Designation: C747 – 16

An American National Standard

Standard Test Method for Moduli of Elasticity and Fundamental Frequencies of Carbon and Graphite Materials by Sonic Resonance[1]

This standard is issued under the fixed designation C747; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

1. Scope*

1.1 This test method covers determination of the dynamic elastic properties of isotropic and near isotropic carbon and graphite materials at ambient temperatures. Specimens of these materials possess specific mechanical resonant frequencies that are determined by the elastic modulus, mass, and geometry of the test specimen. The dynamic elastic properties of a material can therefore be computed if the geometry, mass, and mechanical resonant frequencies of a suitable (rectangular or cylindrical) test specimen of that material can be measured. Dynamic Young's modulus is determined using the resonant frequency in the flexural or longitudinal mode of vibration. The dynamic shear modulus, or modulus of rigidity, is found using torsional resonant vibrations. Dynamic Young's modulus and dynamic shear modulus are used to compute Poisson's ratio.

1.2 This test method determines elastic properties by measuring the fundamental resonant frequency of test specimens of suitable geometry by exciting them mechanically by a singular elastic strike with an impulse tool. Specimen supports, impulse locations, and signal pick-up points are selected to induce and measure specific modes of the transient vibrations. A transducer (for example, contact accelerometer or non-contacting microphone) senses the resulting mechanical vibrations of the specimen and transforms them into electric signals. (See Fig. 1.) The transient signals are analyzed, and the fundamental resonant frequency is isolated and measured by the signal analyzer, which provides a numerical reading that is (or is proportional to) either the frequency or the period of the specimen vibration. The appropriate fundamental resonant frequencies, dimensions, and mass of the specimen are used to calculate dynamic Young's modulus, dynamic shear modulus, and Poisson's ratio. Annex A1 contains an alternative approach using continuous excitation.

1.3 The values stated in SI units are to be regarded as standard. No other units of measurement are included in this standard.

1.4 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:*[2]

- C215 Test Method for Fundamental Transverse, Longitudinal, and Torsional Resonant Frequencies of Concrete Specimens
- C559 Test Method for Bulk Density by Physical Measurements of Manufactured Carbon and Graphite Articles
- C885 Test Method for Young's Modulus of Refractory Shapes by Sonic Resonance
- C1161 Test Method for Flexural Strength of Advanced Ceramics at Ambient Temperature
- E111 Test Method for Young's Modulus, Tangent Modulus, and Chord Modulus
- E177 Practice for Use of the Terms Precision and Bias in ASTM Test Methods
- E228 Test Method for Linear Thermal Expansion of Solid Materials With a Push-Rod Dilatometer
- E691 Practice for Conducting an Interlaboratory Study to Determine the Precision of a Test Method

3. Terminology

3.1 *Definitions:*

3.1.1 *antinodes, n*—two or more locations that have local maximum displacements, called antinodes, in an unconstrained slender rod or bar in resonance. For the fundamental flexure resonance, the antinodes are located at the two ends and the center of the specimen.

3.1.2 *elastic modulus*—the ratio of stress to strain, in the stress range where Hooke's law is valid.

3.1.3 *flexural vibrations, n*—the vibrations that occur when the displacements in a slender rod or bar are in a plane normal to the length dimension.

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[1] This test method is under the jurisdiction of ASTM Committee D02 on Petroleum Products, Liquid Fuels, and Lubricants and is the direct responsibility of Subcommittee D02.F0 on Manufactured Carbon and Graphite Products.

Current edition approved Oct. 1, 2016. Published January 2017. Originally approved in 1974. Last previous edition approved in 2010 as C747 – 93 (2010)$^{\varepsilon 1}$. DOI: 10.1520/C0747-16.

[2] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

***A Summary of Changes section appears at the end of this standard**

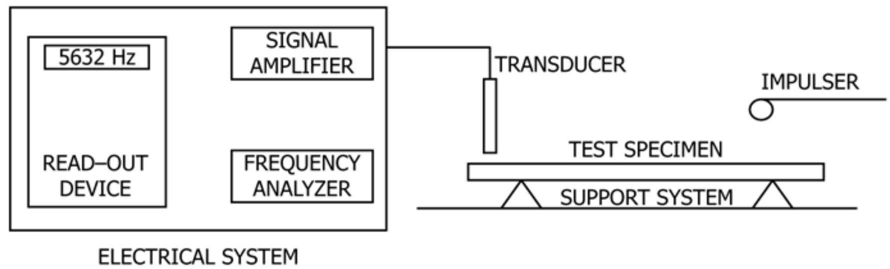


FIG. 1 Block Diagram of Typical Test Apparatus

3.1.4 *homogeneous, adj*—in carbon and graphite technology, the condition of a specimen such that the composition and density are uniform, so that any smaller specimen taken from the original is representative of the whole. Practically, as long as the geometrical dimensions of the test specimen are large with respect to the size of individual grains, crystals, components, pores, or microcracks, the body can be considered homogeneous.

3.1.5 *in-plane flexure, n*—for rectangular parallelepiped geometries, a flexure mode in which the direction of displacement is in the major plane of the test specimen.

3.1.6 *isotropic, adj*—in carbon and graphite technology, having an isotropy ration of 0.9 to 1.1 for a specific property of interest.

3.1.7 *longitudinal vibrations*—when the oscillations in a slender rod or bar are in a plane parallel to the length dimension, the vibrations are said to be in the longitudinal mode.

3.1.8 *nodes, n*—one or more locations in a slender rod or bar in resonance having a constant zero displacement. For the fundamental flexural resonance of such a rod or bar, the nodes are located at 0.224 L from each end, where L is the length of the specimen.

3.1.9 *out-of-plane flexure, n*—for rectangular parallelepiped geometries, a flexure mode in which the direction of displacement is perpendicular to the major plane of the test specimen.

3.1.10 *Poisson's ration (μ), n*—the absolute value of the ratio of transverse strain to the corresponding axial strain resulting from uniformly distributed axial stress below the proportional limit of the material. Young's Modulus (E), shear modulus (G), and Poisson's ratio (μ) are related by the following equation:

$$\mu = (E/2\,G) - 1 \quad (1)$$

3.1.11 *resonant frequency, n*—naturally occurring frequencies of a body driven into flexural, torsional, or longitudinal vibration that are determined by the elastic modulus, mass, and dimensions of the body. The lowest resonant frequency in a given vibrational mode is the fundamental resonant frequency of that mode.

3.1.12 *shear modulus, n*—the elastic modulus in shear or torsion. Also called modulus of rigidity or torsional modulus.

3.1.13 *torsional vibrations, n*—the vibrations that occur when the oscillations in each cross-sectional plane of a slender rod or bar are such that the plane twists around the length dimension axis.

3.1.14 *transverse vibrations, n*—when the oscillations in a slender rod or bar are in a horizontal plane normal to the length dimension, the vibrations are said to be in the transverse mode. This mode is also commonly referred to as the flexural mode when the oscillations are in a vertical plane.

3.1.15 *Young's modulus, n*—the elastic modulus in tension or compression.

4. Summary of Test Method

4.1 This test method measures the fundamental resonant frequency of test specimens of suitable geometry (bar or rod) by exciting them mechanically by a singular elastic strike with an impulse tool. A transducer (for example, contact accelerometer or non-contacting microphone) senses the resulting mechanical vibrations of the specimen and transforms them into electric signals. Specimen supports, impulse locations, and signal pick-up points are selected to induce and measure specific modes of the transient vibrations. The signals are analyzed, and the fundamental resonant frequency is isolated and measured by the signal analyzer, which provides a numerical reading that is (or is proportional to) either the frequency or the period of the specimen vibration. The appropriate fundamental resonant frequencies, dimensions, and mass of the specimen are used to calculate dynamic Young's modulus, dynamic shear modulus, and Poisson's ratio.

5. Significance and Use

5.1 This test method may be used for material development, characterization, design data generation, and quality control purposes.

5.2 This test method is primarily concerned with the room temperature determination of the dynamic moduli of elasticity and rigidity of slender rods or bars composed of homogeneously distributed carbon or graphite particles.

5.3 This test method can be adapted for other materials that are elastic in their initial stress-strain behavior, as defined in Test Method E111.

5.4 This basic test method can be modified to determine elastic moduli behavior at temperatures from –75 °C to +2500 °C. Thin graphite rods may be used to project the specimen extremities into ambient temperature conditions to provide resonant frequency detection by the use of transducers as described in 7.1.

6. Interferences

6.1 The relationships between resonant frequency and dynamic modulus presented herein are specifically applicable to homogeneous, elastic, isotropic materials.

6.1.1 This method of determining the moduli is applicable to inhomogeneous materials only with careful consideration of the effect of inhomogeneities and anisotropy. The character (volume fraction, size, morphology, distribution, orientation, elastic properties, and interfacial bonding) of inhomogeneities in the specimens will have a direct effect on the elastic properties of the specimen as a whole. These effects must be considered in interpreting the test results for composites and inhomogeneous materials.

6.1.2 The procedure involves measuring transient elastic vibrations. Materials with very high damping capacity may be difficult to measure with this technique if the vibration damps out before the frequency counter can measure the signal (commonly within three to five cycles).

6.1.3 If specific surface treatments (coatings, machining,' grinding, etching, etc.) change the elastic properties of the near-surface material, there may be accentuated effects on the properties measured by this flexural method, as compared to static bulk measurements by tensile or compression testing.

6.1.4 The test method is not satisfactory for specimens that have major discontinuities, such as large cracks (internal or surface) or voids.

6.2 This test method for determining moduli is limited to specimens with regular geometries (rectangular parallelepiped and cylinders) for which analytical equations are available to relate geometry, mass, and modulus to the resonant vibration frequencies. The test method is not appropriate for determining the elastic properties of materials that cannot be fabricated into such geometries.

6.2.1 The analytical equations assume parallel and concentric dimensions for the regular geometries of the specimen. Deviations from the specified tolerances for the dimensions of the specimens will change the resonant frequencies and introduce error into the calculations.

6.2.2 Edge treatments such as chamfers or radii are not considered in the analytical equations. Edge chamfers on flexure bars prepared according to Test Method C1161 will change the resonant frequency of the test bars and introduce error into the calculations of the dynamic modulus. It is recommended that specimens for this test method not have chamfered or rounded edges.

6.2.3 For specimens with as-fabricated and rough or uneven surfaces, variations in dimensions can have a significant effect in the calculations. For example, in the calculation of dynamic modulus, the modulus value is inversely proportional to the cube of the thickness. Uniform specimen dimensions and precise measurements are essential for accurate results.

6.3 The test method assumes that the specimen is vibrating freely, with no significant restraint or impediment. Specimen supports should be designed and located properly in accordance with 9.3.1, 9.4.1, and 9.5.1 so the specimen can vibrate freely in the desired mode. In using direct contact transducers, the transducer should be positioned away from antinodes and with minimal force to avoid interference with free vibration. With non-contacting transducers, the maximum sensitivity is accomplished by placing the transducer at an antinode.

6.4 Proper location of the impulse point and transducer is important in introducing and measuring the desired vibration mode. The locations of the impulse point and transducer should not be changed in multiple readings; changes in position may develop and detect alternative vibration modes. In the same manner, the force used in impacting should be consistent in multiple readings.

6.5 If the frequency readings are not repeatable for a specific set of impulse and transducer locations on a specimen, it may be because several different modes of vibration are being developed and detected in the test. The geometry of the test bar and desired vibration mode should be evaluated and used to identify the nodes and antinodes of the desired vibrations. More consistent measurements may be obtained if the impulse point and transducer locations are shifted to induce and measure the single desired mode of vibration.

7. Apparatus

7.1 Apparatus suitable for accurately detecting, analyzing, and measuring the fundamental resonant frequency or period of a vibrating free beam is used. The test apparatus is shown in Fig. 1. It consists of an impulser, a suitable pickup transducer to convert the mechanical vibration into an electrical signal, an electronic system (consisting of a signal conditioner/amplifier, a signal analyzer, and a frequency readout device), and a support system. Commercial instrumentation is available that measures the frequency or period of the vibrating specimen.

7.2 *Impulser*—The exciting impulse is imparted by lightly striking the specimen with a suitable implement. This implement should have most of its mass concentrated at the point of impact and have mass sufficient to induce a measurable mechanical vibration, but not so large as to displace or damage the specimen physically. In practice, the size and geometry of the impulser depends on the size and weightand elastic properties of the specimen and the force needed to produce vibration. For commonly tested geometries (small bars, rods, and discs) in advanced ceramics, an example of such an impulser would be a steel sphere 6 mm in diameter glued to the end of a flexible 10 cm long polymer rod. (See Fig. 2.) An alternative impulser would be a solid metal, ceramic, or

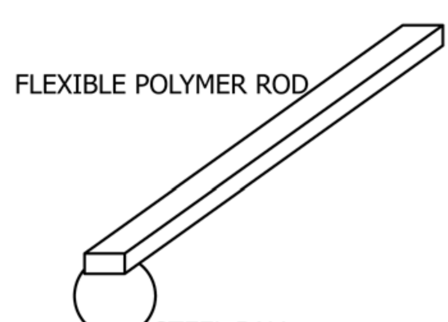


FIG. 2 Diagram of Typical Impulser for Small Specimens

polymer sphere (2 mm to 20 mm in diameter) dropped on the specimen through a guide tube to ensure proper impulse position.

7.3 *Signal Pickup*—Signal detection can be via transducers in direct contact with the specimen or by non-contact transducers. Contact transducers are commonly accelerometers using piezoelectric or strain gage methods to measure the vibration. Non-contact transducers are commonly acoustic microphones, but they may also use laser, magnetic, or capacitance methods to measure the vibration. The frequency range of the transducer shall be sufficient to measure the expected frequencies of the specimens of interest. A suitable range would be 100 Hz to 50 kHz for most graphite test specimens. (Smaller and stiffer specimens vibrate at higher frequencies.) The frequency response of the transducer across the frequency range of interest shall have a bandwidth of at least 10 % of the maximum measured frequency before –3 dB power loss occurs.

7.4 *Electronic System*—The electronic system consists of a signal conditioner/amplifier, signal analyzer, and a frequency readout device. The system should have accuracy and precision sufficient to measure the frequencies of interest to an accuracy of 0.1 %. The signal conditioner/amplifier should be suitable to power the transducer and provide and appropriate amplified signal to the signal analyzer. The signal analysis system consists of a frequency counting device and a readout device. Appropriate devices are frequency counter systems with storage capability or digital storage oscilloscopes with a frequency counter module. With the digital storage oscilloscope, a Fast Fourier Transform signal analysis system may be useful for analyzing more complex waveforms and identifying the fundamental resonant frequency.

7.5 *Support System*—The support shall serve to isolate the specimen from extraneous vibration without restricting the desired mode of specimen vibration. Appropriate materials should be stable at the test temperatures. Support materials can be either soft or rigid for ambient conditions. Examples of soft materials would be a compliant elastomeric material, such as polyurethane foam strips. Such foam strips would have simple flat surfaces for the specimen to rest on. Rigid materials, such as metal or ceramic, should have sharp knife edges or cylindrical surfaces on which the specimen should rest. The rigid supports should be resting on isolation pads to prevent ambient vibrations from being picked up by the transducer. Wire suspension can also be used. Specimens shall be supported along node lines appropriate for the desired vibration in the locations described in Section 9.

8. Test Specimen

8.1 The specimens shall be prepared so that they are either rectangular or circular in cross-section. Either geometry can be used to measure both dynamic Young's modulus and dynamic shear modulus. Although the equations for computing shear modulus with a cylindrical specimen are both simpler and more accurate than those used with a rectangular bar, experimental difficulties in obtaining torsional resonant frequencies for a cylindrical specimen usually preclude its use for determining shear modulus.

8.2 Resonant frequencies for a given specimen are functions of the specimen dimensions as well as its mass and moduli. Dimensions should therefore be selected with this relationship in mind. The selection of size shall be made so that, for an estimated modulus, the resonant frequencies measured will fall within the range of frequency response of the transducers and electronics used. For a slender rod, the ratio of length to minimum cross-sectional dimension should have a value of at least 5. However, a ratio of approximately 10 to 20 is preferred for ease in exciting the fundamental frequency. For shear modulus measurements of rectangular bars, a ratio of width to thickness of 5 or greater is recommended for minimizing experimental difficulties.

8.3 Deviations from the recommended sample ratio range introduce an elevated level of difficulty in obtaining a recorded measurement for fundamental frequency that the operator can have confidence in. For this reason, it is recommended that two additional guidelines be employed as allowable in order to increase the likely accuracy of the frequency being recorded:

8.3.1 Determine the fundamental frequency using specimens that are within the recommended length to width ratio of between 5 and 20, or use progressively larger specimens as necessary, in order to establish baseline frequency characteristics of the material being evaluated. The expected value for fundamental frequency of a non-standard specimen can be calculated based upon the measured geometry and the known fundamental frequency of a standard specimen, and any deviation or shift can be appropriately noted.

8.3.2 Spurious vibration modes are more easily discounted if the test is repeated on the same specimen until ten readings are obtained that lie within ±10 % of the mean. It is acknowledged that for less ideal specimen geometries, the frequency mean that is eventually used for the modulus calculation may require an extended number of measurements until an appropriate group of ten readings is obtained. Operator experience will play a valuable role in the collection of resonant frequency values in non-standard specimen geometries.

8.4 All surfaces on the rectangular specimen shall be flat. Opposite surfaces across the length and width shall be parallel within 0.05 mm or 0.1 %, whichever is greater. Opposite surfaces across the thickness shall be parallel within 0.05 mm or 0.1 %, whichever is greater. The cylindrical specimen shall be round and constant in diameter within 0.05 mm or 0.1 %, whichever is greater.

8.5 Test specimen mass shall be determined within 0.1 % or 10 mg, whichever is greater.

8.6 Test specimen length shall be measured to within 0.025 mm or 0.1 %, whichever is greater. Test specimen cross-sectional dimensions (thickness and width in rectangular beams; diameter in cylindrical rods) shall be measured within 0.1% or 0.025 mm, whichever is greater, at three equally spaced locations along the length and an average value determined.

8.7 Porous materials and those susceptible to hydration should be dried in air at 120 °C in a drying oven until the mass is constant (less than 0.1 % or 10 mg difference in measured mass with 30 min of additional drying).

8.8 It is recommended that the laboratory obtain and maintain an internal reference specimen with known and recorded fundamental resonant frequencies in flexure and torsion. The reference specimen should be used to check and confirm the operation of the test system on a regular basis. It can also be used to train operators in the proper test setup and test procedure. The reference specimen can be a standard ceramic material (alumina, silicon carbide, zirconia, etc.) or it may be of a similar size, composition, and microstructure to the types of specimens commonly tested at the laboratory. The reference specimen must meet the size, dimensional tolerances, and surface finish requirements of Section 8.

9. Procedure

9.1 Activate all electrical equipment and allow it to stabilize according to the manufacturer's recommendations.

9.2 Use a test specimen established as a verification/calibration standard to verify the equipment response and accuracy.

9.3 *Fundamental Flexural Resonance Frequency (Out-of-Plane Flexure):*

9.3.1 Place the specimen on the supports located at the fundamental nodal points (0.224 L from each end; see Fig. 3).

9.3.2 Determine the direction of maximum sensitivity for the transducer. Orient the transducer so that it will detect the desired vibration.

9.3.2.1 *Direct Contact Transducers*—Place the transducer in contact with the test specimen to pick up the desired vibration. If the transducer is placed at an antinode (location of maximum displacement), it may mass load the specimen and modify the natural vibration. The transducer should preferably be placed only as far from the nodal points as necessary to obtain a reading (see Fig. 3). This location will minimize the damping effect from the contacting transducer. The transducer contact force should be consistent, with good response and minimal interference with the free vibration of the specimen.

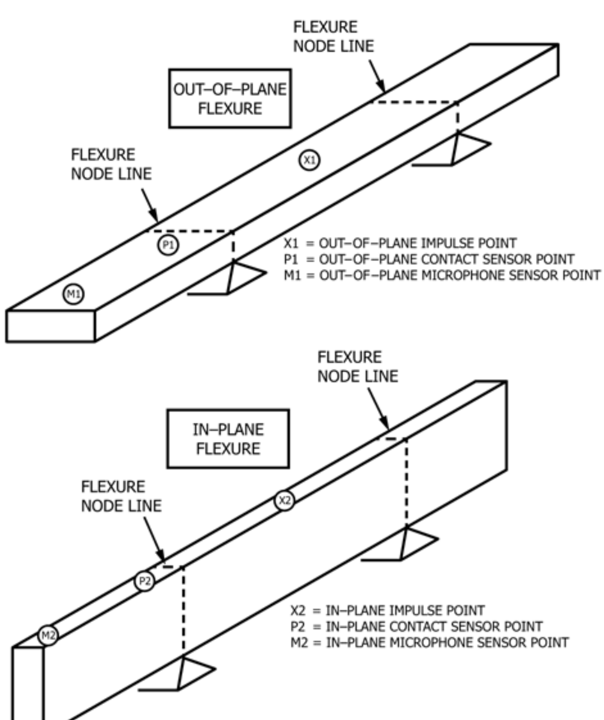


FIG. 3 Rectangular Specimens Tested for In-plane and Out-of-plane Flexure

9.3.2.2 *Non-Contact Transducers*—Place the non-contact transducer over an antinode point and close enough to the test specimen to pick up the desired vibration, but not so close as to interfere with the free vibration (see Fig. 3).

9.3.3 Strike the specimen lightly and elastically, either at the center of the specimen or at the opposite end of the specimen from the detecting transducer (see Fig. 3).

9.3.4 Record the resultant reading, and repeat the test until a recommended ten readings are obtained that lie within 10 % of the mean. The round-robin interlaboratory study (12.2) showed that data points significantly (>10 %) out of range were measurements of spurious vibration modes or secondary harmonics. If ten readings cannot be taken, a minimum of five readings that lie within 10 % of the mean shall be required for estimating the mean. Use the mean of these readings to determine the fundamental resonant frequency in flexure.

9.4 *Fundamental Flexural Resonance Frequency (In-Plane Flexure):*

9.4.1 This procedure is the same as that above (9.3), except that the direction of vibration is in the major plane of the specimen. Rotate the test bar 90° around its long axis and reposition it on the specimen supports (see Fig. 3). Transpose the width and thickness dimensions in the calculations. For homogeneous, isotropic materials, the calculated moduli should be the same as the moduli calculated from the out-of-plane frequency. The comparison of in-plane and out-of-plane frequency measurements can thus be used as a cross check of experimental methods and calculations.

9.5 *Fundamental Torsional Resonance Frequency:*

9.5.1 Support the specimen at the midpoint of its length and width (the torsional nodal planes) (see Fig. 4).

9.5.2 Locate the transducer at one quadrant of the specimen, preferably at approximately 0.224 *L* from one end and toward the edge. This location is a nodal point of flexural vibration and will minimize the possibility of detecting a spurious flexural mode (see Fig. 4).

9.5.3 Strike the specimen on the quadrant diagonally opposite the transducer, again at 0.224 *L* from the end and near the edge. Striking at a flexural nodal point will minimize the possibility of exciting a flexural mode of vibration (see Fig. 4).

9.5.4 Record the resultant reading, and repeat the test until a recommended ten readings are obtained that lie within 10 % of the mean. The round-robin interlaboratory study (12.2) showed that data points significantly (> 10 %) out of range were measurements of spurious vibration modes or secondary harmonics. If ten readings cannot be taken, a minimum of five readings that lie within 10 % of the mean shall be required for estimating the mean. Use the mean of these readings to determine the fundamental resonant frequency in torsion.

9.6 *Longitudinal Fundamental Resonance Frequency*—This procedure is the same as that above (9.3), except that the direction of vibration is longitudinal along the long axis of the

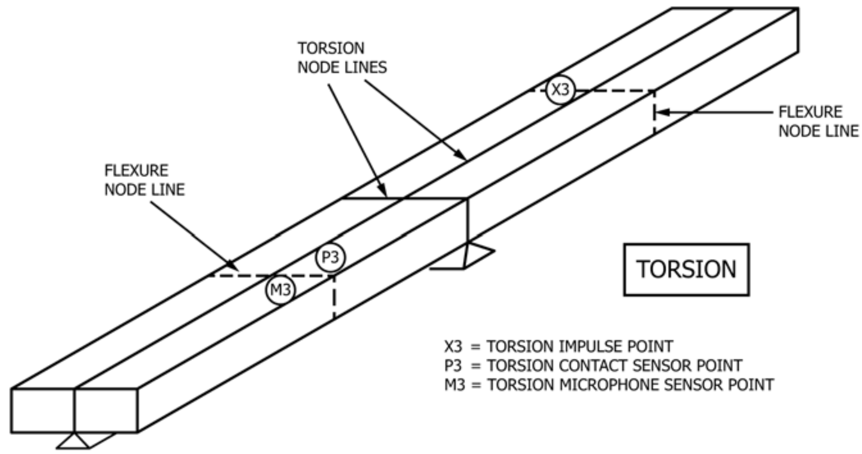


FIG. 4 Rectangular Specimen Tested for Torsional Vibration

specimen. However, the specimen should be supported at locations other than their nodal points, as this will encourage the specimen to vibrate in the flexural mode in addition to the longitudinal mode. Place either a contacting or non-contacting transducer at one end of the specimen and strike the opposite end to induce a vibration parallel to the specimen length. For homogeneous, isotropic materials, the calculated moduli should be the same as the moduli calculated from the in-plane and out-of-plane frequency. A comparison to the in-plane and out-ofplane modulus can thus be used as a cross check of experimental methods and calculations.

10. Calculation

10.1 *Dynamic Young's Modulus*[3,4]*:*

10.1.1 For the fundamental flexure frequency of a rectangular bar,

$$E = 0.9464\left(mf_f^2 / b\right)\left(L^3 / t^3\right)T_1 \quad (2)$$

where:

E = Young's modulus, Pa,
m = mass of the bar, g (see Note 2),
b = width of the bar, mm (see Note 1 and Note 2),
L = length of the bar, mm, (see Note 2),
t = thickness of the bar, mm (see Note 1 and Note 2),
μ = Poisson's Ratio,
f_f = fundamental resonant frequency of bar in flexure, Hz, and
T_1 = correction factor for fundamental flexural mode to account for finite thickness of bar, Poisson's ratio, and so forth.

[3] Spinner, S., Reichard, T. W., and Tefft, W. E., "A Comparison of Experimental and Theoretical Relations Between Young's Modulus and the Flexural and Longitudinal Resonance Frequencies of Uniform Bars," *Journal of Research of the National Bureau of Standards-A. Physics and Chemistry*, Vol 64A, No. 2, March-April, 1960.

[4] Spinner, S., and Tefft, W. E., "A Method for Determining Mechanical Resonance Frequencies and for Calculating Elastic Moduli from These Frequencies," Proceedings, ASTM, 1961, pp. 1221–1238

$$T_1 = 1 + 6.585\left(1 + 0.0752\,\mu + 0.8109\,\mu^2\right)\left(\frac{t}{L}\right)^2 - 0.868\left(\frac{t}{L}\right)^4 - \left[\frac{8.340\left(1 + 0.2023\,\mu + 2.173\,\mu^2\right)\left(\frac{t}{L}\right)^4}{1.000 + 6.338\left(1 + 0.1408\,\mu + 1.536\,\mu^2\right)\left(\frac{t}{L}\right)^2}\right] \quad (3)$$

NOTE 1—The width (b) and thickness (t) values used in the modulus calculations (Eq 2 and Eq 3) for the rectangular specimens will depend on the type of vibration (out-of-plane or in-plane) induced in the specimen. The cross-sectional dimension *t* will always be parallel to the vibrational motion. The dimension *b* will always be perpendicular to the vibrational motion. In effect, the two different flexural modes will give two different fundamental resonant frequencies, but the calculations for the two modes should give the same modulus value, because the values for *b* and *t* are exchanged in the calculations for the two different flexure modes.

NOTE 2—In the modulus equations, the mass and length terms are given in units of grams and millimeters. However, the defined equations can also be used with mass and length terms in units of kilograms and meters with no changes in terms or exponents.

10.1.1.1 If $L / t \geq 10$, T_1 can be simplified to the following:

$$T_1 = 1.000 + 6.585(t / L)^2 \quad (4)$$

and E can be calculated directly.

10.1.1.2 If $L/t < 10$ and Poisson's ratio is known, then T_1, can be calculated directly from Eq 3 and then used to calculate E.

10.1.1.3 If $L/t < 10$ and Poisson's ratio is not known, then an initial Poisson's ratio must be assumed to begin the computations. An iterative process is then used to determine a value of Poisson's ratio, based on experimental Young's modulus and shear modulus. The iterative process is flowcharted in Fig. 5 and described in (*1*) through (*5*) below.

(1) Determine the fundamental flexural and torsional resonant frequency of the rectangular test specimen, as described in Section 9. Using Eq 9 and Eq 10, calculate the dynamic shear modulus of the test specimen for the fundamental torsional resonant frequency.

(2) Using Eq 2 and Eq 3, calculate the dynamic Young's modulus of the rectangular test specimen from the fundamental flexural resonant frequency, dimensions and mass of the specimen, and initial/iterative Poisson's ratio. Care shall be

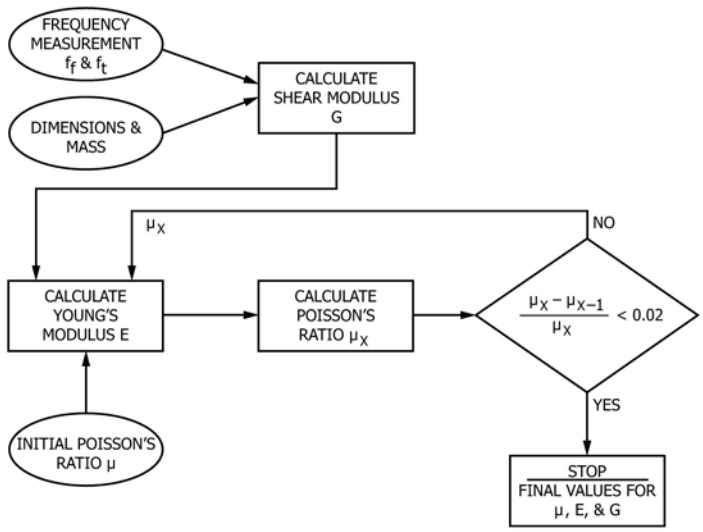


FIG. 5 Process Flowchart for Iterative Determination of Poisson's Ratio

exercised in using consistent units for all of the parameters throughout the computations.

(3) Substitute the dynamic shear modulus and Young's modulus values calculated in steps (*1*) and (*2*) into Eq 12 for Poisson's ratio satisfying isotropic conditions. Calculate a new value for Poisson's ratio for another iteration beginning at Step (*2*).

(4) Repeat Steps (*2*) and (*3*) until no significant difference (2 % or less) is observed between the last iterative value and the final computed value of the Poisson's ratio.

(5) Self-consistent values for the moduli are thus obtained.

10.1.2 For the fundamental flexural frequency of a rod of circular cross-section[3]:

$$E = 1.6067\left(\frac{L^3}{D^4}\right)(mf_f^2)T_1' \tag{5}$$

where:

D = diameter of rod, mm (see Note 2), and
T_1' = correction factor for fundamental flexural mode to account for finite diameter of bar, Poisson's ratio, and so forth.

$$T_1' = 1 + 4.939(1 + 0.0752\,\mu + 0.8109\,\mu^2)\left(\frac{D}{L}\right)^2 - 0.4883\left(\frac{D}{L}\right)^4 - \left[\frac{4.691\left(1 + 0.2023\,\mu + 2.173\,\mu^2\left(\frac{D}{L}\right)^4\right)}{1.000 + 4.754\left(1 + 0.1408\,\mu + 1.536\,\mu^2\left(\frac{D}{L}\right)^2\right)}\right] \tag{6}$$

10.1.2.1 If $L/D \geq 10$, then T_1' can be simplified to the following:

$$T_1' = 1.000 + 4.939(D/L)^2 \tag{7}$$

10.1.2.2 If $L/D < 10$ and Poisson's ratio is known, then T_1' can be calculated directly from Eq 6 and then used to calculate *E*.

10.1.2.3 If $L/D < 10$ and Poisson' a ratio is not known, then an initial Poisson's ratio must be assumed to start the computations. Final values for Poisson's ratio, dynamic Young's modulus, and dynamic shear modulus are determined, using the same method shown in Fig. 5 and described in (*1*) through (*5*) in 10.1.1.3, but using the modulus equations for circular bars (Eq 5, Eq 6, and Eq 7).

10.1.3 For the fundamental longitudinal frequency of a rectangular or circular bar:

$$E = 4.000 f_l^2 L^2 \rho \tag{8}$$

where:

f_l = fundamental resonant frequency of bar vibrating longitudinally, Hz, and
ρ = density of the bar (g/mm^3) (Test Method C559).

10.2 *Dynamic Shear Modulus*[4,5]:

10.2.1 For the fundamental torsional frequency of a rectangular bar.[4]:

$$G = \frac{4Lmf_t^2}{bt}R \tag{9}$$

where:

G = dynamic shear modulus, Pa, and
f_t = fundamental resonant frequency of bar in torsion, Hz.

$$R = \left[\frac{1 + \left(\frac{b}{t}\right)^2}{4 - 2.521\frac{t}{b}\left(1 - \frac{1.991}{e^{\pi\frac{b}{t}} + 1}\right)}\right]\left(1 + \frac{0.00851b^2}{L^2}\right) - 0.060\left(\frac{b}{L}\right)^{\frac{3}{2}}\left(\frac{b}{t} - 1\right)^2 \tag{10}$$

Eq 10 should be accurate to within ~0.2 % for b/L ≤0.3 and b/t ≤10 in the fundamental mode of torsional vibration, otherwise the errors are estimated to be ≤1 %.[4]

10.2.2 For the fundamental torsion frequency of a cylindrical rod[4] :

$$G = 16mf_t^2/(L/\pi D^2) \tag{11}$$

[5] Pickett, G., "Equations for Computing Elastic Constants from Flexural and Torsional Resonant Frequencies of Vibration of Prisms and Cylinders," Proceedings, ASTM, Vol 45, 1945, pp. 846–865.

10.3 *Poisson's Ratio:*

$$\mu = (E/2G) - 1 \quad (12)$$

where:

μ = Poisson's ratio,
E = Young's modulus, and
G = shear modulus.

10.4 If measurements are made at elevated or cryogenic temperatures, the calculated moduli must be corrected for thermal expansion effects using Eq 13.

$$M_T = M_O\left(\frac{f_T}{f_O}\right)^2\left[\frac{1}{(1 + \alpha\,\Delta T)}\right] \quad (13)$$

where:

M_T = modulus at temperature T (either Young's modulus E or shear modulus G),
M_O = modulus at room temperature (either Young's modulus E or shear modulus G),
f_T = resonant frequency in furnace or cryogenic chamber at temperature T,
f_O = resonant frequency at room temperature in furnace or cryogenic chamber,
α = average linear thermal expansion (°C^{-1}) from room temperature to test temperature (Test Method E228 is recommended), and
ΔT = temperature differential in °C between test temperature T and room temperature.

11. Report

11.1 Report the following information:

11.1.1 Identification of specific tests performed, a detailed description of apparatus used (impulser, transducer, electrical system, and support system), and an explanation of any deviations from the described test method.

11.1.2 Description of instrument calibration and functional validation method (refer to 8.8).

11.1.3 Complete identification of the material being tested, including manufacturer, grade number, lot number, grain orientation, and original material size.

11.1.4 Number of specimens tested in each orientation, along with a specimen sampling plan layout.

11.1.5 Specimen dimensions and weight.

11.1.6 Average dynamic modulus for each vibrational mode group.

11.1.7 Standard deviation for each group of specimens.

11.1.8 Environmental conditions of test including temperature, humidity, and special atmosphere (if used).

11.1.9 Date of test and name of the person performing the test.

12. Precision and Bias

12.1 The precision of this test method is based on an interlaboratory study of C747, conducted in 2014. Two materials were analyzed by five participating laboratories. Every test result represents an individual determination, and all labs were asked to report five replicate results. Except for the use of only five laboratories, Practice E691 was followed for the design and analysis of the data; the details are given in the applicable ASTM Research Report.[6]

12.1.1 *Repeatability (r)*—The difference between repetitive results obtained by the same operator in a given laboratory applying the same test method with the same apparatus under constant operating conditions on identical test material within short intervals of time would in the long run, in the normal and correct operation of the test method, exceed the following values only in one case in 20.

12.1.1.1 Repeatability can be interpreted as maximum difference between two results, obtained under repeatability conditions that are accepted as plausible due to random causes under normal and correct operation of the test method

12.1.1.2 Repeatability limits are listed in Tables 1-7.

12.1.2 *Reproducibility (R)*—The difference between two single and independent results obtained by different operators applying the same test method in different laboratories using different apparatus on identical test material would, in the long run, in the normal and correct operation of the test method, exceed the following values only in one case in 20.

12.1.2.1 Reproducibility can be interpreted as maximum difference between two results, obtained under reproducibility conditions that are accepted as plausible due to random causes under normal and correct operation of the test method

12.1.2.2 Reproducibility limits are listed in Tables 1-7.

12.1.3 The above terms (repeatability limit and reproducibility limit) are used as specified in Practice E177.

12.1.4 Any judgment in accordance with 12.1.1 and 12.1.2 would normally have an approximate 95 % probability of being correct, however the precision statistics obtained in this ILS must not be treated as exact mathematical quantities which are applicable to all circumstances and uses. The limited number of materials tested and laboratories reporting results guarantees that there will be times when differences greater than predicted by the ILS results will arise, sometimes with considerably greater or smaller frequency than the 95 % probability limit would imply. The repeatability limit and the reproducibility limit should be considered as general guides,

[6] Supporting data have been filed at ASTM International Headquarters and may be obtained by requesting Research Report RR:D02-1847. Contact ASTM Customer Service at service@astm.org.

TABLE 1 Precision Statistics in Units of GPa for Dynamic Young's Modulus Calculated for a Rectangular Bar L/t ≥ 10 Vibrating in the Flexural Mode (see 10.1.1.1)

Material	Average $\bar{x}$	Repeatability Standard Deviation S_r	Reproducibility Standard Deviation S_R	Repeatability Limit r	Reproducibility Limit R
Graphite A	9.36	0.08	0.10	0.22	0.27
Graphite B	12.27	0.09	0.09	0.24	0.24

TABLE 2 Precision Statistics in Units of GPa for Dynamic Young's Modulus Calculated for a Rectangular Bar L/t ≥ 10 Vibrating in the Flexural Mode (see 10.1.1.3)

Material	Average $\bar{x}$	Repeatability Standard Deviation S_r	Reproducibility Standard Deviation S_R	Repeatability Limit r	Reproducibility Limit R
Graphite A	9.52	0.20	0.36	0.57	1.00
Graphite B	12.41	0.07	0.13	0.20	0.36

TABLE 3 Precision Statistics in Units of GPa for Dynamic Young's Modulus Calculated for a Circular Rod with L/D ≥ 10 Vibrating in the Flexural Mode (see 10.1.2.1)

Material	Average $\bar{x}$	Repeatability Standard Deviation S_r	Reproducibility Standard Deviation S_R	Repeatability Limit r	Reproducibility Limit R
Graphite A	8.98	0.30	0.31	0.84	0.86
Graphite B	12.10	0.08	0.09	0.23	0.26

TABLE 4 Precision Statistics in Units of GPa for Dynamic Young's Modulus Calculated for a Circular Rod with L/D < 10 Vibrating in the Flexural Mode (see 10.1.2.2)

Material	Average $\bar{x}$	Repeatability Standard Deviation S_r	Reproducibility Standard Deviation S_R	Repeatability Limit r	Reproducibility Limit R
Graphite A	8.70	0.04	0.10	0.11	0.29
Graphite B	12.24	0.17	0.17	0.46	0.46

TABLE 5 Precision Statistics in Units of GPa for Dynamic Young's Modulus Calculated for a Rectangular Bar Vibrating in the Longitudinal Mode (see 10.1.3)

Material	Average $\bar{x}$	Repeatability Standard Deviation S_r	Reproducibility Standard Deviation S_R	Repeatability Limit r	Reproducibility Limit R
Graphite A	9.11	0.17	0.17	0.47	0.47
Graphite B	12.09	0.05	0.06	0.15	0.18

TABLE 6 Precision Statistics in Units of GPa for Dynamic Young's Modulus Calculated for a Circular Rod Vibrating in the Longitudinal Mode (see 10.1.3)

Material	Average $\bar{x}$	Repeatability Standard Deviation S_r	Reproducibility Standard Deviation S_R	Repeatability Limit r	Reproducibility Limit R
Graphite A	8.74	0.04	0.07	0.10	0.20
Graphite B	12.33	0.14	0.14	0.38	0.38

TABLE 7 Precision Statistics in Units of GPa for Shear Modulus Calculated for a Rectangular Bar Vibrating in the Torsional Mode (see 10.2.1)

Material	Average $\bar{x}$	Repeatability Standard Deviation S_r	Reproducibility Standard Deviation S_R	Repeatability Limit r	Reproducibility Limit R
Graphite A	3.84	0.17	0.17	0.48	0.48
Graphite B	4.87	0.04	0.19	0.11	0.53

and the associated probability of 95 % as only a rough indicator of what can be expected.

12.2 *Bias*—No reference materials were included in this ILS, therefore no statement on bias is being made.

12.3 The precision statement was determined through statistical examination of all usable data, from five laboratories per analysis, on two different materials, described in Table 8. To judge the equivalency of any two test results, it is recommended to choose the listed material closest in characteristics to the test material.

13. Keywords

13.1 bar; beam; cylindrical rod; disc; dynamic; elastic modulus; elastic properties; flexure; graphite; impulse; Poisson's ratio; resonance; resonant beam; shear modulus; torsion; Young's modulus

TABLE 8 Parameters and Characteristics of the Two Graphite Grades Used in this ILS

Grade	Forming Method	Source Coke	Grain Size (μm)	Ash Content (wppm)	Density (g/cm^3)
Graphite A	iso-static molded	petroleum	20	<10	1.77
Graphite B	vibration molded	coal	300	<10	1.85

ANNEX

(Mandatory Information)

A1. ALTERNATIVE METHOD FOR MODULI OF ELASTICITY AND FUNDAMENTAL FREQUENCIES OF CARBON AND GRAPHITE MATERIALS BY SONIC RESONANCE

A1.1 Scope—This annex covers an alternative method for the measurement of the fundamental transverse, longitudinal, and torsional frequencies of isotropic and anisotropic carbon and graphite materials. These measured resonant frequencies are used to calculate dynamic elastic moduli for any grain orientations.

A1.2 Summary of Test Method—The dynamic methods of determining the elastic moduli are based on the measurement of the fundamental resonant frequencies of a slender rod of circular or rectangular cross-section. The resonant frequencies are related to the specimen dimensions and material properties as follows:

A1.2.1 *Transverse or Flexural Mode*—The equation for the fundamental resonant frequency of the transverse or flexural mode of vibration is as follows:

$$E = CMf^2 \quad \text{(A1.1)}$$

where:

E = elastic modulus, Pa,
C = a dimensional constant that depends upon the shape and size of the specimen, and Poisson's ratio. The units of C are to be consistent with those of E, M, and f,
M = mass of the specimen, kg, and
f = frequency of fundamental transverse or flexural mode of vibration, Hz.

A1.2.2 *Longitudinal Mode*—The equation for the fundamental resonant frequency of the longitudinal mode of variation is as follows:

$$E = Df^2L^2\rho \quad \text{(A1.2)}$$

where:

E = elastic modulus, Pa
D = a constant consistent with the units of E, f, and L,
f = frequency of fundamental longitudinal mode of vibration, Hz,
L = length of the specimen, m, and
ρ = density of the specimen as determined by Test Method C559, kg/m^3.

A1.2.3 *Torsional Mode*—The equation for the fundamental resonant frequency of the torsional mode of vibration is as follows:

$$G = RBf^2L^2\rho \quad \text{(A1.3)}$$

where:

G = modulus of rigidity, Pa,
R = ratio of the polar moment of inertia to the shape factor for torsional rigidity,
B = a constant consistent with the units of G, R, f, L, and p,
f = frequency of fundamental torsional mode of vibration, Hz,
L = length of the specimen, m, and
ρ = density of the specimen as determined by Test Method C559, kg/m^3.

A1.3 Significance and Use

A1.3.1 This test method is primarily concerned with the room temperature determination of the dynamic moduli of elasticity and rigidity of slender rods or bars composed of homogeneously distributed carbon or graphite particles.

A1.3.2 This test method can be adapted for other materials that are elastic in their initial stress-strain behavior, as defined in Test Method E111.

A1.3.3 This basic test method can be modified to determine elastic moduli behavior at temperatures from –75°C to +2500 °C. Thin graphite rods may be used to project the specimen extremities into ambient temperature conditions to provide resonant frequency detection by the use of transducers as described in A1.4.

A1.4 Apparatus—The fundamental resonant frequencies for the different modes of vibration of a test specimen can be determined by several established testing procedures. The apparatus described herein uses phonograph record pickup cartridges as a convenient method of generating and detecting these frequencies. A typical testing apparatus is shown schematically in Fig. A1.1.

A1.4.1 *Driving Circuit*—The driving circuit consists of a variable-frequency oscillator and a record pickup cartridge assembly. It is recommended that a variable-frequency oscillator be used in conjunction with a digital-frequency counter. The oscillator shall have sufficient power output to induce detectable vibrations in the test specimen at frequencies above and below the fundamental frequency under consideration. Means for controlling the output of the oscillator shall be provided. The vibrating needle of the driving unit shall be small in mass as compared to the test specimen, and a means shall be provided to maintain a minimal contact pressure on the

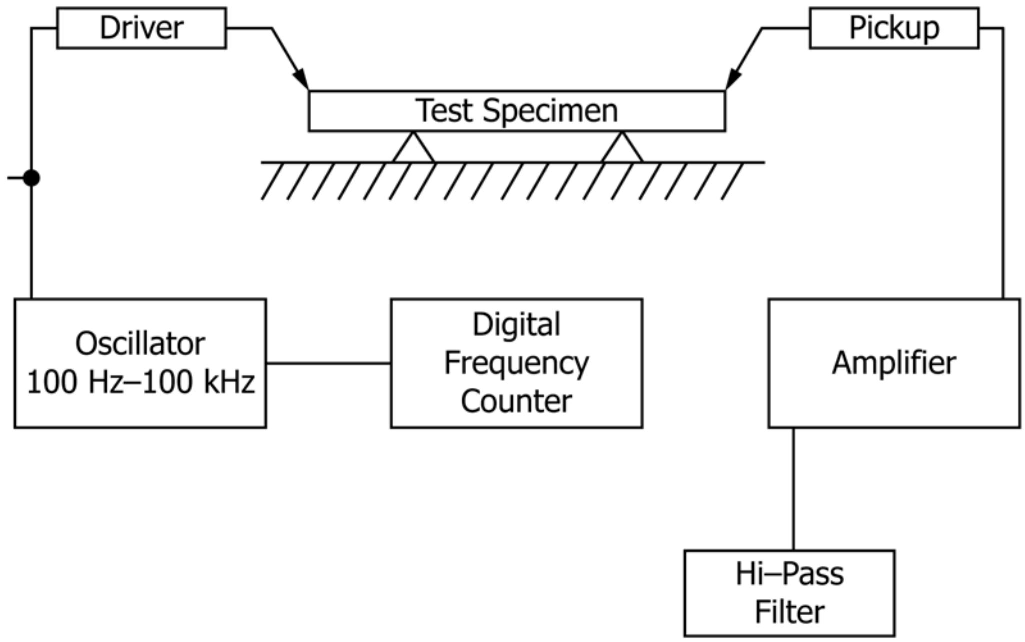


FIG. A1.1 Schematic Diagram of Typical Dynamic Elastic Modulus Detection Apparatus

specimen. Either a piezoelectric or magnetic driving unit meeting these requirements may be used.

A1.4.2 *Pickup Circuit*—The pickup circuit consists of a record pickup cartridge, amplifier, optional high-pass filter, and an indicating meter or cathode-ray oscilloscope. The pickup unit shall generate a voltage proportional to the amplitude, velocity, or acceleration of the test specimen. Either a piezoelectric or magnetic pickup unit meeting these conditions may be used. The amplifier shall have a controllable output of sufficient magnitude to sharply peak out the resonant frequencies on the indicating meter or the cathode-ray oscilloscope display tube. It may be necessary to use a high-pass filter in order to reduce room noise and spurious vibrations. The indicating meter may be a voltmeter, micro ammeter or oscilloscope. An oscilloscope is recommended because it enables the operator to positively identify resonances, including higher order harmonics, by Lissajous figure analysis.

A1.4.3 *Specimen Supports*—The supports shall permit the specimen to oscillate without significant restriction in the desired mode. This is accomplished for all modes by supporting the specimen at its transverse fundamental nodal points (0.224 *L* from each end). The supports should have minimal area in contact with the specimen and shall be of cork, rubber, or similar material. In order to properly identify resonant frequencies, the receiver record pickup cartridge must be movable along the total specimen length. Provisions shall be made to adjust contact pressures of both record pickup cartridges in order to accommodate specimen size variations. The entire specimen support structure shall be mounted on a massive base plate resting on vibration isolators.

A1.5 Test Specimens

A1.5.1 *Selection and Preparation of Specimens*—In the selection and preparation of test specimens, take special care to obtain representative specimens that are straight, uniform in cross-section, and free of extraneous liquids.

A1.5.2 *Measurement of Weight and Dimensions*—Determine the weight and the average length of the specimens within ±0.5 %. Determine average specimen cross-sectional dimension within ±1 %.

A1.5.3 *Limitations on Dimensional Ratio*—Specimens having either very small or very large ratios of length to thickness may be difficult to excite in the fundamental modes of vibration. For this method, the ratio must be between 5 and 20.

A1.6 Procedure

A1.6.1 Switch on all electrical equipment and allow to stabilize in accordance with the manufacturers' recommendations. (Use of a metal bar as a calibration standard is recommended to check equipment response and accuracy. Dimensional measurements and weight shall meet the requirements of A1.5.2.)

A1.6.2 *Transverse Fundamental Resonance Frequency:*

A1.6.2.1 Place the specimen on the supports, which are located at the fundamental transverse nodal points (0.224 *L* from each end). Place the driving and pickup-unit vibrating needles on the specimen center line at its extreme opposite ends with a minimal contact pressure consistent with good response. The vibrating direction of the driving and pickup needles must be perpendicular to the length of the specimen (Fig. A1.2(b)).

A1.6.2.2 Force the test specimen to vibrate at various frequencies and simultaneously observe the amplified output on an indicating meter or oscilloscope. Record the frequency of vibration of the specimen that results in a maximum displacement, having a well-defined peak on the indicator, where nodal point tracking indicates fundamental transverse resonance.

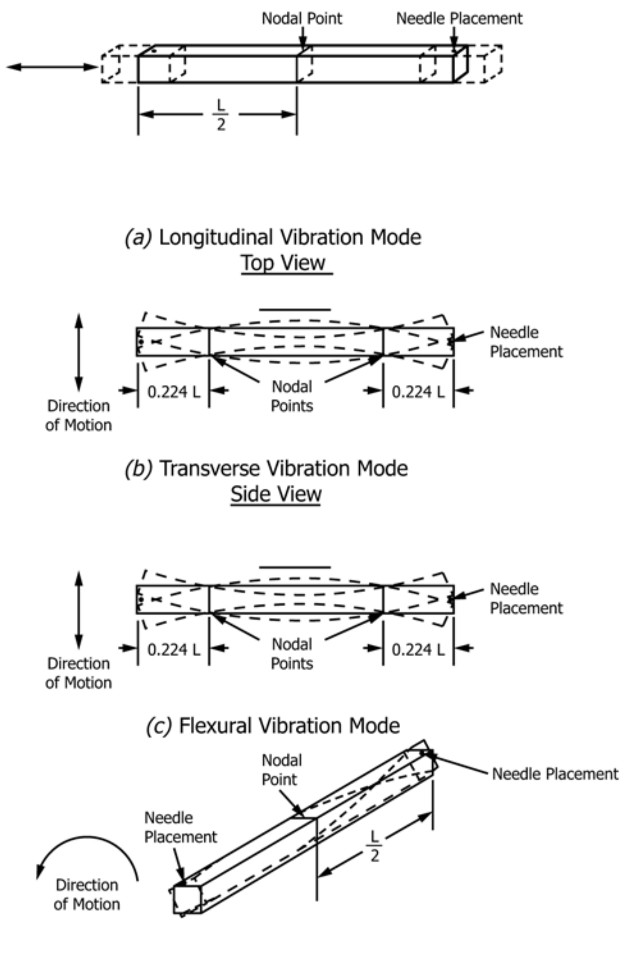


FIG. A1.2 Resonance Modes

TABLE A1.1 Nodal Points Dynamic Modulus Tracking Guide

Vibrational Mode	Harmonic	Nodal Location (Distance from Either End in Fraction of Total Length)
	1F	0.224
	2F	0.132, 0.500
	3F	0.0944, 0.3558
Transverse or Flexural	4F	0.0733, 0.277, 0.500
	5F	0.0600, 0.266, 0.0=409
	6F	0.0508, 0.192, 0.346, 0.500
	1T	0.500
	2T	0.250
Torsional	3T	0.167, 0.500
	4T	0.125, 0.375
	5T	0.100, 0.300, 0.500
	1L	0.500
Longitudinal	2L	0.250
	3L	0.167, 0.500

A1.6.2.3 A basic understanding of Lissajous patterns as displayed on an oscilloscope cathode ray tube (CRT), will aid in the proper identification of the modes of vibration and harmonic frequencies observed. As the oscillator frequency level is increased from a point well below expected resonance, a single closed loop Lissajous pattern tilted from the horizontal reference plane, will eventually be displayed on the CRT. This pattern denotes a resonance mode. The nodal points dynamic modulus tracking guide (Table A1.1) may be used to identify any resonant mode.

A1.6.2.4 Move the pickup cartridge needle slowly toward the specimen center and observe the Lissajous pattern loop. Fundamental transverse resonance is indicated when the following conditions prevail:

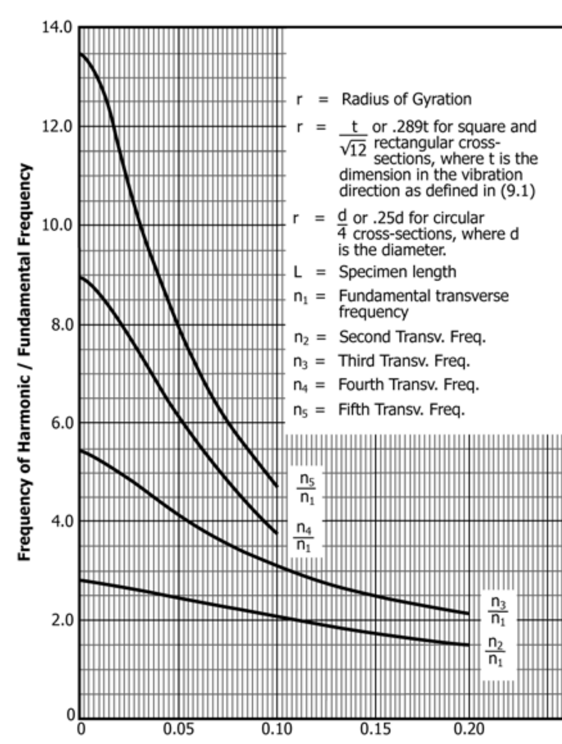
(1) The loop pattern flattens to a horizontal line with the pickup needle over the specimen support.

(2) The CRT pattern opens up to a full loop in a direction normal to its original direction, with the pickup needle over the specimen center.

A1.6.2.5 Return the pickup needle to its original position at the specimen end.

A1.6.2.6 Spurious resonating frequency modes may mask or attenuate the fundamental transverse frequency indication. Investigation of higher order harmonic resonating frequencies by use of the tracking guide (Table A1.1) will help to identify the correct fundamental frequency mode. A plot of the ratio of harmonic to fundamental frequency for transverse mode of vibration (Fig. A1.3) may then be used to calculate the fundamental transverse resonant frequency mode.

A1.6.3 *Longitudinal Fundamental Resonance Frequency:*



NOTE 1—Taken from Pickett, Gerald, "Equations for Computing Elastic Constants from Flexural and Torsional Resonant Frequencies of Vibration of Prisms and Cylinders," Proceedings, ASTM, Vol 45, 1945.

FIG. A1.3 Ratio of Harmonic to Fundamental Frequency for Transverse Mode of Vibration

A1.6.3.1 Leave the specimen supported at the fundamental transverse mode nodal points as in A1.6.2.1. Rotate the driving unit and pickup cartridge needles so as to induce vibrations parallel to the specimen length (Fig. A1.2(a)).

A1.6.3.2 Force the test specimen to vibrate as in A1.6.2.2. Record the frequency of vibration of the test specimen, where nodal point tracking indicates fundamental longitudinal resonance. The second harmonic longitudinal resonant frequency is twice the fundamental longitudinal resonant frequency.

A1.6.4 *Torsional Fundamental Resonance Frequency:*

A1.6.4.1 Leave the specimen supported as in A1.6.2.1. Rotate the driving unit and pickup cartridge needles so as to induce vibrations perpendicular to the length of the sample (Fig. Fig. A1.2(d)).

A1.6.4.2 Force the specimen to vibrate as in A1.6.2.2. Record the frequency of vibration of the test specimen, where nodal point tracking indicates fundamental torsional resonance. The second harmonic torsional resonant frequency is twice the fundamental torsional resonant frequency.

A1.7 Calculation

A1.7.1 Calculate the dynamic modulus of elasticity for the transverse or flexural mode of vibration from the fundamental transverse frequency, weight, and dimensions of the test specimen as follows:

$$\text{Dynamic } E = CMf^2 \tag{A1.4}$$

where units are as defined in A1.2.1. The evaluation of the constant C, because of the complexity of its determination, is in tabular form. Eq A1.4 may be rewritten in the forms:

$$\text{Dynamic } E(\text{Pascals}) = A_C\, M\, f^2\, L\, d \quad \text{for rods with circular cross-sections} \tag{A1.5}$$

where d is the diameter of the rod in meters, and

$$\text{Dynamic E (Pascals)} = ARMf^2/w \quad \text{for bars with square or rectangular cross-sections} \tag{A1.6}$$

where w is the width dimension of the bar in meters.

A1.7.1.1 Values of A_c and A_R are shown in Table A1.2 and Table A1.3. The value of A_c is given as a function of the diameter-to-length ratio of the sample. The value of A_R is given as a function of the ratio of the dimension in the direction of vibration, t, to the length. The dimension, w, is perpendicular to the vibration direction, as shown in Fig. A1.4. Table A1.2 and Table A1.3 have been calculated for three values of Poisson's ratio (μ), The value of (μ) = 0.167 is normally used for carbon-graphite materials.

A1.7.2 The dynamic modulus of elasticity in Pascals may also be calculated from the fundamental longitudinal frequency, weight, and dimensions of the test specimen as follows:

$$\text{Dynamic } E = 4.000 f^2 L^2 \rho \quad \text{for rods and bars} \tag{A1.7}$$

A1.7 where the units are as defined in A1.2.2.

A1.7.3 Calculate the dynamic modulus of rigidity (shear modulus) in Pascals from the fundamental torsional frequency, mass, and dimensions of the test specimen as follows:

$$\text{Dynamic } G = RBf^2L^2\rho \tag{A1.8}$$

where the units are as defined in A1.2.3.

A1.7.3.1 The value of R is equal to 1 for a rod of circular cross-section. R for bars of square cross-section is 1.183. An approximate expression for R in the case of rectangular cross-sections is as follows (see Test Methods C215 and C885):

$$R = \frac{[1 + (a^2/b^2)]}{[4 - 2.52\,(b/a) + 0.21(b/a)^5]} \tag{A1.9}$$

where:

a = large dimension of the rectangular cross-section, and
b = small dimension of the rectangular cross-section.

A1.7.3.2 Eq A1.8 may be rewritten as follows:

$$\text{Dynamic G} = 4.000 f^2 L^2 \rho \quad \text{for circular cylinders} \tag{A1.10}$$

$$\text{Dynamic G} = 4.732 f^2 L^2 \rho \quad \text{for square cross-sections} \tag{A1.11}$$

$$\text{Dynamic G} = 4.732 R\, f^2 L^2 \rho \quad \text{for rectangular cross-sections} \tag{A1.12}$$

A1.8 Report—Report the following information:

A1.8.1 Complete identification of the material being tested, including manufacturer, grade number, lot number, grain orientation, and original material size.

A1.8.2 Number of specimens tested in each orientation, along with a specimen sampling plan layout; Specimen dimensions and weight;.

A1.8.3 Average dynamic modulus for each mode group.

A1.8.4 Standard deviation for each group of specimens; and Environmental conditions of test including temperature, humidity, and special atmosphere (if used).

A1.9 Precision and Bias

A1.9.1 There is no precision and bias data available for this alternative method.

TABLE A1.2 A_c for Rods

d/L	μ = 0	μ = 1⁄6	μ = 1⁄3	*d/L*	μ = 0	μ = 1⁄6	μ = 1⁄3
0.050	13018	13023	13035	0.108	1349	1351	1356
0.051	12273	12278	12291	0.109	1313	1316	1321
0.052	11585	11590	11603	0.110	1280	1283	1288
0.053	10945	10950	10963	0.111	1247	1247	1255
0.054	10356	10358	10371	0.112	1214	1217	1222
0.055	9807	9812	9822	0.113	1184	1186	1191
0.056	9294	9299	9309	0.114	1153	1156	1161
0.057	8819	8824	8834	0.115	1125	1128	1133
0.058	8374	8379	8390	0.116	1097	1100	1105
0.059	7960	7965	7976	0.117	1069	1072	1077
0.060	7574	7577	7590	0.118	1044	1046	1052
0.061	7211	7216	7226	0.119	1019	1021	1026
0.062	6873	6878	6886	0.120	996	998	1003
0.063	6553	6558	6568	0.121	973	975	980
0.064	6256	6259	6269	0.122	950	953	958
0.065	5977	5979	5989	0.123	927	930	935
0.066	5710	5715	5725	0.124	907	909	914
0.067	5464	5466	5476	0.125	886	889	892
0.068	5227	5232	5243	0.126	866	869	874
0.069	5006	5011	5022	0.127	846	848	853
0.070	4798	4803	4813	0.128	828	831	833
0.071	4602	4605	4615	0.129	810	810	815
0.072	4417	4420	4430	0.130	792	792	798
0.073	4239	4244	4252	0.131	775	777	782
0.074	4074	4077	4087	0.132	757	759	765
0.075	3917	3919	3929	0.133	742	744	749
0.076	3764	3769	3777	0.134	726	729	734
0.077	3625	3627	3635	0.135	711	714	716
0.078	3487	3493	3500	0.136	696	699	704
0.079	3360	3366	3371	0.137	681	683	688
0.080	3239	3241	3249	0.138	668	671	673
0.081	3122	3127	3134	0.139	655	655	660
0.082	3012	3015	3023	0.140	643	643	648
0.083	2906	2908	2918	0.141	630	630	635
0.084	2807	2809	2817	0.142	617	617	622
0.085	2710	2713	2720	0.143	605	607	610
0.086	2619	2621	2629	0.144	592	594	599
0.087	2532	2535	2540	0.145	582	582	587
0.088	2449	2451	2459	0.146	569	572	577
0.089	2367	2372	2377	0.147	561	561	564
0.090	2294	2296	2304	0.148	549	551	554
0.091	2220	2223	2230	0.149	538	541	544
0.092	2151	2154	2159	0.150	528	531	533
0.093	2083	2085	2093	0.151	518	521	523
0.094	2019	2022	2027	0.152	508	511	516
0.095	1958	1961	1966	0.153	500	500	505
0.096	1897	1902	1910	0.154	490	493	495
0.097	1842	1847	1852	0.155	483	483	488
0.098	1788	1791	1796	0.156	472	475	478
0.099	1735	1740	1745	0.157	465	467	470
0.100	1687	1689	1694	0.158	457	457	462
0.101	1638	1641	1646	0.159	450	450	455
0.102	1593	1595	1600	0.160	442	442	447
0.103	1547	1549	1557	0.161	434	434	439
0.104	1504	1506	1514	0.162	427	427	432
0.105	1463	1466	1471	0.163	419	419	424
0.106	1422	1427	1433	0.164	411	414	417
0.107	1384	1387	1394	0.165	404	406	409
0.166	399	399	404	0.184	300	302	305
0.167	391	394	396	0.185	295	297	300
0.168	386	386	391	0.186	292	292	295
0.169	378	381	384	0.187	287	287	292
0.170	373	373	378	0.188	282	284	287
0.171	366	368	371	0.189	277	279	282
0.172	361	363	366	0.190	274	277	279
0.173	356	356	361	0.191	272	272	274
0.174	351	353	356	0.192	267	269	272
0.175	343	345	348	0.193	264	264	267
0.176	338	340	343	0.194	259	262	264
0.177	333	335	338	0.195	257	257	262
0.178	328	330	333	0.196	251	254	257
0.179	323	325	328	0.197	249	251	254
0.180	318	320	323	0.198	246	246	249
0.181	312	315	318	0.199	241	244	246
0.182	310	310	312	0.200	239	241	244
0.183	305	305	310				

TABLE A1.3 A_R for Bars

t/L	μ = 0	μ = ⅙	μ = ⅓	t/L	μ = 0	μ = ⅙	μ = ⅓
0.050	7701	7706	7717	0.110	767	770	775
0.051	7262	7267	7277	0.111	747	749	754
0.052	6855	6861	6871	0.112	728	732	734
0.053	6480	6485	6492	0.113	709	711	716
0.054	6129	6134	6144	0.114	693	696	699
0.055	5806	5809	5819	0.115	676	678	683
0.056	5504	5507	5517	0.116	660	660	665
0.057	5225	5227	5237	0.117	643	645	650
0.058	4961	4966	4973	0.118	627	630	635
0.059	4717	4722	4729	0.119	615	615	620
0.060	4488	4493	4501	0.120	599	602	605
0.061	4275	4277	4285	0.121	584	587	592
0.062	4074	4077	4087	0.122	572	574	577
0.063	3886	3889	3899	0.123	559	561	564
0.064	3708	3713	3721	0.124	546	549	551
0.065	3546	3548	3556	0.125	533	536	538
0.066	3388	3393	3399	0.126	523	523	528
0.067	3241	3246	3251	0.127	511	513	516
0.068	3104	3106	3114	0.128	500	500	509
0.069	2974	2979	2985	0.129	488	490	493
0.070	2850	2852	2860	0.130	478	480	483
0.071	2733	2738	2743	0.131	467	470	472
0.072	2624	2626	2634	0.132	457	460	462
0.073	2520	2522	2530	0.133	450	450	455
0.074	2421	2423	2431	0.134	439	439	445
0.075	2327	2332	2337	0.135	429	432	434
0.076	2240	2243	2248	0.136	422	424	427
0.077	2156	2159	2164	0.137	414	414	419
0.078	2075	2078	2085	0.138	406	406	409
0.079	1999	2002	2009	0.139	396	399	401
0.080	1925	1930	1935	0.140	389	391	394
0.081	1859	1862	1867	0.141	381	384	386
0.082	1793	1796	1801	0.142	373	376	378
0.083	1730	1735	1740	0.143	366	368	371
0.084	1671	1674	1679	0.144	358	361	363
0.085	1615	1618	1623	0.145	353	353	358
0.086	1560	1565	1570	0.146	345	348	351
0.087	1509	1511	1516	0.147	340	340	343
0.088	1461	1463	1468	0.148	333	335	338
0.089	1412	1415	1420	0.149	328	328	333
0.090	1367	1369	1377	0.150	320	323	325
0.091	1323	1328	1334	0.151	315	318	320
0.092	1283	1285	1290	0.152	310	310	315
0.093	1245	1247	1252	0.153	305	305	307
0.094	1202	1209	1214	0.154	300	300	302
0.095	1171	1171	1176	0.155	292	295	297
0.096	1135	1135	1143	0.156	287	290	292
0.097	1102	1102	1107	0.157	284	284	287
0.098	1069	1072	1077	0.158	279	282	284
0.099	1041	1041	1046	0.159	274	274	279
0.100	1008	1011	1016	0.160	269	269	274
0.101	980	983	988	0.161	264	267	269
0.102	953	955	960	0.162	259	262	264
0.103	927	930	932	0.163	257	257	259
0.104	902	904	907	0.164	251	251	257
0.105	876	879	884	0.165	246	249	251
0.106	853	856	859	0.166	244	244	246
0.107	831	833	838	0.167	239	241	244
0.108	808	810	815	0.168	236	236	239
0.109	787	790	795	0.169	234	234	236
0.170	229	229	234	0.186	180	180	183
0.171	224	226	229	0.187	178	178	180
0.172	221	221	224	0.188	175	175	178
0.173	218	218	221	0.189	173	173	175
0.174	213	216	218	0.190	170	170	173
0.175	211	213	216	0.191	168	168	170
0.176	208	208	211	0.192	165	165	168
0.177	206	206	208	0.193	163	163	165
0.178	201	203	206	0.194	160	163	165
0.179	198	201	203	0.195	157	160	163
0.180	196	196	201	0.196	157	157	160
0.181	193	193	196	0.197	155	155	157
0.182	191	191	193	0.198	152	152	155
0.183	188	188	191	0.199	150	150	152
0.184	185	185	188	0.200	147	150	152
0.185	183	183	185				

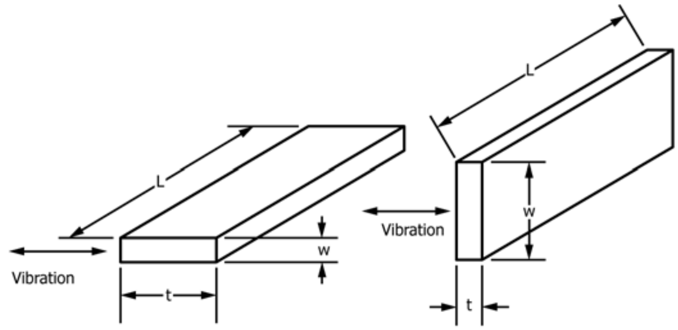


FIG. A1.4 Definition of Length (*L*), Width (*w*), and Thickness (*t*)

SUMMARY OF CHANGES

Subcommittee D02.F0 has identified the location of selected changes to this standard since the last issue (C747 – 93(2010)$^{\varepsilon 1}$) that may impact the use of this standard. (Approved Oct. 1, 2016.)

(1) Major revision/rewrite of entire standard.