

Designation: E2709 –  $14^{\epsilon 1}$ 

# Standard Practice for Demonstrating Capability to Comply with an Acceptance Procedure<sup>1</sup>

This standard is issued under the fixed designation E2709; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon  $(\varepsilon)$  indicates an editorial change since the last revision or reapproval.

ε<sup>1</sup> NOTE—Editorial corrections made throughout in March 2015.

## 1. Scope

- 1.1 This practice provides a general methodology for evaluating single-stage or multiple-stage acceptance procedures which involve a quality characteristic measured on a numerical scale. This methodology computes, at a prescribed confidence level, a lower bound on the probability of passing an acceptance procedure, using estimates of the parameters of the distribution of test results from a sampled population.
- 1.2 For a prescribed lower probability bound, the methodology can also generate an acceptance limit table, which defines a set of test method outcomes (for example, sample averages and standard deviations) that would pass the acceptance procedure at a prescribed confidence level.
- 1.3 This approach may be used for demonstrating compliance with in-process, validation, or lot-release specifications.
  - 1.4 The system of units for this practice is not specified.
- 1.5 This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.

# 2. Referenced Documents

2.1 ASTM Standards:<sup>2</sup>

E456 Terminology Relating to Quality and Statistics E2282 Guide for Defining the Test Result of a Test Method E2586 Practice for Calculating and Using Basic Statistics

# 3. Terminology

3.1 *Definitions*—See Terminology E456 for a more extensive listing of terms in ASTM Committee E11 standards.

- <sup>1</sup> This practice is under the jurisdiction of ASTM Committee E11 on Quality and Statistics and is the direct responsibility of Subcommittee E11.20 on Test Method Evaluation and Quality Control.
- Current edition approved Oct. 1, 2014. Published October 2014. Originally approved in 2009. Last previous edition approved in 2012 as E2709 12. DOI: 10.1520/E2709-14E01.
- <sup>2</sup> For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

- 3.1.1 *characteristic*, *n*—a property of items in a sample or population which, when measured, counted or otherwise observed, helps to distinguish between the items.

  E2282
- 3.1.2 *mean*, n—of a population,  $\mu$ , average or expected value of a characteristic in a population, of a sample  $\bar{x}$ , sum of the observed values in a sample divided by the sample size.

E2586

- 3.1.3 multiple-stage acceptance procedure, n—a procedure that involves more than one stage of sampling and testing a given quality characteristic and one or more acceptance criteria per stage.
- 3.1.4 standard deviation, n—of a population,  $\sigma$ , the square root of the average or expected value of the squared deviation of a variable from its mean of a sample, s, the square root of the sum of the squared deviations of the observed values in the sample divided by the sample size minus 1.
- 3.1.5 *test method*, *n*—a definitive procedure that produces a test result.
  - 3.2 Definitions of Terms Specific to This Standard:
- 3.2.1 acceptable parameter region, n—the set of values of parameters characterizing the distribution of test results for which the probability of passing the acceptance procedure is greater than a prescribed lower bound.
- 3.2.2 acceptance region, n—the set of values of parameter estimates that will attain a prescribed lower bound on the probability of passing an acceptance procedure at a prescribed level of confidence.
- 3.2.3 acceptance limit, n—the boundary of the acceptance region, for example, the maximum sample standard deviation test results for a given sample mean.

# 4. Significance and Use

- 4.1 This practice considers inspection procedures that may involve multiple-stage sampling, where at each stage one can decide to accept or to continue sampling, and the decision to reject is deferred until the last stage.
- 4.1.1 At each stage there are one or more acceptance criteria on the test results; for example, limits on each individual test result, or limits on statistics based on the sample of test results,

such as the average, standard deviation, or coefficient of variation (relative standard deviation).

- 4.2 The methodology in this practice defines an acceptance region for a set of test results from the sampled population such that, at a prescribed confidence level, the probability that a sample from the population will pass the acceptance procedure is greater than or equal to a prespecified lower bound.
- 4.2.1 Having test results fall in the acceptance region is not equivalent to passing the acceptance procedure, but provides assurance that a sample would pass the acceptance procedure with a specified probability.
- 4.2.2 This information can be used for process demonstration, validation of test methods, and qualification of instruments, processes, and materials.
- 4.2.3 This information can be used for lot release (acceptance), but the lower bound may be conservative in some cases.
- 4.2.4 If the results are to be applied to future test results from the same process, then it is assumed that the process is stable and predictable. If this is not the case then there can be no guarantee that the probability estimates would be valid predictions of future process performance.
- 4.3 This methodology was originally developed (1-4)<sup>3</sup> for use in two specific quality characteristics of drug products in the pharmaceutical industry but will be applicable for acceptance procedures in all industries.
- 4.4 Mathematical derivations would be required that are specific to the individual criteria of each test.

#### 5. Methodology

- 5.1 The process for defining the acceptance limits, starting from the definition of the acceptance procedure, is outlined in this section. A computer program is normally required to produce the acceptable parameter region and the acceptance limits.
- 5.1.1 An expression for the exact probability of passing the acceptance procedure might be intractable when the procedure consists of multiple stages with multiple criteria, hence a lower bound for the probability may be used.
- 5.2 Express the probability of passing the acceptance procedure as a function of the parameters characterizing the distribution of the quality characteristic for items in the sampled population.
- 5.2.1 For each stage in the procedure having multiple acceptance criteria, determine the lower bound on the probability of that stage as a function of the probabilities of passing each of the criteria in the stage:

$$P(S_i) = P(C_{i1} \text{ and } C_{i2} \dots \text{ and } C_{im}) \ge 1 - \sum_{i=1}^m (1 - P(C_{ij}))$$
 (1)

where:

 $P(S_i)$  = is the probability of passing stage i,

 $P(C_{ij})$  = is the probability of passing the *j*-th criterion of *m* within the *i*-th stage.

5.2.2 Determine the lower bound on the probability of passing a k-stage procedure as a function of probabilities of passing each of the individual stages:

$$P ext{ (pass } k - \text{ stage procedure)} \ge \max\{P(S_1), P(S_2), \dots, P(S_k)\}$$
 (2)

- 5.3 Determine the contour of the region of parameter values for which the expression for the probability of passing the given acceptance procedure is at least equal to the required lower bound (LB) on the probability of acceptance (*p*). This defines the *acceptable parameter region*.
- 5.4 For each value of a statistic or set of statistics, derive a joint confidence region for the distribution parameters at confidence level, expressed as a percentage, of  $100(1-\alpha)$ . The size of sample to be taken, n, and the statistics to be used, must be predetermined (see 5.6).
- 5.5 Determine the contour of the *acceptance region*, which consists of values of the statistics for which the confidence region at level  $100(1-\alpha)$  is entirely contained in the acceptable parameter region. The *acceptance limits* lie on the contour of the acceptance region.
- 5.6 To select the size of sample, n, to be taken, the probability that sample statistics will lie within acceptance limits should be evaluated over a range of values of n, for values of population parameters of practical interest, and for which probabilities of passing the given acceptance procedure are well above the lower bound. The larger the sample size n that is chosen, the larger will be the acceptance region and the tighter the distribution of the statistics. Choose n so that the probability of passing acceptance limits is greater than a predetermined value.
- 5.7 To use the acceptance limit, sample randomly from the population. Compute statistics for the sample. If statistics fall within the acceptance limits, then there is  $1-\alpha$  confidence that the probability of acceptance is at least p.

# 6. Procedures for Sampling from a Normal Distribution

- 6.1 An important class of procedures is for the case where the quality characteristic is normally distributed. Particular instructions for that case are given in this section, for two sampling methods, simple random and two-stage. In this standard these sampling methods are denoted Sampling Plan 1 and Sampling Plan 2, respectively.
- 6.2 When the characteristic is normally distributed, parameters are the mean  $(\mu)$  and standard deviation  $(\sigma)$  of the population. The acceptable parameter region will be the region under a curve in the half-plane where  $\mu$  is on the horizontal axis,  $\sigma$  on the vertical axis, such as that depicted in Fig. 1.
- 6.3 For simple random sampling from a normal population, the method of Lindgren (5) constructs a simultaneous confidence region of  $(\mu, \sigma)$  values from the sample average  $\bar{X}$  and the sample standard deviation s of n test results.
- 6.3.1 Let  $Z_p$  and  $\chi_p^2$  denote percentiles of the standard normal distribution and of the chi-square distribution with n-1 degrees of freedom, respectively. Given a confidence level  $(1-\alpha)$ , choose  $\delta$  and  $\epsilon$  such that  $(1-\alpha)=(1-2\delta)(1-\epsilon)$ . Although there are many choices for  $\delta$  and  $\epsilon$  that would satisfy this

<sup>&</sup>lt;sup>3</sup> The boldface numbers in parentheses refer to a list of references at the end of this standard.

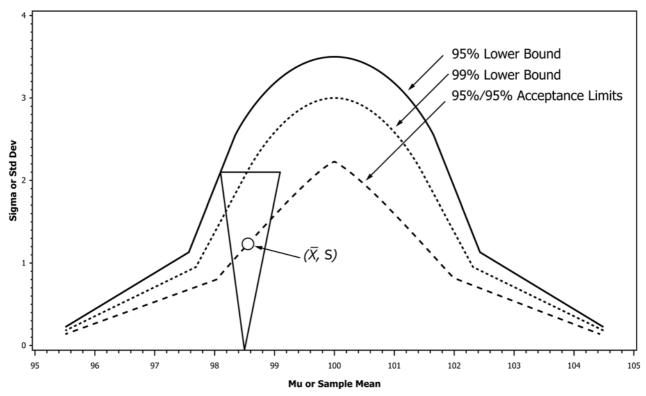


FIG. 1 Example of Acceptance Limit Contour Showing a Simultaneous Confidence Interval With 95 % and 99 % Lower Bound Contours

equation, a reasonable choice is:  $\varepsilon = 1 - \sqrt{1 - \alpha}$  and  $\delta = (1 - \sqrt{1 - \alpha})/2$  which equally splits the overall alpha between estimating  $\mu$  and  $\sigma$ . Then:

$$P\left\{ \left( \frac{\overline{X} - \mu}{\sigma / \sqrt{n}} \right)^2 \le Z_{1-\delta}^2 \right\} P\left\{ \frac{(n-1)s^2}{\sigma^2} \ge \chi_{\varepsilon}^2 \right\}$$

$$= (1 - 2\delta)(1 - \varepsilon)$$

$$= 1 - \alpha$$
(3)

- 6.3.2 The confidence region for  $(\mu, \sigma)$ , two-sided for  $\mu$ , one-sided for  $\sigma$ , is an inverted triangle with a minimum vertex at  $(\bar{X}, 0)$ , as depicted in Fig. 1.
- 6.3.3 The acceptance limit takes the form of a table giving, for each value of the sample mean, the maximum value of the standard deviation (or coefficient of variation) that would meet these requirements. Using a computer program that calculates confidence limits for  $\mu$  and  $\sigma$  given sample mean  $\bar{X}$  and standard deviation s, the acceptance limit can be derived using an iterative loop over increasing values of the sample standard deviation s (starting with s=0) until the confidence limits hit the boundary of the acceptable parameter region, for each potential value of the sample mean.
- 6.4 For two-stage sampling, the population is divided into primary sampling units (locations). L locations are selected and from each of them a subsample of n items is taken. The variance of a single observation,  $\sigma^2$ , is the sum of between-location and within-location variances.
- 6.4.1 A confidence limit for  $\sigma^2$  is given by Graybill and Wang (6) using the between and within location mean squares from analysis of variance. When there are L locations with

subsamples of n items, the mean squares between locations and within locations,  $MS_L$  and  $MS_E$ , have L-1 and L(n-1) degrees of freedom respectively. Express the overall confidence level as a product of confidence levels for the population mean and standard deviation as in 6.3, so that  $(1-\alpha) = (1-2\delta)(1-\epsilon)$ . An upper  $(1-\epsilon)$  confidence limit for  $\sigma^2$  is:

$$[(1/n) MS_L + (1 - 1/n) MS_E] + \{[(1/n)$$

$$((L - 1)/\chi^2_{L-1, 1-\epsilon} - 1) MS_L]^2 + [(1 - 1/n)$$

$$(L(n - 1)/\chi^2_{L(n-1), 1-\epsilon} - 1) MS_E]^2\}^{1/2}$$
(4)

The upper  $(1-\varepsilon)$  confidence limit for  $\sigma$  is the square root of Eq 4. Two sided  $(1-2\delta)$  confidence limits for  $\mu$  are:

$$\bar{X} \pm Z_{1-\delta} \frac{\sigma}{\sqrt{(nL)}}$$
 (5)

- 6.4.2 To verify, at confidence level  $1-\alpha$ , that a sample will pass the original acceptance procedure with probability at least equal to the prespecified lower bound, values of  $(\mu, \sigma)$  defined by the limits given in Eq 4 and Eq 5 should fall within the acceptable parameter region defined in 5.3.
- 6.4.3 An acceptance limit table is constructed by fixing the sample within location standard deviation and the standard deviation of location means and then finding the range of overall sample means such that the confidence interval completely falls below the pre-specified lower bound.

# 7. Examples

7.1 An example of an evaluation of a single-stage lot acceptance procedure is given in Appendix X1. An acceptance

limit table is shown for a sample size of 30, but other sample sizes may be considered.

- 7.2 An example of an evaluation of a two-stage lot acceptance procedure with one or more acceptance criterion at each stage is given in Appendix X2. An acceptance limit table is shown for a sample size of 30.
- 7.3 An example of an evaluation of a two-stage lot acceptance procedure with one or more acceptance criteria at each

stage using Sampling Plan 2 is given in Appendix X3. An acceptance limit table is shown for a sample size of 4 taken at each of 15 locations for a total of 60 units tested.

# 8. Keywords

8.1 acceptance limits; joint confidence regions; multiplestage acceptance procedures; specifications

#### APPENDIXES

(Nonmandatory Information)

#### X1. EXAMPLE: EVALUATION OF A SINGLE STAGE ACCEPTANCE PROCEDURE

- X1.1 A single-stage lot acceptance procedure is stated as follows: Sample five units at random from the lot and measure a numerical quality characteristic ( $X_i$ ) of each unit. Criterion: Pass if all 5 individual units are between 95 and 105; otherwise, fail.
- X1.2 Assume that the test results follow a normal distribution with mean  $\mu$  and standard deviation  $\sigma$ . Let Z denote the standard normal variate, that is, Z is normally distributed with  $\mu = 0$  and  $\sigma = 1$ .
- X1.3 The criterion is  $95 \le X_i \le 105$  for i = 1, ..., 5. Therefore:

$$P(\text{passing test}) = [P((95 - \mu)/\sigma < Z < (105 - \mu)/\sigma)]^5$$
(X1.1)

For any given values of  $\mu$  and  $\sigma,$  the probability of passing Stage 1 can be determined.

X1.4 A simultaneous confidence region for  $\mu$  and  $\sigma$  is generated using the methods of Lindgren (5). See 6.3.1.

X1.5 The acceptance limit table for this example was generated by a computer program and is listed in Table X1.1. The table corresponds to a sample size of 30 using a 95 % confidence interval and a 95 % lower bound, and it lists the output showing the upper bound on the sample standard deviation for sample means between 96 and 104.

TABLE X1.1 Acceptance Limit Table (95 % Confidence Interval/ 95 % Coverage)

	- · · · · · · · · · · · · · · · · · · ·
Mean	Standard Deviation
96.0	0.273
97.0	0.546
98.0	0.819
99.0	1.092
100.0	1.350
101.0	1.092
102.0	0.819
103.0	0.546
104.0	0.273

#### X2. EXAMPLE: EVALUATION OF A MULTIPLE-STAGE ACCEPTANCE PROCEDURE USING SAMPLING PLAN 1

X2.1 A multiple-stage lot acceptance procedure is stated as follows:

Stage 1: Sample five units at random from the lot and measure a numerical quality characteristic  $(X_i)$  of each unit. Criterion: Pass if all 5 individual units are between 95 and 105; otherwise go to Stage 2.

Stage 2: Randomly sample five additional units from the lot and measure a numerical quality characteristic  $(X_i)$  of each unit. Criteria: Pass if the average of the 10 test results is between 97 and 103 and all 10 individual results are between 90 and 110; otherwise fail.

X2.2 To obtain the lower bound on the probability of acceptance, assume that the test results follow a normal distribution with mean  $\mu$  and standard deviation  $\sigma$ . Let Z denote the standard normal variate, that is, Z is normally distributed with  $\mu = 0$  and  $\sigma = 1$ . Let  $S_i$  denote the event that the test results meet the acceptance criteria for Stage i, and let  $C_{ii}$  denote the

event that the *j*-th criterion for the *i*-th stage is met.

X2.2.1 For Stage 1, the criterion  $C_{11}$  is  $95 \le X_i \le 105$  for i = 1, ..., 5. Therefore:

$$P(S_1) = P(C_{11}) = [P((95 - \mu)/\sigma < Z < (105 - \mu)/\sigma)]^5 (X2.1)$$

For any given values of  $\mu$  and  $\sigma$ , the probability of passing Stage 1 can be determined.

X2.2.2 For Stage 2, the criterion  $C_{21}$  is:

$$97 \le \overline{X} \le 103$$

Where  $\bar{X}$  is the average of the 10 test results, and the criterion  $C_{22}$  is  $90 \le X_i \le 110$  for i = 1, ..., 10. Therefore:

$$P(C_{21}) = \left[ P\left( \sqrt{10} (97 - \mu)/\sigma < Z < \sqrt{10} (103 - \mu)/\sigma \right) \right]$$
 (X2.2)

$$P(C_{22}) = [P((90 - \mu)/\sigma < Z < (110 - \mu)/\sigma)]^{10}$$
 (X2.3)

$$P(S_2) \ge P(C_{21}) + P(C_{22}) - 1$$
 (X2.4)

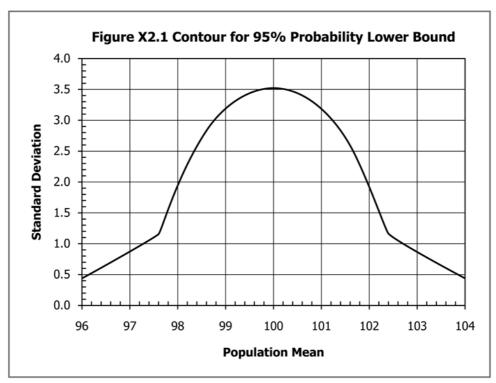


FIG. X2.1 Contour for 95 % Probability Lower Bound

TABLE X2.1 Calculation for 95 % Lower Bound Contour

Рорг	ulation		Probability of Passing							
Mean	Std Dev	$P(S_1)$	P(C <sub>21</sub> )	$P(C_{22})$	$P(S_2)$	LB				
97.0	0.86	0.950	0.500	1.000	0.500	0.950				
98.0	1.92	0.737	0.950	1.000	0.950	0.950				
99.0	3.19	0.485	0.976	0.974	0.950	0.950				
100.0	3.51	0.432	0.993	0.957	0.950	0.950				
101.0	3.19	0.485	0.976	0.974	0.950	0.950				
102.0	1.92	0.737	0.950	1.000	0.950	0.950				
103.0	0.86	0.950	0.500	1.000	0.500	0.950				

TABLE X2.2 Acceptance Limit Table (95 % Confidence Interval/ 95 % Coverage)

Mean	Standard Deviation
97.0	0.546
98.0	0.819
99.0	1.599
100.0	2.240
101.0	1.599
102.0	0.819
103.0	0.546

For any given values of  $\mu$  and  $\sigma,$  the probability of passing Stage 2 can be determined.

X2.3 The lower bound (LB) = Max  $[P(S_1), P(S_2)]$  can be determined for any values of  $\mu$  and  $\sigma$ , and a LB value for  $\sigma$  can be generated for each selected value of  $\mu$  by increasing  $\sigma$  from

0 in increments until the prescribed LB is attained.

X2.3.1 A computer program was used to generate the lower bounds for this example, using values of  $\mu$  ranging from 97.0 to 103.0 in increments of 1.0. Table X2.1 lists the output for selected mean and standard deviation pairs on the lower bound contour. The probabilities of passing the three criteria and the two stages are also listed in the output.

X2.3.2 Since the bound is a lower bound on the probability of passing the procedure, the actual probability is always equal to or greater than the lower bound. A comparison of the lower bound to the actual probability of passing the procedure can be performed using a computer simulation.

X2.4 The acceptance table corresponds to a sample size of 30 using a 95 % confidence interval and a 95 % lower bound. Table X2.2 lists the output showing the upper bound on the sample standard deviation for sample means between 97 and 103.

X2.5 95 % and 99 % contours ( $\mu$ ,  $\sigma$ ) for the acceptance region for this example as well as the acceptance table contour (mean and standard deviation) are depicted in Fig. X2.1. Note that the curve changes shape at the two points where the maximum stage probability for the lower bound changes over from Stage 1 to Stage 2.



## X3. EXAMPLE OF A MULTIPLE-STAGE ACCEPTANCE PROCEDURE USING SAMPLING PLAN 2

X3.1 Using the multiple-stage acceptance procedure given in Appendix X2, Sampling Plan 2 is applied by sampling and testing four units at each of 15 locations throughout a lot. The results and summary statistics are shown in Table X3.1.

X3.2 The overall mean and mean squares between locations and within locations are  $\bar{X} = 98.93$ ,  $MS_L = 4.464$ ,  $MS_E = 1.155$ . The following calculations are for 95 % confidence. The upper 1- $\varepsilon$  confidence limit for the overall population standard deviation  $\sigma$  is calculated using Eq 4 as:

$$\sigma^2 = \left[ \left( \frac{1}{4} \right) 4.464 + \left( 1 - \frac{1}{4} \right) 1.155 \right] + \tag{X3.1}$$

$$\left\{ \left[ \left(\frac{1}{4}\right) \left( \right. \frac{14}{5.644} - \left. 1 \right) \right. 4.464 \right]^2 + \left[ \left. \left( \right. 1 - \frac{1}{4} \right) \left( \right. \frac{45}{28.404} - \left. 1 \right) \right. 1.155 \right]^2 \right\}^{1/2}$$

TABLE X3.1 Sampling Plan 2 Results and Summary Statistics

Location		Re	esult	Summary Statistics				
Location	1	2	3	4	Mean	Variance	Std Dev	
1	97.08	99.72	98.37	97.50	98.17	1.36	1.17	
2	99.72	100.32	101.01	100.29	100.34	0.28	0.53	
3	99.90	98.27	98.88	97.96	98.75	0.73	0.86	
4	98.78	98.17	98.94	97.78	98.42	0.29	0.54	
5	96.32	96.61	99.66	97.20	97.45	2.31	1.52	
6	100.97	102.17	99.06	98.80	100.25	2.57	1.60	
7	97.02	97.35	98.65	99.98	98.25	1.83	1.35	
8	99.39†	98.81	98.63	98.06	98.72	0.30	0.55	
9	99.59	97.80	97.67	98.95	98.50	0.86	0.93	
10	97.97	98.54	100.26	98.74	98.88	0.96	0.98	
11	96.09	98.61	97.49	97.50	97.42	1.07	1.03	
12	98.87	97.81	97.28	98.80	98.19	0.60	0.78	
13	101.10	102.60	100.48	98.62	100.70	2.71	1.65	
14	100.80	100.34	98.49	100.93	100.14	1.27	1.13	
15	99.70	100.09	100.14	99.20	99.78	0.19	0.44	
A THE STATE OF STREET	- 11	- A I						

<sup>†</sup> Editorially corrected

where:

 $\sigma = 1.926$ 

The 1-2 $\delta$  confidence limits for the population mean  $\mu$  are calculated using Eq 5 as:

$$98.93 \pm 2.236 \times \frac{1.926}{\sqrt{60}} = (98.37, 99.48)$$
 (X3.2)

X3.3 Probabilities to pass the acceptance test are calculated using the formulas of Section X2.2. The calculated probability for  $\mu = 98.37$  and  $\sigma = 1.926$  is 0.988. The calculated probability for  $\mu = 99.48$  and  $\sigma = 1.926$  is near 1. Hence, the probability of passing the acceptance test is greater than a lower bound of 0.95 with 95 % confidence.

X3.4 An acceptance limit table is constructed using the overall sample mean, the pooled within-location standard deviation (that is,  $\sqrt{\text{(average of location variances)}}$ ) and the standard deviation of location means. A portion of the acceptance limit tables (90 % and 95 % confidence level with 95 % coverage) associated with this sampling plan are shown in Tables X3.2 and X3.3.

Note X3.1—This is not the between-location variance component but rather just the standard deviation of the location means.

X3.5 The overall sample mean, the pooled within-location standard deviation (that is,  $\sqrt{\text{(average of location variances)}}$ ) and the standard deviation of location means are given in Table X3.4.

Note X3.2—This is not the between-location variance component but rather just the standard deviation of the location means.

X3.6 Using the 95 % confidence limit table, the lower (LL) and upper (UL) acceptance limits for the mean are 98.7 to 101.3. Because 98.93 lies within the interval, the lot meets the acceptance criterion.

## TABLE X3.2 Lower Bound = 95, Confidence Level = 95 Standard Deviation of Location Means

SE	0.	80	0.	0.90 1.00		1.10		1.20		1.30		
SE	LL	UL	LL	UL	LL	UL	LL	UL	LL	UL	LL	UL
0.8	98.2	101.8	98.3	101.7	98.4	101.6	98.6	101.4	98.7	101.3	98.8	101.2
0.9	98.3	101.7	98.4	101.6	98.5	101.5	98.6	101.4	98.7	101.3	98.8	101.2
1.0	98.3	101.7	98.4	101.6	98.5	101.5	98.6	101.4	98.7	101.3	98.9	101.1
1.1	98.4	101.6	98.5	101.5	98.6	101.4	98.7	101.3	98.8	101.2	98.9	101.1
1.2	98.4	101.6	98.5	101.5	98.6	101.4	98.7	101.3	98.8	101.2	98.9	101.1
1.3	98.5	101.5	98.6	101.4	98.7	101.3	98.8	101.2	98.9	101.1	99.0	101.0

## TABLE X3.3 Lower Bound = 95, Confidence Level = 90 Standard Deviation of Location Means

SE	0.	.80	0.	90	1.	.00	1	.10	1.	20	1.	.30
SE	LL	UL										
0.8	98.1	101.9	98.2	101.8	98.3	101.7	98.4	101.6	98.5	101.5	98.6	101.4
0.9	98.1	101.9	98.2	101.8	98.3	101.7	98.4	101.6	98.5	101.5	98.6	101.4
1.0	98.2	101.8	98.3	101.7	98.4	101.6	98.5	101.5	98.4	101.7	98.7	101.3
1.1	98.2	101.8	98.3	101.7	98.4	101.6	98.5	101.5	98.4	101.7	98.7	101.3
1.2	98.3	101.7	98.4	101.6	98.5	101.5	98.5	101.5	98.4	101.7	98.7	101.3
1.3	98.3	101.7	98.4	101.6	98.5	101.5	98.6	101.4	98.3	101.8	98.8	101.2

## **TABLE X3.4 Descriptive Statistics**

Mean	98.93
SE (within-location SD)	1.07
Standard deviation of location means	1.06

## REFERENCES

- (1) Bergum, J. S., "Constructing Acceptance Limits for Multiple Stage Tests," *Drug Development and Industrial Pharmacy*, Vol 16, No 14, 1990, pp. 2153–2166.
- (2) Bergum, J. S., and Utter M. L., "Process Validation," *Encyclopedia of Bio-pharmaceutical Statistics*, 3rd Edition, Informa Health, New York, NY, 2010, pp. 1070–1082.
- (3) Bergum, J. S., and Utter M. L., "Statistical Methods for Uniformity and Dissolution Testing," *Pharmaceutical Process Validation*, 3rd Ed, edited by Robert A. Nash and Alfred H. Watchter, Marcel Dekker,
- New York, NY, 2003, pp. 667-697.
- (4) Bergum, J. S., and Li, H., "Acceptance Limits for the New ICH USP 29 Content Uniformity Test," *Pharmaceutical Technology*, October 2007, pp. 90–100.
- (5) Lindgren, B. W., Statistical Theory, Macmillan, New York, NY, 1968.
- (6) Graybill, F. A., and Wang, C. M., "Confidence Intervals on Nonnegative Linear Combination of Variances," *Journal of the American Statistical Association*, Vol 75, 1980, pp. 869–873.

ASTM International takes no position respecting the validity of any patent rights asserted in connection with any item mentioned in this standard. Users of this standard are expressly advised that determination of the validity of any such patent rights, and the risk of infringement of such rights, are entirely their own responsibility.

This standard is subject to revision at any time by the responsible technical committee and must be reviewed every five years and if not revised, either reapproved or withdrawn. Your comments are invited either for revision of this standard or for additional standards and should be addressed to ASTM International Headquarters. Your comments will receive careful consideration at a meeting of the responsible technical committee, which you may attend. If you feel that your comments have not received a fair hearing you should make your views known to the ASTM Committee on Standards, at the address shown below.

This standard is copyrighted by ASTM International, 100 Barr Harbor Drive, PO Box C700, West Conshohocken, PA 19428-2959, United States. Individual reprints (single or multiple copies) of this standard may be obtained by contacting ASTM at the above address or at 610-832-9585 (phone), 610-832-9555 (fax), or service@astm.org (e-mail); or through the ASTM website (www.astm.org). Permission rights to photocopy the standard may also be secured from the Copyright Clearance Center, 222 Rosewood Drive, Danvers, MA 01923, Tel: (978) 646-2600; http://www.copyright.com/