right)\right] \qquad (26)$$

$$= 1{,}89\cdot 10^{10}\ \text{N/m}^2$$

because

$$\frac{F_{\text{st}}}{n\,A_{\text{s}}} = \frac{302\ \text{N}}{2\cdot 611{,}2\cdot 10^{-6}\ \text{m}^2} = 0{,}247\cdot 10^6\ \text{N/m}^2 \qquad \text{less than} \qquad \sigma_{\text{fin}} = 50\cdot 10^6\ \text{N/m}^2$$

The strain factors are:

$$\varepsilon_{\text{st}} = 1{,}5\frac{F_{\text{st}}\,l_{\text{s}}^2\,N}{(a_{\text{s}} - d)^2}\left(\sin\frac{180^\circ}{n}\right)^2 = 1{,}5\cdot\frac{302\ \text{N}\cdot(13{,}28\ \text{m})^2\cdot 79{,}6\cdot 10^{-8}\ 1/\text{N}}{(0{,}100\ \text{m} - 0{,}0322\ \text{m})^2}\cdot\left(\sin\frac{180^\circ}{2}\right)^2 = 13{,}8 \qquad (56)$$

$$\varepsilon_{\text{pi}} = 0{,}375\,n\frac{F_{\text{v}}\,l_{\text{s}}^3\,N}{(a_{\text{s}} - d)^3}\left(\sin\frac{180^\circ}{n}\right)^3 = 0{,}375\cdot 2\cdot\frac{46{,}4\cdot 10^3\ \text{N}\cdot(13{,}28\ \text{m})^3\cdot 79{,}6\cdot 10^{-8}\ 1/\text{N}}{(0{,}100\ \text{m} - 0{,}0322\ \text{m})^3}\cdot\left(\sin\frac{180^\circ}{2}\right)^3 = 2{,}08\cdot 10^5 \qquad (57)$$

The parameter j is:

$$j = \sqrt{\frac{\varepsilon_{\text{pi}}}{1+\varepsilon_{\text{st}}}} = \sqrt{\frac{2{,}08\cdot 10^5}{1+13{,}8}} = 119 \qquad (58)$$

10.4.3 Pinch force $F_{\text{pi,d}}$

Because

$$j = 119 \qquad \text{greater than} \qquad 1$$

the sub-conductors clash and the tensile forces due to contraction are calculated according to IEC 60865-1:2011, 6.4.2:

$$F_{\text{pi,d}} = F_{\text{st}}\left(1 + \frac{\nu_{\text{e}}}{\varepsilon_{\text{st}}}\xi\right) \qquad (59)$$

According to IEC 60865-1:2011, Figure 11, and with j = 119 and ε_{st} = 13,8 the factor ξ is:

$$\xi = 55{,}0$$

The factor ν_{e} is:

$$\nu_e = \frac{1}{2} + \left[\frac{9}{8} n(n-1) \frac{\mu_0}{2\pi} \left(\frac{I''_k}{n} \right)^2 N \nu_2 \left(\frac{l_s}{a_s - d} \right)^4 \frac{\left(\sin \frac{180°}{n} \right)^4}{\xi^3} \left\{ 1 - \frac{\arctan \sqrt{\nu_4}}{\sqrt{\nu_4}} \right\} - \frac{1}{4} \right]^{1/2}$$

$$= \frac{1}{2} + \left[\frac{9}{8} \cdot 2 \cdot (2-1) \cdot \frac{4\pi \cdot 10^{-7}}{2\pi} \frac{\text{Vs}}{\text{Am}} \cdot \left(\frac{40 \cdot 10^3 \text{ A}}{2} \right)^2 \cdot 79{,}6 \cdot 10^{-8} \frac{1}{\text{N}} \cdot 2{,}11 \cdot \left(\frac{13{,}28 \text{ m}}{0{,}100 \text{ m} - 0{,}0322 \text{ m}} \right)^4 \right. \quad (60)$$

$$\left. \cdot \frac{\left(\sin \frac{180°}{2} \right)^4}{55{,}0^3} \cdot \left\{ 1 - \frac{\arctan \sqrt{2{,}11}}{\sqrt{2{,}11}} \right\} - \frac{1}{4} \right]^{1/2}$$

$$= 1{,}30$$

with the factor ν_4:

$$\nu_4 = \frac{a_s - d}{d} = \frac{0{,}100 \text{ m} - 0{,}0322 \text{ m}}{0{,}0322 \text{ m}} = 2{,}11 \quad (61)$$

With this, the tensile force is:

$$F_{pi,d} = F_{st} \left(1 + \frac{\nu_e}{\varepsilon_{st}} \xi \right) = 302 \text{ N} \cdot \left(1 + \frac{1{,}30}{13{,}8} 55{,}0 \right) = 1878 \text{ N} = 1{,}88 \text{ kN} \quad (59)$$

10.5 Conclusions

According to IEC 60865-1:2011, 6.5.2 and 6.5.3, to the structure, the insulators and the connectors and to the foundations the maximum value of $F_{t,d}$ and $F_{pi,d}$ shall be applied as a static load:

$$\max\{F_{t,d}; F_{pi,d}\} = \max\{3{,}48 \text{ kN}; 1{,}88 \text{ kN}\} = 3{,}48 \text{ kN}$$

given by the short-circuit tensile force $F_{t,d}$.

The maximum horizontal displacement is 1,85 m and the minimum air clearance is 2,3 m.

11 Example 8 – Thermal effect on bare conductors

11.1 General

The basis for the calculation is a three-phase 10 kV busbar with one conductor per phase.

11.2 Data

Initial symmetrical three-phase short-circuit current (r.m.s.)	I''_k	=	24,0 kA
Steady-state short-circuit current (r.m.s.)	I_k	=	19,2 kA
Factor for the calculation of the peak short-circuit current	κ	=	1,8
Duration of short-circuit current	T_k	=	0,8 s
System frequency	f	=	50 Hz

Rectangular conductor of EN AW-6101B with cross-section	A =	600 mm²
Conductor temperature at the beginning of short-circuit	ϑ_b =	65°C
Conductor temperature at the end of short-circuit	ϑ_e =	170°C

11.3 Calculations

For $\vartheta_b = 65°C$ and $\vartheta_e = 170°C$, the rated short-time withstand current density is found from IEC 60865-1:2011, Figure 13b):

$$S_{thr} = 80{,}7\ \text{A/mm}^2$$

The thermal equivalent short-circuit current is according to Equation (103) of IEC 60909-0:2001:

$$I_{th} = I''_k \sqrt{m+n} = 24{,}0\ \text{kA} \cdot \sqrt{0{,}056 + 0{,}86} = 23{,}0\ \text{kA}$$

m and n are found from IEC 60909-0:2001, Figures 21 and 22, for

$$f\,T_k = 50\ 1/\text{s} \cdot 0{,}8\ \text{s} = 40\ ; \qquad \kappa = 1{,}8\ ; \qquad I''_k / I_k = 24{,}0\ \text{kA}/19{,}2\ \text{kA} = 1{,}25$$

For the conductor cross-section $A = 600\ \text{mm}^2$ the thermal equivalent short-circuit current density is:

$$S_{th} = \frac{I_{th}}{A} = \frac{23{,}0 \cdot 10^3\ \text{A}}{600\ \text{mm}^2} = 38{,}3\ \text{A/mm}^2$$

The busbar has sufficient thermal strength if:

$$S_{th} = 38{,}3\ \text{A/mm}^2 \qquad \text{less than} \qquad S_{thr}\sqrt{\frac{T_{kr}}{T_k}} = 80{,}7\ \frac{\text{A}}{\text{mm}^2} \cdot \sqrt{\frac{1\ \text{s}}{0{,}8\ \text{s}}} = 90{,}2\ \text{A/mm}^2 \qquad (65)$$

11.4 Conclusion

The busbas has sufficient thermal strength.

Bibliography

[1] IEC 61936-1, *Power installations exceeding 1 kV a.c. – Part 1: Common rules*

[2] EN 1990, *Eurocode: Basis of structural design*

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