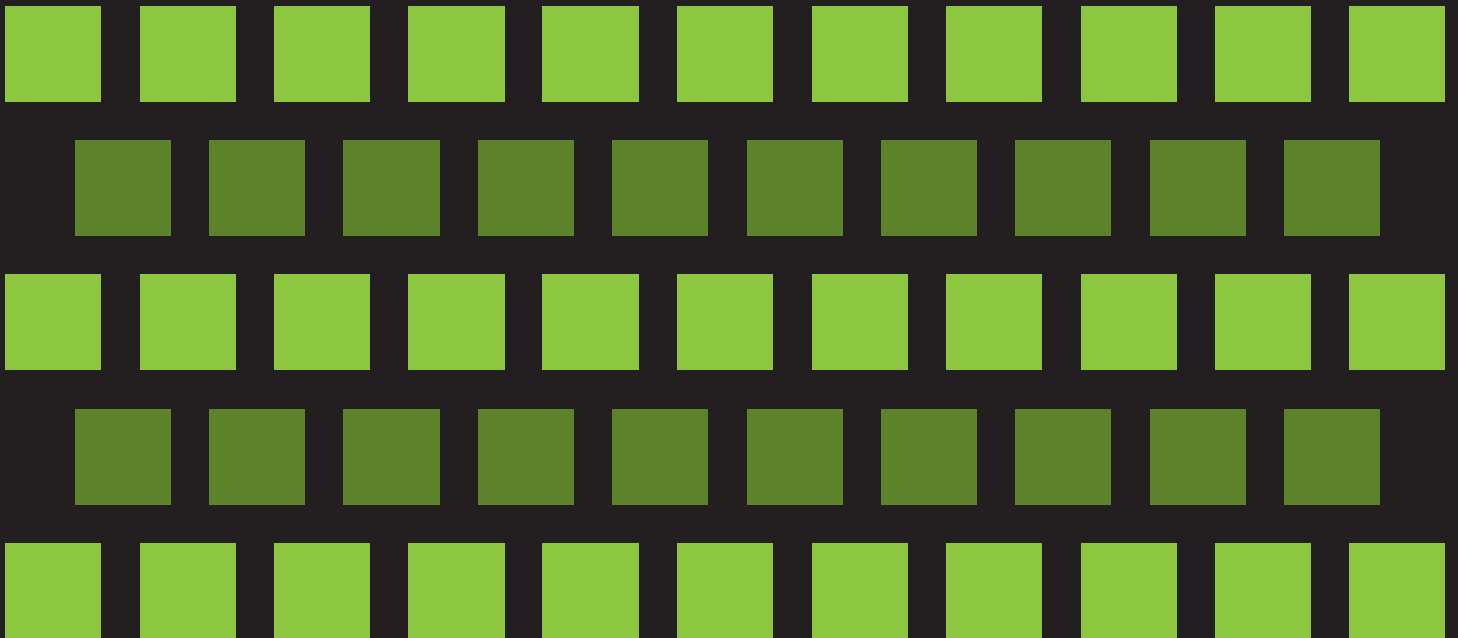


CRITERIA DOCUMENT FOR SHELL-AND-TUBE HEAT EXCHANGERS ACCORDING TO PART UHX OF ASME SECTION VIII DIVISION 1



STP-PT-053

CRITERIA DOCUMENT FOR SHELL-AND-TUBE HEAT EXCHANGERS ACCORDING TO PART UHX OF ASME SECTION VIII DIVISION 1

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FOREWORD

The purpose of this document is to justify and provide technical criteria for the rules of Part UHX of ASME Section VIII Division 1, 2010 Edition, devoted to the design of U-tube, Fixed and Floating head Tubesheet Heat Exchangers. Confirmation and documentation of the basis for UHX-rules is important for the SG-HTE members for future reference and developments or if additional confirmation or comparisons are required. It will be a valuable reference for early career engineers that are using the UHX rules or becoming involved in Code developments of such rules in the future.

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ABSTRACT

The purpose of this document is to justify and provide technical criteria for the rules of Part UHX of ASME Section VIII Division 1, 2010 Edition, devoted to the design of U-tube, Fixed and Floating head Tubesheet Heat Exchangers. The criteria document applies also to paragraph 4.18 of Section VIII, Division 2 which is entirely based on Part UHX.

The analytical treatment of the fixed tubesheet heat exchangers is based on classical discontinuity analysis methods to determine the moments and forces that the tubesheet, tubes, shell and channel must resist.

- The heat exchanger is assumed to be a symmetrical unit with identical tubesheets on both ends.
- The Tubes are replaced by an equivalent elastic foundation.
- The Perforated tubesheet is replaced by an equivalent solid circular plate with effective elastic constants depending on the ligament efficiency of the tubesheet. This equivalent solid circular plate is treated by the theory of thin circular plates subjected to pressure and other applied loads to determine the stresses in the tubesheet
- The unperforated tubesheet rim is treated as a rigid ring.
- Its Connection to the shell and channel accounts for the edge displacements and rotations of these 3 components. Shell and channel are treated by the elastic theory of thin shells of revolution subjected to edge loads to determine the maximum stresses.

The treatment provides, at any radius of the perforated tubesheet, the deflection, the rotation, the bending and shear stresses and the axial stress in the tubes.

- A parametric study permits to determine the maximum stresses in the tubesheet and in the tubes which are given in UHX-13.
- The treatment also determines the loads and displacements acting on the shell and channel at their connection to the tubesheets and then the shell and channel stresses.
- The maximum stresses in tubesheets, tubes, shell and channel so determined are limited to the appropriate allowable stress-based classifications of Section VIII Division 2 Appendix 4, 2004 edition.

The Floating Tubesheet and U-tube Tubesheet heat exchangers are treated as simplified cases of fixed tubesheets heat exchangers.

A checking of the results obtained is provided by comparison with FEA, TEMA, French pressure vessel code CODAP. Applying the appropriate simplifications, the classical formulas for circular plates subjected to pressure have been obtained.

PART 1

Introduction

1 SCOPE

Develop a criteria document covering the development of the TS design rules for the U-tube, Fixed, and Floating Head TS HEs configurations contained in Part UHX of Section VIII Division 1, 2010 Edition. The criteria document will apply also to paragraph 4.18 of Section VIII, Division 2 which is entirely based on Part UHX.

The free body diagram of the HE, the equilibrium and compatibility equations, the solution of resulting differential equations and all intermediate steps are provided to show the derivation of:

- the deflection and the rotation at any radius of the TS,
- the bending and shear stress at any radius of the TS,
- the axial stresses in the tubes at any radius of the TS,
- the axial stretch force acting in the shell,
- the axial displacement of the shell.

The document provides the technical basis of the following items:

- the required loading case combinations,
- the acceptance criteria for each TS configuration, as applicable,
- the TS characteristics including the Effective Elastic Constants,
- the TSs extended as a flange.

The following effects are in addition to the above basic items:

- the effect of different shell material or thickness adjacent to the TS,
- the effect of plasticity at TS-shell-channel joint,
- the effect of radial differential thermal expansion between the TS and integral shell and channel,
- the tubesheet calculated as simply supported TS.

2 HISTORICAL BACKGROUND

The first rules devoted to tubular HEs were developed by TEMA [CS-1] for the first time in 1941 to design U-tube and floating TSs. The design formula was based on the formula for circular plates subject to pressure:

$$T = F \frac{G}{2} \sqrt{\frac{P}{S}}$$

This semi-empirical formula does not account either for the tubes that stiffen the TS, or for the holes that weaken it. Fixed TS HEs were covered in TEMA 1968 5th edition, based on Gardner's work.

Gardner [1][2] in 1948 for floating TSs and in 1952 for fixed TSs was the very first to set-up the basis of a more rational approach by taking into consideration the support afforded by the tubes and the weakening effect of the TS holes. This design method, which involves 15 parameters instead of 3 previously in TEMA, was adopted by TEMA in 1968 in its 5th edition and by the Stoomwezen [CS-2] in 1973.

Simultaneously, and independently, Miller [3] proposed a similar approach that was published in the British Code BS 1515 [CS-3] in 1965. These design rules have the drawback of considering the TS as either simply supported or clamped at its periphery, which compels the designer to make a more or less arbitrary choice between these two extreme cases.

Galletly [4] in 1959 overcame this issue by taking into account the degree of rotational restraint of the TS at its periphery by the shell and channel. This method was adopted by the French Pressure Vessel Code CODAP [CS-4] in 1982 for fixed and floating HEs. CODAP rules were adopted by the European Pressure Vessel Standard EN 13445 [CS-5] published in 2002.

Gardner [5] in 1969 improved his method for U-tube and floating HEs by considering the unperforated solid rim at periphery of the TS. This design method was adopted by BS 5500 [CS-6], CODAP (for U-tube) and ISO [CS-7] in the late seventies and by ASME Section VIII (Appendix AA) in 1982.

Despite these improvements, TEMA rules have been extensively used throughout the world during the last six decades as they have the merit of long satisfactory industrial experience and simplicity. However due to that simplicity (the strengthening of the tubes is assumed to counterbalance the weakening effect of the tubesheet holes), they often lead to TS overthickness if strengthening effect controls or underthickness if weakening effect controls. To-day these disadvantages increase as the chemical and power industries need larger exchangers operating at higher pressures and temperatures. For more details, see Osweiller [6].

In 1975 ASME Subcommittee VIII established a "Subgroup on Heat-Transfer Equipment (SG-HTE)" with the task of developing new rules for the design of TS HEs based on a more rigorous approach than TEMA. This was achieved by considering the perforated TS, the tubes acting as an elastic foundation, the unperforated rim and the connection of the TS with the shell and channel. The analytical treatment is based on Soler's book [7].

In 1992 ASME and CODAP decided to reconcile their rules (scope, TS configurations, loading cases notations, ligament efficiencies, effective elastic constants, design formulas), as they were based on the same approach. For more details, see Osweiller [8]. The 1st ASME draft was issued in 1985.

From 1992 to 2002 ASME tubesheet rules have been published in non-mandatory Appendix AA of Section VIII-Div.1 so that manufacturers can use them as an alternative to TEMA rules.

In 2003 these rules were upgraded to mandatory status and published in 2003 Addenda as a new Part UHX of Section VIII Div. 1, UHX standing for Unfired Heat Exchanger. As of January 1, 2004 a designer does not have the option of using the TEMA rules.

UHX design rules cover essentially the design by formulae of the heat exchanger pressure containing components. For other aspects such as fabrication, tube-to-tubesheet joints, inspection, maintenance, repair, troubleshooting, etc. see Reference [9]. TEMA [CS-1] also covers similar aspects of design such as minimum component thickness (e.g., baffle plates, pass partitions, etc.), tube vibration, etc.

3 TYPES OF HEAT EXCHANGERS COVERED

UHX rules apply to the three typical types of tubular HEx:

- **U-Tube Heat Exchanger:** HE with one stationary TS attached to the shell and channel. The HE contains a bundle of U-tubes attached to the TS, as shown in Figure 1 sketch (a).
- **Fixed Tubesheet Heat Exchanger:** HE with two stationary TSs, each attached to the shell and channel. The HE contains a bundle of straight tubes connecting both TSs, as shown in Figure 1 sketch (b).
- **Floating Tubesheet Heat Exchanger:** HE with one stationary TS attached to the shell and channel, and one floating TS that can move axially. The HE contains a bundle of straight tubes connecting both TSs, as shown in Figure 1 sketch (c).

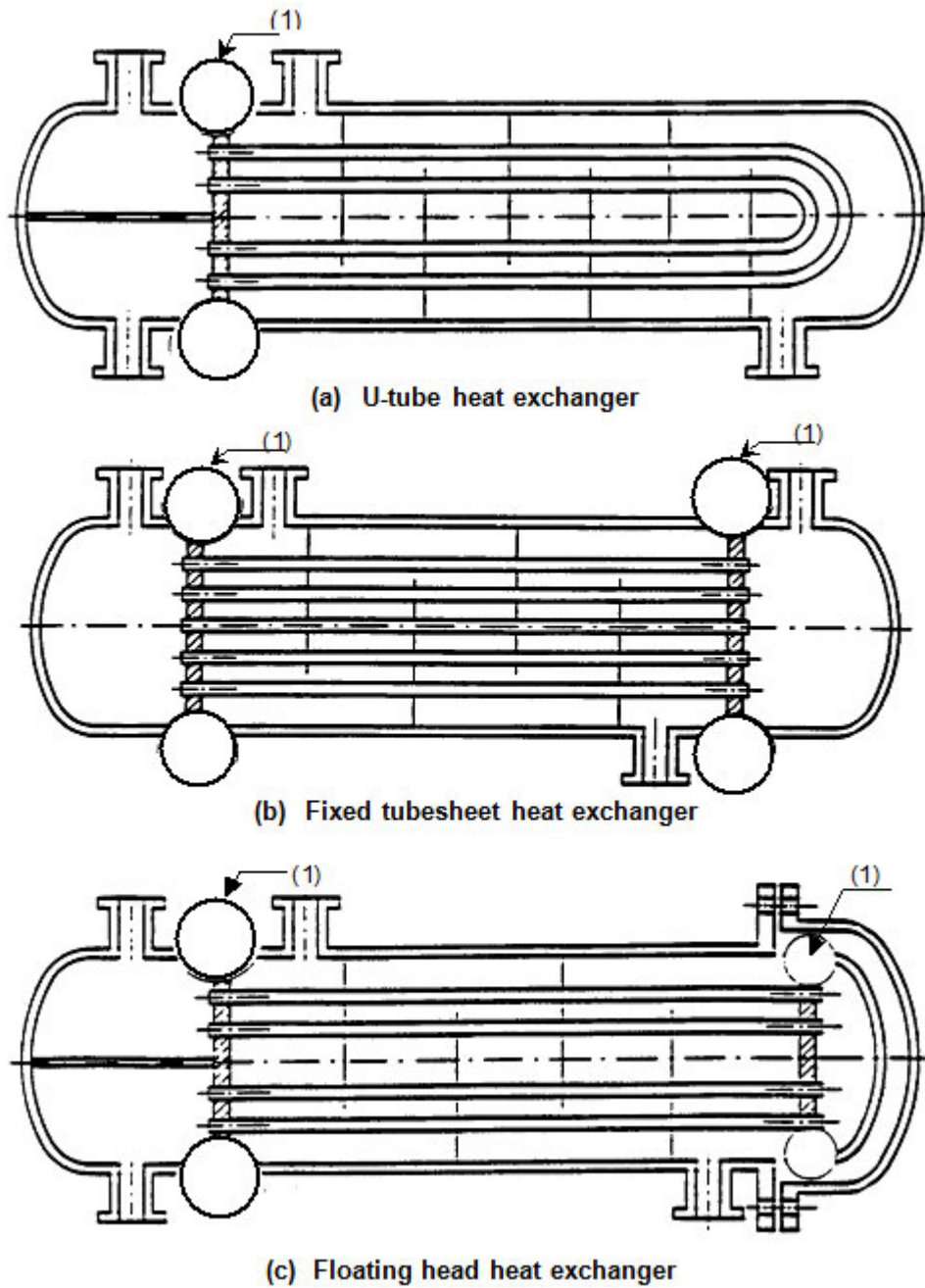


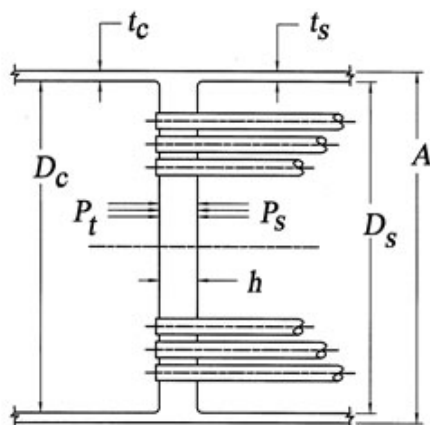
Figure 1—Three types of tubesheet heat exchangers

¹ Configurations of tubesheet – shell – channel connections are detailed in III-2.

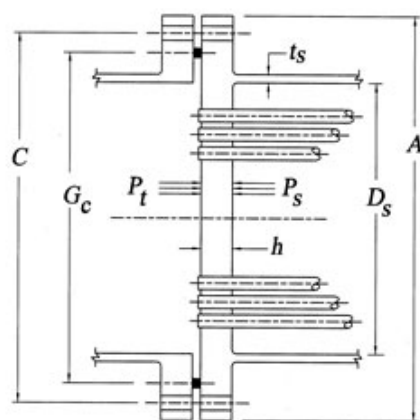
4 TYPES OF TS CONFIGURATIONS

The TS is attached to the shell and channel either by welding (integral TS) or by bolting (gasketed TS) according to 6 configurations encountered in the industry (see Figure 2):

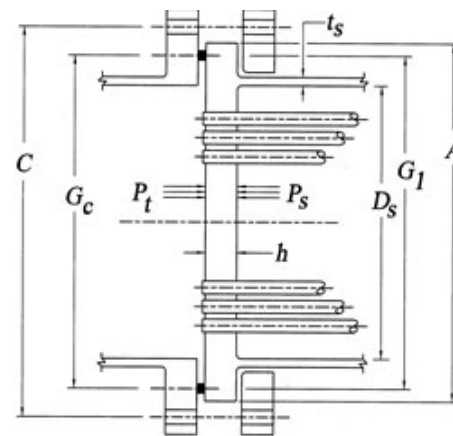
- configuration a: tubesheet integral with shell and channel;
- configuration b: tubesheet integral with shell and gasketed with channel, extended as a flange;
- configuration c: tubesheet integral with shell and gasketed with channel, not extended as a flange;
- configuration d: tubesheet gasketed with shell and channel, extended as a flange or not
- configuration e: tubesheet gasketed with shell and integral with channel, extended as a flange;
- configuration f: tubesheet gasketed with shell and integral with channel, not extended as a flange.



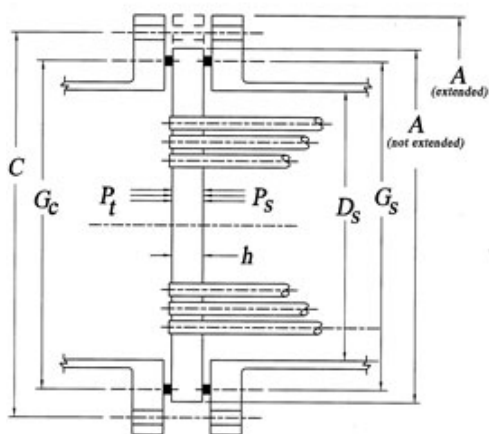
(a) Configuration a:
Tubesheet integral with shell and channel



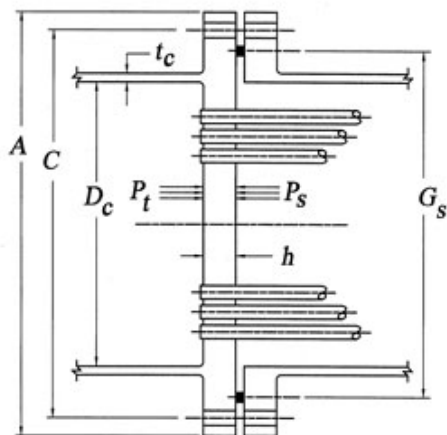
(b) Configuration b:
Tubesheet integral with shell and gasketed with
channel, extended as a flange



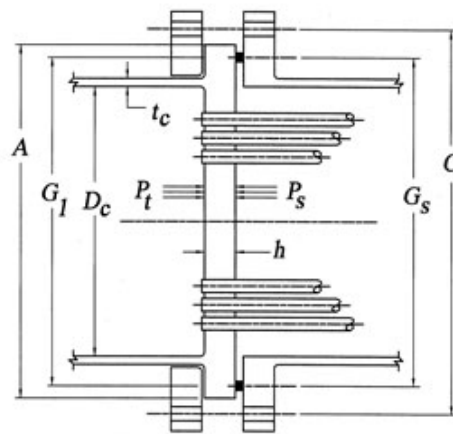
(c) Configuration c:
Tubesheet integral with shell and gasketed with
channel, not extended as a flange



(d) Configuration d:
Tubesheet gasketed with shell and channel



(e) Configuration e:
Tubesheet gasketed with shell and integral with
channel, extended as a flange



(f) Configuration f:
Tubesheet gasketed with shell and integral with
channel, not extended as a flange

Figure 2—Tubesheet configurations

5 LOADING CASES

The normal operating condition of the HE is achieved when the tube side pressure P_t and shell side pressure P_s act simultaneously. However, a loss of pressure or a loss of temperature is always possible. Accordingly, for safety reasons, the designer must always consider the cases where $P_s=0$ and $P_t=0$ for the normal operating conditions.

He must also consider the startup conditions, the shutdown conditions and the upset conditions, if any, which may govern the design.

A TS HE is a statically indeterminate structure for which it is generally not possible to determine the most severe condition of coincident pressure and temperature. Thus, it is necessary to evaluate all the anticipated loading conditions mentioned above to ensure that the worst load combination has been considered in the design.

For each of these conditions, the following 3 pressure loading cases must be considered.

- Loading Case 1: Tube side pressure P_t acting only ($P_s = 0$).
- Loading Case 2: Shell side pressure P_s acting only ($P_t = 0$).
- Loading Case 3: Tube side pressure P_t and shell side pressure P_s acting simultaneously.

For fixed TS HEs, the axial differential thermal expansion between tubes and shell has to be considered and additional thermal loading cases must be added (loading cases 4, 5, 6, 7) as they act simultaneously with the pressure loading cases.

When vacuum exists, each loading case shall be considered with and without the vacuum.

When differential pressure design is specified by the user, the design shall be based only on loading case 3.

As the calculation procedure is iterative, a value h is assumed for the tubesheet thickness to calculate and check that the maximum stresses in tubesheet, tubes, shell, and channel are within the maximum permissible stress limits.

Because any increase of tubesheet thickness may lead to overstresses in the tubes, shell, or channel, a final check shall be performed, using in the formulas the nominal thickness of tubesheet, tubes, shell, and channel, in both corroded and uncorroded conditions.

6 STRUCTURE OF PART UHX

- UHX-1 to UHX-8 provide general considerations (Scope, Material, Fabrication, Terminology,...) which are common to the 3 types of HEs.
- UHX-9 provide design rules for TS flange extension
- UHX-10 (Conditions of Applicability) specifies under which conditions the rules are applicable
- UHX-11 (TS Characteristics) is also common to the 3 types of HEs and provides the design formulae for the ligament efficiency and the effective elastic constants.
- UHX-12, UHX-13 and UHX-14 provide the design rules for U-tube, Fixed and Floating TS HEs.

These 3 chapters are self-supporting and structured in the same way:

- (1) Scope
- (2) Conditions of Applicability
- (3) Notations
- (4) Design Considerations
- (5) Calculation Procedure

Additional rules are provided to cover more specific calculations:

- (1) Effect of different shell material or thickness adjacent to the TS
 - (2) Effect of plasticity at tubesheet-shell-channel joint
 - (3) Effect of radial thermal expansion adjacent to the TS
 - (4) Calculation of TS when considered as simply supported
 - (5) Calculation of TS flange extension
- UHX-15 to UHX-19 provide considerations on Tube-to-Tubesheet Welds, Expansion Bellows, Pressure Tests and Marking.
 - UHX-20 provides design examples for each type of HE.

7 STRUCTURE OF THE DOCUMENT

The report is structured in 6 PARTS.

- PART 1: Introduction (purpose, background, general issues)
- PART 2: Tubesheet Characteristics (ligament efficiencies, effective elastic constants)
- PART 3: Analytical treatment of Fixed TS HEs
- PART 4: Analytical treatment of Floating TS HEs
- PART 5: Analytical treatment of U-tube TS HEs
- PART 6: Conclusions

Each PART is independent with basically the same chapters: Scope, Historical Background, Notations, Configurations covered, Design assumptions, Analytical treatment.

The order of the analytical treatment is based on the complexity of the HE model. The Fixed TS HE is treated first, because it is the most complex. The Floating TS HE treated second, since it is a simplified case of the Fixed TS. The U-Tube TS HE is treated last, since it is a simplified case of the Floating TS.

The detailed structure is given in the Table of Contents.

The report provides the derivation of UHX design rules (UHX-1, UHX-9 to 14, UHX-17 and 20) and refers explicitly to these as necessary. Other UHX rules (UHX-2, 3 and 4, UHX-15 to 19), which are not linked to design formulas are not covered.

8 NOTATIONS

Notations are common to the 3 types of HEs. They are detailed in PART 3, Clause I.2.

REFERENCES—PART 1

Technical Papers

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- [4] GALLETTY “Optimum design of Thin Circular Plates on an Elastic Foundation” , Proceedings of the Institution of Mechanical Engineers”, London Vol. 173 , 1952
- [5] GARDNER “TS Design: a Basis for Standardization” , Pressure Vessel Technology Conference Deft – 1969.
- [6] OSWEILLER “Analysis of TEMA Tubesheet Design Rules – Comparison with-up-to date Code Methods”
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- [8] SOLER “Mechanical Design of Heat Exchangers” – Arcturus publishers -1984 – 1047 pages.
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Codes & Standards (CS)

- [CS-1] TEMA: Standards of Tubular Exchangers Manufacturers Association - 9th Edition 2007– Chapter R7 “TS”
- [CS-2] STOOMWEZEN: Dutch Code for Pressure Vessels – 1985 Edition -Sheet D0403 “Wall thickness calculation of TSs”
- [CS-3] BS 5515: Unfired Fusion Welded Pressure Vessels – 1965 Edition - Chapter 3.9 "Flat HE TSs”
- [CS-4] CODAP: French Code for Unfired pressure Vessels 2005 Edition – Chapter C7 “Design rules for HE TSs”
- [CS-5] EN 13 445: European Standard for Unfired Pressure Vessels – 2002 Edition – Chapter 13 “HE TSs”
- [CS-6] BS 5500: Unfired Fusion Welded Pressure Vessels – 1976 Edition – Chapter 3.9 “Flat HE TSs”
- [CS-7] ISO DIS 2694: Pressure Vessels – Draft 1973 Edition - Chapter 30 “ Flat HE TSs”

PART 2

Tubesheet Characteristics

1 SCOPE (UHX-11.1)

PART 2 provides the technical basis for the determination of:

- the ligament efficiencies μ and μ^*
- the effective elastic constants E^* and

which are given in Clause 11 of Part UHX.

These quantities are important as they enable the replacement of the actual perforated TS by an equivalent solid plate, which is necessary to develop the analytical treatment.

2 NOTATIONS

Notations are taken from UHX-11.3 and are repeated here for convenience.

A_L = total area of untubed lanes = $U_{L1} L_{L1} + U_{L2} L_{L2} + \dots$ limited to $4D_o p$

c_s = tubesheet corrosion allowance on the shell side

c_t = tubesheet corrosion allowance on the tube side

D_o = equivalent diameter of outer tube limit circle [see Figure 6]

d = diameter of tube hole

d_t = nominal outside diameter of tubes

d^* = effective tube hole diameter

E = modulus of elasticity for tubesheet material at tubesheet design temperature

E_{IT} = modulus of elasticity for tube material at tubesheet design temperature

E^* = effective modulus of elasticity of tubesheet in perforated region

h = tubesheet thickness

h_g = tube side pass partition groove depth [see Figure 9]

h'_g = effective tube side pass partition groove depth

$L_{L1}, L_{L2}, \dots$ = length(s) of untubed lane(s) (see Figure 7)

ℓ_{tx} = expanded length of tube in tubesheet ($0 \leq \ell_{tx} \leq h$) [see Figure 8].

p = tube pitch

p^* = effective tube pitch

r_o = radius to outermost tube hole center [see Figure 6]

S = allowable stress for tubesheet material at tubesheet design temperature

S_{IT} = allowable stress for tube material at tubesheet design temperature

t_t = nominal tube wall thickness

$U_{L1}, U_{L2}, \dots$ = center-to-center distance between adjacent tube rows of untubed lane(s), limited to $4p$ (see Figure 7)

μ = basic ligament efficiency for shear

μ^* = effective ligament efficiency for bending

ν^* = effective Poisson's ratio in perforated region of tubesheet

ρ = tube expansion depth ratio = ℓ_{tx}/h , ($0 \leq \rho \leq 1$)

3 DESIGN ASSUMPTIONS (UHX-11.2)

The perforated TS is assumed to be uniformly perforated in triangular or square pattern.

4 LIGAMENT EFFICIENCIES (UHX-11.5.1)

4.1 Introduction

The TS treatment necessitates the replacement of the actual perforated TS by an equivalent unperforated solid plate of same diameter. The ligament efficiency accounts for the fact that the shear load and the bending moment calculated in this unperforated solid plate must be corrected as they apply only to the ligament located between two adjacent holes in the actual TS.

The ligament is represented by the hatched area in Figure 3. Its length varies from $\alpha\alpha' = p - d$ to $\delta\delta' = p$.

Its minimum length $\alpha\alpha' = p - d$ leads to a basic ligament efficiency $\mu = \frac{p - d}{p} = 1 - \frac{d}{p}$

So as to be on the safe side this formula is generally used to calculate the shear stress in the actual TS.

This means that, for the usual pattern ($p = 1.25$ and $d = 1.0$) $\mu = 0.2$, the shear stress in the actual TS will be 5 times the shear stress calculated in the equivalent solid plate.

Among all the ligaments located at radius r of the TS, one can reasonably assume that at least one of length $\alpha\alpha'$ is radially oriented as shown in Figure 4. Therefore the radial bending moment $M(r)$ calculated in the equivalent TS applies only to the ligament between the two holes, which means that the bending moment in the actual TS must be multiplied by the ratio $\omega\omega' / \alpha\alpha'$ or divided by the minimum bending ligament efficiency:

$$\mu^* = \frac{\alpha\alpha'}{\omega\omega'} = \frac{p - d}{p}$$

Its mean length $\beta\beta'$ leads to a higher ligament efficiency value:

$$\mu^* = \frac{pd - \pi d^2 / 4}{pd} = 1 - \frac{\pi}{4} \cdot \frac{d}{p} = 1 - 0.785 \frac{d}{p}$$

For the usual pattern above: $\mu^* = 0.372$, which means that the calculated bending moment in the actual TS would be almost half the value of that using the minimum ligament efficiency of 0.2 above. These two extreme examples show the significant impact of the ligament efficiency on the stresses obtained in the actual TS.

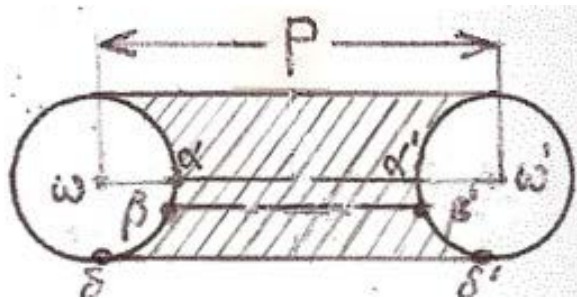


Figure 3—Ligament area in the actual tubesheet

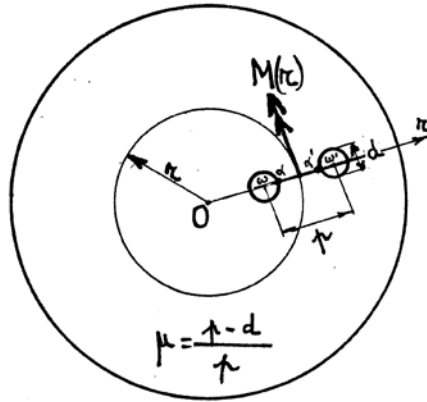


Figure 4—Ligament orientation in the actual tubesheet

4.2 Historical Background

Various formulas have been used for the determination of the bending LE in C&S.

- **BS5500 [CS-1], CODAP [CS-2], EN 13445 [CS-3], ISO [CS-4] and STOOMWEZEN [CS-5]** have used the minimum LE when the tubes are welded only: $\mu^* = \frac{p-d}{p}$.

When the tubes are expanded throughout the full depth of the TS, experimental tests have shown that about half of the thickness participates to the TS strength, which leads to:

$$\mu^* = \frac{p - (d - t_t)}{p}$$

- **TEMA [CS-6] uses the mean LE** based on the ratio of the hole area, $s_0=\pi d^2/4$, to the portion of TS area “s” pertaining to that hole, as shown in Figure 5: $\mu^*=1-\frac{S_o}{S}$ (μ^* quoted η in TEMA). This leads to:

For square pattern: $s = p^2$ and $\mu^* = 1 - \frac{\pi}{4} \left(\frac{d}{p} \right)^2 = 1 - \frac{0.785}{(p/d)^2}$

$$\text{For triangular pattern: } s = p^2 \sin(60^\circ) \quad \text{and} \quad \mu^* = 1 - \frac{\pi}{4} \frac{d^2}{p^2 \sin(60^\circ)} = 1 - \frac{0.907}{(p/d)^2}$$

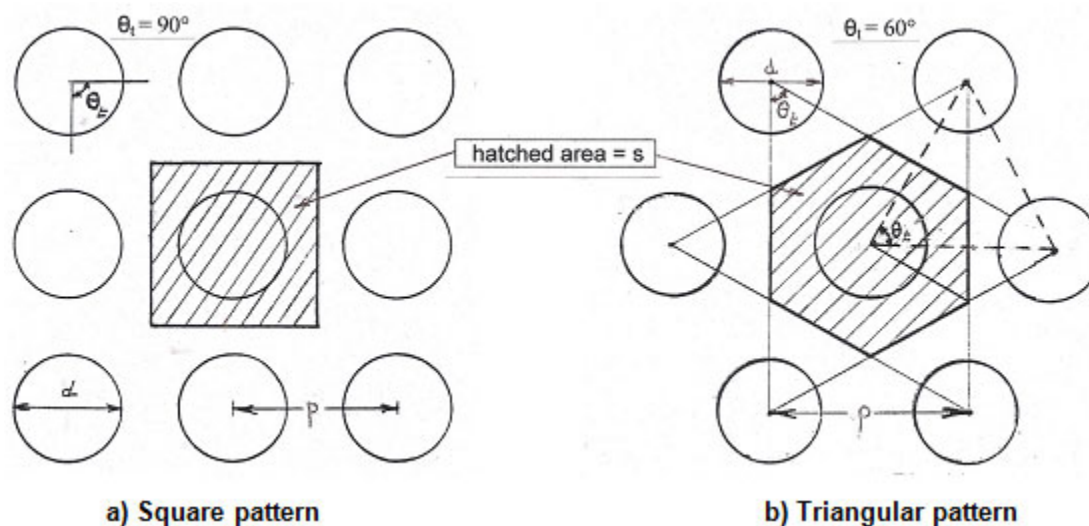


Figure 5—Ligament efficiency used in TEMA

For shear, these C&S use the minimum ligament efficiency: $\mu = \frac{p-d}{p}$

4.3 LE in Part UHX (UHX-11.5.1)

- (a) **Equivalent diameter D_o** of the perforated TS is defined as the equivalent diameter of the outer tube limit circle, calculated from the radius r_o of the outermost tube hole center (see Figure 6):

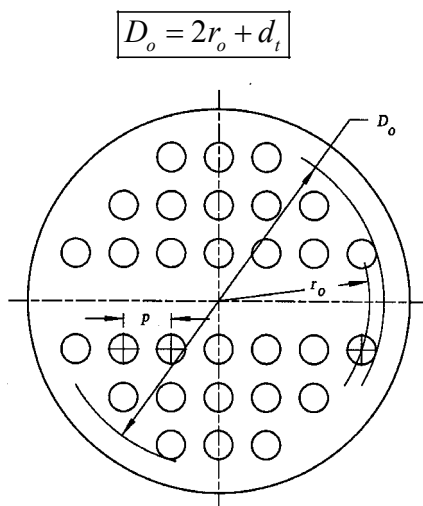


Figure 6—TS equivalent diameter D_o

The diameter D_o corresponds to the similar concept used in TEMA for the equivalent diameter DL used for the determination of shear stress and D_o and DL will have about the same values.

(b) Basic ligament efficiency for shear load

Once the shear load has been determined in the equivalent TS, it must be corrected to account for the holes in the actual TS, by applying the minimum ligament efficiency μ :

$$\mu = \frac{p-d}{p}$$

(c) **Effective ligament efficiency for bending moment**

Once the bending moment has been determined in the equivalent TS, it must be corrected to account for the holes in the actual TS, by applying the bending effective ligament efficiency μ^* defined as follows:

$$\mu^* = \frac{p^* - d^*}{p^*}$$

Effective pitch p^*

- If the TS is uniformly perforated $p^*=p$
- If the TS has an unperforated lane of area $A_L=U_L L_L$ as shown in Figure 7a, the N_t tubes are redistributed

so that the equivalent TS is uniformly perforated over the area $\pi \frac{D_o^2}{4}$ with an equivalent pitch p^* . This is necessary as the analytical treatment is performed for a uniform array of tubes.

Assuming that the portion of TS area pertaining to each hole is p^2 , p^* is obtained from the equation:

$$N_t p^{*2} \pi \frac{D_o^2}{4} = N_t p^2 \left(\pi \frac{D_o^2}{4} - A_L \right)$$

which leads to:

$$p^* = \frac{p}{\sqrt{1 - \frac{4A_L}{\pi D_o^2}}}$$

So as not to deviate too much from the assumption of uniform pattern, the width U_L of the untubed lane in Figure 7a is limited to $4p$ over the diameter D_o , i.e. an area $4pD_o$, which leads to: $A_L \leq 4pD_o$. This condition of applicability appears explicitly in the definition of A_L in UHX-11.3. It has been set-up after discussions among ASME and CODAP experts, based on sound engineering practice, and does not have theoretical basis.

If there are several untubed lanes of widths U_{L1} , U_{L2} , U_{L3} ,... and lengths L_{L1} , L_{L2} , L_{L3} ,... as shown in Figure 7b, the limitation is still maintained to $4pD_o$. Accordingly, p^* becomes:

$$p^* = \frac{p}{\sqrt{1 - \frac{4 \text{MIN}[(A_L), (4pD_o)]}{\pi D_o^2}}}$$

If there is no untubed lane, $A_L=0$ and $p^*=p$.

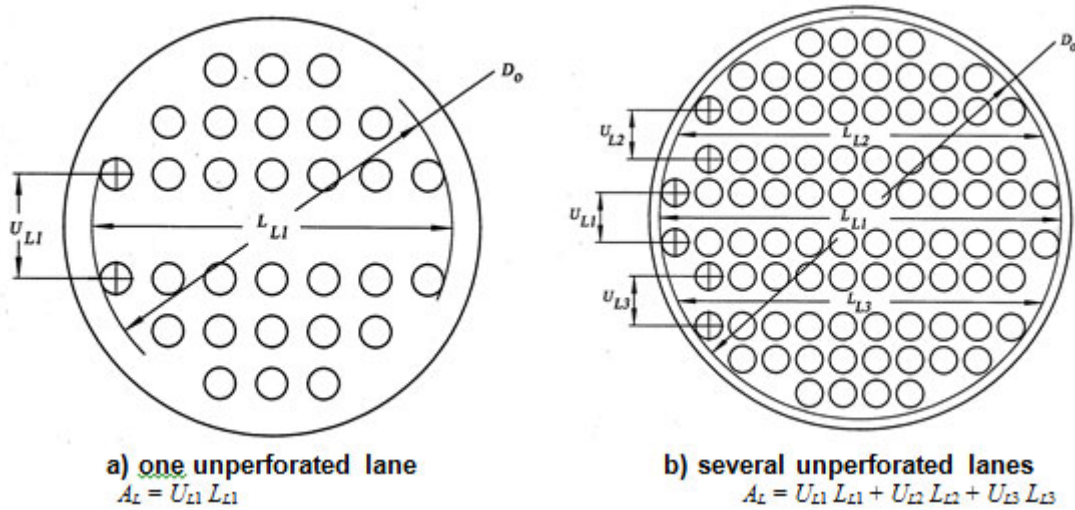


Figure 7—TS with unperforated lanes

Effective tube hole diameter d^*

- When the tubes are welded only: $d^* = d_t$
- When the tubes are expanded into the TS, feedback from HE manufacturers have shown that the bending strength of the TS is increased by the degree of tube expansion ρ (see Figure 8).

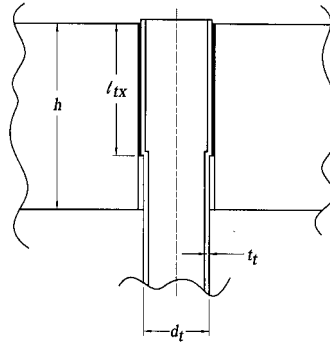


Figure 8—Tube expansion depth ratio $\rho = l_{tx}/h$

The degree of increased strength is also dependent on the difference between the TS and tube material properties. Finally, the effective tube hole diameter writes:

$$d^* = d_t - 2t_t \left(\frac{E_{tT}}{E} \right) \left(\frac{S_{tT}}{S} \right) \rho$$

where ρ is the tube expansion ratio: $\rho = l_{tx} / h$. This formula was proposed for the first time in the 1980 Edition of ASME Section VIII-Division 2.

- If the tubes are welded only: $\rho = 0$ and $d^* = d_t$
- If the tubes are fully expanded: $\rho = 1$ and $d^* = d_t - 2t_t$ if the TS and tubes are made of the same materials.

The effective diameter cannot be less than the inside tube diameter $d_t - 2t_t$, which leads to:

$$d^* = \text{MAX} \left\{ \left[d_t - 2t_t \left(\frac{E_{tT}}{E} \right) \left(\frac{S_{tT}}{S} \right) \rho \right], [d_t - 2t_t] \right\}$$

Values of μ^* are generally comprised between 0.25 and 0.4.

To illustrate the difference between the TEMA method and the Part UHX methods for determining the bending ligament efficiency, consider a tube ($d_t=1.0$, $t_t=0.0625$, $p=1.25$) that is the same material as the tubesheet.

- If the tube is expanded throughout the full depth of the tubesheet, then $\mu^* = 0.304$
- if the tube is welded and not expanded at all, then $\mu^* = 0.20$.

However, the TEMA ligament efficiency is 0.420 and 0.498 for triangular and square pitch layouts respectively, regardless of whether the tubes are expanded or not.

The ligament efficiency has a direct bearing on the calculated tubesheet stress. A smaller ligament efficiency results in a larger predicted tubesheet stress and a larger ligament efficiency results in a smaller predicted tubesheet stress. Thus, as may be seen, if the same basic theory is used to determine the stress in a plate, then the TEMA ligament efficiency would result in a smaller calculated stress as compared to the ASME method, even when the full tube wall is considered. This difference is exacerbated when the tube is not expanded.

(d) Effective tube side pass partition groove

When there is no pass partition groove on tube side of a TS of nominal thickness h_n , the TS corroded thickness $h_{\min}$ is given by: $h_{\min}=h_n-c_s-c_t=h$, as h is the corroded TS thickness obtained by calculation in Part UHX.

When there is a pass partition groove of depth h_g , it is assumed that the bottom of the groove does not corrode.

Two cases are possible:

- If $h_g < c_t$ (see Figure 9a): $h_{\min}=h_n-c_s-c_t=h$
- If $h_g > c_t$ (see Figure 9b), a correction for the groove depth in excess of the tube side corrosion is necessary: $h_{\min}=h_n-c_s-c_t-(h_g-c_t)=h-(h_g-c_t)$

In both cases $h_{\min}$ can be written: $h_{\min}=h-h'_g$ which will be used to calculate the TS bending stress where:

$$h'_g = \text{MAX} \left[(h_g - c_t), (0) \right]$$

The same formula is used in TEMA.

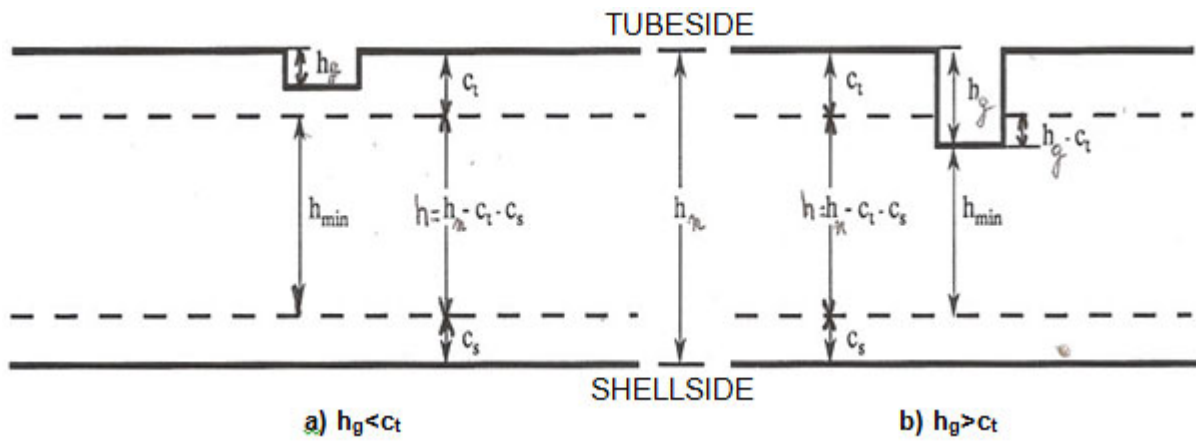


Figure 9—Pass partition groove on tube side of the TS

5 EFFECTIVE ELASTIC CONSTANTS (UHX-11.5.2)

5.1 Introduction

The analytical treatment of TS HEs necessitates the replacement of the actual perforated TS of Modulus of elasticity E and Poisson's ratio ν by an equivalent solid plate of Modulus of elasticity E^* and Poisson's ratio ν^* . These EEC E^* and ν^* are determined so that the equivalent TS has the same mechanical behavior as the actual TS when subjected to the same loading. Due to the weakening effect of the holes, E^* is lower than E and E^*/E is always lower than 1. Values of ν^* may be higher or lower than ν .

The EEC depend on the ligament efficiency μ , ratio h/p and pattern type (triangular or square). They must be as correctly evaluated as possible:

- if they are underestimated, the calculated TS stresses at the junction with shell and channel will be lower than reality.
- if they are overestimated, the calculated TS stresses close to the center of the TS will be lower than reality.

The consequence is that inaccurate estimates of the EEC result in inaccurate stress results.

5.2 Historical Background

During the last decades many authors (about 60 papers) have proposed experimental and theoretical methods to solve the problem. A detailed review of these works has been published in 1989 in a JPVT paper by Osweiller [1]. A short synthesis is provided below.

- Between 1948 and 1958 several authors (Gardner, Miller, Horvay, Duncan, Salerno and Mahoney, ...) proposed various methods for the determination of EECS. These methods having no sound basis and leading to a great disparity of results, PVRC decided in 1960 to undertake theoretical and experimental investigations in order to determine more accurate values for the EEC.
- In 1960 Sampson [2] undertook experimental tests on plastic plates using photo-elastic technique for in-plane and bending loadings.
 - For in-plane loading, values of E^*/E and ν^* are independent of the TS thickness.
 - For bending loading, values of E^*/E and ν^* vary markedly with the TS thickness when $h \leq 2p$.

When $h > 2p$ this variation is very slow and, as the plates gets thicker, the bending values approach the plane stress values.

It appears that that $h=2p$ is a transition zone between thin and thick perforated plates. These results were confirmed by Leven [3] in 1960.

- In 1962 O'Donnell & Langer [4] made a synthesis of these results and proposed a curve for in-plane and bending loading to enables the determination of E^*/E and ν^* as a function of the ligament efficiency for thick plates ($h \geq 2p$) perforated with triangular pitch. This curve was adopted by ASME Section III in 1966 and later in Section VIII-Div. 2.
- In 1963 and later, new theoretical methods were developed on powerful computers to enable the determination of EECS for triangular pattern and square pattern, not previously covered. These methods are based on doubly periodic stress distribution theory induced in an infinite plate evenly perforated in two directions and loaded in plane or bending stress. Two techniques have been used:

- The “direct technique” developed by Meijers [5] in 1969 for thin plates loaded in bending or plane-stress. Grigoljuk and Fil’shtinski [11] obtained the same results using a similar method (see Table 4)

In 1985, Meijers [12] improved his method by proposing for the determination of E^*/E and ν^* :

- an asymptotic solution for thin plates in bending ($h/p \rightarrow 0$) and for thick plates ($h/p \rightarrow \infty$), consistent with 1969 results
- an interpolate solution for intermediate range ($0 < h/p \rightarrow 0$), substantiated by FEA calculations.

See Table 1 for triangular pitch.

These values were adopted by many pressure vessel Codes (BS5500, CODAP, EN 13445, ISO, STOOWEZEN) for treating thin and thick plates loaded in bending.

Table 1—Values for E^*/E and ν^* for triangular pattern from Meijers [12]

μ \ h/P	0	0.25	0.5	1	2	4	∞
0.25	$\frac{E^*_{\Delta}}{E}$	0.348	0.297	0.261	0.230	0.215	0.208
	ν^*_{Δ}	0.032	0.171	0.273	0.359	0.399	0.421
0.33	$\frac{E^*_{\Delta}}{E}$	0.452	0.410	0.365	0.340	0.319	0.310
	ν^*_{Δ}	0.073	0.159	0.250	0.302	0.346	0.364

- The “indirect technique” developed by Bailey and Hicks [6] in 1960 and Slot & O’Donnell [7] in 1971 for thick plates.

These two techniques led to very good agreement both for triangular and square patterns with discrepancies lower than 0.1 % (see Table 4). For more details, see Ref. [1].

- e) These theoretical results have been corroborated by many experimental investigations undertaken by Duncan and Upfold [8] in 1963 and O’Donnell [9][10] in 1967 and 1973 and by F.E.M. calculations from Tran-Huu-Hanh in 1971 and Roberts in 1975.

5.3 The Square Pattern Problem

Contrary to the triangular pattern, the square pattern is characterized by an anisotropic behavior which has been enlightened both by theoretical and experimental investigations. Values of EEC E^* and ν^* are different in the pitch direction P and diagonal direction D (See Figure 10). Anisotropy is more marked for low values of ligament efficiency μ^* (0.2-0.4) than for high values approaching 1 (which corresponds to an unperforated plate), as shown by Slot and O’Donnell [7] (see Table 2).

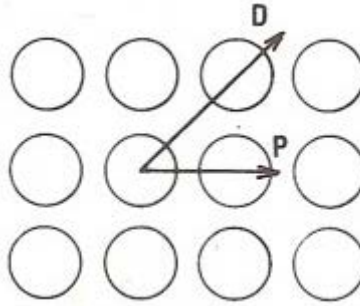


Figure 10—Pitch and diagonal directions for square pattern

Table 2—Values of E^*/E and ν^* for square pattern in pitch and diagonal directions from Slot and O'Donnell [7]

μ	Pitch direction		Diagonal direction		“Isotropic” value	
	$\frac{E_p^*}{E}$	ν_p^*	$\frac{E_d^*}{E}$	ν_d^*	$\frac{E_{\square}^*}{E}$	$\nu_{\square}^*$
0.2	0.311	0.122	0.123	0.654	0.235	0.337
0.4	0.525	0.216	0.380	0.433	0.459	0.316
0.6	0.734	0.275	0.681	0.328	0.708	0.301
0.8	0.918	0.297	0.914	0.301	0.916	0.299

This anisotropy has been confirmed theoretically by O'Donnell [9] and experimentally by Bayley & Hicks [6] and O'Donnell [10].

Due to this anisotropy, the equivalent plate cannot be treated with the classical isotropic solution like in the triangular case. Anisotropic solution should be used.

When the plate is clamped or simply supported, the anisotropic circular plate deflection is given by formulae which can be compared to the classical isotropic formulae. From that comparison, equivalent “isotropic” values for E_{square}^* and ν_{square}^* have been determined by O'Donnell [10], which enables the application of the classical isotropic equations to the equivalent solid plate.

$$E_{\text{square}}^* = E_p^* \frac{1 - \nu_{\text{square}}^*}{1 - \nu_p^*} \quad \nu_{\text{square}}^* = \frac{4}{\frac{3 + \nu_p^*}{1 + \nu_p^*} + \frac{1 - \nu_d^*}{1 + \nu_d^*}} - 1$$

These formulae have been used for the square pattern to calculate the “isotropic” EEC E_{square}^* and ν_{square}^* from the anisotropic values proposed by various authors, as shown in Table 2.

5.4 Synthesis of Results

In 1989, Osweiller [1] made a synthesis of all these experimental and theoretical results for triangular and square patterns which are presented in a graphical form in Figure 55 of Annex A. They give the values of E^*/E and ν^* as a function of the ligament efficiency μ , for various ratios h/p .

This Figure shows that experimental values are available for μ ranging from 0.1 to 0.5, whereas theoretical values cover the full range of μ .

5.5 Determination of EEC for the Full Range of μ^* ($0.1 \leq \mu^* \leq 1.0$)

From this synthesis of results, three ranges of TS thickness were set-up for the determination of E^*/E and ν^* :

- **for thin plates ($h/p \leq 0.1$)** values are taken from theoretical values of Meijers [5][12], combined with experimental values
- **for thick plates ($h/p \geq 2.0$)** values are taken from theoretical values of Slot & O'Donnell [7], combined with experimental values.
- **for the intermediate range ($0.1 < h/p < 2.0$)** values are taken from theoretical values of Meijers [5][12], combined with experimental values of Sampson [2] and O'Donnell [10].

Numerical values and resulting curves are given respectively in Clauses 1 and 2 of Annex B. These curves were initially used in CODAP in 1985 and in Nonmandatory Appendix AA of Section VIII-Div.1, which later became Part UHX.

Note: Experimental and theoretical values of EEC have been obtained from perforated plates for which the ligament efficiency is $\mu = (p-d)/p$. For TSs, the stiffening effect of the tubes mentioned in IV-3c may also be accounted for the determination of the EEC. Accordingly, the EEC are determined using the bending ligament efficiency $\mu^* = (p^*-d^*)/p^*$.

In 1991 Woody Caldwell, as a member of the ASME SG-HTE, provided polynomial approximations for the entire range of these curves (0 $\leq \mu^* \leq 1.0$) ~~degree~~ 7 for ν^* (see Clause 3 of Annex B).

Section VIII Div.2 has retained these curves for the range $0.05 \leq \mu^* \leq 1.0$ for the stress analysis of perforated plates (Annex 5.E-2007 edition).

5.6 Determination of EECS for UHX Rules (UHX-11.5.2)

Numerical values obtained from the polynomials developed for Appendix AA polynomials were not accurate enough for curve $h/p=2.0$ for low values of μ^* : the discrepancy was about 6% on E^*/E for $\mu^*=0.15$ and could reach 15% for $\mu^*=0.1$, with significant effects on calculated tubesheet stresses.

Accordingly, the precision for these polynomials was improved by Osweiller [13] in 1994 by:

- using more points (35 instead of 21 initially)
- exploring several polynomials degrees (degrees 3, 4 and 5 were tested)
- reducing the range of μ^* from 0.1 to 0.6 ($0.1 \leq \mu^* \leq 0.6$), based on the fact that the ligament efficiency of HEs ranges always between these two limits.

The precision targeted for these polynomials was 1%, which corresponds to the reading error of the curves.

Polynomials of degree 4 were finally retained, both for E^*/E and ν^* , which lead to a discrepancy of about 0.2%, with a maximum of 0.77% for the curve $h/p=2.0$.

Figure 11 for Triangular pattern and Figure 12 for Square pattern provide the curves and polynomials for calculating the EECS E^*/E and ν^* for the reduced ligament efficiency range $0.1 \leq \mu^* \leq 0.6$ and fixed values of the ratio h/p :

- **for E^*/E : $h/p = 0.1, 0.25, 0.5, 2.0$.**

Note: For $h/p=0.25$ no curve was available for E^*/E for triangular pattern, due to lack of results. This curve has been added by interpolation between the two adjacent curves relative to $h/p=0.1$ and $h/p=0.6$.

- **for v^* : $h/p=0.1, 0.15, 0.25, 0.5, 1.0, 2.0$.**

Numerical values are available in Clause 1 of Annex B.

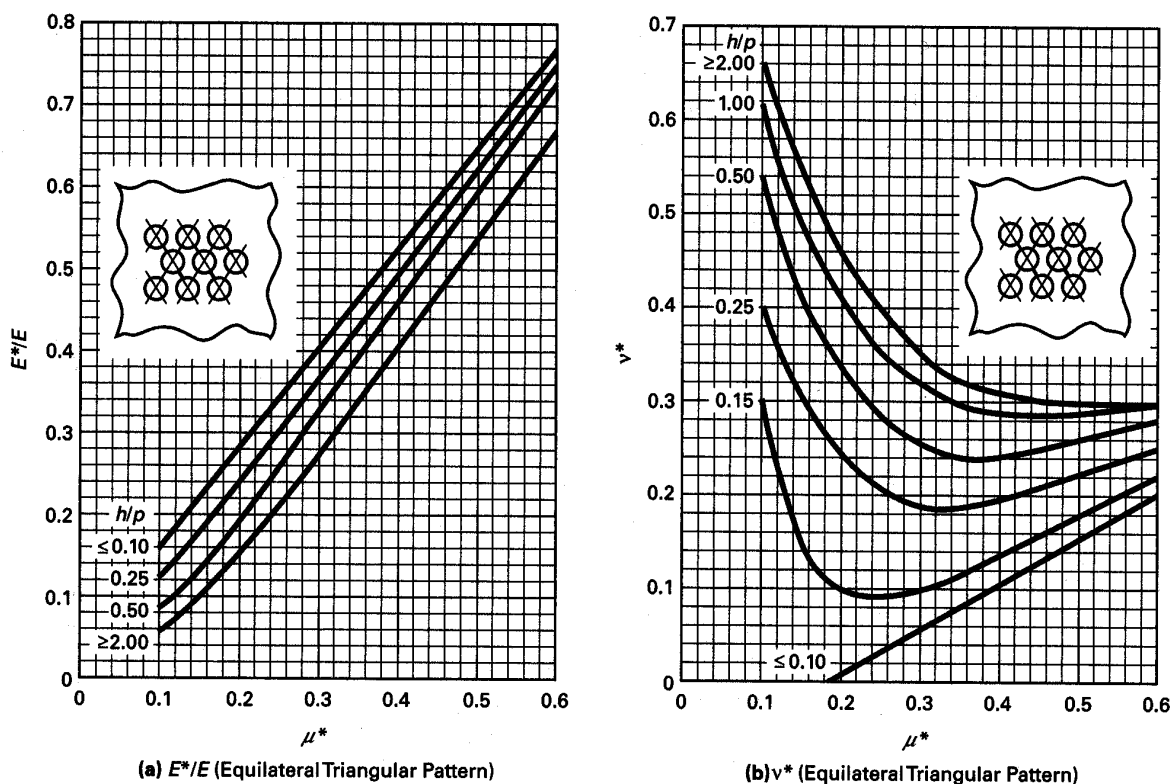
Note: The polynomials cannot be used for ligament efficiencies outside the range $0.1 \leq \eta \leq 0.6$, as shown by Figure 13. In such a case E^*/E and v^* should be determined per Clause V.5 above.

5.7 Conclusion

The perforated tubesheet plate is now replaced by an equivalent solid plate of:

- diameter D_o determined from Clause III.3a,
- equivalent modulus of elasticity E^* and Poisson's ratio v^* determined from IV.6.

The unperforated TS rim extends from diameter D_o to diameter D_s with the basic modulus of elasticity E and Poisson's ratio v .



(a) Equilateral Triangular Pattern: $E^*/E = \alpha_0 + \alpha_1\mu^* + \alpha_2\mu^{*2} + \alpha_3\mu^{*3} + \alpha_4\mu^{*4}$

h/p	α_0	α_1	α_2	α_3	α_4
0.10	0.0353	1.2502	-0.0491	0.3604	-0.6100
0.25	0.0135	0.9910	1.0080	-1.0498	0.0184
0.50	0.0054	0.5279	3.0461	-4.3657	1.9435
2.00	-0.0029	0.2126	3.9906	-6.1730	3.4307

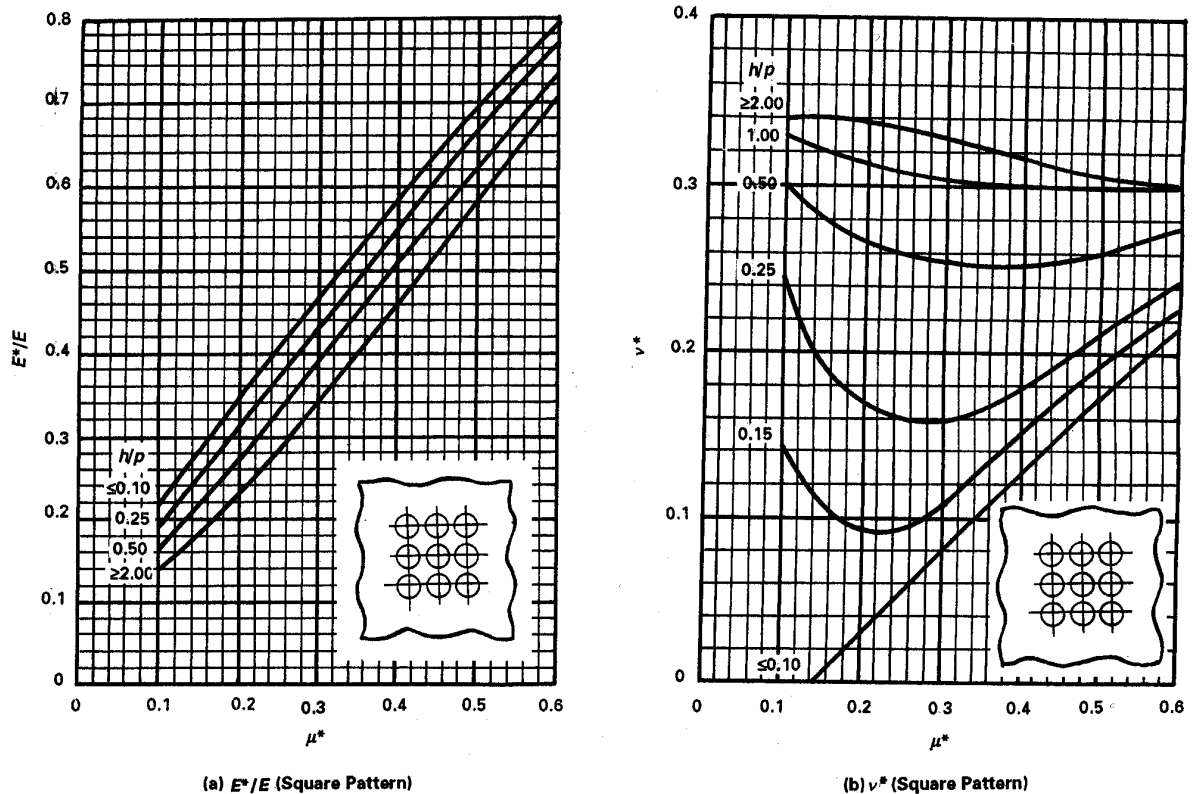
(b) Equilateral Triangular Pattern: $v^* = \beta_0 + \beta_1\mu^* + \beta_2\mu^{*2} + \beta_3\mu^{*3} + \beta_4\mu^{*4}$

h/p	β_0	β_1	β_2	β_3	β_4
0.10	-0.0958	0.6209	-0.8683	2.1099	-1.6831
0.15	0.8897	-9.0855	36.1435	-59.5425	35.8223
0.25	0.7439	-4.4989	12.5779	-14.2092	5.7822
0.50	0.9100	-4.8901	12.4325	-12.7039	4.4298
1.00	0.9923	-4.8759	12.3572	-13.7214	5.7629
2.00	0.9966	-4.1978	9.0478	-7.9955	2.2398

GENERAL NOTES:

- The polynomial equations given in the tabular part of this Figure can be used in lieu of the curves.
- For both parts (a) and (b) in the tabular part of this Figure, these coefficients are only valid for $0.1 \leq \mu^* \leq 0.6$.
- For both parts (a) and (b) in the tabular part of this Figure: for values of h/p lower than 0.1, use $h/p = 0.1$; for values of h/p higher than 2.0, use $h/p = 2.0$.

Figure 11—Curves for the determination of E^*/E and v^* (triangular pattern)



(a) Square Pattern: $E^*/E = \alpha_0 + \alpha_1\mu^* + \alpha_2\mu^{*2} + \alpha_3\mu^{*3} + \alpha_4\mu^{*4}$

h/p	α_0	α_1	α_2	α_3	α_4
0.10	0.0676	1.5756	-1.2119	1.7715	-1.2628
0.25	0.0250	1.9251	-3.5230	6.9830	-5.0017
0.50	0.0394	1.3024	-1.1041	2.8714	-2.3994
2.00	0.0372	1.0314	-0.6402	2.6201	-2.1929

(b) Square Pattern: $v^* = \beta_0 + \beta_1\mu^* + \beta_2\mu^{*2} + \beta_3\mu^{*3} + \beta_4\mu^{*4}$

h/p	β_0	β_1	β_2	β_3	β_4
0.10	-0.0791	0.6008	-0.3468	0.4858	-0.3606
0.15	0.3345	-2.8420	10.9709	-15.8994	8.3516
0.25	0.4296	-2.6350	8.6864	-11.5227	5.8544
0.50	0.3636	-0.8057	2.0463	-2.2902	1.1862
1.00	0.3527	-0.2842	0.4354	-0.0901	-0.1590
2.00	0.3341	0.1260	-0.6920	0.6877	-0.0600

GENERAL NOTES:

- (a) The polynomial equations given in the tabular part of this Figure can be used in lieu of the curves.
- (b) For both parts (a) and (b) in the tabular part of this Figure, these coefficients are only valid for $0.1 \leq \mu^* \leq 0.6$.
- (c) For both parts (a) and (b) in the tabular part of this Figure: for values of h/p lower than 0.1, use $h/p = 0.1$; for values of h/p higher than 2.0, use $h/p = 2.0$.

Figure 12—Curves for the determination of E^*/E and v^* (square pattern)

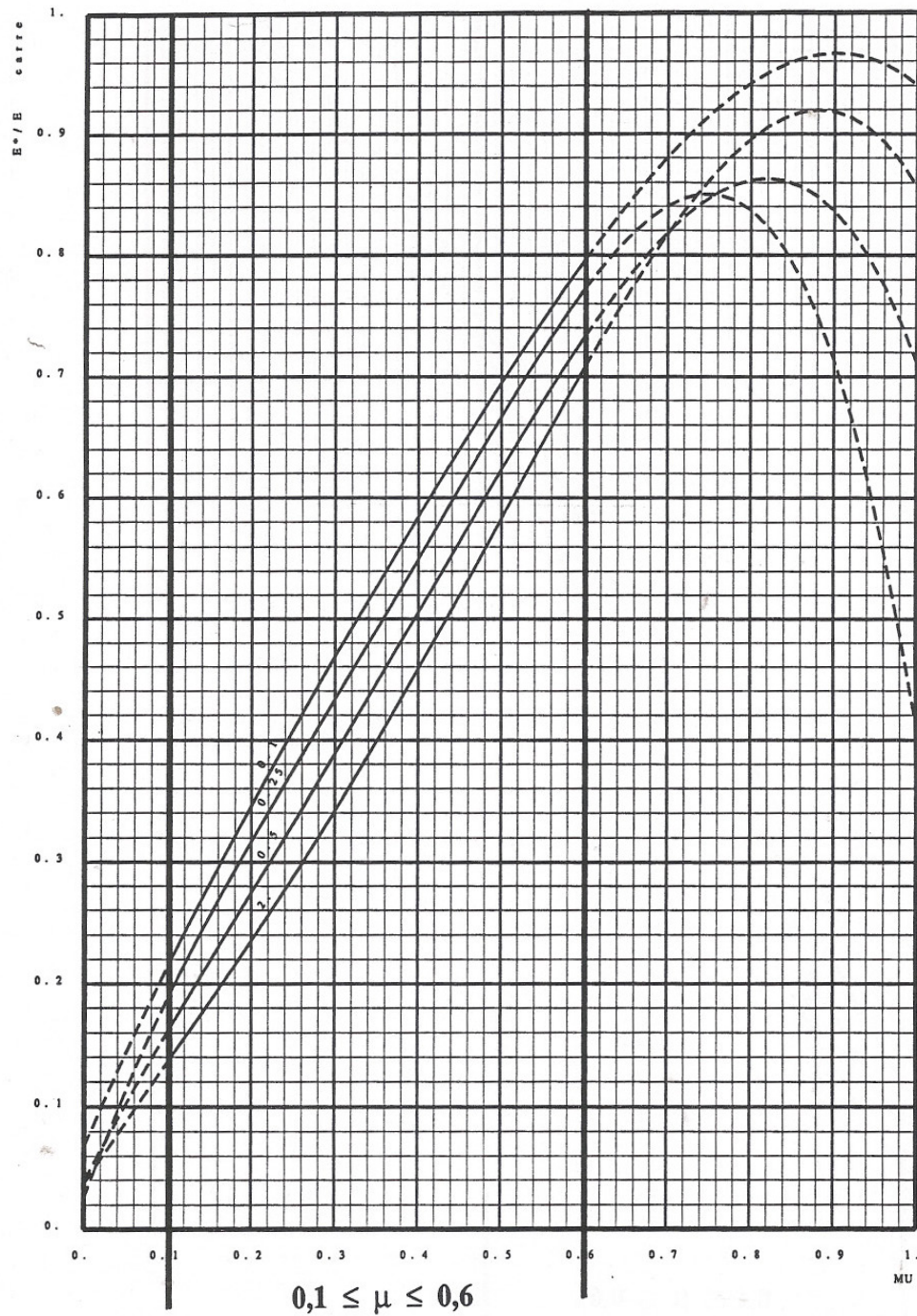


Figure 13—Curves E^*/E for square pattern obtained from polynomial approximation given in Figure 12
 (only valid for $0.1 \leq \mu^* \leq 0.6$) from [13]

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Codes & Standards (CS)

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- [CS-2] **CODAP**: French Code for Unfired pressure Vessels – Chapter C7 “Design rules for HE TSs”
- [CS-3] **EN 13 445**: European Standard for Unfired Pressure Vessels – Chapter 13 “HE TSs”
- [CS-4] **ISO DIS 2694**: Pressure Vessels – Chapter 30 “Flat HE TSs”
- [CS-5] **STOOMWEZEN**: Dutch Code for Pressure Vessels – Chapters D0403 “Wall thickness calculation of TSs”
- [CS-6] **TEMA**: Standards of Tubular Exchangers Manufacturers Association – Chapter R7 “TS”

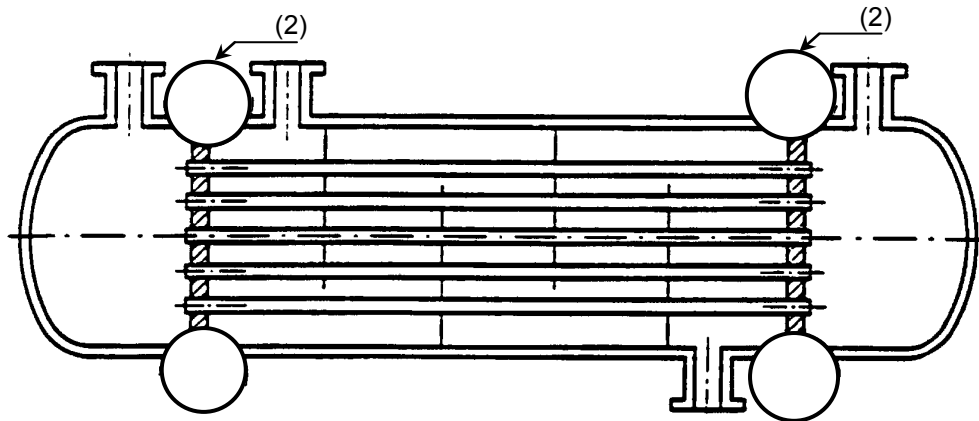
PART 3

Analytical Treatment of Fixed Tubesheet Heat Exchangers

1 SCOPE (UHX-13.1)

PART 3, devoted to Fixed TS HEs (Figure 14), provides the technical basis for the determination of

- the displacement and loads acting on the TS, tubes, shell and channel,
- the stresses in these four components and their relationships with the design rules of UHX-13



2

Figure 14 – Fixed tubesheet heat exchanger

² see TS configurations in Figure 15

2 HISTORICAL BACKGROUND

In the past decades many authors have proposed theoretical methods for the design of fixed TS HEs. The most important contributions are provided below. A more detailed list of technical papers can be found in Ref. [1] .

Gardner [2] in 1952 was the very first to develop an analytical approach by taking into consideration the support afforded by the tubes and the weakening effect of the TS holes. The TS is considered as either simply supported or clamped at its periphery to simulate the rotational restraint afforded by the shell and the channel, which compels the designer to make a more or less arbitrary choice between these two extreme cases.

TEMA adopted this method in its 5th edition published in 1968.

Miller [3] at the same time proposed a similar approach that was published in the British Code BS 1515 in 1965.

Galletly [4] in 1959 improved these design methods by accounting for the degree of rotational restraint of the TS at its periphery by the shell and the channel. This method was adopted by the French Pressure Vessel Code CODAP in 1982 and by the European Pressure Vessel Standard EN13445 in 2002.

Other authors (Yi Yan Yu [5] [6] , Boon and Walsh [7] , Hayashi [8] ,...) have developed more refined methods accounting for the membrane loads acting at mid-surface of the TS, the unperforated rim, the TS-shell-channel connection, the bending effect of the tubes which reinforce the strength of the TS.

Since a solution using these methods requires the use of a computer they were not adopted by Codes.

Soler [9] in 1984 developed a similar method accounting for the unperforated rim and the TS-shell-channel connection. Thanks to a parametric study, it does not need the use of a computer.

The method was adopted by the SG-HTE in the 80's, put into a Code format, and published for the first time in 1995 in Non mandatory Appendix AA of Section VIII Division 1. In 2003 it was published in a new Part UHX of Section VIII Division 1 "Rules for Shell and Tubes Heat Exchangers" which became mandatory in 2004.

This criteria document provides the technical basis of Part UHX-13 of Section VIII Div.1, 2007 Edition [10] , including the 2008 and 2009 Addenda. A few items (analysis of cylindrical and spherical shells, allowable stress limits) are taken from Appendix 4 of Section VIII Div. 2, 2004 Edition [11] .

3 GENERAL

3.1 TS Configurations (UHX-13.1)

The TS is attached to the shell and the channel by welding (integral TS) or by bolting (gasketed TS) in accordance with the following 4 configurations (see Figure 15):

- configuration a: tubesheet integral with shell and channel;
- configuration b: tubesheet integral with shell and gasketed with channel, extended as a flange;
- configuration c: tubesheet integral with shell and gasketed with channel, not extended as a flange;
- configuration d: tubesheet gasketed with shell and channel, extended as a flange or not extended.

An expansion joint can be set-up on the shell as shown on Figure 15 configuration a.

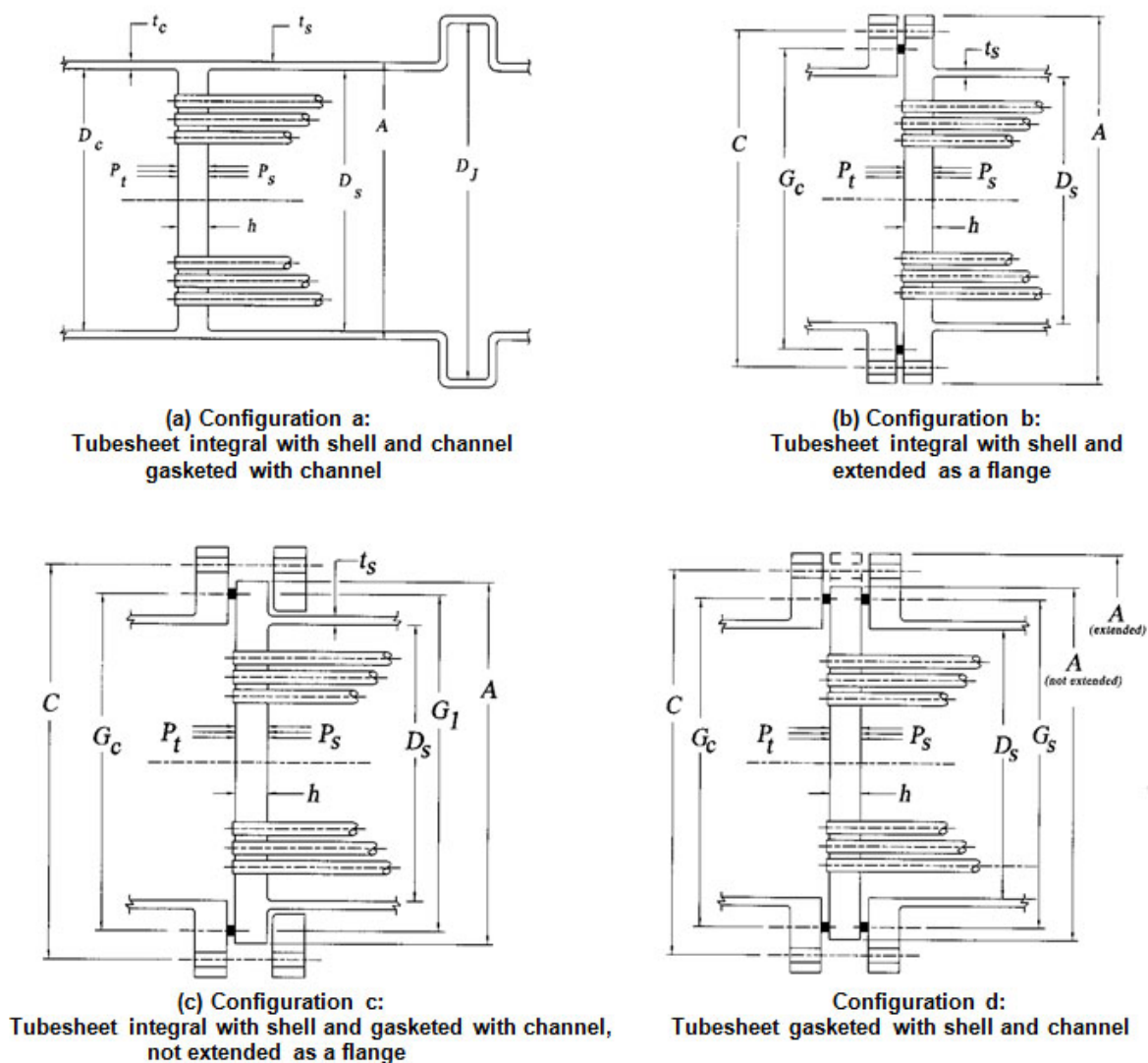


Figure 15—Tubesheet configurations

3.2 Notations (UHX-13.3)

(a) Data for the design of the HE are as follows.

Symbols D_o , E^* , hg' , μ , μ^* and ν^* are defined in Clause II of PART 2.

A = outside diameter of tubesheet

a_c = radial channel dimension

a_o = equivalent radius of outer tube limit circle (see Clause IV.3a of PART 2)

a_s = radial shell dimension

C = bolt circle diameter

D_c = inside channel diameter

D_J = inside diameter of the expansion joint at its convolution height

D_s = Inside shell diameter

d_t = nominal outside diameter of tubes

E = modulus of elasticity for tubesheet material at T

E_c = modulus of elasticity for channel material at Tc

E_s = modulus of elasticity for shell material at Ts

E_t = modulus of elasticity for tube material at Tt

G_c = diameter of channel gasket load reaction

G_s = diameter of shell gasket load reaction

G_1 = midpoint of contact between flange and tubesheet

h = tubesheet thickness

K_J = axial rigidity of expansion joint, total force/elongation

L_t = tube length between outer tubesheet faces

N_t = number of tubes

P_e = effective pressure acting on tubesheet

P_s = shell side internal design pressure. For shell side vacuum use a negative value for Ps.

P_b, P_c = tube side internal design pressure. For tube side vacuum use a negative value for Pt.

Notation P_c , instead of P_t , is used throughout the analytical development so as to maintain the symmetry of the equations involving the shell (subscript s) and the channel (subscript c).

S = allowable stress for tubesheet material at T

S_c = allowable stress for channel material at Tc

S_s = allowable stress for shell material at Ts

S_t = allowable stress for tube material at Tt

$S_{y,c}$ = yield strength for channel material at Tc

$S_{y,s}$ = yield strength for shell material at Ts

$S_{y,t}$ = yield strength for tube material at Tt

S_{PS} = allowable primary plus secondary stress for tubesheet material at T

$S_{PS,c}$ = allowable primary plus secondary stress for channel material at Tc

$S_{PS,s}$ = allowable primary plus secondary stress for shell material at Ts

T = tubesheet design temperature

T_a = ambient temperature, 70oF (20oC)

T_c = channel design temperature

T_s = shell design temperature

T_t = tube design temperature

$T_{s,m}$ = mean shell metal temperature along shell length

$T_{t,m}$ = mean tube metal temperature along tube length

t_c = channel thickness

t_s = shell thickness

t_t = nominal tube wall thickness

W = flange design bolt load

$\alpha_{s,m}$ = mean coefficient of thermal expansion of shell material at $T_{s,m}$

$\alpha_{t,m}$ = mean coefficient of thermal expansion of tube material at $T_{t,m}$

γ = axial differential thermal expansion between tubes and shell

ν = Poisson's ratio of tubesheet material

ν_c = Poisson's ratio of channel material

ν_s = Poisson's ratio of shell material

ν_t = Poisson's ratio of tube material

(b) **Design coefficients** (UHX-13.5.1 to 4)

The following coefficients, specific to each component of the HE, will be used in the analytical treatment.

1) **Perforated TS**

Equivalent diameter of outer tube limit circle (see Clause IV-3.a of PART 2):

$$D_o = 2 r_o + d_t$$

Equivalent radius of outer tube limit circle (see Clause IV-3.a of PART 2): $a_o = \frac{D_o}{2}$

TS coefficients:

- shell side: $x_s = 1 - N_t \left(\frac{d_t}{2a_o} \right)^2$; $1 - x_s = N_t \left(\frac{d_t}{2a_o} \right)^2$
 - tube side: $x_t = 1 - N_t \left(\frac{d_t - 2t_t}{2a_o} \right)^2$; $1 - x_t = N_t \left(\frac{d_t - 2t_t}{2a_o} \right)^2$
 - $x_t - x_s = N_t \left(\frac{d_t^2 - (d_t - 2t_t)^2}{4a_o^2} \right) = \frac{N_t \cdot s_t}{\pi a_o^2} = \frac{N_t \cdot k_t}{E_t} \cdot \frac{l}{\pi a_o^2} = \frac{N_t \cdot K_t}{E_t} \cdot \frac{L}{\pi a_o^2}$
- [III.2.b1]
- Ligament efficiency: $\mu^* = \frac{p^* - d^*}{p^*}$

Effective tube hole diameter d^* and effective pitch p^* are defined in Clause IV-3c of PART 2

- Effective elastic constants E^* and ν^* are given in Clause V-6 of PART 2 as a function of μ^* and h/p (triangular or square pitch).

- Bending stiffness: $D^* = \frac{E^* \cdot h^3}{12(1 - \nu^{*2})}$
- effective tube side pass partition groove depth given in Clause IV-3.d of PART 2: h'_g
- effective pressure acting on tubesheet: P_e

2) Tube Bundle

Tube cross-sectional area:

$$s_t = \frac{\pi}{4} \left[d_t^2 - (d_t - 2t_t)^2 \right] = \pi t_t (d_t - t_t) = \frac{\pi a_o^2}{N_t} (x_t - x_s)$$

Axial stiffness K_t of one tube: $K_t = \frac{E_t s_t}{L} = \frac{\pi t_t (d_t - t_t) E_t}{L}$

axial stiffness k_t of one half tube of length $l=L/2$: $k_t = \frac{E_t s_t}{l} = \frac{2\pi t_t (d_t - t_t) E_t}{L} = 2 K_t$

Effective elastic foundation modulus equivalent to the half tube bundle:

$$k_w = \frac{N_t \cdot k_t}{\pi a_o^2} = \frac{2 N_t \cdot K_t}{\pi a_o^2} = \frac{2 N_t \cdot E_t \cdot t_t (d_t - t_t)}{L a_o^2} = \frac{2 E_t}{L} (x_t - x_s) = \frac{E_t}{l} (x_t - x_s)$$

$$k = \sqrt[4]{\frac{k_w}{D^*}} \quad ; \quad x = k r \quad ; \quad 0 \leq r \leq a_o \quad \Rightarrow \quad 0 \leq x \leq k a_o \quad k a_o = X_a$$

Axial stiffness ratio tubes/TS:

$$X_a = k a_o = \sqrt[4]{\frac{k_w}{D^*}} a_o = \left[24 (1 - \nu^{*2}) N_t \frac{E_t t_t (d_t - t_t) a_o^2}{E^* L h^3} \right]^{1/4}$$

3) Shell

radial shell dimension: $a_s \quad \rho_s = \frac{a_s}{a_o}$

Integral configurations (a, b and c): $a_s = D_s / 2$

Gasketed configuration (d): $a_s = G_s / 2$

mean shell radius: $a'_s = \frac{D_s + t_s}{2}$

Shell cross-sectional area: $s_s = \pi t_s (D_s + t_s)$

axial stiffness K_s of the shell of length L : $K_s = \frac{E_s s_s}{L} = \frac{\pi t_s (D_s + t_s) E_s}{L}$

axial stiffness k'_s of the half shell of length $l=L/2$: $k'_s = \frac{E_s s_s}{l} = \frac{2\pi t_s (D_s + t_s) E_s}{L} = 2 K_s$

Axial stiffness ratio shell/tube bundle:
$$K_{s,t} = \frac{K_s}{N_t \cdot K_t} = \frac{t_s (D_s + t_s) E_s}{N_t \cdot t_t (d_t - t_t) E_t}$$

shell coefficient:
$$\beta_s = \frac{\sqrt[4]{12(1-\nu_s^2)}}{\sqrt{(D_s + t_s) \sqrt[3]{E_s}}}$$

bending stiffness:
$$k_s = \beta_s \frac{E_s \cdot t_s^3}{6(1-\nu_s^2)}$$

4) Channel

radial channel dimension: $a_c \quad \rho_c = \frac{a_c}{a_o}$

Integral configuration (a): $a_c = D_c / 2$

Gasketed configurations (b, c and d): $a_c = G_c / 2$

mean channel radius: $a'_s = \frac{D_c + t_c}{2}$

Channel coefficient:
$$\beta_c = \frac{\sqrt[4]{12(1-\nu_c^2)}}{\sqrt{(D_c + t_c) t_c \sqrt[3]{E_c}}}$$

bending stiffness:
$$k_c = \beta_c \cdot \frac{E_c \cdot t_c^3}{6(1-\nu_c^2)}$$

5) Axial differential thermal expansion between tubes and shell:

$$\gamma = [\alpha_{t,m}(T_{t,m} - T_a) - \alpha_{s,m}(T_{s,m} - T_a)] L$$

6) Expansion joint

D_J = inside diameter of the expansion joint at its convolution height

K_J = axial rigidity of expansion joint, total force/elongation

J = ratio of expansion joint to shell axial rigidity ($J = 1.0$ if no joint):
$$J = \frac{K_J}{K_s + K_J}$$

7) Unperforated rim

D_o = internal diameter

A = external diameter

Diameter ratio: $K = A / D_o$

3.3 Loading Cases (UHX-13.4)

The normal operating condition of the HE is achieved when the tube side pressure P_t , shell side pressure P_s and axial differential thermal expansion between tubes and shell γ act simultaneously.

However, a loss of pressure or a loss of temperature is always possible. Accordingly, for safety reasons, the designer must always consider the cases where $P_s=0$ and $P_t=0$ with and without thermal expansion for the normal operating condition(s).

He must also consider the startup condition(s), the shutdown condition(s) and the upset condition(s), if any, which may govern the design.

A fixed TS HE is a statically indeterminate structure for which it is generally not possible to determine the most severe condition of coincident pressure, temperature and differential thermal expansion. Thus, it is necessary to evaluate all the anticipated loading conditions mentioned above to ensure that the worst load combination has been considered in the design.

For each of these conditions, the following 7 loading cases must be considered.

(a) Pressure loading cases

- Loading Case 1: Tube side pressure P_t acting only ($P_s = 0$), without differential thermal expansion.
- Loading Case 2: Shell side pressure P_s acting only ($P_t = 0$), without differential thermal expansion.
- Loading Case 3: Tube side pressure P_t and shell side pressure P_s acting simultaneously, without differential thermal expansion.

(b) Thermal loading cases

- Loading Case 4: Differential thermal expansion acting only ($P_t = 0, P_s = 0$).
- Loading Case 5: Tube side pressure P_t acting only ($P_s = 0$), with differential thermal expansion.
- Loading Case 6: Shell side pressure P_s acting only ($P_t = 0$), with differential thermal expansion.
- Loading Case 7: Tube side pressure P_t and shell side pressure P_s acting simultaneously, with differential thermal expansion.

When vacuum exists, each loading case shall be considered with and without the vacuum.

When differential pressure design is specified by the user, the design shall be based only on loading cases 3, 4, and 7.

As the calculation procedure is iterative, a value h is assumed for the tubesheet thickness to calculate and check that the maximum stresses in tubesheet, tubes, shell, and channel are within the maximum permissible stress limits.

Because any increase of tubesheet thickness may lead to overstresses in the tubes, shell, or channel, a final check shall be performed, using in the formulas the nominal thickness of tubesheet, tubes, shell, and channel, in both corroded and uncorroded conditions.

3.4 Design Assumptions (UHX-13.2)

A fixed TS HE is a complex structure and several assumptions are necessary to derive a 'design by rules' method. Most of them could be eliminated, but the analytical treatment would lead to 'design by analysis' method requiring the use of a computer.

The design assumptions are as follows.

(a) HE

- The analytical treatment is based on the theory of elasticity applied to the thin shells of revolution.
- The HE is axi-symmetrical.
- The HE is a symmetrical unit with identical TSs so as to analyze a half-unit.

(b) TSs

- The two tubesheets are circular and identical (same diameter, uniform thickness, material, temperature and edge conditions)
- The tubesheets are uniformly perforated over a nominally circular area, in either equilateral triangular or square patterns. This implies that the TSs are fully tubed (no large untubed window)
- Radial displacement at mid-surface of TS is ignored
- Temperature gradient through TS thickness is ignored
- Shear deformation and transverse normal strain in TS are ignored
- The unperforated rim of the TS is treated as a rigid ring without distortion of the cross section

(c) Tubes

- Tubes are assumed identical, straight and at same temperature
- Tubes are uniformly distributed in sufficient density to play the role of an elastic foundation for the TS
- The effect of the rotational stiffness of the tubes is ignored

(d) Shell and channel

- Shell and channel are cylindrical with uniform diameters and thicknesses
- If the channel head is hemispherical, it must be attached directly to the TS, without any cylindrical section between the head and the TS.
- Shell and channel centerlines are the same.

(e) Weights and pressures drops

- Weights and pressures drops are ignored
- Pressures P_s and P_t are assumed uniform

3.5 Basis of Analytical Treatment

3.5.1 General

The design of a fixed TS HEs is complex as the two TSs are connected to each other both through the tube bundle and the shell, and subjected to different temperatures which generate an axial differential thermal expansion. Accordingly the structure is statically indeterminate. Many geometrical, mechanical, thermal and material properties are involved in the design as shown in Clause III-2a which lists the extensive input data.

The analysis includes the effects of the shell and tube side pressures, the axial stiffening effect of the tubes, the stiffening effect of the unperforated ring at the tubesheet edge, and the stiffening effect of the integrally attached channel or shell to the tubesheet. For a tubesheet that is extended as a flange to

which a channel or shell is to be bolted, the bolt load causes an additional moment in the tubesheet which is included in the total stress in the tubesheet in addition to the moments caused by pressure.

The analysis is based on classical discontinuity analysis methods to determine the moments and forces that the tubesheet, tubes, shell and channel must resist. These components are treated using the theory of elasticity applied to the thin shells of revolution.

Because the heat exchanger is assumed to be symmetric about a plane midway between the two tubesheets, only half of the heat exchanger is treated. Main steps of the analytical treatment are as follows.

- Tubesheet is disconnected from the shell and channel. Shear load V_a and moment M_a are applied at the tubesheet edge as shown in Figure 16.
- Perforated tubesheet is replaced by an equivalent solid circular plate of diameter D_o and effective elastic constants E^* (effective modulus of elasticity) and ν^* (effective Poisson's Ratio) depending on the ligament efficiency μ^* of the tubesheet. This equivalent solid plate is treated by the theory of thin circular plates subjected to pressures P_s and P_t and relevant applied loads to determine the maximum stresses.
- Unperforated tubesheet rim is treated as a rigid ring whose cross section does not change under loading.
- Tubes are replaced by an equivalent elastic foundation of modulus k_w .
- Connection of the tubesheet with shell and channel accounts for the edge displacements and rotations of the 3 components.
- Shell and channel are subject to shell side and tube side pressures P_s and P_t and edge loads V_a and M_a to determine the maximum stresses.
- Maximum stresses in tubesheet, tubes, shell and channel are determined and limited to the appropriate allowable stress-based classifications of Section VIII Division 2 PART 4.

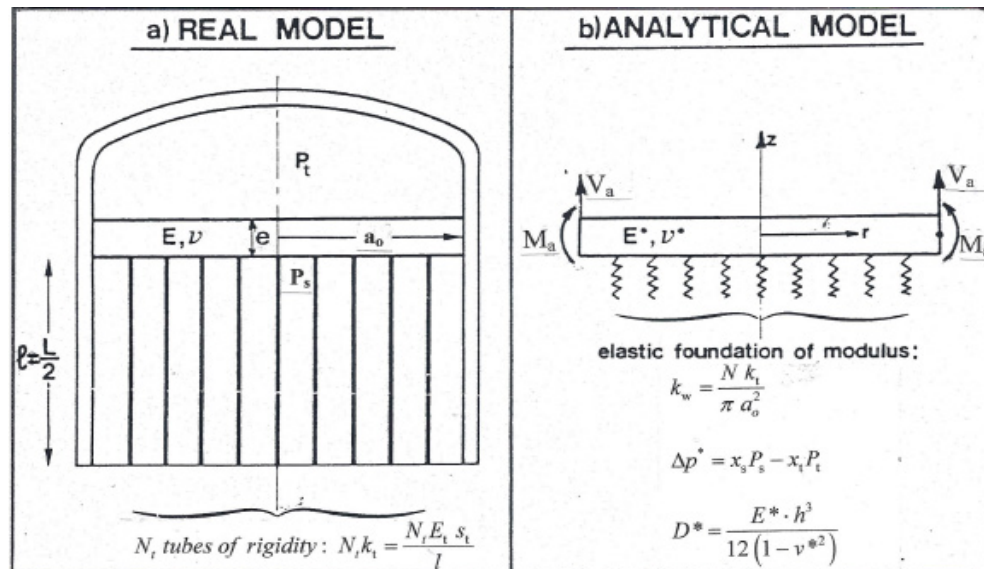


Figure 16—Analytical model used in design method

3.5.2 Free Body Diagram

Figure 17 shows, for a tubesheet integral both sides (configuration a), the free body diagram of the component parts (perforated tubesheet, unperforated tubesheet rim, shell, channel). The Figure details the relevant discontinuity forces (V_a , V_s , Q_s , V_c , Q_c) and moments (M_a , M_s , M_c , M_R) applied on each component, together with edge displacements.

In this Figure:

Forces (V_a , V_s , Q_s , V_c , Q_c) and moments (M_a , M_s , M_c , M_R) are per unit of circumferential length

Following subscripts are used:

- s for shell,
- c for channel,
- R for unperforated rim

No subscript for the perforated TS

Notation P_c instead of P_t (tube side pressure) is used throughout the analytical development so as to maintain the symmetry of the equations involving the shell (subscript s) and the channel (subscript c).

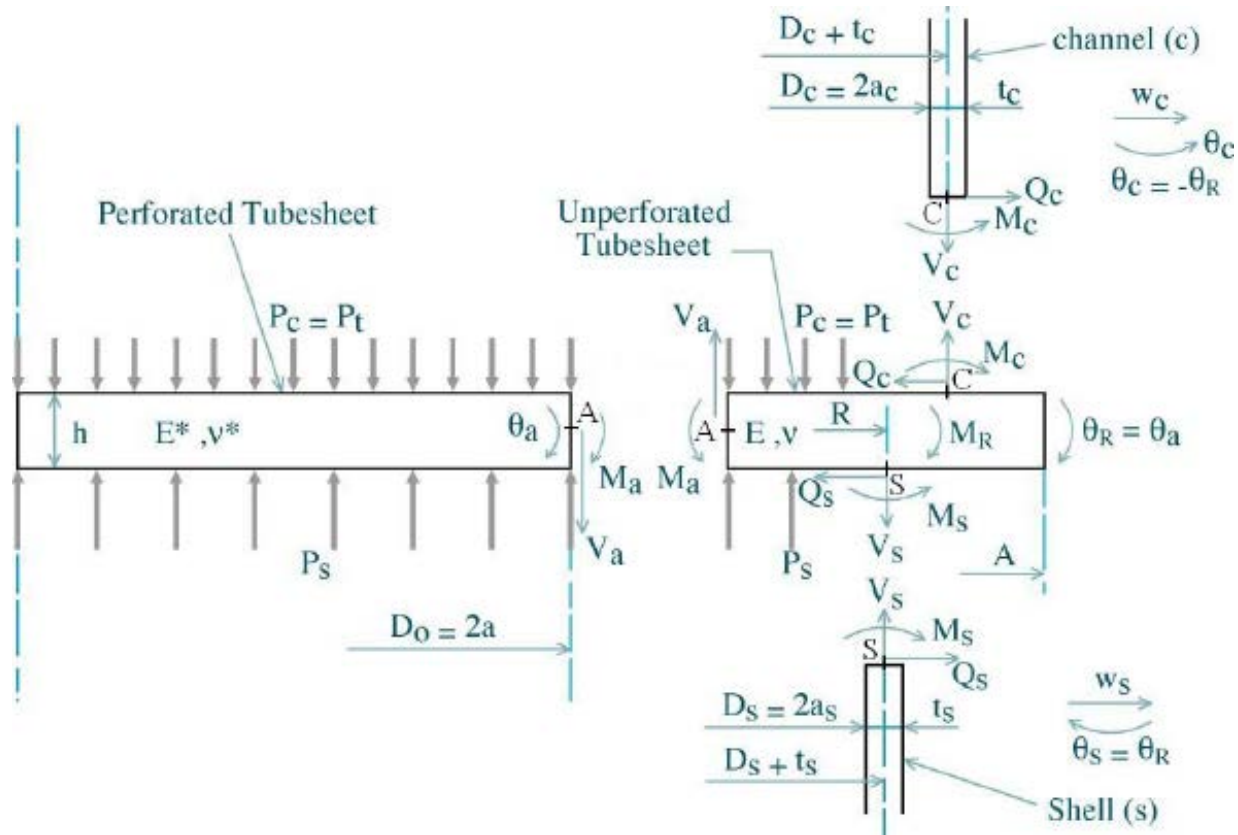


Figure 17—Free body diagram of the analytical model

$$q_t(r) = \frac{-V_t(r)}{\pi a_0^2 / N_t} = -\frac{N_t k_t \delta_t(V_t)}{\pi a_0^2} = -\underbrace{\frac{N_t k_t}{\pi a_0^2}}_{k_w} [\delta_{t,Total}(r) - \delta_t(\theta_t) - \delta_t(v_t)]$$

$$k_w = \frac{N_t k_t}{\pi a_0^2}$$

$$q_t(r) = -k_w [\delta_t(V_t) + \delta_t(\theta_t) + \delta_t(v_t) - \delta_t(\theta_t) - \delta_t(v_t)]$$

4.2 Axial Displacement and Force Acting on the Shell

(a) Axial displacement of the shell and expansion joint due to axial force acting on the shell

- due to axial force V_s acting on the shell:

$$\delta'_s(V_s) = \frac{V_s}{k'_s} \frac{2\pi a'_s}{t_s} = \frac{V_s}{E_s} \frac{2\pi a'_s l}{2\pi a'_s t_s} = \frac{l}{E_s t_s} V_s \quad V_s \text{ per unit of length}$$

$$a'_s = \text{mean shell radius} = \frac{D_s + t_s}{2}$$

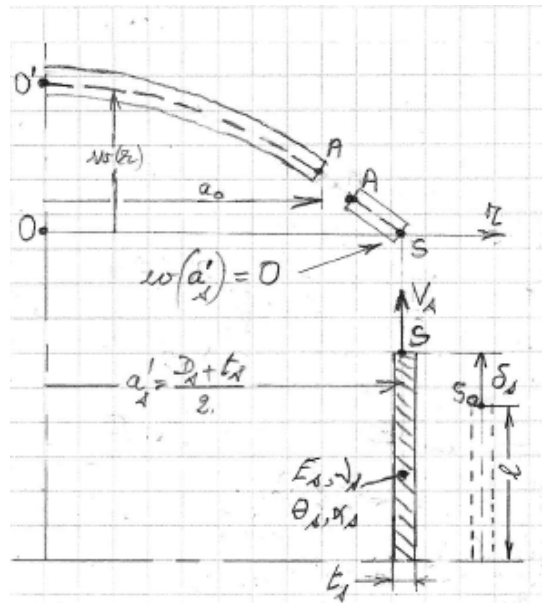


Figure 19—Axial displacement of the shell

- due to axial force V_s acting on the expansion joint J:

$$\delta'_j(V_s) = \frac{V_s}{2K_j} \frac{2\pi a'_s}{1}$$

- axial displacement of shell and expansion joint:

$$\delta_s(V_s) = \delta'_s(V_s) + \delta'_J(V_s) = \frac{V_s}{k'_s} \frac{2\pi a'_s}{2K_I} \left[1 + \frac{k'_s}{2K_I} \right] = \frac{V_s}{k'_s} \frac{2\pi a'_s}{J} = \frac{\delta'_s(V_s)}{J} = \frac{l}{J E_s t_s} V_s$$

$$J = \frac{1}{1 + k'_s / (2 K_j)} \quad k'_s = 2 K_s \quad \boxed{J = \frac{K_j}{K_j + K_s}}$$

Verification: axial rigidity k_s^* shell and expansion joint : $\frac{1}{k_s^*} = \frac{1}{k'_s} + \frac{1}{2 K_j} = \frac{1}{k'_s} \left(1 + \frac{k'_s}{2 K_j} \right) = \frac{1}{J k'_s}$

$$k_s^* = J k'_s \quad \delta_s(V_s) = \frac{V_s 2 \pi a'_s}{k_s^*} = \frac{V_s 2 \pi a'_s}{k'_s} = \frac{\delta'_s(V_s)}{J}$$

(b) **Axial displacement of the shell due to temperature θ_s of shell:**

$$\delta_s(\theta_s) = \alpha_{s,m} \theta_s l \quad \theta_s = T_{s,m} - T_a$$

(c) **Axial displacement of the shell due to Poisson's ratio ν_s of shell (Annex C):**

$$\delta_s(\nu_s) = - \frac{\pi}{2 k'_s} (P_s D_s^2) \nu_s$$

(d) **Axial displacement of the shell due to pressure P_s acting on bellows joint side walls J (Annex D):**

$$\delta_s(J) = P_s \frac{\pi (D_J^2 - D_s^2) / 2}{4 \times 2 K_J} = \frac{\pi}{8} \frac{D_J^2 - D_s^2}{2 K_J} P_s$$

(e) **Total axial displacement of shell:**

$$\delta_{s,\text{Total}} = \underbrace{\delta_s(V_s)}_{\substack{\text{shell displct. due} \\ \text{to axial force} \\ \text{(unknown)}}} + \underbrace{\delta_s(\theta_s) + \delta_s(\nu_s) + \delta_s(J)}_{\substack{\text{free displct.} \\ \text{of shell} \\ \text{(known)}}} \quad [\text{IV.2e}]$$

(f) **Axial force acting on the shell:**

$$\delta_s(V_s) = \delta_{s,\text{Total}} - \delta_s(\theta_s) - \delta_s(\nu_s) - \delta_s(J) = \frac{l}{J E_s t_s} V_s$$

$$V_s = \frac{J E_s t_s}{l} [\delta_{s,\text{Total}} - \delta_s(\theta_s) - \delta_s(\nu_s) - \delta_s(J)]$$

(g) **Axial displacement of tubes at radius r :**

$$\delta_{t,\text{Total}}(r) = \delta_{s,\text{Total}} + w(r) = \delta_s(V_s) + \delta_s(\theta_s) + \delta_s(\nu_s) + \delta_s(J) + w(r)$$

where $w(r)$ is the TS deflection at radius r (see Figure 21)

$$[\text{IV-1d}]: \quad \delta_{t,\text{Total}}(r) = \delta_t(V_t) + \delta_t(\theta_t) + \delta_t(\nu_t)$$

$$\boxed{\delta_t(V_t) = \delta_s(V_s) + w(r) + \underbrace{[\delta_s(\theta_s) - \delta_t(\theta_t)]}_{-\gamma/2} + [\delta_s(\nu_s) - \delta_t(\nu_t)] + [\delta_s(J)]}$$

(h) TS deflection at radius r:

$$w(r) = [\delta_t(V_t) - \delta_s(V_s)] + \left[\underbrace{\delta_t(\theta_t) - \delta_s(\theta_s)}_{\gamma/2} \right] + [\delta_t(v_t) - \delta_s(v_s)] - [\delta_s(J)]$$

5 DEFLECTION AND LOADS ACTING ON THE TUBESHEET

5.1 Equivalent Plate Resting on an Elastic Foundation

(a) Net effective pressure acting on the TS

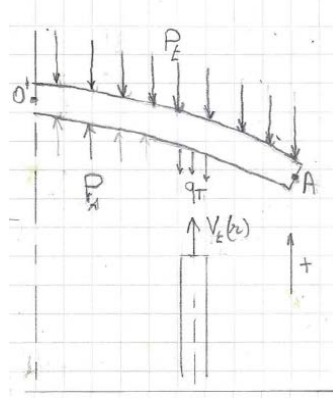


Figure 20—Loads acting on TS

due to tubes: $q_t(r) = -k_w [\delta_t(V_t) + \delta_t(\theta_t) + \delta_t(v_t) - \delta_t(\theta_t) - \delta_t(v_t)]$

Tubes act as an elastic foundation of equivalent modulus given by the axial rigidity of the half- bundle per unit of TS area:

$$k_w = \frac{N_t k_t}{\pi a_o^2}$$

$$q_t(r) = -\frac{V_t(r)}{\pi a_o^2 / N_t} = -k_w [\delta_s(V_s) + w(r) - \delta(\theta) + \delta_s(v_s) - \delta_t(v_t) + \delta_j(P_s)]$$

due to pressures P_s and P_t acting on the equivalent plate (see Annex C Section a):

$$q_p = x_s P_s - x_t P_t = \Delta p^*$$

net effective pressure:

$$q(r) = q_p + q_t(r) = \Delta p^* - \frac{V_t(r)}{\pi a_o^2 / N_t} \quad [\text{V.1a}]$$

$$q(r) = q_p + q_t(r) = \underbrace{\Delta p^* - k_w [\delta_s(\theta_s) - \delta_t(\theta_t) + \delta_s(v_s) - \delta_t(v_t) + \delta_j(P_s) + \delta_s(V_s)]}_{Q} - k_w w(r)$$

- $Q = \Delta p^* + k_w [\delta_t(\theta_t) - \delta_t(\theta_t) + \delta_t(v_t) - \delta_s(v_s) - \delta_j(P_s) - \delta_s(V_s)] \quad [\text{V.1a}]$

In this equation, the displacement $\delta_s(V_s)$ of the shell subjected to axial force V_s is unknown. Other quantities are known for a given HE.

- Annex C: $k_w \delta_t(v_t) = -2 v_t [P_t(1-x_t) - P_s(1-x_s)]$

$$k_w \delta_s(v_s) = -\frac{2 v_s}{k_{s,t}} \left(\frac{D_s}{D_o} \right)^2 P_s$$

- Annex D:

$$k_w \delta_J(P_s) = \frac{\pi k_w}{16 K_J} (D_J^2 - D_s^2) P_s = \frac{\pi N_t k_t}{16 \pi a_o^2 K_J} (D_J^2 - D_s^2) P_s = \frac{N_t k_t}{4 K_J} \frac{D_J^2 - D_s^2}{D_o^2} P_s$$

$$q(r) = Q - k_w w(r)$$

(b) deflection of TS (Figure 21)

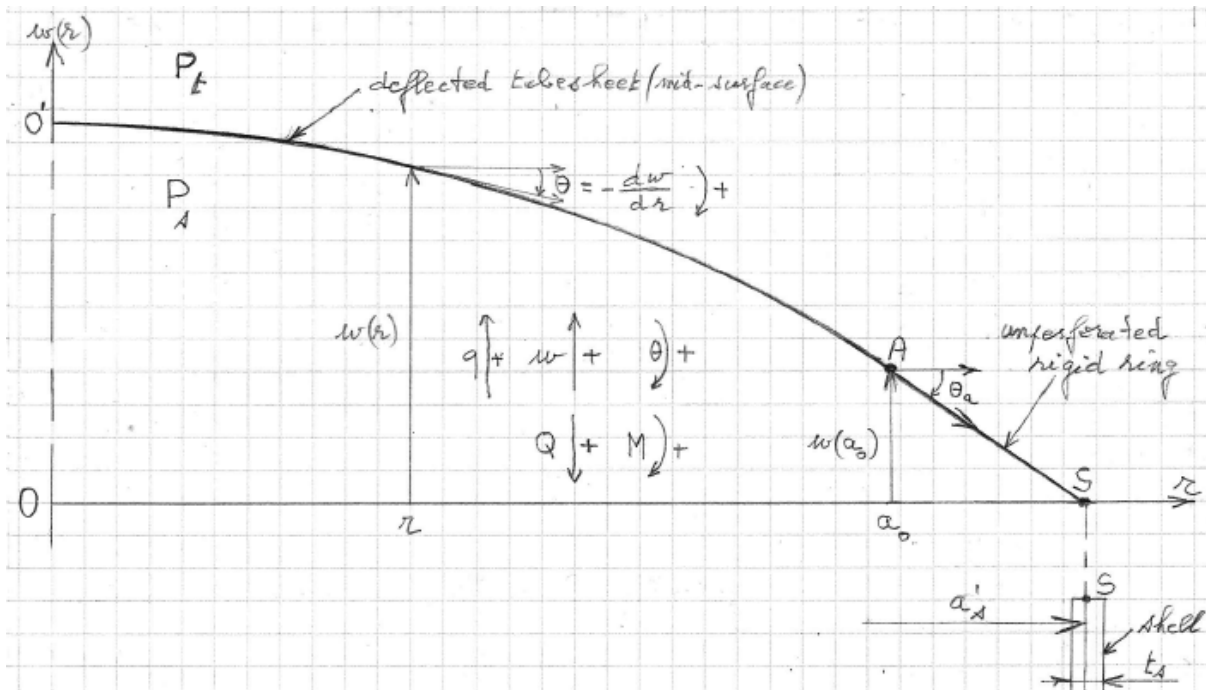


Figure 21—TS displacement

The TS is replaced by an equivalent solid plate:

- of equivalent radius a_o
- having effective elastic constants E^* and ν^* .

- Bending rigidity: $D^* = \frac{E^* h^3}{12(1-\nu^{*2})}$

- resting on an elastic foundation of modulus: $k_w = \frac{N k_t}{\pi a_o^2}$

- subjected to a net effective pressure $q(r)$

- subjected at its periphery to edge loads V_a and M_a

The deflection of such a plate is governed by a differential equation of 4th order:

$$\frac{d^4 w}{dr^4} + \frac{2}{r} \frac{d^3 w}{dr^3} - \frac{1}{r^2} \frac{d^2 w}{dr^2} + \frac{1}{r^3} \frac{dw}{dr} = \frac{q(r)}{D^*} = \frac{Q - k_w w(r)}{D^*}$$

whose solution writes (Annex F): $w(x) = A \operatorname{ber}(x) + B \operatorname{bei}(x) + \frac{Q}{k_w}$

where:

- x is a new dimensionless integration variable which is introduced to solve the differential equation $x = k r = \sqrt[4]{\frac{k_w}{D^*}} r$.

$$\text{At TS periphery (r=a}_o\text{): } X_a = k a_o \quad 0 \leq r \leq a_o \Rightarrow 0 \leq x \leq X_a$$

- $\operatorname{ber}(x)$ and $\operatorname{bei}(x)$ are Kelvin functions of order 0, which will be noted respectively $\operatorname{ber}x$ and beix in the following. Accordingly:

$$w(x) = A \operatorname{ber}x + B \operatorname{beix} + \frac{Q}{k_w}$$

- A, B are integration constants to be determined by boundary conditions at TS periphery ($x = X_a$)

$$\text{At TS periphery (r=a}_o\text{): } X_a = k a_o = \sqrt[4]{\frac{k_w}{D^*}} a_o = \left[24 (1 - \nu^{*2}) N_t \frac{E_t t_t (d_t - t_t) a_o^2}{E^* L h^3} \right]^{1/4}$$

$$\text{net effective pressure: } q(w) = -k_w (A \operatorname{ber}x + B \operatorname{beix})$$

(c) **the rotation**, using the sign conventions shown in Figure 21 ($\theta > 0$ clockwise), writes:

$$\theta(r) = -\frac{dw}{dr} = -k \frac{dw}{dx}$$

$$\theta(x) = -k [A \operatorname{ber}'x + B \operatorname{bei}'x]$$

Note: For $x=0$ $\theta(0)=0$: the rotation at the center of the TS is 0 as expected.

(d) shear force writes:

$$\begin{aligned} Q_r(r) &= \frac{1}{r} \int_0^r \rho q(\rho) d\rho & 0 \leq \rho \leq r & & x = k r \\ &= \frac{1}{r} \int_0^r -k_w \rho [A \operatorname{ber}(k\rho) + B \operatorname{bei}(k\rho)] d\rho & y = k\rho & & 0 < y \leq x \\ &= -\frac{1}{r} \frac{k_w}{k^2} \left[A \int_0^x y \operatorname{ber}y dy + B \int_0^x y \operatorname{bei}y dy \right] \end{aligned}$$

$$y \text{ bery} = + y \text{ bei}'' y + \text{bei}' y = + (y \text{ bei}' y)' \Rightarrow \int y \text{ bery} dy = + y \text{ bei}' y$$

$$y \text{ beiy} = - y \text{ ber}'' y + \text{ber}' y = - (y \text{ ber}' y)' \Rightarrow \int y \text{ beiy} dy = - y \text{ ber}' y$$

$$Q_r(r) = -\frac{1}{r} \frac{k_w}{k^2} [A x \text{ bei}' x - B x \text{ ber}' x]$$

$$Q_r(r) = -\frac{k_w}{k} (A \text{ bei}' x - B \text{ ber}' x)$$

(e) **radial bending moment** writes:

$$M_r(r) = -D^* \left[\frac{d^2 w}{dr^2} + \frac{\nu^*}{r} \frac{dw}{dr} \right] \quad x = k r$$

$$M_r(x) = -D^* k^2 [A \text{ ber}'' x + B \text{ bei}'' x] + \frac{\nu^*}{x} [A \text{ ber}' x + B \text{ bei}' x]$$

with: $\text{ber}'' x = -\text{beix} - \frac{\text{ber}' x}{x} \quad \text{bei}'' x = +\text{berx} - \frac{\text{bei}' x}{x}$

$$M_r(x) = -D^* k^2 \left[-A \left(\underbrace{\text{beix} + \frac{1-\nu^*}{x} \text{ber}' x}_{\Psi_1(x)} \right) + B \left(\underbrace{\text{berx} - \frac{1-\nu^*}{x} \text{bei}' x}_{\Psi_2(x)} \right) \right]$$

$$M_r(x) = -D^* k^2 [-A \Psi_1(x) + B \Psi_2(x)]$$

with:
$$\begin{cases} \Psi_1(x) = \text{beix} + \frac{1-\nu^*}{x} \text{ber}' x & \Psi_1 = \Psi_1(X_a) \\ \Psi_2(x) = \text{berx} + \frac{1-\nu^*}{x} \text{bei}' x & \Psi_2 = \Psi_2(X_a) \end{cases}$$

(f) **circumferential bending moment** writes:

$$M_\theta = -D^* \left[\nu^* \frac{d^2 w}{dr^2} + \frac{1}{r} \frac{dw}{dr} \right]$$

Same calculation leads to:

$$M_\theta(x) = -D^* k^2 [-A \overline{\Psi_1(x)} + B \overline{\Psi_2(x)}]$$

with: $\overline{\Psi_1(x)} = \nu^* \text{beix} - \frac{1-\nu^*}{x} \text{ber}' x \quad \overline{\Psi_2(x)} = \nu^* \text{berx} + \frac{1-\nu^*}{x} \text{bei}' x$

This moment generally does not control the TS design as:

$$|M_r(x)| > |M_\theta(x)|$$

5.2 Determination of Integration Constants A and B

(a) A and B are determined from the boundary conditions at periphery of TS:

$$Q_r(a_o) = V_a \quad M_r(a_o) = M_a$$

where V_a et M_a are the loads applied at periphery of TS (point A in Figure 21).

$$\left\{ \begin{array}{l} Q_r(X_a) = -\frac{k_w}{k} [A \operatorname{bei}'(X_a) - B \operatorname{ber}'(X_a)] \Rightarrow A \operatorname{bei}' - B \operatorname{ber}' = -\frac{k}{k_w} V_a \\ M_r(X_a) = -D^* k^2 [-A \Psi_1(X_a) + B \Psi_2(X_a)] \Rightarrow A \Psi_1 - B \Psi_2 = \frac{M_a}{D^* k^2} = \frac{k^2}{k_w} M_a \end{array} \right\}$$

$$M_r(X_a) = -D^* k^2 [-A \Psi_1(X_a) + B \Psi_2(X_a)] \Rightarrow A \Psi_1 - B \Psi_2 = \frac{M_a}{D^* k^2} = \frac{k^2}{k_w} M_a$$

$$\begin{aligned} \operatorname{ber} = \operatorname{ber}(X_a) \quad \operatorname{bei} = \operatorname{bei}(X_a) \quad \operatorname{ber}' = \operatorname{ber}'(X_a) \quad \operatorname{bei}' = \operatorname{bei}'(X_a) \quad \Psi_1 = \Psi_1(X_a) \\ \Psi_2 = \Psi_2(X_a) \end{aligned}$$

(b) Integration constants A and B are obtained from [V-2a]:

$$\left\{ \begin{array}{l} A = -\frac{k}{k_w} \frac{(k \operatorname{ber}') M_a + \Psi_2 V_a}{Z_a} \\ B = -\frac{k}{k_w} \frac{(k \operatorname{bei}') M_a + \Psi_1 V_a}{Z_a} \end{array} \right\} \quad \text{with: } Z_a = -\Psi_1 \operatorname{ber}' + \Psi_2 \operatorname{bei}'$$

(c) The 3rd boundary condition at the periphery of the unperforated rim, at its connection with the shell of mean radius a'_s (point S in Figure 22) writes: $w(a'_s) = 0$

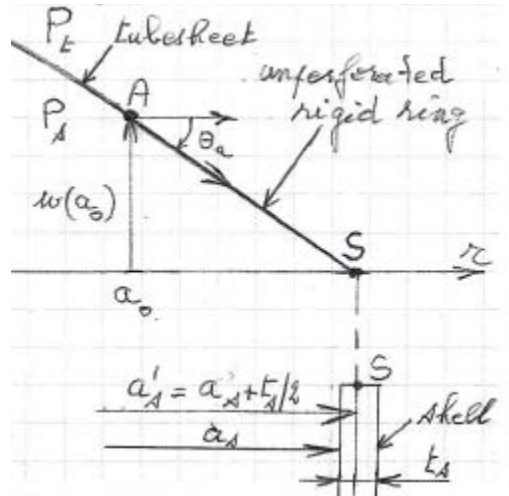


Figure 22

If the shell is integral, the TS rotation at point A writes:

$$\theta_a = -\frac{dw}{dr} = -\frac{\Delta w}{\Delta r} = -\frac{w(a'_s) - w(a_o)}{a'_s - a_o} = \frac{w(a_o)}{a'_s - a_o}$$

Where $\Delta w = w(a'_s) - w(a_o)$ as the unperforated rim is considered as a rigid.

$$w(a_o) = \theta_a (a'_s - a_o) = a_o \theta_a \left(\frac{a'_s}{a_o} - 1 \right)$$

Neglecting shell thickness: $a'_s = a_s$ $w(a_o) = a_o \theta_a (\rho_s - 1)$ [V.2c] with $\rho_s = \frac{a_s}{a_o} = \frac{D_s}{2a_o}$

If the shell is extended as a flange (configuration d): $\rho_s = \frac{G_s}{2a_o} = \frac{a_s}{a_o}$

(d) Substituting the expressions of A and B in V.1b enables to determine $w(x)$, $q(x)$, $\theta(x)$, $Q(x)$, $M(x)$ as functions of x , depending on V_a and M_a which are still unknown.

5.3 Deflection

$$\begin{aligned} w(x) &= \frac{Q}{k_w} - \frac{k}{k_w} \frac{(k \text{ber}' M_a + \Psi_2 V_a) \text{ber } x + (k \text{bei}' M_a + \Psi_1 V_a) \text{bei } x}{Z_a} \\ &= \frac{Q}{k_w} - \frac{1}{D^*} \frac{a_o^3}{X_a^3} \left[k M_a \frac{\text{ber}' \text{ber } x + \text{bei}' \text{bei } x}{Z_a} + V_a \frac{\Psi_2 \text{ber } x + \Psi_1 \text{bei } x}{Z_a} \right] \end{aligned}$$

$$w(x) = \frac{Q}{k_w} - \frac{a_o^2}{D^*} [M_a Z_w(x) + (a_o V_a) Z_d(x)] \quad [V.3]$$

$$Z_w(x) = \frac{\text{ber}' \text{ber } x + \text{bei}' \text{bei } x}{X_a^2 Z_a} \quad Z_d(x) = \frac{\Psi_2 \text{ber } x + \Psi_1 \text{bei } x}{X_a^3 Z_a}$$

Note: Alan SOLER formula is incorrect: $Z_v(x)$ is used instead of $Z_w(x)$. However the UHX rule is correct as it only uses $Z_w(X_a)$ in its formulas and it can be easily shown (see Annex G) that $Z_w(X_a) = Z_v(X_a)$

5.4 Net Effective Pressure

$$q(x) = Q - k_w w(x)$$

$$q(x) = \frac{a_o^2 k_w}{D^*} [M_a Z_w(x) + a_o V_a Z_d(x)] \quad [V.4]$$

5.5 Rotation

$$\theta(x) = -\frac{dw}{dr} = -\frac{dw}{dx} \frac{dx}{dr} = -k \frac{dw}{dx} = k \frac{a_o^2}{D^*} [M_a Z_w'(x) + (a_o V_a) Z_d'(x)]$$

$$\theta(x) = \frac{a_o}{D^*} \left[M_a \underbrace{\frac{\text{ber}' \text{ber}' x + \text{bei}' \text{bei}' x}{X_a Z_a}}_{Z_m(x)} + a_o V_a \underbrace{\frac{\Psi_2 \text{ber}' x + \Psi_1 \text{bei}' x}{X_a^2 Z_a}}_{Z_v(x)} \right]$$

$$\theta(x) = \frac{a_o}{D^*} [M_a Z_m(x) + a_o V_a Z_v(x)] \quad [V.5]$$

$$Z_m(x) = \frac{\text{ber}' \text{ber}' x + \text{bei}' \text{bei}' x}{X_a Z_a} \quad Z_v(x) = \frac{\Psi_2 \text{ber}' x + \Psi_1 \text{bei}' x}{X_a^2 Z_a}$$

Note: for $x=0$: $\begin{cases} Z_m(x)=0 \\ Z_v(x)=0 \end{cases} \theta(0)=0$: the rotation at TS center is 0 as expected.

5.6 Shear Force

$$Q_r(x) = \frac{k \text{ber}' M_a + \Psi_2 V_a}{Z_a} \text{bei}' x - \frac{k \text{bei}' M_a + \Psi_1 V_a}{Z_a} \text{ber}' x \quad k = \frac{X_a}{a_o}$$

$$Q_r(x) = \frac{1}{a_o} \left[M_a \underbrace{\frac{ber' bei' x - bei' ber' x}{Z_a}}_{Q_\alpha(x)} X_a + a_o V_a \underbrace{\frac{\Psi_2 bei' x - \Psi_1 ber'(x)}{Z_a}}_{Q_\beta(x)} \right]$$

$$\boxed{Q_r(x) = \frac{1}{a_o} [M_a Q_\alpha(x) + (a_o V_a) Q_\beta(x)]} \quad [V.6]$$

$$Q_\alpha(x) = \frac{ber' bei' x - bei' ber' x}{Z_a} X_a \quad Q_\beta(x) = \frac{\psi_2 bei' x - \psi_1 ber' x}{Z_a}$$

Note: for $x = X_a$: $\left\{ \begin{array}{l} Q_\alpha(X_a) = 0 \\ Q_\beta(X_a) = 1 \end{array} \right\} \quad Q_r(X_a) = \frac{1}{a_o} a_o V_a \Rightarrow Q(X_a) = V_a$

The boundary condition $Q_r(X_a) = V_a$ is satisfied.

5.7 Bending Moment

$$M_r(x) = \frac{D^* k^3}{k_w} \left[- \left(\frac{k \text{ber}' M_a + \Psi_2 V_a}{Z_a} \right) \Psi_1(x) + \left(\frac{k \text{bei}' M_a + \Psi_1 V_a}{Z_a} \right) \Psi_2(x) \right]$$

$$M_r(x) = M_a \underbrace{\frac{\text{bei}' \Psi_2(x) - \text{ber}' \Psi_1(x)}{Z_a}}_{Q_m(x)} + a_o V_a \underbrace{\frac{\Psi_1 \Psi_2(x) - \Psi_2 \Psi_1(x)}{X_a Z_a}}_{Q_v(x)}$$

$$\boxed{M_r(x) = M_a Q_m(x) + a_o V_a Q_v(x)} \quad [\text{V.7}]$$

$$Q_m(x) = \frac{\text{bei}' \Psi_2(x) - \text{ber}' \Psi_1(x)}{Z_a} \quad Q_v(x) = \frac{\Psi_1 \Psi_2(x) - \Psi_2 \Psi_1(x)}{X_a Z_a}$$

Note: - for $x=X_a$ $\begin{cases} Q_m(X_a)=1 \\ Q_v(X_a)=0 \end{cases} \Rightarrow M_r(X_a)=M_a$: the boundary condition $M(X_a)=M_a$ is satisfied.

5.8 Conclusion

- 1) Coefficients $Z_w(x), Z_d(x); Z_m(x), Z_v(x); Q_\alpha(x), Q_\beta(x); Q_m(x), Q_v(x)$ are combinations of Kelvin functions. They are known for a given HE and for a given value of x varying from 0 at the TS center to X_a at the TS periphery. These coefficients are defined in Table UHX-13.1 of UHX-13. Annex G provides their expressions for $x=0$ and $x=X_a$.
- 2) Quantities $w(x), q(x), \theta(x), Q_r(x), M_r(x)$ depend on moment M_a and force V_a acting at TS periphery.
- 3) Edge loads at TS-shell-connection are still to be determined from the boundary conditions at TS periphery in the following Clause.

6 TREATMENT OF THE UNPERFORATED RIM

6.1 Edge Loads Applied on Shell and Channel at their Connection to the TS

The following equations are developed for integral shell and channel.

(a) **Shell subjected to edge loads Q_s and M_s at its connection to rim** (see Figure 17)

Radial displacement w_s and rotation θ_s are detailed in Annex H. Loads are per unit of length of circumference.

$$\text{Radial displacement: } w_s = \frac{Q_s}{\beta_s^2 k_s} + \frac{M_s}{\beta_s k_s} + \delta_s P_s \quad \text{rotation: } \theta_s = \frac{Q_s}{\beta_s k_s} + \frac{2 M_s}{k_s}$$

Where:

$$\beta_s = \frac{\sqrt[4]{12(1-\nu_s^2)}}{\sqrt{(D_s + t_s)t_s}} \quad \text{is the shell coefficient.}$$

$$k_s = \beta_s \frac{E_s t_s^3}{6(1-\nu_s^2)} \quad \text{is the bending stiffness coefficient of the shell.}$$

$$\delta_s = \frac{D_s^2}{4 E_s t_s} \left(1 - \frac{D_s}{D_s + t_s} \frac{\nu_s}{2} \right) \quad \text{is the coefficient due to pressure acting on the shell.}$$

Note: In UHX, t_s has been neglected compared to D_s and the formula writes:

$$\delta_s = \frac{D_s^2}{4 E_s t_s} \left(1 - \frac{\nu_s}{2} \right)$$

$$\text{Equations for } M_s \text{ and } Q_s \text{ write: } \begin{cases} M_s = k_s \theta_s - \beta_s k_s w_s + \beta_s k_s \delta_s P_s \\ Q_s = -\beta_s k_s \theta_s + 2\beta_s^2 k_s w_s - 2\beta_s k_s \delta_s P_s \end{cases}$$

Neglecting the radial displacement at mid-surface of the ring, compatibility of displacements of the ring and the shell writes (see Annex I): $w_s = -\frac{h}{2} \theta_s$ and loads M_s and Q_s become, using:

$$t'_s = h \beta_s :$$

$$\left. \begin{aligned} M_s &= +k_s \left(1 + \frac{t'_s}{2} \right) \theta_s + \beta_s k_s \delta_s P_s \\ Q_s &= -\beta_s k_s \left(1 + t'_s \right) \theta_s - 2\beta_s^2 k_s \delta_s P_s \end{aligned} \right\} \quad \text{[VI.1a] for an integral shell (configurations a, b, c)}$$

When the shell is not integral with the TS (configuration d), $k_s=0$ and $\delta_s=0$ lead to: $M_s = 0$ and $Q_s = 0$.

Note: These formulas are valid for a shell of sufficient length. Annex J provides the minimum length above which these formulas can be applied.

(b) **Channel subjected to edge loads Q_c and M_c** at its connection to the rim (see Figure 17).

Note: So as to ensure the symmetry of the equations, notation $P_c = P_i$ is used.

Replacing subscript s by subscript c:
$$\begin{cases} w_c = \frac{Q_c}{\beta_c^2 k_c} + \frac{M_c}{\beta_c k_c} + \delta_c P_c \\ \theta_c = \frac{Q_c}{\beta_c k_c} + \frac{2M_c}{k_c} \end{cases} \quad \text{[VI.1b] for an integral}$$

channel (configuration a)

$$\beta_c = \frac{\sqrt[4]{12(1-\nu_c^2)}}{\sqrt{(D_c + t_c) t_c}} \quad \text{channel coefficient.}$$

$$k_c = \beta_c \frac{E_c t_c^3}{6(1-\nu_c^2)} \quad \text{bending stiffness coefficient of channel}$$

$$\delta_c = \frac{D_c^2}{4 E_c t_c} \left(1 - \frac{D_c}{D_c + t_c} \frac{\nu_c}{2} \right) \quad \text{coefficient due to pressure acting on channel.}$$

Equations for M_c and Q_c write
$$\begin{cases} M_c = +k_c \theta_c - \beta_c k_c w_c + \beta_c k_c \delta_c P_c \\ Q_c = -\beta_c k_c \theta_c + 2\beta_c^2 k_c w_c - 2\beta_c k_c \delta_c P_c \end{cases}$$

compatibility of displacements: $w_c = -\frac{h}{2} \theta_c$ lead to equations for M_c and Q_c , using $t'_c = h \beta_c$:

$$\left\{ \begin{array}{l} M_c = +k_c \left(1 + \frac{t'_c}{2} \right) \theta_c + \beta_c k_c \delta_c P_c \\ Q_c = -\beta_c k_c \left(1 + t'_c \right) \theta_c - 2\beta_c^2 k_c \delta_c P_c \end{array} \right\} \quad \text{[VI.1b] for an integral channel (configuration a)}$$

When the channel is not integral with the TS (configurations b, c, d), $k_c = 0$ and $\delta_c = 0$ lead to:
 $M_c = 0$ and $Q_c = 0$

Note 1: These formulas are valid for a channel of sufficient length. Annex J provides the minimum length above which these formulas can be applied.

Note 2: These formulas are valid for a cylindrical channel. If the channel is hemispherical, it must be attached directly to the TS (configurations a, b or c), without any cylindrical section between the head and the TS. Annex K provides the relevant formulas for that case. Only coefficient δ_c is affected:

$$\delta_c = \frac{D_c^2}{4 E_c t_c} \left(\frac{1}{2} - \frac{D_c}{D_c + t_c} \frac{\nu_c}{2} \right)$$

Note 3: In UHX, t_c has been neglected compared to D_a and the formula writes:

$$\delta_c = \frac{D_c^2}{4 E_c t_c} \left(\frac{1}{2} - \frac{\nu_c}{2} \right)$$

6.2 Equilibrium of the Unperforated Rim

(a) due to axial loads

$$\begin{cases} a_s = D_s / 2 \\ a'_s = a_s + \frac{t_s}{2} \end{cases} \quad \begin{cases} a_c = D_c / 2 \\ a'_c = a_c + \frac{t_c}{2} \end{cases}$$

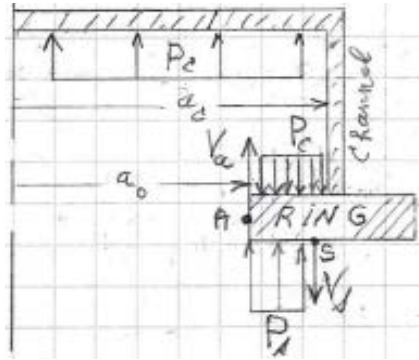


Figure 23—Ring equilibrium of the TS

The axial equilibrium of the ring writes: (see Figure 23):

$$2 \pi a'_s V_s + \pi (a_c^2 - a_o^2) P_c = 2 \pi a'_c V_c + 2 \pi a_o V_a + \pi (a_s^2 - a_o^2) P_s \quad [\text{VI.2a-1}]$$

where: - V_a = axial edge load acting at connection of ring with equivalent plate is still to be determined

- V_s = axial force acting in the shell will be obtained from above equation, once V_a will have been determined. V_s positive when the shell is in tension.

- V_c = axial force acting in the ST channel is known:

$$2 \pi a'_c V_c = \pi a_c^2 P_c \Rightarrow a'_c V_c = \frac{a_c^2}{2} P_c \quad [\text{VI.2a-2}]$$

Axial equilibrium of the ring writes:

$$a'_s V_s = a_o V_a + \frac{a_o^2}{2} P_c + \frac{a_s^2 - a_o^2}{2} P_s \quad [\text{VI.2a-3}]$$

Note: For U-tube and immersed floating head HEs:

$$2 \pi a'_s V_s = \pi a_s^2 P_s \Rightarrow a'_s V_s = \frac{a_s^2}{2} P_s \Rightarrow a_o V_a = \frac{a_o^2}{2} (P_s - P_c) \quad V_a \text{ is known.}$$

(b) due to applied moments

Equilibrium of moments applied to the ring relative to the axis located at radius a_o enables to determine the moment M_R (see Figure 17).

$$R = \text{radius at center of ring} = \frac{A + 2a_o}{4}$$

$$RM_R = -[a_o M_a] + \left[a'_c M_c - a'_c Q_c \frac{h}{2} \right] + \left[M(P_c) - a'_c V_c (a'_c - a_o) \right] - \left[a'_s M_s - a'_s Q_s \frac{h}{2} \right] + \left[M(P_s) - a'_s V_s (a'_s - a_o) \right] \quad [\text{VI.2b}]$$

$$M(P_c) = \text{moment due to pressure } P_c \text{ acting on the ring} = (a_c^2 - a_o^2) \left(\frac{a_c + a_o}{2} - a_o \right) \frac{P_c}{2}$$

$$M(P_s) = \text{moment due to pressure } P_s \text{ acting on the ring} = (a_s^2 - a_o^2) \left(\frac{a_s + a_o}{2} - a_o \right) \frac{P_s}{2}$$

Replacing terms of equation [VI.2b] by their expressions, one obtains for configuration a (see Annex L):

$$RM_R = -a_o M_a + a_o^2 V_a (\rho_s - 1) + P_s \frac{a_o^3}{4} [(\rho_s^2 - 1)(\rho_s - 1)] - P_c \frac{a_o^3}{4} [(\rho_c - 1)(\rho_c^2 + 1) - 2(\rho_s - 1)] - \left[a'_s k_s \left(1 + t'_s + \frac{t_s'^2}{2} \right) + a'_c k_c \left(1 + t'_c + \frac{t_c'^2}{2} \right) \right] \theta_a + a_o [\omega_c P_c - \omega_s P_s] \quad [\text{VI.2b}']$$

$$\boxed{\omega_s = \rho_s \beta_s k_s \delta_s (1 + h \beta_s)} \quad \boxed{\omega_c = \rho_c \beta_c k_c \delta_c (1 + h \beta_c)} \quad [\text{VI.2b}'']$$

Annex L provides the relevant modifications to cover the three other configurations b, c and d.

(c) Rotation of rigid ring

$$K = \frac{A}{D_o} \quad \theta_R = \frac{12}{E h^3} \frac{RM_R}{Ln K} \quad \text{leads to, with } \theta_R = \theta_a :$$

$$RM_R = \left[\frac{E h^3}{12} Ln K \right] \theta_R = \left[\frac{E h^3}{12} Ln K \right] \theta_a$$

Replacing RM_R by its expression [VI.2b'] leads to:

$$\begin{aligned}
& \left. \left[\overbrace{\left[\frac{E h^3}{12} \ln K + a'_s k_s \left(1 + t'_s + \frac{t_s'^2}{2} \right) + a'_c k_c \left(1 + t'_c + \frac{t_c'^2}{2} \right) \right]}^{C1} \right] \theta_a = -a_o M_a + a_o^2 V_a (\rho_s - 1) \right. \\
& \left. + P_s \left\{ \underbrace{\frac{a_o^3}{4} [(\rho_s^2 - 1)(\rho_s - 1)] - a_o \omega_s}_{C2} \right\} - P_c \left\{ \underbrace{\frac{a_o^3}{4} [(\rho_c - 1)(\rho_c^2 + 1) - 2(\rho_s - 1)] - a_o \omega_c}_{C3} \right\} \right] \\
C_1 &= \frac{h^3}{12} \left[\underbrace{\frac{6}{h^3} (D_s + t_s) k_s \left(1 + t'_s + \frac{t_s'^2}{2} \right)}_{\lambda_s} + \underbrace{\frac{6}{h^3} (D_c + t_c) k_c \left(1 + t'_c + \frac{t_c'^2}{2} \right)}_{\lambda_c} + E \ln K \right] = \frac{h^3}{12} [\lambda_s + \lambda_c + E \ln K] \\
\lambda_s &= \frac{6}{h^3} (D_s + t_s) k_s \left(1 + t'_s + \frac{t_s'^2}{2} \right) \quad \lambda_c = \frac{6}{h^3} (D_c + t_c) k_c \left(1 + t'_c + \frac{t_c'^2}{2} \right)
\end{aligned}$$

Note: if $\lambda_s + \lambda_c$ is high (>3) the TS is considered clamped

if $\lambda_s + \lambda_c$ is low (<1) the TS is considered simply supported

$$\begin{aligned}
C2 &= a_o \left\{ a_o^2 \left[\frac{(\rho_s^2 - 1)(\rho_s - 1)}{4} \right] - \omega_s \right\} = a_o \omega_s^* \quad \omega_s^* = a_o^2 \left[\frac{(\rho_s^2 - 1)(\rho_s - 1)}{4} \right] - \omega_s \\
C3 &= a_o \left\{ a_o^2 \left[\frac{(\rho_c^2 + 1)(\rho_c - 1)}{4} - \frac{\rho_s - 1}{2} \right] - \omega_c \right\} = a_o \omega_c^* \\
\omega_c^* &= a_o^2 \left[\frac{(\rho_c^2 + 1)(\rho_c - 1)}{4} - \frac{\rho_s - 1}{2} \right] - \omega_c
\end{aligned}$$

where ω_s and ω_c are given by [VI.2b''].

When the shell and channel are integral with the TS (configuration a), edge loads M_a and V_a , which are still unknown, are linked by equation:

$$\frac{h^3}{12} [\lambda_s + \lambda_c + E \ln K] \theta_a = -a_o M_a + a_o^2 V_a (\rho_s - 1) + a_o (\omega_s^* P_s - \omega_c^* P_c)$$

(Configuration a)

(d) **The generic equation covering the 4 configurations a, b, c and d writes**, accounting for the results obtained at Annex L for configurations b, c and d:

$$\frac{h^3}{12} [\lambda_s + \lambda_c + E \ln K] \theta_a = -a_o M_a + a_o^2 V_a (\rho_s - 1) + a_o (\omega_s^* P_s - \omega_c^* P_c) + \frac{a_o}{2\pi} [W_c \gamma_{bc} - W_s \gamma_{bs}]$$

[VI.2d]

where:

λ_s , λ_c and w_s , w_c are coefficients obtained above. They are known for a given HE.

W_s and W_c are bolt loads applied on shell and channel when the TS is gasketed (configurations b, c, d). Coefficients γ_{bs} and γ_{bc} are defined as follows:

- Configuration a: k_s given in VI.1a k_c given in VI.1b $\gamma_{bs} = 0$ $\gamma_{bc} = 0$
- Configuration b: k_s given in VI.1a $k_c = 0$ $\gamma_{bs} = 0$ $\gamma_{bc} = \frac{G_c - C_c}{D_o}$
- Configuration c: k_s given in VI.1a $k_c = 0$ $\gamma_{bs} = 0$ $\gamma_{bc} = \frac{G_c - G_1}{D_o}$
- Configuration d: $k_s = 0$ $k_c = 0$ $\gamma_{bs} = \frac{G_s - C_s}{D_o}$ $\gamma_{bc} = \frac{G_c - C_c}{D_o}$

UHX-13 covers the usual case $C_s = C_c = C$, $W_s = W_c = W$, which leads to:

$$\begin{aligned} \gamma_b &= 0 && \text{for configuration a,} \\ \gamma_b &= \frac{G_c - C}{D_o} && \text{for configuration b,} \\ \gamma_b &= \frac{G_c - G_1}{D_o} && \text{for configuration c,} \\ \gamma_b &= \frac{G_c - G_s}{2} && \text{for configuration d} \end{aligned}$$

6.3 Edge Loads V_a and M_a Applied to the Tubesheet

(a) Determination of M_a

Quantities θ_a , V_a and M_a at periphery of TS, linked by relation [VI.2d] above, are still unknown.

Equation [V.5] enables to determine:

$$\theta_R = \theta_a = \theta(X_a) = \frac{a_o}{D^*} [M_a Z_m + (a_o V_a) Z_v]$$

which leads to a 1st relationship between V_a and M_a :

$$\frac{h^3}{12} [\lambda_s + \lambda_c + E \ln K] \frac{a_o}{D^*} [M_a Z_m + (a_o V_a) Z_v] = -a_o M_a + a_o^2 V_a (\rho_s - 1) + a_o (\omega_s^* P_s - \omega_c^* P_c) + \frac{a_o}{2\pi} [W_c \gamma_{bc} - W_s \gamma_{bs}]$$

$$\Phi = \frac{h^3}{12} \frac{1}{D^*} [\lambda_s + \lambda_c + E \ln K] = \frac{h^3}{12} \frac{12(1-\nu^{*2})}{E^* h^3} [\lambda_s + \lambda_c + E \ln K]$$

$$\Phi = \frac{(1-\nu^{*2})}{E^*} [\lambda_s + \lambda_c + E \ln K] \quad [\text{VI-3}]$$

Note: Φ denotes the degree of restraint of the TS by the shell and channel

- if Φ is high (>4) the TS can be considered as clamped ($\lambda_s + \lambda_c$ high)
- if Φ is low (<1) the TS can be considered as simply supported ($\lambda_s + \lambda_c$ low)

Using coefficient F relative to U-tube HEs used in UHX-12.5.5:

$$F = \frac{1-\nu^{*2}}{E^*} [\lambda_s + \lambda_c + E \ln K] \quad \Phi = (1+\nu^{*2})F$$

$$\Phi [M_a Z_m + (a_o V_a) Z_v] = -M_a + a_o V_a (\rho_s - 1) + (\omega_s^* P_s - \omega_c^* P_c) + \frac{1}{2\pi} [W_c \gamma_{bc} - W_s \gamma_{bs}]$$

[VI.3a']

$$M_a (1 + \Phi Z_m) = a_o V_a [(\rho_s - 1) - \Phi Z_v] + \left[(\omega_s^* P_s - \omega_c^* P_c) + \left(\frac{W_c \gamma_{bc} - W_s \gamma_{bs}}{2\pi} \right) \right]$$

$$M_a = (a_o V_a) \underbrace{\frac{(\rho_s - 1) - \Phi Z_v}{1 + \Phi Z_m}}_{Q_1} + \underbrace{\frac{(\omega_s^* P_s - \omega_c^* P_c) + \left(\frac{W_c \gamma_{bc} - W_s \gamma_{bs}}{2\pi} \right)}{1 + \Phi Z_m}}_{Q_2} \quad [\text{VI.3a}]$$

$$M_a = (a_o V_a) Q_1 + Q_2 \quad [\text{VI.3a}]$$

$$Q_1 = (a_o V_a) \frac{(\rho_s - 1) - \Phi Z_v}{1 + \Phi Z_m} \quad Q_2 = \frac{(\omega_s^* P_s - \omega_c^* P_c) + \left(\frac{W_c \gamma_{bc} - W_s \gamma_{bs}}{2\pi} \right)}{1 + \Phi Z_m}$$

For the usual case (see VI.2d) covered by UHX-13 ($C_s = C_c = C$):

$$Q_2 = \frac{(\omega_s^* P_s - \omega_c^* P_c) + (W_c - W_s) \frac{\gamma_b}{2\pi}}{1 + \Phi Z_m}$$

Note: In M_a formula:

- term $(a_o V_a) Q_1$ represents the moment due to the equivalent pressure P_e acting on the TS.
- term Q_2 is the sum of the moments acting on the TS, due to pressures P_s, P_c and bolt load W .

(b) 2nd relationship between V_a and M_a

A 2nd relationship between V_a and M_a is obtained from boundary condition [V.2c]:

$$w_a = a_o \theta_a (\rho_s - 1)$$

Replacing w_a and θ_a by their expressions [V.3] and [V.5], [V2c] writes:

$$\frac{Q}{k_w} - \frac{a_o^2}{D^*} [M_a Z_m + (a_o V_a) Z_d] = \frac{a_o^2}{D^*} [M_a Z_m + (a_o V_a) Z_v] (\rho_s - 1)$$

$$\frac{Q}{k_w} = \frac{a_o^2}{D^*} \left\{ M_a \left[\frac{Z_w + (\rho_s - 1) Z_m}{\rho_1} \right] + (a_o V_a) \left[\frac{Z_d + (\rho_s - 1) Z_v}{\rho_2} \right] \right\}$$

$$\frac{Q}{k_w} = \frac{a_o^2}{D^*} [M_a \rho_1 + (a_o V_a) \rho_2] \quad [\text{VI.3b}] \quad \boxed{\begin{cases} \rho_1 = Z_w + (\rho_s - 1) Z_m \\ \rho_2 = Z_d + (\rho_s - 1) Z_v \end{cases}}$$

where Q is given by [V.1a']:

$$Q = \Delta p^* + k_w \left[\underbrace{[\delta_t(\theta_t) - \delta_s(\theta_s)] + [\delta_t(v_t) - \delta_s(v_s)] - [\delta_j(P_s)]}_{\delta} - \delta_s(V_s) \right]$$

$$\delta = [\delta_t(\theta_t) - \delta_s(\theta_s)] + [\delta_t(v_t) - \delta_s(v_s)] - [\delta_j(P_s)]$$

In this equation, all quantities are known except $\delta_s(V_s) = \frac{2\pi a_s'}{J k_s'} V_s$:

$$\text{with: } a_s' V_s = a_o V_a + \frac{a_o^2}{2} P_c + \frac{a_s^2 - a_o^2}{2} P_s \quad [\text{VI.2a-3}]$$

$$\frac{Q}{k_w} = \frac{\Delta p^*}{k_w} + \delta - \frac{2\pi}{J k_s'} \left[(a_o V_a) + \frac{a_o^2}{2} P_c + \frac{a_s^2 - a_o^2}{2} P_s \right]$$

(c) Determination of V_a

Introducing Q/k_w in [VI.3b] leads to a 2nd relationship between V_a and M_a , already linked by relationship [VI.3a] $M_a = (a_o V_a) Q_1 + Q_2$

which enables to determine V_a :

$$\frac{Q}{k_w} = \frac{\Delta p^*}{k_w} + \delta - \frac{2\pi}{J k_s'} \left[(a_o V_a) + \frac{a_o^2}{2} P_c + \frac{a_s^2 - a_o^2}{2} P_s \right] = \frac{a_o^2}{D^*} \{ [(a_o V_a) Q_1 + Q_2] \rho_1 + [(a_o V_a) \rho_2] \}$$

$$a_o V_a \left[1 + J \frac{k_s' a_o^2}{2\pi D^*} (\rho_1 Q_1 + \rho_2) \right] = \frac{J k_s'}{2\pi} \left[\frac{\Delta p^*}{k_w} + \delta \right] - \frac{a_o^2}{2} P_c - \frac{a_s^2 - a_o^2}{2} P_s - \frac{J k_s' a_o^2}{2\pi D^*} \rho_1 Q_2$$

$$\bullet \quad k^4 = \frac{k_w}{D^*} = \frac{N_t k_t}{\pi a_o^2 D^*} \Rightarrow \frac{1}{\pi D^*} = \frac{k^4 a_o^2}{N_t k_t} \Rightarrow \frac{k_s' a_o^2}{2\pi D^*} = \frac{k_s' a_o^4 k^4}{2 N_t k_t} = K_{s,t} \frac{X_a^4}{2}$$

$$\bullet \quad (\rho_1 Q_1 + \rho_2) = Z_m Q_1 + (\rho_s - 1) Q_1 Z_m + Z_d + (\rho_s - 1) Z_v = (Z_w Q_1 + Z_d) + (\rho_s - 1) (Z_w Q_1 + Z_v)$$

$$\bullet \quad J \frac{k_s' a_o^2}{2\pi D^*} (\rho_1 Q_1 + \rho_2) = J K_{s,t} \left[\underbrace{(Z_w Q_1 + Z_d) \frac{X_a^4}{2}}_{Q_{Z1}} + (\rho_s - 1) \underbrace{(Z_w Q_1 + Z_v) \frac{X_a^4}{2}}_{Q_{Z2}} \right] = J K_{s,t} F'_q$$

$$Q_{Z1} = (Z_w Q_1 + Z_d) \frac{X_a^4}{2}$$

$$Q_{Z2} = (Z_m Q_1 + Z_v) \frac{X_a^4}{2}$$

$$F'_q = Q_{Z1} + (\rho_s - 1) Q_{Z2}$$

$$a_o V_a \left(1 + J K_{s,t} F'_q \right) = J \frac{k_s'}{2\pi} \left[\left(\frac{\Delta p^*}{k_w} + \delta \right) - \frac{a_o^2}{D^*} \rho_1 Q_2 \right] - \frac{a_o^2}{2} P_c - \frac{a_s^2 - a_o^2}{2} P_s \quad [\text{VI.3c}]$$

For a given HE all above quantities are known, which enable to calculate V_a and thus M_a :

$$a_o V_a = \frac{J \frac{k_s'}{2\pi} \left[\left(\frac{\Delta p^*}{k_w} + \delta \right) - \frac{a_o^2}{D^*} \rho_1 Q_2 \right] - \frac{a_o^2}{2} P_c - \frac{a_s^2 - a_o^2}{2} P_s}{(1 + J K_{s,t} F'_q)}$$

[VI.3c']

$$M_a = (a_o V_a) Q_1 + Q_2 \quad [\text{VI.3a}]$$

7 EQUIVALENT PRESSURE ACTING ON TUBESHEET

7.1 Definition

(a) A circular plate under uniform pressures P_s and P_t is subjected to a differential pressure

$$P_e = P_s - P_t$$

The axial force V_a at periphery is determined from the plate equilibrium:

$$2\pi a_o V_a = P_e \pi a_o^2 \Rightarrow a_o V_a = \frac{a_o^2 P_e}{2} \quad P_e = \frac{2 V_a}{a_o} \quad [\text{VII.1}]$$

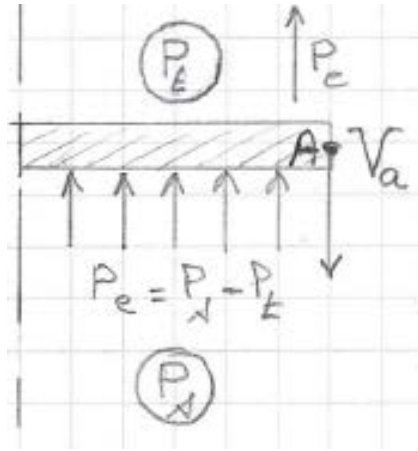


Figure 24

(b) The uniform pressure acting on the equivalent solid plate writes: $P_e = \frac{2 V_a}{a_o}$

This equivalent pressure is due to the various loads acting on the TS:

- Differential pressure: $\Delta p^* = x_s P_s - x_t P_t$
- Axial differential thermal expansion tubes-shell: $\delta(\theta) = \frac{\gamma}{2}$
- Force due to Poisson's ratio of tubes: $\delta_t(v_t) = -\frac{2}{k_w} \left[(1-x_t) P_t - (1-x_s) P_s \right] v_t$
- Force due to Poisson's ratio of shell: $\delta_s(v_s) = -\frac{\pi}{2k_s} \left[P_s D_s^2 \right] v_s$
- Force due to shell pressure acting on the joint: $\delta_j(P_s) = \frac{\pi}{16} \frac{D_j^2 - D_s^2}{K_j} P_s$

(see Annex C, Annex D, and Annex E)

Note: Equilibrium of the equivalent solid plate writes (see [VI.2a-3]):

$$a_o V_a = \frac{a_o^2}{2} (P_s - P_t) - \frac{a_s^2}{2} P_s + a_s' V_s$$

$$\text{Equilibrium of a circular plate writes (see [VII.1]): } a_o V_a = \frac{a_o^2}{2} (P_s - P_t)$$

7.2 Determination of P_e

Equivalent pressure $P_e = \frac{2 V_a}{a_o}$ can be obtained directly from V_a formula [VI.3c]:

$$a_o V_a (1 + J K_{s,t} F_q') = J \frac{k_s'}{2\pi} \left[\left(\frac{\Delta p^*}{k_w} + \delta \right) - \frac{a_o^2}{D^*} \rho_1 Q_2 \right] - \frac{a_o^2}{2} P_t - \frac{a_s^2 - a_o^2}{2} P_s \quad [\text{VI.3c}]$$

All terms appearing in this formula are known for a given HE. However, like TEMA and CODAP did, it is better to write P_e formula in a format which makes appear explicitly the contributions of pressures P_s and P_t , acting directly on the equivalent plate, loads acting on the unperforated rim, bolt load W when the TS is gasketed, and differential thermal expansion tubes-shell. Following manipulations are necessary to achieve this goal.

$$(1 + J K_{s,t} F_q') P_e = \frac{J k_s'}{\pi a_o^2} \left[\frac{1}{k_w} (\Delta p^* + k_w \delta) - \frac{a_o^2}{D^*} \rho_1 Q_2 \right] - P_t - (\rho_s^2 - 1) P_s$$

with:

$$\frac{k_s'}{\pi a_o^2 k_w} = \frac{k_s'}{N_t k_t} = K_{s,t} \quad \frac{k_s'}{\pi D^*} = \frac{X_a^4}{a_o^2} K_{s,t} \quad \delta = [\delta_t(\theta_t) - \delta_s(\theta_s)] + [\delta_t(\nu_t) - \delta_s(\nu_s)] - [\delta_j(P_s)]$$

$$\bullet \quad \frac{k_s'}{\pi a_o^2 k_w} [\Delta p^* + k_w \delta] = K_{s,t} \left[(x_s P_s - x_t P_t) + k_w \frac{\gamma}{2} - 2(1 - x_t) \nu_t P_t + 2(1 - x_s) \nu_t P_s - \frac{\pi k_w}{2 k_s'} P_s D_s^2 \nu_s - \frac{\pi k_w}{16} \frac{D_j^2 - D_s^2}{K_j} \right]$$

$$\frac{\pi k_w}{2 k_s'} P_s D_s^2 \nu_s = \frac{\pi N_t K_t}{2 k_s' \pi a_o^2} D_s^2 \nu_s P_s = \frac{2}{K_{s,t}} \left(\frac{D_s}{D_o} \right)^2 \nu_s P_s$$

$$\frac{\pi k_w}{16 K_j} (D_j^2 - D_s^2) = \frac{k_s'}{a_o^2 K_{s,t} 16 K_j} (D_j^2 - D_s^2) = \frac{1}{2 K_{s,t}} \frac{k_s'}{2 K_j} \frac{D_j^2 - D_s^2}{4 a_o^2} = \frac{1}{2 K_{s,t}} \frac{1 - J}{J} \frac{D_j^2 - D_s^2}{D_o^2}$$

$$\bullet \quad -\frac{k_s'}{\pi D^*} \rho_1 Q_2 = -K_{s,t} \frac{1}{a_o^2} \underbrace{\left[\frac{Z_w + (\rho_s - 1) Z_m}{1 + \Phi Z_m} X_a^4 \right]}_U \left[(\omega_s^* P_s - \omega_c^* P_t) + \frac{\gamma_b}{2\pi} W \right]$$

W is given by [VI.2d]

$$= K_{s,t} \left\{ -\frac{U}{a_o^2} [\omega_s^* P_s - \omega_c^* P_t] - \frac{U}{a_o^2} \left[\frac{\gamma_b}{2\pi} W \right] \right\} \quad \text{where} \quad U = \frac{Z_w + (\rho_s - 1) Z_m}{1 + \Phi Z_m} X_a^4$$

1st term accounts for the effect of moments due to pressures P_s and P_t acting on the unperforated

$$\text{rim: } P_{\text{rim}} = -\frac{U}{a_o^2} \left[\omega_s^* P_s - \omega_c^* P_t \right]$$

2nd term accounts for the effect bolting load W acting on the TS (configurations b, c, d)

$$P_w = -\frac{U}{a_o^2} \frac{\gamma_b}{2\pi} W$$

$$\text{Thus } -\frac{k_s'}{\pi D^*} \rho_1 Q_2 = K_{s,t} [P_{\text{rim}} + P_w]$$

$$(1 + J K_{s,t} F_q') P_e = J K_{s,t} \left\{ \begin{array}{l} P_s \left[x_s + 2(1-x_s) \nu_t + \frac{2}{K_{s,t}} \left(\frac{D_s}{D_o} \right)^2 \nu_s - \frac{\rho_s^2 - 1}{J K_{s,t}} - \frac{1-J}{2 J K_{s,t}} \frac{D_J^2 - D_s^2}{D_o^2} \right] \\ -P_t \left[x_t + 2(1-x_t) \nu_t + \frac{1}{J K_{s,t}} \right] + \gamma \left[\frac{k_w}{2} \right] + [P_w] + [P_{\text{rim}}] \end{array} \right\}$$

$$P_e = \frac{J K_{s,t}}{1 + J K_{s,t} F_q'} \left[P_s' - P_t' + P_\gamma + P_w + P_{\text{rim}} \right] \quad [\text{VII.2}] \quad F_q' = Q_{z1} (\rho_s - 1) Q_{z2}$$

In this formula:

P_s' accounts for the loads due pressure P_s acting on:

- the perforated TS,
- the external wall of the tubes (effect of Poisson's ratio ν_t),
- the external wall of the shell (effect of Poisson's ratio ν_s),
- the unperforated rim,
- the sidewall of the joint.

$$P_s' = P_s \left[x_s + 2(1-x_s) \nu_t + \frac{2}{K_{s,t}} \left(\frac{D_s}{D_o} \right)^2 \nu_s - \frac{\rho_s^2 - 1}{J K_{s,t}} - \frac{1-J}{2 J K_{s,t}} \frac{D_J^2 - D_s^2}{D_o^2} \right]$$

P_t' accounts for the loads due pressure P_t acting on:

- the perforated TS,
- the internal wall of the tubes (effect of Poisson's ratio ν_t),
- the unperforated rim.

$$P'_t = P_t \left[x_t + 2(1-x_t)v_t + \frac{1}{J K_{s,t}} \right]$$

P_γ accounts for the loads due to differential thermal expansion between tubes and shell γ .

$$P_\gamma = \gamma \left[\frac{k_w}{2} \right] = \left[\frac{N_t K_t}{\pi a_o^2} \right] \gamma$$

P_w accounts for the effect bolting load W acting on the TS (configurations b, c, d)

$$P_w = -\frac{U}{a_o^2} \frac{\gamma_b}{2\pi} W$$

P_{rim} accounts for the effect of moments due to pressures P_s and P_t acting on the unperforated rim:

$$P_{rim} = -\frac{U}{a_o^2} \left[\omega_s^* P_s - \omega_c^* P_t \right]$$

Note 1: Different expression of the equivalent pressure. P_e can be formulated differently if it is needed to set J equal to 0 to simulate a shell of rigidity 0. Introducing the numerator $JK_{s,t}$ inside the parenthesis leads to:

$$P_e = \frac{1}{1 + J K_{s,t} F'_q} \left[P'_{s1} - P'_{t1} + P_{\gamma1} + P_{w1} + P_{rim1} \right]$$

$$\text{with: } P'_{s1} = P_s \left[(JK_{s,t})x_s + (JK_{s,t})2(1-x_s)v_t + (J)2\left(\frac{D_s}{D_o}\right)^2 v_s - (\rho_s^2 - 1) - \frac{(1-J)[D_J^2 - D_s^2]}{2 D_o^2} \right]$$

$$P'_{t1} = P_t \left[(JK_{s,t})x_t + (JK_{s,t})2(1-x_t)v_t + 1 \right]$$

$$P_{\gamma1} = JK_{s,t} \left[\frac{N_t K_t}{\pi a_o^2} \gamma \right] \quad P_{w1} = JK_{s,t} \left[-\frac{U}{a_o^2} \frac{\gamma_b}{2\pi} W \right] \quad P_{rim} = JK_{s,t} \left[-\frac{U}{a_o^2} (\omega_s^* P_s - \omega_c^* P_t) \right]$$

This is the way TEMA formula is presented.

Note 2: Externally sealed HE. Setting the bellows rigidity to 0 simulates the case of a HE with an externally sealed floating head which will be covered in PART 4. $K_J=0$ and $D_J=0$ lead to:

$$J = \frac{K_J}{K_s + K_J} = 0 \quad JK_{s,t}=0 \quad P'_{s1} = [-(\rho_s^2 - 1)]P_s = (1 - \rho_s^2)P_s \quad P'_{t1} = [1]P_t = P_t$$

$$P_{\gamma 1} = JK_{s,t} \left[\frac{N_t K_t}{\pi a_o^2} \gamma \right] = 0 \quad P_{W1} = JK_{s,t} \left[-\frac{U}{a_o^2} \frac{\gamma_b}{2\pi} W \right] = 0$$

$$P_{rim1} = JK_{s,t} \left[-\frac{U}{a_o^2} (\omega_s^* P_s - \omega_c^* P_t) \right] = 0$$

$$\boxed{P_e = P'_{s1} - P'_{t1} = (1 - \rho_s^2) P_s - P_t} \quad \text{which is the UHX-14 formula for an externally sealed floating head.}$$

Note 3: Direct determination of the equivalent pressure. Equivalent pressures P'_s , P'_t and P'_γ can be obtained directly by examining the loads applied on the TS as shown in Annex M.

8 STRESSES IN THE HEAT-EXCHANGER COMPONENTS

Deformations (w, θ) and loads (q, Q_r, M_r) applied in the TS have been determined in V.3 to V.7 as a function of the TS radius. These formulas are general as they are valid whether $P_e \neq 0$ or $P_e = 0$. They depend on loads V_a and M_a acting at TS periphery, given by equations [VI.3c'] and [VI.a]:

$$\boxed{a_o V_a = \frac{a_o^2}{2} P_e} \quad \text{with } P_e \text{ given by [VII.2]} \quad \boxed{M_a = (a_o V_a) Q_1 + Q_2}$$

The formulas used in UHX-13 are presented in a different way.

- **If $V_a \neq 0$ ($P_e \neq 0$):** $\boxed{\frac{M_a}{a_o V_a} = Q_1 + \frac{Q_2}{a_o V_a} = Q_3}$ $\boxed{M_a = (a_o V_a) Q_3}$ $\boxed{Q_3 = Q_1 + \frac{2Q_2}{a_o^2 P_e}}$ [VIII]

The advantage of this presentation is that it enables to provide directly the maximum of the TS bending stress and the maximum of the tube stresses, thanks to a parametric study using X_a and Q_3 .

- **If $V_a = 0$ ($P_e = 0$):** Q_3 becomes infinity and equation [VIII] is no longer valid.

Equation [VI.3a] shows that M_a writes: $\boxed{M_a = Q_2}$

See Annex N for relevant equations to be applied in this case.

The following Clauses provide the deformations (w, θ) and loads (q, Q_r, M_r) applied in the TS, using both the general formulas and the UHX-13 formulas

Maximum stresses calculated in the TS, tubes, shell and channel must remain below allowable stress limits which are defined in Clause IX.

8.1 TS Net Effective Pressure

$$[V.4] \quad q(x) = \frac{a_o^2 k_w}{D^*} [M_a Z_w(x) + (a_o V_a) Z_d(x)] = \frac{X_a^4}{a_o^2} (a_o V_a) \left[\frac{M_a}{(a_o V_a)} Z_w(x) + Z_d(x) \right]$$

$$\boxed{q(x) = \frac{X_a^4}{a_o^2} (a_o V_a) [Q_3 Z_w(x) + Z_d(x)] = \frac{X_a^4}{2} \underbrace{[Q_3 Z_w(x) + Z_d(x)]}_{F_t(x)} P_e = F_t(x) P_e} \quad [VIII.1]$$

As shown by Figure 21, a positive value of q denotes a pressure upward.

8.2 TS Axial Displacement

$$[V.3] \quad w(x) = \frac{Q}{k_w} - \frac{q(x)}{k_w} = \frac{Q}{k_w} - \frac{1}{k_w} \frac{X_a^4}{2} \underbrace{[Q_3 Z_w(x) + Z_d(x)]}_{F_t(x)}$$

with [VI.3b]: $\frac{Q}{k_w} = \frac{a_o^2}{D^*} (a_o V_a) [Q_3 \rho_1 + \rho_2]$

$$w(x) = \frac{Q - F_t(x) P_e}{k_w} \quad F_t(x) = \frac{X_a^4}{2} [Q_3 Z_w(x) + Z_d(x)]$$

Expliciting ρ_1 , ρ_2 , V_a , Q and $F_t(x)$, $w(x)$ writes:

$$w(x) = \frac{a_o^4 P_e}{2D^*} \left\{ \left[(Q_3 Z_w + Z_d) - (Q_3 Z_w(x) + Z_d(x)) \right] + \left[(\rho_s - 1)(Q_3 Z_m + Z_v) \right] \right\}$$

The minimum and maximum values of $w(x)$ when x varies from 0 to X_a are obtained from the minimum and maximum values $F_{t,\min}$ and $F_{t,\max}$ of $F_t(x)$.

$F_t(x)$ depends on parameters X_a and Q_3 which are known for a given HE. X_a is usually comprised between 1 and 20, and Q_3 between -0.8 and +0.8. A parametric study has been performed using 20 values for X_a ($X_a = 1; 2; 3; \dots; 20$) and 17 values of Q_3 ($Q_3 = -0.8; -0.7; \dots; -0.1; 0.0; +0.1; \dots; +0.7; +0.8$), which enables to determine, for each couple $[X_a, Q_3]$:

$$F_{t,\min} = \text{MIN} [F_t(x)] \quad \text{and} \quad F_{t,\max} = \text{MAX} [F_t(x)]$$

Annex O provides for $1 \leq X_a \leq 20$ and $-0.8 \leq Q_3 \leq +0.8$:

- values and graphs of $F_t(x)$ for $0 < x < X_a$
- values and graphs of $F_{t,\min}$ and $F_{t,\max}$
- locations of the minimum and maximum of $F_t(x)$: $x_{\min}$ and $x_{\max}$

The minimum and maximum of $w(x)$ are given by:

$$w_1 = \frac{1}{k_w} \left[\frac{X_a^4}{2} (\rho_1 \cdot Q_3 + \rho_2) - F_{t,\min} \right] \cdot P_e \quad \text{and} \quad w_2 = \frac{1}{k_w} \left[\frac{X_a^4}{2} (\rho_1 \cdot Q_3 + \rho_2) - F_{t,\max} \right] \cdot P_e$$

As shown by Figure 21, a positive value of w denotes a deflection upward.

The maximum tubesheet deflection is given by:

$$w_{\max} = \text{MAX} [|w_1|, |w_2|]$$

8.3 TS Rotation

$$[V.5] \quad \theta(x) = \frac{a_o}{D^*} [M_a Z_m(x) + (a_o V_a) Z_v(x)] = \frac{a_o}{D^*} (a_o V_a) [Q_3 Z_m(x) + Z_v(x)]$$

$$\theta(x) = \frac{a_o^3 P_e}{2 D^*} [Q_3 Z_m(x) + Z_v(x)] = \frac{X_a^3}{k^3 2 D^*} [Q_3 Z_m(x) + Z_v(x)] P_e = \frac{k}{k_w} \underbrace{\frac{X_a^3}{2} [Q_3 Z_m(x) + Z_v(x)]}_{F_\theta(x)} P_e$$

$$\boxed{\theta(x) = \frac{k}{k_w} F_\theta(x) P_e} \quad \boxed{F_\theta(x) = \frac{X_a^3}{2} [Q_3 Z_m(x) + Z_v(x)]}$$

Note: for $x=0$: $\begin{cases} Z_m(x)=0 \\ Z_v(x)=0 \end{cases}$ $\theta(0)=0$: the rotation at TS center is 0 as expected.

As shown by Figure 21, a positive value of θ denotes a rotation clockwise.

8.4 Stresses in the Tubesheet

(a) Bending stress

[V.7]

$$M_r(x) = M_a Q_m(x) + (a_o V_a) Q_v(x) = (a_o V_a) [Q_3 Q_m(x) + Q_v(x)] = a_o^2 P_e \underbrace{\left[\frac{Q_3 Q_m(x) + Q_v(x)}{2} \right]}_{F_m(x)}$$

$$M_r(x) = a_o^2 P_e F_m(x) \quad F_m(x) = \frac{Q_3 Q_m(x) + Q_v(x)}{2} \quad \text{with:} \quad \begin{cases} Q_m(x) = \frac{bei' \Psi_2(x) - ber' \Psi_1(x)}{Z_a} \\ Q_v(x) = \frac{\Psi_1 \Psi_2(x) - \Psi_2 \Psi_1(x)}{X_a Z_a} \end{cases}$$

Note: At TS periphery ($x=X_a$): $\begin{cases} Q_m(X_a)=1 \\ Q_v(X_a)=0 \end{cases} \Rightarrow F_m(X_a) = \frac{Q_3}{2} \Rightarrow M_r(X_a) = M_a$:

the boundary condition is satisfied.

The bending stress in the TS is given by: $\sigma_r(x) = \frac{6 M_r(x)}{\mu^* h^2} = \frac{6 a_o^2 P_e F_m(x)}{\mu^* h^2}$

$$\boxed{\sigma_r(x) = \frac{1.5 F_m(x)}{\mu^*} \left(\frac{2 a_o}{h} \right)^2 P_e} \quad \boxed{F_m(x) = \frac{Q_3 Q_m(x) + Q_v(x)}{2}}$$

As shown by Figure 21, a positive value of $\sigma_r(x)$ denotes a bending moment clockwise.

The maximum value of $\sigma_r(x)$ when x varies from 0 to X_a is obtained from the maximum value $F_{m,\max}$ of $F_m(x)$. $F_m(x)$ depends on parameters X_a and Q_3 which are known for a given HE. A parametric study has been performed which enables to determine, for each couple $[X_a, Q_3]$: $F_{m,\max} = \text{MAX} [F_m(x)]$.

Figure 25 shows the bending stress distribution throughout the TS for $Q_3=0.0$ and $X_a=1, 3, 5, 7, 10$ and 15.

Annex P provides for $1 \leq X_a \leq 20$ and $-0.8 \leq Q_3 \leq +0.8$:

- values and graphs of $F_m(x)$ for $0 \leq x \leq X_a$

- values and graphs of $F_{m,\max}$
- location of the maximum of $F_m(x)$: $x_{\max}$

The maximum stress is obtained for: $F_m = |F_{m,\max}| = \text{MAX} [|F_m(x)|]$

The maximum bending stress in the TS writes: $\sigma = \frac{1.5 F_m}{\mu^*} \left(\frac{2 a_o}{h} \right)^2 P_e$ $F_m = \text{MAX} [|F_m(x)|]$

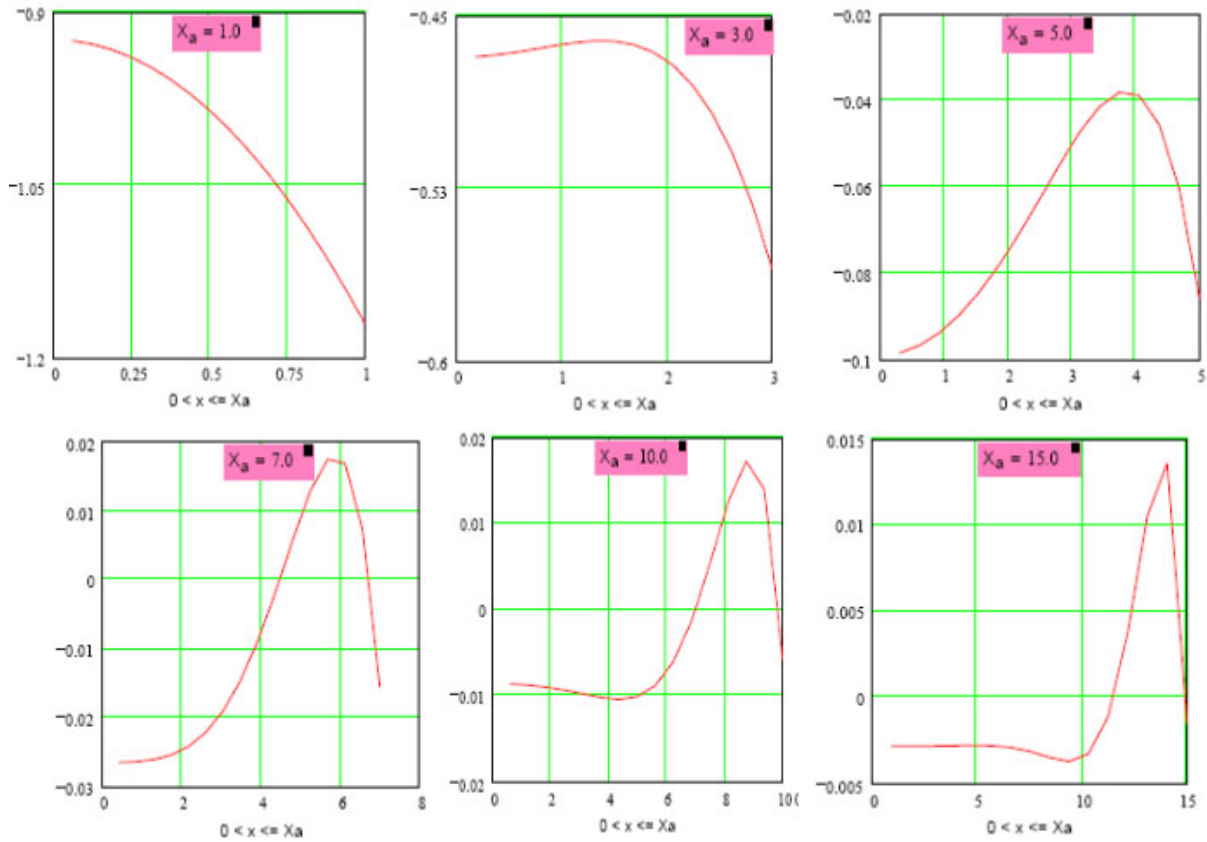


Figure 25—Bending stress distribution throughout the TS for $Q_3=0.0$ and $X_a=1, 3, 5, 7, 10$ and 15

(b) Shear stress

[V.6]

$$Q_r(x) = \frac{1}{a_o} [M_a Q_\alpha(x) + (a_o V_a) Q_\beta(x)] = \frac{(a_o V_a)}{a_o} [Q_3 Q_\alpha(x) + Q_\beta(x)] = \frac{a_o}{2} \underbrace{[Q_3 Q_\alpha(x) + Q_\beta(x)]}_{F_Q(x)} P_e$$

$$Q_r(x) = \frac{a_o}{2} F_Q(x) P_e \quad F_Q(x) = Q_3 Q_\alpha(x) + Q_\beta(x) \quad \text{with:} \quad \begin{cases} Q_\alpha(x) = \frac{\text{ber}' x - \text{bei}' x - \text{ber}' x}{Z_a} X_a \\ Q_\beta(x) = \frac{\psi_2 \text{bei}' x - \psi_1 \text{ber}' x}{Z_a} \end{cases}$$

$$\text{Note: At TS periphery } (x=X_a) \quad \begin{cases} Q_\alpha(X_a) = 0 \\ Q_\beta(X_a) = 1 \end{cases} \quad F_Q(X_a) = 1 \Rightarrow Q_r(X_a) = V_a :$$

the boundary condition is satisfied.

The shear stress in the TS, averaged throughout TS thickness, is given by:

$$\tau(x) = \frac{Q_r(x)}{\mu h} = \frac{a_o}{2 \mu h} F_Q(x) P_e$$

$$\boxed{\tau(x) = \frac{1}{2 \mu h} a_o F_Q(x) P_e} \quad \boxed{F_Q(x) = Q_3 Q_\alpha(x) + Q_\beta(x)}$$

The maximum value of $\tau(x)$ when x varies from 0 to X_a is obtained from the maximum value $F_{Q,\max}$ of $F_Q(x)$. $F_Q(x)$ depends on parameters X_a and Q_3 which are known for a given HE. A parametric study has been performed, which enables to determine, for each couple $[X_a, Q_3]$: $F_{Q,\max} = \text{MAX}[F_Q(x)]$

Annex Q provides for $1 \leq X_a \leq 20$ and $-0.8 \leq Q_3 \leq +0.8$:

- values and graphs of $F_Q(x)$ for $0 \leq x \leq X_a$
- values and graphs of $F_{Q,\max}$
- location of the maximum of $F_Q(x)$: $x_{\max}$

The maximum stress is obtained for: $F_Q = |F_{Q,\max}| = \text{MAX}[|F_Q(x)|]$

Maximum shear stress in the TS writes: $\tau_{\max} = \frac{F_Q}{2 \mu h} \cdot \frac{a_o}{h} \cdot P_e \quad F_Q = \text{MAX}[|F_Q(x)|]$

Shear stress at periphery writes: $\boxed{\tau = \tau(X_a) = \frac{1}{2 \mu h} a_o P_e}$ as $F_Q(X_a) = 1$

This formula is used in Clause UHX-13.5.8. It does not provide necessarily the maximum shear stress as $F_Q(x)$ is not always maximum at periphery. However, a parametric study has shown that the maximum of $\tau(x)$ appears for values of Q_3 and X_a for which the TS bending stress $\sigma(x)$ controls the design.

Note: TEMA rule provides the same formula, but uses the equivalent diameter D_L corresponding to the perimeter of the outermost ligaments, instead of the equivalent diameter D_o of outer tube limit circle.

The equivalent diameter D_L is calculated from the perimeter of the tube layout, $C_p = \pi D_L$, and the area

$$A_p = \pi D_L^2 / 4 \text{ enclosed by this perimeter. This leads to } D_L = 4A_p / C_p$$

$$D_L \text{ is always lower than } D_o \text{ and leads to a lower TS shear stress: } \tau = \frac{1}{4\mu} \frac{D_L}{h} P_e$$

That alternative TEMA formula should be used when the shear stress controls the design, generally in high-pressure cases.

8.5 Axial Membrane Stress in Tubes

Axial force $F_t^*(r)$ in tube row at radius r is obtained from [V.1a]:

$$q(r) = -\frac{V_t(r)}{\pi a_o^2 / N_t} + \Delta p^* \quad \Delta p^* = x_s P_s - x_t P_t$$

$$V_t(x) = \frac{\pi a_o^2}{N_t} [\Delta p^* - q(x)] \quad \text{with [VIII.1]} \quad q(x) = \underbrace{\frac{X_a^4}{2} [Q_3 Z_w(x) + Z_d(x)]}_{F_t(x)} P_e = F_t(x) P_e$$

$$\sigma_t(x) = \frac{V_t(x)}{s_t} = \frac{\pi a_o^2}{N_t s_t} [\Delta p^* - F_t(x) P_e] \quad \text{with [III.4]} \quad \frac{\pi a_o^2}{N_t s_t} = \frac{1}{x_t - x_s}$$

$$\boxed{\sigma_t(x) = \frac{1}{x_t - x_s} [\Delta p^* - F_t(x) P_e]} \quad \boxed{F_t(x) = \frac{X_a^4}{2} [Q_3 Z_w(x) + Z_d(x)]}$$

$\sigma_t(x)$ is either positive (tubes in tension) or negative (tubes in compression). The extreme values (maximum and minimum) of $\sigma_t(x)$ must be determined to obtain the maximum stresses in the tubes. These extreme values of $\sigma_t(x)$ are obtained for the minimum $F_{t,\min}$ and maximum $F_{t,\max}$ values of $F_t(x)$:

$$\boxed{F_{t,\min} = \text{MIN}[F_t(x)] \quad F_{t,\max} = \text{MAX}[F_t(x)]}$$

Annex O provides for $1 \leq X_a \leq 20$ and $-0.8 \leq Q_3 \leq +0.8$:

- values and graphs of $F_t(x)$ for $0 \leq x \leq X_a$
- values and graphs of $F_{t,\min}$ and $F_{t,\max}$
- locations of the minimum and maximum of $F_t(x)$: $x_{\min}$ and $x_{\max}$
- values of $F_t(X_a)$ and F_q

The minimum and maximum of $\sigma_t(x)$ are given by:

$$\boxed{\sigma_{t,1} = \frac{1}{x_t - x_s} [(x_s P_s - x_t P_t) - P_e F_{t,\min}]} \quad \boxed{\sigma_{t,2} = \frac{1}{x_t - x_s} [(x_s P_s - x_t P_t) - P_e F_{t,\max}]}$$

If tubes are in compression ($\sigma_{t,1}$ or $\sigma_{t,1}$ negative), special consideration must be given to their load carrying ability, which could lead to failure by buckling if a substantial number of tubes were above their buckling limit.

See Annex N.5 which provides the allowable buckling stress limit for tubes.

Note: The general of $\sigma_t(x)$ can be written, using the general formula of $q(x)$:

$$q(x) = \frac{X_a^4}{a_o^2} [M_a Z_w(x) + (a_o V_a) Z_d(x)] \quad [V.4]$$

$$\sigma_t(x) = \frac{1}{x_t - x_s} \left\{ \Delta p^* - \frac{X_a^4}{a_o^2} \left[\underbrace{M_a Z_w(x) + (a_o V_a) Z_d(x)}_A \right] \right\} \quad M_a = (a_o V_a) Q_1 + Q_2$$

$$[A] = (a_o V_a) Q_1 Z_w(x) + Q_2 Z_w(x) + (a_o V_a) Z_d(x) = \frac{a_o^2 P_e}{2} [Z_d(x) + Q_1 Z_w(x)] + Q_2 Z_w(x)$$

$$\sigma_t(x) = \frac{1}{x_t - x_s} \left\{ \Delta p^* - X_a^4 \left[\frac{P_e}{2} (Z_d(x) + Q_1 Z_w(x)) + \frac{Q_2}{a_o^2} Z_w(x) \right] \right\}$$

This equation can be used whether $P_e \neq 0$ or $P_e = 0$. However, it does not permit to determine the minimum $F_{t,min}$ and maximum $F_{t,max}$ of $F_t(x)$ using the parametric study of X_a and Q_3 to obtain the curves $F_{t,min}$ and $F_{t,max}$ as a function of X_a and Q_3 .

8.6 Stresses in the Shell

(a) Axial membrane stress

$$[VI.2a-3] a'_s V_s = a_o V_a + \frac{a_o^2}{2} P_t + \frac{a_s^2 - a_o^2}{2} P_s = a_o V_a + \frac{a_s^2 - a_o^2}{2} (P_s - P_t) + \frac{a_s^2}{2} P_t \quad a'_s = \frac{D_s + t_s}{2}$$

- axial force in the shell:

$$V_s = \frac{a_o^2 P_e}{2 a'_s} + \frac{a_o^2}{2 a'_s} (\rho_s^2 - 1) (P_s - P_t) + \frac{a_s^2}{2 a'_s} P_t$$

- axial membrane stress in the shell:

$$\sigma_{s,m} = \frac{V_s}{t_s} = \frac{a_o^2}{(D_s + t_s) t_s} \left[P_e + (\rho_s^2 - 1) (P_s - P_t) \right] + \frac{a_s^2}{(D_s + t_s) t_s} P_t$$

1st term is due to the equivalent pressure P_e acting on the tubesheet

2nd term is due to the pressures P_s and P_t acting on the unperforated ring

3rd term is due to the end load pressure P_t acting on the channel head

Note for the other types of HEs:

U tubes HE: $P_e = P_s - P_t$ (see PART 5)

$$\sigma_{s,m} = \frac{a_o^2}{(D_s + t_s) t_s} \rho_s^2 (P_s - P_t) + \frac{a_s^2 P_t}{(D_s + t_s) t_s} = \frac{a_s^2}{(D_s + t_s) t_s} P_s$$

$$\sigma_{s,m} = \frac{D_s^2}{4(D_s + t_s) t_s} P_s$$

Immersed floating head: $P_e = P_s - P_t$ (see PART 4)

$$\sigma_{s,m} = \frac{D_s^2}{4(D_s + t_s) t_s} P_s$$

In both cases the classical cylinder formula is obtained.

Externally sealed floating head: $P_e = (1 - \rho_s^2) P_s - P_t$ (see PART 4)

$$\sigma_{s,m} = \frac{a_o^2}{(D_s + t_s) t_s} \left[(1 - \rho_s^2) P_s - P_t + (\rho_s^2 - 1) P_s - (\rho_s^2 - 1) P_t \right] + \frac{a_s^2}{(D_s + t_s) t_s} P_t = 0$$

There is no axial force acting in the shell of an externally sealed floating head

Internally sealed floating head: $P_e = (1 - \rho_s^2) (P_s - P_t)$ (see PART 4)

$$\sigma_{s,m} = \frac{a_o^2}{(D_s + t_s) t_s} \left[(1 - \rho_s^2) (P_s - P_t) + (\rho_s^2 - 1) (P_s - P_t) \right] + \frac{a_s^2}{(D_s + t_s) t_s} P_t = 0$$

$$\sigma_{s,m} = \frac{D_s^2}{4(D_s + t_s) t_s} P_t \quad \text{The classical cylinder formula is obtained.}$$

(b) Axial displacement of the shell

For the half-shell of length l [IV.2e]:
$$\delta_{s,\text{Total}} = \frac{l V_s}{J E_s t_s} + \delta_s(\theta_s) + \delta_s(\nu_s) + \delta_J(P_s)$$

$$\delta_{s,\text{Total}} = \frac{l V_s}{J E_s t_s} + l \alpha_{s,m} (T_{s,m} - T_a) - \frac{l D_s^2 P_s}{2 E_s (D_s + t_s) t_s} \nu_s + \frac{\pi}{16} \frac{D_J^2 - D_s^2}{K_J} P_s$$

For the shell of length $L=2l$: $\Delta_s = 2 \delta_{sT}$

$$\Delta_s = \frac{L}{J E_s t_s} V_s + L \alpha_{s,m} (T_{s,m} - T_a) - \frac{L D_s^2 P_s}{2 E_s (D_s + t_s) t_s} \nu_s + \frac{\pi}{8} \frac{D_J^2 - D_s^2}{K_J} P_s$$

1st term is due to the axial force V_s acting in the in the shell

2nd term is due to the displacement of the shell subjected to the mean temperature $T_{s,m}$

3rd term is due to the effect of the Poisson's ratio ν_s of the shell

4th term is due to the pressure P_s acting on the sidewalls of the expansion joint

(c) Bending stress

The bending moment M_s in the shell at its connection with the TS exists only when the shell is integral with the TS (configurations a, b, c). As explained in Annex J, the shell must have a minimum length $l_{s,\min} = 1.8\sqrt{D_s t_s}$ adjacent to the TS.

$$[\text{VI.1a}]: M_s = k_s \left(1 + \frac{t'_s}{2} \right) \theta_s + (k_s \beta_s \delta_s) P_s \quad \text{with } \theta_s = \theta_a \quad \text{given by [V.5]}:$$

$$\theta_a = \frac{a_o}{D^*} [M_a Z_m + (a_o V_a) Z_v] = \frac{a_o}{D^*} [(a_o V_a)(Z_v + Q_1 Z_m) + (Q_2 Z_m)] = \frac{12(1-\nu^{*2})}{E^* h^3} a_o \left[\frac{a_o^2}{2} P_e (Z_v + Q_1 Z_m) + (Q_2 Z_m) \right]$$

$$\theta_a = \frac{6(1-\nu^{*2})}{E^*} \left(\frac{a_o}{h} \right)^3 \left[P_e (Z_v + Q_1 Z_m) + \frac{2}{a_o^2} (Q_2 Z_m) \right] \quad [\text{VIII.7b}]$$

$$M_s = k_s \left\{ \left(1 + \frac{t'_s}{2} \right) \frac{6(1-\nu^{*2})}{E^*} \left(\frac{a_o}{h} \right)^3 \left[P_e (Z_v + Q_1 Z_m) + \frac{2}{a_o^2} (Q_2 Z_m) \right] + (P_s \beta_s \delta_s) \right\}$$

$$t'_s = h \beta_s$$

$$\text{Bending stress } \sigma_{s,b} \text{ writes: } \sigma_{s,b} = 6 \frac{M_s}{t_s^2}$$

$$\sigma_{s,b} = \frac{6}{t_s^2} k_s \left\{ \beta_s \delta_s P_s + \frac{6(1-\nu^{*2})}{E^*} \left(\frac{a_o}{h} \right)^3 \left(1 + \frac{h \beta_s}{2} \right) \left[P_e (Z_v + Q_1 Z_m) + \frac{2}{a_o^2} (Q_2 Z_m) \right] \right\}$$

[VIII.6c]

Note 1: If $P_e \neq 0$ the equation can be written as a function of Q_3 :

$$P_e (Z_v + Q_1 Z_m) + \frac{2}{a_o^2} (Q_2 Z_m) = P_e \left\{ Z_v + Z_m \left[Q_1 + \frac{2}{a_o^2 P_e} Z_m Q_2 \right] \right\} = P_e (Z_v + Q_3 Z_m)$$

$$\text{Note 2: In Clause VII-a: } \delta_s = \frac{a_s^2}{E_s t_s} \left(1 - \frac{\nu_s}{2} \right) = \frac{a_s^2}{E_s t_s} - \frac{\nu_s}{2} \frac{a_s^2}{E_s t_s}$$

The equation used by SOLER [9] in his book, page 429, looks more general:

$$\delta_s^{AS} = \frac{a_s^2}{E_s t_s} - \nu_s \frac{a_s^2}{E_s t_s} \frac{V_s}{P_s} = \frac{a_s^2}{E_s t_s} - \nu_s \frac{a_s^2}{E_s t_s} \frac{t_s}{P_s} \sigma_{s,m}$$

$$\text{where } V_s \text{ is the axial force acting in the shell, which leads to: } \sigma_{s,m} = \frac{V_s}{t_s}$$

In page 447 the term $\nu_s \frac{a_s}{t_s} \frac{\sigma_{s,m}}{P_s}$ is neglected as $\sigma_{s,m}$ is not yet determined. However that term is reintroduced in the final equation giving $\sigma_{s,b}$, which is not correct.

If the classical shell formula is used for $\sigma_{s,m}$: $\sigma_{s,m} = \frac{a_s}{2t_s} P_s$

$$\delta_s^{AS} \text{ writes: } \delta_s^{AS} = \frac{a_s^2}{E_s t_s} - \nu_s \frac{a_s^2}{2 E_s t_s} = \frac{a_s^2}{E_s t_s} \left(1 - \frac{\nu_s}{2}\right) = \delta_s$$

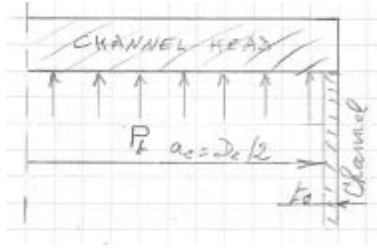
8.7 Stresses in the Channel

(a) axial membrane stress

Integral channel (configuration a):

$$a_c = \frac{D_c}{2}$$

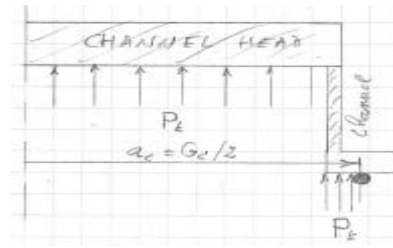
$$\sigma_{c,m} = \frac{\pi \left(\frac{D_c}{2}\right)^2 P_t}{\pi (D_c + t_c) t_c} = \frac{a_c^2}{(D_c + t_c) t_c} P_t$$



Gasketed channel (configurations b, c, d):

$$a_c = \frac{G_c}{2}$$

$$\sigma_{c,m} = \frac{\pi \left(\frac{G_c}{2}\right)^2 P_t}{\pi (D_c + t_c) t_c} = \frac{a_c^2}{(D_c + t_c) t_c} P_t$$



General formula:
$$\sigma_{c,m} = \frac{a_c^2}{(D_c + t_c) t_c} P_t$$

(b) Bending stress

The bending moment M_c in the channel at its connection with the TS exists only when the shell is integral with the TS (configuration a). As explained in Annex J, the channel must have a minimum length $l_{c,min} = 1.8\sqrt{D_c t_c}$ adjacent to the TS.

$$[VI.1b]: M_c = k_c \left(1 + \frac{t'_c}{2}\right) \theta_c + (k_c \beta_c \delta_c) P_c \quad \text{with: } \theta_c = -\theta_a \text{ and } \theta_a \text{ given by [VIII.7b]}$$

$$M_c = k_c \left\{ -\left(1 + \frac{t'_c}{2}\right) \frac{6(1-\nu^{*2})}{E^*} \left(\frac{a_o}{h}\right)^3 \left[P_c (Z_v + Q_1 Z_m) + \frac{2}{a_o^2} (Q_2 Z_m) \right] + (P_c \beta_c \delta_c) \right\} t'_c = h \beta_c$$

$$\sigma_{c,b} = \frac{6}{t_c^2} k_c \left\{ \beta_c \delta_c P_c - \frac{6(1-\nu^{*2})}{E^*} \left(\frac{a_o}{h}\right)^3 \left(1 + \frac{h \beta_c}{2}\right) \left[P_c (Z_v + Q_1 Z_m) + \frac{2}{a_o^2} (Q_2 Z_m) \right] \right\}$$

8.8 How to use the Rules

The calculation procedure can be summarized as follows:

- Set the data listed in III.2a
- Calculate the design coefficients listed in III.2b (x_s, x_t ; p_s, p_c ; K_s, t, J)
- Calculate 1st characteristic parameter X_a and coefficients Q_1, Q_2 ; ω_s^*, ω_c^*
- Calculate coefficients QZ_1, QZ_2, U , the equivalent pressure P_e , and 2nd characteristic parameter Q_3
- Calculate the maximum stresses in TS, tubes, shell and channel and limit their values to the maximum allowable stress limits.

Because of the complexity of the procedure, it is likely that users will computerize the solution. Annex R provides a Mathcad calculation sheet for a fixed TS HE defined in UHX-20.2.3 (Example 3). The calculation sheet is divided in 2 parts:

Part 1 follows strictly the various steps (Steps 1 to 11) of UHX-13.5 calculation procedure.

Part 2 provides the equations developed in VIII, which enable to calculate:

- at any radius of the perforated tubesheet: net pressure, deflection, rotation, bending stress, shear stress
- at any radius of the tube bundle: axial membrane stress.
- at tubesheet-shell-channel connection: loads and displacements acting on the shell.

9 DETERMINATION OF THE ALLOWABLE STRESS LIMITS

9.1 General Considerations

The tubesheet rules of Part UHX are intended to generally follow the stress classification of Appendix 4.1 of Section VIII, Division 2 [11] for primary and primary plus secondary stresses. The scope of Part UHX does not include any consideration of peak stresses or any requirements for fatigue.

(a) pressure loading cases

The bending stress resulting from a pressure loading in a flat plate is a primary bending stress. Any yielding of the plate material results in a permanent deformation, and the deformation may continue to occur until the plate fails (or the deformation is so large that the plate cannot perform its intended function). When the tubesheet is extended as a flange, the loading generated by the bolting moment is considered primary. Thus, primary bending stress limits are appropriate when considering the tubesheet bending stress resulting from pressure loading acting alone or in combination with the bolt loading when applicable, i.e. the so-called "pressure loading cases" 1, 2 and 3 mentioned in Clause III-3a.

The stresses in the shell and channel are somewhat more complex to categorize. The axial membrane stresses in the shell and channel remote from the tubesheet resulting from pressure loadings are primary.

The bending stress at the shell-to-tubesheet junction and the channel-to-tubesheet junction result from restrained differential motion at these junctions. As such, these bending stresses have the basic characteristic of a secondary stress. However, there is a very important distinction that has to be made regarding the status of these stresses, based on footnote 2 of Table 4-120.1 of Section VIII Division 2 [11]. If the discontinuity bending moment at the edge of a flat plate is required to maintain the bending stress elsewhere in the plate to within its allowable stress the shell/channel bending stress is classified as primary bending and should be limited to the primary bending stress limit.

Accordingly, when an elastic stress analysis includes the rotational stiffness of the shell and channel in determining the tubesheet stress under primary loading, the discontinuity bending stress should be categorized as primary bending stress and be limited accordingly.

However, there may be instances where the design need not consider the full strengthening effect of the shell/channel. For example, if one chose to not include the stiffening effect of the shell and channel for the tubesheet analysis, the shell and channel bending stresses could be correctly categorized as secondary and be limited according to the secondary stress considerations.

Or, if the shell/channel bending stresses resulting from pressure/bolting loads do not satisfy the primary bending stress limits, then full credit cannot be taken for the stiffness of that component. In such a case, it is deemed appropriate to apply a "knockdown" factor to the stiffness of the shell or channel component by reducing its modulus of elasticity. The "knockdown" factor used in the UHX "Elastic-Plastic analysis" is based on evaluations of the extent of strengthening offered by the fully plastic moment at the shell and channel junction when the primary bending stress limits are not satisfied.

(b) pressure+thermal loading cases

The stresses resulting from the temperature difference between shell and tubes (thermal loading cases 4, 5, 6 and 7) are secondary in that they are self-limiting. The Code limits on secondary stress are derived to accomplish "shakedown" to elastic action". UHX rules consider the

tubesheet, shell, channel, and tube stresses to be secondary stresses under the action of thermal loads.

Note: It has been a long, standing practice of TEMA to divide the loads resulting from thermal expansion by a factor of two, including the tube loads. Thus, the TEMA allowable tube tensile stress for thermal loading cases is effectively increased by a factor of two. This practice has not led to any noted problems or deficiencies in allowed tube loads, and this practice is continued for the Part UHX rules for allowable tensile stress when considering thermal load conditions.

9.2 Allowable Stress Limit in the Tubesheet

- The bending stress σ due to pressure loading (loading cases 1, 2, 3) is a primary bending stress (P_b) to be limited to $1.5S$.
- The bending stress due to pressure+thermal loading (loading cases 4, 5, 6, 7) is a secondary bending stress (Q) to be limited to S_{PS} .
- The shear stress is limited to $0.8S$, as this is the practice in most Codes (TEMA, CODAP, EN)

9.3 Allowable Stress Limit in the Tubes

- The axial membrane stress σ_t due to pressure loading (loading cases 1, 2, 3) is a primary membrane stress (P_m) to be limited to S_t .
- The axial membrane stress due to pressure+thermal loading (loading cases 4, 5, 6, 7) is strain induced and therefore a secondary stress (Q) to be limited to $S_{PS,t}$.

However, this limit has been lowered to $2S_t$ for two reasons:

- 1) If the stress level in the tubes is too high, it may happen that a substantial number of tubes fail and that the tube bundle could not sustain the required loading, especially for tubes in compression.
- 2) Consistency must be ensured with Appendix A, which limits the maximum axial load in the tube-to-tubesheet joint to $2S_t$.

If the tubes are under compression, tube buckling may restrict the tube's load carrying ability. This is true for either pressure or thermal load conditions. Accordingly, no distinction is made between primary and secondary allowable compressive loads in the tubes.

The tube axial stresses are limited to the maximum buckling stress limit $S_{t,b}$, determined in Annex N.5.

9.4 Allowable Membrane Stress Limit in the Shell

- The axial membrane stress $\sigma_{s,m}$ due to pressure loading (loading cases 1, 2, 3) is a primary membrane stress (P_m) to be limited to S_s .
- The axial membrane stress due to pressure+thermal loading (loading cases 4, 5, 6, 7) is strain induced and therefore is a secondary stress (Q) to be limited to $S_{PS,s}$.

If the shell is under compression, the axial stress must be limited to the maximum buckling stress limit $S_{s,b}$ determined by applying UG-23(b).

9.5 Allowable Membrane+Bending Stress Limit in the Shell

- The membrane+bending stress σ_s of the shell at its junction to the tubesheet due to pressure loading (loading cases 1, 2, 3) is a primary bending stress (P_b) to be limited to $1.5S_s$. If this limit is exceeded, an elastic-plastic analysis may be performed as mentioned in Clause IX.1a.
- The membrane+bending stress due to pressure+thermal loading (loading cases 4, 5, 6, 7) is a secondary stress (Q) to be limited to $S_{PS,s}$.

9.6 Allowable Membrane+Bending Stress Limit in the Channel

Same rules as for the shell above apply.

9.7 Conclusions

The analytical method is based on the concept of the TS replaced by an equivalent solid plate resting on an elastic foundation to which the classical discontinuity analysis is applied.

The analytical development enables to determine:

- at any radius r of the TS, the deflection $w(x)$, rotation $\theta(x)$, net effective pressure $q(x)$, bending stress $\sigma(x)$, shear stress $\tau(x)$ and axial stress in the tubes $\sigma_t(x)$, where $x = r\sqrt[4]{k_w / D^*}$. Their maximum has been obtained thanks to a parametric study performed on the two dimensionless coefficients X_a and Q_3 of the HE.
- in the shell, the axial force and resulting axial displacement and membrane stress, and the bending stress at its connection with TS and channel.
- in the channel, the membrane stress, and the bending stress at its connection with TS and shell.

The results obtained confirm the correctness of the design rules given in UHX-13.5.

Specific rules complete the UHX-13.5 design rules to cover:

- The effect of different shell material or thickness adjacent to the tubesheet (UHX-13.6)
- The effect of plasticity at tubesheet-shell-channel joint (UHX-13.7)
- The effect of radial thermal expansion adjacent to the tubesheet (UHX-13.8)
- The calculation procedure for simply supported tubesheet (UHX-13.9)
- The design of tubesheet flange extension (UHX-9)

10 ADDITIONAL RULES

10.1 Effect of Different Shell Thickness and Material Adjacent to the TS (UHX-13.6)

- (a) **General:** When the shell is integral with the TS (configurations a, b or c), if the stresses are above their allowable stress limits (i.e. $\sigma_s > 1.5S_s$ for pressure loadings or $\sigma_s > S_{PS,s}$ for thermal loadings), the calculated stresses can be reduced by thickening the shell adjacent to the TSs.

As explained in Annex J, the shell must have a minimum length adjacent to the TS of $l_{s,min} = 1.8 \sqrt{D_s t_s}$.

Accordingly, the shell must be thickened over a minimum length $l_{s,min,1} = 1.8 \sqrt{D_{s,1} t_{s,1}}$ adjacent to the TSs.

The thickened TS sections should have the same thickness $t_{s,1}$ and the same material to comply with the assumption that the HE is a symmetrical unit (see III.4-1). However the thickened lengths l_1 and l'_1 adjacent to each TS can be different. The material can be different over these lengths with modulus $E_{s,1}$ and thermal coefficient $\alpha_{s,m,1}$.

Additional notations are as follows, using subscript 1 for the quantities linked to the shell thickening. See Figure 26

$E_{s,1}$	=	modulus of elasticity for shell material adjacent to tubesheets at T_s
l_1, l'_1	=	lengths of shell of thickness $t_{s,1}$ adjacent to tubesheets ($l_1 \geq l_{s,min,1}$ $l'_1 \geq l_{s,min,1}$)
$D_{s,1}$	=	internal shell diameter adjacent to tubesheets
$t_{s,1}$	=	shell thickness adjacent to tubesheets
$S_{s,1}$	=	allowable stress for shell material adjacent to tubesheets at T_s
$S_{y,s,1}$	=	yield strength for shell material adjacent to tubesheets at T_s
$S_{PS,s,1}$	=	allowable primary plus secondary stress for shell material at T_s
$\alpha_{s,m,1}$	=	mean coefficient of thermal expansion of shell material adjacent to tubesheets at $T_{s,m}$

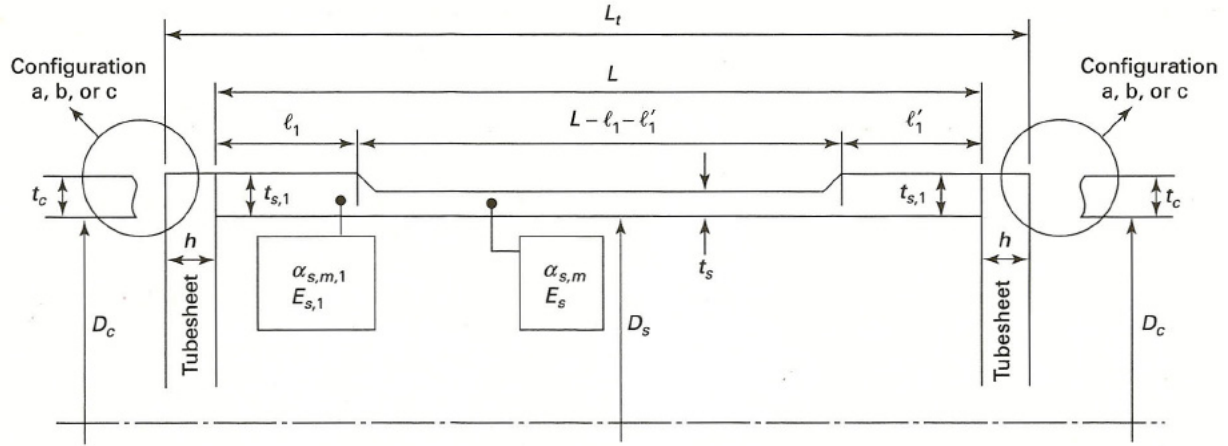


Figure 26—Shell with increased thickness adjacent to TSs

(b) **Shell axial stiffness** K_s^* is determined from:

$$\frac{1}{K_s^*} = \frac{1}{K_{s,0}} + \frac{1}{K_{s,1}} + \frac{1}{K'_{s,1}} \quad \text{where:}$$

$$K_{s,0} = \frac{\pi t_s (D_s + t_s) E_s}{L_o} \quad \text{is the axial stiffness of the basic shell section of length } L_o = L - l_1 - l'_1$$

$$K_{s,1} = \frac{\pi t_{s,1} (D_{s,1} + t_{s,1}) E_{s,1}}{l_1} \quad \text{is the axial stiffness of the shell section of length } l_1$$

$$K'_{s,1} = \frac{\pi t_{s,1} (D_{s,1} + t_{s,1}) E_{s,1}}{l'_1} \quad \text{is the axial stiffness of the shell section of length } l'_1$$

$$\frac{1}{K_s^*} = \frac{L_o}{\pi t_s (D_s + t_s) E_s} + \frac{l_1 + l'_1}{\pi t_{s,1} (D_{s,1} + t_{s,1}) E_{s,1}}$$

$$K_s^* = \frac{\pi}{\frac{L_o}{t_s (D_s + t_s) E_s} + \frac{l_1 + l'_1}{t_{s,1} (D_{s,1} + t_{s,1}) E_{s,1}}}$$

$D_{s,1} + t_{s,1}$ can be assimilated to $D_s + t_s$ for the following reasons:

- $D_{s,1} = D_s$ in most cases
- $t_{s,1}$ is small compared to $D_{s,1}$ and can be replaced by t_s
- calculation performed on Example UHX-20.2.3 (see Annex R), using $t_{s,1} = 1.0$ (instead of 0.625) and $l_1 = l'_1 = 15.0$ (instead of 0.0), shows that K_s^* decreases from 8.866 to 8.860 (0.1 % difference) when $D_{s,1} + t_{s,1}$ is replaced by $D_s + t_s$. There is no effect on TS stress σ and shell bending stress $\sigma_{s,b}$.

Accordingly the formula writes:

$$K_s^* = \frac{\pi(D_s + t_s)}{\frac{L - (l_1 + l_1')}{t_s E_s} + \frac{l_1 + l_1'}{t_{s,1} E_{s,1}}}$$

(c) **Axial differential thermal expansion between tubes and shell** writes:

$$\gamma^* = \alpha_{t,m}(T_{t,m} - T_a)L - \left[\alpha_{s,m}(T_{s,m} - T_a)L_o + \alpha_{s,m,1}(T_{s,m} - T_a)(l_1 + l_1') \right]$$

$$\gamma^* = (T_{t,m} - T_a)\alpha_{t,m}L - (T_{s,m} - T_a) \left[\alpha_{s,m}(L - l_1 - l_1') + \alpha_{s,m,1}(l_1 + l_1') \right]$$

(d) **Design procedure is affected as follows:**

Quantities concerned by the shell thickening are those which are involved by the TS-shell connection and which involve E_s , t_s , K_s , or γ . Accordingly:

- K_s must be replaced by K_s^* , which affects $K_{s,t}$ and J .
- β_s , k_s , δ_s and $\sigma_{s,m}$, $\sigma_{s,b}$ are calculated replacing t_s with $t_{s,1}$ and E_s with $E_{s,1}$.
- must be replaced by γ^* , which affects P_γ

If the material of the thickened shell section is different from the material of the current shell section, use the allowable shell stress limits $S_{s,1}$ and $S_{PS,s,1}$.

10.2 Effect of Plasticity at Tubesheet-Shell-Channel Joint (UHX-13.7)

(a) **General:** If the bending stress in the shell at its junction to the tubesheet exceeds $1.5S_s$ (i.e. yield) for the pressure loadings only, then some type of plastic hinge is developed. The elastic analysis accounts for the edge moment, based only on discontinuity considerations. It does not recognize that the fully plastic moment can never be physically exceeded. Thus the amount of edge restraint that reduces the stress in the center of the tubesheet will be overestimated as the plastic hinge increases.

Taking that into consideration through an elastic-plastic analysis, Note (2) of Table 4-120.1 of ASME Section VIII Div.2 classifies the shell bending stress as a secondary stress (Q) to be limited to $S_{PS,s}$.

Thus the membrane+bending stress σ_s of the shell at its junction to the tubesheet due to pressure loading (loading cases 1, 2, 3) can go up to $S_{PS,s}$, provided that an elastic-plastic analysis is performed.

When the shell/channel bending stresses resulting from pressure loads do not satisfy the primary stress limit, then full credit cannot be taken for the stiffness of that component. In such a case, it is deemed appropriate to apply a "knockdown" factor to the stiffness of the shell or channel component by reducing its modulus of elasticity. This will increase the rotation of the joint up to a value obtained when there is no more support from the shell and channel loading, to the extreme case of a simply supported TS which is covered in Clause X.3. The "knockdown" factor is based on evaluations of the extent of strengthening offered by the fully plastic moment at the shell and channel junction.

In 1985, Soler [13] proposed to use a reduced modulus of elasticity E_s^* for the shell (and E_c^* for the channel) based on the degree of overstress in the shell: $E_s^* = E_s (S_s^* / \sigma_{s,b})$ where S_s^* is the

allowable stress limit. This formula has been modified for use in U-tube TS HEs (UHX-12):
 $E_s^* = E_s \sqrt{S_s^* / \sigma_{s,b}}$. Same formula applies for the channel, using subscript c.

Series of elastic-plastic finite element calculations have been performed in 1990 by Soler [14] on a short cylindrical shell to improve this straightforward formula for fixed TS HEs. The results enable to determine a reduced modulus of elasticity E_s^* which gives the same rotation obtained by the elastic solution at the TS-shell-channel joint:

$$E_s^* = E_s \left[1 - f(\sigma_{s,b} / S_s^*) \right] \text{ where function } f(\sigma_{s,b} / S_s^*) = 0 \text{ when } \sigma_{s,b} = S_s^*$$

and increases when $\sigma_{s,b} \geq S_s^*$.

- (b) **UHX formula:** The above formula has been refined later, using additional elastic-plastic calculations as follows.

$$\text{For the shell: } E_s^* = E_s \left(1.4 - 0.4 \frac{|\sigma_{s,b}|}{S_s^*} \right) \quad \text{For the channel: } E_c^* = E_c \left(1.4 - 0.4 \frac{|\sigma_{c,b}|}{S_c^*} \right)$$

The reduced effective modulus has the effect of reducing the shell and/or channel stresses obtained from the elastic calculation. However, due to load shifting, this usually leads to an increase of the tubesheet stress. Accordingly, this simplified elastic-plastic procedure can only be performed when the TS stress obtained in the elastic calculation is below the allowable stress limit $1.5S$ for pressure loading.

The maximum allowable bending stress limit in the shell and channel for loading cases 1, 2, 3 is defined as:

$$S_s^* = \text{MIN} \left[(S_{y,s}), \left(\frac{S_{PS,s}}{2} \right) \right] \quad S_c^* = \text{MIN} \left[(S_{y,c}), \left(\frac{S_{PS,c}}{2} \right) \right]$$

Above this value, which is more or less the yield stress, the shell and channel start to yield.

- (c) **UHX procedure** is given hereafter for the shell. It applies to the channel, using subscript c instead of s.

- (a) The procedure applies when $|\sigma_{s,b}| > S_s^* \Rightarrow E_s^* < E_s$

Cases where $|\sigma_{s,b}| = S_s^* \Rightarrow E_s^* = E_s$ or when $|\sigma_{s,b}| < S_s^* \Rightarrow E_s^* > E_s$ are covered by the normal elastic procedure and are out of the scope of the elastic-plastic procedure. Therefore E_s^* formula can be written:

$$E_s^* = E_s \text{ fact}_s \quad \text{where} \quad \text{fact}_s = \text{MIN} \left[\left(1.4 - 0.4 \frac{|\sigma_{s,b}|}{S_s^*} \right), (1) \right]$$

Fact_s varies from 1 when $|\sigma_{s,b}| = S_s^*$ to 0.6 when $|\sigma_{s,b}| = S_{PS,s} = 2S_s^*$.

(b) Quantities affected by the elastic-plastic procedure are those which are involved in the TS-shell-channel joint and which involve E_s . Accordingly:

- k_s affects λ_s , which leads to new values for $F, \Phi, Q_1, Q_{Z1}, Q_{Z2}, U$; P_W, P_{rim}, P_e ; Q_2, Q_3, F_m and finally the tubesheet bending stress σ .
- δ_s is not affected because it is used only in $\omega_s = \rho_s \beta_s k_s \delta_s (1 + h \beta_s)$ in which E_s is cancelled out by the product $k_s \delta_s$.

If $|\sigma| \leq 1.5S$, the design is acceptable. Otherwise, the HE geometry must be reconsidered.

For Example 3 given in Annex R, the results for the elastic calculation are as follows for controlling load case 2:

Stiffening coefficient $\Phi=9.0$, $\sigma = 23084 < 1.5S = 23700$, $\sigma_s = 30035 > 1.5S_s = 23700$, but lower than $S_{ps,s} = 47400$

Similar results for the channel.

For the elastic-plastic calculation: the stiffening coefficient Φ decreases to 7.6, and $\sigma = 22205 < 1.5S = 23700$.

In this example the elastic-plastic calculation leads to a decrease of the TS stress. In most cases it is the opposite, where the elastic-plastic calculation increases the TS stress.

10.3 Effect of Radial Thermal Expansion Adjacent to the Tubesheet (UHX-13.8)

When there is a significant temperature gradient at the TS-shell-channel joint, it may be necessary to account for the resulting radial differential thermal expansion at the joint. This occurs when the TS is integral with the shell (configurations a, b, c) or the channel (configuration a).

Additional notations are as follows (see Figure 27):

T' = tubesheet metal temperature at the rim

T'_c = channel metal temperature at the tubesheet

T'_s = shell metal temperature at the tubesheet

α' = mean coefficient of thermal expansion of tubesheet material at T'

α'_c = mean coefficient of thermal expansion of channel material at T'_c

α'_s = mean coefficient of thermal expansion of shell material at T'_s

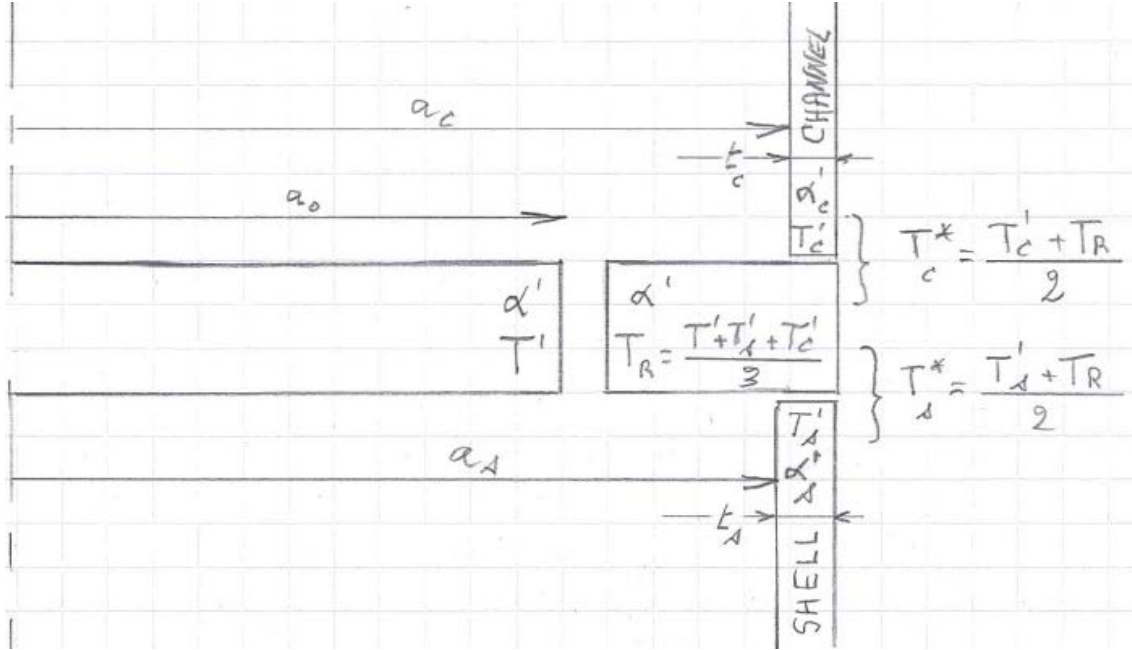


Figure 27—Temperature gradient at TS-shell-channel joint

Average temperature of the unperforated rim T_R :
$$T_R = \frac{T' + T'_s + T'_c}{3}$$

Average temperature of the shell T_s^* and channel T_c^* at their junction to the tubesheet:

$$T_s^* = \frac{T'_s + T_R}{2} \quad T_c^* = \frac{T'_c + T_R}{2}$$

Radial strain of the shell due to T_s^* and T_R :
$$\epsilon_s = \alpha'_s \left(T_s^* - T_a \right) - \alpha'_s \left(T_r - T_a \right)$$

The radial displacement of the shell is given by: $w_s(P_s^*) = a_s \epsilon_s$ which can be written in the same way as $w_s(P_s)$ given by [A-VI.1a-1]:

$$w_s(P_s) = \underbrace{\frac{a_s^2}{E_s t_s} \left(1 - \frac{\nu_s}{2} \right)}_{\delta_s} P_s = \delta_s P_s \quad \text{with} \quad \delta_s = \frac{a_s^2}{E_s t_s} \left(1 - \frac{\nu_s}{2} \right)$$

$$w_s(P_s^*) = \underbrace{\frac{a_s^2}{E_s t_s} \frac{E_s t_s}{a_s}}_{\delta_{s,o}} \underbrace{\epsilon_s}_{P_s^*} = \delta_{s,o} P_s^* \quad \text{with} \quad \boxed{\delta_{s,o} = \frac{a_s^2}{E_s t_s}} \quad \text{and} \quad \boxed{P_s^* = \frac{E_s t_s}{a_s} \epsilon_s}$$

The total radial displacement of the shell w_s given in VI.1a accounting for $w_s(P_s^*)$ becomes:

$$w_s = \frac{Q_s}{\beta_s^2 k_s} + \frac{M_s}{k_s \beta_s} + w_s(P_s) + w_s(P_s^*) = \frac{Q_s}{\beta_s^2 k_s} + \frac{M_s}{k_s \beta_s} + \delta_s P_s + \delta_{s,o} P_s^*$$

with:
$$P_s^* = \frac{E_s t_s}{a_s} \left[a_s' \left(T_s^* - T_a \right) - \alpha' \left(T_r - T_a \right) \right]$$

Thus $\delta_{s,o} P_s^*$ must be added to $\delta_s P_s$ in each equation where $\delta_s P_s$ appears.

The total radial displacement of the channel w_c writes in the same way:

$$w_c = \frac{Q_c}{\beta_c^2 k_c} + \frac{M_c}{k_c \beta_c} + \delta_c P_c + \delta_{c,o} P_c^*$$

with:
$$P_c^* = \frac{E_c t_c}{a_c} \left[a_c' \left(T_c^* - T_a \right) - \alpha' \left(T_r - T_a \right) \right]$$

Thus $\delta_{c,o} P_c^*$ must be added to $\delta_c P_c$ in each equation where $\delta_c P_c$ appears.

Equation [VI.2b'] giving the moment of the ring becomes:

$$R M_R = -a_o M_a + a_o^2 V_a (\rho_s - 1) + P_s \frac{a_o^3}{4} \left[(\rho_s^2 - 1)(\rho_s - 1) \right] - P_c \frac{a_o^3}{4} \left[(\rho_c - 1)(\rho_c^2 + 1) - 2(\rho_s - 1) \right] \\ - \left[a_s' k_s \left(1 + t_s' + \frac{t_s'^2}{2} \right) + a_c' k_c \left(1 + t_c' + \frac{t_c'^2}{2} \right) \right] \theta_a + a_o [\omega_c P_c - \omega_s P_s] + a_o [\omega_{c,o} P_c^* - \omega_{s,o} P_s^*]$$

where:
$$\omega_{s,o} = \rho_s \beta_s k_s \delta_{s,o} (1 + h \beta_s) \quad \omega_{c,o} = \rho_c \beta_c k_c \delta_{c,o} (1 + h \beta_c)$$

Equation giving P_{rim} in VII.2 writes:

$$P_{rim} = -\frac{U}{a_o^2} \left[\frac{a_o^2}{4} \left[(\rho_s^2 - 1)(\rho_s - 1) P_s - ((\rho_c^2 + 1)(\rho_c - 1) - (\rho_s - 1)) P_c \right] \right] + \frac{U}{a_o^2} [\omega_s P_s - \omega_c P_c]$$

A term P_ω must be added to P_{rim} :
$$P_\omega = \frac{U}{a_o^2} (\omega_{s,o} P_s^* - \omega_{c,o} P_c^*)$$

Accordingly the equivalent pressure P_e becomes:
$$P_e = \frac{J K_{s,t}}{1 + J K_{s,t} F_q'} \left[P_s' - P_t' + P_\gamma + P_w + P_{rim} + P_\omega \right]$$

Coefficient Q_2 becomes:
$$Q_2 = \frac{(\omega_s^* P_s - \omega_c^* P_c) - (\omega_{s,o} P_s^* - \omega_{c,o} P_c^*) + \left(W \frac{\gamma_b}{2\pi} \right)}{1 + \Phi Z_m}$$

$\sigma_{s,b}$ and $\sigma_{c,b}$ become:

$$\sigma_{s,b} = \frac{6}{t_s^2} k_s \left\{ \beta_s (\delta_s P_s + \delta_{s,o} P_s^*) + \frac{6(1-\nu^{*2})}{E^*} \left(\frac{a_o}{h} \right)^3 \left(1 + \frac{h\beta_s}{2} \right) \left[P_e (Z_v + Z_m Q_1) + \frac{2}{a_o^2} Q_2 Z_m \right] \right\}$$

$$\sigma_{c,b} = \frac{6}{t_c^2} k_c \left\{ \beta_c (\delta_c P_c + \delta_{c,o} P_c^*) - \frac{6(1-\nu^{*2})}{E^*} \left(\frac{a_o}{h} \right)^3 \left(1 + \frac{h\beta_c}{2} \right) \left[P_e (Z_v + Z_m Q_1) + \frac{2}{a_o^2} Q_2 Z_m \right] \right\}$$

10.4 Calculation Procedure for Simply Supported Tubesheets (UHX-13.9)

- (a) **General:** There are instances where the design does not need to consider the full strengthening effect of the shell/channel when they are integral with the TS. A Code Case was issued in 2005 to answer the concern of a HE manufacturer for externally sealed floating head HEs designed by TEMA. Use of UHX rules would need to increase significantly both the thicknesses of the floating TS and the attached channel, which is not always possible due to diameter and layout constraints. The Code Case proposed to consider the TS as simply supported.

If one chooses to not include the stiffening effect of the shell and channel for the tubesheet analysis when they are overstressed, this assumes that a plastic hinge has formed at the TS-shell-channel junction. Accordingly, the shell and channel bending stresses can be categorized as secondary and be limited according to the secondary stress limitations.

These considerations are a limit case of the elastic-plastic considerations of X-2, using for the shell and channel moduli of elasticity reduced to zero. Accordingly, the design procedure is similar and must be conducted in two phases.

- (b) **Phase 1:** Perform the normal calculation of the TS stress σ and its limitations. Calculate the shell and channel stresses which are considered as secondary and must be limited respectively to $S_{PS,s}$ and $S_{PS,c}$. The shell and channel minimum length limitations $l_{s,min}$ and $l_{c,min}$ no more apply as the stiffening effect due to the bending rigidity of these components will not be considered in Phase 2.

For Example 3 given in Annex R, the results are as follows for controlling load case 2:

Stiffening coefficient $\Phi=9.0$, $\sigma = 23084 < 1.5S = 23700$, $\sigma_s = 30035 > 1.5S_s = 23700$, but lower than $S_{PS,s} = 47400$.

Similar results for the channel.

- (c) **Phase 2:** Ignore the bending rigidity of the shell i.e., from [VI-1a]: $k_s=0$ and $\delta_s=0$, which leads to $M_s=0$. Same for the channel; $k_c=0$ and $\delta_c=0$. Then calculate the TS stress σ for pressure loading (loading cases 1, 2, 3) and check that $\sigma \leq 1.5S$.

For Example 3, the stiffening coefficient Φ decreases to 0.14, and $\sigma = 15170 < 1.5S = 23700$, $\sigma_s = 1561 < 1.5S_s = 23700$

As already mentioned above, the same result would be obtained by imposing $E_s^*=0$ and $E_c^*=0$ in the elastic-plastic method.

In this example the simply supported calculation leads to a decrease of the TS stress. In most cases it is the opposite, where the simply supported calculation increases the TS stress.

10.5 Tubesheet Flange Extension (UHX-9)

- (a) **General:** Rules of UHX-9 provide the determination of the required thickness of the tubesheet flanged extension when bolt loads are transmitted to the extended edge of the tubesheet (all configurations except a and A).

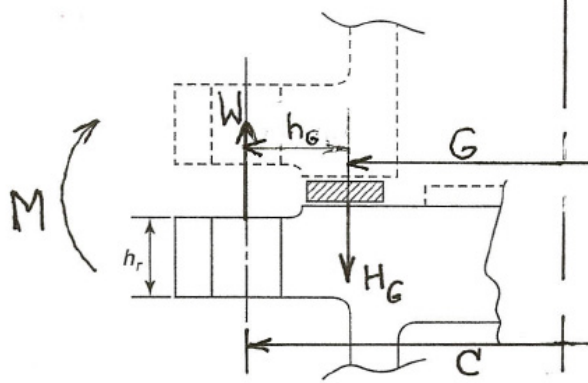


Figure 28

- (b) The moment acting on the TS flanged extension writes: $M = W h_G = W \frac{C - G}{2}$

For operating conditions, W is determined from equation (3) of Appendix 2, Clause 2-5(e) [10] .

For gasket seating condition, W is determined from equation (4) of Appendix 2, Clause 2-5(e) [10] .

The bending stress in the TS flanged extension writes:

$$\sigma = \frac{6 M}{\pi G h_r^2} = \frac{6 W h_G}{\pi G h_r^2} = \frac{1.91 W h_G}{G h_r^2} \quad \text{to be limited to the allowable TS stress.}$$

The minimum required thickness of the tubesheet flanged extension, h_r , is obtained for $\sigma = S$:

$$h_r = \sqrt{\frac{1.91 W h_G}{S G}}$$

The required thickness of the extended portion of the tubesheet differs from that required for the interior of the tubesheet calculated from UHX-12, UHX-13, or UHX-14.

Figure 29 depicts thickness h_r for some representative configurations.

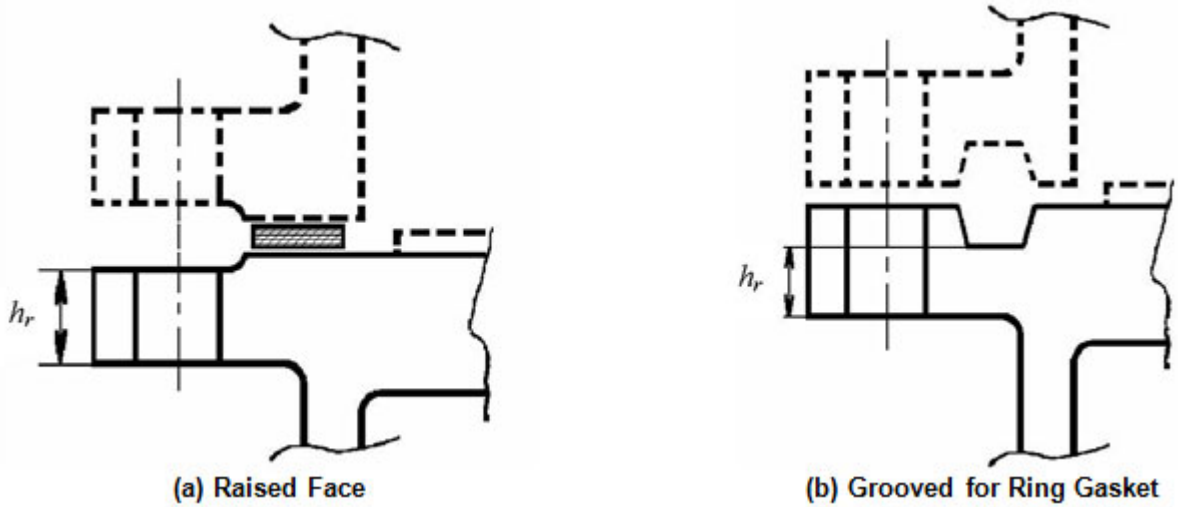


Figure 29—Minimum required thickness of the tubesheet flanged extension, h_r

10.6 HE Set-up with a Thin-Walled Expansion Joint (UHX-13.16)

The joint must comply with the rules of Appendix 26.

10.7 HE Set-up with a Thick-Walled Expansion Joint (UHX-13.17)

The expansion joint must comply with the rules of Appendix 5 which provides allowable stress limits. The design of the joint may be performed using higher allowable stress limits, defined in Table UHX-17, which allow the expansion joint to yield and decreases its stiffness. Accordingly, the HE must be re-designed for pressure loading cases using a zero expansion joint stiffness ($K_J=0$).

The equivalent pressure P_e must be recalculated using $K_J=0$. Using Note 1 of Clause VII.2:

$$P'_{s1} = P_s \left[(JK_{s,t})x_s + (JK_{s,t})2(1-x_s)v_t + (J)2\left(\frac{D_s}{D_o}\right)^2 v_s - (\rho_s^2 - 1) - \frac{(1-J)[D_J^2 - D_s^2]}{2D_o^2} \right]$$

$$P'_{t1} = P_t \left[(JK_{s,t})x_t + (JK_{s,t})2(1-x_t)v_t + 1 \right]$$

Using $J=0$:

$$P'_s = P_s \left[1 - \rho_s^2 - \frac{1}{2} \left[\frac{D_J^2}{D_o^2} - \rho_s^2 \right] \right] = P_s \left[1 - \frac{1}{2} \left[\frac{D_J^2}{D_o^2} + \rho_s^2 \right] \right]$$

$$P'_{t1} = P_t \left[(JK_{s,t})x_t + (JK_{s,t})2(1-x_t)v_t + 1 \right] = P_t$$

$$P_e = \frac{1}{1 + J K_{s,t} F_q} [P'_{s1} - P'_{t1}] = \boxed{P_s \left[1 - \frac{1}{2} \left[\rho_s^2 - \frac{D_J^2}{D_o^2} \right] \right] - P_t}$$

11 CHECKING OF THE RESULTS

11.1 Comparison with FEA

(a) General

SG-HTE proceeded to Finite Element Analysis to check UHX rules. Example UHX-20.2.3 (configuration a) given in Annex R was selected for the comparison.

FEA was performed by Joel Gordon using ANSYS with plate elements for the TS and shell elements for shell and channel. The results are in full agreement with results obtained by Tony Norton using a KSHELL analysis with maximum discrepancies less than 5%.

So as to reflect the UHX assumptions as close as possible, FEA calculation was performed as follows:

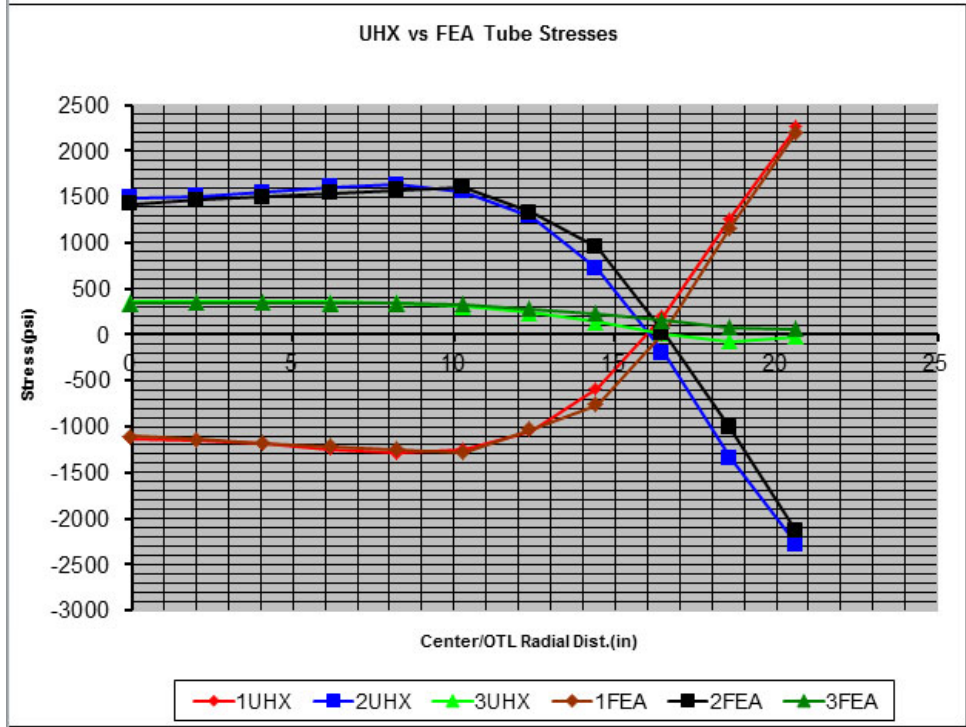
- the TS is replaced by an equivalent unperforated plate with an effective ligament efficiency μ^* and equivalent effective constants E^* and ν^* ,
- The tubes are replaced by an elastic foundation of modulus $k_w = N_t k_t / (\pi a_o^2)$,
- The unperforated rim is narrow enough such that it tends toward a rigid behavior.

(b) Results

- (1) **for tube axial stress** $\sigma_t(r)$, calculated from the TS displacements, FEA and UHX show very good agreement, with discrepancies less than 5% at the center ($r=0$) and periphery of the TS ($r=a_o$). The stress distributions throughout the TS track each other almost perfectly as shown by the shape of the curves given in Figure 30 for loading cases 1, 2 and 3.

It must be noted that the curves meet at the same point ($x=5.5$; $\sigma_t=0$). This surprising result is demonstrated analytically in Annex T.

- (2) **for TS bending stress** $\sigma(r)$ the comparison is not so good: the TS stress obtained by UHX is significantly higher (by about 50%) than the stress obtained by FEA. SG-HTE is still looking for an explanation for this discrepancy. This might be due to the unperforated rim, considered as a rigid ring in the analytical treatment.



	dist. (in)	$\sigma_t(1)$ psi	$\sigma_t(2)$ psi	$\sigma_t(3)$ psi	FEA 1	FEA 2	FEA 3
TS center (r=0)	0.0	-1129	1486	357	-1106	1425	342
	2,06	-1145	1503	358	-1141	1462	344
	4,12	-1191	1551	360	-1176	1499	348
	6,19	-1250	1608	358	-1211	1536	343
	8,25	-1289	1634	345	-1246	1573	338
	10,32	-1249	1558	309	-1281	1610	333
	12,38	-1046	1286	241	-1031	1330	277
	14,44	-581	718	137	-758	955	233
TS periph(r=20.63)	16,50	209	-195	14	23	18	153
	18,57	1265	-1338	-73	1161	-1011	81
	20,63	2259	-2277	-17	2204	-2135	59

Series: 1 (UHX) 2 (UHX) 3 (UHX) 1 (FEA) 2 (FEA) 3 (FEA)

Figure 30—Comparison of tube stresses calculated per UHX and FEA (Example UHX-20.2.3)

11.2 Comparison with CODAP French Rules

(a) General

ASME and CODAP (French Code for Pressure Vessels – Part Design, Section C7.3 “Fixed TS HEs) rules are based on the same analytical approach (TS replaced by an equivalent solid plate of effective elastic constants E^* and ν^* , tube bundle replaced by an equivalent elastic foundation, shell and channel disconnected from the TS with relevant edge loads). In 1992 ASME and CODAP decided to reconcile their rules so as to have unified rules scope, notations, figures, TS configurations, loading cases, ligament efficiencies, effective elastic constants, stress design formulas.

However additional assumptions have been made in the CODAP method, which simplify the analytical treatment:

- TS is assumed to be perforated till the internal shell diameter D_s . Accordingly the unperforated TS rim is ignored
- Treatment of the TS-shell-channel connection is simplified
- Effect of the bolting load (configurations b, c, d) on the TS is ignored. In return, the maximum allowable stress is lowered down.
- TS radial displacement at shell-channel connection is ignored
- Effect of P_s and P_t on shell and channel at their connection with TS is ignored

These simplifications, applied to the UHX method, lead to the CODAP formulas, as shown hereafter.

- (b) **Unperforated rim ignored:** The TS is assumed to be perforated till the internal diameter D_s of the shell. Therefore:

$$\boxed{D_o = D_s = D_c \Rightarrow a_o = a_s = a_c \Rightarrow \rho_s = 1 \quad \rho_c = 1} \quad A = D_o \Rightarrow K = \frac{A}{D_0} = 1$$

$$X_a \text{ writes: } \boxed{X_a = X = \sqrt[4]{24(1-\nu^{*2}) N_t \frac{E_t t_t (d_t - t_t)}{E^* L h^3} \left(\frac{D_s}{2}\right)^2}}$$

X_a , named X in CODAP, is the first characteristic parameter of the HE.

- (c) **effect of P_s and P_t on shell and channel at their connection with TS ignored**

$$\delta_s P_s = 0 \Rightarrow \delta_s = 0 \quad \omega_s = 0 \quad \omega_s^* = 0$$

$$\delta_c P_c = 0 \Rightarrow \delta_c = 0 \quad \omega_c = 0 \quad \omega_c^* = 0$$

- (d) **TS radial displacement at shell-channel connection ignored**

$$\left. \begin{aligned} w_s = 0 &\Rightarrow M_s = k_s \theta_s \\ w_c = 0 &\Rightarrow M_c = k_c \theta_c \end{aligned} \right\} M_s + M_c = k_s \theta_s + k_c \theta_c = (k_s + k_c) \theta_E \quad \text{as } \theta_s = \theta_c = \theta_E$$

Moment M_E acting at TS periphery is such that:

$$M_E = -(M_s + M_c) = -\underbrace{(k_s + k_c)}_{K_\theta} \theta_E = -K_\theta \theta_E$$

The second characteristic parameter of the exchanger is defined as:

$$Z = \frac{K_\theta}{k D^*} = \frac{D_s}{2 X} \frac{k_s + k_c}{D^*}$$

CODAP considers the liaison of the TS with shell and channel as an elastic restraint:

- If $Z = 0$: the TS is simply supported
- If $Z = \infty$: the TS is clamped

(e) Coefficient Φ

$$\left. \begin{aligned} w_s = -\frac{h}{2} \theta_s = 0 \Rightarrow t'_s = 0 \Rightarrow \lambda_s = \frac{6D_s}{h^3} k_s \\ w_c = -\frac{h}{2} \theta_c = 0 \Rightarrow t'_c = 0 \Rightarrow \lambda_c = \frac{6D_c}{h^3} k_c \end{aligned} \right\} \Phi = \frac{1-\nu^{*2}}{E^*} (\lambda_s + \lambda_c) = \frac{1-\nu^{*2}}{E^*} \frac{6}{h^3} (D_s k_s + D_c k_c) = \frac{1}{2D^*} \underbrace{(D_s k_s + D_c k_c)}_{XZ}$$

Accordingly: $\Phi = X Z$ plays the role of Z in CODAP:

- If plate is simply supported (no support from shell and channel): $\Phi=0$ in ASME , $Z=0$ in CODAP
- If plate is clamped (full support from shell and channel): $\Phi=\infty$ in ASME , $Z=\infty$ in CODAP

(f) Second coefficient Q_3 of the HE

$$\text{In ASME: } Q_1 = \frac{(\rho_s - 1) - \Phi Z_v}{1 + \Phi Z_m} \quad Q_2 = \frac{(w_s^* P_s - w_c^* P_c) + \left(\frac{\gamma_b}{2\pi} W \right)}{1 + \Phi Z_m} \quad Q_3 = Q_1 + \frac{2 Q_2}{P_e a_0^2}$$

In CODAP, these formulas become, with $\rho_s=1$, $\omega_s^*=0$, $\omega_c^*=0$, $W=0$:

$$[Q_1]_{CODAP} = \frac{-\Phi Z_v}{1 + \Phi Z_m} \quad [Q_2]_{CODAP} = 0 \quad [Q_3]_{CODAP} = Q_1 = \frac{-\Phi Z_v}{1 + \Phi Z_m} = -\frac{(X Z) Z_v}{1 + (X Z) Z_m}$$

$$Z = \frac{-Q_3}{Q_3 Z_m + Z_v} \frac{1}{X}$$

The couple $[X, Z]$ in CODAP plays the role of couple $[X_a, Q_3]$ in ASME to determine coefficients F_q , F_m , F_Q and F_t .

(g) Equivalent design pressure

$$\text{In ASME: } P_e = \frac{J K_{s,t}}{1 + J K_{s,t} \left[Q_{Z1} + (\rho_s - 1) Q_{Z2} \right]} \left[P'_s - P'_t + P_\gamma + P_w + P_{rim} + P_\omega \right]$$

- $Q_{Z1} = (Z_w Q_1 + Z_d) \frac{X_a^4}{2}$ Substituting $[Q_1]_{CODAP}$, Z_w and Z_d , it can be shown that:

- $Q_{Z1} = \left[F_q \right]_{CODAP}$ Thus, with $\rho_s=1$: $l + J K_{s,t} \left[Q_{Z1} + (\rho_s - 1) Q_{Z2} \right] = l + J K_{s,t} \left[F_q \right]_{CODAP}$

- $P'_s = \left[x_s + 2(1 - x_s) v_t + \frac{2}{K_{s,t}} \left(\frac{D_s}{D_o} \right)^2 v_s - \frac{\rho_s^2 - 1}{J K_{s,t}} - \frac{1 - J}{2 J K_{s,t}} \frac{D_J^2 - D_s^2}{D_o^2} \right] P_s$

with $D_s=D_o$ and $\rho_s=0$:

$$P'_s = \left[x_s + 2(1 - x_s) v_t + \frac{2}{K_{s,t}} v_s - \frac{1 - J}{2 J K_{s,t}} \frac{D_J^2 - D_s^2}{D_o^2} \right] P_s = \left[P'_s \right]_{CODAP}$$

- $P'_t = \left[x_t + 2(1 - x_t) v_t + \frac{1}{J K_{s,t}} \right] P_t = \left[P'_t \right]_{CODAP}$

- $P_\gamma = \frac{N_t K_t}{\pi a_o^2} \gamma = \frac{N_t K_t}{\pi R_e^2} \gamma = K_w \gamma = \left[P_\gamma \right]_{CODAP}$

- $P_w = \frac{U}{a_o^2} \frac{\gamma_b}{2 \pi} W = 0$ due to $W=0$ in CODAP

- $P_{rim} = -\frac{U}{a_o^2} \left[\omega_s^* P_s - \omega_c^* P_t \right] = 0$ due to $\omega_s^*=0$ and $\omega_c^*=0$ in CODAP.

- $P_\omega = -\frac{U}{a_o^2} \left[\omega_s P_s^* - \omega_c P_t^* \right] = 0$ due to $\omega_s=0$ and $\omega_c=0$ in CODAP.

Finally: $P_e = \frac{J K_{s,t}}{1 + J K_{s,t} F_q} \left[P'_s - P'_t + P_\gamma \right] = \left[P_e \right]_{CODAP}$

(h) TS bending stress

In ASME: $F_m(x) = \frac{Q_v(x) + Q_3 Q_m(x)}{2}$

Substituting $Q_v(x)$ and using: $\left[Q_3 \right]_{CODAP} = \frac{\phi Z_v}{1 + \phi Z_m}$

it can be shown that: $F_m(x) = \left[F_m(x) \right]_{CODAP}$ and:

$$\sigma_r(x) = \frac{1,5 F_m(x)}{\mu^*} \left(\frac{2a_o}{h} \right)^2 P_e = [\sigma_r(x)]_{CODAP}$$

(i) **TS shear stress**

$$\text{In ASME: } F_Q(x) = Q_3 Q_\alpha(x) + Q_\beta(x) \quad \tau(x) = \frac{1}{2\mu} \frac{a_o}{h} F_Q(x) P_e$$

The maximum shear stress is at periphery ($x=X_a$), for which:

$$Q_\alpha(X_a) = 0, \quad Q_\beta(X_a) = 1 \Rightarrow F_Q(X_a) = 1$$

$$\text{Thus: } \tau = \frac{1}{2\mu} \frac{a_o}{h} P_e = [\tau]_{CODAP}$$

(j) **Tube axial stress**

$$\text{In ASME: } F_t(x) = \frac{X_a^4}{2} \left[Q_3 Z_w(x) + Z_d(x) \right]$$

$$\text{Substituting } Z_w(x) \text{ and } Z_d(x) \text{ and using } [Q_3]_{CODAP} = \frac{\phi Z_v}{1 + \phi Z_m}$$

$$\text{it can be shown that: } F_t(x) = [F_t(x)]_{CODAP} \text{ and:}$$

$$\sigma_t(x) = \frac{1}{x_t - x_s} [\Delta p^* - F_t(x) P_e] = [\sigma_t(x)]_{CODAP}$$

(k) **Shell stresses**

(1) **Axial membrane stress**

$$\text{In ASME: } \sigma_{s,m} = \frac{a_o^2}{(D_s + t_s) t_s} \left[P_e + (\rho_s^2 - 1)(P_s - P_t) \right] + \frac{a_s^2}{(D_s + t_s) t_s} P_t$$

in CODAP, that formula becomes, with $a_o = \frac{D_s}{2}$ and $\rho_s = 1$:

$$\sigma_{s,m} = \frac{D_s^2}{4 t_s (D_s + t_s)} (P_e + P_t) = [\sigma_{s,m}]_{CODAP} \quad [\text{XI.2.11a}]$$

(2) **Bending stress**

$$\text{In ASME: } M_{s,b} = k_s \left\{ \beta_s \delta_s P_s + \frac{6(1-\nu^{*2})}{E^*} \left(\frac{a_o}{h} \right)^3 \left(1 + \frac{h \beta_s}{2} \right) \left[P_e (Z_v + Q_1 Z_m) + \frac{2}{a_o^2} (Q_2 Z_m) \right] \right\}$$

In CODAP, that formula becomes with $\beta_s = 0$ $\delta_s = 0$ $a_o = \frac{D_s}{2}$

$$[Q_1]_{CODAP} = \frac{-\Phi Z_v}{1 + \Phi Z_m} \quad [Q_2]_{CODAP} = 0 :$$

$$M_{s,b} = \frac{k_s}{2} \frac{1}{D^*} \left(\frac{D_s}{2} \right)^3 \frac{Z_v}{1 + \Phi Z_m} P_e = [M_{s,b}]_{CODAP} \quad \boxed{\sigma_{s,b} = \frac{6}{t_s^2} M_{s,b} = [\sigma_{s,b}]_{CODAP}}$$

(l) **Channel stresses**

(1) **Axial membrane stress:**
$$\sigma_{c,m} = \frac{a_c^2}{(D_c + t_c) t_c} P_t = \frac{D_s^2}{4(D_c + t_c) t_c} P_t = [\sigma_{c,m}]_{CODAP}$$

(2) **Bending stress** is obtained the same way as for the shell:
$$\sigma_{c,b} = \frac{6}{t_c^2} M_{c,b} = [\sigma_{c,b}]_{CODAP}$$

(m) **In conclusion**, all CODAP formulas have been retrieved, although they were obtained at a time where there was no connection between ASME and CODAP committees. This intends to prove that both methods are correct and coherent. This coherency extends to TEMA as shown in next Clause.

In CODAP, the ignorance of unperforated rim and bolting loads are the most important simplifications and have a significant impact on the results. Other simplifications have a less important impact.

Calculation of a HE by CODAP method can be obtained from a UHX-13.3 software, such as Mathcad software shown in Annex R for Example UHX.20.2.3, by imposing the simplifications listed in Clauses 2 to 5 above.

Like ASME, CODAP results for tube axial stress $\sigma_t(r)$ are in good agreement with FEA results as shown by Figure 31 which provides the tube stress distributions throughout the TS.

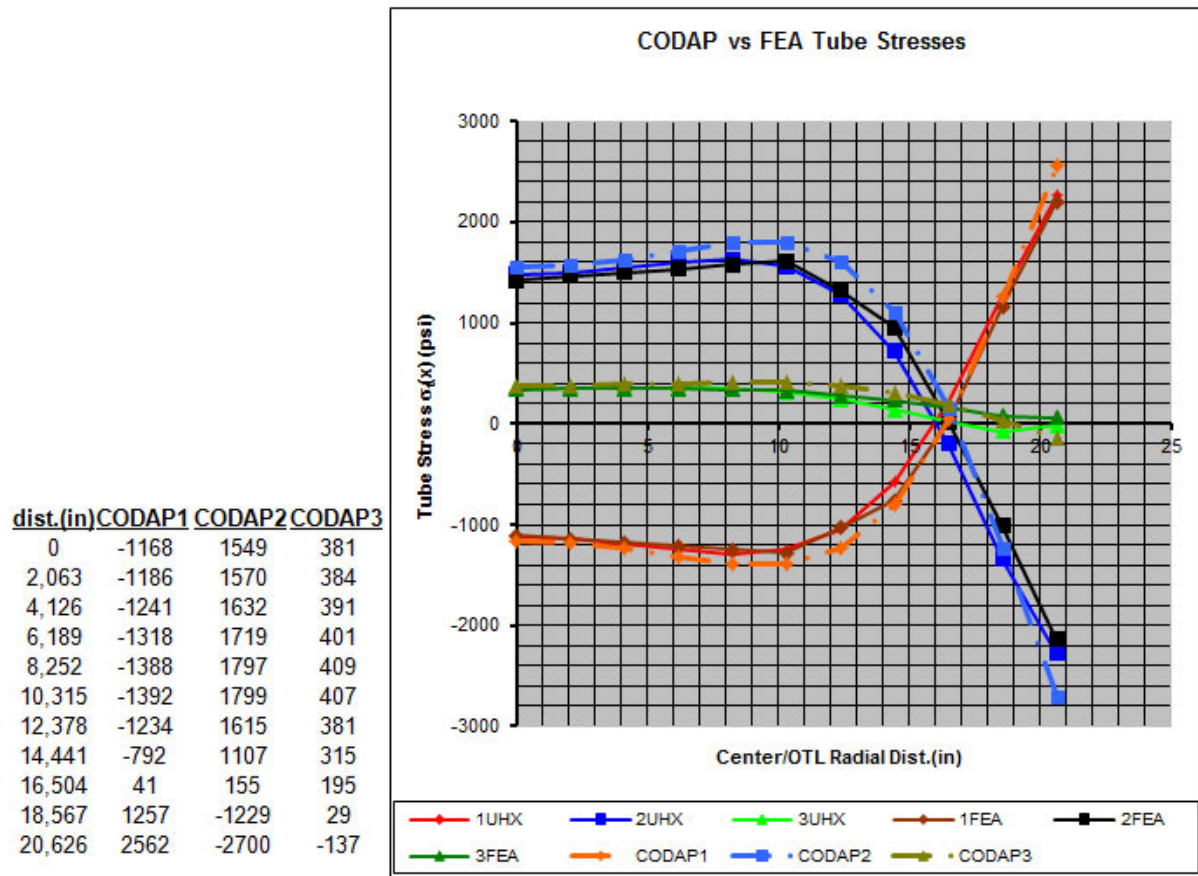


Figure 31—Tube stress distribution obtained by UHX, CODAP and FEA throughout the TS from $r=0$ to $r=ao$

11.3 Comparison with TEMA Rules

- (a) **General:** TEMA rules for fixed TSs have been developed by Gardner, based on its 1952 paper [2]. The analytical approach is the same as for ASME and CODAP with several assumptions to provide more simple rules. CODAP assumptions are used, plus additional simplifications. The TS is assumed to be either Simply Supported (SS) (i.e. no restraint from the shell and channel) or Clamped (CL) (i.e. full restraint from the shell and channel).

The original TEMA formula for the determination of the TS thickness in bending:

$$T = F \frac{G}{2} \sqrt{\frac{P}{S}} \text{ is based on flat circular plates formula, which can be:}$$

either SS:

$$\left. \begin{aligned} h &= F \frac{G}{2} \sqrt{\frac{P}{\sigma_{alw}}} = 1.11 \frac{G}{2} \sqrt{\frac{P}{S}} & \text{if } \sigma_{alw} = S \\ &= 0.91 \frac{G}{2} \sqrt{\frac{P}{S}} & \text{if } \sigma_{alw} = 1.5S \end{aligned} \right\} \text{TEMA uses the mean value } F = 1.0$$

or CL:

$$h = F \frac{G}{2} \sqrt{\frac{P}{\sigma_{alw}}} = 0.87 \frac{G}{2} \sqrt{\frac{P}{S}} \quad \text{if } \sigma_{alw} = S \quad \left. \vphantom{h = F \frac{G}{2} \sqrt{\frac{P}{\sigma_{alw}}}} \right\} \text{TEMA uses the mean value } F = 0.8$$

$$= 0.71 \frac{G}{2} \sqrt{\frac{P}{S}} \quad \text{if } \sigma_{alw} = 1.5S$$

(b) Ligament efficiency

That formula was modified later to introduce a ligament efficiency η based on the mean length of

the ligament $T = F \frac{G}{3} \sqrt{\frac{P}{\eta S}}$, as proposed by Miller [3] (see Clause IV.1 of PART 2):

$$\text{For Triangular pitch: } \eta = 1 - \frac{0.907}{(p/d_i)^2} \quad \text{For Square pitch: } \eta = 1 - \frac{0.785}{(p/d_i)^2}$$

Accordingly TEMA ligament efficiency η is lower than ASME ligament efficiency μ^* .

The new formula was set-up so that the value of T remains approximately the same for the triangular pitch value $\eta=0.42$, obtained from the minimum pitch imposed by TEMA ($p=1.25d_i$).

TEMA simplifications applied to ASME-CODAP methods lead to the TEMA formulas, as shown hereafter.

(c) TEMA bending formula

ASME-CODAP formula for TS bending can be written using the TEMA format:

$$h = \sqrt{\frac{1.5F_m(X)}{\mu^*}} D_o \sqrt{\frac{P_e}{\sigma_{alw}}} = \sqrt{\frac{9 \times 1.5F_m(X)}{\mu^* / \eta}} \frac{D_o}{3} \sqrt{\frac{P_e}{\eta \sigma_{alw}}}$$

In TEMA:

the TS is assumed uniformly perforated till the shell: $D_o=D_s$ (noted G in TEMA for fixed TS).

$\sigma_{alw}=S$ for pressure loadings (loading cases 1, 2, 3), $\sigma=2S$ for thermal loadings (loading cases 4, 5, 6, 7)

Using $\eta/\mu^*=1.33$, which is quite usual:

$$h = \sqrt{12F_m(X)} \frac{D_s}{3} \sqrt{\frac{P}{\eta S}} = F_{CODAP} \frac{D_s}{3} \sqrt{\frac{P}{\eta S}} \quad F_{CODAP} = \sqrt{12F_m(X)}$$

Thus, in CODAP-ASME the TS thickness depends strongly on coefficient $F_m(X)$, whereas TEMA coefficient F has fixed values. Figure 32 gives the variation of F_{CODAP} as a function of X:

- when the TS is SS (obtained for $Z=0$ in CODAP).
- when the TS is CL (obtained for $Z=\infty$ in CODAP).

TEMA considers the TS as SS when $t_s/D_s < 0.02$ with $F=1$, and CL when $t_s/D_s > .05$ with $F=0.8$. In between TEMA uses a linear interpolation (see Figure 33).

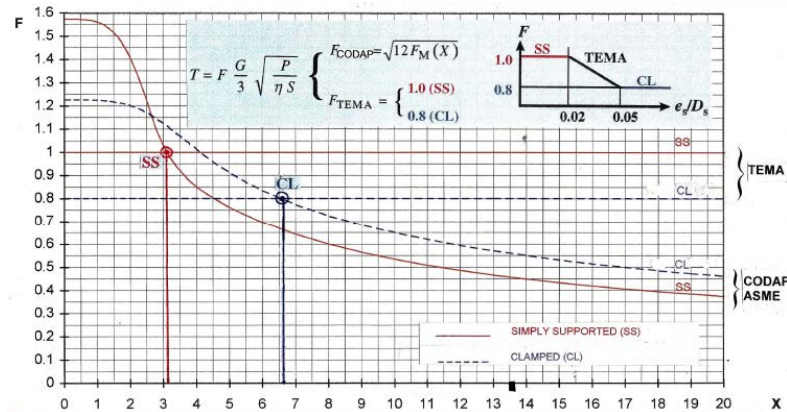


Figure 32—TEMA and ASME-CODAP coefficient F for X varying from $X=0$ to $X=20$

Figure 32 shows that:

- for low values of X_a ($X_a < 3$) TEMA rules generally lead to an unconservative thickness;
- for high values of X_a ($X_a > 6$) TEMA rules generally lead to conservative thickness
- values of X_a between 3 and 6, TEMA rules lead to tubesheet thickness that is close to ASME.

Due to simplifications mentioned above, TEMA does not ensure an overall and consistent design margin for all heat-exchangers. Accordingly, TEMA has imposed limitations on shell diameter D_s and design pressure P : $D_s \leq 60$ inch $P \leq 3000$ Psi $PD_s \leq 60000$ Psi inch (see Figure 34).

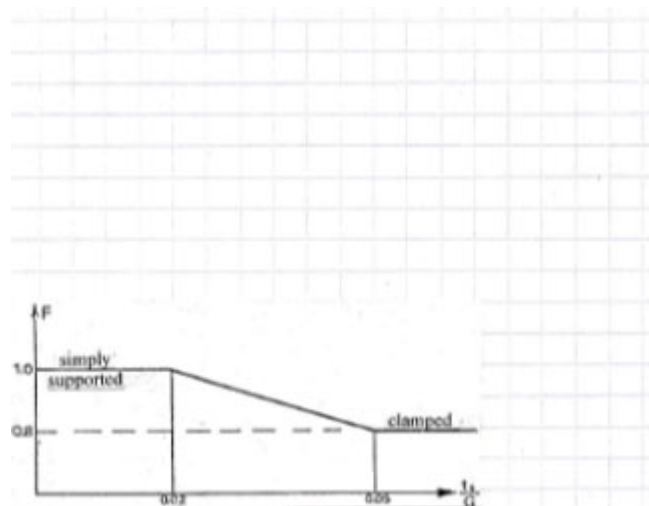


Figure 33—TEMA coefficient F

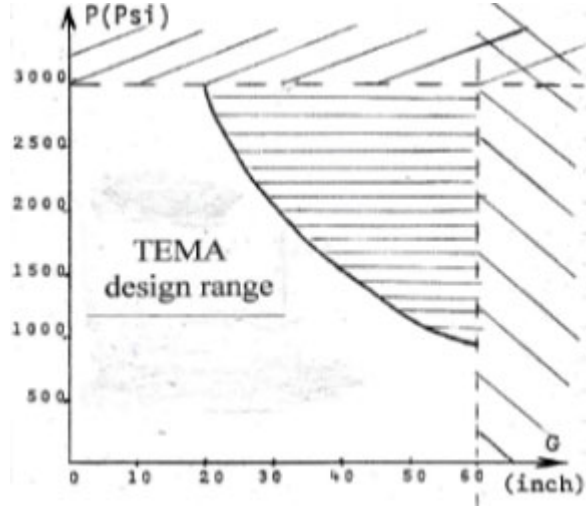


Figure 34—TEMA design range

However, it must be pointed out that the value of coefficient F has been remarkably well chosen as it represents approximately the mean value of coefficient $F(X)$. Finally TEMA formula writes:

$$h = F \frac{D_s}{3} \sqrt{\frac{P_e}{\eta S}} = F \frac{G}{3} \sqrt{\frac{P_{TEMA}}{\eta S}} \quad \text{where: } \begin{cases} P_{TEMA} \text{ is the TEMA effective pressure} \\ G \text{ is the shell internal diameter } D_s \end{cases}$$

(d) **TEMA effective pressure** is given by: $P_{TEMA} = P'_{t,TEMA} - P'_{s,TEMA} + P_{d,TEMA}$ with:

$$P'_{s,TEMA} = \frac{0.4J \left[1.5 + K_{s,t} (1.5 + x_s) \right] - \frac{1-J}{2} \frac{D_J^2 - D_s^2}{D_s^2}}{1 + JK_{s,t} F_q} P_s \quad P'_{t,TEMA} = \frac{1 + 0.4JK_{s,t} (1.5 + x_t)}{1 + JK_{s,t} F_q} P_t$$

$$P_{d,TEMA} = \frac{4JE_s t_s / L}{(D_{s,e} - 3t_s)(1 + JK_{s,t} F_q)} \quad \text{with: } \begin{cases} \Delta L = [\alpha_{s,m}(T_{s,m} - T_a) - \alpha_{t,m}(T_{t,m} - T_a)] L = -\gamma \\ D_{s,e} = \text{external shell diameter} = D_s + 2t_s \end{cases}$$

ASME-CODAP formula for equivalent pressure writes:

$$P_e = \frac{(JK_{s,t})P'_s - (JK_{s,t})P'_t + (JK_{s,t})P_\gamma}{1 + JK_{s,t} F_q} = P'_{s,1} - P'_{t,1} + P_{\gamma,1} \quad \text{with:}$$

$$\begin{aligned} \bullet \quad P'_{s,1} &= \frac{(JK_{s,t})P'_s}{1 + JK_{s,t} F_q} = \frac{(JK_{s,t}) \left[x_s + 2(1-x_s)v_t + \frac{2v_s}{K_{s,t}} - \frac{1-J}{2JK_{s,t}} \frac{D_J^2 - D_s^2}{D_o^2} \right]}{1 + JK_{s,t} F_q} P_s \\ &= \frac{J \left[K_{s,t}x_s + 2K_{s,t}(1-x_s)v_t + 2v_s \right] - \frac{1-J}{2} \frac{D_J^2 - D_s^2}{D_o^2}}{1 + JK_{s,t} F_q} P_s \quad \text{Using } v_s = 0.3 \quad v_t = 0.3 \text{ and } D_o = D_s : \end{aligned}$$

$$P'_{s,1} = \frac{0.4J \left[1.5 + K_{s,t} (1.5 + x_s) \right] - \frac{1-J}{2} \frac{D_J^2 - D_s^2}{D_s^2}}{1 + JK_{s,t} F_q} P = P'_{s,TEMA}$$

$$P'_{s,TEMA} = \frac{JK_{s,t}}{1 + JK_{s,t} F_q} P'_s$$

$$\bullet \quad P'_{t,1} = \frac{(JK_{s,t}) P'_t}{1 + JK_{s,t} F_q} = \frac{(JK_{s,t}) \left[x_t + 2(1-x_t) v_t + \frac{1}{JK_{s,t}} \right]}{1 + JK_{s,t} F_q} P_t = \frac{J \left[K_{s,t} x_t + 2K_{s,t} (1-x_t) v_t + 1 \right]}{1 + JK_{s,t} F_q} P_t \quad \text{Using } v_t = 0.3:$$

$$P'_{t,1} = \frac{1 + 0.4JK_{s,t} (1.5 + x_t)}{1 + JK_{s,t} F_q} P_t = P'_{t,TEMA}$$

$$P'_{t,TEMA} = \frac{JK_{s,t}}{1 + JK_{s,t} F_q} P'_t$$

$$\bullet \quad P_{\gamma,1} = \frac{(JK_{s,t}) F_q N_t K_t}{(1 + JK_{s,t} F_q) \pi a_o^2} \gamma = \frac{4JK_s}{(1 + JK_{s,t} F_q) \pi D_o^2} \gamma = \frac{4JE_s t_s (D_s + t_s)}{(1 + JK_{s,t} F_q) D_s^2} \frac{\gamma}{L}$$

$$\frac{D_s + t_s}{D_s^2} = \frac{D_{s,e} - t_s}{(D_{s,e} - 2t_s)^2} = \frac{1}{D_{s,e}} \frac{1 - \frac{t_s}{D_{s,e}}}{\left(1 - \frac{2t_s}{D_{s,e}}\right)^2} \square \frac{1}{D_{s,e}} \frac{1}{\left(1 - \frac{4t_s}{D_{s,e}}\right) \left(1 + \frac{t_s}{D_{s,e}}\right)} \square \frac{1}{D_{s,e}} \frac{1}{\left(1 - \frac{3t_s}{D_{s,e}}\right)} = \frac{1}{(D_{s,e} - 3t_s)}$$

$$P_{\gamma,1} = \frac{4JE_s t_s}{(1 + JK_{s,t} F_q) (D_{s,e} - 3t_s)} \frac{\gamma}{L} = \frac{4JE_s t_s}{(1 + JK_{s,t} F_q) (D_{s,e} - 3t_s)} \frac{-\Delta L}{L} = -P_d$$

$$P_{d,TEMA} = -\frac{JK_{s,t}}{1 + JK_{s,t} F_q} P_\gamma$$

Finally:

$$P_e = \frac{(JK_{s,t}) P'_s - (JK_{s,t}) P'_t + (JK_{s,t}) P_\gamma}{1 + JK_{s,t} F_q} = P'_{s,1} - P'_{t,1} + P_{\gamma,1} = P'_{s,TEMA} - P'_{t,TEMA} - P_{d,TEMA} = -P_{TEMA}$$

(e) **Coefficient X**

$$X_a = X = \sqrt[4]{24(1-\nu^{*2}) N_t \frac{E_t t_t (d_t - t_t)}{E^* L h^3} \left(\frac{D_s}{2}\right)^2} = \sqrt[4]{\frac{6(1-\nu^{*2})}{E^*} \frac{1}{L} \underbrace{N_t E_t t_t (d_t - t_t)}_{E_s t_s (D_s + t_s) / K_{s,t}} \left(\frac{D_s^2}{h^3}\right)}$$

Introducing the deflection efficiency:

$$\eta^* = \frac{D^*}{D} = \frac{E^*}{1-\nu^{*2}} \frac{1-\nu^2}{E} \quad \text{which leads to: } \frac{1-\nu^{*2}}{E^*} = \frac{1-\nu^2}{E\eta^*}, \text{ it follows:}$$

$$X_a = X = \sqrt[4]{\frac{6(1-\nu^2)}{\eta^* E} \frac{E_s t_s (D_s + t_s)}{LK_{s,t}} \frac{D_s^2}{h^3}} \quad \text{Neglecting } t_s \text{ compared to } D_s : \quad X_{TEMA} = \sqrt[4]{\frac{6(1-\nu^2)}{\eta^*} \frac{E_s t_s}{ELK_{s,t}} \left(\frac{D_s}{h}\right)^3}$$

(f) **coefficient F_q** is given on Figure 35 as a function of X for SS ($Z=0$) and CL ($Z=\infty$) TS.

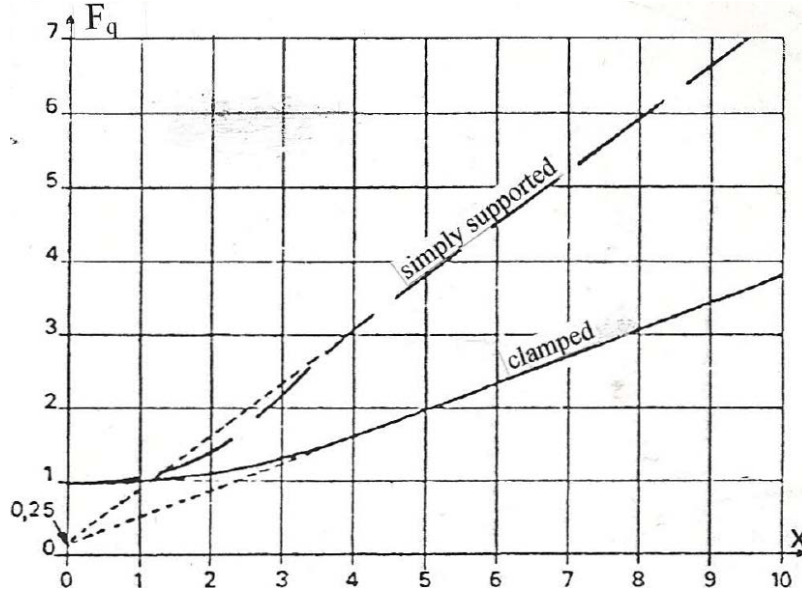


Figure 35—Coefficient F_q as a function of X for SS and CL TS

$$F_q \text{ can be approximated as: } F_q = 0.25 + \alpha X \quad \text{with: } \begin{cases} \alpha = \frac{\sqrt{2}}{2} \text{ when the TS is SS} \\ \alpha = \frac{\sqrt{2}}{4} \text{ when the TS is CL} \end{cases}$$

$$\text{TEMA coefficient } F \text{ equals: } \begin{cases} F = 1.0 \text{ when the TS is SS} \\ F = 0.8 \text{ when the TS is CL} \end{cases}$$

$$\text{Thus, } \alpha \text{ can be written: } \alpha = (F - 0.6) \frac{\sqrt{2}}{0.8} \quad \text{and } F_q \text{ writes:}$$

$$F_q = 0.25 + (F - 0.6) \frac{\sqrt{2}}{0.8} \sqrt[4]{\frac{6(1-\nu^2)}{\eta^*} \frac{E_s t_s}{ELK_{s,t}} \left(\frac{D_s}{h}\right)^3} = 0.25 + (F - 0.6) \underbrace{\sqrt[4]{\frac{24(1-\nu^2)}{0.8^4 \eta^*} \frac{E_s t_s}{ELK_{s,t}} \left(\frac{D_s}{h}\right)^3}}_A$$

For the determination of coefficient A, TEMA uses:

$\nu = 0.3$ and $\eta^* = 0.178$ which leads to $A = 300$ and :

$$F_q = 0.25 + (F - 0.6) \sqrt[4]{\frac{300 E_s t_s}{ELK_{s,t}} \left(\frac{D_s}{T} \right)^3}$$

Figure 35 shows that lowest value of F_q is 1. Thus:

$$F_q = \text{MAX} \left[(1), \left(0.25 + (F - 0.6) \sqrt[4]{\frac{300 E_s t_s}{ELK_{s,t}} \left(\frac{D_s}{T} \right)^3} \right) \right]$$

(g) **TEMA shear formula**

ASME-CODAP formula for shear stress writes: $\tau = \frac{1}{2\mu} \cdot \frac{D_o}{2h} \cdot P_e$ with $\tau \leq 0.8S$

Thus, the minimum TS shear thickness writes: $h = \frac{1}{3.2\mu} \cdot \frac{D_o}{S} \cdot P_e$

TEMA uses for D_o the equivalent diameter D_L of the tube center limit parameter, which leads to:

$$T = \frac{0.31 D_L}{\mu} \frac{P}{S}$$

(h) **Tube longitudinal stress** is calculated by TEMA at periphery of bundle:

$$\sigma_{t,o} = \frac{1}{x_t - x_s} \left[(P_s x_s - P_t x_t) - F_q P_e \right]$$

with:

$$[\text{III.2.b1}] \quad x_t - x_s = \frac{N_t s_t}{\pi a_o^2} = \frac{N_t t_t (d_t - t_t)}{a_o^2} \quad a_o = D_s/2 \quad -P_e = P_{TEMA} = P'_{t,TEMA} - P'_{s,TEMA} + P_{d,TEMA}$$

$$\sigma_{t,o} = \frac{F_q D_s^2}{4 N_t t_t (d_t - t_t)} \left[\frac{x_s P_s - x_t P_t}{F_q} + P'_{t,TEMA} - P'_{s,TEMA} + P_{d,TEMA} \right] = \frac{F_q D_s^2}{4 N_t t_t (d_t - t_t)} \left[\underbrace{\left(P'_{t,TEMA} - \frac{x_t P_t}{F_q} \right)}_{P_2} - \underbrace{\left(P'_{s,TEMA} - \frac{x_s P_s}{F_q} \right)}_{P_3} + P_{d,TEMA} \right]$$

- If $\sigma_{t,o} > 0$ (tubes in traction) $\sigma_{t,o}$ is limited to $2S_t$
- If $\sigma_{t,o} \leq 0$ (tubes in compression) $\sigma_{t,o}$ is limited to S_t and to the maximum permissible buckling stress limit S_{tb} named S_e in TEMA.

(i) **Shell longitudinal stress** is obtained from [XI.2.11a]: $\sigma_{s,m} = \frac{D_s^2}{4 t_s (D_s + t_s)} (P_e + P_t)$

with: $D_s \square D_s + t_s$ and $P_e = -P_{TEMA} = P'_{s,TEMA} - P'_{t,TEMA} - P_{d,TEMA}$

$$\sigma_{s,m} = \frac{D_s^2}{4 t_s (D_s + t_s)} \left(\underbrace{P_t - P'_{t,TEMA}}_{P_1} + P'_{s,TEMA} - P_{d,TEMA} \right)$$

Shell and channel bending stresses are not calculated.

11.4 Comparison with Circular Plates Subject to Pressure

The classical formulas of circular plates subjected to pressure write:

for SS plate:

$$\sigma_r(x) = 1.5 \underbrace{\frac{(3+\nu) \left(1 - \frac{r^2}{R^2}\right)}{16}}_{F^{SS}(r)} \left(\frac{2R}{h}\right)^2 (P_s - P_t) \quad \text{with: } \begin{cases} F^{SS}(0) = \frac{3+\nu}{16} & \text{at TS center } (r=0) \\ F^{SS}(R) = 0 & \text{at TS periphery } (r=R) \end{cases}$$

for CL plate:

$$\sigma_r(x) = 1.5 \underbrace{\frac{(1+\nu) - (3+\nu) \frac{r^2}{R^2}}{16}}_{F^{CL}(r)} \left(\frac{2R}{h}\right)^2 (P_s - P_t) \quad \text{with: } \begin{cases} F^{CL}(0) = \frac{1+\nu}{16} & \text{at TS center } (r=0) \\ F^{CL}(R) = -\frac{1}{8} & \text{at TS periphery } (r=R) \end{cases}$$

The following modifications must be applied to UHX-13.3 rules.

$$(a) \text{ No unperforated rim: } \begin{cases} a_o = a_s = R & D_o = D_s \Rightarrow \rho_s = 1 & \omega_s^* = \omega_s \\ a_o = a_c = R & D_o = D_c \Rightarrow \rho_c = 1 & \omega_c^* = \omega_c \end{cases}$$

$$(b) \text{ No holes and no tubes: } \mu^* = 1 \quad E^* = E \quad \nu^* = \nu$$

$$N_t = 0 \Rightarrow x_s = 1 \quad x_t = 1 \Rightarrow X_a = \sqrt[4]{24(1-\nu^{*2}) N_t \frac{E_t t_t (d_t - t_t)}{E^* L h^3} \left(\frac{D_s}{2}\right)^2} = 0$$

When there no more tubes, there is no more elastic foundation and $X_a=0$. For $X_a=0$ several coefficients go to infinity

(Z_d , Z_w , Q_m , Q_v ,...). Accordingly, these coefficients will be calculated for $X_a \rightarrow 0$, as shown in Annex F.

$$(c) \text{ No bellows: } K_J = 0 \Rightarrow J = \frac{K_J}{K_J + K_s} = 1$$

$$(d) \text{ No bolted flange: } W=0 \quad \text{No differential thermal expansion: } \gamma=0$$

$$(e) \text{ No effect of } P_s \text{ and } P_t \text{ on shell and channel at their connection with TS:}$$

$$\begin{cases} \delta_s P_s = 0 \Rightarrow \delta_s = 0 & \omega_s = 0 & \omega_s^* = 0 \\ \delta_c P_c = 0 \Rightarrow \delta_c = 0 & \omega_c = 0 & \omega_c^* = 0 \end{cases}$$

(f) **Calculation of A and Φ:** $A = D_o \Rightarrow K = \frac{A}{D_o} = 1 \Rightarrow \Phi = \frac{1 - \nu^{*2}}{E^*} (\lambda_s + \lambda_c)$

From [VI-3]:

$$Q_1 = -\frac{1}{4} \frac{\Phi}{(1 + \nu^*) + \Phi} \quad Q_2 = \frac{(\omega_s^* P_s - \omega_c^* P_c) + \left(W \frac{\gamma_b}{2\pi} \right)}{1 + \Phi Z_m} = 0$$

$$Q_3 = Q_1 + \frac{2}{a_0^2 P_e} Q_2 = Q_1 = -\frac{1}{4} \frac{\Phi}{(1 + \nu^*) + \Phi}$$

Using Z_w and Z_d from Annex G:

$$Q_{Z1} = (Z_w Q_1 + Z_d) \frac{X_a^4}{2} = \left[\frac{1}{4(1 + \nu^*)} \frac{-\Phi}{4(1 + \nu^*) + \Phi} + \frac{2}{X_a^4} \right] \frac{X_a^4}{2} = 1$$

Using Z_v and Z_m from Annex G:

$$Q_{Z2} = (Z_v Q_1 + Z_m) \frac{X_a^4}{2} = \left[\frac{1}{4(1 + \nu^*)} \frac{-\Phi}{4(1 + \nu^*) + \Phi} + \frac{1}{1 + \nu^*} \right] \frac{X_a^4}{2} = 0$$

$$U = \frac{Z_w + (\rho_s - 1) Z_m}{1 + \Phi Z_m} X_a^4 = \frac{Z_w}{1 + \Phi Z_m} X_a^4 = \frac{1}{4(1 + \nu^*)} \frac{1}{1 + \Phi Z_m} X_a^4 = 0$$

(g) **Equivalent pressure** writes:

$$P_e = \frac{1}{\frac{1}{JK_{s,t}} + [Q_{Z1} + (\rho_s - 1) Q_{Z2}]} [P_s' - P_t' + P_\gamma + P_w + P_{rim}]$$

Thus: $\boxed{P_e = P_s - P_t}$

- If the plate is simply supported (no support from shell and channel): $\Phi=0$ in ASME , $Z=0$ in CODAP
- If the plate is clamped (full support from shell and channel): $\Phi=\infty$ in ASME , $Z=\infty$ in CODAP

(h) **Coefficient F_m** writes: $F_m(x) = \frac{Q_v(x) + Q_3 Q_m(x)}{2}$

where $Q_v(x)$ and $Q_m(x)$ are given by Annex G when $X_a \rightarrow 0$:

$$Q_m(x \rightarrow 0) = \frac{(1+\nu^*)}{2Z_a} bei' = \frac{(1+\nu^*)}{\frac{1+\nu^*}{2} X_a} \frac{X_a}{2} = 1$$

$$Q_v(x \rightarrow 0) = \frac{(1+\nu^*)}{2Z_a X_a} \Psi_1 = \frac{(1+\nu^*)}{\frac{1+\nu^*}{2} X_a^2} \frac{3+\nu^*}{16} X_a^2 = \frac{3+\nu^*}{8}$$

Coefficient F_m at TS center is given by formula:

$$F_m(x \rightarrow 0) = \frac{Q_3}{2} + \frac{3+\nu^*}{16} = -\frac{1}{8} \frac{\Phi}{(1+\nu^*) + \Phi} + \frac{3+\nu^*}{16}$$

Coefficient F_m at TS periphery is given by formula:

$$F_m(X_a) = \frac{Q_3}{2} = -\frac{1}{8} \frac{\Phi}{(1+\nu^*) + \Phi}$$

(i) **TS bending stress** writes: $\sigma_r(x) = 1.5 F_m(x) \left(\frac{2R}{h} \right)^2 (P_s - P_t)$

If the TS is SS: $\Phi = 0 \Rightarrow Q_3 = 0 \Rightarrow \begin{cases} F_m^{SS}(0) = \frac{3+\nu}{16} & \text{at TS center} \\ F_m^{SS}(R) = 0 & \text{at TS periphery} \end{cases}$

If the TS is CL: $\Phi = \infty \Rightarrow Q_3 = -\frac{1}{4} \Rightarrow \begin{cases} F_m^{CL}(0) = -\frac{1}{8} + \frac{3+\nu}{16} = \frac{1+\nu}{16} & \text{at TS center} \\ F_m^{CL}(R) = -\frac{1}{8} & \text{at TS periphery} \end{cases}$

Accordingly classical formulas for circular plates subjected to pressure given in Clause 1 have been retrieved. Similar calculations can also be performed for the TS deflection $w(x)$ and rotation $\theta(x)$.

These results can be obtained from a UHX-13.3 software, such as Mathcad software shown in Annex R, by imposing the simplifications listed in Clauses 2 to 5 above.

11.5 Conclusions

Applying the relevant simplifications, it has been analytically demonstrated that UHX-13.3 method leads to CODAP, TEMA and circular plates' formulas. This intends to prove the correctness of ASME method, which is confirmed by FEA comparisons for the TS deflection.

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PART 4

Floating Tubesheets

1 SCOPE

PART 4 applies to floating TS HEs that have one stationary tubesheet and one floating tubesheet. Three types of HEs are covered as shown in Figure 36:

- immersed floating head (sketch a)
- externally sealed floating head (sketch b)
- internally sealed floating tubesheet (sketch c)

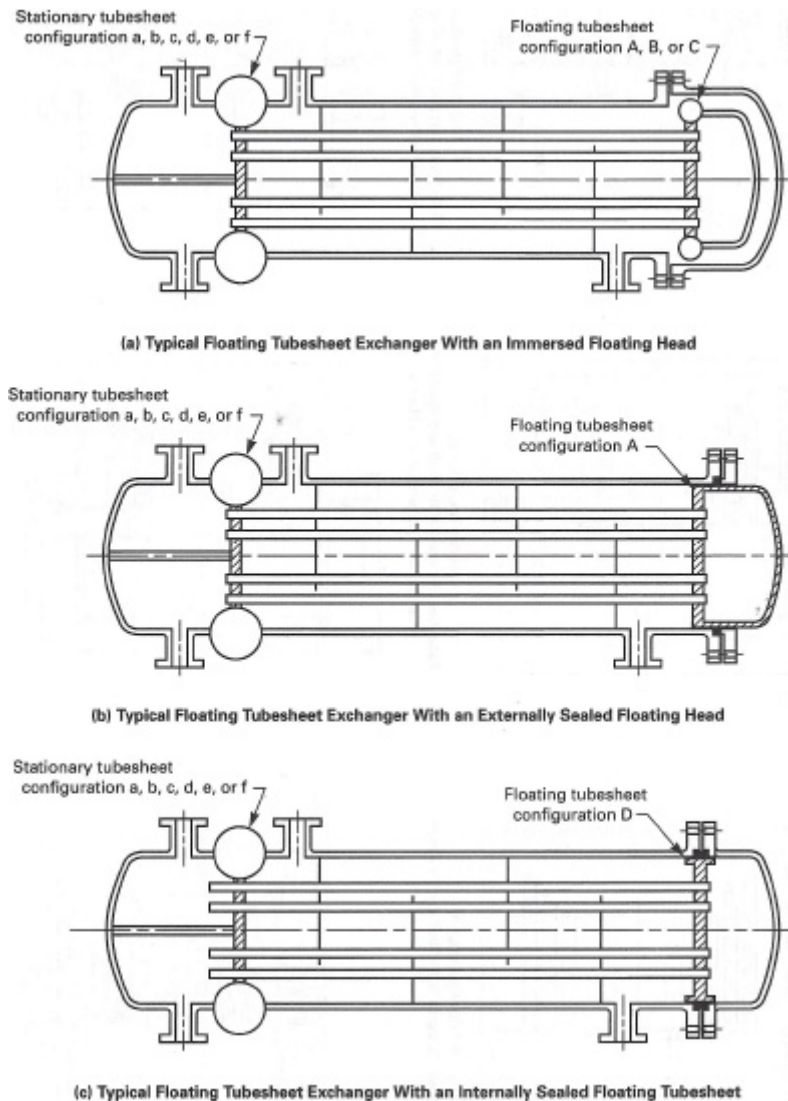


Figure 36—Floating tubesheet heat exchangers

(see TS configurations in Figure 37 and Figure 38)

2 HISTORICAL BACKGROUND

In the past decades many authors have proposed theoretical methods for the design of fixed TS HEs. The most important contributions are provided below.

Gardner [1] in 1948 was the very first to develop an analytical approach by taking into consideration the support afforded by the tubes and the weakening effect of the TS holes. The TS is considered as either simply supported or clamped at its periphery to simulate the rotational restraint afforded by the shell and the channel, which compels the designer to make a more or less arbitrary choice between these two extreme cases. The method was adopted by Dutch code STOOWEZEN in 1975

Miller [2] at the same time proposed a similar approach that was published in the British Code BS 1515 in 1965.

Galletly [3] in 1959 improved these design methods by accounting for the degree of rotational restraint of the TS at its periphery by the shell and the channel. This method was adopted by the French Pressure Vessel Code CODAP in 1982 and by the European Pressure Vessel Standard EN13445 in 2002.

Gardner [4] in 1969 improved his 1942 method by proposing a direct determination of the TS thickness which accounts for the unperforated rim and the TS-shell-channel connection. The method was adopted by ISO/DIS-2694 in 1973, by BS5500 in 1976 and by CODAP in 1995.

Soler [5] in 1984 developed a similar method accounting for the unperforated rim and the TS-shell-channel connection. The method is derived from fixed TS method and published for the first time in Non mandatory Appendix AA of Section VIII Division 1. In 2003 it was published in a new Part UHX of Section VIII Division 1 “Rules for Shell and Tubes Heat Exchangers” which became mandatory in 2004.

3 GENERAL

3.1 TS CONFIGURATIONS (UHX-14.1)

(a) The **stationary TS** is attached to the shell and the channel by welding (integral TS) or by bolting (gasketed TS) in accordance with the following 4 configurations (see Figure 37):

- Configuration a: tubesheet integral with shell and channel;
- Configuration b: tubesheet integral with shell and gasketed with channel, extended as a flange;
- Configuration c: tubesheet integral with shell and gasketed with channel not extended as a flange;
- Configuration d: tubesheet gasketed with shell and channel;
- Configuration e: tubesheet gasketed with shell and integral with channel, extended as a flange;
- Configuration f: tubesheet gasketed with shell and integral with channel not extended as a flange.

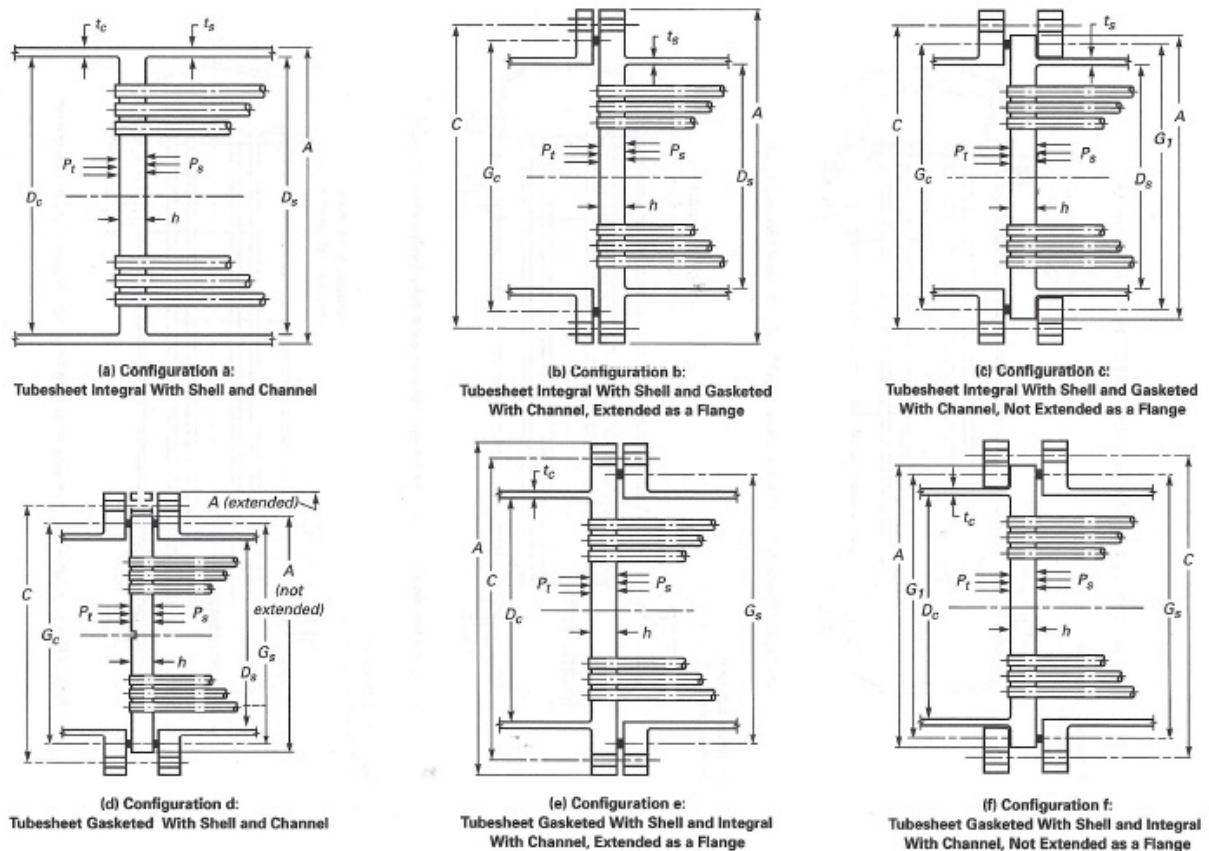


Figure 37—Stationary tubesheet configurations

(b) **The floating TS** is free to move along the HE longitudinal axis in accordance with the following 4 configurations (see Figure 38):

- Configuration A: tubesheet integral;
- Configuration B: tubesheet gasketed, extended as a flange;
- Configuration C: tubesheet gasketed, not extended as a flange;
- Configuration D: tubesheet internally sealed.

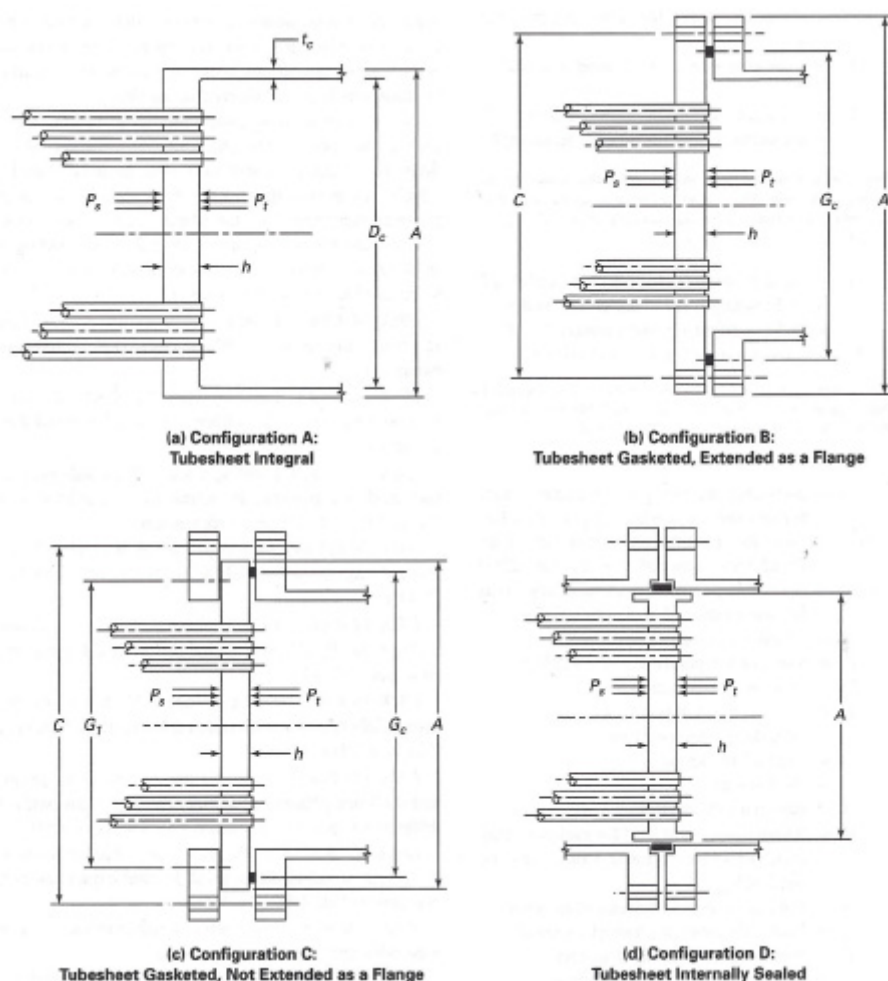


Figure 38—Floating tubesheet configurations

3.2 Notations

(a) **Data for the design of the HE** are as follows (UHX-14.3)

Symbols D_o , E^* , h_g , μ , μ^* and ν^* are defined in Clause II of PART 2.

A = outside diameter of tubesheet

a_c = radial channel dimension

a_o = equivalent radius of outer tube limit circle (see Clause IV.3a of PART 2)

a_s = radial shell dimension

- C = bolt circle diameter
 D_c = inside channel diameter
 D_s = Inside shell diameter
 d_t = nominal outside diameter of tubes
 E = modulus of elasticity for tubesheet material at T
 E_c = modulus of elasticity for channel material at T_c
 E_s = modulus of elasticity for shell material at T_s
 E_t = modulus of elasticity for tube material at T_t
 G_c = diameter of channel gasket load reaction
 G_s = diameter of shell gasket load reaction
 G_1 = midpoint of contact between flange and tubesheet
 h = tubesheet thickness
 L_t = tube length between outer tubesheet faces
 N_t = number of tubes
 P_e = effective pressure acting on tubesheet
 P_s = shell side internal design pressure. For shell side vacuum use a negative value for P_s .
 P_b, P_c = tube side internal design pressure. For tube side vacuum use a negative value for P_t .
Notation P_c , instead of P_b , is used throughout the analytical development so as to maintain the symmetry of the equations involving the shell (subscript s) and the channel (subscript c).
 S = allowable stress for tubesheet material at T
 S_c = allowable stress for channel material at T_c
 S_s = allowable stress for shell material at T_s
 S_t = allowable stress for tube material at T_t
 S_y = yield strength for tubesheet material at T
 $S_{y,c}$ = yield strength for channel material at T_c
 $S_{y,s}$ = yield strength for shell material at T_s
 $S_{y,t}$ = yield strength for tube material at T_t
 S_{PS} = allowable primary plus secondary stress for tubesheet material at T
 $S_{PS,c}$ = allowable primary plus secondary stress for channel material at T_c
 $S_{PS,s}$ = allowable primary plus secondary stress for shell material at T_s
 T = tubesheet design temperature
 T_a = ambient temperature, 70°F (20°C)
 T_c = channel design temperature
 T_s = shell design temperature
 T_t = tube design temperature

- t_c = channel thickness
- t_s = shell thickness
- t_t = nominal tube wall thickness
- W = flange design bolt load
- ν = Poisson's ratio of tubesheet material
- ν_c = Poisson's ratio of channel material
- ν_s = Poisson's ratio of shell material
- ν_t = Poisson's ratio of tube material

(b) **Design coefficients** (UHX-14.5.1 to 4)

The following coefficients, specific to each component of the HE, will be used in the analytical treatment. They complete the data given above.

(1) **Perforated TS**

Equivalent diameter of outer tube limit circle (see Clause IV-3.a of PART 2):

$$D_o = 2 r_o + d_t$$

Equivalent radius of outer tube limit circle (see Clause IV-3.a of PART 2): $a_o = \frac{D_o}{2}$

TS coefficients relating to the tubes:

- shell side: $x_s = 1 - N_t \left(\frac{d_t}{2a_o} \right)^2$; $1 - x_s = N_t \left(\frac{d_t}{2a_o} \right)^2$
- tube side: $x_t = 1 - N_t \left(\frac{d_t - 2t_t}{2a_o} \right)^2$; $1 - x_t = N_t \left(\frac{d_t - 2t_t}{2a_o} \right)^2$
- $x_t - x_s = N_t \left(\frac{d_t^2 - (d_t - 2t_t)^2}{4a_o^2} \right) = \frac{N_t \cdot s_t}{\pi a_o^2} = \frac{N_t \cdot k_t}{E_t} \cdot \frac{l}{\pi a_o^2} = \frac{N_t \cdot K_t}{E_t} \cdot \frac{L}{\pi a_o^2}$
[III.2.b1]
- Ligament efficiency: $\mu^* = \frac{p^* - d^*}{p^*}$
- Effective tube hole diameter d^* and effective pitch p^* are defined in Clause IV-3c of PART 2
- Effective elastic constants E^* and ν^* are given in Clause V-6 of PART 2 as a function of μ^* and h/p (triangular or square pitch).
- Bending stiffness: $D^* = \frac{E^* \cdot h^3}{12(1 - \nu^{*2})}$
- effective tube side pass partition groove depth given in Clause IV-3.d of PART 2: h'_g

- effective pressure acting on tubesheet: P_e

(2) Tube bundle

Tube cross-sectional area: $s_t = \frac{\pi}{4} \left[d_t^2 - (d_t - 2t_t)^2 \right] = \pi t_t (d_t - t_t) = \frac{\pi a_o^2}{N_t} (x_t - x_s)$

Axial stiffness K_t of one tube: $K_t = \frac{E_t s_t}{L} = \frac{\pi t_t (d_t - t_t) E_t}{L}$

axial stiffness k_t of one half tube of length $l=L/2$: $k_t = \frac{E_t s_t}{l} = \frac{2\pi t_t (d_t - t_t) E_t}{L} = 2 K_t$

Effective elastic foundation modulus equivalent to the half tube bundle:

$$k_w = \frac{N_t \cdot k_t}{\pi a_o^2} = \frac{2 N_t \cdot K_t}{\pi a_o^2} = \frac{2 N_t \cdot E_t \cdot t_t (d_t - t_t)}{L a_o^2} = \frac{2 E_t}{L} (x_t - x_s) = \frac{E_t}{l} (x_t - x_s)$$

$$k = \sqrt[4]{\frac{k_w}{D^*}} \quad ; \quad x = k r \quad ; \quad 0 \leq r \leq a_o \quad \Rightarrow \quad 0 \leq x \leq k a_o \quad k a_o = X_a$$

Axial stiffness ratio tubes/TS:

$$X_a = k a_o = \sqrt[4]{\frac{k_w}{D^*}} a_o = \left[24 (1 - \nu^{*2}) N_t \frac{E_t t_t (d_t - t_t) a_o^2}{E^* L h^3} \right]^{1/4}$$

(3) Shell

radial shell dimension: $a_s \quad \rho_s = \frac{a_s}{a_o}$

ST TS Integral configurations (a, b and c): $a_s = D_s / 2$

Gasketed configurations (d): $a_s = G_s / 2$

FL TS configurations (A, B, C and D): $a_s = a_c$

mean shell radius: $a'_s = \frac{D_s + t_s}{2}$

shell cross-sectional area: $s_s = \pi t_s (D_s + t_s)$

axial stiffness k'_s of the half shell of length $l=L/2$: $k'_s = \frac{E_s s_s}{l} = \frac{2\pi t_s (D_s + t_s) E_s}{L} = 2 K_s$

shell coefficient: $\beta_s = \frac{\sqrt[4]{12(1 - \nu_s^2)}}{\sqrt{(D_s + t_s) t_s}}$

$$\text{bending stiffness: } k_s = \beta_s \frac{E_s \cdot t_s^3}{6(1 - \nu_s^2)}$$

(4) Channel

$$\text{radial channel dimension: } a_c \quad \rho_c = \frac{a_c}{a_o}$$

$$\text{Integral configurations (a, e, f, and A): } a_c = D_c / 2$$

$$\text{Gasketed configurations (b, c and d): } a_c = G_c / 2$$

$$\text{Gasketed configuration D: } a_c = A / 2$$

$$\text{mean channel radius: } a'_s = \frac{D_c + t_c}{2}$$

$$\text{Channel coefficient: } \beta_c = \frac{\sqrt[4]{12(1 - \nu_c^2)}}{\sqrt{(D_c + t_c)} \sqrt[4]{E_c}}$$

$$\text{bending stiffness: } k_c = \beta_c \cdot \frac{E_c \cdot t_c^3}{6(1 - \nu_c^2)}$$

(5) Unperforated rim

D_o = internal diameter

A = external diameter

Diameter ratio: $K = A / D_o$

3.3 Loading Cases (UHX-14.4)

The normal operating condition of the HE is achieved when the tube side pressure P_t and shell side pressure P_s act simultaneously. However, a loss of pressure or a loss of temperature is always possible. Accordingly, for safety reasons, the designer must always consider the cases where $P_s=0$ and $P_t=0$ for the normal operating condition(s).

He must also consider the startup condition(s), the shutdown condition(s) and the upset condition(s), if any, which may govern the design.

A floating TS HE is a statically indeterminate structure for which it is generally not possible to determine the most severe condition of coincident pressure and temperature. Thus, it is necessary to evaluate all the anticipated loading conditions mentioned above to ensure that the worst load combination has been considered in the design.

For each of these conditions, the following 3 pressure loading cases must be considered.

- Loading Case 1: Tube side pressure P_t acting only ($P_s = 0$).
- Loading Case 2: Shell side pressure P_s acting only ($P_t = 0$).
- Loading Case 3: Tube side pressure P_t and shell side pressure P_s acting simultaneously.

Additional thermal loading cases (4, 5, 6, 7) must be considered if the effect of the radial thermal expansion adjacent to the tubesheet is accounted for (see section 10.1)

When vacuum exists, each loading case shall be considered with and without the vacuum.

When differential pressure design is specified by the user, the design shall be based only on loading case 3.

As the calculation procedure is iterative, a value h is assumed for the tubesheet thickness to calculate and check that the maximum stresses in tubesheet, tubes, shell, and channel are within the maximum permissible stress limits.

Because any increase of tubesheet thickness may lead to overstresses in the tubes, shell, or channel, a final check shall be performed, using in the formulas the nominal thickness of tubesheet, tubes, shell, and channel, in both corroded and uncorroded conditions.

3.4 Design Assumptions (UHX-14.2)

A FL TS HE is a complex structure and several assumptions are necessary to derive a ‘design by rules’ method. Most of them could be eliminated, but the analytical treatment would lead to ‘design by analysis’ method requiring the use of a computer.

The design assumptions are as follows.

(a) HE

- The analytical treatment is based on the theory of elasticity applied to the thin shells of revolution.
- The HE is axi-symmetrical.
- The HE is a symmetrical unit with identical TSs so as to analyze a half-unit.

(b) TSs

- The two tubesheets are circular and identical (same diameter, uniform thickness, material, temperature and edge conditions). Deviations will be allowed for the FL TS to cover the 3 types of HE.
- The tubesheets are uniformly perforated over a nominally circular area, in either equilateral triangular or square patterns. This implies that each TS is fully tubed (no large untubed window)
- Radial displacement at mid-surface of TS is ignored
- Temperature gradient through TS thickness is ignored
- Shear deformation and transverse normal strain in TS are ignored
- The unperforated rim of each TS is treated as a rigid ring without distortion of the cross section

(c) Tubes

- Tubes are assumed identical, straight and at same temperature
- Tubes are uniformly distributed in sufficient density to play the role of an elastic foundation for the TS
- The effect of the rotational stiffness of the tubes is ignored

(d) Shell and channel

- Shell and channel are cylindrical with uniform diameters and thicknesses
- If the channel head is hemispherical, it must be attached directly to the TS, without any cylindrical section between the head and the TS.
- shell and channel centerlines are the same.

(e) Weights and pressures drops

- Weights and pressures drops are ignored
- Pressures P_s and P_t are assumed uniform

3.5 Basis of Analytical Treatment

3.5.1 General

The design of a FL TS HE is complex as the two TSs are connected to each other through the tube bundle. Accordingly the structure is statically indeterminate. Many geometrical, mechanical and material properties are involved in the design as shown in Clause III-2a which lists the extensive input data.

Although the FL TS differs from the ST TS (smaller channel diameter, edge conditions,...), the two tubesheets are assumed to be identical. This assumption may seem unrealistic, but if the real floating TS geometry is accounted for, the analytical treatment would lead to a 'design by analysis' method requiring the use of a computer.

As for a fixed TS HE, the analysis includes the effects of the shell and tube side pressures, the axial stiffening effect of the tubes, the stiffening effect of the unperforated ring at the tubesheet edge, and the stiffening effect of the integrally attached channel or shell to the tubesheet. For a tubesheet that is extended as a flange to which a channel or shell is to be bolted, the bolt load causes an additional moment in the tubesheet which is included in the total stress in the tubesheet in addition to the moments caused by pressure.

The analysis is based on classical discontinuity analysis methods to determine the moments and forces that the tubesheet, shell and channel must resist. These components are treated using the theory of elasticity applied to the thin shells of revolution.

Because the heat exchanger is assumed to be symmetric, only half of the heat exchanger is treated. Main steps of the ST TS design follow the analytical treatment of fixed TS HEs. The FL TS will be designed the same way in a second step, using its appropriate data (channel thickness and diameter, bolting data, edge conditions,...).

- (a) Tubesheet is disconnected from the shell and channel. Shear load V_a and moment M_a are applied at the tubesheet edge as shown in Figure 39.
- (b) Perforated tubesheet is replaced by an equivalent solid circular plate of diameter D_o and effective elastic constants E^* (effective modulus of elasticity) and ν^* (effective Poisson's Ratio) depending on the ligament efficiency μ^* of the tubesheet. This equivalent solid plate is treated by the theory of thin circular plates subjected to pressures P_s and P_t and relevant applied loads to determine the maximum stresses.
- (c) Unperforated tubesheet rim is treated as a rigid ring whose cross section does not change under loading.
- (d) Tubes are replaced by an equivalent elastic foundation of modulus k_w .

- (e) Connection of the tubesheet with shell and channel accounts for the edge displacements and rotations of the 3 components.
- (f) Shell and channel are subject to shell side and tube side pressures P_s and P_t and edge loads V_a and M_a to determine the maximum stresses.
- (g) Maximum stresses in tubesheet, tubes, shell and channel are determined and limited to the appropriate allowable stress-based classifications of Section VIII Division 2 Part 4.

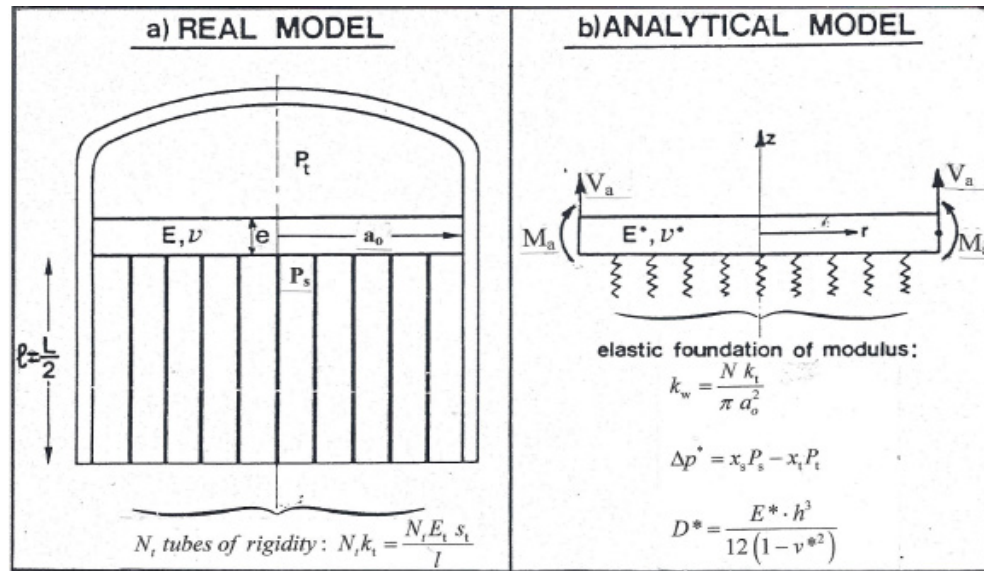


Figure 39—Analytical model used in design method

3.5.2 Free body diagram for ST TS

Figure 40 shows, for a ST TS integral both sides (configuration a), the free body diagram of the component parts (perforated tubesheet, unperforated tubesheet rim, shell, channel). The Figure details the relevant discontinuity forces (V_a , V_s , Q_s , V_c , Q_c) and moments (M_a , M_s , M_c , M_R) applied on each component, together with edge displacements.

In this Figure:

Forces (V_a , V_s , Q_s , V_c , Q_c) and moments (M_a , M_s , M_c , M_R) are per unit of circumferential length

Following subscripts are used:

- s for shell,
- c for channel,
- R for unperforated rim

No subscript for the perforated TS

Notation P_c instead of P_t (tube side pressure) is used throughout the analytical development so as to maintain the symmetry of the equations involving the shell (subscript s) and the channel (subscript c).

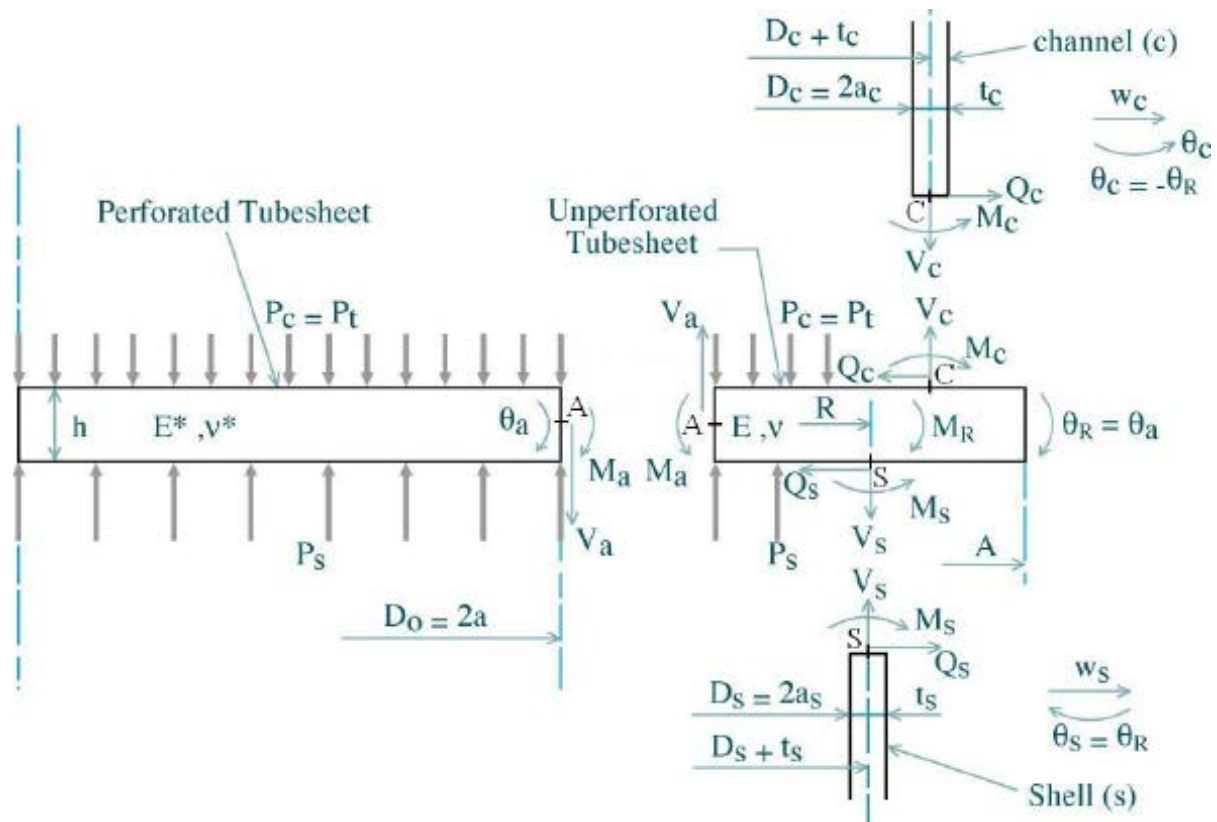


Figure 40—Free body diagram of the analytical model for the ST TS

3.5.3 Free body diagram for FL TS

The FL TS is attached to the FL channel either by welding (integral configuration A) or by bolting (gasketed configurations B or D), but there is no shell, which implies $V_s=0$ and $M_s=0$.

Accordingly, the free body diagram is different from the free body diagram of the ST TS and depends on the HE type shown on Figure 36: immersed, externally sealed, internally sealed. Figure 41 provides the free body diagram of an immersed FL TS with configuration A.

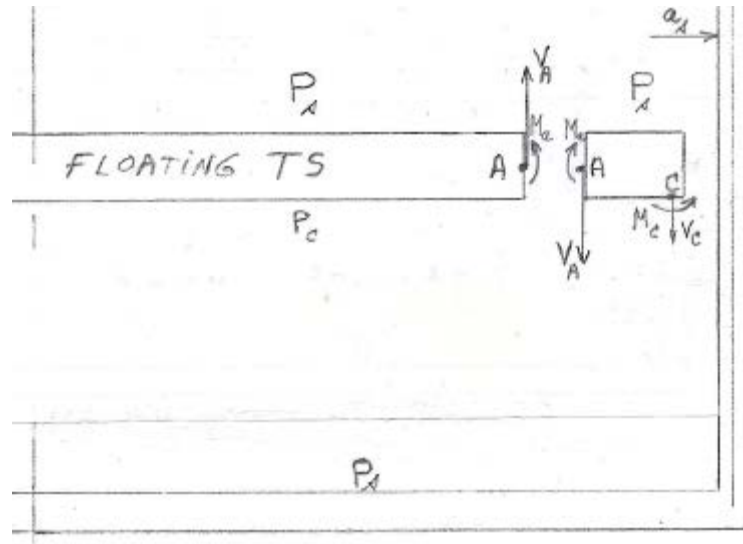


Figure 41—Free body diagram of the analytical model for the FL TS
(floating channel is shown on Figure 48, Figure 49 and Figure 50 for the 3 types of FL TS HEs)

4 AXIAL DISPLACEMENTS AND FORCES ACTING ON THE TUBES AND ON THE SHELL

4.1 Axial Displacement and Force Acting on the Tubes (Figure 42)

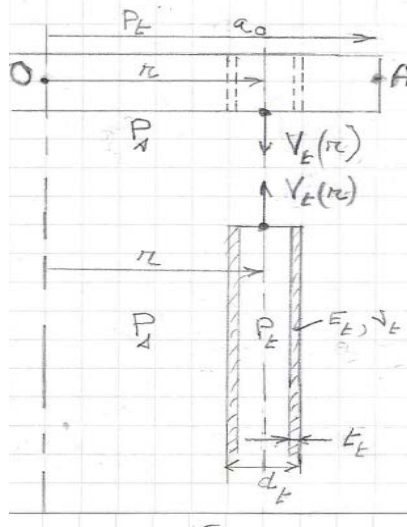


Figure 42—axial displacement of tubes

(a) Axial Displacement of the tubes due to axial force $V_t(r)$ acting on the tube row at radius r :

$$\delta_t(V_t) = \frac{V_t(r)}{k_t} = \frac{V_t(r)l}{E_t s_t} = \frac{l}{\pi E_t (d_o - t_t)} V_t(r)$$

(b) Axial force acting on each tube at radius r : $V_t(r) = k_t [\delta_t(V_t)]$

(c) Net effective pressure acting on the TS due to each tube at radius r of TS area $\pi a_o^2 / N_t$:

$$q_t(r) = \frac{-V_t(r)}{\pi a_o^2 / N_t} = -\frac{N_t k_t \delta_t(V_t)}{\pi a_o^2} = -\underbrace{\frac{N_t k_t}{\pi a_o^2}}_{k_w} \delta_t(V_t) \quad \boxed{k_w = \frac{N_t k_t}{\pi a_o^2}} \quad \boxed{q_t(r) = -k_w \delta_t(V_t)}$$

4.2 Axial Displacement and Force Acting on the Shell (Figure 43)

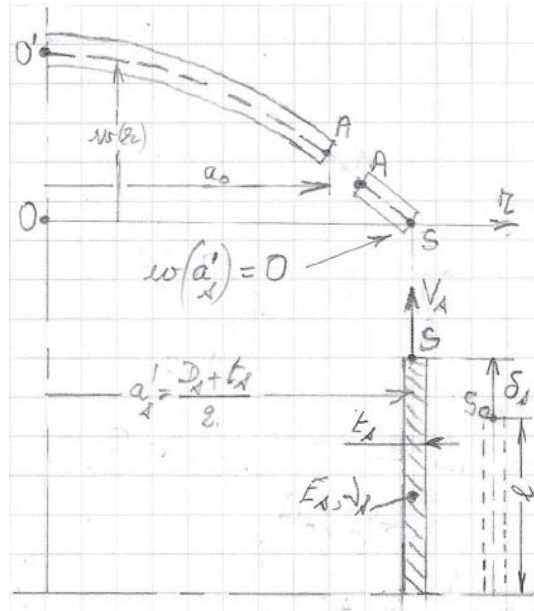


Figure 43—Axial displacement of the shell

(a) Axial displacement of the shell due to axial force V_s acting on the shell:

$$\delta_s(V_s) = \frac{V_s}{k_s} = \frac{V_s}{E_s} \frac{2\pi a_s l}{2\pi a_s t_s} = \frac{l}{E_s t_s} V_s \quad V_s \text{ per unit of length}$$

$$a_s = \text{mean shell radius} = \frac{D_s + t_s}{2}$$

V_s is known and depends on the HE type (see VI.2).

(b) Axial force acting on the shell: $V_s = \frac{E_s t_s}{l} \delta_s(V_s)$

(c) Axial displacement of tubes at radius r :

$$\delta_t(V_t) = \delta_s(V_s) + w(r) \quad \text{where } w(r) \text{ is the TS deflection at radius } r \text{ (see Figure 45)}$$

(d) TS deflection at radius r : $w(r) = \delta_t(V_t) - \delta_s(V_s)$

5 DEFLECTION AND LOADS ACTING ON THE TUBESHEET

5.1 Equivalent Plate Resting on an Elastic Foundation (Figure 44)

(a) Net effective pressure

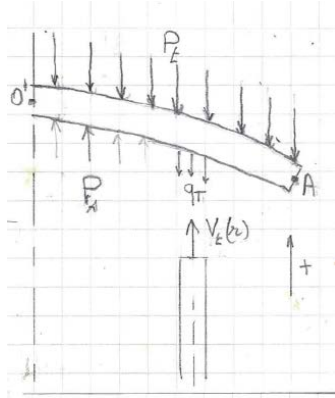


Figure 44—Loads acting on TS

due to tubes: $q_t(r) = -k_w \delta_t(V_t)$

Tubes act as an elastic foundation of equivalent modulus given by the axial rigidity of the half-bundle per unit of TS area:

$$k_w = \frac{N_t k_t}{\pi a_o^2} \quad q_t(r) = -k_w [\delta_s(V_s) + w(r)]$$

due to pressures P_s and P_t acting on the equivalent plate (see Annex E);

$$q_p = x_s P_s - x_t P_t = \Delta p^*$$

net effective pressure:

$$q(r) = q_p + q_t(r) = \Delta p^* - \frac{V_t(r)}{\pi a_o^2 / N_t} \quad [\text{V.1a}]$$

$$q(r) = \underbrace{\Delta p^* + k_w [-\delta_s(V_s)]}_Q - k_w w(r) \quad \boxed{Q = \Delta p^* - k_w \delta_s(V_s)} \quad [\text{V.1a}]$$

In this equation, the displacement δ_s of the shell subjected to axial force V_s is known and

$$\boxed{q(r) = Q - k_w w(r)}$$

(b) Deflection of TS (Figure 45)

The determination of the deflection given in Clause V.1b of PART 3 applies:

$$\boxed{w(x) = A \operatorname{ber} x + B \operatorname{beix} + \frac{Q}{k_w}} \quad \text{where: } x = k r = \sqrt[4]{\frac{k_w}{D^*}} r$$

Same for quantities $q(x)$, $\theta(x)$, $Q_r(x)$ and $M_r(x)$, which depend on integration constants A and B.

$$\text{At TS periphery (r=a}_0\text{): } X_a = k a_o = \sqrt[4]{\frac{k_w}{D^*}} a_o = \left[24 \left(1 - \nu^{*2} \right) N_t \frac{E_t t_t (d_t - t_t) a_o^2}{E^* L h^3} \right]^{1/4}$$

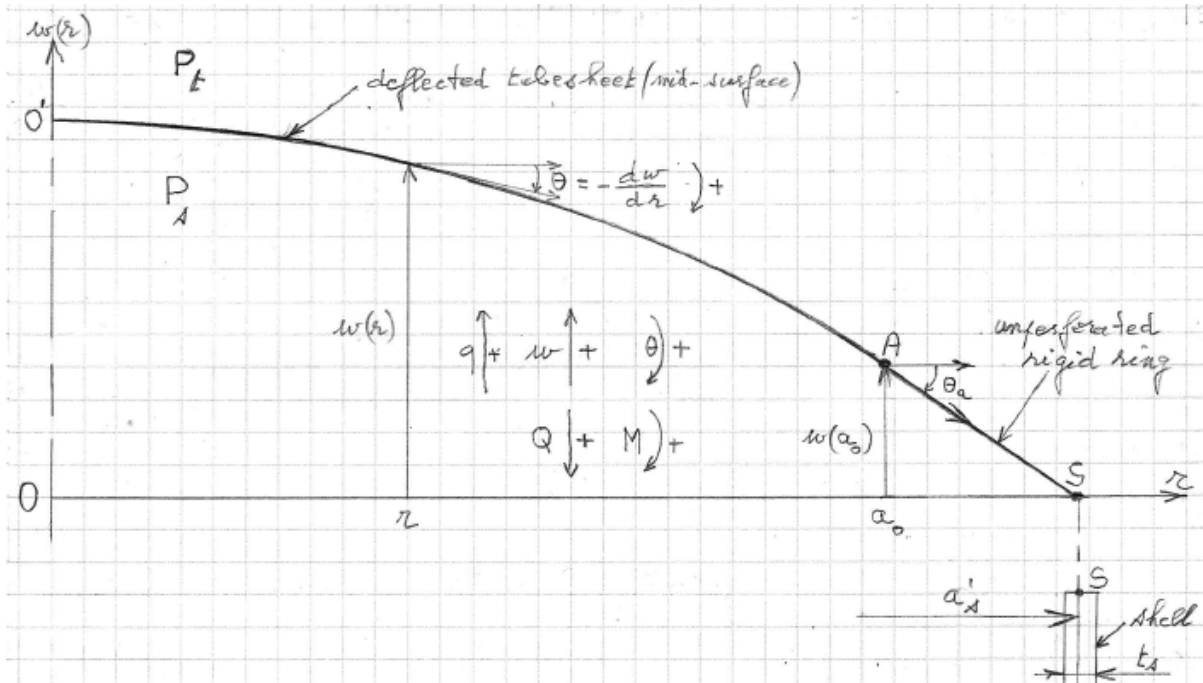


Figure 45—TS displacement

5.2 Determination of Integration Constants A and B

The determination of the integration constants A and B given in Clause V.2 of PART 3 applies.

Substituting the expressions of A and B in V.1b enables to determine:

$$w(x), q(x), \theta(x), Q(x), M(x)$$

as functions of x , depending on V_a and M_a which are still unknown.

6 TREATMENT OF THE UNPERFORATED RIM

6.1 Edge Loads Applied on Shell and Channel at their Connection to the TS

ST TS: Clauses VI.1a and VI.1b of PART 3 apply.

FL TS: Clause VI-1a of PART 3 does not apply as there is no shell.

Accordingly, the shell coefficients β_s , k_s , λ_s , δ_s :

shall be taken equal to 0, and a_s shall be taken equal to a_c .

Clause VI-1b of PART 3 applies.

6.2 Equilibrium of the Unperforated Rim

6.2.1 Due to Axial Loads

(a) The ring equilibrium for the ST TS writes (see Figure 46):

$$2\pi a_o V_a + \pi(a_s^2 - a_o^2)P_s + 2\pi a'_c V_c = \pi(a_c^2 - a_o^2)P_c + 2\pi a'_s V_s \quad [\text{VI.2.1.1}]$$

where:

V_a = axial edge load acting at connection of ring with equivalent plate is still to be determined

V_s = axial force acting in the shell, which depends on the HE type (immersed, externally sealed, internally sealed).

V_c = axial force acting in the stationary channel: $2\pi a'_c V_c = \pi a_c^2 P_c \Rightarrow a'_c V_c = \frac{a_c^2}{2} P_c$

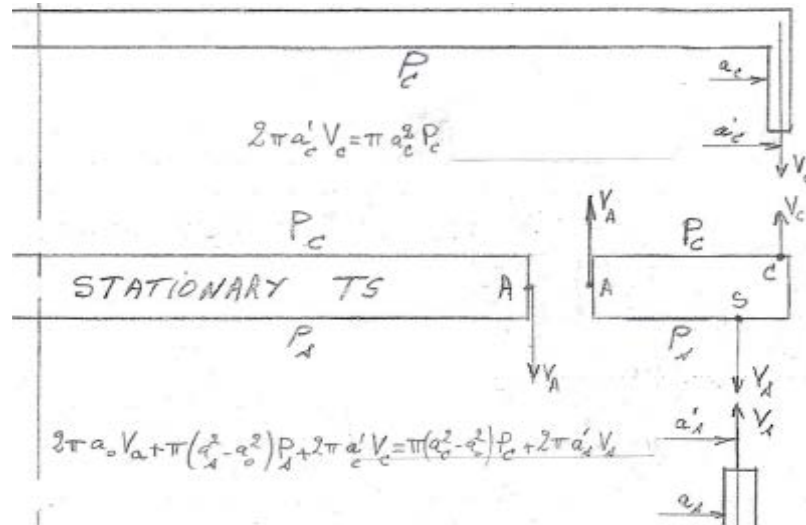


Figure 46—Ring equilibrium of the ST TS

(b) The ring equilibrium for the FL TS writes in the same way (see Figure 47), but there is no shell connected to the TS ($V_s=0$): $2\pi a_o V_a + \pi(a_s^2 - a_o^2)P_s + 2\pi a'_c V_c = \pi(a_c^2 - a_o^2)P_c$ [VI.2.1.2]

where:

V_a = axial edge load acting at connection of ring with equivalent plate is still to be determined

V_c = axial force acting in the floating channel, which depends on the HE type (immersed, externally

sealed, internally sealed).

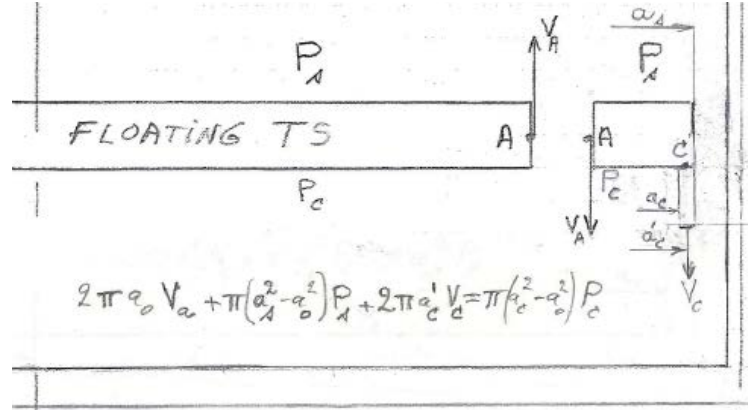


Figure 47—Ring equilibrium of the FL TS

- (c) Accordingly, the free body diagram of the FL TS is different from the free body diagram of the ST TS and depends on the HE type shown on Figure 36: immersed, externally sealed, internally sealed, which must be considered separately to calculate the axial edge load V_a (point A) and the axial force V_E acting at the periphery of the ring (point E).

(1) Immersed FL HE (see Figure 48)

$$\text{For ST TS: } \left. \begin{array}{l} 2\pi a_s' V_s = \pi a_s^2 P_s \\ 2\pi a_c' V_c = \pi a_c^2 P_c \end{array} \right\} \boxed{V_E = 2\pi a_c' V_c - 2\pi a_s' V_s = \pi a_c^2 P_c - \pi a_s^2 P_s}$$

which, combined with [VI.2.1.1], leads to:

$$2\pi a_o V_a = \pi a_o^2 (P_s - P_c) \Rightarrow \boxed{V_a = \frac{a_o}{2} (P_s - P_c)}$$

$$\text{For FL TS: } 2\pi a_c' V_c = \pi a_c^2 P_c - \pi a_s^2 P_s \quad \boxed{V_E = 2\pi a_c' V_c = \pi a_c^2 P_c - \pi a_s^2 P_s}$$

which, combined with [VI.2.1.2], leads to:

$$2\pi a_o V_a = \pi a_o^2 (P_s - P_c) \Rightarrow \boxed{V_a = \frac{a_o}{2} (P_s - P_c)}$$

(2) externally sealed FL HE (see Figure 49)

$$\text{For ST TS: } \left. \begin{array}{l} 2\pi a_s' V_s = 0 \\ 2\pi a_c' V_c = \pi a_c^2 P_c \end{array} \right\} \boxed{V_E = 2\pi a_c' V_c - 2\pi a_s' V_s = \pi a_c^2 P_c}$$

which, combined with [VI.2.1.1], leads to:

$$2\pi a_o V_a = \pi(a_o^2 - a_s^2)P_s - \pi a_o^2 P_c = \pi a_o^2 [(1 - \rho_s^2)P_s - P_c] \Rightarrow V_a = \frac{a_o}{2} [(1 - \rho_s^2)P_s - P_c]$$

For FL TS: $2\pi a_c' V_c = \pi a_c^2 P_c$ $V_E = 2\pi a_c' V_c = \pi a_c^2 P_c$

which, combined with [VI.2.1.2], leads to:

$$2\pi a_o V_a = \pi(a_o^2 - a_s^2)P_s - \pi a_o^2 P_c = \pi a_o^2 [(1 - \rho_s^2)P_s - P_c] \Rightarrow V_a = \frac{a_o}{2} [(1 - \rho_s^2)P_s - P_c]$$

(3) **internally sealed FL HE** (see Figure 50)

For ST TS: $\left. \begin{array}{l} 2\pi a_s' V_s = \pi a_s^2 P_c \\ 2\pi a_c' V_c = \pi a_c^2 P_c \end{array} \right\} V_E = 2\pi a_c' V_c - 2\pi a_s' V_s = 0$ (assuming $a_c' = a_s'$)

which, combined with [VI.2.1.1], leads to:

$$2\pi a_o V_a = \pi(a_o^2 - a_s^2)P_s - \pi(a_o^2 - a_s^2)P_c = \pi a_o^2 (1 - \rho_s^2)(P_s - P_c) \Rightarrow V_a = \frac{a_o}{2} (1 - \rho_s^2)(P_s - P_c)$$

For FL TS: $V_c = 0 \Rightarrow V_E = 2\pi a_c' V_c = 0$

which, combined with [VI.2.1.2], leads to:

$$2\pi a_o V_a = \pi(a_o^2 - a_s^2)P_s - \pi(a_o^2 - a_s^2)P_c = \pi a_o^2 [(1 - \rho_s^2)(P_s - P_c)] \Rightarrow V_a = \frac{a_o}{2} (1 - \rho_s^2)(P_s - P_c)$$

Thus, the axial edge load V_a (point A) and the axial force V_E acting at the periphery of the ring (point E) are the same for the ST TS and the FL TS, which enables the FL HE to be considered as a fixed TS HE for the analytical treatment.

However, the FL TS is different from the ST TS as its internal radius is smaller, it is not connected to the shell and the edge moments are different, except for the externally sealed HE which can be considered as a fixed TS HE with an expansion bellows of rigidity 0 (see Note 2 of Clause VII.2 of PART 3).

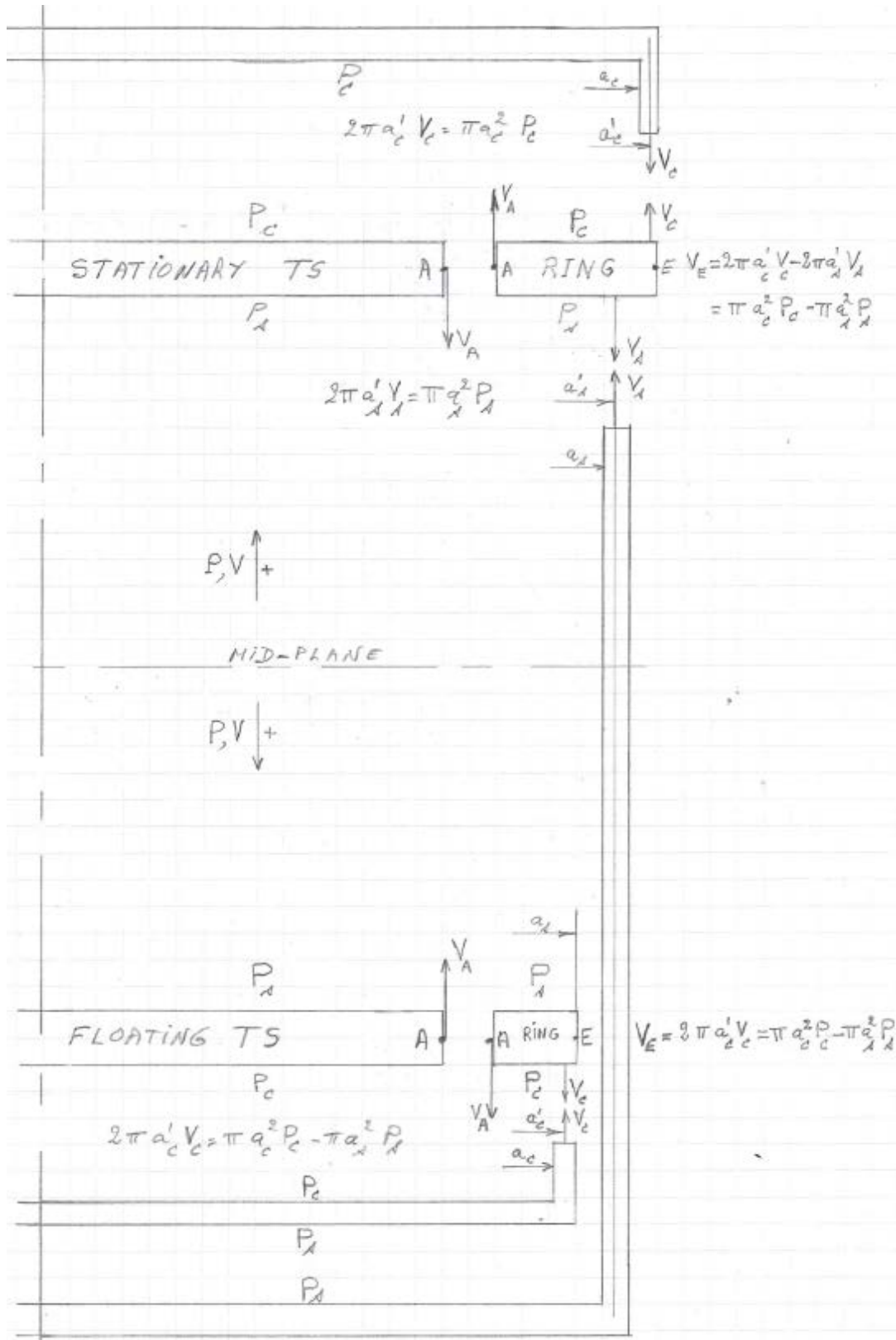


Figure 48—Immersed Floating TS HE

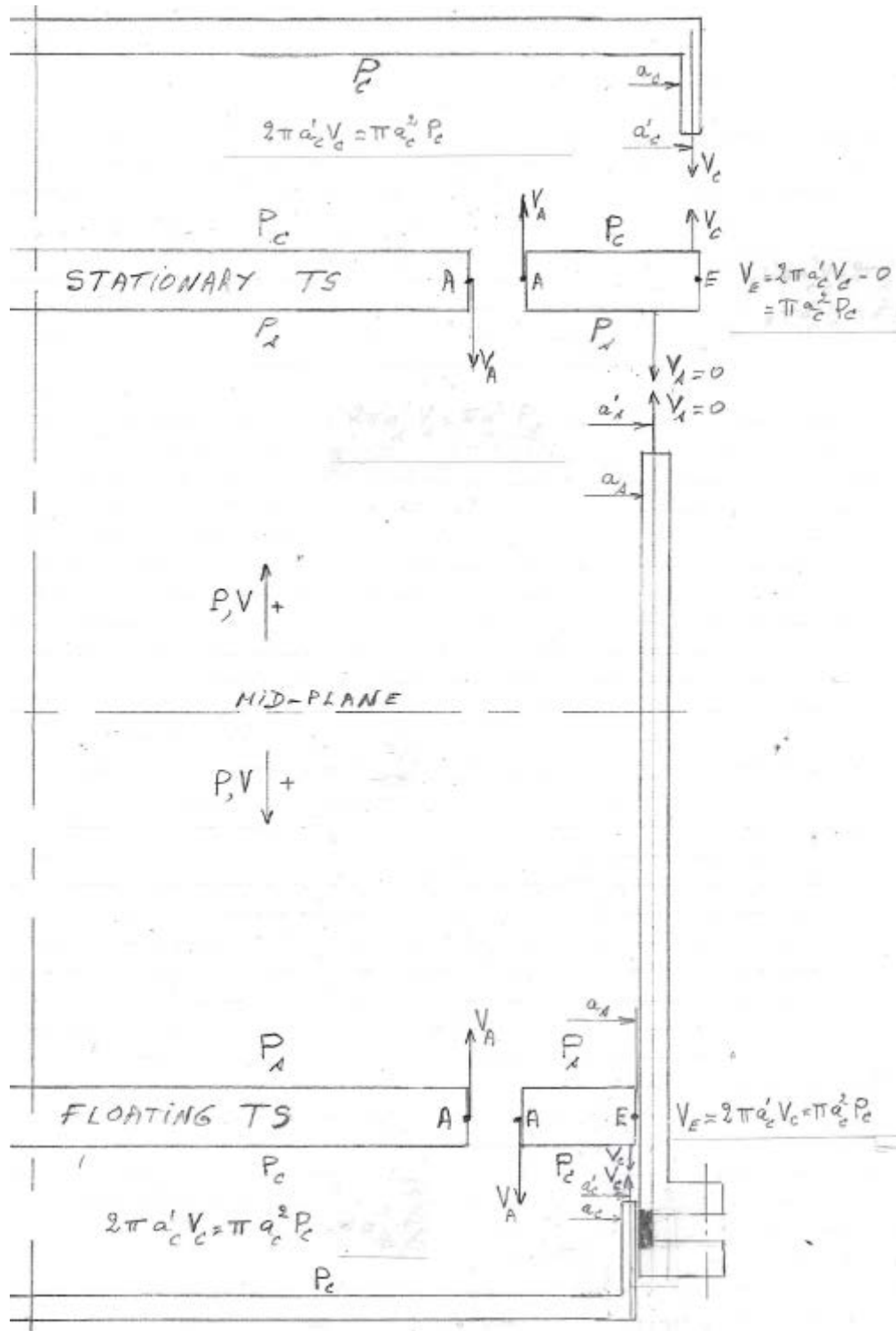


Figure 49—Externally sealed Floating TS HE

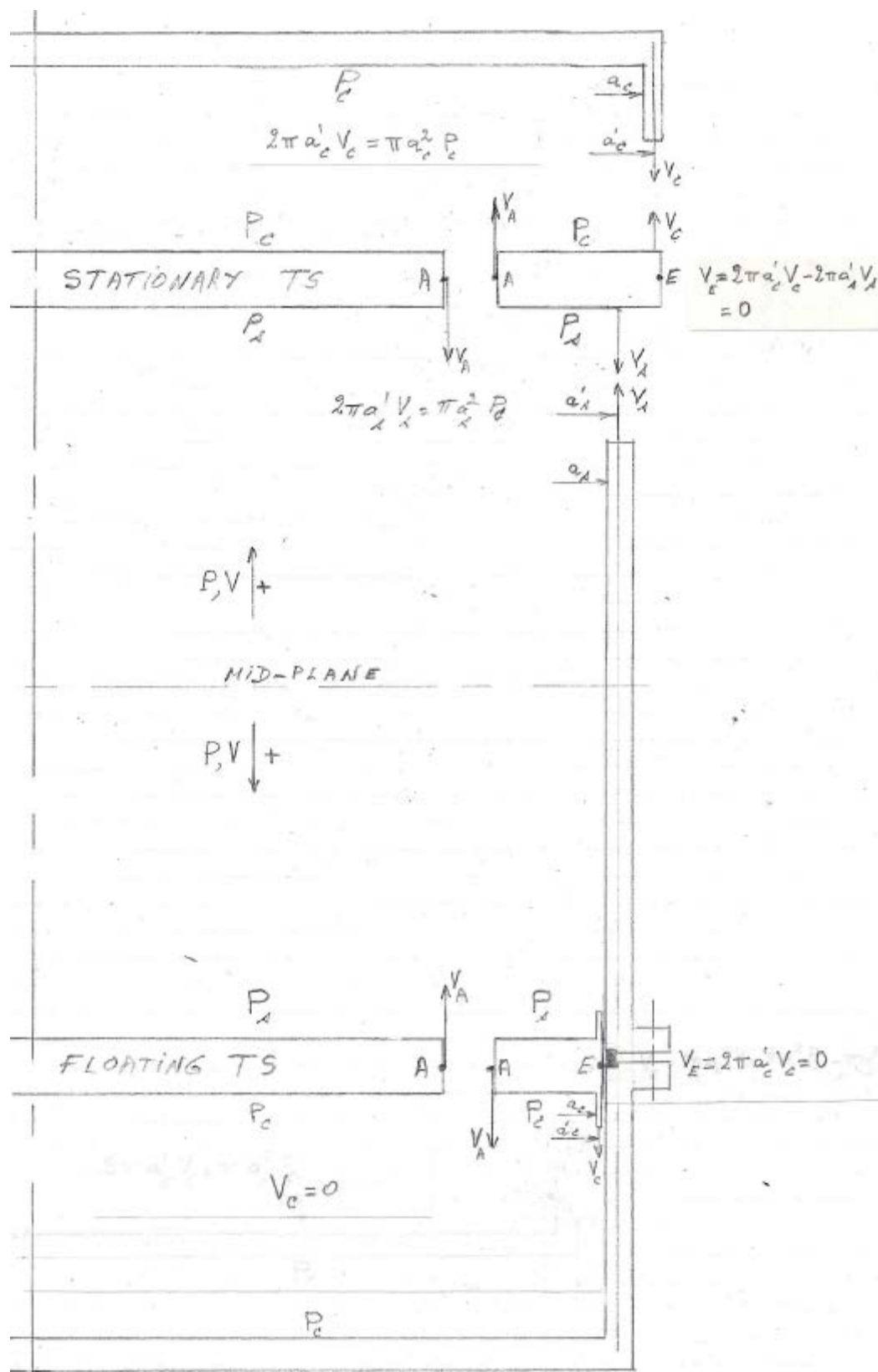


Figure 50—Internally sealed Floating TS HE

Note: The determination of the axial load V_s acting in the shell enables the calculation of the shell membrane stress:

$$\sigma_{s,m} = \frac{V_s}{t_s}$$

- For immersed Floating TS HE: $V_s = \frac{a_s^2 P_s}{2a_s}$ leads to : $\sigma_{s,m} = \frac{a_s^2}{(D_s + t_s) t_s} P_s$
- For internally sealed Floating TS HE: $V_s = 0$ leads to : $\sigma_{s,m} = 0$
- For externally sealed Floating TS HE: $V_s = \frac{a_s^2 P_c}{2a_s}$ leads to : $\sigma_{s,m} = \frac{a_s^2}{(D_s + t_s) t_s} P_c$

These formulas match the general shell membrane stress formula given in Clause VIII.6a of PART 3, using for P_e the relevant formula given in VII hereafter.

6.2.2 Due to Applied Moments

ST TS: Clause VI-2b of PART 3 applies.

FL TS: Clause VI-2b of PART 3 applies using $\beta_s=0$, $k_s=0$, $\lambda_s=0$ and $\delta_s=0$, which leads to:

$$\omega_s=0 \quad \text{and} \quad \omega_s^* = a_o^2 \left[\frac{(\rho_s^2 - 1)(\rho_s - 1)}{4} \right]$$

6.2.3 Edge Loads V_a and M_a Applied to the Tubesheet

(a) **Determination of M_a :** Clause VI-3a of PART 3 applies, which leads to: $M_a = (a_o V_a) Q_1 + Q_2$

(b) **Determination of V_a :** V_a is known and depends on each HE type, as determined in VI.2.1.3.

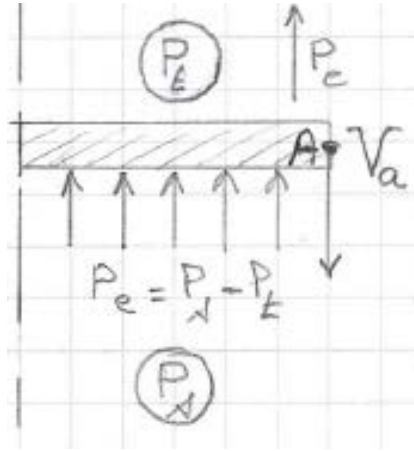
Accordingly, Clauses VI-3b and VI-3c of PART 3 are not relevant for FL TSs and quantities Q_{z1} , Q_{z2} and U do not apply.

7 EQUIVALENT PRESSURE ACTING ON THE TUBESHEET

A circular plate under uniform pressures P_s and P_t is subjected to a differential pressure $P_e = P_s - P_t$

The axial force V_a at periphery is determined from the plate equilibrium:

$$2\pi a_o V_a = P_e \pi a_o^2 \Rightarrow a_o V_a = \frac{a_o^2 P_e}{2} \quad P_e = \frac{2V_a}{a_o} \quad [\text{VII.1}]$$



Where V_a is given in VI.2.1.3 for:

(a) **immersed FL HE:** $V_a = \frac{a_o}{2} (P_s - P_c) \Rightarrow \boxed{P_e = P_s - P_t}$

(b) **externally sealed FL HE:** $V_a = \frac{a_o}{2} \left[(1 - \rho_s^2) P_s - P_c \right] \Rightarrow \boxed{P_e = (1 - \rho_s^2) P_s - P_t}$

(c) **internally sealed FL HE:** $V_a = \frac{a_o}{2} \left[(1 - \rho_s^2) (P_s - P_c) \right] \Rightarrow \boxed{P_e = (1 - \rho_s^2) (P_s - P_t)}$

As shown in Clause VI.2.1.3, these formulas apply both for the ST and FL TSs.

8 STRESSES IN THE HEAT-EXCHANGER COMPONENTS

Stress formulas calculated in Clause VIII of PART 3 apply.

9 DETERMINATION OF ALLOWABLE STRESS LIMITS

The determination of the allowable stress limits developed in Clause IX of PART 3 for pressure loadings cases 1, 2 and 3 apply.

10 ADDITIONAL RULES

The following additional rules apply in the same way as for the fixed TS HE covered in PART 3:

- (1) Effect of radial thermal expansion adjacent to the tubesheet (UHX-14.6), covered in Clause X.4 of PART 3.
- (2) Calculation procedure for simply supported TSs (UHX-14.7), covered in Clause X.3 of PART 3
- (3) Effect of plasticity at tubesheet-shell-channel joint (UHX-14.8), covered in Clause X.2 of PART 3.
- (4) Tubesheet flange extension (UHX-9), covered in Clause X.3 of PART 3.

11 HOW TO USE THE RULES

UHX-14 rules have been developed on the basis that the floating TS HE could be treated as a fixed TS, with some modifications due to the floating TS which is free to move axially inside the shell as outlined previously.

11.1 Stationary TS

The ST TS is designed in the same way as the fixed TSs rules of UHX-13, with the following modifications:

- the floating TS is free to move axially, which implies $K_J=0$ and $J=0$,
- there is no axial differential thermal expansion between tubes and shell, which implies $T_{t,m}=0$, $T_{s,m}=0$ and $\gamma=0$,
- the equivalent pressure depends on the HE type and is defined in Clause VII,
- Coefficients Q_{Z1} , Q_{Z2} and U do not apply.

11.2 Floating TS

The FL TS needs some adaptations as it differs from the ST TS:

- no attached shell,
- channel diameters (A , C , D_s , G_s , G_1) are smaller than the shell diameters,
- channel thickness t_c may differ from the ST channel thickness
- TS configuration is different from the ST TS configuration.

The FL TS shall be designed in the same way as the ST TS, using:

- its own geometrical data (A , C , D_s , G_s , G_1 , t_c),
- its own TS configuration,
- $a_s=a_c$ which is needed for the calculations of Q_1 , ω_s^* and ω_c^* .

So as to maintain a minimum of symmetry between the two TS, the FL TS shall have the same material and the same design temperature as the ST TS, which implies that the FL TS material properties are those of the ST TS (E_c , ν_c , S_c , $S_{y,c}$), and the same thickness as the ST TS.

11.3 Calculation Procedure

Like for the fixed TS HE, the calculation procedure can be summarized as follows:

- Set the data listed in III.2a
- Calculate the design coefficients listed in III.2b (x_s, x_t ; ρ_s, ρ_c)
- Calculate 1st characteristic parameter X_a and coefficients Q_1, Q_2 ; ω_s^*, ω_c^*
- Calculate the equivalent pressure P_e , and 2nd characteristic parameter Q_3
- Calculate the maximum stresses in TS, tubes, shell and channel and limit their values to the maximum allowable stress limits.

Because of the complexity of the procedure, it is likely that users will computerize the solution.

A Mathcad calculation sheet is provided for the immersed floating head TS HE defined in UHX-20.3.1 (Example 1). The stationary TS is gasketed with shell and channel (configuration d) and the floating TS is gasketed with the channel, not extended as a flange (configuration C). The data are shown in the sheet and the calculations follow strictly steps 1 to 10 of UHX-14.5 calculation procedure. The calculation sheet is provided both for the ST and the FL TS. See Annex W and Annex X for UHX14-Example 1 (2010 edition) Stationary and Floating respectively.

11.4 Calculation Using a Fixed TS HE Software

The ST TS can be calculated using a fixed TS HE software, such as the Mathcad software used in Annex R, provided that the 2nd TS is free to move axially by using a bellows of rigidity close to 0, which simulates an externally sealed FL HE. As shown by Clause VII.2-Note 2 of PART 3, $P_e = (1 - \rho_s^2) P_s - P_t$, which is effectively the equivalent pressure formula for this type of HE. For immersed floating head HE and internally sealed HE, it is necessary to replace P_e respectively by $P_e = P_s - P_t$ and $P_e = (1 - \rho_s^2)(P_s - P_t)$ in the software.

REFERENCES—PART 4

- [1] GARDNER “Heat Exchanger Tubesheet Design-2”, ASME Journal of Applied Mechanics, Vol. 19, Trans. ASME Vol. 74, 1948.
- [2] MILLER “The design of TS HEs”, Proceedings of the Institution of Mechanical Engineers”, London, Vol. 18, 1952.
- [3] GALLETLY “Optimum design of Thin Circular Plates on an Elastic Foundation”, Proceedings of the Institution of Mechanical Engineers”, London Vol. 173, 1952.
- [4] GARDNER “Tubesheet design: a basis for standardization.” ASME publication of 1st ICPVT conference—Delft. Part 1 Design and Analysis-1969.
- [5] SOLER “Mechanical Design of Heat Exchangers” – Arcturus publishers -1984 – 1047 pages.

PART 5

Analytical Treatment of U-Tube Tubesheet Heat Exchangers

1 SCOPE

PART 5, devoted to U-tube TS HEs (Figure 51), provides the technical basis for the determination of the stresses acting on the TS, shell and channel and their relationships with the design rules of UHX-12.

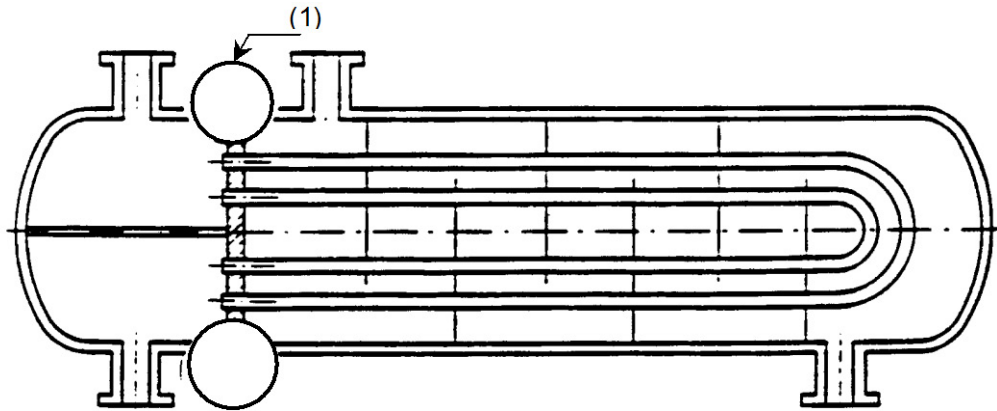


Figure 51—U-tube tubesheet heat exchangers

(1) see TS configurations in Figure 52

2 HISTORICAL BACKGROUND

The first rules for the design of U-tube TSs were published in Appendix AA of Section VIII, Division 1 in 1982. They were based on the 1969 Gardner method [1] to cover configurations a (integral construction) and b (gasketed construction), with some improvements:

- use of an effective pitch p^* to account for unperforated diametral lanes,
- use of an effective tube diameter d^* to account for the tube expansion depth ratio ρ ,
- derivation of a direct formula to determine the tubesheet thickness.

Later on, Miller [2], as a member of the ASME Special Working Group on Heat Transfer Equipment, developed a more refined analytical approach accounting for the unperforated rim for configurations b and e (tubesheet extended as a flange), which were not covered before. The method was adopted in ASME in 1992.

This new set of rules was not totally satisfactory for the following reasons:

- Three different set of rules, based on different analytical approaches, were proposed to cover configurations a, b, d and e.
- Configurations c and f (gasketed tubesheet not extended as a flange) were not covered.
- Rule for configuration "d" covered only the case where the tubesheet was not extended as a flange, with gaskets both sides of same diameter,
- Rule for configuration "a" used the same formula, corrected by a TEMA coefficient F which did not account properly for the degree of restraint of the tubesheet by the shell and channel.

Accordingly, Osweiller in 2002 [3] developed a more refined and unique approach to cover the six tubesheet configurations. This approach is based on Urey Miller's method mentioned above, with the following improvements:

- treatment of configurations c and f where the tubesheet is not extended as a flange.
- accounting for local pressures acting on shell and channel, when integral with the tubesheet.
- use of Poisson's ratio ν in all formulas, rather than using $\nu=0.3$, which leads to inexact coefficients.
- derivation of more condensed formulas providing rules consistent with fixed tubesheet rules.

This method was published for the first time in Nonmandatory Appendix AA of Section VIII Division 1. In 2003 it was published in a new part UHX of Section VIII Division 1 "Rules for Shell and Tubes Heat Exchangers" which became mandatory in 2004.

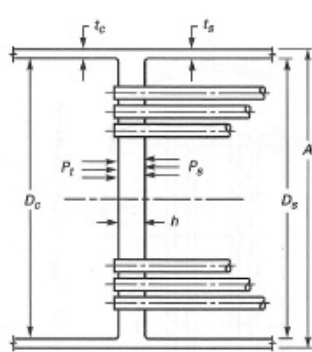
Soler [4] in 1984 developed a similar method accounting for the unperforated rim and the TS-shell-channel connection.

3 GENERAL

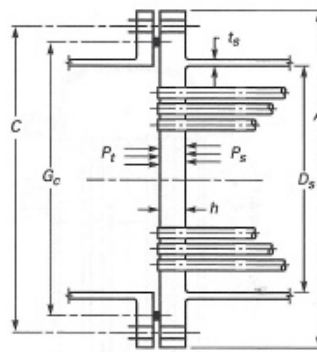
3.1 TS Configurations (UHX-12.1)

(a) The stationary TS is attached to the shell and the channel by welding (integral TS) or by bolting (gasketed TS) in accordance with the following 6 configurations (see Figure 37):

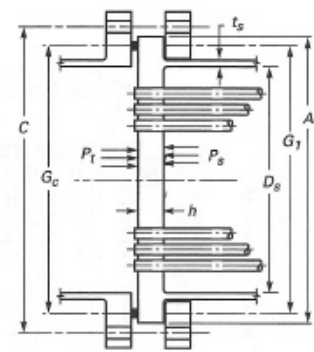
- Configuration a: tubesheet integral with shell and channel;
- Configuration b: tubesheet integral with shell and gasketed with channel, extended as a flange;
- Configuration c: tubesheet integral with shell and gasketed with channel, not extended as a flange;
- Configuration d: tubesheet gasketed with shell and channel;
- Configuration e: tubesheet gasketed with shell and integral with channel, extended as a flange;
- Configuration f: tubesheet gasketed with shell and integral with channel, not extended as a flange.



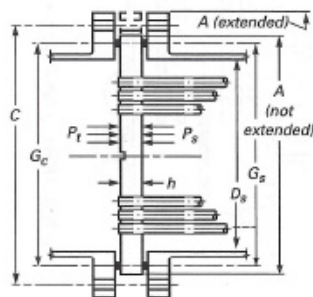
(a) Configuration a:
Tubesheet Integral With Shell and Channel



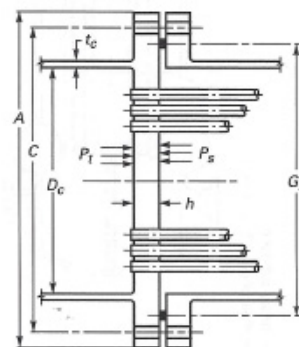
(b) Configuration b:
Tubesheet Integral With Shell and Gasketed
With Channel, Extended as a Flange



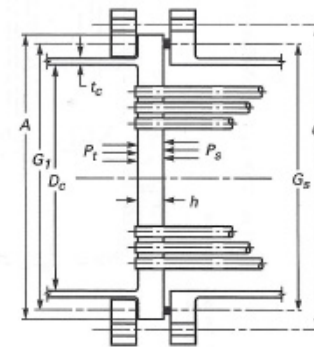
(c) Configuration c:
Tubesheet Integral With Shell and Gasketed
With Channel, Not Extended as a Flange



(d) Configuration d:
Tubesheet Gasketed With Shell and Channel



(e) Configuration e:
Tubesheet Gasketed With Shell and Integral
With Channel, Extended as a Flange



(f) Configuration f:
Tubesheet Gasketed With Shell and Integral
With Channel, Not Extended as a Flange

Figure 52—TS configurations

3.2 Notations

(a) Data for the design of the HE are as follows (UHX-12.3)

Symbols D_o , E^* , h_g , μ , μ^* and ν^* are defined in Clause II of PART 2.

A = outside diameter of tubesheet

a_c = radial channel dimension

a_o = equivalent radius of outer tube limit circle (see Clause IV.3a of PART 2)

a_s = radial shell dimension

C = bolt circle diameter

D_c = inside channel diameter

D_s = Inside shell diameter

E = modulus of elasticity for tubesheet material at T

E_c = modulus of elasticity for channel material at T_c

E_s = modulus of elasticity for shell material at T_s

G_c = diameter of channel gasket load reaction

G_s = diameter of shell gasket load reaction

G_1 = midpoint of contact between flange and tubesheet

h = tubesheet thickness

N_t = number of tubes

P_s = shell side internal design pressure. For shell side vacuum use a negative value for P_s .

P_t = tube side internal design pressure. For tube side vacuum use a negative value for P_t .

Notation P_c , instead of P_b , is used throughout the analytical development so as to maintain the symmetry of the equations involving the shell (subscript s) and the channel (subscript c).

S = allowable stress for tubesheet material at T

S_c = allowable stress for channel material at T_c

S_s = allowable stress for shell material at T_s

$S_{y,c}$ = yield strength for channel material at T_c

$S_{y,s}$ = yield strength for shell material at T_s

$S_{y,t}$ = yield strength for tube material at T_t

S_{PS} = allowable primary plus secondary stress for tubesheet material at T

$S_{PS,c}$ = allowable primary plus secondary stress for channel material at T_c

$S_{PS,s}$ = allowable primary plus secondary stress for shell material at T_s

T = tubesheet design temperature

T_c = channel design temperature

T_c = channel design temperature

T_s = shell design temperature

t_c = channel thickness

t_s = shell thickness

t_t = nominal tube wall thickness

W_c = channel flange design bolt load for the gasket seating condition

W_s = shell flange design bolt load for the gasket seating condition

W_{max} = maximum flange design bolt load

$$= \text{MAX}[(W_s), (W_c)]$$

ν_c = Poisson's ratio of channel material

ν_s = Poisson's ratio of shell material

(b) Design coefficients (UHX-2.5.1 to 5)

The following coefficients, specific to each component of the HE, will be used in the analytical treatment. They complete the data given above.

(1) Perforated TS

- Equivalent diameter of outer tube limit circle (see Clause IV-3.a of PART 2):

$$D_o = 2 r_o + d_t$$

- Equivalent radius of outer tube limit circle (see Clause IV-3.a of PART 2): $a_o = \frac{D_o}{2}$

TS coefficients:

- Ligament efficiency: $\mu^* = \frac{p^* - d^*}{p^*}$
- Effective tube hole diameter d^* and effective pitch p^* are defined in Clause IV-3c of PART 2
- Effective elastic constants E^* and ν^* are given in Clause V-6 of PART 2 as a function of μ^* and h/p (triangular or square pitch).
- Bending stiffness: $D^* = \frac{E^* \cdot h^3}{12(1 - \nu^{*2})}$
- effective tube side pass partition groove depth given in Clause IV-3.d of PART 2: h'_g
- effective pressure acting on tubesheet: P_e

(2) Shell

- radial shell dimension: $a_s \quad \rho_s = \frac{a_s}{a_o}$
- Integral configurations (a, b and c): $a_s = D_s / 2$
- Gasketed configuration (d, e and f): $a_s = G_s / 2$
- mean shell radius: $a'_s = \frac{D_s + t_s}{2}$

- shell coefficient: $\beta_s = \frac{\sqrt[4]{12(1-\nu_s^2)}}{\sqrt{(D_s + t_s) t_s}}$
- bending stiffness: $k_s = \beta_s \frac{E_s \cdot t_s^3}{6(1-\nu_s^2)}$

(3) Channel

- radial channel dimension: $a_c \quad \rho_c = \frac{a_c}{a_o}$
- Integral configuration (a): $a_c = D_c / 2$
- Gasketed configurations (b, c and d): $a_c = G_c / 2$
- mean channel radius: $a'_c = \frac{D_c + t_c}{2}$
- Channel coefficient: $\beta_c = \frac{\sqrt[4]{12(1-\nu_c^2)}}{\sqrt{(D_c + t_c) t_c}}$
- Bending stiffness: $k_c = \beta_c \cdot \frac{E_c \cdot t_c^3}{6(1-\nu_c^2)}$

(4) Unperforated rim

- D_o = internal diameter
- A = external diameter
- Diameter ratio: $K = A / D_o$

3.3 Loading Cases (UHX-12.4)

The normal operating condition of the HE is achieved when the tube side pressure P_t and shell side pressure P_s act simultaneously. However, a loss of pressure is always possible. Accordingly, for safety reasons, the designer must always consider the cases where $P_s=0$ and $P_t=0$ for the normal operating condition(s).

The designer must also consider the startup condition(s), the shutdown condition(s) and the upset condition(s), if any, which may govern the design. For each of these conditions, the following 3 pressure loading cases must be considered.

- Loading Case 1: Tube side pressure P_t acting only ($P_s = 0$).
- Loading Case 2: Shell side pressure P_s acting only ($P_t = 0$).
- Loading Case 3: Tube side pressure P_t and shell side pressure P_s acting simultaneously.

When vacuum exists, each loading case shall be considered with and without the vacuum.

When differential pressure design is specified by the user, the design shall be based only on loading case 3.

As the calculation procedure is iterative, a value h is assumed for the tubesheet thickness to calculate and check that the maximum stresses in tubesheet, shell and channel are within the maximum permissible stress limits.

3.4 Design Assumptions (UHX-12.2)

A U-tube TS HE is a complex structure and several assumptions are necessary to derive a ‘design by rules’ method.

Most of them could be eliminated, but the analytical treatment would lead to ‘design by analysis’ method requiring the use of a computer.

The design assumptions are as follows.

(a) HE

- The analytical treatment is based on the theory of elasticity applied to the thin shells of revolution.
- The HE is axi-symmetrical.

(b) TS

- The tubesheet is circular.
- The tubesheet is uniformly perforated over a nominally circular area, in either equilateral triangular or square patterns. This implies that the TS is fully tubed (no large untubed window)
- Radial displacement at mid-surface of TS is ignored
- Temperature gradient through TS thickness is ignored
- Shear deformation and transverse normal strain in TS are ignored
- The unperforated rim of the TS is treated as a rigid ring without distortion of the cross section

(c) Shell and channel

- Shell and channel are cylindrical with uniform diameters and thicknesses
- Shell and channel centerlines are the same

(d) Weights and pressures drops

- Weights and pressures drops are ignored
- Pressures P_s and P_t are assumed uniform

3.5 Basis of Analytical Treatment

3.5.1 General

Comparison of Figure 36 and Figure 51 of PART 4 shows that the U-tube TS HE can be considered as an immersed floating head TS HE where the floating TS does not exist and the tubes do not play the role of an elastic foundation.

Accordingly, the U-tube HE could be designed as an immersed floating head TS HE where the tubes have no axial rigidity (see Annex A-IX.2). However the analytical treatment presented below is based on the approach developed by Urey Miller [2] as explained in Clause II.

Although the design is less complex, many geometrical, mechanical and material properties are involved in the design as shown in Clause III-2a which lists the extensive input data.

As for a floating TS HE, the analysis includes the effects of the shell and tube side pressures, the stiffening effect of the unperforated ring at the tubesheet edge and the stiffening effect of the integrally attached channel or shell to the tubesheet. When the tubesheet is gasketed with the shell or the channel, the bolt load causes an additional moment in the tubesheet which is included in the total stress in the tubesheet in addition to the moments caused by pressure.

The analysis is based on classical discontinuity analysis methods to determine the moments and forces that the tubesheet, shell and channel must resist. These components are treated using the elastic theory of thin shells of revolution.

Main steps of the U-tube TS design follow the analytical treatment of floating TS HEs.

- (a) Tubesheet is disconnected from the shell and channel. Shear load V_a and moment M_a are applied at the tubesheet edge as shown in Figure 53.
- (b) Perforated tubesheet is replaced by an equivalent solid circular plate of diameter D_o and effective elastic constants E^* (effective modulus of elasticity) and ν^* (effective Poisson's ratio) depending on the ligament efficiency μ^* of the tubesheet. This equivalent solid plate is treated by the theory of thin circular plates subjected to pressures P_s and P_t and relevant applied loads to determine the maximum stresses.
- (c) Unperforated tubesheet rim is treated as a rigid ring whose cross section does not change under loading.
- (d) Connection of the tubesheet with shell and channel accounts for the edge displacements and rotations of the 3 components.
- (e) Shell and channel are subject to shell side and tube side pressures P_s and P_t and edge loads V_a and M_a to determine the maximum stresses.
- (f) Maximum stresses in tubesheet shell and channel are determined and limited to the appropriate allowable stress-based classifications of Section VIII Division 2 Part 4.

3.5.2 Free Body Diagram

Figure 53 shows, for a TS integral both sides (configuration a), the free body diagram of the component parts (perforated tubesheet, unperforated tubesheet rim, shell, channel). The Figure details the relevant discontinuity forces (V_a , V_s , Q_s , V_c , Q_c) and moments (M_a , M_s , M_c , M_R) applied on each component, together with edge displacements.

In this Figure:

Forces (V_a , V_s , Q_s , V_c , Q_c) and moments (M_a , M_s , M_c , M_R) are per unit of circumferential length

Following subscripts are used:

- s for shell,
- c for channel,
- R for unperforated rim

no subscript for the perforated TS

Notation P_c instead of P_t (tube side pressure) is used throughout the analytical development so as to maintain the symmetry of the equations involving the shell (subscript s) and the channel (subscript c).

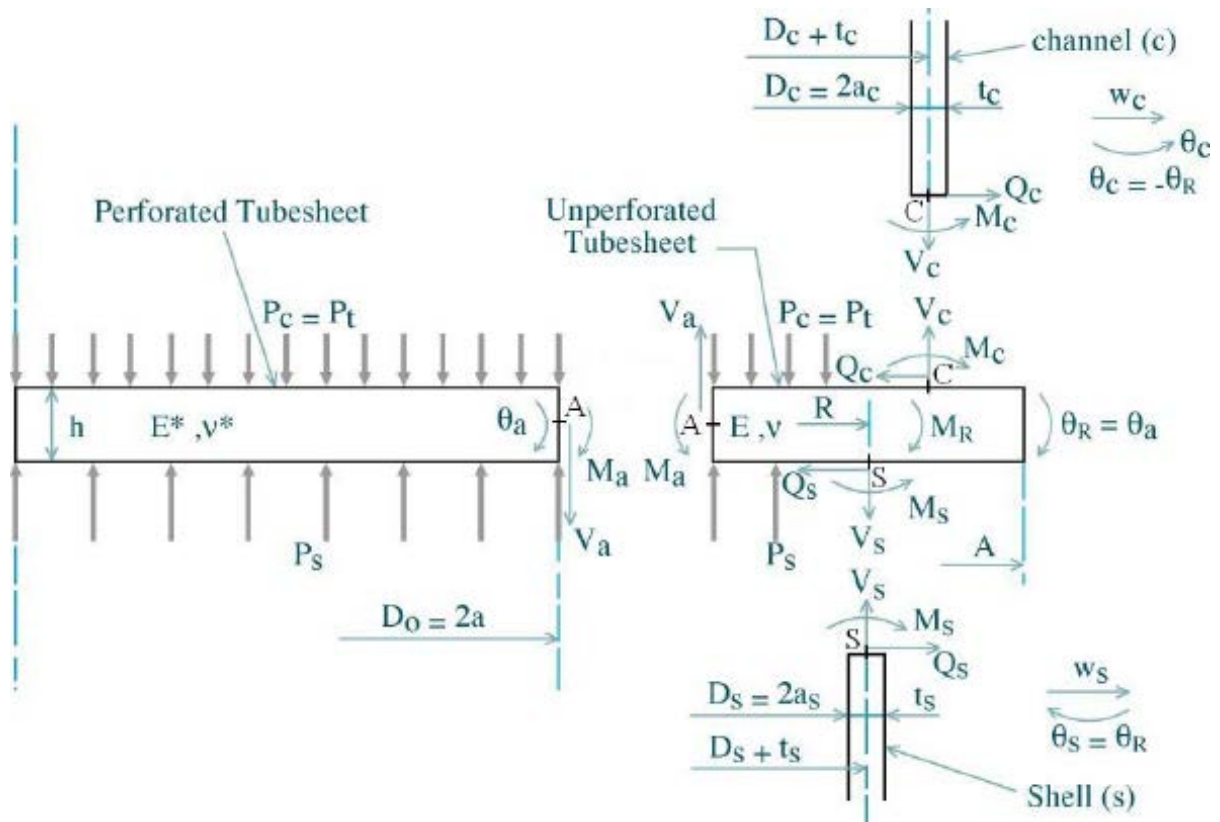


Figure 53—Free body diagram of the analytical model for the TS

4 TREATMENT OF THE PERFORATED TUBESHEET

The perforated tubesheet is treated as an equivalent solid circular plate of radius a_o with effective elastic constants E^* and ν^* , subjected to an equivalent pressure P_e and edge loads V_a and M_a at periphery.

The equilibrium equation of the plate writes: $-2 \pi a_o V_a + \pi a_o^2 P_e = 0$

which leads to: $V_a = \frac{a_o}{2} P_e$ [IV-1]

The rotation of the plate at radius a_o is given by: $\theta_a = \frac{12 (1-\nu^*)}{E^* h^3} \left[a_o M_a + \frac{a_o^3}{8} P_e \right]$ [IV-2]

The radial bending moment at radius r of such a plate is given by the classical formula:

$$M(r) = M_a + (3 + \nu^*) \frac{a_o^2}{16} \left[1 - \left(\frac{r}{a_o} \right)^2 \right] P_e \quad \text{[IV-3]}$$

The maximum bending moment appears

- either at center ($r=0$): $M(r) = M_a + (3 + \nu^*) \frac{a_o^2}{16} P_e$
- or at periphery ($r=a_o$): $M(r) = M_a$

5 TREATMENT OF THE UNPERFORATED RIM

5.1 Edge Loads Applied on Shell and Channel at their Connection to the TS

The following equations are developed for integral shell and channel.

(a) The edge loads Q_s and M_s applied on the shell write (see Clause VI.1a of PART 3):

$$\left\{ \begin{array}{l} M_s = +k_s \left(1 + \frac{t'_s}{2} \right) \theta_s + \beta_s k_s \delta_s P_s \\ Q_s = -\beta_s k_s \left(1 + t'_s \right) \theta_s - 2\beta_s^2 k_s \delta_s P_s \end{array} \right\} \quad [\text{V.1a}] \text{ for an integral shell (configurations a, b, c)}$$

When the shell is not integral with the TS (configurations d, e, f), $k_s=0$ and $\delta_s=0$ lead to:
 $M_s = 0$ and $Q_s = 0$.

Note: These formulas are valid for a shell of sufficient length. Annex J provides the minimum length above which these formulas can be applied.

(b) The edge loads Q_c and M_c applied on the shell write (see Clause VI.1b of PART 3):

$$\left\{ \begin{array}{l} M_c = +k_c \left(1 + \frac{t'_c}{2} \right) \theta_c + \beta_c k_c \delta_c P_c \\ Q_c = -\beta_c k_c \left(1 + t'_c \right) \theta_c - 2\beta_c^2 k_c \delta_c P_c \end{array} \right\} \quad [\text{V.1b}] \text{ for an integral channel (configurations, a e, f)}$$

When the channel is not integral with the TS (configurations b, c, d), $k_c = 0$ and $\delta_c = 0$ lead to:
 $M_c = 0$ and $Q_c = 0$

Note 1: These formulas are valid for a channel of sufficient length. Annex J provides the minimum length above which these formulas can be applied.

Note 2: These formulas are valid for a cylindrical channel. If the channel is hemispherical, it must be attached directly to the TS (configurations a, b or c), without any cylindrical section between the head and the TS. Annex K provides the relevant formulas for that case. Only coefficient δ_c is affected:

$$\delta_c = \frac{D_c^2}{4 E_c t_c} \left(\frac{1 - \nu_c}{2} \right)$$

5.2 Equilibrium of the Unperforated Solid Rim

$$(a) \text{ due to axial loads } \left\{ \begin{array}{l} a_s = D_s / 2 \\ a'_s = a_s + \frac{t_s}{2} \end{array} \right\} \quad \left\{ \begin{array}{l} a_c = D_c / 2 \\ a'_c = a_c + \frac{t_c}{2} \end{array} \right\}$$

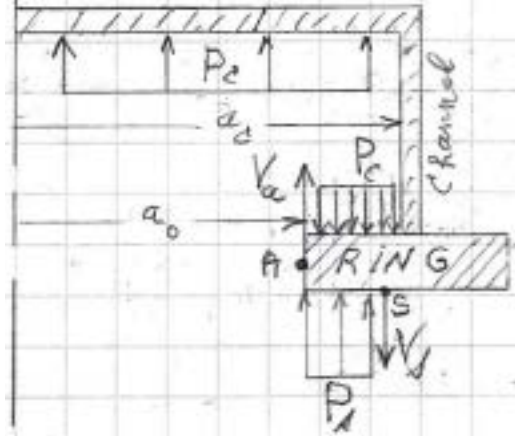


Figure 54—Ring equilibrium of the TS

The axial equilibrium of the ring writes: (see Figure 54):

$$2 \pi a'_s V_s + \pi (a_c^2 - a_o^2) P_c = 2 \pi a'_c V_c + 2 \pi a_o V_a + \pi (a_s^2 - a_o^2) P_s \quad [\text{V.2a-1}]$$

The axial equilibrium of the shell writes: $2 \pi a'_s V_s = \pi a_s^2 P_s$

The axial equilibrium of the channel writes: $2 \pi a'_c V_c = \pi a_c^2 P_c$

This leads to:
$$V_a = \frac{a_o}{2} (P_s - P_c)$$

Comparison with the TS equilibrium in Clause IV shows that $P_c = P_s - P_t$ as for an immersed floating head HE.

(b) due to applied moments

Equilibrium of moments applied to the ring relative to the axis located at radius a_o enables to determine the moment M_R (see Figure 53).

$$R = \text{radius at center of ring} = \frac{A + 2a_o}{4}$$

$$RM_R = -[a_o M_a] + \left[a'_c M_c - a'_c Q_c \frac{h}{2} \right] + \left[M(P_c) - a'_c V_c (a'_c - a_o) \right] - \left[a'_s M_s - a'_s Q_s \frac{h}{2} \right] + \left[M(P_s) - a'_s V_s (a'_s - a_o) \right] \quad [\text{V.2b}]$$

$$M(P_c) = \text{moment due to pressure } P_c \text{ acting on the ring} = (a_c^2 - a_o^2) \left(\frac{a_c + a_o}{2} - a_o \right) \frac{P_c}{2}$$

$$M(P_s) = \text{moment due to pressure } P_s \text{ acting on the ring} = (a_s^2 - a_o^2) \left(\frac{a_s + a_o}{2} - a_o \right) \frac{P_s}{2}$$

From Annex L:

$$\left[a'_c M_c - a'_c Q_c \frac{h}{2} \right] = a'_c k_c \left(1 + t'_c + \frac{t_c'^2}{2} \right) \theta_c + a_o \omega_c P_c \quad \boxed{\omega_c = \rho_c \beta_c k_c \delta_c (1 + h \beta_c)} \quad t'_c = h \beta_c$$

$$\left[a'_s M_s - a'_s Q_s \frac{h}{2} \right] = a'_s k_s \left(1 + t'_s + \frac{t_s'^2}{2} \right) \theta_s + a_o \omega_s P_s \quad \boxed{\omega_s = \rho_s \beta_s k_s \delta_s (1 + h \beta_s)} \quad t'_s = h \beta_s$$

$$\left. \begin{aligned} [M(P_c) - a'_c V_c (a'_c - a_o)] &= \frac{P_c}{4} [(a_c^2 - a_o^2)(a_c - a_o) - 2a_c^2 (a'_c - a_o)] = \frac{P_c}{4} (a_c - a_o)(-a_c^2 - a_o^2) = \frac{P_c}{4} a_o^3 [(\rho_c - 1)(\rho_c^2 + 1)] \\ [M(P_s) - a'_s V_s (a'_s - a_o)] &= \frac{P_s}{4} [(a_s^2 - a_o^2)(a_s - a_o) - 2a_s^2 (a'_s - a_o)] = \frac{P_s}{4} (a_s - a_o)(-a_s^2 - a_o^2) = \frac{P_s}{4} a_o^3 [(\rho_s - 1)(\rho_s^2 + 1)] \\ [R M_R] &= -a_o M_a + \frac{P_s}{4} a_o^3 [(\rho_s - 1)(\rho_s^2 + 1)] - \frac{P_c}{4} a_o^3 [(\rho_c - 1)(\rho_c^2 + 1)] \\ &\quad - \left[a'_s k_s \left(1 + t'_s + \frac{t_s'^2}{2} \right) + a'_c k_c \left(1 + t'_c + \frac{t_c'^2}{2} \right) \right] \theta_a + a_o (\omega_c P_c - \omega_s P_s) \end{aligned} \right\} \quad [V.2b']$$

(c) **Rotation of rigid ring**

$$K = \frac{A}{D_o} \quad \theta_R = \frac{12}{E h^3} \frac{R M_R}{L n K} \text{ leads to, with } \theta_R = \theta_a :$$

$$R M_R = \left[\frac{E h^3}{12} L n K \right] \theta_R = \left[\frac{E h^3}{12} L n K \right] \theta_a$$

Replacing RM_R by its expression [V.2b'] permits to calculate the TS rotation θ_a :

$$\left. \begin{aligned} &\left[\frac{E h^3}{12} L n K + a'_s k_s \left(1 + t'_s + \frac{t_s'^2}{2} \right) + a'_c k_c \left(1 + t'_c + \frac{t_c'^2}{2} \right) \right] \theta_a = -a_o M_a \\ &+ P_s \frac{a_o^3}{4} [(\rho_s - 1)(\rho_s^2 + 1)] - P_c \frac{a_o^3}{4} [(\rho_c - 1)(\rho_c^2 + 1)] + a_o [\omega_c P_c - \omega_s P_s] \end{aligned} \right\}$$

$$C_1 = \frac{h^3}{12} \left[\underbrace{\frac{6}{h^3} (D_s + t_s) k_s \left(1 + t'_s + \frac{t_s'^2}{2} \right)}_{\lambda_s} + \underbrace{\frac{6}{h^3} (D_c + t_c) k_c \left(1 + t'_c + \frac{t_c'^2}{2} \right)}_{\lambda_c} + E L n K \right] = \frac{h^3}{12} [\lambda_s + \lambda_c + E L n K]$$

$$\boxed{\lambda_s = \frac{6}{h^3} (D_s + t_s) k_s \left(1 + t'_s + \frac{t_s'^2}{2} \right)} \quad \boxed{\lambda_c = \frac{6}{h^3} (D_c + t_c) k_c \left(1 + t'_c + \frac{t_c'^2}{2} \right)}$$

Note: if $\lambda_s + \lambda_c$ is high (>3) the TS can be considered as clamped

if $\lambda_s + \lambda_c$ is low (<1) the TS can be considered as simply supported

$$\frac{h^3}{12} [\lambda_s + \lambda_c + E \ln K] \theta_a = -a_o M_a + P_s \frac{a_o^3}{4} [(\rho_s - 1)(\rho_s^2 + 1)] - P_c \frac{a_o^3}{4} [(\rho_c - 1)(\rho_c^2 + 1)] + a_o [\omega_c P_c - \omega_s P_s] \quad [\text{VI.2c}]$$

Where the TS rotation θ_a is given by [IV-2]: $\theta_a = \frac{12 (1-\nu^*)}{E^* h^3} \left[a_o M_a + \frac{a_o^3}{8} P_e \right]$ which permits to calculate M_a :

$$\begin{aligned} \frac{h^3}{12} [\lambda_s + \lambda_c + E \ln K] \frac{12 (1-\nu^*)}{E^* h^3} \left[a_o M_a + \frac{a_o^3}{8} P_e \right] + a_o M_a &= P_s \frac{a_o^3}{4} [(\rho_s - 1)(\rho_s^2 + 1)] - P_c \frac{a_o^3}{4} [(\rho_c - 1)(\rho_c^2 + 1)] + a_o [\omega_c P_c - \omega_s P_s] \\ \underbrace{[\lambda_s + \lambda_c + E \ln K] \frac{(1-\nu^*)}{E^*}}_F \left[M_a + \frac{a_o^2}{8} P_e \right] + M_a &= P_s \frac{a_o^2}{4} [(\rho_s - 1)(\rho_s^2 + 1)] - P_c \frac{a_o^2}{4} [(\rho_c - 1)(\rho_c^2 + 1)] + [\omega_c P_c - \omega_s P_s] \\ [1+F] M_a &= -F \frac{a_o^2}{8} P_e + \underbrace{P_s \frac{a_o^2}{4} [(\rho_s - 1)(\rho_s^2 + 1)] - P_c \frac{a_o^2}{4} [(\rho_c - 1)(\rho_c^2 + 1)]}_{M_{TS}} + [\omega_c P_c - \omega_s P_s] \end{aligned}$$

In this equation:

M_{TS} is the is the moment due to pressures P_s and P_l acting on the rigid ring:

$$M_{TS} = P_s \frac{a_o^2}{4} [(\rho_s - 1)(\rho_s^2 + 1)] - P_c \frac{a_o^2}{4} [(\rho_c - 1)(\rho_c^2 + 1)]$$

F denotes the degree of restrain of the TS by the shell and channel when they are integral with the TS (configuration a):

$$F = [\lambda_s + \lambda_c + E \ln K] \frac{(1-\nu^*)}{E^*}$$

Note:

If there is no bending support from the channel ($k_c = 0$) and the shell ($k_s = 0$): $\lambda_s = 1$ and $\lambda_c = 1$ lead to F close to zero:

$M_a = M_{TS}$ and the tubesheet is almost simply supported. If additionally there is no unperforated rim ($K = 1, \rho_s = 0, \rho_c = 0$):

$F = 0$ and $M_a = 0$: the tubesheet is fully simply supported.

If there is a high bending support from the channel ($k_c = \infty$) and the shell ($k_s = \infty$): $\lambda_c = \infty$ and $\lambda_s = \infty$ lead to $F = \infty$.

The tubesheet is fully clamped: $M_a = -\frac{D_o^2}{32} (P_s - P_c)$.

$$\text{Finally, for configuration a, } M_a \text{ writes: } M_a = \frac{M_{TS} + (\omega_c P_c - \omega_s P_s) - F \left(\frac{a_o^2}{8} P_e \right)}{1 + F} \quad [\text{V.2c}]$$

(d) **Generic equation covering the 6 configurations a to f**

Clause A-VI.2 of PART 3 shows that a term $\frac{a_o}{2\pi} [W_c \gamma_{bc} - W_s \gamma_{bs}]$, which accounts for the flange bolt loads W_s and W_c , must be added in equation [V.2c] to obtain the generic equation for the 6 configurations.

Rules of UHX-12 cover the case where $C_s=C_c=C$. Accordingly, equation [V.2c] giving M_a becomes:

$$M_a = \frac{\overbrace{M_{TS} + (\omega_c P_c - \omega_s P_s) + \frac{\gamma_b}{2\pi} [W_c - W_s]}^{M^*} - F \left(\frac{a_o^2}{8} P_e \right)}{1 + F}$$

Note:

- When the shell is integral with the TS (configurations a, b, c): $W_s=0$
- When the channel is integral with the TS (configurations a, e, f): $W_c=0$
- When the shell and the channel are gasketed with the TS, the highest bolting moment controls and W_c - W_s must be replaced by: $W_{max}=MAX(W_s, W_c)$

$$M^* = M_{TS} + (\omega_c P_c - \omega_s P_s) + \frac{\gamma_b}{2\pi} [W_c - W_s]$$

$$M_{TS} = P_s \frac{a_o^2}{4} [(\rho_s - 1)(\rho_s^2 + 1)] - P_c \frac{a_o^2}{4} [(\rho_c - 1)(\rho_c^2 + 1)]$$

$$F = [\lambda_s + \lambda_c + E \ln K] \frac{(1 - \nu^*)}{E^*}$$

$$M_a = \frac{M^* - F \left(\frac{a_o^2}{8} P_e \right)}{1 + F} \quad [V.2d]$$

The 1st term accounts for the moments due to pressures P_s and P_c and bolting loads W_s and W_c acting on the unperforated rim. The second term accounts for the moment due to pressures P_s and P_c acting on the perforated TS.

This formula applies as follows for the 6 configurations, depending on coefficients λ_s, λ_c , ω_s, ω_c , γ_b .

- For configuration a: λ_s, λ_c and ω_s, ω_c are calculated from the HE data, $\gamma_b = 0$
- For configuration b: λ_s is calculated, $\lambda_c=0$, ω_s is calculated, $\omega_c=0$, $\gamma_b = \frac{G_c - C}{D_o}$
- For configuration c: λ_s is calculated, $\lambda_c=0$, ω_s is calculated, $\omega_c=0$, $\gamma_b = \frac{G_c - G_l}{D_o}$

- For configuration d: $\lambda_s=0$, $\lambda_c=0$, $\omega_s=0$, $\omega_c=0$, $\gamma_b = \frac{G_c - G_s}{2}$
- For configuration e: $\lambda_s=0$, λ_c is calculated, $\omega_s=0$, ω_c is calculated, $\gamma_b = \frac{C - G_s}{D_o}$
- For configuration f: $\lambda_s=0$, λ_c is calculated, $\omega_s=0$, ω_c is calculated, $\gamma_b = \frac{G_1 - G_s}{D_o}$

6 STRESSES IN THE HEAT-EXCHANGER COMPONENTS

6.1 Stresses in the Tubesheet

(a) Bending stress

as explained in Clause IV, the maximum bending moment $M(r)$ appears:

either at periphery ($r=a_o$): $M(r) = M_a$, noted M_p in UHX-12 rules

$$M_p = \frac{M^* - F \left(\frac{a_o^2}{8} P_e \right)}{1 + F}$$

or at center ($r=0$): $M_o = M_p + (3 + \nu^*) \frac{a_o^2}{16} P_e$

The maximum moment $M = \text{MAX} [|M_p|, |M_o|]$

leads to the maximum bending stress in the TS:

$$\sigma = \frac{6M}{(h - h'_g)^2}$$

(b) Shear stress

The shear stress is maximum at periphery: $\tau = \frac{V_a}{\mu h}$

$$\tau = \frac{a_o}{2\mu h} P_e$$

Note: TEMA rule provides the same formula, but uses the equivalent diameter D_L corresponding to the perimeter of the outermost ligaments, instead of the equivalent diameter D_o of outer tube limit circle. The equivalent diameter D_L is calculated from the perimeter of the tube layout, $C_p = \pi D_L$, and the area

$A_p = \pi D_L^2 / 4$ enclosed by this perimeter. This leads to $D_L = 4A_p / C_p$

D_L is always lower than D_o and leads to a lower TS shear stress: $\tau = \frac{1}{4\mu} \frac{D_L}{h} P_e$

That alternative TEMA formula should be used when the shear stress controls the design, generally in high-pressure cases.

6.2 Stresses in the Shell and Channel

(a) Axial membrane stress in the shell is given by the classical formula:

$$\sigma_{s,m} = \frac{a_s^2}{(D_s + t_s) t_s} P_s$$

For the channel:
$$\sigma_{c,m} = \frac{a_c^2}{(D_c + t_c) t_c} P_c$$

(c) Bending stress

The bending moment M_s in the shell at its connection with the TS exists only when the shell is integral with the TS (configurations a, b, c). As explained in Annex J, the shell must have a minimum length $l_{s,min} = 1.8\sqrt{D_s t_s}$ adjacent to the TS.

[V.1a]: $M_s = k_s \left[\beta_s \delta_s P_s + \left(1 + \frac{t'_s}{2} \right) \theta_s \right]$ with $\theta_s = \theta_a$ given by [V.5]:

$$\theta_a = \frac{12(1-\nu^*)}{E^* h^3} \left[a_o M_a + \frac{a_o^3}{8} P_e \right]$$

$$M_s = k_s \left[\beta_s \delta_s P_s + \frac{12(1-\nu^*)}{E^*} \frac{a_o}{h^3} \left(1 + \frac{t'_s}{2} \right) \left(M_a + \frac{a_o^2}{8} P_e \right) \right]$$

The shell bending stress $\sigma_{s,b}$ writes: $\sigma_{s,b} = 6 \frac{M_s}{t_s^2}$

$$\sigma_{s,b} = \frac{6}{t_s^2} k_s \left[\beta_s \delta_s P_s + 6 \frac{1-\nu^*}{E^*} \frac{D_o}{h^3} \left(1 + \frac{h \beta_s}{2} \right) \left(M_p + \frac{D_o^2}{32} (P_s - P_t) \right) \right]$$

The channel bending stress is obtained in the same way , noting that $\theta_c = -\theta_a$:

$$\sigma_{c,b} = \frac{6}{t_c^2} k_c \left[\beta_c \delta_c P_c - 6 \frac{1-\nu^*}{E^*} \frac{D_o}{h^3} \left(1 + \frac{h \beta_c}{2} \right) \left(M_p + \frac{D_o^2}{32} (P_s - P_t) \right) \right]$$

6.3 Determination of Stresses using the Fixed TS Rules

The TS, shell and channel stresses determined here above can also been obtained from the fixed TS analysis of PART 3 as shown in Annex A-VI.3.

7 DETERMINATION OF THE ALLOWABLE STRESS LIMITS

The determination of the allowable stress limits developed in Clause IX of PART 3 for pressure loadings 1, 2 and 3 apply. However, the allowable stress limit for the TS has been upgraded to 2S for the following reasons.

Comparison of TEMA formula with classical plate formula (see section 10) shows that TEMA allows the bending stress in the equivalent solid plate to be about 2S, instead of 1.5 S recommended by ASME Section VIII Div. 2. Nevertheless, for about 40 years, TEMA formula did not lead to failures in U-tube tubesheets. It is likely that a value of allowable stress of 2S could be used without affecting the safety margin. This value was also recommended by GARDNER in 1969 [1] and has been used in most European codes (BS 5500, CODAP) for about 20 years. This is also justified by limit load analysis applied to circular plates, which leads to 1.9S if the tubesheet is simply supported and 2.1S if the tubesheet is clamped.

8 ADDITIONAL RULES

8.1 Effect of Plasticity at Tubesheet-Shell-Channel Joint (UHX-12.5)

In the same way as for fixed TS HE rules, an elastic-plastic analysis is proposed when integral shell and/or channel are overstressed. The concept is explained in Clause X.2.1 of PART 3.

The procedure for the shell applies when $1.5S_s < |\sigma_s| \leq S_{PS}$

The elastic-plastic method is simplified: the reduced modulus of elasticity E_s^* for the shell is based on the degree of overstress in the shell: $E_s^* = E_s \sqrt{1.5S_s / \sigma_s}$.

Quantities affected by the elastic-plastic procedure are those which are involved in the TS-shell-channel joint and which involve E_s , namely k_s which affects λ_s leading to new value for F .

δ_s is not affected because it is used only in $\omega_s = \rho_s \beta_s k_s \delta_s (1 + h \beta_s)$ in which E_s is cancelled out by the product $k_s \delta_s$. Same procedure applies to the channel by replacing subscript “s” by subscript “c”.

If $|\sigma| \leq 1.5S$, the design is acceptable. Otherwise, the HE geometry must be reconsidered.

For Example 4 given in IX, the results for the elastic calculation are as follows for the controlling load case 2:

Stiffening coefficient $F=0.96$, $\sigma = 38175 < S=40000$, $\sigma_c = 56955 > 1.5S_c=30000$, but lower than $S_{PS,c}=65000$

Similar results for the channel.

For the elastic-plastic calculation: the stiffening coefficient F decreases to 0.85, and

$\sigma = 39838 < S=40000$

In this example the elastic-plastic calculation leads to an increase of the TS stress, which happens in most cases.

9 HOW TO USE THE RULES

Similar to the fixed TS HE, the calculation procedure can be summarized as follows:

- Set the data listed in III.2a
- Calculate the shell design coefficients ($\rho_s, k_s, \lambda_s, \delta_s, \omega_s$), and the channel design coefficients ($\rho_c, k_c, \lambda_c, \delta_c, \omega_c$)
- Calculate coefficient F which represents the degree of restrain of the TS by the shell and channel
- Calculate moments M_{TS} and M^*
- Calculate the maximum stresses in TS, shell and channel and limit their values to the maximum allowable stress limits.

Because of the complexity of the procedure, it is likely that users will computerize the solution.

A Mathcad calculation sheet is provided for The U-Tube TS HE defined in UHX-20.1.4 (Example 4). The TS is gasketed with shell and integral with channel (configuration e). The data are shown in the sheet and the calculations follow strictly the steps 1 to 11 of UHX-12.5 calculation procedure. The elastic-plastic procedure is used. See Annex Z for UHX12-Example 4 (2010 edition).

The U-Tube HE can also be calculated using a floating TS HE software, as shown in Annex V.

10 COMPARISON WITH TEMA RULES

10.1 TEMA Formula

The original TEMA formula for the determination of the TS thickness in bending: $T = F \frac{G}{2} \sqrt{\frac{P}{S}}$ is

based on flat circular plates formula: $h_o = C \frac{G}{2} \sqrt{\frac{P}{\mu^* \Omega S}}$

Which writes, for $\mu^*=0.5$ and $\Omega=1.5$: $h_o = \frac{C}{\sqrt{0.75}} \frac{G}{2} \sqrt{\frac{P}{S}}$

where coefficient $\frac{C}{\sqrt{0.75}}$ is equal to:

- 1.285 for simple support, whereas TEMA uses $F=1.25$
- for clamping, which is the value used in TEMA ($F=1.0$)

The TEMA formula was modified later to introduce a ligament efficiency η : $T = F \frac{G}{3} \sqrt{\frac{P}{\eta S}}$,

The ligament efficiency η is based on the mean width of the ligament (see Clause IV.1 of PART 3):

For Triangular pitch: $\eta = 1 - \frac{0.907}{(p/d_t)^2}$ For Square pitch: $\eta = 1 - \frac{0.785}{(p/d_t)^2}$

TEMA mean ligament efficiency η (usually around 0.5) is greater than ASME ligament efficiency μ^* (usually around 0.3) which is based on the minimum width of the ligament. Coefficient 3 in new TEMA formula has been tailored so that old and new formulae give approximately the same results.

10.2 Numerical Comparisons

Numerical comparisons have been performed using the four U-tube tubesheet heat exchangers treated in UHX-20.1 rules of ASME Section VIII (2010 Edition). Table 3 shows the results obtained by ASME and TEMA for the tubesheet bending thickness.

In the 4 examples, TEMA considers the tubesheet as simply supported (TEMA coefficient $F = 1.25$).

However UHX-12 method shows that in example 1 the tubesheet is almost clamped (UHX-12 coefficient $F=9.4$), due to the high bending rigidities of the shell and channel as compared to the tubesheet bending rigidity.

Thicknesses obtained by UHX are slightly less than TEMA thicknesses, except for Example 3 where the ASME ligament efficiency is much smaller than TEMA ligament efficiency.

Table 3—Comparison of TEMA and ASME TS thicknesses for 4 U-tube HEs

EXAMPLE			LIGAMENT EFFICIENCIES			TEMA T (using η)	ASME h (using μ^*)
N ^o	<u>Config.</u>	Pitch	μ (ASME) (TEMA)	μ^* (ASME)	η (TEMA)		
1	a	square	0.25	0.35	0.56	0.61	0.52
2	d	<u>triang.</u>	0.17	0.28	0.37	1.27	1.28
3	d	<u>triang.</u>	0.20	0.24	0.42	3.43	4.15
4	e	square	0.25	0.39	0.56	3.91	3.46

REFERENCES—PART 5

- [1] Karl GARDNER “Tubesheet design: a basis for standardisation”. ASME publication of 1st ICPVT conference-Delft. Part 1 Design and Analysis-1969.
- [2] Urey MILLER “U-Bundle TS extended as flange and integral with shell or channel” – Unpublished report.
- [3] Francis Osweiller “ new common design rules for U-Tube HEs in ASME, CODAP and UPV Codes” – 2002 PVP Conference (Vancouver) Pressure Vessel and Piping C&S volume p.229-240.
- [4] Alan SOLER “Mechanical Design of Heat Exchangers” – Arcturus publishers -1984 – 1047 pages.

PART 6

Summary and Conclusions

1 SUMMARY AND CONCLUSIONS

(a) The purpose of this document is to justify and provide technical criteria for the rules of Part UHX of ASME Section VIII Division 1, 2010 Edition, devoted to the design of U-tube, Fixed and Floating head Tubesheet Heat Exchangers. The document is structured in 5 PARTS.

- PART 1: "Introduction" explains how the UHX rules are structured and provides the heat exchangers types, the loading cases, and the tubesheet configurations covered.
- PART 2: "Tubesheet Characteristics" provides the technical basis of UHX-11 which covers the ligament efficiencies and the effective elastic constants of the tubesheet.
- PART 3: "Fixed Tubesheets Heat Exchangers" provides the technical basis of UHX-13.
- PART 4: "Floating Tubesheet Heat Hexchangers" provides the technical basis of UHX-14.
- PART 5: "U-tube Tubesheet Heat Exchangers" provides the technical basis of UHX-12.

PARTS 3, 4 and 5 are independent and structured in the same way (Scope, Historical background, Notations, Tubesheet Configurations, Loading cases, Design assumptions, Analytical treatment).

Floating and U-tube tubesheet heat exchangers are treated as simplified cases of fixed tubesheet heat exchangers. Accordingly, this type of heat exchanger is treated first in PART 3 which forms the main part of this document.

(b) The analytical treatment is based on classical discontinuity analysis methods to determine the moments and forces that the tubesheet, tubes, shell and channel must resist. These components are treated using the elastic theory of thin shells of revolution.

The heat exchanger is assumed to be a symmetrical unit with identical tubesheets on both ends and the unperforated rim is considered as a rigid circular ring.

The tubes are assumed to be identical and uniformly distributed throughout the tubesheets. Additional assumptions are necessary to perform the analytical treatment and derive the "design by rules" method of UHX.

(c) Fixed Tubesheets heat exchangers are covered in PART 3. Main steps of the analytical treatment are as follows.

- (1) Tubesheet is disconnected from the shell and channel.
- (2) Perforated tubesheet is replaced by an equivalent solid circular plate with effective elastic constants E^* and ν^* which depend on the ligament efficiency μ^* of the tubesheet. This equivalent solid circular plate is treated by the theory of thin circular plates subjected to pressure and other applied loads to determine the maximum stresses.
- (3) Unperforated tubesheet rim is treated as a rigid ring whose cross section does not change under loading.
- (4) Tubes are replaced by an equivalent elastic foundation of modulus k_w .
- (5) Connection of the tubesheet to the shell and channel accounts for the edge displacements and rotations of the 3 components.
- (6) Shell and channel are treated by the elastic theory of thin shells of revolution subjected to edge loads to determine the maximum stresses.

The treatment provides, at any radius r of the perforated tubesheet, the following:

- the deflection $w(r)$

- the rotation $\theta(r)$
- the bending stress $\sigma(r)$
- the shear stress $\tau(r)$
- the axial stress in the tubes $\sigma_t(r)$

These quantities, which are not given in UHX rules, allow one to determine how the tubesheet deflects and the radii corresponding to the locations of the maximum bending stress in the tubesheet and the maximum axial stress in the tube bundle.

These stresses depend on two parameters:

- X_a , which represents the ratio of the tube- bundle axial stiffness to the tubesheet bending stiffness.
- Q_3 , which accounts for the moments due to tube side and shell side pressures and bolt loads acting on the tubesheet.

A parametric study performed with these two parameters permits the derivation of the formulas for the maximum stress in the tubesheet and in the tubes which are given in UHX-13.

The treatment also determines the loads and displacements acting on the shell and channel at their connection to the tubesheets and the loads applied on the unperforated rim. These are not given in UHX rules which provide only the shell and channel stresses.

Maximum stresses in tubesheet, tubes, shell and channel are determined and limited to the appropriate allowable stress-based classifications of Section VIII Division 2 Appendix 4, 2004 edition.

- (d) Floating Tubesheet and U-tube Tubesheet heat exchangers are treated as simplified cases of fixed tubesheets heat exchangers.
- (e) The Criteria Document provides the bases for all UHX formulas, especially the following:
- shell and channel coefficients β , k , λ and δ for the calculation of coefficients ω and ω^*
 - parameter X_a
 - coefficient F for the calculation of coefficients Φ , Q_1 , Q_2 and parameter Q_3 .
 - coefficient J when the shell has an expansion joint.
 - coefficient γ_b when the tubesheet has a gasketed flanged connection to the shell or channel.
 - equivalent pressures P'_s , P'_t , P_γ , P_w , P_{rim} for the calculation of the effective pressure P_e .
 - coefficients F_m and F_t for the calculation of maximum bending stress σ and maximum shear stress τ in the tubesheet
 - maximum axial stress $\sigma_t(r)$ and buckling stress limit S_{tb} in the tubes
 - axial membrane stress $\sigma_{s,m}$ and axial bending stress $\sigma_{s,b}$ in the shell and similar formulas for the channel.

The document provides the mechanical signification basis for the various coefficients and parameters noted above.

The results of the analytical treatment have shown that all UHX formulas were correct, except for four of them which are mentioned in PART 3. These formulas have been corrected in the 2010 edition of UHX.

(f) The basis of the additional UHX rules is provided in the document. They concern:

- The effect of different shell thickness adjacent to the tubesheet
- The effect of plasticity at tubesheet-shell-channel joint
- The effect of radial thermal expansion adjacent to the tubesheet
- The calculation procedure for simply supported tubesheets
- The derivation of the tubesheet flange extension
- The case where the heat exchanger has a thin-walled or thick-walled expansion joint

(g) Checking of the results obtained has been made by comparison with FEA and other Code rules.

FEA results obtained for the axial tube stresses $\sigma_t(r)$ throughout the tubesheet match UHX results with discrepancies of less than 5%. The comparisons were not as close for the tubesheet bending stresses and these discrepancies are still being evaluated by the Sub-Group on Heat Transfer Equipment. They might be due to the unperforated rim considered as a rigid ring in the analytical treatment.

The French pressure vessel Code, CODAP, is based on the same analytical approach as UHX, but additional simplifications have been made (unperforated rim is ignored). When incorporating these simplifications in the UHX method, all CODAP formulas have been retrieved. This demonstrates that both methods are correct and consistent. This consistency extends to TEMA: by implementing the additional TEMA simplifications, all TEMA formulas have been confirmed.

UHX formulas have also been used to simulate circular plates under pressure by ignoring the tubesheet holes, the unperforated rim, the tubes and the connection with the shell and channel. The classical formulas for circular plates subjected to pressure have been obtained.

Thus, applying the relevant simplifications, it has been analytically demonstrated that the UHX-13.3 method leads to CODAP, TEMA and circular plate formulas. This confirms the correctness of the ASME method, which has also been confirmed by FEA comparisons for the tubesheet deflection.

(h) It is thought that justification and documentation of the basis for UHX-rules is important for the SG-HTE members for future reference and developments or if additional confirmation or comparisons are required.

It will be a valuable reference for early career engineers that are using the UHX rules or becoming involved in Code developments of such rules in the future.

ANNEX A—VALUES OF EFFECTIVE ELASTIC CONSTANTS FROM VARIOUS AUTHORS

Table 4—Comparison of Effective Elastic Constants E^* and ν^* values by various theoretical methods for plane stress problem.

(a) Triangular pitch

μ	effective elastic constants	DIRECT METHOD		UNDIRECT METHOD
		Meijers [5]	Grigolyuk and Fil'shtinskii [11]	Slot — O'Donnell [7]
0,1 0,2 0,3 0,4 0,5 0,7 0,8	E_{Δ}^*/E	0,048 2 0,146 2 0,267 5 0,397 0,529 1 0,789 5 0,899	0,048 2 0,146 1 0,396 8 0,899 6	0,048 2 0,146 2 0,267 5 0,396 3 0,529 1 0,789 5 0,898 6
0,1 0,2 0,3 0,4 0,5 0,7 0,8	ν_{Δ}^* ($\nu = 0,3$)	0,684 4 0,488 6 0,384 1 0,337 0,319 4 0,307 0 0,303	0,683 4 0,488 7 0,337 2 0,303 6	0,684 3 0,488 9 0,384 1 0,337 4 0,319 4 0,307 0 0,303 3

(b) Square pitch

μ	effective elastic constants	DIRECT METHOD		UNDIRECT METHOD
		Meijers [5]	Grigolyuk and Fil'shtinskii [11]	Bailey — Hicks [6] Slot — O'Donnell [7]
0.1 0.2 0.3 0.4 0.5 0.7 0.8	E_p^*/E	0.1857 0.3000 0.4069 0.6168 0.8244	0.1856 0.2999 0.5116 0.9139	0.1857 0.3000 0.4069 0.5117 0.6168 0.8244 0.9139
0.1 0.2 0.3 0.4 0.5 0.7 0.8	ν_p^* ($\nu = 0.3$)	0.0956 0.1506 0.1975 0.2665 0.2974	0.0965 0.1509 0.2365 0.3000	0.0956 0.1506 0.1974 0.2365 0.2665 0.2975 0.3010

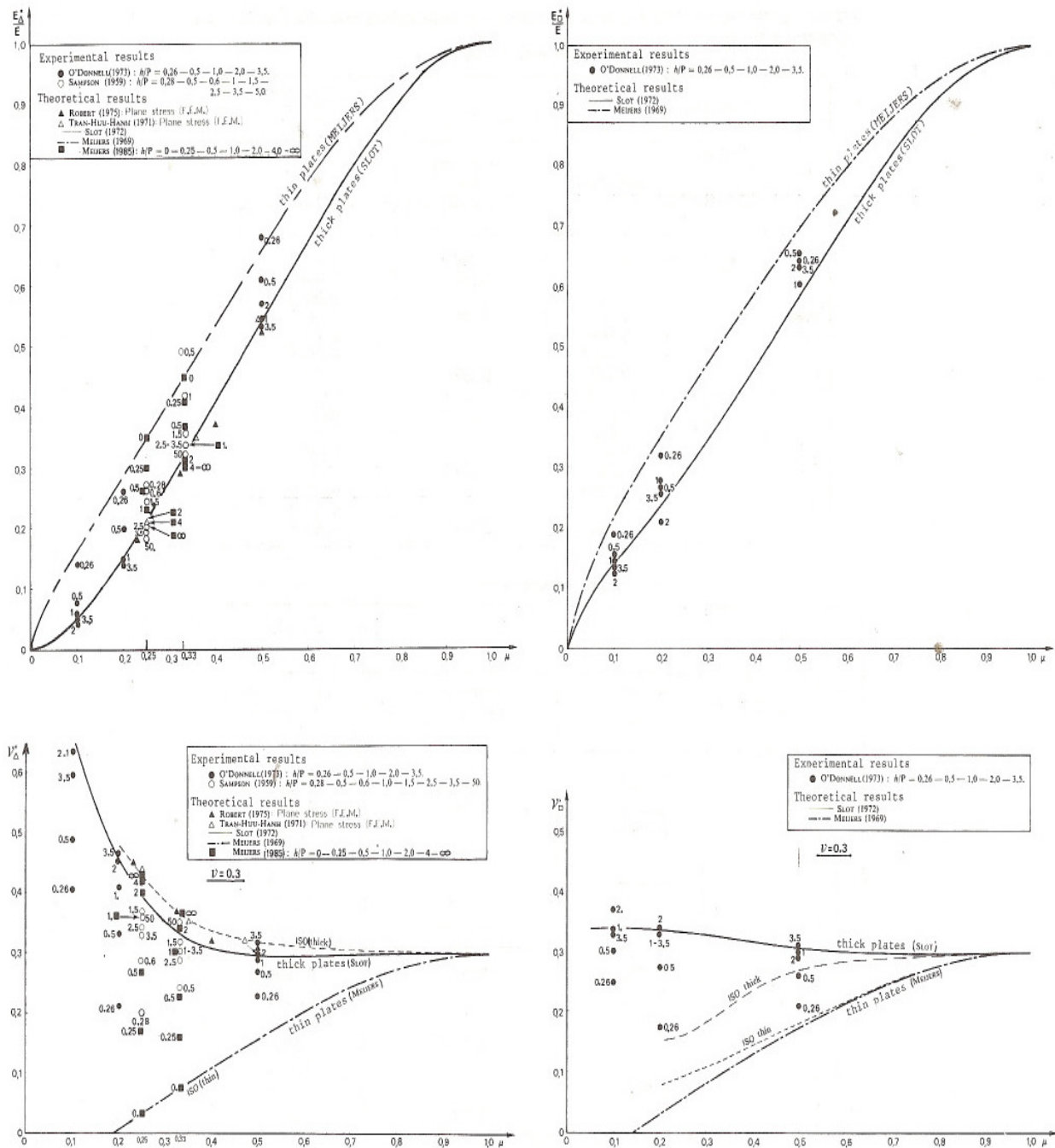


Figure 55—Synthesis of E^*/E and ν^* values from [1], provided by various authors for triangular and square pattern

ANNEX B—VALUES OF EFFECTIVE ELASTIC CONSTANTS FOR THE FULL RANGE OF M ($0.1 \leq M \leq 1.0$)

B.1 Introduction

This Annex provides the curves, numerical values and polynomials to calculate the effective elastic constants E^*/E and ν^* for the full range of the ligament efficiency μ^* ($0.1 \leq \mu^* \leq 1.0$).

B.2 Curves (From [13])

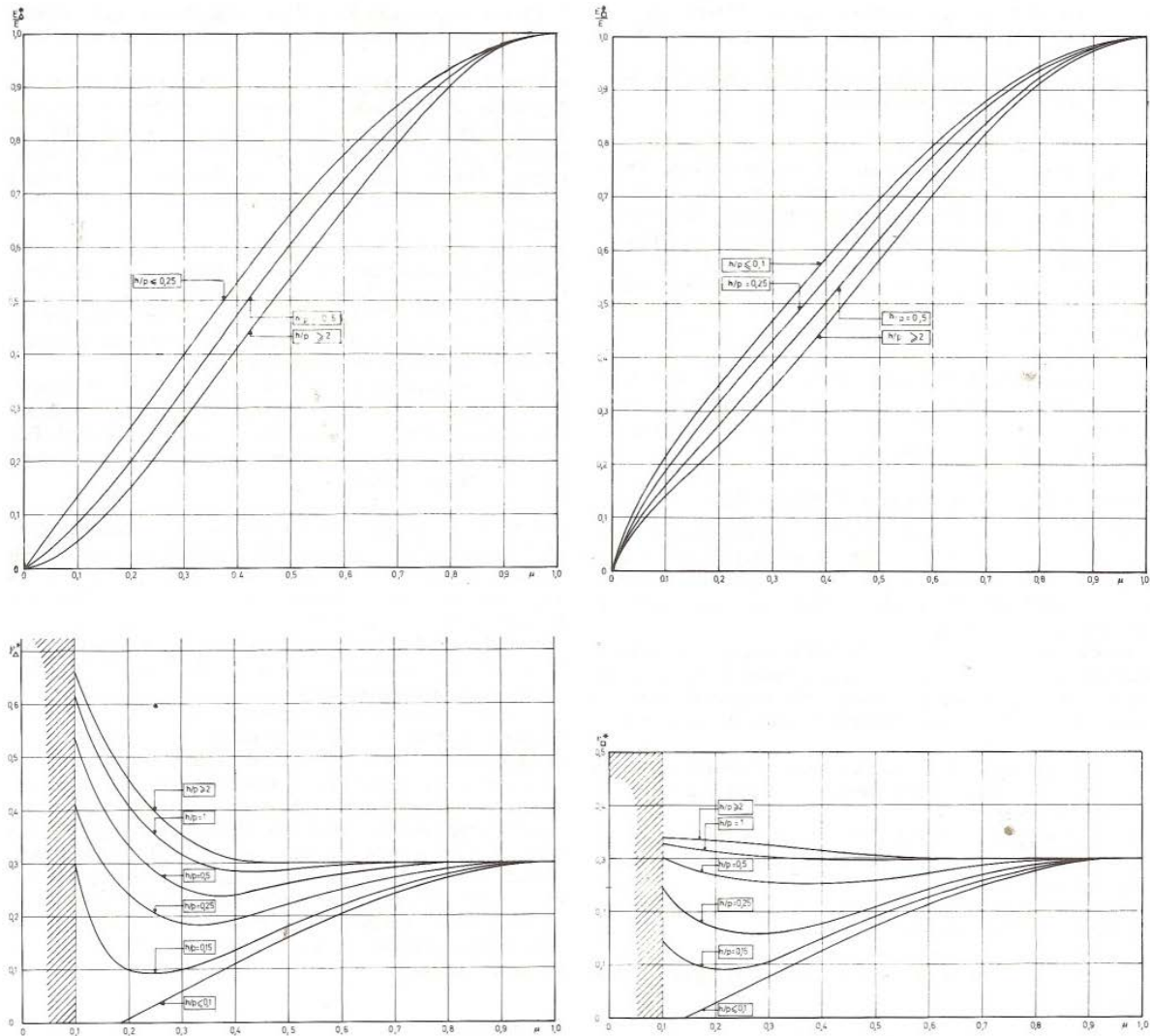


Figure 56—Curves of Effective Elastic Constants for the full range of μ^* ($0.1 \leq \mu^* \leq 1.0$)

B.3 Numerical Values (From [13])

h/p	μ^*	E*/E
0.1	0.0	0.0
0.1	0.02	0.05
0.1	0.05	0.096
0.1	0.1	0.16
0.1	0.15	0.223
0.1	0.2	0.285
0.1	0.22	0.310
0.1	0.25	0.348
0.1	0.28	0.3858
0.1	0.3	0.411
0.1	0.32	0.436
0.1	0.35	0.473
0.1	0.38	0.510
0.1	0.4	0.535
0.1	0.42	0.559
0.1	0.45	0.596
0.1	0.48	0.6318
0.1	0.5	0.656
0.1	0.52	0.678
0.1	0.55	0.712
0.1	0.58	0.7448
0.1	0.6	0.767
0.1	0.626	0.792
0.1	0.64	0.806
0.1	0.657	0.823
0.1	0.675	0.84
0.1	0.7	0.863
0.1	0.719	0.879
0.1	0.75	0.904
0.1	0.777	0.9222
0.1	0.8	0.937
0.1	0.828	0.9523
0.1	0.85	0.964
0.1	0.866	0.9708
0.1	0.882	0.9772
0.1	0.9	0.984
0.1	0.918	0.990
0.1	0.938	0.994
0.1	0.95	0.996
0.1	0.977	0.999
0.1	1.0	1.0

h/p	μ^*	E*/E
0.25	0.0	0.0
0.25	0.1	0.123
0.25	0.15	0.181
0.25	0.2	0.243
0.25	0.25	0.308
0.25	0.3	0.374
0.25	0.35	0.439
0.25	0.4	0.505
0.25	0.45	0.568
0.25	0.5	0.632
0.25	0.55	0.690
0.25	0.6	0.747
0.25	0.7	0.847
0.25	0.75	0.892
0.25	0.8	0.930
0.25	0.825	0.9459
0.25	0.85	0.96
0.25	0.9	0.982
0.25	0.95	0.995
0.25	1.0	1.0

h/p	μ^*	E*/E
0.5	0.0	0.0
0.5	0.02	0.0122
0.5	0.04	0.0287
0.5	0.05	0.0375
0.5	0.0804	0.0653
0.5	0.1	0.0849
0.5	0.12	0.106
0.5	0.14	0.127
0.5	0.15	0.139
0.5	0.184	0.18
0.5	0.2	0.201
0.5	0.219	0.226
0.5	0.25	0.2678
0.5	0.279	0.307
0.5	0.3	0.3361
0.5	0.318	0.361
0.5	0.35	0.4047
0.5	0.4	0.4753
0.5	0.435	0.521
0.5	0.45	0.541
0.5	0.465	0.561
0.5	0.5	0.6075
0.5	0.527	0.641
0.5	0.55	0.6688
0.5	0.6	0.7272
0.5	0.64	0.769
0.5	0.65	0.7794
0.5	0.664	0.794
0.5	0.7	0.8313
0.5	0.718	0.85
0.5	0.739	0.87
0.5	0.75	0.8812
0.5	0.779	0.906
0.5	0.796	0.919
0.5	0.8	0.922
0.5	0.817	0.935
0.5	0.838	0.949
0.5	0.85	0.9563
0.5	0.856	0.96
0.5	0.877	0.971
0.5	0.895	0.979
0.5	0.9	0.9806
0.5	0.917	0.986
0.5	0.938	0.992
0.5	0.95	0.9944
0.5	0.978	0.999
0.5	1.0	1.0

h/p	μ^*	E*/E
2.0	0.0	0.0
2.0	0.0296	0.008
2.0	0.05	0.0172
2.0	0.0596	0.0217
2.0	0.0807	0.0356
2.0	0.1	0.052
2.0	0.12	0.0688
2.0	0.14	0.088
2.0	0.15	0.0994
2.0	0.176	0.129
2.0	0.2	0.1553
2.0	0.22	0.1798
2.0	0.25	0.2165
2.0	0.28	0.255
2.0	0.3	0.2806
2.0	0.333	0.3246
2.0	0.35	0.348
2.0	0.38	0.3862
2.0	0.4	0.412
2.0	0.42	0.4385
2.0	0.45	0.4783
2.0	0.48	0.5181
2.0	0.5	0.5446
2.0	0.52	0.5701
2.0	0.55	0.6084
2.0	0.58	0.6466
2.0	0.6	0.6721
2.0	0.64	0.724
2.0	0.655	0.742
2.0	0.7	0.7996
2.0	0.716	0.82
2.0	0.738	0.845
2.0	0.76	0.869
2.0	0.779	0.888
2.0	0.791	0.9
2.0	0.8	0.908
2.0	0.813	0.92
2.0	0.834	0.938
2.0	0.85	0.95
2.0	0.876	0.966
2.0	0.9	0.978
2.0	0.918	0.986
2.0	0.938	0.992
2.0	0.95	0.996
2.0	0.978	0.999
2.0	1.0	1.0

Table 5—Values of Curves v^* as a function of μ^* for ratios $h/p=0.1, 0.15, 0.25, 0.5, 1.0$ and 2.0 for TRIANGULAR pattern

h/p	μ^*	v^*	h/p	μ^*	v^*	h/p	μ^*	v^*	h/p	μ^*	v^*	h/p	μ^*	v^*	h/p	μ^*	v^*
0.1	0.0	-0.096	0.15	0.1	0.3025	0.25	0.1	0.4135	0.5	0.1	0.5397	1.0	0.1	0.6181	2.0	0.05	0.8078
0.1	0.05	-0.0672	0.15	0.1198	0.2216	0.25	0.1	0.4135	0.5	0.1191	0.4828	1.0	0.1179	0.5675	2.0	0.1	0.6606
0.1	0.1	-0.038	0.15	0.1267	0.2025	0.25	0.1103	0.3817	0.5	0.1305	0.453	1.0	0.1362	0.5224	2.0	0.15	0.5445
0.1	0.15	-0.0158	0.15	0.1345	0.1818	0.25	0.1182	0.3628	0.5	0.139	0.4335	1.0	0.146	0.5031	2.0	0.18	0.49
0.1	0.184	0.0001	0.15	0.1401	0.1685	0.25	0.1292	0.34	0.5	0.1546	0.4029	1.0	0.1575	0.4783	2.0	0.2	0.4575
0.1	0.2	0.0069	0.15	0.155	0.141	0.25	0.1389	0.3224	0.5	0.1675	0.3813	1.0	0.1669	0.4627	2.0	0.22	0.4308
0.1	0.22	0.0166	0.15	0.1682	0.1229	0.25	0.1517	0.3027	0.5	0.1807	0.3616	1.0	0.1779	0.445	2.0	0.25	0.3956
0.1	0.25	0.0312	0.15	0.1748	0.1165	0.25	0.16	0.2904	0.5	0.1963	0.3405	1.0	0.1923	0.422	2.0	0.28	0.3693
0.1	0.28	0.0458	0.15	0.1829	0.1103	0.25	0.1719	0.2741	0.5	0.2	0.336	1.0	0.2	0.4109	2.0	0.3	0.354
0.1	0.3	0.056	0.15	0.1911	0.1052	0.25	0.1844	0.2614	0.5	0.218	0.3152	1.0	0.2174	0.389	2.0	0.31	0.347
0.1	0.32	0.0656	0.15	0.2	0.1011	0.25	0.194	0.2505	0.5	0.2276	0.3062	1.0	0.2259	0.3792	2.0	0.321	0.3405
0.1	0.35	0.0809	0.15	0.21	0.0983	0.25	0.2	0.2449	0.5	0.2384	0.2964	1.0	0.2379	0.3677	2.0	0.333	0.3344
0.1	0.38	0.0961	0.15	0.218	0.0965	0.25	0.2043	0.2409	0.5	0.25	0.2869	1.0	0.247	0.3585	2.0	0.35	0.3263
0.1	0.4	0.106	0.15	0.2291	0.0952	0.25	0.2174	0.2296	0.5	0.2601	0.2786	1.0	0.25	0.3559	2.0	0.36	0.3224
0.1	0.42	0.1162	0.15	0.2382	0.0947	0.25	0.2288	0.2211	0.5	0.2718	0.271	1.0	0.2593	0.3479	2.0	0.38	0.3166
0.1	0.45	0.1311	0.15	0.2498	0.0943	0.25	0.2394	0.2143	0.5	0.2822	0.265	1.0	0.2684	0.34	2.0	0.4	0.312
0.1	0.48	0.146	0.15	0.25	0.0943	0.25	0.25	0.2083	0.5	0.3	0.2562	1.0	0.2787	0.3318	2.0	0.42	0.3082
0.1	0.5	0.156	0.15	0.2604	0.0946	0.25	0.2593	0.2034	0.5	0.3198	0.2486	1.0	0.3	0.3185	2.0	0.45	0.3033
0.1	0.52	0.1653	0.15	0.2708	0.0958	0.25	0.2711	0.198	0.5	0.3323	0.2452	1.0	0.2916	0.3235	2.0	0.48	0.3005
0.1	0.55	0.1793	0.15	0.2787	0.0971	0.25	0.2797	0.1951	0.5	0.3456	0.2417	1.0	0.3051	0.3154	2.0	0.5	0.2994
0.1	0.6	0.203	0.15	0.29	0.0991	0.25	0.2899	0.1914	0.5	0.35	0.241	1.0	0.3191	0.3082	2.0	0.52	0.299
0.1	0.63	0.2165	0.15	0.3	0.1015	0.25	0.3	0.1887	0.5	0.3587	0.2397	1.0	0.3337	0.302	2.0	0.55	0.2986
0.1	0.65	0.2245	0.15	0.3102	0.1038	0.25	0.3104	0.1868	0.5	0.3692	0.2394	1.0	0.3482	0.2969	2.0	0.6	0.2988
0.1	0.67	0.2325	0.15	0.3196	0.1067	0.25	0.3199	0.186	0.5	0.3804	0.2395	1.0	0.35	0.2963	2.0	0.65	0.3
0.1	0.7	0.243	0.15	0.3303	0.1099	0.25	0.3295	0.1854	0.5	0.3914	0.24	1.0	0.3604	0.293	2.0	0.7	0.3
0.1	0.72	0.2501	0.15	0.3394	0.1131	0.25	0.3386	0.1851	0.5	0.4	0.241	1.0	0.3792	0.2893	2.0	0.75	0.3
0.1	0.75	0.2597	0.15	0.35	0.1172	0.25	0.35	0.1857	0.5	0.4129	0.243	1.0	0.4	0.2877	2.0	0.8	0.3

h/p	μ^*	v^*	h/p	μ^*	v^*	h/p	μ^*	v^*	h/p	μ^*	v^*	h/p	μ^*	v^*	h/p	μ^*	v^*
0.1	0.8	0.274	0.15	0.3591	0.1204	0.25	0.3604	0.1873	0.5	0.4198	0.2448	1.0	0.4188	0.2866	2.0	0.85	0.3
0.1	0.83	0.2815	0.15	0.3795	0.1289	0.25	0.3704	0.1891	0.5	0.4394	0.2499	1.0	0.4399	0.2865	2.0	0.9	0.3
0.1	0.85	0.2859	0.15	0.4	0.1376	0.25	0.3805	0.1911	0.5	0.45	0.2525	1.0	0.45	0.2867	2.0	0.95	0.3
0.1	0.875	0.2906	0.15	0.42	0.1464	0.25	0.3908	0.193	0.5	0.5	0.2635	1.0	0.5	0.2893	2.0	1.0	0.3
0.1	0.9	0.295	0.15	0.45	0.1604	0.25	0.4	0.1955	0.5	0.55	0.273	1.0	0.55	0.2935			
0.1	0.92	0.2974	0.15	0.5	0.1834	0.25	0.4099	0.198	0.5	0.6	0.2813	1.0	0.6	0.2976			
0.1	0.95	0.2996	0.15	0.55	0.2038	0.25	0.4197	0.201	0.5	0.65	0.289	1.0	0.65	0.2997			
0.1	1.0	0.3	0.15	0.6	0.2228	0.25	0.4395	0.2067	0.5	0.7	0.295	1.0	0.7	0.3			
			0.15	0.65	0.2413	0.25	0.45	0.2097	0.5	0.75	0.2997	1.0	0.75	0.3			
			0.15	0.7	0.2571	0.25	0.5	0.2246	0.5	0.8	0.3	1.0	0.8	0.3			
			0.15	0.75	0.2709	0.25	0.55	0.2395	0.5	0.85	0.3	1.0	0.85	0.3			
			0.15	0.8	0.2824	0.25	0.6	0.2526	0.5	0.9	0.3	1.0	0.9	0.3			
			0.15	0.8237	0.2872	0.25	0.65	0.2653	0.5	0.95	0.3	1.0	0.95	0.3			
			0.15	0.85	0.2923	0.25	0.7	0.276	0.5	1.0	0.3	1.0	1	0.3			
			0.15	0.8827	0.2971	0.25	0.75	0.2851									
			0.15	0.9	0.2993	0.25	0.8	0.2923									
			0.15	0.9018	0.2995	0.25	0.85	0.2977									
			0.15	0.95	0.3	0.25	0.9	0.3									
			0.15	1.0.0	0.3	0.25	0.95	0.3									
						0.25	1.0	0.3									

Table 6—Values of Curves v^* as a function of μ^* for ratios $h/p=0.1, 0.15, 0.25, 0.5, 1.0$ and 2.0 for SQUARE pattern

h/p	μ^*	v^*	h/p	μ^*	v^*	h/p	μ^*	v^*	h/p	μ^*	v^*	h/p	μ^*	v^*	h/p	μ^*	v^*
0.1	0.1	-0.022	0.15	0.1	0.1465	0.25	0.1	0.2471	0.5	0.1	0.3027	1.0	0.1	0.3283	2.0	0.05	0.339
0.1	0.142	0.0003	0.15	0.1082	0.1357	0.25	0.1183	0.2201	0.5	0.1122	0.2958	1.0	0.1376	0.3216	2.0	0.1	0.34
0.1	0.2	0.031	0.15	0.1179	0.1267	0.25	0.1278	0.2099	0.5	0.1193	0.2925	1.0	0.15	0.3196	2.0	0.125	0.3403
0.1	0.22	0.0408	0.15	0.1281	0.1184	0.25	0.1384	0.2002	0.5	0.1294	0.2881	1.0	0.1784	0.3151	2.0	0.15	0.34
0.1	0.25	0.0555	0.15	0.1379	0.1124	0.25	0.15	0.192	0.5	0.15	0.2809	1.0	0.1984	0.3125	2.0	0.175	0.3387
0.1	0.28	0.0702	0.15	0.15	0.1053	0.25	0.1609	0.185	0.5	0.1775	0.273	1.0	0.2	0.3123	2.0	0.2	0.337
0.1	0.3	0.08	0.15	0.1579	0.1016	0.25	0.1772	0.1767	0.5	0.2	0.2678	1.0	0.2383	0.3082	2.0	0.22	0.3354

h/p	μ^*	v^*		h/p	μ^*	v^*		h/p	μ^*	v^*		h/p	μ^*	v^*		h/p	μ^*	v^*		h/p	μ^*	v^*	
0.1	0.32	0.0896		0.15	0.1694	0.0978		0.25	0.1888	0.1722		0.5	0.2383	0.2611		1.0	0.25	0.307		2.0	0.25	0.333	
0.1	0.35	0.104		0.15	0.1788	0.0951		0.25	0.2	0.1685		0.5	0.25	0.2594		1.0	0.279	0.304		2.0	0.28	0.33	
0.1	0.38	0.1184		0.15	0.1901	0.093		0.25	0.2104	0.165		0.5	0.2784	0.256		1.0	0.3	0.3025		2.0	0.3	0.328	
0.1	0.4	0.128		0.15	0.1984	0.092		0.25	0.2307	0.1602		0.5	0.3	0.2542		1.0	0.3396	0.3005		2.0	0.32	0.3256	
0.1	0.42	0.137		0.15	0.2	0.0919		0.25	0.2391	0.1589		0.5	0.3187	0.253		1.0	0.35	0.3002		2.0	0.35	0.322	
0.1	0.45	0.1505		0.15	0.2099	0.0915		0.25	0.25	0.158		0.5	0.3388	0.2524		1.0	0.4	0.2992		2.0	0.38	0.3184	
0.1	0.48	0.164		0.15	0.2186	0.0914		0.25	0.2529	0.1577		0.5	0.35	0.2522		1.0	0.4386	0.2986		2.0	0.4	0.316	
0.1	0.5	0.173		0.15	0.2292	0.0917		0.25	0.263	0.1573		0.5	0.3581	0.2521		1.0	0.45	0.2984		2.0	0.42	0.314	
0.1	0.52	0.1814		0.15	0.239	0.0926		0.25	0.273	0.1572		0.5	0.3786	0.252		1.0	0.4792	0.298		2.0	0.45	0.3106	
0.1	0.55	0.194		0.15	0.25	0.0937		0.25	0.281	0.1572		0.5	0.3975	0.252		1.0	0.5	0.298		2.0	0.48	0.3076	
0.1	0.58	0.2066		0.15	0.2591	0.0953		0.25	0.2945	0.1573		0.5	0.4	0.252		1.0	0.5198	0.298		2.0	0.5	0.306	
0.1	0.6	0.215		0.15	0.2681	0.0971		0.25	0.3	0.1576		0.5	0.4181	0.2524		1.0	0.55	0.2984		2.0	0.52	0.3044	
0.1	0.65	0.233		0.15	0.2788	0.0999		0.25	0.3105	0.1582		0.5	0.4309	0.2533		1.0	0.6	0.299		2.0	0.55	0.3028	
0.1	0.7	0.25		0.15	0.2917	0.1036		0.25	0.3222	0.1592		0.5	0.45	0.2544		1.0	0.65	0.2995		2.0	0.58	0.3018	
0.1	0.75	0.2644		0.15	0.3	0.1065		0.25	0.3316	0.1604		0.5	0.4802	0.2577		1.0	0.7	0.3		2.0	0.6	0.301	
0.1	0.8	0.277		0.15	0.3181	0.1144		0.25	0.3399	0.1622		0.5	0.5028	0.261		1.0	0.75	0.3		2.0	0.65	0.3002	
0.1	0.85	0.2873		0.15	0.3286	0.1188		0.25	0.35	0.1644		0.5	0.5232	0.2643		1.0	0.8	0.3		2.0	0.7	0.3	
0.1	0.9	0.2956		0.15	0.3395	0.1235		0.25	0.3697	0.1687		0.5	0.5	0.2606		1.0	0.85	0.3		2.0	0.75	0.3	
0.1	0.95	0.2995		0.15	0.35	0.1283		0.25	0.4	0.1762		0.5	0.55	0.268		1.0	0.9	0.3		2.0	0.8	0.3	
0.1	0.98	0.2999		0.15	0.4	0.1498		0.25	0.4121	0.1799		0.5	0.6	0.275		1.0	0.95	0.3		2.0	0.85	0.3	
0.1	1.0	0.3		0.15	0.45	0.1698		0.25	0.4255	0.1843		0.5	0.64	0.28		1.0	1.0	0.3		2.0	0.9	0.3	
				0.15	0.5	0.1903		0.25	0.4386	0.1882		0.5	0.65	0.2812						2.0	0.95	0.3	
				0.15	0.55	0.2092		0.25	0.45	0.1924		0.5	0.7	0.287						2.0	1.0	0.3	
				0.15	0.6	0.2267		0.25	0.4798	0.203		0.5	0.72	0.289									
				0.15	0.65	0.2427		0.25	0.5	0.21		0.5	0.74	0.291									
				0.15	0.7	0.2578		0.25	0.55	0.2276		0.5	0.75	0.2919									
				0.15	0.7196	0.263		0.25	0.6	0.2441		0.5	0.78	0.2948									
				0.15	0.741	0.2682		0.25	0.62	0.25		0.5	0.8	0.296									
				0.15	0.75	0.2702		0.25	0.6461	0.257		0.5	0.82	0.2972									
				0.15	0.7799	0.2773		0.25	0.65	0.2579		0.5	0.84	0.2982									
				0.15	0.8	0.2816		0.25	0.6786	0.2644		0.5	0.85	0.2986									
				0.15	0.821	0.2856		0.25	0.7	0.2692		0.5	0.88	0.2995									
				0.15	0.8316	0.2875		0.25	0.7194	0.2729		0.5	0.9	0.3									
				0.15	0.85	0.2904		0.25	0.7404	0.2771		0.5	0.95	0.3									
				0.15	0.8787	0.2944		0.25	0.75	0.2788		0.5	1.0	0.3									

h/p	μ^*	v^*		h/p	μ^*	v^*		h/p	μ^*	v^*		h/p	μ^*	v^*		h/p	μ^*	v^*		h/p	μ^*	v^*
				0.15	0.9	0.2965		0.25	0.7794	0.2841												
				0.15	0.92	0.2981		0.25	0.8	0.2872												
				0.15	0.94	0.2994		0.25	0.8204	0.29												
				0.15	0.95	0.2997		0.25	0.84	0.2922												
				0.15	1.0.0	0.3		0.25	0.85	0.2932												
								0.25	0.8785	0.2958												
								0.25	0.9	0.2971												
								0.25	0.9206	0.2983												
								0.25	0.94	0.2995												
								0.25	0.95	0.2996												
								0.25	1.0	0.3												

B.4 Polynomials

Table 1: E*/E Coefficients for Triangular Tube Pattern

μ^*	α_0	α_1	α_2	α_3	α_4
0.10	0.002	1.239	0.573	-0.878	0.063
0.50	-0.003	0.728	1.895	-2.051	0.434
2.00	-0.009	0.484	1.957	-1.442	0.016

Table 2: E*/E Coefficients for Square Tube Pattern

μ^*	α_0	α_1	α_2	α_3	α_4
0.10	0.017	2.112	-2.759	3.368	-1.747
0.25	0.013	1.840	-2.111	2.893	-1.642
0.50	0.011	1.567	-1.648	2.747	-1.682
2.00	0.012	1.278	-1.135	2.587	-1.749

Table 3: ν^* Coefficients for Triangular Tube Pattern

μ^*	β_0	β_1	β_2	β_3	β_4	β_5	β_6	β_7
0.10	-0.089	0.450	0.267	-0.325	-0.002	0	0	0
0.15	1.232	-16.249	93.259	-284.35	507.628	-528.539	296.515	-69.197
0.25	0.771	-4.78	13.226	-11.952	-7.503	23.725	-17.848	4.66
0.50	0.989	-6.399	23.448	-53.158	87.214	-96.808	61.016	-16.003
1.00	1.01	-5.142	13.806	-17.444	10.436	-2.366	0	0
2.00	1.018	-4.52	10.821	-12.221	6.421	-1.21	0	0

Table 4: ν^* Coefficients for Square Tube Pattern

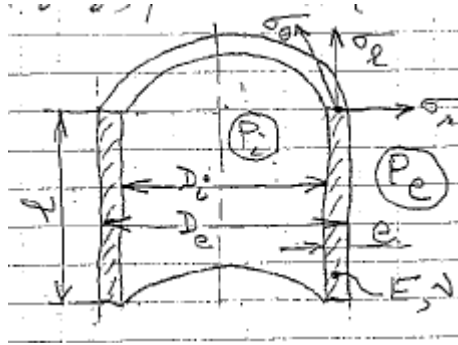
μ^*	β_0	β_1	β_2	β_3	β_4	β_5	β_6	β_7
0.10	-0.067	0.451	0.191	-0.273	0	0	0	0
0.15	0.337	-2.905	11.728	-19.521	15.421	-4.761	0	0
0.25	0.434	-2.632	8.552	-11.617	7.451	-1.889	0	0
0.50	0.400	-1.602	8.636	-29.539	62.382	-74.962	46.713	-11.728
1.00	0.348	-0.201	-0.180	2.124	-4.358	4.145	-1.954	0.374
2.00	0.354	-0.286	2.27	-10.35	22.733	-25.581	14.349	-3.188

Polynomials of Effective Elastic Constants for the full range of μ^* ($0.1 \leq \mu^* \leq 1.0$) initially used for Appendix AA of ASME Section VIII-Div. 1

ANNEX C—POISSON'S RATIO IN TUBES AND SHELL

(a) **Deformation of a shell of length l ; diameters D_i, D_e ; pressures P_i, P_e :**

$$\varepsilon = \frac{1}{E} [\sigma_l - \nu (\sigma_\theta + \sigma_r)] = \varepsilon_l + \varepsilon_v$$



Deformation due to Poisson's ratio: $\varepsilon_v = \frac{\delta_l(\nu)}{l} = -\frac{\nu}{E} (\sigma_\theta + \sigma_r)$

$$\sigma_\theta + \sigma_r = 2 \frac{P_i D_i^2 - P_e D_e^2}{D_e^2 - D_i^2} = \frac{\pi}{2} \frac{P_i D_i^2 - P_e D_e^2}{s} \quad s = \frac{\pi}{4} (D_e^2 - D_i^2)$$

$$\delta_l(\nu) = -\frac{\pi}{2} \frac{l}{E s} [P_i D_i^2 - P_e D_e^2] \nu = -\frac{\pi}{2 k} [P_i D_i^2 - P_e D_e^2] \nu \quad k = \frac{E s}{l}$$

(b) **Application to the tubes:** $D_e = d_t$ $D_i = d_t - 2 t_t$ $P_i = P_t$ $P_e = P_s$

$$\delta_t(\nu_t) = -\frac{\pi}{2 k_t} [P_t (d_t - 2 t_t)^2 - P_s d_t^2] \nu_t$$

$$\begin{cases} x_t = 1 - N_t \frac{(d_t - 2 t_t)^2}{D_o^2} \\ x_s = 1 - N_t \frac{d_t^2}{D_o^2} \end{cases} \quad \begin{cases} 1 - x_t = N_t \frac{(d_t - 2 t_t)^2}{D_o^2} \\ 1 - x_s = N_t \frac{d_t^2}{D_o^2} \end{cases} \quad \begin{cases} (d_t - 2 t_t)^2 = (1 - x_t) \frac{D_o^2}{N_t} \\ d_t^2 = (1 - x_s) \frac{D_o^2}{N_t} \end{cases}$$

$$\delta_t(\nu_t) = -\frac{\pi D_o^2}{2 k_t N_t} [(1 - x_t) P_t - (1 - x_s) P_s] \nu_t \quad k_w = \frac{N_t k_t}{\pi a_o^2} = 4 \frac{N_t k_t}{\pi D_o^2}$$

$$\boxed{\delta_t(\nu_t) = -\frac{2}{k_w} [(1 - x_t) P_t - (1 - x_s) P_s] \nu_t}$$

(c) **Application to the shell:** $D_i = D_s$ $P_i = P_s$ $P_e = 0$ $k'_s = \frac{\pi E_s t_s (D_s + t_s)}{l}$

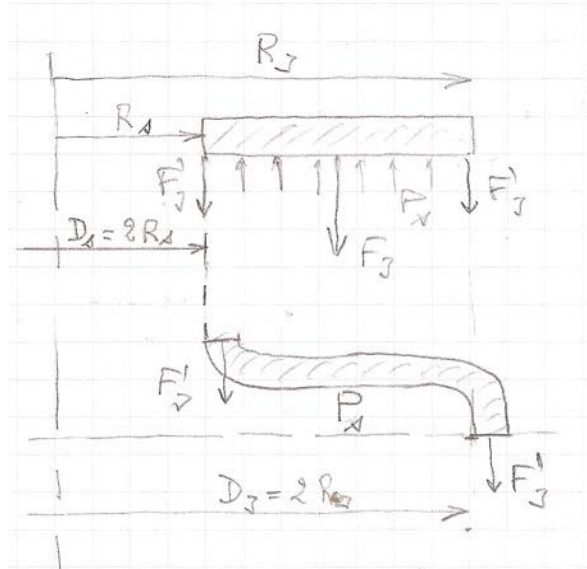
$$\delta_s(v_s) = -\frac{\pi}{2k_s'} [P_s D_s^2] v_s = \frac{-l D_s^2 P_s}{2 E_s t_s (D_s + t_s)} v_s$$

$$k_w \delta_s(v_s) = -\frac{\pi}{2k_s'} \frac{N k_t}{\pi (D_o^2 / 4)} P_s D_s^2 v_s = -\frac{2 v_s}{k_{s,t}} \left(\frac{D_s}{D_o} \right)^2 P_s$$

ANNEX D—SHELL PRESSURE ACTING ON EXPANSION JOINT SIDEWALLS

Axial force acting on the sidewall of one half of the expansion joint due to internal shell pressure P_s :

$$F_J = P_s \pi \left(R_J^2 - R_s^2 \right) \quad R_J = \frac{D_J}{2} \quad R_s = \frac{D_s}{2}$$



Shell pressure acting on the sidewall of the expansion joint creates an axial force. It is assumed that half of this force is resisted by the expansion joint, and the other half of it is carried out by the shell. Accordingly, the axial force acting on the shell is:

$$F_J' = \frac{F_J}{2} = \frac{\pi}{2} (R_J^2 - R_s^2) P_s = \frac{\pi}{8} (D_J^2 - D_s^2) P_s$$

Axial displacement of the half-joint of rigidity k_J , due to F_J' :

$$\delta_J(P_s) = \frac{F_J'}{k_J} = \frac{\pi}{8} \frac{D_J^2 - D_s^2}{k_J} P_s$$

Expansion joint rigidity: $K_J = \frac{k_J}{2}$

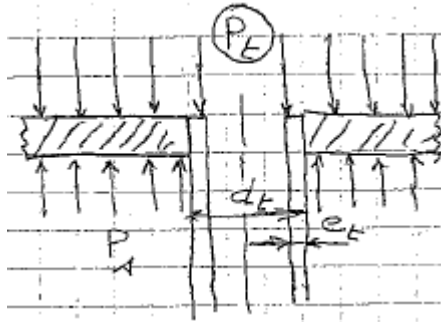
$$\delta_J(P_s) = \frac{\pi}{16} \frac{D_J^2 - D_s^2}{K_J} P_s$$

Note: this formula is still valid if the shell is extended as a flange (D_s should not be replaced by G_s)

ANNEX E—DIFFERENTIAL PRESSURE ACTING ON THE EQUIVALENT SOLID PLATE

(a) Force due to pressure P_s acting on the TS of radius a_o :

$$P_s \left[\pi a_o^2 - N_t \pi \left(\frac{d_t}{2} \right)^2 \right]$$

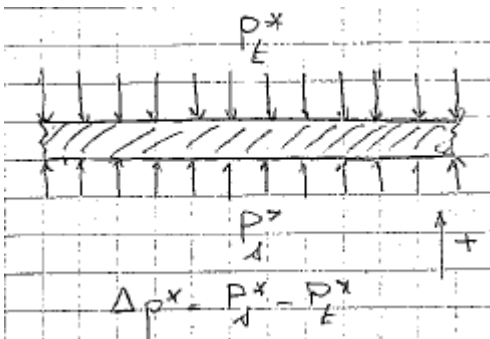


This force leads to a uniform pressure P_s^* acting on the equivalent solid plate given by:

$$P_s^* \pi a_o^2 = P_s \left[\pi a_o^2 - N_t \pi \left(\frac{d_t}{2} \right)^2 \right]$$

$$P_s^* = P_s \left[1 - N_t \left(\frac{d_t}{2 a_o} \right)^2 \right] = P_s x_s \quad D_o = 2 a_o$$

TS coefficient relating to the tubes on shell side: $x_s = 1 - N_t \left(\frac{d_t}{D_o} \right)^2$



(b) Force due to pressure P_t acting on the TS of radius a_o :

$$P_t \left[\pi a_o^2 - N_t \pi \left(\frac{d_t - 2 t_t}{2} \right)^2 \right]$$

This force leads to a uniform pressure P_t^* acting on the equivalent solid plate given by:

$$P_t^* \pi a_o^2 = P_t \left[\pi a_o^2 - N_t \pi \left(\frac{d_t - 2 t_t}{2} \right)^2 \right]$$

$$P_t^* = P_t \left[1 - N_t \left(\frac{d_t - 2 t_t}{2 a_o} \right)^2 \right] = P_t x_t$$

TS coefficient relating to the tubes on tube side: $x_t = 1 - N_t \left(\frac{d_t - 2 t_t}{D_o} \right)^2$

(c) **Differential pressure acting on the equivalent solid plate:**

$$\Delta p^* = x_s P_s - x_t P_t$$

ANNEX F—SOLUTION OF DIFFERENTIAL EQUATION $w(x)$

The deflection of the equivalent solid plate is governed by a differential equation of 4th order:

$$\frac{d^4 w}{dr^4} + \frac{2}{r} \frac{d^3 w}{dr^3} - \frac{1}{r^2} \frac{d^2 w}{dr^2} + \frac{1}{r^3} \frac{dw}{dr} = \frac{q(r)}{D^*} = \frac{Q - k_w w(r)}{D^*}$$

$$\frac{d^4 w}{dr^4} + \frac{2}{r} \frac{d^3 w}{dr^3} - \frac{1}{r^2} \frac{d^2 w}{dr^2} + \frac{1}{r^3} \frac{dw}{dr} + \frac{k_w}{D^*} w(r) = \frac{Q}{D^*}$$

A change of variable is necessary to solve the differential equation: $x = kr$

$$\frac{k_w}{D^*} = k^4 \quad \boxed{k = \sqrt[4]{\frac{k_w}{D^*}}} \quad 0 \leq r \leq a_0 \Rightarrow \boxed{0 \leq x \leq X_a} \quad \boxed{X_a = k a_0}$$

$$\frac{dw}{dr} = \frac{dw}{dx} \frac{dx}{dr} = k \frac{dw}{dx} \quad \frac{d^2 w}{dr^2} = k^2 \frac{d^2 w}{dx^2} \quad \frac{d^3 w}{dr^3} = k^3 \frac{d^3 w}{dx^3} \quad \frac{d^4 w}{dr^4} = k^4 \frac{d^4 w}{dx^4}$$

$$\frac{d^4 w}{dx^4} + \frac{2}{x} \frac{d^3 w}{dx^3} - \frac{1}{x^2} \frac{d^2 w}{dx^2} + \frac{1}{x^3} \frac{dw}{dx} + w(x) = \frac{Q}{k^4 D^*} = \frac{Q}{k_w}$$

- The homogeneous solution is:

$$w(x) = A \operatorname{ber}(x) + B \operatorname{bei}(x) + C \operatorname{ker}(x) + D \operatorname{kei}(x)$$

where:

$\operatorname{ber}(x)$, $\operatorname{bei}(x)$, $\operatorname{ker}(x)$, $\operatorname{kei}(x)$ are Kelvin functions of order 0, which will be noted $\operatorname{ber}x$, beix , $\operatorname{ker}x$, keix

A, B, C, D are integration constants

$$\operatorname{ber}x = \sum_{n=0}^{\infty} (-1)^n \frac{(x/2)^{4n}}{[(2n)!]^2} = 1 - \frac{(x/2)^4}{(2!)^2} + \dots$$

$$\operatorname{beix} = \sum_{n=1}^{\infty} (-1)^{2n-1} \frac{(x/2)^{4n-2}}{[(2n-1)!]^2} = \frac{(x/2)^2}{(1!)^2} - \frac{(x/2)^6}{(3!)^2} + \dots$$

$$\operatorname{ker}x = -\ln(x/2)\operatorname{ber}x + \frac{\pi}{4}\operatorname{beix} + [1] - \left[\frac{(x/2)^4}{(2!)^2} \left(1 + \frac{1}{2}\right) \right] + \left[\frac{(x/2)^8}{(4!)^2} \left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right) \right] - \dots$$

$$\operatorname{keix} = -\ln(x/2)\operatorname{beix} - \frac{\pi}{4}\operatorname{ber}x + [(x/2)^2] - \left[\frac{(x/2)^6}{(3!)^2} \left(1 + \frac{1}{2} + \frac{1}{3}\right) \right] + \left[\frac{(x/2)^{10}}{(5!)^2} \left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}\right) \right] - \dots$$

$$\text{For } x=0: \begin{cases} ber(0) = 1 & bei(0) = 0 & ker(0) \rightarrow \infty & kei(0) = -\frac{\pi}{4} \\ ber'(0) = 1 & bei'(0) = 0 & ker'(0) \rightarrow \infty & kei'(0) = 0 \\ ber''(0) = 0 & bei''(0) = \frac{1}{2} & ker''(0) \rightarrow \infty & kei''(0) \rightarrow \infty \end{cases}$$

The particular solution with 2nd member is: $w(x) = \frac{Q}{k_w}$

General solution: $w(x) = A berx + B beix + C kerx + D keix + \frac{Q}{k_w}$

Determination of constants C and D. Bending moment writes:

$$M_r(r) = -D^* k^2 \left(\frac{d^2 w}{dx^2} + \frac{v^*}{x} \frac{dw}{dx} \right) = -D^* k^2 \left[\begin{aligned} &(Aber''x + Bbei''x + Cker''x + Dkei''x) \\ &+ \frac{v^*}{x} (Aber'x + Bbei'x + Cker'x + Dkei'x) \end{aligned} \right]$$

The bending moment $M_r(0)$ at center of TS must remain finite: $\rightarrow C=0 \quad D=0$

Finally, the solution writes: $w(x) = A berx + B beix + \frac{Q}{k_w}$

ANNEX G—COEFFICIENTS Z_d, Z_v, Z_w, Z_m ; Q_m, Q_v ; Q_α, Q_β ; F_m, F_t

(a) Purpose

This Annex provides the equations for coefficients $Z_d(x), Z_v(x), Z_w(x), Z_m(x)$; $Q_m(x), Q_v(x), Q_\alpha(x), Q_\beta(x)$; $F_m(x), F_t(x)$ and their values for:

- $x=0$ or close to 0 for a given value of X_a .
- $x=X_a$
- $x=X_a$ when X_a is close to 0

Notations are taken from UHX-13:

- ber, bei are Kelvin functions
- $f(x)$ represents the value of function f for x .
- $f(X_a)$ is always noted “ f ”: $f = f(X_a)$. Therefore:

$$ber = ber(X_a) \quad ; \quad ber' = ber'(X_a)$$

$$bei = bei(X_a) \quad ; \quad bei' = bei'(X_a)$$

$$Z_a = Z_a(X_a)$$

$$Z_m = Z_m(X_a) \quad ; \quad Z_v = Z_v(X_a)$$

$$Z_d = Z_d(X_a) \quad ; \quad Z_w = Z_w(X_a)$$

$$\psi_1 = \psi_1(X_a) \quad ; \quad \psi_2 = \psi_2(X_a)$$

(b) Kelvin Functions

$$ber(x) = \sum_{n=0}^{n=m-1} (-1)^n \frac{(x/2)^{4n}}{[(2n)!]^2} = 1 - \frac{(x/2)^4}{(2!)^2} + \frac{(x/2)^8}{(4!)^2} - \frac{(x/2)^{12}}{(6!)^2} + \dots \approx 1 - \frac{x^4}{2^6} \text{ for } x \text{ close to } 0$$

$$bei(x) = \sum_{n=1}^{n=m} (-1)^{n-1} \frac{(x/2)^{4n-2}}{[(2n-1)!]^2} = \frac{(x/2)^2}{(1!)^2} - \frac{(x/2)^6}{(3!)^2} + \frac{(x/2)^{10}}{(5!)^2} - \dots \approx \frac{x^2}{2^2} \text{ for } x \text{ close to } 0$$

$$ber'(x) = \sum_{n=1}^{n=m} (-1)^n \frac{(2n)(x/2)^{4n-1}}{[(2n)!]^2} = -\frac{2(x/2)^3}{(2!)^2} + \frac{4(x/2)^7}{(4!)^2} - \frac{6(x/2)^{11}}{(6!)^2} + \dots \approx -\frac{x^3}{2^4}$$

for x close to 0

$$bei'(x) = \sum_{n=1}^{n=m} (-1)^{n-1} \frac{(2n-1)(x/2)^{4n-3}}{[(2n-1)!]^2} = \frac{(x/2)^1}{(1!)^2} - \frac{3(x/2)^5}{(3!)^2} + \frac{5(x/2)^9}{(5!)^2} - \dots \approx \frac{x}{2}$$

for x close to 0

$$ber''(x) = \sum_{n=1}^{n=m} (-1)^n \frac{(n)(4n-1)(x/2)^{4n-2}}{[(2n)!]^2} = -\frac{3(x/2)^2}{(2!)^2} + \frac{14(x/2)^6}{(4!)^2} - \frac{33(x/2)^{10}}{(6!)^2} + \dots \approx -\frac{3x^2}{2^4}$$

for x close to 0

$$bei''(x) = \sum_{n=1}^{n=m} (-1)^{n-1} \frac{(2n-1)(4n-3)(x/2)^{4n-4}}{2[(2n-1)!]^2} = \frac{1}{2} - \frac{15(x/2)^4}{2(3!)^2} + \frac{45(x/2)^8}{2(5!)^2} - \dots \approx \frac{1}{2}$$

for x close to 0

(c) Coefficients $\Psi_1(x)$, $\Psi_2(x)$, Z_a

$$\psi_1(x) = bei(x) + \frac{1-\nu^*}{x} ber'(x) \approx \frac{3+\nu^*}{2^4} x^2 \quad \text{for } x \text{ close to } 0$$

$$\Psi_1(X_a) \approx \frac{3+\nu^*}{2^4} X_a^2 \quad \text{for } X_a \text{ close to } 0$$

$$\psi_2(x) = ber(x) - \frac{1-\nu^*}{x} bei'(x) \approx \frac{1+\nu^*}{2} \quad \text{for } x \text{ close to } 0$$

$$\Psi_2(X_a) \approx \frac{1+\nu^*}{2} \quad \text{for } X_a \text{ close to } 0$$

$$Z_a = bei' \cdot \Psi_2 - ber' \cdot \Psi_1 = (ber \cdot bei' - ber' \cdot bei) - \frac{1-\nu^*}{X_a} (ber'^2 + bei'^2)$$

$$Z_a \approx \frac{1+\nu^*}{4} X_a \quad \text{for } X_a \text{ close to } 0$$

(d) Coefficients $Z_d(x)$, $Z_v(x)$, $Z_w(x)$, $Z_m(x)$

$$Z_d(x) = \frac{\Xi_2 \cdot (x) \cdot 1 \cdot (x)}{ber + X_a^3 Z_a} = \frac{(ber \cdot ber(x) + bei \cdot bei(x)) + \frac{1-\nu^*}{X_a} (ber' \cdot bei(x) - bei' \cdot ber(x))}{X_a^3 Z_a}$$

$$\approx \frac{4\Psi_2 + \Psi_1 x^2}{4X_a^2 Z_a} \approx \frac{\Psi_2}{X_a^3 Z_a} \quad \text{for } x \text{ close to } 0 \quad Z_d(0) = \frac{\Psi_2}{X_a^3 Z_a}$$

$$Z_d = \frac{\Xi_2 \cdot 1 \cdot (x)}{ber + X_a^3 Z_a} = \frac{(ber^2 + bei^2) + \frac{1-\nu^*}{X_a} (ber' \cdot bei - ber \cdot bei')}{X_a^3 Z_a} \approx \frac{2}{X_a^4} \quad \text{for } X_a \text{ close to } 0$$

$$Z_v(x) = \frac{\Xi_2 \cdot (x) \cdot 1 \cdot (x)}{ber' + X_a^2 Z_a} = \frac{(ber \cdot ber'(x) + bei \cdot bei'(x)) + \frac{1-\nu^*}{X_a} (ber' \cdot bei'(x) - bei' \cdot ber'(x))}{X_a^2 Z_a}$$

$$\approx \frac{8\Psi_1 x - \Psi_2 x^3}{16X_a^2 Z_a} \approx \frac{\Psi_1}{2X_a^2 Z_a} x \quad \text{for } x \text{ close to } 0 \quad Z_v(0) = 0$$

$$Z_v = \frac{\Xi_2 \cdot 1}{\text{ber}' + X_a^2 Z_a} = \frac{\text{ber} \cdot \text{ber}' + \text{bei} \cdot \text{bei}'}{X_a^2 Z_a} \approx \frac{1}{4(1+\nu^*)} \quad \text{for } X_a \text{ close to } 0$$

$$Z_d(x) = \frac{\text{ber}' \cdot \text{ber}(x) + \text{bei}' \cdot \text{bei}(x)}{X_a^2 Z_a}$$

$$\approx \frac{4\text{ber}' + x^2 \text{bei}'}{4X_a^2 Z_a} \approx \frac{\text{ber}'}{X_a^2 Z_a} \quad \text{for } x \text{ close to } 0 \quad Z_w(0) = \frac{\text{ber}'}{X_a^2 Z_a}$$

$$Z_w = \frac{\text{ber} \cdot \text{ber}' + \text{bei} \cdot \text{bei}'}{X_a^2 Z_a} = Z_v \approx \frac{1}{4(1+\nu^*)} \quad \text{for } X_a \text{ close to } 0$$

$$Z_m(x) = \frac{\text{ber}' \cdot \text{ber}'(x) + \text{bei}' \cdot \text{bei}'(x)}{X_a Z_a}$$

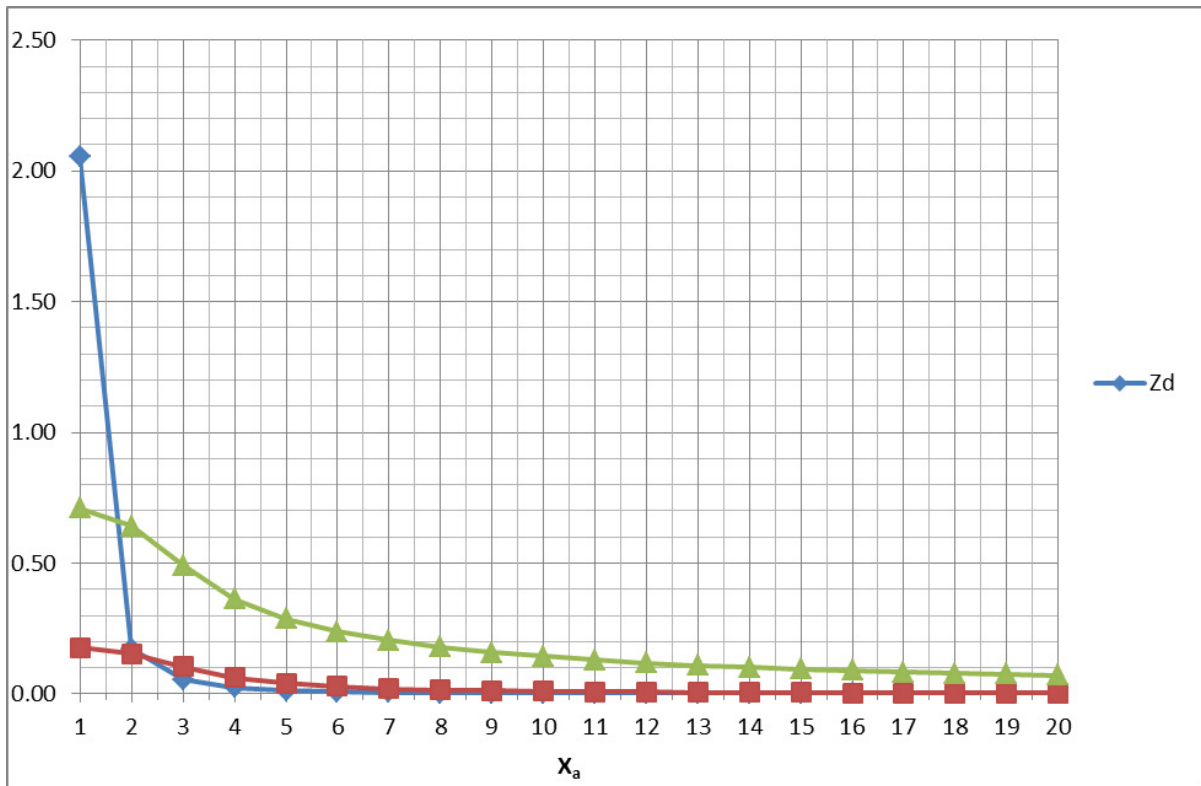
$$\approx \frac{8x\text{bei}' - x^3 \text{ber}'}{16X_a Z_a} \approx \frac{x\text{bei}'}{2X_a Z_a} = \frac{x}{(1+\nu^*)X_a} \quad \text{for } x \text{ close to } 0 \quad Z_m(0) = 0$$

$$Z_m = \frac{\text{ber}'^2 + \text{bei}'^2}{X_a Z_a} \approx \frac{1}{1+\nu^*} \quad \text{for } X_a \text{ close to } 0$$

Below provides the numerical values and graph of Z_d , Z_v , Z_w , Z_m as a function of X_a for $1 \leq X_a \leq 20$.

Coefficients Z_d, Z_v, Z_w, Z_m			
X_a	Z_d	$Z_v = Z_w$	Z_m
1	2.054458	0.176798	0.709032
2	0.171868	0.154393	0.642542
3	0.054471	0.103768	0.491113
4	0.023791	0.062263	0.362879
5	0.012079	0.039878	0.286907
6	0.006924	0.027776	0.238824
7	0.004334	0.020476	0.204651
8	0.002889	0.015704	0.178926
9	0.002021	0.012418	0.158913
10	0.001468	0.010062	0.142915
11	0.001100	0.008317	0.129837
12	0.000845	0.006988	0.118947
13	0.000663	0.005954	0.109738
14	0.000530	0.005133	0.101851
15	0.000430	0.004471	0.095019
16	0.000354	0.003929	0.089046
17	0.000295	0.003480	0.083778
18	0.000248	0.003103	0.079098
19	0.000211	0.002785	0.074912
20	0.000180	0.002513	0.071147

Graph Zd,Zv,Zw,Zm



(e) Coefficients $Q_m(x)$, $Q_v(x)$ and $Q_u(x)$, $Q_\beta(x)$

$$Q_m(x) = \frac{\text{bei}' \cdot \Xi_2(x)}{Z_a} \cdot \Xi_1(x) = \frac{(\text{bei}' \cdot \text{ber}(x) - \text{ber}' \cdot \text{bei}(x)) - \frac{1-\nu^*}{X_a} (\text{ber}' \cdot \text{ber}'(x) + \text{bei}' \cdot \text{bei}'(x))}{Z_a}$$

$$\approx \frac{8(1+\nu^*)\text{bei}' - (3+\nu^*)x^2\text{ber}'}{16Z_a} \approx \frac{(1+\nu^*)\text{bei}'}{2Z_a} \text{ for } x \text{ close to } 0 \quad Q_m(0) = \frac{(1+\nu^*)}{2Z_a} \text{bei}'$$

$$Q_m = \frac{\text{bei}' \cdot \Xi_2}{-b\Xi_1'} \cdot \Xi_1 = \frac{(\text{ber} \cdot \text{bei}' - \text{ber}' \cdot \text{bei}) - \frac{1-\nu^*}{X_a} (\text{ber}'^2 + \text{bei}'^2)}{Z_a} = \frac{Z_a}{Z_a} = 1$$

$$Q_v(x) = \frac{\Xi_1' \cdot \Xi_2(x)}{X_a Z_a} \cdot \Xi_2 \cdot \Xi_1(x) = \frac{\text{ber} \cdot \psi_2(x) - \text{bei} \cdot \psi_1(x) + \frac{1-\nu^*}{X_a} (\text{ber}' \cdot \Psi_2(x) - \text{bei}' \cdot \Psi_1(x))}{X_a Z_a}$$

$$\approx \frac{8(1+\nu^*)\Psi_1 - (3+\nu^*)\Psi_2 x^2}{16X_a Z_a} \approx \frac{(1+\nu^*)\Psi_1}{2X_a Z_a} \text{ for } x \text{ close to } 0$$

$$Q_v(0) = \frac{(1+\nu^*)}{2X_a Z_a} \Psi_1$$

$$Q_v = Q_v(X_a) = \frac{\Xi_1 \cdot \Xi_2}{X_a Z_a} = 0$$

$$Q_d(x) = \frac{\text{ber}' \cdot \text{bei}'(x) - \text{bei}' \cdot \text{ber}'(x)}{Z_a}$$

$$\approx \frac{8x\text{ber}' + x^3\text{bei}'}{16Z_a} = \frac{\text{ber}'}{2Z_a} x \text{ for } x \text{ close to } 0 \quad Q_d(0) = 0$$

$$Q_d = Q_d(X) = \frac{\text{ber}' \cdot \text{bei}' - \text{bei}' \cdot \text{ber}'}{Z_a} = 0$$

$$Q_m(x) = \frac{\Xi_2 \cdot (x) - \Xi_1 \cdot (x)}{\text{bei}' - Z_a} = \frac{(\text{ber} \cdot \text{bei}'(x) - \text{bei} \cdot \text{ber}'(x)) + \frac{1-\nu^*}{X_a}(\text{ber}' \cdot \text{ber}'(x) - \text{bei}' \cdot \text{bei}'(x))}{Z_a}$$

$$\approx \frac{8\Psi_2 x + \Psi_1 x^3}{16Z_a} \approx \frac{\Psi_2}{2Z_a} x \text{ for } x \text{ close to } 0 \quad Q_\beta(0) = 0$$

$$Q_m = Q_m(X) = \frac{\Xi_2 \cdot \Xi_1}{\text{bei}' - Z_a} = \frac{Z_a}{Z_a} = 1$$

(f) Coefficients $F_m(x)$ and $F_t(x)$

$$F_m(x) = \frac{Q_3 \cdot Q_m(x) + Q_v(x)}{2} = \frac{(\Psi_1 + Q_3 X_a \text{bei}')\Psi_2(x) - (\Psi_2 + Q_3 X_a \text{ber}')\Psi_1(x)}{2X_a Z_a}$$

$$F_m(0) = \frac{Q_3 Q_m(0) + Q_v(0)}{2} = \frac{1}{2} \left[Q_3 \frac{1+\nu^*}{2Z_a} \text{bei}' + \frac{1+\nu^*}{2Z_a X_a} \Psi_1 \right] = \frac{1+\nu^*}{4Z_a} \left[Q_3 \text{bei}' + \frac{\Psi_1}{X_a} \right] = \frac{Q_3}{2} + \frac{3+\nu^*}{16}$$

$$F_m(X_a) = \frac{Q_3 Q_m(X_a) + Q_v(X_a)}{2} = \frac{Q_3}{2}$$

Note - When F_m is maximum at periphery:

$$F_m = \text{MAX} = \left[F_m(x) \right] = \left| F_m(X_a) \right| = \frac{Q_3}{2} = \frac{Q_3}{2} \text{ if } Q_3 > 0$$

$$\frac{-Q_3}{2} \text{ if } Q_3 < 0$$

$$F_Q(x) = Q_3 \cdot Q_\alpha(x) + Q_\beta(x) = \frac{(\Psi_2 + Q_3 \text{ber}') \text{bei}'(x) - (\Psi_1 + Q_3 \text{bei}') \text{ber}'(x)}{Z_a}$$

$$F_Q(0) = Q_3 Q_\alpha(0) + Q_\beta(0) = \frac{Q_3 \text{ber}' + \Psi_2}{2Z_a} x \text{ for } x \text{ close to } 0 \rightarrow F_Q(0) = 0$$

$$F_Q(X_a) = Q_3 \cdot Q_\alpha(X_a) + Q_\beta(X_a) = 1$$

$$F_t(x) = [Q_3 \cdot Z_w(x) + Z_d(x)] \frac{X_a^4}{2} = \frac{(\Psi_1 + Q_3 X_a \text{bei}') \text{bei}(x) + (\Psi_2 + Q_3 X_a \text{ber}') \text{ber}(x)}{2Z_a} X_a$$

$$F_t(0) = \frac{Q_3 Q_m(0) + Q_v(0)}{2} = \frac{X_a^4}{2} \left[Q_3 \frac{\text{ber}'}{X_a^2 Z_a} + \frac{\Psi_2}{X_a^3 Z_a} \right] = \frac{X_a^2}{2Z_a} \left[Q_3 \text{ber}' + \frac{\Psi_2}{X_a} \right]$$

$$F_t(X_a) = \left[\frac{Q_3}{4(1+\nu^*)} + \frac{2}{X_a^4} \right] \frac{X_a^4}{2} = \frac{Q_3}{1+\nu^*} \frac{X_a^4}{8} + 1 = 1$$

(g) Parameter Q_3 when $X_a=0$

$$Q_1 = \frac{(\rho_s - 1) - \Phi Z_v}{1 + \Phi Z_m} = \frac{1}{4} \frac{(\rho_s - 1)4(1+\nu^*) - \Phi}{(1+\nu^*) + \Phi} = \frac{(\rho_s - 1) - \frac{F}{4}}{1 + F}$$

$$Q_2 = \frac{\overbrace{(\omega_s^* P_s - \omega_c^* P_c)}^N + \left(W \frac{\gamma_b}{2\pi} \right)}{1 + \Phi Z_m} = \frac{N}{1 + F}$$

$$Q_3 = Q_1 + \frac{2}{a_0^2 P_e} Q_2 = \frac{(\rho_s - 1) - \frac{F}{4}}{1 + F} + \frac{2}{a_0^2 P_e} \frac{N}{1 + F} = \frac{(\rho_s - 1) + \frac{2N}{a_0^2 P_e} - \frac{F}{4}}{1 + F}$$

ANNEX H—RADIAL DISPLACEMENT AND ROTATION OF THE SHELL AT ITS CONNECTION WITH THE RING

H.1 Radial Displacement Due to Internal Pressure P_s

Due to internal pressure P_s , the circumference of the shell internal wall increases from $2\pi a_s$ to $2\pi a'_s$, thus its elongation writes:

$2\pi(a'_s - a_s) = 2\pi w_s(P_s)$ where $w_s(P_s)$ is the radial displacement of the shell due to pressure P_s .

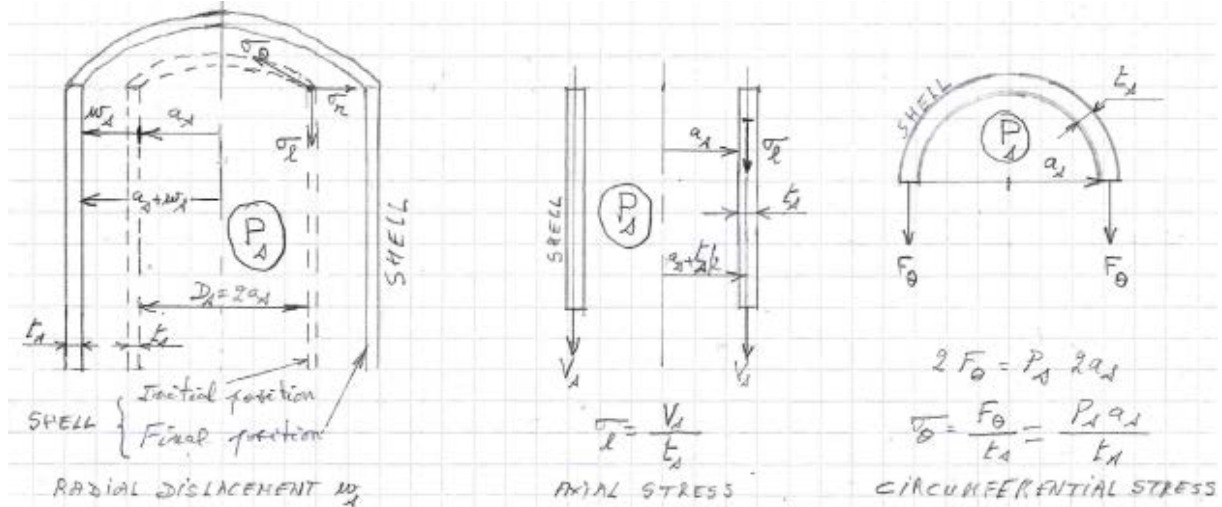


Figure 57

$$\text{Shell strain: } \varepsilon_s(P_s) = \frac{2\pi w_s(P_s)}{2\pi a_s} = \frac{w_s(P_s)}{a_s} = \frac{1}{E_s} [\sigma_\theta - \nu_s (\sigma_l + \sigma_r)] = \frac{1}{E_s} \left[\frac{a_s P_s}{t_s} - \nu_s \frac{V_s}{t_s} \right]$$

where: $\sigma_\theta = \frac{a_s P_s}{t_s}$ is the circumferential stress in the shell

$\sigma_l = \frac{V_s}{t_s}$ is the longitudinal stress in the shell subjected to the longitudinal force V_s

$\sigma_r = -P_s$ is the radial stress in the shell, which can be neglected compared to σ_θ .

Shell radial displacement due to internal pressure:

$$w_s(P_s) = \frac{a_s^2}{E_s t_s} \left[P_s - \frac{V_s}{a_s} \nu_s \right] \quad [\text{A-VI.1a-1}]$$

Note: For a U-tube or floating TS HE, V_s is known:

$$2\pi a_s' V_s = \pi a_s^2 P_s \Rightarrow V_s = \frac{a_s^2 P_s}{2a_s'} \Rightarrow \sigma_l = \frac{a_s^2 P_s}{2a_s' t_s}$$

$$w_s(P_s) = \frac{a_s^2}{E_s t_s} \left[P_s - \frac{a_s P_s \nu_s}{a_s' 2} \right] = \frac{a_s^2}{E_s t_s} \left(1 - \frac{a_s \nu_s}{a_s' 2} \right) P_s = \delta_s P_s$$

$$\text{where: } \delta_s = \frac{a_s^2}{E_s t_s} \left(1 - \frac{a_s \nu_s}{a_s' 2} \right) = \frac{D_s^2}{4 E_s t_s} \left(1 - \frac{D_s \nu_s}{D_s + t_s 2} \right)$$

H.2 Radial Displacement and Rotation due to Edge Loads Q_s and M_s

Formulas provided in this Annex are taken from Appendix 4.2 of ASME Section VIII-Div. 2 [11].

$$w_s(Q_s, M_s) = \frac{Q_s}{2 \beta_s^3 D_s} + \frac{M_s}{2 \beta_s^2 D_s} \quad \theta_s(Q_s, M_s) = \frac{Q_s}{2 \beta_s^2 D_s} + \frac{M_s}{2 \beta_s D_s}$$

$$\beta_s = \frac{\sqrt[4]{12(1-\nu_s^2)}}{\sqrt{(D_s + t_s) t_s}} \quad D_s = \frac{E_s t_s^3}{12(1-\nu_s^2)} \quad k_s = 2 \beta_s D_s = \beta_s \frac{E_s t_s^3}{6(1-\nu_s^2)}$$

$$w_s(Q_s, M_s) = \frac{Q_s}{\beta_s^2 k_s} + \frac{M_s}{k_s} \quad \theta_s(Q_s, M_s) = \frac{Q_s}{\beta_s k_s} + \frac{2 M_s}{k_s}$$

H.3 Radial Displacement due to Internal Pressure and Edge Loads

$$w_s = \frac{Q_s}{\beta_s^2 k_s} + \frac{M_s}{\beta_s k_s} + \delta_s P_s \quad \theta_s = \frac{Q_s}{\beta_s k_s} + \frac{2 M_s}{k_s}$$

With: $\delta_s = \frac{a_s^2}{E_s t_s} \left(1 - \frac{a_s \nu_s}{a_s' 2} \right)$ where ν_s is unknown

At this point of the development, it appears that it is not possible to get a solution if ν_s is unknown.

Thus, the classical shell formula:

$$\sigma_l = \frac{a_s^2 P_s}{2(a_s + t_s/2)t_s} = \frac{D_s}{D_s + t_s} \frac{P_s D_s}{2t_s} \text{ is used, which leads to:}$$

$$\delta_s = \frac{D_s^2}{4 E_s t_s} \left(1 - \frac{D_s \nu_s}{D_s + t_s 2} \right) P_s$$

Note: In UHX-13, t_s has been neglected compared to D_s and:

$$\delta_s = \frac{D_s^2}{4 E_s t_s} \left(1 - \frac{\nu_s}{2} \right) P_s$$

H.4 Channel

Above formulas apply to the channel by replacing subscript s by subscript c, except that the axial force and longitudinal stress in the channel are known:

$$V_c = \frac{a_c P_c}{2} \quad \sigma_l = \frac{a_c P_c}{2 t_c} \quad \text{which leads to:} \quad \boxed{\delta_c = \frac{D_c^2}{4 E_c t_c} \left[1 - \frac{D_c}{D_c + t_c} \frac{\nu_c}{2} \right]}$$

Note: In UHX-13, t_c has been neglected compared to D_c and:

$$\boxed{\delta_c = \frac{D_c^2}{4 E_c t_c} \left[1 - \frac{\nu_c}{2} \right]}$$

ANNEX I—SHELL-RING CONNECTION IN RADIAL DIRECTION

Radial force $Q_r(a_0)$ acting at TS periphery, due to Q_s and Q_c (see Figure 58 below):

$$Q_r(a_o) = Q_s + Q_c$$

TS radial displacement $w_r(a_o)$ at mid-thickness, at point A, due to $Q_r(a_o)$: $w_r(a_o) = \frac{a_o}{h} \frac{1-\nu^*}{E^*} Q_R$

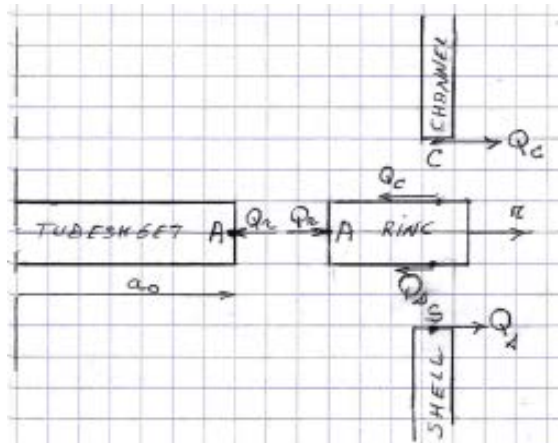


Figure 58

Ring radial displacement w_R at point S (see Figure 59 below):

$$t_g(\theta_R) = \frac{-w_R}{h/2} \approx \theta_R \quad \Rightarrow \quad w_R = -\frac{h}{2}\theta_R$$

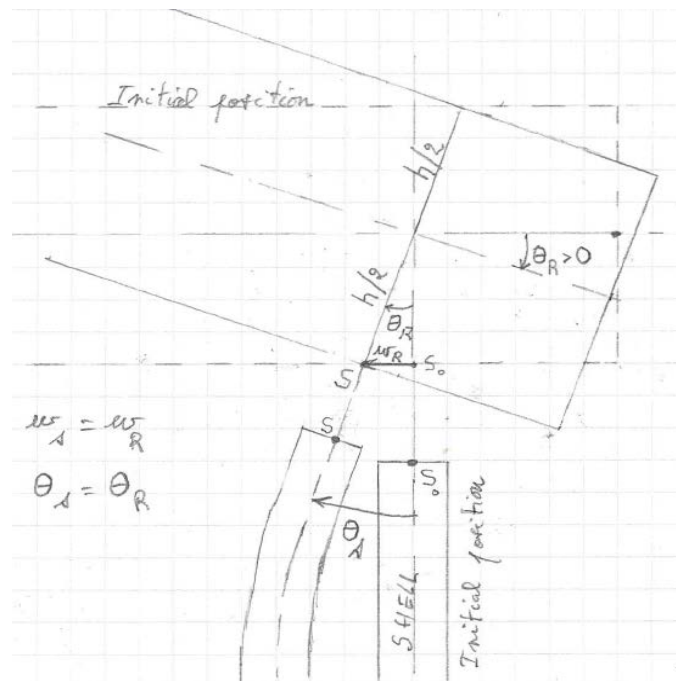


Figure 59

Continuity of displacements at ring-shell connection (point S):

$$w_R = w_s \quad \theta_R = \theta_s \quad \Rightarrow \quad w_s = -\frac{h}{2} \theta_s$$

Total radial displacement w_T of the ring at point S: $w_T = w_r(a_o) + w_R = w_r(a_o) + w_s$

At this point of the development, it appears that it is necessary to ignore the radial displacement at mid thickness $w_r(a_o)$ to get a solution.

Compatibility of shell-ring displacements writes: $w_s = -\frac{h}{2} \theta_s$

ANNEX J—MINIMUM LENGTH OF SHELL AND CHANNEL WHEN INTEGRAL WITH THE TS

The bending moment $M(x)$ located at the distance x of the edge shell (point S in Figure 17), submitted to moment M_s is given by (see Appendix 4.2 of [11]):

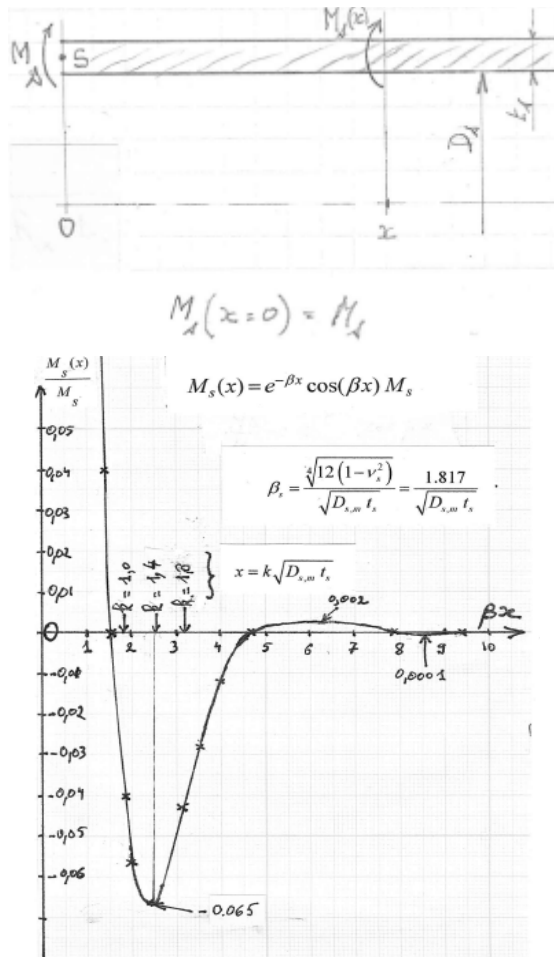
$$M_s(x) = \frac{Q_s}{\beta_s} f_4(\beta x) + M_s f_3(\beta x) \quad \text{with} \quad \beta_s = \frac{\sqrt[4]{12(1-\nu_s^2)}}{\sqrt{(D_{s,m} + t_s) t_s}} = \frac{1.817}{\sqrt{(D_{s,m}) t_s}} \quad \text{for } \nu = 0.3$$

Neglecting the radial displacement of the shell submitted to edge loads M_s and Q_s , given by Clause 2 of Annex H, the following relationship is obtained: $Q_s = -\beta_s M_s$ which leads to:

$$M_s(x) = [f_3(\beta x) - f_4(\beta x)] M_s \quad \text{with:} \quad \left. \begin{aligned} f_3(\beta x) &= e^{-\beta x} [\cos(\beta x) + \sin(\beta x)] \\ f_4(\beta x) &= e^{-\beta x} [\sin(\beta x)] \end{aligned} \right\} f_3(\beta x) - f_4(\beta x) = e^{-\beta x} \cos(\beta x)$$

which leads to: $M_s(x) = e^{-\beta x} \cos(\beta x) M_s$

The moment $M_s(x)$ decreases according to a damped sinusoidal curve represented on Figure below.



The length x can be written as: $x = k\sqrt{D_{s,m} t} \Rightarrow \beta x = \beta k \sqrt{D_{s,m} t} = 1.817 k$

For $k = 1.0$ $x = 1.0 \sqrt{D_{s,m} t_s}$ $M_s(x) = 0.040 M_s$

For $k = 1.4$ $x = 1.4 \sqrt{D_{s,m} t}$ $M_s(x) = 0.065 M_s$

For $k = 1.8$ $x = 1.8 \sqrt{D_{s,m} t}$ $M_s(x) = 0.040 M_s$

For $k = 2.0$ $x = 2.0 \sqrt{D_{s,m} t}$ $M_s(x) = 0.020 M_s$

For $k = 2.5$ $x = 2.5 \sqrt{D_{s,m} t}$ $M_s(x) = 0.002 M_s$

The value $x = 1.8 \sqrt{D_m t}$ has been retained as the remaining bending moment at this distance of the shell edge is only 4%.

Therefore, when they are integral with the TS, the shell and the channel must have a minimum length adjacent to the TS of $l_{s,\min} = 1.8 \sqrt{D_s t_s}$ and $l_{c,\min} = 1.8 \sqrt{D_c t_c}$.

ANNEX K—FORMULAS FOR A HEMISPHERICAL CHANNEL WHEN INTEGRAL WITH THE TS

The formulas given in Annex H are valid for a cylindrical channel. If the channel is hemispherical and attached directly to the TS (configurations a, b or c), without any cylindrical section between the head and the TS (Figure 60), similar formulas can be developed, based on Appendix 4.3 of Section VIII-Div. 2 [11].

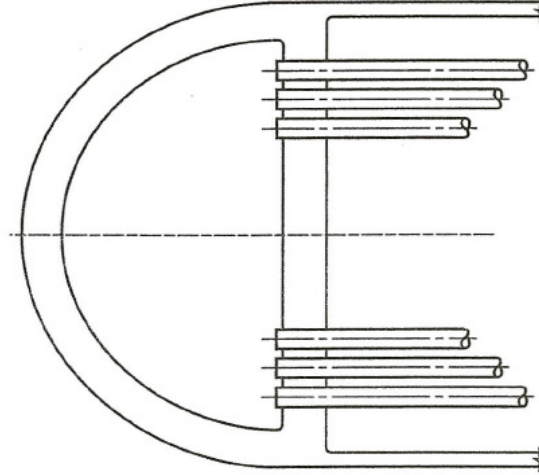


Figure 60—Hemispherical head

K.1 Radial Displacement due to Internal Pressure P_c

Channel strain:

$$\varepsilon_c(P_c) = \frac{2\pi w_c(P_c)}{2\pi a_c} = \frac{w_c(P_c)}{a_c} = \frac{1}{E_c} [\sigma_\theta - \nu_c (\sigma_l + \sigma_r)] = \frac{1}{E_c} \left[\frac{a_c P_c}{2t_c} - \nu_c \frac{a_c^2 P_c}{2(a_c + t_c/2)t_c} \right]$$

where: $\sigma_\theta = \frac{a_c P_c}{t_c}$ is the circumferential stress in the channel

$\sigma_l = \frac{a_c^2 P_c}{2(a_c + t_c/2)t_c}$ is the longitudinal stress in the channel

$\sigma_r = -P_c$ is the radial stress in the channel, which can be neglected compared to σ_θ .

Channel radial displacement due to internal pressure:

$$w_c(P_c) = \underbrace{\frac{a_c^2 P_c}{E_c t_c} \left[\frac{1 - \frac{a_c}{a_c} \nu_c}{2} \right]}_{\delta_c} = \delta_c P_c \quad \text{where: } \delta_c = \frac{a_c^2}{E_c t_c} \left[\frac{1 - \frac{a_c}{a_c} \nu_c}{2} \right] = \frac{D_c^2}{4 E_c t_c} \left[\frac{1 - \frac{D_c}{D_c + t_c} \nu_c}{2} \right]$$

K.2 Radial Displacement and Rotation due to Edge Loads Q_s AND M_s

Radial displacement w_c and rotation θ_c formulas are taken from Clause 4-332(b) of Appendix 4.3 of Section VIII Div. 2 [11] :

$$w_c = \frac{2\beta_c R_m^2}{E_c t_c} Q_c + \frac{2\beta_c^2 R_m^2}{E_c t_c} M_c \quad \theta_c = \frac{2\beta_c^2 R_m^2}{E_c t_c} Q_c + \frac{4\beta_c^3 R_m^2}{E_c t_c} M_c \quad \text{with : } R_m = \frac{D_c + t_c}{2}$$

$$\frac{2\beta_c R_m^2}{E_c t_c} = \frac{1}{2\beta_c^3} \frac{4\beta_c^4 R_m^2}{E_c t_c} = \frac{1}{2\beta_c^3} \frac{12(1-\nu_c^2)}{E_c t_c^3} = \frac{1}{2\beta_c^3} \frac{1}{D_c} = \frac{1}{2\beta_c^3 D_c} = \frac{1}{\beta_c^2 k_c}$$

$$\frac{2\beta_c^2 R_m^2}{E_c t_c} = \beta_c \frac{2\beta_c R_m^2}{E_c t_c} = \frac{1}{2\beta_c^2 D_c} = \frac{1}{\beta_c k_c}$$

$$\frac{4\beta_c^3 R_m^2}{E_c t_c} = 2\beta_c \frac{2\beta_c^2 R_m^2}{E_c t_c} = 2\beta_c \frac{1}{2\beta_c^2 D_c} = \frac{1}{\beta_c D_c} = \frac{2}{k_c}$$

$$w_c = \frac{Q_c}{2\beta_c^3 D_c} + \frac{M_c}{2\beta_c^2 D_c} = \frac{Q_c}{\beta_c^2 k_c} + \frac{M_c}{\beta_c k_c} \quad \theta_c = \frac{Q_c}{2\beta_c^2 D_c} + \frac{M_c}{\beta_c D_c} = \frac{Q_c}{\beta_c k_c} + \frac{2M_c}{k_c}$$

These formulas are the same as for a cylindrical channel (see Clause 4 of Annex H)

K.3 Radial Displacement due to Internal Pressure and Edge Loads

$$w_c = \frac{Q_c}{\beta_c^2 k_c} + \frac{M_c}{\beta_c k_c} + \delta_c P_c \quad \theta_c = \frac{Q_c}{\beta_c k_c} + \frac{2M_c}{k_c} \quad \text{with: } \delta_c = \frac{D_c^2}{4 E_c t_c} \left(\frac{1}{2} - \frac{D_c}{D_c + t_c} \frac{\nu_c}{2} \right)$$

Accordingly, the formulas are the same as for a cylindrical channel, provided in Clause 4 of Annex H,

$$\delta_s = \frac{D_c^2}{4 E_c t_c} \left(1 - \frac{D_c}{D_c + t_c} \frac{\nu_c}{2} \right) P_c \quad \text{is replaced by} \quad \delta_c = \frac{D_c^2}{4 E_c t_c} \left(\frac{1}{2} - \frac{D_c}{D_c + t_c} \frac{\nu_c}{2} \right)$$

Note: In UHX-13, t_c has been neglected compared to D_c :

$$\delta_c = \frac{D_c^2}{4 E_c t_c} \left[\frac{1}{2} - \frac{\nu_c}{2} \right]$$

ANNEX L—EQUILIBRIUM OF RING SUBJECTED TO EDGE MOMENTS

(a) For configuration a

Axial equilibrium of forces

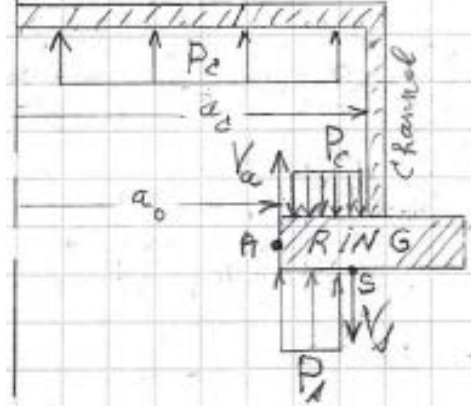


Figure 61—Configuration a

Axial force V_s in shell:

$$2 \pi a'_s V_s + \pi (a_c^2 - a_o^2) P_c = 2 \pi a'_c V_c + 2 \pi a_o V_a + \pi (a_s^2 - a_o^2) P_s \quad [\text{VI.2a-1}]$$

Axial force V_c in channel:

$$2 \pi a'_c V_c = \pi a_c^2 P_c \quad \Rightarrow \quad a'_c V_c = \frac{a_c^2}{2} P_c \quad [\text{VI.2a-2}]$$

Axial equilibrium of the ring writes:
$$a'_s V_s = a_o V_a + \frac{a_o^2}{2} P_c + \frac{a_s^2 - a_o^2}{2} P_s \quad [\text{VI.2a-3}]$$

Axial equilibrium of moments

$$[\text{VI.1a}] \begin{cases} M_s = + \left[k_s \left(1 + \frac{t'_s}{2} \right) \right] \theta_s + [\beta_s k_s] \delta_s P_s \\ Q_s = - \left[\beta_s k_s (1 + t'_s) \right] \theta_s - [2 \beta_s^2 k_s] \delta_s P_s \end{cases}$$

$$[\text{VI.1b}] \begin{cases} M_c = + \left[k_c \left(1 + \frac{t'_c}{2} \right) \right] \theta_c + [\beta_c k_c] \delta_c P_c \\ Q_c = - \left[\beta_c k_c (1 + t'_c) \right] \theta_c - [2 \beta_c^2 k_c] \delta_c P_c \end{cases}$$

From Figure 17 and Figure 61, ring equilibrium writes:

$$\begin{aligned}
 [R M_R]_a = & -\overbrace{a_o M_a}^{A0} + \underbrace{\left[a'_c M_c - a'_c Q_c \frac{h}{2} \right]}_{A1c} + \underbrace{\left[M(P_c) - a'_c V_c (a'_c - a_o) \right]}_{A2c} \\
 & - \underbrace{\left[a'_s M_s - a'_s Q_s \frac{h}{2} \right]}_{A1s} - \underbrace{\left[M(P_s) - a'_s V_s (a'_s - a_o) \right]}_{A2s}
 \end{aligned} \quad [VI.2b]$$

where: R = radius at center of ring $= \frac{A + 2 a_o}{4}$

$M(P_c)$ = moment due to pressure P_c acting on the ring: $(a_c^2 - a_o^2) \left(\frac{a_c + a_o}{2} - a_o \right) \frac{P_c}{2}$

$M(P_s)$ = moment due to pressure P_s acting on the ring: $(a_s^2 - a_o^2) \left(\frac{a_s + a_o}{2} - a_o \right) \frac{P_s}{2}$

Derivations are performed so that the moment M_R can be calculated from V_a, M_a, θ_a and P_s, P_c .

$$\begin{aligned}
 A1c &= \left[a'_c k_c \left(1 + \frac{t'_c}{2} \right) \theta_c + a'_c k_c \beta_c \delta_c P_c \right] + \left[a'_c \beta_c k_c \left(1 + t'_c \right) \frac{h}{2} \theta_c + 2 a'_c k_c \beta_c^2 \delta_c \frac{h}{2} P_c \right] \\
 &= a'_c k_c \left[\left(1 + \frac{t'_c}{2} \right) + \beta_c \left(1 + t'_c \right) \frac{h}{2} \right] \theta_c + a'_c k_c \beta_c \delta_c \left(1 + 2 \beta_c \frac{h}{2} \right) P_c \\
 &= a'_c k_c \left(1 + t'_c + \frac{t'^2_c}{2} \right) \theta_c + a_o \rho_c \beta_c k_c \delta_c (1 + \beta_c h) P_c \\
 &= a'_c k_c \left(1 + t'_c + \frac{t'^2_c}{2} \right) \theta_c + a_o \omega_c P_c \quad \boxed{t'_c = h \beta_c} \quad \boxed{\omega_c = \rho_c \beta_c k_c \delta_c (1 + h \beta_c)}
 \end{aligned}$$

$$A1s = a'_s k_s \left(1 + t'_s + \frac{t'^2_s}{2} \right) \theta_s + a_o \omega_s P_s \quad \boxed{t'_s = h \beta_s} \quad \boxed{\omega_s = \rho_s \beta_s k_s \delta_s (1 + h \beta_s)}$$

$$A1c - A1s = a'_c k_c \left(1 + t'_c + \frac{t'^2_c}{2} \right) \theta_c - a'_s k_s \left(1 + t'_s + \frac{t'^2_s}{2} \right) \theta_s + a_o (\omega_s P_s - \omega_c P_c)$$

Accounting for compatibility of shell and channel rotations: $\begin{cases} \theta_s = \theta_a \\ \theta_c = -\theta_a \end{cases}$

$$A1c - A1s = - \left[a'_s k_s \left(1 + t'_s + \frac{t'^2_s}{2} \right) + a'_c k_c \left(1 + t'_c + \frac{t'^2_c}{2} \right) \right] \theta_a + a_o (\omega_c P_c - \omega_s P_s)$$

$$A2c = (a_c^2 - a_o^2)(a_c - a_o) \frac{P_c}{4} - a_c^2 \frac{P_c}{2} (a'_c - a_o) = \frac{P_c}{4} [(a_c^2 - a_o^2)(a_c - a_o) - 2 a_c^2 (a'_c - a_o)]$$

$$A2s = (a_s^2 - a_o^2)(a_s - a_o) \frac{P_s}{4} - \left[a_o V_a + (a_s^2 - a_o^2) \frac{P_s}{2} + a_o^2 \frac{P_c}{2} \right] (a'_s - a_o)$$

$$= -a_o V_a (a'_s - a_o) + \frac{P_s}{4} [(a_s^2 - a_o^2)(a_s - a_o) - 2(a_s^2 - a_o^2)(a'_s - a_o)] - 2 a_o^2 (a'_s - a_o) \frac{P_c}{4}$$

$$= -a_o V_a (a'_s - a_o) + \frac{P_s}{4} (a_s^2 - a_o^2)(a_s + a_o - 2 a'_s) - \frac{P_c}{4} [2 a_o^2 (a'_s - a_o)]$$

$$A2c - A2s = +a_o V_a (a'_s - a_o) - \frac{P_s}{4} (a_s^2 - a_o^2)(a_s + a_o - 2 a'_s) + \frac{P_c}{4} [(a_c^2 - a_o^2)(a_c - a_o) - 2 a_c^2 (a'_c - a_o) + 2 a_o^2 (a'_s - a_o)]$$

simplification of the calculations: $t_s \ll a_s \rightarrow a'_s = a_s$ $t_c \ll a_c \rightarrow a'_c = a_c$

$$A2c - A2s = +a_o V_a (a_s - a_o) + \frac{P_s}{4} a_o^3 [(\rho_s^2 - 1)(\rho_s - 1)] - \frac{P_c}{4} a_o^3 [(\rho_c^2 + 1)(\rho_c - 1) - 2(\rho_s - 1)]$$

Equation [VI.2b] writes for configuration a:

$$\begin{aligned} [R M_R]_a = & -a_o M_a + a_o^2 V_a (\rho_s - 1) + \frac{P_s}{4} a_o^3 [(\rho_s^2 - 1)(\rho_s - 1)] - \frac{P_c}{4} a_o^3 [(\rho_c^2 + 1)(\rho_c - 1) - 2(\rho_s - 1)] \\ & - \left[a'_s k_s \left(1 + t'_s + \frac{t_s'^2}{2} \right) + a'_c k_c \left(1 + t'_c + \frac{t_c'^2}{2} \right) \right] \theta_a + a_o (\omega_c P_c - \omega_s P_s) \end{aligned}$$

[VI.2b']

(b) For configuration b:

Axial equilibrium of forces

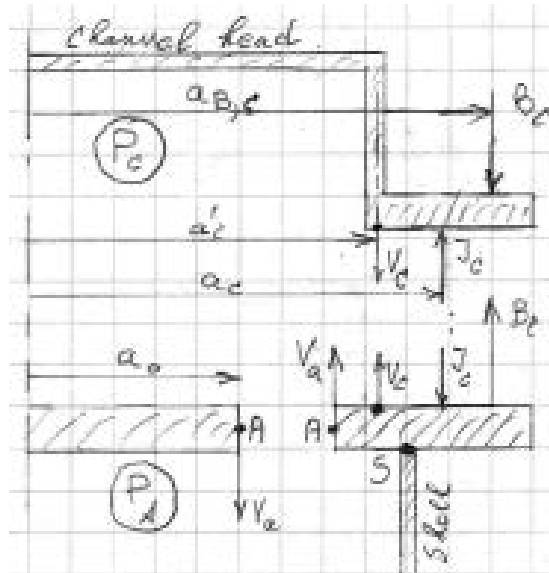


Figure 62—Configuration b

Equilibrium of channel:

$$2 \pi a'_c V_c = 2 \pi a_{Bc} B_c - 2 \pi a_c J_c = \pi a_c^2 P_c \Rightarrow a_{Bc} B_c - a_c J_c = \frac{a_c^2}{2} P_c$$

Axial force V_s in shell:

$$2 \pi a'_s V_s + \pi (a_c^2 - a_o^2) P_c = 2 \pi a_o V_a + \underbrace{2 \pi a_{Bc} B_c - 2 \pi a_c J_c}_{\pi a_c^2 P_c} + \pi (a_s^2 - a_o^2) P_s$$

Axial equilibrium of the ring remains unchanged:

$$a'_s V_s = a_o V_a + \frac{a_o^2}{2} P_c + \frac{a_s^2 - a_o^2}{2} P_s$$

Where (see Figure 62):

$$\begin{cases} a_c = \text{mean gasket radius} = \frac{G_c}{2} & J_c = \text{gasket load per unit of circumference} \\ a_{Bc} = \text{bolt circle radius} = \frac{C_c}{2} & B_c = \text{bolt load per unit of circumference} \end{cases}$$

Axial equilibrium of forces. In equation [VI.2b]:

$$\begin{aligned} [R M_R]_a = & \underbrace{-a_o M_a}_{A0} + \underbrace{\left[a'_c M_c - a'_c Q_c \frac{h}{2} \right]}_{A1c} + \underbrace{\left[M(P_c) - a'_c V_c (a'_c - a_o) \right]}_{A2c} \\ & - \underbrace{\left[a'_s M_s - a'_s Q_s \frac{h}{2} \right]}_{A1s} - \underbrace{\left[M(P_s) - a'_s V_s (a'_s - a_o) \right]}_{A2s} \end{aligned} \quad [VI.2b]$$

- Terms A0, A1s, A2s due to shell remain unchanged.
- Term A1c due to moments applied to the channel disappears, but can be maintained provided that k_c is taken equal to 0: $k_c=0$.
- Term A2c becomes [A2c]_b:

$$[A2c]_b = M(P_c) - a_{Bc} B_c (a_{Bc} - a_o) + a_c J_c (a_c - a_o) = M(P_c) + A_B$$

$$A_B = -a_{Bc} B_c (a_{Bc} - a_o) + \left(a_{Bc} B_c - \frac{a_c^2}{2} P_c \right) (a_c - a_o)$$

Where:

$$= -a_{Bc} B_c a_{Bc} + a_{Bc} B_c a_c - \frac{a_c^2}{2} P_c (a_c - a_o) = a_{Bc} B_c (a_c - a_{Bc}) \frac{a_c^2}{2} (a_c - a_o)$$

$$[A2c]_b = \underbrace{M(P_c) - \frac{a_c^2}{2} (a_c - a_o) P_c + a_{Bc} B_c (a_c - a_{Bc})}_{A3 \text{ with } a'_c = a_c} = A3 + a_{Bc} B_c (a_c - a_{Bc})$$

- $A2c_b - A2s = A2c - A2s + a_{Bc} B_c (a_c - a_{Bc})$ leads to equation $R M_R$ for configuration b:

$$[R M_R]_b = [R M_R]_a + a_{Bc} B_c (a_c - a_{Bc})$$

Using UHX-13 notations:

$$a_{Bc} B_c (a_c - a_{Bc}) = \frac{C_c}{2} \frac{W_c}{\pi C_c} \left(\frac{G_c}{2} - \frac{C_c}{2} \right) = \frac{a_o}{2\pi} W_c \left(\frac{G_c - C_c}{D_o} \right)$$

$$a_{Bc} B_c (a_c - a_{Bc}) = \frac{a_o}{2\pi} W_c \gamma_{bc} \quad \text{where: } \gamma_{bc} = \frac{G_c - C_c}{D_o}$$

$$[R M_R]_b = [R M_R]_a + \frac{a_o}{2\pi} W_c \gamma_{bc} \quad \text{with } k_c = 0 \text{ in } [RM_R]_a \text{ equation}$$

(c) For configuration c:

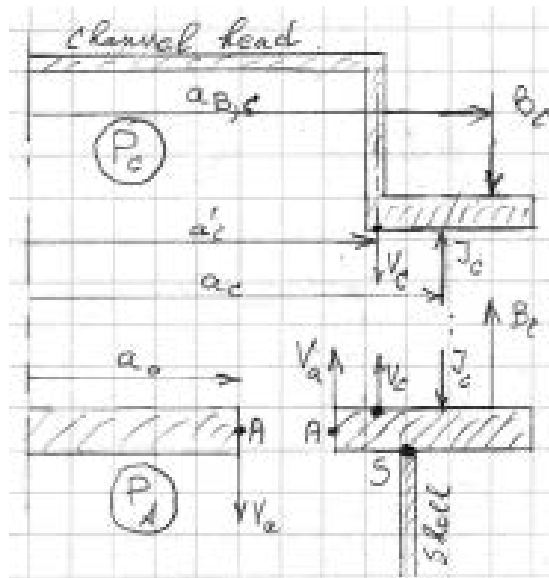


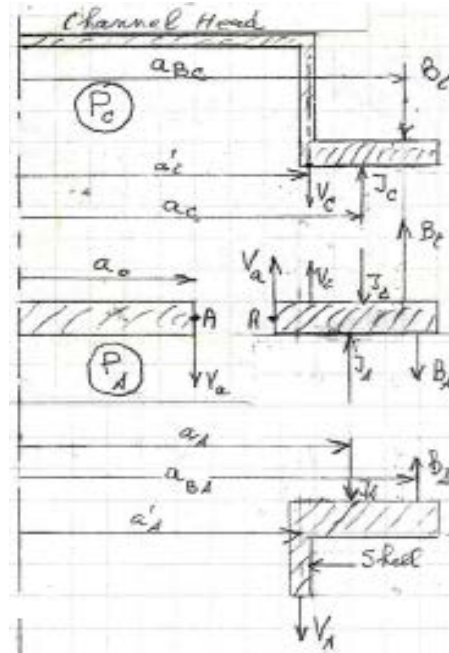
Figure 63—Configuration c

Same applies but the bolting diameter C_c becomes the midpoint of contact between flange and

TS, G_1 (see Figure 63). Thus γ_{bc} becomes: $\gamma_{bc} = \frac{G_c - G_1}{D_o}$

(d) For configuration d:

Axial equilibrium of forces



Axial force V_c in channel remains unchanged:

$$a_c V_c = a_{Bc} B_c - a_c J_c = \frac{a_c^2}{2} P_c \quad a_c = \frac{G_c}{2} \quad a_{Bc} = \frac{C_c}{2}$$

Axial force V_s in shell is given by , with $a_s = \frac{G_s}{2}$, $a_{Bc} = \frac{C_c}{2}$ and $a_{Bs} = \frac{C_s}{2}$

$$2 \pi a'_s V_s + 2 \pi a_s J_s = 2 \pi a_{Bs} B_s \Rightarrow \boxed{a'_s V_s = a_{Bs} B_s - a_s J_s}$$

Axial equilibrium of moments. Comparing to configuration b:

Terms A_0 , A_{1c} and $[A_{2c}]_b$ due to the channel remain unchanged.

Term A_{1s} due to moments applied to the shell disappears, but can be maintained provided that k_s is taken equal to 0: $k_s=0$.

Term A_{2s} becomes $[A_{2s}]_d$, similar to term $[A_{2c}]_b$ of the channel:

$$[A_{2s}]_d = \underbrace{M(P_s) - \frac{a_s^2}{2} (a_s - a_o) P_s}_{A3 \text{ avec } a'_s = a_s} + a_{Bs} B_s (a_s - a_{Bs}) P_c = A_{2s} + a_{Bs} B_s (a_s - a_{Bs})$$

$$\text{Term: } [A_{2c}]_b - [A_{2s}]_d = A_{2c} + a_{Bc} B_c (a_c - a_{Bc}) - A_{2s} - a_{Bs} B_s (a_s - a_{Bs})$$

leads to the equation $[R M_R]_d$ for configuration d:

$$[R M_R]_d = [R M_R]_a + a_{Bc} B_c (a_c - a_{Bc}) - a_{Bs} B_s (a_s - a_{Bs})$$

Using UHX-13 notations:

$$a_{Bs} B_s (a_s - a_{Bs}) = \frac{C_s}{2} \frac{W_s}{\pi C} \left(\frac{G_s}{2} - \frac{C_s}{2} \right) = \frac{a_o}{2\pi} W_s \left(\frac{G_s - C_s}{D_o} \right) = \frac{a_o}{2\pi} W_s \gamma_{bs}$$

$$[R M_R]_d = [R M_R]_a + \frac{a_o}{2\pi} W_c \gamma_{bc} - \frac{a_o}{2\pi} W_s \gamma_{bs} \text{ with: } \begin{cases} k_s = 0 & \text{and} & \gamma_{bs} = \frac{G_s - C_s}{D_o} \\ k_c = 0 & \text{and} & \gamma_{bc} = \frac{G_c - C_c}{D_o} \end{cases}$$

Finally, the generic equation covering the 4 configurations a, b, c and d writes:

$$R M_R = -a_o M_a + a_o^2 V_a (\rho_s - 1) + P_s \frac{a_o^3}{4} [(\rho_s^2 - 1)(\rho_s - 1)] - P_c \frac{a_o^3}{4} [(\rho_c^2 + 1)(\rho_c - 1) - 2(\rho_s - 1)]$$

$$- \left[a'_s k_s \left(1 + t'_s + \frac{t_s'^2}{2} \right) + a'_c k_c \left(1 + t'_c + \frac{t_c'^2}{2} \right) \right] \theta_a + a_o (\omega_c P_c - \omega_s P_s) + \frac{a_o}{2\pi} [W_c \gamma_{bc} - W_s \gamma_{bs}]$$

[VI.2d]

In this equation:

- λ_s , λ_c and w_s , w_c are coefficients obtained above. They are known for a given HE.
- W_s and W_c are bolt loads applied on shell and channel when the TS is gasketed (configurations b, c, d).
- Coefficients γ_{bs} and γ_{bc} are defined for each configuration as follows:

Configuration a: k_s given in VI.1a k_c given in VI.1b $\gamma_{bs} = 0$ $\gamma_{bc} = 0$

Configuration b: k_s given in VI.1a $k_c = 0$ $\gamma_{bs} = 0$ $\gamma_{bc} = \frac{G_c - C_c}{D_o}$

Configuration c: k_s given in VI.1a $k_c = 0$ $\gamma_{bs} = 0$ $\gamma_{bc} = \frac{G_c - G_1}{D_0}$

Configuration d: $k_s = 0$ $k_c = 0$ $\gamma_{bs} = \frac{G_s - C_s}{D_o}$ $\gamma_{bc} = \frac{G_c - C_c}{D_o}$

ANNEX M—DIRECT DETERMINATION OF THE EQUIVALENT PRESSURE

Equivalent pressures P'_s , P'_t and $P'\gamma$ can be obtained directly by examining the loads applied on the TS as follows.

(a) Pressures P_s and P_t acting on the perforated TS (Figure 64)

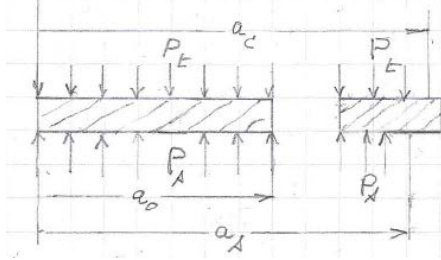


Figure 64

$$\boxed{P_{TS}(P_s, P_t) = x_s P_s - x_t P_t} \quad x_s = 1 - N_t \left(\frac{d_t}{D_0} \right)^2 \quad x_t = 1 - N_t \left(\frac{d_t - 2t_t}{D_0} \right)^2$$

(b) Pressures P_s and P_t acting on the unperforated TS RIM

Load acting on the rim:

$$F_{RIM}(P_s, P_t) = \pi(a_s^2 - a_o^2)P_s - \pi(a_c^2 - a_o^2)P_t = \pi a_o^2 [(\rho_s^2 - 1)P_s - (\rho_c^2 - 1)P_t]$$

$$\text{Part of } F_{rim} \text{ supported by the tubes: } F_{RIM}(P_s, P_t) \frac{N_t K_t}{K_s^*} = F_{RIM}(P_s, P_t) \frac{N_t K_t}{JK_s} = \frac{F_{RIM}(P_s, P_t)}{JK_{s,t}}$$

Equivalent pressure acting on the unperforated RIM:

$$P_{RIM}(P_s, P_t) = -\frac{F_{RIM}(P_s, P_t)}{\pi a_o^2 JK_{s,t}} = -\frac{(\rho_s^2 - 1)P_s - (\rho_c^2 - 1)P_t}{JK_{s,t}}$$

$$P_{RIM}(P_s, P_t) = -\frac{F_{RIM}(P_s, P_t)}{\pi a_o^2 JK_{s,t}} = \boxed{-\frac{(\rho_s^2 - 1)}{JK_{s,t}} P_s + \frac{(\rho_c^2 - 1)}{JK_{s,t}} P_t}$$

(c) Pressure P_s acting on the joint (Figure 65)

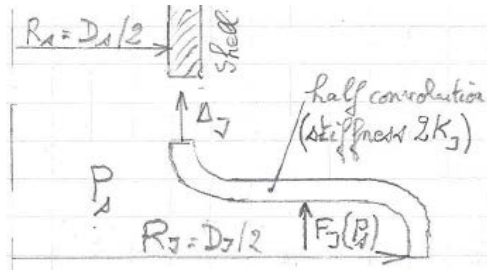


Figure 65

Axial load acting on the shell: $F_J'(P_s) = \frac{F_J(P_s)}{2} = P_s \frac{\pi}{2} (R_J^2 - R_s^2)$

Axial displacement of the half-shell: $\Delta_J(P_s) = \frac{P_s \pi (R_J^2 - R_s^2) / 2}{2K_J}$

Equivalent pressure acting on the perforated TS: using $\frac{k_w}{2K_J} = \frac{1}{\pi a_o^2} \frac{1}{K_{s,t}} \frac{1-J}{J}$ and $K_{s,t} = \frac{k_s}{N_t k_t}$

$$P_J(P_s) = -k_w \Delta_J(P_s) = -\frac{1}{\pi a_o^2} \frac{1-J}{JK_{s,t}} \frac{\pi (R_J^2 - R_s^2)}{2} P_s = \boxed{-\frac{1-J}{2JK_{s,t}} \frac{(D_J^2 - D_s^2)}{D_o^2} P_s}$$

(d) **Effect of v_t due to pressures P_s and P_t acting on the tubes** (Figure 66)

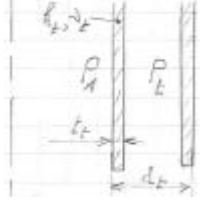


Figure 66

$$\Delta L_T(v_t) = \frac{\pi}{2k_t} \left[P_s d_t^2 - P_t (d_t - 2t_t)^2 \right] v_t$$

Using $k_w = \frac{N_t \cdot k_t}{\pi a_o^2}$:

$$P_T(v_t) = k_w \Delta L_T(v_t) = \frac{\pi}{2k_t} \frac{N_t k_t}{\pi a_o^2} \left[P_s d_t^2 - P_t (d_t - 2t_t)^2 \right] v_t = \frac{2N_t}{4a_o^2} \left[P_s d_t^2 - P_t (d_t - 2t_t)^2 \right] v_t$$

$$\boxed{P_T(v_t) = 2 \left[P_s (1 - x_s) - P_t (1 - x_t) \right] v_t}$$

(e) **Effect of v_s due to pressure P_s acting on the shell:**

In the same way as for the tubes: $\Delta L_S(v_s) = -\frac{\pi}{2k_s} \left[P_s D_s^2 \right] v_s$

Using $k_w = \frac{N_t \cdot k_t}{\pi a_o^2}$ and $K_{s,t} = \frac{k_s}{N_t k_t}$:

$$P_S(v_s) = -k_w \Delta L_S(v_s) = \frac{\pi}{2k_s} \frac{N_t k_t}{\pi a_o^2} \left[P_s D_s^2 \right] v_s = \boxed{\frac{2}{K_{s,t}} \frac{D_s^2}{D_o^2} P_s v_s}$$

$$P'_t = P_t \left[x_t + 2(1 - x_t) \nu_t + \frac{1}{J K_{s,t}} \right] \quad P_\gamma = \left[\frac{N_t K_t}{\pi a_o^2} \right] \gamma$$

Formulas P'_s , P'_t , P_γ match the formulas obtained in VII.2 above. Formulas P_w and P_{rim} cannot be calculated directly as they involve the edge moments at the TS-shell-channel connection.

ANNEX N—FORMULAS TO BE USED WHEN $P_e=0$

The equivalent pressure P_e may be equal to 0 in the following loading cases:

- loading case 1 if $P_t=0$
- loading case 2 if $P_s=0$
- loading case 3 if $P_s=P_t$

When $P_e = 0$, $V_a = 0$ and $Q_3 = Q_1 + \frac{Q_2}{a_o V_a}$ becomes infinity. General formulas depending on V_a and M_a , given in VIII, must be used with $V_a = 0$ to determine the stresses. These formulas can be written in the general symbolic form: $F(x) = K [M_a A(x) + (a_o V_a) B(x)]$

with $M_a = (a_o V_a) Q_1 + Q_2 = Q_2$ Thus $\boxed{M_a = Q_2}$ and $F(x) = K Q_2 A(x)$

Accordingly, the equations giving $q(x)$, $w(x)$, $\theta(x)$, $\sigma(x)$, $\tau(x)$, $\sigma_t(x)$ write as follows.

N.1 Net Effective Pressure: $q(x)$

$$\boxed{q(x) = \frac{X_a^4}{a_o^2} Q_2 Z_w(x)}$$

Maximum of $q(x)$ is obtained for $Z_w(x)$ maximum which is located either inside the TS ($x < X_a$) or at TS periphery ($x = X_a$).

N.2 Axial Displacement: $w(x)$

$$\boxed{w(x) = \frac{Q}{k_w} - \frac{1}{k_w} \frac{X_a^4}{a_o^2} Q_2 Z_w(x)}$$

Maximum of $w(x)$ is obtained for $Z_w(x)$ maximum which is located either inside the TS ($x < X_a$) or at TS periphery ($x = X_a$).

N.3 Rotation: $\theta(x)$

$$\boxed{\theta(x) = \frac{a_o}{D^*} Q_2 Z_m(x)}$$

Maximum of $\theta(x)$ is obtained for $Z_m(x)$ maximum which is located either inside the TS ($x < X_a$) or at TS periphery ($x = X_a$).

N.4 Bending Stress: $\sigma(x)$

$$M_r(x) = Q_2 Q_m(x)$$

$$\sigma_r(x) = \frac{6 Q_2}{\mu^* h^2} Q_m(x)$$

Maximum of $\sigma(x)$ is obtained for $Q_m(x)$. A parametric study performed on $X_a = 1, 2, 3, \dots, 20$ shows that the maximum is always located at TS periphery ($x=X_a$) and is equal to 1 (See Annex A-V.3):

$$\text{MAX} [Q_m(x)] = Q_m(X_a) = 1 \quad \text{Thus} \quad \sigma = \frac{6 Q_2}{\mu^* h^2}$$

N.5 Shear Stress: $\tau(x)$

$$Q(x) = \frac{Q_2}{a_o} Q_\alpha(x)$$

$$\tau(x) = \frac{Q_2}{\mu a_o h} Q_\alpha(x)$$

Maximum of $\tau(x)$ is obtained for $Q_\alpha(x)$ maximum which is located either inside the TS ($x < X_a$) or at TS periphery ($x=X_a$).

N.6 Axial Stress in Tubes: $\sigma_t(x)$

$$\sigma_t(x) = \frac{1}{x_t - x_s} [\Delta p^* - q(x)] \quad q(x) = \frac{X_a^4}{a_o^2} Q_2 Z_w(x)$$

$$\sigma_t(x) = \frac{1}{x_t - x_s} \left[(x_s P_s - x_t P_t) - \frac{X_a^4}{a_o^2} Q_2 Z_w(x) \right]$$

Maximum and minimum values of $\sigma_t(x)$ are obtained for $Z_w(x)$ maximum or minimum which are located either inside the TS ($x < X_a$) or at TS periphery ($x=X_a$).

ANNEX O—TABULAR AND GRAPHICAL REPRESENTATION OF COEFFICIENT $F_t(x)$

Annex O provides for $1 \leq X_a \leq 20$ and $-0.8 \leq Q_3 \leq +0.8$ (see below for the numerical values and graphs of coef. $F_t(x)$):

- values and graphs of $F_t(x)$ for $0 \leq x \leq X_a$
- values and graphs of the minimum and maximum of $F_t(x)$: $F_{t,\min}$ and $F_{t,\max}$
- locations of the minimum and maximum of $F_t(x)$: $x_{\min}$ and $x_{\max}$

F_i(x) for X_A=1,5,7,10,20

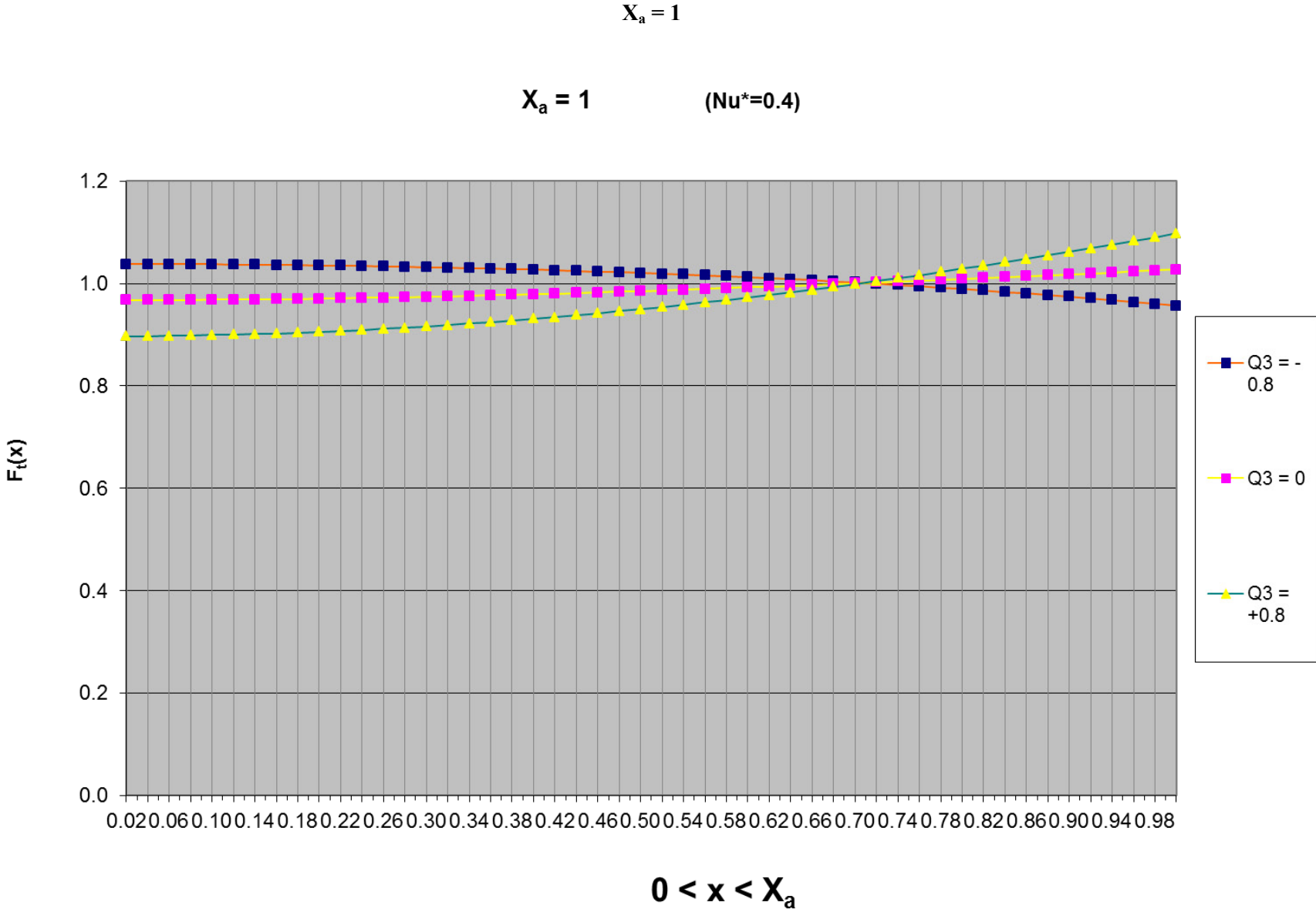
F _i (x) for X _a = 1, 5, 7, 10, 20 and Q ₃ = -0.8, 0.0 , +0.8																(v*=0.4)			
X _a = 1				X _a = 5				X _a = 7				X _a = 10				X _a = 20			
x	Q ₃ = -0.8	Q ₃ = 0	Q ₃ = +0.8	x	Q ₃ = -0.8	Q ₃ = 0	Q ₃ = +0.8	x	Q ₃ = -0.8	Q ₃ = 0	Q ₃ = +0.8	x	Q ₃ = -0.8	Q ₃ = 0	Q ₃ = +0.8	x	Q ₃ = -0.8	Q ₃ = 0	Q ₃ = +0.8
2.00E-02	1.038118	0.9676997	0.8972814	1.00E-01	1.032634	-0.6066762	-2.245986	0.14	-0.8756667	-2.81E-02	0.8194348	0.2	-9.87E-02	4.48E-02	1.88E-01	0.4	7.24E-04	4.65E-05	-6.31E-04
4.00E-02	1.038039	0.9677894	0.8975396	2.00E-01	1.046217	-0.6069373	-2.260092	0.28	-0.8633661	-3.16E-02	0.8000731	0.4	-1.09E-01	4.54E-02	2.00E-01	0.8	5.27E-04	5.92E-05	-4.09E-04
0.06	1.037908	0.9679389	0.8979697	0.3	1.06879	-0.6073345	-2.283459	0.42	-0.8426553	-3.75E-02	0.7676076	0.6	-1.26E-01	4.64E-02	2.19E-01	1.2	1.88E-04	7.96E-05	-2.89E-05
0.08	1.037724	0.9681479	0.8985717	0.4	1.100256	-0.607811	-2.315878	0.56	-0.8132234	-4.57E-02	0.72175	0.8	-1.50E-01	4.76E-02	2.46E-01	1.6	-3.06E-04	1.06E-04	5.18E-04
0.1	1.037487	0.9684163	0.8993454	0.5	1.140476	-0.608287	-2.35705	0.7	10, 20 an	-5.63E-02	0.6621073	1	-1.81E-01	4.90E-02	2.79E-01	2	-9.65E-04	1.37E-04	1.24E-03
0.12	1.037197	0.9687439	0.9002905	0.6	1.189266	-0.6086601	-2.406586	0.84	-0.7263712	-6.91E-02	0.5881936	1.2	-2.17E-01	5.05E-02	3.18E-01	2.4	-1.79E-03	1.68E-04	2.12E-03
0.14	1.036854	0.9691301	0.9014067	0.7	1.246399	-0.6088049	-2.464009	0.98	-0.6677723	-8.42E-02	0.4994436	1.4	-2.60E-01	5.18E-02	3.63E-01	2.8	-2.76E-03	1.93E-04	3.14E-03
0.16	1.036456	0.9695748	0.9026937	0.8	1.311598	-0.6085731	-2.528745	1.12	-0.5981158	-1.01E-01	0.3952311	1.6	-3.06E-01	5.29E-02	4.12E-01	3.2	-3.81E-03	2.04E-04	4.22E-03
0.18	1.036004	0.9700776	0.9041512	0.9	1.384535	-0.6077937	-2.600122	1.26	-0.5165977	-1.21E-01	0.2748903	1.8	-3.57E-01	5.35E-02	4.64E-01	3.6	-4.85E-03	1.91E-04	5.23E-03
0.2	1.035497	0.9706377	0.9057785	1	1.464823	-0.6062728	-2.677369	1.4	-0.4223562	-1.42E-01	0.1377414	2	-4.11E-01	5.33E-02	5.18E-01	4	-5.69E-03	1.40E-04	5.97E-03
0.22	1.034934	0.9712548	0.9075752	1.1	1.552018	-0.6037938	-2.759606	1.54	-0.3144919	-1.66E-01	-0.01688031	2.2	-4.66E-01	5.20E-02	5.70E-01	4.4	-6.10E-03	3.71E-05	6.17E-03
0.24	1.034316	0.9719282	0.9095408	1.2	1.645607	-0.6001173	-2.845841	1.68	-0.1920907	-1.91E-01	-0.189591	2.4	-5.21E-01	4.94E-02	6.19E-01	4.8	-5.71E-03	-1.34E-04	5.44E-03
0.26	1.03364	0.9726573	0.9116746	1.3	1.745007	-0.5949817	-2.93497	1.82	-0.05425141	-2.18E-01	-0.3809201	2.6	-5.72E-01	4.50E-02	6.62E-01	5.2	-4.12E-03	-3.88E-04	3.34E-03
0.28	1.032907	0.9734414	0.913976	1.4	1.849559	-0.5881031	-3.025765	1.96	0.0998848	-2.46E-01	-0.5912681	2.8	-6.18E-01	3.85E-02	6.95E-01	5.6	-7.99E-04	-7.33E-04	-6.68E-04
0.3	1.032115	0.9742795	0.9164441	1.5	1.95852	-0.5791762	-3.116872	2.1	0.2710992	-0.2748804	-8.21E-01	3	-6.56E-01	2.94E-02	7.15E-01	6	4.77E-03	-1.17E-03	-7.10E-03
0.32	1.031264	0.9751709	9.19E-01	1.6	2.071058	-0.5678742	-3.21E+00	2.24	0.4600573	-0.3048178	-1.069693	3.2	-6.80E-01	1.75E-02	7.15E-01	6.4	1.31E-02	-1.67E-03	-1.64E-02
0.34	1.030352	0.9761147	9.22E-01	1.7	2.186247	-0.5538498	-3.29E+00	2.38	6.67E-01	-0.3351072	-1.33748	3.4	-0.6879466	2.12E-03	0.6921918	6.8	2.44E-02	-2.20E-03	-2.88E-02
0.36	1.029379	0.97711	9.25E-01	1.8	2.303055	-0.5367362	-3.38E+00	2.52	0.8930232	-0.3652826	-1.623588	3.6	-0.6736465	-1.70E-02	0.6396902	7.2	3.86E-02	-2.68E-03	-4.40E-02
0.38	1.028343	0.9781556	9.28E-01	1.9	2.420344	-0.5161481	-3.45E+00	2.66	1.137372	-0.3948018	-1.926976	3.8	-0.631919	-4.02E-02	0.5515026	7.6	5.52E-02	-2.97E-03	-6.12E-02
0.4	1.027244	0.9792506	9.31E-01	2	2.536859	-0.4916826	-3.52E+00	2.8	1.400039	-0.42304	-2.246119	4	-0.5567574	-6.79E-02	0.4210006	8	7.26E-02	-2.91E-03	-7.84E-02
0.42	1.02608	0.9803938	9.347077	2.1	2.651222	-0.4629212	-3.577064	2.94	1.680372	-0.4492831	-2.578938	4.2	-0.4417243	-1.00E-01	0.2412963	8.4	8.81E-02	-2.26E-03	-9.26E-02
0.44	1.02485	0.981584	9.383183	2.2	2.761925	-0.4294315	-3.620788	3.08	1.977278	-0.4727224	-2.922723	4.4	-0.2800746	-1.37E-01	0.005432152	8.8	9.73E-02	-7.37E-04	-9.88E-02
0.46	1.023552	0.9828199	9.420877	2.3	2.867325	-0.3907692	-3.648863	3.22	2.28915	-0.4924494	-3.274049	4.6	-0.06492908	-1.79E-01	-0.2933681	9.2	9.44E-02	1.95E-03	-9.05E-02
0.48	1.022186	0.9841002	9.460148	2.4	2.965638	-0.3464808	-3.658599	3.36	2.613796	-0.5074518	-3.6287	4.8	0.2104946	-2.25E-01	-0.6613804	9.6	7.15E-02	6.11E-03	-5.93E-02
0.5	1.020749	0.9854234	0.9500982	2.5	3.054932	-0.2961063	-3.647145	3.5	2.948359	-0.5166103	-3.981579	5	0.5525845	-2.76E-01	-1.103985	10	1.92E-02	1.20E-02	4.76E-03
0.52	1.01924	0.9867884	0.9543367	2.6	3.133124	-0.2391825	-3.611489	3.64	3.289237	-0.5186982	-4.326633	5.2	0.966963	-3.29E-01	-1.625187	10.4	-7.31E-02	1.95E-02	0.1121317
0.54	1.017658	0.9881935	0.9587288	2.7	3.197976	-0.1752467	-3.54847	3.78	3.632005	-0.5123812	-4.656767	5.4	1.458035	-3.85E-01	-2.227052	10.8	-0.2150519	2.86E-02	0.2722115
0.56	1.016001	0.989637	0.963273	2.8	3.247089	-0.103841	-3.454771	3.92	3.971332	-0.4962205	-4.963773	5.6	2.028447	-4.40E-01	-2.909042	11.2	-0.4140377	3.84E-02	0.4907806
0.58	1.014267	0.9911175	0.9679677	2.9	3.277906	-0.02451688	-3.32694	4.06	4.300899	-0.4686789	-5.238256	5.8	2.678461	-0.4944041	-3.667269	11.6	-0.6712334	4.75E-02	0.7662781
0.6	1.012455	0.9926332	0.9728113	3	3.287708	0.06315959	-3.161389	4.2	4.613324	-0.4281282	-5.46958	6	3.40523	-0.5442125	-4.49E+00	12	-0.9773141	5.38E-02	1.084902
0.62	1.010563	0.9941825	0.9778024	3.1	3.273615	0.1596018	-2.954412	4.34	4.900085	-0.3728624	-5.64581	6.2	4.201988	-0.5865092	-5.375007	12.4	-1.306761	5.39E-02	1.414617
0.64	1.008588	0.9957635	0.982939	3.2	3.23259	0.2651963	-2.702198	4.48	5.151461	-0.3011132	-5.753687	6.4	5.057155	-0.6174373	-6.292029	12.8	-1.611355	4.36E-02	1.698476
0.66	1.006529	0.9973744	0.9882194	3.3	3.161443	0.3802958	-2.400851	4.62	5.356469	-0.2110706	-5.77861	6.6	5.953367	-0.6324616	-7.21829	13.2	-1.813493	1.73E-02	1.848088
0.68	1.004385	0.9990131	0.9936417	3.4	3.056837	0.5052112	-2.046414	4.76	5.502823	-0.1009085	-5.70464	6.8	6.866462	-0.6263518	-8.119165	13.6	-1.80055	-3.10E-02	1.738637
0.7	1.002152	1.000678	0.9992042	3.5	2.915298	0.6402039	-1.63489	4.9	5.576911	0.03118399	-5.514544	7	7.764431	-0.5931894	-8.950809	14	-1.422138	-0.107292	1.207554
0.72	0.9998285	1.002367	1.004905	3.6	2.733226	0.7854759	-1.162274	5.04	5.563787	1.87E-01	-5.189855	7.2	8.606379	-0.5264054	-9.659189	14.4	-0.4929101	-0.2165779	5.98E-02
0.74	0.9974129	1.004077	1.010741	3.7	2.50691	0.9411606	-0.6245889	5.18	5.447183	3.68E-01	-4.71098	7.4	9.341547	-0.4188564	-10.17926	14.8	1.195645	-0.360618	-1.916881
0.76	0.9949027	1.005807	1.016712	3.8	2.232545	1.107312	-0.01792095	5.32	5.209565	0.5761069	-4.057351	7.6	9.908447	-0.262948	-10.43434	15.2	3.850281	-0.5355867	-4.921454
0.78	0.9922957	1.007555	1.022814	3.9	1.906255	1.283895	0.6615336	5.46	4.832207	0.8122958	-3.207616	7.8	10.23417	-0.05081466	-10.3358	15.6	7.63344	-0.7287403	-9.09092
0.8	0.9895892	1.009317	1.029046	4	1.524115	1.470769	1.417423	5.6	4.295315	1.077711	-2.139894	8	10.23401	0.2254344	-9.783137	16	12.59882	-0.9145391	-14.4279
0.82	0.9867812	1.011093	1.035405	4.1	1.082182	1.667682	2.253183	5.74	3.57819	1.373051	-0.8320874	8.2	9.811359	0.573393					

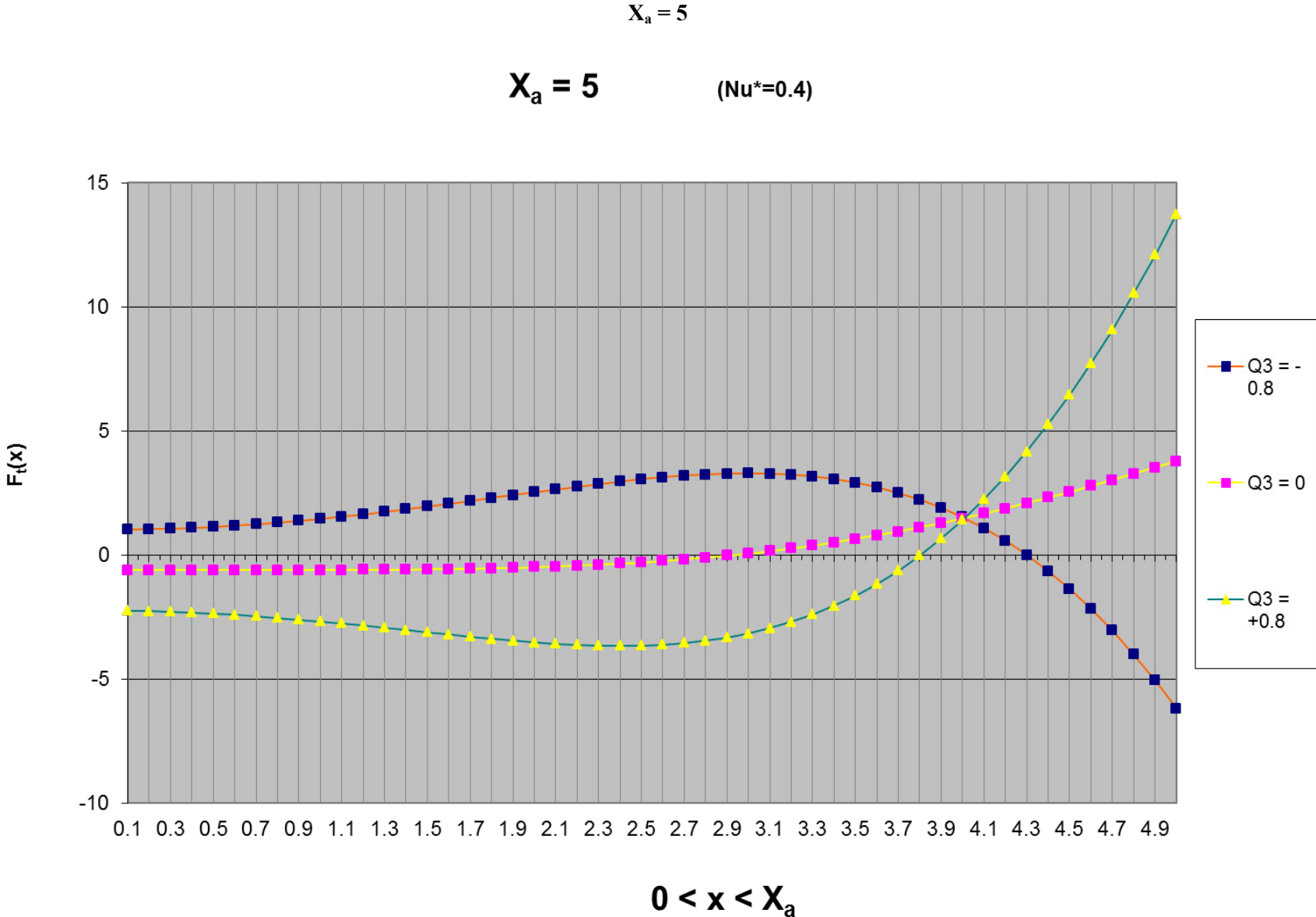
$F_t(X_a)$ ($v^* = 0.4$)

	Fq=Ft(min)					Ft(min)<Fq<Ft(max)						Fq=Ft(max)						
Xa Q3	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	
1	0.9565	0.9653	0.9742	0.9830	0.9919	1.0007	1.0095	1.0184	1.0272	1.0361	1.0449	1.0537	1.0626	1.0714	1.0803	1.0891	1.0979	
2	0.3868	0.5103	0.6339	0.7574	0.8809	1.0044	1.1279	1.2514	1.3749	1.4985	1.6220	1.7455	1.8690	1.9925	2.1160	2.2395	2.3631	
3	-1.1560	-0.7358	-0.3155	0.1048	0.5250	0.9453	1.3655	1.7858	2.2061	2.6263	3.0466	3.4669	3.8871	4.3074	4.7276	5.1479	5.5682	
4	-3.3304	-2.5335	-1.7365	-0.9395	-0.1426	0.6544	1.4514	2.2483	3.0453	3.8423	4.6392	5.4362	6.2332	7.0301	7.8271	8.6241	9.4211	
5	-6.1946	-4.9484	-3.7023	-2.4561	-1.2099	0.0363	1.2825	2.5286	3.7748	5.0210	6.2672	7.5134	8.7595	10.0057	11.2519	12.4981	13.7443	
6	-9.9120	-8.1122	-6.3123	-4.5124	-2.7126	-0.9127	0.8871	2.6870	4.4868	6.2867	8.0866	9.8864	11.6863	13.4861	15.2860	17.0859	18.8857	
7	-14.4624	-12.0043	-9.5461	-7.0880	-4.6299	-2.1718	0.2864	2.7445	5.2026	7.6607	10.1189	12.5770	15.0351	17.4933	19.9514	22.4095	24.8676	
8	-19.8122	-16.5960	-13.3799	-10.1637	-6.9475	-3.7313	-0.5151	2.7010	5.9172	9.1334	12.3496	15.5658	18.7820	21.9981	25.2143	28.4305	31.6467	
9	-25.9594	-21.8858	-17.8122	-13.7385	-9.6649	-5.5913	-1.5177	2.5560	6.6296	10.7032	14.7768	18.8504	22.9241	26.9977	31.0713	35.1449	39.2186	
10	-32.9071	-27.8761	-22.8452	-17.8142	-12.7832	-7.7523	-2.7213	2.3097	7.3406	12.3716	17.4025	22.4335	27.4645	32.4954	37.5264	42.5574	47.5883	
11	-40.6555	-34.5672	-28.4789	-22.3906	-16.3023	-10.2140	-4.1257	1.9626	8.0509	14.1392	20.2275	26.3158	32.4041	38.4924	44.5807	50.6690	56.7573	
12	-49.2043	-41.9587	-34.7131	-27.4675	-20.2219	-12.9763	-5.7307	1.5149	8.7606	16.0062	23.2518	30.4974	37.7430	44.9886	52.2342	59.4798	66.7254	
13	-58.5532	-50.0504	-41.5475	-33.0446	-24.5417	-16.0388	-7.5360	0.9669	9.4698	17.9727	26.4756	34.9784	43.4813	51.9842	60.4871	68.9899	77.4928	
14	-68.7023	-58.8422	-48.9820	-39.1219	-29.2618	-19.4017	-9.5415	0.3186	10.1787	20.0388	29.8989	39.7591	49.6192	59.4793	69.3394	79.1996	89.0597	
15	-79.6514	-68.3341	-57.0167	-45.6994	-34.3820	-23.0647	-11.7473	-0.4300	10.8874	22.2047	33.5220	44.8394	56.1567	67.4741	78.7914	90.1088	101.4261	
16	-91.4006	-78.5261	-65.6515	-52.7770	-39.9024	-27.0279	-14.1533	-1.2788	11.5958	24.4703	37.3449	50.2194	63.0940	75.9685	88.8431	101.7176	114.5922	
17	-103.9499	-89.4181	-74.8864	-60.3546	-45.8229	-31.2912	-16.7594	-2.2277	12.3040	26.8358	41.3675	55.8993	70.4310	84.9627	99.4945	114.0262	128.5579	
18	-117.2992	-101.0102	-84.7213	-68.4324	-52.1435	-35.8546	-19.5657	-3.2767	13.0122	29.3011	45.5900	61.8789	78.1678	94.4567	110.7457	127.0346	143.3235	
19	-131.4485	-113.3024	-95.1563	-77.0102	-58.8642	-40.7181	-22.5720	-4.4259	13.7202	31.8663	50.0123	68.1584	86.3045	104.4506	122.5967	140.7428	158.8888	
20	-146.3978	-126.2946	-106.1914	-86.0881	-65.9849	-45.8816	-25.7784	-5.6752	14.4281	34.5313	54.6346	74.7378	94.8410	114.9443	135.0475	155.1508	175.2540	

$F_q = (Z_d + Q_3 Z_w) X_a^4 / 2$

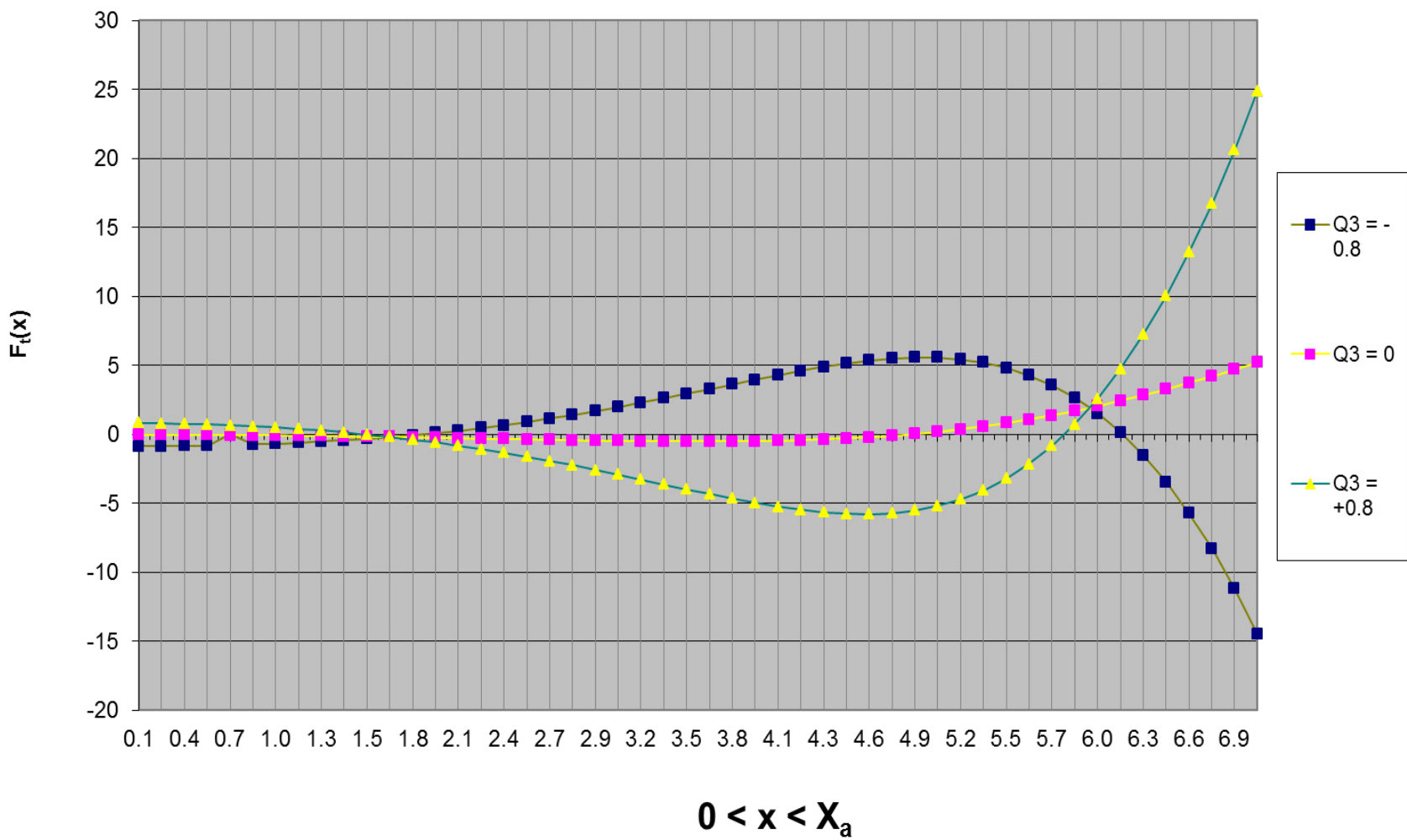
	Z(CODAP)>0																
XaIQ3	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
1	0.9565	0.9653	0.9742	0.9830	0.9919	1.0007	1.0095	1.0184	1.0272	1.0361	1.0449	1.0537	1.0626	1.0714	1.0803	1.0891	1.0979
2	0.3868	0.5103	0.6339	0.7574	0.8809	1.0044	1.1279	1.2514	1.3749	1.4985	1.6220	1.7455	1.8690	1.9925	2.1160	2.2395	2.3631
3	-1.1560	-0.7358	-0.3155	0.1048	0.5250	0.9453	1.3655	1.7858	2.2061	2.6263	3.0466	3.4669	3.8871	4.3074	4.7276	5.1479	5.5682
4	-3.3304	-2.5335	-1.7365	-0.9395	-0.1426	0.6544	1.4514	2.2483	3.0453	3.8423	4.6392	5.4362	6.2332	7.0301	7.8271	8.6241	9.4211
5	-6.1946	-4.9484	-3.7023	-2.4561	-1.2099	0.0363	1.2825	2.5286	3.7748	5.0210	6.2672	7.5134	8.7595	10.0057	11.2519	12.4981	13.7443
6	-9.9120	-8.1122	-6.3123	-4.5124	-2.7126	-0.9127	0.8871	2.6870	4.4868	6.2867	8.0866	9.8864	11.6863	13.4861	15.2860	17.0859	18.8857
7	-14.4624	-12.0043	-9.5461	-7.0880	-4.6299	-2.1718	0.2864	2.7445	5.2026	7.6607	10.1189	12.5770	15.0351	17.4933	19.9514	22.4095	24.8676
8	-19.8122	-16.5960	-13.3799	-10.1637	-6.9475	-3.7313	-0.5151	2.7010	5.9172	9.1334	12.3496	15.5658	18.7820	21.9981	25.2143	28.4305	31.6467
9	-25.9594	-21.8858	-17.8122	-13.7385	-9.6649	-5.5913	-1.5177	2.5560	6.6296	10.7032	14.7768	18.8504	22.9241	26.9977	31.0713	35.1449	39.2186
10	-32.9071	-27.8761	-22.8452	-17.8142	-12.7832	-7.7523	-2.7213	2.3097	7.3406	12.3716	17.4025	22.4335	27.4645	32.4954	37.5264	42.5574	47.5883
11	-40.6555	-34.5672	-28.4789	-22.3906	-16.3023	-10.2140	-4.1257	1.9626	8.0509	14.1392	20.2275	26.3158	32.4041	38.4924	44.5807	50.6690	56.7573
12	-49.2043	-41.9587	-34.7131	-27.4675	-20.2219	-12.9763	-5.7307	1.5149	8.7606	16.0062	23.2518	30.4974	37.7430	44.9886	52.2342	59.4798	66.7254
13	-58.5532	-50.0504	-41.5475	-33.0446	-24.5417	-16.0388	-7.5360	0.9669	9.4698	17.9727	26.4756	34.9784	43.4813	51.9842	60.4871	68.9899	77.4928
14	-68.7023	-58.8421	-48.9820	-39.1219	-29.2618	-19.4017	-9.5415	0.3186	10.1787	20.0388	29.8989	39.7591	49.6192	59.4793	69.3394	79.1996	89.0597
15	-79.6514	-68.3341	-57.0167	-45.6994	-34.3820	-23.0647	-11.7473	-0.4300	10.8874	22.2047	33.5220	44.8394	56.1567	67.4741	78.7914	90.1088	101.4261
16	-91.4006	-78.5261	-65.6515	-52.7770	-39.9024	-27.0279	-14.1533	-1.2788	11.5958	24.4703	37.3449	50.2194	63.0940	75.9685	88.8431	101.7176	114.5922
17	-103.9499	-89.4181	-74.8864	-60.3546	-45.8229	-31.2912	-16.7594	-2.2277	12.3040	26.8358	41.3675	55.8993	70.4310	84.9627	99.4945	114.0262	128.5579
18	-117.2992	-101.0102	-84.7213	-68.4324	-52.1435	-35.8546	-19.5657	-3.2768	13.0122	29.3011	45.5900	61.8789	78.1678	94.4568	110.7457	127.0346	143.3235
19	-131.4485	-113.3024	-95.1563	-77.0102	-58.8642	-40.7181	-22.5720	-4.4259	13.7202	31.8663	50.0123	68.1584	86.3045	104.4506	122.5967	140.7428	158.8888
20	-146.3978	-126.2946	-106.1914	-86.0881	-65.9849	-45.8816	-25.7784	-5.6752	14.4281	34.5313	54.6346	74.7378	94.8410	114.9443	135.0475	155.1508	175.2540





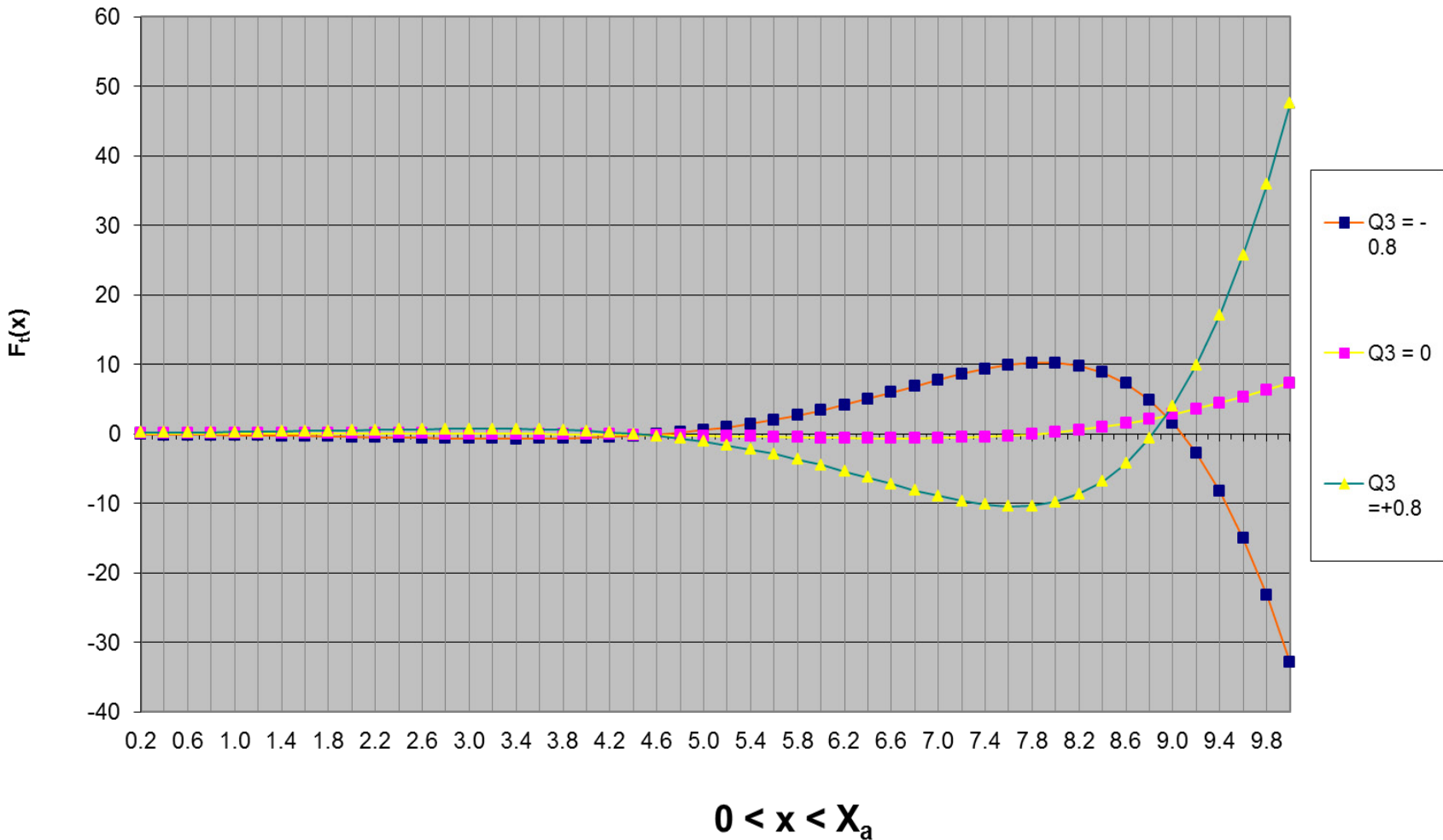
$$X_a = 7$$

$$X_a = 7 \quad (Nu^* = 0.4)$$



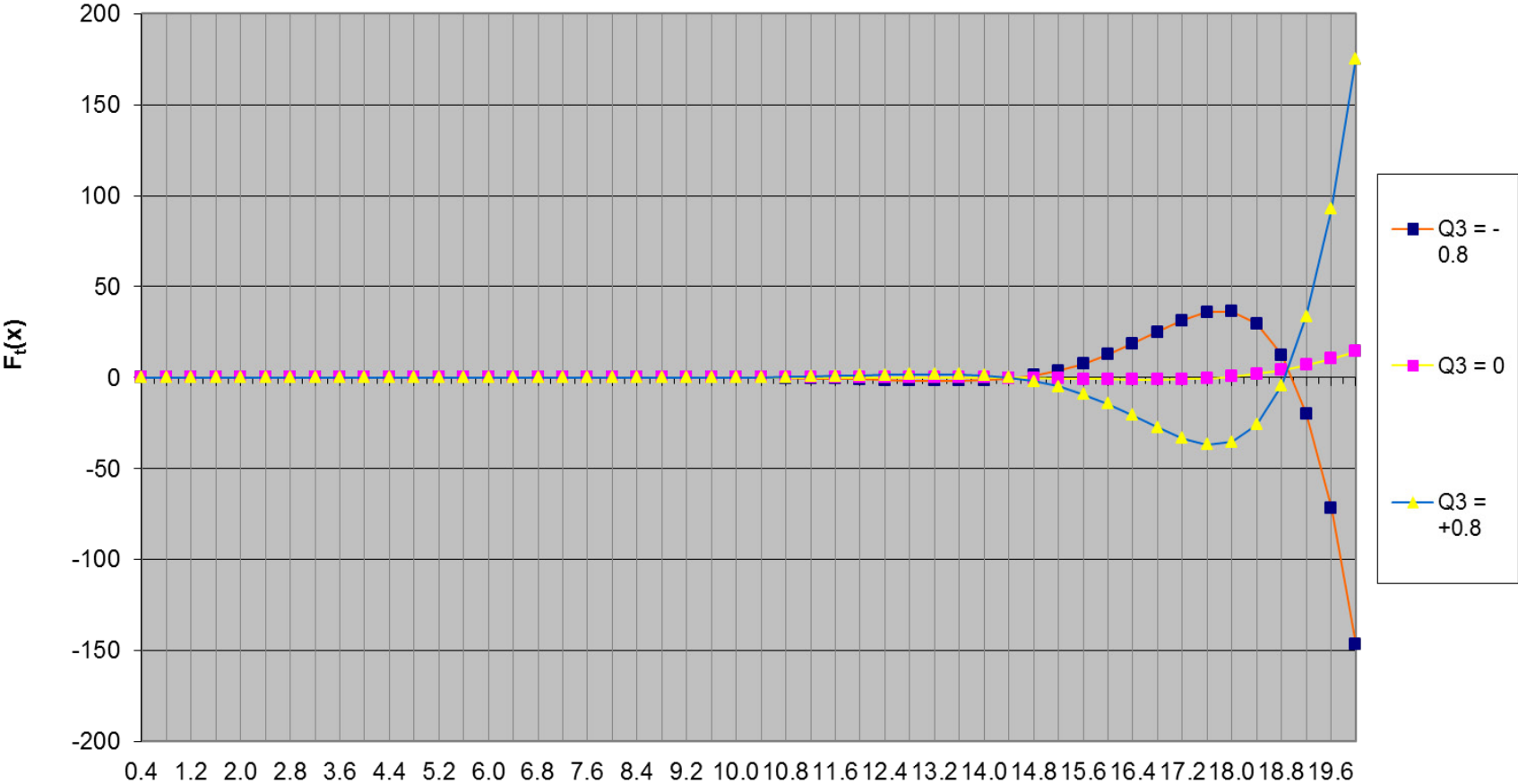
$X_a = 10$

$X_a = 10 \quad (Nu^*=0.4)$



$X_a = 20$

$X_a = 20$ (Nu*=0.4)



$0 < x < X_a$

$F_{t,min}$ vs X_a and Q_3

F _{t,min} =MinF _t (x)																	(v* = 0.4)
	IF _{t,min} < F _{t,max}		IF _{t,min} > F _{t,max}			2 values for F _{inside}					IF _{t,min} < F _{t,max}						
X _a Q ₃	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
1	0.9565	0.9653	0.9742	0.9830	0.9919	0.9941	0.9853	0.9765	0.9677	0.9589	0.9501	0.9413	0.9325	0.9237	0.9149	0.9061	0.8973
2	0.3868	0.5103	0.6339	0.7574	0.8809	0.9159	0.7991	0.6822	0.5654	0.4486	0.3318	0.2150	0.0982	-0.0187	-0.1355	-0.2523	-0.3691
3	-1.1560	-0.7358	-0.3155	0.1048	0.5250	0.6932	0.3721	0.0510	-0.2702	-0.5913	-0.9124	-1.2335	-1.5546	-1.8757	-2.1968	-2.5179	-2.8390
4	-3.3304	-2.5335	-1.7365	-0.9395	-0.1426	0.3679	0.0062	-0.3555	-0.7171	-1.0788	-1.4404	-1.8021	-2.1637	-2.5254	-2.8871	-3.2487	-3.6104
5	-6.1946	-4.9484	-3.7023	-2.4561	-1.2099	0.0081	-0.1968	-0.4018	-0.6088	-0.8977	-1.2503	-1.6307	-2.0251	-2.4271	-2.8343	-3.2446	-3.6586
6	-9.9120	-8.1122	-6.3123	-4.5124	-2.7126	-0.9127	-0.2703	-0.2810	-0.5053	-0.9172	-1.3958	-1.9026	-2.4221	-2.9513	-3.4844	-4.0175	-4.5529
7	-14.4624	-12.0043	-9.5461	-7.0880	-4.6299	-2.1718	-0.2400	-0.2098	-0.5187	-1.0649	-1.6911	-2.3502	-3.0274	-3.7090	-4.3905	-5.0827	-5.7786
8	-19.8122	-16.5960	-13.3799	-10.1637	-6.9475	-3.7313	-0.5151	-0.1858	-0.5545	-1.2450	-2.0372	-2.8708	-3.7126	-4.5685	-5.4348	-6.3010	-7.1672
9	-25.9594	-21.8858	-17.8122	-13.7385	-9.6649	-5.5913	-1.5177	-0.1765	-0.5918	-1.4437	-2.4218	-3.4507	-4.4871	-5.5235	-6.5880	-7.6531	-8.7183
10	-32.9071	-27.8761	-22.8452	-17.8142	-12.7832	-7.7523	-2.7213	-0.1722	-0.6325	-1.6680	-2.8590	-4.0790	-5.3486	-6.6201	-7.8915	-9.1629	-10.4343
11	-40.6555	-34.5672	-28.4789	-22.3906	-16.3023	-10.2140	-4.1257	-0.1715	-0.6747	-1.9039	-3.3266	-4.7998	-6.3010	-7.8021	-9.3032	-10.8043	-12.3054
12	-49.2043	-41.9587	-34.7131	-27.4675	-20.2219	-12.9763	-5.7307	-0.1744	-0.7187	-2.1695	-3.8366	-5.5727	-7.3330	-9.0934	-10.8538	-12.6141	-14.3745
13	-58.5532	-50.0504	-41.5475	-33.0446	-24.5417	-16.0388	-7.5360	-0.1804	-0.7625	-2.4509	-4.4007	-6.3866	-8.4373	-10.4880	-12.5386	-14.5893	-16.6400
14	-68.7023	-58.8422	-48.9820	-39.1219	-29.2618	-19.4017	-9.5415	-0.1883	-0.8077	-2.7544	-5.0134	-7.2927	-9.5720	-11.9125	-14.2749	-16.6373	-18.9996
15	-79.6514	-68.3341	-57.0167	-45.6994	-34.3820	-23.0647	-11.7473	-0.4300	-0.8527	-3.0967	-5.6341	-8.2827	-10.9313	-13.5798	-16.2284	-18.8770	-21.5255
16	-91.4006	-78.5261	-65.6515	-52.7770	-39.9024	-27.0279	-14.1533	-1.2788	-0.8906	-3.4346	-6.3231	-9.2117	-12.2061	-15.2319	-18.2577	-21.2834	-24.3092
17	-103.9499	-89.4181	-74.8864	-60.3546	-45.8229	-31.2912	-16.7594	-2.2277	-0.9428	-3.8003	-7.0446	-10.3976	-13.7506	-17.1037	-20.4567	-23.8098	-27.1628
18	-117.2992	-101.0102	-84.7213	-68.4324	-52.1435	-35.8546	-19.5657	-3.2767	-0.9864	-4.1867	-7.8225	-11.4584	-15.0942	-18.7671	-22.5323	-26.2975	-30.0627
19	-131.4485	-113.3024	-95.1563	-77.0102	-58.8642	-40.7181	-22.5720	-4.4259	-1.0333	-4.6054	-8.5301	-12.7017	-16.8732	-21.0448	-25.2164	-29.3880	-33.5595
20	-146.3978	-126.2946	-106.1914	-86.0881	-65.9849	-45.8816	-25.7784	-5.6752	-1.0736	-4.9554	-9.4954	-14.0354	-18.5754	-23.1154	-27.6554	-32.1954	-36.7354

	x(min)																
			x(min)=X _a						x(min)=0			0<x(min)<X _a					
X _a Q ₃	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
1	1.0000	1.0000	1.0000	1.0000	1.0000	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200
2	2.0000	2.0000	2.0000	2.0000	2.0000	0.0400	0.0400	0.0400	0.0400	0.0400	0.0400	0.0400	0.0400	0.0400	0.0400	0.0400	0.0400
3	3.0000	3.0000	3.0000	3.0000	3.0000	0.0600	0.0600	0.0600	0.0600	0.0600	0.0600	0.0600	0.0600	0.0600	0.0600	0.0600	0.0600
4	4.0000	4.0000	4.0000	4.0000	4.0000	0.0800	0.0800	0.0800	0.0800	0.0800	0.0800	0.0800	0.0800	0.0800	0.0800	0.0800	0.0800
5	5.0000	5.0000	5.0000	5.0000	5.0000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000
6	6.0000	6.0000	6.0000	6.0000	6.0000	0.1200	0.1200	0.1200	0.1200	0.1200	0.1200	0.1200	0.1200	0.1200	0.1200	0.1200	0.1200
7	7.0000	7.0000	7.0000	7.0000	7.0000	0.1400	0.1400	0.1400	0.1400	0.1400	0.1400	0.1400	0.1400	0.1400	0.1400	0.1400	0.1400
8	8.0000	8.0000	8.0000	8.0000	8.0000	0.1600	0.1600	0.1600	0.1600	0.1600	0.1600	0.1600	0.1600	0.1600	0.1600	0.1600	0.1600
9	9.0000	9.0000	9.0000	9.0000	9.0000	0.1800	0.1800	0.1800	0.1800	0.1800	0.1800	0.1800	0.1800	0.1800	0.1800	0.1800	0.1800
10	10.0000	10.0000	10.0000	10.0000	10.0000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000
11	11.0000	11.0000	11.0000	11.0000	11.0000	0.2200	0.2200	0.2200	0.2200	0.2200	0.2200	0.2200	0.2200	0.2200	0.2200	0.2200	0.2200
12	12.0000	12.0000	12.0000	12.0000	12.0000	0.2400	0.2400	0.2400	0.2400	0.2400	0.2400	0.2400	0.2400	0.2400	0.2400	0.2400	0.2400
13	13.0000	13.0000	13.0000	13.0000	13.0000	0.2600	0.2600	0.2600	0.2600	0.2600	0.2600	0.2600	0.2600	0.2600	0.2600	0.2600	0.2600
14	14.0000	14.0000	14.0000	14.0000	14.0000	0.2800	0.2800	0.2800	0.2800	0.2800	0.2800	0.2800	0.2800	0.2800	0.2800	0.2800	0.2800
15	15.0000	15.0000	15.0000	15.0000	15.0000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000
16	16.0000	16.0000	16.0000	16.0000	16.0000	0.3200	0.3200	0.3200	0.3200	0.3200	0.3200	0.3200	0.3200	0.3200	0.3200	0.3200	0.3200
17	17.0000	17.0000	17.0000	17.0000	17.0000	0.3400	0.3400	0.3400	0.3400	0.3400	0.3400	0.3400	0.3400	0.3400	0.3400	0.3400	0.3400
18	18.0000	18.0000	18.0000	18.0000	18.0000	0.3600	0.3600	0.3600	0.3600	0.3600	0.3600	0.3600	0.3600	0.3600	0.3600	0.3600	0.3600
19	19.0000	19.0000	19.0000	19.0000	19.0000	0.3800	0.3800	0.3800	0.3800	0.3800	0.3800	0.3800	0.3800	0.3800	0.3800	0.3800	0.3800
20	20.0000	20.0000	20.0000	20.0000	20.0000	0.4000	0.4000	0.4000	0.4000	0.4000	0.4000	0.4000	0.4000	0.4000	0.4000	0.4000	0.4000

$F_{t,max}$ vs X_a and Q_3

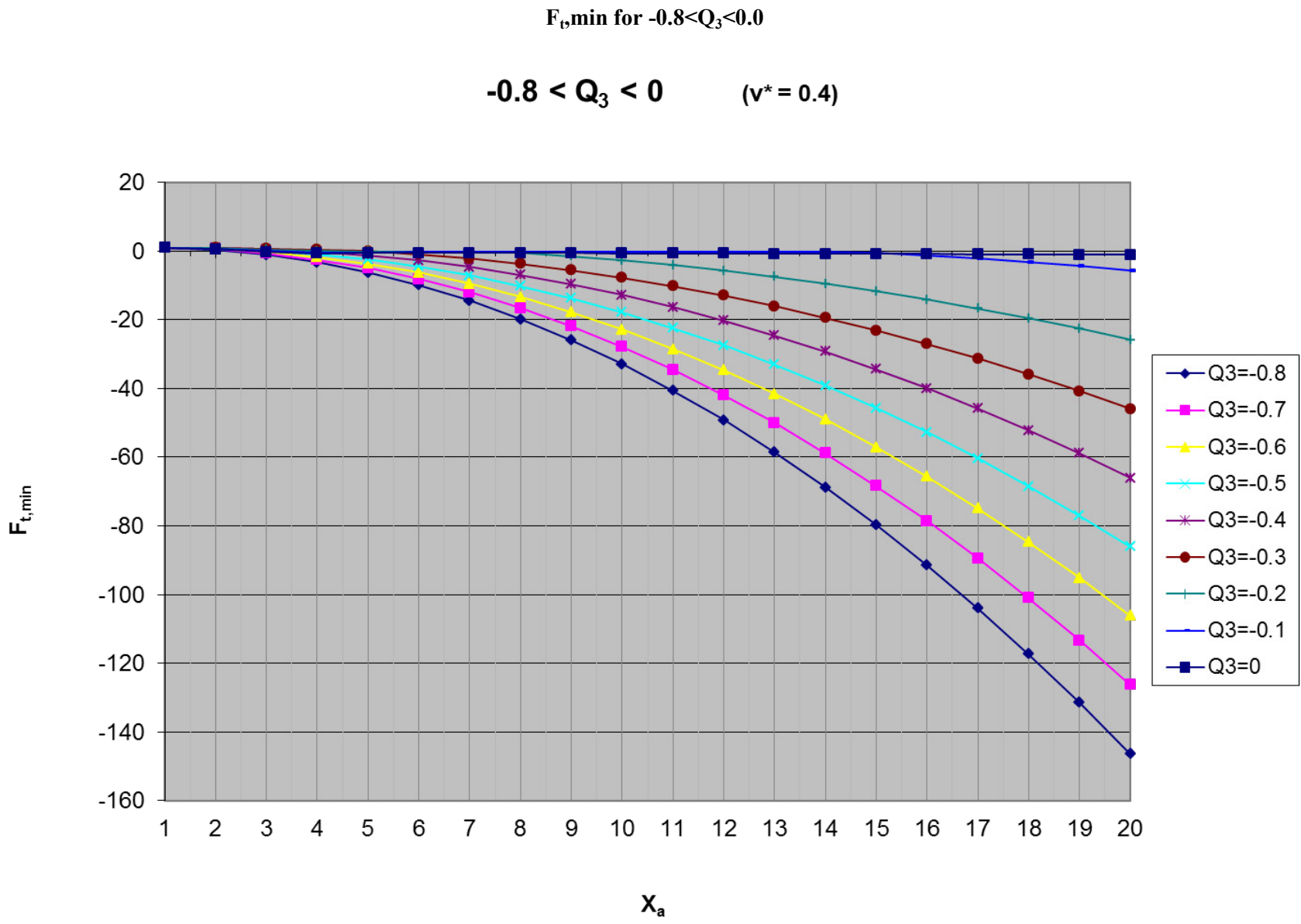
$F_{t,max} = \text{Max} F_t(x)$ $(v^* = 0.4)$																	
	$F_{t,max} > F_{t,min} $		$F_{t,max} < F_{t,min} $			2 values for $F_{t,inside}$						$F_{t,max} > F_{t,min} $					
$X_a \backslash Q_3$	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
1	1.0381	1.0293	1.0205	1.0117	1.0032	1.0020	1.0095	1.0184	1.0272	1.0361	1.0449	1.0537	1.0626	1.0714	1.0803	1.0891	1.0979
2	1.5000	1.3831	1.2663	1.1495	1.0438	1.0284	1.1279	1.2514	1.3749	1.4985	1.6220	1.7455	1.8690	1.9925	2.1160	2.2395	2.3631
3	2.2987	1.9776	1.6569	1.3670	1.1574	1.1085	1.3655	1.7858	2.2061	2.6263	3.0466	3.4669	3.8871	4.3074	4.7276	5.1479	5.5682
4	2.5990	2.2601	1.9361	1.6366	1.3897	1.2671	1.4891	2.2483	3.0453	3.8423	4.6392	5.4362	6.2332	7.0301	7.8271	8.6241	9.4211
5	3.2877	2.8846	2.4951	2.1198	1.7810	1.5293	1.5752	2.5286	3.7748	5.0210	6.2672	7.5134	8.7595	10.0057	11.2519	12.4981	13.7443
6	4.3381	3.8047	3.2892	2.7797	2.3001	1.8883	1.7178	2.6870	4.4868	6.2867	8.0866	9.8864	11.6863	13.4861	15.2860	17.0859	18.8857
7	5.5769	4.8917	4.2196	3.5475	2.9076	2.3198	1.9388	2.7445	5.2026	7.6607	10.1189	12.5770	15.0351	17.4933	19.9514	22.4095	24.8676
8	6.9813	6.1174	5.2535	4.4218	3.5926	2.8188	2.2248	2.7525	5.9172	9.1334	12.3496	15.5658	18.7820	21.9981	25.2143	28.4305	31.6467
9	8.5253	7.4676	6.4331	5.3985	4.3640	3.3954	2.5710	2.7834	6.6296	10.7032	14.7768	18.8504	22.9241	26.9977	31.0713	35.1449	39.2186
10	10.2342	8.9829	7.7319	6.4808	5.2297	4.0376	2.9645	2.8478	7.3406	12.3716	17.4025	22.4335	27.4645	32.4954	37.5264	42.5574	47.5883
11	12.1389	10.6157	9.1361	7.6613	6.1866	4.7219	3.4132	2.9468	8.0509	14.1392	20.2275	26.3158	32.4041	38.4924	44.5807	50.6690	56.7573
12	14.2426	12.4641	10.6857	8.9072	7.2032	5.5109	3.9181	3.0883	8.7606	16.0062	23.2518	30.4974	37.7430	44.9886	52.2342	59.4798	66.7254
13	16.4752	14.4356	12.3960	10.3564	8.3168	6.3145	4.4432	3.2804	9.4698	17.9727	26.4756	34.9784	43.4813	51.9842	60.4871	68.9899	77.4928
14	18.7983	16.4360	14.0736	11.7904	9.5167	7.2430	5.0294	3.5033	10.1787	20.0388	29.8989	39.7591	49.6192	59.4793	69.3394	79.1996	89.0597
15	21.4523	18.7898	16.1272	13.4647	10.8021	8.1396	5.6805	3.7577	10.8874	22.2047	33.5220	44.8394	56.1567	67.4741	78.7914	90.1088	101.4261
16	24.1031	21.0773	18.0516	15.0258	12.0815	9.2086	6.3356	4.0431	11.5958	24.4703	37.3449	50.2194	63.0940	75.9685	88.8431	101.7176	114.5922
17	26.9523	23.6232	20.2940	16.9649	13.6357	10.3065	6.9774	4.3578	12.3040	26.8358	41.3675	55.8993	70.4310	84.9627	99.4945	114.0262	128.5579
18	30.1805	26.4153	22.6501	18.8849	15.1197	11.3545	7.8050	4.6992	13.0122	29.3011	45.5900	61.8789	78.1678	94.4567	110.7457	127.0346	143.3235
19	33.1856	29.0141	24.8425	20.6709	16.5078	12.5729	8.6379	5.0638	13.7202	31.8663	50.0123	68.1584	86.3045	104.4506	122.5967	140.7428	158.8888
20	36.3441	31.8634	27.3828	22.9021	18.4214	13.9407	9.4600	5.4475	14.4281	34.5313	54.6346	74.7378	94.8410	114.9443	135.0475	155.1508	175.2540

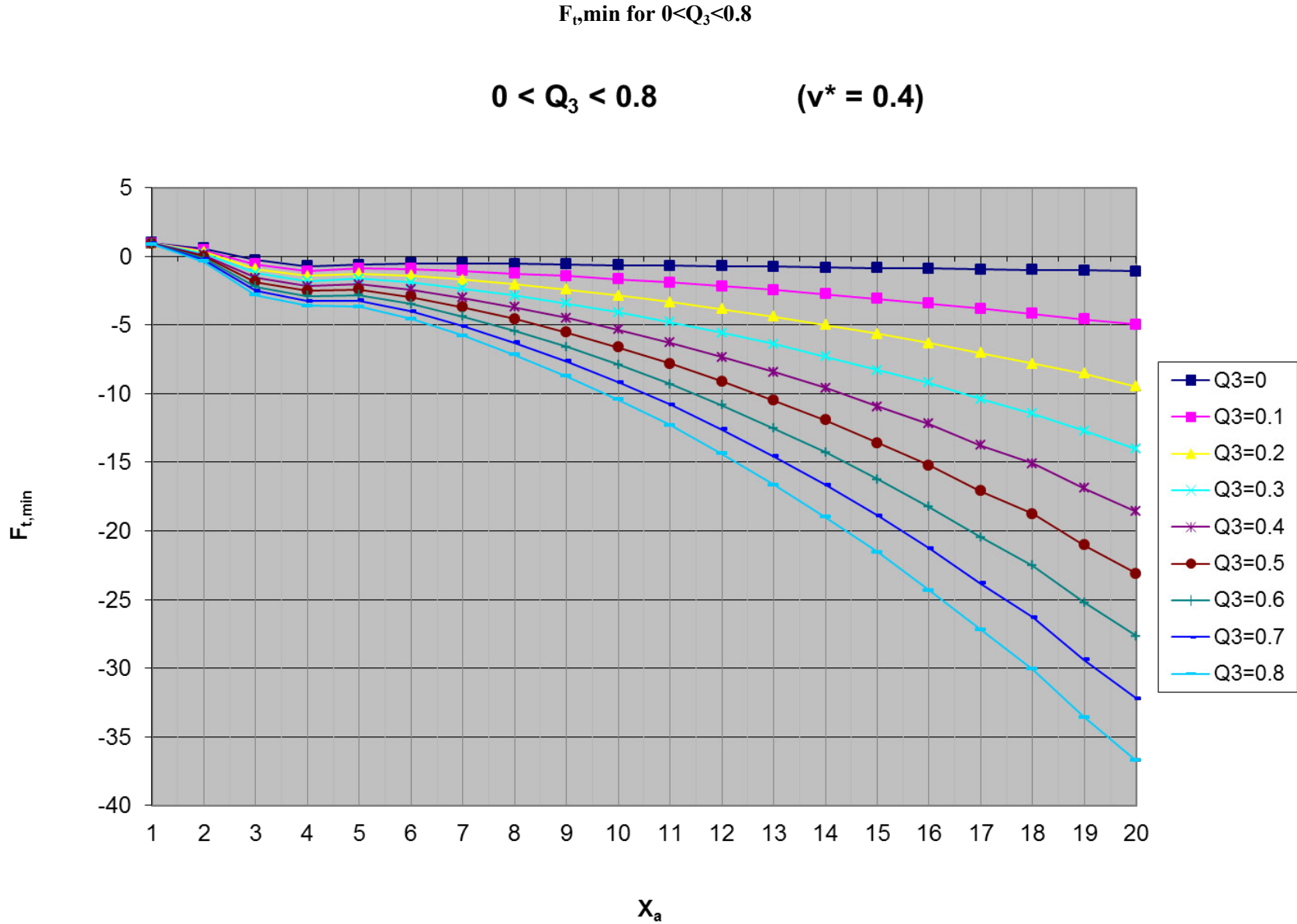
	$x(\text{max})$																	
	$x(\text{max})=0$		$0 < x(\text{max}) < X_a$								$x(\text{max})=X_a$							
$X_a \backslash Q_3$	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	
1	0.0200	0.0200	0.0200	0.0200	0.3800	0.8400	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	
2	0.0400	0.0400	0.0400	0.0400	0.9200	1.6400	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	
3	0.0600	0.0600	0.3600	1.0800	1.7400	2.4000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	
4	1.8400	1.9200	2.0800	2.3200	2.5600	3.0400	3.7600	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	
5	3.0000	3.0000	3.1000	3.2000	3.4000	3.8000	4.4000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	
6	3.9600	4.0800	4.0800	4.2000	4.3200	4.5600	5.1600	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	
7	4.9000	5.0400	5.0400	5.0400	5.1800	5.4600	5.8800	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000	
8	5.9200	5.9200	5.9200	6.0800	6.0800	6.2400	6.7200	7.8400	8.0000	8.0000	8.0000	8.0000	8.0000	8.0000	8.0000	8.0000	8.0000	
9	6.8400	7.0200	7.0200	7.0200	7.0200	7.2000	7.5600	8.6400	9.0000	9.0000	9.0000	9.0000	9.0000	9.0000	9.0000	9.0000	9.0000	
10	7.8000	8.0000	8.0000	8.0000	8.0000	8.2000	8.4000	9.4000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	
11	8.8000	8.8000	9.0200	9.0200	9.0200	9.2400	9.4600	10.3400	11.0000	11.0000	11.0000	11.0000	11.0000	11.0000	11.0000	11.0000	11.0000	
12	9.8400	9.8400	9.8400	9.8400	10.0800	10.0800	10.3200	11.0400	12.0000	12.0000	12.0000	12.0000	12.0000	12.0000	12.0000	12.0000	12.0000	
13	10.9200	10.9200	10.9200	10.9200	10.9200	11.1800	11.1800	11.9600	13.0000	13.0000	13.0000	13.0000	13.0000	13.0000	13.0000	13.0000	13.0000	
14	11.7600	11.7600	11.7600	12.0400	12.0400	12.0400	12.3200	12.8800	14.0000	14.0000	14.0000	14.0000	14.0000	14.0000	14.0000	14.0000	14.0000	
15	12.9000	12.9000	12.9000	12.9000	12.9000	12.9000	13.2000	13.8000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	
16	13.7600	13.7600	13.7600	13.7600	14.0800	14.0800	14.0800	14.7200	16.0000	16.0000	16.0000	16.0000	16.0000	16.0000	16.0000	16.0000	16.0000	
17	14.9600	14.9600	14.9600	14.9600	14.9600	14.9600	14.9600	15.6400	17.0000	17.0000	17.0000	17.0000	17.0000	17.0000	17.0000	17.0000	17.0000	
18	15.8400	15.8400	15.8400	15.8400	15.8400	15.8400	16.2000	16.5600	18.0000	18.0000	18.0000	18.0000	18.0000	18.0000	18.0000	18.0000	18.0000	
19	16.7200	16.7200	16.7200	16.7200	17.1000	17.1000	17.1000	17.4800	19.0000	19.0000	19.0000	19.0000	19.0000	19.0000	19.0000	19.0000	19.0000	
20	18.0000	18.0000	18.0000	18.0000	18.0000	18.0000	18.0000	18.4000	20.0000	20.0000	20.0000	20.0000	20.0000	20.0000	20.0000	20.0000	20.0000	

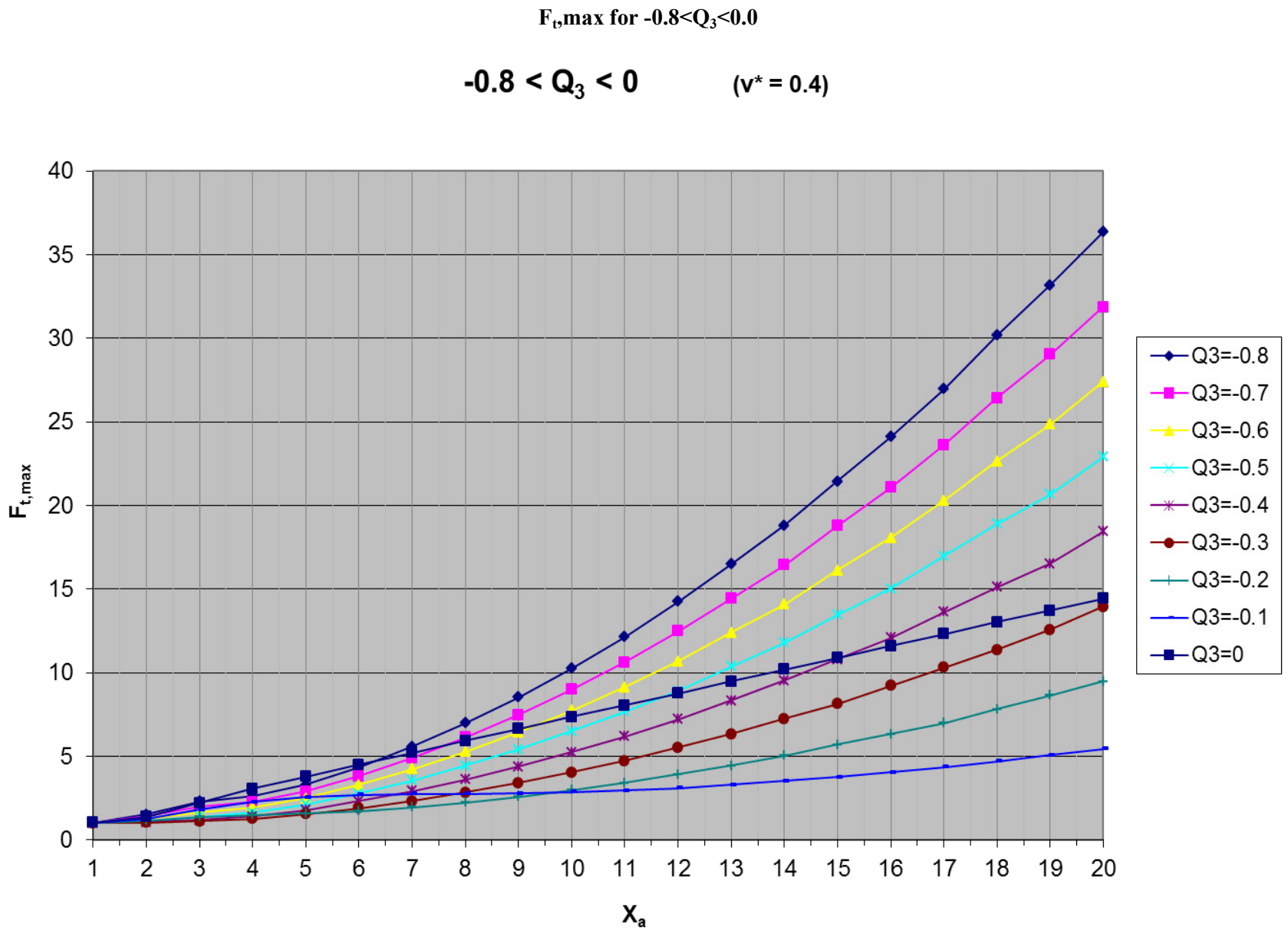
$F_t(X_a)$ and F_q

F _t (X _a)																		(v* = 0.4)
			F _q =F _{t,min}			F _{t,min} <F _q <F _{t,max}						F _q =F _{t,max}						
X _a \Q ₃	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	
1	0.9565	0.9653	0.9742	0.9830	0.9919	1.0007	1.0095	1.0184	1.0272	1.0361	1.0449	1.0537	1.0626	1.0714	1.0803	1.0891	1.0979	
2	0.3868	0.5103	0.6339	0.7574	0.8809	1.0044	1.1279	1.2514	1.3749	1.4985	1.6220	1.7455	1.8690	1.9925	2.1160	2.2395	2.3631	
3	-1.1560	-0.7358	-0.3155	0.1048	0.5250	0.9453	1.3655	1.7858	2.2061	2.6263	3.0466	3.4669	3.8871	4.3074	4.7276	5.1479	5.5682	
4	-3.3304	-2.5335	-1.7365	-0.9395	-0.1426	0.6544	1.4514	2.2483	3.0453	3.8423	4.6392	5.4362	6.2332	7.0301	7.8271	8.6241	9.4211	
5	-6.1946	-4.9484	-3.7023	-2.4561	-1.2099	0.0363	1.2825	2.5286	3.7748	5.0210	6.2672	7.5134	8.7595	10.0057	11.2519	12.4981	13.7443	
6	-9.9120	-8.1122	-6.3123	-4.5124	-2.7126	-0.9127	0.8871	2.6870	4.4868	6.2867	8.0866	9.8864	11.6863	13.4861	15.2860	17.0859	18.8857	
7	-14.4624	-12.0043	-9.5461	-7.0880	-4.6299	-2.1718	0.2864	2.7445	5.2026	7.6607	10.1189	12.5770	15.0351	17.4933	19.9514	22.4095	24.8676	
8	-19.8122	-16.5960	-13.3799	-10.1637	-6.9475	-3.7313	-0.5151	2.7010	5.9172	9.1334	12.3496	15.5658	18.7820	21.9981	25.2143	28.4305	31.6467	
9	-25.9594	-21.8858	-17.8122	-13.7385	-9.6649	-5.5913	-1.5177	2.5560	6.6296	10.7032	14.7768	18.8504	22.9241	26.9977	31.0713	35.1449	39.2186	
10	-32.9071	-27.8761	-22.8452	-17.8142	-12.7832	-7.7523	-2.7213	2.3097	7.3406	12.3716	17.4025	22.4335	27.4645	32.4954	37.5264	42.5574	47.5883	
11	-40.6555	-34.5672	-28.4789	-22.3906	-16.3023	-10.2140	-4.1257	1.9626	8.0509	14.1392	20.2275	26.3158	32.4041	38.4924	44.5807	50.6690	56.7573	
12	-49.2043	-41.9587	-34.7131	-27.4675	-20.2219	-12.9763	-5.7307	1.5149	8.7606	16.0062	23.2518	30.4974	37.7430	44.9886	52.2342	59.4798	66.7254	
13	-58.5532	-50.0504	-41.5475	-33.0446	-24.5417	-16.0388	-7.5360	0.9669	9.4698	17.9727	26.4756	34.9784	43.4813	51.9842	60.4871	68.9899	77.4928	
14	-68.7023	-58.8422	-48.9820	-39.1219	-29.2618	-19.4017	-9.5415	0.3186	10.1787	20.0388	29.8989	39.7591	49.6192	59.4793	69.3394	79.1996	89.0597	
15	-79.6514	-68.3341	-57.0167	-45.6994	-34.3820	-23.0647	-11.7473	-0.4300	10.8874	22.2047	33.5220	44.8394	56.1567	67.4741	78.7914	90.1088	101.4261	
16	-91.4006	-78.5261	-65.6515	-52.7770	-39.9024	-27.0279	-14.1533	-1.2788	11.5958	24.4703	37.3449	50.2194	63.0940	75.9685	88.8431	101.7176	114.5922	
17	-103.9499	-89.4181	-74.8864	-60.3546	-45.8229	-31.2912	-16.7594	-2.2277	12.3040	26.8358	41.3675	55.8993	70.4310	84.9627	99.4945	114.0262	128.5579	
18	-117.2992	-101.0102	-84.7213	-68.4324	-52.1435	-35.8546	-19.5657	-3.2767	13.0122	29.3011	45.5900	61.8789	78.1678	94.4567	110.7457	127.0346	143.3235	
19	-131.4485	-113.3024	-95.1563	-77.0102	-58.8642	-40.7181	-22.5720	-4.4259	13.7202	31.8663	50.0123	68.1584	86.3045	104.4506	122.5967	140.7428	158.8888	
20	-146.3978	-126.2946	-106.1914	-86.0881	-65.9849	-45.8816	-25.7784	-5.6752	14.4281	34.5313	54.6346	74.7378	94.8410	114.9443	135.0475	155.1508	175.2540	

F _q = (Z _d + Q ₃ Z _v) X _a ^{4/2}																	
							Z(CODAP)>0										
X _a \Q ₃	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
1	0.9565	0.9653	0.9742	0.9830	0.9919	1.0007	1.0095	1.0184	1.0272	1.0361	1.0449	1.0537	1.0626	1.0714	1.0803	1.0891	1.0979
2	0.3868	0.5103	0.6339	0.7574	0.8809	1.0044	1.1279	1.2514	1.3749	1.4985	1.6220	1.7455	1.8690	1.9925	2.1160	2.2395	2.3631
3	-1.1560	-0.7358	-0.3155	0.1048	0.5250	0.9453	1.3655	1.7858	2.2061	2.6263	3.0466	3.4669	3.8871	4.3074	4.7276	5.1479	5.5682
4	-3.3304	-2.5335	-1.7365	-0.9395	-0.1426	0.6544	1.4514	2.2483	3.0453	3.8423	4.6392	5.4362	6.2332	7.0301	7.8271	8.6241	9.4210
5	-6.1946	-4.9484	-3.7023	-2.4561	-1.2099	0.0363	1.2825	2.5286	3.7748	5.0210	6.2672	7.5134	8.7595	10.0057	11.2519	12.4981	13.7443
6	-9.9120	-8.1122	-6.3123	-4.5124	-2.7126	-0.9127	0.8871	2.6870	4.4868	6.2867	8.0866	9.8864	11.6863	13.4861	15.2860	17.0859	18.8857
7	-14.4624	-12.0043	-9.5461	-7.0880	-4.6299	-2.1718	0.2864	2.7445	5.2026	7.6607	10.1189	12.5770	15.0351	17.4933	19.9514	22.4095	24.8676
8	-19.8122	-16.5960	-13.3799	-10.1637	-6.9475	-3.7313	-0.5151	2.7010	5.9172	9.1334	12.3496	15.5658	18.7820	21.9981	25.2143	28.4305	31.6467
9	-25.9594	-21.8858	-17.8122	-13.7385	-9.6649	-5.5913	-1.5177	2.5560	6.6296	10.7032	14.7768	18.8504	22.9241	26.9977	31.0713	35.1449	39.2186
10	-32.9071	-27.8761	-22.8452	-17.8142	-12.7832	-7.7523	-2.7213	2.3097	7.3406	12.3716	17.4025	22.4335	27.4645	32.4954	37.5264	42.5574	47.5883
11	-40.6555	-34.5672	-28.4789	-22.3906	-16.3023	-10.2140	-4.1257	1.9626	8.0509	14.1392	20.2275	26.3158	32.4041	38.4924	44.5807	50.6690	56.7573
12	-49.2043	-41.9587	-34.7131	-27.4675	-20.2219	-12.9763	-5.7307	1.5149	8.7606	16.0062	23.2518	30.4974	37.7430	44.9886	52.2342	59.4798	66.7254
13	-58.5532	-50.0504	-41.5475	-33.0446	-24.5417	-16.0388	-7.5360	0.9669	9.4698	17.9727	26.4756	34.9784	43.4813	51.9842	60.4871	68.9899	77.4928
14	-68.7023	-58.8421	-48.9820	-39.1219	-29.2618	-19.4017	-9.5415	0.3186	10.1787	20.0388	29.8989	39.7591	49.6192	59.4793	69.3394	79.1996	89.0597
15	-79.6514	-68.3341	-57.0167	-45.6994	-34.3820	-23.0647	-11.7473	-0.4300	10.8874	22.2047	33.5220	44.8394	56.1567	67.4741	78.7914	90.1088	101.4261
16	-91.4006	-78.5261	-65.6515	-52.7770	-39.9024	-27.0279	-14.1533	-1.2788	11.5958	24.4703	37.3449	50.2194	63.0940	75.9685	88.8431	101.7176	114.5922
17	-103.9499	-89.4181	-74.8864	-60.3546	-45.8229	-31.2912	-16.7594	-2.2277	12.3040	26.8358	41.3675	55.8993	70.4310	84.9627	99.4945	114.0262	128.5579
18	-117.2992	-101.0102	-84.7213	-68.4324	-52.1435	-35.8546	-19.5657	-3.2768	13.0122	29.3011	45.5900	61.8789	78.1678	94.4568	110.7457	127.0346	143.3235
19	-131.4485	-113.3024	-95.1563	-77.0102	-58.8642	-40.7181	-22.5720	-4.4259	13.7202	31.8663	50.0123	68.1584	86.3045	104.4506	122.5967	140.7428	158.8888
20	-146.3978	-126.2946	-106.1914	-86.0881	-65.9849	-45.8816	-25.7784	-5.6752	14.4281	34.5313	54.6346	74.7378	94.8410	114.9443	135.0475	155.1508	175.2540

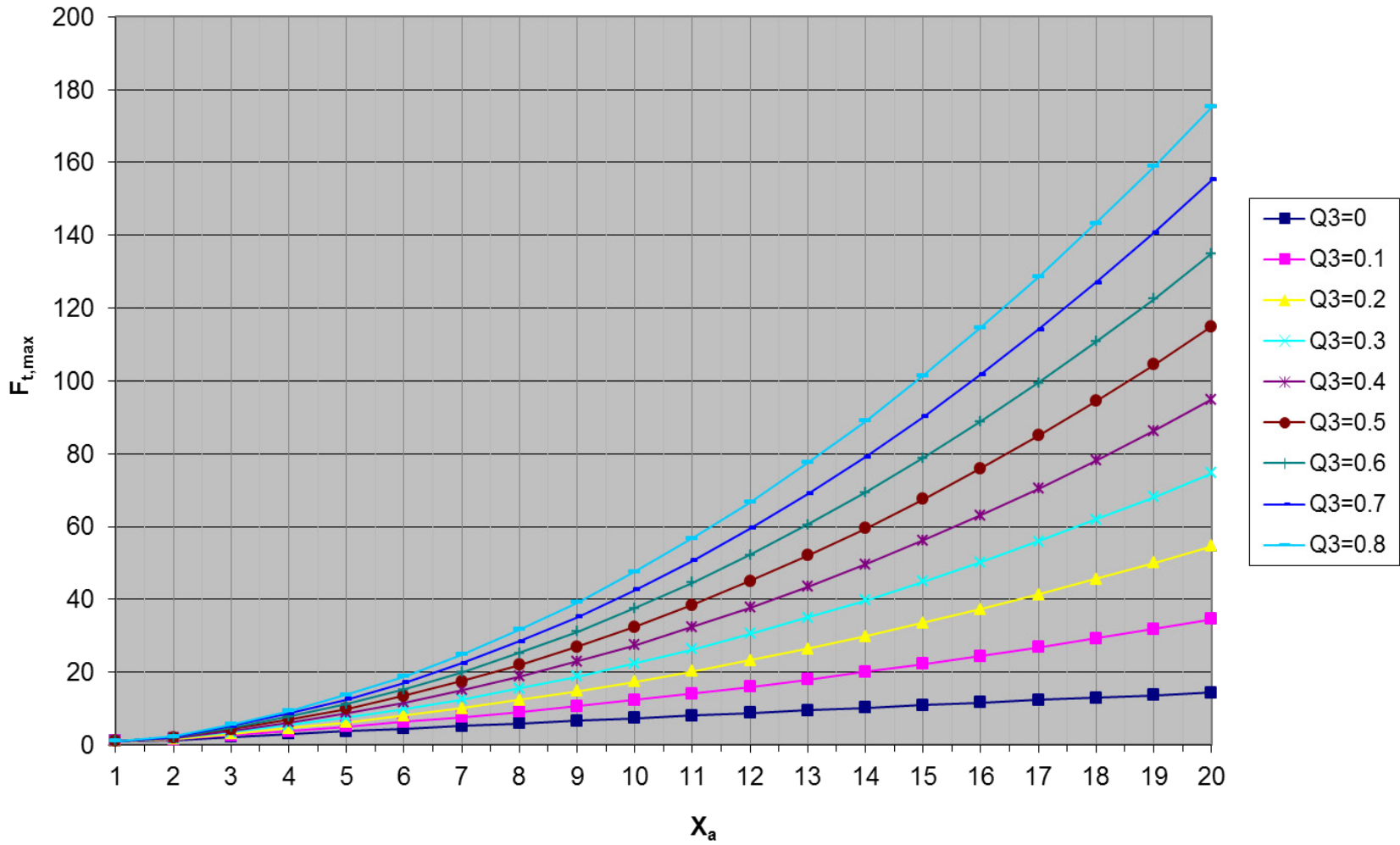


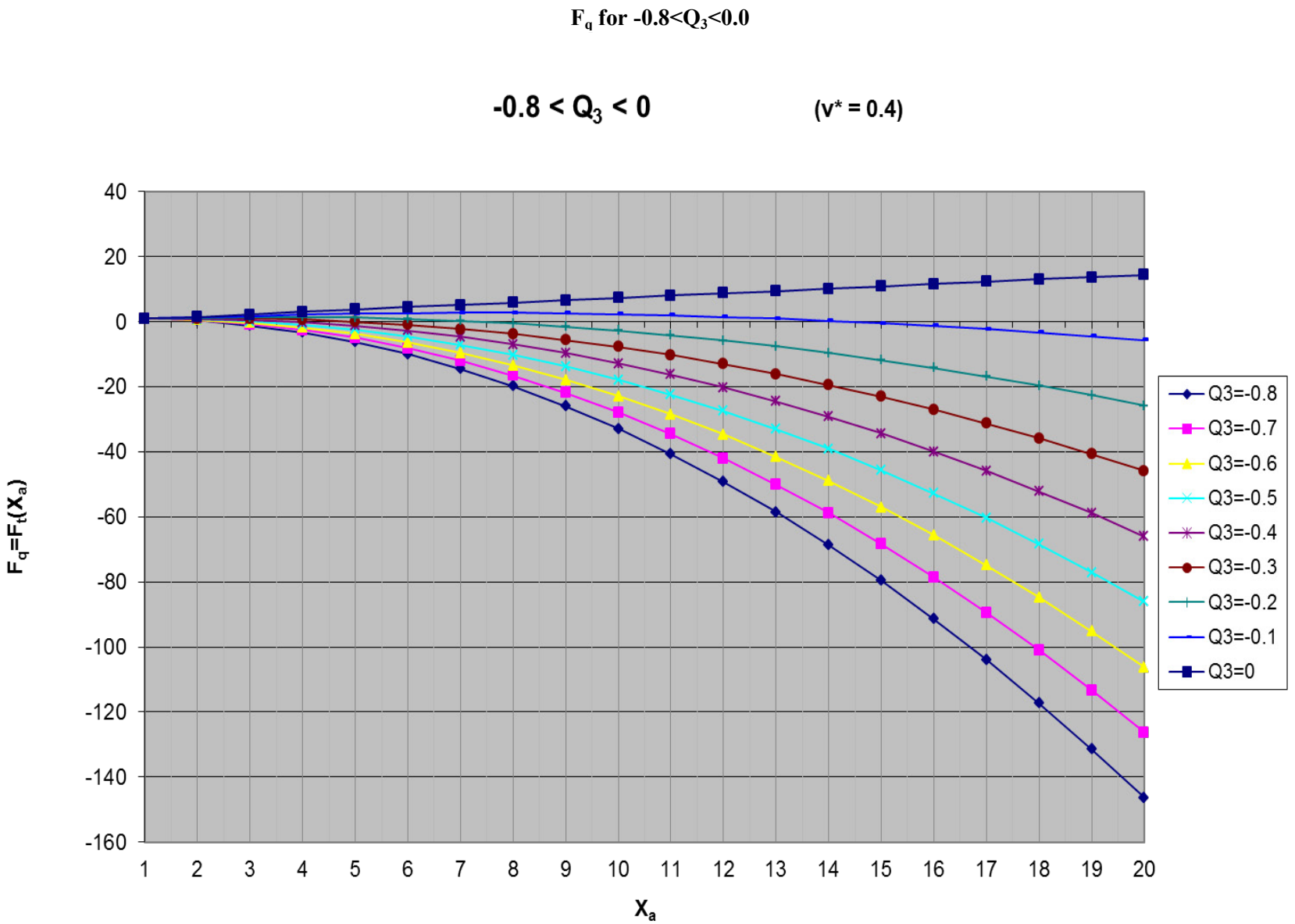


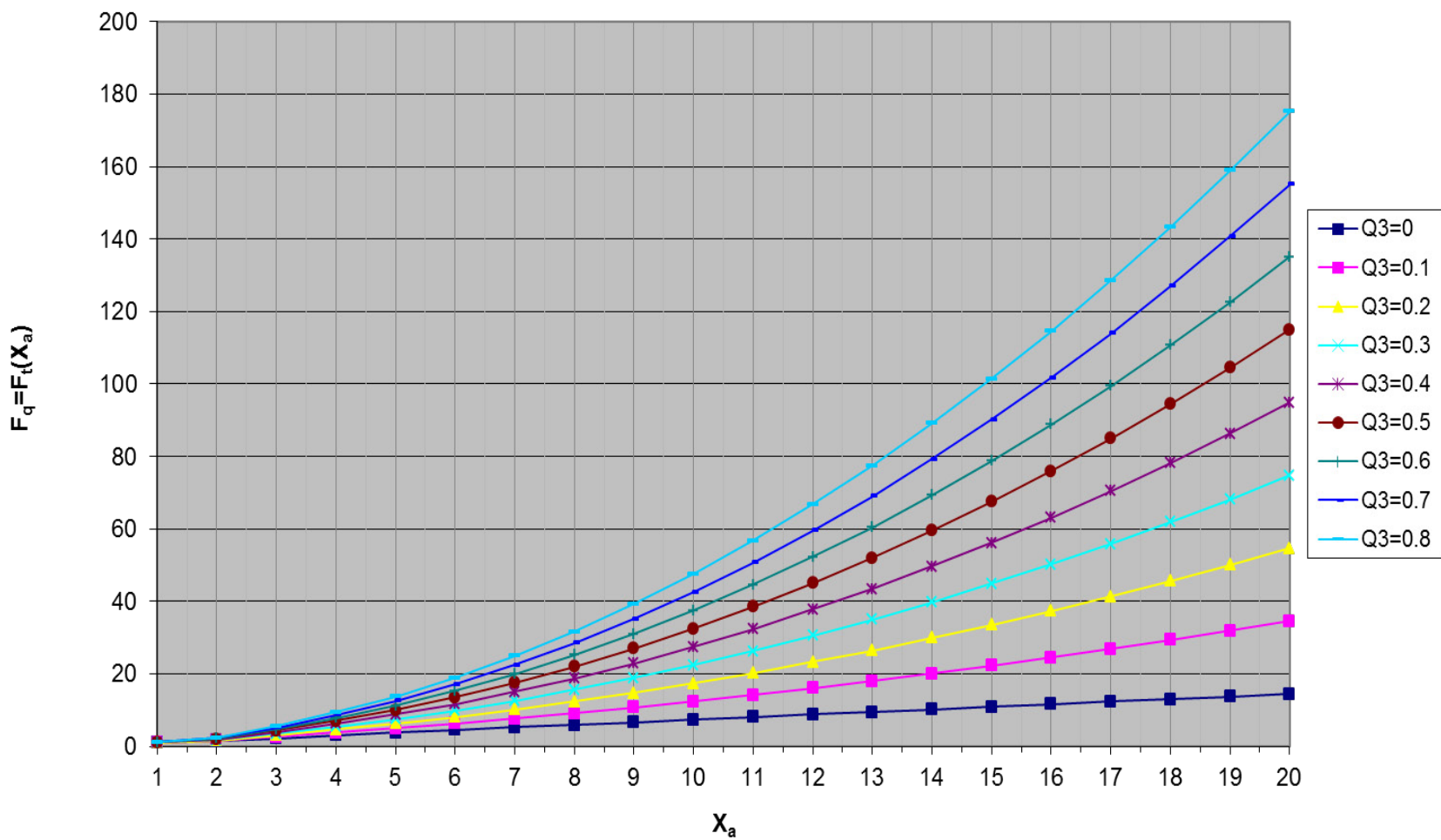


$F_{t,max}$ for $0 < Q_3 < 0.8$

$0 < Q_3 < 0.8$ ($v^* = 0.4$)





F_q for $0 < Q_3 < 0.8$
 $0 < Q_3 < 0.8$
 $(v^* = 0.4)$


ANNEX P—TABULAR AND GRAPHICAL REPRESENTATION OF COEFFICIENT $F_m(x)$

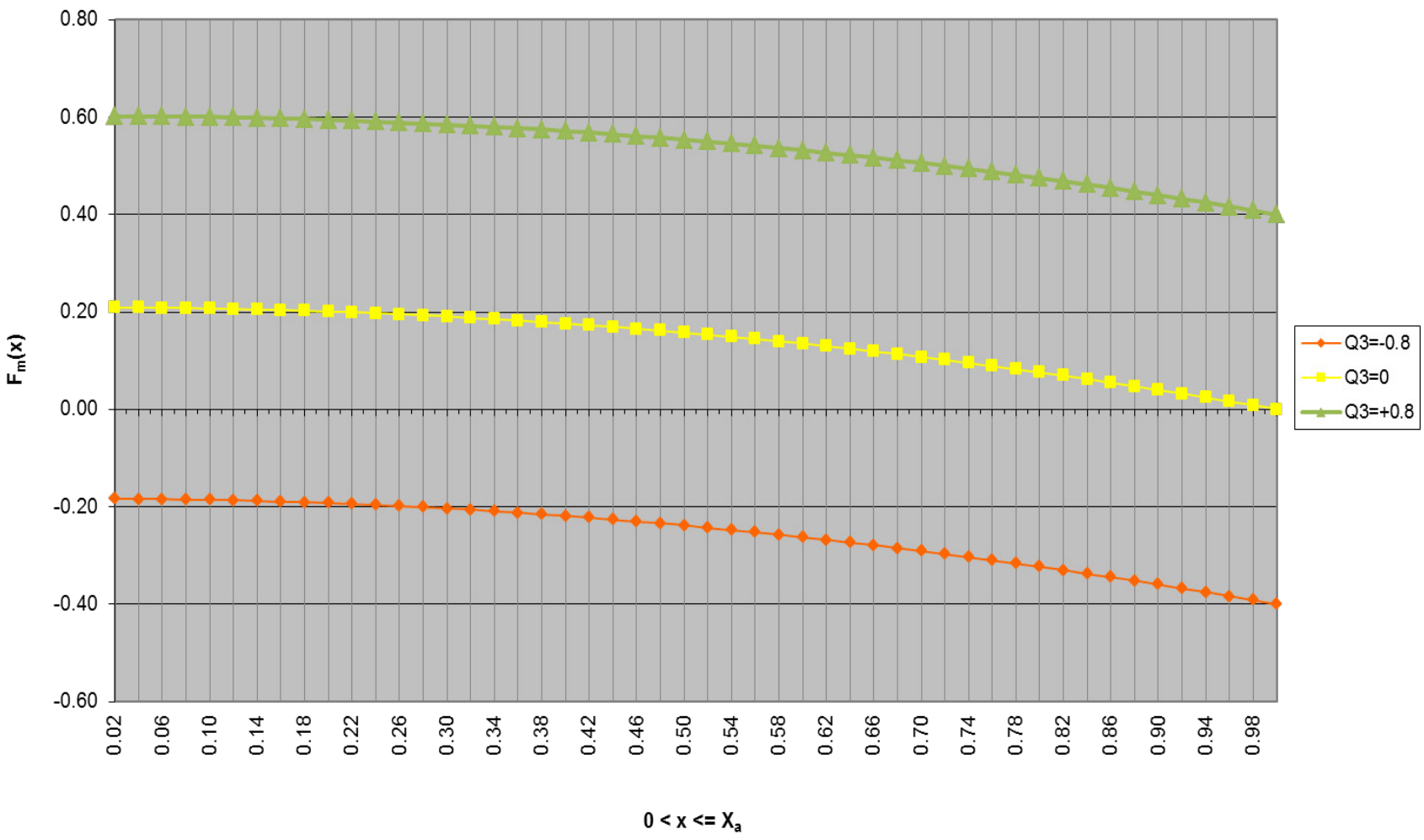
Annex P provides for $1 \leq X_a \leq 20$ and $-0.8 \leq Q_3 \leq +0.8$ (see below for the numerical values and graphs of coef. $F_m(x)$):

- values and graphs of $F_m(x)$ for $0 \leq x \leq X_a$
- values and graphs of the maximum of $F_m(x)$: F_m
- location of the maximum of $F_m(x)$: $x_{\max}$

$F_m(x)$ for $X_s=1,3,5,7,10,20$

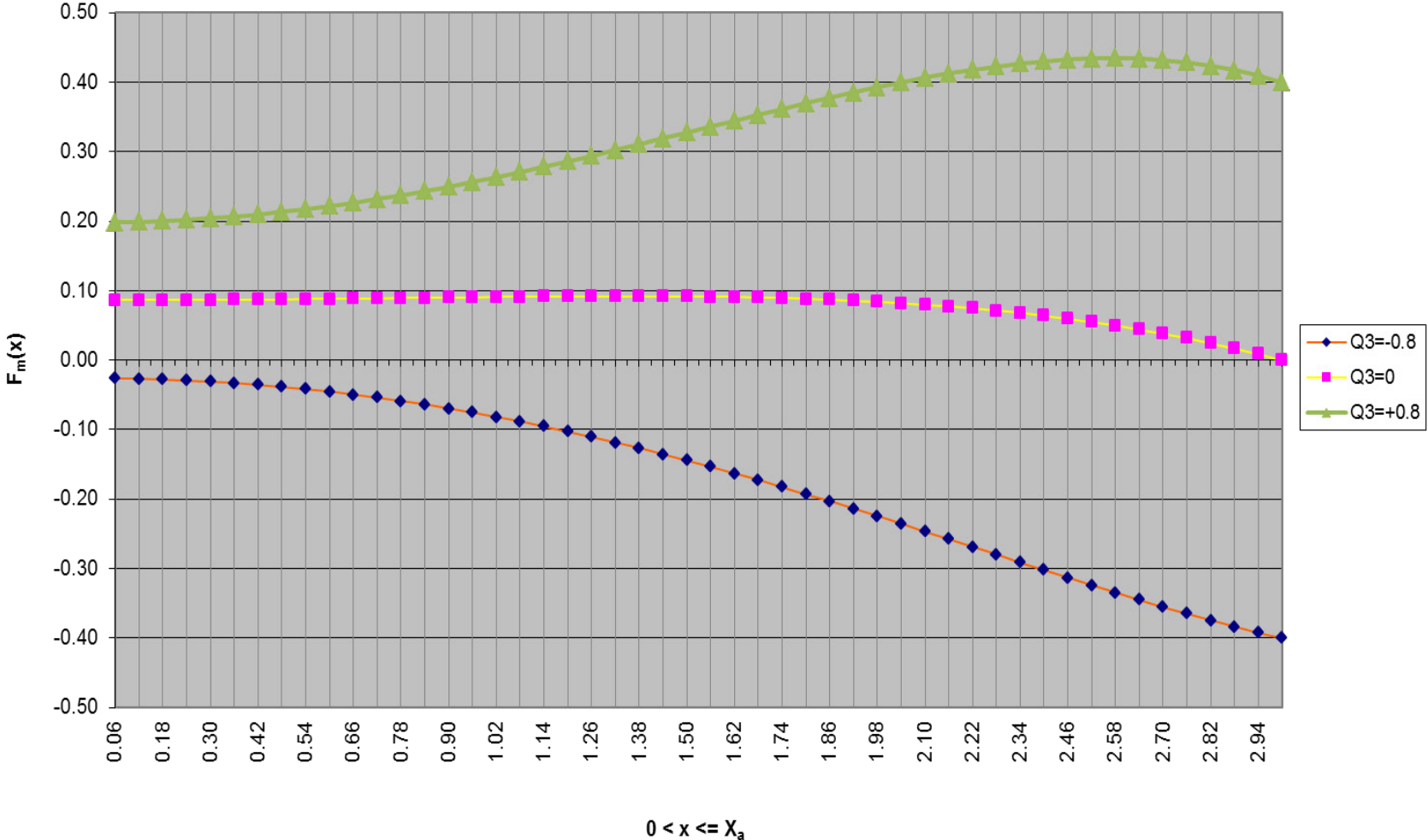
F _m (x) for X _s =1, 3, 5, 7, 10, 20 and Q ₃ =-0.8, 0.0, +0.8 (v*=0.4)																								
X _s =1				X _s =3				X _s =5				X _s =7				X _s =10				X _s =20				
x	Q ₃ =-0.8	Q ₃ =0	Q ₃ =+0.8	x	Q ₃ =-0.8	Q ₃ =0	Q ₃ =+0.8	x	Q ₃ =-0.8	Q ₃ =0	Q ₃ =+0.8	x	Q ₃ =-0.8	Q ₃ =0	Q ₃ =+0.8	x	Q ₃ =-0.8	Q ₃ =0	Q ₃ =+0.8	x	Q ₃ =-0.8	Q ₃ =0	Q ₃ =+0.8	
0.02	-0.18358390	0.20396490	0.60231360	0.06	-0.02616935	0.08616862	0.19850660	0.10	0.05071213	-0.00097628	-0.05266469	0.14	0.01195184	-0.00343116	-0.01881416	0.20	-0.00243681	0.00013826	0.00271334	0.40	-0.00000287	0.00000019	0.00000324	
0.04	-0.18384860	0.20911810	0.60208480	0.12	-0.02675549	0.08623743	0.19923040	0.20	0.05004843	-0.00082157	-0.05209158	0.28	0.01217480	-0.00342389	-0.01902258	0.40	-0.00241136	0.00012682	0.00266499	0.80	-0.00000305	0.00000017	0.00000339	
0.06	-0.18428980	0.20870680	0.60170350	0.18	-0.02773213	0.08635121	0.20043460	0.30	0.05000487	-0.00056365	-0.05113217	0.42	0.01254269	-0.00341072	-0.01936414	0.60	-0.00236581	0.00010756	0.00258092	1.20	-0.00000329	0.00000015	0.00000359	
0.08	-0.18490740	0.20813090	0.60116920	0.24	-0.02909886	0.08650861	0.20211610	0.40	0.04937534	-0.00020240	-0.04978014	0.56	0.01304992	-0.00339006	-0.01983005	0.80	-0.00229549	0.00080824	0.00245598	1.60	-0.00000349	0.00000011	0.00000371	
0.10	-0.18570140	0.20739040	0.60048220	0.30	-0.03085506	0.08670776	0.20427060	0.50	0.04855138	0.00026229	-0.04802680	0.70	0.01368848	-0.00335970	-0.02040788	1.00	-0.00219397	0.00004455	0.00228306	2.00	-0.00000353	0.00000004	0.00000362	
0.12	-0.18667160	0.20648520	0.59964200	0.36	-0.03299997	0.08694620	0.20689240	0.60	0.04752215	0.00083053	-0.04586110	0.84	0.01444788	-0.00331679	-0.02108145	1.20	-0.00205305	0.00000011	0.00205328	2.40	-0.00000322	-0.00000008	0.00000311	
0.14	-0.18781810	0.20541520	0.59864840	0.42	-0.03553258	0.08722100	0.20997460	0.70	0.04627452	0.0150239	-0.04326973	0.98	0.01531492	-0.00325788	-0.02183068	1.40	-0.00186298	-0.00005342	0.00175614	2.80	-0.00000234	-0.00000019	0.00000196	
0.16	-0.18914080	0.20418020	0.59750120	0.48	-0.03845168	0.08752862	0.21350890	0.80	0.04479314	0.00227788	-0.04023738	1.12	0.01627357	-0.00317891	-0.02263138	1.60	-0.00161255	-0.00011637	0.00137980	3.20	-0.00000063	-0.00000038	-0.00000012	
0.18	-0.19063940	0.20278030	0.59620000	0.54	-0.04175581	0.08786500	0.21748580	0.90	0.04306505	0.00315686	-0.03674683	1.26	0.01730466	-0.00307523	-0.02345513	1.80	-0.00128933	-0.00018895	0.00091142	3.60	0.00000219	-0.00000060	-0.00000340	
0.20	-0.19231400	0.20121520	0.59474440	0.60	-0.04544324	0.08822551	0.22189430	1.00	0.04105721	0.00413903	-0.03277915	1.40	0.01838573	-0.00294164	-0.02426902	2.00	-0.00087998	-0.00027119	0.00033759	4.00	0.00000638	-0.00000087	-0.00000811	
0.22	-0.19416430	0.19948480	0.59313400	0.66	-0.04951193	0.08860499	0.22672190	1.10	0.03871618	0.00522384	-0.02831400	1.54	0.01949068	-0.00277239	-0.02503546	2.20	-0.00037059	-0.00036287	-0.00035516	4.40	0.00001210	-0.00000115	-0.00001440	
0.24	-0.19619030	0.19758900	0.59136810	0.72	-0.05395950	0.08899774	0.23135500	1.20	0.03615077	0.00641046	-0.02332985	1.68	0.02058955	-0.00256123	-0.02571201	2.40	0.00025279	-0.00046343	-0.00117965	4.80	0.00001938	-0.00000140	-0.00002219	
0.26	-0.19839170	0.19552740	0.58944650	0.78	-0.05878322	0.08939743	0.23757810	1.30	0.03319965	0.00769765	-0.01780435	1.82	0.02164825	-0.00230147	-0.02625119	2.60	0.00100361	-0.00067184	-0.00214729	5.20	0.00002795	-0.00000158	-0.00003111	
0.28	-0.20076830	0.19300010	0.58736840	0.84	-0.06397994	0.08979724	0.24357440	1.40	0.02982027	0.00908375	-0.01171457	1.96	0.02262828	-0.00198603	-0.02660033	2.80	0.00189401	-0.00068655	-0.00326710	5.60	0.00003712	-0.00000159	-0.00004031	
0.30	-0.20332000	0.19090670	0.58513340	0.90	-0.06954604	0.09018975	0.24952550	1.50	0.02617054	0.01056653	-0.00503749	2.10	0.02348648	-0.00160751	-0.02670149	3.00	0.00293403	-0.00080530	-0.00454464	6.00	0.00004564	-0.00000133	-0.00004829	
0.32	-0.20604660	0.18834690	0.58274040	0.96	-0.07547744	0.09056698	0.25661140	1.60	0.02230660	0.01214312	-0.00224964	2.24	0.02417482	-0.00115829	-0.02684913	3.20	0.00413047	-0.00092508	-0.00598062	6.40	0.00005150	-0.00000064	-0.00005028	
0.34	-0.20894780	0.18562070	0.58018920	1.02	-0.08176948	0.09109207	0.26361020	1.70	0.01745103	0.01380992	0.01016881	2.38	0.02464022	-0.00063063	-0.02690448	3.40	0.00548573	-0.00104190	-0.00756954	6.80	0.00005582	-0.00000062	-0.00005058	
0.36	-0.21202330	0.18272770	0.57747660	1.08	-0.08841692	0.09124079	0.27089850	1.80	0.01238420	0.01556249	0.01874078	2.52	0.02482442	-0.00066802	-0.02485806	3.60	0.00699638	-0.00115077	-0.00929793	7.20	0.00004276	-0.00000061	-0.00003754	
0.38	-0.21527300	0.17965760	0.57460380	1.14	-0.09541386	0.09151853	0.27845090	1.90	0.00860836	0.01739541	0.02798446	2.66	0.02486395	-0.00069070	-0.02328254	3.80	0.00865158	-0.00124551	-0.01114261	7.60	0.00001961	-0.00000056	-0.00000870	
0.40	-0.21869560	0.17644020	0.57157700	1.20	-0.10275370	0.09174329	0.28624030	2.00	0.00688801	0.01930221	0.03791640	2.80	0.02490931	-0.00149915	-0.02109184	4.00	0.01043131	-0.00131868	-0.01306868	8.00	0.00002294	-0.00000022	-0.00004138	
0.42	-0.22229350	0.17304520	0.56838400	1.26	-0.11042900	0.09190422	0.29423750	2.10	-0.00599970	0.02127519	0.04855009	2.94	0.02302905	-0.00241520	-0.01819892	4.20	0.01230449	-0.00136149	-0.01502747	8.40	-0.00009023	-0.00000031	-0.00011785	
0.44	-0.22606380	0.16948230	0.56502840	1.32	-0.11843160	0.09198894	0.30241130	2.20	-0.01328455	0.02330535	0.05898525	3.08	0.02140298	-0.00344472	-0.01451354	4.40	0.01422692	-0.00136373	-0.01695438	8.80	-0.00018654	-0.00000189	-0.00022439	
0.46	-0.23000680	0.16575110	0.56150910	1.38	-0.12675240	0.09198811	0.31072860	2.30	-0.02119274	0.02538221	0.07195716	3.22	0.01912836	-0.00459228	-0.00994320	4.60	0.01613920	-0.00131378	-0.01876677	9.20	-0.00031361	-0.000002395	-0.00036151	
0.48	-0.23412250	0.16185140	0.55782530	1.44	-0.13538120	0.09188642	0.31915400	2.40	-0.02978480	0.02744370	0.08473580	3.36	0.01611890	-0.00586228	-0.00439435	4.80	0.01796455	-0.00119868	-0.02036191	9.60	-0.00046844	-0.000002786	-0.00052415	
0.50	-0.23841020	0.15778280	0.55397570	1.50	-0.14430690	0.09167156	0.32765000	2.50	-0.03897302	0.02962602	0.09822506	3.50	0.01228512	-0.00725566	-0.00222620	5.00	0.01960682	-0.00100418	-0.02161518	10.00	-0.00064056	-0.000002911	-0.00069878	
0.52	-0.24286970	0.15394490	0.54995940	1.56	-0.15351720	0.09132972	0.33617670	2.60	-0.04888477	0.03176349	0.11241170	3.64	0.00753562	-0.00877258	-0.01009950	5.20	0.02094852	-0.00071502	-0.02237857	10.40	-0.00080674	-0.00002562	-0.00085998	
0.54	-0.24750050	0.14913730	0.54577520	1.62	-0.16299870	0.09084656	0.34469180	2.70	-0.05949798	0.03388843	0.12272480	3.78	0.00177833	-0.01041036	-0.01904238	5.40	0.02184935	-0.00031520	-0.02247975	10.80	-0.00093753	-0.00001471	-0.00096695	
0.56	-0.25230220	0.14455980	0.54142190	1.68	-0.17273650	0.09020714	0.35315080	2.80	-0.07082227	0.03598100	0.14278430	3.92	-0.00507792	-0.01216360	-0.02040512	5.60	0.02214497	-0.00021158	-0.02172181	11.20	-0.00097407	-0.00000672	-0.00099604	
0.58	-0.25724740	0.13981190	0.53689830	1.74	-0.18271470	0.08939592	0.36150650	2.90	-0.08286189	0.03801908	0.15890010	4.06	-0.01312157	-0.01402351	-0.04116859	5.80	0.02164662	-0.00088132	-0.01988398	11.60	-0.00084604	-0.000004189	-0.00076226	
0.60	-0.26241660	0.13489320	0.53220300	1.80	-0.19291550	0.08839688	0.36970930	3.00	-0.09561487	0.03997814	0.17557110	4.20	-0.02243614	-0.01597749	-0.05439113	6.00	0.02014153	-0.00170900	-0.01672352	12.00	-0.00046205	-0.000009364	-0.00027478	
0.62	-0.26772820	0.12980340	0.52733490	1.86	-0.20331990	0.08719339	0.37770670	3.10	-0.10907220	0.04183109	0.19273430	4.34	-0.03309772	-0.01800864	-0.06911500	6.20	0.01739438	-0.00270762	-0.01197915	12.40	-0.00028394	-0.00016356	-0.00061106	
0.64	-0.27320870	0.12454200	0.52229270	1.92	-0.21390720	0.08576827	0.38544380	3.20	-0.12321680	0.04354820	0.21031320	4.48	-0.04517204	-0.02009514	-0.08536233	6.40	0.01315036	-0.00388706	-0.00537623	12.80	-0.00150222	-0.00025078	-0.000200378	
0.66	-																							

$X_a = 1$
 $X_a = 1 \quad (v^* = 0.4)$



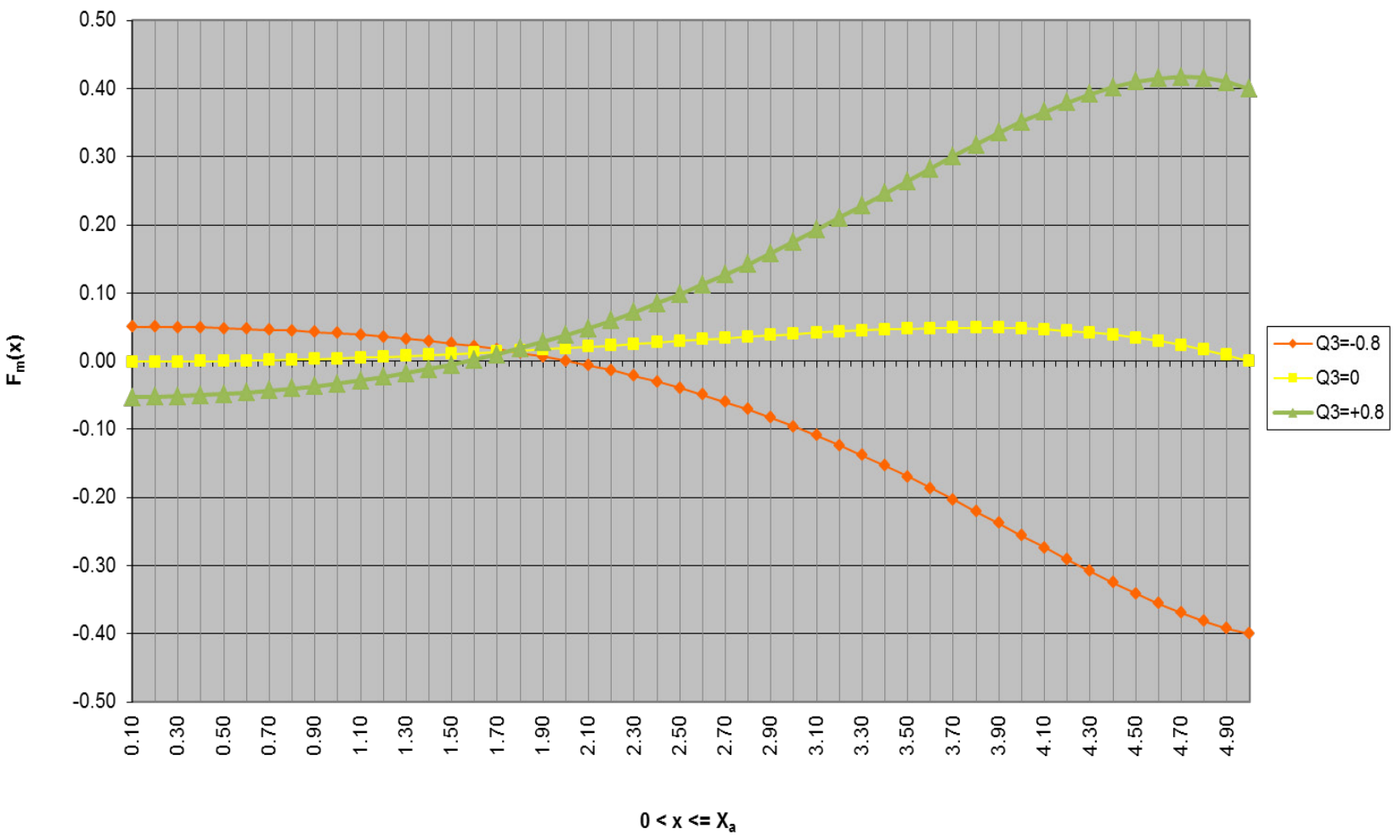
$X_a = 3$

$X_a = 3 \quad (v^* = 0.4)$



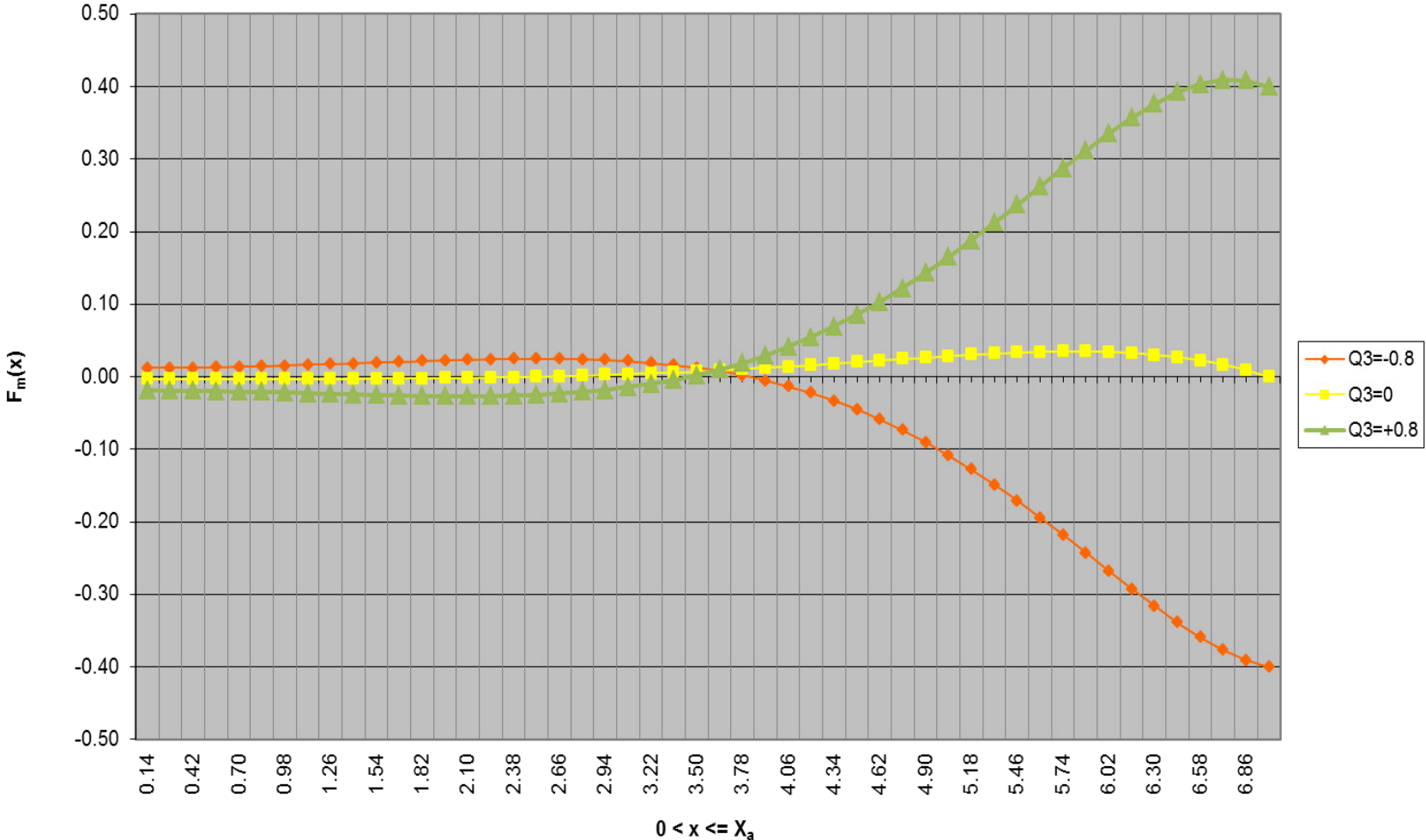
$X_a = 5$

$X_a = 5 \quad (v^* = 0.4)$



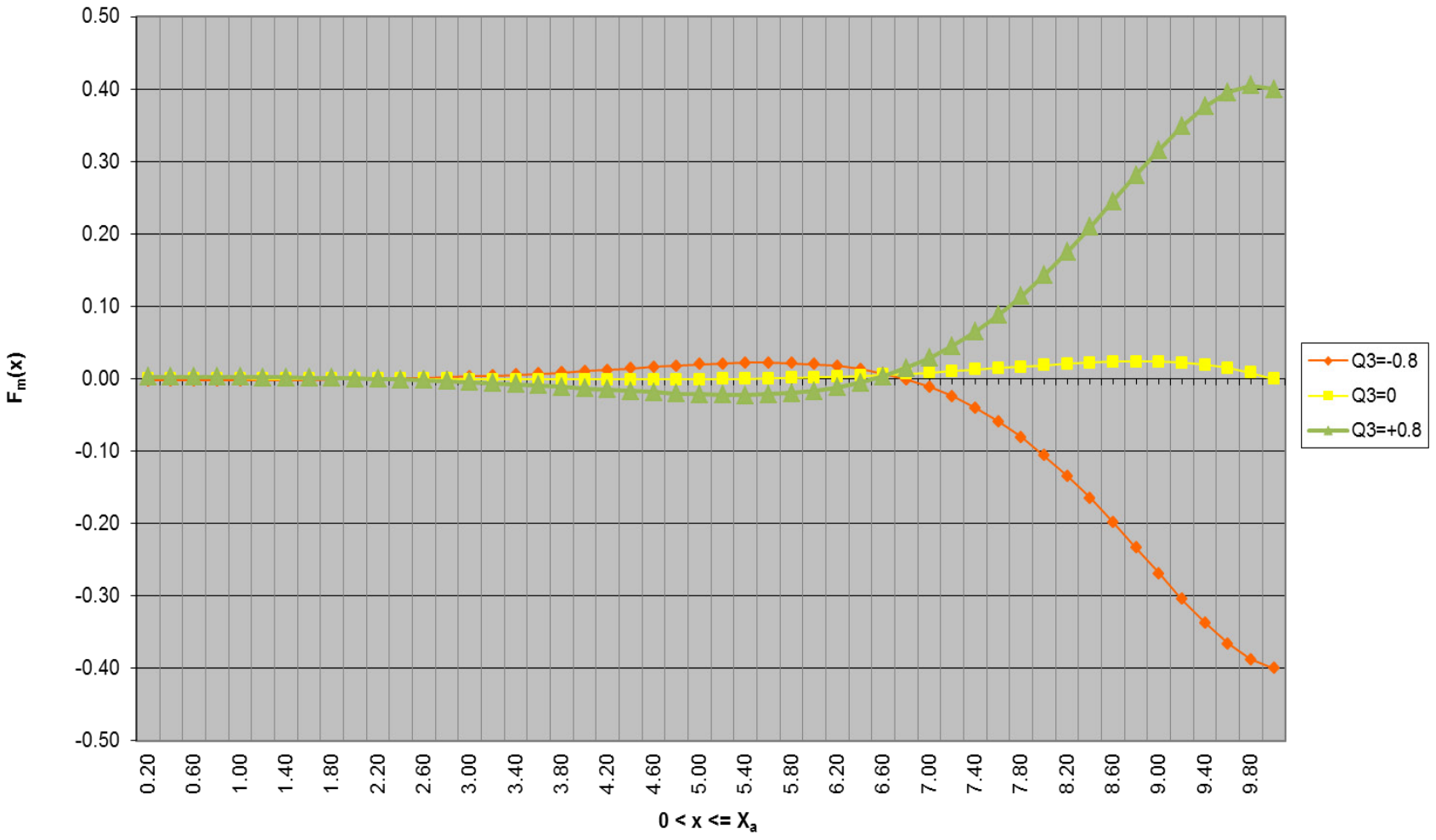
$X_a = 7$

$X_a = 7 \quad (v^* = 0.4)$



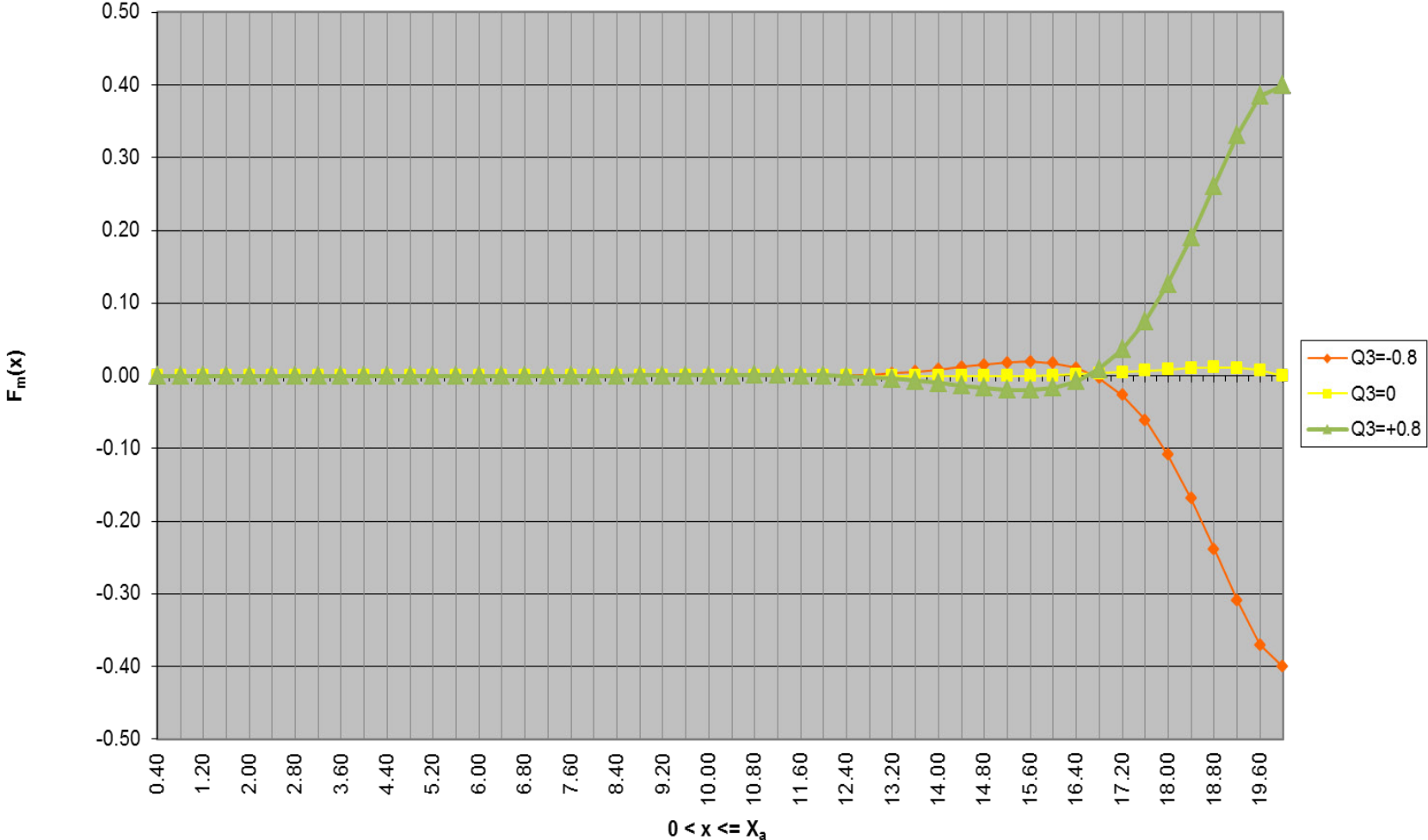
$X_a = 10$

$X_a = 10 \quad (v^* = 0.4)$



$X_a = 20$

$X_a = 20$ $(v^* = 0.4)$

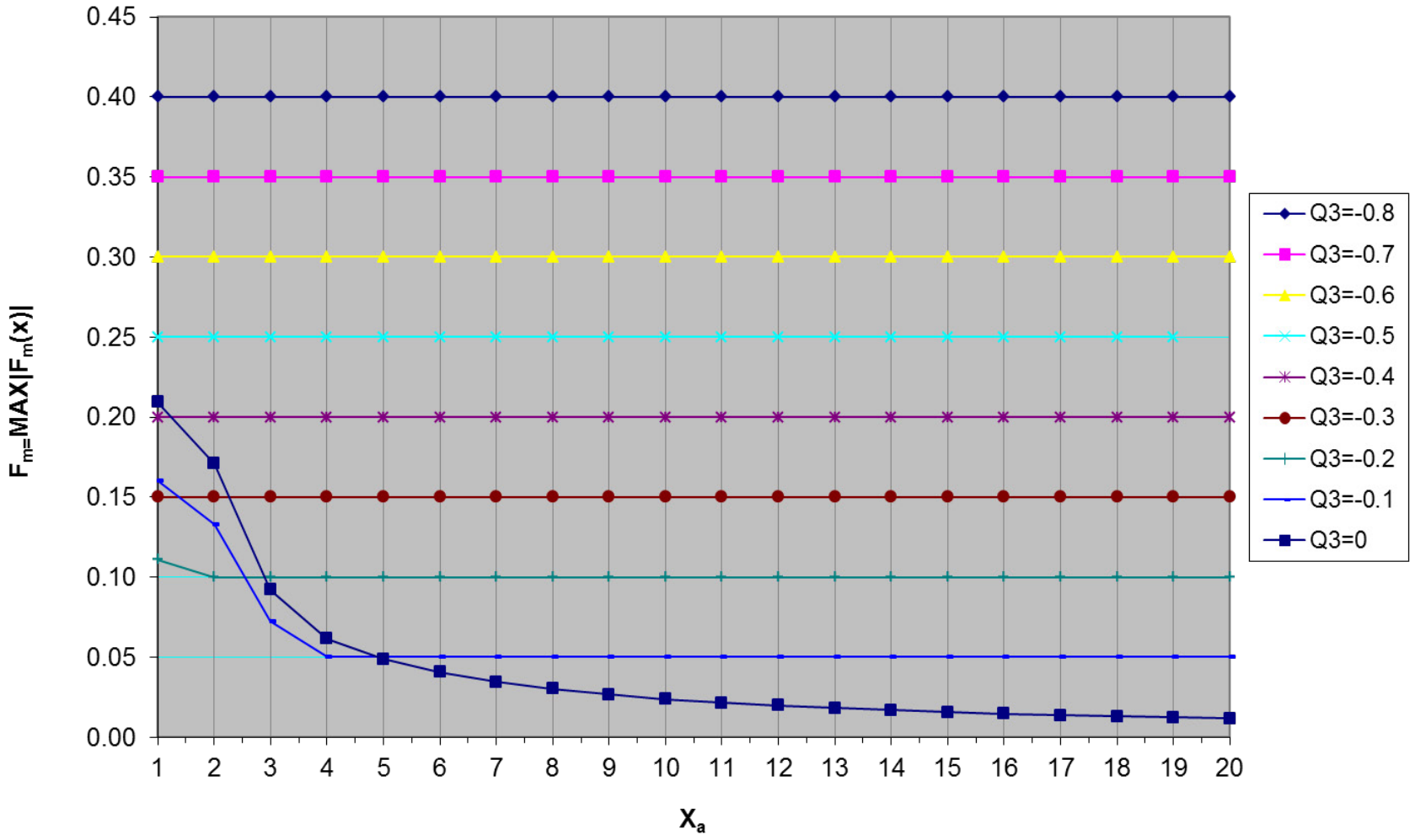


$F_m = \text{Max}|F_m(x)|$ vs X_a and Q_3

			if $x(\text{max})=X_a : F_m(X_a)= Q_3/2 $							$F_m=\text{Max} F_m(x) $							if $x(\text{max})=X_a : F_m(X_a)= Q_3/2 $						
$X_a \backslash Q_3$	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8						
1	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1111	0.1602	0.2094	0.2585	0.3076	0.3567	0.4058	0.4550	0.5041	0.5532	0.6023						
2	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.1329	0.1710	0.2090	0.2471	0.2851	0.3232	0.3613	0.4009	0.4425	0.4855						
3	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0721	0.0920	0.1236	0.1616	0.2035	0.2476	0.2931	0.3396	0.3870	0.4348						
4	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0616	0.0981	0.1400	0.1846	0.2309	0.2782	0.3263	0.3746	0.4234						
5	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0490	0.0866	0.1297	0.1755	0.2225	0.2707	0.3189	0.3681	0.4173						
6	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0407	0.0789	0.1229	0.1694	0.2171	0.2656	0.3141	0.3635	0.4130						
7	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0346	0.0734	0.1181	0.1652	0.2129	0.2621	0.3112	0.3604	0.4096						
8	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0301	0.0694	0.1148	0.1620	0.2107	0.2594	0.3081	0.3579	0.4078						
9	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0266	0.0662	0.1119	0.1600	0.2082	0.2574	0.3071	0.3568	0.4066						
10	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0239	0.0640	0.1103	0.1579	0.2069	0.2565	0.3061	0.3557	0.4052						
11	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0216	0.0620	0.1086	0.1567	0.2061	0.2555	0.3049	0.3543	0.4037						
12	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0198	0.0605	0.1069	0.1561	0.2053	0.2545	0.3037	0.3529	0.4021						
13	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0181	0.0594	0.1063	0.1553	0.2043	0.2533	0.3023	0.3514	0.4004						
14	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0169	0.0583	0.1058	0.1546	0.2033	0.2521	0.3009	0.3500	0.4000						
15	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0157	0.0571	0.1052	0.1537	0.2023	0.2509	0.3000	0.3500	0.4000						
16	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0146	0.0563	0.1046	0.1529	0.2012	0.2500	0.3000	0.3500	0.4000						
17	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0137	0.0559	0.1039	0.1520	0.2000	0.2500	0.3000	0.3500	0.4000						
18	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0130	0.0555	0.1033	0.1511	0.2000	0.2500	0.3000	0.3500	0.4000						
19	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0123	0.0551	0.1026	0.1501	0.2000	0.2500	0.3000	0.3500	0.4000						
20	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0117	0.0547	0.1019	0.1500	0.2000	0.2500	0.3000	0.3500	0.4000						
			$x(\text{max})=X_a$											$x(\text{max})=X_a$									
$X_a \backslash Q_3$	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8						
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200						
2	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	0.0400	0.0400	0.0400	0.0400	0.0400	0.0400	0.0400	0.0400	0.0400	0.0400						
3	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	0.0600	1.3200	1.8000	2.1000	2.2200	2.3400	2.4000	2.4600	2.5200	2.5800						
4	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	2.7200	3.1200	3.2800	3.4400	3.5200	3.5200	3.6000	3.6000	3.6800						
5	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	3.8000	4.2000	4.4000	4.5000	4.6000	4.6000	4.7000	4.7000	4.7000						
6	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	4.8000	5.1600	5.4000	5.5200	5.6400	5.6400	5.6400	5.7600	5.7600						
7	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000	5.7400	6.3000	6.4400	6.5800	6.7200	6.7200	6.7200	6.7200	6.7200						
8	8.0000	8.0000	8.0000	8.0000	8.0000	8.0000	8.0000	8.0000	6.8800	7.3600	7.5200	7.6800	7.8000	7.8000	7.8000	7.8400	7.8400						
9	9.0000	9.0000	9.0000	9.0000	9.0000	9.0000	9.0000	9.0000	7.7400	8.2800	8.6400	8.6400	8.6400	8.8200	8.8200	8.8200	8.8200						
10	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	8.8000	9.4000	9.6000	9.6000	9.8000	9.8000	9.8000	9.8000	9.8000						
11	11.0000	11.0000	11.0000	11.0000	11.0000	11.0000	11.0000	11.0000	9.9000	10.3400	10.5600	10.7800	10.7800	10.7800	10.7800	10.7800	10.7800						
12	12.0000	12.0000	12.0000	12.0000	12.0000	12.0000	12.0000	12.0000	10.8000	11.5200	11.5200	11.7600	11.7600	11.7600	11.7600	11.7600	11.7600						
13	13.0000	13.0000	13.0000	13.0000	13.0000	13.0000	13.0000	13.0000	11.9600	12.4800	12.7400	12.7400	12.7400	12.7400	12.7400	12.7400	12.7400						
14	14.0000	14.0000	14.0000	14.0000	14.0000	14.0000	14.0000	14.0000	12.8800	13.4400	13.7200	13.7200	13.7200	13.7200	13.7200	14.0000	14.0000						
15	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	13.8000	14.4000	14.7000	14.7000	14.7000	14.7000	15.0000	15.0000	15.0000						
16	16.0000	16.0000	16.0000	16.0000	16.0000	16.0000	16.0000	16.0000	14.7200	15.6800	15.6800	15.6800	15.6800	15.6800	16.0000	16.0000	16.0000						
17	17.0000	17.0000	17.0000	17.0000	17.0000	17.0000	17.0000	17.0000	15.9800	16.6600	16.6600	16.6600	16.6600	17.0000	17.0000	17.0000	17.0000						
18	18.0000	18.0000	18.0000	18.0000	18.0000	18.0000	18.0000	18.0000	16.9200	17.6400	17.6400	17.6400	18.0000	18.0000	18.0000	18.0000	18.0000						
19	19.0000	19.0000	19.0000	19.0000	19.0000	19.0000	19.0000	19.0000	17.8600	18.6200	18.6200	18.6200	19.0000	19.0000	19.0000	19.0000	19.0000						
20	20.0000	20.0000	20.0000	20.0000	20.0000	20.0000	20.0000	20.0000	18.8000	19.6000	19.6000	20.0000	20.0000	20.0000	20.0000	20.0000	20.0000						

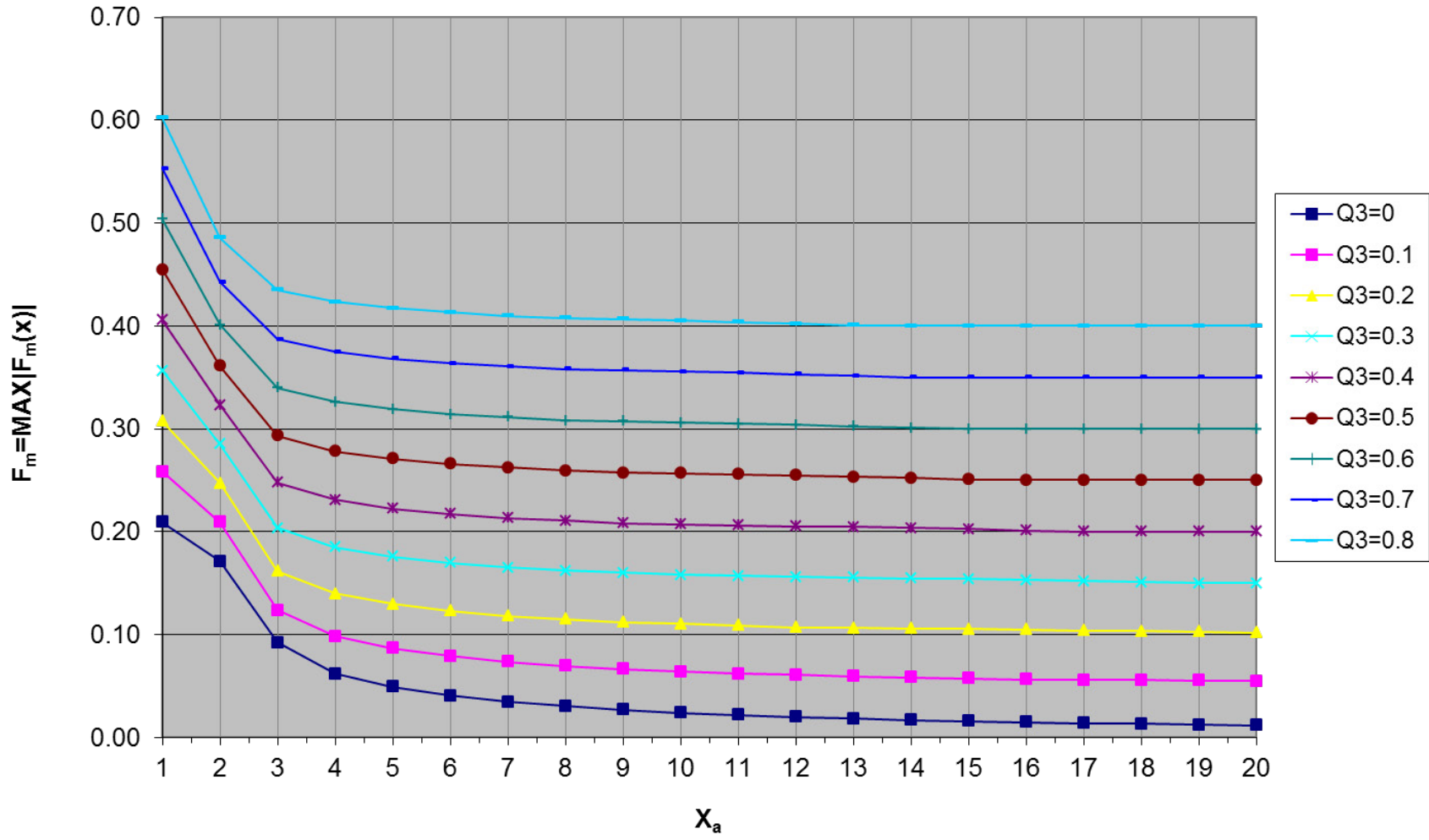
F_m vs X_a and $-0.8 < Q_3 < 0$

$-0.8 < Q_3 < 0.0 \quad (v^* = 0.4)$



F_m vs X_a and $0 < Q_3 < +0.8$

$0.0 < Q_3 < +0.8$ ($v^* = 0.4$)



ANNEX Q—TABULAR AND GRAPHICAL REPRESENTATION OF COEFFICIENT $F_Q(x)$

Annex Q provides for $1 \leq X_a \leq 20$ and $-0.8 \leq Q_3 \leq +0.8$ (see below for the numerical values and graphs of coef. $F_Q(x)$):

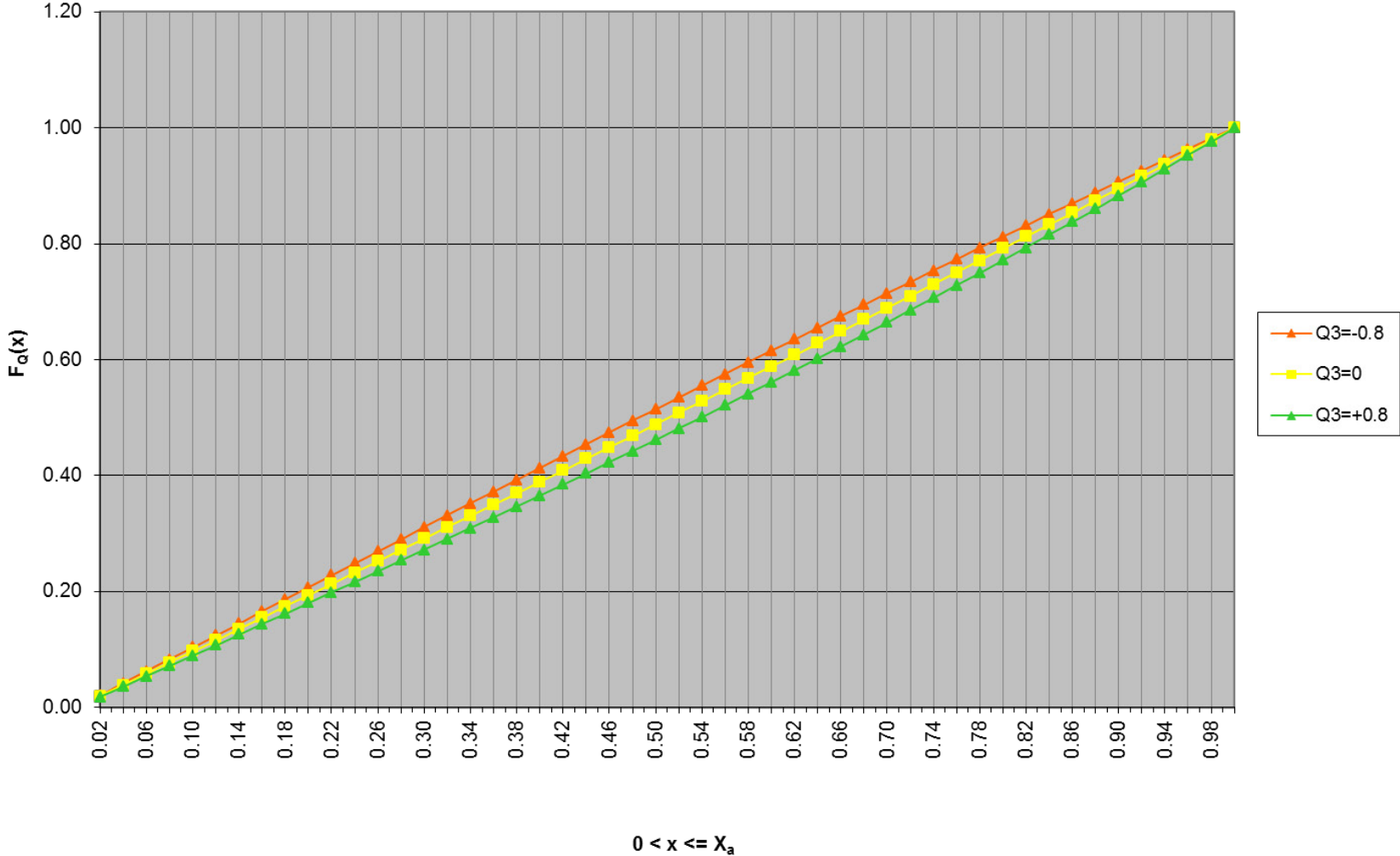
- values and graphs of $F_Q(x)$ for $0 \leq x \leq X_a$
- values and graphs of the maximum of $F_Q(x)$: F_Q
- location of the maximum of $F_Q(x)$: $x_{\max}$

$F_O(x)$ for $X_a=1,3,5,7,10,20$

F _O (x) for X _a = 1, 3, 5, 7, 10, 20 and Q ₃ = -0.8, 0.0, +0.8 (v=0.4)																							
X _a =1				X _a =3				X _a =5				X _a =7				X _a =10				X _a =20			
x	Q ₃ =-0.8	Q ₃ =0	Q ₃ =+0.8	x	Q ₃ =-0.8	Q ₃ =0	Q ₃ =+0.8	x	Q ₃ =-0.8	Q ₃ =0	Q ₃ =+0.8	x	Q ₃ =-0.8	Q ₃ =0	Q ₃ =+0.8	x	Q ₃ =-0.8	Q ₃ =0	Q ₃ =+0.8	x	Q ₃ =-0.8	Q ₃ =0	Q ₃ =+0.8
0.02	0.02072622	0.01935370	0.01794477	0.06	0.04597750	-0.00541298	-0.05680346	0.10	0.02060735	-0.01213262	-0.04487258	0.14	-0.01755414	-0.00055054	0.01645306	0.20	-0.00193894	0.00089464	0.00372823	0.40	0.00001512	0.00000089	-0.00001335
0.04	0.04152367	0.03870919	0.03589477	0.12	0.09193687	-0.01076614	-0.11346920	0.20	0.04148651	-0.02427055	-0.09002762	0.28	-0.03486280	-0.00117172	0.03251937	0.40	-0.00406697	0.00180124	0.00768945	0.80	0.00002634	0.00000203	-0.00002228
0.06	0.06228157	0.05806826	0.05385495	0.18	0.13785960	-0.01599961	-0.16985880	0.30	0.06290802	-0.03641836	-0.13574480	0.42	-0.05167628	-0.00193401	0.04780826	0.60	-0.00665120	0.00273084	0.01211288	1.20	0.00002952	0.00000367	-0.00002218
0.08	0.08303474	0.07743271	0.07183069	0.24	0.18372650	-0.02105345	-0.22583340	0.40	0.08513980	-0.04857909	-0.18229800	0.56	-0.06773657	-0.00290762	0.06192133	0.80	-0.00983450	0.00369271	0.01721991	1.60	0.00002014	0.00000601	-0.00000811
0.10	0.10378160	0.09680434	0.08982707	0.30	0.22951710	-0.02586758	-0.28125230	0.50	0.10844580	-0.06075350	-0.22956280	0.70	-0.08277331	-0.00416227	0.07444877	1.00	-0.01383283	0.00469333	0.02321949	2.00	-0.00000669	0.00000920	-0.00000251
0.12	0.12452060	0.11618490	0.10784920	0.36	0.27520970	-0.03038173	-0.33597320	0.60	0.13308470	-0.07293931	-0.27896330	0.84	-0.09650004	-0.00576892	0.08496620	1.20	-0.01883181	0.00573549	0.03030280	2.40	-0.00005603	0.00001328	-0.00000825
0.14	0.14525000	0.13557610	0.12590230	0.42	0.32078050	-0.03453546	-0.38985140	0.70	0.15930820	-0.08513048	-0.32956910	0.98	-0.10861070	-0.00778948	0.09303176	1.40	-0.02500229	0.00681724	0.03863677	2.80	-0.00013251	0.00001811	-0.00016873
0.16	0.16596330	0.15497980	0.14399140	0.48	0.36620360	-0.03826807	-0.44273970	0.80	0.18735950	-0.09731634	-0.38199220	1.12	-0.11877650	-0.01029634	0.09818387	1.60	-0.03249485	0.00793082	0.04835649	3.20	-0.00023938	0.00002334	-0.00028605
0.18	0.18667380	0.17439770	0.16212160	0.54	0.41145040	-0.04151859	-0.49448750	0.90	0.21747200	-0.10948100	-0.43643390	1.26	-0.12664330	-0.01335183	0.09939367	1.80	-0.04143289	0.00906157	0.05955603	3.60	-0.00037699	0.00002828	-0.00043355
0.20	0.20736500	0.19383160	0.18029810	0.60	0.45648950	-0.04422573	-0.54494090	1.00	0.24966680	-0.12160230	-0.49307130	1.40	-0.13182940	-0.01701756	0.09779428	2.00	-0.05190425	0.01018679	0.07227782	4.00	-0.00054077	0.00003186	-0.00060450
0.22	0.22804000	0.21328300	0.19852600	0.66	0.50128600	-0.04632790	-0.59394180	1.10	0.28475110	-0.13365150	-0.55205400	1.54	-0.13924100	-0.02135156	0.09122102	2.20	-0.06395109	0.01127458	0.08650026	4.40	-0.00071868	0.00003251	-0.00078371
0.24	0.24869730	0.23275380	0.21681020	0.72	0.54580140	-0.04776315	-0.64132770	1.20	0.32231610	-0.14559200	-0.61350010	1.68	-0.13248720	-0.02640728	0.07967261	2.40	-0.07755814	0.01228275	0.10212360	4.80	-0.00088817	0.00002814	-0.00094444
0.26	0.26933520	0.25224550	0.23515590	0.78	0.58999320	-0.04846917	-0.68693160	1.30	0.36273460	-0.15737910	-0.67749280	1.82	-0.12704880	-0.03223242	0.06258393	2.60	-0.09263907	0.01315772	0.11895450	5.20	-0.00101313	0.00001614	-0.00104542
0.28	0.28995180	0.27175990	0.25356800	0.84	0.63381440	-0.04838331	-0.73058190	1.40	0.40615810	-0.16895880	-0.74407580	1.96	-0.11711080	-0.03866747	0.03937585	2.80	-0.10902130	0.01383357	0.13668840	5.60	-0.00104112	-0.00000642	-0.00102829
0.30	0.31054540	0.29129850	0.27205160	0.90	0.67721310	-0.04744254	-0.77208820	1.50	0.45271510	-0.18026740	-0.81325000	2.10	-0.10214920	-0.04634420	0.00946077	3.00	-0.12642890	0.01423129	0.15489150	6.00	-0.00090186	-0.00000456	-0.00081673
0.32	0.33111430	0.31086300	0.29061170	0.96	0.72013270	-0.04558368	-0.81129970	1.60	0.50250690	-0.19123070	-0.86489630	2.24	-0.08161756	-0.05468376	0.02774997	3.20	-0.14446490	0.01425830	0.17298150	6.40	-0.00050796	-0.00000486	-0.00031824
0.34	0.35165660	0.33045490	0.30925310	1.02	0.76251060	-0.04274242	-0.84799550	1.70	0.55605010	-0.20176130	-0.95913120	2.38	-0.05495225	-0.06389466	0.02728370	3.40	-0.16259140	0.01380827	0.19020800	6.80	-0.00024051	-0.00001461	-0.00059973
0.36	0.37217050	0.35007760	0.32799890	1.08	0.80427890	-0.03885531	-0.88198940	1.80	0.61204820	-0.21176730	-1.03556300	2.52	-0.02157894	-0.07397044	0.12636190	3.60	-0.18011110	0.01276156	0.20563420	7.20	0.00144587	-0.00025076	-0.00194738
0.38	0.39265400	0.36972880	0.34739970	1.14	0.84536300	-0.03385787	-0.91307780	1.90	0.67183810	-0.22113350	-1.11410500	2.66	0.01907883	-0.08488717	0.18885320	3.80	-0.19614680	0.01098615	0.21811910	7.60	0.00319507	-0.00034504	-0.00091877
0.40	0.41310520	0.38940900	0.36571460	1.20	0.88588250	-0.02768556	-0.94105360	2.00	0.73493580	-0.22973880	-1.19441300	2.80	0.06758947	-0.09660700	0.26079090	4.00	-0.20962510	0.00833940	0.22630390	8.00	0.00553233	-0.00044706	-0.00642645
0.42	0.43352230	0.40912630	0.38373040	1.26	0.92514960	-0.02027364	-0.96569690	2.10	0.80125750	-0.23744640	-1.27615000	2.94	0.12450080	-0.10940370	0.34258810	4.20	-0.21925940	0.00467078	0.22860100	8.40	0.00842467	-0.00052879	-0.00947825
0.44	0.45390310	0.42887750	0.40385180	1.32	0.96368970	-0.01155732	-0.98678430	2.20	0.87067040	-0.24410590	-1.35888200	3.08	0.19032540	-0.12212230	0.43456990	4.40	-0.22353960	-0.0017432	0.23190990	8.80	0.01169193	-0.00056633	-0.01284599
0.46	0.47424570	0.44866460	0.42303830	1.38	1.00114100	-0.00147171	-1.00408400	2.30	0.94298810	-0.24955200	-1.44209200	3.22	0.26552360	-0.13571290	0.53694950	4.60	-0.22072500	-0.00634949	0.20802600	9.20	0.01498676	-0.00052185	-0.01603048
0.48	0.49454780	0.46848820	0.44243050	1.44	1.03745300	0.01004802	-1.01735700	2.40	1.01796600	-0.25360480	-1.52517500	3.36	0.35048380	-0.14958980	0.64890160	4.80	-0.20884670	-0.01400068	0.18084540	9.60	0.01768924	-0.00034679	-0.01838281
0.50	0.51480750	0.48835230	0.46189720	1.50	1.07248800	0.02306647	-1.02635500	2.50	1.09529500	-0.25606920	-1.60743300	3.50	0.44550030	-0.16378640	0.77303310	5.00	-0.18571850	-0.02325793	0.13920260	10.00	0.01866495	-0.00001631	-0.01832323
0.52	0.53502240	0.50825520	0.48148810	1.56	1.10612000	0.03764805	-1.03082400	2.60	1.17459900	-0.25673520	-1.68806900	3.64	0.55074830	-0.17790120	0.90635060	5.20	-0.14899020	-0.03422524	0.08050972	10.40	0.01712728	-0.00062887	-0.01598954
0.54	0.55519030	0.52819920	0.50120810	1.62	1.13821400	0.05385673	-1.03950100	2.70	1.25542800	-0.25537740	-1.76618300	3.78	0.66625600	-0.19148470	1.04922500	5.40	-0.09603721	-0.04696843	0.02100235	10.80	0.01108357	-0.00154717	-0.00798824
0.56	0.57530900	0.54818520	0.52106140	1.68	1.16862700	0.07175592	-1.02511500	2.80	1.33725200	-0.25175550	-1.84076300	3.92	0.79187370	-0.20449050	1.20085500	5.60	-0.02431677	-0.05150097	0.09865518	11.20	-0.00149866	-0.00280757	0.00711479
0.58	0.59537610	0.56821440	0.54105270	1.74	1.19720700	0.09140831	-1.01439000	2.90	1.41945800	-0.24561470	-1.91068700	4.06	0.92724110	-0.21644110	1.36012300	5.80	0.06885336	-0.07776765	0.22438869	11.60	-0.02262540	-0.00440526	0.03143593
0.60	0.61539920	0.58826780	0.56118630	1.80	1.22379000	0.11287580	-0.99803880	3.00	1.50134100	-0.23668590	-1.97471300	4.20	1.07175000	-0.22690450	1.52555900	6.00	0.18604180	-0.09562604	0.37729390	12.00	-0.05419668	-0.00626580	0.06672829
0.62	0.63534580	0.60840620	0.58146660	1.86	1.24820700	0.13621900	-0.97576930	3.10	1.58210200	-0.22468660	-2.03147500	4.34	1.22450700	-0.23539160	1.69529100	6.20	0.32960290	-0.11482610	0.55925520	12.40	-0.09741537	-0.00821001	0.11383540
0.64	0.65524330	0.62857060	0.60189800	1.92	1.27027600	0.16149750	-0.94728120	3.20	1.66084200	-0.20932150	-2.07948500	4.48	1.38429100	-0.24135410	1.86999600	6.40	0.50151430	-0.13498810	0.77149050	12.80	-0.15204270	-0.00991443	0.17187150
0.66	0.67507920	0.64878190	0.62248460	1.98	1.28980700	0.18876910	-0.91226850																

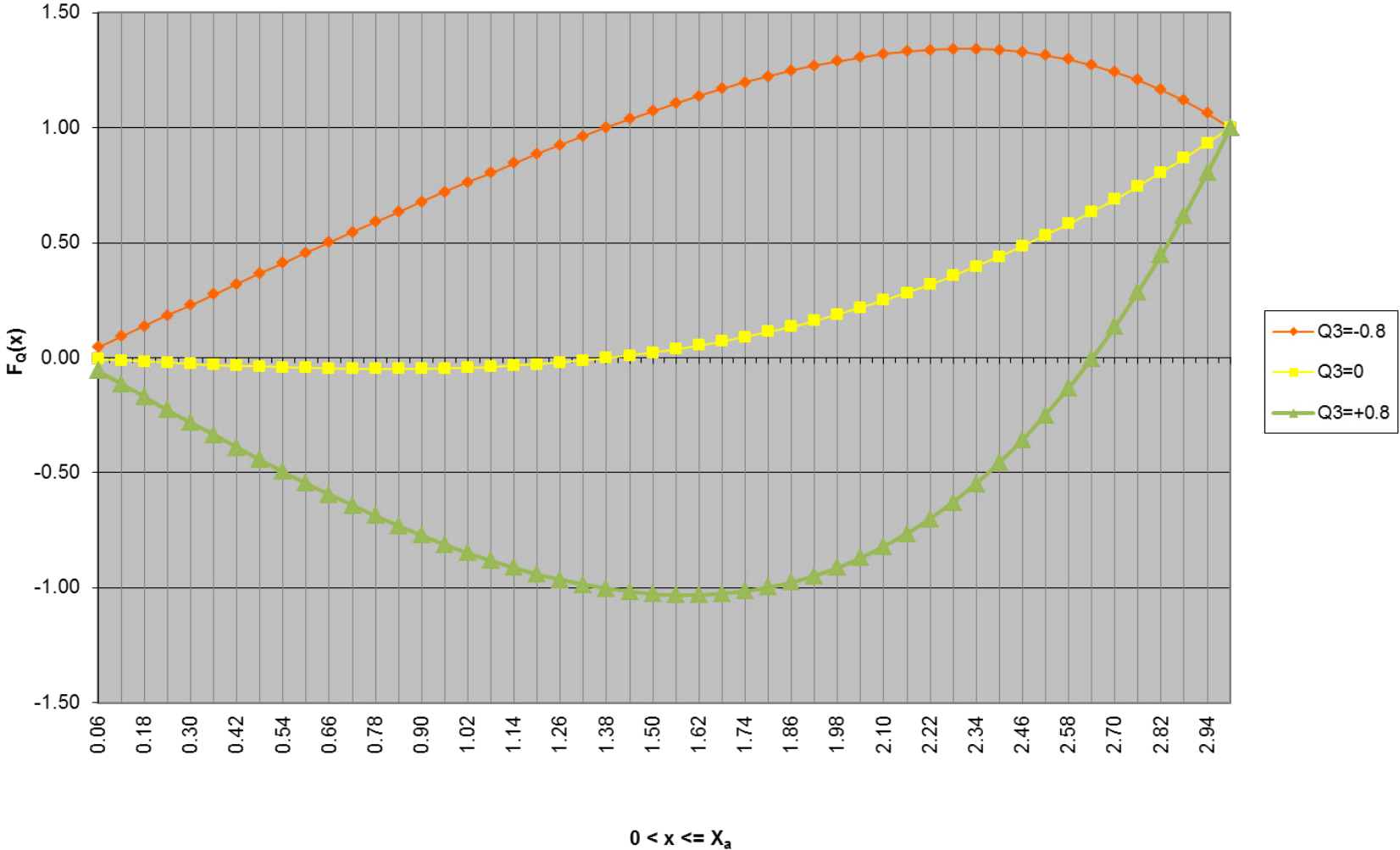
$X_a = 1$

$X_a = 1 \quad (v^* = 0.4)$



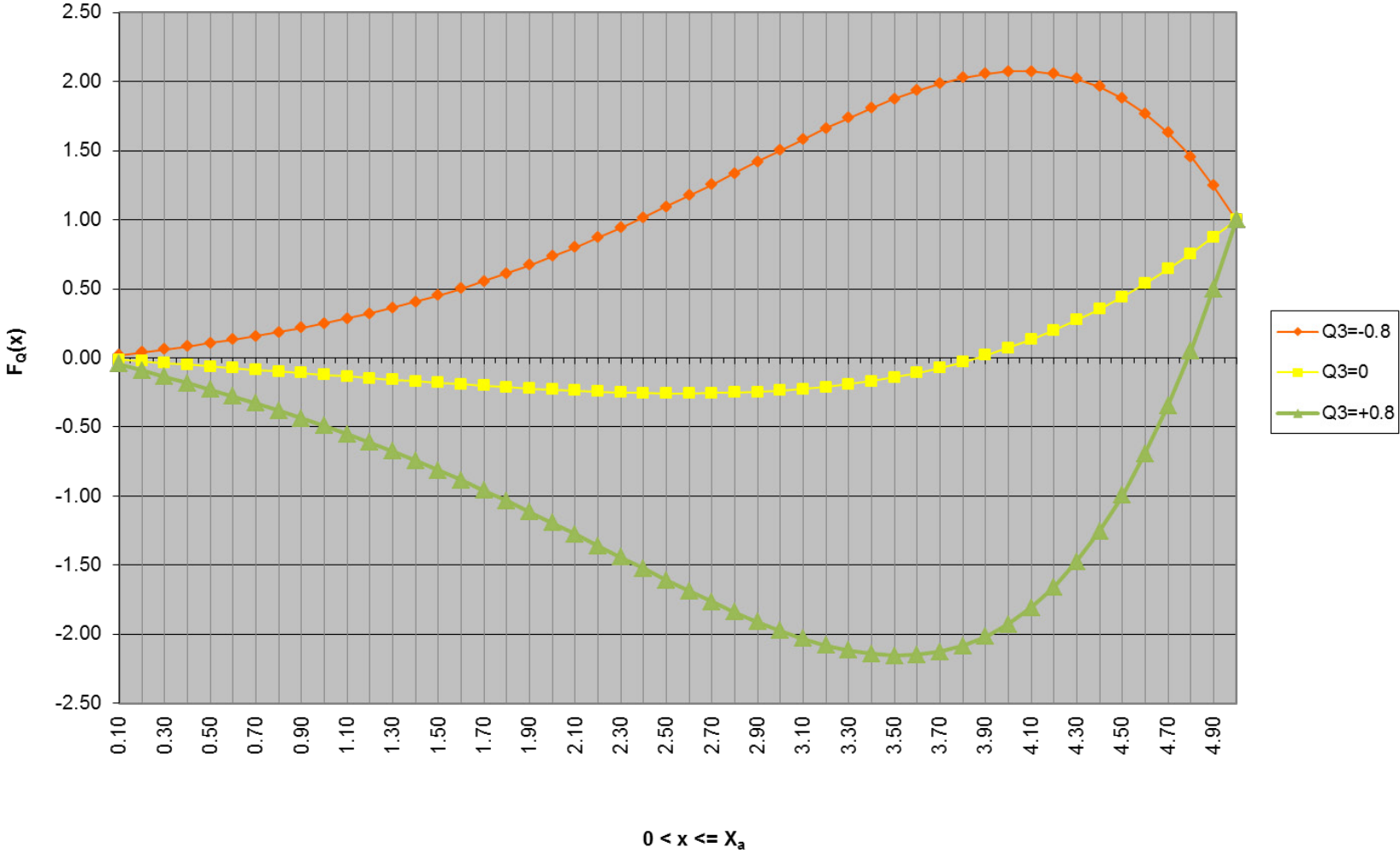
$X_a = 3$

$X_a = 3 \quad (v^* = 0.4)$



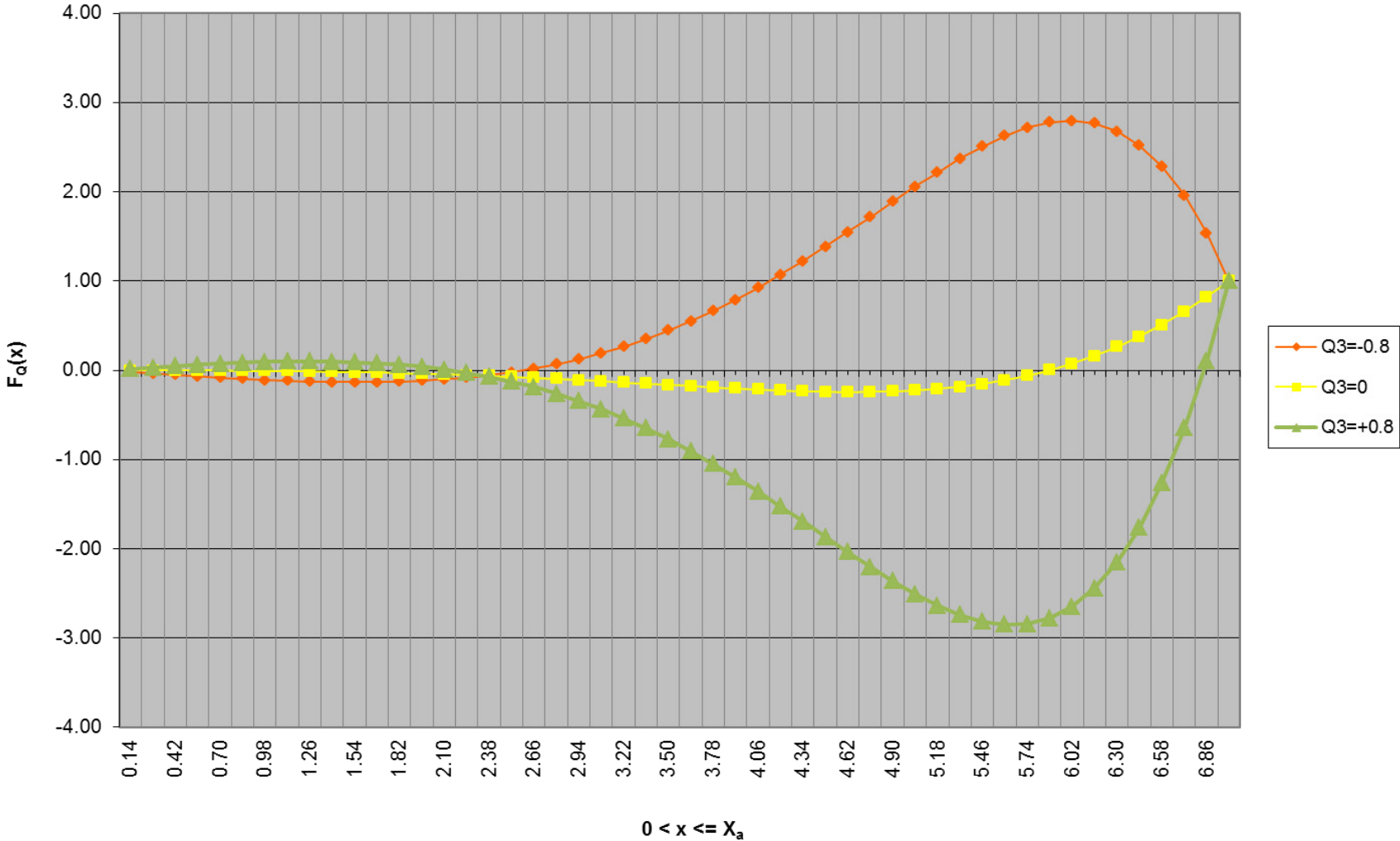
$X_a = 5$

$X_a = 5$ ($v^* = 0.4$)



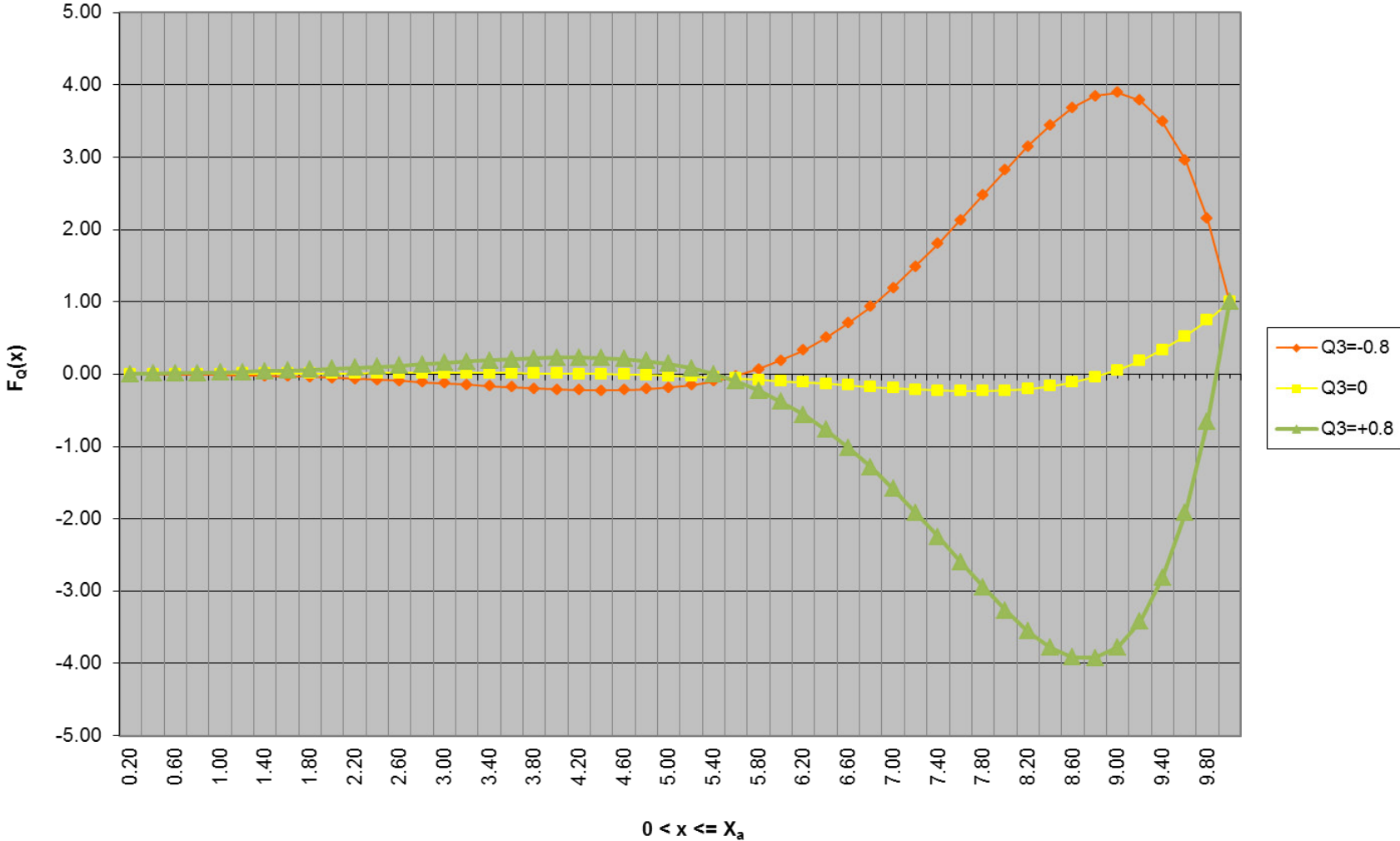
$X_a = 7$

$X_a = 7 \quad (v^* = 0.4)$



$X_a = 10$

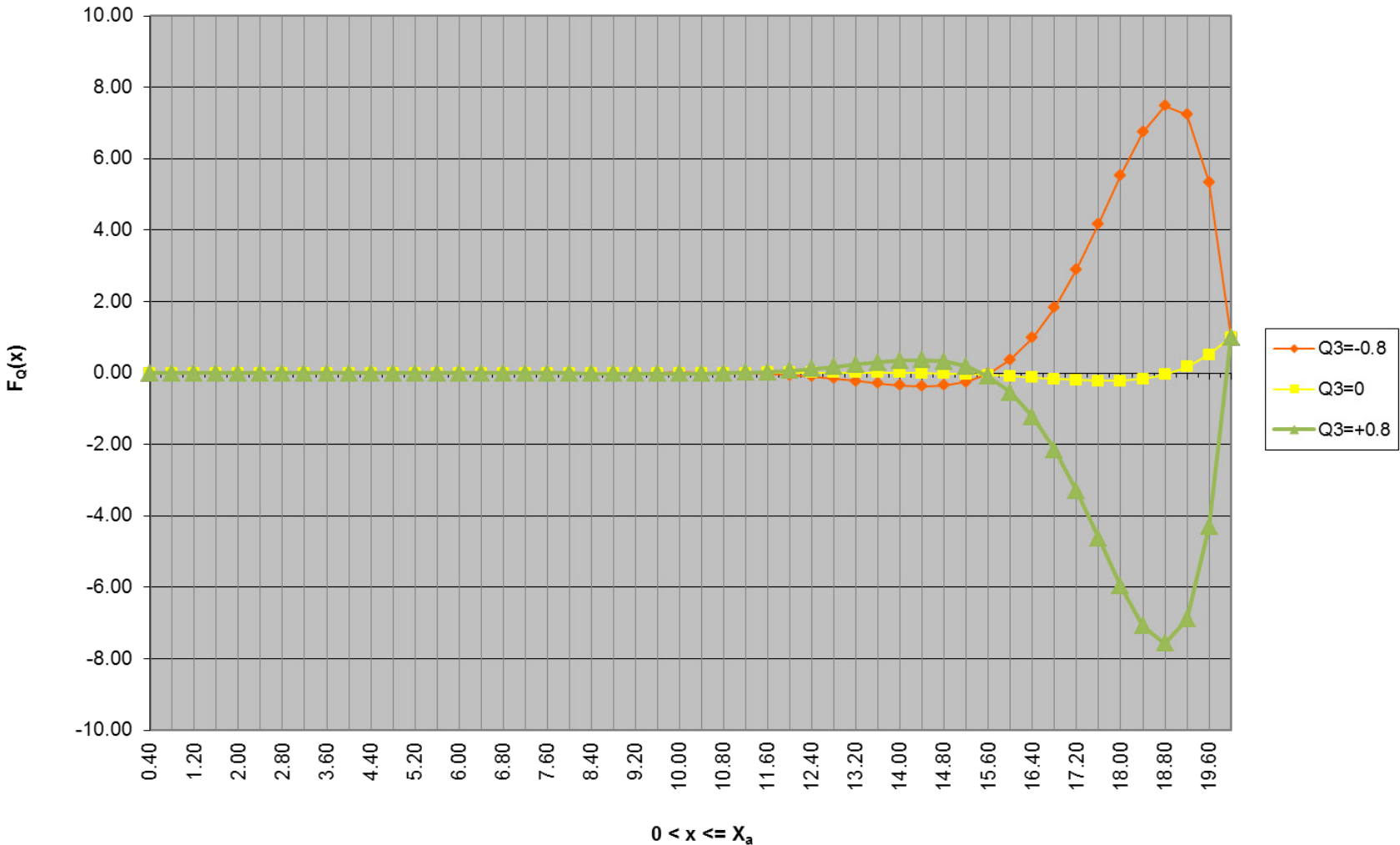
$X_a = 10 \quad (v^* = 0.4)$



$X_a = 20$

$X_a = 20$

$(v^* = 0.4)$

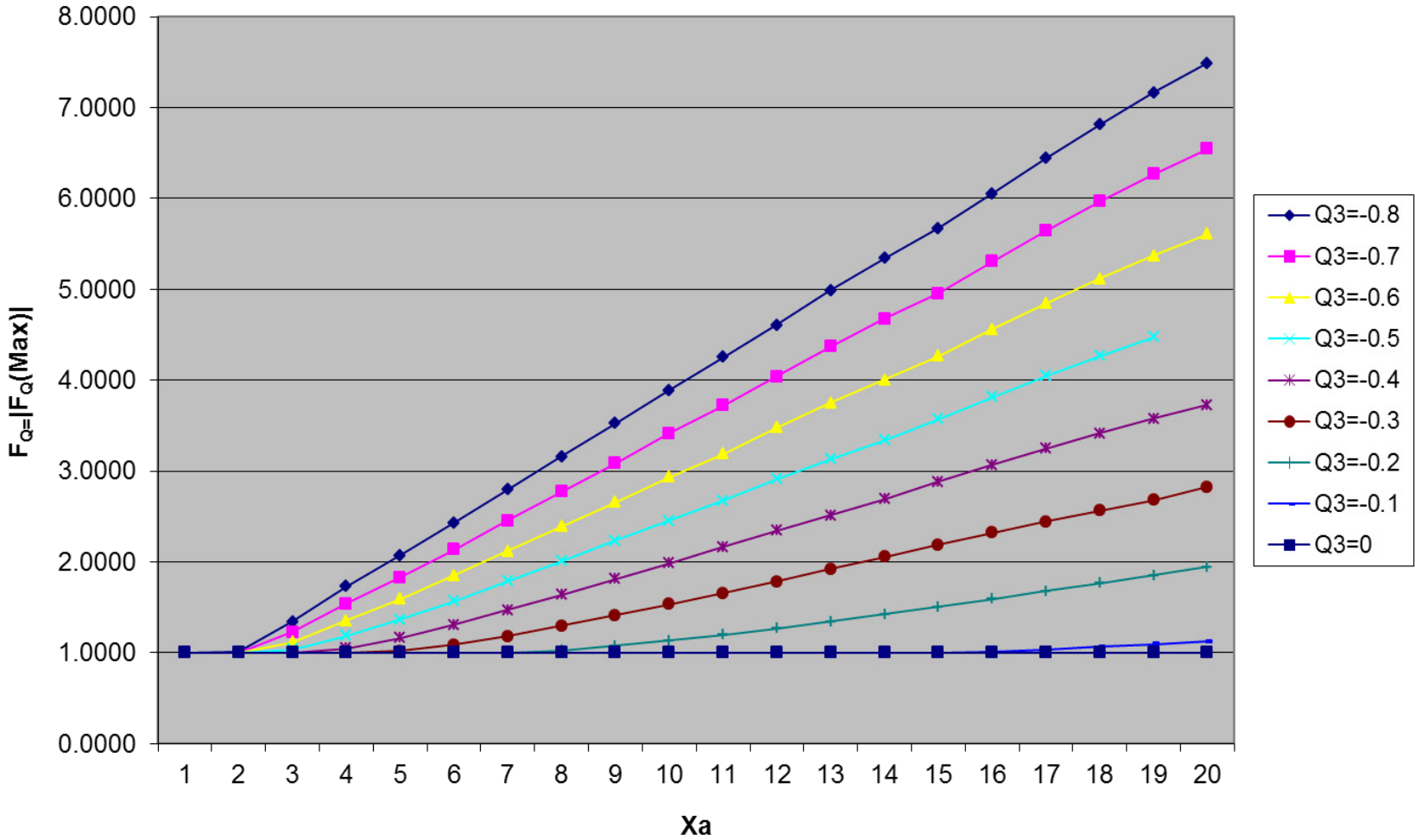


$F_Q = \text{Max}|F_Q(x)|$ vs X_a and Q_3

$X_a \backslash Q_3$	$F_Q = \text{Max} F_Q(x) $										$x(\text{max})=X_a \quad F_Q(X_a)=1$						
	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.0057	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3	1.3420	1.2251	1.1193	1.0353	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0308
4	1.7329	1.5390	1.3538	1.1841	1.0487	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.1542	1.3577	1.5619	1.7685
5	2.0740	1.8314	1.5932	1.3660	1.1614	1.0171	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.1549	1.4019	1.6506	2.1541
6	2.4303	2.1378	1.8505	1.5698	1.3068	1.0852	1.0000	1.0000	1.0000	1.0000	1.0000	1.0261	1.3150	1.6050	1.9012	2.1974	2.4936
7	2.7958	2.4556	2.1155	1.7891	1.4700	1.1820	1.0000	1.0000	1.0000	1.0000	1.0000	1.1517	1.4844	1.8241	2.1664	2.5087	2.8510
8	3.1588	2.7744	2.3900	2.0055	1.6403	1.2933	1.0257	1.0000	1.0000	1.0000	1.0000	1.2779	1.6619	2.0458	2.4297	2.8196	3.2123
9	3.5225	3.0843	2.6566	2.2335	1.8104	1.4120	1.0755	1.0000	1.0000	1.0000	1.0000	1.4094	1.8371	2.2648	2.6966	3.1353	3.5741
10	3.8915	3.4123	2.9331	2.4538	1.9850	1.5343	1.1335	1.0000	1.0000	1.0000	1.0706	1.5393	2.0145	2.4896	2.9648	3.4402	3.9259
11	4.2515	3.7207	3.1899	2.6763	2.1667	1.6570	1.1980	1.0000	1.0000	1.0000	1.1569	1.6651	2.1912	2.7172	3.2433	3.7693	4.2953
12	4.6077	4.0416	3.4755	2.9094	2.3433	1.7842	1.2689	1.0000	1.0000	1.0000	1.2427	1.8012	2.3598	2.9257	3.5027	4.0798	4.6568
13	4.9889	4.3696	3.7504	3.1311	2.5118	1.9196	1.3443	1.0000	1.0000	1.0000	1.3224	1.9318	2.5491	3.1665	3.7838	4.4012	5.0185
14	5.3440	4.6754	4.0067	3.3381	2.6890	2.0548	1.4224	1.0000	1.0000	1.0000	1.4157	2.0670	2.7183	3.3697	4.0210	4.6858	5.3545
15	5.6689	4.9552	4.2629	3.5713	2.8798	2.1882	1.5049	1.0000	1.0000	1.0000	1.4983	2.1818	2.8955	3.6092	4.3229	5.0366	5.7503
16	6.0529	5.3060	4.5591	3.8122	3.0652	2.3183	1.5899	1.0146	1.0000	1.0000	1.5802	2.3343	3.0884	3.8424	4.5965	5.3506	6.1046
17	6.4433	5.6434	4.8435	4.0437	3.2438	2.4439	1.6768	1.0385	1.0000	1.0000	1.6787	2.4682	3.2577	4.0471	4.8366	5.6261	6.4156
18	6.8142	5.9642	5.1141	4.2640	3.4139	2.5639	1.7651	1.0644	1.0000	1.0000	1.7633	2.5831	3.4029	4.2368	5.0868	5.9369	6.7870
19	7.1635	6.2662	5.3689	4.4717	3.5744	2.6783	1.8541	1.0921	1.0000	1.0000	1.8342	2.7064	3.6036	4.5009	5.3982	6.2954	7.1927
20	7.4889	6.5477	5.6065	4.6654	3.7242	2.8262	1.9433	1.1215	1.0000	1.0268	1.9228	2.8640	3.8052	4.7463	5.6875	6.6287	7.5699
$X_a \backslash Q_3$	$x(\text{max})$										$x(\text{max})=X_a \quad F_Q(X_a)=1$						
	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.8800	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000
3	2.2800	2.4000	2.5200	2.7000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	1.5600
4	3.1200	3.2000	3.2800	3.4400	3.6800	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	2.4000	2.4000	2.4800	2.4800
5	4.1000	4.1000	4.2000	4.3000	4.5000	4.8000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	3.4000	3.4000	3.5000	3.5000	3.5000
6	5.0400	5.0400	5.1600	5.1600	5.4000	5.6400	6.0000	6.0000	6.0000	6.0000	6.0000	4.3200	4.4400	4.5600	4.5600	4.5600	4.5600
7	6.0200	6.0200	6.0200	6.1600	6.3000	6.4400	7.0000	7.0000	7.0000	7.0000	7.0000	5.4600	5.4600	5.6000	5.6000	5.6000	5.6000
8	7.0400	7.0400	7.0400	7.0400	7.2000	7.3600	7.8400	8.0000	8.0000	8.0000	8.0000	6.5600	6.5600	6.5600	6.5600	6.7200	6.7200
9	7.9200	7.9200	8.1000	8.1000	8.1000	8.2800	8.6400	9.0000	9.0000	9.0000	9.0000	7.5600	7.5600	7.5600	7.7400	7.7400	7.7400
10	9.0000	9.0000	9.0000	9.0000	9.2000	9.2000	9.6000	10.0000	10.0000	10.0000	8.4000	8.6000	8.6000	8.6000	8.6000	8.8000	8.8000
11	9.9000	9.9000	9.9000	10.1200	10.1200	10.1200	10.5600	11.0000	11.0000	11.0000	9.4600	9.6800	9.6800	9.6800	9.6800	9.6800	9.6800
12	11.0400	11.0400	11.0400	11.0400	11.0400	11.2800	11.2800	12.0000	12.0000	12.0000	10.5600	10.5600	10.5600	10.8000	10.8000	10.8000	10.8000
13	11.9600	11.9600	11.9600	11.9600	11.9600	12.2200	12.2200	13.0000	13.0000	13.0000	11.4400	11.7000	11.7000	11.7000	11.7000	11.7000	11.7000
14	12.8800	12.8800	12.8800	12.8800	13.1600	13.1600	13.4400	14.0000	14.0000	14.0000	12.6000	12.6000	12.6000	12.6000	12.6000	12.8800	12.8800
15	13.8000	13.8000	14.1000	14.1000	14.1000	14.1000	14.4000	15.0000	15.0000	15.0000	13.5000	13.8000	13.8000	13.8000	13.8000	13.8000	13.8000
16	15.0400	15.0400	15.0400	15.0400	15.0400	15.0400	15.3600	15.6800	16.0000	16.0000	14.7200	14.7200	14.7200	14.7200	14.7200	14.7200	14.7200
17	15.9800	15.9800	15.9800	15.9800	15.9800	15.9800	16.3200	16.6600	17.0000	17.0000	15.6400	15.6400	15.6400	15.6400	15.6400	15.6400	15.6400
18	16.9200	16.9200	16.9200	16.9200	16.9200	16.9200	17.2800	17.6400	18.0000	18.0000	16.5600	16.5600	16.5600	16.9200	16.9200	16.9200	16.9200
19	17.8600	17.8600	17.8600	17.8600	17.8600	18.2400	18.2400	18.6200	19.0000	19.0000	17.4800	17.8600	17.8600	17.8600	17.8600	17.8600	17.8600
20	18.8000	18.8000	18.8000	18.8000	18.8000	19.2000	19.2000	19.6000	20.0000	18.4000	18.8000	18.8000	18.8000	18.8000	18.8000	18.8000	18.8000

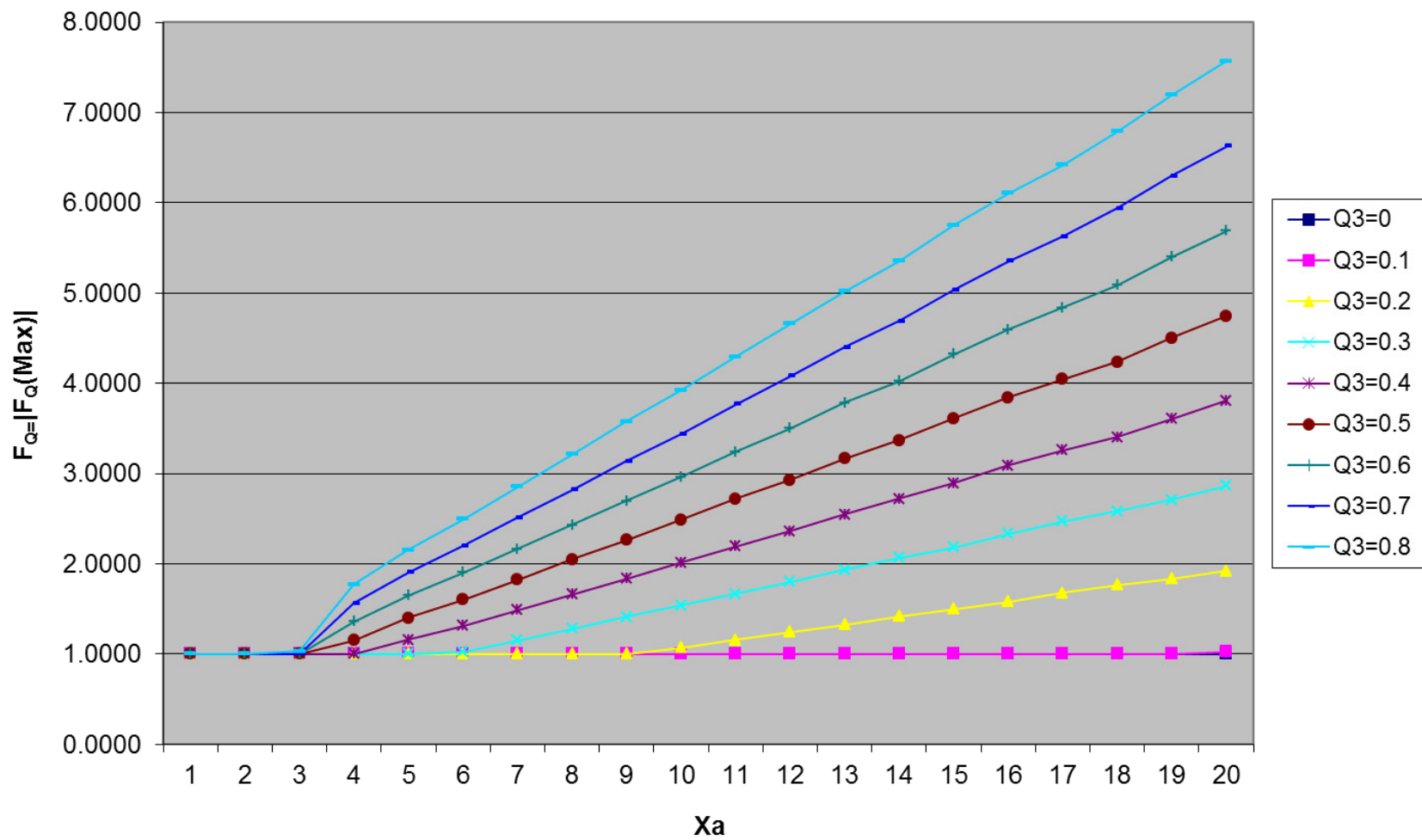
F_Q vs X_a and $-0.8 < Q_3 < 0$

$-0.8 < Q_3 < 0 ; v^* = 0.4$



Graph F_Q vs X_a and $0 < Q_3 < +0.8$

$$0 < Q_3 < 0.8 ; \nu^* = 0.4$$



F_Q/F_m

X_a/Q_3	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
1	2.5	2.857143	3.333333	4	5	6.666667	8.998657	6.240394	4.77635	3.868719	3.250954	2.803314	2.464029	2.198006	1.983826	1.807681	1.660265
2	2.51432	2.857143	3.333333	4	5	6.666667	10	7.523566	5.848846	4.783954	4.047103	3.506946	3.093996	2.767785	2.494105	2.259883	2.05977
3	3.35501	3.500383	3.73087	4.141388	5	6.666666	10	13.86455	10.87076	8.093174	6.187609	4.914857	4.039271	3.412036	2.944396	2.58429	2.371012
4	4.332228	4.397226	4.512783	4.736272	5.24374	6.666666	9.999999	20	16.22084	10.19497	7.142984	5.417763	4.330965	4.148806	4.161247	4.170054	4.176638
5	5.184925	5.232549	5.310653	5.46412	5.80719	6.780873	10	20	20.41941	11.54998	7.711372	5.697706	5.190115	5.178172	5.175274	5.16785	5.162176
6	6.075725	6.108031	6.168247	6.2791	6.534085	7.23438	10	20	24.54934	12.67999	8.133738	6.055543	6.057201	6.043075	6.052838	6.045625	6.037598
7	6.989395	7.016077	7.051653	7.156484	7.35005	7.87972	10	20	28.86764	13.61935	8.465816	6.970071	6.971029	6.959758	6.960386	6.960843	6.96119
8	7.89696	7.926769	7.96651	8.022152	8.20139	8.622153	10.25692	20	33.19143	14.41635	8.714232	7.888013	7.886964	7.886306	7.885861	7.878157	7.877766
9	8.80632	8.812226	8.855463	8.934016	9.051845	9.413327	10.75545	20	37.58715	15.10373	8.940473	8.806101	8.822027	8.798765	8.780189	8.786229	8.790792
10	9.72875	9.749374	9.77687	9.815368	9.92494	10.22883	11.33498	20	41.82752	15.61794	9.707451	9.747581	9.734529	9.705679	9.686176	9.672886	9.688325
11	10.62868	10.63051	10.63296	10.70518	10.83327	11.04675	11.98035	20	46.23938	16.14145	10.64935	10.62359	10.62964	10.63336	10.63587	10.63769	10.63906
12	11.51917	11.54736	11.58494	11.63755	11.71648	11.89494	12.6891	20	50.55819	16.52118	11.62482	11.54198	11.49611	11.49684	11.53397	11.56074	11.58096
13	12.47236	12.48471	12.50117	12.52422	12.5588	12.79733	13.44304	20	55.22265	16.83206	12.438	12.43668	12.47522	12.49884	12.51481	12.52633	12.53502
14	13.36011	13.35825	13.35577	13.3523	13.44501	13.6986	14.22442	20	59.26641	17.16312	13.38468	13.37383	13.3682	13.36474	13.3624	13.38801	13.38615
15	14.17231	14.15778	14.20962	14.28528	14.39875	14.58788	15.04863	20	63.69394	17.51477	14.24479	14.19137	14.31318	14.38781	14.40964	14.39027	14.37574
16	15.13227	15.15997	15.19691	15.24862	15.32618	15.45545	15.89881	20.29252	68.54565	17.7724	15.11083	15.26828	15.3501	15.36974	15.32167	15.28733	15.26158
17	16.10813	16.12396	16.14506	16.1746	16.21892	16.29277	16.76836	20.77014	72.90249	17.8918	16.15014	16.23864	16.28462	16.18856	16.12208	16.07459	16.03898
18	17.0356	17.04047	17.04695	17.05604	17.06966	17.09236	17.65105	21.28718	76.86542	18.01652	17.07245	17.09924	17.01448	16.94706	16.95614	16.96263	16.96749
19	17.90864	17.90342	17.89646	17.88672	17.87211	17.85515	18.54106	21.84126	81.1034	18.14656	17.87644	18.03008	18.01824	18.00362	17.99388	17.98693	17.98171
20	18.72217	18.7077	18.68842	18.66142	18.62093	18.84149	19.43298	22.43002	85.63786	18.77245	18.86903	19.09339	19.02589	18.98539	18.9584	18.93911	18.92465

ANNEX R—CALCULATION SHEET OF A FIXED TS HE (EXAMPLE UHX-20.2.3)

This Annex provides a Mathcad calculation sheet for the fixed TS HE defined in UHX-20.2.3 (Example 3). The TS is integral with shell and channel (configuration a). The data are shown in the calculation sheet, which is divided in 2 parts.

Part 1 follows strictly the steps 1 to 11 of UHX-13.5 calculation procedure. It includes the use of the elastic-plastic procedure at the TS-shell-channel connection.

Part 2 provides the equations which enable to calculate at any radius of the perforated tubesheet:

- the net effective pressure $q(r)$,
- the deflection $w(r)$,
- the rotation $\theta(r)$,
- the bending moment $M_r(r)$, and the bending stress $\sigma(r)$,
- the shear force $Q_r(x)$ and the shear stress $\tau(x)$,
- the tube axial stress $\sigma_t(r)$

These quantities are also given in graphical format. Their maximum values are determined and they match the maximum stress values obtained in Part 1. The positive directions of these quantities are shown on Figure 45.

The equations are taken from Clause VIII and therefore depend on axial load V_a and bending moment M_a acting at the periphery of the tubesheet (see Figure 40) which are determined from the equivalent pressure P_e :

$$V_a = \frac{a_o}{2} \cdot P_e \quad M_a = (a_o V_a) \cdot Q_1 + Q_2$$

These equations are general and do not depend on coefficient Q_3 . Thus, they apply whether $P_e \neq 0$ or $P_e = 0$.

The calculation sheet provides also the determination of the moment M_R acting on the unperforated rim and the edge loads Q_s , Q_c , M_s , M_c and axial force V_s and elastic stretch Δ_s of the shell. The positive directions of these quantities are shown on Figure 40. See Annex Y for UHX13-Example 3 (2010 edition) with General Equations.

ANNEX S—DETERMINATION OF THE ALLOWABLE BUCKLING STRESS LIMITS

If the tubes are under compression, tube buckling may restrict the tube's load carrying ability. If a substantial number of tubes were above their buckling limit, it is possible that the bundle could not sustain the required loading. This is true for either pressure or thermal load conditions. For this reason, no distinction is made between primary and secondary allowable compressive loads in the tubes.

The maximum permissible buckling stress limit in UHX parallels that as given in TEMA [12] .

(a) When tubes are under compression, the axial tube stress $\sigma_t(x)$ is negative and must be limited to:

The critical Euler's stress $\sigma_{cr} = \frac{\pi^2 E_t}{\lambda_t^2}$ where: λ_t is the slenderness ratio of the tube: $\lambda_t = \frac{l_t}{r_t}$

r_t is the tube radius of gyration: $r_t = \frac{\sqrt{d_t^2 + (d_t - 2t_t)^2}}{4}$

$l_t = k l$ is the largest unsupported buckling length of the tube, obtained from the unsupported tube spans, l , and their corresponding method of support k , where:

$k = 0.6$ for unsupported spans between two tubesheets,

$k = 0.8$ for unsupported spans between a tubesheet and a tube support

$k = 1.0$ for unsupported spans between two tube supports.

The yield $S_{y,t}$ of the tube material to prevent entering in the plastic range, which might lead to the failing of the tube bundle.

Figure 68 shows the forbidden range for σ_t represented by the hatched areas.

So as to ensure a smooth transition, TEMA has drawn a line from point A_o (λ_o ; $S_{y,t}/2$) to point A (0 ; $S_{y,t}$) which equation writes:

$$\sigma_{t,bk} = S_{y,t} \left(1 - \frac{\lambda_t}{2\lambda_o} \right) \quad \text{with: } \lambda_o = \sqrt{\frac{2\pi^2 E_t}{S_{y,t}}}$$

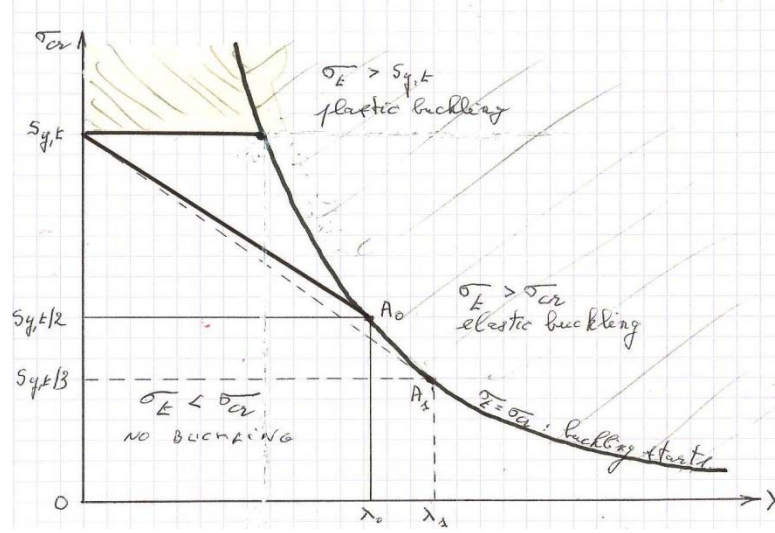


Figure 68—Tube buckling

(b) Accordingly, the buckling stress limit $\sigma_{t,bk}$ is given by:

- $\sigma_{t,bk} = S_{y,t} \left(1 - \frac{\lambda_t}{2\lambda_0} \right)$ if $\lambda_t < \lambda_0$. The tubes have a low slenderness ratio and plastic buckling prevails.
- $\sigma_{t,bk} = \frac{\pi^2 E_t}{\lambda_t^2}$ if $\lambda_t \geq \lambda_0$. The tubes have a high slenderness ratio and elastic buckling prevails.

Note: The slope of line AA_0 writes: $-\frac{S_{y,t}}{2\lambda_0}$ The slope of Euler's curve at point A_0 writes:

$$\left[\frac{d\sigma}{d\lambda} \right]_{A_0} = -\frac{2\pi^2 E_t}{\lambda_0^3} = -\frac{S_{y,t}}{\lambda_0} \text{ which is half of the slope of line } AA_0$$

A smoother transition would be obtained if the line was tangent to the Euler's curve, at point A_1 on Figure 68,

with a slope: $\left[\frac{d\sigma_{cr}}{d\lambda_t} \right]_{A_1} = -\frac{2\pi^2 E_t}{\lambda_1^3}$. Equation of line AA_1 writes: $\sigma_{t,bk} = S_{y,t} - \frac{2\pi^2 E_t}{\lambda_1^3} \lambda$

The abscissa λ_1 of point A_1 is obtained from: $S_{y,t} - \frac{2\pi^2 E_t}{\lambda_1^3} \lambda_1 = \frac{\pi^2 E_t}{\lambda_1^2}$,

which leads to: $\lambda_1 = \sqrt{1.5} \lambda_0$

The ordinate of point A_1 is obtained from: $[\sigma_{cr}]_{A_1} = \frac{\pi^2 E_t}{\lambda_1^2} = \frac{\pi^2 E_t}{1.5\lambda_0^2} = \frac{S_{y,t}}{3}$

Line AA₁, starting at $S_{y,t}/3$ instead of $S_{y,t}/2$, would ensure a better transition between elastic and plastic buckling.

(c) **The safety factor** applied to $\sigma_{t,bk}$ used in UHX is based on TEMA approach as follows.

The tube stress distribution throughout the tube bundle is given in Clause VIII-5:

$$\sigma_t(x) = \frac{1}{x_t - x_s} \left[(x_s P_s - x_t P_t) - F_t(x) P_e \right] \quad \text{with} \quad F_t(x) = \frac{X_a^4}{2} \left[Q_3 Z_w(x) + Z_d(x) \right]$$

[VIII-5] $x=kr$

At periphery ($x=ka_o=X_a$):
$$F_t(X_a) = F_q = \frac{X_a^4}{2} \left[Q_3 Z_w + Z_d \right]$$

Following simplifications are made in TEMA.

(1) No unperforated rim:
$$\left. \begin{array}{l} a_o = a_s \Rightarrow \rho_s = 1 \\ a_o = a_c \Rightarrow \rho_c = 1 \end{array} \right\} a_o = a_s = a_c = a \Rightarrow Q_1 = -\frac{\Phi Z_v}{1 + \Phi Z_m}$$

(2) No flange is considered: $W = 0$, $A = D_o \Rightarrow K = 1 \Rightarrow \Phi = \frac{1 - \nu^{*2}}{E^*} [\lambda_s + \lambda_c]$

(3) Radial displacement due to pressures P_s and P_t acting at TS-shell-channel connection is ignored:

$$\left. \begin{array}{l} w_s = 0 \Rightarrow \delta_s = 0 \Rightarrow \omega_s = 0 \Rightarrow \omega_s^* = 0 \\ w_c = 0 \Rightarrow \delta_c = 0 \Rightarrow \omega_c = 0 \Rightarrow \omega_c^* = 0 \end{array} \right\} \Rightarrow Q_2 = 0 \Rightarrow Q_3 = Q_1 = -\frac{\Phi Z_v}{1 + \Phi Z_m}$$

$$F_t(x) = \frac{X_a^4}{2} \left[-\frac{\Phi Z_v}{1 + \Phi Z_m} Z_w(x) + Z_d(x) \right]$$

(4) The maximum of $F_t(x)$ appears either inside of the tube-bundle ($r < R$) or at its periphery ($r = R$). TEMA considers only the outermost tube row.

At periphery $x=X_a=kR$:
$$F_t(X_a) = F_q = \frac{X_a^4}{2} \left[-\frac{\Phi Z_v}{1 + \Phi Z_m} Z_w + Z_d \right]$$

(5) The TS is considered

- either simply supported: $\lambda_s = 0$, $\lambda_c = 0 \Rightarrow \Phi = 0 \Rightarrow Q_1 = 0$
- either clamped: $\lambda_s = \infty$, $\lambda_c = \infty \Rightarrow \Phi = \infty \Rightarrow Q_1 = -\frac{Z_v}{Z_m}$

Curves $F_t(x)$ as a function of $x=r/R$ for various values of $X_a=kR$ ($X_a=1, 2, 3, \dots, 20$) represent the tube stress distribution $\sigma_t(x)$ throughout the tube bundle given by [VIII-5] above. Examination of these curves shows that:

The stress distribution varies smoothly throughout the tube bundle when X_a is low, i.e. when the tube bundle rigidity is significantly lower than the TS rigidity:

$X_a = 3$ when the TS is simply supported, which leads to $F_q = 2.25$
 $X_a = 7$ when the TS is clamped, which leads to $F_q = 2.7$

} in both cases F_q is about 2.5.

In that case, a large amount of adjacent tube rows may not take over the extra compressive load, which would lead to a general buckling of the tube bundle.

Accordingly, a higher safety factor $F_S=2.0$ is used.

The stress distribution varies significantly throughout the tube bundle when X_a is high, i.e. when the tube bundle rigidity is significantly higher than the TS rigidity:

$X_a = 5$ when the TS is simply supported, which leads to $F_q = 3.79$
 $X_a = 10$ when the TS is clamped, which leads to $F_q = 3.83$

} in both cases F_q is about 4.

In that case, if the outermost tube row buckles, the adjacent tube rows can take over the extra compressive load. Accordingly, a lower safety factor $F_S=1.25$ is used.

In the intermediate range of F_q ($2.5 \leq F_q \leq 4$), a linear interpolation is used (see Figure 69):
 $F_S = 3.25 - 0.5F_q$

Thus, the safety factor writes: $F_S = \text{MAX} \left[(3.25 - 0.5F_q), (1.25) \right]$

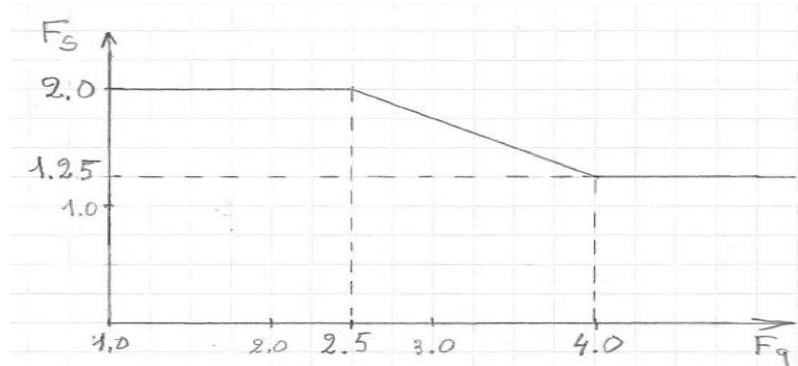


Figure 69—Determination of buckling safety factor F_S

(d) **Moving to UHX/TEMA notations** ($F_t=\lambda_t$; $C_t=\lambda_o$), the maximum permissible buckling stress limit writes, having in mind that it cannot exceed the allowable stress of the tube material:

$$S_{ib} = \text{MIN} \left\{ \left[\frac{1}{F_S} \frac{\pi^2 E_t}{F_t^2} \right], [S_t] \right\} \quad \text{when } C_t \leq F_t$$

$$S_{ib} = \text{MIN} \left\{ \left[\frac{S_{y,t}}{F_S} \left(1 - \frac{F_t}{2C_t} \right) \right], [S_t] \right\} \quad \text{when } C_t > F_t$$

$$\text{with: } F_t = \frac{l_t}{r_t} \quad r_t = \frac{\sqrt{d_t^2 + (d_t - 2t_t)^2}}{4} \quad C_t = \frac{2\pi^2 E_t}{S_{y,t}}$$

$$F_s = MAX \left[(3.25 - 0.5F_q), (1.25) \right] \quad F_q = \frac{X_a^4}{2} [Q_3 Z_w + Z_d]$$

The minimum value of $\sigma_t(x)$, given by: $\sigma_{t,\min} = \text{MIN}[(\sigma_{t,1}), (\sigma_{t,2})]$ must not exceed the buckling stress limit: $|\sigma_{t,\min}| \leq S_{tb}$

ANNEX T—COMMON INTERSECTION OF CURVES $\sigma_t(x)$

T.1 General

Numerical calculations of H.E. according to UHX-13 show that $\sigma_t(x)$ curves ($0 < x < X_a$) intersect at the same point $x = x_0$ for the 7 loading cases, whatever the Q_3 value is for each of these loading cases (see Figure 70).

$$\sigma_t(x) = \frac{1}{x_t - x_s} \left[(P_s \cdot x_s - P_t \cdot x_t) - P_e \cdot F_t(x) \right]$$

$$\sigma_t(x) \text{ is calculated from } F_t(x): F_t(x) = [Z_d(x) + Q_3 \cdot Z_w(x)] \cdot \frac{X_a^4}{2}$$

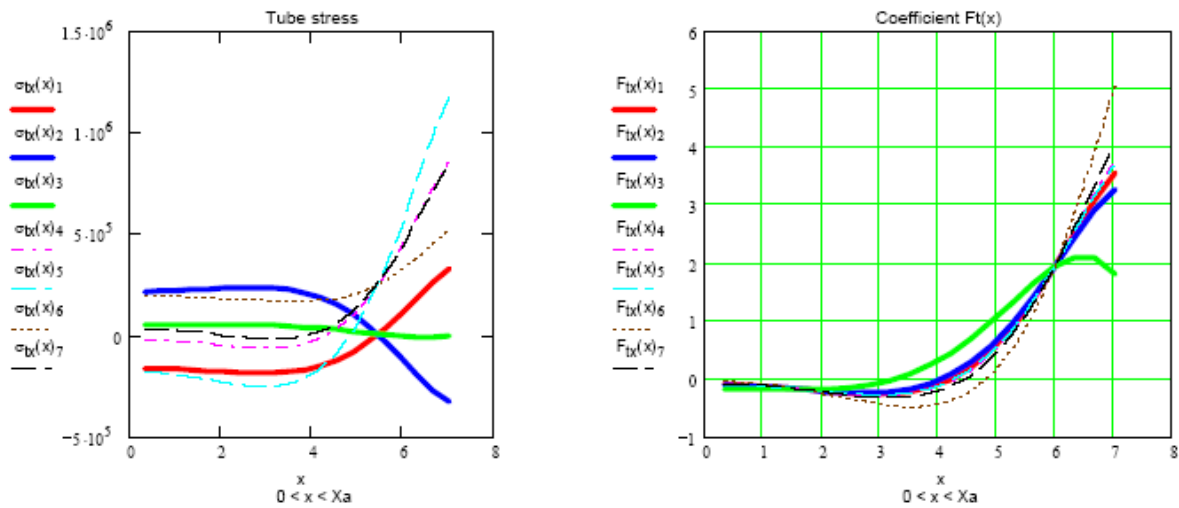


Figure 70—Graphs giving $\sigma_t(x)$ and $F_t(x)$ for the 7 loading cases

T.2 Determination of Common Intersection x_0 for $\sigma_t(x)$

Curves $F_t(x)$ will intersect at the same point of abscissa x_0 , whatever Q_3 value is, if:

$$\frac{2}{X_a^4} F_t(x_0) = Z_d(x_0) + Q_3 \cdot Z_w(x_0) = K$$

Where K is a constant independent of Q_3 . This is only possible if $Z_w(x_0) = 0$, which means that the value x_0 of common intersection is obtained for value(s) of x_0 which make $Z_w(x)$ equal to 0.

Parametric calculations on X_a show that there is always at least 1 value of x_0 which makes $Z_w(x_0) = 0$. For high values of X_a ($X_a > 6$), there are 2 values of x_0 . For example UHX-20.2.3 ($X_a = 7$): $Z_w(x) = 0$ for $x_0 = 1.68$ and $x_0 = 5.97$.

The curves $\sigma_t(x)$ for the 3 pressure loading cases intersect at the same point of abscissa x_1 . This point is different from the intersection point x_0 of $F_t(x)$, due to the presence of P_e .

The curves $\sigma_t(x)$ for the 4 thermal loading cases intersect at the same point of abscissa x_1 , but their values are generally higher than for the 3 pressure loading cases due to higher values of P_e for thermal loading cases.

For example UHX-20.2.3:

- curves relative to pressure loading cases 1, 2 and 3 intersect at the same point ($x_1 = 5.5$; $\sigma_t = 0$)
- curves relative to thermal loading cases 4, 5, 6 and 7 intersect at the same point ($x_1 = 5.5$; $\sigma_t = 1700$ psi)

T.3 Generalization to Other Stresses

The same principle applies also to:

the tubesheet bending stress $\sigma(x)$, calculated from: $F_m(x) = [Q_v(x) + Q_3 \cdot Q_m(x)] \cdot \frac{1}{2}$

Value of intersection is obtained for $Q_m(x_0) = 0$. There is no value of x_0 for $X_a < 5$, and one value of x_0 for $X_a \geq 5$

the tubesheet shear stress $\tau(x)$, calculated from: $F_Q(x) = [Q_\beta(x) + Q_3 \cdot Q_\alpha(x)]$

Value of intersection is obtained for $Q_\alpha(x_0) = 0$. There is always one solution for $x_0 = X_a$ (as $Q_\alpha(X_a) = 0$), and a 2nd value of x_0 for $X_a \geq 7$

the tubesheet slope $\theta(x)$, calculated from: $F_\theta(x) = [Z_v(x) + Q_3 \cdot Z_m(x)] \cdot \frac{X_a^3}{2}$

ANNEX U—DETERMINATION OF STRESSES USING THE FIXED TS RULES

UHX -12 rules can be obtained using the fixed TS analysis developed in PART 3, provided that the tubes have no axial rigidity, which implies that they do not play the role of an elastic foundation.

$$\text{Accordingly: } k_t = 0 \Rightarrow k_w = \frac{N_t k_t}{\pi a_o^2} = 0 \Rightarrow X_a = 0$$

Modifications of PART 3 analysis are as follows.

(a) **Equilibrium of the Unperforated Rim** (see Clauses VI.2 and VI.3 of PART 3)

$$(1) \text{ due to axial loads axial equilibrium of the shell: } 2\pi a_s' V_s = \pi a_s^2 P_s \text{ leads to: } a_o V_a = \frac{a_o^2}{2} P_e$$

(2) **due to applied moments:** expliciting V_a ,

equation [VI.2b'] of PART 3 becomes:

$$\begin{aligned} [R M_R] = & -a_o M_a + \frac{P_s}{4} a_o^3 [(\rho_s - 1)(\rho_s^2 + 1)] - \frac{P_c}{4} a_o^3 [(\rho_c - 1)(\rho_c^2 + 1)] \\ & - \left[a_s' k_s \left(1 + t_s' + \frac{t_s'^2}{2} \right) + a_c' k_c \left(1 + t_c' + \frac{t_c'^2}{2} \right) \right] \theta_a + a_o (\omega_c P_c - \omega_s P_s) \end{aligned}$$

equation [VI.3a'] of PART 3 becomes:

$$\Phi [M_a Z_m + (a_o V_a) Z_v] = -M_a + (\omega_s^* P_s - \omega_c^* P_c) + \frac{1}{2\pi} [W_c \gamma_{bc} - W_s \gamma_{bs}]$$

$$\text{with: } \Phi = (1 + \nu^*) F \left[\omega_s^* = a_o^2 \left[\frac{(\rho_s - 1)(\rho_s^2 + 1)}{4} \right] - \omega_s \right] \left[\omega_c^* = a_o^2 \left[\frac{(\rho_c - 1)(\rho_c^2 + 1)}{4} \right] - \omega_c \right]$$

(b) **Stresses in the Tubesheet** (see Clause VI.3 of PART 3)

(1) **moment at TS periphery**

Equation [VI.3a] of PART 3 becomes:

$$M_a = \underbrace{\left(\frac{a_o^2}{2} P_e \right) \frac{-\Phi Z_v}{1 + \Phi Z_m}}_{Q_1} + \underbrace{\frac{(\omega_s^* P_s - \omega_c^* P_c) + \left(W_c \frac{\gamma_{bc}}{2\pi} - W_s \frac{\gamma_{bs}}{2\pi} \right)}{1 + \Phi Z_m}}_{Q_2}$$

$$M_a = (a_o V_a) Q_1 + Q_2 \quad Q_1 = \frac{-\Phi Z_v}{1 + \Phi Z_m} \quad Q_2 = \frac{(\omega_s^* P_s - \omega_c^* P_c) + (W_c - W_s) \frac{\gamma_b}{2\pi}}{1 + \Phi Z_m}$$

From Annex F, for $X_a=0$:

$$Z_m = \frac{1}{1+\nu^*} \quad \Phi Z_m = F \quad Z_v = \frac{1}{4(1+\nu^*)} \quad \Phi Z_v = \frac{F}{4}$$

$$Q_1 = \frac{\frac{F}{4}}{1+F} \quad Q_2 = \frac{(\omega_s^* P_s - \omega_c^* P_c) + (W_c - W_s) \frac{\gamma_b}{2\pi}}{1+F}$$

Expliciting ω_s^* and ω_c^* , Q_2 writes:

$$Q_2 = \frac{P_s \frac{a_o^2}{4} [(\rho_s - 1)(\rho_s^2 + 1)] - P_c \frac{a_o^2}{4} [(\rho_c - 1)(\rho_c^2 + 1)] + (\omega_c P_c - \omega_s P_s) + (W_c - W_s) \frac{\gamma_b}{2\pi}}{1+F}$$

$$M_a = \frac{\overbrace{P_s \frac{a_o^2}{4} [(\rho_s - 1)(\rho_s^2 + 1)] - P_c \frac{a_o^2}{4} [(\rho_c - 1)(\rho_c^2 + 1)] + (\omega_c P_c - \omega_s P_s) + \frac{\gamma_b}{2\pi} (W_c - W_s) - F \frac{a_o^2}{8} P_e}^{M^*}}{1+F} = \frac{M^* - F \left(\frac{a_o^2}{8} P_e \right)}{1+F}$$

W_s and W_c values to be used in UHX-12 are given in V-2d.

(2) **moment at TS center** is given by Annex F when $x=0$

$$M_0 = M(0) = a_o^2 P_e F_m(0) \quad \text{with :} \quad F_m(0) = \frac{Q_3}{2} + \frac{3+\nu^*}{16} \quad Q_3 = Q_1 + \frac{2}{a_o^2 P_e} Q_2$$

$$\text{Using } Q_1 \text{ and } Q_2 \text{ above: } Q_3 = \frac{-\frac{F}{4}}{1+F} + \frac{2}{(a_o^2 P_e)} \frac{(\omega_s^* P_s - \omega_c^* P_c) + (W_c - W_s) \frac{\gamma_b}{2\pi}}{1+F}$$

$$M_o = \frac{-\frac{F}{4} a_o^2 P_e + (\omega_s^* P_s - \omega_c^* P_c) + (W_c - W_s) \frac{\gamma_b}{2\pi}}{1+F} + \frac{a_o^2}{16} (3+\nu^*) P_e = M_a + \frac{a_o^2}{16} (3+\nu^*) P_e$$

(3) **maximum moment in TS**

Annex P shows that for low values of X_a ($X_a < 1$), the maximum of the TS moment $M(x)$ appears either at the center or at TS periphery: $M = \text{MAX} [|M_p|, |M_o|]$

(c) **Stresses in the Shell and Channel** (see Clause VI.6 of PART 3)

(1) axial membrane stress

From [VIII.6a] of PART 3:

$$\sigma_{s,m} = \frac{a_o^2}{(D_s + t_s)t_s} \left[(P_s - P_t) + (\rho_s^2 - 1)(P_s - P_t) \right] + \frac{a_s^2}{(D_s + t_s)t_s} P_t = \frac{a_o^2}{(D_s + t_s)t_s} \left[\rho_s^2 (P_s - P_t) \right] + \frac{a_s^2}{(D_s + t_s)t_s} P_t$$

$$\sigma_{s,m} = \frac{a_o^2}{(D_s + t_s)t_s} \left[\rho_s^2 (P_s - P_t) \right] + \frac{a_s^2}{(D_s + t_s)t_s} P_t$$

$\sigma_{s,m} = \frac{a_s^2}{(D_s + t_s)t_s} P_s$

 which is the classical formula for cylinders

Axial membrane stress in the channel writes in the same way:

$\sigma_{c,m} = \frac{a_c^2}{(D_c + t_c)t_c} P_t$

(2) axial bending stress

From [VIII.6c] of PART 3:

$$\sigma_{s,b} = \frac{6}{t_s^2} k_s \left\{ \beta_s \delta_s P_s + \frac{6(1-\nu^{*2})}{E^*} \left(\frac{a_o}{h} \right)^3 \left(1 + \frac{h \beta_s}{2} \right) \left[P_e (Z_v + Q_1 Z_m) + \frac{2}{a_o^2} (Q_2 Z_m) \right] \right\}$$

From Annex F, for $X_a=0$: $Z_m = \frac{1}{1+\nu^*}$ $Z_v = \frac{1}{4(1+\nu^*)}$

$$\sigma_{s,b} = \frac{6}{t_s^2} k_s \left\{ \beta_s \delta_s P_s + \frac{6(1-\nu^{*2})}{E^*} \left(\frac{a_o}{h} \right)^3 \left(1 + \frac{h \beta_s}{2} \right) \left[P_e \left(\frac{1}{4(1+\nu^*)} + Q_1 \frac{1}{(1+\nu^*)} \right) + \frac{2}{a_o^2} \left(Q_2 \frac{1}{(1+\nu^*)} \right) \right] \right\}$$

$$\sigma_{s,b} = \frac{6}{t_s^2} k_s \left\{ \beta_s \delta_s P_s + 6 \frac{1-\nu^{*2}}{E^*} \left(\frac{a_o}{h} \right)^3 \left(1 + \frac{h \beta_s}{2} \right) \frac{1}{1+\nu^*} \frac{2}{a_o^2} \left[P_e \frac{a_o^2}{8} + \underbrace{P_e Q_1 \frac{a_o^2}{2} + Q_2}_{M_a} \right] \right\}$$

$\sigma_{s,b} = \frac{6}{t_s^2} k_s \left[\beta_s \delta_s P_s + 6 \frac{1-\nu^{*2}}{E^*} \frac{D_o}{h^3} \left(1 + \frac{h \beta_s}{2} \right) \left(M_a + \frac{D_o^2}{32} P_e \right) \right]$

Axial bending stress in the channel writes in the same way:

$\sigma_{c,b} = \frac{6}{t_c^2} k_c \left[\beta_c \delta_c P_c - 6 \frac{1-\nu^*}{E^*} \frac{D_o}{h^3} \left(1 + \frac{h \beta_c}{2} \right) \left(M_a + \frac{D_o^2}{32} P_e \right) \right]$

- (d) **In conclusion** the general formulas of PART 3, applied to U-Tube TS HE, match the UHX-12 formulas obtained in Clause VII. This implies that UHX-12 rules could be written in the same way as UHX-13 rules, using for (ω_s^*, ω_c^*) and (Q_1, Q_2) the formulas given here above.

ANNEX V—CALCULATION USING A FLOATING OR FIXED TS HE SOFTWARE

The U-Tube HE can be calculated using a floating TS HE software, such as the Mathcad software used in Clause XI-3 of PART 4 as follows:

- use an immersed floating head TS HE as explained in Clause III.5.1
- use $N_t=0$ so that the tubes have no axial rigidity.

Calculation could be also performed using a fixed TS HE software, such as the Mathcad software used in Annex R as follows:

- use a bellows of rigidity close to 0 to simulate an immersed floating TS
- use $L_t=\text{infinity}$ ($L_t=10^{20}$), so that the tubes have no axial rigidity.
- use $P_e=P_s-P_t$

ANNEX W—UHX14 EXAMPLE 1 (2010 EDITION) STATIONARY

STATIONARY Tubesheet Rules acc. to UHX-14(July 2010 Edition)

highlighted in Yellow :most important data and results

CASE 20.3.1 (d + C) STATIONARY Tubesheet

CHANNEL data which are different for Stationary and Floating Tubesheets

SHELL data which are not used for Floating Tubesheet

Geometric Data (from Fig.UHX-14.1)

ExchType := "a" EXCHANGER types : a, b, c

$h := 1.75 \text{ in}$ Tubesheet thickness

StaConfig := "d" STATIONNARY Config. types : a, b, c, d,e,f

FloConfig := "C" FLOATING Config. types : A, B, C, D

Config := StaConfig Config = "d"

Layout := 0 For triangular pitch : "Layout"=0
For square pitch : "Layout"=1

$p := 1.0 \cdot \text{in}$ Tube Pitch Set Farenheit temperatures
degF := R

$r_o := 12.5 \cdot \text{in}$ Radius to outer tube

$N_t := 466$ Number of Tubes
 $d_t := 0.75 \cdot \text{in}$ Tube Diameter

$A := 33.071 \cdot \text{in}$ Outside Dia of Tubesheet

$t_t := 0.083 \cdot \text{in}$ Tube Thickness

$C := 31.417 \cdot \text{in}$ Bolt Circle Diameter

$L_t := 256 \cdot \text{in}$ Tube Length

$A_L := 64.375 \cdot \text{in}^2$ Total Untubed Lanes Area

$\rho := 0.8$ Tube expansion factor
 $l_{tx} := 1.25 \cdot \text{in}$ Length of Expanded Portion of Tube

$h_g := 0.197 \cdot \text{in}$ Groove depth

$c_t := 0 \cdot \text{in}$ Tubesheet Corr. Allow. (Tubeside)

$kl := 15.375 \cdot \text{in}$ $k=0.6$ for spans between Tubesheets
 $k=0.8$ for spans between TS/ support plate
 $k=1.0$ for spans between support plates
 l = unsupported tube span

$c_s := 0 \cdot \text{in}$ Tubesheet Corr. Allow. (Shellside)

SS := "NO" SS="YES" for Simply Supported TS

$D_s := 0 \cdot \text{in}$ Shell ID

$D_c := 0.0 \cdot \text{in}$ Channel ID

$t_s := 0.1 \cdot \text{in}$ Shell Thickness

$t_c := 0.1 \cdot \text{in}$ Channel Thickness

$G_s := 29.375 \cdot \text{in}$ Shell Gasket Diameter

$G_c := 29.375 \cdot \text{in}$ Channel Gasket Diameter

$G_1 := 0 \cdot \text{in}$ shell Contact mid-point TS/Flange

$G_1 := 0 \cdot \text{in}$ Channel Contact mid-point TS/Flange

$C_s := 0 \cdot \text{in}$ Shell Corrosion Allowance

$C_c := 0 \cdot \text{in}$ Channell Corrosion Allowance

CHAN := "CYL" "CYL" for Cylindrical Channel
"HEMI" for Hemispherical Channel

Corroded thicknesses:

Corroded length:

$h := h - c_s - c_t$ Tubesheet thickness

$h = 1.75 \text{ in}$

$L_t := L_t - 2c_t$ $L_t = 256.00 \text{ in}$

$t_s := t_s - C_s$ Shell Thickness away from TS

$t_s = 0.10 \text{ in}$

Corroded diameters:

$t_c := t_c - C_c$ Channel Thickness

$t_c = 0.10 \text{ in}$

$D_s := D_s + 2 \cdot C_s$ Shell ID $D_s = 0.000 \text{ in}$

$D_c := D_c + 2 \cdot C_c$ Channel ID $D_c = 0.000 \text{ in}$

Loading conditions

$$W := 211426 \cdot \text{lb} \quad \text{Shell or Channel Flange bolt load}$$

$$P_{s0} := 250 \cdot \frac{\text{lb}}{\text{in}^2}$$

Shell Design Pressure

$$P_{t0} := 150 \cdot \frac{\text{lb}}{\text{in}^2}$$

Tube Design Pressure

Design Pressures for the Seven Load Cases

$$P_s := \begin{pmatrix} 0 \cdot \frac{\text{lb}}{\text{in}^2} \\ P_{s0} \\ P_{s0} \\ 0 \cdot \frac{\text{lb}}{\text{in}^2} \\ 0 \cdot \frac{\text{lb}}{\text{in}^2} \\ P_{s0} \\ P_{s0} \end{pmatrix}$$

$$P_s = \begin{pmatrix} 0.00 \\ 250.00 \\ 250.00 \\ 0.00 \\ 0.00 \\ 250.00 \\ 250.00 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$P_t := \begin{pmatrix} P_{t0} \\ 0 \cdot \frac{\text{lb}}{\text{in}^2} \\ P_{t0} \\ 0 \cdot \frac{\text{lb}}{\text{in}^2} \\ P_{t0} \\ 0 \cdot \frac{\text{lb}}{\text{in}^2} \\ P_{t0} \end{pmatrix}$$

$$P_t = \begin{pmatrix} 150.00 \\ 0.00 \\ 150.00 \\ 0.00 \\ 150.00 \\ 0.00 \\ 150.00 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

Temperatures

$$T := 70 \cdot \text{degF}$$

Tubesheet design temp.

$$T' := 70 \cdot \text{degF}$$

Tubesheet temp.@ rim

$$T_t := 70 \cdot \text{degF}$$

Tube design temp.

$$T_s := 70 \cdot \text{degF}$$

Shell design temp.

$$T'_s := 70 \cdot \text{degF}$$

Shell temp. @ Tubesheet

$$T_c := 70 \cdot \text{degF}$$

Channel design temp.

$$T'_c := 70 \cdot \text{degF}$$

Channel temp.@ tubesheet

$$T_a := 70 \cdot \text{degF}$$

Ambient temperature

Material Data**Tubesheet Material is SB-564 C276 high**

$$S := 19000 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{TS allowable stress @ T}$$

$$S_y := 31000 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{TS yield stress @T}$$

$$E := 27 \cdot 10^6 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{TS elastic modulus @ T}$$

$$S_{PS} := 33900 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{TS allowable P+S stress @ T}$$

$$\nu := 0.32 \quad \text{TS Poisson's ratio}$$

$$\alpha' := 0 \cdot \frac{\text{in}}{\text{in} \cdot \text{degF}} \quad \text{TS coeff. expansion@ rim}$$

Tube Material is SB-626 C276

$$S_t := 13350 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{Tube allowable stress @ } T_t$$

$$S_{tT} := 13350 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{Tube allowable stress @ } T_{tT}$$

$$E_t := 27 \cdot 10^6 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{Tube elastic modulus @ } T_t$$

$$E_{tT} := 27 \cdot 10^6 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{Tube elastic modulus @ } T_{tT}$$

$$\nu_t := 0.32 \quad \text{Tube Poisson's ratio}$$

$$S_{yt} := 20550 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{Tube yield stress @ } T_t$$

Shell Material is SA-516-70 N

for STATIONARY Tubesheet ONLY

$$S_s := 0 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{Shell allow. stress @Ts}$$

$$S_{ys} := 0 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{Shell yield stress @Ts}$$

$$E_s := 27 \cdot 10^6 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{Shell elast mod. @Ts}$$

$$S_{PS.s} := 0 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{Shell P+S allow. stress @Ts}$$

$$\nu_s := 0 \quad \text{Shell Poisson' ratio}$$

$$\alpha'_s := 0 \cdot \frac{\text{in}}{\text{in} \cdot \text{degF}} \quad \text{Shell coeff. expansion @ T's}$$

Channel Material is SB-575 C276 high

$$S_c := 0 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{Channel allow. stress @ } T_c$$

$$S_{yc} := 0 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{Channel yield stress @ } T_c$$

$$E_c := 27 \cdot 10^6 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{Channel elast. modulus @ } T_c$$

$$S_{PS.c} := 0 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{Channel P+S allow. stress @ } T_c$$

$$\nu_c := 0 \quad \text{Channel Poisson's ratio}$$

$$\alpha'_c := 0 \cdot \frac{\text{in}}{\text{in} \cdot \text{degF}} \quad \text{Channel coeff. expansion @ } T'_c$$

Start of Calculations

$$L := L_t - 2 \cdot h \quad L = 252.50 \text{ in} \quad L = \text{effect. length of shell/tubes}$$

$$h'_g := \max(h_g - c_t, 0) \quad h'_g = 0.20 \text{ in}$$

$$\rho := \frac{l_{tx}}{h} \quad \rho = 0.80 \quad d^* := \max \left[d_t - 2 \cdot t_t \cdot \left(\frac{E_{tT}}{E} \right) \cdot \left(\frac{S_{tT}}{S} \right) \cdot \rho, (d_t - 2t_t) \right] \quad d^* = 0.66 \text{ in}$$

UHX-14.5.1 Step 1 Determine D_o , μ , μ^* and h'_g from UHX-11.5.1 :

$$D_o := 2 \cdot r_o + d_t \quad D_o = 25.75 \text{ in} \quad a_o := \frac{D_o}{2} \quad a_o = 12.88 \text{ in}$$

$$p^* := \frac{p}{\sqrt{1 - \frac{4 \cdot \min(A_L, 4D_o \cdot p)}{\pi \cdot D_o^2}}} \quad p^* = 1.07 \text{ in} \quad \mu^* := \frac{p^* - d^*}{p^*} \quad \mu^* = 0.39$$

$$\mu := \frac{p - d_t}{p} \quad \mu = 0.25$$

Chan Rad. Dim.

$$a_c := \begin{cases} \frac{D_c}{2} & \text{if Config = "a" } \vee \text{ Config = "e" } \vee \text{ Config = "f" } \vee \text{ Config = "A"} \\ \frac{G_c}{2} & \text{if Config = "b" } \vee \text{ Config = "c" } \vee \text{ Config = "d" } \vee \text{ Config = "B" } \vee \text{ Config = "C"} \\ \frac{A}{2} & \text{if Config = "D"} \end{cases} \quad a_c = 14.69 \text{ in}$$

Shell Radial Dim.

$$a_s := \begin{cases} \frac{D_s}{2} & \text{if Config = "a" } \vee \text{ Config = "b" } \vee \text{ Config = "c"} \\ \frac{G_s}{2} & \text{if Config = "d" } \vee \text{ Config = "e" } \vee \text{ Config = "f"} \\ a_c & \text{if Config = "A" } \vee \text{ Config = "B" } \vee \text{ Config = "C" } \vee \text{ Config = "D"} \end{cases} \quad a_s = 14.69 \text{ in}$$

$$\rho_s := \frac{a_s}{a_o} \quad \rho_s = 1.14 \quad \rho_c := \frac{a_c}{a_o} \quad \rho_c = 1.14$$

$$x_t := 1 - N_t \cdot \left(\frac{d_t - 2 \cdot t_t}{2 \cdot a_o} \right)^2 \quad x_t = 0.76 \quad x_s := 1 - N_t \cdot \left(\frac{d_t}{2 \cdot a_o} \right)^2 \quad x_s = 0.60$$

UHX-14.5.2 Step 2 Calculate shell and channel parameters:Use SS=YES for Simply Supported calculation in a 2nd step (see UHX-13.9)

SS = "NO"

$$\beta_S := \begin{cases} \frac{[12 \cdot (1 - \nu_S^2)]^{0.25}}{[(D_S + t_S) \cdot t_S]^{0.5}} & \text{if } SS = \text{"NO"} \wedge (\text{Config} = \text{"a"} \vee \text{Config} = \text{"b"} \vee \text{Config} = \text{"c"}) \\ 0 \cdot \frac{1}{\text{in}} & \text{otherwise} \end{cases} \quad \beta_S = 0.000 \frac{1}{\text{in}}$$

$$\beta_C := \begin{cases} \frac{[12 \cdot (1 - \nu_C^2)]^{0.25}}{[(D_C + t_C) \cdot t_C]^{0.5}} & \text{if } SS = \text{"NO"} \wedge (\text{Config} = \text{"a"} \vee \text{Config} = \text{"e"} \vee \text{Config} = \text{"f"} \vee \text{Config} = \text{"A"}) \\ 0 \cdot \frac{1}{\text{in}} & \text{otherwise} \end{cases} \quad \beta_C = 0.000 \frac{1}{\text{in}}$$

$$k_S := \beta_S \cdot \frac{E_S \cdot t_S^3}{6 \cdot (1 - \nu_S^2)} \quad k_S = 0 \times 10^0 \text{ lb}$$

$$k_C := \beta_C \cdot \frac{E_C \cdot t_C^3}{6 \cdot (1 - \nu_C^2)} \quad k_C = 0 \text{ lb}$$

$$h'_S := h \cdot \beta_S \quad h'_S = 0.00$$

$$h'_C := h \cdot \beta_C \quad h'_C = 0.00$$

$$\lambda_S := \frac{6 \cdot D_S}{h^3} \cdot k_S \cdot \left(1 + h'_S + \frac{h'^2_S}{2}\right) \quad \lambda_S = 0 \times 10^0 \frac{\text{lb}}{\text{in}^2}$$

$$\lambda_C := \frac{6 \cdot D_C}{h^3} \cdot k_C \cdot \left(1 + h'_C + \frac{h'^2_C}{2}\right) \quad \lambda_C = 0 \times 10^0 \frac{\text{lb}}{\text{in}^2}$$

$$\delta_S := \begin{cases} \frac{D_S^2}{4E_S \cdot t_S} \left(1 - \frac{\nu_S}{2}\right) & \text{if } SS = \text{"NO"} \wedge (\text{Config} = \text{"a"} \vee \text{Config} = \text{"b"} \vee \text{Config} = \text{"c"}) \\ 0 \cdot (\text{in}^3 \text{ lb}^{-1}) & \text{otherwise} \end{cases} \quad V := \frac{\lambda_S + \lambda_C}{E} \quad V = 0.00$$

$$\delta_S = 0 \times 10^0 \text{ in}^3 \text{ lb}^{-1}$$

CHAN = "CYL"

SS = "NO"

$$\delta_C := \begin{cases} \frac{D_C^2}{4E_C \cdot t_C} \left(1 - \frac{\nu_C}{2}\right) & \text{if } SS = \text{"NO"} \wedge (\text{Config} = \text{"a"} \vee \text{Config} = \text{"e"} \vee \text{Config} = \text{"f"} \vee \text{Config} = \text{"A"}) \\ \frac{D_C^2}{4E_C \cdot t_C} \left(\frac{1 - \nu_C}{2}\right) & \text{if } SS = \text{"NO"} \wedge \text{CHAN} = \text{"HEMI"} \wedge (\text{Config} = \text{"a"} \vee \text{Config} = \text{"b"} \vee \text{Config} = \text{"c"}) \\ 0 \cdot (\text{in}^3 \text{ lb}^{-1}) & \text{otherwise} \end{cases}$$

$$\delta_C = 0 \times 10^0 \text{ in}^3 \text{ lb}^{-1}$$

UHX-14.5.3 Step 3 Calculate h/p. If ρ changes, recalculate d^* and μ^* from UHX-11.5.1. Determine E^*/E and ν^* relative to h/p from UHX-11.5.2. Calculate X_a

$$\frac{h}{p} = 1.75 \quad \mu^* = 0.39 \quad \frac{E^*}{E} = 0.4039 \quad E^* = 1.09 \times 10^7 \frac{\text{lb}}{\text{in}^2} \quad \nu^* = 0.31 \quad (\text{From pages above, on the right})$$

$$X_a := \left[24 \cdot (1 - \nu^{*2}) \cdot N_t \cdot \frac{E_t \cdot t_t \cdot (d_t - t_t) \cdot a_o^2}{E^* \cdot L \cdot h^3} \right]^{.25} \quad X_a = 3.61$$

UHX-14.5.4 Step 4 Calculate diameter ratio K and coefficient F :

$$K := \frac{A}{D_o} \quad K = 1.28$$

$$F := \frac{1 - \nu^*}{E^*} \cdot (\lambda_s + \lambda_c + E \cdot \ln(K)) \quad F = 0.43$$

$$\Phi := (1 + \nu^*) \cdot F \quad \Phi = 0.56$$

Calculate Z_a, Z_d, Z_v, Z_m

$$N := \text{arrondi} \left(4 + \frac{X_a}{2} \right) + 1 \quad N = 7.00$$

$$\text{ber}_x(x) := \sum_{n=0}^N \left[(-1)^n \cdot \frac{\left(\frac{x}{2}\right)^{4 \cdot n}}{((2 \cdot n)!)^2} \right] \quad \text{ber} := \text{ber}_x(X_a) \quad \text{ber} = -1.46$$

$$\text{bei}_x(x) := \sum_{n=1}^N \left[(-1)^{n-1} \cdot \frac{\left(\frac{x}{2}\right)^{4 \cdot n - 2}}{((2 \cdot n - 1)!)^2} \right] \quad \text{bei} := \text{bei}_x(X_a) \quad \text{bei} = 2.32$$

$$\text{ber}'_x(x) := \sum_{n=1}^N \frac{(-1)^n \cdot (2 \cdot n) \cdot \left(\frac{x}{2}\right)^{4 \cdot n - 1}}{((2 \cdot n)!)^2} \quad \text{ber}' := \text{ber}'_x(X_a) \quad \text{ber}' = -2.52$$

$$\text{bei}'_x(x) := \sum_{n=1}^N \frac{(-1)^{n-1} \cdot (2 \cdot n - 1) \cdot \left(\frac{x}{2}\right)^{4 \cdot n - 3}}{((2 \cdot n - 1)!)^2} \quad \text{bei}' := \text{bei}'_x(X_a) \quad \text{bei}' = 0.28$$

$$\Psi_{1x}(x) := \text{bei}_x(x) + \left(\frac{1 - \nu^*}{x} \right) \cdot \text{ber}'_x(x) \quad \Psi_1 := \Psi_{1x}(X_a) \quad \Psi_1 = 1.84$$

$$\Psi_{2x}(x) := \text{ber}_x(x) - \frac{1 - \nu^*}{x} \cdot \text{bei}'_x(x) \quad \Psi_2 := \Psi_{2x}(X_a) \quad \Psi_2 = -1.51$$

$$Z_a := \text{bei}' \cdot \Psi_2 - \text{ber}' \cdot \Psi_1$$

$$Z_m := \frac{\text{ber}'^2 + \text{bei}'^2}{X_a \cdot Z_a} \quad Z_m = 0.42$$

$$Z_v := \frac{\text{ber}' \cdot \Psi_2 + \text{bei}' \cdot \Psi_1}{X_a^2 \cdot Z_a} \quad Z_v = 0.08$$

$$Z_d := \frac{\text{ber} \cdot \Psi_2 + \text{bei} \cdot \Psi_1}{X_a^3 \cdot Z_a} \quad Z_d = 0.03$$

$$Z_w := \frac{\text{ber}' \cdot \text{ber} + \text{bei}' \cdot \text{bei}}{X_a^2 \cdot Z_a} \quad Z_w = 0.08$$

Calculate Q_1 :

$$Q_1 := \frac{\rho_s - 1 - \Phi \cdot Z_v}{1 + \Phi \cdot Z_m} \quad Q_1 = 0.08$$

UHX-14.5.5 Step 5**a) Calculate parameters ω_s , ω_s^* , ω_c , ω_c^* ,**

$$\omega_s := \rho_s \cdot k_s \cdot \beta_s \cdot \delta_s \cdot (1 + h'_s)$$

$$\omega_s = 0.00 \text{ in}^2$$

$$\omega_s^* := a_o^2 \cdot \frac{(\rho_s^2 - 1) \cdot (\rho_s - 1)}{4} - \omega_s$$

$$\omega_s^* = 1.76 \text{ in}^2$$

$$\omega_c := \rho_c \cdot k_c \cdot \beta_c \cdot \delta_c \cdot (1 + h'_c)$$

$$\omega_c = 0.00 \text{ in}^2$$

$$\omega_c^* := a_o^2 \cdot \left[\frac{(\rho_c^2 + 1) \cdot (\rho_c - 1)}{4} - \frac{\rho_s - 1}{2} \right] - \omega_c$$

$$\omega_c^* = 1.76 \text{ in}^2$$

b) Calculate parameter γ_b :

$$\gamma_b := \begin{cases} 0 & \text{if Config = "a" } \vee \text{ Config = "A" } \vee \text{ Config = "D"} \\ \frac{G_c - C}{D_o} & \text{if Config = "b" } \vee \text{ Config = "B"} \\ \frac{G_c - G_1}{D_o} & \text{if Config = "c" } \vee \text{ Config = "C"} \\ \frac{G_c - G_s}{D_o} & \text{if Config = "d"} \\ \frac{C - G_s}{D_o} & \text{if Config = "e"} \\ \frac{G_1 - G_s}{D_o} & \text{if Config = "f"} \end{cases} \quad \gamma_b = 0.00$$

From UHX-14.6 , Calculate P^*_s and P^*_c : This section is only required for **configurations a,b,or c** when the conditions of §14.6.1 a to d for Radial Differential Thermal Expansion are met.

$$T_r := \begin{cases} \frac{T' + T'_s + T'_c}{3} & \text{if Config = "a"} \\ \frac{T' + T'_s}{2} & \text{if Config = "b" } \vee \text{ Config = "c"} \\ \frac{T' + T'_c}{2} & \text{if Config = "e" } \vee \text{ Config = "f" } \vee \text{ Config = "A"} \\ 0 \cdot \text{degF} & \text{otherwise} \end{cases} \quad T_r = 0.00 \text{ degF}$$

$$T^*_s := \begin{cases} \frac{T'_s + T_r}{2} & \text{if Config = "a" } \vee \text{ Config = "b" } \vee \text{ Config = "c"} \\ 0 \text{ degF} & \text{otherwise} \end{cases} \quad T^*_s = 0.00 \text{ degF}$$

$$T^*_c := \begin{cases} \frac{T'_c + T_r}{2} & \text{if Config = "a" } \vee \text{ Config = "e" } \vee \text{ Config = "f" } \vee \text{ Config = "A"} \\ 0 \text{ degF} & \text{otherwise} \end{cases} \quad T^*_c = 0.00 \text{ degF}$$

$$P^*_{sp1} := \frac{E_s \cdot t_s}{a_s} \cdot [\alpha'_s \cdot (T^*_s - 70 \cdot \text{degF}) - \alpha' \cdot (T_r - 70 \cdot \text{degF})] \quad P^*_{sp1} = 0.00 \frac{\text{lb}}{\text{in}^2}$$

$$P^*_{cp1} := \frac{E_c \cdot t_c}{a_c} \cdot [\alpha'_c \cdot (T^*_c - 70 \cdot \text{degF}) - \alpha' \cdot (T_r - 70 \cdot \text{degF})] \quad P^*_{cp1} = 0.00 \frac{\text{lb}}{\text{in}^2}$$

$$P^*_{sp} := \begin{cases} 0 \frac{\text{lb}}{\text{in}^2} & \text{if Config = "d" } \vee \text{ Config = "A" } \vee \text{ Config = "B" } \vee \text{ Config = "C" } \vee \text{ Config = "D"} \\ P^*_{sp1} & \text{otherwise} \end{cases}$$

$$P^*_{cp} := \begin{cases} 0 \frac{\text{lb}}{\text{in}^2} & \text{if Config = "d" } \vee \text{ Config = "B" } \vee \text{ Config = "C" } \vee \text{ Config = "D"} \\ P^*_{cp1} & \text{otherwise} \end{cases}$$

$$P^*_s := \begin{pmatrix} 0 \cdot \frac{\text{lb}}{\text{in}^2} \\ 0 \cdot \frac{\text{lb}}{\text{in}^2} \\ 0 \cdot \frac{\text{lb}}{\text{in}^2} \\ P^*_{sp} \\ P^*_{sp} \\ P^*_{sp} \\ P^*_{sp} \end{pmatrix} \quad P^*_s = \begin{pmatrix} 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$P^*_c := \begin{pmatrix} 0 \cdot \frac{\text{lb}}{\text{in}^2} \\ 0 \cdot \frac{\text{lb}}{\text{in}^2} \\ 0 \cdot \frac{\text{lb}}{\text{in}^2} \\ P^*_{cp} \\ P^*_{cp} \\ P^*_{cp} \\ P^*_{cp} \end{pmatrix} \quad P^*_c = \begin{pmatrix} 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

UHX-14.5.6 Step 6 For each loading case, calculate the effective pressure P_e

$$P_e := \begin{cases} (P_s - P_t) & \text{if ExchType} = "a" \\ \left[P_s \cdot (1 - \rho_s^2) - P_t \right] & \text{if ExchType} = "b" \\ \left[(P_s - P_t) \cdot (1 - \rho_s^2) \right] & \text{otherwise} \end{cases}$$

$$P_e = \begin{pmatrix} -150.00 \\ 250.00 \\ 100.00 \\ 0.00 \\ -150.00 \\ 250.00 \\ 100.00 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

UHX-14.5.7 Step 7 Determine Q_2 and Q_3 for Loading Cases 1 through 7 :

$$Q_2 := \frac{\left[(\omega_s^* \cdot P_s - \omega_c^* \cdot P_t) - (\omega_s \cdot P_s^* - \omega_c \cdot P_c^*) \right] + \frac{\gamma b}{2 \cdot \pi} \cdot W}{1 + \Phi \cdot Z_m}$$

$$Q_2 = \begin{pmatrix} -213.34 \\ 355.57 \\ 142.23 \\ 0.00 \\ -213.34 \\ 355.57 \\ 142.23 \end{pmatrix} \text{lb}$$

$$Q_{3_1} := \begin{cases} 0 & \text{if } P_{e_1} = 0 \\ \left(Q_1 + \frac{2 \cdot Q_{2_1}}{P_{e_1} \cdot a_0^2} \right) & \text{otherwise} \end{cases}$$

$$Q_{3_2} := \begin{cases} 0 & \text{if } P_{e_2} = 0 \\ \left(Q_1 + \frac{2 \cdot Q_{2_2}}{P_{e_2} \cdot a_0^2} \right) & \text{otherwise} \end{cases}$$

$$Q_{3_3} := \begin{cases} 0 & \text{if } P_{e_3} = 0 \\ \left(Q_1 + \frac{2 \cdot Q_{2_3}}{P_{e_3} \cdot a_0^2} \right) & \text{otherwise} \end{cases}$$

$$Q_{3_4} := \begin{cases} 0 & \text{if } P_{e_4} = 0 \\ \left(Q_1 + \frac{2 \cdot Q_{2_4}}{P_{e_4} \cdot a_0^2} \right) & \text{otherwise} \end{cases}$$

$$Q_{3_5} := \begin{cases} 0 & \text{if } P_{e_5} = 0 \\ \left(Q_1 + \frac{2 \cdot Q_{2_5}}{P_{e_5} \cdot a_0^2} \right) & \text{otherwise} \end{cases}$$

$$Q_{3_6} := \begin{cases} 0 & \text{if } P_{e_6} = 0 \\ \left(Q_1 + \frac{2 \cdot Q_{2_6}}{P_{e_6} \cdot a_0^2} \right) & \text{otherwise} \end{cases}$$

$$Q_{3_7} := \begin{cases} 0 & \text{if } P_{e_7} = 0 \\ \left(Q_1 + \frac{2 \cdot Q_{2_7}}{P_{e_7} \cdot a_0^2} \right) & \text{otherwise} \end{cases}$$

$$Q_1 = 0.08$$

$$Q_2 = \begin{pmatrix} -213.34 \\ 355.57 \\ 142.23 \\ 0.00 \\ -213.34 \\ 355.57 \\ 142.23 \end{pmatrix} \text{lb}$$

$$Q_3 = \begin{pmatrix} 0.10 \\ 0.10 \\ 0.10 \\ 0.00 \\ 0.10 \\ 0.10 \\ 0.10 \end{pmatrix}$$

Determine coefficient F_m for Load Cases 1 through 7:

numpoints := 20

$$j := 1 \dots \text{numpoints} \quad X_j := \frac{j \cdot 1}{\text{numpoints}} \quad x := X \cdot X_a$$

$$Q_V(x) := \frac{\overrightarrow{\Psi_{2x}(x) \cdot \Psi_1 - \Psi_{1x}(x) \cdot \Psi_2}}{X_a \cdot Z_a}$$

$$Q_m(x) := \left(\frac{\overrightarrow{\Psi_{2x}(x) \cdot \text{bei}' - \Psi_{1x}(x) \cdot \text{ber}'}}{Z_a} \right)$$

$$F_{mx1}(x) := \begin{cases} 0 & \text{if } P_{e_1} = 0 \\ \frac{Q_V(x) + Q_{3_1} \cdot Q_m(x)}{2} & \text{otherwise} \end{cases}$$

$$F_{mx4}(x) := \begin{cases} 0 & \text{if } P_{e_4} = 0 \\ \frac{Q_V(x) + Q_{3_4} \cdot Q_m(x)}{2} & \text{otherwise} \end{cases}$$

$$F_{mx2}(x) := \begin{cases} 0 & \text{if } P_{e_2} = 0 \\ \frac{Q_V(x) + Q_{3_2} \cdot Q_m(x)}{2} & \text{otherwise} \end{cases}$$

$$F_{mx5}(x) := \begin{cases} 0 & \text{if } P_{e_5} = 0 \\ \frac{Q_V(x) + Q_{3_5} \cdot Q_m(x)}{2} & \text{otherwise} \end{cases}$$

$$F_{mx3}(x) := \begin{cases} 0 & \text{if } P_{e_3} = 0 \\ \frac{Q_V(x) + Q_{3_3} \cdot Q_m(x)}{2} & \text{otherwise} \end{cases}$$

$$F_{mx6}(x) := \begin{cases} 0 & \text{if } P_{e_6} = 0 \\ \frac{Q_V(x) + Q_{3_6} \cdot Q_m(x)}{2} & \text{otherwise} \end{cases}$$

$$F_{mx7}(x) := \begin{cases} 0 & \text{if } P_{e_7} = 0 \\ \frac{Q_V(x) + Q_{3_7} \cdot Q_m(x)}{2} & \text{otherwise} \end{cases}$$

$$F_{mx}(x) := \begin{pmatrix} F_{mx1}(x) \\ F_{mx2}(x) \\ F_{mx3}(x) \\ F_{mx4}(x) \\ F_{mx5}(x) \\ F_{mx6}(x) \\ F_{mx7}(x) \end{pmatrix} \quad \text{abs}F_{mx}(x) := \begin{pmatrix} |F_{mx1}(x)| \\ |F_{mx2}(x)| \\ |F_{mx3}(x)| \\ |F_{mx4}(x)| \\ |F_{mx5}(x)| \\ |F_{mx6}(x)| \\ |F_{mx7}(x)| \end{pmatrix}$$

$$F_m := \begin{pmatrix} \max(\text{abs}F_{mx}(x)_1) \\ \max(\text{abs}F_{mx}(x)_2) \\ \max(\text{abs}F_{mx}(x)_3) \\ \max(\text{abs}F_{mx}(x)_4) \\ \max(\text{abs}F_{mx}(x)_5) \\ \max(\text{abs}F_{mx}(x)_6) \\ \max(\text{abs}F_{mx}(x)_7) \end{pmatrix} \quad F_m = \begin{pmatrix} 0.10 \\ 0.10 \\ 0.10 \\ 0.00 \\ 0.10 \\ 0.10 \\ 0.10 \end{pmatrix}$$

$$F_{mx}(X_a) = \begin{pmatrix} 0.05 \\ 0.05 \\ 0.05 \\ 0.00 \\ 0.05 \\ 0.05 \\ 0.05 \end{pmatrix}$$

Calculate the Maximum Tubesheet Bending Stress

$$h_{\min} := \begin{pmatrix} h - h'_g \\ h - h'_g \\ h - h'_g \\ h \\ h \\ h \\ h \end{pmatrix} \quad h_{\min} = \begin{pmatrix} 1.55 \\ 1.55 \\ 1.55 \\ 1.75 \\ 1.75 \\ 1.75 \\ 1.75 \end{pmatrix} \text{ in}$$

$$\sigma_1 := \begin{cases} \left[\frac{6 \cdot Q_{2_1}}{\mu^* \cdot (h_{\min_1})^2} \right] & \text{if } P_{e_1} = 0 \\ \left[\frac{1.5 \cdot F_{m_1} \cdot \left(\frac{2 \cdot a_0}{h_{\min_1}} \right)^2 \cdot P_{e_1}}{\mu^*} \right] & \text{otherwise} \end{cases} \quad \sigma_1 = -16389.85 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_2 := \begin{cases} \left[\frac{6 \cdot Q_{2_2}}{\mu^* \cdot (h_{\min_2})^2} \right] & \text{if } P_{e_2} = 0 \\ \left[\frac{1.5 \cdot F_{m_2} \cdot \left(\frac{2 \cdot a_0}{h_{\min_2}} \right)^2 \cdot P_{e_2}}{\mu^*} \right] & \text{otherwise} \end{cases} \quad \sigma_2 = 27316.41 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_3 := \begin{cases} \left[\frac{6 \cdot Q_{2_3}}{\mu^* \cdot (h_{\min_3})^2} \right] & \text{if } P_{e_3} = 0 \\ \left[\frac{1.5 \cdot F_{m_3} \cdot \left(\frac{2 \cdot a_0}{h_{\min_3}} \right)^2 \cdot P_{e_3}}{\mu^*} \right] & \text{otherwise} \end{cases} \quad \sigma_3 = 10926.57 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_4 := \begin{cases} \left[\frac{6 \cdot Q_{2_4}}{\mu^* \cdot (h_{\min_4})^2} \right] & \text{if } P_{e_4} = 0 \\ \left[\frac{1.5 \cdot F_{m_4} \cdot \left(\frac{2 \cdot a_0}{h_{\min_4}} \right)^2 \cdot P_{e_4}}{\mu^*} \right] & \text{otherwise} \end{cases} \quad \sigma_4 = 0.00 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_5 := \begin{cases} \left[\frac{6 \cdot Q_{2_5}}{\mu^* \cdot (h_{\min_5})^2} \right] & \text{if } P_{e_5} = 0 \\ \left[\frac{1.5 \cdot F_{m_5} \cdot \left(\frac{2 \cdot a_0}{h_{\min_5}} \right)^2 \cdot P_{e_5}}{\mu^*} \right] & \text{otherwise} \end{cases} \quad \sigma_5 = -12907.49 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_6 := \begin{cases} \left[\frac{6 \cdot Q_{2_6}}{\mu^* \cdot (h_{\min_6})^2} \right] & \text{if } P_{e_6} = 0 \\ \left[\frac{1.5 \cdot F_{m_6} \cdot \left(\frac{2 \cdot a_0}{h_{\min_6}} \right)^2 \cdot P_{e_6}}{\mu^*} \right] & \text{otherwise} \end{cases} \quad \sigma_6 = 21512.48 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_7 := \begin{cases} \left[\frac{6 \cdot Q_{2_7}}{\mu^* \cdot (h_{\min_7})^2} \right] & \text{if } P_{e_7} = 0 \\ \left[\frac{1.5 \cdot F_{m_7} \cdot \left(\frac{2 \cdot a_0}{h_{\min_7}} \right)^2 \cdot P_{e_7}}{\mu^*} \right] & \text{otherwise} \end{cases} \quad \sigma_7 = 8604.99 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma = \begin{pmatrix} -16389.848 \\ 27316.414 \\ 10926.565 \\ 0 \\ -12907.488 \\ 21512.481 \\ 8604.992 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

- For Pressure only Loading Cases 1, 2, and 3:

$$\sigma_{\max P} := \max(|\sigma_1|, |\sigma_2|, |\sigma_3|) \quad \sigma_{\max P} = 27316.414 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{\text{allow}P} := 1.5 \cdot S \quad \sigma_{\text{allow}P} = 28500.000 \frac{\text{lb}}{\text{in}^2}$$

Calculate the minimum required thickness of the TS flanged extension for configurations b and e:

-For Pressure-Thermal Loading Cases 4, 5, 6 and 7:

$$\sigma_{\max T} := \max(|\sigma_4|, |\sigma_5|, |\sigma_6|, |\sigma_7|) \quad \sigma_{\max T} = 21512.481 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{\text{allow}T} := SPS.s \quad \sigma_{\text{allow}T} = 0.00 \frac{\text{lb}}{\text{in}^2}$$

$$h_r := \begin{cases} \sqrt{\frac{1.9W}{S \cdot G_c} \cdot \frac{C - G_c}{2}} & \text{if Config = "b"} \\ \sqrt{\frac{1.9W}{S \cdot G_s} \cdot \frac{C - G_s}{2}} & \text{if Config = "e"} \\ 0 \cdot \text{in} & \text{otherwise} \end{cases} \quad h_r = 0.000 \text{ in}$$

UHX-13.5.8 Step 8 Determine the maximum tubesheet shear stress :

$$\tau := \left[\left(\frac{1}{2 \cdot \mu} \right) \cdot \left(\frac{D_o}{2 \cdot h} \right) \cdot P_e \right] \rightarrow \tau = \begin{pmatrix} -2207.143 \\ 3678.571 \\ 1471.429 \\ 0.000 \\ -2207.143 \\ 3678.571 \\ 1471.429 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$\tau_{\max} := \max(|\tau|) \quad \tau_{\max} = 3678.571 \frac{\text{lb}}{\text{in}^2}$$

$$\tau_{\text{allow}} := 0.8 \cdot S \quad \tau_{\text{allow}} = 15200.000 \frac{\text{lb}}{\text{in}^2}$$

If $|P_s - P_t| > 3.2 S \mu h / D_o$, the TEMA formula may be used, provided the data A_p and C_p have been defined.

$$A_p := 491 \cdot \text{in}^2$$

Total area enclosed by C_p

$$C_p := 78.5 \cdot \text{in}$$

Perimeter of the tube layout measured stepwise in increment of one tube pitch from the center-to-center of the outermost tubes (see Fig. UHX-12.2)

$$P_s - P_t = \begin{pmatrix} -150.000 \\ 250.000 \\ 100.000 \\ 0.000 \\ -150.000 \\ 250.000 \\ 100.000 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$\tau := \left(\frac{1}{\mu} \right) \cdot \frac{1}{h} \left(\frac{A_p}{C_p} \right) \cdot |P_s - P_t|$$

$$\tau = \begin{pmatrix} 2144.495 \\ 3574.158 \\ 1429.663 \\ 0.000 \\ 2144.495 \\ 3574.158 \\ 1429.663 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$3.2 \cdot S \cdot \mu \cdot \frac{h}{D_o} = 1033.010 \frac{\text{lb}}{\text{in}^2}$$

UHX-13.5.9 Step 9 Determine the minimum and maximum stresses in the tubes for Load Cases 1 to 7

$$Z_{dx}(x) := \frac{\Psi_2 \cdot \text{ber}_x(x) + \Psi_1 \cdot \text{bei}_x(x)}{X_a^3 \cdot Z_a}$$

$$Z_{wx}(x) := \frac{\text{ber}' \cdot \text{ber}_x(x) + \text{bei}' \cdot \text{bei}_x(x)}{X_a^2 \cdot Z_a}$$

Step a) :Determine Coefficients $F_{t,min}$ and $F_{t,max}$ for Load Cases 1 to 7:

$$\begin{aligned}
 F_{tx1}(x) &:= \begin{cases} Z_{wx}(x) \cdot \frac{X_a^4}{2} & \text{if } P_{e_1} = 0 \\ (Z_{dx}(x) + Q_{3_1} \cdot Z_{wx}(x)) \cdot \frac{X_a^4}{2} & \text{otherwise} \end{cases} & F_{tx4}(x) &:= \begin{cases} Z_{wx}(x) \cdot \frac{X_a^4}{2} & \text{if } P_{e_4} = 0 \\ (Z_{dx}(x) + Q_{3_4} \cdot Z_{wx}(x)) \cdot \frac{X_a^4}{2} & \text{otherwise} \end{cases} \\
 F_{tx2}(x) &:= \begin{cases} Z_{wx}(x) \cdot \frac{X_a^4}{2} & \text{if } P_{e_2} = 0 \\ (Z_{dx}(x) + Q_{3_2} \cdot Z_{wx}(x)) \cdot \frac{X_a^4}{2} & \text{otherwise} \end{cases} & F_{tx5}(x) &:= \begin{cases} Z_{wx}(x) \cdot \frac{X_a^4}{2} & \text{if } P_{e_5} = 0 \\ (Z_{dx}(x) + Q_{3_5} \cdot Z_{wx}(x)) \cdot \frac{X_a^4}{2} & \text{otherwise} \end{cases} \\
 F_{tx3}(x) &:= \begin{cases} Z_{wx}(x) \cdot \frac{X_a^4}{2} & \text{if } P_{e_3} = 0 \\ (Z_{dx}(x) + Q_{3_3} \cdot Z_{wx}(x)) \cdot \frac{X_a^4}{2} & \text{otherwise} \end{cases} & F_{tx6}(x) &:= \begin{cases} Z_{wx}(x) \cdot \frac{X_a^4}{2} & \text{if } P_{e_6} = 0 \\ (Z_{dx}(x) + Q_{3_6} \cdot Z_{wx}(x)) \cdot \frac{X_a^4}{2} & \text{otherwise} \end{cases} \\
 & & F_{tx7}(x) &:= \begin{cases} Z_{wx}(x) \cdot \frac{X_a^4}{2} & \text{if } P_{e_7} = 0 \\ (Z_{dx}(x) + Q_{3_7} \cdot Z_{wx}(x)) \cdot \frac{X_a^4}{2} & \text{otherwise} \end{cases}
 \end{aligned}$$

$$F_{tx}(x) := \begin{pmatrix} F_{tx1}(x) \\ F_{tx2}(x) \\ F_{tx3}(x) \\ F_{tx4}(x) \\ F_{tx5}(x) \\ F_{tx6}(x) \\ F_{tx7}(x) \end{pmatrix} \quad F_{tmin} := \begin{pmatrix} \min(F_{tx}(x)_1) \\ \min(F_{tx}(x)_2) \\ \min(F_{tx}(x)_3) \\ \min(F_{tx}(x)_4) \\ \min(F_{tx}(x)_5) \\ \min(F_{tx}(x)_6) \\ \min(F_{tx}(x)_7) \end{pmatrix} \quad F_{tmax} := \begin{pmatrix} \max(F_{tx}(x)_1) \\ \max(F_{tx}(x)_2) \\ \max(F_{tx}(x)_3) \\ \max(F_{tx}(x)_4) \\ \max(F_{tx}(x)_5) \\ \max(F_{tx}(x)_6) \\ \max(F_{tx}(x)_7) \end{pmatrix}$$

$$F_{tx}(X_a) = \begin{pmatrix} 3.42 \\ 3.42 \\ 3.42 \\ 6.69 \\ 3.42 \\ 3.42 \\ 3.42 \end{pmatrix} \quad F_{tmin} = \begin{pmatrix} -1.01 \\ -1.01 \\ -1.01 \\ -3.89 \\ -1.01 \\ -1.01 \\ -1.01 \end{pmatrix} \quad F_{tmax} = \begin{pmatrix} 3.42 \\ 3.42 \\ 3.42 \\ 6.69 \\ 3.42 \\ 3.42 \\ 3.42 \end{pmatrix}$$

Step a') : Determine tube stresses σ_{tmin} and σ_{tmax} for Load Cases 1 to 7 (continued)

$$\sigma_{t1_1} := \begin{cases} \left[\frac{1}{x_t - x_s} \cdot \left[(P_{s_1} \cdot x_s - P_{t_1} \cdot x_t) - \frac{2 \cdot Q_{2_1}}{a_o^2} \cdot F_{tmin_1} \right] \right] & \text{if } P_{e_1} = 0 \\ \left[\frac{1}{x_t - x_s} \cdot \left[(P_{s_1} \cdot x_s - P_{t_1} \cdot x_t) - P_{e_1} \cdot F_{tmin_1} \right] \right] & \text{otherwise} \end{cases} \quad \sigma_{t1_1} = -1710.05 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{t1_2} := \begin{cases} \left[\frac{1}{x_t - x_s} \cdot \left[(P_{s_2} \cdot x_s - P_{t_2} \cdot x_t) - \frac{2 \cdot Q_{2_2}}{a_o^2} \cdot F_{tmin_2} \right] \right] & \text{if } P_{e_2} = 0 \\ \left[\frac{1}{x_t - x_s} \cdot \left[(P_{s_2} \cdot x_s - P_{t_2} \cdot x_t) - P_{e_2} \cdot F_{tmin_2} \right] \right] & \text{otherwise} \end{cases} \quad \sigma_{t1_2} = 2600.08 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{t1_3} := \begin{cases} \left[\frac{1}{x_t - x_s} \cdot \left[(P_{s_3} \cdot x_s - P_{t_3} \cdot x_t) - \frac{2 \cdot Q_{2_3}}{a_o^2} \cdot F_{tmin_3} \right] \right] & \text{if } P_{e_3} = 0 \\ \left[\frac{1}{x_t - x_s} \cdot \left[(P_{s_3} \cdot x_s - P_{t_3} \cdot x_t) - P_{e_3} \cdot F_{tmin_3} \right] \right] & \text{otherwise} \end{cases} \quad \sigma_{t1_3} = 890.03 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{t1_4} := \begin{cases} \left[\frac{1}{x_t - x_s} \cdot \left[(P_{s_4} \cdot x_s - P_{t_4} \cdot x_t) - \frac{2 \cdot Q_{2_4}}{a_o^2} \cdot F_{tmin_4} \right] \right] & \text{if } P_{e_4} = 0 \\ \left[\frac{1}{x_t - x_s} \cdot \left[(P_{s_4} \cdot x_s - P_{t_4} \cdot x_t) - P_{e_4} \cdot F_{tmin_4} \right] \right] & \text{otherwise} \end{cases} \quad \sigma_{t1_4} = 0.00 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{t1_5} := \begin{cases} \left[\frac{1}{x_t - x_s} \cdot \left[(P_{s_5} \cdot x_s - P_{t_5} \cdot x_t) - \frac{2 \cdot Q_{2_5}}{a_o^2} \cdot F_{tmin_5} \right] \right] & \text{if } P_{e_5} = 0 \\ \left[\frac{1}{x_t - x_s} \cdot \left[(P_{s_5} \cdot x_s - P_{t_5} \cdot x_t) - P_{e_5} \cdot F_{tmin_5} \right] \right] & \text{otherwise} \end{cases} \quad \sigma_{t1_5} = -1710.05 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{t1_6} := \begin{cases} \left[\frac{1}{x_t - x_s} \cdot \left[(P_{s_6} \cdot x_s - P_{t_6} \cdot x_t) - \frac{2 \cdot Q_{2_6}}{a_o^2} \cdot F_{tmin_6} \right] \right] & \text{if } P_{e_6} = 0 \\ \left[\frac{1}{x_t - x_s} \cdot \left[(P_{s_6} \cdot x_s - P_{t_6} \cdot x_t) - P_{e_6} \cdot F_{tmin_6} \right] \right] & \text{otherwise} \end{cases} \quad \sigma_{t1_6} = 2600.08 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{t1_7} := \begin{cases} \left[\frac{1}{x_t - x_s} \cdot \left[(P_{s_7} \cdot x_s - P_{t_7} \cdot x_t) - \frac{2 \cdot Q_{2_7}}{a_o^2} \cdot F_{tmin_7} \right] \right] & \text{if } P_{e_7} = 0 \\ \left[\frac{1}{x_t - x_s} \cdot \left[(P_{s_7} \cdot x_s - P_{t_7} \cdot x_t) - P_{e_7} \cdot F_{tmin_7} \right] \right] & \text{otherwise} \end{cases} \quad \sigma_{t1_7} = 890.03 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{t2_1} := \begin{cases} \left[\frac{1}{x_t - x_s} \cdot \left((P_{s_1} \cdot x_s - P_{t_1} \cdot x_t) - \frac{2 \cdot Q_{2_1}}{a_o^2} \cdot F_{tmax_1} \right) \right] & \text{if } P_{e_1} = 0 \\ \left[\frac{1}{x_t - x_s} \cdot \left((P_{s_1} \cdot x_s - P_{t_1} \cdot x_t) - P_{e_1} \cdot F_{tmax_1} \right) \right] & \text{otherwise} \end{cases}$$

$$\sigma_{t2_1} = 2564.13 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{t2_2} := \begin{cases} \left[\frac{1}{x_t - x_s} \cdot \left((P_{s_2} \cdot x_s - P_{t_2} \cdot x_t) - \frac{2 \cdot Q_{2_2}}{a_o^2} \cdot F_{tmax_2} \right) \right] & \text{if } P_{e_2} = 0 \\ \left[\frac{1}{x_t - x_s} \cdot \left((P_{s_2} \cdot x_s - P_{t_2} \cdot x_t) - P_{e_2} \cdot F_{tmax_2} \right) \right] & \text{otherwise} \end{cases}$$

$$\sigma_{t2_2} = -4523.54 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{t2_3} := \begin{cases} \left[\frac{1}{x_t - x_s} \cdot \left((P_{s_3} \cdot x_s - P_{t_3} \cdot x_t) - \frac{2 \cdot Q_{2_3}}{a_o^2} \cdot F_{tmax_3} \right) \right] & \text{if } P_{e_3} = 0 \\ \left[\frac{1}{x_t - x_s} \cdot \left((P_{s_3} \cdot x_s - P_{t_3} \cdot x_t) - P_{e_3} \cdot F_{tmax_3} \right) \right] & \text{otherwise} \end{cases}$$

$$\sigma_{t2_3} = -1959.42 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{t2_4} := \begin{cases} \left[\frac{1}{x_t - x_s} \cdot \left((P_{s_4} \cdot x_s - P_{t_4} \cdot x_t) - \frac{2 \cdot Q_{2_4}}{a_o^2} \cdot F_{tmax_4} \right) \right] & \text{if } P_{e_4} = 0 \\ \left[\frac{1}{x_t - x_s} \cdot \left((P_{s_4} \cdot x_s - P_{t_4} \cdot x_t) - P_{e_4} \cdot F_{tmax_4} \right) \right] & \text{otherwise} \end{cases}$$

$$\sigma_{t2_4} = 0.00 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{t2_5} := \begin{cases} \left[\frac{1}{x_t - x_s} \cdot \left((P_{s_5} \cdot x_s - P_{t_5} \cdot x_t) - \frac{2 \cdot Q_{2_5}}{a_o^2} \cdot F_{tmax_5} \right) \right] & \text{if } P_{e_5} = 0 \\ \left[\frac{1}{x_t - x_s} \cdot \left((P_{s_5} \cdot x_s - P_{t_5} \cdot x_t) - P_{e_5} \cdot F_{tmax_5} \right) \right] & \text{otherwise} \end{cases}$$

$$\sigma_{t2_5} = 2564.13 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{t2_6} := \begin{cases} \left[\frac{1}{x_t - x_s} \cdot \left((P_{s_6} \cdot x_s - P_{t_6} \cdot x_t) - \frac{2 \cdot Q_{2_6}}{a_o^2} \cdot F_{tmax_6} \right) \right] & \text{if } P_{e_6} = 0 \\ \left[\frac{1}{x_t - x_s} \cdot \left((P_{s_6} \cdot x_s - P_{t_6} \cdot x_t) - P_{e_6} \cdot F_{tmax_6} \right) \right] & \text{otherwise} \end{cases}$$

$$\sigma_{t2_6} = -4523.54 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{t2_7} := \begin{cases} \left[\frac{1}{x_t - x_s} \cdot \left((P_{s_7} \cdot x_s - P_{t_7} \cdot x_t) - \frac{2 \cdot Q_{2_7}}{a_o^2} \cdot F_{tmax_7} \right) \right] & \text{if } P_{e_7} = 0 \\ \left[\frac{1}{x_t - x_s} \cdot \left((P_{s_7} \cdot x_s - P_{t_7} \cdot x_t) - P_{e_7} \cdot F_{tmax_7} \right) \right] & \text{otherwise} \end{cases}$$

$$\sigma_{t2_7} = -1959.42 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{to} := \left[\frac{(P_s \cdot x_s - P_t \cdot x_t) - P_e \cdot \frac{(Z_d + Q_3 \cdot Z_v) \cdot x_a^4}{2}}{x_t - x_s} \right]$$

$$\sigma_{t1} = \begin{pmatrix} -1710.05 \\ 2600.08 \\ 890.03 \\ 0.00 \\ -1710.05 \\ 2600.08 \\ 890.03 \end{pmatrix} \frac{\text{lb}}{\text{in}^2} \quad \sigma_{to} = \begin{pmatrix} 2564.13 \\ -4523.54 \\ -1959.42 \\ 0.00 \\ 2564.13 \\ -4523.54 \\ -1959.42 \end{pmatrix} \frac{\text{lb}}{\text{in}^2} \quad \sigma_{t2} = \begin{pmatrix} 2564.13 \\ -4523.54 \\ -1959.42 \\ 0.00 \\ 2564.13 \\ -4523.54 \\ -1959.42 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

- For Pressure Loading Only, Cases 1, 2, and 3 :

$$\begin{aligned} \sigma_{t1P} &:= \max(|\sigma_{t1_1}|, |\sigma_{t1_2}|, |\sigma_{t1_3}|) & \sigma_{t1P} &= 2600.078 \frac{\text{lb}}{\text{in}^2} \\ \sigma_{t2P} &:= \max(|\sigma_{t2_1}|, |\sigma_{t2_2}|, |\sigma_{t2_3}|) & \sigma_{t2P} &= 4523.544 \frac{\text{lb}}{\text{in}^2} \\ \sigma_{tallowP} &:= 1.0 \cdot S_t & \sigma_{tallowP} &= 13350.000 \frac{\text{lb}}{\text{in}^2} \end{aligned}$$

- For Pressure plus Thermal, Loading Cases 4, 5, 6, and 7 :

$$\begin{aligned} \sigma_{t1T} &:= \max(|\sigma_{t1_4}|, |\sigma_{t1_5}|, |\sigma_{t1_6}|, |\sigma_{t1_7}|) & \sigma_{t1T} &= 2600.078 \frac{\text{lb}}{\text{in}^2} \\ \sigma_{t2T} &:= \max(|\sigma_{t2_4}|, |\sigma_{t2_5}|, |\sigma_{t2_6}|, |\sigma_{t2_7}|) & \sigma_{t2T} &= 4523.544 \frac{\text{lb}}{\text{in}^2} \\ \sigma_{tallowT} &:= 2.0 \cdot S_t & \sigma_{tallowT} &= 26700.00 \frac{\text{lb}}{\text{in}^2} \end{aligned}$$

Step (b) : check the tubes for buckling if $\sigma_{t1} < 0$ or $\sigma_{t2} < 0$ **Page 18**

$$l_t := kl \quad r_t := \frac{\sqrt{d_t^2 + (d_t - 2 \cdot t_t)^2}}{4} \quad r_t = 0.24 \text{ in} \quad C_t := \sqrt{\frac{2 \cdot \pi^2 \cdot E_t}{S_{yt}}} \quad C_t = 161.04 \quad F_t := \frac{l_t}{r_t} \quad F_t = 64.70$$

$$F_{S1} := \begin{cases} 1.25 & \text{if } P_{e1} = 0 \\ \max \left[3.25 - .25 \cdot \left[(Z_d + Q_{3_1} \cdot Z_v) \cdot X_a^4 \right], 1.25 \right] & \text{otherwise} \end{cases} \quad F_{S1} := \text{si}(F_{S1} > 2, 2, F_{S1}) \quad F_{S1} = 1.54$$

$$F_{S2} := \begin{cases} 1.25 & \text{if } P_{e2} = 0 \\ \max \left[3.25 - .25 \cdot \left[(Z_d + Q_{3_2} \cdot Z_v) \cdot X_a^4 \right], 1.25 \right] & \text{otherwise} \end{cases} \quad F_{S2} := \text{si}(F_{S2} > 2, 2, F_{S2}) \quad F_{S2} = 1.54$$

$$F_{S3} := \begin{cases} 1.25 & \text{if } P_{e3} = 0 \\ \max \left[3.25 - .25 \cdot \left[(Z_d + Q_{3_3} \cdot Z_v) \cdot X_a^4 \right], 1.25 \right] & \text{otherwise} \end{cases} \quad F_{S3} := \text{si}(F_{S3} > 2, 2, F_{S3}) \quad F_{S3} = 1.54$$

$$F_{S4} := \begin{cases} 1.25 & \text{if } P_{e4} = 0 \\ \max \left[3.25 - .25 \cdot \left[(Z_d + Q_{3_4} \cdot Z_v) \cdot X_a^4 \right], 1.25 \right] & \text{otherwise} \end{cases} \quad F_{S4} := \text{si}(F_{S4} > 2, 2, F_{S4}) \quad F_{S4} = 1.25$$

$$F_{S5} := \begin{cases} 1.25 & \text{if } P_{e5} = 0 \\ \max \left[3.25 - .25 \cdot \left[(Z_d + Q_{3_5} \cdot Z_v) \cdot X_a^4 \right], 1.25 \right] & \text{otherwise} \end{cases} \quad F_{S5} := \text{si}(F_{S5} > 2, 2, F_{S5}) \quad F_{S5} = 1.54$$

$$F_{S6} := \begin{cases} 1.25 & \text{if } P_{e6} = 0 \\ \max \left[3.25 - .25 \cdot \left[(Z_d + Q_{3_6} \cdot Z_v) \cdot X_a^4 \right], 1.25 \right] & \text{otherwise} \end{cases} \quad F_{S6} := \text{si}(F_{S6} > 2, 2, F_{S6}) \quad F_{S6} = 1.54$$

$$F_{S7} := \begin{cases} 1.25 & \text{if } P_{e7} = 0 \\ \max \left[3.25 - .25 \cdot \left[(Z_d + Q_{3_7} \cdot Z_v) \cdot X_a^4 \right], 1.25 \right] & \text{otherwise} \end{cases} \quad F_{S7} := \text{si}(F_{S7} > 2, 2, F_{S7}) \quad F_{S7} = 1.54$$

$$F_s := \begin{pmatrix} F_{S1} \\ F_{S2} \\ F_{S3} \\ F_{S4} \\ F_{S5} \\ F_{S6} \\ F_{S7} \end{pmatrix} \quad F_s = \begin{pmatrix} 1.54 \\ 1.54 \\ 1.54 \\ 1.25 \\ 1.54 \\ 1.54 \\ 1.54 \end{pmatrix}$$

Determine the Maximum Permissible Buckling Stress for Load Cases 1 to 7 (only applicable when the calculated tube compressive stress is negative):

$$S_{tb} := \text{si} \left[C_t > F_t, \frac{S_{yt}}{F_s} \cdot \left(1 - \frac{F_t}{2 \cdot C_t} \right), \frac{1}{F_s} \cdot \pi^2 \cdot \frac{E_t}{F_t^2} \right]$$

Allowable Buckling Stress, S_{tb} Tube Calculated Stress σ_{t1}, σ_{t2}

$$S_{tb} = \begin{pmatrix} 10666.016 \\ 10666.016 \\ 10666.016 \\ 13137.609 \\ 10666.016 \\ 10666.016 \\ 10666.016 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{t1} = \begin{pmatrix} -1710.05 \\ 2600.08 \\ 890.03 \\ 0.00 \\ -1710.05 \\ 2600.08 \\ 890.03 \end{pmatrix} \frac{\text{lb}}{\text{in}^2} \quad \sigma_{t2} = \begin{pmatrix} 2564.13 \\ -4523.54 \\ -1959.42 \\ 0.00 \\ 2564.13 \\ -4523.54 \\ -1959.42 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

UHX 14.5.10 Step 10 Determine the stresses in Shell and Channel when integral with the tubesheet (a,b,c) :**a) Shell****Shell membrane stress :**

$$\sigma_{sm} := \begin{cases} \frac{a_o^2}{(D_s + t_s) \cdot t_s} \cdot \left[P_e + (\rho_s^2 - 1) \cdot (P_s - P_t) \right] + \frac{a_s^2}{(D_s + t_s) t_s} \cdot P_t & \text{if (Config = "a" } \vee \text{ Config = "b" } \vee \text{ Config = "c") } \\ 0 \cdot P_s & \end{cases}$$

$$\sigma_{sm} = \begin{pmatrix} 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

Shell bending stress :

$$l_{smin} := 1.8 \cdot \sqrt{D_s \cdot t_s} \quad l_{smin} = 0.00 \text{ in}$$

$$\sigma_{sb} := \frac{6 \cdot k_s}{t_s^2} \cdot \left[\beta_s \cdot \left(\delta_s \cdot P_s + \frac{a_s^2}{E_s \cdot t_s} \cdot P_s^* \right) + \frac{6 \cdot (1 - \nu^{*2})}{E^*} \cdot \left(\frac{a_o^3}{h^3} \right) \cdot \left(1 + \frac{h'_s}{2} \right) \cdot \left[P_e \cdot (Z_v + Z_m \cdot Q_1) + \frac{2}{a_o^2} \cdot Z_m \cdot Q_2 \right] \right]$$

$$\sigma_{sb} = \begin{pmatrix} 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

Total shell stress:

$$\sigma_s := (|\sigma_{sm}| + |\sigma_{sb}|)$$

$$\sigma_s = \begin{pmatrix} 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

Load cases 1,2,3 : $\sigma_{smaxP} := \max(\sigma_{s_1}, \sigma_{s_2}, \sigma_{s_3})$

$$\sigma_{smaxP} = 0.000 \frac{\text{lb}}{\text{in}^2}$$

$$S_{sallowP} := 1.5 \cdot S_s$$

$$S_{sallowP} = 0.000 \frac{\text{lb}}{\text{in}^2}$$

Load cases 4,5,6,7 : $\sigma_{smaxT} := \max(\sigma_{s_4}, \sigma_{s_5}, \sigma_{s_6}, \sigma_{s_7})$

$$\sigma_{smaxT} = 0.000 \frac{\text{lb}}{\text{in}^2}$$

$$S_{sallowT} := S_{PS.s}$$

$$S_{sallowT} = 0.000 \frac{\text{lb}}{\text{in}^2}$$

b) Channel**Channel membrane stress :**

$$\sigma_{cm} := \begin{cases} \frac{a_c^2}{(D_c + t_c) \cdot t_c} \cdot P_t & \text{if Config = "a" } \vee \text{ Config = "e" } \vee \text{ Config = "f" } \vee \text{ Config = "A"} \\ 0 \cdot P_t & \end{cases}$$

$$\sigma_{cm} = \begin{pmatrix} 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

Channel bending stress :

$$l_{cmin} := 1.8 \cdot \sqrt{D_c \cdot t_c} \quad l_{cmin} = 0.00 \text{ in}$$

$$\sigma_{cb} := \frac{6}{t_c^2} \cdot k_c \cdot \left[\beta_c \cdot \left(\delta_c \cdot P_t + \frac{a_c^2}{E_c \cdot t_c} P_c^* \right) - \frac{6 \cdot (1 - \nu^{*2})}{E^*} \cdot \left(\frac{a_o^3}{h^3} \right) \cdot \left(1 + \frac{h'_c}{2} \right) \cdot \left[P_e \cdot (Z_v + Z_m \cdot Q_1) + \frac{2}{a_o^2} \cdot Z_m \cdot Q_2 \right] \right]$$

$$\sigma_{cb} = \begin{pmatrix} 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

Total channel stress:

$$\sigma_c := \left(|\sigma_{cm}| + |\sigma_{cb}| \right)$$

$$\sigma_c = \begin{pmatrix} 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

Load cases 1,2,3 :

$$\sigma_{cmaxP} := \max(\sigma_{c_1}, \sigma_{c_2}, \sigma_{c_3})$$

$$\sigma_{cmaxP} = 0.000 \frac{\text{lb}}{\text{in}^2}$$

$$S_{callowP} := 1.5 \cdot S_c$$

$$S_{callowP} = 0.000 \frac{\text{lb}}{\text{in}^2}$$

Load cases 4,5,6 and 7:

$$\sigma_{cmaxT} := \max(\sigma_{c_4}, \sigma_{c_5}, \sigma_{c_6}, \sigma_{c_7})$$

$$\sigma_{cmaxT} = 0.000 \frac{\text{lb}}{\text{in}^2}$$

$$S_{callowT} := S_{PS.c}$$

$$S_{callowT} = 0.000 \frac{\text{lb}}{\text{in}^2}$$

UHX13.7 Simplified Elastic Plastic Procedure :

Design Formulas and calculational procedure for the effect of plasticity at the tubesheet, channel or shell joint.

$$S^*_s := \min\left(S_{ys}, \frac{SPS.s}{2}\right)$$

$$S^*_s = 0.00 \frac{\text{lb}}{\text{in}^2}$$

$$S^*_c := \min\left(S_{yc}, \frac{SPS.c}{2}\right)$$

$$S^*_c = 0.00 \frac{\text{lb}}{\text{in}^2}$$

$$\text{fact}_{sv} := \left(1.4 - \frac{0.4 \cdot |\sigma_{sb}|}{S^*_s}\right) \quad \text{fact}_{sv} = \begin{pmatrix} 1.40 \\ 1.40 \\ 1.40 \\ 1.40 \\ 1.40 \\ 1.40 \\ 1.40 \end{pmatrix}$$

$$\text{fact}_s := \begin{pmatrix} \min(\text{fact}_{sv_1}, 1) \\ \min(\text{fact}_{sv_2}, 1) \\ \min(\text{fact}_{sv_3}, 1) \end{pmatrix} \quad \text{fact}_s = \begin{pmatrix} 1.000 \\ 1.000 \\ 1.000 \end{pmatrix}$$

$$\text{fact}_{cv} := \left(1.4 - \frac{0.4 \cdot |\sigma_{cb}|}{S^*_c}\right) \quad \text{fact}_{cv} = \begin{pmatrix} 1.4 \\ 1.4 \\ 1.4 \\ 1.4 \\ 1.4 \\ 1.4 \\ 1.4 \end{pmatrix}$$

$$\text{fact}_c := \begin{pmatrix} \min(\text{fact}_{cv_1}, 1) \\ \min(\text{fact}_{cv_2}, 1) \\ \min(\text{fact}_{cv_3}, 1) \end{pmatrix} \quad \text{fact}_c = \begin{pmatrix} 1.000 \\ 1.000 \\ 1.000 \end{pmatrix}$$

Each Load Case (1, 2, and 3) must be evaluated because "fact" will be different for each case.

$$E^*_s := E_s \cdot \text{fact}_s \quad E^*_s = \begin{pmatrix} 2.70 \times 10^7 \\ 2.70 \times 10^7 \\ 2.70 \times 10^7 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$E^*_c := E_c \cdot \text{fact}_c \quad E^*_c = \begin{pmatrix} 2.70 \times 10^7 \\ 2.70 \times 10^7 \\ 2.70 \times 10^7 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

From Step 2:

$$kk_s := \beta_s \cdot \frac{E^*_s \cdot t_s^3}{6 \cdot (1 - \nu_s^2)} \quad kk_s = \begin{pmatrix} 0.0000 \\ 0.0000 \\ 0.0000 \end{pmatrix} \text{ lb}$$

$$\lambda\lambda_s := \frac{6 \cdot D_s}{h^3} \cdot kk_s \cdot \left(1 + h'_s + \frac{h'^2_s}{2}\right) \quad \lambda\lambda_s = \begin{pmatrix} 0.0000 \\ 0.0000 \\ 0.0000 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$kk_c := \beta_c \cdot \frac{E^*_c \cdot t_c^3}{6 \cdot (1 - \nu_c^2)} \quad kk_c = \begin{pmatrix} 0.0000 \\ 0.0000 \\ 0.0000 \end{pmatrix} \text{ lb}$$

$$\lambda\lambda_c := \frac{6 \cdot D_c}{h^3} \cdot kk_c \cdot \left(1 + h'_c + \frac{h'^2_c}{2}\right) \quad \lambda\lambda_c = \begin{pmatrix} 0.0000 \\ 0.0000 \\ 0.0000 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

From Step 4:

$$F := \frac{1 - v^*}{E^*} \cdot (\lambda \lambda_s + \lambda \lambda_c + E \cdot \ln(K)) \quad F = \begin{pmatrix} 0.43 \\ 0.43 \\ 0.43 \end{pmatrix}$$

$$\Phi := (1 + v^*) \cdot F \quad \Phi = \begin{pmatrix} 0.5606 \\ 0.5606 \\ 0.5606 \end{pmatrix}$$

$$Q_{1e} := \overrightarrow{\left(\frac{\rho_s - 1 - \Phi \cdot Z_v}{1 + \Phi \cdot Z_m} \right)} \quad Q_{1e} = \begin{pmatrix} 0.08 \\ 0.08 \\ 0.08 \end{pmatrix}$$

$$P_{se} := \begin{pmatrix} P_{s_1} \\ P_{s_2} \\ P_{s_3} \end{pmatrix} \quad P_{te} := \begin{pmatrix} P_{t_1} \\ P_{t_2} \\ P_{t_3} \end{pmatrix}$$

$$P'_e := \begin{pmatrix} P_{e_1} \\ P_{e_2} \\ P_{e_3} \end{pmatrix} \quad P'_e = \begin{pmatrix} -150.00 \\ 250.00 \\ 100.00 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

Recalculate the tubesheet bending stress for load cases 1, 2, and 3

$$Q'_2 := \left[\frac{(\omega_s^* \cdot P_{se} - \omega_c^* \cdot P_{te}) + \frac{\gamma_b}{2 \cdot \pi} \cdot W}{1 + \Phi \cdot Z_m} \right]$$

$$Q'_2 = \begin{pmatrix} -213.34 \\ 355.57 \\ 142.23 \end{pmatrix} \text{ lb}$$

$$Q'_3 := \left(Q_1 + \frac{2 \cdot Q'_2}{P'_e \cdot a_o^2} \right)$$

$$Q'_3 = \begin{pmatrix} 0.0953 \\ 0.0953 \\ 0.0953 \end{pmatrix}$$

$$F'm_1(x) := \begin{cases} 0 & \text{if } P'e_1 = 0 \\ \frac{Q_v(x) + Q'_3 \cdot Q_m(x)}{2} & \text{otherwise} \end{cases}$$

$$F'm1_j := |F'm_1(x_j)|$$

$$F'm_2(x) := \begin{cases} 0 & \text{if } P'e_2 = 0 \\ \frac{Q_v(x) + Q'_3 \cdot Q_m(x)}{2} & \text{otherwise} \end{cases}$$

$$F'm2_j := |F'm_2(x_j)|$$

$$F'm_3(x) := \begin{cases} 0 & \text{if } P'e_3 = 0 \\ \frac{Q_v(x) + Q'_3 \cdot Q_m(x)}{2} & \text{otherwise} \end{cases}$$

$$F'm3_j := |F'm_3(x_j)|$$

$$F'm_1 := \max(F'm_1) \quad F'm_1 = 0.102$$

$$F'm_2 := \max(F'm_2) \quad F'm_2 = 0.10$$

$$F'm_3 := \max(F'm_3) \quad F'm_3 = 0.10$$

$$F'_m := \begin{pmatrix} F'm_1 \\ F'm_2 \\ F'm_3 \end{pmatrix} \quad F'_m = \begin{pmatrix} 0.1021 \\ 0.1021 \\ 0.1021 \end{pmatrix}$$

Tubesheet Bending Stress for the Elastic-Plastic Solution

$$\sigma' := \left[\frac{1.5 \cdot F'_m}{\mu^*} \cdot \left(\frac{2 \cdot a_o}{h - h'_g} \right)^2 \cdot P'_e \right]$$

$$\sigma' = \begin{pmatrix} -16389.848 \\ 27316.414 \\ 10926.565 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$\sigma'_{\max} := \max(|\sigma'_1|, |\sigma'_2|, |\sigma'_3|)$$

$$\sigma'_{\max} = 27316.414 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{\text{allow}} := 1.5 \cdot S$$

$$\sigma_{\text{allow}} = 28500.000 \frac{\text{lb}}{\text{in}^2}$$

if $\sigma'_{\max}$ is less than σ_{allow} , then pressure design for the tubesheet, shell and channel is acceptable.

UHX14.7 Simply Supported Tubesheet Procedure

This procedure applies only when the TS is integral with the shell or channel, i.e. :

- shell of configurations a,b,c
- channel of configuration a,e,f,A

SS = "NO"

Calculation must be performed in 2 phases :

Phase 1) Perform Steps 1 to 11 with **SS="NON"** (normal calculation) with the following modifications in Step 11:

- minimum length requirement l_{smin} of shell band for configurations a,b,c do not apply
- minimum length requirement l_{cmin} of channel band for configuration a do not apply

if $\sigma_s \leq S_{PS,s}$ and $\sigma_c \leq S_{PS,c}$, the shell and/or channel designs are acceptable.

Otherwise increase the thickness of the overstressed components (shell and/or channel) and return to Step 1.

Phase 2) Perform Steps 1 to 7 using **SS="OUI"** (Simply Supported calculation) for loading cases 1,2 and 3 only

If $|\sigma| \leq 1.5S$, the calculation procedure is complete.

Otherwise, increase the assumed tubesheet thickness h and repeat Steps 1 to 7.

Note: If $|\sigma| < 1.5S$, the tubesheet thickness can be optimized to a value h_0 till $|\sigma| = 1.5S$

provided that, for that optimized thickness h_0 , the stresses in the tubesheet, shell and channel, calculated by the normal calculation (see Phase 1), remain respectively below S_{PS} , $S_{PS,s}$, $S_{PS,c}$ for each of the 7 loading cases.

Determination of effective elastic properties :

From figure UHX-11.2 - Polynomial equations for the determination of E^*/E and ν^* for triangular pattern

$$\begin{aligned}
 &\text{Equations for } E^* \text{ for triangular pitch} \\
 E^* \text{TriPitch} &:= \begin{pmatrix} 0.0353 & 1.2502 & -0.0491 & 0.3604 & -0.6100 \\ 0.0135 & 0.9910 & 1.0080 & -1.0498 & 0.0184 \\ 0.0054 & 0.5279 & 3.0461 & -4.3657 & 1.9435 \\ -0.0029 & 0.2126 & 3.9906 & -6.1730 & 3.4307 \end{pmatrix} \\
 \nu^* \text{TriPitch} &:= \begin{pmatrix} -0.0958 & 0.6209 & -0.8683 & 2.1099 & -1.6831 \\ 0.8897 & -9.0855 & 36.1435 & -59.5425 & 35.8223 \\ 0.7439 & -4.4989 & 12.5779 & -14.2092 & 5.7822 \\ 0.9100 & -4.8901 & 12.4325 & -12.7039 & 4.4298 \\ 0.9923 & -4.8759 & 12.3572 & -13.7214 & 5.7629 \\ 0.9966 & -4.1978 & 9.0478 & -7.9955 & 2.2398 \end{pmatrix}
 \end{aligned}$$

From figure UHX-11.3 - Polynomial equations for the determination of E^*/E and v^* for square pattern

$$\begin{aligned} \text{Equations for } E^* \text{ for square pitch} \\ E^*_{\text{SquPitch}} := \begin{pmatrix} 0.0676 & 1.5756 & -1.2119 & 1.7715 & -1.2628 \\ 0.0250 & 1.9251 & -3.5230 & 6.9830 & -5.0017 \\ 0.0394 & 1.3024 & -1.1041 & 2.8714 & -2.3994 \\ 0.0372 & 1.0314 & -0.6402 & 2.6201 & -2.1929 \end{pmatrix} v^*_{\text{SquPitch}} := \begin{pmatrix} -0.0791 & 0.6008 & -0.3468 & 0.4858 & -0.3606 \\ 0.3345 & -2.8420 & 10.9709 & -15.8994 & 8.3516 \\ 0.4296 & -2.6350 & 8.6864 & -11.5227 & 5.8544 \\ 0.3636 & -0.8057 & 2.0463 & -2.2902 & 1.1862 \\ 0.3527 & -0.2842 & 0.4354 & -0.0901 & -0.1590 \\ 0.3341 & 0.1260 & -0.6920 & 0.6877 & -0.0600 \end{pmatrix} \\ \text{Elay} := \text{si}(\text{Layout} > 0, E^*_{\text{SquPitch}}, E^*_{\text{TriPitch}}) \end{aligned}$$

$$E^*_1 := \left(\text{Elay}_{1,1} + \text{Elay}_{1,2} \cdot \mu^* + \text{Elay}_{1,3} \cdot \mu^{*2} + \text{Elay}_{1,4} \cdot \mu^{*3} + \text{Elay}_{1,5} \cdot \mu^{*4} \right) \cdot E \quad E^*_1 = 1.40 \times 10^7 \frac{\text{lb}}{\text{in}^2}$$

$$E^*_2 := \left(\text{Elay}_{2,1} + \text{Elay}_{2,2} \cdot \mu^* + \text{Elay}_{2,3} \cdot \mu^{*2} + \text{Elay}_{2,4} \cdot \mu^{*3} + \text{Elay}_{2,5} \cdot \mu^{*4} \right) \cdot E \quad E^*_2 = 1.31 \times 10^7 \frac{\text{lb}}{\text{in}^2}$$

$$E^*_3 := \left(\text{Elay}_{3,1} + \text{Elay}_{3,2} \cdot \mu^* + \text{Elay}_{3,3} \cdot \mu^{*2} + \text{Elay}_{3,4} \cdot \mu^{*3} + \text{Elay}_{3,5} \cdot \mu^{*4} \right) \cdot E \quad E^*_3 = 1.23 \times 10^7 \frac{\text{lb}}{\text{in}^2}$$

$$E^*_4 := \left(\text{Elay}_{4,1} + \text{Elay}_{4,2} \cdot \mu^* + \text{Elay}_{4,3} \cdot \mu^{*2} + \text{Elay}_{4,4} \cdot \mu^{*3} + \text{Elay}_{4,5} \cdot \mu^{*4} \right) \cdot E \quad E^*_4 = 1.06 \times 10^7 \frac{\text{lb}}{\text{in}^2}$$

For h/p lower than 0.1, use $h/p = 0.1$. For values of h/p higher than 2, use $h/p = 2.0$. Otherwise perform a linear interpolation:

$$\text{hpratio} := \begin{pmatrix} 0.1 \\ 0.25 \\ 0.5 \\ 2.0 \end{pmatrix} \quad \text{interE}^* := \begin{pmatrix} E^*_1 \\ E^*_2 \\ E^*_3 \\ E^*_4 \end{pmatrix} \quad \frac{h}{p} = 1.75$$

$$E^* := \text{si}\left(\frac{h}{p} > 2, E^*_4, \text{si}\left(\frac{h}{p} \leq 0.10, E^*_1, \text{interplin}\left(\text{hpratio}, \text{interE}^*, \frac{h}{p}\right)\right)\right) \quad E^* = 1.09 \times 10^7 \frac{\text{lb}}{\text{in}^2} \quad \frac{E^*}{E} = 0.4039$$

$$v_{\text{lay}} := \text{si}(\text{Layout} > 0, v^*_{\text{SquPitch}}, v^*_{\text{TriPitch}})$$

$$v^*_1 := v_{\text{lay}_{1,1}} + v_{\text{lay}_{1,2}} \cdot \mu^* + v_{\text{lay}_{1,3}} \cdot \mu^{*2} + v_{\text{lay}_{1,4}} \cdot \mu^{*3} + v_{\text{lay}_{1,5}} \cdot \mu^{*4} \quad v^*_1 = 0.10$$

$$v^*_2 := v_{\text{lay}_{2,1}} + v_{\text{lay}_{2,2}} \cdot \mu^* + v_{\text{lay}_{2,3}} \cdot \mu^{*2} + v_{\text{lay}_{2,4}} \cdot \mu^{*3} + v_{\text{lay}_{2,5}} \cdot \mu^{*4} \quad v^*_2 = 0.14$$

$$v^*_3 := v_{\text{lay}_{3,1}} + v_{\text{lay}_{3,2}} \cdot \mu^* + v_{\text{lay}_{3,3}} \cdot \mu^{*2} + v_{\text{lay}_{3,4}} \cdot \mu^{*3} + v_{\text{lay}_{3,5}} \cdot \mu^{*4} \quad v^*_3 = 0.19$$

$$v^*_4 := v_{\text{lay}_{4,1}} + v_{\text{lay}_{4,2}} \cdot \mu^* + v_{\text{lay}_{4,3}} \cdot \mu^{*2} + v_{\text{lay}_{4,4}} \cdot \mu^{*3} + v_{\text{lay}_{4,5}} \cdot \mu^{*4} \quad v^*_4 = 0.24$$

$$v^*_5 := v_{\text{lay}_{5,1}} + v_{\text{lay}_{5,2}} \cdot \mu^* + v_{\text{lay}_{5,3}} \cdot \mu^{*2} + v_{\text{lay}_{5,4}} \cdot \mu^{*3} + v_{\text{lay}_{5,5}} \cdot \mu^{*4} \quad v^*_5 = 0.29$$

$$v^*_6 := v_{\text{lay}_{6,1}} + v_{\text{lay}_{6,2}} \cdot \mu^* + v_{\text{lay}_{6,3}} \cdot \mu^{*2} + v_{\text{lay}_{6,4}} \cdot \mu^{*3} + v_{\text{lay}_{6,5}} \cdot \mu^{*4} \quad v^*_6 = 0.31$$

$$\text{interv} := \begin{pmatrix} v^*_1 \\ v^*_2 \\ v^*_3 \\ v^*_4 \\ v^*_5 \\ v^*_6 \end{pmatrix} \quad \text{hpratio} := \begin{pmatrix} 0.1 \\ 0.15 \\ 0.25 \\ 0.50 \\ 1.00 \\ 2.00 \end{pmatrix} \quad \begin{aligned} &\text{For } h/p \text{ lower than } 0.1, \text{ use } h/p = 0.1. \text{ For values of } h/p \text{ higher than } 2, \text{ use } h/p = 2.0 \\ &\text{Otherwise perform a linear interpolation:} \\ &v^* := \text{si}\left(\frac{h}{p} > 2, v^*_6, \text{si}\left(\frac{h}{p} \leq 0.10, v^*_1, \text{interplin}\left(\text{hpratio}, \text{interv}, \frac{h}{p}\right)\right)\right) \\ &\frac{h}{p} = 1.75 \quad v^* = 0.31 \end{aligned}$$

$\frac{1}{2}$

$$A_p := \pi r_o^2 \quad A_p = 490.874 \text{ in}^2$$

$$C_p := 2\pi r_o \quad C_p = 78.540 \text{ in}$$

ANNEX X—UHX14 – EXAMPLE 1 (2010 EDITION) FLOATING

FLOATING Tubesheet Rules acc. to UHX-14 (July 2010 Edition)

highlighted in Yellow :most important data and results

CASE 20.3.1 (d + C) FLOATING Tubesheet

CHANNEL data which are different for Stationary and Floating Tubesheets

SHELL data which are not used for Floating Tubesheet

Geometric Data (from Fig.UHX-14.1)

ExchType := "a"	EXCHANGER types : a, b, c	$h := 1.75 \text{ in}$	Tubesheet thickness
StaConfig := "d"	STATIONNARY Config. types : a, b, c, d,e,f		
FloConfig := "C"	FLOATING Config. types : A, B, C, D		
	Config := FloConfig	Config = "C"	

Layout := 0	For triangular pitch : "Layout"=0 For square pitch : "Layout"=1	$p := 1.0 \cdot \text{in}$	Tube Pitch	Set Farenheit temperatures degF := R
$r_o := 12.5 \cdot \text{in}$	Radius to outer tube	$N_t := 466$	Number of Tubes	
$A := 26.89 \cdot \text{in}$	Outside Dia of Tubesheet	$d_t := 0.75 \cdot \text{in}$	Tube Diameter	
$C := 27.992 \cdot \text{in}$	Bolt Circle Diameter	$t_t := 0.083 \cdot \text{in}$	Tube Thickness	
$A_L := 64.375 \cdot \text{in}^2$	Total Untubed Lanes Area	$L_t := 256 \cdot \text{in}$	Tube Length	
$h_g := 0.0 \cdot \text{in}$	Groove depth	$\rho := 0.8$	Tube expansion factor	
$c_t := 0 \cdot \text{in}$	Tubesheet Corr. Allow. (Tubeside)	$l_{tx} := 1.25 \cdot \text{in}$	Length of Expanded Portion of Tube	
$c_s := 0 \cdot \text{in}$	Tubesheet Corr. Allow. (Shellside)	$kl := 15.375 \cdot \text{in}$	$k=0.6$ for spans between Tubesheets $k=0.8$ for spans between TS/ support plate $k=1.0$ for spans between support plates l = unsupported tube span	

SS := "NO" SS="YES" for Simply Supported TS

$D_s := 0 \cdot \text{in}$	Shell ID	$D_c := 0.0 \cdot \text{in}$	Channel ID
$t_s := 0.1 \cdot \text{in}$	Shell Thickness	$t_c := 0.1 \cdot \text{in}$	Channel Thickness
$G_s := 29.375 \cdot \text{in}$	Shell Gasket Diameter	$G_c := 26.496 \cdot \text{in}$	Channel Gasket Diameter
$G_1 := 0 \cdot \text{in}$	shell Contact mid-point TS/Flange	$G_1 := 26.496 \cdot \text{in}$	Channel Contact mid-point TS/Flange
$C_s := 0 \cdot \text{in}$	Shell Corrosion Allowance	$C_c := 0 \cdot \text{in}$	Channell Corrosion Allowance

CHAN := "CYL" "CYL" for Cylindrical Channel
"HEMI" for Hemispherical Channel

Corroded thicknesses:

Corroded length:

$h := h - c_s - c_t$	Tubesheet thickness	$h = 1.750 \text{ in}$	$L_t := L_t - 2c_t$	$L_t = 256.000 \text{ in}$
$t_s := t_s - C_s$	Shell Thickness away from TS	$t_s = 0.100 \text{ in}$		
$t_c := t_c - C_c$	Channel Thickness	$t_c = 0.100 \text{ in}$		

Corroded diameters:

$D_s := D_s + 2 \cdot C_s$	Shell ID	$D_s = 0.000 \text{ in}$
$D_c := D_c + 2 \cdot C_c$	Channel ID	$D_c = 0.000 \text{ in}$

Loading conditions

$$W := 26225 \cdot \text{lb}$$

Shell or Channel Flange bolt load

$$P_{s0} := 250 \cdot \frac{\text{lb}}{\text{in}^2}$$

Shell Design Pressure

$$P_{t0} := 150 \cdot \frac{\text{lb}}{\text{in}^2}$$

Tube Design Pressure

Design Pressures for the Seven Load Cases

$$P_s := \begin{pmatrix} 0 \cdot \frac{\text{lb}}{\text{in}^2} \\ P_{s0} \\ P_{s0} \\ 0 \cdot \frac{\text{lb}}{\text{in}^2} \\ 0 \cdot \frac{\text{lb}}{\text{in}^2} \\ P_{s0} \\ P_{s0} \end{pmatrix}$$

$$P_s = \begin{pmatrix} 0.000 \\ 250.000 \\ 250.000 \\ 0.000 \\ 0.000 \\ 250.000 \\ 250.000 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$P_t := \begin{pmatrix} P_{t0} \\ 0 \cdot \frac{\text{lb}}{\text{in}^2} \\ P_{t0} \\ 0 \cdot \frac{\text{lb}}{\text{in}^2} \\ P_{t0} \\ 0 \cdot \frac{\text{lb}}{\text{in}^2} \\ P_{t0} \end{pmatrix}$$

$$P_t = \begin{pmatrix} 150.000 \\ 0.000 \\ 150.000 \\ 0.000 \\ 150.000 \\ 0.000 \\ 150.000 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

Temperatures

$$T := 70 \cdot \text{degF}$$

Tubesheet design temp.

$$T' := 70 \cdot \text{degF}$$

Tubesheet temp.@ rim

$$T_t := 70 \cdot \text{degF}$$

Tube design temp.

$$T_s := 70 \cdot \text{degF}$$

Shell design temp.

$$T'_s := 70 \cdot \text{degF}$$

Shell temp. @ Tubesheet

$$T_c := 70 \cdot \text{degF}$$

Channel design temp.

$$T'_c := 70 \cdot \text{degF}$$

Channel temp.@ tubesheet

$$T_a := 70 \cdot \text{degF}$$

Ambient temperature

Material Data**Tubesheet Material is SB-564 C276 high**

$$S := 19000 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{TS allowable stress @ T}$$

$$S_y := 31000 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{TS yield stress @T}$$

$$E := 27 \cdot 10^6 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{TS elastic modulus @ T}$$

$$S_{PS} := 33900 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{TS allowable P+S stress @ T}$$

$$\nu := 0.32 \quad \text{TS Poisson's ratio}$$

$$\alpha' := 0 \cdot \frac{\text{in}}{\text{in} \cdot \text{degF}} \quad \text{TS coeff. expansion@ rim}$$

Tube Material is SB-626 C276

$$S_t := 13350 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{Tube allowable stress @ } T_t$$

$$S_{tT} := 13350 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{Tube allowable stress @ } T_{tT}$$

$$E_t := 27 \cdot 10^6 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{Tube elastic modulus @ } T_t$$

$$E_{tT} := 27 \cdot 10^6 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{Tube elastic modulus @ } T_{tT}$$

$$\nu_t := 0.32 \quad \text{Tube Poisson's ratio}$$

$$S_{yt} := 20550 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{Tube yield stress @ } T_t$$

Shell Material is SA-516-70 N

for STATIONARY Tubesheet ONLY

$$S_s := 0 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{Shell allow. stress @Ts}$$

$$S_{ys} := 0 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{Shell yield stress @Ts}$$

$$E_s := 27 \cdot 10^6 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{Shell elast mod. @Ts}$$

$$S_{PS.s} := 0 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{Shell P+S allow. stress @Ts}$$

$$\nu_s := 0 \quad \text{Shell Poisson' ratio}$$

$$\alpha'_s := 0 \cdot \frac{\text{in}}{\text{in} \cdot \text{degF}} \quad \text{Shell coeff. expansion @ T's}$$

Channel Material is SB-575 C276 high

$$S_c := 0 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{Channel allow. stress @ } T_c$$

$$S_{yc} := 0 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{Channel yield stress @ } T_c$$

$$E_c := 27 \cdot 10^6 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{Channel elast. modulus @ } T_c$$

$$S_{PS.c} := 0 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{Channel P+S allow. stress @ } T_c$$

$$\nu_c := 0 \quad \text{Channel Poisson's ratio}$$

$$\alpha'_c := 0 \cdot \frac{\text{in}}{\text{in} \cdot \text{degF}} \quad \text{Channel coeff. expansion @ } T'_c$$

Start of Calculations

$$L := L_t - 2 \cdot h \quad L = 252.500 \text{ in} \quad L = \text{effect. length of shell/tubes}$$

$$h'_g := \max(h_g - c_t, 0) \quad h'_g = 0.000 \text{ in}$$

$$\rho := \frac{l_{tx}}{h} \quad \rho = 0.800 \quad d^* := \max \left[d_t - 2 \cdot t_t \cdot \left(\frac{E_{tT}}{E} \right) \cdot \left(\frac{S_{tT}}{S} \right) \cdot \rho, (d_t - 2t_t) \right] \quad d^* = 0.657 \text{ in}$$

UHX-14.5.1 Step 1 Determine D_o , μ , μ^* and h'_g from UHX-11.5.1 :

$$D_o := 2 \cdot r_o + d_t \quad D_o = 25.750 \text{ in} \quad a_o := \frac{D_o}{2} \quad a_o = 12.875 \text{ in}$$

$$p^* := \frac{p}{\sqrt{1 - \frac{4 \cdot \min(A_L, 4D_o \cdot p)}{\pi \cdot D_o^2}}} \quad p^* = 1.068 \text{ in} \quad \mu^* := \frac{p^* - d^*}{p^*} \quad \mu^* = 0.385$$

$$\mu := \frac{p - d_t}{p} \quad \mu = 0.250$$

Chan Rad. Dim.

$$a_c := \begin{cases} \frac{D_c}{2} & \text{if Config = "a" } \vee \text{ Config = "e" } \vee \text{ Config = "f" } \vee \text{ Config = "A"} \\ \frac{G_c}{2} & \text{if Config = "b" } \vee \text{ Config = "c" } \vee \text{ Config = "d" } \vee \text{ Config = "B" } \vee \text{ Config = "C"} \\ \frac{A}{2} & \text{if Config = "D"} \end{cases} \quad a_c = 13.248 \text{ in}$$

Shell Radial Dim.

$$a_s := \begin{cases} \frac{D_s}{2} & \text{if Config = "a" } \vee \text{ Config = "b" } \vee \text{ Config = "c"} \\ \frac{G_s}{2} & \text{if Config = "d" } \vee \text{ Config = "e" } \vee \text{ Config = "f"} \\ a_c & \text{if Config = "A" } \vee \text{ Config = "B" } \vee \text{ Config = "C" } \vee \text{ Config = "D"} \end{cases} \quad a_s = 13.248 \text{ in}$$

$$\rho_s := \frac{a_s}{a_o} \quad \rho_s = 1.029 \quad \rho_c := \frac{a_c}{a_o} \quad \rho_c = 1.029$$

$$x_t := 1 - N_t \cdot \left(\frac{d_t - 2 \cdot t_t}{2 \cdot a_o} \right)^2 \quad x_t = 0.760 \quad x_s := 1 - N_t \cdot \left(\frac{d_t}{2 \cdot a_o} \right)^2 \quad x_s = 0.605$$

UHX-14.5.2 Step 2 Calculate shell and channel parameters:

Use SS=YES for Simply Supported calculation in a 2nd step (see UHX-13.9)

SS = "NO"

$$\beta_S := \begin{cases} \frac{[12 \cdot (1 - \nu_S^2)]^{0.25}}{[(D_S + t_S) \cdot t_S]^{0.5}} & \text{if } SS = \text{"NO"} \wedge (\text{Config} = \text{"a"} \vee \text{Config} = \text{"b"} \vee \text{Config} = \text{"c"}) \\ 0 \cdot \frac{1}{\text{in}} & \text{otherwise} \end{cases} \quad \beta_S = 0.000 \frac{1}{\text{in}}$$

$$\beta_C := \begin{cases} \frac{[12 \cdot (1 - \nu_C^2)]^{0.25}}{[(D_C + t_C) \cdot t_C]^{0.5}} & \text{if } SS = \text{"NO"} \wedge (\text{Config} = \text{"a"} \vee \text{Config} = \text{"e"} \vee \text{Config} = \text{"f"} \vee \text{Config} = \text{"A"}) \\ 0 \cdot \frac{1}{\text{in}} & \text{otherwise} \end{cases} \quad \beta_C = 0.000 \frac{1}{\text{in}}$$

$$k_S := \beta_S \cdot \frac{E_S \cdot t_S^3}{6 \cdot (1 - \nu_S^2)} \quad k_S = 0 \times 10^0 \text{ lb}$$

$$k_C := \beta_C \cdot \frac{E_C \cdot t_C^3}{6 \cdot (1 - \nu_C^2)} \quad k_C = 0 \text{ lb}$$

$$h'_S := h \cdot \beta_S \quad h'_S = 0.000$$

$$h'_C := h \cdot \beta_C \quad h'_C = 0.000$$

$$\lambda_S := \frac{6 \cdot D_S}{h^3} \cdot k_S \cdot \left(1 + h'_S + \frac{h'^2_S}{2}\right) \quad \lambda_S = 0 \times 10^0 \frac{\text{lb}}{\text{in}^2}$$

$$\lambda_C := \frac{6 \cdot D_C}{h^3} \cdot k_C \cdot \left(1 + h'_C + \frac{h'^2_C}{2}\right) \quad \lambda_C = 0 \times 10^0 \frac{\text{lb}}{\text{in}^2}$$

$$\delta_S := \begin{cases} \frac{D_S^2}{4E_S \cdot t_S} \left(1 - \frac{\nu_S}{2}\right) & \text{if } SS = \text{"NO"} \wedge (\text{Config} = \text{"a"} \vee \text{Config} = \text{"b"} \vee \text{Config} = \text{"c"}) \\ 0 \cdot (\text{in}^3 \text{ lb}^{-1}) & \text{otherwise} \end{cases} \quad V := \frac{\lambda_S + \lambda_C}{E} \quad V = 0.000$$

$$\delta_S = 0 \times 10^0 \text{ in}^3 \text{ lb}^{-1}$$

CHAN = "CYL"

$$\delta_C := \begin{cases} \frac{D_C^2}{4E_C \cdot t_C} \left(\frac{1 - \nu_C}{2}\right) & \text{if } SS = \text{"NO"} \wedge (\text{Config} = \text{"a"} \vee \text{Config} = \text{"e"} \vee \text{Config} = \text{"f"} \vee \text{Config} = \text{"A"}) \\ \frac{D_C^2}{4E_C \cdot t_C} \left(\frac{1 - \nu_C}{2}\right) & \text{if } SS = \text{"NO"} \wedge \text{CHAN} = \text{"HEMI"} \wedge (\text{Config} = \text{"a"} \vee \text{Config} = \text{"b"} \vee \text{Config} = \text{"c"}) \\ 0 \cdot (\text{in}^3 \text{ lb}^{-1}) & \text{otherwise} \end{cases}$$

$$\delta_C = 0 \times 10^0 \text{ in}^3 \text{ lb}^{-1}$$

UHX-14.5.3 Step 3 Calculate h/p. If ρ changes, recalculate d^* and μ^* from UHX-11.5.1. Determine E^*/E and ν^* relative to h/p from UHX-11.5.2. Calculate X_a

$$\frac{h}{p} = 1.750 \quad \mu^* = 0.385 \quad \frac{E^*}{E} = 0.4039 \quad E^* = 1.091 \times 10^7 \frac{\text{lb}}{\text{in}^2} \quad \nu^* = 0.308 \quad (\text{From pages above, on the right})$$

$$X_a := \left[24 \cdot (1 - \nu^{*2}) \cdot N_t \cdot \frac{E_t \cdot t_t \cdot (d_t - t_t) \cdot a_o^2}{E^* \cdot L \cdot h^3} \right]^{.25} \quad X_a = 3.610$$

UHX-14.5.4 Step 4 Calculate diameter ratio K and coefficient F :

$$K := \frac{A}{D_o} \quad K = 1.044$$

$$F := \frac{1 - \nu^*}{E^*} \cdot (\lambda_s + \lambda_c + E \cdot \ln(K)) \quad F = 0.074$$

$$\Phi := (1 + \nu^*) \cdot F \quad \Phi = 0.097$$

Calculate Z_a, Z_d, Z_v, Z_m

$$N := \text{arrondi} \left(4 + \frac{X_a}{2} \right) + 1 \quad N = 7.000$$

$$\text{ber}_x(x) := \sum_{n=0}^N \left[(-1)^n \cdot \frac{\left(\frac{x}{2}\right)^{4 \cdot n}}{((2 \cdot n)!)^2} \right] \quad \text{ber} := \text{ber}_x(X_a) \quad \text{ber} = -1.461$$

$$\text{bei}_x(x) := \sum_{n=1}^N \left[(-1)^{n-1} \cdot \frac{\left(\frac{x}{2}\right)^{4 \cdot n - 2}}{((2 \cdot n - 1)!)^2} \right] \quad \text{bei} := \text{bei}_x(X_a) \quad \text{bei} = 2.323$$

$$\text{ber}'_x(x) := \sum_{n=1}^N \frac{(-1)^n \cdot (2 \cdot n) \cdot \left(\frac{x}{2}\right)^{4 \cdot n - 1}}{((2 \cdot n)!)^2} \quad \text{ber}' := \text{ber}'_x(X_a) \quad \text{ber}' = -2.515$$

$$\text{bei}'_x(x) := \sum_{n=1}^N \frac{(-1)^{n-1} \cdot (2 \cdot n - 1) \cdot \left(\frac{x}{2}\right)^{4 \cdot n - 3}}{((2 \cdot n - 1)!)^2} \quad \text{bei}' := \text{bei}'_x(X_a) \quad \text{bei}' = 0.278$$

$$\Psi_{1x}(x) := \text{bei}_x(x) + \left(\frac{1 - \nu^*}{x} \right) \cdot \text{ber}'_x(x) \quad \Psi_1 := \Psi_{1x}(X_a) \quad \Psi_1 = 1.841$$

$$\Psi_{2x}(x) := \text{ber}_x(x) - \frac{1 - \nu^*}{x} \cdot \text{bei}'_x(x) \quad \Psi_2 := \Psi_{2x}(X_a) \quad \Psi_2 = -1.515$$

$$Z_a := \text{bei}' \cdot \Psi_2 - \text{ber}' \cdot \Psi_1$$

$$Z_m := \frac{\text{ber}'^2 + \text{bei}'^2}{X_a \cdot Z_a} \quad Z_m = 0.421$$

$$Z_v := \frac{\text{ber}' \cdot \Psi_2 + \text{bei}' \cdot \Psi_1}{X_a^2 \cdot Z_a} \quad Z_v = 0.079$$

$$Z_d := \frac{\text{ber} \cdot \Psi_2 + \text{bei} \cdot \Psi_1}{X_a^3 \cdot Z_a} \quad Z_d = 0.033$$

$$Z_w := \frac{\text{ber}' \cdot \text{ber} + \text{bei}' \cdot \text{bei}}{X_a^2 \cdot Z_a} \quad Z_w = 0.079$$

Calculate Q_1 :

$$Q_1 := \frac{\rho_s - 1 - \Phi \cdot Z_v}{1 + \Phi \cdot Z_m} \quad Q_1 = 0.020$$

UHX-14.5.5 Step 5**a) Calculate parameters $\omega_s, \omega_s^*, \omega_c, \omega_c^*$,**

$$\omega_s := \rho_s \cdot k_s \cdot \beta_s \cdot \delta_s \cdot (1 + h'_s)$$

$$\omega_s = 0.000 \text{ in}^2$$

$$\omega_s^* := a_o^2 \cdot \frac{(\rho_s^2 - 1) \cdot (\rho_s - 1)}{4} - \omega_s$$

$$\omega_s^* = 0.071 \text{ in}^2$$

$$\omega_c := \rho_c \cdot k_c \cdot \beta_c \cdot \delta_c \cdot (1 + h'_c)$$

$$\omega_c = 0.000 \text{ in}^2$$

$$\omega_c^* := a_o^2 \cdot \left[\frac{(\rho_c^2 + 1) \cdot (\rho_c - 1)}{4} - \frac{\rho_s - 1}{2} \right] - \omega_c$$

$$\omega_c^* = 0.071 \text{ in}^2$$

b) Calculate parameter γ_b :

$$\gamma_b := \begin{cases} 0 & \text{if Config = "a" } \vee \text{ Config = "A" } \vee \text{ Config = "D"} \\ \frac{G_c - C}{D_o} & \text{if Config = "b" } \vee \text{ Config = "B"} \\ \frac{G_c - G_1}{D_o} & \text{if Config = "c" } \vee \text{ Config = "C"} \\ \frac{G_c - G_s}{D_o} & \text{if Config = "d"} \\ \frac{C - G_s}{D_o} & \text{if Config = "e"} \\ \frac{G_1 - G_s}{D_o} & \text{if Config = "f"} \end{cases} \quad \gamma_b = 0.000$$

From UHX-14.6 , Calculate P^*_s and P^*_c : This section is only required for **configurations a,b,or c** when the conditions of §14.6.1 a to d for Radial Differential Thermal Expansion are met.

$$T_r := \begin{cases} \frac{T' + T'_s + T'_c}{3} & \text{if Config = "a"} \\ \frac{T' + T'_s}{2} & \text{if Config = "b" } \vee \text{ Config = "c"} \\ \frac{T' + T'_c}{2} & \text{if Config = "e" } \vee \text{ Config = "f" } \vee \text{ Config = "A"} \\ 0 \cdot \text{degF} & \text{otherwise} \end{cases} \quad T_r = 0.000 \text{ degF}$$

$$T^*_s := \begin{cases} \frac{T'_s + T_r}{2} & \text{if Config = "a" } \vee \text{ Config = "b" } \vee \text{ Config = "c"} \\ 0 \text{degF} & \text{otherwise} \end{cases} \quad T^*_s = 0.000 \text{ degF}$$

$$T^*_c := \begin{cases} \frac{T'_c + T_r}{2} & \text{if Config = "a" } \vee \text{ Config = "e" } \vee \text{ Config = "f" } \vee \text{ Config = "A"} \\ 0 \text{degF} & \text{otherwise} \end{cases} \quad T^*_c = 0.000 \text{ degF}$$

$$P^*_{sp1} := \frac{E_s \cdot t_s}{a_s} \cdot [\alpha'_s \cdot (T^*_s - 70 \cdot \text{degF}) - \alpha' \cdot (T_r - 70 \cdot \text{degF})] \quad P^*_{sp1} = 0.000 \frac{\text{lb}}{\text{in}^2}$$

$$P^*_{cp1} := \frac{E_c \cdot t_c}{a_c} \cdot [\alpha'_c \cdot (T^*_c - 70 \cdot \text{degF}) - \alpha' \cdot (T_r - 70 \cdot \text{degF})] \quad P^*_{cp1} = 0.000 \frac{\text{lb}}{\text{in}^2}$$

$$P^*_{sp} := \begin{cases} 0 \frac{\text{lb}}{\text{in}^2} & \text{if Config = "d" } \vee \text{ Config = "A" } \vee \text{ Config = "B" } \vee \text{ Config = "C" } \vee \text{ Config = "D"} \\ P^*_{sp1} & \text{otherwise} \end{cases}$$

$$P^*_{cp} := \begin{cases} 0 \frac{\text{lb}}{\text{in}^2} & \text{if Config = "d" } \vee \text{ Config = "B" } \vee \text{ Config = "C" } \vee \text{ Config = "D"} \\ P^*_{cp1} & \text{otherwise} \end{cases}$$

$$P^*_s := \begin{pmatrix} 0 \cdot \frac{\text{lb}}{\text{in}^2} \\ 0 \cdot \frac{\text{lb}}{\text{in}^2} \\ 0 \cdot \frac{\text{lb}}{\text{in}^2} \\ P^*_{sp} \\ P^*_{sp} \\ P^*_{sp} \\ P^*_{sp} \end{pmatrix} \quad P^*_s = \begin{pmatrix} 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$P^*_c := \begin{pmatrix} 0 \cdot \frac{\text{lb}}{\text{in}^2} \\ 0 \cdot \frac{\text{lb}}{\text{in}^2} \\ 0 \cdot \frac{\text{lb}}{\text{in}^2} \\ P^*_{cp} \\ P^*_{cp} \\ P^*_{cp} \\ P^*_{cp} \end{pmatrix} \quad P^*_c = \begin{pmatrix} 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

UHX-14.5.6 Step 6 For each loading case, calculate the effective pressure P_e

$$P_e := \begin{cases} (P_s - P_t) & \text{if } \text{ExchType} = "a" \\ \left[P_s \cdot (1 - \rho_s^2) - P_t \right] & \text{if } \text{ExchType} = "b" \\ \left[(P_s - P_t) \cdot (1 - \rho_s^2) \right] & \text{otherwise} \end{cases}$$

$$P_e = \begin{pmatrix} -150.000 \\ 250.000 \\ 100.000 \\ 0.000 \\ -150.000 \\ 250.000 \\ 100.000 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

UHX-14.5.7 Step 7 Determine Q_2 and Q_3 for Loading Cases 1 through 7 :

$$Q_2 := \frac{\left[(\omega_s^* \cdot P_s - \omega_c^* \cdot P_t) - (\omega_s \cdot P_s^* - \omega_c \cdot P_c^*) \right] + \frac{\gamma b}{2 \cdot \pi} \cdot W}{1 + \Phi \cdot Z_m}$$

$$Q_2 = \begin{pmatrix} -10.170 \\ 16.950 \\ 6.780 \\ 0.000 \\ -10.170 \\ 16.950 \\ 6.780 \end{pmatrix} \text{lb}$$

$$Q_{3_1} := \begin{cases} 0 & \text{if } P_{e_1} = 0 \\ \left(Q_1 + \frac{2 \cdot Q_{2_1}}{P_{e_1} \cdot a_0^2} \right) & \text{otherwise} \end{cases}$$

$$Q_{3_2} := \begin{cases} 0 & \text{if } P_{e_2} = 0 \\ \left(Q_1 + \frac{2 \cdot Q_{2_2}}{P_{e_2} \cdot a_0^2} \right) & \text{otherwise} \end{cases}$$

$$Q_{3_3} := \begin{cases} 0 & \text{if } P_{e_3} = 0 \\ \left(Q_1 + \frac{2 \cdot Q_{2_3}}{P_{e_3} \cdot a_0^2} \right) & \text{otherwise} \end{cases}$$

$$Q_{3_4} := \begin{cases} 0 & \text{if } P_{e_4} = 0 \\ \left(Q_1 + \frac{2 \cdot Q_{2_4}}{P_{e_4} \cdot a_0^2} \right) & \text{otherwise} \end{cases}$$

$$Q_{3_5} := \begin{cases} 0 & \text{if } P_{e_5} = 0 \\ \left(Q_1 + \frac{2 \cdot Q_{2_5}}{P_{e_5} \cdot a_0^2} \right) & \text{otherwise} \end{cases}$$

$$Q_{3_6} := \begin{cases} 0 & \text{if } P_{e_6} = 0 \\ \left(Q_1 + \frac{2 \cdot Q_{2_6}}{P_{e_6} \cdot a_0^2} \right) & \text{otherwise} \end{cases}$$

$$Q_{3_7} := \begin{cases} 0 & \text{if } P_{e_7} = 0 \\ \left(Q_1 + \frac{2 \cdot Q_{2_7}}{P_{e_7} \cdot a_0^2} \right) & \text{otherwise} \end{cases}$$

$$Q_1 = 0.020$$

$$Q_2 = \begin{pmatrix} -10.170 \\ 16.950 \\ 6.780 \\ 0.000 \\ -10.170 \\ 16.950 \\ 6.780 \end{pmatrix} \text{lb}$$

$$Q_3 = \begin{pmatrix} 0.021 \\ 0.021 \\ 0.021 \\ 0.000 \\ 0.021 \\ 0.021 \\ 0.021 \end{pmatrix}$$

Determine coefficient F_m for Load Cases 1 through 7:

numpoints := 20

$$j := 1 \dots \text{numpoints} \quad X_j := \frac{j \cdot 1}{\text{numpoints}} \quad x := X \cdot X_a$$

$$Q_V(x) := \frac{\overrightarrow{\Psi_{2x}(x) \cdot \Psi_1 - \Psi_{1x}(x) \cdot \Psi_2}}{X_a \cdot Z_a}$$

$$Q_m(x) := \left(\frac{\overrightarrow{\Psi_{2x}(x) \cdot \text{bei}' - \Psi_{1x}(x) \cdot \text{ber}'}}{Z_a} \right)$$

$$F_{mx1}(x) := \begin{cases} 0 & \text{if } P_{e_1} = 0 \\ \frac{Q_V(x) + Q_{3_1} \cdot Q_m(x)}{2} & \text{otherwise} \end{cases}$$

$$F_{mx4}(x) := \begin{cases} 0 & \text{if } P_{e_4} = 0 \\ \frac{Q_V(x) + Q_{3_4} \cdot Q_m(x)}{2} & \text{otherwise} \end{cases}$$

$$F_{mx2}(x) := \begin{cases} 0 & \text{if } P_{e_2} = 0 \\ \frac{Q_V(x) + Q_{3_2} \cdot Q_m(x)}{2} & \text{otherwise} \end{cases}$$

$$F_{mx5}(x) := \begin{cases} 0 & \text{if } P_{e_5} = 0 \\ \frac{Q_V(x) + Q_{3_5} \cdot Q_m(x)}{2} & \text{otherwise} \end{cases}$$

$$F_{mx3}(x) := \begin{cases} 0 & \text{if } P_{e_3} = 0 \\ \frac{Q_V(x) + Q_{3_3} \cdot Q_m(x)}{2} & \text{otherwise} \end{cases}$$

$$F_{mx6}(x) := \begin{cases} 0 & \text{if } P_{e_6} = 0 \\ \frac{Q_V(x) + Q_{3_6} \cdot Q_m(x)}{2} & \text{otherwise} \end{cases}$$

$$F_{mx7}(x) := \begin{cases} 0 & \text{if } P_{e_7} = 0 \\ \frac{Q_V(x) + Q_{3_7} \cdot Q_m(x)}{2} & \text{otherwise} \end{cases}$$

$$F_{mx}(x) := \begin{pmatrix} F_{mx1}(x) \\ F_{mx2}(x) \\ F_{mx3}(x) \\ F_{mx4}(x) \\ F_{mx5}(x) \\ F_{mx6}(x) \\ F_{mx7}(x) \end{pmatrix} \quad \text{abs}F_{mx}(x) := \begin{pmatrix} |F_{mx1}(x)| \\ |F_{mx2}(x)| \\ |F_{mx3}(x)| \\ |F_{mx4}(x)| \\ |F_{mx5}(x)| \\ |F_{mx6}(x)| \\ |F_{mx7}(x)| \end{pmatrix}$$

$$F_m := \begin{pmatrix} \max(\text{abs}F_{mx}(x)_1) \\ \max(\text{abs}F_{mx}(x)_2) \\ \max(\text{abs}F_{mx}(x)_3) \\ \max(\text{abs}F_{mx}(x)_4) \\ \max(\text{abs}F_{mx}(x)_5) \\ \max(\text{abs}F_{mx}(x)_6) \\ \max(\text{abs}F_{mx}(x)_7) \end{pmatrix} \quad F_m = \begin{pmatrix} 0.075 \\ 0.075 \\ 0.075 \\ 0.000 \\ 0.075 \\ 0.075 \\ 0.075 \end{pmatrix}$$

$$F_{mx}(X_a) = \begin{pmatrix} 0.011 \\ 0.011 \\ 0.011 \\ 0.000 \\ 0.011 \\ 0.011 \\ 0.011 \end{pmatrix}$$

Calculate the Maximum Tubesheet Bending Stress

$$h_{\min} := \begin{pmatrix} h - h'_g \\ h - h'_g \\ h - h'_g \\ h \\ h \\ h \\ h \end{pmatrix} \quad h_{\min} = \begin{pmatrix} 1.750 \\ 1.750 \\ 1.750 \\ 1.750 \\ 1.750 \\ 1.750 \\ 1.750 \end{pmatrix} \text{ in}$$

$$\sigma_1 := \begin{cases} \left[\frac{6 \cdot Q_{2_1}}{\mu^* \cdot (h_{\min_1})^2} \right] & \text{if } P_{e_1} = 0 \\ \left[\frac{1.5 \cdot F_{m_1} \cdot \left(\frac{2 \cdot a_0}{h_{\min_1}} \right)^2 \cdot P_{e_1}}{\mu^*} \right] & \text{otherwise} \end{cases} \quad \sigma_1 = -9488.014 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_2 := \begin{cases} \left[\frac{6 \cdot Q_{2_2}}{\mu^* \cdot (h_{\min_2})^2} \right] & \text{if } P_{e_2} = 0 \\ \left[\frac{1.5 \cdot F_{m_2} \cdot \left(\frac{2 \cdot a_0}{h_{\min_2}} \right)^2 \cdot P_{e_2}}{\mu^*} \right] & \text{otherwise} \end{cases} \quad \sigma_2 = 15813.357 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_3 := \begin{cases} \left[\frac{6 \cdot Q_{2_3}}{\mu^* \cdot (h_{\min_3})^2} \right] & \text{if } P_{e_3} = 0 \\ \left[\frac{1.5 \cdot F_{m_3} \cdot \left(\frac{2 \cdot a_0}{h_{\min_3}} \right)^2 \cdot P_{e_3}}{\mu^*} \right] & \text{otherwise} \end{cases} \quad \sigma_3 = 6325.343 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_4 := \begin{cases} \left[\frac{6 \cdot Q_{2_4}}{\mu^* \cdot (h_{\min_4})^2} \right] & \text{if } P_{e_4} = 0 \\ \left[\frac{1.5 \cdot F_{m_4} \cdot \left(\frac{2 \cdot a_0}{h_{\min_4}} \right)^2 \cdot P_{e_4}}{\mu^*} \right] & \text{otherwise} \end{cases} \quad \sigma_4 = 0.000 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_5 := \begin{cases} \left[\frac{6 \cdot Q_{2_5}}{\mu^* \cdot (h_{\min_5})^2} \right] & \text{if } P_{e_5} = 0 \\ \left[\frac{1.5 \cdot F_{m_5} \cdot \left(\frac{2 \cdot a_0}{h_{\min_5}} \right)^2 \cdot P_{e_5}}{\mu^*} \right] & \text{otherwise} \end{cases} \quad \sigma_5 = -9488.014 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_6 := \begin{cases} \left[\frac{6 \cdot Q_{2_6}}{\mu^* \cdot (h_{\min_6})^2} \right] & \text{if } P_{e_6} = 0 \\ \left[\frac{1.5 \cdot F_{m_6} \cdot \left(\frac{2 \cdot a_0}{h_{\min_6}} \right)^2 \cdot P_{e_6}}{\mu^*} \right] & \text{otherwise} \end{cases} \quad \sigma_6 = 15813.357 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_7 := \begin{cases} \left[\frac{6 \cdot Q_{2_7}}{\mu^* \cdot (h_{\min_7})^2} \right] & \text{if } P_{e_7} = 0 \\ \left[\frac{1.5 \cdot F_{m_7} \cdot \left(\frac{2 \cdot a_0}{h_{\min_7}} \right)^2 \cdot P_{e_7}}{\mu^*} \right] & \text{otherwise} \end{cases} \quad \sigma_7 = 6325.343 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma = \begin{pmatrix} -9488.014 \\ 15813.357 \\ 6325.343 \\ 0 \\ -9488.014 \\ 15813.357 \\ 6325.343 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

- For Pressure only Loading Cases 1, 2, and 3:

$$\sigma_{\max P} := \max(|\sigma_1|, |\sigma_2|, |\sigma_3|) \quad \sigma_{\max P} = 15813.357 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{\text{allow}P} := 1.5 \cdot S \quad \sigma_{\text{allow}P} = 28500.000 \frac{\text{lb}}{\text{in}^2}$$

Calculate the minimum required thickness of the TS flanged extension for configurations b and e:

-For Pressure-Thermal Loading Cases 4, 5, 6 and 7:

$$\sigma_{\max T} := \max(|\sigma_4|, |\sigma_5|, |\sigma_6|, |\sigma_7|) \quad \sigma_{\max T} = 15813.357 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{\text{allow}T} := S_{PS.s} \quad \sigma_{\text{allow}T} = 0.000 \frac{\text{lb}}{\text{in}^2}$$

$$h_r := \begin{cases} \sqrt{\frac{1.9W}{S \cdot G_c} \cdot \frac{C - G_c}{2}} & \text{if Config = "b"} \\ \sqrt{\frac{1.9W}{S \cdot G_s} \cdot \frac{C - G_s}{2}} & \text{if Config = "e"} \\ 0 \cdot \text{in} & \text{otherwise} \end{cases} \quad h_r = 0.000 \text{ in}$$

UHX-13.5.8 Step 8 Determine the maximum tubesheet shear stress :

$$\tau := \overrightarrow{\left[\left(\frac{1}{2 \cdot \mu} \right) \cdot \left(\frac{D_o}{2 \cdot h} \right) \cdot P_e \right]} \quad \tau = \begin{pmatrix} -2207.143 \\ 3678.571 \\ 1471.429 \\ 0.000 \\ -2207.143 \\ 3678.571 \\ 1471.429 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$\tau_{\max} := \max(|\tau|) \quad \tau_{\max} = 3678.571 \frac{\text{lb}}{\text{in}^2}$$

$$\tau_{\text{allow}} := 0.8 \cdot S \quad \tau_{\text{allow}} = 15200.000 \frac{\text{lb}}{\text{in}^2}$$

If $|P_s - P_t| > 3.2 S \mu h / D_o$, the TEMA formula may be used, provided the data A_p and C_p have been defined.

$$A_p := 491 \cdot \text{in}^2$$

Total area enclosed by C_p

$$C_p := 78.5 \cdot \text{in}$$

Perimeter of the tube layout measured stepwise in increment of one tube pitch from the center-to-center of the outermost tubes (see Fig. UHX-12.2)

$$\tau := \left(\frac{1}{\mu} \right) \cdot \frac{1}{h} \left(\frac{A_p}{C_p} \right) \cdot |P_s - P_t| \quad \tau = \begin{pmatrix} 2144.495 \\ 3574.158 \\ 1429.663 \\ 0.000 \\ 2144.495 \\ 3574.158 \\ 1429.663 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$P_s - P_t = \begin{pmatrix} -150.000 \\ 250.000 \\ 100.000 \\ 0.000 \\ -150.000 \\ 250.000 \\ 100.000 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$3.2 \cdot S \cdot \mu \cdot \frac{h}{D_o} = 1033.010 \frac{\text{lb}}{\text{in}^2}$$

UHX-13.5.9 Step 9 Determine the minimum and maximum stresses in the tubes for Load Cases 1 to 7

$$Z_{dx}(x) := \frac{\overrightarrow{\Psi_2 \cdot \text{ber}_x(x) + \Psi_1 \cdot \text{bei}_x(x)}}{X_a^3 \cdot Z_a} \quad Z_{wx}(x) := \frac{\overrightarrow{\text{ber}' \cdot \text{ber}_x(x) + \text{bei}' \cdot \text{bei}_x(x)}}{X_a^2 \cdot Z_a}$$

Step a) : Determine Coefficients $F_{t,min}$ and $F_{t,max}$ for Load Cases 1 to 7:

$$\begin{aligned}
 F_{tx1}(x) &:= \begin{cases} Z_{wx}(x) \cdot \frac{X_a^4}{2} & \text{if } P_{e_1} = 0 \\ (Z_{dx}(x) + Q_{3_1} \cdot Z_{wx}(x)) \cdot \frac{X_a^4}{2} & \text{otherwise} \end{cases} & F_{tx4}(x) &:= \begin{cases} Z_{wx}(x) \cdot \frac{X_a^4}{2} & \text{if } P_{e_4} = 0 \\ (Z_{dx}(x) + Q_{3_4} \cdot Z_{wx}(x)) \cdot \frac{X_a^4}{2} & \text{otherwise} \end{cases} \\
 F_{tx2}(x) &:= \begin{cases} Z_{wx}(x) \cdot \frac{X_a^4}{2} & \text{if } P_{e_2} = 0 \\ (Z_{dx}(x) + Q_{3_2} \cdot Z_{wx}(x)) \cdot \frac{X_a^4}{2} & \text{otherwise} \end{cases} & F_{tx5}(x) &:= \begin{cases} Z_{wx}(x) \cdot \frac{X_a^4}{2} & \text{if } P_{e_5} = 0 \\ (Z_{dx}(x) + Q_{3_5} \cdot Z_{wx}(x)) \cdot \frac{X_a^4}{2} & \text{otherwise} \end{cases} \\
 F_{tx3}(x) &:= \begin{cases} Z_{wx}(x) \cdot \frac{X_a^4}{2} & \text{if } P_{e_3} = 0 \\ (Z_{dx}(x) + Q_{3_3} \cdot Z_{wx}(x)) \cdot \frac{X_a^4}{2} & \text{otherwise} \end{cases} & F_{tx6}(x) &:= \begin{cases} Z_{wx}(x) \cdot \frac{X_a^4}{2} & \text{if } P_{e_6} = 0 \\ (Z_{dx}(x) + Q_{3_6} \cdot Z_{wx}(x)) \cdot \frac{X_a^4}{2} & \text{otherwise} \end{cases} \\
 & & F_{tx7}(x) &:= \begin{cases} Z_{wx}(x) \cdot \frac{X_a^4}{2} & \text{if } P_{e_7} = 0 \\ (Z_{dx}(x) + Q_{3_7} \cdot Z_{wx}(x)) \cdot \frac{X_a^4}{2} & \text{otherwise} \end{cases}
 \end{aligned}$$

$$F_{tx}(x) := \begin{pmatrix} F_{tx1}(x) \\ F_{tx2}(x) \\ F_{tx3}(x) \\ F_{tx4}(x) \\ F_{tx5}(x) \\ F_{tx6}(x) \\ F_{tx7}(x) \end{pmatrix} \quad F_{tmin} := \begin{pmatrix} \min(F_{tx}(x)_1) \\ \min(F_{tx}(x)_2) \\ \min(F_{tx}(x)_3) \\ \min(F_{tx}(x)_4) \\ \min(F_{tx}(x)_5) \\ \min(F_{tx}(x)_6) \\ \min(F_{tx}(x)_7) \end{pmatrix} \quad F_{tmax} := \begin{pmatrix} \max(F_{tx}(x)_1) \\ \max(F_{tx}(x)_2) \\ \max(F_{tx}(x)_3) \\ \max(F_{tx}(x)_4) \\ \max(F_{tx}(x)_5) \\ \max(F_{tx}(x)_6) \\ \max(F_{tx}(x)_7) \end{pmatrix}$$

$$F_{tx}(X_a) = \begin{pmatrix} 2.925 \\ 2.925 \\ 2.925 \\ 6.690 \\ 2.925 \\ 2.925 \\ 2.925 \end{pmatrix} \quad F_{tmin} = \begin{pmatrix} -0.726 \\ -0.726 \\ -0.726 \\ -3.890 \\ -0.726 \\ -0.726 \\ -0.726 \end{pmatrix} \quad F_{tmax} = \begin{pmatrix} 2.925 \\ 2.925 \\ 2.925 \\ 6.690 \\ 2.925 \\ 2.925 \\ 2.925 \end{pmatrix}$$

Step a') : Determine tube stresses σ_{tmin} and σ_{tmax} for Load Cases 1 to 7 (continued)

$$\sigma_{t1_1} := \begin{cases} \left[\frac{1}{x_t - x_s} \cdot \left[(P_{s_1} \cdot x_s - P_{t_1} \cdot x_t) - \frac{2 \cdot Q_{2_1}}{a_o^2} \cdot F_{tmin_1} \right] \right] & \text{if } P_{e_1} = 0 \\ \left[\frac{1}{x_t - x_s} \cdot \left[(P_{s_1} \cdot x_s - P_{t_1} \cdot x_t) - P_{e_1} \cdot F_{tmin_1} \right] \right] & \text{otherwise} \end{cases}$$

$$\sigma_{t1_1} = -1432.482 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{t1_2} := \begin{cases} \left[\frac{1}{x_t - x_s} \cdot \left[(P_{s_2} \cdot x_s - P_{t_2} \cdot x_t) - \frac{2 \cdot Q_{2_2}}{a_o^2} \cdot F_{tmin_2} \right] \right] & \text{if } P_{e_2} = 0 \\ \left[\frac{1}{x_t - x_s} \cdot \left[(P_{s_2} \cdot x_s - P_{t_2} \cdot x_t) - P_{e_2} \cdot F_{tmin_2} \right] \right] & \text{otherwise} \end{cases}$$

$$\sigma_{t1_2} = 2137.470 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{t1_3} := \begin{cases} \left[\frac{1}{x_t - x_s} \cdot \left[(P_{s_3} \cdot x_s - P_{t_3} \cdot x_t) - \frac{2 \cdot Q_{2_3}}{a_o^2} \cdot F_{tmin_3} \right] \right] & \text{if } P_{e_3} = 0 \\ \left[\frac{1}{x_t - x_s} \cdot \left[(P_{s_3} \cdot x_s - P_{t_3} \cdot x_t) - P_{e_3} \cdot F_{tmin_3} \right] \right] & \text{otherwise} \end{cases}$$

$$\sigma_{t1_3} = 704.988 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{t1_4} := \begin{cases} \left[\frac{1}{x_t - x_s} \cdot \left[(P_{s_4} \cdot x_s - P_{t_4} \cdot x_t) - \frac{2 \cdot Q_{2_4}}{a_o^2} \cdot F_{tmin_4} \right] \right] & \text{if } P_{e_4} = 0 \\ \left[\frac{1}{x_t - x_s} \cdot \left[(P_{s_4} \cdot x_s - P_{t_4} \cdot x_t) - P_{e_4} \cdot F_{tmin_4} \right] \right] & \text{otherwise} \end{cases}$$

$$\sigma_{t1_4} = 0.000 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{t1_5} := \begin{cases} \left[\frac{1}{x_t - x_s} \cdot \left[(P_{s_5} \cdot x_s - P_{t_5} \cdot x_t) - \frac{2 \cdot Q_{2_5}}{a_o^2} \cdot F_{tmin_5} \right] \right] & \text{if } P_{e_5} = 0 \\ \left[\frac{1}{x_t - x_s} \cdot \left[(P_{s_5} \cdot x_s - P_{t_5} \cdot x_t) - P_{e_5} \cdot F_{tmin_5} \right] \right] & \text{otherwise} \end{cases}$$

$$\sigma_{t1_5} = -1432.482 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{t1_6} := \begin{cases} \left[\frac{1}{x_t - x_s} \cdot \left[(P_{s_6} \cdot x_s - P_{t_6} \cdot x_t) - \frac{2 \cdot Q_{2_6}}{a_o^2} \cdot F_{tmin_6} \right] \right] & \text{if } P_{e_6} = 0 \\ \left[\frac{1}{x_t - x_s} \cdot \left[(P_{s_6} \cdot x_s - P_{t_6} \cdot x_t) - P_{e_6} \cdot F_{tmin_6} \right] \right] & \text{otherwise} \end{cases}$$

$$\sigma_{t1_6} = 2137.470 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{t1_7} := \begin{cases} \left[\frac{1}{x_t - x_s} \cdot \left[(P_{s_7} \cdot x_s - P_{t_7} \cdot x_t) - \frac{2 \cdot Q_{2_7}}{a_o^2} \cdot F_{tmin_7} \right] \right] & \text{if } P_{e_7} = 0 \\ \left[\frac{1}{x_t - x_s} \cdot \left[(P_{s_7} \cdot x_s - P_{t_7} \cdot x_t) - P_{e_7} \cdot F_{tmin_7} \right] \right] & \text{otherwise} \end{cases}$$

$$\sigma_{t1_7} = 704.988 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{t2_1} := \begin{cases} \left[\frac{1}{x_t - x_s} \cdot \left((P_{s_1} \cdot x_s - P_{t_1} \cdot x_t) - \frac{2 \cdot Q_{2_1}}{a_o^2} \cdot F_{tmax_1} \right) \right] & \text{if } P_{e_1} = 0 \\ \left[\frac{1}{x_t - x_s} \cdot \left((P_{s_1} \cdot x_s - P_{t_1} \cdot x_t) - P_{e_1} \cdot F_{tmax_1} \right) \right] & \text{otherwise} \end{cases}$$

$$\sigma_{t2_1} = 2086.845 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{t2_2} := \begin{cases} \left[\frac{1}{x_t - x_s} \cdot \left((P_{s_2} \cdot x_s - P_{t_2} \cdot x_t) - \frac{2 \cdot Q_{2_2}}{a_o^2} \cdot F_{tmax_2} \right) \right] & \text{if } P_{e_2} = 0 \\ \left[\frac{1}{x_t - x_s} \cdot \left((P_{s_2} \cdot x_s - P_{t_2} \cdot x_t) - P_{e_2} \cdot F_{tmax_2} \right) \right] & \text{otherwise} \end{cases}$$

$$\sigma_{t2_2} = -3728.075 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{t2_3} := \begin{cases} \left[\frac{1}{x_t - x_s} \cdot \left((P_{s_3} \cdot x_s - P_{t_3} \cdot x_t) - \frac{2 \cdot Q_{2_3}}{a_o^2} \cdot F_{tmax_3} \right) \right] & \text{if } P_{e_3} = 0 \\ \left[\frac{1}{x_t - x_s} \cdot \left((P_{s_3} \cdot x_s - P_{t_3} \cdot x_t) - P_{e_3} \cdot F_{tmax_3} \right) \right] & \text{otherwise} \end{cases}$$

$$\sigma_{t2_3} = -1641.230 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{t2_4} := \begin{cases} \left[\frac{1}{x_t - x_s} \cdot \left((P_{s_4} \cdot x_s - P_{t_4} \cdot x_t) - \frac{2 \cdot Q_{2_4}}{a_o^2} \cdot F_{tmax_4} \right) \right] & \text{if } P_{e_4} = 0 \\ \left[\frac{1}{x_t - x_s} \cdot \left((P_{s_4} \cdot x_s - P_{t_4} \cdot x_t) - P_{e_4} \cdot F_{tmax_4} \right) \right] & \text{otherwise} \end{cases}$$

$$\sigma_{t2_4} = 0.000 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{t2_5} := \begin{cases} \left[\frac{1}{x_t - x_s} \cdot \left((P_{s_5} \cdot x_s - P_{t_5} \cdot x_t) - \frac{2 \cdot Q_{2_5}}{a_o^2} \cdot F_{tmax_5} \right) \right] & \text{if } P_{e_5} = 0 \\ \left[\frac{1}{x_t - x_s} \cdot \left((P_{s_5} \cdot x_s - P_{t_5} \cdot x_t) - P_{e_5} \cdot F_{tmax_5} \right) \right] & \text{otherwise} \end{cases}$$

$$\sigma_{t2_5} = 2086.845 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{t2_6} := \begin{cases} \left[\frac{1}{x_t - x_s} \cdot \left((P_{s_6} \cdot x_s - P_{t_6} \cdot x_t) - \frac{2 \cdot Q_{2_6}}{a_o^2} \cdot F_{tmax_6} \right) \right] & \text{if } P_{e_6} = 0 \\ \left[\frac{1}{x_t - x_s} \cdot \left((P_{s_6} \cdot x_s - P_{t_6} \cdot x_t) - P_{e_6} \cdot F_{tmax_6} \right) \right] & \text{otherwise} \end{cases}$$

$$\sigma_{t2_6} = -3728.075 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{t2_7} := \begin{cases} \left[\frac{1}{x_t - x_s} \cdot \left((P_{s_7} \cdot x_s - P_{t_7} \cdot x_t) - \frac{2 \cdot Q_{2_7}}{a_o^2} \cdot F_{tmax_7} \right) \right] & \text{if } P_{e_7} = 0 \\ \left[\frac{1}{x_t - x_s} \cdot \left((P_{s_7} \cdot x_s - P_{t_7} \cdot x_t) - P_{e_7} \cdot F_{tmax_7} \right) \right] & \text{otherwise} \end{cases}$$

$$\sigma_{t2_7} = -1641.230 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{to} := \left[\frac{(P_s \cdot x_s - P_t \cdot x_t) - P_e \cdot \frac{(Z_d + Q_3 \cdot Z_v) \cdot x_a^4}{2}}{x_t - x_s} \right]$$

$$\sigma_{t1} = \begin{pmatrix} -1432.482 \\ 2137.470 \\ 704.988 \\ 0.000 \\ -1432.482 \\ 2137.470 \\ 704.988 \end{pmatrix} \frac{\text{lb}}{\text{in}^2} \quad \sigma_{to} = \begin{pmatrix} 2086.845 \\ -3728.075 \\ -1641.230 \\ 0.000 \\ 2086.845 \\ -3728.075 \\ -1641.230 \end{pmatrix} \frac{\text{lb}}{\text{in}^2} \quad \sigma_{t2} = \begin{pmatrix} 2086.845 \\ -3728.075 \\ -1641.230 \\ 0.000 \\ 2086.845 \\ -3728.075 \\ -1641.230 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

- For Pressure Loading Only, Cases 1, 2, and 3 :

$$\begin{aligned} \sigma_{t1P} &:= \max\left(\left|\sigma_{t1_1}\right|, \left|\sigma_{t1_2}\right|, \left|\sigma_{t1_3}\right|\right) & \sigma_{t1P} &= 2137.470 \frac{\text{lb}}{\text{in}^2} \\ \sigma_{t2P} &:= \max\left(\left|\sigma_{t2_1}\right|, \left|\sigma_{t2_2}\right|, \left|\sigma_{t2_3}\right|\right) & \sigma_{t2P} &= 3728.075 \frac{\text{lb}}{\text{in}^2} \\ \sigma_{tallowP} &:= 1.0 \cdot S_t & \sigma_{tallowP} &= 13350.000 \frac{\text{lb}}{\text{in}^2} \end{aligned}$$

- For Pressure plus Thermal, Loading Cases 4, 5, 6, and 7 :

$$\begin{aligned} \sigma_{t1T} &:= \max\left(\left|\sigma_{t1_4}\right|, \left|\sigma_{t1_5}\right|, \left|\sigma_{t1_6}\right|, \left|\sigma_{t1_7}\right|\right) & \sigma_{t1T} &= 2137.470 \frac{\text{lb}}{\text{in}^2} \\ \sigma_{t2T} &:= \max\left(\left|\sigma_{t2_4}\right|, \left|\sigma_{t2_5}\right|, \left|\sigma_{t2_6}\right|, \left|\sigma_{t2_7}\right|\right) & \sigma_{t2T} &= 3728.075 \frac{\text{lb}}{\text{in}^2} \\ \sigma_{tallowT} &:= 2.0 \cdot S_t & \sigma_{tallowT} &= 26700.000 \frac{\text{lb}}{\text{in}^2} \end{aligned}$$

Step (b) : check the tubes for buckling if $\sigma_{t1} < 0$ or $\sigma_{t2} < 0$

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$$l_t := kl \quad r_t := \frac{\sqrt{d_t^2 + (d_t - 2 \cdot t_t)^2}}{4} \quad r_t = 0.238 \text{ in} \quad C_t := \sqrt{\frac{2 \cdot \pi^2 \cdot E_t}{S_{yt}}} \quad C_t = 161.043 \quad F_t := \frac{l_t}{r_t} \quad F_t = 64.699$$

$$F_{S1} := \begin{cases} 1.25 & \text{if } P_{e1} = 0 \\ \max \left[3.25 - .25 \cdot \left[(Z_d + Q_{31} \cdot Z_v) \cdot X_a^4 \right], 1.25 \right] & \text{otherwise} \end{cases} \quad F_{S1} := \text{si}(F_{S1} > 2, 2, F_{S1}) \quad F_{S1} = 1.787$$

$$F_{S2} := \begin{cases} 1.25 & \text{if } P_{e2} = 0 \\ \max \left[3.25 - .25 \cdot \left[(Z_d + Q_{32} \cdot Z_v) \cdot X_a^4 \right], 1.25 \right] & \text{otherwise} \end{cases} \quad F_{S2} := \text{si}(F_{S2} > 2, 2, F_{S2}) \quad F_{S2} = 1.787$$

$$F_{S3} := \begin{cases} 1.25 & \text{if } P_{e3} = 0 \\ \max \left[3.25 - .25 \cdot \left[(Z_d + Q_{33} \cdot Z_v) \cdot X_a^4 \right], 1.25 \right] & \text{otherwise} \end{cases} \quad F_{S3} := \text{si}(F_{S3} > 2, 2, F_{S3}) \quad F_{S3} = 1.787$$

$$F_{S4} := \begin{cases} 1.25 & \text{if } P_{e4} = 0 \\ \max \left[3.25 - .25 \cdot \left[(Z_d + Q_{34} \cdot Z_v) \cdot X_a^4 \right], 1.25 \right] & \text{otherwise} \end{cases} \quad F_{S4} := \text{si}(F_{S4} > 2, 2, F_{S4}) \quad F_{S4} = 1.250$$

$$F_{S5} := \begin{cases} 1.25 & \text{if } P_{e5} = 0 \\ \max \left[3.25 - .25 \cdot \left[(Z_d + Q_{35} \cdot Z_v) \cdot X_a^4 \right], 1.25 \right] & \text{otherwise} \end{cases} \quad F_{S5} := \text{si}(F_{S5} > 2, 2, F_{S5}) \quad F_{S5} = 1.787$$

$$F_{S6} := \begin{cases} 1.25 & \text{if } P_{e6} = 0 \\ \max \left[3.25 - .25 \cdot \left[(Z_d + Q_{36} \cdot Z_v) \cdot X_a^4 \right], 1.25 \right] & \text{otherwise} \end{cases} \quad F_{S6} := \text{si}(F_{S6} > 2, 2, F_{S6}) \quad F_{S6} = 1.787$$

$$F_{S7} := \begin{cases} 1.25 & \text{if } P_{e7} = 0 \\ \max \left[3.25 - .25 \cdot \left[(Z_d + Q_{37} \cdot Z_v) \cdot X_a^4 \right], 1.25 \right] & \text{otherwise} \end{cases} \quad F_{S7} := \text{si}(F_{S7} > 2, 2, F_{S7}) \quad F_{S7} = 1.787$$

$$F_S := \begin{pmatrix} F_{S1} \\ F_{S2} \\ F_{S3} \\ F_{S4} \\ F_{S5} \\ F_{S6} \\ F_{S7} \end{pmatrix} \quad F_S = \begin{pmatrix} 1.787 \\ 1.787 \\ 1.787 \\ 1.250 \\ 1.787 \\ 1.787 \\ 1.787 \end{pmatrix}$$

Determine the Maximum Permissible Buckling Stress for Load Cases 1 to 7 (only applicable when the calculated tube compressive stress is negative):

$$S_{tb} := \text{si} \left[C_t > F_t, \frac{S_{yt}}{F_s} \cdot \left(1 - \frac{F_t}{2 \cdot C_t} \right), \frac{1}{F_s} \cdot \pi^2 \cdot \frac{E_t}{F_t^2} \right]$$

Allowable Buckling Stress, S_{tb}

$$S_{tb} = \begin{pmatrix} 9188.391 \\ 9188.391 \\ 9188.391 \\ 13137.609 \\ 9188.391 \\ 9188.391 \\ 9188.391 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

Tube Calculated Stress σ_{t1}, σ_{t2}

$$\sigma_{t1} = \begin{pmatrix} -1432.482 \\ 2137.470 \\ 704.988 \\ 0.000 \\ -1432.482 \\ 2137.470 \\ 704.988 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}, \quad \sigma_{t2} = \begin{pmatrix} 2086.845 \\ -3728.075 \\ -1641.230 \\ 0.000 \\ 2086.845 \\ -3728.075 \\ -1641.230 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

UHX 14.5.10 Step 10 Determine the stresses in Shell and Channel when integral with the tubesheet (a,b,c) :**a) Shell****Shell membrane stress :**

$$\sigma_{sm} := \begin{cases} \frac{a_o^2}{(D_s + t_s) \cdot t_s} \cdot \left[P_e + (\rho_s^2 - 1) \cdot (P_s - P_t) \right] + \frac{a_s^2}{(D_s + t_s) t_s} \cdot P_t & \text{if (Config = "a" } \vee \text{ Config = "b" } \vee \text{ Config = "c")} \\ 0 \cdot P_s & \end{cases}$$

$$\sigma_{sm} = \begin{pmatrix} 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

Shell bending stress :

$$l_{smin} := 1.8 \cdot \sqrt{D_s \cdot t_s} \quad l_{smin} = 0.000 \text{ in}$$

$$\sigma_{sb} := \frac{6 \cdot k_s}{t_s^2} \cdot \left[\beta_s \cdot \left(\delta_s \cdot P_s + \frac{a_s^2}{E_s \cdot t_s} \cdot P^*_s \right) + \frac{6 \cdot (1 - \nu^{*2})}{E^*} \cdot \left(\frac{a_o^3}{h^3} \right) \cdot \left(1 + \frac{h'_s}{2} \right) \cdot \left[P_e \cdot (Z_v + Z_m \cdot Q_1) + \frac{2}{a_o^2} \cdot Z_m \cdot Q_2 \right] \right]$$

$$\sigma_{sb} = \begin{pmatrix} 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

Total shell stress:

$$\sigma_s := (|\sigma_{sm}| + |\sigma_{sb}|)$$

$$\sigma_s = \begin{pmatrix} 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

Load cases 1,2,3 : $\sigma_{smaxP} := \max(\sigma_{s_1}, \sigma_{s_2}, \sigma_{s_3})$

$$\sigma_{smaxP} = 0.000 \frac{\text{lb}}{\text{in}^2}$$

$$S_{sallowP} := 1.5 \cdot S_s$$

$$S_{sallowP} = 0.000 \frac{\text{lb}}{\text{in}^2}$$

Load cases 4,5,6,7 : $\sigma_{smaxT} := \max(\sigma_{s_4}, \sigma_{s_5}, \sigma_{s_6}, \sigma_{s_7})$

$$\sigma_{smaxT} = 0.000 \frac{\text{lb}}{\text{in}^2}$$

$$S_{sallowT} := S_{PS.s}$$

$$S_{sallowT} = 0.000 \frac{\text{lb}}{\text{in}^2}$$

b) Channel**Channel membrane stress :**

$$\sigma_{cm} := \begin{cases} \frac{a_c^2}{(D_c + t_c) \cdot t_c} \cdot P_t & \text{if Config = "a" } \vee \text{ Config = "e" } \vee \text{ Config = "f" } \vee \text{ Config = "A"} \\ 0 \cdot P_t & \end{cases}$$

$$\sigma_{cm} = \begin{pmatrix} 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

Channel bending stress :

$$l_{cmin} := 1.8 \cdot \sqrt{D_c \cdot t_c} \quad l_{cmin} = 0.000 \text{ in}$$

$$\sigma_{cb} := \frac{6}{t_c^2} \cdot k_c \cdot \left[\beta_c \cdot \left(\delta_c \cdot P_t + \frac{a_c^2}{E_c \cdot t_c} P_c^* \right) - \frac{6 \cdot (1 - \nu^{*2})}{E^*} \cdot \left(\frac{a_o^3}{h^3} \right) \cdot \left(1 + \frac{h'_c}{2} \right) \cdot \left[P_e \cdot (Z_v + Z_m \cdot Q_1) + \frac{2}{a_o^2} \cdot Z_m \cdot Q_2 \right] \right]$$

$$\sigma_{cb} = \begin{pmatrix} 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

Total channel stress:

$$\sigma_c := \left(|\sigma_{cm}| + |\sigma_{cb}| \right)$$

$$\sigma_c = \begin{pmatrix} 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

Load cases 1,2,3 :

$$\sigma_{cmaxP} := \max(\sigma_{c_1}, \sigma_{c_2}, \sigma_{c_3})$$

$$\sigma_{cmaxP} = 0.000 \frac{\text{lb}}{\text{in}^2}$$

$$S_{callowP} := 1.5 \cdot S_c$$

$$S_{callowP} = 0.000 \frac{\text{lb}}{\text{in}^2}$$

Load cases 4,5,6 and 7:

$$\sigma_{cmaxT} := \max(\sigma_{c_4}, \sigma_{c_5}, \sigma_{c_6}, \sigma_{c_7})$$

$$\sigma_{cmaxT} = 0.000 \frac{\text{lb}}{\text{in}^2}$$

$$S_{callowT} := S_{PS.c}$$

$$S_{callowT} = 0.000 \frac{\text{lb}}{\text{in}^2}$$

UHX13.7 Simplified Elastic Plastic Procedure :

Design Formulas and calculational procedure for the effect of plasticity at the tubesheet, channel or shell joint.

$$S^*_s := \min\left(S_{ys}, \frac{SPS.s}{2}\right)$$

$$S^*_s = 0.000 \frac{\text{lb}}{\text{in}^2}$$

$$S^*_c := \min\left(S_{yc}, \frac{SPS.c}{2}\right)$$

$$S^*_c = 0.000 \frac{\text{lb}}{\text{in}^2}$$

$$\text{fact}_{sv} := \left(1.4 - \frac{0.4 \cdot |\sigma_{sb}|}{S^*_s}\right) \quad \text{fact}_{sv} = \begin{pmatrix} 1.400 \\ 1.400 \\ 1.400 \\ 1.400 \\ 1.400 \\ 1.400 \\ 1.400 \end{pmatrix}$$

$$\text{fact}_s := \begin{pmatrix} \min(\text{fact}_{sv_1}, 1) \\ \min(\text{fact}_{sv_2}, 1) \\ \min(\text{fact}_{sv_3}, 1) \end{pmatrix} \quad \text{fact}_s = \begin{pmatrix} 1.000 \\ 1.000 \\ 1.000 \end{pmatrix}$$

$$\text{fact}_{cv} := \left(1.4 - \frac{0.4 \cdot |\sigma_{cb}|}{S^*_c}\right) \quad \text{fact}_{cv} = \begin{pmatrix} 1.4 \\ 1.4 \\ 1.4 \\ 1.4 \\ 1.4 \\ 1.4 \\ 1.4 \end{pmatrix}$$

$$\text{fact}_c := \begin{pmatrix} \min(\text{fact}_{cv_1}, 1) \\ \min(\text{fact}_{cv_2}, 1) \\ \min(\text{fact}_{cv_3}, 1) \end{pmatrix} \quad \text{fact}_c = \begin{pmatrix} 1.000 \\ 1.000 \\ 1.000 \end{pmatrix}$$

Each Load Case (1, 2, and 3) must be evaluated because "fact" will be different for each case.

$$E^*_s := E_s \cdot \text{fact}_s \quad E^*_s = \begin{pmatrix} 2.700 \times 10^7 \\ 2.700 \times 10^7 \\ 2.700 \times 10^7 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$E^*_c := E_c \cdot \text{fact}_c \quad E^*_c = \begin{pmatrix} 2.700 \times 10^7 \\ 2.700 \times 10^7 \\ 2.700 \times 10^7 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

From Step 2:

$$kk_s := \beta_s \cdot \frac{E^*_s \cdot t_s^3}{6 \cdot (1 - \nu_s^2)} \quad kk_s = \begin{pmatrix} 0.0000 \\ 0.0000 \\ 0.0000 \end{pmatrix} \text{ lb}$$

$$\lambda\lambda_s := \frac{6 \cdot D_s}{h^3} \cdot kk_s \cdot \left(1 + h'_s + \frac{h'^2_s}{2}\right) \quad \lambda\lambda_s = \begin{pmatrix} 0.0000 \\ 0.0000 \\ 0.0000 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$kk_c := \beta_c \cdot \frac{E^*_c \cdot t_c^3}{6 \cdot (1 - \nu_c^2)} \quad kk_c = \begin{pmatrix} 0.0000 \\ 0.0000 \\ 0.0000 \end{pmatrix} \text{ lb}$$

$$\lambda\lambda_c := \frac{6 \cdot D_c}{h^3} \cdot kk_c \cdot \left(1 + h'_c + \frac{h'^2_c}{2}\right) \quad \lambda\lambda_c = \begin{pmatrix} 0.0000 \\ 0.0000 \\ 0.0000 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

From Step 4:

$$F := \frac{1 - v^*}{E^*} \cdot (\lambda \lambda_s + \lambda \lambda_c + E \cdot \ln(K)) \quad F = \begin{pmatrix} 0.074 \\ 0.074 \\ 0.074 \end{pmatrix}$$

$$\Phi := (1 + v^*) \cdot F \quad \Phi = \begin{pmatrix} 0.0971 \\ 0.0971 \\ 0.0971 \end{pmatrix}$$

$$Q_{1e} := \overrightarrow{\left(\frac{\rho_s - 1 - \Phi \cdot Z_v}{1 + \Phi \cdot Z_m} \right)} \quad Q_{1e} = \begin{pmatrix} 0.020 \\ 0.020 \\ 0.020 \end{pmatrix}$$

$$P_{se} := \begin{pmatrix} P_{s_1} \\ P_{s_2} \\ P_{s_3} \end{pmatrix} \quad P_{te} := \begin{pmatrix} P_{t_1} \\ P_{t_2} \\ P_{t_3} \end{pmatrix}$$

$$P'_e := \begin{pmatrix} P_{e_1} \\ P_{e_2} \\ P_{e_3} \end{pmatrix} \quad P'_e = \begin{pmatrix} -150.000 \\ 250.000 \\ 100.000 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

Recalculate the tubesheet bending stress for load cases 1, 2, and 3

$$Q'_2 := \left[\frac{(\omega^*_s \cdot P_{se} - \omega^*_c \cdot P_{te}) + \frac{\gamma_b}{2 \cdot \pi} \cdot W}{1 + \Phi \cdot Z_m} \right]$$

$$Q'_2 = \begin{pmatrix} -10.170 \\ 16.950 \\ 6.780 \end{pmatrix} \text{ lb}$$

$$Q'_3 := \left(Q_1 + \frac{2 \cdot Q'_2}{P'_e \cdot a_o^2} \right)$$

$$Q'_3 = \begin{pmatrix} 0.0213 \\ 0.0213 \\ 0.0213 \end{pmatrix}$$

$$F'_{m1}(x) := \begin{cases} 0 & \text{if } P'_e = 0 \\ \frac{Q_v(x) + Q'_3 \cdot Q_m(x)}{2} & \text{otherwise} \end{cases}$$

$$F'_{m1j} := |F'_{m1}(x_j)|$$

$$F'_{m2}(x) := \begin{cases} 0 & \text{if } P'_e = 0 \\ \frac{Q_v(x) + Q'_3 \cdot Q_m(x)}{2} & \text{otherwise} \end{cases}$$

$$F'_{m2j} := |F'_{m2}(x_j)|$$

$$F'_{m3}(x) := \begin{cases} 0 & \text{if } P'_e = 0 \\ \frac{Q_v(x) + Q'_3 \cdot Q_m(x)}{2} & \text{otherwise} \end{cases}$$

$$F'_{m3j} := |F'_{m3}(x_j)|$$

$$F'_{m1} := \max(F'_{m1}) \quad F'_{m1} = 0.075$$

$$F'_{m2} := \max(F'_{m2}) \quad F'_{m2} = 0.075$$

$$F'_{m3} := \max(F'_{m3}) \quad F'_{m3} = 0.075$$

$$F'_m := \begin{pmatrix} F'_{m1} \\ F'_{m2} \\ F'_{m3} \end{pmatrix} \quad F'_m = \begin{pmatrix} 0.075 \\ 0.075 \\ 0.075 \end{pmatrix}$$

Tubesheet Bending Stress for the Elastic-Plastic Solution

$$\sigma' := \left[\frac{1.5 \cdot F'_m}{\mu^*} \cdot \left(\frac{2 \cdot a_o}{h - h'_g} \right)^2 \cdot P'_e \right]$$

$$\sigma' = \begin{pmatrix} -9488.014 \\ 15813.357 \\ 6325.343 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$\sigma'_{\max} := \max(|\sigma'_1|, |\sigma'_2|, |\sigma'_3|)$$

$$\sigma'_{\max} = 15813.357 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{\text{allow}} := 1.5 \cdot S$$

$$\sigma_{\text{allow}} = 28500.000 \frac{\text{lb}}{\text{in}^2}$$

if $\sigma'_{\max}$ is less than σ_{allow} , then pressure design for the tubesheet, shell and channel is acceptable.

UHX14.7 Simply Supported Tubesheet Procedure

This procedure applies only when the TS is integral with the shell or channel, i.e. :

- shell of configurations a,b,c
- channel of configuration a,e,f,A

SS = "NO"

Calculation must be performed in 2 phases :

Phase 1) Perform Steps 1 to 11 with **SS="NON"** (normal calculation) with the following modifications in Step 11:

- minimum length requirement l_{smin} of shell band for configurations a,b,c do not apply
- minimum length requirement l_{cmin} of channel band for configuration a do not apply

if $\sigma_s \leq S_{PS,s}$ and $\sigma_c \leq S_{PS,c}$, the shell and/or channel designs are acceptable.

Otherwise increase the thickness of the overstressed components (shell and/or channel) and return to Step 1.

Phase 2) Perform Steps 1 to 7 using **SS="OUI"** (Simply Supported calculation) for loading cases 1,2 and 3 only

If $|\sigma| \leq 1.5S$, the calculation procedure is complete.

Otherwise, increase the assumed tubesheet thickness h and repeat Steps 1 to 7.

Note: If $|\sigma| < 1.5S$, the tubesheet thickness can be optimized to a value h_0 till $|\sigma| = 1.5S$

provided that, for that optimized thickness h_0 , the stresses in the tubesheet, shell and channel, calculated by the normal calculation (see Phase 1), remain respectively below S_{PS} , $S_{PS,s}$, $S_{PS,c}$ for each of the 7 loading cases.

Determination of effective elastic properties :

From figure UHX-11.2 - Polynomial equations for the determination of E*/E and v* for triangular pattern

Equations for E* for triangular pitch

$$E^*TriPitch := \begin{pmatrix} 0.0353 & 1.2502 & -0.0491 & 0.3604 & -0.6100 \\ 0.0135 & 0.9910 & 1.0080 & -1.0498 & 0.0184 \\ 0.0054 & 0.5279 & 3.0461 & -4.3657 & 1.9435 \\ -0.0029 & 0.2126 & 3.9906 & -6.1730 & 3.4307 \end{pmatrix}$$

$$v^*TriPitch := \begin{pmatrix} -0.0958 & 0.6209 & -0.8683 & 2.1099 & -1.6831 \\ 0.8897 & -9.0855 & 36.1435 & -59.5425 & 35.8223 \\ 0.7439 & -4.4989 & 12.5779 & -14.2092 & 5.7822 \\ 0.9100 & -4.8901 & 12.4325 & -12.7039 & 4.4298 \\ 0.9923 & -4.8759 & 12.3572 & -13.7214 & 5.7629 \\ 0.9966 & -4.1978 & 9.0478 & -7.9955 & 2.2398 \end{pmatrix}$$

From figure UHX-11.3 - Polynomial equations for the determination of E^*/E and v^* for square pattern

$$E^*_{\text{SquPitch}} := \begin{pmatrix} 0.0676 & 1.5756 & -1.2119 & 1.7715 & -1.2628 \\ 0.0250 & 1.9251 & -3.5230 & 6.9830 & -5.0017 \\ 0.0394 & 1.3024 & -1.1041 & 2.8714 & -2.3994 \\ 0.0372 & 1.0314 & -0.6402 & 2.6201 & -2.1929 \end{pmatrix} v^*_{\text{SquPitch}} := \begin{pmatrix} -0.0791 & 0.6008 & -0.3468 & 0.4858 & -0.3606 \\ 0.3345 & -2.8420 & 10.9709 & -15.8994 & 8.3516 \\ 0.4296 & -2.6350 & 8.6864 & -11.5227 & 5.8544 \\ 0.3636 & -0.8057 & 2.0463 & -2.2902 & 1.1862 \\ 0.3527 & -0.2842 & 0.4354 & -0.0901 & -0.1590 \\ 0.3341 & 0.1260 & -0.6920 & 0.6877 & -0.0600 \end{pmatrix}$$

$$\text{Elay} := \text{si}(\text{Layout} > 0, E^*_{\text{SquPitch}}, E^*_{\text{TriPitch}})$$

$$E^*_1 := (\text{Elay}_{1,1} + \text{Elay}_{1,2} \cdot \mu^* + \text{Elay}_{1,3} \cdot \mu^{*2} + \text{Elay}_{1,4} \cdot \mu^{*3} + \text{Elay}_{1,5} \cdot \mu^{*4}) \cdot E \quad E^*_1 = 1.395 \times 10^7 \frac{\text{lb}}{\text{in}^2}$$

$$E^*_2 := (\text{Elay}_{2,1} + \text{Elay}_{2,2} \cdot \mu^* + \text{Elay}_{2,3} \cdot \mu^{*2} + \text{Elay}_{2,4} \cdot \mu^{*3} + \text{Elay}_{2,5} \cdot \mu^{*4}) \cdot E \quad E^*_2 = 1.310 \times 10^7 \frac{\text{lb}}{\text{in}^2}$$

$$E^*_3 := (\text{Elay}_{3,1} + \text{Elay}_{3,2} \cdot \mu^* + \text{Elay}_{3,3} \cdot \mu^{*2} + \text{Elay}_{3,4} \cdot \mu^{*3} + \text{Elay}_{3,5} \cdot \mu^{*4}) \cdot E \quad E^*_3 = 1.226 \times 10^7 \frac{\text{lb}}{\text{in}^2}$$

$$E^*_4 := (\text{Elay}_{4,1} + \text{Elay}_{4,2} \cdot \mu^* + \text{Elay}_{4,3} \cdot \mu^{*2} + \text{Elay}_{4,4} \cdot \mu^{*3} + \text{Elay}_{4,5} \cdot \mu^{*4}) \cdot E \quad E^*_4 = 1.063 \times 10^7 \frac{\text{lb}}{\text{in}^2}$$

For h/p lower than 0.1, use $h/p = 0.1$. For values of h/p higher than 2, use $h/p = 2.0$. Otherwise perform a linear interpolation:

$$\text{hpratio} := \begin{pmatrix} 0.1 \\ 0.25 \\ 0.5 \\ 2.0 \end{pmatrix} \quad \text{interE}^* := \begin{pmatrix} E^*_1 \\ E^*_2 \\ E^*_3 \\ E^*_4 \end{pmatrix} \quad \frac{h}{p} = 1.75$$

$$E^* := \text{si}\left(\frac{h}{p} > 2, E^*_4, \text{si}\left(\frac{h}{p} \leq 0.10, E^*_1, \text{interplin}\left(\text{hpratio}, \text{interE}^*, \frac{h}{p}\right)\right)\right) \quad E^* = 1.091 \times 10^7 \frac{\text{lb}}{\text{in}^2} \quad \frac{E^*}{E} = 0.4039$$

$$v_{\text{lay}} := \text{si}(\text{Layout} > 0, v^*_{\text{SquPitch}}, v^*_{\text{TriPitch}})$$

$$v^*_1 := v_{\text{lay}_{1,1}} + v_{\text{lay}_{1,2}} \cdot \mu^* + v_{\text{lay}_{1,3}} \cdot \mu^{*2} + v_{\text{lay}_{1,4}} \cdot \mu^{*3} + v_{\text{lay}_{1,5}} \cdot \mu^{*4} \quad v^*_1 = 0.098$$

$$v^*_2 := v_{\text{lay}_{2,1}} + v_{\text{lay}_{2,2}} \cdot \mu^* + v_{\text{lay}_{2,3}} \cdot \mu^{*2} + v_{\text{lay}_{2,4}} \cdot \mu^{*3} + v_{\text{lay}_{2,5}} \cdot \mu^{*4} \quad v^*_2 = 0.138$$

$$v^*_3 := v_{\text{lay}_{3,1}} + v_{\text{lay}_{3,2}} \cdot \mu^* + v_{\text{lay}_{3,3}} \cdot \mu^{*2} + v_{\text{lay}_{3,4}} \cdot \mu^{*3} + v_{\text{lay}_{3,5}} \cdot \mu^{*4} \quad v^*_3 = 0.192$$

$$v^*_4 := v_{\text{lay}_{4,1}} + v_{\text{lay}_{4,2}} \cdot \mu^* + v_{\text{lay}_{4,3}} \cdot \mu^{*2} + v_{\text{lay}_{4,4}} \cdot \mu^{*3} + v_{\text{lay}_{4,5}} \cdot \mu^{*4} \quad v^*_4 = 0.242$$

$$v^*_5 := v_{\text{lay}_{5,1}} + v_{\text{lay}_{5,2}} \cdot \mu^* + v_{\text{lay}_{5,3}} \cdot \mu^{*2} + v_{\text{lay}_{5,4}} \cdot \mu^{*3} + v_{\text{lay}_{5,5}} \cdot \mu^{*4} \quad v^*_5 = 0.290$$

$$v^*_6 := v_{\text{lay}_{6,1}} + v_{\text{lay}_{6,2}} \cdot \mu^* + v_{\text{lay}_{6,3}} \cdot \mu^{*2} + v_{\text{lay}_{6,4}} \cdot \mu^{*3} + v_{\text{lay}_{6,5}} \cdot \mu^{*4} \quad v^*_6 = 0.314$$

$$\text{interv} := \begin{pmatrix} v^*_1 \\ v^*_2 \\ v^*_3 \\ v^*_4 \\ v^*_5 \\ v^*_6 \end{pmatrix} \quad \text{hpratio} := \begin{pmatrix} 0.1 \\ 0.15 \\ 0.25 \\ 0.50 \\ 1.00 \\ 2.00 \end{pmatrix} \quad \text{For } h/p \text{ lower than 0.1, use } h/p = 0.1. \text{ For values of } h/p \text{ higher than 2, use } h/p = 2.0. \text{ Otherwise perform a linear interpolation:}$$

$$v^* := \text{si}\left(\frac{h}{p} > 2, v^*_6, \text{si}\left(\frac{h}{p} \leq 0.10, v^*_1, \text{interplin}\left(\text{hpratio}, \text{interv}, \frac{h}{p}\right)\right)\right)$$

$$\frac{h}{p} = 1.750 \quad v^* = 0.308$$

$\frac{1}{2}$

$$A_p := \pi r_o^2 \quad A_p = 490.874 \text{ in}^2$$

$$C_p := 2\pi r_o \quad C_p = 78.540 \text{ in}$$

ANNEX Y—UHX13 – EXAMPLE 3 (2010 EDITION) WITH GENERAL EQUATIONS

FIXED Tubesheet Rules according to UHX-13 (July 2010 Edition)

highlighted in Yellow :most important data and results

Case UHX-20.2.3

Geometric Data (from Fig.UHX-13.1)

$h := 1.375 \cdot \text{in}$ **Tubesheet thickness**

Layout := 0 **For triangular pitch : "Layout"=0**
For square pitch : "Layout"=1

Config := "a"

Configuration types: a, b, c, d

"a" for shell/channel integral both sides

"b" for shell integral channel gask - TS extended

"c" for shell integral channel gask - TS not extended

"d" for gasketed both sides

$r_o := 20.125 \cdot \text{in}$ **Radius to outer tube**

$A := 43.125 \cdot \text{in}$ **Outside Diameter of Tubesheet**

$C := 0 \cdot \text{in}$ **Bolt Circle Diameter**

$A_L := 0 \cdot \text{in}^2$ **Total Untubed Lanes Area**

$h_g := 0 \cdot \text{in}$ **Groove depth**

$c_t := 0 \cdot \text{in}$ **Tubesheet Corr. Allow. (Tubeside)**

$c_s := 0 \cdot \text{in}$ **Tubesheet Corr. Allow. (Shellside)**

$p := 1.25 \cdot \text{in}$ **Tube Pitch**

$N_t := 955$ **Number of Tubes**

$d_t := 1 \cdot \text{in}$ **Tube OutsideDiameter**

$t_t := 0.049 \cdot \text{in}$ **Tube Thickness**

$L_t := 240 \cdot \text{in}$ **Tube Length**

$\rho := 0.909$ **Tube expansion depth ratio**

$l_{tx} := 1.25 \cdot \text{in}$ **Length of Expanded Portion of Tube**

$kl := 48 \cdot \text{in}$ **k=0.6 for spans between Tubesheets**
k=0.8 for spans between TS/support plate
k=1.0 for spans between support plates
l = unsupported tube span

set Farenheit temp.
degF := R

SS := "NO"

SS="YES" for Simply Supported TS calculation (for config. a,b,c only)

$D_s := 42 \cdot \text{in}$

Shell ID

$t_s := 0.5625 \cdot \text{in}$ **Shell Thickness away from TS**

$G_s := 0 \cdot \text{in}$ **Shell Gasket Diameter**

$t_{s1} := 0.5625 \cdot \text{in}$ **Shell Thickness near TS**

$C_s := 0 \cdot \text{in}$ **Shell Corrosion Allowance**

$E_{sw} := 1$ **Shell joint efficiency**
Thick Shell Length @ one end

$l_1 := 0 \cdot \text{in}$ **Thick Shell Length @ other end**

$D_c := 42.125 \cdot \text{in}$

Channel ID

$t_c := 0.375 \cdot \text{in}$ **Channel Thickness**

$G_c := 0 \cdot \text{in}$ **Channel Gasket Diameter**

$G_1 := 0 \cdot \text{in}$ **Contact mid-point TS/Flange**

$C_c := 0 \cdot \text{in}$ **Channel Corrosion Allowance**

CHAN := "CYL" **"CYL" for Cylindrical Channel**
"HEMI" for Hemispherical Channel
(for config. a only)

Corroded thicknesses:

$h := h - c_s - c_t$ **Tubesheet thickness**

$t_s := t_s - C_s$ **Shell Thickness away from TS**

$t_{s1} := t_{s1} - C_s$ **Shell Thickness near TS**

$t_c := t_c - C_c$ **Channel Thickness**

$h = 1.375 \text{ in}$

$t_s = 0.563 \text{ in}$

$t_{s1} = 0.563 \text{ in}$

$t_c = 0.375 \text{ in}$

Corroded length:

$L_t := L_t - 2c_t$ $L_t = 240.000 \text{ in}$

Corroded diameters:

$D_s := D_s + 2 \cdot C_s$ **Shell ID** $D_s = 42.000 \text{ in}$

$D_c := D_c + 2 \cdot C_c$ **Channel ID** $D_c = 42.125 \text{ in}$

Expansion Joint data

$$K_J := 10^{20} \cdot \frac{\text{lb}}{\text{in}}$$

Bellows Axial Rigidity

$$D_J := 0 \cdot \text{in}$$

Inside Diameter of Bellows Convolution**Loading conditions**

$$W := 0.0 \cdot \text{lb}$$

Channel flange design bolt load

$$P_{s0} := 325 \cdot \frac{\text{lb}}{\text{in}^2}$$

Shell Design Pressure

$$P_{t0} := 200 \cdot \frac{\text{lb}}{\text{in}^2}$$

Tube Design Pressure**Design Pressures for the Seven Load Cases**

$$P_t := \begin{pmatrix} P_{t0} \\ 0 \cdot \frac{\text{lb}}{\text{in}^2} \\ P_{t0} \\ 0 \cdot \frac{\text{lb}}{\text{in}^2} \\ P_{t0} \\ 0 \cdot \frac{\text{lb}}{\text{in}^2} \\ P_{t0} \end{pmatrix} \quad P_t = \begin{pmatrix} 200.000 \\ 0.000 \\ 200.000 \\ 0.000 \\ 200.000 \\ 0.000 \\ 200.000 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$P_s := \begin{pmatrix} 0 \cdot \frac{\text{lb}}{\text{in}^2} \\ P_{s0} \\ P_{s0} \\ 0 \cdot \frac{\text{lb}}{\text{in}^2} \\ 0 \cdot \frac{\text{lb}}{\text{in}^2} \\ P_{s0} \\ P_{s0} \end{pmatrix} \quad P_s = \begin{pmatrix} 0.000 \\ 325.000 \\ 325.000 \\ 0.000 \\ 0.000 \\ 325.000 \\ 325.000 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

Temperatures $T := 400 \cdot \text{degF}$ **Tubesheet design temp.** $T' := 70 \cdot \text{degF}$ **Tubesheet temp.@ rim** $T_t := 300 \cdot \text{degF}$ **Tube design temp.** $T_s := 400 \cdot \text{degF}$ **Shell design temp.** $T'_s := 70 \cdot \text{degF}$ **Shell temp. @ Tubesheet** $T_c := 300 \cdot \text{degF}$ **Channel design temp.** $T'_c := 70 \cdot \text{degF}$ **Channel temp.@ tubesheet** $T_{tm} := 113 \cdot \text{degF}$ **Tube mean temp. along L** $T_{sm} := 151 \cdot \text{degF}$ **Shell mean temp. along L** $T_a := 70 \cdot \text{degF}$ **Ambient temperature**

Material Data**Tubesheet Material is SA-XXX**

$$S := 15800 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{TS allowable stress @ T}$$

$$S_y := 17500 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{TS yield stress @ T}$$

$$E := 26.4 \cdot 10^6 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{TS elastic modulus @ T}$$

$$S_{PS} := 47400 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{TS allowable P+S stress @ T}$$

$$\nu := 0.3 \quad \text{TS Poisson's ratio}$$

$$\alpha' := 8.984 \cdot 10^{-6} \cdot \frac{\text{in}}{\text{in} \cdot \text{degF}} \quad \text{TS coeff. expansion @ rim}$$

Tube Material is SA-XXX

$$S_t := 16706 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{Tube allowable stress @ } T_t$$

$$S_{tT} := 15765 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{Tube allowable stress @ } T_t$$

$$E_t := 27.0 \cdot 10^6 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{Tube elastic modulus @ } T_t$$

$$E_{tT} := 26.4 \cdot 10^6 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{Tube elastic modulus @ } T_t$$

$$\nu_t := 0.3 \quad \text{Tube Poisson's ratio}$$

$$\alpha_{tm} := 8.652 \cdot 10^{-6} \cdot \frac{\text{in}}{\text{in} \cdot \text{degF}} \quad \text{Tube coeff. expansion @ } T_{t,m}$$

$$S_{yt} := 19200 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{Tube yield stress @ } T_t$$

Shell Material is SA-XXX

$$S_s := 15800 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{Shell allow. stress @ } T_s$$

$$S_{ys} := 17500 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{Shell yield stress @ } T_s$$

$$E_s := 26.4 \cdot 10^6 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{Thin Shell elast mod. @ } T_s$$

$$S_{PS,s} := 47400 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{Shell allowable P+S stress @ } T_s$$

$$\nu_s := 0.3 \quad \text{Shell Poisson's ratio}$$

$$\alpha_{sm} := 8.802 \cdot 10^{-6} \cdot \frac{\text{in}}{\text{in} \cdot \text{degF}} \quad \text{Shell coeff expansion @ } T_{s,m}$$

Shell Band Material is SA-XXX

$$S_{s1} := 15800 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{Shell allow. stress @ } T'_s$$

$$S_{ys1} := 17500 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{Shell yield stress @ } T'_s$$

$$E_{s1} := 26.4 \cdot 10^6 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{Thick Shell elast mod. @ } T'_s$$

$$\alpha_{sm1} := 8.802 \cdot 10^{-6} \cdot \frac{\text{in}}{\text{in} \cdot \text{degF}} \quad \text{Thick Shell coeff. expansion @ } T'_s$$

$$\nu_{s1} := 0.3 \quad \text{Thin Shell Poisson's ratio}$$

$$\alpha'_s := 8.984 \cdot 10^{-6} \cdot \frac{\text{in}}{\text{in} \cdot \text{degF}} \quad \text{Shell coeff. expansion @ rim}$$

Channel Material is SA-XXX

$$S_c := 20000 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{Channel allow. stress @ } T'_c$$

$$S_{yc} := 33600 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{Channel yield stress @ } T'_c$$

$$E_c := 28.3 \cdot 10^6 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{Channel elast. modulus @ } T'_c$$

$$S_{PS,c} := 67200 \cdot \frac{\text{lb}}{\text{in}^2} \quad \text{Channel allowable P+S stress @ } T'_c$$

$$\nu_c := 0.3 \quad \text{Channel Poisson's ratio}$$

$$\alpha'_c := 6.666 \cdot 10^{-6} \cdot \frac{\text{in}}{\text{in} \cdot \text{degF}} \quad \text{Channel coeff. expansion @ } T'_c$$

Start of Calculations

$$L := L_t - 2 \cdot h \quad L = 237.250 \text{ in}$$

L = effective length of shell/tubes

$$l_{s1} := L - l_1 - l'_1 \quad l_{s1} = 237.250 \text{ in}$$

$$h'_g := \max(h_g - c_t, 0) \quad h'_g = 0.000 \text{ in}$$

$$\rho := \frac{l_{tx}}{h} \quad \rho = 0.909$$

$$d^* := \max \left[d_t - 2 \cdot t_t \cdot \left(\frac{E_{tT}}{E} \right) \cdot \left(\frac{S_{tT}}{S} \right) \cdot \rho, (d_t - 2t_t) \right] \quad d^* = 0.911 \text{ in}$$

UHX-13.5.1 Step 1 Determine D_o , μ , μ^* and h'_g from UHX-11.5.1 :

$$D_o := 2 \cdot r_o + d_t \quad D_o = 41.250 \text{ in}$$

$$a_o := \frac{D_o}{2} \quad a_o = 20.625 \text{ in}$$

$$p^* := \frac{p}{\sqrt{1 - \frac{4 \cdot \min(A_L, 4D_o \cdot p)}{\pi \cdot D_o^2}}} \quad p^* = 1.250 \text{ in}$$

$$\mu^* := \frac{p^* - d^*}{p^*} \quad \mu^* = 0.271$$

$$\mu := \frac{p - d_t}{p} \quad \mu = 0.200$$

Shell Radial Dim.

$$a_s := \begin{cases} \frac{G_s}{2} & \text{if Config = "d"} \\ \frac{D_s}{2} & \text{otherwise} \end{cases} \quad a_s = 21.000 \text{ in}$$

Chan Rad. Dim.

$$a_c := \begin{cases} \frac{D_c}{2} & \text{if Config = "a"} \\ \frac{G_c}{2} & \text{otherwise} \end{cases} \quad a_c = 21.063 \text{ in}$$

$$\rho_s := \frac{a_s}{a_o} \quad \rho_s = 1.018$$

$$\rho_c := \frac{a_c}{a_o} \quad \rho_c = 1.021$$

$$x_t := 1 - N_t \cdot \left(\frac{d_t - 2 \cdot t_t}{2 \cdot a_o} \right)^2 \quad x_t = 0.543$$

$$x_s := 1 - N_t \cdot \left(\frac{d_t}{2 \cdot a_o} \right)^2 \quad x_s = 0.439$$

UHX-13.5.2 Step 2 Calculate the shell axial stiffness K_s , tube axial stiffness K_t and stiffness factors K_{st} and J

$$K_s^* := \frac{\pi \cdot (D_s + t_s)}{\left(\frac{L - l_1 - l'_1}{E_s \cdot t_s} \right) + \frac{l_1 + l'_1}{E_{s1} \cdot t_{s1}}} \quad K_s^* = 8.3695 \times 10^6 \frac{\text{lb}}{\text{in}} \quad K_t := \frac{\pi \cdot t_t \cdot (d_t - t_t) \cdot E_t}{L} \quad K_t = 1.666 \times 10^4 \frac{\text{lb}}{\text{in}}$$

$$K_{st} := \frac{K_s^*}{N_t \cdot K_t} \quad K_{st} = 0.526 \quad J := \frac{1}{1 + \frac{K_s^*}{K_j}} \quad J = 1$$

Calculate shell and channel parameters:Use SS=YES for Simply Supported calculation in a 2nd step (see UHX-13.9)

SS = "NO"

$$\beta_s := \begin{cases} \frac{[12 \cdot (1 - \nu_s^2)]^{0.25}}{[(D_s + t_{s1}) \cdot t_{s1}]^{0.5}} & \text{if } SS = \text{"NO"} \wedge (\text{Config} = \text{"a"} \vee \text{Config} = \text{"b"} \vee \text{Config} = \text{"c"}) \\ 0 \cdot \frac{1}{\text{in}} & \text{otherwise} \end{cases} \quad \beta_s = 0.372 \frac{1}{\text{in}}$$

$$\beta_c := \begin{cases} \frac{[12 \cdot (1 - \nu_c^2)]^{0.25}}{[(D_c + t_c) \cdot t_c]^{0.5}} & \text{if } SS = \text{"NO"} \wedge \text{Config} = \text{"a"} \\ 0 \cdot \frac{1}{\text{in}} & \text{otherwise} \end{cases} \quad \beta_c = 0.455 \frac{1}{\text{in}}$$

$$k_s := \beta_s \cdot \frac{E_{s1} \cdot t_{s1}^3}{6 \cdot (1 - \nu_s^2)} \quad k_s = 3.1971 \times 10^5 \text{ lb} \quad k_c := \beta_c \cdot \frac{E_c \cdot t_c^3}{6 \cdot (1 - \nu_c^2)} \quad k_c = 1.245 \times 10^5 \text{ lb}$$

$$h'_s := h \cdot \beta_s \quad h'_s = 0.511 \quad h'_c := h \cdot \beta_c \quad h'_c = 0.626$$

$$\lambda_s := \frac{6 \cdot D_s}{h^3} \cdot k_s \cdot \left(1 + h'_s + \frac{h'^2_s}{2} \right) \quad \lambda_s = 5.0868 \times 10^7 \frac{\text{lb}}{\text{in}^2} \quad \lambda_c := \frac{6 \cdot D_c}{h^3} \cdot k_c \cdot \left(1 + h'_c + \frac{h'^2_c}{2} \right) \quad \lambda_c = 2.2049 \times 10^7 \frac{\text{lb}}{\text{in}^2}$$

$$\delta_s := \begin{cases} \frac{D_s^2}{4E_{s1} \cdot t_{s1}} \left(1 - \frac{\nu_s}{2} \right) & \text{if } SS = \text{"NO"} \wedge (\text{Config} = \text{"a"} \vee \text{Config} = \text{"b"} \vee \text{Config} = \text{"c"}) \\ 0 \cdot (\text{in}^3 \text{ lb}^{-1}) & \text{otherwise} \end{cases} \quad V := \frac{\lambda_s + \lambda_c}{E} \quad V = 2.762$$

$$\delta_s = 2.5242 \times 10^{-5} \text{ in}^3 \text{ lb}^{-1}$$

$$\text{CHAN} = \text{"CYL"} \quad \text{SS} = \text{"NO"} \quad \delta_c := \begin{cases} \frac{D_c^2}{4E_c \cdot t_c} \left(1 - \frac{\nu_c}{2} \right) & \text{if } SS = \text{"NO"} \wedge \text{CHAN} = \text{"CYL"} \wedge \text{Config} = \text{"a"} \\ \frac{D_c^2}{4E_c \cdot t_c} \left(\frac{1 - \nu_c}{2} \right) & \text{if } SS = \text{"NO"} \wedge \text{CHAN} = \text{"HEMI"} \wedge \text{Config} = \text{"a"} \\ 0 \cdot (\text{in}^3 \text{ lb}^{-1}) & \text{otherwise} \end{cases} \quad \delta_c = 3.5532 \times 10^{-5} \text{ in}^3 \text{ lb}^{-1}$$

UHX-13.5.3 Step 3 Calculate h/p. If ρ changes, recalculate d^* and μ^* from UHX-11.5.1. Determine E^*/E and ν^* relative to h/p from UHX-11.5.2. Calculate X_a

$$\frac{h}{p} = 1.100 \quad \mu^* = 0.271 \quad \frac{E^*}{E} = 0.275 \quad E^* = 7259362.809 \frac{\text{lb}}{\text{in}^2} \quad \nu^* = 0.340 \quad (\text{From right pages above})$$

$$X_a := \left[24 \cdot (1 - \nu^{*2}) \cdot N_t \cdot \frac{E_t \cdot t_t \cdot (d_t - t_t) \cdot a_o^2}{E^* \cdot L \cdot h^3} \right]^{.25} \quad X_a = 7.016$$

UHX-13.5.4 Step 4 Calculate diameter ratio K and coefficient F :

$$K := \frac{A}{D_o} \quad K = 1.045$$

$$F := \frac{1 - \nu^*}{E^*} \cdot (\lambda_s + \lambda_c + E \cdot \ln(K)) \quad F = 6.732$$

$$\Phi := (1 + \nu^*) \cdot F \quad \Phi = 9.024$$

Calculate Z_a, Z_d, Z_v, Z_m $N := \text{arrondi} \left(4 + \frac{X_a}{2} \right) + 1 \quad N = 9.000$

$$\text{ber}_x(x) := \sum_{n=0}^N \left[(-1)^n \cdot \frac{\left(\frac{x}{2}\right)^{4 \cdot n}}{((2 \cdot n)!)^2} \right] \quad \text{ber} := \text{ber}_x(X_a) \quad \text{ber} = -3.432$$

$$\text{bei}_x(x) := \sum_{n=1}^N \left[(-1)^{n-1} \cdot \frac{\left(\frac{x}{2}\right)^{4 \cdot n - 2}}{((2 \cdot n - 1)!)^2} \right] \quad \text{bei} := \text{bei}_x(X_a) \quad \text{bei} = -21.490$$

$$\text{ber}'_x(x) := \sum_{n=1}^N \frac{(-1)^n \cdot (2 \cdot n) \cdot \left(\frac{x}{2}\right)^{4 \cdot n - 1}}{((2 \cdot n)!)^2} \quad \text{ber}' := \text{ber}'_x(X_a) \quad \text{ber}' = 13.069$$

$$\text{bei}'_x(x) := \sum_{n=1}^N \frac{(-1)^{n-1} \cdot (2 \cdot n - 1) \cdot \left(\frac{x}{2}\right)^{4 \cdot n - 3}}{((2 \cdot n - 1)!)^2} \quad \text{bei}' := \text{bei}'_x(X_a) \quad \text{bei}' = -16.061$$

$$\Psi_{1x}(x) := \text{bei}_x(x) + \left(\frac{1 - \nu^*}{x} \right) \cdot \text{ber}'_x(x) \quad \Psi_1 := \Psi_{1x}(X_a) \quad \Psi_1 = -20.261$$

$$\Psi_{2x}(x) := \text{ber}_x(x) - \frac{1 - \nu^*}{x} \cdot \text{bei}'_x(x) \quad \Psi_2 := \Psi_{2x}(X_a) \quad \Psi_2 = -1.921$$

$$Z_a := \text{bei}' \cdot \Psi_2 - \text{ber}' \cdot \Psi_1 \quad Z_a = 295.647$$

$$Z_m := \frac{\text{ber}^2 + \text{bei}^2}{X_a \cdot Z_a} \quad Z_m = 0.207$$

$$Z_v := \frac{\text{ber}' \cdot \Psi_2 + \text{bei}' \cdot \Psi_1}{X_a^2 \cdot Z_a} \quad Z_v = 0.021$$

$$Z_d := \frac{\text{ber} \cdot \Psi_2 + \text{bei} \cdot \Psi_1}{X_a^3 \cdot Z_a} \quad Z_d = 0.004$$

$$Z_w := \frac{\text{ber}' \cdot \text{ber} + \text{bei}' \cdot \text{bei}}{X_a^2 \cdot Z_a} \quad Z_w = 0.021$$

Calculate Q_1 , Q_{z1} , Q_{z2} , and U :

$$Q_1 := \frac{\rho_s - 1 - \Phi \cdot Z_v}{1 + \Phi \cdot Z_m} \quad Q_1 = -0.059$$

$$Q_{z1} := \frac{(Z_d + Q_1 \cdot Z_w) \cdot X_a^4}{2} \quad Q_{z1} = 3.778$$

$$Q_{z2} := \frac{(Z_v + Q_1 \cdot Z_m) \cdot X_a^4}{2} \quad Q_{z2} = 10.312$$

$$U := \frac{[Z_w + (\rho_s - 1) \cdot Z_m] \cdot X_a^4}{1 + \Phi \cdot Z_m} \quad U = 20.625$$

UHX-13.5.5 Step 5

UHX-13.5.5(a) Calculate γ :

$$\gamma_p^* := (T_{tm} - T_a) \cdot \alpha_{tm} \cdot L - (T_{sm} - T_a) \cdot [\alpha_{sm} \cdot (L - l_1 - l'_1) + \alpha_{sm1} \cdot (l_1 + l'_1)]$$

$$\gamma^* := \begin{pmatrix} 0 \cdot \text{in} \\ 0 \cdot \text{in} \\ 0 \cdot \text{in} \\ \gamma_p^* \\ \gamma_p^* \\ \gamma_p^* \\ \gamma_p^* \end{pmatrix} \quad \gamma^* = \begin{pmatrix} 0.000 \\ 0.000 \\ 0.000 \\ -0.081 \\ -0.081 \\ -0.081 \\ -0.081 \end{pmatrix} \text{ in}$$

UHX-13.5.5(b) Calculate parameters ω_s , ω_s^* , ω_c , ω_c^* , and :

$$\omega_s := \rho_s \cdot k_s \cdot \beta_s \cdot \delta_s \cdot (1 + h'_s)$$

$$\omega_c := \rho_c \cdot k_c \cdot \beta_c \cdot \delta_c \cdot (1 + h'_c)$$

$$\omega_s = 4.612 \text{ in}^2$$

$$\omega_c = 3.344 \text{ in}^2$$

$$\omega_s^* := a_o^2 \cdot \frac{(\rho_s^2 - 1) \cdot (\rho_s - 1)}{4} - \omega_s$$

$$\omega_c^* := a_o^2 \cdot \left[\frac{(\rho_c^2 + 1) \cdot (\rho_c - 1)}{4} - \frac{\rho_s - 1}{2} \right] - \omega_c$$

$$\omega_s^* = -4.541 \text{ in}^2$$

$$\omega_c^* = -2.603 \text{ in}^2$$

UHX-13.5.5(c) Calculate γ_b :

$$\gamma_b := \begin{cases} 0 & \text{if Config} = \text{"a"} \\ \frac{G_c - C}{D_o} & \text{if Config} = \text{"b"} \\ \frac{G_c - G_1}{D_o} & \text{if Config} = \text{"c"} \\ \frac{G_c - G_s}{D_o} & \text{if Config} = \text{"d"} \end{cases} \quad \gamma_b = 0.000$$

From UHX-13.8 , Calculate P^*_s and P^*_c : This section is only required for **configurations a,b,or c** when the conditions of §13.8.1 a to d for Radial Differential Thermal Expansion are met.

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$$T_r := \begin{cases} \frac{T' + T'_s + T'_c}{3} & \text{if Config = "a"} \\ \frac{T' + T'_s}{2} & \text{if Config = "b" } \vee \text{ Config = "c"} \\ T_a & \text{if Config = "d"} \end{cases} \quad T_r = 70.000 \text{ degF}$$

$$T^*_s := \frac{T'_s + T_r}{2} \quad T^*_s = 70.000 \text{ degF} \quad T^*_c := \frac{T'_c + T_r}{2} \quad T^*_c = 70.000 \text{ degF}$$

$$P^*_{sp1} := \frac{E_{s1} \cdot t_{s1}}{a_s} \cdot [\alpha'_s \cdot (T^*_s - T_a) - \alpha' \cdot (T_r - T_a)] \quad P^*_{cp1} := \frac{E_c \cdot t_c}{a_c} \cdot [\alpha'_c \cdot (T^*_c - T_a) - \alpha' \cdot (T_r - T_a)]$$

$$P^*_{sp1} = 0.000 \frac{\text{lb}}{\text{in}^2} \quad P^*_{cp1} = 0.000 \frac{\text{lb}}{\text{in}^2}$$

$$P^*_s := \begin{cases} 0 \cdot \frac{\text{lb}}{\text{in}^2} & \text{if Config = "d"} \\ P^*_{sp1} & \text{otherwise} \end{cases} \quad P^*_c := \begin{cases} 0 \cdot \frac{\text{lb}}{\text{in}^2} & \text{if Config = "d" } \vee \text{ Config = "b" } \vee \text{ Config = "c"} \\ P^*_{cp1} & \text{otherwise} \end{cases}$$

$$P^*_s := \begin{pmatrix} 0 \cdot \frac{\text{lb}}{\text{in}^2} \\ 0 \cdot \frac{\text{lb}}{\text{in}^2} \\ 0 \cdot \frac{\text{lb}}{\text{in}^2} \\ P^*_{sp} \\ P^*_{sp} \\ P^*_{sp} \\ P^*_{sp} \end{pmatrix} \quad P^*_s = \begin{pmatrix} 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$P^*_c := \begin{pmatrix} 0 \cdot \frac{\text{lb}}{\text{in}^2} \\ 0 \cdot \frac{\text{lb}}{\text{in}^2} \\ 0 \cdot \frac{\text{lb}}{\text{in}^2} \\ P^*_{cp} \\ P^*_{cp} \\ P^*_{cp} \\ P^*_{cp} \end{pmatrix} \quad P^*_c = \begin{pmatrix} 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

UHX-13.5.6 Step 6 For each loading case, calculate P'_s and P'_t , P_γ , P_γ^* , P_w , P_{rim} , and effective pressure P_e :

$$P'_s := \left[x_s + 2 \cdot (1 - x_s) \cdot v_t + \frac{2 \cdot v_s}{K_{st}} \cdot \left(\frac{D_s}{D_o} \right)^2 - \frac{\rho_s^2 - 1}{J \cdot K_{st}} - \frac{1 - J}{2J \cdot K_{st}} \cdot \frac{D_J^2 - D_s^2}{D_o^2} \right] \cdot P_s$$

$$P'_t := \left[x_t + 2 \cdot (1 - x_t) \cdot v_t + \frac{1}{J \cdot K_{st}} \right] \cdot P_t$$

$$P'_s = \begin{pmatrix} 0.000 \\ 613.671 \\ 613.671 \\ 0.000 \\ 0.000 \\ 613.671 \\ 613.671 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$P'_t = \begin{pmatrix} 543.676 \\ 0.000 \\ 543.676 \\ 0.000 \\ 543.676 \\ 0.000 \\ 543.676 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$P_\gamma := \frac{N_t \cdot K_t}{\pi \cdot a_o^2} \cdot \gamma^*$$

$$P_\omega := \frac{U}{a_o^2} \cdot (\omega_s \cdot P'_s - \omega_c \cdot P'_c)$$

$$P_w := -\frac{U}{a_o^2} \cdot \frac{\gamma_b}{2\pi} W$$

$$P_{rim} := -\frac{U}{a_o^2} \cdot (\omega^*_s \cdot P_s - \omega^*_c \cdot P_t)$$

$$P_\gamma = \begin{pmatrix} 0.000 \\ 0.000 \\ 0.000 \\ -962.977 \\ -962.977 \\ -962.977 \\ -962.977 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$P_\omega = \begin{pmatrix} 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$P_w = 0.000 \frac{\text{lb}}{\text{in}^2}$$

$$P_{rim} = \begin{pmatrix} -25.239 \\ 71.560 \\ 46.321 \\ 0.000 \\ -25.239 \\ 71.560 \\ 46.321 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

Calculate P_e for Loading Cases 1 through 7:

$$P_e := \frac{J \cdot K_{st}}{1 + J \cdot K_{st} \cdot [Q_{z1} + (\rho_s - 1) \cdot Q_{z2}]} \cdot (P'_s - P'_t + P_\gamma + P_\omega + P_w + P_{rim})$$

$$P_e = \begin{pmatrix} -96.973 \\ 116.799 \\ 19.826 \\ -164.142 \\ -261.115 \\ -47.343 \\ -144.316 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

UHX-13.5.7 Step 7 Determine Q_2 and Q_3 for Loading Cases 1 through 7:

$$Q_2 := \frac{\left[(\omega_s^* \cdot P_s - \omega_c^* \cdot P_t) - (\omega_s \cdot P_s^* - \omega_c \cdot P_c^*) \right] + \frac{\gamma_b}{2 \cdot \pi} \cdot W}{1 + \Phi \cdot Z_m} \rightarrow Q_2 = \begin{pmatrix} 181.671 \\ -515.100 \\ -333.429 \\ 0.000 \\ 181.671 \\ -515.100 \\ -333.429 \end{pmatrix} \text{ lb}$$

$$Q_{3_1} := \begin{cases} 0 & \text{if } P_{e_1} = 0 \\ \left(Q_1 + \frac{2 \cdot Q_{2_1}}{P_{e_1} \cdot a_0^2} \right) & \text{otherwise} \end{cases} \quad Q_{3_2} := \begin{cases} 0 & \text{if } P_{e_2} = 0 \\ \left(Q_1 + \frac{2 \cdot Q_{2_2}}{P_{e_2} \cdot a_0^2} \right) & \text{otherwise} \end{cases} \quad Q_{3_3} := \begin{cases} 0 & \text{if } P_{e_3} = 0 \\ \left(Q_1 + \frac{2 \cdot Q_{2_3}}{P_{e_3} \cdot a_0^2} \right) & \text{otherwise} \end{cases}$$

$$Q_{3_4} := \begin{cases} 0 & \text{if } P_{e_4} = 0 \\ \left(Q_1 + \frac{2 \cdot Q_{2_4}}{P_{e_4} \cdot a_0^2} \right) & \text{otherwise} \end{cases} \quad Q_{3_5} := \begin{cases} 0 & \text{if } P_{e_5} = 0 \\ \left(Q_1 + \frac{2 \cdot Q_{2_5}}{P_{e_5} \cdot a_0^2} \right) & \text{otherwise} \end{cases} \quad Q_{3_6} := \begin{cases} 0 & \text{if } P_{e_6} = 0 \\ \left(Q_1 + \frac{2 \cdot Q_{2_6}}{P_{e_6} \cdot a_0^2} \right) & \text{otherwise} \end{cases}$$

$$Q_{3_7} := \begin{cases} 0 & \text{if } P_{e_7} = 0 \\ \left(Q_1 + \frac{2 \cdot Q_{2_7}}{P_{e_7} \cdot a_0^2} \right) & \text{otherwise} \end{cases}$$

$$Q_1 = -0.059 \quad Q_2 = \begin{pmatrix} 181.671 \\ -515.100 \\ -333.429 \\ 0.000 \\ 181.671 \\ -515.100 \\ -333.429 \end{pmatrix} \text{ lb} \quad Q_3 = \begin{pmatrix} -0.067 \\ -0.079 \\ -0.138 \\ -0.059 \\ -0.062 \\ -0.007 \\ -0.048 \end{pmatrix}$$

Determine Coefficient F_m for Load Cases 1 through 7:

$$\text{numpoints} := 20 \quad j := 1 \dots \text{numpoints} \quad X_j := \frac{j \cdot 1}{\text{numpoints}} \quad x := X \cdot X_a$$

$$Q_V(x) := \frac{\overrightarrow{\Psi_{2X}(x) \cdot \Psi_1 - \Psi_{1X}(x) \cdot \Psi_2}}{X_a \cdot Z_a}$$

$$Q_m(x) := \left(\frac{\overrightarrow{\Psi_{2X}(x) \cdot \text{bei}' - \Psi_{1X}(x) \cdot \text{ber}'}}{Z_a} \right)$$

$$F_{mx1}(x) := \begin{cases} 0 & \text{if } P_{e_1} = 0 \\ \frac{Q_V(x) + Q_{3_1} \cdot Q_m(x)}{2} & \text{otherwise} \end{cases}$$

$$F_{mx4}(x) := \begin{cases} 0 & \text{if } P_{e_4} = 0 \\ \frac{Q_V(x) + Q_{3_4} \cdot Q_m(x)}{2} & \text{otherwise} \end{cases}$$

$$F_{mx2}(x) := \begin{cases} 0 & \text{if } P_{e_2} = 0 \\ \frac{Q_V(x) + Q_{3_2} \cdot Q_m(x)}{2} & \text{otherwise} \end{cases}$$

$$F_{mx5}(x) := \begin{cases} 0 & \text{if } P_{e_5} = 0 \\ \frac{Q_V(x) + Q_{3_5} \cdot Q_m(x)}{2} & \text{otherwise} \end{cases}$$

$$F_{mx3}(x) := \begin{cases} 0 & \text{if } P_{e_3} = 0 \\ \frac{Q_V(x) + Q_{3_3} \cdot Q_m(x)}{2} & \text{otherwise} \end{cases}$$

$$F_{mx6}(x) := \begin{cases} 0 & \text{if } P_{e_6} = 0 \\ \frac{Q_V(x) + Q_{3_6} \cdot Q_m(x)}{2} & \text{otherwise} \end{cases}$$

$$F_{mx7}(x) := \begin{cases} 0 & \text{if } P_{e_7} = 0 \\ \frac{Q_V(x) + Q_{3_7} \cdot Q_m(x)}{2} & \text{otherwise} \end{cases}$$

$$F_{mx}(x) := \begin{pmatrix} F_{mx1}(x) \\ F_{mx2}(x) \\ F_{mx3}(x) \\ F_{mx4}(x) \\ F_{mx5}(x) \\ F_{mx6}(x) \\ F_{mx7}(x) \end{pmatrix} \quad \text{abs}F_{mx}(x) := \begin{pmatrix} |F_{mx1}(x)| \\ |F_{mx2}(x)| \\ |F_{mx3}(x)| \\ |F_{mx4}(x)| \\ |F_{mx5}(x)| \\ |F_{mx6}(x)| \\ |F_{mx7}(x)| \end{pmatrix}$$

$$F_m := \begin{pmatrix} \max(\text{abs}F_{mx}(x)_1) \\ \max(\text{abs}F_{mx}(x)_2) \\ \max(\text{abs}F_{mx}(x)_3) \\ \max(\text{abs}F_{mx}(x)_4) \\ \max(\text{abs}F_{mx}(x)_5) \\ \max(\text{abs}F_{mx}(x)_6) \\ \max(\text{abs}F_{mx}(x)_7) \end{pmatrix} \quad F_m = \begin{pmatrix} 0.034 \\ 0.040 \\ 0.069 \\ 0.029 \\ 0.031 \\ 0.032 \\ 0.024 \end{pmatrix}$$

$$F_{mx}(X_a) = \begin{pmatrix} -0.034 \\ -0.040 \\ -0.069 \\ -0.029 \\ -0.031 \\ -0.004 \\ -0.024 \end{pmatrix}$$

Calculate the Maximum Tubesheet Bending Stress

$$h_{\min} := \begin{pmatrix} h - h'_g \\ h - h'_g \\ h - h'_g \\ h \\ h \\ h \\ h \end{pmatrix} \quad h_{\min} = \begin{pmatrix} 1.375 \\ 1.375 \\ 1.375 \\ 1.375 \\ 1.375 \\ 1.375 \\ 1.375 \end{pmatrix} \text{ in}$$

$$\sigma_1 := \begin{cases} \left[\frac{6 \cdot Q_{2_1}}{\mu^* \cdot (h_{\min_1})^2} \right] & \text{if } P_{e_1} = 0 \\ \left[\frac{1.5 \cdot F_{m_1} \cdot \left(\frac{2 \cdot a_0}{h_{\min_1}} \right)^2 \cdot P_{e_1}}{\mu^*} \right] & \text{otherwise} \end{cases} \quad \sigma_1 = -16286.538 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_2 := \begin{cases} \left[\frac{6 \cdot Q_{2_2}}{\mu^* \cdot (h_{\min_2})^2} \right] & \text{if } P_{e_2} = 0 \\ \left[\frac{1.5 \cdot F_{m_2} \cdot \left(\frac{2 \cdot a_0}{h_{\min_2}} \right)^2 \cdot P_{e_2}}{\mu^*} \right] & \text{otherwise} \end{cases} \quad \sigma_2 = 23084.641 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_3 := \begin{cases} \left[\frac{6 \cdot Q_{2_3}}{\mu^* \cdot (h_{\min_3})^2} \right] & \text{if } P_{e_3} = 0 \\ \left[\frac{1.5 \cdot F_{m_3} \cdot \left(\frac{2 \cdot a_0}{h_{\min_3}} \right)^2 \cdot P_{e_3}}{\mu^*} \right] & \text{otherwise} \end{cases} \quad \sigma_3 = 6798.103 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_4 := \begin{cases} \left[\frac{6 \cdot Q_{2_4}}{\mu^* \cdot (h_{\min_4})^2} \right] & \text{if } P_{e_4} = 0 \\ \left[\frac{1.5 \cdot F_{m_4} \cdot \left(\frac{2 \cdot a_0}{h_{\min_4}} \right)^2 \cdot P_{e_4}}{\mu^*} \right] & \text{otherwise} \end{cases} \quad \sigma_4 = -23967.865 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_5 := \begin{cases} \left[\frac{6 \cdot Q_{2_5}}{\mu^* \cdot (h_{\min_5})^2} \right] & \text{if } P_{e_5} = 0 \\ \left[\frac{1.5 \cdot F_{m_5} \cdot \left(\frac{2 \cdot a_0}{h_{\min_5}} \right)^2 \cdot P_{e_5}}{\mu^*} \right] & \text{otherwise} \end{cases} \quad \sigma_5 = -40254.403 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_6 := \begin{cases} \left[\frac{6 \cdot Q_{2_6}}{\mu^* \cdot (h_{\min_6})^2} \right] & \text{if } P_{e_6} = 0 \\ \left[\frac{1.5 \cdot F_{m_6} \cdot \left(\frac{2 \cdot a_0}{h_{\min_6}} \right)^2 \cdot P_{e_6}}{\mu^*} \right] & \text{otherwise} \end{cases} \quad \sigma_6 = -7493.784 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_7 := \begin{cases} \left[\frac{6 \cdot Q_{2_7}}{\mu^* \cdot (h_{\min_7})^2} \right] & \text{if } P_{e_7} = 0 \\ \left[\frac{1.5 \cdot F_{m_7} \cdot \left(\frac{2 \cdot a_0}{h_{\min_7}} \right)^2 \cdot P_{e_7}}{\mu^*} \right] & \text{otherwise} \end{cases} \quad \sigma_7 = -17169.762 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma = \begin{pmatrix} -16286.538 \\ 23084.641 \\ 6798.103 \\ -23967.865 \\ -40254.403 \\ -7493.784 \\ -17169.762 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

- For Pressure only Loading Cases 1, 2, and 3:

$$\sigma_{\max P} := \max(|\sigma_1|, |\sigma_2|, |\sigma_3|) \quad \sigma_{\max P} = 23084.641 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{\text{allow}P} := 1.5 \cdot S \quad \sigma_{\text{allow}P} = 23700.000 \frac{\text{lb}}{\text{in}^2}$$

-For Pressure-Thermal Loading Cases 4, 5, 6 and 7:

$$\sigma_{\max T} := \max(|\sigma_4|, |\sigma_5|, |\sigma_6|, |\sigma_7|) \quad \sigma_{\max T} = 40254.403 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{\text{allow}T} := SPS \quad \sigma_{\text{allow}T} = 47400.000 \frac{\text{lb}}{\text{in}^2}$$

Calculate the minimum required thickness of the TS flanged extension for config. b:

$$h_r := \begin{cases} \sqrt{\frac{1.9W}{S \cdot G_c} \cdot \frac{C - G_c}{2}} & \text{if Config} = "b" \quad h_r = 0.000 \text{ in} \\ 0 \cdot \text{in} & \text{otherwise} \end{cases}$$

UHX-13.5.8 Step 8 Calculate the maximum tubesheet shear stress :

$$\tau := \overrightarrow{\left[\left(\frac{1}{2 \cdot \mu} \right) \cdot \left(\frac{D_o}{2 \cdot h} \right) \cdot P_e \right]} \quad \tau = \begin{pmatrix} -3636.487 \\ 4379.976 \\ 743.489 \\ -6155.322 \\ -9791.809 \\ -1775.346 \\ -5411.833 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$\tau_{\max} := \max(|\tau|) \quad \tau_{\max} = 9791.809 \frac{\text{lb}}{\text{in}^2}$$

$$\tau_{\text{allow}} := 0.8 \cdot S \quad \tau_{\text{allow}} = 12640.000 \frac{\text{lb}}{\text{in}^2}$$

If $|P_s - P_t| > 3.2S\mu h/D_o$, the TEMA formula may be used, provided the data A_p and C_p have been defined.

$$P_s - P_t = \begin{pmatrix} -200.000 \\ 325.000 \\ 125.000 \\ 0.000 \\ -200.000 \\ 325.000 \\ 125.000 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$3.2 \cdot S \cdot \mu \cdot \frac{h}{D_o} = 337.067 \frac{\text{lb}}{\text{in}^2}$$

$$A_p := 1272.4 \cdot \text{in}^2 \quad \text{Total area enclosed by } C_p$$

$$C_p := 126.4 \cdot \text{in} \quad \text{Perimeter of the tube layout measured stepwise in increment of one tube pitch from the center-to-center of the outermost tubes (see Fig. UHX-12.2)}$$

$$\tau := \left(\frac{1}{\mu} \right) \cdot \frac{1}{h} \left(\frac{A_p}{C_p} \right) \cdot \overrightarrow{P_e} \quad \tau = \begin{pmatrix} 3549.725 \\ 4275.475 \\ 725.750 \\ 6008.463 \\ 9558.188 \\ 1732.988 \\ 5282.713 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

UHX-13.5.9 Step 9 Determine the minimum and maximum stresses in the tubes for Load Cases 1 to 7

$$Z_{dx}(x) := \frac{\overrightarrow{\Psi_2 \cdot \text{ber}_x(x) + \Psi_1 \cdot \text{bei}_x(x)}}{X_a^3 \cdot Z_a}$$

$$Z_{wx}(x) := \frac{\overrightarrow{\text{ber}' \cdot \text{ber}_x(x) + \text{bei}' \cdot \text{bei}_x(x)}}{X_a^2 \cdot Z_a}$$

Step a) :Determine Coefficients $F_{t,min}$ and $F_{t,max}$ for Load Cases 1 to 7:

$$\begin{aligned}
 F_{tx1}(x) &:= \begin{cases} Z_{wx}(x) \cdot \frac{X_a^4}{2} & \text{if } P_{e_1} = 0 \\ (Z_{dx}(x) + Q_{3_1} \cdot Z_{wx}(x)) \cdot \frac{X_a^4}{2} & \text{otherwise} \end{cases} & F_{tx4}(x) &:= \begin{cases} Z_{wx}(x) \cdot \frac{X_a^4}{2} & \text{if } P_{e_4} = 0 \\ (Z_{dx}(x) + Q_{3_4} \cdot Z_{wx}(x)) \cdot \frac{X_a^4}{2} & \text{otherwise} \end{cases} \\
 F_{tx2}(x) &:= \begin{cases} Z_{wx}(x) \cdot \frac{X_a^4}{2} & \text{if } P_{e_2} = 0 \\ (Z_{dx}(x) + Q_{3_2} \cdot Z_{wx}(x)) \cdot \frac{X_a^4}{2} & \text{otherwise} \end{cases} & F_{tx5}(x) &:= \begin{cases} Z_{wx}(x) \cdot \frac{X_a^4}{2} & \text{if } P_{e_5} = 0 \\ (Z_{dx}(x) + Q_{3_5} \cdot Z_{wx}(x)) \cdot \frac{X_a^4}{2} & \text{otherwise} \end{cases} \\
 F_{tx3}(x) &:= \begin{cases} Z_{wx}(x) \cdot \frac{X_a^4}{2} & \text{if } P_{e_3} = 0 \\ (Z_{dx}(x) + Q_{3_3} \cdot Z_{wx}(x)) \cdot \frac{X_a^4}{2} & \text{otherwise} \end{cases} & F_{tx6}(x) &:= \begin{cases} Z_{wx}(x) \cdot \frac{X_a^4}{2} & \text{if } P_{e_6} = 0 \\ (Z_{dx}(x) + Q_{3_6} \cdot Z_{wx}(x)) \cdot \frac{X_a^4}{2} & \text{otherwise} \end{cases} \\
 & & F_{tx7}(x) &:= \begin{cases} Z_{wx}(x) \cdot \frac{X_a^4}{2} & \text{if } P_{e_7} = 0 \\ (Z_{dx}(x) + Q_{3_7} \cdot Z_{wx}(x)) \cdot \frac{X_a^4}{2} & \text{otherwise} \end{cases}
 \end{aligned}$$

$$\begin{aligned}
 F_{tx}(x) &:= \begin{pmatrix} F_{tx1}(x) \\ F_{tx2}(x) \\ F_{tx3}(x) \\ F_{tx4}(x) \\ F_{tx5}(x) \\ F_{tx6}(x) \\ F_{tx7}(x) \end{pmatrix} & F_{tmin} &:= \begin{pmatrix} \min(F_{tx}(x)_1) \\ \min(F_{tx}(x)_2) \\ \min(F_{tx}(x)_3) \\ \min(F_{tx}(x)_4) \\ \min(F_{tx}(x)_5) \\ \min(F_{tx}(x)_6) \\ \min(F_{tx}(x)_7) \end{pmatrix} & F_{tmax} &:= \begin{pmatrix} \max(F_{tx}(x)_1) \\ \max(F_{tx}(x)_2) \\ \max(F_{tx}(x)_3) \\ \max(F_{tx}(x)_4) \\ \max(F_{tx}(x)_5) \\ \max(F_{tx}(x)_6) \\ \max(F_{tx}(x)_7) \end{pmatrix} & F_q &:= \frac{(Z_d + Q_3 \cdot Z_v) \cdot X_a^4}{2} \\
 F_{tx}(X_a) &= \begin{pmatrix} 3.558 \\ 3.260 \\ 1.802 \\ 3.778 \\ 3.696 \\ 5.057 \\ 4.050 \end{pmatrix} & F_{tmin} &= \begin{pmatrix} -0.270 \\ -0.242 \\ -0.190 \\ -0.293 \\ -0.282 \\ -0.489 \\ -0.329 \end{pmatrix} & F_{tmax} &= \begin{pmatrix} 3.558 \\ 3.260 \\ 2.097 \\ 3.778 \\ 3.696 \\ 5.057 \\ 4.050 \end{pmatrix} & F_q &= \begin{pmatrix} 3.558 \\ 3.260 \\ 1.802 \\ 3.778 \\ 3.696 \\ 5.057 \\ 4.050 \end{pmatrix}
 \end{aligned}$$

Step a') : Determine tube stresses σ_{tmin} and σ_{tmax} for Load Cases 1 to 7 (continued)

$$\sigma_{t1_1} := \begin{cases} \left[\frac{1}{x_t - x_s} \cdot \left[\left(P_{s_1} \cdot x_s - P_{t_1} \cdot x_t \right) - \frac{2 \cdot Q_{2_1}}{a_o^2} \cdot F_{tmin_1} \right] \right] & \text{if } P_{e_1} = 0 \\ \left[\frac{1}{x_t - x_s} \cdot \left[\left(P_{s_1} \cdot x_s - P_{t_1} \cdot x_t \right) - P_{e_1} \cdot F_{tmin_1} \right] \right] & \text{otherwise} \end{cases} \quad \sigma_{t1_1} = -1288.774 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{t1_2} := \begin{cases} \left[\frac{1}{x_t - x_s} \cdot \left[\left(P_{s_2} \cdot x_s - P_{t_2} \cdot x_t \right) - \frac{2 \cdot Q_{2_2}}{a_o^2} \cdot F_{tmin_2} \right] \right] & \text{if } P_{e_2} = 0 \\ \left[\frac{1}{x_t - x_s} \cdot \left[\left(P_{s_2} \cdot x_s - P_{t_2} \cdot x_t \right) - P_{e_2} \cdot F_{tmin_2} \right] \right] & \text{otherwise} \end{cases} \quad \sigma_{t1_2} = 1633.646 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{t1_3} := \begin{cases} \left[\frac{1}{x_t - x_s} \cdot \left[\left(P_{s_3} \cdot x_s - P_{t_3} \cdot x_t \right) - \frac{2 \cdot Q_{2_3}}{a_o^2} \cdot F_{tmin_3} \right] \right] & \text{if } P_{e_3} = 0 \\ \left[\frac{1}{x_t - x_s} \cdot \left[\left(P_{s_3} \cdot x_s - P_{t_3} \cdot x_t \right) - P_{e_3} \cdot F_{tmin_3} \right] \right] & \text{otherwise} \end{cases} \quad \sigma_{t1_3} = 360.211 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{t1_4} := \begin{cases} \left[\frac{1}{x_t - x_s} \cdot \left[\left(P_{s_4} \cdot x_s - P_{t_4} \cdot x_t \right) - \frac{2 \cdot Q_{2_4}}{a_o^2} \cdot F_{tmin_4} \right] \right] & \text{if } P_{e_4} = 0 \\ \left[\frac{1}{x_t - x_s} \cdot \left[\left(P_{s_4} \cdot x_s - P_{t_4} \cdot x_t \right) - P_{e_4} \cdot F_{tmin_4} \right] \right] & \text{otherwise} \end{cases} \quad \sigma_{t1_4} = -459.772 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{t1_5} := \begin{cases} \left[\frac{1}{x_t - x_s} \cdot \left[\left(P_{s_5} \cdot x_s - P_{t_5} \cdot x_t \right) - \frac{2 \cdot Q_{2_5}}{a_o^2} \cdot F_{tmin_5} \right] \right] & \text{if } P_{e_5} = 0 \\ \left[\frac{1}{x_t - x_s} \cdot \left[\left(P_{s_5} \cdot x_s - P_{t_5} \cdot x_t \right) - P_{e_5} \cdot F_{tmin_5} \right] \right] & \text{otherwise} \end{cases} \quad \sigma_{t1_5} = -1743.539 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{t1_6} := \begin{cases} \left[\frac{1}{x_t - x_s} \cdot \left[\left(P_{s_6} \cdot x_s - P_{t_6} \cdot x_t \right) - \frac{2 \cdot Q_{2_6}}{a_o^2} \cdot F_{tmin_6} \right] \right] & \text{if } P_{e_6} = 0 \\ \left[\frac{1}{x_t - x_s} \cdot \left[\left(P_{s_6} \cdot x_s - P_{t_6} \cdot x_t \right) - P_{e_6} \cdot F_{tmin_6} \right] \right] & \text{otherwise} \end{cases} \quad \sigma_{t1_6} = 1141.628 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{t1_7} := \begin{cases} \left[\frac{1}{x_t - x_s} \cdot \left[\left(P_{s_7} \cdot x_s - P_{t_7} \cdot x_t \right) - \frac{2 \cdot Q_{2_7}}{a_o^2} \cdot F_{tmin_7} \right] \right] & \text{if } P_{e_7} = 0 \\ \left[\frac{1}{x_t - x_s} \cdot \left[\left(P_{s_7} \cdot x_s - P_{t_7} \cdot x_t \right) - P_{e_7} \cdot F_{tmin_7} \right] \right] & \text{otherwise} \end{cases} \quad \sigma_{t1_7} = -129.348 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{t2_1} := \begin{cases} \left[\frac{1}{x_t - x_s} \cdot \left[\left(P_{s_1} \cdot x_s - P_{t_1} \cdot x_t \right) - \frac{2 \cdot Q_{2_1}}{a_o^2} \cdot F_{tmax_1} \right] \right] & \text{if } P_{e_1} = 0 \\ \left[\frac{1}{x_t - x_s} \cdot \left[\left(P_{s_1} \cdot x_s - P_{t_1} \cdot x_t \right) - P_{e_1} \cdot F_{tmax_1} \right] \right] & \text{otherwise} \end{cases}$$

$$\sigma_{t2_1} = 2259.342 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{t2_2} := \begin{cases} \left[\frac{1}{x_t - x_s} \cdot \left[\left(P_{s_2} \cdot x_s - P_{t_2} \cdot x_t \right) - \frac{2 \cdot Q_{2_2}}{a_o^2} \cdot F_{tmax_2} \right] \right] & \text{if } P_{e_2} = 0 \\ \left[\frac{1}{x_t - x_s} \cdot \left[\left(P_{s_2} \cdot x_s - P_{t_2} \cdot x_t \right) - P_{e_2} \cdot F_{tmax_2} \right] \right] & \text{otherwise} \end{cases}$$

$$\sigma_{t2_2} = -2276.568 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{t2_3} := \begin{cases} \left[\frac{1}{x_t - x_s} \cdot \left[\left(P_{s_3} \cdot x_s - P_{t_3} \cdot x_t \right) - \frac{2 \cdot Q_{2_3}}{a_o^2} \cdot F_{tmax_3} \right] \right] & \text{if } P_{e_3} = 0 \\ \left[\frac{1}{x_t - x_s} \cdot \left[\left(P_{s_3} \cdot x_s - P_{t_3} \cdot x_t \right) - P_{e_3} \cdot F_{tmax_3} \right] \right] & \text{otherwise} \end{cases}$$

$$\sigma_{t2_3} = -73.144 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{t2_4} := \begin{cases} \left[\frac{1}{x_t - x_s} \cdot \left[\left(P_{s_4} \cdot x_s - P_{t_4} \cdot x_t \right) - \frac{2 \cdot Q_{2_4}}{a_o^2} \cdot F_{tmax_4} \right] \right] & \text{if } P_{e_4} = 0 \\ \left[\frac{1}{x_t - x_s} \cdot \left[\left(P_{s_4} \cdot x_s - P_{t_4} \cdot x_t \right) - P_{e_4} \cdot F_{tmax_4} \right] \right] & \text{otherwise} \end{cases}$$

$$\sigma_{t2_4} = 5928.058 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{t2_5} := \begin{cases} \left[\frac{1}{x_t - x_s} \cdot \left[\left(P_{s_5} \cdot x_s - P_{t_5} \cdot x_t \right) - \frac{2 \cdot Q_{2_5}}{a_o^2} \cdot F_{tmax_5} \right] \right] & \text{if } P_{e_5} = 0 \\ \left[\frac{1}{x_t - x_s} \cdot \left[\left(P_{s_5} \cdot x_s - P_{t_5} \cdot x_t \right) - P_{e_5} \cdot F_{tmax_5} \right] \right] & \text{otherwise} \end{cases}$$

$$\sigma_{t2_5} = 8187.400 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{t2_6} := \begin{cases} \left[\frac{1}{x_t - x_s} \cdot \left[\left(P_{s_6} \cdot x_s - P_{t_6} \cdot x_t \right) - \frac{2 \cdot Q_{2_6}}{a_o^2} \cdot F_{tmax_6} \right] \right] & \text{if } P_{e_6} = 0 \\ \left[\frac{1}{x_t - x_s} \cdot \left[\left(P_{s_6} \cdot x_s - P_{t_6} \cdot x_t \right) - P_{e_6} \cdot F_{tmax_6} \right] \right] & \text{otherwise} \end{cases}$$

$$\sigma_{t2_6} = 3651.490 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{t2_7} := \begin{cases} \left[\frac{1}{x_t - x_s} \cdot \left[\left(P_{s_7} \cdot x_s - P_{t_7} \cdot x_t \right) - \frac{2 \cdot Q_{2_7}}{a_o^2} \cdot F_{tmax_7} \right] \right] & \text{if } P_{e_7} = 0 \\ \left[\frac{1}{x_t - x_s} \cdot \left[\left(P_{s_7} \cdot x_s - P_{t_7} \cdot x_t \right) - P_{e_7} \cdot F_{tmax_7} \right] \right] & \text{otherwise} \end{cases}$$

$$\sigma_{t2_7} = 5910.832 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{to} := \frac{\left((P_s \cdot x_s - P_t \cdot x_t) - P_e \cdot \frac{(Z_d + Q_3 \cdot Z_v) \cdot x_a^4}{2} \right)}{x_t - x_s}$$

$$\sigma_{t1} = \begin{pmatrix} -1288.774 \\ 1633.646 \\ 360.211 \\ -459.772 \\ -1743.539 \\ 1141.628 \\ -129.348 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{to} = \begin{pmatrix} 2259.342 \\ -2276.568 \\ -17.226 \\ 5928.058 \\ 8187.400 \\ 3651.490 \\ 5910.832 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{t2} = \begin{pmatrix} 2259.342 \\ -2276.568 \\ -73.144 \\ 5928.058 \\ 8187.400 \\ 3651.490 \\ 5910.832 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

- For Pressure Loading Only, Cases 1, 2, and 3 :

$$\sigma_{t1P} := \max\left(\left|\sigma_{t1_1}\right|, \left|\sigma_{t1_2}\right|, \left|\sigma_{t1_3}\right|\right)$$

$$\sigma_{t1P} = 1633.646 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{t2P} := \max\left(\left|\sigma_{t2_1}\right|, \left|\sigma_{t2_2}\right|, \left|\sigma_{t2_3}\right|\right)$$

$$\sigma_{t2P} = 2276.568 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{tallowP} := 1.0 \cdot S_t$$

$$\sigma_{tallowP} = 16706.000 \frac{\text{lb}}{\text{in}^2}$$

- For Pressure plus Thermal, Loading Cases 4, 5, 6, and 7 :

$$\sigma_{t1T} := \max\left(\left|\sigma_{t1_4}\right|, \left|\sigma_{t1_5}\right|, \left|\sigma_{t1_6}\right|, \left|\sigma_{t1_7}\right|\right)$$

$$\sigma_{t1T} = 1743.539 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{t2T} := \max\left(\left|\sigma_{t2_4}\right|, \left|\sigma_{t2_5}\right|, \left|\sigma_{t2_6}\right|, \left|\sigma_{t2_7}\right|\right)$$

$$\sigma_{t2T} = 8187.400 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{tallowT} := 2.0 \cdot S_t$$

$$\sigma_{tallowT} = 33412.000 \frac{\text{lb}}{\text{in}^2}$$

For loading cases 1, 2, and 3, if $|\sigma_t| \leq S_t$, and for loading cases 4, 5, 6, and 7, if $|\sigma_t| \leq 2S_t$, the tube design is acceptable . Otherwise return to Step 1.

Step (b) : check the tubes for buckling if $\sigma_{t1} < 0$ or $\sigma_{t2} < 0$

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$$l_t := kl \quad r_t := \frac{\sqrt{d_t^2 + (d_t - 2 \cdot t_t)^2}}{4} \quad r_t = 0.337 \text{ in} \quad C_t := \sqrt{\frac{2 \cdot \pi^2 \cdot E_t}{S_{yt}}} \quad C_t = 166.608 \quad F_t := \frac{l_t}{r_t} \quad F_t = 142.571$$

$$F_{S1} := \begin{cases} 1.25 & \text{if } P_{e1} = 0 \\ \max \left[3.25 - .25 \cdot \left[(Z_d + Q_{31} \cdot Z_v) \cdot X_a^4 \right], 1.25 \right] & \text{otherwise} \end{cases} \quad F_{S1} := \text{si}(F_{S1} > 2, 2, F_{S1}) \quad F_{S1} = 1.471$$

$$F_{S2} := \begin{cases} 1.25 & \text{if } P_{e2} = 0 \\ \max \left[3.25 - .25 \cdot \left[(Z_d + Q_{32} \cdot Z_v) \cdot X_a^4 \right], 1.25 \right] & \text{otherwise} \end{cases} \quad F_{S2} := \text{si}(F_{S2} > 2, 2, F_{S2}) \quad F_{S2} = 1.620$$

$$F_{S3} := \begin{cases} 1.25 & \text{if } P_{e3} = 0 \\ \max \left[3.25 - .25 \cdot \left[(Z_d + Q_{33} \cdot Z_v) \cdot X_a^4 \right], 1.25 \right] & \text{otherwise} \end{cases} \quad F_{S3} := \text{si}(F_{S3} > 2, 2, F_{S3}) \quad F_{S3} = 2.000$$

$$F_{S4} := \begin{cases} 1.25 & \text{if } P_{e4} = 0 \\ \max \left[3.25 - .25 \cdot \left[(Z_d + Q_{34} \cdot Z_v) \cdot X_a^4 \right], 1.25 \right] & \text{otherwise} \end{cases} \quad F_{S4} := \text{si}(F_{S4} > 2, 2, F_{S4}) \quad F_{S4} = 1.361$$

$$F_{S5} := \begin{cases} 1.25 & \text{if } P_{e5} = 0 \\ \max \left[3.25 - .25 \cdot \left[(Z_d + Q_{35} \cdot Z_v) \cdot X_a^4 \right], 1.25 \right] & \text{otherwise} \end{cases} \quad F_{S5} := \text{si}(F_{S5} > 2, 2, F_{S5}) \quad F_{S5} = 1.402$$

$$F_{S6} := \begin{cases} 1.25 & \text{if } P_{e6} = 0 \\ \max \left[3.25 - .25 \cdot \left[(Z_d + Q_{36} \cdot Z_v) \cdot X_a^4 \right], 1.25 \right] & \text{otherwise} \end{cases} \quad F_{S6} := \text{si}(F_{S6} > 2, 2, F_{S6}) \quad F_{S6} = 1.250$$

$$F_{S7} := \begin{cases} F_{S1} & \text{if } P_{e7} = 0 \\ \max \left[3.25 - .25 \cdot \left[(Z_d + Q_{37} \cdot Z_v) \cdot X_a^4 \right], 1.25 \right] & \text{otherwise} \end{cases} \quad F_{S7} := \text{si}(F_{S7} > 2, 2, F_{S7}) \quad F_{S7} = 1.250$$

$$F_S := \begin{pmatrix} F_{S1} \\ F_{S2} \\ F_{S3} \\ F_{S4} \\ F_{S5} \\ F_{S6} \\ F_{S7} \end{pmatrix} \quad F_S = \begin{pmatrix} 1.471 \\ 1.620 \\ 2.000 \\ 1.361 \\ 1.402 \\ 1.250 \\ 1.250 \end{pmatrix} \quad S_{tb} := \text{si} \left[C_t > F_t, \frac{S_{yt}}{F_s} \cdot \left(1 - \frac{F_t}{2 \cdot C_t} \right), \frac{1}{F_s} \cdot \pi^2 \cdot \frac{E_t}{F_t^2} \right]$$

Tube Calculated Stress σ_{t1}, σ_{t2} **Allowable Buckling Stress, S_{tb}**

$$\sigma_{t1} = \begin{pmatrix} -1288.774 \\ 1633.646 \\ 360.211 \\ -459.772 \\ -1743.539 \\ 1141.628 \\ -129.348 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{t2} = \begin{pmatrix} 2259.342 \\ -2276.568 \\ -73.144 \\ 5928.058 \\ 8187.400 \\ 3651.490 \\ 5910.832 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$S_{tb} = \begin{pmatrix} 7467.826 \\ 6780.719 \\ 5492.523 \\ 8071.901 \\ 7836.484 \\ 8788.037 \\ 8788.037 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

UHX 13.5.10 Step 10 Determine the shell membrane stress :

$$t_{s1} := \begin{cases} 0 \text{ in} & \text{if Config} = "d" \\ t_{s1} & \text{otherwise} \end{cases} \quad t_{s1} = 0.563 \text{ in}$$

$$\sigma_{sm} := \frac{a_o^2}{(D_s + t_{s1}) \cdot t_{s1}} \cdot \left[P_e + (\rho_s^2 - 1) \cdot (P_s - P_t) \right] + \frac{a_s^2}{(D_s + t_{s1}) t_{s1}} \cdot P_t$$

$$\sigma_{sm.allowP} := S_s \quad \sigma_{sm.allowP} = 15800.000 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{sm.allowT} := S_{PS.s} \quad \sigma_{sm.allowT} = 47400.000 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{sm} = \begin{pmatrix} 1830.583 \\ 2287.184 \\ 4117.767 \\ -2916.472 \\ -1085.889 \\ -629.288 \\ 1201.295 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

For loading cases 1, 2, 3 if $|\sigma_{s,m}| > S_s E_{s,w}$ and for loading cases 4, 5, 6, 7 if $|\sigma_{s,m}| > 2S_s$, reconsider the shell design and return to Step 1. If $\sigma_{s,m} < 0$, check the shell against buckling in accordance with UG-23(b).

UHX 13.5.11 Step 11 Determine the shell stresses if integral to the tubesheet :

minimum length of shell band for config. a,b or c

$$l_{smin} := 1.8 \cdot \sqrt{D_s \cdot t_{s1}} \quad l_{smin} = 8.749 \text{ in}$$

Determine shell bending stress:

$$\sigma_{sb} := \frac{6}{t_{s1}^2} \cdot k_s \cdot \left[\beta_s \cdot \left(\delta_s \cdot P_s + \frac{a_s^2}{E_s \cdot t_s} \cdot P^*_s \right) + \frac{6 \cdot (1 - \nu^{*2})}{E^*} \cdot \left(\frac{a_o^3}{h^3} \right) \cdot \left(1 + \frac{h'_s}{2} \right) \cdot \left[P_e \cdot (Z_v + Z_m \cdot Q_1) + \frac{2}{a_o^2} \cdot Z_m \cdot Q_2 \right] \right]$$

$$\sigma_s := \left(|\sigma_{sm}| + |\sigma_{sb}| \right) \sigma_s = \begin{pmatrix} 14014.844 \\ 30035.465 \\ 19681.786 \\ 29150.309 \\ 39503.988 \\ 2143.731 \\ 11871.113 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{sb} = \begin{pmatrix} -12184.262 \\ 27748.281 \\ 15564.019 \\ -26233.837 \\ -38418.099 \\ 1514.444 \\ -10669.818 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

Determine total shell stress:**Determine maximum shell stress:**

Load cases 1,2,3 : $\sigma_{smaxP} := \max(\sigma_{s1}, \sigma_{s2}, \sigma_{s3})$

$$\sigma_{smaxP} = 30035.465 \frac{\text{lb}}{\text{in}^2}$$

$$S_{sallowP} := 1.5 \cdot S_{s1}$$

$$S_{sallowP} = 23700.000 \frac{\text{lb}}{\text{in}^2}$$

Load cases 4,5,6,7 : $\sigma_{smaxT} := \max(\sigma_{s4}, \sigma_{s5}, \sigma_{s6}, \sigma_{s7})$

$$\sigma_{smaxT} = 39503.988 \frac{\text{lb}}{\text{in}^2}$$

$$S_{sallowT} := S_{PS.s}$$

$$S_{sallowT} = 47400.000 \frac{\text{lb}}{\text{in}^2}$$

For loading cases 1, 2, 3 if $\sigma_s \leq 1.5S_s$ and for loading cases 4, 5, 6, 7 if $\sigma_s \leq S_{PS.s}$, the shell design is acceptable. Otherwise return to Step 1 or perform an elastic-plastic calculation if conditions of applicability are satisfied.

UHX-13.5.11 Step 11 (cont'd). Determine the channel stresses if integral to the tubesheet**Determine channel membrane stress:**

minimum length of cylindrical channel band for config. a)

$$\sigma_{cm} := \begin{cases} \frac{a_c^2}{(D_c + t_c) \cdot t_c} P_t & \text{if Config = "a"} \\ 0 \cdot \frac{\text{lb}}{\text{in}^2} & \text{otherwise} \end{cases}$$

$$\sigma_{cm} = \begin{pmatrix} 5567.108 \\ 0.000 \\ 5567.108 \\ 0.000 \\ 5567.108 \\ 0.000 \\ 5567.108 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$l_{cmin} := 1.8 \cdot \sqrt{D_s \cdot t_{s1}} \quad l_{cmin} = 8.749 \text{ in}$$

Determine channel bending stress:

$$S_{PS,c} := \begin{cases} 2 \cdot S_{yc} & \text{if } 2 \cdot S_{yc} > 3 \cdot S_c \\ 3 \cdot S_c & \text{otherwise} \end{cases}$$

$$\sigma_{cb} := \frac{6}{t_c^2} \cdot k_c \cdot \left[\beta_c \cdot \left(\delta_c \cdot P_t + \frac{a_c^2}{E_c \cdot t_c} P_c^* \right) - \frac{6 \cdot (1 - \nu^{*2})}{E^*} \cdot \left(\frac{a_o^3}{h^3} \right) \cdot \left(1 + \frac{h'_c}{2} \right) \cdot \left[P_e \cdot (Z_v + Z_m \cdot Q_1) + \frac{2}{a_o^2} \cdot Z_m \cdot Q_2 \right] \right]$$

$$\sigma_c := \overline{(|\sigma_{cm}| + |\sigma_{cb}|)} \quad \sigma_c = \begin{pmatrix} 33913.053 \\ 8492.394 \\ 25420.659 \\ 24033.188 \\ 57946.241 \\ 15540.794 \\ 49453.847 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{cb} = \begin{pmatrix} 28345.945 \\ -8492.394 \\ 19853.551 \\ 24033.188 \\ 52379.133 \\ 15540.794 \\ 43886.739 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

Determine total channel stress:**Maximum channel stress:**

Load cases 1,2,3 : $\sigma_{cmaxP} := \max(\sigma_{c_1}, \sigma_{c_2}, \sigma_{c_3})$

$$\sigma_{cmaxP} = 33913.053 \frac{\text{lb}}{\text{in}^2}$$

$$S_{callowP} := 1.5 \cdot S_c$$

$$S_{callowP} = 30000.000 \frac{\text{lb}}{\text{in}^2}$$

Load cases 4,5,6 and 7: $\sigma_{cmaxT} := \max(\sigma_{c_4}, \sigma_{c_5}, \sigma_{c_6}, \sigma_{c_7})$

$$\sigma_{cmaxT} = 57946.241 \frac{\text{lb}}{\text{in}^2}$$

$$S_{callowT} := S_{PS,c}$$

$$S_{callowT} = 67200.000 \frac{\text{lb}}{\text{in}^2}$$

For loading cases 1, 2, 3 if $\sigma_c \leq 1.5 S_c$ and for loading cases 4, 5, 6, 7 if $\sigma_c \leq S_{PS,c}$
the channel design is acceptable and the calculation procedure is complete.
Otherwise return to Step 1 or perform an elastic-plastic calculation if conditions of applicability are satisfied.

_UHX13.7 Simplified Elastic Plastic Procedure

Calculation procedure for the effect of plasticity at the tubesheet, channel or shell joint.

This procedure applies only to configurations a,b,c and loading cases 1,2,3 in the following conditions:

-for integral shell (config. a,b,c) when $1.5S_s < \sigma_s \leq S_{PS,s}$ for pressure loading cases 1, 2, and 3.

-for integral channel (config. a) when $1.5S_c < \sigma_c \leq S_{PS,c}$ for pressure loading cases 1, 2, and 3.

$$S^*_s := \min\left(S_{ys1}, \frac{S_{PS,s}}{2}\right)$$

$$S^*_s = 17500.000 \frac{\text{lb}}{\text{in}^2}$$

$$S^*_c := \min\left(S_{yc}, \frac{S_{PS,c}}{2}\right)$$

$$S^*_c = 33600.000 \frac{\text{lb}}{\text{in}^2}$$

$$\text{fact}_{sv} := \left(1.4 - \frac{0.4 \cdot |\sigma_{sb}|}{S^*_s}\right) \quad \text{fact}_{sv} = \begin{pmatrix} 1.122 \\ 0.766 \\ 1.044 \\ 0.800 \\ 0.522 \\ 1.365 \\ 1.156 \end{pmatrix}$$

$$\text{fact}_s := \begin{pmatrix} \min(\text{fact}_{sv1}, 1) \\ \min(\text{fact}_{sv2}, 1) \\ \min(\text{fact}_{sv3}, 1) \end{pmatrix} \quad \text{fact}_s = \begin{pmatrix} 1.000 \\ 0.766 \\ 1.000 \end{pmatrix}$$

$$\text{fact}_{cv} := \left(1.4 - \frac{0.4 \cdot |\sigma_{cb}|}{S^*_c}\right) \quad \text{fact}_{cv} = \begin{pmatrix} 1.0625 \\ 1.2989 \\ 1.1636 \\ 1.1139 \\ 0.7764 \\ 1.215 \\ 0.8775 \end{pmatrix}$$

$$\text{fact}_c := \begin{pmatrix} \min(\text{fact}_{cv1}, 1) \\ \min(\text{fact}_{cv2}, 1) \\ \min(\text{fact}_{cv3}, 1) \end{pmatrix} \quad \text{fact}_c = \begin{pmatrix} 1.000 \\ 1.000 \\ 1.000 \end{pmatrix}$$

Calculate reduced values of E_s and E_c for each loading case:

$$E^*_{s1} := E_{s1} \cdot \text{fact}_s \quad E^*_{s1} = \begin{pmatrix} 26400000.000 \\ 20215894.527 \\ 26400000.000 \end{pmatrix} \frac{\text{lb}}{\text{in}^2} \quad E^*_{c1} := E_c \cdot \text{fact}_c \quad E^*_{c1} = \begin{pmatrix} 28300000.000 \\ 28300000.000 \\ 28300000.000 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

From Step 2 , recalculate:

$$\begin{aligned} \text{kk}_s &:= \beta_s \cdot \frac{E^*_{s1} \cdot t_{s1}^3}{6 \cdot (1 - \nu_s^2)} \quad \text{kk}_s = \begin{pmatrix} 3.1971 \times 10^5 \\ 2.4482 \times 10^5 \\ 3.1971 \times 10^5 \end{pmatrix} \text{lb} & \lambda\lambda_s &:= \frac{6 \cdot D_s}{h^3} \cdot \text{kk}_s \cdot \left(1 + h'_s + \frac{h'^2_s}{2}\right) \quad \lambda\lambda_s = \begin{pmatrix} 5.0868 \times 10^7 \\ 3.8952 \times 10^7 \\ 5.0868 \times 10^7 \end{pmatrix} \frac{\text{lb}}{\text{in}^2} \\ \text{kk}_c &:= \beta_c \cdot \frac{E^*_{c1} \cdot t_c^3}{6 \cdot (1 - \nu_c^2)} \quad \text{kk}_c = \begin{pmatrix} 1.2446 \times 10^5 \\ 1.2446 \times 10^5 \\ 1.2446 \times 10^5 \end{pmatrix} \text{lb} & \lambda\lambda_c &:= \frac{6 \cdot D_c}{h^3} \cdot \text{kk}_c \cdot \left(1 + h'_c + \frac{h'^2_c}{2}\right) \quad \lambda\lambda_c = \begin{pmatrix} 2.2049 \times 10^7 \\ 2.2049 \times 10^7 \\ 2.2049 \times 10^7 \end{pmatrix} \frac{\text{lb}}{\text{in}^2} \end{aligned}$$

From Step 4 , recalculate:

$$F := \frac{1 - v^*}{E^*} \cdot (\lambda \lambda_s + \lambda \lambda_c + E \cdot \ln(K)) \quad F = \begin{pmatrix} 6.732 \\ 5.650 \\ 6.732 \end{pmatrix}$$

$$\Phi := (1 + v^*) \cdot F \quad \Phi = \begin{pmatrix} 9.0238 \\ 7.5726 \\ 9.0238 \end{pmatrix}$$

$$Q_{1e} := \begin{pmatrix} \frac{\rho_s - 1 - \Phi \cdot Z_v}{1 + \Phi \cdot Z_m} \end{pmatrix} \quad Q_{1e} = \begin{pmatrix} -0.059 \\ -0.054 \\ -0.059 \end{pmatrix}$$

$$Q_{z1e} := \frac{(Z_d + Q_{1e} \cdot Z_w) \cdot X_a^4}{2} \quad Q_{z1e} = \begin{pmatrix} 3.778 \\ 3.899 \\ 3.778 \end{pmatrix}$$

$$Q_{z2e} := \frac{(Z_v + Q_{1e} \cdot Z_m) \cdot X_a^4}{2} \quad Q_{z2e} = \begin{pmatrix} 10.312 \\ 11.518 \\ 10.312 \end{pmatrix}$$

$$U_e := \frac{[Z_w + (\rho_s - 1) \cdot Z_m] \cdot X_a^4}{1 + \Phi \cdot Z_m}$$

From Step 6 , recalculate P_w , P_{rim} , and P_e :

$$P'_w := -\frac{U_e}{a_o^2} \cdot \frac{\gamma_b}{2\pi} W \quad P'_w = \begin{pmatrix} 0.000 \\ 0.000 \\ 0.000 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$P_{se} := \begin{pmatrix} P_{s_1} \\ P_{s_2} \\ P_{s_3} \end{pmatrix} \quad P_{te} := \begin{pmatrix} P_{t_1} \\ P_{t_2} \\ P_{t_3} \end{pmatrix} \quad P'_{se} := \begin{pmatrix} P'_{s_1} \\ P'_{s_2} \\ P'_{s_3} \end{pmatrix} \quad P'_{te} := \begin{pmatrix} P'_{t_1} \\ P'_{t_2} \\ P'_{t_3} \end{pmatrix}$$

$$P'_{rim} := \begin{pmatrix} -\frac{U_e}{a_o^2} \cdot (\omega^* s \cdot P_{se} - \omega^* c \cdot P_{te}) \end{pmatrix} \quad P'_{rim} = \begin{pmatrix} -25.239 \\ 79.928 \\ 46.321 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$P'_e := \begin{pmatrix} \frac{J \cdot K_{st}}{1 + J \cdot K_{st} [Q_{z1e} + (\rho_s - 1) \cdot Q_{z2e}]} \cdot (P'_{se} - P'_{te} + P'_w + P'_{rim}) \end{pmatrix} \quad P'_e = \begin{pmatrix} -96.973 \\ 115.426 \\ 19.826 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

From Step 7 , recalculate Q_2 , Q_3 , F_m and the tubesheet bending stress for load cases 1, 2, and 3

$$Q'_2 := \frac{\left(\omega^*_s \cdot P_{se} - \omega^*_c \cdot P_{te} \right) + \frac{\gamma_b}{2 \cdot \pi} \cdot W}{1 + \Phi \cdot Z_m} \quad Q'_2 = \begin{pmatrix} 181.671 \\ -575.336 \\ -333.429 \end{pmatrix} \text{ lb}$$

$$Q'_3 := \begin{pmatrix} Q_{1e} + \frac{2 \cdot Q'_2}{P'_e \cdot a_o^2} \end{pmatrix} \quad Q'_3 = \begin{pmatrix} -0.0675 \\ -0.0773 \\ -0.1377 \end{pmatrix}$$

$$F'_{m1}(x) := \begin{cases} 0 & \text{if } P'_{e1} = 0 \\ \frac{Q_v(x) + Q'_{31} \cdot Q_m(x)}{2} & \text{otherwise} \end{cases} \quad F'_{m1j} := |F'_{m1}(x_j)|$$

$$F'_{m2}(x) := \begin{cases} 0 & \text{if } P'_{e2} = 0 \\ \frac{Q_v(x) + Q'_{32} \cdot Q_m(x)}{2} & \text{otherwise} \end{cases} \quad F'_{m2j} := |F'_{m2}(x_j)|$$

$$F'_{m3}(x) := \begin{cases} 0 & \text{if } P'_{e3} = 0 \\ \frac{Q_v(x) + Q'_{33} \cdot Q_m(x)}{2} & \text{otherwise} \end{cases} \quad F'_{m3j} := |F'_{m3}(x_j)|$$

$$F'_{m1} := \max(F'_{m1}) \quad F'_{m1} = 0.034$$

$$F'_{m2} := \max(F'_{m2}) \quad F'_{m2} = 0.039$$

$$F'_{m3} := \max(F'_{m3}) \quad F'_{m3} = 0.069$$

$$F'_m := \begin{pmatrix} F'_{m1} \\ F'_{m2} \\ F'_{m3} \end{pmatrix} \quad F'_m = \begin{pmatrix} 0.0337 \\ 0.0386 \\ 0.0689 \end{pmatrix}$$

Tubesheet Bending Stress for the Elastic-Plastic Solution

$$\sigma' := \left[\frac{1.5 \cdot F'_m \cdot \left(\frac{2 \cdot a_o}{h - h'_g} \right)^2 \cdot P'_e}{\mu^*} \right] \quad \sigma' = \begin{pmatrix} -16286.538 \\ 22204.961 \\ 6798.103 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$\sigma'_{\max} := \max(|\sigma'_1|, |\sigma'_2|, |\sigma'_3|)$$

$$\sigma'_{\max} = 22204.961 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{\text{allow}} := 1.5 \cdot S$$

$$\sigma_{\text{allow}} = 23700.000 \frac{\text{lb}}{\text{in}^2}$$

**if $\sigma' \leq 1.5S$ the design is acceptable and the calculation procedure is complete.
Otherwise, the unit geometry shall be reconsidered.**

UHX13.9 Simply Supported Tubesheet Procedure

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This procedure applies only when the TS is integral with the shell or channel, i.e. :

- shell of configurations a,b,c
- channel of configuration a

SS = "NO"

Calculation must be performed in 2 phases :

Phase 1) Perform Steps 1 to 11 with **SS="NON"** (normal calculation) with the following modifications in Step 11:

- minimum length requirement l_{smin} of shell band for configurations a,b,c do not apply
- minimum length requirement l_{cmin} of channel band for configuration a do not apply

if $\sigma_s \leq S_{PS,s}$ and $\sigma_c \leq S_{PS,c}$, the shell and/or channel designs are acceptable.

Otherwise increase the thickness of the overstressed components (shell and/or channel) and return to Step 1.

Phase 2) Perform Steps 1 to 7 using **SS="OUI"** (Simply Supported calculation) for loading cases 1,2 and 3 only

If $|\sigma| \leq 1.5S$, the calculation procedure is complete.

Otherwise, increase the assumed tubesheet thickness h and repeat Steps 1 to 7.

Note: If $|\sigma| < 1.5S$, the tubesheet thickness can be optimized to a value h_0 till $|\sigma| = 1.5S$

provided that, for that optimized thickness h_0 , the stresses in the tubesheet, shell and channel, calculated by the normal calculation (see Phase 1), remain respectively below S_{PS} , $S_{PS,s}$, $S_{PS,c}$ for each of the 7 loading cases.

Distribution of Moments and Loads in the Tubesheet

The 2nd part of this Mathcad software provides general basic equations without using coefficient Q_3 . They are taken from Item 04-1401 and enable to calculate $q(x)$, $w(x)$, $\theta(x)$, $\sigma(x)$, $\tau(x)$, $\sigma_t(x)$ at any radius of the Tubesheet, depending on Loads V_a and M_a acting at periphery of the perforated tubesheet ($r = a_o$). It provides also the axial displacement and axial load acting in the shell.

Δ_s = elastic stretch of shell

N_s = axial force in shell (per unit of circumference)

N_c = axial force in channel (per unit of circumference)

K_w = elastic foundation modulus of the heat exchanger (based on L)

$k_w = 2K_w$ = elastic foundation modulus of the half H.E.(based on L/2)

D^* = Tubesheet flexural rigidity

YELLOW :most important data and results

Limit Conditions

Results obtained from UHX-13

Perforated Tubesheet

$$K_w := \frac{N_t \cdot K_t}{\pi a_o^2} \quad k_w := 2 \cdot K_w \quad D^* := \frac{E \cdot h^3}{12 \cdot (1 - \nu^2)} \quad k := \left(\frac{k_w}{D^*} \right)^{0.25} \quad \text{numpoints} = 20$$

$$K_w = 11905.554 \frac{\text{lb}}{\text{in}^3} \quad k_w = 23811.107 \frac{\text{lb}}{\text{in}^3} \quad D^* = 1.779 \times 10^6 \text{ lb} \cdot \text{in} \quad k = 0.340 \text{ in}^{-1}$$

$$Z_{mx}(x) := \frac{\text{ber}' \cdot \text{ber}'_x(x) + \text{bei}' \cdot \text{bei}'_x(x)}{X_a \cdot Z_a}$$

$$Z_{mx}(X_a) = 0.207 \quad Z_m = 0.207$$

$$Z_{dx}(x) := \frac{\Psi_2 \cdot \text{ber}_x(x) + \Psi_1 \cdot \text{bei}_x(x)}{X_a^3 \cdot Z_a}$$

$$Z_{dx}(X_a) = 0.004 \quad Z_d = 0.004$$

$$Z_{vx}(x) := \frac{\Psi_2 \cdot \text{ber}'_x(x) + \Psi_1 \cdot \text{bei}'_x(x)}{X_a^2 \cdot Z_a}$$

$$Z_{vx}(X_a) = 0.021 \quad Z_v = 0.021$$

$$Z_{wx}(x) := \frac{\text{ber}' \cdot \text{ber}_x(x) + \text{bei}' \cdot \text{bei}_x(x)}{X_a^2 \cdot Z_a}$$

$$Z_{wx}(X_a) = 0.021 \quad Z_w := Z_{wx}(X_a) \quad Z_w = 0.021$$

Loads V_a and M_a acting at periphery of perforated tubesheet ($r = a_o$)

$$V_a := \left(\frac{a_o}{2} P_e \right)$$

$$M_a := (a_o \cdot V_a \cdot Q_3)$$

$$V_a = \begin{pmatrix} -1000.034 \\ 1204.493 \\ 204.459 \\ -1692.714 \\ -2692.747 \\ -488.220 \\ -1488.254 \end{pmatrix} \frac{\text{lb}}{\text{in}}$$

$$M_a = \begin{pmatrix} 1391.313 \\ -1972.056 \\ -580.743 \\ 2047.507 \\ 3438.820 \\ 75.451 \\ 1466.764 \end{pmatrix} \text{ lb}$$

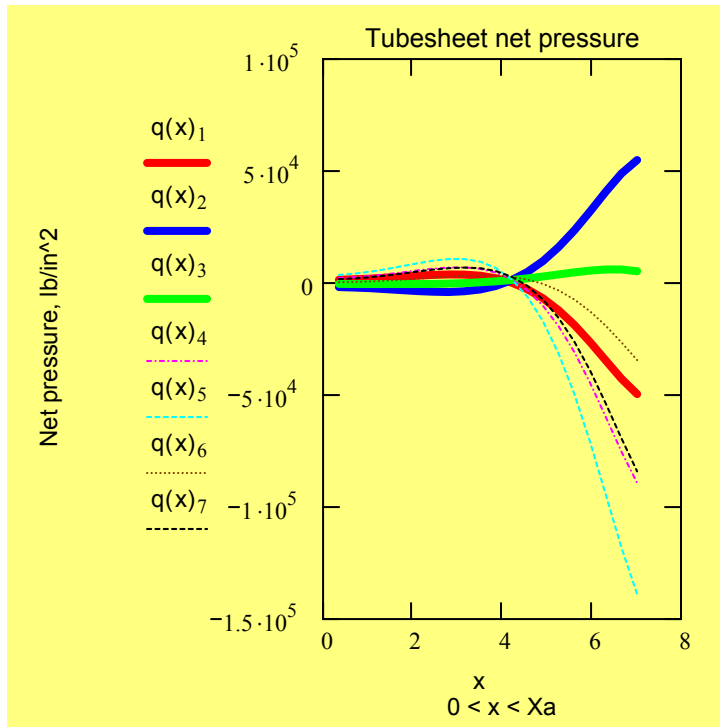
Replace V_a by: $P_e a_o / 2$ and M_a by: $a_o V_a Q_3$ in formulae below to obtain UHX-13 formulae used in previous section

Net Tubesheet Pressure

$$q(x) := \begin{bmatrix} a_o^2 \cdot k^4 \cdot [M_{a_1} \cdot Z_{wx}(x) + (a_o \cdot V_{a_1}) \cdot Z_{dx}(x)] \\ a_o^2 \cdot k^4 \cdot [M_{a_2} \cdot Z_{wx}(x) + (a_o \cdot V_{a_2}) \cdot Z_{dx}(x)] \\ a_o^2 \cdot k^4 \cdot [M_{a_3} \cdot Z_{wx}(x) + (a_o \cdot V_{a_3}) \cdot Z_{dx}(x)] \\ a_o^2 \cdot k^4 \cdot [M_{a_4} \cdot Z_{wx}(x) + (a_o \cdot V_{a_4}) \cdot Z_{dx}(x)] \\ a_o^2 \cdot k^4 \cdot [M_{a_5} \cdot Z_{wx}(x) + (a_o \cdot V_{a_5}) \cdot Z_{dx}(x)] \\ a_o^2 \cdot k^4 \cdot [M_{a_6} \cdot Z_{wx}(x) + (a_o \cdot V_{a_6}) \cdot Z_{dx}(x)] \\ a_o^2 \cdot k^4 \cdot [M_{a_7} \cdot Z_{wx}(x) + (a_o \cdot V_{a_7}) \cdot Z_{dx}(x)] \end{bmatrix}$$

$$q_{\max} := \begin{bmatrix} \max(q(x)_1) \\ \max(q(x)_2) \\ \max(q(x)_3) \\ \max(q(x)_4) \\ \max(q(x)_5) \\ \max(q(x)_6) \\ \max(q(x)_7) \end{bmatrix} \quad q_{\min} := \begin{bmatrix} \min(q(x)_1) \\ \min(q(x)_2) \\ \min(q(x)_3) \\ \min(q(x)_4) \\ \min(q(x)_5) \\ \min(q(x)_6) \\ \min(q(x)_7) \end{bmatrix}$$

$$q_{\max} = \begin{bmatrix} 26.151 \\ 380.756 \\ 41.573 \\ 48.099 \\ 73.726 \\ 23.163 \\ 47.453 \end{bmatrix} \frac{\text{lb}}{\text{in}^2} \quad q_{\min} = \begin{bmatrix} -345.033 \\ -28.309 \\ -3.762 \\ -620.161 \\ -965.194 \\ -239.405 \\ -584.438 \end{bmatrix} \frac{\text{lb}}{\text{in}^2}$$



$$F_q := \frac{(Z_d + Q_3 \cdot Z_v) \cdot X_a^4}{2}$$

$$q(X_a) = \begin{bmatrix} -345.033 \\ 380.756 \\ 35.723 \\ -620.161 \\ -965.194 \\ -239.405 \\ -584.438 \end{bmatrix} \frac{\text{lb}}{\text{in}^2} \quad \overrightarrow{(F_q \cdot P_e)} = \begin{bmatrix} -345.033 \\ 380.756 \\ 35.723 \\ -620.161 \\ -965.194 \\ -239.405 \\ -584.438 \end{bmatrix} \frac{\text{lb}}{\text{in}^2}$$

Deflection $w(x)$ of the perforated tubesheet ($0 \leq x \leq X_a$)

Page 27

$$\rho_1 := Z_w + (\rho_s - 1) \cdot Z_m \quad \rho_1 = 0.024$$

$$\rho_2 := Z_d + (\rho_s - 1) \cdot Z_v \quad \rho_2 = 0.005$$

Determination of Q

$$Q := k_w \cdot \frac{a_o^2}{D^*} \cdot [M_a \cdot \rho_1 + (a_o \cdot V_a) \cdot \rho_2]$$

$$Q = \begin{pmatrix} -359.327 \\ 391.631 \\ 32.304 \\ -650.938 \\ -1010.265 \\ -259.306 \\ -618.633 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$w(x) := \begin{pmatrix} \left(\frac{1}{k_w}\right) \cdot Q_1 - \frac{a_o^2}{D^*} \cdot [M_{a_1} \cdot Z_{wx}(x) + (a_o \cdot V_{a_1}) \cdot Z_{dx}(x)] \\ \left(\frac{1}{k_w}\right) \cdot Q_2 - \frac{a_o^2}{D^*} \cdot [M_{a_2} \cdot Z_{wx}(x) + (a_o \cdot V_{a_2}) \cdot Z_{dx}(x)] \\ \left(\frac{1}{k_w}\right) \cdot Q_3 - \frac{a_o^2}{D^*} \cdot [M_{a_3} \cdot Z_{wx}(x) + (a_o \cdot V_{a_3}) \cdot Z_{dx}(x)] \\ \left(\frac{1}{k_w}\right) \cdot Q_4 - \frac{a_o^2}{D^*} \cdot [M_{a_4} \cdot Z_{wx}(x) + (a_o \cdot V_{a_4}) \cdot Z_{dx}(x)] \\ \left(\frac{1}{k_w}\right) \cdot Q_5 - \frac{a_o^2}{D^*} \cdot [M_{a_5} \cdot Z_{wx}(x) + (a_o \cdot V_{a_5}) \cdot Z_{dx}(x)] \\ \left(\frac{1}{k_w}\right) \cdot Q_6 - \frac{a_o^2}{D^*} \cdot [M_{a_6} \cdot Z_{wx}(x) + (a_o \cdot V_{a_6}) \cdot Z_{dx}(x)] \\ \left(\frac{1}{k_w}\right) \cdot Q_7 - \frac{a_o^2}{D^*} \cdot [M_{a_7} \cdot Z_{wx}(x) + (a_o \cdot V_{a_7}) \cdot Z_{dx}(x)] \end{pmatrix}$$

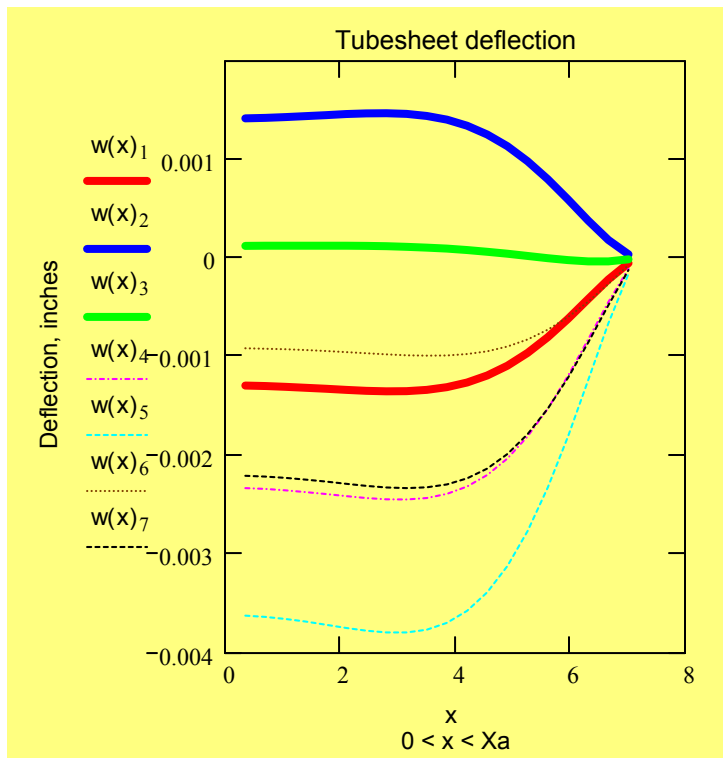
	1
1	0.351
2	0.702
3	1.052
4	1.403
5	1.754
6	2.105
7	2.455
8	2.806
9	3.157
10	3.508
11	3.859
12	4.209
13	4.560
14	4.911
15	5.262
16	5.612
17	5.963
18	6.314
19	6.665
20	7.016

x =

	1
1	$-1.550 \cdot 10^{-2}$
2	$-1.556 \cdot 10^{-2}$
3	$-1.564 \cdot 10^{-2}$
4	$-1.576 \cdot 10^{-2}$
5	$-1.589 \cdot 10^{-2}$
6	$-1.602 \cdot 10^{-2}$
7	$-1.613 \cdot 10^{-2}$
8	$-1.619 \cdot 10^{-2}$
9	$-1.617 \cdot 10^{-2}$
10	$-1.601 \cdot 10^{-2}$
11	$-1.569 \cdot 10^{-2}$
12	$-1.512 \cdot 10^{-2}$
13	$-1.427 \cdot 10^{-2}$
14	$-1.308 \cdot 10^{-2}$
15	$-1.153 \cdot 10^{-2}$
16	$-9.609 \cdot 10^{-3}$
17	$-7.381 \cdot 10^{-3}$
18	$-4.968 \cdot 10^{-3}$
19	$-2.593 \cdot 10^{-3}$
20	$-6.003 \cdot 10^{-4}$

 $w(x)_1 =$

in



$$w(0.0) = \begin{pmatrix} -0.015 \\ 0.017 \\ 0.002 \\ -0.028 \\ -0.043 \\ -0.011 \\ -0.026 \end{pmatrix} \text{ in}$$

$$w(X_a) = \begin{pmatrix} -6.003 \times 10^{-4} \\ 4.567 \times 10^{-4} \\ -1.436 \times 10^{-4} \\ -1.293 \times 10^{-3} \\ -1.893 \times 10^{-3} \\ -8.358 \times 10^{-4} \\ -1.436 \times 10^{-3} \end{pmatrix} \text{ in}$$

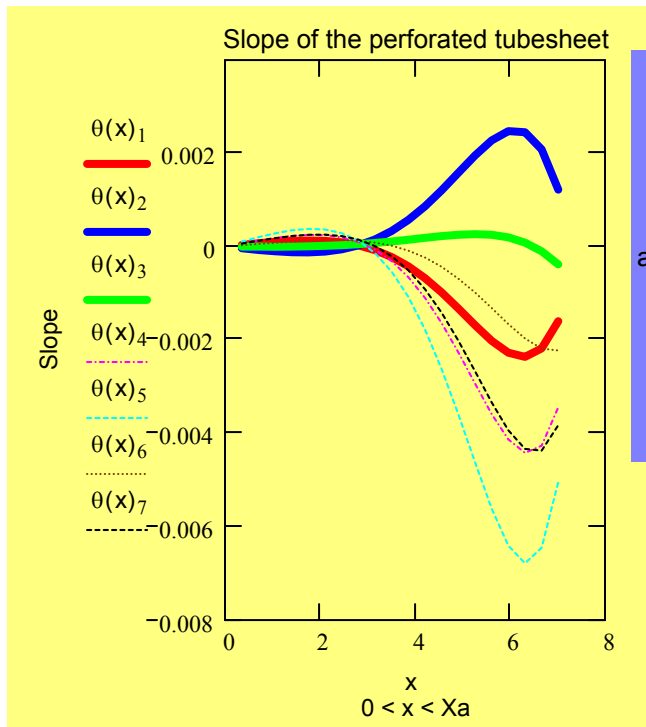
For use on Page 40:

$$ww(x) := w(x)$$

Slope $\theta(x)$ of the perforated tubesheet ($0 \leq x \leq X_a$)

$$\theta(x) := \begin{bmatrix} \frac{a_0}{D^*} \cdot [M_{a_1} \cdot Z_{mx}(x) + (a_0 \cdot V_{a_1}) \cdot Z_{vx}(x)] \\ \frac{a_0}{D^*} \cdot [M_{a_2} \cdot Z_{mx}(x) + (a_0 \cdot V_{a_2}) \cdot Z_{vx}(x)] \\ \frac{a_0}{D^*} \cdot [M_{a_3} \cdot Z_{mx}(x) + (a_0 \cdot V_{a_3}) \cdot Z_{vx}(x)] \\ \frac{a_0}{D^*} \cdot [M_{a_4} \cdot Z_{mx}(x) + (a_0 \cdot V_{a_4}) \cdot Z_{vx}(x)] \\ \frac{a_0}{D^*} \cdot [M_{a_5} \cdot Z_{mx}(x) + (a_0 \cdot V_{a_5}) \cdot Z_{vx}(x)] \\ \frac{a_0}{D^*} \cdot [M_{a_6} \cdot Z_{mx}(x) + (a_0 \cdot V_{a_6}) \cdot Z_{vx}(x)] \\ \frac{a_0}{D^*} \cdot [M_{a_7} \cdot Z_{mx}(x) + (a_0 \cdot V_{a_7}) \cdot Z_{vx}(x)] \end{bmatrix}$$

$$\theta(X_a) = \begin{pmatrix} -1.601 \times 10^{-3} \\ 1.218 \times 10^{-3} \\ -3.829 \times 10^{-4} \\ -3.447 \times 10^{-3} \\ -5.048 \times 10^{-3} \\ -2.229 \times 10^{-3} \\ -3.830 \times 10^{-3} \end{pmatrix}$$



Limit condition : $a_0 \cdot \theta(X_a) \cdot (\rho_s - 1) = w(X_a)$

$$a_0 \cdot \theta(X_a) \cdot (\rho_s - 1) = \begin{pmatrix} -6.003 \times 10^{-4} \\ 4.567 \times 10^{-4} \\ -1.436 \times 10^{-4} \\ -1.293 \times 10^{-3} \\ -1.893 \times 10^{-3} \\ -8.358 \times 10^{-4} \\ -1.436 \times 10^{-3} \end{pmatrix} \quad \text{if } w(X_a) = \begin{pmatrix} -6.003 \times 10^{-4} \\ 4.567 \times 10^{-4} \\ -1.436 \times 10^{-4} \\ -1.293 \times 10^{-3} \\ -1.893 \times 10^{-3} \\ -8.358 \times 10^{-4} \\ -1.436 \times 10^{-3} \end{pmatrix} \quad \text{in}$$

Limit condition : $\theta(0)=0$

$$\theta(0) = \begin{pmatrix} 0.00000 \\ 0.00000 \\ 0.00000 \\ 0.00000 \\ 0.00000 \\ 0.00000 \\ 0.00000 \end{pmatrix}$$

Limit Condition : $w(a_s)=0$

$$w(a_s) := w(X_a) - a_0 \cdot \theta(X_a) \cdot (\rho_s - 1)$$

$$w(a_s) = \begin{pmatrix} 0.00000 \\ 0.00000 \\ 0.00000 \\ 0.00000 \\ 0.00000 \\ 0.00000 \\ 0.00000 \end{pmatrix} \quad \text{in}$$

Bending Moment $M_r(x)$ of the perforated tubesheet ($0 \leq x \leq X_a$)

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$$M_r(x) := \begin{bmatrix} M_{a_1} \cdot Q_m(x) + (a_0 \cdot V_{a_1}) \cdot Q_v(x) \\ M_{a_2} \cdot Q_m(x) + (a_0 \cdot V_{a_2}) \cdot Q_v(x) \\ M_{a_3} \cdot Q_m(x) + (a_0 \cdot V_{a_3}) \cdot Q_v(x) \\ M_{a_4} \cdot Q_m(x) + (a_0 \cdot V_{a_4}) \cdot Q_v(x) \\ M_{a_5} \cdot Q_m(x) + (a_0 \cdot V_{a_5}) \cdot Q_v(x) \\ M_{a_6} \cdot Q_m(x) + (a_0 \cdot V_{a_6}) \cdot Q_v(x) \\ M_{a_7} \cdot Q_m(x) + (a_0 \cdot V_{a_7}) \cdot Q_v(x) \end{bmatrix}$$

$$M_{rmax} := \begin{bmatrix} \max(M_r(x)_1) \\ \max(M_r(x)_2) \\ \max(M_r(x)_3) \\ \max(M_r(x)_4) \\ \max(M_r(x)_5) \\ \max(M_r(x)_6) \\ \max(M_r(x)_7) \end{bmatrix} \quad M_{rmin} := \begin{bmatrix} \min(M_r(x)_1) \\ \min(M_r(x)_2) \\ \min(M_r(x)_3) \\ \min(M_r(x)_4) \\ \min(M_r(x)_5) \\ \min(M_r(x)_6) \\ \min(M_r(x)_7) \end{bmatrix}$$

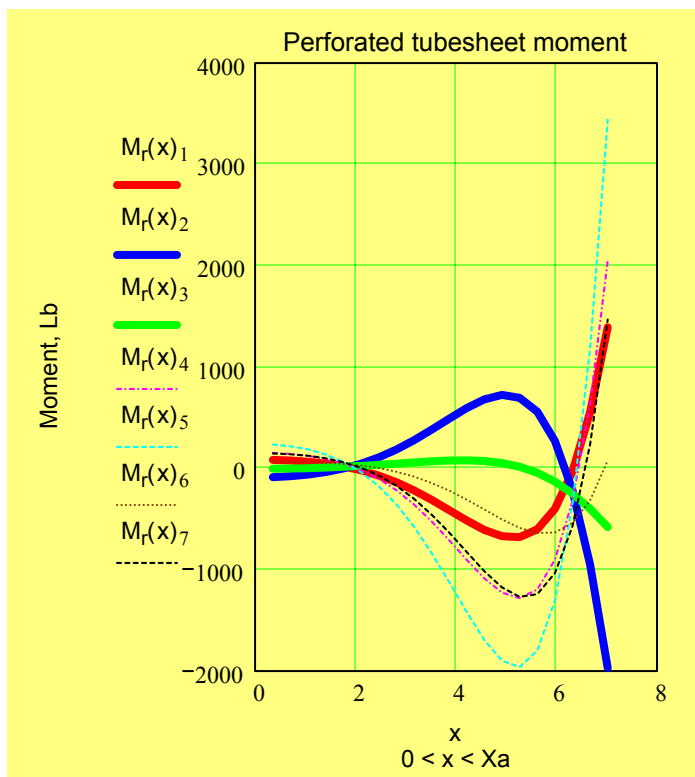
$$M_{rmax} = \begin{bmatrix} 1391.313 \\ 722.596 \\ 76.919 \\ 2047.507 \\ 3438.820 \\ 75.451 \\ 1466.764 \end{bmatrix} \text{ lb} \quad M_{rmin} = \begin{bmatrix} -680.614 \\ -1972.056 \\ -580.743 \\ -1282.601 \\ -1963.215 \\ -640.173 \\ -1269.238 \end{bmatrix} \text{ lb}$$

$$M_r(X_a) = \begin{bmatrix} 1391.313 \\ -1972.056 \\ -580.743 \\ 2047.507 \\ 3438.820 \\ 75.451 \\ 1466.764 \end{bmatrix} \text{ lb}$$

Limit Condition : $M_r(X_a) = M_a$

$$M_r(X_a) = \begin{bmatrix} 1391.313 \\ -1972.056 \\ -580.743 \\ 2047.507 \\ 3438.820 \\ 75.451 \\ 1466.764 \end{bmatrix} \text{ lb}$$

$$M_a = \begin{bmatrix} 1391.313 \\ -1972.056 \\ -580.743 \\ 2047.507 \\ 3438.820 \\ 75.451 \\ 1466.764 \end{bmatrix} \text{ lb}$$



Bending Stress $\sigma_r(x)$ of the perforated tubesheet ($0 \leq x \leq X_a$)

$$\sigma_r(x) := \begin{bmatrix} \frac{6}{\mu^* \cdot (h_{\min_1})^2} \cdot M_r(x)_1 \\ \frac{6}{\mu^* \cdot (h_{\min_2})^2} \cdot M_r(x)_2 \\ \frac{6}{\mu^* \cdot (h_{\min_3})^2} \cdot M_r(x)_3 \\ \frac{6}{\mu^* \cdot (h_{\min_4})^2} \cdot M_r(x)_4 \\ \frac{6}{\mu^* \cdot (h_{\min_5})^2} \cdot M_r(x)_5 \\ \frac{6}{\mu^* \cdot (h_{\min_6})^2} \cdot M_r(x)_6 \\ \frac{6}{\mu^* \cdot (h_{\min_7})^2} \cdot M_r(x)_7 \end{bmatrix}$$

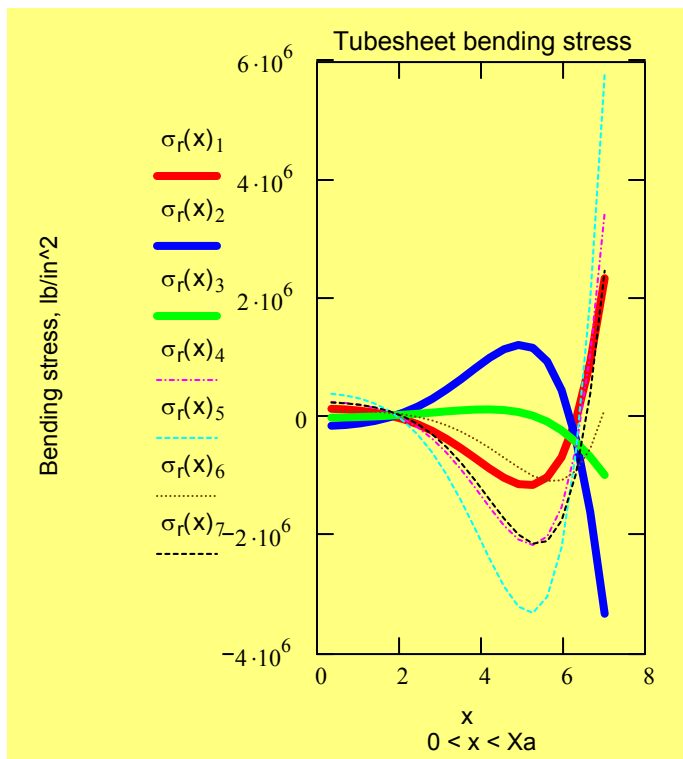
$$\sigma_r(x)_1 =$$

	1
1	963.117
2	885.787
3	744.896
4	523.394
5	199.482
6	-250.984
7	-850.958
8	-1.617·10 ³
9	-2.554·10 ³
10	-3.645·10 ³
11	-4.841·10 ³
12	-6.052·10 ³
13	-7.131·10 ³
14	-7.867·10 ³
15	-7.967·10 ³
16	-7.061·10 ³
17	-4.698·10 ³
18	-368.963
19	6.46·10 ³
20	1.629·10 ⁴

$$\frac{\text{lb}}{\text{in}^2} \quad \sigma_{r\max} := \begin{bmatrix} \max(\sigma_r(x)_1) \\ \max(\sigma_r(x)_2) \\ \max(\sigma_r(x)_3) \\ \max(\sigma_r(x)_4) \\ \max(\sigma_r(x)_5) \\ \max(\sigma_r(x)_6) \\ \max(\sigma_r(x)_7) \end{bmatrix} \quad \sigma_{r\min} := \begin{bmatrix} \min(\sigma_r(x)_1) \\ \min(\sigma_r(x)_2) \\ \min(\sigma_r(x)_3) \\ \min(\sigma_r(x)_4) \\ \min(\sigma_r(x)_5) \\ \min(\sigma_r(x)_6) \\ \min(\sigma_r(x)_7) \end{bmatrix}$$

$$\sigma_{r\max} = \begin{bmatrix} 16286.538 \\ 8458.622 \\ 900.409 \\ 23967.865 \\ 40254.403 \\ 883.224 \\ 17169.762 \end{bmatrix} \frac{\text{lb}}{\text{in}^2} \quad \sigma_{r\min} = \begin{bmatrix} -7967.186 \\ -23084.641 \\ -6798.103 \\ -15013.967 \\ -22981.153 \\ -7493.784 \\ -14857.547 \end{bmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_r(X_a) = \begin{bmatrix} 16286.538 \\ -23084.641 \\ -6798.103 \\ 23967.865 \\ 40254.403 \\ 883.224 \\ 17169.762 \end{bmatrix} \frac{\text{lb}}{\text{in}^2} \quad \sigma = \begin{bmatrix} -16286.538 \\ 23084.641 \\ 6798.103 \\ -23967.865 \\ -40254.403 \\ -7493.784 \\ -17169.762 \end{bmatrix} \frac{\text{lb}}{\text{in}^2}$$



Shear Load $Q(x)$ of the perforated tubesheet ($0 \leq x \leq X_a$)

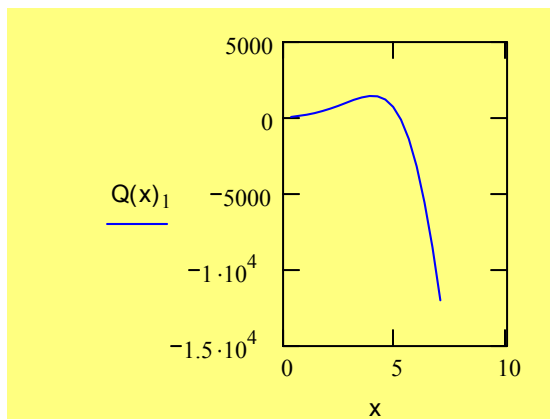
$$Q_{\alpha}(x) := X_a \cdot \frac{\text{ber}' \cdot \text{bei}'_x(x) - \text{bei}' \cdot \text{ber}'_x(x)}{Z_a}$$

$$Q_{\beta}(x) := \frac{\Psi_2 \cdot \text{bei}'_x(x) - \Psi_1 \cdot \text{ber}'_x(x)}{Z_a}$$

$$Q(x) := \begin{bmatrix} \frac{1}{a_0} \cdot [M_{a_1} \cdot Q_{\alpha}(x) + (a_0 \cdot V_{a_1}) \cdot Q_{\beta}(x)] \\ \frac{1}{a_0} \cdot [M_{a_2} \cdot Q_{\alpha}(x) + (a_0 \cdot V_{a_2}) \cdot Q_{\beta}(x)] \\ \frac{1}{a_0} \cdot [M_{a_3} \cdot Q_{\alpha}(x) + (a_0 \cdot V_{a_3}) \cdot Q_{\beta}(x)] \\ \frac{1}{a_0} \cdot [M_{a_4} \cdot Q_{\alpha}(x) + (a_0 \cdot V_{a_4}) \cdot Q_{\beta}(x)] \\ \frac{1}{a_0} \cdot [M_{a_5} \cdot Q_{\alpha}(x) + (a_0 \cdot V_{a_5}) \cdot Q_{\beta}(x)] \\ \frac{1}{a_0} \cdot [M_{a_6} \cdot Q_{\alpha}(x) + (a_0 \cdot V_{a_6}) \cdot Q_{\beta}(x)] \\ \frac{1}{a_0} \cdot [M_{a_7} \cdot Q_{\alpha}(x) + (a_0 \cdot V_{a_7}) \cdot Q_{\beta}(x)] \end{bmatrix}$$

	1
1	4.924
2	10.530
3	17.451
4	26.217
5	37.185
6	50.449
7	65.729
8	82.230
9	98.494
10	112.230
11	120.165
12	117.926
13	100.008
14	59.875
15	-9.746
16	-116.285
17	-266.388
18	-464.532
19	-711.123
20	-1000.034

$Q(x)_1 =$ $\frac{\text{lb}}{\text{in}}$



$$Q_{\min} := \begin{bmatrix} \min(Q(x)_1) \\ \min(Q(x)_2) \\ \min(Q(x)_3) \\ \min(Q(x)_4) \\ \min(Q(x)_5) \\ \min(Q(x)_6) \\ \min(Q(x)_7) \end{bmatrix} \quad Q_{\max} := \begin{bmatrix} \max(Q(x)_1) \\ \max(Q(x)_2) \\ \max(Q(x)_3) \\ \max(Q(x)_4) \\ \max(Q(x)_5) \\ \max(Q(x)_6) \\ \max(Q(x)_7) \end{bmatrix}$$

Limit Condition : $Q(X_a) = V_a$

$$Q(X_a) = \begin{bmatrix} -1000.034 \\ 1204.493 \\ 204.459 \\ -1692.714 \\ -2692.747 \\ -488.220 \\ -1488.254 \end{bmatrix} \frac{\text{lb}}{\text{in}} \quad V_a = \begin{bmatrix} -1000.034 \\ 1204.493 \\ 204.459 \\ -1692.714 \\ -2692.747 \\ -488.220 \\ -1488.254 \end{bmatrix} \frac{\text{lb}}{\text{in}}$$

$$Q_{\min} = \begin{bmatrix} -1000.034 \\ -128.009 \\ -13.919 \\ -1692.714 \\ -2692.747 \\ -488.220 \\ -1488.254 \end{bmatrix} \frac{\text{lb}}{\text{in}} \quad Q_{\max} = \begin{bmatrix} 120.165 \\ 1204.493 \\ 204.459 \\ 224.011 \\ 341.937 \\ 112.016 \\ 223.414 \end{bmatrix} \frac{\text{lb}}{\text{in}}$$

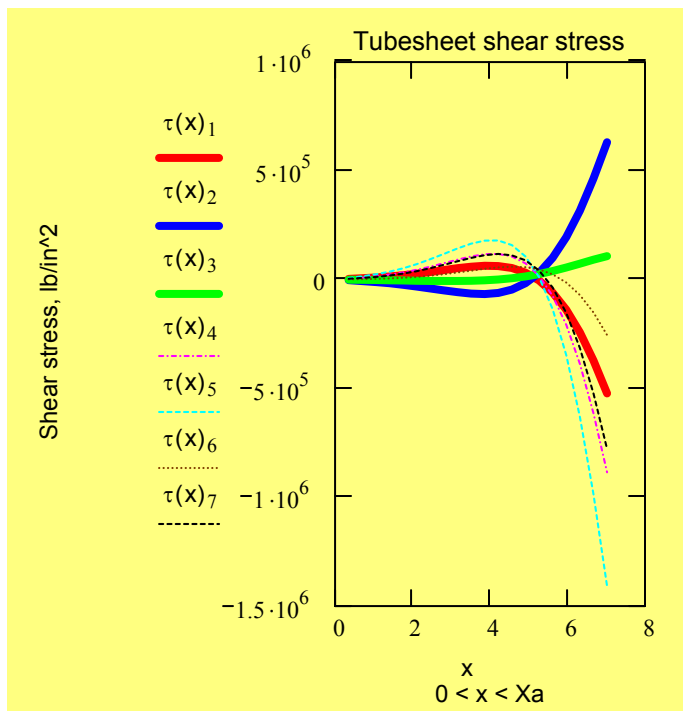
Shear Stress $\tau(x)$ of the perforated tubesheet ($0 \leq x \leq X_a$)

$$\tau_{\text{UHX13}} := \tau_{\text{max}}$$

$$\tau(x) := \begin{pmatrix} \frac{Q(x)_1}{\mu \cdot h} \\ \frac{Q(x)_2}{\mu \cdot h} \\ \frac{Q(x)_3}{\mu \cdot h} \\ \frac{Q(x)_4}{\mu \cdot h} \\ \frac{Q(x)_5}{\mu \cdot h} \\ \frac{Q(x)_6}{\mu \cdot h} \\ \frac{Q(x)_7}{\mu \cdot h} \end{pmatrix} \quad \tau(x)_1 = \begin{array}{|c|c|} \hline & 1 \\ \hline 1 & 17.906 \\ 2 & 38.291 \\ 3 & 63.458 \\ 4 & 95.334 \\ 5 & 135.218 \\ 6 & 183.452 \\ 7 & 239.013 \\ 8 & 299.019 \\ 9 & 358.160 \\ 10 & 408.111 \\ 11 & 436.963 \\ 12 & 428.820 \\ 13 & 363.666 \\ 14 & 217.726 \\ 15 & -35.442 \\ 16 & -422.855 \\ 17 & -968.683 \\ 18 & -1689.208 \\ 19 & -2585.901 \\ 20 & -3636.487 \\ \hline \end{array} \quad \frac{\text{lb}}{\text{in}^2}$$

$$\tau_{\text{max}} := \begin{pmatrix} \max\left(\frac{Q(x)_1}{\mu \cdot h}\right) \\ \max\left(\frac{Q(x)_2}{\mu \cdot h}\right) \\ \max\left(\frac{Q(x)_3}{\mu \cdot h}\right) \\ \max\left(\frac{Q(x)_4}{\mu \cdot h}\right) \\ \max\left(\frac{Q(x)_5}{\mu \cdot h}\right) \\ \max\left(\frac{Q(x)_6}{\mu \cdot h}\right) \\ \max\left(\frac{Q(x)_7}{\mu \cdot h}\right) \end{pmatrix} \quad \tau_{\text{min}} := \begin{pmatrix} \min\left(\frac{Q(x)_1}{\mu \cdot h}\right) \\ \min\left(\frac{Q(x)_2}{\mu \cdot h}\right) \\ \min\left(\frac{Q(x)_3}{\mu \cdot h}\right) \\ \min\left(\frac{Q(x)_4}{\mu \cdot h}\right) \\ \min\left(\frac{Q(x)_5}{\mu \cdot h}\right) \\ \min\left(\frac{Q(x)_6}{\mu \cdot h}\right) \\ \min\left(\frac{Q(x)_7}{\mu \cdot h}\right) \end{pmatrix}$$

$$\tau(X_a) = \begin{pmatrix} -3636.487 \\ 4379.976 \\ 743.489 \\ -6155.322 \\ -9791.809 \\ -1775.346 \\ -5411.833 \end{pmatrix} \frac{\text{lb}}{\text{in}^2} \quad \tau_{\text{max}} = \begin{pmatrix} 436.963 \\ 4379.976 \\ 743.489 \\ 814.586 \\ 1243.406 \\ 407.331 \\ 812.414 \end{pmatrix} \frac{\text{lb}}{\text{in}^2} \quad \tau_{\text{min}} = \begin{pmatrix} -3636.487 \\ -465.488 \\ -50.615 \\ -6155.322 \\ -9791.809 \\ -1775.346 \\ -5411.833 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$



$$\max(|\tau_{\text{max}}|) = 4379.976 \frac{\text{lb}}{\text{in}^2} \quad \max(|\tau_{\text{min}}|) = 9791.809 \frac{\text{lb}}{\text{in}^2}$$

$$\tau_{\text{max}} := \max(\max(|\tau_{\text{max}}|), \max(|\tau_{\text{min}}|))$$

$$\tau_{\text{max}} = 9791.809 \frac{\text{lb}}{\text{in}^2}$$

$$\tau_{\text{UHX13}} = 9791.809 \frac{\text{lb}}{\text{in}^2}$$

Tube Stress $\sigma_t(x)$ of the Tube-Bundle ($0 \leq x \leq X_a$)

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$$\sigma_{tx}(x) := \begin{bmatrix} \frac{1}{x_t - x_s} \cdot [(P_{s_1} \cdot x_s - P_{t_1} \cdot x_t) - q(x)_1] \\ \frac{1}{x_t - x_s} \cdot [(P_{s_2} \cdot x_s - P_{t_2} \cdot x_t) - q(x)_2] \\ \frac{1}{x_t - x_s} \cdot [(P_{s_3} \cdot x_s - P_{t_3} \cdot x_t) - q(x)_3] \\ \frac{1}{x_t - x_s} \cdot [(P_{s_4} \cdot x_s - P_{t_4} \cdot x_t) - q(x)_4] \\ \frac{1}{x_t - x_s} \cdot [(P_{s_5} \cdot x_s - P_{t_5} \cdot x_t) - q(x)_5] \\ \frac{1}{x_t - x_s} \cdot [(P_{s_6} \cdot x_s - P_{t_6} \cdot x_t) - q(x)_6] \\ \frac{1}{x_t - x_s} \cdot [(P_{s_7} \cdot x_s - P_{t_7} \cdot x_t) - q(x)_7] \end{bmatrix}$$

	1
1	-1132.21
2	-1144.74
3	-1164.71
4	-1190.63
5	-1220.14
6	-1249.81
7	-1274.81
8	-1288.77
9	-1283.53
10	-1249.14
11	-1174.08
12	-1045.83
13	-851.86
14	-581.35
15	-227.57
16	208.91
17	716
18	1265.17
19	1805.79
20	2259.34

$$\sigma_{tx}(0) = \begin{pmatrix} -1127.95 \\ 1484.91 \\ 356.96 \\ -135.87 \\ -1263.82 \\ 1349.04 \\ 221.09 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{tx}(0)_1 = -1127.95 \frac{\text{lb}}{\text{in}^2}$$

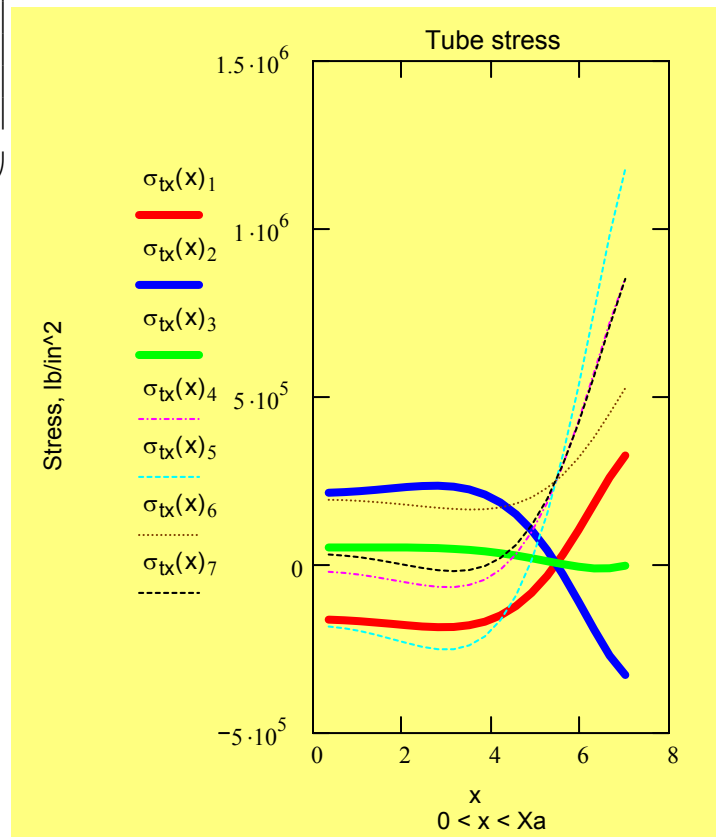
$$\sigma_{txmin} := \begin{pmatrix} \min(\sigma_{tx}(x)_1) \\ \min(\sigma_{tx}(x)_2) \\ \min(\sigma_{tx}(x)_3) \\ \min(\sigma_{tx}(x)_4) \\ \min(\sigma_{tx}(x)_5) \\ \min(\sigma_{tx}(x)_6) \\ \min(\sigma_{tx}(x)_7) \end{pmatrix} \quad \sigma_{txmax} := \begin{pmatrix} \max(\sigma_{tx}(x)_1) \\ \max(\sigma_{tx}(x)_2) \\ \max(\sigma_{tx}(x)_3) \\ \max(\sigma_{tx}(x)_4) \\ \max(\sigma_{tx}(x)_5) \\ \max(\sigma_{tx}(x)_6) \\ \max(\sigma_{tx}(x)_7) \end{pmatrix}$$

$$\sigma_{txmin} = \begin{pmatrix} -1288.774 \\ -2276.568 \\ -73.144 \\ -459.772 \\ -1743.539 \\ 1141.628 \\ -129.348 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{txmax} = \begin{pmatrix} 2259.342 \\ 1633.646 \\ 360.211 \\ 5928.058 \\ 8187.400 \\ 3651.490 \\ 5910.832 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{tx}(X_a) = \begin{pmatrix} 2259.342 \\ -2276.568 \\ -17.226 \\ 5928.058 \\ 8187.400 \\ 3651.490 \\ 5910.832 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{to} = \begin{pmatrix} 2259.342 \\ -2276.568 \\ -17.226 \\ 5928.058 \\ 8187.400 \\ 3651.490 \\ 5910.832 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$



Rigid Ring

$$R := \frac{A + 2 \cdot a_o}{2} \quad R = 42.188 \text{ in}$$

Limit Condition : $\theta_R = \theta(X_a)$

$$\theta_R := \frac{12}{h^3 \cdot (\lambda_s + \lambda_c + E \cdot \ln(K))} \cdot a_o \cdot \left[-M_a + a_o \cdot V_a \cdot (\rho_s - 1) + (\omega^*_s \cdot P_s - \omega^*_c \cdot P_t) + \frac{\gamma_b}{2 \cdot \pi} \cdot W \right]$$

$$\theta_R = \begin{pmatrix} -0.00160 \\ 0.00122 \\ -0.00038 \\ -0.00345 \\ -0.00505 \\ -0.00223 \\ -0.00383 \end{pmatrix}$$

$$\theta(X_a) = \begin{pmatrix} -0.00160 \\ 0.00122 \\ -0.00038 \\ -0.00345 \\ -0.00505 \\ -0.00223 \\ -0.00383 \end{pmatrix}$$

$$M_R := \frac{E \cdot h^3 \cdot \ln(K)}{12 \cdot R} \cdot \theta_R \quad M_R = \begin{pmatrix} -9.647 \\ 7.339 \\ -2.307 \\ -20.770 \\ -30.417 \\ -13.431 \\ -23.078 \end{pmatrix} \text{ lb}$$

Shell and Channel

$$\theta_S := \theta(X_a)$$

$$M_S := \left(1 + \frac{h'_S}{2}\right) \cdot k_S \cdot \theta_S + \beta_S \cdot k_S \cdot \delta_S \cdot P_S \quad M_S = \begin{pmatrix} -642.529 \\ 1463.288 \\ 820.759 \\ -1383.425 \\ -2025.954 \\ 79.863 \\ -562.666 \end{pmatrix} \text{ lb}$$

Verification from σ_{sb}

$$\frac{t_{s1}^2 \cdot \sigma_{sb}}{6} = \begin{pmatrix} -642.529 \\ 1463.288 \\ 820.759 \\ -1383.425 \\ -2025.954 \\ 79.863 \\ -562.666 \end{pmatrix} \text{ lb}$$

$$\theta_C := -\theta(X_a)$$

$$M_C := \left(1 + \frac{h'_C}{2}\right) \cdot k_C \cdot \theta_C + \beta_C \cdot k_C \cdot \delta_C \cdot P_t \quad M_C = \begin{pmatrix} 664.358 \\ -199.040 \\ 465.318 \\ 563.278 \\ 1227.636 \\ 364.237 \\ 1028.595 \end{pmatrix} \text{ lb}$$

Verification from σ_{cb}

$$\frac{t_c^2 \cdot \sigma_{cb}}{6} = \begin{pmatrix} 664.358 \\ -199.040 \\ 465.318 \\ 563.278 \\ 1227.636 \\ 364.237 \\ 1028.595 \end{pmatrix} \text{ lb}$$

$$Q_S := -(1 + h'_S) \cdot \beta_S \cdot k_S \cdot \theta_S - 2\beta_S^2 k_S \cdot \delta_S \cdot P_S \quad Q_S = \begin{pmatrix} 287.278 \\ -942.612 \\ -655.334 \\ 618.537 \\ 905.816 \\ -324.075 \\ -36.797 \end{pmatrix} \text{ lb in}^{-1}$$

$$Q_C := -(1 + h'_C) \cdot \beta_C \cdot k_C \cdot \theta_C - 2\beta_C^2 k_C \cdot \delta_C \cdot P_t \quad Q_C = \begin{pmatrix} -514.307 \\ 112.242 \\ -402.065 \\ -317.640 \\ -831.947 \\ -205.398 \\ -719.705 \end{pmatrix} \text{ lb in}^{-1}$$

$$w_s := \frac{Q_s}{\beta_s^2 \cdot k_s} + \frac{M_s}{\beta_s \cdot k_s} + \delta_s \cdot P_s$$

$$w_s = \begin{pmatrix} 0.0011 \\ -0.0008 \\ 0.0003 \\ 0.0024 \\ 0.0035 \\ 0.0015 \\ 0.0026 \end{pmatrix} \text{ in}$$

$$w_c := \frac{Q_c}{\beta_c^2 \cdot k_c} + \frac{M_c}{\beta_c \cdot k_c} + \delta_c \cdot P_t$$

$$w_c = \begin{pmatrix} -0.0011 \\ 0.0008 \\ -0.0003 \\ -0.0024 \\ -0.0035 \\ -0.0015 \\ -0.0026 \end{pmatrix} \text{ in}$$

VERIFICATIONS for SHELL : $w_s = -(h/2)\theta_s$

$$\theta_s := \frac{Q_s}{\beta_s \cdot k_s} + \frac{2M_s}{k_s}$$

$$\theta_s := \theta(X_a)$$

$$ww_s := \frac{-h}{2} \cdot \theta_s$$

$$\theta_s = \begin{pmatrix} -0.0016 \\ 0.0012 \\ -0.0004 \\ -0.0034 \\ -0.0050 \\ -0.0022 \\ -0.0038 \end{pmatrix}$$

$$\theta_s = \begin{pmatrix} -0.0016 \\ 0.0012 \\ -0.0004 \\ -0.0034 \\ -0.0050 \\ -0.0022 \\ -0.0038 \end{pmatrix}$$

$$ww_s = \begin{pmatrix} 0.0011 \\ -0.0008 \\ 0.0003 \\ 0.0024 \\ 0.0035 \\ 0.0015 \\ 0.0026 \end{pmatrix} \text{ in}$$

VERIFICATIONS for CHANNEL: $w_c = -(h/2)\theta_c$

$$\theta_c := \frac{Q_c}{\beta_c \cdot k_c} + \frac{2M_c}{k_c}$$

$$\theta_c := -\theta(X_a)$$

$$ww_c := \frac{-h}{2} \cdot \theta_c$$

$$\theta_c = \begin{pmatrix} 0.0016 \\ -0.0012 \\ 0.0004 \\ 0.0034 \\ 0.0050 \\ 0.0022 \\ 0.0038 \end{pmatrix}$$

$$\theta_c = \begin{pmatrix} 0.0016 \\ -0.0012 \\ 0.0004 \\ 0.0034 \\ 0.0050 \\ 0.0022 \\ 0.0038 \end{pmatrix}$$

$$ww_c = \begin{pmatrix} -0.0011 \\ 0.0008 \\ -0.0003 \\ -0.0024 \\ -0.0035 \\ -0.0015 \\ -0.0026 \end{pmatrix} \text{ in}$$

$$\sigma_{sb} := \frac{6 \cdot M_s}{t_{s1}^2} \quad \sigma_{sb} = \begin{pmatrix} -12184.262 \\ 27748.281 \\ 15564.019 \\ -26233.837 \\ -38418.099 \\ 1514.444 \\ -10669.818 \end{pmatrix} \text{ lb in}^{-2}$$

Verification from σ_{sb}

$$\sigma_{sb} = \begin{pmatrix} -12184.262 \\ 27748.281 \\ 15564.019 \\ -26233.837 \\ -38418.099 \\ 1514.444 \\ -10669.818 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{sm} = \begin{pmatrix} 1830.583 \\ 2287.184 \\ 4117.767 \\ -2916.472 \\ -1085.889 \\ -629.288 \\ 1201.295 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{cb} := 6 \cdot \frac{M_c}{t_c^2} \quad \sigma_{cb} = \begin{pmatrix} 28345.945 \\ -8492.394 \\ 19853.551 \\ 24033.188 \\ 52379.133 \\ 15540.794 \\ 43886.739 \end{pmatrix} \text{ lb in}^{-2}$$

Verification from σ_{cb}

$$\sigma_{cb} = \begin{pmatrix} 28345.945 \\ -8492.394 \\ 19853.551 \\ 24033.188 \\ 52379.133 \\ 15540.794 \\ 43886.739 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

Axial force in the SHELL

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Calculation from direct formula

$$V_s := \frac{a_o^2 \cdot V_a + \frac{a_o^2}{2} \cdot P_t + \frac{a_s^2 - a_o^2}{2} \cdot P_s}{\frac{D_s + t_{s1}}{2}} \quad V_s = \begin{pmatrix} 1029.703 \\ 1286.541 \\ 2316.244 \\ -1640.515 \\ -610.813 \\ -353.974 \\ 675.728 \end{pmatrix} \frac{\text{lb}}{\text{in}}$$

$$KK := \frac{J \cdot K_{st}}{1 + J \cdot K_{st} \cdot [Q_{z1} + (\rho_s - 1) \cdot Q_{z2}]}$$

Verification from σ_{sm}

$$V_{\sigma sm} := \sigma_{sm} \cdot t_{s1} \quad V_{\sigma sm} = \begin{pmatrix} 1029.703 \\ 1286.541 \\ 2316.244 \\ -1640.515 \\ -610.813 \\ -353.974 \\ 675.728 \end{pmatrix} \frac{\text{lb}}{\text{in}}$$

 V_s due to pressures P_s , P_t and thermal expansion γ

$$V_{s1} := \frac{a_o^2}{D_s + t_{s1}} \cdot \left[KK \cdot \left[x_s + 2 \cdot (1 - x_s) \cdot v_t - \frac{\rho_s^2 - 1}{J \cdot K_{st}} \right] \cdot P_s - P'_t + P_\gamma + P_\omega + P_w + P_{rim} \right] + P_t + (\rho_s^2 - 1) P_s$$

 V_s due to Poisson's ratio v_s of the shell

$$V_{s2} := \frac{a_o^2}{D_s + t_{s1}} \cdot KK \cdot \left[\frac{2}{K_{st}} \cdot \left(\frac{D_s}{D_o} \right)^2 \cdot v_s \right] \cdot P_s$$

 V_s due to pressure P_s acting on Bellows

$$V_{s3} := -\frac{a_o^2}{D_s + t_{s1}} \cdot KK \cdot \frac{1 - J}{2 \cdot J \cdot K_{st}} \cdot \frac{D_J^2 - D_s^2}{D_o^2} \cdot P_s$$

 V_s TOTAL

$$V_{sT} := V_{s1} + V_{s2} + V_{s3}$$

$$V_{s1} = \begin{pmatrix} 1029.703 \\ 631.845 \\ 1661.548 \\ -1640.515 \\ -610.813 \\ -1008.670 \\ 21.032 \end{pmatrix} \frac{\text{lb}}{\text{in}} \quad V_{s2} = \begin{pmatrix} 0.000 \\ 654.696 \\ 654.696 \\ 0.000 \\ 0.000 \\ 654.696 \\ 654.696 \end{pmatrix} \frac{\text{lb}}{\text{in}}$$

$$V_{s3} = \begin{pmatrix} 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \end{pmatrix} \frac{\text{lb}}{\text{in}}$$

$$V_{sT} = \begin{pmatrix} 1029.703 \\ 1286.541 \\ 2316.244 \\ -1640.515 \\ -610.813 \\ -353.974 \\ 675.728 \end{pmatrix} \frac{\text{lb}}{\text{in}}$$

$$\sigma_{sm} = \begin{pmatrix} 1830.583 \\ 2287.184 \\ 4117.767 \\ -2916.472 \\ -1085.889 \\ -629.288 \\ 1201.295 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

Verification σ_{sm}

$$\sigma_{sm} := \frac{V_{sT}}{t_s} \quad \sigma_{sm} = \begin{pmatrix} 1830.583 \\ 2287.184 \\ 4117.767 \\ -2916.472 \\ -1085.889 \\ -629.288 \\ 1201.295 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

SHELL Axial displacement**a) Total Axial displacement Δ_{sTotal} of the shell of length L**

$$T_{sm} := \begin{pmatrix} T_a \\ T_a \\ T_a \\ T_{sm} \\ T_{sm} \\ T_{sm} \\ T_{sm} \end{pmatrix} \quad \theta_s := T_{sm} - T_a \quad \Delta_{sTotal} := \frac{L}{J \cdot E_s \cdot t_s} \cdot V_s - v_s \cdot \frac{L \cdot D_s^2}{2E_s \cdot t_s \cdot (D_s + t_s)} \cdot P_s + \frac{\pi}{8} \frac{D_J^2 - D_s^2}{K_J} \cdot P_s + L \cdot \alpha_{sm} \cdot \theta_s$$

$$\theta_s = \begin{pmatrix} 0.0 \\ 0.0 \\ 0.0 \\ 81.0 \\ 81.0 \\ 81.0 \\ 81.0 \end{pmatrix} \text{ degF} \quad \Delta_{sTotal} = \begin{pmatrix} 0.0165 \\ -0.0117 \\ 0.0047 \\ 0.1429 \\ 0.1594 \\ 0.1312 \\ 0.1477 \end{pmatrix} \text{ in}$$

For half-shell of length $l=L/2$:

$$\delta_{sTotal} := \frac{\Delta_{sTotal}}{2} \quad \delta_{sTotal} = \begin{pmatrix} 0.0082 \\ -0.0059 \\ 0.0024 \\ 0.0715 \\ 0.0797 \\ 0.0656 \\ 0.0738 \end{pmatrix} \text{ in}$$

b) Total Axial displacement of the half-shell of length $l=L/2$: $\delta_{sTotal} = \delta_s(V_s) + \delta_s(\theta_s) + \delta_s(v_s) + \delta_s(J)$

$$l := \frac{L}{2} \quad l = 118.625 \text{ in}$$

 δ_s due to axial load V_s

$$\delta_{sVs} := \frac{l}{J \cdot E_s \cdot t_s} \cdot V_s$$

$$\delta_{sVs} = \begin{pmatrix} 0.008225 \\ 0.010277 \\ 0.018503 \\ -0.013105 \\ -0.004879 \\ -0.002828 \\ 0.005398 \end{pmatrix} \text{ in}$$

 δ_s due to θ_s

$$\delta_{s\theta s} := l \cdot \alpha_{sm} \cdot \theta_s$$

$$\delta_{s\theta s} = \begin{pmatrix} 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0846 \\ 0.0846 \\ 0.0846 \\ 0.0846 \end{pmatrix} \text{ in}$$

 δ_s due to Poisson's ratio v_s

$$\delta_{svs} := -v_s \cdot \frac{l \cdot D_s^2}{2E_s \cdot t_s \cdot (D_s + t_s)} \cdot P_s$$

$$\delta_{svs} = \begin{pmatrix} 0.0000 \\ -0.0161 \\ -0.0161 \\ 0.0000 \\ 0.0000 \\ -0.0161 \\ -0.0161 \end{pmatrix} \text{ in}$$

 δ_s due to pressure P_s acting on Bellows J

$$\delta_{sJ} := \frac{\pi}{8} \frac{D_J^2 - D_s^2}{2K_J} \cdot P_s$$

$$\delta_{sJ} = \begin{pmatrix} 0.0000 \\ -0.0000 \\ -0.0000 \\ 0.0000 \\ 0.0000 \\ -0.0000 \\ -0.0000 \end{pmatrix} \text{ in}$$

TOTAL DISPLACEMENT δ_{sTotal} OF HALF SHELL

$$\delta_{sTotal} := \delta_{sVs} + \delta_{s\theta s} + \delta_{svs} + \delta_{sJ}$$

$$\delta_{sTotal} = \begin{pmatrix} 0.0082 \\ -0.0059 \\ 0.0024 \\ 0.0715 \\ 0.0797 \\ 0.0656 \\ 0.0738 \end{pmatrix} \text{ in}$$

Axial displacement of the half-TUBES of length $l=L/2$

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$$\begin{aligned}
 T_{tm} &:= \begin{pmatrix} T_a \\ T_a \\ T_a \\ T_{tm} \\ T_{tm} \\ T_{tm} \\ T_{tm} \end{pmatrix} \quad T_{tm} = \begin{pmatrix} 70.0 \\ 70.0 \\ 70.0 \\ 113.0 \\ 113.0 \\ 113.0 \\ 113.0 \end{pmatrix} \text{ degF} \quad \theta_t := T_{tm} - T_a \quad \theta_t = \begin{pmatrix} 0.0 \\ 0.0 \\ 0.0 \\ 43.0 \\ 43.0 \\ 43.0 \\ 43.0 \end{pmatrix} \text{ degF} \\
 l &:= \frac{L}{2} \quad l = 118.625 \text{ in} \\
 AA &:= \frac{\pi a_o^2}{N_t} \quad AA = 1.399 \text{ in}^2 \\
 s_t &:= \pi t_t \cdot (d_t - t_t) \quad s_t = 0.1464 \text{ in}^2 \\
 V_t(x) &:= \begin{bmatrix} AA \cdot [(P_{s_1} \cdot x_s - P_{t_1} \cdot x_t) - q(x)_1] \\ AA \cdot [(P_{s_2} \cdot x_s - P_{t_2} \cdot x_t) - q(x)_2] \\ AA \cdot [(P_{s_3} \cdot x_s - P_{t_3} \cdot x_t) - q(x)_3] \\ AA \cdot [(P_{s_4} \cdot x_s - P_{t_4} \cdot x_t) - q(x)_4] \\ AA \cdot [(P_{s_5} \cdot x_s - P_{t_5} \cdot x_t) - q(x)_5] \\ AA \cdot [(P_{s_6} \cdot x_s - P_{t_6} \cdot x_t) - q(x)_6] \\ AA \cdot [(P_{s_7} \cdot x_s - P_{t_7} \cdot x_t) - q(x)_7] \end{bmatrix} \quad V_t(0) = \begin{pmatrix} -165.1261 \\ 217.3833 \\ 52.2572 \\ -19.8908 \\ -185.0169 \\ 197.4924 \\ 32.3663 \end{pmatrix} \text{ lb} \\
 \delta_{tVt}(x) &:= \frac{l}{E_t \cdot s_t} \cdot V_t(x) \quad \delta_{tVt}(0) = \begin{pmatrix} -0.004956 \\ 0.006524 \\ 0.001568 \\ -0.000597 \\ -0.005553 \\ 0.005927 \\ 0.000971 \end{pmatrix} \text{ in} \\
 \text{From Page 33 : } \sigma_{tx}(0)_1 &= -1127.95 \frac{\text{lb}}{\text{in}^2} \\
 \sigma_t(x) &:= \frac{\delta_{tVt}(x)}{l} \cdot E_t \quad \sigma_t(0) = \begin{pmatrix} -1127.95 \\ 1484.91 \\ 356.96 \\ -135.87 \\ -1263.82 \\ 1349.04 \\ 221.09 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}
 \end{aligned}$$

Total Axial displacement of the half-tubes of length $l=L/2$ at TS center ($x=0$) : $\delta_{tTotal} = \delta_t(V_t) + \delta_t(\theta_t) + \delta_t(v_t)$

 δ_t due to axial load V_t δ_t due to θ_t δ_t due to Poisson's ratio v_t

$$\delta_{tVt} := \delta_{tVt}(0)$$

$$\delta_{t\theta t} := l \cdot \alpha_{tm} \cdot \theta_t$$

$$\delta_{tv_t} := \frac{-2}{k_w} [(1 - x_t) \cdot P_t - (1 - x_s) \cdot P_s] v_t$$

$$\delta_{tTotal} := \delta_{tVt} + \delta_{t\theta t} + \delta_{tv_t}$$

$$\delta_{tVt} = \begin{pmatrix} -0.00496 \\ 0.00652 \\ 0.00157 \\ -0.00060 \\ -0.00555 \\ 0.00593 \\ 0.00097 \end{pmatrix} \text{ in}$$

$$\delta_{t\theta t} = \begin{pmatrix} 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0441 \\ 0.0441 \\ 0.0441 \\ 0.0441 \end{pmatrix} \text{ in}$$

$$\delta_{tv_t} = \begin{pmatrix} -0.0023 \\ 0.0046 \\ 0.0023 \\ 0.0000 \\ -0.0023 \\ 0.0046 \\ 0.0023 \end{pmatrix} \text{ in}$$

$$\delta_{tTotal} = \begin{pmatrix} -0.0073 \\ 0.0111 \\ 0.0039 \\ 0.0435 \\ 0.0363 \\ 0.0547 \\ 0.0474 \end{pmatrix} \text{ in}$$

TS DEFLECTION $w(x)$

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TOTAL DISPLACEMENT OF TUBES δ_{tTotal} IS EQUAL TO :
 TOTAL DISPLACEMENT OF SHELL δ_{sTotal} PLUS TS DEFLECTION $w(r)$

$$\delta_{tVt}(x) := \frac{I}{E_t \cdot s_t} \cdot V_t(x)$$

$$\delta_{tTotal}(x) := \delta_{tVt}(x) + \delta_{t\theta t} + \delta_{tvt}$$

$$\delta_{sTotal} := \delta_{sVs} + \delta_{s\theta s} + \delta_{svs} + \delta_{sJ}$$

$$w(x) := \delta_{tTotal}(x) - \delta_{sTotal}$$

Calculated from Page 27

$$w(0) = \begin{pmatrix} -0.015 \\ 0.017 \\ 0.002 \\ -0.028 \\ -0.043 \\ -0.011 \\ -0.026 \end{pmatrix} \text{ in}$$

$$ww(0) = \begin{pmatrix} -0.015 \\ 0.017 \\ 0.002 \\ -0.028 \\ -0.043 \\ -0.011 \\ -0.026 \end{pmatrix} \text{ in}$$

The difference $\delta(x)$ between $w(x)$ and $\delta_{tVt}(x)$ is constant

$$\delta(x) := w(x) - \delta_{tVt}(x)$$

$$\delta_{direct} := \delta_{t\theta t} + \delta_{tvt} - \delta_{sTotal}$$

$$\delta(0) = \begin{pmatrix} -0.011 \\ 0.010 \\ -0.000 \\ -0.027 \\ -0.038 \\ -0.017 \\ -0.027 \end{pmatrix} \text{ in} \quad \delta(7) = \begin{pmatrix} -0.011 \\ 0.010 \\ -0.000 \\ -0.027 \\ -0.038 \\ -0.017 \\ -0.027 \end{pmatrix} \text{ in}$$

$$\delta_{direct} = \begin{pmatrix} -0.011 \\ 0.010 \\ -0.000 \\ -0.027 \\ -0.038 \\ -0.017 \\ -0.027 \end{pmatrix} \text{ in}$$

Determination of effective elastic properties :

From figure UHX-11.2 - Polynomial equations for the determination of E^*/E and ν^* for triangular pattern

$$\begin{aligned}
 &\text{Equations for } E^* \text{ for triangular pitch} \\
 E^* \text{TriPitch} &:= \begin{pmatrix} 0.0353 & 1.2502 & -0.0491 & 0.3604 & -0.6100 \\ 0.0135 & 0.9910 & 1.0080 & -1.0498 & 0.0184 \\ 0.0054 & 0.5279 & 3.0461 & -4.3657 & 1.9435 \\ -0.0029 & 0.2126 & 3.9906 & -6.1730 & 3.4307 \end{pmatrix} \nu^* \text{TriPitch} := \begin{pmatrix} -0.0958 & 0.6209 & -0.8683 & 2.1099 & -1.6831 \\ 0.8897 & -9.0855 & 36.1435 & -59.5425 & 35.8223 \\ 0.7439 & -4.4989 & 12.5779 & -14.2092 & 5.7822 \\ 0.9100 & -4.8901 & 12.4325 & -12.7039 & 4.4298 \\ 0.9923 & -4.8759 & 12.3572 & -13.7214 & 5.7629 \\ 0.9966 & -4.1978 & 9.0478 & -7.9955 & 2.2398 \end{pmatrix}
 \end{aligned}$$

From figure UHX-11.3 - Polynomial equations for the determination of E^*/E and v^* for square pattern

$$\text{Equations for } E^* \text{ for square pitch} \quad \begin{pmatrix} -0.0791 & 0.6008 & -0.3468 & 0.4858 & -0.3606 \\ 0.3345 & -2.8420 & 10.9709 & -15.8994 & 8.3516 \\ 0.4296 & -2.6350 & 8.6864 & -11.5227 & 5.8544 \\ 0.3636 & -0.8057 & 2.0463 & -2.2902 & 1.1862 \\ 0.3527 & -0.2842 & 0.4354 & -0.0901 & -0.1590 \\ 0.3341 & 0.1260 & -0.6920 & 0.6877 & -0.0600 \end{pmatrix} v^* \text{SquPitch} :=$$

$$E^* \text{SquPitch} := \begin{pmatrix} 0.0676 & 1.5756 & -1.2119 & 1.7715 & -1.2628 \\ 0.0250 & 1.9251 & -3.5230 & 6.9830 & -5.0017 \\ 0.0394 & 1.3024 & -1.1041 & 2.8714 & -2.3994 \\ 0.0372 & 1.0314 & -0.6402 & 2.6201 & -2.1929 \end{pmatrix}$$

$$\text{Elay} := \text{si}(\text{Layout} > 0, E^* \text{SquPitch}, E^* \text{TriPitch})$$

$$E^*_1 := (\text{Elay}_{1,1} + \text{Elay}_{1,2} \cdot \mu^* + \text{Elay}_{1,3} \cdot \mu^{*2} + \text{Elay}_{1,4} \cdot \mu^{*3} + \text{Elay}_{1,5} \cdot \mu^{*4}) \cdot E \quad E^*_1 = 9887226.920 \frac{\text{lb}}{\text{in}^2}$$

$$E^*_2 := (\text{Elay}_{2,1} + \text{Elay}_{2,2} \cdot \mu^* + \text{Elay}_{2,3} \cdot \mu^{*2} + \text{Elay}_{2,4} \cdot \mu^{*3} + \text{Elay}_{2,5} \cdot \mu^{*4}) \cdot E \quad E^*_2 = 8855509.805 \frac{\text{lb}}{\text{in}^2}$$

$$E^*_3 := (\text{Elay}_{3,1} + \text{Elay}_{3,2} \cdot \mu^* + \text{Elay}_{3,3} \cdot \mu^{*2} + \text{Elay}_{3,4} \cdot \mu^{*3} + \text{Elay}_{3,5} \cdot \mu^{*4}) \cdot E \quad E^*_3 = 7812059.585 \frac{\text{lb}}{\text{in}^2}$$

$$E^*_4 := (\text{Elay}_{4,1} + \text{Elay}_{4,2} \cdot \mu^* + \text{Elay}_{4,3} \cdot \mu^{*2} + \text{Elay}_{4,4} \cdot \mu^{*3} + \text{Elay}_{4,5} \cdot \mu^{*4}) \cdot E \quad E^*_4 = 6430317.644 \frac{\text{lb}}{\text{in}^2}$$

For h/p lower than 0.1, use $h/p = 0.1$. For values of h/p higher than 2, use $h/p = 2.0$

Otherwise perform a linear interpolation :

$$\text{hpratio} := \begin{pmatrix} 0.1 \\ 0.25 \\ 0.5 \\ 2.0 \end{pmatrix} \quad \text{interE}^* := \begin{pmatrix} E^*_1 \\ E^*_2 \\ E^*_3 \\ E^*_4 \end{pmatrix} \quad \frac{h}{p} = 1.1$$

$$E^* := \text{si}\left(\frac{h}{p} > 2, E^*_4, \text{si}\left(\frac{h}{p} \leq 0.10, E^*_1, \text{interplin}\left(\text{hpratio}, \text{interE}^*, \frac{h}{p}\right)\right)\right) \quad E^* = 7259362.809 \frac{\text{lb}}{\text{in}^2} \quad \frac{E^*}{E} = 0.275$$

$$v\text{lay} := \text{si}(\text{Layout} > 0, v^* \text{SquPitch}, v^* \text{TriPitch})$$

$$v^*_1 := v\text{lay}_{1,1} + v\text{lay}_{1,2} \cdot \mu^* + v\text{lay}_{1,3} \cdot \mu^{*2} + v\text{lay}_{1,4} \cdot \mu^{*3} + v\text{lay}_{1,5} \cdot \mu^{*4} \quad v^*_1 = 0.042$$

$$v^*_2 := v\text{lay}_{2,1} + v\text{lay}_{2,2} \cdot \mu^* + v\text{lay}_{2,3} \cdot \mu^{*2} + v\text{lay}_{2,4} \cdot \mu^{*3} + v\text{lay}_{2,5} \cdot \mu^{*4} \quad v^*_2 = 0.090$$

$$v^*_3 := v\text{lay}_{3,1} + v\text{lay}_{3,2} \cdot \mu^* + v\text{lay}_{3,3} \cdot \mu^{*2} + v\text{lay}_{3,4} \cdot \mu^{*3} + v\text{lay}_{3,5} \cdot \mu^{*4} \quad v^*_3 = 0.197$$

$$v^*_4 := v\text{lay}_{4,1} + v\text{lay}_{4,2} \cdot \mu^* + v\text{lay}_{4,3} \cdot \mu^{*2} + v\text{lay}_{4,4} \cdot \mu^{*3} + v\text{lay}_{4,5} \cdot \mu^{*4} \quad v^*_4 = 0.269$$

$$v^*_5 := v\text{lay}_{5,1} + v\text{lay}_{5,2} \cdot \mu^* + v\text{lay}_{5,3} \cdot \mu^{*2} + v\text{lay}_{5,4} \cdot \mu^{*3} + v\text{lay}_{5,5} \cdot \mu^{*4} \quad v^*_5 = 0.336$$

$$v^*_6 := v\text{lay}_{6,1} + v\text{lay}_{6,2} \cdot \mu^* + v\text{lay}_{6,3} \cdot \mu^{*2} + v\text{lay}_{6,4} \cdot \mu^{*3} + v\text{lay}_{6,5} \cdot \mu^{*4} \quad v^*_6 = 0.376$$

$$\text{interv} := \begin{pmatrix} v^*_1 \\ v^*_2 \\ v^*_3 \\ v^*_4 \\ v^*_5 \\ v^*_6 \end{pmatrix} \quad \text{hpratio} := \begin{pmatrix} 0.1 \\ 0.15 \\ 0.25 \\ 0.50 \\ 1.00 \\ 2.00 \end{pmatrix} \quad \text{For } h/p \text{ lower than 0.1, use } h/p = 0.1. \text{ For values of } h/p \text{ higher than 2, use } h/p = 2.0$$

Otherwise perform a linear interpolation:

$$v^* := \text{si}\left(\frac{h}{p} > 2, v^*_6, \text{si}\left(\frac{h}{p} \leq 0.10, v^*_1, \text{interplin}\left(\text{hpratio}, \text{interv}, \frac{h}{p}\right)\right)\right)$$

$$\frac{h}{p} = 1.100 \quad v^* = 0.340$$

$$A_p := \pi r_o^2 \quad A_p = 1272.394 \text{ in}^2$$

$$C_p := 2\pi r_o \quad C_p = 126.449 \text{ in}$$

ANNEX Z—UHX12 – EXAMPLE 4 (2010 EDITION)

U-TUBE Tubesheet Rules accord. to UHX-12 (July 2010 Edition)

highlighted in Yellow :most important data and results

Case UHX-20.1.4

Geometric Data (from Fig.UHX-13.1)

set Farenheit temp.

degF := R

$$h := 3.5 \cdot \text{in}$$

Layout := 1 For triangular pitch "Layout"=0
For square pitch "Layout"=1

Config := "e"

Configuration types: a, b, c, d, e, f

"a" for shell/channel integral both sides
"b" for shell integral, channel gask - TS extended
"c" for shell integral, channel gask - TS not extended
"d" for gasketed both sides
"e" for channel integral, shell gask - TS extended
"f" for channel integral, shell gask - TS not extended

$$r_o := 12.75 \cdot \text{in} \quad \text{Radius to outer tube}$$

$$A := 37.25 \cdot \text{in} \quad \text{Outside Diameter of Tubesheet}$$

$$C := 35 \cdot \text{in} \quad \text{Bolt Circle Diameter}$$

$$A_L := 36.1 \cdot \text{in}^2 \quad \text{Total Untubed Lanes Area}$$

$$h_g := 0 \cdot \text{in} \quad \text{Tubesheet Corr. Allow. (Tubeside)}$$

$$c_t := 0 \cdot \text{in} \quad \text{Tubesheet Corr. Allow. (Shellside)}$$

$$c_s := 0 \cdot \text{in} \quad \text{Tubesheet Corr. Allow. (Shellside)}$$

$$p := 1 \cdot \text{in} \quad \text{Tube Pitch}$$

$$N_t := 496 \quad \text{Number of Tubes}$$

$$d_t := 0.75 \cdot \text{in} \quad \text{Tube Outside Diameter}$$

$$t_t := 0.085 \cdot \text{in} \quad \text{Tube Thickness}$$

$$\rho := 1 \quad \text{Tube expansion depth ratio}$$

$$l_{tx} := 0 \cdot \text{in} \quad \text{Length of Expanded Portion of Tube}$$

SS := "NO"

SS="YES" for Simply Supported TS calculation (for config. a,b,c,e,f only)

$$D_s := 0 \cdot \text{in}$$

Shell ID

$$t_s := 0.5 \cdot \text{in} \quad \text{Shell Thickness}$$

$$G_s := 32.375 \cdot \text{in} \quad \text{Shell Gasket Diameter}$$

$$C_s := 0 \cdot \text{in} \quad \text{Shell Corrosion Allowance}$$

$$D_c := 31 \cdot \text{in}$$

Channel ID

$$t_c := 0.625 \cdot \text{in} \quad \text{Channel Thickness}$$

$$G_c := 0 \cdot \text{in} \quad \text{Channel Gasket Diameter}$$

$$G_1 := 0 \cdot \text{in} \quad \text{Contact mid-point TS/Flange}$$

$$C_c := 0 \cdot \text{in} \quad \text{Channel Corrosion Allowance}$$

CHAN := "CYL"

"CYL" for Cylindrical Channel
"HEMI" for Hemispherical Channel
(for config. a,e,f,only)

Corroded thicknesses:

$$h := h - c_s - c_t \quad \text{Tubesheet thickness} \quad h = 3.500 \text{ in}$$

$$t_s := t_s - C_s \quad \text{Shell Thickness} \quad t_s = 0.500 \text{ in}$$

$$t_c := t_c - C_c \quad \text{Channel Thickness} \quad t_c = 0.625 \text{ in}$$

Corroded diameters:

$$D_s := D_s + 2 \cdot C_s \quad \text{Shell ID} \quad D_s = 0.000 \text{ in}$$

$$D_c := D_c + 2 \cdot C_c \quad \text{Channel ID} \quad D_c = 31.000 \text{ in}$$

Loading conditions

$$W_s := 656000 \cdot \text{lb}$$

shell flange design bolt load

$$W_c := 0 \cdot \text{lb}$$

channel flange design bolt load

$$W_{\max} := \max(W_s, W_c) \quad W_{\max} = 656000.000 \text{ lb}$$

$$P_{s0} := 650 \cdot \frac{\text{lb}}{\text{in}^2}$$

Shell Design Pressure

$$P_{t0} := 650 \cdot \frac{\text{lb}}{\text{in}^2}$$

Tube Design Pressure

Design Pressures for the Seven Load Cases

$$P_s := \begin{pmatrix} 0 \cdot \frac{\text{lb}}{\text{in}^2} \\ P_{s0} \\ P_{s0} \end{pmatrix} \quad P_s = \begin{pmatrix} 0.000 \\ 650.000 \\ 650.000 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$P_t := \begin{pmatrix} P_{t0} \\ 0 \cdot \frac{\text{lb}}{\text{in}^2} \\ P_{t0} \end{pmatrix} \quad P_t = \begin{pmatrix} 650.000 \\ 0.000 \\ 650.000 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

Temperatures

$$T := 400 \cdot \text{degF}$$

Tubesheet design temp.

$$T_a := 70 \cdot \text{degF}$$

Ambient temperature

$$T_s := 400 \cdot \text{degF}$$

Shell design temp.

$$T_c := 400 \cdot \text{degF}$$

Channel design temp.**Tubesheet Material is SA-516****Material Data**

$$S := 20000 \cdot \frac{\text{lb}}{\text{in}^2}$$

TS allowable stress @ T

$$E := 27.7 \cdot 10^6 \cdot \frac{\text{lb}}{\text{in}^2}$$

TS elastic modulus @ T**Tube Material is SA-179**

$$S_{tT} := 13400 \cdot \frac{\text{lb}}{\text{in}^2}$$

Tube allowable stress @ T_t

$$E_{tT} := 27.7 \cdot 10^6 \cdot \frac{\text{lb}}{\text{in}^2}$$

Tube elastic modulus @ T_t **Shell Material is SA-516**

$$S_s := 0 \cdot \frac{\text{lb}}{\text{in}^2}$$

Shell allow. stress @ T_s

$$S_{ys} := 0 \cdot \frac{\text{lb}}{\text{in}^2}$$

Shell yield stress @ T_s

$$E_s := 0 \cdot \frac{\text{lb}}{\text{in}^2}$$

Thin Shell elast mod. @ T_s

$$S_{PS.s} := 0 \cdot \frac{\text{lb}}{\text{in}^2}$$

Shell allowable P+S stress @ T_s

$$\nu_s := 0.3$$

Shell Poisson' ratio**Channel Material is SA-516**

$$S_c := 20000 \cdot \frac{\text{lb}}{\text{in}^2}$$

Channel allow. stress @ T_c

$$S_{yc} := 32500 \cdot \frac{\text{lb}}{\text{in}^2}$$

Channel yield stress @ T_c

$$E_c := 27.7 \cdot 10^6 \cdot \frac{\text{lb}}{\text{in}^2}$$

Channel elast. modulus @ T_c

$$S_{PS.c} := 65000 \cdot \frac{\text{lb}}{\text{in}^2}$$

Channel allowable P+S stress @ T_c

$$\nu_c := 0.3$$

Channel Poisson's ratio

Start of Calculations

$$h'_g := \max(h_g - c_t, 0) h'_g = 0.000 \text{ in} \quad \rho := \frac{l_{tx}}{h} \quad \rho = 1.000 \quad d^* := \max \left[d_t - 2 \cdot t_t \cdot \left(\frac{E_{tT}}{E} \right) \cdot \left(\frac{S_{tT}}{S} \right) \cdot \rho, (d_t - 2t_t) \right] \quad d^* = 0.636 \text{ in}$$

UHX-12.5.1. Step 1 Determine D_o , μ , μ^* and h'_g from UHX-11.5.1 :

$$D_o := 2 \cdot r_o + d_t \quad D_o = 26.250 \text{ in} \quad p^* := \frac{p}{\sqrt{1 - \frac{4 \cdot \min(A_L, 4D_o \cdot p)}{\pi \cdot D_o^2}}} \quad p^* = 1.035 \times 10^0 \text{ in} \quad \mu^* := \frac{p^* - d^*}{p^*} \quad \mu^* = 0.385$$

$$\mu := \frac{p - d_t}{p} \quad \mu = 0.250$$

$$\rho_s := \begin{cases} \frac{D_s}{D_o} & \text{if } (\text{Config} = \text{"a"}) \vee (\text{Config} = \text{"b"}) \vee (\text{Config} = \text{"c"}) \\ \frac{G_s}{D_o} & \text{if } (\text{Config} = \text{"d"}) \vee (\text{Config} = \text{"e"}) \vee (\text{Config} = \text{"f"}) \end{cases} \quad \rho_s = 1.233$$

$$\rho_c := \begin{cases} \frac{D_c}{D_o} & \text{if } (\text{Config} = \text{"a"}) \vee (\text{Config} = \text{"e"}) \vee (\text{Config} = \text{"f"}) \\ \frac{G_c}{D_o} & \text{if } (\text{Config} = \text{"b"}) \vee (\text{Config} = \text{"c"}) \vee (\text{Config} = \text{"d"}) \end{cases} \quad \rho_c = 1.181$$

$$M_{TS} := \frac{D_o^2}{16} \cdot \left[(\rho_s - 1) \cdot (\rho_s^2 + 1) \cdot P_s - (\rho_c - 1) \cdot (\rho_c^2 + 1) \cdot P_t \right] \quad M_{TS} = \begin{pmatrix} -1.213 \times 10^4 \\ 1.647 \times 10^4 \\ 4.337 \times 10^3 \end{pmatrix} \text{ lb}$$

UHX-12.5.3 Step 3 Calculate h/p. If p changes, recalculate d* and μ* from UHX-11.5.1. Determine E*/E and v* relative to h/p from UHX-11.5.2

$$\frac{h}{p} = 3.500 \quad \mu^* = 0.385 \quad \frac{E^*}{E} = 0.4413 \quad E^* = 1.222 \times 10^7 \frac{\text{lb}}{\text{in}^2} \quad v^* = 0.318 \quad (\text{From right pages above})$$

UHX-12.5.4 Step 4 Calculate coefficients shell and channel parameters: β, k, λ, δ and ω:

Use SS=YES for Simply Supported calculation in a 2nd step (see UHX-13.9)

SS = "NO"

$$\beta_s := \begin{cases} \frac{[12 \cdot (1 - v_s^2)]^{0.25}}{[(D_s + t_s) \cdot t_s]^{0.5}} & \text{if SS = "NO" } \wedge (\text{Config} = \text{"a"} \vee \text{Config} = \text{"b"} \vee \text{Config} = \text{"c"}) \\ 0 \cdot \frac{1}{\text{in}} & \text{otherwise} \end{cases} \quad \beta_s = 0.000 \frac{1}{\text{in}}$$

$$\beta_c := \begin{cases} \frac{[12 \cdot (1 - v_c^2)]^{0.25}}{[(D_c + t_c) \cdot t_c]^{0.5}} & \text{if SS = "NO" } \wedge (\text{Config} = \text{"a"} \vee \text{Config} = \text{"e"} \vee \text{Config} = \text{"f"}) \\ 0 \cdot \frac{1}{\text{in}} & \text{otherwise} \end{cases} \quad \beta_c = 0.409 \frac{1}{\text{in}}$$

$$k_s := \beta_s \cdot \frac{E_s \cdot t_s^3}{6 \cdot (1 - v_s^2)} \quad k_s = 0 \times 10^0 \text{ lb}$$

$$k_c := \beta_c \cdot \frac{E_c \cdot t_c^3}{6 \cdot (1 - v_c^2)} \quad k_c = 5.064 \times 10^5 \text{ lb}$$

$$h'_s := h \cdot \beta_s \quad h'_s = 0.000 \times 10^0$$

$$h'_c := h \cdot \beta_c \quad h'_c = 1.431 \times 10^0$$

$$\lambda_s := \frac{6 \cdot D_s}{h^3} \cdot k_s \cdot \left(1 + h'_s + \frac{h_s'^2}{2}\right) \quad \lambda_s = 0.000 \times 10^0 \frac{\text{lb}}{\text{in}^2}$$

$$\lambda_c := \frac{6 \cdot D_c}{h^3} \cdot k_c \cdot \left(1 + h'_c + \frac{h_c'^2}{2}\right) \quad \lambda_c = 7.591 \times 10^6 \frac{\text{lb}}{\text{in}^2}$$

$$\delta_s := \begin{cases} \frac{D_s^2}{4E_s \cdot t_s} \left(1 - \frac{v_s}{2}\right) & \text{if SS = "NO" } \wedge (\text{Config} = \text{"a"} \vee \text{Config} = \text{"b"} \vee \text{Config} = \text{"c"}) \\ 0 \cdot (\text{in}^3 \text{ lb}^{-1}) & \text{otherwise} \end{cases} \quad \delta_s = 0 \times 10^0 \text{ in}^3 \text{ lb}^{-1}$$

CHAN = "CYL" SS = "NO"

$$\delta_c := \begin{cases} \frac{D_c^2}{4E_c \cdot t_c} \left(1 - \frac{v_c}{2}\right) & \text{if SS = "NO" } \wedge \text{CHAN} = \text{"CYL"} \wedge (\text{Config} = \text{"a"} \vee \text{Config} = \text{"e"} \vee \text{Config} = \text{"f"}) \\ \frac{D_c^2}{4E_c \cdot t_c} \left(\frac{1 - v_c}{2}\right) & \text{if SS = "NO" } \wedge \text{CHAN} = \text{"HEMI"} \wedge (\text{Config} = \text{"a"} \vee \text{Config} = \text{"e"} \vee \text{Config} = \text{"f"}) \\ 0 \cdot (\text{in}^3 \text{ lb}^{-1}) & \text{otherwise} \end{cases}$$

$$\delta_c = 1.18 \times 10^{-5} \text{ in}^3 \text{ lb}^{-1}$$

$$\omega_s := \rho_s \cdot k_s \cdot \beta_s \cdot \delta_s \cdot (1 + h'_s) \quad \omega_s = 0.000 \text{ in}^2$$

$$\omega_c := \rho_c \cdot k_c \cdot \beta_c \cdot \delta_c \cdot (1 + h'_c) \quad \omega_c = 7.013 \text{ in}^2$$

UHX-12.5.5 Step 5 Calculate diameter ratio K and coefficient F :

$$K := \frac{A}{D_o} \quad K = 1.419$$

$$F := \begin{cases} \left[\frac{1 - v^*}{E^*} \cdot (\lambda_s + \lambda_c + E \cdot \ln(K)) \right] & \text{if (Config = "a")} \\ \left[\frac{1 - v^*}{E^*} \cdot (\lambda_s + E \cdot \ln(K)) \right] & \text{if (Config = "b") } \vee \text{ (Config = "c")} \\ \left[\frac{1 - v^*}{E^*} \cdot (E \cdot \ln(K)) \right] & \text{if (Config = "d")} \\ \left[\frac{1 - v^*}{E^*} \cdot (\lambda_c + E \cdot \ln(K)) \right] & \text{if (Config = "e") } \vee \text{ (Config = "f")} \\ 0 & \text{otherwise} \end{cases} \quad F = 0.964$$

UHX-12.5.6 Step 6 Calculate moment M* acting on the unperforated ts rim :

$$M^* := \begin{cases} (M_{TS} + \omega_c \cdot P_t - \omega_s \cdot P_s) & \text{if (Config = "a")} \\ \left(M_{TS} - \omega_s \cdot P_s - \frac{C - G_c}{2 \cdot \pi \cdot D_o} \cdot W_c \right) & \text{if (Config = "b")} \\ \left(M_{TS} - \omega_s \cdot P_s - \frac{G_1 - G_c}{2 \cdot \pi \cdot D_o} \cdot W_c \right) & \text{if (Config = "c")} \\ \left(M_{TS} + \frac{G_c - G_s}{2 \cdot \pi \cdot D_o} \cdot W_{\max} \right) & \text{if (Config = "d")} \\ \left(M_{TS} + \omega_c \cdot P_t + \frac{C - G_s}{2 \cdot \pi \cdot D_o} \cdot W_s \right) & \text{if (Config = "e")} \\ \left(M_{TS} + \omega_c \cdot P_t - \frac{G_1 - G_s}{2 \cdot \pi \cdot D_o} \cdot W_s \right) & \text{if (Config = "f")} \\ 0 \text{ lb} & \text{otherwise} \end{cases} \quad M^* = \begin{pmatrix} 2.869 \times 10^3 \\ 2.691 \times 10^4 \\ 1.934 \times 10^4 \end{pmatrix} \text{ lb}$$

UHX-12.5.7 Step 7 Calculate the max bending moments M_p (at the periphery) , M_o (at the center) :

$$M_p := \frac{M^* - \frac{D_o^2}{32} \cdot F \cdot (P_s - P_t)}{1 + F} \quad M_p = \begin{pmatrix} 8.332 \times 10^3 \\ 6.825 \times 10^3 \\ 9.843 \times 10^3 \end{pmatrix} \text{ lb}$$

$$M_o := M_p + \frac{D_o^2}{64} \cdot (3 + \nu^*) \cdot (P_s - P_t) \quad M_o = \begin{pmatrix} -1.489 \times 10^4 \\ 3.005 \times 10^4 \\ 9.843 \times 10^3 \end{pmatrix} \text{ lb}$$

$$M := \begin{pmatrix} \max(|M_{o1}|, |M_{p1}|) \\ \max(|M_{o2}|, |M_{p2}|) \\ \max(|M_{o3}|, |M_{p3}|) \end{pmatrix} \quad M = \begin{pmatrix} 1.489 \times 10^4 \\ 3.005 \times 10^4 \\ 9.843 \times 10^3 \end{pmatrix} \text{ lb}$$

$$M := \begin{cases} M_o & \text{if SS = "OUI" } \wedge (\text{Config} = \text{"a"} \vee \text{Config} = \text{"b"} \vee \text{Config} = \text{"c"}) \\ M_o & \text{if SS = "OUI" } \wedge (\text{Config} = \text{"a"} \vee \text{Config} = \text{"e"} \vee \text{Config} = \text{"f"}) \\ M & \text{otherwise} \end{cases} \quad M = \begin{pmatrix} 14887.535 \\ 30045.009 \\ 9842.828 \end{pmatrix} \text{ lb}$$

UHX-12.5.8 Step 8 Calculate the tubesheet bending stress σ :

$$\sigma := \frac{6 \cdot M}{\mu^* \cdot (h - h'_g)^2} \quad \sigma = \begin{pmatrix} 18916.215 \\ 38175.417 \\ 12506.372 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{\max} := \max(\sigma) \quad \sigma_{\max} = 38175.417 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{\text{allow}} := 2 \cdot S \quad \sigma_{\text{allow}} = 40000.000 \frac{\text{lb}}{\text{in}^2}$$

Calculate the minimum required thickness of the TS flanged extension for configurations b and e:

$$h_r := \begin{cases} \sqrt{\frac{1.9W_c \cdot C - G_c}{S \cdot G_c} \cdot \frac{C - G_c}{2}} & \text{if Config = "b"} \\ \sqrt{\frac{1.9W_s \cdot C - G_s}{S \cdot G_s} \cdot \frac{C - G_s}{2}} & \text{if Config = "e"} \\ 0 \cdot \text{in} & \text{otherwise} \end{cases} \quad h_r = 1.589 \text{ in}$$

UHX-12.5.9 Step 9 Calculate shear stress in the tubesheet τ :

$$\tau := \left(\frac{1}{4 \cdot \mu} \right) \cdot \left(\frac{D_o}{h} \right) \cdot |P_s - P_t| \quad \tau = \begin{pmatrix} 4875.000 \\ 4875.000 \\ 0.000 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$\tau_{\max} := \max(\tau) \quad \tau_{\max} = 4875.000 \frac{\text{lb}}{\text{in}^2}$$

$$\tau_{\text{allow}} := 0.8 \cdot S \quad \tau_{\text{allow}} = 16000.000 \frac{\text{lb}}{\text{in}^2}$$

If $|P_s - P_t| > 3.2S\mu h / D_o$, the TEMA formula may be used, provided the data A_p and C_p have been defined.

$$P_s - P_t = \begin{pmatrix} -650.000 \\ 650.000 \\ 0.000 \end{pmatrix} \frac{\text{lb}}{\text{in}^2} \quad 3.2 \cdot S \cdot \mu \cdot \frac{h}{D_o} = 2133.333 \frac{\text{lb}}{\text{in}^2}$$

$$A_p := 510.7 \cdot \text{in}^2 \quad \text{Total area enclosed by } C_p$$

$$C_p := 80.1 \cdot \text{in} \quad \text{Perimeter of the tube layout measured stepwise in increment of one tube pitch from the center-to-center of the outermost tubes (see Fig. UHX-12.2)}$$

$$\tau := \left(\frac{1}{\mu} \right) \cdot \frac{1}{h} \left(\frac{A_p}{C_p} \right) \cdot |P_s - P_t| \quad \tau = \begin{pmatrix} 4736.294 \\ 4736.294 \\ 0.000 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

UHX-12.5.10 Step 10 Calculate stresses in the shel and/or channel integral with the tubesheet (config.a,b,c):

$$\sigma_{\text{zero}} := \begin{pmatrix} 0 \cdot \frac{\text{lb}}{\text{in}^2} \\ 0 \cdot \frac{\text{lb}}{\text{in}^2} \\ 0 \cdot \frac{\text{lb}}{\text{in}^2} \end{pmatrix}$$

$$\sigma_{\text{s.m}} := \begin{cases} \left[\frac{D_s^2}{4 \cdot t_s \cdot (D_s + t_s)} \cdot P_s \right] & \text{if (Config = "a") } \vee \text{(Config = "b")} \vee \text{(Config = "c")} \\ \sigma_{\text{zero}} & \text{otherwise} \end{cases}$$

$$\sigma_{\text{s.m}} = \begin{pmatrix} 0.000 \\ 0.000 \\ 0.000 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{\text{sb}} := \frac{6}{t_s} \cdot k_s \cdot \left[\beta_s \cdot \delta_s \cdot P_s + 6 \cdot \frac{1 - \nu^*}{E^*} \cdot \frac{D_o}{h^3} \cdot \left(1 + \frac{h'_s}{2} \right) \cdot \left[M_p + \frac{D_o^2}{32} \cdot (P_s - P_t) \right] \right]$$

$$\sigma_{\text{s.b}} := \begin{cases} \sigma_{\text{sb}} & \text{if (Config = "a") } \vee \text{(Config = "b")} \vee \text{(Config = "c")} \\ \sigma_{\text{zero}} & \text{otherwise} \end{cases}$$

$$\sigma_{\text{s.b}} = \begin{pmatrix} 0.000 \\ 0.000 \\ 0.000 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_s := \overrightarrow{(|\sigma_{\text{s.m}}| + |\sigma_{\text{s.b}}|)} \quad \sigma_s = \begin{pmatrix} 0.000 \\ 0.000 \\ 0.000 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{\text{max}_s} := \max(\sigma_s)$$

$$\sigma_{\text{max}_s} = 0.000 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{\text{allows}} := 1.5 \cdot S_s$$

$$\sigma_{\text{allows}} = 0.000 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{\text{c.m}} := \begin{cases} \left[\frac{D_c^2}{4 \cdot t_c \cdot (D_c + t_c)} \cdot P_t \right] & \text{if (Config = "a") } \vee \text{(Config = "e")} \vee \text{(Config = "f")} \\ \sigma_{\text{zero}} & \text{otherwise} \end{cases}$$

$$\sigma_{\text{c.m}} = \begin{pmatrix} 7900.711 \\ 0.000 \\ 7900.711 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{\text{cb}} := \frac{6}{t_c} \cdot k_c \cdot \left[(\beta_c \cdot \delta_c \cdot P_t) - 6 \cdot \frac{1 - \nu^*}{E^*} \cdot \frac{D_o}{h^3} \cdot \left(1 + \frac{h'_c}{2} \right) \cdot \left[M_p + \frac{D_o^2}{32} \cdot \overrightarrow{(P_s - P_t)} \right] \right]$$

$$\sigma_{\text{c.b}} := \begin{cases} \sigma_{\text{cb}} & \text{if (Config = "a") } \vee \text{(Config = "e")} \vee \text{(Config = "f")} \\ \sigma_{\text{zero}} & \text{otherwise} \end{cases}$$

$$\sigma_{\text{c.b}} = \begin{pmatrix} 39880.909 \\ -56954.890 \\ -2536.635 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_c := \overrightarrow{(|\sigma_{\text{c.m}}| + |\sigma_{\text{c.b}}|)} \quad \sigma_c = \begin{pmatrix} 47781.621 \\ 56954.890 \\ 10437.346 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{\text{max}_c} := \max(\sigma_c)$$

$$\sigma_{\text{max}_c} = 56954.890 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{\text{allowc}} := 1.5 \cdot S_c$$

$$\sigma_{\text{allowc}} = 30000.000 \frac{\text{lb}}{\text{in}^2}$$

UHX-12.5.11 Step 11 Simplified Elastic-Plastic Procedure :

NOTE : This option may be used when : - Configuration a -----> $\sigma_s \leq S_{PS,s}$ and $\sigma_c \leq S_{PS,c}$
 - Configuration b , c -----> $\sigma_s \leq S_{PS,s}$
 - Configuration e , f -----> $\sigma_c \leq S_{PS,c}$

$$E^*_s := E_s \cdot \sqrt{\frac{1.5 \cdot S_s}{\sigma_s}} \quad E^*_s = \begin{pmatrix} 0.000 \times 10^0 \\ 0.000 \times 10^0 \\ 0.000 \times 10^0 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$E^*_c := E_c \cdot \sqrt{\frac{1.5 \cdot S_c}{\sigma_c}} \quad E^*_c = \begin{pmatrix} 2.195 \times 10^7 \\ 2.010 \times 10^7 \\ 4.696 \times 10^7 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$E^*_s := \begin{pmatrix} \min(E^*_{s1}, E_s) \\ \min(E^*_{s2}, E_s) \\ \min(E^*_{s3}, E_s) \end{pmatrix} \quad E^*_s = \begin{pmatrix} 0.000 \times 10^0 \\ 0.000 \times 10^0 \\ 0.000 \times 10^0 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$E^*_c := \begin{pmatrix} \min(E^*_{c1}, E_c) \\ \min(E^*_{c2}, E_c) \\ \min(E^*_{c3}, E_c) \end{pmatrix} \quad E^*_c = \begin{pmatrix} 2.195 \times 10^7 \\ 2.010 \times 10^7 \\ 2.770 \times 10^7 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

From step 4 :

$$k_s := \beta_s \cdot \frac{E^*_s \cdot t_s^3}{6 \cdot (1 - \nu_s^2)} \quad k_s = \begin{pmatrix} 0.000 \times 10^0 \\ 0.000 \times 10^0 \\ 0.000 \times 10^0 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$\lambda_s := \frac{6 \cdot D_s}{h^3} \cdot k_s \cdot \left(1 + h'_s + \frac{h_s'^2}{2}\right) \quad \lambda_s = \begin{pmatrix} 0.000 \times 10^0 \\ 0.000 \times 10^0 \\ 0.000 \times 10^0 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$k_c := \beta_c \cdot \frac{E^*_c \cdot t_c^3}{6 \cdot (1 - \nu_c^2)} \quad k_c = \begin{pmatrix} 4.013 \times 10^5 \\ 3.676 \times 10^5 \\ 5.064 \times 10^5 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$\lambda_c := \frac{6 \cdot D_c}{h^3} \cdot k_c \cdot \left(1 + h'_c + \frac{h_c'^2}{2}\right) \quad \lambda_c = \begin{pmatrix} 6.015 \times 10^6 \\ 5.509 \times 10^6 \\ 7.591 \times 10^6 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

From step 5 :

$$F := \begin{cases} \left[\frac{1 - \nu^*}{E^*} \cdot (\lambda_s + \lambda_c + E \cdot \ln(K)) \right] & \text{if (Config = "a")} \\ \left[\frac{1 - \nu^*}{E^*} \cdot (\lambda_s + E \cdot \ln(K)) \right] & \text{if (Config = "b") } \vee \text{ (Config = "c")} \\ \left[\frac{1 - \nu^*}{E^*} \cdot (\lambda_c + E \cdot \ln(K)) \right] & \text{if (Config = "e") } \vee \text{ (Config = "f")} \\ 0 & \text{otherwise} \end{cases} \quad F = \begin{pmatrix} 0.877 \\ 0.848 \\ 0.964 \end{pmatrix}$$

From step 7 :

$$M_p := \frac{M^* - \frac{D_o^2}{32} \cdot F \cdot (P_s - P_t)}{1 + F} \quad M_p = \begin{pmatrix} 8.332 \times 10^3 \\ 6.825 \times 10^3 \\ 9.843 \times 10^3 \end{pmatrix} \text{lb}$$

$$M_o := \left[M_p + \frac{D_o^2}{64} \cdot (3 + \nu^*) \cdot (P_s - P_t) \right] \quad M_o = \begin{pmatrix} -1.515 \times 10^4 \\ 3.135 \times 10^4 \\ 9.843 \times 10^3 \end{pmatrix} \text{lb}$$

$$M := \begin{pmatrix} \max(|M_{p1}|, |M_{o1}|) \\ \max(|M_{p2}|, |M_{o2}|) \\ \max(|M_{p3}|, |M_{o3}|) \end{pmatrix} \quad M = \begin{pmatrix} 1.515 \times 10^4 \\ 3.135 \times 10^4 \\ 9.843 \times 10^3 \end{pmatrix} \text{lb}$$

From step 8 :

$$\sigma := \frac{6 \cdot M}{\mu^* \cdot (h - h'_g)^2} \quad \sigma = \begin{pmatrix} 19253.502 \\ 39838.016 \\ 12506.372 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{\max} := \max(\sigma) \quad \sigma_{\max} = 39838.016 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{\text{allow}} := 2 \cdot S \quad \sigma_{\text{allow}} = 40000.000 \frac{\text{lb}}{\text{in}^2}$$

UHX12.6 Simply Supported Tubesheet Procedure

This procedure applies only when the TS is integral with the shell or channel, i.e. :

- shell of configurations a,b,c
- channel of configuration a,e,f

SS = "NO"

Calculation must be performed in 2 phases :

Phase 1) Perform Steps 1 to 10 with **SS="NON"** (normal calculation) with the following modifications in Step 10:

- minimum length requirement l_{min} of shell band for configurations a,b,c do not apply
- minimum length requirement l_{cmin} of channel band for configurations a,e,f do not apply

if $\sigma_s \leq S_{PS,s}$ and $\sigma_c \leq S_{PS,c}$, the shell and/or channel designs are acceptable.

Otherwise increase the thickness of the overstressed components (shell and/or channel) and return to Step 1.

Phase 2) Perform Steps 1 to 7 using **SS="OUI"** (Simply Supported calculation) for loading cases 1,2 and 3 only

If $|\sigma| \leq 1.5S$, the calculation procedure is complete.

Otherwise, increase the assumed tubesheet thickness h and repeat Steps 1 to 8, using in Step 7 : $M=|M_0|$

Note: If $|\sigma| < 1.5S$, the tubesheet thickness can be optimized to a value h_0 till $|\sigma|=1.5S$

provided that, for that optimized thickness h_0 , the stresses in the tubesheet, shell and channel calculated by the normal calculation (see Phase 1), remain respectively below S_{PS} , $S_{PS,s}$, $S_{PS,c}$ for each of the 7 loading cases.

Determination of effective elastic properties :

From figure UHX-11.2 - Polynomial equations for the determination of E^*/E and ν^* for triangular pattern

$$\begin{aligned}
 &\text{Equations for } E^* \text{ for triangular pitch} \\
 E^* \text{TriPitch} := &\begin{pmatrix} 0.0353 & 1.2502 & -0.0491 & 0.3604 & -0.6100 \\ 0.0135 & 0.9910 & 1.0080 & -1.0498 & 0.0184 \\ 0.0054 & 0.5279 & 3.0461 & -4.3657 & 1.9435 \\ -0.0029 & 0.2126 & 3.9906 & -6.1730 & 3.4307 \end{pmatrix} \\
 \nu^* \text{TriPitch} := &\begin{pmatrix} -0.0958 & 0.6209 & -0.8683 & 2.1099 & -1.6831 \\ 0.8897 & -9.0855 & 36.1435 & -59.5425 & 35.8223 \\ 0.7439 & -4.4989 & 12.5779 & -14.2092 & 5.7822 \\ 0.9100 & -4.8901 & 12.4325 & -12.7039 & 4.4298 \\ 0.9923 & -4.8759 & 12.3572 & -13.7214 & 5.7629 \\ 0.9966 & -4.1978 & 9.0478 & -7.9955 & 2.2398 \end{pmatrix}
 \end{aligned}$$

Equations for E* for square pitch

$$E^*_{\text{SquPitch}} := \begin{pmatrix} 0.0676 & 1.5756 & -1.2119 & 1.7715 & -1.2628 \\ 0.0250 & 1.9251 & -3.5230 & 6.9830 & -5.0017 \\ 0.0394 & 1.3024 & -1.1041 & 2.8714 & -2.3994 \\ 0.0372 & 1.0314 & -0.6402 & 2.6201 & -2.1929 \end{pmatrix} \quad v^*_{\text{SquPitch}} := \begin{pmatrix} -0.0791 & 0.6008 & -0.3468 & 0.4858 & -0.3606 \\ 0.3345 & -2.8420 & 10.9709 & -15.8994 & 8.3516 \\ 0.4296 & -2.6350 & 8.6864 & -11.5227 & 5.8544 \\ 0.3636 & -0.8057 & 2.0463 & -2.2902 & 1.1862 \\ 0.3527 & -0.2842 & 0.4354 & -0.0901 & -0.1590 \\ 0.3341 & 0.1260 & -0.6920 & 0.6877 & -0.0600 \end{pmatrix}$$

$$\text{Elay} := \text{si}(\text{Layout} > 0, E^*_{\text{SquPitch}}, E^*_{\text{TriPitch}})$$

$$\begin{aligned} E^*_1 &:= \left(\text{Elay}_{1,1} + \text{Elay}_{1,2} \cdot \mu^* + \text{Elay}_{1,3} \cdot \mu^{*2} + \text{Elay}_{1,4} \cdot \mu^{*3} + \text{Elay}_{1,5} \cdot \mu^{*4} \right) \cdot E & E^*_1 &= 4.894 \times 10^5 \frac{\text{sec}^2}{\text{psi}} \\ E^*_2 &:= \left(\text{Elay}_{2,1} + \text{Elay}_{2,2} \cdot \mu^* + \text{Elay}_{2,3} \cdot \mu^{*2} + \text{Elay}_{2,4} \cdot \mu^{*3} + \text{Elay}_{2,5} \cdot \mu^{*4} \right) \cdot E & E^*_2 &= 4.590 \times 10^5 \frac{\text{sec}^2}{\text{psi}} \\ E^*_3 &:= \left(\text{Elay}_{3,1} + \text{Elay}_{3,2} \cdot \mu^* + \text{Elay}_{3,3} \cdot \mu^{*2} + \text{Elay}_{3,4} \cdot \mu^{*3} + \text{Elay}_{3,5} \cdot \mu^{*4} \right) \cdot E & E^*_3 &= 4.209 \times 10^5 \frac{\text{sec}^2}{\text{psi}} \\ E^*_4 &:= \left(\text{Elay}_{4,1} + \text{Elay}_{4,2} \cdot \mu^* + \text{Elay}_{4,3} \cdot \mu^{*2} + \text{Elay}_{4,4} \cdot \mu^{*3} + \text{Elay}_{4,5} \cdot \mu^{*4} \right) \cdot E & E^*_4 &= 3.799 \times 10^5 \frac{\text{sec}^2}{\text{ft}} \end{aligned}$$

For h/p < 0.1, use h/p = 0.1. For values of h/p > 2, use h/p = 2.0. Otherwise perform a linear interpolation

$$\text{hpratio} := \begin{pmatrix} 0.1 \\ 0.25 \\ 0.5 \\ 2.0 \end{pmatrix} \quad \text{interE}^* := \begin{pmatrix} E^*_1 \\ E^*_2 \\ E^*_3 \\ E^*_4 \end{pmatrix} \quad \frac{h}{p} = 3.5$$

$$E^* := \text{si}\left(\frac{h}{p} > 2, E^*_4, \text{si}\left(\frac{h}{p} \leq 0.10, E^*_1, \text{interplin}\left(\text{hpratio}, \text{interE}^*, \frac{h}{p}\right)\right)\right) \quad E^* = 3.799 \times 10^5 \frac{\text{sec}^2}{\text{ft}} \frac{\text{lbf}}{\text{in}^2} \quad \frac{E^*}{E} = 0.4413$$

$$\text{vlay} := \text{si}(\text{Layout} > 0, v^*_{\text{SquPitch}}, v^*_{\text{TriPitch}})$$

$$\begin{aligned} v^*_1 &:= \text{vlay}_{1,1} + \text{vlay}_{1,2} \cdot \mu^* + \text{vlay}_{1,3} \cdot \mu^{*2} + \text{vlay}_{1,4} \cdot \mu^{*3} + \text{vlay}_{1,5} \cdot \mu^{*4} & v^*_1 &= 1.208 \times 10^{-1} \\ v^*_2 &:= \text{vlay}_{2,1} + \text{vlay}_{2,2} \cdot \mu^* + \text{vlay}_{2,3} \cdot \mu^{*2} + \text{vlay}_{2,4} \cdot \mu^{*3} + \text{vlay}_{2,5} \cdot \mu^{*4} & v^*_2 &= 1.429 \times 10^{-1} \\ v^*_3 &:= \text{vlay}_{3,1} + \text{vlay}_{3,2} \cdot \mu^* + \text{vlay}_{3,3} \cdot \mu^{*2} + \text{vlay}_{3,4} \cdot \mu^{*3} + \text{vlay}_{3,5} \cdot \mu^{*4} & v^*_3 &= 1.739 \times 10^{-1} \\ v^*_4 &:= \text{vlay}_{4,1} + \text{vlay}_{4,2} \cdot \mu^* + \text{vlay}_{4,3} \cdot \mu^{*2} + \text{vlay}_{4,4} \cdot \mu^{*3} + \text{vlay}_{4,5} \cdot \mu^{*4} & v^*_4 &= 2.521 \times 10^{-1} \\ v^*_5 &:= \text{vlay}_{5,1} + \text{vlay}_{5,2} \cdot \mu^* + \text{vlay}_{5,3} \cdot \mu^{*2} + \text{vlay}_{5,4} \cdot \mu^{*3} + \text{vlay}_{5,5} \cdot \mu^{*4} & v^*_5 &= 2.992 \times 10^{-1} \\ v^*_6 &:= \text{vlay}_{6,1} + \text{vlay}_{6,2} \cdot \mu^* + \text{vlay}_{6,3} \cdot \mu^{*2} + \text{vlay}_{6,4} \cdot \mu^{*3} + \text{vlay}_{6,5} \cdot \mu^{*4} & v^*_6 &= 3.179 \times 10^{-1} \end{aligned}$$

For h/p lower than 0.1, use h/p = 0.1; for values of h/p higher than 2, use h/p = 2.0, otherwise perform a linear interpolation:

$$\text{interv} := \begin{pmatrix} v^*_1 \\ v^*_2 \\ v^*_3 \\ v^*_4 \\ v^*_5 \\ v^*_6 \end{pmatrix} \quad \text{hpratio} := \begin{pmatrix} 0.1 \\ 0.15 \\ 0.25 \\ 0.50 \\ 1.00 \\ 2.00 \end{pmatrix} \quad \frac{h}{p} = 3.5$$

$$v^* := \text{si}\left(\frac{h}{p} > 2, v^*_6, \text{si}\left(\frac{h}{p} \leq 0.10, v^*_1, \text{interplin}\left(\text{hpratio}, \text{interv}, \frac{h}{p}\right)\right)\right) \quad v^* = 0.318$$

$$A_p := \pi r_o^2 \quad A_p = 510.705 \text{ in}^2$$

$$C_p := 2\pi r_o \quad C_p = 80.111 \text{ in}$$

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ABBREVIATIONS AND ACRONYMS

ASME – American Society of Mechanical Engineers

C&S – Codes and Standards

CL – Clamped

EEC – Effective Elastic Constants

FEA – Finite Element Analysis

FL – Floating

HE(s) – Heat Exchanger(s)

LE – Ligament Efficiency

ST – Stationary

SG-HTE – Subgroup on Heat-Transfer Equipment

TEMA – Tubular Exchanger Manufacturers Association

TS(s) – Tubesheet(s)

UHX – Unfired Heat Exchanger

