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CRITERIA FOR SHELL-AND-TUBE HEAT EXCHANGERS ACCORDING TO PART UHX OF ASME SECTION VIII DIVISION 1

Prepared by:

Francis Osweiller OSWECONSULT



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FOREWORD

The purpose of this document is to justify and provide technical criteria for the rules of Part Unfired Heat Exchanger (UHX) of ASME Section VIII Division 1, 2013 Edition, devoted to the design of U-tube, Fixed and Floating head Tubesheet Heat Exchangers. The criteria document applies also to Paragraph 4.18 of Section VIII, Division 2, 2013 Edition, which is entirely based on Part UHX.

Confirmation and documentation of the basis for UHX-rules is important for the members of the ASME Subgroup on Heat Transfer Equipment to use as a future reference, for confirmation or comparisons of code requirements, and for code development. It will be a valuable reference for both early career and experienced engineers who are using the UHX rules and may become involved in code development of such rules in the future.

The analytical treatment of the fixed tubesheet heat exchangers is based on classical discontinuity analysis methods to determine the moments and forces that the tubesheet, tubes, shell and channel must resist. The treatment provides, at any radius of the perforated tubesheet, the deflection, the rotation, the bending and shear stresses and the axial stress in the tubes. A parametric study permits one to determine the maximum stresses in the tubesheet and in the tubes which are given in UHX-13. The Floating Tubesheet and U-tube Tubesheet heat exchangers are treated as simplified cases of fixed tubesheet heat exchangers. A check of the results obtained is provided by comparing Finite Element Analysis (FEA) results, Tubular Exchanger Manufacturers Association (TEMA) results, and the French pressure vessel code Code Français de Construction des Appareils à Pression (CODAP). Applying the appropriate simplifications, the classical formulas for circular plates subjected to pressure, have been obtained.

The author thanks the members of the peer review committee who sent many valuable comments and provided helpful consulting in the development of this Criteria Document. In particular Ramsey Mahadeen for his support and detailed reviews, Urey Miller for his help in stress classification considerations, Tony Norton for his comments on theoretical issues and performing FEA calculations, Guido Karcher for his support, Anne Chaudouet who spent so much time for checking the development of the formulas and Gabriel Aurioles who supplied the raw Excel spreadsheets and graphs for analysis and was very helpful for computer issues.

The author acknowledges Centre Technique des Industries Mécaniques (CETIM) for its support in the development of the Criteria Document appearing in PART 3, dedicated to fixed tubesheet heat exchangers. The author further acknowledges, with deep appreciation, the activities of ASME ST-LLC and ASME staff and volunteers who have provided valuable technical input, advice and assistance with review and editing of, and commenting on this document.

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ABBREVIATIONS AND ACRONYMS

ASME	American Society of Mechanical Engineers
CL	Clamped
EEC	Effective Elastic Constants
FEA	Finite Element Analysis
FL	Floating
HEs	Heat Exchanger(s)
LE	Ligament Efficiency
SG-HTE	Subgroup on Heat Transfer Equipment
SS	Simply Supported
ST	Stationary
TEMA	Tubular Exchanger Manufacturers Association
TSs	Tubesheet(s)
UHX	Unfired Heat Exchanger

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PART 1 INTRODUCTION

1 SCOPE

This criteria document covers the development of the tubesheet (TS) design rules for the U-tube, Fixed, and Floating Head TS Heat Exchangers (HE) configurations contained in Part UHX of Section VIII Division 1, 2013 Edition. It applies also to Paragraph 4.18 of Section VIII, Division 2 which is entirely based on Part UHX.

The free body diagram of the HE, the equilibrium and compatibility equations, the solution of resulting differential equations and all intermediate steps are provided to show the derivation of:

- the deflection and the rotation at any radius of the TS,
- the bending and shear stress at any radius of the TS,
- the axial stresses in the tubes at any radius of the TS,
- the axial stretch force acting in the shell,
- the axial displacement of the shell.

The document provides the technical basis of the following items:

- the required loading case combinations,
- the acceptance criteria for each TS configuration, as applicable,
- the TS characteristics including the Effective Elastic Constants,
- the TS extended as a flange.

The following effects are in addition to the above basic items:

- the effect of different shell material or thickness adjacent to the TS,
- the effect of plasticity at TS-shell-channel joint,
- the effect of radial differential thermal expansion between the TS and integral shell and channel,
- the tubesheet calculated as a simply supported TS.



2 HISTORICAL BACKGROUND

The first rules devoted to tubular HEs were developed by TEMA [CS-1] for the first time in 1941 to design U-tube and floating TSs. The design formula was based on the formula for circular plates subject to pressure:

$$T = F \frac{G}{2} \sqrt{\frac{P}{S}}$$

This semi-empirical formula does not account for the tubes that stiffen the TS, and for the holes that weaken it. Fixed TS HEs were covered in TEMA 1968 5th edition, based on Gardner's work. Gardner [1][2] in 1948 for floating TSs and in 1952 for fixed TSs was the very first to set-up the basis of a more rational approach by taking into consideration the support afforded by the tubes and the weakening effect of the TS holes. This design method, which involves 15 parameters instead of 3 previously in TEMA, was adopted by TEMA in 1968 in its 5th edition and by the Stoomwezen [CS-2] in 1973.

Simultaneously, and independently, Miller [3] proposed a similar approach that was published in the British Code BS 1515 [CS-3] in 1965. These design rules have the drawback of considering the TS as either simply supported or clamped at its periphery, which compels the designer to make a more or less arbitrary choice between these two extreme cases.

Galletly [4] in 1959 overcame this issue by taking into account the degree of rotational restraint of the TS at its periphery by the shell and channel. This method was adopted by the French pressure vessel code Code Français de Construction des Appareils à Pression (CODAP) [CS-4] in 1982 for fixed and floating HEs. CODAP rules were adopted by the European Pressure Vessel Standard EN 13445 [CS-5] published in 2002.

Gardner [5] in 1969 improved his method for U-tube and floating HEs by considering the unperforated solid rim at the periphery of the TS. This design method was adopted by BS 5500 [CS-6], CODAP (for U-tube) and ISO [CS-7] in the late seventies and by ASME Section VIII (Appendix AA) in 1982. Despite these improvements, TEMA rules have been extensively used throughout the world during the last six decades as they have the merit of long satisfactory industrial experience and simplicity. However due to that simplicity (the strengthening of the tubes is assumed to counterbalance the weakening effect of the tubesheet holes), they often lead to TS over-thickness if the strengthening effect controls or under-thickness if the weakening effect controls. Today these disadvantages increase as the chemical and power industries need larger exchangers operating at higher pressures and temperatures. For more details, see Osweiller [6].

In 1975 ASME Subcommittee VIII established a "Subgroup on Heat-Transfer Equipment (SG-HTE)" with the task of developing new rules for the design of TS HEs based on a more rigorous approach than TEMA. This was achieved by considering the perforated TS, the tubes acting as an elastic foundation, the unperforated rim and the connection of the TS with the shell and channel. The analytical treatment is based on Soler's book [6].

In 1992 ASME and CODAP decided to reconcile their rules (scope, TS configurations, loading cases notations, ligament efficiencies, effective elastic constants, design formulas), as they were based on the same approach. For more details, see Osweiller [7]. The 1st ASME draft was issued in 1985. From 1992 to 2002 the ASME tubesheet rules have been published in non-mandatory Appendix AA of Section VIII-Div. 1 so that manufacturers can use them as an alternative to TEMA rules.

In 2003 these rules were upgraded to mandatory status and published in the 2003 Addenda as a new Part UHX of Section VIII Div. 1, UHX standing for Unfired Heat Exchanger. As of January 1, 2004 a designer does not have the option of using the TEMA rules if the HE needs to be U-stamped.

UHX design rules cover essentially the design by formula of the heat exchanger pressure containing components. For other aspects such as fabrication, tube-to-tubesheet joints, inspection, maintenance, repair, troubleshooting, etc. see Reference [8]. TEMA [CS-1] also covers similar aspects of design such as minimum component thickness (e.g., baffle plates, pass partitions, etc.), tube vibration, etc.



3 TYPES OF HEAT EXCHANGERS COVERED

UHX rules apply to the three typical types of tubular HEs:

- **U-Tube Heat Exchanger:** HE with one stationary TS attached to the shell and channel. The HE contains a bundle of U-tubes attached to the TS, as shown in Figure 1 sketch (a).
- **Fixed Tubesheet Heat Exchanger:** HE with two stationary TSs, each attached to the shell and channel. The HE contains a bundle of straight tubes connecting both TSs, as shown in Figure 1 sketch (b).
- Floating Tubesheet Heat Exchanger: HE with one stationary TS attached to the shell and channel, and one floating TS that can move axially. The HE contains a bundle of straight tubes connecting both TSs, as shown in Figure 1 sketch (c).

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¹ Configurations of tubesheet – shell – channel connections are detailed in 3-2.



4 TYPES OF TS CONFIGURATIONS

The TS is attached to the shell and channel either by welding (integral TS) or by bolting (gasketed TS) according to 6 configurations encountered in the industry (see Figure 2):

- configuration a: tubesheet integral with shell and channel;
- configuration b: tubesheet integral with shell and gasketed with channel, extended as a flange;
- configuration c: tubesheet integral with shell and gasketed with channel, not extended as a flange;
- configuration d: tubesheet gasketed with shell and channel, extended as a flange or not
- configuration e: tubesheet gasketed with shell and integral with channel, extended as a flange;
- configuration f: tubesheet gasketed with shell and integral with channel, not extended as a flange.



Figure 2 — Tubesheet Configurations



5 LOADING CASES

The normal operating condition of the HE is achieved when the tube side pressure P_t and shell side pressure P_s act simultaneously. However, a loss of pressure or a loss of temperature is always possible. Accordingly, for safety reasons, the designer must always consider the cases where $P_s=0$ and $P_t=0$ for the normal operating conditions.

He must also consider the start-up conditions, the shutdown conditions and the upset conditions, if any, which may govern the design.

A TS HE is a statically indeterminate structure for which it is difficult to determine the most severe condition of coincident pressure and temperature. Thus, it is necessary to evaluate all the anticipated loading conditions mentioned above to ensure that the worst load combination has been considered in the design.

For each of these conditions, the following 3 pressure loading cases must be considered.

- Loading Case 1: Tube side pressure P_t acting only ($P_s = 0$).
- Loading Case 2: Shell side pressure P_s acting only ($P_t = 0$).
- Loading Case 3: Tube side pressure P_t and shell side pressure P_s acting simultaneously.

For fixed TS HEs, the axial differential thermal expansion between tubes and shell has to be considered and one set of arbitrary thermal loading cases must be added (loading cases 4, 5, 6, 7) as they act simultaneously with the pressure loading cases.

When vacuum exists, each loading case is considered with and without the vacuum. These loading cases have been traditionally considered in TEMA, French code CODAP, European Standard EN 13445, and earlier editions of UHX.

The 2013 UHX Edition replaces these arbitrary thermal loading cases by actual loading cases accounting for the actual operating pressures and temperatures so that the designer can realistically determine the controlling conditions for each operating loading case considered, including, but not be limited to, normal operating, startup, shutdown, cleaning, and upset conditions. The pressure definitions have been changed to include maximum and minimum design and operating pressures that may be encountered in a particular design. These new loading cases are detailed in Section 3.3 of Part 3.

As the calculation procedure is iterative, a value h is assumed for the tubesheet thickness to calculate and check that the maximum stresses in tubesheet, tubes, shell, and channel are within the maximum permissible stress limits.

Because any increase of tubesheet thickness may lead to over-stress of the tubes, shell, or channel, a final check must be performed, using in the formulas the nominal thickness of tubesheet, tubes, shell, and channel, in both corroded and uncorroded conditions.



6 STRUCTURE OF PART UHX

- UHX-1 to UHX-8 provide general considerations (Scope, Material, Fabrication, Terminology,...) which are common to the three types of HEs.
- UHX-9 provides design rules for the TS flange extension
- UHX-10 (Conditions of Applicability) specifies under which conditions the rules are applicable
- UHX-11 (TS Characteristics) is also common to the three types of HEs and provides the design formula for the ligament efficiency and the effective elastic constants.
- UHX-12, UHX-13 and UHX-14 provide the design rules for U-tube, Fixed and Floating TS HEs.

These three chapters are self-supporting and structured in the same way:

- (1) Scope
- (2) Conditions of Applicability
- (3) Notations
- (4) Design Considerations
- (5) Calculation Procedure

Additional rules are provided to cover more specific calculations:

- (1) Effect of different shell material or thickness adjacent to the TS
- (2) Effect of plasticity at the tubesheet-shell-channel joint
- (3) Effect of radial thermal expansion adjacent to the TS
- (4) Calculation of the TS when considered as simply supported
- (5) Calculation of the TS flange extension
- UHX-15 to UHX-19 provide considerations on Tube-to-Tubesheet Welds, Expansion Bellows, Pressure Tests and Marking.
- ASME PTB-4-2013, ASME Section VIII Division 1 Example Problem Manual (PTB-4), provides design examples for each type of HE.



7 STRUCTURE OF THE DOCUMENT

This document is structured in 6 PARTS.

- PART 1: Introduction (purpose, background, general issues)
- PART 2: Tubesheet Characteristics (ligament efficiencies, effective elastic constants)
- PART 3: Analytical treatment of Fixed TS HEs
- PART 4: Analytical treatment of Floating TS HEs
- PART 5: Analytical treatment of U-tube TS HEs
- PART 6: Conclusions

Each PART is independent with basically the same chapters: Scope, Historical Background, Notations, Configurations covered, Design assumptions, and Analytical treatment.

The order of the analytical treatment is based on the complexity of the HE model. The Fixed TS HE is treated first, because it is the most complex. The Floating TS HE treated second, since it is a simplified case of the Fixed TS. The U-Tube TS HE is treated last, since it is a simplified case of the Floating TS. The detailed structure is given in the Table of Contents.

This document provides the derivation of UHX design rules (UHX-1, UHX-9 to 14, UHX-17 and 20) and refers explicitly to these as necessary. Other UHX rules (UHX-2, 3 and 4, UHX-15 to 19), which are not linked to design formulas are not covered.



8 NOTATIONS

Notations are common to the three (3) types of HEs. They are detailed in PART 3, Section 3.2.

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- [CS-4] CODAP: French Code for Unfired pressure Vessels 2005 Edition Section C7 "Design rules for HE TSs"
- [CS-5] EN 13445: European Standard for Unfired Pressure Vessels 2002 Edition Chapter 13 "HE TSs"
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PART 2 TUBESHEET CHARACTERISTICS



1 SCOPE (UHX-11.1)

PART 2 provides the technical basis for the determination of:

- the ligament efficiencies μ and μ^*
- the effective elastic constants E*;

which are given in Section 11 of Part UHX.

These quantities are important as they enable the replacement of the actual perforated tubesheet (TS) by an equivalent solid plate, which is necessary to develop the analytical treatment.



2 NOTATIONS

Notations are taken from UHX-11.3 and are repeated here for convenience.

A_L	=	total area of untubed lanes = $U_{L1} L_{L1} + U_{L2} L_{L2} + \dots$ limited to $4D_{op}$
C_{S}	=	tubesheet corrosion allowance on the shell side
C_t	=	tubesheet corrosion allowance on the tube side
D_o	=	equivalent diameter of outer tube limit circle (see Figure 6)
d	=	diameter of tube hole
d_t	=	nominal outside diameter of tubes
d^*	=	effective tube hole diameter
E	=	modulus of elasticity for tubesheet material at the tubesheet design temperature
E_{tT}	=	modulus of elasticity for tube material at tubesheet design temperature
E^*	=	effective modulus of elasticity of tubesheet in perforated region
h	=	tubesheet thickness
h_g	=	tube side pass partition groove depth (see Figure 9)
h'_{g}	=	effective tube side pass partition groove depth
$L_{L1}, L_{L2},$	=	length(s) of untubed lane(s) (see Figure 7)
ℓ_{tx}	=	expanded length of tube in tubesheet $(0 \le \ell_{tx} \le h)$ (see Figure 8).
р	=	tube pitch
p^*	=	effective tube pitch
r_o	=	radius to outermost tube hole center (see Figure 6)
S	=	allowable stress for tubesheet material at tubesheet design temperature
S_{tT}	=	allowable stress for tube material at tubesheet design temperature
t_t	=	nominal tube wall thickness
$U_{I1}, U_{I2},$	=	center-to-center distance between adjacent tube rows of untubed lane(s), limited
- 21, - 22,		to $4n$ (see Figure 7)
	=	basic ligament efficiency for shear
μ 11*	=	effective ligament efficiency for bending
μ ν*	_	effective Poisson's ratio in perforated region of the tubesheet
<i>V</i> ·	_	
ρ	=	tube expansion depth ratio = ℓ_{tx}/h , $(0 \le \rho \le 1)$



3 DESIGN ASSUMPTIONS (UHX-11.2)

The perforated TS is assumed to be uniformly perforated in a triangular or square pattern.

4 LIGAMENT EFFICIENCIES (UHX-11.5.1)

4.1 Introduction

The TS treatment necessitates the replacement of the actual perforated TS by an equivalent unperforated solid plate of same diameter. The ligament efficiency accounts for the fact that the shear load and the bending moment calculated in this unperforated solid plate must be corrected as they apply only to the ligament located between two adjacent holes in the actual TS.

The ligament is represented by the hatched area in Figure 3. Its length varies from $\alpha\alpha'=p-d$ to $\delta\delta'=p$.

Its minimum length $\alpha \alpha' = p$ -d leads to a basic ligament efficiency $\mu = \frac{p-d}{p} = 1 - \frac{d}{p}$

So to on the safe side, this formula is generally used to calculate the shear stress in the actual TS. This means that, for the example pattern (p=1.25 and d=1.0) μ = 0.2, the shear stress in the actual TS will be 5 times the shear stress calculated in the equivalent solid plate.

Among all the ligaments located at radius r of the TS, one can reasonably assume that at least one of length $\alpha\alpha'$ is radially oriented as shown in Figure 4. Therefore the radial bending moment M(r) calculated in the equivalent TS applies only to the ligament between the two holes, which means that the bending moment in the actual TS must be multiplied by the ratio $\omega\omega'/\alpha\alpha'$ or divided by the minimum bending ligament efficiency:

$$\mu^* = \frac{\alpha \alpha'}{\omega \omega'} = \frac{p-d}{p}$$

Its mean length $\beta\beta$ ' leads to a higher ligament efficiency value:

$$\mu^* = \frac{pd - \pi d^2 / 4}{pd} = 1 - \frac{\pi}{4} \cdot \frac{d}{p} = 1 - 0.785 \frac{d}{p}$$

For the example pattern above: $\mu^*=0.372$, which means that the calculated bending moment in the actual TS would be almost half the value of that using the minimum ligament efficiency of 0.2 above. These two extreme examples show the significant impact of the ligament efficiency on the stresses obtained in the actual TS.



Figure 3 — Ligament Area in the Actual Tubesheet

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Figure 4 — Ligament Orientation in the Actual Tubesheet

4.2 Historical Background

Various formulas have been used for the determination of the bending LE in codes and standards.

• BS5500 [CS-1], CODAP [CS-2], EN 13445 [CS-3], ISO [CS-4] and STOOMWEZEN [CS-5] have used the minimum LE when the tubes are welded only: $\mu^* = \frac{p-d}{p}$. When the tubes are expanded throughout the full depth of the TS, experimental tests have shown that about half of the thickness participates to the TS strength, which leads to: $\mu^* = \frac{p-(d-t_t)}{p}$

• **TEMA [CS-6] uses the mean LE** based on the ratio of the hole area, $s_o = \pi d^2/4$, to the portion of TS area "s" pertaining to that hole, as shown in Figure 5: $\mu^* = 1 - \frac{S_o}{s}$ (μ^* quoted η in TEMA). This leads to:

For square pattern: $s = p^2$ and $\mu^* = 1 - \frac{\pi}{4} \left(\frac{d}{p}\right)^2 = 1 - \frac{0.785}{(p/d)^2}$ For triangular pattern: $s = p^2 \sin(60^\circ)$ and $\mu^* = 1 - \frac{\pi}{4} \frac{d^2}{p^2 \sin(60^\circ)} = 1 - \frac{0.907}{(p/d)^2}$

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Figure 5 — Ligament Efficiency Used in TEMA

For shear, these codes and standards, use the minimum ligament efficiency: $\mu = \frac{p-d}{p}$

4.3 LE in Part UHX (UHX-11.5.1)

(*a*) **Equivalent diameter D**_o of the perforated TS is defined as the equivalent diameter of the outer tube limit circle, calculated from the radius r_0 of the outermost tube hole center (see Figure 6):



Figure 6 — TS Equivalent Diameter D_o

The diameter D_o corresponds to the similar concept used in TEMA for the equivalent diameter D_L used for the determination of shear stress and D_o and D_L will have about the same values.

(b) Basic ligament efficiency for shear load

Once the shear load has been determined in the equivalent TS, it must be corrected to account for the holes in the actual TS, by applying the minimum ligament efficiency μ :

$$\mu = \frac{p-d}{p}$$

(c) Effective ligament efficiency for bending moment

Once the bending moment has been determined in the equivalent TS, it must be corrected to account for the holes in the actual TS, by applying the bending effective ligament efficiency μ^* defined as follows:

$$\mu^* = \frac{p^* - d^*}{p^*}$$

(d) Effective pitch p*

- If the TS is uniformly perforated p*=p
- If the TS has an unperforated lane of area $A_L=U_LL_L$ as shown in Figure 7(a), the N_t tubes are

redistributed so that the equivalent TS is uniformly perforated over the area $\pi \frac{D_o^2}{4}$ with an

equivalent pitch p*. This is necessary as the analytical treatment is performed for a uniform array of tubes.

Assuming that the portion of TS area pertaining to each hole is p^2 , p^* is obtained from the equation:

$$N_t p^{*^2} \pi \frac{D_o^2}{4} = N_t p^2 (\pi \frac{D_o^2}{4} - A_L)$$

which leads to:

$$p^* = \frac{p}{\sqrt{1 - \frac{4A_L}{\pi D_o^2}}}$$

So as not to deviate too much from the assumption of uniform pattern, the width U_L of the untubed lane in Figure 7(a) is limited to 4p over the diameter D_o , i.e. an area 4p D_o , which leads to: $A_L \leq 4pD_o$. This condition of applicability appears explicitly in the definition of A_L in UHX-11.3. It has been set-up after discussions among ASME and CODAP experts, based on sound engineering practice, and does not have a theoretical basis.

If there are several untubed lanes of widths U_{L1} , U_{L2} , U_{L3} ,... and lengths L_{L1} , L_{L2} , L_{L3} ,... as shown in Figure 7(b), the limit is still 4pD₀. Accordingly, p* becomes:

$$p^{*} = \frac{p}{\sqrt{1 - \frac{4MIN[(A_{L}), (4pD_{o})]}{\pi D_{o}^{2}}}}$$

If there is no untubed lane, $A_L=0$ and $p^*=p$.

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(e) Effective tube hole diameter d*

- When the tubes are welded only: $d^*=d_t$
- When the tubes are expanded into the TS, feedback from HE manufacturers has shown that the bending strength of the TS is increased by the degree of tube expansion ρ (see Figure 8).



Figure 8 — Tube Expansion Depth Ratio ρ=lt,x/h

The degree of increased strength is also dependent on the difference between the TS and tube material properties. Finally, the effective tube hole diameter is written:

$$d^* = d_t - 2t_t \left(\frac{E_{tT}}{E}\right) \left(\frac{S_{tT}}{S}\right) \rho$$

where ρ is the tube expansion ratio: $\rho = l_{t,x} / h$. This formula was proposed for the first time in the 1980 Edition of ASME Section VIII-Division 2.

- If the tubes are welded only: $\rho=0$ and $d^*=d_t$
- If the tubes are fully expanded: ρ=1 and d*=dt-2tt if the TS and tubes are made of the same materials.

The effective diameter cannot be less than the inside tube diameter d_t -2t_t, which leads to:

$$d^* = \operatorname{MAX}\left\{ \left[d_t - 2t_t \left(\frac{E_{tT}}{E} \right) \left(\frac{S_{tT}}{S} \right) \rho \right], \left[d_t - 2t_t \right] \right\}$$

Values of μ^* are generally comprised between 0.25 and 0.4.

To illustrate the difference between the TEMA method and the Part UHX method for determining the bending ligament efficiency, consider a tube (d_t =1.0, t_t =0.0625, p=1.25) that is the same material as the tubesheet.

- If the tube is expanded throughout the full depth of the tubesheet, then $\mu^* = 0.304$
- If the tube is welded and not expanded at all, then $\mu^* = 0.20$.

However, the TEMA ligament efficiency is 0.420 and 0.498 for triangular and square pitch layouts respectively, regardless of whether the tubes are expanded or not.

The ligament efficiency has a direct bearing on the calculated tubesheet stress. A smaller ligament efficiency results in a larger predicted tubesheet stress and a larger ligament efficiency results in a smaller predicted tubesheet stress. Thus, as may be seen, if the same basic theory is used to determine the stress in a plate, then the TEMA ligament efficiency would result in a smaller calculated stress as compared to the ASME method, even when the full tube wall is considered. This difference is exacerbated when the tube is not expanded.

(f) Effective tube side pass partition groove

When there is no pass partition groove on tube side of a TS of nominal thickness h_n , the TS corroded thickness h_{min} is given by: $h_{min}=h_n-c_s-c_t=h$, as h is the corroded TS thickness obtained by calculation in Part UHX.

When there is a pass partition groove of depth h_g , it is assumed that the bottom of the groove does not corrode.

Two cases are possible:

- If $h_g < c_t$ (see Figure 9(a)): $h_{min} = h_n c_s c_t = h_n$
- If h_g>c_t (see Figure 9(b)), a correction for the groove depth in excess of the tube side corrosion is necessary: h_{min}=h_n-c_s-c_t-(h_g-c_t)=h-(h_g-c_t)

In both cases h_{min} can be written: h_{min} =h-h'_g which will be used to calculate the TS bending stress where:

$$\dot{h_g} = \mathrm{MAX}\left[\left(h_g - c_t\right), (0)\right]$$

The same formula is used in TEMA.




Figure 9 — Pass Partition Groove on Tubeside of the TS

5 EFFECTIVE ELASTIC CONSTANTS (UHX-11.5.2)

5.1 Introduction

The analytical treatment of TS HEs necessitates the replacement of the actual perforated TS Modulus of elasticity E and Poisson's ratio v by an equivalent solid plate of Modulus of elasticity E* and Poisson's ratio v*. These EECs E* and v* are determined so that the equivalent TS has the same mechanical behavior as the actual TS when subjected to the same loading. Due to the weakening effect of the holes, E* is lower than E and E*/E is always lower than 1. Values of v* may be higher or lower than v. The EECs depend on the ligament efficiency, μ , ratio h/p and pattern type (triangular or square). They must be as correctly evaluated as possible:

- If they are underestimated, the calculated TS stresses at the junction with shell and channel will be lower than reality.
- If they are overestimated, the calculated TS stresses close to the center of the TS will be lower than reality.

The consequence is that inaccurate estimates of the EECs result in inaccurate stress results.

5.2 Historical Background

During the last decades many authors (about 60 papers) have proposed experimental and theoretical methods to solve the problem. A detailed review of these works was published in 1989 in a JPVT paper by Osweiller [1]. A short synthesis is provided below.

- *a)* Between 1948 and 1958 several authors (Gardner, Miller, Horvay, Duncan, Salerno and Mahoney, ...) proposed various methods for the determination of EECs. These methods had no sound basis and leading to a great disparity of results, PVRC decided in 1960 to undertake theoretical and experimental investigations in order to determine more accurate values for the EEC.
- b) In 1960 Sampson [2] undertook experimental tests on plastic plates using photo-elastic techniques for in-plane and bending loadings.
 - For in-plane loading, values of E^*/E and v^* are independent of the TS thickness.
 - For bending loading, values of E^*/E and v^* vary significantly with the TS thickness when h $\leq 2p$.

When h>2p this variation is very slow, and as the plates gets thicker, the bending values approach the plane stress values.

It appears that that h=2p is a transition zone between thin and thick perforated plates. These results were confirmed by Leven [3] in 1960.

- c) In 1962 O'Donnell & Langer [4] made a synthesis of these results and proposed a curve for in-plane and bending loading that enables the determination of E*/E and v* as a function of the ligament efficiency for thick plates (h≥2p) perforated with triangular pitch. This curve was adopted by ASME Section III in 1966 and later by Section VIII-Div. 2.
- *d)* In 1963 and later, new theoretical methods were developed on powerful computers to enable the determination of EECs for triangular patterns and square patterns. Square patterns were not previously covered. These methods are based on doubly periodic stress distribution theory induced in an infinite plate evenly perforated in two directions and loaded by in-plane or bending stress. Two techniques have been -used:
 - The "direct technique" developed by Meijers [5] in 1969 for thin plates loaded in bending or plane-stress. Grigoljuk and Fil'shtinski [11] obtained the same results using a similar method (see Annex A, Table 4)

In 1985, Meijers [12] improved his method by proposing for the determination of E^*/E and v^* :



- An asymptotic solution for thin plates in bending $(h/p\rightarrow 0)$ and for thick plates $(h/p\rightarrow \infty)$, consistent with the 1969 results
- An interpolated solution for the intermediate range ($0 \le h/p \rightarrow 0$), substantiated by FEA calculations.

See Table 1 for triangular pitch.

These values were adopted by many pressure vessel codes (BS 5500, CODAP, EN 13445, ISO, STOOMWEZEN) for treating thin and thick plates loaded in bending.

Table 1 — Values for E*/E and v* for Triangular Pattern from Meijers [12]

5							
μ h/P	0	0.25	0.5	² 71	2	4	00
$\int \frac{E_{\Delta}^{*}}{E}$	0.348	0.297	0.261	0.230	0.215	0.208	0.205
	0.032	0.171	0.273	0.359	0.399	0.421	0.427
$\int \frac{E_{\Delta}^{*}}{E}$	0.452	0.410	0.365	0.340	0.319	0.313	0.310
0.55 _ ν _Δ *	0.073	0.159	0.250	0.302	0.346	0.357	0.364

• The "indirect technique" developed by Bailey and Hicks [6] in 1960 and Slot & O'Donnell [7] in 1971 for thick plates.

These two techniques led to very good agreement both for triangular and square patterns with discrepancies lower than 0.1 % (see Annex A, Table 4). For more details, see Ref. [1].

e) These theoretical results have been corroborated by many experimental investigations undertaken by Duncan and Upfold [8] in 1963 and O'Donnell [9][10] in 1967 and 1973 and by F.E.M. calculations from Tran-Huu-Hanh in 1971 and Roberts in 1975.

5.3 The Square Pattern Problem

Contrary to the triangular pattern, the square pattern is characterized by an anisotropic behavior which has been enlightened both by theoretical and experimental investigations. Values of EECs E* and v* are different in the pitch direction P and diagonal direction D (See Figure 10). Anisotropy is more marked for low values of ligament efficiency $0.2 \le \mu * \le 0.4$ than for high values approaching 1(which corresponds to an unperforated plate), as shown by Slot and O'Donnell [7] (see Table 2).



Figure 10 — Pitch and Diagonal Directions for Square Pattern

	Pitch direction		Diagonal direction		"Isotropic" value	
μ	$\frac{E_p^*}{E}$	ν_p^*	$\frac{E_d^*}{E}$	ν_d^*	$\frac{E_{\Box}^{*}}{E}$	ν_{\Box}^{*}
0.2	0.311	0.122	0.123	0.654	0.235	0.337
0.4	0.525	0.216	0.380	0.433	0.459	0.316
0.6	0.734	0.275	0.681	0.328	0.708	0.301
0.8	0.918	0.297	0.914	0.301	0.916	0.299

Table 2 — Values of E*/E and v* for Square Pattern in Pitch and Diagonal Directions from Slot and O'Donnell [7]

This anisotropy has been confirmed theoretically by O'Donnell [9] and experimentally by Bayley & Hicks [6] and O'Donnell [10].

Due to this anisotropy, the equivalent plate cannot be treated with the classical isotropic solution, like in the triangular case. The anisotropic solution should be used.

When the plate is clamped or simply supported, the anisotropic circular plate deflection is given by formulaswhich can be compared to the classical isotropic formulae. From that comparison, equivalent "isotropic" values for E^*_{square} and v^*_{square} have been determined by O'Donnell [10], which enables the application of the classical isotropic equations to the equivalent solid plate.

$$E_{square}^{*} = E_{p}^{*} \frac{1 - \upsilon_{square}^{*}}{1 - \upsilon_{p}^{*}} \qquad \qquad \upsilon_{square}^{*} = \frac{4}{\frac{3 + \upsilon_{p}^{*}}{1 + \upsilon_{p}^{*}} + \frac{1 - \upsilon_{d}^{*}}{1 + \upsilon_{p}^{*}}} - 1$$

These formulas have been used for the square pattern to calculate the "isotropic" EECs E^*_{square} and v^*_{square} from the anisotropic values proposed by various authors, as shown in Table 2.

5.4 Synthesis of Results

In 1989, Osweiller [1] made a synthesis of all these experimental and theoretical results for triangular and square patterns which are presented in a graphical form in Figure 55 of Annex A. They give the values of E^*/E and v^* as a function of the ligament efficiency μ , for various ratios h/p.

This figure shows that experimental values are available for μ ranging from 0.1 to 0.5, whereas theoretical values cover the full range of μ .

5.5 Determination of EECs for the Full Range of μ^* (0.1 $\leq \mu^* \leq 1.0$)

From this synthesis of results, three ranges of TS thickness were set-up for the determination of E^*/E and v^* :

- for thin plates (h/p≤0.1) values are taken from theoretical values of Meijers [5][12] and combined with experimental values
- for thick plates (h/p≥2.0) values are taken from theoretical values of Slot & O'Donnell [7] and combined with experimental values.
- for the intermediate range (0.1<h/p<2.0) values are taken from theoretical values of Meijers [5][12] and combined with experimental values of Sampson [2] and O'Donnell [10].

Numerical values and resulting curves are given respectively in Sections 3 and 2 of Annex B. These curves were initially used in CODAP in 1985 and in Nonmandatory Appendix AA of Section VIII-Div. 1, which later became Part UHX.

Note: Experimental and theoretical values of EECs have been obtained from perforated plates for which the ligament efficiency is $\mu = (p-d)/p$. For TSs, the stiffening effect of the tubes mentioned in Section 4.3(e) IV-3c may also be accounted for in the determination of the EECs. Accordingly, the EECs are determined using the bending ligament efficiency $\mu^* = (p^*-d^*)/p^*$.

In 1991 Woody Caldwell, as a member of the ASME SG-HTE, provided polynomial approximations for the entire range of these curves $(0 \le \mu^* \le 1)$ by using polynomials of degree 4 for E*/E and of degree 7 for v* (see Section 4 of Annex B).

Section VIII Div.2 has retained these curves for the range $0.05 \le \mu^* \le 1.0$ for the stress analysis of perforated plates (Annex 5.E of 2013 edition).

5.6 Determination of EECs for the UHX Rules (UHX-11.5.2)

Numerical values obtained from the polynomials developed for Appendix AA polynomials were not accurate enough for curve h/p=2.0 for low values of μ^* . The discrepancy was about 6% on E*/E for $\mu^*=$ 0.15 and could reach 15% for $\mu^*=0.1$. This caused significant effects on calculated tubesheet stresses. Accordingly, the precision for these polynomials was improved by Osweiller [13] in 1994 by:

- using more points (35 instead of 21)
- exploring several polynomials degrees (degrees 3, 4 and 5 were tested)
- reducing the range of μ^* from 0.1 to 0.6 (0.1 $\leq \mu^* \leq 0.6$), based on the fact that the ligament efficiency of HEs is always between these two limits.

The precision targeted for these polynomials was 1%, which corresponds to the reading error of the curves. Polynomials of degree 4 were finally retained, both for E^*/E and for v*, which leads to a discrepancy of about 0.2%, with a maximum of 0.77% for the curve h/p=2.0.

Figure 11 for Triangular patterns and Figure 12 for Square patterns provide the curves and polynomials for calculating the EECs E*/E and v* for the reduced ligament efficiency range $0.1 \le \mu^* \le 0.6$ and for fixed values of the ratio h/p:

- for E*/E: h/p= 0.1, 0.25, 0.5, 2.0.
 Note: For h/p=0.25 no curve was available for E*/E for triangular pattern, due to lack of results. This curve has been added by interpolation between the two adjacent curves relative to h/p=0.1 and h/p=0.6.
- for v*: h/p= 0.1, 0.15, 0.25, 0.5, 1.0, 2.0.

Numerical values are available in Section 3 of Annex B. Note: The polynomials cannot be used for ligament efficiencies outside the range $0.1 \le \mu^* \le 0.6$, as shown by Figure 13. In such a case E^*/E and v^* should be determined per Section 5.5 above.

5.7 Conclusion

The perforated tubesheet plate is now replaced by an equivalent solid plate of:

- diameter D_o determined from Section 4.3(a),
- equivalent modulus of elasticity E^* and Poisson's ratio v^* determined from Section 5.6.

The unperforated TS rim extends from diameter D_o to diameter D_s with the basic modulus of elasticity E and Poisson's ratio ν .

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(b)v* (Equilateral Triangular Pattern)

(a) Equilateral Triangular Pattern: $\mathcal{E}^*/\mathcal{E} = \alpha_0 + \alpha_1 \mu^* + \alpha_2 \mu^{*2} + \alpha_3 \mu^{*3} + \alpha_4 \mu^{*4}$

h∕p	αο	aı	a ₂	<i>a</i> 3	<i>0</i> ′4
0.10	0.0353	1.2502	-0.0491	0.3604	-0.6100
0.25	0.0135	0.9910	1.0080	-1.0498	0.0184
0.50	0.0054	0.5279	3.0461	-4.3657	1.9435
2.00	-0.0029	0.2126	3.9906	-6.1730	3.4307

(b) Equilateral Triangular Pattern: $\nu^* = \beta_0 + \beta_1 \mu^* + \beta_2 \mu^{*2} + \beta_3 \mu^{*3} + \beta_4 \mu^{*4}$

h∕p	β0	β_1	β2	β3	β4
0.10	-0.0958	0.6209	-0.8683	2.1099	-1.6831
0.15	0.8897	-9.0855	36.1435	-59.5425	35.8223
0.25	0.7439	-4.4989	12.5779	-14.2092	5.7822
0.50	0.9100	-4.8901	12.4325	12.7039	4,4298
1.00	0.9923	-4.8759	12.3572	-13.7214	5.7629
2.0	0.9966	-4.1978	9.0478	-7.9955	2.2398

GENERAL NOTES:

(a) The polynomial equations given in the tabular part of this Figure can be used in lieu of the curves.

(b) For both parts (a) and (b) in the tabular part of this Figure, these coefficients are only valid for $0.1 \le$ * ≤ 0.6.

(c) For both parts (a) and (b) in the tabular part of this Figure: for values of h/p lower than 0.1, use h/p= 0.1; for values of h/p higher than 2.0, use h/p = 2.0.

Figure 11 — Curves and Tables for the Determination of E*/E and v* (Triangular Pattern)

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(a) E*/E (Square Pattern)

(b) v# (Square Pattern)

(a) Square Pattern: $E^*/E = \alpha_0 + \alpha_1 \mu^* + \alpha_2 \mu^{*2} + \alpha_3 \mu^{*3} + \alpha_4 \mu^{*4}$

h∕p	αο	aı	α2	<i>a</i> 3	<i>α</i> 4
0.10	0.0676	1.5756	-1.2119	1.7715	-1.2628
0.25	0.0250	1.9251	-3.5230	6.9830	-5.0017
0.50	0.0394	1.3024	-1.1041	2.8714	-2.3994
2.00	0.0372	1.0314	-0.6402	2.6201	-2.1929

(b) Square Pattern: $\nu^* = \beta_0 + \beta_1 \mu^* + \beta_2 \mu^{*2} + \beta_3 \mu^{*3} + \beta_4 \mu^{*4}$

h/p	βo	βι	β2	β3	β4
0.10	-0.0791	0.6008	-0.3468	0.4858	-0.3606
0.15	0.3345	-2.8420	10.9709	-15.8994	8.3516
0.25	0.4296	-2.6350	8.6864	-11.5227	5.8544
0.50	0.3636	-0.8057	2.0463	-2.2902	1.1862
1.00	0.3527	-0.2842	0.4354	-0.0901	-0.1590
2.00	0.3341	0.1260	-0.6920	0.6877	-0.0600

GENERAL NOTES:

(a) The polynomial equations given in the tabular part of this Figure can be used in lieu of the curves.

(b) For both parts (a) and (b) in the tabular part of this Figure, these coefficients are only valid for $0.1 \le \mu^* \le 0.6$.

(c) For both parts (a) and (b) in the tabular part of this Figure: for values of h/p lower than 0.1, use h/p = 0.1; for values of h/p higher than 2.0, use h/p = 2.0.

Figure 12 — Curves and Tables for the Determination of E*/E and v* (Square Pattern)



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Figure 13 — Curves E*/E for Square Pattern Obtained from Polynomial Approximation Given in Figure 12

(Only valid for 0.1≤µ*≤0.6) from [13]

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- [CS-2] CODAP: French Code for Unfired Pressure Vessels Chapter C7 "Design rules for HE TSs"
- [CS-3] EN 13445: European Standard for Unfired Pressure Vessels Chapter 13 "HE TSs"
- [CS-4] ISO DIS 2694: Pressure Vessels Chapter 30 "Flat HE TSs"
- [CS-5] STOOMWEZEN: Dutch Code for Pressure Vessels Chapters D0403 "Wall thickness calculation of TSs"
- [CS-6] TEMA: Standards of Tubular Exchangers Manufacturers Association Chapter R7 "TS"

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PART 3 ANALYICAL TREATMENT OF FIXED TUBESHEET HEAT EXCHANGERS



1 SCOPE (UHX-13.1)

PART 3, devoted to Fixed TS HEs (Figure 14), provides the technical basis for the determination of

- the displacement and loads acting on the TS, tubes, shell and channel,
- the stresses in these four components and their relationships with the design rules of UHX-13



Figure 14 — Fixed Tubesheet Heat Exchanger

2

² see TS configurations in Figure 15



2 HISTORICAL BACKGROUND

In the past decades many authors have proposed theoretical methods for the design of fixed TS HEs. The most important contributions are provided below. A more detailed list of technical papers can be found in Ref. [1].

Gardner [2] in 1952 was the very first to develop an analytical approach by taking into consideration the support afforded by the tubes and the weakening effect of the TS holes. The TS is considered as either simply supported or clamped at its periphery to simulate the rotational restraint afforded by the shell and the channel, which compels the designer to make a more or less arbitrary choice between these two extreme cases.

TEMA adopted this method in its 5th edition published in 1968. K.A.G. Miller [3] at the same time, proposed a similar approach that was published in the British Code BS 1515 in 1965.

Galletly [4] in 1959 improved these design methods by accounting for the degree of rotational restraint of the TS at its periphery by the shell and the channel. This method was adopted by the French Pressure Vessel Code CODAP in 1982 and by the European Pressure Vessel Standard EN13445 in 2002. Other authors (Yi Yan Yu [5] [6], Boon and Walsh [7], Hayashi [8]) have developed more refined methods accounting for the membrane loads acting at mid-surface of the TS, the unperforated rim, the TS-shell-channel connection, and the bending effect of the tubes which reinforces the strength of the TS. Since a solution using these methods requires the use of a computer they were not adopted by Codes. Soler [9] in 1984 developed a similar method accounting for the unperforated rim and the TS-shell-channel connection. Thanks to a parametric study, it does not need the use of a computer.

The method was adopted by the SG-HTE in the 80's, put into a code format, and published for the first time in 1995 in Nonmandatory Appendix AA of Section VIII Division 1. In 2003 it was published in a new Part UHX of Section VIII Division 1 "Rules for Shell and Tubes Heat Exchangers" which became mandatory in 2004.

This criteria document provides the technical basis of Part UHX-13 of Section VIII Div. 1, 2007 Edition [10], including the 2008 and 2009 Addenda. A few items (analysis of cylindrical and spherical shells, allowable stress limits) are taken from Appendix 4 of Section VIII Div. 2, 2004 Edition [11].



3 GENERAL

3.1 TS Configurations (UHX-13.1)

The TS is attached to the shell and the channel by welding (integral TS) or by bolting (gasketed TS) in accordance with the following 4 configurations (see Figure 15):

- configuration a: tubesheet integral with shell and channel;
- configuration b: tubesheet integral with shell and gasketed with channel, extended as a flange;
- configuration c: tubesheet integral with shell and gasketed with channel, not extended as a flange;
- configuration d: tubesheet gasketed with shell and channel, extended as a flange or not extended.

An expansion joint can be set-up on the shell as shown on Figure 15 configuration a.



(a) Configuration a: Tubesheet integral with shell and channel gasketed with channel



(b) Configuration b: Tubesheet integral with shell and extended as a flange



(c) Configuration c: Tubesheet integral with shell and gasketed with channel, not extended as a flange



Configuration d: Tubesheet gasketed with shell and channel



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3.2 Notations (UHX-13.3)

(a) Data for the design of the HE are as follows.

Symb

ols D _o	, E*, ł	n'_{g} , μ , μ^* and ν^* are defined in Section 2 of PART 2.
Α	=	outside diameter of tubesheet
a_c	=	radial channel dimension
a_o	=	equivalent radius of outer tube limit circle
a_s	=	radial shell dimension
С	=	bolt circle diameter
D_c	=	inside channel diameter
D_J	=	inside diameter of the expansion joint at its convolution height
D_s	=	inside shell diameter
d_t	=	nominal outside diameter of tubes
E	=	modulus of elasticity for tubesheet material at T
E_c	=	modulus of elasticity for channel material at T _c
E_s	=	modulus of elasticity for shell material at T _s
E_t	=	modulus of elasticity for tube material at T _t
G_c	=	diameter of channel gasket load reaction
G_s	=	diameter of shell gasket load reaction
G_1	=	midpoint of contact between flange and tubesheet
h	=	tubesheet thickness
K_J	=	axial rigidity of expansion joint, total force/elongation
L_t	=	tube length between outer tubesheet faces
N_t	=	number of tubes
P_{e}	=	effective pressure acting on tubesheet
P.	=	shell side design or operating pressure, as applicable. For shell side vacuum use
- 3		a negative value for P_s .
P_{t}	=	tube side design or operating pressure, as applicable. For tube side vacuum use
- 1		a negative value for P _t .
		Notation P_c , instead of P_t , is used throughout the analytical development so as
		to maintain the symmetry of the equations involving the shell (subscript s) and
C		the channel (subscript c).
S	=	allowable stress for tubesheet material at 1
S_c	=	allowable stress for channel material at Γ_c
S_s	_	allowable stress for tube meterial at T_s
\mathbf{S}_t	_	allowable stress for tube indicitial at T_t
$S_{y,c}$	_	yield strength for shell material at T_c
$S_{y,s}$	_	yield strength for tube material at T_s
$S_{y,t}$	_	allowable primary plus secondary stress for tubesheet material at T
S_{PS}	_	allowable primary plus secondary stress for channel material at T_{a}
SPS c	=	allowable primary plus secondary stress for shell material at T _e
T	=	tubesheet design temperature
T_a	=	ambient temperature, 70°F (20°C)
$\tilde{T_c}$	=	channel design temperature
T_s	=	shell design temperature
T_t	=	tube design temperature
$T_{s,m}$	=	mean shell metal temperature along shell length
$T_{t,m}$	=	mean tube metal temperature along tube length
t_c	=	channel thickness

shell thickness t_s =



t_t	=	nominal tube wall thickness
W_s , W_c	=	shell or channel flange design bolt load for the gasket seating condition
W_{m1s} , W_{m1c}	=	shell or channel flange design bolt load for the operating condition
W_{max}	=	$MAX [(W_c), (W_s)]$
W^*	=	tubesheet effective bolt load determined in accordance with UHX-8
$\alpha_{s,m}$	=	mean coefficient of thermal expansion of shell material at T _{s,m}
$\alpha_{t,m}$	=	mean coefficient of thermal expansion of tube material at T _{t,m}
γ	=	axial differential thermal expansion between tubes and shell
v	=	Poisson's ratio of tubesheet material
Vc	=	Poisson's ratio of channel material
Vs	=	Poisson's ratio of shell material
v _t	=	Poisson's ratio of tube material

(*b*) **Design coefficients** (UHX-13.5.1 to 4)

The following coefficients, specific to each component of the HE, will be used in the analytical treatment.

1) Perforated TS

Equivalent diameter of outer tube limit circle (see Section 4.3(a) of PART 2): $D_o = 2r_o + d_t$

Equivalent radius of outer tube limit circle: $a_o = \frac{D_o}{2}$

TS coefficients:

• Shell side:
$$x_s = 1 - N_t \left(\frac{d_t}{2a_o}\right)^2$$
; $1 - x_s = N_t \left(\frac{d_t}{2a_o}\right)^2$
• Tube side: $x_t = 1 - N_t \left(\frac{d_t - 2t_t}{2a_t}\right)^2$; $1 - x_t = N_t \left(\frac{d_t - 2t_t}{2a_t}\right)^2$

• Tube side:
$$x_t - 1 - N_t \left(\frac{2a_o}{2a_o} \right)^2$$
; $1 - x_t - N_t \left(\frac{2a_o}{2a_o} \right)^2$
• $x_t - x_s = N_t \left(\frac{d_t^2 - (d_t - 2t_t)^2}{4a_o^2} \right) = \frac{N_t \cdot s_t}{\pi a_o^2} = \frac{N_t \cdot k_t}{E_t} \cdot \frac{l}{\pi a_o^2} = \frac{N_t \cdot K_t}{E_t} \cdot \frac{L}{\pi a_o^2}$ [III.2.b1]

• Ligament efficiency:
$$\mu^* = \frac{p^* - d^*}{p^*}$$

Effective tube hole diameter d^* and effective pitch p^* are defined in Section 4.3(d) and (c) of PART 2.

• Effective elastic constants E* and v* are given in Section 5.6 of PART 2 as a function of μ^* and h/p (triangular or square pitch).

• Bending stiffness:
$$D^* = \frac{E^* \cdot h^3}{12(1-v^{*2})}$$

- Effective tube side pass partition groove depth given in Section 4.3(f) of PART 2: h'_g
- Effective pressure acting on tubesheet: P_e

2) **Tube Bundle**

Tube cross-sectional area:

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$$s_{t} = \frac{\pi}{4} \left[d_{t}^{2} - \left(d_{t} - 2t_{t} \right)^{2} \right] = \pi t_{t} \left(d_{t} - t_{t} \right) = \frac{\pi a_{0}^{2}}{N_{t}} \left(x_{t} - x_{s} \right)$$

Axial stiffness K_t of one tube: $K_t = \frac{E_t s_t}{L} = \frac{\pi t_t (d_t - t_t) E_t}{L}$

Axial stiffness k_t of one half tube of length l=L/2: $k_t = \frac{E_t s_t}{l} = \frac{2\pi t_t (d_t - t_t) E_t}{L} = 2 K_t$ Effective elastic foundation modulus equivalent to the half tube bundle:

 $k_{w} = \frac{N_{t} \cdot k_{t}}{\pi a_{o}^{2}} = \frac{2 N_{t} \cdot K_{t}}{\pi a_{o}^{2}} = \frac{2 N_{t} \cdot E_{t} \cdot t_{t} \left(d_{t} - t_{t}\right)}{L a_{o}^{2}} = \frac{2 E_{t}}{L} \left(x_{t} - x_{s}\right) = \frac{E_{t}}{l} \left(x_{t} - x_{s}\right)$ $k = \sqrt[4]{\frac{k_{w}}{D^{*}}} \quad ; \quad x = k r \quad ; \quad 0 \le r \le a_{o} \quad \Rightarrow \quad 0 \le x \le k a_{o} \quad ka_{o} = X_{a}$

Axial stiffness ratio tubes/TS:

$$X_{a} = k a_{o} = \sqrt[4]{\frac{k_{w}}{D^{*}}} a_{o} = \left[24 \left(1 - v^{*2} \right) N_{t} \frac{E_{t} t_{t} \left(d_{t} - t_{t} \right) a_{o}^{2}}{E^{*} L h^{3}} \right]^{\frac{1}{4}}$$

3) Shell

Radial shell dimension: $a_s \quad \rho_s = \frac{a_s}{a_o}$ Integral configurations (a, b and c): $a_s = D_s/2$ Gasketed configuration (d): $a_s = G_s/2$ Mean shell radius: $a'_s = \frac{D_s + t_s}{2}$ Shell cross-sectional area: $s_s = \pi t_s \left(D_s + t_s \right)$ Axial stiffness K_s of the shell of length L: $K_s = \frac{E_s s_s}{L} = \frac{\pi t_s \left(D_s + t_s \right) E_s}{L}$ Axial stiffness k's of the half shell of length l=L/2: $k'_s = \frac{E_s s_s}{l} = \frac{2\pi t_s \left(D_s + t_s \right) E_s}{L} = 2 K_s$ Axial stiffness ratio shell and tube bundle: $K_{s,t} = \frac{K_s}{N_t \cdot K_t} = \frac{t_s \left(D_s + t_s \right) E_s}{N_t \cdot t_t \left(d_t - t_t \right) E_t}$ Shell coefficient: $\beta_s = \frac{\sqrt[4]{12 \left(1 - v_s^2 \right)}}{L}$

$$\beta_{\rm s} = \frac{\sqrt{1-\frac{1}{\sqrt{D_{\rm s} + t_{\rm s}}}}}{\sqrt{D_{\rm s} + t_{\rm s}}} \frac{E_{\rm s} \cdot t_{\rm s}^3}{6\left(1 - v_{\rm s}^2\right)}$$

Bending stiffness:

4) Channel

Radial channel dimension:
$$a_c \qquad \rho_c = \frac{a_c}{a_o}$$

Integral configuration (a): $a_c = D_c / 2$
Gasketed configurations (b, c and d): $a_c = G_c / 2$

Mean channel radius: $a'_s = \frac{D_c + t_c}{2}$

Channel coefficient:

$$\beta_{\rm c} = \frac{\sqrt[4]{12\left(1 - v_{\rm c}^2\right)}}{\sqrt{\left(D_{\rm c} + t_{\rm c}\right)t_{\rm c}}}$$
$$k_{\rm c} = \beta_{\rm c} \cdot \frac{E_{\rm c} \cdot t_{\rm c}^3}{6\left(1 - v_{\rm c}^2\right)}$$

Bending stiffness:

5) Axial differential thermal expansion between the tubes and the shell:

$$\gamma = \left[\alpha_{t,m}(T_{t,m} - T_a) - \alpha_{s,m}(T_{s,m} - T_a)\right]L$$

- 6) Expansion joint
 - D_J = inside diameter of the expansion joint at its convolution height K_J = axial rigidity of the expansion joint, total force/elongation
 - J = ratio of expansion joint to shell axial rigidity (J = 1.0 if no joint):

$$J = \frac{K_J}{K_s + K_J}$$

7) Unperforated rim

 D_o = internal diameter A = external diameter Diameter ratio: $K = A / D_o$

3.3 Loading Cases (UHX-13.4)

The normal operating condition of the HE is achieved when the tube side pressure P_t , shell side pressure P_s and axial differential thermal expansion between tubes and shell γ act simultaneously. However, a loss of pressure or a loss of temperature is always possible. Accordingly, for safety reasons, the designer must always consider the cases where $P_s=0$ or $P_t=0$ with and without thermal expansion for the normal operating condition(s).

The designer must also consider the startup condition(s), the shutdown condition(s) and the upset condition(s), if any, which may govern the design.

A fixed TS HE is a statically indeterminate structure for which it is difficult to determine the most severe condition of coincident pressure, temperature and differential thermal expansion. Thus, it is necessary to evaluate all the anticipated loading conditions mentioned above to ensure that the worst load combination has been considered in the design.

For each of these conditions, ASME, TEMA, and CODAP used to consider the following loading cases. (*a*) Pressure only loading cases

- Loading Case 1: Tube side pressure P_t acting only.
- Loading Case 2: Shell side pressure P_s acting only.

• Loading Case 3: Tube side pressure P_t and shell side pressure P_s acting simultaneously.

(b) Pressure and Thermal loading cases

- Loading Case 4: Differential thermal expansion acting only $(P_t = 0, P_s = 0)$.
- Loading Case 5: Tube side pressure P_t acting only, with differential thermal expansion.
- Loading Case 6: Shell side pressure P_s acting only, with differential thermal expansion.
- Loading Case 7: Tube side pressure P_t and shell side pressure P_s acting simultaneously, with differential thermal expansion.

ASME 2013 Edition provides the detail of the pressure "design loading cases" and "operating (thermal) loading cases" to be considered for each operating condition specified by the user (normal operating conditions, startup conditions, the shutdown conditions, etc.). For the pressure loading cases, a table (table UHX-13.4-1) provides the values to be used for the design pressures P_s and P_t in the formulas, accounting for their maximum and minimum values.

For the operating (thermal) loading cases a second table (table UHX-13.4-2) provides the values to be used for the operating pressures P_s and P_t in the formulas for each operating condition considered. So, this new concept allows the operating pressures (instead of the design pressures to use for operating (thermal) loadings.)

As the calculation procedure is iterative, a value h is assumed for the tubesheet thickness to calculate and check that the maximum stresses in tubesheet, tubes, shell, and channel are within the maximum permissible stress limits.

Because any increase of tubesheet thickness may lead to overstresses in the tubes, shell, or channel, a final check must be performed, using in the formulas the nominal thickness of tubesheet, tubes, shell, and channel, in both corroded and uncorroded conditions.

3.4 Design Assumptions (UHX-13.2)

A fixed TS HE is a complex structure and several assumptions are necessary to derive a 'design by rules' method. Most of them could be eliminated, but the analytical treatment would lead to 'design by analysis' method requiring the use of a computer.

The design assumptions are as follows.

(*a*) HE

- The analytical treatment is based on the theory of elasticity applied to the thin shells of revolution.
- The HE is axi-symmetrical.
- The HE is a symmetrical unit with identical TSs so as to analyze a half-unit.

(b) TSs

- The two tubesheets are circular and identical (same diameter, uniform thickness, material, temperature and edge conditions)
- The tubesheets are uniformly perforated over a nominally circular area, in either equilateral triangular or square patterns. This implies that the TSs are fully tubed (no large untubed window)
- Radial displacement at the mid-surface of the TS is ignored
- Temperature gradient through the TS thickness is ignored
- Shear deformation and transverse normal strain in TS are ignored
- The unperforated rim of the TS is treated as a rigid ring without distortion of the cross section

(c) Tubes

- Tubes are assumed identical, straight and at same temperature
- Tubes are uniformly distributed in sufficient density to play the role of an elastic foundation for the TS
- The effect of the rotational stiffness of the tubes is ignored

(*d*) Shell and channel

- Shell and channel are cylindrical with uniform diameters and thicknesses
- If the channel head is hemispherical, it must be attached directly to the TS, without any cylindrical section between the head and the TS.
- Shell and channel centerlines are the same.



æ)

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(e) Weights and pressures drops

- Weights and pressures drops are ignored
- Pressures P_s and P_t are assumed uniform

3.5 Basis of Analytical Treatment

3.5.1 General

The design of a fixed TS HEs is complex as the two TSs are connected to each other both through the tube bundle and the shell, and subjected to different temperatures which generate an axial differential thermal expansion. Accordingly the structure is statically indeterminate. Many geometrical, mechanical, thermal and material properties are involved in the design as shown in Section 3.2(a) which lists the extensive input data.

The analysis includes the effects of the shell and tube side pressures, the axial stiffening effect of the tubes, the stiffening effect of the unperforated ring at the tubesheet edge, and the stiffening effect of the integrally attached channel or shell to the tubesheet. For a tubesheet that is extended as a flange to which a channel or shell is to be bolted, the bolt load causes an additional moment in the tubesheet which is included in the total stress in the tubesheet in addition to the moments caused by pressure.

The analysis is based on classical discontinuity analysis methods to determine the moments and forces that the tubesheet, tubes, shell and channel must resist. These components are treated using the theory of elasticity applied to the thin shells of revolution.

Because the heat exchanger is assumed to be symmetric about a plane midway between the two tubesheets, only half of the heat exchanger is treated. The main steps of the analytical treatment are as follows.

- (a) The tubesheet is disconnected from the shell and channel. Shear load V_a and moment M_a are applied at the tubesheet edge as shown in Figure 16.
- (b) The perforated tubesheet is replaced by an equivalent solid circular plate of diameter D_o and effective elastic constants E^* (effective modulus of elasticity) and v^* (effective Poisson's Ratio) depending on the ligament efficiency μ^* of the tubesheet. This equivalent solid plate is treated by the theory of thin circular plates subjected to pressures P_s and P_t and relevant applied loads to determine the maximum stresses.
- (c) The unperforated tubesheet rim is treated as a rigid ring whose cross section does not change under loading.
- (d) The tubes are replaced by an equivalent elastic foundation of modulus k_w .
- (e) Connection of the tubesheet with shell and channel accounts for the edge displacements and rotations of the 3 components.
- (f) The shell and channel are treated by the elastic theory of thin shells of revolution subjected to shell side and tube side pressures P_s and P_t and edge loads to determine the maximum stresses.
- (g) The maximum stresses in tubesheet, tubes, shell and channel are determined and limited to the appropriate allowable stress-based classifications of Section VIII Division 2 PART 4.



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Figure 16 — Analytical Model Used in Design Method

3.5.2 Free Body Diagram

Figure 17 shows, for a tubesheet integral both sides (configuration a), the free body diagram of the component parts (perforated tubesheet, unperforated tubesheet rim, shell, channel). The figure details the relevant discontinuity forces (V_a , V_s , Q_s , V_c , Q_c) and moments (M_a , M_s , M_c , M_R) applied on each component, together with edge displacements.

In this figure, forces $(V_a, V_s, Q_s, V_c, Q_c)$ and moments (M_a, M_s, M_c, M_R) are per unit of circumferential length. The following subscripts are used:

- s for shell,
- c for channel,
- R for unperforated rim

No subscript is used for the perforated TS.

Notation P_c instead of P_t (tube side pressure) is used throughout the analytical development so as to maintain the symmetry of the equations involving the shell (subscript s) and the channel (subscript c).



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Figure 17 — Free Body Diagram of the Analytical Model

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4 AXIAL DISPLACEMENTS AND FORCES ACTING ON THE TUBES AND ON THE SHELL

4.1 Axial Displacement and Force Acting on the Tubes (Figure 18)



Figure 18 — Axial Displacement of Tubes

(a) Axial Displacement of the tubes due to axial force $V_t(r)$ acting on the tube row at radius r:

$$\delta_{t}(V_{t}) = \frac{V_{t}(r)}{k_{t}} = \frac{V_{t}(r)l}{E_{t}s_{t}} = \frac{l}{\pi E_{t}(d_{t}-t_{t})t_{t}}V_{t}(r)$$

(b) Axial Displacement of the tubes due to temperature θ_t of tubes:

$$\delta_{t}(\theta_{t}) = \alpha_{t,m} \theta_{t} l \qquad \qquad \theta_{t} = T_{t,m} - T_{a}$$

(c) Axial Displacement of the tubes due to Poisson's ratio v_t of tubes (Annex C):

$$\delta_{\mathrm{t}}(\nu_{\mathrm{t}}) = -\frac{2}{k_{\mathrm{w}}} \left[\left(1 - x_{\mathrm{t}} \right) P_{\mathrm{t}} - \left(1 - x_{\mathrm{s}} \right) P_{\mathrm{s}} \right] \nu_{\mathrm{t}}$$

(d) Total axial displacement of the tube row at radius r:

$$\delta_{t,Total}(r) = \underbrace{\left[\delta_{t}(V_{t})\right]}_{\substack{\text{tube displct.due}\\\text{to axial force}\\(unknown)}} + \underbrace{\left[\delta_{t}(\theta_{t}) + \delta_{t}(V_{t})\right]}_{\substack{\text{free displct.}\\\text{of tube}\\(known)}}$$
[IV.1d]

(e) axial force acting on each tube at radius r:

$$V_{t}(r) = k_{t} \left[\delta_{t}(V_{t}) \right] = k_{t} \left[\delta_{t,\text{Total}}(r) - \delta_{t}(\theta_{t}) - \delta_{t}(v_{t}) \right]$$

(f) net effective pressure acting on the TS due to each tube at radius r of TS area $\pi a_0^2 / N_t$:

$$q_{t}(r) = \frac{-V_{t}(r)}{\pi a_{0}^{2} / N_{t}} = -\frac{N_{t}k_{t} \,\delta_{t}(V_{t})}{\pi a_{0}^{2}} = -\frac{N_{t}k_{t}}{\pi a_{0}^{2}} \left[\delta_{t,Total}(r) - \delta_{t}(\theta_{t}) - \delta_{t}(v_{t})\right] \qquad k_{w} = \frac{N_{t} \,k_{t}}{\pi \,a_{0}^{2}}$$

$$q_{t}(r) = -k_{w} \Big[\delta_{t}(V_{t}) + \delta_{t}(\theta_{t}) + \delta_{t}(v_{t}) - \delta_{t}(\theta_{t}) - \delta_{t}(v_{t}) \Big]$$



4.2 Axial Displacement and Force Acting on the Shell

(a) Axial displacement of the shell and expansion joint due to axial force acting on the shell
due to axial force V_s acting on the shell:



Figure 19 — Axial Displacement of the Shell

• due to axial force V_s acting on the expansion joint J:

$$\delta'_{J}(V_{s}) = \frac{V_{s} \ 2 \ \pi \ a'_{s}}{2 \ K_{J}}$$
 $2K_{J} = \text{axial rigidity of half expansion joint}$

• axial displacement of shell and expansion joint:

Verification: axial rigidity k_s^* shell and expansion joint: $\frac{1}{k_s^*} = \frac{1}{k_s} + \frac{1}{2k_J} = \frac{1}{k_s} \left(1 + \frac{k_s}{2k_J}\right) = \frac{1}{Jk_s}$ $k_s^* = Jk_s' \qquad \delta_s(V_s) = \frac{V_s 2\pi a_s'}{k_s^*} = \frac{V_s 2\pi a_s'}{k_s} = \frac{\delta_s'(V_s)}{J}$

(b) Axial displacement of the shell due to temperature θ_s of shell:

$$\delta_{s}(\theta_{s}) = \alpha_{s,m} \theta_{s} l \qquad \theta_{s} = T_{s,m} - T_{a}$$

(c) Axial displacement of the shell due to Poisson's ratio v_s of shell (Annex C):

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$$\delta_{\rm s}(v_{\rm s}) = -\frac{\pi}{2\,k_{\rm s}^2} \left(P_{\rm s}\,D_{\rm s}^2\right) v_{\rm s}$$

(d) Axial displacement of the shell due to pressure P_s acting on bellows joint side walls J (Annex D):

$$\delta_{s}(J) = P_{s} \frac{\pi \left(D_{J}^{2} - D_{s}^{2}\right)/2}{4 \times 2K_{J}} = \frac{\pi}{8} \frac{D_{J}^{2} - D_{s}^{2}}{2K_{J}} P_{s}$$

(e) Total axial displacement of shell:

$$\delta_{s,Total} = \underbrace{\delta_{s}(V_{s})}_{\substack{shell \ displct. \ due \\ to \ axial \ force \\ (unknown)}} + \underbrace{\delta_{s}(\theta_{s}) + \delta_{s}(v_{s}) + \delta_{s}(J)}_{\substack{free \ displct. \\ of \ shell \\ (known)}}$$
[IV.2e]

(f) Axial force acting on the shell:

$$\delta_{s}(V_{s}) = \delta_{s,\text{Total}} - \delta_{s}(\theta_{s}) - \delta_{s}(v_{s}) - \delta_{s}(J) = \frac{l}{J E_{s} t_{s}} V_{s}$$
$$V_{s} = \frac{J E_{s} t_{s}}{l} \left[\delta_{s,\text{Total}} - \delta_{s}(\theta_{s}) - \delta_{s}(v_{s}) - \delta_{s}(J) \right]$$

(g) Axial displacement of tubes at radius r:

$$\delta_{t,\text{Total}}(r) = \delta_{s,\text{Total}} + w(r) = \delta_s(V_s) + \delta_s(\theta_s) + \delta_s(v_s) + \delta_s(J) + w(r)$$

where w(r) is the TS deflection at radius r (see Figure 21)

$$[\text{IV.1d}]: \quad \delta_{\text{t,Total}}(r) = \delta_{\text{t}}(V_t) + \delta_{\text{t}}(\theta_{\text{t}}) + \delta_{\text{t}}(v_{\text{t}})$$
$$\delta_{\text{t}}(V_t) = \delta_s(V_s) + w(r) + \underbrace{\left[\delta_s(\theta_s) - \delta_{\text{t}}(\theta_{\text{t}})\right]}_{-\gamma/2} + \left[\delta_s(v_s) - \delta_{\text{t}}(v_{\text{t}})\right] + \left[\delta_s(J)\right]$$

(*h*) **TS deflection at radius r:**

$$w(r) = \left[\delta_{t}(V_{t}) - \delta_{s}(V_{s})\right] + \left[\underbrace{\delta_{t}(\theta_{t}) - \delta_{s}(\theta_{s})}_{\gamma/2}\right] + \left[\delta_{t}(v_{t}) - \delta_{s}(v_{s})\right] - \left[\delta_{s}(J)\right]$$

5 DEFLECTION AND LOADS ACTING ON THE TUBESHEET

5.1 Equivalent Plate Resting on an Elastic Foundation

(a) Net effective pressure acting on the TS



Figure 20 — Loads Acting on the TS

due to tubes: $q_t(r) = -k_w \left[\delta_t(V_t) + \delta_t(\theta_t) + \delta_t(v_t) - \delta_t(\theta_t) - \delta_t(v_t) \right]$ Tubes act as an elastic foundation of equivalent modulus given by the axial rigidity of t

Tubes act as an elastic foundation of equivalent modulus given by the axial rigidity of the half-bundle per unit of TS area:

$$k_{\rm w} = \frac{N_t k_t}{\pi a_o^2}$$

$$q_t(r) = -\frac{V_t(r)}{\pi a_o^2 / N_t} = -k_w \left[\delta_s(V_s) + w(r) - \delta(\theta) + \delta_s(v_s) - \delta_t(v_t) + \delta_J(P_s) \right]$$

due to pressures P_s and P_t acting on the equivalent plate (see Annex E):

$$q_{\rm P} = x_{\rm s} P_{\rm s} - x_{\rm t} P_{\rm t} = \Delta p^*$$

net effective pressure:

$$q(r) = q_{\rm P} + q_t(r) = \Delta p^* - \frac{V_t(r)}{\pi a_o^2 / N_t} \quad [V.1a]$$

$$q(r) = q_{\rm P} + q_t(r) = \Delta p^* - k_{\rm w} \left[\left[\delta_s(\theta_s) - \delta_t(\theta_t) \right] + \left[\delta_s(v_s) - \delta_t(v_t) \right] + \left[\delta_J(P_s) \right] + \delta_s(V_s) \right] - k_{\rm w} w(r)$$

$$Q$$

$$Q = \Delta p^* + k_{\rm w} \left[\left[\delta_t(\theta_t) - \delta_t(\theta_t) \right] + \left[\delta_t(v_t) - \delta_s(v_s) \right] - \left[\delta_J(P_s) \right] - \left[\delta_s(V_s) \right] \right] \quad [V.1a']$$

In this equation, the displacement $\delta_s(V_s)$ of the shell subjected to axial force V_s is unknown. Other quantities are known for a given HE.

• Annex C:
$$k_{\rm w} \, \delta_{\rm t} (v_{\rm t}) = -2 \, v_{\rm t} \left[P_{\rm t} (1-x_{\rm t}) - P_{\rm s} (1-x_{\rm s}) \right]$$

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$$k_{\rm w} \,\delta_{\rm s}\left(\nu_{\rm s}\right) = -\frac{2\,\nu_{\rm s}}{k_{\rm s,t}} \left(\frac{D_{\rm s}}{D_{\rm o}}\right)^2 \,P_{\rm s}$$

• Annex D:

$$k_{\rm w}\delta_J(P_{\rm s}) = \frac{\pi}{16}\frac{k_{\rm w}}{K_J}(D_{\rm j}^2 - D_{\rm s}^2)P_{\rm s} = \frac{\pi}{16}\frac{N_t k_{\rm t}}{\pi a_o^2 K_J}(D_J^2 - D_{\rm s}^2)P_{\rm s} = \frac{N_t k_{\rm t}}{4 K_J}\frac{D_J^2 - D_{\rm s}^2}{D_o^2}P_{\rm s}$$

$$\boxed{q(r) = Q - k_{\rm w} w(r)}$$

(b) deflection of TS (Figure 21)



Figure 21 — TS Displacement

The TS is replaced by an equivalent solid plate:

- of equivalent radius a_0
- having effective elastic constants E^* and v^* .
- With bending rigidity:

$$D^* = \frac{E^* h^3}{12(1 - v^{*2})}$$

• resting on an elastic foundation of modulus:

$$k_{\rm w} = \frac{N k_{\rm t}}{\pi a_o^2}$$

- subjected to a net effective pressure q(r)
- subjected at its periphery to edge loads V_a and M_a

The deflection of such a plate is governed by a differential equation of 4th order:

$$\frac{d^4w}{dr^4} + \frac{2}{r}\frac{d^3w}{dr^3} - \frac{1}{r^2}\frac{d^2w}{dr^2} + \frac{1}{r^3}\frac{dw}{dr} = \frac{q(r)}{D^*} = \frac{Q - k_w w(r)}{D^*}$$

whose solution is written (Annex F): $w(x) = A ber(x) + B bei(x) + \frac{Q}{k}$

where:

x is a new dimensionless integration variable which is introduced to solve the differential •

equation
$$x = k \ r = \sqrt[4]{\frac{k_w}{D^*}} \ r$$

At the TS periphery (r =a_o): $X_a = k a_o$ $0 \le r \le a_o \implies 0 \le x \le X_a$

ber(x) and bei(x) are Kelvin functions of order 0, which will be noted respectively berx and beix in the following. Accordingly:

$$w(x) = A berx + B beix + \frac{Q}{k_{w}}$$

A and B are integration constants to be determined by boundary conditions at TS periphery (x = X_a)

At the TS periphery (r =a_o):
$$X_{a} = k a_{o} = \sqrt[4]{\frac{k_{w}}{D^{*}}} a_{o} = \left[24 \left(1 - v^{*2} \right) N_{t} \frac{E_{t} t_{t} \left(d_{t} - t_{t} \right) a_{o}^{2}}{E^{*} L h^{3}} \right]^{\frac{1}{4}}$$

net effective pressure:

$$q(w) = -k_w (A berx + B beix)$$

(c) the rotation, using the sign conventions shown in Figure 21 (θ >0 clockwise), is written:

$$\theta(r) = -\frac{dw}{dr} = -k \frac{dw}{dx}$$
$$\theta(x) = -k \left[A ber' x + B bei' x \right]$$

Note: For x=0 $\theta(0)=0$: the rotation at the center of the TS is 0 as expected. (d) The shear force writes:

$$Q_{r}(r) = \frac{1}{r} \int_{0}^{r} \rho q(\rho) d\rho \qquad 0 \le \rho \le r \qquad x = k r$$

$$= \frac{1}{r} \int_{0}^{r} -k_{w} \rho \left[A \operatorname{ber}(k\rho) + B \operatorname{bei}(k\rho) \right] d\rho \qquad y = k\rho \qquad 0 < y \le x$$

$$= -\frac{1}{r} \frac{k_{w}}{k^{2}} \left[A \int_{0}^{x} y \operatorname{bery} dy + B \int_{0}^{x} y \operatorname{beiy} dy \right]$$

$$y \operatorname{bery} = + y \operatorname{bei}' y + \operatorname{bei}' y = + \left(y \operatorname{bei}' y \right)' \Rightarrow \int y \operatorname{bery} dy = + y \operatorname{bei}' y$$

$$y \operatorname{beiy} = - y \operatorname{ber}' y + \operatorname{ber}' y = - \left(y \operatorname{ber}' y \right)' \Rightarrow \int y \operatorname{beiy} dy = - y \operatorname{ber}' y$$

$$Q_{r}(r) = -\frac{1}{r} \frac{k_{w}}{k^{2}} \left[A x \operatorname{bei}' x - B x \operatorname{ber}' x \right]$$

$$Q_{r}(r) = -\frac{k_{w}}{k} \left(A \operatorname{bei}' x - B \operatorname{ber}' x \right)$$

(e) radial bending moment is written:

$$M_{\rm r}(r) = -D^* \left[\frac{d^2 w}{dr^2} + \frac{v^*}{r} \frac{dw}{dr} \right] \qquad x = k r$$

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$$M_{r}(x) = -D^{*} k^{2} \left[A ber' x + B bei' x \right] + \frac{v^{*}}{x} \left[A ber' x + B bei' x \right]$$
with:

$$ber' x = -beix - \frac{ber' x}{x} \qquad bei' x = + berx - \frac{bei' x}{x}$$

$$M_{r}(x) = -D^{*} k^{2} \left[-A \left(\underbrace{beix + \frac{1 - v^{*}}{x} ber' x}_{\Psi_{1}(x)} \right) + B \left(\underbrace{berx - \frac{1 - v^{*}}{x} bei' x}_{\Psi_{2}(x)} \right) \right]$$

$$M_{r}(x) = -D^{*} k^{2} \left[-A \Psi_{1}(x) + B \Psi_{2}(x) \right]$$
with:

$$\begin{cases} \Psi_{1}(x) = beix + \frac{1 - v^{*}}{x} ber' x \qquad \Psi_{1} = \Psi_{1}(X_{a}) \\ \Psi_{2}(x) = berx + \frac{1 - v^{*}}{x} bei' x \qquad \Psi_{2} = \Psi_{2}(X_{a}) \end{cases}$$

(f) circumferential bending moment is written:

$$M_{\theta} = -D^* \left[v^* \frac{d^2 w}{dr^2} + \frac{1}{r} \frac{dw}{dr} \right]$$

The same calculation leads to:

$$M_{\theta}(x) = -D^{*}k^{2} \left[-A \overline{\Psi_{1}(x)} + B \overline{\Psi_{2}(x)} \right]$$

with: $\overline{\Psi_{1}(x)} = v^{*} beix - \frac{1 - v^{*}}{x} ber'x$ $\overline{\Psi_{2}(x)} = v^{*} berx + \frac{1 - v^{x}}{x} bei'x$
is moment does not generally control the TS design since:

This ient does 0 ge S design since ιy

$$M_{\rm r}(x) > |M_{\theta}(x)|$$

Determination of Integration Constants A and B 5.2

(a) A and B are determined from the boundary conditions at periphery of TS:

$$Q_r(a_o) = V_a \qquad M_r(a_o) = M_a$$

where V_a et M_a are the loads applied at periphery of TS (point A in Figure 21).

$$\begin{cases} Q_r(X_a) = -\frac{k_w}{k} \Big[A \ bei'(X_a) - B \ ber'(X_a) \Big] \qquad \Rightarrow A \ bei' - B \ ber' = -\frac{k}{k_w} V_a \\ M_r(X_a) = -D^* \ k^2 \Big[-A \ \Psi_1(X_a) + B \ \Psi_2(X_a) \Big] \Rightarrow A \ \Psi_1 - B \ \Psi_2 = \frac{M_a}{D^* \ k^2} = \frac{k^2}{k_w} M_a \end{cases}$$
$$M_r(X_a) = -D^* \ k^2 \Big[-A \ \Psi_1(X_a) + B \ \Psi_2(X_a) \Big] \Rightarrow A \ \Psi_1 - B \ \Psi_2 = \frac{M_a}{D^* \ k^2} = \frac{k^2}{k_w} M_a \end{cases}$$
$$ber = ber(X_a) \quad bei = bei(X_a) \quad ber' = ber'(X_a) \quad bei' = bei'(X_a) \qquad \Psi_1 = \Psi_1(X_a)$$

(b) Integration constants A and B are obtained from Section 5.2a:

$$\begin{cases} A = -\frac{k}{k_{w}} \frac{\left(k \ ber'\right) M_{a} + \Psi_{2} V_{a}}{Z_{a}} \\ B = -\frac{k}{k_{w}} \frac{\left(k \ bei'\right) M_{a} + \Psi_{1} V_{a}}{Z_{a}} \end{cases} \text{ with: } Z_{a} = -\Psi_{1} \ ber' + \Psi_{2} \ bei'$$

(c) The 3rd boundary condition at the periphery of the unperforated rim, at its connection with the shell of mean radius a'_{s} (point S in Figure 22) is written: $w(a'_{s}) = 0$



Figure 22 —TS Displacement of the Unperforated Ring and Connection to Shell

If the shell is integral, the TS rotation at point A is written:

$$\theta_{a} = -\frac{dw}{dr} = -\frac{\Delta w}{\Delta r} = -\frac{w(a_{s}) - w(a_{o})}{a_{s} - a_{o}} = \frac{w(a_{o})}{a_{s} - a_{o}}$$

where $\Delta w = w(a_s) - w(a_o)$ as the unperforated rim is considered as rigid.

$$w(a_{o}) = \theta_{a}(a_{s} - a_{o}) = a_{o} \theta_{a}\left(\frac{a_{s}}{a_{o}} - 1\right)$$

Neglecting shell thickness: $a'_s = a_s$ *W*

$$w(a_{o}) = a_{o} \theta_{a} (\rho_{s} - 1) \quad [V.2c] \text{ with } \rho_{s} = \frac{a_{s}}{a_{o}} = \frac{D_{s}}{2a_{o}}$$
(configuration d): $\rho_{s} = \frac{G_{s}}{2a_{o}} = \frac{a_{s}}{a_{o}}$

If the shell is extended as a flange (configuration d): $\rho_s = \frac{G_s}{2 a_o} = \frac{a_s}{a_o}$

(d) Substituting the expressions of A and B in V.1b enables one to determine $w(x), q(x), \theta(x), Q(x), M(x)$ as functions of x, depending on V_a and M_a which are still unknown.

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5.3 Deflection

$$w(x) = \frac{Q}{k_{w}} - \frac{k}{k_{w}} \frac{\left(k \text{ ber' } M_{a} + \Psi_{2} V_{a}\right) \text{ ber } x + \left(k \text{ bei' } M_{a} + \Psi_{1} V_{a}\right) \text{ bei } x}{Z_{a}}$$

$$= \frac{Q}{k_{w}} - \frac{1}{D^{*}} \frac{a_{o}^{3}}{X_{a}^{3}} \left[k M_{a} \frac{\text{ ber' } \text{ ber } x + \text{ bei' } \text{ bei } x}{Z_{a}} + V_{a} \frac{\Psi_{2} \text{ ber } x + \Psi_{1} \text{ bei } x}{Z_{a}}\right]$$

$$w(x) = \frac{Q}{k_{w}} - \frac{a_{o}^{2}}{D^{*}} \left[M_{a} Z_{w}(x) + (a_{o} V_{a}) Z_{d}(x)\right] \qquad [V.3]$$

$$Z_{w}(x) = \frac{\text{ber' } \text{ ber } x + \text{bei' } \text{ bei } x}{X_{a}^{2} Z_{a}} \qquad Z_{d}(x) = \frac{\Psi_{2} \text{ ber } x + \Psi_{1} \text{ bei } x}{X_{a}^{3} Z_{a}}$$

Note: Alan Soler's formula is incorrect: $Z_v(x)$ is used instead of $Z_w(x)$. However the UHX rule is correct as it only uses $Z_w(X_a)$ in its formulas and it can be easily shown (see Annex G) that $Z_w(X_a) = Z_v(X_a)$

5.4 Net Effective Pressure

$$q(x) = Q - k_{w} w(x)$$

$$q(x) = \frac{a_o^2 k_{w}}{D^*} \left[M_a Z_w(x) + a_o V_a Z_d(x) \right] \quad [V.4]$$

5.5 Rotation

$$\theta(x) = -\frac{dw}{dr} = -\frac{dw}{dx}\frac{dx}{dr} = -k\frac{dw}{dx} = k\frac{a_o^2}{D^*} \left[M_A Z_w(x) + (a_o V_A) Z_d(x) \right]$$

$$\theta(x) = \frac{a_o}{D^*} \left[M_a \frac{ber' ber' x + bei' bei' x}{X_a Z_a} + a_o V_a \frac{\Psi_2 ber' x + \Psi_1 bei' x}{X_a^2 Z_a} \right]$$

$$\left[\theta(x) = \frac{a_o}{D^*} \left[M_a Z_m(x) + a_o V_a Z_v(x) \right] \right] \quad [V.5]$$

$$Z_m(x) = \frac{ber' ber' x + bei' bei' x}{X_a Z_a} \qquad Z_v(x) = \frac{\Psi_2 ber' x + \Psi_1 bei' x}{X_a^2 Z_a}$$

Note: for x=0:
$$\begin{cases} Z_m(x) = 0 \\ Z_v(x) = 0 \end{cases} \quad \theta(0) = 0: \text{ the rotation at TS center is 0 as expected.} \end{cases}$$

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5.6 Shear Force

$$Q_{r}(x) = \frac{k \text{ ber}' M_{a} + \Psi_{2} V_{a}}{Z_{a}} \text{ bei } x - \frac{k \text{ bei}' M_{a} + \Psi_{1} V_{a}}{Z_{a}} \text{ ber } x \qquad k = \frac{X_{a}}{a_{o}}$$

$$Q_{r}(x) = \frac{1}{a_{o}} \begin{bmatrix} M_{a} \frac{\text{ber}' \text{ bei } x - \text{bei}' \text{ ber}' x}{Z_{a}} & X_{a} + a_{o} V_{a} \frac{\Psi_{2} \text{ bei } x - \Psi_{1} \text{ ber}'(x)}{Z_{a}} \\ \hline Q_{\beta}(x) \end{bmatrix}$$

$$\boxed{Q_{r}(x) = \frac{1}{a_{o}} \begin{bmatrix} M_{a} Q_{\alpha}(x) + (a_{o} V_{a}) Q_{\beta}(x) \end{bmatrix}}{Q_{\alpha}(x)} \quad [V.6]$$

$$Q_{a}(x) = \frac{\text{ber}' \text{ bei } x - \text{bei}' \text{ ber}' x}{Z_{a}} X_{a} \qquad Q_{\beta}(x) = \frac{\Psi_{2} \text{ bei } x - \Psi_{1} \text{ ber}' x}{Z_{a}}$$

Note: for $x = X_a$: $\begin{cases} Q_{\alpha}(X_a) = 0 \\ Q_{\beta}(X_a) = 1 \end{cases} \quad Q_r(X_a) = \frac{1}{a_o} a_o V_a \implies Q(X_a) = V_a \end{cases}$

The boundary condition $Q_r(X_a) = V_a$ is satisfied.

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5.7 Bending Moment

$$M_{r}(x) = \frac{D^{*}k^{3}}{k_{w}} \left[-\left(\frac{k \text{ ber}' M_{a} + \Psi_{2} V_{a}}{Z_{a}}\right) \Psi_{1}(x) + \left(\frac{k \text{ bei}' M_{a} + \Psi_{1} V_{a}}{Z_{a}}\right) \Psi_{2}(x) \right]$$

$$M_{r}(x) = M_{a} \underbrace{\frac{bei'\Psi_{2}(x) - ber'\Psi_{1}(x)}{Z_{a}}}_{Q_{m}(x)} + a_{o} V_{a} \underbrace{\frac{\Psi_{1}\Psi_{2}(x) - \Psi_{2} \Psi_{1}(x)}{X_{a} Z_{a}}}_{Q_{V}(x)}$$

$$\underbrace{M_{r}(x) = M_{a} Q_{m}(x) + a_{o} V_{a} Q_{v}(x)}_{Q_{v}(x)} \quad [V.7]$$

$$Q_{m}(x) = \frac{bei'\Psi_{2}(x) - ber'\Psi_{1}(x)}{Z_{a}} \qquad Q_{v}(x) = \frac{\Psi_{1}\Psi_{2}(x) - \Psi_{2} \Psi_{1}(x)}{X_{a} Z_{a}}$$
for $x = X_{a} \begin{cases} Q_{m}(X_{a}) = 1 \\ Q_{m}(x) = Q_{m}(x) = M_{a}; \text{ the boundary condition } M(X_{a}) = M_{a}; \text{ satisfies} \end{cases}$

Note: for $x=X_a$ $\begin{cases} Q_m(X_a)=1\\ Q_v(X_a)=0 \end{cases} \implies M_r(X_a)=M_a$: the boundary condition $M(X_a)=M_a$ is satisfied.

5.8 Conclusion

- 1) Coefficients $Z_w(x)$, $Z_d(x)$; $Z_m(x)$, $Z_v(x)$; $Q_a(x)$, $Q_\beta(x)$; $Q_m(x)$, $Q_v(x)$ are combinations of Kelvin functions. They are known for a given HE and for a given value of x varying from 0 at the TS center to X_a at the TS periphery. These coefficients are defined in Table UHX-13.1 of UHX-13. Annex G provides their expressions for x=0 and x=X_a.
- 2) Quantities $w(x), q(x), \theta(x), Q_r(x), M_r(x)$ depend on moment M_a and force V_a acting at TS periphery.
- *3)* Edge loads at TS-shell-connection are still to be determined from the boundary conditions at TS periphery (see Section 6 hereafter).



6 TREATMENT OF THE UNPERFORATED RIM

6.1 Edge Loads Applied on Shell and Channel at their Connection to the TS

The following equations are developed for integral shell and channel.

(a) Shell subjected to edge loads Q_s and M_s at its connection to the rim (see Figure 17) Radial displacement w_s and rotation θ_s are detailed in Annex H. Loads are per unit of length of circumference.

Radial displacement:
$$W_s = \frac{Q_s}{\beta_s^2 k_s} + \frac{M_s}{\beta_s k_s} + \delta_s P_s$$
 rotation: $\theta_s = \frac{Q_s}{\beta_s k_s} + \frac{2M_s}{k_s}$

Where:

$$\beta_{\rm s} = \frac{\sqrt[4]{12\left(1 - v_{\rm s}^2\right)}}{\sqrt{\left(D_{\rm s} + t_{\rm s}\right)t_{\rm s}}}$$
 is the shell coefficient

$$k_{\rm s} = \beta_{\rm s} \frac{E_{\rm s} t_{\rm s}^3}{6(1-v_{\rm s}^2)}$$
 is the bending stiffness coefficient of the shell

$$\delta_{\rm s} = \frac{D_{\rm s}^2}{4 E_{\rm s} t_{\rm s}} \left(1 - \frac{D_{\rm s}}{D_{\rm s} + t_{\rm s}} \frac{v_{\rm s}}{2} \right)$$

is the coefficient due to pressure acting on the shell.

Note: In UHX, t_s has been neglected compared to D_s and the formula is written:

$$\delta_{\rm s} = \frac{D_{\rm s}^2}{4 E_{\rm s} t_{\rm s}} \left(1 - \frac{v_{\rm s}}{2} \right)$$

Equations for M_s and Q_s write:

$$\begin{cases} M_{s} = k_{s} \theta_{s} - \beta_{s} k_{s} w_{s} + \beta_{s} k_{s} \delta_{s} P_{s} \\ Q_{s} = -\beta_{s} k_{s} \theta_{s} + 2\beta_{s}^{2} k_{s} w_{s} - 2\beta_{s} k_{s} \delta_{s} P_{s} \end{cases}$$

Neglecting the radial displacement at mid-surface of the ring, compatibility of displacements of the ring and the shell is written (see Annex I): $w_s = -\frac{h}{2}\theta_s$ and loads M_s and Q_s become, using: $t' = h \beta$:

$$M_{s} = +k_{s}\left(1+\frac{t_{s}}{2}\right)\theta_{s} + \beta_{s}k_{s}\delta_{s}P_{s}$$

$$Q_{s} = -\beta_{s}k_{s}\left(1+t_{s}\right)\theta_{s} - 2\beta_{s}^{2}k_{s}\delta_{s}P_{s}$$
[VI.1a] for an integral shell (configurations a, b)

When the shell is not integral with the TS (configuration d), $k_s=0$ and $\delta s=0$ lead to: $M_s=0$ and $Q_{\rm s}=0$.

Note: These formulas are valid for a shell of sufficient length. Annex J provides the minimum length above which these formulas can be applied.

(b) Channel subjected to edge loads Q_c and M_c at its connection to the rim (see Figure 17). Note: To ensure the symmetry of the equations, notation $P_c=P_t$ is used.

Replacing subscript s by subscript c: $\begin{cases} w_{c} = \frac{Q_{c}}{\beta_{c}^{2} k_{c}} + \frac{M_{c}}{\beta_{c} k_{c}} + \delta_{c} P_{c} \\ \theta_{c} = \frac{Q_{c}}{\beta k} + \frac{2M_{c}}{k_{c}} \end{cases} \text{ for an integral channel (configuration a)}$

$$\beta_c = \frac{\sqrt[4]{12\left(1 - v_c^2\right)}}{\sqrt{\left(D_c + t_c\right)t_c}}$$

channel coefficient.

$$k_c = \beta_c \frac{E_c t_c^3}{6\left(1 - v_c^2\right)}$$

bending stiffness coefficient of channel

 $\left| \delta_c = \frac{D_c^2}{4 E_c t_c} \left(1 - \frac{D_c}{D_c + t_c} \frac{v_c}{2} \right) \right|$

coefficient due to pressure acting on channel.

$$\begin{cases} M_{c} = +k_{c} \theta_{c} - \beta_{c} k_{c} w_{c} + \beta_{c} k_{c} \delta_{c} P_{c} \\ Q_{c} = -\beta_{c} k_{c} \theta_{c} + 2\beta_{c}^{2} k_{c} w_{c} - 2\beta_{c} k_{c} \delta_{c} P_{c} \end{cases}$$

Equations for M_c and Q_c write

compatibility of displacements: $w_c = -\frac{h}{2} \theta_c$ lead to equations for M_c and Q_c , using $t_c = h \beta_c$:

$$\left\{ \begin{matrix} M_{c} = +k_{c} \left(1 + \frac{t_{c}}{2}\right) \theta_{c} + \beta_{c} k_{c} \delta_{c} P_{c} \\ Q_{c} = -\beta_{c} k_{c} \left(1 + t_{c}^{'}\right) \theta_{c} - 2\beta_{c}^{2} k_{c} \delta_{c} P_{c} \end{matrix} \right\}$$
[VI.1b] for an integral channel (configuration a)

When the channel is not integral with the TS (configurations b, c, d), $k_c = 0$ and $\delta_c = 0$ lead to: $M_{\rm c} = 0$ and $Q_{\rm c} = 0$

Note 1: These formulas are valid for a channel of sufficient length. Annex J provides the minimum length above which these formulas can be applied.

Note 2: These formulas are valid for a cylindrical channel. If the channel is hemispherical, it must be attached directly to the TS (configurations a, b or c), without any cylindrical section between the head and the TS. Annex K provides the relevant formulas for that case. Only coefficient δ_c is affected:

$$\delta_c = \frac{D_c^2}{4 E_c t_c} \left(\frac{1}{2} - \frac{D_c}{D_c + t_c} \frac{v_c}{2} \right)$$

Note 3: In UHX, t_c *has been neglected compared to* D_c *and the formula is written:*

$\delta - D_c^2$	(1)	(v_c)
$C_c = 4 E_c t_c$	2	2)

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6.2 Equilibrium of the Unperforated Rim

(a) due to axial loads



Figure 23 — Ring Equilibrium of the TS

The axial equilibrium of the ring is written: (see Figure 23):

$$2 \pi a_{\rm s} V_{\rm s} + \pi \left(a_{\rm c}^2 - a_{\rm o}^2 \right) P_{\rm c} = 2 \pi a_{\rm c} V_{\rm c} + 2 \pi a_{\rm o} V_{\rm a} + \pi \left(a_{\rm s}^2 - a_{\rm o}^2 \right) P_{\rm s} \qquad [\text{VI.2a-1}]$$

where:

- V_a = axial edge load acting at connection of ring with equivalent plate is still to be determined
- $V_s = axial$ force acting in the shell will be obtained from above equation, once V_a will have been determined. V_s positive when the shell is in tension.
- V_c = axial force acting in the ST channel is known.

$$2 \pi a_c^{'} V_c = \pi a_c^2 P_c \implies a_c^{'} V_c = \frac{a_c^2}{2} P_c \quad [VI.2a-2]$$
Axial equilibrium of the ring is written:
$$a_s^{'} V_s = a_o V_a + \frac{a_o^2}{2} P_c + \frac{a_s^2 - a_o^2}{2} P_s \qquad [VI.2a-3]$$

Note: For U-tube and immersed floating head HEs:

$$2\pi a_s' V_s = \pi a_s^2 P_s \Longrightarrow a_s' V_s = \frac{a_s^2}{2} P_s \Longrightarrow a_o V_a = \frac{a_o^2}{2} (P_s - P_c) \quad V_a \text{ is known.}$$

(b) due to applied moments

Equilibrium of moments applied to the ring relative to the axis located at radius a_0 enables to determine the moment M_R (see Figure 17).

$$R = \text{radius at center of ring} = \frac{A + 2a_o}{4}$$

$$RM_{R} = -\left[a_o M_{a}\right] + \left[a_c M_{c} - a_c Q_{c} \frac{h}{2}\right] + \left[M\left(P_{c}\right) - a_c V_{c}\left(a_{c} - a_{o}\right)\right]$$

$$-\left[a_s M_{s} - a_s Q_{s} \frac{h}{2}\right] + \left[M\left(P_{s}\right) - a_s V_{s}\left(a_{s} - a_{o}\right)\right]\right\} \quad [VI.2b]$$

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$$M(P_{\rm c}) = \text{moment due to pressure P}_{\rm c} \text{ acting on the ring} = \left(a_{\rm c}^2 - a_o^2\right) \left(\frac{a_{\rm c} + a_o}{2} - a_o\right) \frac{P_{\rm c}}{2}$$
$$M(P_{\rm s}) = \text{moment due to pressure P}_{\rm s} \text{ acting on the ring} = \left(a_{\rm s}^2 - a_o^2\right) \left(\frac{a_{\rm s} + a_o}{2} - a_o\right) \frac{P_{\rm s}}{2}$$

Replacing terms of equation [VI.2b] by their expressions, one obtains for configuration a (see Annex L):

$$RM_{\rm R} = -a_o M_{\rm a} + a_o^2 V_{\rm a} (\rho_{\rm s} - 1) + P_{\rm s} \frac{a_o^3}{4} \left[\left(\rho_{\rm s}^2 - 1 \right) \left(\rho_{\rm s} - 1 \right) \right] - P_{\rm c} \frac{a_o^3}{4} \left[\left(\rho_{\rm c} - 1 \right) \left(\rho_{\rm c}^2 + 1 \right) - 2 \left(\rho_{\rm s} - 1 \right) \right] \right] - \left[a_s^{\rm c} k_{\rm s} \left(1 + t_{\rm s}^{\rm c} + \frac{t_{\rm s}^{\rm c}}{2} \right) + a_c^{\rm c} k_{\rm c} \left(1 + t_{\rm c}^{\rm c} + \frac{t_{\rm c}^{\rm c}}{2} \right) \right] \theta_{\rm a} + a_o \left[\omega_{\rm c} P_{\rm c} - \omega_{\rm s} P_{\rm s} \right]$$

$$\left[\text{VI.2b'} \right]$$

$$\omega_{\rm s} = \rho_{\rm s} \beta_{\rm s} k_{\rm s} \delta_{\rm s} \left(1 + h \beta_{\rm s}\right) \qquad \omega_{\rm c} = \rho_{\rm c} \beta_{\rm c} k_{\rm c} \delta_{\rm c} \left(1 + h \beta_{\rm c}\right) \qquad [VI.2b'']$$

Annex L provides the relevant modifications to cover the three other configurations b, c and d. *(c)* **Rotation of rigid ring**

$$K = \frac{A}{D_o} \qquad \qquad \theta_{\rm R} = \frac{12}{E h^3} \frac{R M_{\rm R}}{Ln K} \qquad \text{leads to, with } \theta_{\rm R} = \theta_{\rm a}:$$
$$R M_{\rm R} = \left[\frac{E h^3}{12} Ln K\right] \theta_{\rm R} = \left[\frac{E h^3}{12} Ln K\right] \theta_{\rm a}$$

Replacing RM_R by its expression [VI.2b'] leads to:

$$\begin{bmatrix} \frac{C_{1}}{12} Ln K + a_{s}^{'} k_{s} \left(1 + t_{s}^{'} + \frac{t_{s}^{'2}}{2}\right) + a_{c}^{'} k_{c} \left(1 + t_{c}^{'} + \frac{t_{c}^{'2}}{2}\right) \end{bmatrix} \theta_{a} = -a_{o} M_{a} + a_{o}^{2} V_{a} \left(\rho_{s} - 1\right) \\ + P_{s} \underbrace{\left\{\frac{a_{o}^{3}}{4}\left[\left(\rho_{s}^{2} - 1\right)\left(\rho_{s} - 1\right)\right] - a_{o} \omega_{s}\right\}}_{C2} - P_{c} \underbrace{\left\{\frac{a_{o}^{3}}{4}\left[\left(\rho_{c} - 1\right)\left(\rho_{c}^{2} + 1\right) - 2\left(\rho_{s} - 1\right)\right] - a_{o} \omega_{c}\right\}}_{C3}\right\}$$

$$C_{1} = \frac{h^{3}}{12} \left[\underbrace{\frac{6}{h^{3}} (D_{s} + t_{s}) k_{s} \left(1 + t_{s}^{'} + \frac{t_{s}^{'2}}{2}\right)}_{\lambda_{s}} + \underbrace{\frac{6}{h^{3}} (D_{c} + t_{c}) k_{c} \left(1 + t_{c}^{'} + \frac{t_{c}^{'2}}{2}\right)}_{\lambda_{c}} + E Ln K \right] = \frac{h^{3}}{12} \left[\lambda_{s} + \lambda_{c} + E Ln K \right]$$

$$\lambda_{s} = \frac{6}{h^{3}} (D_{s} + t_{s}) k_{s} \left(1 + t_{s} + \frac{t_{s}^{2}}{2} \right) \qquad \lambda_{c} = \frac{6}{h^{3}} (D_{c} + t_{c}) k_{c} \left(1 + t_{c} + \frac{t_{c}^{2}}{2} \right)$$

Note: if $\lambda_s + \lambda_c$ *is high* (>3) *the TS is considered clamped*

if $\lambda_s + \lambda_c$ is low (<1) the TS is considered simply supported
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where ω_s and ω_c are given by [VI.2b''].

When the shell and channel are integral with the TS (configuration a), edge loads M_a and V_a , which are still unknown, are linked by equation:

$$\frac{h^3}{12} \left[\lambda_{\rm s} + \lambda_{\rm c} + E \ln K \right] \theta_{\rm a} = -a_o M_{\rm a} + a_o^2 V_{\rm a} \left(\rho_{\rm s} - 1 \right) + a_o \left(\omega_{\rm s}^* P_{\rm s} - \omega_{\rm c}^* P_{\rm c} \right)$$

(Configuration a)

(d) The generic equation covering the 4 configurations a, b, c and d is written, accounting for the results obtained at Annex L for configurations b, c and d:

$$\frac{h^{3}}{12} \left[\lambda_{s} + \lambda_{c} + E \ln K \right] \theta_{a} = -a_{o} M_{a} + a_{o}^{2} V_{a} \left(\rho_{s} - 1 \right) + a_{o} \left(\omega_{s}^{*} P_{s} - \omega_{c}^{*} P_{c} \right) + \frac{a_{o}}{2\pi} \left[W_{c} \gamma_{bc} - W_{s} \gamma_{bs} \right] \qquad \left[\text{VI.2d} \right]$$

where:

 λ_s , λ_c and ω_s , ω_c are coefficients obtained above. They are known for a given HE.

 $W_{\rm s}$ and $W_{\rm c}$ are bolt loads applied on shell and channel when the TS is gasketed (configurations b, c, d). Coefficients γ_{bs} and γ_{bc} are defined as follows:

- $\gamma_{\rm bs} = 0 \qquad \gamma_{\rm bc} = 0$ Configuration a: k_s given in VI.1a k_c given in VI.1b $\gamma_{\rm bs} = 0 \qquad \gamma_{\rm bc} = \frac{G_{\rm c} - C_{\rm c}}{D_{\rm o}}$
- Configuration b: k_s given in VI.1a $k_c = 0$
- Configuration c: k_s given in VI.1a $k_c = 0$
- Configuration d: $k_s = 0$ $k_{c} = 0$

$$\gamma_{bs} = 0 \qquad \gamma_{bc} = \frac{G_c - G_1}{D_o}$$
$$\gamma_{bs} = \frac{G_s - C_s}{D_o} \qquad \gamma_{bc} = \frac{G_c - C_c}{D_o}$$

UHX-13 covers the usual case where $C_s=C_c=C$, which leads to:

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$$\begin{split} \gamma_{\rm b} &= 0 \qquad for \ configuration \ a \ , \\ \gamma_{\rm b} &= \frac{G_{\rm c} - C}{D_o} \quad for \ configuration \ b \ , \\ \gamma_{\rm b} &= \frac{G_{\rm c} - G_{\rm l}}{D_o} \quad for \ configuration \ c \ , \\ \gamma_{\rm b} &= \frac{G_{\rm c} - G_{\rm s}}{D_o} \quad for \ configuration \ c \ , \\ \gamma_{\rm b} &= \frac{G_{\rm c} - G_{\rm s}}{D_o} \quad for \ configuration \ d \end{split}$$
 $\\ \hline \frac{h^3}{12} \big[\lambda_{\rm s} + \lambda_{\rm c} + E \ Ln \ K \big] \theta_{\rm a} &= -a_o \ M_{\rm a} + a_o^2 \ V_{\rm a} \ (\rho_{\rm s} - 1) + a_o \left(\omega_{\rm s}^* \ P_{\rm s} - \omega_{\rm c}^* \ P_{\rm c} \right) + \frac{a_o}{2\pi} \ \gamma_b \big[W_{\rm c} - W_{\rm s} \ \big] \end{split}$

6.3 Edge Loads V_a and M_a Applied to the Tubesheet

(a) Determination of M_a

Quantities θ_a , V_a and M_a at periphery of TS, linked by relation [VI.2d] above, are still unknown. Equation [V.5] enables to determine:

$$\theta_{\rm R} = \theta_{\rm a} = \theta \left(X_{\rm a} \right) = \frac{a_o}{D^*} \left[M_{\rm a} Z_{\rm m} + \left(a_o V_{\rm a} \right) Z_{\rm v} \right]$$

which leads to a 1st relationship between V_a and M_a:

$$\frac{h^{3}}{12} [\lambda_{s} + \lambda_{c} + E \ln K] \frac{a_{o}}{D^{*}} [M_{a} Z_{m} + (a_{o} V_{a}) Z_{v}] = -a_{o} M_{a} + a_{o}^{2} V_{a} (\rho_{s} - 1) + a_{o} (\omega_{s}^{*} P_{s} - \omega_{c}^{*} P_{c}) + \frac{a_{o}}{2\pi} [W_{c} \gamma_{bc} - W_{s} \gamma_{bs}]$$

$$\Phi = \frac{h^{3}}{12} \frac{1}{D^{*}} [\lambda_{s} + \lambda_{c} + E \ln K] = \frac{h^{3}}{12} \frac{12(1 - v^{*2})}{E^{*} h^{3}} [\lambda_{s} + \lambda_{c} + E \ln K]$$

$$\Phi = \frac{(1 - v^{*2})}{E^{*}} [\lambda_{s} + \lambda_{c} + E \ln K] \qquad [VI.3]$$

Note: Φ denotes the degree of restrain of the TS by the shell and channel

- if Φ is high (>4) the TS can be considered as clamped ($\lambda_s + \lambda_c$ high) •
- if Φ is low (<1) the TS can be considered as simply supported ($\lambda_s + \lambda_c \log \theta$)

Using coefficient F relative to U-tube HEs used in UHX-12.5.5:

$$\frac{F = \frac{1 - v^{*}}{E^{*}} [\lambda_{s} + \lambda_{c} + ELnK]}{\Phi [M_{a} Z_{m} + (a_{o} V_{a}) Z_{v}] = -M_{a} + a_{o} V_{a} (\rho_{s} - 1) + (\omega_{s}^{*} P_{s} - \omega_{c}^{*} P_{c}) + \frac{1}{2\pi} [W_{c} \gamma_{bc} - W_{s} \gamma_{bs}] [VI.3a']}$$

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For the usual case (see VI.2d) covered by UHX-13 ($C_s=C_c=C$ and $W_s=W_c=W^*$):

$Q_2 = -$	$\left(\omega_{\rm s}^* P_{\rm s} - \omega_{\rm c}^* P_{\rm c}\right) + W^* \frac{\gamma_{\rm b}}{2\pi}$
	$1 + \Phi Z_{\rm m}$

Note: In M_a formula:

- term $(a_o V_a) Q_1$ represents the moment due to the equivalent pressure P_e acting on the TS.
- term Q_2 is the sum of the moments acting on the TS, due to pressures P_s , P_c and bolt load W.

(b) 2nd relationship between V_a and M_a

A 2^{nd} relationship between V_a and M_a is obtained from boundary condition [V.2c]:

$$w_{a} = a_{o} \theta_{a} (\rho_{s} - 1)$$

Replacing w_a and θ_a by their expressions [V.3] and [V.5], [V.2c] is written:

$$\frac{Q}{k_{w}} - \frac{a_{o}^{2}}{D^{*}} \left[M_{a} Z_{m} + (a_{o} V_{a}) Z_{d} \right] = \frac{a_{o}^{2}}{D^{*}} \left[M_{a} Z_{m} + (a_{o} V_{a}) Z_{v} \right] (\rho_{s} - 1)$$

$$\frac{Q}{k_{w}} = \frac{a_{o}^{2}}{D^{*}} \left\{ M_{a} \left[\underbrace{Z_{w} + (\rho_{s} - 1) Z_{m}}_{\rho_{1}} \right] + (a_{o} V_{a}) \left[\underbrace{Z_{d} + (\rho_{s} - 1) Z_{v}}_{\rho_{2}} \right] \right\}$$

$$\frac{Q}{k_{w}} = \frac{a_{o}^{2}}{D^{*}} \left[M_{a} \rho_{1} + (a_{o} V_{a}) \rho_{2} \right] \quad [VI.3b]$$

$$\begin{bmatrix} \rho_{1} = Z_{w} + (\rho_{s} - 1) Z_{m} \\ \rho_{2} = Z_{d} + (\rho_{s} - 1) Z_{v} \end{bmatrix}$$

where Q is given by [V.1a']:

$$Q = \Delta p^{*} + k_{w} \left[\underbrace{\left[\delta_{t} \left(\theta_{t} \right) - \delta_{s} \left(\theta_{s} \right) \right] + \left[\delta_{t} \left(v_{t} \right) - \delta_{s} \left(v_{s} \right) \right] - \left[\delta_{J} \left(P_{s} \right) \right]}_{\delta} - \delta_{s} \left(V_{s} \right) \right] \right]$$

$$\delta = \left[\delta_{t}(\theta_{t}) - \delta_{s}(\theta_{s})\right] + \left[\delta_{t}(v_{t}) - \delta_{s}(v_{s})\right] - \left[\delta_{J}(P_{s})\right]$$

In this equation, all quantities are known except $\delta_{s}(V_{s}) = \frac{2\pi a_{s}}{J k_{s}} V_{s}$:

With [VI.2a-3]:
$$a_{s}' V_{s} = a_{o} V_{a} + \frac{a_{o}^{2}}{2} P_{c} + \frac{a_{s}^{2} - a_{o}^{2}}{2} P_{s}$$

$$\frac{Q}{k_{w}} = \frac{\Delta p^{*}}{k_{w}} + \delta - \frac{2\pi}{J k_{s}'} \left[\left(a_{o} V_{a} \right) + \frac{a_{o}^{2}}{2} P_{c} + \frac{a_{s}^{2} - a_{o}^{2}}{2} P_{s} \right]$$

(c) Determination of V_a

Introducing Q/k_w in [VI.3b] leads to a 2nd relationship between V_a and M_a , already linked by relationship [VI.3a] $M_a = (a_o V_a) Q_1 + Q_2$ which enables to determine V_a :

$$\frac{Q}{k_{w}} = \frac{\Delta p^{*}}{k_{w}} + \delta - \frac{2\pi}{J k_{s}} \left[\left(a_{o} V_{a} \right) + \frac{a_{o}^{2}}{2} P_{c} + \frac{a_{s}^{2} - a_{o}^{2}}{2} P_{s} \right] = \frac{a_{o}^{2}}{D^{*}} \left\{ \left[\left(a_{o} V_{a} \right) Q_{1} + Q_{2} \right] \rho_{1} + \left[\left(a_{o} V_{a} \right) \rho_{2} \right] \right\}$$

$$a_{o} V_{a} \left[1 + J \frac{k_{s}^{'} a_{o}^{2}}{2\pi D^{*}} \left(\rho_{1} Q_{1} + \rho_{2} \right) \right] = \frac{J k_{s}^{'}}{2\pi} \left[\frac{\Delta p^{*}}{k_{w}} + \delta \right] - \frac{a_{o}^{2}}{2} P_{c} - \frac{a_{s}^{2} - a_{o}^{2}}{2} P_{s} - \frac{J k_{s}^{'}}{2\pi D^{*}} \frac{a_{o}^{2}}{D^{*}} \rho_{1} Q_{2}$$

$$k^{4} = \frac{k_{w}}{D^{*}} = \frac{N_{t} k_{t}}{\pi a_{o}^{2} D^{*}} \implies \frac{1}{\pi D^{*}} = \frac{k^{4} a_{o}^{2}}{N_{t} k_{t}} \implies \frac{k_{s}^{'} a_{o}^{2}}{2\pi D^{*}} = \frac{k_{s}^{'} a_{o}^{4} k^{4}}{2N_{t} k_{t}} = K_{s,t} \frac{X_{a}^{4}}{2}$$

•
$$(\rho_1 Q_1 + \rho_2) = Z_m Q_1 + (\rho_s - 1) Q_1 Z_m + Z_d + (\rho_s - 1) Z_v = (Z_w Q_1 + Z_d) + (\rho_s - 1) (Z_w Q_1 + Z_v)$$

For a given HE all above quantities are known, which enable to calculate V_a and thus M_a :

$$a_{o} V_{a} = \frac{J \frac{k_{s}}{2\pi} \left[\left(\frac{\Delta p^{*}}{k_{w}} + \delta \right) - \frac{a_{o}^{2}}{D^{*}} \rho_{1} Q_{2} \right] - \frac{a_{o}^{2}}{2} P_{c} - \frac{a_{s}^{2} - a_{o}^{2}}{2} P_{s}}{\left(1 + J K_{s,t} F_{q}^{'} \right)}$$
[VI.3c']

M_a is given by [VI.3a] : $M_a = (a_o V_a) Q_1 + Q_2$

7 EQUIVALENT PRESSURE ACTING ON TUBESHEET

7.1 Definition

(a) A circular plate under uniform pressures P_s and P_t is subjected to a differential pressure $P_e = P_s - P_t$ The axial force V_a at periphery is determined from the plate equilibrium:



Figure 24 — Equivalent Pressure and Axial Force Acting on Plate

(b) The uniform pressure acting on the equivalent solid plate is written: $P_{\rm e} = \frac{2 V_{\rm a}}{a_o}$

This equivalent pressure is due to the various loads acting on the TS:

- Differential pressure: $\Delta p^* = x_s P_s x_t P_t$
- Axial differential thermal expansion tubes-shell:

$$\delta_{t}(v_{t}) = -\frac{2}{k_{w}} \left[(1 - x_{t}) P_{t} - (1 - x_{s}) P_{s} \right] v_{t}$$

 $\delta(\theta) = \frac{\gamma}{2}$

• Force due to Poisson's ratio of shell: $\delta_{s}(v_{s}) = -\frac{\pi}{2k_{s}} \left[P_{s} D_{s}^{2} \right] v_{s}$

• Force due to shell pressure acting on the joint:
$$\delta_{\rm J}(P_{\rm s}) = \frac{\pi}{16} \frac{D_{\rm J}^2 - D_{\rm s}^2}{K_{\rm J}} P_{\rm s}$$

(see Annex C, Annex D, and Annex E) Note: Equilibrium of the equivalent solid plate is written (see [VI.2a-3]):

$$a_{o} V_{a} = \frac{a_{o}^{2}}{2} (P_{s} - P_{t}) - \frac{a_{s}^{2}}{2} P_{s} + a_{s}^{'} V_{s}$$

Equilibrium of a circular plate is written (see [VII.1]): $a_o V_a = \frac{a_o^2}{2} (P_s - P_t)$

7.2 Determination of Pe

Equivalent pressure $P_{\rm e} = \frac{2 V_{\rm a}}{a_{\rm o}}$ can be obtained directly from V_a formula [VI.3c]:

$$a_{o} V_{a} \left(1 + J K_{s,t} F_{q} \right) = J \frac{k_{s}}{2\pi} \left[\left(\frac{\Delta p^{*}}{k_{w}} + \delta \right) - \frac{a_{o}^{2}}{D^{*}} \rho_{1} Q_{2} \right] - \frac{a_{o}^{2}}{2} P_{t} - \frac{a_{s}^{2} - a_{o}^{2}}{2} P_{s}$$

All terms appearing in this formula are known for a given HE. However, like TEMA and CODAP did, it is better to write P_e formula in a format which makes appear explicitly the contributions of pressures P_s and P_t , acting directly on the equivalent plate, loads acting on the unperforated rim, bolt load W* when the TS is gasketed, and differential thermal expansion tubes-shell. Following manipulations are necessary to achieve this goal.

$$\left(1+J K_{s,t} F_{q}^{'}\right) P_{e} = \frac{J k_{s}^{'}}{\pi a_{o}^{2}} \left[\frac{1}{k_{w}} \left(\Delta p^{*} + k_{w} \delta\right) - \frac{a_{o}^{2}}{D^{*}} \rho_{1} Q_{2}\right] - P_{t} - \left(\rho_{s}^{2} - 1\right) P_{s}$$

with:

$$\frac{k_{s}}{\pi a_{o}^{2} k_{w}} = \frac{k_{s}}{N_{t} k_{t}} = K_{s,t} \qquad \frac{k_{s}}{\pi D^{*}} = \frac{X_{a}^{4}}{a_{o}^{2}} K_{s,t} \qquad \delta = \left[\delta_{t} \left(\theta_{t}\right) - \delta_{s} \left(\theta_{s}\right)\right] + \left[\delta_{t} \left(v_{t}\right) - \delta_{s} \left(v_{s}\right)\right] - \left[\delta_{J} \left(P_{s}\right)\right]$$

$$\cdot \frac{k_{s}}{\pi a_{o}^{2} k_{w}} \left[\Delta p^{*} + k_{w} \delta\right] = K_{s,t} \left[(x_{s} P_{s} - x_{t} P_{t}) + k_{w} \frac{\gamma}{2} - 2(1 - x_{t})v_{t} P_{t} + 2(1 - x_{s})v_{t} P_{s} - \frac{\pi k_{w}}{2k_{s}} P_{s} D_{s}^{2} v_{s} - \frac{\pi k_{w}}{16} \frac{D_{J}^{2} - D_{s}^{2}}{K_{J}} \right]$$

$$\frac{\pi k_{w}}{2k_{s}} P_{s} D_{s}^{2} v_{s} = \frac{\pi N_{t} K_{t}}{2k_{s} \pi a_{o}^{2}} D_{s}^{2} v_{s} P_{s} = \frac{2}{K_{s,t}} \left(\frac{D_{s}}{D_{o}}\right)^{2} v_{s} P_{s}$$

$$\frac{\pi k_{w}}{16 K_{J}} \left(D_{J}^{2} - D_{s}^{2}\right) = \frac{k_{s}}{a_{o}^{2} K_{s,t} 16 K_{J}} \left(D_{J}^{2} - D_{s}^{2}\right) = \frac{1}{2 K_{s,t}} \frac{k_{s}}{2 K_{J}} \frac{D_{J}^{2} - D_{s}^{2}}{4 a_{o}^{2}} = \frac{1}{2 K_{s,t}} \frac{1 - J}{J} \frac{D_{J}^{2} - D_{s}^{2}}{D_{o}^{2}}$$

$$\cdot - \frac{k_{s}}{\pi D^{*}} \rho_{1} Q_{2} = -K_{s,t} \frac{1}{a_{o}^{2}} \left[\frac{Z_{w} + \left(\rho_{s} - 1\right) Z_{m}}{U} X_{a}^{4} \right] \left[\left(\omega_{s}^{*} P_{s} - \omega_{c}^{*} P_{t}\right) + \frac{\gamma_{b}}{2 \pi} W^{*} \right]$$

W is given by [VI.2d]

$$=K_{s,t}\left\{-\frac{U}{a_o^2}\left[\omega_s^* P_s - \omega_c^* P_t\right] - \frac{U}{a_o^2}\left[\frac{\gamma_b}{2\pi}W^*\right]\right\} \qquad \text{where} \quad \left[U = \frac{Z_w + (\rho_s - 1)Z_m}{1 + \Phi Z_m}X_a^4\right]$$

1st term accounts for the effect of moments due to pressures Ps and Pt acting on the unperforated rim:

$$P_{\rm rim} = -\frac{U}{a_o^2} \Big[\omega_{\rm s}^* P_{\rm s} - \omega_{\rm c}^* P_{\rm t} \Big]$$

2nd term accounts for the effective bolting load W* acting on the TS (configurations b, c, d)

$$P_{w} = -\frac{U}{a_{o}^{2}} \frac{\gamma_{b}}{2\pi} W^{*}$$
Thus $-\frac{k_{s}}{\pi D^{*}} \rho_{1} Q_{2} = K_{s,t} [P_{rim} + P_{w}]$

$$(1+J K_{s,t} F_{q}^{'}) P_{e} = J K_{s,t} \begin{cases} P_{s} \left[x_{s} + 2(1-x_{s}) v_{t} + \frac{2}{K_{s,t}} \left(\frac{D_{s}}{D_{o}} \right)^{2} v_{s} - \frac{\rho_{s}^{2} - 1}{J K_{s,t}} - \frac{1-J}{2 J K_{s,t}} \frac{D_{J}^{2} - D_{s}^{2}}{D_{o}^{2}} \right] \\ -P_{t} \left[x_{t} + 2(1-x_{t}) v_{t} + \frac{1}{J K_{s,t}} \right] + \gamma \left[\frac{k_{w}}{2} \right] + \left[P_{w} \right] + \left[P_{rim} \right] \end{cases} \end{cases}$$

$$P_{e} = \frac{J K_{s,t}}{1+J K_{s,t} F_{q}^{'}} \left[P_{s}^{'} - P_{t}^{'} + P_{\gamma} + P_{w} + P_{rim} \right] \qquad [VII.2] \qquad F_{q}^{'} = Q_{z1} \left(\rho_{s} - 1 \right) Q_{z2}$$

In this formula:

P_s accounts for the loads due pressure P_s acting on:

- the perforated TS,
- the external wall of the tubes (effect of Poisson's ratio v_t),
- the external wall of the shell (effect of Poisson's ratio v_s),
- the unperforated rim,
- the sidewall of the joint.

$$\left| P_{s}^{'} = P_{s}^{'} \left[x_{s}^{'} + 2\left(1 - x_{s}^{'}\right)v_{t}^{'} + \frac{2}{K_{s,t}} \left(\frac{D_{s}}{D_{o}}\right)^{2}v_{s}^{'} - \frac{\rho_{s}^{2} - 1}{JK_{s,t}} - \frac{1 - J}{2JK_{s,t}} \frac{D_{J}^{2} - D_{s}^{2}}{D_{o}^{2}} \right] \right|$$

- P_t accounts for the loads due pressure P_t acting on:
- the perforated TS,
- the internal wall of the tubes (effect of Poisson's ratio v_{t}),
- the unperforated rim.

$$P_{t}' = P_{t} \left[x_{t} + 2(1 - x_{t}) v_{t} + \frac{1}{J K_{s,t}} \right]$$

 P_{γ} accounts for the loads due to differential thermal expansion between tubes and shell γ .

$$P_{\gamma} = \gamma \left[\frac{k_{w}}{2} \right] = \left[\frac{N_{t} K_{t}}{\pi a_{o}^{2}} \right] \gamma$$

 P_{W} accounts for the effect bolting load W* acting on the TS (configurations b, c, d)

$$P_{\rm w} = -\frac{U}{a_o^2} \frac{\gamma_{\rm b}}{2\,\pi} W^*$$

 P_{rim} accounts for the effect of moments due to pressures P_s and P_t acting on the unperforated rim:

$$P_{\rm rim} = -\frac{U}{a_o^2} \Big[\omega_{\rm s}^* P_{\rm s} - \omega_{\rm c}^* P_{\rm t} \Big]$$

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Note 1: Different expression of the equivalent pressure. P_e can be formulated differently if it is needed to set J equal to 0 to simulate a shell of rigidity 0. Introducing the numerator $JK_{s,t}$ inside the parenthesis leads to:

$$\begin{aligned} \left| P_{e} = \frac{1}{1+J} \frac{1}{K_{s,t}} F_{q}^{'} \left[P_{s1}^{'} - P_{t1}^{'} + P_{\gamma 1} + P_{w1} + P_{rim1} \right] \right| \\ with: P_{s1}^{'} = P_{s} \left[\left(JK_{s,t} \right) x_{s} + (JK_{s,t}) 2 \left(1 - x_{s} \right) v_{t} + (J) 2 \left(\frac{D_{s}}{D_{o}} \right)^{2} v_{s} - (\rho_{s}^{2} - 1) - \frac{\left(1 - J \right) \left[D_{J}^{2} - D_{s}^{2} \right]}{D_{o}^{2}} \right] \\ P_{t1}^{'} = P_{t} \left[\left(JK_{s,t} \right) x_{t} + \left(JK_{s,t} \right) 2 \left(1 - x_{t} \right) v_{t} + 1 \right] \\ P_{\gamma 1} = JK_{s,t} \left[\frac{N_{t}K_{t}}{\pi a_{o}^{2}} \gamma \right] \quad P_{w1} = JK_{s,t} \left[-\frac{U}{a_{o}^{2}} \frac{\gamma_{b}}{2\pi} W^{*} \right] \quad P_{rim} = JK_{s,t} \left[-\frac{U}{a_{o}^{2}} \left(\omega_{s}^{*}P_{s} - \omega_{c}^{*}P_{t} \right) \right] \end{aligned}$$

This is the way TEMA formula is presented.

Note 2: Externally sealed HE. Setting the bellows rigidity to 0 simulates the case of a HE with an externally sealed floating head which will be covered in PART 4. $K_J=0$ and $D_J=0$ lead to:

$$J = \frac{K_J}{K_s + K_J} = 0 \quad JK_{s,t} = 0 \quad P_{s1}' = \left[-(\rho_s^2 - 1) \right] P_s = (1 - \rho_s^2) P_s \qquad P_{t1}' = \left[1 \right] P_t = P_t$$

$$P_{\gamma 1} = JK_{s,t} \left[\frac{N_t K_t}{\pi \ a_o^2} \gamma \right] = 0 \quad P_{W1} = JK_{s,t} \left[-\frac{U}{a_o^2} \frac{\gamma_b}{2\pi} W^* \right] = 0 \qquad P_{rim1} = JK_{s,t} \left[-\frac{U}{a_o^2} \left(\omega_s^* P_s - \omega_c^* P_t \right) \right] = 0$$

$$P_e = P_{s1}' - P_{t1}' = (1 - \rho_s^2) P_s - P_t \qquad \text{which is the UHX-14 formula for an externally sealed floating head.}$$

Note 3: Direct determination of the equivalent pressure. Equivalent pressures P'_s , P'_t and P'_{γ} can be obtained directly by examining the loads applied on the TS as shown in Annex M.



8 STRESSES IN THE HEAT-EXCHANGER COMPONENTS

Deformations (w, θ) and loads (q, Q_r , M_r) applied in the TS have been determined in Sections 5.3 to 5.7 as a function of the TS radius. These formulas are general as they are valid whether $P_e \neq 0$ or Pe=0. They depend on loads V_a and M_a acting at TS periphery, given by equations [VI.3c'] and [VI.3a]:

$$a_o V_a = \frac{a_o^2}{2} P_e$$
 with P_e given by [VII.2] $M_a = (a_o V_a) Q_1 + Q_2$

The formulas used in UHX-13 are presented in a different way.

• If
$$\mathbf{V}_{\mathbf{a}} \neq \mathbf{0}$$
 (P_e $\neq \mathbf{0}$): $\frac{M_{\mathbf{a}}}{a_o V_{\mathbf{a}}} = Q_1 + \frac{Q_2}{a_o V_{\mathbf{a}}} = Q_3$ $M_{\mathbf{a}} = (a_o V_{\mathbf{a}})Q_3$ $Q_3 = Q_1 + \frac{2Q_2}{a_o^2 P_e}$ [VIII]

The advantage of this presentation is that it enables to provide directly the maximum of the TS bending stress and the maximum of the tube stresses, thanks to a parametric study using X_a and Q_3 .

• If $V_a = 0$ (P_e =0): Q₃ becomes infinity and equation [VIII] is no longer valid. Equation [VI.3a] shows that M_a is written: $M_a = Q_2$

See Annex N for relevant equations to be applied in this case.

The following Sections provide the deformations (w, θ) and loads (q, Q_r, M_r) applied in the TS, using both the general formulas and the UHX-13 formulas

Maximum stresses calculated in the TS, tubes, shell and channel must remain below allowable stress limits which are defined in Section 9.

8.1 TS Net Effective Pressure

$$[V.4] \quad q(x) = \frac{a_o^2 k_w}{D^*} \Big[M_a Z_w(x) + (a_o V_a) Z_d(x) \Big] = \frac{X_a^4}{a_o^2} (a_o V_a) \left[\frac{M_a}{(a_o V_a)} Z_w(x) + Z_d(x) \right]$$

$$q(x) = \frac{X_{a}^{4}}{a_{o}^{2}} (a_{o} V_{a}) \Big[Q_{3} Z_{w}(x) + Z_{d}(x) \Big] = \underbrace{\frac{X_{a}^{4}}{2} \Big[Q_{3} Z_{w}(x) + Z_{d}(x) \Big]}_{F_{t}(x)} P_{e} = F_{t}(x) P_{e}$$
[VIII.1]

As shown by Figure 21, a positive value of q denotes a pressure upward.

8.2 TS Axial Displacement

[V.3]
$$w(x) = \frac{Q}{k_{w}} - \frac{q(x)}{k_{w}} = \frac{Q}{k_{w}} - \frac{1}{k_{w}} \frac{X_{a}^{4}}{2} \left[Q_{3} Z_{w}(x) + Z_{d}(x) \right]}{F_{t}(x)}$$



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Developing ρ_1 , ρ_2 , V_a , Q and $F_t(x)$, w(x) is written:

$$w(x) = \frac{a_o^4 P_e}{2D^*} \left\{ \left[\left(Q_3 Z_w + Z_d \right) - \left(Q_3 Z_w \left(x \right) + Z_d \left(x \right) \right) \right] + \left[\left(\rho_s - 1 \right) \left(Q_3 Z_m + Z_v \right) \right] \right\} \right\}$$

The minimum and maximum values of w(x) when x varies from 0 to X_a are obtained from the minimum and maximum values $F_{t,min}$ and $F_{t,max}$ of $F_t(x)$.

 $F_t(x)$ depends on parameters X_a and Q_3 which are known for a given HE. X_a is usually comprised between 1 and 20, and Q_3 between -0.8 and +0.8. A parametric study has been performed using 20 values for X_a (X_a = 1; 2; 3....; 20) and 17 values of Q_3 (Q_3 = -0.8; -0.7; ...-0.1; 0.0; +0.1;; +0.7; +0.8), which enables to determine, for each couple [X_a , Q_3]:

$$F_{t,min} = MIN [F_t(x)]$$
 and $F_{t,max} = MAX [F_t(x)]$

Annex O provides for $1 \le X_a \le 20$ and $-0.8 \le Q_3 \le +0.8$:

- $\bullet \quad \mbox{values and graphs of } Ft(x) \ \ for \quad 0 < x < X_a$
- locations of the minimum and maximum of $F_t(x)$: x_{min} and x_{max}

The minimum and maximum of w(x) are given by:

$$w_{1} = \frac{1}{k_{w}} \left[\frac{X_{a}^{4}}{2} (\rho_{1} \cdot Q_{3} + \rho_{2}) - F_{t,\min} \right] \cdot P_{e} \quad \text{and} \quad w_{2} = \frac{1}{k_{w}} \left[\frac{X_{a}^{4}}{2} (\rho_{1} \cdot Q_{3} + \rho_{2}) - F_{t,\max} \right] \cdot P_{e}$$

As shown by Figure 21, a positive value of w denotes a deflection upward. The maximum tubesheet deflection is given by:

$$w_{\max} = MAX\left[|w_1|, |w_2|\right]$$

8.3 TS Rotation

$$\begin{bmatrix} V.5 \end{bmatrix} \quad \theta(x) = \frac{a_o}{D^*} \Big[M_a Z_m(x) + (a_o V_a) Z_v(x) \Big] = \frac{a_o}{D^*} (a_o V_a) \Big[Q_3 Z_m(x) + Z_v(x) \Big]$$
$$\theta(x) = \frac{a_o^3 P_e}{2 D^*} \Big[Q_3 Z_m(x) + Z_v(x) \Big] = \frac{X_a^3}{k^3 2 D^*} \Big[Q_3 Z_m(x) + Z_v(x) \Big] P_e = \frac{k}{k_w} \underbrace{\frac{X_a^3}{2} \Big[Q_3 Z_m(x) + Z_v(x) \Big]}_{F_a(x)} P_e$$

Note: for x=0: $\begin{cases} Z_m(x) = 0 \\ Z_\nu(x) = 0 \end{cases} \quad \theta(0) = 0: \text{ the rotation at TS center is } 0 \text{ as expected.} \end{cases}$

As shown by Figure 21, a positive value of θ denotes a rotation clockwise.

8.4 Stresses in the Tubesheet

(a) Bending stress

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$$M_{\rm r}(x) = M_{\rm a} Q_{\rm m}(x) + (a_o V_{\rm a}) Q_{\rm v}(x) = (a_o V_{\rm a}) \left[Q_3 Q_{\rm m}(x) + Q_{\rm v}(x) \right] = a_o^2 P_{\rm e} \left[\underbrace{\frac{Q_3 Q_{\rm m}(x) + Q_{\rm v}(x)}{2}}_{F_{m}(x)} \right]$$

$$M_{\rm r}(x) = a_o^2 P_{\rm e} F_{\rm m}(x) \quad F_{\rm m}(x) = \frac{Q_3 Q_{\rm m}(x) + Q_{\rm v}(x)}{2} \text{ with:} \begin{cases} Q_{\rm m}(x) = \frac{bei' \Psi_2(x) - ber' \Psi_1(x)}{Z_{\rm a}} \\ Q_{\rm v}(x) = \frac{\Psi_1 \Psi_2(x) - \Psi_2 \Psi_1(x)}{X_{\rm a} Z_{\rm a}} \end{cases}$$

Note: At TS periphery(x=X_a): $\begin{array}{c} Q_{\rm m}(X_{\rm a}) = 1 \\ Q_{\rm v}(X_{\rm a}) = 0 \end{array} F_{\rm m}(X_{\rm a}) = \frac{Q_3}{2} \Longrightarrow M_{\rm r}(X_{\rm a}) = M_{\rm a}: \end{array}$

the boundary condition is satisfied.

The bending stress in the TS is given by:
$$\sigma_{\rm r}(x) = \frac{6 M_{\rm r}(x)}{\mu^* h^2} = \frac{6 a_o^2 P_{\rm e} F_{\rm m}(x)}{\mu^* h^2}$$

$$\sigma_{\rm r}(x) = \frac{1.5 F_{\rm m}(x)}{\mu^*} \left(\frac{2 a_o}{h}\right)^2 P_{\rm e} \qquad F_{\rm m}(x) = \frac{Q_3 Q_{\rm m}(x) + Q_{\rm v}(x)}{2}$$

As shown by Figure 21, a positive value of $\sigma_r(x)$ denotes a bending moment clockwise.

The maximum value of $\sigma_r(x)$ when x varies from 0 to X_a is obtained from the maximum value $F_{m,max}$ of $F_m(x)$. $F_m(x)$ depends on parameters X_a and Q_3 which are known for a given HE. A parametric study has been performed which enables to determine, for each couple $[X_a, Q_3]$: $F_{m,max} = MAX [F_m(x)]$. Figure 25 shows the bending stress distribution throughout the TS for Q_3 =0.0 and X_a =1, 3, 5, 7, 10 and 15.

Annex P provides for $1 \le X_a \le 20$ and $-0.8 \le Q_3 \le +0.8$:

- values and graphs of $F_m(x)$ for $0 \le x \le X_a$
- values and graphs of F_{m,max}
- location of the maximum of F_m(x): x_{max}

The maximum stress is obtained for:
$$F_{\rm m} = |F_{\rm m,max}| = MAX \left[|F_{\rm m}(x)| \right]$$

The maximum bending stress in the TS is written: $\sigma = \frac{1.5 F_{\rm m}}{\mu^*} \left(\frac{2 a_o}{h} \right)^2 P_{\rm e} \qquad F_{\rm m} = MAX \left[|F_{\rm m}(x)| \right]$

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3, 5, 7, 10 and 15

(b) Shear stress [V.6]

$$Q_{r}(x) = \frac{1}{a_{o}} \Big[M_{a} Q_{a}(x) + (a_{o} V_{a}) Q_{\beta}(x) \Big] = \frac{(a_{o} V_{a})}{a_{o}} \Big[Q_{3} Q_{a}(x) + Q_{\beta}(x) \Big] = \frac{a_{o}}{2} \underbrace{ \left[Q_{3} Q_{a}(x) + Q_{\beta}(x) \right]}_{F_{Q}(x)} P_{c}$$

$$Q_{r}(x) = \frac{a_{o}}{2} F_{Q}(x) P_{e} \qquad F_{Q}(x) = Q_{3} Q_{a}(x) + Q_{\beta}(x) \text{ with:} \begin{cases} Q_{a}(x) = \frac{ber' bei' x - bei' ber' x}{Z_{a}} X_{a} \\ Q_{\beta}(x) = \frac{\psi_{2} bei' x - \psi_{1} ber' x}{Z_{a}} \end{cases}$$

Note: At TS periphery $(x=X_a)$ $Q_{\alpha}(X_a) = 0$ $Q_{\beta}(X_a) = 1$ $F_Q(X_a) = 1 \Rightarrow Q_r(X_a) = V_a$:

the boundary condition is satisfied.

The shear stress in the TS, averaged throughout TS thickness, is given by:

$$\tau(x) = \frac{Q_r(x)}{\mu h} = \frac{a_o}{2 \mu h} F_Q(x) P_e$$

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$$\tau(x) = \frac{1}{2\mu} \frac{a_o}{h} F_Q(x) P_e$$

$$F_{\rm Q}(x) = Q_3 Q_{\alpha}(x) + Q_{\beta}(x)$$

The maximum value of $\tau(x)$ when x varies from 0 to X_a is obtained from the maximum value $F_{Q,max}$ of $F_Q(x)$. $F_Q(x)$ depends on parameters X_a and Q_3 which are known for a given HE. A parametric study has been performed, which enables to determine, for each couple $[X_a, Q_3]$: $F_{Q,max} = MAX [F_Q(x)]$

Annex Q provides for $1 \le X_a \le 20$ and $-0.8 \le Q_3 \le +0.8$:

- $\bullet \quad \text{values and graphs of } F_Q(x) \ \text{ for } \ 0 \leq \ x \leq X_a$
- values and graphs of F_{Q,max}
- location of the maximum of $F_Q(x)$: x_{max}

The maximum stress is obtained for: $F_Q = |F_{Q,\max}| = MAX [|F_Q(x)|]$

Maximum shear stress in the TS is written:
$$\tau_{\max} = \frac{F_Q}{2\mu} \cdot \frac{a_o}{h} \cdot P_e$$
 $F_Q = MAX \left[\left| F_Q(x) \right| \right]$
Shear stress at periphery is written: $\tau = \tau \left(X_a \right) = \frac{1}{4\mu} \frac{D_o}{h} P_e$ as $F_Q(X_a) = 1$

This formula does not provide necessarily the maximum shear stress as $F_Q(x)$ is not always maximum at periphery. However, a parametric study has shown that the maximum of $\tau(x)$ appears for values of Q_3 and X_a for which the TS bending stress $\sigma(x)$ controls the design.

TEMA rules provide the same formula, but use the equivalent diameter D_L corresponding to the perimeter of the outermost ligaments, instead of the equivalent diameter D_0 of outer tube limit circle. The equivalent diameter D_L is calculated from the perimeter of the tube layout, $C_P = \pi D_L$ and the area $A_P = \pi D_L^2 / 4$ enclosed by this perimeter. This leads to $D_L = 4A_P / C_P$

D_L is always lower than D_o and leads to a lower TS shear stress: $\tau = \frac{1}{4\mu} \frac{D_L}{h} P_e$

ASME 2013 edition has adopted that formula under the form $\tau = \left(\frac{1}{4\mu}\right) \left(\frac{1}{h}\right) \left(\frac{4A}{C_p}\right)$

If $\tau \le 0.8S$: $P_e \le 3.2 \mu \frac{h}{D_L}S$ the shear stress does not control and τ does not need to be calculated

8.5 Axial Membrane Stress in Tubes

Axial force $F_t^*(r)$ in tube row at radius r is obtained from [V.1a]:

$$q(r) = -\frac{V_t(r)}{\pi a_o^2 / N_t} + \Delta p^* \qquad \Delta p^* = x_s P_s - x_t P_t$$

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$$V_{t}(x) = \frac{\pi a_{o}^{2}}{N_{t}} \Big[\Delta p^{*} - q(x) \Big] \text{ with [VIII.1] } q(x) = \frac{X_{a}^{4}}{2} \Big[Q_{3} Z_{w}(x) + Z_{d}(x) \Big] P_{e} = F_{t}(x) P_{e}$$

$$\sigma_{t}(x) = \frac{V_{t}(x)}{s_{t}} = \frac{\pi a_{o}^{2}}{N_{t} s_{t}} \Big[\Delta p^{*} - F_{t}(x) P_{e} \Big] \text{ with [III.2.b1] } \frac{\pi a_{o}^{2}}{N_{t} s_{t}} = \frac{1}{x_{t} - x_{s}}$$

$$\overline{\sigma_{t}(x)} = \frac{1}{x_{t} - x_{s}} \Big[\Delta p^{*} - F_{t}(x) P_{e} \Big] \qquad \overline{F_{t}(x)} = \frac{X_{a}^{4}}{2} \Big[Q_{3} Z_{w}(x) + Z_{d}(x) \Big]$$

 $\sigma_t(x)$ is either positive (tubes in tension) or negative (tubes in compression). The extreme values (maximum and minimum) of $\sigma_t(x)$ must be determined to obtain the maximum stresses in the tubes. These extreme values of $\sigma_t(x)$ are obtained for the minimum $F_{t,min}$ and maximum $F_{t,max}$ values of $F_t(x)$:

$$F_{t,min} = MIN[F_t(x)] \qquad F_{t,max} = MAX[F_t(x)]$$

Annex O provides for $1 \le X_a \le 20$ and $-0.8 \le Q_3 \le +0.8$:

- values and graphs of $F_t(x)$ for $0 \le x \le X_a$
- values and graphs of F_{t,min} and F_{t,max}
- locations of the minimum and maximum of $F_t(x)$: x_{min} and x_{max}
- values of $F_t(X_a)$ and F_q

The minimum and maximum of $\sigma_t(x)$ are given by:

$$\sigma_{t,1} = \frac{1}{x_t - x_s} \Big[(x_s P_s - x_t P_t) - P_e F_{t,\min} \Big] \qquad \sigma_{t,2} = \frac{1}{x_t - x_s} \Big[(x_s P_s - x_t P_t) - P_e F_{t,\max} \Big]$$

If tubes are in compression ($\sigma_{t,1}$ or $\sigma_{t,1}$ negative), special consideration must be given to their load carrying ability, which could lead to failure by buckling if a substantial number of tubes were above their buckling limit.

See Annex R which provides the allowable buckling stress limit for tubes.

Note: The general of $\sigma_t(x)$ *can be written, using the general formula of* q(x) *given by* [V.4]*:*

$$q(x) = \frac{X_{a}^{4}}{a_{o}^{2}} \Big[M_{a} Z_{w}(x) + (a_{o} V_{a}) Z_{d}(x) \Big]$$

$$\sigma_{t}(x) = \frac{1}{x_{t} - x_{s}} \left\{ \Delta p^{*} - \frac{X_{a}^{4}}{a_{o}^{2}} \Big[\underbrace{M_{a} Z_{w}(x) + (a_{o} V_{a}) Z_{d}(x)}_{A} \Big] \right\} \qquad M_{a} = (a_{o} V_{a}) Q_{1} + Q_{2}$$

$$[A] = (a_{o} V_{a}) Q_{1} Z_{w}(x) + Q_{2} Z_{w}(x) + (a_{o} V_{a}) Z_{d}(x) = \frac{a_{o}^{2} P_{e}}{2} \Big[Z_{d}(x) + Q_{1} Z_{w}(x) \Big] + Q_{2} Z_{w}(x)$$

$$\sigma_{t}(x) = \frac{1}{x_{t} - x_{s}} \left\{ \Delta p^{*} - X_{a}^{4} \Big[\frac{P_{e}}{2} (Z_{d}(x) + Q_{1} Z_{w}(x)) + \frac{Q_{2}}{a_{o}^{2}} Z_{w}(x) \Big] \right\}$$

This equation can be used whether $P_e \neq 0$ or $P_e = 0$. However, it does not permit to determine the minimum $F_{t,min}$ and maximum $F_{t,max}$ of $F_t(x)$ using the parametric study of X_a and Q_3 to obtain the curves $F_{t,min}$ and $F_{t,max}$ as a function of X_a and Q_3 .

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8.6 Stresses in the Shell

(a) Axial membrane stress

$$[VI.2a-3] a_{s}' V_{s} = a_{o} V_{a} + \frac{a_{o}^{2}}{2} P_{t} + \frac{a_{s}^{2} - a_{o}^{2}}{2} P_{s} = a_{o} V_{a} + \frac{a_{s}^{2} - a_{o}^{2}}{2} (P_{s} - P_{t}) + \frac{a_{s}^{2}}{2} P_{t} \qquad a_{s}' = \frac{D_{s} + t_{s}}{2}$$

• axial force in the shell:

$$V_{\rm s} = \frac{a_o^2 P_{\rm e}}{2 a_{\rm s}^{'}} + \frac{a_o^2}{2 a_{\rm s}^{'}} \left(\rho_{\rm s}^2 - 1\right) \left(P_{\rm s} - P_{\rm t}\right) + \frac{a_s^2}{2 a_{\rm s}^{'}} P_{\rm t} \qquad [VIII.6a]$$

• axial membrane stress in the shell:

$$\sigma_{s,m} = \frac{V_s}{t_s} = \frac{a_o^2}{(D_s + t_s)t_s} \Big[P_e + (\rho_s^2 - 1)(P_s - P_t) \Big] + \frac{a_s^2}{(D_s + t_s)t_s} P_t \Big]$$

 1^{st} term is due to the equivalent pressure P_e acting on the tubesheet 2^{nd} term is due to the pressures P_s and P_t acting on the unperforated ring 3^{rd} term is due to the end load pressure P_t acting on the channel head

Note for the other types of HEs: U tubes HE: $P_e = P_s - P_t$ (see PART 5)

$$\sigma_{s,m} = \frac{a_o^2}{(D_s + t_s)t_s} \rho_s^2 (P_s - P_t) + \frac{a_s^2 P_t}{(D_s + t_s)t_s} = \frac{a_s^2}{(D_s + t_s)t_s} P_s$$

$$\sigma_{s,m} = \frac{D_s^2}{4(D_s + t_s)t_s} P_s$$

Immersed floating head: $P_{e} = P_{s} - P_{t}$ (see PART 4)

$$\sigma_{\rm s,m} = \frac{D_{\rm s}^2}{4\left(D_{\rm s} + t_{\rm s}\right)t_{\rm s}} P_{\rm s}$$

In both cases the classical cylinder formula is obtained. Externally sealed floating head: $P_e = (1 - \rho_s^2)P_s - P_1$ (see PART 4)

$$\sigma_{s,m} = \frac{a_o^2}{(D_s + t_s)t_s} \Big[(1 - \rho_s^2) P_s - P_t + (\rho_s^2 - 1) P_s - (\rho_s^2 - 1) P_t \Big] + \frac{a_s^2}{(D_s + t_s)t_s} P_t = 0$$

There is no axial force acting in the shell of an externally sealed floating head Internally sealed floating head: $P_{\rm e} = (1 - \rho_{\rm s}^2)(P_{\rm s} - P_{\rm t})$ (see PART 4)

$$\sigma_{s,m} = \frac{a_o^2}{\left(D_s + t_s\right)t_s} \left[\left(1 - \rho_s^2\right) \left(P_s - P_t\right) + \left(\rho_s^2 - 1\right) \left(P_s - P_t\right) \right] + \frac{a_s^2}{\left(D_s + t_s\right)t_s} P_t = 0$$

$$\sigma_{s,m} = \frac{D_s^2}{4\left(D_s + t_s\right)t_s} P_t \quad \text{The classical cylinder formula is obtained.}$$

(b) Axial displacement of the shell

For the half-shell of length 1 [IV.2e]: $\delta_{s,Total} = \frac{l V_s}{J E_s t_s} + \delta_s (\theta_s) + \delta_s (v_s) + \delta_J (P_s)$

$$\delta_{\rm s,Total} = \frac{l V_{\rm s}}{J E_{\rm s} t_{\rm s}} + l \alpha_{\rm s,m} \left(T_{\rm s,m} - T_{\rm a}\right) - \frac{l D_{\rm s}^2 P_{\rm s}}{2 E_{\rm s} \left(D_{\rm s} + t_{\rm s}\right) t_{\rm s}} v_{\rm s} + \frac{\pi}{16} \frac{D_{\rm J}^2 - D_{\rm s}^2}{K_{\rm J}} P_{\rm s}$$

For the shell of length L=21: $\Delta_s = 2 \delta_{sT}$

$$\Delta_{\rm s} = \frac{L}{J E_{\rm s} t_{\rm s}} V_{\rm s} + L \alpha_{\rm s,m} \left(T_{\rm s,m} - T_{\rm a} \right) - \frac{L D_{\rm s}^2 P_{\rm s}}{2 E_{\rm s} \left(D_{\rm s} + t_{\rm s} \right) t_{\rm s}} v_{\rm s} + \frac{\pi}{8} \frac{D_{\rm J}^2 - D_{\rm s}^2}{K_{\rm J}} P_{\rm s}$$

 1^{st} term is due to the axial force $V_s \, acting \ in the in the shell$

 2^{nd} term is due to the displacement of the shell subjected to the mean temperature $T_{s,m}$

 3^{rd} term is due to the effect of the Poisson's ratio ν_s of the shell

4th term is due to the pressure Ps acting on the sidewalls of the expansion joint

(c) Bending stress

The bending moment M_s in the shell at its connection with the TS exists only when the shell is integral with the TS (configurations a, b, c). As explained in Annex J, the shell must have a minimum length $l_{s \min} = 1.8 \sqrt{D_s t_s}$ adjacent to the TS.

$$\begin{bmatrix} \text{VI.1a} \end{bmatrix}: \ M_{s} = k_{s} \left(1 + \frac{t_{s}}{2} \right) \theta_{s} + \left(k_{s} \ \beta_{s} \ \delta_{s} \right) P_{s} \quad \text{with } \theta_{s} = \theta_{a} \quad \text{given by [V.5]}:$$

$$\theta_{a} = \frac{a_{o}}{D^{*}} \left[M_{a} \ Z_{m} + \left(a_{o} \ V_{a} \right) Z_{v} \right] = \frac{a_{o}}{D^{*}} \left[\left(a_{o} \ V_{a} \right) \left(Z_{v} + Q_{1} \ Z_{m} \right) + \left(Q_{2} \ Z_{m} \right) \right] = \frac{12 \left(1 - v^{*2} \right)}{E^{*} h^{3}} a_{o} \left[\frac{a_{o}^{2}}{2} \ P_{e} \left(Z_{v} + Q_{1} \ Z_{m} \right) + \left(Q_{2} \ Z_{m} \right) \right] \right]$$

$$\left[\theta_{a} = \frac{6 \left(1 - v^{*2} \right)}{E^{*}} \left(\frac{a_{o}}{h} \right)^{3} \left[P_{e} \left(Z_{v} + Q_{1} \ Z_{m} \right) + \frac{2}{a_{o}^{2}} \left(Q_{2} \ Z_{m} \right) \right] \right] \quad \text{[VIII.6b]}$$

$$M_{s} = k_{s} \left\{ \left(1 + \frac{t_{s}}{2} \right) \frac{6 \left(1 - v^{*2} \right)}{E^{*}} \left(\frac{a_{o}}{h} \right)^{3} \left[P_{e} \left(Z_{v} + Q_{1} \ Z_{m} \right) + \frac{2}{a_{o}^{2}} \left(Q_{2} \ Z_{m} \right) \right] + \left(P_{s} \ \beta_{s} \ \delta_{s} \right) \right\} \quad t_{s} = h \ \beta_{s}$$

Bending stress $\sigma_{s,b}$ is written: $\sigma_{s,b} = 6 \frac{M_s}{t_s^2}$

$$\sigma_{s,b} = \frac{6}{t_s^2} k_s \left\{ \beta_s \,\delta_s \,P_s + \frac{6(1 - \nu^{*2})}{E^*} \left(\frac{a_o}{h}\right)^3 \left(1 + \frac{h \,\beta_s}{2}\right) \left[P_e \left(Z_v + Q_1 \,Z_m\right) + \frac{2}{a_o^2} \left(Q_2 \,Z_m\right) \right] \right\}$$
[VIII.6c']

Note 1: If $P_e \neq 0$ *the equation can be written as a function of* Q_3 *:*

$$P_{\rm e}\left(Z_{\rm v}+Q_{\rm 1}\ Z_{\rm m}\right)+\frac{2}{a_{o}^{2}}\left(Q_{\rm 2}\ Z_{\rm m}\right)=P_{\rm e}\left\{Z_{\rm v}+Z_{\rm m}\left[Q_{\rm 1}+\frac{2}{a_{o}^{2}\ P_{\rm e}}\ Z_{\rm m}\ Q_{\rm 2}\right]\right\}=P_{\rm e}\left(Z_{\rm v}+Q_{\rm 3}\ Z_{\rm m}\right)$$

Note 2: In Section 6.1(a): $\delta_{s} = \frac{a_{s}^{2}}{E_{s}t_{s}} \left(1 - \frac{v_{s}}{2}\right) = \frac{a_{s}^{2}}{E_{s}t_{s}} - \frac{v_{s}}{2}\frac{a_{s}^{2}}{E_{s}t_{s}}$

The equation used by SOLER [9] in his book, page 429, looks more general:

$$\delta_{\rm s}^{AS} = \frac{a_{\rm s}^2}{E_{\rm s} t_{\rm s}} - v_{\rm s} \frac{a_{\rm s}^2}{E_{\rm s} t_{\rm s}} \frac{V_{\rm s}}{P_{\rm s}} = \frac{a_{\rm s}^2}{E_{\rm s} t_{\rm s}} - v_{\rm s} \frac{a_{\rm s}^2}{E_{\rm s} t_{\rm s}} \frac{t_{\rm s}}{P_{\rm s}} \sigma_{\rm s,m}$$

where $V_{\rm s}$ is the axial force acting in the shell, which leads to: $\sigma_{\rm s,m} = \frac{V_{\rm s}}{t}$

In page 447 the term $V_s \frac{a_s}{t_a} \frac{\sigma_{s,m}}{P_a}$ is neglected as $\sigma_{s,m}$ is not yet determined. However that term is

reintroduced in the final equation giving $\sigma_{s,b}$, which is not correct.

If the classical shell formula is used for $\sigma_{s,m}$: $\sigma_{s,m} = \frac{a_s}{2t} P_s$

$$\delta_{s}^{AS} \text{ is written:} \qquad \delta_{s}^{AS} = \frac{a_{s}^{2}}{E_{s} t_{s}} - v_{s} \frac{a_{s}^{2}}{2 E_{s} t_{s}} = \frac{a_{s}^{2}}{E_{s} t_{s}} \left(1 - \frac{v_{s}}{2}\right) = \delta_{s}$$

8.7 Stresses in the Channel

(a) axial membrane stress

Integral channel (configuration a):

Gasketed channel (configurations b, c, d):





General formula:
$$\sigma_{c,m} = \frac{a_c^2}{(D_c + t_c)t_c} P_t$$

(b) Bending stress

The bending moment M_c in the channel at its connection with the TS exists only when the shell is integral with the TS (configuration a). As explained in Annex J, the channel must have a minimum length $l_{c.min} = 1.8 \sqrt{D_c t_c}$ adjacent to the TS. [VI.1b]: $M_{\rm c} = k_{\rm c} \left(1 + \frac{t_{\rm c}}{2} \right) \theta_{\rm c} + \left(k_{\rm c} \beta_{\rm c} \delta_{\rm c} \right) P_{\rm c}$ with: $\theta_{\rm c} = -\theta_{\rm a}$ and $\theta_{\rm a}$ given by [VIII.7b] $M_{\rm c} = k_{\rm c} \left\{ -\left(1 + \frac{t_{\rm c}}{2}\right) \frac{6\left(1 - v^{*2}\right)}{E^*} \left(\frac{a_o}{h}\right)^3 \left[P_{\rm e} \left(Z_{\rm v} + Q_{\rm l} Z_{\rm m}\right) + \frac{2}{a_o^2} \left(Q_2 Z_{\rm m}\right) \right] + \left(P_c \beta_c \delta_c\right) \right\} t_{\rm c} = h \beta_{\rm c}$

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$$\boxed{\sigma_{c,b} = \frac{6}{t_c^2} k_c \left\{ \beta_c \,\delta_c \,P_c - \frac{6(1 - \nu^{*2})}{E^*} \left(\frac{a_o}{h}\right)^3 \left(1 + \frac{h \,\beta_c}{2}\right) \left[P_e \left(Z_v + Q_1 \,Z_m\right) + \frac{2}{a_o^2} \left(Q_2 \,Z_m\right) \right] \right\}}$$

9 DETERMINATION OF THE ALLOWABLE STRESS LIMITS

9.1 General Considerations

The tubesheet rules of Part UHX are intended to generally follow the stress classification of Appendix 4.1 of Section VIII, Division 2 [11] for primary and primary plus secondary stresses. The scope of Part UHX does not include any consideration of peak stresses or any requirements for fatigue.

(a) Design (pressure) loading cases

The bending stress resulting from a pressure loading in a flat plate is a primary bending stress. Any yielding of the plate material results in a permanent deformation, and the deformation may continue to occur until the plate fails (or the deformation is so large that the plate cannot perform its intended function). When the tubesheet is extended as a flange, the loading generated by the bolting moment is considered primary. Thus, primary bending stress limits are appropriate when considering the tubesheet bending stress resulting from pressure loading acting alone or in combination with the bolt loading when applicable, i.e. the so-called "design pressure loading cases".

The stresses in the shell and channel are somewhat more complex to categorize. The axial membrane stresses in the shell and channel remote from the tubesheet resulting from pressure loadings are primary.

The bending stresses at the shell-to-tubesheet junction and the channel-to-tubesheet junction result from restrained differential motion at these junctions. As such, these bending stresses have the basic characteristic of a secondary stress. However, a very important distinction has to be made regarding the status of these stresses, based on footnote 2 of Table 4-120.1 of Section VIII Division 2 [11] . If the discontinuity bending moment at the edge of a flat plate is required to maintain the bending stress elsewhere in the plate to within its allowable stress the shell/channel bending stress is classified as primary bending and should be limited to the primary bending stress limit.

Accordingly, when an elastic stress analysis includes the rotational stiffness of the shell and channel in determining the tubesheet stress under primary loading, the discontinuity bending stress should be categorized as primary bending stress and be limited accordingly.

However, there may be instances where the design needs not consider the full strengthening effect of the shell/channel. For example, if one choses to not include the stiffening effect of the shell and channel for the tubesheet analysis, the shell and channel bending stresses could be correctly categorized as secondary and be limited according to the secondary stress considerations.

Or, if the shell/channel bending stresses resulting from pressure/bolting loads do not satisfy the primary bending stress limits, then full credit cannot be taken for the stiffness of that component. In such a case, it is deemed appropriate to apply a "knockdown" factor to the stiffness of the shell or channel component by reducing its modulus of elasticity. The "knockdown" factor used in the UHX "Elastic-Plastic analysis" is based on evaluations of the extent of strengthening offered by the fully plastic moment at the shell and channel junction when the primary bending stress limits are not satisfied.

(b) Operating (pressure + thermal) loading cases

The stresses resulting from the temperature difference between shell and tubes (operating pressure + thermal loading cases) are secondary in that they are self-limiting. The code limits on secondary stress are derived to accomplish "shakedown" to elastic action". UHX rules consider the tubesheet, shell, channel, and tube stresses to be secondary stresses under the action of thermal loads.

Note: It has been a long, standing practice of TEMA to divide the loads resulting from thermal expansion by a factor of two, including the tube loads. Thus, the TEMA allowable tube tensile stress

for thermal loading cases is effectively increased by a factor of two. This practice has not led to any noted problems or deficiencies in allowed tube loads, and this practice is continued for the Part UHX rules for allowable tensile stress when considering thermal load conditions.

9.2 Allowable Stress Limit in the Tubesheet

- The bending stress σ due to pressure loading (design loading cases) is a primary bending stress (P_b) to be limited to 1.5S.
- The bending stress due to pressure + thermal loading (operating loading cases) is a secondary bending stress (Q) to be limited to S_{PS}.
- The shear stress is limited to 0.8S, as this is the practice in most codes (TEMA, CODAP, EN)

9.3 Allowable Stress Limit in the Tubes

- The axial membrane stress σ_t due to pressure loading (design loading cases) is a primary membrane stress (P_m) to be limited to S_t.
- The axial membrane stress due to pressure + thermal loading (operating loading cases) is strain induced and therefore a secondary stress (Q) to be limited to S_{PS,t} However, this limit has been lowered to 2S_t for two reasons:
 - 1) If the stress level in the tubes is too high, it may happen that a substantial number of tubes fail and that the tube bundle could not sustain the required loading, especially for tubes in compression.
 - 2) Consistency must be ensured with Appendix A, which limits the maximum axial load in the tube-to-tubesheet joint to 2S_t.

If the tubes are under compression, tube buckling may restrict the tubes load carrying ability. This is true for either pressure or thermal load conditions. Accordingly, no distinction is made between primary and secondary allowable compressive loads in the tubes.

The tube axial stresses are limited to the maximum buckling stress limit S_{t,b}, determined in Annex R.

9.4 Allowable Membrane Stress Limit in the Shell

- The axial membrane stress $\sigma_{s,m}$ due to pressure loading (design loading cases) is a primary membrane stress (P_m) to be limited to S_s.
- The axial membrane stress due to pressure + thermal loading (operating loading cases) is strain induced and therefore is a secondary stress (Q) to be limited to S_{PS,s}.

If the shell is under compression, the axial stress must be limited to the maximum buckling stress limit $S_{s,b}$ determined by applying UG-23(b).

9.5 Allowable Membrane + Bending Stress Limit in the Shell

- The membrane + bending stress σ_s of the shell at its junction to the tubesheet due to pressure loading (design loading cases) is a primary bending stress (P_b) to be limited to $1.5S_s$. If this limit is exceeded, an elastic-plastic analysis may be performed as mentioned in Section 9.1a.
- The membrane + bending stress due to pressure + thermal loading (operating loading cases) is a secondary stress (Q) to be limited to S_{PS,s}.

9.6 Allowable Membrane + Bending Stress Limit in the Channel

The same rules as for the shell above apply.



9.7 Conclusions

The analytical method is based on the concept of the TS replaced by an equivalent solid plate resting on an elastic foundation to which the classical discontinuity analysis is applied.

The analytical development enables to determine:

• at any radius r of the TS, the deflection w(x), rotation $\theta(x)$, net effective pressure q(x), bending

stress $\sigma(x)$, shear stress $\tau(x)$ and axial stress in the tubes $\sigma_t(x)$, where $x = r \sqrt[4]{k_w / D^*}$. Their

maximum has been obtained thanks to a parametric study performed on the two dimensionless coefficients X_a and Q_3 of the HE.

- in the shell, the axial force and resulting axial displacement and membrane stress, and the bending stress at its connection with TS and channel.
- in the channel, the membrane stress, and the bending stress at its connection with TS and shell.

The results obtained confirm the correctness of the design rules given in UHX-13.5.

Specific rules complete the UHX-13.5 design rules to cover:

- The effect of different shell material or thickness adjacent to the tubesheet (UHX-13.6)
- The effect of plasticity at tubesheet-shell-channel joint (UHX-13.7)
- The effect of radial thermal expansion adjacent to the tubesheet (UHX-13.8)
- The calculation procedure for simply supported tubesheet (UHX-13.9)
- The design of tubesheet flange extension (UHX-9)

10 ADDITIONAL RULES

10.1 Effect of Different Shell Thickness and Material Adjacent to the TS (UHX-13.6)

(a) General: When the shell is integral with the TS (configurations a, b or c), if the stresses are above their allowable stress limits (i.e. $\sigma_s > 1.5S_s$ for pressure loadings or $\sigma_s > S_{PS,s}$ for thermal loadings), the calculated stresses can be reduced by thickening the shell adjacent to the TSs. As explained in Annex

J, the shell must have a minimum length adjacent to the TS of $l_{s,min} = 1.8 \sqrt{D_s t_s}$.

Accordingly, the shell must be thickened over a minimum length $l_{s,\min,1} = 1.8 \sqrt{D_{s,1} t_{s,1}}$ adjacent to

the TSs. The thickened TS sections should have the same thickness $t_{s,1}$ and the same material to comply with the assumption that the HE is a symmetrical unit (see Section 3.4a). However the thickened lengths l_1 and l'_1 adjacent to each TS can be different. The material can be different over these lengths with modulus $E_{s,1}$ and thermal coefficient $\alpha_{s,m,1}$.

Additional notations are as follows, using subscript 1 for the quantities linked to the shell thickening. See Figure 26

 $E_{s,1}$ = modulus of elasticity for shell material adjacent to tubesheets at T_s

 $l_{l}, l_{l}' =$ lengths of shell of thickness $t_{s,1}$ adjacent to tubesheets $(l_{1} \ge l_{s,\min,1} \quad l_{1}' \ge l_{s,\min,1})$

- $D_{s,1}$ = internal shell diameter adjacent to tubesheets
- $t_{s,1}$ = shell thickness adjacent to tubesheets

 $S_{s,1}$ = allowable stress for shell material adjacent to tubesheets at T_s

 $S_{y,s,1}$ = yield strength for shell material adjacent to tubesheets at T_s .

 $S_{PS,s,1}$ = allowable primary plus secondary stress for shell material at T_s

 $\alpha_{s,m,1}$ = mean coefficient of thermal expansion of shell material adjacent to tubesheets at $T_{s,m}$





(b) Shell axial stiffness K_s* is determined from:

$$\frac{1}{K_s^*} = \frac{1}{K_{s,0}} + \frac{1}{K_{s,1}} + \frac{1}{K_{s,1}}$$
 where:

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$$\begin{split} K_{s,o} &= \frac{\pi t_s \left(D_s + t_s \right) E_s}{L_o} \text{ is the axial stiffness of the basic shell section of length } L_o = L - l_1 - l_1' \\ K_{s,1} &= \frac{\pi t_{s,1} \left(D_{s,1} + t_{s,1} \right) E_{s,1}}{l_1} \text{ is the axial stiffness of the shell section of length } l_1 \\ K_{s,1}' &= \frac{\pi t_{s,1} \left(D_{s,1} + t_{s,1} \right) E_{s,1}}{l_1'} \text{ is the axial stiffness of the shell section of length } l_1' \\ \frac{1}{K_s^*} &= \frac{L_o}{\pi t_s \left(D_s + t_s \right) E_s} + \frac{l_1 + l_1'}{\pi t_{s,1} \left(D_{s,1} + t_{s,1} \right) E_{s,1}} \\ K_s^* &= \frac{\pi t_s \left(D_s + t_s \right) E_s}{L_o + t_s \left(D_s + t_s \right) E_s} + \frac{l_1 + l_1'}{t_s \left(D_s + t_s \right) E_s} + \frac{l_1 + l_1'}{t_s \left(D_s + t_s \right) E_s} \end{split}$$

 $D_{s,1} + t_{s,1}$ can be assimilated to $D_s + t_s$ for the following reasons:

- $D_{s,1}=D_s$ in most cases
- $t_{s,1}$ is small compared to $D_{s,1}$ and can be replaced by t_s
- calculation performed on Example E4.18.7 (see Annex V), using $t_{s,1}=1.0$ (instead of 0.625) and $l_1=l_1$ '=15.0 (instead of 0.0), shows that K_s * decreases from 8.866 to 8.860 (0.1 % difference) when $D_{s,1}+t_{s,1}$, is replaced by D_s+t_s . There is no effect on TS stress σ and shell bending stress $\sigma_{s,b}$.

Accordingly the formula is written:

$$K_{s}^{*} = \frac{\pi(D_{s} + t_{s})}{\frac{L - (l_{1} + l_{1})}{t_{s} E_{s}} + \frac{l_{1} + l_{1}}{t_{s,1} E_{s,1}}}$$

(c) Axial differential thermal expansion between tubes and shell is written:

$$\gamma^{*} = \alpha_{t,m}(T_{t,m} - T_{a})L - \left\lfloor \alpha_{s,m}(T_{s,m} - T_{a})L_{o} + \alpha_{s,m,1}(T_{s,m} - T_{a})(l_{1} + l_{1}) \right\rfloor$$
$$\gamma^{*} = (T_{t,m} - T_{a})\alpha_{t,m}L - (T_{s,m} - T_{a})\left[\alpha_{s,m}(L - l_{1} - l_{1}) + \alpha_{s,m,1}(l_{1} + l_{1})\right]$$

(d) Design procedure is affected as follows:

Quantities concerned by the shell thickening are those which are involved by the TS-shell connection and which involve E_s , t_s , K_s , or γ . Accordingly:

- K_s must be replaced by K_s^* , which affects $K_{s,t}$ and J.
- β_s , k_s , δ_s and $\sigma_{s,m}$, $\sigma_{s,b}$ are calculated replacing t_s with $t_{s,1}$ and E_s with $E_{s,1}$.
- must be replaced by γ^* , which affects P_{γ}

If the material of the thickened shell section is different from the material of the current shell section, use the allowable shell stress limits $S_{s,1}$ and $S_{PS,s,1}$.

10.2 Effect of Plasticity at Tubesheet-Shell-Channel Joint (UHX-13.7)

(1) General: If the bending stress in the shell at its junction to the tubesheet exceeds $1.5S_s$ (i.e. yield) for the pressure loadings only, then some type of plastic hinge is developed. The elastic analysis accounts for the edge moment, based only on discontinuity considerations. It does not recognize that the fully plastic moment can never be physically exceeded. Thus the amount of edge restraint that reduces the stress in the center of the tubesheet will be overestimated as the plastic hinge increases.

Taking that into consideration through an elastic-plastic analysis, Note (2) of Table 4-120.1 of ASME Section VIII Div.2 classifies the shell bending stress as a secondary stress (Q) to be limited to $S_{PS,s}$. Thus the membrane + bending stress σ_s of the shell at its junction to the tubesheet due to pressure loading (design loading cases) can go up to $S_{PS,s}$, provided that an elastic-plastic analysis is performed.

When the shell/channel bending stresses resulting from pressure loads do not satisfy the primary stress limit, then full credit cannot be taken for the stiffness of that component. In such a case, it is deemed appropriate to apply a "knockdown" factor to the stiffness of the shell or channel component by reducing its modulus of elasticity. This will increase the rotation of the joint up to a value obtained when there is no more support from the shell and channel loading, to the extreme case of a simply supported TS which is covered in Section 10.4. The "knockdown" factor is based on evaluations of the extent of strengthening offered by the fully plastic moment at the shell and channel junction.

In 1985, Soler [13] proposed to use a reduced modulus of elasticity E_s^* for the shell (and E_c^* for the channel) based on the degree of overstress in the shell: $E_s^* = E_s (S_s^* / \sigma_{s,b})$ where S_s^* is the allowable stress limit. This formula has been modified for use in U-tube TS HEs (UHX-12); $E_s^* = E_s (S_s^* / \sigma_{s,b})$ The same formula applies for the channel using subscript c_s

 $E_s^* = E_s \sqrt{S_s^* / \sigma_{s,b}}$. The same formula applies for the channel, using subscript c.

Series of elastic-plastic finite element calculations have been performed in 1990 by Soler [14] on a short cylindrical shell to improve this straightforward formula for fixed TS HEs. The results enable to determine a reduced modulus of elasticity E_s^* which gives the same rotation obtained by the elastic solution at the TS-shell-channel joint:

$$E_s^* = E_s \left[1 - f(\sigma_{s,b} / S_s^*) \right] \text{ where function } f(\sigma_{s,b} / S_s^*) = 0 \text{ when } \sigma_{s,b} = S_s^*$$

and increases when $\sigma_{s,b} \ge S_s^*$.

(2) **UHX formula:** The above formula has been refined later, using additional elastic-plastic calculations as follows.

For the shell:
$$E_s^* = E_s \left(1.4 - 0.4 \frac{|\sigma_{s,b}|}{S_s^*} \right)$$
 For the channel: $E_c^* = E_c \left(1.4 - 0.4 \frac{|\sigma_{c,b}|}{S_c^*} \right)$

The reduced effective modulus has the effect of reducing the shell and/or channel stresses obtained from the elastic calculation. However, due to load shifting, this usually leads to an increase of the tubesheet stress. Accordingly, this simplified elastic-plastic procedure can only be performed when the TS stress obtained in the elastic calculation is below the allowable stress limit 1.5S for pressure loading.

The maximum allowable bending stress limit in the shell and channel for design loading cases 1, 2, 3 and 4 is defined as:

$$S_{s}^{*} = \mathrm{MIN}\left[\left(S_{y,s}\right), \left(\frac{S_{PS,s}}{2}\right)\right] \qquad \qquad S_{c}^{*} = \mathrm{MIN}\left[\left(S_{y,c}\right), \left(\frac{S_{PS,c}}{2}\right)\right]$$

Above this value, which is more or less the yield stress, the shell and channel start to yield.

(3) **UHX procedure** is given hereafter for the shell. It applies to the channel, using subscript c instead of s.

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(1) The procedure applies when $\left|\sigma_{s,b}\right| > S_s^* \Longrightarrow E_s^* < E_s$

Cases where $|\sigma_{s,b}| = S_s^* \Rightarrow E_s^* = E_s$ or when $|\sigma_{s,b}| < S_s^* \Rightarrow E_s^* > E_s$ are covered by the normal elastic procedure and are out of the scope of the elastic-plastic procedure. Therefore E_s^* formula can be written:

$$\boxed{E_s^* = E_s \ fact_s} \quad \text{where} \quad \left[fact_s = MIN \left[\left(1.4 - 0.4 \frac{\left| \sigma_{s,b} \right|}{S_s^*} \right), (1) \right] \right]$$

Facts varies from 1 when $|\sigma_{s,b}| = S_s^*$ to 0.6 when $|\sigma_{s,b}| = S_{PS,s} = 2S_s^*$.

- (2) Quantities affected by the elastic-plastic procedure are those which are involved in the TS-shellchannel joint and which involve Es. Accordingly:
 - k_s affects λ_s , which leads to new values for F, Φ , Q_1 , Q_{z1} , Q_{z2} , U; P_W , P_{rim} , P_e ; Q_2 , Q_3 , F_m and finally the tubesheet bending stress σ .
 - δ_s is not affected because it is used only in $\omega_s = \rho_s \beta_s k_s \delta_s (1 + h\beta_s)$ in which E_s is cancelled out by the product $k_s \delta_s$.

If $|\sigma| \le 1.5S$, the design is acceptable. Otherwise, the HE geometry must be reconsidered.

For Example 3 given in Annex V, the results for the elastic calculation are as follows for controlling loading case 2:

Stiffening coefficient Φ =9.0, σ =23084<1.5S=23700, σ_s =30035>1.5Ss=23700, but lower than S_{PS,s}=47400

Similar results for the channel.

For the elastic-plastic calculation: the stiffening coefficient Φ decreases to 7.6, and σ =22205<1.5S=23700.

In this example the elastic-plastic calculation leads to a decrease of the TS stress. In most cases it is the opposite, where the elastic-plastic calculation increases the TS stress.

10.3 Effect of Radial Thermal Expansion Adjacent to the Tubesheet (UHX-13.8)

When there is a significant temperature gradient at the TS-shell-channel joint, it may be necessary to account for the resulting radial differential thermal expansion at the joint. This occurs when the TS is integral with the shell (configurations a, b, c) or the channel (configuration a). Additional notations are as follows (see Figure 27):

Additional notations are as follows (see Figure 2 π'

T' = tubesheet metal temperature at the rim

$$T_c$$
 = channel metal temperature at the tubesheet

 $T_{s}^{'}$ = shell metal temperature at the tubesheet

 α' = mean coefficient of thermal expansion of tubesheet material at T'

 α'_{c} = mean coefficient of thermal expansion of channel material at T'_{c}

 α'_{s} = mean coefficient of thermal expansion of shell material at T'_{s}

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Figure 27 — Temperature Gradient at TS-Shell-Channel Joint

Average temperature of the unperforated rim T_r : $T_R = \frac{T' + T_s' + T_c'}{3}$

Average temperature of the shell T_s^* and channel T_c^* at their junction to the tubesheet:

$$T_{s}^{*} = \frac{T_{s}^{'} + T_{R}}{2}$$
 $T_{c}^{*} = \frac{T_{c}^{'} + T_{R}}{2}$

Radial strain of the shell due to T_s^* and T_R : $\varepsilon_s = \alpha'_s \left(T_s^* - T_a\right) - \alpha' \left(T_r - T_a\right)$

The radial displacement of the shell is given by: $w_s(P_s^*) = a_s \varepsilon_s$ which can be written in the same way as $w_s(P_s)$ given by [A-VI.1a-1]:

$$w_{s}(P_{s}) = \frac{a_{s}^{2}}{\underbrace{E_{s} t_{s}}\left(1 - \frac{v_{s}}{2}\right)}P_{s} = \delta_{s}P_{s} \text{ with } \delta_{s} = \frac{a_{s}^{2}}{E_{s} t_{s}}\left(1 - \frac{v_{s}}{2}\right)$$

$$w_{s}(P_{s}^{*}) = \frac{a_{s}^{2}}{E_{s} t_{s}}\underbrace{E_{s} t_{s}}{a_{s}}\underbrace{E_{s} t_{s}}{e_{s}} = \delta_{s,o} P_{s}^{*} \text{ with } \underbrace{\delta_{s,o} = \frac{a_{s}^{2}}{E_{s} t_{s}}}_{\delta_{s,o} = \frac{\sigma_{s}^{2}}{P_{s}^{*}}} \text{ and } P_{s}^{*} = \frac{E_{s} t_{s}}{a_{s}}\varepsilon_{s}$$

The total radial displacement of the shell w_s given in VI.1a accounting for $w_s(P_s^*)$ becomes:

$$w_{s} = \frac{Q_{s}}{\beta_{s}^{2} k_{s}} + \frac{M_{s}}{k_{s} \beta_{s}} + w_{s}(P_{s}) + w_{s}(P_{s}^{*}) = \frac{Q_{s}}{\beta_{s}^{2} k_{s}} + \frac{M_{s}}{k_{s} \beta_{s}} + \delta_{s}P_{s} + \delta_{s,o}P_{s}^{*}$$

with:
$$\boxed{P_{s}^{*} = \frac{E_{s} t_{s}}{a_{s}} \left[a_{s}^{'} \left(T_{s}^{*} - T_{a}\right) - \alpha^{'} \left(T_{r} - T_{a}\right)\right]}$$

Thus $\delta_{s,o}P_s^*$ must be added to δ_sP_s in each equation where δ_sP_s appears. The total radial displacement of the channel w_c is written in the same way:

$$w_{c} = \frac{Q_{c}}{\beta_{c}^{2} k_{c}} + \frac{M_{c}}{k_{c} \beta_{c}} + \delta_{c} P_{c} + \delta_{c,o} P_{c}^{*}$$

with:
$$P_{c}^{*} = \frac{E_{c} t_{c}}{a_{c}} \left[a_{c}^{\prime} \left(T_{c}^{*} - T_{a} \right) - \alpha^{\prime} \left(T_{r} - T_{a} \right) \right]$$

Thus $\delta_{c,o}P_c^*$ must be added to $\delta_c P_c$ in each equation where $\delta_c P_c$ appears. Equation [VI.2b'] giving the moment of the ring becomes:

$$R M_{\rm R} = -a_o M_{\rm a} + a_o^2 V_{\rm a} \left(\rho_{\rm s} - 1\right) + P_{\rm s} \frac{a_o^3}{4} \left[\left(\rho_{\rm s}^2 - 1\right) \left(\rho_{\rm s} - 1\right) \right] - P_{\rm c} \frac{a_o^3}{4} \left[\left(\rho_{\rm c} - 1\right) \left(\rho_{\rm c}^2 + 1\right) - 2 \left(\rho_{\rm s} - 1\right) \right] - \left[a_s' k_{\rm s} \left(1 + t_{\rm s}' + \frac{t_{\rm s}'^2}{2} \right) + a_c' k_{\rm c} \left(1 + t_{\rm c}' + \frac{t_{\rm c}'^2}{2} \right) \right] \theta_{\rm a} + a_o \left[\omega_{\rm c} P_{\rm c} - \omega_{\rm s} P_{\rm s} \right] + a_o \left[\omega_{\rm c,o} P_{\rm c}^* - \omega_{\rm s,o} P_{\rm s}^* \right]$$

where:
$$\overline{\omega_{s,o} = \rho_s \beta_s k_s \delta_{s,o} \left(1 + h \beta_s \right)} \qquad \overline{\omega_{c,o} = \rho_c \beta_c k_c \delta_{c,o} \left(1 + h \beta_c \right)}$$

Equation giving P_{rim} in VII.2 is written:

$$P_{\rm rim} = -\frac{U}{a_o^2} \left[\frac{a_o^2}{4} \left[\left(\rho_{\rm s}^2 - 1 \right) \left(\rho_{\rm s} - 1 \right) P_{\rm s} - \left(\left(\rho_{\rm c}^2 + 1 \right) \left(\rho_{\rm c} - 1 \right) - \left(\rho_{\rm s} - 1 \right) \right) P_{\rm c} \right] \right] + \frac{U}{a_o^2} \left[\omega_{\rm s} P_{\rm s} - \omega_{\rm c} P_{\rm c} \right]$$

A term P_{ω} must be added to P_{rim} : $P_{\omega} = \frac{U}{a_o^2} \left(\omega_{s,o} P_s^* - \omega_{c,o} P_c^* \right)$

Accordingly the equivalent pressure Pe becomes:

$$P_{e} = \frac{J K_{s,t}}{1 + J K_{s,t} F_{q}^{'}} \left[P_{s}^{'} - P_{t}^{'} + P_{\gamma} + P_{w} + P_{rim} + P_{\omega} \right]$$

$$Q_{2} = \frac{\left(\omega_{s}^{*} P_{s} - \omega_{c}^{*} P_{c} \right) - \left(\omega_{s,o} P_{s}^{*} - \omega_{c,o} P_{c}^{*} \right) + \left(W^{*} \frac{\gamma_{b}}{2\pi} \right)}{1 + \Phi Z}$$

Coefficient Q₂ becomes:

$$\sigma_{s,b}$$
 and $\sigma_{c,b}$ become:

$$\sigma_{s,b} = \frac{6}{t_s^2} k_s \left\{ \beta_s (\delta_s P_s + \delta_{s,o} P_s^*) + \frac{6(1 - v^{*2})}{E^*} \left(\frac{a_o}{h}\right)^3 \left(1 + \frac{h\beta_s}{2}\right) \left[P_e \left(Z_v + Z_m Q_1\right) + \frac{2}{a_o^2} Q_2 Z_m \right] \right\}$$



$$\sigma_{c,b} = \frac{6}{t_c^2} k_c \left\{ \beta_c (\delta_c P_c + \delta_{s,o} P_c^*) - \frac{6(1 - v^{*2})}{E^*} \left(\frac{a_o}{h}\right)^3 \left(1 + \frac{h\beta_c}{2}\right) \left[P_e \left(Z_v + Z_m Q_1\right) + \frac{2}{a_o^2} Q_2 Z_m \right] \right\}$$

10.4 Calculation Procedure for Simply Supported Tubesheets (UHX-13.9)

(*a*) **General:** There are instances where the design does not need to consider the full strengthening effect of the shell/channel when they are integral with the TS. A Code Case was issued in 2005 to answer the concern of a HE manufacturer for externally sealed floating head HEs designed by TEMA. Use of UHX rules would need to increase significantly both the thicknesses of the floating TS and the attached channel, which is not always possible due to diameter and layout constrains. The Code Case proposed to consider the TS as simply supported.

If one chooses to not include the stiffening effect of the shell and channel for the tubesheet analysis when they are overstressed, this assumes that a plastic hinge has formed at the TS-shell-channel junction. Accordingly, the shell and channel bending stresses can be categorized as secondary and be limited according to the secondary stress limitations.

These considerations are a limit case of the elastic-plastic considerations of X-2, using for the shell and channel moduli of elasticity reduced to zero. Accordingly, the design procedure is similar and must be conducted in two phases.

(b) **Phase 1:** Perform the normal calculation of the TS stress σ and its limitations. Calculate the shell and channel stresses which are considered as secondary and must be limited respectively to S_{PS,s} and S_{PS,c}. The shell and channel minimum length limitations l_{s,min} and l_{s,min} no more apply as the stiffening effect due to the bending rigidity of these components will not be considered in Phase 2. For Example E4.18.8 given in Annex V, the results are as follows for controlling loading case 2: Stiffening coefficient Φ =9.0, σ =23084<1.5S=23700, σ s =30035>1.5S_s=23700, but lower than

 $S_{PS,s} = 47400.$

Similar results for the channel.

(c) **Phase 2:** Ignore the bending rigidity of the shell i.e., from [VI.1a]: $k_s=0$ and $\delta_s=0$, which leads to $M_s=0$. Same for the channel; $k_c=0$ and $\delta_c=0$. Then calculate the TS stress σ for pressure loading (design loading cases) and check that $\sigma \leq 1.5S$.

For Example 3, the stiffening coefficient Φ decreases to 0.14, and σ =15170<1.5S=23700 σ_s =1561<1.5S_s=23700

As already mentioned above, the same result would be obtained by imposing $E_s = 0$ and $E_c = 0$ in the elastic-plastic method.

In this example the simply supported calculation leads to a decrease of the TS stress. In most cases it is the opposite, where the simply supported calculation increases the TS stress.

10.5 Tubesheet Effective Bolt Load (UHX-8)

Tubesheets with flanged extensions are affected by the bolt loads in two ways:

- externally (at the flanged end). The design of the tubesheet extension is covered in 10.6

- internally (in the perforated region). The application of the bolt load for a fixed tubesheet exchanger is more complicated than for a regular flange.

The technical basis for the effect of that load on the perforated region is investigated in detail in Parts 3, 4 and 5.

æ)

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For some loading cases, the bolt load is a secondary, self-limiting load. As an example, when considering Loading Case 2 (shell side pressure only) for a configuration b geometry, the bolt load only produces a secondary effect in that it does not directly resist pressure, and if the tubesheet-flange deflects sufficiently, then the bolt load may be totally relieved.

Likewise, for those loading cases where the bolts directly resist the pressure load, the bolt loads used for the design should be consistent with Appendix 2, which requires that the gasket seating bolt load be used to check the flange at ambient temperature for protection against damage due to over bolting. The "primary" bolt load, W_{ml} is used in Appendix 2 for designing the flange under operating conditions. In order to be consistent with Appendix 2, W_{ml} must be used as the bolt load for loading cases where primary stresses are determined i.e. Pressure Design Loading Cases.

Pressure and Thermal Operating Loading Cases require consideration of secondary stresses for which it is appropriate to include the design bolt load from Appendix 2. A new term "tubesheet effective bolt load W*", is introduced in 2013 Edition and selected from a new table UHX-8.1 where the designer can select the appropriate bolt load to be used for the respective Configuration and Loading Case combinations that accounts for the above considerations. So, part UHX is consistent with the philosophy of Appendix 2 bolt loads.

Table UHX-8.1

	Loading Case				
	1	2	3	4-7	
Configuration					
a	0	0	0	0	
b	W _{m1c}	0	W _{m1c}	Wc	
с	W _{m1c}	0	W _{m1c}	Wc	
d	W _{m1c}	W _{m1s}	W _{m1max}	W _{max}	
e	0	W _{m1s}	W _{m1s}	Ws	
f	0	W _{m1s}	W _{m1s}	Ws	
Α	0	0	0	0	
В	W _{m1c}	0	W _{m1c}	Wc	
C	W _{m1c}	0	W _{m1c}	Wc	
D	0	0	0	0	

TUBESHEET EFFECTIVE BOLT LOAD, W*

Where:

 W_c = channel flange design bolt load for the gasket seating condition

 W_s = shell flange design bolt load for the gasket seating condition

 $W_{max} = MAX [(W_c), (W_s)]$

 W_{mlc} = channel flange design bolt load for the operating condition

 W_{m1s} = shell flange design bolt load for the operating condition

 $W_{m1max} = MAX [(W_{m1c}), (W_{m1s})]$

 W^* = tubesheet effective bolt load selected from Table UHX-8.1 for the respective Configuration and Loading Case

10.6 Tubesheet Flange Extension (UHX-9)

(*a*) **General:** UHX-9 provides the rules to determine the required thickness of flanged TS extensions when bolt loads are transmitted to them (configurations b, d, e and B) and to flanged and unflanged

TS extensions when no bolt loads are applied to them (configurations c, d, f and C). The rules in UHX-9 are not applicable to Configurations a, A and D.

The required thickness of the flanged TS extension calculated in accordance with UHX-9 may differ from that required for the interior of the TS calculated in accordance with UHX-12, UHX-13, or UHX-14.

Figure 28 depicts thickness h_r for some representative configurations.



Figure 28 — Tubesheet Flanged Extension

(b) Flanged TS extension with bolt loads applied:

The moment acting on the TS flanged extension is written:

$$M = W h_G = W \frac{C - G}{2}$$

The bending stress in the TS flanged extension is written:

$$\sigma = \frac{6M}{\pi G h_r^2} = \frac{6W h_G}{\pi G h_r^2} = \frac{1.91W h_G}{G h_r^2} \quad \text{and is limited to the allowable TS stress.}$$

For the gasket seating condition, the minimum required thickness of the tubesheet flanged extension, h_r , is obtained for $\sigma = S_a$:

$$h_r = \sqrt{\frac{1.9W h_G}{S_a G}}$$

A similar formula must be applied for the operating condition (W=Wm1, S). Accordingly:

$$h_r = \text{MAX}\left[\sqrt{\frac{1.9Wh_G}{S_aG}}, \sqrt{\frac{1.9W_{m1}h_G}{SG}}\right]$$

For design conditions, $W = W_{m1}$ from equation (1) of Appendix 2, Section 2-5(c)(1) [10]. For gasket seating condition, W = W from equation (5) of Appendix 2, Section 2-5(e) [10]. S = allowable stress for the material of the tubesheet extension at design temperature $S_a =$ allowable stress for the material of the tubesheet extension at ambient temperature



(c) Flanged and Unflanged TS extension with no bolt load applied:

For flanged Configuration d and for unflanged Configurations D and C having no bolt loads applied to the extension, the shear load due to effect of pressure at the gasket ID must be considered. The derivation of the shear stress equation is shown in Section 8.4(b). The minimum required thickness of the extension, h_r , shall be the maximum of the values determined for each design loading case as follows:

$$h_r = \left(\frac{D_E}{3.2S}\right) \left| P_s - P_t \right|$$

 D_E = maximum of the shell and channel gasket inside diameters, but not less than the maximum of the shell and channel flange inside diameters

(d) Unflanged TS extension with bolt loads applied:

The calculation procedure for unflanged Configurations c and f are not provided in UHX-9, but the minimum required thickness of the extension, h_r , shall be calculated in accordance with ASME Section VIII, Division 1,Mandatory Appendix 2, 2-8(c) for loose type flanges with laps.



Figure 29 — Minimum Required Thickness of the Tubesheet Flanged Extension

10.7 HE Set-up with a Thin-Walled Expansion Joint (UHX-13.16)

The joint must comply with the rules of Appendix 26.

10.8 HE Set-up with a Thick-Walled Expansion Joint (UHX-13.17)

The expansion joint must comply with the rules of Appendix 5 which provides allowable stress limits. The design of the joint may be performed using higher allowable stress limits, defined in Table UHX-17, which allow the expansion joint to yield and decrease its stiffness. Accordingly, the HE must be redesigned for design (pressure) loading cases using a zero expansion joint stiffness (K_J =0). The equivalent pressure P_e must be recalculated using K_J =0. Using Note 1 of Section 7.2:

$$P_{s1} = P_{s} \left[\left(JK_{s,t} \right) x_{s} + (JK_{s,t}) 2 \left(1 - x_{s} \right) v_{t} + (J) 2 \left(\frac{D_{s}}{D_{o}} \right)^{2} v_{s} - (\rho_{s}^{2} - 1) - \frac{\left(1 - J \right)}{2} \frac{\left[D_{J}^{2} - D_{s}^{2} \right]}{D_{o}^{2}} \right]$$

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$$P_{t1}' = P_{t} \left[\left(JK_{s,t} \right) x_{t} + \left(JK_{s,t} \right) 2 \left(1 - x_{t} \right) v_{t} + 1 \right]$$
Using J=0:

$$P_{s}' = P_{s} \left[1 - \rho_{s}^{2} - \frac{1}{2} \left[\frac{D_{J}^{2}}{D_{o}^{2}} - \rho_{s}^{2} \right] \right] = P_{s} \left[1 - \frac{1}{2} \left[\frac{D_{J}^{2}}{D_{o}^{2}} + \rho_{s}^{2} \right] \right]$$

$$P_{t1}' = P_{t} \left[\left(JK_{s,t} \right) x_{t} + \left(JK_{s,t} \right) 2 \left(1 - x_{t} \right) v_{t} + 1 \right] = P_{t}$$

$$P_{e} = \frac{1}{1 + J K_{s,t} F_{q}'} \left[P_{s1}' - P_{t1}' \right] = \left[P_{s} \left[1 - \frac{1}{2} \left[\rho_{s}^{2} - \frac{D_{J}^{2}}{D_{o}^{2}} \right] \right] - P_{t} \right]$$



11 HOW TO USE THE RULES

The calculation procedure can be summarized as follows:

- Set the data listed in Section 3.2a
- Calculate the design coefficients listed in Section 3.2b ($x_s, x_t; \rho_s, \rho_c; K_{s,t}; J$)
- Calculate first characteristic parameter X_a and coefficients Q_1, Q_2 ; ω_s^*, ω_c^*
- Calculate coefficients Q_{Z1}, Q_{Z2}, U, the equivalent pressure P_e, and second characteristic parameter Q₃
- Calculate the maximum stresses in TS, tubes, shell and channel and limit their values to the maximum allowable stress limits.

Because of the complexity of the procedure, it is likely that users will computerize the solution. Annex V provides a Mathcad calculation sheet for a fixed TS HE defined in PTB-4 Example E4.18.7. The calculation sheet is divided in 2 parts:

Part 1 follows strictly the various steps (Steps 1 to 11) of UHX-13.5 calculation procedure. Part 2 provides the equations developed in VIII, which enable to calculate:

- at any radius of the perforated tubesheet: net pressure, deflection, rotation, bending stress, shear stress
- at any radius of the tube bundle: axial membrane stress.
- at tubesheet-shell-channel connection: loads and displacements acting on the shell.

12 CHECKING OF THE RESULTS

12.1 Comparison with FEA

(a) General

SG-HTE proceeded to Finite Element Analysis to check UHX rules. Example E4.18.7 (configuration a) given inAnnex V was selected for the comparison.

FEA was performed by Joel Gordon, member of SG-HTE, using ANSYS with plate elements for the TS and shell elements for shell and channel. The results are in full agreement with results obtained by Tony Norton, member of SG-THE, using a KSHELL analysis with maximum discrepancies less than 5%.

So as to reflect the UHX assumptions as closely as possible, FEA calculation was performed as follows:

- The TS is replaced by an equivalent unperforated plate with an effective ligament efficiency μ^* and equivalent effective constants E* and v*,
- The tubes are replaced by an elastic foundation of modulus $k_w = N_t k_t / (\pi a_a^2)$,
- The unperforated rim is narrow enough such that it tends toward a rigid behavior.

(b) Results

- (1) for tube axial stress $\sigma_t(\mathbf{r})$, calculated from the TS displacements, FEA and UHX show very good agreement, with discrepancies less than 5% at the center (r=0) and at theperiphery of the TS (r=a_o). The stress distributions throughout the TS track each other almost perfectly as shown by the shape of the curves given in
- (2) Figure 30 for design (pressure) loading cases 1, 2 and 3. It must be noted that the curves meet at the same point (x =5.5; σ_t =0). This surprising result is demonstrated analytically in Annex S.
- (3) For TS bending stress $\sigma(r)$ the comparison is not so good: the TS stress obtained by UHX is significantly higher (by about 50%) than the stress obtained by FEA. SG-HTE is still looking for an explanation for this discrepancy. This might be due to the unperforated rim, considered as a rigid ring in the analytical treatment.

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Figure 30 — Comparison of Tube Stresses Calculated Per UHX and FEA (Example E4.18.7)

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12.2 Comparison with CODAP French Rules

(a) General

ASME and CODAP (French Code for Pressure Vessels – Part Design, Chapter C7.3 "Fixed TS HEs) rules are based on the same analytical approach (TS replaced by an equivalent solid plate of effective elastic constants E^* and v^* , tube bundle replaced by an equivalent elastic foundation, shell and channel disconnected from the TS with relevant edge loads). In 1992 ASME and CODAP decided to reconcile their rules so as to have unified rules: scope, notations, figures, TS configurations, loading cases, ligament efficiencies, effective elastic constants, stress design formulas.

However additional assumptions have been maintained in the basic rules of CODAP (sections C7.3 devoted to Fixed TS HEs and C7.4 devoted to FL TS HEs), which simplify the analytical treatment:

- TS is assumed to be perforated up to the internal shell diameter D_s. Accordingly the unperforated TS rim is ignored
- Treatment of the TS-shell-channel connection is simplified
- Effect of the bolting load (configurations b, c, d) on the TS is ignored. In return, the maximum allowable stress is lowered down.
- TS radial displacement at shell-channel connection is ignored
- Effect of P_s and P_t on shell and channel at their connection with TS is ignored

Note: In June 2012, new chapters C7.5 and C7.6, based on UHX-13 and UHX-14, have been introducted in CODAP code as alternative rules to sections C7.3 and C7.4.

These simplifications, applied to the UHX method, lead to the CODAP formulas, as shown hereafter.
 (b) Unperforated rim ignored: The TS is assumed to be perforated till the internal diameter D_s of the shell. Therefore:

$$\boxed{D_o = D_s = D_c \implies a_o = a_s = a_c \implies \rho_s = 1 \quad \rho_c = 1} \qquad A = D_o \implies K = \frac{A}{D_0} = 1$$

$$X_a \text{ is written:} \qquad \boxed{X_a = X = \sqrt[4]{24 \left(1 - v^{*2}\right) N_t \frac{E_t t_t \left(d_t - t_t\right)}{E^* L h^3} \left(\frac{D_s}{2}\right)^2}}$$

X_a, named X in CODAP, is the first characteristic parameter of the HE.

(c) effect of P_s and P_t on shell and channel at their connection with TS ignored

- $\delta_s P_s = 0 \implies \delta_s = 0 \quad \omega_s = 0 \quad \omega_s^* = 0$ $\delta_c P_c = 0 \implies \delta_c = 0 \quad \omega_c = 0 \quad \omega_c^* = 0$
- (d) TS radial displacement at shell-channel connection ignored

$$\begin{split} w_s &= 0 \implies M_s = k_s \theta_s \\ w_c &= 0 \implies M_c = k_c \theta_c \end{split} \\ M_s + M_c &= k_s \theta_s + k_c \theta_c = (k_s + k_c) \theta_E \quad as \ \theta_s = \theta_c = \theta_E \end{split}$$

Moment M_E acting at TS periphery is such that:

$$M_{E} = -(M_{s} + M_{c}) = -\underbrace{(k_{s} + k_{c})}_{K_{\theta}} \theta_{E} = -K_{\theta} \theta_{E}$$

The second characteristic parameter of the exchanger is defined as:

$$Z = \frac{K_{\theta}}{k D^*} = \frac{D_s}{2 X} \frac{k_s + k_c}{D^*}$$

CODAP considers the liaison of the TS with shell and channel as an elastic restraint:

• If Z = 0: the TS is simply supported


• If $Z = \infty$: the TS is clamped

(e) Coefficient Φ

$$w_{s} = -\frac{h}{2}\theta_{s} = 0 \Rightarrow t_{s}^{'} = 0 \Rightarrow \lambda_{s} = \frac{6D_{s}}{h^{3}}k_{s}$$

$$w_{c} = -\frac{h}{2}\theta_{c} = 0 \Rightarrow t_{c}^{'} = 0 \Rightarrow \lambda_{c} = \frac{6D_{c}}{h^{3}}k_{c}$$

$$\Phi = \frac{1-\nu^{*2}}{E^{*}}(\lambda_{s} + \lambda_{c}) = \frac{1-\nu^{*2}}{E^{*}}\frac{6}{h^{3}}(D_{s}k_{s} + D_{c}k_{c}) = \frac{1}{2D^{*}}(D_{s}k_{s} + D_{c}k_{c})$$

$$XZ$$

Accordingly: $\Phi = X Z$ plays the role of Z in CODAP:

- If the plate is simply supported (no support from shell and channel): $\Phi=0$ in ASME, Z=0 in • CODAP
- If the plate is clamped (full support from shell and channel): $\Phi = \infty$ in ASME, $Z = \infty$ in • CODAP

(f) Second coefficient Q₃ of the HE

In ASME:
$$Q_1 = \frac{(\rho_s - 1) - \Phi Z_v}{1 + \Phi Z_m}$$
 $Q_2 = \frac{(w_s^* P_s - w_c^* P_c) + (\frac{\gamma_b}{2\pi} W^*)}{1 + \Phi Z_m}$ $Q_3 = Q_1 + \frac{2Q_2}{P_e a_0^2}$

In CODAP, these formulas become, with $\rho_s=1$, $\omega^*_s=0$, $\omega^*_c=0$, $W^*=0$:

$$[Q_{1}]_{CODAP} = \frac{-\Phi Z_{v}}{1 + \Phi Z_{m}} \qquad [Q_{2}]_{CODAP} = 0 \qquad [Q_{3}]_{CODAP} = Q_{1} = \frac{-\Phi Z_{v}}{1 + \Phi Z_{m}} = -\frac{(X Z) Z_{v}}{1 + (X Z) Z_{m}}$$
$$\boxed{Z = \frac{-Q_{3}}{Q_{3} Z_{m} + Z_{v}} \frac{1}{X}}$$

The couple [X,Z] in CODAP plays the role of couple [X_a,Q₃] in ASME to determine coefficients F_q, F_m, F₀ and F_t.

(g) Equivalent design pressure

In ASME:
$$P_{\rm e} = \frac{J K_{\rm s,t}}{1 + J K_{\rm s,t} \left[Q_{Z1} + (\rho_{\rm s} - 1) Q_{Z2} \right]} \left[P_{\rm s}' - P_{\rm t}' + P_{\gamma} + P_{\rm W} + P_{\rm rim} + P_{\omega} \right]$$

- $Q_{Z1} = (Z_w Q_1 + Z_d) \frac{X_a^4}{2}$ Substituting $[Q_1]_{CODAP}$, Z_w and Z_d , it can be shown that: $Q_{Z1} = \begin{bmatrix} F_q \end{bmatrix}_{CODAP}$ Thus, with $\rho_s = 1$: $l + J K_{s,t} \begin{bmatrix} Q_{Z1} + (\rho_s 1)Q_{Z2} \end{bmatrix} = l + J K_{s,t} \begin{bmatrix} F_q \end{bmatrix}_{CODAP}$

•
$$P_{s}' = \left[x_{s} + 2(1 - x_{s}) v_{t} + \frac{2}{K_{s,t}} \left(\frac{D_{s}}{D_{o}} \right)^{2} v_{s} - \frac{\rho_{s}^{2} - 1}{J K_{s,t}} - \frac{1 - J}{2 J K_{s,t}} \frac{D_{J}^{2} - D_{s}^{2}}{D_{o}^{2}} \right] P_{s}$$

with
$$D_s=D_o$$
 and $\rho_s=0$:

$$P_{s}^{'} = \left[x_{s} + 2(1 - x_{s})v_{t} + \frac{2}{K_{s,t}}v_{s} - \frac{1 - J}{2JK_{s,t}}\frac{D_{J}^{2} - D_{s}^{2}}{D_{o}^{2}} \right]P_{s} = \left[P_{s}^{'}\right]_{CODAP}$$

$$P_{t}^{'} = \left[x_{t} + 2(1 - x_{t})v_{t} + \frac{1}{JK_{s,t}} \right]P_{t} = \left[P_{t}^{'}\right]_{CODAP}$$

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•
$$P_{\gamma} = \frac{N_t K_t}{\pi a_o^2} \gamma = \frac{N_t K_t}{\pi R_e^2} \gamma = K_w \gamma = [P_{\gamma}]_{CODAP}$$
•
$$P_w = \frac{U}{a_o^2} \frac{\gamma_b}{2\pi} W^* = 0$$
 due to W*=0 in CODAP
•
$$P_{rim} = -\frac{U}{a_o^2} [\omega_s^* P_s - \omega_c^* P_t] = 0$$
 due to $\omega^*_s = 0$ and $\omega^*_c = 0$ in CODAP.
•
$$P_{\omega} = -\frac{U}{a_o^2} [\omega_s P_s^* - \omega_c P_t^*] = 0$$
 due to $\omega = 0$ and $\omega_c = 0$ in CODAP.
•
$$P_{\omega} = -\frac{U}{a_o^2} [\omega_s P_s^* - \omega_c P_t^*] = 0$$
 due to $\omega = 0$ and $\omega_c = 0$ in CODAP.
•
$$P_{\omega} = -\frac{U}{a_o^2} [\omega_s P_s^* - \omega_c P_t^*] = 0$$
 due to $\omega = 0$ and $\omega_c = 0$ in CODAP.
•
$$P_{\omega} = -\frac{U}{a_o^2} [\omega_s P_s^* - \omega_c P_t^*] = 0$$
 due to $\omega = 0$ and $\omega_c = 0$ in CODAP.
•
$$P_{\omega} = -\frac{U}{a_o^2} [\omega_s P_s^* - \omega_c P_t^*] = 0$$
 due to $\omega = 0$ and $\omega_c = 0$ in CODAP.
•
$$P_{\omega} = -\frac{U}{a_o^2} [\omega_s P_s^* - \omega_c P_t^*] = 0$$
 due to $\omega = 0$ and $\omega_c = 0$ in CODAP.
•
$$P_{\omega} = -\frac{U}{a_o^2} [\omega_s P_s^* - \omega_c P_t^*] = 0$$
 due to $\omega = 0$ and $\omega_c = 0$ in CODAP.
•
$$P_{\omega} = -\frac{U}{a_o^2} [\omega_s P_s^* - \omega_c P_t^*] = 0$$
 due to $\omega = 0$ and $\omega_c = 0$ in CODAP.
•
$$P_{\omega} = -\frac{U}{a_o^2} [\omega_s P_s^* - \omega_c P_t^*] = 0$$
 due to $\omega = 0$ and $\omega_c = 0$ in CODAP.
•
$$P_{\omega} = -\frac{U}{a_o^2} [\omega_s P_s^* - \omega_c P_t^*] = 0$$
 due to $\omega = 0$ and $\omega_c = 0$ in CODAP.
•
$$P_{\omega} = -\frac{U}{a_o^2} [\omega_s P_s^* - \omega_c P_s^*] = 0$$
 due to $\omega = 0$ and $\omega_c = 0$ in CODAP.
•
$$P_{\omega} = -\frac{U}{a_o^2} [\omega_s P_s^* - \omega_c P_s^*] = 0$$
 due to $\omega = 0$ and $\omega_c = 0$ in CODAP.
•
$$P_{\omega} = -\frac{U}{a_o^2} [\omega_s P_s^* - \omega_c P_s^*] = 0$$
 due to $\omega = 0$ and $\omega_c = 0$ in CODAP.
•
$$P_{\omega} = -\frac{U}{a_o^2} [\omega_s P_s^* - \omega_c P_s^*] = 0$$
 due to $\omega = 0$ and $\omega_c = 0$ in CODAP.
•
$$P_{\omega} = -\frac{U}{a_o^2} [\omega_s P_s^* - \omega_c P_s^*] = 0$$
 due to $\omega = 0$ and $\omega_c = 0$ in CODAP.
•
$$P_{\omega} = -\frac{U}{a_o^2} [\omega_s P_s^* - \omega_c P_s^*] = 0$$
 due to $\omega = 0$ and $\omega_c = 0$ in CODAP.
•
$$P_{\omega} = -\frac{U}{a_o^2} [\omega_s P_s^* - \omega_c P_s^*] = 0$$
 due to $\omega = 0$ and $\omega_c = 0$ in CODAP.
•
$$P_{\omega} = -\frac{U}{a_o^2} [\omega_s P_s^* - \omega_c P_s^*] = 0$$
 due to $\omega = 0$ and $\omega_c = 0$ in CODAP.
•
$$P_{\omega} = -\frac{U}{a_o^2} [\omega_s P_s^* - \omega_c P_s^*] = 0$$
 due to $\omega = 0$ and $\omega_c = 0$ in CODAP.
•
$$P_{\omega} = -\frac{U}{a_o^2} [\omega_s P$$

it can be shown that: $\left|F_{m}(x) = \left[F_{m}(x)\right]_{CODAP}\right|$ and:

$$\sigma_{\rm r}(x) = \frac{1.5F_{\rm m}(x)}{\mu^*} \left(\frac{2a_o}{h}\right)^2 P_{\rm e} = \left[\sigma_{\rm r}(x)\right]_{CODAP}$$

(i) TS shear stress

(*h*)

In ASME:
$$F_Q(x) = Q_3 Q_\alpha(x) + Q_\beta(x)$$
 $\tau(x) = \frac{1}{2\mu} \frac{a_o}{h} F_Q(x) P_e$

The maximum shear stress is at periphery (x=X_a), for which: $Q_{\alpha}(X_{a}) = 0$, $Q_{\beta}(X_{a}) = 1 \implies F_{Q}(X_{a}) = 1$

Thus:
$$\tau = \frac{1}{2\mu} \frac{a_o}{h} P_e = [\tau]_{CODAP}$$

(*j*) **Tube axial stress**

In ASME:
$$F_t\left(x\right) = \frac{X_a^4}{2} \left[Q_3 Z_w\left(x\right) + Z_d\left(x\right) \right]$$

Substituting $Z_w(x)$ and $Z_d(x)$ and using $\left[Q_3 \right]_{CODAP} = \frac{\phi Z_v}{1 + \phi Z_m}$
it can be shown that: $F_t\left(x\right) = \left[F_t\left(x\right) \right]_{CODAP}$ and:
 $\sigma_t\left(x\right) = \frac{1}{x_t - x_s} \left[\Delta p^* - F_t\left(x\right) P_e \right] = \left[\sigma_t\left(x\right) \right]_{CODAP}$

(k) Shell stresses

(1) Axial membrane stress

In ASME:
$$\sigma_{s,m} = \frac{a_o^2}{(D_s + t_s)t_s} \Big[P_e + (\rho_s^2 - 1)(P_s - P_t) \Big] + \frac{a_s^2}{(D_s + t_s)t_s} P_t$$

In CODAP, that formula becomes, with $a_0 = \frac{D_s}{2}$ and $\rho_s = 1$:
$$\sigma_{s,m} = \frac{D_s^2}{4t_s(D_s + t_s)} (P_e + P_t) = \Big[\sigma_{s,m} \Big]_{CODAP} [XI.2.11a]$$

(2) Bending stress

In ASME:
$$M_{s,b} = k_s \left\{ \beta_s \,\delta_s \,P_s + \frac{6(1-\nu^{*2})}{E^*} \left(\frac{a_o}{h}\right)^3 \left(1 + \frac{h \,\beta_s}{2}\right) \left[P_e \left(Z_v + Q_1 \,Z_m\right) + \frac{2}{a_o^2} \left(Q_2 \,Z_m\right) \right] \right\}$$

In CODAP, that formula becomes with, $\beta_s = 0$ $\delta_s = 0$ $a_o = \frac{D_s}{2}$

$$\begin{bmatrix} Q_1 \end{bmatrix}_{CODAP} = \frac{-\Phi Z_v}{1 + \Phi Z_m} \quad \begin{bmatrix} Q_2 \end{bmatrix}_{CODAP} = 0:$$

$$M_{s,b} = \frac{k_s}{2} \frac{1}{D^*} \left(\frac{D_s}{2}\right)^3 \frac{Z_v}{1 + \Phi Z_m} P_e = \begin{bmatrix} M_{s,b} \end{bmatrix}_{CODAP} \quad \sigma_{s,b} = \frac{6}{t_s^2} M_{s,b} = \begin{bmatrix} \sigma_{s,b} \end{bmatrix}_{CODAP}$$

(*l*) Channel stresses

(1) Axial membrane stress:
$$\sigma_{c,m} = \frac{a_c^2}{\left(D_c + t_c\right)t_c} P_t = \frac{D_s^2}{4\left(D_c + t_c\right)t_c} P_t = \left[\sigma_{c,m}\right]_{CODAP}$$
(2) Bending stress is obtained the same way as for the shell:
$$\sigma_{c,b} = \frac{6}{t_c^2} M_{c,b} = \left[\sigma_{c,b}\right]_{CODAP}$$

(m) In conclusion, all CODAP formulas have been retrieved, although they were obtained at a time where there was no connection between ASME and CODAP committees. This tends to prove that both methods are correct and coherent. This coherency extends to TEMA as shown in Section 12.3. In CODAP, the ignorance of unperforated rim and bolting loads are the most important simplifications and have a significant impact on the results. Other simplifications have a less important impact.

Calculation of a HE by CODAP method can be obtained from a UHX-13.3 software, such as Mathcad software shown in Annex V for Example UHX.20.2.3, by imposing the simplifications listed in Sections 12.2(b) to (e) above.

Like ASME, CODAP results for tube axial stress $\sigma_t(r)$ are in good agreement with FEA results as shown by Figure 31 which provides the tube stress distributions throughout the TS.

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Figure 31 — Tube Stress Distribution Obtained by UHX, CODAP and FEA throughout the TS from r = 0 to r = ao

12.3 Comparison with TEMA Rules

(a) General: TEMA rules for fixed TSs have been developed by Gardner, based on its 1952 paper [2]. The analytical approach is the same as for ASME and CODAP with several assumptions to provide more simple rules. CODAP assumptions are used, plus additional simplifications. The TS is assumed to be either Simply Supported (SS) (i.e. no restraint from the shell and channel) or Clamped (CL) (i.e. full restraint from the shell and channel).

The original TEMA formula for the determination of the TS thickness in bending:

$$T = F \frac{G}{2} \sqrt{\frac{P}{S}} \text{ is based on flat circular plates formula, which can be either SS:}$$
$$h = F \frac{G}{2} \sqrt{\frac{P}{\sigma_{alw}}} = 1.11 \frac{G}{2} \sqrt{\frac{P}{S}} \quad if \ \sigma_{alw} = S$$
$$= 0.91 \frac{G}{2} \sqrt{\frac{P}{S}} \quad if \ \sigma_{alw} = 1.5S$$
$$TEMA \text{ uses the mean value } F = 1.0$$

or CL:

$$\begin{split} h &= F \frac{G}{2} \sqrt{\frac{P}{\sigma_{alw}}} = 0.87 \frac{G}{2} \sqrt{\frac{P}{S}} \qquad if \ \sigma_{alw} = S \\ &= 0.71 \frac{G}{2} \sqrt{\frac{P}{S}} \qquad if \ \sigma_{alw} = 1.5S \end{split}$$
 TEMA uses the mean value $F = 0.8$

(b) Ligament efficiency

That formula was modified later to introduce a ligament efficiency η based on the mean length of the

ligament
$$T = F \frac{G}{3} \sqrt{\frac{P}{\eta S}}$$
, as proposed by Miller [3] (see Section 4.1 of PART 2):

For Triangular pitch:
$$\eta = 1 - \frac{0.907}{(p/d_t)^2}$$
 For Square pitch: $\eta = 1 - \frac{0.785}{(p/d_t)^2}$

Accordingly TEMA ligament efficiency η is lower than ASME ligament efficiency μ^* . The new formula was set-up so that the value of T remains approximately the same for the triangular pitch value η =0.42, obtained from the minimum pitch imposed by TEMA (p=1.25d_t). TEMA simplifications applied to ASME-CODAP methods lead to the TEMA formulas, as shown hereafter.

(c) TEMA bending formula

ASME-CODAP formula for TS bending can be written using the TEMA format:

$$h = \sqrt{\frac{1.5F_{\rm m}(X)}{\mu^*}} D_o \sqrt{\frac{P_e}{\sigma_{alw}}} = \sqrt{\frac{9 \times 1.5F_{\rm m}(X)}{\mu^* / \eta}} \frac{D_o}{3} \sqrt{\frac{P_e}{\eta \sigma_{alw}}}$$

In TEMA, the TS is assumed uniformly perforated up to the shell: $D_0=D_s$ (noted G in TEMA for fixed TS).

 σ_{alw} =S for design (pressure) loadings (loading cases 1, 2, 3 and 4), σ =2S for operating (thermal + pressure) loadings (loading cases 1, 2, 3 and 4)

Using $\eta/\mu^*=1.33$, which is quite usual:

$$h = \sqrt{12F_{\rm m}(X)} \frac{D_s}{3} \sqrt{\frac{P}{\eta S}} = F_{CODAP} \frac{D_s}{3} \sqrt{\frac{P}{\eta S}} \qquad F_{CODAP} = \sqrt{12F_{\rm m}(X)}$$

Thus, in CODAP-ASME the TS thickness depends strongly on coefficient $F_m(X)$, whereas TEMA coefficient F has fixed values. Figure 32 gives the variation of F_{CODAP} as a function of X:

- when the TS is SS (obtained for Z=0 in CODAP).
- when the TS is CL (obtained for $Z=\infty$ in CODAP).

TEMA considers the TS as SS when $t_s/D_s < 0.02$ with F=1, and CL when $t_s/D_s > .05$ with F=0.8. In between TEMA uses a linear interpolation (see Figure 33).

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Figure 32 shows that:

- for low values of $X_a(X_a < 3)$ TEMA rules generally lead to an unconservative thickness;
- for high values of $X_a(X_a > 6)$ TEMA rules generally lead to conservative thickness
- values of X_{a} between 3 and 6, TEMA rules lead to tubesheet thickness that is close to ASME.

Due to simplifications mentioned above, TEMA does not ensure an overall and consistent design margin for all heat-exchangers. Accordingly, TEMA has imposed limitations on shell diameter D_s and design pressure P: $D_s \le 60$ inch $P \le 3,000$ Psi $PD_s \le 60,000$ Psi inch (see Figure 34).



Figure 34 — TEMA Design Range



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However, it must be pointed out that the value of coefficient F has been remarkably well chosen as it represents approximately the mean value of coefficient F(X). Finally TEMA formula is written:

$$h = F \frac{D_s}{3} \sqrt{\frac{P_e}{\eta S}} = F \frac{G}{3} \sqrt{\frac{P_{TEMA}}{\eta S}} \quad \text{where:} \begin{cases} P_{TEMA} \text{ is the TEMA effective pressure} \\ G \text{ is the shell internal diameter } D_s \end{cases}$$

(d) **TEMA effective pressure** is given by: $P_{TEMA} = P_{t,TEMA} - P_{s,TEMA} + P_{d,TEMA}$ with:

$$P_{s,TEMA}^{'} = \frac{0.4J \left[1.5 + K_{s,t} \left(1.5 + x_{s} \right) \right] - \frac{1-J}{2} \frac{D_{J}^{-} - D_{s}^{-}}{D_{s}^{-}}}{1 + JK_{s,t}F_{q}} P_{s} \qquad P_{t,TEMA}^{'} = \frac{1 + 0.4JK_{s,t} \left(1.5 + x_{t} \right)}{1 + JK_{s,t}F_{q}} P_{t}$$

$$P_{d,TEMA} = \frac{4JE_s t_s / L}{\left(D_{s,e} - 3t_s\right)\left(1 + JK_{s,t}F_q\right)} \quad \text{with:} \quad \begin{cases} \Delta L = \left[\alpha_{s,m}(T_{s,m} - T_a) - \alpha_{t,m}(T_{t,m} - T_a)\right]L = -\gamma \\ D_{s,e} = \text{external shell diameter} = D_s + 2t_s \end{cases}$$

ASME-CODAP formula for equivalent pressure is written:

$$P_{e} = \frac{\left(JK_{s,t}\right)P_{s}' - \left(JK_{s,t}\right)P_{t}' + \left(JK_{s,t}\right)P_{\gamma}}{1 + JK_{s,t}F_{q}} = P_{s,1}' - P_{t,1}' + P_{\gamma,1} \quad \text{with:}$$

$$P_{s,1}' = \frac{\left(JK_{s,t}\right)P_{s}'}{1 + JK_{s,t}F_{q}} = \frac{\left(JK_{s,t}\right)\left[x_{s} + 2(1 - x_{s})v_{t} + \frac{2v_{s}}{K_{s,t}} - \frac{1 - J}{2JK_{s,t}}\frac{D_{s}^{2} - D_{s}^{2}}{D_{o}^{2}}\right]}{1 + JK_{s,t}F_{q}}P_{s}$$

$$= \frac{J\left[K_{s,t}x_{s} + 2K_{s,t}(1 - x_{s})v_{t} + 2v_{s}\right] - \frac{1 - J}{2}\frac{D_{s}^{2} - D_{s}^{2}}{D_{o}^{2}}P_{s}}{1 + JK_{s,t}F_{q}}P_{s} \quad Using v_{s} = 0.3 v_{t} = 0.3 and D_{o} = D_{s}:$$

$$P_{s,1}^{'} = \frac{0.4J \left[1.5 + K_{s,t} \left(1.5 + x_{s} \right) \right] - \frac{1 - J}{2} \frac{D_{J}^{2} - D_{s}^{2}}{D_{s}^{2}}}{1 + JK_{s,t}F_{q}} P = P_{s,TEMA}^{'} = \frac{JK_{s,t}}{1 + JK_{s,t}F_{q}} P_{s}^{'}$$

•
$$P_{t,1} = \frac{\left(JK_{s,t}\right)P_{t}}{1 + JK_{s,t}F_{q}} = \frac{\left(JK_{s,t}\right)\left[x_{t} + 2(1 - x_{t})v_{t} + \frac{1}{JK_{s,t}}\right]}{1 + JK_{s,t}F_{q}}P_{t} = \frac{J\left[K_{s,t}x_{t} + 2K_{s,t}(1 - x_{t})v_{t} + 1\right]}{1 + JK_{s,t}F_{q}}P_{t}$$
 Using $v_{t} = 0.3$:

$$P_{r,1} = \frac{1 + 0.4JK_{s,t}(1.5 + x_t)}{1 + JK_{s,t}F_q}P_t = P_{t,TEMA}$$

$$P_{r,1} = \frac{(JK_{s,t})F_qN_tK_t}{(1 + JK_{s,t}F_q)\pi a_o^2}\gamma = \frac{4JK_s}{(1 + JK_{s,t}F_q)\pi D_o^2}\gamma = \frac{4JE_st_s(D_s + t_s)}{(1 + JK_{s,t}F_q)D_s^2}\frac{\gamma}{L}$$

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$$\frac{D_{s} + t_{s}}{D_{s}^{2}} = \frac{D_{s,e} - t_{s}}{\left(D_{s,e} - 2t_{s}\right)^{2}} = \frac{1}{D_{s,e}} \frac{1 - \frac{t_{s}}{D_{s,e}}}{\left(1 - \frac{2t_{s}}{D_{s,e}}\right)^{2}} - \frac{1}{D_{s,e}} \frac{1}{\left(1 - \frac{4t_{s}}{D_{s,e}}\right)} \left(1 + \frac{t_{s}}{D_{s,e}}\right) - \frac{1}{D_{s,e}} \frac{1}{\left(1 - \frac{3t_{s}}{D_{s,e}}\right)} = \frac{1}{\left(D_{s,e} - 3t_{s}\right)}$$

$$\frac{P_{\gamma,1}}{\left(1 + JK_{s,t}F_{q}\right)\left(D_{s,e} - 3t_{s}\right)} \frac{\gamma}{L} = \frac{4JE_{s}t_{s}}{\left(1 + JK_{s,t}F_{q}\right)\left(D_{s,e} - 3t_{s}\right)} - \frac{\Delta L}{L} = -P_{d}}$$
Finally:
$$P_{e} = \frac{\left(JK_{s,t}\right)P_{s}^{'} - \left(JK_{s,t}\right)P_{t}^{'} + \left(JK_{s,t}\right)P_{\gamma}}{1 + JK_{s,t}F_{q}} = P_{s,1}^{'} - P_{t,1}^{'} + P_{\gamma,1} = P_{s,TEMA}^{'} - P_{t,TEMA}^{'} - P_{d,TEMA}^{'} = -P_{TEMA}^{'}$$

$$= \frac{1}{1 + JK_{s,t}F_q} = P_{s,1} - P_{t,1} + P_{\gamma,1} = P_{s,TEMA} - P_{t,TEMA} - P_{d,TEMA}$$

(e) Coefficient X

$$X_{a} = X = \sqrt[4]{24(1-\nu^{*2})N_{t} \frac{E_{t}t_{t}(d_{t}-t_{t})}{E^{*}Lh^{3}}\left(\frac{D_{s}}{2}\right)^{2}} = \sqrt[4]{\frac{6(1-\nu^{*2})}{E^{*}}\frac{1}{L}\sum_{s,t_{s}(D_{s}+t_{s})/K_{s,t}}^{N_{t}}\left(\frac{D_{s}}{h^{3}}\right)^{2}}$$

Introducing the deflection efficiency:

$$\eta^{*} = \frac{D^{*}}{D} = \frac{E^{*}}{1 - v^{*2}} \frac{1 - v^{2}}{E} \quad \text{which leads to} : \frac{1 - v^{*2}}{E^{*}} = \frac{1 - v^{2}}{E\eta^{*}} \quad \text{, it follows:}$$

$$X_{a} = X = \sqrt[4]{\frac{6(1 - v^{2})}{\eta * E} \frac{E_{s}t_{s}(D_{s} + t_{s})}{LK_{s,t}} \frac{D_{s}^{2}}{h^{3}}} \qquad \text{Neglecting } t_{s} \text{ compared to } D_{s} : \left[X_{TEMA} = \sqrt[4]{\frac{6(1 - v^{2})}{\eta * E} \frac{E_{s}t_{s}}{ELK_{s,t}} \left(\frac{D_{s}}{h}\right)^{3}}}\right]$$

(f) Coefficient F_q is given on Figure 35 as a function of X for SS (Z=0) and CL (Z= ∞) TS.

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$$F_{q} = MAX\left[(1), \left(0.25 + (F - 0.6)\sqrt[4]{\frac{300E_{s}t_{s}}{ELK_{s,t}}}\left(\frac{D_{s}}{T}\right)^{3}\right)\right]$$

(g) TEMA shear formula

CODAP formula for shear stress is written: $\tau = \frac{1}{2\mu} \cdot \frac{D_o}{2h} \cdot P_e$ with $\tau \le 0.8S$

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Thus, the minimum TS shear thickness is written: $h = \frac{1}{3.2\mu} \cdot \frac{D_o}{S} \cdot P_e$

ASME 2013 uses for Do the equivalent diameter DL of the tube center limit parameter, which leads to:

$$T = \frac{0.31D_L}{\mu} \frac{P}{S}$$

(*h*) **Tube longitudinal stress** is calculated by TEMA at periphery of bundle:

$$\sigma_{t,o} = \frac{1}{x_t - x_s} \Big[\big(P_s x_s - P_t x_t \big) - F_q P_e \Big]$$

with:

with:

 $\sigma_{\scriptscriptstyle s,m}$

 σ_{t}

$$[\text{III.2.b1}] \quad x_{t} - x_{s} = \frac{N_{t}s_{t}}{\pi a_{o}^{2}} = \frac{N_{t}t_{t}\left(d_{t} - t_{t}\right)}{a_{o}^{2}} \qquad \text{a}_{o} = D_{s}/2 \qquad -P_{e} = P_{TEMA} = P_{t,TEMA}^{'} - P_{s,TEMA}^{'} + P_{d,TEMA}$$
$$= \frac{F_{q}D_{s}^{2}}{4N_{t}t_{t}\left(d_{t} - t_{t}\right)} \left[\frac{x_{s}P_{s} - x_{t}P_{t}}{F_{q}} + P_{t,TEMA}^{'} - P_{s,TEMA}^{'} + P_{d,TEMA}^{'}\right] = \frac{F_{q}D_{s}^{2}}{4N_{t}t_{t}\left(d_{t} - t_{t}\right)} \left[\frac{\left(P_{t,TEMA}^{'} - \frac{x_{t}P_{t}}{F_{q}}\right) - \left(P_{s,TEMA}^{'} - \frac{x_{s}P_{s}}{F_{q}}\right) + P_{d,TEMA}^{'}\right]}{P_{o}} + P_{d,TEMA}^{'}$$

- If $\sigma_{t,o} > 0$ (tubes in traction) $\sigma_{t,o}$ is limited to $2S_t$
- If $\sigma_{t,o} \leq 0$ (tubes in compression) $\sigma_{t,o}$ is limited to S_t and to the maximum permissible buckling stress limit S_{tb} named S_c in TEMA.

bingitudinal stress is obtained from [XI.2.11a]:
$$\sigma_{s,m} = \frac{D_s^2}{4 t_s (D_s + t_s)} (P_e + P_t)$$
$$D_s \cong D_s + t_s \quad and \quad P_e = -P_{TEMA} = P_{s,TEMA} - P_{t,TEMA} - P_{d,TEMA}$$
$$= \frac{D_s^2}{4 t_s (D_s + t_s)} \left(\underbrace{P_t - P_{t,TEMA}}_{P_1} + P_{s,TEMA} - P_{d,TEMA} \right)$$

Shell and channel bending stresses are not calculated.

12.4 Comparison with Circular Plates Subject to Pressure

The classical formulas of circular plates subjected to pressure for SS plates are:

$$\sigma_{r}(x) = 1.5 \underbrace{\frac{(3+\upsilon)\left(1-\frac{r^{2}}{R^{2}}\right)}{\underbrace{\frac{16}{F^{SS}(r)}}} \left(\frac{2R}{h}\right)^{2} \left(P_{s}-P_{t}\right) \qquad \text{with} : \begin{cases} F^{SS}(0) = \frac{3+\upsilon}{16} & \text{at TS center } (r=0) \\ F^{SS}(R) = 0 & \text{at TS periphery } (r=R) \end{cases}$$

for CL plates:

$$\sigma_{r}(x) = 1.5 \underbrace{\frac{(1+\upsilon) - (3+\upsilon)\frac{r^{2}}{R^{2}}}_{F^{CL}(r)}}_{(r)} \underbrace{\left(\frac{2R}{h}\right)^{2}}_{2} \left(P_{s} - P_{t}\right) \quad with: \begin{cases} F^{CL}(0) = \frac{1+\upsilon}{16} & \text{at TS center } (r=0) \\ F^{CL}(R) = -\frac{1}{8} & \text{at TS periphery } (r=R) \end{cases}$$

The following modifications must be applied to UHX-13.3 rules.

(a) No unperforated rim:
$$\begin{cases} a_o = a_s = R & D_o = D_s \implies \rho_s = 1 & \omega_s^* = \omega_s \\ a_o = a_c = R & D_o = D_c \implies \rho_c = 1 & \omega_c^* = \omega_c \end{cases}$$

(b) No holes and no tubes: $\mu^* = 1$ $E^* = E$ $\upsilon^* = \upsilon$

$$N_{t} = 0 \implies x_{s} = 1 \quad x_{t} = 1 \Longrightarrow X_{a} = \sqrt[4]{24 \left(1 - v^{*2}\right) N_{t} \frac{E_{t} t_{t} \left(d_{t} - t_{t}\right)}{E^{*} L h^{3}} \left(\frac{D_{s}}{2}\right)^{2}} = 0$$

When there no more tubes, there is no more elastic foundation and $X_a=0$. For $X_a=0$ several coefficients are to infinity

 $(Z_d, Z_w, Q_m, Q_v,...)$. Accordingly, these coefficients will be calculated for $X_a \rightarrow 0$, as shown in Annex F.

(c) No bellows:
$$K_J = 0 \implies J = \frac{K_J}{K_J + K_s} = 1$$

- (d) No bolted flange: $W^*=0$ No differential thermal expansion: $\gamma = 0$
- (e) No effect of P_s and P_t on shell and channel at their connection with TS:

$$\begin{cases} \delta_s P_s = 0 \implies \delta_s = 0 \quad \omega_s = 0 \quad \omega_s^* = 0 \\ \delta_c P_c = 0 \implies \delta_c = 0 \quad \omega_c = 0 \quad \omega_c^* = 0 \end{cases}$$

(f) Calculation of A and Φ : $A = D_o \implies K = \frac{A}{D_0} = 1 \implies \Phi = \frac{1 - v^{*2}}{E^*} (\lambda_s + \lambda_c)$

From [VI.3]:

$$Q_{1} = -\frac{1}{4} \frac{\Phi}{(1+\nu^{*})+\Phi} \qquad \qquad Q_{2} = \frac{\left(\omega_{s}^{*}P_{s} - \omega_{c}^{*}P_{c}\right) + \left(W\frac{\gamma_{b}}{2\pi}\right)}{1+\Phi Z_{m}} = 0$$
$$Q_{3} = Q_{1} + \frac{2}{a_{0}^{2}P_{e}}Q_{2} = Q_{1} = -\frac{1}{4}\frac{\Phi}{(1+\nu^{*})+\Phi}$$

Using Z_w and Z_d from Annex G:

$$Q_{Z1} = (Z_{w}Q_{1} + Z_{d})\frac{X_{a}^{4}}{2} = \left[\frac{1}{4(1+\upsilon^{*})}\frac{-\Phi}{4(1+\upsilon^{*})+\Phi} + \frac{2}{X_{a}^{4}}\right]\frac{X_{a}^{4}}{2} = 1$$

Using Z_v and Z_m from Annex G:

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$$Q_{Z2} = (Z_{\nu}Q_{1} + Z_{m})\frac{X_{a}^{4}}{2} = \left[\frac{1}{4(1+\nu^{*})}\frac{-\Phi}{4(1+\nu^{*})+\Phi} + \frac{1}{1+\nu^{*}}\right]\frac{X_{a}^{4}}{2} = 0$$
$$U = \frac{Z_{w} + (\rho_{s} - 1)Z_{m}}{1+\Phi Z_{m}}X_{a}^{4} = \frac{Z_{w}}{1+\Phi Z_{m}}X_{a}^{4} = \frac{1}{4(1+\nu^{*})}\frac{1}{1+\Phi Z_{m}}X_{a}^{4} = 0$$

(g) Equivalent pressure is written:

$$P_{e} = \frac{1}{\frac{1}{JK_{s,t}} + \left[Q_{Z1} + (\rho_{s} - 1)Q_{Z2}\right]} \left[P_{s}' - P_{t}' + P_{\gamma} + P_{w} + P_{rim}\right]$$

Thus: $P_e = P_s - P_t$

- If the plate is simply supported (no support from shell and channel): $\Phi=0$ in ASME , Z=0 in CODAP
- If the plate is clamped (full support from shell and channel): $\Phi = \infty$ in ASME, $Z = \infty$ in CODAP

(*h*) Coefficient
$$\mathbf{F}_{\mathbf{m}}$$
 is written: $F_m(x) = \frac{Q_v(x) + Q_3 Q_m(x)}{2}$

where $Q_v(x)$ and $Q_m(x)$ are given by Annex G when $X_a \rightarrow 0$:

$$Q_{m}(x \to 0) = \frac{(1+\upsilon^{*})}{2Z_{a}}bei' = \frac{(1+\upsilon^{*})}{\frac{1+\upsilon^{*}}{2}X_{a}}\frac{X_{a}}{2} = 1$$

$$Q_{\nu}(x \to 0) = \frac{(1+\upsilon^{*})}{2Z_{a}X_{a}}\Psi_{1} = \frac{(1+\upsilon^{*})}{\frac{1+\upsilon^{*}}{2}X_{a}^{2}}\frac{3+\upsilon^{*}}{16}X_{a}^{2} = \frac{3+\upsilon^{*}}{8}$$

Coefficient F_m at TS center is given by formula:

$$F_m(x \to 0) = \frac{Q_3}{2} + \frac{3 + \upsilon^*}{16} = -\frac{1}{8} \frac{\Phi}{(1 + \upsilon^*) + \Phi} + \frac{3 + \upsilon^*}{16}$$

Coefficient F_m at TS periphery is given by formula:

$$F_{m}(X_{a}) = \frac{Q_{3}}{2} = -\frac{1}{8} \frac{\Phi}{(1+\nu^{*}) + \Phi}$$

(*i*) **TS bending stress** is written: $\sigma_r(x) = 1.5 F_m(x) \left(\frac{2R}{h}\right)^2 \left(P_s - P_t\right)$

If the TS is SS:
$$\Phi = 0 \implies Q_3 = 0 \implies \begin{cases} F_m^{SS}(0) = \frac{3+\upsilon}{16} & at TS center \\ F_m^{SS}(R) = 0 & at TS periphery \end{cases}$$

If the TS is CL:
$$\Phi = \infty \implies Q_3 = -\frac{1}{4} \implies \begin{cases} F_m^{CL}(0) = -\frac{1}{8} + \frac{3+\upsilon}{16} = \frac{1+\upsilon}{16} & \text{at TS center} \\ F_m^{CL}(R) = -\frac{1}{8} & \text{at TS periphery} \end{cases}$$

Accordingly classical formulas for circular plates subjected to pressure given in this section have been retrieved. Similar calculations can also be performed for the TS deflection w(x) and rotation $\theta(x)$.

These results can be obtained from a UHX-13.3 software, such as Mathcad software shown in Annex V, by imposing the simplifications listed in (a) to (e) above.

12.5 Conclusions

Applying the relevant simplifications, it has been analytically demonstrated that UHX-13.3 method leads to CODAP, TEMA and circular plates' formulas. This intends to prove the correctness of ASME method, which is confirmed by FEA comparisons for the TS deflection.



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PART 4 FLOATING TUBESHEETS



1 SCOPE

PART 4 applies to floating tubesheet (TS) Heat Exchangers (HE)s that have one stationary tubesheet and one floating tubesheet. Three types of HEs are covered as shown in Figure 36:

- immersed floating head (sketch a)
- externally sealed floating head (sketch b)
- internally sealed floating tubesheet (sketch c)



(a) Typical Floating Tubesheet Exchanger With an Immersed Floating Head



(b) Typical Floating Tubesheet Exchanger With an Externally Sealed Floating Head



(c) Typical Floating Tubesheet Exchanger With an Internally Sealed Floating Tubesheet Figure 36 — Floating Tubesheet Heat Exchangers

(see TS configurations in Figure 37 and Figure 38)

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2 HISTORICAL BACKGROUND

In the past decades many authors have proposed theoretical methods for the design of floating TS HEs. The most important contributions are provided below.

Gardner [1] in 1948 was the very first to develop an analytical approach by taking into consideration the support afforded by the tubes and the weakening effect of the TS holes. The TS is considered as either simply supported or clamped at its periphery to simulate the rotational restraint afforded by the shell and the channel, which compels the designer to make a more or less arbitrary choice between these two extreme cases. The method was adopted by Dutch code STOOWEZEN in 1975

K.A.G. Miller [2] at the same time, proposed a similar approach that was published in the British Code BS 1515 in 1965.

Galletly [3] in 1959 improved these design methods by accounting for the degree of rotational restraint of the TS at its periphery by the shell and the channel. This method was adopted by the French Pressure Vessel Code CODAP in 1982 and by the European Pressure Vessel Standard EN13445 in 2002.

Gardner [4] in 1969 improved his 1942 method by proposing a direct determination of the TS thickness which accounts for the unperforated rim and the TS-shell-channel connection. The method was adopted by ISO/DIS-2694 in 1973, by BS5500 in 1976 and by CODAP in 1995.

Soler [5] in 1984 developed a similar method accounting for the unperforated rim and the TS-shellchannel connection. The method is derived from fixed TS method and published for the first time in Non mandatory Appendix AA of Section VIII Division 1. In 2003 it was published in a new Part UHX of Section VIII Division 1 "Rules for Shell and Tubes Heat Exchangers" which became mandatory in 2004.

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3 GENERAL

3.1 TS Configurations (UHX-14.1)

(*a*) **The stationary TS** is attached to the shell and the channel by welding (integral TS) or by bolting (gasketed TS) in accordance with the following 6 configurations (see Figure 37):

- Configuration a: tubesheet integral with shell and channel;
- Configuration b: tubesheet integral with shell and gasketed with channel, extended as a flange;
- Configuration c: tubesheet integral with shell and gasketed with channel not extended as a flange;
- Configuration d: tubesheet gasketed with shell and channel extended as a flange or not;
- Configuration e: tubesheet gasketed with shell and integral with channel, extended as a flange;
- Configuration f: tubesheet gasketed with shell and integral with channel not extended as a flange.



(a) Configuration a: Tubesheet Integral With Shell and Channel



(d) Configuration d: Tubesheet Gasketed With Shell and Channel



(b) Configuration b: Tubesheet Integral With Shell and Gasketed With Channel, Extended as a Flange



Tubesheet Gasketed With Shell and Integral With Channel, Extended as a Flange



(c) Configuration c: Tubesheet Integral With Shell and Gasketed With Channel, Not Extended as a Flange



(f) Configuration f: Tubesheet Gasketed With Shell and Integral With Channel, Not Extended as a Flange

Figure 37 — Stationary Tubesheet Configurations

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- (*b*) **The floating TS** is free to move along the HE longitudinal axis in accordance with the following 4 configurations (see Figure 38):
 - Configuration A: integral tubesheet;
 - Configuration B: gasketed, tubesheet extended as a flange;
 - Configuration C: gasketed, tubesheet not extended as a flange;
 - Configuration D: internally sealed tubesheet.



Figure 38 — Floating Tubesheet Configurations

3.2 Notations

(a) Data for the design of the HE are as follows (UHX-14.3)

Symbols D_o , E*, h_g , μ , μ^* and ν^* are defined in Section 2 of PART 2.

- A = outside diameter of tubesheet
- a_c = radial channel dimension
- a_o = equivalent radius of outer tube limit circle
- a_s = radial shell dimension
- C = bolt circle diameter
- D_c = inside channel diameter
- D_s = Inside shell diameter
- d_t = nominal outside diameter of tubes
- E = modulus of elasticity for tubesheet material at T

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- E_c = modulus of elasticity for channel material at T_c
- E_s = modulus of elasticity for shell material at T_s
- E_t = modulus of elasticity for tube material at T_t
- G_c = diameter of channel gasket load reaction
- G_s = diameter of shell gasket load reaction
- G_1 = midpoint of contact between flange and tubesheet
- h = tubesheet thickness
- L_t = tube length between outer tubesheet faces
- N_t = number of tubes
- P_e = effective pressure acting on tubesheet
- P_s = shell side design or operating pressure. For shell side vacuum use a negative value for P_s .

 $P_t = \frac{\text{tube side design or operating pressure. For tube side vacuum use a negative value for }{P_t}$

Notation P_c , instead of P_t , is used throughout the analytical development so as to maintain the symmetry of the equations involving the shell (subscript s) and the channel (subscript c).

- S = allowable stress for tubesheet material at T
- S_c = allowable stress for channel material at T_c
- S_s = allowable stress for shell material at T_s
- S_t = allowable stress for tube material at T_t
- S_y = yield strength for tubesheet material at T
- $S_{y,c}$ = yield strength for channel material at T_c
- $S_{y,s}$ = yield strength for shell material at T_s
- $S_{y,t}$ = yield strength for tube material at T_t
- S_{PS} = allowable primary plus secondary stress for tubesheet material at T
- $S_{PS,c}$ = allowable primary plus secondary stress for channel material at T_c
- $S_{PS,s}$ = allowable primary plus secondary stress for shell material at T_s
- T = tubesheet design temperature
- T_a = ambient temperature, 70°F (20°C)
- T_c = channel design temperature
- T_s = shell design temperature
- T_t = tube design temperature
- t_c = channel thickness
- t_s = shell thickness
- t_t = nominal tube wall thickness
- W^* = tubesheet effective bolt load determined in accordance with UHX-8
- v = Poisson's ratio of tubesheet material
- v_c = Poisson's ratio of channel material
- v_s = Poisson's ratio of shell material
- v_t = Poisson's ratio of tube material

(*b*) **Design coefficients** (UHX-14.5.1 to 4)

The following coefficients, specific to each component of the HE, will be used in the analytical treatment. They complete the data given above.

(1) Perforated TS

Equivalent diameter of outer tube limit circle (see Section 4.3(a) of PART 2): $D_o = 2r_o + d_t$

Equivalent radius of outer tube limit circle: $a_o = \frac{D_o}{2}$

TS coefficients relating to the tubes:

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• Shell side:
$$x_s = 1 - N_t \left(\frac{d_t}{2a_o}\right)^2$$
; $1 - x_s = N_t \left(\frac{d_t}{2a_o}\right)^2$
• Tube side: $x_t = 1 - N_t \left(\frac{d_t - 2t_t}{2a_o}\right)^2$; $1 - x_t = N_t \left(\frac{d_t - 2t_t}{2a_o}\right)^2$

•
$$x_{t} - x_{s} = N_{t} \left(\frac{d_{t}^{2} - (d_{t} - 2t_{t})^{2}}{4 a_{o}^{2}} \right) = \frac{N_{t} \cdot s_{t}}{\pi a_{o}^{2}} = \frac{N_{t} \cdot k_{t}}{E_{t}} \cdot \frac{l}{\pi a_{o}^{2}} = \frac{N_{t} \cdot K_{t}}{E_{t}} \cdot \frac{L}{\pi a_{o}^{2}}$$

• Ligament efficiency:
$$\mu^* = \frac{p^* - d^*}{p^*}$$

- Effective tube hole diameter d* and effective pitch p* are defined in Section 4.3(d) and (c) of PART 2
- Effective elastic constants E* and v* are given in Section 5.6 of PART 2 as a function of μ* and h/p (triangular or square pitch).

• Bending stiffness:
$$D^* = \frac{E^* \cdot h^3}{12(1-v^{*2})}$$

- Effective tube side pass partition groove depth given in Section 4.3(f) of PART 2: h'_{g}
- Effective pressure acting on tubesheet: P_e

(2) **Tube bundle**

Tube cross-sectional area:
$$s_{t} = \frac{\pi}{4} \left[d_{t}^{2} - (d_{t} - 2t_{t})^{2} \right] = \pi t_{t} (d_{t} - t_{t}) = \frac{\pi a_{o}^{2}}{N_{t}} (x_{t} - x_{s})$$

Axial stiffness K_t of one tube: $K_{t} = \frac{E_{t} s_{t}}{L} = \frac{\pi t_{t} (d_{t} - t_{t}) E_{t}}{L}$
Axial stiffness k_t of one half tube of length l=L/2: $k_{t} = \frac{E_{t} s_{t}}{l} = \frac{2\pi t_{t} (d_{t} - t_{t}) E_{t}}{L} = 2 K_{t}$
Effective elastic foundation modulus equivalent to the half tube bundle:
 $k_{w} = \frac{N_{t} \cdot k_{t}}{\pi a_{o}^{2}} = \frac{2 N_{t} \cdot K_{t}}{\pi a_{o}^{2}} = \frac{2 N_{t} \cdot E_{t} \cdot t_{t} (d_{t} - t_{t})}{L a_{o}^{2}} = \frac{2 E_{t}}{L} (x_{t} - x_{s}) = \frac{E_{t}}{l} (x_{t} - x_{s})$
 $k = 4\sqrt{\frac{k_{w}}{D^{*}}}$; $x = k r$; $0 \le r \le a_{o} \implies 0 \le x \le k a_{o}$ $ka_{o} = X_{a}$
Axial stiffness ratio tubes/TS:

$$X_{a} = k a_{o} = \sqrt[4]{\frac{k_{w}}{D^{*}}} a_{o} = \left[24 \left(1 - v^{*2} \right) N_{t} \frac{E_{t} t_{t} \left(d_{t} - t_{t} \right) a_{o}^{2}}{E^{*} L h^{3}} \right]^{\frac{1}{4}}$$

(3) Shell

Radial shell dimension: a_s $\rho_s = \frac{a_s}{a_o}$ ST TS Integral configurations (a, b and c): $a_s = D_s / 2$ Gasketed configurations (d): $a_s = G_s / 2$

FL TS configurations (A, B, C and D): $a_s = a_c$ $a_s = \frac{D_s + t_s}{2}$ Mean shell radius: $s_s = \pi t_s \left(D_s + t_s \right)$ Shell cross-sectional area: Axial stiffness k's of the half shell of length l=L/2: $k_s = \frac{E_s s_s}{I} = \frac{2\pi t_s (D_s + t_s) E_s}{I} = 2 K_s$ Shell coefficient: $\beta_{\rm s} = \frac{\sqrt[4]{12(1-v_{\rm s}^2)}}{\sqrt{(D_{\rm s}+t_{\rm s})t_{\rm s}}}$ Bending stiffness: $k_{\rm s} = \beta_{\rm s} \frac{E_{\rm s} \cdot t_{\rm s}^3}{6(1 - v_{\rm s}^2)}$ (4) Channel $a_c \qquad \rho_c = \frac{a_c}{a}$ Radial channel dimension: Integral configurations (a, e, f, and A): $a_c = D_c / 2$ Gasketed configurations (b, c and d): $a_c = G_c / 2$ Gasketed configuration D: $a_{c} = A / 2$ $a_{s}' = \frac{D_{c} + t_{c}}{2}$ Mean channel radius: $\beta_{\rm c} = \frac{\sqrt[4]{12(1-v_{\rm c}^2)}}{\sqrt{(D_{\rm c}+t_{\rm c})t_{\rm c}}}$ Channel coefficient: $k_{\rm c} = \beta_{\rm c} \cdot \frac{E_{\rm c} \cdot t_{\rm c}^3}{6(1-v^2)}$ Bending stiffness: (5) Unperforated rim D_0 = internal diameter A = external diameterDiameter ratio: $K = A / D_{a}$

3.3 Loading Cases (UHX-14.4)

The normal operating condition of the HE is achieved when the tube side pressure P_t and shell side pressure P_s act simultaneously. However, a loss of pressure is always possible. Accordingly, for safety reasons, the designer must always consider the cases where $P_s=0$ or $P_t=0$ for the normal operating condition(s).

The designer must also consider the startup condition(s), the shutdown condition(s) and the upset condition(s), if any, which may govern the design.

A floating TS HE is a statically indeterminate structure for which it is generally not possible to determine the most severe condition of coincident pressure and temperature. Thus, it is necessary to evaluate all the anticipated loading conditions mentioned above to ensure that the worst load combination has been considered in the design.

For each of these conditions, ASME, TEMA and CODAP used to consider the following pressure loading cases.



- Loading Case 1: Tube side pressure P_t acting only ($P_s = 0$).
- Loading Case 2: Shell side pressure P_s acting only ($P_t = 0$).
- Loading Case 3: Tube side pressure P_t and shell side pressure P_s acting simultaneously.

ASME 2013 Edition provides the detail of the pressure "design loading cases" to be considered for each operating condition specified by the user (normal operating conditions, startup conditions, the shutdown conditions,...). For the pressure loading cases, a table (table UHX-14.4-1) provides the values to be used for the design pressures P_s and P_t in the formulas, accounting for their maximum and minimum values. Additional operating (thermal + pressure) loading cases must be considered if the effect of the radial thermal expansion adjacent to the tubesheet is accounted for (see Section 10.1)

As the calculation procedure is iterative, a value h is assumed for the tubesheet thickness to calculate and check that the maximum stresses in tubesheet, tubes, shell, and channel are within the maximum permissible stress limits.

Because any increase of tubesheet thickness may lead to overstresses in the tubes, shell, or channel, a final check must be performed, using in the formulas the nominal thickness of tubesheet, tubes, shell, and channel, in both corroded and uncorroded conditions.

3.4 Design Assumptions (UHX-14.2)

A FL TS HE is a complex structure and several assumptions are necessary to derive a 'design by rules' method. Most of them could be eliminated, but the analytical treatment would lead to 'design by analysis' method requiring the use of a computer.

The design assumptions are as follows.

(*a*) HE

- The analytical treatment is based on the theory of elasticity applied to the thin shells of revolution.
- The HE is axi-symmetrical.
- The HE is a symmetrical unit with identical TSs so as to analyze a half-unit.
- (b) TSs
 - The two tubesheets are circular and identical (same diameter, uniform thickness, material, temperature and edge conditions). Deviations will be allowed for the FL TS to cover the 3 types of HE.
 - The tubesheets are uniformly perforated over a nominally circular area, in either equilateral triangular or square patterns. This implies that each TS is fully tubed (no large untubed window)
 - Radial displacement at the mid-surface of the TS is ignored
 - Temperature gradient through TS thickness is ignored
 - Shear deformation and transverse normal strain in the TS are ignored
 - The unperforated rim of each TS is treated as a rigid ring without distortion of the cross section
- (c) Tubes
 - Tubes are assumed identical, straight and at same temperature
 - Tubes are uniformly distributed in sufficient density to play the role of an elastic foundation for the TS
 - The effect of the rotational stiffness of the tubes is ignored
- (*d*) Shell and channel
 - Shell and channel are cylindrical with uniform diameters and thicknesses
 - If the channel head is hemispherical, it must be attached directly to the TS, without any cylindrical section between the head and the TS.



- shell and channel centerlines are the same.
- (e) Weights and pressures drops
 - Weights and pressures drops are ignored
 - Pressures P_s and P_t are assumed uniform

3.5 Basis of Analytical Treatment

3.5.1 General

The design of a FL TS HE is complex as the two TSs are connected to each other through the tube bundle. Accordingly the structure is statically indeterminate. Many geometrical, mechanical and material properties are involved in the design as shown in Section 3.2(a) which lists the extensive input data. Although the FL TS differs from the ST TS (smaller channel diameter, edge conditions,...), the two tubesheets are assumed to be identical. This assumption may seem unrealistic, but if the real floating TS geometry is accounted for, the analytical treatment would lead to a 'design by analysis' method requiring the use of a computer.

As for a fixed TS HE, the analysis includes the effects of the shell and tube side pressures, the axial stiffening effect of the tubes, the stiffening effect of the unperforated ring at the tubesheet edge, and the stiffening effect of the integrally attached channel or shell to the tubesheet. For a tubesheet that is extended as a flange to which a channel or shell is to be bolted, the bolt load causes an additional moment in the tubesheet which is included in the total stress in the tubesheet in addition to the moments caused by pressure.

The analysis is based on classical discontinuity analysis methods to determine the moments and forces that the tubesheet, shell and channel must resist. These components are treated using the theory of elasticity applied to the thin shells of revolution.

Because the heat exchanger is assumed to be symmetric, only half of the heat exchanger is treated. The main steps of the ST TS design follow the analytical treatment of fixed TS HEs. The FL TS will be designed the same way in a second step, using its appropriate data (channel thickness and diameter, bolting data, edge conditions,...).

- (a) The tubesheet is disconnected from the shell and channel. The shear load V_a and moment M_a are applied at the tubesheet edge as shown in Figure 39.
- (b) The perforated tubesheet is replaced by an equivalent solid circular plate of diameter D_o and effective elastic constants E^* (effective modulus of elasticity) and v^* (effective Poisson's Ratio) depending on the ligament efficiency μ^* of the tubesheet. This equivalent solid plate is treated by the theory of thin circular plates subjected to pressures P_s and P_t and relevant applied loads to determine the maximum stresses.
- (c) The unperforated tubesheet rim is treated as a rigid ring whose cross section does not change under loading.
- (d) The tubes are replaced by an equivalent elastic foundation of modulus k_w .
- (e) The connection of the tubesheet with shell and channel accounts for the edge displacements and rotations of the 3 components.
- (f) The shell and channel are subject to shell side and tube side pressures P_s and P_t and edge loads V_a and M_a to determine the maximum stresses.
- (g) The maximum stresses in tubesheet, tubes, shell and channel are determined and limited to the appropriate allowable stress-based classifications of Section VIII Division 2 Part 4.



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Figure 39 — Analytical Model Used in Design Method

3.5.2 Free Body Diagram for ST TS

Figure 40 shows, for a ST TS integral both sides (configuration a), the free body diagram of the component parts (perforated tubesheet, unperforated tubesheet rim, shell, channel). The figure details the relevant discontinuity forces (V_a , V_s , Q_s , V_c , Q_c) and moments (M_a , M_s , M_c , M_R) applied on each component, together with edge displacements.

In this figure:

Forces $(V_a, V_s, Q_s, V_c, Q_c)$ and moments (M_a, M_s, M_c, M_R) are per unit of circumferential length The following subscripts are used:

- s for shell,
- c for channel,
- R for unperforated rim

No subscript for the perforated TS

Notation P_c instead of P_t (tube side pressure) is used throughout the analytical development so as to maintain the symmetry of the equations involving the shell (subscript s) and the channel (subscript c).



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Figure 40 — Free Body Diagram of the Analytical Model for the ST TS

3.5.3 Free Body Diagram for FL TS

The FL TS is attached to the FL channel either by welding (integral configuration A) or by bolting (gasketed configurations B or C), but there is no shell, which implies $V_s=0$ and $M_s=0$.

Accordingly, the free body diagram is different from the free body diagram of the ST TS and depends on the HE type shown on Figure 36: immersed, externally sealed, internally sealed. Figure 41 provides the free body diagram of an immersed FL TS with configuration A.

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Figure 41 — Free Body Diagram of the Analytical Model for the FL TS (floating channel is shown on Figure 48, Figure 49 and Figure 50 for the 3 types of FL TS HEs)

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4 AXIAL DISPLACEMENTS AND FORCES ACTING ON THE TUBES AND ON THE SHELL

4.1 Axial Displacement and Force Acting on the Tubes (Figure 42)



Figure 42 — Axial Displacement of Tubes

(a) Axial Displacement of the tubes due to axial force $V_t(r)$ acting on the tube row at radius r:

$$\delta_{t}\left(V_{t}\right) = \frac{V_{t}\left(r\right)}{k_{t}} = \frac{V_{t}\left(r\right)l}{E_{t} s_{t}} = \frac{l}{\pi E_{t} \left(d_{t}-t_{t}\right)t_{t}} V_{t}\left(r\right)$$

(b) Axial force acting on each tube at radius r: $V_t(r) = k_t[\delta_t(V_t)]$

(c) Net effective pressure acting on the TS due to each tube at radius r of TS area $\pi a_0^2 / N_t$:

$$q_{t}(r) = \frac{-V_{t}(r)}{\pi a_{0}^{2} / N_{t}} = -\frac{N_{t}k_{t} \delta_{t}(V_{t})}{\pi a_{0}^{2}} = -\frac{N_{t}k_{t}}{\pi a_{0}^{2}} \delta_{t}(V_{t}) \qquad k_{w} = \frac{N_{t}k_{t}}{\pi a_{0}^{2}} \qquad \boxed{q_{t}(r) = -k_{w} \delta_{t}(V_{t})}$$

4.2 Axial Displacement and Force Acting on the Shell (Figure 43)





(a) Axial displacement of the shell due to axial force V_s acting on the shell:

$$\delta_{s}(V_{s}) = \frac{V_{s} \ 2\pi \ a_{s}}{k_{s}} = \frac{V_{s} \ 2\pi \ a_{s}}{E_{s} \ 2\pi \ a_{s}} t_{s}} = \frac{l}{E_{s} \ t_{s}} V_{s} \qquad V_{s} \text{ is per unit of length}$$

$$a_{s} = mean \ shell \ radius = \frac{D_{s} + t_{s}}{2}$$

$$V_{s} \text{ is known and depends on the HE type (see VI.2).}$$

(b) Axial force acting on the shell: $V_s = \frac{E_s t_s}{l} \delta_s(V_s)$

(c) Axial displacement of tubes at radius r:

 $\delta_t(V_t) = \delta_s(V_s) + w(r)$ where w(r) is the TS deflection at radius r (see Figure 45)

(d) **TS deflection at radius r:** $w(r) = \delta_t(V_t) - \delta_s(V_s)$

5 DEFLECTION AND LOADS ACTING ON THE TUBESHEET

5.1 Equivalent Plate Resting on an Elastic Foundation (Figure 44)

(a) Net effective pressure



Figure 44 — Loads Acting on TS

due to tubes: $q_{t}(r) = -k_{w} \delta_{t}(V_{t})$

Tubes act as an elastic foundation of equivalent modulus given by the axial rigidity of the halfbundle per unit of TS area:

$$k_{w} = \frac{N_{t} k_{t}}{\pi a_{o}^{2}} \qquad q_{t}(r) = -k_{w} \left[\delta_{s}(V_{s}) + w(r)\right]$$

due to pressures P_s and P_t acting on the equivalent plate (see Annex E);

 $q_{\rm P} = x_{\rm s} P_{\rm s} - x_{\rm t} P_{\rm t} = \Delta p^*$

net effective pressure:

$$q(r) = q_{\rm P} + q_t(r) = \Delta p^* - \frac{V_t(r)}{\pi a_o^2 / N_t} \quad [V.1a]$$

$$q(r) = \underbrace{\Delta p^* + k_{\rm w} \left[-\delta_{\rm s}(V_s) \right]}_{Q} - k_{\rm w} w(r) \qquad \boxed{Q = \Delta p^* - k_{\rm w} \delta_{\rm s}(V_s)} \quad [V.1a']$$

In this equation, the displacement δ_s of the shell subjected to axial force V_s is known and $q(r) = Q - k_w w(r)$

(*b*) **Deflection of TS** (Figure 45)

The determination of the deflection given in Section 5.1(b) of PART 3 applies:

$$w(x) = A \ berx + B \ beix + \frac{Q}{k_w} \quad \text{where:} \ x = k \ r = \sqrt[4]{\frac{k_w}{D^*}} \ r$$

Quantities q(x), $\theta_r(x)$, $Q_r(x)$ and $M_r(x)$ are respectively given in Section 5.1(a), 5.1(c), 5.1(d) and 5.1(e) of PART 3

At TS periphery (r =a_o):
$$X_{a} = k a_{o} = \sqrt[4]{\frac{k_{w}}{D^{*}}} a_{o} = \left[24 \left(1 - v^{*2} \right) N_{t} \frac{E_{t} t_{t} \left(d_{t} - t_{t} \right) a_{o}^{2}}{E^{*} L h^{3}} \right]^{\frac{1}{4}}$$



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Figure 45 — TS Displacement

5.2 Determination of Integration Constants A and B

The determination of the integration constants A and B given in Section 5.2 of PART 3 applies. Substituting the expressions of A and B in V.1b enables to determine:

 $w(x), q(x), \theta(x), Q(x), M(x)$

as functions of x, depending on V_a and M_a which are still unknown.



6 TREATMENT OF THE UNPERFORATED RIM

6.1 Edge Loads Applied on Shell and Channel at their Connection to the TS

ST TS: Section 6.1(a) and 6.1(b) of PART 3 apply.

FL TS: Section 6.1(a) of PART 3 does not apply as there is no shell.

Accordingly, the shell coefficients β_s , k_s , λ_s , δ_s : shall be taken equal to 0, and a_s shall be taken equal to a_c .

Section 6.1(b) of PART 3 applies.

6.2 Equilibrium of the Unperforated Rim

6.2.1 Due to Axial Loads

(a) The ring equilibrium for the ST TS is written (see Figure 46):

$$2\pi a_o V_a + \pi \left(a_s^2 - a_o^2\right) P_s + 2\pi a_c V_c = \pi \left(a_c^2 - a_o^2\right) P_c + 2\pi a_s V_s \qquad [VI.2.1.1]$$

where:

 V_a = axial edge load acting at connection of ring with equivalent plate is still to be determined V_s = axial force acting in the shell, which depends on the HE type (immersed, externally sealed, internally sealed).

 $V_c = axial \text{ force acting in the stationary channel: } 2 \pi a_c V_c = \pi a_c^2 P_c \implies a_c V_c = \frac{a_c^2}{2} P_c$



Figure 46 — Ring Equilibrium of the ST TS

(b) The ring equilibrium for the FL TS is written in the same way (see Figure 47), but there is no shell connected to the TS (V_s=0): $2\pi a_o V_a + \pi (a_s^2 - a_o^2) P_s + 2\pi a_c V_c = \pi (a_c^2 - a_o^2) P_c$ [VI.2.1.2] where:

V_a = axial edge load acting at connection of ring with equivalent plate is still to be determined

 V_c = axial force acting in the floating channel, which depends on the HE type (immersed, externally sealed, internally sealed).





- (c) Accordingly, the free body diagram of the FL TS is different from the free body diagram of the ST TS and depends on the HE type shown on Figure 36: immersed, externally sealed, internally sealed, which must be considered separately to calculate the axial edge load V_a (point A) and the axial force V_E acting at the periphery of the ring (point E).
 - (1) Immersed FL HE (see Figure 48)

For ST TS:
$$2\pi a_{s}^{'}V_{s} = \pi a_{s}^{2}P_{s} \\ 2\pi a_{c}^{'}V_{c} = \pi a_{c}^{2}P_{c} \end{bmatrix} \underbrace{V_{E} = 2\pi a_{c}^{'}V_{c} - 2\pi a_{s}^{'}V_{s} = \pi a_{c}^{2}P_{c} - \pi a_{s}^{2}P_{s}}$$

which, combined with [VI.2.1.1], leads to:

$$2\pi a_{o}V_{a} = \pi a_{o}^{2}(P_{s} - P_{c}) \implies V_{a} = \frac{a_{o}}{2}(P_{s} - P_{c})$$

For FL TS: $2\pi a_{c}V_{c} = \pi a_{c}^{2}P_{c} - \pi a_{s}^{2}P_{s}$ $V_{E} = 2\pi a_{c}V_{c} = \pi a_{c}^{2}P_{c} - \pi a_{s}^{2}P_{s}$

which, combined with [VI.2.1.2], leads to:

$$2\pi a_o V_{\rm a} = \pi a_o^2 (P_{\rm s} - P_{\rm c}) \implies V_{\rm a} = \frac{a_o}{2} (P_{\rm s} - P_{\rm c})$$

(2) externally sealed FL HE (see Figure 49)

For ST TS:
$$2\pi a_{s}^{'}V_{s} = 0$$
$$2\pi a_{c}^{'}V_{c} = \pi a_{c}^{2}P_{c}$$
$$V_{E} = 2\pi a_{c}^{'}V_{c} - 2\pi a_{s}^{'}V_{s} = \pi a_{c}^{2}P_{c}$$

which, combined with [VI.2.1.1], leads to:

$$2\pi a_o V_a = \pi \left(a_o^2 - a_s^2\right) P_s - \pi a_o^2 P_c = \pi a_o^2 \left[\left(1 - \rho_s^2\right) P_s - P_c \right] \Rightarrow V_a = \frac{a_o}{2} \left[\left(1 - \rho_s^2\right) P_s - P_c \right]$$

For FL TS: $2\pi a_c^2 V_c = \pi a_c^2 P_c$ $V_E = 2\pi a_c^2 V_c = \pi a_c^2 P_c$
which, combined with [VI.2.1.2], leads to:

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$$2\pi a_{o}V_{a} = \pi \left(a_{o}^{2} - a_{s}^{2}\right)P_{s} - \pi a_{o}^{2}P_{c} = \pi a_{o}^{2}\left[\left(1 - \rho_{s}^{2}\right)P_{s} - P_{c}\right] \implies V_{a} = \frac{a_{o}}{2}\left[\left(1 - \rho_{s}^{2}\right)P_{s} - P_{c}\right]$$

(3) internally sealed FL HE (see Figure 50)

For ST TS:
$$2\pi a_{s}^{'}V_{s} = \pi a_{s}^{2}P_{c}^{'}\\2\pi a_{c}^{'}V_{c} = \pi a_{c}^{2}P_{c}^{'} \} \underbrace{V_{E} = 2\pi a_{c}^{'}V_{c} - 2\pi a_{s}^{'}V_{s} = 0}_{(assuming \ a_{c}^{'} = a_{s}^{'})}$$

which, combined with [VI.2.1.1], leads to:

$$2\pi a_o V_a = \pi \left(a_o^2 - a_s^2\right) P_s - \pi \left(a_o^2 - a_s^2\right) P_c = \pi a_o^2 \left(1 - \rho_s^2\right) \left(P_s - P_c\right) \Rightarrow V_a = \frac{a_o}{2} \left(1 - \rho_s^2\right) \left(P_s - P_c\right)$$

For FL TS:
$$V_c = 0 \implies V_E = 2\pi a_c V_c = 0$$

which, combined with [VI.2.1.2], leads to:

$$2\pi a_o V_{\rm a} = \pi \left(a_o^2 - a_s^2 \right) P_s - \pi \left(a_o^2 - a_s^2 \right) P_{\rm c} = \pi a_o^2 \left[\left(1 - \rho_s^2 \right) \left(P_s - P_c \right) \right] \Rightarrow V_{\rm a} = \frac{a_o}{2} \left(1 - \rho_s^2 \right) \left(P_s - P_c \right) \right]$$

Thus, the axial edge load V_a (point A) and the axial force V_E acting at the periphery of the ring (point E) are the same for the ST TS and the FL TS, which enables the FL HE to be considered as a fixed TS HE for the analytical treatment.

However, the FL TS is different from the ST TS as its internal radius is smaller, it is not connected to the shell and the edge moments are different, except for the externally sealed HE which can be considered as a fixed TS HE with an expansion bellows of rigidity (see Note 2 of Section 7.2 of PART 3).

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Figure 48 — Immersed Floating TS HE



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Figure 50 — Internally Sealed Floating TS HE

Note: The determination of the axial load V_s acting in the shell enables the calculation of the shell membrane stress:

 $\sigma_{s,m} = \frac{V_s}{t}$

• For immersed Floating TS HE:

$$V_{s} = \frac{a_{s}^{2}P_{s}}{2a_{s}} \quad leads \ to \ : \sigma_{s,m} = \frac{a_{s}^{2}}{\left(D_{s} + t_{s}\right)t_{s}} P_{s}$$

- For internally sealed Floating TS HE: $V_s = 0$ leads to : $\sigma_{s,m} = 0$
- For externally sealed Floating TS HE: $V_s = \frac{a_s^2 P_c}{2a_s}$ leads to $:\sigma_{s,m} = \frac{a_s^2}{(D_s + t_s)t_s} P_c$

These formulas match the general shell membrane stress formula given in Section 8.6(a) of PART 3, using for P_e the relevant formula given in VII hereafter.

6.2.2 Due to Applied Moments

ST TS: Section 6.2(b) of PART 3 applies.

FL TS: Section 6.2(b) of PART 3 applies using $\beta_s=0$, $k_s=0$, $\lambda_s=0$ and $\delta s=0$, which leads to:

$$\omega_{s}=0$$
 and $\omega_{s}^{*}=a_{o}^{2}\left[\frac{\left(\rho_{s}^{2}-1\right)\left(\rho_{s}-1\right)}{4}\right]$

6.2.3 Edge Loads V_a and M_a Applied to the Tubesheet

(a) **Determination of Ma:** Section 6.3(a) of PART 3 applies, which leads to: $M_a = (a_o V_a)Q_1 + Q_2$

(b) **Determination of V**_a: V_a is known and depends on each HE type, as determined in VI.2.1.3. Accordingly, Section 6.3(b) and 6.3(c) of PART 3 are not relevant for FL TSs and quantities Q_{Z1} , Q_{Z2} and U do not apply.

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7 EQUIVALENT PRESSURE ACTING ON THE TUBESHEET

A circular plate under uniform pressures P_s and P_t is subjected to a differential pressure $P_e = P_s - P_t$ The axial force V_a at periphery is determined from the plate equilibrium:

$$2\pi a_o V_a = P_e \pi a_o^2 \implies a_o V_a = \frac{a_o^2 P_e}{2} \qquad P_e = \frac{2V_a}{a_o}$$

$$P_t$$

$$P_t$$

$$P_e = P_s - P_t$$

$$P_s$$

Where V_a is given in Section 6.2.1(c) for:

- (a) immersed FL HE: $V_{a} = \frac{a_{o}}{2}(P_{s} P_{c}) \Rightarrow P_{e} = P_{s} P_{t}$ (b) externally sealed FL HE: $V_{a} = \frac{a_{o}}{2} \left[\left(1 - \rho_{s}^{2} \right) P_{s} - P_{c} \right] \Rightarrow P_{e} = \left(1 - \rho_{s}^{2} \right) P_{s} - P_{t}$
- (c) internally sealed FL HE: $V_a = \frac{a_o}{2} \left[\left(1 \rho_s^2 \right) \left(P_s P_c \right) \right] \Rightarrow \left[P_e = \left(1 \rho_s^2 \right) \left(P_s P_t \right) \right]$

As shown in Section 6.2.1(c), these formulas apply both for the ST and FL TSs.



8 STRESSES IN THE HEAT-EXCHANGER COMPONENTS

Stress formulas calculated in Section 8 of PART 3 apply.



9 DETERMINATION OF ALLOWABLE STRESS LIMITS

The determination of the allowable stress limits developed in Section 9 of PART 3 apply.



10 ADDITIONAL RULES

The following additional rules apply in the same way as for the fixed TS HE covered in PART 3:

- (1) Effect of radial thermal expansion adjacent to the tubesheet (UHX-14.6), covered in Section 10.3 of PART 3.
- (2) Calculation procedure for simply supported TSs (UHX-14.7), covered in Section 10.4 of PART 3
- (3) Effect of plasticity at tubesheet-shell-channel joint (UHX-14.8), covered in Section 10.2 of PART 3.
- (4) Tubesheet flange extension (UHX-9), covered in Section 10.5 of PART 3.



11 HOW TO USE THE RULES

UHX-14 rules have been developed on the basis that the floating TS HE could be treated as a fixed TS, with some modifications due to the floating TS which is free to move axially inside the shell as outlined previously.

11.1 Stationary TS

The ST TS is designed in the same way as the fixed TSs rules of UHX-13, with the following modifications:

- the floating TS is free to move axially, which implies $K_J=0$ and J=0,
- there is no axial differential thermal expansion between tubes and shell, which implies $T_{t,m}=0$, $T_{s,m}=0$ and $\gamma=0$,
- the equivalent pressure depends on the HE type and is defined in Section 7,
- Coefficients Q_{Z1}, Q_{Z2} and U do not apply.

11.2 Floating TS

The FL TS needs some adaptations as it differs from the ST TS:

- no attached shell,
- channel diameters (A, C, D_s, G_s, G₁) are smaller than the shell diameters,
- channel thickness t_c may differ from the ST channel thickness
- TS configuration is different from the ST TS configuration.

The FL TS shall be designed in the same way as the ST TS, using:

- its own geometrical data (A, C, D_s, G_s,G₁, t_c),
- its own TS configuration,
- $a_s=a_c$ which is needed for the calculations of Q_1 , ω_s^* and ω_c^* .

So as to maintain a minimum of symmetry between the two TS, the FL TS shall have the same material and the same design temperature as the ST TS , which implies that the FL TS material properties are those of the ST TS (E_c , v_c , $S_{y,c}$), and the same thickness as the ST TS.

11.3 Calculation Procedure

Like for the fixed TS HE, the calculation procedure can be summarized as follows:

- Set the data listed in Section 3.2(a)
- Calculate the design coefficients listed in Section 3.2(b) ($x_s, x_t; \rho_s, \rho_c$)
- Calculate first characteristic parameter X_a and coefficients Q_1, Q_2 ; ω_s^*, ω_c^*
- Calculate the equivalent pressure Pe, and second characteristic parameter Q3
- Calculate the maximum stresses in TS, tubes, shell and channel and limit their values to the maximum allowable stress limits.

Because of the complexity of the procedure, it is likely that users will computerize the solution. A Mathcad calculation sheet is provided for the immersed floating head TS HE defined in PTB-4 Example E4.18.8.

The stationary TS is gasketed with shell and channel (configuration d) and the floating TS is gasketed with the channel, not extended as a flange (configuration C). The data are shown in the sheet and the calculations follow strictly steps 1 to 10 of UHX-14.5 calculation procedure. The calculation sheet is



provided both for the ST and the FL TS. See Annex W and Annex X for UHX-14-Example E4.18.8 (PTB-4 2013 Edition) Stationary and Floating TS respectively.

11.4 Calculation Using a Fixed TS HE Software

The ST TS can be calculated using a fixed TS HE software, such as the Mathcad software used in Annex V, provided that the other TS is free to move axially by using a bellows of rigidity close to 0, which simulates an externally sealed FL HE. As shown by Section 7.2-Note 2 of PART 3, $P_e = (1 - \rho_s^2)P_s - P_t$, which is effectively the equivalent pressure formula for this type of HE. For immersed floating head HE and internally sealed HE, it is necessary to replace P_e respectively by $P_e = P_s - P_t$ and

 $P_e = (1 - \rho_s^2)(P_s - P_t)$ in the software.



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- [3] GALLETLY "Optimum design of Thin Circular Plates on an Elastic Foundation", Proceedings of the Institution of Mechanical Engineers", London Vol. 173, 1952.
- [4] GARDNER "Tubesheet design: a basis for standardization." ASME publication of 1st ICPVT conference–Delft. Part 1 Design and Analysis-1969.
- [5] SOLER "Mechanical Design of Heat Exchangers" Arcturus publishers -1984 1047 pages.

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PART 5 ANALYTICAL TREATMENT OF U-TUBE TUBESHEET HEAT EXCHANGERS



1 SCOPE

PART 5, devoted to U-tube TS HEs (Figure 51), provides the technical basis for the determination of the stresses acting on the TS, shell and channel and their relationships with the design rules of UHX-12.





2 HISTORICAL BACKGROUND

The first rules for the design of U-tube TSs were published in Appendix AA of Section VIII, Division 1 in 1982. They were based on the 1969 Gardner method [1] to cover configurations a (integral construction) and b (gasketed construction), with some improvements:

- use of an effective pitch p* to account for unperforated diametral lanes,
- use of an effective tube diameter d^* to account for the tube expansion depth ratio ρ ,
- derivation of a direct formula to determine the tubesheet thickness.

Later on, Urey Miller [2], as a member of the ASME Special Working Group on Heat Transfer Equipment, developed a more refined analytical approach accounting for the unperforated rim for configurations b and e (tubesheet extended as a flange), which were not covered before. The method was adopted in ASME in 1992.

This new set of rules was not totally satisfactory for the following reasons:

- Three different set of rules, based on different analytical approaches, were proposed to cover configurations a, b, d and e.
- Configurations c and f (gasketed tubesheet not extended as a flange) were not covered.
- Rule for configuration "d" covered only the case were the tubesheet was not extended as a flange, with gaskets both sides of same diameter,
- Rule for configuration "a" used the same formula, corrected by a TEMA coefficient F which did not account properly for the degree of restraint of the tubesheet by the shell and channel.

Accordingly, Osweiller in 2002 [3] developed a more refined and unique approach to cover the six tubesheet configurations. This approach is based on Urey Miller's method mentioned above, with the following improvements:

- treatment of configurations c and f where the tubesheet is not extended as a flange.
- accounting for local pressures acting on shell and channel, when integral with the tubesheet.
- use of Poisson's ratio v in all formulas, rather than using v=0.3, which leads to inexact coefficients.
- derivation of more condensed formulas providing rules consistent with fixed tubesheet rules.

This method was published for the first time in Nonmandatory Appendix AA of Section VIII Division 1. In 2003 it was published in a new part UHX of Section VIII Division 1 "Rules for Shell and Tubes Heat Exchangers" which became mandatory in 2004.

Soler [4] in 1984 developed a similar method accounting for the unperforated rim and the TS-shellchannel connection.



3 GENERAL

3.1 TS Configurations (UHX-12.1)

(*a*) **The stationary TS** is attached to the shell and the channel by welding (integral TS) or by bolting (gasketed TS) in accordance with the following 6 configurations (see Figure 3752):

- Configuration a: tubesheet integral with shell and channel;
- Configuration b: tubesheet integral with shell and gasketed with channel, extended as a flange;
- Configuration c: tubesheet integral with shell and gasketed with channel, not extended as a flange;
- Configuration d: tubesheet gasketed with shell and channel extends as a flange or not;
- Configuration e: tubesheet gasketed with shell and integral with channel, extended as a flange;
- Configuration f: tubesheet gasketed with shell and integral with channel, not extended as a flange.



(a) Configuration a: Tubesheet Integral With Shell and Channel





(b) Configuration b: Tubesheet Integral With Shell and Gasketed With Channel, Extended as a Flange



(e) Configuration e: Tubesheet Gasketed With Shell and Integral With Channel, Extended as a Flange

Figure 52 — TS Configurations



(c) Configuration c: Tubesheet Integral With Shell and Gasketed With Channel, Not Extended as a Flange



(f) Configuration f: Tubesheet Gasketed With Shell and Integral With Channel, Not Extended as a Flange

3.2 Notations

(a) Data for the design of the HE are as follows (UHX-12.3)

Symbols D_o , E^* , h_g , μ , μ^* and ν^* are defined in Section 2 of PART 2.

- A = outside diameter of tubesheet
- a_c = radial channel dimension
- a_o = equivalent radius of outer tube limit circle
- a_s = radial shell dimension
- C = bolt circle diameter
- D_c = inside channel diameter

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- D_s = Inside shell diameter
- E = modulus of elasticity for tubesheet material at T
- E_c = modulus of elasticity for channel material at T_c
- E_s = modulus of elasticity for shell material at T_s
- G_c = diameter of channel gasket load reaction
- G_s = diameter of shell gasket load reaction
- G_1 = midpoint of contact between flange and tubesheet
- h =tubesheet thickness
- N_t = number of tubes
- P_s = shell side design pressure. For shell side vacuum use a negative value for P_s .
- P_t = tube side design pressure. For tube side vacuum use a negative value for P_t . Notation P_c , instead of P_t , is used throughout the analytical development so as to maintain the symmetry of the equations involving the shell (subscript s) and the channel (subscript c).
- S = allowable stress for tubesheet material at T
- S_c = allowable stress for channel material at T_c
- S_s = allowable stress for shell material at T_s
- $S_{y,c}$ = yield strength for channel material at T_c
- $S_{y,s}$ = yield strength for shell material at T_s
- $S_{y,t}$ = yield strength for tube material at T_t
- S_{PS} = allowable primary plus secondary stress for tubesheet material at T
- $S_{PS,c}$ = allowable primary plus secondary stress for channel material at T_c
- $S_{PS,s}$ = allowable primary plus secondary stress for shell material at T_s
 - T = tubesheet design temperature
 - T_c = channel design temperature
 - T_c = channel design temperature
 - T_s = shell design temperature
 - t_c = channel thickness
 - t_s = shell thickness
 - t_t = nominal tube wall thickness

 W_s , W_c = shell or channel flange design bolt load for the gasket seating condition

- W^* = tubesheet effective bolt load determined in accordance with UHX-8
- v_c = Poisson's ratio of channel material
- v_s = Poisson's ratio of shell material

(b) Design coefficients (UHX-12.5.1 to 5)

The following coefficients, specific to each component of the HE, will be used in the analytical treatment. They complete the data given above.

- (1) Perforated TS
 - Equivalent diameter of outer tube limit circle (see Section 4.3(a) of PART 2): $D_o = 2r_o + d_t$
 - Equivalent radius of outer tube limit circle: $a_o = \frac{D_o}{2}$

TS coefficients:

- Ligament efficiency: $\mu^* = \frac{p^* d^*}{p^*}$
- Effective tube hole diameter d* and effective pitch p* are defined in Section 4.3(c) and (d) of PART 2
- Effective elastic constants E* and v* are given in Section 5.6 of PART 2 as a function of μ^* and h/p (triangular or square pitch).

• Bending stiffness:
$$D^* = \frac{E^* \cdot h^3}{12(1-v^{*2})}$$

- Effective tube side pass partition groove depth given in Section 4.3(f) of PART 2: h'_{g} •
- Effective pressure acting on tubesheet: P_e •
- (2) Shell
 - $a_s \quad \rho_s = \frac{a_s}{a_s}$ Radial shell dimension: •
 - Integral configurations (a, b and c): $a_s = D_s/2$ ٠
 - Gasketed configuration (d, e and f): $a_s = G_s / 2$
 - $a_{s}' = \frac{D_{s} + t_{s}}{2}$ Mean shell radius: • $\beta_{\rm s} = \frac{\sqrt[4]{12(1-v_{\rm s}^2)}}{\sqrt{(D_{\rm s}+t_{\rm s})} t_{\rm s}}}$
 - Shell coefficient: •
 - Bending stiffness: $k_s = \beta_s \frac{E_s \cdot t_s^3}{6(1 v_s^2)}$

(3) Channel

- $a_c \qquad \rho_c = \frac{a_c}{a_o}$ Radial channel dimension: •
- Integral configuration (a): •
- Gasketed configurations (b, c and d): •
- Mean channel radius: •
 - Channel coefficient:

$$a_{c}^{'} = \frac{D_{c} + t_{c}}{2}$$

$$\beta_{c} = \frac{\sqrt[4]{12(1 - v_{c}^{2})}}{\sqrt{(D_{c} + t_{c})t_{c}}}$$

 $a_c = D_c / 2$

 $a_c = G_c / 2$

• Bending stiffness:
$$k_{\rm c} = \beta_{\rm c} \cdot \frac{E_{\rm c} \cdot t_{\rm c}^3}{6(1 - v_{\rm c}^2)}$$

(4) Unperforated rim

- D_0 = internal diameter
- A = external diameter
- Diameter ratio: $K = A / D_a$

3.3 Loading Cases (UHX-12.4)

The normal operating condition of the HE is achieved when the tube side pressure P_t and shell side pressure P_s act simultaneously. However, a loss of pressure is always possible. Accordingly, for safety reasons, the designer must always consider the cases where $P_s=0$ and $P_t=0$ for the normal operating condition(s).

The designer must also consider the start-up condition(s), the shut-down condition(s) and the upset condition(s), if any, which may govern the design.

For each of these conditions, ASME, TEMA and CODAP used to consider the following pressure loading cases.

- Loading Case 1: Tube side pressure P_t acting only ($P_s = 0$).
- Loading Case 2: Shell side pressure P_s acting only ($P_t = 0$).
- Loading Case 3: Tube side pressure Pt and shell side pressure Ps acting simultaneously.

ASME 2013 Edition provides the detail of the pressure "design loading cases" to be considered for each operating condition specified by the user (normal operating conditions, startup conditions, the shutdown conditions,...).A table (table UHX-12.4-1) provides the values to be used for the design pressures P_s and P_t in the formulas, accounting for their maximum and minimum values.

As the calculation procedure is iterative, a value h is assumed for the tubesheet thickness to calculate and check that the maximum stresses in tubesheet, shell and channel are within the maximum permissible stress limits.

3.4 Design Assumptions (UHX-12.2)

A U-tube TS HE is a complex structure and several assumptions are necessary to derive a 'design by rules' method.

Most of them could be eliminated, but the analytical treatment would lead to 'design by analysis' method requiring the use of a computer.

The design assumptions are as follows.

(*a*) HE

- The analytical treatment is based on the theory of elasticity applied to the thin shells of revolution.
- The HE is axi-symmetrical.

(*b*) TS

- The tubesheet is circular.
- The tubesheet is uniformly perforated over a nominally circular area, in either equilateral triangular or square patterns. This implies that the TS is fully tubed (no large untubed window)
- Radial displacement at the mid-surface of the TS is ignored
- Temperature gradient through TS thickness is ignored
- Shear deformation and transverse normal strain in the TS are ignored
- The unperforated rim of the TS is treated as a rigid ring without distortion of the cross section
- (c) Shell and channel
 - Shell and channel are cylindrical with uniform diameters and thicknesses
 - Shell and channel centerlines are the same
- (d) Weights and pressures drops
 - Weights and pressures drops are ignored
 - Pressures P_s and P_t are assumed uniform

3.5 Basis of Analytical Treatment

3.5.1 General

Comparison of Figure 3651 and Figure 5136 of PART 4 shows that the U-tube TS HE can be considered as an immersed floating head TS HE where the floating TS does not exist and the tubes do not play the role of an elastic foundation.

Accordingly, the U-tube HE could be designed as an immersed floating head TS HE where the tubes have no axial rigidity (see Annex U). However the analytical treatment presented below is based on the approach developed by Urey Miller [2] as explained in Section 2.

Although the design is less complex, many geometrical, mechanical and material properties are involved in the design as shown in Section 3.2(a) which lists the extensive input data.

As for a floating TS HE, the analysis includes the effects of the shell and tube side pressures, the stiffening effect of the unperforated ring at the tubesheet edge and the stiffening effect of the integrally attached channel or shell to the tubesheet. When the tubesheet is gasketed with the shell or the channel, the bolt load causes an additional moment in the tubesheet which is included in the total stress in the tubesheet in addition to the moments caused by pressure.

The analysis is based on classical discontinuity analysis methods to determine the moments and forces that the tubesheet, shell and channel must resist. These components are treated using the elastic theory of thin shells of revolution.

The main steps of the U-tube TS design follow the analytical treatment of floating TS HEs.

- (a) Tubesheet is disconnected from the shell and channel. Shear load V_a and moment M_a are applied at the tubesheet edge as shown in Figure 53.
- (b) Perforated tubesheet is replaced by an equivalent solid circular plate of diameter D_o and effective elastic constants E^* (effective modulus of elasticity) and v^* (effective Poisson's ratio) depending on the ligament efficiency μ^* of the tubesheet. This equivalent solid plate is treated by the theory of thin circular plates subjected to pressures P_s and P_t and relevant applied loads to determine the maximum stresses.
- (c) Unperforated tubesheet rim is treated as a rigid ring whose cross section does not change under loading.
- (d) Connection of the tubesheet with shell and channel accounts for the edge displacements and rotations of the 3 components.
- (e) Shell and channel are subject to shell side and tube side pressures P_s and P_t and edge loads V_a and M_a to determine the maximum stresses.
- (f) Maximum stresses in tubesheet shell and channel are determined and limited to the appropriate allowable stress-based classifications of Section VIII Division 2 Part 4.

3.5.2 Free Body Diagram

Figure 53 shows, for a TS integral both sides (configuration a), the free body diagram of the component parts (perforated tubesheet, unperforated tubesheet rim, shell, channel). The figure details the relevant discontinuity forces (V_a , V_s , Q_s , V_c , Q_c) and moments (M_a , M_s , M_c , M_R) applied on each component, together with edge displacements.

In this figure: Forces $(V_a, V_s, Q_s, V_c, Q_c)$ and moments (M_a, M_s, M_c, M_R) are per unit of circumferential length

The following subscripts are used:

- s for shell,
- c for channel,
- R for unperforated rim

No subscript is used for the perforated TS



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Notation P_c instead of P_t (tube side pressure) is used throughout the analytical development so as to maintain the symmetry of the equations involving the shell (subscript s) and the channel (subscript c).



Figure 53 — Free Body Diagram of the Analytical Model for the TS



4 TREATMENT OF THE PERFORATED TUBESHEET

The perforated tubesheet is treated as an equivalent solid circular plate of radius a_0 with effective elastic constants E^* and ν^* , subjected to an equivalent pressure P_e and edge loads V_a and M_a at periphery. The equilibrium equation of the plate is written: $-2 \pi a_o V_a + \pi a_o^2 P_e = 0$

which leads to: $V_{a} = \frac{a_{o}}{2} P_{e}$ [IV-1]

The rotation of the plate at radius a_o is given by: $\theta_a = \frac{12(1-\nu^*)}{E^*h^3} \left[a_o M_a + \frac{a_o^3}{8} P_e \right]$ [IV-2]

The radial bending moment at radius r of such a plate is given by the classical formula:

$$M(r) = M_{a} + (3 + v^{*}) \frac{a_{0}^{2}}{16} \left[1 - \left(\frac{r}{a_{o}}\right)^{2} \right] P_{0}$$

The maximum bending moment appears

- either at center (r=0): $M(r) = M_a + (3 + v^*) \frac{a_0^2}{16} P_e$
- or at periphery (r=a_o): $M(r) = M_a$



5 TREATMENT OF THE UNPERFORATED RIM

5.1 Edge Loads Applied on Shell and Channel at their Connection to the TS

The following equations are developed for integral shell and channel.

(a) The edge loads Q_s and M_s applied on the shell write (see Section 6.1(a) of PART 3):

 $\left\{ \begin{array}{l} M_{s} = +k_{s} \left(1 + \frac{t_{s}}{2}\right) \theta_{s} + \beta_{s} k_{s} \delta_{s} P_{s} \\ Q_{s} = -\beta_{s} k_{s} \left(1 + t_{s}\right) \theta_{s} - 2\beta_{s}^{2} k_{s} \delta_{s} P_{s} \end{array} \right\}$ [V.1a] for an integral shell (configurations a, b, c)

When the shell is not integral with the TS (configurations d, e, f), $k_s=0$ and $\delta s=0$ lead to: $M_s=0$ and $Q_s=0$.

Note: These formulas are valid for a shell of sufficient length. Annex J provides the minimum length above which these formulas can be applied.

(b) The edge loads Q_c and M_c applied on the channel write (see Section 6.1(b) of PART 3):

 $\left\{ \begin{array}{l} M_{c} = +k_{c} \left(1 + \frac{t_{c}}{2}\right) \theta_{c} + \beta_{c} k_{c} \delta_{c} P_{c} \\ Q_{c} = -\beta_{c} k_{c} \left(1 + t_{c}\right) \theta_{c} - 2\beta_{c}^{2} k_{c} \delta_{c} P_{c} \end{array} \right\} \quad [V.1b] \text{ for an integral channel (configurations, a e, f)}$

When the channel is not integral with the TS (configurations b, c, d), $k_c = 0$ and $\delta_c = 0$ lead to: $M_c = 0$ and $Q_c = 0$

Note 1: These formulas are valid for a channel of sufficient length. Annex J provides the minimum length above which these formulas can be applied.

Note 2: These formulas are valid for a cylindrical channel. If the channel is hemispherical, it must be attached directly to the TS (configurations a, b or c), without any cylindrical section between the head and the TS. Annex K provides the relevant formulas for that case. Only coefficient δ_c is affected:

$\delta_c =$	D_c^2	$\left(\underline{1-v_c}\right)$		
	$\overline{4 E_c t_c}$	$\left(\begin{array}{c} 2 \end{array}\right)$		

5.2 Equilibrium of the Unperforated Solid Rim

(a) due to axial loads
$$\begin{cases} a_{s} = D_{s} / 2 \\ a_{s} = a_{s} + \frac{t_{s}}{2} \end{cases} \quad \begin{cases} a_{c} = D_{c} / 2 \\ a_{c} = a_{c} + \frac{t_{c}}{2} \end{cases}$$

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Figure 54 — Ring Equilibrium of the TS

 $2\pi a_s V_s = \pi a_s^2 P_s$

The axial equilibrium of the ring is written: (see Figure 54):

$$2\pi a_{s}' V_{s} + \pi \left(a_{c}^{2} - a_{o}^{2}\right) P_{c} = 2\pi a_{c}' V_{c} + 2\pi a_{o} V_{a} + \pi \left(a_{s}^{2} - a_{o}^{2}\right) P_{s}$$

The axial equilibrium of the shell is written:

The axial equilibrium of the channel is written: $2\pi a_c V_c = \pi a_c^2 P_c$

This leads to:
$$V_{\rm a} = \frac{a_o}{2} (P_{\rm s} - P_{\rm c})$$

Comparison with equation [IV-1] shows that $P_e = P_s - P_t$

as for an immersed floating head HE.

(b) due to applied moments

Equilibrium of moments applied to the ring relative to the axis located at radius a_0 enables to determine the moment M_R (see Figure 53).

$$R = \text{radius at center of ring} = \frac{A + 2a_o}{4}$$

$$RM_{R} = -\left[a_o M_{a}\right] + \left[a'_c M_c - a'_c Q_c \frac{h}{2}\right] + \left[M\left(P_c\right) - a'_c V_c\left(a'_c - a_o\right)\right]$$

$$-\left[a'_s M_s - a'_s Q_s \frac{h}{2}\right] + \left[M\left(P_s\right) - a'_s V_s\left(a'_s - a_o\right)\right]\right]$$

 $M(P_{\rm c}) =$ moment due to pressure P_c acting on the ring = $\left(a_{\rm c}^2 - a_o^2\right) \left(\frac{a_{\rm c} + a_o}{2} - a_o\right) \frac{P_{\rm c}}{2}$

 $M(P_{\rm s}) =$ moment due to pressure P_s acting on the ring $= \left(a_{\rm s}^2 - a_{\rm o}^2\right) \left(\frac{a_{\rm s} + a_{\rm o}}{2} - a_{\rm o}\right) \frac{P_{\rm s}}{2}$

From Annex L:

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$$\begin{bmatrix} a'_{s} M_{s} - a'_{s} Q_{s} \frac{h}{2} \end{bmatrix} = a'_{s} k_{s} \left(1 + t'_{s} + \frac{t'^{2}}{2} \right) \theta_{s} + a_{o} \omega_{s} P_{s} \qquad \boxed{\omega_{s} = \rho_{s} \beta_{s} k_{s} \delta_{s} (1 + h \beta_{s})} \qquad t'_{s} = h \beta_{s}$$

$$\begin{bmatrix} M(P_{c}) - a'_{c} V_{c} (a'_{c} - a_{o}) \end{bmatrix} = \frac{P_{c}}{4} \left[\left(a^{2}_{c} - a^{2}_{o} \right) (a_{c} - a_{o}) - 2a^{2}_{c} (a'_{c} - a_{o}) \right] = \frac{P_{c}}{4} (a_{c} - a_{o}) (-a^{2}_{c} - a^{2}_{o}) = \frac{P_{c}}{4} a^{3}_{o} \left[(\rho_{c} - 1) (\rho^{2}_{c} + 1) \right]$$

$$\begin{bmatrix} M(P_{s}) - a'_{s} V_{s} (a'_{s} - a_{o}) \end{bmatrix} = \frac{P_{s}}{4} \left[\left(a^{2}_{s} - a^{2}_{o} \right) (a_{s} - a_{o}) - 2a^{2}_{s} (a'_{s} - a_{o}) \right] = \frac{P_{s}}{4} (a_{s} - a_{o}) (-a^{2}_{s} - a^{2}_{o}) = \frac{P_{s}}{4} a^{3}_{o} \left[(\rho_{s} - 1) (\rho^{2}_{s} + 1) \right]$$

$$\begin{bmatrix} R M_{R} \end{bmatrix} = -a_{o} M_{a} + \frac{P_{s}}{4} a^{3}_{o} \left[(\rho_{s} - 1) (\rho^{2}_{s} + 1) \right] - \frac{P_{c}}{4} a^{3}_{o} \left[(\rho_{c} - 1) (\rho^{2}_{c} + 1) \right]$$

$$- \left[a'_{s} k_{s} \left(1 + t'_{s} + \frac{t'^{2}}{2} \right) + a'_{c} k_{c} \left(1 + t'_{c} + \frac{t'^{2}}{2} \right) \right] \theta_{a} + a_{o} \left(\omega_{c} P_{c} - \omega_{s} P_{s} \right) \right]$$

$$\begin{bmatrix} V.2b' \end{bmatrix}$$

(c) Rotation of rigid ring

$$K = \frac{A}{D_o} \quad \theta_{\rm R} = \frac{12}{E h^3} \frac{R M_{\rm R}}{Ln K} \text{ leads to, with } \theta_{\rm R} = \theta_{\rm a} :$$
$$R M_{\rm R} = \left[\frac{E h^3}{12} Ln K\right] \theta_{\rm R} = \left[\frac{E h^3}{12} Ln K\right] \theta_{\rm a}$$

Replacing RM_R by its expression [V.2b'] permits to calculate the TS rotation θ_a :

$$\begin{bmatrix} \frac{E}{12} h^{3} \ln K + a_{s}^{'} k_{s} \left(1 + t_{s}^{'} + \frac{t_{s}^{'2}}{2}\right) + a_{c}^{'} k_{c} \left(1 + t_{c}^{'} + \frac{t_{c}^{'2}}{2}\right) \right] \theta_{a} = -a_{o} M_{a} \\ + P_{s} \frac{a_{o}^{3}}{4} \left[\left(\rho_{s} - 1\right) \left(\rho_{s}^{2} + 1\right) \right] - P_{c} \frac{a_{o}^{3}}{4} \left[\left(\rho_{c} - 1\right) \left(\rho_{c}^{2} + 1\right) \right] + a_{o} \left[\omega_{c} P_{c} - \omega_{s} P_{s} \right] \right] \\ C_{1} = \frac{h^{3}}{12} \left[\frac{6}{h^{3}} \left(D_{s} + t_{s}\right) k_{s} \left(1 + t_{s}^{'} + \frac{t_{s}^{'2}}{2}\right) + \frac{6}{h^{3}} \left(D_{c} + t_{c}\right) k_{c} \left(1 + t_{c}^{'} + \frac{t_{c}^{'2}}{2}\right) + E \ln K \right] = \frac{h^{3}}{12} \left[\lambda_{s} + \lambda_{c} + E \ln K \right] \\ \frac{\lambda_{s}}{\lambda_{s}} = \frac{6}{h^{3}} \left(D_{s} + t_{s}\right) k_{s} \left(1 + t_{s}^{'} + \frac{t_{s}^{'2}}{2}\right) \left[\lambda_{c} = \frac{6}{h^{3}} \left(D_{c} + t_{c}\right) k_{c} \left(1 + t_{c}^{'} + \frac{t_{c}^{'2}}{2}\right) \right] \\ Note: \text{ if } \lambda + \lambda \text{ is high } (>3) \text{ the TS can be considered as clamped}$$

Where the TS rotation
$$\theta_{a}$$
 is given by [IV-2]: $\theta_{a} = \frac{12}{E*h^{3}} \left[a_{o} M_{a} + \frac{a_{o}^{3}}{8} P_{e} \right]$ which permits to calculate M_{a} :

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$$\frac{h^{3}}{12} [\lambda_{s} + \lambda_{c} + ELnK] \frac{12(1-\nu^{*})}{E^{*}h^{3}} \left[a_{o}M_{a} + \frac{a_{o}^{3}}{8} P_{e} \right] + a_{o}M_{a} = P_{s} \frac{a_{o}^{3}}{4} \left[(\rho_{s} - 1)(\rho_{s}^{2} + 1) \right] - P_{c} \frac{a_{o}^{3}}{4} \left[(\rho_{c} - 1)(\rho_{c}^{2} + 1) \right] + a_{o} \left[\omega_{c}P_{c} - \omega_{s}P_{s} \right]$$

$$\underbrace{\left[\lambda_{s} + \lambda_{c} + ELnK \right] \frac{(1-\nu^{*})}{E^{*}} \left[M_{a} + \frac{a_{o}^{2}}{8} P_{e} \right] + M_{a} = P_{s} \frac{a_{o}^{2}}{4} \left[(\rho_{s} - 1)(\rho_{s}^{2} + 1) \right] - P_{c} \frac{a_{o}^{2}}{4} \left[(\rho_{c} - 1)(\rho_{c}^{2} + 1) \right] + \left[\omega_{c}P_{c} - \omega_{s}P_{s} \right]$$

$$\underbrace{\left[1+F \right] M_{a} = -F \frac{a_{o}^{2}}{8} P_{e} + \underbrace{P_{s} \frac{a_{o}^{2}}{4} \left[(\rho_{s} - 1)(\rho_{s}^{2} + 1) \right] - P_{c} \frac{a_{o}^{2}}{4} \left[(\rho_{c} - 1)(\rho_{c}^{2} + 1) \right] + \left[\omega_{c}P_{c} - \omega_{s}P_{s} \right]$$

$$\underbrace{M_{TS}}$$

In this equation:

 M_{TS} is the the moment due to pressures P_s and P_t acting on the rigid ring:

$$M_{TS} = P_{\rm s} \frac{a_o^2}{4} \Big[\big(\rho_{\rm s} - 1\big) \big(\rho_{\rm s}^2 + 1\big) \Big] - P_{\rm c} \frac{a_o^2}{4} \Big[\big(\rho_{\rm c} - 1\big) \big(\rho_{\rm c}^2 + 1\big) \Big]$$

F denotes the degree of restrain of the TS by the shell and channel when they are integral with the TS (configuration a):

$$F = \left[\lambda_{\rm s} + \lambda_{\rm c} + ELnK\right] \frac{\left(1 - \nu^*\right)}{E^*}$$

Note:

If there is no bending support from the channel $(k_c = 0)$ and the shell $(k_s = 0)$: $\lambda_s = 1$ and $\lambda_c = 1$ lead to *F* close to zero:

 $M_a=M_{TS}$ and the tubesheet is almost simply supported. If additionally there is no unperforated rim $(K=1, \rho_s=0, \rho_c=0)$:

F = 0 and $M_a=0$: the tubesheet is fully simply supported. If there is a high bending support from the channel $(k_c = \infty)$ and the shell $(k_s = \infty)$: $\lambda_c = \infty$ and $\lambda_s = \infty$ lead to $F = \infty$.

The tubesheet is fully clamped:
$$M_a = -\frac{D_o^2}{32} (P_s - P_c)$$

Finally, for configuration a, M_a is written:
$$M_a = \frac{M_{TS} + (\omega_c P_c - \omega_s P_s) - F\left(\frac{a_o^2}{8} P_e\right)}{1 + F}$$
 [V.2c]

(d) Generic equation covering the 6 configurations a to f

Equation [VI.2d] of PART 3 shows that a term $\frac{a_o}{2\pi} \left[W_c \gamma_{bc} - W_s \gamma_{bs} \right]$, which accounts for the

flange bolt loads W_s and W_c , must be added in equation [V.2c] to obtain the generic equation for the 6 configurations.

Rules of UHX-12 cover the case where $C_s=C_c=C$. Accordingly, equation [V.2c] giving M_a becomes:

$$M_{a} = \frac{M^{*}}{M_{TS} + (\omega_{c}P_{c} - \omega_{s}P_{s}) + \frac{\gamma_{b}}{2\pi}[W_{c} - W_{s}] - F\left(\frac{a_{o}^{2}}{8}P_{e}\right)}{1 + F}$$

$$M_{a}^{*} = \frac{M_{TS}^{*} + (\omega_{c}P_{c} - \omega_{s}P_{s}) + \frac{\gamma_{b}}{2\pi}W^{*} - F\left(\frac{a_{o}^{2}}{8}P_{e}\right)}{1+F}$$
$$M^{*} = M_{TS} + (\omega_{c}P_{c} - \omega_{s}P_{s}) + \frac{\gamma_{b}}{2\pi}[W_{c} - W_{s}]$$

- When the shell is integral with the TS (configurations a, b, c): $W_s=0$
- When the channel is integral with the TS (configurations a, e, f): $W_c=0$
- When the shell and the channel are gasketed with the TS, the highest bolting moment controls and W_c- W_s must be replaced by: W_{max}=MAX (W_s,W_c) and W* = W_{max}

UHX-12 covers these configurations through the TS effective bolt load W* which is explicated in UHX-12.5.6 for each configuration a to f.

$$M^* = M_{TS} + (\omega_c P_c - \omega_s P_s) + \frac{\gamma_b}{2\pi} W^*$$

$$M_{TS} = P_s \frac{a_o^2}{4} \Big[(\rho_s - 1) (\rho_s^2 + 1) \Big] - P_c \frac{a_o^2}{4} \Big[(\rho_c - 1) (\rho_c^2 + 1) \Big]$$

$$F = [\lambda_s + \lambda_c + ELnK] \frac{(1 - \nu^*)}{E^*}$$

$$M_a = \frac{M^* - F \Big(\frac{a_o^2}{8} P_e \Big)}{1 + F}$$

The 1^{st} term accounts for the moments due to pressures P_s and P_c and bolting loads W_s and W_c acting on the unperforated rim. The second term accounts for the moment due to pressures P_s and P_c acting on the perforated TS.

This formula applies as follows for the 6 configurations, depending on coefficients λ_s , λ_c , ω_s , ω_c , γ_b .

- For configuration a: λ_s , λ_c and ω_s , ω_c are calculated from the HE data, $\gamma_b = 0$
- For configuration b: λ_s is calculated, $\lambda_c=0$, ω_s is calculated, $\omega_c=0$, $\gamma_b = \frac{G_c C}{D_a}$
- For configuration c: λ_s is calculated, $\lambda_c=0$, ω_s is calculated, $\omega_c=0$, $\gamma_b = \frac{G_c G_1}{D_o}$
- For configuration d: $\lambda_s=0$ $\lambda_c=0$, $\omega_s=0$, $\omega_c=0$, $\gamma_b = \frac{G_c G_s}{2}$
- For configuration e: $\lambda_s=0$, λ_c is calculated, $\omega_s=0$, ω_c is calculated, $\gamma_b = \frac{C-G_s}{D_o}$
- For configuration f: $\lambda_s=0$, λ_c is calculated, $\omega_s=0$, ω_c is calculated, $\gamma_b = \frac{G_1 G_s}{D_o}$

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6 STRESSES IN THE HEAT-EXCHANGER COMPONENTS

6.1 Stresses in the Tubesheet

(a) **Bending stress**

as explained in Section 4, the maximum bending moment M(r) appears:

either at periphery (r=a_o):
$$M(r) = M_a$$
, noted M_P in UHX-12 rules



or at center (r=0): $M_o = M_P + (3 + v^*) \frac{a_0^2}{16} P_e$

The maximum moment $M = MAX[|M_p|, |M_o|]$

leads to the maximum bending stress in the TS:

(b) Shear stress

The shear stress is maximum at periphery:
$$\tau = \frac{V_a}{\mu h}$$
 $\tau = \frac{a_o}{2\mu h} P_a$

Note: ASME and TEMA rules provide the same formula, but they use the equivalent diameter D_L corresponding to the perimeter of the outermost ligaments, instead of the equivalent diameter D_o of outer tube limit circle. The equivalent diameter D_L is calculated from the perimeter of the tube layout, $C_p = \pi D_L$, and the area $A_p = \pi D_L^2/4$ enclosed by this perimeter. This leads to $D_L = 4A_p/C_p$

 D_L is always lower than D_o and leads to a lower TS shear stress: $\tau = \frac{1}{4\mu} \frac{D_L}{h} P_e$

The ASME formula should be used when the shear stress controls the design, generally in highpressure cases.

6.2 Stresses in the Shell and Channel

(a) Axial membrane stress in the shell is given by the classical formula:

$$\sigma_{\rm s,m} = \frac{a_{\rm s}^2}{\left(D_{\rm s} + t_{\rm s}\right)t_{\rm s}} P_{\rm s}$$

For the channel

$$: \sigma_{c,m} = \frac{a_c^2}{\left(D_c + t_c\right)t_c} P_c$$

(c) Bending stress

The bending moment M_s in the shell at its connection with the TS exists only when the shell is integral with the TS (configurations a, b, c). As explained in Annex J, the shell must have a minimum length $l_{s,\min} = 1.8\sqrt{D_s t_s}$ adjacent to the TS.

[V.1a]:
$$M_s = k_s \left[\beta_s \delta_s P_s + \left(1 + \frac{t_s}{2}\right) \theta_s \right]$$
 with $\theta_s = \theta_a$ given by [V.5]:

$$\theta_{a} = \frac{12(1-\nu^{*})}{E^{*}h^{3}} \left[a_{o}M_{a} + \frac{a_{o}^{3}}{8}P_{e} \right]$$
$$M_{s} = k_{s} \left[\beta_{s}\delta_{s}P_{s} + \frac{12(1-\nu^{*})}{E^{*}}\frac{a_{o}}{h^{3}} \left(1 + \frac{t_{s}}{2}\right) \left(M_{a} + \frac{a_{o}^{2}}{8}P_{e}\right) \right]$$

The shell bending stress $\sigma_{s,b}$ is written: $\sigma_{s,b} = 6 \frac{M_s}{t_c^2}$

$$\sigma_{s,b} = \frac{6}{t_s^2} k_s \left[\beta_s \delta_s P_s + 6 \frac{1 - \nu^*}{E^*} \frac{D_o}{h^3} \left(1 + \frac{h \beta_s}{2} \right) \left(M_p + \frac{D_o^2}{32} \left(P_s - P_t \right) \right) \right]$$

The channel bending stress is obtained in the same way, noting that $\theta_c = -\theta_a$:

$$\sigma_{c,b} = \frac{6}{t_c^2} k_c \left[\beta_c \delta_c P_c - 6 \frac{1 - \nu^*}{E^*} \frac{D_o}{h^3} \left(1 + \frac{h \beta_c}{2} \right) \left(M_p + \frac{D_o^2}{32} \left(P_s - P_t \right) \right) \right]$$

6.3 Determination of Stresses using the Fixed TS Rules

The TS, shell and channel stresses determined here above can also been obtained from the fixed TS analysis of PART 3 as shown in Annex T.



7 DETERMINATION OF THE ALLOWABLE STRESS LIMITS

The determination of the allowable stress limits developed in Section 9 of PART 3 for the design loading cases apply. However, the allowable stress limit for the TS has been upgraded to 2S for the following reasons.

Comparison of TEMA formula with classical plate formula (see Section 10) shows that TEMA allows the bending stress in the equivalent solid plate to be about 2S, instead of 1.5 S recommended by ASME Section VIII Div. 2. Nevertheless, for about 40 years, TEMA formula did not lead to failures in U-tube tubesheets. It is likely that a value of allowable stress of 2S could be used without affecting the safety margin. This value was also recommended by GARDNER in 1969 [1] and has been used in most European codes (BS 5500, CODAP) for about 20 years. This is also justified by limit load analysis applied to circular plates, which leads to 1.9S if the tubesheet is simply supported and 2.1S if the tubesheet is clamped.

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8 ADDITIONAL RULES

8.1 Effect of Plasticity at the Tubesheet-Shell-Channel Joint (UHX-12.5)

In the same way as for fixed TS HE rules, an elastic-plastic analysis is proposed when integral shell and/or channel are overstressed. The concept is explained in Section 10.2(a) of PART 3.

The procedure for the shell applies when $1.5S_s < |\sigma_s| \le S_{PS}$

The elastic-plastic method is simplified: the reduced modulus of elasticity E_s^* for the shell is based on the degree of overstress in the shell: $E_s^* = E_s \sqrt{1.5S_s / \sigma_s}$.

Quantities affected by the elastic-plastic procedure are those which are involved in the TS-shell-channel joint and which involve E_s , namely k_s which affects λ_s leading to new value for F.

 δ_s is not affected because it is used only in $\omega_s = \rho_s \beta_s k_s \delta_s (1 + h\beta_s)$ in which E_s is cancelled out by the product $k_s \delta_s$. Same procedure applies to the channel by replacing subscript "s" by subscript "c".

If $|\sigma| \le 1.5S$, the design is acceptable. Otherwise, the HE geometry must be reconsidered. For Example E4.18.4 given in Section 9, the results for the elastic calculation are as follows for the controlling loading case 2:

Stiffening coefficient F=0.96, σ =38175<2S=40000, σ_c =56955>1.5S_c=30000, but lower than S_{PS,c}=65000 Similar results for the channel.

For the elastic-plastic calculation: the stiffening coefficient F decreases to 0.85, and σ =39838<2S=40000

In this example the elastic-plastic calculation leads to an increase of the TS stress, which happens in most cases.



9 HOW TO USE THE RULES

Similar to the fixed TS HE, the calculation procedure can be summarized as follows:

- Set the data listed in Section 3.2(a)
- Calculate the shell design coefficients (ρ_s, k_s, λ_s, δ_s, ω_s), and the channel design coefficients (ρ_c, k_c, λ_c, δ_c, ω_c)
- Calculate coefficient F which represents the degree of restrain of the TS by the shell and channel
- Calculate moments M_{TS} and M*
- Calculate the maximum stresses in TS, shell and channel and limit their values to the maximum allowable stress limits.

Because of the complexity of the procedure, it is likely that users will computerize the solution. A Mathcad calculation sheet is provided for The U-Tube TS HE defined in PTB-4 Example E4.18.4.The TS is gasketed with shell and integral with channel (configuration e). The data are shown in the sheet and the calculations follow strictly the steps 1 to 11 of UHX-12.5 calculation procedure. The elastic-plastic procedure is used. See Annex Y for UHX-12-Example E4.18. (PTB-4 2013 Edition).

The U-Tube HE can also be calculated using floating TS HE software, as shown in Annex U.

10 COMPARISON WITH TEMA RULES

10.1 TEMA Formula

The original TEMA formula for the determination of the TS thickness in bending: $T = F \frac{G}{2} \sqrt{\frac{P}{S}}$ is

based on flat circular plates formula: $h_0 = C \frac{G}{2} \sqrt{\frac{P}{\mu^* \Omega S}}$

Which is written, for $\mu^*=0.5$ and $\Omega=1.5$: $h_0 = \frac{C}{\sqrt{0.75}} \frac{G}{2} \sqrt{\frac{P}{S}}$

where coefficient $\frac{C}{\sqrt{0.75}}$ is equal to:

- 1.285 for simple support, whereas TEMA uses F=1.25
- for clamping, which is the value used in TEMA (F=1.0)

The TEMA formula was modified later to introduce a ligament efficiency η :

$$T = F \frac{G}{3} \sqrt{\frac{P}{\eta S}}$$

The ligament efficiency η is based on the mean width of the ligament (see Section 4.2 of PART 2):

For Triangular pitch:
$$\eta = 1 - \frac{0.907}{(p/d_t)^2}$$
 For Square pitch: $\eta = 1 - \frac{0.785}{(p/d_t)^2}$

TEMA mean ligament efficiency η (usually around 0.5) is greater than ASME ligament efficiency μ^* (usually around 0.3) which is based on the minimum width of the ligament. Coefficient 3 in new TEMA formula has been tailored so that old and new formulasgive approximately the same results.

10.2 Numerical Comparisons

Numerical comparisons have been performed using the four U-tube tubesheet heat exchangers treated in PTB-4 (2013 Edition) Examples E4.18.1 thru E4.18.4. Table 3 shows the results obtained by ASME and TEMA for the tubesheet bending thickness. Examples 1 thru 4 in Table 3 correspond to Examples E4.18.1 thru E4.18.4 respectively.

In the 4 examples, TEMA considers the tubesheet as simply supported (TEMA coefficient F = 1.25). However UHX-12 method shows that in Example 1 the tubesheet is almost clamped (UHX-12 coefficient F=9.4), due to the high bending rigidities of the shell and channel as compared to the tubesheet bending rigidity.

Thicknesses obtained by UHX are slightly less than TEMA thicknesses, except for Example 3 where the ASME ligament efficiency is much smaller than TEMA ligament efficiency.

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EXAMPLE		LIGAMENT EFFICIENCIES		TEMA	ASME		
N°	Config.	Pitch	μ (ASME) (TEMA)	µ* (ASME)	η (TEMA)	Τ (using η)	h (using μ*)
1	а	square	0.25	0.35	0.56	0.61	0.52
2	d	triang.	0.17	0.28	0.37	1.27	1.28
3	d	triang.	0.20	0.24	0.42	3.43	4.15
4	e	square	0.25	0.39	0.56	3.91	3.46

Table 3 — Comparison of TEMA and ASME TS Thicknesses for 4 U-tube HEs



REFERENCES—PART 5

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- [3] Francis Osweiller "New common design rules for U-Tube HEs in ASME, CODAP and UPV Codes" 2002 PVP Conference (Vancouver) Pressure Vessel and Piping Codes and Standards volume p.229-240.
- [4] Alan SOLER "Mechanical Design of Heat Exchangers" Arcturus publishers -1984 1047 pages.

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PART 6 SUMMARY AND CONCLUSIONS



1 SUMMARY AND CONCLUSIONS

- (*a*) The purpose of this document is to justify and provide technical criteria for the rules of Part UHX of ASME Section VIII Division 1, 2013 Edition, devoted to the design of U-tube, Fixed and Floating head Tubesheet Heat Exchangers. The document is structured in 5 PARTS.
 - PART 1: "Introduction" explains how the UHX rules are structured and provides the heat exchangers types, the loading cases, and the tubesheet configurations covered.
 - PART 2: "Tubesheet Characteristics" provides the technical basis of UHX-11 which covers the ligament efficiencies and the effective elastic constants of the tubesheet.
 - PART 3: "Fixed Tubesheets Heat Exchangers" provides the technical basis of UHX-13.
 - PART 4: "Floating Tubesheet Heat Exchangers" provides the technical basis of UHX-14.
 - PART 5: "U-tube Tubesheet Heat Exchangers" provides the technical basis of UHX-12.

PARTS 3, 4 and 5 are independent and structured in the same way (Scope, Historical background, Notations, Tubesheet Configurations, Loading cases, Design assumptions, and Analytical treatment). Floating and U-tube tubesheet heat exchangers are treated as simplified cases of fixed tubesheet heat exchangers. Accordingly, this type of heat exchanger is treated first in PART 3 which forms the main part of this document.

(b) The analytical treatment is based on classical discontinuity analysis methods to determine the moments and forces that the tubesheet, tubes, shell and channel must resist. These components are treated using the elastic theory of thin shells of revolution.

The heat exchanger is assumed to be a symmetrical unit with identical tubesheets on both ends and the unperforated rim is considered as a rigid circular ring.

The tubes are assumed to be identical and uniformly distributed throughout the tubesheets. Additional assumptions are necessary to perform the analytical treatment and derive the "design by rules" method of UHX.

- (c) Fixed Tubesheets heat exchangers are covered in PART 3. Main steps of the analytical treatment are as follows.
 - (1) Tubesheet is disconnected from the shell and channel.
 - (2) Perforated tubesheet is replaced by an equivalent solid circular plate with effective elastic constants E^* and v^* which depend on the ligament efficiency μ^* of the tubesheet. This equivalent solid circular plate is treated by the theory of thin circular plates subjected to pressure and other applied loads to determine the maximum stresses.
 - (3) Unperforated tubesheet rim is treated as a rigid ring whose cross section does not change under loading.
 - (4) Tubes are replaced by an equivalent elastic foundation of modulus k_w .
 - (5) Connection of the tubesheet to the shell and channel accounts for the edge displacements and rotations of the 3 components.
 - (6) Shell and channel are treated by the elastic theory of thin shells of revolution subjected to edge loads to determine the maximum stresses.

The treatment provides, at any radius r of the perforated tubesheet:

- the deflection w(r)
- the rotation $\theta(\mathbf{r})$
- the bending stress $\sigma(\mathbf{r})$
- the shear stress $\tau(\mathbf{r})$
- the axial stress in the tubes $\sigma_t(r)$

These quantities, which are not given in UHX rules, allow one to determine how the tubesheet deflects and the radii corresponding to the locations of the maximum bending stress in the tubesheet and the maximum axial stress in the tube bundle.

These stresses depend on two parameters:

- X_a, which represents the ratio of the tube- bundle axial stiffness to the tubesheet bending stiffness.
- Q₃, which accounts for the moments due to tube side and shell side pressures and bolt loads acting on the tubesheet.

A parametric study performed with these two parameters permits the derivation of the formulas for the maximum stress in the tubesheet and in the tubes which are given in UHX-13.

The treatment also determines the loads and displacements acting on the shell and channel at their connection to the tubesheets and the loads applied on the unperforated rim. These are not given in UHX rules which provide only the shell and channel stresses.

Maximum stresses in tubesheet, tubes, shell and channel are determined and limited to the appropriate allowable stress-based classifications of Section VIII Division 2 Appendix 4, 2004 edition.

- (*d*) Floating Tubesheet and U-tube Tubesheet heat exchangers are treated as simplified cases of fixed tubesheets heat exchangers.
- (e) This document provides the bases for all UHX formulas, especially the following:
 - shell and channel coefficients β , k, λ and δ for the calculation of coefficients ω and ω *
 - parameter X_a
 - coefficient F for the calculation of coefficients Φ , Q_1 , Q_2 and parameter Q_3 .
 - coefficient J when the shell has an expansion joint.
 - coefficient γ_b when the tubesheet has a gasketed flanged connection to the shell or channel.
 - equivalent pressures P'_{s} , P'_{t} , P_{γ} , P_{w} , P_{rim} for the calculation of the effective pressure P_{e} .
 - coefficients F_m and F_t for the calculation of maximum bending stress σ and maximum shear stress τ in the tubesheet
 - maximum axial stress $\sigma_t(r)$ and buckling stress limit S_{tb} in the tubes
 - axial membrane stress $\sigma_{s,m}$ and axial bending stress $\sigma_{s,b}$ in the shell and similar formulas for the channel.

This document provides the mechanical signification basis for the various coefficients and parameters noted above.

The results of the analytical treatment have shown that all UHX formulas were correct, except for four of them which are mentioned in PART 3. These formulas have been corrected in the 2010 and subsequent editions of UHX.

- (f) The basis of the additional UHX rules is provided in this document. They concern:
 - The effect of different shell thickness adjacent to the tubesheet
 - The effect of plasticity at tubesheet-shell-channel joint
 - The effect of radial thermal expansion adjacent to the tubesheet
 - The calculation procedure for simply supported tubesheets
 - The derivation of the tubesheet flange extension
 - The case where the heat exchanger has a thin-walled or thick-walled expansion joint
- (g) Checking of the results obtained has been made by comparison with FEA and other code rules. FEA results obtained for the axial tube stresses $\sigma_t(r)$ throughout the tubesheet match UHX results with discrepancies of less than 5%. The comparisons were not as close for the tubesheet bending stresses

and these discrepancies are still being evaluated by the Sub-Group on Heat Transfer Equipment. They might be due to the unperforated rim considered as a rigid ring in the analytical treatment.

The French pressure vessel code, CODAP, is based on the same analytical approach as UHX, but additional simplifications have been made (unperforated rim is ignored). When incorporating these simplifications in the UHX method, all CODAP formulas have been retrieved. This demonstrates that both methods are correct and consistent. This consistency extends to TEMA: by implementing the additional TEMA simplifications, all TEMA formulas have been confirmed.

UHX formulas have also been used to simulate circular plates under pressure by ignoring the tubesheet holes, the unperforated rim, the tubes and the connection with the shell and channel. The classical formulas for circular plates subjected to pressure have been obtained.

Thus, applying the relevant simplifications, it has been analytically demonstrated that the UHX-13.3 method leads to CODAP, TEMA and circular plate formulas. This confirms the correctness of the ASME method, which has also been confirmed by FEA comparisons for the tubesheet deflection.

(*h*) It is thought that justification and documentation of the basis for UHX-rules is important for the SG-HTE members for future reference and developments or if additional confirmation or comparisons are required.

It will be a valuable reference for early career engineers that are using the UHX rules or becoming involved in code developments of such rules in the future.
ANNEX A — VALUES OF EFFECTIVE ELASTIC CONSTANTS FROM VARIOUS AUTHORS

Table 4 — Comparison of Effective Elastic Constants E* and v* Values by Various Theoretical Methods for Plane Stress Problem

	effective	DIREC	T METHOD	UNDIRECT METHOD
μ	elastic constants	Meijers [5]	Grigolyuk and Fil'shtinskii [11]	Slot – O'Donnell [7]
0,1 0,2 0,3 0.4 0,5 0,7 0,8	E _A */E	0,048 2 0,146 2 0,267 5 0,397 0,529 1 0,789 5 0,899	0,048 2 0,146 1 0,396 8 0,899 6	0,048 2 0,146 2 0,267 5 0,396 3 0,529 1 0,789 5 0,898 6
0,1 0,2 0,3 0,4 0,5 0,7 0,8	(v = 0,3)	0,684 4 0,488 6 0,384 1 0,337 0,319 4 0,307 0 0,303	0,683 4 0,488 7 0,337 2 0,303 6	0,684 3 0,488 9 0,384 1 0,337 4 0,319 4 0,307 0 0,303 3

(a) Triangular pitch

(b) Square pitch

	effective	DIREC	T METHOD	UNDIRECT METHOD
μ	constants	Meijers [5]	Grigolyuk and Fil'shtinskii [11]	Bailey – Hicks [6] Slot – O'Donnell [7]
0.1	E_*/E	0.1857	0,1856	0 1857
0.2	⁻ P / -	0.3000	0.2999	0.3000
0.3		0.4069		0,4069
0.4			0.5116	0.5117
0.5		0.6168		0.6168
0.7		0.8244	1	0.8244
0.8			0.9139	0.9139
0.1	V.,*	0.0956	0.0965	0.0956
0.2	(v= 0.3)	0.1506	0.1509	0.1506
0.3	A	0.1975	-	0.1974
0.4			0.2365	0.2365
0.5		0.2665		0.2665
0.7		0.2974		0.2975
0.8			0.3000	0.3010

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Figure 55 — Synthesis of E*/E and v* Values from [1], Provided by Various Authors for Triangular and Square Pattern

ANNEX B — VALUES OF EFFECTIVE ELASTIC CONSTANTS FOR THE FULL RANGE OF M (0.1 \leq M* \leq 1.0)

1 Introduction

This Annex provides the curves, numerical values and polynomials to calculate the effective elastic constants E*/E and v* for the full range of the ligament efficiency μ^* (0.1 $\leq \mu^* \leq 1.0$).

2 Curves (From [13])



Figure 56 — Curves of Effective Elastic Constants for the Full Range of μ^* (0.1 $\leq \mu^* \leq 1.0$)



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3 Numerical Values (From [13])

h/p	μ*	E*/E
0.1	0.0	0.0
0.1	0.02	0.05
0.1	0.05	0.096
0.1	0.1	0.16
0.1	0.15	0.223
0.1	0.2	0.285
0.1	0.22	0.310
0.1	0.25	0.348
0.1	0.28	0.3858
0.1	0.3	0.411
0.1	0.32	0.436
0.1	0.35	0.473
0.1	0.38	0.510
0.1	0.4	0.535
0.1	0.42	0.559
0.1	0.45	0.596
0.1	0.48	0.6318
0.1	0.5	0.656
0.1	0.52	0.678
0.1	0.55	0.712
0.1	0.58	0.7448
0.1	0.6	0.767
0.1	0.626	0.792
0.1	0.64	0.806
0.1	0.657	0.823
0.1	0.675	0.84
0.1	0.7	0.863
0.1	0.719	0.879
0.1	0.75	0.904
0.1	0.777	0.9222
0.1	0.8	0.937
0.1	0.828	0.9523
0.1	0.85	0.964
0.1	0.866	0.9708
0.1	0.882	0.9772
0.1	0.9	0.984
0.1	0.918	0.990
0.1	0.938	0.994
0.1	0.95	0.996
0.1	0.977	0.999
0.1	1.0	1.0

h/p	μ*	E*/E
0.25	0.0	0.0
0.25	0.1	0.123
0.25	0.15	0.181
0.25	0.2	0.243
0.25	0.25	0.308
0.25	0.3	0.374
0.25	0.35	0.439
0.25	0.4	0.505
0.25	0.45	0.568
0.25	0.5	0.632
0.25	0.55	0.690
0.25	0.6	0.747
0.25	0.7	0.847
0.25	0.75	0.892
0.25	0.8	0.930
0.25	0.825	0.9459
0.25	0.85	0.96
0.25	0.9	0.982
0.25	0.95	0.995
0.25	1.0	1.0

h/p	μ*	E*/E
0.5	0.0	0.0
0.5	0.02	0.0122
0.5	0.04	0.0287
0.5	0.05	0.0375
0.5	0.0804	0.0653
0.5	0.1	0.0849
0.5	0.12	0.106
0.5	0.14	0.127
0.5	0.15	0.139
0.5	0.184	0.18
0.5	0.2	0.201
0.5	0.219	0.226
0.5	0.25	0.2678
0.5	0.279	0.307
0.5	0.3	0.3361
0.5	0.318	0.361
0.5	0.35	0.4047
0.5	0.4	0.4753
0.5	0.435	0.521
0.5	0.45	0.541
0.5	0.465	0.561
0.5	0.5	0.6075
0.5	0.527	0.641
0.5	0.55	0.6688
0.5	0.6	0.7272
0.5	0.64	0.769
0.5	0.65	0.7794
0.5	0.664	0.794
0.5	0.7	0.8313
0.5	0.718	0.85
0.5	0.739	0.87
0.5	0.75	0.8812
0.5	0.779	0.906
0.5	0.796	0.919
0.5	0.8	0.922
0.5	0.817	0.935
0.5	0.838	0.949
0.5	0.85	0.9563
0.5	0.856	0.96
0.5	0.877	0.971
0.5	0.895	0.979
0.5	0.9	0.9806
0.5	0.917	0.986
0.5	0.938	0.992
0.5	0.95	0.9944
0.5	0.978	0.999
0.5	1.0	1.0

h/p	μ*	E*/E
2.0	0.0	0.0
2.0	0.0296	0.008
2.0	0.05	0.0172
2.0	0.0596	0.0217
2.0	0.0807	0.0356
2.0	0.1	0.052
2.0	0.12	0.0688
2.0	0.14	0.088
2.0	0.15	0.0994
2.0	0.176	0.129
2.0	0.2	0.1553
2.0	0.22	0.1798
2.0	0.25	0.2165
2.0	0.28	0.255
2.0	0.3	0.2806
2.0	0.333	0.3246
2.0	0.35	0.348
2.0	0.38	0.3862
2.0	0.4	0.412
2.0	0.42	0.4385
2.0	0.45	0.4783
2.0	0.48	0.5181
2.0	0.5	0.5446
2.0	0.52	0.5701
2.0	0.55	0.6084
2.0	0.58	0.6466
2.0	0.6	0.6721
2.0	0.64	0.724
2.0	0.655	0.742
2.0	0.7	0.7996
2.0	0.716	0.82
2.0	0.738	0.845
2.0	0.76	0.869
2.0	0.779	0.888
2.0	0.791	0.9
2.0	0.8	0.908
2.0	0.813	0.92
2.0	0.834	0.938
2.0	0.85	0.95
2.0	0.876	0.966
2.0	0.9	0.978
2.0	0.918	0.986
2.0	0.938	0.992
2.0	0.95	0.996
2.0	0.978	0.999
2.0	1.0	1.0

Ia	ble 5	— Value	es of (Curves	v* as a	Func	tion of	pat	Ratios tern	s h/p=0.	.1, 0.15,	0.25,	0.5, 1.0	0 and 2.	U for	Iriang	jular
h/n	*	*	h/n	*	*	h/n	*	*	h/n	*	*	h/n	*	*	h/n	*	*
<u>n/p</u>	μ	V 0.006	0.45	μ	V 0.2025	0.25	μ	V 0.4425		μ	V	100	μ	V	20	μ	V
0.1	0.0	-0.090	0.15	0.1	0.3025	0.20	0.1	0.4135	0.5	0.1	0.5397	1.0	0.1	0.0101	2.0	0.05	0.6076
0.1	0.05	-0.0672	0.15	0.1190	0.2210	0.20	0.1	0.4133	0.5	0.1191	0.4620	1.0	0.1179	0.5075	2.0	0.1	0.6606
0.1	0.1	-0.036	0.15	0.1207	0.2025	0.20	0.1103	0.3017	0.5	0.1305	0.455	1.0	0.1362	0.5224	2.0	0.15	0.5445
0.1	0.15	-0.0156	0.15	0.1345	0.1010	0.20	0.1102	0.3020	0.5	0.139	0.4335	1.0	0.140	0.3031	2.0	0.10	0.49
0.1	0.104	0.0001	0.15	0.1401	0.1000	0.25	0.1292	0.34	0.5	0.1540	0.4029	1.0	0.1575	0.4703	2.0	0.2	0.4375
0.1	0.2	0.0009	0.15	0.155	0.141	0.25	0.1509	0.3224	0.5	0.1075	0.3013	1.0	0.1009	0.4027	2.0	0.22	0.4306
0.1	0.22	0.0100	0.15	0.1002	0.1229	0.25	0.1517	0.3027	0.5	0.1007	0.3010	1.0	0.1779	0.445	2.0	0.20	0.3950
0.1	0.20	0.0312	0.15	0.1740	0.1103	0.25	0.10	0.2904	0.5	0.1903	0.3405	1.0	0.1923	0.422	2.0	0.20	0.3093
0.1	0.20	0.0456	0.15	0.1029	0.1103	0.25	0.1719	0.2741	0.5	0.2	0.330	1.0	0.2	0.4109	2.0	0.3	0.354
0.1	0.3	0.050	0.15	0.1911	0.1052	0.25	0.1044	0.2014	0.5	0.210	0.3152	1.0	0.2174	0.309	2.0	0.31	0.347
0.1	0.32	0.0000	0.15	0.2	0.1011	0.25	0.194	0.2505	0.5	0.2270	0.3002	1.0	0.2259	0.3792	2.0	0.321	0.3405
0.1	0.33	0.0003	0.15	0.21	0.0303	0.25	0.2	0.2449	0.5	0.2304	0.2304	1.0	0.2373	0.3585	2.0	0.35	0.3263
0.1	0.30	0.0301	0.15	0.210	0.0303	0.25	0.2043	0.2403	0.5	0.20	0.2003	1.0	0.247	0.3559	2.0	0.35	0.3203
0.1	0.42	0.100	0.15	0.2201	0.0002	0.25	0.2174	0.2230	0.5	0.2001	0.2700	1.0	0.20	0.3379	2.0	0.38	0.3166
0.1	0.45	0.1102	0.15	0.2002	0.0047	0.25	0.2200	0.2211	0.5	0.2710	0.271	1.0	0.2684	0.0473	2.0	0.00	0.312
0.1	0.40	0.1011	0.15	0.2400	0.0040	0.25	0.2004	0.2140	0.5	0.2022	0.2562	1.0	0.2004	0.3318	2.0	0.4	0.3082
0.1	0.40	0.140	0.15	0.2604	0.0040	0.25	0.2593	0.2000	0.5	0.3198	0.2002	1.0	0.2707	0.3185	2.0	0.42	0.0002
0.1	0.52	0.1653	0.15	0.2708	0.0040	0.25	0.2000	0.198	0.5	0.3323	0.2452	1.0	0.2916	0.3235	2.0	0.40	0.3005
0.1	0.55	0 1793	0.15	0 2787	0.0971	0.25	0.2797	0 1951	0.5	0.3456	0.2102	1.0	0.3051	0.3154	2.0	0.10	0 2994
0.1	0.6	0.203	0.15	0.29	0.0991	0.25	0 2899	0 1914	0.5	0.35	0.241	1.0	0.3191	0.3082	2.0	0.52	0.299
0.1	0.63	0.2165	0.15	0.3	0 1015	0.25	0.3	0 1887	0.5	0.3587	0.2397	1.0	0.3337	0.302	2.0	0.55	0.2986
0.1	0.65	0.2245	0.15	0.3102	0.1038	0.25	0.3104	0.1868	0.5	0.3692	0.2394	1.0	0.3482	0.2969	2.0	0.6	0.2988
0.1	0.67	0.2325	0.15	0.3196	0.1067	0.25	0.3199	0.186	0.5	0.3804	0.2395	1.0	0.35	0.2963	2.0	0.65	0.3
0.1	0.7	0.243	0.15	0.3303	0.1099	0.25	0.3295	0.1854	0.5	0.3914	0.24	1.0	0.3604	0.293	2.0	0.7	0.3
0.1	0.72	0.2501	0.15	0.3394	0.1131	0.25	0.3386	0.1851	0.5	0.4	0.241	1.0	0.3792	0.2893	2.0	0.75	0.3
0.1	0.75	0.2597	0.15	0.35	0.1172	0.25	0.35	0.1857	0.5	0.4129	0.243	1.0	0.4	0.2877	2.0	0.8	0.3
							0										
0.1	0.8	0.274	0.15	0.3591	0.1204	0.25	.3604	0.1873	0.5	0.4198	0.2448	1.0	0.4188	0.2866	2.0	0.85	0.3

h/p	μ*	ν*	h/p	μ*	ν*	h/p	μ*	ν*	h/p	μ*	ν*	h/p	μ*	ν*	h/p	μ*	ν*
0.1	0.83	0.2815	0.15	0.3795	0.1289	0.25	0.3704	0.1891	0.5	0.4394	0.2499	1.0	0.4399	0.2865	2.0	0.9	0.3
0.1	0.85	0.2859	0.15	0.4	0.1376	0.25	0.3805	0.1911	0.5	0.45	0.2525	1.0	0.45	0.2867	2.0	0.95	0.3
0.1	0.875	0.2906	0.15	0.42	0.1464	0.25	0.3908	0.193	0.5	0.5	0.2635	1.0	0.5	0.2893	2.0	1.0	0.3
0.1	0.9	0.295	0.15	0.45	0.1604	0.25	0.4	0.1955	0.5	0.55	0.273	1.0	0.55	0.2935			
0.1	0.92	0.2974	0.15	0.5	0.1834	0.25	0.4099	0.198	0.5	0.6	0.2813	1.0	0.6	0.2976			
0.1	0.95	0.2996	0.15	0.55	0.2038	0.25	0.4197	0.201	0.5	0.65	0.289	1.0	0.65	0.2997			
0.1	1.0	0.3	0.15	0.6	0.2228	0.25	0.4395	0.2067	0.5	0.7	0.295	1.0	0.7	0.3			
			0.15	0.65	0.2413	0.25	0.45	0.2097	0.5	0.75	0.2997	1.0	0.75	0.3			
			0.15	0.7	0.2571	0.25	0.5	0.2246	0.5	0.8	0.3	1.0	0.8	0.3			
			0.15	0.75	0.2709	0.25	0.55	0.2395	0.5	0.85	0.3	1.0	0.85	0.3			
			0.15	0.8	0.2824	0.25	0.6	0.2526	0.5	0.9	0.3	1.0	0.9	0.3			
			0.15	0.8237	0.2872	0.25	0.65	0.2653	0.5	0.95	0.3	1.0	0.95	0.3			
			0.15	0.85	0.2923	0.25	0.7	0.276	0.5	1.0	0.3	1.0	1	0.3			
			0.15	0.8827	0.2971	0.25	0.75	0.2851									
			0.15	0.9	0.2993	0.25	0.8	0.2923									
			0.15	0.9018	0.2995	0.25	0.85	0.2977									
			0.15	0.95	0.3	0.25	0.9	0.3									
			0.15	1.0.0	0.3	0.25	0.95	0.3									
						0.25	1.0	0.3									

Table 6 — Values of Curves v^* as a function of μ^* for ratios h/p=0.1, 0.15, 0.25, 0.5, 1.0 and 2.0 for square pattern

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h/p	μ*	ν*	h/p	μ*	ν*	h/p	μ*	v*	h/p	μ*	v*	h/p	μ*	ν*	h/p	μ*	ν*
0.1	0.1	-0.022	0.15	0.1	0.1465	0.25	0.1	0.2471	0.5	0.1	0.3027	1.0	0.1	0.3283	2.0	0.05	0.339
0.1	0.142	0.0003	0.15	0.1082	0.1357	0.25	0.1183	0.2201	0.5	0.1122	0.2958	1.0	0.1376	0.3216	2.0	0.1	0.34
0.1	0.2	0.031	0.15	0.1179	0.1267	0.25	0.1278	0.2099	0.5	0.1193	0.2925	1.0	0.15	0.3196	2.0	0.125	0.3403
0.1	0.22	0.0408	0.15	0.1281	0.1184	0.25	0.1384	0.2002	0.5	0.1294	0.2881	1.0	0.1784	0.3151	2.0	0.15	0.34
0.1	0.25	0.0555	0.15	0.1379	0.1124	0.25	0.15	0.192	0.5	0.15	0.2809	1.0	0.1984	0.3125	2.0	0.175	0.3387
0.1	0.28	0.0702	0.15	0.15	0.1053	0.25	0.1609	0.185	0.5	0.1775	0.273	1.0	0.2	0.3123	2.0	0.2	0.337
0.1	0.3	0.08	0.15	0.1579	0.1016	0.25	0.1772	0.1767	0.5	0.2	0.2678	1.0	0.2383	0.3082	2.0	0.22	0.3354
0.1	0.32	0.0896	0.15	0.1694	0.0978	0.25	0.1888	0.1722	0.5	0.2383	0.2611	1.0	0.25	0.307	2.0	0.25	0.333
0.1	0.35	0.104	0.15	0.1788	0.0951	0.25	0.2	0.1685	0.5	0.25	0.2594	1.0	0.279	0.304	2.0	0.28	0.33
0.1	0.38	0.1184	0.15	0.1901	0.093	0.25	0.2104	0.165	0.5	0.2784	0.256	1.0	0.3	0.3025	2.0	0.3	0.328

h/p	μ*	ν*	h/p	μ*	ν*	h/p	μ*	v*	h/p	μ*	ν*	h/p	μ*	ν*	h/p	μ*	v*	
0.1	0.4	0.128	0.15	0.1984	0.092	0.25	0.2307	0.1602	0.5	0.3	0.2542	1.0	0.3396	0.3005	2.0	0.32	0.3256	
0.1	0.42	0.137	0.15	0.2	0.0919	0.25	0.2391	0.1589	0.5	0.3187	0.253	1.0	0.35	0.3002	2.0	0.35	0.322	
0.1	0.45	0.1505	0.15	0.2099	0.0915	0.25	0.25	0.158	0.5	0.3388	0.2524	1.0	0.4	0.2992	2.0	0.38	0.3184	
0.1	0.48	0.164	0.15	0.2186	0.0914	0.25	0.2529	0.1577	0.5	0.35	0.2522	1.0	0.4386	0.2986	2.0	0.4	0.316	
0.1	0.5	0.173	0.15	0.2292	0.0917	0.25	0.263	0.1573	0.5	0.3581	0.2521	1.0	0.45	0.2984	2.0	0.42	0.314	
0.1	0.52	0.1814	0.15	0.239	0.0926	0.25	0.273	0.1572	0.5	0.3786	0.252	1.0	0.4792	0.298	2.0	0.45	0.3106	
0.1	0.55	0.194	0.15	0.25	0.0937	0.25	0.281	0.1572	0.5	0.3975	0.252	1.0	0.5	0.298	2.0	0.48	0.3076	
0.1	0.58	0.2066	0.15	0.2591	0.0953	0.25	0.2945	0.1573	0.5	0.4	0.252	1.0	0.5198	0.298	2.0	0.5	0.306	
0.1	0.6	0.215	0.15	0.2681	0.0971	0.25	0.3	0.1576	0.5	0.4181	0.2524	1.0	0.55	0.2984	2.0	0.52	0.3044	
0.1	0.65	0.233	0.15	0.2788	0.0999	0.25	0.3105	0.1582	0.5	0.4309	0.2533	1.0	0.6	0.299	2.0	0.55	0.3028	
0.1	0.7	0.25	0.15	0.2917	0.1036	0.25	0.3222	0.1592	0.5	0.45	0.2544	1.0	0.65	0.2995	2.0	0.58	0.3018	
0.1	0.75	0.2644	0.15	0.3	0.1065	0.25	0.3316	0.1604	0.5	0.4802	0.2577	1.0	0.7	0.3	2.0	0.6	0.301	
0.1	0.8	0.277	0.15	0.3181	0.1144	0.25	0.3399	0.1622	0.5	0.5028	0.261	1.0	0.75	0.3	2.0	0.65	0.3002	
0.1	0.85	0.2873	0.15	0.3286	0.1188	0.25	0.35	0.1644	0.5	0.5232	0.2643	1.0	0.8	0.3	2.0	0.7	0.3	
0.1	0.9	0.2956	0.15	0.3395	0.1235	0.25	0.3697	0.1687	0.5	0.5	0.2606	1.0	0.85	0.3	2.0	0.75	0.3	
0.1	0.95	0.2995	0.15	0.35	0.1283	0.25	0.4	0.1762	0.5	0.55	0.268	1.0	0.9	0.3	2.0	0.8	0.3	
0.1	0.98	0.2999	0.15	0.4	0.1498	0.25	0.4121	0.1799	0.5	0.6	0.275	1.0	0.95	0.3	2.0	0.85	0.3	
0.1	1.0	0.3	0.15	0.45	0.1698	0.25	0.4255	0.1843	0.5	0.64	0.28	1.0	1.0	0.3	2.0	0.9	0.3	
			0.15	0.5	0.1903	0.25	0.4386	0.1882	0.5	0.65	0.2812				2.0	0.95	0.3	
			0.15	0.55	0.2092	0.25	0.45	0.1924	0.5	0.7	0.287				 2.0	1.0	0.3	
			0.15	0.6	0.2267	0.25	0.4798	0.203	0.5	0.72	0.289							
			0.15	0.65	0.2427	0.25	0.5	0.21	0.5	0.74	0.291							
			 0.15	0.7	0.2578	 0.25	0.55	0.2276	0.5	0.75	0.2919							
			 0.15	0.7196	0.263	 0.25	0.6	0.2441	0.5	0.78	0.2948							
			 0.15	0.741	0.2682	 0.25	0.62	0.25	0.5	0.8	0.296							
			 0.15	0.75	0.2702	 0.25	0.6461	0.257	0.5	0.82	0.2972							
			0.15	0.7799	0.2773	0.25	0.65	0.2579	0.5	0.84	0.2982							
			0.15	0.8	0.2816	0.25	0.6786	0.2644	0.5	0.85	0.2986							
			0.15	0.821	0.2856	0.25	0.7	0.2692	0.5	0.88	0.2995							
			0.15	0.8316	0.2875	0.25	0.7194	0.2729	0.5	0.9	0.3							
			0.15	0.85	0.2904	0.25	0.7404	0.2771	0.5	0.95	0.3							
			0.15	0.8787	0.2944	0.25	0.75	0.2788	0.5	1.0	0.3						\mid	
			0.15	0.9	0.2965	0.25	0.7794	0.2841										
			0.15	0.92	0.2981	0.25	0.8	0.2872										
			0.15	0.94	0.2994	0.25	0.8204	0.29										
			0.15	0.95	0.2997	0.25	0.84	0.2922										
			0.15	1.0.0	0.3	0.25	0.85	0.2932										

*

h/p	μ*	ν*	h/p	μ*	ν*	h/p	μ*	ν*	h/p	μ*	ν*	h/p	µ*	ν*	h/p	μ*	v*
						0.25	0.8785	0.2958									
						0.25	0.9	0.2971									
						0.25	0.9206	0.2983									
						0.25	0.94	0.2995									
						0.25	0.95	0.2996									
						0.25	1.0	0.3									

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4 Polynomials

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1	Table	1: E*/E Coefficients	for Triang	gular Tube Pattern	
۳*	x,	R'1	×2	×3	ø4
0.10	0.002	1.239	0.573	-0.878	0.063
0.50 2.00	-0.003	0.728	1.895 1.957	-2.051 -1.442	0.434

	Table	2:E*/E Coefficients	for Square	Tube Pattern	
μ*	No	R'A	×2	Ø3	K4
0.10	0.017	2.112 -	2.759	3.368	-1.747
0.25	0.013	1.840 -	2.111	2.893	-1.642
0.50	0.011	1.567 -	1.648	2.747	-1.682
2.00	0.012	1.278 -	1.135	2.587	-1.749

Table 3: v* Coefficients for Triangular Tube Pattern

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pr*	Po	Pa	β2	P3	6.	65	Рe	Ęz
0.10	-0.089	0.450	0.267	-0.325	-0.002	0	0	0
0.15	1.232	-16.249	93.259	-284.35	507.628	-528.539	296.515	-69.197
0.25	0.771	-4.78	13.226	-11.952	-7.503	23.725	-17.848	4.66
0.50	0.989	-6.399	23.448	-53.158	87.214	-96.808	61.016	-16.003
1.00	1.01	-5.142	13.806	-17.444	10.436	-2.366	0	0
2.00	1.018	-4.52	10.821	-12.221	6.421	-1.21	0	0

Table 4: v* Coefficients for Square Tube Pattern

٣*	(ª o	(31	62	P3	C4	ß5	Co	C7
0.10	-0.067	0.451	0.191	-0.273	0	0	0	0
0.15	0.337	-2.905	11.728	-19.521	15.421	-4.761	0	0
0.25	0.434	-2.632	8.552	-11.617	7.451	-1.889	0	0
0.50	0.400	-1.602	8.636	-29.539	62.382	-74.962	46.713	-11.728
1.00	0.348	-0.201	-0.180	2.124	-4.358	4.145	-1.954	0.374
2.00	0.354	-0.286	2.27	-10.35	22.733	-25.581	14.349	-3.188

Polynomials of Effective Elastic Constants for the full range of µ* (0.1≤µ*≤1.0) initially used for Appendix AA of ASME Section VIII-Div. 1

ANNEX C — POISSON'S RATIO IN TUBES AND SHELL

(a) **Deformation of a shell of length l**; diameters D_i ,D_e ; pressures P_i, P_e:



Deformation due to Poisson's ratio: $\varepsilon_{v} = \frac{\delta_{l}(v)}{l} = -\frac{v}{E}(\sigma_{\theta} + \sigma_{r})$

$$\sigma_{\theta} + \sigma_{r} = 2 \frac{P_{i} D_{i}^{2} - P_{e} D_{e}^{2}}{D_{e}^{2} - D_{i}^{2}} = \frac{\pi}{2} \frac{P_{i} D_{i}^{2} - P_{e} D_{e}^{2}}{s} \qquad s = \frac{\pi}{4} \left(D_{e}^{2} - D_{i}^{2} \right)$$
$$\delta_{l} \left(\nu \right) = -\frac{\pi}{2} \frac{l}{E s} \left[P_{i} D_{i}^{2} - P_{e} D_{e}^{2} \right] \nu = -\frac{\pi}{2 k} \left[P_{i} D_{i}^{2} - P_{e} D_{e}^{2} \right] \nu \qquad k = \frac{E s}{l}$$

$$(b) \text{ Application to the tubes: } D_{e} = d_{t} \qquad D_{i} = d_{t} - 2t_{t} \qquad P_{i} = P_{t} \qquad P_{e} = P_{s}$$

$$\delta_{t}(v_{t}) = -\frac{\pi}{2k_{t}} \Big[P_{t} \left(d_{t} - 2t_{t} \right)^{2} - P_{s} d_{t}^{2} \Big] v_{t}$$

$$\begin{cases} x_{t} = 1 - N_{t} \frac{\left(d_{t} - 2t_{t} \right)^{2}}{D_{o}^{2}} \\ x_{s} = 1 - N_{t} \frac{d_{t}^{2}}{D_{o}^{2}} \\ x_{s} = 1 - N_{t} \frac{d_{t}^{2}}{D_{o}^{2}} \\ z_{s} = 1 - N_{t} \frac{d_{$$

(c) Application to the shell: $D_{\rm i} = D_{\rm s}$ $P_{\rm i} = P_{\rm s}$ $P_{\rm e} = 0$ $k_{\rm s}' = \frac{\pi E_{\rm s} t_{\rm s} (D_{\rm s} + t_{\rm s})}{l}$

$$\delta_{\rm s}\left(v_{\rm s}\right) = -\frac{\pi}{2\,k_{\rm s}}\left[P_{\rm s}\,D_{\rm s}^2\right]v_{\rm s} = \frac{-l\,D_{\rm s}^2\,P_{\rm s}}{2\,E_{\rm s}\,t_{\rm s}\left(D_{\rm s}+t_{\rm s}\right)}\,v_{\rm s}$$

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$$k_{\rm w} \,\delta_{\rm s}\left(v_{s}\right) = -\frac{\pi}{2 \,k_{\rm s}} \frac{N \,k_{\rm t}}{\pi \left(D_{o}^{2} / 4\right)} P_{\rm s} \,D_{s}^{2} \,v_{s} = -\frac{2 \,v_{\rm s}}{k_{\rm s,t}} \left(\frac{D_{\rm s}}{D_{o}}\right)^{2} P_{\rm s}$$

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ANNEX D — SHELL PRESSURE ACTING ON THE EXPANSION JOINT SIDEWALLS

Axial force acting on the sidewall of one half of the expansion joint due to internal shell pressure Ps:



The shell pressure acting on the sidewall of the expansion joint creates an axial force. It is assume that half of this force is resisted by the expansion joint, and the other half of it is carried out by the shell. Accordingly, the axial force acting on the shell is:

$$F_{J} = \frac{F_{J}}{2} = \frac{\pi}{2} \left(R_{J}^{2} - R_{s}^{2} \right) P_{s} = \frac{\pi}{8} \left(D_{J}^{2} - D_{s}^{2} \right) P_{s}$$

Axial displacement of the half-joint of rigidity k_J, due to F'_J:

$$\delta_J(P_s) = \frac{F_J}{k_J} = \frac{\pi}{8} \frac{D_J^2 - D_s^2}{k_J} P_s$$

Expansion joint rigidity: $K_J = \frac{k_J}{2}$

$$\delta_{J}\left(P_{s}\right) = \frac{\pi}{16} \frac{D_{J}^{2} - D_{s}^{2}}{K_{J}} P_{s}$$

Note: this formula is still valid if the shell is extended as a flange (D_s should not be replaced by G_s)

ANNEX E — DIFFERENTIAL PRESSURE ACTING ON THE EQUIVALENT SOLID PLATE

(a) Force due to pressure P_s acting on the TS of radius a_0 :



This force leads to a uniform pressure P_s* acting on the equivalent solid plate given by:

$$P_s^* \pi a_o^2 = P_s \left[\pi a_o^2 - N_t \pi \left(\frac{d_t}{2} \right)^2 \right]$$
$$P_s^* = P_s \left[1 - N_t \left(\frac{d_t}{2 a_o} \right)^2 \right] = P_s x_s \qquad D_o = 2 a_o$$

TS coefficient relating to the tubes on shell side: $x_s = 1 - 1$



(b) Force due to pressure Pt acting on the TS of radius a_o:

$$P_t\left[\pi a_o^2 - N_t \pi \left(\frac{d_t - 2t_t}{2}\right)^2\right]$$

This force leads to a uniform pressure P_t^* acting on the equivalent solid plate given by:

$$P_t^* \pi a_o^2 = P_t \left[\pi a_o^2 - N_t \pi \left(\frac{d_t - 2t_t}{2} \right)^2 \right]$$

$$P_t^* = P_t \left[1 - N_t \left(\frac{d_t - 2t_t}{2a_o} \right)^2 \right] = P_t x_t$$

TS coefficient relating to the tubes on tube side: x_t

$$=1-N_t\left(\frac{d_t-2t_t}{D_o}\right)$$

(c) Differential pressure acting on the equivalent solid plate:

$$\Delta p^* = x_s P_s - x_t P_t$$

ANNEX F — SOLUTION OF DIFFERENTIAL EQUATION W(X)

The deflection of the equivalent solid plate is governed by a differential equation of 4th order:

$$\frac{d^4w}{dr^4} + \frac{2}{r}\frac{d^3w}{dr^3} - \frac{1}{r^2}\frac{d^2w}{dr^2} + \frac{1}{r^3}\frac{dw}{dr} = \frac{q(r)}{D^*} = \frac{Q - k_w w(r)}{D^*}$$
$$\frac{d^4w}{dr^4} + \frac{2}{r}\frac{d^3w}{dr^3} - \frac{1}{r^2}\frac{d^2w}{dr^2} + \frac{1}{r^3}\frac{dw}{dr} + \frac{k_w}{D^*}w(r) = \frac{Q}{D^*}$$

A change of variable is necessary to solve the differential equation: x = kr

$$\frac{k_{w}}{D^{*}} = k^{4} \qquad k = \sqrt[4]{\frac{k_{w}}{D^{*}}} \qquad 0 \le r \le a_{0} \implies 0 \le x \le X_{a} \qquad X_{a} = k a_{o}$$

$$\frac{d}{dr} = \frac{d}{dr} \frac{w}{dr} \frac{d}{dr} = k \frac{d}{dr} \frac{w}{dr} \qquad \frac{d^{2} w}{dr^{2}} = k^{2} \frac{d^{2} w}{dr^{2}} \qquad \frac{d^{3} w}{dr^{3}} = k^{3} \frac{d^{3} w}{dr^{3}} \qquad \frac{d^{4} w}{dr^{4}} = k^{4} \frac{d^{4} w}{dr^{4}}$$

$$\frac{d^{4} w}{dr^{4}} + \frac{2}{x} \frac{d^{3} w}{dr^{3}} - \frac{1}{x^{2}} \frac{d^{2} w}{dr^{2}} + \frac{1}{x^{3}} \frac{dw}{dr} + w(x) = \frac{Q}{k^{4} D^{*}} = \frac{Q}{k_{w}}$$

• The homogeneous solution is:

$$w(x) = A ber(x) + B bei(x) + C ker(x) + D kei(x)$$

where:

ber(x), bei(x), ker(x), kei(x) are Kelvin functions of order 0, which will be noted berx, beix, kerx, keix

A, B, C, D are integration constants

$$berx = \sum_{0}^{\infty} (-1)^{n} \frac{(x/2)^{4n}}{[(2n)!]^{2}} = 1 - \frac{(x/2)^{4}}{(2!)^{2}} + \cdots$$

$$beix = \sum_{1}^{\infty} (-1)^{2n-1} \frac{(x/2)^{4n-2}}{[(2n-1)!]^{2}} = \frac{(x/2)^{2}}{(1!)^{2}} - \frac{(x/2)^{6}}{(3!)^{2}} + \cdots$$

$$kerx = -ln(x/2)berx + \frac{\pi}{4}beix + [1] - \left[\frac{(x/2)^{4}}{(2!)^{2}}(1+\frac{1}{2})\right] + \left[\frac{(x/2)^{8}}{(4!)^{2}}(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4})\right] - \cdots$$

$$keix = -ln(x/2)beix - \frac{\pi}{4}berx + \left[(x/2)^{2}\right] - \left[\frac{(x/2)^{6}}{(3!)^{2}}(1+\frac{1}{2}+\frac{1}{3})\right] + \left[\frac{(x/2)^{10}}{(5!)^{2}}(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5})\right] - \cdots$$
For x=0:
$$\begin{cases} ber(0) = 1 \quad bei(0) = 0 \quad ker(0) = \rightarrow \infty \quad kei(0) = -\frac{\pi}{4} \\ ber'(0) = 1 \quad bei'(0) = 0 \quad ker'(0) = \rightarrow \infty \quad kei'(0) = -\infty \\ ber''(0) = 0 \quad bei''(0) = \frac{1}{2} \quad ker''(0) = \rightarrow \infty \quad kei''(0) = \rightarrow \infty \end{cases}$$

The particular solution with 2nd member is: $w(x) = \frac{Q}{k_w}$

General solution: $w(x) = A berx + B beix + C kerx + D keix + \frac{Q}{k_w}$



Determination of constants C and D. Bending moment is written:

$$M_r(r) = -D^*k^2 \left(\frac{d^2w}{dx^2} + \frac{v^*}{x}\frac{dw}{dx}\right) = -D^*k^2 \left[\frac{(Aber''x + Bbei''x + C\ker''x + Dkei''x)}{\frac{v^*}{x}(Aber'x + Bbei'x + C\ker''x + Dkei''x)}\right]$$

The bending moment $M_r(0)$ at center of TS must remain finite: \rightarrow C=0 D=0

Finally, the solution is written:

$$w(x) = A berx + B beix + \frac{Q}{k_w}$$

ANNEX G - COEFFICIENTS ZD, ZV, ZW, ZM; QM, QV; QA, QB; FM, FT

(a) **Purpose**

•

This Annex provides the equations for coefficients $Z_d(x)$, $Z_v(x)$, $Z_w(x)$, $Z_m(x)$; $Q_m(x)$, $Q_v(x)$, $Q_\alpha(x)$, $Q_{\beta}(x)$; $F_{m}(x)$, $F_{t}(x)$ and their values for:

- x=0 or close to 0 for a given value of X_a.
- x=X_a •
- $x=X_a$ when X_a is close to 0 •

Notations are taken from UHX-13:

- ber, bei are Kelvin functions •
- f(x) represents the value of function f for x. •
 - $f(X_a)$ is always noted "f": $f = f(X_a)$. Therefore: $ber = ber(X_a)$; $ber' = ber'(X_a)$ bei' = bei'(X_a) $bei = bei(X_a)$; $Z_a = Z_a(X_a)$ $Z_v = Z_v(X_a)$ $Z_m = Z_m(X_a) \quad ; \quad$ $Z_w = Z_w(X_a)$ $Z_d = Z_d(X_a) \quad ; \quad$ $\psi_2 = \psi_2(X_a)$ $\psi_1 = \psi_1(X_a)$;

(b) Kelvin Functions

$$ber(x) = \sum_{n=0}^{n=m-1} (-1)^n \frac{(x/2)^{4n}}{\left[(2n)!\right]^2} = 1 - \frac{(x/2)^4}{(2!)^2} + \frac{(x/2)^8}{(4!)^2} - \frac{(x/2)^{12}}{(6!)^2} + \dots \approx 1 - \frac{x^4}{2^6} \text{ for x close to } 0$$

$$bei(x) = \sum_{n=1}^{n=m} (-1)^{n-1} \frac{(x/2)^{4n-2}}{\left[(2n-1)!\right]^2} = \frac{(x/2)^2}{(1!)^2} - \frac{(x/2)^6}{(3!)^2} + \frac{(x/2)^{10}}{(5!)^2} - \dots \approx \frac{x^2}{2^2} \text{ for x close to } 0$$

$$ber'(x) = \sum_{n=1}^{n=m} (-1)^n \frac{(2n)(x/2)^{4n-1}}{\left[(2n)!\right]^2} = -\frac{2(x/2)^3}{(2!)^2} + \frac{4(x/2)^7}{(4!)^2} - \frac{6(x/2)^{11}}{(6!)^2} + \dots \approx -\frac{x^3}{2^4}$$

for x close to 0

$$bei'(x) = \sum_{n=1}^{n=m} (-1)^{n-1} \frac{(2n-1)(x/2)^{4n-3}}{\left[(2n-1)!\right]^2} = \frac{(x/2)^1}{(1!)^2} - \frac{3(x/2)^5}{(3!)^2} + \frac{5(x/2)^9}{(5!)^2} - \dots \approx \frac{x}{2}$$
for x close to 0.

for x close to 0

$$ber''(x) = \sum_{n=1}^{n=m} (-1)^n \frac{(n)(4n-1)(x/2)^{4n-2}}{\left[(2n)!\right]^2} = -\frac{3(x/2)^2}{(2!)^2} + \frac{14(x/2)^6}{(4!)^2} - \frac{33(x/2)^{10}}{(6!)^2} + \dots \approx -\frac{3x^2}{2^4}$$

for x close to 0

$$bei''(x) = \sum_{n=1}^{n=m} (-1)^{n-1} \frac{(2n-1)(4n-3)(x/2)^{4n-4}}{2[(2n-1)!]^2} = \frac{1}{2} - \frac{15(x/2)^4}{2(3!)^2} + \frac{45(x/2)^8}{2(5!)^2} - \dots \approx \frac{1}{2}$$

for x close to 0

for x close to 0

(c) Coefficients $\Psi_1(x)$, $\Psi_2(x)$, Z_a

$$\psi_1(x) = bei(x) + \frac{1 - \upsilon^*}{x} ber'(x) \approx \frac{3 + \upsilon^*}{2^4} x^2 \quad \text{for x close to } 0$$

$$\Psi_1(X_a) \approx \frac{3 + \upsilon^*}{2^4} X_a^2 \quad \text{for } X_a \text{ close to } 0$$

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$$\psi_{2}(x) = ber(x) - \frac{1 - v^{*}}{x} bei'(x) \approx \frac{1 + v^{*}}{2} \text{ for x close to } 0$$

$$\Psi_{2}(X_{a}) \approx \frac{1 + v^{*}}{2} \text{ for } X_{a} \text{ close to } 0$$

$$Z_{a} = bei' \cdot \Psi_{2} - ber' \cdot \Psi_{1} = (ber \cdot bei' - ber' \cdot bei) - \frac{1 - v^{*}}{X_{a}} (ber'^{2} + bei'^{2})$$

$$Z_{a} \approx \frac{1 + v^{*}}{4} X_{a} \text{ for } X_{a} \text{ close to } 0$$

(d) Coefficients
$$Z_d(x)$$
, $Z_v(x)$, $Z_w(x)$, $Z_m(x)$

$$\begin{aligned} \mathbf{Z}(\mathbf{p}) & \frac{\mathbf{\Psi}_{2} \cdot \mathbf{ber}(x) + \mathbf{\Psi}_{1} \cdot \mathbf{bei}(x)}{\mathbf{X}_{a}^{3} \mathbf{Z}_{a}} = \frac{(\mathbf{ber} \cdot \mathbf{ber}(x) + \mathbf{bei} \cdot \mathbf{bei}(x)) + \frac{1 - U}{X_{a}} (\mathbf{ber}^{1} \cdot \mathbf{bei}(x) - \mathbf{bei}^{1} \cdot \mathbf{ber}(x))}{X_{a}^{3} \mathbf{Z}_{a}} \\ \approx \frac{4 \Psi_{2} + \Psi_{1} x^{2}}{4 X_{a}^{2} \mathbf{Z}_{a}} \approx \frac{\Psi_{2}}{\mathbf{X}_{a}^{3} \mathbf{Z}_{a}} \text{ for x close to } 0 & Z_{d}(0) = \frac{\Psi_{2}}{\mathbf{X}_{a}^{3} \mathbf{Z}_{a}} \\ \mathbf{Z}_{d} &= \frac{\Psi_{2} \cdot \mathbf{ber} + \Psi_{1} \cdot \mathbf{bei}}{\mathbf{X}_{a}^{3} \mathbf{Z}_{a}} = \frac{(\mathbf{ber}^{2} + \mathbf{bei}^{2}) + \frac{1 - U^{*}}{X_{a}} (\mathbf{ber}^{1} \cdot \mathbf{bei} - \mathbf{ber} \cdot \mathbf{bei})}{X_{a}^{3} \mathbf{Z}_{a}} \approx \frac{2}{\mathbf{X}_{a}^{4}} \text{ for } \mathbf{X}_{a} \text{ close to } 0 \\ \mathbf{Z}(\mathbf{p}) & \frac{\Psi_{2} \cdot \mathbf{ber}'(x) + \Psi_{1} \cdot \mathbf{bei}'(x)}{\mathbf{X}_{a}^{2} \mathbf{Z}_{a}} = \frac{(\mathbf{ber} \cdot \mathbf{ber}'(x) + \mathbf{bei} \cdot \mathbf{bei} \cdot (x)) + \frac{1 - U^{*}}{X_{a}} (\mathbf{ber} \cdot \mathbf{bei} \cdot (x) - \mathbf{bei} \cdot \mathbf{ber} \cdot (x))}{\mathbf{X}_{a}^{2} \mathbf{Z}_{a}} \\ \approx \frac{8 \Psi_{1} x - \Psi_{2} x^{3}}{16 \mathbf{X}_{a}^{2} \mathbf{Z}_{a}} \approx \frac{\Psi_{1}}{2 \mathbf{X}_{a}^{2} \mathbf{Z}_{a}} x \text{ for x close to } 0 & \mathbf{Z}_{v}(0) = 0 \\ \mathbf{Z}_{v} &= \frac{\Psi_{2} \cdot \mathbf{ber}' + \Psi_{1} \cdot \mathbf{bei}}{\mathbf{X}_{a}^{2} \mathbf{Z}_{a}} x \text{ for x close to } 0 & \mathbf{Z}_{v}(0) = 0 \\ \mathbf{Z}_{v} &= \frac{\Psi_{2} \cdot \mathbf{ber}'(x) + \mathbf{bei} \cdot \mathbf{bei}(x)}{\mathbf{X}_{a}^{2} \mathbf{Z}_{a}} = \frac{\mathbf{ber} \cdot \mathbf{ber}' + \mathbf{bei} \cdot \mathbf{bei}}{\mathbf{X}_{a}^{2} \mathbf{Z}_{a}} \approx \frac{1}{\mathbf{X}_{a}^{2} \mathbf{Z}_{a}} \text{ for x close to } 0 \\ \mathbf{Z}_{v}(\mathbf{0}) &= \frac{\mathbf{ber}'}{\mathbf{X}_{a}^{2} \mathbf{Z}_{a}} \text{ for x close to } 0 \\ \mathbf{Z}_{w}(\mathbf{0}) &= \frac{\mathbf{ber}'}{\mathbf{X}_{a}^{2} \mathbf{Z}_{a}} \text{ for x close to } 0 \\ \mathbf{Z}_{w} &= \frac{\mathbf{ber}' \cdot \mathbf{ber}'(x) + \mathbf{bei} \cdot \mathbf{bei}!}{\mathbf{X}_{a}^{2} \mathbf{Z}_{a}} \text{ for x close to } 0 \\ \mathbf{Z}_{w}(\mathbf{0}) &= \frac{\mathbf{ber}'}{\mathbf{X}_{a}^{2} \mathbf{Z}_{a}} \text{ for x close to } 0 \\ \mathbf{Z}_{w}(\mathbf{0}) &= \frac{\mathbf{ber}' \cdot \mathbf{ber}'(x) + \mathbf{bei} \cdot \mathbf{bei}!}{\mathbf{X}_{a}^{2} \mathbf{Z}_{a}} \text{ for x close to } 0 \\ \mathbf{Z}_{w}(\mathbf{0}) &= 0 \\ \mathbf{Z}_{w} &= \frac{\mathbf{ber}' \cdot \mathbf{ber}'(x) + \mathbf{bei} \cdot \mathbf{bei}!}{\mathbf{X}_{a}^{2}} = \frac{\mathbf{1}}{(1 + v^{*})} \text{ for } \mathbf{X}_{a} \text{ close to } 0 \\ \mathbf{Z}_{w}(\mathbf{0}) &= 0 \\ \mathbf{Z}_{w} &= \frac{\mathbf{ber}'^{2} + \mathbf{bei}!^{2}}{\mathbf{X}_{a}^{2} \mathbf{Z}_{a}} \approx \frac{\mathbf{1}}{(1 + v^{*})} \text{ for } \mathbf{X}_{a} \text{ close to } 0 \\ \mathbf{Z}_{w}(\mathbf{0}) &= 0 \\ \mathbf{Z}_$$



The numerical values and graph of Z_d , Z_v , Z_w , Z_m as a function of X_a for $1 \le X_a \le 20$ are provided below.

Coefficients Z_d, Z_v, Z_w, Z_m						
Ха	Z _d	$Z_v = Z_w$	Z _m			
1	2.054458	0.176798	0.709032			
2	0.171868	0.154393	0.642542			
3	0.054471	0.103768	0.491113			
4	0.023791	0.062263	0.362879			
5	0.012079	0.039878	0.286907			
6	0.006924	0.027776	0.238824			
7	0.004334	0.020476	0.204651			
8	0.002889	0.015704	0.178926			
9	0.002021	0.012418	0.158913			
10	0.001468	0.010062	0.142915			
11	0.001100	0.008317	0.129837			
12	0.000845	0.006988	0.118947			
13	0.000663	0.005954	0.109738			
14	0.000530	0.005133	0.101851			
15	0.000430	0.004471	0.095019			
16	0.000354	0.003929	0.089046			
17	0.000295	0.003480	0.083778			
18	0.000248	0.003103	0.079098			
19	0.000211	0.002785	0.074912			
20	0.000180	0.002513	0.071147			

Table Zd,Zv,Zw,Zm



(e) Coefficients $Q_m(x),\,Q_v(x)\,$ and $\,Q_{\alpha}(x),\,Q_{\beta}(x)$

$$\mathbf{Q}_{m}(\mathbf{x}) = \frac{\mathbf{bei'} \cdot \mathbf{\psi}_{2}(x) - \mathbf{ber'} \cdot \mathbf{\psi}_{1}(x)}{\mathbf{Z}_{a}} = \frac{(bei' \cdot ber(x) - ber' \cdot bei(x)) - \frac{1 - \upsilon^{*}}{X_{a}}(ber' \cdot ber'(x) + bei' \cdot bei'(x))}{Z_{a}}$$

$$\approx \frac{8(1 + \upsilon^{*})bei' - (3 + \upsilon^{*})x^{2}ber'}{16Z_{a}} \approx \frac{(1 + \upsilon^{*})bei'}{2Z_{a}} \text{ for x close to } 0 \qquad Q_{m}(0) = \frac{(1 + \upsilon^{*})}{2Z_{a}}bei'$$

$$\mathbf{Q}_{m} = \frac{\mathbf{bei'} \cdot \mathbf{\psi}_{2} - \mathbf{ber'} \cdot \mathbf{\psi}_{1}}{\mathbf{Z}_{a}} = \frac{(ber \cdot bei' - ber' \cdot bei) - \frac{1 - \upsilon^{*}}{X_{a}}(ber'^{2} + bei'^{2})}{Z_{a}} = \frac{Z_{a}}{Z_{a}} = 1$$

$$\begin{aligned} \mathbf{Q}_{\mathbf{v}}(x) &= \frac{\mathbf{\Psi}_{1} \cdot \mathbf{\Psi}_{2}(x) - \mathbf{\Psi}_{2} \cdot \mathbf{\Psi}_{1}(x)}{\mathbf{X}_{\mathbf{a}} \mathbf{Z}_{\mathbf{a}}} = \frac{ber \cdot \mathbf{\Psi}_{2}(x) - bei \cdot \mathbf{\Psi}_{1}(x) + \frac{1 - \upsilon^{*}}{X_{a}} (ber^{*} \cdot \mathbf{\Psi}_{2}(x) - bei^{*} \cdot \mathbf{\Psi}_{1}(x))}{X_{a} Z_{a}} \\ &\approx \frac{8(1 + \upsilon^{*}) \mathbf{\Psi}_{1} - (3 + \upsilon^{*}) \mathbf{\Psi}_{2} x^{2}}{16X_{a} Z_{a}} \approx \frac{(1 + \upsilon^{*}) \mathbf{\Psi}_{1}}{2X_{a} Z_{a}} \text{ for x close to 0} \qquad Q_{v}(0) = \frac{(1 + \upsilon^{*})}{2X_{a} Z_{a}} \mathbf{\Psi}_{1} \\ &\mathbf{Q}_{v} = \mathbf{Q}_{v} \mathbf{X} = \frac{\mathbf{h} \cdot \mathbf{\Psi}_{1} \cdot \mathbf{\Psi}_{2} - \mathbf{\Psi}_{2} \cdot \mathbf{\Psi}_{1}}{\mathbf{X}_{a} \mathbf{Z}_{a}} = 0 \\ &\mathbf{Q}_{a}(x) = \frac{\mathbf{b} \mathbf{e} \mathbf{r} \cdot \mathbf{b} \mathbf{e} \mathbf{i}'(x) - \mathbf{b} \mathbf{e} \mathbf{i} \cdot \mathbf{b} \mathbf{e} \mathbf{r}'(x)}{\mathbf{Z}_{a}} \\ &\approx \frac{8xber^{*} + x^{3}bei^{*}}{16Z_{a}} = \frac{ber^{*}}{2Z_{a}} x \quad \text{for x close to 0} \qquad Q_{a}(0) = 0 \\ &\mathbf{Q}_{a} = \mathbf{Q}_{a} \mathbf{X} = \frac{\mathbf{h} \mathbf{e} \mathbf{r}' \cdot \mathbf{b} \mathbf{e} \mathbf{i}' - \mathbf{b} \mathbf{e} \mathbf{i}' \cdot \mathbf{b} \mathbf{e} \mathbf{i}'(x) - bei^{*} \cdot \mathbf{b} \mathbf{e} \mathbf{r}'(x) + \frac{1 - \upsilon^{*}}{X_{a}} (ber^{*} \cdot ber^{*}(x) - bei^{*} \cdot bei^{*}(x))}{\mathbf{Z}_{a}} \\ &\approx \frac{8xber^{*} + x^{3}bei^{*}}{16Z_{a}} = \frac{ber^{*} \cdot \mathbf{b} \mathbf{e} \mathbf{i}' - \mathbf{b} \mathbf{e} \mathbf{i}' \cdot \mathbf{b} \mathbf{e} \mathbf{i}'}{\mathbf{Z}_{a}} = 0 \\ &\mathbf{Q}_{\mu}(x) = \frac{\mathbf{\Psi}_{2} \cdot \mathbf{b} \mathbf{e} \mathbf{i}'(x) - \mathbf{\Psi}_{1} \cdot \mathbf{b} \mathbf{e} \mathbf{r}'(x)}{\mathbf{Z}_{a}} = \frac{(ber \cdot bei^{*} \cdot bei^{*} \cdot ber^{*}(x)) + \frac{1 - \upsilon^{*}}{X_{a}} (ber^{*} \cdot ber^{*}(x) - bei^{*} \cdot bei^{*}(x))}{Z_{a}}}{\mathbf{Z}_{a}} \\ &\approx \frac{8\Psi_{2} x + \Psi_{1} x^{3}}{16Z_{a}} \approx \frac{\Psi_{2}}{2Z_{a}} x \quad \text{for x close to 0} \qquad Q_{\mu}(0) = 0 \\ &\mathbf{Q}_{\mu} = \mathbf{Q}_{\mu} \mathbf{X} = \frac{\Psi_{2} \cdot \mathbf{b} \mathbf{e} \mathbf{i}' - \mathbf{\Psi}_{1} \cdot \mathbf{b} \mathbf{e} \mathbf{r}'}{\mathbf{Z}_{a}}} \qquad \mathbf{Z}_{a} = 1 \end{aligned}$$

(f) Coefficients
$$\mathbf{F}_{m}(\mathbf{x})$$
 and $\mathbf{F}_{t}(\mathbf{x})$

$$F_{m}(\mathbf{x}) = \frac{\mathbf{Q}_{3} \cdot \mathbf{Q}_{m}(\mathbf{x}) + \mathbf{Q}_{v}(\mathbf{x})}{2} = \frac{(\Psi_{1} + Q_{3}X_{a}bei')\Psi_{2}(\mathbf{x}) - (\Psi_{2} + Q_{3}X_{a}ber')\Psi_{1}(\mathbf{x})}{2X_{a}Z_{a}}$$

$$F_{m}(0) = \frac{Q_{3}Q_{m}(0) + Q_{v}(0)}{2} = \frac{1}{2} \left[Q_{3}\frac{1 + \upsilon^{*}}{2Z_{a}}bei' + \frac{1 + \upsilon^{*}}{2Z_{a}}\Psi_{1} \right] = \frac{1 + \upsilon^{*}}{4Z_{a}} \left[Q_{3}bei' + \frac{\Psi_{1}}{X_{a}} \right] = \frac{Q_{3}}{2} + \frac{3 + \upsilon^{*}}{16}$$

$$F_{m}(X_{a}) = \frac{Q_{3}Q_{m}(X_{a}) + Q_{v}(X_{a})}{2} = \frac{Q_{3}}{2}$$

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Note - When F_m is maximum at periphery: $F_m = MAX = \left[\left| F_m(x) \right| \right] = \left| F_m(X_a) \right| = \frac{Q_3}{2} = \frac{Q_3}{2} \text{ if } Q_3 > 0$ $\frac{-Q_3}{2} \text{ if } Q_3 < 0$ $F_Q(x) = \mathbf{Q}_3 \cdot \mathbf{Q}_a(x) + \mathbf{Q}_\beta(x) = \frac{(\Psi_2 + Q_3 \text{ ber'})\text{bei'}(x) - (\Psi_1 + Q_3 \text{ bei'})\text{ber'}(x)}{Z_a}$ $F_Q(0) = Q_3 Q_a(0) + Q_\beta(0) = \frac{Q_3 \text{ ber'} + \Psi_2}{2Z_a} x \text{ for x close to } 0 \rightarrow F_Q(0) = 0$ $F_Q(X_a) = \mathbf{Q}_3 \cdot \mathbf{Q}_a(X_a) + \mathbf{Q}_\beta(X_a) = 1$ $F_t(x) = \left[\mathbf{Q}_3 \cdot \mathbf{Z}_w(x) + \mathbf{Z}_d(x) \right] \frac{\mathbf{X}_a^4}{2} = \frac{(\Psi_1 + Q_3 X_a \text{bei'})\text{bei}(x) + (\Psi_2 + Q_3 X_a \text{ber'})\text{ber}(x))}{2Z_a} X_a$ $F_t(0) = \frac{Q_3 Q_m(0) + Q_v(0)}{2} = \frac{X_a^4}{2} \left[Q_3 \frac{\text{ber'}}{X_a^2 Z_a} + \frac{\Psi_2}{X_a^3} \right] = \frac{X_a^2}{2Z_a} \left[Q_3 \text{ ber'} + \frac{\Psi_2}{X_a} \right]$ $F_t(X_a) = \left[\frac{Q_3}{4(1 + v^*)} + \frac{2}{X_a^4} \right] \frac{X_a^4}{2} = \frac{Q_3}{1 + v^*} \frac{X_a^4}{8} + 1 = 1$

(g) Parameter Q₃ when X_a=0

$$Q_{1} = \frac{(\rho_{s} - 1) - \Phi Z_{v}}{1 + \Phi Z_{m}} = \frac{1}{4} \frac{(\rho_{s} - 1)4(1 + v^{*}) - \Phi}{(1 + v^{*}) + \Phi} = \frac{(\rho_{s} - 1) - \frac{F}{4}}{1 + F}$$

$$Q_{2} = \frac{N}{(\omega_{s}^{*} P_{s} - \omega_{c}^{*} P_{c}) + (W^{*} \frac{\gamma_{b}}{2\pi})}{1 + \Phi Z_{m}} = \frac{N}{1 + F}$$

$$Q_{3} = Q_{1} + \frac{2}{a_{0}^{2} P_{s}} Q_{2} = \frac{(\rho_{s} - 1) - \frac{F}{4}}{1 + F} + \frac{2}{a_{0}^{2} P_{s}} \frac{N}{1 + F} = \frac{(\rho_{s} - 1) + \frac{2N}{a_{0}^{2} P_{s}} - \frac{F}{4}}{1 + F}$$

ANNEX H — RADIAL DISPLACEMENT AND ROTATION OF THE SHELL AT ITS CONNECTION WITH THE RING

1 Radial Displacement Due to Internal Pressure Ps

Due to internal pressure P_s , the circumference of the shell internal wall increases from $2\pi a_s$ to $2\pi a'_s$, thus its elongation is written: $2\pi(a'_s-a_s)=2\pi w_s(P_s)$ where $w_s(P_s)$ is the radial displacement of the shell due to pressure P_s .





Shell strain: $\varepsilon_s \left(P_s\right) = \frac{2\pi w_s \left(P_s\right)}{2\pi a_s} = \frac{w_s \left(P_s\right)}{a_s} = \frac{1}{E_s} \left[\sigma_\theta - v_s \left(\sigma_l + \sigma_r\right] = \frac{1}{E_s} \left[\frac{a_s P_s}{t_s} - v_s \frac{V_s}{t_s}\right]$ where: $\sigma_\theta = \frac{a_s P_s}{t_s}$ is the circumferential stress in the shell $\sigma_l = \frac{V_s}{t_s}$ is the longitudinal stress in the shell subjected to the longitudinal force V_s $\sigma_r = -P_s$ is the radial stress in the shell, which can be neglected compared to σ_θ .

Shell radial displacement due to internal pressure:

$$w_{s}\left(P_{s}\right) = \frac{a_{s}^{2}}{E_{s}t_{s}}\left[P_{s} - \frac{V_{s}}{a_{s}}v_{s}\right] \quad [\text{A-VI.1a-1}]$$

Note: For a U-tube or floating TS HE, V_s is known:

$$2\pi a_s' V_s = \pi a_s^2 P_s \implies V_s = \frac{a_s^2 P_s}{2a_s'} \implies \sigma_l = \frac{a_s^2 P_s}{2a_s' t_s}$$
$$w_s \left(P_s\right) = \frac{a_s^2}{E_s t_s} \left[P_s - \frac{a_s P_s}{a_s'} \frac{V_s}{2}\right] = \frac{a_s^2}{E_s t_s} \left(1 - \frac{a_s}{a_s'} \frac{V_s}{2}\right) P_s = \delta_s P_s$$

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where:
$$\delta_{s} = \frac{a_{s}^{2}}{E_{s}t_{s}} \left(1 - \frac{a_{s}}{a_{s}} \frac{v_{s}}{2} \right) = \frac{D_{s}^{2}}{4E_{s}t_{s}} \left(1 - \frac{D_{s}}{D_{s} + t_{s}} \frac{v_{s}}{2} \right)$$

2 Radial Displacement and Rotation Due to Edge Loads Qs and Ms

Formulas provided in this Annex are taken from Appendix 4.2 of ASME Section VIII-Div. 2 [11] .

$$w_{s}(Q_{s}, M_{s}) = \frac{Q_{s}}{2\beta_{s}^{3}D_{s}} + \frac{M_{s}}{2\beta_{s}^{2}D_{s}} \qquad \theta_{s}(Q_{s}, M_{s}) = \frac{Q_{s}}{2\beta_{s}^{2}D_{s}} + \frac{M_{s}}{2\beta_{s}D_{s}}$$
$$\beta_{s} = \frac{\sqrt[4]{12(1-v_{s}^{2})}}{\sqrt{(D_{s}+t_{s})t_{s}}} \qquad D_{s} = \frac{E_{s}t_{s}^{3}}{12(1-v_{s}^{2})} \qquad k_{s} = 2\beta_{s}D_{s} = \beta_{s}\frac{E_{s}t_{s}^{3}}{6(1-v_{s}^{2})}$$
$$w_{s}(Q_{s}, M_{s}) = \frac{Q_{s}}{\beta_{s}^{2}k_{s}} + \frac{M_{s}}{k_{s}} \qquad \theta_{s}(Q_{s}, M_{s}) = \frac{Q_{s}}{\beta_{s}k_{s}} + \frac{2M_{s}}{k_{s}}$$

3 Radial Displacement Due to Internal Pressure and Edge Loads

$$w_{s} = \frac{Q_{s}}{\beta_{s}^{2} k_{s}} + \frac{M_{s}}{\beta_{s} k_{s}} + \delta_{s} P_{s}$$
$$\theta_{s} = \frac{Q_{s}}{\beta_{s} k_{s}} + \frac{2M_{s}}{k_{s}}$$
With: $\delta_{s} = \frac{a_{s}^{2}}{E_{s} t_{s}} \left(1 - \frac{a_{s}}{a_{s}} \frac{v_{s}}{2}\right)$ where V_{s} is unknown

At this point of the development, it appears that it is not possible to get a solution if V_s is unknown. Thus, the classical shell formula:

$$\sigma_l = \frac{a_s^2 P_s}{2(a_s + t_s/2)t_s} = \frac{D_s}{D_s + t_s} \frac{P_s D_s}{2t_s}$$
 is used, which leads to:
$$\delta_s = \frac{D_s^2}{4 E_s t_s} \left(1 - \frac{D_s}{D_s + t_s} \frac{V_s}{2}\right) P_s$$

Note: In UHX-13, t_s has been neglected compared to D_s and:

$$\delta_s = \frac{D_s^2}{4 E_s t_s} \left(1 - \frac{v_s}{2} \right) P_s$$

4 Channel

Above formulas apply to the channel by replacing subscript s by subscript c, except that the axial force and longitudinal stress in the channel are known:

$$V_{c} = \frac{a_{c} P_{c}}{2} \qquad \sigma_{l} = \frac{a_{c} P_{c}}{2 t_{c}} \quad \text{which leads to:} \qquad \delta_{c} = \frac{D_{c}^{2}}{4 E_{c} t_{c}} \left[1 - \frac{D_{c}}{D_{c} + t_{c}} \frac{v_{c}}{2} \right]$$
Note: In UHX-13, t_{c} has been neglected compared to D_c and:
$$\delta_{c} = \frac{D_{c}^{2}}{4 E_{c} t_{c}} \left[1 - \frac{v_{c}}{2} \right]$$

ANNEX I — SHELL-TO-RING CONNECTION IN RADIAL DIRECTION

Radial force $Q_r(a_o)$ acting at TS periphery, due to Q_s and Q_c (see Figure 58 below): $Q_r(a_o) = Q_s + Q_c$ TS radial displacement $w_r(a_o)$ at mid-thickness, at point A, due to $Q_r(a_o)$: $w_r(a_o) = \frac{a_o}{h} \frac{1 - v^*}{E^*} Q_R$





Ring radial displacement w_R at point S (see Figure 59 below):



Figure 59 — Ring Radial Displacement



Continuity of displacements at ring-shell connection (point S):

$$w_{\rm R} = w_{\rm s}$$
 $\theta_{\rm R} = \theta_{\rm s}$ \Rightarrow $w_{\rm s} = -\frac{h}{2}\theta_{\rm s}$

Total radial displacement w_T of the ring at point S: $w_T = w_r(a_o) + w_R = w_r(a_o) + w_s$

At this point of the development, it appears that it is necessary to ignore the radial displacement at mid thickness $w_r(a_0)$ to get a solution.

Compatibility of shell-ring displacements is written:

$$w_{s} = -\frac{h}{2} \theta_{s}$$

ANNEX J — MINIMUM LENGTH OF SHELL AND CHANNEL WHEN INTEGRAL WITH THE TS

The bending moment M(x) located at the distance x of the edge of the shell (point S in Figure 17), submitted to moment M_s is given by (see Appendix 4.2 of [11]):

$$M_{s}(x) = \frac{Q_{s}}{\beta_{s}} f_{4}(\beta x) + M_{s} f_{3}(\beta x) \qquad \text{with} \quad \beta_{s} = \frac{\sqrt[4]{12(1-v_{s}^{2})}}{\sqrt{(D_{s}+t_{s})}t_{s}} = \frac{1.817}{\sqrt{(D_{s,m})t_{s}}} \quad \text{for } v = 0.3$$

Neglecting the radial displacement of the shell submitted to edge loads M_s and Q_s, given by Section H.2 of Annex H, the following relationship is obtained: $Q_s = -\beta_s M_s$ which leads to:

$$\begin{split} M_s(x) = & \left[f_3(\beta x) - f_4(\beta x) \right] M_s \qquad \text{with}: \quad f_3(\beta x) = e^{-\beta x} \left[\cos(\beta x) + \sin(\beta x) \right] \\ & f_4(\beta x) = e^{-\beta x} \left[\sin(\beta x) \right] \end{split} \right\} f_3(\beta x) - f_4(\beta x) = e^{-\beta x} \cos(\beta x) \end{split}$$

which leads to: $M_s(x) = e^{-\beta x} \cos(\beta x) M_s$

The moment M_s(x) decreases according to a damped sinusoidal curve represented on figure below.



The length x can be written as: $x = k\sqrt{D_{s,m} t} \implies \beta x = \beta k \sqrt{D_{s,m} t} = 1.817 k$ For k = 1.0 $x = 1.0 \sqrt{D_{s,m} t_s}$ M_s(x) = 0.040 M_s For k = 1.4 $x = 1.4 \sqrt{D_{s,m} t}$ M_s(x) = 0.065 M_s For k = 1.8 $x = 1.8 \sqrt{D_{s,m} t}$ M_s(x) = 0.040 M_s For k = 2.0 $x = 2.0 \sqrt{D_{s,m} t}$ M_s(x) = 0.020 M_s For k = 2.5 $x = 2.5 \sqrt{D_{s,m} t}$ M_s(x) = 0.002 M_s

The value $x = 1.8 \sqrt{D_m t}$ has been retained as the remaining bending moment at this distance of the shell edge is only 4%.

Therefore, when they are integral with the TS, the shell and the channel must have a minimum length adjacent to the TS of $l_{s,\min} = 1.8 \sqrt{D_s t_s}$ and $l_{c,\min} = 1.8 \sqrt{D_c t_c}$.

ANNEX K — FORMULAS FOR A HEMISPHERICAL CHANNEL WHEN INTEGRAL WITH THE TS

The formulas given in Annex H are valid for a cylindrical channel. If the channel is hemispherical and attached directly to the TS (configurations a, b or c), without any cylindrical section between the head and the TS (Figure 60), similar formulas can be developed, based on Appendix 4.3 of Section VIII-Div. 2 [11].



Figure 60 — Hemispherical Head

1 Radial Displacement Due to Internal Pressure Pc

Channel strain:

$$\varepsilon_{c}(P_{c}) = \frac{2\pi w_{c}(P_{c})}{2\pi a_{c}} = \frac{w_{c}(P_{c})}{a_{c}} = \frac{1}{E_{c}} \left[\sigma_{\theta} - v_{c} (\sigma_{l} + \sigma_{r}) \right] = \frac{1}{E_{c}} \left[\frac{a_{c} P_{c}}{2t_{c}} - v_{c} \frac{a_{c}^{2} P_{c}}{2(a_{c} + t_{c}/2)t_{c}} \right]$$

where: $\sigma_{\theta} = \frac{a_{c} P_{c}}{t_{c}}$ is the circumferential stress in the channel

$$\sigma_l = \frac{a_c^2 P_c}{2(a_c + t_c / 2)t_c}$$
 is the longitudinal stress in the channel

 $\sigma_r = -P_c$ is the radial stress in the channel, which can be neglected compared to σ_{θ} . Channel radial displacement due to internal pressure:

$$w_{c}(P_{c}) = \underbrace{\frac{a_{c}^{2}P_{c}}{E_{c}t_{c}}}_{\delta_{c}} \left[\frac{1 - \frac{a_{c}}{a_{c}}v_{c}}{2} \right]_{\delta_{c}} = \delta_{c}P_{c} \quad \text{where:} \quad \delta_{c} = \frac{a_{c}^{2}}{E_{c}t_{c}} \left[\frac{1 - \frac{a_{c}}{a_{c}}v_{c}}{2} \right]_{\delta_{c}} = \frac{D_{c}^{2}}{4E_{c}t_{c}} \left[\frac{1 - \frac{D_{c}}{D_{c} + t_{c}}v_{c}}{2} \right]_{\delta_{c}}$$

2 Radial Displacement and Rotation Due to Edge Loads Qs and Ms

Radial displacement w_c and rotation θ_c formulas are taken from Clause 4-332(b) of Appendix 4.3 of Section VIII Div. 2 [11] :

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$$\begin{split} w_{c} &= \frac{2\beta_{c} R_{m}^{2}}{E_{c} t_{c}} Q_{c} + \frac{2\beta_{c}^{2} R_{m}^{2}}{E_{c} t_{c}} M_{c} \qquad \qquad \theta_{c} = \frac{2\beta_{c}^{2} R_{m}^{2}}{E_{c} t_{c}} Q_{c} + \frac{4\beta_{c}^{3} R_{m}^{2}}{E_{c} t_{c}} M_{c} \qquad \qquad with: R_{m} = \frac{D_{c} + t_{c}}{2} \\ \frac{2\beta_{c} R_{m}^{2}}{E_{c} t_{c}} &= \frac{1}{2\beta_{c}^{3}} \frac{4\beta_{c}^{4} R_{m}^{2}}{E_{c} t_{c}} = \frac{1}{2\beta_{c}^{3}} \frac{12(1-\upsilon_{c}^{2})}{E_{c} t_{c}^{3}} = \frac{1}{2\beta_{c}^{3}} \frac{1}{D_{c}} = \frac{1}{2\beta_{c}^{3} D_{c}} = \frac{1}{\beta_{c}^{2} k_{c}} \\ \frac{2\beta_{c}^{2} R_{m}^{2}}{E_{c} t_{c}} &= \beta_{c} \frac{2\beta_{c} R_{m}^{2}}{E_{c} t_{c}} = \frac{1}{2\beta_{c}^{2} D_{c}} = \frac{1}{\beta_{c} k_{c}} \\ \frac{4\beta_{c}^{3} R_{m}^{2}}{E_{c} t_{c}} &= 2\beta_{c} \frac{2\beta_{c}^{2} R_{m}^{2}}{E_{c} t_{c}} = 2\beta_{c} \frac{1}{2\beta_{c}^{2} D_{c}} = \frac{1}{\beta_{c} k_{c}} \\ w_{c} &= \frac{Q_{c}}{2\beta_{c}^{3} D_{c}} + \frac{M_{c}}{2\beta_{c}^{2} D_{c}} = \frac{Q_{c}}{\beta_{c}^{2} k_{c}} + \frac{M_{c}}{\beta_{c} k_{c}} \qquad \qquad \theta_{c} = \frac{Q_{c}}{2\beta_{c}^{2} D_{c}} = \frac{Q_{c}}{\beta_{c} k_{c}} + \frac{2M_{c}}{k_{c}} \end{split}$$

These formulas are the same as for a cylindrical channel (see Section 4 of Annex H)

3 Radial Displacement Due to Internal Pressure and Edge Loads

Accordingly, the formulas are the same as for a cylindrical channel, provided in Section 4 of Annex H,

$$\delta_s = \frac{D_c^2}{4E_c t_c} \left(1 - \frac{D_c}{D_c + t_c} \frac{V_c}{2} \right) P_c \quad \text{is replaced by} \quad \left| \delta_c = \frac{D_c^2}{4E_c t_c} \left(\frac{1}{2} - \frac{D_c}{D_c + t_c} \frac{V_c}{2} \right) \right|$$

Note: In UHX-13, t_c has been neglected compared to D_c:

$$\delta_c = \frac{D_c^2}{4E_c t_c} \left[\frac{1}{2} - \frac{v_c}{2} \right]$$

ANNEX L — EQUILIBRIUM OF RING SUBJECTED TO EDGE MOMENTS

(a) For configuration a

Axial equilibrium of forces



Figure 61 — Configuration a

Axial force V_s in the shell:

[VI.2a-1] $2 \pi a_{s} V_{s} + \pi (a_{c}^{2} - a_{o}^{2}) P_{c} = 2 \pi a_{c} V_{c} + 2 \pi a_{o} V_{a} + \pi (a_{s}^{2} - a_{o}^{2}) P_{s}$ Axial force V_c in the channel:

[VI.2a-2]
$$2 \pi a_c V_c = \pi a_c^2 P_c \implies a_c V_c = \frac{a_c^2}{2} P_c$$

Axial equilibrium of the ring is written: [VI.2a-3] $a_s V_s = a_o V_a + \frac{a_o^2}{2} P_c + \frac{a_s^2 - a_o^2}{2} P_s$

Axial equilibrium of moments

$$[VI.1a] \begin{cases} M_{s} = +\left[k_{s}\left(1+\frac{t_{s}}{2}\right)\right]\theta_{s} + \left[\beta_{s} k_{s}\right]\delta_{s} P_{s} \\ Q_{s} = -\left[\beta_{s} k_{s}\left(1+t_{s}\right)\right]\theta_{s} - \left[2\beta_{s}^{2} k_{s}\right]\delta_{s} P_{s} \end{cases}$$
$$[VI.1b] \begin{cases} M_{c} = +\left[k_{c}\left(1+\frac{t_{c}}{2}\right)\right]\theta_{c} + \left[\beta_{c} k_{c}\right]\delta_{c} P_{c} \\ Q_{c} = -\left[\beta_{c} k_{c}\left(1+t_{c}\right)\right]\theta_{c} - \left[2\beta_{c}^{2} k_{c}\right]\delta_{c} P_{c} \end{cases}$$

From Figure 17 and Figure 61, ring equilibrium is written:

$$\begin{bmatrix} R M_{R} \end{bmatrix}_{a} = -\overrightarrow{a_{o} M_{a}} + \begin{bmatrix} \overrightarrow{a_{c} M_{c} - a_{c} Q_{c} \frac{h}{2}} \end{bmatrix} + \begin{bmatrix} M(P_{c}) - \overrightarrow{a_{c} V_{c} (a_{c} - a_{o})} \end{bmatrix}$$

$$- \begin{bmatrix} \overrightarrow{a_{s} M_{s} - a_{s} Q_{s} \frac{h}{2}} \end{bmatrix} - \begin{bmatrix} M(P_{s}) - \overrightarrow{a_{s} V_{s} (a_{s} - a_{o})} \end{bmatrix}$$

$$A1s$$
[VI.2b]

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where:
$$R = \text{radius at center of ring} = \frac{A+2a_0}{4}$$

 $M(P_c) = \text{moment due to pressure P_c acting on the ring: } (a_c^2 - a_o^2) \left(\frac{a_c + a_o}{2} - a_o\right) \frac{P_c}{2}$
 $M(P_s) = \text{moment due to pressure P_s acting on the ring: } (a_s^2 - a_o^2) \left(\frac{a_s + a_o}{2} - a_o\right) \frac{P_s}{2}$
Derivations are performed so that the moment M_R can be calculated from V_a, M_a, 0a and P_s, P_c.
 $Alc = \left[a_c^{-}k_c\left(1 + \frac{t_c}{2}\right)\theta_c + a_c^{-}k_c\beta_c\delta_cP_c\right] + \left[a_c^{-}\beta_c k_c\left(1 + t_c^{-}\right)\frac{h}{2}\theta_c + 2a_c^{-}k_c\beta_c^2\delta_c\frac{h}{2}P_c\right]$
 $= a_c^{-}k_c\left[\left(1 + \frac{t_c}{2}\right) + \beta_c\left(1 + t_c^{-}\right)\frac{h}{2}\right]\theta_c + a_c^{-}k_c\beta_c\delta_c\left(1 + 2\beta_c\frac{h}{2}\right)P_c$
 $= a_c^{-}k_c\left(1 + t_c^{-} + \frac{t_c^{-2}}{2}\right)\theta_c + a_o\beta_c\beta_ck_c\delta_c\left(1 + \beta_ch\right)P_c$
 $= a_c^{-}k_c\left(1 + t_c^{-} + \frac{t_c^{-2}}{2}\right)\theta_c + a_o\beta_c\beta_cR_c\left(1 + \frac{h}{2}ch\right)$
 $Als = a_s^{-}k_s\left(1 + t_s^{-} + \frac{t_s^{-2}}{2}\right)\theta_s + a_o\beta_sR_s\left(1 + t_s^{-} + \frac{t_s^{-2}}{2}\right)\theta_s + a_o\left(\omega_s P_s - \omega_c P_c\right)$
Accounting for compatibility of shell and channel rotations: $\begin{cases} \theta_s = \theta_a \\ \theta_c = -\theta_a \end{cases}$
 $Alc - Als = -\left[a_s^{-}k_s\left(1 + t_s^{-} + \frac{t_s^{-2}}{2}\right) + a_c^{-}k_c\left(1 + t_c^{-} + \frac{t_c^{-2}}{2}\right)\right]\theta_a + a_o\left(\omega_c P_c - \omega_s P_s\right)$

$$\begin{aligned} A2c &= \left(a_{c}^{2} - a_{o}^{2}\right)\left(a_{c} - a_{o}\right)\frac{P_{c}}{4} - a_{c}^{2}\frac{P_{c}}{2}\left(a_{c}^{2} - a_{o}\right) = \frac{P_{c}}{4}\left[\left(a_{c}^{2} - a_{o}^{2}\right)\left(a_{c} - a_{o}\right) - 2a_{c}^{2}\left(a_{c}^{2} - a_{o}\right)\right]\right] \\ A2s &= \left(a_{s}^{2} - a_{o}^{2}\right)\left(a_{s} - a_{o}\right)\frac{P_{s}}{4} - \left[a_{o}V_{a} + \left(a_{s}^{2} - a_{o}^{2}\right)\frac{P_{s}}{2} + a_{o}^{2}\frac{P_{c}}{2}\right]\left(a_{s}^{2} - a_{o}\right)\right] \\ &= -a_{o}V_{a}\left(a_{s}^{2} - a_{o}\right) + \frac{P_{s}}{4}\left[\left(a_{s}^{2} - a_{o}^{2}\right)\left(a_{s} - a_{o}\right) - 2\left(a_{s}^{2} - a_{o}^{2}\right)\left(a_{s}^{2} - a_{o}\right)\right] - 2a_{0}^{2}\left(a_{s}^{2} - a_{o}\right)\frac{P_{c}}{4} \\ &= -a_{o}V_{a}\left(a_{s}^{2} - a_{o}\right) + \frac{P_{s}}{4}\left(a_{s}^{2} - a_{o}^{2}\right)\left(a_{s} + a_{o} - 2a_{s}^{2}\right) - \frac{P_{c}}{4}\left[2a_{o}^{2}\left(a_{s}^{2} - a_{o}\right)\right] \\ A2c - A2s = +a_{o}V_{a}\left(a_{s}^{2} - a_{o}\right) - \frac{P_{s}}{4}\left(a_{s}^{2} - a_{o}^{2}\right)\left(a_{s} + a_{o} - 2a_{s}^{2}\right) + \frac{P_{c}}{4}\left[\left(a_{c}^{2} - a_{o}^{2}\right)\left(a_{c} - a_{o}\right) - 2a_{0}^{2}\left(a_{s}^{2} - a_{o}\right)\frac{P_{c}}{4}\right] \\ A2c - A2s = +a_{o}V_{a}\left(a_{s}^{2} - a_{o}^{2}\right)\left(a_{s} + a_{o} - 2a_{s}^{2}\right) + \frac{P_{c}}{4}\left[\left(a_{c}^{2} - a_{o}^{2}\right)\left(a_{c} - a_{o}\right) - 2a_{0}^{2}\left(a_{s}^{2} - a_{o}\right)\frac{P_{c}}{4}\left(a_{s}^{2} - a_{o}^{2}\right)\left(a_{s} - a_{o}\right)\frac{P_{c}}{4}\right] \\ A2c - A2s = +a_{o}V_{a}\left(a_{s}^{2} - a_{o}^{2}\right)\left(a_{s} + a_{o} - 2a_{s}^{2}\right)\left(a_{s} - a_{o}^{2}\right)\left(a_{c}^{2} - a_$$



Equation [VI.2b] is written for configuration a:

(*b*) For configuration b:

Axial equilibrium of forces



Figure 62 — Configuration b

Equilibrium of the channel:

 $2 \pi a_{c}^{'} V_{c} = 2 \pi a_{Bc} B_{c} - 2 \pi a_{c} J_{c} = \pi a_{c}^{2} P_{c} \implies a_{Bc} B_{c} - a_{c} J_{c} = \frac{a_{c}^{2}}{2} P_{c}$ Axial force V_s in the shell: $2 \pi a_{s}^{'} V_{s} + \pi \left(a_{c}^{2} - a_{o}^{2}\right) P_{c} = 2 \pi a_{o} V_{a} + \underbrace{2 \pi a_{Bc} B_{c} - 2 \pi a_{c} J_{c}}_{\pi a_{c}^{2}} + \pi \left(a_{s}^{2} - a_{o}^{2}\right) P_{s}$

Axial equilibrium of the ring remains unchanged:

$$a_{s}' V_{s} = a_{o} V_{a} + \frac{a_{o}^{2}}{2} P_{c} + \frac{a_{s}^{2} - a_{o}^{2}}{2} P_{s}$$

Where (see Figure 62):

 $\begin{cases} a_c = mean \ gasket \ radius = \frac{G_c}{2} & J_c = gasket \ load \ per \ unit \ of \ circumference \\ a_{Bc} = bolt \ circle \ radius = \frac{C_c}{2} & B_c = bolt \ load \ per \ unit \ of \ circumference \\ Arried \ acciliation \ for \ circumference \\ \end{cases}$

Axial equilibrium of moments. In equation [VI.2b]:

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$$\begin{bmatrix} R M_{R} \end{bmatrix}_{a} = \overbrace{-a_{o} M_{a}}^{A0} + \overbrace{\begin{bmatrix} a_{c} M_{c} - a_{c} Q_{c} \frac{h}{2} \end{bmatrix}}^{A1c} + \overbrace{\begin{bmatrix} M (P_{c}) - a_{c} V_{c} (a_{c} - a_{o}) \end{bmatrix}}^{A2c} - \underbrace{\begin{bmatrix} a_{s} M_{s} - a_{s} Q_{s} \frac{h}{2} \end{bmatrix}}_{A1s} - \underbrace{\begin{bmatrix} M (P_{s}) - a_{s} V_{s} (a_{s} - a_{o}) \end{bmatrix}}_{A2s}$$
[VI.2b]

- Terms A0, A1s, A2s due to shell remain unchanged.
- Term A1c due to moments applied to the channel disappears, but can be maintained provided that k_c is taken equal to 0: k_c=0.
- Term A2c becomes [A2c]_b:

$$\begin{bmatrix} A2c \end{bmatrix}_{b} = M(P_{c}) - a_{Bc} B_{c} (a_{Bc} - a_{o}) + a_{c} J_{c} (a_{c} - a_{o}) = M(P_{c}) + A_{B}$$

$$A_{B} = -a_{Bc} B_{c} (a_{Bc} - a_{o}) + \left(a_{Bc} B_{c} - \frac{a_{c}^{2}}{2} P_{c}\right) (a_{c} - a_{o})$$
Where:
$$B_{C} = -a_{Bc} B_{c} (a_{Bc} - a_{o}) + \left(a_{Bc} B_{c} - \frac{a_{c}^{2}}{2} P_{c}\right) (a_{c} - a_{o})$$

$$= -a_{Bc} B_{c} a_{Bc} + a_{Bc} B_{c} a_{c} - \frac{a_{c}^{2}}{2} P_{c} (a_{c} - a_{o}) = a_{Bc} B_{c} (a_{c} - a_{Bc}) \frac{a_{c}^{2}}{2} (a_{c} - a_{o})$$

$$[A2c]_{b} = \underbrace{M(P_{c}) - \frac{a_{c}^{2}}{2} (a_{c} - a_{o}) P_{c} + a_{Bc} B_{c} (a_{c} - a_{Bc})}_{A3 \text{ with}: a_{c}' = a_{c}} = A3 + a_{Bc} B_{c} (a_{c} - a_{Bc})$$

•
$$A2c_b - A2s = A2c - A2s + a_{Bc} B_c (a_c - a_{Bc})$$
 leads to equation $R M_R$ for configuration b:
 $\begin{bmatrix} R M_R \end{bmatrix}_b = \begin{bmatrix} R M_R \end{bmatrix}_a + a_{Bc} B_c (a_c - a_{Bc})$
Using UHX-13 notations:
 $a_{Bc} B_c (a_c - a_{Bc}) = \frac{C_c}{2} \frac{W_c}{\pi C_c} \left(\frac{G_c}{2} - \frac{C_c}{2}\right) = \frac{a_o}{2\pi} W_c \left(\frac{G_c - C_c}{D_o}\right)$
 $a_{Bc} B_c (a_c - a_{Bc}) = \frac{a_o}{2\pi} W_c \gamma_{bc}$ where: $\gamma_{bc} = \frac{G_c - C_c}{D_o}$

- $\left[RM_{R}\right]_{b} = \left[RM_{R}\right]_{a} + \frac{u_{o}}{2\pi}W_{c}\gamma_{bc} \text{ with } k_{c} = 0 \text{ in } [RM_{R}]_{a} \text{ equation}$
- (c) For configuration c:

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Figure 63 — Configuration c

The same applies but the bolting diameter Cc becomes the midpoint of contact between flange and

FS, G₁ (see Figure 63). Thus
$$\gamma_{bc}$$
 becomes: $\gamma_{bc} = \frac{G_c - G_1}{D_o}$

(d) For configuration d: Axial equilibrium of forces

•



Figure 64 — Configuration d

Axial force V_c in the channel remains unchanged:

$$a_c V_c = a_{Bc} B_c - a_c J_c = \frac{a_c^2}{2} P_c \qquad a_c = \frac{G_c}{2} \qquad a_{Bc} = \frac{C_c}{2}$$

Axial force V_s in the shell is given by, with $a_s = \frac{G_s}{2}$, $a_{Bc} = \frac{C_c}{2}$ and $a_{Bs} = \frac{C_s}{2}$

$$2 \pi a_s V_s + 2 \pi a_s J_s = 2 \pi a_{Bs} B_s \implies \boxed{a_s V_s = a_{Bs} B_s - a_s J_s}$$

Axial equilibrium of moments. Comparing to configuration b:

Terms A0, A1c and [A2c]_b due to the channel remain unchanged.

Term A1s due to moments applied to the shell disappears, but can be maintained provided that k_s is taken equal to 0: $k_s=0$.

Term A2s becomes [A2s]_d, similar to term [A2c]_b of the channel:

$$[A2s]_{d} = \underbrace{M(P_{s}) - \frac{a_{s}^{2}}{2}(a_{s} - a_{o})}_{A3 \text{ avec } a_{s} = a_{s}} P_{s} + a_{Bs} B_{s} (a_{s} - a_{Bs}) P_{c} = A2s + a_{Bs} B_{s} (a_{s} - a_{Bs})$$

$$Torm: [A2c] - [A2s] = A2c + a_{Bs} B_{s} (a_{s} - a_{Bs}) - A2s = a_{Bs} B_{s} (a_{s} - a_{Bs})$$

Term:
$$[A2c]_{b} - [A2s]_{d} = A2c + a_{Bc} B_{c} (a_{c} - a_{Bc}) - A2s - a_{Bs} B_{s} (a_{s} - a_{Bs})$$

leads to the equation
$$\begin{bmatrix} R & M_R \end{bmatrix}_d$$
 for configuration d:

$$\begin{bmatrix} R M_{R} \end{bmatrix}_{a} = \begin{bmatrix} R M_{R} \end{bmatrix}_{a} + a_{Bc} B_{c} (a_{c} - a_{Bc}) - a_{Bs} B_{s} (a_{s} - a_{Bs})$$

Using UHX-13 notations:

$$a_{Bs} B_{s} \left(a_{s}-a_{Bs}\right) = \frac{C_{s}}{2} \frac{W_{s}}{\pi C} \left(\frac{G_{s}}{2}-\frac{C_{s}}{2}\right) = \frac{a_{o}}{2 \pi} W_{s} \left(\frac{G_{s}-C_{s}}{D_{o}}\right) = \frac{a_{o}}{2 \pi} W_{s} \gamma_{bs}$$

$$\begin{bmatrix} R M_{R} \end{bmatrix}_{a} = \begin{bmatrix} R M_{R} \end{bmatrix}_{a} + \frac{a_{o}}{2 \pi} W_{c} \gamma_{bc} - \frac{a_{o}}{2 \pi} W_{s} \gamma_{bs} \text{ with:} \begin{cases} k_{s}=0 \quad and \quad \gamma_{bs} = \frac{G_{s}-C_{s}}{D_{o}} \\ k_{c}=0 \quad and \quad \gamma_{bc} = \frac{G_{c}-C_{c}}{D_{o}} \end{cases}$$

Finally, the generic equation covering the 4 configurations a, b, c and d is written:

$$\begin{bmatrix} R M_{R} = -a_{o} M_{a} + a_{o}^{2} V_{a} (\rho_{s} - 1) + P_{s} \frac{a_{o}^{3}}{4} \left[\left(\rho_{s}^{2} - 1 \right) \left(\rho_{s} - 1 \right) \right] - P_{c} \frac{a_{o}^{3}}{4} \left[\left(\rho_{c}^{2} + 1 \right) \left(\rho_{c} - 1 \right) - 2 \left(\rho_{s} - 1 \right) \right] \\ - \left[a_{s}^{'} k_{s} \left(1 + t_{s}^{'} + \frac{t_{s}^{'2}}{2} \right) + a_{c}^{'} k_{c} \left(1 + t_{c}^{'} + \frac{t_{c}^{'2}}{2} \right) \right] \theta_{a} + a_{o} \left(\omega_{c} P_{c} - \omega_{s} P_{s} \right) + \frac{a_{o}}{2 \pi} \left[W_{c} \gamma_{bc} - W_{s} \gamma_{bs} \right] \end{bmatrix} \begin{bmatrix} \text{VI.2d} \end{bmatrix}$$

In this equation:

- λ_s , λ_c and w_s , w_c are coefficients obtained above. They are known for a given HE.
- $W_{\rm s}$ and $W_{\rm c}$ are bolt loads applied on shell and channel when the TS is gasketed (configurations b, c, d).
- Coefficients γ_{bs} and γ_{bc} are defined for each configuration as follows:

Configuration a:
$$k_s$$
 given in VI.1a k_c given in VI.1b $\gamma_{bs} = 0$ $\gamma_{bc} = 0$ Configuration b: k_s given in VI.1a $k_c = 0$ $\gamma_{bs} = 0$ $\gamma_{bc} = \frac{G_c - C_c}{D_o}$

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Configuration c: k_{s} given in VI.1a $k_{c} = 0$ $\gamma_{bs} = 0$ $\gamma_{bc} = \frac{G_{c} - G_{1}}{D_{0}}$ Configuration d: $k_{s} = 0$ $k_{c} = 0$ $\gamma_{bs} = \frac{G_{s} - C_{s}}{D_{0}}$ $\gamma_{bc} = \frac{G_{c} - C_{c}}{D_{0}}$


ANNEX M — DIRECT DETERMINATION OF THE EQUIVALENT PRESSURE

Equivalent pressures P's, P't and P' γ can be obtained directly by examining the loads applied on the TS as follows.

(a) Pressures P_s and P_t acting on the perforated TS (Figure 65)



Figure 65 — Pressures P_s and P_t Acting on TS

$$\boxed{P_{TS}(P_s, P_t) = x_s P_s - x_t P_t} \qquad x_s = 1 - N_t \left(\frac{d_t}{D_0}\right)^2 \qquad x_t = 1 - N_t \left(\frac{d_t - 2t_t}{D_0}\right)^2$$

(b) Pressures P_s and P_t acting on the unperforated TS RIM Load acting on the rim:

$$F_{RIM}(P_s, P_t) = \pi (a_s^2 - a_o^2) P_s - \pi (a_c^2 - a_o^2) P_t = \pi a_o^2 \Big[(\rho_s^2 - 1) P_s - (\rho_c^2 - 1) P_t \Big]$$

Part of F_{rim} supported by the tubes: $F_{RIM}(P_s, P_t) \frac{N_t K_t}{K_s^*} = F_{RIM}(P_s, P_t) \frac{N_t K_t}{JK_s} = \frac{F_{RIM}(P_s, P_t)}{JK_{s,t}}$

Equivalent pressure acting on the unperforated RIM:

$$P_{RIM}(P_s, P_t) = -\frac{F_{RIM}(P_s, P_t)}{\pi a_o^2 J K_{s,t}} = -\frac{(\rho_s^2 - 1)P_s - (\rho_c^2 - 1)P_t}{J K_{s,t}}$$
$$P_{RIM}(P_s, P_t) = -\frac{F_{RIM}(P_s, P_t)}{\pi a_o^2 J K_{s,t}} = \boxed{-\frac{(\rho_s^2 - 1)P_s + (\rho_c^2 - 1)P_s}{J K_{s,t}}}$$

(c) **Pressure P**_s acting on the joint (Figure 66)

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Figure 66 — Pressure Ps Acting on Bellows Joint

Axial load acting on the shell: $F_{J}(P_{s}) = \frac{F_{J}(P_{s})}{2} = P_{s} \frac{\pi}{2} \left(R_{J}^{2} - R_{s}^{2}\right)$ Axial displacement of the half-shell: $\Delta_{J}(P_{s}) = \frac{P_{s} \pi \left(R_{J}^{2} - R_{s}^{2}\right)/2}{2K_{J}}$ Equivalent pressure acting on the perforated TS: using $\frac{k_{w}}{2K_{J}} = \frac{1}{\pi a_{o}^{2}} \frac{1}{K_{s,t}} \frac{1-J}{J} \text{ and } K_{s,t} = \frac{k_{s}}{N_{t}k_{t}}$ $P_{J}(P_{s}) = -k_{w} \Delta_{J}(P_{s}) = -\frac{1}{\pi a_{o}^{2}} \frac{1-J}{JK_{s,t}} \frac{\pi \left(R_{J}^{2} - R_{s}^{2}\right)}{2} P_{s} = \left[-\frac{1-J}{2JK_{s,t}} \frac{\left(D_{J}^{2} - D_{s}^{2}\right)}{D_{o}^{2}} P_{s}\right]$

(d) Effect of v_t due to pressures P_s and P_t acting on the tubes (Figure 67)



Figure 67 — Effect of v_t Due to Pressures P_s and P_t

$$\Delta L_{T}(v_{t}) = \frac{\pi}{2 k_{t}} \Big[P_{s} d_{t}^{2} - P_{t} (d_{t} - 2 t_{t})^{2} \Big] v_{t}$$
Using $k_{w} = \frac{N_{t} \cdot k_{t}}{\pi a_{0}^{2}}$:
$$P_{T}(v_{t}) = k_{w} \Delta L_{T}(v_{t}) = \frac{\pi}{2k_{t}} \frac{N_{t}k_{t}}{\pi a_{0}^{2}} \Big[P_{s} d_{t}^{2} - P_{t} (d_{t} - 2t_{t})^{2} \Big] v_{t} = \frac{2N_{t}}{4a_{0}^{2}} \Big[P_{s} d_{t}^{2} - P_{t} (d_{t} - 2t_{t})^{2} \Big] v_{t}$$

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$$P_{T}(v_{t}) = 2 \left[P_{s} (1 - x_{s}) - P_{t} (1 - x_{t}) \right] v_{t}$$

(e) Effect of v_s due to pressure P_s acting on the shell:

In the same way as for the tubes: $\Delta L_s(v_s) = -\frac{\pi}{2k_s} \left[P_s D_s^2 \right] v_s$

Using
$$k_{\rm w} = \frac{N_{\rm t} \cdot k_{\rm t}}{\pi a_o^2}$$
 and $K_{s,t} = \frac{k_s}{N_t k_t}$:

$$P_{s}(\upsilon_{s}) = -k_{w}\Delta L_{s}(\upsilon_{s}) = \frac{\pi}{2k_{s}} \frac{N_{t}k_{t}}{\pi a_{o}^{2}} \left[P_{s}D_{s}^{2} \right] v_{s} = \left[\frac{2}{K_{s,t}} \frac{D_{s}^{2}}{D_{o}^{2}} P_{s}v_{s} \right]$$

(f) Pressure Pt acting on the Channel Head (Figure 68)



Figure 68 — Pressure Pt Acting on the Channel Head

Load acting at periphery of TS: $F_C(P_t) = \pi a_c^2 P_t$

Part of
$$F_C(P_t)$$
 supported by the tubes: $F_C(P_t) \frac{N_t K_t}{K_s^*} = F_C(P_t) \frac{N_t K_t}{JK_s} = \frac{F_C(P_t)}{JK_{s,t}}$

Equivalent pressure acting on the perforated TS:

$$P_{C}(P_{t}) = -\frac{F_{C}(P_{t})}{\pi a_{o}^{2} J K_{s,t}} = -\frac{a_{c}^{2}}{a_{o}^{2} J K_{s,t}} P_{t} = -\frac{\rho_{c}^{2}}{J K_{s,t}} P_{t}$$

(g) Effect of γ due to differential thermal expansion between tubes and shell $\gamma = \left[\alpha_{t,m}(T_{t,m} - T_a) - \alpha_{s,m}(T_{s,m} - T_a) \right] L$

Equivalent pressure acting on the perforated TS: $P_{\gamma} = k_w \gamma = \left| \frac{N_t k_t}{\pi a_o^2} \gamma \right|$

(*h*) The total pressure acting on the TS is written:

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$$P_{T} = P_{TS}(P_{s}, P_{t}) + P_{RIM}(P_{s}, P_{t}) + P_{J}(P_{s}) + P(\upsilon_{t}) + P(\upsilon_{s}) + P_{C}(P_{t})$$

$$P_{T} = x_{s}P_{s} - x_{t}P_{t} + -\frac{\rho_{s}^{2} - 1}{JK_{s,t}}P_{s} + \frac{\rho_{c}^{2} - 1}{JK_{s,t}}P_{t} - \frac{1 - J}{2JK_{s,t}}\frac{(D_{J}^{2} - D_{s}^{2})}{D_{o}^{2}}P_{s} + 2(1 - x_{s})\nu_{t}P_{s} - 2(1 - x_{t})\nu_{t}P_{t} + \frac{2}{K_{s,t}}\frac{D_{s}^{2}}{D_{o}^{2}}\nu_{s}P_{s}$$

$$-\frac{\rho_{c}^{2}}{JK_{s,t}}P_{t} + \frac{N_{t}k_{t}}{\pi a_{o}^{2}}\gamma$$
Pr can be written:
$$P_{T} = P_{s} - P_{t} + P_{\gamma}$$
with:
$$P_{s} = P_{s} \left[x_{s} + 2(1 - x_{s})\nu_{t} + \frac{2}{K_{s,t}}\left(\frac{D_{s}}{D_{o}}\right)^{2}\nu_{s} - \frac{\rho_{s}^{2} - 1}{JK_{s,t}} - \frac{1 - J}{2JK_{s,t}}\frac{D_{J}^{2} - D_{s}^{2}}{D_{o}^{2}}\right]$$

 $\left| P_{t} = P_{t} \left[x_{t} + 2\left(1 - x_{t}\right)v_{t} + \frac{1}{J K_{s,t}} \right] \right| \qquad \left| P_{\gamma} = \left[\frac{2v_{t} K_{t}}{\pi a_{o}^{2}} \right] \gamma \right|$ Formulas P's, P't, P_γ match the formulas obtained in VII.2 above. Formulas P_w and P_{rim} cannot be calculated directly as they involve the edge moments at the TS-shell-channel connection.

ANNEX N — FORMULAS TO BE USED WHEN PE=0

The equivalent pressure Pe may be equal to 0 in the following loading cases:

- loading case 1 if $P_t=0$
- loading case 2 if $P_s=0$
- loading case 3 if $P_s=P_t$
- loading case 4 if $P_s=P_t$

When $P_e = 0$, $V_a = 0$ and $Q_3 = Q_1 + \frac{Q_2}{a_o V_a}$ becomes infinity. General formulas depending on V_a and

 M_a , given in VIII, must be used with $V_a = 0$ to determine the stresses. These formulas can be written in the general symbolic form: $F(x) = K [M_a A(x) + (a_0 V_a) B(x)]$

with
$$M_a = (a_o V_a) Q_1 + Q_2 = Q_2$$
 Thus $M_a = Q_2$ and $F(x) = K Q_2 A(x)$

Accordingly, the equations giving q(x), w(x), $\theta(x)$, $\sigma(x)$, $\tau(x)$, $\sigma_t(x)$ write as follows.

1 Net Effective Pressure: q(x)

$$q(x) = \frac{X_a^4}{a_o^2} Q_2 Z_w(x)$$

Maximum of q(x) is obtained for $Z_w(x)$ maximum which is located either inside the TS (x<X_a) or at TS periphery (x=X_a).

2 Axial Displacement: w(x)

$$w(x) = \frac{Q}{k_{w}} - \frac{1}{k_{w}} \frac{X_{a}^{4}}{a_{o}^{2}} Q_{2} Z_{w}(x)$$

Maximum of w(x) is obtained for $Z_w(x)$ maximum which is located either inside the TS (x<X_a) or at TS periphery (x=X_a).

3 Rotation: $\theta(x)$

$$\theta(x) = \frac{a_o}{D^*} Q_2 Z_m(x)$$

Maximum of θ (x) is obtained for $Z_m(x)$ maximum which is located either inside the TS (x<X_a) or at TS periphery (x=X_a).

4 Bending Stress: $\sigma(x)$

$$M_r(x) = Q_2 Q_m(x)$$
$$\sigma_r(x) = \frac{6 Q_2}{\mu^* h^2} Q_m(x)$$

Maximum of $\sigma(x)$ is obtained for $Q_m(x)$. A parametric study performed on $X_a = 1, 2, 3..., 20$ shows that the maximum is always located at TS periphery (x=X_a) and is equal to 1 (See Annex G):

MAX
$$[Q_m(x)] = Q_m(X_a) = 1$$
 Thus $\sigma = \frac{6Q_2}{\mu^* h^2}$

5 Shear Stress: $\tau(x)$

$$Q(x) = \frac{Q_2}{a_o} Q_\alpha(x)$$
$$\overline{\tau(x)} = \frac{Q_2}{\mu a_o h} Q_\alpha(x)$$

Maximum of $\tau(x)$ is obtained for $Q_{\alpha}(x)$ maximum which is located either inside the TS (x<X_a) or at TS periphery (x=X_a).

6 Axial Stress in Tubes: $\sigma_t(x)$

$$\sigma_{t}(x) = \frac{1}{x_{t} - x_{s}} \left[\Delta p^{*} - q(x) \right] \qquad q(x) = \frac{X_{a}^{4}}{a_{o}^{2}} Q_{2} Z_{w}(x)$$
$$\sigma_{t}(x) = \frac{1}{x_{t} - x_{s}} \left[(x_{s} P_{s} - x_{t} P_{t}) - \frac{X_{a}^{4}}{a_{o}^{2}} Q_{2} Z_{w}(x) \right]$$

Maximum and minimum values of $\sigma_t(x)$ are obtained for $Z_w(x)$ maximum or minimum which are located either inside the TS (x<X_a) or at TS periphery (x=X_a).



ANNEX O — TABULAR AND GRAPHICAL REPRESENTATION OF COEFFICIENT $F_t(\textbf{x})$

Annex O provides for 1≤X_a≤20 and -0.8≤Q₃≤+0.8:

- values and graphs of $F_t(x)$ for $0 \le x \le X_a$
- values and graphs of the minimum and maximum of $F_t(x)$: $F_{t,min}$ and $F_{t,max}$
- locations of the minimum and maximum of $F_t(x)$: x_{min} and x_{max}

$F_t(x)$ for $X_A=1,5,7,10,20$

						F _t (x) for	· X _a = 1,	5,7,10),20 a	and C	Q ₃ = -0.8	, 0.0 ,	+0.8			(v*=0	.4)		
	Xa	= 1			Xa	= 5			Xa	= 7			Xa	= 10			Xa	= 20	
x	Q ₃ = -0.8	Q ₃ = 0	Q ₃ = +0.8	x	Q ₃ = -0.8	Q ₃ = 0	Q ₃ = +0.8	x	Q ₃ = -0.8	$Q_3 = 0$	Q ₃ = +0.8	x	Q ₃ = -0.8	Q ₃ = 0	Q ₃ =+0.8	x	Q ₃ = -0.8	$Q_3 = 0$	Q ₃ = +0.8
2.00E-02	1.038118	0.9676997	0.8972814	1.00E-01	1.032634	-0.6066762	-2.245986	0.14	-0.8756667	-2.81E-02	0.8194348	0.2	-9.87E-02	4.48E-02	1.88E-01	0.4	7.24E-04	4.65E-05	-6.31E-04
4.00E-02	1.038039	0.9677894	0.8975396	2.00E-01	1.046217	-0.6069373	-2.260092	0.28	-0.8633661	-3.16E-02	0.8000731	0.4	-1.09E-01	4.54E-02	2.00E-01	0.8	5.27E-04	5.92E-05	-4.09E-04
0.06	1.037908	0.9679389	0.8979697	0.3	1.06879	-0.6073345	-2.283459	0.42	-0.8426553	-3.75E-02	0.7676076	0.6	-1.26E-01	4.64E-02	2.19E-01	1.2	1.88E-04	7.96E-05	-2.89E-05
0.08	1.037724	0.9681479	0.8985717	0.4	1.100256	-0.607811	-2.315878	0.56	-0.8132234	-4.57E-02	0.72175	0.8	-1.50E-01	4.76E-02	2.46E-01	1.6	-3.06E-04	1.06E-04	5.18E-04
0.1	1.037487	0.9684163	0.8993454	0.5	1,140476	-0.608287	-2.35705	0.7	10.20 an	-5.63E-02	0.6621073	1	-1.81E-01	4.90E-02	2.79E-01	2	-9.65E-04	1.37E-04	1.24E-03
0.12	1.037197	0.9687439	0.9002905	0.6	1,189266	-0.6086601	-2,406586	0.84	-0.7263712	-6.91E-02	0.5881936	1.2	-2.17E-01	5.05E-02	3.18E-01	2.4	-1.79E-03	1.68E-04	2.12E-03
0.14	1.036854	0.9691301	0.9014067	0.7	1,246399	-0.6088049	-2,464009	0.98	-0.6677723	-8.42E-02	0.4994436	1.4	-2.60E-01	5.18E-02	3.63E-01	2.8	-2.76E-03	1.93E-04	3.14E-03
0.16	1.036456	0.9695748	0.9026937	0.8	1.311598	-0.6085731	-2.528745	1.12	-0.5981158	-1.01E-01	0.3952311	1.6	-3.06E-01	5.29E-02	4,12E-01	3.2	-3.81E-03	2.04E-04	4.22E-03
0.18	1 036004	0.9700776	0 9041512	0.9	1 384535	-0.6077937	-2 600122	1.26	-0.5165977	-1 21E-01	0 2748903	1.8	-3.57E-01	5.35E-02	4 64E-01	3.6	-4 85E-03	1.91E-04	5 23E-03
0.2	1 035497	0.9706377	0.9057785	1	1 464823	-0.6062728	-2 677369	1.4	-0.4223562	-1 42E-01	0 1377414	2	-4 11E-01	5 33E-02	5 18E-01	4	-5.69E-03	1.40E-04	5.97E-03
0.22	1 034934	0.9712548	0.9075752	11	1.552018	-0.6037938	-2 759606	1.54	-0 3144919	-1.66E-01	-0.01688031	22	-4.66E-01	5 20E-02	5.70E-01	44	-6 10E-03	3 71E-05	6 17E-03
0.24	1.004004	0.0710292	0.0005409	1.1	1.645607	0.6001172	2.705000	1.69	0.1020007	1.00E 01	0.190501	2.2	5 21E 01	4.04E.02	6 10E 01	4.4	5.71E.02	1.24E.04	5.44E 02
0.24	1.034310	0.0726572	0.0116746	1.2	1.745007	0.5040917	2.040041	1.00	0.05425141	2.19E.01	0.2900201	2.9	5.21E-01	4.540-02	6 62E 01		4.12E.02	2 99E 04	2.24E.02
0.20	1.00004	0.9720373	0.9110740	1.3	1.745007	-0.5949817	-2.33437	1.02	0.0008848	-2.10E-01	-0.3809201	2.0	-5.72E-01	4.00E-02	6.02E-01	5.2	-4.12E-03	7.000-04	5.54E=03
0.26	1.032907	0.9734414	0.913976	1.4	1.649559	-0.5661031	-3.025765	1.90	0.0996646	-2.40E-01	-0.5912001	2.0	-0.10E-01	3.00E-02	0.95E-01	5.6	-7.99E-04	-7.33E-04	-0.00E-04
0.3	1.032115	0.9742795	0.9164441	1.5	1.95852	-0.5791762	-3.1168/2	2.1	0.2710992	-0.2748804	-8.21E-01	3	-6.56E-01	2.94E-02	7.15E-01	6	4.77E-03	-1.17E-03	-7.10E-03
0.32	1.031264	0.9751709	9.19E-01	1.6	2.071058	-0.5678742	-3.21E+00	2.24	0.4600573	-0.3048178	-1.069693	3.2	-6.80E-01	1.75E-02	7.15E-01	6.4	1.31E-02	-1.67E-03	-1.64E-02
0.34	1.030352	0.9761147	9.22E-01	1.7	2.186247	-0.5538498	-3.29E+00	2.38	6.67E-01	-0.3351072	-1.33748	3.4	-0.6879466	2.12E-03	0.6921918	6.8	2.44E-02	-2.20E-03	-2.88E-02
0.36	1.029379	0.97711	9.25E-01	1.8	2.303055	-0.5367362	-3.38E+00	2.52	0.8930232	-0.3652826	-1.623588	3.6	-0.6736465	-1.70E-02	0.6396902	7.2	3.86E-02	-2.68E-03	-4.40E-02
0.38	1.028343	0.9781556	9.28E-01	1.9	2.420344	-0.5161481	-3.45E+00	2.66	1.137372	-0.3948018	-1.926976	3.8	-0.631919	-4.02E-02	0.5515026	7.6	5.52E-02	-2.97E-03	-6.12E-02
0.4	1.027244	0.9792506	9.31E-01	2	2.536859	-0.4916826	-3.52E+00	2.8	1.400039	-0.42304	-2.246119	4	-0.5567574	-6.79E-02	0.4210006	8	7.26E-02	-2.91E-03	-7.84E-02
0.42	1.02608	0.9803938	0.9347077	2.1	2.651222	-0.4629212	-3.577064	2.94	1.680372	-0.4492831	-2.578938	4.2	-0.4417243	-1.00E-01	0.2412963	8.4	8.81E-02	-2.26E-03	-9.26E-02
0.44	1.02485	0.981584	0.9383183	2.2	2.761925	-0.4294315	-3.620788	3.08	1.977278	-0.4727224	-2.922723	4.4	-0.2800746	-1.37E-01	0.005432152	8.8	9.73E-02	-7.37E-04	-9.88E-02
0.46	1.023552	0.9828199	0.9420877	2.3	2.867325	-0.3907692	-3.648863	3.22	2.28915	-0.4924494	-3.274049	4.6	-0.06492908	-1.79E-01	-0.2933681	9.2	9.44E-02	1.95E-03	-9.05E-02
0.48	1.022186	0.9841002	0.9460148	2.4	2.965638	-0.3464808	-3.658599	3.36	2.613796	-0.5074518	-3.6287	4.8	0.2104946	-2.25E-01	-0.6613804	9.6	7.15E-02	6.11E-03	-5.93E-02
0.5	1.020749	0.9854234	0.9500982	2.5	3.054932	-0.2961063	-3.647145	3.5	2.948359	-0.5166103	-3.981579	5	0.5525845	-2.76E-01	-1.103985	10	1.92E-02	1.20E-02	4.76E-03
0.52	1.01924	0.9867884	0.9543367	2.6	3.133124	-0.2391825	-3.611489	3.64	3.289237	-0.5186982	-4.326633	5.2	0.966963	-3.29E-01	-1.625187	10.4	-7.31E-02	1.95E-02	0.1121317
0.54	1.017658	0.9881935	0.9587288	2.7	3.197976	-0.1752467	-3.54847	3.78	3.632005	-0.5123812	-4.656767	5.4	1.458035	-3.85E-01	-2.227052	10.8	-0.2150519	2.86E-02	0.2722115
0.56	1.016001	0.989637	0.963273	2.8	3.247089	-0.103841	-3.454771	3.92	3.971332	-0.4962205	-4.963773	5.6	2.028447	-4.40E-01	-2.909042	11.2	-0.4140377	3.84E-02	0.4907806
0.58	1.014267	0.9911175	0.9679677	2.9	3.277906	-0.02451688	-3.32694	4.06	4.300899	-0.4686789	-5.238256	5.8	2.678461	-0.4944041	-3.667269	11.6	-0.6712334	4.75E-02	0.7662781
0.6	1.012455	0.9926332	0.9728113	3	3.287708	0.06315959	-3.161389	4.2	4.613324	-0.4281282	-5.46958	6	3.40523	-0.5442125	-4.49E+00	12	-0.9773141	5.38E-02	1.084902
0.62	1.010563	0.9941825	0.9778024	3.1	3.273615	0.1596018	-2.954412	4.34	4.900085	-0.3728624	-5.64581	6.2	4.201988	-0.5865092	-5.375007	12.4	-1.306761	5.39E-02	1.414617
0.64	1.008588	0.9957635	0.982939	3.2	3.23259	0.2651963	-2.702198	4.48	5.151461	-0.3011132	-5.753687	6.4	5.057155	-0.6174373	-6.292029	12.8	-1.611355	4.36E-02	1.698476
0.66	1.006529	0.9973744	0.9882194	3.3	3.161443	0.3802958	-2.400851	4.62	5.356469	-0.2110706	-5.77861	6.6	5.953367	-0.6324616	-7.21829	13.2	-1.813493	1.73E-02	1.848088
0.68	1.004385	0.9990131	0.9936417	3.4	3.056837	0.5052112	-2.046414	4.76	5.502823	-0.1009085	-5.70464	6.8	6.866462	-0.6263518	-8.119165	13.6	-1.80055	-3.10E-02	1.738637
0.7	1.002152	1.000678	0.9992042	3.5	2.915298	0.6402039	-1.63489	4.9	5.576911	0.03118399	-5.514544	7	7.764431	-0.5931894	-8.950809	14	-1.422138	-0.107292	1.207554
0.72	0.9998285	1.002367	1,004905	3.6	2,733226	0.7854759	-1,162274	5.04	5,563787	1.87E-01	-5,189855	7.2	8,606379	-0.5264054	-9.659189	14.4	-0.4929101	-0.2165779	5.98E-02
0.74	0.9974129	1.004077	1.010741	3.7	2,50691	0.9411606	-0.6245889	5.18	5,447183	3.68E-01	-4,71098	7.4	9.341547	-0.4188564	-10,17926	14.8	1,195645	-0.360618	-1.916881
0.76	0.9949027	1 005807	1 016712	3.8	2 232545	1 107312	-0.01792095	5.32	5 209565	0.5761069	-4 057351	7.6	9 908447	-0 262948	-10 43434	15.2	3 850281	-0.5355867	-4 921454
0.78	0.9922957	1 007555	1 022814	3.9	1 906255	1 283895	0.6615336	5.46	4 832207	0.8122958	-3 207616	7.8	10 23417	-0.05081466	-10 3358	15.6	7 63344	-0 7287403	-9 09092
0.18	0.0022007	1 009317	1 029046	4	1.524115	1.200000	1 417423	5.40	4 295315	1 077711	-2 139894	9.0	10 23401	0.2254344	-9 783137	10.0	12 59882	-0.9145391	-14 4279
0.0	0.9867812	1 011003	1.025/05	4 1	1.024113	1.667682	2 253182	5.74	3 57810	1 373051	-0.8320874	20	9,811350	0.5733036	-8 664572	16.4	18 60214	-1.050520	-20 7032
0.84	0.0000012	1.011093	1.030400	4.1	0.5765282	1.007082	2.200100	5.74	3.57819	1.075001	-0.8320874	0.2	9.811339	0.0000507	6 959227	10.4	25 10700	1.030328	-20.7032
0.84	0.9030091	1.012079	1.041088	4.2	0.0700283	2.090057	4 176607	5.88	2.009440	2.054070	2 500994	6.4	7.2562242	1 510705	-0.000337	10.8	23.10708	-1.073037	-21.33435
0.00	0.9000903	1.014073	1.040495	4.3	0.00327703	2.009957	4.170037	0.02	0.1209050	2.054079	4.747540	0.6	1.200321	2.400700	0.6501700	47.0	31.45123	-0.0979306	-33.24709
0.88	0.9717222	1.0104/2	1.055221	4.4	-0.0413519	2.314114	5.209579	0.16	0.1298059	2.438061	4.747516	8.8	4.87876	2.109792	4.002000	17.6	35.90466	-0.4153511	-30.73536
0.9	0.9744825	1.018274	1.062065	4.5	-1.360997	2.0408/1	0.452/4	6.3	-1.524521	2.850749	7.226019	9	0.705551	2.798285	4.003605	18	30.34414	0.4985645	-35.34701
0.92	0.971128	1.020076	1.069023	4.6	-2.159101	2.78419	7.727481	6.44	-3.466443	3.28792	10.04228	9.2	-2.735554	3.574282	9.884117	18.4	29.77993	1.9/148/	-25.83695
0.94	0.9676566	1.021875	1.076094	4.7	-3.03885	3.027827	9.094502	6.58	-5.715328	3.74679	13.20891	9.4	-8.238123	4.431479	17.10108	18.8	12.46808	4.110861	-4.246357
0.96	0.9640648	1.023669	1.083273	4.8	-4.003114	3.275318	10.55375	6.72	-8.288511	4.222885	16.73428	9.6	-15.03634	5.358265	25.75287	19.2	-19.88116	6.96749	33.81614
0.98	0.9603503	1.025455	1.090559	4.9	-5.054369	3.524966	12.1043	6.86	-11.20056	4.710511	20.62158	9.8	-23.23396	6.336559	35.90708	19.6	-71.63274	10.48407	92.60088

						-	$\mathbf{F}_{\mathbf{t}}(\mathbf{X}_{\mathbf{a}})$		$(\mathbf{v}^* = 0.$	4)							
			Fq=Ft(m	iin)		Ft(min)	<fq<ft(max)< th=""><th></th><th></th><th></th><th></th><th>Fq</th><th>=Ft(max)</th><th></th><th></th><th></th><th></th></fq<ft(max)<>					Fq	=Ft(max)				
Xa\Q3	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
1	0.9565	0.9653	0.9742	0.9830	0.9919	1.0007	1.0095	1.0184	1.0272	1.0361	1.0449	1.0537	1.0626	1.0714	1.0803	1.0891	1.097
2	0.3868	0.5103	0.6339	0.7574	0.8809	1.0044	1.1279	1.2514	1.3749	1.4985	1.6220	1.7455	1.8690	1.9925	2.1160	2.2395	2.363
3	-1.1560	-0.7358	-0.3155	0.1048	0.5250	0.9453	1.3655	1.7858	2.2061	2.6263	3.0466	3.4669	3.8871	4.3074	4.7276	5.1479	5.568
4	-3.3304	-2.5335	-1.7365	-0.9395	-0.1426	0.6544	1.4514	2.2483	3.0453	3.8423	4.6392	5.4362	6.2332	7.0301	7.8271	8.6241	9.421
5	-6.1946	-4.9484	-3.7023	-2.4561	-1.2099	0.0363	1.2825	2.5286	3.7748	5.0210	6.2672	7.5134	8.7595	10.0057	11.2519	12.4981	13.744
6	-9.9120	-8.1122	-6.3123	-4.5124	-2.7126	-0.9127	0.8871	2.6870	4.4868	6.2867	8.0866	9.8864	11.6863	13.4861	15.2860	17.0859	18.885
7	-14.4624	-12.0043	-9.5461	-7.0880	-4.6299	-2.1718	0.2864	2.7445	5.2026	7.6607	10.1189	12.5770	15.0351	17.4933	19.9514	22.4095	24.867
8	-19.8122	-16.5960	-13.3799	-10.1637	-6.9475	-3.7313	-0.5151	2.7010	5.9172	9.1334	12.3496	15.5658	18.7820	21.9981	25.2143	28.4305	31.646
9	-25.9594	-21.8858	-17.8122	-13.7385	-9.6649	-5.5913	-1.5177	2.5560	6.6296	10.7032	14.7768	18.8504	22.9241	26.9977	31.0713	35.1449	39.218
10	-32.9071	-27.8761	-22.8452	-17.8142	-12.7832	-7.7523	-2.7213	2.3097	7.3406	12.3716	17.4025	22.4335	27.4645	32.4954	37.5264	42.5574	47.588
11	-40.6555	-34.5672	-28.4789	-22.3906	-16.3023	-10.2140	-4.1257	1.9626	8.0509	14.1392	20.2275	26.3158	32.4041	38.4924	44.5807	50.6690	56.757
12	-49.2043	-41.9587	-34.7131	-27.4675	-20.2219	-12.9763	-5.7307	1.5149	8.7606	16.0062	23.2518	30.4974	37.7430	44.9886	52.2342	59.4798	66.725
13	-58.5532	-50.0504	-41.5475	-33.0446	-24.5417	-16.0388	-7.5360	0.9669	9.4698	17.9727	26.4756	34.9784	43.4813	51.9842	60.4871	68.9899	77.492
14	-68.7023	-58.8422	-48.9820	-39.1219	-29.2618	-19.4017	-9.5415	0.3186	10.1787	20.0388	29.8989	39.7591	49.6192	59.4793	69.3394	79.1996	89.059
15	-79.6514	-68.3341	-57.0167	-45.6994	-34.3820	-23.0647	-11.7473	-0.4300	10.8874	22.2047	33.5220	44.8394	56.1567	67.4741	78.7914	90.1088	101.426
16	-91.4006	-78.5261	-65.6515	-52.7770	-39.9024	-27.0279	-14.1533	-1.2788	11.5958	24.4703	37.3449	50.2194	63.0940	75.9685	88.8431	101.7176	114.592
17	-103.9499	-89.4181	-74.8864	-60.3546	-45.8229	-31.2912	-16.7594	-2.2277	12.3040	26.8358	41.3675	55.8993	70.4310	84.9627	99.4945	114.0262	128.557
18	-117.2992	-101.0102	-84.7213	-68.4324	-52.1435	-35.8546	-19.5657	-3.2767	13.0122	29.3011	45.5900	61.8789	78.1678	94.4567	110.7457	127.0346	143.323
19	-131.4485	-113.3024	-95.1563	-77.0102	-58.8642	-40.7181	-22.5720	-4.4259	13.7202	31.8663	50.0123	68.1584	86.3045	104.4506	122.5967	140.7428	158.888
20	-146.3978	-126.2946	-106.1914	-86.0881	-65.9849	-45.8816	-25.7784	-5.6752	14.4281	34.5313	54.6346	74.7378	94.8410	114.9443	135.0475	155.1508	175.254

$\mathbf{F}(\mathbf{V})$ (--* 0 1)

$F_q = (Z_d + Q_3 Z_w) X_a^4 / 2$

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							Z(CODAP)>0										
Xa\Q3	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
1	0.9565	0.9653	0.9742	0.9830	0.9919	1.0007	1.0095	1.0184	1.0272	1.0361	1.0449	1.0537	1.0626	1.0714	1.0803	1.0891	1.0979
2	0.3868	0.5103	0.6339	0.7574	0.8809	1.0044	1.1279	1.2514	1.3749	1.4985	1.6220	1.7455	1.8690	1.9925	2.1160	2.2395	2.3631
3	-1.1560	-0.7358	-0.3155	0.1048	0.5250	0.9453	1.3655	1.7858	2.2061	2.6263	3.0466	3.4669	3.8871	4.3074	4.7276	5.1479	5.5682
4	-3.3304	-2.5335	-1.7365	-0.9395	-0.1426	0.6544	1.4514	2.2483	3.0453	3.8423	4.6392	5.4362	6.2332	7.0301	7.8271	8.6241	9.4210
5	-6.1946	-4.9484	-3.7023	-2.4561	-1.2099	0.0363	1.2825	2.5286	3.7748	5.0210	6.2672	7.5134	8.7595	10.0057	11.2519	12.4981	13.7443
6	-9.9120	-8.1122	-6.3123	-4.5124	-2.7126	-0.9127	0.8871	2.6870	4.4868	6.2867	8.0866	9.8864	11.6863	13.4861	15.2860	17.0859	18.8857
7	-14.4624	-12.0043	-9.5461	-7.0880	-4.6299	-2.1718	0.2864	2.7445	5.2026	7.6607	10.1189	12.5770	15.0351	17.4933	19.9514	22.4095	24.8676
8	-19.8122	-16.5960	-13.3799	-10.1637	-6.9475	-3.7313	-0.5151	2.7010	5.9172	9.1334	12.3496	15.5658	18.7820	21.9981	25.2143	28.4305	31.6467
9	-25.9594	-21.8858	-17.8122	-13.7385	-9.6649	-5.5913	-1.5177	2.5560	6.6296	10.7032	14.7768	18.8504	22.9241	26.9977	31.0713	35.1449	39.2186
10	-32.9071	-27.8761	-22.8452	-17.8142	-12.7832	-7.7523	-2.7213	2.3097	7.3406	12.3716	17.4025	22.4335	27.4645	32.4954	37.5264	42.5574	47.5883
11	-40.6555	-34.5672	-28.4789	-22.3906	-16.3023	-10.2140	-4.1257	1.9626	8.0509	14.1392	20.2275	26.3158	32.4041	38.4924	44.5807	50.6690	56.7573
12	-49.2043	-41.9587	-34.7131	-27.4675	-20.2219	-12.9763	-5.7307	1.5149	8.7606	16.0062	23.2518	30.4974	37.7430	44.9886	52.2342	59.4798	66.7254
13	-58.5532	-50.0504	-41.5475	-33.0446	-24.5417	-16.0388	-7.5360	0.9669	9.4698	17.9727	26.4756	34.9784	43.4813	51.9842	60.4871	68.9899	77.4928
14	-68.7023	-58.8421	-48.9820	-39.1219	-29.2618	-19.4017	-9.5415	0.3186	10.1787	20.0388	29.8989	39.7591	49.6192	59.4793	69.3394	79.1996	89.0597
15	-79.6514	-68.3341	-57.0167	-45.6994	-34.3820	-23.0647	-11.7473	-0.4300	10.8874	22.2047	33.5220	44.8394	56.1567	67.4741	78.7914	90.1088	101.4261
16	-91.4006	-78.5261	-65.6515	-52.7770	-39.9024	-27.0279	-14.1533	-1.2788	11.5958	24.4703	37.3449	50.2194	63.0940	75.9685	88.8431	101.7176	114.5922
17	-103.9499	-89.4181	-74.8864	-60.3546	-45.8229	-31.2912	-16.7594	-2.2277	12.3040	26.8358	41.3675	55.8993	70.4310	84.9627	99.4945	114.0262	128.5579
18	-117.2992	-101.0102	-84.7213	-68.4324	-52.1435	-35.8546	-19.5657	-3.2768	13.0122	29.3011	45.5900	61.8789	78.1678	94.4568	110.7457	127.0346	143.3235
19	-131.4485	-113.3024	-95.1563	-77.0102	-58.8642	-40.7181	-22.5720	-4.4259	13.7202	31.8663	50.0123	68.1584	86.3045	104.4506	122.5967	140.7428	158.8888
20	-146.3978	-126.2946	-106.1914	-86.0881	-65.9849	-45.8816	-25.7784	-5.6752	14.4281	34.5313	54.6346	74.7378	94.8410	114.9443	135.0475	155.1508	175.2540

 $X_a = 1$



 $0 < x < X_{a}$

 $F_t(x)$

 $X_a = 5$



 $0 < x < X_{a}$

 $F_t(x)$

 $X_a = 7$



 $0 < x < X_{a}$

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 $X_a = 10$





 $0 < x < X_{a}$

 $F_t(x)$



 $0 < x < X_{a}$

F_t,min vs X_a and Q₃

	F _{t,min} =MinF _t (x) (v*= IF _{t,min} I < F _{t.max} IF _{t.min} I > F _{t.max} 2 values for F _{inside} IF _{t.min} KF _{t.max} 0 0.1 0.2 0.3 0.4 0.5 0.6 0.5														(v* = 0.4)		
	IF _{t,min} I< F	t,max	I F _{t,min}	I> F _{t,max}		2	values for I	F _{inside}				IF	_{t,min} I <f<sub>t,max</f<sub>				
X _a \Q ₃	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
1	0.9565	0.9653	0.9742	0.9830	0.9919	0.9941	0.9853	0.9765	0.9677	0.9589	0.9501	0.9413	0.9325	0.9237	0.9149	0.9061	0.8973
2	0.3868	0.5103	0.6339	0.7574	0.8809	0.9159	0.7991	0.6822	0.5654	0.4486	0.3318	0.2150	0.0982	-0.0187	-0.1355	-0.2523	-0.3691
3	-1.1560	-0.7358	-0.3155	0.1048	0.5250	0.6932	0.3721	0.0510	-0.2702	-0.5913	-0.9124	-1.2335	-1.5546	-1.8757	-2.1968	-2.5179	-2.8390
4	-3.3304	-2.5335	-1.7365	-0.9395	-0.1426	0.3679	0.0062	-0.3555	-0.7171	-1.0788	-1.4404	-1.8021	-2.1637	-2.5254	-2.8871	-3.2487	-3.6104
5	<u>-6.1946</u>	-4.9484	-3.7023	-2.4561	-1.2099	0.0081	-0.1968	-0.4018	-0.6088	-0.8977	-1.2503	-1.6307	-2.0251	-2.4271	-2.8343	-3.2446	-3.6586
6	-9.9120	-8.1122	<u>-6.3123</u>	-4.5124	-2.7126	-0.9127	-0.2703	-0.2810	-0.5053	-0.9172	-1.3958	-1.9026	-2.4221	-2.9513	-3.4844	-4.0175	-4.5529
7	-14.4624	-12.0043	-9.5461	-7.0880	-4.6299	-2.1718	-0.2400	-0.2098	-0.5187	-1.0649	-1.6911	-2.3502	-3.0274	-3.7090	-4.3905	-5.0827	-5.7786
8	<u>-19.8122</u>	-16.5960	-13.3799	-10.1637	-6.9475	-3.7313	-0.5151	-0.1858	-0.5545	-1.2450	-2.0372	-2.8708	-3.7126	-4.5685	-5.4348	-6.3010	-7.1672
9	-25.9594	-21.8858	-17.8122	-13.7385	-9.6649	-5.5913	-1.5177	-0.1765	-0.5918	-1.4437	-2.4218	-3.4507	-4.4871	-5.5235	-6.5880	-7.6531	-8.7183
10	-32.9071	-27.8761	-22.8452	-17.8142	-12.7832	-7.7523	-2.7213	-0.1722	-0.6325	-1.6680	-2.8590	-4.0790	-5.3486	-6.6201	-7.8915	-9.1629	-10.4343
11	-40.6555	-34.5672	-28.4789	-22.3906	-16.3023	-10.2140	-4.1257	-0.1715	-0.6747	-1.9039	-3.3266	-4.7998	-6.3010	-7.8021	-9.3032	-10.8043	-12.3054
12	-49.2043	-41.9587	-34.7131	-27.4675	-20.2219	-12.9763	-5.7307	-0.1744	-0.7187	-2.1695	-3.8366	-5.5727	-7.3330	-9.0934	-10.8538	-12.6141	-14.3745
13	-58.5532	-50.0504	-41.5475	-33.0446	-24.5417	-16.0388	-7.5360	-0.1804	-0.7625	-2.4509	-4.4007	-6.3866	-8.4373	-10.4880	-12.5386	-14.5893	-16.6400
14	-68.7023	-58.8422	-48.9820	-39.1219	-29.2618	-19.4017	-9.5415	-0.1883	-0.8077	-2.7544	-5.0134	-7.2927	-9.5720	-11.9125	-14.2749	-16.6373	-18.9996
15	-79.6514	-68.3341	-57.0167	-45.6994	-34.3820	-23.0647	-11.7473	-0.4300	-0.8527	-3.0967	-5.6341	-8.2827	-10.9313	-13.5798	-16.2284	-18.8770	-21.5255
16	-91.4006	-78.5261	-65.6515	-52.7770	-39.9024	-27.0279	-14.1533	-1.2788	-0.8906	-3.4346	-0.3231	-9.2117	-12.2061	-15.2319	-18.2577	-21.2834	-24.3092
10	-103.9499	-09.4101	-/4.0004	-60.3546	-40.0229	-31.2912	10 5657	-2.2211	-0.9420	-3.0003	-7.0440	-10.3970	-13.7506	-17.1037	-20.4507	-23.0090	-27.1020
10	121 4495	-101.0102	-04.7213	77 0102	-52.1435	-35.0540	-19.5057	-3.2707	-0.9004	-4.1007	-7.0223	12 7017	-15.0942	-10.7071	-22.0020	-20.2975	-30.0027
20	146 2079	126 2046	106 1014	96 0991	65 0940	40.7 181	22.3720	-4.4239	1.0333	4.0054	-0.001	14.0254	19 5754	-21.0440	-25.2104	-29.3000	-33.0090
20	-140.0070	= 120.2040	-100.1014	-00.0001	-00.0040	-40.00101	-23.1104	-0.07.02	-1.07.301	-4.0004	-3.4334		-10.07.04	-20.1104	-21.0004	-32.1334	-30.7334
								x(min)									
			x(min)	=X.				x(min)	x(mir	i)=0		0 <x(mi< th=""><th>n)<x<sub>2</x<sub></th><th></th><th></th><th></th><th></th></x(mi<>	n) <x<sub>2</x<sub>				
X_\Q2	-0.8	-0.7	x(min) -0.6	=X _a -0.5	-0.4	-0.3	-0.2	x(min)	x(mir 0	i)=0 0.1	0.2	0 <x(mi< td=""><td>n)<x<sub>a 0.4</x<sub></td><td>0.5</td><td>0.6</td><td>0.7</td><td>0.8</td></x(mi<>	n) <x<sub>a 0.4</x<sub>	0.5	0.6	0.7	0.8
X _a \Q ₃ 1	-0.8 1 0000	-0.7	x(min) -0.6	=X _a -0.5	- 0.4	-0.3	-0.2	x(min) -0.1	x(mir 0 0.0200	i)=0 0.1	0.2	0 <x(mi 0.3</x(mi 	n) <x<sub>a 0.4</x<sub>	0.5	0.6	0.7	0.8
X _a \Q ₃ 1 2	-0.8 1.0000 2.0000	-0.7 1.0000 2.0000	x(min) -0.6 1.0000 2.0000	=X _a -0.5 1.0000 2.0000	-0.4 1.0000 2.0000	-0.3 0.0200 0.0400	-0.2 0.0200 0.0400	-0.1 0.0200	x(mir 0 0.0200 0.0400	0.1 0.0200 0.0400	0.2 0.0200 0.0400	0 <x(mi 0.3 0.0200 0.0400</x(mi 	n) <x<sub>a 0.4 0.0200 0.0400</x<sub>	0.5 0.0200 0.0400	0.6 0.0200 0.0400	0.7 0.0200 0.0400	0.8 0.0200 0.0400
X _a \Q ₃ 1 2 3	-0.8 1.0000 2.0000 3.0000	-0.7 1.0000 2.0000 3.0000	x(min) -0.6 1.0000 2.0000 3.0000	=X _a -0.5 1.0000 2.0000 3.0000	-0.4 1.0000 2.0000 3.0000	-0.3 0.0200 0.0400 0.0600	-0.2 0.0200 0.0400 0.0600	x(min) -0.1 0.0200 0.0400 0.0600	x(mir 0 0.0200 0.0400 0.0600	0)=0 0.1 0.0200 0.0400 0.0600	0.2 0.0200 0.0400 0.0600	0 <x(mi 0.3 0.0200 0.0400 0.0600</x(mi 	n) <x<sub>a 0.4 0.0200 0.0400 0.0600</x<sub>	0.5 0.0200 0.0400 0.0600	0.6 0.0200 0.0400 0.0600	0.7 0.0200 0.0400 0.0600	0.8 0.0200 0.0400 0.0600
X _a \Q ₃ 1 2 3 4	-0.8 1.0000 2.0000 3.0000 4.0000	-0.7 1.0000 2.0000 3.0000 4.0000	x(min) -0.6 1.0000 2.0000 3.0000 4.0000	=X _a -0.5 1.0000 2.0000 3.0000 4.0000	-0.4 1.0000 2.0000 3.0000 4.0000	-0.3 0.0200 0.0400 0.0600 0.0800	-0.2 0.0200 0.0400 0.0600 0.0800	x(min) -0.1 0.0200 0.0400 0.0600 0.0800	x(mir 0 0.0200 0.0400 0.0600 0.0800)=0 0.1 0.0200 0.0400 0.0600 0.0800	0.2 0.0200 0.0400 0.0600 0.0800	0 <x(mi 0.3 0.0200 0.0400 0.0600 0.0800</x(mi 	n) <x<sub>a 0.4 0.0200 0.0400 0.0600 0.0800</x<sub>	0.5 0.0200 0.0400 0.0600 0.0800	0.6 0.0200 0.0400 0.0600 0.0800	0.7 0.0200 0.0400 0.0600 0.0800	0.8 0.0200 0.0400 0.0600 0.0800
X _a \Q ₃ 1 2 3 4 5	-0.8 1.0000 2.0000 3.0000 4.0000 5.0000	-0.7 1.0000 2.0000 3.0000 4.0000 5.0000	x(min) -0.6 1.0000 2.0000 3.0000 4.0000 5.0000	=X _a -0.5 1.0000 2.0000 3.0000 4.0000 5.0000	-0.4 1.0000 2.0000 3.0000 4.0000 5.0000	-0.3 0.0200 0.0400 0.0600 0.0800 0.1000	-0.2 0.0200 0.0400 0.0600 0.0800 0.1000	x(min) -0.1 0.0200 0.0400 0.0600 0.0800 0.1000	x(mir 0 0.0200 0.0400 0.0600 0.0800 0.7000	0)=0 0.1 0.0200 0.0400 0.0600 0.0800 1.6000	0.2 0.0200 0.0400 0.0600 0.0800 1.9000	0 <x(mi) 0.3 0.0200 0.0400 0.0600 0.0800 2.1000</x(mi) 	n) <x<sub>a 0.4 0.0200 0.0400 0.0600 0.0800 2.2000</x<sub>	0.5 0.0200 0.0400 0.0600 0.0800 2.3000	0.6 0.0200 0.0400 0.0600 0.0800 2.3000	0.7 0.0200 0.0400 0.0600 0.0800 2.4000	0.8 0.0200 0.0400 0.0600 0.0800 2.4000
X _a \Q ₃ 1 2 3 4 5 6	-0.8 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000	-0.7 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000	x(min) -0.6 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000	=X _a -0.5 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000	-0.4 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000	-0.3 0.0200 0.0400 0.0600 0.0800 0.1000 6.0000	-0.2 0.0200 0.0400 0.0600 0.0800 0.1000 0.1200	x(min) -0.1 0.0200 0.0400 0.0600 0.0800 0.1000 0.9600	x(mir 0 0.0200 0.0400 0.0600 0.0800 0.7000 2.5200	a)=0 0.1 0.0200 0.0400 0.0600 0.0800 1.6000 3.0000	0.2 0.0200 0.0400 0.0600 0.0800 1.9000 3.2400	0 <x(mi 0.3 0.0200 0.0400 0.0600 0.0800 2.1000 3.3600</x(mi 	n) <x<sub>a 0.4 0.0200 0.0400 0.0600 0.0800 2.2000 3.3600</x<sub>	0.5 0.0200 0.0400 0.0600 0.0800 2.3000 3.4800	0.6 0.0200 0.0400 0.0600 0.0800 2.3000 3.4800	0.7 0.0200 0.0400 0.0600 0.0800 2.4000 3.4800	0.8 0.0200 0.0400 0.0600 0.0800 2.4000 3.6000
X _a \Q ₃ 1 2 3 4 5 6 7	-0.8 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000	-0.7 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000	x(min) -0.6 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000	=X _a -0.5 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000	-0.4 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000	-0.3 0.0200 0.0400 0.0600 0.0800 0.1000 6.0000 7.0000	-0.2 0.0200 0.0400 0.0600 0.0800 0.1000 0.1200 0.1400	X(min) -0.1 0.0200 0.0400 0.0600 0.0800 0.1000 0.9600 2.3800	x(mir 0 0.0200 0.0400 0.0600 0.0800 0.7000 2.5200 3.6400)=0 0.1 0.0200 0.0400 0.0600 0.0800 1.6000 3.0000 4.0600	0.2 0.0200 0.0400 0.0600 0.0800 1.9000 3.2400 4.3400	0 <x(mi 0.3 0.0200 0.0400 0.0600 0.0800 2.1000 3.3600 4.3400</x(mi 	n) <x<sub>a 0.4 0.0200 0.0400 0.0600 0.0800 2.2000 3.3600 4.4800</x<sub>	0.5 0.0200 0.0400 0.0600 0.0800 2.3000 3.4800 4.4800	0.6 0.0200 0.0400 0.0600 0.0800 2.3000 3.4800 4.4800	0.7 0.0200 0.0400 0.0600 0.0800 2.4000 3.4800 4.6200	0.8 0.0200 0.0400 0.0600 0.0800 2.4000 3.6000 4.6200
X _a \Q ₃ 1 2 3 4 5 6 7 8	-0.8 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000	-0.7 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000	x(min) -0.6 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000	=X _a -0.5 1.0000 2.0000 4.0000 5.0000 6.0000 7.0000 8.0000	-0.4 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000	-0.3 0.0200 0.0400 0.0600 0.0800 0.1000 6.0000 7.0000 8.0000	-0.2 0.0200 0.0400 0.0600 0.0800 0.1000 0.1200 0.1400 8.0000	X(MIN) -0.1 0.0200 0.0400 0.0800 0.1000 0.9600 2.3800 3.2000	x(mir 0 0.0200 0.0400 0.0600 0.7000 0.7000 2.5200 3.6400 4.6400)=0 0.1 0.0200 0.0400 0.0600 0.0800 1.6000 3.0000 4.0600 5.1200	0.2 0.0200 0.0400 0.0600 1.9000 3.2400 4.3400 5.2800	0 <x(mi 0.3 0.0200 0.0400 0.0600 0.0800 2.1000 3.3600 4.3400 5.4400</x(mi 	n) <x<sub>a 0.4 0.0200 0.0400 0.0600 0.0800 2.2000 3.3600 4.4800 5.4400</x<sub>	0.5 0.0200 0.0400 0.0600 2.3000 3.4800 4.4800 5.6000	0.6 0.0200 0.0400 0.0600 2.3000 3.4800 4.4800 5.6000	0.7 0.0200 0.0400 0.0600 2.4000 3.4800 4.6200 5.6000	0.8 0.0200 0.0400 0.0600 2.4000 3.6000 4.6200 5.6000
X _a \Q ₃ 1 2 3 4 5 6 7 8 9	-0.8 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000	-0.7 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000	x(min) -0.6 1.0000 2.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000	=X _a -0.5 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000	-0.4 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000	-0.3 0.0200 0.0400 0.0600 0.0800 0.1000 6.0000 7.0000 8.0000 9.0000	-0.2 0.0200 0.0400 0.0600 0.1000 0.1200 0.1400 8.0000 9.0000	X(min) -0.1 0.0200 0.0400 0.0600 0.0800 0.1000 0.9600 2.3800 3.2000 4.1400	x(mir 0 0.0200 0.0400 0.0600 0.0800 0.7000 2.5200 3.6400 4.6400 5.5800)=0 0.1 0.0200 0.0400 0.0600 1.6000 3.0000 4.0600 5.1200 6.1200	0.2 0.0200 0.0400 0.0600 1.9000 3.2400 4.3400 5.2800 6.3000	0 <x(mi 0.3 0.0200 0.0400 0.0600 0.0800 2.1000 2.1000 3.3600 4.3400 5.4400 6.4800</x(mi 	n) <x<sub>a 0.4 0.0200 0.0400 0.0600 0.0800 2.2000 3.3600 4.4800 5.4400 6.4800</x<sub>	0.5 0.0200 0.0400 0.0800 2.3000 3.4800 4.4800 5.6000 6.4800	0.6 0.0200 0.0400 0.0800 2.3000 3.4800 4.4800 5.6000 6.6600	0.7 0.0200 0.0400 0.0600 2.4000 3.4800 4.6200 5.6000 6.6600	0.8 0.0200 0.0400 0.0600 2.4000 3.6000 4.6200 5.6000 6.6600
X _a \Q ₃ 1 2 3 4 5 6 7 8 9 9	-0.8 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000	-0.7 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000	x(min) -0.6 1.0000 2.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000	=X _a -0.5 1.0000 2.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000	-0.4 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000	-0.3 0.0200 0.0400 0.0600 0.1000 6.0000 7.0000 8.0000 9.0000 10.0000	-0.2 0.0200 0.0400 0.0600 0.1000 0.1200 0.1400 8.0000 9.0000 10.0000	X(min) -0.1 0.0200 0.0400 0.0600 0.0800 0.1000 0.9600 2.3800 3.2000 4.1400 5.0000	x(mir 0 0.0200 0.0400 0.0600 0.7000 2.5200 3.6400 4.6400 5.5800 6.6000)=0 0.1 0.0200 0.0400 0.0800 1.6000 3.0000 4.0600 5.1200 6.1200 7.2000	0.2 0.0200 0.0400 0.0600 1.9000 3.2400 4.3400 5.2800 6.3000 7.4000	0 <x(mi 0.3 0.0200 0.0400 0.0600 2.1000 2.1000 3.3600 4.3400 5.4400 6.4800 7.4000</x(mi 	n) <x<sub>a 0.4 0.0200 0.0400 0.0600 2.2000 3.3600 4.4800 5.4400 6.4800 7.6000</x<sub>	0.5 0.0200 0.0400 0.0800 2.3000 3.4800 4.4800 5.6000 6.4800 7.6000	0.6 0.0200 0.0400 0.0800 2.3000 3.4800 4.4800 5.6000 6.6600 7.6000	0.7 0.0200 0.0400 0.0600 2.4000 3.4800 4.6200 5.6000 6.6600 7.6000	0.8 0.0200 0.0400 0.0600 2.4000 3.6000 4.6200 5.6000 6.6600 7.6000
X _a \Q ₃ 1 2 3 4 5 6 7 8 9 10 11	-0.8 1.0000 2.0000 3.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000	-0.7 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000	x(min) -0.6 1.0000 2.0000 4.0000 5.0000 6.0000 7.0000 7.0000 8.0000 9.0000 10.0000 11.0000	=X _a -0.5 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000	-0.4 1.0000 2.0000 3.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000	-0.3 0.0200 0.0400 0.0600 0.0800 0.1000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000	-0.2 0.0200 0.0600 0.0800 0.1000 0.1200 0.1400 8.0000 9.0000 10.0000 11.0000	X(min) -0.1 0.0200 0.0400 0.0600 0.0800 0.1000 0.9600 2.3800 3.2000 4.1400 5.0000 5.7200	x(mir 0 0.0200 0.0600 0.0800 0.7000 2.5200 3.6400 4.6400 5.5800 6.6000 7.7000)=0 0.1 0.0200 0.0600 0.0800 1.6000 3.0000 4.0600 5.1200 5.1200 6.1200 7.2000 8.1400	0.2 0.0200 0.0600 0.0800 1.9000 3.2400 4.3400 5.2800 6.3000 7.4000 8.3600	0 <x(mi 0.3 0.0200 0.0400 0.0600 0.0800 2.1000 3.3600 4.3400 5.4400 6.4800 7.4000 8.5800</x(mi 	n) <x<sub>a 0.4 0.0200 0.0400 0.0600 2.2000 3.3600 4.4800 5.4400 6.4800 7.6000 8.5800</x<sub>	0.5 0.0200 0.0600 0.0800 2.3000 3.4800 4.4800 5.6000 6.4800 7.6000 8.5800	0.6 0.0200 0.0600 2.3000 3.4800 4.4800 5.6000 6.6600 7.6000 8.5800	0.7 0.0200 0.0600 0.0800 2.4000 3.4800 4.6200 5.6000 6.6600 7.6000 8.5800	0.8 0.0200 0.0600 0.0800 2.4000 3.6000 4.6200 5.6000 6.6600 7.6000 8.5800
X _a \Q ₃ 1 2 3 4 5 6 7 8 9 10 11 12	-0.8 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000	-0.7 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000	x(min) -0.6 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000	=X _a -0.5 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000	-0.4 1.0000 2.0000 3.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000	-0.3 0.0200 0.0400 0.0600 0.0800 0.1000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000	-0.2 0.0200 0.0600 0.0800 0.1200 0.1200 0.1400 8.0000 9.0000 10.0000 11.0000 12.0000	X(min) -0.1 0.0200 0.0400 0.0600 0.0800 0.1000 0.9600 2.3800 3.2000 4.1400 5.0000 5.7200 6.7200	x(mir 0 0.0200 0.0600 0.0800 0.7000 2.5200 3.6400 4.6400 5.5800 6.6000 7.7000 8.6400))=0 0.1 0.0200 0.0600 0.0800 1.6000 3.0000 4.0600 5.1200 6.1200 7.2000 8.1400 9.3600	0.2 0.0400 0.0600 0.0800 1.9000 3.2400 4.3400 5.2800 6.3000 7.4000 8.3600 9.3600	0 <x(mi 0.3 0.0200 0.0600 0.0800 2.1000 3.3600 4.3400 5.4400 6.4800 7.4000 8.5800 9.6000</x(mi 	n) <x<sub>a 0.4 0.0200 0.0600 0.0800 2.2000 3.3600 4.4800 5.4400 6.4800 7.6000 8.5800 9.6000</x<sub>	0.5 0.0200 0.0400 0.0600 2.3000 3.4800 4.4800 5.6000 6.4800 7.6000 8.5800 9.6000	0.6 0.0200 0.0600 0.0800 2.3000 3.4800 4.4800 5.6000 6.6600 7.6000 8.5800 9.6000	0.7 0.0200 0.0600 0.0800 2.4000 3.4800 4.6200 5.6000 6.6600 7.6000 8.5800 9.6000	0.8 0.0200 0.0600 0.0800 2.4000 3.6000 4.6200 5.6000 6.6600 7.6000 8.5800 9.6000
X _a \Q ₃ 1 2 3 4 5 6 7 8 9 10 11 12 13	-0.8 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000	-0.7 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000	x(min) -0.6 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000	=X _a -0.5 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000	-0.4 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000	-0.3 0.0200 0.0400 0.0600 0.1000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 13.0000	-0.2 0.0200 0.0600 0.0800 0.1200 0.1400 8.0000 9.0000 10.0000 11.0000 13.0000	X(min) -0.1 0.0200 0.0400 0.0600 0.0800 0.0800 2.3800 3.2000 4.1400 5.0000 5.7200 6.7200 7.5400	x(mir 0 0.0200 0.0400 0.0800 0.7000 2.5200 3.6400 4.6400 5.5800 6.6000 6.6000 7.7000 8.6400 9.6200))=0 0.1 0.0200 0.0400 0.0800 1.6000 3.0000 4.0600 5.1200 6.1200 7.2000 8.1400 9.3600 10.4000	0.2 0.0200 0.0400 0.0800 1.9000 3.2400 4.3400 5.2800 6.3000 7.4000 8.3600 9.3600 10.4000	0 <x(mi 0.3 0.0200 0.0600 0.0800 2.1000 3.3600 4.3400 5.4400 6.4800 7.4000 8.5800 9.6000 10.6600</x(mi 	n) <x<sub>a 0.4 0.0200 0.0600 0.0800 2.2000 3.3600 4.4800 5.4400 6.4800 7.6000 7.6000 8.5800 9.6000 10.6600</x<sub>	0.5 0.0200 0.0400 0.0800 2.3000 3.4800 4.4800 5.6000 6.4800 7.6000 8.5800 9.6000 10.6600	0.6 0.0200 0.0600 0.0800 2.3000 3.4800 4.4800 5.6000 6.6600 7.6000 8.5800 9.6000 10.6600	0.7 0.0200 0.0400 0.0800 2.4000 3.4800 4.6200 5.6000 6.6600 7.6000 8.5800 9.6000 10.6600	0.8 0.0200 0.0600 0.0800 2.4000 3.6000 4.6200 5.6000 6.6600 7.6000 8.5800 9.6000 10.6600
X _a \Q ₃ 1 2 3 4 5 6 7 8 9 10 11 12 13 14	-0.8 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000 14.0000	-0.7 1.0000 2.0000 3.0000 4.0000 5.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000 14.0000	x(min) -0.6 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 11.0000 13.0000 14.0000	=X _a -0.5 1.0000 2.0000 3.0000 4.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000 14.0000	-0.4 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000 14.0000	-0.3 0.0200 0.0400 0.0600 0.0800 0.1000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 11.0000 13.0000 14.0000	-0.2 0.0200 0.0600 0.0800 0.1000 0.1200 0.1400 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000	X(min) -0.1 0.0200 0.0400 0.0600 0.0800 0.0800 0.1000 0.9600 2.3800 3.2000 4.1400 5.7200 6.7200 7.5400 8.4000	x(mir 0 0.0200 0.0400 0.0600 0.7000 2.5200 3.6400 4.6400 5.5800 6.6000 7.7000 8.6400 9.6200 10.6400)=0 0.1 0.0200 0.0400 0.0800 1.6000 3.0000 4.0600 5.1200 6.1200 7.2000 8.1400 9.3600 10.4000 11.2000	0.2 0.0200 0.0400 0.0800 1.9000 3.2400 4.3400 5.2800 6.3000 7.4000 8.3600 9.3600 10.4000 11.4800	0 <x(mi 0.3 0.0200 0.0400 0.0600 2.1000 3.3600 4.3400 5.4400 6.4800 7.4000 8.5800 9.6000 10.6600 11.4800</x(mi 	n) <x<sub>a 0.4 0.0200 0.0400 0.0800 2.2000 3.3600 4.4800 5.4400 6.4800 7.6000 8.5800 9.6000 10.6600 11.4800</x<sub>	0.5 0.0200 0.0400 0.0600 2.3000 3.4800 4.4800 5.6000 6.4800 7.6000 8.5800 9.6000 10.6600 11.7600	0.6 0.0200 0.0600 0.0800 2.3000 3.4800 4.4800 5.6000 6.6600 7.6000 8.5800 9.6000 10.6600 11.7600	0.7 0.0200 0.0400 0.0800 2.4000 3.4800 4.6200 5.6000 6.6600 7.6000 8.5800 9.6000 10.6600 11.7600	0.8 0.0200 0.0400 0.0800 2.4000 3.6000 4.6200 5.6000 6.6600 7.6000 8.5800 9.6000 10.6600 11.7600
X _a \Q ₃ 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15	-0.8 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000 14.0000 15.0000	-0.7 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000 14.0000 15.0000	x(min) -0.6 1.0000 2.0000 3.0000 4.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000 14.0000	=X _a -0.5 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 14.0000 15.0000	-0.4 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000 14.0000 15.0000	-0.3 0.0200 0.0400 0.0600 0.0800 0.1000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000 14.0000	-0.2 0.0200 0.0400 0.0800 0.1000 0.1200 0.1400 8.0000 9.0000 11.0000 11.0000 12.0000 13.0000	X(min) -0.1 0.0200 0.0400 0.0600 0.0800 0.1000 0.9600 2.3800 3.2000 4.1400 5.0000 6.7200 6.7200 8.4000 15.0000	x(mir 0 0.0200 0.0400 0.0600 0.7000 2.5200 3.6400 4.6400 5.5800 6.6000 7.7000 8.6400 9.6200 10.6400 11.7000)=0 0.1 0.0200 0.0400 0.0800 1.6000 3.0000 4.0600 5.1200 6.1200 7.2000 8.1400 9.3600 10.4000 11.2000 12.3000	0.2 0.0200 0.0400 0.0600 1.9000 3.2400 4.3400 5.2800 6.3000 7.4000 8.3600 9.3600 10.4000 11.4800 12.6000	0 <x(mi 0.3 0.0200 0.0400 0.0600 2.1000 3.3600 4.3400 5.4400 6.4800 7.4000 8.5800 9.6000 10.6600 11.4800 12.6000</x(mi 	n) <x<sub>a 0.4 0.0200 0.0400 0.0600 2.2000 3.3600 4.4800 5.4400 6.4800 7.6000 8.5800 9.6000 10.6600 11.4800 12.6000</x<sub>	0.5 0.0200 0.0400 0.0600 2.3000 3.4800 4.4800 5.6000 6.4800 7.6000 8.5800 9.6000 10.6600 11.7600 12.6000	0.6 0.0200 0.0400 0.0800 2.3000 3.4800 4.4800 5.6000 6.6600 7.6000 8.5800 9.6000 10.6600 11.7600	0.7 0.0200 0.0400 0.0800 2.4000 3.4800 4.6200 5.6000 6.6600 7.6000 8.5800 9.6000 10.6600 11.7600 12.6000	0.8 0.0200 0.0400 0.0800 2.4000 3.6000 4.6200 5.6000 6.6600 7.6000 8.5800 9.6000 10.6600 11.7600
X _a \Q ₃ 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16	-0.8 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000 14.0000 15.0000	-0.7 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000 14.0000 15.0000	x(min) -0.6 1.0000 2.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000 14.0000 15.0000	=X _a -0.5 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 14.0000 15.0000 16.0000	-0.4 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000 14.0000 15.0000	-0.3 0.0200 0.0400 0.0600 0.0800 0.1000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000 14.0000 15.0000	-0.2 0.0200 0.0400 0.0600 0.1000 0.1200 0.1400 8.0000 10.0000 11.0000 12.0000 13.0000 14.0000 15.0000	X(min) -0.1 0.0200 0.0400 0.0600 0.0800 0.1000 0.9600 2.3800 3.2000 4.1400 5.0000 6.7200 6.7200 0.7.5400 8.4000 15.0000 16.0000	x(mir 0 0.0200 0.0400 0.0600 0.7000 2.5200 3.6400 4.6400 5.5800 6.6000 7.7000 8.6400 9.6200 10.6400 11.7000)=0 0.1 0.0200 0.0400 0.0600 1.6000 3.0000 4.0600 5.1200 6.1200 7.2000 8.1400 9.3600 10.4000 11.2000 12.3000 13.4400	0.2 0.0200 0.0400 0.0600 1.9000 3.2400 4.3400 5.2800 6.3000 7.4000 8.3600 9.3600 10.4000 11.4800 12.6000 13.4400	0 <x(mi 0.3 0.0200 0.0400 0.0600 2.1000 3.3600 4.3400 5.4400 6.4800 7.4000 8.5800 9.6000 10.6600 11.4800 12.6000 13.4400</x(mi 	n) <x<sub>a 0.4 0.0200 0.0400 0.0600 0.0800 2.2000 3.3600 4.4800 5.4400 6.4800 7.6000 8.5800 9.6000 9.6000 10.6600 11.4800 12.6000 13.7600</x<sub>	0.5 0.0200 0.0400 0.0600 2.3000 3.4800 4.4800 5.6000 6.4800 7.6000 8.5800 9.6000 10.6600 11.7600 12.6000 13.7600	0.6 0.0200 0.0400 0.0800 2.3000 3.4800 4.4800 5.6000 6.6600 7.6000 8.5800 9.6000 10.6600 11.7600 12.6000	0.7 0.0200 0.0400 0.0600 2.4000 3.4800 4.6200 5.6000 6.6600 7.6000 8.5800 9.6000 10.6600 11.7600 12.6000 13.7600	0.8 0.0200 0.0400 0.0600 2.4000 3.6000 4.6200 5.6000 6.6600 7.6000 9.6000 9.6000 10.6600 11.7600 12.6000
X _a \Q ₃ 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17	-0.8 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 11.0000 13.0000 14.0000 15.0000 17.0000	-0.7 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 10.0000 11.0000 11.0000 13.0000 14.0000 15.0000 17.0000	x(min) -0.6 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 11.0000 13.0000 14.0000 15.0000 17.0000	=X _a -0.5 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 13.0000 13.0000 14.0000 15.0000 17.0000	-0.4 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 10.0000 11.0000 11.0000 13.0000 14.0000 15.0000 16.0000 17.0000	-0.3 0.0200 0.0400 0.0800 0.1000 6.0000 7.0000 8.0000 10.0000 11.0000 11.0000 13.0000 14.0000 15.0000 16.0000 17.0000	-0.2 0.0200 0.0600 0.0800 0.1000 0.1200 0.1400 8.0000 10.0000 11.0000 11.0000 12.0000 14.0000 15.0000 16.0000 17.0000	X(min) -0.1 0.0200 0.0400 0.0800 0.1000 0.32000 4.1400 5.0000 5.7200 6.7200 6.7200 0.7.5400 8.4000 15.0000 15.0000 15.0000	x(mir 0 0.0200 0.0400 0.0800 0.7000 2.5200 3.6400 4.6400 5.5800 6.6000 7.7000 8.6400 9.6200 10.6400 11.7000 12.8000)=0 0.1 0.0200 0.0400 0.0800 1.6000 3.0000 4.0600 5.1200 6.1200 7.2000 8.1400 9.3600 10.4000 10.4000 12.3000 13.4400 14.2800	0.2 0.0200 0.0400 0.0600 1.9000 3.2400 4.3400 5.2800 6.3000 7.4000 8.3600 9.3600 10.4000 11.4800 12.6000 13.4400	0 <x(mi 0.3 0.0200 0.0400 0.0600 2.1000 3.3600 4.3400 5.4400 6.4800 7.4000 8.5800 9.6000 10.6600 11.4800 12.6000 13.4400 14.6200</x(mi 	n) <x<sub>a 0.4 0.0200 0.0400 0.0600 2.2000 2.2000 3.3600 4.4800 5.4400 6.4800 7.6000 8.5800 9.6000 10.6600 11.4800 12.6000 13.7600</x<sub>	0.5 0.0200 0.0400 0.0800 2.3000 3.4800 5.6000 6.4800 7.6000 8.5800 9.6000 10.6600 11.7600 12.6000 13.7600	0.6 0.0200 0.0400 0.0800 2.3000 3.4800 4.4800 5.6000 6.6600 7.6000 8.5800 9.6000 10.6600 11.7600 12.6000 13.7600	0.7 0.0200 0.0400 0.0600 2.4000 3.4800 4.6200 5.6000 6.6600 7.6000 8.5800 9.6000 10.6600 11.7600 12.6000 13.7600 14.6200	0.8 0.0200 0.0400 0.0800 2.4000 3.6000 4.6200 5.6000 6.6600 7.6000 8.5800 9.6000 10.6600 11.7600 12.6000 13.7600
X _a \Q ₃ 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18	-0.8 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000 15.0000 16.0000 17.0000 18.0000	-0.7 1.0000 2.0000 3.0000 6.0000 7.0000 8.0000 9.0000 10.0000 12.0000 13.0000 13.0000 15.0000 15.0000 16.0000 17.0000 18.0000	x(min) -0.6 1.0000 2.0000 3.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000 15.0000 15.0000 16.0000 17.0000 18.0000	=X _a -0.5 1.0000 2.0000 3.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000 15.0000 15.0000 16.0000 17.0000 18.0000	-0.4 1.0000 2.0000 3.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000 15.0000 15.0000 16.0000 17.0000 18.0000	-0.3 0.0200 0.0400 0.0600 0.0800 0.1000 6.0000 7.0000 8.0000 10.0000 11.0000 12.0000 13.0000 14.0000 15.0000 16.0000 18.0000 18.0000	-0.2 0.0200 0.0400 0.0600 0.1200 0.1200 0.1400 8.0000 10.0000 11.0000 12.0000 13.0000 14.0000 15.0000 15.0000 17.0000 18.0000	X(min) -0.1 0.0200 0.0400 0.0600 0.0800 0.1000 0.9600 2.3800 3.2000 4.1400 5.7200 6.7200 6.7200 6.7200 15.0000 15.0000 15.0000 16.0000 17.0000 18.0000	x(mir 0 0.0200 0.0600 0.0800 0.7000 2.5200 3.6400 4.6400 5.5800 6.6000 7.7000 8.6400 9.6200 10.6400 11.7000 112.8000 13.6000))=0 0.1 0.0200 0.0600 0.0800 1.6000 3.0000 4.0600 5.1200 6.1200 7.2000 8.1400 9.3600 10.4000 11.2000 11.2000 13.4400 14.2800 15.4800	0.2 0.0200 0.0600 0.0800 1.9000 3.2400 4.3400 5.2800 6.3000 7.4000 8.3600 9.3600 10.4000 11.4800 13.4400 14.6200	0 <x(mi 0.3 0.0200 0.0600 0.0800 2.1000 3.3600 4.3400 6.4800 7.4000 8.5800 9.6000 10.6600 11.4800 13.4400 13.4400 15.4800</x(mi 	n) <x<sub>a 0.4 0.0200 0.0600 0.0800 2.2000 3.3600 4.4800 5.4400 6.4800 9.6000 10.6600 11.4800 12.6000 13.7600 14.6200 15.4800</x<sub>	0.5 0.0200 0.0600 2.3000 3.4800 4.4800 5.6000 6.4800 7.6000 10.6600 10.6600 11.7600 12.6000 13.7600 14.6200	0.6 0.0200 0.0600 2.3000 3.4800 4.4800 5.6000 6.6600 7.6000 8.5800 9.6000 10.6600 11.7600 11.7600 11.7600 11.7600 11.7600	0.7 0.0200 0.0600 0.0800 2.4000 3.4800 4.6200 5.6000 6.6600 7.6000 8.5800 9.6000 10.6600 11.7600 12.6000 13.7600 14.6200	0.8 0.0200 0.0600 0.0800 2.4000 3.6000 4.6200 5.6000 6.6600 7.6000 8.58800 9.6000 10.6600 11.76000 12.6000 13.7600 14.6200
X _a \Q ₃ 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19	-0.8 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000 14.0000 15.0000 16.0000 17.0000 18.0000 19.0000	-0.7 1.0000 2.0000 3.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000 14.0000 15.0000 16.0000 17.0000 18.0000 19.0000	x(min) -0.6 1.0000 2.0000 3.0000 5.0000 6.0000 7.0000 8.0000 10.0000 11.0000 11.0000 12.0000 13.0000 14.0000 15.0000 17.0000 18.0000 19.0000	=X _a -0.5 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 10.0000 11.0000 12.0000 13.0000 14.0000 15.0000 16.0000 17.0000 18.0000	-0.4 1.0000 2.0000 3.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000 14.0000 15.0000 17.0000 18.0000 19.0000	-0.3 0.0200 0.0400 0.0600 0.0800 0.1000 6.0000 7.0000 8.0000 10.0000 11.0000 12.0000 13.0000 14.0000 15.0000 16.0000 18.0000 19.0000	-0.2 0.0200 0.0600 0.0800 0.1200 0.1400 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000 14.0000 15.0000 16.0000 17.0000 18.0000	X(min) -0.1 0.0200 0.0400 0.0600 0.0800 0.0800 2.3800 3.2000 4.1400 5.7200 6.7200 6.7200 6.7200 8.4000 15.0000 17.0000 18.0000 19.0000	x(mir 0 0.0200 0.0400 0.0800 0.7000 2.5200 3.6400 4.6400 5.5800 6.6000 7.7000 8.6400 9.6200 10.6400 11.7000 12.8000 13.6000 14.7600))=0 0.1 0.0200 0.0400 0.0800 1.6000 3.0000 4.0600 5.1200 6.1200 6.1200 0.1200 0.3.0000 1.2400 1.2400 1	0.2 0.0400 0.0600 0.0800 3.2400 4.3400 5.2800 6.3000 7.4000 7.4000 9.3600 10.4000 11.4800 12.6000 13.4400 13.4400 13.4400 13.4400	0 <x(mi 0.3 0.0200 0.0400 0.0800 2.1000 3.3600 4.3400 5.4400 6.4800 7.4000 8.5800 9.6000 10.6600 11.4800 12.6000 13.4400 13.4400 15.4800 16.7200</x(mi 	n) <x<sub>a 0.4 0.0200 0.0600 0.0800 2.2000 3.3600 4.4800 5.4400 6.4800 7.6000 7.6000 7.6000 10.6600 11.4800 12.6000 13.7600 14.6200 15.4800 16.7200</x<sub>	0.5 0.0200 0.0400 0.0800 2.3000 3.4800 4.4800 5.6000 6.4800 7.6000 7.6000 10.6600 11.7600 12.6000 13.7600 13.7600 13.7600 13.7600	0.6 0.0400 0.0600 0.0800 2.3000 3.4800 4.4800 5.6000 6.6600 7.6000 8.5800 9.6000 10.6600 11.7600 12.6000 13.7600 13.7600 13.7600 14.6200	0.7 0.0200 0.0400 0.0800 2.4000 3.4800 4.6200 5.6000 6.6600 7.6000 7.6000 10.6600 11.7600 12.6000 13.7600 13.7600 13.7600 14.6200	0.8 0.0200 0.0600 0.0800 2.4000 3.6000 4.6200 5.6000 6.6600 7.6000 7.6000 9.6000 10.6600 11.7600 12.6000 13.7600 13.7600 13.7600 14.6200

PTB-7-2014: Criteria for Shell-and-Tube Heat Exchangers According to Part UHX of ASME Section VIII Division 1

Ft,max vs Xa and Q3

									F _{t.}	_{max} =Ma	axF _t (x)			(v* =	0.4)		
	Ft may>	Ft min		Frank < Frank		2 v	alues for F.	incido				F	France France				
X_\Q2	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
1	1 0381	1 0293	1 0205	1 0117	1.0032	1.0020	1 0095	1 0184	1 0272	1.0361	1 0449	1.0537	1.0626	1 0714	1 0803	1 0891	1 0979
2	1.5000	1.3831	1.2663	1.1495	1.0438	1.0284	1,1279	1.2514	1.3749	1,4985	1.6220	1,7455	1.8690	1,9925	2,1160	2.2395	2,3631
3	2.2987	1.9776	1.6569	1.3670	1.1574	1.1085	1.3655	1.7858	2.2061	2.6263	3.0466	3.4669	3.8871	4.3074	4.7276	5.1479	5.5682
4	2.5990	2.2601	1.9361	1.6366	1.3897	1.2671	1.4891	2.2483	3.0453	3.8423	4.6392	5.4362	6.2332	7.0301	7.8271	8.6241	9.4211
5	3.2877	2.8846	2.4951	2.1198	1.7810	1.5293	1.5752	2.5286	3.7748	5.0210	6.2672	7.5134	8.7595	10.0057	11.2519	12.4981	13.7443
6	4.3381	3.8047	3.2892	2.7797	2.3001	1.8883	1.7178	2.6870	4.4868	6.2867	8.0866	9.8864	11.6863	13.4861	15.2860	17.0859	18.8857
7	5.5769	4.8917	4.2196	3.5475	2.9076	2.3198	1.9388	2.7445	5.2026	7.6607	10.1189	12.5770	15.0351	17.4933	19.9514	22.4095	24.8676
8	6.9813	6.1174	5.2535	4.4218	3.5926	2.8188	2.2248	2.7525	5.9172	9.1334	12.3496	15.5658	18.7820	21.9981	25.2143	28.4305	31.6467
9	8.5253	7.4676	6.4331	5.3985	4.3640	3.3954	2.5710	2.7834	6.6296	10.7032	14.7768	18.8504	22.9241	26.9977	31.0713	35.1449	39.2186
10	10.2342	8.9829	7.7319	6.4808	5.2297	4.0376	2.9645	2.8478	7.3406	12.3716	17.4025	22.4335	27.4645	32.4954	37.5264	42.5574	47.5883
11	12.1389	10.6157	9.1361	7.6613	6.1866	4.7219	3.4132	2.9468	8.0509	14.1392	20.2275	26.3158	32.4041	38.4924	44.5807	50.6690	56.7573
12	14.2426	12.4641	10.6857	8.9072	7.2032	5.5109	3.9181	3.0883	8.7606	16.0062	23.2518	30.4974	37.7430	44.9886	52.2342	59.4798	66.7254
13	16.4752	14.4356	12.3960	10.3564	8.3168	6.3145	4.4432	3.2804	9.4698	17.9727	26.4756	34.9784	43.4813	51.9842	60.4871	68.9899	77.4928
14	18.7983	16.4360	14.0736	11.7904	9.5167	7.2430	5.0294	3.5033	10.1787	20.0388	29.8989	39.7591	49.6192	59.4793	69.3394	79.1996	89.0597
15	21.4523	18.7898	16.1272	13.4647	10.8021	8.1396	5.6805	3.7577	10.8874	22.2047	33.5220	44.8394	56.1567	67.4741	78.7914	90.1088	101.4261
16	24.1031	21.0773	18.0516	15.0258	12.0815	9.2086	6.3356	4.0431	11.5958	24.4703	37.3449	50.2194	63.0940	75.9685	88.8431	101.7176	114.5922
17	26.9523	23.6232	20.2940	16.9649	13.6357	10.3065	6.9774	4.3578	12.3040	26.8358	41.3675	55.8993	70.4310	84.9627	99.4945	114.0262	128.5579
18	30.1805	26.4153	22.6501	18.8849	15.1197	11.3545	7.8050	4.6992	13.0122	29.3011	45.5900	61.8789	78.1678	94.4567	110.7457	127.0346	143.3235
19	33.1856	29.0141	24.8425	20.6709	16.5078	12.5729	8.6379	5.0638	13.7202	31.8663	50.0123	68.1584	86.3045	104.4506	122.5967	140.7428	158.8888
20	36 3//1	31 8634	27 2020	22 0024	10/01/	12 0407	0.4600	E 447E	11 1001	24 5242	E4 CO 4C	74 7070	04 0 44 0	4440440	405 0475	455 4500	475 0540
	30.3441	51.0054	21.3020	22.9021	10.4214	13.9407	9.4000	0.4470	14.4201	34.3313	54.6346	14.1318	94.8410	114.9443	135.0475	100.1008	175.2540
	30.3441	51.0054	21.3020	22.9021	10.4214	13.9407	9.4000	5.4475	x(max)	34.5313	54.0340	74.7378	94.8410	114.9443	135.0475	100.1008	1/5.2540
	x(max	:)=0	21.3020	0	<x(max)<x<sub>a</x(max)<x<sub>	13.9407	9.4000	5.4475	x(max)	34.5315	54.6346	x(max)=X _a	94.8410	114.9443	135.0475	155.1508	175.2540
X _a \Q ₃	x(max -0.8	:)=0 -0.7	-0.6	0 -0.5	<x(max)<x<sub>a -0.4</x(max)<x<sub>	-0.3	-0.2	-0.1	x(max)	0.1	0.2	14.7378 x(max)=X _a 0.3	94.8410 0.4	0.5	0.6	0.7	0.8
X _a \Q ₃ 1	x(max -0.8 0.0200	;)=0 - 0.7 0.0200	-0.6 0.0200	0 -0.5 0.0200	<x(max)<x<sub>a -0.4 0.3800</x(max)<x<sub>	-0.3 0.8400	-0.2 1.0000	-0.1 1.0000	x(max) 0 1.0000	0.1 1.0000	0.2 1.0000	74.7378 x(max)=X _a 0.3 1.0000	94.8410 0.4 1.0000	0.5 1.0000	0.6 1.0000	0.7 1.0000	0.8 1.0000
X _a \Q ₃ 1 2	x(max -0.8 0.0200 0.0400	.)=0 -0.7 0.0200 0.0400	-0.6 0.0200 0.0400	0 -0.5 0.0200 0.0400	<x(max)<x<sub>a -0.4 0.3800 0.9200</x(max)<x<sub>	-0.3 0.8400 1.6400	-0.2 1.0000 2.0000	-0.1 1.0000 2.0000	0 1.0000 2.0000	0.1 2.0000	54.6346 0.2 1.0000 2.0000	x(max)=X _a 0.3 1.0000 2.0000	94.8410 0.4 1.0000 2.0000	0.5 1.0000 2.0000	0.6 1.0000 2.0000	0.7 1.0000 2.0000	0.8 2.0000
X _a \Q ₃ 1 2 3	x(max -0.8 0.0200 0.0400 0.0600	-0.7 0.0200 0.0400 0.0600	-0.6 0.0200 0.0400 0.3600	0 -0.5 0.0200 0.0400 1.0800	<x(max)<x<sub>a -0.4 0.3800 0.9200 1.7400</x(max)<x<sub>	-0.3 0.8400 1.6400 2.4000	-0.2 1.0000 2.0000 3.0000	-0.1 1.0000 2.0000 3.0000	0 1.0000 2.0000 3.0000	0.1 1.0000 2.0000 3.0000	0.2 1.0000 2.0000 3.0000	x(max)=X _a 0.3 1.0000 2.0000 3.0000	0.4 1.0000 2.0000 3.0000	0.5 1.0000 2.0000 3.0000	0.6 1.0000 2.0000 3.0000	0.7 1.0000 2.0000 3.0000	0.8 1.0000 2.0000 3.0000
X _a \Q ₃ 1 2 3 4	x(max -0.8 0.0200 0.0400 0.0600 1.8400	-0.7 0.0200 0.0400 0.0600 1.9200	-0.6 0.0200 0.0400 0.3600 2.0800	0 -0.5 0.0200 0.0400 1.0800 2.3200	<x(max)<x<sub>a -0.4 0.3800 0.9200 1.7400 2.5600</x(max)<x<sub>	-0.3 0.8400 1.6400 2.4000 3.0400	-0.2 1.0000 2.0000 3.0000 3.7600	-0.1 1.0000 2.0000 3.0000 4.0000	0 1.0000 2.0000 3.0000 4.0000	0.1 1.0000 2.0000 3.0000 4.0000	0.2 1.0000 2.0000 3.0000 4.0000	x(max)=X _a 0.3 1.0000 2.0000 3.0000 4.0000	0.4 1.0000 2.0000 3.0000 4.0000	0.5 1.0000 2.0000 3.0000 4.0000	0.6 1.0000 2.0000 3.0000 4.0000	0.7 1.0000 2.0000 3.0000 4.0000	0.8 1.0000 2.0000 3.0000 4.0000
X _a \Q ₃ 1 2 3 4 5	x(max -0.8 0.0200 0.0400 0.0600 1.8400 3.0000	-0.7 0.0200 0.0400 0.0600 1.9200 3.0000	-0.6 0.0200 0.0400 0.3600 2.0800 3.1000	0 -0.5 0.0200 0.0400 1.0800 2.3200 3.2000	<x(max)<x<sub>a -0.4 0.3800 0.9200 1.7400 2.5600 3.4000</x(max)<x<sub>	-0.3 0.8400 1.6400 2.4000 3.0400 3.8000	-0.2 1.0000 2.0000 3.0000 3.7600 4.4000	-0.1 1.0000 2.0000 3.0000 4.0000 5.0000	0 1.0000 2.0000 3.0000 4.0000 5.0000	0.1 1.0000 2.0000 3.0000 4.0000 5.0000	54.6346 0.2 1.0000 2.0000 3.0000 4.0000 5.0000	x(max)=X _a 0.3 1.0000 2.0000 3.0000 4.0000 5.0000	0.4 1.0000 2.0000 3.0000 4.0000 5.0000	0.5 1.0000 2.0000 3.0000 4.0000 5.0000	0.6 1.0000 2.0000 3.0000 4.0000 5.0000	0.7 1.0000 2.0000 3.0000 4.0000 5.0000	0.8 1.0000 2.0000 3.0000 4.0000 5.0000
X _a \Q ₃ 1 2 3 4 5 6	x(max -0.8 0.0200 0.0400 0.0600 1.8400 3.0000 3.9600)=0 -0.7 0.0200 0.0400 0.0600 1.9200 3.0000 4.0800	-0.6 0.0200 0.0400 0.3600 2.0800 3.1000 4.0800	0 -0.5 0.0200 0.0400 1.0800 2.3200 3.2000 4.2000	(max) <x<sub>a -0.4 0.3800 0.9200 1.7400 2.5600 3.4000 4.3200</x<sub>	-0.3 0.8400 1.6400 2.4000 3.0400 3.8000 4.5600	-0.2 1.0000 2.0000 3.0000 3.7600 4.4000 5.1600	-0.1 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000	0 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000	0.1 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000	54.6340 0.2 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000	x(max)=X _a 0.3 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000	94.8410 0.4 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000	0.5 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000	0.6 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000	0.7 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000	0.8 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000
X _a \Q ₃ 1 2 3 4 5 6 7	x(max -0.8 0.0200 0.0400 0.0600 1.8400 3.0000 3.9600 4.9000)=0 -0.7 0.0200 0.0400 0.0600 1.9200 3.0000 4.0800 5.0400	-0.6 0.0200 0.0400 0.3600 2.0800 3.1000 4.0800 5.0400	0 -0.5 0.0200 0.0400 1.0800 2.3200 3.2000 4.2000 5.0400	(max) <x<sub>a -0.4 0.3800 0.9200 1.7400 2.5600 3.4000 4.3200 5.1800</x<sub>	-0.3 0.8400 1.6400 2.4000 3.0400 3.8000 4.5600 5.4600	-0.2 1.0000 2.0000 3.7600 4.4000 5.1600 5.8800	-0.1 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000	0 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000	0.1 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000	54.6340 0.2 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000	x(max)=X _a 0.3 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000	94.8410 0.4 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000	0.5 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000	0.6 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000	0.7 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000	0.8 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000
X _a \Q ₃ 1 2 3 4 5 6 7 8	x(max -0.8 0.0200 0.0400 1.8400 3.0000 4.9000 5.9200)=0 -0.7 0.0200 0.0400 0.0600 1.9200 3.0000 4.0800 5.0400 5.9200	-0.6 0.0200 0.0400 0.3600 2.0800 3.1000 4.0800 5.0400 5.9200	0 -0.5 0.0200 0.0400 1.0800 2.3200 3.2000 4.2000 5.0400 6.0800	(max) <x<sub>a -0.4 0.3800 0.9200 1.7400 2.5600 3.4000 4.3200 5.1800 6.0800</x<sub>	-0.3 0.8400 1.6400 2.4000 3.8000 4.5600 5.4600 6.2400	-0.2 1.0000 2.0000 3.7600 4.4000 5.1600 5.8800 6.7200	-0.1 1.0000 2.0000 3.0000 5.0000 6.0000 7.0000 7.8400	0 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000	0.1 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000	54.6346 0.2 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000	x(max)=X _a 0.3 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000	94.8410 0.4 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000	0.5 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000	0.6 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000	0.7 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000	0.8 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000
X _a \Q ₃ 1 2 3 4 5 6 7 7 8 9	x(max -0.8 0.0200 0.0400 0.0600 1.8400 3.9600 3.9600 4.9000 5.9200 6.8400)=0 -0.7 0.0200 0.0400 0.0600 1.9200 3.0000 4.0800 5.0400 5.9200 7.0200	-0.6 0.0200 0.0400 0.3600 2.0800 3.1000 4.0800 5.0400 5.9200 7.0200	0 -0.5 0.0200 0.0400 1.0800 2.3200 3.2000 4.2000 5.0400 6.0800 7.0200	(max)=Xa -0.4 -0.3800 0.9200 1.7400 2.5600 3.4000 4.3200 5.1800 6.0800 7.0200	-0.3 0.8400 1.6400 2.4000 3.0400 3.8000 4.5600 5.4600 6.2400 7.2000	-0.2 1.0000 2.0000 3.0000 3.7600 4.4000 5.1600 5.8800 6.7200 7.5600	-0.1 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 7.8400 8.6400	0 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 6.0000 8.0000 9.0000	0.1 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000	54.6346 0.2 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000	<pre>/4./3/8 (max)=Xa 0.3 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 6.0000 7.0000 8.0000 9.0000</pre>	94.8410 0.4 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000	0.5 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000	0.6 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000	0.7 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000	0.8 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000
X _a \Q ₃ 1 2 3 4 5 6 7 8 9 10	x(max -0.8 0.0200 0.0400 0.0600 1.8400 3.0000 3.9600 4.9000 5.9200 6.8400 7.8000)=0 -0.7 0.0200 0.0400 0.0600 1.9200 3.0000 4.0800 5.0400 5.9200 7.0200 8.0000	-0.6 0.0200 0.0400 0.3600 2.0800 3.1000 4.0800 5.0400 5.9200 7.0200 8.0000	0 -0.5 0.0200 1.0800 2.3200 3.2000 4.2000 5.0400 6.0800 7.0200 8.0000	(max) <x<sub>a -0.4 0.3800 0.9200 1.7400 2.5600 3.4000 4.3200 5.1800 6.0800 7.0200 8.0000</x<sub>	-0.3 0.8400 1.6400 2.4000 3.0400 3.8000 4.5600 5.4600 6.2400 7.2000 8.2000	-0.2 1.0000 2.0000 3.0000 3.7600 4.4000 5.1600 5.8800 6.7200 7.5600 8.4000	-0.1 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 7.8000 8.6400 9.4000	0 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000	0.1 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000	54.6340 0.2 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000	74.7378 0.3 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000	94.8410 0.4 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000	0.5 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000	0.6 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000	0.7 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000	0.8 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000
X _a \Q ₃ 1 2 3 4 5 6 7 8 9 10 11	x(max -0.8 0.0200 0.0400 0.0600 1.8400 3.0000 3.9600 4.9000 5.9200 6.8400 7.8000 8.8000)=0 -0.7 0.0200 0.0400 0.0600 1.9200 3.0000 4.0800 5.9200 7.0200 8.0000 8.8000	-0.6 0.0200 0.0400 0.3600 2.0800 3.1000 4.0800 5.0400 5.0400 7.0200 8.0000 9.0200	0 -0.5 0.0200 1.0800 2.3200 3.2000 4.2000 5.0400 6.0800 7.0200 8.0000 9.0200	(max) <x<sub>a -0.4 0.3800 0.9200 1.7400 2.5600 3.4000 4.3200 5.1800 6.0800 7.0200 8.0000 9.0200</x<sub>	-0.3 0.8400 1.6400 2.4000 3.0400 3.8000 4.5600 5.4600 6.2400 7.2000 8.2000 9.2400	-0.2 1.0000 2.0000 3.0000 3.7600 4.4000 5.1600 5.8800 6.7200 7.5600 8.4000 9.4600	-0.1 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 7.8400 8.6400 9.4000 10.3400	0 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 9.0000 10.0000 11.0000	0.1 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000	54.6346 0.2 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000	74.7378 0.3 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000	94.8410 0.4 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000	0.5 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000	0.6 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000	0.7 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000	0.8 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000
X _a \Q ₃ 1 2 3 4 5 6 7 8 9 10 11 12	x(max -0.8 0.0200 0.0400 0.0600 1.8400 3.9600 4.9000 5.9200 6.8400 7.8000 8.8000 9.8400)=0 -0.7 0.0200 0.0400 0.0600 1.9200 3.0000 4.0800 5.9200 7.0200 8.0000 8.8000 9.8400	-0.6 0.0200 0.0400 0.3600 2.0800 3.1000 4.0800 5.0400 5.0400 5.9200 7.0200 8.0000 9.0200 9.8400	0 -0.5 0.0200 0.0400 1.0800 2.3200 3.2000 4.2000 5.0400 6.0800 7.0200 8.0000 9.0200 9.8400	(max) <x<sub>9 -0.4 0.3800 0.9200 1.7400 2.5600 3.4000 4.3200 5.1800 6.0800 7.0200 8.0000 9.0200 10.0800</x<sub>	-0.3 0.8400 1.6400 2.4000 3.0400 3.8000 4.5600 5.4600 6.2400 7.2000 8.2000 9.2400 10.0800	-0.2 1.0000 2.0000 3.0000 3.7600 4.4000 5.1600 5.8800 6.7200 6.7200 8.4000 9.4600 10.3200	-0.1 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 7.8400 8.6400 9.4000 10.3400 11.0400	0 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 10.0000 11.0000 12.0000	0.1 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000	54.6346 0.2 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000	<pre>/4./3/8 (max)=Xa 0.3 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000</pre>	94.8410 0.4 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000	0.5 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000	0.6 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000	0.7 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000	0.8 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 9.0000 10.0000 11.0000 12.0000
X _a \Q ₃ 1 2 3 4 5 6 7 7 8 9 10 11 12 13	x(max -0.8 0.0200 0.0400 0.0600 1.8400 3.0000 3.9600 4.9000 5.9200 6.8400 7.8000 8.8000 9.8400 10.9200)=0 -0.7 0.0200 0.0400 0.0600 1.9200 3.0000 4.0800 5.0400 5.9200 7.0200 8.0000 8.8000 9.8400 10.9200	-0.6 0.0200 0.0400 0.3600 2.0800 3.1000 4.0800 5.0400 5.9200 7.0200 8.0000 9.0200 9.8400 10.9200	0 -0.5 0.0200 0.0400 1.0800 2.3200 3.2000 4.2000 5.0400 6.0800 7.0200 8.0000 9.0200 9.8400 10.9200	(max) <x<sub>a -0.4 0.3800 0.9200 1.7400 2.5600 3.4000 4.3200 5.1800 6.0800 7.0200 8.0000 9.0200 10.0800 10.0800 10.9200</x<sub>	-0.3 0.8400 1.6400 2.4000 3.0400 3.8000 4.5600 5.4600 6.2400 7.2000 8.2000 9.2400 10.0800 11.1800	-0.2 1.0000 2.0000 3.0000 3.7600 4.4000 5.1600 5.8800 6.7200 7.5600 8.4000 9.4600 10.3200 11.1800	-0.1 1.0000 2.0000 3.0000 4.0000 5.0000 7.0000 7.8400 7.8400 9.4000 10.3400 11.9600 11.9600	0 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 10.0000 11.0000 11.0000 12.0000 13.0000 12.0000 13.0000 13.0000 13.0000 13.0000 13.0000 13.0000 14.00000 14.0000 14.0000 14.0000 14.0000 14.0000 14.00000 14.000	0.1 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000 13.0000	54.6340 0.2 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000 13.0000	x(max)=X _a 0.3 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 11.0000 112.0000	94.8410 0.4 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 11.0000 13.0000 13.0000	0.5 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 11.0000 13.0000 13.0000	0.6 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 11.0000 13.0000 13.0000	0.7 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 11.0000 12.0000 13.0000	0.8 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 10.0000 11.0000 11.0000 12.0000 13.0000 13.0000
X _a \Q ₃ 1 2 3 4 5 6 7 8 9 10 11 12 13 14	x(max -0.8 0.0200 0.0400 0.0600 1.8400 3.9600 4.9000 5.9200 6.8400 7.8000 8.8000 9.8400 10.9200 11.7600)=0 -0.7 0.0200 0.0400 0.0600 1.9200 3.0000 4.0800 5.9200 7.0200 8.0000 8.8000 9.8400 10.9200 11.7600	-0.6 0.0200 0.0400 0.3600 2.0800 3.1000 4.0800 5.9200 7.0200 8.0000 9.0200 9.8400 10.9200 11.7600	0 -0.5 0.0200 0.0400 1.0800 2.3200 3.2000 4.2000 5.0400 6.0800 7.0200 8.0000 9.0200 9.8400 10.9200 12.0400	(max) <x<sub>a -0.4 0.3800 0.9200 1.7400 2.5600 3.4000 4.3200 5.1800 6.0800 7.0200 8.0000 8.0000 9.0200 10.0800 10.0800 10.9200 12.0400</x<sub>	-0.3 0.8400 1.6400 2.4000 3.0400 3.8000 4.5600 5.4600 6.2400 7.2000 8.2000 9.2400 10.0800 11.1800 12.0400	-0.2 1.0000 2.0000 3.0000 3.7600 4.4000 5.1600 5.8800 6.7200 7.5600 8.4000 9.4600 10.3200 11.1800 12.3200	-0.1 1.0000 2.0000 3.0000 4.0000 5.0000 7.0000 7.8400 8.6400 9.4000 10.3400 11.0400 11.9600 12.8800 12.8800	0 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 10.0000 11.0000 11.0000 11.0000 11.0000 11.0000 11.0000 11.0000 11.0000 11.0000 11.0000 11.0000 11.0000 11.0000 11.0000 11.0000 11.0000 11.0000 11.0000 11.0000 10.00000 10.0000 10.0000 10.0000 10.0000 10.0000 10.00000 10.000	0.1 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000 14.00000 14.00000 14.0000 14.0000 14.0000 14.0000 14.00000000 1	54.6340 0.2 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 11.0000 13.0000 14.0000 13.0000 14.00000 14.00000 14.0000 14.0000 14.0000 14.0000 14.00	x(max)=X _a 0.3 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 11.0000 13.0000	94.8410 0.4 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 11.0000 13.0000 14.0000 14.0000 13.0000 14.00000 14.00000 14.0000 14.0000 14.0000 14.0000 14.00	0.5 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000 14.0000 14.0000 14.0000 14.0000 14.0000 14.0000 14.0000 14.0000 14.0000 14.0000 14.0000 14.0000 14.0000 14.0000 14.0000 14.0000 14.0000 14.0000 10.00000 10.00000 10.0000 10.0000 10.0000 10.0000 10.00000000 1	0.6 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 11.0000 13.0000 13.0000 14.0000 13.0000 14.00000 14.00000 14.0000 14.0000 14.0000 14.0000 14.00000000 1	0.7 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 11.0000 13.0000 14.0000 13.0000 14.00000 14.00000 14.0000 14.0000 14.0000 14.0000 14.0000000 14	0.8 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000 13.0000 14.0000
X _a \Q ₃ 1 2 3 4 5 6 6 7 8 9 10 11 11 12 13 14 15	x(max -0.8 0.0200 0.0400 0.0600 1.8400 3.9600 4.9000 5.9200 6.8400 7.8000 8.8000 9.8400 10.9200 11.7600 12.9000)=0 -0.7 0.0200 0.0400 0.0600 1.9200 3.0000 4.0800 5.9200 7.0200 8.0000 8.8000 9.8400 10.9200 11.7600 12.9000	-0.6 0.0200 0.0400 0.3600 2.0800 3.1000 4.0800 5.9200 7.0200 8.0000 9.0200 9.8400 10.9200 11.7600 12.9000 12.9000	0 -0.5 0.0200 0.0400 1.0800 2.3200 3.2000 4.2000 5.0400 6.0800 7.0200 8.0000 9.0200 9.8400 10.9200 12.0400 12.9000	<x(max)<x<sub>a -0.4 0.3800 0.9200 1.7400 2.5600 3.4000 4.3200 5.1800 6.0800 7.0200 8.0000 9.0200 10.0800 10.0800 10.9200 12.0400 12.9000 2001</x(max)<x<sub>	-0.3 0.8400 1.6400 2.4000 3.0400 3.8000 6.2400 7.2000 8.2000 9.2400 10.0800 11.1800 12.0400 12.0400 12.9000	-0.2 1.0000 2.0000 3.0000 3.7600 4.4000 5.1600 5.8800 6.7200 7.5600 8.4000 9.4600 9.4600 10.3200 11.1800 12.3200 13.2000	-0.1 1.0000 2.0000 3.0000 4.0000 5.0000 7.0000 7.8400 8.6400 9.4000 10.3400 11.9600 11.9600 12.8800 13.8000 14.90000 14.90000 14.9000 14.9000 14.9000 14.9000 14.90000 14.	0 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 10.0000 10.0000 11.0000 11.0000 11.0000 11.0000 11.0000 11.0000 11.0000 11.0000 11.0000 12.0000 13.0000 14.0000 14.0000 14.0000 14.0000 14.0000 14.0000 14.0000 14.0000 14.0000 14.0000 14.0000 14.0000 14.0000 14.0000 14.0000 14.0000 14.0000 15.00000 15.0000000 15	0.1 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 10.0000 11.0000 11.0000 11.0000 12.0000 14.00000 14.0000 14.0000 14.0000 14.0000 14.0000 14.0000000 14	54.6346 0.2 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 10.0000 11.0000 11.0000 11.0000 11.0000 13.0000 14.0000	<pre>/4./3/8 (max)=Xa 0.3 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 6.0000 9.0000 10.0000 11.0000 11.0000 12.0000 14.0000 14.0000 14.0000 16.0000</pre>	94.8410 0.4 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 9.0000 10.0000 11.0000 11.0000 12.0000 14.0000 14.0000 14.0000 15.0000	0.5 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 10.0000 11.0000 11.0000 12.0000 14.00000 14.0000 14.0000 14.0000 14.0000 14.0000 14.0000000 14	0.6 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 10.0000 11.0000 11.0000 12.0000 14.0000 14.0000 15.0000 14.0000 14.0000 14.0000 14.0000 14.0000 15.00000 15.0000 15.0000 15.0000 15.0000 15.0000 15.00000000 1	0.7 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 10.0000 11.0000 11.0000 12.0000 14.00000 14.0000 14.0000 14.0000 14.0000 14.0000 14.0000000 14	0.8 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 10.0000 11.0000 12.0000 13.0000 14.0000 14.0000 15.0000 14.00000 14.0000 14.0000 14.0000 14.0000 14.0000 14.0000000 14
X _a \Q ₃ 1 2 3 4 5 6 7 7 8 9 10 11 12 12 14 14 15 16	x(max -0.8 0.0200 0.0400 0.0600 1.8400 3.9600 3.9600 5.9200 6.8400 7.8000 8.8000 9.8400 10.9200 11.7600 12.9000 13.7600)=0 -0.7 0.0200 0.0400 0.0600 1.9200 3.0000 4.0800 5.9200 7.0200 8.0000 8.8000 9.8400 9.8400 9.8400 1.9200 11.7600 12.9000 13.7600 14.9605	-0.6 0.0200 0.0400 0.3600 2.0800 3.1000 4.0800 5.0400 5.9200 7.0200 8.0000 9.0200 9.0200 9.0200 11.7600 12.9000 13.7600 12.9000 14.9655	0 -0.5 0.0200 0.0400 1.0800 2.3200 3.2000 4.2000 5.0400 6.0800 7.0200 8.0000 9.0200 9.8400 10.9200 12.0400 12.9000 13.76000	<x(max)<x<sub>a -0.4 0.3800 0.9200 1.7400 2.5600 3.4000 4.3200 5.1800 6.0800 7.0200 8.0000 9.0200 10.0800 10.9200 10.0800 10.9200 12.0400 12.9000 14.0800 14.0800</x(max)<x<sub>	-0.3 0.8400 1.6400 2.4000 3.0400 3.8000 4.5600 6.2400 7.2000 8.2000 9.2400 10.0800 11.1800 12.0400 12.9000 14.0800 14.0800	-0.2 1.0000 2.0000 3.0000 3.7600 4.4000 5.8800 6.7200 7.5600 8.4000 9.4600 10.3200 11.1800 12.3200 13.2000 14.0800	-0.1 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.8400 8.6400 9.4000 10.3400 11.9600 11.9600 11.9600 13.8000 13.8000 14.7200	0 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 10.0000 11.0000 11.0000 13.0000 13.0000 14.0000 15.0000 16.0000 10.00000 10.0000 10.0000 10.0000 10.0000 10.0000 10.0000000 10.0	0.1 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 10.0000 11.0000 11.0000 13.0000 13.0000 14.0000 15.0000 16.0000 10.00000 10.0000 10.0000 10.0000 10.0000 10.0000000 10.00000	54.6346 0.2 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 11.0000 13.0000 13.0000 14.0000 15.0000 16.0000 10.00000 10.00000 10.0000 10.0000 10.0000 10.0000 10.00	x(max)=X _a 0.3 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 10.0000 11.0000 11.0000 13.0000 13.0000 14.0000 15.0000	94.8410 0.4 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 11.0000 13.0000 13.0000 14.0000 15.0000 16.0000 10.00000 10.00000 10.0000 10.0000 10.0000 10.0000 10.00	0.5 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 10.0000 11.0000 11.0000 13.0000 13.0000 14.0000 15.0000 16.0000 10.00000 10.0000 10.0000 10.0000 10.0000 10.0000000 10.00000	0.6 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 10.0000 11.0000 11.0000 13.0000 13.0000 14.0000 15.0000 16.0000 10.00000 10.0000 10.0000 10.0000 10.0000 10.0000 10.0000000 10	0.7 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 10.0000 11.0000 11.0000 13.0000 13.0000 14.0000 15.0000 16.0000 10.00000 10.0000 10.0000 10.0000 10.0000 10.0000000 10.00000	0.8 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 10.0000 11.0000 11.0000 13.0000 14.0000 15.0000 16.0000 10.00000 10.0000 10.0000 10.0000 10.0000 10.0000000 10.00000
X _a \Q ₃ 1 2 3 4 5 6 7 7 8 9 10 11 12 13 14 15 16 17	x(max -0.8 0.0200 0.0400 0.0600 1.8400 3.0000 3.9600 4.9000 5.9200 6.8400 7.8000 8.8000 9.8400 10.9200 11.7600 12.9000 13.7600)=0 -0.7 0.0200 0.0400 0.0600 1.9200 3.0000 4.0800 5.9200 7.0200 8.0000 8.8000 9.8400 10.9200 11.7600 12.9000 13.7600 14.9600	-0.6 0.0200 0.0400 0.3600 2.0800 3.1000 4.0800 5.9200 7.0200 8.0000 9.0200 9.8400 10.9200 11.7600 12.9000 13.7600 14.9600	0 -0.5 0.0200 1.0800 2.3200 3.2000 4.2000 4.2000 5.0400 6.0800 7.0200 8.0000 9.0200 9.8400 10.9200 12.0400 12.9000 13.7600 14.9600	<x(max)<x<sub>a -0.4 0.3800 0.9200 1.7400 2.5600 3.4000 4.3200 5.1800 6.0800 7.0200 8.0000 9.0200 10.0800 10.9200 10.9200 12.0400 12.9000 14.0800 14.9600 14.9600</x(max)<x<sub>	-0.3 0.8400 1.6400 2.4000 3.0400 3.8000 4.5600 6.2400 7.2000 8.2000 9.2400 10.0800 11.0800 12.0400 12.9000 14.0800 14.0800 14.9600	-0.2 1.0000 2.0000 3.0000 3.7600 4.4000 5.1600 5.8800 6.7200 7.5600 8.4000 9.4600 10.3200 11.1800 11.3200 11.2000 13.2000 14.0800 14.0800 14.9600	-0.1 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 7.8400 8.6400 9.4000 10.3400 11.9600 11.9600 13.8000 13.8000 14.7200 15.6400 9.9001 10.34000 10.9000 10.90000 10.90000 10.90000 10.90000 10.90000 10.90000 10.90000 10.90000 10.900000 10.90000 10.90000 10.90000 10.90000 10.90000 10.900000 10.900000 10.900000 10.900000000000000000000000000000000000	0 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 10.0000 11.0000 11.0000 11.0000 11.0000 11.0000 14.0000 15.0000 10.0000 11.0000 11.0000 11.0000 11.0000 11.0000 14.0000 15.0000 10.00000 10.0000 10.0000 10.0000 10.0000 10.0000000 10.00000 10	0.1 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 10.0000 11.0000 11.0000 11.0000 11.0000 15.0000 16.0000 10.00000 10.0000 10.0000 10.0000 10.0000 10.0000000 10.00000	54.6346 0.2 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 10.0000 11.0000 11.0000 11.0000 11.0000 15.0000 14.0000 15.0000 16.0000 10.0000 11.0000 11.0000 11.0000 11.0000 11.0000 11.0000 11.0000 11.0000 11.0000 11.0000 11.0000 11.0000 11.0000 11.0000 11.0000 11.0000 11.0000 11.0000 10.00000 10.0000 10.0000 10.0000 10.0000 10.00000000	x(max)=X _a 0.3 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 7.0000 10.0000 11.0000 11.0000 11.0000 11.0000 15.0000 16.0000 16.0000	94.8410 0.4 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 10.0000 11.0000 11.0000 11.0000 11.0000 11.0000 15.0000 16.0000 10.000 10	0.5 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 10.0000 11.0000 11.0000 11.0000 11.0000 15.0000 16.0000 10.00000 10.0000 10.0000 10.0000 10.0000 10.00000000 10.00000	0.6 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 10.0000 11.0000 11.0000 11.0000 11.0000 13.0000 14.0000 14.0000 15.0000 16.0000 10.0000 11.0000 11.0000 11.0000 11.0000 11.0000 11.0000 11.0000 11.0000 11.0000 11.0000 11.0000 11.0000 11.0000 11.0000 11.0000 11.0000 11.0000 11.0000 10.00000 10.0000 10.0000 10.0000 10.0000 10.0000000 10.00000	0.7 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 10.0000 11.0000 11.0000 11.0000 11.0000 14.0000 15.0000 16.0000 10.00000 10.0000 10.0000 10.0000 10.0000 10.00000000 10.000000	0.8 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 10.0000 11.0000 11.0000 11.0000 14.0000 14.0000 15.0000 16.0000 17.0000 16.0000 10.00000 10.0000 10.0000 10.0000 10.0000 10.0000000 10.00000
X _a \Q ₃ 1 2 3 4 5 6 6 7 8 9 10 11 12 13 14 15 16 17 18	x(max -0.8 0.0200 0.0400 0.0600 1.8400 3.0000 3.9600 4.9000 5.9200 6.8400 7.8000 8.8000 9.8400 10.9200 11.7600 11.7600 13.7600 14.9600 15.8400)=0 -0.7 0.0200 0.0400 0.0600 1.9200 3.0000 4.0800 5.9200 7.0200 8.0000 8.8000 9.8400 10.9200 11.7600 11.7600 13.7600 14.9600 15.8400 0.9201 14.9600 15.8400 14.9600 15.8400 14.9600 15.8400 14.9600 15.8400 14.9600 15.84000 15.84000 15.8400 15	-0.6 0.0200 0.0400 0.3600 2.0800 3.1000 4.0800 5.9200 7.0200 8.0000 9.8400 10.9200 11.7600 11.7600 13.7600 14.9600 15.8400 0.9201 14.9600 15.8400 14.9600 15.8400 14.9601 15.8400 14.9601 15.84000 15.8400 15.8400 15.8400	0 -0.5 0.0200 0.0400 1.0800 2.3200 3.2000 4.2000 5.0400 6.0800 7.0200 8.0000 9.0200 9.8400 10.9200 12.0400 12.9000 13.7600 14.9600 14.9600 15.8400 0 15.8400 14.95000 14.95000 14.95000 14.95000 14.95000000 14.9500000000	<x(max)<x<sub>a -0.4 0.3800 0.9200 1.7400 2.5600 3.4000 4.3200 4.3200 5.1800 6.0800 7.0200 8.0000 9.0200 10.0800 10.0800 10.0800 10.2000 10.0800 10.2000 10.0800 10.2000 10.4000 12.0400 12.0400 14.0800 14.9600 15.8400 14.9600</x(max)<x<sub>	-0.3 0.8400 1.6400 2.4000 3.0400 3.8000 4.5600 5.4600 5.4600 9.2400 10.0800 11.1800 12.0400 12.9000 14.0800 14.0800 14.9600 15.8400 17.4655	-0.2 1.0000 2.0000 3.0000 3.7600 4.4000 5.1600 5.1600 5.8800 6.7200 7.5600 8.4000 9.4600 10.3200 11.1800 12.3200 13.2000 14.08000 14.08000 14.0800 14.0800	-0.1 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.8400 8.6400 9.4000 10.3400 11.0400 11.9600 12.8800 13.8000 14.7200 15.6400 16.5600 16.5600 14.7600	0 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 10.0000 11.0000 11.0000 11.0000 11.0000 14.0000 15.0000 16.0000 17.0000 18.0000 16.0000 10.00000 10.0000 10.0000 10.0000 10.0000 10.0000000 10.00000 10	0.1 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 10.0000 11.0000 11.0000 11.0000 14.0000 15.0000 16.0000 17.0000 18.0000 10.00000 10.0000 10.0000 10.0000 10.0000 10.0000000 10.00000	54.6346 0.2 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 10.0000 11.0000 11.0000 11.0000 14.0000 15.0000 16.0000 17.0000 18.0000 16.0000 10.00000 10.0000 10.0000 10.0000 10.0000 10.0000000 1	x(max)=X _a 0.3 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 10.0000 11.0000 11.0000 11.0000 11.0000 14.0000 15.0000 15.0000 15.0000 16.0000	94.8410 0.4 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 10.0000 11.0000 11.0000 12.0000 14.0000 15.0000 16.0000 17.0000 18.0000 16.000	0.5 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 10.0000 11.0000 11.0000 12.0000 13.0000 14.0000 15.0000 16.0000 17.0000 18.0000 10.00000 10.0000 10.0000 10.0000 10.0000 10.0000000 10.00000	0.6 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 10.0000 11.0000 11.0000 11.0000 14.0000 15.0000 16.0000 17.0000 18.0000 16.0000 10.00000 10.0000 10.0000 10.0000 10.0000 10.0000000 10.00000	0.7 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 10.0000 11.0000 11.0000 11.0000 14.0000 15.0000 15.0000 16.0000 17.0000 18.0000 16.0000 10.00000 10.0000 10.0000 10.0000 10.0000 10.0000000 10.00000	0.8 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 10.0000 11.0000 11.0000 12.0000 13.0000 14.0000 15.0000 16.0000 17.0000 16.0000 17.0000 16.0000 17.0000 16.0000 17.0000 16.0000 10.00000 10.0000 10.0000 10.0000 10.0000 10.0000000 10.00000
XaVQ3 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19	x(max -0.8 0.0200 0.0400 0.0600 1.8400 3.0000 3.9600 4.9000 5.9200 6.8400 7.8000 8.8000 9.8400 10.9200 11.7600 11.7600 13.7600 14.9600 15.8400 16.7200)=0 -0.7 0.0200 0.0400 0.0600 1.9200 3.0000 4.0800 5.0400 5.0400 5.0400 5.9200 7.0200 8.0000 8.8000 9.8400 10.9200 11.7600 11.7600 11.7600 14.9600 15.8400 16.7200 10.524	-0.6 0.0200 0.0400 0.3600 2.0800 3.1000 4.0800 5.9200 7.0200 8.0000 9.0200 9.8400 10.9200 11.7600 11.7600 13.7600 14.9600 15.8400 16.7200	0 -0.5 0.0200 0.0400 1.0800 2.3200 3.2000 4.2000 5.0400 5.0400 5.0400 5.0400 9.0200 9.0200 9.8400 10.9200 12.9000 12.9000 13.7600 14.9600 15.8400 16.7200	<x(max)<x<sub>a -0.4 0.3800 0.9200 1.7400 2.5600 3.4000 4.3200 5.1800 6.0800 7.0200 8.0000 9.0200 10.0800 10.9200 10.0800 10.9200 12.9000 14.0800 14.9600 15.8400 17.1000</x(max)<x<sub>	-0.3 0.8400 1.6400 2.4000 3.0400 3.8000 4.5600 5.4600 6.2400 7.2000 8.2000 9.2400 10.0800 11.1800 12.9000 14.0800 14.9600 15.8400 17.1000 12.9000 14.9600 15.8400 14.9600 15.8400 14.9600 15.8400 14.9600 15.84000 15.84000 15.8400 15.8400 15.8400 15.840	-0.2 1.0000 2.0000 3.0000 3.7600 4.4000 5.1600 5.8800 6.7200 7.5600 8.4000 9.4600 10.3200 11.1800 11.2200 13.2000 14.9600 14.9600 16.2000 17.1000	-0.1 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.8400 7.8400 10.3400 11.0400 11.9600 11.9600 13.8000 13.8000 13.8000 14.7200 15.6400 16.5600 17.4800 16.5600 17.4800 16.5600 17.4800 16.5600 17.4800 16.5600 17.4800 16.5600 17.4800 16.5600 17.4800 16.5600 17.4800 16.5600 17.4800 16.5600 17.4800 16.5600 17.4800 16.5600 17.4800 16.5600 17.4800 16.5600 17.4800 10.5600 10.50000 10.50000 10.5000 10.5000 10.5000 10.50000 10.500000	14.420 (max) 0 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 10.0000 11.0000 11.0000 11.0000 13.0000 14.0000 15.0000 15.0000 16.0000 17.0000 18.0000 19.0000 19.0000 19.0000 19.0000 19.0000 19.0000 19.0000 10.00000 10.0000 10.0000 10.0000 10.0000 10.000000	0.1 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 10.0000 11.0000 11.0000 11.0000 13.0000 14.0000 15.0000 15.0000 16.0000 17.0000 18.0000 19.0000 10.00000 10.0000 10.0000 10.0000 10.0000 10.0000000 10.00000	54.6346 0.2 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 10.0000 11.0000 11.0000 11.0000 13.0000 14.0000 15.0000 15.0000 16.0000 17.0000 18.0000 19.0000 10.00000 10.0000 10.0000 10.0000 10.0000 10.0000 10.00	x(max)=X _a 0.3 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 10.0000 11.0000 11.0000 11.0000 11.0000 11.0000 11.0000 15.0000 15.0000 17.0000	94.8410 0.4 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 10.0000 11.0000 11.0000 13.0000 14.0000 15.0000 16.0000 17.0000 18.0000 19.0000 18.0000 19.0000 19.0000 19.0000 10.00000 10.0000 10.0000 10.0000 10.000	0.5 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 10.0000 11.0000 11.0000 13.0000 13.0000 14.0000 15.0000 16.0000 17.0000 18.0000 19.0000 10.00000 10.0000 10.0000 10.0000 10.0000 10.0000 10.0000000 10	0.6 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 10.0000 11.0000 11.0000 11.0000 13.0000 14.0000 15.0000 15.0000 16.0000 17.0000 18.0000 19.0000 10.00000 10.0000 10.0000 10.0000 10.0000 10.0000000 10.00000	0.7 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 10.0000 11.0000 11.0000 11.0000 13.0000 14.0000 15.0000 15.0000 16.0000 17.0000 18.0000 19.0000 10.00000 10.0000 10.0000 10.0000 10.0000 10.0000 10.0000000 10	0.8 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 10.0000 11.0000 11.0000 12.0000 13.0000 14.0000 15.0000 15.0000 16.0000 17.0000 18.0000 19.0000 10.00000 10.0000 10.0000 10.0000 10.0000 10.0000000 10.00000

$F_t(X_a)$ and F_q

									F _t ()	K _a)						(v* =	: 0.4)
			F _q =l	F _{t,min}		F,	_{,min} <fq<f<sub>t,r</fq<f<sub>	nax					F _q =F _{t,max}				
X _a \Q ₃	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
1	0.9565	0.9653	0.9742	0.9830	0.9919	1.0007	1.0095	1.0184	1.0272	1.0361	1.0449	1.0537	1.0626	1.0714	1.0803	1.0891	1.0979
2	0.3868	0.5103	0.6339	0.7574	0.8809	1.0044	1.1279	1.2514	1.3749	1.4985	1.6220	1.7455	1.8690	1.9925	2.1160	2.2395	2.3631
3	-1.1560	-0.7358	-0.3155	0.1048	0.5250	0.9453	1.3655	1.7858	2.2061	2.6263	3.0466	3.4669	3.8871	4.3074	4.7276	5.1479	5.5682
4	-3.3304	-2.5335	-1.7365	-0.9395	-0.1426	0.6544	1.4514	2.2483	3.0453	3.8423	4.6392	5.4362	6.2332	7.0301	7.8271	8.6241	9.4211
5	-6.1946	-4.9484	-3.7023	-2.4561	-1.2099	0.0363	1.2825	2.5286	3.7748	5.0210	6.2672	7.5134	8.7595	10.0057	11.2519	12.4981	13.7443
6	-9.9120	-8.1122	-6.3123	-4.5124	-2.7126	-0.9127	0.8871	2.6870	4.4868	6.2867	8.0866	9.8864	11.6863	13.4861	15.2860	17.0859	18.8857
7	-14.4624	-12.0043	-9.5461	-7.0880	-4.6299	-2.1718	0.2864	2.7445	5.2026	7.6607	10.1189	12.5770	15.0351	17.4933	19.9514	22.4095	24.8676
8	-19.8122	-16.5960	-13.3799	-10.1637	-6.9475	-3.7313	-0.5151	2.7010	5.9172	9.1334	12.3496	15.5658	18.7820	21.9981	25.2143	28.4305	31.6467
9	-25.9594	-21.8858	-17.8122	-13.7385	-9.6649	-5.5913	-1.5177	2.5560	6.6296	10.7032	14.7768	18.8504	22.9241	26.9977	31.0713	35.1449	39.2186
10	-32.9071	-27.8761	-22.8452	-17.8142	-12.7832	-7.7523	-2.7213	2.3097	7.3406	12.3716	17.4025	22.4335	27.4645	32.4954	37.5264	42.5574	47.5883
11	-40.6555	-34.5672	-28.4789	-22.3906	-16.3023	-10.2140	-4.1257	1.9626	8.0509	14.1392	20.2275	26.3158	32.4041	38.4924	44.5807	50.6690	56.7573
12	-49.2043	-41.9587	-34.7131	-27.4675	-20.2219	-12.9763	-5.7307	1.5149	8.7606	16.0062	23.2518	30.4974	37.7430	44.9886	52.2342	59.4798	66.7254
13	-58.5532	-50.0504	-41.5475	-33.0446	-24.5417	-16.0388	-7.5360	0.9669	9.4698	17.9727	26.4756	34.9784	43.4813	51.9842	60.4871	68.9899	77.4928
14	-68.7023	-58.8422	-48.9820	-39.1219	-29.2618	-19.4017	-9.5415	0.3186	10.1787	20.0388	29.8989	39.7591	49.6192	59.4793	69.3394	79.1996	89.0597
15	-79.6514	-68.3341	-57.0167	-45.6994	-34.3820	-23.0647	-11.7473	-0.4300	10.8874	22.2047	33.5220	44.8394	56.1567	67.4741	78.7914	90.1088	101.4261
16	-91.4006	-78.5261	-65.6515	-52.7770	-39.9024	-27.0279	-14.1533	-1.2788	11.5958	24.4703	37.3449	50.2194	63.0940	75.9685	88.8431	101.7176	114.5922
17	-103.9499	-89.4181	-74.8864	-60.3546	-45.8229	-31.2912	-16.7594	-2.2277	12.3040	26.8358	41.3675	55.8993	70.4310	84.9627	99.4945	114.0262	128.5579
18	-117.2992	-101.0102	-84.7213	-68.4324	-52.1435	-35.8546	-19.5657	-3.2767	13.0122	29.3011	45.5900	61.8789	78.1678	94.4567	110.7457	127.0346	143.3235
19	-131.4485	-113.3024	-95.1563	-77.0102	-58.8642	-40.7181	-22.5720	-4.4259	13.7202	31.8663	50.0123	68.1584	86.3045	104.4506	122.5967	140.7428	158.8888
20	-146.3978	-126.2946	-106.1914	-86.0881	-65.9849	-45.8816	-25.7784	-5.6752	14.4281	34.5313	54.6346	74.7378	94.8410	114.9443	135.0475	155.1508	175.2540

$F_q = (Z_d + Q_3 Z_v) X_a^4/2$

										.,	-						
							2	Z(CODAP)>0									
X _a \Q ₃	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
1	0.9565	0.9653	0.9742	0.9830	0.9919	1.0007	1.0095	1.0184	1.0272	1.0361	1.0449	1.0537	1.0626	1.0714	1.0803	1.0891	1.0979
2	0.3868	0.5103	0.6339	0.7574	0.8809	1.0044	1.1279	1.2514	1.3749	1.4985	1.6220	1.7455	1.8690	1.9925	2.1160	2.2395	2.3631
3	-1.1560	-0.7358	-0.3155	0.1048	0.5250	0.9453	1.3655	1.7858	2.2061	2.6263	3.0466	3.4669	3.8871	4.3074	4.7276	5.1479	5.5682
4	-3.3304	-2.5335	-1.7365	-0.9395	-0.1426	0.6544	1.4514	2.2483	3.0453	3.8423	4.6392	5.4362	6.2332	7.0301	7.8271	8.6241	9.4210
5	-6.1946	-4.9484	-3.7023	-2.4561	-1.2099	0.0363	1.2825	2.5286	3.7748	5.0210	6.2672	7.5134	8.7595	10.0057	11.2519	12.4981	13.7443
6	-9.9120	-8.1122	-6.3123	-4.5124	-2.7126	-0.9127	0.8871	2.6870	4.4868	6.2867	8.0866	9.8864	11.6863	13.4861	15.2860	17.0859	18.8857
7	-14.4624	-12.0043	-9.5461	-7.0880	-4.6299	-2.1718	0.2864	2.7445	5.2026	7.6607	10.1189	12.5770	15.0351	17.4933	19.9514	22.4095	24.8676
8	-19.8122	-16.5960	-13.3799	-10.1637	-6.9475	-3.7313	-0.5151	2.7010	5.9172	9.1334	12.3496	15.5658	18.7820	21.9981	25.2143	28.4305	31.6467
9	-25.9594	-21.8858	-17.8122	-13.7385	-9.6649	-5.5913	-1.5177	2.5560	6.6296	10.7032	14.7768	18.8504	22.9241	26.9977	31.0713	35.1449	39.2186
10	-32.9071	-27.8761	-22.8452	-17.8142	-12.7832	-7.7523	-2.7213	2.3097	7.3406	12.3716	17.4025	22.4335	27.4645	32.4954	37.5264	42.5574	47.5883
11	-40.6555	-34.5672	-28.4789	-22.3906	-16.3023	-10.2140	-4.1257	1.9626	8.0509	14.1392	20.2275	26.3158	32.4041	38.4924	44.5807	50.6690	56.7573
12	-49.2043	-41.9587	-34.7131	-27.4675	-20.2219	-12.9763	-5.7307	1.5149	8.7606	16.0062	23.2518	30.4974	37.7430	44.9886	52.2342	59.4798	66.7254
13	-58.5532	-50.0504	-41.5475	-33.0446	-24.5417	-16.0388	-7.5360	0.9669	9.4698	17.9727	26.4756	34.9784	43.4813	51.9842	60.4871	68.9899	77.4928
14	-68.7023	-58.8421	-48.9820	-39.1219	-29.2618	-19.4017	-9.5415	0.3186	10.1787	20.0388	29.8989	39.7591	49.6192	59.4793	69.3394	79.1996	89.0597
15	-79.6514	-68.3341	-57.0167	-45.6994	-34.3820	-23.0647	-11.7473	-0.4300	10.8874	22.2047	33.5220	44.8394	56.1567	67.4741	78.7914	90.1088	101.4261
16	-91.4006	-78.5261	-65.6515	-52.7770	-39.9024	-27.0279	-14.1533	-1.2788	11.5958	24.4703	37.3449	50.2194	63.0940	75.9685	88.8431	101.7176	114.5922
17	-103.9499	-89.4181	-74.8864	-60.3546	-45.8229	-31.2912	-16.7594	-2.2277	12.3040	26.8358	41.3675	55.8993	70.4310	84.9627	99.4945	114.0262	128.5579
18	-117.2992	-101.0102	-84.7213	-68.4324	-52.1435	-35.8546	-19.5657	-3.2768	13.0122	29.3011	45.5900	61.8789	78.1678	94.4568	110.7457	127.0346	143.3235
19	-131.4485	-113.3024	-95.1563	-77.0102	-58.8642	-40.7181	-22.5720	-4.4259	13.7202	31.8663	50.0123	68.1584	86.3045	104.4506	122.5967	140.7428	158.8888
20	-146.3978	-126.2946	-106.1914	-86.0881	-65.9849	-45.8816	-25.7784	-5.6752	14.4281	34.5313	54.6346	74.7378	94.8410	114.9443	135.0475	155.1508	175.2540

F_t,min for -0.8<Q₃<0.0

(v* = 0.4)

-0.8 < Q₃ < 0



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Xa

F_t,min for 0<Q₃<0.8



Xa



Ft,max for -0.8<Q3<0.0



F_t,max for 0<Q₃<0.8





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 $\mathsf{F}_{\mathsf{q}}=\mathsf{F}_{\mathsf{t}}(\mathsf{X}_{\mathsf{a}})$

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ANNEX P — TABULAR AND GRAPHICAL REPRESENTATION OF COEFFICIENT $F_m(x)$

Annex P provides for $1 \le X_a \le 20$ and $-0.8 \le Q_3 \le +0.8$:

- values and graphs of $F_m(x)$ for $0 \le x \le X_a$
- values and graphs of the maximum of $F_m(x)$: F_m
- location of the maximum of F_m(x): x_{max}



8F_m(x) for X_a=1,3,5,7,10,20

								F _m (x) for	X _a = 1, 3 , 5 , 7	7,10,20	and Q ₃ =	-0.8 , 0.0 , +	0.8	(v*=0.4)		
	X	=1			X _a =3			X _a =5			X _a =7			X _a =10		X _a = 20
x	Q ₃ =-0.8	Q ₃ =0	Q ₃ =+0.8	x	Q ₃ =-0.8 Q ₃ =0	Q ₃ =+0.8	x	Q3=-0.8 Q	3=0 Q3=+0.8	x	Q ₃ =-0.8 Q ₃	=0 Q ₃ =+0.8	x	Q3=-0.8 Q3=0 Q3=+0.8	X	Q3=-0.8 Q3=0 Q3=+0.8
0.02	-0.18358390	0.20936490	0.60231360	0.06	-0.02616935 0.0861686	2 0.19850660	0.10	0.05071213 -0.00	097628 -0.0526646	9 0.14	0.01195184 -0.003	43116 -0.01881416	0.20	-0.00243681 0.00013826 0.002713	34 0.40	-0.00000287 0.00000019 0.00000324
0.04	-0.18384860	0.20911810	0.60208480	0.12	-0.02675549 0.0862374	3 0.19923040	0.20	0.05044843 -0.00	082157 -0.0520915	8 0.28	0.01217480 -0.003	42389 -0.01902258	0.40	-0.00241136 0.00012682 0.002664	99 0.80	-0.00000305 0.00000017 0.00000339
0.06	-0.18428980	0.20870680	0.60170350	0.18	-0.02773213 0.0863512	1 0.20043460	0.30	0.05000487 -0.00	056365 -0.0511321	7 0.42	0.01254269 -0.003	41072 -0.01936414	0.60	-0.00236581 0.00010756 0.002580	92 1.20	-0.00000329 0.00000015 0.00000359
0.08	-0.18490740	0.20813090	0.60116920	0.24	-0.02909886 0.0865086	1 0.20211610	0.40	0.04937534 -0.00	020240 -0.0497801	4 0.56	0.01304992 -0.003	39006 -0.01983005	0.80	-0.00229549 0.00008024 0.002455	98 1.60	-0.00000349 0.00000011 0.00000371
0.10	-0.18570140	0.20739040	0.60048220	0.30	-0.03085506 0.0867077	6 0.20427060	0.50	0.04855138 0.00	026229 -0.0480268	0 0.70	0.01368848 -0.003	35970 -0.02040788	1.00	-0.00219397 0.00004455 0.002283	06 2.00	-0.00000353 0.00000004 0.00000362
0.12	-0.18667160	0.20648520	0.59964200	0.36	-0.03299997 0.0869462	0 0.20689240	0.60	0.04752215 0.00	083053 -0.0458611	0 0.84	0.01444788 -0.003	31679 -0.02108145	1.20	-0.00205305 0.00000011 0.002053	28 2.40	-0.00000322 -0.00000006 0.00000311
0.14	-0.18781810	0.20541520	0.59864840	0.42	-0.03553258 0.0872210	0 0.20997460	0.70	0.04627452 0.00	0150239 -0.0432697	3 0.98	0.01531492 -0.003	25788 -0.02183068	1.40	-0.00186298 -0.00005342 0.001756	14 2.80	-0.00000234 -0.00000019 0.00000196
0.16	-0.18914080	0.20418020	0.59750120	0.48	-0.03845168 0.0875286	2 0.21350890	0.80	0.04479314 0.00	0227788 -0.0402373	8 1.12	0.01627357 -0.003	0.02263138	1.60	-0.00161255 -0.00011637 0.001379	80 3.20	-0.00000063 -0.0000038 -0.00000012
0.18	-0.19063940	0.20278030	0.59620000	0.54	-0.04175581 0.0878650	0 0.21748580	0.90	0.04306055 0.00	0315686 -0.0367468	3 1.26	0.01730466 -0.003	07523 -0.02345513	1.80	-0.00128933 -0.00018895 0.000911	42 3.60	0.00000219 -0.00000060 -0.00000340
0.20	-0.19231400	0.20121520	0.59474440	0.60	-0.04544324 0.0882255	1 0.22189430	1.00	0.04105721 0.00	0413903 -0.0327791	<mark>5</mark> 1.40	0.01838573 -0.002	94164 -0.02426902	2.00	-0.00087998 -0.00027119 0.000337	59 4.00	0.00000638 -0.00000087 -0.00000811
0.22	-0.19416430	0.19948480	0.59313400	0.66	-0.04951193 0.0886049	9 0.22672190	1.10	0.03876168 0.00	522384 -0.0283140	0 1.54	0.01949068 -0.002	0.02503546	2.20	-0.00037059 -0.00036287 -0.000355	16 4.40	0.00001210 -0.00000115 -0.00001440
0.24	-0.19619030	0.19758900	0.59136810	0.72	-0.05395950 0.0889977	4 0.23195500	1.20	0.03615077 0.00	641046 -0.0233298	5 1.68	0.02058955 -0.002	56123 -0.02571201	2.40	0.00025279 -0.00046343 -0.001179	65 4.80	0.00001938 -0.00000140 -0.00002219
0.26	-0.19839170	0.19552740	0.58944650	0.78	-0.05878322 0.0893974	3 0.23757810	1.30	0.03319965 0.00	0769765 -0.0178043	5 1.82	0.02164825 -0.002	30147 -0.02625119	2.60	0.00100361 -0.00057184 -0.002147	29 5.20	0.00002795 -0.00000158 -0.00003111
0.28	-0.20076830	0.19330010	0.58736840	0.84	-0.06397994 0.0897972	4 0.24357440	1.40	0.02988207 0.00	908375 -0.0117145	7 1.96	0.02262828 -0.001	98603 -0.02660033	2.80	0.00189401 -0.00068655 -0.003267	10 5.60	0.00003712 -0.00000159 -0.00004031
0.30	-0.20332000	0.19090670	0.58513340	0.90	-0.06954604 0.0901897	5 0.24992550	1.50	0.02617054 0.01	056653 -0.0050374	<mark>9 2.10</mark>	0.02348648 -0.001	60751 -0.02670149	3.00	0.00293403 -0.00080530 -0.004544	64 <u>6.00</u>	0.00004564 -0.00000133 -0.00004829
0.32	-0.20604660	0.18834690	0.58274040	0.96	-0.07547744 0.0905669	8 0.25661140	1.60	0.02203660 0.01	214312 0.0022496	4 2.24	0.02417482 -0.001	15829 -0.02649139	3.20	0.00413047 -0.00092508 -0.005980	62 6.40	0.00005150 -0.00000064 -0.00005278
0.34	-0.20894780	0.18562070	0.58018920	1.02	-0.08176948 0.0909203	7 0.26361020	1.70	0.01745103 0.01	380992 0.0101688	1 2.38	0.02464022 -0.000	63063 -0.02590148	3.40	0.00548573 -0.00104190 -0.007569	54 6.80	0.00005182 0.0000062 -0.00005058
0.36	-0.21202330	0.18272770	0.57747860	1.08	-0.08841692 0.0912407	9 0.27089850	1.80	0.01238420 0.01	556249 0.0187407	8 2.52	0.02482442 -0.000	01682 -0.02485806	3.60	0.00699638 -0.00115077 -0.009297	93 7.20	0.00004276 0.00000261 -0.00003754
0.38	-0.21527300	0.17966760	0.57460830	1.14	-0.09541386 0.0915185	3 0.27845090	1.90	0.00680636 0.01	739541 0.0279844	6 2.66	0.02466395 0.000	69070 -0.02328254	3.80	0.00865158 -0.00124551 -0.011142	61 7.60	0.00001961 0.00000546 -0.00000870
0.40	-0.21869650	0.17644020	0.57157700	1.20	-0.10275370 0.0917432	9 0.28624030	2.00	0.00068801 0.01	930221 0.0379164	0 2.80	0.02409015 0.001	49915 -0.02109184	4.00	0.01043131 -0.00131868 -0.013068	68 8.00	-0.00002294 0.00000922 0.00004138
0.42	-0.22229350	0.17304520	0.56838400	1.26	-0.11042900 0.0919042	2 0.29423750	2.10	-0.00599970 0.02	0.0485500	9 2.94	0.02302931 0.002	41520 -0.01819892	4.20	0.01230449 -0.00136149 -0.015027	47 <u>8.40</u>	-0.00009023 0.00001381 0.00011785
0.44	-0.22606380	0.16948230	0.56502840	1.32	-0.11843160 0.0919898	4 0.30241130	2.20	-0.01328455 0.02	0.0598952	5 3.08	0.02140298 0.003	44472 -0.01451354	4.40	0.01422692 -0.00136373 -0.016954	38 8.80	-0.00018654 0.00001892 0.00022439
0.46	-0.23000680	0.16575110	0.56150910	1.38	-0.12675240 0.0919881	1 0.31072860	2.30	-0.02119274 0.02	0.0719571	6 3.22	0.01912836 0.004	59258 -0.00994320	4.60	0.01613920 -0.00131378 -0.018766	77 <u>9.20</u>	-0.00031361 0.00002395 0.00036151
0.48	-0.23412250	0.16185140	0.55782530	1.44	-0.13538120 0.0918864	2 0.31915400	2.40	-0.02974840 0.02	0.0847358	0 3.36	0.01611890 0.005	86228 -0.00439435	4.80	0.01796455 -0.00119868 -0.020361	91 9.60	-0.00046844 0.00002786 0.00052415
0.50	-0.23841020	0.15778280	0.55397570	1.50	-0.14430690 0.0916715	6 0.32765000	2.50	-0.03897302 0.02	962602 0.0982250	<u>6 3.50</u>	0.01228512 0.007	25566 0.00222620	5.00	0.01960682 -0.00100418 -0.021615	18 10.00	-0.00064056 0.00002911 0.00069879
0.52	-0.24286970	0.15354490	0.54995940	1.56	-0.15351720 0.0913297	2 0.33617670	2.60	-0.04888477 0.03	0.1124117	0 3.64	0.00753562 0.008	0.01000950	5.20	0.02094852 -0.00071502 -0.022378	57 <u>10.40</u>	-0.00080874 0.00002562 0.00085998
0.54	-0.24750050	0.14913730	0.54577520	1.62	-0.16299870 0.0908465	6 0.34469180	2.70	-0.05949798 0.03	3388843 0.1272748	0 3.78	0.00177833 0.010	41036 0.01904238	5.40	0.02184935 -0.00031520 -0.022479	75 10.80	-0.00093753 0.00001471 0.00096695
0.56	-0.25230220	0.14455980	0.54142190	1.68	-0.17273650 0.0902071	4 0.35315080	2.80	-0.07082227 0.03	0.1427843	0 3.92	-0.00507792 0.012	16360 0.02940512	5.60	0.02214497 0.00021158 -0.021721	81 11.20	-0.00097407 -0.00000672 0.00096064
0.58	-0.25727440	0.13981190	0.53689830	1.74	-0.18271470 0.0893959	2 0.36150650	2.90	-0.08286189 0.03	801908 0.1589001	0 4.06	-0.01312157 0.014	02351 0.04116859	5.80	0.02164662 0.00088132 -0.019883	98 11.60	-0.00084604 -0.00004189 0.00076226
0.60	-0.26241660	0.13489320	0.53220300	1.80	-0.19291550 0.0883968	8 0.36970930	3.00	-0.09561487 0.03	8997814 0.1755711	0 4.20	-0.02243614 0.015	97749 0.05439113	6.00	0.02014153 0.00170900 -0.016723	52 12.00	-0.00046205 -0.00009364 0.00027478
0.62	-0.26772820	0.12980340	0.52733490	1.86	-0.20331990 0.0871933	9 0.37770670	3.10	-0.10907220 0.04	0.1927343	0 4.34	-0.03309772 0.018	00864 0.06911500	6.20	0.01739438 0.00270762 -0.011979	15 12.40	0.00028394 -0.00016356 -0.00061106
0.64	-0.27320870	0.12454200	0.52229270	1.92	-0.21390720 0.0857682	7 0.38544380	3.20	-0.12321680 0.04	354820 0.2103132	0 4.48	-0.0451/204 0.020	09514 0.08536233	6.40	0.01315036 0.00388706 -0.005376	23 12.80	0.00150222 -0.00025078 -0.00200378
0.66	-0.27885770	0.11910860	0.51707480	1.98	-0.22465500 0.0841039	0 0.39286280	3.30	-0.13802260 0.04	0.2282165	0 4.62	-0.05871119 0.022	20979 0.10313080	6.60	0.00713974 0.00525278 0.003365	82 13.20	0.00328846 -0.00035039 -0.00398925
0.08	-0.28467440	0.11350270	0.51167990	2.04	-0.23553890 0.0821820	4 0.39990300	3.40	-0.15345380 0.04	0.2463377	0 4.76	-0.07374990 0.024	0.12238860	0.80	-0.00091539 0.00680417 0.014523	73 13.60	0.00569418 -0.00045152 -0.00659722
0.70	-0.29065840	0.10/72410	0.50010660	2.10	-0.24003290 0.0799840	0.40650100	3.50	-0.10940340 0.04	0.2645532	0 5.04	-0.09030136 0.026	0.14306940	7.00	-0.01129201 0.00853281 0.028357	16 14.00	0.000000400 -0.00053518 -0.00975521
0.72	-0.29080910	0.00564647	0.00030330	2.10	-0.25700890 0.0774907	0.41259040	3.00	-0.18599230 0.04	030432 0.2827213	0 5.04	-0.10835280 0.028	0.10006660	7.20	-0.02425436 0.01042040 0.045095	14.40	0.01208474 -0.00057224 -0.01322922
0.74	-0.30312500	0.09034665	0.49920000	2.22	-0.20073030 0.0740823	0.41810170	3.70	-0.20290820 0.04		0 5.18	-0.12700010 0.030	0.00000 0.010022700	7.40	0.05993651 0.01453633 0.004912	07 14.80	0.01001022 -0.00002192 -0.01000400
0.78	-0.30900760	0.08384005	0.48100950	2.28	-0.279883300 0.0715395	6 0.42290260	3.80	-0.22030430 0.04	037301 0.3162503	0 5.32	-0.14874310 0.031	00000 0.21234480	7.00	-0.03003031 0.01433028 0.087909	70 15.20	0.01029042 -0.00033153 -0.01895948
0.70	-0.31023400	0.06267223	0.40199000	2.34	0.20200460 0.0644667	0.42709730	3.90	-0.23769630 0.04	0.3332202	0 5.60	-0.17067690 0.033	13792 0.23713470	9.00		70 15.00	0.01342719 0.00000138 -0.01930402
0.80	-0.32300440	0.07022273	0.47550990	2.40	-0.31308220 0.0508051	4 0.43297250	4.00	-0.22336580 0.04	0.3013620	0 5.00	-0.19409030 0.034	64086 0.28745090	8.20	-0.13403100 0.02061670 0.175265	50 16.00	0.01064304 0.00072466 -0.01003326
0.02	0.33003730	0.06339774	0.400000040	2.40	0.0000220 0.0000000	4 0.43207230	4.10	0.20004570 0.04	0.3004034	0 5.99	0.24280050 0.034	04000 0.20743030	0.20	0.16501380 0.02001073 0.173203	20 16.90	0.00225002 0.00208572 0.00020403
0.04	-0.33/1/3/0	0.05521042	0.40190730	2.52	-0.32393010 0.0502030	2 0.43476440	4.20	-0.29094570 0.04	1205725 0.3002110	0 0.00	-0.24200300 0.034	03068 0.33554020	9.60	0.10301300 0.02222224 0.209458	70 17.00	0.00323992 0.00300373 0.00943139
0.88	-0.35102010	0.04786516	0.44765040	2.50	-0.34506110 0.0300740	0.43402650	4.00	-0.32491280 0.03	876032 0.4024234	0 6.16	-0.2020000 0.033	48701 0 35726400	8.80	0.23339890 0.02390770 0.243124	30 17.60	0.02000107 0.00401720 0.03010043
0.00	-0.35054600	0.04033350	0.44021440	2.04	-0.35523470 0.0394067	3 0.43204840	4.40	0.02491200 0.03	0.4024334	0 6.20	-0.31619470 -0.032	13407 0.33720490	0.00	0.26015610 0.02350162 0.246220	20 18.00	0.00120100 0.00002311 0.01492000
0.90	-0.30904090	0.04033359	0.44021410	2.70	-0.35523470 0.0384067	2 0.43204810	4.50	-0.34080000 0.03	073473 0 4152220	0 6.44	-0.31010470 0.030	75002 0 39222820	9.00		80 18.40	0.16870540 0.01072779 0.40016400
0.02	-0.37525920	0.02473663	0.42473160	2.70	-0.37453870 0.0247152	4 0.42396010	4.00	-0.36955910 0.02	2386778 0.4172046	0 6.58	-0.35925150 0.020	10172 0.40363500	9.40	0.33732350 0.01043421 0.276103	18.20	0.23800230 0.01167708 0.26125640
0.94	-0.38334000	0.01667024	0.41660060	2.02	-0.38355400 0.0170557	3 0.41766540	4.80	-0.38170490 0.01	700389 0 4157127	0 6.72	-0.37602320 -0.046	31409 0.40955140	0.40	-0.36601350 0.01502885 0.296074	20 10.00	0.20000200 0.01107708 0.20130040
0.00	-0 30150750	0.00842497	0.40844740	2.00	-0.30206200 0.0170337	3 0.40971200	4.00	-0.39196810 0.00	007170 0.4101115	0 6.86	-0.39084540 0.008	0.40878010	9.00	0.38790400 0.00865981 0.405223	60 19.60	0.37015030 0.00749342 0.38513720
1.00	-0.40000000	0.00000000	0.40000000	3.00	-0.40000000 0.0000240	0 4000000	5.00	-0.40000000 0.00	000000 0.400000	0 7.00	-0.40000000 -0.000		10.00	-0.4000000 0.0000000 0.400000	20.00	-0.40000000 0.0000000 0.40000000
1.00	-0.40000000	0.00000000	0.40000000	3.00	-0.4000000 0.000000	0.40000000	0.00	-0.4000000 0.00	0.400000	0.00	-0.4000000 0.000	0.4000000	10.00	-0.4000000 0.0000000 0.400000	20.00	-0.40000000 0.0000000 0.40000000

$$X_a = 1$$
 (v* = 0.4)



0 < x <= X_a

$$X_a = 3$$
 (v* = 0.4)



0 < x <= X_a

 $X_a = 5$ (v* = 0.4)



0 < x <= X_a

$$X_a = 7$$
 (v* = 0.4)



 $X_a = 10$

$$X_a = 10$$
 (v* = 0.4)



 $X_a = 20$ (v* = 0.4)



F_m(x)

$F_m=Max|F_m(x)|$ vs X_a and Q_3

			if x(max))=X _a : F _m (X	a)= Q ₃ /2			F _m =	Max F _m ([x)			if x(max)	=X _a : F _m (X _a	a)= Q₃/2		
X_\Q2	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
1	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1111	0.1602	0.2094	0.2585	0.3076	0.3567	0.4058	0.4550	0.5041	0.5532	0.6023
2	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.1329	0.1710	0.2090	0.2471	0.2851	0.3232	0.3613	0.4009	0.4425	0.4855
3	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0721	0.0920	0.1236	0.1616	0.2035	0.2476	0.2931	0.3396	0.3870	0.4348
4	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0616	0.0981	0.1400	0.1846	0.2309	0.2782	0.3263	0.3746	0.4234
5	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0490	0.0866	0.1297	0.1755	0.2225	0.2707	0.3189	0.3681	0.4173
6	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0407	0.0789	0.1229	0.1694	0.2171	0.2656	0.3141	0.3635	0.4130
7	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0346	0.0734	0.1181	0.1652	0.2129	0.2621	0.3112	0.3604	0.4096
8	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0301	0.0694	0.1148	0.1620	0.2107	0.2594	0.3081	0.3579	0.4078
9	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0266	0.0662	0.1119	0.1600	0.2082	0.2574	0.3071	0.3568	0.4066
10	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0239	0.0640	0.1103	0.1579	0.2069	0.2565	0.3061	0.3557	0.4052
11	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0216	0.0620	0.1086	0.1567	0.2061	0.2555	0.3049	0.3543	0.4037
12	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0198	0.0605	0.1069	0.1561	0.2053	0.2545	0.3037	0.3529	0.4021
13	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0181	0.0594	0.1063	0.1553	0.2043	0.2533	0.3023	0.3514	0.4004
14	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0169	0.0583	0.1058	0.1546	0.2033	0.2521	0.3009	0.3500	0.4000
15	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0157	0.0571	0.1052	0.1537	0.2023	0.2509	0.3000	0.3500	0.4000
16	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0146	0.0563	0.1046	0.1529	0.2012	0.2500	0.3000	0.3500	0.4000
17	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0137	0.0559	0.1039	0.1520	0.2000	0.2500	0.3000	0.3500	0.4000
18	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0130	0.0555	0.1033	0.1511	0.2000	0.2500	0.3000	0.3500	0.4000
19	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0123	0.0551	0.1026	0.1501	0.2000	0.2500	0.3000	0.3500	0.4000
20	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0117	0.0547	0.1019	0.1500	0.2000	0.2500	0.3000	0.3500	0.4000
				x(max)=X _a										x(max)=X _a			
X _a \Q ₃	-0.8	-0.7	-0.6	<mark>x(max)=X</mark> a -0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	<mark>x(max)=X_a</mark> 0.5	0.6	0.7	0.8
X _a \Q ₃ 1	-0.8 1.0000	-0.7 1.0000	-0.6 1.0000	<mark>x(max)=X</mark> a -0.5 1.0000	-0.4 1.0000	-0.3 1.0000	-0.2 0.0200	-0.1 0.0200	0 0.0200	0.1 0.0200	0.2 0.0200	0.3 0.0200	0.4 0.0200	x(max)=X _a 0.5 0.0200	0.6 0.0200	0.7 0.0200	0.8 0.0200
X _a \Q ₃ 1 2	-0.8 1.0000 2.0000	-0.7 1.0000 2.0000	-0.6 1.0000 2.0000	<mark>x(max)=X_a -0.5 1.0000 2.0000</mark>	-0.4 1.0000 2.0000	-0.3 1.0000 2.0000	-0.2 0.0200 2.0000	-0.1 0.0200 0.0400	0 0.0200 0.0400	0.1 0.0200 0.0400	0.2 0.0200 0.0400	0.3 0.0200 0.0400	0.4 0.0200 0.0400	x(max)=X _a 0.5 0.0200 0.2800	0.6 0.0200 0.6800	0.7 0.0200 0.8800	0.8 0.0200 1.0000
X _a \Q ₃ 1 2 3	-0.8 1.0000 2.0000 3.0000	-0.7 1.0000 2.0000 3.0000	-0.6 1.0000 2.0000 3.0000	x(max)=X _a -0.5 1.0000 2.0000 3.0000	-0.4 1.0000 2.0000 3.0000	-0.3 1.0000 2.0000 3.0000	-0.2 0.0200 2.0000 3.0000	-0.1 0.0200 0.0400 0.0600	0 0.0200 0.0400 1.3200	0.1 0.0200 0.0400 1.8000	0.2 0.0200 0.0400 2.1000	0.3 0.0200 0.0400 2.2200	0.4 0.0200 0.0400 2.3400	x(max)=X _a 0.5 0.0200 0.2800 2.4000	0.6 0.0200 0.6800 2.4600	0.7 0.0200 0.8800 2.5200	0.8 0.0200 1.0000 2.5800
X _a \Q ₃ 1 2 3 4	-0.8 1.0000 2.0000 3.0000 4.0000	-0.7 1.0000 2.0000 3.0000 4.0000	-0.6 1.0000 2.0000 3.0000 4.0000	x(max)=X _a -0.5 1.0000 2.0000 3.0000 4.0000	-0.4 1.0000 2.0000 3.0000 4.0000	-0.3 1.0000 2.0000 3.0000 4.0000	-0.2 0.0200 2.0000 3.0000 4.0000	-0.1 0.0200 0.0400 0.0600 4.0000	0 0.0200 0.0400 1.3200 2.7200	0.1 0.0200 0.0400 1.8000 3.1200	0.2 0.0200 0.0400 2.1000 3.2800	0.3 0.0200 0.0400 2.2200 3.4400	0.4 0.0200 0.0400 2.3400 3.5200	x(max)=X _a 0.5 0.0200 0.2800 2.4000 3.5200	0.6 0.0200 0.6800 2.4600 3.6000	0.7 0.0200 0.8800 2.5200 3.6000	0.8 0.0200 1.0000 2.5800 3.6800
X _a \Q ₃ 1 2 3 4 5	-0.8 1.0000 2.0000 3.0000 4.0000 5.0000	-0.7 1.0000 2.0000 3.0000 4.0000 5.0000	-0.6 1.0000 2.0000 3.0000 4.0000 5.0000	x(max)=X _a -0.5 1.0000 2.0000 3.0000 4.0000 5.0000	-0.4 1.0000 2.0000 3.0000 4.0000 5.0000	-0.3 1.0000 2.0000 3.0000 4.0000 5.0000	-0.2 0.0200 2.0000 3.0000 4.0000 5.0000	-0.1 0.0200 0.0400 0.0600 4.0000 5.0000	0 0.0200 0.0400 1.3200 2.7200 3.8000	0.1 0.0200 0.0400 1.8000 3.1200 4.2000	0.2 0.0200 0.0400 2.1000 3.2800 4.4000	0.3 0.0200 0.0400 2.2200 3.4400 4.5000	0.4 0.0200 0.0400 2.3400 3.5200 4.6000	x(max)=X _a 0.5 0.0200 0.2800 2.4000 3.5200 4.6000	0.6 0.0200 0.6800 2.4600 3.6000 4.7000	0.7 0.0200 0.8800 2.5200 3.6000 4.7000	0.8 0.0200 1.0000 2.5800 3.6800 4.7000
X _a \Q ₃ 1 2 3 4 5 6	-0.8 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000	-0.7 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000	-0.6 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000	x(max)=X _a -0.5 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000	-0.4 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000	-0.3 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000	-0.2 0.0200 2.0000 3.0000 4.0000 5.0000 6.0000	-0.1 0.0200 0.0400 0.0600 4.0000 5.0000 6.0000	0 0.0200 0.0400 1.3200 2.7200 3.8000 4.8000	0.1 0.0200 0.0400 1.8000 3.1200 4.2000 5.1600	0.2 0.0200 0.0400 2.1000 3.2800 4.4000 5.4000	0.3 0.0200 0.0400 2.2200 3.4400 4.5000 5.5200	0.4 0.0200 0.0400 2.3400 3.5200 4.6000 5.6400	x(max)=X _a 0.5 0.0200 0.2800 2.4000 3.5200 4.6000 5.6400	0.6 0.0200 0.6800 2.4600 3.6000 4.7000 5.6400	0.7 0.0200 0.8800 2.5200 3.6000 4.7000 5.7600	0.8 0.0200 1.0000 2.5800 3.6800 4.7000 5.7600
X _a \Q ₃ 1 2 3 4 5 6 7	-0.8 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000	-0.7 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000	-0.6 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000	x(max)=X _a -0.5 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000	-0.4 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000	-0.3 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000	-0.2 0.0200 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000	-0.1 0.0200 0.0400 4.0000 5.0000 6.0000 7.0000	0 0.0200 0.0400 1.3200 2.7200 3.8000 4.8000 5.7400	0.1 0.0200 0.0400 1.8000 3.1200 4.2000 5.1600 6.3000	0.2 0.0200 0.0400 2.1000 3.2800 4.4000 5.4000 6.4400	0.3 0.0200 0.0400 2.2200 3.4400 4.5000 5.5200 6.5800	0.4 0.0200 0.0400 2.3400 3.5200 4.6000 5.6400 6.7200	x(max)=X _a 0.5 0.0200 0.2800 2.4000 3.5200 4.6000 5.6400 6.7200	0.6 0.0200 0.6800 2.4600 3.6000 4.7000 5.6400 6.7200	0.7 0.0200 0.8800 2.5200 3.6000 4.7000 5.7600 6.7200	0.8 0.0200 1.0000 2.5800 3.6800 4.7000 5.7600 6.7200
X _a \Q ₃ 1 2 3 4 5 6 7 8	-0.8 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000	-0.7 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000	-0.6 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000	x(max)=X _a -0.5 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000	-0.4 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000	-0.3 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000	-0.2 0.0200 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000	-0.1 0.0200 0.0400 4.0000 5.0000 6.0000 7.0000 8.0000	0 0.0200 0.0400 1.3200 2.7200 3.8000 4.8000 5.7400 6.8800	0.1 0.0200 0.0400 1.8000 3.1200 4.2000 5.1600 6.3000 7.3600	0.2 0.0200 2.1000 3.2800 4.4000 5.4000 6.4400 7.5200	0.3 0.0200 0.0400 2.2200 3.4400 4.5000 5.5200 6.5800 7.6800	0.4 0.0200 2.3400 3.5200 4.6000 5.6400 6.7200 7.6800	x(max)=X _a 0.5 0.0200 0.2800 2.4000 3.5200 4.6000 5.6400 6.7200 7.6800	0.6 0.0200 0.6800 2.4600 3.6000 4.7000 5.6400 6.7200 7.6800	0.7 0.0200 0.8800 2.5200 3.6000 4.7000 5.7600 6.7200 7.8400	0.8 0.0200 1.0000 2.5800 3.6800 4.7000 5.7600 6.7200 7.8400
X _a \Q ₃ 1 2 3 4 5 6 7 8 9 9	-0.8 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000	-0.7 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000	-0.6 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000	x(max)=X _a -0.5 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000	-0.4 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000	-0.3 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000	-0.2 0.0200 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000	-0.1 0.0200 0.0400 0.0600 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000	0 0.0200 1.3200 2.7200 3.8000 4.8000 5.7400 6.8800 7.7400	0.1 0.0200 0.0400 1.8000 3.1200 4.2000 5.1600 6.3000 7.3600 8.2800	0.2 0.0200 2.1000 3.2800 4.4000 5.4000 6.4400 7.5200 8.6400	0.3 0.0200 2.2200 3.4400 4.5000 5.5200 6.5800 7.6800 8.6400	0.4 0.0200 0.0400 2.3400 3.5200 4.6000 5.6400 6.7200 7.6800 8.6400 0.000	x(max)=X _a 0.5 0.2800 2.4000 3.5200 4.6000 5.6400 6.7200 7.6800 8.8200	0.6 0.0200 0.6800 2.4600 3.6000 4.7000 5.6400 6.7200 7.6800 8.8200	0.7 0.0200 0.8800 2.5200 3.6000 4.7000 5.7600 6.7200 7.8400 8.8200	0.8 0.0200 1.0000 2.5800 3.6800 4.7000 5.7600 6.7200 7.8400 8.8200
X _a \Q ₃ 1 2 3 4 5 6 7 8 9 9 10	-0.8 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000	-0.7 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000	-0.6 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000	x(max)=X _a -0.5 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000	-0.4 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 10.0000	-0.3 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 14.0000	-0.2 0.0200 2.0000 3.0000 4.0000 5.0000 7.0000 8.0000 9.0000 10.0000	-0.1 0.0200 0.0400 0.0600 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000	0 0.0200 1.3200 2.7200 3.8000 5.7400 6.8800 7.7400 8.8000	0.1 0.0200 0.0400 1.8000 3.1200 4.2000 5.1600 6.3000 7.3600 8.2800 9.4000	0.2 0.0200 2.1000 3.2800 4.4000 5.4000 6.4400 7.5200 8.6400 9.6000	0.3 0.0200 0.0400 2.2200 3.4400 4.5000 5.5200 6.5800 7.6800 8.6400 9.6000	0.4 0.0200 0.0400 2.3400 3.5200 4.6000 6.7200 7.6800 8.6400 9.8000	x(max)=X _a 0.5 0.0200 0.2800 2.4000 3.5200 4.6000 5.6400 6.7200 7.6800 8.8200 9.8000	0.6 0.0200 2.4600 3.6000 4.7000 5.6400 6.7200 7.6800 8.8200 9.8000	0.7 0.0200 0.8800 2.5200 3.6000 4.7000 5.7600 6.7200 7.8400 8.8200 9.8000	0.8 0.0200 1.0000 2.5800 3.6800 4.7000 5.7600 6.7200 7.8400 8.8200 9.8000
X _a \Q ₃ 1 2 3 4 5 6 7 7 8 9 9 10 11	-0.8 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 10.0000	-0.7 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000	-0.6 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 10.0000	x(max)=X _a -0.5 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000	-0.4 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 10.0000	-0.3 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 14.0000	-0.2 0.0200 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000	-0.1 0.0200 0.0400 0.0600 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000	0 0.0200 1.3200 2.7200 3.8000 4.8000 5.7400 6.8800 7.7400 8.8000 9.9000	0.1 0.0200 0.0400 1.8000 3.1200 4.2000 5.1600 5.1600 5.1600 7.3600 8.2800 9.4000 10.3400	0.2 0.0200 2.1000 3.2800 4.4000 5.4000 6.4400 7.5200 8.6400 9.6000 10.5600	0.3 0.0200 2.2200 3.4400 4.5000 5.5200 6.5800 7.6800 8.6400 9.6000 10.7800	0.4 0.0200 2.3400 3.5200 4.6000 5.6400 5.6400 7.6800 8.6400 9.8000 10.7800	x(max)=X _a 0.5 0.0200 0.2800 2.4000 3.5200 4.6000 5.6400 6.7200 7.6800 8.8200 9.8000 10.7800	0.6 0.0200 2.4600 3.6000 4.7000 5.6400 5.6400 7.6800 8.8200 9.8000 10.7800	0.7 0.0200 0.8800 2.5200 3.6000 4.7000 5.7600 6.7200 7.8400 8.8200 9.8000 10.7800	0.8 0.0200 1.0000 2.5800 3.6800 4.7000 5.7600 6.7200 7.8400 8.8200 9.8000 10.7800
X _a \Q ₃ 1 2 3 4 5 6 7 8 9 10 11 12 12	-0.8 1.0000 2.0000 3.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000	-0.7 1.0000 2.0000 3.0000 4.0000 5.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 12.0000	-0.6 1.0000 2.0000 3.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000	x(max)=Xa -0.5 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 11.0000	-0.4 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 12.0000	-0.3 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 12.0000	-0.2 0.0200 2.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000	-0.1 0.0200 0.0400 4.0000 5.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000	0 0.0200 1.3200 2.7200 3.8000 4.8000 5.7400 6.8800 7.7400 8.8000 9.9000 10.8000	0.1 0.0200 0.0400 1.8000 3.1200 4.2000 5.1600 6.3000 7.3600 8.2800 9.4000 10.3400 11.5200	0.2 0.0200 2.1000 3.2800 4.4000 5.4000 6.4400 7.5200 8.6400 9.6000 10.5600 11.5200	0.3 0.0200 0.0400 2.2200 3.4400 4.5000 5.5200 6.5800 7.6800 7.6800 9.6000 10.7800 11.7600	0.4 0.0200 0.0400 2.3400 3.5200 4.6000 5.6400 6.7200 7.6800 8.6400 9.8000 10.7800 11.7600 4.7400 10.7800 11.7600 10.78000 10.7800 10.7800 10.7800	x(max)=X _a 0.5 0.0200 0.2800 2.4000 3.5200 4.6000 5.6400 6.7200 7.6800 7.6800 9.8000 10.7800 11.7600 11.7600	0.6 0.0200 0.6800 2.4600 3.6000 4.7000 5.6400 6.7200 7.6800 9.8000 10.7800 11.7600	0.7 0.0200 0.8800 2.5200 3.6000 4.7000 5.7600 6.7200 7.8400 8.8200 9.8000 10.7800 11.7600	0.8 0.0200 1.0000 2.5800 3.6800 4.7000 5.7600 6.7200 7.8400 8.8200 9.8000 10.7800 11.7600
X _a \Q ₃ 1 2 3 4 5 6 7 8 9 10 11 12 13 14	-0.8 1.0000 2.0000 3.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000 14.0000	-0.7 1.0000 2.0000 3.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 12.0000 13.0000 14.0000	-0.6 1.0000 2.0000 3.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000 14.0000	x(max)=Xa -0.5 1.0000 2.0000 3.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 11.0000 12.0000 13.0000	-0.4 1.0000 2.0000 3.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 11.0000 12.0000 13.0000 14.0002	-0.3 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000 14.0000	-0.2 0.0200 2.0000 3.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 11.0000 12.0000 13.0000	-0.1 0.0200 0.0400 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000 14.0000	0 0.0200 1.3200 2.7200 3.8000 4.8000 5.7400 6.8800 7.7400 8.8000 9.9000 10.8000 11.9600 14.9000	0.1 0.0200 1.8000 3.1200 4.2000 5.1600 6.3000 7.3600 8.2800 9.4000 10.3400 11.5200 12.4800	0.2 0.0200 0.0400 2.1000 3.2800 4.4000 5.4000 6.4400 7.5200 8.6400 9.6000 10.5600 11.5200 11.5200 11.5200	0.3 0.0200 0.0400 2.2200 3.4400 4.5000 5.5200 6.5800 7.6800 8.6400 9.6000 10.7800 11.7600 11.7600	0.4 0.0200 0.0400 2.3400 3.5200 4.6000 5.6400 6.7200 7.6800 8.6400 9.8000 10.7800 11.7600 12.7400 4.27202	x(max)=X _a 0.5 0.0200 0.2800 2.4000 3.5200 4.6000 5.6400 6.7200 7.6800 0.88200 9.8000 10.7800 11.7600 12.7400 4.27002	0.6 0.0200 0.6800 2.4600 3.6000 4.7000 5.6400 6.7200 7.6800 8.8200 9.8000 10.7800 11.7600 11.7600	0.7 0.0200 0.8800 2.5200 3.6000 4.7000 5.7600 6.7200 7.8400 8.8200 9.8000 10.7800 11.7600 12.7400	0.8 0.0200 1.0000 2.5800 3.6800 4.7000 5.7600 6.7200 7.8400 8.8200 9.8000 10.7800 11.7600 12.7400
X _a \Q ₃ 1 2 3 4 5 6 7 8 9 10 11 12 13 14 45	-0.8 1.0000 2.0000 3.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000 14.00000 14.0000 14.0000 14.0000 14.0000 14.0000 14.0000000 1	-0.7 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 10.0000 11.0000 11.0000 12.0000 13.0000 14.0000	-0.6 1.0000 2.0000 3.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000 14.0000 14.0000	x(max)=Xa -0.5 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000 14.0000	-0.4 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 11.0000 13.0000 14.0000 14.0000	-0.3 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000 14.0000	-0.2 0.0200 2.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 11.0000 112.0000 13.0000	-0.1 0.0200 0.0400 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000 14.0000	0 0.0200 0.0400 1.3200 2.7200 3.8000 4.8000 5.7400 6.8800 7.7400 8.8000 9.9000 10.8000 11.9600 12.8800 12.8800	0.1 0.0200 0.0400 1.8000 3.1200 5.1600 6.3000 7.3600 8.2800 9.4000 10.3400 11.5200 12.4800 13.4400	0.2 0.0200 2.1000 2.1000 3.2800 4.4000 5.4000 6.4400 7.5200 8.6400 9.6000 10.5600 11.5200 11.5200 12.7400 13.7200	0.3 0.0200 0.0400 2.2200 3.4400 4.5000 5.5200 6.5800 7.6800 8.6400 9.6000 10.7800 11.7600 12.7400 13.7200	0.4 0.0200 0.0400 2.3400 3.5200 4.6000 5.6400 6.7200 7.6800 8.6400 9.8000 10.7800 11.7600 12.7400 13.7200 14.7200	x(max)=X _a 0.5 0.0200 0.2800 2.4000 3.5200 4.6000 5.6400 6.7200 7.6800 8.8200 9.8000 10.7800 11.7600 12.7400 13.7200 4.47002	0.6 0.0200 2.4600 3.6000 4.7000 5.6400 6.7200 7.6800 8.8200 10.7800 11.7600 11.7600 12.7400 13.7200	0.7 0.0200 0.8800 2.5200 3.6000 4.7000 5.7600 6.7200 7.8400 8.8200 9.8000 10.7800 11.7600 12.7400 14.0000	0.8 0.0200 1.0000 2.5800 3.6800 4.7000 5.7600 6.7200 7.8400 8.8200 9.8000 10.7800 11.7600 12.7400 12.7400
X _a \Q ₃ 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16	-0.8 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000 14.0000 15.0000	-0.7 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 10.0000 11.0000 11.0000 13.0000 14.0000 14.0000 16.0000	-0.6 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000 14.0000 15.0000	x(max)=Xa -0.5 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 11.0000 11.0000 13.0000 14.0000	-0.4 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 10.0000 11.0000 11.0000 13.0000 14.0000 15.0000	-0.3 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 10.0000 11.0000 11.0000 13.0000 14.0000 14.0000	-0.2 0.0200 2.0000 3.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 11.0000 13.0000 14.0000 14.0000	-0.1 0.0200 0.0400 4.0000 5.0000 6.0000 7.0000 8.0000 10.0000 11.0000 12.0000 13.0000 14.0000 16.0000	0 0.0200 1.3200 2.7200 3.8000 4.8000 5.7400 6.8800 7.7400 8.8000 7.7400 8.8000 10.8000 11.9600 11.9600 12.8800 13.8000	0.1 0.0200 0.0400 1.8000 3.1200 4.2000 5.1600 6.3000 7.3600 8.2800 9.4000 10.3400 11.5200 12.4800 13.4400 14.4000 14.4000	0.2 0.0200 2.1000 3.2800 4.4000 5.4000 6.4400 7.5200 8.6400 9.6600 10.5600 11.5200 11.5200 12.7400 13.7200 14.7000	0.3 0.0200 0.0400 2.2200 3.4400 5.5200 6.5800 7.6800 8.6400 9.6000 10.7800 11.7600 12.7400 13.7200 14.7000	0.4 0.0200 0.0400 2.3400 3.5200 4.6000 5.6400 6.7200 7.6800 8.6400 9.8000 10.7800 11.7600 12.7400 13.7200 14.7000 15.6800	x(max)=X _a 0.5 0.0200 0.2800 2.4000 3.5200 4.6000 5.6400 6.7200 7.6800 8.8200 9.8000 10.7800 11.7600 12.7400 13.7200 14.7000 14.7000	0.6 0.0200 0.6800 2.4600 3.6000 4.7000 5.6400 6.7200 7.6800 8.8200 9.8000 10.7800 11.7600 12.7400 13.7200 15.0000 16.0000	0.7 0.0200 0.8800 2.5200 3.6000 4.7000 5.7600 6.7200 7.8400 8.8200 9.8000 10.7800 11.7600 12.7400 14.0000 15.0000 16.0000	0.8 0.0200 1.0000 2.5800 3.6800 4.7000 5.7600 6.7200 7.8400 8.8200 9.8000 10.7800 11.7600 12.7400 12.7400 14.0000
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X _a \Q ₃ 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 19	-0.8 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 10.0000 11.0000 11.0000 12.0000 14.0000 15.0000 16.0000 17.0000 18.0000	-0.7 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 10.0000 11.0000 11.0000 13.0000 14.0000 15.0000 16.0000 17.0000 18.0000	-0.6 1.0000 2.0000 3.0000 4.0000 5.0000 7.0000 8.0000 9.0000 10.0000 11.0000 11.0000 12.0000 14.0000 15.0000 16.0000 17.0000 18.0000	x(max)=X _a -0.5 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 11.0000 11.0000 13.0000 14.0000 15.0000 16.0000 17.0000	-0.4 1.0000 2.0000 3.0000 4.0000 5.0000 7.0000 8.0000 9.0000 11.0000 11.0000 12.0000 13.0000 14.0000 15.0000 16.0000 18.0000	-0.3 1.0000 2.0000 3.0000 4.0000 5.0000 7.0000 8.0000 9.0000 11.0000 11.0000 13.0000 14.0000 15.0000 16.0000 17.0000 18.0000 19.0000 10.00000 10.0000 10.0000 10.0000 10.0000 10.0000 10.00000 10.	-0.2 0.0200 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 10.0000 11.0000 11.0000 13.0000 14.0000 15.0000 16.0000 18.0000 19.0000	-0.1 0.0200 0.0400 0.0600 4.0000 5.0000 7.0000 8.0000 1.0000 11.0000 11.0000 13.0000 14.0000 15.0000 16.0000 18.0000	0 0.0200 1.3200 2.7200 3.8000 4.8000 5.7400 6.8800 7.7400 8.8000 9.9000 10.8000 11.9600 11.9600 11.8000 11.9600 13.8000	0.1 0.0200 0.0400 1.8000 3.1200 4.2000 5.1600 6.3000 7.3600 8.2800 9.4000 10.3400 11.5200 12.4800 13.4400 15.6800 16.6600 17.6400	0.2 0.0200 2.1000 3.2800 4.4000 5.4000 6.4400 7.5200 8.6400 9.6000 10.5600 11.5200 13.7200 14.7000 15.6800 16.6600	0.3 0.0200 0.0400 2.2200 3.4400 4.5000 5.5200 6.5800 7.6800 8.6400 9.6000 10.7800 11.7600 11.7600 13.7200 14.7000 15.6800 16.6600	0.4 0.0200 0.0400 2.3400 3.5200 4.6000 5.6400 0.76800 8.6400 9.8000 10.7800 11.7600 11.7600 13.7200 14.7000 15.6800 16.6600 18.0000	x(max)=X _a 0.5 0.0200 0.2800 2.4000 3.5200 4.6000 5.6400 6.7200 7.6800 8.8200 9.8000 10.7800 11.7600 11.7600 11.7400 13.7200 14.7000 16.0000 17.0000	0.6 0.0200 0.6800 2.4600 3.6000 4.7000 5.6400 6.7200 7.6800 8.8200 9.8000 10.7800 11.7600 11.7600 13.7200 15.0000 16.0000 17.0000 18.0000	0.7 0.0200 0.8800 2.5200 3.6000 4.7000 5.7600 6.7200 7.8400 8.8200 9.8000 10.7800 10.7800 11.7600 11.7600 12.7400 14.0000 15.0000 16.0000 17.0000 18.0000	0.8 0.0200 1.0000 2.5800 3.6800 5.7600 6.7200 7.8400 8.8200 9.8000 10.7800 10.7800 11.7600 12.7400 14.0000 15.0000 16.0000 17.0000
X _a \Q ₃ 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19	-0.8 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 10.0000 11.0000 11.0000 11.0000 14.0000 14.0000 15.0000 16.0000 17.0000 18.0000 19.0000	-0.7 1.0000 2.0000 3.0000 4.0000 5.0000 7.0000 8.0000 10.0000 11.0000 11.0000 12.0000 14.0000 14.0000 15.0000 16.0000 17.0000 18.0000 19.0000	-0.6 1.0000 2.0000 3.0000 5.0000 6.0000 7.0000 8.0000 10.0000 11.0000 11.0000 12.0000 14.0000 14.0000 15.0000 16.0000 18.0000 19.0000	x(max)=Xa -0.5 1.0000 2.0000 3.0000 5.0000 6.0000 7.0000 8.0000 10.0000 11.0000 11.0000 11.0000 11.0000 14.0000 15.0000 16.0000 17.0000 18.0000	-0.4 1.0000 2.0000 3.0000 4.0000 5.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000 14.0000 15.0000 16.0000 17.0000 18.0000 19.0000	-0.3 1.0000 2.0000 3.0000 4.0000 5.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000 14.0000 15.0000 16.0000 17.0000 18.0000 19.0000	-0.2 0.0200 2.0000 3.0000 5.0000 6.0000 7.0000 8.0000 10.0000 11.0000 11.0000 12.0000 14.0000 14.0000 15.0000 16.0000 17.0000 18.0000 19.0000	-0.1 0.0200 0.0400 0.0600 4.0000 5.0000 7.0000 8.0000 10.0000 11.0000 12.0000 13.0000 14.0000 15.0000 16.0000 17.0000 18.0000	0 0.0200 1.3200 2.7200 3.8000 4.8000 5.7400 6.8800 7.7400 8.8000 9.9000 10.8000 11.9600 12.8800 13.8000 14.7200 15.9800 16.9200	0.1 0.0200 0.0400 1.8000 3.1200 4.2000 5.1600 6.3000 7.3600 8.2800 9.4000 10.3400 11.5200 12.4800 13.4400 13.4400 15.6800 16.6600 17.6400 18.6200	0.2 0.0200 0.0400 2.1000 3.2800 4.4000 5.4000 6.4400 7.5200 8.6400 9.6000 10.5600 11.5200 12.7400 13.7200 14.7000 15.6800 16.6600 17.6400 18.6200	0.3 0.0200 0.0400 2.2200 3.4400 4.5000 5.5200 6.5800 7.6800 9.6000 10.7800 11.7600 12.7400 13.7200 14.7000 15.6800 16.6600 17.6400 18.6200	0.4 0.0200 0.0400 2.3400 3.5200 4.6000 5.6400 6.7200 7.6800 10.7800 10.7800 11.7600 12.7400 13.7200 14.7000 15.6800 16.6600 18.0000 19.0000	x(max)=X _a 0.5 0.0200 0.2800 2.4000 3.5200 4.6000 5.6400 6.7200 7.6800 9.8000 10.7800 11.7600 11.7600 13.7200 14.7000 16.0000 17.0000 18.0000 19.0000	0.6 0.0200 0.6800 2.4600 3.6000 4.7000 5.6400 6.7200 7.6800 9.8000 10.7800 11.7600 11.7600 13.7200 13.7200 15.0000 16.0000 17.0000 19.0000	0.7 0.0200 0.8800 2.5200 3.6000 4.7000 5.7600 6.7200 7.8400 8.8200 9.8000 10.7800 11.7600 12.7400 14.0000 15.0000 16.0000 17.0000 18.0000 19.0000	0.8 0.0200 1.0000 2.5800 3.6800 4.7000 5.7600 6.7200 7.8400 7.8400 9.8000 10.7800 11.7600 12.7400 12.7400 14.0000 15.0000 16.0000 17.0000 18.0000
X _a \Q ₃ 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20	-0.8 1.0000 2.0000 3.0000 5.0000 6.0000 7.0000 8.0000 10.0000 11.0000 11.0000 12.0000 13.0000 14.0000 15.0000 16.0000 17.0000 18.0000 19.0000 19.0000	-0.7 1.0000 2.0000 3.0000 4.0000 5.0000 7.0000 8.0000 10.0000 11.0000 11.0000 11.0000 14.0000 15.0000 16.0000 17.0000 18.0000 19.0000	-0.6 1.0000 2.0000 3.0000 4.0000 5.0000 7.0000 8.0000 10.0000 11.0000 11.0000 12.0000 13.0000 14.0000 15.0000 15.0000 16.0000 17.0000 18.0000 19.0000	x(max)=Xa -0.5 1.0000 2.0000 3.0000 5.0000 6.0000 7.0000 8.0000 10.0000 11.0000 11.0000 11.0000 13.0000 14.0000 15.0000 16.0000 17.0000 18.0000 19.0000	-0.4 1.0000 2.0000 3.0000 5.0000 6.0000 7.0000 8.0000 10.0000 11.0000 11.0000 13.0000 14.0000 15.0000 15.0000 16.0000 17.0000 18.0000 19.0000 10.00000 10.0000 10.0000 10.0000 10.0000 10.0000000 10.00000	-0.3 1.0000 2.0000 3.0000 5.0000 6.0000 7.0000 8.0000 10.0000 11.0000 12.0000 13.0000 14.0000 15.0000 16.0000 17.0000 18.0000 19.0000	-0.2 0.0200 2.0000 3.0000 5.0000 6.0000 7.0000 8.0000 10.0000 11.0000 11.0000 11.0000 13.0000 14.0000 15.0000 15.0000 16.0000 17.0000 18.0000 19.0000	-0.1 0.0200 0.0400 0.0600 5.0000 6.0000 7.0000 8.0000 10.0000 11.0000 11.0000 13.0000 14.0000 15.0000 16.0000 17.0000 18.0000 19.0000	0 0.0200 1.3200 2.7200 3.8000 4.8000 5.7400 6.8800 7.7400 8.8000 9.9000 10.8000 11.9600 11.9600 13.8000 13.8000 14.7200 15.9800 16.9200 17.8600	0.1 0.0200 0.0400 1.8000 3.1200 4.2000 5.1600 6.3000 7.3600 9.4000 10.3400 11.5200 12.4800 13.4400 13.6800 14.4000 15.6800 16.6600 17.6400 18.6200 19.6000	0.2 0.0200 0.0400 2.1000 3.2800 4.4000 5.4000 6.4400 7.5200 10.5600 10.5600 11.5200 12.7400 13.7200 13.7200 14.7000 15.6800 16.6600 17.6400 18.6200	0.3 0.0200 0.0400 2.2200 3.4400 4.5000 5.5200 6.5800 7.6800 7.6800 10.7800 11.7600 12.7400 13.7200 14.7000 15.6800 16.6600 17.6400 18.6200 0.0000	0.4 0.0200 0.0400 2.3400 3.5200 4.6000 5.6400 6.7200 7.6800 10.7800 10.7800 11.7600 12.7400 13.7200 14.7000 15.6800 16.6600 18.0000 19.0000	x(max)=X _a 0.5 0.0200 0.2800 2.4000 3.5200 4.6000 5.6400 6.7200 7.6800 9.8000 10.7800 11.7600 12.7400 13.7200 14.7000 14.7000 14.0000 17.0000 18.0000 19.0000	0.6 0.0200 0.6800 2.4600 3.6000 4.7000 5.6400 6.7200 7.6800 9.8000 10.7800 11.7600 12.7400 13.7200 15.0000 15.0000 16.0000 17.0000 18.0000 19.0000	0.7 0.0200 0.8800 2.5200 3.6000 4.7000 5.7600 6.7200 7.8400 9.8000 10.7800 11.7600 12.7400 14.0000 15.0000 15.0000 16.0000 17.0000 18.0000 19.0000	0.8 0.0200 1.0000 2.5800 3.6800 4.7000 5.7600 6.7200 7.8400 7.8400 8.8200 9.8000 10.7800 10.7800 11.7600 11.7600 11.7600 11.76000 15.0000 15.0000 17.0000 18.0000 9.0000



 $-0.8 < Q_3 < 0.0$ (v* = 0.4)





 $F_{m} vs X_{a} and 0 < Q_{3} < +0.8$

(v* = 0.4)

 $0.0 < Q_3 < +0.8$

0.70 0.60 -------Q3=0 0.50 ----Q3=0.1 ▲Q3=0.2 0.40 Q3=0.3 --Q3=0.5 0.30 ----Q3=0.6 -Q3=0.7 0.20 Q3=0.8 0.10 0.00 2 5 12 15 16 17 18 19 20 1 3 6 7 8 13 14 4 9 10 11

Xa

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F_m=MAX|F_m(x)|

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ANNEX Q—TABULAR AND GRAPHICAL REPRESENTATION OF COEFFICIENT $F_{\mbox{\scriptsize Q}}(x)$

Annex Q provides for $1 \le X_a \le 20$ and $-0.8 \le Q_3 \le +0.8$:

- values and graphs of $F_Q(x)$ for $0 \le x \le X_a$
- values and graphs of the maximum of $F_Q(x)$: F_Q
- location of the maximum of $F_Q(x)$: x_{max}



F_Q(x) for X_a=1,3,5,7,10,20

								F _Q ((x) for X _a	= 1, 3 , 5	,7,10,2	20 and	Q ₃ = -0.8	, 0.0 , +0.	8		(v*=0.	.4)				
	X _a =1			Xa	=3			Xa	=5			X	₃ =7			X _a =1	10			Xa =	20	
x	Q ₃ =-0.8 Q ₃ =0	Q ₃ =+0.8	x	Q ₃ =-0.8	Q ₃ =0	Q ₃ =+0.8	x	Q ₃ =-0.8	Q ₃ =0	Q ₃ =+0.8	x	Q ₃ =-0.8	Q ₃ =0	Q ₃ =+0.8	x	Q ₃ =-0.8	Q ₃ =0	Q ₃ =+0.8	x	Q3=-0.8	Q3=0	Q3=+0.8
0.02	0.02076262 0.0193537	0 0.0179447	0.06	0.04597750	-0.00541298	-0.05680346	0.10	0.02060735	-0.01213262	-0.04487258	0.14	-0.01755414	-0.00055054	0.01645306	0.20	-0.00193894	0.00089464	0.00372823	0.40	0.00001512	0.0000089	-0.0000133
0.04	0.04152367 0.0387091	9 0.03589470	0.12	0.09193687	-0.01076614	-0.11346920	0.20	0.04148651	-0.02427055	-0.09002762	0.28	-0.03486280	-0.00117172	0.03251937	0.40	-0.00408697	0.00180124	0.00768945	0.80	0.00002634	0.00000203	-0.00002228
0.06	0.06228157 0.0580682	6 0.05385495	0.18	0.13785960	-0.01599961	-0.16985880	0.30	0.06290802	-0.03641836	-0.13574480	0.42	-0.05167628	-0.00193401	0.04780826	0.60	-0.00665120	0.00273084	0.01211288	1.20	0.00002952	0.00000367	-0.00002218
0.08	0.08303474 0.0774327	1 0.07183069	0.24	0.18372650	-0.02105345	-0.22583340	0.40	0.08513980	-0.04857909	-0.18229800	0.56	-0.06773657	-0.00290762	0.06192133	0.80	-0.00983450	0.00369271	0.01721991	1.60	0.00002014	0.00000601	-0.0000081
0.10	0.10378160 0.0968043	4 0.08982707	0.30	0.22951710	-0.02586758	-0.28125230	0.50	0.10844580	-0.06075350	-0.22995280	0.70	-0.08277331	-0.00416227	0.07444877	1.00	-0.01383283	0.00469333	0.02321949	2.00	-0.00000669	0.00000920	0.00002510
0.12	0.12452060 0.1161849	0 0.10784920	0.36	0.27520970	-0.03038173	-0.33597320	0.60	0.13308470	-0.07293931	-0.27896330	0.84	-0.09650004	-0.00576692	0.08496620	1.20	-0.01883181	0.00573549	0.03030280	2.40	-0.00005603	0.00001328	0.00008259
0.14	0.14525000 0.1355761	0 0.12590230	0.42	0.32078050	-0.03453546	-0.38985140	0.70	0.15930820	-0.08513048	-0.32956910	0.98	-0.10861070	-0.00778948	0.09303176	1.40	-0.02500229	0.00681724	0.03863677	2.80	-0.00013251	0.00001811	0.00016873
0.16	0.16596830 0.1549798	0 0.14399140	0.48	0.36620360	-0.03826807	-0.44273970	0.80	0.18735950	-0.09731634	-0.38199220	1.12	-0.11877650	-0.01029634	0.09818387	1.60	-0.03249485	0.00793082	0.04835649	3.20	-0.00023938	0.00002334	0.00028605
0.18	0.18667380 0.1743977	0 0.16212160	0.54	0.41145040	-0.04151859	-0.49448750	0.90	0.21747200	-0.10948100	-0.43643390	1.26	-0.12664330	-0.01335183	0.09993967	1.80	-0.04143289	0.00906157	0.05955603	3.60	-0.00037699	0.00002828	0.00043355
0.20	0.20736500 0.1938316	0 0.18029810	0.60	0.45648950	-0.04422573	-0.54494090	1.00	0.24986680	-0.12160230	-0.49307130	1.40	-0.13182940	-0.01701756	0.09779428	2.00	-0.05190425	0.01018679	0.07227782	4.00	-0.00054077	0.00003186	0.00060450
0.22	0.22804000 0.2132830	0 0.19852600	0.66	0.50128600	-0.04632790	-0.59394180	1.10	0.28475110	-0.13365150	-0.55205400	1.54	-0.13392410	-0.02135156	0.09122102	2.20	-0.06395109	0.01127458	0.08650026	4.40	-0.00071868	0.00003251	0.0007837
0.24	0.24869730 0.2327538	0 0.21681020	0.72	0.54580140	-0.04776315	-0.64132770	1.20	0.32231610	-0.14559200	-0.61350010	1.68	-0.13248720	-0.02640728	0.07967261	2.40	-0.07755814	0.01228275	0.10212360	4.80	-0.00088817	0.00002814	0.00094444
0.26	0.26933520 0.2522455	0 0.23515590	0.78	0.58999320	-0.04846917	-0.68693160	1.30	0.36273460	-0.15737910	-0.67749280	1.82	-0.12704880	-0.03223242	0.06258393	2.60	-0.09263907	0.01315772	0.11895450	5.20	-0.00101313	0.00001614	0.00104542
0.28	0.28995180 0.2717599	0 0.25356800	0.84	0.63381440	-0.04838331	-0.73058100	1.40	0.40615810	-0.16895880	-0.74407580	1.96	-0.11711080	-0.03886747	0.03937585	2.80	-0.10902130	0.01383357	0.13668840	5.60	-0.00104112	-0.00000642	0.0010282
0.30	0.31054540 0.2912985	0 0.27205160	0.90	0.67721310	-0.04744254	-0.77209820	1.50	0.45271510	-0.18026740	-0.81325000	2.10	-0.10214920	-0.04634420	0.00946077	3.00	-0.12642890	0.01423129	0.15489150	6.00	-0.00090186	-0.00004256	0.00081673
0.32	0.33111430 0.3108630	0 0.29061170	0.96	0.72013270	-0.04558348	-0.81129970	1.60	0.50250690	-0.19123070	-0.88496830	2.24	-0.08161756	-0.05468376	-0.02774997	3.20	-0.14446490	0.01425830	0.17298150	6.40	-0.00050796	-0.00009486	0.00031824
0.34	0.35165660 0.3304549	0 0.30925310	1.02	0.76251060	-0.04274242	-0.84799550	1.70	0.55560510	-0.20176310	-0.95913120	2.38	-0.05495225	-0.06389466	-0.07283707	3.40	-0.16259140	0.01380827	0.19020800	6.80	0.00024051	-0.00016461	-0.00056973
0.36	0.37217050 0.3500756	0 0.32798080	1.08	0.80427890	-0.03885531	-0.88198940	1.80	0.61204820	-0.21176730	-1.03558300	2.52	-0.02157894	-0.07397044	-0.12636190	3.60	-0.18011100	0.01276156	0.20563420	7.20	0.00144587	-0.00025076	-0.0019473
0.38	0.39265400 0.3697268	0 0.34679970	1.14	0.84536300	-0.03385787	-0.91307870	1.90	0.67183810	-0.22113350	-1.11410500	2.66	0.01907883	-0.08488717	-0.18885320	3.80	-0.19614680	0.01098615	0.21811910	7.60	0.00319507	-0.00034840	-0.0038918
0.40	0.41310520 0.3894099	0 0.36571460	1.20	0.88568250	-0.02768556	-0.94105360	2.00	0.73493580	-0.22973880	-1.19441300	2.80	0.06758947	-0.09660070	-0.26079090	4.00	-0.20962510	0.00833940	0.22630390	8.00	0.00553233	-0.00044706	-0.0064264
0.42	0.43352230 0.4091263	0 0.38473040	1.26	0.92514960	-0.02027364	-0.96569690	2.10	0.80125750	-0.23744640	-1.27615000	2.94	0.12450080	-0.10904370	-0.34258810	4.20	-0.21925940	0.00467078	0.22860100	8.40	0.00842067	-0.00052879	-0.0094782
0.44	0.45390310 0.4288775	0 0.40385180	1.32	0.96366970	-0.01155732	-0.98678430	2.20	0.87067040	-0.24410590	-1.35888200	3.08	0.19032540	-0.12212230	-0.43456990	4.40	-0.22353960	-0.00017432	0.22319090	8.80	0.01169193	-0.00056633	-0.01282459
0.46	0.47424570 0.4486646	0 0.42308360	1.38	1.00114100	-0.00147171	-1.00408400	2.30	0.94298810	-0.24955200	-1.44209200	3.22	0.26552360	-0.13571290	-0.53694950	4.60	-0.22072500	-0.00634949	0.20802600	9.20	0.01498676	-0.00052185	-0.01603046
0.48	0.49454780 0.4684892	0 0.44243050	1.44	1.03745300	0.01004802	-1.01735700	2.40	1.01796600	-0.25360480	-1.52517500	3.36	0.35048380	-0.14965890	-0.64980160	4.80	-0.20884670	-0.01400068	0.18084540	9.60	0.01768924	-0.00034679	-0.0183828
0.50	0.51480750 0.4883523	0 0.46189720	1.50	1.07248800	0.02306647	-1.02635500	2.50	1.09529500	-0.25606920	-1.60743300	3.50	0.44550030	-0.16376640	-0.77303310	5.00	-0.18571850	-0.02325793	0.13920260	10.00	0.01886495	0.00001631	-0.01883233
0.52	0.53502240 0.5082552	0 0.48148810	1.56	1.10612000	0.03764805	-1.03082400	2.60	1.17459900	-0.25673520	-1.68806900	3.64	0.55074830	-0.17780120	-0.90635060	5.20	-0.14896020	-0.03422524	0.08050973	10.40	0.01721728	0.00062887	-0.01595954
0.54	0.55519030 0.5281992	0 0.50120810	1.62	1.13821400	0.05385673	-1.03050100	2.70	1.25542800	-0.25537740	-1.76618300	3.78	0.66625600	-0.19148470	-1.04922500	5.40	-0.09603721	-0.04696843	0.00210035	10.80	0.01108357	0.00154717	-0.00798924
0.56	0.57530900 0.5481852	0 0.52106140	1.68	1.16862700	0.07175592	-1.02511500	2.80	1.33725200	-0.25175550	-1.84076300	3.92	0.79187370	-0.20449050	-1.20085500	5.60	-0.02431677	-0.06150097	-0.09868518	11.20	-0.00149966	0.00280757	0.00711479
0.58	0.59537610 0.5682144	0 0.54105270	1.74	1.19720700	0.09140831	-1.01439000	2.90	1.41945800	-0.24561470	-1.91068700	4.06	0.92724110	-0.21644110	-1.36012300	5.80	0.06885336	-0.07776765	-0.22438860	11.60	-0.02262540	0.00440526	0.03143593
0.60	0.61538920 0.5882878	0 0.56118630	1.80	1.22379000	0.11287580	-0.99803880	3.00	1.50134100	-0.23668590	-1.97471300	4.20	1.07175000	-0.22690450	-1.52555900	6.00	0.18604180	-0.09562604	-0.37729390	12.00	-0.05419668	0.00626580	0.06672829
0.62	0.63534580 0.6084062	0 0.58146660	1.86	1.24820700	0.13621900	-0.97576930	3.10	1.58210200	-0.22468660	-2.03147500	4.34	1.22450700	-0.23539160	-1.69529100	6.20	0.32960290	-0.11482610	-0.55925520	12.40	-0.09741537	0.00821001	0.11383540
0.64	0.65524330 0.6285706	0 0.60189800	1.92	1.27027600	0.16149750	-0.94728120	3.20	1.66084200	-0.20932150	-2.07948500	4.48	1.38429100	-0.24135410	-1.86699900	6.40	0.50151430	-0.13498810	-0.77149050	12.80	-0.15204270	0.00991443	0.17187150
0.66	0.67507920 0.6487819	0 0.62248460	1.98	1.28980700	0.18876910	-0.91226850	3.30	1.73655300	-0.19028430	-2.11712200	4.62	1.54951000	-0.24418280	-2.03787500	6.60	0.70318190	-0.15557860	-1.01433900	13.20	-0.21541390	0.01087201	0.23715790
0.68	0.69485090 0.6690407	0 0.64323060	2.04	1.30659800	0.21809000	-0.87041780	3.40	1.80812000	-0.16725850	-2.14263700	4.76	1.71815600	-0.24320700	-2.20457000	6.80	0.93520770	-0.17588700	-1.28698200	13.60	-0.28122570	0.01036090	0.30194750
0.70	0.71455550 0.6893478	0 0.66414010	2.10	1.32043900	0.24951430	-0.82141050	3.50	1.87431000	-0.13991930	-2.15414800	4.90	1.88776300	-0.23769520	-2.36315400	7.00	1.19712200	-0.19500020	-1.58712200	14.00	-0.33817440	0.00743362	0.35304160
0.72	0.73419030 0.7097038	0 0.68521730	2.16	1.33110900	0.28309390	-0.76492160	3.60	1.93377000	-0.10793560	-2.14964100	5.04	2.05535600	-0.22685640	-2.50906900	7.20	1.48707700	-0.21178020	-1.91063700	14.40	-0.36860900	0.00094339	0.37049570
0.74	0.75375250 0.7301093	0 0.70646620	2.22	1.33837800	0.31887790	-0.70062230	3.70	1.98502300	-0.07097274	-2.12696900	5.18	2.21740700	-0.20984350	-2.63709400	7.40	1.80150500	-0.22484280	-2.25119000	14.80	-0.34747830	-0.01037070	0.32673690
0.76	0.77323900 0.7505648	0.72789060	2.28	1.34200400	0.35691240	-0.62817870	3.80	2.02646800	-0.02869451	-2.08385700	5.32	2.36979200	-0.18575810	-2.74130800	7.60	2.13474700	-0.23254080	-2.59982900	15.20	-0.24199930	-0.02771412	0.18657110
0.78	0.79264680 0.7710706	0 0.74949440	2.34	1.34173500	0.39724070	-0.54725330	3.90	2.05637100	0.01923318	-2.01790500	5.46	2.50774900	-0.15365740	-2.81506300	7.80	2.47865200	-0.23295290	-2.94455700	15.60	-0.01263246	-0.05195559	-0.09127872
0.80	0.81197300 0.7916272	0 0.77128130	2.40	1.33731000	0.43990170	-0.45750610	4.00	2.07287000	0.07313900	-1.92659200	5.60	2.62583700	-0.11256380	-2.85096500	8.00	2.82215600	-0.22388120	-3.26991900	16.00	0.38387920	-0.08320964	-0.55029850
0.82	0.83121420 0.8122347	0 0.79325520	2.46	1.32845700	0.48493080	-0.35859530	4.10	2.07397000	0.13334220	-1.80728600	5.74	2.71791200	-0.06147613	-2.84086500	8.20	3.15087000	-0.20285870	-3.55658700	16.40	0.98852070	-0.12026780	-1.22905600
0.84	0.85036740 0.8328934	0.81541950	2.52	1.31489500	0.53235890	-0.25017740	4.20	2.05754600	0.20014810	-1.65724900	5.88	2.77709500	0.00061549	-2.77586400	8.40	3.44665700	-0.16717170	-3.78100000	16.80	1.83012500	-0.15988300	-2.14989100
0.86	0.86942890 0.8536034	0 0.83777790	2.58	1.29633300	0.58221160	-0.13190980	4.30	2.02134200	0.27384340	-1.47365500	6.02	2.79575800	0.07471172	-2.64633400	8.60	3.68726400	-0.11389870	-3.91506100	17.20	2.91076000	-0.19594480	-3.30265000
0.88	0.88839570 0.8743647	0.86033380	2.64	1.27246900	0.63450960	-0.00345021	4.40	1.96297800	0.35469060	-1.25359700	6.16	2.76552400	0.16178290	-2.44195900	8.80	3.84599800	-0.03997031	-3.92593800	17.60	4.18589200	-0.21863160	-4.62315500
0.90	0.90726390 0.8951772	0.88309060	2.70	1.24299500	0.68926760	0.13554060	4.50	1.87995400	0.44292210	-0.99410990	6.30	2.67727600	0.26274380	-2.15178800	9.00	3.89150000	0.05774586	-3.77600800	18.00	5.53950500	-0.21369580	-5.96689700
0.92	0.92603020 0.9160409	0.90605160	2.76	1.20759000	0.74649410	0.28539830	4.60	1.76965500	0.53873340	-0.69218810	6.44	2.52118100	0.37842480	-1.76433100	9.20	3.78764400	0.18233180	-3.42298000	18.40	6.75556100	-0.16212130	-7.07980400
0.94	0.94469080 0.9369554	0.92922000	2.82	1.16592700	0.80619030	0.44645350	4.70	1.62936100	0.64227670	-0.34480800	6.58	2.28673900	0.50953830	-1.26766300	9.40	3.49360700	0.33667290	-2.82026100	18.80	7.48886700	-0.04049700	-7.56986100
0.96	0.96324190 0.9579204	0 0.95259890	2.88	1.11767100	0.86835070	0.61903010	4.80	1.45626000	0.75365280	0.05104512	6.72	1.96285600	0.65664090	-0.64957460	9.60	2.96417000	0.52327460	-1.91762100	19.20	7.24084700	0.17744890	-6.88594900
0.98	0.98167960 0.9789354	0.97619120	2.94	1.06247900	0.93296140	0.80344380	4.90	1.24745900	0.87290290	0.49834700	6.86	1.53793500	0.82009080	0.10224680	9.80	2.15028600	0.74403930	-0.66220770	19.60	5.34871100	0.51761380	-4.31348300
1.00	0.99999990 0.9999999	0.99999990	3.00	0.99999990	0.99999990	0.99999990	5.00	1.00000000	1.00000000	1.0000000	7.00	1.00000000	1.00000000	1.00000000	10.00	1.00000000	0.99999990	0.99999950	20.00	1.00000200	1.00000000	0.99999840








 $X_a = 3$

 $X_a = 3$ (v* = 0.4)



 $X_a = 5$ (v* = 0.4)





$$X_a = 7$$
 (v* = 0.4)



$$X_a = 10$$
 (v* = 0.4)



0 < x <= X_a

 $X_a = 20$

$$X_a = 20$$
 (v* = 0.4)



0 < x <= X_a

F_a(x)

 $F_Q=Max|F_Q(x)|$ vs X_a and Q_3

								Fo	₂ =Max F	[q (x)]	x(max)=X _a	F _Q (X _a)=1					
X _a \Q ₃	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.0057	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3	1.3420	1.2251	1.1193	1.0353	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0308
4	1.7329	1.5390	1.3538	1.1841	1.0487	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.1542	1.3577	1.5619	1.7685
5	2.0740	1.8314	1.5932	1.3660	1.1614	1.0171	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.1549	1.4019	1.6506	1.9024	2.1541
6	2.4303	2.1378	1.8505	1.5698	1.3068	1.0852	1.0000	1.0000	1.0000	1.0000	1.0000	1.0261	1.3150	1.6050	1.9012	2.1974	2.4936
7	2.7958	2.4556	2.1155	1.7891	1.4700	1.1820	1.0000	1.0000	1.0000	1.0000	1.0000	1.1517	1.4844	1.8241	2.1664	2.5087	2.8510
8	3.1588	2.7744	2.3900	2.0055	1.6403	1.2933	1.0257	1.0000	1.0000	1.0000	1.0000	1.2779	1.6619	2.0458	2.4297	2.8196	3.2123
9	3.5225	3.0843	2.6566	2.2335	1.8104	1.4120	1.0755	1.0000	1.0000	1.0000	1.0000	1.4094	1.8371	2.2648	2.6966	3.1353	3.5741
10	3.8915	3.4123	2.9331	2.4538	1.9850	1.5343	1.1335	1.0000	1.0000	1.0000	1.0706	1.5393	2.0145	2.4896	2.9648	3.4402	3.9259
11	4.2515	3.7207	3.1899	2.6763	2.1667	1.6570	1.1980	1.0000	1.0000	1.0000	1.1569	1.6651	2.1912	2.7172	3.2433	3.7693	4.2953
12	4.6077	4.0416	3.4755	2.9094	2.3433	1.7842	1.2689	1.0000	1.0000	1.0000	1.2427	1.8012	2.3598	2.9257	3.5027	4.0798	4.6568
13	4.9889	4.3696	3.7504	3.1311	2.5118	1.9196	1.3443	1.0000	1.0000	1.0000	1.3224	1.9318	2.5491	3.1665	3.7838	4.4012	5.0185
14	5.3440	4.6754	4.0067	3.3381	2.6890	2.0548	1.4224	1.0000	1.0000	1.0000	1.4157	2.0670	2.7183	3.3697	4.0210	4.6858	5.3545
15	5.6689	4.9552	4.2629	3.5713	2.8798	2.1882	1.5049	1.0000	1.0000	1.0000	1.4983	2.1818	2.8955	3.6092	4.3229	5.0366	5.7503
16	6.0529	5.3060	4.5591	3.8122	3.0652	2.3183	1.5899	1.0146	1.0000	1.0000	1.5802	2.3343	3.0884	3.8424	4.5965	5.3506	6.1046
17	6.4433	5.6434	4.8435	4.0437	3.2438	2.4439	1.6768	1.0385	1.0000	1.0000	1.6787	2.4682	3.2577	4.0471	4.8366	5.6261	6.4156
18	6.8142	5.9642	5.1141	4.2640	3.4139	2.5639	1.7651	1.0644	1.0000	1.0000	1.7633	2.5831	3.4029	4.2368	5.0868	5.9369	6.7870
19	7.1635	6.2662	5.3689	4.4717	3.5744	2.6783	1.8541	1.0921	1.0000	1.0000	1.8342	2.7064	3.6036	4.5009	5.3982	6.2954	7.1927
20	7.4889	6.5477	5.6065	4.6654	3.7242	2.8262	1.9433	1.1215	1.0000	1.0268	1.9228	2.8640	3.8052	4.7463	5.6875	6.6287	7.5699
			x(max)=X _a	F _Q (X _a)=1					x(max)			x(max)=X _a I	F _Q (X _a)=1				
X _a \Q ₃	-0.8	-0.7	<mark>x(max)=X_a</mark> -0.6	F _Q (X _a)=1 -0.5	-0.4	-0.3	-0.2	-0.1	x(max)	0.1	0.2	x(max)=X _a I 0.3	F _Q (X _a)=1 0.4	0.5	0.6	0.7	0.8
X _a \Q ₃	-0.8 1.0000	- 0.7 1.0000	x(max)=X _a -0.6 1.0000	F _Q (X _a)=1 -0.5 1.0000	- 0.4 1.0000	-0.3 1.0000	-0.2 1.0000	-0.1 1.0000	x(max) 0 1.0000	0.1 1.0000	0.2 1.0000	x(max)=X _a 0.3 1.0000	F _Q (X _a)=1 0.4 1.0000	0.5 1.0000	0.6 1.0000	0.7 1.0000	0.8 <u>1.0000</u>
X _a \Q ₃ 1 2	-0.8 1.0000 1.8800	-0.7 1.0000 2.0000	x(max)=X _a -0.6 1.0000 2.0000	F _Q (X _a)=1 -0.5 1.0000 2.0000	-0.4 1.0000 2.0000	-0.3 1.0000 2.0000	-0.2 1.0000 2.0000	-0.1 1.0000 2.0000	x(max) 0 1.0000 2.0000	0.1 1.0000 2.0000	0.2 1.0000 2.0000	x(max)=X _a 0.3 1.0000 2.0000	F _Q (X _a)=1 0.4 1.0000 2.0000	0.5 1.0000 2.0000	0.6 1.0000 2.0000	0.7 <u>1.0000</u> <u>2.0000</u>	0.8 1.0000 2.0000
X _a \Q ₃ 1 2 3	-0.8 1.0000 1.8800 2.2800	-0.7 1.0000 2.0000 2.4000	x(max)=X _a -0.6 1.0000 2.0000 2.5200	F _Q (X _a)=1 -0.5 1.0000 2.0000 2.7000	-0.4 1.0000 2.0000 3.0000	-0.3 1.0000 2.0000 3.0000	-0.2 1.0000 2.0000 3.0000	-0.1 1.0000 2.0000 3.0000	x(max) 0 1.0000 2.0000 3.0000	0.1 1.0000 2.0000 3.0000	0.2 1.0000 2.0000 3.0000	x(max)=X _a I 0.3 1.0000 2.0000 3.0000	F _Q (X _a)=1 0.4 1.0000 2.0000 3.0000	0.5 1.0000 2.0000 3.0000	0.6 1.0000 2.0000 3.0000	0.7 1.0000 2.0000 3.0000	0.8 1.0000 2.0000 1.5600
X _a \Q ₃ 1 2 3 4	-0.8 1.0000 1.8800 2.2800 3.1200	-0.7 1.0000 2.0000 2.4000 3.2000	x(max)=X a -0.6 1.0000 2.0000 2.5200 3.2800	F _Q (X _a)=1 -0.5 1.0000 2.0000 2.7000 3.4400	-0.4 1.0000 2.0000 3.0000 3.6800	-0.3 1.0000 2.0000 3.0000 4.0000	-0.2 1.0000 2.0000 3.0000 4.0000	-0.1 1.0000 2.0000 3.0000 4.0000	x(max) 0 1.0000 2.0000 3.0000 4.0000	0.1 1.0000 2.0000 3.0000 4.0000	0.2 1.0000 2.0000 3.0000 4.0000	x(max)=X _a I 0.3 1.0000 [2.0000] 3.0000 [4.0000]	F _Q (X _a)=1 0.4 1.0000 2.0000 3.0000 4.0000	0.5 1.0000 2.0000 3.0000 2.4000	0.6 1.0000 2.0000 3.0000 2.4000	0.7 1.0000 2.0000 3.0000 2.4800	0.8 1.0000 2.0000 1.5600 2.4800
X _a \Q ₃ 1 2 3 4 5	-0.8 1.0000 1.8800 2.2800 3.1200 4.1000	-0.7 1.0000 2.0000 2.4000 3.2000 4.1000	x(max)=X _a -0.6 1.0000 2.5200 3.2800 4.2000	F _Q (X _a)=1 -0.5 1.0000 2.0000 2.7000 3.4400 4.3000	-0.4 1.0000 2.0000 3.0000 3.6800 4.5000	-0.3 1.0000 2.0000 3.0000 4.0000 4.8000	-0.2 1.0000 2.0000 3.0000 4.0000 5.0000	-0.1 1.0000 2.0000 3.0000 4.0000 5.0000	x(max) 0 1.0000 2.0000 3.0000 4.0000 5.0000	0.1 1.0000 2.0000 3.0000 4.0000 5.0000	0.2 1.0000 2.0000 3.0000 4.0000 5.0000	x(max)=X _a 1 0.3 1.0000 0 2.0000 0 3.0000 0 4.0000 0 5.0000	F _Q (X _a)=1 0.4 1.0000 2.0000 3.0000 4.0000 3.4000	0.5 1.0000 2.0000 3.0000 2.4000 3.4000	0.6 1.0000 2.0000 3.0000 2.4000 3.5000	0.7 1.0000 2.0000 3.0000 2.4800 3.5000	0.8 1.0000 2.0000 1.5600 2.4800 3.5000
X _a \Q ₃ 1 2 3 4 5 6	-0.8 1.0000 1.8800 2.2800 3.1200 4.1000 5.0400	-0.7 1.0000 2.0000 2.4000 3.2000 4.1000 5.0400	x(max)=X _a -0.6 1.0000 2.0000 2.5200 3.2800 4.2000 5.1600	F _Q (X _a)=1 -0.5 1.0000 2.0000 2.7000 3.4400 4.3000 5.1600	-0.4 1.0000 2.0000 3.0000 3.6800 4.5000 5.4000	-0.3 1.0000 2.0000 3.0000 4.0000 4.8000 5.6400	-0.2 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000	-0.1 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000	x(max) 0 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000	0.1 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000	0.2 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000	x(max)=X _a I 0.3 1.0000 2.0000 3.0000 4.0000 5.0000 4.3200	F _Q (X _a)=1 0.4 1.0000 2.0000 3.0000 4.0000 3.4000 4.4400	0.5 1.0000 2.0000 3.0000 2.4000 3.4000 4.5600	0.6 1.0000 2.0000 3.0000 2.4000 3.5000 4.5600	0.7 1.0000 2.0000 3.0000 2.4800 3.5000 4.5600	0.8 1.0000 2.0000 1.5600 2.4800 3.5000 4.5600
X _a \Q ₃ 1 2 3 4 5 6 7	-0.8 1.0000 1.8800 2.2800 3.1200 4.1000 5.0400 6.0200	-0.7 1.0000 2.0000 2.4000 3.2000 4.1000 5.0400 6.0200	x(max)=X _a -0.6 2.0000 2.5200 3.2800 4.2000 5.1600 6.0200	F _Q (X _a)=1 -0.5 1.0000 2.0000 2.7000 3.4400 4.3000 5.1600 6.1600	-0.4 1.0000 2.0000 3.0000 3.6800 4.5000 5.4000 6.3000	-0.3 1.0000 2.0000 3.0000 4.0000 4.8000 5.6400 6.4400	-0.2 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000	-0.1 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000	x(max) 0 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000	0.1 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000	0.2 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000	x(max)=Xa 1 0.3 1.0000 2.0000 3.0000 4.0000 5.0000 4.3200 5.4600	F _Q (X _a)=1 0.4 1.0000 2.0000 3.0000 4.0000 3.4000 4.4400 5.4600	0.5 1.0000 2.0000 3.0000 2.4000 3.4000 4.5600 5.6000	0.6 1.0000 2.0000 3.0000 2.4000 3.5000 4.5600 5.6000	0.7 1.0000 2.0000 3.0000 2.4800 3.5000 4.5600 5.6000	0.8 1.0000 2.0000 1.5600 2.4800 3.5000 4.5600 5.6000
X _a \Q ₃ 1 2 3 4 5 6 7 8	-0.8 1.0000 1.8800 2.2800 3.1200 4.1000 5.0400 6.0200 7.0400	-0.7 1.0000 2.0000 2.4000 3.2000 4.1000 5.0400 6.0200 7.0400	x(max)=X _a -0.6 1.0000 2.5200 3.2800 4.2000 5.1600 6.0200 7.0400	F _Q (X _a)=1 -0.5 1.0000 2.7000 3.4400 4.3000 5.1600 6.1600 7.0400	-0.4 1.0000 2.0000 3.0000 3.6800 4.5000 5.4000 6.3000 7.2000	-0.3 1.0000 2.0000 3.0000 4.8000 5.6400 6.4400 7.3600	-0.2 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 7.8400	-0.1 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000	x(max) 0 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000	0.1 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000	0.2 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000	x(max)=Xa 1 0.3 1.0000 2.0000 3.0000 4.0000 4.3200 5.4600 6.5600	F _Q (X _a)=1 0.4 1.0000 2.0000 3.0000 4.0000 3.4000 4.4400 5.4600 6.5600	0.5 1.0000 2.0000 3.0000 2.4000 3.4000 4.5600 5.6000 6.5600	0.6 1.0000 2.0000 3.0000 2.4000 3.5000 4.5600 5.6000 6.5600	0.7 1.0000 2.0000 3.0000 2.4800 3.5000 4.5600 5.6000 6.7200	0.8 1.0000 2.0000 1.5600 2.4800 3.5000 4.5600 5.6000 6.7200
X _a \Q ₃ 1 2 3 4 5 6 7 8 9	-0.8 1.0000 1.8800 3.1200 4.1000 5.0400 6.0200 7.0400 7.9200	-0.7 1.0000 2.0000 2.4000 3.2000 4.1000 5.0400 6.0200 7.0400 7.9200	x(max)=X _a -0.6 1.0000 2.5200 3.2800 4.2000 5.1600 6.0200 7.0400 8.1000	F _Q (X _a)=1 -0.5 1.0000 2.7000 3.4400 4.3000 5.1600 6.1600 7.0400 8.1000	-0.4 1.0000 2.0000 3.0000 3.6800 4.5000 5.4000 6.3000 7.2000 8.1000	-0.3 1.0000 2.0000 3.0000 4.0000 5.6400 6.4400 7.3600 8.2800	-0.2 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 7.8400 8.6400	-0.1 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000	x(max) 0 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000	0.1 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000	0.2 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000	x(max)=X _a 1 0.3 1.0000 2.0000 3.0000 4.0000 5.0000 5.4600 5.4600 6.5600 7.5600	F _Q (X _a)=1 0.4 1.0000 2.0000 3.0000 4.0000 3.4000 3.4000 5.4600 6.5600 7.5600	0.5 1.0000 2.0000 3.0000 2.4000 3.4000 4.5600 5.6000 6.5600 7.5600	0.6 1.0000 2.0000 3.0000 2.4000 3.5000 4.5600 5.6000 6.5600 7.7400	0.7 1.0000 2.0000 3.0000 2.4800 3.5000 4.5600 5.6000 6.7200 7.7400	0.8 1.0000 2.0000 1.5600 2.4800 3.5000 4.5600 5.6000 6.7200 7.7400
X _a \Q ₃ 1 2 3 4 5 6 7 8 9 9	-0.8 1.0000 1.8800 2.2800 3.1200 4.1000 5.0400 6.0200 7.0400 7.9200 9.0000	-0.7 1.0000 2.0000 3.2000 4.1000 5.0400 6.0200 7.0400 7.9200 9.0000	x(max)=X _a -0.6 1.0000 2.5200 3.2800 4.2000 5.1600 6.0200 7.0400 8.1000 9.0000	F _Q (X _a)=1 -0.5 1.0000 2.0000 2.7000 3.4400 4.3000 5.1600 6.1600 7.0400 8.1000 9.0000	-0.4 1.0000 2.0000 3.0000 3.6800 4.5000 5.4000 6.3000 7.2000 8.1000 9.2000	-0.3 1.0000 2.0000 3.0000 4.0000 4.8000 5.6400 6.4400 7.3600 8.2800 9.2000	-0.2 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 7.8400 8.6400 9.6000	-0.1 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000	x(max) 0 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000	0.1 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000	0.2 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 8.4000	x(max)=X _a 0.3 1.0000 2.0000 3.0000 4.0000 5.0000 5.4600 6.5600 7.5600 8.6000	F _Q (X _a)=1 0.4 1.0000 2.0000 3.0000 4.0000 3.4000 5.4600 6.5600 7.5600 8.6000	0.5 1.0000 2.0000 3.0000 2.4000 3.4000 4.5600 5.6000 6.5600 7.5600 8.6000	0.6 1.0000 2.0000 3.0000 2.4000 3.5000 4.5600 5.6000 6.5600 7.7400 8.6000	0.7 1.0000 2.0000 3.0000 2.4800 3.5000 4.5600 5.6000 6.7200 7.7400 8.8000	0.8 1.0000 2.0000 1.5600 2.4800 3.5000 4.5600 5.6000 6.7200 7.7400 8.8000
X _a \Q ₃ 1 2 3 4 5 6 7 8 9 10 11	-0.8 1.0000 1.8800 2.2800 3.1200 4.1000 5.0400 6.0200 7.0400 7.9200 9.0000 9.9000	-0.7 1.0000 2.0000 3.2000 4.1000 5.0400 6.0200 7.0400 7.9200 9.0000 9.9000	x(max)=X _a -0.6 1.0000 2.5200 3.2800 4.2000 5.1600 6.0200 7.0400 8.1000 9.0000 9.9000	F _Q (X _a)=1 -0.5 1.0000 2.7000 3.4400 4.3000 5.1600 6.1600 7.0400 8.1000 9.0000 10.1200	-0.4 1.0000 2.0000 3.0800 4.5000 5.4000 6.3000 7.2000 8.1000 9.2000 10.1200	-0.3 1.0000 2.0000 3.0000 4.0000 4.8000 5.6400 6.4400 7.3600 8.2800 9.2000 10.1200	-0.2 1.0000 2.0000 3.0000 4.0000 5.0000 7.0000 7.8400 8.6400 9.6000 10.5600	-0.1 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000	x(max) 0 1.0000 2.0000 3.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000	0.1 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000	0.2 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 8.4000 9.4600	x(max)=X _a 1 0.3 1.0000 2.0000 3.0000 4.0000 5.0000 4.3200 5.4600 6.5600 7.5600 8.6000 9.6800	F _Q (X _a)=1 0.4 1.0000 2.0000 3.0000 4.0000 3.4000 4.4400 5.4600 6.5600 7.5600 8.6000 9.6800	0.5 1.0000 2.0000 3.0000 2.4000 3.4000 4.5600 5.6000 7.5600 8.6000 9.6800	0.6 1.0000 2.0000 3.0000 2.4000 3.5000 4.5600 5.6000 5.6000 7.7400 8.6000 9.6800	0.7 1.0000 2.0000 3.0000 2.4800 3.5000 4.5600 5.6000 5.6000 6.7200 7.7400 8.8000 9.6800	0.8 1.0000 2.0000 1.5600 2.4800 3.5000 4.5600 5.6000 6.7200 7.7400 8.8000 9.6800
X _a \Q ₃ 1 2 3 4 5 6 7 8 9 10 11 12	-0.8 1.0000 1.8800 2.2800 3.1200 4.1000 5.0400 6.0200 7.0400 7.9200 9.0000 9.9000 11.0400	-0.7 1.0000 2.0000 3.2000 4.1000 5.0400 6.0200 7.0400 7.0400 9.9000 9.9000 11.0400	x(max)=X _a -0.6 1.0000 2.0000 2.5200 3.2800 4.2000 5.1600 6.0200 7.0400 7.0400 9.0000 9.9000 11.0400	F _Q (X _a)=1 -0.5 1.0000 2.7000 3.4400 4.3000 5.1600 6.1600 7.0400 8.1000 9.0000 10.1200 11.0400	-0.4 1.0000 2.0000 3.0800 4.5000 5.4000 6.3000 7.2000 8.1000 9.2000 10.1200 11.0400	-0.3 1.0000 2.0000 3.0000 4.0000 4.8000 5.6400 6.4400 7.3600 8.2800 9.2000 10.1200 11.2800	-0.2 1.0000 2.0000 3.0000 4.0000 5.0000 7.0000 7.8400 8.6400 9.6000 10.5600 11.2800	-0.1 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000	x(max) 0 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000	0.1 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000	0.2 1.0000 2.0000 3.0000 5.0000 6.0000 7.0000 8.0000 9.0000 8.4000 9.4600 10.5600	x(max)=X _a 1 0.3 1.0000 2.0000 4.0000 5.0000 4.3200 5.4600 6.5600 7.5600 8.6000 9.6800 10.5600	F _Q (X _a)=1 0.4 1.0000 2.0000 3.0000 4.0000 3.4000 4.4400 5.4600 6.5600 7.5600 7.5600 8.6000 9.6800 10.5600	0.5 1.0000 2.0000 3.0000 2.4000 3.4000 4.5600 6.5600 0.56000 7.5600 8.6000 9.6800 10.8000	0.6 1.0000 2.0000 3.0000 2.4000 3.5000 4.5600 5.6000 6.5600 7.7400 8.6000 9.6800 10.8000	0.7 1.0000 2.0000 3.0000 2.4800 3.5000 4.5600 5.6000 6.7200 7.7400 7.7400 9.6800 10.8000	0.8 1.0000 2.0000 1.5600 2.4800 3.5000 4.5600 5.6000 6.7200 7.7400 8.8000 9.6800 10.8000
X _a \Q ₃ 1 2 3 4 5 6 7 8 9 10 11 12 13	-0.8 1.0000 1.8800 2.2800 3.1200 4.1000 5.0400 6.0200 7.9200 7.9200 9.0000 9.9000 11.0400 11.9600	-0.7 1.0000 2.0000 2.4000 3.2000 4.1000 5.0400 6.0200 7.0400 7.9200 9.0000 9.9000 11.0400 11.9600	x(max)=X _a -0.6 1.0000 2.5200 3.2800 4.2000 5.1600 6.0200 7.0400 8.1000 9.0000 9.9000 11.0400 11.9600	F _Q (X _a)=1 -0.5 1.0000 2.7000 3.4400 4.3000 5.1600 6.1600 7.0400 8.1000 9.0000 10.1200 11.0400 11.9600	-0.4 1.0000 2.0000 3.0000 3.6800 4.5000 5.4000 6.3000 7.2000 8.1000 9.2000 10.1200 11.0400 11.9600	-0.3 1.0000 2.0000 3.0000 4.0000 4.8000 5.6400 6.4400 7.3600 8.2800 9.2000 10.1200 11.2800 12.2200	-0.2 1.0000 2.0000 3.0000 4.0000 5.0000 7.8400 8.6400 9.6000 10.5600 11.2800 12.2200	-0.1 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000	x(max) 0 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 112.0000 13.0000	0.1 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000	0.2 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 9.4600 10.5600 11.4400	x(max)=X _a 1 0.3 1.0000 2.0000 3.0000 4.0000 5.0000 4.3200 5.4600 6.5600 7.5600 7.5600 9.6800 10.5600 11.7000	F _Q (X _a)=1 0.4 1.0000 2.0000 3.0000 4.0000 4.4000 5.4600 6.5600 7.5600 7.5600 9.6800 10.5600 11.7000	0.5 1.0000 2.0000 3.4000 4.5600 5.6000 6.56000 7.5600 8.6000 9.6800 10.8000 11.7000	0.6 1.0000 2.0000 3.0000 2.4000 3.5000 4.5600 5.6000 6.5600 7.7400 8.6000 9.6800 10.8000 11.7000	0.7 1.0000 2.0000 2.4800 3.5000 4.5600 6.7200 7.7400 8.8000 9.6800 10.8000 11.7000	0.8 1.0000 2.0000 1.5600 2.4800 3.5000 4.5600 6.7200 7.7400 8.8000 9.6800 10.8000 11.7000
X _a \Q ₃ 1 2 3 4 5 6 7 8 9 10 11 12 13 14	-0.8 1.0000 1.8800 3.1200 4.1000 5.0400 6.0200 7.0400 7.9200 9.0000 9.9000 11.0400 11.9600 12.8800	-0.7 1.0000 2.0000 2.4000 3.2000 4.1000 5.0400 6.0200 7.0400 7.9200 9.0000 9.9000 11.0400 11.9600 12.8800	x(max)=X _a -0.6 1.0000 2.5200 3.2800 4.2000 5.1600 6.0200 7.0400 8.1000 9.0000 9.9000 11.0400 11.9600 12.8800	F _Q (X _a)=1 -0.5 1.0000 2.7000 3.4400 4.3000 5.1600 6.1600 7.0400 8.1000 9.0000 10.1200 11.0400 11.9600 12.8800	-0.4 1.0000 2.0000 3.0000 3.6800 4.5000 5.4000 6.3000 7.2000 8.1000 9.2000 10.1200 11.0400 11.9600 13.1600	-0.3 1.0000 2.0000 3.0000 4.0000 5.6400 6.4400 7.3600 8.2800 9.2000 10.1200 11.2800 12.2200 13.1600	-0.2 1.0000 2.0000 3.0000 4.0000 5.0000 7.8400 8.6400 9.6000 10.5600 11.2800 12.2200 13.4400	-0.1 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000 14.0000	x(max) 0 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 11.0000 13.0000 14.0000	0.1 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000 14.0000	0.2 1.0000 2.0000 3.0000 5.0000 6.0000 7.0000 8.0000 9.0000 8.4000 9.4600 10.5600 11.4400 12.6000	x(max)=Xa 1 0.3 1.0000 2.0000 3.0000 4.0000 5.0000 4.3200 5.4600 6.5600 7.5600 8.6000 9.6800 10.5600 11.7000 12.6000	F _Q (X _a)=1 0.4 1.0000 2.0000 3.0000 4.0000 4.4400 5.4600 6.5600 7.5600 8.6000 9.6800 10.5600 11.7000 12.6000	0.5 1.0000 2.0000 3.4000 4.5600 5.6000 6.56000 7.5600 8.6000 9.6800 10.8000 11.7000 12.6000	0.6 1.0000 2.0000 3.0000 2.4000 3.5000 4.5600 7.7400 8.6000 9.6800 10.8000 11.7000 12.6000	0.7 1.0000 2.0000 3.0000 2.4800 3.5000 4.5600 5.6000 6.7200 7.7400 8.8000 9.6800 10.8000 11.7000 12.8800	0.8 1.0000 2.0000 1.5600 2.4800 3.5000 4.5600 6.7200 7.7400 8.8000 9.6800 10.8000 11.7000 12.8800
X _a \Q ₃ 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15	-0.8 1.0000 1.8800 2.2800 3.1200 4.1000 5.0400 6.0200 7.0400 7.9200 9.0000 9.0000 9.9000 11.0400 11.9600 12.8800 13.8000	-0.7 1.0000 2.0000 2.4000 3.2000 4.1000 5.0400 6.0200 7.0400 7.9200 9.0000 9.9000 11.0400 11.9600 12.8800 13.8000	x(max)=X _a -0.6 1.0000 2.5200 3.2800 4.2000 5.1600 6.0200 7.0400 8.1000 9.9000 11.0400 11.9600 12.8800 14.1000	F _Q (X _a)=1 -0.5 1.0000 2.7000 3.4400 4.3000 5.1600 6.1600 7.0400 8.1000 9.0000 10.1200 11.0400 11.9600 12.8800 14.1000	-0.4 1.0000 2.0000 3.0000 3.6800 4.5000 6.3000 7.2000 8.1000 9.2000 10.1200 11.0400 11.9600 13.1600 14.1000	-0.3 1.0000 2.0000 3.0000 4.8000 5.6400 6.4400 7.3600 8.2800 9.2000 10.1200 11.2800 11.2800 13.1600 14.1000	-0.2 1.0000 2.0000 3.0000 4.0000 5.0000 7.0000 7.8400 8.6400 9.6000 10.5600 11.2800 12.2200 13.4400 14.4000	-0.1 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 13.0000 14.0000 15.0000	x(max) 0 1.0000 2.0000 3.0000 4.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 11.0000 13.0000 14.0000 15.0000	0.1 1.0000 2.0000 3.0000 4.0000 5.0000 7.0000 8.0000 9.0000 10.0000 11.0000 11.0000 13.0000 14.0000 15.0000	0.2 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 8.4000 9.4600 10.5600 11.4400 12.6000 13.5000	x(max)=Xa 1 0.3 1.0000 2.0000 3.0000 4.0000 4.3200 5.4600 6.5600 7.5600 8.6000 9.6800 9.6800 10.5600 11.7000 12.6000 13.8000	F _Q (X _a)=1 0.4 1.0000 2.0000 3.0000 4.0000 4.4000 5.4600 5.4600 6.5600 7.5600 8.6000 9.6800 9.6800 10.5600 11.7000 12.6000	0.5 1.0000 2.0000 3.0000 2.4000 3.4000 5.6000 6.5600 7.5600 8.6000 9.6800 10.8000 11.7000 12.6000 13.8000	0.6 1.0000 2.0000 3.0000 2.4000 3.5000 4.5600 5.6000 6.5600 7.7400 8.6000 9.6800 10.8000 11.7000 12.6000 13.8000	0.7 1.0000 2.0000 3.0000 2.4800 3.5000 4.5600 5.6000 6.7200 7.7400 8.8000 9.6800 10.8000 11.7000 12.8800 13.8000	0.8 1.0000 2.0000 1.5600 2.4800 3.5000 4.5600 5.6000 6.7200 7.7400 8.8000 9.6800 10.8000 11.7000 12.8800 13.8000
X _a \Q ₃ 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16	-0.8 1.0000 1.8800 2.2800 3.1200 4.1000 5.0400 7.0400 7.0400 7.9200 9.0000 9.9000 11.0400 11.9600 12.8800 13.8000 15.0400	-0.7 1.0000 2.0000 3.2000 4.1000 5.0400 6.0200 7.0400 7.9200 9.0000 9.0000 11.0400 11.9600 12.8800 13.8000 15.0400	x(max)=X _a -0.6 1.0000 2.5200 3.2800 4.2000 5.1600 6.0200 7.0400 8.1000 9.0000 9.90000 11.0400 11.9600 12.8800 14.1000 15.0400	F _Q (X _a)=1 -0.5 1.0000 2.7000 3.4400 4.3000 5.1600 6.1600 7.0400 8.1000 9.0000 10.1200 11.0400 11.9600 12.8800 14.1000 15.0400	-0.4 1.0000 2.0000 3.0000 3.6800 4.5000 6.3000 7.2000 8.1000 9.2000 10.1200 11.0400 11.9600 13.1600 14.1000 15.0400	-0.3 1.0000 2.0000 3.0000 4.8000 5.6400 6.4400 7.3600 8.2800 9.2000 10.1200 11.280 11.280	-0.2 1.0000 2.0000 3.0000 4.0000 5.0000 7.0000 7.8400 8.6400 9.6000 10.5600 11.2800 11.2800 13.4400 14.4000 15.3600	-0.1 1.0000 2.0000 3.0000 4.0000 5.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000 14.0000 15.6800	x(max) 0 1.0000 2.0000 3.0000 4.0000 5.0000 7.0000 8.0000 9.0000 10.0000 11.0000 11.0000 13.0000 14.0000 15.0000	0.1 1.0000 2.0000 3.0000 4.0000 5.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000 14.0000 15.0000	0.2 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 8.4000 9.4600 9.4600 10.5600 11.4400 12.6000 13.5000 14.7200	x(max)=X _a 1 0.3 1.0000 2.0000 3.0000 4.0000 4.3000 5.4600 5.4600 6.5600 7.5600 8.6000 9.6800 9.6800 10.5600 11.7000 12.6000 13.8000 14.7200	F _Q (X _a)=1 0.4 1.0000 2.0000 3.0000 4.0000 3.4000 3.4000 5.4600 6.5600 7.5600 8.6000 9.6800 10.5600 11.7000 11.7000 13.8000 14.7200	0.5 1.0000 2.0000 3.0000 2.4000 4.5600 5.6000 6.5600 7.5600 8.6000 9.6800 10.8000 11.7000 13.8000 14.7200	0.6 1.0000 2.0000 3.0000 2.4000 3.5000 4.5600 5.6000 6.5600 7.7400 8.6000 9.6800 10.8000 11.7000 12.6000 13.8000 14.7200	0.7 1.0000 2.0000 3.0000 2.4800 3.5000 4.5600 5.6000 6.7200 7.7400 8.8000 9.6800 10.8000 11.7000 12.8800 13.8000 14.7200	0.8 1.0000 2.0000 1.5600 2.4800 3.5000 5.6000 5.6000 6.7200 7.7400 8.8000 9.6800 10.8000 11.7000 12.8800 13.8000 14.7200
X _a \Q ₃ 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17	-0.8 1.0000 1.8800 2.2800 3.1200 4.1000 5.0400 6.0200 7.0400 7.9200 9.0000 9.9000 11.0400 11.9600 12.8800 13.8000 15.0400 15.9800	-0.7 1.0000 2.0000 3.2000 4.1000 5.0400 6.0200 7.0400 7.9200 9.0000 11.0400 11.9600 12.8800 13.8000 15.0400 15.9800	x(max)=X _a -0.6 1.0000 2.0000 3.2800 4.2000 5.1600 6.0200 7.0400 8.1000 9.0000 9.9000 11.0400 11.9600 12.8800 14.1000 15.0400 15.9800	F _Q (X _a)=1 -0.5 1.0000 2.7000 3.4400 4.3000 5.1600 6.1600 7.0400 8.1000 9.0000 10.1200 11.0400 11.9600 12.8800 14.1000 15.0400 15.9800	-0.4 1.0000 2.0000 3.0800 4.5000 5.4000 6.3000 7.2000 10.1200 11.0400 11.9600 13.1600 14.1000 15.0400 15.9800	-0.3 1.0000 2.0000 3.0000 4.8000 5.6400 6.4400 7.3600 8.2800 9.2000 10.1200 11.2800 12.2200 13.1600 14.1000 15.0400 15.9800	-0.2 1.0000 2.0000 3.0000 5.0000 6.0000 7.0000 7.8400 9.6000 10.5600 11.2800 11.2800 12.2200 13.4400 13.4400 15.3600 16.3200	-0.1 1.0000 2.0000 3.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 11.0000 13.0000 14.0000 15.6800 16.6600	x(max) 0 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 10.0000 11.0000 11.0000 11.0000 14.0000 15.0000 16.0000 17.0000	0.1 1.0000 2.0000 3.0000 5.0000 6.0000 7.0000 8.0000 10.0000 11.0000 12.0000 14.0000 14.0000 15.0000 17.0000	0.2 1.0000 2.0000 3.0000 5.0000 6.0000 7.0000 8.0000 9.0000 8.4000 9.4600 10.5600 11.4400 12.6000 13.5000 14.7200 15.6400	x(max)=X _a 1 0.3 1.0000 2.0000 3.0000 4.0000 5.0000 4.3200 5.4600 6.5600 7.5600 7.5600 10.5600 11.7000 12.6000 13.8000 14.7200 15.6400	F _Q (X _a)=1 0.4 1.0000 2.0000 3.0000 4.0000 3.4000 4.4400 5.4600 6.5600 7.5600 7.5600 10.5600 11.7000 11.6000 13.8000 14.7200 15.6400	0.5 1.0000 2.0000 3.0000 3.4000 4.5600 6.5600 6.5600 7.5600 8.6000 9.6800 10.8000 11.7000 12.6000 14.7200 15.6400	0.6 1.0000 2.0000 3.0000 2.4000 3.5000 4.5600 6.5600 6.5600 7.7400 8.6000 9.6800 10.8000 11.7000 12.6000 14.7200 15.6400	0.7 1.0000 2.0000 3.0000 2.4800 3.5000 4.5600 5.6000 6.7200 7.7400 8.8000 9.6800 10.8000 11.7000 12.8800 13.8000 14.7200 15.6400	0.8 1.0000 2.0000 1.5600 3.5000 4.5600 5.6000 6.7200 7.7400 8.8000 9.6800 10.8000 11.7000 12.8800 13.8000 14.7200 15.6400
X _a \Q ₃ 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18	-0.8 1.0000 1.8800 2.2800 3.1200 4.1000 5.0400 6.0200 7.0400 7.9200 9.0000 9.9000 11.0400 11.9600 12.8800 13.8000 15.9400 15.9800 16.9200	-0.7 1.0000 2.0000 3.2000 4.1000 5.0400 6.0200 7.0400 7.0400 7.9200 9.0000 9.9000 11.0400 11.9600 12.8800 13.8000 15.9400 15.9800 16.9200	x(max)=X _a -0.6 1.0000 2.0000 2.5200 3.2800 4.2000 5.1600 6.0200 7.0400 7.0400 9.9000 9.9000 11.0400 11.9600 12.8800 14.1000 15.9800 16.9200	F _Q (X _a)=1 -0.5 1.0000 2.7000 3.4400 4.3000 5.1600 6.1600 7.0400 10.1200 10.1200 11.0400 11.9600 12.8800 14.1000 15.9400 15.9800 16.9200	-0.4 1.0000 2.0000 3.0000 3.6800 4.5000 5.4000 6.3000 7.2000 8.1000 9.2000 10.1200 11.0400 11.9600 13.1600 14.1000 15.9400 15.9800 16.9200	-0.3 1.0000 2.0000 3.0000 4.0000 4.8000 5.6400 6.4400 7.3600 8.2800 9.2000 10.1200 11.2800 12.2200 13.1600 14.1000 15.9400 15.9800 16.9200	-0.2 1.0000 2.0000 3.0000 5.0000 6.0000 7.0000 7.8400 8.6400 9.6000 10.5600 11.2800 11.2800 13.4400 14.4000 15.3600 16.3200 17.2800	-0.1 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 11.0000 13.0000 14.0000 15.6800 16.6600 17.6400	x(max) 0 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 11.0000 13.0000 14.0000 15.0000 16.0000 17.0000 18.0000	0.1 1.0000 2.0000 3.0000 5.0000 6.0000 7.0000 8.0000 10.0000 11.0000 12.0000 13.0000 14.0000 15.0000 16.0000 17.0000 18.0000	0.2 1.0000 2.0000 3.0000 6.0000 7.0000 8.0000 9.0000 8.4000 9.4600 10.5600 11.4400 12.6000 13.5000 14.7200 15.6400 16.5600	x(max)=X _a 1 0.3 1.0000 2.0000 3.0000 4.0000 5.0000 4.3200 6.5600 7.5600 7.5600 10.5600 10.5600 11.7000 12.6000 13.8000 14.7200 15.6400 16.5600	F _Q (X _a)=1 0.4 1.0000 2.0000 3.0000 4.0000 3.4000 4.4400 5.4600 6.5600 7.5600 7.5600 10.5600 10.5600 11.7000 13.8000 13.8000 14.7200 15.6400 16.5600	0.5 1.0000 2.0000 3.4000 4.5600 5.6000 6.5600 7.5600 9.6800 10.8000 11.7000 12.6000 13.8000 14.7200 15.6400 16.9200	0.6 1.0000 2.0000 3.0000 2.4000 3.5000 4.5600 6.5600 7.7400 8.6000 9.6800 10.8000 11.7000 12.6000 13.8000 14.7200 15.6400 16.9200	0.7 1.0000 2.0000 2.4800 3.5000 4.5600 6.7200 7.7400 9.6800 10.8000 11.7000 12.8800 13.8000 13.8000 14.7200 15.6400 16.9200	0.8 1.0000 2.0000 1.5600 2.4800 3.5000 4.5600 6.7200 7.7400 8.8000 9.6800 10.8000 11.7000 11.7000 13.8000 13.8000 14.7200 15.6400 16.9200
X _a \Q ₃ 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19	-0.8 1.0000 1.8800 2.2800 3.1200 4.1000 5.0400 6.0200 7.0400 9.9000 9.9000 11.0400 11.9600 12.8800 13.8000 15.0400 15.9800 16.9200 17.8600	-0.7 1.0000 2.0000 2.4000 3.2000 4.1000 5.0400 6.0200 7.9200 9.9000 11.0400 11.9600 11.9600 13.8000 15.0400 16.9200 17.8600	x(max)=X _a -0.6 1.0000 2.5200 3.2800 4.2000 5.1600 6.0200 7.0400 8.1000 9.9000 11.0400 11.9600 11.9800 14.1000 15.9800 16.9200 17.8600	F _Q (X _a)=1 -0.5 1.0000 2.7000 3.4400 4.3000 5.1600 6.1600 7.0400 8.1000 9.0000 10.1200 11.0400 11.9600 14.1000 15.9800 16.9200 17.8600	-0.4 1.0000 2.0000 3.0000 3.6800 4.5000 5.4000 6.3000 7.2000 8.1000 9.2000 10.1200 11.0400 11.9600 13.1600 14.1000 15.9800 16.9200 17.8600	-0.3 1.0000 2.0000 3.0000 4.0000 5.6400 6.4400 7.3600 8.2800 9.2000 10.1200 11.2800 12.2200 13.1600 14.1000 15.9800 16.9200 18.2400	-0.2 1.0000 2.0000 3.0000 5.0000 6.0000 7.8400 8.6400 9.6000 10.5600 11.2800 11.2800 12.2200 13.4400 14.4000 15.3600 16.3200 17.2800 18.2400	-0.1 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 10.0000 11.0000 11.0000 13.0000 14.0000 15.6800 15.6800 17.6400 18.6200	x(max) 0 1.0000 2.0000 3.0000 4.0000 6.0000 7.0000 8.0000 10.0000 11.0000 12.0000 13.0000 14.0000 15.0000 15.0000 15.0000 16.0000 17.0000 18.0000 19.0000	0.1 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000 10.0000 11.0000 12.0000 13.0000 14.0000 15.0000 16.0000 17.0000 18.0000 19.0000	0.2 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.4000 9.4600 10.5600 11.4400 12.6000 13.5000 14.7200 15.6400 16.5600 17.4800	x(max)=X _a 1 0.3 1.0000 2.0000 3.0000 4.0000 5.0000 4.3200 5.4600 6.5600 7.5600 7.5600 10.5600 10.5600 11.7000 12.6000 14.7200 15.6400 15.6400 16.5600 17.8600	F _Q (X _a)=1 0.4 1.0000 2.0000 3.0000 4.0000 4.4000 5.4600 6.5600 7.5600 7.5600 9.6800 10.5600 11.7000 13.8000 14.7200 15.6400 16.5600 17.8600	0.5 1.0000 2.0000 3.4000 4.5600 5.6000 6.56000 7.5600 8.6000 9.6800 10.8000 11.7000 12.6000 13.8000 14.7200 15.6400 15.6400 16.9200 17.8600	0.6 1.0000 2.0000 3.0000 2.4000 3.5000 4.5600 5.6000 7.7400 8.6000 9.6800 10.8000 11.7000 12.6000 13.8000 14.7200 15.6400 16.9200 17.8600	0.7 1.0000 2.0000 2.4800 3.5000 4.5600 6.7200 7.7400 9.6800 10.8000 11.7000 12.8800 13.8000 14.7200 15.6400 15.6400 16.9200 17.8600	0.8 1.0000 2.0000 1.5600 2.4800 3.5000 4.5600 6.7200 7.7400 9.6800 10.8000 11.7000 12.8800 13.8000 14.7200 15.6400 16.9200 17.8600

 $-0.8 < Q_3 < 0$; v* = 0.4



Graph F_Q vs X_a and 0<Q₃<+0.8

$$0 < Q_3 < 0.8$$
; v* = 0.4



X _a \Q ₃	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
1	2.5	2.857143	3.333333	4	5	6.666667	8.998657	6.240394	4.77635	3.868719	3.250954	2.803314	2.464029	2.198006	1.983826	1.807681	1.660265
2	2.51432	2.857143	3.333333	4	5	6.666667	10	7.523566	5.848846	4.783954	4.047103	3.506946	3.093996	2.767785	2.494105	2.259883	2.05977
3	3.35501	3.500383	3.73087	4.141388	5	6.666666	10	13.86455	10.87076	8.093174	6.187609	4.914857	4.039271	3.412036	2.944396	2.58429	2.371012
4	4.332228	4.397226	4.512783	4.736272	5.24374	6.666666	9.999999	20	16.22084	10.19497	7.142984	5.417763	4.330965	4.148806	4.161247	4.170054	4.176638
5	5.184925	5.232549	5.310653	5.46412	5.80719	6.780873	10	20	20.41941	11.54998	7.711372	5.697706	5.190115	5.178172	5.175274	5.16785	5.162176
6	6.075725	6.108031	6.168247	6.2791	6.534085	7.23438	10	20	24.54934	12.67999	8.133738	6.055543	6.057201	6.043075	6.052838	6.045625	6.037598
7	6.989395	7.016077	7.051653	7.156484	7.35005	7.87972	10	20	28.86764	13.61935	8.465816	6.970071	6.971029	6.959758	6.960386	6.960843	6.96119
8	7.89696	7.926769	7.96651	8.022152	8.20139	8.622153	10.25692	20	33.19143	14.41635	8.714232	7.888013	7.886964	7.886306	7.885861	7.878157	7.877766
9	8.80632	8.812226	8.855463	8.934016	9.051845	9.413327	10.75545	20	37.58715	15.10373	8.940473	8.806101	8.822027	8.798765	8.780189	8.786229	8.790792
10	9.72875	9.749374	9.77687	9.815368	9.92494	10.22883	11.33498	20	41.82752	15.61794	9.707451	9.747581	9.734529	9.705679	9.686176	9.672886	9.688325
11	10.62868	10.63051	10.63296	10.70518	10.83327	11.04675	11.98035	20	46.23938	16.14145	10.64935	10.62359	10.62964	10.63336	10.63587	10.63769	10.63906
12	11.51917	11.54736	11.58494	11.63755	11.71648	11.89494	12.6891	20	50.55819	16.52118	11.62482	11.54198	11.49611	11.49684	11.53397	11.56074	11.58096
13	12.47236	12.48471	12.50117	12.52422	12.5588	12.79733	13.44304	20	55.22265	16.83206	12.438	12.43668	12.47522	12.49884	12.51481	12.52633	12.53502
14	13.36011	13.35825	13.35577	13.3523	13.44501	13.6986	14.22442	20	59.26641	17.16312	13.38468	13.37383	13.3682	13.36474	13.3624	13.38801	13.38615
15	14.17231	14.15778	14.20962	14.28528	14.39875	14.58788	15.04863	20	63.69394	17.51477	14.24479	14.19137	14.31318	14.38781	14.40964	14.39027	14.37574
16	15.13227	15.15997	15.19691	15.24862	15.32618	15.45545	15.89881	20.29252	68.54565	17.7724	15.11083	15.26828	15.3501	15.36974	15.32167	15.28733	15.26158
17	16.10813	16.12396	16.14506	16.1746	16.21892	16.29277	16.76836	20.77014	72.90249	17.8918	16.15014	16.23864	16.28462	16.18856	16.12208	16.07459	16.03898
18	17.0356	17.04047	17.04695	17.05604	17.06966	17.09236	17.65105	21.28718	76.86542	18.01652	17.07245	17.09924	17.01448	16.94706	16.95614	16.96263	16.96749
19	17.90864	17.90342	17.89646	17.88672	17.87211	17.85515	18.54106	21.84126	81.1034	18.14656	17.87644	18.03008	18.01824	18.00362	17.99388	17.98693	17.98171
20	18.72217	18.7077	18.68842	18.66142	18.62093	18.84149	19.43298	22.43002	85.63786	18.77245	18.86903	19.09339	19.02589	18.98539	18.9584	18.93911	18.92465

ANNEX R — DETERMINATION OF THE ALLOWABLE BUCKLING STRESS LIMITS

If the tubes are under compression, tube buckling may restrict the tube's load carrying ability. If a substantial number of tubes are above their buckling limit, it is possible that the bundle cannot sustain the required loading. This is true for either pressure or thermal load conditions. For this reason, no distinction is made between primary and secondary allowable compressive loads in the tubes. The maximum permissible buckling stress limit in UHX parallels that as given in TEMA [12].

- (a) When tubes are under compression, the axial tube stress $\sigma_t(x)$ is negative and must be limited to:
 - The critical Euler's stress $\sigma_{cr} = \frac{\pi^2 E_t}{\lambda_t^2}$ where: λ_t is the slenderness ratio of the tube: $\lambda_t = \frac{l_t}{r_t}$

$$r_t$$
 is the tube radius of gyration: $r_t = \frac{\sqrt{d_t^2 + (d_t - 2t_t)^2}}{4}$

 $l_t = k l$ is the largest unsupported buckling length of the tube, obtained from the unsupported tube

spans, l, and their corresponding method of support k, where:

k=0.6 for unsupported spans between two tubesheets,

k=0.8 for unsupported spans between a tubesheet and a tube support

k=1.0 for unsupported spans between two tube supports.

• The yield strength, $S_{y,t}$ of the tube material to prevent entering in the plastic range, which might lead to the failing of the tube bundle.

Figure 69 shows the forbidden range for σ_t represented by the hatched areas.

So as to ensure a smooth transition, TEMA has drawn a line from point A_o (λ_o ; $S_{y,t}/2$) to point A (0; $S_{y,t}$) which equation is written:

$$\sigma_{t,bk} = S_{y,t} \left(1 - \frac{\lambda_t}{2\lambda_o} \right) \qquad \text{with: } \lambda_o = \sqrt{\frac{2\pi^2 E_t}{S_{y,t}}}$$

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Figure 69 — Tube Buckling

- (b) Accordingly, the buckling stress limit $\sigma_{t,bk}$ is given by:
 - $\sigma_{t,bk} = S_{y,t} \left(1 \frac{\lambda_t}{2\lambda_o} \right)$ if $\lambda_t < \lambda_o$. The tubes have a low slenderness ratio and plastic buckling prevails

prevails.

• $\sigma_{t,bk} = \frac{\pi^2 E_t}{\lambda_t^2}$ if $\lambda_{t \ge} \lambda_{o}$. The tubes have a high slenderness ratio and elastic buckling prevails.

Note: The slope of line AA_o is written: $-\frac{S_{y,t}}{2\lambda_o}$ The slope of Euler's curve at point A_o is written:

$$\left[\frac{d\sigma}{d\lambda}\right]_{A_o} = -\frac{2\pi^2 E_t}{\lambda_o^3} = -\frac{S_{y,t}}{\lambda_o} \quad \text{which is half of the slope of line AAo}$$

A smoother transition would be obtained if the line was tangent to the Euler's curve, at point A_1 on Figure 69,

with a slope:
$$\left[\frac{d\sigma_{cr}}{d\lambda_t}\right]_{A_1} = -\frac{2\pi^2 E_t}{\lambda_1^3}$$
. Equation of line AA₁ is written: $\sigma_{t,bk} = S_{y,t} - \frac{2\pi^2 E_t}{\lambda_1^3} \lambda$

The abscissa λ_1 of point $\lambda_1 A_1$ is obtained from: $S_{y,t} - \frac{2\pi^2 E_t}{\lambda_1^3} \lambda_1 = \frac{\pi^2 E_t}{\lambda_1^2}$,

which leads to: $\lambda_1 = \sqrt{1.5}\lambda_o$

The ordinate of point A₁ is obtained from: $[\sigma_{cr}]_{A_1} = \frac{\pi^2 E_t}{\lambda_1^2} = \frac{\pi^2 E_t}{1.5\lambda_o^2} = \frac{S_{y,t}}{3}$

Line AA₁, starting at $S_{y,t}/3$ instead of $S_{y,t}/2$, would ensure a better transition between elastic and plastic buckling.

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(c) The safety factor applied to $\sigma_{t,bk}$ used in UHX is based on TEMA approach as follows. The tube stress distribution throughout the tube bundle is given in Section 8.5 of PART 3:

$$\sigma_{t}(x) = \frac{1}{x_{t} - x_{s}} \left[\left(x_{s} P_{s} - x_{t} P_{t} \right) - F_{t}(x) P_{e} \right] \text{ with } F_{t}(x) = \frac{X_{a}^{4}}{2} \left[Q_{3} Z_{w}(x) + Z_{d}(x) \right] \text{ [VIII-5]}$$

x=kr

At periphery (x=ka₀=Xa):
$$F_t(X_a) = F_q = \frac{X_a^4}{2} [Q_3 Z_w + Z_d]$$

The following simplifications are made in TEMA.

(1) No unperforated rim:
$$\begin{array}{c} a_o = a_s \Rightarrow \rho_s = 1 \\ a_o = a_c \Rightarrow \rho_c = 1 \end{array} a_o = a_s = a_c = a \Rightarrow Q_1 = -\frac{\Phi Z_v}{1 + \Phi Z_m} \end{array}$$

(2) No flange is considered: W = 0, A = D_o \Rightarrow K = 1 \Rightarrow $\Phi = \frac{1 - \upsilon^{*2}}{E^*} [\lambda_s + \lambda_c]$

(3) Radial displacement due to pressures P_s and P_t acting at TS-shell-channel connection is ignored:

$$w_{s} = 0 \Rightarrow \delta_{s} = 0 \Rightarrow \omega_{s} = 0 \Rightarrow \omega_{s}^{*} = 0$$

$$w_{c} = 0 \Rightarrow \delta_{c} = 0 \Rightarrow \omega_{c} = 0 \Rightarrow \omega_{c}^{*} = 0$$

$$F_{t}(x) = \frac{X_{a}^{4}}{2} \left[-\frac{\Phi Z_{v}}{1 + \Phi Z_{m}} Z_{w}(x) + Z_{d}(x) \right]$$

(4) The maximum of $F_t(x)$ appears either inside of the tube-bundle (r<R) or at its periphery (r=R). TEMA considers only the outermost tube row.

At periphery x=X_a=kR:
$$F_t(X_a) = F_q = \frac{X_a^4}{2} \left[-\frac{\Phi Z_v}{1 + \Phi Z_m} Z_w + Z_d \right]$$

- (5) The TS is considered
 - either simply supported: $\lambda_s = 0$, $\lambda_c = 0 \Longrightarrow \Phi = 0 \Longrightarrow Q_1 = 0$
 - or clamped: $\lambda_s = \infty$, $\lambda_c = \infty \Longrightarrow \Phi = \infty \Longrightarrow Q_1 = -\frac{Z_v}{Z_m}$

Curves $F_t(x)$ as a function of x=r/R for various values of X_a=kR (X_a=1, 2, 3, ..., 20) represent the tube stress distribution $\sigma_t(x)$ throughout the tube bundle given by [VIII-5] above. Examination of these curves shows that:

The stress distribution varies smoothly throughout the tube bundle when X_a is low, i.e. when the tube bundle rigidity is significantly lower than the TS rigidity:

$$X_a = 3$$
 when the TS is simply supported, which leads to $F_q = 2.25$
 $X_a = 7$ when the TS is clamped, which leads to $F_q = 2.7$ in both cases F_q is about 2.5.

In that case, a large amount of adjacent tube rows may not take over the extra compressive load, which would lead to a general buckling of the tube bundle. Accordingly, a higher safety factor $F_s=2.0$ is used.

The stress distribution varies significantly throughout the tube bundle when X_a is high, i.e. when the tube bundle rigidity is significantly higher than the TS rigidity:

 $X_a = 5$ when the TS is simply supported, which leads to $F_q = 3.79$ $X_a = 10$ when the TS is clamped, which leads to $F_q = 3.83$ in both cases F_q is about 4.

In that case, if the outermost tube row buckles, the adjacent tube rows can take over the extra compressive load. Accordingly, a lower safety factor $F_s=1.25$ is used.

In the intermediate range of F_q (2.5 \leq $F_q \leq$ 4), a linear interpolation is used (see Figure 70): $F_s = 3.25 - 0.5F_q$

Thus, the safety factor is written: $F_s = MAX \left[(3.25 - 0.5F_q), (1.25) \right]$



Figure 70 — Determination of Buckling Safety Factor, FS

(*d*) Moving to UHX/TEMA notations ($F_t = \lambda_t$; $C_t = \lambda_o$), the maximum permissible buckling stress limit is written, having in mind that it cannot exceed the allowable stress of the tube material:

$$S_{tb} = MIN\left\{\left[\frac{1}{F_s}\frac{\pi^2 E_t}{F_t^2}\right], [S_t]\right\} \text{ when } C_t \leq F_t$$

$$S_{tb} = MIN\left\{\left[\frac{S_{y,t}}{F_s}\left(1 - \frac{F_t}{2C_t}\right)\right], [S_t]\right\} \text{ when } C_t > F_t$$
with: $F_t = \frac{l_t}{r_t}$ $r_t = \frac{\sqrt{d_t^2 + (d_t - 2t_t)^2}}{4}$ $C_t = \frac{2\pi^2 E_t}{S_{y,t}}$
 $F_s = MAX\left[\left(3.25 - 0.5F_q\right), (1.25)\right]$ $F_q = \frac{X_a^4}{2}\left[Q_3 Z_w + Z_d\right]$
The minimum value of $\sigma_t(\mathbf{x})$, given by: $\sigma_{t-t} = \text{MIN}\left[(\sigma_{t-t}), (\sigma_{t-t})\right]$ must not

The minimum value of $\sigma_t(\mathbf{x})$, given by: $\sigma_{t,\min} = \text{MIN}\lfloor (\sigma_{t,1}), (\sigma_{t,2}) \rfloor$ must not exceed the

buckling stress limit: $\sigma_{t,\min} \leq S_{tb}$

ANNEX S — COMMON INTERSECTION OF CURVES $\Sigma_t(x)$

1 General

Numerical calculations of H.E. according to UHX-13 show that $\sigma_t(x)$ curves (0<x<X_a) intersect at the same point x=x_o for the 7 loading cases (ASME 2013), whatever the Q₃ value is for each of these loading cases (see Figure 71).

$$\sigma_{t}(x) = \frac{1}{x_{t} - x_{s}} \left[\left(P_{s} \cdot x_{s} - P_{t} \cdot x_{t} \right) - P_{e} \cdot F_{t}(x) \right]$$

 $\sigma_t(x)$ is calculated from $F_t(x)$:

$$F_{t}(x) = [Z_{d}(x) + Q_{3} \cdot Z_{w}(x)] \cdot \frac{X_{a}^{4}}{2}$$



Figure 71 — Graphs Giving $\sigma_t(x)$ and $F_t(x)$ for the 7 Loading Cases (ASME 2013)

2 Determination of Common Intersection x_0 for $\sigma_t(x)$

Curves $F_t(x)$ will intersect at the same point of abscissa x_0 , whatever Q_3 value is, if:

$$\frac{2}{X_{c}^{4}}F_{t}\left(x_{o}\right) = Z_{d}\left(x_{o}\right) + Q_{3} \cdot Z_{w}\left(x_{o}\right) = K$$

Where K is a constant independent of Q_3 . This is only possible if $Z_w(x_o) = 0$, which means that the value x_o of common intersection is obtained for value(s) of x_o which make $Z_w(x)$ equal to 0. Parametric calculations on X_a show that there is always at least 1 value of x_o which makes $Z_w(x_o) = 0$. For high values of X_a ($X_a > 6$), there are 2 values of x_o . For Example E4.18.7 ($X_a = 7$): $Z_w(x) = 0$ for $x_o = 1.68$ and $x_o = 5.97$.

The curves $\sigma_t(x)$ for the 3 pressure loading cases (ASME 2013) intersect at the same point of abscissa x_1 . This point is different from the intersection point x_0 of $F_t(x)$, due to the presence of P_e .

The curves $\sigma_t(x)$ for the 4 pressure and thermal loading cases (ASME 2013) intersect at the same point of abscissa x_1 , but their values are generally higher than for the 3 pressure loading cases due to higher values of P_e for thermal loading cases.

For Example E4.18.7 (2013):

• curves relative to pressure loading cases 1, 2 and 3 intersect at the same point ($x_1 = 5.5$; $\sigma_t = 0$)



• curves relative to pressure and thermal loading cases 4, 5, 6 and 7 intersect at the same point ($x_1 = 5.5$; $\sigma_t = 1700$ psi)

3 Generalization to Other Stresses

The same principle applies also to:

the tubesheet bending stress $\sigma(\mathbf{x})$, calculated from: $F_m(x) = [Q_v(x) + Q_3 \cdot Q_m(x)] \cdot \frac{1}{2}$

Value of intersection is obtained for $Q_m(x_o)=0$. There is no value of x_o for $X_a<5$, and one value of x_o for $X_a\geq 5$

the tubesheet shear stress $\tau(\mathbf{x})$, calculated from: $F_o(x) = [Q_{\beta}(x) + Q_3 \cdot Q_{\alpha}(x)]$

Value of intersection is obtained for $Q_{\alpha}(x_o)=0$. There is always one solution for $x_o=X_a$ (as $Q_{\alpha}(X_a)=0$), and a 2nd value of x_o for $X_a \ge 7$

the tubesheet slope $\theta(\mathbf{x})$, calculated from: $F_{\theta}(x) = [Z_v(x) + Q_3 \cdot Z_m(x)] \cdot \frac{X_a^3}{2}$

ANNEX T — DETERMINATION OF STRESSES IN U-TUBE TS HES USING THE FIXED TS RULES

UHX -12 rules can be obtained using the fixed TS analysis developed in PART 3, provided that the tubes have no axial rigidity, which implies that they do not play the role of an elastic foundation.

Accordingly:
$$k_t = 0 \implies k_w = \frac{N_t k_t}{\pi a_o^2} = 0 \implies X_a = 0$$

Modifications of PART 3 analysis are as follows.

- (a) Equilibrium of the Unperforated Rim (see Sections 6.2 and 6.3 of PART 3)
 - (1) **due to axial loads** axial equilibrium of the shell: $2\pi a_s^2 V_s = \pi a_s^2 P_s$ leads to: $a_o V_a = \frac{a_o^2}{2} P_e$
 - (2) **due to applied moments:** explicating V_a, equation [VI.2b'] of PART 3 becomes:

$$\begin{bmatrix} R M_{R} \end{bmatrix} = -a_{o} M_{a} + \frac{P_{s}}{4} a_{o}^{3} \begin{bmatrix} (\rho_{s} - 1)(\rho_{s}^{2} + 1) \end{bmatrix} - \frac{P_{c}}{4} a_{o}^{3} \begin{bmatrix} (\rho_{c} - 1)(\rho_{c}^{2} + 1) \end{bmatrix} \\ - \begin{bmatrix} a_{s} K_{s} \begin{bmatrix} 1 + t_{s} + \frac{t_{s}^{2}}{2} \end{bmatrix} + a_{c} K_{c} \begin{bmatrix} 1 + t_{c} + \frac{t_{c}^{2}}{2} \end{bmatrix} \end{bmatrix} \theta_{a} + a_{o} (\omega_{c} P_{c} - \omega_{s} P_{s})$$

equation [VI.3a'] of PART 3 becomes:

$$\Phi \left[M_{a} Z_{m} + (a_{o} V_{a}) Z_{v} \right] = -M_{a} + (\omega_{s}^{*} P_{s} - \omega_{c}^{*} P_{c}) + \frac{1}{2\pi} \left[W_{c} \gamma_{bc} - W_{s} \gamma_{bs} \right]$$

with: $\Phi = (1 + \upsilon^{*}) F \left[\omega_{s}^{*} = a_{o}^{2} \left[\frac{(\rho_{s} - 1)(\rho_{s}^{2} + 1)}{4} \right] - \omega_{s} \right] \left[\omega_{c}^{*} = a_{o}^{2} \left[\frac{(\rho_{c} - 1)(\rho_{c}^{2} + 1)}{4} \right] - \omega_{c} \right]$

(b) Stresses in the Tubesheet (see Section 6.3 of PART 3)

(1) moment at TS periphery

Equation [VI.3a] of PART 3 becomes:

$$M_{a} = \left(\frac{a_{o}^{2}}{2}P_{e}\right) \underbrace{\frac{-\Phi Z_{v}}{1+\Phi Z_{m}}}_{Q_{1}} + \underbrace{\frac{\left(\omega_{s}^{*}P_{s}-\omega_{c}^{*}P_{c}\right) + \left(W_{c}\frac{\gamma_{bc}}{2\pi} - W_{s}\frac{\gamma_{bs}}{2\pi}\right)}{Q_{2}}}_{Q_{2}}$$

$$V_{v} Q_{1} + Q_{2} \qquad Q_{1} = \underbrace{-\Phi Z_{v}}_{Q_{v}} \qquad Q_{2} - \underbrace{\left(\omega_{s}^{*}P_{s}-\omega_{c}^{*}P_{c}\right) + \left(W_{c}-W_{s}\right)\frac{\gamma_{b}}{2\pi}}_{Q_{2}}$$

$$M_{\rm a} = (a_{\rm o} V_{\rm a}) Q_{\rm l} + Q_{\rm 2} \qquad Q_{\rm l} = \frac{-\Phi Z_{\rm v}}{1 + \Phi Z_{\rm m}} \qquad Q_{\rm 2} = \frac{(\omega_{\rm s} T_{\rm s} - \omega_{\rm c} T_{\rm c}) + (w_{\rm c} - w_{\rm s}) - 2}{1 + \Phi Z_{\rm m}}$$

From Annex F, for $X_a=0$:

$$Z_{m} = \frac{1}{1 + \upsilon^{*}} \qquad \Phi Z_{m} = F \qquad Z_{\nu} = \frac{1}{4(1 + \upsilon^{*})} \qquad \Phi Z_{\nu} = \frac{F}{4}$$
$$Q_{1} = \frac{\frac{F}{4}}{1 + F} \qquad Q_{2} = \frac{\left(\omega_{s}^{*} P_{s} - \omega_{c}^{*} P_{c}\right) + \left(W_{c} - W_{s}\right)\frac{\gamma_{b}}{2\pi}}{1 + F}$$



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Explicating ω_s^* and ω_c^* , Q₂ is written:

$$Q_{2} = \frac{P_{s} \frac{a_{o}^{2}}{4} \Big[(\rho_{s} - 1) (\rho_{s}^{2} + 1) \Big] - P_{c} \frac{a_{o}^{2}}{4} \Big[(\rho_{c} - 1) (\rho_{c}^{2} + 1) \Big] + (\omega_{c} P_{c} - \omega_{s} P_{s}) + (W_{c} - W_{s}) \frac{\gamma_{b}}{2\pi}}{1 + F}$$

$$M_{a} = \frac{M_{rs}^{*}}{\frac{P_{s} \frac{a_{o}^{2}}{4} \left[\left(\rho_{s} - 1 \right) \left(\rho_{s}^{2} + 1 \right) \right] - P_{c} \frac{a_{o}^{2}}{4} \left[\left(\rho_{c} - 1 \right) \left(\rho_{c}^{2} + 1 \right) \right]} + \left(\omega_{c} P_{c} - \omega_{s} P_{s} \right) + \frac{\gamma_{b}}{2\pi} \left(W_{c} - W_{s} \right) - F \frac{a_{o}^{2}}{8} P_{e}}{1 + F} = \frac{M^{*} - F \left(\frac{a_{o}^{2}}{8} P_{e} \right)}{1 + F}$$

 W_s and W_c values to be used in UHX-12 are given in Section 5.2(d) of PART 5.

(2) moment at TS center is given by Annex F when x=0

$$M_{0} = M(0) = a_{o}^{2} P_{e} F_{m}(0) \text{ with: } F_{m}(0) = \frac{Q_{3}}{2} + \frac{3 + \upsilon^{*}}{16} \qquad Q_{3} = Q_{1} + \frac{2}{a_{0}^{2} P_{e}} Q_{2}$$

Using Q₁ and Q₂ above:
$$Q_{3} = \frac{-\frac{F}{4}}{1+F} + \frac{2}{(a_{o}^{2} P_{e})} \frac{(\omega_{s}^{*} P_{s} - \omega_{c}^{*} P_{c}) + (W_{c} - W_{s}) \frac{\gamma_{b}}{2\pi}}{1+F}$$

$$M_{o} = \frac{-\frac{F}{4}a_{o}^{2}P_{e} + \left(\omega_{s}^{*}P_{s} - \omega_{c}^{*}P_{c}\right) + \left(W_{c} - W_{s}\right)\frac{\gamma_{b}}{2\pi}}{1+F} + \frac{a_{o}^{2}}{16}\left(3+\upsilon^{*}\right)P_{e} = M_{a} + \frac{a_{o}^{2}}{16}\left(3+\upsilon^{*}\right)P_{e}$$

(3) maximum moment in TS

Annex P shows that for low values of X_a ($X_a < 1$), the maximum of the TS moment M(x) appears either at the center or at TS periphery: $M = MAX [|M_p|, |M_o|]$

(c) Stresses in the Shell and Channel (see Section 8.6 an 8.7 of PART 3)

(1) axial membrane stress

From [VIII.6a] of PART 3:

$$\sigma_{s,m} = \frac{a_o^2}{(D_s + t_s)t_s} \Big[(P_s - P_t) + (\rho_s^2 - 1)(P_s - P_t) \Big] + \frac{a_s^2}{(D_s + t_s)t_s} P_t = \frac{a_o^2}{(D_s + t_s)t_s} \Big[\rho_s^2 (P_s - P_t) \Big] + \frac{a_s^2}{(D_s + t_s)t_s} \sigma_{s,m} = \frac{a_o^2}{(D_s + t_s)t_s} \Big[\rho_s^2 (P_s - P_t) \Big] + \frac{a_s^2}{(D_s + t_s)t_s} P_t$$

 $\left|\sigma_{s,m} = \frac{a_s}{\left(D_s + t_s\right)t_s}P_s\right|$ which is the classical formula for cylinders

Axial membrane stress in the channel is written in the same way:

$$\sigma_{\rm c,m} = \frac{a_c^2}{\left(D_c + t_c\right)t_c} P_t$$

(2) **bending stress** From [VIII.6c] of PART 3:

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$$\sigma_{s,b} = \frac{6}{t_s^2} k_s \left\{ \beta_s \,\delta_s \,P_s + \frac{6(1 - v^{*2})}{E^*} \left(\frac{a_o}{h}\right)^3 \left(1 + \frac{h \,\beta_s}{2}\right) \left[P_e \left(Z_v + Q_1 \,Z_m\right) + \frac{2}{a_o^2} \left(Q_2 \,Z_m\right) \right] \right\}$$

From Annex F, for X_a=0:
$$Z_m = \frac{1}{1+\upsilon^*}$$
 $Z_v = \frac{1}{4(1+\upsilon^*)}$

$$\sigma_{s,b} = \frac{6}{t_s^2} k_s \left\{ \beta_s \,\delta_s \, P_s + \frac{6(1-\upsilon^{*2})}{E^*} \left(\frac{a_o}{h}\right)^3 \left(1 + \frac{h \,\beta_s}{2}\right) \left[P_e \left(\frac{1}{4(1+\upsilon^*)} + Q_1 \,\frac{1}{(1+\upsilon^*)}\right) + \frac{2}{a_o^2} \left(Q_2 \,\frac{1}{(1+\upsilon^*)}\right) \right] \right\}$$

$$\sigma_{s,b} = \frac{6}{t_s^2} k_s \left\{ \beta_s \,\delta_s \,P_s + 6 \,\frac{1 - v^{*2}}{E^*} \left(\frac{a_o}{h}\right)^3 \left(1 + \frac{h \,\beta_s}{2}\right) \frac{1}{1 + v^*} \frac{2}{a_o^2} \left[P_e \frac{a_o^2}{8} + \underbrace{P_e Q_1 \frac{a_o^2}{2} + Q_2}_{M_a} \right] \right\}$$

$$\boxed{\sigma_{s,b} = \frac{6}{2} k_s \left[\beta_s \delta_s P_s + 6 \frac{1 - v^{*2}}{E^*} \frac{D_o}{4^2} \left(1 + \frac{h \beta_s}{2}\right) \left(M_a + \frac{D_o^2}{E^*} P_e\right) \right]}$$

$$\left[\sigma_{s,b} = \frac{6}{t_s^2} k_s \left[\beta_s \delta_s P_s + 6\frac{1-v}{E^*} \frac{D_o}{h^3} \left(1 + \frac{n\rho_s}{2}\right) \left(M_a + \frac{D_o}{32} P_e\right)\right]$$

Axial bending stress in the channel is written in the same way:

$$\sigma_{c,b} = \frac{6}{t_c^2} k_c \left[\beta_c \delta_c P_c - 6 \frac{1 - \nu^*}{E^*} \frac{D_o}{h^3} \left(1 + \frac{h \beta_c}{2} \right) \left(M_a + \frac{D_o^2}{32} P_e \right) \right]$$

(d) In conclusion the general formulas of PART 3, applied to U-Tube TS HE, match the UHX-12 formulas obtained in Section 6 of PART 5. This implies that UHX-12 rules could be written in the same way as UHX-13 rules, using for (ω_s^*, ω_c^*) and (Q1, Q2) the formulas given here above.

ANNEX U — CALCULATION OF A U-TUBE TS USING FLOATING OR FIXED TS HE SOFTWARE

A U-Tube HE can be calculated using a floating TS HE software, such as the Mathcad software used in Section 11.3 of PART 4 as follows:

- use an immersed floating head TS HE as explained in Section 3.5.1 of PART 4.
- use N_t=0 so that the tubes have no axial rigidity.

Calculation could be also performed using a fixed TS HE software, such as the Mathcad software used in Annex V as follows:

- use a bellows of rigidity close to 0 to simulate an immersed floating TS
- use L_t =infinity (Lt=10²⁰), so that the tubes have no axial rigidity.
- use $P_e = P_s P_t$

ANNEX V — UHX-13 – EXAMPLE E4.18.7 (PTB-4 2013 EDITION) WITH GENERAL EQUATIONS

This Annex provides a Mathcad calculation sheet for the fixed TS HE defined in Example E4.18.7 (PTB-4 2013 Edition). The TS is integral with shell and channel (configuration a). The data are shown in the calculation sheet, which is divided in 2 parts.

Part 1 follows strictly the steps 1 to 11 of UHX-13.5 calculation procedure. It includes the use of the elastic-plastic procedure at the TS-shell-channel connection.

Part 2 provides the equations which enable to calculate at any radius of the perforated tubesheet:

- the net effective pressure q(r),
- the deflection w(r),
- the rotation $\theta(\mathbf{r})$,
- the bending moment $M_r(r)$, and the bending stress $\sigma(r)$,
- the shear force $Q_r(x)$ and the shear stress $\tau(x)$,
- the tube axial stress $\sigma_t(\mathbf{r})$

These quantities are also given in graphical format. Their maximum values are determined and they match the maximum stress values obtained in Part 1. The positive directions of these quantities are shown on Figure 45.

The equations are taken from Section 8 of PART 3 and therefore depend on axial load V_a and bending moment M_a acting at the periphery of the tubesheet (see Figure 40) which are determined from the equivalent pressure P_e :

$$V_a = \frac{a_o}{2} \cdot P_e \qquad M_a = (a_o V_a) \cdot Q_1 + Q_2$$

These equations are general and do not depend on coefficient Q₃. Thus, they apply whether $P_e \neq 0$ or $P_e=0$. The calculation sheet provides also the determination of the moment M_R acting on the unperforated rim and the edge loads Q_s, Q_c, M_s, M_c and axial force V_s and elastic stretch Δ_s of the shell. The positive directions of these quantities are shown on Figure 40. See Annex Y for UHX-13- Example E4.18.7 (PTB-4 2013 Edition) with General Equations.

A fixed tubesheet heat exchanger with the tubesheet construction in accordance with Configuration a as shown in VIII-1, Figure UHX-13.1, Configuration a.

- For the Design Condition, the shell side design pressure is 325 psig at 400°F, and the tube side design pressure is 200 psig at 300°F.
- There is one operating condition. For Operating Condition 1, the shell side design pressure is 325 psig at 400°F, the tube side design pressure is 200 psig at 300°F, the shell mean metal temperature is 151°F, and the tube mean metal temperature is 113°F. For this example, the operating pressures and operating metal temperatures are assumed to be the same as the design values.
- The tube material is SA-249, Type 304L (S30403). The tubes are 1 in. outside diameter and are 0.049 in. thick.
- The tubesheet material is SA-240, Type 304L (S30403). The tubesheet outside diameter is 43.125 in. There are 955 tube holes on a 1.25 in. triangular pattern. There is no pass partition lane and the outermost tube radius from the tubesheet center is 20.125 in. The distance between the outer tubesheet faces is 240 in. The option for the effect of differential radial expansion is not required. There is no corrosion allowance on the tubesheet.



- The shell material is SA-240, Type 304L (S30403). The shell inside diameter is 42 in. and the thickness is 0.5625 in. There is no corrosion allowance on the shell and no expansion joint in the shell. The efficiency of shell circumferential welded joint (Category B) is 0.85.
- The channel material is SA-516, Grade 70 (K02700). The inside diameter of the channel is 42.125 in. and the channel is 0.375 in. thick. There is no corrosion allowance on the channel.

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FIXED TUBESHEET RULES according to UHX-13 (July 2013 Edition)

Example E4.18.7 (PTB- 4 2013 edition) Fixed Tubesheet configuration "a"

1 - GEOMETRIC Data (from Fig.UHX-13.1)

Type:	s of Operating Conditions	Configuration types: a, b, c, d					
x=1 NOF x=2 ST/ x=3 SHU x=4 UPS x=5 CLE	RMAL operating condition ARTUP operating condition JTDOWN operating condition GET operating condition SARING operating condition	Config := "a" for shell/channel integral both sides "b" for shell integral channel gask - TS extended "c" for shell integral channel gask - TS not extended "d" for gasketed both sides					
x=6 OTH	IER operating condition YELLO	W :most important data and results					
Tubeshee	et Data (from Fig.UHX-13.1)	Tube Data (from Fig.UHX-11.1)					
<mark>h := 1.375 in</mark>	Tubesheet thickness	p := 1.25 in Tube Pitch					
Layout := 0	For triangular pitch : "Layout"=0 For square pitch : "Layout"=1	N _t := 955 Number of Tubes					
r _o := 20.125∙in	Radius to outer tube	$d_t := 1 \cdot m$ Tube Outside Diameter					
<mark>A := 43.125 in</mark>	Outside Diameter of Tubesheet	q = 0.043 m Tube Length					
C := 0⋅in	Bolt Circle Diameter	$L_{1} = 2.6$ $L_{2} = 237,250$ in Effective length of tubes					
C _p := 126.4 ·in	Perimeter of the tube layout	$c_1 = c_1 = 2.0$ Tube expansion depth ratio					
$A_p := 1272.4 \cdot \text{in}^2$	Total area enclosed by C _p	$I_{tx} := 1.25$ in Length of Expanded Portion of Tube					
$A_1 := 0 \cdot in^2$	Total Untubed Lanes Area	kl := 48 in k=0.6 for spans between Tubesheets					
_ c _t := 0⋅in	Tubesheet Corr. Allow. (Tubeside)	k=0.0 for spans between support plates					
c _s := 0⋅in	Tubesheet Corr. Allow. (Shellside)	I unsupported tube span					
h _g := 0∙in Gr o	ss := "NO" SS="YES "	for TS calculated as Simply Supported acc. to UHX-13.9					
Shell	Data (from Fig.UHX-13.1)	Channel Data (from Fig.UHX-13.1)					
D _s := 42·in	Shell ID	D _c := 42.125 in Channel ID					
$\textbf{t}_{s} := 0.5625 {\cdot} \text{in}$	Shell Thickness away from TS	t _c := 0.375·in Channel Thickness					
<mark>G_s := 0∙in</mark>	Shell Gasket Diameter	G _c := 0.1 in Channel Gasket Diameter					
$\textbf{t_{s1}} \coloneqq 0.5625{\cdot}\text{in}$	Shell Thickness near TS	$G_1 := 0.1 \cdot in$ Channel Contact mid-point TS/Flange					
$\boldsymbol{C_s} \coloneqq \boldsymbol{0}{\cdot}\text{in}$	Shell Corrosion Allowance	C _c := 0·in Channel Corrosion Allowance					
E _{sw} := 0.85	Shell joint efficiency	CHAN := "CYL" "CYL" for Cylindrical Channel					
$I_1 := 0 \cdot in$	Thick Shell Length @ one end	"HEMI" for Hemisherical Channel					
$I'_1 := 0 \cdot in$	Thick Shell Length @ other end						
$h := h - c_s - c_t$	Corroded thicknesses: Tubesheet thickness	$\label{eq:corroded length:} \begin{array}{llllllllllllllllllllllllllllllllllll$					
$t_s := t_s - C_s$	Shell Thickness away from TS	t _s = 0.563 in Corroded diameters					
$\mathbf{t_{s1}} := \mathbf{t_{s1}} - \mathbf{C_s}$	Shell Thickness near TS	$t_{s1} = 0.563 \text{ in}$ $D_s := D_s + 2 \cdot C_s$ Shell ID $D_s = 42.000 \text{ in}$					
$t_c := t_c - C_c$	Channel Thickness	$t_c = 0.375 \text{ in}$ $D_c := D_c + 2 \cdot C_c$ Channel ID $D_c = 42.125 \text{ in}$					

ExampleE4.18.7-FIXED TS(AnnexV) from PTB4

ExampleE4.18.7-FIXED TS(AnnexV) from PTB4

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 $K_{J} := 10^{20} \cdot \frac{lb}{lc}$ Bellows Axial Rigidity D₁:= 0·in Inside Diameter of Bellows **Expansion Joint data** 2 - Design (D) and Operating (O) PRESSURES data (from UHX-13.3) Maximum and Minimum DESIGN PRESSURES (D) **OPERATING PRESSURES** (O) for Operating Condition X $P_{sO_x} := 325 \cdot \frac{lb}{lc^2}$ Shellside Operating Pressure $P_{sD_max} := 325 \cdot \frac{lb}{...2}$ maximum Shellside Design Pressure $P_{sD_{min}} := 0 \cdot \frac{lb}{ln^2}$ minimum Shellside Design Pressure $P_{tD_{max}} := 200 \cdot \frac{lb}{c_{n}^{2}}$ maximumTubeside Design Pressure $P_{tO_x} := 200 \cdot \frac{lb}{la^2}$ Tubeside Operating Pressure $P_{tD_{min}} := 0 \cdot \frac{lb}{ln^2}$ minimum Tubeside Design Pressure DESIGN PRESSURES P_{sD} and P_{tD} (from Table UHX-13.4-1) $P_{sD} := \begin{vmatrix} P_{sD_{max}} \\ P_{sD_{max}} \\ P_{sD_{max}} \end{vmatrix} \qquad P_{sD} = \begin{vmatrix} 0.000 \\ 325.000 \\ 325.000 \\ 0.000 \end{vmatrix} \frac{lb}{in^{2}}$ $\mathsf{P}_{\mathsf{tD}} \coloneqq \left| \begin{array}{c} \mathsf{P}_{\mathsf{tD_min}} \\ \mathsf{P}_{\mathsf{tD_max}} \end{array} \right| \qquad \mathsf{P}_{\mathsf{tD}} = \left| \begin{array}{c} 200,000 \\ 0.000 \\ 200,000 \end{array} \right| \frac{\mathsf{lb}}{\mathsf{in}^2}$ OPERATING PRESSURES Pso.x and Pto.x for oper.cond. x (from Table UHX-13.4-2) $P_{sO} := \begin{vmatrix} 0 \cdot \frac{1}{n^{2}} \\ P_{sO_{x}} \\ P_{sO_{x}} \\ 0 \cdot \frac{|b|}{n^{2}} \end{vmatrix} \qquad P_{sO} = \begin{pmatrix} 0.000 \\ 325.000 \\ 325.000 \\ 0.000 \end{pmatrix}$ $P_{tO} := \begin{vmatrix} 0 \cdot \frac{lb}{in^2} \\ P_{tO_x} \\ 0 \cdot \frac{lb}{2} \\ 0 \cdot \frac{lb}{2} \end{vmatrix} \qquad P_{tO} = \begin{pmatrix} 200.000 \\ 0.000 \\ 200.000 \\ 0.000 \end{pmatrix} \frac{lb}{in^2}$ Determination of DESIGN and OPERATING PRESSURES \mathbf{P}_{s} and \mathbf{P}_{t} $\begin{pmatrix} \mathsf{P}_{\mathsf{sD_min}} \\ \mathsf{P}_{\mathsf{sD_max}} \\ \mathsf{P}_{\mathsf{-}} \end{pmatrix} \begin{pmatrix} 0.0 \\ \mathsf{-} \end{pmatrix} \begin{pmatrix} \mathsf{P}_{\mathsf{tD_max}} \\ \mathsf{P}_{\mathsf{tD_min}} \\ \mathsf{P}_{\mathsf{tD_max}} \end{pmatrix} \begin{pmatrix} 200.0 \\ \mathsf{0.0} \end{pmatrix}$ 2013 Design LC1= 2010 Design LC1 2013 Design LC2= 2

$$\begin{array}{c} 100 \text{ Design LC2} \\ 010 \text{ Design LC3} \\ 010 \text{ Design LC4} \\ 01$$

lb in²

2013 Design LC3= 2 2013 Design LC4= r

2013 Operating LC5 2013 Operating LC6 2013 Operating LC7 2013 Operating LC8

ExampleE4.18.7-FIXED TS(AnnexV) from PTB4

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1	3 - TEMPE	RATURE Data for D	esign (D) and O	perating (O) cond	itions
	D	esign conditions	Ope	erating conditions	set Farenheit temp. degF := R
Tubesheet	T _D := 400 ⋅ degF	Tubesheet Design Temp.	$T_{O_x} := 400 \cdot degF$	TS Oper.Temp. for (Oper. Cond. x
Tubes	T _{tD} := 300⋅degF	Tube Design Temp.	$T_{tO_x} := 300 \cdot deg$	F Tube Oper. Temp.	for Oper. Cond. x
Shell	T _{sD} := 400 ⋅ degF	Shell Design Temp.	$T_{sO_x} := 400 \cdot deg$	F Shell Oper.Temp. fo	or Oper. Cond. x
Channel	$T_{cD} := 300 \cdot degF$	Channel Design Temp.	$T_{cO_x} := 300 \cdot deg$	F Channel Oper.Tem	p. for Oper. Cond. x
		T _a := 70⋅0	degF Ambient tempe	rature	
T _{tm}	_x := 113 degF N	lean Tube temp. along L	T _{tm.} := 1	T _{tm x} T _{tm.} = 113.0 degl	=
T _{sm_}	_ _x := 151 ⋅ degF N	lean Shell temp. along L	T _{sm.} := ⁻	Γ _{sm_x} T _{sm.} = 151.0 deg	F
	Additiona	al Temperature Data for Rad	ial Thermal Expansion	from UHX-13.8.4 (if req	uired)
Tubes	<mark>sheet</mark> T' _x := 70⋅c	^{degF} TS temp.@ rim		$T' := T'_{X}$ $T' = 70.0$	degF
Shell	T' _{sx} := 70∙	degF Shell temp. @ Tubes	sheet	$T'_{s} := T'_{sx} T'_{s} = 70.0$	degF
Chanr	nel T' _{cx} := 70∙	degF Channel temp.@ tub	esheet	$T'_{c} := T'_{cx} T'_{c} = 70.0$) degF
		_			
		4	- MATERIAL Da	ita	
		TUBES	HEET Material is SA-	240/304L	
S _D := 15800	$\frac{10}{10^2}$ TS allo	owable stress @ T _D	S _{PS} := 47400·	$\frac{10}{10}$ TS allowable P-	+S stress @ T _{O_x}
S _a := 20000	$\frac{1b}{\ln^2}$ TS allo	owable stress @ Ta			
E _D := 26.4 · 1	$10^6 \cdot \frac{\text{lb}}{\text{in}^2}$ TS ela st	stic modulus @ T _D	E _O := 26.4·10	$\frac{10}{100}$ TS elastic mod	ulus @ T _{O_x}
v := 0.3	TS Poi	isson's ratio	α' := 8.984·10	^{−6} . in TS coeff . in degF	expansion@ rim
		TUB	E Material is SA-249/3	304L	
S _{tD} := 16700	$6 \frac{\text{lb}}{\text{in}^2}$ Tube a	allowable stress @ T _{tD}	$S_{tO} := 16706 \cdot \frac{lb}{in^2}$	Tube allowable stress	s @ T _{tO_x}
E _{tD} := 27.0-	$10^{6} \cdot \frac{\text{lb}}{\text{in}^{2}}$ Tube e	elastic modulus @T _{tD}	$E_{tO} \coloneqq 27.0 \cdot 10^6 \cdot \frac{lb}{in^2}$	Tube elastic modulus	@ T _{tO_x}
S _{ytD} := 1920	$\frac{10}{100} \cdot \frac{10}{100}$ Tube y	vield stress @T _{tD}	$S_{ytO} \coloneqq 19200 \cdot \frac{lb}{in^2}$	Tube yield stress @ T	tO_x
S _{tT} := 15765	$5 \cdot \frac{\text{lb}}{\text{in}^2}$ Tube a	allowable stress @ T _D	$E_{tT} \coloneqq 26.4 \cdot 10^6 \cdot \frac{lb}{in^2}$	Tube elastic modulus	@ T _D
v _t := 0.3	Tube F	Poisson's ratio	$\alpha_{tm} \coloneqq 8.652 \cdot 10^{-6} \cdot \frac{1}{10^{-6}} \cdot \frac$	in degF Tube coeff. e	expansion@T _{t,m}
Europe 1- E 4 40 Z		TT DTD 4	e (

ExampleE4.18.7-FIXED TS(AnnexV) from PTB4

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ExampleE4.18.7-FIXED TS(AnnexV) from PTB4

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SHELL Material is SA-240/304L

$$\begin{split} S_{sD} &\coloneqq 15800 \frac{lb}{in^2} & \text{Shell allow. stress } @\ \textbf{T}_{sD} & \text{S}_{PSs} \coloneqq 47400 \frac{lb}{in^2} & \text{Shell allowable P+S stress } @\ \textbf{T}_{sO_x} \\ E_{sD} &\coloneqq 26.4 \cdot 10^6 \frac{lb}{in^2} & \text{Thin Shell elast mod. } @\ \textbf{T}_{sD} & E_{sO} \coloneqq 26.4 \cdot 10^6 \frac{lb}{in^2} & \text{Thin Shell elast mod. } @\ \textbf{T}_{sO_x} \\ S_{ysD} &\coloneqq 17500 \frac{lb}{in^2} & \text{Shell yield stress } @\ \textbf{T}_{sD} & S_{ysO} \coloneqq 17500 \frac{lb}{in^2} & \text{Shell yield stress } @\ \textbf{T}_{sO_x} \\ v_s &\coloneqq 0.3 & \text{Shell Poisson' ratio} & \alpha_{sm} \coloneqq 8.802 \cdot 10^{-6} \frac{in}{in \cdot \deg F} & \text{Shell coeff expansion } @\ \textbf{T}_{s,m} \\ \hline & \frac{SHELL Band Material is SA-240/304L}{in^2} \\ S_{s1} &\coloneqq 15800 \frac{lb}{in^2} & \text{Shell allow. stress } @\ \textbf{T}_s & S_{ys1} \coloneqq 17500 \frac{lb}{in^2} & \text{Shell yield stress } @\ \textbf{T}_s \\ E_{s1} &\coloneqq 26.4 \cdot 10^6 \frac{lb}{in^2} & \text{Thick Shell elast mod. } @\ \textbf{T}_s, & \alpha_{sm1} \coloneqq 8.802 \cdot 10^{-6} \frac{in}{in \cdot \deg F} & \text{Thick Shell coeff. expansion } @\ \textbf{T}_s \\ \hline & E_{s1} &\coloneqq 26.4 \cdot 10^6 \frac{lb}{in^2} & \text{Thick Shell elast mod. } @\ \textbf{T}_s, & \alpha_{sm1} \coloneqq 8.802 \cdot 10^{-6} \frac{in}{in \cdot \deg F} & \text{Thick Shell coeff. expansion } @\ \textbf{T}_s \\ \hline & F_{s1} &\coloneqq 26.4 \cdot 10^6 \frac{lb}{in^2} & \text{Thick Shell elast mod. } @\ \textbf{T}_s, & \alpha_{sm1} \coloneqq 8.802 \cdot 10^{-6} \frac{in}{in \cdot \deg F} & \text{Thick Shell coeff. expansion } @\ \textbf{T}_s \\ \hline & F_{s1} &\coloneqq 26.4 \cdot 10^6 \frac{lb}{in^2} & \text{Thick Shell elast mod. } @\ \textbf{T}_s, & \alpha_{sm1} \coloneqq 8.802 \cdot 10^{-6} \frac{in}{in \cdot \deg F} & \text{Thick Shell coeff. expansion } @\ \textbf{T}_s \\ \hline & F_{s1} &\coloneqq 26.4 \cdot 10^6 \frac{lb}{in^2} & \text{Thick Shell elast mod. } @\ \textbf{T}_s, & \alpha_{sm1} \coloneqq 8.802 \cdot 10^{-6} \frac{in}{in \cdot \deg F} & \text{Thick Shell coeff. expansion } @\ \textbf{T}_s \\ \hline & F_{s1} &\coloneqq 26.4 \cdot 10^6 \frac{lb}{in^2} & \text{Thick Shell elast mod. } @\ \textbf{T}_s & F_{s1} &\coloneqq 1500 \frac{b}{in^2} & \text{Thick Shell coeff. expansion } @\ \textbf{T}_s \\ \hline & F_{s1} &\coloneqq 1500 \frac{b}{in^2} & \text{Thick Shell coeff. expansion } @\ \textbf{T}_s \\ \hline & F_{s1} &\coloneqq 1500 \frac{b}{in^2} & \text{Thick Shell coeff. expansion } @\ \textbf{T}_s \\ \hline & F_{s1} &\coloneqq 1500 \frac{b}{in^2} & \text{Thick Shell coeff. expansion } @\ \textbf{T}_s \\ \hline & F_{s1} &\coloneqq 1500 \frac{b}{in^2} & \text{Thick Shell coeff. expansion } @\ \textbf$$

$$v_{s1} := 0.3$$
 Thick Shell Poisson' ratio $\alpha'_s := 8.984 \cdot 10^{-6} \cdot \frac{\text{in}}{\text{in} \cdot \text{degF}}$ Shell coeff. expansion @ rim

CHANNEL Material is SA-516/grade70

$$\begin{split} & S_{cD} \coloneqq 20000 \cdot \frac{lb}{in^2} & \text{Channel allow. stress } @\ \textbf{T}_{cD} & S_{PSc} \coloneqq 67200 \cdot \frac{lb}{in^2} & \text{Channel allowable P+S stress } @\ \textbf{T}_{cO_x} \\ & E_{cD} \coloneqq 28.3 \cdot 10^6 \cdot \frac{lb}{in^2} & \text{Channel elast. modulus } @\ \textbf{T}_{cD} & E_{cO} \coloneqq 28.3 \cdot 10^6 \cdot \frac{lb}{in^2} & \text{Channel elast. modulus } @\ \textbf{T}_{cO_x} \\ & S_{ycD} \coloneqq 33600 \cdot \frac{lb}{in^2} & \text{Channel yield stress } @\ \textbf{T}_{cD} \\ & v_c \coloneqq 0.3 & \text{Channel Poisson's ratio} & \alpha'_c \coloneqq 6.666 \cdot 10^{-6} \cdot \frac{in}{in \cdot degF} & \text{Channel coeff. expansion } @\ \textbf{T}_c \end{split}$$

ExampleE4.18.7-FIXED TS(AnnexV) from PTB4

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5 - Flange Design and Operating BOLT LOADS data (from UHX-13.3)

Maximum and Minim	um Flange D	ESIGN B		DS Fla	nge BOI	LT LOA	DS fo	r GASKE	T SEATING Condi	tion
$W_{m1s} := 0.0 \cdot lb$	V _{m1s} := 0.0·lb Shell flange Design bolt load					Shell flange bolt load for Gasket Seating				
$W_{m1c} := 0.0 \cdot lb$ C	Channel flang	e Design	h bolt load	$W_c := 0$).0·lb	Chan	nel flar	nge bolt	load for Gasket Se	ating
$W_{m1max} := max(W_{m1s})$	s, W _{m1c})	W _{m1max}	_x = 0.000 lb		W	_{max} := m	ax(W _s ,	W _c)	$W_{\text{max}}=0.000\text{lb}$	
Deter	rmination of I	EFFECTI		LOAD W* f	or each	Config	uration	na,b,c	, d	
W* _a :=	(0.0·lb) 0.0·lb 0.0·lb 0.0·lb 0.0·lb 0.0·lb 0.0·lb 0.0·lb	W* _b :=	(W _{m1c}) 0.0·lb W _{m1c} 0.0·lb W _c W _c W _c	W* _c ≔	(W _{m1c}) 0.0·lb W _{m1c} 0.0·lb W _c W _c W _c W _c		W* _d :=	(Wm1c Wm1s Wm1max 0.0·lb Wmax Wmax Wmax Wmax		
	Minimu	W* := V V V V	V* _a if Cor V* _b if Cor V* _c if Con V* _d if Cor	fig = "a" fig = "b" fig = "c" fig = "d"	W* =	(0.000) 0.000 0.000 0.000 0.000 0.000 0.000	lb	(from)		
	<mark>Minimu</mark>	m requir	ed thickne	ess h _r of th	e TS fla	nged ex	xtensio	on (from	UHX-9)	

For flanged Cor	nfig. b , d (extended as a flange)	For unflanged Config.c	For unflanged Config.d
	from UHX-9.5a	See UHX-9.5b	See UHX-9.5c
$h_G := \frac{C - G_c}{2}$ Gasket	moment arm $h_G = -0.050$ in		
$h_{rG} := \sqrt{\frac{1.9W_c}{S_a \cdot G_c}} \cdot h_G$	h _{rG} = 0.000 in		
$h_{rO} := \sqrt{\frac{1.9W_{m1c}}{S_D \cdot G_c}} \cdot h_G$	h _{rO} = 0.000 in		
$\mathbf{h}_{r} \coloneqq max \left(\mathbf{h}_{rG}, \mathbf{h}_{rO} \right)$	$h_{r} = 0.000 \text{ in}$		

 $L:=L_t-2{\cdot}h\qquad L=237.250\,\text{in}$

ExampleE4.18.7-FIXED TS(AnnexV) from PTB4

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Start of Calculations

L = effective length of shell/tubes

 $D_o =$ equivalent diameter of outer tubes $D_o := 2 \cdot r_o + d_t$ $D_o = 41.250$ in

$$a_0 := \frac{D_0}{2} a_0 = 20.625$$
 in

 $I_{s1} := L - I_1 - I_1' \quad I_{s1} = 237.250 \, \text{in}$

UHX-13.5.1 Step 1 Determine D₀, μ, μ* and h'g from UHX-11.5.1 :

$$\rho := \frac{l_{tx}}{h} \quad \rho = 0.909 \qquad \qquad d^* := \max\left[d_t - 2 \cdot t_t \cdot \left(\frac{E_{tT}}{E_D}\right) \cdot \left(\frac{S_{tT}}{S_D}\right) \cdot \rho , \left(d_t - 2t_t\right)\right] \quad d^* = 0.9111 \text{ in}$$

$$p^* := \frac{p}{\sqrt{1 - \frac{4 \cdot \min(A_L, 4D_0 \cdot p)}{\pi \cdot D_0^2}}} \quad p^* = 1.250 \text{ in}$$

$$\mu := \frac{p - d_t}{p} \qquad \mu = 0.200 \qquad \qquad \mu^* := \frac{p^* - d^*}{p^*} \qquad \mu^* = 0.271$$

Shell Radial Dim.Chan Rad. Dim.
$$a_s := \begin{bmatrix} \frac{G_s}{2} & \text{if Config = "d"} \\ \frac{D_s}{2} & \text{otherwise} \end{bmatrix}$$
 $a_c := \begin{bmatrix} \frac{D_c}{2} & \text{if Config = "a"} \\ \frac{G_c}{2} & \text{otherwise} \end{bmatrix}$

$$\begin{split} \rho_{s} &\coloneqq \frac{a_{s}}{a_{o}} \quad \rho_{s} = 1.018 & x_{s} &\coloneqq 1 - N_{t} \cdot \left(\frac{d_{t}}{2 \cdot a_{o}}\right)^{2} \quad x_{s} = 0.439 \\ \rho_{c} &\coloneqq \frac{a_{c}}{a_{o}} \quad \rho_{c} = 1.021 & x_{t} &\coloneqq 1 - N_{t} \cdot \left(\frac{d_{t} - 2 \cdot t_{t}}{2 \cdot a_{o}}\right)^{2} \quad x_{t} = 0.543 \end{split}$$

ExampleE4.18.7-FIXED TS(AnnexV) from PTB4

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UHX-13.5.2 Step 2 Calculate the shell axial stiffness Ks, tube axial stiffness Kt, and stiffness factors Kst and J

$$K_{s}^{*} := \frac{\pi \cdot (D_{s} + t_{s})}{\frac{L - (I_{1} + I'_{1})}{E_{sD} \cdot t_{s}} + \frac{I_{1} + I'_{1}}{E_{s1} \cdot t_{s1}}} \qquad K_{s}^{*} = 8.3695 \times 10^{6} \frac{\text{lb}}{\text{in}} \qquad K_{t} := \frac{\pi \cdot t_{t} \cdot (d_{t} - t_{t}) \cdot E_{tD}}{L} \qquad K_{t} = 1.666 \times 10^{4} \frac{\text{lb}}{\text{in}} \qquad K_{t} := \frac{\pi \cdot t_{t} \cdot (d_{t} - t_{t}) \cdot E_{tD}}{L} \qquad K_{t} = 1.666 \times 10^{4} \frac{\text{lb}}{\text{in}} \qquad K_{t} = 1.6$$

Calculate shell and channel parameters:

$$\beta_{c} := \frac{\left\lfloor 12 \cdot \left(1 - v_{c}^{2}\right) \right\rfloor}{\left[\left(D_{c} + t_{c}\right) \cdot t_{c} \right]^{0.5}} \quad \text{if } SS = "NO" \land \text{Config} = "a"$$
$$\beta_{c} = 0.455 \frac{1}{100}$$
$$\beta_{c} = 0.455 \frac{1}{100}$$

$$k_{s} := \beta_{s} \cdot \frac{E_{s1} \cdot t_{s1}^{3}}{6 \cdot \left(1 - \nu_{s}^{2}\right)} \qquad k_{s} = 3.1971 \times 10^{5} \text{ lb} \qquad \qquad k_{c} := \beta_{c} \cdot \frac{E_{cD} \cdot t_{c}^{3}}{6 \cdot \left(1 - \nu_{c}^{2}\right)} \qquad \qquad k_{c} = 1.245 \times 10^{5} \text{ lb}$$

$$\begin{aligned} h_{s}' &:= h \cdot \beta_{s} \quad h_{s}' = 0.511 \\ \lambda_{s} &:= \frac{6 \cdot D_{s}}{h^{3}} \cdot k_{s} \cdot \left(1 + h_{s}' + \frac{h_{s}'^{2}}{2}\right) \\ \lambda_{s} &= 5.0868 \times 10^{7} \frac{lb}{in^{2}} \\ \lambda_{c} &:= \frac{6 \cdot D_{c}}{h^{3}} \cdot k_{c} \cdot \left(1 + h_{c}' + \frac{h_{c}'^{2}}{2}\right) \\ \lambda_{c} &= 2.2049 \times 10^{7} \frac{lb}{in^{2}} \\ \lambda_{c} &:= \frac{2.2049 \times 10^{7} \frac{lb}{in^{2}} \\ \lambda_{c} &:= \frac{1}{2} \frac{1}{4E_{s1} \cdot t_{s1}} \left(1 - \frac{v_{s}}{2}\right) \\ \frac{1}{4E_{s1} \cdot t_{s1}} \left(1 - \frac{v_{s}}{2}\right) \quad \text{if } SS = "NO" \land (Config = "a" \lor Config = "b" \lor Config = "c") \\ 0 \cdot \left(\ln^{3} lb^{-1}\right) \quad \text{otherwise} \end{aligned}$$

$$\begin{aligned} CHAN &= "CYL" \\ SS &= "NO" \end{aligned}$$

$$\begin{aligned} \delta_{c} &:= \frac{1}{4E_{cD} \cdot t_{c}} \left(1 - \frac{v_{c}}{2}\right) \quad \text{if } SS = "NO" \land CHAN = "CYL" \land Config = "a" \\ \frac{1}{4E_{cD} \cdot t_{c}} \left(1 - \frac{v_{c}}{2}\right) \quad \text{if } SS = "NO" \land CHAN = "CYL" \land Config = "a" \\ \frac{1}{4E_{cD} \cdot t_{c}} \left(1 - \frac{v_{c}}{2}\right) \quad \text{if } SS = "NO" \land CHAN = "CYL" \land Config = "a" \\ \frac{1}{4E_{cD} \cdot t_{c}} \left(1 - \frac{v_{c}}{2}\right) \quad \text{if } SS = "NO" \land CHAN = "CYL" \land Config = "a" \\ \frac{1}{4E_{cD} \cdot t_{c}} \left(1 - \frac{v_{c}}{2}\right) \quad \text{if } SS = "NO" \land CHAN = "CYL" \land Config = "a" \\ \frac{1}{4E_{cD} \cdot t_{c}} \left(1 - \frac{v_{c}}{2}\right) \quad \text{if } SS = "NO" \land CHAN = "CYL" \land Config = "a" \\ \frac{1}{4E_{cD} \cdot t_{c}} \left(1 - \frac{v_{c}}{2}\right) \quad \text{if } SS = "NO" \land CHAN = "HEM!" \land Config = "a" \\ \frac{1}{4E_{cD} \cdot t_{c}} \left(1 - \frac{v_{c}}{2}\right) \quad \text{if } SS = "NO" \land CHAN = "HEM!" \land Config = "a" \\ \frac{1}{4E_{cD} \cdot t_{c}} \left(1 - \frac{v_{c}}{2}\right) \quad \text{if } SS = "NO" \land CHAN = "HEM!" \land Config = "a" \\ \frac{1}{4E_{cD} \cdot t_{c}} \left(1 - \frac{v_{c}}{2}\right) \quad \text{if } SS = "NO" \land CHAN = "HEM!" \land Config = "a" \\ \frac{1}{4E_{cD} \cdot t_{c}} \left(1 - \frac{v_{c}}{2}\right) \quad \text{if } SS = "NO" \land CHAN = "HEM!" \land Config = "a" \\ \frac{1}{4E_{cD} \cdot t_{c}} \left(1 - \frac{v_{c}}{2}\right) \quad \text{if } SS = "NO" \land CHAN = "HEM!" \land Config = "a" \\ \frac{1}{4E_{cD} \cdot t_{c}} \left(1 - \frac{v_{c}}{2}\right) \quad \text{if } SS = "NO" \land CHAN = "HEM!" \land Config = "a" \\ \frac{1}{4E_{cD} \cdot t_{c}} \left(1 - \frac{v_{c}}{2}\right) \quad \text{if } SS = "NO" \land CHAN = "HEM!" \land Config = "a" \\ \frac{1}{4E_{cD} \cdot t_{c}} \left(1 - \frac{v_{c}}{2}\right) \quad \frac{1}{4E_{cD} \cdot t_{c}} \left(1 - \frac{v_{c}}{2}\right) \quad \frac{1}{4E_{cD} \cdot t_{c}} \left(1 - \frac{v_{c}}{2}\right) \quad \frac{1}{4E_{cD} \cdot t_{c}}$$

ExampleE4.18.7-FIXED TS(AnnexV) from PTB4

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UHX-13.5.3 Step 3 Determine E*/E and v* relative to h/p from UHX-11.5.2. Calculate Xa

$$\frac{h}{p} = 1.100 \quad \mu^* = 0.271 \qquad \qquad \frac{E^*}{E_D} = 0.275 \qquad E^* = 7259614.115 \frac{lb}{in^2}$$

 $v^* = 0.3404$ (From right pages above)

$$X_{a} := \left[24 \cdot \left(1 - v^{*2}\right) \cdot N_{t} \cdot \frac{E_{tD} \cdot t_{t} \cdot \left(d_{t} - t_{t}\right) \cdot a_{o}^{2}}{E^{*} \cdot L \cdot h^{3}} \right]^{25} \quad X_{a} = 7.016$$

UHX-13.5.4 Step 4 Calculate diameter ratio K and coefficient F :

$$\begin{split} \mathsf{K} &:= \frac{\mathsf{A}}{\mathsf{D}_{\mathsf{0}}} & \mathsf{K} = 1.045 \\ \mathsf{F} &:= \frac{1 - v^{*}}{\mathsf{E}^{*}} \cdot \left(\lambda_{\mathsf{S}} + \lambda_{\mathsf{C}} + \mathsf{E}_{\mathsf{D}} \cdot \mathsf{In}(\mathsf{K})\right) & \mathsf{F} = 6.732 \\ \Phi &:= (1 + v^{*}) \cdot \mathsf{F} & \Phi = 9.024 \end{split}$$

$$\begin{aligned} \text{Calculate } Z_{a}, Z_{d}, Z_{v}, Z_{m} & \text{N} := \operatorname{arrondi}\left(4 + \frac{X_{a}}{2}\right) + 1 & \text{N} = 9.000 \end{aligned}$$

$$\operatorname{ber}_{x}(x) := \sum_{n=0}^{N} \left[(-1)^{n} \cdot \frac{\left(\frac{x}{2}\right)^{4 \cdot n}}{\left((2 \cdot n)!\right)^{2}} \right] & \operatorname{ber} := \operatorname{ber}_{x}(X_{a}) & \operatorname{bei}_{x}(x) := \sum_{n=1}^{N} \left[(-1)^{n-1} \cdot \frac{\left(\frac{x}{2}\right)^{4 \cdot n-2}}{\left((2 \cdot n-1)!\right)^{2}} \right] & \operatorname{bei} := \operatorname{bei}_{x}(X_{a}) \\ \operatorname{ber} := -3.432 & \operatorname{bei}_{x}(x) := \sum_{n=1}^{N} \left[(-1)^{n-1} \cdot \frac{\left(\frac{x}{2}\right)^{4 \cdot n-2}}{\left((2 \cdot n-1)!\right)^{2}} \right] & \operatorname{bei} := \operatorname{bei}_{x}(X_{a}) \\ \operatorname{bei} := -21.489 \end{aligned}$$

$$ber'_{x}(x) := \sum_{n=1}^{N} \frac{(-1)^{n} \cdot (2 \cdot n) \cdot \left(\frac{x}{2}\right)^{4 \cdot n - 1}}{\left((2 \cdot n)!\right)^{2}} \quad ber' := ber'_{x}(X_{a}) \qquad bei'_{x}(x) := \sum_{n=1}^{N} \frac{(-1)^{n-1} \cdot (2 \cdot n - 1) \cdot \left(\frac{x}{2}\right)^{4 \cdot n - 3}}{\left((2 \cdot n - 1)!\right)^{2}} \quad bei' := bei'_{x}(X_{a}) \\ bei' = -16.061 \quad bei'_{x}(x) := bei_{x}(x) + \left(\frac{1 - v^{*}}{x}\right) \cdot ber'_{x}(x) \quad \Psi_{1} := \Psi_{1x}(X_{a}) \\ \Psi_{1} = -20.260 \quad \Psi_{2x}(x) := ber_{x}(x) - \frac{1 - v^{*}}{x} \cdot bei'_{x}(x) \quad \Psi_{2} := \Psi_{2x}(X_{a}) \\ \Psi_{2} = -1.922 \quad \Psi_{2}(x) := ber_{x}(x) - \frac{1 - v^{*}}{x} \cdot bei'_{x}(x) \quad \Psi_{2} := \Psi_{2x}(X_{a})$$

 $Z_a := bei' \cdot \Psi_2 - ber' \cdot \Psi_1 \qquad \quad Z_a = 295.626$

$$Z_{m} := \frac{ber'^{2} + bei'^{2}}{X_{a} \cdot Z_{a}}$$
 $Z_{m} = 0.207$

$$Z_d := \frac{ber \cdot \Psi_2 + bei \cdot \Psi_1}{X_a^3 \cdot Z_a} \qquad Z_d = 0.004$$

$$Z_v := \frac{ber' \cdot \Psi_2 + bei' \cdot \Psi_1}{X_a^2 \cdot Z_a} \qquad Z_v = 0.021$$

$$Z_{w} := \frac{ber' \cdot ber + bei' \cdot bei}{X_{a}^{2} \cdot Z_{a}} \qquad Z_{w} = 0.021$$

ExampleE4.18.7-FIXED TS(AnnexV) from PTB4

ExampleE4.18.7-FIXED TS(AnnexV) from PTB4

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Calculate Q_1, Q_{z1}, Q_{z2} , and U:

$$\begin{split} & \mathsf{Q}_{1} := \frac{\rho_{s} - 1 - \Phi \cdot Z_{v}}{1 + \Phi \cdot Z_{m}} \qquad \qquad \mathsf{Q}_{1} = -0.059 \\ & \mathsf{Q}_{z1} := \frac{\left(Z_{d} + \mathsf{Q}_{1} \cdot Z_{w}\right) \cdot X_{a}^{-4}}{2} \qquad \qquad \mathsf{Q}_{z1} = 3.778 \\ & \mathsf{Q}_{z2} := \frac{\left(Z_{v} + \mathsf{Q}_{1} \cdot Z_{m}\right) \cdot X_{a}^{-4}}{2} \qquad \qquad \mathsf{Q}_{z2} = 10.312 \end{split}$$

$$U := \frac{\left[Z_w + \left(\rho_s - 1\right) \cdot Z_m\right] \cdot X_a^4}{1 + \Phi \cdot Z_m} \quad U = 20.625$$

UHX-13.5.5 Step 5 UHX-13.5.5.(a) Calculate γ:

$$\gamma^{*}{}_{p} := (T_{tm.} - T_{a}) \cdot \alpha_{tm} \cdot L - (T_{sm.} - T_{a}) \cdot [\alpha_{sm} \cdot (L - I_{1} - I'_{1}) + \alpha_{sm1} \cdot (I_{1} + I'_{1})] \qquad \gamma^{*} := \begin{pmatrix} 0 \cdot in \\ \gamma^{*}{}_{p} \\ \gamma^{*}{}_{p} \\ \gamma^{*}{}_{p} \\ \gamma^{*}{}_{p} \\ \gamma^{*}{}_{p} \end{pmatrix} \qquad \gamma^{*} := \begin{pmatrix} 0 \cdot in \\ 0 \cdot in \\ 0 \cdot in \\ 0 \cdot in \\ \gamma^{*}{}_{p} \\ \gamma^{*}{}_{p} \\ \gamma^{*}{}_{p} \\ \gamma^{*}{}_{p} \end{pmatrix} \qquad \gamma^{*} := \begin{pmatrix} 0 \cdot in \\ \gamma^{*}{}_{p} \\ \gamma^{*}{}_{p} \\ \gamma^{*}{}_{p} \end{pmatrix} \qquad \gamma^{*} := \begin{pmatrix} 0 \cdot in \\ 0 \cdot$$

UHX-13.5.5(b) Calculate parameters $\omega_{s}^{}, \omega_{s}^{*}, \omega_{c}^{}, \omega_{c}^{*}$, and :

$$\begin{split} \omega_{s} &\coloneqq \rho_{s} \cdot k_{s} \cdot \beta_{s} \cdot \delta_{s} \cdot \left(1 + h'_{s}\right) & \omega_{c} &\coloneqq \rho_{c} \cdot k_{c} \cdot \beta_{c} \cdot \delta_{c} \cdot \left(1 + h'_{c}\right) \\ \omega_{s} &= 4.612 \, \text{in}^{2} & \omega_{c} &= 3.344 \, \text{in}^{2} \end{split}$$

UHX-13.5.5(c) Calculate γ_b:

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Calculate P*s and P*c from UHX-13.8 , if required by the user (for configurations a,b,or c only)

$$T_{r} := \begin{cases} \frac{T' + T'_{s} + T'_{c}}{3} & \text{if Config = "a"} \\ \frac{T' + T'_{s}}{2} & \text{if Config = "b"} \lor \text{Config = "c"} \end{cases} T_{r} = 70.0 \text{ degF}$$

$$T_{a} \text{ if Config = "d"}$$

$$T_{s}^{*} := \frac{T_{s}^{*} + T_{r}}{2} T_{s}^{*} = 70.0 \text{ degF} \qquad T_{c}^{*} := \frac{T_{c}^{*} + T_{r}}{2} T_{c}^{*} = 70.0 \text{ degF}$$

$$P^{*}_{sp1} := \frac{E_{s1} \cdot t_{s1}}{a_{s}} \cdot \left[\alpha'_{s} \cdot (T^{*}_{s} - T_{a}) - \alpha' \cdot (T_{r} - T_{a}) \right]$$

$$P^{*}_{cp1} := \frac{E_{cD} \cdot t_{c}}{a_{c}} \cdot \left[\alpha'_{c} \cdot (T^{*}_{c} - T_{a}) - \alpha' \cdot (T_{r} - T_{a}) \right]$$

$$P^{*}_{sp1} = 0.000 \frac{lb}{in^{2}}$$

$$P^{*}_{cp1} = 0.001 \frac{lb}{in^{2}}$$

$$\mathbf{P}^{*}\mathbf{s} := \begin{pmatrix} \mathbf{0} \cdot \frac{\mathbf{lb}}{\mathbf{in}^{2}} \\ \mathbf{0} \cdot \frac{\mathbf{lb}}{\mathbf{in}^{2}$$

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UHX-13.5.6 Step 6 For each loading case, calculate P's and P't, P_γ, P_γ*, P_w, P_{rim}, and effective pressure Pe

$$P_{s}^{*} := \begin{bmatrix} x_{s} + 2 \cdot (1 - x_{s}) \cdot v_{t} + \frac{2 \cdot v_{s}}{K_{st}} \left(\frac{D_{s}}{D_{0}} \right)^{2} - \frac{\rho_{s}^{2} - 1}{J \cdot K_{st}} - \frac{1 - J}{2J \cdot K_{st}} \cdot \frac{D_{J}^{2} - D_{s}^{2}}{D_{0}^{2}} \end{bmatrix} \cdot P_{s} \qquad P_{t}^{*} := \begin{bmatrix} x_{t} + 2 \cdot (1 - x_{t}) \cdot v_{t} + \frac{1}{J \cdot K_{st}} \end{bmatrix} \cdot P_{t}$$

$$P_{s}^{*} := \begin{bmatrix} 0.000 \\ 613.671 \\ 613.671 \\ 0.000 \\ 613.671 \\ 613.671 \\ 613.671 \\ 0.000 \end{bmatrix}$$

$$P_{t}^{*} := \frac{N_{t} \cdot K_{t}}{\pi \cdot a_{0}^{2}} \cdot \gamma^{*} \qquad P_{w}^{*} := \frac{U}{a_{0}^{2}} \cdot \frac{\gamma_{b}}{2\pi} W^{*} \qquad P_{\omega} := \frac{U}{a_{0}^{2}} \cdot (\omega_{s} \cdot P^{*}_{s} - \omega_{c} \cdot P^{*}_{c}) \qquad P_{rim}^{*} := \frac{U}{a_{0}^{2}} \cdot (\omega^{*}_{s} \cdot P_{s} - \omega^{*}_{c} \cdot P_{t})$$

$$P_{y}^{*} := \frac{0.000}{0.000} \frac{0.000}{0.000} \frac{1}{\ln^{2}} \qquad P_{w}^{*} := \frac{U}{a_{0}^{2}} \cdot \frac{\gamma_{b}}{2\pi} W^{*} \qquad P_{\omega}^{*} := \frac{U}{a_{0}^{2}} \cdot (\omega_{s} \cdot P^{*}_{s} - \omega_{c} \cdot P^{*}_{c}) \qquad P_{rim}^{*} := \frac{U}{a_{0}^{2}} \cdot (\omega^{*}_{s} \cdot P_{s} - \omega^{*}_{c} \cdot P_{t})$$

$$P_{y}^{*} := \frac{0.000}{0.000} \frac{0.000}{0.000} \frac{1}{\ln^{2}} \qquad P_{w}^{*} := \frac{0.000}{0.000} \frac{0.000}{0.000} \frac{1}{\ln^{2}} \qquad P_{\omega}^{*} := \frac{0.000}{0.000} \frac{0.000}{0.000} \frac{1}{\ln^{2}} \qquad P_{rim}^{*} := \frac{-2.238}{0.238} \frac{71.560}{71.560} \frac{1}{10} \frac$$

Calculate $\rm P_{e}$ for Pressure and Operating Loading Cases 1 through 8:

$$\mathsf{P}_{e} := \frac{J \cdot \mathsf{K}_{st}}{1 + J \cdot \mathsf{K}_{st} \cdot \left[\mathsf{Q}_{z1} + \left(\rho_{s} - 1 \right) \cdot \mathsf{Q}_{z2} \right]} \cdot \left(\mathsf{P}'_{s} - \mathsf{P}'_{t} + \mathsf{P}_{\gamma} + \mathsf{P}_{\omega} + \mathsf{P}_{w} + \mathsf{P}_{rim} \right)$$

PRESSURE DESIGN Loading cases : terms 1,2 3, 4
P_e =
$$\begin{pmatrix}
-96.973 \\
116.800 \\
19.826 \\
0.000 \\
-261.115 \\
-47.343 \\
-144.316 \\
-164.142
\end{pmatrix}$$

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UHX-13.5.7 Step 7 Determine Q₂ and Q₃ for Loading Cases 1 through 7:

$$Q_{2} := \frac{\overbrace{\left[\left(\omega^{*}{}_{s} \cdot P_{s} - \omega^{*}{}_{c} \cdot P_{t}\right) - \left(\omega_{s} \cdot P^{*}{}_{s} - \omega_{c} \cdot P^{*}{}_{c}\right)\right] + \frac{\gamma_{b}}{2 \cdot \pi} \cdot W^{*}}{1 + \Phi \cdot Z_{m}} \qquad \qquad Q_{2} := \left(\begin{array}{c}181.674\\-515.107\\-333.433\\0.000\\181.674\\-515.107\\-333.433\\0.000\end{array}\right)$$

$$\begin{aligned} \mathbf{Q}_{3_{1}} &:= \begin{bmatrix} 0 & \text{if } \mathbf{P}_{e_{1}} = 0 \\ \left(\mathbf{Q}_{1} + \frac{2 \cdot \mathbf{Q}_{2_{1}}}{\mathbf{P}_{e_{1}} \cdot \mathbf{a}_{0}^{2}} \right) & \text{otherwise} \\ \left(\mathbf{Q}_{1} + \frac{2 \cdot \mathbf{Q}_{2_{2}}}{\mathbf{P}_{e_{2}} \cdot \mathbf{a}_{0}^{2}} \right) & \text{otherwise} \\ \mathbf{Q}_{3_{4}} &:= \begin{bmatrix} 0 & \text{if } \mathbf{P}_{e_{4}} = 0 \\ \left(\mathbf{Q}_{1} + \frac{2 \cdot \mathbf{Q}_{2_{3}}}{\mathbf{P}_{e_{3}} \cdot \mathbf{a}_{0}^{2}} \right) & \text{otherwise} \\ \left(\mathbf{Q}_{1} + \frac{2 \cdot \mathbf{Q}_{2_{3}}}{\mathbf{P}_{e_{3}} \cdot \mathbf{a}_{0}^{2}} \right) & \text{otherwise} \\ \left(\mathbf{Q}_{1} + \frac{2 \cdot \mathbf{Q}_{2_{3}}}{\mathbf{P}_{e_{3}} \cdot \mathbf{a}_{0}^{2}} \right) & \text{otherwise} \\ \left(\mathbf{Q}_{1} + \frac{2 \cdot \mathbf{Q}_{2_{3}}}{\mathbf{P}_{e_{3}} \cdot \mathbf{a}_{0}^{2}} \right) & \text{otherwise} \\ \left(\mathbf{Q}_{1} + \frac{2 \cdot \mathbf{Q}_{2_{3}}}{\mathbf{P}_{e_{3}} \cdot \mathbf{a}_{0}^{2}} \right) & \text{otherwise} \\ \mathbf{Q}_{3_{7}} &:= \begin{bmatrix} 0 & \text{if } \mathbf{P}_{e_{7}} = 0 \\ \mathbf{Q}_{1} + \frac{2 \cdot \mathbf{Q}_{2_{7}}}{\mathbf{P}_{e_{7}} \cdot \mathbf{a}_{0}^{2}} \right) & \text{otherwise} \\ \left(\mathbf{Q}_{1} + \frac{2 \cdot \mathbf{Q}_{2_{7}}}{\mathbf{P}_{e_{8}} \cdot \mathbf{a}_{0}^{2}} \right) & \text{otherwise} \\ \left(\mathbf{Q}_{1} + \frac{2 \cdot \mathbf{Q}_{2_{7}}}{\mathbf{P}_{e_{8}} \cdot \mathbf{a}_{0}^{2}} \right) & \text{otherwise} \\ \left(\mathbf{Q}_{1} + \frac{2 \cdot \mathbf{Q}_{2_{8}}}{\mathbf{P}_{e_{8}} \cdot \mathbf{a}_{0}^{2}} \right) & \text{otherwise} \\ \left(\mathbf{Q}_{1} + \frac{2 \cdot \mathbf{Q}_{2_{7}}}{\mathbf{P}_{e_{7}} \cdot \mathbf{a}_{0}^{2}} \right) & \text{otherwise} \\ \left(\mathbf{Q}_{1} + \frac{2 \cdot \mathbf{Q}_{2_{8}}}{\mathbf{P}_{e_{8}} \cdot \mathbf{a}_{0}^{2}} \right) & \text{otherwise} \\ \left(\mathbf{Q}_{1} + \frac{2 \cdot \mathbf{Q}_{2_{8}}}{\mathbf{P}_{e_{8}} \cdot \mathbf{a}_{0}^{2}} \right) & \text{otherwise} \\ \left(\mathbf{Q}_{1} + \frac{2 \cdot \mathbf{Q}_{2_{8}}}{\mathbf{P}_{e_{8}} \cdot \mathbf{a}_{0}^{2}} \right) & \text{otherwise} \\ \left(\mathbf{Q}_{1} + \frac{2 \cdot \mathbf{Q}_{2_{8}}}{\mathbf{P}_{e_{8}} \cdot \mathbf{a}_{0}^{2}} \right) & \text{otherwise} \\ \left(\mathbf{Q}_{1} + \frac{2 \cdot \mathbf{Q}_{2_{8}}}{\mathbf{P}_{e_{8}} \cdot \mathbf{a}_{0}^{2}} \right) & \text{otherwise} \\ \left(\mathbf{Q}_{1} + \frac{2 \cdot \mathbf{Q}_{2_{8}}}{\mathbf{P}_{e_{8}} \cdot \mathbf{a}_{0}^{2}} \right) & \text{otherwise} \\ \left(\mathbf{Q}_{1} + \frac{2 \cdot \mathbf{Q}_{2_{8}}}{\mathbf{P}_{e_{8}} \cdot \mathbf{A}_{0}^{2}} \right) & \text{otherwise} \\ \left(\mathbf{Q}_{1} + \frac{2 \cdot \mathbf{Q}_{2_{8}}}{\mathbf{P}_{e_{8}} \cdot \mathbf{A}_{0}^{2}} \right) & \text{otherwise} \\ \left(\mathbf{Q}_{1} + \frac{2 \cdot \mathbf{Q}_{2_{8}}}{\mathbf{P}_{e_{8}} \cdot \mathbf{A}_{0}^{2}} \right) & \text{otherwise} \\ \left(\mathbf{Q}_{1} + \frac{2 \cdot \mathbf{Q}_{2_{8}}}{\mathbf{P}_{e_{8}} \cdot \mathbf{A}_{0}^{2}} \right) & \text{otherwise} \\ \left(\mathbf{Q}_{1} + \frac{2 \cdot \mathbf{Q}_{2_{8}}}{\mathbf{P}_{e_{8}} \cdot \mathbf{A}_{0}^{2}} \right) & \text{ot$$

$$Q_{1} = -0.059 \qquad Q_{2} = \begin{pmatrix} 181.674 \\ -515.107 \\ -333.433 \\ 0.000 \\ 181.674 \\ -515.107 \\ -333.433 \\ 0.000 \end{pmatrix} | b \qquad Q_{3} = \begin{pmatrix} -0.06746 \\ -0.07938 \\ -0.13772 \\ 0.00000 \\ -0.06192 \\ -0.00749 \\ -0.00749 \\ -0.05865 \end{pmatrix}$$

ExampleE4.18.7-FIXED TS(AnnexV) from PTB4

ExampleE4.18.7-FIXED TS(AnnexV) from PTB4

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Determine Coefficient F_m for Load Cases 1 through 7:

$$\begin{split} \text{numpoints} & = 20 \qquad \text{j} \coloneqq 1 \text{ ... numpoints} \qquad X_{j} \coloneqq \frac{j 1}{\text{numpoints}} \qquad x \coloneqq X_{a} \\ Q_{q}(x) \coloneqq \frac{\overline{\Psi_{2q}(x), \Psi_{1} - \Psi_{1q}(x), \Psi_{2}}{X_{a}, Z_{a}} \qquad Q_{m}(x) \coloneqq \left(\frac{\Psi_{2q}(x), bei - \Psi_{1q}(x), b$$

ExampleE4.18.7-FIXED TS(AnnexV) from PTB4

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Calculate the Maximum Tubesheet Bending Stress

$$Effective Groove depth h'_{g} := max(h_{g} - Q_{1}, 0) \qquad h'_{g} = 0.000 \text{ in } \qquad h_{min} := \begin{pmatrix} h - h'_{g} \\ h - h'_{g}$$
×

ExampleE4.18.7-FIXED TS(AnnexV) from PTB4

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TUBESHEET MAXIMUM STRESS for DESIGN LOADING CASES 1, 2, 3, 4 $\sigma_{\mathsf{D}} \coloneqq \max(|\sigma_1|, |\sigma_2|, |\sigma_3|, |\sigma_4|) \quad \sigma_{\mathsf{D}} = 23084.1 - 1000$

$$\sigma_{\text{allowD}} \coloneqq 1.5 \cdot S_{\text{D}} \qquad \sigma_{\text{allowD}} = 23700.0 - \frac{\text{lb}}{\text{m}}$$

TUBESHEET MAXIMUM STRESS For OPERATING LOADING CASES 5, 6, 7, 8

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$$\sigma_{O} := \max(|\sigma_{5}|, |\sigma_{6}|, |\sigma_{7}|, |\sigma_{8}|) \quad \sigma_{O} = 40253.4 \frac{\text{lb}}{\text{in}^{2}}$$
$$\sigma_{\text{allowO}} := S_{PS} \quad \sigma_{\text{allowO}} = 47400.0 \frac{\text{lb}}{\text{in}^{2}}$$

UHX-13.5.8 Step 8 Calculate the maximum tubesheet shear stress



UHX-13.5.9 Step 9 Determine the minimum and maximum stresses in the tubes for Load Cases 1 to 8

$$Z_{dx}(x) := \frac{\overbrace{\Psi_2 \cdot ber_x(x) + \Psi_1 \cdot bei_x(x)}}{X_a^3 \cdot Z_a} \qquad \qquad Z_{wx}(x) := \frac{\overbrace{ber' \cdot ber_x(x) + bei' \cdot bei_x(x)}}{X_a^2 \cdot Z_a}$$

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a) :Determine Coefficients
$$F_{t,min}$$
 and $F_{t,max}$ for Load Cases 1 to 8:

$$F_{bx1}(x) := \begin{vmatrix} z_{wx}(x) \cdot \frac{x_a^4}{2} & \text{if } P_{e_1} = 0 \\ (Z_{dx}(x) + Q_{3_1} \cdot Z_{wx}(x)) \cdot \frac{x_a^4}{2} & \text{otherwise} \end{vmatrix}$$

$$F_{bx2}(x) := \begin{vmatrix} z_{wx}(x) \cdot \frac{x_a^4}{2} & \text{if } P_{e_2} = 0 \\ (Z_{dx}(x) + Q_{3_2} \cdot Z_{wx}(x)) \cdot \frac{x_a^4}{2} & \text{otherwise} \end{vmatrix}$$

$$F_{bx2}(x) := \begin{vmatrix} z_{wx}(x) \cdot \frac{x_a^4}{2} & \text{if } P_{e_2} = 0 \\ (Z_{dx}(x) + Q_{3_2} \cdot Z_{wx}(x)) \cdot \frac{x_a^4}{2} & \text{otherwise} \end{vmatrix}$$

$$F_{bx3}(x) := \begin{vmatrix} z_{wx}(x) \cdot \frac{x_a^4}{2} & \text{if } P_{e_3} = 0 \\ (Z_{dx}(x) + Q_{3_2} \cdot Z_{wx}(x)) \cdot \frac{x_a^4}{2} & \text{otherwise} \end{vmatrix}$$

$$F_{bx3}(x) := \begin{vmatrix} z_{wx}(x) \cdot \frac{x_a^4}{2} & \text{if } P_{e_3} = 0 \\ (Z_{dx}(x) + Q_{3_2} \cdot Z_{wx}(x)) \cdot \frac{x_a^4}{2} & \text{otherwise} \end{vmatrix}$$

$$F_{bx3}(x) := \begin{vmatrix} z_{wx}(x) \cdot \frac{x_a^4}{2} & \text{if } P_{e_3} = 0 \\ (Z_{dx}(x) + Q_{3_2} \cdot Z_{wx}(x)) \cdot \frac{x_a^4}{2} & \text{otherwise} \end{vmatrix}$$

$$F_{bx7}(x) := \begin{vmatrix} z_{wx}(x) \cdot \frac{x_a^4}{2} & \text{if } P_{e_3} = 0 \\ (Z_{dx}(x) + Q_{3_2} \cdot Z_{wx}(x)) \cdot \frac{x_a^4}{2} & \text{otherwise} \end{vmatrix}$$

$$F_{bx7}(x) := \begin{vmatrix} z_{wx}(x) \cdot \frac{x_a^4}{2} & \text{if } P_{e_3} = 0 \\ (Z_{dx}(x) + Q_{3_2} \cdot Z_{wx}(x)) \cdot \frac{x_a^4}{2} & \text{otherwise} \end{vmatrix}$$

$$F_{bx7}(x) := \begin{vmatrix} z_{wx}(x) \cdot \frac{x_a^4}{2} & \text{if } P_{e_3} = 0 \\ (Z_{dx}(x) + Q_{3_2} \cdot Z_{wx}(x)) \cdot \frac{x_a^4}{2} & \text{otherwise} \end{vmatrix}$$

$$F_{bx7}(x) := \begin{vmatrix} z_{wx}(x) \cdot \frac{x_a^4}{2} & \text{if } P_{e_3} = 0 \\ (Z_{dx}(x) + Q_{3_2} \cdot Z_{wx}(x)) \cdot \frac{x_a^4}{2} & \text{otherwise} \end{vmatrix}$$

$$F_{bx7}(x) := \begin{vmatrix} F_{bx7}(x) \\ F_{bx8}(x) \\ F_{bx7}(x) \\ F_{bx8}(x) \\$$

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*

$$\begin{split} \sigma_{11_{1}} &:= \begin{bmatrix} \frac{1}{x_{1} - x_{5}} \left[\left(P_{s_{1}} \cdot x_{5} - P_{t_{1}} \cdot x_{1} \right) - \frac{2 \cdot Q_{2}}{a_{0}^{-2}} \cdot F_{tmin_{1}} \right] & \text{if } P_{e_{1}} = 0 \\ \sigma_{11_{1}} = -1288.776 \frac{lb}{in^{2}} \\ \hline \frac{1}{x_{1} - x_{5}} \left[\left(P_{s_{1}} \cdot x_{5} - P_{t_{1}} \cdot x_{1} \right) - P_{e_{1}} \cdot F_{tmin_{1}} \right] & \text{otherwise} \\ \sigma_{11_{2}} &:= \begin{bmatrix} \frac{1}{x_{1} - x_{5}} \left[\left(P_{s_{2}} \cdot x_{5} - P_{t_{2}} \cdot x_{1} \right) - \frac{2 \cdot Q_{2}}{a_{0}^{-2}} \cdot F_{tmin_{2}} \right] & \text{if } P_{e_{2}} = 0 \\ \hline \frac{1}{x_{1} - x_{5}} \left[\left(P_{s_{3}} \cdot x_{5} - P_{t_{3}} \cdot x_{1} \right) - \frac{2 \cdot Q_{2}}{a_{0}^{-2}} \cdot F_{tmin_{2}} \right] & \text{otherwise} \\ \sigma_{11_{3}} &:= \begin{bmatrix} \frac{1}{x_{1} - x_{5}} \left[\left(P_{s_{3}} \cdot x_{5} - P_{t_{3}} \cdot x_{1} \right) - \frac{2 \cdot Q_{2}}{a_{0}^{-2}} \cdot F_{tmin_{3}} \right] & \text{if } P_{e_{3}} = 0 \\ \hline \frac{1}{x_{1} - x_{5}} \left[\left(P_{s_{3}} \cdot x_{5} - P_{t_{3}} \cdot x_{1} \right) - \frac{2 \cdot Q_{2}}{a_{0}^{-2}} \cdot F_{tmin_{3}} \right] & \text{otherwise} \\ \sigma_{11_{3}} &:= \begin{bmatrix} \frac{1}{x_{1} - x_{5}} \left[\left(P_{s_{3}} \cdot x_{5} - P_{t_{3}} \cdot x_{1} \right) - \frac{2 \cdot Q_{2}}{a_{0}^{-2}} \cdot F_{tmin_{3}} \right] & \text{otherwise} \\ \sigma_{11_{3}} &:= \begin{bmatrix} \frac{1}{x_{1} - x_{5}} \left[\left(P_{s_{4}} \cdot x_{5} - P_{t_{3}} \cdot x_{1} \right) - \frac{2 \cdot Q_{2}}{a_{0}^{-2}} \cdot F_{tmin_{3}} \right] & \text{otherwise} \\ \sigma_{11_{4}} &:= 0.000 \frac{lb}{ln^{2}} \\ \hline \frac{1}{x_{1} - x_{5}} \left[\left(P_{s_{6}} \cdot x_{5} - P_{t_{5}} \cdot x_{1} \right) - \frac{2 \cdot Q_{2}}{a_{0}^{-2}} \cdot F_{tmin_{3}} \right] & \text{otherwise} \\ \sigma_{11_{5}} &:= 1743.547 \frac{lb}{ln^{2}} \\ \hline \frac{1}{x_{1} - x_{5}} \left[\left(P_{s_{5}} \cdot x_{5} - P_{t_{5}} \cdot x_{1} \right) - \frac{2 \cdot Q_{2}}{a_{0}^{-2}} \cdot F_{tmin_{5}} \right] & \text{otherwise} \\ \sigma_{11_{5}} &:= 1743.547 \frac{lb}{ln^{2}} \\ \hline \frac{1}{x_{1} - x_{5}} \left[\left(P_{s_{5}} \cdot x_{5} - P_{t_{5}} \cdot x_{1} \right) - \frac{2 \cdot Q_{2}}{a_{0}^{-2}} \cdot F_{tmin_{5}} \right] & \text{otherwise} \\ \sigma_{11_{5}} &:= 1743.547 \frac{lb}{ln^{2}} \\ \hline \frac{1}{x_{1} - x_{5}} \left[\left(P_{s_{5}} \cdot x_{5} - P_{t_{5}} \cdot x_{1} \right) - \frac{2 \cdot Q_{2}}{a_{0}^{-2}} \cdot F_{tmin_{5}} \right] & \text{otherwise} \\ \sigma_{11_{5}} &:= 1743.547 \frac{lb}{ln^{2}} \\ \hline \frac{1}{x_{1} - x_{5}} \left[\left(P_{s_{7}} \cdot x_{5} - P_{t_{7}} \cdot x_{1} \right) - \frac{2 \cdot Q_{2}}{a_{0}^{-2}} \cdot F_{tmin_{5}} \right] & \text{otherwise} \\ \sigma_{11_{5}} &:= 174$$

b) Determine tube stresses $~\sigma_{tmin}$ and σ_{tmax} for Load Cases 1 to 8 (continued)

$$\sigma_{11_{8}} := \begin{bmatrix} \frac{1}{x_{1} - x_{s}} \left[\left(P_{s_{8}} \cdot x_{s} - P_{t_{8}} \cdot x_{t} \right) - \frac{2 \cdot Q_{2_{8}}}{a_{0}^{2}} \cdot F_{train_{8}} \right] & \text{if } P_{e_{8}} = 0 \\ \sigma_{11_{8}} := -459.775 \frac{b}{in^{2}} \\ \sigma_{12_{1}} := \begin{bmatrix} \frac{1}{x_{1} - x_{s}} \left[\left(P_{s_{1}} \cdot x_{s} - P_{t_{1}} \cdot x_{t} \right) - \frac{2 \cdot Q_{2}}{a_{0}^{2}} \cdot F_{train_{1}} \right] & \text{otherwise} \\ \sigma_{12_{1}} := \begin{bmatrix} \frac{1}{x_{1} - x_{s}} \left[\left(P_{s_{1}} \cdot x_{s} - P_{t_{1}} \cdot x_{t} \right) - \frac{2 \cdot Q_{2}}{a_{0}^{2}} \cdot F_{train_{1}} \right] & \text{otherwise} \\ \sigma_{12_{2}} := \begin{bmatrix} \frac{1}{x_{1} - x_{s}} \left[\left(P_{s_{1}} \cdot x_{s} - P_{t_{1}} \cdot x_{t} \right) - \frac{2 \cdot Q_{2}}{a_{0}^{2}} \cdot F_{train_{1}} \right] & \text{otherwise} \\ \sigma_{12_{2}} := \begin{bmatrix} \frac{1}{x_{1} - x_{s}} \left[\left(P_{s_{2}} \cdot x_{s} - P_{t_{2}} \cdot x_{t} \right) - \frac{2 \cdot Q_{2}}{a_{0}^{2}} \cdot F_{train_{2}} \right] & \text{otherwise} \\ \sigma_{12_{2}} := \begin{bmatrix} \frac{1}{x_{1} - x_{s}} \left[\left(P_{s_{2}} \cdot x_{s} - P_{t_{2}} \cdot x_{t} \right) - P_{e_{1}} \cdot F_{train_{2}} \right] & \text{otherwise} \\ \sigma_{12_{2}} := \begin{bmatrix} \frac{1}{x_{1} - x_{s}} \left[\left(P_{s_{3}} \cdot x_{s} - P_{t_{3}} \cdot x_{t} \right) - P_{e_{3}} \cdot F_{train_{3}} \right] & \text{otherwise} \\ \sigma_{12_{3}} := \left[\frac{1}{x_{1} - x_{s}} \left[\left(P_{s_{3}} \cdot x_{s} - P_{t_{3}} \cdot x_{t} \right) - P_{e_{3}} \cdot F_{train_{3}} \right] & \text{otherwise} \\ \sigma_{12_{3}} := -73.142 \frac{b}{in^{2}} \\ \hline \left[\frac{1}{x_{1} - x_{s}} \left[\left(P_{s_{3}} \cdot x_{s} - P_{t_{3}} \cdot x_{t} \right) - P_{e_{3}} \cdot F_{train_{3}} \right] & \text{otherwise} \\ \sigma_{12_{4}} := \left[\frac{1}{x_{1} - x_{s}} \left[\left(P_{s_{3}} \cdot x_{s} - P_{t_{3}} \cdot x_{t} \right) - P_{e_{3}} \cdot F_{train_{3}} \right] & \text{otherwise} \\ \sigma_{12_{4}} := \left[\frac{1}{x_{1} - x_{s}} \left[\left(P_{s_{4}} \cdot x_{s} - P_{t_{3}} \cdot x_{t} \right) - P_{e_{3}} \cdot F_{train_{3}} \right] & \text{otherwise} \\ \sigma_{12_{5}} := \left[\frac{1}{x_{1} - x_{s}} \left[\left(P_{s_{5}} \cdot x_{s} - P_{t_{5}} \cdot x_{t} \right) - P_{e_{5}} \cdot F_{train_{3}} \right] & \text{otherwise} \\ \sigma_{12_{5}} := \left[\frac{1}{x_{1} - x_{s}} \left[\left(P_{s_{6}} \cdot x_{s} - P_{t_{5}} \cdot x_{t} \right) - P_{e_{5}} \cdot F_{train_{5}} \right] & \text{otherwise} \\ \sigma_{12_{5}} := 8187.390 \frac{b}{in^{2}} \\ \left[\frac{1}{x_{1} - x_{s}} \left[\left(P_{s_{6}} \cdot x_{s} - P_{t_{5}} \cdot x_{t} \right) - P_{e_{5}} \cdot F_{train_{5}} \right] & \text{otherwise} \\ \sigma_{12_{6}} := \left[\frac{1}{x_{1} - x_{$$

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lb in²

280

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-0 or -

-0

$$\begin{split} \text{Step (b) : check the tubes for buckling if } \sigma_{t_1} < 0 \text{ or } \sigma_{t_2} < 0 \\ t_1 = kl + 48.000 \text{ in} \\ r_1 := \sqrt{\frac{4(\frac{2}{4} + (4(-2+4)^2)}{4}}{r_1 = 0.337 \text{ in}} \\ r_2 = 0.337 \text{ in} \\ r_3 := 0 \\ max[3.25 - 25[(24 + 0_{3_1} Z_3) \times x_3^4], 1.25] \text{ otherwise} \\ r_{51} := si(F_{51} > 2.2, F_{51}) \\ r_{52} := 125 \text{ if } P_{6_2} = 0 \\ max[3.25 - 25[(24 + 0_{3_2} Z_3) \times x_3^4], 1.25] \text{ otherwise} \\ r_{53} := 1.25 \text{ if } P_{6_2} = 0 \\ max[3.25 - 25[(24 + 0_{3_2} Z_3) \times x_3^4], 1.25] \text{ otherwise} \\ r_{53} := 1.25 \text{ if } P_{6_3} = 0 \\ max[3.25 - 25[(24 + 0_{3_3} Z_3) \times x_3^4], 1.25] \text{ otherwise} \\ r_{54} := 1.25 \text{ if } P_{6_3} = 0 \\ max[3.25 - 25[(24 + 0_{3_3} Z_3) \times x_3^4], 1.25] \text{ otherwise} \\ r_{54} := 1.25 \text{ if } P_{6_3} = 0 \\ max[3.25 - 25[(24 + 0_{3_3} Z_3) \times x_3^4], 1.25] \text{ otherwise} \\ r_{55} := 1.25 \text{ if } P_{6_5} = 0 \\ max[3.25 - 25[(24 + 0_{3_6} Z_3) \times x_3^4], 1.25] \text{ otherwise} \\ r_{56} := 1.25 \text{ if } P_{6_5} = 0 \\ max[3.25 - 25[(24 + 0_{3_6} Z_3) \times x_3^4], 1.25] \text{ otherwise} \\ r_{56} := 1.25 \text{ if } P_{6_5} = 0 \\ max[3.25 - 25[(24 + 0_{3_6} Z_3) \times x_3^4], 1.25] \text{ otherwise} \\ r_{56} := 1.25 \text{ if } P_{6_5} = 0 \\ max[3.25 - 25[(24 + 0_{3_6} Z_3) \times x_3^4], 1.25] \text{ otherwise} \\ r_{57} := 1.25 \text{ if } P_{6_5} = 0 \\ max[3.25 - 25[(24 + 0_{3_6} Z_3) \times x_3^4], 1.25] \text{ otherwise} \\ r_{57} := 1.25 \text{ if } P_{6_5} = 0 \\ max[3.25 - 25[(24 + 0_{3_6} Z_3) \times x_3^4], 1.25] \text{ otherwise} \\ r_{57} := 1.25 \text{ if } P_{6_5} = 0 \\ max[3.25 - 25[(24 + 0_{3_6} Z_3) \times x_3^4], 1.25] \text{ otherwise} \\ r_{58} := \frac{1.25 \text{ if } P_{6_5} = 0 \\ max[3.25 - 25[(24 + 0_{3_6} Z_3) \times x_3^4], 1.25] \text{ otherwise} \\ r_{58} := \frac{1.25 \text{ if } P_{6_5} = 0 \\ max[3.25 - 25[(24 + 0_{3_6} Z_3) \times x_3^4], 1.25] \text{ otherwise} \\ r_{58} := \frac{1.25 \text{ if } P_{6_5} = 0 \\ max[3.25 - 25[(24 + 0_{3_6} Z_3) \times x_3^4], 1.25] \text{ otherwise} \\ r_{58} := \frac{1.25 \text{ if } P_{6_5} = 0 \\ max[3.25 - 25[(24 + 0_{3_6} Z_3) \times x_3^4], 1.25] \text{ otherwise} \\ r_{58} := \frac{1.25 \text{ if } P_{6_5} = 0 \\ max[3.25 - 25[(24 + 0_{5_5} Z_3) \times x_3^4], 1.25] \text{ otherwis$$

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ExampleE4.18.7-FIXED TS(AnnexV) from PTB4

UHX 13.5.10 Etep 10 Determine the shell membrane stress in the main shell
$$a_{mn} := \frac{a_0^2}{(D_n + l_n) + l_n} \left[P_n + (p_n^2 - 1) \cdot (P_n - P_i) \right] + \frac{a_n^2}{(D_n + l_n) + l_n} P_t - a_{mn} = \frac{a_0^2}{(D_n + l_n) + l_n} \left[\frac{b}{(D_n^2 + l_n) + l_n} \right] + \frac{a_n^2}{(D_n^2 + l_n) + l_n} P_t - a_{mn} = \frac{a_0^2}{(D_n^2 + l_n) + l_n} \left[\frac{b}{(D_n^2 + l_n) + l_n} \right] + \frac{a_n^2}{(D_n^2 + l_n) + l_n} P_t - a_{mn} = \frac{a_0^2}{(D_n^2 + l_n) + l_n} \left[\frac{b}{(D_n^2 + l_n) + l_n} \right] + \frac{a_n^2}{(D_n^2 + l_n) + l_n} P_t - a_{mn} = \frac{a_0^2}{(D_n^2 + l_n) + l_n} \left[\frac{b}{(D_n^2 + l_n) + l_n} \right] + \frac{a_n^2}{(D_n^2 + l_n) + l_n} P_t - a_{mn} = \frac{a_0^2}{(D_n^2 + l_n) + l_n} \left[\frac{b}{(D_n^2 + l_n) + l_n} \right] + \frac{a_n^2}{(D_n^2 + l_n) + l_n} \left[\frac{b}{(D_n^2 + l_n) + l_n} \right] + \frac{a_n^2}{(D_n^2 + l_n) + l_n} \left[\frac{b}{(D_n^2 + l_n) + l_n} \right] + \frac{a_n^2}{(D_n^2 + l_n) + l_n} \left[\frac{b}{(D_n^2 + l_n) + l_n} \right] + \frac{a_n^2}{(D_n^2 + l_n) + l_n} \left[\frac{b}{(D_n^2 + l_n) + l_n} \right] + \frac{a_n^2}{(D_n^2 + l_n) + l_n} \left[\frac{b}{(D_n^2 + l_n) + l_n} \right] + \frac{a_n^2}{(D_n^2 + l_n) + l_n} \left[\frac{b}{(D_n^2 + l_n) + l_n} \right] + \frac{a_n^2}{(D_n^2 + l_n) + l_n} \left[\frac{b}{(D_n^2 + l_n) + l_n} \right] + \frac{a_n^2}{(D_n^2 + l_n) + l_n} \left[\frac{b}{(D_n^2 + l_n) + l_n} \right] + \frac{a_n^2}{(D_n^2 + l_n) + l_n} \left[\frac{b}{(D_n^2 + l_n) + l_n} \right] + \frac{a_n^2}{(D_n^2 + l_n) + l_n} \left[\frac{b}{(D_n^2 + l_n) + l_n} \right] + \frac{a_n^2}{(D_n^2 + l_n) + l_n} \left[\frac{b}{(D_n^2 + l_n) + l_n} \right] + \frac{a_n^2}{(D_n^2 + l_n) + l_n} \left[\frac{b}{(D_n^2 + l_n) + l_n} \right] + \frac{a_n^2}{(D_n^2 + l_n) + l_n} \left[\frac{a_n^2 + l_n}{(D_n^2 + l_n) + l_n} \right] + \frac{a_n^2}{(D_n^2 + l_n) + l_n} \left[\frac{b}{(D_n^2 + l_n) + l_n} \right] + \frac{a_n^2}{(D_n^2 + l_n) + l_n} \left[\frac{b}{(D_n^2 + l_n) + l_n} \right] + \frac{a_n^2}{(D_n^2 + l_n) + l_n} \left[\frac{b}{(D_n^2 + l_n) + l_n} \right] + \frac{a_n^2}{(D_n^2 + l_n) + l_n} \left[\frac{b}{(D_n^2 + l_n) + l_n} \right] + \frac{a_n^2}{(D_n^2 + l_n) + l_n} \left[\frac{b}{(D_n^2 + l_n) + l_n} \right] + \frac{a_n^2}{(D_n^2 + l_n) + l_n} \left[\frac{b}{(D_n^2 + l_n) + l_n} \right] + \frac{a_n^2}{(D_n^2 + l_n) + l_n} \left[\frac{b}{(D_n^2 + l_n) + l_n} \right] + \frac{a_n^2}{(D_n^2 + l_n) + l_n} \left[\frac{b}{(D_n^2 + l_n) + l_n} \right]$$



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 $S_{c}^{*} := \min\left(S_{ycD}, \frac{S_{PSc}}{2}\right)$

 $S^{*}_{c} = 33600.000 \frac{lb}{in^{2}}$

 $fact_{cv} := \overbrace{\left(1.4 - \frac{0.4 \cdot \left|\sigma_{cb}\right|}{S_{c}^{*}}\right)}^{\bullet} \qquad fact_{cv} =$

 $fact_{c} := \begin{pmatrix} \min(fact_{cv_{1}}, 1) \\ \min(fact_{cv_{2}}, 1) \\ \min(fact_{cv_{3}}, 1) \\ \min(fact_{cv_{3}}, 1) \\ \min(fact_{cv_{3}}, 1) \\ 1.000 \\ 1.000 \end{pmatrix}$

min(fact_{cv}

(1.0625) 1.2989 1.1636

1.4

0.7764 1.215 0.8775 (1.1139)

ExampleE4.18.7-FIXED TS(AnnexV) from PTB4

UHX13.7 Simplified Elastic Plastic Procedure

Calculation procedure for the effect of plasticity at the tubesheet, channel or shell joint.

This procedure applies only to Configurations a,b,c and Design Loading Cases 1, 2, 3, 4 in the following conditions: -for integral shell (config. a,b,c) when 1.5S $_{s} < \sigma_{s} \le S_{PS,s}$ -for integral channel (config. a) when $1.5S_c < \sigma_c \le S_{PS,c}$

$$\begin{split} \mathbf{S^{*}_{s}} &:= \min \left(\mathbf{S}_{ys1}, \frac{\mathbf{S}_{PSs}}{2} \right) \\ \mathbf{S^{*}_{s}} &= 17500.000 \frac{lb}{in^{2}} \\ fact_{sv} &:= \overline{\left(1.4 - \frac{0.4 \cdot \left| \sigma_{sb1} \right|}{\mathbf{S^{*}_{s}}} \right)} \\ fact_{sv} &:= \overline{\left(1.4 - \frac{0.4 \cdot \left| \sigma_{sb1} \right|}{\mathbf{S^{*}_{s}}} \right)} \\ fact_{sv} &= \left(\frac{1.122}{0.766} \\ 1.044 \\ 1.400 \\ 0.522 \\ 1.365 \\ 1.156 \\ 0.800 \right) \\ fact_{s} &:= \left(\frac{\min \left(fact_{sv_{1}}, 1 \right)}{\min \left(fact_{sv_{2}}, 1 \right)} \\ \min \left(fact_{sv_{3}}, 1 \right) \\ \min \left(fact_{sv_{4}}, 1 \right) \right) \\ fact_{s} &= \left(\frac{1.000}{0.766} \\ 1.000 \\ 1.000 \\ 1.000 \right) \\ \end{split}$$

Calculate reduced values of $\rm E_{s}$ and $\rm E_{c}$ for each loading case:

$$E^{*}_{s1} := E_{s1} \cdot fact_{s} \qquad E^{*}_{s1} := E_{s1} \cdot fact_{s} \qquad E^{*}_{s1} = \begin{pmatrix} 2.6400 \times 10^{7} \\ 2.0216 \times 10^{7} \\ 2.6400 \times 10^{7} \\ 1.2448 \times 10^{5} \\ 3.1971 \times 10^{5} \\ 1.2446 \times 10^{5} \\ 1.246 \times$$

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(6.732)

5.649

6.732 6.732

F =

From Step 4, recalculate:

$$\mathsf{F} := \frac{1 - \nu^{\star}}{\mathsf{E}^{\star}} \cdot \left(\lambda \lambda_{\mathsf{S}} + \lambda \lambda_{\mathsf{C}} + \mathsf{E}_{\mathsf{D}} \cdot \mathsf{In}(\mathsf{K}) \right)$$

$$Q_{1e} := \overbrace{\left(\frac{\rho_{s} - 1 - \Phi \cdot Z_{v}}{1 + \Phi \cdot Z_{m}}\right)}^{(-0.059)} \qquad \qquad Q_{1e} = \begin{cases} -0.059 \\ -0.054 \\ -0.059 \\ -0.059 \end{cases}$$

$$Q_{z1e} := \frac{(Z_d + Q_{1e} \cdot Z_w) \cdot X_a^4}{2} \qquad \qquad Q_{z1e} = \begin{pmatrix} 3.778 \\ 3.899 \\ 3.778 \\ 3.778 \end{pmatrix}$$

$$U_{e} := \frac{\left[Z_{w} + \left(\rho_{s} - 1\right) \cdot Z_{m}\right] \cdot X_{a}^{4}}{1 + \Phi \cdot Z_{m}} \qquad \qquad U_{e} = \begin{pmatrix} 20.625 \\ 23.037 \\ 20.625 \\ 20.625 \end{pmatrix}$$

From Step 6 , recalculate $\mathbf{P}_{\mathbf{w}}, \mathbf{P}_{\mathrm{rim}}, \mathrm{and} \ \mathbf{P}_{\mathrm{e}\,:}$

$$P'_{w} := \begin{pmatrix} \overline{U_{e_{1}}} & \overline{Y_{b}} \\ \overline{u_{o}^{2}} & \overline{2\pi} W^{*} \\ \overline{u_{o}^{2}} & \overline{u_{o}^{2}} \\ \overline{u_{o}^{2}} \\$$

(9.024) $\Phi := (1 + v^*) \cdot \mathsf{F} \qquad \Phi = \left(\begin{array}{c} 9.024 \\ 7.572 \\ 9.024 \\ 9.024 \end{array} \right)$ (9.024)

*

$$Q_{z2e} := \frac{(Z_v + Q_{1e} \cdot Z_m) \cdot X_a^4}{2} \qquad Q_{z2e} = \begin{pmatrix} 10.312 \\ 11.518 \\ 10.312 \\ 10.312 \end{pmatrix}$$

$$W^{*'} := \begin{pmatrix} W^{*}_{1} \\ W^{*}_{2} \\ W^{*}_{3} \\ W^{*}_{4} \end{pmatrix} \qquad W^{*'} = \begin{pmatrix} 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \end{pmatrix} | Ib$$

$$\begin{pmatrix} W^*_1 \\ W^*_2 \end{pmatrix} \qquad \begin{pmatrix} 0.000 \\ 0.000 \end{pmatrix}$$

$$W^{*'} := \begin{pmatrix} W^{*}_{1} \\ W^{*}_{2} \\ W^{*}_{3} \end{pmatrix} \qquad W^{*'} = \begin{pmatrix} 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \end{pmatrix}$$

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From Step 7 , recalculate $\rm Q_2$, $\rm Q_3$, $\rm F_m$ and the tubesheet bending stress for load cases 1, 2, and 3

$$Q'_{2} := \begin{bmatrix} \left(\omega^{*}{}_{s} \cdot P_{se} - \omega^{*}{}_{c} \cdot P_{te} \right) + \frac{\gamma_{b}}{2 \cdot \pi} \cdot W^{*'} \\ 1 + \Phi \cdot Z_{m} \end{bmatrix} \qquad \qquad Q'_{2} = \begin{bmatrix} 181.674 \\ -575.344 \\ -333.433 \\ 0.000 \end{bmatrix} \\ Q'_{3} := \boxed{Q'_{3} := \left(Q_{1e} + \frac{2 \cdot Q'_{2}}{P'_{e} \cdot a_{0}^{2}} \right)} \qquad \qquad Q'_{3} = \begin{bmatrix} -0.0675 \\ -0.0773 \\ -0.1377 \\ -0.0586 \end{bmatrix}$$

$$F'_{m1}(x) := \begin{bmatrix} 0 & \text{if } P'_{e_1} = 0 \\ \frac{Q_v(x) + Q'_{3_1} \cdot Q_m(x)}{2} & F'm1_j := |F'_{m1}(x_j)| \\ F'_{m1} := max(F'm1) & F'_{m1} = 0.034 \end{bmatrix}$$

$$F'_{m2}(x) := \begin{vmatrix} 0 & \text{if } P'_{e_2} = 0 \\ \frac{Q_v(x) + Q'_{3_2} \cdot Q_m(x)}{2} & F'_{m2} := |F'_{m2}(x_j)| \\ F'_{m2} := \max(F'_{m2}) & F'_{m2} = 0.039 \end{vmatrix}$$

$$F'_{m4}(x) := \begin{bmatrix} 0 & \text{if } P'_{e_4} = 0 \\ \\ \frac{Q_v(x) + Q'_{3_4} \cdot Q_m(x)}{2} & \text{otherwise} \end{bmatrix}$$

$$F'_{m4} := \left|F'_{m4}(x_j)\right| \qquad F'_{m4} := \max(F'm4) \qquad F'_{m4} = 0.000$$

$$\mathbf{F'_m} := \begin{pmatrix} \mathbf{F_{m1}} \\ \mathbf{F'_{m2}} \\ \mathbf{F'_{m3}} \\ \mathbf{F'_{m4}} \end{pmatrix} \qquad \mathbf{F'_m} = \begin{pmatrix} 0.0337 \\ 0.0386 \\ 0.0689 \\ 0 \end{pmatrix}$$

Tubesheet Bending Stress for the Elastic-Plastic Solution

$$\sigma' := \boxed{\left[\frac{1.5 \cdot F'_{m}}{\mu^{*}} \cdot \left(\frac{2 \cdot a_{o}}{h - h'_{g}}\right)^{2} \cdot P'_{e}\right]} \qquad \sigma' = \begin{pmatrix} -16286.147 \\ 22204.445 \\ 6797.980 \\ 0.000 \end{pmatrix} \cdot \boxed{\frac{lb}{in^{2}}} \qquad \sigma'_{max} := max(\left|\sigma'_{1}\right|, \left|\sigma'_{2}\right|, \left|\sigma'_{3}\right|) \qquad \sigma'_{max} = 22204.445 \frac{lb}{in^{2}} \\ \sigma'_{max} := 1.5 \cdot S_{D} \qquad \sigma_{allow} := 1.5 \cdot S_{D} \qquad \sigma_{allow} = 23700.0 \frac{lb}{in^{2}}$$

ExampleE4.18.7-FIXED TS(AnnexV) from PTB4

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Calculation must be performed in 2 phases :

Phase 1) Perform Steps 1 to 11 with SS="NON" (normal calculation) with the following modifications in Step 11:

- minimum length requirement Ismin of shell band for configurations a,b,c do not apply

- minimum length requirement $\mathbf{I}_{\rm cmin}$ of channel band for configuration a do not apply

if $\sigma_s \leq S_{PS,s}$ and $\sigma_c \leq S_{PS,c}$, the shell and/or channel designs are acceptable. Otherwise increase the thickness of the overstressed components (shell and/or channel) and return to Step 1.

Phase 2) Perform Steps 1 to 7 using SS="OUI" (Simply Supported calculation) for loading cases 1,2 and 3 only

If $|\sigma| \le 1.5S$, the calculation procedure is complete. Otherwise, increase the assumed tubesheet thickness *h* and repeat Steps 1 to 7.

Note: If $|\sigma|<1.5S$, the tubesheet thickness can be optimized to a value h_0 till $|\sigma|=1.5S$ provided that, for that optimized thickness h_0 , the stresses in the tubesheet, shell and channel , calculated by the normal calculation (see Phase 1), remain respectively below $S_{PS, S}$, $S_{PS,c}$ for each of the 8 loading cases.

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The 2nd part of this Mathcad software provides general basic equations without using coefficient Q_3 They are taken from Item 04-1401 and enable to calculate q(x), w(x), $\theta(x)$, $\sigma(x)$, $\tau(x)$, $\sigma_t(x)$ at any radius of the Tubesheet, depending on Loads V_a and M_a acting at periphery of the perforated tubesheet (r = a_0). It provides also the axial deplacement and axial load acting in the shell.

 Δ_s = elastic stretch of shell N_s = axial force in shell (per unit of circumference) N_c = axial force in channel (per unit of circumference) K_w = elastic foundation modulus of the heat exchanger (based on L) $k_w = 2K_w = elastic foundation modulus of the half H.E.(based on L/2)$ D^{*} = Tubesheet flexural rigidity YELLOW :most important data and results Limit Conditions **Results obtained fromUHX-13** Perforated Tubesheet $K_{w} := \frac{N_{t} \cdot K_{t}}{\pi a_{o}^{2}} \qquad \qquad k_{w} := 2 \cdot K_{w} \qquad D^{*} := \frac{E^{*} h^{3}}{12 \cdot (1 - v^{*2})} \qquad \qquad k := \left(\frac{k_{w}}{D^{*}}\right)^{0.25} \qquad \qquad numpoints = 20$ $K_{w} = 11905.554 \frac{lb}{in^{3}} \qquad \qquad k_{w} = 23811.107 \frac{lb}{in^{3}} \qquad D^{*} = 1.779 \times 10^{6} \, lb \cdot in \qquad \qquad k = 0.340 \, in^{-1}$ $\mathsf{Z}_{\mathsf{VX}}(\mathsf{x}) \coloneqq \frac{\Psi_2 \cdot \mathsf{ber'}_\mathsf{X}(\mathsf{x}) + \Psi_1 \cdot \mathsf{bei'}_\mathsf{X}(\mathsf{x})}{\mathsf{X_a}^2 \cdot \mathsf{Z_a}}$ $Z_{mx}(x) := \frac{ber' \cdot ber'_{x}(x) + bei' \cdot bei'_{x}(x)}{X_{a} \cdot Z_{a}}$ $Z_{mx}(X_a) = 0.207$ $Z_m = 0.207$ $Z_{vx}(X_{a}) = 0.021$ $Z_{v} = 0.021$ $Z_{dx}(x) := \frac{\Psi_2 \cdot ber_x(x) + \Psi_1 \cdot bei_x(x)}{X_a^3 \cdot Z_a} \qquad \qquad Z_{wx}(x) := \frac{ber' \cdot ber_x(x) + bei' \cdot bei_x(x)}{X_a^2 \cdot Z_a} \\ Z_{dx}(X_a) = 0.004 \qquad \qquad Z_d = 0.004 \qquad \qquad Z_{wx}(X_a) = 0.021 \qquad \qquad Z_w := Z_{wx}(X_a) \qquad \qquad Z_w = 0.021 \qquad \qquad Z_w$ Loads V_a and M_a acting at periphery of perforated tubesheet (r = a_o) $V_a := \left(\frac{a_0}{2}P_e\right)$ $M_a := (a_0 \cdot V_a \cdot Q_3)$ -1000.036 1391.316 1204.496 -1972.064 204.460 -580.747 0.000 0.000 lb 3438.824 -488.221 75.445



1466.761

2047.508)

-1488.257

-1692.717

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ExampleE4.18.7-FIXED TS(AnnexV) from PTB4

Net Tubesheet Pressure

$$q_{max} := \begin{pmatrix} \max(q(x)_{1}) \\ \max(q(x)_{2}) \\ \max(q(x)_{3}) \\ \max(q(x)_{3}) \\ \max(q(x)_{4}) \\ \max(q(x)_{5}) \\ \max(q(x)_{5}) \\ \max(q(x)_{7}) \\ \max(q(x)_{8}) \end{pmatrix} \qquad q_{min} := \begin{pmatrix} \min(q(x)_{1}) \\ \min(q(x)_{2}) \\ \min(q(x)_{3}) \\ \min(q(x)_{5}) \\ \min(q(x)_{5}) \\ \min(q(x)_{7}) \\ \min(q(x)_{8}) \end{pmatrix}$$

$$q_{max} = \begin{pmatrix} 26.152 \\ 380.756 \\ 41.573 \\ 0.000 \\ 73.727 \\ 23.163 \\ 47.453 \\ 48.099 \end{pmatrix} \qquad q_{min} = \begin{pmatrix} -345.033 \\ -28.309 \\ -3.762 \\ 0.000 \\ -965.193 \\ -239.405 \\ -584.437 \\ -620.160 \end{pmatrix}$$

$$F_{q} := \frac{(Z_{d} + Q_{3} \cdot Z_{v}) \cdot X_{a}^{4}}{2}$$

$$\left(\begin{pmatrix} -345.033 \\ 380.756 \\ 35.723 \\ \end{pmatrix} \right) \qquad F_{q} := \frac{(Z_{d} + Q_{3} \cdot Z_{v}) \cdot X_{a}^{4}}{2}$$

$$q(X_{a}) = \begin{pmatrix} 380.756\\ 35.723\\ 0.000\\ -965.193\\ -239.405\\ -584.437 \\ -620.160 \end{pmatrix} \xrightarrow{|b|} (F_{q} \cdot P_{e}) = \begin{pmatrix} 380.756\\ 35.723\\ 0.000\\ -965.193\\ -239.405\\ -584.437 \\ -620.160 \end{pmatrix}$$

$$q(\mathbf{x}) := \begin{bmatrix} a_0^2 \cdot \mathbf{k}^4 \cdot \left[\mathbf{M}_{a_1} \cdot \mathbf{Z}_{wx}(\mathbf{x}) + \left(a_0 \cdot \mathbf{V}_{a_1} \right) \cdot \mathbf{Z}_{dx}(\mathbf{x}) \right] \\ a_0^2 \cdot \mathbf{k}^4 \cdot \left[\mathbf{M}_{a_2} \cdot \mathbf{Z}_{wx}(\mathbf{x}) + \left(a_0 \cdot \mathbf{V}_{a_2} \right) \cdot \mathbf{Z}_{dx}(\mathbf{x}) \right] \\ a_0^2 \cdot \mathbf{k}^4 \cdot \left[\mathbf{M}_{a_3} \cdot \mathbf{Z}_{wx}(\mathbf{x}) + \left(a_0 \cdot \mathbf{V}_{a_3} \right) \cdot \mathbf{Z}_{dx}(\mathbf{x}) \right] \\ a_0^2 \cdot \mathbf{k}^4 \cdot \left[\mathbf{M}_{a_4} \cdot \mathbf{Z}_{wx}(\mathbf{x}) + \left(a_0 \cdot \mathbf{V}_{a_4} \right) \cdot \mathbf{Z}_{dx}(\mathbf{x}) \right] \\ a_0^2 \cdot \mathbf{k}^4 \cdot \left[\mathbf{M}_{a_5} \cdot \mathbf{Z}_{wx}(\mathbf{x}) + \left(a_0 \cdot \mathbf{V}_{a_5} \right) \cdot \mathbf{Z}_{dx}(\mathbf{x}) \right] \\ a_0^2 \cdot \mathbf{k}^4 \cdot \left[\mathbf{M}_{a_6} \cdot \mathbf{Z}_{wx}(\mathbf{x}) + \left(a_0 \cdot \mathbf{V}_{a_6} \right) \cdot \mathbf{Z}_{dx}(\mathbf{x}) \right] \\ a_0^2 \cdot \mathbf{k}^4 \cdot \left[\mathbf{M}_{a_7} \cdot \mathbf{Z}_{wx}(\mathbf{x}) + \left(a_0 \cdot \mathbf{V}_{a_7} \right) \cdot \mathbf{Z}_{dx}(\mathbf{x}) \right] \\ a_0^2 \cdot \mathbf{k}^4 \cdot \left[\mathbf{M}_{a_8} \cdot \mathbf{Z}_{wx}(\mathbf{x}) + \left(a_0 \cdot \mathbf{V}_{a_8} \right) \cdot \mathbf{Z}_{dx}(\mathbf{x}) \right] \end{bmatrix}$$





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Slope $\theta(x)$ of the perforated tubesheet (0 <= x <= Xa)

$$\theta(\mathbf{x}) := \begin{bmatrix} \frac{\mathbf{a}_{o}}{\mathbf{D}^{\star}} \cdot \left[\mathbf{M}_{a_{1}} \cdot \mathbf{Z}_{m\mathbf{x}}(\mathbf{x}) + \left(\mathbf{a}_{o} \cdot \mathbf{V}_{a_{1}}\right) \cdot \mathbf{Z}_{v\mathbf{x}}(\mathbf{x}) \right] \\ \frac{\mathbf{a}_{o}}{\mathbf{D}^{\star}} \cdot \left[\mathbf{M}_{a_{2}} \cdot \mathbf{Z}_{m\mathbf{x}}(\mathbf{x}) + \left(\mathbf{a}_{o} \cdot \mathbf{V}_{a_{2}}\right) \cdot \mathbf{Z}_{v\mathbf{x}}(\mathbf{x}) \right] \\ \frac{\mathbf{a}_{o}}{\mathbf{D}^{\star}} \cdot \left[\mathbf{M}_{a_{3}} \cdot \mathbf{Z}_{m\mathbf{x}}(\mathbf{x}) + \left(\mathbf{a}_{o} \cdot \mathbf{V}_{a_{3}}\right) \cdot \mathbf{Z}_{v\mathbf{x}}(\mathbf{x}) \right] \\ \frac{\mathbf{a}_{o}}{\mathbf{D}^{\star}} \cdot \left[\mathbf{M}_{a_{4}} \cdot \mathbf{Z}_{m\mathbf{x}}(\mathbf{x}) + \left(\mathbf{a}_{o} \cdot \mathbf{V}_{a_{4}}\right) \cdot \mathbf{Z}_{v\mathbf{x}}(\mathbf{x}) \right] \\ \frac{\mathbf{a}_{o}}{\mathbf{D}^{\star}} \cdot \left[\mathbf{M}_{a_{5}} \cdot \mathbf{Z}_{m\mathbf{x}}(\mathbf{x}) + \left(\mathbf{a}_{o} \cdot \mathbf{V}_{a_{5}}\right) \cdot \mathbf{Z}_{v\mathbf{x}}(\mathbf{x}) \right] \\ \frac{\mathbf{a}_{o}}{\mathbf{D}^{\star}} \cdot \left[\mathbf{M}_{a_{6}} \cdot \mathbf{Z}_{m\mathbf{x}}(\mathbf{x}) + \left(\mathbf{a}_{o} \cdot \mathbf{V}_{a_{6}}\right) \cdot \mathbf{Z}_{v\mathbf{x}}(\mathbf{x}) \right] \\ \frac{\mathbf{a}_{o}}{\mathbf{D}^{\star}} \cdot \left[\mathbf{M}_{a_{7}} \cdot \mathbf{Z}_{m\mathbf{x}}(\mathbf{x}) + \left(\mathbf{a}_{o} \cdot \mathbf{V}_{a_{7}}\right) \cdot \mathbf{Z}_{v\mathbf{x}}(\mathbf{x}) \right] \\ \frac{\mathbf{a}_{o}}{\mathbf{D}^{\star}} \cdot \left[\mathbf{M}_{a_{8}} \cdot \mathbf{Z}_{m\mathbf{x}}(\mathbf{x}) + \left(\mathbf{a}_{o} \cdot \mathbf{V}_{a_{8}}\right) \cdot \mathbf{Z}_{v\mathbf{x}}(\mathbf{x}) \right] \\ \end{bmatrix}$$



$$\Theta(X_a) = \begin{pmatrix} -1.601 \times 10^{-3} \\ 1.218 \times 10^{-3} \\ -3.829 \times 10^{-4} \\ 0.000 \\ -5.048 \times 10^{-3} \\ -2.229 \times 10^{-3} \\ -3.830 \times 10^{-3} \\ -3.447 \times 10^{-3} \end{pmatrix}$$



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ExampleE4.18.7-FIXED TS(AnnexV) from PTB4

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ExampleE4.18.7-FIXED TS(AnnexV) from PTB4

Moment, Lb

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ExampleE4.18.7-FIXED TS(AnnexV) from PTB4



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Shear Load Q(x) of the perform	ited tubesheet (0 <= x <= Xa)
ber'·bei' _x (x) – bei'·ber' _x (x)	$\Psi_2 \cdot bei'_x(x) - \Psi_1 \cdot ber'_x(x)$
$Z_{\alpha}(x) = x_{a} = Z_{a}$	$Z_{\beta}(x) = Z_{a}$
$Q(x) := \begin{bmatrix} \frac{1}{a_0} \cdot \left[M_{a_1} \cdot Q_\alpha(x) + \left(a_0 \cdot V_{a_1}\right) \cdot Q_\beta(x) \right] \\ \frac{1}{a_0} \cdot \left[M_{a_2} \cdot Q_\alpha(x) + \left(a_0 \cdot V_{a_2}\right) \cdot Q_\beta(x) \right] \\ \frac{1}{a_0} \cdot \left[M_{a_3} \cdot Q_\alpha(x) + \left(a_0 \cdot V_{a_3}\right) \cdot Q_\beta(x) \right] \\ \frac{1}{a_0} \cdot \left[M_{a_4} \cdot Q_\alpha(x) + \left(a_0 \cdot V_{a_4}\right) \cdot Q_\beta(x) \right] \\ \frac{1}{a_0} \cdot \left[M_{a_5} \cdot Q_\alpha(x) + \left(a_0 \cdot V_{a_5}\right) \cdot Q_\beta(x) \right] \\ \frac{1}{a_0} \cdot \left[M_{a_6} \cdot Q_\alpha(x) + \left(a_0 \cdot V_{a_6}\right) \cdot Q_\beta(x) \right] \\ \frac{1}{a_0} \cdot \left[M_{a_7} \cdot Q_\alpha(x) + \left(a_0 \cdot V_{a_7}\right) \cdot Q_\beta(x) \right] \\ \frac{1}{a_0} \cdot \left[M_{a_8} \cdot Q_\alpha(x) + \left(a_0 \cdot V_{a_8}\right) \cdot Q_\beta(x) \right] \end{bmatrix}$	$Q(x)_{1} = \begin{bmatrix} 1 \\ 1 \\ 4.925 \\ 2 \\ 10.531 \\ 3 \\ 17.452 \\ 4 \\ 26.218 \\ 5 \\ 37.187 \\ 6 \\ 50.451 \\ 7 \\ 65.731 \\ 17 \\ 65.731 \\ 17 \\ 65.731 \\ 10 \\ 112.232 \\ 11 \\ 120.166 \\ 12 \\ 117.926 \\ 13 \\ 100.008 \\ 14 \\ 59.874 \\ 15 \\ -9.748 \\ 16 \\ -116.288 \\ \end{bmatrix} \frac{lb}{in}$
$ \begin{array}{c} 5000 \\ 0 \\ 0 \\ -1.5000 \\ -1.5 \cdot 10^4 \\ 0 \\ x \end{array} $	$\mathbf{Q}_{rmin} := \begin{pmatrix} \min(\mathbf{Q}(\mathbf{x})_{1}) \\ \min(\mathbf{Q}(\mathbf{x})_{2}) \\ \min(\mathbf{Q}(\mathbf{x})_{3}) \\ \min(\mathbf{Q}(\mathbf{x})_{3}) \\ \min(\mathbf{Q}(\mathbf{x})_{4}) \\ \min(\mathbf{Q}(\mathbf{x})_{5}) \\ \min(\mathbf{Q}(\mathbf{x})_{6}) \\ \min(\mathbf{Q}(\mathbf{x})_{7}) \\ \min(\mathbf{Q}(\mathbf{x})_{8}) \end{pmatrix} \qquad \mathbf{Q}_{rmax} := \begin{pmatrix} \max(\mathbf{Q}(\mathbf{x})_{1}) \\ \max(\mathbf{Q}(\mathbf{x})_{2}) \\ \max(\mathbf{Q}(\mathbf{x})_{3}) \\ \max(\mathbf{Q}(\mathbf{x})_{4}) \\ \max(\mathbf{Q}(\mathbf{x})_{5}) \\ \max(\mathbf{Q}(\mathbf{x})_{6}) \\ \max(\mathbf{Q}(\mathbf{x})_{7}) \\ \max(\mathbf{Q}(\mathbf{x})_{8}) \end{pmatrix}$
Limit Condition : $Q(X_a)=V_a$	
$X_{a} = \begin{pmatrix} -1000.036 \\ 1204.496 \\ 204.460 \\ 0.000 \\ -2692.753 \\ -488.221 \\ -1488.257 \\ -1692.717 \end{pmatrix} V_{a} = \begin{pmatrix} -1000.036 \\ 1204.496 \\ 204.460 \\ 0.000 \\ -2692.753 \\ -488.221 \\ -1488.257 \\ -1692.717 \end{pmatrix}$	$\mathbf{Q}_{\text{rmin}} = \begin{pmatrix} -1000.036 \\ -128.010 \\ -13.919 \\ 0.000 \\ -2692.753 \\ -488.221 \\ -1488.257 \\ -1692.717 \end{pmatrix} \qquad \mathbf{Q}_{\text{rmax}} = \begin{pmatrix} 120.166 \\ 1204.496 \\ 204.460 \\ 0.000 \\ 341.939 \\ 112.018 \\ 223.416 \\ 224.013 \end{pmatrix}$

Q

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ExampleE4.18.7-FIXED TS(AnnexV) from PTB4

35/44 $\frac{1}{x_t - x_s} \cdot \left[\left(P_{s_1} \cdot x_s - P_{t_1} \cdot x_t \right) - q(x)_1 \right] ^{Tube Sress \sigma_t(x) of the Tube-Bundle (0 <= x <= Xa)}$

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ExampleE4.18.7-FIXED TS(AnnexV) from PTB4

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Rigid Ring

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$$R := \frac{A + 2 \cdot a_o}{2} \qquad R = 42.188 \text{ in}$$

Limit Condition : $\theta_{R} = \theta(X_{a})$



$$M_{R} := \frac{E_{D} \cdot h^{3} \cdot \ln(K)}{12 \cdot R} \cdot \theta_{R} \qquad M_{R} = \begin{bmatrix} -2.307 \\ 0.000 \\ -30.417 \\ -13.431 \\ -23.078 \\ -20.770 \end{bmatrix} lb$$

ExampleE4.18.7-FIXED TS(AnnexV) from PTB4

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Shell and Channel Verification from σ_{sh} -642.531) -642.531 1463.293 1463.293 $\theta_{s} := \theta(X_{a})$ 820.761 820.761 $M_{s} := \left(1 + \frac{h'_{s}}{2}\right) \cdot k_{s} \cdot \theta_{s} + \beta_{s} \cdot k_{s} \cdot \delta_{s} \cdot P_{s}$ _{s1}-σ_{sb1} 0.000 0.000 M_s = lb lb -2025.958 -2025.958 79.866 79.866 -562.665 -562.665 -1383.426*)* 1383.426 Verification from σ_{ch} 664.359 664.359 -199.042 -199.042 $\theta_{c} := -\theta(X_{a})$ 465.317 465.317 $M_{c} := \left(1 + \frac{h'_{c}}{2}\right) \cdot k_{c} \cdot \theta_{c} + \beta_{c} \cdot k_{c} \cdot \delta_{c} \cdot P_{t}$ 0.000 0.000 t_c⁻.σ_{cb} $M_{c} =$ lb lb 1227.637 6 1227.637 364.236 364.236 1028.595 1028.595 563.278 563.278 $Q_{s} := -(1 + h'_{s}) \cdot \beta_{s} \cdot k_{s} \cdot \theta_{s} - 2\beta_{s}^{2} k_{s} \cdot \delta_{s} \cdot P_{s}$ $Q_{c} := -(1 + h'_{c}) \cdot \beta_{c} \cdot k_{c} \cdot \theta_{c} - 2\beta_{c}^{2} k_{c} \cdot \delta_{c} \cdot P_{t}$ 287.279 (-514.307) -942.614 112.243 -655.335 -402.065 0.000 0.000 lb in⁻¹ lb in⁻¹ $Q_s =$ Q_c = 905.817 -831.948 -324.077 -205.397 -36.797 -719.705

ExampleE4.18.7-FIXED TS(AnnexV) from PTB4

618.538

-317.640

 $w_{s} := \frac{Q_{s}}{\beta_{s}^{2} \cdot k_{s}} + \frac{M_{s}}{\beta_{s} \cdot k_{s}} + \delta_{s} \cdot P_{s}$

 $\theta_s :=$

 $\theta_s =$



0.0011

-0.0008

0.0003

0.0000

0.0035 0.0015

0.0026

 $W_S =$



×

(-0.0011)

0.0008

-0.0003

0.0000

-0.0035

-0.0015

-0.0026

in



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ExampleE4.18.7-FIXED TS(AnnexV) from PTB4

Axial force in the SHELL





$$V_{s1} \coloneqq \frac{a_0^2}{D_s + t_{s1}} \cdot \left[KK \cdot \left[x_s + 2 \cdot \left(1 - x_s\right) \cdot v_t - \frac{\rho_s^2 - 1}{J \cdot K_{st}} \right] \cdot P_s - P'_t + P_\gamma + P_\omega + P_w + P_{rim} \right] + P_t + \left(\rho_s^2 - 1\right) P_s - P'_s + P_w + P_w + P_{rim} = 0$$



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ExampleE4.18.7-FIXED TS(AnnexV) from PTB4



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TS DEFLECTION w(x)

TOTAL DISPLACEMENT OF TUBES $\delta_{tTotal}(x)$ IS EQUAL TO : TOTAL DISPLACEMENT OF SHELL δ_{sTotal} + TS DEFLECTION w(r)



The difference $\delta(x)$ between w(x) and $\delta_{tVt}\!(x)$ is constant

$$\delta(0) = \begin{pmatrix} -0.011 \\ 0.010 \\ -0.000 \\ 0.000 \\ -0.038 \\ -0.017 \\ -0.027 \\ -0.027 \end{pmatrix} \text{ in } \delta(7) = \begin{pmatrix} -0.011 \\ 0.010 \\ -0.000 \\ 0.000 \\ -0.038 \\ -0.017 \\ -0.027 \\ -0.027 \end{pmatrix} \text{ in } \delta\text{direct} = \begin{pmatrix} -0.011 \\ 0.010 \\ -0.000 \\ 0.000 \\ 0.000 \\ -0.038 \\ -0.017 \\ -0.027 \\ -0.027 \end{pmatrix} \text{ in } \delta\text{direct} = \begin{pmatrix} -0.011 \\ 0.010 \\ -0.000 \\ 0.000 \\ -0.038 \\ -0.017 \\ -0.027 \\ -0.027 \end{pmatrix}$$

Determination of effective elastic properties :

ExampleE4.18.7-FIXED TS(AnnexV) from PTB4

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-1.6831)

2.2398)

-0.3606)

-0.0600)

Fr	om figure	UHX-11	<mark>.2 - Polyr</mark>	omial eq	uations for	the determina	tion of E*/	E and v*	for triang	ular patter	n	
E	Equations	for E* fo	r triangula	ar pitch			(_0.0958	0 6209	-0.8683	2 1099	-1 6831	
	0.0353	1.2502	-0.0491	0.3604	–0.6100 \ 0.0184		0.8897	-9.0855	36.1435	-59.5425	35.8223	
E*TriPitch ·	0.0135	0.9910	1.0080	-1.0498			0.7439	-4.4989	12.5779	-14.2092	5.7822	
	0.0054	0.5279	3.0461	-4.3657	1.9435	$v^*TriPitch :=$	0.9100	-4.8901	12.4325	-12.7039	4.4298	
	(-0.0029	0.2126	126 3.9906 -	-6.1730 3.4307)		0.9923	-4.8759	12.3572	-13.7214	5.7629		
							0.9966	-4.1978	9.0478	-7.9955	2.2398	
Fr	om figure	UHX-11	<mark>.3 - Polyr</mark>	nomial equ	uations for	the determina	tion of E*/	E and v*	for square	<mark>e patter</mark> n		
F	auations	for F* fo	r square i	nitch			(-0.0791	0.6008	-0.3468	0.4858	-0.3606	
L		1 5756	1 2110	1 7715	1 2629)		0.3345	-2.8420	10.9709	-15.8994	8.3516	
	0.0070	1.0750	-3.5230	6.9830	-5.0017	*0 Ditala	0.4296	-2.6350	8.6864	-11.5227	5.8544	
E*SquPitch :	=	1 302/	-1 10/1	2 8714	-2 30017	v [*] SquPitch :=	0.3636	-0.8057	2.0463	-2.2902	1.1862	
	0.0372	1.0024	-0.6402	2 6201	-2.333 4 -2.1929		0.3527	-0.2842	0.4354	-0.0901	-0.1590	
	(0.0072	1.0014	0.0402	2.0201	2.1020)		0.3341	0.1260	-0.6920	0.6877	-0.0600	
		El	ay := si(La	ayout > 0,	E*SquPitch	n, E*TriPitch)						
I	E* ₁ := (Ela	y _{1,1} + Ela	ay _{1,2} .µ*+	- Elay _{1,3} .⊧	ι* ² + Elay _{1,}	_{.4} ·μ ^{∗³} + Elay _{1,5}	· µ* ⁴)·E _D	E*,	₁ = 98874	62.429		
	E*2 := (Ela	V2 1 + Eli	aVっ っ·u* +	Elav _{2 2} .	$\iota^{*2} + Elav_2$	$_{4} \cdot {\mu^{*}}^{3} + Elav_{2} =$	·u* ⁴)·En			in ²		
I	E* ₃ ≔ (Ela	y _{3 1} + Ela	ay _{3 2} ·μ* +	Elay _{3 3} .	$\iota^{*2} + Elay_3$	₄ ·μ ^{*3} + Elay _{3,5}	·μ* ⁴)·E _D	E*2	₂ = 88557	55.287 <u>lb</u> in ²		
	E*⊿ := (Ela	1V4 4 + Eli	aV₄ ₂·μ* +	Elav₄ ₂∙u	ı* ² + Elav₄	4·μ* ³ + Elav _{4 5}	· "* ⁴)·ED	E*,	o = 78123	17.136 ^{lb}		
/	4 · (· ·	54,1	-94,21	574,57		,4 12	~) U	,)	in ²		
(E*1)			$\frac{n}{n} = 1.1$					1 = 64305	59.584 <mark>lb</mark>		
interE* :=	E*2	For h/p	lower tha	n 0.1,use	h/p = 0.1		$ \text{tratio} := \begin{bmatrix} 0.1 \\ 0.25 \\ 0.5 \end{bmatrix} \text{in}^2 $					
	E*3	For valu	ies of h/p	higher th	an 2, use	h/p = 2.0 hp						
l	E*4)	Otherwi	se perfor	m a lineai	r interpolat	lion :		20				
(h	(ĥ				h)))	(•	,	D E*			
$E^* := si \left(\frac{-}{p} \right)$	2, E* ₄ , si	— ≤ 0.10 ∖p), E* ₁ , inte	rplin hpra	tio, interE*,		E* = 72590		2 E _D	= 0.275		
	vla	ay := si(L	ayout > 0	, v*SquPite	ch, v*TriPito	ch)						
ν*	₁ ≔ vlay _{1,}	₁ + vlay _{1,}	₂ ·μ* + νla	y _{1,3} ·μ* ² +	vlay _{1,4} ⋅µ*	3 + vlay _{1,5} · μ^{*}^{4}	v* ₁ =	= 0.042				
ν*	a := vlava	$1 + v a v_2$	₂·u* + vla	V2 2·4* ² +	vlav₂⊿∙u*	$^{3} + v a v_{2.5} \cdot u^{*}$	V*2 =	= 0.090				
ν*	₂ := vlay ₃	$_1 + v lay_3$	2 · μ* + vla	y _{3 3} ·μ* ² +	vlay _{3 4} ·µ*	$^{3} + v lay_{3} 5 \cdot \mu^{*}$	v*3 =	= 0.197				
v*	⊿ := vlav⊿	₁ + vlav₄	$2 \cdot u^* + v a$	V4 2·4* ² +	vlav₄⊿·u*	3 + vlav ₄ 5·u [*]	v*4 =	= 0.269				
v*	r ≔ vlavr	₁ + vlav _⊏	2 π 2 π	Vr. 2011* ² +	vlav _{e 4} .u*	³ + vlave e u* ⁴	v*r =	- 0.336				
۰ *	$c = v a v_c$	4 + vlava	2 m + vla	× × × × × × × × × × × × × × × × × × ×	vlavo <u>4</u>	³ + vlavo cur ⁴	v*o -	= 0.376				
v	6 - viciy6,	1 [,] viay6,	2 μ + via	96,3°μ +	νια 9 6,4 [.] μ	¹ νια96,5 [.] μ	v 6 -	- 0.070				
xampleE4.18.7-F	IXED TS(Ann	nexV) from F	PTB4			43/44						





PTB-7-2014: Criteria for Shell-and-Tube Heat Exchangers According to Part UHX of ASME Section VIII Division 1

ANNEX W — UHX-14 – EXAMPLE E4.18.8 (PTB-4 2013 EDITION) STATIONARY

The floating tubesheet exchanger with an immersedThe floating head is to be designed as shown in VIII-1, Figure UHX-14.1, Configuration a. The stationary tubesheet is gasketed with the shell and channel in accordance with configuration d as shown in VIII-1, Figure UHX-14.2, sketch (d). The floating tubesheet is not extended as a flange in accordance with configuration C as shown in VIII-1, Figure UHX-14.3, sketch (c). There is no allowance for corrosion.

ExampleE4.18.8-STATIONARY TS(AnnexW) from PTB4

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FLOATING TUBESHEET RULES accord. to UHX-14 (July 2013 Edition)

Example E4.18.8 (PTB- 4 2013 edition) Floating Head H. E. - STATIONARY TS config. "d"

1 - GEOMETRIC Data (from Fig.UHX-14.1)

Тур	oes of Oper	rating Co	nditions			<mark>Туре</mark>	es of H	leat Exc	:han	ger		
<mark>x=1 N</mark>	ORMAL	operating	g condition		ExchT	ype := "a"	EXCH	IANGEF	<mark>R typ</mark>	es : a, b, c		
x=2 S x ⋅= 1 x=3 S	TARTUP HUTDOWN	operating operatin	g condition a condition		STAco	onfig := "d"	STAT	IONNA	ry C	onfig. type	<mark>s : a</mark> ,	, b, c, d,e,f
x=4 U	PSET	operating	condition		FLOco	onfig := "C"	FLO	DATING	С	<mark>onfig. type</mark>	<mark>s : A</mark> ,	, B, C, D
x=5 C x=6 O	LEANING THER	operatin	g condition		Config	:= STAcor	nfig <mark>C</mark>	onfig = "	'd"			
<u></u>		oporadin	goonanion	highlig	hted in y	ellow :mos	st impor	tant data	and	results		
Tubes	sheet Data	(from Fig	.UHX-14.1)			Tube Dat	i ta (fror	m Fig.Ul	HX-1	1.1)		
n := 1.75·m	For t	riangula	r pitch : "La	yout"=0	<mark>p :=</mark>	<mark>⊧ 1.0∙in</mark>	Tul	be Pitch	า			
Layout := 0	For s	square pi	tch : "La	iyout"=1	N _t :	= 466	Nu	mber of	f Tuk	bes		
r _o := 12.5∙in	Radi	us to out	ter tube		d _t :₌	<mark>= 0.75∙in</mark>	Tul	be Outs	<mark>ide</mark> [Diameter		
A := 33.071·	in Outs	ide Diam	eter of Tub	esheet		0.000 in	т.,,	ha Thia	knoc			
C := 31.417·	in Bolt	Circle Di	ameter		ել :=	: 0.083·IN	Tu			5		
C _p := 78∙in	Perin	neter of t	the tube lay	out	L _t :=	= 256.0∙in	<mark>i u</mark>	be Leng	<u>jtn</u>			
ر ۵ := 400 in	² Total	l area en	closed by (L :=	ELt – 2∙h	L =	= 252.500	0 in	Effective ler	ngth c	of tubes
Ap .= 490·11	- Total			р	ρ:=	= 0.8	Tul	be expa	insio	on depth rat	tio	
A _L := 64.375	in ² Total	I Untube	d Lanes Ar	ea	I _{tx} :=	= 1.40 in	Ler	ngth of	Expa	anded Port	ion o	f Tube
$c_t := 0.0 \cdot in$	Tube	esheet Co	orr. Allow. (Tubeside)	kl :=	= 15.375∙in	n k=0	.6 for s	pans	between T	lubes	sheets
c _s := 0.0∙in	Tube	esheet Co	orr. Allow. (Shellside)			k=0 k-1	8 for sl 0 for si	pans	between T	S/su	pport plate
h _g := 0.197∙i	n Groo	ve depth	1				I	unsup	ppor	ted tube sp	ban	
SS := "NO"	SS="YES"	' for Tub	esheet calc	ulated as S	Simply S	Supported	d acc.	to UHX-	-13.9	(config. a,l	b,c or	nly)
She	ell Data (fro	om Fig.Ul	HX-14.1)			С	hanne	el Data	(fron	n Fig.UHX-1	14.1)	
SHEL used f	L data whic or Floating	ch are not Tubeshee	t et			(for	CHANI r Statio	NEL dat onary an	a wh d Flo	ich are diffe bating Tube	erent sheet	S
$D_s := 0 \cdot in$	Shell ID				[D _c := 0∙in		Chann	nel ID)		
t _s := 0.1 ⋅ in	Shell Thi	ickness a	away from 1	S	t	t _c := 0.1∙in		Chann	nel Tl	hickness		
G _s := 29.375∙i	n Shell Ga a	sket Diar	neter		C	G _c := 29.37	75∙in	Chann	nel G	asket Diam	neter	
$G_1 := 0 \cdot in$	Shell Co	ntact mic	d-point TS/F	lange	C	G ₁ := 0∙in		Chann	nel C	ontact mid	-poin	t TS/Flange
$\boldsymbol{C}_{s} := \boldsymbol{0}.{\cdot}\text{in}$	Shell Co	rrosion A	llowance		C	C _c := 0.0⋅in	า	Chann	nel C	orrosion A	llowa	ince
	Corrodeo	d lenath:				CHAN	:= "CY	′L" "C "H	YL" IEMI'	for Cylind " for Hemis	rical herio	Channel cal Channel
$L_t :=$	L _t – 2c _t	L _t = 2	56.000 in				Corr	roded th	nickr	nesses:		
C	Corroded d	liameters		h	ı := h − c	$s - c_t$	Tubes	sheet th	nickn	ess		h = 1.750 in
$D_s := D_s + 2$	·C _s She	ell ID	D _s = 0.000 ir	n t _s	_s := t _s – C	C _s	Shell	Thickne	ess a	away from	TS	$t_{s} = 0.100 \text{ in}$
$D_c := D_c + 2$	C _c Char	nnel ID	D _c = 0.000 ir	ı t _o	_c := t _c - C	Cc	Chan	nel Thic	ckne	SS		$t_{c} = 0.100 \text{ in}$

ExampleE4.18.8-STATIONARY TS(AnnexW) from PTB4

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2 - Design (D) and Operating (O) PRESSURES data (from UHX-14.3)

Maximum and Minimum DESIGN PRESSURES (D) **OPERATING PRESSURES** (O) for Operating Condition x $P_{sD_max} := 250.0 \cdot \frac{lb}{{}_{...}2}$ maximum Shellside Design Pressure $P_{sO_x} := 250.0 \cdot \frac{lb}{c^2}$ Shellside Operating Pressure $P_{sD_{min}} := 0.0 \cdot \frac{lb}{ln^2}$ minimum Shellside Design Pressure $P_{tD_{max}} = 150.0 \cdot \frac{lb}{m^2}$ maximumTubeside Design Pressure $P_{tO_x} := 150.0 \cdot \frac{lb}{in^2}$ Tubeside Operating Pressure $P_{tD_{min}} := 0.0 \cdot \frac{lb}{in^2}$ minimum Tubeside Design Pressure DESIGN PRESSURES P_{sD} and P_{tD} (from Table UHX-14.4-1) $P_{sD} := \begin{vmatrix} P_{sD}_{max} \\ P_{sD}_{max} \\ P_{sD}_{max} \end{vmatrix} \qquad P_{sD} = \begin{pmatrix} 0.000 \\ 250.000 \\ 250.000 \\ 0.000 \\ 0 \end{vmatrix} \begin{vmatrix} Ib \\ in^{2} \\ 0.000 \\ 0 \end{vmatrix} \qquad P_{tD} := \begin{vmatrix} P_{tD}_{min} \\ P_{tD}_{max} \\ P_{tD}_{min} \\ P_{tD}_{max} \end{vmatrix} \qquad P_{tD} = \begin{pmatrix} 150.000 \\ 0.000 \\ 150.000 \\ 150.000 \\ 0.000 \\ 0 \end{vmatrix}$ OPERATING PRESSURES P so,x and P to,x for oper.cond. x (from Table UHX-14.4-2 $P_{sO} := \begin{pmatrix} 0 \cdot \frac{lb}{in^{2}} \\ P_{sO_{x}} \\ P_{sO_{x}} \\ 0 \cdot \frac{lb}{in} \\ \end{pmatrix} \qquad P_{sO} = \begin{pmatrix} 0.000 \\ 250.000 \\ 250.000 \\ 0.000 \\ \end{pmatrix} \frac{lb}{in^{2}} \qquad P_{tO} := \begin{pmatrix} 0 \cdot \frac{lb}{in^{2}} \\ P_{tO_{x}} \\ 0 \cdot \frac{lb}{in^{2}} \\ P_{tO_{x}} \\ 0 \cdot \frac{lb}{in^{2}} \\ 0 \cdot$ Determination of DESIGN and OPERATING PRESSURES P_s and P_t $P_{s} := \begin{pmatrix} P_{sD_max} \\ P_{sD_max} \\ P_{sD_min} \\ 0.\frac{lb}{in^{2}} \\ P_{sO_x} \\ 0.\frac{lb}{in^{2}} \\ 0.\frac$ ´Ρ_{sD_min} ∖ P_{tD max} 2013 Design LC1= 2010 Design LC1 2013 Design LC2= 2010 Design LC2 2013 Design LC3= 2010 Design LC3 2013 Design LC4= not explicited in 2010 2013 Operating LC5= 2010 Operating LC5 2013 Operating LC6= 2010 Operating LC6 2013 Operating LC7= 2010 Operating LC7 2013 Operating LC8= 2010 Operating LC4

ExampleE4.18.8-STATIONARY TS(AnnexW) from PTB4

ExampleE4.18.8-STATIONARY TS(AnnexW) from PTB4

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CHANNEL Material is SA-516/grade70

CHANNEL data which are different for Stationary and Floating Tubesheets



ExampleE4.18.8-STATIONARY TS(AnnexW) from PTB4

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4 - Flange Design and Operating BOLT LOADS data (from Table UHX-8.1)

Maximum and Minimum Flange DESIGN BOLT LOADS Flange BOLT LOADS for GASKET SEATING Condition

Shell flange Design bolt load $W_s := 0.0 \cdot lb$ Shell flange bolt load for Gasket Seating

Wm1s := 0·lbShell flange Design bolt loadWm1c := 211426·lbChannel flange Design bolt load

W_c := 211426·lb Channel flange bolt load for Gasket Seating

 $W_{m1max} := max(W_{m1s}, W_{m1c})$ $W_{m1max} = 211426.0 lb$

 $W_{max} := max(W_s, W_c)$ $W_{max} = 211426.0 lb$

Determination of EFFECTIVE BOLT LOAD W* for each Configuration a , b , c , d

Configuration a		a	Configuration b		Config	Configuration c Conf			Config	Configuration e	
	(0.0·lb)			(W _{m1c})		(W _{m1c})		(W _{m1c})		(0.0·lb)	
	0.0·lb			0.0·lb		0.0·lb		W _{m1s}		W _{m1s}	
	0.0·lb			W _{m1c}		W _{m1c}		W _{m1max}		W _{m1s}	
W*a :=	0.0·lb		0.0·lb		0.0·lb		0.0·lb		0.0·lb		
·· a·	0.0·lb		W* _b :=	W _c	W* _c :=	W _c	W* _d :=	W _{max}	W* _e :=	Ws	
	0.0·lb			W _c		W _c		W _{max}		Ws	
	0.0·lb			W _c		W _c		W _{max}		Ws	
	(0.0·lb)			w _c		w _c		W _{max}		w _s	
W* _A	:= W* _a		\٨/*_	· \\\/*.	\\/*_	· \\\/*			\۸/*.	· \\\/*	
W* _D := W* _a		•• B	w b			(211426.000)	.— 🖤 е		
	\ \/ * ·	۱۸/* if	Config	- "a" \/ Cor	ofic = "A" > C	onfia – "D"		0.000			
	VV	vva "	Config	= a ♥ 001	ing = A ⊽ O	oning = D		211426.000			
		W [*] _b if Config = "b" ∨ Config = "B"				14/*	0.000				
	W_{c}^{*} if Config = "c" \lor Config = "C"				VV." =	211426.000	D				
	W* _d if Config = "d"				211426.000						
	W_e^* if Config = "e" \vee Config = "f"					211426.000					
							l	211426.000	J		

Minimum required thickness h, of the TS flanged extension (from UHX-9)

For flanged Configurati	ions b , d (extended as a flange) , e	For unflanged Config.c, f	For unflanged Config.d, C
from	UHX-9.5a	See UHX-9.5b	See UHX-9.5c
$h_G := \frac{C - G_c}{2}$ Gast	ket moment arm $h_G = 1.021$ in		
$h_{rD} := \sqrt{\frac{1.9W_{m1c}}{S_D \cdot G_c}} \cdot h_G$	$h_{rD} = 0.857 \text{ in}$		
$h_{rG} := \sqrt{\frac{1.9W_c}{S_a \cdot G_c} \cdot h_G}$	$h_{rG} = 0.836$ in		
$\textbf{h}_r := max \left(\textbf{h}_{rD}, \textbf{h}_{rG} \right)$	$h_r = 0.857$ in		
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Start of Calculations

L = effective length of shell/tubes $L := L_t - 2 \cdot h$ L = 252.500 in

 D_o = equivalent diameter of outer tubes $D_o := 2 \cdot r_o + d_t$ $D_o = 25.750 \text{ in}$ $a_o := \frac{D_o}{2}$ $a_o = 12.875 \text{ in}$

E

UHX-14.5.1 Step 1 Determine D₀, μ, μ* and h'g from UHX-11.5.1 :

$$\rho := \frac{l_{tx}}{h} \quad \rho = 0.800 \qquad d^* := \max \left[d_t - 2 \cdot t_t \cdot \left(\frac{E_{tT}}{E_D} \right) \cdot \left(\frac{S_{tT}}{S_D} \right) \cdot \rho , \left(d_t - 2t_t \right) \right] \qquad d^* = 0.657 \text{ in}$$

$$p^* := \frac{p}{\sqrt{1 - \frac{4 \cdot \min(A_L, 4D_0 \cdot p)}{\pi \cdot D_0^2}}} \qquad p^* = 1.068 \text{ in}$$

$$\mu^* := \frac{p^* - d^*}{p^*} \quad \mu^* = 0.385 \qquad \mu := \frac{p - d_t}{p} \quad \mu = 0.250$$

Chan Rad. Dim. $a_c := \begin{bmatrix} \frac{D_c}{2} & \text{if } Config = "a" \lor Config = "e" \lor Config = "f" \lor Config = "A" \\ \frac{G_c}{2} & \text{if } Config = "b" \lor Config = "c" \lor Config = "d" \lor Config = "B" \lor Config = "C" \\ \frac{A}{2} & \text{if } Config = "D" \end{bmatrix}$

Shell Radial Dim.

$$a_{s} := \begin{bmatrix} \frac{D_{s}}{2} & \text{if } Config = "a" \lor Config = "b" \lor Config = "c" \\ \\ \frac{G_{s}}{2} & \text{if } Config = "d" \lor Config = "e" \lor Config = "f" \\ \\ a_{c} & \text{if } Config = "A" \lor Config = "B" \lor Config = "C" \lor Config = "D" \\ \end{bmatrix}$$

$$\rho_{s} := \frac{a_{s}}{a_{o}} \quad \rho_{s} = 1.141 \qquad \qquad x_{s} := 1 - N_{t} \cdot \left(\frac{d_{t}}{2 \cdot a_{o}}\right)^{2} \quad x_{s} = 0.605$$

$$\rho_{c} := \frac{a_{c}}{a_{o}} \quad \rho_{c} = 1.141 \qquad \qquad x_{t} := 1 - N_{t} \cdot \left(\frac{d_{t} - 2 \cdot t_{t}}{2 \cdot a_{o}}\right)^{2} \quad x_{t} = 0.760$$

ExampleE4.18.8-STATIONARY TS(AnnexW) from PTB4

ExampleE4.18.8-STATIONARY TS(AnnexW) from PTB4

otherwise

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UHX-14.5.2 Step 2 Calculate shell and channel parameters:

 $SS = "NO" \quad Use SS=YES \text{ for Simply Supported calculation in a 2nd step (see UHX-13.9)} \\ \beta_{s} := \begin{bmatrix} \frac{\left[12 \cdot \left(1 - v_{s}^{2}\right)\right]^{0.25}}{\left[\left(D_{s} + t_{s}\right) \cdot t_{s}\right]^{0.5}} & \text{if } SS = "NO" \land (Config = "a" \lor Config = "b" \lor Config = "c")} \\ 0.\frac{1}{in} & \text{otherwise} \end{bmatrix} \\ \beta_{c} := \begin{bmatrix} \frac{\left[12 \cdot \left(1 - v_{c}^{2}\right)\right]^{0.25}}{\left[\left(D_{c} + t_{c}\right) \cdot t_{c}\right]^{0.5}} & \text{if } SS = "NO" \land (Config = "a" \lor Config = "f" \lor Config = "A")} \\ \beta_{c} := \begin{bmatrix} \frac{\left[12 \cdot \left(1 - v_{c}^{2}\right)\right]^{0.25}}{\left[\left(D_{c} + t_{c}\right) \cdot t_{c}\right]^{0.5}} & \text{if } SS = "NO" \land (Config = "a" \lor Config = "f" \lor Config = "A")} \\ \beta_{c} = 0.000 \cdot 1 \\ \frac{1}{in} & \frac{1}{in} & \frac{1}{in} & \frac{1}{in} & \frac{1}{in} \\ \frac{1}{in} & \frac{1}{in} \\ \frac{1}{in} & \frac{1}{in}$

$$\begin{split} & SS = "NO" \qquad CHAN = "CYL" \\ & \delta_{S} \coloneqq \left[\begin{array}{c} \frac{D_{s}^{-2}}{4E_{sD}\cdot t_{s}} \left(1 - \frac{v_{s}}{2} \right) & \text{if } SS = "NO" \land (Config = "a" \lor Config = "b" \lor Config = "c") \\ 0.(in^{3} lb^{-1}) & \text{otherwise} \end{array} \right] \\ & \delta_{s} = 0 \times 10^{0} in^{3} lb^{-1} \\ & \frac{D_{c}^{-2}}{4E_{cD}\cdot t_{c}} \left(1 - \frac{v_{c}}{2} \right) & \text{if } SS = "NO" \land (Config = "a" \lor Config = "e" \lor Config = "f" \lor Config = "A") \\ & \frac{D_{c}^{-2}}{4E_{cD}\cdot t_{c}} \left(\frac{1 - v_{c}}{2} \right) & \text{if } SS = "NO" \land (Config = "a" \lor Config = "a" \lor Config = "f" \lor Config = "A") \\ & \frac{D_{c}^{-2}}{4E_{cD}\cdot t_{c}} \left(\frac{1 - v_{c}}{2} \right) & \text{if } SS = "NO" \land CHAN = "HEMI" \land (Config = "a" \lor Config = "b" \lor Config = "c") \\ & 0.(in^{3} lb^{-1}) & \text{otherwise} \end{split}$$

δ_c:=

ExampleE4.18.8-STATIONARY TS(AnnexW) from PTB4

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ExampleE4.18.8-STATIONARY TS(AnnexW) from PTB4

ExampleE4.18.8-STATIONARY TS(AnnexW) from PTB4

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UHX-14.5.5 Step 5

a) Calculate parameters $\omega_{s}^{}, \omega_{s}^{*}, \omega_{c}^{}, \omega_{c}^{*}$,

b) Calculate parameter γ_{b} :

$$\begin{split} \gamma_{b} &\coloneqq 0 \quad \text{if } Config = \text{"a"} \lor Config = \text{"A"} \lor Config = \text{"D"} \\ & \frac{G_{c} - C}{D_{0}} \quad \text{if } Config = \text{"b"} \lor Config = \text{"B"} \\ & \frac{G_{c} - G_{1}}{D_{0}} \quad \text{if } Config = \text{"c"} \lor Config = \text{"C"} \\ & \frac{G_{c} - G_{s}}{D_{0}} \quad \text{if } Config = \text{"d"} \\ & \frac{C - G_{s}}{D_{0}} \quad \text{if } Config = \text{"d"} \\ & \frac{G_{1} - G_{s}}{D_{0}} \quad \text{if } Config = \text{"f"} \end{split}$$

ExampleE4.18.8-STATIONARY TS(AnnexW) from PTB4

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Calculate P*_s and P*_c from UHX-14.8, if required by the user (for configurations a,b,or c only) Page10

$$\begin{split} T_r &:= \begin{array}{l} \displaystyle \frac{T'+T'_s+T'_c}{3} & \text{if } Config = "a" \\ \\ \displaystyle \frac{T'+T'_s}{2} & \text{if } Config = "b" \lor Config = "c" \\ \\ \displaystyle \frac{T'+T'_c}{2} & \text{if } Config = "e" \lor Config = "f" \lor Config = "A" \\ \\ \displaystyle 0.\text{degF } \text{ otherwise} \end{split}$$

$$T^{*}_{s} := \begin{cases} \frac{T'_{s} + T_{r}}{2} & \text{if Config = "a"} \lor Config = "b" \lor Config = "c" \\ 0 \text{degF otherwise} \end{cases} T^{*}_{s} = 0.000 \text{ degF}$$

$$T^{*}_{c} := \begin{cases} \frac{T'_{c} + T_{r}}{2} & \text{if } Config = "a" \lor Config = "e" \lor Config = "f" \lor Config = "A" \\ 0 \text{degF } \text{otherwise} \end{cases} T^{*}_{c} = 0.000 \text{ degF}$$

$$P^{*}_{sp1} := \frac{E_{sD} \cdot t_{s}}{a_{s}} \cdot \left[\alpha'_{s} \cdot \left(T^{*}_{s} - 70 \cdot degF\right) - \alpha' \cdot \left(T_{r} - 70 \cdot degF\right) \right] \qquad P^{*}_{sp1} = 0.000 \frac{lb}{in^{2}}$$

$$P^{*}_{cp1} := \frac{E_{cD} \cdot t_{c}}{a_{c}} \cdot \left[\alpha'_{c} \cdot \left(T^{*}_{c} - 70 \cdot degF\right) - \alpha' \cdot \left(T_{r} - 70 \cdot degF\right) \right] \qquad P^{*}_{cp1} = 0.000 \frac{lb}{in^{2}}$$

$$P^{*}_{sp} := \begin{bmatrix} 0 \frac{lb}{in^{2}} & \text{if Config} = "d" \lor \text{Config} = "A" \lor \text{Config} = "B" \lor \text{Config} = "C" \lor \text{Config} = "D" \\ P^{*}_{cp} := \begin{bmatrix} 0 \frac{lb}{in^{2}} & \text{if Config} = "d" \lor \text{Config} = "B" \lor \text{Config} = "C" \lor \text{Config} = "D" \\ P^{*}_{cp} := \begin{bmatrix} 0 \frac{lb}{in^{2}} & \text{if Config} = "d" \lor \text{Config} = "B" \lor \text{Config} = "C" \lor \text{Config} = "D" \\ P^{*}_{cp} := \begin{bmatrix} 0 \frac{lb}{in^{2}} & \text{if Config} = "d" \lor \text{Config} = "B" \lor \text{Config} = "C" \lor \text{Config} = "D" \\ P^{*}_{cp} := \begin{bmatrix} 0 \frac{lb}{in^{2}} & \text{if Config} = "d" \lor \text{Config} = "B" \lor \text{Config} = "D" \\ P^{*}_{cp} := \begin{bmatrix} 0 \frac{lb}{in^{2}} & \text{if Config} = "d" \lor \text{Config} = "B" \lor \text{Config} = "C" \lor \text{Config} = "D" \\ P^{*}_{cp} := \begin{bmatrix} 0 \frac{lb}{in^{2}} & \text{if Config} = "d" \lor \text{Config} = "B" \lor \text{Config} = "C" \lor \text{Config} = "D" \\ P^{*}_{cp} := \begin{bmatrix} 0 \frac{lb}{in^{2}} & \text{if Config} = "d" \lor \text{Config} = "B" \lor \text{Config} = "D" \\ P^{*}_{cp} := \begin{bmatrix} 0 \frac{lb}{in^{2}} & \text{if Config} = "d" \lor \text{Config} = "B" \lor \text{Config} = "C" \lor \text{Config} = "D" \\ P^{*}_{cp} := \begin{bmatrix} 0 \frac{lb}{in^{2}} & \text{if Config} = "d" \lor \text{Config} = "B" \lor \text{Config} = "C" \lor \text{Config} = "D" \\ P^{*}_{cp} := \begin{bmatrix} 0 \frac{lb}{in^{2}} & \text{if Config} = "d" \lor \text{Config} = "B" \lor \text{Config} = "C" \lor \text{Config} = "D" \\ P^{*}_{cp} := \begin{bmatrix} 0 \frac{lb}{in^{2}} & \text{if Config} = "d" \lor \text{Config} = "B" \lor \text{Config} = "C" \lor \text{Config} = "D" \\ P^{*}_{cp} := \begin{bmatrix} 0 \frac{lb}{in^{2}} & \text{if Config} = "C' \lor \text{Config} = "D" \\ P^{*}_{cp} := \begin{bmatrix} 0 \frac{lb}{in^{2}} & \text{if Config} = "C' \lor \text{Config} = "D" \\ P^{*}_{cp} := \begin{bmatrix} 0 \frac{lb}{in^{2}} & \text{if Config} = "C' \lor \text{Config} = "D" \\ P^{*}_{cp} := \begin{bmatrix} 0 \frac{lb}{in^{2}} & \text{if Config} = "C' \lor \text{Config} = "D" \\ P^{*}_{cp} := \begin{bmatrix} 0 \frac{lb}{in^{2}} & \text{if Config} = "C' \lor \text{Config} = "D" \\ P^{*}_{cp} := \begin{bmatrix} 0 \frac{lb}{in^{2}} & \text{if Config} = "C' \lor \text{Config} = "D" \\ P^{*}_{cp} := \begin{bmatrix}$$

$$P^{*}_{cp1} \text{ otherwise}$$

$$P^{*}_{cp1} = \begin{pmatrix} 0 \cdot \frac{lb}{in^{2}} \\ 0 \cdot \frac{lb}{in^{2}} \\$$

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UHX-14.5.6 Step 6 For each loading case, calculate effective pressure Pe

Calculate P_e for Pressure and Operating Loading Cases 1 through 8:

$$\begin{split} \mathsf{P}_{e} &\coloneqq \begin{bmatrix} \left(\mathsf{P}_{s}-\mathsf{P}_{t}\right) \;\; \text{if ExchType} = "a" \\ & \begin{bmatrix} \mathsf{P}_{s} \cdot \left(1-\rho_{s}^{-2}\right)-\mathsf{P}_{t} \end{bmatrix} \;\; \text{if ExchType} = "b" \\ & \begin{bmatrix} \left(\mathsf{P}_{s}-\mathsf{P}_{t}\right) \cdot \left(1-\rho_{s}^{-2}\right) \end{bmatrix} \;\; \text{otherwise} \end{split}$$

PRESSURE DESIGN Loading cases : terms 1,2 3, 4

PRESSURE OPERATING Loading cases : terms 5,6,7, 8



UHX-14.5.7 Step 7 Determine Q₂ and Q₃ for Loading Cases 1 through 8:

$$\begin{aligned} \mathbf{Q}_{3_{1}} &\coloneqq \begin{bmatrix} 0 & \text{if } \mathbf{P}_{e_{1}} = 0 \\ \left(\mathbf{Q}_{1} + \frac{2 \cdot \mathbf{Q}_{2_{1}}}{\mathbf{P}_{e_{1}} \cdot \mathbf{a}_{0}^{2}} \right) & \text{otherwise} \\ \left(\mathbf{Q}_{1} + \frac{2 \cdot \mathbf{Q}_{2_{2}}}{\mathbf{P}_{e_{2}} \cdot \mathbf{a}_{0}^{2}} \right) & \text{otherwise} \\ \mathbf{Q}_{3_{4}} &\coloneqq \begin{bmatrix} 0 & \text{if } \mathbf{P}_{e_{4}} = 0 \\ \left(\mathbf{Q}_{1} + \frac{2 \cdot \mathbf{Q}_{2_{3}}}{\mathbf{P}_{e_{3}} \cdot \mathbf{a}_{0}^{2}} \right) & \text{otherwise} \\ \left(\mathbf{Q}_{1} + \frac{2 \cdot \mathbf{Q}_{2_{4}}}{\mathbf{P}_{e_{4}} \cdot \mathbf{a}_{0}^{2}} \right) & \text{otherwise} \\ \left(\mathbf{Q}_{1} + \frac{2 \cdot \mathbf{Q}_{2_{5}}}{\mathbf{P}_{e_{5}} \cdot \mathbf{a}_{0}^{2}} \right) & \text{otherwise} \\ \mathbf{Q}_{3_{7}} &\coloneqq \begin{bmatrix} 0 & \text{if } \mathbf{P}_{e_{7}} = 0 \\ \left(\mathbf{Q}_{1} + \frac{2 \cdot \mathbf{Q}_{2_{6}}}{\mathbf{P}_{e_{5}} \cdot \mathbf{a}_{0}^{2}} \right) & \text{otherwise} \\ \left(\mathbf{Q}_{1} + \frac{2 \cdot \mathbf{Q}_{2_{6}}}{\mathbf{P}_{e_{5}} \cdot \mathbf{a}_{0}^{2}} \right) & \text{otherwise} \\ \left(\mathbf{Q}_{1} + \frac{2 \cdot \mathbf{Q}_{2_{6}}}{\mathbf{P}_{e_{6}} \cdot \mathbf{a}_{0}^{2}} \right) & \text{otherwise} \\ \mathbf{Q}_{3_{7}} &\coloneqq \begin{bmatrix} 0 & \text{if } \mathbf{P}_{e_{7}} = 0 \\ \left(\mathbf{Q}_{1} + \frac{2 \cdot \mathbf{Q}_{2_{7}}}{\mathbf{P}_{e_{7}} \cdot \mathbf{a}_{0}^{2}} \right) & \text{otherwise} \\ \left(\mathbf{Q}_{1} + \frac{2 \cdot \mathbf{Q}_{2_{8}}}{\mathbf{P}_{e_{8}} \cdot \mathbf{a}_{0}^{2}} \right) & \text{otherwise} \\ \left(\mathbf{Q}_{1} + \frac{2 \cdot \mathbf{Q}_{2_{7}}}{\mathbf{P}_{e_{7}} \cdot \mathbf{a}_{0}^{2}} \right) & \text{otherwise} \\ \left(\mathbf{Q}_{1} + \frac{2 \cdot \mathbf{Q}_{2_{8}}}{\mathbf{P}_{e_{8}} \cdot \mathbf{a}_{0}^{2}} \right) & \text{otherwise} \\ \left(\mathbf{Q}_{1} + \frac{2 \cdot \mathbf{Q}_{2_{7}}}{\mathbf{P}_{e_{8}} \cdot \mathbf{a}_{0}^{2}} \right) & \text{otherwise} \\ \left(\mathbf{Q}_{1} + \frac{2 \cdot \mathbf{Q}_{2_{8}}}{\mathbf{P}_{e_{8}} \cdot \mathbf{a}_{0}^{2}} \right) & \text{otherwise} \\ \left(\mathbf{Q}_{1} + \frac{2 \cdot \mathbf{Q}_{2_{7}}}{\mathbf{P}_{e_{8}} \cdot \mathbf{a}_{0}^{2}} \right) & \text{otherwise} \\ \left(\mathbf{Q}_{1} + \frac{2 \cdot \mathbf{Q}_{2_{8}}}{\mathbf{P}_{e_{8}} \cdot \mathbf{a}_{0}^{2}} \right) & \text{otherwise} \\ \end{array} \right)$$

$$Q_{1} = 0.078 \qquad Q_{2} = \begin{pmatrix} -213.344 \\ 355.573 \\ 142.229 \\ 0.000 \\ -213.344 \\ 355.573 \\ 142.229 \\ 0.000 \end{pmatrix} b \qquad Q_{3} = \begin{pmatrix} 0.09533 \\ 0.09533 \\ 0.09533 \\ 0.09533 \\ 0.09533 \\ 0.09533 \\ 0.09533 \\ 0.09533 \\ 0.09533 \\ 0.09533 \\ 0.09533 \\ 0.09533 \\ 0.0000 \end{pmatrix}$$

ExampleE4.18.8-STATIONARY TS(AnnexW) from PTB4

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Determine Coefficient F_m for Load Cases 1 through 8:

$$\begin{split} \text{numpoints} := 20 \qquad \text{j} := 1 \dots \text{numpoints} \qquad X_{j} := \frac{j \cdot 1}{\text{numpoints}} \qquad x := X \cdot X_{a} \\ \hline Q_{i}(x) := \overline{\frac{\Psi_{2x}(x) \cdot \Psi_{1} - \Psi_{1x}(x) \cdot \Psi_{2}}{X_{a} \cdot Z_{a}}} \qquad Q_{m}(x) := \overline{\left(\frac{\Psi_{2x}(x) \cdot bei' - \Psi_{1x}(x) \cdot ber'}{Z_{a}}\right)} \\ F_{mx1}(x) := \begin{bmatrix} 0 \quad \text{if } P_{e_{1}} = 0 \\ \frac{Q_{i}(x) + Q_{3_{1}} \cdot Q_{m}(x)}{2} & \text{otherwise} \end{bmatrix} \qquad F_{mx5}(x) := \begin{bmatrix} 0 \quad \text{if } P_{e_{5}} = 0 \land \left(|P^{*}_{s}|\right) = 0 \land |P^{*}_{c}| = 0\right) \\ \frac{Q_{i}(x) + Q_{3_{2}} \cdot Q_{m}(x)}{2} & \text{otherwise} \end{bmatrix} \\ F_{mx2}(x) := \begin{bmatrix} 0 \quad \text{if } P_{e_{2}} = 0 \\ \frac{Q_{i}(x) + Q_{3_{2}} \cdot Q_{m}(x)}{2} & \text{otherwise} \end{bmatrix} \qquad F_{mx6}(x) := \begin{bmatrix} 0 \quad \text{if } P_{e_{6}} = 0 \land \left(|P^{*}_{s}|\right) = 0 \land |P^{*}_{c}| = 0\right) \\ \frac{Q_{i}(x) + Q_{3_{2}} \cdot Q_{m}(x)}{2} & \text{otherwise} \end{bmatrix} \\ F_{mx3}(x) := \begin{bmatrix} 0 \quad \text{if } P_{e_{3}} = 0 \\ \frac{Q_{i}(x) + Q_{3_{3}} \cdot Q_{m}(x)}{2} & \text{otherwise} \end{bmatrix} \qquad F_{mx7}(x) := \begin{bmatrix} 0 \quad \text{if } P_{e_{7}} = 0 \land \left(|P^{*}_{s}|\right) = 0 \land |P^{*}_{c}| = 0\right) \\ \frac{Q_{i}(x) + Q_{3_{2}} \cdot Q_{m}(x)}{2} & \text{otherwise} \end{bmatrix} \\ F_{mx4}(x) := \begin{bmatrix} 0 \quad \text{if } P_{e_{4}} = 0 \\ \frac{Q_{i}(x) + Q_{3_{4}} \cdot Q_{m}(x)}{2} & \text{otherwise} \end{bmatrix} \qquad F_{mx8}(x) := \begin{bmatrix} 0 \quad \text{if } P_{e_{8}} = 0 \land \left(|P^{*}_{s}|\right) = 0 \land |P^{*}_{c}| = 0\right) \\ \frac{Q_{i}(x) + Q_{3_{4}} \cdot Q_{m}(x)}{2} & \text{otherwise} \end{bmatrix} \\ F_{mx4}(x) := \begin{bmatrix} 0 \quad \text{if } P_{e_{4}} = 0 \\ \frac{Q_{i}(x) + Q_{3_{4}} \cdot Q_{m}(x)}{2} & \text{otherwise} \end{bmatrix} \qquad F_{mx8}(x) := \begin{bmatrix} 0 \quad \text{if } P_{e_{8}} = 0 \land \left(|P^{*}_{s}|\right) = 0 \land |P^{*}_{c}| = 0\right) \\ \frac{Q_{i}(x) + Q_{3_{4}} \cdot Q_{m}(x)}{2} & \text{otherwise} \end{bmatrix} \\ \left(\begin{bmatrix} F_{mx1}(x) \\ F_{mx2}(x) \end{bmatrix} \qquad (max(absF_{mx}(x)_{1}) \end{pmatrix} \qquad (0.1021) \qquad (0)$$

$$F_{mx}(x) := \begin{pmatrix} F_{mx1}(x) \\ F_{mx2}(x) \\ F_{mx3}(x) \\ F_{mx3}(x) \\ F_{mx4}(x) \\ F_{mx5}(x) \\ F_{mx5}(x) \\ F_{mx7}(x) \\ F_{mx8}(x) \end{pmatrix} absF_{mx}(x) := \begin{vmatrix} \overrightarrow{|F_{mx2}(x)|} \\ \overrightarrow{|F_{mx3}(x)|} \\ \overrightarrow{|F_{mx4}(x)|} \\ \overrightarrow{|F_{mx4}(x)|} \\ \overrightarrow{|F_{mx5}(x)|} \\ F_{mx7}(x) \\ F_{mx8}(x) \end{pmatrix} \\ F_{mx8}(x) \end{pmatrix} F_{mx}(absF_{mx}(x)_{3}) \\ F_{mx}(absF_{mx}(x)_{3}) \\ max(absF_{mx}(x)_{3}) \\ max(absF_{mx}(x)_{5}) \\ max(absF_{mx}(x)_{5}) \\ max(absF_{mx}(x)_{7}) \\ max(absF_{mx}(x)_{7}) \\ max(absF_{mx}(x)_{8}) \end{pmatrix} \\ F_{mx}(absF_{mx}(x)_{8}) \end{pmatrix} F_{mx}(x) = \begin{pmatrix} 0.1021 \\ 0.1021 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \end{pmatrix} \\ F_{mx}(x) = \begin{pmatrix} 0.048 \\ 0.048 \\ 0.048 \\ 0.000 \\ 0.000 \\ 0.0000 \\$$

ExampleE4.18.8-STATIONARY TS(AnnexW) from PTB4

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Calculate the Maximum Tubesheet Bending Stress

Effective Groove depth
$$h'_g := max(h_g - c_t, 0)$$
 $h'_g = 0.197$ in $h_{min} := \begin{pmatrix} h - h'_g \\ h - h'_g \\ h - h'_g \\ h - h'g \\ h \\ h \\ h \\ h \end{pmatrix}$ $h_{min} = \begin{pmatrix} 1.5530 \\ 1.5530 \\ 1.5530 \\ 1.5530 \\ 1.7500 \\ 1.7500 \\ 1.7500 \\ 1.7500 \\ 1.7500 \\ 1.7500 \\ 1.7500 \end{pmatrix}$

.

$$\sigma_{1} := \begin{bmatrix} \overline{6 \cdot Q_{2_{1}}} \\ \mu^{*} (h_{\min_{1}})^{2} \end{bmatrix} \text{ if } P_{e_{1}} = 0 \qquad \sigma_{2} := \begin{bmatrix} \overline{6 \cdot Q_{2_{2}}} \\ \mu^{*} (h_{\min_{2}})^{2} \end{bmatrix} \text{ if } P_{e_{2}} = 0 \qquad \sigma_{1} = -16389.848 \frac{lb}{ln^{2}}$$

$$\sigma_{1} := -16389.848 \frac{lb}{ln^{2}}$$

$$\sigma_{2} := \begin{bmatrix} \overline{1.5 \cdot F_{m_{1}}} (2 \cdot a_{0})^{2} \cdot P_{e_{1}} \end{bmatrix} \text{ otherwise } \sigma_{2} = 27316.414 \frac{lb}{ln^{2}}$$

$$\sigma_{3} := \begin{bmatrix} \overline{6 \cdot Q_{2_{3}}} \\ \mu^{*} (h_{\min_{3}})^{2} \end{bmatrix} \text{ if } P_{e_{3}} = 0 \qquad \sigma_{4} := \begin{bmatrix} \overline{6 \cdot Q_{2_{4}}} \\ \mu^{*} (h_{\min_{4}})^{2} \end{bmatrix} \text{ if } P_{e_{4}} = 0 \qquad \sigma_{3} = 10926.565 \frac{lb}{ln^{2}}$$

$$\overline{\left[\frac{1.5 \cdot F_{m_{3}}}{\mu^{*}} (\frac{2 \cdot a_{0}}{h_{\min_{3}}})^{2} \cdot P_{e_{3}} \end{bmatrix} \text{ otherwise } \sigma_{4} := \begin{bmatrix} \overline{6 \cdot Q_{2_{4}}} \\ \mu^{*} (h_{\min_{4}})^{2} \end{bmatrix} \text{ if } P_{e_{4}} = 0 \qquad \sigma_{3} = 10926.565 \frac{lb}{ln^{2}}$$

$$\sigma_{5} := \begin{bmatrix} \overline{6 \cdot Q_{2_{5}}} \\ \frac{1.5 \cdot F_{m_{3}}}{\mu^{*}} (\frac{2 \cdot a_{0}}{h_{\min_{3}}})^{2} \cdot P_{e_{3}} \end{bmatrix} \text{ otherwise } \sigma_{6} := \begin{bmatrix} \overline{6 \cdot Q_{2_{6}}} \\ \mu^{*} (h_{\min_{6}})^{2} \end{bmatrix} \text{ if } P_{e_{6}} = 0 \qquad \sigma_{5} = 0.000 \frac{lb}{ln^{2}}$$

$$\sigma_{7} := \begin{bmatrix} \overline{6 \cdot Q_{2_{7}}} \\ \frac{1.5 \cdot F_{m_{5}}}{\mu^{*}} (\frac{2 \cdot a_{0}}{h_{\min_{5}}})^{2} \cdot P_{e_{5}} \end{bmatrix} \text{ otherwise } \sigma_{8} := \begin{bmatrix} \overline{6 \cdot Q_{2_{6}}} \\ \mu^{*} (h_{\min_{6}})^{2} \end{bmatrix} \text{ if } P_{e_{8}} = 0 \qquad \sigma_{6} := \begin{bmatrix} \overline{6 \cdot Q_{2_{6}}} \\ \frac{1.5 \cdot F_{m_{6}}}{\mu^{*} (h_{\min_{6}})^{2}} \end{bmatrix} \text{ otherwise } \sigma_{6} = 0.000 \frac{lb}{ln^{2}}$$

$$\sigma_{7} := \begin{bmatrix} \overline{6 \cdot Q_{2_{7}}} \\ \frac{1.5 \cdot F_{m_{7}}}{\mu^{*} (h_{\min_{7}})^{2}} \end{bmatrix} \text{ if } P_{e_{7}} = 0 \qquad \sigma_{8} := \begin{bmatrix} \overline{6 \cdot Q_{2_{8}}} \\ \frac{1.5 \cdot F_{m_{8}}}{\mu^{*} (h_{\min_{8}})^{2}} \end{bmatrix} \text{ otherwise } \sigma_{7} = 0.000 \frac{lb}{ln^{2}}$$

$$\sigma_{7} := 0.000 \frac{lb}{ln^{2}} \qquad \sigma_{8} = 0.000 \frac{lb}{ln^{2}} \qquad \sigma_{8} = 0.000 \frac{lb}{ln^{2}}$$

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ExampleE4.18.8-STATIONARY TS(AnnexW) from PTB4

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TUBESHEET MAXIMUM STRESS for DESIGN LOADING CASES 1, 2, 3, 4 $\sigma_{\rm D} = 27316.4 \frac{\rm lb}{\rm in^2}$ $\sigma_{\mathsf{D}} \coloneqq \mathsf{max}(|\sigma_1|, |\sigma_2|, |\sigma_3|, |\sigma_4|)$ σ_{allowD} = 28500.0 $\sigma_{allowD} := 1.5 \cdot S_D$ TUBESHEET MAXIMUM STRESS For OPERATING LOADING CASES 5,6,7,8 $\sigma_{O} = 0.0 \frac{\text{lb}}{\text{in}^2}$ $\sigma_{\mathsf{O}} \coloneqq \max(\left|\sigma_{\mathsf{5}}\right|, \left|\sigma_{\mathsf{6}}\right|, \left|\sigma_{\mathsf{7}}\right|, \left|\sigma_{\mathsf{8}}\right|)$ $\sigma_{\text{allowO}} = 0.0 \frac{\text{lb}}{\text{in}^2}$ $\sigma_{\text{allowO}} \coloneqq \textbf{S}_{\text{PS}}$ UHX-13.5.8 Step 8 Calculate the maximum tubesheet shear stress P_e1 P_{e_2} 〔150.000〕 P_e3 250.000 100.000 P_{e4} P_{e5} P_{e6} If $|Pe|<3.2S\mu h/D_o$, the TEMA formula $1.6 \cdot S_D \cdot \mu \cdot \frac{h}{a_o} = 1033.010 \frac{lb}{in^2}$ $absP_e := is not required to be calculated$ 0.000 lb absP_e = in² 150.000 250.000 100.000 0.000) $\tau := \left(\frac{1}{\mu}\right) \cdot \frac{1}{h} \left(\frac{A_p}{C_p}\right) \cdot \overrightarrow{|P_e|}$ 2010 results(for information only) -2207.1) (2153.8) 3678.6 3589.7 $\begin{aligned} \tau_{max} \coloneqq max \left(\overrightarrow{|\tau|} \right) & \tau_{max} = 3589.7 \frac{lb}{dt} \\ \hline \frac{lb}{in^2} & \tau_{allow} \coloneqq 0.8 \cdot S_D & \tau_{allow} = 15200.0 - 1000 \\ \hline \tau_{allow} = 15200.0 - 1000 \\ \hline \tau_{allow} = 1$ 1471.4 1435.9 0.0 0.0 ^t2010 = $\tau =$ $\tau_{max2010} := max(\tau_{2010})$ 2153.8 -2207.1 3589.7 3678.6 $\tau_{max2010} = 3678.6$ 1435.9 1471.4 0.0 0.0 /

<u>UHX-13.5.9</u> Step 9 Determine the minimum and maximum stresses in the tubes for Load Cases 1 to 8

$$Z_{dx}(x) := \frac{\Psi_2 \cdot ber_x(x) + \Psi_1 \cdot bei_x(x)}{X_a^3 \cdot Z_a} \qquad \qquad Z_{wx}(x) := \frac{ber' \cdot ber_x(x) + bei' \cdot bei_x(x)}{X_a^2 \cdot Z_a}$$

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a) :Determine Coefficients ${\rm F}_{\rm t,min}$ and ${\rm F}_{\rm t,max}$ for Load Cases 1 to 8:

$$\begin{split} F_{1x1}(x) &:= \left| \begin{array}{c} Z_{ux}(x) - \frac{x_{0}^{-4}}{2} & \text{if } P_{e_{\frac{1}{2}}} = 0 \\ \left(Z_{dx}(x) + Q_{3_{\frac{1}{2}}} Z_{ux}(x) \right) \frac{x_{0}^{-4}}{2} & \text{otherwise} \end{array} \right| F_{1x5}(x) &:= \left| \begin{array}{c} Z_{ux}(x) - \frac{x_{0}^{-4}}{2} & \text{if } P_{e_{\frac{1}{2}}} = 0 \\ \left(Z_{dx}(x) + Q_{3_{\frac{1}{2}}} Z_{ux}(x) \right) \frac{x_{0}^{-4}}{2} & \text{otherwise} \end{array} \right| F_{1x5}(x) &:= \left| \begin{array}{c} Z_{ux}(x) - \frac{x_{0}^{-4}}{2} & \text{if } P_{e_{\frac{1}{2}}} = 0 \\ \left(Z_{dx}(x) + Q_{3_{\frac{1}{2}}} Z_{ux}(x) \right) \frac{x_{0}^{-4}}{2} & \text{otherwise} \end{array} \right| F_{1x5}(x) &:= \left| \begin{array}{c} Z_{ux}(x) - \frac{x_{0}^{-4}}{2} & \text{if } P_{e_{\frac{1}{2}}} = 0 \\ \left(Z_{dx}(x) + Q_{3_{\frac{1}{2}}} Z_{ux}(x) \right) \frac{x_{0}^{-4}}{2} & \text{otherwise} \end{array} \right| F_{1x7}(x) &:= \left| \begin{array}{c} Z_{ux}(x) - \frac{x_{0}^{-4}}{2} & \text{if } P_{e_{\frac{1}{2}}} = 0 \\ \left(Z_{dx}(x) + Q_{3_{\frac{1}{2}}} Z_{ux}(x) \right) \frac{x_{0}^{-4}}{2} & \text{otherwise} \end{array} \right| F_{1x7}(x) &:= \left| \begin{array}{c} Z_{ux}(x) - \frac{x_{0}^{-4}}{2} & \text{if } P_{e_{\frac{1}{2}}} = 0 \\ \left(Z_{dx}(x) + Q_{3_{\frac{1}{2}}} Z_{ux}(x) \right) \frac{x_{0}^{-4}}{2} & \text{otherwise} \end{array} \right| F_{1x6}(x) &:= \left| \begin{array}{c} Z_{ux}(x) - \frac{x_{0}^{-4}}{2} & \text{if } P_{e_{\frac{1}{2}}} = 0 \\ \left(Z_{dx}(x) + Q_{3_{\frac{1}{2}}} Z_{ux}(x) \right) \frac{x_{0}^{-4}}{2} & \text{otherwise} \end{array} \right| F_{1x6}(x) &:= \left| \begin{array}{c} Z_{ux}(x) - \frac{x_{0}^{-4}}{2} & \text{if } P_{e_{\frac{1}{2}}} = 0 \\ \left(Z_{dx}(x) + Q_{3_{\frac{1}{2}}} Z_{ux}(x) \right) \frac{x_{0}^{-4}}{2} & \text{otherwise} \end{array} \right| F_{1x6}(x) &:= \left| \begin{array}{c} Z_{ux}(x) - \frac{x_{0}^{-4}}{2} & \text{if } P_{e_{\frac{1}{2}}} = 0 \\ \left(Z_{dx}(x) + Q_{3_{\frac{1}{2}}} Z_{ux}(x) \right) \frac{x_{0}^{-4}}{2} & \text{otherwise} \end{array} \right| F_{1x6}(x) &:= \left| \begin{array}{c} Z_{ux}(x) - \frac{x_{0}^{-4}}{2} & \text{if } P_{e_{\frac{1}{2}}} = 0 \\ \left(Z_{dx}(x) + Q_{3_{\frac{1}{2}}} Z_{ux}(x) \right) \frac{x_{0}^{-4}}{2} & \text{otherwise} \end{array} \right| F_{1x6}(x) &:= \left| \begin{array}{c} Z_{ux}(x) - \frac{x_{0}^{-4}}{2} & \text{if } P_{e_{\frac{1}{2}}} = 0 \\ \left(Z_{dx}(x) + Q_{3_{\frac{1}{2}}} Z_{ux}(x) \right) \frac{x_{0}^{-4}}{2} & \text{otherwise} \end{array} \right| F_{1x6}(x) &:= \left| \begin{array}{c} Z_{ux}(x) - \frac{x_{0}^{-4}}{2} & \text{if } P_{e_{\frac{1}{2}}} = 0 \\ \left(Z_{ux}(x) + Q_{3_{\frac{1}{2}}} Z_{ux}(x) \right) \frac{x_{0}^{-4}}{2} & \text{otherwise} \end{array} \right| F_{1x6}(x) &:= \left| \begin{array}{c} Z_{ux}(x) - \frac{x_{0}^{-4}}{2} & \text{if } P_{e_{\frac{1}{2}$$

ExampleE4.18.8-STATIONARY TS(AnnexW) from PTB4

σ_{t11} :=

b) Determine tube stresses $~\sigma_{tmin}$ and σ_{tmax} for Load Cases 1 to 8 (continued) $\begin{bmatrix} \frac{1}{x_t - x_s} \cdot \left[\left(P_{s_1} \cdot x_s - P_{t_1} \cdot x_t \right) - \frac{2 \cdot Q_{2_1}}{a_o^2} \cdot F_{tmin_1} \right] \end{bmatrix} \text{ if } P_{e_1} = 0 \qquad \sigma_{t1_1} = -1710.047 \frac{\text{lb}}{\text{in}^2}$ $\begin{bmatrix} \frac{1}{x_t - x_s} \cdot \left[\left(P_{s_1} \cdot x_s - P_{t_1} \cdot x_t \right) - P_{e_1} \cdot F_{tmin_1} \right] \end{bmatrix} \text{ otherwise}$ $\sigma_{t1_{2}} := \left[\underbrace{\frac{1}{x_{t} - x_{s}} \cdot \left[\left(\mathsf{P}_{s_{2}} \cdot x_{s} - \mathsf{P}_{t_{2}} \cdot x_{t} \right) - \frac{2 \cdot \mathsf{Q}_{2}}{\mathsf{a}_{0}^{2}} \cdot \mathsf{F}_{tmin_{2}} \right]}_{\left[\frac{1}{x_{t} - x_{s}} \cdot \left[\left(\mathsf{P}_{s_{2}} \cdot x_{s} - \mathsf{P}_{t_{2}} \cdot x_{t} \right) - \mathsf{P}_{e_{2}} \cdot \mathsf{F}_{tmin_{2}} \right]} \right] \text{ otherwise } \right]$

$$\sigma_{t1_3} := \begin{bmatrix} \frac{1}{x_t - x_s} \cdot \left[\left(\mathsf{P}_{s_3} \cdot x_s - \mathsf{P}_{t_3} \cdot x_t \right) - \frac{2 \cdot \mathsf{Q}_{2_3}}{a_0^2} \cdot \mathsf{F}_{tmin_3} \right] \end{bmatrix} \text{ if } \mathsf{P}_{e_3} = 0$$

$$\begin{bmatrix} \frac{1}{x_t - x_s} \cdot \left[\left(\mathsf{P}_{s_3} \cdot x_s - \mathsf{P}_{t_3} \cdot x_t \right) - \mathsf{P}_{e_3} \cdot \mathsf{F}_{tmin_3} \right] \end{bmatrix} \text{ otherwise}$$

$$\sigma_{t1_4} := \left[\frac{1}{x_t - x_s} \cdot \left[\left(\mathsf{P}_{\mathsf{s}_4} \cdot x_s - \mathsf{P}_{\mathsf{t}_4} \cdot x_t \right) - \frac{2 \cdot \mathsf{Q}_{\mathsf{2}_4}}{\mathsf{a}_0^2} \cdot \mathsf{F}_{\mathsf{tmin}_4} \right] \right] \text{ if } \mathsf{P}_{\mathsf{e}_4} = 0$$

$$\left[\frac{1}{x_t - x_s} \cdot \left[\left(\mathsf{P}_{\mathsf{s}_4} \cdot x_s - \mathsf{P}_{\mathsf{t}_4} \cdot x_t \right) - \mathsf{P}_{\mathsf{e}_4} \cdot \mathsf{F}_{\mathsf{tmin}_4} \right] \right] \text{ otherwise}$$

$$\sigma_{t1_{5}} \coloneqq \left[\left[\frac{1}{x_{t} - x_{s}} \cdot \left[\left(\mathsf{P}_{s_{5}} \cdot x_{s} - \mathsf{P}_{t_{5}} \cdot x_{t} \right) - \frac{2 \cdot \mathsf{Q}_{2_{5}}}{a_{0}^{2}} \cdot \mathsf{F}_{tmin_{5}} \right] \right] \text{ if } \mathsf{P}_{e_{5}} = 0$$

$$\left[\frac{1}{x_{t} - x_{s}} \cdot \left[\left(\mathsf{P}_{s_{5}} \cdot x_{s} - \mathsf{P}_{t_{5}} \cdot x_{t} \right) - \mathsf{P}_{e_{5}} \cdot \mathsf{F}_{tmin_{5}} \right] \right] \text{ otherwise}$$

$$\sigma_{t1_{6}} \coloneqq \left[\underbrace{\frac{1}{x_{t} - x_{s}} \cdot \left[\left(\mathsf{P}_{s_{6}} \cdot x_{s} - \mathsf{P}_{t_{6}} \cdot x_{t} \right) - \frac{2 \cdot \mathsf{Q}_{2_{6}}}{a_{0}^{2}} \cdot \mathsf{F}_{tmin_{6}} \right] \right]}_{\left[\underbrace{\frac{1}{x_{t} - x_{s}} \cdot \left[\left(\mathsf{P}_{s_{6}} \cdot x_{s} - \mathsf{P}_{t_{6}} \cdot x_{t} \right) - \mathsf{P}_{e_{6}} \cdot \mathsf{F}_{tmin_{6}} \right] \right]}_{otherwise} \right]$$

$$\sigma_{t1_{7}} \coloneqq \left[\frac{1}{x_{t} - x_{s}} \cdot \left[\left(\mathsf{P}_{s_{7}} \cdot x_{s} - \mathsf{P}_{t_{7}} \cdot x_{t} \right) - \frac{2 \cdot \mathsf{Q}_{2_{7}}}{a_{o}^{2}} \cdot \mathsf{F}_{tmin_{7}} \right] \right] \text{ if } \mathsf{P}_{e_{7}} = \mathsf{Q}_{1_{7}}$$
$$\left[\frac{1}{x_{t} - x_{s}} \cdot \left[\left(\mathsf{P}_{s_{7}} \cdot x_{s} - \mathsf{P}_{t_{7}} \cdot x_{t} \right) - \mathsf{P}_{e_{7}} \cdot \mathsf{F}_{tmin_{7}} \right] \right] \text{ otherwise}$$

$$\sigma_{t1_2} = 2600.078 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{t1_3} = 890.031 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{t1_4} = 0.000 \frac{lb}{ln^2}$$

$$\sigma_{t1_{5}} := \begin{bmatrix} 0 & \text{if } \left(\left| \mathsf{P}^{*}_{s} \right| = 0 \land \left| \mathsf{P}^{*}_{c} \right| = 0 \right) \\ \sigma_{t1_{5}} & \text{otherwise} \end{bmatrix}$$
$$\sigma_{t1_{5}} = 0.000 \frac{\text{lb}}{\text{in}^{2}}$$

$$\begin{split} \sigma_{t1_{6}} &\coloneqq \begin{bmatrix} 0 & \text{if } \left(\left| \mathsf{P}^{\star}_{s} \right| = 0 \land \left| \mathsf{P}^{\star}_{c} \right| = 0 \right) \\ \sigma_{t1_{6}} & \text{otherwise} \end{bmatrix} \\ \sigma_{t1_{6}} &= 0.000 \frac{\text{lb}}{\text{in}^{2}} \end{split}$$

$$\sigma_{t1_{7}} \coloneqq \begin{bmatrix} 0 & \text{if } \left(\left| \mathsf{P}^*_{s} \right| = 0 \land \left| \mathsf{P}^*_{c} \right| = 0 \right) \\ \sigma_{t1_{7}} & \text{otherwise} \end{bmatrix}$$
$$\sigma_{t1_{7}} = 0.000 \frac{\text{lb}}{\text{in}^2}$$

ExampleE4.18.8-STATIONARY TS(AnnexW) from PTB4

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0)

$$\sigma_{18} := \begin{bmatrix} \frac{1}{x_1 - x_5} \left[\left(P_{8_8} \cdot x_5 - P_{1_8} \cdot x_1 \right) - \frac{2 \cdot Q_{2_8}}{a_0^2} F_{train_8} \right] & \text{if } P_{e_8} = 0 & \sigma_{18} := \begin{bmatrix} 0 & \text{if } (|P^*_{8}| = 0 \land |P^*_{6}| = 0 \\ \sigma_{18_8} \cdot \sigma_{18_8} \cdot$$

ExampleE4.18.8-STATIONARY TS(AnnexW) from PTB4

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$$\begin{split} \text{Size} (b): \text{check the tubes for buckling if } a_{t} < 0 \text{ or } a_{2} < 0 \\ b_{1} := bl \quad l_{t} = 15.375 \text{ in } \quad r_{t} := \frac{\sqrt{4t_{t}^{2} + (4_{t} - 2\cdot t_{t})^{2}}{4}}{r_{t} = 0.238 \text{ in } \quad C_{t} := \sqrt{\frac{2\pi^{2} + E_{0}}{S_{yD}}} \quad C_{t} := 161.043 \quad F_{t} := \frac{4}{r_{t}} \quad F_{t} = 64.699 \\ F_{t} := sl(F_{51} > 2.2, F_{51}) \quad F_{t} = 1.540 \\ mas[325 - 25[(Z_{4} + O_{3}, Z_{7}) \times a_{t}^{4}] \cdot 125] \quad \text{otherwise}} \\ F_{52} := 125 \quad \text{if } P_{e_{3}} = 0 \\ mas[325 - 25[(Z_{4} + O_{3}, Z_{7}) \times a_{t}^{4}] \cdot 125] \quad \text{otherwise}} \\ F_{53} := 125 \quad \text{if } P_{e_{3}} = 0 \\ max[325 - 25[(Z_{4} + O_{3}, Z_{7}) \times a_{t}^{4}] \cdot 125] \quad \text{otherwise}} \\ F_{53} := 125 \quad \text{if } P_{e_{3}} = 0 \\ max[325 - 25[(Z_{4} + O_{3}, Z_{7}) \times a_{t}^{4}] \cdot 125] \quad \text{otherwise}} \\ F_{54} := 125 \quad \text{if } P_{e_{3}} = 0 \\ max[325 - 25[(Z_{4} + O_{3}, Z_{7}) \times a_{t}^{4}] \cdot 125] \quad \text{otherwise}} \\ F_{55} := 125 \quad \text{if } P_{e_{3}} = 0 \\ max[325 - 25[(Z_{4} + O_{3}, Z_{7}) \times a_{t}^{4}] \cdot 125] \quad \text{otherwise}} \\ F_{56} := 125 \quad \text{if } P_{e_{3}} = 0 \\ max[325 - 25[(Z_{4} + O_{3}, Z_{7}) \times a_{t}^{4}] \cdot 125] \quad \text{otherwise}} \\ F_{57} := 125 \quad \text{if } P_{e_{3}} = 0 \\ max[325 - 25[(Z_{4} + O_{3}, Z_{7}) \times a_{t}^{4}] \cdot 125] \quad \text{otherwise}} \\ F_{57} := 125 \quad \text{if } P_{e_{3}} = 0 \\ max[325 - 25[(Z_{4} + O_{3}, Z_{7}) \times a_{t}^{4}] \cdot 125] \quad \text{otherwise}} \\ F_{57} := 125 \quad \text{if } P_{e_{3}} = 0 \\ max[325 - 25[(Z_{4} + O_{3}, Z_{7}) \times a_{t}^{4}] \cdot 125] \quad \text{otherwise}} \\ F_{58} := 125 \quad \text{if } P_{e_{3}} = 0 \\ max[325 - 25[(Z_{4} + O_{3}, Z_{7}) \times a_{t}^{4}] \cdot 125] \quad \text{otherwise}} \\ F_{58} := 125 \quad \text{if } P_{e_{3}} = 0 \\ max[325 - 25[(Z_{4} + O_{3}, Z_{7}) \times a_{t}^{4}] \cdot 125] \quad \text{otherwise}} \\ F_{58} := 125 \quad \text{if } P_{e_{3}} = 0 \\ max[325 - 25[(Z_{4} + O_{3}, Z_{7}) \times a_{t}^{4}] \cdot 125] \quad \text{otherwise}} \\ F_{58} := 125 \quad \text{if } P_{e_{3}} = 0 \\ max[325 - 25[(Z_{4} + O_{3}, Z_{7}) \times a_{t}^{4}] \cdot 125] \quad \text{otherwise}} \\ F_{58} := 125 \quad \text{if } P_{e_{3}} = 0 \\ max[325 - 25[(Z_{4} + O_{3}, Z_{7}) \times a_{t}^{4}] \cdot 125] \quad \text{otherwise}} \\ F_{58} := 125 \quad \text{if } P_{e_{3}} = 0 \\$$

ExampleE4.18.8-STATIONARY TS(AnnexW) from PTB4

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UHX 14.5.10 Step 10 Determine the shell stresses

a) shell membrane stress

$$\sigma_{sm} := \left[\frac{a_0^2}{\left(D_s + t_s\right) \cdot t_s} \cdot \left[P_e + \left(\rho_s^2 - 1\right) \cdot \left(P_s - P_t\right) \right] + \frac{a_s^2}{\left(D_s + t_s\right) t_s} \cdot P_t \text{ if } (Config = "a" \lor Config = "a$$

0.Ps otherwise

$$\sigma_{\text{SM}} = \begin{pmatrix} 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \end{pmatrix} \frac{\text{lb}}{\text{in}^2}$$

SHELL MAXIMUM MEMBRANE STRESS for DESIGN LOADING CASES 1, 2, 3, 4



"b" \vee Config = "c")

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ExampleE4.18.8-STATIONARY TS(AnnexW) from PTB4

UHX 14.5.10 Step 10 Determine the channel stresses

a) channel membrane stress:

$$\sigma_{cm} \coloneqq \left\{ \begin{array}{l} a_c^2 \\ \hline \left(D_c + t_c \right) \cdot t_c \end{array} \right. P_t \quad \text{if } Config = "a" \lor Config = "e" \lor Config = "f" \lor Config = "A" \\ \hline 0 \cdot P_t \quad \text{otherwise} \end{array} \right.$$

$$\sigma_{cm} = \begin{pmatrix} 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \end{pmatrix} \frac{lb}{in^2}$$

b) channel bending stress:

$$I_{cmin} := 1.8 \cdot \sqrt{D_c \cdot t_c} \qquad I_{cmin} = 0.000 \text{ in}$$

$$\sigma_{cb} := \frac{6}{t_c^2} \cdot k_c \left[\beta_c \cdot \left(\delta_c \cdot P_t + \frac{a_c^2}{E_{cD} \cdot t_c} P^*_c \right) - \frac{6 \cdot \left(1 - v^{*^2}\right)}{E^*} \cdot \left(\frac{a_0^3}{h^3} \right) \cdot \left(1 + \frac{h'_c}{2} \right) \cdot \left[P_e \cdot \left(Z_v + Z_m \cdot Q_1\right) + \frac{2}{a_0^2} \cdot Z_m \cdot Q_2 \right] \right] \right]$$

$$\sigma_{cb} = \begin{pmatrix} 0.000 \\$$

CHANNEL MAXIMUM STRESS for DESIGN LOADING CASES 1, 2, 3, 4

$$\sigma_{cmaxD} \coloneqq max(\sigma_{c_1}, \sigma_{c_2}, \sigma_{c_3}, \sigma_{c_4}) \qquad \sigma_{cmaxD} = 0.000 \frac{lb}{ln^2} \qquad \sigma_{callowD} \coloneqq 1.5 \cdot S_{cD} \qquad \sigma_{callowD} = 0.0 \frac{lb}{ln^2}$$

CHANNEL MAXIMUM STRESS for OPERATING LOADING CASES 5, 6, 7, 8

$$\sigma_{cmaxO} := max \left(\sigma_{c_5}, \sigma_{c_6}, \sigma_{c_7}, \sigma_{c_8} \right) \qquad \sigma_{cmaxO} = 0.000 \frac{lb}{in^2} \qquad \sigma_{callowO} := S_{PSc} \qquad \sigma_{callowO} = 0.0 \frac{lb}{in^2}$$

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ExampleE4.18.8-STATIONARY TS(AnnexW) from PTB4

_UHX13.7 Simplified Elastic Plastic Procedure

Calculation procedure for the effect of plasticity at the tubesheet, channel or shell joint.

 $S_{c}^{*} := \min\left(S_{ycD}, \frac{S_{PSc}}{2}\right)$

 $S_c^* = 0.000 \frac{lb}{lo^2}$

 $(\min(fact_{cv_1}, 1))$

 $\min(fact_{cv_A}, 1)$

 $\begin{array}{c|c} \min\left(fact_{cv_{2}},1\right) \\ \min\left(fact_{cv_{3}},1\right) \end{array} \qquad \qquad fact_{c} = \end{array}$

This procedure applies only to Configurations a,b,c and Design Loading Cases 1, 2, 3, 4 in the following conditions: -for integral shell (config. a,b,c) when 1.5S $_{s} < \sigma_{s} \le S_{PS,s}$ -for integral channel (config. a) when $1.5S_c < \sigma_c \le S_{PS,c}$

$$\begin{split} \mathbf{S}^{*}_{s} &:= \min\left(\mathbf{S}_{ys\mathbf{D}}, \frac{\mathbf{S}_{PSc}}{2}\right) \\ \mathbf{S}^{*}_{s} &:= \min\left(\mathbf{S}_{ys\mathbf{D}}, \frac{\mathbf{S}_{PSc}}{2}\right) \\ \mathbf{S}^{*}_{s} &= 0.000 \frac{lb}{in^{2}} \\ \mathbf{fact}_{sv} &:= \overbrace{\left(1.4 - \frac{0.4 \cdot |\sigma_{sb}|}{\mathbf{S}^{*}_{s}}\right)}^{\mathsf{fact}_{sv}} \quad \mathbf{fact}_{sv} = \overbrace{\left(\begin{array}{c}1.400\\1.400\\1.400\\1.400\\1.400\\1.400\\1.400\end{array}\right)}^{\mathsf{fact}_{sv} &:= \overbrace{\left(1.4 - \frac{0.4 \cdot |\sigma_{cb}|}{\mathbf{S}^{*}_{c}}\right)}^{\mathsf{fact}_{sv}} \quad \mathbf{fact}_{cv} = \overbrace{\left(\begin{array}{c}1.400\\1.400\\1.400\\1.400\\1.400\right)}^{\mathsf{fact}_{sv}} &:= \overbrace{\left(\begin{array}{c}1.4 - \frac{0.4 \cdot |\sigma_{cb}|}{\mathbf{S}^{*}_{c}}\right)}^{\mathsf{fact}_{sv}} \quad \mathbf{fact}_{cv} = \overbrace{\left(\begin{array}{c}1.000\\1.400\\1.400\\1.400\right)}^{\mathsf{fact}_{sv}} &:= \overbrace{\left(\begin{array}{c}1.4 - \frac{0.4 \cdot |\sigma_{cb}|}{\mathbf{S}^{*}_{c}}\right)}^{\mathsf{fact}_{sv}} \quad \mathbf{fact}_{cv} = \overbrace{\left(\begin{array}{c}1.400\\1.400\\1.400\\1.400\right)}^{\mathsf{fact}_{sv}} &:= \overbrace{\left(\begin{array}{c}1.4 - \frac{0.4 \cdot |\sigma_{cb}|}{\mathbf{S}^{*}_{c}}\right)}^{\mathsf{fact}_{sv}} &:= \overbrace{\left(\begin{array}{c}1.4 - \frac{0.4 \cdot |\sigma_{cb}|}{\mathbf{S}^{*}_{sv}}\right)}^{\mathsf{fact}_{sv}} &:= \overbrace{\left(\begin{array}{c}1.4 - \frac$$

Calculate reduced values of E_s and E_c for each loading case:

$$E_{sD}^{*} := E_{sD}^{*} \cdot fact_{s} \quad E_{sD}^{*} = \begin{pmatrix} 1.0000 \times 10^{0} \\ 1.0000 \times 10^{0} \\ 1.0000 \times 10^{0} \\ 1.0000 \times 10^{0} \\ 1.0000 \times 10^{0} \end{pmatrix} \qquad E_{cD}^{*} := E_{cD}^{*} \cdot fact_{c} \quad E_{cD}^{*} = \begin{pmatrix} 1.000 \times 10^{0} \\ 1.000 \times 10^{0} \\ 1.000 \times 10^{0} \\ 1.000 \times 10^{0} \end{pmatrix} \qquad Ib$$

From Ste

$$kk_{s} := \beta_{s} \cdot \frac{E_{sD}^{*} \cdot t_{s}^{3}}{6 \cdot (1 - v_{s}^{2})} \qquad kk_{s} = \begin{pmatrix} 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \end{pmatrix} | b \qquad \qquad \lambda\lambda_{s} := \frac{6 \cdot D_{s}}{h^{3}} \cdot kk_{s} \cdot \left(1 + h'_{s} + \frac{h'_{s}^{2}}{2}\right) \qquad \lambda\lambda_{s} = \begin{pmatrix} 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \end{pmatrix} | \frac{lb}{ln^{2}} = \frac{h'_{s}^{2}}{h^{3}} \cdot kk_{s} \cdot \left(1 + h'_{s} + \frac{h'_{s}^{2}}{2}\right) \qquad \lambda\lambda_{s} = \begin{pmatrix} 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \end{pmatrix} | \frac{lb}{ln^{2}} = \frac{h'_{s}^{2}}{h^{3}} \cdot kk_{s} \cdot \left(1 + h'_{s} + \frac{h'_{s}^{2}}{2}\right) \qquad \lambda\lambda_{s} = \begin{pmatrix} 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \end{pmatrix} | \frac{lb}{ln^{2}} = \frac{h'_{s}^{2}}{h^{3}} \cdot kk_{s} \cdot \left(1 + h'_{s} + \frac{h'_{s}^{2}}{2}\right) \qquad \lambda\lambda_{s} = \begin{pmatrix} 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \end{pmatrix} | \frac{lb}{ln^{2}} = \frac{h'_{s}^{2}}{h^{3}} \cdot kk_{s} \cdot \left(1 + h'_{s} + \frac{h'_{s}^{2}}{2}\right) \qquad \lambda\lambda_{s} = \begin{pmatrix} 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \end{pmatrix} | \frac{lb}{ln^{2}} = \frac{h'_{s}^{2}}{h^{3}} \cdot kk_{s} \cdot \left(1 + h'_{s} + \frac{h'_{s}^{2}}{2}\right) \qquad \lambda\lambda_{s} = \begin{pmatrix} 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \end{pmatrix} | \frac{lb}{ln^{2}} = \frac{h'_{s}^{2}}{h^{3}} \cdot kk_{s} \cdot \left(1 + h'_{s} + \frac{h'_{s}^{2}}{2}\right) \qquad \lambda\lambda_{s} = \begin{pmatrix} 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \end{pmatrix} | \frac{h'_{s}}{ln^{2}} + \frac{h'_{s}^{2}}{h^{3}} \cdot kk_{s} \cdot \left(1 + h'_{s} + \frac{h'_{s}^{2}}{h^{3}}\right) = \frac{h'_{s}^{2}}{h^{3}} \cdot \frac{h'_{s}^{2}}{h^{3}$$

$$kk_{c} := \beta_{c} \cdot \frac{E_{cD}^{*} \cdot t_{c}^{3}}{6 \cdot (1 - v_{c}^{2})} \qquad kk_{c} = \begin{pmatrix} 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \end{pmatrix} \qquad \qquad \lambda\lambda_{c} := \frac{6 \cdot D_{c}}{h^{3}} \cdot kk_{c} \cdot \left(1 + h'_{c} + \frac{h'_{c}^{2}}{2}\right) \qquad \lambda\lambda_{c} = \begin{pmatrix} 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \end{pmatrix}$$

ExampleE4.18.8-STATIONARY TS(AnnexW) from PTB4

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(1.4)1.4 1.4

1.4 1.4 1.4 1.4 (1.4)

(1.000)

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1.000 1.000

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(0.561)

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From Step 4, recalculate:

$$\mathsf{F} := \frac{1 - v^{\star}}{\mathsf{E}^{\star}} \cdot \left(\lambda \lambda_{\mathsf{S}} + \lambda \lambda_{\mathsf{C}} + \mathsf{E}_{\mathsf{D}} \cdot \mathsf{In}(\mathsf{K}) \right) \qquad \mathsf{F} = \begin{pmatrix} 0.428 \\ 0.428 \\ 0.428 \\ 0.428 \end{pmatrix}$$

$$Q_{1e} := \overbrace{\left(\frac{\rho_{s} - 1 - \Phi \cdot Z_{v}}{1 + \Phi \cdot Z_{m}}\right)}^{\text{Q}_{1e}} \qquad Q_{1e} = \begin{cases} 0.078 \\ 0.078 \\ 0.078 \\ 0.078 \\ 0.078 \end{cases} \qquad W^{*'} :=$$

$$\mathbf{P}_{se} \coloneqq \begin{pmatrix} \mathbf{P}_{s_1} \\ \mathbf{P}_{s_2} \\ \mathbf{P}_{s_3} \\ \mathbf{P}_{s_4} \end{pmatrix} \quad \mathbf{P}_{te} \coloneqq \begin{pmatrix} \mathbf{P}_{t_1} \\ \mathbf{P}_{t_2} \\ \mathbf{P}_{t_2} \\ \mathbf{P}_{t_3} \\ \mathbf{P}_{t_4} \end{pmatrix}$$

$$W^{*'} := \begin{pmatrix} W^{*}_{1} \\ W^{*}_{2} \\ W^{*}_{3} \\ W^{*}_{4} \end{pmatrix} \qquad W^{*'} = \begin{pmatrix} 211426.000 \\ 0.000 \\ 211426.000 \\ 0.000 \end{pmatrix} lb$$

 $\Phi := (1 + v^*) \cdot \mathsf{F} \qquad \Phi = \begin{pmatrix} 0.561 \\ 0.561 \\ 0.561 \\ 0.561 \end{pmatrix}$



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From Step 7 , recalculate $Q_2^{}$, $Q_3^{}$, $F_m^{}$ and the tubesheet bending stress for load cases 1, 2, and 3

$$Q'_{2} := \boxed{\frac{\left(\omega^{*} \cdot P_{se} - \omega^{*} \cdot e^{\cdot} P_{te}\right) + \frac{\gamma_{b}}{2 \cdot \pi} \cdot W^{*'}}{1 + \Phi \cdot Z_{m}}} \qquad \qquad Q'_{2} = \begin{pmatrix} -213.344 \\ 355.573 \\ 142.229 \\ 0.000 \end{pmatrix}}$$
$$Q'_{3} := \boxed{Q'_{3} := \left(Q_{1e} + \frac{2 \cdot Q'_{2}}{P'_{e} \cdot a_{0}^{2}}\right)} \qquad \qquad Q'_{3} = \begin{pmatrix} 0.0953 \\ 0.0953 \\ 0.0953 \\ 0.0953 \\ 0.0782 \end{pmatrix}}$$

$$F'_{m1}(x) := \begin{bmatrix} 0 & \text{if } P'_{e_1} = 0 \\ \\ \frac{Q_v(x) + Q'_{3_1} \cdot Q_m(x)}{2} & F'_{m1} := |F'_{m1}(x_j)| \\ F'_{m1} := \max(F'_{m1}) & F'_{m1} = 0.102 \end{bmatrix}$$

$$F'_{m2}(x) := \begin{vmatrix} 0 & \text{if } P'_{e_2} = 0 \\ \frac{Q_v(x) + Q'_{3_2} \cdot Q_m(x)}{2} & F'_{m2} := |F'_{m2}(x_j)| \\ F'_{m2} := \max(F'_{m2}) & F'_{m2} = 0.102 \end{vmatrix}$$

$$F'_{m} := \begin{pmatrix} F'_{m1} \\ F'_{m2} \\ F'_{m3} \\ F'_{m4} \end{pmatrix} \qquad F'_{m} = \begin{pmatrix} 0.1021 \\ 0.1021 \\ 0.1021 \\ 0.1021 \\ 0 \end{pmatrix}$$

Tubesheet Bending Stress for the Elastic-Plastic Solution

$$\sigma' := \left[\frac{1.5 \cdot F'_{m}}{\mu^{*}} \cdot \left(\frac{2 \cdot a_{0}}{h - h'_{g}} \right)^{2} \cdot P'_{e} \right] \qquad \sigma' = \left(\begin{array}{c} -16389.848 \\ 27316.414 \\ 10926.565 \\ 0.000 \end{array} \right) \frac{|h|}{|h|} \qquad \sigma'_{max} := max(|\sigma'_{1}|, |\sigma'_{2}|, |\sigma'_{3}|)$$

$$\sigma'_{\text{max}} = 27316.414 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{\text{allow}} \coloneqq 1.5 \cdot \text{S}_{\text{D}} \qquad \sigma_{\text{allow}} = 28500.0 \frac{\text{lb}}{\text{in}^2}$$

ExampleE4.18.8-STATIONARY TS(AnnexW) from PTB4

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Calculation must be performed in 2 phases :

Phase 1) Perform Steps 1 to 11 with **SS="NON"** (normal calculation) with the following modifications in Step 11:

- minimum length requirement Ismin of shell band for configurations a,b,c do not apply

- minimum length requirement I_{cmin} of channel band for configuration a do not apply

if $\sigma_s \leq S_{PS,s}$ and $\sigma_c \leq S_{PS,c}$, the shell and/or channel designs are acceptable. Otherwise increase the thickness of the overstressed components (shell and/or channel) and return to Step 1.

Phase 2) Perform Steps 1 to 7 using SS="OUI" (Simply Supported calculation) for loading cases 1,2 and 3 only

If $|\sigma| \le 1.5S$, the calculation procedure is complete. Otherwise, increase the assumed tubesheet thickness *h* and repeat Steps 1 to 7.

Note: If $|\sigma|<1.5S$, the tubesheet thickness can be optimized to a value h_0 till $|\sigma|=1.5S$ provided that, for that optimized thickness h_0 , the stresses in the tubesheet,shell and channel , calculated by the normal calculation (see Phase 1), remain respectively below $S_{PS, S}$, $S_{PS,c}$ for each of the 8 loading cases.

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Determination of effective elastic properties :

From figure UHX-11.2 - Polynomial equations for the determination of E*/E and v* for triangular pattern

Equations for E* for triangular pitch

	quations		r thangu	ai pitch			(-0.0958	0.6209	-0.8683	2.1099	-1.6831)
E*TriPitch :=	0.0353	1.2502	-0.0491	0.3604	-0.6100)		0.8897	-9.0855	36.1435	-59.5425	35.8223
	0.0135	0.9910	1.0080	-1.0498	0.0184		0.7439	-4.4989	12.5779	-14.2092	5.7822
	0.0054	0.5279	3.0461	-4.3657	1.9435	v*TriPitch :=	0.9100	-4.8901	12.4325	-12.7039	4.4298
	(-0.0029	0.2126	3.9906	-6.1730			0.9923	-4.8759	12.3572	-13.7214	5.7629
							0.9966	-4.1978	9.0478	-7.9955	2.2398 <i>)</i>

From figure UHX-11.3 - Polynomial equations for the determination of E*/E and v* for square pattern

Eq	ations for E* for square pitch	(-0.0791	0.6008	-0.3468	0.4858	-0.3606)	
		28)	0.3345	-2.8420	10.9709	-15.8994	8.3516
E*SquPitch :=	0 0250 1 9251 -3 5230 6 9830 -5 00		0.4296	-2.6350	8.6864	-11.5227	5.8544
	0 0394 1 3024 -1 1041 2 8714 -2 39	94	0.3636	-0.8057	2.0463	-2.2902	1.1862
	0.0372 1.0314 -0.6402 2.6201 -2.19	29	0.3527	-0.2842	0.4354	-0.0901	-0.1590
	0.0072 1.0014 0.0402 2.0201 2.10	20)	0.3341	0.1260	-0.6920	0.6877	-0.0600)

Elay := si(Layout > 0, E*SquPitch, E*TriPitch)

$$E^{*}_{1} := \left(Elay_{1,1} + Elay_{1,2} \cdot \mu^{*} + Elay_{1,3} \cdot \mu^{*2} + Elay_{1,4} \cdot \mu^{*3} + Elay_{1,5} \cdot \mu^{*4}\right) \cdot E_{D}$$

$$E^{*}_{2} := \left(Elay_{2,1} + Elay_{2,2} \cdot \mu^{*} + Elay_{2,3} \cdot \mu^{*2} + Elay_{2,4} \cdot \mu^{*3} + Elay_{2,5} \cdot \mu^{*4}\right) \cdot E_{D}$$

$$E^{*}_{3} := \left(Elay_{3,1} + Elay_{3,2} \cdot \mu^{*} + Elay_{3,3} \cdot \mu^{*2} + Elay_{3,4} \cdot \mu^{*3} + Elay_{3,5} \cdot \mu^{*4}\right) \cdot E_{D}$$

$$E^{*}_{4} := \left(Elay_{4,1} + Elay_{4,2} \cdot \mu^{*} + Elay_{4,3} \cdot \mu^{*2} + Elay_{4,4} \cdot \mu^{*3} + Elay_{4,5} \cdot \mu^{*4}\right) \cdot E_{D}$$

$$E^{*}_{3} = 12259057.430 \frac{lb}{in^{2}}$$

$$E^{*}_{4} = 10634573.132 \frac{lb}{in^{2}}$$

$$E^{*}_{4} = 10634573.132 \frac{lb}{in^{2}}$$

$$\begin{array}{c} \left(E^{*}_{4} \right) & \text{Otherwise perform a linear interpolation :} \\ E^{*}_{4} \left(E^{*}_{4} \right) & \left(\frac{h}{p} > 2, E^{*}_{4}, si \left(\frac{h}{p} \le 0.10, E^{*}_{1}, interplin \left(\frac{h}{pratio}, interE^{*}, \frac{h}{p} \right) \right) \right) \\ & \text{vlay := si(Layout > 0, v^{*}SquPitch, v^{*}TriPitch)} \end{array}$$

$$v_{1}^{*} := v lay_{1,1} + v lay_{1,2} \cdot \mu^{*} + v lay_{1,3} \cdot \mu^{*2} + v lay_{1,4} \cdot \mu^{*3} + v lay_{1,5} \cdot \mu^{*4}$$

$$v_{1}^{*} = 0.098$$

$$v_{2}^{*} := v lay_{2,1} + v lay_{2,2} \cdot \mu^{*} + v lay_{2,3} \cdot \mu^{*2} + v lay_{2,4} \cdot \mu^{*3} + v lay_{2,5} \cdot \mu^{*4}$$

$$v_{2}^{*} = 0.138$$

$$v_{3}^{*} := v lay_{3,1} + v lay_{3,2} \cdot \mu^{*} + v lay_{3,3} \cdot \mu^{*2} + v lay_{3,4} \cdot \mu^{*3} + v lay_{3,5} \cdot \mu^{*4}$$

$$v_{3}^{*} = 0.192$$

$$v_{4}^{*} := v lay_{4,1} + v lay_{4,2} \cdot \mu^{*} + v lay_{4,3} \cdot \mu^{*2} + v lay_{4,4} \cdot \mu^{*3} + v lay_{4,5} \cdot \mu^{*4}$$

$$v_{4}^{*} = 0.242$$

$$v_{5}^{*} := v lay_{5,1} + v lay_{5,2} \cdot \mu^{*} + v lay_{5,3} \cdot \mu^{*2} + v lay_{5,4} \cdot \mu^{*3} + v lay_{5,5} \cdot \mu^{*4}$$

$$v_{5}^{*} = 0.290$$

$$v_{6}^{*} := v lay_{6,1} + v lay_{6,2} \cdot \mu^{*} + v lay_{6,3} \cdot \mu^{*2} + v lay_{6,4} \cdot \mu^{*3} + v lay_{6,5} \cdot \mu^{*4}$$

$$v_{6}^{*} = 0.314$$

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s:

PTB-7-2014: Criteria for Shell-and-Tube Heat Exchangers According to Part UHX of ASME Section VIII Division 1

ANNEX X — UHX-14 – EXAMPLE E4.18.8 (PTB-4 2013 EDITION) FLOATING

The floating tubesheet exchanger with an immersed floating head is to be designed as shown in VIII-1, Figure UHX-14.1, Configuration a. The stationary tubesheet is gasketed with the shell and channel in accordance with configuration d as shown in VIII-1, Figure UHX-14.2, sketch (d). The floating tubesheet is not extended as a flange in accordance with configuration C as shown in VIII-1, Figure UHX-14.3, sketch(c).There is no allowance for corrosion.

ExampleE4.18.8-FLOATING TS(AnnexX) from PTB4

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FLOATING TUBESHEET RULES accord. to UHX-14 (July 2013 Edition)

Example E4.18.8 (PTB- 4 2013 edition) Floating Head H. E. - FLOATING TS config. "C"

1 - GEOMETRIC Data (from Fig.UHX-14.1)

Types o	f Operating Conditions		Туре	s of Heat Excl	nanger	
<mark>x=1 NORM</mark>	AL operating conditio	<mark>n</mark>	ExchType := "a"	EXCHANGER	types : a, b, c	
	TUP operating condition	<mark>n</mark>	STAconfig := "d"	STATIONNAR	Y Config. types :	<mark>a, b, c, d,e,f</mark>
x=4 UPSE1	operating condition	n l	FLOconfig := "C"	FLOATING	Config. types :	A, B, C, D
x=5 CLEAN	NING operating condition	on	Config := FLOcor	nfia Confia = "(
x=6 OTHE	R operating condition	n hiahliah	ted in vellow :most	t important data	and results	
Tubesheet	: Data (from Fig.UHX-14.	.1)	Tube Dat	ta (from Fig UF	IX-11 1)	
<mark>h := 1.75 in</mark>	Tubesheet thickness	'L avout"-0	$p := 1.0 \cdot in$	Tube Pitch		
Layout := 0	For square pitch : '	Layout =0	N _t := 466	Number of	Tubes	
r _o := 12.5∙in	Radius to outer tube	-	$d_{t} = 0.75$ in	Tube Outsi	deDiameter	
A := 26.89 · in	Outside Diameter of T	ubesheet				
C := 27.992.in	Bolt Circle Diameter		$t_t := 0.083 \cdot in$	Tube Thick	ness	
C _n := 78⋅in	Perimeter of the tube	layout	L _t := 256.0∙in	Tube Lengt	t <mark>h</mark>	
P 2			$L := L_t - 2 \cdot h$	L = 252.500	in Effective length	of tubes
A _p := 490 · in [−]	Total area enclosed b	уС _р	ρ := 0.8	Tube expar	nsion depth ratio	
$A_L := 64.375 \cdot \text{in}^2$	Total Untubed Lanes	Area	l _{tx} := 1.40 in	Length of E	Expanded Portion	of Tube
$c_t := 0.0 \cdot in$	Tubesheet Corr. Allow	v. (Tubeside)	kl := 15.375∙in	k=0.6 for sp	ans between Tub	esheets
c _s := 0.0 ⋅ in	Tubesheet Corr. Allow	v. (Shellside)		k=0.8 for sp	ans between TS/s	support plate
h _g := 0.0∙in	Groove depth			l unsup	ported tube span	port plates
SS := "NO" SS =	"YES" for Tubesheet ca	alculated as Si	imply Supported	acc. to UHX-	13.9 (config. a,b,c	only)
Shell Da	ata (from Fig.UHX-14.1)		C	hannel Data (from Fig.UHX-14.1)
SHELL dat used for Flo	ta which are not pating Tubesheet		(for	CHANNEL data	which are differer Floating Tubeshe	it ets
$D_s := 0 \cdot in$ Sh	ell ID		$D_c := 0 \cdot in$	Channe	el ID	
$t_s := 0.1 \cdot in$ Sh	ell Thickness away fror	n TS	$t_c := 0.1 \cdot in$	Channe	el Thickness	
$G_s := 0 \cdot in$ Sh	ell Gasket Diameter		G _c := 26.49	6-in Channe	el Gasket Diamete	er
G ₁ := 0 · in Sh	ell Contact mid-point T	S/Flange	G ₁ := 26.49	6.in Channe	el Contact mid-po	int TS/Flange
C _s := 0.∙in Sh	ell Corrosion Allowanc	e	C _c := 0.0 ⋅ in	Channe	el Corrosion Allov	vance
			CHAN	:= "CYL" "CY	L" for Cylindrica	I Channel
, Co	rroded length:			10		icai channel
$L_{t} := L_{t} - 2$	$L_t = 256.000 \text{ in}$			Corroded thi	icknesses:	
Corro	ded diameters:	h	$:= h - c_s - c_t$	i upesneet thi	CKNESS	h = 1.750 in
$D_{S} := D_{S} + 2 \cdot C_{S}$	Shell ID $D_s = 0.00$	0 in t _s	$:= t_s - C_s$	Shell Thickne	ss away from TS	$t_{s} = 0.100 \text{ in}$
$D_C := D_C + 2 \cdot C_C$	Channel ID $D_c = 0.00$	0 in t _c	$:= t_c - C_c$	Channel Thicl	kness	$t_{c} = 0.100 \text{ in}$

ExampleE4.18.8-FLOATING TS(AnnexX) from PTB4

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2 - Design (D) and Operating (O) PRESSURES data (from UHX-14.3)

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Maximum and Minimum DESIGN PRESSURES (b)OPERATING PRESSURES (c) for Operating Condition X
$$P_{sD_{a},max} := 250.0 \frac{lb}{ln^2}$$
maximum Shellside Design Pressure $P_{sD_{a},max} := 250.0 \frac{lb}{ln^2}$ Shellside Operating Pressure $P_{sD_{a},max} := 0.0 \frac{lb}{ln^2}$ maximum Tubeside Design Pressure $P_{o_{a},x} := 250.0 \frac{lb}{ln^2}$ Tubeside Operating Pressure $P_{D_{a},max} := 150.0 \frac{lb}{ln^2}$ maximum Tubeside Design Pressure $P_{O_{a},x} := 150.0 \frac{lb}{ln^2}$ Tubeside Operating Pressure $P_{D_{a},max} := 0.0 \frac{lb}{ln^2}$ minimum Tubeside Design Pressure $P_{O_{a},x} := 150.0 \frac{lb}{ln^2}$ Tubeside Operating Pressure $P_{D_{a},max} := 0.0 \frac{lb}{ln^2}$ minimum Tubeside Design Pressure $P_{O_{a},x} := 150.0 \frac{lb}{ln^2}$ Tubeside Operating Pressure $P_{B_{a},max} := 0.0 \frac{lb}{ln^2}$ minimum Tubeside Design Pressure $P_{O_{a},x} := 150.0 \frac{lb}{ln^2}$ Tubeside Operating Pressure $P_{B_{a},max} := 0.0 \frac{lb}{ln^2}$ minimum Tubeside Design Pressure $P_{D_{a},max} := \frac{lb}{lb} = \frac{lb}{lb} = \frac{lb}{lb}$ $P_{D_{a},max} := \frac{lb}{lb} = \frac{lb}{lb} =$

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ExampleE4.18.8-FLOATING TS(AnnexX) from PTB4

 $T_D := 70.0 \cdot degF$

 $T_{tD} := 70.0 \cdot degF$

 $T_{sD} := 70.0 \cdot degF$

 $T_{cD} := 70.0 \cdot degF$

Tubesheet

Tubes

Shell

Channel

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set Farenheit temp.

degF := R

3 - TEMPERATURE Data for Design (D) and Operating (O) conditions CHANNEL data which are different for Stationary and Floating Tubesheets SHELL data which are not used for Floating Tubesheet **Design conditions** Operating conditions $T_{O_x} := 70.0 \cdot degF$ TS Oper.Temp. for Oper. Cond. x **Tubesheet Design Temp.** Tube Oper. Temp. for Oper. Cond. x Tube Design Temp. $T_{tO x} := 70.0 \cdot degF$ Shell Design Temp. Shell Oper.Temp. for Oper. Cond. x $T_{sO x} := 70.0 \cdot degF$ Channel Design Temp. Channel Oper.Temp. for Oper. Cond. x T_{cO} x := 70.0 degF T_a := 70 degF Ambient temperature Additional Data for Radial Thermal Expansion from UHX-13.8.4 (if required)

Tubesheet	$T'_x := 70.0 \cdot degF$	TS temp.@ rim	$T':=T'_{X}$	$T'_{x} = 70.0 degF$
Shell	$T'_{sx} := 70.0 \cdot degF$	Shell temp. @ Tubesheet	$T'_{s} := T'_{sx}$	$T'_{s} = 70.0 degF$
Channel	$T'_{cx} := 70.0 \cdot degF$	Channel temp.@ tubesheet	$T'_{C} := T'_{CX}$	$T'_c = 70.0 degF$

MATERIAL Data Δ

TUBESHEET Material is SA-516/gr70

$S_D := 19000.0 \cdot \frac{lb}{in^2}$	TS allowable stress @ T_D	$S_{PS} := 0 \cdot \frac{lb}{in^2}$	TS allowable P+S stress @ T_{0_x}
$S_a := 20000.0 \cdot \frac{lb}{in^2}$	TS extension allowable stress @ TS ext	tension design temp	erature
$E_D \coloneqq 27 \cdot 10^6 \cdot \frac{lb}{in^2}$	TS elastic modulus @ T _D	$E_{O} \coloneqq 27 \cdot 10^6 \cdot \frac{lb}{in^2}$	TS elastic modulus @ T _{O_x}
$\nu := 0.32$	TS Poisson's ratio	$\alpha' \coloneqq 0 \cdot \frac{\text{in}}{\text{in} \cdot \text{degF}}$	TS coeff. expansion@ rim
	TUBE	Material is SA-214 w	/elded
$S_{tD} \coloneqq 13350 \cdot \frac{lb}{in^2}$	Tube allowable stress @ T _{tD}	$S_{tO} \coloneqq 13350 \cdot \frac{lb}{in^2}$	Tube allowable stress @ T _{tO_x}
$E_{tD} \coloneqq 27 \cdot 10^6 \cdot \frac{lb}{in^2}$	Tube elastic modulus @ T _{tD}	$E_{tO} \coloneqq 27 \cdot 10^6 \cdot \frac{lb}{in^2}$	Tube elastic modulus@ T _{tO_x}
$S_{ytD} \coloneqq 20550 \cdot \frac{lb}{in^2}$	Tube yield stress @ T _{tD}	$S_{ytO} := 20550 \cdot \frac{lb}{in^2}$	Tube yield stress @ T _{tO_x}
$S_{tT} \coloneqq 13350 \cdot \frac{\text{lb}}{\text{in}^2}$	Tube allowable stress @ T _D	$E_{tT} \coloneqq 27 \cdot 10^6 \cdot \frac{lb}{in^2}$	Tube elastic modulus @ T _{tD}
v _t := 0.32	Tube Poisson's ratio		

ExampleE4.18.8-FLOATING TS(AnnexX) from PTB4

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CHANNEL Material is SA-516/grade70

CHANNEL data which are different for Stationary and Floating Tubesheets



ExampleE4.18.8-FLOATING TS(AnnexX) from PTB4

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4 - Flange Design and Operating BOLT LOADS data (from Table UHX-8.1)

Maximum and Minimum Flange DESIGN BOLT LOADS Flange BOLT LOADS for GASKET SEATING Condition

$W_{m1s} := 26225 {\cdot} \text{lb}$	Shell flange Design bolt load	$W_{\rm S}:=26225{\cdot}{\rm lb}$	Shell flange bolt load for Gasket Seating
W _{m1c} := 26225⋅lb	Channel flange Design bolt load	$W_c := 26225 \cdot lb$	Channel flange bolt load for Gasket Seating

 $W_{m1max} := max \Big(W_{m1s}, W_{m1c} \Big) \qquad W_{m1max} = 26225.0 \, \text{lb}$

 $W_{max} := max(W_s, W_c)$ $W_{max} = 26225.0 \, lb$

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Determination of EFFECTIVE BOLT LOAD W* for each Configuration a , b , c , d

Config	uration	a	Config	uration b	Config	uration c	Con	figuration o	d Config	juration e
	(0.0·lb)			(W _{m1c})		(W _{m1c})		(W _{m1c})		(0.0·lb)
	0.0·lb			0.0·lb		0.0·lb		W _{m1s}		W _{m1s}
	0.0·lb			W _{m1c}		W _{m1c}		W _{m1max}		W _{m1s}
W*a :=	0.0·lb		0.0·lb		0.0·lb		0.0·lb		0.0·lb	
·· a·	0.0·lb		W* _b :=	W _c	W* _c :=	W _c	W* _d :=	W _{max}	W* _e :=	Ws
	0.0·lb			W _c		W _c		W _{max}		Ws
	0.0·lb			W _c		w _c		Wmax		W _s
	(0.0·lb)			W _c		W _c		Wmax /		W _s
W* _A	:= W* _a		\۸/*	· \\/*	\۸/*	. \\/*			\\/*	. \\/*
W* _D	:= W* _a		vv B	.= vv b	vv C	;.= VV _C		26225.000	vv f	.= vv e
	\ \/ * ·	\\/* ;f	Config	- "a" v Co	ofia – "A" v C	onfia – "D"		0.000		
	vv		Connig		iiig = A ∨ C			26225.000		
		vv _{°b} if	Config	= "b" ∨ Coi	ntig = "B"		14/4	0.000		
		W* _c if	Config	= "c" ∨ Cor	nfig = "C"		VV* =	26225.000	di	
		W* _d if	Config	= "d"				26225.000		
		W* _e if	Config	= "e" ∨ Cor	nfig = "f"			26225.000		
								26225.000	J	

Minimum required thickness h, of the TS flanged extension (from UHX-9)

For flanged Configurat	ions b , d (extended as a flange) , e	For unflanged Config.c , f	For unflanged Config.d, C
from	UHX-9.5a	See UHX-9.5b	See UHX-9.5c
$h_{\rm G} := \frac{{\rm C} - {\rm G}_{\rm c}}{2} \qquad {\rm Gas}$	ket moment arm $h_G = 0.748 in$		
$h_{rD} := \sqrt{\frac{1.9W_{m1c}}{S_D \cdot G_c}} \cdot h_G$	$h_{rD} = 0.272 \text{ in}$		
$h_{rG} := \sqrt{\frac{1.9W_c}{S_a \cdot G_c} \cdot h_G}$	$h_{rG} = 0.265 \text{ in}$		
$\textbf{h}_r := max \big(\textbf{h}_{rD} , \textbf{h}_{rG} \big)$	$h_{r} = 0.272 \text{ in}$		

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ExampleE4.18.8-FLOATING TS(AnnexX) from PTB4

Start of Calculations

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 $L = effective length of shell/tubes \qquad L := L_t - 2 \cdot h \qquad L = 252.500 \text{ in}$ $D_o = equivalent \text{ diameter of outer tubes } D_o := 2 \cdot r_o + d_t \qquad D_o = 25.750 \text{ in} \qquad a_o := \frac{D_o}{2} \qquad a_o = 12.875 \text{ in}$

<u>UHX-14.5.1</u> Step 1 <u>Determine D₀, μ, μ* and h'g from UHX-11.5.1 :</u>

$$\rho := \frac{I_{tx}}{h} \quad \rho = 0.800 \qquad d^* := \max \left[d_t - 2 \cdot t_t \cdot \left(\frac{E_{tT}}{E_D} \right) \cdot \left(\frac{S_{tT}}{S_D} \right) \cdot \rho , \left(d_t - 2t_t \right) \right] \qquad d^* = 0.657 \text{ in}$$

$$p^* := \frac{p}{\sqrt{1 - \frac{4 \cdot \min(A_L, 4D_0 \cdot p)}{\pi \cdot D_0^2}}} \qquad p^* = 1.068 \text{ in}$$

$$\mu^* := \frac{p^* - d^*}{p^*} \quad \mu^* = 0.385 \qquad \mu := \frac{p - d_t}{p} \quad \mu = 0.250$$

Chan Rad. Dim.

$$a_{c} := \begin{bmatrix} \frac{D_{c}}{2} & \text{if } Config = "a" \lor Config = "e" \lor Config = "f" \lor Config = "A" \\ \frac{G_{c}}{2} & \text{if } Config = "b" \lor Config = "c" \lor Config = "d" \lor Config = "B" \lor Config = "C" \quad a_{c} = 13.248 \text{ in} \\ \frac{A}{2} & \text{if } Config = "D" \end{bmatrix}$$

Shell Radial Dim.

$$a_{s} := \begin{bmatrix} D_{s} \\ 2 \end{bmatrix} \text{ if } Config = "a" \lor Config = "b" \lor Config = "c" \\ \frac{G_{s}}{2} \quad \text{if } Config = "d" \lor Config = "e" \lor Config = "f" \\ a_{c} \quad \text{if } Config = "A" \lor Config = "B" \lor Config = "C" \lor Config = "D" \\ \end{bmatrix}$$

$$\rho_{s} := \frac{a_{s}}{a_{0}} \quad \rho_{s} = 1.029 \qquad \qquad x_{s} := 1 - N_{t} \cdot \left(\frac{d_{t}}{2 \cdot a_{0}}\right)^{2} \quad x_{s} = 0.605$$

$$\rho_{c} := \frac{a_{c}}{a_{0}} \quad \rho_{c} = 1.029 \qquad \qquad x_{t} := 1 - N_{t} \cdot \left(\frac{d_{t} - 2 \cdot t_{t}}{2 \cdot a_{0}}\right)^{2} \quad x_{t} = 0.760$$

ExampleE4.18.8-FLOATING TS(AnnexX) from PTB4

ExampleE4.18.8-FLOATING TS(AnnexX) from PTB4

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(Config = "a" ∨ Config = "b" ∨ Config = "c")

UHX-14.5.2 Step 2 Calculate shell and channel parameters:

$$\beta_{c} := \frac{\left[12 \cdot \left(1 - v_{c}^{2}\right)\right]^{0.25}}{\left[\left(D_{c} + t_{c}\right) \cdot t_{c}\right]^{0.5}} \quad \text{if } SS = "NO" \land (Config = "a" \lor Config = "e" \lor Config = "f" \lor Config = "A")} \\ \frac{0 \cdot 1}{1} \quad \text{otherwise}$$

$$\begin{split} k_{s} &\coloneqq \beta_{s} \cdot \frac{E_{sD} \cdot t_{s}^{3}}{6 \cdot \left(1 - v_{s}^{2}\right)} & k_{s} = 0 \text{ lb} \\ n_{s} &\coloneqq n \cdot \beta_{s} \\ h_{s}^{\prime} &\coloneqq n \cdot \beta_{c} \\ h_{s}^{\prime} &\mapsto n \cdot \beta_{c} \\ h_{s}^{\prime} &\coloneqq n \cdot \beta_{c} \\ h_{s}^{\prime} &\mapsto n \cdot \beta_{c} \\ h_{s}^{\prime} &\coloneqq n \cdot \beta_{c} \\ h_{s}^{\prime} &\mapsto n \cdot \beta_{c} \\ h_{s}^{\prime} &\coloneqq n \cdot \beta_{c} \\ h_{s}^{\prime} &\coloneqq n \cdot \beta_{c} \\ h_{s}^{\prime} &\mapsto n \cdot \beta_{c} \\ h_{s}^{\prime} &\coloneqq n \cdot \beta_{c} \\ h_{s}^{\prime} &\mapsto n \cdot \beta_{c} \\ h_{s}^{\prime} &\mapsto$$

$$SS = "NO" \qquad CHAN = "CYL"$$

$$\delta_{S} := \begin{bmatrix} \frac{D_{s}^{2}}{4E_{sD} \cdot t_{s}} \left(1 - \frac{v_{s}}{2}\right) & \text{if } SS = "NO" \land (Config = "a" \lor Config = "b" \lor Config = "c") \\ 0 \cdot \left(\ln^{3} lb^{-1}\right) & \text{otherwise} \end{bmatrix} \qquad \delta_{s} = 0 \times 10^{0} \ln^{3} lb^{-1}$$

$$\delta_{s} = 0 \times 10^{0} \ln^{3} lb^{-1}$$

$$\frac{D_{c}^{2}}{4E_{cD} \cdot t_{c}} \left(1 - \frac{v_{c}}{2}\right) & \text{if } SS = "NO" \land (Config = "a" \lor Config = "e" \lor Config = "f" \lor Config = "A") \\ \frac{D_{c}^{2}}{4E_{cD} \cdot t_{c}} \left(\frac{1 - v_{c}}{2}\right) & \text{if } SS = "NO" \land (ChAN = "HEMI" \land (Config = "a" \lor Config = "b" \lor Config = "c") \\ 0 \cdot \left(\ln^{3} lb^{-1}\right) & \text{otherwise} \end{bmatrix} \qquad \delta_{c} = 0 \times 10^{0} \ln^{3} lb^{-1}$$

 δ_{c} :=

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 $\beta_{s} = 0.000$

 $\beta_{c} = 0.000$

ExampleE4.18.8-FLOATING TS(AnnexX) from PTB4

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UHX-14.5.3. Step 3 Determine E'/E and v* relative to h/p from UHX-11.5.2. Calculate X₁

$$\frac{h}{p} = 1.750 \quad \mu^{*} = 0.385 \qquad \frac{E^{*}}{E_{D}} = 0.4039 \quad E^{*} = 10905320.515 \frac{h}{h^{2}} \quad v^{*} = 0.3084 \quad (From right pages above) \\
x_{n} := \left[24 \left(1 - v^{2} \right) N_{1} \cdot \frac{E_{10} \cdot v \left((4 - t_{1}) \cdot a_{0}^{2} \right)^{2}}{E^{*} \cdot L \cdot h^{3}} \right]^{25} \quad X_{n} = 3.610 \\
UHX-14.5.4 \quad Step 4 \quad Calculate diameter ratio K and coefficient 0
K := $\frac{A}{D_{0}} \qquad K = 1.044 \\
F := \frac{1 - v^{*}}{E^{*}} \cdot (\lambda_{0} + \lambda_{0} + E_{D} \cdot ln(K)) \qquad F = 0.074 \\
\phi := (1 + v^{*}) \cdot F \qquad \phi = 0.097 \\
Calculate Z_{n} \cdot Z_{n} \cdot Z_{n} \qquad N := \operatorname{amond} \left(4 + \frac{X_{0}}{2} \right) + 1 \quad N = 7 \\
\text{ber}_{x}(x) := \sum_{n=0}^{N} \left[\left(-1 \right)^{n} \frac{\left(\frac{x}{2} \right)^{4n}}{\left((2 \cdot n)! \right)^{2}} \right] \qquad \text{ber} := \operatorname{ber}_{x}(X_{0}) \qquad \operatorname{bel}_{x}(x) := \sum_{n=1}^{N} \frac{\left(-1 \right)^{n-1} \cdot \left(\frac{x}{2} \right)^{4n-2}}{\left((2 \cdot n)! \right)^{2}} \right] \qquad \text{ber} := \operatorname{ber}_{x}(X_{0}) \\
\text{ber}_{x}(x) := \sum_{n=1}^{N} \frac{\left(-1 \right)^{n} \cdot \left(\frac{x}{2} \right)^{4n-1}}{\left((2 \cdot n)! \right)^{2}} \qquad \text{ber} := \operatorname{ber}_{x}(X_{0}) \qquad \operatorname{bel}_{x}(x) := \sum_{n=1}^{N} \frac{\left(-1 \right)^{n-1} \cdot \left((2 \cdot n - 1) \right) \left(\frac{x}{2} \right)^{4n-3}}{\left((2 \cdot n - 1)! \right)^{2}} \qquad \text{ber} := \operatorname{bel}_{x}(X_{0}) \\
\text{ber}_{x}(x) := \sum_{n=1}^{N} \frac{\left(-1 \right)^{n} \cdot \left(\frac{x}{2} \right)^{4n-1}}{\left((2 \cdot n)! \right)^{2}} \qquad \operatorname{ber} := \operatorname{ber}_{x}(X_{0}) \qquad \operatorname{bel}_{x}(x) := \sum_{n=1}^{N} \frac{\left(-1 \right)^{n-1} \cdot \left((2 \cdot n - 1)! \left(\frac{x}{2} \right)^{4n-3}}{\left((2 \cdot n - 1)! \right)^{2}} \qquad \operatorname{ber} := \operatorname{bel}_{x}(X_{0}) \\ \text{ber}_{x}(x) := \sum_{n=1}^{N} \frac{\left(-1 \right)^{n} \cdot \left(\frac{x}{2} \right)^{4n-3}}{\left((2 \cdot n)! \right)^{2}} \qquad \operatorname{ber} := \operatorname{bel}_{x}(X_{0}) \\ \text{ber}_{x}(x) := \operatorname{bel}_{x}(x) + \left(\frac{1 - v^{*}}{x} \right) \operatorname{ber}_{x}(x) \qquad \Psi_{1} := \Psi_{1}(X_{0}) \\ \Psi_{1}(x) := \operatorname{bel}_{x}(x) + \left(\frac{1 - v^{*}}{x} \right) \operatorname{ber}_{x}(x) \qquad \Psi_{1} := \Psi_{1}(X_{0}) \\ \Psi_{1}(x) := \operatorname{bel}_{x}(x) + \left(\frac{1 - v^{*}}{x} \right) \operatorname{ber}_{x}(x) \qquad \Psi_{1} := \Psi_{1}(X_{0}) \\ \Psi_{1}(x) := \operatorname{bel}_{x}(x) + \left(\frac{1 - v^{*}}{x} \right) \operatorname{ber}_{x}(x) \qquad \Psi_{1} := \Psi_{1}(X_{0}) \\ \Psi_{2}(x) := \operatorname{bel}_{x}(x) = \frac{1 - v^{*}}{x} \cdot \operatorname{bel}_{x}(x) \qquad \Psi_{2} := \Psi_{2}(X_{0}) \\ Z_{n} := \frac{\operatorname{bel}_{x}^{2} + \operatorname{bel}_{x}^{2} \\$$$

Calculate
$$Q_1 := \frac{\rho_s - 1 - \Phi \cdot Z_v}{1 + \Phi \cdot Z_m}$$
 $Q_1 = 0.020$

ExampleE4.18.8-FLOATING TS(AnnexX) from PTB4

ExampleE4.18.8-FLOATING TS(AnnexX) from PTB4

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UHX-14.5.5 Step 5

a) Calculate parameters $\omega_{s}^{}, \omega_{s}^{*}, \omega_{c}^{}, \omega_{c}^{*}$,

b) Calculate parameter $\gamma_{\rm b}$:

$$\begin{split} \gamma_{b} &\coloneqq 0 \quad \text{if } Config = \text{"a"} \lor Config = \text{"A"} \lor Config = \text{"D"} \\ & \frac{G_{c} - C}{D_{0}} \quad \text{if } Config = \text{"b"} \lor Config = \text{"B"} \\ & \frac{G_{c} - G_{1}}{D_{0}} \quad \text{if } Config = \text{"c"} \lor Config = \text{"C"} \\ & \frac{G_{c} - G_{s}}{D_{0}} \quad \text{if } Config = \text{"d"} \\ & \frac{C - G_{s}}{D_{0}} \quad \text{if } Config = \text{"e"} \\ & \frac{G_{1} - G_{s}}{D_{0}} \quad \text{if } Config = \text{"f"} \end{split}$$

ExampleE4.18.8-FLOATING TS(AnnexX) from PTB4

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Calculate P*s and P*c from UHX-14.8, if required by the user (for configurations a,b,or c only)

$$\begin{split} \mathsf{T}_r &\coloneqq \left\{ \begin{array}{l} \frac{\mathsf{T}' + \mathsf{T'}_s + \mathsf{T'}_c}{3} \quad \text{if } \ \ \text{Config} = \texttt{"a"} \\ \\ \frac{\mathsf{T}' + \mathsf{T'}_s}{2} \quad \text{if } \ \ \text{Config} = \texttt{"b"} \lor \text{Config} = \texttt{"c"} \\ \\ \frac{\mathsf{T}' + \mathsf{T'}_c}{2} \quad \text{if } \ \ \text{Config} = \texttt{"e"} \lor \text{Config} = \texttt{"f"} \lor \text{Config} = \texttt{"A"} \\ \\ 0 \cdot \text{degF} \quad \text{otherwise} \\ \end{split} \end{split}$$

$$T^{*}_{s} := \begin{bmatrix} \frac{T'_{s} + T_{r}}{2} & \text{if Config} = "a" \lor \text{Config} = "b" \lor \text{Config} = "c" \\ 0 \text{degF otherwise} \end{bmatrix} T^{*}_{s} = 0.000 \text{ degF}$$

$$T_{c}^{*} := \begin{cases} \frac{T_{c}^{*} + T_{r}}{2} & \text{if } Config = "a" \lor Config = "e" \lor Config = "f" \lor Config = "A" \\ 0 \text{degF} & \text{otherwise} \end{cases} T_{c}^{*} = 0.000 \text{degF}$$

$$P^{*}_{sp1} := \frac{E_{sD} \cdot t_{s}}{a_{s}} \cdot \left[\alpha'_{s} \cdot \left(T^{*}_{s} - 70 \cdot degF\right) - \alpha' \cdot \left(T_{r} - 70 \cdot degF\right) \right] \qquad P^{*}_{sp1} = 0.000 \frac{lb}{ln^{2}}$$

$$P^{*}_{cp1} := \frac{E_{cD} \cdot t_{c}}{a_{c}} \cdot \left[\alpha'_{c} \cdot \left(T^{*}_{c} - 70 \cdot degF\right) - \alpha' \cdot \left(T_{r} - 70 \cdot degF\right) \right] \qquad P^{*}_{cp1} = 0.000 \frac{lb}{ln^{2}}$$

$$P^{*}_{sp} := \begin{bmatrix} 0 \frac{lb}{ln^{2}} & \text{if Config} = "d" \vee \text{Config} = "A" \vee \text{Config} = "B" \vee \text{Config} = "C" \vee \text{Config} = "D" \\ P^{*}_{sp1} & \text{otherwise} \\ P^{*}_{cp} := \begin{bmatrix} 0 \frac{lb}{ln^{2}} & \text{if Config} = "d" \vee \text{Config} = "B" \vee \text{Config} = "C" \vee \text{Config} = "D" \\ P^{*}_{cp1} & \text{otherwise} \\ P^{*}_{cp1} & \text{otherwise} \\ \begin{bmatrix} 0 \cdot \frac{lb}{ln^{2}} \\ P^{*}_{cp1} & \text{otherwise} \\ \end{bmatrix} \qquad \left(\begin{array}{c} 0 \cdot \frac{lb}{ln^{2}} \\ P^{*}_{cp1} & \text{otherwise} \\ \end{bmatrix} \right)$$

$$P^{*}_{s} := \begin{vmatrix} in^{2} \\ 0.\frac{lb}{in^{2}} \\ 0.000$$

ExampleE4.18.8-FLOATING TS(AnnexX) from PTB4

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UHX-14.5.6 Step 6 For each loading case, calculate effective pressure Pe

Calculate $\rm P_{e}$ for Pressure and Operating Loading Cases 1 through 8:

$$\begin{split} \mathsf{P}_{e} &\coloneqq \begin{bmatrix} \left(\mathsf{P}_{s} - \mathsf{P}_{t}\right) \text{ if } \mathsf{ExchType} = "a" \\ \begin{bmatrix} \mathsf{P}_{s} \cdot \left(1 - \rho_{s}^{2}\right) - \mathsf{P}_{t} \end{bmatrix} \text{ if } \mathsf{ExchType} = "b" \\ \begin{bmatrix} \left(\mathsf{P}_{s} - \mathsf{P}_{t}\right) \cdot \left(1 - \rho_{s}^{2}\right) \end{bmatrix} \text{ otherwise} \end{split}$$

PRESSURE DESIGN Loading cases : terms 1,2 3, 4

PRESSURE OPERATING Loading cases : terms 5,6,7, 8



ExampleE4.18.8-FLOATING TS(AnnexX) from PTB4

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UHX-14.5.7 Step 7 Determine Q₂ and Q₃ for Loading Cases 1 through 8:

$$Q_{1} = 0.020 \qquad Q_{2} = \begin{pmatrix} -10.170 \\ 16.950 \\ 6.780 \\ 0.000 \\ -10.170 \\ 16.950 \\ 6.780 \\ 0.000 \end{pmatrix} | b \qquad Q_{3} = \begin{pmatrix} 0.02131 \\ 0.02131 \\ 0.0000 \\ 0.02131 \\ 0.02131 \\ 0.02131 \\ 0.02131 \\ 0.02131 \\ 0.02131 \\ 0.0000 \end{pmatrix}$$

ExampleE4.18.8-FLOATING TS(AnnexX) from PTB4

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otherwise

otherwise
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Determine Coefficient F_m for Load Cases 1 through 8:

$$\begin{split} \textbf{numpoints} &= 20 \qquad \textbf{j} := 1 \dots \textbf{numpoints} \qquad \textbf{X}_{\textbf{j}} := \frac{\textbf{j} \cdot \textbf{1}}{\textbf{numpoints}} \qquad \textbf{x} := X \cdot \textbf{X}_{\textbf{a}} \\ \hline \textbf{Q}_{q}(\textbf{x}) := \frac{\overline{\Psi_{2q}(\textbf{x}) \cdot \Psi_{1} - \Psi_{1x}(\textbf{x}) \cdot \Psi_{2}}{\textbf{X}_{a} \cdot \textbf{Z}_{\textbf{a}}} \qquad \textbf{Q}_{m}(\textbf{x}) := \left[\frac{\Psi_{2q}(\textbf{x}) \cdot \textbf{ber} - \Psi_{1x}(\textbf{x}) \cdot \textbf{ber}}{\textbf{Z}_{\textbf{a}}} \right] \\ F_{mxt}(\textbf{x}) := \left[\begin{array}{c} 0 \quad \text{if } P_{\textbf{e}_{1}} = 0 \\ \frac{Q_{q}(\textbf{x}) + Q_{3} \cdot Q_{m}(\textbf{x})}{2} & \text{otherwise} \end{array} \right] \\ F_{mxz}(\textbf{x}) := \left[\begin{array}{c} 0 \quad \text{if } P_{\textbf{e}_{2}} = 0 \\ \frac{Q_{q}(\textbf{x}) + Q_{3} \cdot Q_{m}(\textbf{x})}{2} & \text{otherwise} \end{array} \right] \\ F_{mxd}(\textbf{x}) := \left[\begin{array}{c} 0 \quad \text{if } P_{\textbf{e}_{2}} = 0 \\ \frac{Q_{q}(\textbf{x}) + Q_{3} \cdot Q_{m}(\textbf{x})}{2} & \text{otherwise} \end{array} \right] \\ F_{mxd}(\textbf{x}) := \left[\begin{array}{c} 0 \quad \text{if } P_{\textbf{e}_{2}} = 0 \\ \frac{Q_{q}(\textbf{x}) + Q_{3} \cdot Q_{m}(\textbf{x})}{2} & \text{otherwise} \end{array} \right] \\ F_{mxd}(\textbf{x}) := \left[\begin{array}{c} 0 \quad \text{if } P_{\textbf{e}_{3}} = 0 \\ \frac{Q_{q}(\textbf{x}) + Q_{3} \cdot Q_{m}(\textbf{x})}{2} & \text{otherwise} \end{array} \right] \\ F_{mxd}(\textbf{x}) := \left[\begin{array}{c} 0 \quad \text{if } P_{\textbf{e}_{3}} = 0 \\ \frac{Q_{q}(\textbf{x}) + Q_{3} \cdot Q_{m}(\textbf{x})}{2} & \text{otherwise} \end{array} \right] \\ F_{mxd}(\textbf{x}) := \left[\begin{array}{c} 0 \quad \text{if } P_{\textbf{e}_{3}} = 0 \\ \frac{Q_{q}(\textbf{x}) + Q_{3} \cdot Q_{m}(\textbf{x})}{2} & \text{otherwise} \end{array} \right] \\ F_{mxd}(\textbf{x}) := \left[\begin{array}{c} 0 \quad \text{if } P_{\textbf{e}_{3}} = 0 \land |P^{*}\textbf{c}| = 0 \right] \\ \frac{Q_{q}(\textbf{x}) + Q_{3} \cdot Q_{m}(\textbf{x})}{2} & \text{otherwise} \end{array} \right] \\ F_{mxd}(\textbf{x}) := \left[\begin{array}{c} 0 \quad \text{if } P_{\textbf{e}_{3}} = 0 \land |P^{*}\textbf{c}| = 0 \right] \\ \frac{Q_{q}(\textbf{x}) + Q_{3} \cdot Q_{m}(\textbf{x})}{2} & \text{otherwise} \end{array} \right] \\ F_{mxd}(\textbf{x}) := \left[\begin{array}{c} 0 \quad \text{if } P_{\textbf{e}_{3}} = 0 \land |P^{*}\textbf{c}| = 0 \right] \\ \frac{Q_{q}(\textbf{x}) + Q_{3} \cdot Q_{m}(\textbf{x})}{2} & \text{otherwise} \end{array} \right] \\ F_{mxd}(\textbf{x}) := \left[\begin{array}{c} 0 \quad \text{if } P_{\textbf{e}_{3}} = 0 \land |P^{*}\textbf{c}| = 0 \right] \\ \frac{Q_{q}(\textbf{x}) + Q_{3} \cdot Q_{m}(\textbf{x})}{2} & \text{otherwise} \end{array} \right] \\ F_{mxd}(\textbf{x}) := \left[\begin{array}{c} 0 \quad \text{if } P_{\textbf{e}_{3}} = 0 \land |P^{*}\textbf{c}| = 0 \right] \\ \frac{Q_{q}(\textbf{x}) + Q_{3} \cdot Q_{m}(\textbf{x})}{2} & \text{otherwise} \end{array} \right] \\ F_{mxd}(\textbf{x}) := \left[\begin{array}{c} 0 \quad \text{if } P_{\textbf{e}_{3}} = 0 \land |P^{*}\textbf{c}| = 0 \right] \\ \frac{Q_{q}(\textbf{x}) + Q_{3} \cdot Q_{m}(\textbf{x})}{2} & \text{otherwise} \end{array} \right] \\ F_{mxd}(\textbf{x}) := \left[\begin{array}{c} P_{mxd}(\textbf{x}) = P_{mxd}(\textbf{x}) = P_{mxd}(\textbf{x}) \\ P_{mxd}(\textbf{x}) = P_{$$

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(F_{mx8}(x))

(0.0000)

(0.000)

 $\left| \overrightarrow{|\mathsf{F}_{\mathsf{mx7}}(\mathsf{x})|} \right|$ $\overrightarrow{|\mathsf{F}_{\mathsf{mx8}}(\mathsf{x})|}$

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ExampleE4.18.8-FLOATING TS(AnnexX) from PTB4

Calculate the Maximum Tubesheet Bending Stress

Effective Groove depth
$$h'_g := max(h_g - c_t, 0)$$
 $h'_g = 0.000 \text{ in}$ $h_{min} := \begin{pmatrix} h - h'_g \\ h \\ h \\ h \end{pmatrix}$ $h_{min} = \begin{pmatrix} 1.7500 \\ 1.7500 \\ 1.7500 \\ 1.7500 \\ 1.7500 \\ 1.7500 \\ 1.7500 \\ 1.7500 \\ 1.7500 \\ 1.7500 \\ 1.7500 \\ 1.7500 \\ 1.7500 \\ 1.7500 \end{pmatrix}$

$$\sigma_{1} := \begin{bmatrix} \overline{6 \cdot Q_{2}} \\ \mu^{*}(h_{min_{2}})^{2} \end{bmatrix} \text{ if } P_{e_{1}} = 0 \qquad \sigma_{2} := \begin{bmatrix} \overline{6 \cdot Q_{2}} \\ \mu^{*}(h_{min_{2}})^{2} \end{bmatrix} \text{ if } P_{e_{2}} = 0 \qquad \sigma_{1} = -9488.014 \frac{\text{lb}}{\text{in}^{2}}$$

$$\sigma_{3} := \begin{bmatrix} \overline{6 \cdot Q_{2}} \\ \mu^{*}(h_{min_{3}})^{2} \end{bmatrix} \text{ if } P_{e_{3}} = 0 \qquad \sigma_{4} := \begin{bmatrix} \overline{6 \cdot Q_{2}} \\ \mu^{*}(h_{min_{4}})^{2} \end{bmatrix} \text{ if } P_{e_{4}} = 0 \qquad \sigma_{3} = 6325.343 \frac{\text{lb}}{\text{in}^{2}}$$

$$\sigma_{5} := \begin{bmatrix} \overline{6 \cdot Q_{2}} \\ \mu^{*}(h_{min_{3}})^{2} \end{bmatrix} \text{ if } P_{e_{5}} = 0 \qquad \sigma_{6} := \begin{bmatrix} \overline{6 \cdot Q_{2}} \\ \mu^{*}(h_{min_{4}})^{2} \end{bmatrix} \text{ if } P_{e_{6}} = 0 \qquad \sigma_{5} = 0.000 \frac{\text{lb}}{\text{in}^{2}}$$

$$\sigma_{7} := \begin{bmatrix} \overline{6 \cdot Q_{2}} \\ \mu^{*}(h_{min_{5}})^{2} \end{bmatrix} \text{ if } P_{e_{7}} = 0 \qquad \sigma_{8} := \begin{bmatrix} \overline{6 \cdot Q_{2}} \\ \mu^{*}(h_{min_{6}})^{2} \end{bmatrix} \text{ if } P_{e_{8}} = 0 \qquad \sigma_{6} = 0 \qquad \sigma_{7} = 0.000 \frac{\text{lb}}{\text{in}^{2}}$$

$$\sigma_{7} := \begin{bmatrix} \overline{6 \cdot Q_{2}} \\ \mu^{*}(h_{min_{5}})^{2} \end{bmatrix} \text{ if } P_{e_{7}} = 0 \qquad \sigma_{8} := \begin{bmatrix} \overline{6 \cdot Q_{2}} \\ \mu^{*}(h_{min_{6}})^{2} \end{bmatrix} \text{ if } P_{e_{8}} = 0 \qquad \sigma_{6} = 0 \qquad \sigma_{6} = 0 \qquad \sigma_{6} = 0.000 \frac{\text{lb}}{\text{in}^{2}}$$

$$\sigma_{7} := \begin{bmatrix} \overline{6 \cdot Q_{2}} \\ \mu^{*}(h_{min_{5}})^{2} \end{bmatrix} \text{ otherwise} \qquad \sigma_{8} := \begin{bmatrix} \overline{6 \cdot Q_{2}} \\ \mu^{*}(h_{min_{6}})^{2} \end{bmatrix} \text{ otherwise} \qquad \sigma_{6} = 0.000 \frac{\text{lb}}{\text{in}^{2}}$$

$$\sigma_{7} = 0.000 \frac{\text{lb}}{\text{in}^{2}} \qquad \sigma_{8} = 0 \qquad \sigma_{8} = 0.000 \frac{\text{lb}}{\text{in}^{2}}$$



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lb

in²

0.000)

Pe

TUBESHEET MAXIMUM STRESS for DESIGN LOADING CASES 1, 2, 3, 4

$$\sigma_{D} \coloneqq \max(|\sigma_{1}|, |\sigma_{2}|, |\sigma_{3}|, |\sigma_{4}|) \qquad \sigma_{D} = 15813.4 \frac{\text{lb}}{\text{in}^{2}}$$
$$\sigma_{\text{allow}D} \coloneqq 1.5 \cdot \text{S}_{D} \qquad \sigma_{\text{allow}D} = 28500.0 \frac{\text{lb}}{\text{in}^{2}}$$

TUBESHEET MAXIMUM STRESS For OPERATING LOADING CASES 5,6,7,8

$$\sigma_{O} := \max(|\sigma_{5}|, |\sigma_{6}|, |\sigma_{7}|, |\sigma_{8}|) \qquad \sigma_{O} = 0.0 \frac{lb}{ln^{2}}$$
$$\sigma_{allowO} := S_{PS} \qquad \sigma_{allowO} = 0.0 \frac{lb}{ln^{2}}$$

UHX-13.5.8 Step 8 Calculate the maximum tubesheet shear stress If $|Pe|<3.2S\mu h/D_o$, the TEMA formula $1.6 \cdot S_D \cdot \mu \cdot \frac{h}{a_o} = 1033.010 \frac{lb}{in^2}$ $absP_e := \begin{pmatrix} |Pe_1| \\ |Pe_2| \\ |Pe_3| \\ |Pe_4| \\ |Pe_5| \\ |Pe_6| \\ |Pe_6$

$$\tau_{2010} := \boxed{\left(\frac{1}{2}, \mu\right) \cdot \left(\frac{D_{o}}{2 \cdot h}, P_{e}\right)}{\tau_{max2010} := max(\tau_{2010})}$$

$$\tau_{2010} := \boxed{\left(\frac{1}{2}, \mu\right) \cdot \left(\frac{D_{o}}{2 \cdot h}, P_{e}\right)}{\tau_{max2010} := max(\tau_{2010})}$$

$$\tau_{2010} := \frac{\left(\frac{1}{2}, \mu\right) \cdot \left(\frac{D_{o}}{2 \cdot h}, P_{e}\right)}{\tau_{2010}}$$

$$\tau_{2010} := \frac{\left(\frac{-2207.1}{3678.6}\right)}{1471.4}$$

$$\tau_{2010} := \frac{\left(\frac{-2207.1}{3678.6}\right)}{1471.4}$$

$$\tau_{2010} := \frac{\left(\frac{1}{2}, 2207.1\right)}{16}$$

$$\tau_{2010} := \frac{$$

UHX-13.5.9 Step 9 Determine the minimum and maximum stresses in the tubes for Load Cases 1 to 8

$$Z_{dx}(x) := \frac{\overline{\Psi_2 \cdot \text{ber}_x(x) + \Psi_1 \cdot \text{bei}_x(x)}}{X_a^3 \cdot Z_a} \qquad \qquad Z_{wx}(x) := \frac{\overline{\text{ber}' \cdot \text{ber}_x(x) + \text{bei}' \cdot \text{bei}_x(x)}}{X_a^2 \cdot Z_a}$$

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a) :Determine Coefficients $\mathbf{F}_{t,min}$ and $\mathbf{F}_{t,max}$ for Load Cases 1 to 8:

$$\begin{split} F_{tx1}(x) &:= \begin{vmatrix} Z_{ux}(x) \frac{X_{a}^{4}}{2} & \text{if } P_{e_{\frac{1}{2}}} = 0 \\ (Z_{dx}(x) + O_{3_{\frac{1}{2}}}, Z_{ux}(x)) \frac{X_{a}^{4}}{2} & \text{otherwise} \end{vmatrix} \quad F_{tx5}(x) &:= \begin{vmatrix} Z_{ux}(x) \frac{X_{a}^{4}}{2} & \text{if } P_{e_{\frac{5}{5}}} = 0 \\ (Z_{dx}(x) + O_{3_{\frac{1}{2}}}, Z_{ux}(x)) \frac{X_{a}^{4}}{2} & \text{otherwise} \end{vmatrix} \quad F_{tx6}(x) &:= \begin{vmatrix} Z_{ux}(x) \frac{X_{a}^{4}}{2} & \text{if } P_{e_{\frac{5}{5}}} = 0 \\ (Z_{dx}(x) + O_{3_{\frac{1}{2}}}, Z_{ux}(x)) \frac{X_{a}^{4}}{2} & \text{otherwise} \end{vmatrix} \quad F_{tx6}(x) &:= \begin{vmatrix} Z_{ux}(x) \frac{X_{a}^{4}}{2} & \text{if } P_{e_{\frac{5}{5}}} = 0 \\ (Z_{dx}(x) + O_{3_{\frac{1}{2}}}, Z_{ux}(x)) \frac{X_{a}^{4}}{2} & \text{otherwise} \end{vmatrix} \quad F_{tx7}(x) &:= \begin{vmatrix} Z_{ux}(x) \frac{X_{a}^{4}}{2} & \text{if } P_{e_{\frac{5}{5}}} = 0 \\ (Z_{dx}(x) + O_{3_{\frac{1}{2}}}, Z_{ux}(x)) \frac{X_{a}^{4}}{2} & \text{otherwise} \end{vmatrix} \quad F_{tx7}(x) &:= \begin{vmatrix} Z_{ux}(x) \frac{X_{a}^{4}}{2} & \text{if } P_{e_{\frac{5}{5}}} = 0 \\ (Z_{dx}(x) + O_{3_{\frac{1}{2}}}, Z_{ux}(x)) \frac{X_{a}^{4}}{2} & \text{otherwise} \end{vmatrix} \quad F_{tx7}(x) &:= \begin{vmatrix} Z_{ux}(x) \frac{X_{a}^{4}}{2} & \text{if } P_{e_{\frac{5}{5}}} = 0 \\ (Z_{dx}(x) + O_{3_{\frac{1}{2}}}, Z_{ux}(x)) \frac{X_{a}^{4}}{2} & \text{otherwise} \end{vmatrix} \quad F_{tx7}(x) &:= \begin{vmatrix} Z_{ux}(x) \frac{X_{a}^{4}}{2} & \text{if } P_{e_{\frac{5}{5}}} = 0 \\ (Z_{dx}(x) + O_{3_{\frac{1}{2}}}, Z_{ux}(x)) \frac{X_{a}^{4}}{2} & \text{otherwise} \end{vmatrix} \quad F_{tx7}(x) &:= \begin{vmatrix} Z_{ux}(x) \frac{X_{a}^{4}}{2} & \text{if } P_{e_{\frac{5}{5}}} = 0 \\ (Z_{dx}(x) + O_{3_{\frac{1}{2}}}, Z_{ux}(x)) \frac{X_{a}^{4}}{2} & \text{otherwise} \end{vmatrix} \quad F_{tx6}(x) := \begin{vmatrix} Z_{ux}(x) \frac{X_{a}^{4}}{2} & \text{if } P_{e_{\frac{5}{5}}} = 0 \\ (Z_{dx}(x) + O_{3_{\frac{1}{2}}}, Z_{ux}(x)) \frac{X_{a}^{4}}{2} & \text{otherwise} \end{cases} \quad F_{tx6}(x) := \begin{vmatrix} Z_{ux}(x) \frac{X_{a}^{4}}{2} & \text{if } P_{e_{\frac{5}{5}}} = 0 \\ (Z_{dx}(x) + O_{3_{\frac{1}{2}}}, Z_{ux}(x)) \frac{X_{a}^{4}}{2} & \text{otherwise} \end{cases} \quad F_{tx6}(x) := \begin{vmatrix} Z_{ux}(x) \frac{X_{a}^{4}}{2} & \text{if } P_{e_{\frac{5}{5}}} = 0 \\ (Z_{dx}(x) + O_{3_{\frac{1}{2}}}, Z_{ux}(x)) \frac{X_{a}^{4}}{2} & \text{otherwise} \end{cases} \quad F_{tx6}(x) := \begin{vmatrix} Z_{ux}(x) \frac{X_{a}^{4}}{2} & \text{if } P_{e_{\frac{5}{5}}} = 0 \\ (Z_{dx}(x) + O_{3_{\frac{1}{2}}, Z_{ux}(x)) \frac{X_{a}^{4}}{2} & \text{otherwise} \end{cases} \quad F_{tx6}(x) := \begin{vmatrix} Z_{ux}(x) \frac{X_{a}^{4}}{2} & \text{if } P_{e_{\frac{5}{5}}} = 0 \\ (Z_{dx}(x) + O_{3_{\frac{1}{$$

ExampleE4.18.8-FLOATING TS(AnnexX) from PTB4

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$$\begin{aligned} \textbf{b} \ \textbf{Determine tube stresses } \quad \textbf{g}_{tmin} \text{ and } \quad \textbf{g}_{tmax} \text{ for Load Cases 1 to 8 (continued)} \\ \sigma_{t1_{1}} := \begin{bmatrix} \frac{1}{x_{t} - x_{s}} \left[\left(P_{s_{1}} x_{s} - P_{t_{1}} x_{t} \right) - \frac{2 \cdot Q_{2_{1}}}{a_{o}^{2}} F_{tmin_{1}} \right] \end{bmatrix} \text{ if } P_{e_{1}} = 0 & \sigma_{t1_{1}} = -1432 \cdot 482 \frac{lb}{ln^{2}} \\ \hline \begin{bmatrix} \frac{1}{x_{t} - x_{s}} \left[\left(P_{s_{1}} x_{s} - P_{t_{1}} x_{t} \right) - P_{e_{1}} F_{tmin_{1}} \right] \end{bmatrix} \text{ otherwise} \\ \sigma_{t1_{2}} := \begin{bmatrix} \frac{1}{x_{t} - x_{s}} \left[\left(P_{s_{2}} x_{s} - P_{t_{2}} x_{t} \right) - \frac{2 \cdot Q_{2_{2}}}{a_{o}^{2}} F_{tmin_{2}} \right] \end{bmatrix} \text{ otherwise} \\ \sigma_{t1_{2}} := \begin{bmatrix} \frac{1}{x_{t} - x_{s}} \left[\left(P_{s_{3}} x_{s} - P_{t_{3}} x_{t} \right) - \frac{2 \cdot Q_{2_{3}}}{a_{o}^{2}} F_{tmin_{2}} \right] \end{bmatrix} \text{ otherwise} \\ \sigma_{t1_{3}} := \begin{bmatrix} \frac{1}{x_{t} - x_{s}} \left[\left(P_{s_{3}} x_{s} - P_{t_{3}} x_{t} \right) - \frac{2 \cdot Q_{2_{3}}}{a_{o}^{2}} F_{tmin_{3}} \right] \end{bmatrix} \text{ otherwise} \\ \sigma_{t1_{3}} := \begin{bmatrix} \frac{1}{x_{t} - x_{s}} \left[\left(P_{s_{4}} x_{s} - P_{t_{3}} x_{t} \right) - P_{o_{3}} F_{tmin_{3}} \right] \end{bmatrix} \text{ otherwise} \\ \sigma_{t1_{3}} := \begin{bmatrix} \frac{1}{x_{t} - x_{s}} \left[\left(P_{s_{5}} x_{s} - P_{t_{3}} x_{t} \right) - P_{o_{3}} F_{tmin_{3}} \right] \end{bmatrix} \text{ otherwise} \\ \sigma_{t1_{4}} := \begin{bmatrix} \frac{1}{x_{t} - x_{s}} \left[\left(P_{s_{5}} x_{s} - P_{t_{3}} x_{t} \right) - P_{o_{4}} F_{tmin_{3}} \right] \end{bmatrix} \text{ otherwise} \\ \sigma_{t1_{5}} := \begin{bmatrix} 0 \quad \text{if } \left(|P_{s_{5}}| = 0 \land |P_{s}| = 0 \right) \\ \frac{1}{n_{1}} \frac{1}{x_{t} - x_{s}} \left[\left(P_{s_{5}} x_{s} - P_{t_{5}} x_{t} \right) - P_{o_{5}} F_{tmin_{5}} \right] \end{bmatrix} \text{ otherwise} \\ \sigma_{t1_{5}} := 0 \quad 0 \quad \textbf{if } \left(|P_{s_{5}}| = 0 \land |P_{s}| = 0 \right) \\ \frac{1}{n_{1}} \frac{1}{x_{t} - x_{s}} \left[\left(P_{s_{5}} x_{s} - P_{t_{5}} x_{t} \right) - P_{s_{5}} F_{tmin_{5}} \right] \end{bmatrix} \text{ otherwise} \\ \sigma_{t1_{5}} := 0 \quad 0 \quad \textbf{if } \left(|P_{s_{5}}| = 0 \land |P_{s}| = 0 \right) \\ \frac{1}{n_{1}} \frac{1}{x_{t} - x_{s}} \left[\left(P_{s_{5}} x_{s} - P_{t_{5}} x_{t} \right) - P_{s_{5}} F_{tmin_{5}} \right] \end{bmatrix} \text{ otherwise} \\ \sigma_{t1_{5}} := 0 \quad 0 \quad \textbf{if } \left(|P_{s_{5}}| = 0 \land |P_{s}| = 0 \right) \\ \frac{1}{n_{1}} \frac{1}{x_{t} - x_{s}} \left[\left(P_{s_{5}} x_{s} - P_{t_{5}} x_{t} \right) - P_{s_{5}} F_{tmin_{5}} \right] \end{bmatrix} \text{ otherwise} \\ \sigma_{t1_{5}} := 0 \quad 0 \quad \textbf{if } \left[\frac{1}{n_{1} - x_{s}} \left[\left(P_{s_{5}} x_$$

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$$\begin{split} \sigma_{11_{g}} &:= \left[\begin{array}{c} \frac{1}{x_{1} - x_{5}} \left[\left(P_{s_{g}} x_{5} - P_{t_{g}} x_{1} \right) - \frac{2 \cdot Q_{2_{g}}}{a_{0}^{-2}} F_{min_{g}} \right] \right] \text{ if } P_{e_{g}} = 0 & \sigma_{11_{g}} &:= \left[\begin{array}{c} 0 & \text{if } \left(\left| P^{*}_{s} \right| = 0 \land \left| P^{*}_{c} \right| = 0 \right) \right. \\ \left. \frac{1}{x_{1} - x_{5}} \left[\left(P_{s_{g}} x_{5} - P_{t_{g}} x_{1} \right) - P_{e_{g}} F_{min_{g}} \right] \right] & \text{otherwise} & \sigma_{11_{g}} = 0 \\ \sigma_{01_{g}} &:= \left[\begin{array}{c} \frac{1}{x_{1} - x_{5}} \left[\left(P_{s_{g}} x_{5} - P_{t_{g}} x_{1} \right) - P_{e_{g}} F_{min_{g}} \right] & \text{if } P_{e_{g}} = 0 \\ \left. \frac{1}{x_{1} - x_{5}} \left[\left(P_{s_{g}} x_{5} - P_{t_{g}} x_{1} \right) - P_{e_{g}} F_{min_{g}} \right] & \text{if } P_{e_{g}} = 0 \\ \left. \frac{1}{x_{1} - x_{5}} \left[\left(P_{s_{g}} x_{5} - P_{t_{g}} x_{1} \right) - P_{e_{g}} F_{min_{g}} \right] & \text{if } P_{e_{g}} = 0 \\ \left. \frac{1}{x_{1} - x_{5}} \left[\left(P_{s_{g}} x_{5} - P_{t_{g}} x_{1} \right) - P_{e_{g}} F_{min_{g}} \right] & \text{otherwise} \\ \end{array} \right] & \sigma_{02_{g}} &:= \left[\frac{1}{\frac{1}{x_{1} - x_{5}} \left[\left(P_{s_{g}} x_{5} - P_{t_{g}} x_{1} \right) - P_{e_{g}} F_{max_{g}} \right] & \text{otherwise} \\ \sigma_{02_{g}} &:= \left[\frac{1}{\frac{1}{x_{1} - x_{5}} \left[\left(P_{s_{g}} x_{5} - P_{t_{g}} x_{1} \right) - P_{e_{g}} F_{max_{g}} \right] & \text{otherwise} \\ \sigma_{02_{g}} &:= \left[\frac{1}{\frac{1}{x_{1} - x_{5}} \left[\left(P_{s_{g}} x_{5} - P_{t_{g}} x_{1} \right) - P_{e_{g}} F_{max_{g}} \right] & \text{otherwise} \\ \sigma_{02_{g}} &:= \left[\frac{1}{\frac{1}{x_{1} - x_{5}} \left[\left(P_{s_{g}} x_{5} - P_{t_{g}} x_{1} \right) - P_{e_{g}} F_{max_{g}} \right] & \text{otherwise} \\ \sigma_{02_{g}} &:= \left[\frac{1}{\frac{1}{x_{1} - x_{5}} \left[\left(P_{s_{g}} x_{5} - P_{t_{g}} x_{1} \right) - P_{e_{g}} F_{max_{g}} \right] & \text{otherwise} \\ \sigma_{02_{g}} &:= \left[\frac{1}{\frac{1}{x_{1} - x_{5}} \left[\left(P_{s_{g}} x_{5} - P_{t_{g}} x_{1} \right) - P_{e_{g}} F_{max_{g}} \right] & \text{otherwise} \\ \sigma_{02_{g}} &= 0 & \sigma_{02_{g}} &:= \left[0 & \text{if } \left(|P^{*}_{g}| = 0 \land |P^{*}_{g}| = 0 \right) \\ \frac{1}{\sigma_{02_{g}}} & \frac{1}{\sigma_{0}} F_{max_{g}} \right] & \text{otherwise} \\ \sigma_{02_{g}} &:= \left[\frac{1}{\frac{1}{x_{1} - x_{5}} \left[\left(P_{s_{g}} x_{5} - P_{t_{g}} x_{1} \right) - P_{e_{g}} F_{max_{g}} \right] & \text{otherwise} \\ \sigma_{02_{g}} &= 0 & \sigma_{02_{g}} &:= \left[0 & \text{if } \left(|P^{*}_{g}| = 0 \land |P^{*}_{g}| = 0 \right) \\ \frac{1}{\sigma_{02_{g}}} & \frac{1}{$$

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Step (b) : check the tubes for buckling $I_t := kI \quad I_t = 15.375 \text{ in } \qquad r_t := \frac{\sqrt{d_t^2 + (d_t - 2 \cdot \overline{t_t})^2}}{4} \qquad r_t = 0.238 \text{ in }$ $F_{S1} := \begin{bmatrix} 1.25 & \text{if } P_{e_1} = 0 \end{bmatrix}$ $max \left[3.25 - .25 \cdot \left[\left(Z_d + Q_{3_1} \cdot Z_v \right) \cdot X_a^4 \right], 1.25 \right] \text{ otherwise}$ $F_{S2} := 1.25$ if $P_{e_2} = 0$ $max \left[3.25 - .25 \cdot \left[\left(Z_d + Q_{3_2} \cdot Z_v \right) \cdot X_a^4 \right], 1.25 \right] \text{ otherwise}$ $F_{S3} := 1.25$ if $P_{e_3} = 0$ $max \left[3.25 - .25 \cdot \left[\left(Z_d + Q_{3_3} \cdot Z_v \right) \cdot X_a^4 \right], 1.25 \right] \text{ otherwise}$ $F_{S4} := 1.25$ if $P_{e_4} = 0$ $max \left[3.25 - .25 \cdot \left[\left(Z_d + Q_{3_4} \cdot Z_v \right) \cdot X_a^4 \right], 1.25 \right] \text{ otherwise}$ $F_{S5} := 1.25 \text{ if } P_{e_5} = 0$ $max \left[3.25 - .25 \cdot \left[\left(Z_d + Q_{3_5} \cdot Z_v \right) \cdot X_a^4 \right], 1.25 \right] \text{ otherwise}$ $F_{S6} := 1.25 \text{ if } P_{e_6} = 0$ $max \left[3.25 - .25 \cdot \left[\left(Z_d + Q_{3_6} \cdot Z_v \right) \cdot X_a^4 \right], 1.25 \right] \text{ otherwise}$ $F_{S7} := 1.25 \text{ if } P_{e_7} = 0$ $F_{S8} := \begin{bmatrix} 1.25 & \text{if } P_{e_8} \\ max \begin{bmatrix} 3.25 - .2 \\ 1.25 & \text{if } P_{e_8} \\ max \begin{bmatrix} 3.25 - .2 \\ F_{s} \end{bmatrix} \\ F_{s} := \begin{bmatrix} F_{s1} \\ F_{s2} \\ F_{s3} \\ F_{s4} \\ F_{s5} \end{bmatrix}_{8} \cdot Z_{v} \\ F_{s} = \begin{bmatrix} 1.787 \\ 1.787 \\ 1.787 \\ 1.250 \\ 1.787 \\ 1.787 \\ 1.787 \\ 1.787 \\ 1.787 \\ 1.787 \\ 1.787 \\ 1.787 \\ 1.787 \end{bmatrix} \text{ therwise}$ F_{s6} F_{s7} 1.787 1.787 1.250 F_{s8} / $\sigma_{tmin} =$

kling if
$$\sigma_{t1} < 0$$
 or $\sigma_{t2} < 0$
 $C_t := \sqrt{\frac{2 \cdot \pi^2 \cdot E_{tD}}{S_{ytD}}} \quad C_t = 161.043 \quad F_t := \frac{I_t}{r_t} \quad F_t = 64.699$
 $F_{s1} := si(F_{S1} > 2, 2, F_{S1}) \quad F_{s1} = 1.787$
 $F_{s2} := si(F_{S2} > 2, 2, F_{S2}) \quad F_{s2} = 1.787$
 $F_{s3} := si(F_{S3} > 2, 2, F_{S3}) \quad F_{s3} = 1.787$
 $F_{s4} := si(F_{S4} > 2, 2, F_{S4}) \quad F_{s4} = 1.250$
 $F_{s5} := si(F_{S5} > 2, 2, F_{S5}) \quad F_{s5} = 1.787$
 $F_{s6} := si(F_{S6} > 2, 2, F_{S6}) \quad F_{s6} = 1.787$
 $F_{s7} := si(F_{S7} > 2, 2, F_{S7}) \quad F_{s7} = 1.787$
 $F_{s8} := si(F_{S8} > 2, 2, F_{S8}) \quad F_{s8} = 1.250$
 $S_{tb} := si(C_{t} > F_{t}, \frac{S_{ytD}}{F_{s}} \cdot (1 - \frac{F_{t}}{2 \cdot C_{t}}), \frac{1}{F_{s}} \cdot \pi^2 \cdot \frac{E_{tD}}{F_{t}^2}$



-1432.5

-3728.1

<u>–1641.2</u>

0.0

0.0

0.0

0.0

0.0

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UHX 14.5.10 Step 10 Determine the shell stresses

a) shell membrane stress

$$\sigma_{am} := \begin{bmatrix} \frac{a_o^2}{(D_a + t_b)t_b} \left[P_o + \left(p_o^2 - 1 \right) \cdot \left(P_b - P_b \right) \right] + \frac{a_s^2}{(D_a + t_b)t_b} P_t & \text{if } (Config = "a" \lor Config = "b" \lor Config = "c") \\ 0 \cdot P_s & \text{otherwise} \end{bmatrix}$$

$$\sigma_{am} := \begin{bmatrix} 0.000\\ 0.000\\ 0.000\\ 0.000\\ 0.000\\ 0.000 \end{bmatrix} \frac{lb}{ln^2}$$

$$\sigma_{am} = \begin{bmatrix} 0.000\\ 0.000\\ 0.000\\ 0.000\\ 0.000 \end{bmatrix} \frac{lb}{ln^2}$$

$$\sigma_{anlowD} := S_{aD} \quad \sigma_{anlowD} = 0.0 \frac{lb}{ln^2}$$

$$\sigma_{anlowD} := S_{aD} \quad \sigma_{anlowD} = 0.0 \frac{lb}{ln^2}$$

$$\sigma_{anlowD} := S_{aD} \quad \sigma_{anlowD} = 0.0 \frac{lb}{ln^2}$$

$$\sigma_{anlowD} := max \left(\left| \sigma_{am}_1 \right| \cdot \left| \sigma_{am}_2 \right| \cdot \left| \sigma_{am}_3 \right| \cdot \left| \sigma_{am}_4 \right| \right) \quad \sigma_{amD} = 0 \frac{lb}{ln^2} \quad \sigma_{anlowD} := S_{aD} \quad \sigma_{anlowD} = 0.0 \frac{lb}{ln^2}$$

$$shell haximum membrane stress for OPERATINE LOADING CASES 1, 2, 3, 4$$

$$\sigma_{amD} := max \left(\left| \sigma_{am}_1 \right| \cdot \left| \sigma_{am}_2 \right| \cdot \left| \sigma_{am}_3 \right| \cdot \left| \sigma_{am}_3 \right| \right| \cdot \left| \sigma_{am}_3 \right| \cdot \left| \sigma_{am}_3 \right| \right| \quad \sigma_{am} = 0.00 \frac{lb}{ln^2}$$

$$\sigma_{anlowO} := S_{PS} \quad \sigma_{anlowO} - 0.0 \frac{lb}{ln^2}$$

$$\sigma_{anlowO} := S_{anlowO} := S_{anno} := S_{anno} := S_{anno}$$

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b) channel bending stress:

UHX 14.5.10 Step 10 Determine the channel stresses

a) channel membrane stress:

$$\sigma_{cm} \coloneqq \left[\frac{a_c^2}{\left(D_c + t_c\right) \cdot t_c} \cdot P_t \text{ if } Config = "a" \lor Config = "e" \lor Config = "f" \lor Config = "A"} \right]$$

$$\sigma_{cm} = \left[\frac{a_c^2}{\left(D_c + t_c\right) \cdot t_c} \cdot P_t \text{ if } Config = "a" \lor Config = "e" \lor Config = "f" \lor Config = "A"} \right]$$

b) channel bending stress:

$$I_{cmin} := 1.8 \cdot \sqrt{D_c \cdot t_c} \qquad I_{cmin} = 0.000 \text{ in}$$

$$\sigma_{cb} := \frac{6}{t_c^{-2}} \cdot K_c \left[\beta_c \cdot \left(\delta_c \cdot P_t + \frac{a_c^2}{E_{cD} \cdot t_c} P^*_c \right) - \frac{6 \cdot (1 - v^{*2})}{E^*} \cdot \left(\frac{a_0^{-3}}{h^3} \right) \cdot \left(1 + \frac{h'_c}{2} \right) \cdot \left[P_e \cdot (Z_v + Z_m \cdot Q_1) + \frac{2}{a_0^{-2}} \cdot Z_m \cdot Q_2 \right] \right]$$

$$\sigma_{cb} = \begin{pmatrix} 0.000 \\ 0.000$$

CHANNEL MAXIMUM STRESS for DESIGN LOADING CASES 1, 2, 3, 4

 $\sigma_{cmaxD} \coloneqq max \left(\sigma_{c_1}, \sigma_{c_2}, \sigma_{c_3}, \sigma_{c_4} \right) \qquad \sigma_{cmaxD} = 0.000 \frac{lb}{in^2}$ σ_{callowD} := 1.5·S_{cD} σ_{callowD} = 0.0 in CHANNEL MAXIMUM STRESS for OPERATING LOADING CASES 5,6,7,8

$$\sigma_{cmaxO} := max \left(\sigma_{c_5}, \sigma_{c_6}, \sigma_{c_7}, \sigma_{c_8} \right) \qquad \sigma_{cmaxO} = 0.000 \frac{lb}{in^2} \qquad \sigma_{callowO} := S_{PSc} \qquad \sigma_{callowO} = 0.0 \frac{lb}{in^2}$$

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_UHX13.7 Simplified Elastic Plastic Procedure

Calculation procedure for the effect of plasticity at the tubesheet, channel or shell joint.

This procedure applies only to Configurations a,b,c and Design Loading Cases 1, 2, 3, 4 in the following conditions: -for integral shell (config. a,b,c) when $1.5S_s < \sigma_s \le S_{PS,s}$ -for integral channel (config. a) when $1.5S_c < \sigma_c \le S_{PS,c}$

$$\begin{split} \mathbf{S}^{*}_{s} &:= \min \left(\mathbf{S}_{ysD}, \frac{\mathbf{S}_{PSc}}{2} \right) \\ \mathbf{S}^{*}_{s} &:= \min \left(\mathbf{S}_{ycD}, \frac{\mathbf{S}_{PSc}}{2} \right) \\ \mathbf{S}^{*}_{s} &= 0.000 \frac{lb}{in^{2}} \\ \mathbf{fact}_{sv} &:= \overbrace{\left(1.4 - \frac{0.4 \cdot \left| \sigma_{sb} \right|}{\mathbf{S}^{*}_{s}} \right)}^{\mathsf{fact}_{sv}} \quad \mathbf{fact}_{sv} = \left(\begin{array}{c} 1.400 \\ 1.600 \\ 1.000 \\$$

Calculate reduced values of E_s and E_c for each loading case:

$$E^{*}{}_{sD} := E_{sD} \cdot fact_{s} \quad E^{*}{}_{sD} = \begin{pmatrix} 1.0000 \times 10^{0} \\ 1.0000 \times 10^{0} \\ 1.0000 \times 10^{0} \\ 1.0000 \times 10^{0} \\ 1.0000 \times 10^{0} \end{pmatrix} \qquad E^{*}{}_{cD} := E_{cD} \cdot fact_{c} \quad E^{*}{}_{cD} = \begin{pmatrix} 1.000 \times 10^{0} \\ 1.000 \times 10^{0} \\ 1.000 \times 10^{0} \\ 1.000 \times 10^{0} \end{pmatrix}$$
From Step 2, recalculate:
$$E^{*}{}_{cD} := E_{cD} \cdot fact_{c} \quad E^{*}{}_{cD} = \begin{pmatrix} 1.000 \times 10^{0} \\ 1.000 \times 10^{0} \\ 1.000 \times 10^{0} \end{pmatrix}$$

$$kk_{s} := \beta_{s} \cdot \frac{E_{sD}^{*} \cdot t_{s}^{3}}{6 \cdot \left(1 - v_{s}^{2}\right)} \qquad kk_{s} = \begin{vmatrix} 0.000 \\ 0.0000 \\ 0.0000 \end{vmatrix} lb \qquad \qquad \lambda\lambda_{s} := \frac{6 \cdot D_{s}}{h^{3}} \cdot kk_{s} \cdot \left(1 + h'_{s} + \frac{h'_{s}^{2}}{2}\right) \qquad \lambda\lambda_{s} = \begin{vmatrix} 0.000 \\ 0.0000 \\ 0.0000 \end{vmatrix} \frac{lb}{ln^{2}}$$

$$kk_{c} := \beta_{c} \cdot \frac{E_{cD}^{*} \cdot t_{c}^{3}}{6 \cdot \left(1 - v_{c}^{2}\right)} \qquad kk_{c} = \begin{pmatrix} 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \end{pmatrix} \qquad \qquad \lambda\lambda_{c} := \frac{6 \cdot D_{c}}{h^{3}} \cdot kk_{c} \cdot \left(1 + h'_{c} + \frac{h'_{c}^{2}}{2}\right) \qquad \lambda\lambda_{c} = \begin{pmatrix} 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \end{pmatrix} \frac{|b|}{|a|^{2}}$$

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$$F := \frac{1 - v^{*}}{E^{*}} \cdot \left(\lambda \lambda_{s} + \lambda \lambda_{c} + E_{D} \cdot \ln(K) \right) \qquad F = \begin{pmatrix} 0.074 \\ 0.074 \\ 0.074 \\ 0.074 \\ 0.074 \end{pmatrix} \qquad \Phi := (1 + v^{*}) \cdot F \qquad \Phi = \begin{pmatrix} 0.097 \\ 0.097 \\ 0.097 \\ 0.097 \\ 0.097 \end{pmatrix}$$

$$Q_{1e} := \overbrace{\left(\frac{\rho_{s} - 1 - \Phi \cdot Z_{v}}{1 + \Phi \cdot Z_{m}}\right)}^{Q_{1e}} \qquad Q_{1e} = \begin{cases} 0.020 \\ 0.020 \\ 0.020 \\ 0.020 \end{pmatrix} \qquad W^{*'} := \begin{cases} W^{*}_{1} \\ W^{*}_{2} \\ W^{*}_{3} \\ W^{*}_{4} \end{pmatrix} \qquad W^{*'} = \begin{cases} W^{*}_{1} \\ W^{*}_{2} \\ W^{*}_{3} \\ W^{*}_{4} \end{pmatrix}$$

$$\mathbf{P}_{se} := \begin{pmatrix} \mathbf{P}_{s_1} \\ \mathbf{P}_{s_2} \\ \mathbf{P}_{s_3} \\ \mathbf{P}_{s_4} \end{pmatrix} \quad \mathbf{P}_{te} := \begin{pmatrix} \mathbf{P}_{t_1} \\ \mathbf{P}_{t_2} \\ \mathbf{P}_{t_3} \\ \mathbf{P}_{t_3} \\ \mathbf{P}_{t_4} \end{pmatrix}$$

$$P'_{e} := \begin{pmatrix} P_{e_{1}} \\ P_{e_{2}} \\ P_{e_{3}} \\ P_{e_{3}} \\ P_{e_{4}} \end{pmatrix} \qquad P'_{e} = \begin{pmatrix} -150.000 \\ 250.000 \\ 100.000 \\ 0.000 \end{pmatrix} \frac{lb}{in^{2}}$$

ExampleE4.18.8-FLOATING TS(AnnexX) from PTB4



(26225.000) 0.000

26225.000 0.000

lb

)

From Step 7 , recalculate $Q_2^{}$, $Q_3^{}$, $F_m^{}$ and the tubesheet bending stress for load cases 1, 2, and 3

$$Q'_{2} := \boxed{\frac{\left(\left(\omega^{*}{}_{s} \cdot P_{se} - \omega^{*}{}_{c} \cdot P_{te} \right) + \frac{\gamma_{b}}{2 \cdot \pi} \cdot W^{*'} \right)}{1 + \Phi \cdot Z_{m}}} \qquad \qquad Q'_{2} = \begin{pmatrix} -10.170 \\ 16.950 \\ 6.780 \\ 0.000 \end{pmatrix} | lb \\ 0.000 \end{pmatrix}$$
$$Q'_{3} := \boxed{\left(Q_{1e} + \frac{2 \cdot Q'_{2}}{P'_{e} \cdot a_{0}^{2}} \right)} \qquad \qquad Q'_{3} = \begin{pmatrix} 0.0213 \\ 0.0213 \\ 0.0213 \\ 0.0205 \end{pmatrix}$$

$$F'_{m2}(x) := \begin{bmatrix} 0 & \text{if } P'_{e_2} = 0 \\ \\ \frac{Q_v(x) + Q'_{3_2} \cdot Q_m(x)}{2} & F'_{m2} := |F'_{m2}(x_j)| \\ \hline F'_{m2} := \max(F'_{m2}) & F'_{m2} = 0.075 \end{bmatrix}$$

$$F'_{m} := \begin{pmatrix} F'_{m1} \\ F'_{m2} \\ F'_{m3} \\ F'_{m4} \end{pmatrix} \qquad F'_{m} = \begin{pmatrix} 0.075 \\ 0.075 \\ 0.075 \\ 0.075 \\ 0 \end{pmatrix}$$

Tubesheet Bending Stress for the Elastic-Plastic Solution

$$\sigma' := \left[\underbrace{\frac{1.5 \cdot F'_{m}}{\mu^{*}} \cdot \left(\frac{2 \cdot a_{o}}{h - h'_{g}}\right)^{2} \cdot P'_{e}}_{\text{add}} \right] \qquad \sigma' = \left[\underbrace{\frac{15813.357}{6325.343}}_{0.000} \right] \frac{|b|}{|n|^{2}} \qquad \sigma'_{max} := max(|\sigma'_{1}|, |\sigma'_{2}|, |\sigma'_{3}|) \qquad \sigma_{allow} := 1.5 \cdot S_{D}$$

$$\sigma'_{\text{max}} = 15813.357 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{\text{allow}} = 28500.0 \frac{\text{lb}}{\text{in}^2}$$

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UHX13.9 Simply Supported Tubesheet Procedure This procedure applies only when the TS is integral with the shell or channel, i.e. : - shell of configurations a, b or c - channel of configuration a To perform this calculation, use the option : SS = "NO"

Calculation must be performed in 2 phases :

Phase 1) Perform Steps 1 to 11 with **SS="NON"** (normal calculation) with the following modifications in Step 11:

- minimum length requirement Ismin of shell band for configurations a,b,c do not apply

- minimum length requirement I_{cmin} of channel band for configuration a do not apply

if $\sigma_s \leq S_{PS,s}$ and $\sigma_c \leq S_{PS,c}$, the shell and/or channel designs are acceptable. Otherwise increase the thickness of the overstressed components (shell and/or channel) and return to Step 1.

Phase 2) Perform Steps 1 to 7 using SS="OUI" (Simply Supported calculation) for loading cases 1,2 and 3 only

If $|\sigma| \le 1.5S$, the calculation procedure is complete. Otherwise, increase the assumed tubesheet thickness *h* and repeat Steps 1 to 7.

Note: If $|\sigma|<1.5S$, the tubesheet thickness can be optimized to a value h_0 till $|\sigma|=1.5S$ provided that, for that optimized thickness h_0 , the stresses in the tubesheet,shell and channel , calculated by the normal calculation (see Phase 1), remain respectively below $S_{PS, S}$, $S_{PS,c}$ for each of the 8 loading cases.

*

-1.6831)

2.2398)

-0.3606

1.1862

-0.1590

-0.0600)

ExampleE4.18.8-FLOATING TS(AnnexX) from PTB4

E*TriPitch :=

interE* :=

 $E^* := si\left(\frac{h}{p} > \right)$

Е

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Determination of effective elastic properties :

From figure UHX-11.2 - Polynomial equations for the determination of EVE and v⁺ for triangular pattern
Equations for E⁺ for triangular pitch

$$\left[\begin{array}{c} 0.0353 & 1.2502 - 0.0491 & 0.3604 & -0.6100\\ 0.0354 & 0.5279 & 3.0461 & -4.3657 & 1.9435\\ -0.0029 & 0.2126 & 3.9906 & -6.1730 & 3.4307 \end{array} \right]$$

 $v^+TriPitch = \left[\begin{array}{c} 0.0058 & 0.6209 & -0.8683 & 2.1099 & -1.6831\\ 0.0897 & -0.0855 & 36.1435 & 53.5425 & 53.5425 & 53.5225 \\ -0.0029 & 0.2126 & 3.9906 & -6.1730 & 3.4307 \end{array} \right]$
 $v^+TriPitch = \left[\begin{array}{c} 0.0076 & 1.5756 & -1.2119 & 1.7715 & -1.2628 \\ 0.0256 & 1.2517 & -3.520 & 6.9830 & -5.0017 \\ 0.0394 & 1.3024 & -1.1041 & 2.8714 & -2.3994 \\ 0.0372 & 1.0314 & -0.6402 & 2.6201 & -2.1929 \end{array} \right]$
 $v^+SquiPitch := \left[\begin{array}{c} 0.0771 & 0.6008 & -0.4980 & 0.4858 & -0.3606 \\ 0.4296 & -2.6350 & 8.6864 & -11.5227 & 5.844 \\ 0.3636 & -0.0657 & 2.0463 & -2.2902 & 1.1682 \\ 0.3527 & -0.2842 & 0.4345 & -0.0901 & -0.1590 \\ 0.3374 & 1.3024 & -1.1041 & 2.8714 & -2.3994 \\ 0.0372 & 1.0314 & -0.6402 & 2.6201 & -2.1929 \end{array} \right]$
 $v^+SquiPitch := \left[\begin{array}{c} 0.0791 & 0.6008 & -0.6920 & 0.6877 & -0.0600 \\ E^+s_1 & = (Elay_{2,1} + Elay_{2,2}u^+ + Elay_{2,3}u^2^+ + Elay_{2,4}u^{-3} + Elay_{1,5}u^4 \right)$
 $E^+ := i(Elay_{2,1} + Elay_{2,2}u^+ + Elay_{2,3}u^2 + Elay_{2,4}u^{-3} + Elay_{2,5}u^4 \right)$
 $E^+ := i(Elay_{2,1} + Elay_{2,2}u^+ + Elay_{2,3}u^2 + Elay_{2,4}u^{-3} + Elay_{2,5}u^4 \right)$
 $E^+ := i\left[\begin{array}{c} 0.1 \\ 0.25 \\ 0.5 \\$

ExampleE4.18.8-FLOATING TS(AnnexX) from PTB4

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PTB-7-2014: Criteria for Shell-and-Tube Heat Exchangers According to Part UHX of ASME Section VIII Division 1

ANNEX Y — UHX-12 – EXAMPLE E4.18.4 (PTB-4 2013 EDITION)

The U-tube heat exchanger is to be designed with the tubesheet construction in accordance with configuration as shown in VIII-1, Figure UHX-12.1, Configuration e.

- The shell side design condition is 650 psig at 400°F.
- The tube side design condition is 650 psig at 400°F.
- The tube material is SA-179 (K10200). The tubes are 0.75 in. outside diameter and 0.085 in. thick and are to be expanded for the full thickness of the tubesheet.
- The tubesheet material is SA-516, Grade 70 (K02700) with a 0.125 in. corrosion allowance on the tube side and no pass partition grooves. The tubesheet outside diameter is 37.25 in. The tubesheet has 496 tube holes on a 1.0 in. square pattern with one centerline pass lane. The largest center-to-center distance between adjacent tube rows is 1.375 in., and the radius to the outermost tube hole center is 12.75 in.
- The diameter of the shell gasket load reaction is 32.375 in., the shell flange bolt circle is 35 in., and the shell flange design bolt load is 656,000 lb.
- The channel material is SA-516, Grade 70, (K02700). The channel inside diameter is 31 in. and the channel thickness is 0.625 in.

ExampleE4.18.4-U TUBE TS(AnnexY) from PTB4

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U-TUBE TUBESHEET RULES according to UHX-12 (July 2013 Edition)

Example E4.18.4 (PTB-4 2013 edition) : U-Tube Tubesheet configuration "e"

1 - GEOMETRIC Data (from Fig.UHX-14.1)

	Types	of Operating Conditions	Config	juration types: a, b, c, d, e, f	
<mark>x := 1</mark>	x=1 NOR x=2 STA x=3 SHU ⁻ x=4 UPSI x=5 CLE/ x=6 OTH	MAL operating condition RTUP operating condition IDOWN operating condition Config := Config condition ANING operating condition ER operating condition	"a" for shell/ "b" for shell "c" for shell "d" for gaske "e" for chann "f" for chann	channel integral both sides integral, channel gask - TS extended integral, channel gask - TS not extended eted both sides nel integral, shell gask - TS extended nel integral, shell gask - TS not extended	
Τι	Ibesheet	Data (from Fig.UHX-14.1)	Tube	Data (from Fig.UHX-11.1)	
h := 3.	<mark>5∙in</mark>	Tubesheet thickness	<mark>p := 1.0∙in</mark>	Tube Pitch	
Lavout	:= 1	For triangular pitch : "Layout"=0	N _t := 496	Number of Tubes	
r _o := 12	r _o := 12.75 in Radius to outer tube		d _t := 0.75 in	Tube OutsideDiameter	
A := 37	<mark>7.25∙in</mark>	Outside Diameter of Tubesheet	$\textbf{t}_t := 0.085 {\cdot} \text{in}$	Tube Thickness	
C := 35	ō∙in	Bolt Circle Diameter	ρ := 1.0	Tube expansion depth ratio	
C _p := 8	30.1in	Perimeter of the tube layout	$I_{tx} := 3.5 \cdot in$	Length of Expanded Portion of Tube	
A _p := 5	510.7∙in ²	Total area enclosed by C _p			
A _L := 3	86.0938∙in ²	Total Untubed Lanes Area			
c _t := 0.	0∙in	Tubesheet Corr. Allow. (Tubeside)			
c _s := 0.	.0∙in	Tubesheet Corr. Allow. (Shellside)			
h _g := 0	.0∙in	Groove depth			
SS := "	'NO" SS=	"YES" for Tubesheet calculated as Simply S	Supported acc. t	o UHX-13.9 (config. a,b,c only)	
Shell Data (from Fig.UHX-12.1) Channel Data (from Fig.UHX-12.1)					
D _s := 0)∙in	Shell ID	D _c := 31.0 in	Channel ID	
$t_s := 0.5$	5∙in	Shell Thickness away from TS	t _c := 0.625 · in	Channel Thickness	
G _s := 3	32.375∙in	Shell Gasket Diameter	G _c := 0⋅in	Channel Gasket Diameter	
G ₁ := 0)∙in	Shell Contact mid-point TS/Flange	G ₁ := 0⋅in	Channel Contact mid-point TS/Flange	
C _s := 0).0∙in	Shell Corrosion Allowance	C _c := 0.0⋅in	Channel Corrosion Allowance	

Corroded thickr	nesses:
-----------------	---------

$:= h - c_s - c_t$	Tubesheet thickness	h = 3.500 in	$D_{S} := D_{S} + 2 \cdot C_{S}$	Shell ID	$D_s = 0.000$ in
$:= t_s - C_s$	Shell Thickness away from TS	$t_{s} = 0.500 \text{ in}$	$D_c := D_c + 2 \cdot C_c$	Channel ID	$D_{c} = 31.000 \text{ in}$
$:= t_c - C_c$	Channel Thickness	t _c = 0.625 in			

CHAN := "CYL"

"CYL" for Cylindrical Channel

Corroded diameters:

"HEMI" for Hemisherical Channel

h t_s

t_c

ExampleE4.18.4-U TUBE TS(AnnexY) from PTB4

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2 - Design (D) PRESSURES data (from UHX-12.3)

Maximum and Minimum DESIGN PRESSURES (D)

$$\frac{P_{sD_max} := 650.0 \cdot \frac{lb}{in^2}}{P_{sD_min} := 0.0 \cdot \frac{lb}{in^2}}$$
 maximum Shellside Design Pressure
$$\frac{P_{tD_max} := 650.0 \cdot \frac{lb}{in^2}}{P_{tD_min} := 0.0 \cdot \frac{lb}{in^2}}$$
 maximum Tubeside Design Pressure
$$P_{tD_min} := 0.0 \cdot \frac{lb}{in^2}$$
 minimum Tubeside Design Pressure

DESIGN PRESSURES P_s and P_t (from Table UHX-12.4-1)

$$P_{s} := \begin{pmatrix} P_{sD_min} \\ P_{sD_max} \\ P_{sD_max} \\ P_{sD_min} \end{pmatrix} \qquad P_{s} = \begin{pmatrix} 0.000 \\ 650.000 \\ 650.000 \\ 0.000 \end{pmatrix} \frac{|lb|}{|lb|} \\ 0.000 \end{pmatrix} \qquad P_{t} := \begin{pmatrix} P_{tD_max} \\ P_{tD_min} \\ P_{tD_max} \\ P_{tD_min} \end{pmatrix} \qquad P_{t} = \begin{pmatrix} 650.000 \\ 0.000 \\ 650.000 \\ 0.000 \end{pmatrix} \frac{|lb|}{|lb|} \\ 0.000 \end{pmatrix}$$

$$P_{e} := P_{s} - P_{t} \quad P_{e} = \begin{pmatrix} -650.000 \\ 650.000 \\ 0.000 \\ 0.000 \end{pmatrix} \frac{|lb|}{|la|^{2}}$$



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ExampleE4.18.4-U TUBE TS(AnnexY) from PTB4

 $W_{m1c} := 0.0 \cdot lb$

5 - Flange BOLT LOADS data (from Table UHX-8.1)

Maximum and Minimum Flange DESIGN BOLT LOADS

W_{m1s} := 656000.0 lb Shell flange Design bolt load

Channel flange Design bolt load

$$\label{eq:Ws} \begin{split} & \mathsf{W}_{s} \coloneqq 656000 \cdot \mathsf{lb} \ \ \text{Shell flange bolt load for Gasket Seating} \\ & \mathsf{W}_{c} \coloneqq 0.0 \cdot \mathsf{lb} \ \ \ \text{Channel flange bolt load for Gasket Seating} \end{split}$$

Flange BOLT LOADS for GASKET SEATING Condition

×

 $W_{m1max} := max \Big(W_{m1s}, W_{m1c} \Big) \ \ W_{m1max} = 656000.0 \, \text{lb} \label{eq:wm1}$

 $W_{max} := max(W_s, W_c)$ $W_{max} = 656000.0 \text{ lb}$

Determination of EFFECTIVE BOLT LOAD W* for each Configuration a , b , c , d

Configuration a	Configurat	ion b and c	Config	uration d	Configuration	on e and f
$W_{a}^{*} := \begin{pmatrix} 0.0 \cdot lb \\ 0.0 \cdot lb \\ 0.0 \cdot lb \\ 0.0 \cdot lb \end{pmatrix}$	W* _b := W* _c	$ \begin{cases} W_{m1c} \\ 0.0 \cdot Ib \\ W_{m1c} \\ 0.0 \cdot Ib \end{pmatrix} \\ := W_{b}^{*} $	W* _d :=	W _{m1c} W _{m1s} W _{m1max} 0.0·lb	W* _e := W* _f	$ \begin{pmatrix} 0.0 \cdot lb \\ W_{m1s} \\ W_{m1s} \\ 0.0 \cdot lb \end{pmatrix} $:= W_e^*

W* :=	W* _a	if	Config = "a"	(0.000
	W^*b	if	Config = "b" \lor Config = "c"	14/*	656000.000
	W* _d	if	Config = "d"	VV" =	656000.000 ^{ID}
	W*e	if	Config = "e" ∨ Config = "f"		0.000

Minimum required thickness h_r of the TS flanged extension (from UHX-9)

Gasket moment arm $h_G := \frac{C - G_c}{2}$ $h_G = 17.500$ in

For flanged Configurations b, d (extended as a flange), e

from UHX-9.5a

For unflanged Config.c , f

For unflanged Config.d , C See UHX-9.5c

$$\begin{split} h_{rD} &:= max \Biggl(\sqrt{\frac{1.9W_{m1s}}{S \cdot G_s}} \cdot h_G, \sqrt{\frac{1.9W_{m1c}}{S \cdot G_c}} \cdot h_G \Biggr) \quad h_{rD} = 5.804 \text{ in} \\ h_{rG} &:= \sqrt{\frac{1.9W_c}{S_a \cdot G_c}} \cdot h_G \quad h_{rG} = 0.000 \text{ in} \end{split}$$

$$h_r := max(h_{rD}, h_{rG})$$
 $h_r = 5.804$ in

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ExampleE4.18.4-U TUBE TS(AnnexY) from PTB4

Start of Calculations

*

UHX-12.5.1 Step 1 Determine D_0 , μ , μ^* and h'_{g} from UHX-11.5.1 :

$$\begin{split} D_{o} &:= 2 \cdot r_{o} + d_{t} \qquad D_{o} = 26.250 \text{ in} \\ d^{*} &:= max \Bigg[d_{t} - 2 \cdot t_{t} \cdot \left(\frac{E_{tT}}{E}\right) \cdot \left(\frac{S_{tT}}{S}\right) \cdot \rho , \left(d_{t} - 2t_{t}\right) \Bigg] \qquad d^{*} = 0.636 \text{ in} \\ p^{*} &:= \frac{p}{\sqrt{1 - \frac{4 \cdot min(A_{L}, 4D_{o} \cdot p)}{\pi \cdot D_{o}^{2}}}} \quad p^{*} = 1.035 \text{ in} \end{split}$$

$$\mu := \frac{p - d_t}{p} \quad \mu = 0.250 \qquad \qquad \mu^* := \frac{p^* - d^*}{p^*} \quad \mu^* = 0.385$$

UHX-12.5.2 Step 2 Calculate coefficients ρ_s and ρ_c and moment M_{TS}

$$\rho_{s} := \begin{vmatrix} \frac{D_{s}}{D_{0}} & \text{if } (\text{Config} = "a") \lor (\text{Config} = "b") \lor (\text{Config} = "c") \\ \frac{G_{s}}{D_{0}} & \text{if } (\text{Config} = "d") \lor (\text{Config} = "e") \lor (\text{Config} = "f") \end{vmatrix}$$

$$\rho_{c} := \begin{vmatrix} \frac{D_{c}}{D_{0}} & \text{if } (\text{Config} = "a") \lor (\text{Config} = "e") \lor (\text{Config} = "f") \\ \frac{G_{c}}{D_{0}} & \text{if } (\text{Config} = "b") \lor (\text{Config} = "c") \lor (\text{Config} = "d") \end{vmatrix}$$

$$\rho_{c} = 1.181$$

$$M_{TS} := \frac{D_{o}^{2}}{16} \cdot \left[\left(\rho_{s} - 1 \right) \cdot \left(\rho_{s}^{2} + 1 \right) \cdot P_{s} - \left(\rho_{c} - 1 \right) \cdot \left(\rho_{c}^{2} + 1 \right) \cdot P_{t} \right] \qquad M_{TS} = \begin{pmatrix} -12129.924 \\ 16467.238 \\ 4337.314 \\ 0.000 \end{pmatrix}$$

ExampleE4.18.4-U TUBE TS(AnnexY) from PTB4

ExampleE4.18.4-U TUBE TS(AnnexY) from PTB4

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*

UHX-12.5.3 Step 3 Determine E*/E and v* relative to h/p from UHX-11.5.2

$$\frac{h}{p} = 3.500 \qquad \mu^{*} = 0.385 \qquad \frac{E^{*}}{E} = 0.441 \qquad E^{*} = 1.222 \times 10^{7} \frac{lb}{in^{2}} \qquad \nu^{*} = 0.318 \qquad (\mbox{ From right pages above })$$

$$\begin{array}{l} \textbf{UHX-12.5.4.} \quad \boxed{\textbf{Step 4}} \quad \boxed{\textbf{Calculate coefficients shell and channel parameters:} \underbrace{b}_{1}, k, \lambda, \delta \text{ and } \underline{w}_{1}}{k} \\ \hline \textbf{SS} = \texttt{NO} \quad \boxed{\textbf{Use SS=YES for Simply Supported calculation in a 2^{nd} step (see UHX-13.9)}{\left[\left(D_{n}+t_{n}\right) t_{n}^{2}\right]^{0.25}} \quad \text{if SS = `NO' ~ (Config = `a' ~ Config = `b' ~ Config = `c')} \quad \boxed{p_{a} = 0.000 \frac{1}{in}} \\ \hline \textbf{g}_{c} := \begin{bmatrix} \frac{\left[12\left(1-v_{c}^{2}\right)\right]^{0.25}}{\left[\left(D_{c}+t_{c}\right) t_{c}^{2}\right]^{0.5}} & \text{if SS = `NO' ~ (Config = `a' ~ Config = `b' ~ Config = `t')} \\ \hline \textbf{g}_{c} := \left[\frac{\left[12\left(1-v_{c}^{2}\right)\right]^{0.25}}{\left[\left(D_{c}+t_{c}\right) t_{c}^{2}\right]^{0.5}} & \text{if SS = `NO' ~ (Config = `a' ~ Config = `e' ~ Config = `t')} \\ \hline \textbf{g}_{c} := \left[\frac{\left[12\left(1-v_{c}^{2}\right)\right]^{0.25}}{\left[\left(D_{c}+t_{c}\right) t_{c}^{2}\right]^{0.5}} & \text{if SS = `NO' ~ (Config = `a' ~ Config = `e' ~ Config = `t')} \\ \hline \textbf{g}_{c} := 0.000 \frac{1}{in} \\ \hline \textbf{g}_{c} := 0.000 & \textbf{h}_{c} := b_{c} \frac{E_{c} t_{c}^{3}}{6\left(1-v_{c}^{2}\right)} & \textbf{k}_{c} = 5.064 \times 10^{5} \text{ lb} \\ \hline \textbf{h}_{s} := h \cdot \beta_{s} \quad \textbf{h}_{s} = 0.000 & \textbf{h}_{c} := h \cdot \beta_{c} \quad \textbf{h}_{c} = 1.431 \\ \hline \textbf{h}_{s} := \frac{6 \cdot b_{s}}{h^{3}} \cdot \textbf{k}_{s} \left(1+h_{s}^{2}+\frac{h_{s}^{2}}{2}\right) & \lambda_{s} = 0 \times 10^{0} \frac{1}{in^{2}} & \lambda_{c} := \frac{6 \cdot D_{c}}{h^{3}} \cdot \textbf{k}_{c} \left(1+h_{c}^{2}+\frac{h_{c}^{2}}{2}\right) & \lambda_{c} = 7.591 \times 10^{6} \frac{1}{in^{2}} \\ \hline \textbf{a}_{s} := \frac{D_{s}^{2}}{4E_{s} t_{s}} \left(1+\frac{v_{s}}{2}\right) & \textbf{i} \quad \textbf{SS = `NO' ~ (Config = `a' ~ Config = `b' ~ Config = `c')} \\ \hline \textbf{a}_{s} := 0 \times 10^{0} \text{ in}^{3} \text{ lb}^{-1} \\ \hline \textbf{a}_{s} := \frac{D_{s}^{2}}{4E_{c} t_{s}} \left(1-\frac{v_{c}}{2}\right) & \textbf{i} \quad \textbf{SS = `NO' ~ (Config = `a' ~ Config = `b' ~ Config = `c')} \\ \hline \textbf{a}_{s} := 0 \times 10^{0} \text{ in}^{3} \text{ lb}^{-1} \\ \hline \textbf{a}_{c} := \frac{D_{c}^{2}}{4E_{c} t_{s}} \left(1-\frac{v_{c}}{2}\right) & \textbf{i} \quad \textbf{SS = `NO' ~ (CHAN = "CYL^{*} ~ (Config = `a' ~ Config = `e' ~ Config = `t')} \\ \hline \textbf{a}_{c} := 1.18 \times 10^{-5} \text{ in}^{3} \text{ lb}^{-1} \\ \hline \textbf{a}_{c} := \rho_{s} k_{s} \beta_{s} \delta_{s} \left(1+h_{s}^{*}\right) & \textbf{a}_{s} = 0.000 \text{ in}^{2} \quad \textbf{a}_{c} := \rho_{c} k_{c} \beta_{c} \delta_{c} \left(1+h_{c}^{*}\right) \\ \hline \textbf{a}_{c} := 1.18 \times 10^{-5} \text{ in}^{3} \text{ lb}^{-1} \\ \hline$$

ExampleE4.18.4-U TUBE TS(AnnexY) from PTB4

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K = 1.419

$$\begin{split} \mathsf{M}^{\star} &:= \left[\begin{pmatrix} \mathsf{M}_{\mathsf{TS}} + \omega_c \cdot \mathsf{P}_t - \omega_s \cdot \mathsf{P}_s \end{pmatrix} \text{ if } (\mathsf{Config} = "a") \\ \begin{pmatrix} \mathsf{M}_{\mathsf{TS}} - \omega_s \cdot \mathsf{P}_s - \frac{\mathsf{C} - \mathsf{G}_c}{2 \cdot \pi \cdot \mathsf{D}_o} \cdot \mathsf{W}^{\star} \end{pmatrix} \text{ if } (\mathsf{Config} = "b") \\ \begin{pmatrix} \mathsf{M}_{\mathsf{TS}} - \omega_s \cdot \mathsf{P}_s - \frac{\mathsf{G}_1 - \mathsf{G}_c}{2 \cdot \pi \cdot \mathsf{D}_o} \cdot \mathsf{W}^{\star} \end{pmatrix} \text{ if } (\mathsf{Config} = "c") \\ \begin{pmatrix} \mathsf{M}_{\mathsf{TS}} + \frac{\mathsf{G}_c - \mathsf{G}_s}{2 \cdot \pi \cdot \mathsf{D}_o} \cdot \mathsf{W}^{\star} \end{pmatrix} \text{ if } (\mathsf{Config} = "d") \\ \begin{pmatrix} \mathsf{M}_{\mathsf{TS}} + \frac{\mathsf{G}_c - \mathsf{G}_s}{2 \cdot \pi \cdot \mathsf{D}_o} \cdot \mathsf{W}^{\star} \end{pmatrix} \text{ if } (\mathsf{Config} = "d") \\ \begin{pmatrix} \mathsf{M}_{\mathsf{TS}} + \omega_c \cdot \mathsf{P}_t + \frac{\mathsf{C} - \mathsf{G}_s}{2 \cdot \pi \cdot \mathsf{D}_o} \cdot \mathsf{W}^{\star} \end{pmatrix} \text{ if } (\mathsf{Config} = "e") \\ \begin{pmatrix} \mathsf{M}_{\mathsf{TS}} + \omega_c \cdot \mathsf{P}_t - \frac{\mathsf{G}_1 - \mathsf{G}_s}{2 \cdot \pi \cdot \mathsf{D}_o} \cdot \mathsf{W}^{\star} \end{pmatrix} \text{ if } (\mathsf{Config} = "f") \\ \end{pmatrix} \text{ othe attaces is a } \end{split}$$

0lb otherwise

UHX-12.5.7 Step 7 Calculate the max bending moments M_p (at the periphery) , M_o (at the center) :

$$\begin{split} M_{p} &:= \frac{M^{*} - \frac{D_{0}^{-2}}{32} \cdot F \cdot \left(P_{s} - P_{t}\right)}{1 + F} \quad M_{p} = \begin{pmatrix} 3017.568 \\ 6825.214 \\ 9842.782 \\ 0.000 \end{pmatrix} \\ M_{max} &:= \begin{pmatrix} max \left(\left| M_{0_{1}} \right|, \left| M_{p_{1}} \right| \right) \\ max \left(\left| M_{0_{2}} \right|, \left| M_{p_{2}} \right| \right) \\ max \left(\left| M_{0_{3}} \right|, \left| M_{p_{3}} \right| \right) \\ max \left(\left| M_{0_{3}} \right|, \left| M_{p_{3}} \right| \right) \\ max \left(\left| M_{0_{4}} \right|, \left| M_{p_{4}} \right| \right) \end{pmatrix} \\ M_{max} &:= \begin{pmatrix} max \left(\left| M_{0_{4}} \right|, \left| M_{p_{4}} \right| \right) \\ max \left(\left| M_{0_{4}} \right|, \left| M_{p_{4}} \right| \right) \\ max \left(\left| M_{0_{4}} \right|, \left| M_{p_{4}} \right| \right) \end{pmatrix} \\ M_{max} &:= \begin{pmatrix} max \left(\left| M_{0_{4}} \right|, \left| M_{p_{4}} \right| \right) \\ max \left(\left| M_{0_{4}} \right|, \left| M_{p_{4}} \right| \right) \end{pmatrix} \\ M_{max} &:= \begin{pmatrix} 20202.132 \\ 30044.913 \\ 9842.782 \\ 0.000 \end{pmatrix} \\ M_{max} &: \begin{pmatrix} M_{0} & \text{if } SS = "OUI" \land (Config = "a" \lor Config = "b" \lor Config = "c"} \\ M_{0} & \text{if } SS = "OUI" \land (Config = "a" \lor Config = "e" \lor Config = "f"} \end{pmatrix} \\ M_{max} & \text{otherwise} \end{pmatrix} \\ M_{max} & \text{otherwise} \end{pmatrix}$$

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ExampleE4.18.4-U TUBE TS(AnnexY) from PTB4

0.000 (-7571.674) –12129.924) 656000.000 | 16467.238 26907.802 lb $M_{TS} =$ M* = lb lb 4337.314 656000.000 |" 19336.128 0.000 J 0.000 0.000 / Summary : (3017.568) (-20202.132) (20202.132) 6825.214 9842.782 30044.913 30044.913 $M_p =$ lb $M_o =$ lb M = lb 9842.782 9842.782 0.000 J 0.000 0.000

UHX-12.5.8 Step 8 Calculate the tubesheet bending stress σ :

Effective Groove depth
$$h'_{g} := max(h_{g} - c_{t}, 0)$$
 $h'_{g} = 0.000 \text{ in}$ $h_{min} := \begin{pmatrix} h - h'_{g} \\ h - h'_{g} \\ h - h'_{g} \\ h - h'_{g} \end{pmatrix}$ $h_{min} = \begin{pmatrix} 3.5000 \\ 3.5000 \\ 3.5000 \\ 3.5000 \end{pmatrix}$

(

a) Maximum tubesheet stress for loading cases 1,2,3,4

$$\sigma := \frac{6 \cdot M}{\mu^* \cdot (h - h'g)^2} \quad \sigma = \begin{pmatrix} 25669.233 \\ 38175.669 \\ 12506.435 \\ 0.000 \end{pmatrix} \frac{1b}{in^2}$$
$$\sigma_{max} := max(|\sigma_1|, |\sigma_2|, |\sigma_3|, |\sigma_4|) \quad \sigma_{max} = 38175.7 \frac{1b}{in^2}$$

$$\sigma_{\text{allow}} \coloneqq 2 \cdot \text{S}$$
 $\sigma_{\text{allow}} = 40000.0 \frac{\text{lb}}{12}$

Tubesheet_bending_Design := if $(\sigma_{max} > \sigma_{allow}, "NOT OKAY", "OKAY")$

Tubesheet_bending_Design = "OKAY"

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b) Minimum required thickness of the TS flanged extension for configurations b and e:

$$h_{r} := \begin{cases} \sqrt{\frac{1.9W_{c}}{S \cdot G_{c}}} \cdot \frac{C - G_{c}}{2} & \text{if Config = "b"} \\ \sqrt{\frac{1.9W_{s}}{S \cdot G_{s}}} \cdot \frac{C - G_{s}}{2} & \text{if Config = "e"} \end{cases} \quad h_{r} = 1.589 \text{ in} \\ 0 \cdot \text{in otherwise} \end{cases}$$

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UHX 12.5.10 Step 10 Determine the shell stresses

a) shell membrane stress

$$\sigma_{zero} \coloneqq \begin{pmatrix} 0 \cdot \frac{lb}{in^2} \\ 0 \cdot \frac{lb}{in^2} \end{pmatrix} \qquad \sigma_{sm} \coloneqq \begin{bmatrix} \frac{D_s^2}{4 \cdot t_s \cdot (D_s + t_s)} \cdot P_s \\ \sigma_{zero} \text{ otherwise} \end{bmatrix} \text{ if } (Config = "a") \lor (Config = "b") \lor (Config = "c") \qquad \sigma_{sm} = \begin{pmatrix} 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \end{pmatrix}$$

b) shell bending stress:

$$\sigma_{sbb} := \frac{6}{t_s^2} \cdot k_s \cdot \left[\beta_s \cdot \delta_s \cdot P_s + 6 \cdot \frac{1 - v^*}{E^*} \cdot \frac{D_o}{h^3} \cdot \left(1 + \frac{h'_s}{2} \right) \cdot \left(M_p + \frac{D_o^2}{32} \cdot P_e \right) \right]$$

$$\sigma_{sb} := \left[\begin{array}{c} \sigma_{sbb} & \text{if (Config = "a")} \lor (Config = "b") \lor (Config = "c") \\ \sigma_{zero} & \text{otherwise} \end{array} \right] \sigma_{sb} = \left[\begin{array}{c} 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \end{array} \right] \frac{1b}{n^2}$$

c) total shell stress:

$$\sigma_{s} := \overrightarrow{\left(\left| \sigma_{sm} \right| + \left| \sigma_{sb} \right| \right)} \quad \sigma_{s} = \begin{pmatrix} 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \end{pmatrix} \frac{lb}{in^{2}} \qquad \sigma_{smax} := max(\sigma_{s}) \qquad \sigma_{smax} = 0.0 \frac{lb}{in^{2}} \\ \sigma_{sallow} := 1.5 \cdot S_{s} \qquad \sigma_{sallow} = 30000.0 \frac{lb}{in^{2}} \\ \text{Shell_design} := if(\sigma_{smax} > \sigma_{sallow}, "NOT OKAY", "OKAY") \qquad \text{Shell_design} = "OKAY"$$

Note : If the shell stress is greater than σ_{allows} , but is less than S_{PSs} , then Elastic/Plastic procedure "EP" can be used

$$S_{PSs} = 65000 \frac{lb}{in^2}$$

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UHX 12.5.10 Step 10 Determine the channel stresses

a) channel membrane stress:

$$\sigma_{cm} \coloneqq \begin{bmatrix} D_c^2 \\ 4 \cdot t_c \cdot (D_c + t_c) \\ \sigma_{zero} \text{ otherwise} \end{bmatrix} \text{ if } (Config = "a") \lor (Config = "e") \lor (Config = "f") \\ \sigma_{cm} = \begin{bmatrix} 7900.711 \\ 0.000 \\ 7900.711 \\ 1n^2 \\ 0.000 \end{bmatrix}$$

b) channel bending stress:

$$\sigma_{cbb} \coloneqq \frac{6}{t_c^2} \cdot k_c \cdot \left[\left(\beta_c \cdot \delta_c \cdot P_t \right) - 6 \cdot \frac{1 - v^*}{E^*} \cdot \frac{D_o}{h^3} \cdot \left(1 + \frac{h'_c}{2} \right) \cdot \left(M_p + \frac{D_o^2}{32} \cdot P_e \right) \right] \\ \sigma_{cb} \coloneqq \left[\sigma_{cbb} \quad \text{if (Config = "a")} \lor (Config = "e") \lor (Config = "f") \\ \sigma_{zero} \quad \text{otherwise} \right]$$

c) total channel stress:

$$\sigma_{c} := \overrightarrow{\left(\left|\sigma_{cm}\right| + \left|\sigma_{cb}\right|\right)} \quad \sigma_{c} = \begin{pmatrix} 62319.114 \\ 56955.170 \\ 10437.478 \\ 0.000 \end{pmatrix} \begin{vmatrix} lb \\ in^{2} \\ \sigma_{callow} := max(\sigma_{c}) \\ \sigma_{cmax} = 62319.1 \frac{lb}{in^{2}} \\ \sigma_{callow} = 30000.0 \frac{lb}{in^{2}} \\ \sigma_{callow} = 30000.0 \frac{lb}{in^{2}} \\ channel_design := if(\sigma_{cmax} > \sigma_{callow}, "NOT OKAY", "OKAY") \\ \end{bmatrix}$$

Note : If the channel stress is greater than σ_{allowc} , but is less than S_{PSc} , then Elastic/Plastic procedure "EP" can be used

$$S_{PSc} = 65000 \frac{lb}{in^2}$$

 $\text{EPchannel} \coloneqq \text{si} \Big(\sigma_{\text{callow}} < \sigma_{\text{cmax}} \leq \text{S}_{\text{PSc}}, \text{"EP May Be Applied"}, \text{"EP is not Applicable"} \Big)$

EPchannel = "EP May Be Applied"

$$\sigma_{zero} := \begin{pmatrix} 0 \cdot \frac{lb}{in^2} \\ 0 \cdot \frac{lb}{in^2} \end{pmatrix} \qquad \sigma_{zero1} := \begin{pmatrix} 1 \cdot \frac{lb}{in^2} \\ 1 \cdot \frac{lb}{in^2} \end{pmatrix}$$

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ExampleE4.18.4-U TUBE TS(AnnexY) from PTB4

From step 4 :

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UHX12.6 Simply Supported Tubesheet Procedure

This procedure applies only when the TS is integral with the shell or channel, i.e. :

- shell of configurations a,b,c

- channel of configuration a,e,f

SS = "NO"

Calculation must be performed in 2 phases :

Phase 1) Perform Steps 1 to 10 with **SS="NON"** (normal calculation) with the following modifications in Step 10:

- minimum length requirement I_{smin} of shell band for configurations a,b,c do not apply
- minimum length requirement I cmin of channel band for configurations a,e,f do not apply

if $\sigma_s \leq S_{PS,s}$ and $\sigma_c \leq S_{PS,c}$, the shell and/or channel designs are acceptable.

Otherwise increase the thickness of the overstressed components (shell and/or channel) and return to Step 1.

Phase 2) Perform Steps 1 to 7 using SS="OUI" (Simply Supported calculation) for loading cases 1,2 and 3 only

If $|\sigma| \le 1.5S$, the calculation procedure is complete. Otherwise, increase the assumed tubesheet thickness *h* and repeat Steps 1 to 8, using in Step 7 : M=|M_o|

Note: If $|\sigma|<1.5S$, the tubesheet thickness can be optimized to a value h_0 till $|\sigma|=1.5S$ provided that, for that optimized thickness h_0 , the stresses in the tubesheet,shell and channel calculated by the normal calculation (see Phase 1), remain respectively below $S_{PS, S}$, $S_{PS,s}$, $S_{PS,c}$ for each of the 7 loading cases.

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E

2.1099 -1.6831) -59.5425 35.8223

$$E^{*}_{2} := \left(E \operatorname{lay}_{2,1} + E \operatorname{lay}_{2,2} \cdot \mu^{*} + E \operatorname{lay}_{2,3} \cdot \mu^{*2} + E \operatorname{lay}_{2,4} \cdot \mu^{*3} + E \operatorname{lay}_{2,5} \cdot \mu^{*4}\right) \cdot E \qquad E^{*}_{2} = 458997.963 \operatorname{sec}^{2} \operatorname{ft}^{-1} \operatorname{psi}_{2} = 420895.830 \operatorname{sec}^{2} \operatorname{ft}^{-1} \operatorname{psi}_{2} = 42089$$

For h/p < 0.1, use h/p = 0.1. For values of h/p > 2, use h/p = 2.0. Otherwise perform a linear interpolation

$$\begin{aligned} & \text{hpratio} := \begin{pmatrix} 0.1 \\ 0.25 \\ 0.5 \\ 2.0 \end{pmatrix} & \text{interE}^* := \begin{pmatrix} \mathsf{E}^*_1 \\ \mathsf{E}^*_2 \\ \mathsf{E}^*_3 \\ \mathsf{E}^*_4 \end{pmatrix} & \frac{h}{p} = 3.5 \end{aligned}$$

$$\begin{aligned} & \mathsf{E}^* := \mathsf{si} \bigg(\frac{h}{p} > 2, \mathsf{E}^*_4, \mathsf{si} \bigg(\frac{h}{p} \le 0.10, \mathsf{E}^*_1, \mathsf{interplin} \bigg(\mathsf{hpratio}, \mathsf{interE}^*, \frac{h}{p} \bigg) \bigg) \bigg) & \mathsf{E}^* = 1.222 \times 10^7 \frac{\mathsf{lb}}{\mathsf{in}^2} - \frac{\mathsf{E}^*}{\mathsf{E}} = 0.4413 \end{aligned}$$

$$\begin{aligned} & \mathsf{vlay} := \mathsf{si} (\mathsf{Layout} > 0, \mathsf{v}^* \mathsf{SquPitch}, \mathsf{v}^* \mathsf{TriPitch}) & \\ & \mathsf{v}^*_1 := \mathsf{vlay}_{1,1} + \mathsf{vlay}_{1,2} \cdot \mu^* + \mathsf{vlay}_{1,3} \cdot \mu^{*2} + \mathsf{vlay}_{1,4} \cdot \mu^{*3} + \mathsf{vlay}_{1,5} \cdot \mu^{*4} & \mathsf{v}^*_1 = 0.121 \end{aligned}$$

$$\begin{aligned} & \mathsf{v}^*_2 := \mathsf{vlay}_{2,1} + \mathsf{vlay}_{2,2} \cdot \mu^* + \mathsf{vlay}_{2,3} \cdot \mu^{*2} + \mathsf{vlay}_{2,4} \cdot \mu^{*3} + \mathsf{vlay}_{2,5} \cdot \mu^{*4} & \mathsf{v}^*_2 = 0.143 \end{aligned}$$

$$\begin{aligned} & \mathsf{v}^*_3 := \mathsf{vlay}_{3,1} + \mathsf{vlay}_{3,2} \cdot \mu^* + \mathsf{vlay}_{3,3} \cdot \mu^{*2} + \mathsf{vlay}_{4,4} \cdot \mu^{*3} + \mathsf{vlay}_{4,5} \cdot \mu^{*4} & \mathsf{v}^*_4 = 0.252 \end{aligned}$$

$$& \mathsf{v}^*_5 := \mathsf{vlay}_{5,1} + \mathsf{vlay}_{5,2} \cdot \mu^* + \mathsf{vlay}_{5,3} \cdot \mu^{*2} + \mathsf{vlay}_{5,4} \cdot \mu^{*3} + \mathsf{vlay}_{5,5} \cdot \mu^{*4} & \mathsf{v}^*_5 = 0.299 \end{aligned}$$

$$& \mathsf{v}^*_6 := \mathsf{vlay}_{6,1} + \mathsf{vlay}_{6,2} \cdot \mu^* + \mathsf{vlay}_{6,3} \cdot \mu^{*2} + \mathsf{vlay}_{6,4} \cdot \mu^{*3} + \mathsf{vlay}_{6,5} \cdot \mu^{*4} & \mathsf{v}^*_6 = 0.318 \end{aligned}$$

For h/p lower than 0.1, use h/p = 0.1; for values of h/p higher than 2, use h/p = 2.0, otherwise perform a linear interpolation:

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