Standard Practice for Determining Sandwich Beam Flexural and Shear Stiffness[1]

This standard is issued under the fixed designation D7250/D7250M; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

1. Scope

1.1 This practice covers determination of the flexural and transverse shear stiffness properties of flat sandwich constructions subjected to flexure in such a manner that the applied moments produce curvature of the sandwich facing planes. Permissible core material forms include those with continuous bonding surfaces (such as balsa wood and foams) as well as those with discontinuous bonding surfaces (such as honeycomb). The calculation methods in this practice are limited to sandwich beams exhibiting linear force-deflection response. This practice uses test results obtained from Test Methods C393/C393M and/or D7249/D7249M.

1.2 The values stated in either SI units or inch-pound units are to be regarded separately as standard. The values stated in each system may not be exact equivalents; therefore, each system shall be used independently of the other. Combining values from the two systems may result in non-conformance with the standard.

1.2.1 Within the text the inch-pound units are shown in brackets.

1.3 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:*[2]

C273 Test Method for Shear Properties of Sandwich Core Materials
C393/C393M Test Method for Core Shear Properties of Sandwich Constructions by Beam Flexure
D883 Terminology Relating to Plastics
D3878 Terminology for Composite Materials
D7249/D7249M Test Method for Facing Properties of Sandwich Constructions by Long Beam Flexure
E6 Terminology Relating to Methods of Mechanical Testing
E122 Practice for Calculating Sample Size to Estimate, With Specified Precision, the Average for a Characteristic of a Lot or Process
E177 Practice for Use of the Terms Precision and Bias in ASTM Test Methods
E456 Terminology Relating to Quality and Statistics

3. Terminology

3.1 *Definitions*—Terminology D3878 defines terms relating to high-modulus fibers and their composites, as well as terms relating to sandwich constructions. Terminology D883 defines terms relating to plastics. Terminology E6 defines terms relating to mechanical testing. Terminology E456 and Practice E177 define terms relating to statistics. In the event of a conflict between terms, Terminology D3878 shall have precedence over the other terminologies.

3.2 *Symbols:* b = sandwich width, mm [in.]
c = core thickness, mm [in.]
d = sandwich thickness, mm [in.]
D = flexural stiffness, N-mm^2 [lb-in.^{2}]
Δ = beam mid-span deflection, mm [in.]
G = core shear modulus, MPa [psi]
S = support span length, mm [in.]
L = load span length, mm [in.] (L = 0.0 for 3-point mid-span loading configuration)
n = number of specimens
P = total applied force, N [lb]
t = facing thickness, mm [in.]
U = transverse shear rigidity, N [lb]

4. Summary of Practice

4.1 This practice consists of calculating the flexural stiffness, transverse (through-thickness) shear rigidity and core shear modulus of a sandwich beam using deflection and/or strain data from two or more flexure tests of different loading configurations conducted under Test Methods C393/C393M and/or D7249/D7249M. This practice also includes equations for calculating the shear rigidity and core shear modulus of a

[1] This practice is under the jurisdiction of ASTM Committee D30 on Composite Materials and is the direct responsibility of Subcommittee D30.09 on Sandwich Construction.

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[2] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

sandwich beam using deflection data from a single flexure test conducted under Test Method C393/C393M when the facing modulus is known.

5. Significance and Use

5.1 Flexure tests on flat sandwich constructions may be conducted to determine the sandwich flexural stiffness, the core shear strength and shear modulus, or the facings compressive and tensile strengths. Tests to evaluate core shear strength may also be used to evaluate core-to-facing bonds.

5.2 This practice provides a standard method of determining sandwich flexural and shear stiffness and core shear modulus using calculations involving measured deflections of sandwich flexure specimens. Tests can be conducted on short specimens and on long specimens (or on one specimen loaded in two ways), and the flexural stiffness, shear rigidity and core shear modulus can be determined by simultaneous solution of the complete deflection equations for each span or each loading. If the facing modulus values are known, a short span beam can be tested and the calculated bending deflection subtracted from the beam's total deflection. This gives the shear deflection from which the transverse shear modulus can be determined.

NOTE 1—Core shear strength and shear modulus are best determined in accordance with Test Method C273 provided bare core material is available.

NOTE 2—For cores with high shear modulus, the shear deflection will be quite small and ordinary errors in deflection measurements will cause considerable variations in the calculated shear modulus.

NOTE 3—To insure that simple sandwich beam theory is valid, a good rule of thumb for a four-point bending test is the span length divided by the sandwich thickness should be greater than 20 ($L_1/d > 20$) with the ratio of facing thickness to core thickness less than 0.1 ($t/c < 0.1$).

6. Interferences

6.1 *Material and Specimen Preparation*—Important aspects of sandwich core specimen preparation that contribute to data scatter include the existence of joints, voids or other core discontinuities, out-of-plane curvature, and surface roughness.

6.2 *Geometry*—Specific geometric factors that affect sandwich facing stiffness and thereby the sandwich flexural stiffness include facing thickness, core cell geometry, and facing surface flatness (toolside or bagside surface in compression).

6.3 *Environment*—Results are affected by the environmental conditions under which specimens are conditioned, as well as the conditions under which the tests are conducted. Specimens tested in various environments can exhibit significant differences in stiffness. Critical environments must be assessed independently for each specific combination of core material, facing material, and core-to-facing interfacial adhesive (if used) that is tested.

6.4 *Core Material*—For some core materials, the core shear modulus is a function of the direction that the core is oriented relative to the length of the specimen. Another material factor that affects sandwich core stiffness is variability in core density.

7. Sampling and Test Specimens

7.1 *Sampling*—Test at least five specimens per test condition unless valid results can be gained through the use of fewer specimens, as in the case of a designed experiment. For statistically significant data, consult the procedures outlined in Practice E122. Report the method of sampling.

7.2 *Specimen Geometry*—The test specimens shall be rectangular in cross section. The depth of the specimens shall be equal to the thickness of the sandwich construction, and the width shall be not less than twice the total thickness, not less than three times the dimension of a core cell, nor greater than one half the span length. The specimen length shall be equal to the span length plus 50 mm [2 in.] or plus one half the sandwich thickness whichever is the greater.

7.3 *Loading Configurations, Unknown Facing Modulus*—For cases where the facing modulus is not known, a minimum of two loading configurations must be selected. Refer to Test Methods C393/C393M and D7249/D7249M for the equations used to size the specimen lengths and loading configurations so that facing failure and core shear failures do not occur below the desired maximum applied force level. It is recommended that one loading configuration use a short support span and specimen and the other loading configuration use a long support span and specimen. The purpose of this recommendation is to obtain force-deflection data for one test with relatively high shear deflection and one test with relatively high flexural deformation. If two short configurations or two long configurations are tested, measurement errors may be large relative to the difference in shear and flexural deflections between the two tests and may lead to significant errors in the calculated flexural and shear stiffness values.

7.4 *Loading Configurations, Known Facing Modulus*—For cases where the facing modulus is known for sandwich beams with identical facings, a short support span loading configuration test should be conducted per Test Method C393/C393M.

8. Procedure

8.1 *Unknown Facing Modulus*—Conduct tests on sandwich beam specimens per Test Methods C393/C393M and/or D7249/D7249M using two or more different loading configurations; Fig. 1. It is preferable to conduct each of the loading conditions on each test specimen. This requires that the applied forces for all but the last loading condition to be kept sufficiently low to avoid failure and permanent deformations of the specimen.

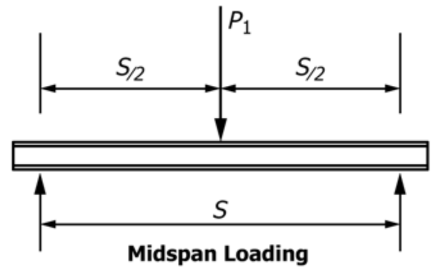
8.2 *Known Facing Modulus*—Conduct tests on sandwich beam specimens per Test Methods C393/C393M using a single short support span loading configuration.

8.3 *Data Recording*—Record force-deflection curves for each test specimen using a transducer, deflectometer, or dial gage to measure the mid-span deflection.

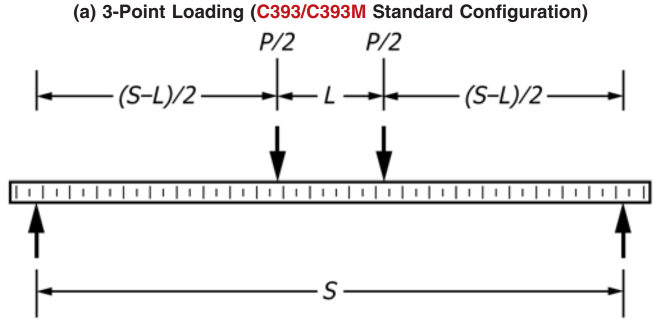
NOTE 4—The use of crosshead or actuator displacement for the beam mid-span deflection produces inaccurate results; the direct measurement of the deflection of the mid-span of the beam must be made by a suitable instrument.

9. Validation

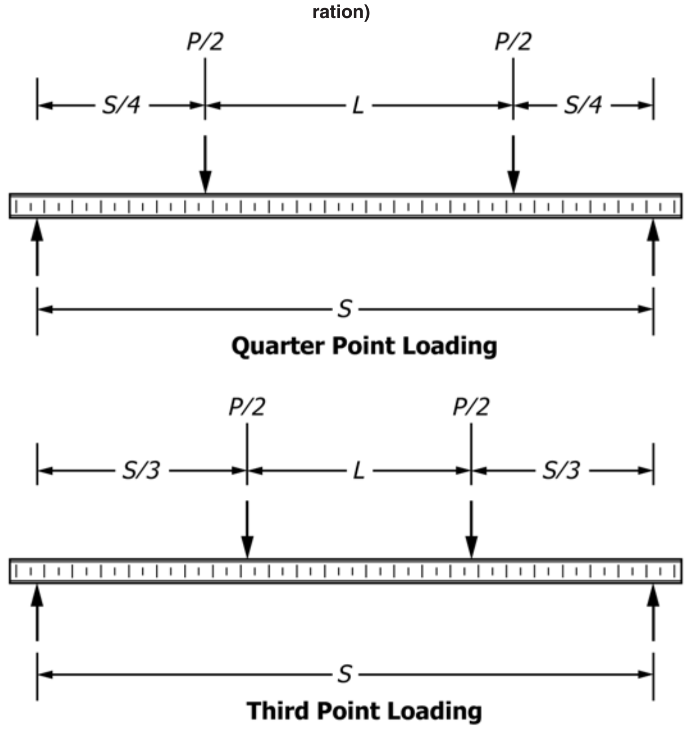
9.1 Values for stiffness properties shall not be calculated at any applied force level above or beyond the point of initial specimen failure, or above a point where the specimen exhibits obvious non-linear deflection response due to excessive local



(a) 3-Point Loading (C393/C393M Standard Configuration)



(b) 4-Point Loading (D7249/D7249MLong Beam Flexure Standard Configuration)



(c) 4-Point Loading (C393/C393M and D7249/D7249M Non-Standard Configuration)

FIG. 1 Loading Configurations

or overall deflection. Retests shall be performed for any specimen on which values are not calculated.

10. Calculation

10.1 *General Instructions, Unknown Facing Modulus*—Calculation procedures for flexural stiffness and transverse shear rigidity for cases where the facing modulus values are not known are given in 10.1 – 10.2. Calculation procedures for transverse shear rigidity and core shear modulus for cases where the facing modulus is known are given in 10.3.

NOTE 5—The equations in this section assume linear force-deflection response for both the facing and core materials. If the force-deflection response is non-linear, the extraction of non-linear flexural and shear stiffnesses is significantly more complicated and is beyond the scope of this standard practice.

10.1.1 *Criteria for Force-Deflection Linearity*—For purposes of validating the force-displacement linearity assumption, determine the maximum offset from a linear force-displacement curve over the range of applied forces to be used to calculate the stiffnesses. Determine the offset from linearity using the method shown in Fig. 2. The maximum offset shall be less than 10 % for the linearity assumption to be valid.

10.1.2 *Results from Tests Using Two Loading Configurations on the Same Test Specimen*—For each specimen, calculate the flexural stiffness, shear rigidity and core shear modulus using the equations in 10.2 for a series of applied forces up to the lowest maximum applied force of the two loading configurations. Values should be calculated for a minimum of ten (10) force levels evenly spaced over the force range. Calculate the average value and statistics of the flexural stiffness, shear rigidity and core shear modulus using the values calculated at each force level for each specimen replicate. The result is a set of stiffness values as a function of force level. If the sandwich response is linear, then calculate an overall average flexural stiffness, shear rigidity and core shear modulus using the values from all force levels. Report all of the individual and average calculated stiffness values.

10.1.3 *Results from Tests Using Three or More Loading Configurations on the Same Test Specimen*—For each specimen calculate the flexural stiffness, shear rigidity and core shear modulus using the equations in 10.2 for each pair of loading configurations for a series of applied forces up to the lowest maximum applied force of the loading configurations. Values should be calculated for a minimum of ten (10) force levels evenly spaced over the force range. Next, calculate the average value of flexural stiffness, shear rigidity and core shear modulus for each force level for each specimen. Then calculate the average values of the flexural stiffness, shear rigidity and core shear modulus using the values calculated at each force level for each specimen replicate. The result is a set of stiffness values as a function of force level. If the sandwich response is linear, then calculate an overall average flexural stiffness, shear rigidity and core shear modulus using the values from all force levels. Report all of the individual and average calculated stiffness values.

10.1.4 *Results from Tests Using Two Loading Configurations on Different Test Specimens*—In some cases it may not be possible or desired to conduct tests using two or more loading conditions on the same specimen. In this case, a continuous force-displacement curve must be calculated for each loading configuration so that displacement values for the two specimens can be determined at the same force value. To calculate the continuous force-displacement curve, perform a linear regression analysis on the force-displacement data for each loading condition for each specimen using a linear function in displacement for the regression analysis. If the linear curve gives a poor fit to the data, the regression and stiffness calculations should only be performed over the linear range of the force-deflection curve. Once the force-displacement curves are calculated for each loading condition, use the curves and the procedure of 10.1.1 to calculate the flexural stiffness, shear rigidity and core shear modulus.

10.1.5 *Results from Tests Using Three or More Loading Configurations on Different Test Specimens*—For this case, combine the calculation procedures of 10.1.3 and 10.1.4.

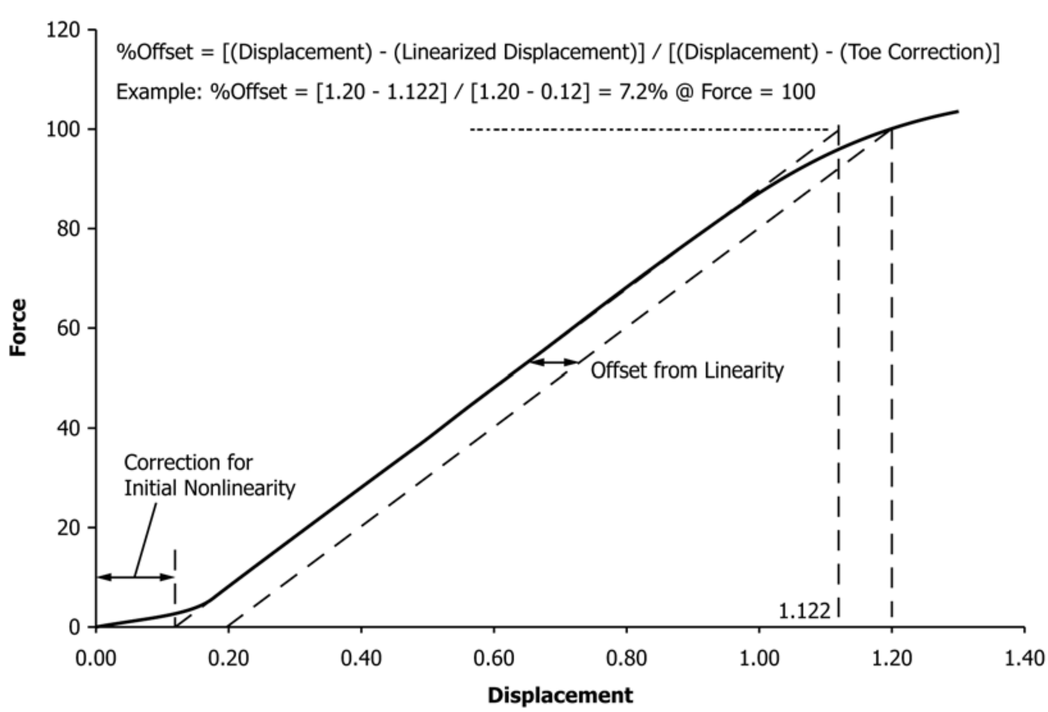


FIG. 2 Determination of Non-Linearity Offset

10.2 *Calculation of Flexural Stiffness, Transverse Shear Rigidity, and Core Shear Modulus*—The mid-span deflection of a beam with identical facings in flexure is given by Eq 1. Given deflections and applied forces from results of testing the same sandwich beam with two different loading configurations, the flexural stiffness, D, and the transverse shear rigidity, U, may be determined from simultaneous solution of the deflection equation (Eq 1) for the two loading cases. The core shear modulus, G, may then be calculated using Eq 2. Due to the length of the equations, the solution for the general loading configuration case is not given here. The following subsections give the solution for common combinations of loading conditions.

$$\Delta = \frac{P(2S^3 - 3SL^2 + L^3)}{96D} + \frac{P(S - L)}{4U} \tag{1}$$

$$G = \frac{U(d - 2t)}{(d - t)^2 b} \tag{2}$$

where:

Δ = beam mid-span deflection, mm [in.],
P = total applied force, N [lbf],
d = sandwich thickness, mm [in.],
b = sandwich width, mm [in.],
t = facing thickness, mm [in.],
S = support span length, mm [in.],
L = load span length, mm [in.] (L = 0.0 for 3-point mid-span loading configuration),
G = core shear modulus, MPa [psi],
D = flexural stiffness, N-mm^2 [lb-in.^{2}], and
U = transverse shear rigidity, N [lb].

10.2.1 *One D7249/D7249M Standard 4-Point Loading Configuration and One 3-Point Mid-Span Loading Configuration*—The solution for this case ($L_1 = 2\,S_1/11$, $L_2 = 0$) is given by Eq 3 and 4. Per the procedures of 10.1, calculate the flexural stiffness, shear rigidity, and core shear modulus for each selected value of P_1.

$$D = \frac{3P_1S_1^3(141 - 121S_2^2/S_1^2)}{1936\Delta_1(11 - 9P_1S_1\Delta_2/P_2S_2\Delta_1)} \tag{3}$$

$$U = \frac{9P_1S_1(141S_1^2/S_2^2 - 121)}{4\Delta_1(1269(P_1S_1^3\Delta_2/P_2S_3^3\Delta_1) - 1331)} \tag{4}$$

where:

P_1 = total applied force, N [lb],
S_1 = support span length (configuration #1), mm [in.],
L_1 = load span length (configuration #1), mm [in.],
S_2 = support span length (configuration #2), mm [in.],
L_2 = load span length (configuration #2), mm [in.],
Δ_1 = beam mid-span deflection (configuration #1), mm [in.] corresponding to force P_1, and
Δ_2 = beam mid-span deflection (configuration #2), mm [in.] corresponding to force P_1.

10.2.2 *Two 3-Point Mid-Span Loading Configurations*—The solution for this case ($L_1 = L_2 = 0$) is given by Eq 5 and 6. Per the procedures of 10.1, calculate the flexural stiffness, shear rigidity, and core shear modulus for each selected value of P_1.

$$D = \frac{P_1S_1^3(1 - S_2^2/S_1^2)}{48\Delta_1(1 - P_1S_1\Delta_2/P_2S_2\Delta_1)} \tag{5}$$

$$U = \frac{P_1S_1(S_1^2/S_2^2 - 1)}{4\Delta_1((P_1S_1^3\Delta_2/P_2S_2^3\Delta_1) - 1)} \tag{6}$$

10.2.3 *One 3-Point Mid-Span Loading Configuration and One 4-Point Quarter-Span Loading Configuration*—The solution for this case ($L_1 = 0$, $L_2 = S_2/2$) is given by Eq 7 and 8. Per the procedures of 10.1, calculate the flexural stiffness, shear rigidity, and core shear modulus for each selected value of P_1.

$$D = \frac{P_1S_1^3(1 - 11S_2^2/8S_1^2)}{48\Delta_1(1 - 2P_1S_1\Delta_2/P_2S_2\Delta_1)} \tag{7}$$

$$U = \frac{P_1S_1(8S_1^2/11S_2^2 - 1)}{4\Delta_1((16P_1S_1^3\Delta_2/11P_2S_2^3\Delta_1) - 1)} \tag{8}$$

10.2.4 *One 3-Point Mid-Span Loading Configuration and One 4-Point Third-Span Loading Configuration*—The solution for this case ($L_1 = 0$, $L_2 = S_2/3$) is given by Eq 9 and 10. Per the procedures of 10.1, calculate the flexural stiffness, shear rigidity, and core shear modulus for each selected value of P_1.

$$D = \frac{P_1S_1^3(1 - 23S_2^2/18S_1^2)}{48\Delta_1(1 - 3P_1S_1\Delta_2/2P_2S_2\Delta_1)} \tag{9}$$

$$U = \frac{P_1S_1(18S_1^2/23S_2^2 - 1)}{4\Delta_1((27P_1S_1^3\Delta_2/23P_2S_2^3\Delta_1) - 1)} \tag{10}$$

10.2.5 *One 4-Point Quarter-Span Loading Configuration and One 4-Point Third-Span Loading Configuration*—The solution for this case ($L_1 = S_1/4$, $L_2 = S_2/3$) is given by Eq 11 and 12. Per the procedures of 10.1, calculate the flexural stiffness, shear rigidity, and core shear modulus for each selected value of P_1.

$$D = \frac{99P_1S_1^3(1 - 92S_2^2/99S_1^2)}{6912\Delta_1(1 - 3P_1S_1\Delta_2/4P_2S_2\Delta_1)} \tag{11}$$

$$U = \frac{P_1S_1(99S_1^2/92S_2^2 - 1)}{2\Delta_1((297P_1S_1^3\Delta_2/368P_2S_2^3\Delta_1) - 1)} \tag{12}$$

10.2.6 *One D7249/D7249M Standard 4-Point Loading Configuration and One 4-Point Third-Span Loading Configuration*—The solution for this case ($L_1 = 2S_1/11$, $L_2 = S_2/3$) is given by Eq 13 and 14. Per the procedures of 10.1, calculate the flexural stiffness, shear rigidity, andcore shear modulus for each selected value of P_1.

$$D = \frac{P_1S_1^3(2538 - 2783S_2^2/S_1^2)}{5808\Delta_1(22 - 27P_1S_1\Delta_2/P_2S_2\Delta_1)} \tag{13}$$

$$U = \frac{9P_1S_1(2538S_1^2/S_2^2 - 2783)}{4\Delta_1(34263(P_1S_1^3\Delta_2/P_2S_2^3\Delta_1) - 30613)} \tag{14}$$

10.2.7 *One D7249/D7249M Standard 4-Point Loading Configuration and One 4-Point Quarter-Span Loading Configuration*—The solution for this case ($L_1 = 2S_1/11$, $L_2 = S_2/2$) is given by Eq 15 and 16. Per the procedures of 10.1, calculate the flexural stiffness, shear rigidity, and core shear modulus for each selected value of P_1.

$$D = \frac{3P_1S_1^3(1128 - 1331S_2^2/S_1^2)}{15488\Delta_1(11 - 18P_1S_1\Delta_2/P_2S_2\Delta_1)} \tag{15}$$

$$U = \frac{9P_1S_1(1128S_1^2/S_2^2 - 1331)}{4\Delta_1(20304(P_1S_1^3\Delta_2/P_2S_2^3\Delta_1) - 14641)} \tag{16}$$

10.3 *Calculation of Core Shear Modulus Using Known Facing Modulus*—If the modulus of the sandwich facings are known and the facings are identical, the sandwich transverse shear rigidity, andcore shear modulus can be calculated from the results of a single loading configuration test on the sandwich using Eq 17-19. First calculate the flexural stiffness. Then for each specimen, calculate the shear rigidity and core shear modulus for a series of applied forces up to the maximum applied force. Values should be calculated for a minimum of ten (10) force levels evenly spaced over the force range. Calculate the average value and statistics of the shear rigidity and core shear modulus using the values calculated at each force level for each specimen replicate. The result is a set of stiffness values as a function of force level. If the sandwich response is linear (per 10.1.1), then calculate an overall average flexural stiffness, shear rigidity and core shear modulus using the values from all force levels. Report all of the individual and average calculated stiffness values.

$$D = \frac{E(d^3 - c^3)b}{12} \tag{17}$$

$$U = \frac{P(S_1 - L_1)}{4\left[\Delta - \frac{P(2S_1^3 - 3S_1L_1^2 + L_1^3)}{96D}\right]} \tag{18}$$

$$G = \frac{U(d - 2t)}{(d - t)^2 b} \tag{19}$$

where:

E = facing modulus, MPa [psi], and
c = core thickness = $d - 2t$, mm [in.].

10.4 *Statistics*—For each series of tests calculate the average value, standard deviation, and coefficient of variation (in percent) for flexural stiffness, transverse shear rigidity and core shear modulus:

$$\bar{x} = \left(\sum_{i=1}^{n} x_i\right)/n \tag{20}$$

$$S_{n-1} = \sqrt{\left(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right)/(n-1)} \tag{21}$$

$$CV = 100 \times S_{n-1}/\bar{x} \tag{22}$$

where:

$\bar{x}$ = sample mean (average),
S_{n-1} = sample standard deviation,
CV = sample coefficient of variation, %,
n = number of specimens, and
x_i = measured or derived property.

11. Report

11.1 Report the following information, or references pointing to other documentation containing this information, to the maximum extent applicable (reporting of items beyond the control of a given testing laboratory, such as might occur with material details or panel fabrication parameters, shall be the responsibility of the requestor):

11.1.1 The revision level or date of issue of this practice.

11.1.2 The name(s) of the test operator(s).

11.1.3 Any variations to this practice.

11.1.4 Identification of all the materials constituent to the sandwich panel specimen tested (including facing, adhesive and core materials), including for each: material specification, material type, manufacturer's material designation, manufacturer's batch or lot number, source (if not from manufacturer), date of certification, and expiration of certification. Description of the core orientation.

11.1.5 Description of the fabrication steps used to prepare the sandwich panel including: fabrication start date, fabrication end date, process specification, and a description of the equipment used.

11.1.6 Method of preparing the test specimen, including specimen labeling scheme and method, specimen geometry, sampling method, and specimen cutting method.

11.1.7 Results of any nondestructive evaluation tests.

11.1.8 Measured lengths, widths and thicknesses for each specimen.

11.1.9 Number of specimens tested.

11.1.10 Facing thicknesses used in the calculations.

11.1.11 Facing modulus used in the calculations (for known modulus cases).

11.1.12 Calculated sandwich flexural stiffnesses and average value, standard deviation, and coefficient of variation (in percent) for the population.

11.1.13 Calculated sandwich transverse shear rigidities and average value, standard deviation, and coefficient of variation (in percent) for the population.

11.1.14 Calculated sandwich core shear moduli and average value, standard deviation, and coefficient of variation (in percent) for the population.

12. Keywords

12.1 core shear modulus; flexural stiffness; sandwich construction; sandwich deflection; shear rigidity; shear stiffness

ANNEX

(Mandatory Information)

A1. TOE COMPENSATION

A1.1 In a typical force versus displacement curve (see Fig. A1.1) there is a toe region, AC, that does not represent a property of the material. It is an artifact caused by a take-up of slack and alignment or seating of the specimen. In order to obtain correct force-displacement slope values, this artifact must be compensated for to give the corrected zero point on the displacement axis.

A1.2 For sandwich materials exhibiting a region of Hookean (linear) behavior (see Fig. A1.1), a continuation of the linear (CD) region of the curve may be constructed through the zero-force axis. This intersection (B) is the corrected zero displacement point (δ = 0.000) from which all displacements must be measured.

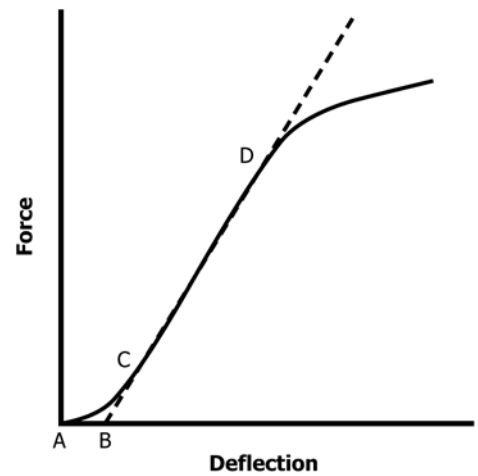


FIG. A1.1 Sandwich Material with Hookean Response Region