ASME PTC 11-2008
(Revision of ASME PTC 11-1984)

Fans

Performance Test Codes

AN AMERICAN NATIONAL STANDARD

The American Society of Mechanical Engineers

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SETTING THE STANDARD

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Date of Issuance: December 8, 2008

This Code will be revised when the Society approves the issuance of a new edition. There will be no addenda issued to PTC 11-2008.

ASME issues written replies to inquiries concerning interpretations of technical aspects of this document. Periodically certain actions of the ASME PTC Committee may be published as Code Cases. Code Cases and interpretations are published on the ASME Web site under the Committee Pages at http://cstools.asme.org as they are issued.

The American Society of Mechanical Engineers
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NOTICE

All Performance Test Codes must adhere to the requirements of ASME PTC 1, General Instructions. The following information is based on that document and is included here for emphasis and for the convenience of the user of the Code. It is expected that the Code user is fully cognizant of Sections 1 and 3 of ASME PTC 1 and has read them prior to applying this Code.

ASME Performance Test Codes provide test procedures that yield results of the highest level of accuracy consistent with the best engineering knowledge and practice currently available. They were developed by balanced committees representing all concerned interests and specify procedures, instrumentation, equipment-operating requirements, calculation methods, and uncertainty analysis.

When tests are run in accordance with a code, the test results themselves, without adjustment for uncertainty, yield the best available indication of the actual performance of the tested equipment. ASME Performance Test Codes do not specify means to compare those results with contractual guarantees. Therefore, it is recommended that the parties to a commercial test agree before starting the test and preferably before signing the contract on the method to be used for comparing the test results with the contractual guarantees. It is beyond the scope of any code to determine or interpret how such comparisons shall be made.

FOREWORD

PTC 11-1946, entitled Test Code for Fans, was published by the Society in 1946. As noted in its Foreword, the personnel of the committee that developed the Code consisted of members of the American Society of Heating and Ventilating Engineers, the National Association of Fan Manufacturers, and the American Society of Mechanical Engineers. The Code, as written, was a laboratory test standard in that it provided instructions for arrangement of test equipment, such as ducts, plenum chamber, and flow straighteners, as well as instruments. It even stated that the test could be conducted in the manufacturer's shops, the customer's premises, or elsewhere.

Most ASME Power Test Codes (later called Performance Test Codes) provided instructions for testing equipment after it was installed. Since PTC 11-1946 was a laboratory standard, it was allowed to go out of print with the expectation that a revised code would be written that would provide directions for site testing of fans.

In July of 1961, a new PTC 11 Committee was formed. Several drafts were prepared, but all of them essentially provided laboratory directions. This Committee still considered field or site testing to be impractical unless laboratory conditions could be duplicated.

The PTC 11 Committee was reorganized in 1971. It initially attempted to resolve the difficulties of site testing by resorting to model testing. This was not acceptable to the Society. Ultimately, procedures were developed that could be used in the field without the need to modify the installation so as to condition the flow for measurement. The Committee performed tests to determine the acceptability of these procedures. These tests included full-scale field tests of two large mechanical-draft fans, as well as various laboratory tests of various probes for measuring flow angles and pressures. Subsequent tests [3] performed independently of the Committee have demonstrated the practicability of this Code with regard to both manpower and equipment in a large power-plant situation.

The Committee also monitored the progress of an International Committee that was writing test codes for fans. While this Committee, ISO 117, had not completed its work, it was obvious that several things they were doing should be incorporated in PTC 11. The major item contributed by ISO 117 is the concept of specific energy (also called work per unit mass), which, when combined with mass flow rate, provides an approach to fan performance that can be used instead of the volume flow rate/pressure approach. ISO also recognizes the distributionality of velocity across the measuring plane, and PTC 11 incorporates provisions to account for this. This resulted in the second edition, published in 1984.

Work on the current revision began on January 17, 2002. The goal for this effort was to revise and update several sections to make the Code more universally accepted and user friendly. For example, additional points of agreement between parties to the test were developed. The number and geometry of the traverse grid elements were changed to allow greater variation in the aspect parameter. A statistical procedure was developed to guide the user in selection of traverse planes for defining fan flow. Greater emphasis was placed on the use of five-hole (three-dimensional) probes to completely characterize flow at the traverse plane(s). Guidance was included for establishing fan operation at test conditions so that it would be near specified conditions after all corrections have been applied. A procedure was developed to correct fan power from test conditions to specified conditions.

Historically, fan performance was typically based on design, or test block, conditions that represent the fan's ability to move a specific amount of gas at a specific system resistance. It is generally taken to be the fan's maximum performance capability. More recently, however, there has been increased emphasis in demonstrating fan performance at a power guarantee point usually corresponding to part load on a fan. This presents some unique testing challenges.

There have also been significant advancements in electronic technology. Readily available portable computers are now able to support off-the-shelf data acquisition systems to monitor key parameters and provide real-time trends of operational steadiness during a test. This capability extends to traverse data as well, where key pressures are electronically monitored to determine the alignment of directionally sensitive probes with flow, to average all pressures, and to archive all information. Repeatability of results is greatly improved because mental averaging and manual data logging are eliminated. Finally, data reduction turnaround time is greatly shortened, which increases the productivity of test personnel when multiple test runs are required or where test time may be limited.

While some installations may not meet ideal inlet and/or outlet conditions for flow distribution or geometry, the objective of this test code is to determine a fan's installed performance without listing any criteria for disqualification of this test procedure. The subcommittee has made every effort to include test and data reduction methods that will lead to results that will be acceptable to all parties to the test.

This Code was approved by the Council as a Standard practice of the Society by action of the Board on Standardization and Testing on April 7, 2008. It was also approved as an American National Standard by the ANSI Board of Standards Review on July 15, 2008.

ASME PTC COMMITTEE
PERFORMANCE TEST CODES

(The following is the roster of the Committee at the time of approval of this Code.)

PTC 11 COMMITTEE — FANS

S. P. Nuspl, *Chair*, The Babcock & Wilcox Co.
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C. W. Carr, Southern Company Services
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M. J. Magill, Howden Buffalo, Inc.
R. A. Moczadlo, TLT-Babcock, Inc.
R. T. Noble, Southern Company Services

CORRESPONDENCE WITH THE PTC 11 COMMITTEE

General. ASME Codes are developed and maintained with the intent to represent the consensus of concerned interests. As such, users of this Code may interact with the Committee by requesting interpretations, proposing revisions, and attending Committee meetings. Correspondence should be addressed to:

Secretary, PTC 11 Standards Committee
The American Society of Mechanical Engineers
Three Park Avenue
New York, NY 10016-5990

Proposing Revisions. Revisions are made periodically to the Code to incorporate changes that appear necessary or desirable, as demonstrated by the experience gained from the application of the Code. Approved revisions will be published periodically.

The Committee welcomes proposals for revisions to this Code. Such proposals should be as specific as possible, citing the paragraph number(s), the proposed wording, and a detailed description of the reasons for the proposal, including any pertinent documentation.

Proposing a Case. Cases may be issued for the purpose of providing alternative rules when justified, to permit early implementation of an approved revision when the need is urgent, or to provide rules not covered by existing provisions. Cases are effective immediately upon ASME approval and shall be posted on the ASME Committee Web page.

Requests for Cases shall provide a Statement of Need and Background Information. The request should identify the Code, the paragraph, figure or table number(s), and be written as a Question and Reply in the same format as existing Cases. Requests for Cases should also indicate the applicable edition(s) of the Code to which the proposed Case applies.

Interpretations. Upon request, the PTC 11 Committee will render an interpretation of any requirement of the Code. Interpretations can only be rendered in response to a written request sent to the Secretary of the PTC 11 Standards Committee.

The request for interpretation should be clear and unambiguous. It is further recommended that the inquirer submit his request in the following format:

Subject: Cite the applicable paragraph number(s) and a concise description.

Edition: Cite the applicable edition of the Code for which the interpretation is being requested.

Question: Phrase the question as a request for an interpretation of a specific requirement suitable for general understanding and use, not as a request for an approval of a proprietary design or situation. The inquirer may also include any plans or drawings that are necessary to explain the question; however, they should not contain proprietary names or information.

Requests that are not in this format will be rewritten in this format by the Committee prior to being answered, which may inadvertently change the intent of the original request.

ASME procedures provide for reconsideration of any interpretation when or if additional information that might affect an interpretation is available. Further, persons aggrieved by an interpretation may appeal to the cognizant ASME Committee. ASME does not “approve,” “certify,” “rate,” or “endorse” any item, construction, proprietary device, or activity.

Attending Committee Meetings. The PTC 11 Standards Committee holds meetings or telephone conferences, which are open to the public. Persons wishing to attend any meeting or telephone conference should contact the Secretary of the PTC 11 Standards Committee or check our Web site at http://cstools.asme.org.

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FANS

Section 1
Object and Scope

1-1 OBJECT

This Code provides standard procedures for conducting and reporting tests on fans, including those of the centrifugal, axial, and mixed flow types.

1-1.1 Objectives

The objectives of this Code are to provide:

(a) the rules for testing fans to determine performance under actual operating conditions

(b) additional rules for converting measured performance to that which would prevail under specified operating conditions

(c) methods for comparing measured or converted performance with specified performance

1-1.2 Principal Quantities

The principal quantities that can be determined are

(a) fan mass flow rate or, alternatively, fan volume flow rate

(b) fan specific energy or, alternatively, fan pressure

(c) fan input power

Henceforth, these parameters shall be inclusively covered by the term "performance."

1-1.3 Additional Quantities

Additional quantities that can be determined are

(a) gas properties at the fan inlet

(b) fan speed

Henceforth, these parameters shall be inclusively covered by the term "operating conditions."

1-1.4 Other Quantities

Various other quantities can be determined, including

(a) fan output power

(b) compressibility coefficient

(c) fan efficiency

(d) inlet flow conditions

1-2 SCOPE

The scope of this Code is limited to the testing of fans after they have been installed in the systems for which they were intended. However, the same directions can be followed in a laboratory test. (The laboratory test performance may not be duplicated by a test after installation because of system effects.) The term "fan" implies that the machine is used primarily for moving air or gas rather than compression. The distinction between fans, blowers, exhausters, and compressors in common practice is rather vague; accordingly, machines that bear any of these names may be tested under the provisions of this Code. (It is conceivable that these machines can also be tested under the provisions of PTC 10, Compressors and Exhausters.)

This Code does not include procedures for determining fan mechanical and acoustical characteristics.

1-3 APPLICABILITY

A fan test is considered an ASME Code test only if the test procedures comply with procedures and allowed variations specified by this Code.

1-4 UNCERTAINTY

The uncertainties of fan test results depend on features of the fan installation, such as duct configuration, and on parameters of the performance test, such as instruments selected, their locations, and number and frequency of readings. This Code requires a post-test uncertainty analysis as described herein and in accordance with PTC 19.1. The pretest uncertainty analysis, although nonmandatory, may be used to develop specific test procedures that result in meeting an agreed upon uncertainty. For a typical fan installation and performance test in accordance with this Code, the following uncertainties can be realized:

(a) fan flow rate, 2%

(b) fan specific energy/fan pressure, 1%

(c) fan input power, 1½%

Section 2
Definitions of Terms, Symbols, and Their Descriptions

2-1 SYMBOLS

The symbols tabulated below are shown with their primary definitions. However, some are redefined in the text for other purposes.

2-1.1 Symbols and Subscripted Symbols

Symbol	Description	Unit/Value	
		U.S. Customary	SI
A	Cross-sectional area of duct	ft^2	m^2
a	Parameter in eq. (5-11-13)	Dimensionless	Dimensionless
B_X	Systematic uncertainty in X	Same as X	Same as X
b	Parameter in eq. (5-10-7)	Dimensionless	Dimensionless
C	Cross-sectional area of calibration jet or wind tunnel	ft^2	m^2
C_1, C_2, etc.	See para. 2-1.3		
C_D	Drag coefficient of probe section	Dimensionless	Dimensionless
C_ϕ	Pitch pressure coefficient	Dimensionless	Dimensionless
c_p	Specific heat at constant pressure	Btu/lbm · °F	J/kg · K
c_v	Specific heat at constant volume	Btu/lbm · °F	J/kg · K
D	Duct diameter	ft	m
d	Duct cross-sectional dimension parallel to the fan shaft	ft	m
d_p	Probe diameter	ft	m
E	Electric potential (voltage)	V	V
e_K	Specific kinetic energy	ft · lb/lbm	J/kg
F_n	Number of points factor	Dimensionless	Dimensionless
f	Frequency	Hz	Hz
g	Local acceleration due to gravity	ft/sec^2	m/s^2
g_c	See para. 2-1.3		
H	Humidity ratio	Dimensionless	Dimensionless
I	Electric current (amperage)	A	A
J	See para. 2-1.3		
K_p	Compressibility coefficient (volume flow – pressure approach)	Dimensionless	Dimensionless

Symbol	Description	Unit/Value U.S. Customary	Unit/Value SI
K_t	Probe total pressure coefficient	Dimensionless	Dimensionless
K_v	Probe velocity pressure coefficient	Dimensionless	Dimensionless
K_ρ	Compressibility coefficient (mass flow – specific energy approach)	Dimensionless	Dimensionless
k	Ratio of specific heats (c_p/c_v)	Dimensionless	Dimensionless
M	Mach number	Dimensionless	Dimensionless
M	Molecular weight	lbm/lbm-mol	kg/kg-mol
$\dot{m}$	Mass flow rate	lbm/sec	kg/s
$\dot{m}_F$	Fan mass flow rate	lbm/sec	kg/s
N	Rotational speed	rpm	rev/s
N_s	Specified rotational speed	rpm	rev/s
n	Count or number	Dimensionless	Dimensionless
n_p	Number of poles	Dimensionless	Dimensionless
P_I	Fan input power	hp	kW
P_O	Fan output power	hp	kW
p_b	Barometric pressure	in. Hg	kPa
p_e	Saturated vapor pressure	in. Hg	kPa
p_{Fs}	Fan static pressure	in. wg [Note (1)]	kPa
p_{Ft}	Fan total pressure	in. wg	kPa
p_{Fv}	Fan velocity pressure	in. wg	kPa
p_p	Partial pressure of water vapor	in. Hg	kPa
p_s	Static pressure	in. wg	kPa
p_{sa}	Absolute static pressure	in. wa [Note (2)]	kPa
p_t	Total pressure	in. wg	kPa
p_{ta}	Absolute total pressure	in. wa	kPa
p_v	Velocity pressure	in. wg	kPa
Δp	Differential pressure	in.wg	kPa
Q_F	Fan volume flow rate	cfm	m^3/s
Re_p	Probe Reynolds number	Dimensionless	Dimensionless
R	Specific gas constant	ft · lb/lbm · °R	J/kg · K
R_O	See para. 2-1.3		
S	Aspect parameter	Dimensionless	Dimensionless

Symbol	Description	Unit/Value	
		U.S. Customary	SI
S_p	Frontal area of probe exposed to calibration stream	ft^2	m^2
S_X	Random uncertainty in X	Same as X	Same as X
s	Specific humidity	lbm vapor/lbm dry gas	kg vapor/kg dry gas
s_w	Specific humidity at saturation	lbm vapor/lbm dry gas	kg vapor/kg dry gas
T_s	Absolute static temperature	°R	K
T_t	Absolute total temperature	°R	K
t	Time	sec	s
t_d	Dry-bulb temperature	°F	°C
t_s	Static temperature	°F	°C
t_t	Total temperature	°F	°C
t_w	Wet-bulb temperature	°F	°C
U_X	Absolute uncertainty in X	Same as X	Same as X
$\hat{U}_X$	Random or systematic absolute uncertainty in X (for convenience in derivations)		
u_X	Relative uncertainty in X	per unit	per unit
$\hat{u}_X$	Random or systematic relative uncertainty in X (for convenience in derivations)		
V	Velocity	fpm	m/s
$\widehat{V}_a$	Axial distortion parameter	fpm	m/s
$\widehat{V}_r$	Velocity ratio	Dimensionless	Dimensionless
$\widehat{V}_s$	Shear parameter	fpm	m/s
$\widehat{V}_t$	Transverse distortion parameter	fpm	m/s
W	Electrical power input to motor	kW	kW
w	Duct cross-sectional dimension perpendicular to the fan shaft	ft	m
(X)	Volume fraction of gas constituent whose chemical symbol is X	ft^3/ft^3	m^3/m^3
x	Function used to determine K_p	Dimensionless	Dimensionless
y_F	Fan specific energy	ft·lb/lbm	J/kg
z	Function used to determine K_p	Dimensionless	Dimensionless

2-1.2 Greek Symbols

Symbol	Description	Unit/Value U.S. Customary	Unit/Value SI
$1-\varepsilon_p$	Compressibility correction factor for velocity pressure	Dimensionless	Dimensionless
$1+\varepsilon_T$	Compressibility correction factor for absolute temperature	Dimensionless	Dimensionless
α	Kinetic energy correction factor	Dimensionless	Dimensionless
β	Parameter used to correct probe calibration for blockage	Dimensionless	Dimensionless
$\hat{\varepsilon}_a$	Axial offset parameter	Dimensionless	Dimensionless
$\hat{\varepsilon}_t$	Transverse offset parameter	Dimensionless	Dimensionless
η	Fan efficiency	Percent or per unit	Percent or per unit
η_M	Motor efficiency	Percent or per unit	Percent or per unit
η_s	Fan static efficiency	Percent or per unit	Percent or per unit
η_t	Fan total efficiency	Percent or per unit	Percent or per unit
θ	Power factor	Dimensionless	Dimensionless
θ_i	Sensitivity coefficient	Various	Various
μ	Dynamic viscosity	lbm/ft · sec	Pa · s
ρ	Density	lbm/ft^3	kg/m^3
ρ_F	Fan gas density	lbm/ft^3	kg/m^3
ρ_m	Fan mean density	lbm/ft^3	kg/m^3
$\sum_{j=1}^{n}$	Summation of corrected values over n observations	----	----
τ	Torque	lb · ft	N · m
ϕ	Pitch angle	deg	deg
$\bar{\phi}$	Average pitch angle	deg	deg
ψ	Yaw angle	deg	deg
$\bar{\psi}$	Average yaw angle	deg	deg

2-1.3 Subscripts

Symbol	Description	Unit/Value U.S. Customary	Unit/Value SI
c	Converted value	----	----
da	Dry air	---	---
dg	Dry gas	---	---
i	Indicated value at a point	---	---
j	Corrected value at a point	---	---
ma	Moist air	---	---
mg	Moist gas	---	---
o	Ambient	---	---
R	Reference measurement	---	---
ref	Value for calibration reference probe	---	---
t	Turbine and drive train	---	---
wv	Water vapor	---	---
x	Total value at plane x for A, $\dot{m}$, and Q_F or average value at plane x for c_p, e_k, M, p_s, p_t, T, t_s, V, (X), a, and p	---	---
1	Plane 1 (fan inlet)	---	---
2	Plane 2 (fan outlet)	---	---
3	Plane 3 (alternate velocity traverse station)	---	---

2-1.4 Unit Conversions and Dimensional Constants

Symbol	Description	Unit/Value U.S. Customary	Unit/Value SI
C_1	---	459.7°F	273.2°C
C_2	---	60 sec/min	1.0 s/s
C_3	---	1.0	1.8 °R/K
C_4	---	0.672 lbm/ft · sec	1.0 Pa · s
C_5	---	1.0 Btu/lbm · °F	4186 J/kg · °C
C_8	---	2,830°F	1 572°C
C_9	---	1.44	0.80
C_{10}	---	70.77 lb/ft^2-in. Hg	10^3 J/m^3-kPa
C_{11}	---	5.193 lb/ft^2-in. wg	10^3 J/m^3-kPa

Symbol	Description	Unit/Value	
		U.S. Customary	SI
C_{12}	---	1,097 (lbm/ft-min^2-in. wg)$^{1/2}$	[2000 (m^2/s^2-Pa)]$^{1/2}$
C_{13}	---	13.62 in. wg/in. Hg	1.0 kPa/kPa
C_{14}	---	745.7 W/hp	10^3 W/kW
C_{15}	---	5,252 ft-lb-rev/hp-min	(10^3/2π) N-m-rev/kW-s
C_{16}	---	550 ft-lb/hp-sec	N-m/kW-s
C_{17}	---	6,354 ft^3-in. wg/hp-min	1.0 kJ/kW-s
C_{18}	---	0.03635549	0.610588882
C_{19}	---	0.002799407	0.044635714
C_{20}	---	2.07899E-05	0.001401772
C_{21}	---	9.66602E-07	2.79728E-05
C_{22}	---	-1.05944E-10	2.53599E-07
C_{23}	---	4.52482E-11	2.89536E-09
g_c	---	32.17 ft · lbm/lb · sec^2	1.0 kg · m/N · s^2
J	---	778.2 ft · lb/Btu	1.0 J/J
R_o	---	1,545 ft · lb/lbm-mol ·°R	8314 J/kg-mol · K

NOTES:
(1) The term "in. wg" stands for inches water gage.
(2) The term "in. wa" stands for inches water absolute.

2-2 DEFINITIONS

2-2.1 Temperature

absolute temperature, T: the value of temperature when the datum is absolute zero. It is measured in kelvins or degrees Rankine. The absolute temperature in degrees Rankine is the temperature in degrees Fahrenheit plus 459.7, and the absolute temperature in kelvins is the temperature in degrees Celsius plus 273.2.

dry-bulb temperature, t_d: the temperature measured by a dry thermometer or other dry sensor.

static temperature, t_s, T_s: the temperature measured in such a way that no effect is produced by the velocity of the flowing fluid. It would be shown by a measuring instrument moving at the same velocity as the moving fluid. Absolute static temperature is used as a property in defining the thermodynamic state of the fluid.

total temperature, t_t, T_t: sometimes called *stagnation temperature*, the temperature that would be measured when a moving fluid is brought to rest and its kinetic potential energies are converted to an enthalpy rise by an isoenergetic compression from the flow condition to the stagnation condition. At any point in a stationary body of fluid, the static and total temperatures are numerically equal.

wet-bulb depression, $t_d - t_w$: the difference between the dry-bulb and wet-bulb temperatures at the same location.

wet-bulb temperature, t_w: the temperature measured by a thermometer or other sensor covered by a water-moistened wick and exposed to gas in motion. When properly measured, it is a close approximation to the temperature of adiabatic saturation.

2-2.2 Specific Energy and Pressure

absolute pressure, p_a: the value of a pressure when the datum is absolute zero. It is always positive.

barometric pressure, p_b: the absolute pressure exerted by the atmosphere.

differential pressure, Δp: the difference between any two pressures.

gage pressure, p: the value of a pressure when the datum is the barometric pressure at the point of measurement. It is the difference between the absolute pressure at a point and the pressure of the ambient atmosphere in which the measuring gage is located. It may be positive or negative.

pressure, p: normal force per unit area. Since pressure divided by density may appear in energy balance equations, it is sometimes convenient to consider pressure as a type of energy per unit volume.

specific energy, y: energy per unit mass. Specific kinetic energy is kinetic energy per unit mass and is equal to one-half the square of the fluid velocity. Specific potential energy is potential energy per unit mass and is equal to the gravitational acceleration multiplied by the elevation above a specified datum. Fluid pressure divided by density is sometimes called "specific pressure energy" and is considered a type of specific energy; however, this term is more properly called specific flow work.

static pressure, p_s, p_{sa}: the pressure measured in such a manner that no effect is produced by the velocity of the flowing fluid. Similar to the static temperature, it would be sensed by a measuring instrument moving at the same velocity as the fluid. Static pressure may be expressed as either an absolute or gage pressure. Absolute static pressure is used as a property in defining the thermodynamic state of the fluid.

total pressure, p_t, p_{ta}: sometimes called the *stagnation pressure*, would be measured when a moving fluid is brought to rest and its kinetic and potential energies are converted to an enthalpy rise by an isentropic compression from the flow condition to the stagnation condition. It is the pressure sensed by an impact tube or by the impact hole of a Pitot-static tube when the tube is aligned with the local velocity vector. Total pressure may be expressed as either an absolute or gage pressure. In a stationary body of fluid, the static and total pressures are numerically equal.

velocity pressure, p_v: sometimes called "dynamic pressure," is defined as the product of fluid density and specific kinetic energy. Hence, velocity pressure is kinetic energy per unit volume. If compressibility can be neglected, it is equal to the difference of the total pressure and the static pressure at the same point in a fluid and is the differential pressure, which would be sensed by a properly aligned Pitot-static tube. In this Code, the indicated velocity pressure, p_{vi}, shall be corrected for probe calibration, probe blockage, and compressibility before it can be called velocity pressure.

2-2.3 Density and Specific Humidity

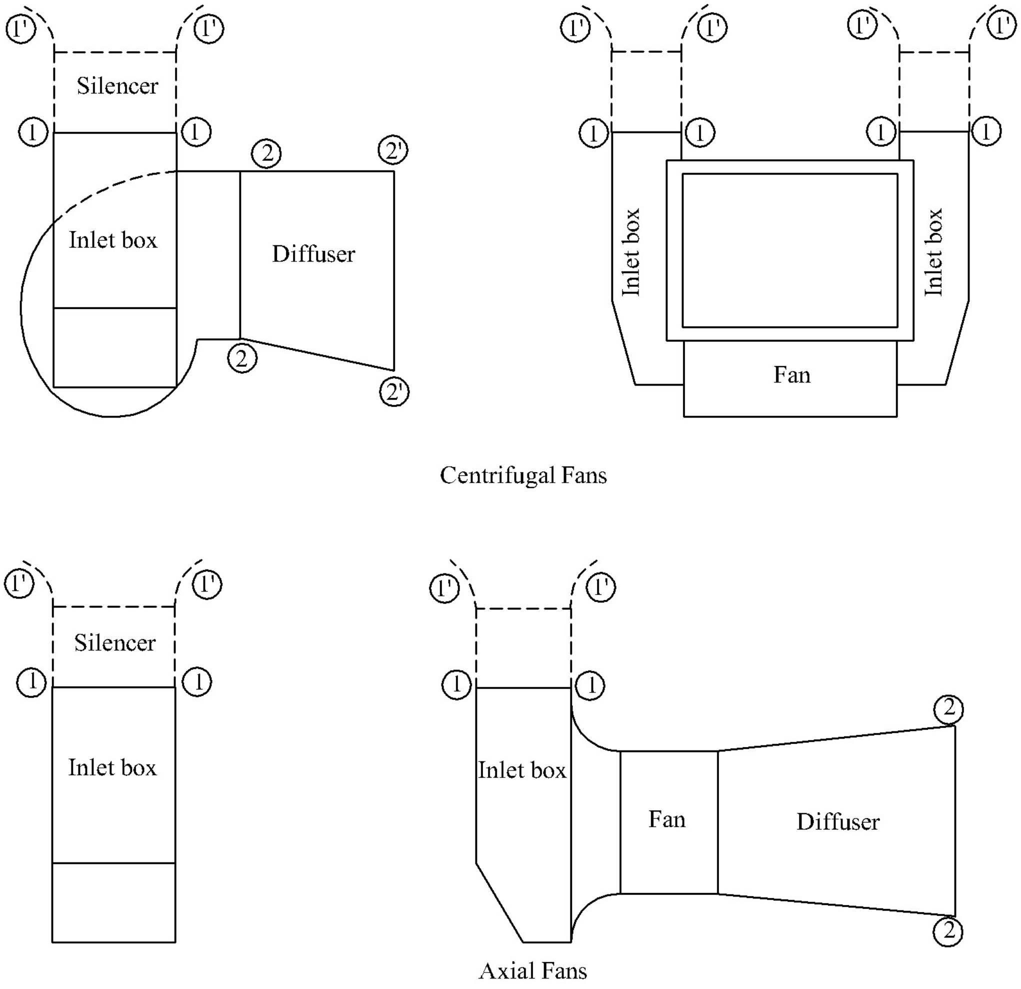
density, ρ: mass per unit volume of a fluid. The density can be given static and total values in a fashion similar to pressure and temperature. If the gas is at rest, static and total densities are equal.

specific humidity, s: the mass of water vapor per unit mass of dry gas.

2-2.4 Fan Boundaries

The fan boundaries are defined as the interface between the fan and the remainder of the system. These boundaries may differ slightly from fan to fan. The fan accepts power at its input power boundary and moves a quantity of gas from its inlet boundary to its outlet boundary and in the process increases the specific energy and pressure of this gas. The inlet boundary may be specified to include inlet boxes, silencers, rain hoods, or debris screens as a part of the fan. The outlet boundary may be specified to include dampers or a diffuser as a part of the fan. The input power boundary may be specified to include the fan-to-motor coupling or a speed reducer as part of the fan. See Figs. 2-2.4-1 and 2-2.4-2.

Fig. 2-2.4-1 Typical Input and Outlet Boundaries



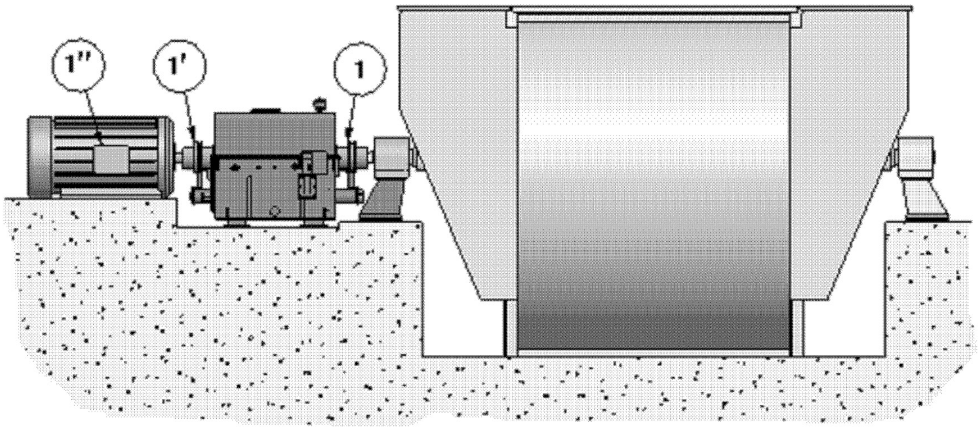
GENERAL NOTES:

(a) The inlet boundary is at 1 1 for centrifugal or axial fan furnished with an inlet box or at 1' 1' if a silencer is considered a part of the fan.

(b) The outlet boundary is at 2 2 for a centrifugal fan without a diffuser or at 2' 2' if a diffuser is part of the fan.

(c) An axial fan is usually furnished with a diffuser.

Fig. 2-2.4-2 Typical Input Power Boundaries



GENERAL NOTES:

(a) The input boundary is normally at 1, the point of coupling between the drive train and fan.

(b) The input power boundary may be at 1', the point of coupling between the motor and an intermediate drive element, e.g., a variable speed coupling. The drive element is considered to be a part of the fan.

(c) The input power boundary may be at 1'', the electrical interface if the entire drive train is considered to be a part of the fan.

2-2.5 Fan Performance

2-2.5.1 General. Fan performance can be expressed in terms of different sets of parameters. This Code provides the user with two choices. One set uses mass flow rate and specific energy. The other uses volume flow rate and pressure. The product of mass flow rate and specific energy and the product of volume flow rate, pressure, and a compressibility coefficient are each designated fan output power. However, values of output power calculated by the two methods are slightly different [1].

2-2.5.2 Mass Flow Rate–Specific Energy Approach. The fan performance parameters that are associated with this approach are defined as follows:

compressibility coefficient, K_ρ: the ratio of the fan inlet density to the fan mean density; is useful in this approach.

fan efficiency, η: the ratio of the fan output power to the fan input power. In this approach, there is only one definition of fan output power, so there is only one definition of fan efficiency.

fan mass flow rate, $\dot{m}_F$: the mass of fluid passing through the fan per unit time.

fan mean density, ρ_m: the ratio of the pressure change across the fan to the thermodynamic path integral of the differential of the pressure divided by the density.

$$\rho_m \equiv \frac{p_2 - p_1}{\int_1^2 \frac{dp}{\rho}} \tag{2-2-1}$$

In this approach, mean density is approximated by the arithmetic mean of inlet and outlet densities.

$$\rho_m \approx \left(\frac{\rho_1 + \rho_2}{2}\right) \tag{2-2-2}$$

fan output power, P_o: the product of fan mass flow rate and fan specific energy. Since mass flow rate equals the product of volume flow rate and density at a particular plane, fan output power can also be expressed as the product of fan inlet density, fan inlet volume flow rate, and fan specific energy.

fan specific energy, y_F: the work per unit mass that would be done on the gas in an ideal (frictionless) transition between the actual inlet and outlet states. The ideal work done on a unit mass of fluid is equal to the integral of the static pressure differential divided by the fluid density for the fan flow process plus changes of specific kinetic energy and specific potential energy across the fan. The fan specific energy is the average of the ideal work for all fluid particles passing through the fan. Refer to subsection 5-7 for appropriate averages.

Only the component of velocity in the nominal direction of flow shall be taken into account when determining the specific kinetic energy. It is customary to assume that changes in potential energy are negligible in fans.

$$y_F = \int_1^2 \frac{dp}{\rho} + e_{K2} - e_{K1} \tag{2-2-3}$$

For an incompressible flow process, the product of fan specific energy and fluid density is equal to the fan total pressure. For a nonconstant density process, fan specific energy can be approximated by assuming some thermodynamic process within the fan in order to perform the pressure-density integration.

kinetic energy correction factor, α: a dimensionless factor used to account for the difference between the true average kinetic energy of the fluid and the kinetic energy calculated as one half the square of the average velocity.

2-2.5.3 Volume Flow Rate–Pressure Approach. The fan performance parameters associated with this approach are defined as follows.

compressibility coefficient, K_p: a dimensionless coefficient used to account for compressibility effects [2] and is calculated according to the procedure given in para. 5-11.4 [3].

fan efficiency, η: In this approach, fan efficiency is expressed as either fan total efficiency or fan static efficiency.

fan static efficiency, η_s: the ratio of fan output power to fan input power, in which the fan output power is modified by deleting the fan velocity pressure. This may also be called total-to-static efficiency.

fan total efficiency, η_t: the ratio of fan output power to fan input power. This may also be called total-to-total efficiency.

fan gas density, ρ_F: the total density of the gas at fan inlet conditions.

fan output power, P_o: the product of fan volume flow rate, fan total pressure, and compressibility coefficient K_p.

fan pressure: in this approach, three fan pressures are defined as follows:

fan static pressure, p_{Fs}: the difference between the fan total pressure and the fan velocity pressure. Therefore, fan static pressure is the difference between the average static pressure at the fan outlet and the average total pressure at the fan inlet. Refer to subsection 5-7 for appropriate averages.

fan total pressure, p_{Ft}: the difference between the average total pressure at the fan outlet and the average total pressure at the fan inlet. Only the component of velocity in the nominal direction of flow shall be taken into account when determining fan total pressure. Refer to subsection 5-7 for appropriate averages. It is customary to assume that pressure changes due to elevation changes are negligible in fans.

fan velocity pressure, p_{Fv}: the product of the average density and average specific kinetic energy at the fan outlet. Refer to subsection 5-7 for the appropriate averages. This corresponds to the velocity pressure corresponding to the average velocity at the fan outlet as defined in the ASHRAE Standard 51 and AMCA Standard 210 [2].

fan volume flow rate, Q_F: the fan mass flow rate divided by the fan gas density.

2-2.5.4 Fan Input Power. P_I, fan input power, is the power required to drive the fan and any elements in the drive train that are considered to be within the fan boundaries.

2-2.6 Fan Operating Conditions

Fan operating conditions are specified by the speed of rotation of the fan and sufficient information to determine the average gas properties, including pressure, temperature, density, viscosity, gas constants, and specific heats at the fan inlet.

2-2.7 Errors and Uncertainties

confidence level, L_c: a percentage value such that if a very large number of determinations of a variable are made, there is an L_c percent probability that the true value will fall within the interval defined by the mean plus or minus the uncertainty. A value for uncertainty is meaningful only if it is associated with a specific confidence level. As used in this Code, all uncertainties are assumed to be at the 95% confidence level. If the number of determinations of a variable is large and if the values are normally distributed, the uncertainty at the 95% confidence level is approximately twice the standard deviation of the mean of the values.

error: the difference between the true value of a quantity and the measured value. The true value of an error cannot be determined.

random uncertainty, $S_{\bar{X}}, S_{\bar{X}} / \bar{X}$: uncertainty due to numerous small independent influences that prevent a measurement system from delivering the same reading when supplied with the same input. Random uncertainties can be reduced by replication and averaging [4]. Random uncertainty is often calculated as the standard deviation of the mean for a particular set of measurements. Hence, the symbol used for random uncertainty is the same as that typically used for standard deviation of the mean.

sensitivity coefficient, θ_i: also called "sensitivity factor," the ratio of the change in a result to a unit change in a parameter. Influence coefficients have been utilized in the derivations of the uncertainties equations in this Code.

systematic uncertainty, B_X, B_X / X: uncertainty due to such things as instrument and operator bias and changes in ambient conditions for the instruments. Systematic uncertainty is essentially "frozen" in the measurement system and cannot be reduced by increasing the number of measurements if the equipment and conditions of measurements remain unchanged [4].

total uncertainty, U_X, U_X / X: of a result is obtained by combining the random and systematic uncertainties of that result in a manner that reflects the confidence level. In this Code, random and systematic uncertainties are combined using a "root sum square (RSS) model." See eqs. (5-13-1) and (5-13-2).

uncertainty: a possible value for the error [5]. It is also the interval within which the true value can be expected to lie with a stated probability [4]. The uncertainty is used to estimate the error.

absolute uncertainty (U): has the same units as the variable in question.

relative uncertainty (u): absolute uncertainty divided by the magnitude of the variable and is dimensionless; also called "per unit uncertainty."

2-2.8 General Definitions

acceptance test: the evaluating action(s) to determine if a new or modified piece of equipment satisfactorily meets its performance criteria, permitting the purchaser to "accept" it from the supplier.

calibration: the process of comparing the response of an instrument or measurement system with a standard instrument or measurement system over some measurement range and adjusting the instrument or measurement system to match the standard if appropriate.

instrument: a tool or device used to measure the physical value of a variable. These values can include size, weight, pressure, temperature, velocity, fluid flow, voltage, electric current, density, viscosity, gas composition, and power. Sensors are included that may not, by themselves, incorporate a display but transmit signals to remote computer type devices for display, processing, or process control. Also included are items of ancillary equipment directly affecting the display of the primary instrument (e.g., ammeter shunt). Also included are tools or fixtures used as the basis for determining part acceptability.

parties to a test: those persons and companies interested in the results.

serialize: to permanently mark an instrument so that it can be identified and tracked.

test boundary: see Fan Boundaries, Figs. 2-2.4-1 and 2-2.4-2.

test reading: one recording of all required test instrumentation.

test run: a group of test readings.

traceable: records are available demonstrating that the instrument can be traced through a series of calibrations to an appropriate ultimate reference, such as National Institute for Standards and Technology (NIST).

Section 3
Guiding Principles

3-1 INTRODUCTION

In applying this Code to a specific fan test, various decisions must be made. This Section explains what decisions shall be made and gives general guidelines for performing a Code test.

Any test shall be performed only after the fan has been found by inspection to be in a satisfactory condition to undergo the test. The parties to the test shall mutually decide when the test is to be performed and shall be entitled to have present such representatives as are required for them to be assured that the test is conducted in accordance with this Code and with any written agreements made prior to the test.

3-2 PRIOR AGREEMENTS

Prior to conducting a Code test, written agreement shall be reached by the parties to the test on the following items:

(a) object of test
(b) duration of operation under test conditions
(c) test personnel and assignments
(d) person in charge of test
(e) test methods to be used
(f) test instrumentation and methods of calibration
(g) locations for taking measurements and orientation of traverse ports
(h) number and frequency of observations, including reference measurements
(i) method of computing results
(j) values or methods for calculation of primary uncertainties
(k) arbitrator to be used if one becomes necessary
(l) applicable performance curves and/or the specified performance and operating conditions
(m) fan boundaries
(n) number of test runs
(o) pretest uncertainty analysis
(p) uncertainty targets
(q) permissible limits of inlet flow distortion

3-3 CODE PHILOSOPHY

3-3.1 Fan Performance

This Code offers the user the choice of expressing fan performance in terms of mass flow rate and specific energy or volume flow rate and pressure. After reviewing both methods, the parties to the test shall decide which method they intend to use. Once a method is selected, then the principles and procedures for only that method shall be adhered to throughout the test, rather than commingling the various aspects of the two methods [1].

3-3.2 Methods for Determining Fan Performance

The methods of this Code are based on the assumption that fan pressures or specific energies are measured sufficiently close to the fan boundaries that corrections for losses between the measurement planes and fan boundaries are not required. It is not feasible to include methods for such corrections in this Code; therefore, if such corrections are necessary, the test cannot be a Code test.

For the purpose of determining proper average values of pressure, temperature, and density, it is always necessary to measure point velocities at the fan boundaries. However, only the point velocities measured at traverse planes conforming to the requirements of this Code (see para. 4-2.3) shall be used for fan flow rate. If the conditions at the fan boundaries do not meet the criteria given in this Code for a suitable flow traverse, then point velocity measurements made at the fan boundaries shall be used only for determining average values of pressure, temperature, density, and specific kinetic energy and not for fan flow rate. If this condition exists, then the fan flow rate may be determined at a plane other than the fan boundary, provided that no fluid enters or leaves the duct between the fan boundary and measurement plane. Although the point velocities measured at the fan boundaries may not conform to the requirements for a valid flow traverse, they can provide a useful statistical basis for substantiating the fan flow rate.

3-3.3 Flow Measurement Methods

For large ducts handling gas flows, often the only practicable method of gas flow measurement is the velocity traverse method. This method shall be considered the primary method for measuring flows of the type addressed by this Code. Other methods of determining flow, including but not limited to stoichiometric methods (where applicable), ultrasonic methods, and methods using such devices as flow nozzles, may be permitted if it can be shown that the accuracy of the proposed method is at least equal to that of the primary method.

In the velocity traverse method, the duct is subdivided into a number of elemental areas and, using a suitable probe, the velocity is measured at a point in each elemental area. The total flow is then obtained by summing the contributions of each elemental area (some methods use different weighting factors for different areas). Within the framework of the velocity traverse method, many different techniques have been proposed for selecting the number of points at which velocity is measured, for establishing the size and geometry of the elemental areas, and for summing (theoretically integrating) the contributions of each elemental area. Options that have been proposed include the placing of points based on an assumed (log-linear, Legendre polynomial, or Chebyschev polynomial) velocity distribution [2, 6], the use of graphical or numerical techniques to integrate the velocity distribution over the duct cross section [6, 7], the use of equal elemental areas with simple arithmetic summing of the contribution of each area to the total flow [6, 8, 9], and the use of boundary layer corrections to account for the thin layer of slow-moving fluid near a wall. As a general rule, accuracy of flow measurement can be increased by either increasing the number of points in the traverse plane or by using more sophisticated mathematical techniques (e.g., interpolation polynomials, boundary layer corrections) [6, 8]. PTC 19.5 recommends either a Gaussian or Chebyschev integration scheme. Investigations performed by the PTC 11 committee using different velocity distributions similar to those that actually occur in the field have shown that no particular technique is always more accurate.

Considering the requirements of field testing and the varied velocity distributions that may occur in the field, this Code specifies flow measurements at a relatively large number of points in lieu of assuming velocity distributions or using corrections for boundary layer effects. It is usually desirable to have a large number of points (elemental areas) so that the complete velocity profile can be characterized. Accordingly, this Code adopts the equal-area method with measurement at a relatively large number of points. Investigations of flow measurement under conditions similar to those expected in application of this Code have demonstrated the validity of this approach [8–10]. In some circumstances, it may be desirable to use Gaussian or Chebyschev schemes because they require a smaller number of measurement points. PTC 19.5 may be consulted for details on these methods.

3-3.4 Flow at the Fan Boundaries

Due to the highly disturbed flow at the fan boundaries and the errors obtained when making measurements with probes unable to distinguish directionality, probes capable of indicating gas direction and speed, hereinafter referred to as directional probes, are generally required. Only the component of velocity normal to the elemental area is pertinent to the calculation of flow. Measurement of this component cannot be accomplished by simply aligning a nondirectional probe parallel to the duct axis, since such probes only indicate the correct velocity pressure when aligned with the velocity vector. Errors are generally due to undeterminable effects on the static (and, to a lesser degree, total) pressure-sensing holes. Therefore, adequate flow measurements in a highly disturbed region can only be made by measuring speed and direction at each point and then calculating the component of velocity parallel to the duct axis. Only in some circumstances (see subsection 4-7) may nondirectional probes be used.

3-3.5 Averaging Methods

Various methods of averaging are required to calculate the appropriate values of the parameters that determine fan performance. These methods, along with the large number of traverse points, the directional probe, and requirements for measurements at the fan boundaries, make it possible to conduct an accurate field test for most fan installations.

3-3.6 Compressibility Effects

The instruments and methods of measurement specified in this Code are selected on the premise that only mild compressibility effects are present in the flow. The velocity, pressure, and temperature determinations provided for in this Code are limited to situations in which the gas is moving with a Mach number less than 0.4. This corresponds to a value of ($K_{vj}p_{vi}$ / p_{sai}) of approximately 0.1 (see para. 5-2.2).

3-3.7 Test Speed Versus Specified Speed

Although this Code provides methods for conversion of measured fan performance variables to specified operating conditions, such conversions shall not be permitted if the test speed differs by more than 10% from the specified speed or if the test values of the fan inlet density, ρ_1, or fan gas density, ρ_F, differ by more than 20% from specified values.

3-3.8 Accuracy of Results

A question that invariably arises in connection with any test is, “How accurate are the results?” [5]. This question is addressed in this Code by the inclusion of a complete procedure for the evaluation of uncertainties. It is believed that all significant sources of error in a fan test have been identified and addressed in this procedure. Since in fact any results based on measurements are of little value without an accompanying statement of their expected accuracy, uncertainty evaluation is made a mandatory part of this Code.

3-3.9 Inlet Flow Distortion

Fan performance is typically predicted assuming that a uniform flow velocity profile at the fan inlet plane and equal flow at each inlet, in the case of double inlet fans, will be present. Laboratory test conditions ensure that such a uniform profile exists. When a fan is installed in a system, the fan may be subjected to a distorted inlet profile because of upstream ductwork geometry or, for open inlet fans, the geometry of the space in which the fan is installed. Experience shows that inlet flow distortion or imbalance can exist and can often affect fan performance. Wright et al. [11, 12] have measured the effects of inlet flow distortion on a single-inlet centrifugal fan. This is the only published information on distortion known to the PTC 11 Committee.

Inlet flow distortion can be quantified by various velocity profile parameters: velocity ratio, transverse distortion, axial distortion, transverse shear, transverse offset, axial offset, average yaw, and average pitch. The term “transverse” refers to the direction perpendicular to the fan shaft, and the term “axial” refers to the direction parallel to the fan shaft. This Code provides equations for computing these parameters. Specification of acceptable levels for these parameters or methods for accounting for the effects of distortion on fan performance is beyond the scope of this Code.

3-3.10 Laboratory Versus In Situ Tests

Commercially quoted fan performance is usually based on measurements made under laboratory conditions. In a laboratory test, a fan is operated in a system specifically designed to facilitate accurate measurement of fan performance parameters and to minimize those system effects that can degrade fan performance [2, 13]. Comparative fan tests conducted according to a laboratory standard [2] and procedures of this Code have demonstrated that similar performance ratings can be obtained if the fan is operated under laboratory conditions [14].

The user of this Code should be aware that application of the procedures contained herein will reveal the performance of the test fan as it is affected by the system in which it is installed. These in situ performance ratings and ratings of the same fan based on laboratory tests or ratings of a model fan based on laboratory tests may not be the same due to various effects generally called "system effects" [13]. Any methods for reconciliation of in situ performance ratings and laboratory-based ratings are beyond the scope of this Code.

3-4 SYSTEM DESIGN CONSIDERATIONS

There are field situations where it is not possible to obtain sufficiently accurate measurements to conform with this Code. Consideration of a few simple concepts when a new system is designed will facilitate fan testing as well as improve the fan system performance.

3-4.1 Fan Flow Rate

Generally, the most difficult parameter to determine during a field test is the fan flow rate. If the following considerations can be made during the design of the fan and duct system, fan flow rates will be easier to determine:

(a) Design of inlet and outlet ducts should avoid internal stiffeners for three equivalent diameters both upstream and downstream of the fan boundaries.

(b) Abrupt changes in direction should not be located at the fan boundaries.

(c) All transitions in duct size should be smooth.

(d) A duct length of approximately 3 ft (1 m) should be allowed at the fan boundaries for inserting probes. This section should be free of internal obstructions that would affect the flow measurement and external obstructions that would impede probe maneuverability, such as structural steel, walkways, handrails, etc. Ideally, the area of the measuring section, A_{2duct}, should be the same as that of the fan, A_{2fan}. If not, the fan velocity pressure shall be corrected as indicated below. Differences in density may be ignored.

3-4.2 Fan Input Power

Considerations to be observed that will aid the determination of fan input power are

(a) installing a calibrated drive train or

(b) allowing sufficient shaft length at the fan for the installation of a torque meter

3-5 INTERNAL INSPECTION AND MEASUREMENT OF CROSS SECTION

An internal inspection of the ductwork, at planes where velocity and/or pressure measurements are to be made, shall be conducted by the parties to the test to ensure that no obstructions will affect the measurements. Areas where there is an accumulation of dust such that the duct area is significantly reduced shall be avoided as this indicates that the velocities are inadequate to prevent entrained dust from settling. This dust settlement will in effect cause the duct cross-sectional area to decrease during the test. Where this situation exists, it is recommended that velocity measurements be made in vertical runs.

The internal cross-sectional area shall be based on the average of at least four equally spaced measurements across each duct dimension for nominally rectangular ducts and on the basis of the average of at least four equally spaced diametral measurements for nominally circular ducts. Sufficient equally spaced measurements shall be used to limit the uncertainty in the area to 0.3%. If the duct area is measured under conditions different from operating conditions, suitable expansion or contraction corrections for temperature and pressure shall be made.

3-6 TEST PERSONNEL

3-6.1 Test Team

A test team shall be selected that includes a sufficient number of test personnel to record the various readings in the allotted time. Test personnel shall have the experience and training necessary to obtain accurate and reliable records. All data sheets shall be signed by the observers. The use of automatic data recording systems can reduce the number of people required.

3-6.2 Person-in-Charge

The person in charge of the test shall direct the test and shall exercise authority over all observers. This person shall certify that the test is conducted in accordance with this Code and with all written agreements made prior to the test. This person may be required to be a registered professional engineer.

3-7 POINT OF OPERATION

This Code describes a method for determining the performance of a fan at a single point of operation. If more than one point of operation is required, a test shall be made for each. The parties to the test must agree prior to the tests on the method of varying the system resistance to obtain the various points of operation. If performance curves are desired, then the parties to the test shall agree beforehand as to the number and location of points required to construct the curves.

3-8 METHOD OF OPERATION DURING TEST

3-8.1 Manual Mode Operation

When a system contains fans operating in parallel, the fan to be tested shall be operated in the manual mode during the test and the remaining fans in the system used to follow load variations. The fan to be tested shall be operated at a constant speed with constant damper and vane positions. Various positions may be required for part-load tests.

3-8.2 Constant Conditions

The system shall be operated to maintain conditions at constant gas flows and other operating conditions. For example, for draft fans, the boiler load should be steady. Soot blowers should not be cycled on and off during the test. If soot blowing is necessary, it should be used throughout the test. The operation of pulverizers, stokers, baghouses, scrubbers, air heaters, etc., shall not be allowed to affect the results of the test.

3-8.3 Records

Adequate records of the position of variable vanes, variable blades, dampers, or other control devices shall be maintained.

3-9 INSPECTION, ALTERATIONS, AND ADJUSTMENTS

Prior to the test, the manufacturer or supplier shall have reasonable opportunity to inspect the fan and appurtenances for correction of noted defects, for normal adjustments to meet specifications and contract agreements, and to otherwise place the equipment in condition to undergo further operation and testing. The parties to the test shall not alter or change the equipment or appurtenances in such a manner as to modify or void specifications or contract agreements or prevent continuous and reliable operation of the equipment at all capacities and outputs under all specified operating conditions. Adjustments to the fan that may affect test results are not permitted once the test has started. Should such adjustments be deemed necessary, prior test runs shall be voided and the test restarted. Any readjustments and reruns shall be agreed to by the parties to the test.

3-10 INCONSISTENCIES

If inconsistencies in the measurements are observed during the conduct of the test, the person in charge of the test shall be permitted to take steps to remedy the inconsistency and continue the test. Any actions in this regard must be noted and are subject to approval by the parties to the test. Any such action shall be fully documented in the test report.

3-11 MULTIPLE INLETS OR DUCTS

If there is more than one fan inlet, measurements shall be obtained at each inlet or in each inlet duct. It is not permissible to measure the conditions at one inlet and assume the conditions are the same for all the inlets. Similarly, if the discharge duct from a fan splits into two or more ducts and it is more practical to measure the conditions downstream of the split, then the conditions in each branch of the duct shall be measured to determine the total flow.

3-12 PRELIMINARY TEST

Prior to performing a Code test, a preliminary test shall be made. The purpose of the preliminary test is to train the observers, determine if all instruments are functioning properly, and verify that the system and fan are in proper order to permit a valid Code test. The preliminary test can be considered a Code test if agreed to by the parties to the test and all requirements of this Code are met.

In fans with multiple inlets or ducts, it may be desirable to calculate parameters such as flow rate, density, gas composition, and parameters used to describe inlet flow distortion separately for each inlet or duct. If this is done, then the total mass flow rate shall be calculated as the sum of the separate inlet or duct mass flow rates, and the inlet static pressure, temperature, specific kinetic energy, and gas composition shall be calculated as the mass flow-weighted average of the values determined in the separate inlets or ducts.

3-13 REFERENCE MEASUREMENTS

For the purposes of determining that the system has reached steady state, verifying the constancy of operating conditions, and verifying that the fan performs at a constant point of operation during the test, the following reference measurements shall be made:

(a) speed, N_R

(b) driver power, or some quantity proportional to driver power (e.g., I_R, τ_R, W_R, etc.)

(c) fan inlet static pressure, p_{1sR}

(d) fan outlet static pressure, p_{2sR}

(e) static pressure at traverse plane (if used), p_{3sR}

(f) fan inlet temperature, T_{1R}

(g) fan outlet temperature, T_{2R}

(h) temperature at traverse plane (if used), T_{3R}

(i) total pressure rise across the fan, p_{tR}

(j) velocity pressure in either inlet or outlet or traverse plane, p_{vR}

The measurement of speed and power made in accordance with the requirements of Section 4 for determining fan performance shall be used for reference purposes. The reference measurements for pressure and temperature shall be in accordance with Section 4 except a single point measurement shall be used for each parameter instead of the sampling grid. For purposes of reference measurements, probes capable of sensing total pressure, static pressure, velocity pressure, and temperature connected to appropriate indicators shall be permanently fixed at central locations in the inlet and outlet planes. These need not be directional probes nor do they have to be calibrated, since measurements taken from these probes are for reference purposes only. To facilitate uncertainty analysis, at least 30 sets of reference measurements shall be taken during a test run. Reference measurements shall be taken at regular intervals and shall be averaged over a time period of at least 15 sec. For example, for a 1-hr test, reference measurements would be made at 2-min intervals and recorded as averages over 15-sec periods. This may be done manually or automatically. It would be useful to record the trend on a graph.

The following test shall be used to determine if the test conditions are sufficiently steady. For each reference measurement, a test parameter equal to twice the standard deviation of the mean divided by the mean of the measurements $\left(2S_{\bar{X}}/\bar{X}\right)$ is calculated. If the value of the test parameter exceeds 0.01 (1%) for any reference measurement, the test shall be invalidated because of unsteadiness.

The person in charge of the test shall be solely responsible for deciding when operating conditions are sufficiently constant to begin and continue the test.

Section 4
Instruments and Methods of Measurement

4-1 GENERAL CONSIDERATIONS

4-1.1 Accuracy

The specifications for selection and calibration of instruments that follow include accuracy requirements. Unless otherwise stated, specified accuracies are expressed in terms of the maximum uncertainty in any reading due to the instrument based on a minimum confidence level of 95%.

It is a requirement of this Code that the parties to the test agree in advance on the limits of possible measurement errors and test uncertainties. The parties should base their judgments of possible error on the references cited for each instrument, any records pertaining to the instrument to be used, and their collective experience with similar measurements.

Certain instruments are specified in this Code. However, it should be understood that other instruments for the same purpose may be used, provided their accuracies are equal to or better than those of the specified instruments.

4-1.2 Instrument Calibration

All instruments used in a Code test shall be calibrated. It is not necessary to calibrate all instruments specifically for the test if the parties to the test agree on the validity of previous calibrations.

The calibration data for an instrument shall be represented as a continuous function that may be determined by graphically fairing a smooth curve among the calibration points or by fitting, using the least squares methods, a mathematical curve that has a number of fitting parameters less than or equal to one-half of the number of calibration points. In a polynomial, the fitting parameters are the undetermined coefficients. In a power law equation, e.g., $y = ax^b$, where a and b are the fitting parameters. The fitting parameters for other cases may be determined in a similar manner.

Where the physical facts dictate, the calibration function may be extrapolated to the origin. Calibration data should cover the entire range of instrument readings, except where extrapolation to zero is indicated. Any other extrapolation requires agreement among the parties.

4-1.3 Monitoring Operational Steadiness

It is a requirement of this Code (see subsection 3-13) that operating conditions and point of operation be held steady during the test. Readings for some of the test parameters, such as rotational speed and input power, can be monitored for operational steadiness. Other test variables, such as velocity and pressure, are not uniformly distributed; therefore, test readings should not be used to monitor operational steadiness. Separate instruments shall, therefore, be used. Such monitoring instruments shall be held in a fixed position rather than used to traverse the plane.

Monitoring instruments shall be sensitive to changes in the monitored variables that would affect results. However, the accuracy and calibration requirements for the measuring instruments that follow can be relaxed or eliminated for instruments used only for monitoring purposes. It may even be desirable to use instruments with appreciably more damping than would be acceptable for measuring instruments as long as the response is fast enough to adequately indicate departures from operational steadiness.

4-2 TRAVERSE SPECIFICATIONS

4-2.1 Quantities Measured by Traverse

Because the distributions of velocity, pressure, temperature, gas composition, and moisture across the duct cross section are nonuniform, each quantity shall be measured at a sufficient number of points to facilitate the calculation of a proper average value. Point values of all of these quantities are theoretically required at every traverse plane, but this Code recognizes that the distributions of gas composition and moisture are generally much more uniform than the distributions of velocity, pressure, and temperature. Accordingly, the Code does not require that gas composition and moisture be measured at every point in a traverse plane. Similarly, the Code does not require that these quantities be measured at all traverse planes if there are sound reasons to believe that there will be no change between planes. There may also be cases where the distribution of temperature is quite uniform. The parties may, therefore, agree to relax the requirement for temperature measurements if they are convinced this will have a negligible effect on the results.

4-2.2 Number of Traverse Planes

Two traverse planes are required to determine specific output (fan pressure or fan specific energy), except for the case mentioned below. The preferred locations for the traverse planes are at the fan inlet and outlet boundaries. However, a slight offset, upstream or downstream, is usually required so that heavy flanges or stiffeners do not have to be penetrated. Similarly, when dampers are located at the fan boundaries, it is more desirable to traverse slightly upstream of these dampers than downstream of them.

Only one traverse plane is required to determine flow rate, but if both the inlet plane and outlet plane qualify, each should be used. If neither the inlet plane nor outlet plane qualifies, a third plane will be required for the velocity traverse to determine flow rate.

If at its inlet boundary the fan draws gas from an essentially quiescent region of large volume and the inlet flow path is free from obstructions (e.g., a fan drawing air from the atmosphere or a fan located inside a large room), it is not necessary to traverse the inlet to determine specific output. The inlet total pressure, inlet static pressure, and inlet velocity pressure are all zero if the inlet region pressure is selected as the datum. If the inlet region pressure is not the datum, then the inlet velocity pressure is zero, and the inlet total and inlet static pressures are each equal to the inlet region pressure (see Fig. 4-2.2-1). However, if such fans are equipped with inlet boxes, the flow can be expected to be quite uniform at the entrance to the inlet box, particularly if equipped with an inlet bell, and this may be the optimum location for a velocity traverse to determine the flow rate.

4-2.3 Qualified Velocity Traverse Planes

To qualify for a velocity traverse for purposes of determining fan flow rate (see para. 3-3.2), a plane shall meet the following specifications:

(a) There shall be no internal stiffeners or other internal obstructions.

(b) There shall be no accumulation of dust or debris.

(c) The traverse plane shall be at least one damper blade width upstream or ten damper blade widths downstream of a damper.

(d) A preliminary velocity traverse shall show that the flow is reversed or essentially stagnant at no more than 15% (preferably 0%) of the elemental areas.

(e) There shall be no sudden change in either cross-sectional area or duct direction.

4-2.4 Determination of Sampling Grid

Measurements shall be taken at centroids of equal elemental areas. However, allowing for probe stem droop and the need to avoid duct bracing, the probe tip shall be located within a central area the sides of which are no more than 30% of the corresponding dimensions of the elemental area. Similarly, the probe tip may be outside the traverse plane by no more than 30% of the largest elemental area dimension and then only if the duct area is the same as at the traverse plane. Refer to Figs. 4-2.4-1 and 4-2.4-2. The minimum number of test points shall be per Fig. 4-2.4-3.

Fig. 4-2.2-1 Fan Room Pressure

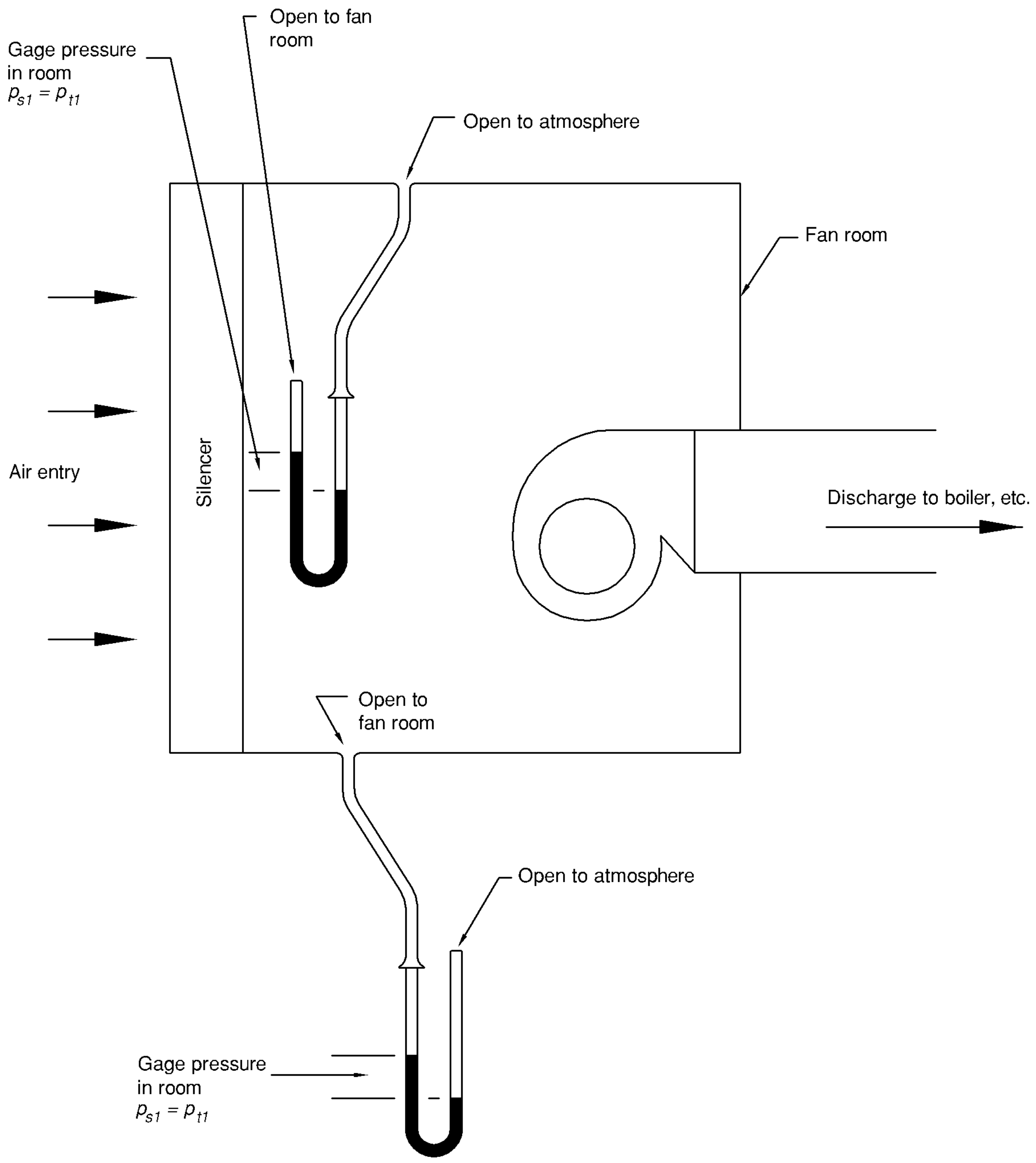
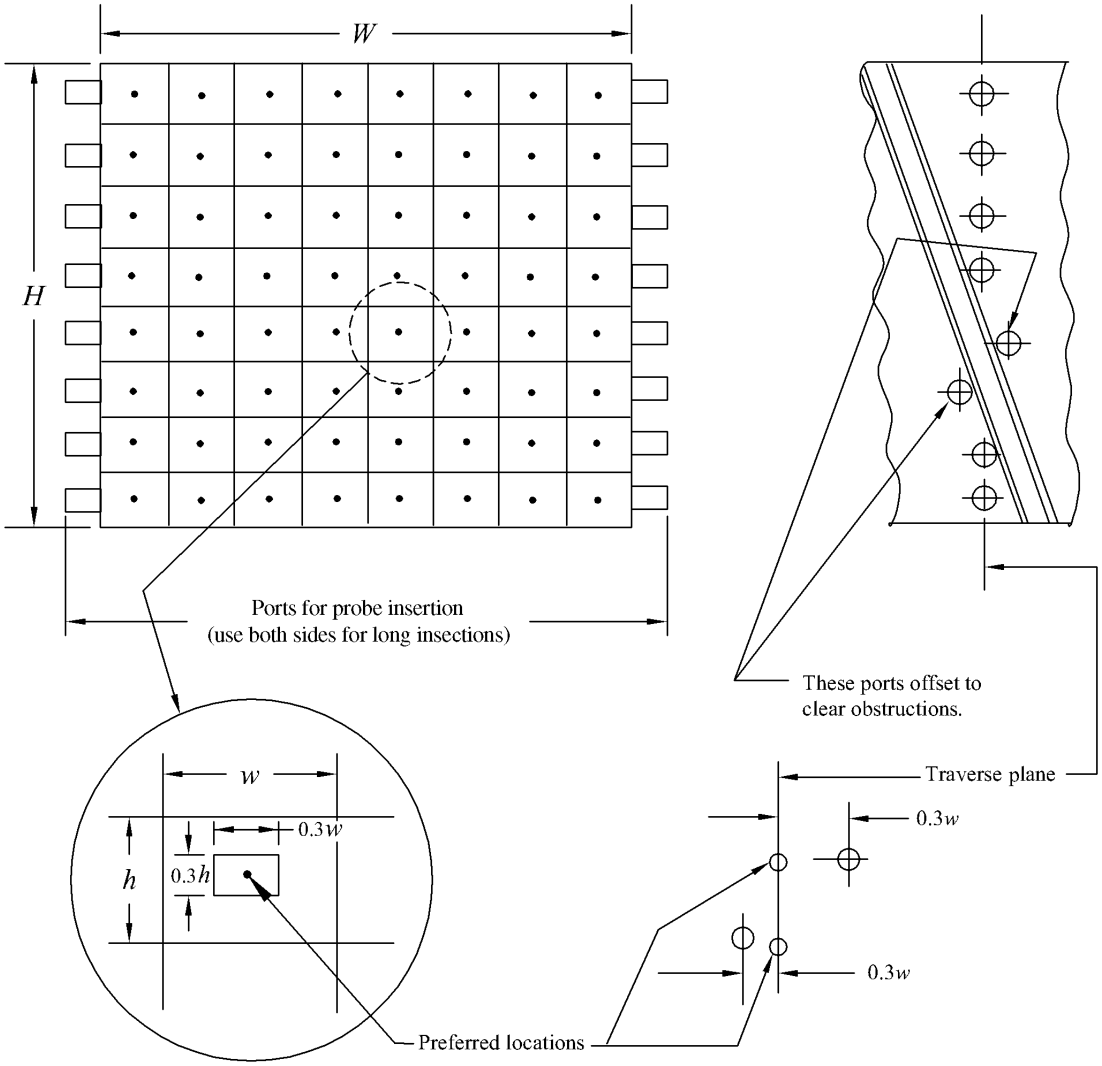


Fig. 4-2.4-1 Sampling Point Details (Rectangular Duct)



For measurement planes of rectangular and square cross section, the aspect parameter *S* shall be between 0.5 and 2.0. The long dimension of the elemental area shall align with the long dimension of the duct cross section.

The intent of this specification is to make the elemental areas closely geometrically similar to the duct cross section (see [8] and Fig. 4-2.4-1).

For measurement planes of circular cross section, there shall be a minimum of eight equally spaced radial traverse lines (eight radii or four diameters), and the distance between adjacent points on any radial line shall not be less than 0.5 ft (0.15 m). It may be necessary to increase the number of radial lines to meet this requirement. Refer to Fig. 4-2.4-2.

Fig. 4-2.4-2 Sampling Point Details (Circular Duct)

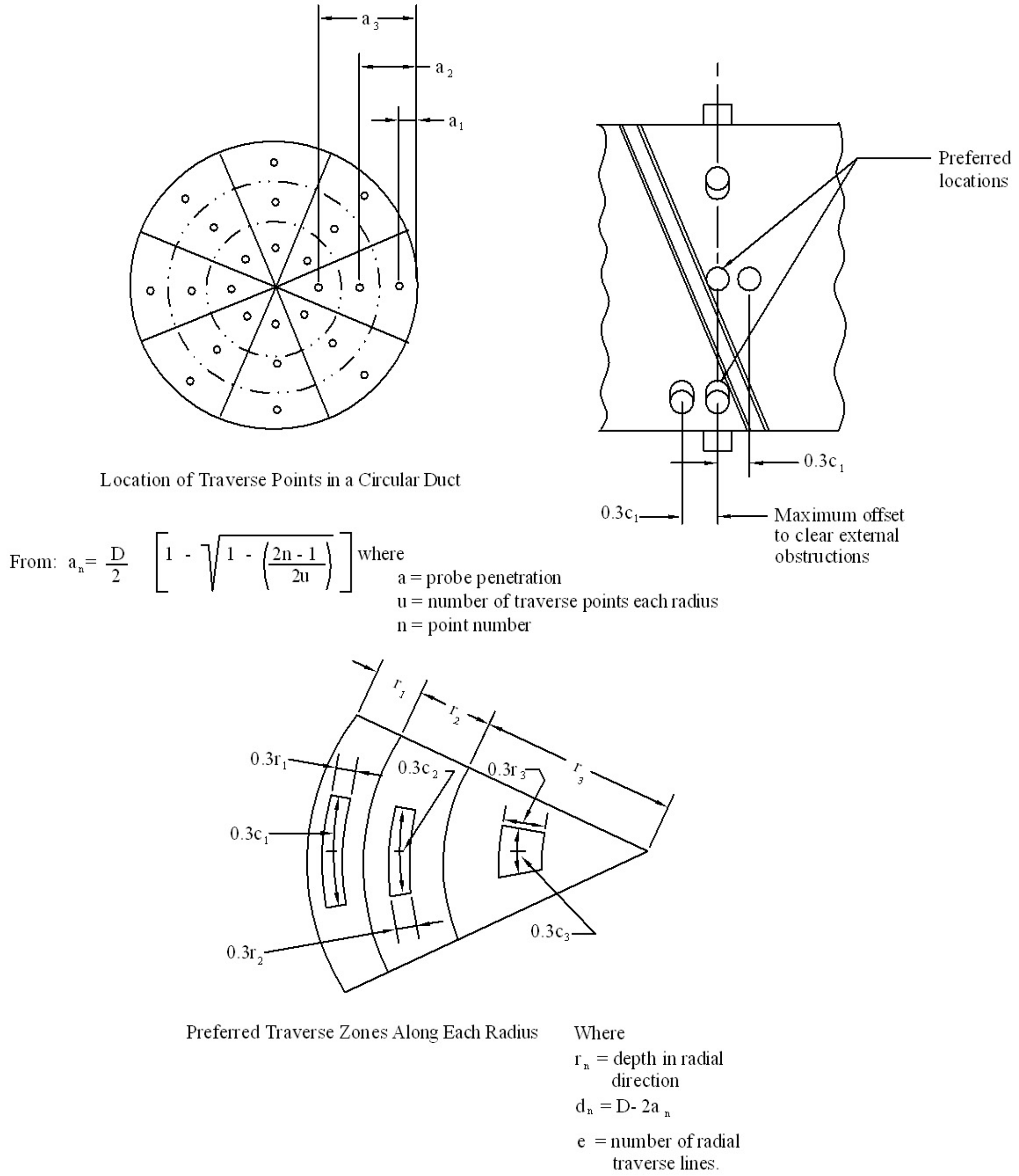
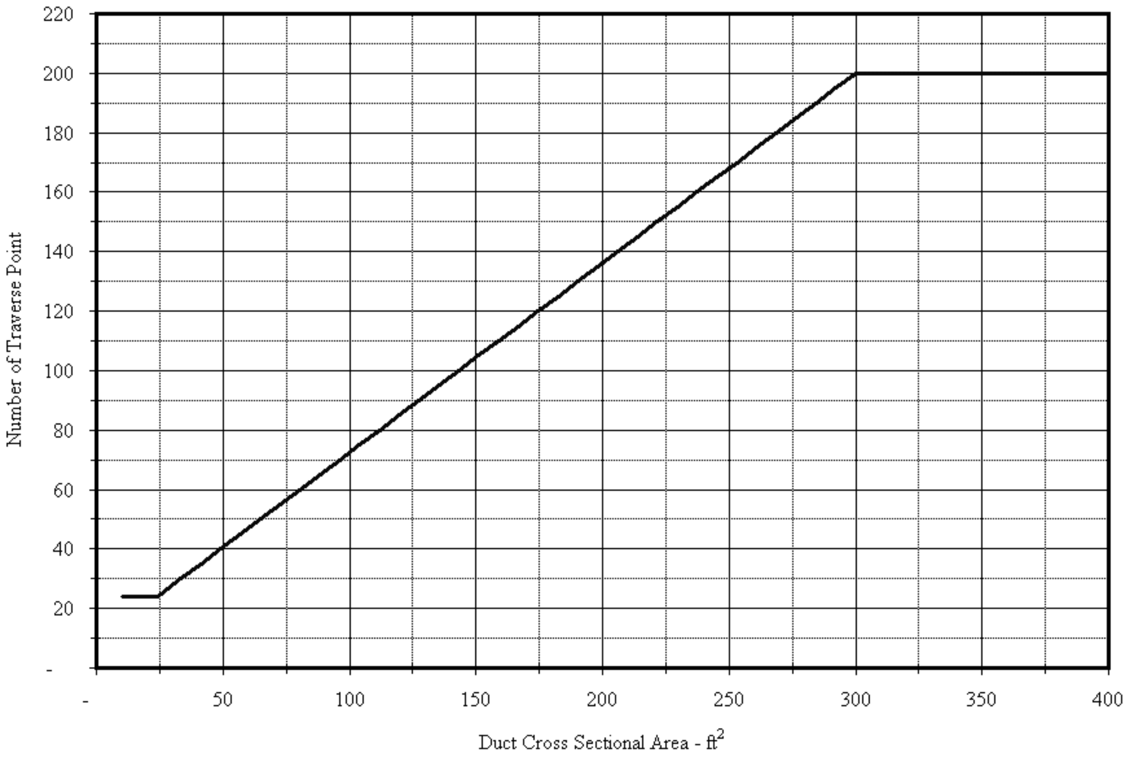


Fig. 4-2.4-3 Number of Traverse Points



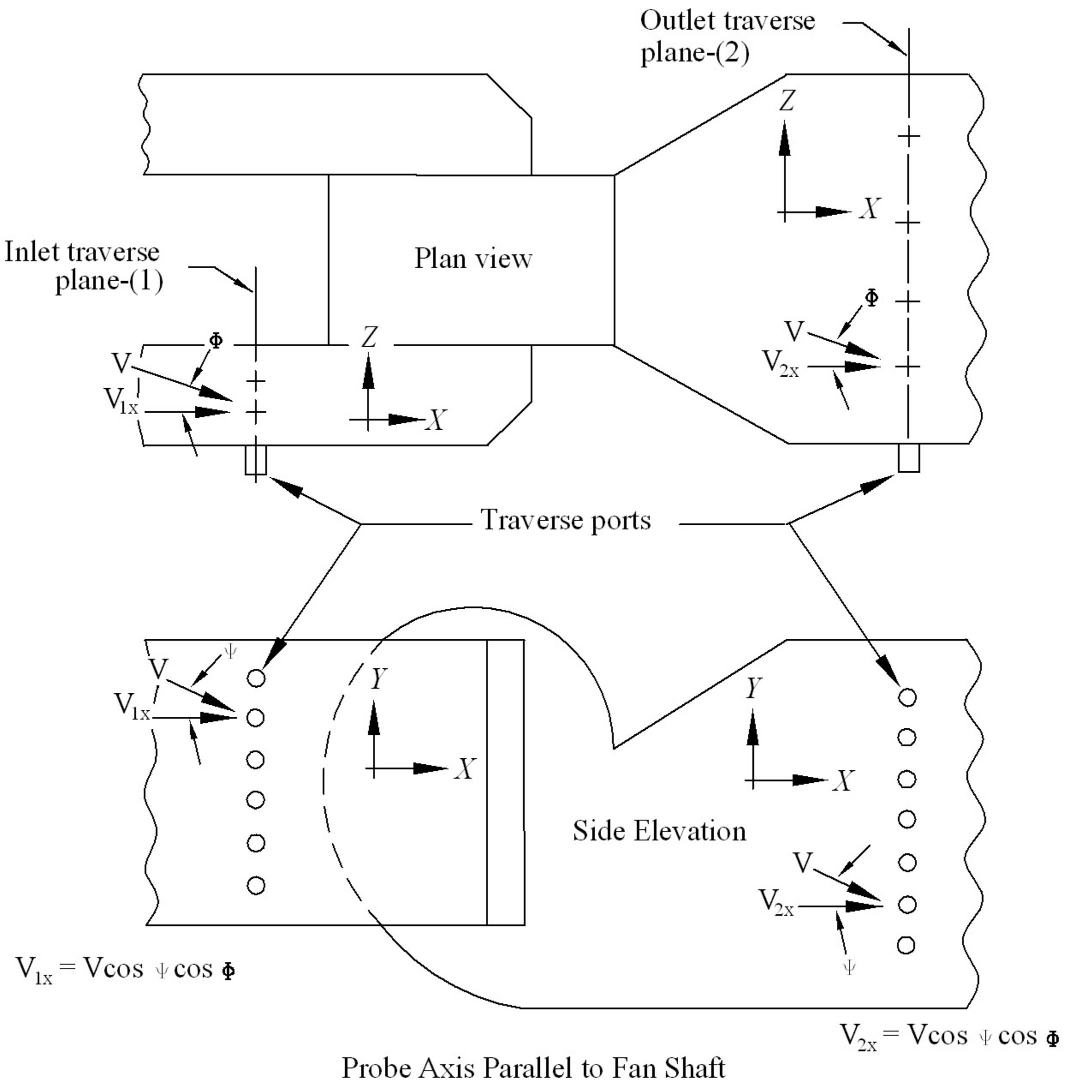
4-2.5 Orientation of Traverse Ports

Yaw and pitch are the two angles necessary to orient the velocity vector with respect to the nominal direction of flow (normal to the measurement plane). It is desirable, when measuring both yaw and pitch, to measure the larger angle by rotating the probe as explained in para. 4-9.5. For this reason, the traverse ports should be located in the duct wall or walls to orient the probes accordingly.

For measurement planes of circular cross section, the traverse ports should be oriented so that the probe stem will be inserted radially.

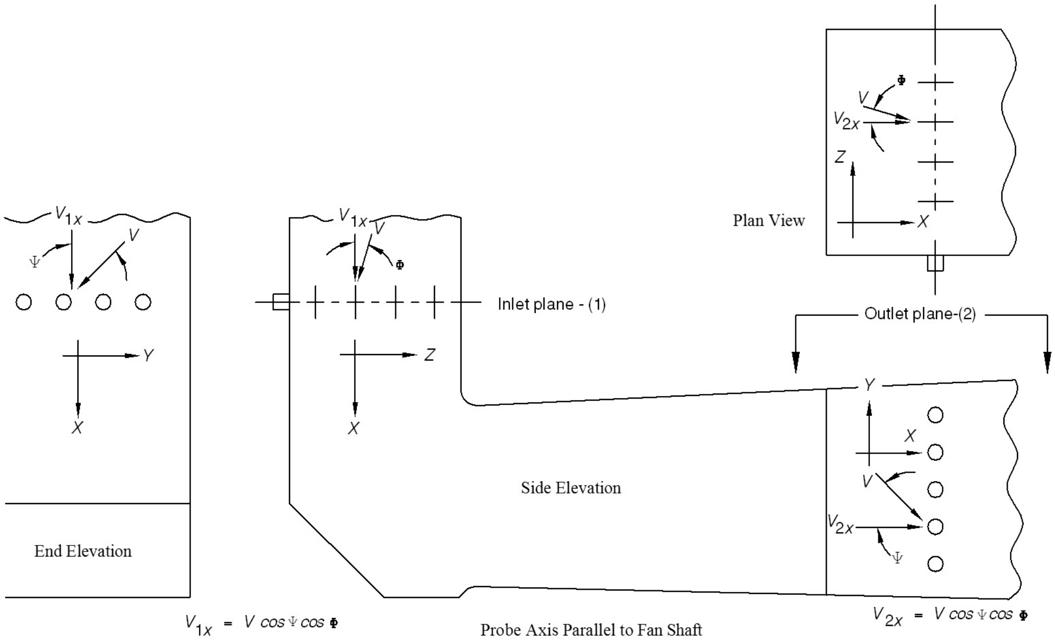
For measurement planes of rectangular cross section, the traverse ports should generally be oriented so that the probe stem is parallel to the fan shaft. This is particularly appropriate for inlet measurements on either axial or centrifugal fans with inlet boxes. It is also appropriate for outlet measurements on centrifugal fans, unless the geometry of the diffuser would suggest otherwise. In any case, the parties should agree in advance to the orientation of the traverse ports. Refer to Figs. 4-2.5-1 and 4-2.5-2.

Fig. 4-2.5-1 Probe Orientation: Centrifugal Fans



Probe Axis Parallel to Fan Shaft

Fig. 4-2.5-2 Probe Orientation: Axial Fans



4-3 BAROMETRIC PRESSURE

The barometric pressure may vary significantly between two locations, both of which are in the vicinity of the test. For example, if the fan is installed in a room and the air is drawn through silencers or heaters, the pressure in the room will be lower than that outside (see Fig. 4-2.2-1). Therefore, care must be taken to apply the correct local barometric pressure to measured static pressure for density calculations.

The wording implies that the barometric pressure will vary whether the measurement is made inside or outside of the duct. The barometric pressure will only vary with elevation.

4-3.1 Instruments

The atmospheric pressure shall be measured with a barometer. A Fortin type barometer is generally preferred, but an aneroid type can be acceptable.

4-3.2 Accuracy

The barometer shall have a demonstrated accuracy of ±0.05 in. Hg (±170 Pa). Readings shall be corrected for temperature and gravity (elevation) according to the procedures given in PTC 19.2 in the section on barometers.

4-3.3 Calibration

The barometer shall be calibrated in accordance with the section on barometer calibration in PTC 19.2.

4-3.4 Number of Readings

Measurements shall be made in the test vicinity at the beginning of the test and repeated every 15 min until the test is completed. These readings shall be used not only for calculation of results but also for monitoring operational steadiness.

4-3.5 Operation

The method of using a barometer is covered in the section on barometers in PTC 19.2.

4-4 TEMPERATURE

4-4.1 Instruments

Various temperature-measuring systems, including thermometers, thermocouples, RTDs, thermistors, and others, may be used.

4-4.2 Accuracy

The temperature-measuring system shall have a demonstrated accuracy of ±2.0°F (±1.0°C). Readings shall be corrected for emergent stem, reference junction temperature, and any other condition that might affect the reading as noted in the appropriate paragraphs of PTC 19.3.

4-4.3 Calibration

Instruments shall be calibrated in accordance with the chapter on calibration of instruments in PTC 19.3.

4-4.4 Number of Readings

Temperature measurements shall be made at each traverse point for each traverse plane. Temperatures can be measured simultaneously with pressures if the thermocouple is attached to the pressure probe so that it does not interfere with other measurements.

If the fan handles ambient air, the air temperature shall be measured in the test vicinity at the beginning of the test and every 15 min until the test is completed. These measurements are used to monitor the operational steadiness and calculate the results.

4-4.5 Operation

The operation of various temperature-measuring systems shall conform to PTC 19.3.

4-5 MOISTURE

4-5.1 Instruments

The moisture content of ambient air shall be measured using a psychrometer or other humidity-measuring system. A simple sling psychrometer is generally preferred.

The moisture content of other gases shall be measured using a condensation/desiccation sampling train or other moisture-measuring system. Stoichiometric methods can also be used in some cases. The condensation/desiccation method is generally preferred, because it does not require fuel sampling and analysis.

4-5.2 Accuracy

The humidity-measuring system for air shall have a demonstrated accuracy of 0.001 mass units of water vapor per unit mass of dry air. For other gases, the measuring system shall have a demonstrated accuracy of 0.5% by volume.

4-5.3 Calibration

The various elements in the moisture-measuring system shall each be calibrated according to the procedure for that element in ASME PTC 19.3, Temperature Measurement Supplement.

4-5.4 Number of Readings

If the fan handles ambient air, the ambient air measurements shall be made in the test vicinity at the beginning of the test and repeated every 15 min until the test is completed. These readings shall be used to monitor operational steadiness and calculate results. Moisture measurements in other gases shall be made at a minimum of five locations within the test plane. This requirement can be reduced to a single point sample if the parties agree that the preliminary test shows the distribution of moisture is sufficiently uniform.

4-5.5 Operation

If used, the moisture sampling train shall conform to the latest revision of the Code of Federal Regulations, Title 40, Chapter I, Part 60, Appendix A, Method 4 (40 CFR, Ch. I, Pt. 60, App. A-3, Meth. 4), "Determination of Moisture Content in Stack Gases."

4-6 GAS COMPOSITION

4-6.1 Instruments

The composition of air can generally be assumed to be that of normal atmospheric air, and measurements need not be made. The composition of other gases shall be measured by using a sampling train containing a gas analysis system. Electronic analyzers should be used to measure flue gas composition.

4-6.2 Accuracy

The gas composition-measuring system shall have a demonstrated accuracy of 0.1% by volume for each major constituent (e.g., 5% ± 0.1% for oxygen).

4-6.3 Calibration

The various elements of the gas composition-measuring system shall be calibrated against appropriate standards. Certified standard gas samples are available commercially.

4-6.4 Number of Readings

Gas composition measurements shall be made at a minimum of five locations within the test plane. This requirement can be reduced to a single point sample if the parties agree that the preliminary test shows the distribution of gas composition is sufficiently uniform.

4-6.5 Operation

Operation of flue and exhaust gas analysis systems shall conform to PTC 19.10.

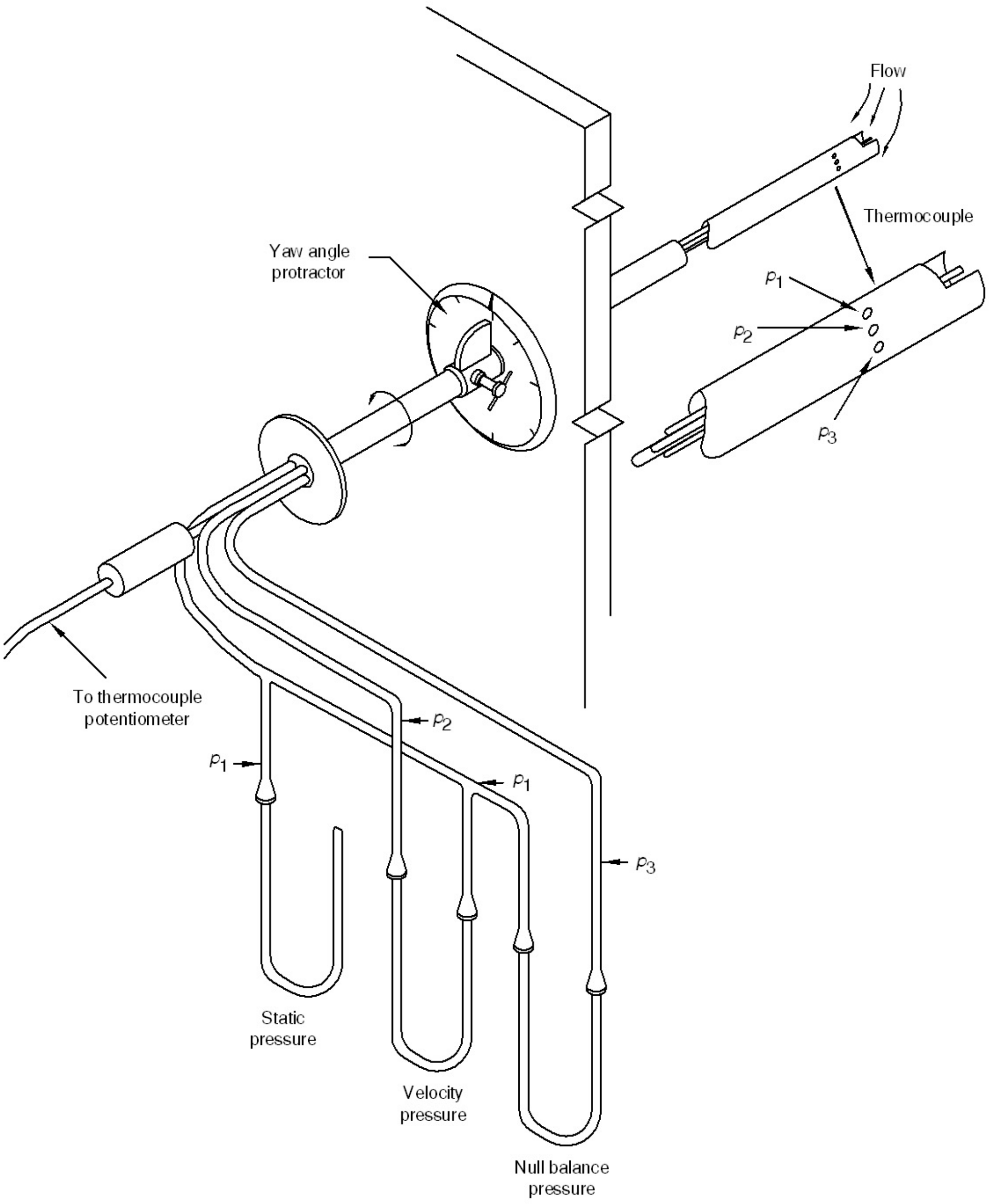
4-7 PRESSURE SENSING

Point values of pressure (velocity and total or static pressure) shall be measured using a probe that can be positioned at the appropriate points by insertion through one or more ports as required. A probe capable of measuring static pressure, total pressure, their differential, yaw angle, and pitch angle is preferred (see Figs. 4-7-1 through 4-7-4). A probe with only yaw-measuring capability can only be used if a preliminary test gives good evidence that the average of absolute values of pitch angle does not exceed 5 deg. A nondirectional probe may only be used where the preliminary test gives good evidence that the average of the absolute values of neither yaw angle nor pitch angle exceeds 5 deg.

4-7.1 Instruments

Nondirectional probes include Pitot-static tubes and Stauschiebe tubes. The latter are also called type S or forward-reverse tubes. Direction-finding probes include the Fechheimer probe, which has two holes and is capable of determining yaw angles and static pressure only. A three-hole version of the Fechheimer probe, also called a three-hole cylindrical yaw probe, can be used to determine total pressure (and therefore indicated velocity pressure), as well as the static pressure and yaw (see Fig. 4-7-1). A five-hole probe is generally required to determine pitch angles, as well as the various pressures and yaw angles. See Fig. 4-7.1-1,

Fig. 4-7-1 Fechheimer Probe



illustrations (a) through (c). Probes with wedge shapes (see Fig. 4-7.1-2) where the holes are located on flat surfaces are slightly preferred over probes with spherical shapes throughout, because they are easier to null-balance (see para. 4-9.5). Total probe blockage shall not exceed 5% of the duct cross-sectional area.

4-7.2 Accuracy

Refer to subsection 4-8 for accuracy of pressure readings and subsection 4-9 for accuracy of angularity readings.

Fig. 4-7-2 Five-Hole Probe

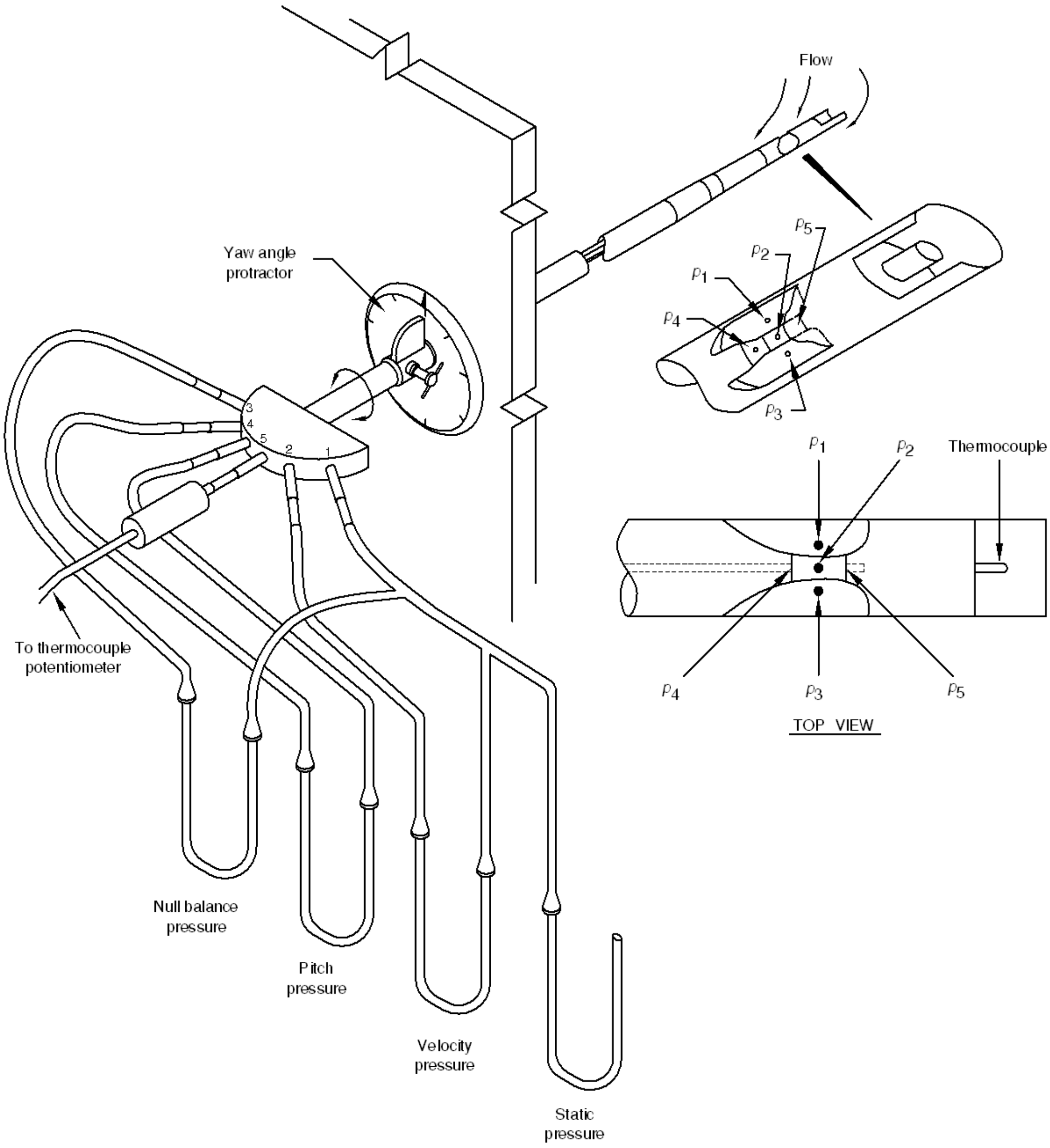


Fig. 4-7-3 Yaw and Pitch Planes

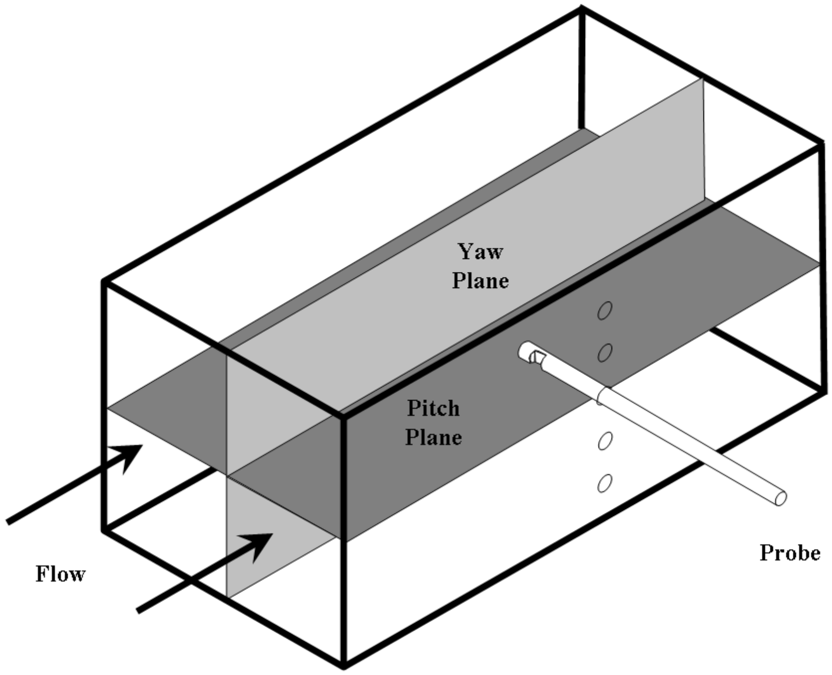


Fig. 4-7-4 Yaw and Pitch Convention

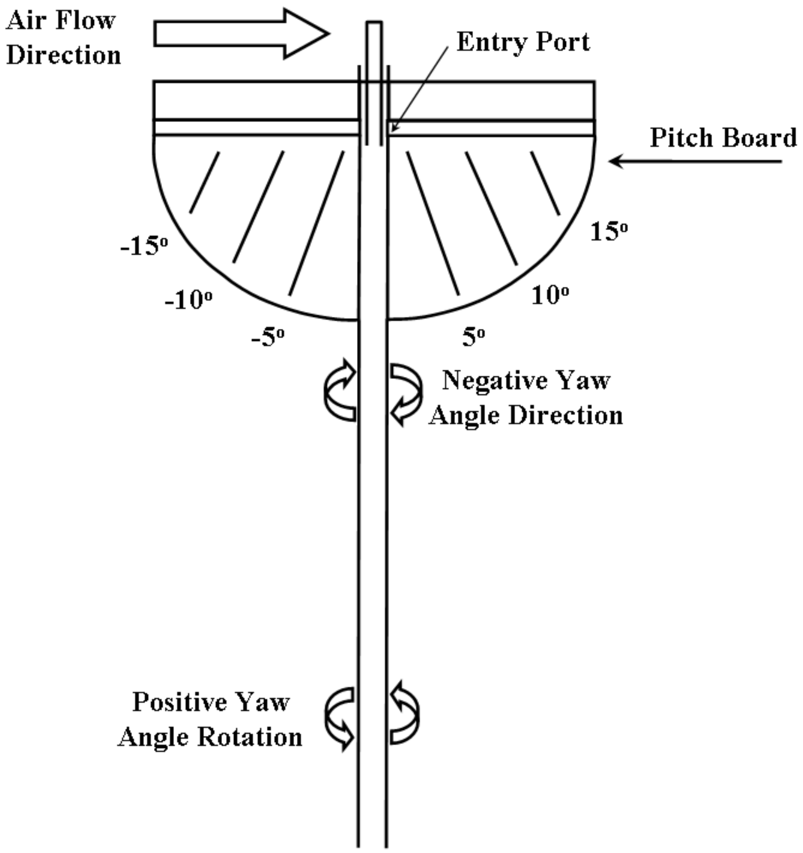


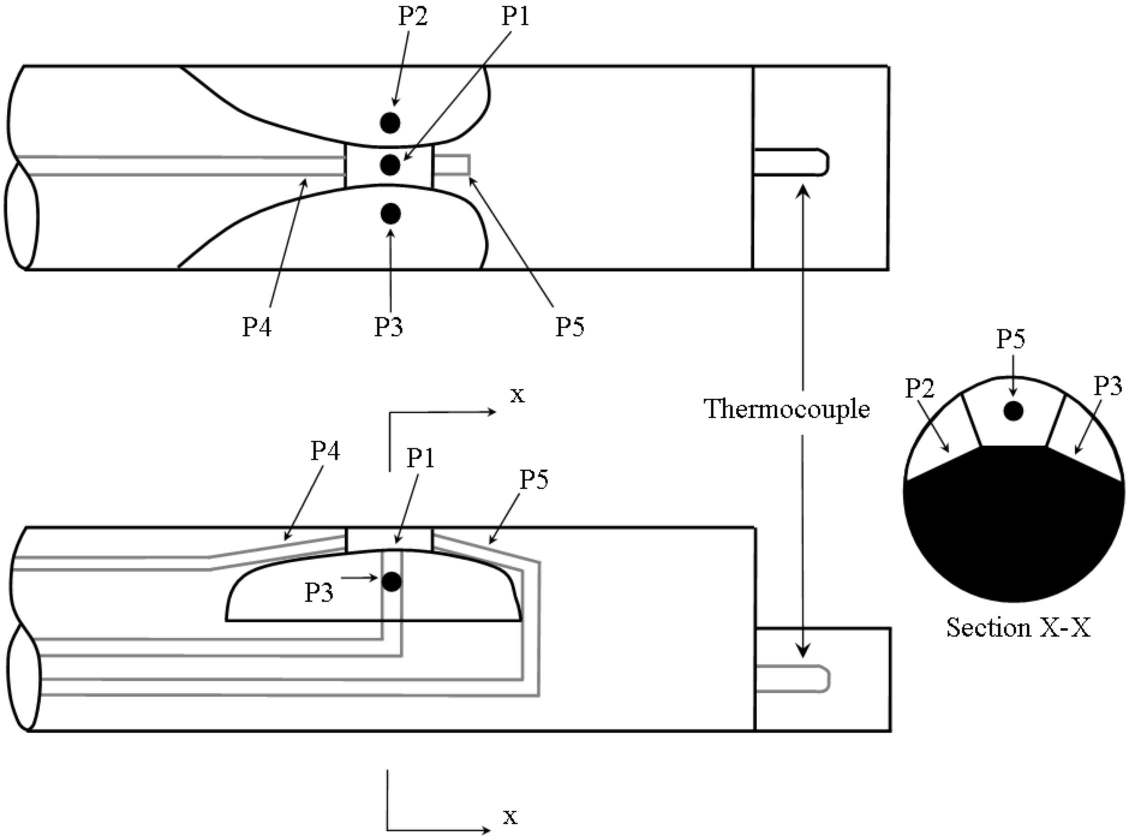
Fig. 4-7.1-1 Five-Hole Probe Photos

(a)

(b)

(c)

Fig. 4-7.1-2 Prism Probe Cut-Away



4-7.3 Probe Calibration

All probes except Pitot-static tubes shall be calibrated. Pitot-static tubes are considered primary instruments and need not be calibrated, provided they are maintained in the specified condition described in. reference [2]. The calibration procedures specified in this paragraph apply to pressure measurement only. Calibration of probes for direction sensing is usually carried out simultaneously with calibration for pressure. See para. 4-9.3 for calibration procedures for direction sensing.

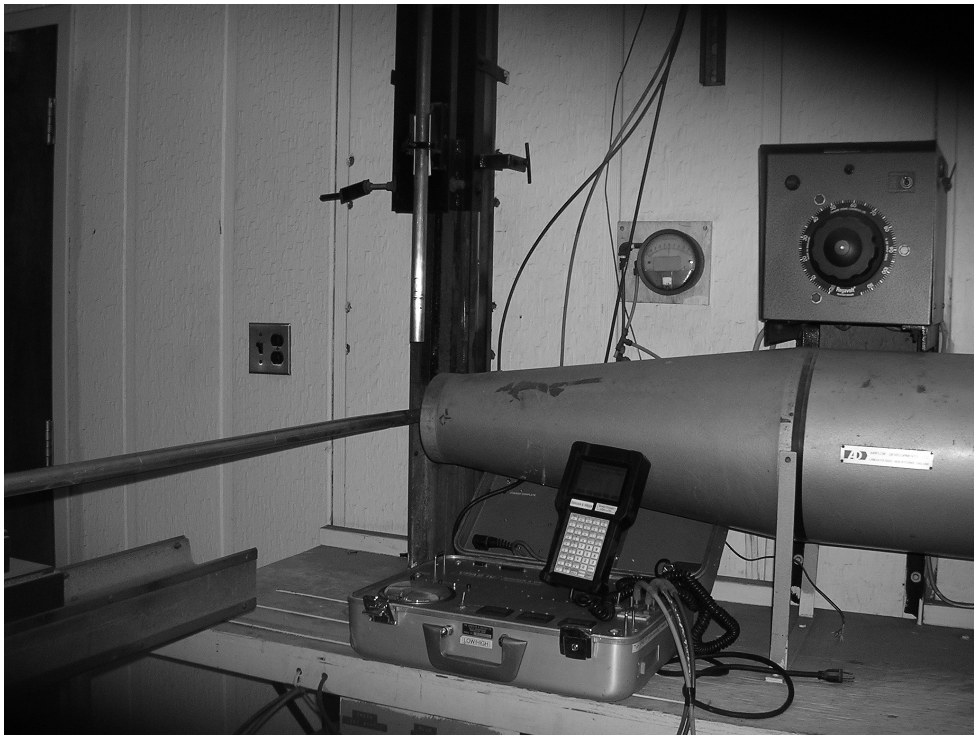
Probe calibration may be carried out in a free stream nozzle jet or a closed wind tunnel (see Figs. 4-7.3-1 through 4-7.3-3). In either case, the probe blockage shall be less than 5% of the cross-sectional area. Preferably, the probe blockage should be as small as possible. The flow should be adjusted to produce equally spaced calibration points. For two- and three-hole probes, a minimum of eight points between the range of 30 ft/sec and 100 ft/sec nominal velocity is required. For five-hole probes, calibration points are required at a minimum of three points, typically 40 ft/sec, 70 ft/sec, and 100 ft/sec nominal velocity. Application of calibration data is described in subsection 5-2.

The calibration reference may be a standard Pitot-static tube (preferred) or a previously calibrated reference probe of another type. The blockage of the reference probe should be as small as possible. In no case shall the blockage of the reference probe exceed 5% of the cross-sectional area.

Fig. 4-7.3-1 Free Stream Nozzle Jet

Fig. 4-7.3-2 Wind Tunnel

Fig. 4-7.3-3 Free Stream



The reference probe and test probe shall each be mounted so that they can be placed in the stream alternately, and their positions in the stream will be the same and firmly held, or the test probe and the reference probe can be placed side by side if it can be demonstrated that there is no difference in flow conditions between the two locations, the total blockage does not exceed 5%, and there is no interference between the test probe and reference probe. When calibrating directional probes, the probe shall be aligned with the stream to eliminate yaw according to the null-balance principle described in para. 4-9.5. Any offset of the null position with respect to jet or tunnel axis shall be recorded. Positive yaw angles are associated with probe rotation clockwise to achieve null-balance position and negative yaw angles with counterclockwise rotation. Static pressure indication shall be from the appropriate static pressure hole(s) of the reference probe and test probe and not from wall taps (wind tunnel), nor shall it be assumed equal to ambient pressure (free jet). The test probe and reference probe shall be connected to appropriate indicators so that the indicated static pressure, p_{si}; indicated total pressure, pi_t; and indicated velocity pressure, p_{vi}, can each be recorded for each probe. The static pressure hole that is used to obtain indicated velocity pressure during the calibration should be noted and the same hole used for subsequent tests.

Probe calibration shall be expressed in terms of a probe total pressure coefficient, K_t, and a probe velocity coefficient, K_v. The probe total pressure coefficient is calculated from the calibration data by

$$K_t = \frac{\left(p_{ti}\right)_{ref}}{\left(p_{ti}\right)_{test}} \tag{4-7-1}$$

The probe velocity pressure coefficient is calculated from the test data by

$$K_v = \frac{\left(\frac{(K_v)_{ref}}{1+(K_v)_{ref}\,\beta_{ref}}\right)\left(\frac{(p_{v1})_{ref}}{(p_{v1})_{test}}\right)}{1-\left(\frac{\beta_{test}\,(K_v)_{ref}}{1+(K_v)_{ref}\,\beta_{ref}}\right)\left(\frac{(p_{v1})_{ref}}{(p_{v1})_{test}}\right)} \tag{4-7-2}$$

where

$$\beta = \pm\frac{(C_D)(1-\varepsilon_p)}{4(1-\varepsilon_p)-3}\left(\frac{S_p}{C}\right) \tag{4-7-3}$$

and

$$(1-\varepsilon_p) = 1-\left[\frac{(K_v)_{ref}}{2k}\right]\left[\frac{(p_{vi})_{ref}}{(p_{sa})_{ref}}\right] \tag{4-7-4}$$

NOTE: It is recognized that C_D is usually not known to a high degree of accuracy. Lacking specific information, $C_D \approx 1.2$ for probes of cylindrical shape. For a closed wind tunnel, β will be positive; for a free jet, β will be negative.

The equation for K_v includes a correction for probe blockage derived from the analysis presented in references [15] and [16]. If the reference probe is a Pitot-static tube $K_{v,ref} = 1.0$ and the blockage of both the reference probe and test probe is negligible (S_p/C<0.005), the equation for K_v assumes the simplified form

$$K_v = \frac{(p_{vi})_{ref}}{(p_{vi})_{test}} \tag{4-7-5}$$

Generally, the probe total pressure coefficient and probe velocity pressure coefficient are functions of Reynolds number, Re_p, for nondirectional and three-hole probes and functions of pitch pressure coefficient, C_ϕ, and Reynolds number for five-hole probes. For probes of highly angular shape, such as the prismatic five-hole probe shown in Fig. 4-7.1-2, the coefficients may be expected to be independent of Reynolds number for values of Reynolds number above roughly 10^4. For such probes, Reynolds number effects on the coefficients may be ignored. See para. 4-1.2 regarding calibration function.

Calibrated probes should be handled with care because large scratches or nicks near the pressure taps will invalidate the calibration. Probe recalibration should be performed on a regular basis but shall be performed if damage near the sensing holes is noted.

4-7.4 Number of Readings

Pressure measurements shall be made at each traverse point for each traverse plane. The indicated velocity pressure and either the total pressure or static pressure shall be measured. The remaining pressure can be determined arithmetically.

Pressures can be obtained at two or more locations, simultaneously, by using two or more probes as appropriate. It may be desirable to traverse both inlet boxes of a double inlet fan and to traverse from both sides of the outlet, all simultaneously. This would require four probes and four probe crews, but it would significantly reduce the total elapsed time required for a test.

4-7.5 Operation

Refer to paras. 4-8.5 and 4-9.5.

4-8 PRESSURE INDICATING

4-8.1 Instruments

Manometers or other pressure-indicating systems shall be connected to the appropriate taps of the pressure-sensing probes to measure point values of pressure. A five-hole probe requires one indicator for velocity pressure, one indicator for static pressure or total pressure, and additional indicators for nulling and pitch determination (see subsection 4-9 for the latter). A three-hole probe requires the same indicators, except for pitch determination. A nondirectional probe requires indicators only for velocity pressure and either static or total pressure. Pressure transducers are generally preferred, but inclined manometers, U-tube manometers, and other indicators are acceptable if they meet the following specifications.

4-8.2 Accuracy

Pressure-measuring systems including the sensor and indicator shall have a demonstrated accuracy of ±1% of the reading or 0.01 in. wg (2.5 Pa), whichever is larger. Readings shall be corrected for any difference from calibration conditions in specific weight of manometer fluid, gas column balancing effect, or any change in length of the graduated scale due to temperature. However, corrections may be omitted for temperature changes less than 10ºF (5ºC) from calibration and elevation changes less than 5,000 ft (1 500 m).

4-8.3 Calibration

Pressure-indicating instruments shall be calibrated against a suitable standard for pressures from 0 in. wg to 10 in. wg (0 kPa to 2.5 kPa), gage of the micrometer type, or a precision micromanometer. When the pressure is above 10 in. wg (2.5 kPa), calibration shall be against a water-filled hook gage of the micrometer type, a precision micromanometer, or water-filled U-tube. Pressure-indicating instruments should preferably be calibrated in place, but the parties may agree to a remote calibration in a more suitable laboratory environment. In the latter case, extreme care should be taken to mount the pressure-indicating instrument in exactly the same manner for calibration as it is mounted for the test. Calibration points shall be selected to fall at both ends of the expected range and at sufficient intermediate points so that no reading will be more than 0.25 in. wg (60 Pa) removed from a calibration point for pressures from 0 in. wg to 10 in. wg (0 kPa to 2.5 kPa) or more than 1 in. wg (250 Pa) removed for pressures above 10 in. wg (2.5 kPa).

4-8.4 Averaging Fluctuating Readings

Pressure-measuring instruments shall be read at each position of the probe as outlined in para. 4-7.4. Since pressures are seldom strictly steady, the pressure indicated on any instrument will fluctuate with time. To obtain a reading, either the instrument shall be damped or the readings shall be averaged in a suitable manner. Averaging can be accomplished mentally, if the fluctuations are small and regular. If the fluctuations are large and irregular, more sophisticated methods shall be used. It is possible to obtain a temporal average electronically when an electrical pressure transducer is the primary element. Even though the spatial average velocity is obtained from the square roots of the temporal average velocity pressures, it is not proper to take the square root of the raw data before temporal averaging, as this may introduce a bias into the average values [10].

4-8.5 Operation

For many of the principles of operation, refer to PTC 19.2. Refer to Figs. 4-7-1 and 4-7-2 for the proper hose connecting arrangements for probes and indicators. Precautions should be taken to protect the indicator from the effects of wind, sun, and radiant heat. Periodically during the test, probes, hoses, and indicators should be checked for leaks or plugging. Plugging can result from either particulate buildup in the probe or condensation in a portion of the system.

Indicators used for static or total pressure measurement have one tap open to atmosphere. If the indicator is not located in the same atmosphere as the barometer, an additional measurement to determine the difference in pressure is required.

4-9 YAW AND PITCH

4-9.1 Instruments

Yaw angle shall be measured using a directional probe equipped with a suitable indicating device. Pitch shall be determined from directional probe calibration. A five-hole probe is preferred as noted in para. 4-7.1. A three-hole probe may be suitable in some cases (see Figs.4-7-1 and 4-7-2).

4-9.2 Accuracy

The yaw- and pitch-measuring systems shall have demonstrated accuracies of ±2 deg.

4-9.3 Calibration

A reference line shall be scribed along the probe axis prior to calibration for pressure response. This reference line is typically aligned with, or 180 deg from, the total pressure-sensing hole. The scribe is used as a reference position for installation of a yaw angle-measuring device. The relationship of the reference line to null-balance position shall be known as determined in para. 4-7.3. The probe is then equipped with a protractor scale that can be checked against any high-quality protractor used as a reference. As noted below, the protractor arrangement is only used to measure yaw.

Calibration for pitch can be performed in a free stream nozzle jet or in a wind tunnel and is usually completed during calibrations outlined in para. 4-7.3. The facility should be equipped to allow the test probe to be positioned at various pitch angles. The mounting apparatus should firmly hold the test probe at each location along the pitch arc. Probe sensing head location should remain in the same position within the flow stream as the probe pitch angle is varied.

The probe shall be precision aligned at various pitch angles, null-balanced, and the pressure difference across the taps for the fourth and fifth holes recorded along with pressures and pressure differences required in para. 4-7.3 and any null-balance offset. Pressure data shall be recorded at pitch angles from –30 deg to +30 deg in 5-deg increments at each of three nominal velocities as described in para. 4-7.3. The calibration facility flow should be set at one nominal velocity and data recorded at each required pitch angle before proceeding to subsequent nominal velocities and repeating. Alternatively, the nominal velocity can be set at required values for each probe pitch position to develop the data set.

Calibration functions, which represent pitch angle and probe coefficient(s) as a function of pitch pressure coefficient, C_{Φ} ($\equiv$ pitch pressure difference/indicated velocity pressure), and Reynolds number may be derived. For probes of highly angular shape, such as the prismatic five-hole probe, the pitch angle-pitch pressure coefficient relationship may be expected to be independent of Reynolds number for values of Reynolds number above roughly 10^4. For such probes, Reynolds number effects may be ignored (see Figs. 4-9.3-1 through 4-9.3-3).

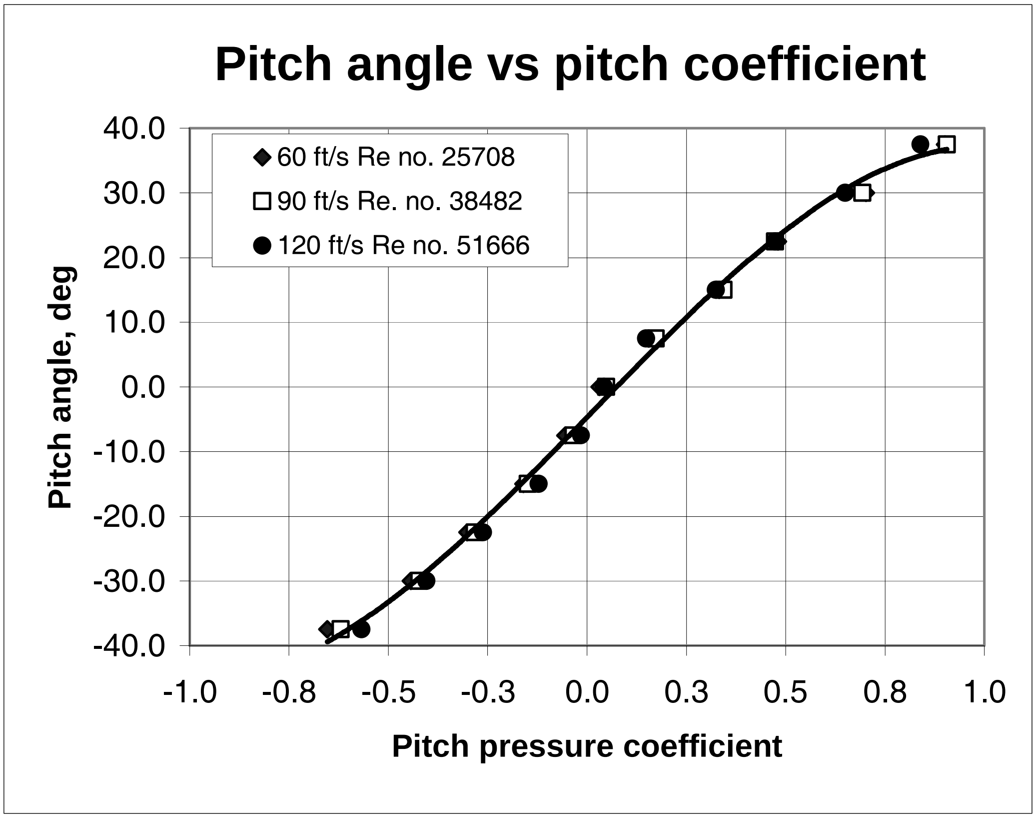
4-9.4 Number of Readings

Yaw and pitch angles shall be determined at each traverse point for each traverse plane. This is the same requirement as for pressures that should be measured simultaneously.

4-9.5 Operation

In operation, a five-hole probe is inserted in the proper port to the proper depth for each traverse point. The probe should be rigid enough over its inserted length to avoid any droop beyond the permissible amount as noted in para. 4-2.4. The reference line on the probe should be used to orient the probe in such a way that when the total pressure hole is pointing upstream perpendicular to the measuring plane, the indicated yaw angle is zero. The probe is then rotated about its own axis until a null balance is obtained

Fig. 4-9.3-1 Pitch Angle, Φ, Versus Pitch Coefficient, C_Φ



across the taps of the static pressure holes. The angle of probe rotation from the zero yaw reference direction is measured with an appropriate indicator and is reported as the yaw angle. Without changing the angularity of the probe, the pressure difference across the taps for the fourth and fifth holes shall also be recorded and used with the indicated velocity pressure and pitch pressure coefficient to determine pitch angle. Measurements of indicated velocity pressure and static pressure or indicated velocity pressure and total pressure as outlined in para. 4-7.4 shall be recorded with the probe in the proper null-balance position. (Note that a null balance can be obtained at four different positions, but only one is correct. Incorrect null positions usually correspond to negative velocity pressures.)

When a directional probe cannot be nulled, velocity pressure shall be recorded as zero. A three-hole probe is operated in a similar manner, except that the pitch pressure difference is omitted.

Fig. 4-9.3-2 Velocity Pressure Coefficient, K_v, Versus Pitch Pressure Coefficient, C_Φ

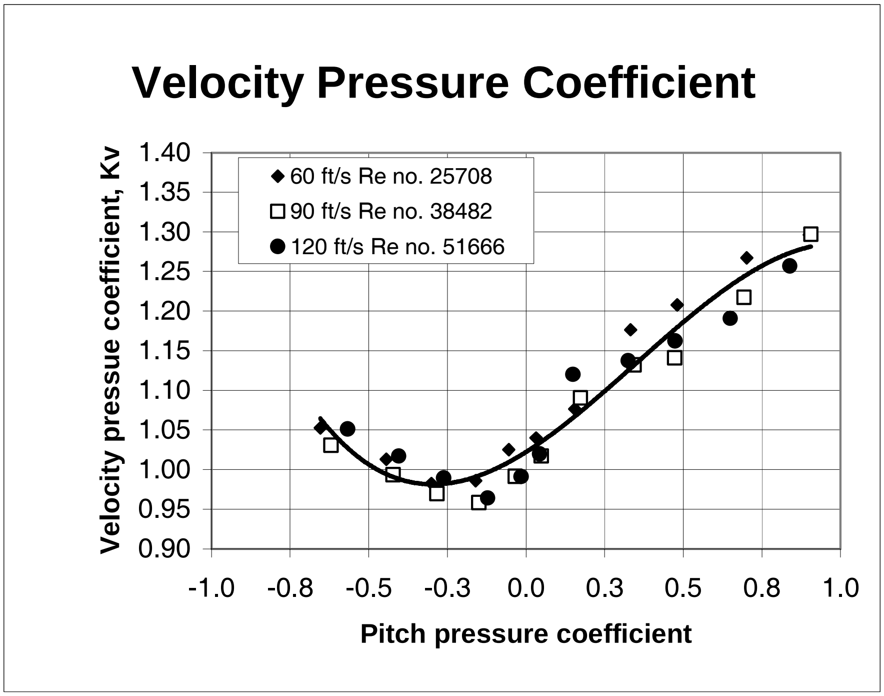
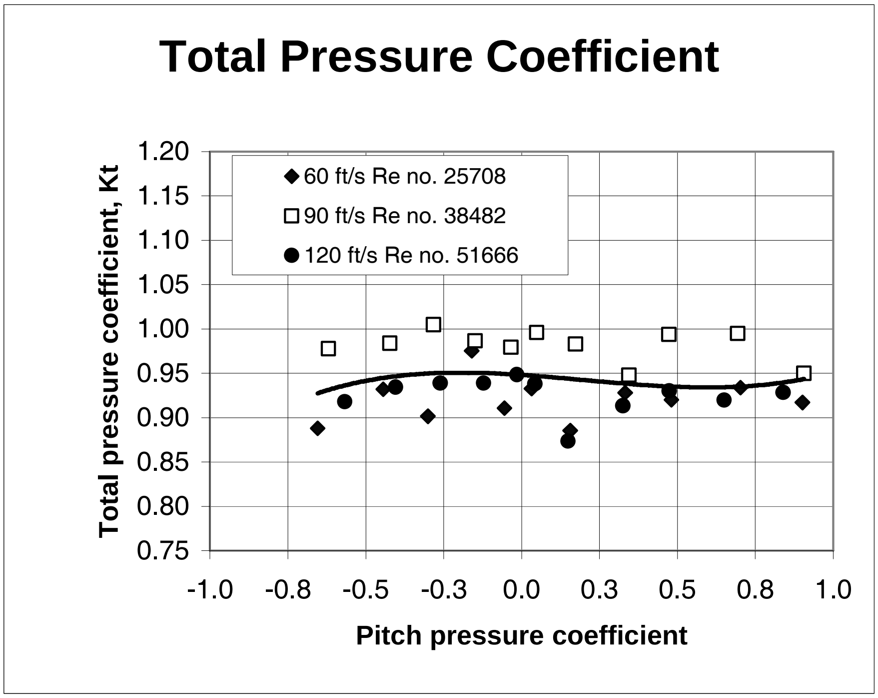


Fig. 4-9.3-3 Total Pressure Coefficient, K_t, Versus Pitch Pressure Coefficient, C_Φ



4-10 ROTATIONAL SPEED

4-10.1 Instruments

The speed of the fan shall be measured with a speed-measuring system. An electronic counter actuated by a magnetic pulse generator or photoelectric pickup is preferred. Slip counting with stroboscopic light may be acceptable for speeds close to line frequency synchronous speeds. Hand tachometers, mechanical revolution counters, and vibrating-reed tachometers are unacceptable.

4-10.2 Accuracy

Speed-measuring instruments shall be calibrated against the line frequency of a suitable major power circuit or other frequency standard.

4-10.3 Number of Readings

Fan speed shall be measured at the beginning of the test and every 15 min until the conclusion of the test. These readings shall be used to monitor operational steadiness as well as for calculations.

4-10.4 Operation

The electronic counter should be equipped with a digital readout and may be equipped with a recorder and an automatic average.

With the slip method, the shaft must be marked with a reference line or other mark that is easily visible under stroboscopic light flashing at line frequency. The mark will appear to slowly rotate opposite shaft rotation and permit visual observation of the slip frequency. A stopwatch shall be used to measure the time for at least ten rotations of the mark. Average slip frequency is derived by dividing the total number of mark rotations by the measured time interval for which the counts were made.

See PTC 19.13 for further information on the measurement of rotary speed.

4-11 INPUT POWER

4-11.1 Instruments

The fan input power shall be derived from measurements of torque with a torque meter, measurements of electrical input when a calibrated electric motor is used, or other suitable measurements if the fan is driven by some other calibrated prime mover and drive train. Both the torque meter and calibrated prime mover measurements qualify as preferred methods. If a torque meter cannot be used and if the drive train is not calibrated prior to installation, the parties to the test must agree upon a method of estimating the drive train losses. Also, it must be noted that various methods and procedures for calibrating the drive train may result in accuracies that are unacceptable for this Code. The parties to the test and party responsible for the calibration must agree beforehand to the method of calibration and expected accuracy. (PTC 19.7-1980 and relevant IEEE standards, such as IEEE 112, may offer some insight.)

Since the temperature rise through a fan is generally not large enough to permit accurate measurement and heat transfer losses through the casing are indeterminate, the heat balance method is not acceptable for determining fan input power.

4-11.2 Accuracy

The input power-measuring system shall have a demonstrated accuracy of ±1%.

4-11.3 Calibration

A torque meter shall be calibrated in accordance with the provisions of PTC 19.7. The drive train in the context of this Code includes the driver, whether it is an electric motor, steam turbine, or other prime mover, and any in intermediate elements, such as gear boxes and variable speed drives. The drive train may be calibrated as a unit, or the driver and any intermediate elements may be separately calibrated. Calibration procedures as given in ASME PTC 19.7 and the following IEEE standards should be followed.

Designator	Title
IEEE 112	Test procedure for polyphase induction motors and generators
IEEE 113	Test procedure for DC machines
IEEE 114	Standard test procedures for single-phase induction motors
IEEE 115	Test procedure for synchronous machines

Calibration shall be performed under specified operating conditions and a range of loads sufficient to cover the anticipated test conditions.

4-11.4 Number of Readings

Torque or electrical input shall be measured at the start of the test and at least every 15 min until the conclusion of the test. These readings shall be used to monitor operational steadiness, as well as for calculations.

4-11.5 Operation

Operation of prime movers is covered in the various Standards listed in para. 4-11.3. Operation of the instruments for measuring the output of these prime movers is covered in various supplements on instruments and apparatuses. Electrical instruments shall conform to ASME PTC 19.22. A wattmeter and voltmeter or an ammeter, voltmeter, and power factor meter may be used together with the necessary instrument transformers. Of the above-mentioned devices, a wattmeter with appropriate current and voltage transformers is preferred. Refer to PTC 19.6, Electrical Power Measurements, for instructions. Meter ranges and transformer ratio shall be such as to produce readings above one-third full scale. Instruments shall have full-scale accuracy of 0.5% or better. They shall be used in the same position as rated (usually horizontal). Care should be taken to maintain instruments at a uniform and constant temperature near the calibration temperature; otherwise, corrections shall be made according to manufacturer’s instructions regarding lead wires, waveform, etc.

The preferred location for taking electrical measurement is at the terminals of the motor. If this is not possible, then allowance shall be made for the drop in potential between the point of measurement and the motor terminals. Care shall be taken to measure motor power only and not include any auxiliary’s power.

When fan speed is controlled by a variable frequency drive or a hydraulic coupling, accurate determination of fan input power by electrical means is impractical, and a torque meter is the preferred method of power measurement.

For fans driven by steam or gas turbines, or other nonelectric means, the torque meter is again the preferred method of power measurement.

For a summary of instrumentation requirements, see Table 4-11.5-1.

Table 4-11.5-1 Summary of Instrumentation Requirements

Measurement	Instrument	Accuracy	Frequency of Readings	PTC 11 Subsection
Atmospheric pressure	Barometer	±0.05 in. Hg ±170 Pa	15 min	4-3
Temperature	Thermometer or thermocouple	±2ºF ±1ºC	Each traverse point	4-4
Moisture	Psychrometer or condensation/ desiccation	0.001 lbm/lbm air 0.001 kg/kg air 0.5% by volume gas	Air: 15 min Gas: 5 points	4-5 4-5
Gas analysis	Electronic analyzers	0.1% by volume	5 points	4-6
Pressure	Manometer or pressure indicator	Larger of ±1.0% or ±0.01 in. wg ±2.5 Pa	Each traverse point	4-8
Yaw angle	Protractor	±2 deg	Each traverse point	4-9
Pitch angle	(See Pressure)	N/A	Each traverse point	4-8 and 4-9
Speed	Magnetic pulse fiber optic or slip	Smaller of ±0.1% or ±1 rpm	15 min	4-10
Power	Torque meter or calibrated drive	±1.0 %	15 min	4-11

Section 5
Computation of Results

5-1 GENERAL CONSIDERATIONS

The results of the test shall be calculated in accordance with the appropriate paragraphs of this Section and any prior agreement reached by the parties regarding computation of results. The following paragraphs are intended to cover all possible cases, but it is not necessary to use every paragraph for any particular case (i.e., it is not necessary to refer to the paragraphs on products of combustion if the test gas is air). Similarly, only the paragraphs on computing power that correspond to the method of power measurement shall be used. Various other calculations may be omitted depending on whether mass flow rate and specific energy or volume flow rate and fan total pressure are used to express fan performance. The data to be used in the calculations are the measured values of pressure and temperature at various planes, the fan input power measurements, various geometric information (primarily duct areas at measurement planes), and information used to determine gas composition.

This Section provides the equations for calculating test results and uncertainty. These equations can be used directly; however, incorporating them into a spreadsheet or other computer program together with the test data is recommended because of the complexity involved.

5-1.1 Calibration Corrections

Temporal averaging shall be performed prior to correcting for calibrations. Calibration corrections shall be applied to individual readings before spatial averaging or other calculations.

5-1.2 Average Values

Nonuniform velocity distribution and temperature or composition stratification are normal on large fans. Therefore, the appropriate volume-flow-weighted or mass-flow-weighted average values at the traverse planes must be used for determination of fan performance. [17]

5-2 CORRECTION OF TRAVERSE DATA

Difficulties arise in using traverse data in calculations as these data usually must be corrected for probe calibration and possibly for blockage and compressibility as well. The probe calibration coefficients K_t and K_v are sometimes functions of the probe Reynolds number Re_p, which is determined by actual gas velocity V, density ρ, and viscosity μ at the probe location. They are also slightly dependent upon specific heat ratio k. As these four quantities are determined only from the measurements themselves, an iteration procedure may be necessary. Such a procedure would be as follows:

(a) Select provisional values of K_{tj}, K_{vj}, and k (see para. 5-2.1).

(b) Correct the traverse readings for calibration, and, if necessary, probe blockage and compressibility (see para. 5-2.2)

(c) Proceed with calculations.

(d) After determining gas composition (see subsection 5-3), densities (see subsection 5-4), and velocities (see para. 5-5.1) at all points in a traverse plane, calculate Reynolds number (see para. 5-2.2) at all points, and determine new values of K_{tj} and K_{vj}.

(e) If new values of K_{tj} and K_{vj} are significantly different from the old values, the process must be repeated.

The probe calibration coefficients are also a function of pitch pressure coefficient, C_ϕ; however, this dependency does not affect the iteration process.

5-2.1 Guideline for Initial Estimation of Probe Coefficient

To begin calculations, initial values of K_{tj} and K_{vj} must be selected. The selection of an appropriate value makes the calculation procedure converge more rapidly, often making iteration unnecessary. The following are guidelines to help the initial selection of K_{tj} and K_{vj}:

(a) For Pitot-static probe, K_{tj} and K_{vj} = 1.0 and need not be changed.

(b) For other probes, the K_{tj} and K_{vj} versus Re_p curves should be relatively flat in the range of interest; hence, any reasonable first estimates of K_{tj} and K_{vj} should produce satisfactory results. The following ideas are suggested:

(1) Select the values of K_{tj} and K_{vj} at the middle of the range of calibration data

(2) Use an average K_{tj} and K_{vj} value based on the calibration data

(3) Estimate Re_p from specified fan conditions, and use corresponding K_{tj} and K_{vj} values, or

(4) Estimate Re_p from a typical point in the traverse data, and use the corresponding K_{tj} and K_{vj} values

5-2.2 Correction for Probe Coefficient and Probe Blockage

Measured values from traverse are t_i, p_{vi}, and p_{si} or p_{ti}. The remaining pressures can be calculated from $p_{ti} = p_{si} + p_{vi}$. Corrected values (subscript j) at each point shall be obtained from the measured values (subscript i) at that point and probe coefficients K_{tj} and K_{vj} using

$$p_{tj} = K_{tj} p_{ti} \tag{5-2-1}$$

$$K_{vjc} = \frac{K_{vj}}{1+\beta_j K_{vj}} \tag{5-2-2}$$

$$p_{sj} = K_{tj} p_{ti} - K_{vjc} p_{vi} \quad \text{or}$$

$$p_{sj} = K_{vjc} p_{si} - (K_{vjc} - K_{tj}) p_{ti} \tag{5-2-3}$$

$$p_{saj} = p_{sj} + C_{13} p_b \tag{5-2-4}$$

$$p_{vj} = K_{vjc}(1-\varepsilon_p) p_{vi} \tag{5-2-5}$$

$$T_{sj} = T_i / (1+\varepsilon_T)$$

where

$$T_i = t_i + C_1 \tag{5-2-6}$$

β_j is used to correct for probe blockage and is calculated by

$$\beta_j = \frac{C_D(1-\varepsilon_p)}{4(1-\varepsilon_p)-3}\frac{S_{pj}}{A} \tag{5-2-7}$$

In these equations, $\left(1-\varepsilon_p\right)$ and $\left(1+\varepsilon_t\right)$ are compressibility corrections and are calculated by

$$(1-\varepsilon_p) = 1 - \frac{1}{2k}\left(\frac{K_{vjc}P_{vi}}{P_{saj}}\right) \tag{5-2-8}$$

and

$$(1+\varepsilon_T) = 1 + 0.85\frac{k-1}{k}\left(\frac{K_{vjc}P_{vi}}{p_{saj}}\right) \tag{5-2-9}$$

provided that ($K_{vjc}p_{vi} / p_{saj}$) does not exceed 0.1 (see para. 3-3.6).

NOTE: The recovery factor of the temperature sensor is assumed to be 0.85 [18].

5-3 GAS COMPOSITION

For the purpose of this Code, it is sufficient to use a uniform gas composition and uniform values of molecular weight, specific heats, and viscosity to characterize any particular plane. These values shall be determined by arithmetic averages of gas composition data and the use of arithmetic averages of measured temperatures in the plane in question where temperatures are needed to determine the appropriate gas properties.

5-3.1 Arithmetic Average of Composition and Property Data

The average volume fraction of constituent $(X)_x$ at plane x shall be calculated from the point values $(X)_j$ using

$$(X)_x = \frac{1}{n}\sum_{j=1}^{n}(X)_j \tag{5-3-1}$$

The average temperature t_x at plane x (to be used only for purposes of defining gas composition and properties) shall be calculated from point values t_j using

$$t_x = \frac{1}{n}\sum_{j=1}^{n}t_j \tag{5-3-2}$$

5-3.2 Molecular Weight and Humidity Ratio

The molecular weight of air is 28.965. The molecular weight of any dry gas, including flue gas M_{dg}, shall be calculated from the average volume fractions $(X)_x$ using

$$M_{dg} = 44.01(CO_2) + 28.01(CO) + 32(O_2) + 28.02(N_2) + \cdots \tag{5-3-3}$$

The molecular weight of moist gas M_{mg} shall be calculated from

$$M_{mg} = \frac{1+H}{\dfrac{H}{18.02}+\dfrac{1}{M_{dg}}} \tag{5-3-4}$$

The humidity ratio H shall be calculated from the following equations unless a condensation/desiccation method is used to measure moisture content. In that event, calculations appropriate to the method shall be used.

Saturation vapor pressure (p_{ew}) at t_w for t_w between 32°F and 140°F (0°C to 60°C)

$$p_{ew} = C_{18} + C_{19}\ t_w + C_{20}\ t_w^2 + C_{21}t_w^3 + C_{22}t_w^4 + C_{23}t_w^5 \tag{5-3-5}$$

Partial pressure of water vapor in air (p_p)

$$p_p = p_{ew} - \frac{(p_b - p_{ew})\ (t_d - t_w)}{(C_8 - C_9 t_w)} \quad (5\text{-}3\text{-}6)$$

Humidity Ratio (H)

$$H = 0.622 \frac{p_p}{\left(p_b - p_p\right)} \quad (5\text{-}3\text{-}7)$$

5-3.3 Specific Heat [19]

The specific heats of dry air, water vapor, and moist air shall be calculated from the following equations:

Specific heat of dry air (c_{pda})

$$c_{pda} = C_5 \left(0.343 - \frac{1.253}{(C_3 T)^{0.5}} - \frac{83.76}{(C_3 T)} + \frac{3.087 \times 10^4}{(C_3 T^2)} \right) \quad (5\text{-}3\text{-}8)$$

Specific heat of water vapor (c_{pwv})

$$c_{pwv} = \frac{C_5}{18} \left(19.86 - \frac{597}{(C_3 T)^{0.5}} + \frac{7500}{(C_3 T)} \right) \quad (5\text{-}3\text{-}9)$$

Specific heat of moist air (c_{pma})

$$c_{pma} = \frac{c_{pda} + c_{pwv} H}{1 + H} \quad (5\text{-}3\text{-}10)$$

The specific heats of other gases, including flue gas, shall be calculated from their component specific heats and volume fractions using the following equations:

Specific heat of CO_2 (c_{pCO_2})

$$c_{pCO_2} = \frac{C_5}{44.01} \left(16.2 - \frac{6.53 \times 10^3}{(C_3 T)} + \frac{1.4 \times 10^6}{(C_3 T)^2} \right) \quad (5\text{-}3\text{-}11)$$

Specific heat of O_2 (c_{pO_2})

$$c_{pO_2} = \frac{C_5}{32} \left(11.515 - \frac{172}{(C_3 T)^{0.5}} + \frac{1530}{(C_3 T)}) \right) \quad (5\text{-}3\text{-}12)$$

Specific heat of N_2 (c_{pN_2})

$$c_{pN_2} = \frac{C_5}{28.02} \left(9.47 - \frac{3470}{(C_3 T)} + \frac{1.16 \times 10^6}{(C_3 T)^2} \right) \quad (5\text{-}3\text{-}13)$$

Specific heat of CO (c_{pCO})

$$c_{pCO} = \frac{C_5}{28.01} \left(9.46 - \frac{3290}{(C_3 T)} + \frac{1.07 \times 10^6}{(C_3 T)^2} \right) \quad (5\text{-}3\text{-}14)$$

Specific heat of dry gas (c_{pdg})

$$c_{pdg} = \frac{44.01(CO_2)c_{pCO2} + 32.00(O_2)c_{pO_2} + 28.02(N_2)c_{pN_2} + 28.01(CO)c_{pCO} \cdots}{M_{dg}} \tag{5-3-15}$$

Specific heat of moist gas (c_{pmg})

$$c_{pmg} = \frac{c_{pdg} + c_{pwv}H}{1+H} \tag{5-3-16}$$

5-3.4 Specific Gas Constant (*R*) and Specific Heat Ratio (*k*)

$$R = R_o / M \tag{5-3-17}$$

$$k = \frac{c_p}{c_v} = \frac{c_p}{\left(c_p - \frac{R}{J}\right)} \tag{5-3-18}$$

5-3.5 Viscosity [20]

The viscosities of dry air (μ_{da}), water vapor (μ_{wv}), and moist air (μ_{ma}) shall be calculated from

$$\mu_{da} = C_4 \frac{10.874(C_3T)^{3/2}}{C_3T + 199} \times 10^{-7} \tag{5-3-19}$$

$$\mu_{wv} = C_4 \frac{12.03(C_3T)^{3/2}}{C_3T + 987.4} \times 10^{-7} \tag{5-3-20}$$

$$\mu_{ma} = \frac{\sqrt{28.965}\mu_{da} + \sqrt{18.02}\frac{28.965H}{18.02}\mu_{wv}}{\sqrt{28.965} + \sqrt{18.02}\frac{28.956H}{18.02}} \tag{5-3-21}$$

The viscosity of any moist gas, μ_{mg}, including flue gas, shall be calculated from the component viscosities and the volume fractions using the following equations:

$$\mu_{CO_2} = C_4 \frac{12.721(C_3T)^{3/2}}{C_3T + 515.04} \times 10^{-7} \tag{5-3-22}$$

$$\mu_{O_2} = C_4 \frac{13.11(C_3T)^{3/2}}{C_3T + 238.54} \times 10^{-7} \tag{5-3-23}$$

$$\mu_{N_2} = C_4 \frac{10.75(C_3T)^{3/2}}{C_3T + 204.67} \times 10^{-7} \tag{5-3-24}$$

$$\mu_{CO} = C_4 \frac{10.86(C_3T)^{3/2}}{C_3T + 214.72} \times 10^{-7} \tag{5-3-25}$$

$$\mu_{mg} = \frac{\sqrt{44.01}(CO_2)\mu_{CO_2} + \sqrt{32.00}(O_2)\mu_{O_2} + \sqrt{28.02}(N_2)\mu_{N_2} + \sqrt{28.01}(CO)\mu_{CO} + \cdots + \sqrt{18.02}\left(\frac{M_{dg}H}{18.02}\right)\mu_{wv}}{\sqrt{44.01}(CO_2) + \sqrt{32.00}(O_2) + \sqrt{28.02}(N_2) + \sqrt{28.01}(CO) + \cdots + \sqrt{18.02}\left(\frac{M_{dg}H}{18.02}\right)}$$

(5-3-26)

5-3.6 Combustion Calculations

Combustion calculations may be used for determining gas constituents but shall not be used for determining gas flow rate. Fuel analysis and measured parameters such as O_2 or CO_2 may be used to calculate the gas constituents. A sample combustion calculation is provided in Nonmandatory Appendix B.

5-4 DENSITY

5-4.1 Atmospheric Air

The density of atmospheric air-vapor mixture, ρ_o, shall be calculated using the ideal gas relationship.

$$\rho_o = \frac{C_{10}(p_b - 0.378 p_p)}{R(t_d + C_1)} \qquad (5\text{-}4\text{-}1)$$

The point values of density, ρ_j, shall be calculated from

$$\rho_j = \rho_o \frac{(t_d + C_1) p_{saj}}{C_{13} T_{sj} p_b} \qquad (5\text{-}4\text{-}2)$$

5-4.2 Gas Products of Combustion

The density of products of combustion, ρ_j, at each point shall be calculated from the absolute pressure, p_{sa}, absolute temperature, T_{sj}, and specific gas constant, *R*, using the ideal gas relationship.

$$\rho_j = \frac{C_{11} p_{saj}}{R T_{sj}} \qquad (5\text{-}4\text{-}3)$$

5-4.3 Other Gases

For gases other than air, or products of combustion with air, parties to the test shall agree on a method for determining the necessary gas properties.

5-5 FLUID VELOCITY

5-5.1 Point Velocities

The velocity, *V*, at each point in the traverse plane shall be calculated from

$$V_j = C_{12}\sqrt{\frac{p_{vj}}{\rho_j}} \qquad (5\text{-}5\text{-}1)$$

5-5.2 Correction for Point Calibration Coefficients

This procedure is intended only for probes whose calibration has Reynolds number dependence. This does not apply to five-hole prism probes. For each point, j, calculate the probe Reynolds number, Re_{pj}, using

$$\mathrm{Re}_{pj} = \frac{\rho_j V_j d}{\mu C_2} \tag{5-5-2}$$

Using the probe calibration, obtain new values of K_{tj} and K_{vj} at each point. Recompute P_{ti}, K_{vjc}, P_{sj}, P_{saj}, P_{vj}, and T_{sj} at each point using new K_{tj} and K_{vj} in eqs. (5-2-1), (5-2-2), (5-2-3), (5-2-4), (5-2-5), and (5-2-6). Recompute velocity at each point V_j using new P_{vi} in eq. (5-2-1). At any point at which the value of K_{tj} and K_{vj} has changed by more than 0.1%, it will be necessary to repeat the calculations of subsections 5-2 through 5-5 using corrected values of measured pressures and temperatures. If no points have K_{tj} and K_{vj} changed by more than 0.1%, calculations may proceed using the latest values of V_j, p_{ti}, K_{vjc}, p_{vj}, and T_{sj}.

5-6 MASS FLOW RATE

5-6.1 Mass Flow Rate at Plane *x*, $\dot{m}_x$

$$\dot{m}_x = \sum_{j=1}^{n} (\dot{m}_j)_x = \frac{A_x}{C_2}\frac{1}{n}\sum_{j=1}^{n} (\rho_j V_j \cos\psi_j \cos\phi_j) \tag{5-6-1}$$

5-6.2 Fan Mass Flow Rate, $\dot{m}_F$

If the mass flow rate is measured at only one plane

$$\dot{m}_F = \dot{m}_1 \tag{5-6-2}$$

or

$$\dot{m}_F = \dot{m}_2 \tag{5-6-3}$$

or

$$\dot{m}_F = \dot{m}_3 \text{ as appropriate} \tag{5-6-4}$$

If the mass flow rate is measured at all three planes, the fan mass flow rate shall be determined from the uncertainties weighted average of the measured mass flow rates using

$$\dot{m}_F = w_1\dot{m}_1 + w_2\dot{m}_2 + w_3\dot{m}_3 \tag{5-6-5}$$

where

$$w_1 = \left(\frac{1}{U_{\dot{m}_1}}\right)^2 \Bigg/ \left[\left(\frac{1}{U_{\dot{m}_1}}\right)^2 + \left(\frac{1}{U_{\dot{m}_2}}\right)^2 + \left(\frac{1}{U_{\dot{m}_3}}\right)^2\right] \tag{5-6-6}$$

$$w_2 = \left(\frac{1}{U_{\dot{m}_2}}\right)^2 \Bigg/ \left[\left(\frac{1}{U_{\dot{m}_1}}\right)^2 + \left(\frac{1}{U_{\dot{m}_2}}\right)^2 + \left(\frac{1}{U_{\dot{m}_3}}\right)^2\right] \tag{5-6-7}$$

$$w_3 = \left(\frac{1}{U_{\dot{m}_3}}\right)^2 \Bigg/ \left[\left(\frac{1}{U_{\dot{m}_1}}\right)^2 + \left(\frac{1}{U_{\dot{m}_2}}\right)^2 + \left(\frac{1}{U_{\dot{m}_3}}\right)^2\right] \tag{5-6-8}$$

However, if the mass flow rate is not measured in one of the three planes, w for that plane shall be taken as zero (0), and the reciprocal of its uncertainty shall be taken as zero (0) in eqs. (5-6-5) and (5-6-6).

Subsection 7-4 discusses uncertainty weighting and equations for the uncertainty, U.

5-7 FLOW-WEIGHTED AVERAGES

The averages that properly represent the mass and energy flows through the fan shall be calculated as shown in paras. 5-7.1 through 5-7.8. In the case of uniform, parallel, constant density gas motion, the average parameters reduce to the customary one-dimensional values [17].

5-7.1 Average Static Pressure at Plane *x*, p_{sx}

$$p_{sx} \equiv \frac{\sum_{j=1}^{n} (p_{sj} V_j \cos\psi_j \cos\phi_j)}{\sum_{j=1}^{n} (V_j \cos\psi_j \cos\phi_j)} \tag{5-7-1}$$

5-7.2 Average Density at Plane *x*, ρ_x

$$\rho_x \equiv \frac{\sum_{j=1}^{n} (\rho_j V_j \cos\psi_j \cos\phi_j)}{\sum_{j=1}^{n} (V_j \cos\psi_j \cos\phi_j)} \tag{5-7-2}$$

5-7.3 Average Temperature at Plane *x*, T_{sx}

$$T_{sx} \equiv \frac{\sum_{j=1}^{n} (T_{sj} \rho_j V_j \cos\psi_j \cos\phi_j)}{\sum_{j=1}^{n} (\rho_j V_j \cos\psi_j \cos\phi_j)} \tag{5-7-3}$$

5-7.4 Average Specific Kinetic Energy at Plane *x*, e_{Kx}

$$e_{Kx} \equiv \frac{\sum_{j=1}^{n} (\rho_j V_j^3 \cos^3\psi_j \cos^3\phi_j)}{2 g_c C_2^2 \sum_{j=1}^{n} (\rho_j V_j \cos\psi_j \cos\phi_j)} \tag{5-7-4}$$

5-7.5 Kinetic Energy Correction Factor at Plane *x*, α_x

$$\alpha_x \equiv \frac{2 g_c \rho_x^2 A_x^2 e_{Kx}}{m_x^2} \tag{5-7-5}$$

5-7.6 Average Velocity Pressure at Plane *x*, p_{vx}

$$p_{vx} = \frac{\rho_x e_{Kx}}{C_{11}} \tag{5-7-6}$$

5-7.7 Average Total Pressure at Plane *x*, p_{tx}

$$p_{tx} = p_{sx} + p_{vx} \tag{5-7-7}$$

5-7.8 Average Absolute Pressure at Plane *x*, p_{sax}, p_{tax}

$$p_{sax} = p_{sx} + C_{13} p_b \tag{5-7-8}$$

$$p_{tax} = p_{tx} + C_{13} p_b \tag{5-7-9}$$

5-8 FAN INPUT POWER

The fan input power, P_I, shall be calculated from one of the following (paras. 5-8.1 through 5-8.3) as appropriate.

5-8.1 AC Motors (Three Phase)

$$P_I = \frac{10^3 W \eta_M}{C_{14}} \tag{5-8-1}$$

5-8.2 DC Motors (Calibrated)

$$P_I = \frac{EI \eta_M}{C_{14}} \tag{5-8-2}$$

5-8.3 Torque Meters

$$P_I = \frac{\tau N}{C_{15}} \tag{5-8-3}$$

5-9 FAN SPEED (SLIP METHOD)

When the fan speed is measured by the slip method, the stroboscope is operated on line frequency, the slip is determined by measuring the period of time, *t*, that a single mark on the shaft passes a fixed reference mark illuminated by the strobe light a set number, *n*, times (e.g., ten times). Fan speed, *N*, shall be calculated using

$$slip = \frac{120n}{tn_p} \tag{5-9-1}$$

$$synchronous\,.\,speed = \frac{120f}{n_p} \tag{5-9-2}$$

$$N = (\text{synchronous speed}) - (\text{slip}) \tag{5-9-3}$$

5-10 MASS FLOW RATE: SPECIFIC ENERGY APPROACH

When the mass flow rate, $\dot{m}_F$, – specific energy approach, y_F, [1] is selected, the following calculations shall be performed.

5-10.1 Fan Mass Flow Rate, $\dot{m}_F$

Refer to para. 5-6.2.

5-10.2 Fan Mean Density, ρ_m

$$\rho_m = \frac{\rho_1 + \rho_2}{2} \tag{5-10-1}$$

5-10.3 Fan Specific Energy, y_F

$$y_F = \frac{C_{11}(p_{s2} - p_{s1})}{\rho_m} + e_{K_2} - e_{K_1} \tag{5-10-2}$$

5-10.4 Fan Output Power, P_O

$$P_O = \frac{\dot{m}_F \, y_F}{C_{16}} \tag{5-10-3}$$

5-10.5 Compressibility Coefficient, K_ρ

$$K_\rho \equiv \frac{\rho_1}{\rho_m} = \frac{2\rho_1}{\rho_2 + \rho_1} \tag{5-10-4}$$

5-10.6 Fan Efficiency, η

$$\eta = \frac{P_O}{P_I} \tag{5-10-5}$$

5-10.7 Conversion Calculations for $\dot{m}_F$ and y_F [21]

$$b = \left(\frac{N_c}{N}\right)^2 \left(\frac{T_1}{T_{1c}}\right) \tag{5-10-6}$$

$$K_{\rho c} = 1 - b(1 - K_\rho)\frac{\eta k_c - (k_c - 1)(1 + b\left[1 + K_p\right])}{\eta k - (k - 1)(1 + \left[1 + K_p\right])} \tag{5-10-7}$$

$$\rho_{mc} = \frac{\rho_{1c}}{K_{\rho c}} \tag{5-10-8}$$

$$\dot{m}_{Fc} = \dot{m}_F \left(\frac{\rho_{1c}}{\rho_1}\right)\left(\frac{N_c}{N}\right)\left(\frac{K_\rho}{K_{\rho c}}\right) \tag{5-10-9}$$

$$y_{Fc} = y_F \left(\frac{N_c}{N}\right)^2 \tag{5-10-10}$$

$$P_{Oc} = \frac{\dot{m}_{Fc} \, y_{Fc}}{C_{16}} \tag{5-10-11}$$

$$P_{Ic} = P_I \left(\frac{N_c}{N} \right)^3 \left(\frac{\rho_{1c}}{\rho_1} \right) \left(\frac{K_\rho}{K_{\rho c}} \right) \tag{5-10-12}$$

$$\eta_c = \eta \tag{5-10-13}$$

5-11 VOLUME FLOW RATE: PRESSURE APPROACH

When the volume flow rate, Q_F, minus pressure, p_F, approach [1] is selected, the following calculations (paras. 5-11.1 through 5-11.7) shall be performed.

5-11.1 Fan Gas Density, ρ_F

$$\rho_F = \rho_1 \frac{p_{ta1}}{p_{sa1}\left[1 + \dfrac{e_{K1}}{Jc_{p1}T_{s1}}\right]} \tag{5-11-1}$$

5-11.2 Fan Volume Flow Rate, Q_F

$$Q_F = \frac{C_2 \dot{m}_F}{\rho_F} \tag{5-11-2}$$

5-11.3 Fan Pressures

(a) fan total pressure, p_{Ft}

$$p_{Ft} = p_{t2} - p_{t1} \tag{5-11-3}$$

(b) fan velocity pressure, p_{Fv}

$$p_{Fv} = \frac{\rho_2 e_{K2}}{C_{11}} \tag{5-11-4}$$

(c) fan static pressure, p_{Fs}

$$p_{Fs} = p_{Ft} - p_{Fv} \tag{5-11-5}$$

5-11.4 Compressibility Coefficient, K_p

$$z = \left(\frac{k-1}{k} \right) \frac{P_I C_{17}}{Q_F \, p_{ta1}} \tag{5-11-6}$$

$$x = \frac{p_{Ft}}{p_{ta1}} \tag{5-11-7}$$

$$K_p = \frac{z \ln(1+x)}{x \ln(1+z)} \tag{5-11-8}$$

5-11.5 Fan Output Power, P_O

$$P_O = \frac{Q_F \, p_{Ft} K_p}{C_{17}} \tag{5-11-9}$$

5-11.6 Fan Efficiency

(a) fan total efficiency, η_t

$$\eta_t = \frac{P_O}{P_I} \tag{5-11-10}$$

(b) fan static efficiency, η_s

$$\eta_s = \eta_t \frac{p_{Fs}}{p_{Ft}} \tag{5-11-11}$$

5-11.7 Conversion Calculations for Q_F and p_{Ft}

$$\frac{z}{z_c} = \left(\frac{k-1}{k}\right)\left(\frac{k_c}{k_c-1}\right)\left(\frac{p_{ta1c}}{p_{ta1}}\right)\left(\frac{N}{N_c}\right)^2\left(\frac{\rho_F}{\rho_{Fc}}\right) \tag{5-11-12}$$

$$a = \ln(1+x_c) = \ln(1+x)\frac{\ln(1+z_c)}{\ln(1+z)}\left(\frac{k-1}{k}\right)\left(\frac{k_c}{k_c-1}\right) \tag{5-11-13}$$

$$x_c = e^a - 1 \tag{5-11-14}$$

$$K_{pr} = \frac{K_p}{K_{pc}} = \left(\frac{z}{z_c}\right)\left(\frac{x_c}{x}\right)\left(\frac{k}{k-1}\right)\left(\frac{k_c-1}{k_c}\right) \tag{5-11-15}$$

$$K_{pc} = \frac{K_p}{K_{pr}} \tag{5-11-16}$$

$$Q_{Fc} = Q_F\left(\frac{N_c}{N}\right)\left(\frac{K_p}{K_{pc}}\right) \tag{5-11-17}$$

$$p_{Ftc} = p_{Ft}\left(\frac{\rho_{Fc}}{\rho_F}\right)\left(\frac{N_c}{N}\right)^2\left(\frac{K_p}{K_{pc}}\right) \tag{5-11-18}$$

$$p_{Fvc} = p_{Fv}\left(\frac{N_c}{N}\right)^2\left(\frac{\rho_{Fc}}{\rho_F}\right) \tag{5-11-19}$$

$$p_{Fsc} = p_{Ftc} - p_{Fvc} \tag{5-11-20}$$

$$P_{Oc} = \frac{Q_{Fc}\, p_{Ftc} K_{pc}}{C_{17}} \tag{5-11-21}$$

$$P_{Ic} = P_I\left(\frac{\rho_{Fc}}{\rho_F}\right)\left(\frac{N_c}{N}\right)^3\left(\frac{K_p}{K_{pc}}\right) \tag{5-11-22}$$

$$\eta_{tc} = \eta_t \tag{5-11-23}$$

5-12 INLET FLOW DISTORTION

Inlet flow distortion [11, 12] may include nonuniform velocity profiles along the axial dimension of the inlet plane or along the transverse dimension. It may also include vorticity or combinations of the three flow patterns. Contrary to the main body of this Code, these equations are written in two-dimensional format reflecting the nature of the flow across the measuring plane. This also facilitates calculation on a spreadsheet. The number of points in the x direction is n_x and in the z direction is n_z. The total number of points is $n_x n_z$.

Inlet flow distortion is quantified by the following parameters.

5-12.1 Velocity Ratio, $\hat{V}_r$

Velocity ratio is the standard deviation of the mean velocity and is a measure of the overall amplitude of the disturbance compared with uniform flow.

$$\hat{V}_r = \frac{\left(\dfrac{\sum_{xj=1}^{n_x}\sum_{zj=1}^{n_z}\left((V\cos\psi\cos\varphi)_{xj,zj} - \bar{V}\right)^2}{n_x n_z}\right)^{1/2}}{\bar{V}} \qquad (5\text{-}12\text{-}1)$$

where

$$\bar{V} = \frac{\sum_{xj=1}^{n_x}\sum_{zj=1}^{n_z}(V\cos\psi\cos\varphi)_{xj,zj}}{n_x n_z} \qquad (5\text{-}12\text{-}2)$$

Velocity ratio resembles standard deviation of the mean velocity but does not have statistical significance.

5-12.2 Mean Velocity for Each Line of Traverse Points Along the Transverse Direction, $\bar{V}_{zj}$

$$\bar{V}_{zj} = \frac{\sum_{zj=1}^{n_z}(V\cos\psi\cos\varphi)_{xj.zj}}{n_z} \qquad (5\text{-}12\text{-}3)$$

5-12.3 Mean Velocity for Each Line of Traverse Points Along the Axial Direction, $\bar{V}_{xj}$

$$\bar{V}_{xj} = \frac{\sum_{xj=1}^{n_x}(V\cos\psi\cos\varphi)_{xj.zj}}{n_x} \qquad (5\text{-}12\text{-}4)$$

5-12.4 Transverse Distortion Parameter, $\hat{V}_t$

$$\hat{V}_t = \frac{\left(\dfrac{\sum_{zj=1}^{n_z}\left(\hat{V}_{zj} - \bar{V}\right)}{n_z}\right)^{1/2}}{\bar{V}} \qquad (5\text{-}12\text{-}5)$$

5-12.5 Axial Distortion Parameter, $\widehat{V}_a$

$$\widehat{V}_a = \frac{\left(\dfrac{\sum_{xj=1}^{n_x} \left(\widehat{V}_{xj} - \bar{V} \right)}{n_x} \right)^{1/2}}{\bar{V}} \tag{5-12-6}$$

5-12.6 Shear Parameter, $\widehat{V}_s$

$$\widehat{V}_s = \frac{\left(\sum_{xj=1}^{n_x-1} \sum_{zj=1}^{n_z} \left((V\cos\psi\cos\varphi)_{xj+1,zj} - (V\cos\psi\cos\varphi)_{xj,jz} \right)^2 + \sum_{xj=1}^{n_x} \sum_{zj=1}^{n_z-1} \left((V\cos\psi\cos\varphi)_{zj+1,xj} - (V\cos\psi\cos\varphi)_{xj,zj} \right)^2 \right)^{1/2}}{\left[(n_x - 1)(n_z - 1)\bar{V} \right]} \tag{5-12-7}$$

5-12.7 Transverse Offset Parameter, $\widehat{\varepsilon}_t$

$$\widehat{\varepsilon}_t = 2\left[\left[\frac{\sum_{xj=1}^{n_x} \sum_{zj=1}^{n_z} (V\cos\psi\cos\phi)_{xj,zj}\, z_{xj,zj}}{\sum_{j=1}^{n} (V\cos\psi\cos\phi)_{xj,zj}} \right] \frac{1}{w} - \frac{1}{2} \right] \tag{5-12-8}$$

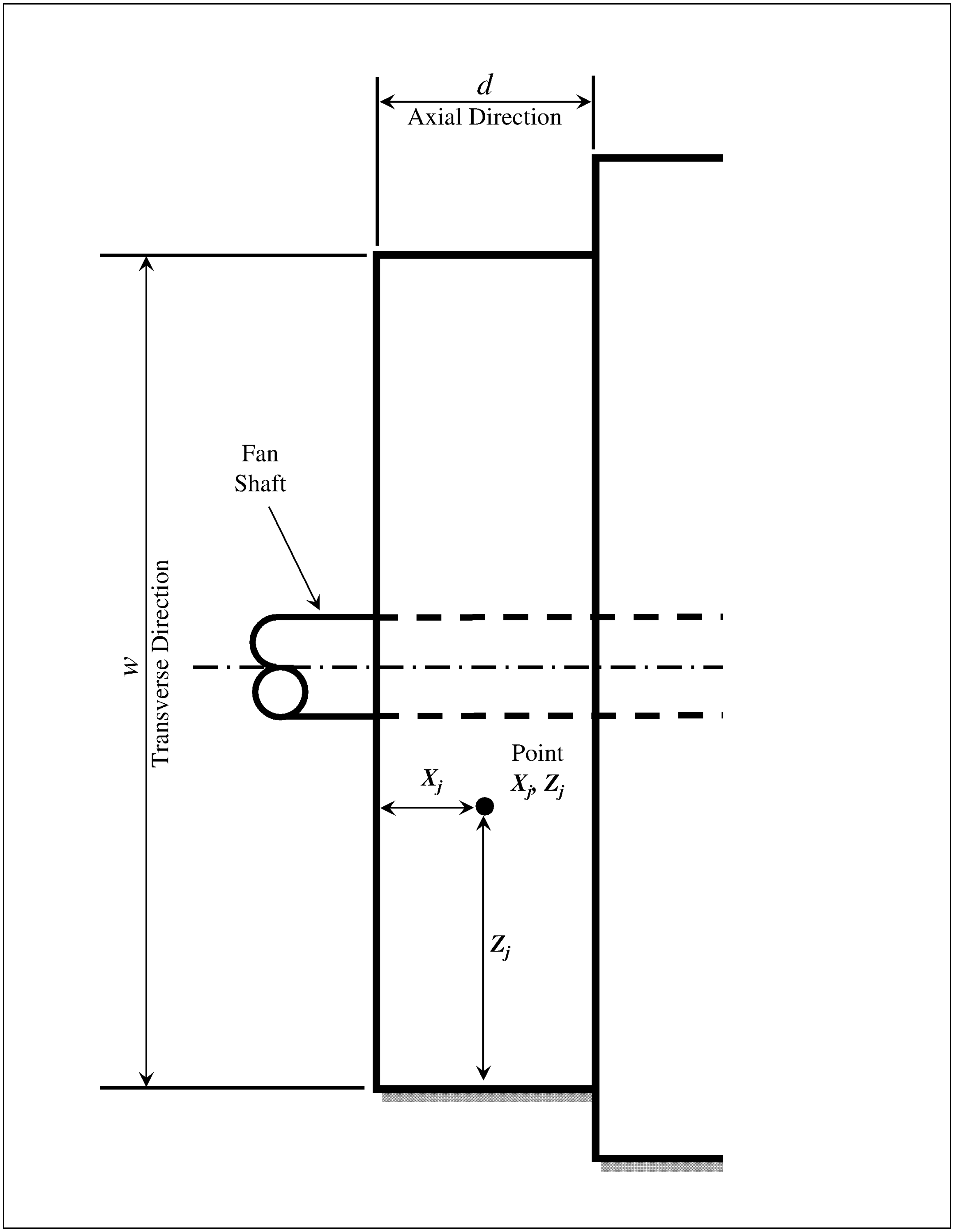
where w is the duct cross-sectional dimension perpendicular to the fan shaft (see Fig. 5-12.7-1).

5-12.8 Axial Offset Parameter, $\widehat{\varepsilon}_a$

$$\widehat{\varepsilon}_a = 2\left[\left[\frac{\sum_{xj=1}^{n_x} \sum_{zj=1}^{n_z} (V\cos\psi\cos\phi)_{xj,zj}\, x_{xj,zj}}{\sum_{xj=1}^{n_x} \sum_{zj=1}^{n_z} (V\cos\psi\cos\phi)_{xj,zj}} \right] \frac{1}{d} - \frac{1}{2} \right] \tag{5-12-9}$$

where d is the duct cross-sectional dimension parallel to the fan shaft (see Fig. 5-12.7-1).

Fig. 5-12.7-1 Traverse Point Geometry



5-12.9 Average Yaw Angle, $\bar{\psi}$

$$\bar{\psi} = \frac{\sum_{1}^{n} \left|\psi_j\right|}{n} \tag{5-12-10}$$

5-12.10 Average Pitch Angle, $\bar{\varphi}$

$$\bar{\phi} = \frac{\sum_{1}^{n} \left|\phi_j\right|}{n} \tag{5-12-11}$$

5-13 UNCERTAINTIES

The equations in this subsection shall be used to propagate the uncertainties of the test measurements into the uncertainties of the various results. The symbols used for uncertainties in these equations are $\hat{U}$ and $\hat{u}$ to distinguish them from the symbols for total absolute uncertainty, U, and total relative uncertainty, u. Each equation shall be used twice for each variable, once to determine random uncertainty and once to determine systematic uncertainty. The random uncertainty in X is designated $S_{\bar{X}}$. The systematic uncertainty in X is designated B_X. The calculated random and systematic uncertainties for each variable shall be combined using

$$U_X = 2\left(\left(\frac{B_X}{2}\right)^2 + \left(S_{\bar{X}}\right)^2\right)^{1/2} \tag{5-13-1}$$

or

$$u_X = 2\left(\left(\frac{B_X / X}{2}\right)^2 + \left(S_{\bar{X}} / X\right)^2\right)^{1/2} \tag{5-13-2}$$

These equations are based on a Student's t value of 2 and provide a confidence level of 95%.

Paragraphs 5-13.1 through 5-13.11 apply to both the mass flow–specific energy approach and the volume flow rate–pressure approach. Paragraphs 5-13.12 through 5-13.16 apply only to the mass flow rate–specific energy approach. Paragraphs 5-13.17 through 5-13.22 apply only to the volume flow rate–pressure approach.

5-13.1 Mass Flow Rate at Plane *x*, $\dot{m}_x$

$$\hat{u}_{\dot{m}_x}^2 = \hat{u}_{F_{nx}}^2 + \hat{u}_{A_x}^2$$
$$+\sum_{j=1}^{n}\left(\frac{\dot{m}_j}{\dot{m}_x}\right)^2\left[\frac{1}{4}\left(\left(\frac{\hat{U}_{p_{sj}}^2 + C_{13}^2\hat{U}_{p_b}^2}{p_{saj}^2}\right) + \hat{u}_R^2 + \hat{u}_{T_{sj}}^2 + \hat{u}_{p_{vj}}^2\right) + \left(\frac{\tan^2\psi_j\hat{U}_{\psi_j}^2 + \tan^2\phi_j\hat{U}_{\phi_j}^2}{57.30^2}\right)\right]_x \tag{5-13-3}$$

5-13.2 Fan Mass Flow Rate, $\dot{m}_F$

If the mass flow rate is measured at only one plane

$$\hat{u}_{\dot{m}_F} = \hat{u}_{\dot{m}_1} \tag{5-13-4}$$

or

$$\hat{u}_{\dot{m}_F}^2 = \hat{u}_{\dot{m}_2}^2 \tag{5-13-5}$$

or

$$\hat{u}_{\dot{m}_F}^2 = \hat{u}_{\dot{m}_3}^2 \tag{5-13-6}$$

as appropriate (see para. 5-6.2).

If the mass flow rate is measured at all three planes, the uncertainty in the fan mass flow rate shall be determined from the following:

$$\hat{u}_{\dot{m}_F}^2 = w_1^2 \hat{u}_{\dot{m}_1}^2 + w_2^2 \hat{u}_{\dot{m}_2}^2 + w_3^2 \hat{u}_{\dot{m}_3}^2 \tag{5-13-7}$$

where

$$w_1 = \left(\frac{1}{U_{\dot{m}_1}}\right)^2 \Bigg/ \left[\left(\frac{1}{U_{\dot{m}_1}}\right)^2 + \left(\frac{1}{U_{\dot{m}_2}}\right)^2 + \left(\frac{1}{U_{\dot{m}_3}}\right)^2\right] \tag{5-6-6}$$

$$w_2 = \left(\frac{1}{U_{\dot{m}_2}}\right)^2 \Bigg/ \left[\left(\frac{1}{U_{\dot{m}_1}}\right)^2 + \left(\frac{1}{U_{\dot{m}_2}}\right)^2 + \left(\frac{1}{U_{\dot{m}_3}}\right)^2\right] \tag{5-6-7}$$

$$w_3 = \left(\frac{1}{U_{\dot{m}_3}}\right)^2 \Bigg/ \left[\left(\frac{1}{U_{\dot{m}_1}}\right)^2 + \left(\frac{1}{U_{\dot{m}_2}}\right)^2 + \left(\frac{1}{U_{\dot{m}_3}}\right)^2\right] \tag{5-6-8}$$

However, if the mass flow rate is not measured in one of the three planes, w for that plane shall be taken as zero, and the reciprocal of its uncertainty shall be deleted in eqs. (5-6-6) through (5-6-8).

The weighting factors w are also used in calculating uncertainties in other results [see eqs. (5-2-25), (5-13-33), and (5-13-37)].

5-13.3 Average Static Pressure at Plane *x*, p_{s_x}

$$\hat{u}_{p_{sx}}^2 = \frac{1}{n^2}\sum_{i=1}^{n}\left(\frac{p_{sj}}{p_{sx}}\right)^2 \hat{u}_{p_{sj}}^2 \tag{5-13-8}$$

5-13.4 Average Density at Plane *x*, ρ_x

$$\hat{u}_{\rho_x}^2 = \frac{1}{n^2}\sum_{j=1}^{n}\left(\frac{\rho_j}{\rho_x}\right)^2 \left[\hat{u}_R^2 + \hat{u}_{T_{sj}}^2 + \left(\frac{\hat{U}_{p_{sj}}^2 + C_{13}^2 \hat{U}_{p_b}^2}{p_{saj}^2}\right)\right]_x \tag{5-13-9}$$

5-13.5 Average Absolute Static Temperature at Plane *x*, T_{s_x}

$$\hat{u}_{T_{sx}}^2 = \frac{1}{n^2}\sum_{i=1}^{n}\left(\frac{T_{sj}}{T_{sx}}\right)^2 \hat{u}_{T_{sj}}^2 \tag{5-13-10}$$

5-13.6 Average Specific Kinetic Energy at Plane *x*, e_{K_x}

$$\hat{u}_{e_{Kx}}^2 = \frac{1}{n^2}\sum_{i=1}^{n}\left(\frac{e_{Kj}}{e_{Kx}}\right)^2 \left[\hat{u}_R^2 + \hat{u}_{T_{sj}}^2 + \left(\frac{\hat{U}_{p_{sj}}^2 + C_{13}^2 \hat{U}_{p_b}^2}{p_{saj}^2}\right) + 4\left(\frac{\tan^2\psi_j \hat{U}_{\psi_j}^2 + \tan^2\phi_j \hat{U}_{\phi_j}^2}{57.30^2}\right)\right]_x \tag{5-13-11}$$

where

$$e_{Kx} = \frac{1}{2}V_j^2 \cos^2\psi_j \cos^2\phi_j \tag{5-13-12}$$

5-13.7 Average Velocity Pressure at Plane *x*, p_{v_x}

$$\hat{u}_{p_{vx}}^2 = \frac{1}{n^2}\sum_{i=1}^{n}\left(\frac{p_{vj}\cos^2\psi_j\cos^2\phi_j}{p_{vx}}\right)^2\left[\hat{u}_{p_{vj}}^2 + 4\left(\frac{\tan\psi_j\hat{U}_{\psi_j}^2 + \tan\phi_j\hat{U}_{\phi_j}^2}{57.30^2}\right)\right]_x \qquad (5\text{-}13\text{-}13)$$

5-13.8 Average Total Pressure at Plane *x*, p_{t_x}

$$\hat{u}_{p_{tx}}^2 = \frac{1}{n^2}\left\{\sum_{i=1}^{n}\left(\frac{p_{sj}}{p_{tx}}\right)^2\hat{u}_{p_{vj}}^2 + \sum_{i=1}^{n}\left(\frac{p_{vj}\cos^2\psi_j\cos^2\phi_j}{p_{vx}}\right)^2\left[\hat{u}_{p_{vj}}^2 + 4\left(\frac{\tan^2\psi_j\hat{U}_{\psi_j}^2 + \tan^2\phi_j\hat{U}_{\phi_j}^2}{57.30^2}\right)\right]_x\right\} \qquad (5\text{-}13\text{-}14)$$

5-13.9 Average Absolute Pressure at Plane *x*, p_{sa_x}

$$\hat{u}_{p_{sax}}^2 = \frac{\hat{U}_{p_{sx}}^2 + C_{13}^2\hat{U}_{p_b}^2}{p_{sax}^2} \qquad (5\text{-}13\text{-}15)$$

5-13.10 Fan Input Power, P_I

$$\hat{u}_{P_I}^2 = \hat{u}_{\eta_M}^2 + \hat{u}_W^2 \text{ for AC motors} \qquad (5\text{-}13\text{-}16)$$

$$\hat{u}_{P_I}^2 = \hat{u}_{\eta_M}^2 + \hat{u}_E^2 + \hat{u}_I^2 \text{ for DC motors} \qquad (5\text{-}13\text{-}17)$$

$$\hat{u}_{P_I}^2 = \hat{u}_\tau^2 + \hat{u}_N^2 \text{ for torque meters} \qquad (5\text{-}13\text{-}18)$$

$$\hat{u}_{P_I}^2 = \hat{u}_{P_t}^2 \text{ for turbines} \qquad (5\text{-}13\text{-}19)$$

5-13.11 F an Speed, *N*

$$\hat{u}_N^2 = \hat{u}_N^2 \text{ for electronic counters} \qquad (5\text{-}13\text{-}20)$$

$$\hat{u}_N^2 = \hat{u}_n^2 + \hat{u}_t^2 \text{ for slip method} \qquad (5\text{-}13\text{-}21)$$

5-13.12 Fan Mean Density, ρ_m

$$\hat{u}_{\rho_m}^2 = \frac{\hat{U}_{\rho_1}^2 + \hat{U}_{\rho_2}^2}{(\rho_1+\rho_2)^2} \qquad (5\text{-}13\text{-}22)$$

5-13.13 Fan Specific Energy, y_F

$$\hat{u}_{y_F}^2 = \hat{u}_R^2 + \left(\frac{C_{11}}{y_F^2}\right)\left[\begin{aligned}&\left(\frac{\rho_1\left(p_{s2}-p_{s1}\right)}{2\rho_m^2}-\frac{p_{v1}}{\rho_1}\right)^2\hat{u}_{T_{s1}}^2+\left(\frac{\rho_2\left(p_{s2}-p_{s1}\right)}{2\rho_m^2}+\frac{p_{v2}}{\rho_2}\right)^2\hat{u}_{T_{s2}}^2\\&+\left(\frac{p_{v1}p_b}{\rho_1 p_{sa1}}-\frac{\left(p_{s2}-p_{s1}\right)}{2\rho_m^2}\left(\frac{p_b}{RT_{s1}}+\frac{p_b}{RT_{s2}}\right)-\frac{p_{v2}p_b}{\rho_2 p_{sa2}}\right)^2\hat{u}_{p_b}^2\\&+\left(\frac{p_{v1}}{\rho_1}\frac{p_{s1}}{p_{sa1}}-\frac{\rho_1\left(p_{s2}-p_{s1}\right)}{2\rho_m^2}\frac{p_{s1}}{p_{sa1}}-\frac{p_{s1}}{\rho_m}\right)^2\hat{u}_{ps1}^2\\&+\left(\frac{p_{s2}}{\rho_m}-\frac{\rho_2\left(p_{s2}-p_{s1}\right)}{2\rho_m^2}\frac{p_{s2}}{p_{sa2}}-\frac{p_{v2}}{\rho_2}\frac{p_{s2}}{p_{sa2}}\right)^2\hat{u}_{p_{s2}}^2\\&+\left(\frac{p_{v1}}{\rho_1}\right)^2\hat{u}_{p_{v1}}^2+\left(\frac{p_{v2}}{\rho_1}\right)^2\hat{u}_{p_{v2}}^2\end{aligned}\right] \qquad (5\text{-}13\text{-}23)$$

5-13.14 Fan Output Power, P_O

$$\hat{u}_{P_O}^2 = \left[\begin{aligned}&\frac{1}{4}\hat{u}_R^2+\left(\frac{w_1\dot{m}_1}{\dot{m}_F}\right)^2\hat{u}_{F_{n1}}^2+\left(\frac{w_2\dot{m}_2}{\dot{m}_F}\right)^2\hat{u}_{F_{n2}}^2+\left(\frac{w_3\dot{m}_3}{\dot{m}_F}\right)^2\hat{u}_{F_{n3}}^2\\&+\left(\frac{w_1\dot{m}_1}{\dot{m}_F}\right)^2\hat{u}_{A_1}^2+\left(\frac{w_2\dot{m}_2}{\dot{m}_F}\right)^2\hat{u}_{A_2}^2+\left(\frac{w_3\dot{m}_3}{\dot{m}_F}\right)^2\hat{u}_{A_3}^2\\&+\left(\frac{w_1m_1}{2\dot{m}_F}-\frac{C_{11}}{y_F}\frac{\rho_1\left(p_{s2}-p_{s1}\right)}{2\rho_m^2}-\frac{e_{K1}}{y_F}\right)^2\hat{u}_{T_{s1}}^2\\&+\left(\frac{w_2m_2}{2\dot{m}_F}-\frac{C_{11}}{y_F}\frac{\rho_2\left(p_{s2}-p_{s1}\right)}{2\rho_m^2}+\frac{e_{K2}}{y_F}\right)^2\hat{u}_{T_{s2}}^2+\left(\frac{w_3m_3}{2\dot{m}_F}\right)^2\hat{u}_{T_{s3}}^2\\&+\left(\begin{aligned}&\frac{w_1m_1}{2\dot{m}_F}\frac{C_{13}p_b}{p_{sa1}}+\frac{w_2m_2}{2\dot{m}_F}\frac{C_{13}p_b}{p_{sa2}}+\frac{w_3m_3}{2\dot{m}_F}\frac{C_{13}p_b}{p_{sa3}}\\&+\frac{C_{11}}{y_F}\left(\frac{p_{v1}C_{13}p_b}{\rho_1 p_{sa1}}-\frac{C_{11}\left(p_{s2}-p_{s1}\right)}{2\rho_m^2}\left(\frac{C_{13}p_b}{RT_{s1}}+\frac{C_{13}p_b}{RT_{s2}}\right)-\frac{p_{v2}C_{13}p_b}{\rho_2 p_{sa2}}\right)\end{aligned}\right)^2\hat{u}_{p_b}^2\\&+\left(\frac{w_1\dot{m}_1}{2\dot{m}_F}\frac{p_{s1}}{p_{sa1}}+\frac{C_{11}}{y_F}\left(\frac{p_{v1}}{\rho_1}\frac{p_{s1}}{p_{sa1}}-\frac{\rho_1\left(p_{s2}-p_{s1}\right)}{2\rho_m^2}\frac{p_{s1}}{p_{sa1}}-\frac{p_{s1}}{\rho_m}\right)\right)^2\hat{u}_{p_{s1}}^2\\&+\left(\frac{w_2\dot{m}_2}{2\dot{m}_F}\frac{p_{s2}}{p_{sa2}}+\frac{C_{11}}{y_F}\left(\frac{p_{s2}}{\rho_m}-\frac{\rho_2\left(p_{s2}-p_{s1}\right)}{2\rho_m^2}\frac{p_{s2}}{p_{sa2}}-\frac{p_{v2}}{\rho_2}\frac{p_{s2}}{p_{sa2}}\right)\right)^2\hat{u}_{p_{s2}}^2\\&+\left(\frac{w_3\dot{m}_3}{2\dot{m}_F}\frac{p_{s3}}{p_{sa3}}\right)^2\hat{u}_{p_{s3}}^2\\&+\left(\frac{w_1\dot{m}_1}{2\dot{m}_F}-\frac{e_{K1}}{y_F}\right)^2\hat{u}_{pv1}^2+\left(\frac{w_2\dot{m}_2}{2\dot{m}_F}+\frac{e_{K2}}{y_F}\right)^2\hat{u}_{p_{v2}}^2+\left(\frac{w_3\dot{m}_3}{2\dot{m}_F}\right)^2\hat{u}_{pv3}^2\end{aligned}\right] \qquad (5\text{-}13\text{-}24)$$

5-13.15 Fan Efficiency, η_F

$$\hat{u}_{\eta}^2 = \hat{u}_{P_O}^2 + \hat{u}_{P_I}^2 \tag{5-13-25}$$

5-13.16 Conversions ($\dot{m}_{Fc}$, y_{Fc}, P_{Oc}, P_{Ic}, η_c)

$$\hat{u}_{\dot{m}_{Fc}}^2 = \hat{u}_{\dot{m}_F}^2 + \hat{u}_N^2 + \hat{u}_{\rho_1}^2 \tag{5-13-26}$$

$$\hat{u}_{y_{Fc}}^2 = \hat{u}_{y_F}^2 + 4\hat{u}_N^2 \tag{5-13-27}$$

$$\hat{u}_{P_{Oc}}^2 = \hat{u}_{P_O}^2 + 9\hat{u}_N^2 + \hat{u}_{\rho_1}^2 \tag{5-13-28}$$

$$\hat{u}_{P_{Ic}}^2 = \hat{u}_{P_I}^2 + 9\hat{u}_N^2 + \hat{u}_{\rho_1}^2 \tag{5-13-29}$$

$$\hat{u}_{\eta_c}^2 = \hat{u}_{\eta}^2 \tag{5-13-30}$$

5-13.17 Fan Gas Density, ρ_F

$$\hat{u}_{\rho_F}^2 = \hat{u}_{\rho_1}^2 \tag{5-13-31}$$

5-13.18 Fan Volume Flow Rate, Q_F

$$\hat{u}_{Q_F}^2 = \left[\begin{array}{l}
\frac{1}{4}\hat{u}_R^2 + \left(\frac{w_1\dot{m}_1}{\dot{m}_F}\right)^2 \hat{u}_{F_{n1}}^2 + \left(\frac{w_2\dot{m}_2}{\dot{m}_F}\right)^2 \hat{u}_{F_{n2}}^2 + \left(\frac{w_3\dot{m}_3}{\dot{m}_F}\right)^2 \hat{u}_{F_{n3}}^2 \\
+\left(\frac{w_1\dot{m}_1}{\dot{m}_F}\right)^2 \hat{u}_{A_1}^2 + \left(\frac{w_2\dot{m}_2}{\dot{m}_F}\right)^2 \hat{u}_{A_2}^2 + \left(\frac{w_3\dot{m}_3}{\dot{m}_F}\right)^2 \hat{u}_{A_3}^2 \\
+\left(1-\frac{w_1\dot{m}_1}{2\dot{m}_F}\right)^2 u_{T_{s1}}^2 + \left(\frac{w_2\dot{m}_2}{2\dot{m}_F}\right)^2 u_{T_{s2}}^2 + \left(\frac{w_3\dot{m}_3}{2\dot{m}_F}\right)^2 u_{T_{s3}}^2 \\
+\left(\frac{w_1\dot{m}_1}{2\dot{m}_F}\frac{C_{13}p_b}{p_{sa1}} + \frac{w_2\dot{m}_2}{2\dot{m}_F}\frac{C_{13}p_b}{p_{sa2}} + \frac{w_3\dot{m}_3}{2\dot{m}_F}\frac{C_{13}p_b}{p_{sa3}} - \frac{C_{13}p_b}{p_{ta1}}\right)^2 \hat{u}_{p_b}^2 \\
+\left(\frac{p_{s1}}{p_{sa1}}\frac{w_1\dot{m}_1}{2\dot{m}_F} - \frac{p_{s1}}{p_{ta1}}\right)^2 \hat{u}_{p_{s1}}^2 + \left(\frac{w_2\dot{m}_2}{2\dot{m}_F}\frac{p_{s2}}{p_{sa2}}\right)^2 \hat{u}_{p_{s2}}^2 + \left(\frac{w_3\dot{m}_3}{2\dot{m}_F}\frac{p_{s3}}{p_{sa3}}\right)^2 \hat{u}_{p_{s3}}^2 \\
+\left(\frac{w_1\dot{m}_1}{2\dot{m}_F} - \frac{p_{v1}}{p_{ta1}}\right)^2 \hat{u}_{pv1}^2 + \left(\frac{w_2\dot{m}_2}{2\dot{m}_F}\right)^2 \hat{u}_{p_{v2}}^2 + \left(\frac{w_3\dot{m}_3}{2\dot{m}_F}\right)^2 \hat{u}_{pv3}^2
\end{array}\right] \tag{5-13-32}$$

5-13.19 Fan Pressure (p_{F_t}, p_{F_v}, p_{F_s})

$$\hat{u}_{p_{Ft}}^2 = \frac{\hat{U}_{p_{t2}}^2 + \hat{U}_{p_{t1}}^2}{p_{F_t}^2} \tag{5-13-33}$$

$$\hat{u}_{p_{Fv}}^2 = \hat{u}_{p_{v2}}^2 \tag{5-13-34}$$

$$\hat{u}_{p_{Fs}}^2 = \frac{\hat{U}_{p_{Ft}}^2 + \hat{U}_{p_{Fv}}^2}{p_{Fs}^2} \tag{5-13-35}$$

5-13.20 Fan Output Power, P_O

$$\hat{u}_{P_O}^2 = \left[\begin{array}{l} \frac{1}{4}\hat{u}_R^2 + \left(\frac{w_1\dot{m}_1}{\dot{m}_F}\right)^2 \hat{u}_{F_{n1}}^2 + \left(\frac{w_2\dot{m}_2}{\dot{m}_F}\right)^2 \hat{u}_{F_{n2}}^2 + \left(\frac{w_3\dot{m}_3}{\dot{m}_F}\right)^2 \hat{u}_{F_{n3}}^2 \\ + \left(\frac{w_1\dot{m}_1}{\dot{m}_F}\right)^2 \hat{u}_{A_1}^2 + \left(\frac{w_2\dot{m}_2}{\dot{m}_F}\right)^2 \hat{u}_{A_2}^2 + \left(\frac{w_3\dot{m}_3}{\dot{m}_F}\right)^2 \hat{u}_{A_3}^2 \\ + \left(1 - \frac{w_1 m_1}{2\dot{m}_F}\right)^2 \hat{u}_{T_{s1}}^2 + \left(\frac{w_2 m_2}{2\dot{m}_F}\right)^2 \hat{u}_{T_{s2}}^2 + \left(\frac{w_3 m_3}{2\dot{m}_F}\right)^2 \hat{u}_{T_{s3}}^2 \\ + \left(\frac{w_1\dot{m}_1}{2\dot{m}_F}\frac{C_{13}p_b}{p_{sa1}} + \frac{w_2\dot{m}_2}{2\dot{m}_F}\frac{C_{13}p_b}{p_{sa2}} + \frac{w_3\dot{m}_3}{2\dot{m}_F}\frac{C_{13}p_b}{p_{sa3}} - \frac{C_{13}p_b}{p_{ta1}}\right)^2 u_{p_b}^2 \\ + \left(\frac{w_1\dot{m}_1}{2\dot{m}_F}\frac{p_{s1}}{p_{sa1}} - \frac{p_{s1}}{p_{ta1}} - \frac{p_{s1}}{p_{Ft}}\right)^2 \hat{u}_{p_{s1}}^2 + \left(\frac{w_2\dot{m}_2}{2\dot{m}_F}\frac{p_{s2}}{p_{sa2}} + \frac{p_{s2}}{p_{Ft}}\right)^2 \hat{u}_{p_{s2}}^2 + \left(\frac{w_3\dot{m}_3}{2\dot{m}_F}\frac{p_{s3}}{p_{sa3}}\right)^2 \hat{u}_{p_{s3}}^2 \\ + \left(\frac{w_1\dot{m}_1}{2\dot{m}_F} - \frac{p_{v1}}{p_{ta1}} - \frac{p_{v1}}{p_{Ft}}\right)^2 \hat{u}_{pv1}^2 + \left(\frac{w_2\dot{m}_2}{2\dot{m}_F} + \frac{p_{v2}}{p_{Ft}}\right)^2 \hat{u}_{p_{v2}}^2 + \left(\frac{w_3\dot{m}_3}{2\dot{m}_F}\right)^2 \hat{u}_{p_{v3}}^2 \end{array}\right] \tag{5-13-36}$$

5-13.21 Fan Efficiency (η_t, η_s)

$$\hat{u}_{\eta_t}^2 = \hat{u}_{P_O}^2 + \hat{u}_{P_I}^2 \tag{5-13-37}$$

$$\hat{u}_{\eta_s}^2 = \hat{u}_{\eta_t}^2 \tag{5-13-38}$$

5-13.22 Conversions (Q_{F_c}, $p_{F_{tc}}$, $p_{F_{vc}}$, $p_{F_{sc}}$, $P_{F_{Oc}}$, $P_{F_{Ic}}$, η_{t_c})

$$\hat{u}_{Q_{Fc}}^2 = \hat{u}_{Q_F}^2 + \hat{u}_N^2 \tag{5-13-39}$$

$$\hat{u}_{p_{Ftc}}^2 = \hat{u}_{p_{Ft}}^2 + 4\hat{u}_N^2 + \hat{u}_{\rho_1}^2 \tag{5-13-40}$$

$$\hat{u}_{p_{F\,vc}}^2 = \hat{u}_{p_{Fv}}^2 + 4\hat{u}_N^2 + \hat{u}_{\rho_1}^2 \tag{5-13-41}$$

$$\hat{u}_{p_{Fsc}}^2 = \hat{u}_{p_{Fs}}^2 + \hat{u}_N^2 + \hat{u}_{\rho_1}^2 \tag{5-13-42}$$

$$\hat{u}_{P_{Oc}}^2 = \hat{u}_{P_O}^2 + 9\hat{u}_N^2 + \hat{u}_{\rho_1}^2 \tag{5-13-43}$$

$$\hat{u}_{P_{Ic}}^2 = \hat{u}_{P_I}^2 + 9\hat{u}_N^2 + \hat{u}_{\rho_1}^2 \tag{5-13-44}$$

$$\hat{u}_{\eta_{tc}}^2 = \hat{u}_{\eta_t}^2 \tag{5-13-45}$$

5-13.23 Computation

Because of the complexities of the uncertainties calculations, it is recommended that the equations be incorporated into an electronic spreadsheet or other computer program.

Section 6
Report of Results

6-1 GENERAL REQUIREMENTS

The results of the test shall be presented in a written report. The preparation of the report shall be the responsibility of the person in charge of the test who shall certify its correctness.

Prior to writing the report, the parties shall decide whether to use SI units, U.S. Customary units, or both. This selection will generally depend upon the units in which the fan performance is specified.

The Test Report shall include the information specified in subsections 6-2 through 6-7.

6-2 EXECUTIVE SUMMARY

The following information is to be included in the executive summary:

(a) general information about the fan and the test, such as the fan type and operating configuration, and the test objective

(b) date and time of the test

(c) summary of the results of the test, including uncertainty and conclusions reached

(d) comparison with the specified performance, if any

(e) any agreements among the parties to the test to allow any deviations from the test requirements

6-3 INTRODUCTION

The following information is to be included in the introduction:

(a) authorization for the tests, their object, specified performance, stipulated agreements, by whom the test is directed, and the representative parties to the test

(b) any additional general information about the fan and the test not included in the executive summary, such as

(1) a historical perspective, if appropriate

(2) an equipment diagram showing the test boundary

(3) description of the equipment tested and any other auxiliary apparatus, the operation of which may influence the test result

(c) a listing of the representatives of the parties to the test

(d) any pretest agreements that were not tabulated in the executive summary

(e) the organization of the test personnel

(f) test goal per Sections 3 and 5 of this Code

6-4 CALCULATIONS AND RESULTS

The following information is to be included in the calculations and results:

(a) method of the test and operating conditions

(b) tabular summary of measurements and observations, including the reduced data necessary to calculate the results and a summary of additional operating conditions not part of such reduced data

(c) step-by-step calculation of test results from the reduced data, including the uncertainty analysis

(d) any calculations showing elimination of data for outlier reason or for any other reason

(e) comparison of repeatability of test runs

(f) correction factors to be applied because of deviations, if any, of test conditions from those specified

(g) primary measurement uncertainties, including method of application

(h) the test performances stated under the following headings:

(1) test results computed on the basis of the test operating conditions, instrument calibrations only having been applied

(2) test results corrected to specified conditions if test operating conditions have deviated from those specified

(i) tabular and graphical presentation of the test results

(j) discussion and details of the test results uncertainties

(k) discussion of the test and its results

A copy of the computer program used to determine results will be included as an appendix to the report.

6-5 INSTRUMENTATION

The following information is to be included in the instrumentation:

(a) tabulation of instrumentation used, including make, model number, etc.

(b) description of the instrumentation location

(c) means of data collection for each data point, such as temporary data acquisition system printout, plant control computer printout, or manual data sheet, and any identifying tag number and/or address of each

(d) identification of the instrument that was used as backup

(e) description of data acquisition system(s) used

(f) summary of pretest and post-test calibration

6-6 CONCLUSIONS

The following information is to be included in the conclusions:

(a) A statement as to whether the test met the test objectives. If it does not meet the test objectives, the reasons shall be stated.

(b) A statement as to whether the test met the requirements of PTC 11. If it does not meet the requirements of PTC 11, the reasons shall be stated.

(c) A statement as to whether the fan met the specified performance (if applicable).

6-7 APPENDICES

The following information is to be included in the appendices:

(a) copies of original data sheets and/or data acquisition system(s) printouts

(b) copies of operator logs or other recording of operating activity during each test

(c) instrumentation calibration results from laboratories, certification from manufacturers

(d) copies of fuel analysis (if applicable)

Section 7
Uncertainty Analysis

7-1 INTRODUCTION

Uncertainty analysis is a procedure by which the accuracy of test results can be quantified. Because it is required that the parties to the test agree to the quality of the test (measured by test uncertainty), pretest and post-test uncertainty analyses are an indispensable part of a meaningful performance test.

In planning a test, a pretest uncertainty analysis allows corrective action to be taken prior to the test, either to decrease the uncertainty to a level consistent with the overall objective of the test or to reduce the cost of the test while still attaining the objective. An uncertainty analysis is useful to determine the number of observations.

A post-test uncertainty analysis determines the uncertainty intervals for the actual test. This analysis should confirm the pretest systematic and random uncertainty estimates. It serves to validate the quality of the test results or to expose problems.

PTC 19.1, Test Uncertainty, is the primary reference for uncertainty calculations, and any uncertainty analysis method that conforms to PTC 19.1 is acceptable.

Before an uncertainties analysis can be performed, both the random uncertainty, $S_{\bar{X}}$, and the systematic uncertainty, B_X, in each of the test measurements X must be established. For this Code, the random and systematic uncertainties for an additional factor must also be determined. This is called the number-of-points factor, F_n. Next, the appropriate test measurement uncertainties must be combined appropriately for each test result, a process called propagating the test uncertainties into the uncertainty of the results. The random and systematic uncertainties must be propagated individually for each result. Finally, these random and systematic uncertainties are combined to yield the total absolute uncertainty, U_X, or the total relative uncertainty, u_X, in each of the results. See eq. (5-13-1) or (5-13-2).

The confidence level for uncertainties calculated in this manner will be 95%. Subsection 7-2 explains how this Code propagates the test uncertainties into the results. Subsection 7-3 describes how to determine the random and systematic uncertainties in the basic measured parameters and number of points factor. Subsection 7-4 discusses the uncertainty when mass flow is determined at multiple traverse planes. Subsection 5-13 of this Code provides the equations for calculating the uncertainty in each result. These equations can be used directly; however, incorporating them into a spreadsheet or other computer program together with the test data is recommended because of the complexity involved.

An alternative to the use of the equations of subsection 5-13 is to use numerical evaluation of sensitivity coefficients, as described in PTC 19.1 This method is sometimes called "dithering" or "jittering." This method is especially useful when a computer code or spreadsheet is being used to calculate test results because the computer can be easily used to calculate the results that would occur with a small perturbation of the input (test) data.

7-2 UNCERTAINTY PROPAGATION EQUATIONS

The uncertainty propagation equations are given in subsection 5-13. These equations are derived in Nonmandatory Appendix E. All the derivations follow the approach suggested by Kline and McClintock [5]. This approach is equivalent to the "Taylor Series" method described in PTC 19.1.

The uncertainty propagation equations derived and used in this Code assume that there are no correlations between systematic uncertainties. This would be the case when different measuring systems, calibrated against different standards, are used at each point in each traverse plane. This approach is taken because it is not possible to identify in general all cases wherein correlated systematic uncertainties may occur. For example, in one test, the same instrument system may be used to traverse both the inlet and outlet of a particular fan, so all instrument-associated systematic errors may be correlated, while in another test, different instrument systems may be used, but the two probes may have been calibrated against a common standard so that only a portion of the systematic uncertainties is correlated. A special case occurs when all traverse points in a particular plane are measured with the same instrument system. This case will be dealt with in this Section. If there are other correlations between systematic uncertainties, then the methods described in PTC 19.1 shall be used to account for them.

7-3 ASSIGNING VALUES TO PRIMARY UNCERTAINTIES

The equations in subsection 5-13 give the uncertainties of the various results of the test in terms of the uncertainties in the test measurements and in certain other factors. These measurement and factor uncertainties, herein called "primary uncertainties," should reflect the circumstances of the test. Some of the circumstances that affect the primary uncertainties are discussed in this Section. Some typical values of the primary uncertainties are also suggested here. Values are suggested for both the systematic and random components of the uncertainties where appropriate. Generally speaking, primary random uncertainties are calculated by statistical analysis, and primary systematic uncertainties are estimated by the person(s) responsible for the test, using experience, special studies and models, or engineering judgment. Methods for estimating random uncertainties from test data are described. Also, some typical values for the primary systematic uncertainties are suggested here; however, they should be used only upon agreement of all parties to a test.

7-3.1 Calculating Primary Random Uncertainties

Generally, the random uncertainty of a measured parameter is determined from the standard deviation of the mean of the parameter measurements. Following PTC 19.1, this Code assumes that a significantly large number[1] of readings is available so that the value of Student's t can be taken as 2.

There are two types of measurements made in a fan test. The first type is "single value" or "integrated" measurements. Typical "single value" measurements include electrical power input to a motor, motor speed, and wet- and dry-bulb temperatures. For these measurements, the calculation of the standard deviation of the mean is straightforward, as illustrated by the following equation for motor input power:

$$S_{\overline{W}} = \frac{S_W}{\sqrt{N}} = \left[\frac{\sum_{1}^{N}(W_i - \overline{W})^2}{N(N-1)}\right]^{1/2} \tag{7-3-1}$$

where

N = number of measurements

$\overline{W}$ = average value

W_i = measured values

The other type of measurements in a fan test are multiple point measurements made during a traverse. Velocity pressure, gas temperature, and static pressure are examples of this type. The desired input to the uncertainty analysis is the standard deviation of the mean for each variable at each traverse point. It is generally quite impractical to attempt to obtain sufficient data to calculate these standard deviations as it would require a large number of readings to be made at each traverse point. To overcome this difficulty, in this Code, sufficient measurements to determine the standard deviation of the mean at a single reference

[1] The word "large" is ambiguous. The number of readings should be greater than ten, so that a value of 2 is satisfactory for Student's t. A very large number of readings, say in the hundreds or thousands such as might be made by an automatic data logging system, is to be avoided, as such large numbers can overlook significant trends in the data.

point are made. The standard deviation of the mean calculated from the reference point data is then taken as the standard deviation of the mean for each point in the traverse plane containing the reference point.

As an example, the evaluation for $S_{\bar{p}_{vj}}$ is obtained as follows:

(a) Obtain data for p_{vR} for each window of time.

(b) Calculate the mean, $\bar{p}_{vR}$; standard deviation, S_{pvR}; and standard deviation of the mean, $S_{\bar{p}_{vR}}$, for all p_{vR} (i.e., for all windows of time).

(c) Then $S_{\bar{p}_{vj}} = S_{\bar{p}_{vR}}$ for all *j* (i.e., for all points in the traverse plane).

The standard deviations of the mean for other traverse point parameters, namely $p_{s_j}, p_{t_j}, T_{s_j}$, are required, and an identical procedure can be used.

There are two further traverse point variables to address, namely the pitch angle, ϕ_j, and the yaw angle, ψ_j. In a typical test, there are no data from which to reliably calculate the standard deviation of the mean. In this case, it is necessary to estimate a value. As the yaw data are often obtained by a human operator who rotates the probe to obtain a null balance, it may be possible to determine a reliable estimate for the uncertainty of the pitch angle from the probe operator. Estimates for the pitch angle uncertainty may be made from observations of pitch pressure fluctuation together with the pitch angle calibration curve. If neither of these approaches is feasible, then the following estimates are suggested. These estimates shall be used only if all parties to the test agree.

$$2S_{\bar{\phi}_j} \approx 1^\circ + 2^\circ \left(\frac{\phi^\circ}{45^\circ} \right) \tag{7-3-2}$$

$$2S_{\bar{\psi}_j} \approx 1^\circ + 2^\circ \left(\frac{\psi^\circ}{45^\circ} \right) \tag{7-3-3}$$

The random uncertainty in the gas constant, *R*, is assumed to be insignificant. The number of points factor, F_n, has no random uncertainty.

7-3.2 Estimating Primary Systematic Uncertainties

Unlike random uncertainties, which are usually evaluated from test data, systematic uncertainties are assigned using estimates or models based on special studies. Estimates are usually based on the judgment of experts. Estimated values for systematic uncertainties should be made with a 95% confidence limit. (Following Kline and McClintock, the estimator should be willing to bet at 19:1 odds that the true systematic error lies within plus or minus the estimate. Perhaps more importantly, it is not required to estimate the largest possible value.) For a Code test, all parties to the test must reach agreement on the values to be used for the systematic uncertainties.

It is important to note that systematic uncertainties may be correlated. If correlations exist between systematic uncertainties, they shall be accounted for using the methods specified in PTC 19.1. With a single exception, described in the next paragraph, this Code assumes that systematic uncertainties are uncorrelated.

7-3.2.1 Systematic Uncertainty for Points in a Traverse Plane. In many fan tests, all points in a traverse plane are measured with the same probe and instrument system. In this case, it is logical to assume that the systematic uncertainty is the same for each point. Then the systematic uncertainty for the integrated (e.g., $\dot{m}_x$) or average (e.g., p_{s_x}) value cannot be calculated by the formulas derived in subsection 7-2, because these equations assume no correlation. In this case, the systematic uncertainty of the integrated or average parameter is equal to the systematic uncertainty for a single point, thus

$$B_{\dot{m}_x} = B_{\dot{m}_j} \tag{7-3-4}$$

and

$$B_{p_{sx}} = B_{p_{sj}} \tag{7-3-5}$$

etc.

7-3.2.2 Systematic Uncertainty for Number of Points Factor, F_n. The factor F_n was introduced in subsection 7-2 in the derivation of the uncertainty in $\dot{m}_x$. The factor F_n itself is assumed equal to unity and is dropped from the final equations for $\dot{m}_x$ and $u_{\dot{m}_x}$. The relative systematic uncertainty in F_n is calculated from a model based on M. J. Dorsey's master's thesis [22] and is

$$B_{F_n} = \frac{0.45(\alpha - 1)^{0.33}}{n^{0.67}} \tag{7-3-6}$$

where

n = the number of points in the traverse plane

Some estimates for other systematic uncertainties are shown in Table 7-3.2.2-1. The various systematic uncertainties that are listed in Table 7-3.2.2-1 are based on the assumption that instruments are selected for the test in accordance with the specifications in this Code. The values shown are based on estimates of the residual uncertainty remaining after calibration, on estimates of the effects of temperature and other changes not included in the calibration, and on estimates of operator bias.

Table 7-3.2.2-1 Typical Values for Primary Systematic Uncertainty

Measurement	Systematic Uncertainty
A_x	$B_{A_x} / A_x = 0.007$
R	$B_R / R = 0.002$
T_{s_j}	$B_{T_{sj}} = 2°\text{F} = 1°\text{C}$
p_{v_j}	$B_{p_{vj}} / p_{vj} = 0.011$
p_{s_j}	$B_{p_{sj}} / p_{sj} = 0.011$
p_b	$B_{p_b} = 0.05$ in.Hg $= 0.2$ kPa
ψ_j	$B_{\psi_j} = 2°$
ϕ_j	$B_{\phi_j} = 2°$
η_M	$B_{\eta_M} / \eta_M = 0.010$
W	$B_W / W = 0.010$
E	$B_E / E = 0.010$
I	$B_I / I = 0.010$
τ	$B_\tau / \tau = 0.010$
N	$B_N / N = 0.001$
P_I	$B_{P_I} / P_I = 0.010$
n	$B_n =$ nil
t	$B_t = 1$ sec

7-4 FAN MASS FLOW AND UNCERTAINTY FOR MULTIPLE TRAVERSE PLANES

This Code encourages traversing to determine mass flow and average gas properties at multiple traverse planes (as many as three planes may be used in certain circumstances). While the fan mass flow rate is properly evaluated by averaging values from all traverse planes, it is recognized that a measured/calculated mass flow rate at various planes may be of greater or lesser accuracy. Following PTC 19.1, it is recommended that the fan mass flow be calculated as a weighted average using the uncertainties (actually, the variances) as weighting factors. The average mass flow rate is calculated from

$$\bar{\dot{m}}_F = \sum w_i \dot{m}_i \tag{7-4-1}$$

where

$$w_i = \frac{\left(\frac{1}{U_{\dot{m}_i}}\right)^2}{\sum\left(\frac{1}{U_{\dot{m}_i}}\right)^2} \tag{7-4-2}$$

$U_{\dot{m}_i}$ is the total uncertainty for mass flow at plane i,

$$U_{\dot{m}_i} = 2\left[\left(\frac{B_{\dot{m}_i}}{2}\right)^2 + \left(S_{\bar{\dot{m}}_i}\right)^2\right]^{\frac{1}{2}} \tag{7-4-3}$$

The uncertainties for the weighted average mass flow rate are

$$B_{\bar{\dot{m}}_F} = \left(\sum w_i^2 B_{\dot{m}_i}^2\right)^{1/2} \tag{7-4-4}$$

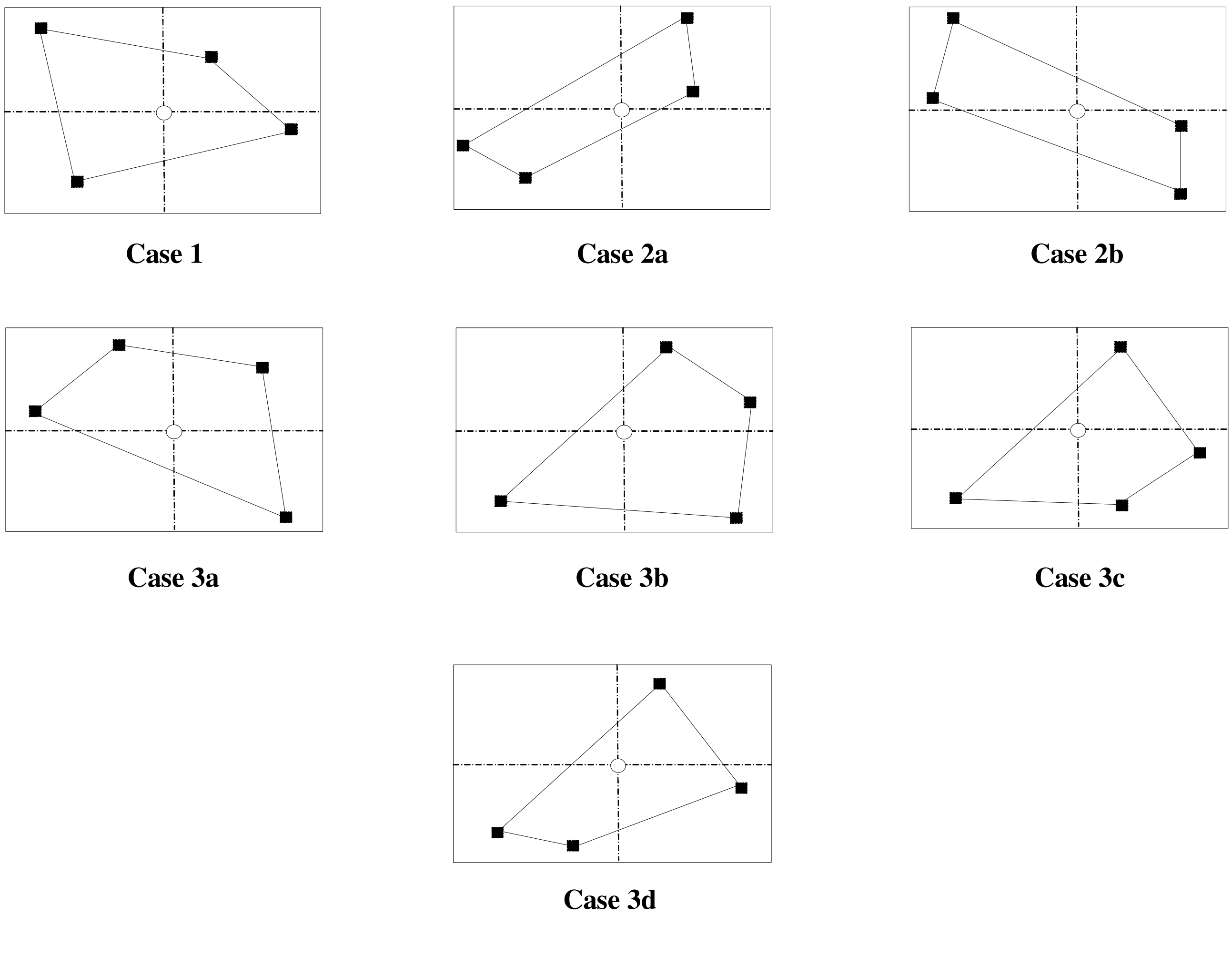
$$S_{\bar{\dot{m}}_F} = \left(\sum w_i^2 S_{\bar{\dot{m}}_i}^2\right)^{1/2} \tag{7-4-5}$$

MANDATORY APPENDIX I
REDUCED LOAD FAN INPUT POWER DETERMINATION

To determine the in situ power requirements of a fan at a specified point of flow, fan pressure, inlet density, and shaft speed, a procedure known as the distance-weighted interpolation method, as described below, shall be used. This procedure only applies to load reduction achieved by closure of inlet vanes, inlet dampers, or variable blade pitch. Recognizing that it is unlikely a single field test can be run at the specified point, the interpolation method requires that four test runs be conducted. The results of each test run shall be corrected for specified inlet conditions and fall within a test window whose width is defined as ±3% of the specified flow and whose height is defined as ±2% of the specified pressure rise.

When a rectangular coordinate system with its origin point is superimposed on the test window, the preferred point locations are depicted in Fig. I-1. There are seven different cases depicted in Fig. I-1. The cases are arranged from least to greater uncertainty; however, the uncertainty for all seven cases is an order of magnitude smaller than the uncertainty of the underlying power measurements.

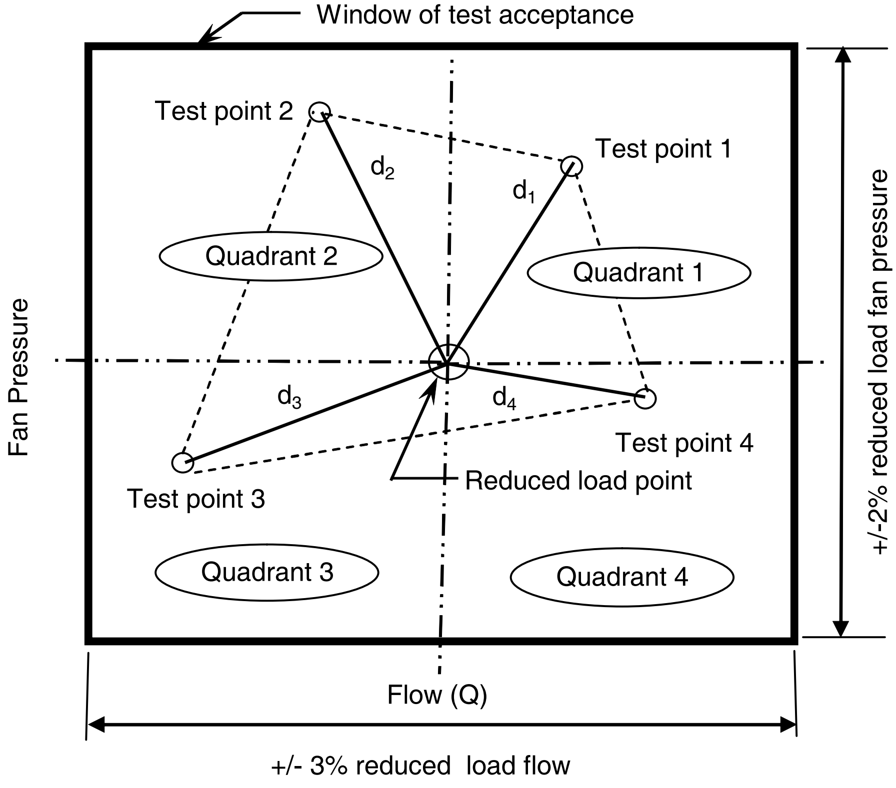
Fig. I-1 Preferred Point Locations



Individual points may fall anywhere within a quadrant with the only requirement being that straight lines connecting all points encircle the specified point located at the center.

A four-point, distance-weighted interpolation method, as described below, shall be used to determine the fan power required to deliver the specified flow and pressure rise (see Fig. I-2 and the equations below).

Fig. I-2 Four-Point, Distance-Weighted Interpolation Method



$$d_i = \sqrt{\left(\frac{Q_i - Q_d}{Q_d}\right)^2 + \left(\frac{p_i - p_d}{p_d}\right)^2}$$

Where:

d_i = distance from test point to the specified point

Q_i = flow at i[th] test condition, acfm

Q_d = flow at the specified condition, acfm

p_i = pressure rise at i[th] test condition, in. wg

p_d = pressure rise at the specified condition, in. wg

$$P = \frac{\frac{1}{d_1} \times P_1 + \frac{1}{d_2} \times P_2 + \frac{1}{d_3} \times P_3 + \frac{1}{d_4} \times P_4}{\frac{1}{d_1} + \frac{1}{d_2} + \frac{1}{d_3} + \frac{1}{d_4}}$$

Where:

P = indicated power at specified conditions, bhp

P_i = tested power at i[th] test condition, bhp

NONMANDATORY APPENDIX A
DATA SHEETS

SAMPLE DATA SHEET
DUCT MEASUREMENTS

SAMPLE DATA SHEET
DUCT MEASUREMENTS

Test ______ Date ____________ Time ______ to ____________ Page ______ of ____________

User ____________________ Plant Name/ Unit No. ______________

Fan: Function ______________ Identification No. ________________ No. Ports ____________

Recorded by ______________ Witnessed by ________________ No. Points/Port ____________

Test Location ______________

DUCT WIDTH* ______ ________ ______ ______

DUCT HEIGHT* ______ ________ ______ ______

TEMPERATURE DURING MEASUREMENT [ ]

PORT SPACING [ ]

* Identify measurement units

SAMPLE DATA SHEET
FAN TEST

Test ______ Date ____________ Time ______ to ____________ Page ______ of ____________

User ____________________ Plant Name/ Unit No. ____________________

Fan: Function ____________________ Identification No. ____________________ No. Ports ____________

Recorded by ____________________ Witnessed by ____________________ No. Points/Port ____________

Probe Type/No. ____________________ Test Location ____________________

VELOCITY PRESSURE MAN. IDENT. ________ MAN. UNITS ____________

STATIC PRESSURE MAN. IDENT. ________ MAN. UNITS ____________

TEMPERATURE T.C. IDENT. ________ T.C. UNITS ____________

PITCH PRESSURE MAN. IDENT. ________ MAN. UNITS ____________

PORT #/DEPTH	VELOCITY PRESSURE	STATIC PRESSURE	TEMP.	YAW ANGLE	PITCH PRESSURE (ΔP)	TIME

PORT #/DEPTH	VELOCITY PRESSURE	STATIC PRESSURE	TEMP.	YAW ANGLE	PITCH PRESSURE (ΔP)	TIME

SAMPLE DATA SHEET
REFERENCE PROBE

Test ______ Date ____________ Time ______ to ____________ Page ______ of ____________
User ________________________ Plant Name/ Unit No. __________________
Fan: Function __________________ Identification No. __________________ Port # ____________
Recorded by __________________ Witnessed by __________________ Point # ____________
Probe Type/No. __________________ Test Location __________________

VELOCITY PRESSURE MAN. IDENT. ________ MAN. UNITS ____________
STATIC PRESSURE MAN. IDENT. ________ MAN. UNITS ____________
TEMPERATURE T.C. IDENT. ________ T.C. UNITS ____________

REFERENCE PROBE			
VELOCITY PRESSURE	STATIC PRESSURE	TEMP.	TIME

REFERENCE PROBE			
VELOCITY PRESSURE	STATIC PRESSURE	TEMP.	TIME

SAMPLE DATA SHEET
GAS ANALYSIS AND AMBIENT CONDITIONS

Test ____ Date __________ Time ____ to _______ Page _______ of ___________

User ____________________ Plant Name/ Unit No. ______________

Fan: Function ______________ Identification No. ____________________ Port # ___________

Recorded by ______________ Witnessed by ____________________ Point # ___________

Analyzer Type/No. ______________ Test Location ______________

	INBOARD			OUTBOARD			AMBIENT CONDITIONS		
Time	CO_2	O_2	CO	CO_2	O_2	CO	Dry Bulb	Wet Bulb	Barometric Pressure
Average									

NOTE: Inboard and outboard gas analyses are averaged together for data processing. Separate analyses for each inlet are recommended for informational purposes in order to explain temperature differences for fans handling products of combustion where infiltration may occur.

$$\text{Speed} = \frac{\text{Pulse freq}^{(1)} \text{ (measured in cps)}}{60 \times \text{no. pulses/rev}}, \quad \text{rpm}$$

$$\text{Power} = \frac{\text{Torque}^{(1)} \text{ (measured in ft - lb)} \times \text{rpm}}{5252}, \quad \text{hp}$$

$$\text{Power} = \frac{\sqrt{3} \times \text{volts}^{(1)} \times \text{amps}^{(1)} \times \text{power factor}^{(1,2)} \times \text{motor eff} \times \text{meter calib coeff}}{745.7}, \quad \text{hp}$$

(1) Average quantities

(2) Power factor = cos(average phase angle between volts and amps)

NONMANDATORY APPENDIX B
SAMPLE CALCULATIONS

The code recommends that a spreadsheet or other computer program be used for the calculations of results and uncertainties. The following tables are extracted from a spreadsheet program.

These tables are of two types. The first type shows what data have been entered, as well as the results of the calculations. Data entry cells are shaded; results cells are not. The second type of table, designated by an F after the table number, shows the formulas for the various results in the corresponding first type of table.

The tables with the formulas show the variable names, values for constants and input, formulas for calculations, and names of the cell or range. Each cell or range has a name that is as close to the symbols used in this Code, with the limits of EXCEL. If the formulas are entered as shown, and the cells and ranges are named as indicated, the results should be the same as the sample.

To minimize the number of pages devoted to this example, neither the input data nor the calculations are complete. Traverse data and traverse point calculations are shown for only a limited number of points at Plane 1A and completely omitted for Planes 1B and 2. The results are given for all three planes.

This example uses U.S. Customary units and simulates a test of a double inlet-induced draft fan using five-hole probes in rectangular ducts. Power is measured by wattmeter.

Table B-1 lists the values of the various constants used throughout the program.

Table B-2 is for the specified operating conditions.

Table B-1 Unit Conversions and Constants

C_1	459.7	°F
C_2	60	sec/min
C_3	1	
C_4	0.672	lbm/ft-sec
C_5	1	Btu/lbm-°F
C_6	57.296	degrees/radian
C_7		
C_8	2830	°F
C_9	1.44	
C_{10}	70.77	lb/ft^2-in.Hg
C_{11}	5.193	lb/ft^2-in.wg
C_{12}	1097	(lbm/ft-min^2-in.wg)$^{1/2}$
C_{13}	13.62	in.wg/in.Hg
C_{14}	745.7	W/hp
C_{15}	5252	ft-lb-in.wg/hp-min
C_{16}	550	ft-lb/hp-sec
C_{17}	6354	ft^3-in.wg/hp-min
C_{18}	0.036355	in.Hg
C_{19}	0.002799	in.Hg/°F
C_{20}	2.08E-05	in.Hg/°F^2
C_{21}	9.67E-07	in.Hg/°F^3
C_{22}	-1.06E-10	in.Hg/°F^4
C_{23}	4.52E-11	in.Hg/°F^5
g_c	32.17	ft-lbm/lb-sec^2
J	778.2	ft-lb/Btu
R_o	1545	ft-lb/lbm-mol-°R

Table B-2 Specified Operating Conditions

N_c	654	rpm
p_{bc}	29.921	in.Hg
p_{t1}	-16	in.wg
p_{ta1}	391.524	in.wg
t_{1c}	290	°F
ρ_{1c}	0.0524	lbm/ft^3
k_c	1.33	
$(k_c-1)/k_c$	0.248	

Table B-1F Unit Conversions and Constants		
Variable	**Formula/Value**	**Name**
C_1	459.7	Const1
C_2	60	Const2
C_3	1	Const3
C_4	0.672	Const4
C_5	1	Const5
C_6	57.296	Const6
C_7		
C_8	2830	Const8
C_9	1.44	Const9
C_{10}	70.77	Const10
C_{11}	5.193	Const11
C_{12}	1097	Const12
C_{13}	13.62	Const13
C_{14}	745.7	Const14
C_{15}	5252	Const15
C_{16}	550	Const16
C_{17}	6354	Const17
C_{18}	0.03635549	Const18
C_{19}	0.002799407	Const19
C_{20}	0.000020788990	Const20
C_{21}	9.66602E-07	Const21
C_{22}	-1.05944E-10	Const22
C_{23}	4.52482E-11	Const23
g_c	32.17	gc
J	778.2	J
R_o	1545	Ro

Table B-2F Specified Operating Conditions		
Variable	**Formula/Value**	**Name**
N_c	654	Nc_s
p_{bc}	29.921	pbc_s
p_{t1}	-16	pt1_s
p_{ta1}	=pt1_s+pbc_s*Const13	pta1
t_{1c}	290	t1c_s
ρ_{1c}	0.0524	ρ1c_s
k_c	1.33	kc_s
$(k_c-1)/k_c$	=(kc_s-1)/kc_s	kc_1__kc_s

Table B-3 is for duct measurements and calculations.

Table B-4 is for probe data.

Table B-5 is for weighting factors. If more than one plane is used for flow rate determination, the weighting factors are calculated using equations 5-6-6 through 5-6-8. In this sample calculation, only Plane 1 is used for flow rate determination; thus, the weighting factor for Plane 1 is 1, and the weighting factor for any other plane is 0.

Table B-6 is for power calculations.

Table B-7 lists some of the intermediate results of gas properties calculations.

Table B-3 Rectangular Duct Plane 1A

d	7	ft
w	18	ft
n_x	9	
n_z	8	
A_{1A}	98	ft^2

Table B-6 AC Motor - Wattmeter

W	1875.4	kW
η_M	0.9497	per uniit
P_I	2388.5	hp

Table B-4 Probe Plane 1A

d_p	0.083333333	ft
C_D	1.2	
L_{head}	0.2	ft

Table B-5 Weighting Factors

Plane	Factor
1	1
2	0

Table B-7 Specific Heat and Visocity

Plane 1A	c_p	μ
O_2	0.227990071	1.80161E-05
CO_2	0.228450685	1.41231E-05
N_2	0.246818853	1.57567E-05
CO	0.249458383	1.57546E-05
H_2O	0.44882041	9.75496E-06

Table B-3F Rectangular Duct Plane 1A		
Variable	**Formula/Value**	**Name**
d	7	width1A
w	18	height1A
nx	9	Nports
nz	8	Npoints
A1A	=width1A*height1A	A1Ar

Table B-4F Probe Plane 1A		
Variable	**Value**	**Name**
d_p	0.083333333	diameter1A
C_D	1.2	CD
L_{head}		*Lhead*

Table B-5F Weighting Factors		
Plane	**Value**	**Name**
1	1	w1_un

Table B-6F AC Motor - Wattmeter		
Variable	**Formula/Value**	**Name**
W	=avg_W	W
η_M	0.9497	ηM
P_I	=1000*W*ηM/Const14	Pinput

Table B-7F Specific Heat c_p Plane 1A		
Variable	**Formula**	**Name**
O_2	=Const5/32*(11.515-172/(Const3*Td_1A)^0.5+1530/(Const3*Td_1A))	O2_Cp_1A
CO_2	=Const5/44.01*(16.2-6530/(Const3*Td_1A)+1400000/(Const3*Td_1A)^2)	CO2_Cp_1A
N_2	=Const5/28.02*(9.47-3470/(Const3*Td_1A)+1160000/(Const3*Td_1A)^2)	N2_Cp_1A
CO	=Const5/28.01*(9.46-3290/(Const3*Td_1A)+1070000/(Const3*Td_1A)^2)	CO_μ_1A
H_2O	=Const5/18*(19.86-597/(Const3*Td_1A)^0.5+7500/(Const3*Td_1A))	H2O_Cp_1A

Table B-7F Viscosity μ Plane 1A		
Variable	**Formula**	**Name**
O_2	=Const4*12.721*(Const3*Td_1A)^1.5*10^-7/(Const3*Td_1A+238.54)	O2_μ_1A
CO_2	=Const4*12.721*(Const3*Td_1A)^1.5*10^-7/(Const3*Td_1A+515.04)	CO2_μ_1A
N_2	=Const4*10.75*(Const3*Td_1A)^1.5*10^-7/(Const3*Td_1A+204.67)	N2_μ_1A
CO	=Const4*10.86*(Const3*Td_1A)^1.5*10^-7/(Const3*Td_1A+214.72)	CO_μ_1A
H_2O	=Const4*12.03*(Const3*Td_1A)^1.5*10^-7/(Const3*Td_1A+987.4)	H2O_μ_1A

Table B-8 gives the results and uncertainties from atmospheric measurements. Room air measurements are only used with open inlet fans.

Table B-8 Results from Atmospheric or Fan Room Air Measurements

Variable	Value	Unit	*B*	*S*	*U*	*U/X*
t_d	63.4	°F		0.014503		
t_w	54.43	°F		0.031128		
t_d-t_w	9	°F				
T_d	523.1	°R	2	0.014503	2.0002	0.38%
p_b	29.49	in.Hg	0.01	0.001671	0.0105	0.04%
p_{ew}	0.4269	in.Hg				
p_p	0.3318	in.Hg				
H	0.0071	lbm_{wv}/lbm_{da}				
c_{pda}	0.2409	Btu/lbm-°F				
c_{pwv}	0.4497	Btu/lbm-°F				
c_{pma}	0.2424	Btu/lbm-°F				
k	1.3943					
$(k-1)/k$	0.2828					
M_{da}	28.9650	lbm/lbm-mol				
R	53.3402	ft-lb/lbm-mol-°R	0.10668	0	0.1067	0.20%
ρ_0	0.07447	lbm/ft^3	0.000322	0.000322	0.0007	0.97%
μ_{da}	0.00001211	lbm/ft-s				
μ_{wv}	0.00000640	lbm/ft-s				

Table 8F Uncertainties in Atmospheric Mesurements		
Variable	**Formula**	**Name**
	Absolute Systematic Uncertainty *B*	
T_d	=Tsj1Bx	TdabsB
p_b	=pbBx	pbB
R	=R_*RSx_X	RB
ρ_0	=((RB/R_)^2+(TdabsB/Tdabs)^2+(pbB/pb)^2)^0.5*ρ0	ρ0B
	Absolute Random Uncertainty *S*	
T_d	=SmeanX_td	TdabsS
p_b	=Smeanx_pb	pbS
R	=R_*RSx_X	RS
ρ_0	=((RB/R_)^2+(TdabsB/Tdabs)^2+(pbB/pb)^2)^0.5*ρ0	ρ0S
	Absolute Total Uncertainty *U*	
T_d	=2*((TdabsB/2)^2+TdabsS^2)^0.5	tdabsU
p_b	=2*((pbB/2)^2+pbS^2)^0.5	pbU
R	=2*((RB/2)^2+RS^2)^0.5	RU
ρ_0	=2*((ρ0B/2)^2+ρ0S^2)^0.5	ρ0U
	Relative Total Uncertainty *U/X*	
T_d	=tdabsU/Tdabs	tdabsU_X
p_b	=pbU/pb	pbU_X
R	=RU/R_	RU_X
ρ_0	=ρ0U/ρ0	ρ0U_X

Table B-9 lists the systematic and random measurement uncertainties.

Table B-9 Measurement Uncertainties						
X	**Unit**		B_X	B_X/X	S_X	S_X/X
F_n			Calculated			
A	ft^2			0.007		0.007
R	ft-lb/lbm-°R			0.002		0.000
T_{sj1}	°R		2		0.458258	
p_{vj1}	in.wg			0.011		0.007412
p_{sj1}	in.wg			0.011		-0.00198
p_b	in.Hg		0.01		0.001671	
ψ_{j1}	deg		2		Calculated	
φ_{j1}	deg		2		Calculated	
η_M	per unit			0.010		0.000000
W	W			0.010		0.000312
N	rpm			0.001		0.000024

Table B-9F Systematic Uncertainties in Measurements				
X	B_X	Formula	B_X/X	Name
F_n		see Abs. Sys. Uncert. in n_{1A}		n1A_B
A		established from experience	0.007	ABx_X
R		established from experience	0.002	RBx_X
T_{sj1}	2	established from experience		Tsj1Bx
p_{vj1}		established from experience	0.011	pvj1Bx_X
p_{sj1}		established from experience	0.011	psj1Bx_X
p_b	0.01	established from experience		pbBx
ψ_{j1}	2	established from experience		ψj1Bx
φ_{j1}	2	established from experience		φj1Bx
η_M		established from experience	0.010	ηMBx_X
W		established from experience	0.010	WBx_X
N		established from experience	0.001	NBx_X

Table B-9F Random Uncertainties in Measurements				
X	S_X	Formula	S_X/X	Name
R		assumed zero	0	RSx_X
T_{sj1}	0.458258	=SmeanX_t1R		Tsj1Sx
p_{vj1}		=SmeanX_pvR/avg_pvR	0.0074122	pvj1Sx_X
p_{sj1}		=SmeanX_ps1R/avg_ps1R	-0.00198	psj1Sx_X
p_b	0.001671	=Smeanx_pb		pbSx
η_M		assumed zero	0	ηMSx_X
W		=SmeanX_W/avg_W	0.000015	WSx_X
N		=Smeanx_N/avg_N	0.000024	NSx_X

Table B-10 lists the calculated gas properties for Duct 1A.

Table B-11 lists the calculated gas properties for Duct 1B.

Table B-12 lists the gas properties for Duct 1, which are calculated as the mass flow averages of the properties in Ducts 1A and 1B.

Table B-10 Gas Properties at Plane 1A

Variable	Value	Unit
T_d	764.5555556	°R
O_2	0.0537	per unit
CO_2	0.1314	per unit
N_2	0.8149	per unit
CO	0.0000	per unit
H	0.0414	lbm_{wv}/lbm_{dg}
c_{pdg}	0.24424	Btu/lbm-°F
c_{pwv}	0.44882	Btu/lbm-°F
c_{pmg}	0.2621	Btu/lbm-°F
k	1.3450	
$(k-1)/k$	0.2565	
M_{dg}	30.3349	lbm/lbm-mol
R_{mg}	52.3152	ft-lb/lbm-mol-°R
M_{mg}	29.5325	lbm/lbm-mol
μ_{dg}	0.00001562	lbm/ft-s
μ_{wv}	0.000009755	lbm/ft-s
μ_{mg}	0.00001615	lbm/ft-s

Table B-11 Gas Properties at Plane 1B

Variable	Value	Unit
T_d	751.584	°R
O_2	0.0539	per unit
CO_2	0.1312	per unit
N_2	0.8149	per unit
CO	0.0000	per unit
H	0.0411	lbmwv/lbmdg
c_{pdg}	0.2417	Btu/lbm-°F
c_{pwv}	0.4479	Btu/lbm-°F
c_{pmg}	0.2594	Btu/lbm-°F
k	1.3375	
$(k-1)/k$	0.2524	
M_{dg}	30.3319	lbm/lbm-mol
R_{mg}	50.9365	ft-lb/lbm-mol-°R
M_{mg}	29.5353	lbm/lbm-mol
μ_{dg}	0.00001543	lbm/ft-s
μ_{wv}	0.00000958	lbm/ft-s
μ_{mg}	0.00001594	lbm/ft-s

Table B-12 Gas Properties at Plane 1

Variable	Value	Unit
c_{pmg}	0.260727743	Btu/lbm-°F
$(k-1)/k$	0.254429573	

Table B-10F Gas Properties at Plane 1A		
Variable	**Formula**	**Name**
T_d	=avg_tdi+Const1	Td_1A
	#VALUE!	
O_2	=avg_O2j_1A	O2_1A
CO_2	=avg_CO2j_1A	CO2_1A
N_2	=avg_N2j_1A	N2_1A
CO	=avg_COj_1A	CO_1A
H	0.0414	H_1A
c_{pdg}	=(44.01*CO2_1A*CO2_Cp_1A+32*O2_1A*O21A+28.02*N2_1A *CO_Cp_1A+28.01*CO_1A*N2_Cp_1A)/Mdg_1A	cpdg_1A
c_{pwv}	=Const5/18*(19.86-597/(Const3*Td_1A)^0.5+7500/(Const3*Td_1A))	cpwv_1A
c_{pmg}	=cpdg_1A+cpwv_1A*H_1A/(1+H_1A)	cpmg_1A
k	=cpmg_1A/(cpmg_1A-Rmg_1A/J)	K_1A
(k-1)/k	=(K_1A-1)/K_1A	k_1__k_1A
M_{dg}	=44.01*CO2_1A+28.01*CO_1A+32*O2_1A+28.02*N2_1A	Mdg_1A
R_{dg}	=Ro/Mmg_1A	Rdg_1A
M_{mg}	=IF(H_1A=0, Mdg_1A, ((1+H_1A)/(H_1A/18.02+1/Mdg_1A)))	Mmg_1A
μ_{dg}	=(44.01^0.5*CO2_1A*CO2_µ_1A+32^0.5*O2_1A*O2_µ_1A+28.02^0.5*N2_1A*N2_µ_1A+28.01^0.5*CO_1A*CO_µ_1A)/(44.01^0.5*CO2_1A+32^0.5*O2_1A+28.02^0.5*N2_1A+28.01^0.5*CO_1A)	µdg_1A
μ_{wv}	=Const4*12.03*(Const3*Td_1A)^1.5*10^-7/(Const3*Td_1A+987.4)	µwv_1A
μ_{mg}	=(44.01^0.5*CO2_1A*CO2_µ_1A+32^0.5*O2_1A*O2_µ_1A+28.02^0.5*N2_1A*N2_µ_1A+28.01^0.5*CO_1A*CO_µ_1A+18.02^0.5*Mdg_1A*H_1A*µwv_1A/18.02)/(44.01^0.5*CO2_1A+32^0.5*O2_1A+28.02^0.5*N2_1A+28.01^0.5*CO_1A+18.02^0.5*µwv_1A*H_1A/18.02)	µmg_1A

Table B-12F Gas Properties at Plane 1		
Variable	**Formula**	**Name**
c_{pmg}	=(cpmg_1A*m1A+cpmg_1B*m1B)/m1p	cpmg_1
(k-1)/k	=(k_1__k_1A*m1A+k_1__k_1B*m1B)/m1p	K_1_K_1

Table B-13 gives the results of the traverse at Plane 1A.

Table B-14 gives the results of the traverse at Plane 1B.

Table B-13 Results from Plane 1A Measurements						
Variable	**Value**	**Unit**	***B***	***S***	***U***	***U/X***
A_{1A}	98	ft^2	0.6860	0.6860	1.5339	1.57%
$\boldsymbol{n_{1A}}$	**72**		0.0135		0.0135	
m_{1A}	324.7	lbm/sec	2.3064	2.2922	5.1318	1.58%
Q_{1A}	390124	cfm	2775.5	2754.3	6168.3	1.58%
p_{s1A}	-16.864	in.wg	-0.0219	-0.0039	0.0233	-0.14%
ρ_{1A}	0.0499	lbm/ft^3	0.000020	0.000004	0.00002	0.04%
T_{s1A}	764.8	°R	0.2357	0.0540	0.3	0.03%
e_{K1A}	78.26	ft-lb/lbm	0.1705	0.1265	0.3051	0.39%
α_{1A}	1.1439					
p_{v1A}	0.75258	in.wg	0.0016	0.0012	0.0029	0.39%
p_{t1A}	-16.11125	in.wg	-0.0219	-0.0041	0.0234	-0.15%
p_{sa1A}	384.7	in.wg	0.0271	0.0027	0.0277	0.01%
p_{ta1A}	385.5	in.wg				

Table B-14 Results from Plane 1B Measurements						
Variable	**Value**	**Unit**	***B***	***S***	***U***	***U/X***
A_{1B}	126	ft^2	0.8820	0.8820	1.9722	1.57%
n_{1B}	75		0.0086		0.0086	
					7.7362	1.64%
Q_{1B}	556223	cfm	4213.7	4054.8	9138.9	1.64%
p_{s1B}	-17.28	in.wg	-0.0220	-0.0040	0.0234	-0.14%
ρ_{1B}	0.0508	lbm/ft3	0.00002	0.00000	0.00002	0.04%
T_{s1B}	750.9	°R	0.2309	0.0529	0.2540	0.03%
e_{K1B}	87.48	ft-lb/lbm	0.4774	0.2707	0.7217	0.83%
α_{1B}	1.0398					
p_{v1B}	0.856	in.wg	0.0047	0.0026	0.0070	0.82%
p_{t1B}	-16.43	in.wg	-0.0225	-0.0048	0.0244	-0.15%
p_{sa1B}	384.3	in.wg	0.0271	0.00264	0.0276	0.01%
p_{ta1B}	385.2	in.wg				

Table B-13F Results from Plane 1A Measurements		
Variable	**Formula**	**Name**
A_{1A}	*=A1Ar*	A1A
n_{1A}	*=Nports*Npoints*	n1A
m_{1A}	*=sum_mj*	m1A
Q_{1A}	*=m1A*Const2/ρ1A*	Q1A
p_{s1A}	*=sum_psjVjcosψjcosφj/sum_Vjcosψjcosφj*	ps1A
ρ_{1A}	*=sum_ρjVjcosψjcosφj/sum_Vjcosψjcosφj*	ρ1A
T_{s1A}	*=sum_TsjρjVjcosψjcosφj/sum_ρjVjcosψjcosφj*	Ts1A
e_{K1A}	*=sum_ρjV3jcos3ψjcos3φj/sum_ρjVjcosψjcosφj/(2*gc*Const2^2)*	eK1A
α_{1A}	*=2*gc*eK1A*A1A^2*ρ1A^2/m1A^2*	α1A
p_{v1A}	*=ρ1A*eK1A/Const11*	pv1A
p_{t1A}	*=ps1A+pv1A*	pt1A
p_{sa1A}	*=ps1A+pb*Const13*	psa1A
p_{ta1A}	*=pt1A+pb*Const13*	pta1A

Table B-13F Absolute Systematic Uncertainty *B*		
Variable	**Formula**	**Name**
n_{1A}	=0.45*(α1A-1)^0.33/n1A^0.67	n1A_B
m_{1A}	=m1A*((n1A_B/n1A)^2+ABx_X^2+sum_Σmj)^0.5	m1A_B
Q_{1A}	=((m1A_B/m1A)^2+(ρ1A_B/ρ1A)^2)^0.5*Q1A	Q1A_B
p_{s1A}	=ps1A/n1A*sum_Σpsj^0.5	ps1A_B
ρ_{1A}	=ρ1A/n1A*sum_Σρj^0.5	ρ1A_B
T_{s1A}	=Ts1A/n1A*sum_ΣTsj^0.5	Ts1A_B
e_{K1A}	=eK1A/n1A*sum_ΣeKj^0.5	eK1A_B
α_{1A}		
p_{v1A}	=pv1A/n1A*sum_Σpvj^0.5	pv1A_B
p_{t1A}	=pt1A/n1A*sum_Σptj^0.5	pt1A_B
p_{sa1A}	=psa1A/n1A*sum_upsaj_2^0.5	psa1A_B
p_{ta1A}		

Table B-13F Absolute Random Uncertainty *S*		
Variable	**Formula**	**Name**
A_{1A}	=ASx_X*A1A	A1A_S
n_{1A}		
m_{1A}	=m1A*(ASx_X^2+sum_Σmj_S)^0.5	m1A_S
Q_{1A}	=((m1A_S/m1A)^2+(ρ1A_S/ρ1A)^2)^0.5*Q1A	Q1A_S
p_{s1A}	=ps1A/n1A*sum_Σpsj_S^0.5	ps1A_S
ρ_{1A}	=ρ1A/n1A*sum_Σρj_S^0.5	ρ1A_S
T_{s1A}	=Ts1A/n1A*sum_ΣTsj_S^0.5	Ts1A_S
e_{K1A}	=eK1A/n1A*sum_ΣeKj_S^0.5	eK1A_S
α_{1A}		
p_{v1A}	=pv1A/n1A*sum_Σpvj_S^0.5	pv1A_S
p_{t1A}	=pt1A/n1A*sum_Σptj_S^0.5	pt1A_S
p_{sa1A}	=psa1A/n1A*sum_upsaj_2_S^0.5	psa1A_S

Table B-13F Absolute Total Uncertainty *U*		
Variable	**Formula**	**Name**
A_{1A}	=2*((A1A_B/2)^2+A1A_S^2)^0.5	A1A_U
n_{1A}	=2*((n1A_B/2)^2+AI5^2)^0.5	n1A_U
m_{1A}	=2*((m1A_B/2)^2+m1A_S^2)^0.5	m1A_U
Q_{1A}	=2*((Q1A_B/2)^2+Q1A_S^2)^0.5	Q1A_U
p_{s1A}	=2*((ps1A_B/2)^2+ps1A_S^2)^0.5	ps1A_U
ρ_{1A}	=2*((ρ1A_B/2)^2+ρ1A_S^2)^0.5	ρ1A_U
T_{s1A}	=2*((Ts1A_B/2)^2+Ts1A_S^2)^0.5	Ts1A_U
e_{K1A}	=2*((eK1A_B/2)^2+eK1A_S^2)^0.5	eK1A_U
α_{1A}		
p_{v1A}	=2*((pv1A_B/2)^2+pv1A_S^2)^0.5	pv1A_U
p_{t1A}	=2*((pt1A_B/2)^2+pt1A_S^2)^0.5	pt1A_U
p_{sa1A}	=2*((psa1A_B/2)^2+psa1A_S^2)^0.5	psa1A_U
p_{ta1A}		

Table B-13F Relative Total Uncertainty *U/X*		
Variable	**Formula**	**Name**
A_{1A}	=A1A_U/A1A	A1A_U_X
n_{1A}		
m_{1A}	=m1A_U/m1A	m1A_U_X
Q_{1A}	=Q1A_U/Q1A	Q1A_U_X
p_{s1A}	=ps1A_U/ps1A	ps1A_U_X
ρ_{1A}	=ρ1A_U/ρ1A	ρ1A_U_X
T_{s1A}	=Ts1A_U/Ts1A	Ts1A_U_X
e_{K1A}	=eK1A_U/eK1A	eK1A_U_X
α_{1A}		
p_{v1A}	=pv1A_U/pv1A	pv1A_U_X
p_{t1A}	=pt1A_U/pt1A	pt1A_U_X
p_{sa1A}	=psa1A_U/psa1A	psa1A_U_X
p_{ta1A}		

Table B-15 gives the combined results for Plane 1, which in this instance is used to determine the fan flow rate.

Table B-16 gives the results from the Plane 2 traverse. They do not qualify for flow measurement due to the small number of measurements. Plane 1 and 2 results are used to determine the other fan performance parameters.

Table B-15 Combined Results for Plane 1

Variable	Value	Unit	*B*	*S*	*U*	*U/X*
A_1	126	ft^2	0.8820	0.8820	1.9722	1.57%
$\boldsymbol{n_1}$	150		0.0082		0.0082	
m_1	889.4	lbm/sec	6.3896	6.3103	14.1459	1.59%
Q_1	1059196	cfm	7610.7	7515.0	16847.2	1.59%
p_{s1}	-17.08	in.wg	-0.0077	-0.0014	0.0082	-0.05%
ρ_1	0.0504	lbm/ft3	0.00001	0.00000	0.00001	0.02%
T_{s1}	757.6	°R	0.0818	0.0188	0.0900	0.01%
e_{K1}	83.22	ft-lb/lbm	0.1273	0.0749	0.1966	0.24%
α_1	1.1399					
p_{v1}	0.807	in.wg	0.0012	0.0007	0.0019	0.24%
p_{t1}	-16.27	in.wg	-0.0078	-0.0016	0.0084	-0.05%
p_{sa1}	384.5	in.wg	0.0003	0.0000	0.0003	0.00%
p_{ta1}	385.3	in.wg				

Table B-16 Results from Plane 2 Measurements

Variable	Value	Unit	*B*	*S*	*U*	*U/X*
A_2	201.6	ft^2	1.4112	1.4112	3.1555	1.57%
n_2	30		0.0046		0.0046	
m_1	648.0	lbm/sec			13.0129	1.57%
Q_2	780002	cfm	6881.1	6811.1	15261.4	1.58%
p_{s2}	-1.77	in.wg	-0.0036	0.0000	0.0036	-0.20%
ρ_2	0.0512	lbm/ft3	0.00003	0.000001	0.0000	0.06%
T_{s2}	775.2	°R	0.3651	0.0000	0.3651	0.05%
e_{K2}	99.73	ft-lb/lbm	0.2090	0.1350	0.3414	0.34%
α_2	1.0009					
p_{v2}	0.983	in.wg	0.0020	0.0013	0.0033	0.34%
p_{t2}	-0.789	in.wg	0.0000	0.0000	0.0000	0.00%
p_{sa2}	399.8	in.wg	0.0251	0.0042	0.0265	0.01%
p_{ta2}	400.8	in.wg				

Table B-15F		
Plane 1		
Variable	Formula	Name
A_1	=A1A+A1B	A1p
n_1	=n1A+n1B	n1p
m_1	=m1A+m1B	m1p
Q_1	=m1p*Const2/ρ1p	Q1p
p_{s1}	=(Q1A*ps1A+Q1B*ps1B)/Q1p	ps1p
ρ_1	=(Q1A*ρ1A+Q1B*ρ1B)/(Q1A+Q1B)	ρ1p
T_{s1}	=(m1A*Ts1A+m1B*Ts1B)/m1p	Ts1p
e_{K1}	=(m1A*eK1A+m1B*eK1B)/m1p	eK1p
α_1	=(m1A*α1A+m1B*α1B)/m1p	α1p
p_{v1}	=ρ1p*eK1p/Const11	pv1p
p_{t1}	=ps1p+pv1p	pt1p
p_{sa1}	=ps1p+pb*Const13	pt1p
p_{ta1}	=pt1p+pb*Const13	pta1p

Formulas same as for Plane 1A			
Plane 1B		Plane 2	
Variable	Name	Variable	Name
A_{1B}	A1B	A_2	A2p
n_{1B}	n1B	n_2	n2p
m_{1B}	m1B	m_2	m2p
Q_{1B}	Q1B	Q_2	Q2p
p_{s1B}	ps1B	p_{s2}	ps2p
ρ_{1B}	ρ1B	ρ_2	ρ2p
T_{s1B}	Ts1B	T_{s2}	Ts2p
e_{K1B}	eK1B	e_{K2}	eK2p
α_{1B}	α1B	α_2	α2p
p_{v1B}	pv1B	p_{v2}	pv2p
p_{t1B}	pt1B	p_{t2}	pt2p
p_{sa1B}	psa_B	p_{sa2}	psa2p
p_{ta1B}	pta1B	p_{ta2}	pta2p

Formulas for the results for plane 1B and plane 2 similar to those for Plane 1A

Table B-17 gives the final results for mass flow-specific energy performance. Note that the performance converted at specified conditions is also shown.

Table B-17 Results for Mass Flow - Specific Energy Approach						
Variable	Value	Unit	***B***	***S***	***U***	***U/X***
m_F	649.686	lbm/sec	6.390	6.310	14.146	2.18%
ρ_m	0.051	lbm/ft^3	0.00002	0.00000	0.00002	0.03%
y_F	1581.533	ft-lb/lbm	3.706	1.606	4.905	0.31%
P_O	1868.182	hp	13.449	13.235	29.690	1.59%
P_I	2388.450	hp	33.778	0.744	33.811	1.42%
K_ρ	0.992					
η	0.782	per unit	0.012	0.006	0.017	2.13%
N	672.293	rpm	0.672	0.016	0.673	0.10%
b	0.956					
$K_{\rho c}$	0.968					
ρ_{mc}	0.0541	lbm/ft^3				
m_{Fc}	673.350	lbm/sec	6.657	6.540	14.677	2.18%
y_{Fc}	1496.636	ft-lb/lbm	4.611	1.522	5.525	0.37%
P_{Oc}	1832.291	hp	14.292	12.984	29.641	1.62%
P_{Ic}	2342.565	hp	33.868	0.815	33.907	1.45%
η_c	0.782	per unit	0.012	0.006	0.017	2.13%

Table B-17F Results for Mass Flow - Specific Energy Approach		
Variable	Formula	Name
m_F	=m1p	mF
ρ_m	=(ρ1p+ρ2p)/2	ρm
y_F	=Const11*(ps2p-ps1p)/ρm+eK2p-eK1p	yF
P_O	=mF*yF/Const16	PO
P_I	=Pinput	PI
K_ρ	=ρ1p/ρm	Kρ
η	=Pomf/Pimf	η
N	=avg_N	N
b	=(Nc_s/Nmf)^2*Ts1p/(t1c_s+Const1)	b
$K_{\rho c}$	=1-b*(1-Kρ)*(η*kc_s-(kc_s-1)*(1+b*(1+Kρ)))/(η*K_1A-(K_1A-1)*(1+(1+Kρ)))	Kρc
ρ_{mc}	=ρ1c_s/Kρc	ρmc
m_{Fc}	=mF*(ρ1c_s/ρ1p)*(Nc_s/Nmf)*(Kρ/Kρc)	mFc
y_{Fc}	=yF*(Nc_s/Nmf)^2	yFc
P_{Oc}	=mFc*yFc/Const16	POc
P_{Ic}	=Pinput*(Nc_s/Nmf)^3*(ρ1c_s/ρ1p)*(Kρ/Kρc)	PIc
η_c	=η	ηc

Table B-17F Absolute Systematic Uncertainty *B*

Variable	Formula	Name
m_F	=m1p_B	mF_B
ρ_m	=ρm*((ρ1p_B^2+ρ2p_B^2)/(ρ1p+ρ2p)^2)^0.5	ρm_B
y_F	=yF*(((Ts1p_B/Ts1p)^2*((ps2p-ps1p)/(2*ρm^2)*ρ1p-pv1p/ρ1p)^2+(Ts2p_B/Ts2p)^2*((ps2p-ps1p)/(2*ρm^2)*ρ2p+pv2p/ρ2p)^2+(pbB/pb)^2*(pv1p/ρ1p*Const13*pb/psa1p-Const11*(ps2p-ps1p)/(2*ρm^2)*(Const13*pb/R_/Ts1p+Const13*pb/R_/Ts2p)-pv2p/ρ2p*Const13*pb/psa2p)^2+(ps1p_B/ps1p)^2*(pv1p/ρ1p*ps1p/psa1p-(ps2p-ps1p)/(2*ρm^2)*ρ1p*ps1p/psa1p-ps1p/ρm)^2+(m2p_B/m2p)^2*(ps2p/ρm-(ps2p-ps1p)/(2*ρm^2)*ρ2p*ps2p/psa2p-pv2p/ρ2p*ps2p/psa2p)^2+(pv1p_B/pv1p)^2*(pv1p/ρ1p)^2+(pv2p_B/pv2p)^2*(pv2p/ρ2p)^2)*(Const11/yF)^2+RBx_X^2)^0.5	yF_B
P_O	=Pomf*(RBx_X^2/4+(n1p_b/n1p)^2*(w1_un*m1p/mF)^2+(w1_un*m1p/mF)^2*ABx_X^2+(Ts1p_B/Ts1p)^2*((w1_un*m1p/(2*mF))-Const11/yF*(ps2p-ps1p)/(2*ρm^2)*ρ1p-eK1p/yF)^2+(Ts2p_B/Ts2p)^2*(-Const11/yF*(ps2p-ps1p)/(2*ρm^2)*ρ2p+eK2p/yF)^2+(pbB/pb)^2*((w1_un*m1p/(2*mF))*Const13*pb/psa1p+Const11/yF*(pv1p/ρ1p*Const13*pb/psa1p-Const11*(ps2p-ps1p)/(2*ρm^2)*(Const13*pb/R_/Ts1p+Const13*pb/R_/Ts2p)-pv2p/ρ2p*Const13*pb/psa2p))^2+(ps1p_B/ps1p)^2*((w1_un*m1p/(2*mF))*ps1p/psa1p+Const11/yF*((pv1p/ρ1p*ps1p/psa1p-(ps2p-ps1p)/(2*ρm^2)*ρ1p*ps1p/psa1p-ps1p/ρm)))^2+(m2p_B/m2p)^2*(Const11/yF*(ps2p/ρm-(ps2p-ps1p)/(2*ρm^2)*ρ2p*ps2p/psa2p-pv2p/ρ2p*ps2p/psa2p))^2+(pv1p_B/pv1p)^2*((w1_un*m1p/(2*mF))-eK1p/yF)^2+(pv2p_B/pv2p)^2*(eK2p/yF)^2)^0.5	Pomf_B
P_I	=(WBx_X^2+ηMBx_X^2)^0.5*Pimf	Pimf_B
K_ρ		H29
η	=η*((Pomf_B/Pomf)^2+(Pimf_B/Pimf)^2)^0.5	η_B
N	=NBx_X*Nmf	Ns_B
b		H32
$K_{\rho c}$		H33
ρ_{mc}		H34
m_{Fc}	=mFc*((mF_B/mF)^2+NBx_X^2+(ρ1p_B/ρ1p)^2)^0.5	mFc_B
yFc	=yFc*((yF_B/yF)^2+4*NBx_X^2)^0.5	yFc_B
P_{Oc}	=POcmf*((Pomf_B/Pomf)^2+(ρ1p_B/ρ1p)^2+9*NBx_X^2)^0.5	Pocmf_B
P_{Ic}	=Picmf*((Pimf_B/Pimf)^2+(ρ1p_B/ρ1p)^2+9*NBx_X^2)^0.5	Picmf_B
η_c	=η_B	ηc_B

Table B-17F Absolute Random Uncertainty *S*

Variable	Formula	Name
m_F	=m1p_S	mF_S
ρ_m	=ρm*((ρ1p_S^2+ρ2p_S^2)/(ρ1p+ρ2p)^2)^0.5	ρm_S
y_F	=yF*(((Ts1p_S/Ts1p)^2*((ps2p-ps1p)/(2*ρm^2)*ρ1p-pv1p/ρ1p)^2+(Ts2p_S/Ts2p)^2*((ps2p-ps1p)/(2*ρm^2)*ρ2p+pv2p/ρ2p)^2+(pbS/pb)^2*(pv1p/ρ1p*Const13*pb/psa1p-Const11*(ps2p-ps1p)/(2*ρm^2)*(Const13*pb/R_/Ts1p+Const13*pb/R_/Ts2p)-pv2p/ρ2p*Const13*pb/psa2p)^2+(ps1p_S/ps1p)^2*(pv1p/ρ1p*ps1p/psa1p-(ps2p-ps1p)/(2*ρm^2)*ρ1p*ps1p/psa1p-ps1p/ρm)^2+(m2p_S/m2p)^2*(ps2p/ρm-(ps2p-ps1p)/(2*ρm^2)*ρ2p*ps2p/psa2p-pv2p/ρ2p*ps2p/psa2p)^2+(pv1p_S/pv1p)^2*(pv1p/ρ1p)^2+(pv2p_S/pv2p)^2*(pv2p/ρ2p)^2)*(Const11/yF)^2+RSx_X^2)^0.5	yF_S
P_O	=Pomf*((w1_un*m1p/mF)^2*ASx_X^2+(Ts1p_S/Ts1p)^2*((w1_un*m1p/(2*mF))-Const11/yF*(ps2p-ps1p)/(2*ρm^2)*ρ1p-eK1p/yF)^2+(Ts2p_S/Ts2p)^2*(-Const11/yF*(ps2p-ps1p)/(2*ρm^2)*ρ2p+eK2p/yF)^2+(pbS/pb)^2*((w1_un*m1p/(2*mF))*Const13*pb/psa1p+Const11/yF*(pv1p/ρ1p*Const13*pb/psa1p-Const11*(ps2p-ps1p)/(2*ρm^2)*(Const13*pb/R_/Ts1p+Const13*pb/R_/Ts2p)-pv2p/ρ2p*Const13*pb/psa2p))^2+(ps1p_S/ps1p)^2*((w1_un*m1p/(2*mF))*ps1p/psa1p+Const11/yF*((pv1p/ρ1p*ps1p/psa1p-(ps2p-ps1p)/(2*ρm^2)*ρ1p*ps1p/psa1p-ps1p/ρm)))^2+(m2p_S/m2p)^2*(Const11/yF*(ps2p/ρm-(ps2p-ps1p)/(2*ρm^2)*ρ2p*ps2p/psa2p-pv2p/ρ2p*ps2p/psa2p))^2+(pv1p_S/pv1p)^2*((w1_un*m1p/(2*mF))-eK1p/yF)^2+(pv2p_S/pv2p)^2*(eK2p/yF)^2)^0.5	Pomf_S
P_I	=(WSx_X^2+ηMSx_X^2)^0.5*Pimf	Pimf_S
K_ρ		H49
η	=η*((Pomf_S/Pomf)^2+(Pimf_S/Pimf)^2)^0.5	η_S
N	=NSx_X*Nmf	Ns_S
b		H52
$K_{\rho c}$		H53
ρ_{mc}		H54
m_{Fc}	=mFc*((mF_S/mF)^2+NSx_X^2+(ρ1p_S/ρ1p)^2)^0.5	mFc_S
y_{Fc}	=yFc*((yF_S/yF)^2+4*NSx_X^2)^0.5	yFc_S
P_{Oc}	=POcmf*((Pomf_S/Pomf)^2+(ρ1p_B/ρ1p)^2+9*NSx_X^2)^0.5	Pocmf_S
P_{Ic}	=Picmf*((Pimf_S/Pimf)^2+(ρ1p_B/ρ1p)^2+9*NSx_X^2)^0.5	Picfm_S
η_c	=η_S	ηc_S

Table B-17F Absolute Total Uncertainty *U*		
Variable	**Formula**	**Name**
m_F	=2*((mF_B/2)^2+mF_S^2)^0.5	mF_U
ρ_m	=2*((ρm_B/2)^2+ρm_S^2)^0.5	ρm_U
y_F	=2*((yF_B/2)^2+yF_S^2)^0.5	yF_U
P_O	=2*((Pomf_B/2)^2+Pomf_S^2)^0.5	Pomf_U
P_I	=2*((Pimf_B/2)^2+Pimf_S^2)^0.5	Pimf_U
K_ρ		
η	=2*((η_B/2)^2+η_S^2)^0.5	η_U
N	=2*((Ns_B/2)^2+Ns_S^2)^0.5	Ns_U
b		
$K_{\rho c}$		
ρ_{mc}		
m_{Fc}	=2*((mFc_B/2)^2+mFc_S^2)^0.5	mFc_U
y_{Fc}	=2*((yFc_B/2)^2+yFc_S^2)^0.5	yFc_U
P_{Oc}	=2*((Pocmf_B/2)^2+Pocmf_S^2)^0.5	Pocmf_U
P_{Ic}	=2*((Picmf_B/2)^2+AQ18^2)^0.5	Picfm_U
η_c	=2*((ηc_B/2)^2+ηc_S^2)^0.5	ηc_U

Table B-17F Relative Total Uncertainty *U/X*		
Variable	**Formula**	**Name**
m_F	=mF_U/mF	mF_U_X
ρ_m	=ρm_U/ρm	ρm_U_X
y_F	=yF_U/yF	yF_U_X
P_O	=Pomf_U/Pomf	Pomf_U_X
P_I	=Pimf_U/Pimf	Pimf_U_X
K_ρ		
η	=η_U/η	η_U_X
N	=Ns_U/Nmf	Ns_U_X
b		
$K_{\rho c}$		
ρ_{mc}		
m_{Fc}	=mFc_U/mFc	mFc_U_X
y_{Fc}	=yFc_U/yFc	yFc_U_X
P_{Oc}	=Pocmf_U/POcmf	Pocmf_U_X
P_{Ic}	=AR18/Picmf	Picfm_U_X
η_c	=ηc_U/ηc	ηc_U_X

Table B-18 gives the final results for volume flow-pressure performance. Results at specified conditions are also shown.

Table B-18 Results for Volume Flow - Pressure Approach

Variable	Value	Unit	*B*	*S*	*U*	*U/X*
ρ_F	0.05046	lbm/ft^3	0.00001	0.00000	0.00001	0.02%
Q_F	772498	cfm	5495.7	5418.6	12150.9	1.57%
p_{Ft}	15.482	in.wg	0.0078	0.0016	0.0084	0.05%
p_{Fv}	0.983	in.wg	0.0020	0.0013	0.0033	0.34%
p_{Fs}	14.499	in.wg	0.0080	0.0021	0.0090	0.06%
z	0.013					
x	0.040					
K_p	0.987					
P_O	1857.4	hp	13.242	13.025	29.223	1.57%
P_I	2388.5	hp	33.778	0.7444	33.811	1.42%
η_t	0.778	per unit	0.0123	0.0055	0.0165	2.12%
η_s	0.728	per unit	0.0115	0.0051	0.0154	2.12%
N	672.293	rpm	0.6723	0.016	0.6730	0.10%
z/z_c	1.069					
z_c	0.0121					
a	0.038					
x_c	0.039					
Kpr	1.000					
K_{pc}	0.987					
Q_{Fc}	751306	cfm	5397.5	5269.9	11841.5	1.58%
p_{Ftc}	15.211	in.wg	0.0314	0.0019	0.0317	0.21%
p_{Fvc}	0.966	in.wg	0.0027	0.0013	0.0038	0.39%
p_{Fsc}	14.245	in.wg	0.0296	0.0022	0.0300	0.21%
P_{Oc}	1775.2	hp	13.7323	12.4495	28.4348	1.60%
P_{Ic}	2282.7	hp	33.0020	0.5741	33.0220	1.45%
η_{tc}	0.728	per unit	0.0123	0.0055	0.0165	2.26%

Table B-18F Results for Volume Flow - Pressure Approach		
Variable	**Formula**	**Name**
ρ_F	=ρ1p*pta1p/psa1p/(1+eK1p/(J*cpmg_1*Ts1p))	ρF
Q_F	=Const2*m1p/ρF	QF
p_{Ft}	=pt2p-pt1p	pFt
p_{Fv}	=pv2p	pFv
p_{Fs}	=pFt-pFv	pFs
z	=K_1_K_1*PI/QF*Const17/pta1p	z
x	=pFt/pta1p	x
K_p	=z/x*LN(1+x)/LN(1+z)	Kp
P_O	=QF*pFt*Kp/Const17	PO
P_I	=Pinput	PI
η_t	=PO/PI	ηt
η_s	=ηt*pFs/pFt	ηs
N	=avg_N	Nv
z/z_c	=k_1__k_1A/kc_1__kc_s*pta1/pta1p*(Nv/Nc_s)^2*ρF/ρ1c_s	z_zc
z_c	=z/z_zc	zc
a	=LN(1+x)*LN(1+zc)/LN(1+z)*k_1__k_1A/kc_1__kc_s	a
x_c	=EXP(a)-1	xc
K_{pr}	=z/zc*xc/x*kc_1__kc_s/k_1__k_1A	Kpr
K_{pc}	=Kp/Kpr	Kpc
Q_{Fc}	=QF*Nc_s/Nv*Kp/Kpc	QFc
p_{Ftc}	=pFt*ρ1c_s/ρF*(Nc_s/Nv)^2*Kp/Kpc	pFtc
p_{Fvc}	=pFv*(Nc_s/Nv)^2*ρ1c_s/ρF	pFvc
p_{Fsc}	=pFtc-pFvc	pFsc
P_{Oc}	=QFc*pFtc*Kpc/Const17	POc
P_{Ic}	=PI*ρ1c_s/ρF*(Nc_s/Nv)^3*Kp/Kpc	PIc
η_{tc}	=ηs	ηtc

Table B-18 F Absolute Systematic Uncertainty ***B***		
Variable	**Formula**	**Name**
ρ_F	=ρ1p_B	ρF_B
Q_F	=QF*(RBx_X^2/4+(n1p_b/n1p)^2*(w1_un*m1p/mF)^2+(w1_un*m1p/mF)^2*ABx_X^2+(Ts1p_B/Ts1p)^2*(1-(w1_un*m1p/(2*mF)))^2+(pbB/pb)^2*((w1_un*m1p/(2*mF))*Const13*pb/psa1p-Const13*pb/pta1p)^2+(ps1p_B/ps1p)^2*((w1_un*m1p/(2*mF))*ps1p/psa1p-ps1p/pta1p)^2+(pv1p_B/pv1p)^2*((w1_un*m1p/(2*mF))-pv1p/pta1p)^2)^0.5	QF_B
p_{Ft}	=pFt*((pt2p_B^2+pt1p_B^2)/pFt^2)^0.5	pft_B
p_{Fv}	=pv2p_B	pfv_B
p_{Fs}	=pFs*((pft_B^2+pfv_B^2)/pFs^2)^0.5	pfs_B
P_O	=(RBx_X^2/4+(n1p_b/n1p)^2*(w1_un*m1p/mF)^2+(w1_un*m1p/mF)^2*ABx_X^2+(Ts1p_B/Ts1p)^2*(1-(w1_un*m1p/(2*mF)))^2+(pbB/pb)^2*((w1_un*m1p/(2*mF))*Const13*pb/psa1p-Const13*pb/pta1p)^2+(ps1p_B/ps1p)^2*((w1_un*m1p/(2*mF))*ps1p/psa1p-ps1p/pta1p-ps1p/pFt)^2+(ps2p_B/ps2p)^2*(ps2p/pFt)^2+(pv1p_B/pv1p)^2*((w1_un*m1p/(2*mF))-pv1p/pta1p-pv1p/pFt)^2+(pv2p_B/pv2p)^2*(pv2p/pFt)^2)^0.5*PO	PO_B
P_I	=(WBx_X^2+ηMBx_X^2)^0.5*PI	PI_B
η_t	=ηt*((PI_B/PI)^2+(PO_B/PO)^2)^0.5	ηt_B
η_s	=ηs*((PI_B/PI)^2+(PO_B/PO)^2)^0.5	ηs_B
N	=NBx_X*Nv	N_B
Q_{Fc}	=QFc*((QF_B/QF)^2+(Ns_B/Nmf)^2)^0.5	QFc_B
p_{Ftc}	=pFtc*((pft_B/pFt)^2+4*(Ns_B/Nmf)^2+(ρ1p_B/ρ1p)^2)^0.5	pFtc_B
p_{Fvc}	=pFvc*((pfv_B/pFv)^2+4*(Ns_B/Nmf)^2+(ρ1p_B/ρ1p)^2)^0.5	pFvc_B
p_{Fsc}		
P_{Oc}		
P_{Ic}		
η_{tc}	=ηt_B	ηtc_B

Table B-18 F Absolute Random Uncertainty ***S***		
Variable	**Formula**	**Name**
ρ_F	=ρ1p_S	ρF_S
Q_F	=QF*((w1_un*m1p/mF)^2*ASx_X^2+(Ts1p_S/Ts1p)^2*(1-(w1_un*m1p/(2*mF)))^2+(pbS/pb)^2*((w1_un*m1p/(2*mF))*Const13*pb/psa1p-Const13*pb/pta1p)^2+(ps1p_S/ps1p)^2*((w1_un*m1p/(2*mF))*ps1p/psa1p-ps1p/pta1p)^2+(pv1p_S/pv1p)^2*((w1_un*m1p/(2*mF))-pv1p/pta1p)^2)^0.5	QF_S
p_{Ft}	=pFt*((pt2p_S^2+pt1p_S^2)/pFt^2)^0.5	pft_S
p_{Fv}	=pv2p_S	pfv_S
p_{Fs}	=pFs*((pft_S^2+pfv_S^2)/pFs^2)^0.5	pfs_S
P_O	=((w1_un*m1p/mF)^2*ASx_X^2+(Ts1p_S/Ts1p)^2*(1-(w1_un*m1p/(2*mF)))^2+(pbS/pb)^2*((w1_un*m1p/(2*mF))*Const13*pb/psa1p-Const13*pb/pta1p)^2+(ps1p_S/ps1p)^2*((w1_un*m1p/(2*mF))*ps1p/psa1p-ps1p/pta1p-ps1p/pFt)^2+(ps2p_S/ps2p)^2*(ps2p/pFt)^2+(pv1p_S/pv1p)^2*((w1_un*m1p/(2*mF))-pv1p/pta1p-pv1p/pFt)^2+(pv2p_S/pv2p)^2*(pv2p/pFt)^2)^0.5*PO	PO_S
P_I	=(WSx_X^2+ηMSx_X^2)^0.5*PI	PI_S
η_t	=ηt*((PI_S/PI)^2+(PO_S/PO)^2)^0.5	ηt_S
η_s	=ηs*((PI_S/PI)^2+(PO_S/PO)^2)^0.5	ηs_S
N	=NSx_X*Nv	N_S
Q_{Fc}	=QFc*((QF_S/QF)^2+(Ns_S/Nmf)^2)^0.5	QFc_S
p_{Ftc}	=pFtc*((pft_S/pFt)^2+4*(Ns_S/Nmf)^2+(ρ1p_S/ρ1p)^2)^0.5	pFtc_S
p_{Fvc}	=pFvc*((pfv_S/pFv)^2+4*(Ns_S/Nmf)^2+(ρ1p_S/ρ1p)^2)^0.5	pFvc_S
p_{Fsc}	=pFsc*((pfs_S/pFs)^2+4*(Ns_S/Nmf)^2+(ρ1p_S/ρ1p)^2)^0.5	pFsc_S
P_{Oc}	=POc*((PO_S/PO)^2+9*(Ns_S/Nmf)^2+(ρ1p_S/ρ1p)^2)^0.5	POc_S
P_{Ic}	=POc*((PI_S/PI)^2+9*(Ns_S/Nmf)^2+(ρ1p_S/ρ1p)^2)^0.5	PIc_S
η_{tc}	=ηt_S	ηtc_S

Table B-18F Absolute Total Uncertainty *U*		
Variable	**Formula**	**Name**
ρ_F	=2*((ρF_B/2)^2+ρF_S^2)^0.5	ρF_U
Q_F	=2*((QF_B/2)^2+QF_S^2)^0.5	QF_U
p_{Ft}	=2*((pft_B/2)^2+pft_S^2)^0.5	pft_U
p_{Fv}	=2*((pfv_B/2)^2+pfv_S^2)^0.5	pfv_U
p_{Fs}	=2*((pfs_B/2)^2+pfs_S^2)^0.5	pfs_U
P_O	=2*((PO_B/2)^2+PO_S^2)^0.5	PO_U
P_I	=2*((PI_B/2)^2+PI_S^2)^0.5	PI_U
η_t	=2*((ηt_B/2)^2+ηt_S^2)^0.5	ηt_U
η_s	=2*((ηs_B/2)^2+ηs_S^2)^0.5	ηs_U
N	=2*((N_B/2)^2+N_S^2)^0.5	N_U
Q_{Fc}	=2*((QFc_B/2)^2+QFc_S^2)^0.5	QFc_U
p_{Ftc}	=2*((pFtc_B/2)^2+pFtc_S^2)^0.5	pFtc_U
p_{Fvc}	=2*((pFvc_B/2)^2+pFvc_S^2)^0.5	pFvc_U
p_{Fsc}	=2*((pFsc_B/2)^2+pFsc_S^2)^0.5	pFsc_U
P_{Oc}	=2*((POc_B/2)^2+POc_S^2)^0.5	POc_U
P_{Ic}	=2*((PIc_B/2)^2+PIc_S^2)^0.5	PIc_U
η_{tc}	=2*((ηtc_B/2)^2+ηtc_S^2)^0.5	ηtc_U

Table B-18F Relative Total Uncertainty *U/X*		
Variable	**Formula**	**Name**
ρ_F	=ρF_U/ρF	ρF_U_X
Q_F	=QF_U/QF	QF_U_X
p_{Ft}	=pft_U/pFt	pft_U_X
p_{Fv}	=IF(pFv=0, 0, pfv_U/pFv)	pfv_U_X
p_{Fs}	=pfs_U/pFs	pfs_U_X
P_O	=PO_U/PO	PO_U_X
P_I	=PI_U/PI	PI_U_X
η_t	=ηt_U/ηt	ηt_U_X
η_s	=ηs_U/ηs	ηs_U_X
N	=N_U/Nv	N_U_X
Q_{Fc}	=QFc_U/QFc	QFc_U_X
p_{Ftc}	=pFtc_U/pFtc	pFtc_U_X
p_{Fvc}	=IF(pFvc=0, 0, pFvc_U/pFvc)	pFvc_U_X
p_{Fsc}	=pFsc_U/pFsc	pFsc_U_X
P_{Oc}	=POc_U/POc	POc_U_X
P_{Ic}	=PIc_U/PIc	PIc_U_X
η_{tc}	=ηtc_U/ηtc	ηtc_U_X

Table B-19 gives the test data and statistical results for atmospheric parameters, speed, and power measurements.

Table B-19 Ambient Conditions , Power, & Speed Measurements

rdg	t_d	t_w	p_b	W	N
1	63.5	54.5	29.49	3476	672.3
2	63.4	54.4	29.48	3476	672.4
3	63.5	54.6	29.48	3476	672.3
4	63.3	54.3	29.50	3476	672.2
5	63.3	54.5	29.50	3476	672.2
6	63.4	54.6	29.48	3476	672.3
7	63.5	54.4	29.49	3476	672.1
8	63.5	54.5	29.47	3476	672.4
9	63.4	54	29.49	3476	672.2
10	63.5	54.5	29.49	3476	672.4
11	63.5	54.5	29.49	3476	672.3
12	63.4	54.4	29.48	3476	672.4
13	63.5	54.6	29.48	3476	672.3
14	63.3	54.3	29.50	3476	672.2
15	63.3	54.5	29.50	3476	672.2
16	63.4	54.6	29.48	3476	672.3
17	63.5	54.4	29.49	3476	672.3
18	63.5	54.5	29.47	3476.6	672.4
19	63.4	54	29.49	3476	672.2
20	63.5	54.5	29.49	3476	672.4
21	63.5	54.5	29.49	3476	672.3
22	63.4	54.4	29.48	3476	672.4
23	63.5	54.6	29.48	3475	672.3
24	63.3	54.3	29.50	3476	672.2
25	63.3	54.5	29.50	3476	672.2
26	63.4	54.6	29.48	3476	672.3
27	63.5	54.4	29.49	3476	672.3
28	63.5	54.5	29.47	3475	672.4
29	63.4	54	29.49	3476	672.2
30	63.5	54.5	29.49	3476	672.4
count	30	30	30	30	30
sum	1902.9	1632.9	884.61	56262	20168.8
avg	63.43	54.43	29.487	1875.4	672.2933
S_X	0.079438	0.170496	0.009154	3.201293	0.086834
S_{meanX}	0.014503	0.031128	0.001671	0.584473	0.015854
	t_d	t_w	p_b	W	N
***$2S_{meanX}/X_{avg}$* (check anything over 1%)**					
S/X	0.0%	0.1%	0.0%	0.1%	0.0%

S/X = $2S_{meanX}/X_{avg}$

Table B-19F Calculations for Ambient Conditions

Range	t_d		t_w	
Name	td_amb		tw_amb	
Calculation	formula	name	formula	name
count	=COUNT(td_amb)	count_td	=COUNT(tw_amb)	count_tw
sum	=SUM(td_amb)	sum_td	=SUM(tw_amb)	sum_tw
avg	=AVERAGE(td_amb)	avg_td	=AVERAGE(tw_amb)	avg_tw
S_X	=STDEV(td_amb)	SX_td	=STDEV(tw_amb)	Sx_tw
S_{meanX}	=SX_td/count_td^0.5	SmeanX_td	=Sx_tw/count_tw^0.5	Smeanx_tw
S/X	=2*SmeanX_td/avg_td	S_X_td	=2*Smeanx_tw/avg_tw	S_X_tw

Range	p_b		W	
Name	pb_amb		W_pow	
Calculation	formula	name	formula	name
count	=COUNT(pb_amb)	count_pb	=COUNT(W_pow)	count_W
sum	=SUM(pb_amb)	sum_pb	=SUM(W_pow)	sum_W
avg	=AVERAGE(pb_amb)	avg_pb	=AVERAGE(W_pow)	avg_W
S_X	=STDEV(pb_amb)	Sx_pb	=STDEV(W_pow)	Sx_W
S_{meanX}	=Sx_pb/count_pb^0.5	Smeanx_pb	=Sx_W/count_W^0.5	SmeanX_W
S/X	=2*Smeanx_pb/avg_pb	S_X_pb	=2*SmeanX_W/avg_W	S_X_W

Range	N	
Name	N_spd	
Calculation	formula	name
count	=COUNT(N_spd)	count_N
sum	=SUM(N_spd)	sum_N
avg	=AVERAGE(N_spd)	avg_N
SX	=STDEV(N_spd)	Sx_N
S_{meanX}	=Sx_N/count_N^0.5	Smeanx_N
S/X	=2*Smeanx_N/avg_N	S_X_N

Table B-20 gives the test data and statistical results for reference measurements required by this Code.

Table B-20 Reference Measurements						
rdg	t_{1R}	t_{2R}	p_{s1R}	p_{s2R}	p_{tR}	p_{vR}
1	312	315	-16.79	-1.73	14.9	1.1
2	308	317	-16.42	-1.77	15.6	1.12
3	312	310	-16.60	-1.73	14.8	1.08
4	311	317	-16.35	-1.74	15.4	1.13
5	310	317	-16.46	-1.82	15.2	1.09
6	309	316	-16.21	-1.8	15	1.15
7	307	317	-16.55	-1.79	14.6	1.07
8	311	320	-16.73	-1.79	14.9	1.03
9	304	320	-16.65	-1.77	15.3	1.11
10	307	311	-16.72	-1.78	15.1	1.2
11	312	315	-16.79	-1.73	14.9	1.1
12	308	317	-16.42	-1.77	15.6	1.12
13	312	310	-16.60	-1.73	14.8	1.08
14	311	317	-16.35	-1.74	15.4	1.13
15	310	317	-16.46	-1.82	15.2	1.09
16	309	316	-16.21	-1.8	15	1.15
17	307	317	-16.55	-1.79	14.6	1.07
18	311	320	-16.73	-1.79	14.9	1.03
19	304	320	-16.65	-1.77	15.3	1.11
20	307	311	-16.72	-1.78	15.1	1.2
21	312	315	-16.79	-1.73	14.9	1.1
22	308	317	-16.42	-1.77	15.6	1.12
23	312	310	-16.60	-1.73	14.8	1.08
24	311	317	-16.35	-1.74	15.4	1.13
25	310	317	-16.46	-1.82	15.2	1.09
26	309	316	-16.21	-1.8	15	1.15
27	307	317	-16.55	-1.79	14.6	1.07
28	311	320	-16.73	-1.79	14.9	1.03
29	304	320	-16.65	-1.77	15.3	1.11
30	307	311	-16.72	-1.78	15.1	1.2
count	30	30	30	30	30	30
sum	9273	9480	-496.44	-53.16	452.4	33.24
avg	309.1	316	-16.548	-1.772	15.08	1.108
S_X	2.50998	3.184012	0.179643	0.029408	0.290541	0.044983
S_{meanX}	0.458258	0.581318	0.032798	0.005369	0.053045	0.008213
	t_{1R}	t_{2R}	p_{s1R}	p_{s2R}	p_{tR}	p_{vR}
$2S_{meanX}/X_{avg}$ (check anything over 1%)						
S/X	0.3%	0.4%	-0.4%	-0.6%	0.7%	1.5%

S/X $= 2S_{meanX}/X_{avg}$

Table B-20F Calculations for Reference Measurements

Range	t_{1R}		t_{2R}
Name	t1R		t2R
Calculation	**Formula**	**Name**	**Formula**
S/X	=COUNT(t1R)	count_t1R	=COUNT(t2R)
sum	=SUM(t1R)	sum_t1R	=SUM(t2R)
avg	=AVERAGE(t1R)	avg_t1R	=AVERAGE(t2R)
Sx	=STDEV(t1R)	SX_t1R	=STDEV(t2R)
S_{meanX}	=SX_t1R/count_t1R^0.5	SmeanX_t1R	=SX_t2R/count_t2R^0.5
S/X	=2*SmeanX_t1R/avg_t1R	S_Avg_t1R	=2*SmeanX_t2R/avg_t2R

Range	p_{s1R}		p_{s2R}
Name	ps1R		ps2R
Calculation	**Formula**	**Name**	**Formula**
count	=COUNT(ps1R)	count_ps1R	=COUNT(ps2R)
sum	=SUM(ps1R)	sum_ps1R	=SUM(ps2R)
avg	=AVERAGE(ps1R)	avg_ps1R	=AVERAGE(ps2R)
Sx	=STDEV(ps1R)	Sx_ps1R	=STDEV(ps2R)
S_{meanX}	=Sx_ps1R/count_ps1R^0.5	SmeanX_ps1R	=Sx_ps2R/count_ps2R^0.5
S/X	=2*SmeanX_ps1R/avg_ps1R	S_avg_ps1R	=2*SmeanX_ps2R/avg_Ps2R

Range	p_{tR}		p_{vR}
Name	ptR		pvR
Calculation	**Formula**	**Name**	**Formula**
count	=COUNT(ptR)	count_PtR	=COUNT(pvR)
sum	=SUM(ptR)	sum_ptR	=SUM(pvR)
avg	=AVERAGE(ptR)	avg_ptR	=AVERAGE(pvR)
Sx	=STDEV(ptR)	Sx_ptR	=STDEV(pvR)
S_{meanX}	=Sx_ptR/count_PtR^0.5	SmeanX_ptR	=Sx_pvR/count_pvR^0.5
S/X	=2*SmeanX_ptr/avg_ptR	S_avg_ptR	=2*SmeanX_pvR/avg_pvR

Table B-21 gives the test data and statistical results for gas measurements required by this Code.

Table B-21 Gas Measurements at Plane 1A					
rdg	O_2	CO	CO_2	N_2	
1	0.0537	0	0.1313	0.815	
2	0.0534	0	0.131	0.8156	
3	0.0539	0	0.1315	0.8146	
4	0.0536	0	0.1316	0.8148	
5	0.054	0	0.1313	0.8147	
6	0.053	0	0.1314	0.8156	
7	0.0535	0	0.1318	0.8147	
8	0.0538	0	0.1313	0.8149	
9	0.0541	0	0.1316	0.8143	
10	0.0537	0	0.1313	0.815	
11	0.0537	0	0.1313	0.815	
12	0.0534	0	0.131	0.8156	
13	0.0539	0	0.1315	0.8146	
14	0.0536	0	0.1316	0.8148	
15	0.054	0	0.1313	0.8147	
16	0.053	0	0.1314	0.8156	
17	0.0535	0	0.1318	0.8147	
18	0.0538	0	0.1313	0.8149	
19	0.0541	0	0.1316	0.8143	
19	0.0541	0	0.1316	0.8143	
21	0.0537	0	0.1313	0.815	
22	0.0534	0	0.131	0.8156	
23	0.0539	0	0.1315	0.8146	
24	0.0536	0	0.1316	0.8148	
25	0.054	0	0.1313	0.8147	
26	0.053	0	0.1314	0.8156	
27	0.0535	0	0.1318	0.8147	
28	0.0538	0	0.1313	0.8149	
29	0.0541	0	0.1316	0.8143	
30	0.0537	0	0.1313	0.815	
n	30	30	30	30	
Σ	*1.6101*	*0*	*3.9423*	*24.4476*	
avg	0.05367	0	0.13141	0.81492	
S_X	0.000309	0	0.000216	0.000399	
S_{meanX}	5.64E-05	0	3.93E-05	7.28E-05	
	O_2	CO	CO_2	N_2	

Table B-21F Calculations for Gas Measurements at Plane 1A

Range	O_2	
Name	O2j_1A	
Calculation	**Formula**	**Name**
n	=COUNT(O2j_1A)	count_O2j_1A
Σ	=SUM(O2j_1A)	sum_O2j_1A
avg	=AVERAGE(O2j_1A)	avg_O2j_1A
S_X	=STDEV(O2j_1A)	SX_O2_1A
S_{meanX}	=SX_O2_1A/count_O2j_1A^0.5	SmeanX_O2j_1A
Range	CO	
Name	CO2j_1A	
Calculation	**Formula**	**Name**
n	=COUNT(COj_1A)	count_COj_1A
Σ	=SUM(COj_1A)	sum_COj_1A
avg	=AVERAGE(COj_1A)	avg_COj_1A
S_X	=STDEV(COj_1A)	Sx_COj_1A
S_{meanX}	=Sx_COj_1A/count_COj_1A^0.5	SmeanX_COj_1A
Range	CO_2	
Name	CO2j_1A	
Calculation	**Formula**	**Name**
n	=COUNT(CO2j_1A)	count_CO2j_1A
Σ	=SUM(CO2j_1A)	sum_CO2j_1A
avg	=AVERAGE(CO2j_1A)	avg_CO2j_1A
S_X	=STDEV(CO2j_1A)	Sx_CO2j_1A
S_{meanX}	=Sx_CO2j_1A/count_CO2j_1A^0.5	SmeanX_CO2j_1A
Range	N_2	
Name	N2j_1A	
Calculation	**Formula**	**Name**
n	=COUNT(N2j_1A)	count_N2j_1A
Σ	=SUM(N2j_1A)	sum_N2j_1A
avg	=AVERAGE(N2j_1A)	avg_N2j_1A
S_X	=STDEV(N2j_1A)	Sx_N2j_1A
S_{meanX}	=Sx_N2j_1A/count_N2j_1A^0.5	SmeanX_N2j_1A

Table B-22 is devoted to the traverse point measurements and the calculations of the various point quantities needed to produce the final results. In spreadsheet terminology, each row is dedicated to a particular traverse point. The columns contain either measurements or calculations. Because so many columns are needed, this table is presented in several individual sections.

Table B-22.1 lists 30 of the 72 traverse measurements made at Plane 1A. The others are not shown simply to save space. At the bottom of each column, the program calculates the number of readings, the total, and the average, also not shown.

Table B-22.1 Traverse Measurements at Plane 1A												From Probe Calibration			
rdg	Port	Point	x_j	z_j	L_p	S_p	t_{di}	p_{vi}	p_{si}	ψ	Δp_φ	C_φ	φ	K_{vj}	K_{tj}
1	1	1	0.375	1	1.2	0.1000	308.3	0.51	-16.45	0	-0.05	-0.10	9.04	1.068081	1.025908
2	1	2	1.125	1	1.2	0.1000	309.9	0.65	-16.47	0	-0.14	-0.22	16.52	1.044756	1.025563
3	1	3	1.875	1	1.2	0.1000	310.7	0.77	-16.66	0	-0.12	-0.16	12.73	1.056405	1.025723
4	1	4	2.625	1	1.2	0.1000	311.4	0.8	-16.59	0	0.25	0.31	-14.61	1.24406	1.025913
5	1	5	3.375	1	1.2	0.1000	299.2	0.6	-16.76	0	-0.23	-0.38	26.68	0.999506	1.02579
6	1	6	4.125	1	1.2	0.1000	303.7	0.59	-16.44	10	0.12	0.20	-9.03	1.173432	1.026205
7	1	7	4.875	1	1.2	0.1000	309.3	0.69	-16.21	10	-0.14	-0.20	15.73	1.047237	1.025591
8	1	8	5.625	1	1.2	0.1000	310.2	0.71	-16.09	0	-0.06	-0.08	8.17	1.070996	1.025951
9	2	1	0.375	3	3.2	0.2667	311	0.91	-16	0	-0.11	-0.12	10.49	1.063345	1.025834
10	2	2	1.125	3	3.2	0.2667	304.7	0.41	-16.81	0	0.06	0.15	-5.87	1.144869	1.026284
11	2	3	1.875	3	3.2	0.2667	305.3	0.36	-16.63	0	0.03	0.08	-2.22	1.118864	1.026294
12	2	4	2.625	3	3.2	0.2667	306.8	0.44	-16.65	12	0.07	0.16	-6.59	1.150813	1.026272
13	2	5	3.375	3	3.2	0.2667	307.9	0.51	-16.62	10	0.07	0.14	-5.36	1.14078	1.026291
14	2	6	4.125	3	3.2	0.2667	308.8	0.54	-16.67	0	0.09	0.17	-7.02	1.154461	1.026263
15	2	7	4.875	3	3.2	0.2667	310.4	0.74	-16.21	0	0.01	0.01	2.01	1.095761	1.026207
16	2	8	5.625	3	3.2	0.2667	303.8	0.21	-16.1	0	0.08	0.38	-17.78	1.30129	1.025694
17	3	1	0.375	5	5.2	0.4333	304.4	0.26	-16.28	0	0.05	0.19	-8.43	1.167476	1.026226
18	3	2	1.125	5	5.2	0.4333	305.9	0.26	-16.2	0	0.12	0.46	-21.19	1.384444	1.025495
19	3	3	1.875	5	5.2	0.4333	307.8	0.46	-16.13	0	0.09	0.20	-8.61	1.169252	1.02622
20	3	4	2.625	5	5.2	0.4333	309.7	0.71	-16.28	0	0.07	0.10	-3.12	1.124674	1.026299
21	3	5	3.375	5	5.2	0.4333	299	0.23	-16.17	10	0.01	0.04	0.18	1.105016	1.026256
22	3	6	4.125	5	5.2	0.4333	301.9	0.34	-16.27	10	-0.03	-0.09	8.41	1.070184	1.025939
23	3	7	4.875	5	5.2	0.4333	304.5	0.42	-16.48	0	-0.02	-0.05	5.83	1.079474	1.026061
24	3	8	5.625	5	5.2	0.4333	307.1	0.53	-16.4	0	0.01	0.02	1.68	1.097348	1.026217
25	4	1	0.375	7	7.2	0.6000	291.6	0.5	-16.44	0	0.08	0.16	-6.64	1.151246	1.026271
26	4	2	1.125	7	7.2	0.6000	302	0.45	-16.38	0	0.01	0.02	1.48	1.098356	1.026223
27	4	3	1.875	7	7.2	0.6000	301.7	0.4	-16.38	0	0.14	0.35	-16.38	1.274028	1.025792
28	4	4	2.625	7	7.2	0.6000	304.1	0.63	-16.75	0	-0.01	-0.02	3.84	1.087535	1.026144
29	4	5	3.375	7	7.2	0.6000	306	0.77	-16.14	0	0.26	0.34	-15.80	1.263811	1.025832
30	4	6	4.125	7	7.2	0.6000	308	0.88	-16.36	0	-0.15	-0.17	13.66	1.053564	1.025679

additional rows for data, count, sum and avg not shown

Table B-22.2 gives the corrections for probe calibration and blockage for each traverse point.

Table B-22.2 Corrections for Probe Calibration and Blockage Plane 1A

p_{ti}	p_{sai}	β_j	K_{vjc}	K_{vjc_e}	$(1+\varepsilon_T)$	T_i	T_{sj}	p_{tj}	p_{sj}	p_{vj}	p_{saj}
-15.94	385.1629	0.001226	1.066684	1.067266	1.000308	768.0	767.7633	-16.3530	-16.89769	0.543723	384.7152
-15.82	385.1429	0.001227	1.043419	1.042764	1.000384	769.6	769.3043	-16.2244	-16.90349	0.677778	384.7094
-15.89	384.9529	0.001227	1.055038	1.053131	1.000461	770.4	770.0453	-16.2987	-17.11217	0.811741	384.5008
-15.79	385.0229	0.001228	1.242162	1.237809	1.000564	771.1	770.6656	-16.1992	-17.19442	0.992775	384.4185
-16.16	384.8529	0.001227	0.998282	0.99437	1.000340	758.9	758.6423	-16.5768	-17.17647	0.598623	384.4365
-15.85	385.1729	0.001227	1.171745	1.172448	1.000392	763.4	763.101	-16.2654	-16.95768	0.690868	384.6553
-15.52	385.4029	0.001227	1.045893	1.045235	1.000409	769.0	768.6858	-15.9172	-16.63977	0.721163	384.9732
-15.38	385.5229	0.001227	1.069591	1.067631	1.000430	769.9	769.5691	-15.7791	-16.53953	0.758853	385.0734
-15.09	385.6129	0.003274	1.059655	1.058775	1.000546	770.7	770.2795	-15.4798	-16.44748	0.963389	385.1655
-16.4	384.8029	0.00327	1.1406	1.138138	1.000265	764.4	764.1973	-16.8311	-17.30046	0.467434	384.3125
-16.27	384.9829	0.003269	1.114786	1.117969	1.000228	765.0	764.826	-16.6978	-17.10059	0.401167	384.5124
-16.21	384.9629	0.00327	1.146498	1.148397	1.000286	766.5	766.2808	-16.6359	-17.14223	0.504213	384.4707
-16.11	384.9929	0.003271	1.136539	1.136965	1.000329	767.6	767.3478	-16.5335	-17.11534	0.57931	384.4976
-16.13	384.9429	0.003271	1.150117	1.149081	1.000352	768.5	768.2294	-16.5536	-17.17704	0.62069	384.4359
-15.47	385.4029	0.003273	1.091845	1.089587	1.000458	770.1	769.7477	-15.8754	-16.68629	0.807335	384.9266
-15.89	385.5129	0.003268	1.29578	1.300081	1.000154	763.5	763.3824	-16.2983	-16.57155	0.272042	385.0414
-16.02	385.3329	0.005311	1.160282	1.164991	1.000171	764.1	763.9694	-16.4401	-16.74368	0.301585	384.8693
-15.94	385.4129	0.005312	1.374338	1.378832	1.000202	765.6	765.4451	-16.3464	-16.70634	0.357204	384.9066
-15.67	385.4829	0.005314	1.162031	1.163736	1.000303	767.5	767.2677	-16.0809	-16.61872	0.534258	384.9942
-15.57	385.3329	0.005318	1.117987	2.163736	1.000450	769.4	769.0541	-15.9795	-16.778	0.793162	384.8349
-15.94	385.4429	0.00531	1.09857	3.163736	1.000143	758.7	758.5915	-16.3585	-16.61268	0.25261	385.0003
-15.93	385.3429	0.005312	1.064135	4.163736	1.000205	761.6	761.4439	-16.3432	-16.70707	0.361679	384.9059
-16.06	385.1329	0.005313	1.073318	1.076058	1.000256	764.2	764.0048	-16.4785	-16.93192	0.450597	384.681
-15.87	385.2129	0.005315	1.090984	1.092487	1.000328	766.8	766.5488	-16.2861	-16.86766	0.577899	384.7453
-15.94	385.1729	0.007359	1.141574	1.144438	1.000324	751.3	751.057	-16.3588	-16.93439	0.570472	384.6786
-15.93	385.2329	0.007357	1.089551	1.097494	1.000278	761.7	761.4884	-16.3477	-16.842	0.490066	384.7709
-15.98	385.2329	0.007358	1.262196	1.271067	1.000286	761.4	761.1822	-16.3922	-16.90177	0.504632	384.7112
-16.12	384.8629	0.007361	1.078898	1.084066	1.000386	763.8	763.5056	-16.5414	-17.22659	0.679259	384.3864
-15.37	385.4729	0.007368	1.252152	1.257361	1.000546	765.7	765.282	-15.7670	-16.74017	0.963259	384.8728
-15.48	385.2529	0.007367	1.04545	1.047853	1.000521	767.7	767.3	-15.8775	-16.80465	0.919178	384.8083

additional rows for data, count, sum and avg not shown

Table B-22.3 shows the results of calculating the corrected point values from the traverse measurements.

Table B-22.3 Point Values Plane 1A						
ρ_j	V_j	Re_{pj}	$\cos \psi_j$	$\cos \varphi_j$	m_j	e_{Kj}
0.04974	3626.973	3.03E-06	1	0.987591	4.041719	55.39359
0.049639	4053.571	3.38E-06	1	0.95871	4.37615	65.20279
0.049565	4439.456	3.7E-06	1	0.975415	4.868921	80.95707
0.049514	4912.106	4.09E-06	1	0.967668	5.339064	97.54497
0.050301	3784.376	3.2E-06	1	0.893551	3.858635	49.36779
0.050036	4076.281	3.43E-06	0.984808	0.987601	4.500071	67.85949
0.049713	4178.184	3.49E-06	0.984808	0.962546	4.466574	67.72316
0.049669	4287.876	3.58E-06	1	0.989844	4.782306	77.77396
0.049635	4832.956	4.03E-06	1	0.983272	5.350792	97.4966
0.049919	3356.853	2.82E-06	1	0.994749	3.781442	48.14021
0.049904	3110.287	2.61E-06	1	0.999252	3.518486	41.70302
0.049804	3490.446	2.92E-06	0.978148	0.993386	3.831874	49.66188
0.049738	3743.833	3.13E-06	0.984808	0.995632	4.14191	58.17681
0.049673	3877.774	3.24E-06	1	0.99251	4.336934	63.95153
0.049639	4424.086	3.69E-06	1	0.999382	4.978717	84.39699
0.050067	2557.096	2.15E-06	1	0.952238	2.765606	25.59772
0.050007	2694.003	2.26E-06	1	0.989193	3.023076	30.6602
0.049915	2934.602	2.46E-06	1	0.9324	3.098314	32.32358
0.049808	3592.804	3.01E-06	1	0.988723	4.013728	54.47943
0.049672	4383.629	3.66E-06	1	0.998521	4.932203	82.71761
0.050378	2456.461	2.08E-06	0.984808	0.999995	2.76468	25.2659
0.050177	2945.202	2.48E-06	0.984808	0.989246	3.266029	35.54334
0.04998	3293.846	2.77E-06	1	0.994822	3.715233	46.35687
0.049822	3736.118	3.13E-06	1	0.999568	4.220842	60.21192
0.050841	3674.652	3.14E-06	1	0.993284	4.209661	57.51688
0.050157	3429.014	2.89E-06	1	0.999667	3.900285	50.73013
0.050169	3479.172	2.93E-06	1	0.959423	3.798959	48.10478
0.049974	4044.374	3.4E-06	1	0.997756	4.574715	70.30205
0.049921	4818.76	4.04E-06	1	0.962196	5.250822	92.8142
0.049782	4713.807	3.94E-06	1	0.971695	5.172655	90.57741

additional rows for data, count, sum and avg not shown

Table B-22.4 shows the calculations for flow weighted averages.

Table B-22.4 Calculations for Flow Weighted Averages

$p_{vj}cos^2\psi_j cos^2\varphi_j$	$V_j cos\psi_j cos\varphi_j$	$p_{sj}V_j cos\psi_j cos\varphi_j$	$\rho_j V_j cos\psi_j cos\varphi_j$	$T_{sj}\rho_j V_j cos\psi_j cos\varphi_j$	$\rho_j V^3_j cos^3\psi_j cos^3\varphi_j$
0.530312787	3581.966498	-60526.96354	178.1655919	136789.0011	2285950775
0.622961838	3886.197538	-65690.30586	192.9078169	148404.812	2913396343
0.772318227	4330.311737	-74101.02879	214.6299761	165274.7977	4024655404
0.929616323	4753.288952	-81730.04705	235.3546484	181379.7404	5317545466
0.477960275	3381.533029	-58082.80831	170.0949515	129041.2228	1944995905
0.653522923	3964.579005	-67230.06743	198.3704616	151376.6914	3117964439
0.648006503	3960.594535	-65903.37754	196.8938883	151349.5403	3088538387
0.743516619	4244.327449	-70199.17567	210.8118663	162234.2974	3797631468
0.931427401	4752.110349	-78160.24252	235.8716517	181687.1065	5326584022
0.46253818	3339.225766	-57770.13676	166.6921159	127385.6566	1858688555
0.400567571	3107.96094	-53147.95857	155.1006042	118624.9673	1498182065
0.476057108	3391.590134	-58139.41865	168.9152832	129436.5357	1943012847
0.556944404	3670.85075	-62827.86903	182.5821735	140104.026	2460321304
0.611426874	3848.728301	-66109.75508	191.1791493	146869.4436	2831881208
0.806337943	4421.353748	-73775.99556	219.4699616	168936.5032	4290279787
0.2466761	2434.963135	-40351.11026	121.9124162	93065.78956	722824258.8
0.295102427	2664.889883	-44620.05835	133.2621355	101808.1986	946379457.3
0.310542701	2736.223049	-45712.2739	136.5787345	104543.5184	1022553591
0.522276371	3552.287025	-59034.46185	176.9316727	135753.9632	2232655326
0.790817265	4377.143423	-73439.69417	217.4195639	167207.416	4165625032
0.244990046	2419.129836	-40188.22437	121.871601	92450.75473	713215662.7
0.343269623	2869.26649	-47937.0356	143.9719019	109626.5328	1185276064
0.445942987	3276.791661	-55482.37165	163.7735504	125123.7775	1758496157
0.577399625	3734.504688	-62992.36668	186.0615968	142625.2978	2594912761
0.562834828	3649.971286	-61810.02352	185.5687193	139372.6893	2472200365
0.489739876	3427.873246	-57732.23152	171.9309122	130923.4014	2020242375
0.464509851	3337.996485	-56418.04192	167.464327	127470.8645	1865924463
0.676214173	4035.299531	-69514.44435	201.6609213	153969.2497	3283774310
0.891806578	4636.593196	-77617.37442	231.4647882	177135.8367	4976029199
0.867879988	4580.382335	-76971.70188	228.019085	174959.0506	4783818133

additional rows for data, count, sum and avg not shown

Table B-22.5 shows some of the intermediate calculations for determining uncertainties.

Table B-22.5 Calculations for Uncertainties

$(m_j/m_x)^2$	$(p_{sj}/p_{sx})^2$	$(\rho_j/\rho_x)^2$	$(T_{sj}/T_{sx})^2$	$(e_{Kj}/e_{Kx})^2$	$(p_{vj}cos^2\psi_j cos^2\varphi_j/p_{vx})^2$	$(p_{sj}/p_{tx})^2$	$(p_{vj}cos^2\psi_j cos^2\varphi_j/p_{tx})^2$	$\tan^2\psi_j$	$\tan^2\varphi_j$
0.000155	1.004019	0.992154	1.00773	0.500954	0.49654023	1.10000892	0.001083442	0	0.025287
0.000182	1.004709	0.988154	1.01178	0.694083	0.685193333	1.10076419	0.00149508	0	0.087993
0.000225	1.029669	0.985183	1.01373	1.070012	1.053132003	1.12811045	0.002297915	0	0.051044
0.00027	1.039591	0.983177	1.015364	1.553419	1.525800399	1.13898115	0.00332927	0	0.067941
0.000141	1.037422	1.014683	0.983929	0.397893	0.403342371	1.13660462	0.000880086	0	0.252453
0.000192	1.011161	1.004002	0.995528	0.751796	0.7540704	1.10783329	0.001645368	0.031091	0.025267
0.000189	0.973603	0.991102	1.010154	0.748779	0.741393836	1.06668466	0.001617708	0.031091	0.079337
0.000217	0.961908	0.989343	1.012476	0.987523	0.976048958	1.05387167	0.002129722	0	0.020626
0.000272	0.951231	0.987991	1.014347	1.551879	1.531751316	1.042174	0.003342254	0	0.034315
0.000136	1.052453	0.99934	0.998391	0.378351	0.377733424	1.15307272	0.000824208	0	0.010585
0.000117	1.028275	0.998735	1.000034	0.283932	0.283296895	1.1265839	0.000618149	0	0.001497
0.000139	1.033289	0.994731	1.003842	0.402648	0.400136666	1.13207738	0.000873091	0.04518	0.01336
0.000163	1.030051	0.992106	1.00664	0.552559	0.547663669	1.12852887	0.001194992	0.031091	0.008794
0.000178	1.03749	0.989512	1.008954	0.667699	0.660053728	1.13667958	0.001440226	0	0.01515
0.000235	0.979055	0.98813	1.012946	1.162875	1.147953866	1.07265757	0.002504815	0	0.001237
7.26E-05	0.965636	1.005277	0.996263	0.106975	0.107434654	1.05795615	0.000234421	0	0.102832
8.67E-05	0.985801	1.002835	0.997796	0.153472	0.153757321	1.08004836	0.000335496	0	0.021969
9.11E-05	0.981409	0.999166	1.001654	0.170576	0.170267945	1.07523682	0.000371522	0	0.150259
0.000153	0.971141	0.994878	1.00643	0.484556	0.481605012	1.06398765	0.001050854	0	0.022942
0.000231	0.989846	0.989442	1.011122	1.117056	1.104186737	1.08448022	0.002409316	0	0.002965
7.25E-05	0.970435	1.017797	0.983797	0.104219	0.105971021	1.06321416	0.000231227	0.031091	9.71E-06
0.000101	0.981495	1.009691	0.99121	0.206251	0.208046775	1.0753307	0.000453954	0.031091	0.02186
0.000131	1.008091	1.001762	0.997888	0.350838	0.351114625	1.1044698	0.000766126	0	0.010437
0.000169	1.000454	0.995456	1.004545	0.591894	0.588631145	1.09610284	0.001284383	0	0.000864
0.000168	1.008385	1.036586	0.964352	0.540094	0.559309465	1.10479167	0.001220403	0	0.01357
0.000144	0.997412	1.008865	0.991325	0.420156	0.423468563	1.09276972	0.000924001	0	0.000665
0.000137	1.004504	1.009363	0.990528	0.377794	0.380960632	1.10053976	0.000831249	0	0.086375
0.000199	1.043484	1.001536	0.996585	0.806891	0.807344293	1.14324689	0.001761611	0	0.004502
0.000262	0.985388	0.999416	1.001227	1.406397	1.404208459	1.07959628	0.003063958	0	0.080121
0.000254	0.992993	0.993833	1.006515	1.339426	1.329871231	1.08792808	0.002901756	0	0.059108

additional rows for data, count, sum and avg not shown

Table B-22.6 shows the remainder of the calculations needed to determine systematic uncertainties for each point.

Table B-22.6 Systematic Uncertainties

$(U_{psj})^2$	$(u_{psaj})^2$	$(u_{Tsj})^2$	*tan* term*	Σm_j	$\Sigma p_{sj}\ldots$	$\Sigma \rho_j\ldots$	$\Sigma T_{sj}\ldots$	$\Sigma eK_j\ldots$	$\Sigma p_{vj}\ldots$	$\Sigma p_{tj\ldots}$
0.034549	3.59E-07	6.79E-06	3.08071E-05	9.89E-09	0.000121	1.11E-05	6.84E-06	0.000128	0.000121	0.000133
0.034573	3.59E-07	6.76E-06	0.0001072	2.55E-08	0.000122	1.1E-05	6.84E-06	0.000389	0.000377	0.000134
0.035432	3.65E-07	6.75E-06	6.2187E-05	2.14E-08	0.000125	1.09E-05	6.84E-06	0.000408	0.000389	0.000137
0.035773	3.68E-07	6.73E-06	8.27713E-05	3.13E-08	0.000126	1.09E-05	6.84E-06	0.000720	0.00069	0.000139
0.035699	3.67E-07	6.95E-06	0.00030756	4.81E-08	0.000126	1.15E-05	6.84E-06	0.000542	0.000545	0.000139
0.034795	3.61E-07	6.87E-06	6.86608E-05	1.95E-08	0.000122	1.13E-05	6.84E-06	0.000306	0.000298	0.000135
0.033503	3.51E-07	6.77E-06	0.000134534	3.17E-08	0.000118	1.1E-05	6.84E-06	0.000502	0.000489	0.00013
0.0331	3.48E-07	6.75E-06	2.51288E-05	1.26E-08	0.000116	1.1E-05	6.84E-06	0.000230	0.000216	0.000128
0.032733	3.46E-07	6.74E-06	4.18052E-05	2.03E-08	0.000115	1.1E-05	6.84E-06	0.000464	0.000441	0.000127
0.036216	3.71E-07	6.85E-06	1.28961E-05	6.23E-09	0.000127	1.12E-05	6.84E-06	0.000070	6.52E-05	0.00014
0.035384	3.65E-07	6.84E-06	1.82418E-06	4.1E-09	0.000124	1.12E-05	6.84E-06	0.000040	3.63E-05	0.000136
0.035557	3.66E-07	6.81E-06	7.13193E-05	1.45E-08	0.000125	1.11E-05	6.84E-06	0.000168	0.000163	0.000137
0.035445	3.65E-07	6.79E-06	4.85912E-05	1.33E-08	0.000125	1.11E-05	6.84E-06	0.000180	0.000173	0.000137
0.035701	3.67E-07	6.78E-06	1.84577E-05	9.19E-09	0.000126	1.1E-05	6.84E-06	0.000138	0.000129	0.000138
0.03369	3.53E-07	6.75E-06	1.50647E-06	8.12E-09	0.000118	1.1E-05	6.84E-06	0.000161	0.000146	0.00013
0.033229	3.49E-07	6.86E-06	0.000125279	1.15E-08	0.000117	1.13E-05	6.84E-06	0.000068	6.68E-05	0.000128
0.033922	3.54E-07	6.85E-06	2.67641E-05	5.19E-09	0.000119	1.12E-05	6.84E-06	0.000037	3.51E-05	0.000131
0.033771	3.53E-07	6.83E-06	0.000183059	1.97E-08	0.000119	1.12E-05	6.84E-06	0.000147	0.000145	0.00013
0.033418	3.51E-07	6.79E-06	2.79501E-05	9.32E-09	0.000118	1.11E-05	6.84E-06	0.000118	0.000112	0.000129
0.034062	3.55E-07	6.76E-06	3.61273E-06	8.46E-09	0.00012	1.1E-05	6.84E-06	0.000164	0.00015	0.000132
0.033394	3.5E-07	6.95E-06	3.78899E-05	5.15E-09	0.000117	1.15E-05	6.84E-06	0.000030	2.89E-05	0.000129
0.033774	3.53E-07	6.9E-06	6.451E-05	9.87E-09	0.000119	1.14E-05	6.84E-06	0.000080	7.89E-05	0.00013
0.034689	3.6E-07	6.85E-06	1.27148E-05	5.99E-09	0.000122	1.12E-05	6.84E-06	0.000064	6.03E-05	0.000134
0.034427	3.58E-07	6.81E-06	1.05301E-06	5.76E-09	0.000121	1.11E-05	6.84E-06	0.000081	7.37E-05	0.000133
0.0347	3.6E-07	7.09E-06	1.65316E-05	8.35E-09	0.000122	1.19E-05	6.84E-06	0.000107	0.000105	0.000134
0.034322	3.57E-07	6.9E-06	8.10724E-07	4.89E-09	0.000121	1.14E-05	6.84E-06	0.000057	5.26E-05	0.000132
0.034566	3.59E-07	6.9E-06	0.00010523	1.89E-08	0.000122	1.14E-05	6.84E-06	0.000209	0.000206	0.000134
0.035907	3.69E-07	6.86E-06	5.48528E-06	7.65E-09	0.000126	1.12E-05	6.84E-06	0.000124	0.000115	0.000139
0.033908	3.54E-07	6.83E-06	9.7611E-05	3.42E-08	0.000119	1.12E-05	6.84E-06	0.000735	0.000718	0.000132
0.03417	3.56E-07	6.79E-06	7.20102E-05	2.67E-08	0.00012	1.11E-05	6.84E-06	0.000563	0.000544	0.000133

additional rows for data, count, sum and avg not shown

*tan term = $(\tan^2 \psi B_\psi^2 + \tan^2 \varphi B_\varphi^2)/C_6$

Table B-22.7 shows the remainder of the calculations needed to determine random uncertainties for each point.

Table B-22.7 Random Uncertainties

$(U_{psj})^2$	$(u_{psaj})^2$	$(u_{Tsj})^2$	*tan* term*	Σm_j	Σp_{sj}...	$\Sigma \rho_j$...	ΣT_{sj}...	ΣeK_j...	Σp_{vj}...	$\Sigma p_{tj...}$
3.8669E-06	3.5269E-09	3.5626E-07	1.5129E-05	4.4866E-09	3.9441E-06	3.5978E-07	3.5901E-07	5.802E-05	5.733E-05	4.4463E-06
3.8358E-06	3.5268E-09	3.5483E-07	8.0611E-05	1.7155E-08	3.9468E-06	3.5836E-07	3.5901E-07	0.00026219	0.00025858	4.8884E-06
3.8128E-06	3.5305E-09	3.5415E-07	3.8118E-05	1.168E-08	4.0449E-06	3.5768E-07	3.5901E-07	0.00022232	0.00021843	4.9082E-06
3.7973E-06	3.5319E-09	3.5358E-07	2.545E-06	4.4262E-09	4.0839E-06	3.5711E-07	3.5901E-07	0.00010171	9.9361E-05	4.6911E-06
4.0446E-06	3.5332E-09	3.6488E-07	0.00036731	5.3829E-08	4.0754E-06	3.6841E-07	3.5901E-07	0.0006066	0.00061476	5.8064E-06
3.9599E-06	3.5286E-09	3.6062E-07	2.2515E-05	6.9808E-09	3.9722E-06	3.6415E-07	3.5901E-07	0.00010928	0.00010934	4.5905E-06
3.8587E-06	3.5221E-09	3.554E-07	8.9522E-05	1.9558E-08	3.8247E-06	3.5893E-07	3.5901E-07	0.00030953	0.00030622	4.8585E-06
3.8451E-06	3.5202E-09	3.5459E-07	1.1675E-05	5.532E-09	3.7787E-06	3.5811E-07	3.5901E-07	0.00010	9.9206E-05	4.3564E-06
3.8346E-06	3.5184E-09	3.5393E-07	2.2475E-05	9.8583E-09	3.7368E-06	3.5745E-07	3.5901E-07	0.00022533	0.00022186	4.5781E-06
3.9232E-06	3.5347E-09	3.5959E-07	1.7603E-06	2.1141E-09	4.1344E-06	3.6313E-07	3.5901E-07	2.3588E-05	2.3413E-05	4.5808E-06
3.9184E-06	3.531E-09	3.59E-07	3.7064E-07	1.6671E-09	4.0394E-06	3.6253E-07	3.5901E-07	1.6123E-05	1.5984E-05	4.4605E-06
3.8871E-06	3.5315E-09	3.5764E-07	3.4387E-05	6.7151E-09	4.0591E-06	3.6117E-07	3.5901E-07	7.765E-05	7.7021E-05	4.6153E-06
3.8666E-06	3.5309E-09	3.5664E-07	2.1312E-05	5.718E-09	4.0464E-06	3.6017E-07	3.5901E-07	7.7662E-05	7.6776E-05	4.6008E-06
3.8464E-06	3.5319E-09	3.5583E-07	2.185E-06	2.8565E-09	4.0756E-06	3.5936E-07	3.5901E-07	4.2759E-05	4.2033E-05	4.557E-06
3.8356E-06	3.5228E-09	3.5442E-07	4.4706E-07	3.3557E-09	3.8461E-06	3.5795E-07	3.5901E-07	6.6384E-05	6.5122E-05	4.3559E-06
3.9699E-06	3.5216E-09	3.6036E-07	1.3785E-06	1.1031E-09	3.7934E-06	3.6388E-07	3.5901E-07	6.506E-06	6.4949E-06	4.1702E-06
3.9507E-06	3.5246E-09	3.5981E-07	2.6161E-06	1.4254E-09	3.8726E-06	3.6333E-07	3.5901E-07	1.0094E-05	1.0056E-05	4.2647E-06
3.9218E-06	3.5238E-09	3.5842E-07	1.5562E-07	1.2731E-09	3.8553E-06	3.6194E-07	3.5901E-07	9.5394E-06	9.4606E-06	4.2445E-06
3.8882E-06	3.5219E-09	3.5672E-07	2.6618E-06	2.5195E-09	3.815E-06	3.6024E-07	3.5901E-07	3.1955E-05	3.1587E-05	4.2486E-06
3.8458E-06	3.5246E-09	3.5506E-07	6.7027E-07	3.3448E-09	3.8885E-06	3.5859E-07	3.5901E-07	6.4767E-05	6.3625E-05	4.399E-06
4.0694E-06	3.523E-09	3.6492E-07	1.976E-05	2.4352E-09	3.8122E-06	3.6845E-07	3.5901E-07	1.4002E-05	1.4198E-05	4.2077E-06
4.0049E-06	3.5243E-09	3.622E-07	3.2323E-05	4.6696E-09	3.8557E-06	3.6572E-07	3.5901E-07	3.8074E-05	3.8329E-05	4.3079E-06
3.9422E-06	3.528E-09	3.5977E-07	5.0405E-06	2.4702E-09	3.9601E-06	3.633E-07	3.5901E-07	2.6476E-05	2.6369E-05	4.3963E-06
3.8927E-06	3.5265E-09	3.5739E-07	3.0413E-07	2.3878E-09	3.9301E-06	3.6091E-07	3.5901E-07	3.3453E-05	3.3056E-05	4.378E-06
4.221E-06	3.53E-09	3.7228E-07	2.0524E-06	2.6697E-09	3.9613E-06	3.7581E-07	3.5901E-07	3.431E-05	3.532E-05	4.4171E-06
3.9983E-06	3.5268E-09	3.6215E-07	2.3018E-07	2.0284E-09	3.9182E-06	3.6568E-07	3.5901E-07	2.3624E-05	2.3655E-05	4.3444E-06
4.0023E-06	3.5279E-09	3.6244E-07	1.9477E-06	2.1595E-09	3.946E-06	3.6597E-07	3.5901E-07	2.3838E-05	2.3898E-05	4.3754E-06
3.9404E-06	3.5334E-09	3.6024E-07	1.8792E-06	3.1178E-09	4.0992E-06	3.6378E-07	3.5901E-07	5.069E-05	5.0424E-05	4.6011E-06
3.9238E-06	3.5244E-09	3.5857E-07	2.1609E-06	4.181E-09	3.8709E-06	3.621E-07	3.5901E-07	8.9933E-05	8.9285E-05	4.4358E-06
3.88E-06	3.5253E-09	3.5669E-07	4.6509E-05	1.5313E-08	3.9008E-06	3.6021E-07	3.5901E-07	0.00032325	0.00032047	4.973E-06

additional rows for data, count, sum and avg not shown

**tan* term = $(\tan^2\psi S_\psi^{\,2}+\tan^2\varphi S_\varphi^{\,2})/C_6$

Tables B-22.8 shows the calculations needed to determine some of the distortion parameters.

Eqs. 7-3.3	Eqs. 7-3.2
$U_{\psi j}$	$U_{\varphi j}$
1	1.40157687
1	1.73432088
1	1.56582696
1	0.35069732
1	2.18564809
1.44444444	0.59857948
1.44444444	1.69915058
1	1.36323778
1	1.46642833
1	0.7389232
1	0.90151233
1.53333333	0.70696204
1.44444444	0.7619029
1	0.68812966
1	1.0895088
1	0.20979083
1	0.62529417
1	0.05831414
1	0.61720334
1	0.86146689
1.44444444	1.00793684
1.44444444	1.37379254
1	1.25924754
1	1.07484374
1	0.70469584
1	1.06567575
1	0.27209303
1	1.17061361
1	0.29757612
1	1.60731848

Table B-22.8 Distortion				
$(V_j \cos\psi_j \cos\varphi_{j\_}$ 1A-$Vavg)^2$	$V_j \cos\psi_j \cos\varphi_j y_j$	$V_j \cos\psi_j \cos\varphi_j x_j$	abs ψj	abs φj
125231.7768	3581.966498	1343.237437	0.00	9.04
5287.885769	3886.197538	4371.97223	0.00	16.52
210315.7576	4330.311737	8119.334507	0.00	12.73
867231.4042	4753.288952	12477.3835	0.00	14.61
38603.4863	3381.533029	11412.67397	0.00	26.68
9106.418143	3964.579005	16353.8884	10.00	9.03
38939.26168	3960.594535	19307.89836	10.00	15.73
94262.99038	4244.327449	23874.3419	0.00	8.17
726078.9023	14256.33105	1782.041381	0.00	10.49
389377.0531	10017.6773	3756.628986	0.00	5.87
757886.4019	9323.88282	5827.426762	0.00	2.22
240499.8429	10174.7704	8902.924102	12.00	6.59
56178.97295	11012.55225	12389.12128	10.00	5.36
10625.46395	11546.1849	15876.00424	0.00	7.02
196455.4036	13264.06124	21554.09952	0.00	2.01
2027085.23	7304.889406	13696.66764	0.00	17.78
1655985.111	13324.44942	999.3337063	0.00	8.43
1094641.754	13681.11524	3078.25093	0.00	21.19
150582.1561	17761.43513	6660.538172	0.00	8.61
162227.7213	21885.71711	11490.00148	0.00	3.12
2323773.579	12095.64918	8164.563195	10.00	0.18
1072573.546	14346.33245	11835.72427	10.00	8.41
471978.7737	16383.95831	15974.35935	0.00	5.83
59895.38325	18672.52344	21006.58887	0.00	1.68
93759.39729	25549.799	1368.739232	0.00	6.64
304527.37	23995.11272	3856.357402	0.00	1.48
251684.8097	23365.97539	6258.743409	0.00	16.38
4034.800601	28247.09672	10592.66127	0.00	3.84
702086.3237	32456.15237	15648.50204	0.00	15.80
537220.1955	32062.67634	18894.07713	0.00	13.66

additional rows for data, count, sum and avg not shown

Table B-23 shows the two-dimensional calculations needed for the remainder of the distortion parameters.

Table B-23A Transverse & Axial Distortion Parameter Calculation

	Ports										
Points	1	2	3	4	5	6	7	8	9		$(V_x - V_{mean})^2$
1	3581.97	4752.11	2664.89	3649.97	3991.81	3494.18	5007.30	4427.65	5231.38		11702.00729
2	3886.20	3339.23	2736.22	3427.87	4647.00	2941.13	3849.97	3421.08	4521.59		115402.4387
3	4330.31	3107.96	3552.29	3338.00	4879.85	4651.05	3179.94	5303.46	4710.06		18533.33994
4	4753.29	3391.59	4377.14	4035.30	976.93	4651.68	3841.73	5259.02	4729.11		436.9117501
5	3381.53	3670.85	2419.13	4636.59	3583.51	5051.32	4376.89	4286.42	4715.74		1069.359797
6	3964.58	3848.73	2869.27	4580.38	4820.64	3451.63	4819.87	4613.29	4675.09		40704.45597
7	3960.59	4421.35	3276.79	2735.75	5107.98	3750.60	503.91	4737.09	4163.09		124100.8265
8	4244.33	2434.96	3734.50	3802.32	5189.45	4542.07	3963.71	5306.53	4340.69		36986.53447
$(V_z - V_{mean})^2$	1023.759575	129604.1	603844.1	42058.09234	28491.58	7370.9	82908.781	473982.5	429013.1		

Table B-23B Shear Parameter Calculation

	Ports								
Points	1	2	3	4	5	6	7	8	9
1		1372304	4356510	973507.0556	117523.5	247282.6	2299116.6	335704.2	650610.1
2		298269.5	362603.2	479534.5119	1491683	2914645	829799.3	183038.6	1216279
3		1492587	198475.2	45488.97663	2383966	52028.92	2168063.2	4530977	352307.6
4		1852568	974231.7	116398.9957	9371168	13544574	656107.4	2018820	281049.8
5		84316.93	1566672	4931274.436	1109735	2161656	454571.8	8021.419	185701.5
6		13196.3	958964.8	2935164.098	58434.27	1876404	1880921.1	42087.83	3920.302
7		213397.2	1309285	292540.0442	5644300	1844641	10568173.5	17985099	329292.7
8		3271890	1692085	4745.089395	1930987	418869.7	334055.4	1812439	934552.7

Table B-23C Shear Parameter Calculation

	Ports								
Points	1	2	3	4	5	6	7	8	9
1									
2	92575.98571	1997506	5144.967	49484.83871	430435.4	306785.5	1343820.7	1015385	506113.8
3	197299.6811	53541.3	666314.8	8007.509115	54373.1	2932023	450311.69	3554683	35713.92
4	179063.8921	80532.98	681135.8	486391.8984	15271121	0.319998	439563.13	2003.607	347.9021
5	1882660.554	78044.13	3838997	362987.567	6813568	160194.4	287326.11	949428.6	177.2623
6	340144.4138	31673.35	202833.6	3254.105928	1532958	2566242	196902.3	107679.8	1613.851
7	16.42675795	328067.2	166300.2	3408833.023	82793.85	89630.05	18687955	15148.84	263550.8
8	80535.10228	3948165	209803	1140190.114	6662.311	628210.1	12009652	325902.6	31682.75

Table B-23AF Transverse & Axial Distortion Parameter Calculation

	Ports	V_{a1}	V_{a2}	V_{a3}	*	V_x-V_{mean}
Points		1	2	3		$(V_x\text{-}V_{mean})^2$
V_{t1}	1	=INDEX(Vjcosψjcosφj_1A, 1, 1)	=INDEX(Vjcosψjcosφj_1A, 9, 1)	=INDEX(Vjcosψjcosφj_1A, 17, 1)		=(AVERAGE(Vt1)-Vavg)^2
V_{t2}	2	=INDEX(Vjcosψjcosφj_1A, 2, 1)	=INDEX(Vjcosψjcosφj_1A, 10, 1)	=INDEX(Vjcosψjcosφj_1A, 18, 1)		=(AVERAGE(Vt2)-Vavg)^2
V_{t3}	3	=INDEX(Vjcosψjcosφj_1A, 3, 1)	=INDEX(Vjcosψjcosφj_1A, 11, 1)	=INDEX(Vjcosψjcosφj_1A, 19, 1)		=(AVERAGE(Vt3)-Vavg)^2
V_{t4}	4	=INDEX(Vjcosψjcosφj_1A, 4, 1)	=INDEX(Vjcosψjcosφj_1A, 12, 1)	=INDEX(Vjcosψjcosφj_1A, 20, 1)		=(AVERAGE(Vt4)-Vavg)^2
V_{t5}	5	=INDEX(Vjcosψjcosφj_1A, 5, 1)	=INDEX(Vjcosψjcosφj_1A, 13, 1)	=INDEX(Vjcosψjcosφj_1A, 21, 1)		=(AVERAGE(Vt5)-Vavg)^2
V_{t6}	6	=INDEX(Vjcosψjcosφj_1A, 6, 1)	=INDEX(Vjcosψjcosφj_1A, 14, 1)	=INDEX(Vjcosψjcosφj_1A, 22, 1)		=(AVERAGE(Vt6)-Vavg)^2
V_{t7}	7	=INDEX(Vjcosψjcosφj_1A, 7, 1)	=INDEX(Vjcosψjcosφj_1A, 15, 1)	=INDEX(Vjcosψjcosφj_1A, 23, 1)		=(AVERAGE(Vt7)-Vavg)^2
V_{t8}	8	=INDEX(Vjcosψjcosφj_1A, 8, 1)	=INDEX(Vjcosψjcosφj_1A, 16, 1)	=INDEX(Vjcosψjcosφj_1A, 24, 1)		=(AVERAGE(Vt8)-Vavg)^2
V_z-V_{mean}	$(V_z\text{-}V_{mean})^2$	=(AVERAGE(Va1)-Vavg)^2	=(AVERAGE(Va2)-Vavg)^2	=(AVERAGE(Va3)-Vavg)^2		

Table B-23BF Shear Parameter Calculation

	Ports	V_{a1}	V_{a2}	V_{a3}	*
Points		1	2	3	
V_{t1}	1		=(Va2-Va1)^2	=(Va3-Va2)^2	
V_{t2}	2		=(Va2-Va1)^2	=(Va3-Va2)^2	
V_{t3}	3		=(Va2-Va1)^2	=(Va3-Va2)^2	
V_{t4}	4		=(Va2-Va1)^2	=(Va3-Va2)^2	
V_{t5}	5		=(Va2-Va1)^2	=(Va3-Va2)^2	
V_{t6}	6		=(Va2-Va1)^2	=(Va3-Va2)^2	
V_{t7}	7		=(Va2-Va1)^2	=(Va3-Va2)^2	
V_{t8}	8		=(Va2-Va1)^2	=(Va3-Va2)^2	

Table B-23CF Shear Parameter Calculation

	Ports	V_{a1}	V_{a2}	V_{a3}	*
Points		1	2	3	
V_{t1}	1				
V_{t2}	2	=(Vt2-Vt1)^2	=(Vt2-Vt1)^2	=(Vt2-Vt1)^2	
V_{t3}	3	=(Vt3-Vt2)^2	=(Vt3-Vt2)^2	=(Vt3-Vt2)^2	
V_{t4}	4	=(Vt4-Vt3)^2	=(Vt4-Vt3)^2	=(Vt4-Vt3)^2	
V_{t5}	5	=(Vt5-Vt4)^2	=(Vt5-Vt4)^2	=(Vt5-Vt4)^2	
V_{t6}	6	=(Vt6-Vt5)^2	=(Vt6-Vt5)^2	=(Vt6-Vt5)^2	
V_{t7}	7	=(Vt7-Vt6)^2	=(Vt7-Vt6)^2	=(Vt7-Vt6)^2	
V_{t8}	8	=(Vt8-Vt7)^2	=(Vt8-Vt7)^2	=(Vt8-Vt7)^2	

Table B-24 shows distortion results.

Table B-24 Inlet Flow Distortion Results			
Plane 1A		Plane 1B	
V_r	0.2305	Vr	0.2350
V_t	0.1123	V_t	0.1200
V_a	0.0525	V_a	0.0800
V_s	693.5	V_s	589.2
ε_t	0.3408	ε_t	0.0329
ε_a	-0.1404	ε_a	0.1563
ψ_{mean}	5.65	ψ_{mean}	4.32
φ_{mean}	8.41	φ_{mean}	12.01

Table B-24F Inlet Flow Distortion		
Plane 1A		
Variable	Formula	Name
V_r	=(sum_Vj_Vavg_2/n1A)^0.5/Vavg	Vr_1A
V_t	=SQRT(SUM(Vz_V)/Nports)/Vavg	Vt_1A
V_a	=SQRT(SUM(Vx_V)/Npoints)/Vavg	Va_1A
V_s	=(SUM(Shear1)^2+SUM(Shear2)^2)^0.5/(Nports-1)/(Npoints-1)/Vavg	Vs_1A
ε_t	=2*((sum_Vjcosψjcosφjyj/sum_Vjcosψjcosφj)/height1A-0.5)	εa_1A
ε_a	=2*((sum_Vjcosψjcosφjxj/sum_Vjcosψjcosφj)/width1A-0.5)	σt_1A
ψ_{mean}	=avg_abs_ψ_1A	ψmean
φ_{mean}	=avg_abs_φ_1A	φmean

Tables B-22.1 F through B22.9 F show the formulas for the traverse calculations in Table B-22. Note that the format has been transposed for easier printing. Equations 5-2-2, 5-2-7, and 5-2-8 in this Code result in a circular calculation of the probe calibration coefficient, $K_{vj.}$

Table B-22.1 F Traverse Measurements and Probe Calibration Calculations at Plane 1A

Variable	Formula/Value	Name
rdg	1	
Port	1	
Point	0.375	
x_j	1	xj_1A
y_j	0.7	yj_1A
L_p	'=zj_1A+Lhead	Lp_1A
S_p	=Lp_1A*diameter1A	Sp_1A
t_{di}	308.3	tdi_1A
p_{vi}	0.51	pvi_1A
p_{si}	-16.45	psi_1A
ψ	0	ψ_1A
Δp_{φ}	-0.05	Δpφ_1A
C_{φ}	=IF(pvi_1A=0, 0, Δpφ_1A/pvi_1A)	Cφ_1A
φ	-7.016794552	φ_1A
K_{vj}	=1.09188+0.28085*Cφ_1A+0.46032*Cφ_1A^2+0.68812*Cφ_1A^3-0.37163*Cφ_1A^4+0.673*Cφ_1A^5	Kvj_1A
K_{tj}	=1.02618+0.00214*Cφ_1A-0.00832*Cφ_1A^2-0.01503*Cφ_1A^3+0.03617*Cφ_1A^4-0.00307*Cφ_1A^5	Ktj_1A

Table B-22.2 F Corrections for Probe Calibration and Blockage Plane 1A

Variable	Formula	Name
p_{ti}	=psi_1A+pvi_1A	pti_1A
p_{sai}	=psi_1A+pb*Const13	psai_1A
β_j	=CD*Sp_1A/A1A*_1_εp_1A/(4*_1_εp_1A-3)	βj_1A
K_{vjc}	=Kvj_1A/(1+βj_1A*Kvj_1A)	Kvjc_1A
$(1-\varepsilon_p)$	=1-1/(2*K_1A)*Kvjc_1A*pvi_1A/psaj_1A	_1_εp_1A
$(1+\varepsilon_T)$	=1+0.85*(K_1A-1)/K_1A*Kvjc_1A*pvi_1A/psaj_1A	_1_εT_1A
T_i	=tdi_1A+Const1	Ti_1A
T_{sj}	=Ti_1A/_1_εT_1A	Tsj_1A
p_{tj}	=Ktj_1A*pti_1A	ptj_1A
p_{sj}	=ptj_1A-Kvj_1A*pvi_1A	psj_1A
p_{vj}	=Kvjc_1A*_1_εp_1A*pvi_1A	pvj_1A
p_{saj}	=psj_1A+Const13*pb	psaj_1A
$K_{vjc}p_{vj}/p_{saj}$	=Kvjc_1A*pvj_1A/psai_1A	Kvjcpvj_psaj_1A

Table B-22.3 F Calculating Point Values Plane 1A		
Variable	**Formula**	**Name**
ρ_j	=Const11*psaj_1A/Rdg_1A/Tsj_1A	ρj_1A
V_j	=Const12*(pvj_1A/ρj_1A)^0.5	Vj_1A
Re_{pj}	=ρj_1A*Vj_1A*diameter1A*μma/Const2	Repj_1A
$\cos\psi_j$	=COS(ABS(ψ_1A)*PI()/180)	cos_ψj_1A
$\cos\varphi_j$	=COS(ABS(φ_1A)*PI()/180)	cos_φj_1A
m_j	=ρj_1A*Vj_1A*cos_ψj_1A*cos_φj_1A*A1Ar/Const2/count_pvi_1A	mj_1A
e_{Kj}	=Vjcosψjcosφj_1A^2/2/gc/Const2^2	eKj_1A

Table B-22.4 F Calculations for Flow Weighted Averages Plane 1A		
Variable	**Formula**	**Name**
$p_{vj}\cos^2\psi_j\cos^2\varphi_j$	=pvj_1A*cos_ψj_1A^2*cos_φj_1A^2	pvjcos2ψjcos2φj_1A
$V_j\cos\psi_j\cos\varphi_j$	=Vj_1A*cos_ψj_1A*cos_φj_1A	Vjcosψjcosφj_1A
$p_{sj}V_j\cos\psi_j\cos\varphi_j$	=psj_1A*cos_ψj_1A*cos_φj_1A*Vj_1A	psjVjcosψjcosφj_1A
$\rho_j V_j\cos\psi_j\cos\varphi_j$	=ρj_1A*Vj_1A*cos_ψj_1A*cos_φj_1A	ρjVjcosψjcosφj
$T_{sj}\rho_j V_j\cos\psi_j\cos\varphi_j$	=ρjVjcosψjcosφj*Tsj_1A	TsjρjVjcosψjcosφj_1A
$\rho_j V^3_j\cos^3\psi_j\cos^3\varphi_j$	=ρj_1A*Vj_1A^3*cos_ψj_1A^3*cos_φj_1A^3	ρjV3jcos3ψjcos3φj_1A

Table B-22.5 F Calculations for Uncertainties Plane 1A		
Variable	**Formula**	**Name**
$(m_j/m_x)^2$	=(mj_1A/m1A)^2	mj_mx_2_1A
$(p_{sj}/p_{sx})^2$	=(psj_1A/ps1A)^2	psj_psx_2_1A
$(\rho_j/\rho_x)^2$	=(ρj_1A/ρ1A)^2	ρj_ρx_2_1A
$(T_{sj}/T_{sx})^2$	=(Tsj_1A/Ts1A)^2	Tsj_Tsx_2_1A
$(e_{Kj}/e_{Kx})^2$	=(eKj_1A/eK1A)^2	eKj_eKx_2_1A
$(p_{vj}\cos^2\psi_j\cos^2\varphi_j/p_{vx})^2$	=(pvjcos2ψjcos2φj_1A/pv1A)^2	pvjcos2ψjcos2φj_pvx_2
$(p_{sj}/p_{tx})^2$	=(psj_1A/pt1A)^2	psj_ptx_2_1A
$(p_{vj}\cos^2\psi_j\cos^2\varphi_j/p_{tx})^2$	=(pvjcos2ψjcos2φj_1A/pt1A)^2	pvjcos2ψjcos2φj_ptx_2_1A
$\tan^2\psi_j$	=TAN(ψ_1A*PI()/180)^2	tan2ψj_1A
$\tan^2\varphi_j$	=TAN(φ_1A*PI()/180)^2	tan2φj_1A

Table B-22.6 F Calculating Systematic Uncertainties Plane 1A		
Variable	**Formula**	**Name**
$(U_{psj})^2$	=(psj1Bx_X*psj_1A)^2	Upsj_2_1A
$(u_{psaj})^2$	=(Upsj_2_1A+Const13^2*pbBx^2)/psaj_1A^2	upsaj_2_1A
$(u_{Tsj})^2$	=(Tsj1Bx/Tsj_1A)^2	uTsj_2_1A
$\tan^2\psi\varphi$jU2	=tan2ψj_1A*ψj1Bx^2/57.3^2+tan2φj_1A*φj1Bx^2/57.3^2	tan2ψφjU2
Σm_j	=mj_mx_2_1A*0.25*(upsaj_2_1A+RBx_X^2+uTsj_2_1A+pvj1Bx_X^2+4*tan2ψφjU2)	Σmj_1A
Σp_{sj}.. .	=psj_psx_2_1A*psj1Bx_X^2	Σpsj_1A
$\Sigma\rho_j$...	=ρj_ρx_2_1A*(RBx_X^2+uTsj_2_1A+upsaj_2_1A)	Σρj_1A
ΣT_{sj}...	=Tsj_Tsx_2_1A*uTsj_2_1A	ΣTsj_1A
ΣeK_j...	=eKj_eKx_2_1A*(RBx_X^2+pvj1Bx_X^2+uTsj_2_1A+upsaj_2_1A+4*tan2ψφjU2)	ΣeKj_1A
Σp_{vj}...	=eKj_eKx_2_1A*(RBx_X^2+pvj1Bx_X^2+uTsj_2_1A+upsaj_2_1A+4*tan2ψφjU2)	Σpvj_1A
$\Sigma p_{tj...}$	=psj_ptx_2_1A*psj1Bx_X^2+pvjcos2ψjcos2φj_ptx_2_1A*(pvj1Bx_X^2+4*tan2ψφjU2)	Σptj_1A

Table B-22.7 F Calculating Random Uncertainties Plane 1A		
Variable	**Formula**	**Name**
$(U_{psj})^2$	=(psj1Sx_X*ρj_ρx_2_1A)^2	Upsj_2_S_1A
$(u_{psaj})^2$	=(Upsj_2_S_1A+Const13^2*pbSx^2)/psaj_1A^2	upsaj_2_S_1A
$(u_{Tsj})^2$	=(Tsj1Sx/Tsj_1A)^2	uTsj_2_S_1A
$\tan^2 \psi$ φjU2_S	=tan2ψj_1A*Uψj_S_1A^2/57.3^2+tan2φj_1A*Uφj_S_1A^2/57.3^2	tan2ψφjU2_S
Σm_j	=mj_mx_2_1A*0.25*(upsaj_2_S_1A+RSx_X^2+uTsj_2_S_1A+pvj1Sx_X^2+4*tan2ψφjU2_S)	Σmj_S_1A
Σp_{sj}. . .	=psj_psx_2_1A*psj1Sx_X^2	Σpsj_S_1A
$\Sigma \rho_j$...	=RSx_X^2+uTsj_2_S_1A+upsaj_2_S_1A	Σρj_S_1A
ΣT_{sj}...	=Tsj_Tsx_2_1A*uTsj_2_S_1A	ΣTsj_S_1A
ΣeK_j...	=eKj_eKx_2_1A*(RSx_X^2+pvj1Sx_X^2+uTsj_2_S_1A+upsaj_2_S_1A+4*tan2ψφjU2_S)	ΣeKj_S_1A
Σp_{vj}...	#NAME?	Σpvj_S_1A
$\Sigma p_{tj...}$	4.46049E-06	Σptj_S_1A

Equations 7-3.2 & 3.3 and Table B-22.8 F Distortion Plane 1A		
Variable	**Formula**	**Name**
$U_{\psi j}$	=1+2*ψ_1A/45	Uψj_S_1A
$U_{\varphi j}$	=1+2*φ_1A/45	Uφj_S_1A
$(V_j \cos\psi_j \cos\varphi_{j\_}$ 1A-$Vavg)2$	=(Vjcosψjcosφj_1A-Vavg)^2	$(V_j \cos\psi_j \cos\varphi_{j\_}$ 1A-$Vavg)2$
$V_j \cos\psi_j \cos\varphi_j y_j$	=Vjcosψjcosφj_1A*yj_1A	Vjcosψjcosφjyj_1A
$V_j \cos\psi_j \cos\varphi_j x_j$	=Vjcosψjcosφj_1A*xj_1A	Vjcosψjcosφjxj_1A
$abs\ \psi_j$	'=AVERAGE(Abs_ψj_1A)	avg_abs_ψ_1A
$abs\ \varphi_j$	'=AVERAGE(Abs_φj_1A)	avg_abs_φ_1A

Table B-22.9 F Average, Count and Sums of Traverse Calculations	
Formula	**Name**
=AVERAGE(tdi_1A)	avg_tdi
=COUNT(pvi_1A)	count_pvi_1A
=SUM(Vj_1A)	sum_Vj
=SUM(mj_1A)	sum_mj
=SUM(Vjcosψjcosφj_1A)	sum_Vjcosψjcosφj
=AVERAGE(Vjcosψjcosφj_1A)	Vavg
=SUM(psjVjcosψjcosφj_1A)	sum_psjVjcosψjcosφj
=SUM(ρjVjcosψjcosφj)	sum_ρjVjcosψjcosφj
=SUM(TsjρjVjcosψjcosφj_1A)	sum_TsjρjVjcosψjcosφj
=SUM(ρjV3jcos3ψjcos3φj_1A)	sum_ρjV3jcos3ψjcos3φj
=SUM(upsaj_2_1A)	sum_upsaj_2
=SUM(Σmj_1A)	sum_Σmj
=SUM(Σpsj_1A)	sum_Σpsj
=SUM(Σρj_1A)	sum_Σρj
=SUM(ΣTsj_1A)	sum_ΣTsj
=SUM(ΣeKj_1A)	sum_ΣeKj
=SUM(Σpvj_1A)	sum_Σpvj
=SUM(Σptj_1A)	sum_Σptj
=SUM(upsaj_2_S_1A)	sum_upsaj_2_S
=SUM(Σmj_S_1A)	sum_Σmj_S
=SUM(Σpsj_S_1A)	sum_Σpsj_S
=SUM(Σρj_S_1A)	sum_Σρj_S
=SUM(ΣTsj_S_1A)	sum_ΣTsj_S
=SUM(ΣeKj_S_1A)	sum_ΣeKj_S
=SUM(Σpvj_S_1A)	sum_Σpvj_S
=SUM(Σptj_S_1A)	sum_Σptj_S
=SUM(Vj_Vmean_2_1A)	sum_Vj_Vmean_2
=SUM(Vjcosψjcosφjyj_1A)	sum_Vjcosψjcosφjyj
=SUM(Vjcosψjcosφjxj_1A)	sum_Vjcosψjcosφjxj
=SUM(Vjsinψj_1A)	sum_Vjsinψj
=SUM(Vjsinφj_1A)	sum_Vjsinφj

NONMANDATORY APPENDIX C
METHOD OF APPROACHING A SPECIFIED POINT OF OPERATION

C-1 INTRODUCTION

Testing a part load on a fan is often required to demonstrate performance at a specified point of operation that is other than "test block." This presents a challenge to the test engineer because a specific combination of flow and static pressure rise needs to be established at existing barometric pressure, density, inlet temperature, and shaft speed that may differ from design. Fan performance will depend on fan flow control devices (vane position, speed, blade pitch, etc.) and system resistance that act interdependently.

Flow rate adjustments affect pressure rise, and pressure rise adjustments affect flow rate. Frequently, it is necessary to establish a test, or operating, condition that can be corrected to a specified point of operation using the fan laws.

The following procedure is intended as a guide that can be used for quickly establishing fan test conditions.

C-2 FLOW MEASUREMENT

Figures C-2-1 through C-2-3 show three locations where pressure differentials representative of fan flow rate can be measured.

Fig. C-2-1 Typical Centrifugal Fan Arrangement Showing Flow and Differential Pressure Measurement Locations

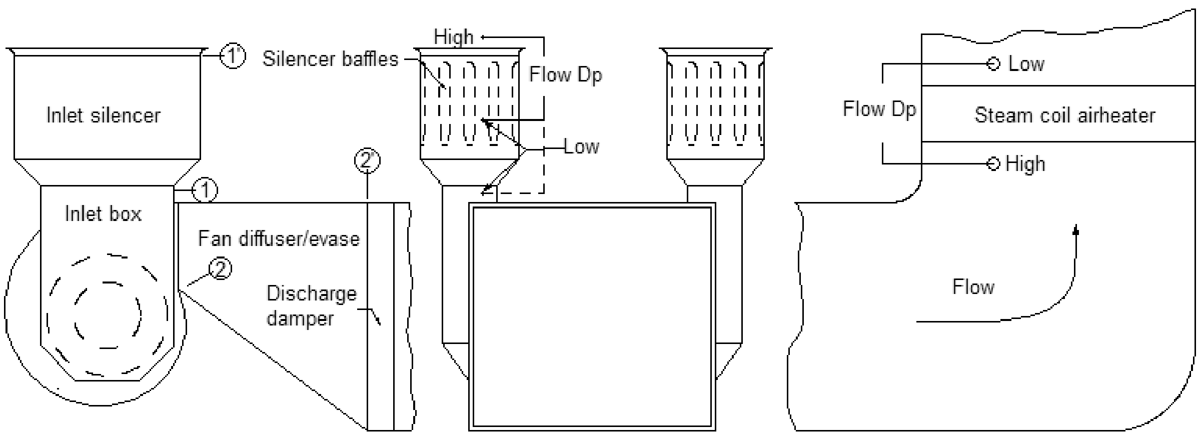


Fig. C-2-2 Axial Fan Arrangement Showing Where Flow Rate Can Be Monitored

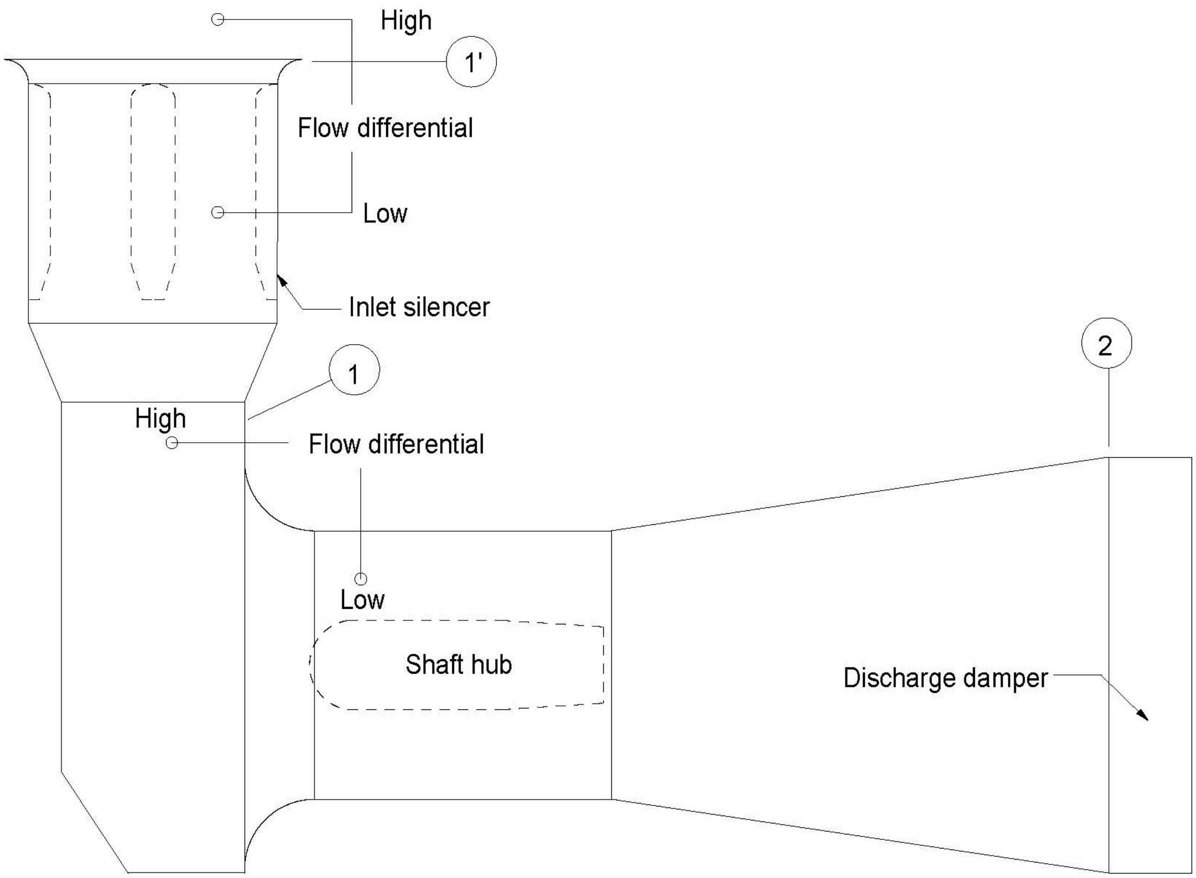
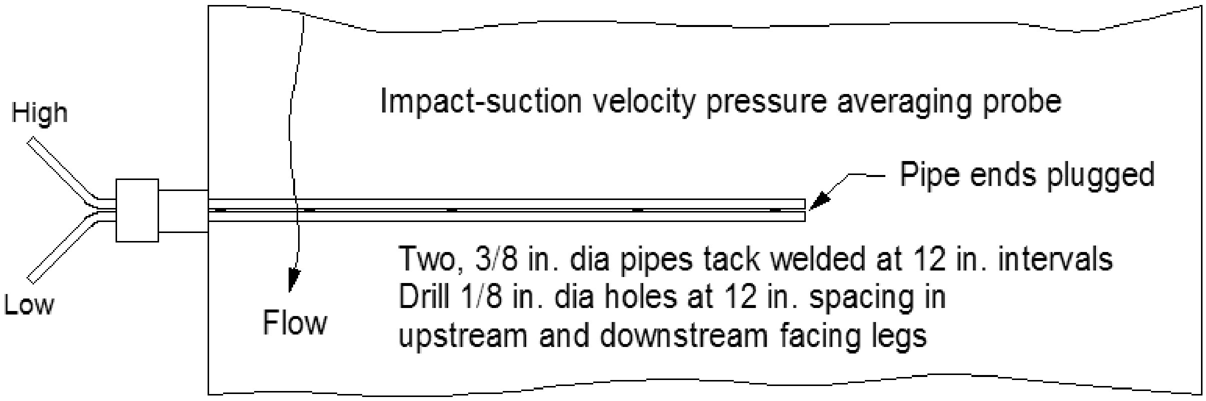


Fig. C-2-3 Averaging Impact-Suction Flow-Monitoring Probe



Regardless of where pressure differentials are measured, it is important that this be done with good accuracy. Differential pressure measurements should be made to within 1.0% of the nearest whole number reading.

C-2.1 Flow Calibration

The above referenced flow measurement devices can be calibrated using the following equation:

$$C = \frac{\dot{m}}{\sqrt{\Delta p \times \rho}}$$

where

C = flow coefficient, lbm/min (in. wg)$^{-0.5}$(lbm/ft^3)$^{-0.5}$

$\dot{m}$ = flow rate measured by traverse, lbm/min

Δp = differential pressure, in. wg

ρ = density, lbm/ft^3

Multiple traverses, even if at slightly different flow rates, should produce relatively constant flow coefficients. It is recommended that a flow coefficient be determined for each traverse and averaged with previous values. Individual coefficients should not deviate more than about 2% from the average. Once a representative flow coefficient is determined, the above equation can be rearranged to determine flow rate as a function of C, Δp, and ρ as follows:

$$\dot{m} = C \times \sqrt{\Delta p \times \rho}$$

It should be pointed out that the flow rate determined from the above procedure is intended to act as an estimate of the actual flow rate to help expedite the testing process. The actual flow rate at any test condition must be determined via test plane traverse as discussed in this Code.

C-3 TEST PROCEDURE

The test procedure is a multistep process as follows:

(a) Establish pressure connections at, or as close as possible to, the fan inlet/outlet boundaries as defined in this Code.

(b) When possible, place the fan in manual control, and for the first test, adjust the flow rate at 60% to 70% of test block rating and at a corresponding pressure rise that falls on a parabolic system resistance line drawn through the test block performance point.

(c) Conduct a preliminary test to determine representative values for pressure, temperature, and flow. Use traverse data to determine a flow coefficient in accordance with the equation above.

(d) Enter values for flow and pressure rise at the specified point of operation in the calculation sheet provided (see Table C-3-1), and note the offset between the operating and target points.

(e) Adjust flow and pressure rise to achieve offset values to within agreed-upon limits between target and test values. Retest.

(f) Enter new differential and static pressures into a worksheet, and check flow and pressure rise offsets. If values are within agreed-upon limits, conduct another complete test. One test point should lie in each quadrant of a rectangular coordinate system, centered on the target point that lies within a box defined by the above limits.

Table C-3-1 Test Procedure

Worksheet to Determine Part Load Fan Operating Conditions

Line No.		Equations	Variable	Value	
	SPECIFIED FAN PERFORMANCE				
1	Flow		Qspecified		10^3 acfm
2	Fan total pressure		ptspecified		in. wg
3	Base speed - specified conditions		SpecRpm		rpm
4	Base density - specified conditions		SpecDensity		lbm/ft^3
	TEST DATA				**Units**
5	**New speed - test conditions**		**TestRpm**		**rpm**
6	**New density - test conditions**		**TestDensity**		**lbm/ft^3**
7	**Barometric pressure**		**Pbar**		**in. Hg**
8	**Inlet static pressure - Plane 1**		**ps1**		**in. wg**
9	**Inlet temperature - Plane 1**		**Temp1**		**^{o} F**
10	**Outlet static pressure - Plane 2**		**ps2**		**in. H_2O**
11	**Outlet temperature - Plane 2**		**Temp2**		**^{o}F**
12	**Flow differential - Δp**		**Dp**		**in. H_2O**
13	**Flow coefficient**		**C**		**lbm/min (in. wg)$^{-0.5}$ (lbm/cuft)$^{-0.5}$**
14	**Molecular weight of gas/air**		**Mwgas**		**lbm/lbm-mol**
15	**Inlet area - Plane 1**		**Area1**		**ft^2**
16	**Outlet area - Plane 2**		**Area2**		**ft^2**
	TARGET VALUES AT TEST CONDITIONS				
17	Fan flow	Qspecified*(TestRpm/SpecRpm) =	Qtarget		10^3 acfm
18	Fan pressure	ptspecified*(TestRpm/SpecRpm)^2*(TestDensity/SpecDensity) =	Pttarget		in. wg
	CALCULATIONS				
	At test conditions				
19	Fan flow	Q1 =	QTest		10^3 acfm
20	Fan pressure	pt2 - pt1 =	pfTest		in. H_2O
	Percentage of offset between operating and target points				
21	Flow	(QTest-QTarget)/QTarget x 100 =	FlowOffset		%
22	Fan pressure	(pfTest-ptTarget)/ptTarget x 100 =	PresOffset		%
	Plane 1 - Fan inlet:				
23	Area Plane 1		Area1		ft^2
24	Density	((Pbar/2.04 + ps1/27.7) x 144)/(1545/Mwgas x (460 + Temp1)) =	Den1		lbm/ft^3
25	Mass flow rate	C x SQRT(Dp x Den1) =	M1_		lbm/min
26	Volumetric flow rate	M1/ Den1 =	Q1		10^3 acfm
27	Velocity	Q1_/Area1 x 1000 =	Vel1		ft/min
		ps1 =	ps1		
28	Velocity pressure	(Vel1/1097)^2 x Den1 =	pv1		in. wg
29	Total pressure	ps1+ pv1 =	pt1		in. wg
	Plane 2 - Fan outlet:				
30	Area Plane 2		Area2		ft^2
31	Density	((Pbar/2.04 + Ps2/27.7) x 144)/(1545/Mwgas x (460 + Temp2)) =	Den2		lbm/ft^3
32	Mass flow rate	Conservation of mass flow M2 = M1	M2		10^3 lbm/min
33	Volumetric flow rate	M2/Den2 =	Q2		10^3 acfm
34	Velocity	Q2/Area_2 x 1000 =	Vel2		ft/min
		ps2 =	ps2		in. wg
35	Velocity pressure	(Vel2/1097)^2 x Den2 =	pv2		in. wg
36	Total pressure	ps2 + pv2 =	pt2		in. wg

NOTE: Values shown in bold face with shaded background represent data that needs to be changed as necessary for each test run

NONMANDATORY APPENDIX D[1] DERIVATIONS OF UNCERTAINTIES EQUATIONS

D-1 UNCERTAINTY IN THE MASS FLOW RATE, $\dot{m}_x$, AT PLANE x

The equation for $\dot{m}_x$ is given in Section 5 as

$$\dot{m}_x = \frac{A_x}{C_2}\frac{1}{n}\sum_{j=1}^{n}\left(\rho_j V_j \cos\psi_j \cos\phi_j\right)_x \tag{5-6-1}$$

Not all of the variables in this equation are direct test measurements. We can get closer to measurements by substituting for ρ_j and V_j.

$$\rho_j = \frac{C_{11} p_{saj}}{RT_{sj}} = \frac{C_{11}\left(p_{sj} + C_{13} p_b\right)}{RT_{sj}} \tag{5-4-3}$$

$$V_j = C_{12}\sqrt{\frac{p_{vj}}{\rho_j}} \tag{5-5-1}$$

We can also improve this analysis by adding a factor F_n to the original equation. This number of points factor, F_n, is assumed equal to unity; therefore, it does not change the original equation. However, it will provide a basis for evaluating the uncertainties due to the number of points (that is, the uncertainty associated with numerical integration over the measurement plane). Substituting for ρ_j and V_j and inserting F_n gives

$$\dot{m}_x = \frac{A_x}{C_2}\frac{1}{n}F_n\sum_{j=1}^{n}\left(C_{11}^{1/2}C_{12}\frac{\left(p_{sj}+C_{13}p_b\right)^{1/2}}{R^{1/2}T_{sj}^{\;1/2}}p_{vj}^{1/2}\cos\psi_j\cos\phi_j\right)_x \tag{D-1-1}$$

It will be helpful to introduce A_j, which is equal to A_x/n, and substitute

$$\dot{m}_x = \frac{C_{11}^{1/2}C_{12}}{C_2}F_n\sum_{j=1}^{n}\left(A_j\frac{\left(p_{sj}+C_{13}p_b\right)^{1/2}}{R^{1/2}T_{sj}^{\;1/2}}p_{vj}^{1/2}\cos\psi_j\cos\phi_j\right)_x \tag{D-1-2}$$

defining the flow through A_j as $\dot{m}_j$.

$$\dot{m}_j = \frac{C_{11}^{1/2}C_{12}}{C_2}\left(A_j\frac{\left(p_{sj}+C_{13}p_b\right)^{1/2}}{R^{1/2}T_{sj}^{\;1/2}}p_{vj}^{1/2}\cos\psi_j\cos\phi_j\right) \tag{D-1-3}$$

The constants C_{11}, C_{12}, and C_2 can be considered exact and, therefore, have no effect in the uncertainty analysis. It follows that

$$\dot{m}_x = F_n\sum_{j=1}^{n}\dot{m}_j \tag{D-1-4}$$

[1] In this Appendix, equations from other parts of the book are sometimes repeated for reference. These equations retain their original numbering when cited in this Appendix.

differentiating

$$d\dot{m}_x = \frac{\partial \dot{m}_x}{\partial F_n} dF_n + \frac{\partial \dot{m}_x}{\partial \sum_{j=1}^{n} \dot{m}_j} d\sum_{j=1}^{n} \dot{m}_j \tag{D-1-5}$$

[In this derivation, the differential notation (e.g., $d\dot{m}$, dF_n) is used to represent uncertainties. In PTC 19.1, σ is used.]

$$\frac{\partial \dot{m}_x}{\partial F_n} = \sum_{j=1}^{n} \dot{m}_j = \frac{\dot{m}_x}{F_n} \tag{D-1-6}$$

$$\frac{\partial \dot{m}_x}{\partial \sum_{j=1}^{n} \dot{m}_j} = F_n = \frac{\dot{m}_x}{\sum_{j=1}^{n} \dot{m}_j} \tag{D-1-7}$$

Kline and McClintock [5] and PTC 19.1 recommend a second power equation for combining uncertainties.

$$\left(d\dot{m}_x\right)^2 = \left(\frac{\dot{m}_x}{F_n} dF_n\right)^2 + \left(\frac{\dot{m}_x}{\sum_{j=1}^{n} \dot{m}_j} d\sum_{j=1}^{n} \dot{m}_j\right)^2 + \text{cross product terms} \tag{D-1-8}$$

Assuming complete independence of the individual terms, the cross product terms can be dropped.

Similarly,

$$\sum_{j=1}^{n} \dot{m}_j = \dot{m}_1 + \dot{m}_2 + \cdots + \dot{m}_n \tag{D-1-9}$$

$$d\sum_{j=1}^{n} \dot{m}_j = d\dot{m}_1 + d\dot{m}_2 \cdots + d\dot{m}_n \tag{D-1-10}$$

$$\left(d\sum_{j=1}^{n} \dot{m}_j\right)^2 = \left(d\dot{m}_1\right)^2 + \left(d\dot{m}_2\right)^2 + \cdots + \left(d\dot{m}_n\right)^2 + \text{cross product terms} \tag{D-1-11}$$

Hence, also dropping the cross product terms,

$$\left(d\dot{m}_x\right)^2 = \left(\frac{\dot{m}_x}{F_n} dF_n\right)^2 + \left(\frac{\dot{m}_x}{\sum_{j=1}^{n} \dot{m}_j}\right)^2 \sum_{j=1}^{n} \left(d\dot{m}_j\right)^2 \tag{D-1-12}$$

Dividing by $(\dot{m}_x)^2$

$$\left(\frac{d\dot{m}_x}{\dot{m}_x}\right)^2 = \left(\frac{dF_n}{F_n}\right)^2 + \frac{\sum_{j=1}^{n} \left(d\dot{m}_j\right)^2}{\left(\sum_{j=1}^{n} \dot{m}_j\right)^2} \tag{D-1-13}$$

To develop a compact notation, let

$$\hat{U}_{\dot{m}_x} = d\dot{m}_x \quad \hat{u}_{\dot{m}_x} = \frac{d\dot{m}_x}{\dot{m}_x} \quad \hat{U}_{F_n} = dF_n \quad \hat{u}_{F_n} = \frac{dF_n}{F_n} \quad \text{etc.}$$

where $\hat{U}$ is either the absolute random or absolute systematic uncertainty and $\hat{u}$ is either the relative random or relative systematic uncertainty in the subscripted quantity.

It is also useful to denote the partial derivative of a result with respect to a particular variable as the sensitivity factor θ. For example,

$$\theta_{F_n} = \frac{\partial \dot{m}_x}{\partial F_n} \text{ etc.} \tag{D-1-14}$$

To develop a compact notation, let

$$\theta_{i,j} = \frac{\partial \dot{m}_j}{\partial v_{i,j}} \text{ for variables } v_{i,j} \text{ in } \dot{m}_j$$

The sensitivity factors for the variables in $\dot{m}$ are

$$\theta_{A_j} = \frac{\partial \dot{m}_j}{\partial A_j} = \frac{\dot{m}_j}{A_j} \tag{D-1-15}$$

$$\theta_{p_{sj}} = \frac{\partial \dot{m}_j}{\partial p_{sj}} = \frac{\dot{m}_j}{2\left(p_{sj} + C_{13} p_b\right)} \tag{D-1-16}$$

$$\theta_{p_b} = \frac{\partial \dot{m}_j}{\partial p_b} = \frac{\dot{m}_j}{2\left(p_{sj} + C_{13} p_b\right)} \tag{D-1-17}$$

$$\theta_R = \frac{\partial \dot{m}_j}{\partial R} = \frac{\dot{m}_j}{-2R} \tag{D-1-18}$$

$$\theta_{T_{sj}} = \frac{\partial \dot{m}_j}{\partial T_{sj}} = \frac{\dot{m}_j}{-2T_{sj}} \tag{D-1-19}$$

$$\theta_{p_{vj}} = \frac{\partial \dot{m}_j}{\partial p_{vj}} = \frac{\dot{m}_j}{-2p_{vj}} \tag{D-1-20}$$

$$\theta_{\psi_j} = \frac{\partial \dot{m}_j}{\partial \psi_j} = -\tan\psi_j \dot{m}_j \tag{D-1-21}$$

$$\theta_{\phi_j} = \frac{\partial \dot{m}_j}{\partial \phi_j} = -\tan\phi_j \dot{m}_j \tag{D-1-22}$$

All of these sensitivity factors have the general form

$$\theta_{i,j} = \frac{\dot{m}_j}{g\left(v_{i,j}\right)} \text{ where } g\left(v_{i,j}\right) \text{ is a function of } v_{i,j}$$

We can also let

$$\sum_{j=1}^{n}\left(d\dot{m}_j\right)^2 = \sum_{j=i}^{n}\hat{U}_{\dot{m}_j}^2 \tag{D-1-23}$$

However,

$$\hat{U}_{\dot{m}_j}^2 = \sum_{i=1}^{k}\left(\theta_{i,j}\hat{U}_i\right)^2 \tag{D-1-24}$$

where $\hat{U}_i$ is the uncertainty in variable i, and where $i = A_j, p_{sj}, p_{b,}\ etc.$ It follows that

$$\sum_{j=1}^{n}\hat{U}_{\dot{m}_j}^2 = \sum_{j=1}^{n}\sum_{i=1}^{k}\left(\theta_{i,j}\hat{U}_i\right)^2 \tag{D-1-25}$$

Also that

$$\sum_{j=1}^{n}\hat{U}_{\dot{m}_j}^2 = \sum_{j=1}^{n}\sum_{i=1}^{k}\left(\frac{\dot{m}_j\hat{U}_i}{g\left(v_{i,j}\right)}\right)^2 = \sum_{j=1}^{n}\left(\dot{m}_j\right)^2\sum_{i=1}^{k}\left(\frac{\hat{U}_i}{g\left(v_{i,j}\right)}\right)^2 \tag{D-1-26}$$

Rearranging equation D-1-4 for $\dot{m}_x$ gives

$$\sum_{j=1}^{n}\left(\dot{m}_j\right)^2 = \frac{\dot{m}_x^{\ 2}}{F_n^2} \tag{D-1-27}$$

Also

$$\begin{aligned}\sum_{i=1}^{k}\left(\frac{\hat{U}_i}{g\left(v_{i,j}\right)}\right)^2 = {} & \left(\frac{\hat{U}_{A_j}}{A_j}\right)^2 + \frac{1}{4}\left[\left(\frac{\hat{U}_{p_{sj}}^2 + C_{13}^2\hat{U}_{p_b}^2}{p_{saj}^2}\right) + \left(\frac{\hat{U}_R}{R}\right)^2 + \left(\frac{\hat{U}_{T_{sj}}}{T_{sj}}\right)^2 + \left(\frac{\hat{U}_{p_{vj}}}{p_{vj}}\right)^2\right] \\ & + \left(\frac{\left(\tan^2\psi_j\right)\hat{U}_{\psi_j}^2 + \left(\tan^2\phi_j\right)\hat{U}_{\phi_j}^2}{57.30^2}\right)\end{aligned} \tag{D-1-28}$$

Therefore

$$\begin{aligned}\left(\frac{\hat{U}_{\dot{m}_x}}{\dot{m}_x}\right)^2 = \left(\frac{\hat{U}_{F_n}}{F_n}\right)^2 + \frac{F_n^2}{\dot{m}_x^2}\sum_{j=1}^{n}\left(\dot{m}_j\right)^2 \\ \left(\left(\frac{\hat{U}_{A_j}}{A_j}\right)^2 + \frac{1}{4}\left[\left(\frac{\hat{U}_{p_{sj}}^2 + C_{13}^2\hat{U}_{p_b}^2}{p_{saj}^2}\right) + \left(\frac{\hat{U}_R}{R}\right)^2 + \left(\frac{\hat{U}_{T_{sj}}}{T_{sj}}\right)^2 + \left(\frac{\hat{U}_{p_{vj}}}{p_{vj}}\right)^2 + 4\left(\frac{\left(\tan^2\psi_J\right)\hat{U}_{\psi_j}^2 + \left(\tan^2\phi_j\right)\hat{U}_{\phi_j}^2}{57.30^2}\right)\right]\right)\end{aligned} \tag{D-1-29}$$

Setting F_n equal to unity, rearranging, and substituting relative uncertainties where possible yields the following result:

$$\hat{u}_{\dot{m}_x}^2 = \hat{u}_{F_n}^2 + \hat{u}_{A_x}^2 + \sum_{j=1}^{n}\left(\frac{\dot{m}_j}{\dot{m}_x}\right)^2\left[\frac{1}{4}\left(\left(\frac{\hat{U}_{p_{sj}}^2 + C_{13}^2\hat{U}_{p_b}^2}{p_{saj}^2}\right) + \hat{u}_R^2 + \hat{u}_{T_{sj}}^2 + \hat{u}_{p_{vj}}^2 + 4\left(\frac{\left(\tan^2\psi_j\right)\hat{U}_{\psi_j}^2 + \left(\tan^2\phi_j\right)\hat{U}_{\phi_j}^2}{57.30^2}\right)\right)\right] \tag{5-13-3}$$

D-2 UNCERTAINTY IN p_{sx}, THE AVERAGE STATIC PRESSURE, AT PLANE *x*

The equation for p_{sx} is given in Section 5 as

$$p_{sx} = \frac{\sum_{j=1}^{n}\left(p_{sj}V_j\cos\psi_j\cos\phi_j\right)_x}{\sum_{j=1}^{n}\left(V_j\cos\psi_j\cos\phi_j\right)_x} \tag{5-7-1}$$

The $V_j\cos\psi_j\cos\phi_j$ terms in both the numerator and denominator are weighting factors in the averaging process. We will assume that the contributions of these weighting factors to uncertainty are negligible and approximate eq. (5-7-1) by

$$p_{sx} \cong \frac{1}{n}\sum_{j=1}^{n} p_{sj} \tag{D-2-1}$$

only for the purpose of uncertainty evaluation.

Differentiating

$$dp_{sx} = \frac{1}{n}d\sum_{j=1}^{n} p_{sj} \tag{D-2-2}$$

Noting that

$$d\sum_{j=1}^{n} p_{sj} = \sum_{j=1}^{n} dp_{sj} \tag{D-2-3}$$

and

$$\left(d\sum_{j=1}^{n} p_{sj}\right)^2 = \sum_{j=1}^{n}\left(dp_{sj}\right)^2 \tag{D-2-4}$$

Dropping the cross product terms, i.e., assuming no correlated uncertainties

$$\left(dp_{sx}\right)^{2} = \frac{1}{n^2}\sum_{j=1}^{n}\left(dp_{sj}\right)^2 \tag{D-2-5}$$

Dividing by p_{sx}^2

$$\left(\frac{dp_{sx}}{p_{sx}}\right)^2 = \frac{1}{n^2}\frac{\sum_{j=1}^{n}\left(dp_{sj}\right)^2}{p_{sx}^2} \tag{D-2-6}$$

Multiplying by $\left(p_{sj}^2 / p_{sj}^2\right)$

$$\left(\frac{dp_{sx}}{p_{sx}}\right)^2 = \frac{1}{n^2}\sum_{j=1}^{n}\left(\frac{p_{sj}}{p_{sx}}\right)^2\left(\frac{dp_{sj}}{p_{sj}}\right)^2 \tag{D-2-7}$$

Since $dp_{sj} \big/ p_{sj} = \hat{u}_{p_{sj}}$, the final equation is

$$\hat{u}^2_{p_{sx}} = \frac{1}{n^2} \sum_{j=1}^{n} \left(\frac{p_{sj}}{p_{sx}} \right)^2 \hat{u}^2_{p_{sj}} \tag{5-13-8}$$

Equation 5-13-11 is derived in a similar manner.

D-3 UNCERTAINTY IN THE AVERAGE DENSITY, ρ_x , AT PLANE *x*

The equation for ρ_x is given in Section 5 as

$$\rho_x = \frac{\sum_{j=1}^{n} \left(\rho_j V_j \cos\psi_j \cos\phi_j \right)_x}{\sum_{j=1}^{n} \left(V_j \cos\psi_j \cos\phi_j \right)_x} \tag{5-7-2}$$

Using the p_{sx} approach regarding the $V_j \cos\psi_j \cos\phi_j$ terms

$$\rho_x = \frac{1}{n} \sum_{j=1}^{n} \rho_j \tag{D-3-1}$$

Differentiating

$$d\rho_x = \frac{1}{n} d \sum_{j=1}^{n} \rho_j = \frac{1}{n} \sum_{j=1}^{n} d\rho_j \tag{D-3-2}$$

$$\left(d\rho_x \right)^2 = \frac{1}{n^2} \sum_{j=1}^{n} \left(d\rho_j \right)^2 \tag{D-3-3}$$

Dividing by $\left(\rho_x \right)^2$

$$\left(\frac{d\rho_x}{\rho_x} \right)^2 = \frac{1}{n^2} \sum_{j=1}^{n} \frac{\left(d\rho_j \right)^2}{\rho_x^2} \tag{D-3-4}$$

Multiplying by ρ_j^2 / ρ_j^2

$$\left(\frac{d\rho_x}{\rho_x} \right)^2 = \frac{1}{n^2} \sum_{j=1}^{n} \frac{\left(d\rho_j \right)^2}{\rho_x^2} \frac{\rho_j^2}{\rho_j^2} = \frac{1}{n^2} \sum_{j=1}^{n} \left(\frac{\rho_j}{\rho_x} \right)^2 \left(\frac{d\rho_j}{\rho} \right)^2 = \frac{1}{n^2} \sum_{j=1}^{n} \left(\frac{\rho_j}{\rho_x} \right)^2 u^2_{\rho_j} \tag{D-3-5}$$

Reducing ρ_j to primary variables

$$\rho_j = \frac{p_{saj}}{RT_{sj}} = \frac{\left(p_{sj} + p_b \right)}{RT_{sj}} \tag{D-3-6}$$

Combining uncertainties per Kline and McClintock

$$\hat{u}_{\rho_x}^2 = \left(\frac{\hat{U}_{\rho_x}}{\rho_x}\right)^2 = \left(\frac{1}{\rho_x^2}\right)\left[\left(\frac{\delta\rho_x}{\delta R}\right)^2 \hat{U}_R^2 + \left(\frac{\delta\rho_x}{\delta T_{sj}}\right)^2 \hat{U}_{T_{sj}}^2 + \left(\frac{\delta\rho_x}{\delta p_{sj}}\right)^2 \hat{U}_{p_{sj}}^2 + \left(\frac{\delta\rho_x}{\delta p_b}\right)^2 \hat{U}_{p_b}^2\right] \tag{D-3-7}$$

$$\left(\frac{1}{\rho_x^2}\right)\left(\frac{\delta\rho_x}{\delta R}\right)^2 \hat{U}_R^2 = \left(\frac{-\frac{\left(p_{sj}+p_b\right)}{R^2 T_{sj}}}{\frac{\left(p_{sj}+p_b\right)}{RT_{sj}}}\right)^2 \hat{U}_R^2 = \left(\frac{\hat{U}_R}{R}\right)^2 = \hat{u}_R^2 \tag{D-3-8}$$

$$\left(\frac{1}{\rho_x^2}\right)\left(\frac{\delta\rho_x}{\delta T_{sj}}\right)^2 \hat{U}_{T_{sj}}^2 = \left(\frac{-\frac{\left(p_{sj}+p_b\right)}{RT_{sj}^2}}{\frac{\left(p_{sj}+p_b\right)}{RT_{sj}}}\right)^2 \hat{U}_{T_{sj}}^2 = \left(\frac{\hat{U}_{T_{sj}}}{T_{sj}}\right)^2 = \hat{u}_{T_{sj}}^2 \tag{D-3-9}$$

$$\left(\frac{1}{\rho_x^2}\right)\left(\frac{\delta\rho_x}{\delta p_{sj}}\right)^2 \hat{U}_{p_{sj}}^2 = \left(\frac{\frac{1}{RT_{sj}}}{\frac{\left(p_{sj}+p_b\right)}{RT_{sj}}}\right)^2 \hat{U}_{p_{sj}}^2 = \left(\frac{\hat{U}_{p_{sj}}}{\left(p_{sj}+p_b\right)}\right)^2 = \frac{\hat{U}_{p_{sj}}^2}{p_{saj}^2} \tag{D-3-10}$$

$$\left(\frac{1}{\rho_x^2}\right)\left(\frac{\delta\rho_x}{\delta p_b}\right)^2 \hat{U}_{p_b}^2 = \left(\frac{\frac{1}{RT_{sj}}}{\frac{\left(p_{sj}+p_b\right)}{RT_{sj}}}\right)^2 \hat{U}_{p_b}^2 = \left(\frac{\hat{U}_{p_b}}{\left(p_{sj}+p_b\right)}\right)^2 = \frac{\hat{U}_{p_b}^2}{p_{saj}^2} \tag{D-3-11}$$

Substituting in eq. (D-3-5)

$$\hat{u}_{\rho_x}^2 = \frac{1}{n^2}\sum_{j=1}^{n}\left(\frac{\rho_j}{\rho_x}\right)^2 \hat{u}_{\rho_j}^2 = \frac{1}{n^2}\sum_{j=1}^{n}\left(\frac{\rho_j}{\rho_x}\right)^2 \left(\hat{u}_R^2 + \hat{u}_{T_{sj}}^2 + \frac{\hat{U}_{p_{sj}}^2 + \hat{U}_{p_b}^2}{p_{saj}^2}\right) \tag{D-3-12}$$

Inserting unit conversions

$$\hat{u}_{\rho_x}^2 = \frac{1}{n^2}\sum_{j=1}^{n}\left(\frac{\rho_j}{\rho_x}\right)^2 \left(\hat{u}_R^2 + \hat{u}_{T_{sj}}^2 + \frac{\hat{U}_{p_{sj}}^2 + C_{13}^2\hat{U}_{p_b}^2}{p_{saj}^2}\right) \tag{5-13-9}$$

D-4 UNCERTAINTY IN THE AVERAGE SPECIFIC KINETIC ENERGY, e_{Kx}, AT PLANE *x*

The equation for e_{Kx} is given in Section 5 as

$$e_{Kx} = \frac{\sum_{j=1}^{n}\left(\rho_j V_j^3 \cos^3\psi_j \cos^3\phi_j\right)_x}{\sum_{j=1}^{n}\left(\rho_j V_j \cos\psi_j \cos\phi_j\right)_x} \tag{5-7-4}$$

Using the p_{sx} approach regarding the $V_j \cos\psi_j \cos\phi_j$ terms

$$e_{Kx} = \frac{1}{n}\sum_{j=1}^{n}\left(\rho_j V_j^2 \cos^2\psi_j \cos^2\phi_j\right)_x = \frac{1}{n}\sum_{j=1}^{n} e_{Kj} \quad \text{(D-4-1)}$$

Differentiating

$$de_{Kx} = \frac{1}{n} d\sum_{j=1}^{n} e_{Kj} = \frac{1}{n}\sum_{j=1}^{n} de_{Kj} \quad \text{(D-4-2)}$$

$$\left(de_{Kx}\right)^2 = \frac{1}{n^2}\sum_{j=1}^{n}\left(de_{Kj}\right)^2 \quad \text{(D-4-3)}$$

Dividing by $\left(e_{Kx}\right)^2$

$$\frac{\left(de_{Kx}\right)}{e_{Kx}^2} = \frac{1}{n^2}\sum_{j=1}^{n}\frac{\left(de_{Kj}\right)^2}{e_{Kx}^2} \quad \text{(D-4-4)}$$

Multiplying by e_{Kj}^2 / e_{Kj}^2

$$\left(\frac{de_{Kx}}{e_{Kx}}\right)^2 = \frac{1}{n^2}\sum_{j=1}^{n}\frac{\left(de_{Kj}\right)^2}{e_{K_x}^2}\frac{e_{Kj}^2}{e_{Kj}^2} = \frac{1}{n^2}\sum_{j=1}^{n}\left(\frac{e_{Kj}}{e_{Kj}}\right)^2\left(\frac{de_{Kj}}{e_{Kj}}\right)^2 = \frac{1}{n^2}\sum_{j=1}^{n}\left(\frac{e_{Kj}}{e_{Kx}}\right)^2 u_{e_{Kj}}^2 \quad \text{(D-4-5)}$$

Reducing e_{Kj} to primary variables

$$e_{Kj} = V_j^2 \cos^2\psi_j \cos^2\phi_j = \frac{p_{vj} R T_{sj}}{p_{sj} + p_b}\cos^2\psi_j \cos^2\phi_j \quad \text{(D-4-6)}$$

Combining uncertainties per Kline and McClintock

$$\hat{u}_{e_{Kx}}^2 = \left(\frac{\hat{U}_{e_{Kx}}}{e_{Kx}}\right)^2 = \left(\frac{1}{e_{Kx}^2}\right)\left[\left(\frac{\partial e_{Kx}}{\partial p_{vj}}\right)^2 \hat{U}_{p_{vj}}^2 + \left(\frac{\partial e_{Kx}}{\partial R}\right)^2 \hat{U}_R^2 + \left(\frac{\partial e_{Kx}}{\partial T_{sj}}\right)^2 \hat{U}_{T_{sj}}^2 + \left(\frac{\partial e_{Kx}}{\partial p_{sj}}\right)^2 \hat{U}_{p_{sj}}^2 + \left(\frac{\partial e_{Kx}}{\partial p_b}\right)^2 \hat{U}_{p_b}^2\right] \quad \text{(D-4-7)}$$

$$\left(\frac{1}{e_{Kx}^2}\right)\left(\frac{\partial e_{Kj}}{\partial R}\right)^2 \hat{U}_{p_{vj}}^2 = \left(\frac{\dfrac{R T_{sj}}{p_{sj} + p_b}\cos^2\psi_j \cos^2\phi_j}{\dfrac{p_{vj} R T_{sj}}{p_{sj} + p_b}\cos^2\psi_j \cos^2\phi_j}\right)^2 \hat{U}_{p_{vj}}^2 = \left(\frac{\hat{U}_{p_{vj}}}{p_{vj}}\right)^2 = \hat{u}_{p_{vj}}^2 \quad \text{(D-4-8)}$$

$$\left(\frac{1}{e_{Kx}^2}\right)\left(\frac{\partial e_{Kj}}{\partial R}\right)^2 \hat{U}_R^2 = \left(\frac{\dfrac{p_{vj} T_{sj}}{p_{sj} + p_b}\cos^2\psi_j \cos^2\phi_j}{\dfrac{p_{vj} R T_{sj}}{p_{sj} + p_b}\cos^2\psi_j \cos^2\phi_j}\right)^2 \hat{U}_R^2 = \left(\frac{\hat{U}_R}{R}\right)^2 = \hat{u}_R^2 \quad \text{(D-4-9)}$$

$$\left(\frac{1}{e_{Kx}^2}\right)\left(\frac{\partial e_{Kj}}{\partial T_{sj}}\right)^2 \hat{U}_{T_{sj}}^2 = \left(\frac{\frac{p_{vj}R}{p_{sj}+p_b}\cos^2\psi_j\cos^2\phi_j}{\frac{p_{vj}RT_{sj}}{p_{sj}+p_b}\cos^2\psi_j\cos^2\phi_j}\right)^2 \hat{U}_{T_{sj}}^2 = \left(\frac{\hat{U}_{T_{sj}}}{T_{sj}}\right)^2 = \hat{u}_{T_{sj}}^2 \quad \text{(D-4-10)}$$

$$\left(\frac{1}{e_{Kx}^2}\right)\left(\frac{\partial e_{Kj}}{\partial p_{sj}}\right)^2 \hat{U}_{p_{sj}}^2 = \left(\frac{\frac{p_{vj}RT_{sj}}{\left(p_{sj}+p_b\right)^2}\cos^2\psi_j\cos^2\phi_j}{\frac{p_{vj}RT_{sj}}{p_{sj}+p_b}\cos^2\psi_j\cos^2\phi_j}\right)^2 \hat{U}_{p_{sj}}^2 = \left(\frac{\hat{U}_{p_{sj}}}{\left(p_{sj}+p_b\right)}\right)^2 = \frac{\hat{U}_{p_{sj}}^2}{p_{saj}^2} \quad \text{(D-4-11)}$$

$$\left(\frac{1}{e_{Kx}^2}\right)\left(\frac{\partial e_{Kj}}{\partial p_b}\right)^2 \hat{U}_{p_b}^2 = \left(\frac{\frac{p_{vj}RT_{sj}}{\left(p_{sj}+p_b\right)^2}\cos^2\psi_j\cos^2\phi_j}{\frac{p_{vj}RT_{sj}}{p_{sj}+p_b}\cos^2\psi_j\cos^2\phi_j}\right)^2 \hat{U}_{p_b}^2 = \left(\frac{\hat{U}_{p_b}}{\left(p_{sj}+p_b\right)}\right)^2 = \frac{\hat{U}_{p_b}^2}{p_{saj}^2} \quad \text{(D-4-12)}$$

$$\left(\frac{1}{e_{Kx}^2}\right)\left(\frac{\partial e_{K_j}}{\partial \psi_j}\right)^2 \hat{U}_{\psi_j}^2 = \left(\frac{-\frac{p_{vj}RT_{sj}}{p_{sj}+p_b}2\sin\psi_j\cos\psi_j\cos^2\phi_j}{\frac{p_{vj}RT_{sj}}{p_{sj}+p_b}\cos^2\psi_j\cos^2\phi_j}\right)^2 \hat{U}_{\psi_j}^2 = \left(\frac{-2\sin\psi_j}{\cos\psi_j}\right)^2 \hat{U}_{\psi_j}^2 = 4\tan^2\psi_j\hat{U}_{\psi_j}^2 \quad \text{(D-4-13)}$$

$$\left(\frac{1}{e_{Kx}^2}\right)\left(\frac{\partial e_{K_j}}{\partial \phi_j}\right)^2 \hat{U}_{\phi_j}^2 = \left(\frac{-\frac{p_{vj}RT_{sj}}{p_{sj}+p_b}2\sin\phi_j\cos\phi_j\cos^2\psi_j}{\frac{p_{vj}RT_{sj}}{p_{sj}+p_b}\cos^2\psi_j\cos^2\phi_j}\right)^2 \hat{U}_{\phi_j}^2 = \left(\frac{-2\sin\phi_j}{\cos\phi_j}\right)^2 \hat{U}_{\phi_j}^2 = 4\tan^2\phi_j\hat{U}_{\phi_j}^2 \quad \text{(D-4-14)}$$

Substituting in eq. (D-4-5)

$$\hat{u}_{e_{Kx}}^2 = \frac{1}{n^2}\sum_{j=1}^{n}\left(\frac{e_{Kj}}{e_{Kx}}\right)^2 \hat{u}_{\rho_j}^2 = \frac{1}{n^2}\sum_{j=1}^{n}\left(\frac{e_{Kj}}{e_{Kx}}\right)^2 \left(u_{p_{vj}}^2 + \hat{u}_R^2 + \hat{u}_{T_{sj}}^2 + \frac{\hat{U}_{p_{sj}}^2 + \hat{U}_{p_b}^2}{p_{saj}^2} + 4\tan^2\psi U_{\psi_j}^2 + 4\tan^2\phi U_{\phi_j}^2\right) \quad \text{(D-4-15)}$$

Inserting unit conversions

$$\hat{u}_{e_{Kx}}^2 = \frac{1}{n^2}\sum_{j=1}^{n}\left(\frac{e_{Kj}}{e_{Kx}}\right)^2 \hat{u}_{\rho_j}^2 = \frac{1}{n^2}\sum_{j=1}^{n}\left(\frac{e_{Kj}}{e_{Kx}}\right)^2 \left[u_{p_{vj}}^2 + \hat{u}_R^2 + \hat{u}_{T_{sj}}^2 + \frac{\hat{U}_{p_{sj}}^2 + C_{13}^2\hat{U}_{p_b}^2}{p_{saj}^2} + 4\left(\frac{\tan^2\psi U_{\psi_j}^2 + \tan^2\phi U_{\phi_j}^2}{57.30^2}\right)\right] \quad \text{(5-13-11)}$$

D-5 UNCERTAINTY IN THE AVERAGE VELOCITY PRESSURE, p_{vx}, AT PLANE x

The equation for p_{vx} is given in Section 5 as

$$p_{vx} = \rho_x e_{Kx} \quad \text{(5-7-6)}$$

Reducing to primary variables

$$\rho_x = \frac{\sum_{j=1}^{n}\left(\rho_j V_j \cos\psi_j \cos\phi_j\right)_x}{\sum_{j=1}^{n}\left(V_j \cos\psi_j \cos\phi_j\right)_x} \tag{D-5-1}$$

$$e_{Kx} = \frac{\sum_{j=1}^{n}\left(\rho_j V_j^3 \cos\psi_j^3 \cos\phi_j^3\right)_x}{\sum_{j=1}^{n}\left(\rho_j V_j \cos\psi_j \cos\phi_j\right)_x} \tag{D-5-2}$$

$$p_{vx} = \frac{\sum_{j=1}^{n}\left(\rho_j V_j \cos\psi_j \cos\phi_j\right)_x}{\sum_{j=1}^{n}\left(V_j \cos\psi_j \cos\phi_j\right)_x} \frac{\sum_{j=1}^{n}\left(\rho_j V_j^3 \cos\psi_j^3 \cos\phi_j^3\right)_x}{\sum_{j=1}^{n}\left(\rho_j V_j \cos\psi_j \cos\phi_j\right)_x} = \frac{\sum_{j=1}^{n}\left(\rho_j V_j^3 \cos\psi_j^3 \cos\phi_j^3\right)_x}{\sum_{j=1}^{n}\left(V_j \cos\psi_j \cos\phi_j\right)_x} \tag{D-5-3}$$

Using the p_{sx} approach regarding the $V_j \cos\psi_j \cos\phi_j$ terms

$$p_{vx} = \frac{1}{n}\sum_{j=1}^{n}\left(\rho_j V_j^2 \cos^2\psi_j \cos^2\phi_j\right)_x = \frac{1}{n}\sum_{j=1}^{n}\left(p_{vj} \cos^2\psi_j \cos^2\phi_j\right)_x \tag{D-5-4}$$

Differentiating

$$dp_{vx} = \frac{1}{n} d\sum_{j=1}^{n}\left(p_{vj} \cos^2\psi_j \cos^2\phi_j\right)_x = \sum_{j=1}^{n} d\left(p_{vj} \cos^2\psi_j \cos^2\phi_j\right)_x \tag{D-5-5}$$

$$\left(dp_{vx}\right)^2 = \frac{1}{n^2}\left(\sum_{j=1}^{n} d\left(p_{vj} \cos^2\psi_j \cos^2\phi_j\right)\right)_x^2 \tag{D-5-6}$$

Dividing by $\left(p_{vx}\right)^2$

$$\left(\frac{dp_{vx}}{p_{vx}}\right)^2 = \frac{1}{n^2}\left(\frac{\sum_{j=1}^{n} d\left(p_{vj} \cos^2\psi_j \cos^2\phi_j\right)}{p_{vx}}\right)^2 \tag{D-5-7}$$

Multiplying by $\left(p_{vj} \cos^2\psi_j \cos^2\phi_j\right)^2 / \left(p_{vj} \cos^2\psi_j \cos^2\phi_j\right)^2$

$$\left(\frac{dp_{vx}}{p_{vx}}\right)^2 = \frac{1}{n^2}\sum_{j=1}^{n}\left(\frac{p_{vj} \cos^2\psi_j \cos^2\phi_j}{p_{vx}}\right)^2 \left(\frac{d\left(p_{vj} \cos^2\psi_j \cos^2\phi_j\right)}{p_{vj} \cos^2\psi_j \cos^2\phi_j}\right)^2 \tag{D-5-8}$$

For convenience let

$$\overline{p}_{vj} = p_{vj} \cos^2\psi_j \cos^2\phi_j \tag{D-5-9}$$

$$\left(\frac{dp_{vx}}{p_{vx}}\right)^2 = \frac{1}{n^2}\sum_{j=1}^{n}\left(\frac{p_{vj}\cos^2\psi_j\cos^2\phi_j}{p_{vx}}\right)^2\left(\frac{U_{\bar{p}_{vx}}}{\bar{p}_{vx}}\right)^2 \tag{D-5-10}$$

Combining uncertainties per Kline and McClintock

$$\hat{u}^2_{\bar{p}_{vx}} = \left(\frac{\hat{U}_{\bar{p}_{vx}}}{\bar{p}_{vx}}\right)^2 = \left(\frac{1}{\bar{p}^2_{vx}}\right)\left[\left(\frac{\partial\bar{p}_{vx}}{\partial p_{vj}}\right)^2\hat{U}^2_{p_{vj}} + \left(\frac{\partial\bar{p}_{vx}}{\partial\psi_j}\right)^2\hat{U}^2_{\psi_j} + \left(\frac{\partial\bar{p}_{vx}}{\partial\phi_j}\right)^2\hat{U}^2_{\phi_j}\right] \tag{D-5-11}$$

$$\left(\frac{1}{\bar{p}^2_{vx}}\right)\left(\frac{\partial\bar{p}_{vx}}{\partial p_{vj}}\right)^2\hat{U}^2_{p_{vj}} = \left(\frac{\cos^2\psi_j\cos^2\phi_j}{p_{vj}\cos^2\psi_j\cos^2\phi_j}\right)^2\hat{U}^2_{p_{vj}} = \left(\frac{\hat{U}_{p_{vj}}}{p_{vj}}\right)^2 = \hat{u}^2_{p_{vj}} \tag{D-5-12}$$

$$\left(\frac{1}{\bar{p}^2_{vx}}\right)\left(\frac{\partial\bar{p}_{vx}}{\partial\psi_j}\right)^2\hat{U}^2_{\psi_j} = \left(\frac{-2p_{vj}\sin\psi_j\cos\psi_j\cos^2\phi_j}{p_{vj}\cos^2\psi_j\cos^2\phi_j}\right)^2\hat{U}^2_{\psi_j} = \left(-2\tan\psi_j\right)^2\hat{U}^2_{\psi_j} = 4\tan^2\psi_j\hat{U}^2_{\psi_j} \tag{D-5-13}$$

$$\left(\frac{1}{\bar{p}^2_{vx}}\right)\left(\frac{\partial\bar{p}_{vx}}{\partial\phi_j}\right)^2\hat{U}^2_{\phi_j} = \left(\frac{-2p_{vj}\sin\phi_j\cos\phi_j\cos^2\psi_j}{p_{vj}\cos^2\psi_j\cos^2\phi_j}\right)^2\hat{U}^2_{\phi_j} = \left(-2\tan\phi_j\right)^2\hat{U}^2_{\phi_j} = 4\tan^2\phi_j\hat{U}^2_{\phi_j} \tag{D-5-14}$$

Substituting in equation (D-5-6)

$$\left(\frac{dp_{vx}}{p_{vx}}\right)^2 = \frac{1}{n^2}\sum_{j=1}^{n}\left(\frac{p_{vj}\cos^2\psi_j\cos^2\phi_j}{p_{vx}}\right)^2\left(\hat{u}^2_{p_{vj}} + 4\tan^2\psi_j\hat{U}^2_{\psi_j} + 4\tan^2\phi_j\hat{U}^2_{\phi_j}\right) \tag{D-5-15}$$

Inserting unit conversions

$$\left(\hat{u}_{p_{vx}}\right)^2 = \frac{1}{n^2}\sum_{j=1}^{n}\left(\frac{p_{vj}\cos^2\psi_j\cos^2\phi_j}{p_{vx}}\right)^2\left[\hat{u}^2_{p_{vj}} + 4\left(\frac{\tan^2\psi_j\hat{U}^2_{\psi_j} + \tan^2\phi_j\hat{U}^2_{\phi_j}}{57.30^2}\right)\right] \tag{5-13-13}$$

D-6 UNCERTAINTY IN THE AVERAGE TOTAL PRESSURE, p_{tx}, AT PLANE x

The equation for p_{tx} is given in Section 5 as

$$p_{tx} = p_{sx} + p_{vx} \tag{5-7-7}$$

From the derivation of $u_{p_{sx}}$

$$\left(dp_{sx}\right)^2 = \frac{1}{n^2}\sum_{j=1}^{n}\left(dp_{sj}\right)^2 \tag{D-6-1}$$

From the derivation of $u_{p_{vx}}$

$$\left(dp_{vx}\right)^2 = \frac{1}{n^2}\sum_{j=1}^{n}\left(d\left(p_{vj}\cos^2\psi_j\cos^2\phi_j\right)\right)^2 \tag{D-6-2}$$

Combining uncertainties per Kline and McClintock

$$\left(dp_{tx}\right)^2 = \left(dp_{sx}\right)^2 + \left(dp_{vx}\right)^2 \tag{D-6-3}$$

$$\left(dp_{tx}\right)^2 = \frac{1}{n^2}\sum_{j=1}^{n}\left(dp_{sj}\right)^2 + \frac{1}{n^2}\sum_{j=1}^{n}\left(d\left(p_{vj}\cos^2\psi_j\cos^2\phi_j\right)\right)^2 \qquad \text{(D-6-4)}$$

Dividing by $\left(p_{tx}\right)^2$

$$\left(\frac{dp_{tx}}{p_{tx}}\right)^2 = \frac{1}{n^2}\sum_{j=1}^{n}\left(\frac{dp_{sj}}{p_{tx}}\right)^2 + \frac{1}{n^2}\sum_{j=1}^{n}\left(\frac{d\left(p_{vj}\cos^2\psi_j\cos^2\phi_j\right)}{p_{tx}}\right)^2 \qquad \text{(D-6-5)}$$

Expanding

$$\left(\frac{dp_{tx}}{p_{tx}}\right)^2 = \frac{1}{n^2}\sum_{j=1}^{n}\left(\frac{dp_{sj}}{p_{tx}}\right)^2\left(\frac{p_{sx}}{p_{sx}}\right)^2 + \frac{1}{n^2}\sum_{j=1}^{n}\left(\frac{d\left(p_{vj}\cos^2\psi_j\cos^2\phi_j\right)}{p_{tx}}\right)^2\left(\frac{p_{vx}}{p_{vx}}\right)^2 \qquad \text{(D-6-6)}$$

$$\left(\frac{dp_{tx}}{p_{tx}}\right)^2 = \frac{1}{n^2}\sum_{j=1}^{n}\left(\frac{p_{sx}}{p_{tx}}\right)^2\left(\frac{dp_{sx}}{p_{sx}}\right)^2 + \frac{1}{n^2}\sum_{j=1}^{n}\left(\frac{p_{vx}}{p_{tx}}\right)^2\left(\frac{d\left(p_{vj}\cos^2\psi_j\cos^2\phi_j\right)}{p_{vx}}\right)^2 \qquad \text{(D-6-7)}$$

From the derivation of $u_{p_{vx}}^2$

$$\left(\frac{d\left(p_{vj}\cos^2\psi_j\cos^2\phi_j\right)}{p_{vx}}\right)^2 = \hat{u}_{p_{vj}}^2 + 4\left(\frac{\tan^2\psi_j\hat{U}_{\psi_j}^2 + \tan^2\phi_j\hat{U}_{\phi_j}^2}{57.30^2}\right) \qquad \text{(D-6-8)}$$

$$\left(\hat{u}_{p_{tx}}\right)^2 = \frac{1}{n^2}\left\{\sum_{j=1}^{n}\left(\frac{p_{sx}}{p_{tx}}\right)^2\left(\hat{u}_{p_{sx}}\right)^2 + \frac{1}{n^2}\sum_{j=1}^{n}\left(\frac{p_{vx}}{p_{tx}}\right)^2\left[\hat{u}_{p_{vj}}^2 + 4\left(\frac{\tan^2\psi_j\hat{U}_{\psi_j}^2 + \tan^2\phi_j\hat{U}_{\phi_j}^2}{57.30^2}\right)\right]\right\} \qquad \text{(5-13-14)}$$

D-7 UNCERTAINTY IN THE AVERAGE ABSOLUTE STATIC PRESSURE, p_{sax}, AT PLANE *x*

The equation for p_{sax} is given in Section 5 as

$$p_{sax} = p_{sx} + C_{13}p_b \qquad \text{(5-7-8)}$$

Differentiating

$$dp_{sax} = dp_{sx} + C_{13}dp_b \qquad \text{(D-7-1)}$$

Combining uncertainties per Kline and McClintock

$$\left(dp_{sax}\right)^2 = \left(dp_{sx}\right)^2 + \left(C_{13}dp_b\right)^2 \qquad \text{(D-7-2)}$$

Dividing by $\left(p_{sax}\right)^2$

$$\frac{\left(dp_{sax}\right)^2}{\left(p_{sax}\right)^2} = \frac{\left(dp_{sx}\right)^2 + \left(C_{13}dp_b\right)^2}{\left(p_{sax}\right)^2} \qquad \text{(D-7-3)}$$

In uncertainties terms

$$\hat{u}_{psax}^2 = \frac{\hat{U}_{p_{sx}}^2 + C_{13}^2 \hat{U}_{p_b}^2}{p_{sax}^2} \tag{5-13-15}$$

Similarly

$$\hat{u}_{ptax}^2 = \frac{\hat{U}_{p_{tx}}^2 + C_{13}^2 \hat{U}_{p_b}^2}{p_{tax}^2}$$

D-8 UNCERTAINTY IN THE INPUT POWER, P_I, FOR A CALIBRATED AC MOTOR

The equation for P_I is given in Section 5 as

$$P_I = \frac{10^3 W \eta_M}{C_{14}} \tag{5-8-1}$$

Differentiating

$$dP_I = \left(W d\eta_M + \eta_M dW\right)\frac{10^3}{C_{14}} \tag{D-8-1}$$

Substituting for W and η_M

$$dP_I = \left(\frac{P_I}{\eta_M} d\eta_M + \frac{P_I}{W} dW\right) \tag{D-8-2}$$

Dividing by P_I, squaring, and setting cross product terms to zero

$$\left(\frac{dP_I}{P_I}\right)^2 = \left[\left(\frac{d\eta_M}{\eta_M}\right)^2 + \left(\frac{dW}{W}\right)^2\right] \tag{D-8-3}$$

In terms of relative uncertainties, the result is

$$\hat{u}_{P_I}^2 = \hat{u}_{\eta_M}^2 + \hat{u}_W^2 \tag{5-13-16}$$

Equations (5-13-17) through (5-13-22) are derived in a similar manner as are eqs. (5-13-26) through (5-13-32), (5-13-34) through (5-13-36), and (5-13-38) through (5-13-46).

D-9 UNCERTAINTY IN THE FAN MEAN DENSITY, ρ_m

The equation for ρ_m is given in Section 5 as

$$\rho_m = \frac{\rho_1 + \rho_2}{2} \tag{5-10-1}$$

Differentiating

$$d\rho_m = \frac{1}{2}\left(d\rho_1 + d\rho_2\right) \tag{D-9-1}$$

Squaring and dropping cross product terms

$$d\rho_m^2 = \frac{1}{4}\left(d\rho_1^2 + d\rho_2^2\right) \tag{D-9-2}$$

Dividing by ρ_m^2

$$\left(\frac{d\rho_m}{\rho_m}\right)^2 = \frac{d\rho_1^2 + d\rho_2^2}{\left(\rho_1 + \rho_2\right)^2} \tag{D-9-3}$$

Writing in terms of uncertainties

$$\hat{u}_{\rho_m}^2 = \frac{\hat{U}_{\rho_1}^2 + \hat{U}_{\rho_2}^2}{\left(\rho_1 + \rho_2\right)^2} \tag{5-13-21}$$

D-10 UNCERTAINTY IN THE FAN SPECIFIC ENERGY, y_F

The equation for y_F is given in Section 5 as

$$y_F = \frac{p_{s2} - p_{s1}}{\rho_m} + e_{K2} - e_{K1} \tag{5-10-2}$$

Reducing to primary values

$$\rho_m = \frac{\rho_2 + \rho_1}{2} \tag{D-10-1}$$

$$y_F = \frac{2\left(p_{s2} - p_{s1}\right)}{\rho_2 + \rho_1} + \frac{p_{v2}}{\rho_2} - \frac{p_{v1}}{\rho_1} = \frac{2R\left(p_{s2} - p_{s1}\right)}{\dfrac{p_{s2} + p_b}{T_{s2}} + \dfrac{p_{s1} + p_b}{T_{s1}}} + \frac{p_{v2} R T_{s2}}{p_{s2} + p_b} - \frac{p_{v1} R T_{s1}}{p_{s1} + p_b} \tag{D-10-2}$$

Combining uncertainties per Kline and McClintock

$$\hat{u}_{y_F}^2 = \left(\frac{1}{y_F^2}\right)\left[\begin{array}{l} \left(\dfrac{\partial y_F}{\partial R}\right)^2 \hat{U}_R^2 + \left(\dfrac{\partial y_F}{\partial T_{s1}}\right)^2 \hat{U}_{T_1}^2 + \left(\dfrac{\partial y_F}{\partial T_{s2}}\right)^2 \hat{U}_{T_2}^2 \\ + \left(\dfrac{\partial y_F}{\partial p_b}\right)^2 \hat{U}_{p_b}^2 + \left(\dfrac{\partial y_F}{\partial p_{s1}}\right)^2 \hat{U}_{p_{s1}}^2 + \left(\dfrac{\partial y_F}{\partial p_{s2}}\right)^2 \hat{U}_{p_{s2}}^2 \\ + \left(\dfrac{\partial y_F}{\partial p_{v1}}\right)^2 \hat{U}_{p_{v1}}^2 + \left(\dfrac{\partial y_F}{\partial p_{v1}}\right)^2 \hat{U}_{p_{v1}}^2 \end{array}\right] \tag{D-10-3}$$

Since y_F is directly proportional to R

$$\left(\frac{1}{y_F^2}\right)\left(\frac{\partial y_F}{\partial R}\right)^2 \hat{U}_R^2 = \hat{u}_R^2 \tag{D-10-4}$$

For T_{s1} and T_{s2}

$$\left(\frac{\partial y_F}{\partial T_{s1}}\right)^2 = \left(\frac{2R\left(p_{s2}-p_{s1}\right)}{\left(\frac{p_{s2}+p_b}{T_{s2}}+\frac{p_{s1}+p_b}{T_{s1}}\right)^2}\left(\frac{\left(p_{s1}+p_b\right)}{T_{s1}}\right)-\frac{p_{v1}R_1}{\left(p_{s1}+p_b\right)}\right)^2 \quad \text{(D-10-5)}$$

Combining and multiplying by $\left(T_{s1}/T_{s1}\right)^2$, $\left(R/R\right)$, and $\left(2/2\right)$ where appropriate

$$\left(\frac{\partial y_F}{\partial T_{s1}}\right)^2 = \left(\frac{1}{T_{s1}}\right)^2\left(\frac{4\left(p_{s2}-p_{s1}\right)}{2\left(\frac{p_{s2}+p_b}{RT_{s2}}+\frac{p_{s1}+p_b}{RT_{s1}}\right)^2}\left(\frac{\left(p_{s1}+p_b\right)}{RT_{s1}^2}\right)T_{s1}-\frac{p_{v1}R_1T_{s1}}{\left(p_{s1}+p_b\right)}\right)^2 \quad \text{(D-10-6)}$$

Expanding and combining

$$\left(\frac{1}{y_F^2}\right)\left(\frac{\partial y_F}{\partial T_{s1}}\right)^2\hat{U}_{T_{s1}}^2 = \left(\frac{1}{y_F}\right)^2\left(\frac{\hat{U}_{T_{s1}}}{T_{s1}}\right)^2\left(\frac{\left(p_{s2}-p_{s1}\right)}{2\rho_m^2}\left(\rho_1\right)-\frac{p_{v1}}{\rho_1}\right)^2 \quad \text{(D-10-7)}$$

$$\left(\frac{1}{y_F^2}\right)\left(\frac{\partial y_F}{\partial T_{s1}}\right)^2\hat{U}_{T_{s1}}^2 = \left(\frac{1}{y_F}\right)^2\left(\frac{\rho_1\left(p_{s2}-p_{s1}\right)}{2\rho_m^2}-\frac{p_{v1}}{\rho_1}\right)^2\hat{u}_{T_{s1}}^2 \quad \text{(D-10-8)}$$

Similarly,

$$\left(\frac{1}{y_F^2}\right)\left(\frac{\partial y_F}{\partial T_{s2}}\right)^2\hat{U}_{T_{s2}}^2 = \left(\frac{1}{y_F}\right)^2\left(\frac{\rho_2\left(p_{s2}-p_{s1}\right)}{2\rho_m^2}+\frac{p_{v2}}{\rho_2}\right)^2\hat{u}_{T_{s2}}^2 \quad \text{(D-10-9)}$$

For p_b

$$\left(\frac{\partial y_F}{\partial p_b}\right)^2 = \left(\frac{-2R\left(p_{s2}-p_{s1}\right)}{\left(\frac{p_{s2}+p_b}{T_{s2}}+\frac{p_{s1}+p_b}{T_{s1}}\right)^2}\left(\frac{1}{T_{s2}}+\frac{1}{T_{s1}}\right)-\left(\frac{p_{v2}RT_{s2}}{\left(p_{s2}+p_b\right)^2}\right)+\left(\frac{p_{v1}RT_{s1}}{\left(p_{s1}+p_b\right)^2}\right)\right)^2 \quad \text{(D-10-10)}$$

Multiplying by $\left(p_b/p_b\right)^2$, $\left(R/R\right)$, and $\left(p_{sax}/p_{sax}\right)$ where appropriate

$$\left(\frac{\partial P_O}{\partial p_b}\right)^2 = \left(\frac{1}{p_b}\right)^2\left(-\frac{\left(p_{s2}-p_{s1}\right)}{2\rho_m^2}\left(\frac{p_b}{RT_{s2}}+\frac{p_b}{RT_{s1}}\right)-\left(\frac{p_{v2}p_b}{\rho_2 p_{sa2}}\right)+\left(\frac{p_{v1}p_b}{\rho_1 p_{sa1}}\right)\right)^2 \quad \text{(D-10-11)}$$

Combining

$$\left(\frac{\partial y_F}{\partial p_b}\right)^2 = \left(\frac{1}{p_b}\right)^2\left(-\frac{\left(p_{s2}-p_{s1}\right)}{2\rho_m^2}\left(\frac{p_b}{RT_{s2}}+\frac{p_b}{RT_{s1}}\right)-\left(\frac{p_{v2}p_b}{\rho_2 p_{sa2}}\right)+\left(\frac{p_{v1}p_b}{\rho_1 p_{sa1}}\right)\right)^2 \quad \text{(D-10-12)}$$

Expanding and combining

$$\left(\frac{1}{y_F^2}\right)\left(\frac{\partial y_F}{\partial p_b}\right)^2\hat{U}_{p_b}^2 = \left(\frac{\hat{U}_{p_b}^2}{p_b}\right)^2\left(\frac{1}{y_F}\right)^2\left(-\frac{\left(p_{s2}-p_{s1}\right)}{2\rho_m^2}\left(\frac{p_b}{RT_{s2}}+\frac{p_b}{RT_{s1}}\right)-\left(\frac{p_{v2}p_b}{\rho_2 p_{sa2}}\right)+\left(\frac{p_{v1}p_b}{\rho_1 p_{sa1}}\right)\right)^2 \quad \text{(D-10-13)}$$

$$\left(\frac{1}{y_F^2}\right)\left(\frac{\partial y_F}{\partial p_b}\right)^2\hat{U}_{p_b}^2 = \left(\frac{1}{y_F}\right)^2\left(-\frac{\left(p_{s2}-p_{s1}\right)}{2\rho_m^2}\left(\frac{p_b}{RT_{s2}}+\frac{p_b}{RT_{s1}}\right)-\left(\frac{p_{v2}p_b}{\rho_2 p_{sa2}}\right)+\left(\frac{p_{v1}p_b}{\rho_1 p_{sa1}}\right)\right)^2\hat{u}_{p_b}^2 \quad \text{(D-10-14)}$$

For p_{s1} and p_{s2}

$$\left(\frac{\partial y_F}{\partial p_{s1}}\right)^2=\left(\frac{-2R}{\frac{p_{s2}+p_b}{T_{s2}}+\frac{p_{s1}+p_b}{T_{s1}}}-\frac{2R\left(p_{s2}-p_{s1}\right)}{\left(\frac{p_{s2}+p_b}{T_{s2}}+\frac{p_{s1}+p_b}{T_{s1}}\right)^2}\left(\frac{1}{T_{s1}}\right)+\frac{p_{v1}RT_{s1}}{\left(p_{s1}+p_b\right)^2}\right)^2 \tag{D-10-15}$$

Multiplying by $\left(p_{s1}/p_{s1}\right)^2$, $\left(p_{sa1}/p_{sa1}\right)$, $\left(R/R\right)$, and $\left(2/2\right)$ where appropriate

$$\left(\frac{\partial y_F}{\partial p_{s1}}\right)^2=\frac{1}{p_{s1}^2}\left(\frac{-2Rp_{s1}}{\frac{p_{s2}+p_b}{T_{s2}}+\frac{p_{s1}+p_b}{T_{s1}}}-\frac{p_{sa1}4\left(p_{s2}-p_{s1}\right)p_{s1}}{p_{sa1}RT_{s1}2\left(\frac{p_{s2}+p_b}{RT_{s2}}+\frac{p_{s1}+p_b}{RT_{s1}}\right)^2}+\frac{p_{sa1}}{p_{sa1}}\frac{p_{v1}RT_{s1}p_{s1}}{\left(p_{s1}+p_b\right)^2}\right)^2 \tag{D-10-16}$$

Combining

$$\left(\frac{\partial y_F}{\partial p_{s1}}\right)^2=\frac{1}{p_{s1}^2}\left(-\frac{p_{s1}}{\rho_m}-\frac{\rho_1\left(p_{s2}-p_{s1}\right)p_{s1}}{2\rho_m^2 p_{sa1}}+\frac{p_{s1}}{p_{sa1}}\frac{p_{v1}}{\rho_1}\right)^2 \tag{D-10-17}$$

Expanding and combining further

$$\left(\frac{1}{y_F^2}\right)\left(\frac{\partial y_F}{\partial p_{s1}}\right)^2\hat{U}_{p_{s1}}^2=\left(\frac{1}{y_F^2}\right)\frac{\hat{U}_{p_{s1}}^2}{p_{s1}^2}\left(-\frac{p_{s1}}{\rho_m}-\frac{\rho_1\left(p_{s2}-p_{s1}\right)}{2\rho_m^2}\frac{p_{s1}}{p_{sa1}}+\frac{p_{v1}}{\rho_1}\frac{p_{s1}}{p_{sa1}}\right)^2 \tag{D-10-18}$$

$$\left(\frac{1}{y_F^2}\right)\left(\frac{\partial y_F}{\partial p_{s1}}\right)^2\hat{U}_{p_{s1}}^2=\left(\frac{1}{y_F^2}\right)\left(-\frac{p_{s1}}{\rho_m}-\frac{\rho_1\left(p_{s2}-p_{s1}\right)}{2\rho_m^2}\frac{p_{s1}}{p_{sa1}}+\frac{p_{v1}}{\rho_1}\frac{p_{s1}}{p_{sa1}}\right)^2\hat{u}_{ps1}^2 \tag{D-10-19}$$

Similarly,

$$\left(\frac{1}{y_F^2}\right)\left(\frac{\partial y_F}{\partial p_{s2}}\right)^2\hat{U}_{p_{s2}}^2=\left(\frac{1}{y_F^2}\right)\left(\frac{p_{s2}}{\rho_m}-\frac{\rho_2\left(p_{s2}-p_{s1}\right)}{2\rho_m^2}\frac{p_{s2}}{p_{sa2}}-\frac{p_{v2}}{\rho_2}\frac{p_{s2}}{p_{sa2}}\right)^2\hat{u}_{p_{s2}}^2 \tag{D-10-20}$$

For p_{v1} and p_{v2}

$$\left(\frac{\partial y_F}{\partial p_{v1}}\right)^2=\left(\frac{1}{\rho_1}\right)^2 \tag{D-10-21}$$

Multiplying by $\left(p_{v1}/p_{v1}\right)^2$ and expanding

$$\left(\frac{1}{y_F^2}\right)\left(\frac{\partial y_F}{\partial p_{v1}}\right)^2\hat{U}_{p_{v1}}^2=\left(\frac{1}{y_F^2}\right)\left(\frac{\hat{U}_{p_{v1}}}{p_{v1}}\right)^2\left(\frac{p_{v1}}{\rho_1}\right)^2=\left(\frac{1}{y_F^2}\right)\left(\frac{p_{v1}}{\rho_1}\right)^2\hat{u}_{p_{v1}}^2 \tag{D-10-22}$$

Similarly

$$\left(\frac{1}{y_F^2}\right)\left(\frac{\partial y_F}{\partial p_{v2}}\right)^2\hat{U}_{p_{v2}}^2=\left(\frac{1}{y_F^2}\right)\left(\frac{p_{v2}}{\rho_1}\right)^2\hat{u}_{p_{v2}}^2 \tag{D-10-23}$$

Gathering terms

$$\hat{u}_{y_F}^2 = \hat{u}_R^2 + \left(\frac{1}{y_F^2}\right) \left[\begin{aligned} &\left(\frac{\rho_1 (p_{s2} - p_{s1})}{2\rho_m^2} - \frac{p_{v1}}{\rho_1}\right)^2 \hat{u}_{T_{s1}}^2 + \left(\frac{\rho_2 (p_{s2} - p_{s1})}{2\rho_m^2} + \frac{p_{v2}}{\rho_2}\right)^2 \hat{u}_{T_{s2}}^2 \\ &+ \left(\frac{p_{v1} p_b}{\rho_1 p_{sa1}} - \frac{(p_{s2} - p_{s1})}{2\rho_m^2}\left(\frac{p_b}{RT_{s1}} + \frac{p_b}{RT_{s2}}\right) - \frac{p_{v2} p_b}{\rho_2 p_{sa2}}\right)^2 \hat{u}_{p_b}^2 \\ &+ \left(\frac{p_{v1}}{\rho_1}\frac{p_{s1}}{p_{sa1}} - \frac{\rho_1 (p_{s2} - p_{s1})}{2\rho_m^2}\frac{p_{s1}}{p_{sa1}} - \frac{p_{s1}}{\rho_m}\right)^2 \hat{u}_{ps1}^2 \\ &+ \left(\frac{p_{s2}}{\rho_m} - \frac{\rho_2 (p_{s2} - p_{s1})}{2\rho_m^2}\frac{p_{s2}}{p_{sa2}} - \frac{p_{v2}}{\rho_2}\frac{p_{s2}}{p_{sa2}}\right)^2 \hat{u}_{p_{s2}}^2 \\ &+ \left(\frac{p_{v1}}{\rho_1}\right)^2 \hat{u}_{p_{v1}}^2 + \left(\frac{p_{v2}}{\rho_1}\right)^2 \hat{u}_{p_{v2}}^2 \end{aligned} \right] \quad \text{(D-10-24)}$$

Inserting unit conversions

$$\hat{u}_{y_F}^2 = \hat{u}_R^2 + \left(\frac{C_{11}}{y_F}\right)^2 \left[\begin{aligned} &\left(\frac{\rho_1 (p_{s2} - p_{s1})}{2\rho_m^2} - \frac{p_{v1}}{\rho_1}\right)^2 \hat{u}_{T_{s1}}^2 + \left(\frac{\rho_2 (p_{s2} - p_{s1})}{2\rho_m^2} + \frac{p_{v2}}{\rho_2}\right)^2 \hat{u}_{T_{s2}}^2 \\ &+ \left(\frac{p_{v1} C_{13} p_b}{\rho_1 p_{sa1}} - \frac{C_{11} (p_{s2} - p_{s1})}{2\rho_m^2}\left(\frac{C_{13} p_b}{RT_{s1}} + \frac{C_{13} p_b}{RT_{s2}}\right) - \frac{p_{v2} C_{13} p_b}{\rho_2 p_{sa2}}\right)^2 \hat{u}_{p_b}^2 \\ &+ \left(\frac{p_{v1}}{\rho_1}\frac{p_{s1}}{p_{sa1}} - \frac{\rho_1 (p_{s2} - p_{s1})}{2\rho_m^2}\frac{p_{s1}}{p_{sa1}} - \frac{p_{s1}}{\rho_m}\right)^2 u_{ps1}^2 \\ &+ \left(\frac{p_{s2}}{\rho_m} - \frac{\rho_2 (p_{s2} - p_{s1})}{2\rho_m^2}\frac{p_{s2}}{p_{sa2}} - \frac{p_{v2}}{\rho_2}\frac{p_{s2}}{p_{sa2}}\right)^2 \hat{u}_{p_{s2}}^2 \\ &+ \left(\frac{p_{v1}}{\rho_1}\right)^2 \hat{u}_{p_{v1}}^2 + \left(\frac{p_{v2}}{\rho_1}\right)^2 \hat{u}_{p_{v2}}^2 \end{aligned} \right] \quad \text{(5-13-23)}$$

D-11 UNCERTAINTY IN THE FAN OUTPUT POWER, P_O

The equation for P_O is given in Section 5 as

$$P_O = \dot{m}_F y_F \quad \text{(5-10-3)}$$

Reducing to primary variables

$$\dot{m}_F = w_1 \dot{m}_1 + w_2 \dot{m}_2 + w_3 \dot{m}_3 \quad \text{(D-11-1)}$$

$$\dot{m}_1 = F_{n1} A_1 \rho_1 V_1 = F_{n1} A_1 \rho_1 \left(\frac{p_{v1}}{\rho_1}\right)^{1/2} = F_{n1} A_1 (\rho_1 p_{v1})^{1/2} \quad \text{(D-11-2)}$$

$\dot{m}_2$ and $\dot{m}_3$ are the same except for subscripts.

$$\rho_1 = \frac{p_{sa1}}{RT_{s1}} = \frac{(p_{s1} + p_b)}{RT_{s1}} \quad \text{(D-11-3)}$$

ρ_2 and ρ_3 are the same except for subscripts.

$$w_1 \dot{m}_1 = w_1 F_{n1} A_1 \left(\frac{p_{v1}\left(p_{s1} + p_b\right)}{RT_{s1}} \right)^{1/2} \tag{D-11-4}$$

$w_2 \dot{m}_2$ and $w_3 \dot{m}_3$ are the same except for subscripts.

$$y_F = \frac{\left(p_{s2} - p_{s1}\right)}{\rho_m} + e_{K1} - e_{K2} = \frac{2R\left(p_{s2} - p_{s1}\right)}{\frac{p_{s2} + p_b}{T_{s2}} + \frac{p_{s1} + p_b}{T_{s1}}} + \frac{p_{v2} RT_{s2}}{p_{s2} + p_b} - \frac{p_{v1} RT_{s1}}{p_{s1} + p_b} \tag{D-11-5}$$

$$P_O = \left[\begin{array}{c} \left(w_1 F_{n1} A_1 \left(\frac{p_{v1}\left(p_{s1} + p_b\right)}{RT_{s1}} \right)^{1/2} + w_2 F_{n2} A_2 \left(\frac{p_{v2}\left(p_{s2} + p_b\right)}{RT_{s2}} \right)^{1/2} + F_{n3} w_3 A_3 \left(\frac{p_{v3}\left(p_{s3} + p_b\right)}{RT_{s3}} \right)^{1/2} \right) \\ \left(\frac{2R\left(p_{s2} - p_{s1}\right)}{\frac{p_{s2} + p_b}{T_{s2}} + \frac{p_{s1} + p_b}{T_{s1}}} + \frac{p_{v2} RT_{s2}}{p_{s2} + p_b} - \frac{p_{v1} RT_{s1}}{p_{s1} + p_b} \right) \end{array} \right] \tag{D-11-6}$$

Combining uncertainties per Kline and McClintock

$$\hat{u}_{P_O}^2 = \left(\frac{C_{11}}{P_O^2} \right) \left[\begin{array}{l} \left(\frac{\partial P_O}{\partial R} \right)^2 \hat{U}_R^2 + \left(\frac{\partial P_O}{\partial F_{n1}} \right)^2 \hat{U}_{F_{n1}}^2 + \left(\frac{\partial P_O}{\partial F_{n2}} \right)^2 \hat{U}_{F_{n2}}^2 + \left(\frac{\partial P_O}{\partial F_{n3}} \right)^2 \hat{U}_{F_{n3}}^2 \\ \left(\frac{\partial P_O}{\partial A_1} \right)^2 \hat{U}_{A_1}^2 + \left(\frac{\partial P_O}{\partial A_2} \right)^2 \hat{U}_{A_2}^2 + \left(\frac{\partial P_O}{\partial A_3} \right)^2 \hat{U}_{A_3}^2 \\ + \left(\frac{\partial P_O}{\partial T_{s1}} \right)^2 \hat{U}_{T_{s1}}^2 + \left(\frac{\partial P_O}{\partial T_{s2}} \right)^2 \hat{U}_{T_{s2}}^2 + \left(\frac{\partial P_O}{\partial T_{s3}} \right)^2 \hat{U}_{T_{s3}}^2 \\ + \left(\frac{\partial P_O}{\partial p_b} \right)^2 \hat{U}_{p_b}^2 + \left(\frac{\partial P_O}{\partial p_{s1}} \right)^2 \hat{U}_{p_{s1}}^2 + \left(\frac{\partial P_O}{\partial p_{s2}} \right)^2 \hat{U}_{p_{s2}}^2 + \left(\frac{\partial P_O}{\partial p_{s3}} \right)^2 \hat{U}_{p_{s3}}^2 \\ + \left(\frac{\partial P_O}{\partial p_{v1}} \right)^2 \hat{U}_{p_{v1}}^2 + \left(\frac{\partial P_O}{\partial p_{v2}} \right)^2 \hat{U}_{p_{v2}}^2 + \left(\frac{\partial P_O}{\partial p_{v3}} \right)^2 \hat{U}_{p_{v3}}^2 \end{array} \right] \tag{D-11-7}$$

For R

$$\frac{\partial P_O}{\partial R} = -\left(w_1 F_{n1} A_1 \frac{1}{2} \frac{1}{R^{3/2}} \left(\frac{p_{v1}\left(p_{s1} + p_b\right)}{T_{s1}} \right)^{1/2} + \cdots \right) y_F + \dot{m}_F \left(\frac{2\left(p_{s2} - p_{s1}\right)}{\frac{p_{s2} + p_b}{T_{s2}} + \frac{p_{s1} + p_b}{T_{s1}}} + \frac{p_{v2} T_{s2}}{p_{s2} + p_b} - \frac{p_{v1} T_{s1}}{p_{s1} + p_b} \right) \tag{D-11-8}$$

Multiplying by (R/R) and combining

$$\frac{\partial P_O}{\partial R} = -\left(w_1 F_{n1} A_1 \frac{1}{2} \frac{1}{R} \left(\frac{p_{v1}\left(p_{s1} + p_b\right)_1}{RT_{s1}} \right)^{1/2} + \cdots \right) y_F + \dot{m}_F \left(\frac{1}{R} \right) \left(\frac{2R\left(p_{s2} - p_{s1}\right)}{\frac{p_{s2} + p_b}{T_{s2}} + \frac{p_{s1} + p_b}{T_{s1}}} + \frac{p_{v2} RT_{s2}}{p_{s2} + p_b} - \frac{p_{v1} RT_{s1}}{p_{s1} + p_b} \right) \tag{D-11-9}$$

Combining

$$\frac{\partial P_O}{\partial R} = -\frac{1}{2}\left(w_1 F_{n1} A_1 \left(\rho_1 p_{v1}\right)^{1/2} + \cdots\right)\left(\frac{1}{R}\right) y_F + \dot{m}_F \left(\frac{1}{R}\right) y_F \tag{D-11-10}$$

Expanding and combining

$$\left(\frac{1}{P_O}\right)\left(\frac{\partial P_O}{\partial R}\right)\hat{U}_R = -\frac{1}{2}\left(\frac{1}{P_O}\right)\dot{m}_F y_F \left(\frac{\hat{U}_R}{R}\right) + \left(\frac{1}{P_O}\right)\dot{m}_F y_F \left(\frac{\hat{U}_R}{R}\right) \tag{D-11-11}$$

$$\left(\frac{1}{P_O}\right)\left(\frac{\partial P_O}{\partial R}\right)\hat{U}_R = -\frac{1}{2}\hat{u}_R + \hat{u}_R = \frac{1}{2}\hat{u}_R \tag{D-11-12}$$

$$\left(\frac{1}{P_O}\right)^2 \left(\frac{\partial P_O}{\partial R}\right)^2 \hat{U}_R^2 = \frac{1}{4}\hat{u}_R^2 \tag{D-11-13}$$

For A_1, A_2, and A_3

$$\frac{\partial P_O}{\partial A_1} = w_1 F_{n1}\left(\frac{p_{v1}\left(p_{s1} + p_b\right)}{RT_{s1}}\right)^{1/2} y_F \tag{D-11-14}$$

Multiplying by $\left(A_1/A_1\right)$ and combining

$$\frac{\partial P_O}{\partial A_1} = \frac{1}{A_1} w_1 F_{n1} A_1 \left(\rho_1 p_{v1}\right)^{1/2} y_F = \frac{1}{A_1} w_1 m_1 y_F \tag{D-11-15}$$

Expanding and combining

$$\left(\frac{1}{P_O}\right)\left(\frac{\partial P_O}{\partial A_1}\right)\hat{U}_{A_1} = \frac{\hat{U}_{A_1}}{A_1}\left(\frac{w_1 \dot{m}_1}{\dot{m}_F}\right) \tag{D-11-16}$$

$$\left(\frac{1}{P_O^2}\right)\left(\frac{\partial P_O}{\partial A_1}\right)^2 \hat{U}_{A_1}^2 = \left(\frac{w_1 \dot{m}_1}{\dot{m}_F}\right)^2 \hat{u}_{A_1}^2 \tag{D-11-17}$$

Similarly,

$$\left(\frac{1}{P_O^2}\right)\left(\frac{\partial P_O}{\partial A_2}\right)^2 \hat{U}_{A_2}^2 = \left(\frac{w_2 \dot{m}_2}{\dot{m}_F}\right)^2 \hat{u}_{A_2}^2 \tag{D-11-18}$$

and

$$\left(\frac{1}{P_O^2}\right)\left(\frac{\partial P_O}{\partial A_3}\right)^2 \hat{U}_{A_3}^2 = \left(\frac{w_3 \dot{m}_3}{\dot{m}_F}\right)^2 \hat{u}_{A_3}^2 \tag{D-11-19}$$

For F_{n1}, F_{n2}, and F_{n3} using analysis similar to that for A_1, etc.

$$\left(\frac{1}{P_O^2}\right)\left(\frac{\partial P_O}{\partial F_{n1}}\right)^2 \hat{U}_{F_{n1}}^2 = \left(\frac{w_1 \dot{m}_1}{\dot{m}_F}\right)^2 \hat{u}_{F_{n1}}^2 \tag{D-11-20}$$

$$\left(\frac{1}{P_O^2}\right)\left(\frac{\partial P_O}{\partial F_{n2}}\right)^2 \hat{U}_{F_{n2}}^2 = \left(\frac{w_2 \dot{m}_2}{\dot{m}_F}\right)^2 \hat{u}_{F_{n2}}^2 \tag{D-11-21}$$

$$\left(\frac{1}{P_O^2}\right)\left(\frac{\partial P_O}{\partial F_{n3}}\right)^2 \hat{U}_{F_{n3}}^2 = \left(\frac{w_3 \dot{m}_3}{\dot{m}_F}\right)^2 \hat{u}_{F_{n3}}^2 \tag{D-11-22}$$

For T_{s1}, T_{s2}, and T_{s3}

$$\frac{\partial P_O}{\partial T_{s1}} = \left(\begin{array}{c} \left(-w_1 F_{n1} A_1 \frac{1}{2}\left(\frac{p_{v1}\left(p_{s1}+p_b\right)}{R}\right)^{1/2} \frac{1}{T_{s1}^{3/2}}\right) y_F \\ +\dot{m}_F \left(\frac{-2R\left(p_{s2}-p_{s1}\right)}{\left(\frac{p_{s2}+p_b}{T_{s2}}+\frac{p_{s1}+p_b}{T_{s1}}\right)^2} \left(\frac{\left(p_{s1}+p_b\right)}{T_{s1}}\right) - \frac{p_{v1} R_1}{\left(p_{s1}+p_b\right)} \right) \end{array} \right) \tag{D-11-23}$$

Multiplying by $\left(T_{s1}/T_{s1}\right)$, etc.

$$\frac{\partial P_O}{\partial T_{s1}} = \left(\frac{1}{T_{s1}}\right) \left(\begin{array}{c} \left(-w_1 F_{n1} A_1 \frac{1}{2}\left(\frac{p_{v1}\left(p_{s1}+p_b\right)}{RT_{s1}}\right)^{1/2} \frac{T_{s1}}{T_{s1}}\right) y_F \\ +\dot{m}_F \left(\frac{-4\left(p_{s2}-p_{s1}\right)}{2\left(\frac{p_{s2}+p_b}{RT_{s2}}+\frac{p_{s1}+p_b}{RT_{s1}}\right)^2} \left(\frac{\left(p_{s1}+p_b\right)}{RT_{s1}^2}\right) T_{s1} - \frac{p_{v1} RT_{s1}}{\left(p_{s1}+p_b\right)} \right) \end{array} \right) \tag{D-11-24}$$

Expanding and combining

$$\left(\frac{1}{P_O^2}\right)\left(\frac{\partial P_O}{\partial T_{s1}}\right)^2 \hat{U}_{T_{s1}}^2 = \left(\frac{\hat{U}_{t_{s1}}}{T_{s1}}\right)^2 \left(\frac{1}{P_O^2}\right) \left(-\frac{1}{2} w_1 \dot{m}_1 y_F + \dot{m}_F \left(-\frac{\left(p_{s2}-p_{s1}\right)}{2\rho_m^2}\left(\rho_1\right) - \frac{p_{v1}}{\rho_1}\right)\right)^2 \tag{D-11-25}$$

$$\left(\frac{1}{P_O^2}\right)\left(\frac{\partial P_O}{\partial T_{s1}}\right)^2 \hat{U}_{T_{s1}}^2 = \left(\frac{w_1 \dot{m}_1}{2\dot{m}_F} + \frac{1}{y_F}\left(-\frac{\rho_1\left(p_{s2}-p_{s1}\right)}{2\rho_m^2} - e_{K1}\right)\right)^2 \hat{u}_{T_{s1}}^2 \tag{D-11-26}$$

Similarly,

$$\left(\frac{1}{P_O^2}\right)\left(\frac{\partial P_O}{\partial T_{s2}}\right)^2 \hat{U}_{T_{s2}}^2 = \left(\frac{w_2 \dot{m}_2}{2\dot{m}_F} + \frac{1}{y_F}\left(-\frac{\rho_2\left(p_{s2}-p_{s1}\right)}{2\rho_m^2} + e_{K2}\right)\right)^2 \hat{u}_{T_{s2}}^2 \tag{D-11-27}$$

$$\left(\frac{1}{P_O^2}\right)\left(\frac{\partial P_O}{\partial T_{s3}}\right)^2 \hat{U}_{T_{s3}}^2 = \left(\frac{w_3 \dot{m}_3}{2\dot{m}_F}\right)^2 \hat{u}_{T_{s3}}^2 \tag{D-11-28}$$

For p_b

$$\frac{\partial P_O}{\partial p_b} = \left(\begin{array}{c} \left(w_1 F_{n1} A_1 \frac{1}{2}\left(\frac{p_{v1}}{RT_{s1}\left(p_{s1}+p_b\right)}\right)^{1/2} + w_2 F_{n2} A_2 \frac{1}{2}\left(\frac{p_{v2}}{RT_{s2}\left(p_{s2}+p_b\right)}\right)^{1/2} + w_3 F_{n3} A_3 \frac{1}{2}\left(\frac{p_{v3}}{RT_{s3}\left(p_{s3}+p_b\right)}\right)^{1/2} \right) y_F \\ +\dot{m}_F \left(\frac{-2R\left(p_{s2}-p_{s1}\right)}{\left(\frac{p_{s2}+p_b}{T_{s2}}+\frac{p_{s1}+p_b}{T_{s1}}\right)^2} \left(\frac{1}{T_{s1}}+\frac{1}{T_{s2}}\right) + \left(\frac{p_{v1} RT_{s1}}{\left(p_{s1}+p_b\right)^2}\right) - \left(\frac{p_{v2} RT_{s2}}{\left(p_{s2}+p_b\right)^2}\right) \right) \end{array} \right) \tag{D-11-29}$$

Multiplying by $\left(p_b/p_b\right)$, $\left(R/R\right)$, and $\left(p_{sax}/p_{sax}\right)$ where appropriate

$$\frac{\partial P_O}{\partial p_b}=\left(\frac{1}{p_b}\right)\left(\begin{gathered}\left(w_1F_{n1}A_1\frac{1}{2}\left(\frac{p_{v1}\left(p_{s1}+p_b\right)}{RT_{s1}\left(p_{s1}+p_b\right)^2}\right)^{1/2}p_b+w_2F_{n2}A_2\frac{1}{2}\left(\frac{p_{v2}\left(p_{s2}+p_b\right)}{RT_{s2}\left(p_{s2}+p_b\right)^2}\right)^{1/2}p_b+w_3F_{n3}A_3\frac{1}{2}\left(\frac{p_{v3}\left(p_{s3}+p_b\right)}{RT_{s3}\left(p_{s3}+p_b\right)^2}\right)^{1/2}p_b\right)y_F\\+\dot{m}_F\left(-\frac{\left(p_{s2}-p_{s1}\right)}{2\rho_m^2}\left(\frac{p_b}{RT_{s1}}+\frac{p_b}{RT_{s2}}\right)+\left(\frac{p_{v1}p_b}{\rho_1p_{sa1}}\right)-\left(\frac{p_{v2}p_b}{\rho_2p_{sa2}}\right)\right)\end{gathered}\right)\qquad\text{(D-11-30)}$$

Expanding and combining

$$\left(\frac{\partial P_O}{\partial p_b}\right)^2=\left(\frac{1}{p_b}\right)^2\left(\begin{gathered}\left(w_1F_{n1}A_1\frac{1}{2}\left(\frac{p_{v1}\rho_1}{p_{sa1}^2}\right)^{1/2}p_b+w_2F_{n2}A_2\frac{1}{2}\left(\frac{p_{v2}\rho_2}{p_{sa2}^2}\right)^{1/2}p_b+w_3F_{n3}A_3\frac{1}{2}\left(\frac{p_{v3}\rho_3}{p_{sa3}^2}\right)^{1/2}p_b\right)y_F\\+\dot{m}_F\left(-\frac{\left(p_{s2}-p_{s1}\right)}{2\rho_m^2}\left(\frac{p_b}{RT_{s2}}+\frac{p_b}{RT_{s1}}\right)+\left(\frac{p_{v2}p_b}{\rho_2p_{sa2}}\right)-\left(\frac{p_{v1}p_b}{\rho_1p_{sa1}}\right)\right)\end{gathered}\right)^2\qquad\text{(D-11-31)}$$

$$\left(\frac{1}{P_O^2}\right)\left(\frac{\partial P_O}{\partial p_b}\right)^2\hat{U}_{p_b}^2=\left(\frac{\hat{U}_{p_b}}{p_b}\right)^2\left(\frac{1}{P_O^2}\right)\left(\begin{gathered}\left(w_1\dot{m}_1\frac{1}{2}\frac{p_b}{p_{sa1}}+w_2\dot{m}_2\frac{1}{2}\frac{p_b}{p_{sa2}}+w_3\dot{m}_3\frac{1}{2}\frac{p_b}{p_{sa3}}\right)y_F\\+\dot{m}_F\left(-\frac{\left(p_{s2}-p_{s1}\right)}{2\rho_m^2}\left(\frac{p_b}{RT_{s2}}+\frac{p_b}{RT_{s1}}\right)+\left(\frac{p_{v2}p_b}{\rho_2p_{sa2}}\right)-\left(\frac{p_{v1}p_b}{\rho_1p_{sa1}}\right)\right)\end{gathered}\right)^2\qquad\text{(D-11-32)}$$

$$\left(\frac{1}{P_O^2}\right)\left(\frac{\partial P_O}{\partial p_b}\right)^2\hat{U}_{p_b}^2=\left(\begin{gathered}\left(\frac{w_1\dot{m}_1}{2\dot{m}_F}\frac{p_b}{p_{sa1}}+\frac{w_2\dot{m}_2}{2\dot{m}_F}\frac{p_b}{p_{sa2}}+\frac{w_3\dot{m}_3}{2\dot{m}_F}\frac{p_b}{p_{sa3}}\right)\\+\frac{1}{y_F}\left(-\frac{\left(p_{s2}-p_{s1}\right)}{2\rho_m^2}\left(\frac{p_b}{RT_{s2}}+\frac{p_b}{RT_{s1}}\right)+\left(\frac{p_{v2}p_b}{\rho_2p_{sa2}}\right)-\left(\frac{p_{v1}p_b}{\rho_1p_{sa1}}\right)\right)\end{gathered}\right)^2\hat{u}_{p_b}^2\qquad\text{(D-11-33)}$$

For p_{s1}, p_{s2}, and p_{s3}

$$\frac{\partial P_O}{\partial p_{s1}}=\left(\begin{gathered}w_1F_{n1}A_1\frac{1}{2}\left(\frac{p_{v1}}{RT_{s1}\left(p_{s1}+p_b\right)}\right)^{1/2}y_F\\+\dot{m}_F\left(\frac{-2R}{\frac{p_{s2}+p_b}{T_{s2}}+\frac{p_{s1}+p_b}{T_{s1}}}-\frac{2R\left(p_{s2}-p_{s1}\right)}{\left(\frac{p_{s2}+p_b}{T_{s2}}+\frac{p_{s1}+p_b}{T_{s1}}\right)^2}\left(\frac{1}{T_{s1}}\right)+\left(\frac{p_{v1}RT_{s1}}{\left(p_{s1}+p_b\right)^2}\right)\right)\end{gathered}\right)\qquad\text{(D-11-34)}$$

Multiplying by $\left(p_{s1}/p_{s1}\right)$, $\left(p_{sa1}/p_{sa1}\right)$, $\left(R/R\right)$, and $\left(2/2\right)$ where appropriate

$$\frac{\partial P_O}{\partial p_{s1}}=\frac{1}{p_{s1}}\left(\begin{gathered}w_1F_{n1}A_1p_{s1}\frac{1}{2}\left(\frac{p_{v1}}{RT_{s1}p_{sa1}}\right)^{1/2}\frac{p_{sa1}}{p_{sa1}}y_F\\+\dot{m}_F\left(\frac{-2Rp_{s1}}{\frac{p_{s2}+p_b}{T_{s2}}+\frac{p_{s1}+p_b}{T_{s1}}}-\frac{p_{sa1}4\left(p_{s2}-p_{s1}\right)p_{s1}}{p_{sa1}RT_{s1}2\left(\frac{p_{s2}+p_b}{RT_{s2}}+\frac{p_{s1}+p_b}{RT_{s1}}\right)^2}+\frac{p_{sa1}}{p_{sa1}}\left(\frac{p_{v1}RT_{s1}p_{s1}}{\left(p_{s1}+p_b\right)^2}\right)\right)\end{gathered}\right)\qquad\text{(D-11-35)}$$

Combining

$$\frac{\partial P_O}{\partial p_{s1}}=\frac{1}{p_{s1}}\left(w_1F_{n1}A_1\frac{1}{2}\left(\rho_1p_{v1}\right)^{1/2}\frac{p_{s1}}{p_{sa1}}y_F+\dot{m}_F\left(-\frac{p_{s1}}{\rho_m}-\frac{\rho_1\left(p_{s2}-p_{s1}\right)p_{s1}}{2\rho_m^2p_{sa1}}+\frac{p_{s1}}{p_{sa1}}\frac{p_{v1}}{\rho_1}\right)\right)\qquad\text{(D-11-36)}$$

Expanding and combining further

$$\left(\frac{1}{P_O^2}\right)\left(\frac{\partial P_O}{\partial p_{s1}}\right)^2 \hat{U}_{p_{s1}}^2 = \left(\frac{1}{P_O^2}\right)\frac{\hat{U}_{p_{s1}}^2}{p_{s1}^2}\left(w_1\dot{m}_1\frac{1}{2}\frac{p_{s1}}{p_{sa1}}y_F + \dot{m}_F\left(-\frac{p_{s1}}{\rho_m} - \frac{\rho_1\left(p_{s2}-p_{s1}\right)p_{s1}}{2\rho_m^2 p_{sa1}} + \frac{p_{s1}}{p_{sa1}}\frac{p_{v1}}{\rho_1}\right)\right)^2 \qquad \text{(D-11-37)}$$

$$\left(\frac{1}{P_O^2}\right)\left(\frac{\partial P_O}{\partial p_{s1}}\right)^2 \hat{U}_{p_{s1}}^2 = \left(\frac{1}{\dot{m}_F y_F}\right)^2\left(w_1\dot{m}_1\frac{1}{2}\frac{p_{s1}}{p_{sa1}}y_F + \dot{m}_F\left(-\frac{p_{s1}}{\rho_m} - \frac{\rho_1\left(p_{s2}-p_{s1}\right)}{2\rho_m^2}\frac{p_{s1}}{p_{sa1}} + \frac{p_{v1}}{\rho_1}\frac{p_{s1}}{p_{sa1}}\right)\right)^2 \hat{u}_{p_{s1}}^2 \qquad \text{(D-11-38)}$$

$$\left(\frac{1}{P_O^2}\right)\left(\frac{\partial P_O}{\partial p_{s1}}\right)^2 \hat{U}_{p_{s1}}^2 = \left(\frac{w_1\dot{m}_1}{2\dot{m}_F}\frac{p_{s1}}{p_{sa1}} + \frac{1}{y_F}\left(-\frac{p_{s1}}{\rho_m} - \frac{\rho_1\left(p_{s2}-p_{s1}\right)}{2\rho_m^2}\frac{p_{s1}}{p_{sa1}} + \frac{p_{v1}}{\rho_1}\frac{p_{s1}}{p_{sa1}}\right)\right)^2 \hat{u}_{p_{s1}}^2 \qquad \text{(D-11-39)}$$

Similarly,

$$\left(\frac{1}{P_O^2}\right)\left(\frac{\partial P_O}{\partial p_{s2}}\right)^2 \hat{U}_{p_{s2}}^2 = \left(\frac{w_2\dot{m}_2}{2\dot{m}_F}\frac{p_{s2}}{p_{sa2}} + \frac{1}{y_F}\left(\frac{p_{s2}}{\rho_m} - \frac{\rho_2\left(p_{s2}-p_{s1}\right)}{2\rho_m^2}\frac{p_{s2}}{p_{sa2}} - \frac{p_{v2}}{\rho_2}\frac{p_{s2}}{p_{sa2}}\right)\right)^2 \hat{u}_{p_{s2}}^2 \qquad \text{(D-11-40)}$$

For p_{v1}, p_{v2}, and p_{v3}

$$\frac{\partial P_O}{\partial p_{v1}} = \frac{1}{2}w_1F_{n1}A_1\left(\frac{\rho_1}{p_{v1}}\right)^{1/2} y_F - \dot{m}_F\left(\frac{1}{\rho_1}\right) \qquad \text{(D-11-41)}$$

Multiplying by $\left(p_{v1}/p_{v1}\right)$

$$\frac{\partial P_O}{\partial p_{v1}} = \frac{1}{p_{v1}}\left(\frac{1}{2}w_1F_{n1}A_1\left(\rho_1 p_{v1}\right)^{1/2} y_F - \dot{m}_F\left(\frac{p_{v1}}{\rho_1}\right)\right) \qquad \text{(D-11-42)}$$

Expanding and combining

$$\left(\frac{1}{P_O^2}\right)\left(\frac{\partial P_O}{\partial p_{v1}}\right)^2 \hat{U}_{p_{v1}}^2 = \left(\frac{1}{P_O^2}\right)\left(\frac{1}{2}w_1\dot{m}_1 y_F - \dot{m}_F e_{K1}\right)^2\left(\frac{\hat{U}_{p_{v1}}}{p_{v1}}\right)^2 \qquad \text{(D-11-43)}$$

$$\left(\frac{1}{P_O^2}\right)\left(\frac{\partial P_O}{\partial p_{v1}}\right)^2 \hat{U}_{p_{v1}}^2 = \left(\frac{w_1\dot{m}_1}{2\dot{m}_F} - \frac{e_{K1}}{y_F}\right)^2 \hat{u}_{pv1}^2 \qquad \text{(D-11-44)}$$

Similarly,

$$\left(\frac{1}{P_O^2}\right)\left(\frac{\partial P_O}{\partial p_{v2}}\right)^2 \hat{U}_{p_{v2}}^2 = \left(\frac{w_2\dot{m}_2}{2\dot{m}_F} + \frac{e_{K2}}{y_F}\right)^2 \hat{u}_{p_{v2}}^2 \qquad \text{(D-11-45)}$$

and

$$\left(\frac{1}{P_O^2}\right)\left(\frac{\partial P_O}{\partial p_{v3}}\right)^2 \hat{U}_{p_{v3}}^2 = \left(\frac{w_3\dot{m}_3}{2\dot{m}_F}\right)^2 \hat{u}_{p_{v3}}^2 \qquad \text{(D-11-46)}$$

Gathering terms

$$\hat{u}_{P_O}^2 = \left[\begin{aligned} & \frac{1}{4}\hat{u}_R^2 + \left(\frac{w_1\dot{m}_1}{\dot{m}_F}\right)^2 \hat{u}_{F_{n1}}^2 + \left(\frac{w_2\dot{m}_2}{\dot{m}_F}\right)^2 \hat{u}_{F_{n2}}^2 + \left(\frac{w_3\dot{m}_3}{\dot{m}_F}\right)^2 \hat{u}_{F_{n3}}^2 \\ & + \left(\frac{w_1\dot{m}_1}{\dot{m}_F}\right)^2 \hat{u}_{A_1}^2 + \left(\frac{w_2\dot{m}_2}{\dot{m}_F}\right)^2 \hat{u}_{A_2}^2 + \left(\frac{w_3\dot{m}_3}{\dot{m}_F}\right)^2 \hat{u}_{A_3}^2 \\ & + \left(\frac{w_1 m_1}{2\dot{m}_F} - \frac{1}{y_F}\frac{\rho_1\left(p_{s2}-p_{s1}\right)}{2\rho_m^2} - \frac{e_{K1}}{y_F}\right)^2 \hat{u}_{T_{s1}}^2 \\ & + \left(\frac{w_2 m_2}{2\dot{m}_F} - \frac{1}{y_F}\frac{\rho_2\left(p_{s2}-p_{s1}\right)}{2\rho_m^2} + \frac{e_{K2}}{y_F}\right)^2 \hat{u}_{T_{s2}}^2 + \left(\frac{w_3 m_3}{2\dot{m}_F}\right)^2 \hat{u}_{T_{s3}}^2 \\ & + \left(\begin{aligned} & \frac{w_1 m_1}{2\dot{m}_F}\frac{p_b}{p_{sa1}} + \frac{w_2 m_2}{2\dot{m}_F}\frac{p_b}{p_{sa2}} + \frac{w_3 m_3}{2\dot{m}_F}\frac{p_b}{p_{sa3}} \\ & + \frac{1}{y_F}\left(\frac{p_{v1}p_b}{\rho_1 p_{sa1}} - \frac{\left(p_{s2}-p_{s1}\right)}{2\rho_m^2}\left(\frac{p_b}{RT_{s1}} + \frac{p_b}{RT_{s2}}\right) - \frac{p_{v2}p_b}{\rho_2 p_{sa2}}\right) \end{aligned}\right)^2 \hat{u}_{p_b}^2 \\ & + \left(\frac{w_1\dot{m}_1}{2\dot{m}_F}\frac{p_{s1}}{p_{sa1}} + \frac{1}{y_F}\left(\frac{p_{v1}}{\rho_1}\frac{p_{s1}}{p_{sa1}} - \frac{\rho_1\left(p_{s2}-p_{s1}\right)}{2\rho_m^2}\frac{p_{s1}}{p_{sa1}} - \frac{p_{s1}}{\rho_m}\right)\right)^2 u_{p_{s1}}^2 \\ & + \left(\frac{w_2\dot{m}_2}{2\dot{m}_F}\frac{p_{s2}}{p_{sa2}} + \frac{1}{y_F}\left(\frac{p_{s2}}{\rho_m} - \frac{\rho_2\left(p_{s2}-p_{s1}\right)}{2\rho_m^2}\frac{p_{s2}}{p_{sa2}} - \frac{p_{v2}}{\rho_2}\frac{p_{s2}}{p_{sa2}}\right)\right)^2 \hat{u}_{p_{s2}}^2 \\ & + \left(\frac{w_3\dot{m}_3}{2\dot{m}_F}\frac{p_{s3}}{p_{sa3}}\right)^2 \hat{u}_{p_{s3}}^2 \\ & + \left(\frac{w_1\dot{m}_1}{2\dot{m}_F} - \frac{e_{K1}}{y_F}\right)^2 \hat{u}_{pv1}^2 + \left(\frac{w_2\dot{m}_2}{2\dot{m}_F} + \frac{e_{K2}}{y_F}\right)^2 \hat{u}_{p_{v2}}^2 + \left(\frac{w_3\dot{m}_3}{2\dot{m}_F}\right)^2 \hat{u}_{pv3}^2 \end{aligned} \right] \tag{D-11-47}$$

Inserting unit conversions

$$\hat{u}_{P_O}^2 = \left[\begin{array}{l} \frac{1}{4}\hat{u}_R^2 + \left(\frac{w_1\dot{m}_1}{\dot{m}_F}\right)^2 \hat{u}_{F_{n1}}^2 + \left(\frac{w_2\dot{m}_2}{\dot{m}_F}\right)^2 \hat{u}_{F_{n2}}^2 + \left(\frac{w_3\dot{m}_3}{\dot{m}_F}\right)^2 \hat{u}_{F_{n3}}^2 \\ + \left(\frac{w_1\dot{m}_1}{\dot{m}_F}\right)^2 \hat{u}_{A_1}^2 + \left(\frac{w_2\dot{m}_2}{\dot{m}_F}\right)^2 \hat{u}_{A_2}^2 + \left(\frac{w_3\dot{m}_3}{\dot{m}_F}\right)^2 \hat{u}_{A_3}^2 \\ + \left(\frac{w_1 m_1}{2\dot{m}_F} - \frac{C_{11}}{y_F}\frac{\rho_1\left(p_{s2}-p_{s1}\right)}{2\rho_m^2} - \frac{e_{K1}}{y_F}\right)^2 \hat{u}_{T_{s1}}^2 \\ + \left(\frac{w_2 m_2}{2\dot{m}_F} - \frac{C_{11}}{y_F}\frac{\rho_2\left(p_{s2}-p_{s1}\right)}{2\rho_m^2} + \frac{e_{K2}}{y_F}\right)^2 \hat{u}_{T_{s2}}^2 + \left(\frac{w_3 m_3}{2\dot{m}_F}\right)^2 \hat{u}_{T_{s3}}^2 \\ + \left(\begin{array}{l} \frac{w_1 m_1}{2\dot{m}_F}\frac{C_{13}p_b}{p_{sa1}} + \frac{w_2 m_2}{2\dot{m}_F}\frac{C_{13}p_b}{p_{sa2}} + \frac{w_3 m_3}{2\dot{m}_F}\frac{C_{13}p_b}{p_{sa3}} \\ + \frac{C_{11}}{y_F}\left(\frac{p_{v1}C_{13}p_b}{\rho_1 p_{sa1}} - \frac{C_{11}\left(p_{s2}-p_{s1}\right)}{2\rho_m^2}\left(\frac{C_{13}p_b}{RT_{s1}} + \frac{C_{13}p_b}{RT_{s2}}\right) - \frac{p_{v2}C_{13}p_b}{\rho_2 p_{sa2}}\right) \end{array}\right)^2 \hat{u}_{p_b}^2 \\ + \left(\frac{w_1\dot{m}_1}{2\dot{m}_F}\frac{p_{s1}}{p_{sa1}} + \frac{C_{11}}{y_F}\left(\frac{p_{v1}}{\rho_1}\frac{p_{s1}}{p_{sa1}} - \frac{\rho_1\left(p_{s2}-p_{s1}\right)}{2\rho_m^2}\frac{p_{s1}}{p_{sa1}} - \frac{p_{s1}}{\rho_m}\right)\right)^2 \hat{u}_{p_{s1}}^2 \\ + \left(\frac{w_2\dot{m}_2}{2\dot{m}_F}\frac{p_{s2}}{p_{sa2}} + \frac{C_{11}}{y_F}\left(\frac{p_{s2}}{\rho_m} - \frac{\rho_2\left(p_{s2}-p_{s1}\right)}{2\rho_m^2}\frac{p_{s2}}{p_{sa2}} - \frac{p_{v2}}{\rho_2}\frac{p_{s2}}{p_{sa2}}\right)\right)^2 \hat{u}_{p_{s2}}^2 \\ + \left(\frac{w_3\dot{m}_3}{2\dot{m}_F}\frac{p_{s3}}{p_{sa3}}\right)^2 \hat{u}_{p_{s3}}^2 \\ + \left(\frac{w_1\dot{m}_1}{2\dot{m}_F} - \frac{e_{K1}}{y_F}\right)^2 \hat{u}_{pv1}^2 + \left(\frac{w_2\dot{m}_2}{2\dot{m}_F} + \frac{e_{K2}}{y_F}\right)^2 \hat{u}_{p_{v2}}^2 + \left(\frac{w_3\dot{m}_3}{2\dot{m}_F}\right)^2 \hat{u}_{pv3}^2 \end{array}\right] \qquad (5\text{-}13\text{-}24)$$

D-12 UNCERTAINTY IN THE FAN VOLUME FLOW RATE, Q_F

The equation for Q_F is given in Section 5 as

$$Q_F = \frac{\dot{m}_F}{\rho_F} \qquad (5\text{-}11\text{-}2)$$

Reducing to primary variables

$$\dot{m}_F = w_1\dot{m}_1 + w_2\dot{m}_2 + w_3\dot{m}_3 \qquad (\text{D-12-1})$$

$$\dot{m}_1 = F_{n1}A_1\rho_1 V_1 = F_{n1}A_1\rho_1\left(\frac{p_{v1}}{\rho_1}\right)^{1/2} = F_{n1}A_1\left(\rho_1 p_{v1}\right)^{1/2} \qquad (\text{D-12-2})$$

$\dot{m}_2$ and $\dot{m}_3$ are the same except for subscripts.

$$\rho_1 = \frac{p_{sa1}}{RT_{s1}} = \frac{\left(p_{s1}+p_b\right)}{RT_{s1}} \qquad (\text{D-12-3})$$

ρ_2 and ρ_3 are the same except for subscripts.

$$w_1\dot{m}_1 = w_1 F_{n1} A_1 \left(\frac{p_{v1}\left(p_{s1}+p_b\right)}{RT_{s1}} \right)^{1/2} \tag{D-12-4}$$

$w_2\dot{m}_2$ and $w_3\dot{m}_3$ are the same except for subscripts.

$$T_{t1} = T_{s1} + \frac{V_1^2}{2g_c c_p J} \approx T_{s1} \tag{D-12-5}$$

$$\rho_F = \frac{p_{ta1}}{RT_{t1}} = \frac{\left(p_{v1}+p_{s1}+p_b\right)}{RT_{t1}} \tag{D-12-6}$$

$$Q_F = \left[w_1 F_{n1} A_1 \left(\frac{p_{v1}\left(p_{s1}+p_b\right)}{RT_{s1}} \right)^{1/2} + w_2 F_{n2} A_2 \left(\frac{p_{v2}\left(p_{s2}+p_b\right)}{RT_{s2}} \right)^{1/2} + w_3 F_{n3} A_3 \left(\frac{p_{v3}\left(p_{s3}+p_b\right)}{RT_{s3}} \right)^{1/2} \right] \left(\frac{RT_{t1}}{p_{ta1}} \right) \tag{D-12-7}$$

Combining uncertainties per Kline and McClintock

$$\hat{u}_{Q_F}^2 = \hat{u}_{Fn}^2 + \left(\frac{1}{Q_F^2}\right) \left[\begin{array}{l} \left(\frac{\partial Q_F}{\partial R}\right)^2 \hat{U}_R^2 + \left(\frac{\partial Q_F}{\partial F_{n1}}\right)^2 \hat{U}_{F_{n1}}^2 + \left(\frac{\partial Q_F}{\partial F_{n2}}\right)^2 \hat{U}_{F_{n2}}^2 + \left(\frac{\partial Q_F}{\partial F_{n3}}\right)^2 \hat{U}_{F_{n3}}^2 \\ + \left(\frac{\partial Q_F}{\partial A_1}\right)^2 \hat{U}_{A_1}^2 + \left(\frac{\partial Q_F}{\partial A_2}\right)^2 \hat{U}_{A_2}^2 + \left(\frac{\partial Q_F}{\partial A_3}\right)^2 \hat{U}_{A_3}^2 \\ + \left(\frac{\partial Q_F}{\partial T_{s1}}\right)^2 \hat{U}_{T_{s1}}^2 + \left(\frac{\partial Q_F}{\partial T_{s2}}\right)^2 \hat{U}_{T_{s2}}^2 + \left(\frac{\partial Q_F}{\partial T_{s3}}\right)^2 \hat{U}_{T_{s3}}^2 + \left(\frac{\partial Q_F}{\partial p_b}\right)^2 \hat{U}_{p_b}^2 \\ + \left(\frac{\partial Q_F}{\partial p_{s1}}\right)^2 \hat{U}_{p_{s1}}^2 + \left(\frac{\partial Q_F}{\partial p_{s2}}\right)^2 \hat{U}_{p_{s2}}^2 + \left(\frac{\partial Q_F}{\partial p_{s3}}\right)^2 \hat{U}_{p_{s3}}^2 \\ + \left(\frac{\partial Q_F}{\partial p_{v1}}\right)^2 \hat{U}_{p_{v1}}^2 + \left(\frac{\partial Q_F}{\partial p_{v1}}\right)^2 \hat{U}_{p_{v1}}^2 + \left(\frac{\partial Q_F}{\partial p_{v3}}\right)^2 \hat{U}_{p_{v3}}^2 \end{array} \right] \tag{D-12-8}$$

For R

$$\frac{\partial Q_F}{\partial R} = -\left(\frac{1}{2} w_1 F_{n1} A_1 \frac{1}{R^{3/2}} \left(\frac{p_{v1}\left(p_{s1}+p_b\right)}{T_{s1}} \right)^{1/2} + \cdots \right) \frac{1}{\rho_F} + \dot{m}_F \left(\frac{T_{t1}}{p_{ta1}} \right) \tag{D-12-9}$$

Multiplying by $\left(R/R\right)$

$$\frac{\partial Q_F}{\partial R} = -\frac{1}{R}\frac{1}{2} \left(w_1 F_{n1} A_1 \left(\frac{p_{v1}\left(p_{s1}+p_b\right)}{RT_{s1}} \right)^{1/2} + \cdots \right) \frac{1}{\rho_F} + \dot{m}_F \frac{1}{R} \left(\frac{RT_{t1}}{p_{ta1}} \right) \tag{D-12-10}$$

Combining

$$\frac{\partial Q_F}{\partial R} = \frac{1}{R} \left(-\frac{1}{2}\left(w_1\dot{m}_1 + \cdots\right)\frac{1}{\rho_F} + \frac{\dot{m}_F}{\rho_F} \right) \tag{D-12-11}$$

Expanding and combining

$$\left(\frac{1}{Q_F^2}\right)\left(\frac{\partial Q_F}{\partial R}\right)^2 \hat{U}_R^2 = \left(\frac{\hat{U}_R}{R}\right)^2 \left(\frac{1}{Q_F^2}\right)\left(-\frac{1}{2}(\dot{m}_F)\frac{1}{\rho_F} + \frac{\dot{m}_F}{\rho_F}\right)^2 \tag{D-12-12}$$

$$\left(\frac{1}{Q_F^2}\right)\left(\frac{\partial Q_F}{\partial R}\right)^2 \hat{U}_R^2 = \left(\frac{1}{Q_F^2}\right)\left(-\frac{1}{2}Q_F + Q_F\right)^2 \hat{u}_R^2 = \frac{1}{4}\hat{u}_R^2 \tag{D-12-13}$$

For A_1, A_2, and A_3

$$\frac{\partial Q_F}{\partial A_1} = \left[w_1 F_{n1}\left(\frac{p_{v1}(p_{s1}+p_b)}{RT_{s1}}\right)^{1/2}\right]\left(\frac{1}{\rho_F}\right) + \dot{m}_F(0) \tag{D-12-14}$$

Multiplying by (A_1/A)

$$\frac{\partial Q_F}{\partial A_1} = \frac{1}{A_1}\left[w_1 F_{n1} A_1 \left(\frac{p_{v1}(p_{s1}+p_b)}{RT_{s1}}\right)^{1/2} \frac{1}{\rho_F}\right] \tag{D-12-15}$$

Expanding and combining

$$\left(\frac{1}{Q_F^2}\right)\left(\frac{\partial Q_F}{\partial A_1}\right)^2 \hat{U}_{A_1}^2 = \left(\frac{1}{Q_F^2}\right)\left(\frac{\hat{U}_{A_1}}{A_1}\right)^2 \left[w_1 \dot{m}_1 \frac{1}{\rho_F}\right]^2 \tag{D-12-16}$$

$$\left(\frac{1}{Q_F^2}\right)\left(\frac{\partial Q_F}{\partial A_1}\right)^2 \hat{U}_{A_1}^2 = \left(\frac{w_1 \dot{m}_1}{\dot{m}_F}\right)^2 \hat{u}_{A_1}^2 \tag{D-12-17}$$

Similarly

$$\left(\frac{1}{Q_F^2}\right)\left(\frac{\partial Q_F}{\partial A_2}\right)^2 \hat{U}_{A_2}^2 = \left(\frac{w_2 \dot{m}_2}{\dot{m}_F}\right)^2 \hat{u}_{A_2}^2 \tag{D-12-18}$$

$$\left(\frac{1}{Q_F^2}\right)\left(\frac{\partial Q_F}{\partial A_3}\right)^2 \hat{U}_{A_3}^2 = \left(\frac{w_3 \dot{m}_3}{\dot{m}_F}\right)^2 \hat{u}_{A_3}^2 \tag{D-12-19}$$

For T_{s1}, T_{s2}, and T_{s3}

$$\frac{\partial Q_F}{\partial T_{s1}} = \left(-w_1 F_{n1} A_1 \frac{1}{2}\left(\frac{p_{v1}(p_{s1}+p_b)}{R}\right)^{1/2} \frac{1}{T_{s1}^{3/2}}\frac{1}{\rho_F} + \dot{m}_F\left(\frac{R}{p_{ta1}}\right)\right) \tag{D-12-20}$$

Multiplying by (T_{s1}/T_{s1}), etc.

$$\frac{\partial Q_F}{\partial T_{s1}} = \left(\frac{1}{T_{s1}}\right)\left(-w_1 F_{n1} A_1 \frac{1}{2}\left(\frac{p_{v1}(p_{s1}+p_b)}{RT_{s1}}\right)^{1/2} \frac{T_{s1}}{T_{s1}} + \dot{m}_F\left(\frac{RT_{t1}}{p_{ta1}}\right)\right) \tag{D-12-21}$$

Expanding and combining

$$\left(\frac{1}{Q_F^2}\right)\left(\frac{\partial Q_F}{\partial T_{s1}}\right)^2 \hat{U}_{T_1}^2 = \left(\frac{\hat{U}_{t_1}}{T_{s1}}\right)^2 \left(\frac{1}{Q_F^2}\right)\left(-\frac{1}{2} w_1 \dot{m}_1 \frac{1}{\rho_F} + \dot{m}_F\left(\frac{RT_{t1}}{p_{ta1}}\right)\right)^2 \tag{D-12-22}$$

$$\left(\frac{1}{Q_F^2}\right)\left(\frac{\partial Q_F}{\partial T_{s1}}\right)^2 \hat{U}_{T_{s1}}^2 = \frac{1}{Q_F^2}\left(-\frac{w_1 \dot{m}_1}{2}\frac{1}{\rho_F} + \dot{m}_F \frac{1}{\rho_F}\right)^2 \hat{u}_{T_{s1}}^2 = \left(1 - \frac{w_1 \dot{m}_1}{2\dot{m}_F}\right)^2 \hat{u}_{T_{s1}}^2 \tag{D-12-23}$$

Similarly,

$$\left(\frac{1}{Q_F^2}\right)\left(\frac{\partial Q_F}{\partial T_{s2}}\right)^2 \hat{U}_{T_{s2}}^2 = \left(\frac{w_2 m_2}{2\dot{m}_F}\right)^2 \hat{u}_{T_{s2}}^2 \tag{D-12-24}$$

$$\left(\frac{1}{Q_F^2}\right)\left(\frac{\partial Q_F}{\partial T_{s3}}\right)^2 \hat{U}_{T_{s3}}^2 = \left(\frac{w_3 m_3}{2\dot{m}_F}\right)^2 \hat{u}_{T_{s3}}^2 \tag{D-12-25}$$

For p_b

$$\frac{\partial Q_F}{\partial p_b} = \left(\begin{array}{c}\left(w_1 F_{n1} A_1 \frac{1}{2}\left(\frac{p_{v1}}{RT_{s1}\left(p_{s1}+p_b\right)}\right)^{1/2} + w_2 F_{n2} A_2 \frac{1}{2}\left(\frac{p_{v2}}{RT_{s2}\left(p_{s2}+p_b\right)}\right)^{1/2} + w_3 F_{n3} A_3 \frac{1}{2}\left(\frac{p_{v3}}{RT_{s1}\left(p_{s3}+p_b\right)}\right)^{1/2}\right)\frac{1}{\rho_F} \\ + \dot{m}_F\left(\frac{-RT_{t1}}{\left(p_{v1}+p_{s1}+p_b\right)}\right)\end{array}\right) \tag{D-12-26}$$

Multiplying by $\left(p_b/p_b\right)$, $\left(R/R\right)$, $\left(p_{sax}/p_{sax}\right)$, and $\left(p_{ta1}/p_{ta1}\right)$ where appropriate

$$\frac{\partial Q_F}{\partial p_b} = \left(\frac{1}{p_b}\right)\left(\begin{array}{c}\left(w_1 F_{n1} A_1 \frac{1}{2}\left(\frac{p_{v1}\left(p_{s1}+p_b\right)}{RT_{s1}\left(p_{s1}+p_b\right)^2}\right)^{1/2} p_b + w_2 F_{n2} A_2 \frac{1}{2}\left(\frac{p_{v2}\left(p_{s2}+p_b\right)}{RT_{s2}\left(p_{s2}+p_b\right)^2}\right)^{1/2} p_b + w_3 F_{n3} A_3 \frac{1}{2}\left(\frac{p_{v3}\left(p_{s3}+p_b\right)}{RT_{s1}\left(p_{s3}+p_b\right)^2}\right)^{1/2} p_b\right)\frac{1}{\rho_F} \\ + \dot{m}_F \frac{p_b}{p_{ta1}}\left(\frac{-RT_{t1}}{\left(p_{v1}+p_{s1}+p_b\right)^2}\right)\end{array}\right) \tag{D-12-27}$$

Expanding and combining

$$\left(\frac{\partial Q_F}{\partial p_b}\right)^2 = \left(\frac{1}{p_b}\right)^2\left(\begin{array}{c}\left(w_1 F_{n1} A_1 \frac{1}{2}\left(p_{v1}\rho_1\right)^{1/2}\frac{p_b}{p_{sa1}} + w_2 F_{n2} A_2 \frac{1}{2}\left(p_{v2}\rho_2\right)^{1/2}\frac{p_b}{p_{sa2}} + w_3 F_{n3} A_3 \frac{1}{2}\left(p_{v3}\rho_3\right)^{1/2}\frac{p_b}{p_{sa3}}\right)\frac{1}{\rho_F} \\ - \frac{p_b}{\left(p_{v1}+p_{s1}+p_b\right)}\frac{\dot{m}_F}{\rho_F}\end{array}\right)^2 \tag{D-12-28}$$

$$\left(\frac{1}{Q_F^2}\right)\left(\frac{\partial Q_F}{\partial p_b}\right)^2 U_{p_b}^2 = \left(\frac{U_{p_b}}{p_b}\right)^2\left(\frac{1}{Q_F^2}\right)\left(\left(\frac{w_1\dot{m}_1}{2}\frac{p_b}{p_{sa1}} + \frac{w_2\dot{m}_2}{2}\frac{p_b}{p_{sa2}} + \frac{w_3\dot{m}_3}{2}\frac{p_b}{p_{sa3}}\right)\frac{1}{\rho_F} - \frac{p_b}{p_{ta1}}\frac{\dot{m}_F}{\rho_F}\right)^2 \tag{D-12-29}$$

$$\left(\frac{1}{Q_F^2}\right)\left(\frac{\partial Q_F}{\partial p_b}\right)^2 U_{p_b}^2 = \left(\frac{w_1\dot{m}_1}{2\dot{m}_F}\frac{p_b}{p_{sa1}} + \frac{w_2\dot{m}_2}{2\dot{m}_F}\frac{p_b}{p_{sa2}} + \frac{w_3\dot{m}_3}{2\dot{m}_F}\frac{p_b}{p_{sa3}} - \frac{p_b}{p_{ta1}}\right)^2 u_{p_b}^2 \tag{D-12-30}$$

For p_{s1}, p_{s2}, and p_{s3}

$$\frac{\partial Q_F}{\partial p_{s1}} = w_1 F_{n1} A_1 \frac{1}{2}\left(\frac{p_{v1}}{RT_1\left(p_{s2}-p_{s1}\right)}\right)^2 \frac{1}{\rho_F} + \dot{m}_F\left(\frac{-RT_{t1}}{\left(p_{v1}+p_{s1}+p_b\right)^2}\right) \tag{D-12-31}$$

Multiplying by $\left(p_{s1}/p_{s1}\right)$, $\left(p_{sa1}/p_{sa1}\right)$ and $\left(p_{ta1}/p_{ta1}\right)$ where appropriate

$$\frac{\partial Q_F}{\partial p_{s1}} = \frac{1}{p_{s1}}\left(w_1 F_{n1} A_1 \frac{1}{2} p_{s1} \left(\frac{p_{v1}}{RT_{s1} p_{sa1}} \right)^{1/2} \frac{p_{sa1}}{p_{sa1}} \frac{1}{\rho_F} + \dot{m}_F p_{s1} \frac{1}{p_{ta1}} \left(\frac{-RT_{t1}}{\left(p_{v1} + p_{s1} + p_b\right)} \right) \right) \quad \text{(D-12-32)}$$

Combining

$$\frac{\partial Q_F}{\partial p_{s1}} = \frac{1}{p_{s1}}\left(w_1 F_{n1} A_1 \frac{1}{2} \left(\rho_1 p_{v1}\right)^{1/2} \frac{p_{s1}}{p_{sa1}} \frac{1}{\rho_F} - \frac{p_{s1}}{p_{ta1}} \frac{\dot{m}_F}{\rho_F} \right) \quad \text{(D-12-33)}$$

Expanding and combining further

$$\left(\frac{1}{Q_F^2}\right)\left(\frac{\partial Q_F}{\partial p_{s1}}\right)^2 \hat{U}_{p_{s1}}^2 = \left(\frac{1}{Q_F^2}\right) \frac{\hat{U}_{p_{s1}}^2}{p_{s1}^2} \left(\frac{1}{2} \frac{p_{s1}}{p_{sa1}} \frac{w_1 m_1}{\rho_F} - \frac{p_{s1}}{p_{ta1}} \frac{\dot{m}_F}{\rho_F} \right)^2 \quad \text{(D-12-34)}$$

$$\left(\frac{1}{Q_F^2}\right)\left(\frac{\partial Q_F}{\partial p_{s1}}\right)^2 \hat{U}_{p_{s1}}^2 = \left(\frac{\rho_F}{\dot{m}_F}\right)^2 \left(\frac{1}{2} \frac{p_{s1}}{p_{sa1}} \frac{w_1 m_1}{\rho_F} - \frac{p_{s1}}{p_{ta1}} \frac{\dot{m}_F}{\rho_F} \right)^2 \hat{u}_{p_{s1}}^2 \quad \text{(D-12-35)}$$

$$\left(\frac{1}{Q_F^2}\right)\left(\frac{\partial Q_F}{\partial p_{s1}}\right)^2 \hat{U}_{p_{s1}}^2 = \left(\frac{p_{s1}}{p_{sa1}} \frac{w_1 m_1}{2\dot{m}_F} - \frac{p_{s1}}{p_{ta1}} \right)^2 \hat{u}_{p_{s1}}^2 \quad \text{(D-12-36)}$$

Similarly,

$$\left(\frac{1}{Q_F^2}\right)\left(\frac{\partial Q_F}{\partial p_{s2}}\right)^2 \hat{U}_{p_{s2}}^2 = \left(\frac{w_2 \dot{m}_2}{2\dot{m}_F} \frac{p_{s2}}{p_{sa2}} \right)^2 \hat{u}_{p_{s2}}^2 \quad \text{(D-12-37)}$$

and

$$\left(\frac{1}{Q_F^2}\right)\left(\frac{\partial Q_F}{\partial p_{s3}}\right)^2 \hat{U}_{p_{s3}}^2 = \left(\frac{w_3 \dot{m}_3}{2\dot{m}_F} \frac{p_{s3}}{p_{sa3}} \right)^2 \hat{u}_{p_{s3}}^2 \quad \text{(D-12-38)}$$

For p_{v1}, p_{v2}, and p_{v3}

$$\frac{\partial Q_F}{\partial p_{v1}} = \frac{1}{2} w_1 F_{n1} A_1 \left(\frac{\rho_1}{p_{v1}}\right)^{1/2} \frac{1}{\rho_F} + \dot{m}_F \left(\frac{-RT_{t1}}{\left(p_{v1} + p_{s1} + p_b\right)^2} \right) \quad \text{(D-12-39)}$$

Multiplying by $\left(p_{v1}/p_{v1}\right)$ and $\left(p_{ta1}/p_{ta1}\right)$ where appropriate

$$\frac{\partial Q_F}{\partial p_{v1}} = \frac{1}{p_{v1}}\left(\frac{1}{2} w_1 F_{n1} A_1 \left(\rho_1 p_{v1}\right)^{1/2} \frac{1}{\rho_F} + \dot{m}_F \frac{p_{v1}}{p_{ta1}} \left(\frac{-RT_{t1}}{\left(p_{v1} + p_{s1} + p_b\right)} \right) \right) \quad \text{(D-12-40)}$$

Expanding and combining

$$\left(\frac{1}{Q_F^2}\right)\left(\frac{\partial Q_F}{\partial p_{v1}}\right)^2 \hat{U}_{p_{v1}}^2 = \left(\frac{1}{Q_F^2}\right) \left(\frac{1}{2} w_1 \dot{m}_1 \frac{1}{\rho_F} - \frac{p_{v1}}{p_{ta1}} \frac{\dot{m}_F}{\rho_F} \right)^2 \left(\frac{\hat{U}_{p_{v1}}}{p_{v1}} \right)^2 \quad \text{(D-12-41)}$$

$$\left(\frac{1}{Q_F^2}\right)\left(\frac{\partial Q_F}{\partial p_{v1}}\right)^2 \hat{U}_{p_{v1}}^2 = \left(\frac{w_1 \dot{m}_1}{2\dot{m}_F} - \frac{p_{v1}}{p_{ta1}} \right)^2 \hat{u}_{pv1}^2 \quad \text{(D-12-42)}$$

Similarly,

$$\left(\frac{1}{Q_F^2}\right)\left(\frac{\partial Q_F}{\partial p_{v2}}\right)^2 \hat{U}^2_{p_{v2}} = \left(\frac{w_2\dot{m}_2}{2\dot{m}_F}\right)^2 \hat{u}^2_{p_{v2}} \tag{D-12-43}$$

and

$$\left(\frac{1}{Q_F^2}\right)\left(\frac{\partial Q_F}{\partial p_{v3}}\right)^2 \hat{U}^2_{p_{v3}} = \left(\frac{w_3\dot{m}_3}{2\dot{m}_F}\right)^2 \hat{u}^2_{p_{v3}} \tag{D-12-44}$$

Gathering terms

$$\hat{u}^2_{Q_F} = \left[\begin{array}{l}
\frac{1}{4}\hat{u}^2_R + \left(\frac{w_1\dot{m}_1}{\dot{m}_F}\right)^2 \hat{u}^2_{F_{n1}} + \left(\frac{w_2\dot{m}_2}{\dot{m}_F}\right)^2 \hat{u}^2_{F_{n2}} + \left(\frac{w_3\dot{m}_3}{\dot{m}_F}\right)^2 \hat{u}^2_{F_{n3}} \\
+\left(\frac{w_1\dot{m}_1}{\dot{m}_F}\right)^2 \hat{u}^2_{A_1} + \left(\frac{w_2\dot{m}_2}{\dot{m}_F}\right)^2 \hat{u}^2_{A_2} + \left(\frac{w_3\dot{m}_3}{\dot{m}_F}\right)^2 \hat{u}^2_{A_3} \\
+\left(1-\frac{w_1\dot{m}_1}{2\dot{m}_F}\right)^2 \hat{u}^2_{T_{s1}} + \left(\frac{w_2\dot{m}_2}{2\dot{m}_F}\right)^2 \hat{u}^2_{T_{s2}} + \left(\frac{w_3\dot{m}_3}{2\dot{m}_F}\right)^2 \hat{u}^2_{T_{s3}} \\
+\left(\frac{w_1\dot{m}_1}{2\dot{m}_F}\frac{p_b}{p_{sa1}} + \frac{w_2\dot{m}_2}{2\dot{m}_F}\frac{p_b}{p_{sa2}} + \frac{w_3\dot{m}_3}{2\dot{m}_F}\frac{p_b}{p_{sa3}} - \frac{p_b}{p_{ta1}}\right)^2 \hat{u}^2_{p_b} \\
+\left(\frac{p_{s1}}{p_{sa1}}\frac{w_1\dot{m}_1}{2\dot{m}_F} - \frac{p_{s1}}{p_{ta1}}\right)^2 \hat{u}^2_{p_{s1}} + \left(\frac{w_2\dot{m}_2}{2\dot{m}_F}\frac{p_{s2}}{p_{sa2}}\right)^2 \hat{u}^2_{p_{s2}} + \left(\frac{w_3\dot{m}_3}{2\dot{m}_F}\frac{p_{s3}}{p_{sa3}}\right)^2 \hat{u}^2_{p_{s3}} \\
+\left(\frac{w_1\dot{m}_1}{2\dot{m}_F} - \frac{p_{v1}}{p_{ta1}}\right)^2 \hat{u}^2_{pv1} + \left(\frac{w_2\dot{m}_2}{2\dot{m}_F}\right)^2 \hat{u}^2_{p_{v2}} + \left(\frac{w_3\dot{m}_3}{2\dot{m}_F}\right)^2 \hat{u}^2_{pv3}
\end{array}\right] \tag{D-12-45}$$

Inserting unit conversions

$$\hat{u}^2_{Q_F} = \left[\begin{array}{l}
\frac{1}{4}\hat{u}^2_R + \left(\frac{w_1\dot{m}_1}{\dot{m}_F}\right)^2 \hat{u}^2_{F_{n1}} + \left(\frac{w_2\dot{m}_2}{\dot{m}_F}\right)^2 \hat{u}^2_{F_{n2}} + \left(\frac{w_3\dot{m}_3}{\dot{m}_F}\right)^2 \hat{u}^2_{F_{n3}} \\
+\left(\frac{w_1\dot{m}_1}{\dot{m}_F}\right)^2 \hat{u}^2_{A_1} + \left(\frac{w_2\dot{m}_2}{\dot{m}_F}\right)^2 \hat{u}^2_{A_2} + \left(\frac{w_3\dot{m}_3}{\dot{m}_F}\right)^2 \hat{u}^2_{A_3} \\
+\left(1-\frac{w_1\dot{m}_1}{2\dot{m}_F}\right)^2 u^2_{T_{s1}} + \left(\frac{w_2\dot{m}_2}{2\dot{m}_F}\right)^2 u^2_{T_{s2}} + \left(\frac{w_3\dot{m}_3}{2\dot{m}_F}\right)^2 u^2_{T_{s3}} \\
+\left(\frac{w_1\dot{m}_1}{2\dot{m}_F}\frac{C_{13}p_b}{p_{sa1}} + \frac{w_2\dot{m}_2}{2\dot{m}_F}\frac{C_{13}p_b}{p_{sa2}} + \frac{w_3\dot{m}_3}{2\dot{m}_F}\frac{C_{13}p_b}{p_{sa3}} - \frac{C_{13}p_b}{p_{ta1}}\right)^2 \hat{u}^2_{p_b} \\
+\left(\frac{p_{s1}}{p_{sa1}}\frac{w_1\dot{m}_1}{2\dot{m}_F} - \frac{p_{s1}}{p_{ta1}}\right)^2 \hat{u}^2_{p_{s1}} + \left(\frac{w_2\dot{m}_2}{2\dot{m}_F}\frac{p_{s2}}{p_{sa2}}\right)^2 \hat{u}^2_{p_{s2}} + \left(\frac{w_3\dot{m}_3}{2\dot{m}_F}\frac{p_{s3}}{p_{sa3}}\right)^2 \hat{u}^2_{p_{s3}} \\
+\left(\frac{w_1\dot{m}_1}{2\dot{m}_F} - \frac{p_{v1}}{p_{ta1}}\right)^2 \hat{u}^2_{pv1} + \left(\frac{w_2\dot{m}_2}{2\dot{m}_F}\right)^2 \hat{u}^2_{p_{v2}} + \left(\frac{w_3\dot{m}_3}{2\dot{m}_F}\right)^2 \hat{u}^2_{pv3}
\end{array}\right] \tag{5-13-32}$$

D-13 UNCERTAINTY IN THE FAN OUTPUT POWER, P_O

The equation for P_O is given in Section 5 as

$$P_O = Q_F p_{Ft} K_p \tag{5-11-9}$$

Reducing to primary variables

$$Q_F = \frac{\dot{m}_F}{\rho_F} \tag{D-13-1}$$

$$\dot{m}_F = w_1 \dot{m}_1 + w_2 \dot{m}_2 + w_3 \dot{m}_3 \tag{D-13-2}$$

$$\dot{m}_1 = F_{n1} A_1 \rho_1 V_1 = F_{n1} A_1 \rho_1 \left(\frac{p_{v1}}{\rho_1} \right)^{1/2} = F_{n1} A_1 \left(\rho_1 p_{v1} \right)^{1/2} \tag{D-13-3}$$

$\dot{m}_2$ and $\dot{m}_3$ are the same except for subscripts.

$$\rho_1 = \frac{p_{sa1}}{RT_{s1}} = \frac{\left(p_{s1} + p_b \right)}{RT_{s1}} \tag{D-13-4}$$

ρ_2 and ρ_3 are the same except for subscripts.

$$w_1 \dot{m}_1 = w_1 F_{n1} A_1 \left(\frac{p_{v1} \left(p_{s1} + p_b \right)}{RT_{s1}} \right)^{1/2} \tag{D-13-5}$$

$w_2 \dot{m}_2$ and $w_3 \dot{m}_3$ are the same except for subscripts.

For the purpose of this calculation, let

$$\rho_F = \frac{p_{ta1}}{RT_{t1}} = \frac{\left(p_{s1} + p_{v1} + p_b \right)}{RT_{t1}} \tag{D-13-6}$$

$$Q_F = \left[w_1 F_{n1} A_1 \left(\frac{p_{v1} \left(p_{s1} + p_b \right)}{RT_{s1}} \right)^{1/2} + w_2 F_{n2} A_2 \left(\frac{p_{v2} \left(p_{s2} + p_b \right)}{RT_{s2}} \right)^{1/2} + w_3 F_{n3} A_3 \left(\frac{p_{v3} \left(p_{s3} + p_b \right)}{RT_{s3}} \right)^{1/2} \right] \left(\frac{RT_{t1}}{p_{ta1}} \right) \tag{D-13-7}$$

$$p_{Ft} = p_{t2} - p_{t1} = p_{s2} - p_{s1} + p_{v2} - p_{v1} \tag{D-13-8}$$

$$P_O = \left(w_1 F_{n1} A_1 \left(\frac{p_{v1} \left(p_{s1} + p_b \right)}{RT_{s1}} \right)^{1/2} + w_2 F_{n2} A_2 \left(\frac{p_{v2} \left(p_{s2} + p_b \right)}{RT_{s2}} \right)^{1/2} + w_3 F_{n3} A_3 \left(\frac{p_{v3} \left(p_{s3} + p_b \right)}{RT_{s3}} \right)^{1/2} \right) \left(\frac{RT_{t1}}{p_{ta1}} \right) \left(p_{s2} - p_{s1} + p_{v2} - p_{v1} \right) \tag{D-13-9}$$

Combining uncertainties per Kline and McClintock

$$\hat{u}_{P_O}^2 = \left(\frac{1}{P_O^2}\right)\begin{bmatrix} \left(\frac{\partial P_O}{\partial R}\right)^2 \hat{U}_R^2 + \left(\frac{\partial P_O}{\partial F_{n1}}\right)^2 \hat{U}_{F_{n1}}^2 + \left(\frac{\partial P_O}{\partial F_{n2}}\right)^2 \hat{U}_{F_{n2}}^2 + \left(\frac{\partial P_O}{\partial F_{n3}}\right)^2 \hat{U}_{F_{n3}}^2 \\ + \left(\frac{\partial P_O}{\partial A_1}\right)^2 \hat{U}_{A_1}^2 + \left(\frac{\partial P_O}{\partial A_2}\right)^2 \hat{U}_{A_2}^2 + \left(\frac{\partial P_O}{\partial A_3}\right)^2 \hat{U}_{A_3}^2 \\ + \left(\frac{\partial P_O}{\partial T_{s1}}\right)^2 \hat{U}_{T_{s1}}^2 + \left(\frac{\partial P_O}{\partial T_{s2}}\right)^2 \hat{U}_{T_{s2}}^2 + \left(\frac{\partial P_O}{\partial T_{s3}}\right)^2 \hat{U}_{T_{s3}}^2 \\ + \left(\frac{\partial P_O}{\partial p_b}\right)^2 \hat{U}_{p_b}^2 + \left(\frac{\partial P_O}{\partial p_{s1}}\right)^2 \hat{U}_{p_{s1}}^2 + \left(\frac{\partial P_O}{\partial p_{s2}}\right)^2 \hat{U}_{p_{s2}}^2 + \left(\frac{\partial P_O}{\partial p_{s3}}\right)^2 \hat{U}_{p_{s3}}^2 \\ + \left(\frac{\partial P_O}{\partial p_{v1}}\right)^2 \hat{U}_{p_{v1}}^2 + \left(\frac{\partial P_O}{\partial p_{v2}}\right)^2 \hat{U}_{p_{v2}}^2 + \left(\frac{\partial P_O}{\partial p_{v3}}\right)^2 \hat{U}_{p_{v3}}^2 \end{bmatrix} \tag{D-13-10}$$

For R

$$\frac{\partial P_O}{\partial R} = \left(-w_1 F_{n1} A_1 \left(\frac{1}{2}\frac{1}{R^{3/2}}\left(\frac{p_{v1}\left(p_{s1}+p_b\right)}{T_{s1}}\right)^{1/2} + \cdots\right)\frac{1}{\rho_F} + \dot{m}_F\left(\frac{T_{t1}}{p_{ta1}}\right)\right)p_{Ft} + Q_F\left(0\right) \tag{D-13-11}$$

Multiplying by (R/R)

$$\frac{\partial P_O}{\partial R} = \left(-w_1 F_{n1} A_1 \frac{1}{R}\left(\frac{1}{2}\left(\frac{p_{v1}\left(p_{s1}+p_b\right)}{RT_{s1}}\right)^{1/2} + \cdots\right)\frac{1}{\rho_F} + \dot{m}_F\left(\frac{RT_{t1}}{p_{ta1}}\right)\right)p_{Ft} \tag{D-13-12}$$

Combining

$$\frac{\partial P_O}{\partial R} = \frac{1}{R}\left(\frac{1}{2}\left(-w_1\dot{m}_1 + \cdots\right)\frac{1}{\rho_F} + \frac{\dot{m}_F}{\rho_F}\right)p_{Ft} \tag{D-13-13}$$

Expanding and combining

$$\left(\frac{1}{P_O^2}\right)\left(\frac{\partial P_O}{\partial R}\right)^2 \hat{U}_R^2 = \left(\frac{\hat{U}_R}{R}\right)^2\left(\frac{1}{P_O^2}\right)\left(\frac{1}{2}\left(-\dot{m}_F\right)\frac{1}{\rho_F} + \frac{\dot{m}_F}{\rho_F}\right)^2 p_{Ft}^2 \tag{D-13-14}$$

$$\left(\frac{1}{P_O^2}\right)\left(\frac{\partial P_O}{\partial R}\right)^2 \hat{U}_R^2 = \left(\frac{1}{P_O^2}\right)\left(-\frac{1}{2}Q_F p_{Ft}^2 + Q_F p_{Ft}^2\right)^2 \hat{u}_R^2 = \frac{1}{4}\hat{u}_R^2 \tag{D-13-15}$$

For A_1, A_2, and A_3

$$\frac{\partial P_O}{\partial A_1} = \left(w_1 F_{n1}\left(\frac{p_{v1}\left(p_{s1}+p_b\right)}{RT_{s1}}\right)^{1/2}\right)p_{Ft}^2 + Q_F\left(0\right) \tag{D-13-16}$$

Multiplying by $\left(A_1/A\right)$

$$\frac{\partial P_O}{\partial A_1}=\frac{1}{A_1}\left(w_1F_{n1}A_1\left(\frac{p_{v1}\left(p_{s1}+p_b\right)}{RT_{s1}}\right)^{1/2}p_{Ft}^2\right) \tag{D-13-17}$$

Expanding and combining

$$\left(\frac{1}{P_O^2}\right)\left(\frac{\partial P_O}{\partial A_1}\right)^2\hat{U}_{A_1}^2=\left(\frac{1}{P_O^2}\right)\left(\frac{\hat{U}_{A_1}}{A_1}\right)^2\left(w_1\dot{m}_1\right)^2p_{Ft}^2 \tag{D-13-18}$$

$$\left(\frac{1}{P_O^2}\right)\left(\frac{\partial P_O}{\partial A_1}\right)^2\hat{U}_{A_1}^2=\left(\frac{w_1\dot{m}_1}{\dot{m}_F}\right)^2\hat{u}_{A_1}^2 \tag{D-13-19}$$

Similarly

$$\left(\frac{1}{P_O^2}\right)\left(\frac{\partial P_O}{\partial A_2}\right)^2\hat{U}_{A_2}^2=\left(\frac{w_2\dot{m}_2}{\dot{m}_F}\right)^2\hat{u}_{A_2}^2 \tag{D-13-20}$$

$$\left(\frac{1}{P_O^2}\right)\left(\frac{\partial P_O}{\partial A_3}\right)^2\hat{U}_{A_3}^2=\left(\frac{w_3\dot{m}_3}{\dot{m}_F}\right)^2\hat{u}_{A_3}^2 \tag{D-13-21}$$

For T_{s1}, T_{s2}, and T_{s3}

$$\frac{\partial P_O}{\partial T_1}=\left(-w_1F_{n1}A_1\frac{1}{2}\left(\frac{p_{v1}\left(p_{s1}+p_b\right)}{R}\right)^{1/2}\frac{1}{T_{s1}^{3/2}}\frac{1}{\rho_F}+\dot{m}_F\frac{R}{p_{ta1}}\right)p_{Ft}+Q_F\left(0\right) \tag{D-13-22}$$

Multiplying by $\left(T_{s1}/T_{s1}\right)$, etc.

$$\frac{\partial P_O}{\partial T_1}=\left(\frac{1}{T_{s1}}\right)\left(-w_1F_{n1}A_1\frac{1}{2}\left(\frac{p_{v1}\left(p_{s1}+p_b\right)}{RT_{s1}}\right)^{1/2}\frac{T_{s1}}{T_{s1}}\frac{1}{\rho_F}+\dot{m}_F\frac{RT_{t1}}{p_{ta1}}\right)p_{Ft} \tag{D-13-23}$$

Expanding and combining

$$\left(\frac{1}{P_O^2}\right)\left(\frac{\partial P_O}{\partial T_{s1}}\right)^2\hat{U}_{T_{s1}}^2=\left(\frac{\hat{U}_{T_{s1}}}{T_{s1}}\right)^2\left(\frac{1}{P_O}\right)^2\left(-\frac{1}{2}w_1\dot{m}_1\frac{p_{Ft}}{\rho_F}+\dot{m}_F\frac{1}{\rho_F}p_{Ft}\right)^2 \tag{D-13-24}$$

$$\left(\frac{1}{P_O^2}\right)\left(\frac{\partial P_O}{\partial T_{s1}}\right)^2\hat{U}_{T_{s1}}^2=\left(\frac{\hat{U}_{T_{s1}}}{T_{s1}}\right)^2\left(-\frac{1}{2}w_1\dot{m}_1\frac{1}{\dot{m}_F}+1\right)^2 \tag{D-13-25}$$

$$\left(\frac{1}{P_O^2}\right)\left(\frac{\partial P_O}{\partial T_{s1}}\right)^2\hat{U}_{T_{s1}}^2=\left(1-\frac{w_1\dot{m}_1}{2\dot{m}_F}\right)^2\hat{u}_{T_{s1}}^2 \tag{D-13-26}$$

Similarly,

$$\left(\frac{1}{P_O^2}\right)\left(\frac{\partial P_O}{\partial T_{s2}}\right)^2\hat{U}_{T_{s2}}^2=\left(\frac{w_2\dot{m}_2}{2\dot{m}_F}\right)^2\hat{u}_{T_{s2}}^2 \tag{D-13-27}$$

$$\left(\frac{1}{P_O^2}\right)\left(\frac{\partial P_O}{\partial T_{s3}}\right)^2 \hat{U}_{T_{s3}}^2 = \left(\frac{w_3 \dot{m}_3}{2\dot{m}_F}\right)^2 \hat{u}_{T_{s3}}^2 \tag{D-13-28}$$

For p_b

$$\frac{\partial P_O}{\partial p_b} = \left(\begin{array}{c} \left(w_1 F_{n1} A_1 \frac{1}{2}\left(\frac{p_{v1}}{RT_{s1}(p_{s1}+p_b)}\right)^{1/2} + w_2 F_{n2} A_2 \frac{1}{2}\left(\frac{p_{v2}}{RT_{s2}(p_{s2}+p_b)}\right)^{1/2} + w_3 F_{n3} A_3 \frac{1}{2}\left(\frac{p_{v3}}{RT_{s3}(p_{s3}+p_b)}\right)^{1/2}\right)\frac{1}{\rho_F} \\ + \dot{m}_F \left(\frac{-RT_{t1}}{(p_{s1}+p_{v1}+p_b)^2}\right) \end{array} \right) p_{Ft} + Q_F(0) \tag{D-13-29}$$

Multiplying by (p_b/p_b), (R/R), (p_{sax}/p_{sax}), and (p_{ta1}/p_{ta1}) where appropriate

$$\frac{\partial P_O}{\partial p_b} = \left(\frac{1}{p_b}\right) \left(\begin{array}{c} \left(w_1 F_{n1} A_1 \frac{1}{2}\left(\frac{p_{v1}(p_{s1}+p_b)}{RT_{s1}(p_{s1}+p_b)^2}\right)^{1/2} p_b + w_2 F_{n2} A_2 \frac{1}{2}\left(\frac{p_{v2}(p_{s2}+p_b)}{RT_{s2}(p_{s2}+p_b)^2}\right)^{1/2} p_b + w_3 F_{n3} A_3 \frac{1}{2}\left(\frac{p_{v3}(p_{s3}+p_b)}{RT_{s3}(p_{s3}+p_b)^2}\right)^{1/2} p_b \right)\frac{p_{Ft}}{\rho_F} \\ + \dot{m}_F \frac{p_b}{p_{ta1}}\left(\frac{-RT_{t1}}{((p_{s1}+p_{v1}+p_b))}\right) p_{Ft} \end{array} \right) \tag{D-13-30}$$

Expanding and combining

$$\left(\frac{\partial P_O}{\partial p_b}\right)^2 = \left(\frac{1}{p_b}\right)^2 \left(\left(w_1 F_{n1} A_1 \frac{1}{2}(p_{v1}\rho_1)^{1/2} \frac{p_b}{p_{sa1}} + w_2 F_{n2} A_2 \frac{1}{2}(p_{v2}\rho_2)^{1/2} \frac{p_b}{p_{sa2}} + w_3 F_{n3} A_3 \frac{1}{2}(p_{v3}\rho_3)^{1/2} \frac{p_b}{p_{sa3}} \right)\frac{1}{\rho_F} - \frac{p_b}{p_{ta1}}\frac{\dot{m}_F}{\rho_F}\right)^2 p_{Ft}^2$$

(D-13-31)

$$\left(\frac{1}{P_O^2}\right)\left(\frac{\partial P_O}{\partial p_b}\right)^2 U_{p_b}^2 = \left(\frac{U_{p_b}}{p_b}\right)^2 \left(\frac{1}{Q_F p_{Ft}}\right)^2 \left(\left(\frac{w_1 \dot{m}_1}{2}\frac{p_b}{p_{sa1}} + \frac{w_2 \dot{m}_2}{2}\frac{p_b}{p_{sa2}} + \frac{w_3 \dot{m}_3}{2}\frac{p_b}{p_{sa3}} \right)\frac{p_{Ft}}{\rho_F} - \frac{p_b}{p_{ta1}}\frac{\dot{m}_F}{\rho_F} p_{Ft}\right)^2 \tag{D-13-32}$$

$$\left(\frac{1}{P_O^2}\right)\left(\frac{\partial P_O}{\partial p_b}\right)^2 U_{p_b}^2 = \left(\frac{w_1 \dot{m}_1}{2\dot{m}_F}\frac{p_b}{p_{sa1}} + \frac{w_2 \dot{m}_2}{2\dot{m}_F}\frac{p_b}{p_{sa2}} + \frac{w_3 \dot{m}_3}{2\dot{m}_F}\frac{p_b}{p_{sa3}} - \frac{p_b}{p_{ta1}}\right)^2 u_{p_b}^2 \tag{D-13-33}$$

For p_{s1} and p_{s2}

$$\frac{\partial P_O}{\partial p_{s1}} = \left(w_1 F_{n1} A_1 \frac{1}{2}\left(\frac{p_{v1}}{RT_{s1}(p_{s2}-p_{s1})}\right)^2 \frac{1}{\rho_F} + \dot{m}_F \frac{-RT_{t1}}{(p_{s1}+p_1+p_b)^2}\right) p_{Ft} + Q_F(-1) \tag{D-13-34}$$

Multiplying by (p_{s1}/p_{s1}), (p_{sa1}/p_{sa1}), and (p_{ta1}/p_{ta1}) where appropriate

$$\frac{\partial P_O}{\partial p_{s1}} = \frac{1}{p_{s1}} \left(\left(w_1 F_{n1} A_1 \frac{1}{2} p_{s1} \left(\frac{p_{v1}}{RT_{s1} p_{sa1}}\right)^{1/2} \frac{p_{sa1}}{p_{sa1}}\frac{1}{\rho_F} + \dot{m}_F p_{s1} \frac{1}{p_{ta1}} \frac{-RT_{t1}}{(p_{s1}+p_1+p_b)}\right) p_{Ft} - p_{s1} Q_F \right) \tag{D-13-35}$$

Combining

$$\frac{\partial P_O}{\partial p_{s1}} = \frac{1}{p_{s1}} \left(w_1 F_{n1} A_1 \frac{1}{2}(\rho_1 p_{v1})^{1/2} \frac{p_{s1}}{p_{sa1}}\frac{1}{\rho_F} - \frac{p_{s1}}{p_{ta1}}\frac{\dot{m}_F}{\rho_F}\right) p_{Ft} - p_{s1} Q_F \tag{D-13-36}$$

Expanding and combining further

$$\left(\frac{1}{P_O^2}\right)\left(\frac{\partial P_O}{\partial p_{s1}}\right)^2 \hat{U}_{p_{s1}}^2 = \left(\frac{1}{P_O^2}\right)\frac{\hat{U}_{p_{s1}}^2}{p_{s1}^2}\left(\left(\frac{w_1\dot{m}_1}{2}\frac{p_{s1}}{p_{sa1}}\frac{1}{\rho_F} - \frac{p_{s1}}{p_{ta1}}Q_F\right)p_{Ft} - p_{s1}Q_F\right)^2 \quad \text{(D-13-37)}$$

$$\left(\frac{1}{P_O^2}\right)\left(\frac{\partial P_O}{\partial p_{s1}}\right)^2 \hat{U}_{p_{s1}}^2 = \left(\frac{1}{Q_F\, p_{Ft}}\right)^2\left(\left(\frac{w_1\dot{m}_1}{2}\frac{p_{s1}}{p_{sa1}}\frac{p_{Ft}}{\rho_F} - \frac{p_{s1}}{p_{ta1}}Q_F\, p_{Ft}\right) - p_{s1}Q_F\right)^2 \hat{u}_{p_{s1}}^2 \quad \text{(D-13-38)}$$

$$\left(\frac{1}{P_O^2}\right)\left(\frac{\partial P_O}{\partial p_{s1}}\right)^2 \hat{U}_{p_{s1}}^2 = \left(\frac{w_1\dot{m}_1}{2\dot{m}_F}\frac{p_{s1}}{p_{sa1}} - \frac{p_{s1}}{p_{ta1}} - \frac{p_{s1}}{p_{Ft}}\right)^2 \hat{u}_{p_{s1}}^2 \quad \text{(D-13-39)}$$

Similarly,

$$\left(\frac{1}{P_O^2}\right)\left(\frac{\partial P_O}{\partial p_{s2}}\right)^2 \hat{U}_{p_{s2}}^2 = \left(\frac{w_2\dot{m}_2}{2\dot{m}_F}\frac{p_{s2}}{p_{sa2}} + \frac{p_{s2}}{p_{Ft}}\right)^2 \hat{u}_{p_{s2}}^2 \quad \text{(D-13-40)}$$

and

$$\left(\frac{1}{P_O^2}\right)\left(\frac{\partial P_O}{\partial p_{s3}}\right)^2 \hat{U}_{p_{s3}}^2 = \left(\frac{w_3\dot{m}_3}{2\dot{m}_F}\frac{p_{s3}}{p_{sa3}}\right)^2 \hat{u}_{p_{s3}}^2 \quad \text{(D-13-41)}$$

For p_{v1}, p_{v2}, and p_{v3}

$$\frac{\partial P_O}{\partial p_{v1}} = \left(\frac{1}{2}w_1F_{n1}A_1\left(\frac{\rho_1}{p_{v1}}\right)^{1/2}\frac{1}{\rho_F} + \dot{m}_F\frac{-T_{t1}}{\left(p_{s1}+p_{v1}+p_b\right)^2}\right)p_{Ft} + Q_F\left(-1\right) \quad \text{(D-13-42)}$$

Multiplying by $\left(p_{v1}/p_{v1}\right)$ and $\left(p_{ta1}/p_{ta1}\right)$ where appropriate

$$\frac{\partial P_O}{\partial p_{v1}} = \frac{1}{p_{v1}}\left(\left(\frac{1}{2}w_1F_{n1}A_1\left(\rho_1 p_{v1}\right)^{1/2}\frac{1}{\rho_F} + \dot{m}_F\frac{p_{v1}}{p_{ta1}}\frac{-RT_{t1}}{\left(p_{v1}+p_{s1}+p_b\right)}\right)p_{Ft} - p_{v1}Q_F\right) \quad \text{(D-13-43)}$$

Expanding and combining

$$\left(\frac{1}{P_O^2}\right)\left(\frac{\partial P_O}{\partial p_{v1}}\right)^2 \hat{U}_{p_{v1}}^2 = \left(\frac{1}{P_O}\right)^2\left(\frac{1}{2}w_1\dot{m}_1\frac{1}{\rho_F} - \frac{p_{v1}}{p_{ta1}}\frac{\dot{m}_F}{\rho_F} - p_{v1}Q_F\right)^2\left(\frac{\hat{U}_{p_{v1}}}{p_{v1}}\right)^2 \quad \text{(D-13-44)}$$

$$\left(\frac{1}{P_O^2}\right)\left(\frac{\partial P_O}{\partial p_{v1}}\right)^2 \hat{U}_{p_{v1}}^2 = \left(\frac{w_1\dot{m}_1}{2\dot{m}_F} - \frac{p_{v1}}{p_{ta1}} - \frac{p_{v1}}{p_{Ft}}\right)^2 \hat{u}_{pv1}^2 \quad \text{(D-13-45)}$$

Similarly,

$$\left(\frac{1}{P_O^2}\right)\left(\frac{\partial P_O}{\partial p_{v2}}\right)^2 \hat{U}_{p_{v2}}^2 = \left(\frac{w_2\dot{m}_2}{2\dot{m}_F} + \frac{p_{v2}}{p_{Ft}}\right)^2 \hat{u}_{p_{v2}}^2 \quad \text{(D-13-46)}$$

and

$$\left(\frac{1}{P_O^2}\right)\left(\frac{\partial P_O}{\partial p_{v3}}\right)^2 \hat{U}_{p_{v3}}^2 = \left(\frac{w_3\dot{m}_3}{2\dot{m}_F}\right)^2 \hat{u}_{p_{v3}}^2 \quad \text{(D-13-47)}$$

Gathering terms

$$\hat{u}_{P_O}^2 = \left[\begin{array}{l} \frac{1}{4}u_R^2 + \left(\frac{w_1\dot{m}_1}{\dot{m}_F}\right)^2 \hat{u}_{F_{n1}}^2 + \left(\frac{w_2\dot{m}_2}{\dot{m}_F}\right)^2 \hat{u}_{F_{n2}}^2 + \left(\frac{w_3\dot{m}_3}{\dot{m}_F}\right)^2 \hat{u}_{F_{n3}}^2 \\ + \left(\frac{w_1\dot{m}_1}{\dot{m}_F}\right)^2 \hat{u}_{A_1}^2 + \left(\frac{w_2\dot{m}_2}{\dot{m}_F}\right)^2 \hat{u}_{A_2}^2 + \left(\frac{w_3\dot{m}_3}{\dot{m}_F}\right)^2 \hat{u}_{A_3}^2 \\ + \left(1 - \frac{w_1 m_1}{2\dot{m}_F}\right)^2 \hat{u}_{T_{s1}}^2 + \left(\frac{w_2 m_2}{2\dot{m}_F}\right)^2 \hat{u}_{T_{s2}}^2 + \left(\frac{w_3 m_3}{2\dot{m}_F}\right)^2 \hat{u}_{T_{s3}}^2 \\ + \left(\frac{w_1\dot{m}_1}{2\dot{m}_F}\frac{p_b}{p_{sa1}} + \frac{w_2\dot{m}_2}{2\dot{m}_F}\frac{p_b}{p_{sa2}} + \frac{w_3\dot{m}_3}{2\dot{m}_F}\frac{p_b}{p_{sa3}} - \frac{p_b}{p_{ta1}}\right)^2 \hat{u}_{p_b}^2 \\ + \left(\frac{w_1\dot{m}_1}{2\dot{m}_F}\frac{p_{s1}}{p_{sa1}} - \frac{p_{s1}}{p_{ta1}} - \frac{p_{s1}}{p_{Ft}}\right)^2 \hat{u}_{p_{s1}}^2 + \left(\frac{w_2\dot{m}_2}{2\dot{m}_F}\frac{p_{s2}}{p_{sa2}} + \frac{p_{s2}}{p_{Ft}}\right)^2 \hat{u}_{p_{s2}}^2 + \left(\frac{w_3\dot{m}_3}{2\dot{m}_F}\frac{p_{s3}}{p_{sa3}}\right)^2 \hat{u}_{p_{s3}}^2 \\ + \left(\frac{w_1\dot{m}_1}{2\dot{m}_F} - \frac{p_{v1}}{p_{ta1}} - \frac{p_{v1}}{p_{Ft}}\right)^2 \hat{u}_{pv1}^2 + \left(\frac{w_2\dot{m}_2}{2\dot{m}_F} + \frac{p_{v2}}{p_{Ft}}\right)^2 \hat{u}_{p_{v2}}^2 + \left(\frac{w_3\dot{m}_3}{2\dot{m}_F}\right)^2 \hat{u}_{p_{v3}}^2 \end{array} \right] \quad \text{(D-13-48)}$$

Inserting unit conversions

$$\hat{u}_{P_O}^2 = \left[\begin{array}{l} \frac{1}{4}\hat{u}_R^2 + \left(\frac{w_1\dot{m}_1}{\dot{m}_F}\right)^2 \hat{u}_{F_{n1}}^2 + \left(\frac{w_2\dot{m}_2}{\dot{m}_F}\right)^2 \hat{u}_{F_{n2}}^2 + \left(\frac{w_3\dot{m}_3}{\dot{m}_F}\right)^2 \hat{u}_{F_{n3}}^2 \\ + \left(\frac{w_1\dot{m}_1}{\dot{m}_F}\right)^2 \hat{u}_{A_1}^2 + \left(\frac{w_2\dot{m}_2}{\dot{m}_F}\right)^2 \hat{u}_{A_2}^2 + \left(\frac{w_3\dot{m}_3}{\dot{m}_F}\right)^2 \hat{u}_{A_3}^2 \\ + \left(1 - \frac{w_1 m_1}{2\dot{m}_F}\right)^2 \hat{u}_{T_{s1}}^2 + \left(\frac{w_2 m_2}{2\dot{m}_F}\right)^2 \hat{u}_{T_{s2}}^2 + \left(\frac{w_3 m_3}{2\dot{m}_F}\right)^2 \hat{u}_{T_{s3}}^2 \\ + \left(\frac{w_1\dot{m}_1}{2\dot{m}_F}\frac{C_{13}p_b}{p_{sa1}} + \frac{w_2\dot{m}_2}{2\dot{m}_F}\frac{C_{13}p_b}{p_{sa2}} + \frac{w_3\dot{m}_3}{2\dot{m}_F}\frac{C_{13}p_b}{p_{sa3}} - \frac{C_{13}p_b}{p_{ta1}}\right)^2 u_{p_b}^2 \\ + \left(\frac{w_1\dot{m}_1}{2\dot{m}_F}\frac{p_{s1}}{p_{sa1}} - \frac{p_{s1}}{p_{ta1}} - \frac{p_{s1}}{p_{Ft}}\right)^2 \hat{u}_{p_{s1}}^2 + \left(\frac{w_2\dot{m}_2}{2\dot{m}_F}\frac{p_{s2}}{p_{sa2}} + \frac{p_{s2}}{p_{Ft}}\right)^2 \hat{u}_{p_{s2}}^2 + \left(\frac{w_3\dot{m}_3}{2\dot{m}_F}\frac{p_{s3}}{p_{sa3}}\right)^2 \hat{u}_{p_{s3}}^2 \\ + \left(\frac{w_1\dot{m}_1}{2\dot{m}_F} - \frac{p_{v1}}{p_{ta1}} - \frac{p_{v1}}{p_{Ft}}\right)^2 \hat{u}_{pv1}^2 + \left(\frac{w_2\dot{m}_2}{2\dot{m}_F} + \frac{p_{v2}}{p_{Ft}}\right)^2 \hat{u}_{p_{v2}}^2 + \left(\frac{w_3\dot{m}_3}{2\dot{m}_F}\right)^2 \hat{u}_{p_{v3}}^2 \end{array} \right] \quad \text{(5-13-36)}$$

NONMANDATORY APPENDIX E
REFERENCES AND FURTHER READING

E-1 REFERENCES

[1] Gerhart, P. M., Jorgensen, R., and Kroll, J. "A Comparison of Two Alternative Methods for Defining Fan Performance," Journal of Engineering for Power, January 1982.

[2] ASHRAE Standard 51-75/AMCA Standard 210-74, "Laboratory Methods of Testing Fans for Rating."

[3] Yost, John G. "Field Performance Testing of Large Power Plant Fans," MSME Thesis, University of Akron, Akron, Ohio.

[4] ISO Standard 5168, "Measurement of Fluid Flow-Estimation of Uncertainty of a Flow-Rate Measurement."

[5] Kline, S. J., and McClintock, F. A. "Describing Uncertainties in Single-Sample Experiments," Mechanical Engineering, January 1953.

[6] Brown, N. "A Mathematical Evaluation of Pitot Tube Traverse Methods," ASHRAE Paper 2325, 1975.

[7] "Draft Proposal for an ISO Standard: Measurement of Fluid Flow in Closed Conduits by the Velocity Area Method Using Pitot-Static Tubes," ISO/TC-30/SC-3, February 1974.

[8] "Particulate Sampling Strategies for Large Power Plants Including Non-uniform Flow," EPA Report PB-257-090, June 1976.

[9] Gerhart, P. M., Nuspl, S., Wood, C., and Lovejoy, S. "An Evaluation of Velocity Probes for Measuring Non-uniform Gas Flow in Large Ducts," Journal of Engineering for Power, October 1979.

[10] Gerhart, P. M., and Dorsey, M. J. "Investigation of Field Test Procedures for Large Fans," EPRI Report CS 1651, December 1980.

[11] Madhaven, S., DiRe, J., and Wright, T. "Inlet Flow Distortion in Centrifugal Fans," ASME Paper 84-JPGC-GT-4.

[12] Wright, T., Madhaven, S., and DiRe, J. "Centrifugal Fan Performance With Disturbed Inflows," ASME Paper 83-JPGC-GT-5.

[13] AMCA Publication 201, Fans and Systems.

[14] Clarke, M. S. "The Implementation and Analysis of a PTC 11 Test Program," 1982 AMCA Engineering Conference.

[15] Wyler, J. S. "Probe Blockage Effects in Free Jets and Closed Tunnels," Journal of Engineering for Power, October 1975.

[16] Benedict, R. P. Fundamentals of Temperature, Pressure, and Flow Measurements, 2nd Edition, Wiley-lnterscience, 1977, pp. 356–359.

[17] Gerhart, P. M. "Averaging Methods for Determining the Performance of Large Fans from Field Measurements," Journal of Engineering for Power, April 1981.

[18] Dean, R. C. (ed.) Aerodynamic Measurements, MIT Gas Turbine Lab Report, 1953.

[19] Obert, E., and Gaggoli, R. Thermodynamics, 2nd Edition, McGraw-Hill Book Co., 1963.

[20] Perry and Chilton. Chemical Engineers Handbook, 5th Edition, McGraw-Hill Book Co., 1973, pp. 3–248.

[21] "Compressibility Effects for Industrial Fans," ISO/TC 117/SC-1, January 1982.

[22] Dorsey, M. J. "Minimizing and Estimation of Uncertainty in Numerical Integration by Controlling the Number and Location of Integration Data Points," MSME Thesis, August 1986, University of Akron.

E-2 FURTHER READING

Henry, R., Jorgensen, R., and Gerhart, P.M. Uncertainty Analysis in Fan Testing, POWER 2007-22099.

Kaufman, S., and Falk, M. Fan Performance Test and Evaluation Methodology: Addressing the Difficulties of Full and Partial Capacity Testing When the Specified and Site Conditions Don't Match, POWER 2007-22110.

Nuspl, S., and Gerhart, P. M. Reduced Load Fan Performance Testing for Input Power Determination, POWER 2007-22087.

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