



Standard Test Method for Determination of Bulk Density of Coal Using Nuclear Backscatter Depth Density Methods¹

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INTRODUCTION

Data obtained from the density of coal stockpiles is combined with volume determinations per Test Method **D6172** to complete the physical inventory of total tons. The book inventory is compared to the physical inventory for determining the accumulation of measurement difference that have occurred since the last physical inventory.

Since the physical inventory is the reference value used to compare and adjust the book inventory, it is important that the standard methods for conducting the density and volume measurements introduce the least possible error. Close adherence to the details of the procedures described in this standard is a prerequisite to the objective of this test method.

This standard test method for determination of bulk density of stockpiled coal is used for all ranks of coal. Proper density determination involves an understanding of the physical characteristics of the coal types and the stockpiling facilities.

1. Scope

1.1 This test method covers procedures for determining the bulk density of coal using instrumentation that measures the relative backscatter of nuclear gamma radiation throughout the depth of the stockpile under test.

1.2 This procedure is applicable to all ranks of coal.

1.3 The values stated in either SI units or inch-pound units are to be regarded separately as standard. The values stated in each system may not be exact equivalents; therefore, each system shall be used independently of the other. Combining values from the two systems may result in non-conformance with the standard.

1.4 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use. The equipment uses*

radioactive materials, which may be hazardous to the health of users, unless proper precautions are taken.

2. Referenced Documents

2.1 *ASTM Standards:*²

D1586 Test Method for Penetration Test (SPT) and Split-Barrel Sampling of Soils

D6172 Test Method for Determining the Volume of Bulk Materials Using Contours or Cross Sections Created by Direct Operator Compilation Using Photogrammetric Procedures

2.2 *NIST Standard:*

National Institute for Standards and Technology Handbook 44, Specifications, Tolerances, and Other Technical Requirements for Weighing and Measuring Devices³

2.3 *ASME Standard:*

ASME Pipe Codes⁴

¹ This test method is under the jurisdiction of ASTM Committee **D05** on Coal and Coke and is the direct responsibility of Subcommittee **D05.07** on Physical Characteristics of Coal.

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² For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

³ Available from National Institute of Standards and Technology (NIST), 100 Bureau Dr., Stop 1070, Gaithersburg, MD 20899-1070, <http://www.nist.gov>.

⁴ Available from American Society of Mechanical Engineers (ASME), ASME International Headquarters, Three Park Ave., New York, NY 10016-5990, <http://www.asme.org>.

2.4 Code of Federal Regulations:

Title 10, Parts 19 and 20⁵

Title 49⁵

3. Terminology

3.1 Definitions of Terms:

3.1.1 *counting period*—the period of time, set using the scaler time switch, during which the readout device accumulates pulses.

3.1.2 *test counts*—the recording of pulses accumulated by the readout device during a counting period. The test count is initiated by pushing a start button located on the scaler, which resets the accumulator and starts a new counting period.

3.1.3 *standard count*—a test count taken with the probe positioned inside the shield and standard assembly.

3.1.4 *reference standard count*—a series of 30 consecutive standard counts, taken on the occasions specified in 9.1.5.

3.1.5 *stability check standard count*—a series of five consecutive standard counts, taken on the occasions specified in 9.1.6.

3.1.6 *access hole*—used loosely in this standard to denote the opening made into the stockpile for insertion of the probe access tube.

3.1.7 *penetration*—inserting the probe access tube in a selected position for collecting test count readings.

3.1.8 *vertical position*—the position of the probe where a test count reading is taken, measured from a reference elevation.

3.1.9 *vertical interval*—the vertical interval moved by the test probe when repositioned from one vertical position to the next test vertical position in the same access hole. For this test method, a vertical interval is equal to 30 in. [76 cm].

4. Summary of Test Method

4.1 The area of the stockpile accessible for stockpile penetration is first identified. An access (sampling) plan for the accessible area is determined, and the depth of the stockpile is accessed at locations within the area according to plan. The probe is used to obtain test count readings at vertical intervals within each access hole. Test count readings of compacted coal in field calibration vessels are taken using coal from the stockpile so as to convert stockpile test count readings to bulk density using a calibration curve. The mean bulk density of the stockpile is estimated using the test count reading and calibration data.

4.2 The procedures appear in the following order:

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| | |
|---|-------|
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5. Significance and Use

5.1 The measured mean bulk density of a coal stockpile is used with a measurement of the stockpile volume per Test Method D6172. Procedures to determine the quantity of stockpiled coal. This measure of quantity is often used as a reference value for adjusting inventory records.

6. Apparatus

6.1 Field-test location preparation equipment consisting of:

6.1.1 *Drilling Rig*—mobile equipment capable of boring, driving, or hydraulically forcing, on its own power, access holes through the entire depth of the stockpile.

6.1.2 *Access Tube*—hollow tubing with an inside diameter allowing clearance between 0.035 in. [0.90 mm] and 0.060 in. [1.50 mm] for the nuclear depth probe to be positioned at specified vertical intervals. The tubing shall be sufficiently durable to withstand the applied forces of insertion into the access hole. The inside diameter of the access tube shall provide a smooth path with an annular space and wall thickness sufficient that there is no deformation of shape under normal use. ASME Schedule 80 thickness PVC pipe is the most commonly used access tube material. Other materials may be used, but their use may require a different tube wall thickness or a different counting period, or both, to obtain an equivalent test count.

6.1.3 *Small Auger*—a boring apparatus with connecting sections used to provide an access hole the entire depth of the stockpile. The outside diameter of the auger is sized to produce an access hole with a diameter equal to the outside diameter of the access tube. To obtain reliable test count readings, it is essential that the access tube fit snugly into the access hole with no empty spaces at the junction between the coal and the access tube.

6.1.4 *Split-Barrel Sampler*—a hollow device with a cutting edge, which, when driven into coal, can be extracted and leave an access hole. Drill rod extensions provide for sampling the entire depth of the stockpile.

6.1.5 *Hollow Stem Auger*—an auger with a hollow central shaft having an inside diameter larger than the access tube. This auger is used in the split-barrel access technique to continue the access hole through previously tested depths of the stockpile.

⁵ Available from the U.S. Government Printing Office, Superintendent of Documents, Washington, DC 20402.



6.1.6 *Nuclear Depth Density Probe*—The device required for collecting the test count readings from the stockpile. It is equipped with the following components:

6.1.7 *Gamma Source*—an encapsulated and sealed radioactive source that meets the specific form requirements of Title 49 of the Code of Federal Regulations. A Cesium 137, nominal 8-milli-curie source is used.

6.1.8 *Gamma Detector*—a Geiger-Mueller tube housed in the probe is an unlimited life, metal wall gamma detector. A preamplifier is included to amplify and send the electrical signal to the readout device or scaler.

6.1.9 *Shield and Standard*—an assembly, which protects the probe from mechanical damage, provides the necessary shielding of the radioactive source when not in use and provides a means to obtain repeatable reference and stability check standard counts.

6.1.10 *Read-Out Device*—a scaler with a resolution and range to display counts over the density for which the apparatus will be used. The scaler contains other electronic components and an electrical power supply. Units with a maximum error of timing ± 0.005 and $\pm 0.002\%$ stability.

6.1.11 *Electrical Cable*—to connect the probe to the readout device. The cable shall be of sufficient length to allow test counts to be taken the entire depth of coal being tested.

6.1.12 *Transport Case*—a sampling case used for storing and transporting the nuclear depth probe. The transport case shall be constructed and labeled to meet the U.S. Department of Transportation requirements in Title 49 of the Code of Federal Regulations.

6.2 *Field Calibration Equipment:*

6.2.1 *Field Calibration Vessel*—a vessel with a minimum inside diameter of 23 in. [58 cm] and a minimum inside height of 32 in. [81 cm]. The volume of the vessel is measured and certified semiannually using the temperature-corrected water method. A weighing device that has been certified as accurate by a metrology laboratory determines certification weights. A certificate of calibration is provided in accordance with 13.1.12. The vessel can be made of metal or polymer material, provided the vessel retains constant volume during compaction and weighing. However, the coefficient of thermal expansion for polymer material, which can be eleven times greater than for steel over the same temperature ranges, can result in calibration error.

6.2.2 *Field Calibration Access Tube*—is identical to that used to access the stockpile and specified in 6.1.2.

6.2.3 *Field Calibration Coal Retrieval Auger*—an auger, with a minimum outside diameter of 2.5 times of top size of the coal, used to collect calibration coal from the stockpile.

6.2.4 *Field Compaction Hammer Assembly*—an apparatus for compacting the coal uniformly in the calibration vessel to the required range of densities. Manual devices are not recommended for compaction.

6.2.5 *Field Calibration Scale*—a weighing device meeting National Institute for Standards and Technology Handbook 44, Type II requirements. The weighing device shall have the capacity of weighing the calibration vessel empty or filled with coal in a single weighment. The weighing device shall be

certified accurate to $\pm 0.1\%$ of applied load and calibrated semiannually. Provide a certificate of calibration in accordance with 13.1.12.

6.2.6 *Access Stabilization/Sample Collection Device*—a sample collection pan with an opening placed over the top of the calibration vessel to restrict the lateral movement of the access equipment.

7. Precautions

7.1 Operators of the equipment shall obtain a license for the use of the equipment from the U.S. Nuclear Regulatory commission or state regulatory agencies, or both. The operator takes proper precautions to ensure that the use of this equipment complies with applicable sections of Title 10, Code of Federal Regulations, Parts 19 and 20.

7.2 Backfill the access holes to lessen the risk of stockpile spontaneous combustion.

8. Preparation of the Sampling Plan

8.1 Use experienced personnel familiar with stockpiling and reclaiming procedures used at the location. Determine the surface area and the nominal depth of the stockpile to be sampled. Prepare a drawing that clearly identifies areas that are not accessible for penetration or else not intended for access, such as areas covering underground hoppers and reclaiming facilities, or heavily compacted haul roads.

8.1.1 It is good practice to use laboratory analysis from samples collected at each vertical test interval to identify foreign material from the base of the stockpile and to convert bulk density to other than in-situ moisture basis.

8.1.2 Determine the number (n) of test counts required to represent the stockpile. For stockpiles with a prior history of density testing, this data will prove helpful in making this determination. If no prior history is available, it is recommended that a minimum of 100 test counts and a minimum of 8 access holes be used to represent the stockpile.

NOTE 1—The minimum test counts designated for stockpiles with no history available are based on a standard deviation taken from a Z table. Eight borings were designated to assure reasonable representation of the stockpile.

8.1.3 Using the accessible surface area and the nominal depth of the stockpile determine the number of access holes required to provide the number (n) of test counts selected in 8.1.2, allowing one vertical interval per test count. Divide the accessible area into the same number of grid openings as the number of access holes. The grid openings should represent as near equal volume as feasible. The openings need not be identical in shape.

8.1.4 Select one access hole site within each grid opening and mark the site locations on the plot plan. For the selection process, use a random location in each grid.

8.1.5 For penetration and access of the stockpile, choose one of the procedures described as Procedures A, B, and C in Section 10.

8.1.6 Treat areas not included in Section 8 such as heavily compacted haul roads, uncompacted slope areas, and large ready piles, separately.

9. Standardization of the Nuclear Depth Density Gauge

9.1 *Field Standardization*—use these standardization procedures to protect against instrument fault or drift and assure consistent probe response.

9.1.1 Allow the electrical equipment to stabilize in accordance with the manufacturer's recommendation when activated.

9.1.2 Ensure that the probe remains in the “power on” or “stand by” position while fieldwork is being conducted.

9.1.3 Use the same counting period for test counts throughout the bulk density test, including calibration procedures.

9.1.4 Take all standard counts with the probe located at least 3 ft [90 cm] from other radioactive sources, large masses of metal, or materials other than coal, to assure that standard counts are not affected by these influences.

9.1.5 Take a reference standard count at the beginning of work at each stockpile test site. Calculate the mean \bar{y}_{rc} and sample standard deviation S_{rc} of the 30 test counts comprising the standard reference count.

9.1.6 A stability check standard count is taken, (1) at the beginning of each day's work, (2) at each stockpile access hole site immediately before taking test count readings from within the hole, and (3) at the beginning of each calibration vessel immediately before taking test count readings within the vessel. Calculate the mean \bar{y}_{sc} of the five stability standard test counts.

9.1.7 The probe is considered to be in stable condition and ready for use if:

$$|\bar{y}_{sc} - \bar{y}_{rc}| < 1.3 \quad (1)$$

where:

\bar{y}_{sc} = the mean of five stability standard test counts,

\bar{y}_{rc} = the mean of 30 reference standard reference counts, and

S_{rc} = the sample standard deviation of the 30 standard test counts taken in the standard reference count.

9.2 When the comparison does not meet the criterion in 9.1.7, take another stability check standard count. If the results of the second run meet the criterion, the probe is considered to be in stable condition and ready for use. If the second stability check standard count does not meet the criterion of 9.1.7, repair or replace the nuclear depth density probe.

9.2.1 If the nuclear depth density probe is repaired, reconstruct the stability check standard count using the reference standard count taken at the beginning of the stockpile site work. If the criterion in 9.1.7 is met with the repaired probe, testing may continue and readings taken before repair may be included in the density calculation for the stockpile.

9.2.2 If a new reference standard count is run to establish stability criterion in 9.1.7 for the repaired nuclear depth density probe, all readings taken before repair will not be used in the density calculations for the stockpile.

9.2.3 If the nuclear depth density probe is replaced, all readings taken before replacement will not be used in the density calculations for the stockpile.

10. Stockpile Penetration And Access Procedures

10.1 Standardize the nuclear depth density depth probe in accordance with Section 9.

10.2 Using the procedure selected in 8.1.5, penetrate the stockpile at each access site defined on the plot plan.

10.3 Measure and record the depth of the access holes. If data are available, compare original base elevations to base elevations determined by access holes and record differences.

10.4 Place the access stabilization/sample collection device over each access site before penetration.

NOTE 2—It is good practice to use laboratory analysis of samples collected at each vertical test interval to identify foreign material from the base of the stockpile and thus assist in defining the useful base of the stockpile.

10.5 Stockpile Procedures A, B, and C require taking replicate test counts at each vertical position. To minimize data recording errors, compare the difference between the two test counts. If this difference exceeds preselected criteria (based on previous experience with the probe being used), repeat the counts.

10.6 *Procedure A—Split-Barrel Sampler/Large Auger Procedure:*

10.6.1 Select the first access site. Drive the split-barrel sampler, following Test Method D1586, to the depth of one vertical interval in a manner that least disturbs the coal in the stockpile.

10.6.2 Remove the split-barrel sampler. Collect, package, and label the coal sample within the split-barrel sampler as the quality sample for this vertical interval.

10.6.3 Insert an access tube of the same outside diameter as the split-barrel sampler into the access hole.

10.6.4 Lower the nuclear depth density probe, one vertical interval, into the access tube. Record replicate test count readings for this vertical interval.

10.6.5 Remove the nuclear depth density probe and the access tube.

10.6.6 Drill to the top the next vertical interval using a hollow stem auger. The auger has an outside diameter of at least five 5 in. [127 mm] and an inside diameter large enough to allow insertion of the access tube.

10.6.7 Penetrate to the base material except where liners separate base material from the stockpile.

10.6.8 Repeat 10.6.2 – 10.6.6 until reaching the base of the stockpile.

10.6.9 Penetrate the depth of the stockpile with the hollow stem auger in the same location where the split-barrel sampler has been driven for the tests. Collect sufficient auger cuttings for use as calibration coal (seal to prevent moisture loss and label for location and depth). Repeat at each access hole, collecting in total sufficient coal for a minimum of eight calibration vessels.

10.6.10 Repeat the procedure from Steps 10.6.1 – 10.6.8 for each access site.

10.7 *Procedure B—Small Auger Procedure:*

10.7.1 Select the first access site. Using a small diameter auger, penetrate the entire depth of the stockpile in a manner

that least disturbs the coal. Collect, package, and label auger cuttings for each predetermined vertical interval. Penetration should reach the base material except where liners separate the base material from the stockpile.

10.7.2 Insert an access tube with the same nominal outside diameter as the small diameter auger into the access hole to the base of the stockpile.

10.7.3 Lower the nuclear depth density probe into the access tube and record replicate test count readings for each predetermined vertical interval.

10.7.4 Remove the nuclear depth density probe and the access tube.

10.7.5 Penetrate the depth of the stockpile with an auger of minimum 5-in. (127-mm) outside diameter in the same location of the penetration of the small diameter auger for the tests. Collect sufficient auger cuttings for use as calibration coal (seal to prevent moisture loss and label for location and depth). Repeat at each access hole, collecting sufficient coal for a minimum of eight calibration vessels.

10.7.6 Repeat the procedure in [10.7.1 – 10.7.5](#) for each access site.

10.8 *Procedure C—Hydraulic Access Procedure:*

10.8.1 Select the first access site. Use hydraulic force to insert the access tube the entire depth of the stockpile (or until refusal) in a manner that least disturbs the coal in the stockpile. Penetration should reach the base material except where liners separate the base material from the stockpile.

10.8.2 Lower the nuclear depth density probe into the access tube and record replicate test count readings for each predetermined vertical interval.

10.8.3 Remove the nuclear depth density probe and the access tube.

10.8.4 Penetrate the stockpile at the same depth as the access tube is inserted. Use an auger with a minimum 5-in. [127-mm] outside diameter. Collect sufficient auger cuttings for use as calibration coal (seal to prevent moisture loss and label for location and depth). Repeat at each access hole collecting sufficient coal for a minimum of eight calibration vessels.

10.8.5 Repeat the procedure from steps [10.8.1 – 10.8.3](#) for each access site.

11. Field Calibration of The Nuclear Depth Density Gauge

11.1 The count rate of the nuclear depth density probe is influenced by the elemental composition of the material. A probe measurement of two materials of the same bulk density but with different elemental analysis will yield different count rates. The nature of the disturbed surface interface between the access tube and the walls of the access hole can affect the nuclear depth density probe response and, consequently, a different response may result from fine coal than from coarser coal. There is insufficient data to quantify these factors to enable a mathematical correction of the nuclear depth density probe response based on independent measures of these factors. Therefore, it is necessary that field calibrations be performed using coal from the stockpile being tested.

11.2 Perform calibration procedures on-site simulating actual field test conditions using coal collected from stockpile access sites.

11.3 A minimum of eight pairs of values shall be used (compaction of eight separate calibration vessels) for field calibration of the nuclear gauge. In planning compaction of calibration vessels, cover as much of the range of test counts determined in the stockpile as is feasible. The minimum coverage is the difference between the mean test counts of the lowest and highest access holes tested in the stockpile. Distribute the calibration vessel test counts uniformly within that range. The same coal sample shall not be used for more than one calibration vessel.

11.4 Weigh and record the weight of the empty calibration vessels before beginning each calibration vessel compaction. An example calibration vessel log is shown in [Annex A4](#).

11.5 Compact the coal in the calibration vessel by placing a uniform layer (lift) of coal in the vessel and striking the layer a number of times using the field compaction hammer and compaction plate. Repeat the procedure, compacting successive lifts equally until the coal overflows the vessel. Level the coal to the top of the vessel by carefully striking off the overflowing coal. Calibration vessels with different densities are achieved by changing the thickness of the lift and the number of times each lift is struck.

11.6 Weigh and record the compacted coal and the calibration vessel. An example calibration vessel log is shown in [Annex A4](#).

11.7 Penetrate the depth of the compacted coal within the calibration vessel. Use the same procedure that was used to penetrate the depth of the stockpile. Use access tubing identical to that used for the stockpile. Place the access stabilization device on the compacted coal in the calibration vessel before penetration.

11.8 Take and record a minimum of 15 replicate test counts (30 total), with the spatial positions of the replicate counts distributed uniformly by moving the probe in 1-in. [25.4-mm] vertical intervals. An example calibration vessel log is shown in [Annex A4](#).

11.9 Conduct a uniform compaction check using the procedures given in [Annex A2](#) to assure uniform compaction within the vessel.

11.10 The data values, which are used for each compaction vessel in establishing the calibration curve, are the mean of the 15 replicate test counts (the probe response) and the compaction vessel density. The compaction vessel density is derived from the volume and the net weight of the coal in the vessel (the bulk density). See the example calibration vessel log in [Annex A4](#).

11.11 Develop a calibration curve using the mathematical process given in [Annex A4](#).

11.12 Develop a separate calibration curve for each access method used in the coal stockpile.

12. Interpretation of Results

12.1 **Annex A1**, **Annex A2**, and **Annex A4** give an example of a typical coal pile bulk density survey. **Annex A1** shows the calculations for determining the pile test count grand average. **Annex A2** shows a calibration vessel uniform compaction check, **Annex A3** shows an alternate calibration vessel uniform compaction check, **Annex A4** shows the computations for the calibration curve, and **Annex A5** shows the Jackknife method to determine the precision at two standard deviations of the mean density estimate; the confidence interval at 95 % of the mean density estimate in pounds per cubic foot (pcf) and in percent of the mean density.

13. Report

13.1 Prepare a report for each stockpile promptly after completion of field work and include the following information:

13.1.1 The owner, location of the stockpile, the party authorizing the work, identification of the contractors, and its representative. Identify responsible field personnel, personnel preparing the report, and subcontractors.

13.1.2 The dates of performance of all major work functions.

13.1.3 A statement of the mean bulk density estimate, the precision at two standard deviations of that estimate and the confidence limits at the 95 % confidence level applicable to the volume of the stockpile covered by the density survey.

13.1.4 List alternative and optional procedures covered by this standard when used. Also list all exceptions to the standard procedure including authorization. Include all relevant procedures and formulas of statistical methods applied for authorized exceptions.

13.1.5 A test site location plan developed from a contour map of the stockpile including the vertical and horizontal coordinates of the access locations, if available. Submit the plot plan required in Section 8 as a minimum requirement.

13.1.6 All raw data and results of statistical calculations. Organize data logically by the test location and elevation. Report nuclear depth density probe calibration data for each

calibration point and vertical position within each test vessel. Record the quantity of water added in the calibration procedure, if any.

13.1.7 A clear description of the calculations and results of the uniform compaction tests required in 11.9 and developed in accordance with **Annex A2**.

13.1.8 A summary chart showing the results of the linear calibration procedures and a graph of the calibration curve required in 11.10 and developed in accordance with **Annex A4**.

13.1.9 A summary chart showing the results of the collection of stockpile test count data and a graph of test count distribution within the stockpile specified in Section 8 and developed in accordance with **Annex A1**.

13.1.10 The results of investigation of any aberrant results and the disposition thereof.

13.1.11 Verify adherence to all ASTM methods or authorized exceptions or both. Include a description of all relevant formulas of statistical methods applied for authorized exceptions.

13.1.12 Certificates of calibration for the calibration vessel and scale.

13.1.13 Operators' Nuclear Regulatory Commission (or equivalent) authorized license or technicians training certification.

13.1.14 Confirmation of Nuclear Regulatory Commission authorization or reciprocity compliance between jurisdictions to operate nuclear apparatus within a state.

14. Precision and Bias

14.1 The method for determining the precision of the density survey results is defined in **Annex A5** and included in the report (see 13.1.3).

14.2 There is no accepted reference method for determining the bulk density of coal in stockpile, thus bias has not been determined.

15. Keywords

15.1 bulk density; coal; nuclear backscatter; stockpile

ANNEXES

(Mandatory Information)

A1. ESTIMATING THE AVERAGE STOCKPILE TEST COUNT

A1.1 *Assignment of Data to Strata:*

A1.1.1 Prepare a table for each access hole as shown in **Table A1.1** and **Table A1.2**. Beginning with the lowest vertical interval, record the replicate test counts and the average of those test counts in serial order for each interval.

A1.1.2 Determine the total number (n) of replicate test counts, the summation (Σ) of replicate test counts, and the mean test count (y_o) for the access hole.

A1.1.3 Prepare a table for each stockpile as shown in **Table A1.3**. Enter each access location in serial order as row numbers. Align the average test count readings for the corresponding vertical intervals from all access locations as columns labeled j , where $j = 1, 2, \dots, m$ is the total number of stratum, that is, all No.1 vertical intervals in Column 1, all No. 2 vertical intervals in Column 2, and so forth.

A1.1.4 Identify in the column heading the elevation represented by each stratum.

A1.1.5 Enter the number n_j , of average replicate test count readings, that is, vertical intervals in each column (stratum).

A1.1.6 Determine and record the grand average test count given by

where:

n = total number of observations,

i = row index,

j = column index, and

$\bar{y}_o = 352\,012/131 = 2687.1145$.

$$\bar{y}_o = \frac{1}{n} \sum_i \sum_j y_{ij} \quad (\text{A1.1})$$

TABLE A1.1 Access Location No. 05

| Vertical Interval No. | Depth, ft | Elevation, ft | Test 1 (Counts) | Test 2 (Counts) | Replicate Average (Counts) |
|-----------------------|-----------|---|-----------------|-----------------|----------------------------|
| Base | ... | 440.0 | ... | ... | ... |
| 1 | 27.5 | 442.5 | 2378 | 2367 | 2372.5 |
| 2 | 25.0 | 445.0 | 2781 | 2780 | 2780.5 |
| 3 | 22.5 | 447.5 | 2715 | 2711 | 2713.0 |
| 4 | 20.0 | 450.0 | 2636 | 2621 | 2628.5 |
| 5 | 17.5 | 452.5 | 2546 | 2534 | 2540.0 |
| 6 | 15.0 | 455.0 | 2758 | 2725 | 2741.5 |
| 7 | 12.5 | 457.5 | 2650 | 2629 | 2639.5 |
| 8 | 10.0 | 460.0 | 2676 | 2646 | 2661.0 |
| 9 | 7.5 | 462.5 | 2507 | 2539 | 2523.0 |
| 10 | 5.0 | 465.0 | 2758 | 2793 | 2775.5 |
| 11 | 2.5 | 467.5 | 2648 | 2645 | 2646.5 |
| Surface | ... | 468.1 | ... | ... | ... |
| 11 | | $y_o = \Sigma \text{ counts} = 2638.3182$ | | | 29 021.5 |
| n counts | | n | | | $\Sigma \text{ counts}$ |

TABLE A1.2 Access Location No. 02

| Vertical Interval No. | Depth, ft | Elevation, ft | Test 1 (Counts) | Test 2 (Counts) | Replicate Average (Counts) |
|-----------------------|-----------|---|-----------------|-----------------|----------------------------|
| Base | | 441.0 | | | |
| 1 | 30.0 | 443.5 | 2443 | 2424 | 2433.5 |
| 2 | 27.5 | 446.0 | 2979 | 3009 | 2994.0 |
| 3 | 25.0 | 448.5 | 2954 | 2971 | 2962.5 |
| 4 | 22.5 | 451.0 | 2862 | 2893 | 2877.5 |
| 5 | 20.0 | 453.5 | 2842 | 2840 | 2841.0 |
| 6 | 17.5 | 456.0 | 2726 | 2731 | 2728.5 |
| 7 | 15.0 | 458.5 | 2855 | 2846 | 2850.5 |
| 8 | 12.5 | 461.0 | 2660 | 2680 | 2670.0 |
| 9 | 10.0 | 463.5 | 2698 | 2699 | 2698.5 |
| 10 | 7.5 | 466.0 | 2648 | 2632 | 2640.0 |
| 11 | 5.0 | 468.5 | 2595 | 2562 | 2578.5 |
| | 2.5 | 471.0 | 2604 | 2638 | 2621.0 |
| Surface | | 471.2 | | | |
| 12 | | $y_o = \Sigma \text{ counts} = 2741.2917$ | | | 32 895.5 |
| n counts | | n | | | $\Sigma \text{ counts}$ |

TABLE A1.3 Vertical Interval Test Count Data by Elevation, Stratum j^A

| Access Location No. | 1 442.5 | 2 445.0 | 3 447.5 | 4 450.0 | 5 452.5 | 6 455.0 | 7 457.5 | 8 460.0 | 9 462.5 | 10 465.0 | 11 467.5 | 12 470.0 | 13 472.5 | 14 SUM TC |
|---------------------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|-------------|-------------|-------------|-------------|--------------|
| 1 | 2550.0 | 2505.0 | 2676.0 | 2604.0 | 2726.5 | 2808.0 | 2815.0 | 2756.0 | 2705.5 | 2666.0 | 2608.0 | | | 29 420.0 |
| 2 | 2433.5 | 2994.0 | 2962.5 | 2877.5 | 2841.0 | 2728.5 | 2850.5 | 2670.0 | 2698.5 | 2640.0 | 2578.5 | 2621.0 | | 32 895.5 |
| 3 | 2337.0 | 2527.0 | 2734.0 | 2748.0 | 2757.0 | 2820.0 | 2808.5 | 2735.5 | 2535.0 | 2670.0 | 2596.5 | 2640.0 | | 31 908.5 |
| 4 | 2364.5 | 2394.0 | 2611.0 | 2605.5 | 2699.5 | 2680.5 | 2613.0 | 2569.0 | 2506.0 | 2541.0 | | | | 25 584.0 |
| 5 | 2372.5 | 2780.5 | 2713.0 | 2628.5 | 2540.0 | 2741.5 | 2639.5 | 2661.0 | 2523.0 | 2775.5 | 2646.5 | | | 29 021.5 |
| 6 | 2306.5 | 2800.0 | 2895.0 | 2786.5 | 2752.5 | 2691.0 | 2566.0 | 2506.0 | 2565.5 | 2546.5 | | | | 26 415.5 |
| 7 | 2439.0 | 2610.5 | 2758.5 | 2680.5 | 2623.5 | 2690.0 | 2619.5 | 2474.5 | 2774.5 | 2691.5 | | | | 26 362.0 |
| 8 | 2439.5 | 2499.5 | 2942.0 | 2902.0 | 2927.5 | 2861.0 | 2762.0 | 2739.0 | 2657.0 | 2681.0 | | | | 27 410.5 |
| 9 | 2456.5 | 2727.0 | 2680.0 | 2770.0 | 2848.0 | 2784.0 | 2811.5 | 2896.0 | 2776.5 | 2837.5 | | | | 27 587.0 |
| 10 | 2353.0 | 2825.0 | 2862.5 | 2856.5 | 2820.0 | 2777.0 | 2781.5 | 2863.5 | 2829.5 | 2903.5 | 2800.0 | 2775.0 | 2727.0 | 36 174.0 |
| 11 | 2465.5 | 2778.0 | 2845.0 | 2816.5 | 2763.5 | 2699.0 | 2516.5 | 2743.0 | 2603.5 | 2644.0 | 2643.5 | 2666.5 | | 32 184.5 |
| 12 | 2647.0 | 2704.5 | 2748.0 | 2731.5 | 2585.0 | 2876.0 | 2735.0 | 2630.0 | 2716.0 | 2676.0 | | | | 27 049.0 |
| Σ | 29 164.5 | 32 145.0 | 33 427.5 | 33 007.0 | 32 884.0 | 33 156.5 | 32 518.5 | 32 243.5 | 31 890.5 | 32 272.5 | 15 873.0 | 10 702.5 | 2727.0 | 352 012.0 |
| n_j | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 6 | 4 | 1 | ... |

^AWhere $j = 1, 2, \dots, m$ and $m =$ total number of strata.

A2. CALIBRATION VESSEL UNIFORM COMPACTION CHECK

A2.1 Compute and record the average of the replicate test count measurements for each vertical position and place in serial order, 1 to 15, beginning from the bottom of the calibration vessel. Denote the i th replicate average as the i th vertical measurement.

A2.2 Determine the mean of the vertical measurement:

$$y = \Sigma y_i / n \quad (\text{A2.1})$$

where:

y = mean of vertical measurements,
 Σy_i = sum of vertical measurements, and
 n = number of vertical measurements.

A2.3 Determine the standard deviation of the vertical measurements

$$\text{Standard deviation} = S_y = \sqrt{\frac{\Sigma(y - \bar{y})^2}{n - 1}} \text{ or } \sqrt{\frac{n(\Sigma y^2) - (\Sigma y)^2}{n(n - 1)}} \quad (\text{A2.2})$$

where:

S_y = individual vertical measurements,
 y = mean of vertical measurements, and
 n = number of vertical measurements.

A2.4 Determine the coefficient of variation measurements in percent

$$\text{CV in \%} = 100 \Sigma S_y / y \quad (\text{A2.3})$$

If the CV expressed as a percentage exceeds either historical values or 2 %, the calibration vessel is suspect. Recheck the source of the data and the calibration coal used for the vessel before including data to determine the calibration curve. If the source of the high CV value cannot be reconciled or if the calibration point appears to be an outlier as determined in [A2.1](#), the vessel shall not be used to determine the calibration curve.

A3. ALTERNATE CALIBRATION VESSEL UNIFORM COMPACTION CHECK

A3.1 Compute and record the average of the replicate test count measurement for each vertical position, and place in serial order, 1 to 15, beginning from the bottom of the calibration vessel (see [Table A3.1](#)). Denote the i th replicate average as the i th vertical measurement.

A3.2 Form three groups of five vertical measurements, with the first group including vertical measurements 1 through 5, the second group including measurements 6 through 10, and the third group including measurements 11 through 15.

A3.3 Let y_{ij} denote the i th measurement of the j th group, where $i = 1, 2, 3, 4, 5$ and $j = 1, 2, 3$. For each of the three groups formed, calculate the group average y_j and the sample variance s_{ij} where $j = 1, 2, 3$ and $i = 1, 2, 3, 4, 5$ using [Eq A2.2](#).

TABLE A3.1 Example Calibration Vessel Log

| Calibration Vessel Log Stockpile Main Point No. 1 | | | | | |
|---|---------------------------------------|-----------------|---------------------|-----------------|---------------------------|
| Vessel I.D. | Vessel Volume 8.83 ft ³ | Scale I.D. | Weight, lb Gross | Tare | Net |
| | Test Counts | 1005.0 | 471.0 | 534.0 | |
| Test Strata No. (n) | y ₁ | y ₂ | y | y ² | Comments/ Observations |
| 1 | 2896 | 2901 | 2898.5 | 8 401 308.25 | |
| 2 | 2909 | 2905 | 2907.0 | 8 450 649.00 | |
| 3 | 2902 | 2872 | 2887.0 | 8 334 769.00 | |
| 4 | 2892 | 2884 | 2888.0 | 8 340 544.00 | |
| 5 | 2896 | 2885 | 2890.5 | 8 354 990.35 | |
| 6 | 2880 | 2873 | 2876.5 | 8 274 252.25 | |
| 7 | 2871 | 2857 | 2864.0 | 8 202 496.00 | |
| 8 | 2870 | 2873 | 2871.5 | 8 245 512.35 | |
| 9 | 2886 | 2856 | 2871.0 | 8 242 641.00 | |
| 10 | 2861 | 2890 | 2875.5 | 8 268 500.25 | |
| 11 | 2836 | 2860 | 2848.0 | 8 111 104.00 | |
| 12 | 2841 | 2885 | 2863.0 | 8 196 769.00 | |
| 13 | 2844 | 2848 | 2846.0 | 8 099 716.00 | |
| 14 | 2824 | 2839 | 2931.5 | 8 017 392.25 | |
| 15 | 2838 | 2824 | 2831.0 | 8 014 561.00 | |
| 15 | 43 046.0 | 43 052.0 | 43 049.0 | 123 555 198.50 | |
| n | Σy ₁ | Σy ₂ | Σy | Σy ² | |

$$\bar{y}_j = \frac{1}{5} \sum_{i=1}^5 y_{ij}, \text{ and } s_j^2 = \frac{1}{y-1} \sum_{i=1}^5 (y_{ij} - \bar{y}_j)^2, \text{ for } j = 1, 2, 3 \quad (\text{A3.1})$$

$$Q_{12} = \frac{|\bar{y}_1 - \bar{y}_2|}{S_{\bar{y}}}, Q_{13} = \frac{|\bar{y}_1 - \bar{y}_3|}{S_{\bar{y}}}, Q_{23} = \frac{|\bar{y}_2 - \bar{y}_3|}{S_{\bar{y}}} \quad (\text{A3.3})$$

A3.4 Compute the estimate $S_{\bar{y}}^2$ of the variance of the group averages and the corresponding standard deviation $S_{\bar{y}}$ as:

$$s_{\bar{y}}^2 = \frac{1}{9} \sum_{j=1}^3 s_j^2 \text{ and } s_{\bar{y}} = \sqrt{s_{\bar{y}}^2} \quad (\text{A3.2})$$

NOTE A3.1—If Q_{12} , Q_{13} , or Q_{23} exceeds the critical studentized range value 5.05, the compaction vessel is suspect. Recheck the source of the data and the calibration coal used for the vessel before including data to determine the calibration curve. If the source of the high value cannot be reconciled or if the calibration point appears as an outlier as determined in A2.1, the vessel will be void and not used to determine the calibration curve.

A3.5 Compute the three statistics Q_{12} , Q_{13} , and Q_{23} , using

A4. DETERMINATION OF THE CALIBRATION CURVE

A4.1 The data values, which are used for each compaction vessel in establishing the calibration curve, are the mean of the 15 replicate test counts (the probe response) and the compaction vessel density. The compaction vessel density is derived from the volume and the net weight of the coal in the vessel (the bulk density) in accordance with Section 11 (see Table A4.1).

$$Y \text{ Average} = (\Sigma y)/n = (43\,049.0)/15 = 2870 \quad (\text{A4.1})$$

$$\frac{\text{Net Weight, lb}}{\text{Vessel Volume, ft}^3} = \frac{534.0}{8.83} = 60.48 \quad (\text{A4.2})$$

$$SS(y) = \Sigma y^2 - (\Sigma y)^2/n = 7438.4333 \quad (\text{A4.3})$$

$$S_y = SS(y)/n - 1 = 23 \quad (\text{A4.4})$$

Calibration point completion check: passed.

TABLE A4.1 Example Calibration Data

| Calibration Points | Number of Tests | Density, lb/ft ³ x | Test Counts y | x ² | y ² | xy |
|--------------------|-----------------|-------------------------------|---------------|------------------|------------------|--------------|
| 1 | 15 | 60.48 | 2870 | 3657.8304 | 8 236 900 | 173 577.60 |
| 2 | 15 | 65.69 | 2756 | 4315.1761 | 7 595 536 | 181 041.64 |
| 3 | 15 | 69.82 | 2698 | 4874.8324 | 7 279 204 | 188 374.36 |
| 4 | 15 | 69.99 | 2691 | 4898.6001 | 7 241 481 | 188 343.09 |
| 5 | 15 | 70.27 | 2660 | 4937.8729 | 7 075 600 | 186 918.20 |
| 6 | 15 | 71.52 | 2 632 | 5115.1104 | 6 927 424 | 188 240.64 |
| 7 | 15 | 72.42 | 2625 | 5244.6564 | 6 890 625 | 190 102.50 |
| 8 | 15 | 76.44 | 2558 | 5843.0736 | 6 543 364 | 195 533.52 |
| p points | Σ tests | Σ x | Σ y | Σ x ² | Σ y ² | Σ xy |
| 8 | 120 | 556.63 | 21 490 | 38 887.1523 | 57 790 134 | 1 492 131.55 |

A4.2 Plot the calibration point (x_i, y_i) on a scatter diagram (Fig. A4.1), for the i th compaction vessel, letting y_i represent the vessel mean test count and x_i the independently measured coal density. The relationship should appear to be a linear straight line with close correlation. If there is an outlying point that does not appear to correlate with the common linear relationship, recheck the source data and reconcile or identify and justify the cause for the outlier before proceeding.

A4.3 The following straight-line relationship is assumed:

$$y = b_o + b_1 x \quad (\text{A4.5})$$

Where the intercept constant (b_o) and the regression line slope coefficient (b_1) are estimated from the calibration data using the least squares method.

A4.4 Determine the test count mean (y) and the density mean (x) for the calibration points.

$$\bar{y} = \Sigma y_i / p \quad (\text{A4.6})$$

$$\bar{x} = \Sigma x_i / p \quad (\text{A4.7})$$

where:

Σy_i = sum of calibration point test counts,

Σx_i = sum of calibration point density value, and

p = the number of calibration points.

Examples:

$$\bar{y} = \frac{21490}{8} = 2686.2500 \quad (\text{A4.8})$$

$$\bar{x} = \frac{556.63}{8} = 69.5788 \quad (\text{A4.9})$$

A4.5 Determine the sum of squares $S(xx)$, $S(yy)$, and $S(xy)$ and the linear correlation coefficient r .

$$S(xx) = \Sigma x^2 - \frac{(\Sigma x)^2}{p} \quad (\text{A4.10})$$

$$S(yy) = \Sigma y^2 - \frac{(\Sigma y)^2}{p} \quad (\text{A4.11})$$

$$S(xy) = \Sigma xy - \frac{(\Sigma x)(\Sigma y)}{p} \quad (\text{A4.12})$$

$$r = \frac{S(xy)}{\sqrt{S(xx)S(yy)}} \quad (\text{A4.13})$$

where:

Σx^2 = sum of squared values of x ,

$(\Sigma x)^2$ = sum of the values of x squared,

Σy^2 = sum of squared values of y ,

$(\Sigma y)^2$ = sum of the values of y squared,

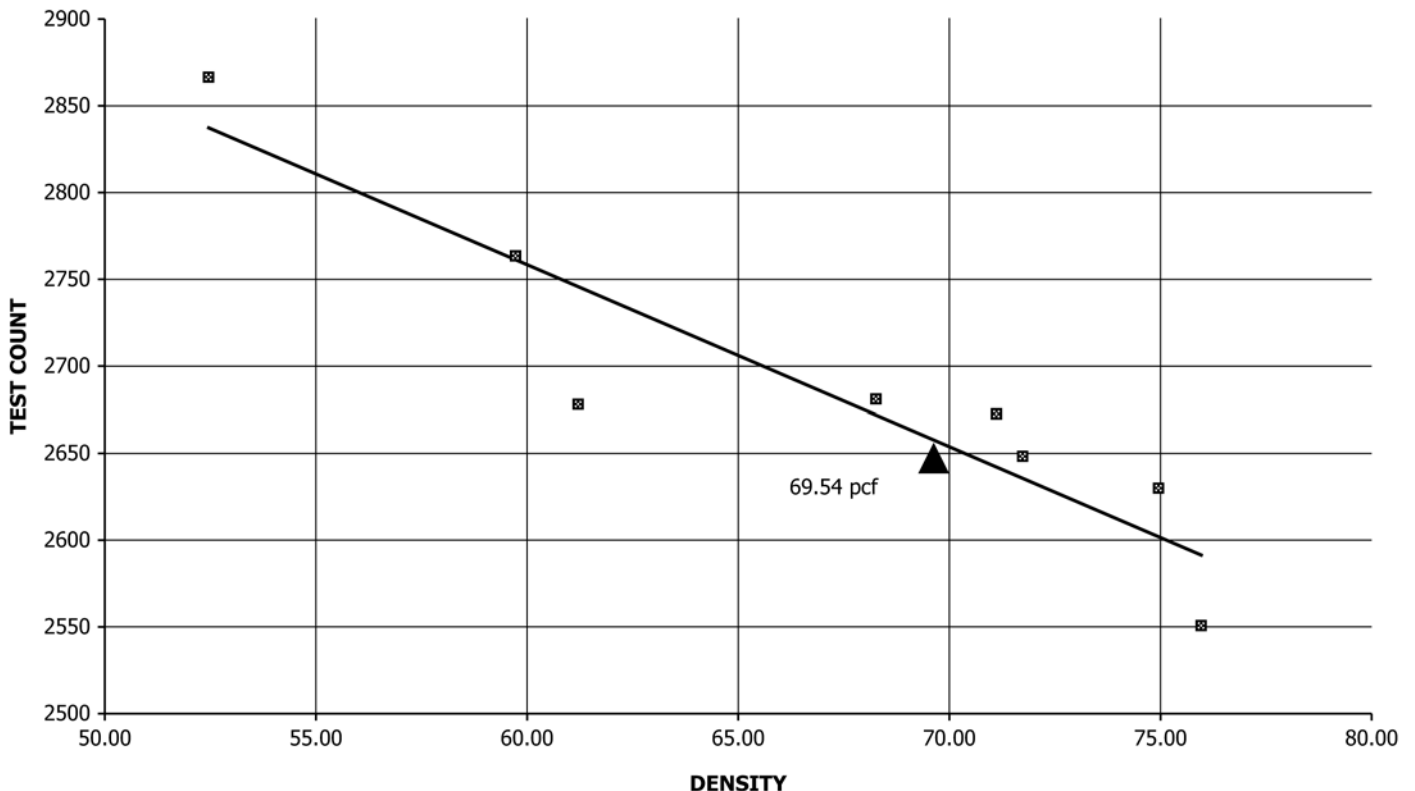
Σxy = sum of the products of x and y , and

p = number of calibration points.

Examples:

$$S(xx) = 38887.1523 - \frac{309836.9569}{8} = 157.5327 \quad (\text{A4.14})$$

$$S(yy) = 57790134 - \frac{461820800}{8} = 62621.5 \quad (\text{A4.15})$$



NOTE 1—See A4.3 and A4.10.

FIG. A4.1 Example Calibration Point Scatter Diagram

$$S(xy) = 1\,492\,131.55 - \frac{(556.63)(21\,490)}{8} = -3115.7875 \quad (\text{A4.16})$$

$$r = \frac{3115.7875}{\sqrt{(157.5327)(62\,621.5)}} = -0.9920 \quad (\text{A4.17})$$

A4.6 Determine the percentage of the variation accounted for by the regression by squaring the correlation coefficient determined in A4.4 and multiplying by 100. This value is expected to exceed 95 % for most calibrations.

$$\text{Percentage } r^2 = 100(r^2) \quad (\text{A4.18})$$

Example:

$$100(0.9841) = 98.41 \% \quad (\text{A4.19})$$

A4.7 Determine the regression line slope coefficient (b_1) and the intercept constant (b_o).

$$b_1 = \frac{S(xy)}{S(xx)} \quad (\text{A4.20})$$

$$b_o = \bar{y} - b_1\bar{x} \quad (\text{A4.21})$$

Example:

$$\begin{aligned} b_1 &= -3115.7875/157.5327 = -19.7787 \\ b_o &= 2686.2500 - (-19.7787 \times 69.5788) = 4062.4253 \end{aligned}$$

A4.8 The regression line $y' = b_o - b_1x$ is the calibration curve. Where y' is the predicted test count, values for the measured densities forming the regression line. Plot this line on the scatter chart. Plot the point (\bar{x}, \bar{y}) determined in A4.3 on the scatter chart. By least squares criterion this point always falls on the regression line.

A4.9 Determine the stockpile mean test count (\bar{y}_o) from A1.1.

A4.10 Determine the stockpile mean density.

$$\bar{x}_o = \frac{\bar{y}_o - b_o}{b_1} \quad (\text{A4.22})$$

Example:

$$\bar{X}_o = 2687.1145 - 4062.4253/-19.7787 = 69.54 \text{ lb/ft}^3$$

A5. DETERMINE THE PRECISION AND THE CONFIDENCE INTERVAL OF THE STOCKPILE MEAN BULK DENSITY

A5.1 Use the Jackknife Method to determine the precision, pcf, at 2 standard deviations and the confidence interval, pcf, and in percentage of the mean bulk density at the 95 % confidence level.

NOTE A5.1—The Jackknife Method assigns the original stockpile test count data to subsets equal to the number (p) of original calibration points. The estimates of density of the subsets average test count values are determined from additional calibrations (1 through (p)) that alternately use $p-1$ of the original calibration points. The variance of these pseudo density values (PV) about the original estimate of the stockpile density is the basis for determining the precision and confidence interval.

A5.2 From Table A1.3, assign all stockpile test counts to subgroups into Table A5.1:

A5.2.1 List test counts by strata in serial order 1 through n , and

A5.2.2 Assign group numbers 1 through p to all test counts.

A5.3 In Table A5.2, list all test counts from Table A5.1 by group numbers 1 through P . Perform the following calculations:

A5.3.1 Determine the test count sum for each subgroup 1 through p ,

A5.3.2 Subtract each subgroup sum from the stockpile test count grand total from Table A1.3,

A5.3.3 Determine the remainder to be the subset total test count, and

A5.3.4 Use these average test counts to estimate density values in pseudo calibrations A5.3.2 through A5.3.p, respectively.

A5.4 List the following values from the pseudo calibrations 2 through 9 from A5.3 in Tables A5.3-A5.10:

A5.4.1 b_1 slope coefficients in column 2 rows 2 through p ,

A5.4.2 b_o Intercept in column 3 rows 2 through p ,

A5.4.3 Subset mean test counts in column 4 rows 2 through p ,

A5.4.4 Pseudo estimates of mean densities in column 5 rows 2 through p , and

A5.4.5 List the original calibration results for the above values in row 1 of Table A5.3 columns 2 through 5.

A5.5 In Table A5.3 column 6, calculate the pseudo density values (PV) for rows 2 through p using the following equation:

$$PV = p - (p - 1) \quad (\text{A5.1})$$

where:

p = original estimated mean density and
 $p - 1$ = estimate of density from pseudo calibration.

A5.5.1 List the square of each pseudo value (PV^2) in column 7.

A5.6 Determine the variance of $PV = S_{PV}^2$ using the following equation:

$$S_{PV}^2 = (\text{Sum } PV^2 - (\text{Sum } PV)^2/p)/p - 1 \quad (\text{A5.2})$$

A5.7 Determine the standard deviation of $PV = S_{PV}$ using the following equation:

$$S_{PV} = (S_{PV}^2)^{1/2} \quad (\text{A5.3})$$

TABLE A5.1 Stockpile Test Count Assignment to Subgroups by Strata

| Serial Number | Group Number | Test Count | Serial Number | Group Number | Test Count | Serial Number | Group Number | Test Count | Serial Number | Group Number | Test Count |
|---------------|--------------|------------|---------------|--------------|------------|---------------|--------------|------------|---------------|--------------|------------|
| 1 | 1 | 2550.0 | 41 | 1 | 2628.5 | 81 | 1 | 2811.5 | 121 | 1 | 2608.0 |
| 2 | 2 | 2433.5 | 42 | 2 | 2786.5 | 82 | 2 | 2781.5 | 122 | 2 | 2578.5 |
| 3 | 3 | 2337.0 | 43 | 3 | 2680.5 | 83 | 3 | 2516.5 | 123 | 3 | 2596.5 |
| 4 | 4 | 2364.5 | 44 | 4 | 2902.0 | 84 | 4 | 2735.0 | 124 | 4 | 2646.5 |
| 5 | 5 | 2372.5 | 45 | 5 | 2770.0 | 85 | 5 | 2756.0 | 125 | 5 | 2800.0 |
| 6 | 6 | 2306.5 | 46 | 6 | 2856.5 | 86 | 6 | 2670.0 | 126 | 6 | 2643.5 |
| 7 | 7 | 2439.0 | 47 | 7 | 2816.5 | 87 | 7 | 2735.5 | 127 | 7 | 2621.0 |
| 8 | 8 | 2439.5 | 48 | 8 | 2731.5 | 88 | 8 | 2569.0 | 128 | 8 | 2640.0 |
| 9 | 1 | 2456.5 | 49 | 1 | 2726.5 | 89 | 1 | 2661.0 | 129 | 1 | 2775.0 |
| 10 | 2 | 2353.0 | 50 | 2 | 2841.0 | 90 | 2 | 2506.0 | 130 | 2 | 2666.5 |
| 11 | 3 | 2465.5 | 51 | 3 | 2757.0 | 91 | 3 | 2474.5 | 131 | 3 | 2727.0 |
| 12 | 4 | 2647.0 | 52 | 4 | 2699.5 | 92 | 4 | 2739.0 | | | |
| 13 | 5 | 2505.0 | 53 | 5 | 2540.0 | 93 | 5 | 2896.0 | | | |
| 14 | 6 | 2994.0 | 54 | 6 | 2752.5 | 94 | 6 | 2863.5 | | | |
| 15 | 7 | 2527.0 | 55 | 7 | 2623.5 | 95 | 7 | 2743.0 | | | |
| 16 | 8 | 2394.0 | 56 | 8 | 2927.5 | 96 | 8 | 2630.0 | | | |
| 17 | 1 | 2780.5 | 57 | 1 | 2848.0 | 97 | 1 | 2705.5 | | | |
| 18 | 2 | 2800.0 | 58 | 2 | 2820.0 | 98 | 2 | 2698.5 | | | |
| 19 | 3 | 2610.5 | 59 | 3 | 2763.5 | 99 | 3 | 2535.0 | | | |
| 20 | 4 | 2499.5 | 60 | 4 | 2585.0 | 100 | 4 | 2506.0 | | | |
| 21 | 5 | 2727.0 | 61 | 5 | 2808.0 | 101 | 5 | 2523.0 | | | |
| 22 | 6 | 2825.0 | 62 | 6 | 2728.5 | 102 | 6 | 2565.5 | | | |
| 23 | 7 | 2778.0 | 63 | 7 | 2820.0 | 103 | 7 | 2774.5 | | | |
| 24 | 8 | 2704.5 | 64 | 8 | 2680.5 | 104 | 8 | 2657.0 | | | |
| 25 | 1 | 2676.0 | 65 | 1 | 2741.5 | 105 | 1 | 2776.5 | | | |
| 26 | 2 | 2962.5 | 66 | 2 | 2691.0 | 106 | 2 | 2829.5 | | | |
| 27 | 3 | 2734.0 | 67 | 3 | 2690.0 | 107 | 3 | 2603.5 | | | |
| 28 | 4 | 2611.0 | 68 | 4 | 2861.0 | 108 | 4 | 2716.0 | | | |
| 29 | 5 | 2713.0 | 69 | 5 | 2784.0 | 109 | 5 | 2666.0 | | | |
| 30 | 6 | 2895.0 | 70 | 6 | 2777.0 | 110 | 6 | 2640.0 | | | |
| 31 | 7 | 2758.5 | 71 | 7 | 2699.0 | 111 | 7 | 2670.0 | | | |
| 32 | 8 | 2942.0 | 72 | 8 | 2876.0 | 112 | 8 | 2541.0 | | | |
| 33 | 1 | 2680.0 | 73 | 1 | 2815.0 | 113 | 1 | 2775.5 | | | |
| 34 | 2 | 2862.5 | 74 | 2 | 2850.5 | 114 | 2 | 2546.5 | | | |
| 35 | 3 | 2845.0 | 75 | 3 | 2808.5 | 115 | 3 | 2691.5 | | | |
| 36 | 4 | 2748.0 | 76 | 4 | 2613.0 | 116 | 4 | 2681.5 | | | |
| 37 | 5 | 2604.0 | 77 | 5 | 2639.5 | 117 | 5 | 2837.5 | | | |
| 38 | 6 | 2877.5 | 78 | 6 | 2566.0 | 118 | 6 | 2903.5 | | | |
| 39 | 7 | 2748.0 | 79 | 7 | 2619.5 | 119 | 7 | 2644.0 | | | |
| 40 | 8 | 2605.5 | 80 | 8 | 2762.0 | 120 | 8 | 2676.0 | | | |

A5.8 Determine the standard error of mean SE_{PV} using the following equation:

$$SE_{PV} = S_{PV}/p^{1/2} \quad (\text{A5.4})$$

A5.9 Precision at 2 standard deviations = $2(SE)$.

A5.10 Calculate the confidence interval (CI) at 95 % confidence level, lb/ft³, at $p - 2$ degrees of freedom, using the following equation:

$$CI = (t \text{ alpha } 2 \text{ at } p - 2)(SE) \quad (\text{A5.5})$$

A5.11 Calculate the confidence interval, lb/ft³, as a percentage of the mean density, lb/ft³, using the following equation:

$$CI = (100 \times CI)/(\text{mean density}) \quad (\text{A5.6})$$

TABLE A5.2 Jackknife—Strata Subsets

| <i>n_j</i> | Subgroup and Subset I.D. and Test Counts | | | | | | | |
|-----------------------------|--|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| | 1 131 | 2 131 | 3 131 | 4 131 | 5 131 | 6 131 | 7 131 | 8 131 |
| Sum TC Subgroup | 352 012 | 352 012 | 352 012 | 352 012 | 352 012 | 352 012 | 352 012 | 352 012 |
| 1 | 2550.0 | 2433.5 | 2337.0 | 2364.5 | 2372.5 | 2306.5 | 2439.0 | 2439.5 |
| 2 | 2456.5 | 2353.0 | 2465.5 | 2647.0 | 2505.0 | 2994.0 | 2527.0 | 2394.0 |
| 3 | 2780.5 | 2800.0 | 2610.5 | 2499.5 | 2727.0 | 2825.0 | 2778.0 | 2704.5 |
| 4 | 2676.0 | 2962.5 | 2734.0 | 2611.0 | 2713.0 | 2895.0 | 2758.5 | 2942.0 |
| 5 | 2680.0 | 2862.5 | 2845.0 | 2748.0 | 2604.0 | 2877.5 | 2748.0 | 2605.5 |
| 6 | 2628.5 | 2786.5 | 2680.5 | 2902.0 | 2770.0 | 2856.5 | 2816.5 | 2731.5 |
| 7 | 2726.5 | 2841.0 | 2757.0 | 2699.5 | 2540.0 | 2752.5 | 2623.5 | 2927.5 |
| 8 | 2848.0 | 2820.0 | 2763.5 | 2585.0 | 2808.0 | 2728.5 | 2820.0 | 2680.5 |
| 9 | 2741.5 | 2691.0 | 2690.0 | 2861.0 | 2784.0 | 2777.0 | 2699.0 | 2876.0 |
| 10 | 2815.0 | 2850.5 | 2808.5 | 2613.0 | 2639.5 | 2566.0 | 2619.5 | 2762.0 |
| 11 | 2811.5 | 2781.5 | 2516.5 | 2735.0 | 2756.0 | 2670.0 | 2735.5 | 2569.0 |
| 12 | 2661.0 | 2506.0 | 2474.5 | 2739.0 | 2896.0 | 2863.5 | 2743.0 | 2630.0 |
| 13 | 2705.5 | 2698.5 | 2535.0 | 2506.0 | 2523.0 | 2565.5 | 2774.5 | 2657.0 |
| 14 | 2776.5 | 2829.5 | 2603.5 | 2716.0 | 2666.0 | 2640.0 | 2670.0 | 2541.0 |
| 15 | 2775.5 | 2546.5 | 2691.5 | 2681.0 | 2837.5 | 2903.5 | 2644.0 | 2676.0 |
| 16 | 2608.0 | 2578.5 | 2596.5 | 2646.5 | 2800.0 | 2643.5 | 2621.0 | 2640.0 |
| 17 | 2775.0 | 2666.5 | 2727.0 | | | | | |
| Sum | 46 015.5 | 46 007.5 | 44 835.0 | 42 554.0 | 42 941.5 | 43 864.5 | 43 017.0 | 42 776.0 |
| Subset TC Sum | 305 996.5 | 306 004.5 | 307 176.0 | 309 458.0 | 309 070.5 | 308 147.5 | 308 995.0 | 309 236.0 |
| Subset <i>n_j</i> | 114 | 114 | 114 | 115 | 115 | 115 | 115 | 115 |
| Subset Avg. TC | 2684.1798 | 2684.2500 | 2694.5263 | 2690.9391 | 2687.5696 | 2679.5435 | 2686.9130 | 2689.0087 |

TABLE A5.3 Stockpile I.D.^A

NOTE 1—Row 1 = original calibration data; rows 2 through 27 = subset data.

PV = number of original calibration points * original estimate of mean density (row 1 column 5) minus *P* - 1 * jackknife estimate of mean densities (column 5 rows 2 through 9).

| Jackknife Calculations | | | Strata | | | |
|------------------------|-------------------|----------------|--------------------|---|--|---|
| 1 | 2 Slope Coeff. | 3 Intercept | 4 Mean Test Ct. | 5 Est. Mean Density, lb/ft ³ | 6 Est. Pseudo Density, <i>PV</i> | 7 <i>PV</i> Squared <i>PV</i> ² |
| 1 | -19.7787 | 4062.4253 | 2687.1145 | 69.5350 | | |
| 2 | -19.1525 | 4017.5017 | 2684.1798 | 69.6161 | 68.9673 | 4756.488 469 29 |
| 3 | -20.0057 | 4079.3711 | 2684.2500 | 69.7362 | 68.1266 | 4641.233 627 56 |
| 4 | -19.8076 | 4062.0769 | 2694.5263 | 69.0417 | 72.9881 | 5327.262 741 61 |
| 5 | -19.8172 | 4063.2604 | 2690.9391 | 69.2491 | 71.5363 | 5117.442 217 69 |
| 6 | -19.7154 | 4059.8244 | 2687.5696 | 69.6033 | 69.0569 | 4768.855 437 61 |
| 7 | -19.5491 | 4048.7814 | 2679.5435 | 70.0409 | 65.9937 | 4355.168 439 69 |
| 8 | -19.6680 | 4055.4931 | 2686.9130 | 69.5840 | 69.1920 | 4787.532 864 00 |
| 9 | -20.3423 | 4100.0265 | 2689.0087 | 69.3636 | 70.7348 | 5003.411 931 04 |
| 10 | | | | | | |
| Sum | | | | | 556.5957 | 38 757.395 728 49 |
| Mean | | | | | 69.5745 | |

^A

$$\begin{aligned}
 \text{Variance } PV &= S^2_{PV} = \frac{\sum PV^2 - (\sum PV)^2}{p - 1} \\
 &= \frac{38\,757.395\,728\,49 - 38\,724.846\,657\,31}{(8 - 1)} = 4.6499 \\
 \text{Std. Dev. } S_{PV} &= \sqrt{S^2_{PV}} = \sqrt{4.649867\,31} = 2.1564 \\
 \text{Std. error of mean } PV(Se) &= S_{PV}/\sqrt{P} \\
 &= 2.156355\,098\,598/\sqrt{8} = 0.7624 \\
 \text{Precision at 2 Std. Dev.} &= 2(Se) = \pm 1.52 \text{ lb/ft}^3 \\
 \text{Confidence interval at 95 \% confidence level in lb/ft}^3 &= t \times 2 \text{ at } p - 2 \text{ degrees of freedom} \times PV(Se) \\
 &= 2.447 \times 0.762\,386\,656\,43 = \pm 1.87 \text{ lb/ft}^3 \\
 \text{Confidence as a percentage of mean density} &= \frac{100 \times \text{confidence interval lb/ft}^3}{\text{mean density lb/ft}^3} = \pm 2.68 \%
 \end{aligned}$$

TABLE A5.4 STOCKPILE I.D. Jackknife Pseudo Cal. Pt. No. 2^{A,B}

| Cal. Pts. | No. Tests | Density, lb/ft x | Test Cts., y | Density Squared, x ² | Test Cts. Squared, y ² | Product x and y, xy |
|----------------|--------------|--|--------------|---------------------------------|-----------------------------------|---------------------|
| 1 | | | | 0 | 0 | 0 |
| 2 | 15 | 65.69 | 2756 | 4315.1761 | 7 595 536 | 181 041.64 |
| 3 | 15 | 69.82 | 2698 | 4874.8324 | 7 279 204 | 188 374.36 |
| 4 | 15 | 69.99 | 2691 | 4898.6001 | 7 241 481 | 188 343.09 |
| 5 | 15 | 70.27 | 2660 | 4937.8729 | 7 075 600 | 186 918.20 |
| 6 | 15 | 71.52 | 2632 | 5115.1104 | 6 927 424 | 188 240.64 |
| 7 | 15 | 72.42 | 2625 | 5244.6564 | 6 890 625 | 190 102.50 |
| 8 | 15 | 76.44 | 2558 | 5843.0736 | 6 543 364 | 195 533.52 |
| Sum | 105 Tests | 496.15 x | 18 620 y | 35 229.3219 x ² | 49 553 234 y ² | 1 318 553.95 xy |
| No. pts. P = 7 | | Density mean = (\bar{x}) = 70.8786 | | Test count mean = (\bar{y}) | | 2660.0000 |

A

$$S(xx) = \sum x^2 - \frac{(\sum x)^2}{p} = 62.9187$$

$$S(yy) = \sum y^2 - \frac{(\sum y)^2}{p} = 24\,034.0000$$

$$S(xy) = \sum xy - \frac{(\sum x)(\sum y)}{p} = -1205.0500$$

$$\text{slope coefficient } b_1 = \frac{S(xy)}{S(xx)} = -19.1525$$

$$\text{intercept } b_0 = \bar{y} - (b_1 * \bar{x}) = 4017.5017$$

subset estimate of stockpile mean test count (\bar{y}_0) = sum of TC/n = 305 996.5/114 = 2684.1798
subset estimate of stockpile mean density (\bar{x}_0) = $\bar{y} - b_0/b_1$ = 69.6161

^BTransfer the following data to Table A5.3 on the row number corresponding to the pseudo calibration point number above:

slope coefficient to column 2,
intercept to column 3,
subset estimate of mean test count to column 4, and
subset estimate of mean density to column 5.

TABLE A5.5 STOCKPILE I.D. Jackknife Pseudo Cal. Pt. No. 3^{A,B}

| Cal. Pts. | No. Tests | Density, lb/ft, x | Test Cts., y | Density Squared, x ² | Test Cts. Squared, y ² | Product x and y, xy |
|----------------|--------------|--|--------------|---------------------------------|-----------------------------------|---------------------|
| 1 | 15 | 60.48 | 2870 | 3 657.8304 | 8 236 900 | 173 577.60 |
| 2 | 15 | | | 0 | 0 | 0.00 |
| 3 | 15 | 69.82 | 2698 | 4 874.8324 | 7 279 204 | 188 374.36 |
| 4 | 15 | 69.99 | 2691 | 4 898.6001 | 7 241 481 | 188 343.09 |
| 5 | 15 | 70.27 | 2660 | 4 937.8729 | 7 075 600 | 186 918.20 |
| 6 | 15 | 71.52 | 2632 | 5 115.1104 | 6 927 424 | 188 240.64 |
| 7 | 15 | 72.42 | 2625 | 5 244.6564 | 6 890 625 | 190 102.50 |
| 8 | 15 | 76.44 | 2558 | 5 843.0736 | 6 543 364 | 195 533.52 |
| Sum | 105 Tests | 490.94 x | 18734 y | 34 571.9762 x ² | 50 194 598 y ² | 1 311 089.91 xy |
| No. pts. P = 7 | | Density mean = (\bar{x}) = 70.1343 | | Test count mean = (\bar{y}) | | 2676.2857 |

A

$$S(xx) = \sum x^2 - \frac{(\sum x)^2}{p} = 140.2500$$

$$S(yy) = \sum y^2 - \frac{(\sum y)^2}{p} = 57\,061.4286$$

$$S(xy) = \sum xy - \frac{(\sum x)(\sum y)}{p} = -2805.7986$$

$$\text{slope coefficient } b_1 = \frac{S(xy)}{S(xx)} = -20.0057$$

$$\text{intercept } b_0 = \bar{y} - (b_1 * \bar{x}) = 4079.3711$$

subset estimate of stockpile mean test count (\bar{y}_0) = sum of TC/n = 306 004.5/114 = 2684.2500
subset estimate of stockpile mean density (\bar{x}_0) = $\bar{y}_0 - b_0/b_1$ = 69.7362

^BTransfer the following data to Table A5.3 on the row number corresponding to the pseudo calibration point number above:

slope coefficient to column 2,
intercept to column 3,
subset estimate of mean test count to column 4, and
subset estimate of mean density to column 5.

TABLE A5.6 STOCKPILE I.D. Jackknife Pseudo Cal. Pt. No. 4^{A,B}

| Cal. Pts. | No. Tests | Density, lb/ft x | Test Cts., y | Density Squared, x ² | Test Cts. Squared, y ² | Product x and y, xy |
|----------------|-----------|--------------------------------------|--------------|---------------------------------|-----------------------------------|---------------------|
| 1 | 15 | 60.48 | 2 870 | 3 657.8304 | 8 236 900 | 173 577.60 |
| 2 | 15 | 65.69 | 2 756 | 4 315.1761 | 7 595 536 | 181 041.64 |
| 3 | 15 | | | 0 | 0 | 0.00 |
| 4 | 15 | 69.99 | 2 691 | 4 898.6001 | 7 241 481 | 188 343.09 |
| 5 | 15 | 70.27 | 2 660 | 4 937.8729 | 7 075 600 | 186 918.20 |
| 6 | 15 | 71.52 | 2 632 | 5 115.1104 | 6 927 424 | 188 240.64 |
| 7 | 15 | 72.42 | 2 625 | 5 244.6564 | 6 890 625 | 190 102.50 |
| 8 | 15 | 76.44 | 2 558 | 5 843.0736 | 6 543 364 | 195 533.52 |
| Sum | 105 | 486.81 | 18 792 | 34 012.3199 | 50 510 930 | 1 303 757.19 |
| | Tests | x | y | x ² | y ² | xy |
| No. pts. P = 7 | | Density mean = $(\bar{x}) = 69.5443$ | | Test count mean = (\bar{y}) | | 2684.5714 |

A

$$S(xx) = \sum x^2 - \frac{(\sum x)^2}{p} = 157.4662$$

$$S(yy) = \sum y^2 - \frac{(\sum y)^2}{p} = 62\,463.7143$$

$$S(xy) = \sum xy - \frac{(\sum x)(\sum y)}{p} = -3119.0271$$

$$\text{slope coefficient } b_1 = \frac{S(xy)}{S(xx)} = -19.8076$$

$$\text{intercept } b_0 = \bar{y} - (b_1 * \bar{x}) = 4062.0769$$

$$\text{subset estimate of stockpile mean test count } (\bar{y}_0) = \text{sum of TC}/n = 307\,176/114 = 2694.5263$$

$$\text{subset estimate of stockpile mean density } (\bar{x}_0) = \bar{y}_0 - b_0/b_1 = 69.0417$$

^BTransfer the following data to Table A5.3 on the row number corresponding to the pseudo calibration point number above:

slope coefficient to column 2,

intercept to column 3,

subset estimate of mean test count to column 4, and

subset estimate of mean density to column 5.

TABLE A5.7 STOCKPILE I.D. Jackknife Pseudo Cal. Pt. No. 5^{A,B}

| Cal. Pts. | No. Tests | Density, lb/ft, x | Test Cts., y | Density Squared, x ² | Test Cts. Squared, y ² | Product x and y, xy |
|----------------|-----------|--------------------------------------|--------------|---------------------------------|-----------------------------------|---------------------|
| 1 | 15 | 60.48 | 2 870 | 3 657.8304 | 8 236 900 | 173 577.60 |
| 2 | 15 | 65.69 | 2 756 | 4 315.1761 | 7 595 536 | 181 041.64 |
| 3 | 15 | 69.82 | 2 698 | 4 874.8324 | 7 279 204 | 188 374.36 |
| 4 | | | | 0 | 0 | 0.00 |
| 5 | 15 | 70.27 | 2 660 | 4 937.8729 | 7 075 600 | 186 918.20 |
| 6 | 15 | 71.52 | 2 632 | 5 115.1104 | 6 927 424 | 188 240.64 |
| 7 | 15 | 72.42 | 2 625 | 5 244.6564 | 6 890 625 | 190 102.50 |
| 8 | 15 | 76.44 | 2 558 | 5 843.0736 | 6 543 364 | 195 533.52 |
| Sum | 105 | 486.64 | 18 799 | 33 988.5522 | 50 548 653 | 1303 788.46 |
| | Tests | x | y | x ² | y ² | xy |
| No. pts. P = 7 | | Density mean = $(\bar{x}) = 69.5200$ | | Test count mean = (\bar{y}) | | 2685.5714 |

A

$$S(xx) = \sum x^2 - \frac{(\sum x)^2}{p} = 157.3394$$

$$S(yy) = \sum y^2 - \frac{(\sum y)^2}{p} = 62\,595.7143$$

$$S(xy) = \sum xy - \frac{(\sum x)(\sum y)}{p} = -3118.0200$$

$$\text{slope coefficient } b_1 = \frac{S(xy)}{S(xx)} = -19.8172$$

$$\text{intercept } b_0 = \bar{y} - (b_1 * \bar{x}) = 4063.2604$$

$$\text{subset estimate of stockpile mean test count } (\bar{y}_0) = \text{sum of TC}/n = 309\,458/115 = 2690.9391$$

$$\text{subset estimate of stockpile mean density } (\bar{x}_0) = \bar{y}_0 - b_0/b_1 = 69.2491$$

^BTransfer the following data to Table A5.3 on the row number corresponding to the pseudo calibration point number above:

slope coefficient to column 2,

intercept to column 3,

subset estimate of mean test count to column 4, and

subset estimate of mean density to column 5.

TABLE A5.8 STOCKPILE I.D. Jackknife Pseudo Cal. Pt. No. 6^{A,B}

| Cal. Pts. | No. Tests | Density, lb/ft, x | Test Cts., y | Density Squared, x^2 | Test Cts. Squared, y^2 | Product x and y , xy |
|------------------|-----------|--------------------------------------|----------------|-------------------------------|--------------------------|----------------------------|
| 1 | 15 | 60.48 | 2 870 | 3 657.8304 | 8 236 900 | 173 577.60 |
| 2 | 15 | 65.69 | 2 756 | 4 315.1761 | 7 595 536 | 181 041.64 |
| 3 | 15 | 69.82 | 2 698 | 4 874.8324 | 7 279 204 | 188 374.36 |
| 4 | 15 | 69.99 | 2 691 | 4 898.6001 | 7 241 481 | 188 343.09 |
| 5 | | | | 0 | 0 | 0.00 |
| 6 | 15 | 71.52 | 2 632 | 5 115.1104 | 6 927 424 | 188 240.64 |
| 7 | 15 | 72.42 | 2 625 | 5 244.6564 | 6 890 625 | 190 102.50 |
| 8 | 15 | 76.44 | 2 558 | 5 843.0736 | 6 543 364 | 195 533.52 |
| Sum | 105 | 486.36 | 18 830 | 33 949.2794 | 50 714 534 | 1 305 213.35 |
| | Tests | x | y | x^2 | y^2 | xy |
| No. pts. $P = 7$ | | Density mean $= (\bar{x}) = 69.4800$ | | Test count mean $= (\bar{y})$ | | 2690.0000 |

A

$$S(xx) = \sum x^2 - \frac{(\sum x)^2}{p} = 156.9866$$

$$S(yy) = \sum y^2 - \frac{(\sum y)^2}{p} = 61\,834.0000$$

$$S(xy) = \sum xy - \frac{(\sum x)(\sum y)}{p} = -3095.0500$$

$$\text{slope coefficient } b_1 = \frac{S(xy)}{S(xx)} = -19.7154$$

$$\text{intercept } b_0 = \bar{y} - (b_1 * \bar{x}) = 4059.8244$$

$$\text{subset estimate of stockpile mean test count } (\bar{y}_0) = \text{sum of TC}/n = 309\,070.5/115 = 2687.5696$$

$$\text{subset estimate of stockpile mean density } (\bar{x}_0) = \bar{y}_0 - b_0/b_1 = 69.6033$$

^BTransfer the following data to Table A5.3 on the row number corresponding to the pseudo calibration point number above:

slope coefficient to column 2,

intercept to column 3,

subset estimate of mean test count to column 4, and

subset estimate of mean density to column 5.

TABLE A5.9 STOCKPILE I.D. Jackknife Pseudo Cal. Pt. No. 7^{A,B}

| Cal. Pts. | No. Tests | Density, lb/ft, x | Test Cts., y | Density Squared, x^2 | Test Cts. Squared, y^2 | Product x and y , xy |
|------------------|-----------|--------------------------------------|----------------|-------------------------------|--------------------------|----------------------------|
| 1 | 15 | 60.48 | 2 870 | 3 657.8304 | 8 236 900 | 173 577.60 |
| 2 | 15 | 65.69 | 2 756 | 4 315.1761 | 7 595 536 | 181 041.64 |
| 3 | 15 | 69.82 | 2 698 | 4 874.8324 | 7 279 204 | 188 374.36 |
| 4 | 15 | 69.99 | 2 691 | 4 898.6001 | 7 241 481 | 188 343.09 |
| 5 | 15 | 70.27 | 2 660 | 4 937.8729 | 7 075 600 | 186 918.20 |
| 6 | | | | 0 | 0 | 0.00 |
| 7 | 15 | 72.42 | 2 625 | 5 244.6564 | 6 890 625 | 190 102.50 |
| 8 | 15 | 76.44 | 2 558 | 5 843.0736 | 6 543 364 | 195 533.52 |
| Sum | 105 | 485.11 | 18 858 | 33 772.0419 | 50 862 710 | 1 303 890.91 |
| | Tests | x | y | x^2 | y^2 | xy |
| No. pts. $P = 7$ | | Density mean $= (\bar{x}) = 69.3014$ | | Test count mean $= (\bar{y})$ | | 2694.0000 |

A

$$S(xx) = \sum x^2 - \frac{(\sum x)^2}{p} = 153.2259$$

$$S(yy) = \sum y^2 - \frac{(\sum y)^2}{p} = 59\,258.0000$$

$$S(xy) = \sum xy - \frac{(\sum x)(\sum y)}{p} = -2995.4300$$

$$\text{slope coefficient } b_1 = \frac{S(xy)}{S(xx)} = -19.5491$$

$$\text{intercept } b_0 = \bar{y} - (b_1 * \bar{x}) = 4048.7814$$

$$\text{subset estimate of stockpile mean test count } (\bar{y}_0) = \text{sum of TC}/n = 308\,147.5/115 = 2679.5435$$

$$\text{subset estimate of stockpile mean density } (\bar{x}_0) = \bar{y}_0 - b_0/b_1 = 70.0409$$

^BTransfer the following data to Table A5.3 on the row number corresponding to the pseudo calibration point number above:

slope coefficient to column 2,

intercept to column 3,

subset estimate of mean test count to column 4, and

subset estimate of mean density to column 5.

TABLE A5.10 STOCKPILE I.D. Jackknife Pseudo Cal. Pt. No. 8^{A,B}

| Cal. Pts. | No. Tests | Density, lb/ft, x | Test Cts., y | Density Squared, x^2 | Test Cts. Squared, y^2 | Product x and y , xy |
|------------------|-----------|--------------------------------------|---------------------|-------------------------------|---------------------------|----------------------------|
| 1 | 15 | 60.48 | 2 870 | 3 657.8304 | 8 236 900 | 173 577.60 |
| 2 | 15 | 65.69 | 2 756 | 4 315.1761 | 7 595 536 | 181 041.64 |
| 3 | 15 | 69.82 | 2 698 | 4 874.8324 | 7 279 204 | 188 374.36 |
| 4 | 15 | 69.99 | 2 691 | 4 898.6001 | 7 241 481 | 188 343.09 |
| 5 | 15 | 70.27 | 2 660 | 4 937.8729 | 7 075 600 | 186 918.20 |
| 6 | 15 | 71.52 | 2 632 | 5 115.1104 | 6 927 424 | 188 240.64 |
| 7 | | | | 0 | 0 | 0.00 |
| 8 | 15 | 76.44 | 2 558 | 5 843.0736 | 6 543 364 | 195 533.52 |
| Sum | 105 Tests | 484.21 \bar{x} | 18 865 \bar{y} | 33 642.4959 \bar{x}^2 | 50 899 509 \bar{y}^2 | 1 302 029.05 \bar{xy} |
| No. pts. $P = 7$ | | Density mean $= (\bar{x}) = 69.1729$ | | Test count mean $= (\bar{y})$ | | 2695.0000 |

A

$$S(xx) = \sum x^2 - \frac{(\sum x)^2}{p} = 148.3067$$

$$S(yy) = \sum y^2 - \frac{(\sum y)^2}{p} = 58\,334.0000$$

$$S(xy) = \sum xy - \frac{(\sum x)(\sum y)}{p} = -2916.9000$$

$$\text{slope coefficient } b_1 = \frac{S(xy)}{S(xx)} = -19.6680$$

$$\text{intercept } b_0 = \bar{y} - (b_1 * \bar{x}) = 4055.4931$$

$$\text{subset estimate of stockpile mean test count } (\bar{y}_0) = \text{sum of TC}/n = 308\,995/115 = 2686.9130$$

$$\text{subset estimate of stockpile mean density } (\bar{x}_0) = \bar{y}_0 - b_0/b_1 = 69.5840$$

^BTransfer the following data to Table A5.3 on the row number corresponding to the pseudo calibration point number above:

slope coefficient to column 2,

intercept to column 3,

subset estimate of mean test count to column 4, and

subset estimate of mean density to column 5.

TABLE A5.11 STOCKPILE I.D. Jackknife Pseudo Cal. Pt. No. 9^{A,B}

| Cal. Pts. | No. Tests | Density, lb/ft, x | Test Cts., y | Density Squared, x^2 | Test Cts. Squared, y^2 | Product x and y , xy |
|------------------|-----------|--------------------------------------|---------------------|-------------------------------|---------------------------|----------------------------|
| 1 | 15 | 60.48 | 2 870 | 3 657.8304 | 8 236 900 | 173 577.60 |
| 2 | 15 | 65.69 | 2 756 | 4 315.1761 | 7 595 536 | 181 041.64 |
| 3 | 15 | 69.82 | 2 698 | 4 874.8324 | 7 279 204 | 188 374.36 |
| 4 | 15 | 69.99 | 2 691 | 4 898.6001 | 7 241 481 | 188 343.09 |
| 5 | 15 | 70.27 | 2 660 | 4 937.8729 | 7 075 600 | 186 918.20 |
| 6 | 15 | 71.52 | 2 632 | 5 115.1104 | 6 927 424 | 188 240.64 |
| 7 | 15 | 72.42 | 2 625 | 5 244.6564 | 6 890 625 | 190 102.50 |
| 8 | | | | 0 | 0 | 0.00 |
| Sum | 105 Tests | 480.19 \bar{x} | 18 932 \bar{y} | 33 044.0787 \bar{x}^2 | 51 246 770 \bar{y}^2 | 1 296 598.03 \bar{xy} |
| No. pts. $P = 7$ | | Density mean $= (\bar{x}) = 68.5986$ | | Test count mean $= (\bar{y})$ | | 2704.5714 |

A

$$S(xx) = \sum x^2 - \frac{(\sum x)^2}{p} = 103.7307$$

$$S(yy) = \sum y^2 - \frac{(\sum y)^2}{p} = 48\,823.7143$$

$$S(xy) = \sum xy - \frac{(\sum x)(\sum y)}{p} = -2110.1243$$

$$\text{slope coefficient } b_1 = \frac{S(xy)}{S(xx)} = -20.3423$$

$$\text{intercept } b_0 = \bar{y} - (b_1 * \bar{x}) = 4100.0265$$

$$\text{subset estimate of stockpile mean test count } (\bar{y}_0) = \text{sum of TC}/n = 309\,236/115 = 2689.0087$$

$$\text{subset estimate of stockpile mean density } (\bar{x}_0) = \bar{y}_0 - b_0/b_1 = 69.3636$$

^BTransfer the following data to Table A5.3 on the row number corresponding to the pseudo calibration point number above:

slope coefficient to column 2,

intercept to column 3,

subset estimate of mean test count to column 4, and

subset estimate of mean density to column 5.

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