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Standard Test Method for Determination of Slow Crack Growth Parameters of Advanced Ceramics by Constant Stress Flexural Testing (Stress Rupture) at Ambient Temperature[1]

This standard is issued under the fixed designation C1576; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

1. Scope

1.1 This standard test method covers the determination of slow crack growth (SCG) parameters of advanced ceramics by using constant stress flexural testing in which time to failure of flexure test specimens is determined in *four-point* flexure as a function of constant applied stress in a given environment at ambient temperature. In addition, test specimen fabrication methods, test stress levels, data collection and analysis, and reporting procedures are addressed. The decrease in time to failure with increasing applied stress in a specified environment is the basis of this test method that enables the evaluation of slow crack growth parameters of a material. The preferred analysis in the present method is based on a power law relationship between crack velocity and applied stress intensity; alternative analysis approaches are also discussed for situations where the power law relationship is not applicable.

NOTE 1—The test method in this standard is frequently referred to as "static *fatigue*" or stress-rupture testing (**1-3**)[2] in which the term "*fatigue*" is used interchangeably with the term "slow crack growth." To avoid possible confusion with the "*fatigue*" phenomenon of a material that occurs exclusively under cyclic loading, as defined in Terminology E1823, this test method uses the term "constant stress testing" rather than "static *fatigue*" testing.

1.2 This test method applies primarily to monolithic advanced ceramics that are macroscopically homogeneous and isotropic. This test method may also be applied to certain whisker- or particle-reinforced ceramics as well as certain discontinuous fiber-reinforced composite ceramics that exhibit macroscopically homogeneous behavior. Generally, continuous fiber ceramic composites do not exhibit macroscopically isotropic, homogeneous, continuous behavior, and the application of this test method to these materials is not recommended.

1.3 This test method is intended for use with various test environments such as air, other gaseous environments, and liquids.

1.4 The values stated in SI units are to be regarded as the standard and in accordance with IEEE/ASTM SI 10 Standard.

1.5 *This test method may involve hazardous materials, operations, and equipment. This standard does not purport to address all of the safety concerns associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:*[3]

C1145 Terminology of Advanced Ceramics
C1161 Test Method for Flexural Strength of Advanced Ceramics at Ambient Temperature
C1322 Practice for Fractography and Characterization of Fracture Origins in Advanced Ceramics
C1368 Test Method for Determination of Slow Crack Growth Parameters of Advanced Ceramics by Constant Stress-Rate Strength Testing at Ambient Temperature
C1465 Test Method for Determination of Slow Crack Growth Parameters of Advanced Ceramics by Constant Stress-Rate Flexural Testing at Elevated Temperatures
E4 Practices for Force Verification of Testing Machines
E6 Terminology Relating to Methods of Mechanical Testing
E112 Test Methods for Determining Average Grain Size
E337 Test Method for Measuring Humidity with a Psychrometer (the Measurement of Wet- and Dry-Bulb Temperatures)
E399 Test Method for Linear-Elastic Plane-Strain Fracture Toughness K_{Ic} of Metallic Materials
E1823 Terminology Relating to Fatigue and Fracture Testing

3. Terminology

3.1 *Definitions:*

[1] This practice is under the jurisdiction of ASTM Committee C28 on Advanced Ceramics and is the direct responsibility of Subcommittee C28.01 on Mechanical Properties and Performance.

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[2] The boldface numbers in parentheses refer to a list of references at the end of this standard.

[3] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

3.1.1 The terms described in Terminology C1145, Terminology E6, and Terminology E1823 are applicable to this test standard. Specific terms relevant to this test method are as follows:

3.1.2 *advanced ceramic, n*—a highly engineered, high performance, predominately non-metallic, inorganic, ceramic material having specific functional attributes. **C1145**

3.1.3 *constant applied stress, $\sigma[FL^{-2}]$, n*—a constant maximum flexural stress applied to a specified beam test specimen by using a constant static force with a test machine or a test fixture.

3.1.4 *'constant applied stress-time to failure' diagram*—a plot of constant applied stress against time to failure. Constant applied stress and time to failure are both plotted on logarithmic scales.

3.1.5 *'constant applied stress-time to failure' curve*—a curve fitted to the values of time to failure at each of several applied stresses.

NOTE 2—In the ceramics literature, this is often called a "static fatigue" curve.

3.1.6 *test environment, n*—the aggregate of chemical species and energy that surrounds a test specimen. **E1823**

3.1.7 *test environmental chamber, n*—a container surrounding the test specimen that is capable of providing controlled local environmental condition. **C1368, C1465**

3.1.8 *flexural strength, σ_f $[FL^{-2}]$, n*—a measure of the ultimate strength of a specified beam test specimen in flexure determined at a given stress rate in a particular environment.

3.1.9 *fracture toughness, (critical stress intensity factor) K_{IC} $[FL^{-3/2}]$, n*—a generic term for measures of resistance to extension of a crack. **E1823, E399**

3.1.10 *inert flexural strength $[FL^{-2}]$, n*—the flexural strength of a specified beam as determined in an inert condition whereby no slow crack growth occurs.

NOTE 3—An inert condition may be obtained by using vacuum, low temperature, very fast test rate, or an inert environment such as silicone oil or high purity dry N_2.

3.1.11 *R-curve, n*—a plot of crack-extension resistance as a function of stable crack extension. **C1145**

3.1.12 *run-out, n*—a test specimen that does not fail before a prescribed test time.

3.1.13 *slow crack growth (SCG), n*—subcritical crack growth (extension) which may result from, but is not restricted to, such mechanisms as environmentally assisted stress corrosion or diffusive crack growth. **C1368, C1465**

3.1.14 *slow crack growth (SCG) parameters*—the parameters estimated as constants in the log (*time to failure*) versus log (*constant applied stress*), which represent a measure of susceptibility to slow crack growth of a material (see Appendix X1).

3.1.15 *stress intensity factor, K_I $[FL^{-3/2}$, n*—the magnitude of the ideal-crack-tip stress field stress field singularity) subjected to mode I loading in a homogeneous, linear elastic body. **E1823**

3.1.16 *time to failure, t_f [t], n*—total elapsed time from test initiation to test specimen failure.

4. Significance and Use

4.1 The service life of many structural ceramic components is often limited by the subcritical growth of cracks. This test method provides an approach for appraising the relative slow crack growth susceptibility of ceramic materials under specified environments at ambient temperature. Furthermore, this test method may establish the influences of processing variables and composition on slow crack growth as well as on strength behavior of newly developed or existing materials, thus allowing tailoring and optimizing material processing for further modification. In summary, this test method may be used for material development, quality control, characterization, design code or model verification, and limited design data generation purposes.

NOTE 4—Data generated by this test method do not necessarily correspond to crack velocities that may be encountered in service conditions. The use of data generated by this test method for design purposes, depending on the range and magnitude of applied stresses used, may entail extrapolation and uncertainty.

4.2 This test method is related to Test Method C1368 ("constant stress-rate flexural testing"), however, C1368 uses constant stress rates to determine corresponding flexural strengths whereas this test method employs constant stress to determine corresponding times to failure. In general, the data generated by this test method may be more representative of actual service conditions as compared with those by constant stress-rate testing. However, in terms of test time, constant stress testing is inherently and significantly more time consuming than constant stress rate testing.

4.3 The flexural stress computation in this test method is based on simple elastic beam theory, with the assumptions that the material is isotropic and homogeneous, the moduli of elasticity in tension and compression are identical, and the material is linearly elastic. The grain size should be no greater than one-fiftieth (1/50) of the beam depth as measured by the mean linear intercept method (Test Methods E112). In cases where the material grain size is bimodal or the grain size distribution is wide, the limit should apply to the larger grains.

4.4 The test specimen sizes and test fixtures have been selected in accordance with Test Methods C1161 and C1368, which provides a balance between practical configurations and resulting errors, as discussed in Ref (**4, 5**).

4.5 The data are evaluated by regression of log applied stress versus log time to failure to the experimental data. The recommendation is to determine the slow crack growth parameters by applying the power law crack velocity function. For derivation of this, and for alternative crack velocity functions, see Appendix X1.

NOTE 5—A variety of crack velocity functions exist in the literature. A comparison of the functions for the prediction of long-term static fatigue data from short-term dynamic fatigue data (**6**) indicates that the exponential forms better predict the data than the power-law form. Further, the exponential form has a theoretical basis (**7-10**), however, the power law form is simpler mathematically. Both have been shown to fit short-term test data well.

4.6 The approach used in this method assumes that the material displays no rising R-curve behavior, that is, no increasing fracture resistance (or crack-extension resistance) with increasing crack length. The existence of such behavior cannot be determined from this test method. The analysis further assumes that the same flaw type controls all times-to-failure.

4.7 Slow crack growth behavior of ceramic materials can vary as a function of mechanical, material, thermal, and environmental variables. Therefore, it is essential that test results accurately reflect the effects of specific variables under study. Only then can data be compared from one investigation to another on a valid basis, or serve as a valid basis for characterizing materials and assessing structural behavior.

4.8 Like strength, time to failure of advanced ceramics subjected to slow crack growth is probabilistic in nature. Therefore, slow crack growth that is determined from times to failure under given constant applied stresses is also a probabilistic phenomenon. The scatter in time to failure in constant stress testing is much greater than the scatter in strength in constant stress-rate (or any strength) testing (**1, 11-13**), see Appendix X2. Hence, a proper range and number of constant applied stresses, in conjunction with an appropriate number of test specimens, are required for statistical reproducibility and reliable design data generation (**1-3**). This standard provides guidance in this regard.

4.9 The time to failure of a ceramic material for a given test specimen and test fixture configuration is dependent on its inherent resistance to fracture, the presence of flaws, applied stress, and environmental effects. Fractographic analysis to verify the failure mechanisms has proven to be a valuable tool in the analysis of SCG data to verify that the same flaw type is dominant over the entire test range Ref (**14, 15**), and it is to be used in this standard (refer to Practice C1322).

5. Interferences

5.1 Slow crack growth may be the product of both mechanical and chemical driving forces. The chemical driving force for a given material can vary strongly with the composition and temperature of a test environment. Testing is conducted in environments representative of service conditions so as to evaluate material performance under use conditions. Note that slow crack growth testing, particularly constant stress testing, is very time consuming. The overall test time is considerably greater in constant stress testing than in constant stress-rate testing. Because of this longer test time, the chemical variables of the test environment must be prevented from changing significantly throughout all test times. Inadequate control of these chemical variables may result in inaccurate time-to-failure data, especially for materials that are more sensitive to the test environment.

5.2 Depending on the degree of SCG susceptibility of a material, the linear relationship between log (*constant applied stress*) and log (*time to failure*) may start to deviate at a certain high applied stress where the crack velocity increases rapidly with a subsequently short test duration, that is, the applied stress approaches the strength, see Fig. 1. This is analogous to

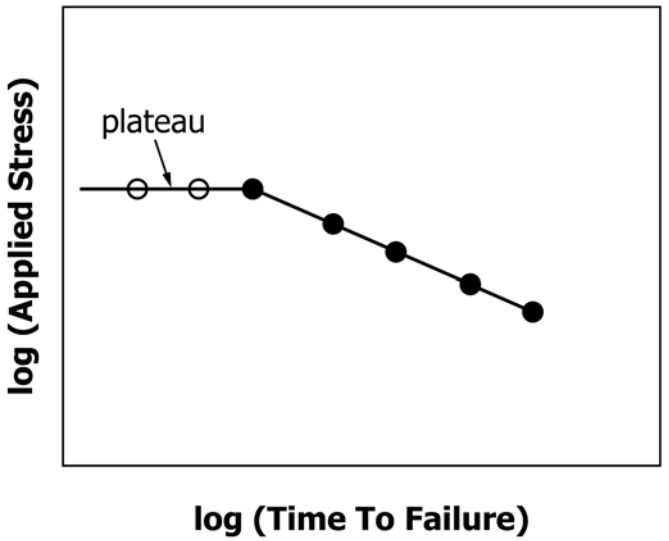

FIG. 1 Schematic Diagram Showing Unacceptable (Average) Data Points (With an "Open" Symbol) in the Plateau Region in Determining Slow Crack Growth (SCG) Parameters

the occurrence of a strength plateau observed at higher test rates in constant stress-rate testing (**16**). If the time-to-failure data determined in this plateau region are included in the analysis, a misleading estimate of the SCG parameters will be obtained (**17**). Therefore, the strength data in the plateau shall be excluded as data points in estimating the SCG parameters of the material. Similarly, a plateau can also exist at the fatigue limit end of the curve, and these data points shall also be excluded in estimating the SCG parameters.

NOTE 6—There are no simple guidelines in determining whether a plateau region is reached, however with knowledge of the inert strength and the fracture toughness of the test material, the slow crack growth rate – applied stress intensity (v-K) curve may be determined. Evaluating this will help determine where the experimental conditions fall.

5.3 When testing a material exhibiting a high SCG resistance (typically SCG parameter $n > 70$) an unrealistically large number of test specimens may be required in a small range of applied stresses since a significant number of test specimens may be expected to fail while loading. Furthermore, if lower stresses are to be used, unrealistically long test times are to be expected. As a result, *practical*, *specific*, *quantitative* values of SCG parameters required for life prediction can only with great difficulty be determined for this type of material (**18**). In this case, a companion test method—constant stress-rate testing, Test Method C1368—may be utilized instead to determine the corresponding SCG parameters of the material. The constant stress-rate test may be used provided the same flaw types are activated in both stress states.

5.4 Surface preparation of test specimens can introduce flaws that may have pronounced effects on flexural strength and thus time to failure. Machining damage imposed during test specimen preparation can be either a random interfering factor, or an inherent part of the strength characteristics to be measured. Surface preparation can also lead to residual stress. It should be understood that the final machining steps may or may not negate machining damage introduced during the

earlier coarse or intermediate machining steps. In some cases, test specimens need to be tested in the as-processed condition to simulate a specific service condition. Test specimen fabrication history may play an important role in strength as well as time-to-failure behavior, which consequently may affect the values of the SCG parameters to be determined. Therefore, the test specimen fabrication history shall be reported. In addition the nature of fabrication used for certain advanced ceramic components may require testing of specimens with surfaces in the as-fabricated condition (that is, it may not be possible, desired, or required to machine some test specimens directly in contact with test fixture components). In such cases, a fully articulated test fixture is required. However, for very rough or wavy as-fabricated surfaces, eccentricities in the stress state due to non-symmetric cross sections as well as variations in the cross-sectional dimensions may also interfere with the strength measurement.

5.5 Premature fracture may be initiated at surface flaws (for example, scratches, edge chips) introduced while handling the specimens.

5.6 Fractures that consistently initiate near or just outside the load pins may be due to factors such as friction or contact stresses introduced by the load fixtures, or via misalignment of the test specimen load pins. Failure of test specimens initiated consistently from their edges may be due to poor specimen preparation (for example, severe grinding or very poor edge preparation) or excessive twisting stresses at the specimen edges Ref (**4**, **5**, **19**).

5.7 Fractures may initiate from different flaw types (for example, surface flaws like scratches and machining flaws, or pores and agglomerates that may be located in the volume or at the surface of the specimens). The analysis performed in this standard assumes that all failures initiate from similar types of flaws as confirmed by fractography according to Practice C1322.

6. Apparatus

6.1 *Test Machine*—Dead weight or universal test machines capable of maintaining a constant force may be used for constant stress testing. The variations in the selected force shall not exceed ±1.0 % of the nominal value at any given time during the test. The force must be monitored and the variations in the selected force shall not exceed the ±1.0 % limit at any given time during the test. Test machines used for this test method shall conform to the requirements of Practices E4.

6.2 *Test Fixtures*—The configurations and mechanical properties of test fixtures shall be in accordance with Test Method C1161. The materials from which the test fixtures, including bearing cylinders, are fabricated shall be effectively inert to the test environment so that they do not significantly react with or contaminate either the test specimen or the test environment.

NOTE 7—For testing in distilled water, for example, it is recommended that the test fixture be fabricated from stainless steel. The bearing cylinders may be machined from hardenable stainless steel (for example, 316 SS) or a ceramic material such as silicon nitride, silicon carbide or alumina.

6.2.1 *Four-Point Flexure*—The four-point-¼-point fixture configuration as described in Test Method C1161 shall be used in this test method. Three-point flexure shall not be used.

6.2.2 *Bearing Cylinders*—The requirements of dimensions and mechanical properties of bearing cylinders as described in Test Method C1161 shall be used in this test method. The bearing cylinders shall be free to roll in order to relieve frictional constraints, as described in Test Method C1161.

6.2.3 *Semiarticulating Four-Point Fixture*—The semiarticulating four-point fixture as described in Test Method C1161 may be used in this test method. This fixture shall be used when the parallelism requirements of test specimens are met according to Test Method C1161.

6.2.4 *Fully Articulating Four-Point Fixture*—The fully articulating four-point fixture as described in Test Method C1161 may be used in this test method. Specimens that do not meet the parallelism requirements in Test Method C1161, due to the nature of fabrication process (as-fired, heat treated, or oxidized), shall be tested in this fully articulating fixture.

6.3 *Environmental Facility*—For testing in an environment other than ambient air, use a chamber that is inert to the test environment, capable of safely containing the environment and allowing monitoring of environments to ensure consistency. The chamber shall be sufficiently large to immerse the test specimen in the test medium. A circulation or mixing system may be desirable depending on the conditions to be simulated. Additionally, the facility shall be able to safely contain the test environment. If it is necessary to direct force through bellows, fittings, or seals, it shall be verified that force losses or errors do not exceed 1 % of the prospective applied force. If ambient temperature tests are conducted under constant environmental conditions, then control the temperature and relative humidity to within ± 3 °C and ± 10 % of the set humidity level, respectively.

6.4 *Data Acquisition*—Accurate determination of time to failure (or test time in case of run-out) is important since time to failure is the only dependent variable in this test method. This is particularly important when time to failure is relatively short (<10 s) when a higher applied stress is used. Devices to measure time to failure may be either digital or analog and incorporate a switching mechanism to stop the device at test specimen failure. The recording device shall be accurate to within ±1 % of the selected range. If universal test machines are used, at the minimum, an autographic record of applied force versus time shall be determined during testing. Either analog chart recorders or digital data acquisition systems can be used for this purpose. Recording devices shall be accurate to 1.0 % of the recording range and shall have a minimum data acquisition rate sufficient to adequately describe the whole test series. The appropriate data acquisition rate depends on the actual time to failure (that is, magnitude of applied stress), but should preferably be in the 0.2 to 50 Hz range (50 Hz for times less than 5 s, 10 Hz for times between 5 s and 10 min, 1 Hz for times between 10 min and 5 h, and 0.2 Hz for times over 5 h).

6.5 *Dimension-Measuring Devices*—Micrometers and other devices used for measuring test specimen dimensions shall have a resolution of 0.002 mm or smaller. To avoid damage in

the gage section area, depth measurements should be made using a flat, anvil-type micrometer. Ball-tipped or sharp anvil micrometers should not be used because localized damage (for example, cracking) can be induced.

7. Test Specimen

7.1 *Specimen Size*—The types and dimensions of rectangular beam specimens as described in Test Method C1161 shall be used in this test method.

7.2 *Specimen Preparation*—Specimen fabrication and preparation methods as described in Test Method C1161 shall be used in this test method.

7.3 *Specimen Dimensions*—Determine the width and depth of each test specimen as described in Test Method C1161, either optically or mechanically using a flat, anvil-type micrometer. Exercise extreme caution to prevent damage to the critical area of the test specimen. Record and report the measured dimensions and locations of the measurements. Use the average of the multiple measurements in the stress calculation.

7.4 *Handling and Cleaning*—Exercise care in handling and storing specimens in order to avoid introducing random and severe flaws, which might occur if the specimens were allowed to impact or scratch each other. Clean the test specimens with an appropriate medium such as methanol or high-purity (>99 %) isopropyl alcohol to avoid contamination of the test environment by residual machining or processing fluids. After cleaning and drying, store the test specimens in a controlled environment such as a vacuum or a dessicator in order to avoid exposure to moisture. This is necessary if testing is to be carried out in an environment other than ambient air or water. Adsorbed moisture on the test specimen surfaces can change crack growth rates.

7.5 *Number of Test Specimens*—At least ten specimens per applied stress shall be used. The total number of test specimens shall be at least 40, with at least four different applied stresses (see 8.3.1). The numbers of test specimens and applied stresses in this test method have been established with the intent of determining reasonable confidence limits on both time-to-failure distribution and SCG parameters.

NOTE 8—Refer to Ref (**11**) when a specific purpose is sought for the statistical reproducibility of SCG parameters in terms of several variables.

7.6 *Randomization of Test Specimens*—Since a somewhat large number of test specimens (a minimum of 40) with at least four different applied stresses is used in this test method, it is highly recommended that all the test specimens be randomized prior to testing in order to reduce any systematic error associated with material fabrication and/or specimen preparation. Randomize the test specimens (using, for example, a random number generator) in groups equal to the number of applied stresses to be employed. Complete randomization may not be appropriate if the specimens stem from different billets. Trace the origin of the test specimens and use an appropriate statistical blocking scheme for distributing the specimens.

8. Procedure

8.1 *Test Specimen and Load Fixture Dimensions*—Choose the appropriate fixture in the specific test configurations. A fully articulating fixture is required if the specimen parallelism requirements cannot be met. Conduct 100 % inspection/measurements of the test specimens and test specimen dimensions to assure compliance with the specifications in this test method. Measure the test specimen width, b, and depth, d. Exercise extreme caution to prevent damage to the test specimen.

8.2 Measurement of surface finish is not required, however, such information would be helpful. Methods such as contact profilometry can be used to determine the surface roughness of the test specimen faces. When quantified, report surface roughness, test conditions, and the direction of the measurement with respect to the test specimen long axis.

8.3 *Applied Stresses:*

8.3.1 *Range and Number of Applied Stress Levels*—The choice of range and number of applied stress levels (or applied force levels) not only depends on test material but also affects the statistical reproducibility of SCG parameters. Time to failure of advanced monolithic ceramics in constant stress testing is probabilistic. Furthermore, the scatter in time to failure is significantly greater than that in strength (**11-13**), typically (n+1) times the Weibull modulus of strength distribution, see Appendix X2. Hence, unlike metallic or polymeric materials, a considerable increase in the scatter of time to failure is expected for advanced monolithic ceramics, attributed to both a large strength scatter (Weibull modulus of about 10 to 15) and a typically high SCG parameter $n \geq 20$. As a consequence, testing a few test specimens at each applied stress using a few stress levels may not be sufficient to produce statistically reliable design data. On the contrary, the use of many test specimens with many applied stresses is quite time consuming or even unrealistic in some cases. In general, choose the upper limit of applied stresses that would result in corresponding time to failure ≥10 s. The choice of the lower limit of applied stresses depends on run-out times, where some of test specimens would not fail within a prescribed length of test time. The run-out time needs to be determined in the particular test program; however experience has shown that run-out times up to 10 days are reasonable in *laboratory* test conditions. Choose at least four applied stresses covering at least four orders of magnitude in time. See also Appendix X3.

NOTE 9—If SCG parameters are available from constant stress-rate testing (Test Method C1368), time to failure in constant stress testing can be estimated as a function of applied stress from a prediction shown in Appendix X3. This approach, although theoretical, allows one to quickly find the range and magnitude of stresses and the run-out time to be applied. There might be some discrepancies in the prediction; however, use of this prediction can significantly reduce many uncertainties and trial-and-errors associated with selecting stresses and run-out time. If no SCG data for the test material is available, run simplified constant stress-rate testing using both high (around 10 MPa/s) and low (around 0.01 MPa/s) stress rates with at least five test specimens at each stress rate to determine fracture strengths. Then determine the corresponding SCG parameters (n and D_d) based on the procedure in Test Method C1368. Use these simplified SCG data to select applied stresses and run-out time to be used in constant stress testing by following the prediction described in Appendix X3.

8.4 *Assembling Test Fixture/Specimen:*

8.4.1 Examine the bearing cylinders to make sure that they are undamaged, and that there are no reaction products (corrosion products or oxidation) that could result in uneven line loading of the test specimen or prevent the bearing cylinders from rolling. Remove and clean, or replace, the bearing cylinders, if necessary. Avoid any undesirable dimensional changes in the bearing cylinders, for example, by inadvertently forming a small flat on the cylinder surface when abrasion (for example, abrasive paper) is used to remove the reaction products from the cylinders. The same care should be directed toward the contact surfaces in the loading and support members of the test fixture that are in contact with the bearing cylinders.

8.4.2 Carefully place each test specimen into the test fixture to avoid possible damage and contamination and to ensure alignment of the test specimen relative to the test fixture. There should be an equal amount of overhang of the test specimen beyond the outer bearing cylinders and the test specimen shall be directly centered below the axis of the applied force. Provide a way (for example, pencil marking in the test specimen or known positioning of the test specimen relative to a reference point or surface of the test fixture) to determine the fracture location of the test specimen upon fracture.

8.5 *Loading the Test Fixture/Specimen Assembly into Test Machine*—Mount the test fixture/test specimen assembly in the load train of the test machine. If necessary, slowly (~1MPa/s) apply a preload of no more than 25 % of (fast) fracture force to maintain system alignment.

8.6 *Environment*—Choose the test environment as appropriate to the test program. If the test environment is other than ambient air, supply the environmental chamber with the test medium so that the test specimen is completely exposed to the test environment. The immersion or exposure time for equilibration of the test specimen in the test environment should be determined by agreement between the parties involved in the test program. Consistent conditions (composition, supply rate, etc.) of the test environment should be maintained throughout the test series (also refer to 6.3). When a corrosive liquid environment is used, put a proper protective cover onto the environment chamber (or container) to keep the test environment from splashing out of the chamber (container) upon fracture. If the tests are carried out in a humid atmosphere, the relative humidity shall not vary more than 10 % of the set humidity level during the entire test series. Determine the relative humidity in accordance with Test Method E337. Allow a sufficient period for equilibration of the test specimen in the environment. The equilibration time should be based on agreement between the parties involved in the test program and be consistent for the entire test program. This is particularly important for an environment that is chemically corrosive. When tests are conducted in ambient air, put cotton, tissues, or other appropriate material to prevent broken pieces of test specimens flying out of the test fixtures upon fracture.

8.7 *Conducting the Test*—Initiate the data acquisition. Start the test by applying a selected applied force (applied stress) with an accuracy of ±1.0 %. Time-measuring devices, particularly when used with dead-weight test machines, should be synchronized upon the application of a test force to the test specimen. Time shall be measured at an accuracy of ±1 % of the actual value. Record time to failure. If failure does not occur within the specific time agreed upon in the test program, record this as run-out.

8.7.1 *Recording*—Record a force-versus-time curve for each test in order to check the requirement of force variation of testing machines. Care should be taken in recording adequate response-rate capacity of the recorder, as described in 6.4.

8.8 *Post-Test Treatments:*

8.8.1 Carefully collect as many fragments as possible. Clean the fragments if necessary and store in a protective container for further analysis, including fractography.

8.8.2 *Fractography*—Fractographic analysis of fractured test specimens shall be employed to ensure that all the fracture origins are from the same population. Additional fractography may be performed to characterize the types, locations and sizes of fracture origins as well as the flaw extensions due to slow crack growth. Follow the guidance established in Practice C1322. See also 5.7.

9. Calculation

9.1 *Applied Stress:*

9.1.1 Calculate the flexural strength according to the formula for the strength of a beam in four-point ¼-point flexure:

$$\sigma = \frac{3PL}{4\,bd^2} \tag{1}$$

where:

σ = applied stress, MPa,
P = applied force, N,
L = outer (support) span, mm,
b = test specimen width, mm, and
d = test specimen depth, mm.

9.2 *Determining the Fatigue Curve and the Slow Crack Growth Parameters n and D:*

9.2.1 Use each individual time to failure, not averaged per applied stress, to determine the fatigue curve. This can be done by linear regression or maximum likelihood regression. If the data contains specimens that failed upon loading a censored analysis must be performed (left-hand censoring), if the data contains run-outs, a right-hand censoring must be performed. Datasets that contain both failures upon loading and run-outs must be analyzed by a two-sided censoring technique The censoring can be performed by an iterative least squares procedure or by a maximum likelihood analysis. Several commercial statistics analysis programs and certain freeware contain censored analyses as an option, **(20-22)**.

Determination of SCG parameters depends on which crack velocity relationship is selected. The approach based on a power law relationship between crack velocity and applied stress intensity is given as the preferred method in this standard. See Appendix X1 for derivations and alternative methods.

Use each individual time to failure, not averaged per applied stress, to determine the SCG parameters. Plot log (*applied stress, in MPa*) against log (*time to failure, in s*). The SCG

parameters n and D_s can be determined by a linear regression analysis using all log t_f over the complete range of individual log σ, based on the following equation (see Appendix X1 for derivation):

$$\log t_f = -n\log\sigma + \log D_s \quad (2)$$

Include in the diagram all the data points determined as valid tests. However, do not include the run-outs or the data points in the plateau regions (see Fig. 1) in calculating SCG parameters. A typical example of a plot of log (*applied stress*) against log (*time to failure*) is shown in Fig. 2.

NOTE 10—It seems to be more logical to plot the dependent variable, log (t_f), as a function of the independent variable, log (σ), however, it has been a long practice to plot log (σ) versus log (t_f) such as in Fig. 2. This type of diagram when determined under cyclic loading is called *S-N* curve (Terminology E1823). This test method follows such a common convention in plotting data points. However, the regression must be performed as defined in Eq 2.

NOTE 11—This test method is intended to determine only slow crack growth parameters n and D. The calculation of the parameter A (in $v = A[K_I/K_{IC}]$") requires knowledge of other material parameters, and is beyond the scope of this test method (see Appendix X1).

NOTE 12—This test method is primarily for test specimens with intrinsic flaws. If test specimens, however, possess any residual stresses produced by localized contact damage (for example, particle impact or indents) or any other treatments, the estimated SCG parameters will be different and shall be denoted as such. Refer to Ref **(24)** for more detailed information on the analysis of slow crack growth behavior of a material containing a localized residual stress field.

9.2.1.1 Calculate the slope of the linear regression line as follows:

$$\alpha = \frac{K\sum_{j=1}^{K}(\log\sigma_j \log t_j) - \left(\sum_{j=1}^{K}\log\sigma_j \sum_{j=1}^{K}\log t_j\right)}{K\sum_{j=1}^{K}(\log\sigma_j)^2 - \left(\sum_{j=1}^{K}\log\sigma_j\right)^2} \quad (3)$$

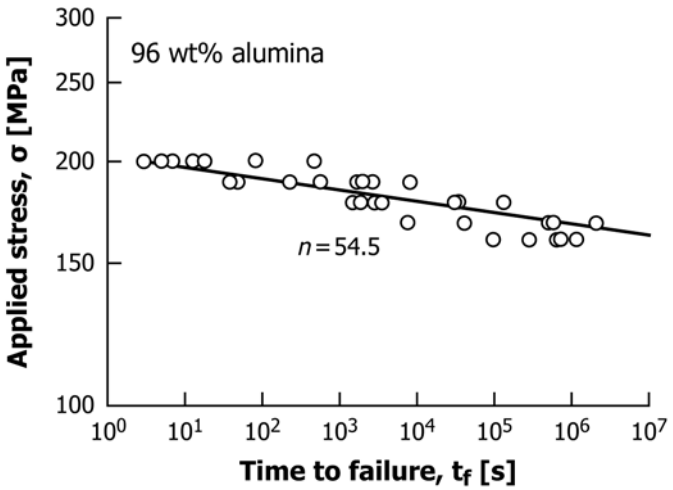

FIG. 2 Example of an Applied Stress-Time to Failure Diagram Determined for 96 wt% Alumina in Distilled Water at Ambient Temperature (23)

where:

α = slope,
σ_j = the jth applied stress, MPa,
t_j = the jth measured time to failure, s, and
K = total number of test specimens tested validly for the whole series of tests excluding the run-out test specimens.

9.2.1.2 Calculate the SCG parameter n as follows:

$$n = -\alpha \quad (4)$$

9.2.1.3 Calculate the intercept of the linear regression line as follows:

$$\beta = \frac{\left(\sum_{j=1}^{K}\log t_j\right)\sum_{j=1}^{K}(\log\sigma_j)^2 - \left(\sum_{j=1}^{K}\log\sigma_j \log t_j\right)\left(\sum_{j=1}^{K}\log\sigma_j\right)}{K\sum_{j=1}^{K}(\log\sigma_j)^2 - \left(\sum_{j=1}^{K}\log\sigma_j\right)^2} \quad (5)$$

where:

β = intercept.

9.2.1.4 Calculate the SCG parameter D_S as follows:

$$D_S = 10^{\beta} \quad (6)$$

9.2.1.5 Calculate the standard deviations of the slope α and of the SCG parameter n as follows:

$$SD_\alpha = \sqrt{\frac{K}{K-2}\,\frac{\sum_{j=1}^{K}(\alpha\log\sigma_j + \beta - \log t_j)^2}{K\sum_{j=1}^{K}(\log\sigma_j)^2 - \left(\sum_{j=1}^{K}\log\sigma_j\right)^2}} \quad (7)$$

$$SD_n = SD_\alpha \quad (8)$$

where:

SD_α = standard deviation of the slope, α and
SD_n = standard deviation of the SCG parameter n.

9.2.1.6 Calculate the standard deviations of the intercept ß and of the SCG parameter D_S as follows:

$$SD_\beta = \sqrt{\frac{\sum_{j=1}^{K}(\alpha\log\sigma_j + \beta - \log t_j)^2 \sum_{j=1}^{K}(\log\sigma_j)^2}{(K-2)\left[K\sum_{j=1}^{K}(\log\sigma_j)^2 - \left(\sum_{j=1}^{K}\log\sigma_j\right)^2\right]}} \quad (9)$$

$$SD_{D_S} = 2.3026\,(SD_\beta)(10^\beta) \quad (10)$$

where:

SD_β = standard deviation of the intercept β, and
SD_{D_S} = standard deviation of the SCG parameter D_S.

9.2.1.7 Calculate the coefficients of variation of the SCG parameter n and of the SCG parameter D_S as follows:

$$CV_n\,(\%) = \frac{100\,(SD_n)}{n} \quad (11)$$

$$CV_{D_S}\,(\%) = \frac{100\,(SD_{D_S})}{D_S} \quad (12)$$

where:

CV_n = coefficient of variation of the SCG parameter n, and

CV_{D_S} = coefficient of variation of the SCG parameter D_S.

9.2.1.8 Calculate the square of correlation coefficient (r) of the linear regression line as follows:

$$r^2 = \frac{\left[K\sum_{j=1}^{K}(\log\sigma_j \log t_j) - \left(\sum_{j=1}^{K}\log\sigma_j \sum_{j=1}^{K}\log t_j\right)\right]^2}{\left[K\sum_{j=1}^{K}(\log\sigma_j)^2 - \sum_{j=1}^{K}(\log\sigma_j)^2\right]\left[K\sum_{j=1}^{K}(\log t_f)^2 - \sum_{j=1}^{K}(\log t_f)^2\right]} \quad (13)$$

where:

r^2 = square of the correlation coefficient.

9.2.1.9 (Optional) The mean time to failure is not used in this method to calculate SCG parameters. If desired for a specific purpose, calculate for each applied stress the corresponding mean time to failure, standard deviation, and coefficient of variation as follows:

$$\bar{t}_f = \frac{\sum_{j=1}^{N} t_j}{N} \quad (14)$$

$$SD_{t_f} = \sqrt{\frac{\sum_{j=1}^{N}\left(t_j - \bar{t}_f\right)^2}{N-1}} \quad (15)$$

$$CV_{t_f}(\%) = \frac{100(SD_{t_f})}{\bar{t}_f} \quad (16)$$

where:

$\bar{t}_f$ = mean time to failure, s,
t_j = the jth measured time-to-failure value, s,
N = number of test specimens tested validly at each applied stress excluding the run-out specimens and specimens that failed upon loading test, if any. When there is no run-out test specimen, the minimum number of test specimens is 10.
SD_{t_f} = standard deviation, and
CV_{t_f} = coefficient of variation.

10. Report

10.1 *Test Specimens, Equipments, and Test Conditions*—Report the following information for the test specimens, equipment and test conditions. Note in the report any deviations and alterations from the procedures and requirements described in this test method.

10.1.1 Date and location of the testing.

10.1.2 Specimen geometry type and specimen dimensions.

10.1.3 Test fixture dimensions (inner and outer span).

10.1.4 The number of test specimens tested at each stress level.

10.1.5 All relevant material data including vintage data or billet identification data.

10.1.6 Exact method of test specimen preparation, including all stages of machining.

10.1.7 Heat treatments or heat exposures, if any. Any environmental preconditioning of the test specimens.

10.1.8 Relevant information on randomization of the test specimens.

10.1.9 Methods of test specimen cleaning and storage.

10.1.10 All preconditioning of test specimens prior to testing, if any.

10.1.11 Type and configuration of the test machine including the load cell.

10.1.12 Type, configuration, and material of the test fixture with degree of articulation.

10.1.13 Type and configuration of the data acquisition system.

10.1.14 Test temperature and test environment (type, conditions, and application method).

10.1.15 Ambient conditions such as temperature and humidity.

10.1.16 Method and magnitude of preloading for each test specimen, if any.

10.1.17 Magnitude of applied stresses.

10.2 *Test Results*—Report the following information for the test results. Note in the report any deviations and alterations from the procedures and requirements described in this test method.

10.2.1 Number of the valid tests, (for example, fracture in the inner span) as well as of the invalid tests (for example, fracture outside the inner span).

10.2.2 Equations used for stress calculation.

10.2.3 Applied stresses to three significant figures.

10.2.4 Time to failure of each test specimen to one decimal point when $t < 10$ s.

10.2.5 Mean time to failure, standard deviation, and coefficient of variation determined at each applied stress, if determined (optional).

10.2.6 Graphical representation (Fig. 2) of test results showing log (*applied stress*) against log (*time to failure*) using all data points including the run-outs. Include in the figure the determined best-fit line together with the estimated value of SCG parameter n. Include, if desired, in the figure some key information on test material, test temperature, test specimen size, test fixture, and test environment, etc., as shown in Fig. 2.

10.2.7 Fractography information including type, location and size of fracture origin as well as the degree of slow crack growth, if possible.

11. Precision and Bias

11.1 The time to failure of an advanced ceramic for a given applied stress is not a deterministic quantity, but will vary from test specimen to test specimen. Weibull statistics may model this variability Ref (**3, 12, 13, 25**). This test method has been devised so that the precision is high and the bias is low compared to the inherent variability of time to failure of the material.

11.2 The experimental stress errors, as well as the error due to cross section reduction associated with chamfering the edges, have been analyzed in detail in Ref (**4**) and described in terms of precision and bias in Test Method C1161. Test Method C1161 also includes chamfer correction factors that shall be used if necessary.

11.3 The statistical reproducibility of slow crack growth parameters determined from constant stress testing has been analyzed (**1**). The degree of reproducibility of SCG parameters

depends on not only the number of test specimens but also on other experimental test variables. These variables include the SCG parameters, Weibull modulus, and the number and range of test stresses.

11.4 Bias may result from inadequate use and/or treatments of the test environment, particularly in terms of its composition, aging and contamination.

11.5 Because of the nature of the materials and lack of a wide database on a variety of applicable advanced ceramics tested in constant stress testing, no definitive statement can be made at this time concerning precision and bias of this test method.

12. Keywords

12.1 advanced ceramics; constant stress testing; flexural testing; four-point flexure; slow crack growth; slow crack growth parameters; time to failure

APPENDIXES

(Nonmandatory Information)

X1. TIME TO FAILURE AS A FUNCTION OF APPLIED STRESS IN CONSTANT STRESS ("STATIC FATIGUE") TESTING

The SCG behavior of glass and ceramics can be described in terms of so-called *v-K* diagrams, which establish the relationship between the applied stress intensity, *K*, and the growth velocity of cracks, *v*, in a given environment (**26**). If the *v-K* curve is known, lifetime prediction can be made through the use of fracture mechanics. Some materials may not exhibit a threshold stress intensity (K_{th}) below which no SCG occurs, whereas others may not have measurable stage II or III regimes before fast fracture occurs. In determination of the SCG parameters for material comparison and life time predictions, it is therefore imperative to establish the entire *v-K* curve rather than to just determine the slope, *n*, for stage I (**27**). Several test methods assumes *a priori* knowledge of the *v-K* relationship, and much research has been focused on exploring the fundamental mechanisms governing subcritical crack growth behavior to establish a universal relationship between crack growth and applied stress intensity. Other test methods involve a direct measurement of the growing crack as a function of a well-defined applied *K*, and hence, no assumptions on the functional relationship need to be made.

Fracture Mechanics Equations

The Mode I stress intensity factor, K_{Ia}, for a flaw of size *a* (*a* represents the depth of a surface flaw or radius of a volume flaw) subjected to a remote applied stress of σ*a* is given by:

$$K_{Ia} = Y\,\sigma_a\sqrt{a} \tag{X1.1}$$

where *Y* is a crack geometry factor dependent on the flaw shape (**28**). By rearranging and differentiating with respect to time, the relationship between the applied stress (or stress intensity) and the change in crack size (crack velocity) may be obtained:

$$v = \frac{da}{dt} = \frac{2K_{Ia}}{Y^2\,\sigma_a^2}\frac{dK_{Ia}}{dt} - \frac{2K_{Ia}^2}{Y^2\,\sigma_a^3}\frac{d\sigma_a}{dt} \tag{X1.2}$$

In order to integrate Eq X1.2 and obtain the strength in the degrading environment, an assumption of the relationship between the crack velocity *v* and the applied stress intensity K_{Ia} must be made.

Power Law Formulation

The relationship most commonly used is a power-law representation and this is recommended as the preferred method in this standard. This approach introduces mathematical simplicity, and has been shown to empirically fit most SCG data well (**2**, **26**, **29**, **30**). The power law has also been adopted in several design codes for advanced ceramics. The crack velocity during subcritical crack growth is given as:

$$v = A\left(\frac{K_{Ia}}{K_{IC}}\right)^n. \tag{X1.3}$$

The constants *A* and *n* are the fatigue parameters, dependent on material and environment, and K_{IC} is the material's Mode I plane strain fracture toughness. Often it is observed that the fatigue behavior is temperature dependent, and the power-law relationship may be modified to take this into account by introducing a term containing temperature dependence:

$$v = v_0'\left(\frac{K_{Ia}}{K_{IC}}\right)^n \exp\left[-\left(\frac{E^*}{RT}\right)\right], \tag{X1.4}$$

where v_0' and *n* are the fatigue parameters, E^* is the activation energy, *R* is the gas constant, and *T* is absolute temperature.

The strength σ_i in the inert environment and σ_f in the strength reducing environment are given by:

$$K_{IC} = Y\,\sigma_i\sqrt{a_i} \tag{X1.5}$$

and

$$K_{IC} = Y\,\sigma_f\sqrt{a_f}, \tag{X1.6}$$

respectively, with a_i and a_f representing the initial and final crack lengths. Using the power-law relation in Eq X1.3 in Eq X1.2 and utilizing the expressions in Eq X1.5, the following expression for the reduced fatigue strength (σ_f) as a function of applied stress is obtained:

$$\sigma_f^{n-2} = \sigma_i^{n-2} - \frac{1}{B}\int_o^t [\sigma_a(t)]^n\,dt, \tag{X1.7}$$

where

$$B = \frac{2K_{IC}^2}{v_0\,Y^2(n-2)}. \tag{X1.8}$$

In the case of static fatigue (that is, constant applied stress σ_a) (Eq X1.6) may be integrated to determine the time to failure:

$$t_f = B\,\sigma_f^{-n}\left(\sigma_i^{n-2} - \sigma_f^{n-2}\right), \tag{X1.9}$$

and this may be further simplified to:

$$t_f = B\,\sigma_i^{n-2}\,\sigma_f^{-n}, \tag{X1.10}$$

under the assumption that $\sigma_i / \sigma_f >> 1$ (that is, that the inert strength is much higher than the strength in a corrosive environment). Rearranging and taking logarithms, it is found that:

$$\log t_f = -n\log\sigma_f + \log B + (n-2)\log\sigma_i \tag{X1.11}$$

or simplified to [Eq 2]:

$$\log t_f = -n\log\sigma_f + \log D_S\,. \tag{X1.12}$$

NOTE X1.1—For constant stress testing σ_f is identical to σ (the applied stress at failure), and these are used interchangeably.

The fatigue parameters n and D_s may be obtained from the slope and intercept of the failure time as a function of fatigue strength in a log-log plot. For comparing various materials and conditions, Eq X1.11 is often rearranged in the following way (**31**):

$$\log(t\sigma_f^2) = \log B + (n-2)\log\left(\frac{\sigma_i}{\sigma_j}\right). \tag{X1.13}$$

Similarly the modified power law Eq X1.4 can be used to yield the following expression for the time to failure:

$$t_f = \left[\frac{2}{AY^n(n-2)}\right]\sigma_f^{-n}\,a_i^{\frac{2-n}{2}}\exp\left(\frac{Q}{RT}\right) \tag{X1.14}$$

Taking logarithms and rearranging Eq X1.14 may be used to determine the fatigue parameter n. Notice that in this formulation the intercept determined by regression analysis will contain different parameters than the D_S determined above.

Exponential v-K Relationship

Alternatively an exponential relationship between v and K, which is easier to reconcile with fundamental aspects of SCG is given by (**32**):

$$v = A\exp\left[n\left(\frac{K_{Ia}}{K_{IC}}\right)\right], \tag{X1.15}$$

or in a more detailed version (**33**):

$$v = a'\exp\left[-\left(\frac{E^*}{RT}\right)\right]\exp\left[b\left(\frac{K_{Ia}}{RTK_{IC}}\right)\right], \tag{X1.16}$$

where a' and b are the material-dependent fatigue parameters.

The necessary time-to-failure equations may be developed using this exponential relationship, see Ref (**33**). For the static fatigue case, the resulting equation is:

$$t_f\,\frac{2a}{K_i^2\,d}\exp\left(-\frac{E^*}{RT}\right)\lim_{K_i}^{K_{IC}} K_{Ia}\exp\left(-\frac{bK_{Ia}}{RT}\right)dK, \tag{X1.17}$$

where a' and b are the fatigue parameters previously defined, a is the final crack length, and K_i is the initial stress intensity factor calculated from the initial crack length and applied load (**31**). The necessary time-to-failure equations may be developed using numerical solutions of these exponential relationships (**23**). The resulting equation for the crack velocity expression of Eq X1.15 is:

$$\ln t_f = -\left[\frac{n}{\sigma_i}\right]\sigma_a + \chi \tag{X1.18}$$

where $\chi = \ln\frac{a_i}{A} + \beta$ with β being a weak function of n.

In the same way, the resulting time to failure for the crack velocity equation of Eq X1.16 is:

$$\ln t_f = -\left[\frac{b}{RT\,\sigma_i}\right]\sigma_a + \chi' \tag{X1.19}$$

where

$$\chi' = \ln\left[\frac{a_i}{a'}\right] + \frac{E^*}{RT} + \beta \tag{X1.20}$$

Therefore, SCG parameters can be conveniently determined from slope and intercept through a linear regression analysis of $\ln t_f$ versus σ_a together with known parameters. However, the above approach requires that the inert strength be known *priori* to determine the major SCG parameter n or b (see Eq X1.17 or Eq X1.18), which is a significant drawback as compared with the power-law formulation (**33**).

No a Priori Assumption of the v-K Relationship

Recently Gupta, et al., (**34**) citing early unpublished work by Fuller, presented an analysis deriving the *v-K* relationship from the applied stress and the time to failure without any prior assumption on the functional form. The approach was necessitated for the extrapolation of static fatigue data for optical glass fibers into a region of long failure times or low stresses, in which the power law and the exponential law diverge by several orders of magnitude (**35**).

Acknowledging this analysis, Eq X1.2 may be rewritten as:

$$\frac{dt}{dK} = \frac{K_{Ia}}{(Y\sigma)^2\,v}\,, \tag{X1.21}$$

and the time to failure can be determined as:

$$t_f = \frac{2}{(Y\sigma)^2}\lim_{K_i}^{K_{IC}}\left(\frac{K}{V}\right)dK. \tag{X1.22}$$

Gupta, et al. obtained $v(K)$ by taking the partial derivative of this expression with respect to K_i at fixed a_i, with the result being:

$$v(K_i) = \frac{\left[\frac{-2}{t_f}\right]\left[\frac{K_{IC}}{Y\sigma_i}\right]^2}{2 + \frac{d(\ln t_f)}{d(\ln\sigma_f)}}\,. \tag{X1.23}$$

This approach requires the measurement of the inert strength and the fracture toughness, and then applying these, the crack velocity v can be obtained for measuring the time to failure at different applied stresses.

X2. ESTIMATION OF SCATTER IN TIME TO FAILURE IN CONSTANT STRESS ("STATIC FATIGUE") TESTING WITH RESPECT TO ESTIMATED SCATTER IN STRENGTH IN CONSTANT STRESS-RATE ("DYNAMIC FATIGUE") TESTING (1-3)

Strength distribution of most advanced ceramics can be described typically with the two-parameter Weibull function as follows:

$$\ln\ln\frac{1}{1-F} = m\ln\sigma_f - m\ln\sigma_o \quad \text{(X2.1)}$$

where:

F = failure probability,
m = Weibull modulus,
σ_f = fracture strength, and
σ_o = characteristic strength.

Solving for $\ln\sigma_i$ in Eq X2.1 with $\sigma_f \equiv \sigma_i$ (for inert flexure strength) and substituting into Eq X1.11 after taking natural logarithms of both sides of Eq X1.11 yields

$$\ln\ln\frac{1}{1-F} = \left[\frac{m}{n-2}\right]\ln t_f - \frac{m}{n-2}\ln\left[B\sigma_0^{n-2}\,\sigma^{-n}\right] \quad \text{(X2.2)}$$

In the same way, for constant stress-rate testing, solving for $\ln\sigma_i$ with $\sigma_f \equiv \sigma_i$ (for inert strength) the following equation is obtained:

$$\ln\ln\frac{1}{1-F} = \left[\frac{m(n+1)}{n-2}\right]\ln\sigma_f - \frac{m(n+1)}{n-2}\ln\left[B(n+1)\sigma_0^{n-2}\,\sigma\right]^{\frac{1}{n+1}} \quad \text{(X2.3)}$$

Therefore, both the *equivalent* Weibull modulus (m_{es}) of the plot of failure probability versus time to failure in constant stress testing (Eq X2.2) and the *equivalent* Weibull modulus (m_{ed}) of the plot of failure probability versus fracture strength in constant stress-rate testing (Eq X2.3) are:

$$m_{es} = \frac{m}{n-2}\,;\, m_{ed} = \frac{m(n+1)}{n-2} \quad \text{(X2.4)}$$

Therefore, from the *equivalent* Weibull moduli of Eq X2.2, a relationship can be found

$$\frac{m_{es}}{m_{ed}} = \frac{1}{n+1} \quad \text{(X2.5)}$$

Therefore, equivalent Weibull modulus of time to failure in constant stress testing is expected to be $1/(n+1)$ times that of fracture strength in constant stress-rate testing. In other words, for a given material/environment the scatter in time to failure would be $(n+1)$ times greater than the scatter (typically m = 10 to 15 for most advanced ceramics) in fracture strength. The scatter in time to failure would be significantly amplified since the SCG parameter n is typically greater than 20 for most advanced ceramics. The relation of Eq X2.5 thus verifies that a significant variation in time to failure is exacerbated in constant stress testing for advanced monolithic ceramics, as observed experimentally.

X3. A SIMPLIFIED PREDICTION OF TIME TO FAILURE IN STRESS ("STATIC FATIGUE") TESTING BASED ON SCG DATA OBTAINED FROM CONSTANT STRESS-RATE ("DYNAMIC FATIGUE") TESTING

In this appendix, a simplified prediction of time to failure as a function of applied stress in constant stress testing is made based on the SCG data determined from constant stress-rate testing. This prediction, although theoretical, allows one to help finding an approximated relationship between time to failure and applied stresses so that the range and number of applied stresses can be quickly and reasonably well selected together with to-be-prescribed run-out times.

From dynamic fatigue testing it was obtained: (See Test Method C1368)

$$B\sigma_i^{n-2} = \frac{D_d^{n+1}}{n+1} \quad \text{(X3.1)}$$

Substitute Eq X3.1 into the time to failure (t_{fs}) equation to yield

$$t_f = \left[\frac{D_d^{n+1}}{n+1}\right]\sigma^{-n} \quad \text{(X3.2)}$$

where:

$$Log\ D_d = \log(B\sigma_i^{n-2}) \quad \text{(X3.3)}$$

Therefore, from the relationship in Eq X3.2, once the parameters n and Dd of a test material for a given environment are known from constant stress-rate testing, time to failure in constant stress testing can be easily estimated as a function of stress to be applied in the same environment. It would be more convenient to use Eq X3.2 if the equation is plotted. There might be some discrepancies in the estimation; however, this prediction still can significantly reduce many uncertainties and trial-and-errors, associated with choices of the run-out time and the number and range of applied stresses to be employed.

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