**ASME PTB-3-2013** 

# ASME Section VIII – Division 2 Example Problem Manual



## ASME Section VIII - Division 2 Example Problem Manual

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## TABLE OF CONTENTS

Forewor	d	vi
ACKNOW	eagements	VIII
PART 1		1
1.1	Introduction	1
1.2	Scope	1
1.3	Organization and Use	1
1.4	References	1
PART 2		2
2.1	General	2
2.2	Example Problem Format	2
2.3	Calculation Precision	2
PART 3		3
2.1	Example E2.1 Line of MDMT Exampliping Curves	
3.1	Example E3.1 – Use of MDMT Exemption Curves with Stress Reduction	3
33	Example E3.2 – Ose of MDMT Liging Fracture Mechanics (API 570 1/ASME EES 1)	 6
5.5 DADT (		0
PARI 4		19
4.1	General Requirements	19
4.1.	1 Example E4.1.1 – Review of General Requirements for a Vessel Design	19
4.1.	2 Example E4.1.2 – Required Wall Thickness of a Hemispherical Head	20 wth
4.1.	Material	21
4.0	Waldad Jointa	Z I
4.2	1 Example E4.2.1 – Nondestructive Examination Requirement for Vessel Design	22
4.2.	2 Example E4.2.2 – Nozzle Detail and Weld Sizing	25
4.2.	3 Example E4.2.3 – Nozzle Detail with Reinforcement Pad and Weld Sizing	26
4.3	Internal Design Pressure	28
4.3.	1 Example E4.3.1 – Cylindrical Shell	28
4.3.	2 Example E4.3.2 – Conical Shell	29
4.3.	3 Example E4.3.3 – Spherical Shells	30
4.3.4	4 Example E4.3.4 – Torispherical Head	31
4.3.	5 Example E4.3.5 – Elliptical Head	34
4.3.	6 Example E4.3.6 – Combined Loadings and Allowable Stresses	37
4.3.	<ul> <li>Example E4.3.7 – Conical Transitions Without a Knuckle</li> <li>Example E4.3.8 – Conical Transitions with a Knuckle.</li> </ul>	43
4.3.	Challe Under Externel Deserves and Allewable Compressive Otherses	04
4.4 4 4	Shells Under External Pressure and Allowable Compressive Stresses	69 69
+. 4 4 1	<ul> <li>Example E4.4.2 – Conical Shell</li> </ul>	71
4.4.3	3 Example E4.4.3 – Spherical Shell and Hemispherical Head	74
4.4.4	4 Example E4.4.4 – Torispherical Head	75
4.4.	5 Example E4.4.5 – Elliptical Head	77
4.4.	6 Example E4.4.6 – Combined Loadings and Allowable Compressive Stresses	79
4.4.	7 Example E4.4.7 – Conical Transitions without a Knuckle	95
4.4.	8 Example E4.4.8 – Conical Transitions with a Knuckle	116

4.5 She	ells Openings in Shells and Heads Example F4 5.1 – Radial Nozzle in Cylindrical Shell and Weld Strength Analysis	. 122
4.5.2	Example E4.5.2 – Hillside Nozzle in Cylindrical Shell and Weld Strength Analysis	. 128
4.5.3	Example E4.5.3 – Radial Nozzle in Elliptical Head and Weld Strength Analysis	. 134
4.6 Fla	t Heads	140
4.6.1	Example E4.6.1 – Flat Un-staved Circular Heads.	. 140
4.6.2	Example E4.6.2 – Flat Un-staved Non-Circular Heads Attached by Welding	. 142
17 Snł	perically Dished Bolted Covers	1/13
4.7.1	Example E4.7.1 – Thickness Calculation for a Type D Head	. 143
4.7.2	Example E4.7.2 – Thickness Calculation for a Type D Head Using the Alternative R	ule
	in Paragraph 4.7.5.3	. 153
4.8 Out	ick-Actuating (Quick Opening) Closures	163
4.8.1	Example E4.8.1 – Review of Requirements for Quick-Actuating Closures	. 163
4.0 Pro	and and Stoved Surfaces	165
4.9 Dia 4.91	Example F4.9.1 – Braced and Staved Surfaces	165
4 10 Lia		160
4.10 Lig	aments Example F4 10 1 - Ligaments	168
4.10.1		. 100
4.11 Jac	Keted Vessels	. 170
4.11.1	Example E4.11.1 - Jackeled Vessel	. 170
4.11.2		. 175
4.12 NO	nCircular Vessels	. 176
4.12.1	Example E4.12.1 – Type 1	101
4.12.2		. 101
4.13 Lay	/ered Vessels	. 190
4.13.1	Example E4.13.1 – Layered Cylindrical Shell	100
4.13.2	Example E4.13.2 – Layeleu Hemispherical Heau	101
4.13.3		. 191
4.14 EVa	aluation of Vessels Outside of Tolerance	. 192
4.14.1	Example E4.14.1 - Shell Tolerances and Estique Evaluation	104
4.14.2	Example E4.14.2 – Shell Tolerances and Taligue Evaluation	200
4.45 0		. 200
4.15 Sup	pports and Attachments	203
4 15 2	Example E4 15.2 – Vertical Vessel, Skirt Design	205
4.10.2	nand lainta	. 211
4.16 Fla	nged Joints	219
4 16 2	Example E4 16.2 $-1$ cose Type	220
4.47 Cla	mad Connections	. 223
4.17 Cla	Example E4 17.1 Elange and Clamp Design Procedure	. 238
T. 17. 1	Example L4. 17.1 - Trange and Gramp Design Trocedure	. 200
4.18 IU	Desneets In Shell and Tube Heat Exchangers	. 249 240
4.10.1	Example E4.18.1 – 0-1 ube Tubesheet Casketed With Shell and Channel	. 249 253
ד. וט.∠ ע 1פ פ	Example E4.18.3 – U-Tube Tubesheet Gasketed With Shell and Channel	256
4 18 4	Example E4 18 4 – U-Tube Tubesheet Gasketed With Shell and Integral with Chan	. 200 1el
ч. то. <del>ч</del>	Extended as a Flance	250
4 18 5	Example F4 18 5 – Fixed Tubesheet Exchanger Configuration b. Tubesheet Integra	. 200 al
	with Shell. Extended as a Flange and Gasketed on the Channel Side	. 263

4.18	8.6 Example E4.18.6 – Fixed Tubesheet Exchanger, Configuration b, Tubesheet Interview with Shell, Extended as a Elange and Gasketed on the Channel Side	gral 275
4 18	8.7 Example F4 18.7 – Fixed Tubesheet Exchanger Configuration a	288
4.10	8.8 Example E4 18.8 – Stationary Tubesheet Gasketed With Shell and Channel: Flo	ating
1.10	Tubesheet Gasketed. Not Extended as a Flange	
4.18	8.9 Example E4.18.9 – Stationary Tubesheet Gasketed With Shell and Channel: Flore	ating
	Tubesheet Integral	
4.18	8.10 Example E4.18.10 – Stationary Tubesheet Gasketed With Shell and Channel; Fl	oating
	Tubesheet Internally Sealed	
4.19	Bellows Expansion Joints	324
4.19	9.1 Example E4.19.1 – U-Shaped Un-reinforced Bellows Expansion Joint and Fatigu	е
	Evaluation	324
4.19	9.2 Example E4.19.2 – Toroidal Bellows Expansion Joint and Fatigue Evaluation	332
PART 5		338
5.1	General Requirements	338
5.2	Protection Against Plastic Collapse	338
5.2.	1 Example E5.2.1 – Elastic Stress Analysis	
5.2.	2 Example E5.2.2 – Limit Load Analysis	
5.2.		
5.3	Protection Against Local Failure	
53	2 Example E5.3.2 Elastic Analysis	356
53	3 Example E5.3.2 - Elastic Analysis	357
54	Example E5.4 Protection Against Collapse from Puckling	
5.4	Example E5.4 – Protection Against Collapse from Bucking	
5.5	1 Overview	
5.5.	2 Example E5.5.2 – Fatigue Screening	
5.5.	.3 Example E5.5.3 – Elastic Stress Analysis, and Equivalent Stresses	369
5.5.	4 Example E5.5.4 – Elastic-Plastic Stress Analysis, and Equivalent Strains	376
5.5.	5 Example E5.5.5 – Elastic Stress Analysis, and Structural Stress	384
5.5.	.6 Example E5.5.6 – Protection Against Ratcheting Using Elastic Stress Analysis	391
5.5.	.7 Example E5.5.7 – Protection Against Ratcheting Using Elastic-Plastic Stress Ana	alysis
		396
PART 6		399
6.1	Example E6.1 – Postweld Heat Treatment of a Pressure Vessel	399
6.2	Example E6.2 – Out-of-Roundness of a Cylindrical Forged Vessel	403
PART 7		405
7.1	Example E7.1 – NDE Requirements: Vessel with One Examination Group Designation	on406
7.2	Example E7.2 – NDE Requirements: Vessel with Two Examination Group Designation	ons 410
PART 8		419
8 1	Example F8.1 – Determination of a Hydrostatic Test Pressure	410
8.2	Example E8.2 – Determination of a Pneumatic Test Pressure	

## FOREWORD

This document is the second edition of the ASME Section VIII – Division 2 example problem manual. The purpose of this second edition is to update the example problems to keep current with the changes incorporated into the 2013 edition of the ASME B&PV Code, Section VIII, Division 2. The example problems included in the first edition of the manual were based on the contents of the 2010 edition of the B&PV Code. In 2011, ASME transitioned to a two year publishing cycle for the B&PV Code without the release of addenda. The release of the 2011 addenda to the 2010 edition was the last addenda published by ASME and numerous changes to the Code were since adopted.

Known corrections to design equations and results have also been made in this second edition. Additionally, some formatting modifications were made to facilitate better use of the example manual, as applicable.

In 1998 the ASME Boiler and Pressure Vessel Standards Committee authorized a project to rewrite the ASME B&PV Code, Section VIII, Division 2. This decision was made shortly after the design margin on specified minimum tensile strength was lowered from 4.0 to 3.5 in Section I and Section VIII, Division 1. ASME saw the need to update Section VIII, Division 2 to incorporate the latest technologies and to be more competitive. In lieu of revising the existing standard, the decision was made to perform a clean sheet rewrite. By doing so it was felt that, not only could the standard be modernized with regard to the latest technical advances in pressure vessel construction, but it could be structured in a way to make it more user-friendly for both users and the committees that maintain it.

Much new ground was broken in the development of the new Section VIII, Division 2, including the process taken to write the new standard. Traditionally, development of new standards by ASME is carried out by volunteers who serve on the different committees responsible for any given standard. Depending upon the complexity of the standard, the development of the first drafts may take up to 15 years to complete based on past history. The prospect of taking 15 or more years to develop VIII-2 was unacceptable to ASME and the volunteer leadership. The decision was made to subcontract the development of the draft to the Pressure Vessel Research Council (PVRC) who in turn formed the Task Group on Continued Modernization of Codes to oversee the development of the new Section VIII, Division 2 Code. PVRC utilized professionals with both engineering and technical writing expertise to develop new technology and the initial drafts of the new Section VIII, Division 2.

A Steering Committee made up of ASME Subcommittee VIII members was formed to provide technical oversight and direction to the development team with the goal of facilitating the eventual balloting and approval process. ASME also retained a Project Manager to manage all the activities required to bring this new standard to publication.

The project began with the development of a detailed table of contents containing every paragraph heading that would appear in the new standard and identifying the source for the content that would be placed in this paragraph. In preparing such a detailed table of contents, the lead authors were able to quickly identify areas where major development effort was required to produce updated rules. A list of some of the new technology produced for VIII-2 rewrite includes:

- Adoption of a design margin on specified minimum tensile strength of 2.4,
- Toughness requirements,
- Design-by-rule for the creep range,
- Conical transition reinforcement requirements,
- Opening reinforcement rules,
- Local strain criteria for design-by-analysis using elastic-plastic analysis,
- Limit load and plastic collapse analysis for multiple loading conditions,
- Fatigue design for welded joints based on structural stress method, and
- Ultrasonic examination in lieu of radiographic examination.

Users of the Section VIII, Division 2 Code (manufacturers and owner/operators) were surveyed at the beginning of the project to identify enhancements that they felt the industry wanted and would lead to increased use of the standard. Since the initial focus of the Code was for the construction of pressure equipment for the chemical and petrochemical industry, the people responsible for specifying equipment for this sector were very much interested in seeing that common requirements that are routinely found in vessel specifications would become a requirement within this standard. This was accomplished by close participation of the petrochemical industry during the development of this standard. Some of the enhancements included:

- Alternatives provided for U.S. and Canadian Registered Professional Engineer certification of the User Design Specification and Manufacturers Design Report,
- Consolidation of weld joint details and design requirements,
- Introduction of a weld joint efficiency and the use of partial radiographic and ultrasonic examination,
- Introduction of the concept of a Maximum Allowable Working Pressure (MAWP) identical to VIII-1,
- Significant upgrade to the design-by-rule and design-by-analysis procedures,
- Extension of the time-independent range for low chrome alloys used in heavy wall vessels,
- Extension of fatigue rules to 900°F (400°C) for low-chrome alloys used in heavy wall vessels,
- Adoption of new examination requirements and simplification of presentation of the rules,
- User-friendly extensive use of equations, tables, and figures to define rules and procedures, and
- ISO format; logical paragraph numbering system and single column format,
- Many of these enhancements identified by users were included in the first release of Section VIII, Division 2 in 2007.

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We would also like to commend the efforts of Allison Bradfield, Jeffrey Gifford, and Tiffany Shaughnessy for their documentation control and preparation skills in the publication of this manual.

## **GENERAL REQUIREMENTS**

## PART CONTENTS

## 1.1 Introduction

ASME B&PV Code, Section VIII, Division 2 contains mandatory requirements, specific prohibitions, and non-mandatory guidance for the design, materials, fabrication, examination, inspection, testing, and certification of pressure vessels and their associated pressure relief devices. The 2007 edition of the code has been re-written and reorganized, and incorporates the latest technologies for pressure vessel design. Since this initial release the code has undergone further development in all of its Parts, including refinement of its Part 4 design-by-rule procedures and Part 5 design-by-analysis methods. These modifications are captured in this PTB document.

## 1.2 Scope

Example problems illustrating the use of the design-by-rule and design-by-analysis methods in ASME B&PV Code, Section VIII, Division 2 are provided in this document. Example problems are provided for all calculation procedures primarily in US Customary units, however select problems are shown using SI units.

## 1.3 Organization and Use

An introduction to the example problems is described in Part 2 of this document. The remaining Parts of this document contain the example problems. The Parts 3, 4, and 5 in this document coincide with the Parts 3, 4 and 5 in the ASME B&PV Code, Section VIII, Division 2. For example, example problems illustrating the design-by-rule calculations contained in Part 4 of Section VIII, Division 2 are provided in Part 4 of this document. All paragraph references are to the ASME B&PV Code, Section VIII, Division 2, 2013 Edition. [1].

The example problems in this manual follow the calculation procedures in ASME B&PV Code, Section VIII, Division 2. It is recommended that users of this manual obtain a copy of ASME PTB-1-2013 [2] that contains criteria and commentary on the use of the design rules.

It should be noted that VIII-2 permits the use of API 579-1/ASME FFS-1 [3] for some calculation procedures. When reviewing certain example problems in this manual, it is recommended that users obtain a copy of this standard.

## 1.4 References

- 1. ASME B&PV Code, Section VIII, Division 2, 2013, ASME, New York, New York, 2013.
- 2. Osage, D., *ASME Section VIII Division 2 Criteria and Commentary*, PTB-1-2013, ASME, New York, New York, 2013.
- **3.** API, API 579-1/ASME FFS-1 2007 *Fitness-For-Service*, American Petroleum Institute, Washington, D.C., 2007.

## EXAMPLE PROBLEM DESCRIPTIONS PART CONTENTS

## 2.1 General

Example problems are provided for the following parts of the document;

- Part 3 Materials Requirements
- Part 4 Design By Rule Requirements
- Part 5 Design By Analysis Requirements
- Part 6 Fabrication Requirements
- Part 7 Examination Requirements
- Part 8 Pressure Testing Requirements

A summary of the example problems provided is contained in the Table of Contents.

## 2.2 Example Problem Format

In all of the example problems, with the exception of tubesheet design rules in paragraph 4.18, the code equations are shown with symbols and with substituted numerical values to fully illustrate the use of the code rules. Because of the complexity of the tubesheet rules, only the results for each step in the calculation producer is shown.

## 2.3 Calculation Precision

The calculation precision used in the example problems is intended for demonstration proposes only; an intended precision is not implied. In general, the calculation precision should be equivalent to that obtained by computer implementation, rounding of calculations should only be done on the final results.

## MATERIALS REQUIREMENTS

## **PART CONTENTS**

## 3.1 Example E3.1 – Use of MDMT Exemptions Curves

Determine if Impact Testing is required for the proposed shell section, using only the rules of paragraph 3.11.2.3. The shell is cylindrical with all Category A joints made up of Type 1 butt welds which have been 100% radiographically examined.

Vessel Data:

•	Material	=	<i>SA</i> –516, <i>Grade</i> 70, <i>Norm</i> .
•	Nominal Thickness	=	1.8125 in
•	PWHT	=	Yes
•	MDMT	=	-20.0°F
•	Corrosion Allowance	=	0.125 in

Per paragraph 3.11.2.3 for Carbon and Low Alloy Steel Except Bolting.

- a) Since the vessel has been PWHT, Figure 3.8 (or Table 3.15) shall be used to establish impact testing exemptions based on the impact test exemption curve for the subject material specification, MDMT, and governing thickness of a welded part.
- b) As noted in Figure 3.8, from the Material Assignment Table, a material specification of SA-516, *Grade* 70, *Norm.* is designated a Curve D material.
- c) The governing thickness  $t_g$  of a welded part is determined from the criteria of paragraph 3.11.2.3.b. For a butt joint in a cylindrical shell,  $t_g$  is equal to the nominal thickness of the thickest weld joint, see Figure 3.9 Sketch (a).

 $t_{o} = 1.8125$  in

d) If an MDMT and thickness combination for the subject material is on or above the applicable impact test exemption curve, then impact testing is not required for base metal. Requirements for weld metal and heat affected zones are provided in paragraph 3.11.8.

Interpreting the value of MDMT from Figure 3.8 is performed as follows. Enter the figure along the abscissa with a nominal governing thickness of  $t_g = 1.8125$  *in* and project upward until an intersection with the Curve D material is achieved. Project this point left to the ordinate and interpret the MDMT. This results in an approximate value of  $MDMT = -19.0^{\circ}F$ .

A more accurate value for MDMT can be achieved by using the tabular values found in Table 3.15. Linear interpolation between thicknesses for a  $t_g = 1.8125$  *in* and a Curve D material results in the following value for MDMT.

$$y = \left(\frac{x - x_1}{x_2 - x_1}\right) (y_2 - y_1) + y_1$$
  
MDMT =  $\left(\frac{1.8125 - 1.75}{2.0 - 1.75}\right) [(-15.8) - (-20.2)] + (-20.2) = -19.1°F$ 

Since the calculated MDMT of  $-19.1^{\circ}F$  is warmer than the required MDMT of  $-20.0^{\circ}F$ , impact testing is required using only the rules in 3.11.2.3. However, impact testing may still be avoided by applying the rules of paragraph 3.11.2.5 or 3.11.2.8.

#### 3.2 Example E3.2 – Use of MDMT Exemption Curves with Stress Reduction

Determine if impact testing is required for the proposed shell section in E3.1, using the rules of paragraph 3.11.2.5. The shell is cylindrical with all Category A joints made up of Type 1 butt welds which have been 100% radiographically examined.

#### Vessel Data:

•	Material	=	SA-516, Grade 70, Norm
•	Design Conditions	=	356 <i>psi</i> @ 300°F
•	Inside Diameter	=	150 in
•	Nominal Thickness	=	1.8125 in
•	PWHT	=	Yes
•	MDMT	=	$-20.0^{\circ}F$
•	Weld Joint Efficiency	=	1.0
•	Corrosion Allowance	=	0.125 in
•	Allowable Stress at Ambient Temperature	=	22400 psi
•	Allowable Stress at Design Temperature	=	22400 psi
•	Yield Strength at Ambient Temperature	=	38000 psi

In accordance with paragraph 3.11.2.5, the procedure that is used to determine the exemption from impact testing based on design stress values is shown below.

a) STEP 1 – For the welded part under consideration, determine the nominal thickness of the part,  $t_n$ , and the required governing thickness  $t_g$  using paragraph 3.11.2.3.

 $t_n = t_g = 1.8125$  in

- b) STEP 2 Determine the applicable material toughness curve to be used. Since the vessel has been PWHT, Figure 3.8 (or Table 3.15) shall be used to establish impact testing exemptions based on the impact test exemption curve for the subject material specification, MDMT, and governing thickness of a welded part. As noted in Figure 3.8, from the Material Assignment Table, a material specification of SA-516, *Grade* 70, *Norm.* is designated a Curve D material.
- c) STEP 3 Determine the MDMT from Figure 3.8 based on the applicable toughness curve and the governing thickness,  $t_{a}$ , From E3.1,

 $MDMT = -19.1^{\circ}F$ 

d) STEP 4 – Based on the design loading conditions at the MDMT, determine the stress ratio,  $R_{ts}$ , using the thickness basis, Equation (3.1).

$$R_{ts} = \frac{t_r E^*}{t_n - CA}$$

Where,  $t_r$  is the required thickness of the cylindrical shell at the specified MDMT of  $-20.0^{\circ}F$ , using Equation 4.3.1 from Part 4.

$$t_r = \frac{D}{2} \left( \exp\left[\frac{P}{SE}\right] - 1 \right)$$

where,

 $D = 150.0 + 2(Corrosion \ Allowance) = 150.0 + 2(0.125) = 150.25 \ in$ 

$$t_r = \frac{150.25}{2} \left( \exp\left[\frac{356}{22400(1.0)}\right] - 1 \right) = 1.2035 \text{ in}$$

The variables  $E^*$ ,  $t_n$ , and CA are defined as follows:

$$E^* = \max[E, 0.80] = \max[1.0, 0.8] = 1.0$$
  
 $t_n = 1.8125$  in  
 $CA = 0.125$  in

Therefore,

$$R_{ts} = \frac{t_r E^*}{t_n - CA} = \frac{1.2035(1.0)}{1.8125 - 0.125} = 0.7132$$

e) STEP 5 - Determine the final value of the MDMT and evaluate the results. Since the computed value of the ratio  $R_{ts} > 0.24$  from STEP 4, the specified minimum yield strength,  $S_{y} \le 50 \ ksi$ , and the shell was subject to PWHT, then the reduction in MDMT based on available thickness  $T_{R}$  is computed using Figure 3.13 (or Table 3.17).

Interpreting the value of  $T_R$  from Figure 3.13 is performed as follows. Enter the figure along the ordinate with a value of  $R_{ts} = 0.7132$  project horizontally until an intersection with the  $S_y \leq 50 \ ksi$  curve is achieved. Project this point downward to the abscissa and interpret  $T_R$ . This results in an approximate value of  $T_R = -28.0^{\circ}F$ .

A more accurate value for  $T_R$  can be achieved by using the tabular values found in Table 3.17. Linear interpolation between ratios  $R_{ts} = 0.7132$  and  $S_y \le 50 \ ksi$  results in the following value of  $T_R$ .

$$T_{R} = \left(\frac{0.7132 - 0.690}{0.734 - 0.690}\right) (25.8 - 31.1) + (31.1) = 28.3^{\circ}F$$

The final computed value of the MDMT is determined using Equation (3.5).

$$MDMT = MDMT_{STEP3} - T_R = -19.1^{\circ}F - 28.3^{\circ}F = -47.4^{\circ}F$$

Since the final value of MDMT is colder than the proposed MDMT, impact testing is not required.

## 3.3 Example E3.3 – Develop MDMT Using Fracture Mechanics (API 579-1/ASME FFS-1)

Using the fracture mechanics approach for determination of MDMT given in paragraph 3.11.2.8, determine if an MDMT of  $-20^{\circ}F$  is acceptable for the vessel shell shown below. Assume that the crack is located in the heat affected zone of a longitudinal double-v groove weld seam, oriented parallel to the weld seam.

Vessel Data:

•	Material	=	SA-516, Grade 70, Norm.
•	Design Conditions	=	356 psig @300°F
•	Inside Radius	=	75.125 in
•	Nominal Thickness	=	1.6875 in
•	PWHT	=	Yes
•	MDMT	=	-20.0°F
•	Longitudinal Weld Joint Efficiency	=	1.0
•	Ambient Yield Strength	=	38000 <i>psi</i>
•	Flaw Type per paragraph 3.11.2.8	=	Long, semi–ellip crack, long weld, inside surface
•	Probability of Failure	=	10 <sup>-6</sup>
•	Coefficient of Variation	=	0.10 (Stresses well known)
•	Loads	=	Pressure Only

In accordance with paragraph 3.11.2.8, evaluate an assessment temperature of  $-20^{\circ}F$  using a fracture mechanics methodology. NOTE – for a typical fitness-for-service assessment involving crack-like flaws, the component under consideration shall be evaluated in its current state, where future corrosion allowance is not considered.

In accordance with paragraph 3.11.2.8.c, determine the values for a and 2c. From the vessel data provided,

$$a = \frac{1}{4}t = \frac{1}{4}(1.6875) = 0.4219 \text{ in}$$
$$2c = 6a = 6(0.4219) = 2.5314 \text{ in}$$

Using API 579-1/ASME FFS-1, Part 9 Paragraph 9.4.3 Level 2 Assessment

a) STEP 1 – Evaluate operating conditions and determine the pressure, temperature and supplemental loading combinations to be evaluated.

P = 356 psig @ 300°FNo Supplemental Loads

- STEP 2 Determine the stress distributions at the location of the flaw based on the applied loads in STEP 1 and classify the resulting stresses into the following stress categories: Primary Stress; Secondary Stress; and Residual Stress.
  - Primary Stress The flaw is located away from all structural discontinuities. Therefore, the primary stress at the weld joint perpendicular to the crack face is a membrane hoop stress. From Annex C, Table C.1, the flaw geometry, component geometry, and loading condition correspond to KCSCLE2 and RCSCLE2, i.e. Cylinder - Surface Crack - Longitudinal Direction - Semi-Elliptical Shape - Internal Pressure. The stress intensity factor solution for KCSCLE2 is provided in Annex C, paragraph C.5.10. The reference stress solution for RCSCLE2 is provided in Annex D, paragraph D.5.10. The membrane and bending components of the primary stress for the calculation of the reference stress are given by Equations (D.47) and (D.48).

$$P_{m} = \frac{pR_{i}}{t} = \frac{356(75.125)}{1.6875} = 15848.5926 \ psi$$

$$P_{b} = \frac{pR_{o}^{2}}{R_{o}^{2} - R_{i}^{2}} \left[ \frac{t}{R_{i}} - \frac{3}{2} \left( \frac{t}{R_{i}} \right)^{2} + \frac{9}{5} \left( \frac{t}{R_{i}} \right)^{3} \right]$$

$$P_{b} = \frac{356(76.8125)^{2}}{(76.8125)^{2} - (75.125)^{2}} \left[ \frac{1.6875}{75.125} - \frac{3}{2} \left( \frac{1.6875}{75.125} \right)^{2} + \frac{9}{5} \left( \frac{1.6875}{75.125} \right)^{3} \right]$$

$$P_{b} = 177.9865 \ psi$$

- Secondary Stress Thermal gradients do not exist in the vessel at the location of the flaw, and the flaw is located away from all major structural discontinuities. Therefore, there are no secondary stresses.
- 3) Residual Stress The flaw is located at a weld in a vessel that was subject to PWHT at the time of fabrication. From Annex E, paragraph E.3.2, to estimate the magnitude of the residual stress distribution at a weld joint, an estimate of the actual yield strength of the material must be made. The elevation of the effective yield strength above the specified minimum yield strength accounts for the typical elevation of actual properties above minimum requirements and is calculated per Equation (E.2)

$$\sigma_{ys}^{r} = \sigma_{ys} + 10 \ ksi = 38.0 + 10.0 = 48.0 \ ksi$$

For a longitudinal double-v weld seam subject to PWHT, the residual stress distribution perpendicular to the weld seam is determined in Annex E, paragraph E.4.4.1 per Equation (E.65). The value is considered to be a constant through-thickness distribution.

$$\sigma^{r}(x) = 0.2\sigma^{r}_{ys} = 0.2(48.0) = 9.6 \ ksi$$

c) STEP 3 – Determine material properties; yield strength, tensile strength and fracture toughness,

 $K_{mat}$ , for the conditions being evaluated in STEP 1. The actual material properties for the plate are not available; therefore, the specified minimum yield strength and tensile strength are used.

$$\sigma_{ys} = 38.0 \, ksi$$
  
 $\sigma_{uts} = 70.0 \, ksi$ 

The material fracture toughness is established using the MPC Charpy impact energy correlation found in Annex F, paragraph F.4.5.3, Equation (F.84).

$$K_{mat} = K_{IC} = \sigma_{ys} \left\{ 1.7 + \left( 1.7 - \frac{27}{\sigma_{ys}} \right) \cdot \tanh\left[ \frac{T - (T_o - 75)}{C} \right] \right\}$$
$$K_{mat} = K_{IC} = 38.0 \left\{ 1.7 + \left( 1.7 - \frac{27}{38.0} \right) \cdot \tanh\left[ \frac{(-20 - (12 - 75))}{66} \right] \right\} = 86.1331 \ ksi \sqrt{in}$$

Where,

 $T_o = 12^{\circ}F$   $T = -20^{\circ}F$  (Assessment Temperature)  $C = 66^{\circ}F$ 

 $\sigma_{ys}$  yield stress in *ksi* 

d) STEP 4 – Determine crack-like flaw dimensions in accordance with paragraph 3.11.2.8.c.

$$a = 0.4219$$
 in  
 $2c = 2.5314$  in

e) STEP 5 – Modify the primary stress, material fracture toughness, and flaw size using the Partial Safety Factors, *PSF*. If a given input value is known to be a conservative estimate (e.g. upper-bound stresses, lower-bound fracture toughness, or upper -bound flaw size), then an applicable *PSF* equal to 1.0 may be used in the assessment.

From Part 9, Table 9.3, modify the primary stress, material fracture toughness, and flaw size using the applicable PSF values. Based on an assumed depth of flaw, a coefficient of variation,  $COV_s$ , which is used to define the uncertainty in the primary stress distribution, and the probability of failure,  $p_t$ , the variable  $R_c$  used in Table 9.3 is determined as shown.

$$\begin{bmatrix} a = 0.4219 \ in \\ COV_s = 0.10 \\ p_f = 10^{-6} \end{bmatrix} \rightarrow R_c = 1.8 \ \sqrt{in}$$

Calculation of the variable,  $R_{_{kv}}$ , is also required to determine the values of PSF.

$$R_{ky} = \frac{K_{mat}^{mean}}{\sigma_{ys}} C_u = \left(\frac{86.1331}{38.0}\right) \cdot 1.0 = 2.2667 \ \sqrt{in}$$

Where,

$$K_{mat}^{mean} = K_{IC} = 86.1331 \text{ ksi}\sqrt{in}$$
  

$$C_u = 1.0 \text{ (if units of } K_{mat}^{mean} \text{ are } \text{ ksi}\sqrt{in} \text{ and } \sigma_{ys} \text{ are } \text{ ksi}\text{)}$$

Therefore, the values of PSF can now be determined from Table 9.3.

$$\begin{vmatrix} a = 0.4219 \ in \\ COV_s = 0.10 \\ p_f = 10^{-6} \\ R_c = 1.8 \ \sqrt{in} \\ R_{ky} = 2.2667 \ \sqrt{in} \end{vmatrix} \rightarrow \begin{bmatrix} PSF_s = 2.0 \\ PSF_k = 1.0 \\ PSF_a = 1.0 \end{bmatrix}$$

1) Primary Membrane and Bending Stress - Modify the primary membrane and bending stress components determined in STEP 2.

$$P_m = P_m \cdot PSF_s = 15848.5926(2.0) = 31697.1852 \ psi$$
  
 $P_b = P_b \cdot PSF_s = 177.9865(2.0) = 355.9730 \ psi$ 

2) Material Toughness - Modify the mean value of the material fracture toughness determined in STEP 3.

$$K_{mat} = \frac{K_{mat}}{PSF_{\nu}} = \frac{86.1331}{1.0} = 86.1331 \ ksi\sqrt{in}$$

3) Flaw Size - Modify the flaw size determined in STEP 4.

$$a = a \cdot PSF_a = 0.4219(1.0) = 0.4219$$
 in

f) STEP 6 – Compute the reference stress for primary stresses,  $\sigma_{ref}^{P}$  based on the modified primary stress distribution and modified flaw size from STEP 5 and the reference stress solution in Annex D.

The reference stress solution for RCSCLE2 is provided in Annex D, paragraph D.5.10, Equation (D.74).

$$\sigma_{ref}^{P} = \frac{gP_{b} + \left[ \left(gP_{b}\right)^{2} + 9\left(M_{s}P_{m}\left(1-\alpha\right)^{2}\right)^{2} \right]^{0.5}}{3\left(1-\alpha\right)^{2}}$$

$$\sigma_{ref}^{P} = \frac{0.9948(355.9730) + \left[ \left(0.9948(355.9730)\right)^{2} + 9\left(1.0106\left(31697.1852\right)\left(1-0.1071\right)^{2}\right)^{2} \right]^{0.5}}{3\left(1-0.1071\right)^{2}}$$

$$\sigma_{ref}^{P} = 32181.5735 \ psi$$

Where, g is calculated using Annex D Equation (D.31) and  $\alpha$  is calculated using Annex D Equation (D.75)

$$g = 1 - 20 \left(\frac{a}{2c}\right)^{0.75} \alpha^3 = 1 - 20 \left(\frac{0.4219}{2.5314}\right)^{0.75} (0.1071)^3 = 0.9948$$
$$\alpha = \left(\frac{\frac{a}{t}}{1 + \frac{t}{c}}\right) = \left(\frac{\left(\frac{0.4219}{1.6875}\right)}{1 + \left(\frac{1.6875}{1.2657}\right)}\right) = 0.1071$$

Where,  $M_s$  is the surface correction factor and is calculated per Annex D, paragraph D.2.3.3.b.1, Equation (D.18).

$$M_{s} = \frac{1}{1 - \frac{a}{t} + \frac{a}{t} \left(\frac{1}{M_{t}(\lambda_{a})}\right)} = \frac{1}{1 - \frac{0.4219}{1.6875} + \frac{0.4219}{1.6875} \left(\frac{1}{1.0436}\right)} = 1.0106$$

In the above equation,  $M_t(\lambda_a)$  is the surface correction factor for a cylindrical shell containing a longitudinal through-wall crack per Annex D, Equation (D.8), as a function of  $\lambda_a$  per Equation (D.19).

$$\begin{split} M_{t}(\lambda_{a}) &= \left(\frac{1.02 + 0.4411\lambda_{a}^{2} + 0.006124\lambda_{a}^{4}}{1.0 + 0.02642\lambda_{a}^{2} + 1.533(10)^{-6}\lambda_{a}^{4}}\right)^{0.5} \\ M_{t}(\lambda_{a}) &= \left(\frac{1.02 + 0.4411(0.4087)^{2} + 0.006124(0.4087)^{4}}{1.0 + 0.02642(0.4087)^{2} + 1.533(10)^{-6}(0.4087)^{4}}\right)^{0.5} = 1.0436 \\ \lambda_{a} &= \frac{1.818c}{\sqrt{R_{i}a}} = \frac{1.818(1.2657)}{\sqrt{75.125(0.4219)}} = 0.4087 \end{split}$$

g) STEP 7 – Compute the Load Ratio or the abscissa of the FAD using the reference stress for primary loads from STEP 6 and the yield strength from STEP 3.

$$L_r^P = \frac{\sigma_{ref}^P}{\sigma_{vs}} = \frac{32181.5735}{38000.0} = 0.8469$$

h) STEP 8 – Compute the stress intensity attributed to the primary loads  $K_I^P$ , using the modified primary stress distribution and modified flaw size from STEP 5, and the stress intensity factor solutions in Annex C.

The stress intensity factor solution for KCSCLE2 is provided in Annex C, paragraph C.5.11, Equation (C.192), with reference to paragraph C.5.10. For an equivalent membrane and bending stress distribution of a fourth order polynomial stress distribution, the higher order (non-linear) terms  $G_2 \rightarrow G_4$  and  $\sigma_2 \rightarrow \sigma_4$  can be eliminated from Equation (C.192). Therefore, the stress intensity factor solution from Equation (C.192) reduces to the following.

$$K_{I}^{P} = \left[G_{o}\left\{\sigma_{o}+p_{c}\right\}+G_{1}\sigma_{1}\left(\frac{a}{t}\right)+G_{2}\sigma_{2}\left(\frac{a}{t}\right)^{2}+G_{3}\sigma_{3}\left(\frac{a}{t}\right)^{3}+G_{4}\sigma_{4}\left(\frac{a}{t}\right)^{4}\right]\sqrt{\frac{\pi a}{Q}}$$
$$K_{I}^{P} = \left[G_{o}\left\{\sigma_{o}+p_{c}\right\}+G_{1}\sigma_{1}\left(\frac{a}{t}\right)\right]\sqrt{\frac{\pi a}{Q}}$$

The value of  $K_I^P$  is calculated at the base of the flaw and at the surface of the flaw.

At the base of the flaw,  $\varphi = 90^{\circ}$ :

$$K_{I}^{P} = \left[G_{o}\left\{\sigma_{o} + p_{c}\right\} + G_{1}\sigma_{1}\left(\frac{a}{t}\right)\right]\sqrt{\frac{\pi a}{Q}}$$

$$K_{I}^{P} = \left[1.1512828\left\{32053.1582 + 356\right\} + 0.6975812\left(-711.946\right)\left(0.25\right)\right]\sqrt{\frac{\pi(0.4219)}{1.2389}}$$

$$K_{I}^{P} = 38.4648 \ ksi\sqrt{in}$$

At the surface of the flaw,  $\varphi = 0^{\circ}$  :

$$K_{I}^{P} = \left[G_{o}\left\{\sigma_{o} + p_{c}\right\} + G_{1}\sigma_{1}\left(\frac{a}{t}\right)\right]\sqrt{\frac{\pi a}{Q}}$$

$$K_{I}^{P} = \left[0.7418361\left\{32053.1582 + 356\right\} + 0.1159489\left(-711.946\right)\left(0.25\right)\right]\sqrt{\frac{\pi(0.4219)}{1.2389}}$$

$$K_{I}^{P} = 24.8464 \ ksi\sqrt{in}$$

Per Annex D, paragraph C.5.10.2, the influence coefficients  $G_0$  and  $G_1$  are determined using Equations (C.188) and (C.189), respectively.

At the base of the flaw,  $\varphi = 90^{\circ}$ :

$$G_{0} = A_{0,0} + A_{1,0}\beta + A_{2,0}\beta^{2} + A_{3,0}\beta^{3} + A_{4,0}\beta^{4} + A_{5,0}\beta^{5} + A_{6,0}\beta^{6}$$

$$G_{0} = \begin{pmatrix} 0.7418361 + (-0.450705)(1) + 6.1501137(1)^{2} + (-15.50397)(1)^{3} \\ +19.978669(1)^{4} + (-13.3919)(1)^{5} + 3.627239(1)^{6} \end{pmatrix} = 1.1512828$$

$$G_{1} = A_{0,1} + A_{1,1}\beta + A_{2,1}\beta^{2} + A_{3,1}\beta^{3} + A_{4,1}\beta^{4} + A_{5,1}\beta^{5} + A_{6,1}\beta^{6}$$

$$G_{1} = \begin{pmatrix} 0.1159489 + 0.1836592(1) + 2.2622005(1)^{2} + (-4.094396)(1)^{3} + \\ 4.6541091(1)^{4} + (-3.530137)(1)^{5} + 1.1061961(1)^{6} \end{pmatrix} = 0.6975812$$

At the surface of the flaw,  $\varphi = 0^\circ$  :

$$G_{0} = A_{0,0} + A_{1,0}\beta + A_{2,0}\beta^{2} + A_{3,0}\beta^{3} + A_{4,0}\beta^{4} + A_{5,0}\beta^{5} + A_{6,0}\beta^{6}$$

$$G_{0} = A_{0,0} = 0.7418361$$

$$G_{1} = A_{0,1} + A_{1,1}\beta + A_{2,1}\beta^{2} + A_{3,1}\beta^{3} + A_{4,1}\beta^{4} + A_{5,1}\beta^{5} + A_{6,1}\beta^{6}$$

$$G_{1} = A_{0,1} = 0.1159489$$

Where,  $\beta$  is given by Annex C, Equation (C.96) and the parameters  $A_{i,j}$  are provided in Annex C, Table C.12 for an inside surface crack. For a surface crack with semi-elliptical shape, the elliptic angle  $\varphi$  is measured from the surface of the crack; see Annex C, Figure C.2.

At the base of the flaw,  $\varphi = 90^{\circ} = \frac{\pi}{2} rad$  .

$$\beta = \frac{2\varphi}{\pi} = \frac{2\left(\frac{\pi}{2}\right)}{\pi} = 1$$

At the surface of the flaw,  $\varphi = 0^{\circ} = 0 \ rad$ .

$$\beta = \frac{2\varphi}{\pi} = \frac{2(0)}{\pi} = 0$$

The coefficients  $A_{0,0} \rightarrow A_{6,0}$  and  $A_{0,1} \rightarrow A_{6,1}$  are linearly interpolated from the values found in Annex C, Table C.12. The flaw ratios and parameters used in Table C.12 are as follows:

$$\begin{cases} \frac{t}{R_i} = \frac{1.6875}{75.125} = 0.0225 \\ \frac{a}{c} = \frac{0.4219}{1.2657} = 0.3333 \\ \frac{a}{t} = 0.25 \end{cases} \Rightarrow \begin{cases} A_{0,0} = 0.7418361 & A_{0,1} = 0.1159489 \\ A_{1,0} = -0.450705 & A_{1,1} = 0.1836592 \\ A_{2,0} = 6.1501137 & A_{2,1} = 2.2622005 \\ A_{3,0} = -15.50397 & A_{3,1} = -4.094396 \\ A_{4,0} = 19.978669 & A_{4,1} = 4.6541091 \\ A_{5,0} = -13.3919 & A_{5,1} = -3.530137 \\ A_{6,0} = 3.627239 & A_{6,1} = 1.1061961 \end{cases}$$

The calculation procedure to determine coefficient  $A_{0,0}$  for  $G_0$  is shown for reference. The procedure requires 3 levels of interpolation within Table C.12.

- 1) Interpolate for  $t/R_i = 0.0225$  between the values of 0.05 and 0.01667
- 2) Interpolate for a/c = 0.333 between the values of 0.25 and 0.50
- 3) Interpolate for a/t = 0.25 between the values of 0.2 and 0.4

The calculations begin with the interpolation for a/t = 0.25, which must be done for all conditions.

$t / R_i = 0.05, a / c = 0.5$		$t/R_i=0.05,$	a / c = 0.25	$t/R_{_{I}}=0.016$	67, $a/c = 0.5$	$t/R_{i} = 0.0166$	$t/R_i = 0.01667, a/c = 0.25$	
a/t = 0.2	0.8998006	a/t = 0.2	0.6229529	a/t = 0.2	0.8987625	a/t = 0.2	0.6254105	
a/t = 0.4	0.9875950	a/t = 0.4	0.7303090	a/t = 0.4	0.9959920	a/t = 0.4	0.7330250	
a/t = 0.25	0.9217492	a/t = 0.25	0.6497919	a/t = 0.25	0.9230699	a/t = 0.25	0.6523141	

The resulting values are used to interpolate for a/c = 0.333.

$t/R_i =$	= 0.05	$t/R_i = 0.01667$		
<i>a</i> / <i>c</i> = 0.25	0.6497919	a/c = 0.25	0.6523141	
a/c = 0.5	0.9217492	a/c = 0.5	0.9230699	
a/c = 0.333	0.7400817	a/c = 0.333	0.7422050	

The final interpolation is for  $t/R_i = 0.0225$ .

$t / R_i = 0.01667$	0.7422050
$t / R_i = 0.05$	0.7400817
$t/R_i = 0.0225$	0.7418361

This results in a value of  $A_0 = 0.7418361$  for  $G_0$ .

Per Annex D, paragraph C.5.10.2, the influence coefficients  $G_2$ ,  $G_3$ , and  $G_4$  are determined using paragraph C.14.3, Equations (C.274), (C.275), and (C.276), respectively. The value Q is calculated using Annex C, Equation (C.15) or (C.16). Crack and geometry dimensional limits are as follows:

$0.0 \le a/t \le 0.8$	a/t = 0.25	satisfied
$0.03125 \le a/c \le 2.0$	a/c = 0.333	satisfied
$0 \le \varphi \le \pi$	$\varphi = \pi/2$	satisfied
$0.0 \le t/R_i \le 1.0$	$t/R_i = 0.0225$	satisfied

As noted above, the higher order (non-linear) terms  $G_2 \rightarrow G_4$  are eliminated from Equation (C.192); therefore, are not required to be determined.

Where, Q is calculated per Equation (C.15), since  $a/c \le 1.0$ .

$$Q = 1.0 + 1.464 \left(\frac{a}{c}\right)^{1.65} = 1.0 + 1.464 \left(0.333\right)^{1.65} = 1.2389$$

The stress coefficients for a fourth-order polynomial distribution are described in Annex C, paragraph C.2.2.3, Equations (C.3), (C.4), and (C.5). For an equivalent membrane and bending stress distribution, the fourth order polynomial stress distributions reduce to the following for an inside surface crack.

$$\begin{cases} \sigma_{m} = \sigma_{0} + \frac{\sigma_{1}}{2} \\ \sigma_{b} = -\frac{\sigma_{1}}{2} \end{cases} \Rightarrow \begin{cases} \sigma_{0} = \sigma_{m} - \frac{\sigma_{1}}{2} \\ \sigma_{1} = -2\sigma_{b} \end{cases} \Rightarrow \begin{cases} \sigma_{0} = \sigma_{m} + \sigma_{b} \\ \sigma_{1} = -2\sigma_{b} \end{cases}$$

Therefore, for a known membrane and bending stress, Equation (C.3) reduces to the following.

$$\sigma\left(\frac{x}{t}\right) = \left(\sigma_m + \sigma_b\right) - 2\sigma_b\left(\frac{x}{t}\right)$$

With x/t = 1,

$$\sigma_0 = (\sigma_m + \sigma_b) = (31697.1852 + 355.973) = 32053.1582 \text{ psi}$$
  
$$\sigma_1 = -2\sigma_b (1.0) = -2(355.973) = -711.946 \text{ psi}$$

i) STEP 9 – Compute the reference stress for secondary and residual stresses,  $\sigma_{ref}^{SR}$  based on the secondary and residual stress distributions from STEP 2, the modified flaw size from STEP 5, and the reference stress solutions in Annex D.

The reference stress solution for RCSCLE2 is provided in Annex D, paragraph D.5.10, Equation (D.74).

$$\sigma_{ref}^{SR} = \frac{gQ_b + \left[ (gQ_b)^2 + 9(M_s\sigma_r(1-\alpha)^2)^2 \right]^{0.5}}{3(1-\alpha)^2}$$
  
$$\sigma_{ref}^{SR} = \frac{(0.9948)(0.0) + \left[ (0.9948(0.0))^2 + 9(1.0106(9600.0)(1-0.1071)^2)^2 \right]^{0.5}}{3(1-0.1071)^2} = 9701.76 \text{ psi}$$

Where, the following variables were determined in STEP 6.

g = 0.9948  $\alpha = 0.1071$  $M_s = 1.0106$ 

Additionally, the residual stress  $\sigma^r$  calculated in STEP 2 is substituted for  $P_m$  and the secondary bending stress  $Q_b$  is substituted for  $P_b$ .

$$\sigma^r = 9600.0 \ psi$$
$$Q_b = 0.0$$

j) STEP 10 – Compute the stress intensity attributed to the secondary and residual stress,  $K_I^{SR}$ , using the secondary and residual stress distributions from STEP 2, the modified flaw size from STEP 5, and the stress intensity factor solutions in Annex C.

The stress intensity factor solution for KCSCLE2 is provided in Annex C, paragraph C.5.11, Equation (C.192), with reference to paragraph C.5.10. For an equivalent membrane and bending stress distribution of a fourth order polynomial stress distribution, the higher order (non-linear) terms  $G_2 \rightarrow G_4$  and  $\sigma_2 \rightarrow \sigma_4$  can be eliminated from Equation (C.192). Further, the residual stress is a constant value through-thickness; therefore,  $\sigma_1$  can also be eliminated. This reduces the stress intensity factor solution from Equation (C.192) to the following.

$$\begin{split} K_{I}^{SR} = & \left[ G_{o} \left\{ \sigma_{r} + p_{c} \right\} + G_{I} \sigma_{I} \left( \frac{a}{t} \right) + G_{2} \sigma_{2} \left( \frac{a}{t} \right)^{2} + G_{3} \sigma_{3} \left( \frac{a}{t} \right)^{3} + G_{4} \sigma_{4} \left( \frac{a}{t} \right)^{4} \right] \sqrt{\frac{\pi a}{Q}} \\ K_{I}^{SR} = & \left[ G_{o} \left\{ \sigma_{r} + p_{c} \right\} \right] \sqrt{\frac{\pi a}{Q}} \end{split}$$

The value of  $K_{I}^{SR}$  is calculated at the base of the flaw and at the surface of the flaw.

At the base of the flaw,  $\varphi = 90^{\circ}$ :

$$K_{I}^{SR} = \left[G_{o}\left\{\sigma_{r} + p_{c}\right\}\right] \sqrt{\frac{\pi a}{Q}}$$
$$K_{I}^{SR} = \left[1.1512828\left\{9600.0 + 0.0\right\}\right] \sqrt{\frac{\pi \left(0.4219\right)}{1.2389}} = 11.4318 \ ksi\sqrt{in}$$

At the surface of the flaw,  $\varphi = 0^{\circ}$ :

$$K_{I}^{SR} = \left[G_{o}\left\{\sigma_{r} + p_{c}\right\}\right] \sqrt{\frac{\pi a}{Q}}$$
$$K_{I}^{SR} = \left[0.7418361\left\{9600.0 + 0.0\right\}\right] \sqrt{\frac{\pi (0.4219)}{1.2389}} = 7.3662 \ ksi\sqrt{in}$$

Where, the following variables were determined in STEP 8.

At the base of the flaw,  $\varphi = 90^{\circ} \rightarrow G_0 = 1.1512828$ 

At the surface of the flaw,  $\varphi = 0^{\circ} \rightarrow G_0 = 0.7418361$ 

Additionally, the residual stress  $\sigma^r$  calculated in STEP 2 is substituted for  $\sigma_0$  and the residual stress does not act on the crack face pressure.

$$\sigma^r = 9600 \ psi$$
$$p_c = 0.0$$

k) STEP 11 – Compute the plasticity interaction factor,  $\Phi$ , using the following procedure.

1) STEP 11.1 - Since  $K_I^{SR} \neq 0.0$  compute  $L_r^{SR}$ 

$$L_r^{SR} = \frac{\sigma_{ref}^{SR}}{\sigma_{ys}} = \frac{9701.76}{38000.0} = 0.2553$$

2) STEP 11.2, Determine  $\psi$  and  $\phi$  using Tables 9.4 and 9.6, and compute  $\frac{\Phi}{\Phi_0}$  using Equation (9.19). The parameter  $L_r^P$  used to calculate  $\psi$  and  $\phi$  shall be from STEP 7.

Using linear interpolation between the values of  $L_r^P$  and  $L_r^{SR}$  from Tables 9.4 and 9.6 produces the following.

$$\begin{cases} L_r^P = 0.8469 \\ L_r^{SR} = 0.2553 \end{cases} \Longrightarrow \begin{cases} \psi = 0.0258 \\ \phi = 0.1351 \end{cases}$$

Therefore, per Equation (9.19),

$$\frac{\Phi}{\Phi_0} = 1 + \frac{\psi}{\phi} = 1 + \frac{0.0258}{0.1351} = 1.1910$$

3) STEP 11.3, Compute the plasticity interaction factor,  $\Phi$ .

If 
$$0 < L_r^{SR} \le 4.0$$
, then  $\Phi_0 = 1.0$ . Since  $L_r^{SR} = 0.2553$   
$$\Phi = 1 + \frac{\psi}{\phi} = 1 + \frac{0.0258}{0.1351} = 1.1910$$

I) STEP 12 – Determine toughness ratio or ordinate of the FAD assessment point where  $K_I^P$  is the applied stress intensity due to the primary stress distribution from STEP 8,  $K_I^{SR}$  is the applied stress intensity due to the secondary and residual stress intensity from STEP 10,  $K_{mat}$  is the modified material toughness for STEP 5, and  $\Phi$  is the plasticity correction factor from STEP 11.

At the base of the flaw,  $\varphi = 90^{\circ}$ :

$$K_r = \left(\frac{K_I^P + \Phi K_I^{SR}}{K_{mat}}\right) = \left(\frac{38.4648 + (1.1910)(11.4318)}{86.1331}\right) = 0.6046$$

At the surface of the flaw,  $\varphi = 0^{\circ}$ :

$$K_r = \left(\frac{K_I^P + \Phi K_I^{SR}}{K_{mat}}\right) = \left(\frac{24.8464 + (1.1910)(7.3662)}{86.1331}\right) = 0.3903$$

m) STEP 13 – Evaluate results; the FAD Assessment point for the current flaw size and operating conditions is defined as  $(K_r, L_r^P)$ . Plot these points on the Failure Assessment Diagram (FAD), Fig. E3.3.1. The  $L_r$  cut-off for the material use in this assessment is 1.0

At the base of the flaw,  $\varphi = 90^{\circ} \rightarrow (K_r, L_r^p) = (0.6046, 0.8469)$ :

At the surface of the flaw,  $\varphi = 0^{\circ} \rightarrow (K_r, L_r^P) = (0.3903, 0.8469)$ :



Figure E3.3.1 - The Failure assessment diagram (Fig. 9.20 of API 579-1)

Since both of the assessment points falls within the FAD envelop, the component containing this postulated flaw size is acceptable at the assessment temperature  $-20^{\circ}F$  per the Level 2 Assessment procedure.

## DESIGN BY RULE REQUIREMENTS PART CONTENTS

#### 4.1 General Requirements

## 4.1.1 Example E4.1.1 – Review of General Requirements for a Vessel Design

a) General Requirements

An engineer is tasked with developing a design specification for a new pressure vessel that is to be constructed in accordance with ASME B&PV Code, Section VIII, Division 2 (VIII-2). The VIII-2 Code is being considered because the vessel in question is to be constructed of carbon steel with a specified corrosion allowance and a design pressure and temperature of 1650 psig at 200°F. As part of developing the design specification, the following items need to be evaluated.

- b) Scope
  - 1) The vessel may be designed using the design-by-rule procedures of Part 4, using the design-by-analysis rules of Part 5, or a combination of both Parts 4 and 5. Some limitations may apply to either design method.
  - 2) The user of the vessel is responsible for defining all applicable loads and operating conditions that the vessel will be subject to. All loads and conditions must be specified on the User's Design Specification, see Part 2 paragraph 2.2.
  - 3) A fatigue screening shall be applied to all vessel part designed in accordance with this Division to determine if a formal fatigue analysis is required, see Part 5 paragraph 5.5.2.
- c) Minimum Thickness Requirements

Based on product form and process service, the parts of the vessel must meet the minimum thickness requirements presented in Part 4, paragraph 4.1.2.

d) Material Thickness Requirements

Fabrication tolerances must be considered in the design of the vessel components, based on forming, heat treatment and product form.

e) Corrosion Allowance in Design Equations

The equations used in a design-by-rule procedure of Part 4 or the thicknesses used in a designby-analysis of Part 5 must be performed in a corroded condition. The term corrosion allowance is representative of loss of metal due to corrosion, erosion, mechanical abrasion, or other environmental effects. The corrosion allowance must be documented in the User's Design Specification.

- f) Design Basis
  - The pressure used in the design of a vessel component together with the coincident design metal temperature must be specified. Where applicable, the pressure resulting from static head and other static or dynamic loads shall be included in addition to the specified design pressure. The typical loads that need to be considered in the design of a vessel are shown in Part 5, Table 5.1.
  - 2) The specified design temperature shall not be less than the mean metal temperature expected coincidentally with the corresponding maximum pressure.

- 3) A minimum design metal temperature shall be determined and shall consider the coldest operating temperature, operational upsets, auto refrigeration, atmospheric temperature, and any other source of cooling.
- 4) All applicable loads and load case combinations shall be considered in the design to determine the minimum required wall thickness for a vessel part, see Part 4, Table 4.1.1.
- g) Design Allowable Stress

Specifications for all materials of construction are provided in Part 3, Annex 3.A.

## 4.1.2 Example E4.1.2 – Required Wall Thickness of a Hemispherical Head

Determine the required thickness for a hemispherical head at the bottom of a vertical vessel considering the following design conditions. All Category A and B joints are Type 1 butt welds and have been 100% radiographically examined.

Vessel Data:

•	Material	=	SA-516, Grade 70
•	Design Conditions	=	1650 <i>psig</i> @300°F
•	Liquid Head	=	60 <i>ft</i>
•	Liquid Specific Gravity	=	0.89
•	Inside Diameter	=	96.0 in
•	Corrosion Allowance	=	0.125 in
•	Allowable Stress	=	22400 psi
•	Weld Joint Efficiency	=	1.0

The design pressure used to establish the wall thickness must be adjusted for the liquid head in accordance with Part 4, paragraph 4.1.5.2.a

Design Pressure = Specified Design Pressure +  $\gamma gh$ 

Design Pressure = 
$$1650 + \frac{0.89(62.4)(60)}{144} = 1673.140 \text{ psig}$$

In accordance with Part 4, paragraph 4.3.5, determine the required thickness of the bottom hemispherical head.

in

$$t = \frac{D}{2} \left( \exp\left[\frac{0.5P}{SE}\right] - 1 \right)$$
  

$$D = 96.0 + 2 (Corrosion \ Allowance) = 96.0 + 2 (0.125) = 96.25$$
  

$$t = \frac{96.25}{2} \left( \exp\left[\frac{0.5(1673.14)}{22400(1.0)}\right] - 1 \right) = 1.8313 \ in$$
  

$$t = 1.8313 + Corrosion \ Allowance = 1.8313 + 0.125 = 1.9563 \ in$$

The required thickness of the bottom head is 1.9563 in

## 4.1.3 Example E4.1.3 – Required Wall Thickness of a Hemispherical Head - Higher Strength Material

Determine the required thickness for a hemispherical head at the bottom of a vertical vessel in example E.4.1.2 considering the following design conditions. Note that a higher strength material is being used. All Category A and B joints are Type 1 butt welds and have been 100% radiographically examined.

Vessel Data:		
Material	=	SA-537, Class 1
Design Conditions	=	1650 <i>psig</i> @300°F
Liquid Head	=	60 <i>ft</i>
Liquid Specific Gravity	=	0.89
Inside Diameter	=	96.0 in
Corrosion Allowance	=	0.125 in
Allowable Stress	=	29000 psi
Weld Joint Efficiency	=	1.0

The design pressure used to establish the wall thickness must be adjusted for the liquid head in accordance with Part 4, paragraph 4.1.5.2.a

Design Pressure = Specified Design Pressure +  $\gamma gh$ 

Design Pressure =  $1650 + \frac{0.89(62.4)(60)}{144} = 1673.140 \text{ psig}$ 

In accordance with Part 4, paragraph 4.3.5, determine the required thickness of the bottom hemispherical head.

in

$$t = \frac{D}{2} \left( \exp\left[\frac{0.5P}{SE}\right] - 1 \right)$$
  

$$D = 96.0 + 2 \left( Corrosion \ Allowance \right) = 96.0 + 2 \left(0.125\right) = 96.25$$
  

$$t = \frac{96.25}{2} \left( \exp\left[\frac{0.5(1673.14)}{29000(1.0)}\right] - 1 \right) = 1.4085 \ in$$
  

$$t = 1.4085 + Corrosion \ Allowance = 1.4085 + 0.125 = 1.5335 \ in$$

The required thickness of the bottom head constructed with a stronger material is 1.5335 *in*. This represents a savings in material costs of approximately 22%. Additional costs in welding and NDE are also expected. Similar cost savings can be achieved by using a stronger material for the cylinder shell. The design margins in Section VIII, Division 2 will typically result in a more efficient design when higher strength materials are used as shown in this example. For many fluid service environments, higher strength materials may be prone to cracking. However, if PWHT is specified for fluid service, as opposed to wall thickness requirements in accordance with Part 6, the use of higher strength materials may be justified and result in significant cost savings.

## 4.2 Welded Joints

## 4.2.1 Example E4.2.1 – Nondestructive Examination Requirement for Vessel Design

A significant change to the new VIII-2 is the introduction of rules for the examination of welded joints, which introduce Examination Groups. These Examination Groups permit partial radiography and also require surface examination as part of the nondestructive examination (NDE) requirements. Part 7, paragraph 7.4.2 and Table 7.1 of VIII-2 define the different Examination Groups assigned to welded joints based on the manufacturing complexity of the material group, the maximum thickness, the welding process, and the selected joint efficiency. Once the Examination Group is selected, Table 7.2 provides the required NDE, joint category designation, joint efficiency, and acceptable joint types.

An engineer is tasked with developing a design specification for a new pressure vessel that is to be constructed in accordance with ASME B&PV Code, Section VIII, Division 2 (VIII-2). Based on the anticipated fabrication data provided, the engineer compares the Examination Groups to aid in the decision for NDE requirements for vessel design as follows.

#### Vessel Data:

•	Material	=	<i>SA</i> -516, <i>Grade</i> 70
•	Welding Processes	=	SAW and SMAW
•	Weld Categories	=	A, B, C, D
•	Weld Joint Types	=	<i>Type</i> $1 \rightarrow Category A, B, and C$
		=	<i>Type</i> 7 $\rightarrow$ <i>Category D</i>

The definitions of Weld Categories and Weld Joint Types are provided in Part 4, Tables 4.2.1 and 4.2.2, respectively. Some acceptable weld joints for main shell weld seams, formed heads, flange attachments, and nozzles are provided in Tables 4.2.4, 4.2.5, 4.2.9, and 4.2.10, respectively.

Per Part 7, Table 7.1, a comparison of Examination Group 1b and Examination Group 3b for carbon steel material, SA-516, *Grade* 70 is as follows.

Deremeter	Examination Group		
Palameter	1b	3b	
Permitted Material	P-No. 1 Gr 1 and 2	P-No. 1 Gr 1 and 2	
Maximum Thickness Unlimited		50 mm (2 inches)	
Welding Process	Unrestricted	Unrestricted	
Design Basis	Part 4 or Part 5	Part 4	

From the results of the comparison above, there are two parameters that will require a decision to be made by the engineer prior to assigning an Examination Group, maximum thickness of the vessel components and design basis. A preliminary check of the required wall thickness for the main cylinder and heads can be performed in accordance with the rules of Part 4. However, if there is one or several components that may require their design to be based on numerical analysis, i.e. finite element analysis per Part 5, only Examination Group 1b would be permitted.

Per Part 7, Table 7.2, a comparison	of the required NDE for Examination	Group 1b and Examination
Group 3b for carbon steel material,	SA-516, <i>Grade</i> 70 is as follows.	

Examination Group			1b	3b
Joint Efficiency			1.0	0.85
Joint Category	bint Category Type of Weld Type of NDE		Extent of NDE	
A	Type 1: Full Penetration Longitudinal Seam	RT or UT MT or PT	100% 10%	10% 10%
В	Type 1: Full Penetration Circumferential Seam	RT or UT MT or PT	100% 10%	10% 10%
С	Type 1: Full Penetration Flange/Nozzle Attachment	RT or UT MT or PT	100% 10%	10% 10%
P	Type 7: Full Penetration Corner Joint, Nozzle d > 150 mm(NPS 6)or t > 16 mm(0.625 in)	RT or UT MT or PT	100% 10%	10% 10%
U	Type 7: Full Penetration Corner Joint, Nozzle $d \le 150 mm (NPS \ 6) and$ $t \le 16 mm (0.625 in)$	MT or PT	10%	10%

A review of the above table indicates that more inspection is required for Examination Group 1b when compared to 3b. However, the increased costs for examination may be offset by the materials and fabrication savings. Consider the following comparison for a cylindrical shell.

#### Vessel Data:

•	Material	=	SA-516, Grade 70
•	Design Conditions	=	725 <i>psig</i> @300°F
•	Inside Diameter	=	60.0 in
•	Corrosion Allowance	=	0.125 in
•	Allowable Stress	=	22400 <i>psi</i>

For examination Group 1b, consider the requirements for a Category A Type 1 weld. The required wall thickness in accordance with Part 4, paragraph 4.3.3 is computed as shown below.

$$t = \frac{D}{2} \left( \exp\left[\frac{P}{SE}\right] - 1 \right)$$
  

$$D = 60.0 + 2 (Corrosion \ Allowance) = 60.0 + 2 (0.125) = 60.25 \ in$$
  

$$t = \frac{60.25}{2} \left( \exp\left[\frac{725}{22400(1.0)}\right] - 1 \right) = 0.9910 \ in$$
  

$$t = 0.9910 + Corrosion \ Allowance = 0.9910 + 0.125 = 1.1160 \ in$$

Alternatively, for examination Group 3b, the required wall thickness for a Category A Type 1 weld in accordance with Part 4, paragraph 4.3.3 is computed as shown below.

$$t = \frac{D}{2} \left( \exp\left[\frac{P}{SE}\right] - 1 \right)$$
  

$$D = 60.0 + 2 \left( Corrosion \ Allowance \right) = 60.0 + 2 \left( 0.125 \right) = 60.25 \ in$$
  

$$t = \frac{60.25}{2} \left( \exp\left[\frac{725}{22400(0.85)}\right] - 1 \right) = 1.1692 \ in$$
  

$$t = 1.1692 + Corrosion \ Allowance = 1.1692 + 0.125 = 1.2942 \ in$$

Examination Group 1b when compared to 3b results in an approximate 14% reduction in wall thickness. Cost savings for this reduction in wall thickness will include less material and less welding, and these reductions may offset the increased examination costs. It should also be noted that many refining and petrochemical companies invoke additional examination requirements in their associated corporate standards based on fluid service. Therefore, in some cases the increased examination requirements for Examination Group 1b may already be required based on fluid service.

## 4.2.2 Example E4.2.2 – Nozzle Detail and Weld Sizing

Determine the required fillet weld size and inside corner radius of a set-in type nozzle as shown in Table 4.2.10, Detail 4. The vessel and nozzle were designed such that their nominal thicknesses were established as follows.

Vessel Data:

•	Cylinder Thickness	=	0.625 inches
•	Nozzle Diameter	=	NPS 10
•	Nozzle Thickness	=	Schedule $XS \rightarrow 0.500$ inches
•	Corrosion Allowance	=	0.125 inches

The minimum fillet weld throat dimension,  $t_c$ , is calculated as follows.

$$t_{c} \geq \min[0.7t_{n}, 6\,mm \ (0.25 \ in)]$$
  

$$t_{n} = 0.500 - Corrosion \ Allowance = 0.5 - 0.125 = 0.375 \ in$$
  

$$t_{c} \geq \min[0.70(0.375), 0.25]$$
  

$$t_{c} \geq 0.25 \ in$$

The resulting fillet weld leg size is determined as,  $\frac{t_c}{0.7} = 0.357 \text{ in}$ . Therefore, a fillet weld leg size of 0.375 in would be acceptable.

The minimum inside corner radius,  $r_1$ , is calculated as follows.

$$0.125t \le r_1 \le 0.5t$$
  

$$t = 0.625 - Corrosion \ Allowance = 0.625 - 0.125 = 0.500 \ in$$
  

$$0.125(0.500) \le r_1 \le 0.5(0.500)$$
  

$$0.0625 \le r_1 \le 0.250 \ in$$



Figure E4.2.2 - Nozzle Detail

## 4.2.3 Example E4.2.3 – Nozzle Detail with Reinforcement Pad and Weld Sizing

Determine the required fillet weld sizes and inside corner radius of a set-in type nozzle with added reinforcement pad as shown in Table 4.2.11, Detail 2. The vessel and nozzle were designed such that their nominal thicknesses were established as follows.

Vessel Data:

•	Cylinder Thickness	=	0.625 inches
•	Nozzle Diameter	=	NPS 10
•	Nozzle Thickness	=	Schedule $XS \rightarrow 0.500$ inches
•	Reinforcement Pad Thickness	=	0.625 inches
•	Corrosion Allowance	=	0.125 inches

The minimum fillet weld throat dimension,  $t_c$ , is calculated as follows.

$$t_{c} \ge \min[0.7t_{n}, 6 mm \ (0.25 \ in)]$$
  
$$t_{n} = 0.500 - Corrosion \ Allowance = 0.5 - 0.125 = 0.375 \ in$$
  
$$t_{c} \ge \min[0.7(0.375), 0.25]$$
  
$$t_{c} \ge 0.25 \ in$$

The resulting fillet weld leg size is determined as,  $\frac{t_c}{0.7} = 0.357$  *in*. Therefore, a fillet weld leg size of 0.375 *in* would be acceptable.

The minimum fillet weld throat dimension,  $t_{f1}$ , is calculated as follows.

$$t_{f1} \ge \min[0.6t_e, 0.6t]$$
  

$$t = 0.625 - Corrosion \ Allowance = 0.625 - 0.125 = 0.500 \ in$$
  

$$t_{f1} \ge \min[0.6(0.625), 0.6(0.500)]$$
  

$$t_{f1} \ge 0.300 \ in$$

The resulting fillet weld leg size is determined as,  $\frac{t_{f1}}{0.7} = 0.429$  *in*. Therefore, a fillet weld leg size of 0.4375 *in* would be acceptable.

The minimum inside corner radius,  $r_1$ , is calculated as follows.

$$0.125t \le r_1 \le 0.5t$$
  

$$t = 0.625 - Corrosion \ Allowance = 0.625 - 0.125 = 0.500 \ in$$
  

$$0.125(0.500) \le r_1 \le 0.5(0.500)$$
  

$$0.0625 \le r_1 \le 0.250 \ in$$



Figure E4.2.3 - Nozzle Detail
# 4.3 Internal Design Pressure

# 4.3.1 Example E4.3.1 – Cylindrical Shell

Determine the required thickness for a cylindrical shell considering the following design conditions. All Category A and B joints are Type 1 butt welds and have been 100% radiographically examined.

Vessel Data:

•	Material	=	<i>SA</i> –516, <i>Grade</i> 70, <i>Norm</i> .
•	Design Conditions	=	356 psig@300°F
•	Inside Diameter	=	90.0 in
•	Corrosion Allowance	=	0.125 in
•	Allowable Stress	=	22400 psi
•	Weld Joint Efficiency	=	1.0

Adjust for corrosion allowance.

$$D = 90.0 + 2(Corrosion \ Allowance) = 90.0 + 2(0.125) = 90.25 \ in$$

Evaluate per paragraph 4.3.3.

$$t = \frac{D}{2} \left( \exp\left[\frac{P}{SE}\right] - 1 \right)$$
  
$$t = \frac{90.25}{2} \left( \exp\left[\frac{356}{22400(1.0)}\right] - 1 \right) = 0.7229 \text{ in}$$
  
$$t = 0.7229 + Corrosion \text{ Allowance} = 0.7229 + 0.125 = 0.8479 \text{ in}$$

The required thickness is 0.8479 in.

Combined Loadings – cylindrical shells subject to internal pressure and other loadings shall satisfy the requirements of paragraph 4.3.10. In this example problem, the cylindrical shell is only subject to internal pressure.

## 4.3.2 Example E4.3.2 – Conical Shell

Determine the required thickness for a conical shell considering the following design conditions. All Category A and B joints are Type 1 butt welds and have been 100% radiographically examined.

Vessel Data:

•	Material	=	SA-516, Grade 70, Norm.
•	Design Conditions	=	356 psig@300°F
•	Inside Diameter (Large End)	=	150.0 in
•	Inside Diameter (Small End)	=	90.0 in
•	Length of Conical Section	=	78.0 in
•	Corrosion Allowance	=	0.125 in
•	Allowable Stress	=	22400 psi
•	Weld Joint Efficiency	=	1.0

Adjust for corrosion allowance and determine the cone angle.

$$D_{L} = 150.0 + 2(Corrosion \ Allowance) = 150.0 + 2(0.125) = 150.25 \ in$$
$$D_{s} = 90.0 + 2(Corrosion \ Allowance) = 150.0 + 2(0.125) = 90.25 \ in$$
$$L_{C} = 78.0$$
$$\alpha = \arctan\left[\frac{0.5(D_{L} - D_{s})}{L_{C}}\right] = \arctan\left[\frac{0.5(150.25 - 90.25)}{78.0}\right] = 21.0375 \ deg$$

Evaluate per paragraph 4.3.4 using the large end diameter of the conical shell.

$$t = \frac{D}{2\cos[\alpha]} \left( \exp\left[\frac{P}{SE}\right] - 1 \right)$$
  
$$t = \frac{150.25}{2\cos[21.0375]} \left( \exp\left[\frac{356}{22400(1.0)}\right] - 1 \right) = 1.2894 \text{ in}$$
  
$$t = 1.2894 + Corrosion \text{ Allowance} = 1.2894 + 0.125 = 1.4144 \text{ in}$$

The required thickness is 1.4144 in.

Combined Loadings – conical shells subject to internal pressure and other loadings shall satisfy the requirements of paragraph 4.3.10. In this example problem, the conical shell is only subject to internal pressure.

# 4.3.3 Example E4.3.3 – Spherical Shells

Determine the required thickness for a spherical shell considering the following design conditions. All Category A joints are Type 1 butt welds and have been 100% radiographically examined.

Vessel Data:

•	Material	=	SA-542, Type D, Class 4a
•	Design Conditions	=	2080 psig@850°F
•	Inside Diameter	=	149.0 in
•	Corrosion Allowance	=	0.0 <i>in</i>
•	Allowable Stress	=	28900 psi
•	Weld Joint Efficiency	=	1.0

Evaluate per paragraph 4.3.5.

$$t = \frac{D}{2} \left( \exp\left[\frac{0.5P}{SE}\right] - 1 \right) = \frac{149.0}{2} \left( \exp\left[\frac{0.5(2080)}{28900(1.0)}\right] - 1 \right) = 2.7298 \text{ in}$$

The required thickness is 2.7298 in.

Combined Loadings – spherical shells and hemispherical heads subject to internal pressure and other loadings shall satisfy the requirements of paragraph 4.3.10. In this example problem, the spherical shell is only subject to internal pressure.

## 4.3.4 Example E4.3.4 – Torispherical Head

Determine the maximum allowable working pressure (MAWP) for the proposed seamless torispherical head. The Category B joint joining the head to the shell is a Type 1 butt weld and has been 100% radiographically examined.

Vessel Data:

•	Material	=	SA-387, Grade 11, Class 1
•	Design Temperature	=	650°F
•	Inside Diameter	=	72.0 in
•	Crown Radius	=	72.0 in
•	Knuckle Radius	=	4.375 in
•	Thickness	=	0.625 in
•	Corrosion Allowance	=	0.125 in
•	Allowable Stress	=	18000 <i>psi</i>
•	Weld Joint Efficiency	=	1.0
•	Modulus of Elasticity at Design Temperature	=	$26.55E + 06 \ psi$
•	Yield Strength at Design Temperature	=	26900 psi

Evaluate per paragraph 4.3.6.

a) STEP 1 – Determine, D, assume values for L, r and t (known variables from above) and adjust for corrosion.

$$D = 72.0 + 2(Corrosion \ Allowance) = 72.0 + 2(0.125) = 72.25 \ in$$
  

$$L = 72.0 + Corrosion \ Allowance = 72.0 + 0.125 = 72.125 \ in$$
  

$$r = 4.375 + Corrosion \ Allowance = 4.375 + 0.125 = 4.5 \ in$$
  

$$t = 0.625 - Corrosion \ Allowance = 0.625 - 0.125 = 0.5 \ in$$

b) STEP 2 – Compute the head L/D, r/D, and L/t ratios and determine if the following equations are satisfied.

$$0.7 \le \left\{ \frac{L}{D} = \frac{72.125}{72.25} = 0.9983 \right\} \le 1.0 \qquad True$$
$$\left\{ \frac{r}{D} = \frac{4.5}{72.25} = 0.0623 \right\} \ge 0.06 \qquad True$$

$$20 \le \left\{ \frac{L}{t} = \frac{72.125}{0.5} = 144.25 \right\} \le 2000 \qquad True$$

c) STEP 3 – Calculate the geometric constants  $\beta_{th}$  ,  $\phi_{th}$  ,  $R_{th}$ 

$$\beta_{th} = \arccos\left[\frac{0.5D - r}{L - r}\right] = \arccos\left[\frac{0.5(72.25) - 4.5}{72.125 - 4.5}\right] = 1.0842 \ rad$$
$$\phi_{th} = \frac{\sqrt{Lt}}{r} = \frac{\sqrt{72.125(0.5)}}{4.5} = 1.3345 \ rad$$

Since  $\phi_{th} \ge \beta_{th}$ , calculate  $R_{th}$  as follows:

$$R_{th} = 0.5D = 0.5(72.25) = 36.125$$
 in

d) STEP 4 – Compute the coefficients  $C_1$  and  $C_2$ 

Since 
$$\frac{r}{D} = 0.0623 \le 0.08$$
, calculate  $C_1$  and  $C_2$  as follows:  
 $C_1 = 9.31 \left(\frac{r}{D}\right) - 0.086 = 9.31 (0.0623) - 0.086 = 0.4940$   
 $C_2 = 1.25$ 

e) STEP 5 – Calculate the value of internal pressure expected to produce elastic buckling of the knuckle,  $P_{eth}$ .

$$P_{eth} = \frac{C_1 E_T t^2}{C_2 R_{th} \left(\frac{R_{th}}{2} - r\right)} = \frac{(0.4940)(26.55E + 06)(0.5)^2}{1.25(36.125)\left(\frac{36.125}{2} - 4.5\right)} = 5353.9445 \ psi$$

f) STEP 6 – Calculate the value of internal pressure that will result in a maximum stress in the knuckle equal to the material yield strength,  $P_{y}$ .

$$P_{y} = \frac{C_{3}t}{C_{2}R_{th}\left(\frac{R_{th}}{2r} - 1\right)}$$

Since the allowable stress at design temperature is governed by time-independent properties,  $C_3$  is the material yield strength at the design temperature, or  $C_3 = S_y$ .

$$P_{y} = \frac{26900(0.5)}{1.25(36.125)\left(\frac{36.125}{2(4.5)} - 1\right)} = 98.8274 \ psi$$

g) STEP 7 – Calculate the value of internal pressure expected to result in a buckling failure of the knuckle,  $P_{ck}$ .

Calculate variable, G:

$$G = \frac{P_{eth}}{P_y} = \frac{5353.9445}{98.8274} = 54.1747$$

Since G > 1.0, calculate  $P_{ck}$  as follows:

$$P_{ck} = \left(\frac{0.77508G - 0.20354G^{2} + 0.019274G^{3}}{1 + 0.19014G - 0.089534G^{2} + 0.0093965G^{3}}\right) (P_{y})$$

$$P_{ck} = \left(\frac{0.77508(54.1747) - 0.20354(54.1747)^{2} +}{0.019274(54.1747)^{3}} + \frac{0.019274(54.1747)^{3}}{1 + 0.19014(54.1747) - 0.089534(54.1747)^{2} +} \right) (98.8274)$$

$$P_{ck} = 199.5671 \, psi$$

h) STEP 8 – Calculate the allowable pressure based on a buckling failure of the knuckle,  $P_{ak}$ .

$$P_{ak} = \frac{P_{ck}}{1.5} = \frac{199.5671}{1.5} = 133.0447 \ psi$$

i) STEP 9 – Calculate the allowable pressure based on rupture of the crown,  $P_{ac}$ .

$$P_{ac} = \frac{2SE}{\frac{L}{t} + 0.5} = \frac{2(18000)(1.0)}{\frac{72.125}{0.5} + 0.5} = 248.7047 \ psi$$

j) STEP 10 – Calculate the maximum allowable internal pressure,  $P_a$ .

$$P_a = \min[P_{ak}, P_{ac}] = \min[133.0447, 248.7047] = 133.0 \ psi$$

k) STEP 11 – If the allowable internal pressure computed from STEP 10 is greater than or equal to the design pressure, then the design is complete. If the allowable internal pressure computed from STEP 10 is less than the design pressure, then increase the head thickness and repeat steps 2 through 10.

The MAWP is 133.0 psi.

Combined Loadings – torispherical heads subject to internal pressure and other loadings shall satisfy the requirements of paragraph 4.3.10. In this calculation, the torispherical head shall be approximated as an equivalent spherical shell with a radius equal to L. In this example problem, the torisphercial head is only subject to internal pressure.

# 4.3.5 Example E4.3.5 – Elliptical Head

Determine the maximum allowable working pressure (MAWP) for the proposed seamless 2:1 elliptical head. The Category B joint joining the head to the shell is a Type 1 butt weld and has been 100% radiographically examined.

Vessel Data:

•	Material	=	SA-516, Grade 70, Norm.
•	Design Temperature	=	300°F
•	Inside Diameter	=	90.0 in
•	Thickness	=	1.125 in
•	Corrosion Allowance	=	0.125 in
•	Allowable Stress	=	22400 psi
•	Weld Joint Efficiency	=	1.0
•	Modulus of Elasticity at Design Temperature	=	$28.3E + 06 \ psi$
•	Yield Strength at Design Temperature	=	33600 <i>psi</i>

Evaluate per paragraph 4.3.7 and paragraph 4.3.6.

Calculate k, using the uncorroded inside diameter, D and depth of head, h.

$$k = \frac{D}{2h} = \frac{90.0}{2(22.5)} = 2.0$$

Verify variable k is within the established limits, permitting the use of the rules of paragraph 4.3.7.

$$1.7 \le \{k = 2\} \le 2.2$$
 True

Determine the variables r and L using the uncorroded inside diameter, D.

$$r = D\left(\frac{0.5}{k} - 0.08\right) = 90.0\left(\frac{0.5}{2.0} - 0.08\right) = 15.3 \text{ in}$$
$$L = D\left(0.44k + 0.02\right) = 90.0\left(0.44(2.0) + 0.02\right) = 81.0 \text{ in}$$

Proceed with the design following the steps outlined in paragraph 4.3.6.

a) STEP 1 – Determine, D, assume values for L, r and t (determined from paragraph 4.3.7) and adjust for corrosion.

$$D = 90.0 + 2(Corrosion \ Allowance) = 90.0 + 2(0.125) = 90.25 \ in$$
  

$$L = 81.0 + Corrosion \ Allowance = 81.0 + 0.125 = 81.125 \ in$$
  

$$r = 15.3 + Corrosion \ Allowance = 15.3 + 0.125 = 15.425 \ in$$
  

$$t = 1.125 - Corrosion \ Allowance = 1.125 - 0.125 = 1.0 \ in$$

b) STEP 2 – Compute the head L/D, r/D, and L/t ratios and determine if the following equations are satisfied.

$$0.7 \le \left\{ \frac{L}{D} = \frac{81.125}{90.25} = 0.8989 \right\} \le 1.0 \qquad True$$
$$\left\{ \frac{r}{D} = \frac{15.425}{90.25} = 0.1709 \right\} \ge 0.06 \qquad True$$
$$20 \le \left\{ \frac{L}{t} = \frac{81.125}{1.000} = 81.125 \right\} \le 2000 \qquad True$$

c) STEP 3 – Calculate the geometric constants  $\beta_{th}$  ,  $\phi_{th}$  ,  $R_{th}$ 

$$\beta_{th} = \arccos\left[\frac{0.5D - r}{L - r}\right] = \arccos\left[\frac{0.5(90.25) - 15.425}{81.125 - 15.425}\right] = 1.1017 \ rad$$
$$\phi_{th} = \frac{\sqrt{Lt}}{r} = \frac{\sqrt{81.125(1.0)}}{15.425} = 0.5839 \ rad$$

Since  $\phi_{th} < \beta_{th}$ , calculate  $R_{th}$  as follows:

$$R_{th} = \frac{0.5D - r}{\cos[\beta_{th} - \phi_{th}]} + r = \frac{0.5(90.25) - 15.425}{\cos[1.1017 - 0.5839]} + 15.425 = 49.6057 \text{ in}$$

d) STEP 4 – Compute the coefficients  $C_1$  and  $C_2$ 

Since 
$$\frac{r}{D} = 0.1709 > 0.08$$
, calculate  $C_1$  and  $C_2$  as follows:  
 $C_1 = 0.692 \left(\frac{r}{D}\right) + 0.605 = 0.692 \left(0.1709\right) + 0.605 = 0.7233$   
 $C_2 = 1.46 - 2.6 \left(\frac{r}{D}\right) = 1.46 - 2.6 \left(0.1709\right) = 1.0157$ 

e) STEP 5 – Calculate the value of internal pressure expected to produce elastic buckling of the knuckle,  $P_{\it eth}$  .

$$P_{eth} = \frac{C_1 E_T t^2}{C_2 R_{th} \left(\frac{R_{th}}{2} - r\right)} = \frac{\left(0.7233\right) \left(28.3E + 06\right) \left(1.0\right)^2}{1.0157 \left(49.6057\right) \left(\frac{49.6057}{2} - 15.425\right)} = 43321.6096 \ psi$$

f) STEP 6 – Calculate the value of internal pressure that will result in a maximum stress in the knuckle equal to the material yield strength,  $P_y$ .

$$P_{y} = \frac{C_{3}t}{C_{2}R_{th}\left(\frac{R_{th}}{2r} - 1\right)}$$

Since the allowable stress at design temperature is governed by time-independent properties,  $C_3$  is the material yield strength at the design temperature, or  $C_3 = S_y$ .

$$P_{y} = \frac{33600(1.0)}{1.0157(49.6057) \left(\frac{49.6057}{2(15.425)} - 1\right)} = 1096.8927 \ psi$$

g) STEP 7 – Calculate the value of internal pressure expected to result in a buckling failure of the knuckle,  $P_{ck}$ .

Calculate variable, G:

$$G = \frac{P_{eth}}{P_{v}} = \frac{43321.6096}{1096.8927} = 39.4948$$

Since G > 1.0, calculate  $P_{ck}$  as follows:

$$P_{ck} = \left(\frac{0.77508G - 0.20354G^{2} + 0.019274G^{3}}{1 + 0.19014G - 0.089534G^{2} + 0.0093965G^{3}}\right) (P_{y})$$

$$P_{ck} = \left(\frac{0.77508(39.4948) - 0.20354(39.4948)^{2} + 0.019274(39.4948)^{3}}{1 + 0.19014(39.4948) - 0.089534(39.4948)^{2} + 0.0093965(39.4948)^{3}}\right) (1096.8927)$$

$$P_{ck} = 2206.1634 \ psi$$

h) STEP 8 – Calculate the allowable pressure based on a buckling failure of the knuckle,  $P_{ak}$ .

$$P_{ak} = \frac{P_{ck}}{1.5} = \frac{2206.1634}{1.5} = 1470.8 \ psi$$

i) STEP 9 – Calculate the allowable pressure based on rupture of the crown,  $P_{ac}$ .

$$P_{ac} = \frac{2SE}{\frac{L}{t} + 0.5} = \frac{2(22400)(1.0)}{\frac{81.125}{1.0} + 0.5} = 548.9 \ psi$$

j) STEP 10 – Calculate the maximum allowable internal pressure,  $P_a$ .

$$P_a = \min[P_{ak}, P_{ac}] = \min[1470.8, 548.9] = 548.9 \ psi$$

k) STEP 11 – If the allowable internal pressure computed from STEP 10 is greater than or equal to the design pressure, then the design is complete. If the allowable internal pressure computed from STEP 10 is less than the design pressure, then increase the head thickness and repeat STEPs 2 through 10. The MAWP is 548.9 psi.

Combined Loadings – ellipsoidal heads subject to internal pressure and other loadings shall satisfy the requirements of paragraph 4.3.10. In this calculation, the ellipsoidal head shall be approximated as an equivalent spherical shell with a radius equal to L. In this example problem, the ellipsoidal head is only subject to internal pressure.

# 4.3.6 Example E4.3.6 – Combined Loadings and Allowable Stresses

Determine the maximum tensile stress of the proposed cylindrical shell section considering the following design conditions and specified applied loadings. All Category A and B joints are Type 1 butt welds and have been 100% radiographically examined.

Vessel Data:

•	Material	=	SA-516, Grade 70, Norm
•	Design Conditions	=	356 psig@300°F
•	Inside Diameter	=	90.0 in
•	Thickness	=	1.125 in
•	Corrosion Allowance	=	0.125 in
•	Allowable Stress	=	22400 psi
•	Weld Joint Efficiency	=	1.0
•	Axial Force	=	-66152.5 <i>lbs</i>
•	Net Section Bending Moment	=	3.048E + 06 in - lbs
•	Torsional Moment	=	0.0 in-lbs

Adjust variables for corrosion and determine outside dimensions.

$$D = 90.0 + 2(Corrosion \ Allowance) = 90.0 + 2(0.125) = 90.25 \ in$$
  
$$D_o = 90.0 + 2(Uncorroded \ Thickness) = 90.0 + 2(1.125) = 92.25 \ in$$
  
$$t = 1.125 - Corrosion \ Allowance = 1.125 - 0.125 = 1.0 \ in$$

Evaluate per paragraph 4.3.10.

The loads transmitted to the cylindrical shell are given in the Table E4.3.6.2. Note that this table is given in terms of the load parameters and load combinations shown in Part 4, Table 4.1.1 and Table 4.1.2 (Table E4.3.6.1 of this example). As shown in Table E4.3.6.1, the acceptance criteria is that the general primary membrane stress for each load case must be less than or equal to the allowable stress at the specified design condition.

Determine applicability of the rules of paragraph 4.3.10 based on satisfaction of the following requirements.

The section of interest is at least  $2.5\sqrt{Rt}$  away from any major structural discontinuity.

$$2.5\sqrt{Rt} = 2.5\sqrt{\left(\frac{90.25}{2}\right)(1.125)} = 16.7938 \text{ in}$$

Shear force is not applicable.

The shell R/t ratio is greater than 3.0, or:

$$\left\{ R / t = \frac{90.25 / 2}{1.0} = 45.125 \right\} > 3.0$$
 True

Paragraph 4.3.10.2, the following procedure shall be used to design cylindrical, spherical, and conical shells subjected to internal pressure plus supplemental loads of applied net section axial force, bending moment, and torsional moment as shown below. By inspection of the results shown in Table E4.3.6.3, Load Case 5 is determined to be the governing load case. The pressure and net section axial force, bending moment, and torsional moment at the location of interest for Load Case 5 are:

$$P = 320.4 \ psi$$
  
 $F_5 = -66152.5 \ lbs$   
 $M_5 = 1828800 \ in - lbs$   
 $M_{15} = 0.0 \ in - lbs$ 

a) STEP 1 – Calculate the membrane stress for the cylindrical shell. Note that the maximum bending stress occurs at  $\theta = 0.0 \ deg$ .

$$\sigma_{\theta m} = \frac{PD}{E(D_o - D)} = \frac{320.4(90.25)}{1.0(92.25 - 90.25)} = 14458.05 \ psi$$

$$\sigma_{sm} = \frac{1}{E} \left( \frac{PD^2}{D_o^2 - D^2} + \frac{4F}{\pi (D_o^2 - D^2)} + \frac{32MD_o \cos[\theta]}{\pi (D_o^4 - D^4)} \right)$$

$$\sigma_{sm} = \frac{1}{1.0} \left( \frac{320.4(90.25)^2}{(92.25)^2 - (90.25)^2} + \frac{32(1828800)(92.25)\cos[0.0]}{\pi [(92.25)^4 - (90.25)^4]} \right)$$

$$\sigma_{sm} = \begin{cases} 7149.8028 + (-230.7616) + 282.6779 = 7201.7191 \ psi}{7149.8028 + (-230.7616) - 282.6779 = 6636.3633 \ psi} \end{cases}$$

$$\tau = \frac{16M_i D_o}{\pi (D_o^4 - D^4)} = \frac{16(0.0)(92.25)}{\pi [(92.25)^4 - (90.25)^4]} = 0.0 \ psi$$

b) STEP 2 – Calculate the principal stresses.

$$\sigma_{1} = 0.5 \left( \sigma_{\theta m} + \sigma_{sm} + \sqrt{\left( \sigma_{\theta m} - \sigma_{sm} \right)^{2} + 4\tau^{2}} \right)$$
  
$$\sigma_{1} = \begin{cases} 0.5 \left( 14458.05 + 7201.7191 + \sqrt{\left( 14458.05 - 7201.7191 \right)^{2} + 4\left( 0.0 \right)^{2}} \right) = 14458.05 \ psi \\ 0.5 \left( 14458.05 + 6636.3633 + \sqrt{\left( 14458.05 - 6636.3633 \right)^{2} + 4\left( 0.0 \right)^{2}} \right) = 14458.05 \ psi \end{cases}$$

$$\sigma_{2} = 0.5 \left( \sigma_{\theta m} + \sigma_{sm} - \sqrt{\left( \sigma_{\theta m} - \sigma_{sm} \right)^{2} + 4\tau^{2}} \right)$$

$$\sigma_{2} = \begin{cases} 0.5 \left( 14458.05 + 7201.7191 - \sqrt{\left( 14458.05 - 7201.7191 \right)^{2} + 4\left( 0.0 \right)^{2}} \right) \right] \\ 0.5 \left( 14458.05 + 6636.3633 - \sqrt{\left( 14458.05 - 6636.3633 \right)^{2} + 4\left( 0.0 \right)^{2}} \right) \right] \\ \sigma_{2} = \begin{cases} 7201.7191 \ psi \\ 6636.3633 \ psi \end{cases}$$

$$\sigma_3 = \sigma_r = -0.5P = -0.5(320.4) = -160.2 \ psi$$

c) STEP 3 – At any point on the shell, the following limit shall be satisfied.

$$\begin{split} \sigma_{e} &= \frac{1}{\sqrt{2}} \Big[ \left( \sigma_{1} - \sigma_{2} \right)^{2} + \left( \sigma_{2} - \sigma_{3} \right)^{2} + \left( \sigma_{3} - \sigma_{1} \right)^{2} \Big]^{0.5} \leq SE \\ \sigma_{e} &= \begin{cases} \frac{1}{\sqrt{2}} \Bigg[ \left( 14458.05 - 7201.7191 \right)^{2} + \\ \left( 7201.7191 - \left( -160.2 \right) \right)^{2} + \left( \left( -160.2 \right) - 14458.05 \right)^{2} \Bigg]^{0.5} = 12659.9 \ psi \\ \frac{1}{\sqrt{2}} \Bigg[ \left( 14458.05 - 6636.3633 \right)^{2} + \\ \left( 6636.3633 - \left( -160.2 \right) \right)^{2} + \left( \left( -160.2 \right) - 14458.05 \right)^{2} \Bigg]^{0.5} = 12670.1 \ psi \\ \begin{cases} \sigma_{e} = 12659.9 \ psi \\ \sigma_{e} = 12670.1 \ psi \end{cases} \leq \{SE = 22400 \ psi \} \end{split}$$

Since the maximum tensile stress is less than the acceptance criteria, the shell section is adequately designed.

d) STEP 4 – For cylindrical and conical shells, if the meridional stress,  $\sigma_{sm}$  is compressive, then Equation (4.3.45) shall be satisfied where  $F_{xa}$  is evaluated using paragraph 4.4.12.2 with  $\lambda = 0.15$ .

Step 4 is not necessary in this example because the meridional stress,  $\sigma_{\rm sm}$  , is tensile.

Table 4.1.1 – Design Loads				
Design Load Parameter	Description			
P Internal or External Specified Design Pressure (see para 4.1.5.2.a)				
<i>P<sub>s</sub></i> Static head from liquid or bulk materials (e.g. catalyst)				
D	<ul> <li>Dead weight of the vessel, contents, and appurtenances at the location of interest, including the following:</li> <li>Weight of vessel including internals, supports (e.g. skirts, lugs, saddles, and legs), and appurtenances (e.g. platforms, ladders, etc.)</li> <li>Weight of vessel contents under operating and test conditions</li> <li>Refractory linings, insulation</li> <li>Static reactions from the weight of attached equipment, such as motors, machinery, other vessels, and piping</li> <li>Transportation Loads (The static forces obtained as equivalent to the dynamic loads experienced during normal operation of a transport vessel – see paragraph 1.2.1.2.b)</li> </ul>			
L	<ul> <li>Appurtenance Live loading</li> <li>Effects of fluid flow, steady state or transient</li> <li>Loads resulting from wave action</li> </ul>			
E	Earthquake loads (see ASCE 7 for the specific definition of the earthquake load, as applicable)			
W	Wind Loads (See 4.1.5.3.b)			
S	Snow Loads			
F	<i>F</i> Loads due to Deflagration			

Table E4.3.6.1 - Desig	n Loads an	d Load Combina	tions from VIII-2
	<b>_</b> oaao a		

Table 4.1.2 – Design Load Combinations			
Design Load Combination (1)	General Primary Membrane Allowable Stress (2)		
$P + P_s + D$	S		
$P + P_s + D + L$	S		
$P + P_s + D + S$	S		
$0.9P + P_s + D + 0.75L + 0.75S$	S		
$0.9P + P_s + D + (0.6W \text{ or } 0.7E)$	S		
$0.9P + P_{S} + D + 0.75(0.6W \text{ or } 0.7E) + 0.75L + 0.75S$	S		
0.6D + (0.6W  or  0.7E) (3)	S		
$P_s + D + F$	See Annex 4.D		
Notes			

1) The parameters used in the Design Load Combination column are defined in Table 4.1.1.

2) S is the allowable stress for the load case combination (see paragraph 4.1.5.3.c)

3) This load combination addresses an overturning condition for foundation design. It does not apply to design of anchorage (if any) to the foundation. Refer to ASCE/SEI 7-10, 2.4.1 Exception 2 for an additional reduction to W that may be applicable.

## PTB-3-2013

Design Load Parameter	Description	Magnitude of Pressure, Force and Moment
Р	Internal or External Specified Design Pressure (see paragraph 4.1.5.2.a)	<i>P</i> = 356.0
$P_s$	Static head from liquid or bulk materials (e.g. catalyst)	$P_{s} = 0.0$
D	The dead weight of the vessel including skirt, contents, and appurtenances at the location of interest	$D_F = -66152.5 \ lbs$ $D_M = 0.0 \ in - lbs$
L	Appurtenance live loading and effects of fluid flow	$L_F = 0.0 \ lbs$ $L_M = 0.0 \ in - lbs$
Ε	Earthquake loads	$E_F = 0.0 \ lbs$ $E_M = 0.0 \ in - lbs$
W	Wind Loads	$W_F = 0.0 \ lbs$ $W_M = 3.048E + 06 \ in - lbs$
S	Snow Loads	$S_F = 0.0 \ lbs$ $S_M = 0.0 \ in - lbs$
F	Loads due to Deflagration	$F_F = 0.0 \ lbs$ $F_M = 0.0 \ in - lbs$

# Table E4.3.6.2 - Design Loads (Net-Section Axial Force and Bending Moment) at the Location of Interest

Based on these loads, the shell is required to be designed for the load case combinations shown in Table E4.3.6.3. Note that this table is given in terms of the load combinations shown in VIII-2, Table 4.1.2 (Table E4.3.6.1 of this example).

# PTB-3-2013

Load Case	Design Load Combination	Magnitude of Pressure, Force and Moment	General Primary Membrane Allowable Stress
1	$P + P_s + D$	$P + P_s = 356.0 \ psi$ $F_1 = -66152.5 \ lbs$ $M_1 = 0.0 \ in - lbs$	S
2	$P + P_s + D + L$	$P + P_s = 356.0 \ psi$ $F_2 = -66152.5 \ lbs$ $M_2 = 0.0 \ in - lbs$	S
3	$P + P_s + D + S$	$P + P_s = 356.0 \ psi$ $F_3 = -66152.5 \ lbs$ $M_3 = 0.0 \ in - lbs$	S
4	$0.9P + P_s + D + 0.75L + 0.75S$	$0.9P + P_s = 320.4 \ psi$ $F_4 = -66152.5 \ lbs$ $M_4 = 0.0 \ in - lbs$	S
5	$0.9P + P_s + D + (0.6W \text{ or } 0.7E)$	$0.9P + P_s = 320.4 \ psi$ $F_5 = -66152.5 \ lbs$ $M_5 = 1828800 \ in - lbs$	S
6	$\begin{pmatrix} 0.9P + P_{S} + D + 0.75(0.6W \text{ or } 0.7E) + \\ 0.75L + 0.75S \end{pmatrix}$	$0.9P + P_s = 320.4 \ psi$ $F_6 = -66152.5 \ lbs$ $M_6 = 1371600 \ in - lbs$	S
7	0.6D + (0.6W  or  0.7E) Anchorage is included in the design. Therefore, consideration of this load combination is not required.	$F_7 = -39691.5 \ lbs$ $M_7 = 1828800 \ in - lbs$	S
8	$P_s + D + F$	$P_{s} = 0.0 \ psi$ $F_{8} = -66152.5 \ lbs$ $M_{8} = 0.0 \ in - lbs$	See Annex 4.D

Table E4.3.6.3 - Load Case Combination at the Location of Interest

# 4.3.7 Example E4.3.7 – Conical Transitions Without a Knuckle

Determine if the proposed large and small end cylinder-to-cone transitions are adequately designed considering the following design conditions and applied forces and moments. Evaluate the stresses in the cylinder and cone at both the large and small end junction.

Vessel Data:

•	Material		=	SA-516, Grade 70, Norm.
•	Design Conditions		=	356 psig@300°F
•	Inside Radius (Large End)		=	75.0 in
•	Thickness (Large End)		=	1.8125 in
•	Cylinder Length (Large End)		=	60.0 in
•	Inside Radius (Small End)		=	45.0 in
•	Thickness (Small End)		=	1.125 in
•	Cylinder Length (Small End)		=	48.0 in
•	Thickness (Conical Section)		=	1.9375 in
•	Length of Conical Section		=	78.0 in
•	Corrosion Allowance		=	0.125 in
•	Allowable Stress		=	22400 psi
•	Weld Joint Efficiency		=	1.0
•	One-Half Apex Angle (See Figure E4.3.7)		=	21.0375 <i>deg</i>
•	Axial Force (Large End)		=	–99167 <i>lbs</i>
•	Net Section Bending Moment (Large End)		=	5.406E + 06 in - lbs
•	Axial Force (Small End)		=	-78104 <i>lbs</i>
•	Net Section Bending Moment (Small End	)	=	4.301E + 06 in - lbs

Adjust variables for corrosion.

 $\begin{aligned} R_L &= 75.0 + Corrosion \ Allowance = 75.0 + 0.125 = 75.125 \ in \\ R_S &= 45.0 + Corrosion \ Allowance = 45.0 + 0.125 = 45.125 \ in \\ t_L &= 1.8125 - Corrosion \ Allowance = 1.8125 - 0.125 = 1.6875 \ in \\ t_S &= 1.125 - Corrosion \ Allowance = 1.125 - 0.125 = 1.0 \ in \\ t_C &= 1.9375 - Corrosion \ Allowance = 1.9375 - 0.125 = 1.8125 \ in \\ \alpha &= 21.0375 \ deg \end{aligned}$ 

Evaluate per paragraph 4.3.11.

Per paragraph 4.3.11.3, the length of the conical shell, measured parallel to the surface of the cone, shall be equal to or greater than the following:

$$\begin{split} & L_{c} \geq 2.0 \sqrt{\frac{R_{L}t_{c}}{\cos[\alpha]}} + 1.4 \sqrt{\frac{R_{s}t_{c}}{\cos[\alpha]}} \\ & 2.0 \sqrt{\frac{75.125(1.8125)}{\cos[21.0375]}} + 1.4 \sqrt{\frac{45.125(1.8125)}{\cos[21.0375]}} = 37.2624 \ in \\ & L_{c} = 78.0 \geq 37.2624 \qquad True \end{split}$$

Evaluate the Large End cylinder-to-cone junction per VIII-2, paragraph 4.3.11.4.

a) STEP 1 – Compute the large end cylinder thickness, *t<sub>L</sub>*, using paragraph 4.3.3., (as specified in design conditions).

$$t_L = 1.6875$$
 in

b) STEP 2 – Determine the cone half-apex angle,  $\alpha$ , and compute the cone thickness,  $t_c$ , at the large end using paragraph 4.3.4., (as specified in design conditions).

$$\alpha = 21.0375 \ deg$$
  
 $t_c = 1.8125 \ in$ 

c) STEP 3 – Proportion the cone geometry such that the following equations are satisfied. If all of these equations are not satisfied, then the cylinder-to-cone junction shall be designed in accordance with Part 5. In the calculations, if  $0 \deg < \alpha \le 10 \deg$ , then use  $\alpha = 10 \deg$ .

$$20 \le \left\{ \frac{R_L}{t_L} = \frac{75.125}{1.6875} = 44.5185 \right\} \le 500 \qquad True$$
$$1 \le \left\{ \frac{t_C}{t_L} = \frac{1.8125}{1.6875} = 1.0741 \right\} \le 2 \qquad True$$
$$\{ \alpha = 21.0375 \ deg \} \le \{ 60 \ deg \} \qquad True$$

d) STEP 4 – Determine the net section axial force,  $F_L$ , and bending moment,  $M_L$ , applied to the conical transition (as specified in design conditions). The thrust load due to pressure shall not be included as part of the axial force,  $F_L$ . Calculate the equivalent line load,  $X_L$ , using the specified net section axial force,  $F_L$ , and bending moment,  $M_L$ .

$$X_{L} = \frac{F_{L}}{2\pi R_{L}} \pm \frac{M_{L}}{\pi R_{L}^{2}} = \begin{cases} \frac{-99167}{2\pi (75.125)} + \frac{5406000}{\pi (75.125)^{2}} = 94.8111 \frac{lbs}{in} \\ \frac{-99167}{2\pi (75.125)} - \frac{5406000}{\pi (75.125)^{2}} = -514.9886 \frac{lbs}{in} \end{cases}$$

#### PTB-3-2013

e) STEP 5 – Compute the junction transition design parameters (the normalized resultant moment,  $M_{sN}$ , and shear force,  $Q_N$ ) for the internal pressure and equivalent line load per Table 4.3.3 and Table 4.3.4, respectively. For calculated values of *n* other than those presented in Table 4.3.3 and Table 4.3.4, linear interpolation of the equation coefficients,  $C_i$ , is permitted.

$$n = \frac{t_C}{t_L} = \frac{1.8125}{1.6875} = 1.0741$$
$$H = \sqrt{\frac{R_L}{t_L}} = \sqrt{\frac{75.125}{1.6875}} = 6.6722$$
$$B = \tan[\alpha] = \tan[21.0375] = 0.3846$$

# PTB-3-2013

Linear interpolation of the equation coefficients,	$C_i$ in Table 4.3.3 and Table 4.3.4 is required.
The results of the interpolation are summarized w	vith the following values for $ C_i^{}$ .

	VIII-2, Table 4.3.3		VIII-2, Table 4.3.4	
Equation Coefficients $C_i$	Pressure Applied Junction Moment Resultant	Pressure Applied Junction Shear Force Resultant	Equivalent Line Load Junction Moment Resultant	Equivalent Line Load Junction Shear Force Resultant
	$M_{_{SN}}$	$\mathcal{Q}_{\scriptscriptstyle N}$	$M_{_{sN}}$	$Q_{\scriptscriptstyle N}$
1	-3.079751	-1.962370	-5.706141	-4.878520
2	3.662099	2.375540	0.004705	0.006808
3	0.788301	0.582454	0.474988	-0.018569
4	-0.226515	-0.107299	-0.213112	-0.197037
5	-0.080019	-0.103635	2.241065	2.033876
6	0.049314	0.151522	0.000025	-0.000085
7	0.026266	0.010704	0.002759	-0.000109
8	-0.015486	-0.018356	-0.001786	-0.004071
9	0.001773	0.006551	-0.214046	-0.208830
10	-0.007868	-0.021739	0.000065	-0.000781
11			-0.106223	0.004724

For the applied pressure case both  $M_{\rm sN}$  and  $Q_{\rm N}$  are calculated using the following equation.

$$M_{sN}, Q_{N} = -\exp\begin{bmatrix}C_{1} + C_{2}\ln[H] + C_{3}\ln[B] + C_{4}(\ln[H])^{2} + C_{5}(\ln[B])^{2} + C_{6}\ln[H]\ln[B] + C_{7}(\ln[H])^{3} + C_{8}(\ln[B])^{3} + C_{9}\ln[H](\ln[B])^{2} + C_{10}(\ln[H])^{2}\ln[B]$$

This results in

$$M_{sN} = -\exp \begin{bmatrix} -3.079751 + 3.662099 \cdot \ln [6.6722] + 0.788301 \cdot \ln [0.3846] + \\ (-0.226515)(\ln [6.6722])^2 + (-0.080019)(\ln [0.3846])^2 + \\ 0.049314 \cdot \ln [6.6722] \cdot \ln [0.3846] + \\ 0.026266(\ln [6.6722])^3 + (-0.015486)(\ln [0.3846])^3 + \\ 0.001773 \cdot \ln [6.6722] \cdot (\ln [0.3846])^2 + \\ (-0.007868)(\ln [6.6722])^2 \cdot \ln [0.3846] \end{bmatrix} = -10.6148$$

$$Q_{N} = -\exp \begin{bmatrix} -1.962370 + 2.375540 \cdot \ln [6.6722] + 0.582454 \cdot \ln [0.3846] + \\ (-0.107299) (\ln [6.6722])^{2} + (-0.103635) (\ln [0.3846])^{2} + \\ 0.151522 \cdot \ln [6.6722] \cdot \ln [0.3846] + \\ 0.010704 (\ln [6.6722])^{3} + (-0.018356) (\ln [0.3846])^{3} + \\ 0.006551 \cdot \ln [6.6722] \cdot (\ln [0.3846])^{2} + \\ (-0.021739) (\ln [6.6722])^{2} \cdot \ln [0.3846] \end{bmatrix}^{2} + \end{bmatrix} = -4.0925$$

For the Equivalent Line Load case,  $M_{\rm sN}$  and  $Q_{\rm N}$  are calculated using the following equation.

$$M_{sN}, Q_{N} = -\exp\left[\frac{\left(C_{1} + C_{3} \ln\left[H^{2}\right] + C_{5} \ln\left[\alpha\right] + C_{7} \left(\ln\left[H^{2}\right]\right)^{2} + \left(C_{9} \left(\ln\left[\alpha\right]\right)^{2} + C_{11} \ln\left[H^{2}\right] \ln\left[\alpha\right]\right)\right)}{\left(1 + C_{2} \ln\left[H^{2}\right] + C_{4} \ln\left[\alpha\right] + C_{6} \left(\ln\left[H^{2}\right]\right)^{2} + \left(C_{8} \left(\ln\left[\alpha\right]\right)^{2} + C_{10} \ln\left[H^{2}\right] \ln\left[\alpha\right]\right)\right)\right)}\right]$$

This results in  

$$M_{sN} = -\exp \begin{bmatrix} \begin{pmatrix} -5.706141 + 0.474988 \cdot \ln[6.6722^{2}] + \\ 2.241065 \cdot \ln[21.0375] + 0.002759 (\ln[6.6722^{2}])^{2} + \\ (-0.214046) (\ln[21.0375])^{2} + \\ (-0.214046) (\ln[21.0375]]^{2} + \\ (-0.106223) \ln[6.6722^{2}] \cdot \ln[21.0375] \\ \hline \\ 1 + 0.004705 \cdot \ln[6.6722^{2}] + (-0.213112) \ln[21.0375] + \\ 0.000025 (\ln[6.6722^{2}])^{2} + (-0.001786) (\ln[21.0375])^{2} + \\ 0.000065 \cdot \ln[6.6722^{2}] \cdot \ln[21.0375] \\ \hline \\ 0.000065 \cdot \ln[6.6722^{2}] \cdot \ln[21.0375] \\ \hline \\ (-2.038376 \cdot \ln[21.0375] + (-0.000109) (\ln[6.6722^{2}])^{2} + \\ (-0.208830) (\ln[21.0375])^{2} + \\ 0.004724 \cdot \ln[6.6722^{2}] \cdot \ln[21.0375] \\ \hline \\ \hline \\ 1 + 0.006808 \cdot \ln[6.6722^{2}] \cdot \ln[21.0375] \\ \hline \\ (-0.00085) (\ln[6.6722^{2}])^{2} + (-0.004071) (\ln[21.0375])^{2} + \\ (-0.000781) \ln[6.6722^{2}] \cdot \ln[21.0375] \\ \hline \\ \end{bmatrix} = -0.1845$$

Summarizing, the normalized resultant moment  $M_{\rm sN}$ , and shear force  $Q_{\rm N}$  for the internal pressure and equivalent line load are as follows:

Internal Pressure :	$M_{_{sN}} = -10.6148,$	$Q_N = -4.0925$
Equivalent Line Load :	$M_{sN} = -0.4912,$	$Q_N = -0.1845$

f) STEP 6 – Compute the stresses in the cylinder and cone at the junction using the equations in Table 4.3.1 for the Large End Junction.

Evaluate the Cylinder at the Large End:

Stress Resultant Calculations:

$$\begin{split} M_{sP} &= Pt_{L}^{2}M_{sN} = 356 (1.6875)^{2} (-10.6148) = -10760.9194 \frac{in - lbs}{in} \\ M_{sX} &= X_{L}t_{L}M_{sN} = \begin{cases} 94.8111 (1.6875) (-0.4912) = -78.5889 \frac{in - lbs}{in} \\ -514.9886 (1.6875) (-0.4912) = 426.8741 \frac{in - lbs}{in} \end{cases} \\ M_{s} &= M_{sP} + M_{sX} = \begin{cases} -10760.9194 + (-78.5889) = -10839.5083 \frac{in - lbs}{in} \\ -10760.9194 + 426.8741 = -10334.0453 \frac{in - lbs}{in} \end{cases} \\ Q_{P} &= Pt_{L}Q_{N} = 356 (1.6875) (-4.0925) = -2458.5694 \frac{lbs}{in} \\ -514.9886 (-0.1845) = -17.4926 \frac{lbs}{in} \\ -514.9886 (-0.1845) = 95.0154 \frac{lbs}{in} \end{cases} \\ Q_{Z} &= X_{L}Q_{N} = \begin{cases} 94.8111 (-0.1845) = -17.4926 \frac{lbs}{in} \\ -514.9886 (-0.1845) = 95.0154 \frac{lbs}{in} \\ -2458.5694 + (-17.4926) = -2476.0620 \frac{lbs}{in} \\ -2458.5694 + 95.0154 = -2363.5540 \frac{lbs}{in} \end{cases} \end{cases} \\ \mathcal{B}_{cy} &= \left[ \frac{3(1 - v^{2})}{R_{L}^{2}t_{L}^{2}} \right]^{0.25} = \left[ \frac{3(1 - (0.3)^{2})}{(75.125)^{2} (1.6875)^{2}} \right]^{0.25} = 0.1142 in^{-1} \\ N_{s} &= \frac{PR_{L}}{2} + X_{L} = \begin{cases} \frac{356(75.125)}{2} + 94.8111 = 13467.0611 \frac{lbs}{in} \\ \frac{356(75.125)}{2} + (-514.9886) = 12857.2614 \frac{lbs}{in} \end{cases} \end{cases} \end{cases}$$

$$\begin{split} N_{\theta} &= PR_{L} + 2\beta_{cy}R_{L}\left(-M_{s}\beta_{cy} + Q\right) \\ N_{\theta} &= \begin{cases} 356(75.125) + 2(0.1142)(75.125)\left(-(-10839.5083)(0.1142) + (-2476.0620)\right) \\ 356(75.125) + 2(0.1142)(75.125)\left(-(-10334.0453)(0.1142) + (-2363.553)\right) \end{cases} \\ N_{\theta} &= \begin{cases} 5498.9524 \frac{lbs}{in} \\ 6438.9685 \frac{lbs}{in} \\ \end{cases} \\ K_{pc} &= 1.0 \end{split}$$

Stress Calculations: Determine the meridional and circumferential membrane and bending stresses.

$$\sigma_{sm} = \frac{N_s}{t_L} = \begin{cases} \frac{13467.0611}{1.6875} = 7980.4807 \ psi \\ \frac{12857.2614}{1.6875} = 7619.1179 \ psi \end{cases}$$

$$\sigma_{sb} = \frac{6M_s}{t_L^2 K_{pc}} = \begin{cases} \frac{6(-10839.5083)}{(1.6875)^2 (1.0)} = -22838.7994 \ psi \\ \frac{6(-10334.0453)}{(1.6875)^2 (1.0)} = -21773.7909 \ psi \end{cases}$$

$$\sigma_{\theta m} = \frac{N_{\theta}}{t_L} = \begin{cases} \frac{5498.9524}{1.6875} = 3258.6385 \ psi \\ \frac{6438.9685}{1.6875} = 3815.6850 \ psi \end{cases}$$

$$\sigma_{\theta b} = \frac{6vM_s}{t_L^2 K_{pc}} = \begin{cases} \frac{6(0.3)(-10839.5083)}{(1.6875)^2 (1.0)} = -6851.6398 \ psi \\ \frac{6(0.3)(-10334.0453)}{(1.6875)^2 (1.0)} = -6532.1373 \ psi \end{cases}$$

Check Acceptance Criteria:

$$\begin{cases} \sigma_{sm} = 7980.4807 \ psi \\ \sigma_{sm} = 7619.1179 \ psi \end{cases} \leq \{1.5S = 1.5(22400) = 33600 \ psi \}$$

$$True$$

$$\begin{cases} \sigma_{sm} + \sigma_{sb} = 7980.4807 + (-22838.7994) = |-14858.3 \ psi | \\ \sigma_{sm} - \sigma_{sb} = 7980.4807 - (-22838.7994) = 30819.3 \ psi | \\ \sigma_{sm} - \sigma_{sb} = 7619.1179 + (-21773.7909) = |-14154.7 \ psi | \\ \sigma_{sm} - \sigma_{sb} = 7619.1179 - (-21773.7909) = 29392.9 \ psi \end{cases} \leq \{S_{PS} = 67200 \ psi \}$$

$$True$$

$$\begin{cases} \sigma_{\theta m} = 3258.6385 \ psi \\ \sigma_{\theta m} = 3815.6850 \ psi \} \leq \{1.5S = 1.5(22400) = 33600 \ psi \}$$

$$True$$

$$\begin{cases} \sigma_{\theta m} + \sigma_{\theta b} = 3258.6385 + (-6851.6398) = |-3593.0 \ psi | \\ \sigma_{\theta m} - \sigma_{\theta b} = 3258.6385 - (-6851.6398) = 10110.3 \ psi | \\ \sigma_{\theta m} - \sigma_{\theta b} = 3815.6850 + (-6532.1373) = |-2716.5 \ psi | \\ \sigma_{\theta m} - \sigma_{\theta b} = 3815.6850 - (-6532.1373) = 10347.8 \ psi \end{cases} \leq \{S_{PS} = 67200 \ psi \}$$

$$True$$

Since the longitudinal membrane stress,  $\sigma_{sm}$  and the circumferential membranes stress,  $\sigma_{\theta m}$  are both tensile, the condition of local buckling need not be considered. Therefore, the cylinder at the cylinder-to-cone junction at the large end is adequately designed.

## Evaluate the Cone at the Large End:

Stress Resultant Calculations - as determined above:

$$\begin{split} M_{csP} &= M_{sP} = -10760.9194 \; \frac{in - lbs}{in} \\ M_{csX} &= M_{sX} = \begin{cases} -78.5889 \; \frac{in - lbs}{in} \\ 426.8741 \; \frac{in - lbs}{in} \end{cases} \\ 426.8741 \; \frac{in - lbs}{in} \end{cases} \\ M_{cs} &= M_{csP} + M_{csX} = \begin{cases} -10760.9194 + (-78.5889) = -10839.5083 \; \frac{in - lbs}{in} \\ -10760.9194 + 426.8741 = -10334.0453 \; \frac{in - lbs}{in} \end{cases} \\ Q_c &= Q \cos[\alpha] + N_s \sin[\alpha] \\ Q_c &= \begin{cases} ((-2476.0620) \cos[21.0375] + (13467.0611) \sin[21.0375]) = 2523.3690 \; \frac{lbs}{in} \\ ((-2363.5540) \cos[21.0375] + (12857.2614) \sin[21.0375]) = 2409.4726 \; \frac{lbs}{in} \end{cases} \end{split}$$

$$\begin{split} R_{c} &= \frac{R_{L}}{\cos[\alpha]} = \frac{75.125}{\cos[21.0375]} = 80.4900 \ in \\ \beta_{co} &= \left[\frac{3(1-v^{2})}{R_{c}^{2}t_{c}^{2}}\right]^{0.25} = \left[\frac{3(1-(0.3)^{2})}{(80.4900)^{2}(1.8125)^{2}}\right]^{0.25} = 0.1064 \ in^{-1} \\ N_{cs} &= N_{s} \cos[\alpha] - Q\sin[\alpha] \\ N_{cs} &= \begin{cases} (13467.0611)\cos[21.0375] - (-2476.0620)\sin[21.0375] = 13458.2772 \ \frac{lbs}{in} \\ (12857.2614)\cos[21.0375] - (-2363.5540)\sin[21.0375] = 12848.7353 \ \frac{lbs}{in} \end{cases} \\ N_{c\theta} &= \frac{PR_{L}}{\cos[\alpha]} + 2\beta_{c\sigma}R_{c} \left(-M_{cs}\beta_{c\sigma} - Q_{c}\right) \\ N_{c\theta} &= \begin{cases} \frac{356(75.125)}{\cos[21.0375]} + 2(0.1064)(80.4900)(-(-10839.5083)(0.1064) - 2523.3690) \\ \frac{356(75.125)}{\cos[21.0375]} + 2(0.1064)(80.4900)(-(-10334.0453)(0.1064) - 2409.4726) \\ \frac{5187.9337}{in} \\ 6217.6021 \ \frac{lbs}{in} \\ K_{cpc} &= 1.0 \end{split}$$

Stress Calculations: Determine the meridional and circumferential membrane and bending stresses.

$$\sigma_{sm} = \frac{N_{cs}}{t_c} = \begin{cases} \frac{13458.2772}{1.8125} = 7425.2564 \ psi \\ \frac{12848.7353}{1.8125} = 7088.9574 \ psi \end{cases}$$

$$\sigma_{sb} = \frac{6M_{cs}}{t_c^2 K_{cpc}} = \begin{cases} \frac{6(-10839.5083)}{(1.8125)^2 (1.0)} = -19797.2470 \ psi \\ \frac{6(-10334.0453)}{(1.8125)^2 (1.0)} = -18874.0708 \ psi \end{cases}$$

$$\sigma_{\theta m} = \frac{N_{c\theta}}{t_c} = \begin{cases} \frac{5187.9337}{1.8125} = 2862.3082 \ psi \\ \frac{6217.6021}{1.8125} = 3430.4012 \ psi \end{cases}$$

$$\sigma_{\theta b} = \frac{6\nu M_{cs}}{t_c^2 K_{cpc}} = \begin{cases} \frac{6(0.3)(-10839.5083)}{(1.8125)^2 (1.0)} = -5939.1741 \ psi \\ \frac{6(0.3)(-10334.0453)}{(1.8125)^2 (1.0)} = -5662.2213 \ psi \end{cases}$$

Check Acceptance Criteria:

$$\begin{cases} \sigma_{sm} = 7425.2564 \ psi \\ \sigma_{sm} = 7088.9574 \ psi \end{cases} \le \{1.5S = 1.5(22400) = 33600 \ psi \}$$

$$True$$

$$\begin{cases} \sigma_{sm} + \sigma_{sb} = 7425.2564 + (-19797.2470) = |-12371.9906 \ psi | \\ \sigma_{sm} - \sigma_{sb} = 7425.2564 - (-19797.2470) = 27222.5034 \ psi | \\ \sigma_{sm} - \sigma_{sb} = 7088.9574 + (-18874.0708) = |-11785.1 \ psi | \\ \sigma_{sm} - \sigma_{sb} = 7088.9574 - (-18874.0708) = 25963.0 \ psi \\ \end{cases} \le \{S_{PS} = 67200 \ psi \}$$

$$\begin{cases} \sigma_{\theta m} = 2862.3082 \ psi \\ \sigma_{\theta m} = 3430.4012 \ psi \\ \end{cases} \le \{1.5S = 1.5(22400) = 33600 \ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = 2862.3082 + (-5939.1741) = |-3076.8659 \ psi | \\ \sigma_{\theta m} - \sigma_{\theta b} = 2862.3082 - (-5939.1741) = 8801.4823 \ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = 3430.4012 + (-5662.2213) = |-2231.8 \ psi | \\ \sigma_{\theta m} - \sigma_{\theta b} = 3430.4012 - (-5662.2213) = 9092.6 \ psi \\ \end{cases} \le \{S_{PS} = 67200 \ psi \}$$

$$True$$

Since the longitudinal membrane stress,  $\sigma_{sm}$  and the circumferential membranes stress,  $\sigma_{\theta m}$  are both tensile, the condition of local buckling need not be considered. Therefore, the cone at the cylinder-to-cone junction at the large end is adequately designed.

g) STEP 7 – The stress acceptance criterion in STEP 6 is satisfied for both the cylinder and cone. Therefore the design is complete.

Evaluate the Small End cylinder-to-cone junction per paragraph 4.3.11.5.

- a) STEP 1 Compute the small end cylinder thickness,  $t_s$ , using paragraph 4.3.3., (as specified in design conditions).
- b) STEP 2 Determine the cone half-apex angle,  $\alpha$ , and compute the cone thickness,  $t_c$ , at the small end using paragraph 4.3.4., (as specified in design conditions
- c) STEP 3 Proportion the cone geometry such that the following equations are satisfied. If all of these equations are not satisfied, then the cylinder-to-cone junction shall be designed in accordance with Part 5. In the calculations, if  $0 \deg < \alpha \le 10 \deg$ , then use  $\alpha = 10 \deg$ .

$$20 \le \left(\frac{R_s}{t_s} = \frac{45.125}{1.0} = 45.125\right) \le 500 \qquad True$$
$$1 \le \left(\frac{t_c}{t_s} = \frac{1.8125}{1.0} = 1.8125\right) \le 2 \qquad True$$
$$\{\alpha = 21.0375 \ deg\} \le \{60 \ deg\} \qquad True$$

d) STEP 4 – Calculate the equivalent line load,  $X_s$ , given the net section axial force,  $F_s$ , and bending moment,  $M_s$ , applied at the conical transition.

$$X_{s} = \frac{F_{s}}{2\pi R_{s}} \pm \frac{M_{s}}{\pi R_{s}^{2}} = \begin{cases} \frac{-78104.2}{2\pi (45.125)} + \frac{4301000}{\pi (45.125)^{2}} = 396.8622 \frac{lbs}{in} \\ \frac{-78104.2}{2\pi (45.125)} - \frac{4301000}{\pi (45.125)^{2}} = -947.8060 \frac{lbs}{in} \end{cases}$$

e) STEP 5 – Compute the junction transition design parameters (the normalized resultant moment,  $M_{sN}$ , and shear force,  $Q_N$ ) for the internal pressure and equivalent line load per Table 4.3.5 and Table 4.3.6, respectively. For calculated values of *n* other than those presented in Table 4.3.5 and Table 4.3.6, linear interpolation of the equation coefficients,  $C_i$ , is permitted.

$$n = \frac{t_C}{t_S} = \frac{1.8125}{1.000} = 1.8125$$
$$H = \sqrt{\frac{R_S}{t_S}} = \sqrt{\frac{45.125}{1.000}} = 6.7175$$
$$B = \tan[\alpha] = \tan[21.0375] = 0.3846$$

# PTB-3-2013

VIII-2, Table 4.3.5		VIII-2, Table 4.3.6		
Equation Coefficients $C_i$	Pressure Applied Junction Moment Resultant	Pressure Applied Junction Shear Force Resultant	Equivalent Line Load Junction Moment Resultant	Equivalent Line Load Junction Shear Force Resultant
	$M_{_{SN}}$	$Q_{\scriptscriptstyle N}$	$M_{_{sN}}$	$Q_{\scriptscriptstyle N}$
1	-15.144683	0.569891	0.006792	-0.408044
2	3.036812	-0.000027	0.000290	0.021200
3	6.460714	0.008431	-0.000928	-0.325518
4	-0.155909	0.002690	0.121611	-0.003988
5	-1.462862	-0.002884	0.010581	-0.111262
6	-0.369444	0.000000	-0.000011	0.002204
7	0.007742	-0.000005	-0.00008	0.000255
8	0.143191	-0.000117	0.005957	-0.014431
9	0.040944	-0.000087	0.001310	0.000820
10	0.007178	0.000001	0.000186	0.000106
11		-0.003778	0.194433	

Linear interpolation of the equation coefficients,  $C_i$  in Table 4.3.5 and Table 4.3.6 is required. The results of the interpolation is summarized with the following values for  $C_i$ .

For the applied pressure case  $\,M_{\rm \scriptscriptstyle SN}\,$  is calculated using the following equation

$$M_{sN} = \exp \begin{bmatrix} C_1 + C_2 \ln \left[ H^2 \right] + C_3 \ln \left[ \alpha \right] + C_4 \left( \ln \left[ H^2 \right] \right)^2 + C_5 \left( \ln \left[ \alpha \right] \right)^2 + C_6 \ln \left[ H^2 \right] \ln \left[ \alpha \right] + C_7 \left( \ln \left[ H^2 \right] \right)^3 + C_8 \left( \ln \left[ \alpha \right] \right)^3 + C_9 \ln \left[ H^2 \right] \left( \ln \left[ \alpha \right] \right)^2 + C_{10} \left( \ln \left[ H^2 \right] \right)^2 \ln \left[ \alpha \right] \end{bmatrix}$$

This results in

$$M_{sN} = \exp \begin{bmatrix} -15.144683 + 3.036812 \cdot \ln \left[ 6.7175^{2} \right] + 6.460714 \cdot \ln \left[ 21.0375 \right] + \\ (-0.155909) \left( \ln \left[ 6.7175^{2} \right] \right)^{2} + (-1.462862) \left( \ln \left[ 21.0375 \right] \right)^{2} + \\ (-0.369444) \ln \left[ 6.7175^{2} \right] \cdot \ln \left[ 21.0375 \right] + \\ 0.007742 \left( \ln \left[ 6.7175^{2} \right] \right)^{3} + 0.143191 \left( \ln \left[ 21.0375 \right] \right)^{3} + \\ 0.040944 \cdot \ln \left[ 6.7175^{2} \right] \cdot \left( \ln \left[ 21.0375 \right] \right)^{2} + \\ 0.007178 \left( \ln \left[ 6.7175^{2} \right] \right)^{2} \cdot \ln \left[ 21.0375 \right] \end{bmatrix}^{2} + \\ \end{bmatrix} = 9.2135$$

For the applied pressure case  ${\it Q}_{\scriptscriptstyle N}$  is calculated using the following equation

$$Q_{N} = \left(\frac{C_{1} + C_{3}H^{2} + C_{5}\alpha + C_{7}H^{4} + C_{9}\alpha^{2} + C_{11}H^{2}\alpha}{1 + C_{2}H^{2} + C_{4}\alpha + C_{6}H^{4} + C_{8}\alpha^{2} + C_{10}H^{2}\alpha}\right)$$

This results in

$$Q_{N} = \begin{pmatrix} \left( \begin{array}{c} 0.569891 + 0.008431(6.7175)^{2} + (-0.002884)(21.0375) + \\ (-0.000005)(6.7175)^{4} + (-0.000087)(21.0375)^{2} + \\ (-0.003778)(6.7175)^{2}(21.0375) \\ \end{array} \right) \\ \hline \left( \begin{array}{c} 1 + (-0.000027)(6.7175)^{2} + 0.002690(21.0375) + \\ 0.000000(6.7175)^{4} + (-0.000117)(21.0375)^{2} + \\ 0.000001(6.7175)^{2}(21.0375) \\ \end{array} \right) \\ \end{array} \right) = -2.7333$$

For the Equivalent Line Load case,  $\,M_{\rm \scriptscriptstyle SN}\,$  is calculated using the following equation

$$M_{sN} = \left(\frac{C_1 + C_3 H + C_5 B + C_7 H^2 + C_9 B^2 + C_{11} H B}{1 + C_2 H + C_4 B + C_6 H^2 + C_8 B^2 + C_{10} H B}\right)$$

This results in

$$M_{sN} = \begin{pmatrix} (0.006792 + (-0.000928)(6.7175) + 0.010581(0.3846) + \\ (-0.000008)(6.7175)^2 + 0.001310(0.3846)^2 + \\ 0.194433(6.7175)(0.3846) \\ \hline \\ (1+0.000290(6.7175) + 0.121611(0.3846) + \\ (-0.00011)(6.7175)^2 + 0.005957(0.3846)^2 + \\ 0.000186(6.7175)(0.3846) \end{pmatrix} = 0.4828$$

For the Equivalent Line Load case,  ${\it Q}_{\scriptscriptstyle N}$  is calculated using the following equation

$$Q_{N} = \begin{pmatrix} C_{1} + C_{2} \ln[H] + C_{3} \ln[B] + C_{4} (\ln[H])^{2} + C_{5} (\ln[B])^{2} + C_{6} \ln[H] \ln[B] + \\ C_{7} (\ln[H])^{3} + C_{8} (\ln[B])^{3} + C_{9} \ln[H] (\ln[B])^{2} + C_{10} (\ln[H])^{2} \ln[B] \end{pmatrix}$$

This results in

$$Q_{N} = \begin{pmatrix} -0.408044 + 0.021200 \cdot \ln[6.7175] + (-0.325518) \ln[0.3846] + \\ (-0.003988) (\ln[6.7175])^{2} + (-0.111262) (\ln[0.3846])^{2} + \\ 0.002204 \cdot \ln[6.7175] \cdot \ln[0.3846] + 0.000255 \cdot (\ln[6.7175])^{3} + \\ (-0.014431) (\ln[0.3846])^{3} + 0.000820 \cdot \ln[6.7175] \cdot (\ln[0.3846])^{2} + \\ 0.000106 (\ln[6.7175])^{2} \cdot \ln[0.3846] \end{pmatrix} = -0.1613$$

Summarizing, the normalized resultant moment  $M_{\rm sN}$ , and shear force  $Q_{\rm N}$  for the internal pressure and equivalent line load are as follows:

Internal Pressure :	$M_{sN} = 9.2135,$	$Q_N = -2.7333$
Equivalent Line Load :	$M_{sN} = 0.4828,$	$Q_N = -0.1613$

f) STEP 6 – Compute the stresses in the cylinder and cone at the junction using the equations in Table 4.3.2 for the Small End Junction.

Evaluate the Cylinder at the Small End:

Stress Resultant Calculations:

$$\begin{split} M_{sP} &= Pt_{s}^{2}M_{sN} = 356(1.000)^{2} (9.2135) = 3280.0060 \frac{in - lbs}{in} \\ M_{sX} &= X_{s}t_{s}M_{sN} = \begin{cases} 396.8622(1.0000)(0.4828) = 191.6051 \frac{in - lbs}{in} \\ -947.8060(1.0000)(0.4828) = -457.6007 \frac{in - lbs}{in} \\ -947.8060(1.0000)(0.4828) = -457.6007 \frac{in - lbs}{in} \\ 3280.0060 + (191.6051) = 3471.6111 \frac{in - lbs}{in} \\ 3280.0060 + (-457.6007) = 2822.4053 \frac{in - lbs}{in} \\ 3280.0060 + (-457.6007) = 2822.4053 \frac{in - lbs}{in} \\ Q_{P} &= Pt_{s}Q_{N} = 356(1.0000)(-2.7333) = -973.0548 \frac{lbs}{in} \\ Q_{X} &= X_{s}Q_{N} = \begin{cases} 396.8622(-0.1613) = -64.0139 \frac{lbs}{in} \\ -947.8060(-0.1613) = 152.8811 \frac{lbs}{in} \\ -947.8060(-0.1613) = 152.8811 \frac{lbs}{in} \\ -973.0548 + (52.8811 = -820.1737 \frac{lbs}{in} \\ -973.0548 + 152.8811 = -820.1737 \frac{lbs}{in} \\ \end{cases} \end{split}$$

$$\begin{split} \mathcal{B}_{cy} &= \left[ \frac{3(1 - \nu^{2})}{R_{s}^{2}t_{s}^{2}} \right]^{0.25} = \left[ \frac{3(1 - (0.3)^{2})}{(45.1250)^{2}(1.000)^{2}} \right]^{0.25} = 0.1914 in^{-1} \\ N_{s} &= \frac{PR_{s}}{2} + X_{s} = \begin{cases} \frac{356(45.125)}{2} + (-947.8060) = 7084.4440 \frac{lbs}{in} \\ \frac{356(45.125)}{2} + (-947.8060) = 7084.4440 \frac{lbs}{in} \\ \end{cases} \end{split}$$

$$\begin{split} N_{\theta} &= PR_{s} + 2\beta_{cy}R_{s}\left(-M_{s}\beta_{cy} - Q\right) \\ N_{\theta} &= \begin{cases} 356(45.125) + 2(0.1914)(45.125)\left(-(3471.6111)(0.1914) - (-1037.0687)\right)\right) \\ 356(45.125) + 2(0.1914)(45.125)\left(-(2822.4053)(0.1914) - (-820.1737)\right) \end{cases} \\ N_{\theta} &= \begin{cases} 22500.7769 \frac{lbs}{in} \\ 20900.5790 \frac{lbs}{in} \end{cases} \\ K_{pc} &= 1.0 \end{split}$$

Stress Calculations: Determine the meridional and circumferential membrane and bending stresses:

$$\sigma_{sm} = \frac{N_s}{t_s} = \begin{cases} \frac{8429.1122}{1.0000} = 8429.1122 \ psi \\ \frac{7084.4440}{1.0000} = 7084.4440 \ psi \end{cases}$$

$$\sigma_{sb} = \frac{6M_s}{t_s^2 K_{pc}} = \begin{cases} \frac{6(3471.6111)}{(1.0000)^2(1.0)} = 20829.66666 \ psi \\ \frac{6(2822.4053)}{(1.0000)^2(1.0)} = 16934.4318 \ psi \end{cases}$$

$$\sigma_{\theta m} = \frac{N_{\theta}}{t_s} = \begin{cases} \frac{22500.7769}{1.0000} = 22500.7769 \ psi \\ \frac{20900.5790}{1.0000} = 20900.5790 \ psi \end{cases}$$

$$\sigma_{\theta b} = \frac{6\nu M_s}{t_s^2 K_{pc}} = \begin{cases} \frac{6(0.3)(3471.6111)}{(1.0000)^2(1.0)} = 6248.8999 \ psi \\ \frac{6(0.3)(2822.4053)}{(1.0000)^2(1.0)} = 5080.3295 \ psi \end{cases}$$

## PTB-3-2013

Check Acceptance Criteria:

$$\begin{cases} \sigma_{sm} = 8429.1122 \ psi \\ \sigma_{sm} = 7084.4440 \ psi \end{cases} \le \{1.5S = 1.5(22400) = 33600 \ psi \}$$

$$True$$

$$\begin{cases} \sigma_{sm} + \sigma_{sb} = 8429.1122 + (20829.6666) = 29258.8 \ psi \\ \sigma_{sm} - \sigma_{sb} = 8429.1122 - (20829.6666) = |-12400.6 \ psi | \\ \sigma_{sm} + \sigma_{sb} = 7084.4440 + (16934.4318) = 24018.9 \ psi \\ \sigma_{sm} - \sigma_{sb} = 7084.4440 - (16934.4318) = |-9850.0 \ psi | \\ \end{cases} \le \{S_{PS} = 67200 \ psi \}$$

$$True$$

$$\begin{cases} \sigma_{\theta m} = 22500.7769 \ psi \\ \sigma_{\theta m} = 20900.5790 \ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = 22500.7769 - (6248.8999) = 28749.7 \ psi \\ \sigma_{\theta m} + \sigma_{\theta b} = 22500.7769 - (6248.8999) = 16251.9 \ psi \\ \sigma_{\theta m} + \sigma_{\theta b} = 20900.5790 + (5080.3295) = 25981.0 \ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = 20900.5790 - (5080.3295) = 15820.2 \ psi \\ \end{cases} \le \{S_{PS} = 67200 \ psi \}$$

$$True$$

Since the longitudinal membrane stress,  $\sigma_{sm}$  and the circumferential membranes stress,  $\sigma_{\theta m}$  are both tensile, the condition of local buckling need not be considered. Therefore, the cylinder at the cylinder-to-cone junction at the small end is adequately designed.

## Evaluate the Cone at the Small End:

Stress Resultant Calculations:

$$\begin{split} M_{csP} &= M_{sP} = 3280.0060 \; \frac{in - lbs}{in} \\ M_{csX} &= M_{sX} = \begin{cases} 191.6051 \; \frac{in - lbs}{in} \\ -457.6007 \; \frac{in - lbs}{in} \end{cases} \\ M_{cs} &= M_{csP} + M_{csX} = \begin{cases} 3280.0060 + 191.6051 = 3471.6111 \; \frac{in - lbs}{in} \\ 3280.0060 + (-457.6007) = 2822.4053 \; \frac{in - lbs}{in} \end{cases} \\ Q_c &= Q \cos[\alpha] + N_s \sin[\alpha] \\ Q_c &= \begin{cases} -1037.0687 \cos[21.0375] + 8429.1122 \sin[21.0375] = 2057.9298 \; \frac{lbs}{in} \\ -820.1737 \cos[21.0375] + 7084.4440 \sin[21.0375] = 1777.6603 \; \frac{lbs}{in} \end{cases} \end{split}$$

$$\begin{split} R_{c} &= \frac{R_{c}}{\cos[\alpha]} = \frac{45.1250}{\cos[21.0375]} = 48.3476 \ in \\ \beta_{co} &= \left[\frac{3(1-v^{2})}{R_{c}^{2}t_{c}^{2}}\right]^{0.25} = \left[\frac{3(1-(0.3)^{2})}{(48.3476)^{2}(1.8125)^{2}}\right]^{0.25} = 0.1373 \ in^{-1} \\ N_{cs} &= N_{s} \cos[\alpha] - Q \sin[\alpha] \\ N_{cs} &= \begin{cases} (8429.1122 \cos[21.0375] - (-1037.0687) \sin[21.0375]) = 8239.5612 \ \frac{lbs}{in} \\ (7084.4440 \cos[21.0375] - (-820.1737) \sin[21.0375]) = 6906.6602 \ \frac{lbs}{in} \\ \end{cases} \\ N_{c\theta} &= \frac{PR_{s}}{\cos[\alpha]} + 2\beta_{co}R_{c} \left(-M_{cs}\beta_{co} + Q_{c}\right) \\ N_{c\theta} &= \begin{cases} \frac{356(45.125)}{\cos[21.0375]} + 2(0.1373)(48.3476)(-(3471.6111)(0.1373) + 2057.9298) \\ \frac{356(45.125)}{\cos[21.0375]} + 2(0.1373)(48.3476)(-(2822.4053)(0.1373) + 1777.6603) \\ N_{c\theta} &= \begin{cases} \frac{38205.1749}{lbs} \\ \frac{lbs}{in} \\ 35667.6380 \ \frac{lbs}{in} \\ \end{cases} \\ K_{cpc} &= 1.0 \end{cases} \end{split}$$

Stress Calculations: Determine the meridional and circumferential membrane and bending stresses:

$$\sigma_{sm} = \frac{N_{cs}}{t_c} = \begin{cases} \frac{8239.5612}{1.8125} = 4545.9648 \ psi \\ \frac{6906.6602}{1.8125} = 3810.5711 \ psi \end{cases}$$

$$\sigma_{sb} = \frac{6M_{cs}}{t_c^2 K_{cpc}} = \begin{cases} \frac{6(3471.6111)}{(1.8125)^2(1.0)} = 6340.5406 \ psi \\ \frac{6(2822.4053)}{(1.8125)^2(1.0)} = 5154.8330 \ psi \end{cases}$$

$$\sigma_{\theta m} = \frac{N_{c\theta}}{t_c} = \begin{cases} \frac{38205.1749}{1.8125} = 21078.7172 \ psi \\ \frac{35667.6380}{1.8125} = 19678.6968 \ psi \end{cases}$$

$$\sigma_{\theta b} = \frac{6\nu M_{cs}}{t_c^2 K_{cpc}} = \begin{cases} \frac{6(0.3)(3471.6111)}{(1.8125)^2(1.0)} = 1902.1622 \ psi \\ \frac{6(0.3)(2822.4053)}{(1.8125)^2(1.0)} = 1546.4499 \ psi \end{cases}$$

Check Acceptable Criteria:

$$\begin{cases} \sigma_{sm} = 4545.9648 \ psi \\ \sigma_{sm} = 3810.5711 \ psi \end{cases} \le \{1.5S = 1.5(22400) = 33600 \ psi \}$$

$$True$$

$$\begin{cases} \sigma_{sm} + \sigma_{sb} = 4545.9648 + (6340.5406) = 10886.5 \ psi \\ \sigma_{sm} - \sigma_{sb} = 4545.9648 - (6340.5406) = |-1794.6 \ psi | \\ \sigma_{sm} + \sigma_{sb} = 3810.5711 + (5154.8330) = 8965.4 \ psi \\ \sigma_{sm} - \sigma_{sb} = 3810.5711 - (5154.8330) = |-1344.3 \ psi | \end{cases} \le \{S_{PS} = 67200 \ psi \}$$

$$True$$

$$\begin{cases} \sigma_{\theta m} = 21078.7172 \ psi \\ \sigma_{\theta m} = 19678.6968 \ psi \end{cases} \le \{1.5S = 1.5(22400) = 33600 \ psi \}$$

$$True$$

$$\begin{cases} \sigma_{\theta m} + \sigma_{\theta b} = 21078.7172 + (1902.1622) = 22980.9 \ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = 21078.7172 - (1902.1622) = 19176.6 \ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = 19678.6968 + (1546.4499) = 21225.1 \ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = 19678.6968 - (1546.4499) = 18132.2 \ psi \end{cases} \le \{S_{PS} = 67200 \ psi \}$$

$$True$$

Since the longitudinal membrane stress,  $\sigma_{sm}$  and the circumferential membranes stress,  $\sigma_{\theta m}$  are both tensile, the condition of local buckling need not be considered. Therefore, the cone at the cylinder-to-cone junction at the small end is adequately designed.

g) STEP 7 – The stress acceptance criterion in STEP 6 is satisfied for both the cylinder and cone. Therefore the design is complete.



Figure E4.3.7 - Conical Transition
# 4.3.8 Example E4.3.8 – Conical Transitions with a Knuckle

Determine if the proposed design for the large end of a cylinder-to-cone junction with a knuckle is adequately designed considering the following design conditions and applied forces and moments. See Figure 4.3.8 for details

Vessel Data:

•	Material	=	SA-516, Grade 70, Norm.
•	Design Conditions	=	280 psig@300°F
•	Inside Diameter (Large End)	=	120.0 in
•	Inside Radius (Large End)	=	60.0 in
•	Knuckle Radius	=	10.0 in
•	Large End Thickness	=	1.0 <i>in</i>
•	Cone Thickness	=	1.0 <i>in</i>
•	Knuckle Thickness	=	1.0 <i>in</i>
•	Corrosion Allowance	=	0.0 <i>in</i>
•	Allowable Stress	=	22400 psi
•	Weld Joint Efficiency	=	1.0
•	One-Half Apex Angle	=	30.0 <i>deg</i>
•	Axial Force (Large End)	=	-10000 <i>lbs</i>
•	Net Section Bending Moment (Large End)	=	2.0E + 06 in - lbs

Evaluate per paragraph 4.3.12.

a) STEP 1 – Compute the large end cylinder thickness,  $t_L$ , using paragraph 4.3.3.

$$t_L = \frac{D}{2} \left( \exp\left[\frac{P}{SE}\right] - 1 \right) = \frac{120.0}{2} \left( \exp\left[\frac{280.0}{22400.0(1.0)}\right] - 1 \right) = 0.7547 \text{ in}$$

As specified in the design conditions,

 $t_L = 1.0$  in

Since the required thickness is less than the design thickness, the cylinder is adequately designed for internal pressure.

b) STEP 2 – Determine the cone half-apex angle,  $\alpha$ , and compute the cone thickness,  $t_c$ , at the large end using paragraph 4.3.4.

$$\alpha = 30.0 \ deg$$
$$t_{C} = \frac{D_{i}}{2\cos[\alpha]} \left( \exp\left[\frac{P}{SE}\right] - 1 \right) = \frac{117.3205}{2\cos[30.0]} \left( \exp\left[\frac{280.0}{22400.0(1.0)}\right] - 1 \right) = 0.8520 \ in$$

Where, the value of  $D_i$  is substituted for D and is calculated as follows,

$$D_i = D - 2r(1 - \cos[\alpha]) = 120.0 - 2(10.0)(1 - \cos[30.0]) = 117.3205 \text{ in}$$

As specified in the design conditions,

 $t_{C} = 1.0 in$ 

Since the required thickness is less than the design thickness, the cone is adequately designed for internal pressure.

c) STEP 3 – Proportion the transition geometry by assuming a value for the knuckle radius,  $r_k$ , and knuckle thickness,  $t_k$ , such that the following equations are satisfied. If all of these equations cannot be satisfied, the cylinder-to-cone junction shall be designed in accordance with Part 5.

$$\{t_{k} = 1.0 \text{ in}\} \ge \{t_{L} = 1.0 \text{ in}\} \qquad True$$

$$\{r_{k} = 10.0 \text{ in}\} > \{3t_{k} = 3.0 \text{ in}\} \qquad True$$

$$\{\frac{r_{k}}{R_{L}} = \frac{10.0}{60.0} = 0.1667\} > \{0.03\} \qquad True$$

$$\{\alpha = 30 \text{ deg}\} \le \{60 \text{ deg}\} \qquad True$$

d) STEP 4 – Determine the net section axial force,  $F_L$ , and bending moment,  $M_L$ , applied to the conical transition at the location of the knuckle. The thrust load due to pressure shall not be included as part of the axial force,  $F_L$ .

 $F_L = 10000 \ lbs$  $M_L = 2.0E + 06 \ in - lbs$ 

e) STEP 5 – Compute the stresses in the cylinder, knuckle and cone at the junction using the equations in Table 4.3.7.

Determine if the knuckle is considered to be compact or non-compact.

$$\alpha r_{k} < 2K_{m} \left( \left\{ R_{k} \left( \alpha^{-1} \tan \left[ \alpha \right] \right)^{0.5} + r_{k} \right\} t_{k} \right)^{0.5}$$

$$\left\{ 0.5236(10.0) \right\} = \left\{ 2(0.7) \left( \left\{ 50.0 \left( (0.5236)^{-1} \tan \left[ 0.5236 \right] \right)^{0.5} + 10 \right\} 1 \right)^{0.5} \right\}$$

$$\left\{ 5.2360 \text{ in} \right\} < \left\{ 11.0683 \text{ in} \right\} \qquad True$$

Where,

$$K_m = 0.7$$
  

$$\alpha = \frac{30.0}{180} \pi = 0.5236 \ rad$$
  

$$R_k = R_L - r_k = 60.0 - 10.0 = 50.0 \ in$$

Therefore, analyze the knuckle junction as a compact knuckle.

Stress Calculations: Determine the hoop and axial membrane stresses at the knuckle:

$$\sigma_{\theta m} = \frac{PK_m \left( R_L \sqrt{R_L t_L} + L_k \sqrt{L_k t_C} \right) + \alpha \left( PL_{1k} r_k - 0.5 P_e L_{1k}^2 \right)}{K_m \left( t_L \sqrt{R_L t_L} + t_C \sqrt{L_k t_C} \right) \alpha t_k r_k}$$
$$\sigma_{sm} = \frac{P_e L_{1k}}{2t_k}$$

Where,

$$\begin{split} L_{1k} &= R_k \left( \alpha^{-1} \tan\left[\alpha\right] \right)^{0.5} + r_k = 50.0 \left( \left( 0.5236 \right)^{-1} \tan\left[ 0.5236 \right] \right)^{0.5} + 10.0 = 62.5038 \ in \\ L_k &= \frac{R_k}{\cos\left[\alpha\right]} + r_k = \frac{50.0}{\cos\left[ 0.5236 \right]} + 10.0 = 67.7351 \ in \\ P_e &= P + \frac{F_L}{\pi L_{1k}^2 \cos^2\left[\frac{\alpha}{2}\right]} \pm \frac{2M_L}{\pi L_{1k}^3 \cos^3\left[\frac{\alpha}{2}\right]} \\ &= \begin{cases} 280 + \frac{-10000.0}{\pi (62.5038)^2 \cos^2\left[\frac{0.5236}{2}\right]} + \frac{2(2.0E + 06)}{\pi (62.5038)^3 \cos^3\left[\frac{0.5236}{2}\right]} \\ 280 + \frac{-10000.0}{\pi (62.5038)^2 \cos^2\left[\frac{0.5236}{2}\right]} - \frac{2(2.0E + 06)}{\pi (62.5038)^3 \cos^3\left[\frac{0.5236}{2}\right]} \end{cases} \\ P_e &= \begin{cases} 284.9125 \ psi \\ 273.3410 \ psi \end{cases} \end{split}$$

Therefore,

$$\sigma_{\theta m} = \begin{cases} \left[ \frac{280(0.7)(60.0\sqrt{60.0(1.0)} + 67.7351\sqrt{67.7351(1.0)}) + }{0.5236(280(62.5038)(10.0) - 0.5(284.9125)(62.5038)^2))} \\ - \frac{1000}{0.7(1.0\sqrt{60.0(1.0)} + 1.0\sqrt{67.7351(1.0)}) + 0.5236(1.0)(10.0)} \\ - \frac{1000}{0.5236(280(62.5038)(10.0) - 0.5(273.3410)(62.5038)^2))} \\ - \frac{1000}{0.7(1.0\sqrt{60.0(1.0)} + 1.0\sqrt{67.7351(1.0)}) + 0.5236(1.0)(10.0)} \\ - \frac{1000}{0.7(1.0\sqrt{60.0(1.0)} + 0.5236(1.0)(10.0)} \\ - \frac{1000}{0.5(1.0)} \\ - \frac{1000}{$$

And,

$$\sigma_{sm} = \begin{cases} \frac{284.9125(62.5038)}{2(1.0)} = 8904.0570 \ psi \\ \frac{273.3410(62.5038)}{2(1.0)} = 8542.4256 \ psi \end{cases}$$

Check Acceptable Criteria:

$$\begin{cases} \sigma_{\theta m} = 35.9 \ psi \\ \sigma_{\theta m} = 756.7 \ psi \end{cases} \le \{S = 22400 \ psi\}$$
True  
$$\begin{cases} \sigma_{sm} = 8904.1 \ psi \\ \sigma_{sm} = 8542.4 \ psi \end{cases} \le \{S = 22400 \ psi\}$$
True

Since the longitudinal membrane stress,  $\sigma_{sm}$  and the circumferential membranes stress,  $\sigma_{\theta m}$  in the knuckle are both tensile, the condition of local buckling need not be considered. Therefore, the knuckle at the cylinder-to-cone junction at the large end is adequately designed.

STEP 6 – The stress acceptance criterion in STEP 5 is satisfied for the knuckle. Therefore, the design is complete.



Figure E4.3.8 - Knuckle Detail

# 4.4 Shells Under External Pressure and Allowable Compressive Stresses

## 4.4.1 Example E4.4.1 – Cylindrical Shell

Determine the maximum allowable external pressure (MAEP) for a cylindrical shell considering the following design conditions.

Vessel Data:

•	Material	=	SA-516, Grade 70, Norm.
•	Design Temperature	=	300°F
•	Inside Diameter	=	90.0 in
•	Thickness	=	1.125 in
•	Corrosion Allowance	=	0.125 in
•	Unsupported Length	=	636.0 in
•	Modulus of Elasticity at Design Temperature	=	$28.3E + 06 \ psi$
•	Yield Strength	=	33600 psi

Evaluate per paragraph 4.4.5.

a) STEP 1 – Assume an initial thickness, t, and unsupported length, L.

$$t = t - Corrosion \ Allowance = 1.125 - 0.125 = 1.0 \ in L = 636.0 \ in$$

b) STEP 2 – Calculate the predicted elastic buckling stress,  $F_{he}$ .

$$F_{he} = \frac{1.6C_h E_y t}{D_o} = \frac{1.6(0.0092)(28.3E+06)(1.0)}{92.25} = 4515.7290 \ psi$$

Where,

$$D_o = D + 2(Uncorroded Thickness) = 90.0 + 2(1.125) = 92.25$$
 in

$$M_x = \frac{L}{\sqrt{R_o t}} = \frac{636.0}{\sqrt{\left(\frac{92.25}{2}\right)1.0}} = 93.6459$$

$$2\left(\frac{D_o}{t}\right)^{0.94} = 2\left(\frac{92.25}{1.0}\right)^{0.94} = 140.6366$$

Since  $13 < M_x < 2 \left(\frac{D_o}{t}\right)^{0.94}$ , calculate  $C_h$  as follows:

$$C_h = 1.12 M_x^{-1.058} = 1.12 (93.6459)^{-1.058} = 0.0092$$

c) STEP 3 – Calculate the predicted buckling stress,  $F_{ic}$ .

$$\frac{F_{he}}{S_y} = \frac{4515.7290}{33600.0} = 0.1344$$
  
Since  $\frac{F_{he}}{S_y} \le 0.552$ , calculate  $F_{ic}$  as follows:

$$F_{ic} = F_{he} = 4515.7290 \ psi$$

d) STEP 4 – Calculate the value of design factor, FS, per paragraph 4.4.2.

$$0.55S_y = 0.55(33600.0) = 18480.0 \ psi$$

Since  $F_{ic} \leq 0.55S_v$ , calculate *FS* as follows:

$$FS = 2.0$$

e) STEP 5 – Calculate the allowable external pressure,  $P_a$ .

$$P_{a} = 2F_{ha} \left(\frac{t}{D_{o}}\right) = 2(2257.8645) \left(\frac{1.0}{92.25}\right) = 48.9 \text{ psi}$$
$$F_{ha} = \frac{F_{ic}}{FS} = \frac{4515.7290}{2.0} = 2257.8645 \text{ psi}$$

f) STEP 6 – If the allowable external pressure,  $P_a$ , is less than the design external pressure, increase the shell thickness or reduce the unsupported length of the shell (i.e. by the addition of a stiffening rings) and go to STEP 2. Repeat this process until the allowable external pressure is equal to or greater than the design external pressure.

The allowable external pressure is  $P_a = 48.9 \ psi$ 

Combined Loadings – cylindrical shells subject to external pressure and other loadings shall satisfy the requirements of paragraph 4.4.12. In this example problem, the cylindrical shell is only subject to external pressure.

# 4.4.2 Example E4.4.2 – Conical Shell

Determine the maximum allowable external pressure (MAEP) for a conical shell considering the following design conditions.

Vessel Data:

•	Material	=	SA-516, Grade 70, Norm
•	Design Temperature	=	300°F
•	Inside Diameter (Large End)	=	150.0 in
•	Thickness (Large End)	=	1.8125 in
•	Inside Diameter (Small End)	=	90.0 in
•	Thickness (Small End)	=	1.125 in
•	Thickness (Conical Section)	=	1.9375 in
•	Axial Cone Length	=	78.0 in
•	One-Half Apex Angle	=	21.0375 deg
•	Corrosion Allowance	=	0.125 in
•	Modulus of Elasticity at Design Temperature	=	$28.3E + 06 \ psi$
•	Yield Strength	=	33600 <i>psi</i>

Evaluate per paragraph 4.4.6. and 4.4.5.

The required thickness of a conical shell subjected to external pressure loading shall be determined using the equations for a cylinder by making the following substitutions:

a) The value of  $t_c$  is substituted for t in the equations in paragraph 4.4.5.

 $t_c = t = 1.9375 - Corrosion Allowance = 1.9375 - 0.125 = 1.8125$  in

b) For offset cones, the cone angle,  $\alpha$ , shall satisfy the requirements of paragraph 4.3.4.

The conical shell in this example problem is not of the offset type. Therefore, no additional requirements are necessary.

c) The value of  $0.5(D_L + D_S)/\cos[\alpha]$  is substituted for  $D_o$  in the equations in paragraph 4.4.5, (concentric cone design with common center line per Figure 4.4.7 Sketch (a)).

$$D_o = \frac{0.5(D_L + D_s)}{\cos[\alpha]} = \frac{0.5[(150.0 + 2(1.8125)) + (90.0 + 2(1.125))]}{\cos[21.0375]} = 131.7170 \text{ in}$$

d) The value of  $L_{ce}/\cos[\alpha]$  is substituted for *L* in the equations in paragraph 4.4.5 where  $L_{ce}$  is determined as shown below. For Sketches (a) and (e) in Figure 4.4.7:

$$L_{ce} = L_c$$
  
$$L = \frac{L_{ce}}{\cos[\alpha]} = \frac{78.0}{\cos[21.0375]} = 83.5703 \text{ in}$$

e) Note that the half-apex angle of a conical transition can be computed knowing the shell geometry with the following equations. These equations were developed with the assumption that the conical transition contains a cone section, knuckle, or flare. If the transition does not contain a knuckle or flare, the radii of these components should be set to zero when computing the half-apex angle (see Figure 4.4.7).

$$If \ (R_{L} - r_{k}) \ge (R_{S} + r_{f}):$$

$$\alpha = \beta + \phi = 0.3672 - 0 = 0.3672 \ rad = 21.0375 \ deg$$

$$\beta = \arctan\left[\frac{(R_{L} - r_{k}) - (R_{S} + r_{f})}{L_{c}}\right] = \arctan\left[\frac{(75.0 - 0) - (45.0 + 0)}{78.0}\right] = 0.3672 \ rad$$

$$\phi = \arcsin\left[\frac{(r_{f} + r_{k})\cos[\beta]}{L_{c}}\right] = \arcsin\left[\frac{(0.0 + 0.0)\cos[0.3672]}{78.0}\right] = 0.0 \ rad$$

Proceed with the design following the steps outlined in paragraph 4.4.5.

a) STEP 1 – Assume an initial thickness, t, and unsupported length, L (see Figures 4.4.1 and 4.4.2).

$$t = 1.8125$$
 in  
 $L = 83.5703$  in

b) STEP 2 – Calculate the predicted elastic buckling stress,  $F_{he}$ .

$$F_{he} = \frac{1.6C_h E_y t}{D_o} = \frac{1.6(0.1301)(28.3E + 06)(1.8125)}{131.7170} = 81062.4824 \ psi$$

Where,

$$M_{x} = \frac{L}{\sqrt{R_{o}t}} = \frac{83.5703}{\sqrt{\left(\frac{131.7170}{2.0}\right)} 1.8125}} = 7.6490$$
$$2\left(\frac{D_{o}}{t}\right)^{0.94} = 2\left(\frac{131.7170}{1.8125}\right)^{0.94} = 112.3859$$

Since  $1.5 < M_x < 13$ , calculate  $C_h$  as follows:

$$C_h = \frac{0.92}{M_x - 0.579} = \frac{0.92}{7.6490 - 0.579} = 0.1301$$

c) STEP 3 – Calculate the predicted buckling stress,  $F_{ic}$ .

$$\frac{F_{he}}{S_y} = \frac{81062.4824}{33600.0} = 2.4126$$

Since 
$$0.552 < \left(\frac{F_{he}}{S_y}\right) < 2.439$$
, calculate  $F_{ic}$  as follows:

$$F_{ic} = 0.7S_y \left(\frac{F_{he}}{S_y}\right)^{0.4} = 0.7(33600.0) \left(\frac{81062.4824}{33600.0}\right)^{0.4} = 33452.5760 \text{ psi}$$

d) STEP 4 – Calculate the value of design factor, FS, per paragraph 4.4.2.

$$0.55S_y = 0.55(33600.0) = 18480.0 \ psi$$

Since  $0.55S_y < F_{ic} < S_y$ , calculate FS as follows:

$$FS = 2.407 - 0.741 \left(\frac{F_{ic}}{S_y}\right) = 2.407 - 0.741 \left(\frac{33452.5760}{33600.0}\right) = 1.6693$$

e) STEP 5 – Calculate the allowable external pressure,  $P_a$ .

$$P_{a} = 2F_{ha} \left(\frac{t}{D_{o}}\right) = 2(20039.8826) \left(\frac{1.8125}{131.7170}\right) = 551.5 \text{ psi}$$
$$F_{ha} = \frac{F_{ic}}{FS} = \frac{33452.5760}{1.6693} = 20039.8826 \text{ psi}$$

f) STEP 6 – If the allowable external pressure,  $P_a$ , is less than the design external pressure, increase the shell thickness or reduce the unsupported length of the shell (i.e. by the addition of a stiffening rings) and go to STEP 2. Repeat this process until the allowable external pressure is equal to or greater than the design external pressure.

The maximum allowable external pressure  $P_a = 551.5 \ psi$ 

Combined Loadings – conical shells subject to external pressure and other loadings shall satisfy the requirements of paragraph 4.4.12. In this example problem, the conical shell is only subject to external pressure.

# 4.4.3 Example E4.4.3 – Spherical Shell and Hemispherical Head

Determine the maximum allowable external pressure (MAEP) for a hemispherical head considering the following design conditions.

Vessel Data:

•	Material	=	SA-542, Type D, Class 4a
•	Design Temperature	=	350°F
•	Inside Diameter	=	149.0 in
•	Thickness	=	2.8125 in
•	Corrosion Allowance	=	0.0 <i>in</i>
•	Modulus of Elasticity at Design Temperature	=	$29.1E + 06 \ psi$
•	Yield Strength	=	58000 <i>psi</i>

Evaluate per paragraph 4.4.7.

a) STEP 1 – Assume an initial thickness, *t* for the spherical shell.

$$t = 2.8125$$
 in

b) STEP 2 – Calculate the predicted elastic buckling stress,  $F_{he}$ .

$$F_{he} = 0.075E_y \left(\frac{t}{R_o}\right) = 0.075(29.1E + 06) \left(\frac{2.8125}{\frac{149.0}{2} + 2.8125}\right) = 79395.7154 \text{ psi}$$

c) STEP 3 – Calculate the predicted buckling stress,  $F_{ic}$ .

$$\frac{F_{he}}{S_v} = \frac{79395.7154}{58000.0} = 1.3689$$

Since  $0.55 < \left(\frac{F_{he}}{S_y}\right) < 1.6$ , calculate  $F_{ic}$  as follows:

$$F_{ic} = 0.18F_{he} + 0.45S_y = 0.18(79395.7154) + 0.45(58000.0) = 40391.2288 \ psi$$

d) STEP 4 – Calculate the value of design margin, *FS* , per paragraph 4.4.2.

$$0.55S_y = 0.55(58000.0) = 31900.0 \ psi$$

Since  $0.55S_y < F_{ic} < S_y$ , calculate the FS as follows:

$$FS = 2.407 - 0.741 \left(\frac{F_{ic}}{S_y}\right) = 2.407 - 0.741 \left(\frac{40391.2288}{58000.0}\right) = 1.8910$$

e) STEP 5 – Calculate the allowable external pressure,  $P_a$ .

$$P_{a} = 2F_{ha} \left(\frac{t}{R_{o}}\right) = 2\left(21359.7191\right) \left(\frac{2.8125}{\frac{149.0}{2} + 2.8125}\right) = 1554.1 \text{ psi}$$
$$F_{ha} = \frac{F_{ic}}{FS} = \frac{40391.2288}{1.8910} = 21359.7191 \text{ psi}$$

f) STEP 6 – If the allowable external pressure,  $P_a$ , is less than the design external pressure, increase the shell thickness and go to STEP 2. Repeat this process until the allowable external pressure is equal to or greater than the design external pressure.

The maximum allowable external pressure  $P_a = 1554.1 \ psi$ 

Combined Loadings – spherical shells and hemispherical heads subject to external pressure and other loadings shall satisfy the requirements of paragraph 4.4.12. In this example problem, the hemispherical head is only subject to external pressure.

## 4.4.4 Example E4.4.4 – Torispherical Head

Determine the maximum allowable external pressure (MAEP) for a torispherical head considering the following design conditions.

Vessel Data:

•	Material	=	SA-387, Grade 11, Class 1
•	Design Temperature	=	650°F
•	Inside Diameter	=	72.0 in
•	Crown Radius	=	72.0 <i>in</i>
•	Knuckle Radius	=	4.375 in
•	Thickness	=	0.625 in
•	Corrosion Allowance	=	0.125 in
•	Modulus of Elasticity at Design Temperature	=	$26.55E + 06 \ psi$
•	Yield Strength at Design Temperature	=	26900 psi

Evaluate per paragraph 4.4.8 and 4.4.7.

The required thickness of a torispherical head subjected to external pressure loading shall be determined using the equations for a spherical shell in paragraph 4.4.7 by substituting the outside crown radius for  $R_a$ .

$$R_o = 72.0 + 0.625 = 72.625$$
 in

Restrictions on Torispherical Head Geometry – the restriction of paragraph 4.3.6 shall apply. See paragraph 4.3.6.1.b, STEP 2 of E4.3.4.

Torispherical heads With Different Dome and Knuckle Thickness – heads with this configuration shall be designed in accordance with Part 5. In this example problem, the dome and knuckle thickness are the same.

Proceed with the design following the steps outlined in paragraph 4.4.7.

a) STEP 1 – Assume an initial thickness, t for the torispherical head.

t = 0.625 - Corrosion Allowance = 0.625 - 0.125 = 0.500 in

b) STEP 2 – Calculate the predicted elastic buckling stress,  $F_{he}$ .

$$F_{he} = 0.075E_y \left(\frac{t}{R_o}\right) = 0.075(26.55E + 06) \left(\frac{0.500}{72.625}\right) = 13709.1222 \ psi$$

c) STEP 3 – Calculate the predicted buckling stress,  $F_{ic}$ .

$$\frac{F_{he}}{S_v} = \frac{13709.1222}{26900.0} = 0.5096$$

Since  $\frac{F_{he}}{S_y} \le 0.55$  , calculate  $F_{ic}$  as follows:

$$F_{ic} = F_{he} = 13709.1222 \ psi$$

d) STEP 4 – Calculate the value of design margin, FS, per paragraph 4.4.2.

$$0.55S_y = 0.55(26900.0) = 14795.0 \ psi$$

Since  $F_{ic} \leq 0.55S_v$ , calculate the *FS* as follows:

$$FS = 2.0$$

e) STEP 5 – Calculate the allowable external pressure,  $P_a$ .

$$P_{a} = 2F_{ha} \left(\frac{t}{R_{o}}\right) = 2(6854.5611) \left(\frac{0.500}{72.625}\right) = 94.4 \text{ psi}$$
$$F_{ha} = \frac{F_{ic}}{FS} = \frac{13709.1222}{2.0} = 6854.5611 \text{ psi}$$

f) STEP 6 – If the allowable external pressure,  $P_a$ , is less than the design external pressure, increase the shell thickness and go to STEP 2. Repeat this process until the allowable external pressure is equal to or greater than the design external pressure.

The maximum allowable external pressure  $P_a = 94.4 \ psi$ 

Combined Loadings – torispherical heads subject to external pressure and other loadings shall satisfy the requirements of paragraph 4.4.12. In this example problem, the torispherical head is only subject to external pressure.

# 4.4.5 Example E4.4.5 – Elliptical Head

Determine the maximum allowable external pressure (MAEP) for a 2:1 elliptical head considering the following design conditions.

Vessel Data:

•	Material	=	SA-516, Grade 70, Norm.
•	Design Temperature	=	300°F
•	Inside Diameter	=	90.0 in
•	Thickness	=	1.125 in
•	Corrosion Allowance	=	0.125 in
•	Modulus of Elasticity at Design Temperature	=	$28.3E + 06 \ psi$
•	Yield Strength	=	33600 <i>psi</i>

## Evaluate per paragraph 4.4.9 and 4.4.7.

The required thickness of an elliptical head subjected to external pressure loading shall be determined using the equations for a spherical shell in paragraph 4.4.7 by substituting  $K_o D_o$  for  $R_o$  where  $K_o$  is given by the following equation.

$$\begin{split} K_o &= 0.25346 + 0.13995 \left(\frac{D_o}{2h_o}\right) + 0.12238 \left(\frac{D_o}{2h_o}\right)^2 - 0.015297 \left(\frac{D_o}{2h_o}\right)^3 \\ K_o &= \left(\begin{array}{c} 0.25346 + 0.13995 \left(\frac{92.25}{2(23.0625)}\right) + 0.12238 \left(\frac{92.25}{2(23.0625)}\right)^2 - \\ 0.015297 \left(\frac{92.25}{2(23.0625)}\right)^3 \end{array}\right) = 0.9005 \\ D_o &= 90.0 + 2(1.125) = 92.25 \\ h_o &= \left(\frac{D_o}{4}\right) = \frac{92.25}{4} = 23.0625 \ in \end{split}$$

Therefore,

 $R_o = K_o D_o = 0.9005(92.25) = 83.0711$  in

Proceed with the design following the steps outlined in paragraph 4.4.7.

a) STEP 1 – Assume an initial thickness, t for the elliptical head.

 $t = 1.125 - Corrosion \ Allowance = 1.125 - 0.125 = 1.0 \ in$ 

b) STEP 2 – Calculate the predicted elastic buckling stress,  $F_{he}$ .

$$F_{he} = 0.075E_y \left(\frac{t}{R_o}\right) = 0.075(28.3E + 06) \left(\frac{1.0}{83.0711}\right) = 25550.4020 \ psi$$

c) STEP 3 – Calculate the predicted buckling stress,  $F_{ic}$ .

$$\frac{F_{he}}{S_y} = \frac{25550.4020}{33600} = 0.7604$$

Since 
$$0.55 < \left(\frac{F_{he}}{S_y}\right) \le 1.6$$
, calculate  $F_{ic}$  as follows:  
 $F_{ic} = 0.18F_{he} + 0.45S_y$ 

$$F_{ic} = 0.18(25550.4020) + 0.45(33600.0) = 19719.0724 \ psi$$

d) STEP 4 – Calculate the value of design margin, FS, per paragraph 4.4.2.

$$0.55S_y = 0.55(33600.0) = 18480.0 \ psi$$

Since  $0.55S_v < F_{ic} < S_v$ , calculate the FS as follows:

$$FS = 2.407 - 0.741 \left(\frac{F_{ic}}{S_y}\right) = 2.407 - 0.741 \left(\frac{19719.0724}{33600.0}\right) = 1.9721$$

e) STEP 5 – Calculate the allowable external pressure,  $P_a$ .

$$P_{a} = 2F_{ha}\left(\frac{t}{R_{o}}\right) = 2(9999.0226)\left(\frac{1.0}{83.0711}\right) = 240.7 \ psi$$
$$F_{ha} = \frac{F_{ic}}{FS} = \frac{19719.0724}{1.9721} = 9999.0226 \ psi$$

f) STEP 6 – If the allowable external pressure,  $P_a$ , is less than the design external pressure, increase the shell thickness and go to STEP 2. Repeat this process until the allowable external pressure is equal to or greater than the design external pressure.

The maximum allowable external pressure  $P_a = 240.7 \ psi$ 

Combined Loadings – ellipsoidal heads subject to external pressure and other loadings shall satisfy the requirements of paragraph 4.4.12. In this example problem, the ellipsoidal head is only subject to external pressure.

# 4.4.6 Example E4.4.6 – Combined Loadings and Allowable Compressive Stresses

Determine the allowable compressive stresses of the proposed cylindrical shell section considering the following design conditions and specified applied loadings.

Vessel Data:

•	Material	=	SA-516, Grade 70, Norm.
•	Design Conditions	=	–14.7 psig @ 300°F
•	Inside Diameter	=	90.0 in
•	Thickness	=	1.125 in
•	Corrosion Allowance	=	0.125 in
•	Unsupported Length	=	636.0 in
•	Modulus of Elasticity at Design Temperature	=	$28.3E + 06 \ psi$
•	Yield Strength	=	33600 <i>psi</i>
•	Axial Force	=	-66152.5 <i>lbs</i>
•	Net Section Bending Moment	=	3.048E + 06 in - lbs
•	Shear Force	=	11257.6 <i>lbs</i>

Adjust variables for corrosion and determine outside dimensions.

$$D_o = 90.0 + 2(Uncorroded Thickness) = 90.0 + 2(1.125) = 92.25 in$$

$$R_o = 0.5D_o = 0.5(92.25) = 46.125 in$$

$$D_i = 90.0 + 2(Corrosion Allowance) = 90.0 + 2(0.125) = 90.25 in$$

$$t = 1.125 - Corrosion Allowance = 1.125 - 0.125 = 1.0 in$$

Evaluate per paragraph 4.4.12.2

The loads transmitted to the cylindrical shell are given in the Table E4.4.6.2. Note that this table is given in terms of the load parameters shown in Part 4, Table 4.1.1 (Table E4.4.6.1 of this example). As shown in Table E4.4.6.1, the acceptance criteria is that the general primary membrane stress for each load case must be less than or equal to the allowable stress at the specified design condition

Paragraph 4.4.12.2, the following procedure shall be used to determine the allowable compressive stresses for cylindrical shells that are based on loading conditions. By inspection of the results shown in Table E4.4.6.3, Load Case 5 is determined to be the governing load case. The pressure and net section axial force, bending moment, and radial shear force at the location of interest for Load Case 5 are:

 $0.9P + P_s = -14.7 \ psi$  (Conservatively)  $F_5 = -66152.5 \ lbs$   $M_5 = 1828800 \ in - lbs$  $V_5 = 6754.6 \ lbs$ 

Common parameters used in each of the loading conditions are given in paragraph 4.4.12.2.k.

Per paragraph 4.4.12.2.k:

$$A = \frac{\pi \left( D_o^2 - D_i^2 \right)}{4} = \frac{\pi \left( 92.25^2 - 90.25^2 \right)}{4} = 286.6703 \ in^2$$

$$S = \frac{\pi \left( D_o^4 - D_i^4 \right)}{32D_o} = \frac{\pi \left( 92.25^4 - 90.25^4 \right)}{32(92.25)} = 6469.5531 \ in^3$$

$$f_h = \frac{PD_o}{2t} = \frac{14.7(92.25)}{2(1.0)} = 678.0375 \ psi$$

$$f_b = \frac{M}{S} = \frac{1828800}{6469.5531} = 282.6779 \ psi$$

$$f_a = \frac{F}{A} = \frac{66152.5}{286.6703} = 230.7616 \ psi$$

$$f_q = \frac{P\pi D_i^2}{4A} = \frac{14.7(\pi)(90.25)^2}{4(286.6703)} = 328.0341 \ psi$$

$$f_v = \frac{V \sin \left[\phi\right]}{A} = \frac{6754.6 \cdot \sin \left[90.0\right]}{286.6703} = 23.5623 \ psi$$

Note:  $\phi$  is defined as the angle measured around the circumference from the direction of the applied shear force to the point under consideration. For this example problem,  $\phi = 90^{\circ}$  to maximize the shear force.

$$r_g = 0.25\sqrt{D_o^2 + D_i^2} = 0.25\sqrt{(92.25^2 + 90.25^2)} = 32.2637 \text{ in}$$
$$M_x = \frac{L}{\sqrt{R_o t}} = \frac{636.0}{\sqrt{(46.125)1.0}} = 93.6459$$

The value of the slenderness factor for column buckling,  $\lambda_c$  is calculated in paragraph 4.4.12.2.b.

Per paragraph 4.4.12.2:

a) <u>External Pressure Acting Alone</u> – the allowable hoop compressive membrane stress of a cylinder subject to external pressure acting alone,  $F_{ha}$ , is computed using the equations in paragraph 4.4.5.

From Example E4.4.1,

$$F_{ha} = 2257.8645 \ psi$$

b) <u>Axial Compressive Stress Acting Alone</u> – the allowable axial compressive membrane stress of a cylinder subject to an axial compressive load acting alone,  $F_{xa}$ , is computed using the following equations:

The value of the slenderness factor for column buckling,  $\lambda_c$  is dependent on the calculated value of  $F_{xa}$ , defined as the allowable compressive membrane stress of a cylinder due to an axial compressive load, with  $\lambda_c \leq 0.15$ . The value of  $\lambda_c$  determines the procedure to be used in obtaining the allowable axial compressive stress, either due to local buckling,  $\lambda_c \leq 0.15$ , or column buckling,  $\lambda_c > 0.15$ . Therefore, an initial calculation is required to determine the value of  $F_{xa}$  with an assumed value of  $\lambda_c \leq 0.15$ . The actual value of  $\lambda_c$  is then calculated and the procedure to obtain the allowable axial compressive stress is determined.

The design factor *FS* used in paragraph 4.4.12.2.b is dependent on the predicted buckling stress  $F_{ic}$  and the material's yield strength,  $S_y$  as shown in paragraph 4.4.2. An initial calculation is required to determine the value of  $F_{xa}$  by setting FS = 1.0, with  $F_{ic} = F_{xa}$ . The initial value of  $F_{ic}$  is then compared to  $S_y$  as shown in paragraph 4.4.2 and the value of *FS* is determined. This computed value of *FS* is then used in paragraph 4.4.12.2.b.

1) For  $\lambda_c \leq 0.15$ , (Local Buckling)

$$F_{xa} = \min[F_{xa1}, F_{xa2}]$$
$$\frac{D_o}{t} = \frac{92.25}{1.0} = 92.25$$

Since  $\frac{D_o}{t} \le 135$ , calculate  $F_{xa1}$  as follows with an initial value of FS = 1.0.

$$F_{xa1} = \frac{S_y}{FS} = \frac{33600}{1.0} = 33600 \ psi$$

The value of  $F_{xa2}$  is calculated as follows with an initial value of FS = 1.0.

$$F_{xa2} = \frac{F_{xe}}{FS}$$
$$F_{xe} = \frac{C_x E_y t}{D_o}$$

Since  $\frac{D_o}{t} \le 1247$ , calculate  $C_x$  as follows:

$$C_x = \min\left[\frac{409\overline{c}}{\left(389 + \frac{D_o}{t}\right)}, \ 0.9\right]$$

Since  $M_x \ge 15$ , calculate  $\overline{c}$  as follows:

$$\overline{c} = 1.0$$

$$C_x = \min\left[\frac{409(1.0)}{389 + \frac{92.25}{1.0}}, \ 0.9\right] = 0.8499$$

Therefore,

$$F_{xe} = \frac{0.8499(28.3E + 06)(1.0)}{92.25} = 260728.1301 \text{ psi}$$
$$F_{xa2} = \frac{F_{xe}}{FS} = \frac{260728.1301}{1.0} = 260728.1301 \text{ psi}$$
$$F_{xa} = \min[33600, 260728] = 33600 \text{ psi}$$

With a value of  $F_{ic} = F_{xa} = 33600$ , in accordance with paragraph 4.4.2, it is determined the value of FS = 1.667 since  $\{F_{ic} = 33600\} = \{S_y = 33600\}$ . Using this computed value of FS = 1.667 in paragraph 4.4.12.2.b,  $F_{xa}$  is calculated as follows.

$$F_{xa1} = \frac{S_y}{FS} = \frac{33600}{1.667} = 20155.9688 \ psi$$
  
$$F_{xa2} = \frac{F_{xe}}{FS} = \frac{260728.1301}{1.667} = 156405.5969 \ psi$$
  
$$F_{xa} = \min[20155.9688, 156405.5969] = 20155.9688 \ psi$$

With  $F_{xa}$  calculated, determine the value of  $\lambda_c$  from paragraph 4.4.12.2.k. For a cylinder with end conditions with one end free and the other end fixed,  $K_u = 2.1$ .

$$\lambda_{c} = \frac{K_{u}L_{u}}{\pi r_{g}} \left(\frac{F_{xa} \cdot FS}{E_{y}}\right)^{0.5} = \frac{2.1(636.0)}{\pi (32.2637)} \left(\frac{20155.9688(1.667)}{28.3E + 06}\right)^{0.5} = 0.4540$$

Since  $\lambda_c > 0.15$ , the allowable axial compressive membrane stress of the cylinder is due to Column Buckling, per paragraph 4.4.12.2.b.2.

2) For  $\lambda_c > 0.15$  and  $\frac{K_u L_u}{r_g} < 200$  (Column Buckling)  $\left\{ \frac{K_u L_u}{r_g} = \frac{2.1(636.0)}{32.2637} = 41.3964 \right\} < \{200\}$  True

Since  $0.15 < \lambda_c < 1.147$  , calculate  $F_{ca}$  as follows:

$$F_{ca} = F_{xa} \left[ 1 - 0.74 (\lambda_c - 0.15) \right]^{0.3}$$
  
$$F_{ca} = 20155.9688 \left[ 1 - 0.74 (0.4540 - 0.15) \right]^{0.3} = 18672.4331 \text{ psi}$$

c) <u>Compressive Bending Stress</u> – the allowable axial compressive membrane stress of a cylindrical shell subject to a bending moment acting across the full circular cross section,  $F_{ba}$ , is computed using the following equations:

Similar to the procedure used in paragraph 4.4.12.2.b, the design factor *FS* used in paragraph 4.4.12.2.c is dependent on the predicted buckling stress  $F_{ic}$  and the material's yield strength,  $S_y$  as shown in paragraph 4.4.2. An initial calculation is required to determine the value of  $F_{ba}$  by setting FS = 1.0, with  $F_{ic} = F_{ba}$ . The initial value of  $F_{ic}$  is then compared to  $S_y$  as shown in paragraph 4.4.2 and the value of *FS* is determined. This computed value of *FS* is then used in paragraph 4.4.12.2.c.

$$\frac{D_o}{t} = \frac{92.25}{1.0} = 92.25$$
$$\gamma = \frac{S_y D_o}{E_y t} = \frac{33600(92.25)}{(28.3E + 06)(1.0)} = 0.1095$$

Since  $\frac{D_o}{t} \le 100$  and  $\gamma < 0.11$ , calculate  $F_{ba}$  as follows with an initial value of FS = 1.0:

$$F_{ba} = \frac{S_{y} \left(1.4 - 2.9\gamma\right)}{FS} = \frac{33600 \left(1.4 - 2.9 \left(0.1095\right)\right)}{1.0} = 36370.32 \ psi$$

With a value of  $F_{ic} = F_{ba} = 36370.32$ , in accordance with paragraph 4.4.2, it is determined the value of FS = 1.667 since  $\{F_{ic} = 36370.32\} \ge \{S_y = 33600\}$ . Using this computed value of FS = 1.667 in paragraph 4.4.12.2.c,  $F_{ba}$  is calculated as follows.

$$F_{ba} = \frac{S_{y} \left(1.4 - 2.9\gamma\right)}{FS} = \frac{33600 \left(1.4 - 2.9 \left(0.1095\right)\right)}{1.667} = 21817.8284 \ psi$$

d) <u>Shear Stress</u> – the allowable shear stress of a cylindrical shell,  $F_{va}$ , is computed using the following equations:

Similar to the procedure used in paragraph 4.4.12.2.b, the design factor *FS* used in paragraph 4.4.12.2.d is dependent on the predicted buckling stress  $F_{ic}$  and the material's yield strength,  $S_y$  as shown in paragraph 4.4.2. An initial calculation is required to determine the value of  $F_{va}$  by setting FS = 1.0, with  $F_{ic} = F_{va}$ . The initial value of  $F_{ic}$  is then compared to  $S_y$  as shown in paragraph 4.4.2 and the value of *FS* is determined. This computed value of *FS* is then used in paragraph 4.4.12.2.d.

The value of  $F_{va}$  is calculated as follows with an initial value of FS = 1.0.

$$F_{va} = \frac{\eta_v F_{ve}}{FS}$$

$$F_{ve} = \alpha_v C_v E_y \left(\frac{t}{D_o}\right)$$

For a value of  $M_{\rm x}=93.6459$  ,

$$4.347 \left(\frac{D_o}{t}\right) = 4.347 \left(\frac{95.25}{1.0}\right) = 401.0108$$

Since  $26 < M_x < 4.347 \left(\frac{D_o}{t}\right)$ , calculate  $C_v$  as follows:

$$C_{v} = \frac{1.492}{M_{x}^{0.5}} = \frac{1.492}{(93.6459)^{0.5}} = 0.1542$$

Since  $\left(\frac{D_o}{t}\right) \le 500$  , calculate  $\alpha_v$  as follows:  $\alpha_v = 0.8$ 

It follows then,

$$F_{ve} = 0.8(0.1542)(28.3E + 06)\left(\frac{1.0}{92.25}\right) = 37843.7724 \text{ psi}$$
$$\frac{F_{ve}}{S_y} = \frac{37843.7724}{33600} = 1.1263$$
Since  $0.48 < \left(\frac{F_{ve}}{S_y}\right) < 1.7$ , calculate  $\eta_v$  as follows:

$$\eta_{v} = 0.43 \left(\frac{S_{v}}{F_{ve}}\right) + 0.1 = 0.43 \left(\frac{33600}{37843.7724}\right) + 0.1 = 0.4818$$

Therefore,

$$F_{va} = \frac{0.4818(37843.7724)}{1.0} = 18233.1295 \ psi$$

With a value of  $F_{ic} = F_{va} = 18233.1295$ , in accordance with VIII-2, paragraph 4.4.2, it is determined the value of FS = 2.0 since  $\{F_{ic} = 18233.1295\} \le \{0.55S_y = 18480\}$ . Using this computed value of FS = 2.0 in paragraph 4.4.12.2.d,  $F_{va}$  is calculated as follows.

$$F_{va} = \frac{0.4818(37843.7724)}{2.0} = 9116.5648 \, psi$$

- e) <u>Axial Compressive Stress and Hoop Compression</u> the allowable compressive stress for the combination of uniform axial compression and hoop compression,  $F_{xha}$ , is computed using the following equations:
  - 1) For  $\lambda_c \leq 0.15$ ,  $F_{xha}$  is computed using the following equation with  $F_{ha}$  and  $F_{xa}$  evaluated using the equations in paragraphs 4.4.12.2.a and 4.4.12.2.b.1, respectively.

Although,  $0.15 < \lambda_c \le 1.2$ , the procedure in VIII-2, paragraph 4.4.12.2.e.1 to calculate  $F_{xha}$  is required per paragraph 4.4.12.2.e.2 with the modifications noted, (see next step in procedure).

$$F_{xha} = \left[ \left( \frac{1}{F_{xa}^2} \right) - \left( \frac{C_1}{C_2 F_{xa} F_{ha}} \right) + \left( \frac{1}{C_2^2 F_{ha}^2} \right) \right]^{-0.5}$$

$$F_{xha} = \left[ \left( \frac{1}{(20155.9688)^2} \right) - \left( \frac{0.1344}{0.8241(20155.9688)(2257.8645)} \right) + \left[ \frac{1}{(0.8241)^2(2257.8645)^2} \right]^{-0.5} \right]^{-0.5}$$

Where,

$$C_{1} = \frac{\left(F_{xa} \cdot FS + F_{ha} \cdot FS\right)}{S_{y}} - 1.0 = \frac{20155.9688(1.667) + 2257.8645(2.0)}{33600} - 1.0$$

$$C_{1} = 0.1344$$

$$C_{2} = \frac{f_{x}}{f_{h}} = \frac{558.7957}{678.0375} = 0.8241$$

$$f_{x} = f_{a} + f_{q} = 230.7616 + 328.0341 = 558.7957 \ psi$$

2) For  $0.15 < \lambda_c \le 1.2$ ,  $F_{xha}$ , is computed from the following equation with  $F_{ah1} = F_{xha}$  evaluated using the equations in paragraph 4.4.12.2.e.1, and  $F_{ah2}$  using the following procedure. The value of  $F_{ca}$  used in the calculation for  $F_{ah2}$  is evaluated using the equation in VIII-2, paragraph 4.4.12.2.b.2 with  $F_{xa} = F_{xha}$  as determined in VIII-2, paragraph 4.4.12.2.e.1. As noted, the load on the end of a cylinder due to external pressure does not contribute to column buckling and therefore  $F_{ah1}$  is compared with  $f_a$  rather than  $f_x$ . The stress due to the pressure load does, however, lower the effective yield stress and the quantity in  $(1 - f_q / S_y)$  accounts for this reduction.

$$F_{xha} = \min[F_{ah1}, F_{ah2}] = \min[1864.3312, 1710.2496] = 1710.2496 \text{ psi}$$
$$F_{ah1} = F_{xha} = 1864.3312 \text{ psi}$$

$$F_{ah2} = F_{ca} \left( 1 - \frac{f_q}{S_y} \right) = 1727.1112 \left( 1 - \frac{328.0341}{33600} \right) = 1710.2496 \ psi$$

Where,

$$F_{ca} = F_{xha} \left[ 1 - 0.74 (\lambda_c - 0.15) \right]^{0.3}$$
  
$$F_{ca} = 1864.3312 \left[ 1 - 0.74 (0.4540 - 0.15) \right]^{0.3} = 1727.1112 \text{ psi}$$

3) For  $\lambda_c \leq 0.15$ , the allowable hoop compressive membrane stress,  $F_{hxa}$ , is given by the following equation.

$$F_{hxa} = \frac{F_{xha}}{C_2}$$

Note: this step is not required since  $\lambda_c > 0.15$  .

- f) <u>Compressive Bending Stress and Hoop Compression</u> the allowable compressive stress for the combination of axial compression due to a bending moment and hoop compression,  $F_{bha}$ , is computed using the following equations.
  - 1) An iterative solution procedure is utilized to solve these equations for  $C_3$  with  $F_{ha}$  and  $F_{ba}$  evaluated using the equations in paragraphs 4.4.12.2.a and 4.4.12.2.c, respectively.

$$F_{bha} = C_3 C_4 F_{ba} = 0.9968 (0.0431) (21817.8284) = 937.3393 \ psi$$

Where,

$$C_{4} = \left(\frac{f_{b}}{f_{h}}\right) \left(\frac{F_{ha}}{F_{ba}}\right) = \left(\frac{282.6779}{678.0375}\right) \left(\frac{2257.8645}{21817.8284}\right) = 0.0431$$
$$C_{3}^{2} \left(C_{4}^{2} + 0.6C_{4}\right) + C_{3}^{2n} - 1 = 0$$
$$n = 5 - \frac{4F_{ha} \cdot FS}{S_{v}} = 5 - \frac{4(2257.8645)(2.0)}{33600} = 4.4624$$

 $1^{st}$  attempt at solving for  $C_3$ , using an interval halving approach, with an initial guess at  $C_3$  as follows:

$$C_3 = \frac{Upper Bound + Lower Bound}{2} = \frac{1.0 + 0.0}{2} = 0.5$$

The following results are obtained:

$$0.5^{2} \left( \left( 0.0431 \right)^{2} + 0.6 \left( 0.0431 \right) \right) + 0.5^{2(4.4624)} - 1 = -0.9910$$

 $2^{nd}$  attempt at solving for  $C_3$ , with an second guess of  $C_3$  as follows:

$$C_3 = \frac{1.0 + 0.5}{2} = 0.75$$

The following results are obtained:

$$0.75^{2} \left( \left( 0.0431 \right)^{2} + 0.6 \left( 0.0431 \right) \right) + 0.75^{2(4.4624)} - 1 = -0.9077$$

Successive iterations are performed at solving for  $C_3$  until the following value is obtained.

 $C_3 = 0.9968$ 

The following results are obtained which satisfy the equation within a tolerance of  $\pm 0.001$ :

$$0.9968^{2} \left( \left( 0.0431 \right)^{2} + 0.6 \left( 0.0431 \right) \right) + 0.9968^{2(4.4624)} - 1 = -0.0007$$

2) The allowable hoop compressive membrane stress,  $F_{hba}$ , is given by the following equation.

$$F_{hba} = F_{bha} \left(\frac{f_h}{f_b}\right) = 937.3393 \left(\frac{678.0375}{282.6779}\right) = 2248.3229 \text{ psi}$$

g) <u>Shear Stress and Hoop Compression</u> – the allowable compressive stress for the combination of shear,  $F_{vha}$ , and hoop compression is computed using the following equations.

Note: This load combination is only applicable for shear stress and hoop compression, in the absence of axial compressive stress and compressive bending stress. It is shown in this example problem for informational purposes only. The effect of shear is accounted for in the interaction equations of paragraphs 4.4.12.2.h and 4.4.12.2.i through the variable  $K_{e}$ .

1) The allowable shear stress is given by the following equation with  $F_{ha}$  and  $F_{va}$  evaluated using the equations in paragraphs 4.4.12.2.a and 4.4.12.2.d, respectively.

$$F_{vha} = \left[ \left( \frac{F_{va}^2}{2C_5 F_{ha}} \right)^2 + F_{va}^2 \right]^{0.5} - \frac{F_{va}^2}{2C_5 F_{ha}}$$

$$F_{vha} = \left[ \left( \frac{(9116.5648)^2}{2(0.0348)(2257.8645)} \right)^2 + (9116.5648)^2 \right]^{0.5} - \left[ \frac{(9116.5648)^2}{2(0.0348)(2257.8645)} \right]$$

$$F_{vha} = 78.5678 \ psi$$

Where,

$$C_5 = \frac{f_v}{f_h} = \frac{23.5623}{678.0375} = 0.0348$$

2) The allowable hoop compressive membrane stress,  $F_{hva}$ , is given by the following equation.

$$F_{hva} = \frac{F_{vha}}{C_5} = \frac{78.5678}{0.0348} = 2257.6954 \ psi$$

- Axial Compressive Stress, Compressive Bending Stress, Shear Stress, and Hoop Compression

   the allowable compressive stress for the combination of uniform axial compression, axial compression due to a bending moment, and shear in the presence of hoop compression is computed using the following interaction equations.
  - 1) The shear coefficient is determined using the following equation with  $F_{va}$  from paragraph 4.4.12.2.d.

$$K_{s} = 1.0 - \left(\frac{f_{v}}{F_{va}}\right)^{2} = 1.0 - \left(\frac{23.5623}{9116.5648}\right)^{2} = 0.9999$$

2) For  $\lambda_c \leq 0.15$  the acceptability of a member subject to compressive axial and bending stresses,  $f_a$  and  $f_b$ , respectively, is determined using the following interaction equation with  $F_{xha}$  and  $F_{bha}$  evaluated using the equations in paragraphs 4.4.12.2.e.1 and 4.4.12.2.f.1, respectively.

$$\left(\frac{f_a}{K_s F_{sha}}\right)^{1.7} + \left(\frac{f_b}{K_s F_{bha}}\right) \le 1.0$$

Note: this step is not required since  $\lambda_c > 0.15$ .

3) For  $0.15 < \lambda_c \le 1.2$  the acceptability of a member subject to compressive axial and bending stresses,  $f_a$  and  $f_b$ , respectively, is determined using the following interaction equation with  $F_{xha}$  and  $F_{bha}$  evaluated using the equations in paragraphs 4.4.12.2.e.2 and 4.4.12.2.f.1, respectively.

$$\frac{f_a}{K_s F_{xha}} = \frac{230.7616}{0.9999(1710.2496)} = 0.1349$$

Since  $\frac{f_a}{K_s F_{xha}}$  < 0.2 , the following equation shall be used:

$$\left(\frac{f_a}{2K_s F_{xha}}\right) + \left(\frac{\Delta f_b}{K_s F_{bha}}\right) \le 1.0$$

$$\left\{ \left(\frac{230.7616}{2(0.9999)(1710.2496)}\right) + \left(\frac{1.0024(282.6779)}{0.9999(937.3393)}\right) = 0.3698 \right\} \le \{1.0\}$$
 True

Where,

$$\Delta = \frac{C_m}{1 - \left(\frac{f_a \cdot FS}{F_e}\right)} = \frac{1.0}{1 - \left(\frac{230.7616(1.667)}{162990.2785}\right)} = 1.0024$$

$$F_{e} = \frac{\pi^{2} E_{y}}{\left(\frac{K_{u} L_{u}}{r_{g}}\right)^{2}} = \frac{\pi^{2} \left(28.3E + 06\right)}{\left(\frac{2.1(636.0)}{32.2637}\right)^{2}} = 162990.2785 \ psi$$

- Axial Compressive Stress, Compressive Bending Stress, and Shear Stress the allowable compressive stress for the combination of uniform axial compression, axial compression due to a bending moment, and shear in the absence of hoop compression is computed using the following interaction equations.
  - 1) The shear coefficient is determined using the equation in paragraph 4.4.12.2.h.1 with  $F_{va}$  from paragraph 4.4.12.2.d.

$$K_s = 1.0 - \left(\frac{f_v}{F_{va}}\right)^2 = 1.0 - \left(\frac{23.5623}{9116.5648}\right)^2 = 0.9999$$

2) For  $\lambda_c \leq 0.15$  the acceptability of a member subject to compressive axial and bending stresses,  $f_a$  and  $f_b$ , respectively, is determined using the following interaction equation with  $F_{xa}$  and  $F_{ba}$  evaluated using the equations in paragraphs 4.4.12.2.b.1 and 4.4.12.2.c, respectively.

$$\left(\frac{f_a}{K_s F_{xa}}\right)^{1.7} + \left(\frac{f_b}{K_s F_{ba}}\right) \le 1.0$$

Note: this step is not required since  $\lambda_c > 0.15$  .

3) For  $0.15 < \lambda_c \le 1.2$  the acceptability of a member subject to compressive axial and bending stresses,  $f_a$  and  $f_b$ , respectively, is determined using the following interaction equation with  $F_{ca}$  and  $F_{ba}$  evaluated using the equations in paragraphs 4.4.12.2.b.2 and 4.4.12.2.c, respectively. The coefficient  $\Delta$  is evaluated using the equations in paragraph 4.4.12.2.h.3.

$$\frac{f_a}{K_s F_{ca}} = \frac{230.7616}{0.9999(18672.4331)} = 0.0124$$

Since  $\frac{f_a}{K_s F_{ca}} < 0.2$  , the following equation shall be used:

$$\left(\frac{f_a}{2K_sF_{ca}}\right) + \left(\frac{\Delta f_b}{K_sF_{ba}}\right) \le 1.0$$

$$\left\{ \left(\frac{230.7616}{2(0.9999)(18672.4331)}\right) + \left(\frac{1.0024(282.6779)}{0.9999(21817.8284)}\right) = 0.0192 \right\} \le \{1.0\} \quad True$$

From paragraph 4.4.12.2.h.3:

$$\Delta = 1.0024$$

$$F_{e} = 162990.2785 \ psi$$

j) The maximum deviation , e, may exceed the value  $e_x$  given in paragraph 4.4.4.2 if the maximum axial stress is less than  $F_{xa}$  for shells designed for axial compression only, or less than  $F_{xha}$  for shells designed for combinations of axial compression and external pressure. The change in buckling stress,  $F_{xe}$ , is given by Equation (4.4.114). The reduced allowable buckling stress,  $F_{xa(reduced)}$ , is determined using Equation (4.4.115) where e is the new maximum deviation,  $F_{xa}$  is determined using Equation 4.4.61, and  $FS_{xa}$  is the value of the stress reduction factor used to determine  $F_{xa}$ .

$$F_{xe}' = \left(0.944 - \left| 0.286 \log \left[ \frac{0.0005e}{e_x} \right] \right| \right) \left( \frac{E_y t}{R} \right)$$

$$F_{xe}' = \left(0.944 - \left| 0.286 \log \left[ \frac{0.0005(0.2501)}{(0.0913)} \right] \right| \right) \left( \frac{(28.3E + 06)(1.0)}{46.125} \right) = 76737.5098 \text{ psi}$$

$$F_{xa(reduced)} = \frac{F_{xa} \cdot FS_{xa} - F_{xe}'}{FS_{xa}}$$

$$F_{xa(reduced)} = \frac{20155.9688(1.667) - (76737.5098)}{1.667} = -25877.3304 \text{ psi}$$

From paragraph 4.4.4.1, assuming the measurements are taken using the outside radius:

$$e = \min[e_c, 2t] = \min[0.2501, 2(1.0)] = 0.2501 \text{ in}$$

$$e_c = 0.0165t \left(\frac{L_{ec}}{\sqrt{Rt}} + 3.25\right)^{1.069} = 0.0165(1.0) \left(\frac{64.3134}{\sqrt{46.125(1.0)}} + 3.25\right)^{1.069} = 0.2501 \text{ in}$$

$$L_{ec} = 2R \sin\left[\frac{\pi}{2n}\right] = 2(46.125) \sin\left(\frac{\pi}{2(2.0362)}\right) = 64.3134 \text{ in}$$

$$n = \xi \left(\sqrt{\frac{R}{t}} \cdot \left(\frac{R}{L}\right)\right)^{\psi} = (2.80) \left(\sqrt{\frac{46.125}{1.0}} \left(\frac{46.125}{636.0}\right)\right)^{0.4498} = 2.0362$$

$$\xi = \min\left[2.28 \left(\frac{R}{t}\right)^{0.54}, 2.80\right] = \min\left[2.28 \left(\frac{46.125}{1.0}\right)^{0.54}, 2.80\right]$$

$$\xi = \min[18.05, 2.80]$$

$$\xi = 2.80$$

$$\psi = \min\left[0.38 \left(\frac{R}{t}\right)^{0.044}, 0.485\right] = \min\left[0.38 \left(\frac{46.125}{1.0}\right)^{0.044}, 0.485\right]$$
$$\psi = \min\left[0.4498, 0.485\right]$$
$$\psi = 0.4498$$

From paragraph 4.4.4.2:

$$e_x = 0.002R_m = 0.002(45.625) = 0.0913$$
 in  
 $R_m = \frac{(D_o + D_i)}{4} = \frac{(92.25 + 90.25)}{4} = 45.625$  in

## A summary of the allowable compressive stresses are as follows:

Paragraph 4.4.12.2.a, External Pressure Acting Alone

 $F_{ha} = 2257.8645 \ psi$ 

Paragraph 4.4.12.2.b, Axial Compressive Stress Acting Alone

 $F_{xa} = 20155.9688 \ psi$ 

 $F_{ca} = 18672.4331 \ psi$ 

Paragraph 4.4.12.2.c, Compressive Bending Stress

 $F_{ba} = 21817.8284 \ psi$ 

Paragraph 4.4.12.2.d, Shear Stress

 $F_{va} = 9116.5648 \ psi$ 

Paragraph 4.4.12.2.e, Axial Compressive Stress and Hoop Compression

 $F_{xha} = 1710.2496 \ psi$ 

Paragraph 4.4.12.2.f, Compressive Bending Stress and Hoop Compression

 $F_{bha} = 937.3393 \ psi$ 

 $F_{hba} = 2248.3229 \ psi$ 

Paragraph 4.4.12.2.g, Shear Stress and Hoop Compression

 $F_{vha} = 78.5678 \ psi$ 

$$F_{hva} = 2257.6954 \ psi$$

Table 4.1.1 – Design Loads				
Design Load Parameter	oad Description ter			
P Internal or External Specified Design Pressure (see paragraph 4.1.5.2.a)				
$P_s$	Static head from liquid or bulk materials (e.g. catalyst)			
D	<ul> <li>Dead weight of the vessel, contents, and appurtenances at the location of interest, including the following:</li> <li>Weight of vessel including internals, supports (e.g. skirts, lugs, saddles, and legs), and appurtenances (e.g. platforms, ladders, etc.)</li> <li>Weight of vessel contents under operating and test conditions</li> <li>Refractory linings, insulation</li> <li>Static reactions from the weight of attached equipment, such as motors, machinery, other vessels, and piping</li> <li>Transportation Loads (The static forces obtained as equivalent to the dynamic loads experienced during normal operation of a</li> </ul>			
L	<ul> <li>Appurtenance Live loading</li> <li>Effects of fluid flow, steady state or transient</li> <li>Loads resulting from wave action</li> </ul>			
E	<i>E</i> Earthquake loads (see ASCE 7 for the specific definition of the earthquake load, as applicable)			
W	W Wind Loads (See 4.1.5.3.b)			
S	Snow Loads			
F	<i>F</i> Loads due to Deflagration			

Table E4.4.6.1 ·	- Design Loads	s and Load	Combinations	from VIII-2

Table 4.1.2 – Design Load Combinations				
Design Load Combination (1)	General Primary Membrane Allowable Stress (2)			
$P + P_s + D$	S			
$P + P_s + D + L$	S			
$P + P_s + D + S$	S			
$0.9P + P_s + D + 0.75L + 0.75S$	S			
$0.9P + P_s + D + (0.6W \text{ or } 0.7E)$	S			
$0.9P + P_{S} + D + 0.75(0.6W \text{ or } 0.7E) + 0.75L + 0.75S$	S			
0.6D + (0.6W  or  0.7E) (3)	S			
$P_s + D + F$	See Annex 4.D			
Notes				

1) The parameters used in the Design Load Combination column are defined in Table 4.1.1.

2) S is the allowable stress for the load case combination (see paragraph 4.1.5.3.c)

3) This load combination addresses an overturning condition for foundation design. It does not apply to design of anchorage (if any) to the foundation. Refer to ASCE/SEI 7-10, 2.4.1 Exception 2 for an additional reduction to W that may be applicable.

Design Load Parameter	Description	Magnitude of Pressure, Force and Moment
Р	Internal or External Specified Design Pressure (see paragraph 4.1.5.2.a)	P = -14.7
$P_s$	Static head from liquid or bulk materials (e.g. catalyst)	$P_{s} = 0.0$
D	The dead weight of the vessel including skirt, contents, and appurtenances at the location of interest	$D_F = -66152.5 \ lbs$ $D_M = 0.0 \ in - lbs$
L	Appurtenance live loading and effects of fluid flow	$L_F = 0.0 \ lbs$ $L_M = 0.0 \ in - lbs$
Ε	Earthquake loads	$E_F = 0.0 \ lbs$ $E_M = 0.0 \ in - lbs$
W	Wind Loads	$W_F = 0.0 \ lbs$ $W_M = 3.048E + 06 \ in - lbs$ $W_V = 11257.6 \ lbs$
S	Snow Loads	$S_F = 0.0 \ lbs$ $S_M = 0.0 \ in - lbs$
F	Loads due to Deflagration	$F_F = 0.0 \ lbs$ $F_M = 0.0 \ in - lbs$

# Table E4.4.6.2 - Design Loads (Net-Section Axial Force and Bending Moment) at the Location of Interest

Based on these loads, the shell is required to be designed for the load case combinations shown in Table E4.4.6.3. Note that this table is given in terms of the load combinations shown in VIII-2, Table 4.1.2 (Table E4.4.6.1 of this example).

Load Case	Design Load Combination	Magnitude of Pressure, Force and Moment	General Primary Membrane Allowable Stress
1	$P + P_s + D$	$P + P_s = -14.7 \ psi$ $F_1 = -66152.5 \ lbs$ $M_1 = 0.0 \ in - lbs$	S
2	$P + P_s + D + L$	$P + P_s = -14.7 \ psi$ $F_2 = -66152.5 \ lbs$ $M_2 = 0.0 \ in - lbs$	S
3	$P + P_s + D + S$	$P + P_s = -14.7 \ psi$ $F_3 = -66152.5 \ lbs$ $M_3 = 0.0 \ in - lbs$	S
4	$0.9P + P_s + D + 0.75L + 0.75S$	$0.9P + P_s = -13.2 \ psi$ $F_4 = -66152.5 \ lbs$ $M_4 = 0.0 \ in - lbs$	S
5	$0.9P + P_s + D + (0.6W \text{ or } 0.7E)$	$0.9P + P_s = -13.4 \text{ psi}$ $F_5 = -66152.5 \text{ lbs}$ $M_5 = 1828800 \text{ in} - \text{lbs}$ $V_5 = 6754.6 \text{ lbs}$	S
6	$\begin{pmatrix} 0.9P + P_s + D + 0.75(0.6W \text{ or } 0.7E) + \\ 0.75L + 0.75S \end{pmatrix}$	$0.9P + P_{s} = 320.4 \text{ psi}$ $F_{6} = -66152.5 \text{ lbs}$ $M_{6} = 1371600 \text{ in} - \text{lbs}$ $V_{6} = 5065.9 \text{ lbs}$	S
7	0.6D + (0.6W  or  0.7E) Anchorage is included in the design. Therefore, consideration of this load combination is not required.	$F_7 = -39691.5 \ lbs$ $M_7 = 1828800 \ in - lbs$ $V_7 = 6754.6 \ lbs$	S
8	$P_s + D + F$	$P_{s} = 0.0 \ psi$ $F_{8} = -66152.5 \ lbs$ $M_{8} = 0.0 \ in - lbs$	See Annex 4.D

Table E4.4.6.3 - Load Case Combination at the Location of Interest

# 4.4.7 Example E4.4.7 – Conical Transitions without a Knuckle

Determine if the proposed large and small end cylinder-to-cone transitions are adequately designed considering the following design conditions and applied forces and moments.

Vessel Data:

•	Material	=	SA-516, Grade 70, Norm.
•	Design Conditions	=	-14.7 <i>psig</i> @300°F
•	Inside Radius (Large End)	=	75.0 in
•	Thickness (Large End)	=	1.8125 in
•	Inside Radius (Small End)	=	45.0 <i>in</i>
•	Thickness (Small End)	=	1.125 in
•	Thickness (Conical Section)	=	1.9375 in
•	Length of Conical Section	=	78.0 in
•	Unsupported Length of Large Cylinder	=	732.0 in
•	Unsupported Length of Small Cylinder	=	636.0 in
•	Corrosion Allowance	=	0.125 in
•	Allowable Stress	=	22400 psi
•	Yield Strength	=	33600 <i>psi</i>
•	Modulus of Elasticity at Design Temperature	=	$28.3E + 06 \ psi$
•	Weld Joint Efficiency	=	1.0
•	One-Half Apex Angle (See E4.3.2)	=	21.0375 <i>deg</i>
•	Axial Force (Large End)	=	–99167 <i>lbs</i>
•	Net Section Bending Moment (Large End)	=	5.406E + 06 in - lbs
•	Axial Force (Small End)	=	-78104 <i>lbs</i>
•	Net Section Bending Moment (Small End)	=	4.301E + 06 in - lbs

Adjust variables for corrosion.

 $R_L = 75.0 + Corrosion \ Allowance = 75.0 + 0.125 = 75.125 \ in$   $R_S = 45.0 + Corrosion \ Allowance = 45.0 + 0.125 = 45.125 \ in$   $t_L = 1.8125 - Corrosion \ Allowance = 1.8125 - 0.125 = 1.6875 \ in$   $t_S = 1.125 - Corrosion \ Allowance = 1.125 - 0.125 = 1.0 \ in$  $t_C = 1.9375 - Corrosion \ Allowance = 1.9375 - 0.125 = 1.8125 \ in$ 

Evaluate per paragraphs 4.4.13 and 4.3.11.

The design rules in paragraph 4.3.11 shall be satisfied. In these calculations, a negative value of pressure shall be used in all applicable equations.

Proceed with the design following the steps outlined in paragraph 4.3.11.3.

The length of the conical shell, measured parallel to the surface of the cone, shall be equal to or greater than the following:

$$\begin{split} L_{c} &\geq 2.0 \sqrt{\frac{R_{L}t_{c}}{\cos[\alpha]}} + 1.4 \sqrt{\frac{R_{s}t_{c}}{\cos[\alpha]}} \\ &2.0 \sqrt{\frac{75.125(1.8125)}{\cos[21.0375]}} + 1.4 \sqrt{\frac{45.125(1.8125)}{\cos[21.0375]}} = 37.2624 \ in \\ L_{c} &= 78.0 \geq 37.2624 \qquad \qquad True \end{split}$$

Evaluate the Large End cylinder-to-cone junction per paragraph 4.3.11.4.

a) STEP 1 – Compute the large end cylinder thickness, *t<sub>L</sub>*, using paragraph 4.3.3., (as specified in design conditions).

$$t_{I} = 1.6875$$
 in

b) STEP 2 – Determine the cone half-apex angle,  $\alpha$ , and compute the cone thickness,  $t_c$ , at the large end using paragraph 4.3.4., (as specified in design conditions).

 $\alpha = 21.0375 \ deg$  $t_c = 1.8125 \ in$ 

c) STEP 3 – Proportion the cone geometry such that the following equations are satisfied. If all of these equations are not satisfied, then the cylinder-to-cone junction shall be designed in accordance with Part 5. In the calculations, if  $0 \deg < \alpha \le 10 \deg$ , then use  $\alpha = 10 \deg$ .

$$20 \le \left\{ \frac{R_L}{t_L} = \frac{75.125}{1.6875} = 44.5185 \right\} \le 500 \qquad True$$
$$1 \le \left\{ \frac{t_C}{t_L} = \frac{1.8125}{1.6875} = 1.0741 \right\} \le 2 \qquad True$$
$$\{ \alpha = 21.0375 \ deg \} \le 60 \ deg \qquad True$$

d) STEP 4 – Determine the net section axial force,  $F_L$ , and bending moment,  $M_L$ , applied to the conical transition (as specified in design conditions). The thrust load due to pressure shall not be included as part of the axial force,  $F_L$ . Calculate the equivalent line load,  $X_L$ , using the specified net section axial force,  $F_L$ , and bending moment,  $M_L$ .

$$X_{L} = \frac{F_{L}}{2\pi R_{L}} \pm \frac{M_{L}}{\pi R_{L}^{2}} = \begin{cases} \frac{-99167}{2\pi (75.125)} + \frac{5.406E + 06}{\pi (75.125)^{2}} = 94.8111 \frac{lbs}{in} \\ \frac{-99167}{2\pi (75.125)} - \frac{5.406E + 06}{\pi (75.125)^{2}} = -514.9886 \frac{lbs}{in} \end{cases}$$

e) STEP 5 – Compute the junction transition design parameters (the normalized resultant moment,  $M_{sN}$ , and shear force,  $Q_N$ ) for the internal pressure and equivalent line load per Table 4.3.3 and Table 4.3.4, respectively. For calculated values of *n* other than those presented in Table 4.3.3 and Table 4.3.4, linear interpolation of the equation coefficients,  $C_i$ , is permitted.

$$n = \frac{t_C}{t_L} = \frac{1.8125}{1.6875} = 1.0741$$
$$H = \sqrt{\frac{R_L}{t_L}} = \sqrt{\frac{75.125}{1.6875}} = 6.6722$$
$$B = \tan[\alpha] = \tan[21.0375] = 0.3846$$

Linear interpolation of the equation coefficients,  $C_i$  in Table 4.3.3 and Table 4.3.4 is required. The results of the interpolation are summarized with the following values for  $C_i$  (see paragraph 4.3.11.4, STEP 5 of E4.3.7).

For the applied pressure case both  $M_{_{s\!N}}$  and  $Q_{_N}$  are calculated using the following equation

$$M_{sN}, Q_{N} = -\exp \begin{bmatrix} C_{1} + C_{2} \ln[H] + C_{3} \ln[B] + C_{4} (\ln[H])^{2} + C_{5} (\ln[B])^{2} + \\ C_{6} \ln[H] \ln[B] + C_{7} (\ln[H])^{3} + C_{8} (\ln[B])^{3} + \\ C_{9} \ln[H] (\ln[B])^{2} + C_{10} (\ln[H])^{2} \ln[B] \end{bmatrix}$$

This results in the following (see paragraph 4.3.11.4, STEP 5 of E4.3.7):

$$M_{sN} = -10.6148$$
  
 $Q_N = -4.0925$ 

For the Equivalent Line Load case,  $M_{\rm sN}$  and  $Q_{\rm N}$  are calculated using the following equation.

$$M_{sN}, Q_{N} = -\exp\left[\frac{\left[\binom{C_{1} + C_{3} \ln\left[H^{2}\right] + C_{5} \ln\left[\alpha\right] + C_{7} \left(\ln\left[H^{2}\right]\right)^{2} + C_{9} \left(\ln\left[\alpha\right]\right)^{2} + C_{11} \ln\left[H^{2}\right] \ln\left[\alpha\right]\right)}{\left[1 + C_{2} \ln\left[H^{2}\right] + C_{4} \ln\left[\alpha\right] + C_{6} \left(\ln\left[H^{2}\right]\right)^{2} + C_{8} \left(\ln\left[\alpha\right]\right)^{2} + C_{10} \ln\left[H^{2}\right] \ln\left[\alpha\right]\right)}\right]$$

This results in the following (see paragraph 4.3.11.4, STEP 5 of E4.3.7):

$$M_{sN} = -0.4912$$
  
 $Q_N = -0.1845$ 

Summarizing, the normalized resultant moment  $M_{sN}$ , and shear force  $Q_N$  for the internal pressure and equivalent line load are as follows:

Internal Pressure :	$M_{sN} = -10.6148,$	$Q_N = -4.0925$
Equivalent Line Load :	$M_{sN} = -0.4912,$	$Q_N = -0.1845$

f) STEP 6 – Compute the stresses in the cylinder and cone at the junction using the equations in Table 4.3.1 for the Large End Junction.

Evaluate the Cylinder at the Large End:

Stress Resultant Calculations:

$$\begin{split} M_{sP} &= Pt_{L}^{2}M_{sN} = -14.7 (1.6875)^{2} (-10.6148) = 444.3413 \frac{in-lb}{in} \\ M_{sX} &= X_{L}t_{L}M_{sN} = \begin{cases} 94.8111 (1.6875) (-0.4912) = -78.5889 \frac{in-lbs}{in} \\ -514.9886 (1.6875) (-0.4912) = 426.8741 \frac{in-lbs}{in} \end{cases} \\ M_{s} &= M_{sP} + M_{sX} = \begin{cases} 444.3413 + (-78.5889) = 365.7524 \frac{in-lbs}{in} \\ 444.3413 + 426.8741 = 871.2154 \frac{in-lbs}{in} \end{cases} \end{cases}$$

$$\begin{aligned} Q_{P} &= Pt_{L}Q_{N} = -14.7 (1.6875) (-4.0925) = 101.5196 \frac{lb}{in} \\ -514.9886 (-0.1845) = -17.4926 \frac{lbs}{in} \\ -514.9886 (-0.1845) = 95.0154 \frac{lbs}{in} \end{cases} \end{aligned}$$

$$\begin{aligned} Q &= Q_{P} + Q_{X} = \begin{cases} 101.5196 + (-17.4926) = 84.0270 \frac{lbs}{in} \\ 101.5196 + 95.0154 = 196.5350 \frac{lbs}{in} \\ 101.5196 + 95.0154 = 196.5350 \frac{lbs}{in} \end{cases} \end{aligned}$$

$$\begin{aligned} \beta_{cy} &= \left[ \frac{3(1-v^{2})}{R_{L}^{2}t_{L}^{2}} \right]^{0.25} = \left[ \frac{3(1-(0.3)^{2})}{(75.125)^{2} (1.6875)^{2}} \right]^{0.25} = 0.1142 in^{-1} \\ N_{s} &= \frac{PR_{L}}{2} + X_{L} = \begin{cases} \frac{-14.7 (75.125)}{2} + 94.8111 = -457.3577 \frac{lbs}{in} \\ \frac{-14.7 (75.125)}{2} + (-514.9886) = -1067.1574 \frac{lbs}{in} \end{cases} \end{aligned}$$

$$\begin{split} N_{\theta} &= PR_{L} + 2\beta_{cy}R_{L}\left(-M_{s}\beta_{cy} + Q\right) \\ N_{\theta} &= \begin{cases} -14.7(75.125) + 2(0.1142)(75.125)\left(-(365.7524)(0.1142) + 84.0270\right) \\ -14.7(75.125) + 2(0.1142)(75.125)\left(-(871.2154)(0.1142) + 196.5350\right) \end{cases} \\ N_{\theta} &= \begin{cases} -379.2502 \frac{lbs}{in} \\ 560.7660 \frac{lbs}{in} \end{cases} \\ K_{pc} &= 1.0 \end{split}$$

Stress Calculations: Determine the meridional and circumferential membrane and bending stresses.

$$\sigma_{sm} = \frac{N_s}{t_L} = \begin{cases} \frac{-457.3577}{1.6875} = -271.0268 \ psi \\ \frac{-1067.1574}{1.6875} = -632.3895 \ psi \end{cases}$$

$$\sigma_{sb} = \frac{6M_s}{t_L^2 K_{pc}} = \begin{cases} \frac{6(365.7524)}{(1.6875)^2(1.0)} = 770.6388 \ psi \\ \frac{6(871.2154)}{(1.6875)^2(1.0)} = 1835.6472 \ psi \end{cases}$$

$$\sigma_{\theta m} = \frac{N_{\theta}}{t_L} = \begin{cases} \frac{-379.2502}{1.6875} = -224.7409 \ psi \\ \frac{560.7660}{1.6875} = 332.3058 \ psi \end{cases}$$

$$\sigma_{\theta b} = \frac{6\nu M_s}{t_L^2 K_{pc}} = \begin{cases} \frac{6(0.3)(365.7524)}{(1.6875)^2(1.0)} = 231.1916 \ psi \\ \frac{6(0.3)(871.2154)}{(1.6875)^2(1.0)} = 550.6942 \ psi \end{cases}$$
Check Acceptance Criteria:

$$\begin{cases} \sigma_{sm} = -271.0268 \ psi \\ \sigma_{sm} = -632.3895 \ psi \end{cases} \leq \{1.5S, \ not \ applicable \ due \ to \ compressive \ stress\} \\ \begin{cases} \sigma_{sm} + \sigma_{sb} = -271.0268 + 770.6388 = 499.6 \ psi \\ \sigma_{sm} - \sigma_{sb} = -271.0268 - 770.6388 = |-1041.7| \ psi \\ \sigma_{sm} - \sigma_{sb} = -632.3895 + 1835.6472 = 1203.3 \ psi \\ \sigma_{sm} - \sigma_{sb} = -632.3895 - 1835.6472 = |-2468.0| \ psi \end{cases} \leq \{S_{PS} = 67200 \ psi\}$$

$$True \\ \begin{cases} \sigma_{\thetam} = -224.7409 \\ \sigma_{\thetam} = 332.3058 \end{cases} \leq \{1.5S, \ not \ applicable \ due \ to \ compressive \ stress \\ 1.5S = 1.5(22400) = 33600 \ psi \end{cases}$$

$$True \\ \begin{cases} \sigma_{\thetam} + \sigma_{\thetab} = -224.7409 + 231.1916 = 6.5 \ psi \\ \sigma_{\thetam} - \sigma_{\thetab} = -224.7409 - 231.1916 = |-455.9| \ psi \\ \sigma_{\thetam} - \sigma_{\thetab} = 332.3058 + 550.6942 = 883.0 \ psi \\ \sigma_{\thetam} - \sigma_{\thetab} = 332.3058 - 550.6942 = |-218.4| \ psi \end{cases}$$

Since the hoop membrane stress,  $\sigma_{\theta m}$  and the axial membrane stress,  $\sigma_{sm}$  are compressive, the condition of local buckling shall be considered. Local buckling is not a concern if the following limit is satisfied.

$$\sigma_{\theta m} \leq F_{ha}$$
$$\sigma_{sm} \leq F_{xa}$$

 $F_{ha}$  is evaluated using paragraph 4.4.5.1, but substituting  $F_{he}$  with the following equation.

$$F_{he} = \frac{0.4E_y t}{D_o}$$

In accordance with paragraph 4.4.5.1, the value of  $F_{ha}$  is calculated as follows.

1) STEP 1 – Assume an initial thickness, t and unsupported length, L.

t = 1.6875 in  $L \rightarrow Not$  required, as the equation for  $F_{he}$  is independent of L

2) STEP 2 – Calculate the predicted elastic buckling stress,  $F_{he}$ 

$$F_{he} = \frac{0.4E_{y}t}{D_{o}} = \frac{0.4(28.3E + 06)(1.6875)}{153.625} = 124344.9959 \ psi$$

3) STEP 3 – Calculate the predicted buckling stress,  $F_{ic}$ 

$$\frac{F_{he}}{S_y} = \frac{124344.9959}{33600} = 3.7007$$

Since 
$$\frac{F_{he}}{S_y} \ge 2.439$$
, calculate  $F_{ic}$  as follows:  
 $F_{ic} = S_y = 33600 \ psi$ 

4) STEP 4 – Calculate the value of design factor, *FS* per paragraph 4.4.2. Since  $F_{ic} = S_y = 33600 \ psi$ , calculate *FS* as follows:

FS = 1.667

5) STEP 5 – Calculate the allowable hoop compressive membrane stress as follows:

$$F_{ha} = \frac{F_{ic}}{FS} = \frac{33600}{1.667} = 20155.9688 \ psi$$

6) STEP 6 – Compare the calculated hoop compressive membrane stress,  $\sigma_{\theta m}$  to the allowable hoop compressive membrane stress,  $F_{ha}$  per following criteria.

$$\{\sigma_{\theta m} = 224.7 \ psi\} \le \{F_{ha} = 20156.0 \ psi\}$$
 True

Therefore, local buckling due to hoop compressive membrane stress is not a concern.

 $F_{xa}$  is evaluated using paragraph 4.4.12.2.b with  $\lambda = 0.15$ . The design factor *FS* used in paragraph 4.4.12.2.b is dependent on the predicted buckling stress  $F_{ic}$  and the material's yield strength,  $S_y$  as shown in paragraph 4.4.2. An initial calculation is required to determine the value of  $F_{xa}$  by setting FS = 1.0, with  $F_{ic} = F_{xa}$ . The initial value of  $F_{ic}$  is then compared to  $S_y$  as shown in paragraph 4.4.2 and the value of *FS* is determined. This computed value of *FS* is then used in paragraph 4.4.12.2.b.

For  $\lambda_c = 0.15$  , (Local Buckling)

$$F_{xa} = \min[F_{xa1}, F_{xa2}]$$

$$\frac{D_o}{t} = \frac{153.625}{1.6875} = 91.0370$$

$$M_x = \frac{L}{\sqrt{R_o t}} = \frac{732.0}{\sqrt{76.8125(1.6875)}} = 64.2944$$

Since  $\frac{D_o}{t} \le 135$ , calculate  $F_{xa1}$  as follows with an initial value of FS = 1.0.

$$F_{xa1} = \frac{S_y}{FS} = \frac{33600}{1.0} = 33600 \ psi$$

The value of  $F_{\rm xa2}$  is calculated as follows with an initial value of FS = 1.0.

$$F_{xa2} = \frac{F_{xe}}{FS}$$
$$F_{xe} = \frac{C_x E_y t}{D_o}$$

Since  $\frac{D_o}{t} \le 1247$ , calculate  $C_x$  as follows:

$$C_{x} = \min\left[\frac{409\overline{c}}{\left(389 + \frac{D_{o}}{t}\right)}, \ 0.9\right]$$

Since  $M_x \ge 15$ , calculate  $\overline{c}$  as follows:

$$\overline{c} = 1.0$$
  
 $C_x = \min\left[\frac{409(1.0)}{389 + \frac{153.625}{1.6875}}, 0.9\right] = 0.8520$ 

Therefore,

$$F_{xe} = \frac{0.8520(28.3E + 06)(1.6875)}{153.625} = 264854.8413 \text{ psi}$$

$$F_{xa2} = \frac{F_{xe}}{FS} = \frac{264854.8413}{1.0} = 264854.8413 \text{ psi}$$

$$F_{xa} = \min[33600, 264855] = 33600 \text{ psi}$$

With a value of  $F_{ic} = F_{xa} = 33600$ , in accordance with paragraph 4.4.2, it is determined the value of FS = 1.667 since  $\{F_{ic} = 33600\} = \{S_y = 33600\}$ . Using this computed value of FS = 1.667 in paragraph 4.4.12.2.b,  $F_{xa}$  is calculated as follows.

$$F_{xa1} = \frac{S_y}{FS} = \frac{33600}{1.667} = 20155.9688 \ psi$$
  

$$F_{xa2} = \frac{F_{xe}}{FS} = \frac{264854.8413}{1.667} = 158881.1286 \ psi$$
  

$$F_{xa} = \min[20155.9688, 158881.1286] = 20155.9688 \ psi$$

Compare the calculated axial compressive membrane stress,  $\sigma_{sm}$  to the allowable axial compressive membrane stress,  $F_{xa}$  per following criteria.

$$\{\sigma_{sm} = 632.4 \ psi\} \le \{F_{xa} = 20156.0 \ psi\}$$
 True

Therefore, local buckling due to axial compressive membrane stress is not a concern. The cylinder at the cylinder-to-cone junction at the large end is adequately designed.

# Evaluate the Cone at the Large End:

Stress Resultant Calculations, as determined above.

$$\begin{split} M_{csP} &= M_{sP} = 444.3413 \; \frac{in - lbs}{in} \\ M_{csX} &= M_{sX} = \begin{cases} -78.5889 \; \frac{in - lbs}{in} \\ 426.8741 \; \frac{in - lbs}{in} \end{cases} \\ M_{cs} &= M_{csP} + M_{csX} = \begin{cases} 444.3413 + (-78.5889) = 365.7524 \; \frac{in - lbs}{in} \\ 444.3413 + 426.8741 = 871.2154 \; \frac{in - lbs}{in} \end{cases} \end{split}$$

$$Q_{c} = Q\cos[\alpha] + N_{s}\sin[\alpha]$$

$$Q_{c} = \begin{cases} 84.0270(\cos[21.0375]) + (-457.3577)\sin[21.0375] = -85.7555 \frac{lbs}{in} \\ 196.5350(\cos[21.0375]) + (-1067.1574)\sin[21.0375] = -199.6519 \frac{lbs}{in} \end{cases}$$

$$R_{C} = \frac{R_{L}}{\cos[\alpha]} = \frac{75.125}{\cos[21.0375]} = 80.4900 \text{ in}$$
  
$$\beta_{co} = \left[\frac{3(1-\nu^{2})}{R_{C}^{2}t_{C}^{2}}\right]^{0.25} = \left[\frac{3(1-(0.3)^{2})}{(80.4900)^{2}(1.8125)^{2}}\right]^{0.25} = 0.1064 \text{ in}^{-1}$$
  
$$N_{cs} = N_{s} \cos[\alpha] - Q \sin[\alpha]$$

$$N_{cs} = \begin{cases} -457.3577 \left( \cos[21.0375] \right) - 84.0270 \sin[21.0375] = -457.0368 \frac{lbs}{in} \\ -1067.1574 \left( \cos[21.0375] \right) - 196.5350 \sin[21.0375] = -1066.5786 \frac{lbs}{in} \end{cases}$$

$$\begin{split} N_{c\theta} &= \frac{PR_L}{\cos[\alpha]} + 2\beta_{co}R_C \left(-M_{cs}\beta_{co} - Q_c\right) \\ N_{c\theta} &= \begin{cases} \frac{-14.7(75.125)}{\cos[21.0375]} + 2(0.1064)(80.4900) \left(-(365.7524)(0.1064) - (-85.7555)\right) \\ \frac{-14.7(75.125)}{\cos[21.0375]} + 2(0.1064)(80.4900) \left(-(871.2154)(0.1064) - (-199.6519)\right) \end{cases} \\ N_{c\theta} &= \begin{cases} -380.9244 \frac{lbs}{in} \\ 648.7441 \frac{lbs}{in} \\ 648.7441 \frac{lbs}{in} \end{cases} \end{split}$$

Stress Calculations: Determine the meridional and circumferential membrane and bending stresses.

$$\sigma_{sm} = \frac{N_{cs}}{t_c} = \begin{cases} \frac{-457.0368}{1.8125} = -252.1582 \ psi \\ \frac{-1066.5786}{1.8125} = -588.4572 \ psi \end{cases}$$

$$\sigma_{sb} = \frac{6M_{cs}}{t_c^2 K_{cpc}} = \begin{cases} \frac{6(365.7524)}{(1.8125)^2(1.0)} = 668.0091 \ psi \\ \frac{6(871.2154)}{(1.8125)^2(1.0)} = 1591.1853 \ psi \end{cases}$$

$$\sigma_{\theta m} = \frac{N_{c\theta}}{t_c} = \begin{cases} \frac{-380.9244}{1.8125} = -210.1652 \ psi \\ \frac{648.7441}{1.8125} = 357.9278 \ psi \end{cases}$$

$$\sigma_{\theta b} = \frac{6\nu M_{cs}}{t_c^2 K_{cpc}} = \begin{cases} \frac{6(0.3)(365.7524)}{(1.8125)^2(1.0)} = 200.4027 \ psi \\ \frac{6(0.3)(871.2154)}{(1.8125)^2(1.0)} = 477.3556 \ psi \end{cases}$$

Check Acceptance Criteria:

$$\begin{cases} \sigma_{sm} = -252.1582 \ psi \\ \sigma_{sm} = -588.4572 \ psi \end{cases} \leq \{1.5S, \ not \ applicable \ due \ to \ compressive \ stress\} \\ \begin{cases} \sigma_{sm} + \sigma_{sb} = -252.1582 + 668.0091 = 415.6 \ psi \\ \sigma_{sm} - \sigma_{sb} = -252.1582 - 668.0091 = |-920.2| \ psi \\ \sigma_{sm} - \sigma_{sb} = -588.4572 + 1591.1853 = 1002.7 \ psi \\ \sigma_{sm} - \sigma_{sb} = -588.4572 - 1591.1853 = |-2179.6| \ psi \end{cases} \leq \{S_{PS} = 67200 \ psi\}$$

$$True \begin{cases} \sigma_{\theta m} = -210.1652 \\ \sigma_{\theta m} = 357.9278 \end{cases} \leq \begin{cases} 1.5S, \ not \ applicable \ due \ to \ compressive \ stress} \\ 1.5S = 1.5(22400) = 33600 \ psi \end{cases}$$

$$True \begin{cases} \sigma_{\theta m} + \sigma_{\theta b} = -210.1652 + 200.4027 = |-9.7| \ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = -210.1652 - 200.4027 = -410.6 \ psi \\ \sigma_{\theta m} + \sigma_{\theta b} = 357.9278 + 477.3556 = 835.3 \ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = 357.9278 - 477.3556 = |-119.4| \ psi \end{cases} \leq \{S_{PS} = 60000 \ psi \}$$

$$True$$

Since the hoop membrane stress,  $\sigma_{\theta m}$  and the axial membrane stress,  $\sigma_{sm}$  are compressive, the condition of local buckling shall be considered. Local buckling is not a concern if the following limits are satisfied.

Using the procedure shown above for the cylindrical shell and substituting the cone thickness,  $t_c$  for the cylinder thickness, t, the allowable compressive hoop membrane and axial membrane stresses,  $F_{ha}$  and  $F_{xa}$ , respectively, are calculated as follows.

 $F_{ha} = 20156.0 \ psi$  $F_{xa} = 20156.0 \ psi$ 

Compare the calculated hoop compressive membrane stress,  $\sigma_{\theta m}$  and axial compressive membrane stress,  $\sigma_{sm}$ , to the allowable hoop compressive membrane stress,  $F_{ha}$  and axial compressive membrane stress,  $F_{xa}$  per following criteria.

$$\{\sigma_{\theta m} = 210.2 \ psi\} \le \{F_{ha} = 20156.0 \ psi\}$$
True  
$$\{\sigma_{sm} = 588.5 \ psi\} \le \{F_{xa} = 20156.0 \ psi\}$$
True

Therefore, local buckling due to hoop and axial compressive membrane stress is not a concern.

The cone at the cylinder-to-cone junction at the large end is adequately designed.

g) STEP 7 – The stress acceptance criterion in STEP 6 is satisfied for both the cylinder and cone. Therefore, the design is complete.

#### Evaluate the Small End cylinder-to-cone junction per paragraph 4.3.11.5.

a) STEP 1 – Compute the small end cylinder thickness, t<sub>s</sub>, using paragraph 4.3.3., (as specified in design conditions)

 $t_{s} = 1.0 in$ 

b) STEP 2 – Determine the cone half-apex angle,  $\alpha$ , and compute the cone thickness,  $t_c$ , at the small end using paragraph 4.3.4., (as specified in design conditions)

 $\alpha = 21.0375 \ deg$ 

 $t_{C} = 1.8125 in$ 

c) STEP 3 – Proportion the cone geometry such that the following equations are satisfied. If all of these equations are not satisfied, then the cylinder-to-cone junction shall be designed in accordance with Part 5. In the calculations, if  $0 \deg < \alpha \le 10 \deg$ , then use  $\alpha = 10 \deg$ .

$$20 \le \left\{ \frac{R_s}{t_s} = \frac{45.125}{1.0} = 45.125 \right\} \le 500 \qquad True$$
$$1 \le \left( \frac{t_c}{t_s} = \frac{1.8125}{1.0} = 1.8125 \right) \le 2 \qquad True$$
$$\{ \alpha = 21.0375 \ deg \} \le 60 \ deg \qquad True$$

d) STEP 4 – Determine the net section axial force,  $F_s$ , and bending moment,  $M_s$ , applied to the conical transition (as specified in design conditions). The thrust load due to pressure shall not be included as part of the axial force,  $F_s$ . Calculate the equivalent line load,  $X_s$ , using the specified net section axial force,  $F_s$ , and bending moment,  $M_s$ .

$$X_{s} = \frac{F_{s}}{2\pi R_{s}} \pm \frac{M_{s}}{\pi R_{s}^{2}} = \begin{cases} \frac{-78104}{2\pi (45.125)} + \frac{4.301E + 06}{\pi (45.125)^{2}} = 396.8629 \frac{lbs}{in} \\ \frac{-78104}{2\pi (45.125)} - \frac{4.301E + 06}{\pi (45.125)^{2}} = -947.8053 \frac{lbs}{in} \end{cases}$$

e) STEP 5 – Compute the junction transition design parameters (the normalized resultant moment,  $M_{sN}$ , and shear force,  $Q_N$ ) for the internal pressure and equivalent line load per Table 4.3.5 and Table 4.3.6, respectively. For calculated values of *n* other than those presented in Table 4.3.5 and Table 4.3.6, linear interpolation of the equation coefficients,  $C_i$ , is permitted.

$$n = \frac{t_{c}}{t_{s}} = \frac{1.8125}{1.0} = 1.8125$$
$$H = \sqrt{\frac{R_{s}}{t_{s}}} = \sqrt{\frac{45.125}{1.0}} = 6.7175$$
$$B = \tan[\alpha] = \tan[21.0375] = 0.3846$$

Linear interpolation of the equation coefficients,  $C_i$  in Table 4.3.5 and Table 4.3.6 is required. The results of the interpolation are summarized with the following values for  $C_i$  (see paragraph 4.3.11.5, STEP 5 of E4.3.7)

For the applied pressure case  $M_{_{\rm NN}}$  is calculated using the following equation

$$M_{sN} = \exp \begin{bmatrix} C_1 + C_2 \ln \left[ H^2 \right] + C_3 \ln \left[ \alpha \right] + C_4 \left( \ln \left[ H^2 \right] \right)^2 + C_5 \left( \ln \left[ \alpha \right] \right)^2 + C_6 \ln \left[ H^2 \right] \ln \left[ \alpha \right] + C_7 \left( \ln \left[ H^2 \right] \right)^3 + C_8 \left( \ln \left[ \alpha \right] \right)^3 + C_9 \ln \left[ H^2 \right] \left( \ln \left[ \alpha \right] \right)^2 + C_{10} \left( \ln \left[ H^2 \right] \right)^2 \ln \left[ \alpha \right] \end{bmatrix}$$

This results in the following (see paragraph 4.3.11.5, STEP 5 of E4.3.7)

$$M_{sN} = 9.2135$$

For the applied pressure case  $Q_{\scriptscriptstyle N}$  is calculated using the following equation

$$Q_{N} = \left(\frac{C_{1} + C_{3}H^{2} + C_{5}\alpha + C_{7}H^{4} + C_{9}\alpha^{2} + C_{11}H^{2}\alpha}{1 + C_{2}H^{2} + C_{4}\alpha + C_{6}H^{4} + C_{8}\alpha^{2} + C_{10}H^{2}\alpha}\right)$$

This results in the following (see paragraph 4.3.11.5, STEP 5 of E4.3.7)

$$Q_N = -2.7333$$

For the Equivalent Line Load case,  $M_{\rm \tiny SN}$  is calculated using the following equation

$$M_{sN} = \left(\frac{C_1 + C_3 H + C_5 B + C_7 H^2 + C_9 B^2 + C_{11} HB}{1 + C_2 H + C_4 B + C_6 H^2 + C_8 B^2 + C_{10} HB}\right)$$

This results in the following (see paragraph 4.3.11.5, STEP 5 of E4.3.7)

$$M_{sN} = 0.4828$$

For the Equivalent Line Load case,  $\mathcal{Q}_{\scriptscriptstyle N}$  is calculated using the following equation

$$Q_{N} = \begin{pmatrix} C_{1} + C_{2} \ln[H] + C_{3} \ln[B] + C_{4} (\ln[H])^{2} + C_{5} (\ln[B])^{2} + C_{6} \ln[H] \ln[B] + \\ C_{7} (\ln[H])^{3} + C_{8} (\ln[B])^{3} + C_{9} \ln[H] (\ln[B])^{2} + C_{10} (\ln[H])^{2} \ln[B] \end{pmatrix}$$

This results in the following (see paragraph 4.3.11.5, STEP 5 of E4.3.7)

 $Q_N = -0.1613$ 

Summarizing, the normalized resultant moment  $M_{sN}$ , and shear force  $Q_N$  for the internal pressure and equivalent line load are as follows:

Internal Pressure :	$M_{sN} = 9.2135,$	$Q_N = -2.7333$
Equivalent Line Load :	$M_{sN} = 0.4828,$	$Q_N = -0.1613$

f) STEP 6 – Compute the stresses in the cylinder and cone at the junction using the equations in Table 4.3.2. for the Small End Junction

Evaluate the Cylinder at the Small End.

Stress Resultant Calculations.

$$M_{sP} = Pt_{s}^{2}M_{sN} = -14.7(1.0)^{2}(9.2135) = -135.4385 \frac{in-lbs}{in}$$

$$M_{sX} = X_{s}t_{s}M_{sN} = \begin{cases} 396.8629(1.0)(0.4828) = 191.6054 \frac{in-lbs}{in} \\ -947.8053(1.0)(0.4828) = -457.6004 \frac{in-lbs}{in} \end{cases}$$

$$M_{s} = M_{sP} + M_{sX} = \begin{cases} -135.4385 + (191.6054) = 56.1669 \frac{in-lbs}{in} \\ -135.4385 + (-457.6004) = -593.0389 \frac{in-lbs}{in} \end{cases}$$

$$Q_P = Pt_S Q_N = -14.7(1.0)(-2.7333) = 40.1795 \frac{lbs}{in}$$

$$Q_X = X_S Q_N = \begin{cases} 396.8629(-0.1613) = -64.0140 \frac{lbs}{in} \\ -947.8053(-0.1613) = 152.8810 \frac{lbs}{in} \end{cases}$$

$$Q = Q_P + Q_X = \begin{cases} 40.1795 + (-64.0140) = -23.8345 \frac{105}{in} \\ 40.1795 + 152.8810 = 193.0605 \frac{lbs}{in} \end{cases}$$

$$\beta_{cy} = \left[\frac{3(1-\nu^2)}{R_s^2 t_s^2}\right]^{0.25} = \left[\frac{3(1-(0.3)^2)}{(45.1250)^2 (1.000)^2}\right]^{0.25} = 0.1914 \text{ in}^{-1}$$

$$N_{s} = \frac{PR_{s}}{2} + X_{s} = \begin{cases} \frac{-14.7(45.125)}{2} + 396.8629 = 65.1942 \frac{lbs}{in} \\ \frac{-14.7(45.125)}{2} + (-947.8053) = -1279.4741 \frac{lbs}{in} \end{cases}$$

$$\begin{split} N_{\theta} &= PR_{s} + 2\beta_{cy}R_{s}\left(-M_{s}\beta_{cy} - Q\right) \\ N_{\theta} &= \begin{cases} -14.7(45.125) + 2(0.1914)(45.125)(-(56.1669)(0.1914) - (-23.8345))) \\ -14.7(45.125) + 2(0.1914)(45.125)(-(-593.0389)(0.1914) - 193.0605) \end{cases} \\ N_{\theta} &= \begin{cases} -437.3238 \frac{lbs}{in} \\ -2037.5216 \frac{lbs}{in} \\ \end{bmatrix} \\ K_{pc} &= 1.0 \end{split}$$

Stress Calculations: Determine the meridional and circumferential membrane and bending stresses:

$$\sigma_{sm} = \frac{N_s}{t_s} = \begin{cases} \frac{65.1942}{1.0} = 65.1942 \ psi \\ \frac{-1279.4741}{1.0} = -1279.4741 \ psi \end{cases}$$

$$\sigma_{sb} = \frac{6M_s}{t_s^2 K_{pc}} = \begin{cases} \frac{6(56.1669)}{(1.0)^2 (1.0)} = 337.0014 \ psi \\ \frac{6(-593.0389)}{(1.0)^2 (1.0)} = -3558.2334 \ psi \end{cases}$$

$$\sigma_{\theta m} = \frac{N_{\theta}}{t_s} = \begin{cases} \frac{-437.3238}{1.0} = -437.3238 \ psi \\ \frac{-2037.5216}{1.0} = -2037.5216 \ psi \end{cases}$$

$$\sigma_{\theta b} = \frac{6vM_s}{t_s^2 K_{pc}} = \begin{cases} \frac{6(0.3)(56.1669)}{(1.0)^2 (1.0)} = 101.1004 \ psi \\ \frac{6(0.3)(-593.0389)}{(1.0)^2 (1.0)} = -1067.4700 \ psi \end{cases}$$

Check Acceptance Criteria:

$$\begin{cases} \sigma_{sm} = 65.1942 \ psi \\ \sigma_{sm} = -1279.4741 \ psi \end{cases} \le \begin{cases} 1.5S = 1.5(22400) = 33600 \ psi \\ 1.5S, \ not \ applicable \ due \ to \ compressive \ stress \end{cases}$$

$$True \\ \begin{cases} \sigma_{sm} + \sigma_{sb} = 65.1942 + 337.0014 = 402.2 \ psi \\ \sigma_{sm} - \sigma_{sb} = 65.1942 - 337.0014 = |-271.8| \ psi \\ \sigma_{sm} - \sigma_{sb} = -1279.4741 + (-3558.2334) = |-4837.7| \ psi \\ \sigma_{sm} - \sigma_{sb} = -1279.4741 - (-3558.2334) = 2278.8 \ psi \end{cases} \le \{S_{PS} = 67200 \ psi\}$$

$$True \\ \begin{cases} \sigma_{\theta m} = -437.3238 \\ \sigma_{\theta m} = -2037.5216 \end{bmatrix} \le \{1.5S, \ not \ applicable \ due \ to \ compressive \ stress \}$$

$$\begin{cases} \sigma_{\theta m} + \sigma_{\theta b} = -437.3238 + 101.1004 = |-336.2| \ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = -437.3238 - 101.1004 = |-538.4| \ psi \\ \sigma_{\theta m} + \sigma_{\theta b} = -2037.5216 + (-1067.4700) = |-3105.0| \ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = -2037.5216 - (-1067.4700) = |-970.1| \ psi \end{cases} \le \{S_{PS} = 67200 \ psi \}$$

$$True$$

Since the hoop membrane stress,  $\sigma_{\theta m}$  and the axial membrane stress,  $\sigma_{sm}$  are compressive, the condition of local buckling shall be considered. Local buckling is not a concern if the following limit is satisfied.

$$\sigma_{\theta m} \leq F_{ha}$$
$$\sigma_{sm} \leq F_{xa}$$

 $F_{ha}$  is evaluated using paragraph 4.4.5.1, but substituting  $F_{he}$  with the following equation.

$$F_{he} = \frac{0.4E_y t}{D_o}$$

In accordance with paragraph 4.4.5.1, the value of  $F_{ha}$  calculated as follows.

1) STEP 1 – Assume an initial thickness, t and unsupported length, L.

$$t = 1.0$$
 in  
  $L \rightarrow Not$  required, as the equation for  $F_{he}$  is independent of L

2) STEP 2 – Calculate the predicted elastic buckling stress,  $F_{he}$ 

$$F_{he} = \frac{0.4E_y t}{D_o} = \frac{0.4(28.3E + 06)(1.0)}{92.25} = 122710.0271 \text{ psi}$$

3) STEP 3 – Calculate the predicted buckling stress,  $F_{ic}$ 

$$\frac{F_{he}}{S_y} = \frac{122710.0271}{33600} = 3.6521$$
  
Since  $\frac{F_{he}}{S_y} \ge 2.439$ , calculate  $F_{ic}$  as follows:  
 $F_{ic} = S_y = 33600 \ psi$ 

4) STEP 4 – Calculate the value of design factor, *FS* per paragraph 4.4.2. Since  $F_{ic} = S_y = 33600 \ psi$ , calculate *FS* as follows:

$$FS = 1.667$$

5) STEP 5 – Calculate the allowable hoop compressive membrane stress as follows:

$$F_{ha} = \frac{F_{ic}}{FS} = \frac{33600}{1.667} = 20155.9688 \ psi$$

6) STEP 6 – Compare the calculated hoop compressive membrane stress,  $\sigma_{\theta m}$  to the allowable hoop compressive membrane stress,  $F_{ha}$  per following criteria.

$$\{\sigma_{\theta m} = 2037.5 \ psi\} \le \{F_{ha} = 20156.0 \ psi\}$$
 True

Therefore, local buckling due to hoop compressive membrane stress is not a concern.

 $F_{xa}$  is evaluated using paragraph 4.4.12.2.b with  $\lambda = 0.15$ . The design factor *FS* used in paragraph 4.4.12.2.b is dependent on the predicted buckling stress  $F_{ic}$  and the material's yield strength,  $S_y$  as shown in paragraph 4.4.2. An initial calculation is required to determine the value of  $F_{xa}$  by setting FS = 1.0, with  $F_{ic} = F_{xa}$ . The initial value of  $F_{ic}$  is then compared to  $S_y$  as shown in paragraph 4.4.2 and the value of *FS* is determined. This computed value of *FS* is then used in paragraph 4.4.12.2.b.

For  $\lambda_c = 0.15$  , (Local Buckling)

$$F_{xa} = \min[F_{xa1}, F_{xa2}]$$

$$\frac{D_o}{t} = \frac{92.25}{1.0} = 92.25$$

$$M_x = \frac{L}{\sqrt{R_o t}} = \frac{636.0}{\sqrt{46.125(1.0)}} = 93.6459$$

Since  $\frac{D_o}{t} \le 135$ , calculate  $F_{xa1}$  as follows with an initial value of FS = 1.0.

$$F_{xa1} = \frac{S_y}{FS} = \frac{33600}{1.0} = 33600 \ psi$$

The value of  $F_{\rm xa2}$  is calculated as follows with an initial value of FS=1.0.

$$F_{xa2} = \frac{F_{xe}}{FS}$$
$$F_{xe} = \frac{C_x E_y t}{D_a}$$

Since  $\frac{D_o}{t} \le 1247$ , calculate  $C_x$  as follows:

$$C_x = \min\left[\frac{409\overline{c}}{\left(389 + \frac{D_o}{t}\right)}, \ 0.9\right]$$

Since  $M_x \ge 15$ , calculate  $\overline{c}$  as follows:

$$\overline{c} = 1.0$$
  
 $C_x = \min\left[\frac{409(1.0)}{389 + \frac{92.25}{1.0}}, 0.9\right] = 0.8499$ 

Therefore,

$$F_{xe} = \frac{0.8520(28.3E + 06)(1.0)}{92.25} = 260728.1301 \text{ psi}$$
$$F_{xa2} = \frac{F_{xe}}{FS} = \frac{260728.1301}{1.0} = 260728.1301 \text{ psi}$$
$$F_{xa} = \min[33600, 260728] = 33600 \text{ psi}$$

With a value of  $F_{ic} = F_{xa} = 33600$ , in accordance with paragraph 4.4.2, it is determined the value of FS = 1.667 since  $\{F_{ic} = 33600\} = \{S_y = 33600\}$ . Using this computed value of FS = 1.667 in paragraph 4.4.12.2.b,  $F_{xa}$  is calculated as follows.

$$F_{xa1} = \frac{S_y}{FS} = \frac{33600}{1.667} = 20155.9688 \ psi$$
  
$$F_{xa2} = \frac{F_{xe}}{FS} = \frac{260728.1301}{1.6670} = 156405.5969 \ psi$$
  
$$F_{xa} = \min[20155.9688, 156405.5969] = 20155.9688 \ psi$$

Compare the calculated axial compressive membrane stress,  $\sigma_{sm}$  to the allowable axial compressive membrane stress,  $F_{xa}$  per following criteria

$$\{\sigma_{sm} = 1279.5 \ psi\} \le \{F_{xa} = 20156.0 \ psi\}$$
 True

Therefore, local buckling due to axial compressive membrane stress is not a concern.

The cylinder at the cylinder-to-cone junction at the small end is adequately designed.

# Evaluate the Cone at the Small End.

Stress Resultant Calculations as determined above.

$$\begin{split} M_{csP} &= M_{sP} = -135.4385 \; \frac{in - lbs}{in} \\ M_{csX} &= M_{sX} = \begin{cases} 191.6054 \; \frac{in - lbs}{in} \\ -457.6004 \; \frac{in - lbs}{in} \end{cases} \\ M_{cs} &= M_{csP} + M_{csX} = \begin{cases} -135.4385 + 191.6054 = 56.1669 \; \frac{in - lbs}{in} \\ -135.4385 + (-457.6004) = -593.0389 \; \frac{in - lbs}{in} \end{cases} \\ Q_{c} &= Q \cos[\alpha] + N_{s} \sin[\alpha] \\ Q_{c} &= \begin{cases} (-23.8345) \cos[21.0375] + 65.1942 \sin[21.0375] = 1.1575 \; \frac{lbs}{in} \\ 193.0605 \cos[21.0375] + (-1279.4741) \sin[21.0375] = -279.1120 \; \frac{lbs}{in} \end{cases} \\ R_{c} &= \frac{R_{c}}{\cos[\alpha]} = \frac{45.1250}{\cos[21.0375]} = 48.3476 \; in \end{cases} \\ \beta_{co} &= \left[ \frac{3(1 - v^{2})}{R_{c}^{2} t_{c}^{2}} \right]^{0.25} = \left[ \frac{3(1 - (0.3)^{2})}{(48.3476)^{2}(1.8125)^{2}} \right]^{0.25} = 0.1373 \; in^{-1} \end{split}$$

$$\begin{split} N_{cs} &= N_{s} \cos[\alpha] - Q \sin[\alpha] \\ N_{cs} &= \begin{cases} 65.1942 \cos[21.0375] - (-23.8345) \sin[21.0375] = 69.4048 \frac{lbs}{in} \\ (-1279.4741) \cos[21.0375] - 193.0605 \sin[21.0375] = -1263.4963 \frac{lbs}{in} \end{cases} \\ N_{c\theta} &= \frac{PR_{s}}{\cos[\alpha]} + 2\beta_{co}R_{c} \left(-M_{cs}\beta_{co} + Q_{c}\right) \\ N_{c\theta} &= \begin{cases} \frac{-14.7(45.125)}{\cos[21.0375]} + 2(0.1373)(48.3476)(-(56.1669)(0.1373) + 1.1575) \\ \frac{-14.7(45.125)}{\cos[21.0375]} + 2(0.1373)(48.3476)(-(-593.0389)(0.1373) + (-279.1120)) \end{cases} \\ N_{c\theta} &= \begin{cases} -797.7248 \frac{lbs}{in} \\ -3335.2619 \frac{lbs}{in} \end{cases} \\ K_{cpc} &= 1.0 \end{split}$$

Stress Calculations: Determine the meridional and circumferential membrane and bending stresses:

$$\sigma_{sm} = \frac{N_{cs}}{t_{c}} = \begin{cases} \frac{69.4048}{1.8125} = 38.2923 \ psi \\ \frac{-1263.4963}{1.8125} = -697.1014 \ psi \end{cases}$$

$$\sigma_{sb} = \frac{6M_{cs}}{t_{c}^{2}K_{cpc}} = \begin{cases} \frac{6(56.1669)}{(1.8125)^{2}(1.0)} = 102.5831 \ psi \\ \frac{6(-593.0389)}{(1.8125)^{2}(1.0)} = -1083.1246 \ psi \end{cases}$$

$$\sigma_{\theta m} = \frac{N_{c\theta}}{t_{c}} = \begin{cases} \frac{-797.7248}{1.8125} = -440.1240 \ psi \\ \frac{-3335.2619}{1.8125} = -1840.1445 \ psi \end{cases}$$

$$\sigma_{\theta b} = \frac{6vM_{cs}}{t_{c}^{2}K_{cpc}} = \begin{cases} \frac{6(0.3)(56.1669)}{(1.8125)^{2}(1.0)} = 30.7749 \ psi \\ \frac{6(0.3)(-593.0389)}{(1.8125)^{2}(1.0)} = -324.9374 \ psi \end{cases}$$

Check Acceptable Criteria:

$$\begin{cases} \sigma_{sm} = 38.2923 \ psi \\ \sigma_{sm} = -697.1014 \ psi \end{cases} \le \begin{cases} 1.5S = 1.5(22400) = 33600 \ psi \\ 1.5S, \ not \ applicable \ due \ to \ compressive \ stress \end{cases}$$

$$True$$

$$\begin{cases} \sigma_{sm} + \sigma_{sb} = 38.2923 + 102.5831 = 140.9 \ psi \\ \sigma_{sm} - \sigma_{sb} = 38.2923 - 102.5831 = |-64.3| \ psi \\ \sigma_{sm} - \sigma_{sb} = -697.1014 + (-1083.1246) = |-1780.2| \ psi \\ \sigma_{sm} - \sigma_{sb} = -697.1014 - (-1083.1246) = 386.0 \ psi \end{cases}$$

$$\begin{cases} \sigma_{\thetam} = -440.1240 \\ \sigma_{\thetam} = -1840.1445 \end{cases} \le \{1.5S, \ not \ applicable \ due \ to \ compressive \ stress \}$$

$$\begin{cases} \sigma_{\thetam} + \sigma_{\thetab} = -440.1240 + 30.7749 = |-409.3| \ psi \\ \sigma_{\thetam} + \sigma_{\thetab} = -1840.1445 + (-324.9374) = |-2164.9| \ psi \\ \sigma_{\thetam} - \sigma_{\thetab} = -1840.1445 - (-324.9374) = |-1515.1| \ psi \end{cases}$$

Since the hoop membrane stress,  $\sigma_{\theta m}$  and the axial membrane stress,  $\sigma_{sm}$  are compressive, the condition of local buckling shall be considered. Local buckling is not a concern if the following limits are satisfied.

Using the procedure shown above for the cylindrical shell and substituting the cone thickness,  $t_c$  for the cylinder thickness, t, the allowable compressive hoop membrane and axial membrane stresses,  $F_{ha}$  and  $F_{xa}$ , respectively, are calculated as follows.

 $F_{ha} = 20156.0 \ psi$  $F_{xa} = 20156.0 \ psi$ 

Compare the calculated hoop compressive membrane stress,  $\sigma_{\theta m}$  and axial compressive membrane stress,  $\sigma_{sm}$ , to the allowable hoop compressive membrane stress,  $F_{ha}$  and axial compressive membrane stress,  $F_{xa}$  per following criteria.

$$\{\sigma_{\theta m} = 1840.1 \ psi\} \le \{F_{ha} = 20156.0 \ psi\}$$
True  
$$\{\sigma_{sm} = 697.1 \ psi\} \le \{F_{xa} = 20156.0 \ psi\}$$
True

Therefore, local buckling due to hoop and axial compressive membrane stress is not a concern.

The cone at the cylinder-to-cone junction at the small end is adequately designed.

g) STEP 7 – The stress acceptance criterion in STEP 6 is satisfied for both the cylinder and cone. Therefore, the design is complete.

# 4.4.8 Example E4.4.8 – Conical Transitions with a Knuckle

Determine if the proposed design for the large end of a cylinder-to-cone junction with a knuckle is adequately designed considering the following design conditions and applied forces and moments.

Vessel Data:

•	Material	=	<i>SA</i> –516, <i>Grade</i> 70, <i>Norm</i> .
•	Design Conditions	=	-14.7 psig@300°F
•	Inside Diameter (Large End)	=	120.0 in
•	Large End Thickness	=	1.0 <i>in</i>
•	Inside Diameter (Small End)	=	60.0 in
•	Small End Thickness	=	1.0 <i>in</i>
•	Knuckle Radius	=	10.0 <i>in</i>
•	Cone Thickness	=	1.0 <i>in</i>
•	Knuckle Thickness	=	1.0 <i>in</i>
•	Length of Conical Section	=	73.0 in
•	Unsupported Length of Large Cylinder	=	240.0 in
•	Unsupported Length of Small Cylinder	=	360.0 in
•	Corrosion Allowance	=	0.0 <i>in</i>
•	Allowable Stress	=	22400 psi
•	Yield Strength	=	33600 <i>psi</i>
•	Modulus of Elasticity	=	$28.3E + 06 \ psi$
•	Weld Joint Efficiency	=	1.0
•	One-Half Apex Angle	=	30.0 <i>deg</i>
•	Axial Force (Large End)	=	$-10000 \ lbs$
•	Net Section Bending Moment (Large End)	=	2.0E + 06 in - lbs

Evaluate per paragraphs 4.4.14 and 4.3.12.

The design rules in paragraph 4.3.12 shall be satisfied. In these calculations, a negative value of pressure shall be used in all applicable equations.

a) STEP 1 – Compute the large end cylinder thickness,  $t_L$ , using paragraph 4.4.5, (as specified in design conditions)

 $t_L = 1.0 in$ 

b) STEP 2 – Determine the cone half-apex angle,  $\alpha$ , and compute the cone thickness,  $t_c$ , at the large end using paragraph 4.4.5, (as specified in design conditions).

$$\alpha = 30 \ deg$$
  
 $t_c = 1.0 \ in$ 

c) STEP 3 – Proportion the transition geometry by assuming a value for the knuckle radius,  $r_k$ , and knuckle thickness,  $t_k$ , such that the following equations are satisfied. If all of these equations cannot be satisfied, the cylinder-to-cone junction shall be designed in accordance with Part 5.

$$\{t_{k} = 1.0 \text{ in}\} \ge \{t_{L} = 1.0 \text{ in}\} \qquad True$$

$$\{r_{k} = 10.0 \text{ in}\} > \{3t_{k} = 3.0 \text{ in}\} \qquad True$$

$$\{\frac{r_{k}}{R_{L}} = \frac{10.0}{60.0} = 0.1667\} > \{0.03\} \qquad True$$

$$\{\alpha = 30 \text{ deg}\} \le \{60 \text{ deg}\} \qquad True$$

d) STEP 4 – Determine the net section axial force,  $F_L$ , and bending moment,  $M_L$ , applied to the conical transition at the location of the knuckle. The thrust load due to pressure shall not be included as part of the axial force,  $F_L$ .

$$F_L = -10000 \ lbs$$
$$M_L = 2.0E + 06 \ in - lbs$$

e) STEP 5 – Compute the stresses in the knuckle at the junction using the equations in Table 4.3.7.
 Determine if the knuckle is considered to be compact or non-compact.

$$\alpha r_{k} < 2K_{m} \left( \left\{ R_{k} \left( \alpha^{-1} \tan \left[ \alpha \right] \right)^{0.5} + r_{k} \right\} t_{k} \right)^{0.5}$$

$$\left\{ 0.5236(10.0) \right\} < \left\{ 2(0.7) \left( \left\{ 50.0 \left( (0.5236)^{-1} \cdot \tan \left[ 0.5236 \right] \right)^{0.5} + 10 \right\} 1 \right)^{0.5} \right\}$$

$$\left\{ 5.2360 \text{ in} \right\} < \left\{ 11.0683 \text{ in} \right\} \qquad True$$

Where,

$$K_m = 0.7$$
  

$$\alpha = \frac{30.0}{180} \pi = 0.5236 \ rad$$
  

$$R_k = R_L - r_k = 60.0 - 10.0 = 50.0 \ in$$

Therefore, analyze the knuckle junction as a compact knuckle.

Stress Calculations: Determine the circumferential and longitudinal membrane and bending stresses:

$$\sigma_{\theta m} = \frac{PK_m \left( R_L \sqrt{R_L t_L} + L_k \sqrt{L_k t_C} \right) + \alpha \left( PL_{1k} r_k - 0.5 P_e L_{1k}^2 \right)}{K_m \left( t_L \sqrt{R_L t_L} + t_C \sqrt{L_k t_C} \right) \alpha t_k r_k}$$

$$\sigma_{sm} = \frac{P_e L_{1k}}{2t_k}$$

Where,

$$\begin{split} &L_{1k} = R_k \left( \alpha^{-1} \tan\left[\alpha\right] \right)^{0.5} + r_k = 50.0 \left( \left(0.5236\right)^{-1} \tan\left[0.5236\right] \right)^{0.5} + 10.0 = 62.5038 \text{ in} \\ &L_k = \frac{R_k}{\cos\left[\alpha\right]} + r_k = \frac{50.0}{\cos\left[0.5236\right]} + 10.0 = 67.7351 \text{ in} \\ &P_e = P + \frac{F_L}{\pi L_{1k}^2 \cos^2\left[\frac{\alpha}{2}\right]} \pm \frac{2M_L}{\pi L_{1k}^3 \cos^3\left[\frac{\alpha}{2}\right]} \\ &= \begin{cases} -14.7 + \frac{-10000.0}{\pi (62.5038)^2 \cdot \cos^2\left[\frac{0.5236}{2}\right]} + \frac{2(2.0E + 06)}{\pi (62.5038)^3 \cdot \cos^3\left[\frac{0.5236}{2}\right]} \\ -14.7 + \frac{-10000.0}{\pi (62.5038)^2 \cdot \cos^2\left[\frac{0.5236}{2}\right]} - \frac{2(2.0E + 06)}{\pi (62.5038)^3 \cdot \cos^3\left[\frac{0.5236}{2}\right]} \end{cases} \\ &P_e = \begin{cases} -9.7875 \ psi \\ -21.3590 \ psi \end{cases} \end{split}$$

Therefore,

$$\sigma_{\theta m} = \begin{cases} \left( (-14.7)(0.7)(60.0\sqrt{60.0(1.0)} + 67.7351\sqrt{67.7351(1.0)}) + \\ 0.5236((-14.7)(62.5038)(10.0) - 0.5(-9.7875)(62.5038)^2)) \right) \\ \hline 0.7(1.0\sqrt{60.0(1.0)} + 1.0\sqrt{67.7351(1.0)}) + 0.5236(1.0)(10.0) \\ \hline (-14.7)(0.7)(60.0\sqrt{60.0(1.0)} + 67.7351\sqrt{67.7351(1.0)}) + \\ \hline 0.5236((-14.7)(62.5038)(10.0) - 0.5(-21.3590)(62.5038)^2)) \\ \hline 0.7(1.0\sqrt{60.0(1.0)} + 1.0\sqrt{67.7351(1.0)}) + 0.5236(1.0)(10.0) \\ = 396.8501 \ psi \end{cases}$$

And,

$$\sigma_{sm} = \begin{cases} \frac{-9.7875(62.5038)}{2(1.0)} = -305.8780 \ psi \\ \frac{-21.3590(62.5038)}{2(1.0)} = -667.5093 \ psi \end{cases}$$

Check Acceptable Criteria:

$$\begin{cases} \sigma_{\theta m} = -324.0 \ psi \\ \sigma_{\theta m} = 396.9 \ psi \end{cases} \leq \begin{cases} S, \ not \ applicable \ due \ to \ compressive \ stress \\ S = 22400 \ psi \end{cases} True \end{cases}$$
$$\begin{cases} \sigma_{sm} = -305.9 \ psi \\ \sigma_{sm} = -667.5 \ psi \end{cases} \leq \{S, \ not \ applicable \ due \ to \ compressive \ stress \}$$

Since the hoop membrane stress  $\sigma_{\theta m}$  and axial membrane stress  $\sigma_{sm}$  are compressive, the condition of local buckling shall be considered. Local buckling is not a concern if the following limits are satisfied.

$$\sigma_{\theta m} \leq F_{ha}$$
$$\sigma_{sm} \leq F_{xa}$$

 $F_{ha}$  is evaluated using paragraph 4.4.5.1, but substituting  $F_{he}$  with the following equation.

$$F_{he} = \frac{0.4E_y t}{D_o}$$

In accordance with paragraph 4.4.5.1, the value of  $F_{ha}$  is calculated as follows.

1) STEP 1 – Assume an initial thickness, t and unsupported length, L.

t = 1.0 in  $L \rightarrow Not$  required, as the equation for  $F_{he}$  is independent of L

2) STEP 2 – Calculate the predicted elastic buckling stress,  $F_{he}$ 

$$F_{he} = \frac{0.4E_y t}{D_o} = \frac{0.4(28.3E + 06)(1.0)}{122.0} = 92786.8853 \ psi$$

3) STEP 3 – Calculate the predicted buckling stress,  $F_{ic}$ 

$$\frac{F_{he}}{S_y} = \frac{92786.8853}{33600} = 2.7615$$

Since 
$$\frac{F_{he}}{S_y} \ge 2.439$$
, calculate  $F_{ic}$  as follows:

$$F_{ic} = S_v = 33600 \ psi$$

4) STEP 4 – Calculate the value of design factor, *FS* per paragraph 4.4.2. Since  $F_{ic} = S_y = 33600 \ psi$ , calculate *FS* as follows:

$$FS = 1.667$$

5) STEP 5 – Calculate the allowable hoop compressive membrane stress as follows:

$$F_{ha} = \frac{F_{ic}}{FS} = \frac{33600}{1.667} = 20155.9688 \ psi$$

6) STEP 6 – Compare the calculated hoop compressive membrane stress,  $\sigma_{\theta m}$  to the allowable hoop compressive membrane stress,  $F_{ha}$  per following criteria.

$$\{\sigma_{\theta m} = -324.0 \ psi\} \le \{F_{ha} = 20156.0 \ psi\}$$
 True

Therefore, local buckling due to hoop compressive membrane stress is not a concern.

 $F_{xa}$  is evaluated using paragraph 4.4.12.2.b with  $\lambda = 0.15$ . The design factor FS used in paragraph 4.4.12.2.b is dependent on the predicted buckling stress  $F_{ic}$  and the material's yield strength,  $S_y$  as shown in VIII-2, paragraph 4.4.2. An initial calculation is required to determine the value of  $F_{xa}$  by setting FS = 1.0, with  $F_{ic} = F_{xa}$ . The initial value of  $F_{ic}$  is then compared to  $S_y$  as shown in paragraph 4.4.2 and the value of FS is determined. This computed value of FS is then used in paragraph 4.4.12.2.b.For  $\lambda_c = 0.15$ , (Local Buckling)

$$F_{xa} = \min[F_{xa1}, F_{xa2}]$$

$$\frac{D_o}{t} = \frac{122.0}{1.0} = 122.0$$

$$M_x = \frac{L}{\sqrt{R_o t}} = \frac{240.0}{\sqrt{61.0(1.0)}} = 30.7289$$

Since  $\frac{D_o}{t} \le 135$ , calculate  $F_{xa1}$  as follows with an initial value of FS = 1.0.

$$F_{xa1} = \frac{S_y}{FS} = \frac{33600}{1.0} = 33600 \ psi$$

The value of  $F_{xa2}$  is calculated as follows with an initial value of FS = 1.0.

$$F_{xa2} = \frac{F_{xe}}{FS}$$
$$F_{xe} = \frac{C_x E_y t}{D_o}$$

Since  $\frac{D_o}{t} \le 1247$ , calculate  $C_x$  as follows:

$$C_{x} = \min\left[\frac{409\overline{c}}{\left(389 + \frac{D_{o}}{t}\right)}, \ 0.9\right]$$

Since  $M_x \ge 15$ , calculate  $\overline{c}$  as follows:

$$\overline{c} = 1.0$$
  
 $C_x = \min\left[\frac{409(1.0)}{389 + \frac{122.0}{1.0}}, \ 0.9\right] = 0.8004$ 

Therefore,

$$F_{xe} = \frac{0.8004(28.3E+06)(1.0)}{122.0} = 185666.5574 \text{ psi}$$
$$F_{xa2} = \frac{F_{xe}}{FS} = \frac{185666.5574}{1.0} = 185666.5574 \text{ psi}$$
$$F_{xa} = \min[33600, 185666.5574] = 33600 \text{ psi}$$

With a value of  $F_{ic} = F_{xa} = 33600$ , in accordance with VIII-2, paragraph 4.4.2, it is determined the value of FS = 1.667 since  $\{F_{ic} = 33600\} = \{S_y = 33600\}$ . Using this computed value of FS = 1.667 in paragraph 4.4.12.2.b,  $F_{xa}$  is calculated as follows.

$$F_{xa1} = \frac{S_y}{FS} = \frac{33600}{1.667} = 20155.9688 \ psi$$
  

$$F_{xa2} = \frac{F_{xe}}{FS} = \frac{185666.5574}{1.6670} = 111377.6589 \ psi$$
  

$$F_{xa} = \min[20155.9688, 111377.6589] = 20155.9688 \ psi$$

Compare the calculated axial compressive membrane stress,  $\sigma_{sm}$  to the allowable axial compressive membrane stress,  $F_{xa}$  per the following criteria:

$$\begin{cases} \sigma_{sm} = 305.9 \ psi \\ \sigma_{sm} = 667.5 \ psi \end{cases} \leq \{F_{xa} = 20156.0 \ psi \}$$
True

Therefore, local buckling due to axial compressive membrane stress is not a concern.

The cylinder-to-cone junction with a knuckle at the large end is adequately designed.

STEP 6 – The stress acceptance criterion in STEP 6 is satisfied. Therefore, the design is complete.

# 4.5 Shells Openings in Shells and Heads

# 4.5.1 Example E4.5.1 – Radial Nozzle in Cylindrical Shell and Weld Strength Analysis

Design an integral nozzle and perform a weld strength analysis. The parameters used in this design procedure are shown in Figure E4.5.1.

Vessel and Nozzle Data:

•	Design Conditions	=	356 psig@300°F
٠	Corrosion Allowance	=	0.125 in
•	Weld Joint Efficiency	=	1.0
•	Shell Material	=	SA-516, Grade 70, Norm.
٠	Shell Allowable Stress	=	22400 psi
•	Nozzle Material	=	SA-105
•	Nozzle Allowable Stress	=	21200 <i>psi</i>
•	Shell Inside Diameter	=	150.0 in
•	Shell Thickness	=	1.8125 in
•	Nozzle Outside Diameter	=	19.0 in
•	Nozzle Hub Outside Diameter	=	25.5 in
•	Nozzle Hub Height	=	7.1875 in
•	Nozzle Thickness	=	4.75 in
•	External Nozzle Projection	=	14.1875 in
•	Internal Nozzle Projection	=	0.0 <i>in</i>

The nozzle is inserted through the shell, i.e. set-in type nozzle, see Figure 4.5.13.

Establish the corroded dimensions.

Shell:

$$D_i = 150.0 + 2(Corrosion Allowance) = 150.0 + 2(0.125) = 150.25$$
 in  
t = 1.8125 - Corrosion Allowance = 1.8125 - 0.125 = 1.6875 in

Nozzle:

$$t_n = 4.75 - Corrosion Allowance = 4.75 - 0.125 = 4.625$$
 in  
 $R_n = \frac{D - 2(t_n)}{2} = \frac{25.5 - 2(4.625)}{2} = 8.125$  in

The procedure, per paragraph 4.5.5, to design a radial nozzle in a cylindrical shell subject to pressure loading is shown below.

Note: This is an analysis of a single nozzle; therefore, the spacing criterion is automatically satisfied. If there were multiple nozzles in the shell, the spacing requirements for nozzles in paragraph 4.5.13 would need to be checked.

a) STEP 1 – Determine the effective radius of the shell as follows

$$R_{eff} = 0.5D_i = 0.5(150.25) = 75.125$$
 in

b) STEP 2 – Calculate the limit of reinforcement along the vessel wall.

For set-in, integrally reinforced nozzles:

$$L_{R} = \min\left[\sqrt{R_{eff}t}, 2R_{n}\right]$$
$$L_{R} = \min\left[\sqrt{(75.125)(1.6875)}, 2(8.125)\right] = \min[11.2594, 16.25] = 11.2594 \text{ in}$$

c) STEP 3 – Calculate the limit of reinforcement along the nozzle wall projecting outside the vessel surface.

For set-in nozzles:

$$L_{H1} = \min[1.5t, t_e] + \sqrt{R_n t_n} = \min[1.5(1.6875), 0.0] + \sqrt{8.125(4.625)} = 6.1301 \text{ in}$$

$$L_{H2} = L_{pr1} = 14.1875 \text{ in}$$

$$L_{H3} = 8(t + t_e) = 8(1.6875 + 0.0) = 13.5 \text{ in}$$

$$L_{H} = \min[L_{H1}, L_{H2}, L_{H3}] + t = \min[6.1301, 15.875, 13.5] + 1.6875 = 7.8176 \text{ in}$$

d) STEP 4 – Calculate the limit of reinforcement along the nozzle wall projecting inside the vessel surface, if applicable

$$L_{I1} = \sqrt{R_n t_n} = \sqrt{8.125(4.625)} = 6.1301 \text{ in}$$
$$L_{I2} = L_{pr2} = 0.0$$
$$L_{I3} = 8(t + t_e) = 8(1.6875 + 0.0) = 13.5 \text{ in}$$
$$L_I = \min[L_{I1}, L_{I2}, L_{I3}] = \min[6.1301, 0.0, 13.5] = 0.0$$

e) STEP 5 – Determine the total available area near the nozzle opening, see Figure 4.5.2. Do not include any area that falls outside of the limits defined by  $L_H$ ,  $L_R$ , and  $L_I$ . For variable thickness nozzles, see Figure 4.5.13 for metal area definitions of  $A_2$ .

For set-in nozzles:

$$A_{T} = A_{1} + f_{rn} \left( A_{2} + A_{3} \right) + A_{41} + A_{42} + A_{43} + f_{rp} A_{5}$$
$$A_{1} = \left( tL_{R} \right) \cdot \max \left[ \left( \frac{\lambda}{5} \right)^{0.85}, \ 1.0 \right] = 1.6875 \left( 11.2594 \right) \cdot \max \left[ \left( \frac{1.3037}{5} \right)^{0.85}, \ 1.0 \right]$$
$$A_{1} = 19.0002 \ in^{2}$$

$$\begin{split} \lambda &= \min\left[\left\{\frac{\left(2R_{n}+t_{n}\right)}{\sqrt{\left(D_{i}+t_{eff}\right)t_{eff}}}\right\}, 12.0\right] = \min\left[\frac{2\left(8.125\right)+4.625}{\sqrt{150.25+1.6875}\left(1.6875\right)}, 12.0\right] = 1.3037\\ t_{eff} &= t + \left(\frac{A_{5}f_{rp}}{L_{R}}\right) = 1.6875 + \left(\frac{0.0\left(0.0\right)}{11.2594}\right) = 1.6875 \text{ in}\\ f_{rn} &= \min\left[\frac{S_{n}}{S}, 1\right] = \frac{21200}{22400} = 0.9464\\ f_{rp} &= \min\left[\frac{S_{p}}{S}, 1\right] = 0.0 \end{split}$$

Since  $\{L_{H} = 7.8176 \ in\} \le \{L_{x3} = L_{pr3} + t = 7.1875 + 1.6875 = 8.875 \ in\}$ , calculate  $A_{2}$  as follows, see Figure 4.5.13:

$$A_{2} = t_{n}L_{H} = 4.625(7.8176) = 36.1564 in^{2}$$

$$A_{3} = t_{n}L_{I} = 0.0$$

$$A_{41} = 0.5L_{41}^{2} = 0.5(0.375)^{2} = 0.0703 in^{2}$$

$$A_{42} = 0.5L_{42}^{2} = 0.0$$

$$A_{43} = 0.5L_{43}^{2} = 0.0$$

$$A_{5} = \min[A_{5a}, A_{5b}]$$

$$A_{5a} = Wt_{e} = 0.0$$

$$A_{5b} = L_{R}t_{e} = 0.0$$

$$A_{5} = 0.0$$

$$A_{5} = 0.0$$

$$A_{7} = 19.0002 + 0.9464(36.1564 + 0.0) + 0.0703 + 0.0 + 0.0 + 0.0 = 53.2889 in^{2}$$

f) STEP 6 – Determine the applicable forces

For set-in nozzles:

$$f_{N} = PR_{xn}L_{H} = 356(10.2644)(7.8176) = 28566.4985 \ lbs$$

$$R_{xn} = \frac{t_{n}}{\ln\left[\frac{R_{n} + t_{n}}{R_{n}}\right]} = \frac{4.625}{\ln\left[\frac{8.125 + 4.625}{8.125}\right]} = 10.2644 \ in$$

$$f_{S} = PR_{xs}(L_{R} + t_{n}) = 356(75.9656)(11.2594 + 4.625) = 429573.7997 \ lbs$$

$$R_{xs} = \frac{t_{eff}}{\ln\left[\frac{R_{eff} + t_{eff}}{R_{eff}}\right]} = \frac{1.6875}{\ln\left[\frac{75.125 + 1.6875}{75.125}\right]} = 75.9656 \text{ in}$$
  
$$f_{Y} = PR_{xs}R_{nc} = 356(75.9656)(8.125) = 219730.4980 \text{ lbs}$$
  
Note, for radial nozzles,  $R_{nc} = R_{r}$ .

g) STEP 7 – Determine the average local primary membrane stress and the general primary membrane stress at the nozzle intersection.

$$\sigma_{avg} = \frac{\left(f_N + f_S + f_Y\right)}{A_T} = \frac{28566.4985 + 429573.7997 + 219730.4980}{53.2889} = 12720.6753 \ psi$$
  
$$\sigma_{circ} = \frac{PR_{xs}}{t_{eff}} = \frac{356(75.9656)}{1.6875} = 16025.9281 \ psi$$

h) STEP 8 – Determine the maximum local primary membrane stress at the nozzle intersection.

$$P_{L} = \max\left[\left\{2\sigma_{avg} - \sigma_{circ}\right\}, \sigma_{circ}\right]$$
$$P_{L} = \max\left[\left\{2(12720.6753) - 16025.9281\right\}, 16025.9281\right] = 16025.9281 \text{ psi}$$

i) STEP 9 – The calculated maximum local primary membrane stress should satisfy Equation 4.5.56. If the nozzle is subjected to internal pressure, then the allowable stress,  $S_{allow}$ , is given by Equation 4.5.57. If the nozzle is subjected to external pressure, then the allowable stress is given by Equation 4.5.58 where  $F_{ha}$  is evaluated in paragraph 4.4 for the shell geometry being evaluated (e.g. cylinder, spherical shell, or formed head). The allowable stress shall be the minimum of the shell or nozzle material evaluated at the design temperature.

$$\{P_L = 16025.9281 \ psi\} \le \{S_{allow} = 1.5SE = 1.5(21200)(1.0) = 31800 \ psi\}$$

j) STEP 10 – Determine the maximum allowable working pressure at the nozzle intersection.

$$P_{\max 1} = \frac{S_{allow}}{\frac{2A_p}{A_T} - \frac{R_{xs}}{t_{eff}}} = \frac{1.5(21200)(1.0)}{\left(\frac{2(1904.1315)}{53.2889}\right) - \left(\frac{75.9656}{1.6875}\right)} = 1202.3676 \ psi$$

$$A_p = \frac{f_N + f_S + f_Y}{P} = \frac{28566.4985 + 429573.7997 + 219730.4980}{356.0} = 1904.1315 \ in^2$$

$$P_{\max 2} = S\left(\frac{t}{R_{xs}}\right) = 22400\left(\frac{1.6875}{75.9656}\right) = 497.5936 \ psi$$

$$P_{\max 3} = \min[P_{\max 1}, P_{\max 2}] = \min[1202.3676, \ 497.5936] = 497.5936 \ psi$$

The nozzle is acceptable because  $P_{max} = 497.6 \ psi$  is greater than the specified design pressure of 356 psig.

## Weld Strength Analysis

The procedure to evaluate attachment welds of nozzles in a cylindrical, conical, or spherical shell or formed head subject to pressure loading per paragraph 4.5.14.2 is shown below.

a) STEP 1 – Determine the discontinuity force factor,

For set-in nozzles:

$$k_{y} = \frac{R_{nc} + t_{n}}{R_{nc}} = \frac{8.125 + 4.625}{8.125} = 1.5692$$

Note, for radial nozzles,  $R_{nc} = R_n$ .

b) STEP 2 – Calculate weld length resisting continuity force,

Weld length of nozzle to shell weld, for radial nozzles:

$$L_{\tau} = \frac{\pi}{2} (R_n + t_n) = \frac{\pi}{2} (8.125 + 4.625) = 20.0277 \text{ in}$$

Weld length of pad to shell weld, for radial nozzles:

$$L_{\tau p} = \frac{\pi}{2} \left( R_n + t_n + W \right) \qquad \text{Not Applicable}$$

c) STEP 3 – Compute the weld throat dimensions, as applicable

$$\begin{split} L_{_{41T}} &= 0.7071 L_{_{41}} = 0.7071 \big( 0.375 \big) = 0.2652 ~in \\ L_{_{42T}} &= 0.0 \\ L_{_{43T}} &= 0.0 \end{split}$$

d) STEP 4 – Determine if the weld sizes are acceptable. If the nozzle is integrally reinforced, and the computed shear stress in the weld given by Equation (4.5.182) satisfies Equation (4.5.183), then the design is complete. If the shear stress in the weld does not satisfy Equation (4.5.183), increase the weld size and return to Step 3.

$$\tau = \frac{f_{welds}}{L_{\tau} \left( 0.49L_{41T} + 0.6t_{w1} + 0.49L_{43T} \right)}$$
  
$$\tau = \frac{45450.9764}{20.0277 \left( 0.49 \left( 0.2652 \right) + 0.6 \left( 1.6875 \right) + 0.49 \left( 0.0 \right) \right)} = 1986.4411 \ psi$$

Where,

$$f_{welds} = \min\left[f_Y k_y, 1.5S_n \left(A_2 + A_3\right), \frac{\pi}{4} P R_n^2 k_y^2\right]$$

$$f_{welds} = \min\left[\left\{219730.4980(1.5692) = 344801.0975\right\} \\ \left\{1.5(21200)(36.1564 + 0) = 1149773.52\right\} \\ \left\{\frac{\pi}{4}(356)(8.125)^2(1.5692)^2 = 45450.9764\right\}\right] = 45450.9764 \ lbs$$

$$\left\{\tau = 1986.4 \ psi\right\} \le \left\{S = 22400 \ psi\right\}$$

The weld strength is acceptable.



Figure E4.5.1 - Nozzle Detail

# 4.5.2 Example E4.5.2 – Hillside Nozzle in Cylindrical Shell and Weld Strength Analysis

Design an integral hillside nozzle in a cylindrical shell and perform a weld strength analysis. The parameters used in this design procedure are shown in Figure E4.5.2.

Vessel and Nozzle Data:

•	Design Conditions	=	356 psig@300°F
•	Corrosion Allowance	=	0.125 in
•	Weld Joint Efficiency	=	1.0
•	Shell Material	=	SA-516, Grade 70, Norm.
•	Shell Allowable Stress	=	22400 psi
•	Nozzle Material	=	SA-105
•	Nozzle Allowable Stress	=	21200 <i>psi</i>
•	Shell Inside Diameter	=	150.0 in
•	Shell Thickness	=	1.8125 in
•	Nozzle Outside Diameter	=	11.56 in
•	Nozzle Thickness	=	1.97 <i>in</i>
•	External Nozzle Projection	=	19.0610 in
•	Internal Nozzle Projection	=	0.0 <i>in</i>
•	Distance from cylinder/nozzle centerlines	=	34.875 in

The hillside nozzle is inserted through the shell, i.e. set-in type nozzle, see Figure 4.5.4.

Establish the corroded dimensions.

Shell:

$$D_i = 150.0 + 2(Corrosion Allowance) = 150.0 + 2(0.125) = 150.25$$
 in  
t = 1.8125 - Corrosion Allowance = 1.8125 - 0.125 = 1.6875 in

Nozzle:

$$t_n = 1.97 - Corrosion \ Allowance = 1.97 - 0.125 = 1.845 \ in$$
$$R_n = \frac{D - 2(t_n)}{2} = \frac{11.56 - 2(1.845)}{2} = 3.935 \ in$$

For a hillside nozzle in a cylindrical shell (see Figure 4.5.4), the design procedure in paragraph 4.5.5 shall be used with the following substitutions from paragraph 4.5.6.

$$R_{nc} = \max\left[\left(\frac{R_{ncl}}{2}\right), R_n\right]$$

Where,

$$R_{ncl} = R_{eff} \left(\theta_1 - \theta_2\right)$$
  

$$\theta_1 = \cos^{-1} \left\lfloor \frac{D_X}{R_{eff}} \right\rfloor = \cos^{-1} \left[ \frac{34.875}{75.125} \right] = 62.3398 \ deg = 1.0880 \ rad$$
  

$$\theta_2 = \cos^{-1} \left\lfloor \frac{D_X + R_n}{R_{eff}} \right\rfloor = \cos^{-1} \left[ \frac{34.875 + 3.935}{75.125} \right] = 58.8952 \ deg = 1.0279 \ rad$$
  

$$R_{ncl} = 75.125 (1.0880 - 1.0279) = 4.5150 \ in$$
  

$$R_{nc} = \max \left[ \left( \frac{4.5150}{2} \right), 3.935 \right] = 3.935 \ in$$

The procedure in paragraph 4.5.5 is shown below.

Note: This is an analysis of a single nozzle; therefore, the spacing criterion is automatically satisfied. If there were multiple nozzles in the shell, the spacing requirements for nozzles in paragraph 4.5.13 would need to be checked.

a) STEP 1 – Determine the effective radius of the shell as follows:

$$R_{eff} = 0.5D_i = 0.5(150.25) = 75.125$$
 in

b) STEP 2 – Calculate the limit of reinforcement along the vessel wall:

For set-in, integrally reinforced nozzles:

$$L_R = \min\left[\sqrt{R_{eff}t}, 2R_n\right] = \min\left[\sqrt{75.125(1.6875)}, 2(3.935)\right] = 7.8700 \text{ in}$$

c) STEP 3 – Calculate the limit of reinforcement along the nozzle wall projecting outside the vessel surface.

For set-in nozzles:

$$L_{H1} = \min[1.5t, t_e] + \sqrt{R_n t_n} = \min[1.5(1.6875), 0.0] + \sqrt{3.935(1.845)} = 2.6945 \text{ in}$$

$$L_{H2} = L_{pr1} = 19.0610 \text{ in}$$

$$L_{H3} = 8(t + t_e) = 8(1.6875 + 0.0) = 13.5 \text{ in}$$

$$L_H = \min[L_{H1}, L_{H2}, L_{H3}] + t = \min[2.6945, 19.0610, 13.5] + 1.6875 = 4.3820 \text{ in}$$

 d) STEP 4 – Calculate the limit of reinforcement along the nozzle wall projecting inside the vessel surface, if applicable:

$$L_{I1} = \sqrt{R_n t_n} = \sqrt{3.935(1.845)} = 2.6945$$
$$L_{I2} = L_{pr2} = 0.0$$

$$L_{I3} = 8(t + t_e) = 8(1.6875 + 0.0) = 13.5 in$$
$$L_I = \min[L_{I1}, L_{I2}, L_{I3}] = 0.0$$

e) STEP 5 – Determine the total available area near the nozzle opening, see Figures 4.5.1. Do not include any area that falls outside of the limits defined by  $L_H$ ,  $L_R$ , and  $L_I$ .

For set-in nozzles:

$$\begin{aligned} A_{T} &= A_{1} + f_{rn}(A_{2} + A_{3}) + A_{41} + A_{42} + A_{43} + f_{rp}A_{5} \\ A_{1} &= \left(tL_{R}\right) \cdot \max\left[\left(\frac{\lambda}{5}\right)^{0.85}, 1.0\right] = 1.6875\left(7.8700\right) \cdot \max\left[\left(\frac{0.6067}{5}\right)^{0.85}, 1.0\right] = 13.2806 \\ \lambda &= \min\left[\left\{\frac{\left(2R_{n} + t_{n}\right)}{\sqrt{\left(D_{i} + t_{eff}\right)t_{eff}}}\right\}, 12.0\right] = \min\left[\left\{\frac{2\left(3.935\right) + 1.845}{\sqrt{\left(150.25 + 1.6875\right)\left(1.6875\right)}}\right\}, 12.0\right] = 0.6067 \\ t_{eff} &= t + \left(\frac{A_{5}f_{rp}}{L_{R}}\right) = 1.6875 + \left(\frac{0.0(0.0)}{7.87}\right) = 1.6875 \text{ in} \\ f_{rn} &= \frac{S_{n}}{S} = \frac{21200}{22400} = 0.9464 \\ f_{rp} &= \frac{S_{p}}{S} = 0.0 \end{aligned}$$

Since  $\{t_n = 1.845 \ in\} = \{t_{n2} = 1.845 \ in\}$ , calculate  $A_2$  as follows:

$$A_{2} = t_{n}L_{H} = 1.845(4.3820) = 8.0848 \ in^{2}$$

$$A_{3} = t_{n}L_{I} = 1.845(0.0) = 0.0$$

$$A_{41} = 0.5L_{41}^{2} = 0.5(0.375) = 0.0703 \ in^{2}$$

$$A_{42} = 0.5L_{42}^{2} = 0.0$$

$$A_{43} = 0.5L_{43}^{2} = 0.0$$

$$A_{5} = \min[A_{5a}, A_{5b}]$$

$$A_{5a} = Wt_{e} = 0.0$$

$$A_{5b} = L_{R}t_{e} = 0.0$$

$$A_{5} = 0.0$$

$$A_{7} = 13.2806 + 0.9464(8.0848 + 0.0) + 0.0 + 0.0703 + 0.0 + 0.0 = 21.0024 \ in^{2}$$

f) STEP 6 – Determine the applicable forces:

For set-in nozzles:

$$f_{N} = PR_{xn}L_{H} = 356(4.7985)(4.3820) = 7485.6216 \ lbs$$

$$R_{xn} = \frac{t_{n}}{\ln\left[\frac{R_{n} + t_{n}}{R_{n}}\right]} = \frac{1.845}{\ln\left[\frac{3.935 + 1.845}{3.935}\right]} = 4.7985 \ in$$

$$f_{S} = PR_{xs}(L_{R} + t_{n}) = 356(75.9656)(7.87 + 1.845) = 262730.0662 \ lbs$$

$$R_{xs} = \frac{t_{eff}}{\ln\left[\frac{R_{eff} + t_{eff}}{R_{eff}}\right]} = \frac{1.6875}{\ln\left[\frac{75.125 + 1.6875}{75.125}\right]} = 75.9656 \ in$$

$$f_{Y} = PR_{xs}R_{nc} = 356(75.9656)(3.935) = 106417.1704 \ lbs$$

g) STEP 7 – Determine the average local primary membrane stress and the general primary membrane stress at the nozzle intersection:

$$\sigma_{avg} = \frac{\left(f_N + f_S + f_Y\right)}{A_T} = \frac{\left(7485.6216 + 262730.0662 + 106417.1704\right)}{21.0024} = 17932.8485 \text{ psi}$$
  
$$\sigma_{circ} = \frac{PR_{xs}}{t_{eff}} = \frac{356(75.9656)}{1.6875} = 16025.9281 \text{ psi}$$

h) STEP 8 – Determine the maximum local primary membrane stress at the nozzle intersection:

$$P_{L} = \max\left[\left\{2\sigma_{avg} - \sigma_{circ}\right\}, \sigma_{circ}\right]$$
$$P_{L} = \max\left[\left\{2(17932.8485) - 16025.9281\right\}, 16025.9281\right] = 19839.7689 \ psi$$

i) STEP 9 – The calculated maximum local primary membrane stress should satisfy Equation 4.5.56. If the nozzle is subjected to internal pressure, then the allowable stress,  $S_{allow}$ , is given by Equation 4.5.57. If the nozzle is subjected to external pressure, then the allowable stress is given by Equation 4.5.58 where  $F_{ha}$  is evaluated in paragraph 4.4 for the shell geometry being evaluated (e.g. cylinder, spherical shell, or formed head). The allowable stress shall be the minimum of the shell or nozzle material evaluated at the design temperature.

$$\{P_L = 19839.7689 \ psi\} \le \{S_{allow} = 1.5SE = 1.5(21200)(1.0) = 31800 \ psi\}$$

j) STEP 10 – Determine the maximum allowable working pressure of the nozzle:

$$P_{\max 1} = \frac{S_{allow}}{\frac{2A_p}{A_T} - \frac{R_{xs}}{t_{eff}}} = \frac{1.5(21200)(1.0)}{\frac{2(1057.9575)}{21.0024} - \frac{75.9656}{1.6875}} = 570.6114 \, psi$$
$$A_p = \frac{\left(f_N + f_S + f_Y\right)}{A_T} = \frac{\left(7485.6216 + 262730.0662 + 106417.1704\right)}{356} = 1057.9575 \, in^2$$

$$P_{\max 2} = S\left(\frac{t}{R_{xs}}\right) = 22400 \left(\frac{1.6875}{75.9656}\right) = 497.5936 \ psi$$
$$P_{\max} = \min[P_{\max 1}, P_{\max 2}] = \min[570.6114, 497.5936] = 497.5936 \ psi$$

The nozzle is acceptable because  $P_{max} = 497.6 \ psi$  is greater than the specified design pressure of 356 psig.

## Weld Strength Analysis.

The procedure to evaluate attachment welds of nozzles in a cylindrical, conical, or spherical shell or formed head subject to pressure loading per paragraph 4.5.14 is shown below.

a) STEP 1 – Determine the discontinuity force factor,

For set-in Nozzles:

$$k_{y} = \frac{R_{nc} + t_{n}}{R_{nc}} = \frac{3.935 + 1.845}{3.935} = 1.4689$$

b) STEP 2 – Calculate weld length resisting continuity force,

Weld length of nozzle to shell weld, for non-radial nozzles:

$$L_{\tau} = \frac{\pi}{2} \sqrt{\frac{\left(R_{nc} + t_n\right)^2 + \left(R_n + t_n\right)^2}{2}} = \frac{\pi}{2} \sqrt{\frac{\left(3.935 + 1.845\right)^2 + \left(3.935 + 1.845\right)^2}{2}} = 9.0792 \text{ in}$$

Weld length of pad to shell weld, for non-radial nozzles:

$$L_{\tau p} = \frac{\pi}{2} \sqrt{\frac{\left(R_{nc} + t_n + W\right)^2 + \left(R_n + t_n + W\right)^2}{2}}$$
 Not Applicable

c) STEP 3 – Compute the weld throat dimensions, as applicable

$$\begin{split} L_{41T} &= 0.7071 L_{41} = 0.7071 \big( 0.375 \big) = 0.2652 ~in \\ L_{42T} &= 0.0 \\ L_{43T} &= 0.0 \end{split}$$

d) STEP 4 – Determine if the weld sizes are acceptable. If the nozzle is integrally reinforced, and the computed shear stress in the weld given by Equation (4.5.182) satisfies Equation (4.5.183), then the design is complete. If the shear stress in the weld does not satisfy Equation (4.5.183), increase the weld size and return to Step 3.

$$\tau = \frac{f_{welds}}{L_{\tau} \left( 0.49L_{41T} + 0.6t_{w1} + 0.49L_{43T} \right)}$$
  
$$\tau = \frac{9341.4397}{9.0792 \left( 0.49 \left( 0.2652 \right) + 0.6 \left( 1.6875 \right) + 0.49 \left( 0.0 \right) \right)} = 900.5955 \ psi$$

Where,

$$f_{welds} = \min\left[f_Y k_y, 1.5S_n (A_2 + A_3), \frac{\pi}{4} P R_n^2 k_y^2\right]$$

$$f_{welds} = \min\left[\left\{106417.1704(1.4689) = 156316.1816\right\} \\ \left\{1.5(21200)(8.0848 + 0) = 257096.64\right\} \\ \left\{\frac{\pi}{4}(356)(3.935)^2(1.4689)^2 = 9341.4397\right\}\right] = 9341.4397 \ lbs$$

$$\{\tau = 900.6 \ psi\} \le \{S = 22400 \ psi\}$$

The weld strength is acceptable.



Figure E4.5.2 - Nozzle Detail

# 4.5.3 Example E4.5.3 – Radial Nozzle in Elliptical Head and Weld Strength Analysis

Design an integral radial nozzle centered in a 2:1 elliptical head and perform a weld strength analysis. The parameters used in this design procedure are shown in Figure E4.5.3.

Vessel and Nozzle Data:

•	Design Conditions	=	356 <i>psig</i> @300°F
•	Corrosion Allowance	=	0.125 in
•	Weld Joint Efficiency	=	1.0
•	Head Material	=	SA-516, Grade 70, Norm.
•	Head Allowable Stress	=	22400 psi
•	Nozzle Material	=	SA-105
•	Nozzle Allowable Stress	=	21200 psi
•	Head Inside Diameter	=	90.0 in
•	Height of the 2:1 Elliptical Head	=	22.625 in
•	Head Thickness	=	1.0 <i>in</i>
•	Nozzle Outside Diameter	=	15.94 in
•	Nozzle Thickness	=	2.28 in
•	External Nozzle Projection	=	13.5 in
•	Internal Nozzle Projection	=	0.0 <i>in</i>
•	Distance from head/nozzle centerlines	=	0.0 <i>in</i>

The nozzle is inserted centrally through the head, i.e. set-in type nozzle, see Figure 4.5.9.

Establish the corroded dimensions.

Head:

$$D_{i} = 90.0 + 2(Corrosion \ Allowance) = 90.0 + 2(0.125) = 90.25 \ in$$
$$R = \frac{D_{i}}{2} = \frac{90.25}{2} = 45.125 \ in$$
$$t = 1.0 - Corrosion \ Allowance = 1.0 - 0.125 = 0.875 \ in$$

Nozzle:

$$t_n = 2.28 - Corrosion \ Allowance = 2.28 - 0.125 = 2.155 \ in$$
  
 $R_n = \frac{D - 2(t_n)}{2} = \frac{15.94 - 2(2.155)}{2} = 5.815 \ in$ 

The procedure, per paragraph 4.5.10, to design a radial nozzle in an elliptical head subject to pressure loading is shown below.

Note: This is an analysis of a single nozzle; therefore, the spacing criterion is automatically satisfied. If there were multiple nozzles in the shell, the spacing requirements for nozzles in paragraph 4.5.13 would need to be checked.

a) STEP 1 – Determine the effective radius of the shell or formed head as follows.

For ellipsoidal heads:

$$R_{eff} = \frac{0.9D_i}{6} \left[ 2 + \left(\frac{D_i}{2h}\right)^2 \right] = \frac{0.9(90.25)}{6} \left[ 2 + \left(\frac{90.25}{2(22.625)}\right)^2 \right] = 80.9262 \text{ in}$$

b) STEP 2 – Calculate the limit of reinforcement along the vessel wall.

For set-in, integrally reinforced nozzles in ellipsoidal heads,

$$L_R = \min\left[\sqrt{R_{eff}t}, 2R_n\right] = \min\left[\sqrt{80.9262(0.875)}, 2(5.8150)\right] = 8.4149 \text{ in}$$

c) STEP 3 – Calculate the limit of reinforcement along the nozzle wall projecting outside the vessel surface, see Figures 4.5.9 and 4.5.10.

For set-in nozzles,

$$L_{H} = \min\left[t + t_{e} + F_{p}\sqrt{R_{n}t_{n}}, L_{pr1} + t\right]$$

For ellipsoidal heads,

$$X_{o} = \min\left[D_{R} + (R_{n} + t_{n}) \cdot \cos\left[\theta\right], \frac{D_{i}}{2}\right]$$

$$X_{o} = \min\left[0.0 + (5.8150 + 2.1550) \cdot \cos\left[0.0\right], \frac{90.25}{2}\right] = 7.9700 \text{ in}$$

$$\theta = \arctan\left[\left(\frac{h}{R}\right) \cdot \left(\frac{D_{R}}{\sqrt{R^{2} - D_{R}^{2}}}\right)\right] = \arctan\left[\left(\frac{22.625}{45.125}\right) \cdot \left(\frac{0.0}{\sqrt{45.125^{2} - 0.0^{2}}}\right)\right] = 0.0 \text{ rad}$$

Since  $\{X_o = 7.9700 \ in\} \le \{0.35D_i = 0.35(90.25) = 31.5875 \ in\}$ , calculate  $F_p$  as follows:

$$F_{p} = \left\{ C_{n} = \min\left[\left(\frac{t+t_{e}}{t_{n}}\right)^{0.35}, 1.0\right] = \min\left[\left(\frac{0.875+0.0}{2.1550}\right)^{0.35}, 1.0\right] = 0.7295 \right\}$$
$$L_{H} = \min\left[0.875+0.0+(0.7295)\sqrt{5.8150(2.1550)}, 13.5+0.875\right] = 3.4574 \text{ in}$$

 d) STEP 4 – Calculate the limit of reinforcement along the nozzle wall projecting inside the vessel surface, if applicable.

$$L_{pr2} = 0.0$$
$$L_{I} = \min\left[F_{p}\sqrt{R_{n}t_{n}}, L_{pr2}\right] = 0.0$$

e) STEP 5 – Determine the total available area near the nozzle opening, see Figures 4.5.1, where  $f_{rn}$  and  $f_{rp}$  are given by Equations (4.5.21) and (4.5.22) respectively. Do not include any area that falls outside of the limits defined by  $L_{H}$ ,  $L_{R}$ , and  $L_{I}$ .
For set-in nozzles:

$$A_{T} = A_{1} + f_{rn}(A_{2} + A_{3}) + A_{41} + A_{42} + A_{43} + f_{rp}A_{5}$$
$$A_{1} = tL_{R} = 0.875(8.4149) = 7.3630 \ in^{2}$$

Since  $\{t_n = 2.1550 \ in\} = \{t_{n2} = 2.1550 \ in\}$ , calculate  $A_2$  as follows:

$$A_{2} = t_{n}L_{H} = 2.1550(3.4574) = 7.4507 \ in^{2}$$

$$A_{3} = t_{n}L_{I} = 0.0$$

$$A_{41} = 0.5L_{41}^{2} = 0.5(0.375)^{2} = 0.0703 \ in^{2}$$

$$A_{42} = 0.5L_{42}^{2} = 0.0$$

$$A_{43} = 0.5L_{43}^{2} = 0.0$$

$$t_{e} = 0.0 \ in$$

$$A_{5a} = Wt_{e} = 0.0$$

$$A_{5b} = (L_{R} - t_{n})t_{e} = 0.0$$

$$A_{5} = \min[A_{5a}, A_{5b}] = 0.0$$

$$f_{rn} = \frac{S_{n}}{S} = \frac{21200}{22400} = 0.9464$$

$$f_{rp} = \frac{S_{p}}{S} = 0.0$$

$$A_T = 7.363 + 0.9464 (7.4507 + 0.0) + 0.0703 + 0.0 + 0.0 + 0.0 (0.0) = 14.4846 \text{ in}^2$$

f) STEP 6 – Determine the applicable forces.

For set-in nozzles,

$$f_{N} = PR_{xn}L_{H} = 356(6.8360)(3.4572) = 8413.4972 \ lbs$$

$$R_{xn} = \frac{t_{n}}{\ln\left[1 + \frac{t_{n}}{R_{n}}\right]} = \frac{2.1550}{\ln\left[1 + \frac{2.1550}{5.8150}\right]} = 6.8360 \ in$$

$$f_{S} = \frac{PR_{xs}\left(L_{R} + t_{n}\right)}{2} = \frac{356(81.3629)(8.4149 + 2.1550)}{2} = 153079.5936 \ lbs$$

$$R_{xs} = \frac{t_{eff}}{\ln\left[1 + \frac{t_{eff}}{R_{eff}}\right]} = \frac{0.875}{\ln\left[1 + \frac{0.875}{80.9262}\right]} = 81.3629 \ in$$

$$t_{eff} = t + \left(\frac{A_5 f_{rp}}{L_R}\right) = 0.875 + \left(\frac{0.0}{8.4149}\right) = 0.875 \text{ in}$$
$$f_Y = \frac{PR_{xs}R_{nc}}{2} = \frac{356(81.3629)(5.8150)}{2} = 84216.2969 \text{ lbs}$$

g) STEP 7 – Determine the average local primary membrane stress and the general primary membrane stress at the nozzle intersection.

$$\sigma_{avg} = \frac{\left(f_N + f_S + f_Y\right)}{A_T} = \frac{8413.4972 + 153079.5936 + 84216.2969}{14.4846} = 16963.4914 \ psi$$

$$\sigma_{circ} = \frac{PR_{xs}}{2t_{eff}} = \frac{356(81.3629)}{2(0.875)} = 16551.5385 \ psi$$

h) STEP 8 – Determine the maximum local primary membrane stress at the nozzle intersection.

$$P_{L} = \max\left[\left\{2\sigma_{avg} - \sigma_{circ}\right\}, \sigma_{circ}\right]$$
$$P_{L} = \max\left[\left\{2(16963.4914) - 16551.5385\right\}, 16551.5385\right] = 17375.4443 \ psi$$

i) STEP 9 – The calculated maximum local primary membrane stress should satisfy Equation 4.5.146. If the nozzle is subjected to internal pressure, then the allowable stress,  $S_{allow}$ , is given by Equation 4.5.57. If the nozzle is subjected to external pressure, then the allowable stress is given by Equation 4.5.58.

$$\{P_L = 17375.4443\} \le \{S_{allow} = 1.5SE = 1.5(21200)(1.0) = 31800 \ psi\}$$

j) STEP 10 – Determine the maximum allowable working pressure of the nozzle.

$$P_{\max 1} = \frac{S_{allow}}{\left(\frac{2A_p}{A_T}\right) - \left(\frac{R_{xs}}{2t_{eff}}\right)} = \frac{1.5(21200)(1.0)}{\left(\frac{2(690.1949)}{14.4846}\right) - \left(\frac{81.3629}{2(0.875)}\right)} = 651.5402 \ psi$$

$$A_p = \frac{\left(f_N + f_S + f_Y\right)}{P}$$

$$A_p = \frac{8413.4972 + 153079.5936 + 84216.2969}{356} = 690.1949 \ in^2$$

$$P_{\max 2} = 2S\left(\frac{t}{R_{xs}}\right) = 2(22400)\left(\frac{0.875}{81.3629}\right) = 481.7921 \ psi$$

$$P_{\max 3} = \min\left[P_{\max 1}, P_{\max 2}\right] = \min\left[651.5402, 481.7921\right] = 481.7921 \ psi$$

The nozzle is acceptable because  $P_{max} = 481.7921 \ psi$  is greater than the specified design pressure of 356 psig.

#### Weld Strength Analysis

The procedure to evaluate attachment welds of nozzles in a cylindrical, conical, or spherical shell or formed head subject to pressure loading per paragraph 4.5.14.2 is shown below.

a) STEP 1 – Determine the discontinuity force factor,

For set-in Nozzles:

$$k_{y} = \frac{R_{nc} + t_{n}}{R_{nc}} = \frac{5.8150 + 2.155}{5.8150} = 1.3706$$

Note, for radial nozzles,  $R_{nc} = R_n$ .

b) STEP 2 – Calculate weld length resisting continuity force,

Weld length of nozzle to shell weld, for radial nozzles:

$$L_{\tau} = \frac{\pi}{2} \left( R_n + t_n \right) = \frac{\pi}{2} \left( 5.8150 + 2.155 \right) = 12.5192 \text{ in}$$

Weld length of pad to shell weld, for radial nozzles:

$$L_{\tau p} = \frac{\pi}{2} \left( R_n + t_n + W \right) \qquad \text{Not Applicable}$$

c) STEP 3 – Compute the weld throat dimensions, as applicable.

$$L_{41T} = 0.7071L_{41} = 0.7071(0.375) = 0.2652 \text{ in}$$
$$L_{42T} = 0.7071L_{42} = 0.7071(0.0) = 0.0$$
$$L_{43T} = 0.7071L_{43} = 0.7071(0.0) = 0.0$$

e) STEP 4 – Determine if the weld sizes are acceptable. If the nozzle is integrally reinforced, and the computed shear stress in the weld given by Equation (4.5.182) satisfies Equation (4.5.183), then the design is complete. If the shear stress in the weld does not satisfy Equation (4.5.183), increase the weld size and return to Step 3. For nozzles on heads,  $A_2$  and  $A_3$  are to be calculated using  $F_p = 1.0$ , when computing  $f_{welds}$  using Equation (4.5.184).

From STEP 3 of paragraph 4.5.10, re–calculate the limit of reinforcement along the nozzle wall projecting outside the vessel surface using  $F_p = 1.0$ .

$$L_{H} = \min\left[t + t_{e} + F_{p}\sqrt{R_{n}t_{n}}, L_{pr1} + t\right]$$
  
$$L_{H} = \min\left[0.875 + 0.0 + (1.0)\sqrt{5.8150(2.1550)}, 13.5 + 0.875\right] = 4.4150 \text{ in}$$

Re–calculate the values of  $A_2$  using the new value of  $L_{\!_H}$  .

 $A_2 = t_n L_H = 2.1550(4.4150) = 9.5143 in^2$ 

#### PTB-3-2013

From STEP 4 of paragraph 4.5.10, re–calculate the limit of reinforcement along the nozzle wall projecting inside the vessel surface using  $F_p = 1.0$ . Since the nozzle does not have an internal projection, the value of  $L_I = 0.0$ ; therefore,  $A_3 = 0.0 in^2$ .

$$\tau = \frac{f_{welds}}{L_{\tau} \left( 0.49L_{41T} + 0.6t_{w1} + 0.49L_{43T} \right)}$$
  
$$\tau = \frac{17760.7284}{12.5192 \left( 0.49 \left( 0.2652 \right) + 0.6 \left( 0.875 \right) + 0.49 \left( 0.0 \right) \right)} = 2166.0944 \ psi$$

Where,

$$f_{welds} = \min\left[f_Y k_y, 1.5S_n \left(A_2 + A_3\right), \frac{\pi}{4} P R_n^2 k_y^2\right]$$

$$f_{welds} = \min\left[\left\{219730.4980(1.3706) = 301162.6206\right\}, \\ \left\{1.5(21200)(9.5143 + 0) = 302554.74\right\}, \\ \left\{\frac{\pi}{4}(356)(5.8150)^2(1.3706)^2 = 17760.7284\right\}\right] = 17760.7284 \ lbs$$

 $\{\tau = 2166.1 \ psi\} \le \{S = 22400 \ psi\}$ 

The weld strength is acceptable.



Figure E4.5.3 - Nozzle Details

## 4.6 Flat Heads

## 4.6.1 Example E4.6.1 – Flat Un-stayed Circular Heads

Determine the required thickness for a heat exchanger blind flange.

Blind Flange Data:

•	Material	=	SA-105
•	Design Conditions	=	135 psig @ 650°F
•	Flange Bolt-Up Temperature	=	100°F
•	Corrosion Allowance	=	0.125 in
•	Allowable Stress	=	17800 <i>psi</i>
•	Allowable Stress at Flange Bolt-Up Temp.	=	24000 psi
•	Weld Joint Efficiency	=	1.0

• Mating flange information and gasket details are provided in Example Problem E4.16.1.

Using the procedure in paragraph 4.6.2.3, calculate the minimum required thickness for the heat exchanger blind flange.

The minimum required thickness of a flat unstayed non-circular head, cover, or blind flange that is attached with bolting that results in an edge moment (see Table 4.6.1, Detail 7) shall be calculated by the equations shown below. The operating and gasket seating bolt loads,  $W_o$  and  $W_g$ , and the moment arm of this load,  $h_G$ , in these equations shall be computed based on the flange geometry and gasket material as described in paragraph 4.16.

a) STEP 1 – Calculate the gasket moment arm,  $h_G$ , and the diameter of the gasket load reaction d in accordance with paragraph 4.16, as demonstrated in Example Problem E4.16.1.

Flange Design Procedure, STEP 6:  $h_G = 0.875$  in

Design Bolt Loads, STEP 3: d = G = 29.5 in

b) STEP 2 – Calculate the operating and gasket seating bolt loads,  $W_o$  and  $W_g$ , in accordance with paragraph 4.16, as demonstrated in Example Problem E4.16.1.

Design Bolt Loads, STEP 4:  $W_a = 111329.5 \ lbs$ 

Design Bolt Loads, STEP 5:  $W_g = 237626.3 \ lbs$ 

c) STEP 3 – Identify the appropriate attachment factor, C, from Table 4.6.1.

Per Detail 7: C = 0.3

d) STEP 4 - The required thickness of the blind flange is the maximum of the thickness required for the operating and gasket seating conditions.

$$t = \max\left[t_o, t_g\right]$$

1) The required thickness in the operating condition is in accordance with Equation (4.6.3):

$$t_{o} = d \sqrt{\left(\frac{CP}{S_{ho}E}\right) + \left(\frac{1.9W_{o}h_{G}}{S_{ho}Ed^{3}}\right)} + CA$$
  
$$t_{o} = (29.5) \sqrt{\left(\frac{0.3(135)}{17800(1.0)}\right) + \left(\frac{1.9(111329.5)(0.875)}{17800(1.0)(29.5)^{3}}\right)} + 0.125 = 1.6523 \text{ in}$$

2) The required thickness in the gasket seating condition is in accordance with Equation (4.6.4):

$$t_{g} = d \sqrt{\frac{1.9W_{g}h_{G}}{S_{hg}Ed^{3}}} + CA$$
  
$$t_{g} = (29.5) \sqrt{\frac{1.9(237626.3)(0.875)}{24000(1.0)(29.5)^{3}}} + 0.125 = 0.8720 \text{ in}$$

$$t = \max[1.6523, 0.8720] = 1.6523 in$$

The required thickness is 1.6523 in.

### 4.6.2 Example E4.6.2 – Flat Un-stayed Non-Circular Heads Attached by Welding

Determine the required thickness for an air-cooled heat exchanger end plate. The end plate is welded to the air-cooled heat exchanger box with full penetration Category C, Type 7 corner joints.

End Plate Data:

•	Material	=	<i>SA</i> -516, <i>Grade</i> 70
•	Design Conditions	=	400 psig @ 500°F
•	Short Span Length	=	7.125 in
•	Long Span Length	=	9.25 in
•	Corrosion Allowance	=	0.125 in
•	Allowable Stress	=	20600 psi
•	Weld Joint Efficiency	=	1.0

Using the procedure in paragraph 4.6.3.1, calculate the minimum required thickness for the end plate.

The minimum required thickness of a flat unstayed non-circular head or cover that is not attached with bolting that results in an edge moment shall be calculated by the following equations.

a) STEP 1 – Determine the short and long span dimensions of the non-circular plate, d and D, respectively (in the corroded state) as demonstrated in Example Problem E4.12.1.

$$d = 7.125 + 2(0.125) = 7.375 in$$

$$D = 9.250 + 2(0.125) = 9.500$$
 in

Note, the variables d and D used in paragraph 4.6.3 are denoted as H and h, respectively in paragraph 4.12.

b) STEP 2 – Calculate the Z factor in accordance with Equation (4.6.6).

$$Z = \min\left[2.5, \left(3.4 - \left(\frac{2.4d}{D}\right)\right)\right] = \min\left[2.5, \left(3.4 - \left(\frac{2.4(7.375)}{9.5}\right)\right)\right] = 1.5368 \text{ in}$$

c) STEP 3 - The appropriate attachment factor, C, is taken from paragraph 4.12.2.6. For end closures of non-circular vessels constructed of flat plate, the design rules of paragraph 4.6 shall be used except that 0.20 shall be used for the value of C in all of the calculations.

$$C = 0.2$$

d) STEP 4 - Calculate the required thickness, t using Equation (4.6.5).

$$t = d\sqrt{\frac{ZCP}{S_{ho}E}} + CA = 7.375\sqrt{\frac{1.5368(0.2)(400)}{20600(1.0)}} + 0.125 = 0.6947 \text{ in}$$

The required thickness is 0.6947 in

## 4.7 Spherically Dished Bolted Covers

### 4.7.1 Example E4.7.1 – Thickness Calculation for a Type D Head

Determine if the proposed Type D spherically dished bolted cover is adequately designed, considering the following design conditions. The spherically dished head is seamless. See Figure E4.7.1 for details.

## Tubeside Data:

•	Design Conditions	=	213 psig @ 400°F
•	Corrosion Allowance (CAT)	=	0.125 in
•	Weld Joint Efficiency	=	1.0
<u>Sh</u>	ellside Data:		
•	Design Conditions	=	305 psig @ 250°F
•	Corrosion Allowance (CAS)	=	0.125 in
•	Weld Joint Efficiency	=	1.0
Fla	nge Data:		
•	Material	=	SA-105
•	Allowable Stress at Ambient Temperature	=	24000 psi
•	Allowable Stress at Tubeside Design Temperature	=	20500 psi
•	Allowable Stress at Shellside Design Temperature	=	21600 psi
<u>He</u>	ad Data:		
•	Material	=	<i>SA</i> -515, <i>Grade</i> 60
•	Allowable Stress At Ambient Temperature	=	21300 psi
•	Allowable Stress at Tubeside Design Temperature	=	18200 <i>psi</i>
•	Allowable Stress at Shellside Design Temperature	=	19200 psi
•	Yield Stress at Shellside Design Temperature	=	28800 psi
•	Modulus of Elasticity at Shellside Design Temp.	=	$28.55E + 06 \ psi$
<u>Bo</u>	<u>t Data:</u>		
•	Material	=	SA-193, Grade B7
•	Diameter	=	0.75 in
•	Cross-Sectional Root Area	=	$0.302 in^2$
•	Number of Bolts	=	20
•	Allowable Stress at Ambient Temperature	=	25000 psi
•	Allowable Stress at Tubeside Design Temperature	=	25000 psi
•	Allowable Stress at Shellside Design Temperature	=	25000 psi

### Gasket Data

•	Material	=	Solid Flat Metal (Iron/Soft Steel)
•	Gasket Factor	=	5.5
•	Gasket Seating Factor	=	18000 psi
•	Inside Diameter	=	16.1875 in
•	Outside Diameter	=	17.0625 in

Per paragraph 4.7.1.3, calculations shall be performed using dimensions in the corroded condition and the uncorroded condition, and the more severe case shall control. This example only evaluates the spherically dished bolted cover in the corroded condition.

Per paragraph 4.7.5.1, the thickness of the head for a Type D Head Configuration (see Figure 4.7.4) shall be determined by the following equations.

a) Internal pressure (pressure on the concave side) – the head thickness shall be determined using Equation (4.7.2).

$$t = \left(\frac{5PL}{6S}\right) = \frac{5(213)(16.125)}{6(18200)} = 0.1573 \text{ in}$$

Where,

$$L = 16.0 + CAT = 16.0 + 0.125 = 16.125$$
 in

This thickness is increased for the corrosion allowance on both the shell and tube side.

$$t = t + CAS + CAT = 0.1573 + 0.125 + 0.125 = 0.4073$$
 in

b) External pressure (pressure on the convex side) – the head thickness shall be determined in accordance with the ruled in paragraph 4.4.

Per paragraph 4.4.7.1, the required thickness of a spherical shell or hemispherical head subjected to external pressure loading shall be determined using the following procedure

1) STEP 1 - Assume an initial thickness, *t*, for the spherical shell.

The specified head thickness shall consider corrosion from tubeside and shellside, resulting in the following.

$$t = t - CAS - CAT$$
  
$$t = 0.875 - 0.125 - 0.125 = 0.625 in$$

2) STEP 2 - Calculate the predicted elastic buckling stress,  $F_{he}$ .

$$F_{he} = 0.075E_y \left(\frac{t}{R_o}\right) = 0.075(28.55E + 06)\left(\frac{0.625}{16.75}\right) = 79897.3881 \ psi$$

Where,

$$R_o = L + t = 16.125 + 0.625 = 16.75$$
 in

3) STEP 3 - Calculate the predicted buckling stress,  $F_{ic}$ .

$$\frac{F_{he}}{S_y} = \frac{79897.3881}{28800} = 2.7742$$
  
Since  $1.6 < \frac{F_{he}}{S_y} < 6.25$ , use Equation (4.4.55), to calculate  $F_{ic}$   
$$F_{ic} = \frac{1.31S_y}{\left(1.15 + \frac{S_y}{F_{he}}\right)} = \frac{1.31(28800)}{\left(1.15 + \frac{28800}{79897.3881}\right)} = 24977.7825 \text{ psi}$$

4) STEP 4 - Calculate the value of the design margin, FS per paragraph 4.4.2.

 $0.55S_v = 0.55(28800) = 15840 \ psi$ 

Since  $0.55S_y < F_{ic} < S_y$  , use Equation (4.4.2) to calculate  $\,FS$  .

$$FS = 2.407 - 0.741 \left(\frac{F_{ic}}{S_y}\right) = 2.407 - 0.741 \left(\frac{24977.7825}{28800}\right) = 1.7643$$

5) STEP 5 - Calculate the allowable external pressure,  $P_a$ .

$$P_a = 2F_{ha}\left(\frac{t}{R_o}\right) = 2(14157.3329)\left(\frac{0.625}{16.75}\right) = 1056.5 \ psi$$

Where,

$$F_{ha} = \frac{F_{ic}}{FS} = \frac{24977.7825}{1.7643} = 14157.3329 \ psi$$

6) STEP 6 - If the allowable external pressure,  $P_a$ , is less than the design external pressure, increase the shell thickness and go to STEP 2.

Since  $\{P_a = 1056.5 \ psi\} > \{P = 305 \ psi\}$ , the specified head thickness is acceptable for external pressure.

The flange thickness of the head for a Type D Head Configuration is determined per paragraph 4.7.5.2. To compute the flange thickness calculations, the flange operating and gasket seating moments are determined using the flange design procedure from paragraphs 4.16.6 and 4.16.7.

<u>Paragraph 4.16.6: Design Bolt Loads</u>. The procedure to determine the bolt loads for the operating and gasket seating conditions is shown below.

a) STEP 1 – Determine the design pressure and temperature of the flanged joint.

Tubeside Conditions: P = 213 psig at  $400^{\circ}F$ 

b) STEP 2 – Select a gasket and determine the gasket factors m and y from Table 4.16.1.

m = 5.5 $y = 18000 \ psi$ 

c) STEP 3 – Determine the width of the gasket, N, basic gasket seating width,  $b_o$ , the effective gasket seating width, b, and the location of the gasket reaction, G.

$$N = 0.5(GOD - GID) = 0.5(17.0625 - 16.1875) = 0.4375$$
 in

From Table 4.16.3, Facing Sketch Detail 2, Column I,

$$b_o = \frac{w+N}{4} = \frac{(0.125+0.4375)}{4} = 0.1406 \text{ in}$$

Where,

w = raised nubbin width = 0.125 in

For  $b_o \leq 0.25 \ in$  ,

$$b = b_o = 0.1406$$
 in  
 $G = mean \ diameter \ of \ the \ gasket \ contact \ face$   
 $G = 0.5(17.0625+16.1875) = 16.625$  in

d) STEP 4 – Determine the design bolt load for the operating condition.

$$W_{o} = \frac{\pi}{4}G^{2}P + 2b\pi GmP \qquad for \ non-self-energized \ gaskets$$
$$W_{o} = \frac{\pi}{4}(16.625)^{2}(213) + 2(0.1406)\pi(16.625)(5.5)(213) = 63442.9 \ lbs$$

e) STEP 5 – Determine the design bolt load for the gasket seating condition.

$$W_{g} = \left(\frac{A_{m} + A_{b}}{2}\right)S_{bg} = \left(\frac{5.2872 + 6.04}{2}\right)25000 = 141590.0 \ lbs$$

Where,

$$A_{b} = (Number of bolts)(Root area of one bolt) = (20)(0.302) = 6.04 in^{2}$$

$$A_{m} = \max\left[\left(\frac{W_{o} + F_{A} + \frac{4M_{E}}{G}}{S_{bo}}\right), \left(\frac{W_{gs}}{S_{bg}}\right)\right] = \max\left[\left(\frac{63442.9 + 0 + 0}{25000}\right), \left(\frac{132181.1}{25000}\right)\right]$$

$$A_{m} = \max\left[2.5377, 5.2872\right] = 5.2872 in^{2}$$

Note,  $F_A = 0$  and  $M_E = 0$  since there are no externally applied net-section axial forces or bending moments.

And,

$$W_{gs} = \pi b G(C_{us}y) \qquad for \ non-self-energized \ gaskets$$
$$W_{gs} = \pi (0.1406)(16.625)(1.0(18000)) = 132181.1 \ lbs$$

<u>Paragraph 4.16.7: Flange Design Procedure</u>. The procedure in this paragraph can be used to design circular integral, loose or reverse flanges, subject to internal or external pressure, and external loadings.

a) STEP 1 – Determine the design pressure and temperature of the flanged joint and the external net-section axial force,  $F_A$ , and bending moment,  $M_E$ .

Tubeside Conditions : P = 213 psig at  $400^{\circ}F$ Shellside Conditions : P = 305 psig at  $250^{\circ}F$  $F_A = 0$  $M_E = 0$ 

b) STEP 2 – Determine the design bolt loads for operating condition  $W_o$ , and the gasket seating condition  $W_g$ , and the corresponding actual bolt load area  $A_b$ , from paragraph 4.16.6.

$$W_o = 63442.9 \ lbs$$
  
 $W_g = 141590.0 \ lbs$   
 $A_b = 6.04 \ in^2$ 

- c) STEP 3 Determine an initial flange geometry (see Figure E4.7.1), in addition to the information required to determine the bolt load, the following geometric parameters are required.
  - 1) Flange bore

$$B = [16.25 + 2(CAT)] = [16.25 + 2(0.125)] = 16.50 \text{ in}$$

2) Bolt circle diameter

C = 18.125 in

3) Outside diameter of the flange

$$A = [19.625 - 2(CAS)] = [19.625 - 2(0.125)] = 19.375 in$$

4) Flange thickness, (see Figure E4.7.1)

$$T = 2.375 - 2(CAS) = 2.375 - 2(0.125) = 2.125$$
 in

5) Thickness of the hub at the large end

Not Applicable

6) Thickness of the hub at the small end

Not Applicable

7) Hub length

Not Applicable

- d) STEP 4 Determine the flange stress factors using the equations in Table 4.16.4 and 4.16.5. *Not Applicable*
- e) STEP 5 Determine the flange forces.

**Tubeside Conditions:** 

$$H_{D} = \frac{\pi}{4}B^{2}P = \frac{\pi}{4}(16.5)^{2}(213) = 45544.7 \ lbs$$
$$H = \frac{\pi}{4}G^{2}P = \frac{\pi}{4}(16.625)^{2}(213) = 46237.3 \ lbs$$
$$H_{T} = H - H_{D} = 46237.3 - 45544.7 = 692.6 \ lbs$$
$$H_{G} = W_{o} - H = 63442.9 - 46237.3 = 17205.6 \ lbs$$

Shellside Conditions:

 $H_G = Not Applicable$ 

$$H_{D} = \frac{\pi}{4}B^{2}P = \frac{\pi}{4}(16.5)^{2}(305) = 65216.5 \ lbs$$
$$H = \frac{\pi}{4}G^{2}P = \frac{\pi}{4}(16.625)^{2}(305) = 66208.4 \ lbs$$
$$H_{T} = H - H_{D} = 66208.4 - 65216.5 = 991.9 \ lbs$$

f) STEP 6 – Determine the flange moment for the operating condition using Equation (4.16.14) or Equation (4.16.15), as applicable. When specified by the user or his designated agent, the maximum bolt spacing  $(B_{smax})$  and the bolt spacing correction factor  $(B_{sc})$  shall be applied in calculating the flange moment for internal pressure using the equations in Table 4.16.11. The flange moment  $M_o$  for the operating condition and flange moment  $M_g$  for the gasket seating condition without correction for bolt spacing  $B_{sc} = 1$  is used for the calculation of the rigidity index in Step 10. In these equations,  $h_D$  is determined from Equation (4.7.21), as referenced in paragraph 4.7.5.2, and  $h_T$  and  $h_G$  are determined from Table 4.16.6. Since  $F_A = 0$  and  $M_E = 0$ , the flange cross-section bending moment of inertia, I, and polar moment of inertia,  $I_p$ , need not be calculated; and the flange design moment calculation for net-section bending moment and axial force supplemental loads,  $M_{oe} = 0$ . Additionally,  $F_s = 1.0$  for non-split rings.

For internal pressure (Tubeside Conditions):

$$M_{o} = abs \left[ \left( \left( H_{D}h_{D} + H_{T}h_{T} + H_{G}h_{G} \right) B_{sc} + M_{oe} \right) F_{s} \right]$$
  

$$M_{o} = abs \left[ \left( \left( 45544.7 \left( 0.8125 \right) + 692.6 \left( 0.7813 \right) + 17205.6 \left( 0.75 \right) \right) 1.0 + 0 \right) 1.0 \right]$$
  

$$M_{o} = 50450.4 \ in - lbs$$

For external pressure (Shellside Conditions):

$$M_{o} = abs \left[ \left( \left( H_{D}h_{D} + H_{T}h_{T} + H_{G}h_{G} \right) B_{sc} + M_{oe} \right) F_{s} \right]$$
  

$$M_{o} = abs \left[ \left( \left( 65216.5 \left( 0.8125 - 0.75 \right) + 991.9 \left( 0.7813 - 0.75 \right) \right) 1.0 + 0 \right) 1.0 \right]$$
  

$$M_{o} = 4107.1 \text{ in} - lbs$$

From Equation (4.7.21),

$$h_D = \frac{C-B}{2} = \frac{18.125 - 16.50}{2} = 0.8125$$
 in

From Table 4.16.6, for loose type flanges,

$$h_G = \frac{C-G}{2} = \frac{18.125 - 16.625}{2} = 0.75 \text{ in}$$
$$h_T = \frac{h_D + h_T}{2} = \frac{0.8125 + 0.75}{2} = 0.7813 \text{ in}$$

g) STEP 7 – Determine the flange moment for the gasket seating condition using Equation (4.16.17) or Equation (4.16.18), as applicable.

For internal pressure (Tubeside Conditions):

$$M_g = \frac{W_g (C-G) B_{sc} F_s}{2} = \frac{(141590.0) (18.125 - 16.625) (1.0) (1.0)}{2} = 106192.5 \text{ in - lbs}$$

For external pressure (Shellside Conditions):

$$M_g = W_g h_G F_s = (141590.0)(0.75)(1.0) = 106192.5 \text{ in} - lbs$$

Per paragraph 4.7.5.2, the flange thickness of the head for a Type D Head Configuration shall be determined by the following equations. When determining the flange design moment for the design condition,  $M_o$ , an additional moment term,  $M_r$ , computed using Equation (4.7.22) shall be added to  $M_o$  as defined in paragraph 4.16. The term  $M_{oe}$  in the equation for  $M_o$  as defined in paragraph 4.16. The term  $M_{oe}$  in the equation for  $M_o$  as defined in paragraph 4.16 shall be set to zero in this calculation. Note that this term may be positive or negative depending on the location of the head-to-flange ring intersection with relation to the flange ring centroid. Since the head-to-flange ring intersection is above the flange centroid, the sign of the  $M_r$  value is negative.

$$T = \max\left[T_g, T_o\right] = \max\left[T_g, \max\left[T_{o(tubeside)}, T_{o(shellside)}\right]\right]$$

Where,

$$T_g = \sqrt{\frac{M_g}{S_{fg}B} \left(\frac{A+B}{A-B}\right)} + CAS + CAS$$

$$T_o = Q + \sqrt{Q^2 + \frac{M_o}{S_{fo}B} \left(\frac{A+B}{A-B}\right)} + CAS + CAS$$

And,

$$Q = \frac{\left|P\right|B\sqrt{4L^2 - B^2}}{8S_{fo}\left(A - B\right)}$$

Gasket Seating Conditions:

$$T_g = \sqrt{\left(\frac{106192.5}{(24000)(16.5)}\right) \cdot \left(\frac{(19.375 + 16.5)}{(19.375 - 16.5)}\right)} + 0.125 + 0.125 = 2.0793 \text{ in}$$

Tubeside Conditions:

$$T_{o} = 0.2065 + \sqrt{\left(0.2065\right)^{2} + \left(\frac{30838.6}{(20500)(16.5)}\right) \cdot \left(\frac{(19.375 + 16.5)}{(19.375 - 16.5)}\right)} + 0.125 + 0.125$$
  
$$T_{o} = 1.5429 \text{ in}$$

Where,

$$Q = \frac{|P|B\sqrt{4L^2 - B^2}}{8S_{fo}(A - B)} = \frac{|213|(16.5)\sqrt{(4(16.125)^2 - (16.5)^2)}}{8(20500)(19.375 - 16.5)} = 0.2065$$

$$M_o = M_o \pm M_r F_s = 50450.4 - 19611.8(1.0) = 30838.6 \text{ in} - lbs$$

$$M_r = H_r h_r = 78447.1(0.25) = 19611.8 \text{ in} - lbs$$

$$H_r = (0.785B^2P \cot[\beta]) = \left[(0.785)(16.5)^2(213)\cot[30.1259]\right] = 78447.1 \text{ lbs}$$

$$\beta = \arcsin\left[\frac{B}{2L + t}\right] = \arcsin\left[\frac{(16.5)}{2(16.125) + (0.625)}\right] = \begin{cases} 0.5258 \text{ rad} \\ 30.1259 \text{ deg} \end{cases}$$

$$h_r = \frac{T}{2} - (X - CAS) = \frac{2.125}{2} - (0.9375 - 0.125) = 0.25 \text{ in}$$

Shellside Conditions:

$$T_{o} = 0.2807 + \sqrt{\left(0.2807\right)^{2} + \left(\frac{23975.5}{\left(21600\right)\left(16.5\right)}\right) \cdot \left(\frac{\left(19.375 + 16.5\right)}{\left(19.375 - 16.5\right)}\right)} + 0.125 + 0.125$$
  
$$T_{o} = 1.2389 \text{ in}$$

Where,

$$Q = \frac{|P|B\sqrt{4L^2 - B^2}}{8S_{fo}(A - B)} = \frac{|305|(16.5)\sqrt{(4(16.125)^2 - (16.5)^2)}}{8(21600)(19.375 - 16.5)} = 0.2807$$

$$M_o = M_o \pm M_r F_s = 4107.1 - 28082.6(1.0) = -23975.5 \text{ in} - lbs$$

$$M_r = H_r h_r = 112330.3(0.25) = 28082.6 \text{ in} - lbs$$

$$H_r = (0.785B^2P \cot[\beta]) = \left[(0.785)(16.5)^2(305)\cot[30.1259]\right] = 112330.3 \text{ lbs}$$

Therefore,

$$T = \max[T_g, T_o] = \max[T_g, \max[T_{o(tubeside)}, T_{o(shellside)}]]$$
$$T = \max[2.0793, \max[1.5429, 1.2389]]$$
$$T = 2.0793 in$$

Since the specified head thickness,  $\{t = 0.875 \ in\} > \{t_{req} = 0.4073 \ in\}$ , and the specified flange thickness,  $\{T = 2.375 \ in\} > \{T_{req} = 2.0793 \ in\}$  are shown to be greater than the required thickness for both internal pressure (tubeside conditions) and external pressure (shellside conditions), the proposed Type D spherically dished bolted cover is adequately designed.



Figure E4.7.1 - Spherically Dished Bolted Cover

# 4.7.2 Example E4.7.2 – Thickness Calculation for a Type D Head Using the Alternative Rule in Paragraph 4.7.5.3

Determine if the proposed Type D spherically dished bolted cover is adequately designed, considering the following design conditions. The spherically dished head is seamless. Evaluate using the alternative procedure in paragraph 4.7.5.3.

#### Tubeside Data:

•	Design Conditions	=	213 psig @ 400°F
•	Corrosion Allowance (CAT)	=	0.125 in
•	Weld Joint Efficiency	=	1.0
<u>Sh</u>	ellside Data:		
•	Design Conditions	=	305 psig @ 250°F
•	Corrosion Allowance (CAS)	=	0.125 in
•	Weld Joint Efficiency	=	1.0
<u>Fla</u>	nge Data:		
•	Material	=	SA-105
•	Allowable Stress at Ambient Temperature	=	24000 psi
•	Allowable Stress at Tubeside Design Temperature	=	20500 psi
•	Allowable Stress at Shellside Design Temperature	=	21600 psi
He	ad Data:		
•	Material	=	<i>SA</i> -515, <i>Grade</i> 60
•	Allowable Stress At Ambient Temperature	=	21300 psi
•	Allowable Stress at Tubeside Design Temperature	=	18200 <i>psi</i>
•	Allowable Stress at Shellside Design Temperature	=	19200 psi
•	Yield Stress at Shellside Design Temperature	=	28800 psi
•	Modulus of Elasticity at Shellside Design Temp.	=	$28.55E + 06 \ psi$
<u>Bo</u>	It Data:		
•	Material	=	<i>SA</i> –193, <i>Grade B</i> 7
•	Diameter	=	0.75 in
•	Cross-Sectional Root Area	=	$0.302 in^2$
•	Number of Bolts	=	20
•	Allowable Stress at Ambient Temperature	=	25000 psi
•	Allowable Stress at Tubeside Design Temperature	=	25000 psi
•	Allowable Stress at Shellside Design Temperature	=	25000 psi

#### Gasket Data

•	Material	=	Solid Flat Metal (Iron/Soft Steel)
•	Gasket Factor	=	5.5
•	Gasket Seating Factor	=	18000 <i>psi</i>
•	Inside Diameter	=	16.1875 in
•	Outside Diameter	=	17.0625 in

Per paragraph 4.7.5.3, as an alternative to the rules in paragraph 4.7.5.1 and 4.7.5.2, the following procedure can be used to determine the required head and flange thickness of a Type D head. This procedure accounts for the continuity between the flange ring and the head, and represents a more accurate method of analysis.

a) STEP 1 – Determine the design pressure and temperature of the flange joint. If the pressure is negative, a negative value must be used for P in all of the equations of this procedure, and

$$P_e = 0.0$$
for internal pressure $P_e = P$ for external pressureTubeside Conditions :  $P = 213$  psig at 400°FShellside Conditions :  $P = 305$  psig at 250°F

- b) STEP 2 Determine an initial Type D head configuration geometry (see Figure E4.7.1). The following geometry parameters are required.
  - 1) Flange bore

$$B = [16.25 + 2(CAT)] = [16.25 + 2(0.125)] = 16.50 \text{ in}$$

2) Bolt circle diameter

C = 18.125 in

3) Outside diameter of the flange

$$A = [19.625 - 2(CAS)] = [19.625 - 2(0.125)] = 19.375$$
 in

4) Flange thickness, (see Figure E4.7.1)

$$T = T - 2(CAS) = 2.375 - 2(0.125) = 2.125$$
 in

5) Mean head radius, (see Figure 4.7.5)

$$R = \frac{(L + t_{uncorroded} - CAS) + (L + CAT)}{2}$$
$$R = \frac{(16.0 + 0.875 - 0.125) + (16.0 + 0.125)}{2} = 16.4375 \text{ in}$$

6) Head thickness

$$t = t - CAT - CAS = 0.875 - 0.125 - 0.125 = 0.625$$
 in

7) Inside depth of flange to the base of the head, (see Figure 4.7.5) q = q - CAS = 1.0762 - 0.125 = 0.9512 in c) STEP 3 – Select a gasket configuration and determine the location of the gasket reaction, G, and the design bolt loads for the gasket seating,  $W_g$ , and operating conditions,  $W_o$ , using the rules of paragraph 4.16. Computations for the following parameters are shown in E4.7.1.

$$G = 16.625 \text{ in}$$
  
 $W_g = 141590.0 \text{ lbs}$   
 $W_{gs} = 132181.1 \text{ lbs}$   
 $W_o = 63442.9 \text{ lbs}$ 

d) STEP 4 – Determine the geometry parameters

$$h_{1} = \frac{(C-G)}{2} = \frac{(18.125 - 16.625)}{2} = 0.75 \text{ in}$$

$$h_{2} = \frac{(G-B)}{2} = \frac{(16.625 - 16.5)}{2} = 0.0625 \text{ in}$$

$$d = \frac{(A-B)}{2} = \frac{(19.375 - 16.5)}{2} = 1.4375 \text{ in}$$

$$n = \frac{T}{t} = \frac{2.125}{0.625} = 3.4$$

$$K = \frac{A}{B} = \frac{19.375}{16.5} = 1.1742$$

$$\phi = \arcsin\left[\frac{B}{2R}\right] = \arcsin\left[\frac{16.5}{2(16.4375)}\right] = \begin{cases} 0.5258 \text{ rad} \\ 30.1259 \text{ deg} \end{cases}$$

$$e = q - \frac{1}{2} \left[T - \frac{t}{\cos[\phi]}\right] = 0.9512 - \frac{1}{2} \left[2.125 - \frac{0.625}{\cos[30.1259]}\right] = 0.25 \text{ in}$$

$$k_{1} = 1 - \left(\frac{1-2\nu}{2\lambda}\right) \cot[\phi] = 1 - \left[\frac{1-2(0.3)}{2(6.5920)}\right] \cot[30.1259] = 0.9477$$

$$k_{2} = 1 - \left(\frac{1+2\nu}{2\lambda}\right) \cot[\phi] = 1 - \left[\frac{1+2(0.3)}{2(6.5920)}\right] \cot[30.1259] = 0.7907$$

Where,

$$\lambda = \left[ 3\left(1 - v^2\right) \left(\frac{R}{t}\right)^2 \right]^{0.25} = \left\{ 3\left(1 - 0.3^2\right) \left(\frac{16.4375}{0.625}\right)^2 \right\}^{0.25} = 6.5920$$

e) STEP 5 – Determine the shell discontinuity geometry factors

$$C_{1} = \frac{0.275n^{3}t \cdot \ln[K]}{k_{1}} - e = \left(\frac{0.275(3.4)^{3}(0.625) \cdot \ln[1.1742]}{0.9477}\right) - (0.25) = 0.8947$$

$$C_{2} = \frac{1.1\lambda n^{3}t \ln[K]}{Bk_{1}} + 1 = \left(\frac{1.1(6.5920)(3.4)^{3}(0.625) \cdot \ln[1.1742]}{(16.5)(0.9477)}\right) + 1 = 2.8293$$

$$C_{4} = \frac{\lambda \sin[\phi]}{2} \left(k_{2} + \frac{1}{k_{1}}\right) + \frac{B}{4nd} + \frac{1.65e}{tk_{1}}$$

$$C_{4} = \left[\frac{(6.5920)\sin[30.1259]}{2} \left(0.7907 + \frac{1}{0.9477}\right) + \frac{1}{0.9477}\right] = 4.5940$$

$$C_{5} = \frac{1.65}{tk_{1}} \left(1 + \frac{4\lambda e}{B}\right) = \left(\frac{1.65}{(0.625)(0.9477)}\right) \left(1 + \frac{4(6.5920)(0.25)}{(16.5)}\right) = 3.8986$$

- f) STEP 6 Determine the shell discontinuity load factors for the operating and gasket seating conditions.
  - Operating Condition Tubeside:

$$\begin{split} C_{3o} &= \frac{\pi B^2 P}{4} \bigg[ e \cot[\phi] + \frac{2q(T-q)}{B} - h_2 \bigg] - W_o h_1 \\ C_{3o} &= \bigg( \frac{\pi (16.5)^2 (213)}{4} \bigg) \bigg[ \frac{(0.25) \cot[30.1259] +}{2(0.9512)(2.125 - 0.9512)} - 0.0625 \bigg] - 63442.9(0.75) \\ C_{3o} &= -24643.1908 \ in - lbs \\ C_{6o} &= \frac{\pi B^2 P}{4} \bigg( \frac{4q - B \cot[\phi]}{4nd} - \frac{0.35}{\sin[\phi]} \bigg) \\ C_{6o} &= \frac{\pi (16.5)^2 (213)}{4} \bigg( \frac{4(0.9512) - (16.5) \cot[30.1259]}{4(3.4)(1.4375)} - \frac{0.35}{\sin[30.1259]} \bigg) \\ C_{6o} &= -89138.7025 \ lbs \end{split}$$

Operating Condition - Shellside:

$$C_{3o} = \frac{\pi B^2 P}{4} \left[ e \cot[\phi] + \frac{2q(T-q)}{B} - h_2 \right] - W_o h_1$$

For shellside pressure (external pressure) the operating bolt load,  $W_o$ , is set equal to the bolt load required to seat the gasket, defined as  $W_{gs}$ . See STEP 3.

$$C_{3o} = \left(\frac{\pi (16.5)^{2} (-305)}{4}\right) \left[\frac{(0.25) \cot [30.1259] +}{2(0.9512)(2.125 - 0.9512)} - 0.0625\right] - 132181.1(0.75)$$

$$C_{3o} = -131982.7272 \text{ in} - lbs$$

$$C_{6o} = \frac{\pi B^{2} P}{4} \left(\frac{4q - B \cot [\phi]}{4nd} - \frac{0.35}{\sin [\phi]}\right)$$

$$C_{6o} = \frac{\pi (16.5)^{2} (-305)}{4} \left(\frac{4(0.9512) - (16.5) \cot [30.1259]}{4(3.4)(1.4375)} - \frac{0.35}{\sin [30.1259]}\right)$$

$$C_{6o} = 127639.9260 \text{ lbs}$$

Gasket Seating Condition:

$$C_{3g} = -W_g h_1 = -(141590.0)(0.75) = -106192.5 \text{ in} - lbs$$
$$C_{6g} = 0.0$$

g) STEP 7 – Determine the shell discontinuity force and moment for the operating and gasket condition.

Operating Condition - Tubeside:

$$\begin{split} V_{do} &= \frac{C_2 C_{6o} - C_{3o} C_5}{C_2 C_4 - C_1 C_5} \\ V_{do} &= \frac{\left(2.8293 \left(-89138.7025\right)\right) - \left(-24643.1908 \left(3.8986\right)\right)}{\left(2.8293 \left(4.5940\right)\right) - \left(0.8947 \left(3.8986\right)\right)} = -16417.5261 \, lbs \\ M_{do} &= \frac{C_1 C_{6o} - C_{3o} C_4}{C_2 C_4 - C_1 C_5} \\ M_{do} &= \frac{\left(0.8947 \left(-89138.7025\right)\right) - \left(-24643.1908 \left(4.5940\right)\right)}{\left(2.8293 \left(4.5940\right)\right) - \left(0.8947 \left(3.8986\right)\right)} = 3518.3368 \, in - lbs \end{split}$$

Operating Condition - Shellside:

$$\begin{split} V_{do} &= \frac{C_2 C_{6o} - C_{3o} C_5}{C_2 C_4 - C_1 C_5} \\ V_{do} &= \frac{\left(2.8293 (127639.9260)\right) - \left(-131982.7272 (3.8986)\right)}{\left(2.8293 (4.5940)\right) - \left(0.8947 (3.8986)\right)} = 92082.5091 \, lbs \\ M_{do} &= \frac{C_1 C_{6o} - C_{3o} C_4}{C_2 C_4 - C_1 C_5} \\ M_{do} &= \frac{\left(0.8947 (127639.9260)\right) - \left(-131982.7272 (4.5940)\right)}{\left(2.8293 (4.5940)\right) - \left(0.8947 (3.8986)\right)} = 75767.4860 \, in - lbs \end{split}$$

Gasket Seating Condition:

$$V_{dg} = \frac{C_2 C_{6g} - C_{3g} C_5}{C_2 C_4 - C_1 C_5} = \frac{(2.8293(0.0)) - (-106192.5(3.8986))}{(2.8293(4.5940)) - (0.8947(3.8986))} = 43534.5925 \ lbs$$

$$M_{dg} = \frac{C_1 C_{6g} - C_{3g} C_4}{C_2 C_4 - C_1 C_5} = \frac{(0.8947(0.0)) - (-106192.5(4.5940))}{(2.8293(4.5940)) - (0.8947(3.8986))} = 51299.9328 \ in - lbs$$

h) STEP 8 – Calculate the stresses in the head and at the head to flange junction using Table 4.7.1 and check the stress criteria for both the operating and gasket conditions.

Calculated Stresses - Operating Conditions - Tubeside:

$$S_{hm} = \frac{PR}{2t} + P_e = \frac{213(16.4375)}{2(0.625)} + 0.0 = 2801.0 \text{ psi}$$

$$S_{hl} = \frac{PR}{2t} + \frac{V_{do} \cos[\phi]}{\pi Bt} + P_e$$

$$S_{hl} = \frac{213(16.4375)}{2(0.625)} + \frac{(-16417.5261)\cos[30.1259]}{\pi (16.5)(0.625)} + 0.0 = 2362.6 \text{ psi}$$

$$S_{hb} = \frac{6M_{do}}{\pi Bt^2} = \frac{6(3518.3368)}{\pi (16.5)(0.625)^2} = 1042.5 \text{ psi}$$

$$S_{hlbi} = S_{hl} - S_{hb} = 2362.6 - 1042.5 = 1320.1 \text{ psi}$$

$$S_{hlbo} = S_{hl} + S_{hb} = 2362.6 + 1042.5 = 3405.1 \text{ psi}$$

$$S_{fm} = \frac{1}{\pi BT} \left( \frac{\pi B^2 P}{4} \left( \frac{4q}{B} - \cot[\phi] \right) - V_{do} \right) \left( \frac{K^2 + 1}{K^2 - 1} \right) + P_e$$

$$S_{fm} = \left( \frac{1}{\pi (16.5)(2.125)} \right) \left( \frac{\left( \frac{\pi (16.5)^2 (213)}{4} \right)}{(16.5) - \cot[30.1259]} - \left( \frac{\left( \frac{1.1742}{(1.1742)^2 + 1} \right) + 0.0}{(-16417.5261)} \right) \right)$$

$$\begin{split} S_{fm} &= -2940.2 \ psi \\ S_{fb} &= \frac{0.525n}{Btk_1} \left( V_{do} - \frac{4M_{do}\lambda}{B} \right) \\ S_{fb} &= \frac{0.525(3.4)}{(16.5)(0.625)(0.9477)} \left( (-16417.5261) - \frac{4(3518.3368)(6.5920)}{(16.5)} \right) = -4025.5 \ psi \\ S_{fmbo} &= S_{fm} - S_{fb} = -2940.2 - (-4025.5) = 1085.3 \ psi \\ S_{fmbi} &= S_{fm} + S_{fb} = -2940.2 - (-4025.5) = -6965.7 \ psi \end{split}$$

Acceptance Criteria – Operating Conditions – Tubeside:

$$\{S_{hm} = 2801.0 \ psi\} \le \{S_{ho} = 18200 \ psi\}$$
 True

$$\{S_{hl} = 2362.6 \ psi\} \le \{1.5S_{ho} = 1.5(18200) = 27300 \ psi\}$$
True

$$\begin{cases} S_{hlbi} = 1320.1 \ psi \\ S_{hlbo} = 3405.1 \ psi \end{cases} \le \{1.5S_{ho} = 1.5(18200) = 27300 \ psi \}$$
True

$$\left\{ S_{fm} = -2940.2 \ psi \right\} \le \left\{ S_{fo} = 20500 \ psi \right\}$$
True

$$\begin{cases} S_{finbo} = 1085.3 \ psi \\ S_{finbi} = -6965.7 \ psi \end{cases} \le \{1.5S_{fo} = 1.5(20500) = 30750 \ psi \}$$
True

Calculated Stresses - Operating Conditions - Shellside:

$$S_{hm} = \frac{PR}{2t} + P_e = \frac{(-305)(16.4375)}{2(0.625)} + (-305) = -4315.8 \text{ psi}$$

$$\begin{split} S_{hl} &= \frac{PR}{2t} + \frac{V_{do} \cos[\phi]}{\pi Bt} + P_e \\ S_{hl} &= \frac{(-305)(16.4375)}{2(0.625)} + \frac{(92082.5091)\cos[30.1259]}{\pi(16.5)(0.625)} + (-305) = -1857.4 \ psi \\ S_{hb} &= \frac{6M_{do}}{\pi Bt^2} = \frac{6(75767.4860)}{\pi(16.5)(0.625)^2} = 22451.2 \ psi \\ S_{hbbo} &= S_{hl} - S_{hb} = -1857.4 - 22451.2 = -24308.6 \ psi \\ S_{hbbo} &= S_{hl} - S_{hb} = -1857.4 - 22451.2 = 20593.8 \ psi \\ S_{fibo} &= S_{hl} - S_{hb} = -1857.4 - 22451.2 = 20593.8 \ psi \\ S_{fibo} &= S_{hl} - S_{hb} = -1857.4 - 22451.2 = 20593.8 \ psi \\ S_{fib} &= \frac{1}{\pi BT} \left( \frac{\pi B^2 P}{4} \left( \frac{4q}{B} - \cot[\phi] \right) - V_{do} \right) \left( \frac{K^2 + 1}{K^2 - 1} \right) + P_e \\ &= \left( \frac{1}{\pi(16.5)(2.125)} \right) \left( \frac{4(0.9512)}{(16.5)} - \cot[30.1259] \right) - \left( \frac{(1.1742)^2 + 1}{(1.1742)^2 - 1} \right) + (-305) \\ (92082.5091) \\ S_{fim} &= -4.7 \ psi \\ S_{fib} &= \frac{0.525n}{Btk_1} \left( V_{do} - \frac{4M_{do}\lambda}{B} \right) \\ S_{fib} &= \frac{0.525(3.4)}{(16.5)(0.625)(0.9477)} \left( 92082.5091 - \frac{4(75767.4860)(6.5920)}{(16.5)} \right) = -5296.4 \ psi \end{split}$$

$$S_{finbo} = S_{fin} - S_{fb} = (-4.7) - (-5296.4) = 5291.7 \ psi$$
$$S_{finbi} = S_{fin} + S_{fb} = (-4.7) + (-5296.4) = -5301.1 \ psi$$

Acceptance Criteria – Operating Conditions – Shellside:

$$\{S_{hm} = -4315.8 \ psi\} \le \{S_{ho} = 18200 \ psi\}$$
True

$$\{S_{hl} = -1857.4 \ psi\} \le \{1.5S_{ho} = 1.5(18200) = 27300 \ psi\}$$
True

$$\begin{cases} S_{hlbi} = -24308.6 \ psi \\ S_{hlbo} = 20593.8 \ psi \end{cases} \le \{1.5S_{ho} = 1.5(18200) = 27300 \ psi \}$$
True
$$\{S_{fm} = -4.7 \ psi \} \le \{S_{fo} = 20500 \ psi \}$$
True

### PTB-3-2013

$$\begin{cases} S_{fmbo} = 5291.7 \ psi \\ S_{fmbi} = -5301.1 \ psi \end{cases} \le \{1.5S_{fo} = 1.5(20500) = 30750 \ psi \}$$
True

Calculated Stresses – Gasket Seating Conditions:

$$\begin{split} S_{hm} &= 0.0 \\ S_{hl} &= \frac{V_{dg} \cos[\phi]}{\pi Bt} = \frac{(43534.5925)\cos[30.1259]}{\pi(16.5)(0.625)} = 1162.2 \ psi \\ S_{hb} &= \frac{6M_{dg}}{\pi Bt^2} = \frac{6(51299.9328)}{\pi(16.5)(0.625)^2} = 15201.1 \ psi \\ S_{hlbi} &= S_{hl} - S_{hb} = 1162.2 - 15201.1 = -14038.9 \ psi \\ S_{hlbo} &= S_{hl} + S_{hb} = 1162.2 + 15201.1 = 16363.3 \ psi \\ S_{fm} &= \frac{1}{\pi BT} \left( -V_{dg} \right) \left( \frac{K^2 + 1}{K^2 - 1} \right) \\ S_{fm} &= \left( \frac{1}{\pi(16.5)(2.125)} \right) \left( -43534.5925 \right) \left( \frac{(1.1742)^2 + 1}{(1.1742)^2 - 1} \right) = -2482.2 \ psi \\ S_{fb} &= \frac{0.525n}{Btk_1} \left( V_{dg} - \frac{4M_{dg}\lambda}{B} \right) \\ S_{fb} &= \frac{0.525(3.4)}{(16.5)(0.625)(0.9477)} \left( 43534.5925 - \frac{4(51299.9328)(6.5920)}{(16.5)} \right) = -7021.9 \ psi \\ S_{fmbo} &= S_{fm} - S_{fb} = -2482.2 - (-7021.9) = 4539.7 \ psi \\ S_{fmbi} &= S_{fm} + S_{fb} = -2482.2 + (-7021.9) = -9504.1 \ psi \end{split}$$

Acceptance Criteria – Gasket Seating Conditions:

$$\{S_{hm} = 0.0 \ psi\} \le \{S_{hg} = 18200 \ psi\}$$
 True

$$\{S_{hl} = 1162.2 \ psi\} \le \{1.5S_{hg} = 1.5(21300 \ psi) = 31950 \ psi\}$$
True

$$\begin{cases} S_{hlbi} = -14038.9 \ psi \\ S_{hlbo} = 16363.3 \ psi \end{cases} \le \{1.5S_{hg} = 1.5(21300 \ psi) = 31950 \ psi \} \quad True$$

$$\left\{S_{fm} = -2482.2 \ psi\right\} \le \left\{S_{fg} = 24000 \ psi\right\}$$
True

## PTB-3-2013

$$\begin{cases} S_{finbo} = 4539.7 \ psi \\ S_{finbi} = -9504.1 \ psi \end{cases} \le \{1.5S_{fg} = 1.5(24000) = 36000 \ psi \}$$
True

Since the calculated stresses in both the head and flange ring are shown to be within the acceptance criteria, for both internal pressure (tubeside conditions) and external pressure (shellside conditions), and gasket seating, the proposed Type D spherically dished bolted cover is adequately designed.

## 4.8 Quick-Actuating (Quick Opening) Closures

#### 4.8.1 Example E4.8.1 – Review of Requirements for Quick-Actuating Closures

A plant engineer is tasked with developing a design specification for an air filter vessel to be equipped with a quick-actuating closure that is to be constructed in accordance with ASME B&PV Code, Section VIII, Division 2 (VIII-2). As part of developing the design specification, the following items need to be considered.

a) Scope

Specific calculation methods are not given in paragraph 4.8. However, both general and specific design requirements are provided.

b) General Design Requirements

Quick-actuating closures shall be designed such that:

- 1) The locking elements will be engaged prior to or upon application of the pressure and will not disengage until the pressure is released.
- 2) The failure of a single locking component while the vessel is pressurized will not:
  - i) Cause or allow the closure to be opened or leaked; or
  - ii) Result in the failure of any other locking component or holding element; or
  - iii) Increase the stress in any other locking or holding element by more than 50% above the allowable stress of the component.
- 3) All locking components can be verified to be fully engaged by visual observation or other means prior to application if pressure to the vessel.
- 4) When installed:
  - i) It may be determined by visual external observation that the holding elements are in satisfactory condition.
  - ii) All vessels shall be provided with a pressure-indicating device visible from the operating area and suitable to detect pressure at the closure.
- c) Specific Design Requirements

Quick-actuating closures that are held in position by positive locking devices and that are fully released by partial rotation or limited movement of the closure itself or the locking mechanism and any closure that is other than manually operates shall be so designed that when the vessel is installed the following conditions are met:

- 1) The closure and its holding elements are fully engaged in their intended operating position before pressure can be applied in the vessel.
- 2) Pressure tending to force the closure open or discharge the contents clear of the vessel shall be released before the closure can be fully opened for access.

The designer shall consider the effects of cyclic loading, other loadings, and mechanical wear on the holding and locking components.

d) Alternative Designs for Manually Operated Closures

Quick-actuating closures that are held in position by a locking mechanism designed for manual operation shall be designed such that if an attempt is made to open the closure when the vessel is under pressure, the closure will leak prior to full disengagement of the locking components and release of the closure. Any leakage shall be directed away from the normal position of the operator.

## e) Supplementary Requirements

Annex 4.B provides additional design information for the Manufacturer and provides installation.

### 4.9 Braced and Stayed Surfaces

#### 4.9.1 Example E4.9.1 – Braced and Stayed Surfaces

Determine the required thickness for a flat plate with welded staybolts considering the following design condition. Verify that the welded staybolts are adequately designed. See Figure E4.9.1

Vessel Data:

•	Plate Material	=	<i>SA</i> -516, <i>Grade</i> 70
•	Design Conditions	=	100 psig@300°F
•	Staybolt Material	=	<i>SA</i> -675, <i>Grade</i> 70
•	Staybolt Diameter	=	1.5 <i>in</i>
•	Corrosion Allowance	=	0.0 <i>in</i>
•	Allowable Stress Plate Material	=	22400 psi@300°F
•	Allowable Stress Staybolt Material	=	20600 psi@300°F
•	Staybolt Pattern	=	Equilateral Triangle
•	Staybolt Pitch	=	$p_s = p_{horizontal} = p_{diagonal} = 15.0$ in

Using the procedure in paragraph 4.9, calculate the required thickness of the flat plate, the load carried by each staybolt, and the required diameter of the staybolt.

Paragraph 4.9.2, the minimum required thickness for braced and stayed flat plates and those parts that, by these rules, require staying as flat plates or staybolts of uniform diameter symmetrically spaced, shall be calculated by the following equation.

Assume, C = 2.2 from Table 4.9.1 with the Welded Staybolt Construction per Figure 4.9.1 Detail (c).

$$t = p_s \sqrt{\frac{P}{SC}} = 15.0 \sqrt{\frac{100.0}{22400(2.2)}} = 0.6757 \text{ in}$$

Paragraph 4.9.3, the required area of a staybolt or stay as its minimum cross section, usually located at the root of the thread, exclusive of any corrosion allowance, shall be obtained by dividing the load on the staybolt computed in accordance with paragraph 4.9.3.2 by the allowable tensile stress value for the staybolt material, multiplying the result by 1.10.

The area supported by a staybolt or stay shall be computed on the basis of the full pitch dimensions, with a deduction for the area occupied by the stay. The load carried by a stay is the product of the area supported by the stay and the maximum allowable working pressure.

a) The area of the flat plate supported by the staybolt,  $A_p$ , is calculated as follows.

$$A_{p} = (p_{horizontal} \cdot p_{diagonal} \cdot \cos[\theta]) - A_{sb} = (15.0(15.0) \cdot \cos[30]) - 1.7671 = 193.0886 \ in^{2}$$

Where,

$$\theta = 30 \ deg,$$
 See Figure E4.9.1  
 $A_{sb} = \frac{\pi}{4} (1.5)^2 = 1.7671 \ in^2$ 

b) The load carried by the staybolt,  $L_{sb}$ , is calculated as follows.

$$L_{sb} = A_n \cdot P = 193.0886(100.0) = 19308.9 \ lbs$$

c) The required area of the staybolt,  $A_{\rm rsb}$ , is calculated as follows.

$$A_{rsb} = 1.10 \left(\frac{L_{sb}}{S_{sb}}\right) = 1.10 \left(\frac{19308.9}{20600}\right) = 1.0311 \text{ in}^2$$

Since  $\{A_{sb} = 1.7671 \ in^2\} > \{A_{rsb} = 1.0311 \ in^2\}$ , the staybolt is adequately designed.

Paragraph 4.9.4.1, welded-in staybolts may be use provided the following requirements are satisfied.

d) The configuration is in accordance with the typical arrangements shown in Figure 4.9.1.

Construction per Figure 
$$4.9.1(c)$$
 satisfied

e) The required thickness of the plate shall not exceed 38 mm (1.5 in).

$$t \le 1.5$$
 in  $t = 0.6757$  in satisfied

f) The maximum pitch shall not exceed 15 times the diameter of the staybolt.

$$p_s \le 15(d_{sb})$$
 15.0  $\le 15(1.5) = 22.5$  in satisfied

g) The size of the attachment welds is not less than that shown in Figure 4.9.1.

Full Penetration Weld per Figure 
$$4.9.1(c)$$
 satisfied

h) The allowable load on the welds shall not exceed the product of the weld area (based on the weld dimension parallel to the staybolt), the allowable tensile stress of the material being welded, and a weld joint factor of 60%.

$$\{L_{sb} = 19308.9 \ lbs\} \le \{L_a = 39356.2 \ lbs\}$$
 satisfied

Where,

$$L_a = E(t \cdot \pi d_{sb})S_{sb} = 0.6(0.6757 \cdot (\pi(1.5)))20600 = 39356.2 \ lbs$$





### 4.10 Ligaments

#### 4.10.1 Example E4.10.1 – Ligaments

Determine the ligament efficiency and corresponding efficiency to be used in the design equations of paragraph 4.3 for a group of tube holes in a cylindrical shell as shown in Figure E4.10.1.

Using the procedure in paragraph 4.10, calculate the ligament efficiency for the group of tube holes. As shown in Figure E4.10.1, three ligaments are produced; longitudinal, circumferential, and diagonal.

Paragraph 4.10.2.1.c, when the adjacent longitudinal rows are drilled as described in paragraph (b), diagonal and circumferential ligaments shall also be examined. The least equivalent longitudinal ligament efficiency shall be used to determine the minimum required thickness and the maximum allowable working pressure.

Considering only pressure loading, the circumferential ligament can be half as strong as the longitudinal ligament. This is because the circumferential ligament is subject to longitudinal stress which is essentially half of circumferential stress. By inspection, the circumferential ligament is greater than the longitudinal ligament and thus will the not govern the design. Therefore, the circumferential ligament efficiency is not explicitly calculated.

Paragraph 4.10.2.1.d, when a cylindrical shell is drilled for holes so as to form diagonal ligaments, as shown in Figure E4.10.1, the efficiency of these ligaments shall be determined by paragraph 4.10, Figures 4.10.5 or 4.10.6. Figure 4.10.5 is used when either or both longitudinal and circumferential ligaments exist with diagonal ligaments. The procedure to determine the ligament efficiency is as follows.

a) STEP 1 – Compute the value of  $p^*/p_1$ .

Diagonal Pitch,  $p^* = 3.75$  in

Unit Length of Ligament,  $p_1 = 4.5$  in

$$\frac{p^*}{p_1} = \frac{3.75}{4.5} = 0.8333$$

b) STEP 2 – Compute the efficiency of the longitudinal ligament in accordance with Figure 4.10.5, Note 4.

$$E_{long} = 100 \left(\frac{p_1 - d}{p_1}\right) = 100 \left(\frac{4.5 - 2.25}{4.5}\right) = 50\%$$

Where,

Diameter of Tube Holes, d = 2.25 in

c) STEP 3 – Compute the diagonal efficiency in accordance with Figure 4.10.5, Note 2.

$$\begin{split} E_{diag} &= \frac{J + 0.25 - \left(1 - 0.01 \cdot E_{long}\right) \sqrt{0.75 + J}}{0.00375 + 0.005J} \\ E_{diag} &= \frac{0.6944 + 0.25 - \left(1 - 0.01(50)\right) \sqrt{(0.75 + 0.6944)}}{0.00375 + 0.005(0.6944)} = 47.56\% \end{split}$$

Where, 
$$J = \left(\frac{p^*}{p_1}\right)^2 = \left(\frac{3.75}{4.5}\right)^2 = 0.6944$$

Alternatively, STEP 3 can be replaced with the following procedure.

STEP 3 (Alternate) – Enter Figure 4.10.5 at the vertical line corresponding to the value of the longitudinal efficiency,  $E_{long}$ , and follow this line vertically to the point where it intersects the diagonal line representing the ratio of the value of  $p^*/p_1$ . Then project this point horizontally to the left, and read the diagonal efficiency of the ligament on the scale at the edge of the diagram.

$$E_{diag} \approx 47.5\%$$

d) STEP 4 – The minimum shell thickness and the maximum allowable working pressure shall be based on the ligament that has the lower efficiency.

$$E = \min\left[E_{long}, E_{diag}\right] = \min\left[50\%, 47.5\%\right] = 47.5\%$$

Paragraph 4.10.3, when ligaments occur in cylindrical shells made from welded pipe or tubes and their calculated efficiency is less than 85% (longitudinal) or 50% (circumferential), the efficiency to be used in paragraph 4.3 to determine the minimum required thickness is the calculated ligament efficiency. In this case, the appropriate stress value in tension may be multiplied by the factor 1.18.



• All Finished Hole Diameters are 2.25 in.

Figure E4.10.1 - Installation Pattern

## 4.11 Jacketed Vessels

## 4.11.1 Example E4.11.1 – Jacketed Vessel

Design a jacketed vessel to be installed on the outside diameter of a section of a tower in accordance with Figure 4.11.1, Type 1.

Vessel Data:

•	Material	=	<i>SA</i> –516, <i>Grade</i> 70
•	Design Conditions	=	350 psig@300°F
•	Vessel ID	=	90.0 in
•	Nominal Thickness	=	1.125 in
•	Allowable Stress	=	22400 psi
•	Corrosion Allowance	=	0.125 in
•	Weld Joint Efficiency	=	1.0
Jac	ket Data:		
•	Jacket Type	=	Figure 4.11.1, Type 1
•	Material	=	<i>SA</i> –516, <i>Grade</i> 70
•	Design Conditions	=	150 psig@400°F
•	Yield Stress at Design Temperature	=	32500 <i>psi</i>
•	Minimum Ultimate Tensile Strength	=	70000 <i>psi</i>
•	Jacket ID	=	96.0 in
•	Allowable Stress	=	22400 psi
•	Corrosion Allowance	=	0.125 in
•	Weld Joint Efficiency	=	1.0

Notes:

1. Jacket closure will be made using closure members in per Table 4.11.2, Detail 6.

2. Full penetration welds will be used in the closure, satisfying paragraph 4.11.3.2.

Establish the corroded dimensions.

$$\begin{split} R_{j} &= 48.0 + Corrosion \ Allowance = 48.0 + 0.125 = 48.125 \ in \\ ID \ of \ Jacket = 2(48.125) = 96.25 \ in \\ OD \ of \ Inner \ Shell = 90 + 2(1.125 - 0.125) = 92.0 \ in \\ t_{s} &= 1.125 - 2(Corrosion \ Allowance) = 1.125 - 2(0.125) = 0.8750 \ in \\ R_{s} &= \frac{OD \ of \ Inner \ Shell}{2} = \frac{92.0}{2} = 46.0 \ in \end{split}$$

- a) Paragraph 4.11.2: Design of Jacketed Shells and Jacketed Heads
  - 1) Paragraph 4.11.2.1, determine the required thickness of the jacket using Equation (4.3.1).

$$t_{rj} = R_j \left( \exp\left[\frac{P_j}{S_j E}\right] - 1 \right) = 48.125 \left( \exp\left[\frac{150}{22400(1.0)}\right] - 1 \right) = 0.3233 \text{ in}$$

 $t_{ri} = t_{ri} + Corrosion \ Allowance = 0.3233 + 0.125 = 0.4483 \ in$ 

Therefore, use a jacket plate with a wall thickness of  $t_i = 0.5 in$ .

- b) Paragraph 4.11.3: Design of Closure Member of Jacket to Vessel
  - 1) Paragraph 4.11.3.1, the design of jacket closure members shall be in accordance with Table 4.11.1 and the additional requirements of paragraph 4.11.3.
  - 2) Paragraph 4.11.3.2, radial welds in closure members shall be butt-welded joints through the full thickness of the member.
  - Paragraph 4.11.3.3, partial penetration and fillet welds are permitted when both of the following requirements are satisfied.
    - iii) The material of construction satisfies the following equation,

$$\left\{\frac{S_{yT}}{S_u} = \frac{32500}{70000} = 0.464\right\} \le 0.625$$
 True

- iv) The component is not in cyclic service.
- 4) Determine maximum jacket space, j, to ensure that proposed jacket is acceptable.

$$i_{specified} = \frac{(ID \text{ of } Jacket) - (OD \text{ of } Inner \text{ Shell})}{2} = \frac{(96.25 - 92.0)}{2} = 2.125 \text{ in}$$

The maximum of j is determined from Table 4.11.1, Detail 6.

$$j = \left(\frac{2S_c t_s^2}{P_j R_j}\right) - \left(\frac{\left(t_s + t_j\right)}{2}\right) = \left(\frac{2(22400)(0.875)^2}{150(48.125)}\right) - \left(\frac{0.875 + 0.5}{2}\right) = 4.0640 \text{ in}$$

Since,  $\{j_{specified} = 2.125 \ in\} \le \{j = 4.0640 \ in\}$ , the design is acceptable.

5) Determine thickness of jacket closures. From Table 4.11.1, Detail 6, Figure (c).

$$t_{rc} = 1.414 \sqrt{\frac{P_j R_s j}{S_c}} = 1.414 \sqrt{\frac{150(48.125)(2.125)}{22400}} = 1.1701 \text{ in}$$

$$t_{rc} = t_{rc} + Corrosion \ Allowance = 1.1701 + 0.125 = 1.2951 \ in$$

Therefore, use an end closure plate with a wall thickness of  $t_c = 1.3125 \text{ in}$ .

- 6) Determine minimum required weld sizes, see Table 4.11.1 Detail 6(c). Jacket to closure weld Table 4.11.1 Detail (c):
  - To be full penetration with backing strip.
  - Fillet weld to be equal to  $t_i$  as a minimum.
Closure to shell weld (a full penetration weld is to be used), see Table 4.11.1 Detail 5(c).

$$t_{c} = t_{rc} - Corrosion \ Allowance = 1.3125 - 0.125 = 1.3 \ in$$
  
$$t_{s} = 0.875$$
  
$$\{Y = a + b\} \ge \left\{\min\left[1.5t_{c}, 1.5t_{s}\right] = \min\left[1.5(1.3), 1.5(0.875)\right] = 1.3125\right\}$$
  
$$Z = Y - \frac{t_{s}}{2} = 1.3125 - \frac{0.875}{2} = 0.8750 \ in$$

And,

$$\{a, b\} \ge \{\min[6 \ mm(1/4 \ in), t_c, t_s] = \min[0.25, 1.25, 0.875] = 0.25 \ in\}$$

# 4.11.2 Example E4.11.2 – Half-Pipe Jacket

Design a half-pipe jacket for a section of a tower in accordance with paragraph 4.11.6 using the information shown below.

Vessel Data:

•	Material	=	<i>SA</i> -516, <i>Grade</i> 70
•	Design Conditions	=	350 psig@300°F
•	Vessel ID	=	90.0 <i>in</i>
•	Nominal Thickness	=	1.125 in
•	Allowable Stress	=	22400 psi
•	Corrosion Allowance	=	0.125 in
•	Applied Axial Force	=	78104.2 <i>lbs</i>
•	Applied Net Section Bending Moment	=	4.301E + 06 in - lbs
Ha	lf-Pipe Jacket Data:		
•	Material	=	SA-106, Grade B
•	Design Conditions	=	150 psig@400°F
•	Yield Stress at Design Temperature	=	29900 psi
•	Minimum Ultimate Tensile Strength	=	60000 <i>psi</i>
•	Jacket ID	=	NPS 4 (STD WT) $\rightarrow$ 0.237 in
•	Allowable Stress	=	20600 psi
•	Weld Joint Efficiency	=	1.0
•	Corrosion Allowance	=	0.0 <i>in</i>
•	Fillet Weld Leg	=	0.375 in

Establish the corroded dimensions.

Vessel:

$$D_0 = 90.0 + 2t_s = 90.0 + 2(0.125) = 90.25 \text{ in}$$
  

$$t_s = 1.125 - Corrosion \text{ Allowance} = 1.125 - 0.125 = 1.0 \text{ in}$$
  

$$D = 90.0 + 2(Corrosion \text{ Allowance}) = 90.0 + 2(0.125) = 0.25 \text{ in}$$

Half-Jacket:

$$D_{pj} = 4.5 - 2t_j = 4.5 - 2(0.237) = 4.026 \text{ in}$$
$$r_p = \frac{D_{pj}}{2} = \frac{4.026}{2} = 2.013 \text{ in}$$

Evaluate half-pipe jacket per paragraph 4.11.6.

a) Paragraph 4.11.6.1, the rules in this section are applicable for the design of half-pipe jackets constructed of NPS 2, NPS 3, or NPS 4 pipes and subject to internal pressure loading.

$$Half - Pipe \ Jacket \rightarrow NPS \ 4 \qquad True$$

- b) Paragraph 4.11.6.2:
  - 1) The fillet weld attaching the half-pipe jacket to the vessel shall have a throat thickness not less than the smaller of the jacket or shell thickness.

$$throat_{f} = 0.707 \cdot leg = 0.707(0.375) = 0.265 \ in$$
$$\{throat_{f} = 0.265 \ in\} \ge \{\min[0.237, (1.125 - 0.125)] \ in = 0.237 \ in\} \qquad True$$

- 2) The requirements of paragraph 4.11.3.3 shall be satisfied. Paragraph 4.11.3.3, partial penetration and fillet welds are permitted when both of the following requirements are satisfied.
  - i) The material of construction satisfies the following equation, SA-106, Grade B

$$\left\{\frac{S_{yT}}{S_u} = \frac{32500}{70000} = 0.464\right\} \le 0.625$$
 True

- ii) The component is not in cyclic service. *True*
- c) Calculate the minimum required thickness for the NPS 4 STD WT half-pipe jacket.

$$t_{rp} = \frac{P_j r_p}{0.85S_j - 0.6P_j} = \frac{150(2.0130)}{0.85(20600) - 0.6(150)} = 0.0173 \text{ in}$$

Since  $\{t_j = 0.237 \ in\} \ge \{t_{rp} = 0.0173 \ in\}$ , the thickness of STD WT pipe is acceptable for the half-pipe jacket.

d) Calculate maximum permissible pressure in the half-pipe,  $P_{iom}$ , to verify that  $P_i < P_{iom}$ .

$$P_{jpm} = \frac{F_p}{K_p}$$

$$F_p = \min\left[\left(1.5S - S^*\right), 1.5S\right]$$

$$K_p = C_1 + C_2 D^{0.5} + C_3 D + C_4 D^{1.5} + C_5 D^2 + C_6 D^{2.5} + C_7 D^3 + C_8 D^{3.5} + C_9 D^4 + C_{10} D^{4.5}$$

In order to compute  $P_{jpm}$ , the parameter  $S^*$  defined as the actual longitudinal stress in the shell must be computed. This stress may be computed using the equations in paragraph 4.3.10.2. However, since this is a thin shell, the thin-wall equations for a cylindrical shell will be used.

$$S^* = Pressure \ Stress + Axial \ Stress \pm Bending \ Stress$$
$$S^* = \frac{PD}{4t_s} + \frac{F}{A} \pm \frac{Mc}{I}$$

$$S^{*} = \begin{cases} \frac{350(90.25)}{4(1.0)} + \frac{-78104.2}{286.6703} + \frac{4.301E + 06\left(\frac{92.25}{2}\right)}{298408.1359} = 8269.2283 \ psi \\ \frac{350(90.25)}{4(1.0)} + \frac{-78104.2}{286.6703} - \frac{4.301E + 06\left(\frac{92.25}{2}\right)}{298408.1359} = 6959.6156 \ psi \end{cases}$$

Where,

$$I = \frac{\pi}{64} \left( D_o^4 - D^4 \right) = \frac{\pi}{64} \left( \left( 92.25 \right)^4 - \left( 90.25 \right)^4 \right) = 298408.1359 \text{ in}^4$$
$$A = \frac{\pi}{4} \left( D_o^2 - D^2 \right) = \frac{\pi}{4} \left( \left( 92.25 \right)^2 - \left( 90.25 \right)^2 \right) = 286.6703 \text{ in}^2$$

Therefore,

$$F_{p} = \min\left[\left(1.5S - S^{*}\right), \ 1.5S\right]$$
  
$$F_{p} = \min\left[\left(1.5 \cdot 22400 - 8269.2283\right), \ 1.5 \cdot 22400\right] = 25330.7717 \ psi$$

Coefficients for  $K_p$  formula obtained from Table 4.11.3 for *NPS* 4 and shell thickness,  $t_s = 1.0 \ in$ .

$$\begin{array}{ll} C_1 = -2.5016604E + 02, & C_2 = 1.7178270E + 02, & C_3 = -4.6844914E + 01 \\ C_4 = 6.6874346E + 00, & C_5 = -5.2507555E - 01, & C_6 = 2.1526948E - 02 \\ C_7 = -3.6091550E - 04, & C_8 = C_9 = C_{10} = 0.0 \end{array}$$

With the a vessel diameter,  $D\,{=}\,90.25~in\,{\rm ,}$  the value of  $\,K_p^{}\,$  is calculated as,

$$\begin{split} K_{p} &= C_{1} + C_{2}D^{0.5} + C_{3}D + C_{4}D^{1.5} + C_{5}D^{2} + C_{6}D^{2.5} + C_{7}D^{3} + C_{8}D^{3.5} + C_{9}D^{4} + C_{10}D^{4.5} \\ K_{p} &= 11.2903 \end{split}$$

Therefore, the maximum permissible pressure in the half-pipe is calculated as,

$$P_{jpm} = \frac{F_p}{K_p} = \frac{25330.7717}{11.2903} = 2243.6 \ psi$$

Since  $\{P_{jpm} = 2243.6 \ psi\} \ge \{P_j = 150 \ psi\}$ , the half-pipe design is acceptable.

# 4.12 NonCircular Vessels

# 4.12.1 Example E4.12.1 – Type 1

Using the data shown below, design a Type 1 non-circular pressure vessel per paragraph 4.12.7.

Vessel Data:

•	Material	=	SA-516, Grade 70
•	Design Conditions	=	400 <i>psig</i> @500°F
•	Inside Length (Short Side)	=	7.125 in
•	Inside Length (Long Side)	=	9.25 in
•	Overall Vessel Length	=	40.0 in
•	Thickness (Short Side)	=	1.0 <i>in</i>
•	Thickness (Long Side)	=	1.0 <i>in</i>
•	Thickness (End Plate)	=	0.75 in
•	Corrosion Allowance	=	0.125 in
•	Allowable Stress	=	20600 psi
•	Weld Joint Efficiency (Corner Joint)	=	1.0
•	Tube Outside Diameter	=	1.0000 in
•	Tube Pitch	=	2.3910 in

Adjust variable for corrosion.

 $h = 9.25 + 2(Corrosion \ Allowance) = 9.25 + 2(0.125) = 9.50 \ in$   $H = 7.125 + 2(Corrosion \ Allowance) = 7.125 + 2(0.125) = 7.375 \ in$   $t_1 = 1.0 - Corrosion \ Allowance = 1.0 - 0.125 = 0.875 \ in$   $t_2 = 1.0 - Corrosion \ Allowance = 1.0 - 0.125 = 0.875 \ in$  $t_5 = 0.75 - Corrosion \ Allowance = 0.75 - 0.125 = 0.625 \ in$ 

Evaluate per paragraph 4.12.

## Paragraph 4.12.2, General Design Requirements

Paragraph 4.12.2.7 – The design equations in this paragraph are based on vessels in which the ratio of the length of the vessel to the long side or short side lengths (aspect ratio) is greater than four. Vessels with aspect ratios less than four may be designed in accordance with the provisions of Part 5.

Aspect Ratio = 
$$\frac{L_v}{h} = \frac{40.0}{9.5} = 4.21$$

Paragraph 4.12.2.9

The openings in this noncircular vessel meet the requirements of paragraph 4.5.2.

#### Paragraphs 4.12.3, 4.12.4.4, and 4.12.5.5

These paragraphs are not applicable to this design.

#### Paragraph 4.12.6, Weld Joint Factors and Ligament Efficiency

Paragraph 4.12.6.1 – The non-circular vessel is constructed with corner joints typical of paragraph 4.2. Therefore, the weld joint efficiencies  $E_m$  and  $E_b$  are set to 1.0 at stress calculation locations in the corners of the vessel. Since there are no welds or hole pattern in the short side plates of the vessel, the weld joint efficiencies  $E_m$  and  $E_b$  are set to 1.0 for these stress calculation locations. For the stress calculation locations on the long side plates that do not contain welded joints, but do contain a hole pattern, the weld joint efficiencies  $E_m$  and  $E_b$  are set equal to the ligament efficiencies  $e_m$  and  $e_b$ , respectively.

Paragraph 4.12.6.3 – It is assumed that the holes drilled in the long side plates (tube sheet and plug sheet) are of uniform diameter. Therefore,  $e_m$  and  $e_b$  shall be the same value and calculated in accordance with paragraph 4.10.

$$e_m = e_b = \frac{p-d}{p} = \frac{2.3910 - 1.0}{2.3910} = 0.5818$$

Paragraph 4.12.7, Design Procedure

- a) STEP 1 The design pressure and temperature are listed in the information given above.
- b) STEP 2 The vessel to be designed is a Type 1 vessel, see Figure 4.12.1.
- c) STEP 3 The vessel configuration and wall thicknesses of the pressure containing plates are listed in the information given above.
- d) STEP 4 Determine the location of the neutral axis from the inside and outside surfaces. Since the section under evaluation does not have stiffeners, but has uniform diameter holes, then  $c_i = c_o = t/2$  where *t* is the thickness of the plate.

$$c_i = c_o = \frac{t_1}{2} = \frac{t_2}{2} = \frac{0.875}{2} = 0.4375$$
 in

- e) STEP 5 Determine the weld joint factor and ligaments efficiencies, as applicable (see paragraph 4.12.6) and determine the factors  $E_m$  and  $E_b$ .
- STEP 6 Complete the stress calculation for the selected noncircular vessel Type (see Table 4.12.1), and check the acceptance criteria.

For non-circular vessel Type 1, the applicable table for stress calculations is Table 4.12.2 and the corresponding details are shown in Figure 4.12.1.

Calculate the equation constants:

$$b = 1.0$$
 (unit width)  
 $J_{2s} = 1.0$   
 $J_{3s} = 1.0$   
 $J_{2l} = 1.0$ 

$$J_{3l} = 1.0$$

$$I_1 = \frac{bt_1^3}{12} = \frac{1.0(0.875)^3}{12} = 0.0558 \text{ in}^4$$

$$I_2 = \frac{bt_2^3}{12} = \frac{1.0(0.875)^3}{12} = 0.0558 \text{ in}^4$$

$$\alpha = \frac{H}{h} = \frac{7.375}{9.5} = 0.7763$$

$$K = \frac{I_2}{I_1} \alpha = \left(\frac{0.0558}{0.0558}\right) 0.7763 = 0.7763$$

Calculate the membrane and bending stresses at the Critical Locations of Maximum Stress.

The membrane stress on the short side plate:

$$S_m^s = \frac{Ph}{2(t_1)E_m} = \frac{400(9.5)}{2(0.875)(1.0)} = 2171.4 \text{ psi}$$

The bending stress at Location C on the short side plate:

$$S_{bi}^{sC} = -S_{bo}^{sC} \left(\frac{c_i}{c_o}\right) = \frac{PbJ_{2s}c_i}{12I_1E_b} \left[ -1.5H^2 + h^2 \left(\frac{1+\alpha^2 K}{1+K}\right) \right]$$
$$S_{bi}^{sC} = \frac{400(1.0)(1.0)(0.4375)}{12(0.0558)(1.0)} \left[ -1.5(7.375)^2 + (9.5)^2 \left(\frac{1+(0.7763)^2(0.7763)}{1+0.7763}\right) \right]$$
$$S_{bi}^{sC} = -1831.7 \text{ psi}$$

$$S_{bi} = -1831.7 \ pst$$
$$S_{bo}^{sC} = -S_{bi}^{sC} \left(\frac{c_o}{c_i}\right) 1831.7 = -(-1831.7) \left(\frac{0.4375}{0.4375}\right) = 1831.7 \ psi$$

The bending stress at Location B on the short side plate:

$$S_{bi}^{sB} = -S_{bo}^{sB} \left(\frac{c_i}{c_o}\right) = \frac{Pbh^2 J_{3s} c_i}{12I_1 E_b} \left[\frac{1+\alpha^2 K}{1+K}\right]$$
$$S_{bi}^{sB} = \frac{400(1.0)(9.5)^2(1.0)(0.4375)}{12(0.0558)(1.0)} \left[\left(\frac{1+(0.7763)^2(0.7763)}{1+0.7763}\right)\right]$$

$$S_{bi}^{sB} = 19490.8 \ psi$$
$$S_{bo}^{sB} = -S_{bi}^{sB} \left(\frac{c_o}{c_i}\right) = -19490.8 \left(\frac{0.4375}{0.4375}\right) = -19490.8 \ psi$$

The membrane stress on the long side plate:

$$S_m^l = \frac{PH}{2t_2 E_m} = \frac{400(7.375)}{2(0.875)(0.5818)} = 2897.4 \text{ psi}$$

The bending stress at Location A on the long side plate:

$$S_{bi}^{lA} = -S_{bo}^{lA} \left(\frac{c_i}{c_o}\right) = \frac{Pbh^2 J_{2l} c_i}{12I_2 E_b} \left[ -1.5 + \left(\frac{1+\alpha^2 K}{1+K}\right) \right]$$
$$S_{bi}^{lA} = \frac{400(1.0)(9.5)^2 (1.0)(0.4375)}{12(1.0)(0.0558)(0.5818)} \left[ -1.5 + \left(\frac{1+(0.7763)^2 (0.7763)}{1+0.7763}\right) \right]$$
$$S_{bi}^{lA} = -27310.9 \ psi$$

$$S_{bo}^{lA} = -S_{bi}^{lA} \left(\frac{c_o}{c_i}\right) = -(-27310.9) \left(\frac{0.4375}{0.4375}\right) = 27310.9 \text{ psi}$$

The bending stress at Location B on the long side plate:

$$S_{bi}^{IB} = -S_{bo}^{IB} \left(\frac{c_i}{c_o}\right) = \frac{Pbh^2 J_{3l} c_i}{12I^2 E_b} \left[\frac{1+\alpha^2 K}{1+K}\right]$$
$$S_{bi}^{IB} = \frac{400(1.0)(9.5)^2(1.0)(0.4375)}{12(0.0558)(1.0)} \left[\left(\frac{1+(0.7763)^2(0.7763)}{1+0.7763}\right)\right]$$
$$S_{bi}^{IB} = 19490.8 \ psi$$
$$S_{bo}^{IB} = -S_{bi}^{IB} \left(\frac{c_o}{c_i}\right) = -19490.8 \left(\frac{0.4375}{0.4375}\right) = -19490.8 \ psi$$

Acceptance Criteria - Critical Locations of Maximum Stress.

$$\begin{cases} S_m^s = 2171.4 \ psi \end{cases} \leq \{S = 20600 \ psi \} & True \\ \begin{cases} S_m^s + S_{bi}^{sC} = 2171.4 + (-1831.7) = 339.7 \ psi \\ S_m^s + S_{bo}^{sC} = 2171.4 + 1831.7 = 4003.1 \ psi \end{cases} \\ \begin{cases} S_m^s + S_{bo}^{sC} = 339.7 \ psi \\ S_m^s + S_{bo}^{sC} = 4003.1 \ psi \end{cases} \leq \{1.5S = 1.5(20600) = 30900 \ psi \} & True \\ \begin{cases} S_m^s + S_{bo}^{sB} = 2171.4 + 19490.8 = 21662.2 \ psi \\ S_m^s + S_{bo}^{sB} = 2171.4 + (-19490.8) = -17319.4 \ psi \end{cases} \\ \begin{cases} S_m^s + S_{bo}^{sB} = 21662.2 \ psi \\ S_m^s + S_{bo}^{sB} = -17319.4 \ psi \end{cases} \leq \{1.5S = 1.5(20600) = 30900 \ psi \} & True \\ \begin{cases} S_m^s + S_{bo}^{sB} = -17319.4 \ psi \end{cases} \leq \{1.5S = 1.5(20600) = 30900 \ psi \} & True \\ \begin{cases} S_m^s + S_{bo}^{sB} = -17319.4 \ psi \end{cases} \leq \{1.5S = 1.5(20600) = 30900 \ psi \} & True \end{cases}$$

$$\begin{cases} S_m^l + S_{bi}^{lA} = 2897.4 + (-27310.9) = -24413.5 \ psi \\ S_m^l + S_{bo}^{lA} = 2897.4 + 27310.9 = 30208.3 \ psi \end{cases}$$

$$\begin{cases} S_m^l + S_{bi}^{lA} = -24413.5 \ psi \\ S_m^l + S_{bo}^{lA} = 30208.3 \ psi \end{cases} \leq \{1.5S = 1.5(20600) = 30900 \ psi \}$$

$$True$$

$$\begin{cases} S_m^l + S_{bo}^{lB} = 2897.4 + 19490.8 = 22388.2 \ psi \\ S_m^l + S_{bo}^{lB} = 2897.4 + (-19490.8) = -16593.4 \ psi \end{cases}$$

$$\begin{cases} S_m^l + S_{bi}^{lB} = 22388.2 \ psi \\ S_m^l + S_{bi}^{lB} = 22388.2 \ psi \\ S_m^l + S_{bo}^{lB} = -16593.4 \ psi \end{cases} \leq \{1.5S = 1.5(20600) = 30900 \ psi \}$$

$$True$$

Since the acceptance criteria are satisfied, the design is complete. The vessel meets the requirements as designed and no further iterations are necessary.

# 4.12.2 Example E4.12.2 – Type 4

Using the data shown below, design a Type 4 non-circular pressure vessel per paragraph 4.12.7. The stiffeners are attached with continuous fillets welds on both sides of the member (Category E, Type 10) and satisfy the requirements of paragraph 4.2, Figure 4.2.2.

Vessel Data:

•	Material	=	SA-516, Grade 70
•	Design Conditions	=	50 <i>psig</i> @200°F
•	Inside Length (Short Side)	=	30.0 <i>in</i>
•	Inside Length (Long Side)	=	60.0 in
•	Overall Vessel Length	=	240.0 in
•	Unstiffened Span Length	=	12.0 in
•	Thickness (Short Side)	=	0.4375 in
•	Thickness (Long Side)	=	0.4375 in
•	Corrosion Allowance	=	0.0 <i>in</i>
•	Allowable Stress	=	23200 psi
•	Weld Joint Efficiency	=	1.0
•	Yield Stress at Design Temperature	=	34800 <i>psi</i>
•	Modulus of Elasticity at Design Temperature	=	$28.8E + 06 \ psi$
•	Modulus of Elasticity at Ambient Temperature	=	$29.4E + 06 \ psi$
<u>Sti</u>	ffener Data:		
•	Material	=	SA-36
•	Allowable Stress	=	22000 psi
•	Stiffener Yield Stress at Design Temperature	=	33000 <i>psi</i>
•	Modulus of Elasticity at Design Temperature	=	$28.8E + 06 \ psi$
•	Modulus of Elasticity at Ambient Temperature	=	$29.4E + 06 \ psi$
•	Stiffener Cross Sectional Area	=	$3.83 in^2$
•	Stiffener Moment of Inertia	=	11.3 $in^4$
•	Stiffener Height	=	4.125 in
•	Stiffener Centerline Distance (Short Side)	=	34.125 in
•	Stiffener Centerline Distance (Long Side)	=	64.125 in

Required variables.

$$h = 60.0$$
 in  
 $H = 30.0$  in  
 $t_1 = 0.4375$  in  
 $t_2 = 0.4375$  in

Evaluate per paragraph 4.12.

Paragraph 4.12.2, General Design Requirements

Paragraph 4.12.2.3.c – For a vessel with reinforcement, when the reinforcing member does not have the same allowable stress as the vessel, the total stress shall be determined at the inside and outside surfaces of each component of the composite section. The total stresses at the inside and outside surfaces shall be compared to the allowable stress.

- i) For locations of stress below the neutral axis, the bending equation used to compute the stress shall be that considered acting on the inside surface.
- ii) For locations of stress above the neutral axis, the bending equation used to compute the stress shall be that considered acting on the outside surface.

Paragraph 4.12.2.7 – The design equations in this paragraph are based on vessels in which the ratio of the length of the vessel to the long side or short side lengths (aspect ratio) is greater than four. Vessels with aspect ratios less than four may be designed in accordance with the provisions of Part 5.

Aspect Ratio = 
$$\frac{L_v}{h} = \frac{240.0}{60.0} = 4.0$$

Paragraph 4.12.2.9

There are no specified openings for this example problem.

Paragraph 4.12.3, Requirements for Vessels with Reinforcement

Paragraph 4.12.3.1 – Design rules are provided for Type 4 configurations where the welded-on reinforcement members are in a plane perpendicular to the long axis of the vessel. All reinforcement members attached to two opposite plates shall have the same moment of inertia.

Paragraph 4.12.3.5 – Reinforcing members shall be placed on the outside of the vessel and shall be attached to the plates of the vessel by welding on each side of the reinforcing member. For continuous reinforcement, the welding may be continuous or intermittent.

Paragraph 4.12.3.6 – The maximum distance between reinforcing members is computed in paragraph 4.12.3 and are covered in STEP 3 of the Design Procedure in paragraph 4.12.7.

### Paragraphs 4.12.4 and 4.12.5

These paragraphs are not applicable to this design.

#### Paragraph 4.12.6, Weld Joint Factors and Ligament Efficiency

Paragraph 4.12..1 – The non-circular vessel is constructed with corner joints typical of paragraph 4.2. Therefore, the weld joint efficiencies  $E_m$  and  $E_b$  are set to 1.0 at stress calculation locations in the corners of the vessel. Since there are no welds or hole pattern in either the short side plates or long side plates of the vessel, the weld joint efficiencies  $E_m$  and  $E_b$  are set to 1.0 for these stress calculation locations.

### Paragraph 4.12.7, Design Procedure

- a) STEP 1 The design pressure and temperature are listed in the information given above.
- b) STEP 2 The vessel to be designed is a Type 4 vessel, see Figure 4.12.4.
- c) STEP 3 The vessel configuration and wall thicknesses of the pressure containing plates are listed in the information given above. The vessel has stiffeners; therefore, calculate the maximum spacing and size of the stiffeners per paragraph 4.12.3.

Paragraph 4.12.3.6.a – The maximum distance between and reinforcing member centerlines is given by Equation (4.12.1). In the equations for calculating stresses for reinforced noncircular vessels, the value of p shall be the sum of one-half the distances to the next reinforcing member on each side.

For the short side plate, where  $\{H = 30.0 \text{ in}\} \ge \{p = 12.0 \text{ in}\}$ ,

$$p_1 = t_1 \sqrt{\frac{SJ_1}{P}} = 0.4375 \sqrt{\frac{(22000)2.2647}{50}} = 13.8105 \text{ in}$$

Where,

$$J_{1} = -0.26667 + \frac{24.222}{(\beta_{1\max})} - \frac{99.478}{(\beta_{1\max})^{2}} + \frac{194.59}{(\beta_{1\max})^{3}} - \frac{169.99}{(\beta_{1\max})^{4}} + \frac{55.822}{(\beta_{1\max})^{5}}$$
$$J_{1} = -0.26667 + \frac{24.222}{(2.2558)} - \frac{99.478}{(2.2558)^{2}} + \frac{194.59}{(2.2558)^{3}} - \frac{169.99}{(2.2558)^{4}} + \frac{55.822}{(2.2558)^{4}}$$
$$J_{1} = 2.2647$$
$$\beta_{1\max} = \min\left[\max\left[\beta_{1}, \frac{1}{\beta_{1}}\right], 4.0\right] = \min\left[\max\left[2.2558, \frac{1}{2.2558}\right], 4.0\right] = 2.2558$$

$$\beta_{1} = \frac{H}{p_{b1}} = \frac{30.0000}{13.2988} = 2.2558 \qquad (for rectangular vessels)$$

$$r_{b1} = \frac{1}{p_{b1}} = \frac{1}{2.15} + \frac{1}{2.15} = 0.4275 = \frac{1}{2.1(22000)} = 12.2088 \text{ in}$$

$$p_{b1} = t_1 \sqrt{\frac{2.1S}{P}} = t_1 \sqrt{\frac{2.1S}{P}} = 0.4375 \sqrt{\frac{2.1(22000)}{50}} = 13.2988 \text{ in}$$

For the long side plate, where  $\{h = 60.0 \ in\} \ge \{p = 12.0 \ in\}$ ,

$$p_2 = t_2 \sqrt{\frac{SJ_2}{P}} = 0.4375 \sqrt{\frac{(22000)2.0000}{50}} = 12.9783$$
 in

Where,

$$J_{2} = -0.26667 + \frac{24.222}{(\beta_{1max})} - \frac{99.478}{(\beta_{1max})^{2}} + \frac{194.59}{(\beta_{1max})^{3}} - \frac{169.99}{(\beta_{1max})^{4}} + \frac{55.822}{(\beta_{1max})^{5}}$$

$$J_{2} = -0.26667 + \frac{24.222}{(4.0)} - \frac{99.478}{(4.0)^{2}} + \frac{194.59}{(4.0)^{3}} - \frac{169.99}{(4.0)^{4}} + \frac{55.822}{(4.0)^{5}}$$

$$J_{2} = 2.0024 \rightarrow However, J_{2} \text{ is limited to } 2.0000$$

$$\beta_{2max} = \min\left[\max\left[\beta_{2}, \frac{1}{\beta_{2}}\right], 4.0\right] = \min\left[\max\left[4.5117, \frac{1}{4.5117}\right], 4.0\right] = 4.0$$

$$\beta_{2} = \frac{h}{p_{b2}} = \frac{60.0000}{13.2988} = 4.5117 \qquad (for \ rectangular \ vessels)$$

$$p_{b2} = t_{2}\sqrt{\frac{2.1S}{P}} = t_{2}\sqrt{\frac{2.1S}{P}} = 0.4375\sqrt{\frac{2.1(22000)}{50}} = 13.2988 \ in$$

Therefore,

$$p = \min[p_1, p_2] = \min[13.8105, 12.9783] = 12.9783$$
 in

Since  $\{p_{design} = 12.0 \ in\} < \{p_{allow} = 12.9783 \ in\}$ , the design is acceptable.

Paragraph 4.12.3.6.b – The allowable effective widths of shell plate,  $w_1$  and  $w_2$  shall not be greater than the value given by equation (4.12.16) or Equation (4.12.17), nor greater than the actual value of p if this value is less than that computed in paragraph 4.12.3.6.a. One half of w shall be considered to be effective on each side of the reinforcing member centerline, but the effective widths shall not overlap. The effective width shall not be greater than the actual width available.

$$w_{1} = \min[p, \min[w_{\max}, p_{1}]] = \min[12.0, \min[14.1552, 13.8105]] = 12.0 in$$
$$w_{2} = \min[p, \min[w_{\max}, p_{2}]] = \min[12.0, \min[14.1552, 12.9783]] = 12.0 in$$

Where,

$$w_{\max} = \frac{t_1 \Delta}{\sqrt{S_y}} \left( \frac{E_y}{E_{ya}} \right) = \frac{t_2 \Delta}{\sqrt{S_y}} \left( \frac{E_y}{E_{ya}} \right) = \frac{0.4375(6000)}{\sqrt{33000}} \left( \frac{28.8E + 06}{29.4E + 06} \right) = 14.1552 \text{ in}$$
  
$$\Delta = 6000 \sqrt{psi} \qquad From Table \ 4.12.14$$

Paragraph 4.12.3.6.c – At locations, other than in the corner regions where the shell plate is in tension, the effective moments of inertia,  $I_{11}$  and  $I_{21}$ , of the composite section (reinforcement and shell plate acting together) shall be computed based on the values of  $w_1$  and  $w_2$  computed in paragraph 4.12.3.6.b.

#### PTB-3-2013

NOTE – A composite structure may include the use of two or more different materials, each carrying a part of the load. Unless all the various materials used have the same Modulus of Elasticity, the evaluation of the composite section will need to consider the ratio of the moduli. Although the material specifications for the shell plate and stiffeners are different, their Moduli of Elasticity are the same; therefore, no adjustment to the procedure to calculate the composite section moment of inertia is required.

Calculate the short side stiffener/plate composite section neutral axis as follows see Figure E4.12.2.

$$\overline{y} = \frac{A_{stif}\left(t_1 + \frac{h_s}{2}\right) + A_{plate}\left(\frac{t_1}{2}\right)}{\left(A_{stif} + A_{plate}\right)}$$
$$\overline{y} = \frac{3.83\left(0.4375 + \frac{4.125}{2}\right) + 0.4375(12.0)\left(\frac{0.4375}{2}\right)}{\left(3.83 + 0.4375(12.0)\right)} = 1.1810 \text{ in}$$

Calculate the short side composite section moment of inertia,  $I_{11}$ , using parallel axis theorem.

$$I_{11} = I_{stif} + A_{stif} \left( t_1 + \frac{h_s}{2} - \overline{y} \right)^2 + \frac{w_1(t_1)^3}{12} + w_1(t_1) \left( \overline{y} - \frac{t_1}{2} \right)^2$$

$$I_{11} = \begin{cases} 11.3 + 3.83 \left( 0.4375 + \frac{4.125}{2} - 1.1810 \right)^2 + \\ \frac{12.0(0.4375)^3}{12} + 12.0(0.4375) \left( 1.1810 - \frac{0.4375}{2} \right)^2 \end{cases} = 22.9081 \ in^4$$

Since the stiffener is continuous around the vessel with a consistent net section, the plate thicknesses of the short side and long side are equal,  $t_1 = t_2$ , the pitch of stiffeners are equal,  $w_1 = w_2$ , it follows that  $\overline{y}$  for the short side and long side plates are equal and  $I_{11} = I_{21}$ .

d) STEP 4 – Determine the location of the neutral axis from the inside and outside surfaces. If the section under evaluation has stiffeners, then  $c_i$  and  $c_o$  are determined from the cross section of the combined plate and stiffener section using strength of materials concepts.

For the short side plate,

$$c_i = \overline{y} = 1.1810$$
 in  
 $c_o = t_1 + h_s - \overline{y} = 0.4375 + 4.125 - 1.1810 = 3.3815$  in

For the long side plate,

$$c_i = \overline{y} = 1.1810$$
 in  
 $c_o = t_2 + h_s - \overline{y} = 0.4375 + 4.125 - 1.1810 = 3.3815$  in

#### PTB-3-2013

The reinforcing member does not have the same allowable stress as the vessel; therefore, the stress at the interface of the components of the composite section shall be determined. Since the interface between components is oriented below the composite section neutral axis, the bending equation used to compute the stress shall be that considered acting on the inside surface. The distance between the composite section neutral axis and the interface of the components is calculated as follows.

For the short side and long side plates, respectively,

$$c_{i(interface)} = \overline{y} - t_1 = 1.1810 - 0.4375 = 0.7435 \text{ in}$$
$$c_{i(interface)} = \overline{y} - t_2 = 1.1810 - 0.4375 = 0.7435 \text{ in}$$

e) STEP 5 – Determine the weld joint factor and ligaments efficiencies, as applicable (see paragraph 4.12.6), and determine the factors  $E_m$  and  $E_b$ .

$$E_m = E_b = 1.0$$

STEP 6 – Complete the stress calculation for the selected noncircular vessel Type (see Table 4.12.1), and check the acceptance criteria.

For non-circular vessel Type 4, the applicable table for stress calculations is Table 4.12.5 and the corresponding details are shown in Figure 4.12.4.

Calculate the equation constants:

$$\alpha_{1} = \frac{H_{1}}{h_{1}} = \frac{H + 2(t_{1}) + h_{s}}{h + 2(t_{2}) + h_{s}} = \frac{30 + 2(0.4375) + 4.125}{60 + 2(0.4375) + 4.125} = 0.5385$$
$$k = \frac{I_{21}}{I_{11}}\alpha_{1} = \left(\frac{22.9081}{22.9081}\right)0.5385 = 0.5385$$

Calculate the Composite Section membrane and bending stresses at the Critical Locations of Maximum Stress.

The membrane stress on the short side plate:

$$S_m^s = \frac{Php}{2(A_1 + t_1p)E_m} = \frac{50(60.0)12.0}{2(3.83 + 0.4375(12.0))1.0} = 1982.4 \ psi$$

The bending stress at Location C, on the short side plate:

$$S_{bi}^{sC} = -S_{bo}^{sC} \left(\frac{c_i}{c_o}\right) = \frac{Ppc_i}{24I_{11}E_b} \left[-3H^2 + 2h^2 \left(\frac{1+\alpha_1^2 k}{1+k}\right)\right]$$
$$S_{bi}^{sC} = \frac{50(12.0)1.1810}{24(22.9081)1.0} \left[-3(30.0)^2 + 2(60.0)^2 \left(\frac{1+(0.5385)^2(0.5385)}{1+0.5385}\right)\right]$$
$$S_{bi}^{sC} = 3493.6 \ psi$$
$$S_{bo}^{sC} = -S_{bi}^{sC} \left(\frac{c_o}{c_i}\right) = -3493.6 \left(\frac{3.3815}{1.1810}\right) = -10003.1 \ psi$$

The bending stress at Location B, on the short side plate:

$$S_{bi}^{sB} = -S_{bo}^{sB} \left(\frac{c_i}{c_o}\right) = \frac{Ph^2 pc_i}{12I_{11}E_b} \left[\frac{1+\alpha_1^2 k}{1+k}\right]$$

$$S_{bi}^{sB} = \frac{50(60.0)^2 (12.0)(1.1810)}{12(22.9081)1.0} \left[\frac{1+(0.5385)^2 (0.5385)}{1+0.5385}\right]$$

$$S_{bi}^{sB} = 6973.5 \ psi$$

$$S_{bo}^{sB} = -S_{bi}^{sB} \left(\frac{c_o}{c_i}\right) = -6973.5 \left(\frac{3.3815}{1.1810}\right) = -19966.9 \ psi$$
The membrane stress on the large side plate:

The membrane stress on the long side plate:

$$S_m^l = \frac{PHp}{2(A_2 + t_2p)E_m} = \frac{50(30.0)(12.0)}{2(3.83 + 0.4375(12.0))1.0} = 991.2 \text{ psi}$$

The bending stress at Location A, on the long side plate:

$$S_{bi}^{lA} = -S_{bo}^{lA} \left(\frac{c_i}{c_o}\right) = \frac{Ph^2 pc_i}{24I_{21}E_b} \left[-3 + 2\left(\frac{1+\alpha_1^2 k}{1+k}\right)\right]$$
$$S_{bi}^{lA} = \frac{50(60.0)^2 (12.0)(1.1810)}{24(22.9081)1.0} \left[-3 + 2\left(\frac{1+(0.5385)^2 (0.5385)}{1+0.5385}\right)\right]$$

$$S_{bi}^{lA} = -6946.0 \ psi$$

$$S_{bo}^{lA} = -S_{bi}^{lA} \left(\frac{c_o}{c_i}\right) = -(-6946.0) \left(\frac{3.3815}{1.1810}\right) = 19888.2 \ psi$$

The bending stress at Location B, on the long side plate:

$$S_{bi}^{IB} = -S_{bo}^{IB} \left(\frac{c_i}{c_o}\right) = \frac{Ph^2 pc_i}{12I_{21}E_b} \left[\frac{1+\alpha_1^2 k}{1+k}\right]$$
$$S_{bi}^{IB} = \frac{50(60.0)^2 (12.0)(1.1810)}{12(22.9081)1.0} \left[\frac{1+(0.5385)^2 (0.5385)}{1+0.5385}\right]$$
$$S_{bi}^{IB} = 6973.5 \ psi$$
$$S_{bo}^{IB} = -S_{bi}^{IB} \left(\frac{c_o}{c_i}\right) = -6973.5 \left(\frac{3.3815}{1.1810}\right) = -19966.9 \ psi$$

Calculate the bending stresses at the interface of the shell plate and stiffener at the Critical Locations of Maximum Stress.

The bending stress at Location C, on the short side plate:

$$S_{bi}^{sC} = \frac{Ppc_i}{24I_{11}E_b} \left[ -3H^2 + 2h^2 \left( \frac{1+\alpha_1^2 k}{1+k} \right) \right]$$
$$S_{bi}^{sC} = \frac{50(12.0)(0.7435)}{24(22.9081)1.0} \left[ -3(30.0)^2 + 2(60.0)^2 \left( \frac{1+(0.5385)^2(0.5385)}{1+0.5385} \right) \right]$$

 $S_{bi}^{sC} = 2199.4 \ psi$ The bending stress at Location B, on the short side plate:

$$S_{bi}^{sB} = \frac{Ph^2 pc_i}{12I_{11}E_b} \left[ \frac{1 + \alpha_1^2 k}{1 + k} \right]$$
$$S_{bi}^{sB} = \frac{50(60.0)^2 (12.0)(0.7435)}{12(22.9081)1.0} \left[ \frac{1 + (0.5385)^2 (0.5385)}{1 + 0.5385} \right] = 4390.2 \ psi$$

The bending stress at Location A, on the long side plate:

$$S_{bi}^{lA} = \frac{Ph^2 pc_i}{24I_{21}E_b} \left[ -3 + 2\left(\frac{1+\alpha_1^2 k}{1+k}\right) \right]$$
$$S_{bi}^{lA} = \frac{50(60.0)^2 (12.0)(0.7435)}{24(22.9081)1.0} \left[ -3 + 2\left(\frac{1+(0.5385)^2 (0.5385)}{1+0.5385}\right) \right] = -4372.9 \ psi$$

The bending stress at Location B, on the long side plate:

$$S_{bi}^{IB} = \frac{Ph^2 pc_i}{12I_{21}E_b} \left[ \frac{1 + \alpha_1^2 k}{1 + k} \right]$$
  
$$S_{bi}^{IB} = \frac{50(60.0)^2 (12.0)(0.7435)}{12(22.9081)1.0} \left[ \frac{1 + (0.5385)^2 (0.5385)}{1 + 0.5385} \right] = 4390.2 \text{ psi}$$

<u>Acceptance Criteria – Critical Locations of Maximum Stress:</u> The stiffener allowable stress,  $S_{stif}$ , is used for the membrane stress and membrane plus bending stress for the outside fiber stress acceptance criteria, while the plate allowable stress, S, is used for the membrane plus bending stress for inside fiber allowable stress criteria.

$$\begin{cases} S_m^s = 1982.4 \ psi \end{cases} \le \{S = 22000 \ psi \} & True \\ \begin{cases} S_m^s + S_{bi}^{sC} = 1982.4 + 3493.5 = 5476.0 \ psi \\ S_m^s + S_{bo}^{sC} = 1982.4 + (-10003.1) = -8020.7 \ psi \end{cases} \le \begin{cases} 1.5S = 1.5(23200) = 34800 \ psi \\ 1.5S = 1.5(22000) = 33000 \ psi \end{cases} True \\ \begin{cases} S_m^s + S_{bi}^{sB} = 1982.4 + (6973.5 = 8955.9 \ psi \\ S_m^s + S_{bo}^{sB} = 1982.4 + (-19966.9) = -17984.5 \ psi \end{cases} \le \begin{cases} 1.5S = 1.5(23200) = 34800 \ psi \\ 1.5S = 1.5(22000) = 33000 \ psi \end{cases} True \\ \begin{cases} S_m^l = 991.2 \ psi \\ S_m^l = 991.2 \ p$$

### PTB-3-2013

$$\begin{cases} S_{m}^{l} + S_{bi}^{lA} = 991.2 + (-6946.0) = -5954.8 \ psi \\ S_{m}^{l} + S_{bo}^{lA} = 991.2 + 19888.2 = 20879.4 \ psi \end{cases} \leq \begin{cases} 1.5S = 1.5(23200) = 34800 \ psi \\ 1.5S = 1.5(22000) = 33000 \ psi \end{cases} True \\ \begin{cases} S_{m}^{l} + S_{bi}^{lB} = 991.2 + 6973.5 = 7964.7 \ psi \\ S_{m}^{l} + S_{bo}^{lB} = 991.2 + (-19966.9) = -18975.7 \ psi \end{cases} \leq \begin{cases} 1.5S = 1.5(23200) = 34800 \ psi \\ 1.5S = 1.5(23200) = 34800 \ psi \\ 1.5S = 1.5(23200) = 34800 \ psi \\ 1.5S = 1.5(22000) = 33000 \ psi \end{cases} True$$

The allowable stress of the shell plate and stiffener is limited by the stiffener. Therefore, at the interface of the shell plate and stiffener, the allowable stress used in the acceptance criteria is that of the stiffener.

$$\left\{S_m^s + S_{bi}^{sC} = 1982.4 + 2199.4 = 4181.8 \ psi\right\} \le \left\{1.5S = 1.5(22000) = 33000 \ psi\right\}$$
True

$$\left\{S_m^s + S_{bi}^{sB} = 1982.4 + 4390.2 = 6372.6 \ psi\right\} \le \left\{1.5S = 1.5(22000) = 33000 \ psi\right\} \qquad True$$

$$\left\{S_m^l + S_{bi}^{lA} = 991.2 + \left(-4372.9\right) = -3381.7 \ psi\right\} \le \left\{1.5S = 1.5(22000) = 33000 \ psi\right\} \ True$$

$$\left\{S_m^l + S_{bi}^{lB} = 991.2 + 4390.2 = 5381.4 \text{ psi}\right\} \le \left\{1.5S = 1.5(22000) = 33000 \text{ psi}\right\}$$
True

The acceptance criteria for membrane and membrane plus bending stresses are satisfied at all locations; therefore the design is complete.



Figure E4.12.2 - Composite Section Details

## 4.13 Layered Vessels

## 4.13.1 Example E4.13.1 – Layered Cylindrical Shell

Determine the required total thickness of the layered cylindrical shell for the following design conditions. The vessel has a corrosion resistant internal liner. Examination requirements are to be in accordance with Part 7, paragraph 7.4.11.

## Vessel Data:

•	Material	=	SA-724, Grade B
•	Design Conditions	=	5400 psig@300°F
•	Inside Diameter	=	84.0 in
•	Corrosion Allowance	=	0.0 <i>in</i>
•	Allowable Stress	=	39600 psi
•	Weld Joint Efficiency	=	1.0
•	Thickness of each layer	=	0.3125 in

In accordance with paragraph 4.13.4.1, determine the total thickness of the layered cylindrical shell using paragraph 4.3.3.

$$t = \frac{D}{2} \left( \exp\left[\frac{P}{SE}\right] - 1 \right) = \frac{84}{2} \left( \exp\left[\frac{5400}{39600(1.0)}\right] - 1 \right) = 6.1361 \text{ in}$$

The required thickness for all layers is 6.1361 in

## 4.13.2 Example E4.13.2 – Layered Hemispherical Head

Determine the required total thickness of the layered hemispherical head for the following design conditions. The vessel has a corrosion resistant internal liner. Examination requirements are to be in accordance with Part 7, paragraph 7.4.11.

Vessel Data:

•	Material	=	SA-724, Grade B
•	Design Conditions	=	5400 psig@300°F
•	Inside Diameter	=	84.0 in
•	Corrosion Allowance	=	0.0 <i>in</i>
•	Allowable Stress	=	39600 psi
•	Weld Joint Efficiency	=	1.0
•	Thickness of each layer	=	0.3125 in

In accordance with paragraph 4.13.4.1, determine the total thickness of the layered hemispherical head using paragraph 4.3.5.

$$t = \frac{D}{2} \left( \exp\left[\frac{0.5P}{SE}\right] - 1 \right) = \frac{84}{2} \left( \exp\left[\frac{0.5(5400)}{39600(1.0)}\right] - 1 \right) = 2.9635 \text{ in}$$

The required thickness for all layers is 2.9635 in

# 4.13.3 Example E4.13.3 – Maximum Permissible Gap in a Layered Cylindrical Shell

Determine the maximum permissible gap between any two layers in accordance with paragraph 4.13.12.3 for the cylindrical shell in Example Problem E4.13.1. The vessel has a corrosion resistant internal liner. Examination requirements are to be in accordance with Part 7, paragraph 7.4.11.

Vessel Data:

•	Material	=	SA-302, Grade B
•	Design Conditions	=	4800 psi @ 300° F
•	Inside Diameter	=	84.0 in
•	Corrosion Allowance	=	0.0 <i>in</i>
•	Allowable Stress	=	30300 <i>psi</i>
•	Weld Joint Efficiency	=	1.0
•	Thickness of each layer	=	0.3125 in
•	Number of layers	=	20
•	Specified design cycles in the UDS	=	1000
•	Stress amplitude at 1000 cycles	=	80600 <i>psi</i>
•	Elastic modulus	=	$28.3E + 06 \ psi$

In accordance with paragraph 4.13.12.3, determine the maximum permissible gap between any two layers, consider the outermost layer for the value of  $R_{\sigma}$ .

$$h = 0.55 \left( N - 0.5 - \frac{P}{S_m} \right) \frac{R_g S_m}{E_y}$$
  
$$h = 0.55 \left( 3.3324 - 0.5 - \frac{5400}{39600} \right) \frac{(45.125)(5400)}{28.3E + 06} = 0.0128 \text{ in}$$

Where,

$$N = \frac{2}{K_c} \left(\frac{S_a}{S_m}\right) = \frac{2}{1.2216} \left(\frac{80600}{39600}\right) = 3.3324$$

With,

$$K_{c} = \sqrt{\frac{4S_{a}}{3S_{m}} + 0.25} - 0.5 = \sqrt{\frac{4(80600)}{3(39600)} + 0.25} - 0.5 = 1.2216$$
$$R_{g} = \frac{84 + 20(0.3125)}{2} = 45.125 \text{ in}$$

The maximum permissible gap is 0.0128 in.

## 4.14 Evaluation of Vessels Outside of Tolerance

### 4.14.1 Example E4.14.1 – Shell Tolerances

A pressure vessel is constructed from NPS 30 long seam welded pipe. During construction, examination of the vessel shell indicates peaking at the long seam weld. The shell tolerances do not satisfy the fabrication tolerances given in paragraph 4.3.2 and 4.4.4. The Determine if the design may be qualified using paragraph 4.14.1.

#### Vessel Data

•	Material	=	SA–106, Grade B
•	Design Conditions	=	325 <i>psig</i> @600°F
•	Pipe Outside Diameter	=	30 <i>in</i>
•	Wall Thickness	=	0.5 <i>in</i>
•	Joint Efficiency	=	100 %
•	Corrosion Allowance	=	0.063 in
•	Allowable Stress	=	17900 psi@600°F
•	Material Yield Strength	=	26800 psi@600°F

### Examination Data

• Peaking distortion  $\delta$  = 0.33 in

The vessel is not in cyclic operation based on the screening criteria performed in accordance with paragraph 5.5.2.

The user has agreed to permit the assessment procedures in API 579-1/ASME FFS-1 to be used to qualify the design. When API 579-1/ASME FFS-1 is used for the assessment, a Remaining Strength Factor of 0.95 shall be used in the calculations unless another value is agreed to by the user. However, the Remaining Strength Factor shall not be less than 0.90. A fatigue analysis is not required.

The assessment procedure for evaluation peaking at a weld joint is provided in API 579-1/ASME FFS-1, Part 8, paragraph 8.3.4.2. The step-by-step procedure from this paragraph is shown below.

a) STEP 1 – Identify the component and weld misalignment type (see Part 8 Table 8.10 API 579-1/ASME FFS-1) and determine the following variables as applicable (see Figures 8.2, 8.3, and 8.4 Part 8 API 579-1/ASME FFS-1). The weld misalignment is identified as peaking which occurs on a longitudinal weld seam. The following data is required for the assessment:

$D_o = 30$ in	LOSS = 0.0 in
$t_{nom} = 0.5$ in	FCA = 0.063 in
$P = 325 \ psig$	$\delta = 0.33$ in
$E_y = 26.5E + 06 \ psi$	$S_a = 17900 \ psi$
v = 0.3	$H_{f} = 3.0$

b) STEP 2 – Determine the wall thickness to be used in the assessment

$$t_c = t_{nom} - LOSS - FCA = 0.5 - 0.0 - 0.063 = 0.437$$
 in

c) STEP 3 – Determine the membrane stress based on the current design pressure, see API 579-1/ASME FFS-1 Annex A, Equation (A.11). Note that API 579-1/ASME FFS-1 still uses the membrane stress equation from Section VIII, Division 1.

$$R = \frac{D_o}{2} - t_c = \frac{30}{2} - 0.437 = 14.563 \text{ in}$$
  

$$MA = 0.0 \text{ in}$$
  

$$Y_{B31} = 0.4$$
  

$$\sigma_m^c = \frac{P}{E} \left(\frac{R}{t_c} + 0.6\right) = \frac{325}{1.0} \left(\frac{14.563}{0.437} + 0.6\right) = 11025.606 \text{ psi}$$

d) STEP 4 – Calculate the ratio of the induced bending stress to the applied membrane stress using the equations in Part 8 Table 8.10 of API 579-1/ASME FFS-1 based on local peaking.

$$S_{p} = \sqrt{\frac{12(1-v^{2})PR^{3}}{E_{y}t_{c}^{3}}} = \sqrt{\frac{12(1-(0.3)^{2})(325)(14.563)^{3}}{(26.5E+06)(0.437)^{3}}} = 2.2263$$

$$\frac{\delta}{R} = \frac{0.33}{(14.563)} = 0.02266$$
From Figure 8.13, with  $\begin{cases} S_{p} = 2.2263\\ \frac{\delta}{R} = 0.02266 \end{cases} \Rightarrow C_{f} \approx 0.84$ 

$$R_{b}^{clja} = \frac{(6)(0.33)}{(0.437)}(0.84) = 3.81$$

$$R_{b} = R_{b}^{cljc} + R_{b}^{clja} = 0.0+3.81$$

$$R_{bs} = -1.0$$

e) STEP 5 – Determine the remaining strength factors, use  $H_f = 3.0$  (the bending stress due to the peaking is evaluated as a secondary stress).

$$RSF = \min\left[\frac{(3.0)(17900)}{(11025.606)(1+3.81) + (0.0)(1+(-1.0))}, 1.0\right] = \min\left[1.01257, 1.0\right] = 1$$

f) STEP 6 – Evaluate the results.

$$\{RSF = 1.0\} \ge \{RSF_a = 0.95\}$$
 True

The Level 2 Assessment Criterion is satisfied; therefore the peaking angle is acceptable for the specified design conditions.

## 4.14.2 Example E4.14.2 – Shell Tolerances and Fatigue Evaluation

Determine if the vessel in the Example Problem E4.14.2 can operate for 1200 cycles at 325 psig.

In accordance with paragraph 4.14.1, a fatigue analysis may be performed using API 579-1/ASME FFS-1. The fatigue analysis procedure is given in Part 8, paragraph 8.4.3.8. A fatigue analysis may be performed since the vessel satisfies the Level 2 criterion for the assessment of the weld misalignment as shown in Example Problem E4.14.2.

The step-by-step procedure from API 579-1/ASME FFS-1, Part 8, paragraph 8.3.4.2 is shown below.

- a) STEP 1 Determine the nature of the loading, the associated membrane stress and the number of operating cycles.
  - The loading consists of pressure loading.
  - From Example E4.14.2, the circumferential membrane stress is  $\sigma_m = 11025.606$  psi.
  - The desired number of operating cycles is 1500.
- b) STEP 2 Determine the ratio of the induced bending stress to the membrane stress,  $R_b$  resulting from weld misalignment.

$$\begin{aligned} R_b^{cljc} &= 0.0 & (since \ centerline \ offset \ is \ not \ present) \\ R_b^{clja} &= 3.81 \\ R_b^{or} &= 0.0 & (since \ neither \ general \ or \ arbitrary \ out-of-roundness \ is \ present) \\ R_b &= R_b^{cljc} + R_b^{clja} + R_b^{or} = 0.0 + 3.81 + 0.0 = 3.81 \end{aligned}$$

c) STEP 3 – Using the loading history and membrane stress from STEP 1 and  $R_b$  from STEP 2, calculate the stress range for the fatigue analysis using Part 8 Table 8.12 of API 579-1/ASME FFS-1.

From Table 8.12, for a cylinder with a longitudinal weld joint with weld misalignment:

$$\Delta \sigma_m = \sigma_m = 11498 \ psi$$
$$\Delta \sigma_b = \sigma_m \left( R_b^{cljc} + R_b^{clja} + R_b^{or} \right) = (11025.606)(0.0 + 3.81 + 0.0) = 42007 \ psi$$

The fatigue strength reduction factor will be applied when computing the alternating stress range; therefore, set  $K_f = 1.0$  in the equation for  $\Delta S_P$ .

$$\Delta S_P = \sigma_m \left( 1 + R_b^{cljc} + R_b^{clja} + R_b^{or} \right) \left( K_f \right)$$
  
$$\Delta S_P = (11025.606) \left( 1 + 0.0 + 3.81 + 0.0 \right) (1.0) = 55033 \ psi$$

- d) STEP 4 Compute the number of allowed cycles using the stress range determined in STEP 3.
   Table 8.12 references Annex B1, paragraph B1.5 of API 579-1/ASME FFS-1. Paragraph B1.5 provides three methods for determining the permissible number of cycles:
  - 1) Elastic Stress Analysis and Equivalent Strength in accordance with paragraph B1.5.3
  - 2) Elastic-Plastic Stress Analysis and Equivalent Strain in accordance with paragraph B1.5.4
  - 3) Elastic Stress Analysis and Structural Stress in accordance with paragraph B1.5.5

Since an elastic-plastic stress analysis has not been conducted, the permitted number of cycles will be determined using Methods 1 and 3. In both cases the stresses considered consist of those due to pressure loading, stresses from supplementary loads and thermal gradients are considered negligible.

#### Method 1 per API 579-1/ASME FFS-1, Annex B1, paragraph B1.5.3

For a fatigue assessment using an elastic stress analysis and equivalent stresses, STEPS 1 through 3 in paragraph B1.5.3.1 of API 579-1/ASME FFS-1 are similar to STEPS 1 through 3 in paragraph 8.4.3.8 with the exception that the elastic stress range is calculated from the stress tensors and that the stress state from both mechanical and thermal loading are considered. For this example problem the stress range due to thermal loading is considered negligible and the mechanical loading consists of internal pressure. Thus the stress range is given by STEP 3 and is 54372 psi.

Paragraph B1.5.3, STEP 4 – Determine the effective alternating stress from Equation (B1.30), modified to ignore cyclic thermal stress, (i.e.,  $\Delta S_{LT} = 0.0$ ):

$$S_{alt} = \frac{K_f \cdot K_e \cdot \Delta S_P}{2}$$

The fatigue strength reduction factor,  $K_f$ , is determined from Table B1.10 based on type of weld and the quality level determined from Table B.11. The quality level in Table B1.11 is based on the type of inspection performed on the weld. For the vessel material, the specification called for full volumetric and full visual examination, but neither MT nor PT were performed on the weld. Thus from Table B1.11 the quality level is 4. The weld being assessed is a full penetration weld. For a full penetration weld inspected to quality level 4, Table B1.10 stipulated a weld fatigue reduction factor of  $K_f = 2.0$ .

The factor  $K_e$  is a fatigue penalty factor that may be determined from Equations (B1.31) to (B1.33) depending on the value of the stress range  $\Delta S_P$  compared to the permitted primary plus secondary stress range,  $S_{PS}$ . The value of  $S_{PS}$  is the larger of three times the allowable stress at temperature or two times the material yield strength at the average temperature during a stress cycle. The allowable stress at temperature,  $S_a$ , equals 17900 psi and the yield strength for the SA-106 Grade B material,  $S_y$ , equals 26800 psi at 600° F and 35,000 at ambient temperature. The average yield stress during the cycle is thus 30,900 psi.

$$S_{PS} = \max\left[3.0S_a, 2S_y\right] = \max\left[(3)(17900), (2)(30900)\right] = 61800 \ psi$$

Compare the value of  $\Delta S_P$  to  $S_{PS}$ :

$$\{\Delta S_P = 55033\} \le \{S_{PS} = 61800\}$$
 (*True*)

Therefore from Equation (B1.31),  $K_e = 1$ 

$$S_{alt} = \frac{K_f \cdot K_e \cdot \Delta S_P}{2} = \frac{(2)(1)(55033)}{2} = 55033 \ psi$$

Paragraph B1.5.3, STEP 5 – Determine the permitted number of cycles, N, for the alternating stress computed in STEP 4 and the smooth bar fatigue curves as provided in Annex F, paragraph F.6.2.1 of API 579-1/ASME FFS-1. For temperatures not in the creep range, the permitted number of cycles is given by the following equations.

$$N = (10)^X$$

where

$$X = \frac{C_1 + C_3 Y + C_5 Y^2 + C_7 Y^3 + C_9 Y^4 + C_{11} Y^5}{1 + C_2 Y + C_4 y^2 + C_6 Y^3 + C_8 Y^4 + C_{10} Y^5}$$
$$Y = \left(\frac{S_{alt,k}}{C_{us}}\right) \left(\frac{E_{FC}}{E_T}\right)$$

The values of the coefficients  $C_i$  are given in Table F.13 for low allow steels where  $\sigma_{UTS} \le 80$  ksi. Examining Table F.13, it is noted that the values of  $C_6$  through  $C_{11}$  all equal zero.

Substituting the values for  $S_{alt} = 55.305 \ ksi$ ,  $C_{us} = 1$ ,  $E_T = 28.3E + 03 \ ksi$  (a)  $335^{\circ}F$ , and  $E_{FC} = 28.3E + 03 \ ksi$ :

$$Y = \left(\frac{S_{alt,k}}{C_{us}}\right) \left(\frac{E_{FC}}{E_T}\right) = \left(\frac{55.033}{1.0}\right) \left(\frac{28.3E + 03}{28.3E + 03}\right) = 55.033 \ ksi = 1000$$

Substituting the values for  $C_1$  through  $C_5$ ,  $S_{alt} = 54.372$ :

$$X = \frac{7.999502 + (1.50085E - 01)(55.033) + (-5.263661E - 05)(55.033)^2}{1 + (5.832491E - 02)(55.033) + (1.273659E - 04)(55.033)^2} = 3.5519$$

And the number of permissible cycles is:

$$N = (10)^{3.5519} = 3564 \ cycles$$

Method 3 per API 579-1/ASME FFS-1, Annex B1, paragraph B1.5.5

Paragraph B1.5.3, STEP 1 – Determine the load history for the component, considering all significant operating loads. The load applied to the pipe consists of internal pressure, P, of 325 psig.

Paragraph B1.5.3, STEP 2 – For the weld joint subject to fatigue evaluation determine the individual number of stress-strain cycles. The desired number of cycles, N, is 1500.

Paragraph B1.5.3, STEP 3 – Determine the elastically calculated membrane and bending stress normal to the hypothetical crack plane at the start and end of the cycle. Using this data calculate the membrane and bending stress ranges between the time of maximum and minimum stress for the cycle. From Example Problem 1 the maximum membrane stress for the cycle occurs at a pressure of 325 psig, and the minimum membrane stress for the cycle occurs at zero pressure. Similarly, the maximum bending stress for the cycle occurs at a pressure of 325 psig, and the

minimum bending stress for the cycle occurs at zero pressure. The values of the two stress ranges given by Equations (B1.46) through (B1.50) are:

$$\Delta \sigma_{m} = {}^{m} \sigma_{m}^{e} - {}^{n} \sigma_{m}^{e} = 11.025 - 0 = 11.025 \ ksi$$

$$\Delta \sigma_{b} = {}^{m} \sigma_{b}^{e} - {}^{n} \sigma_{b}^{e} = (R_{b}) \left( {}^{m} \sigma_{m}^{e} - {}^{n} \sigma_{m}^{e} \right) = (3.81)(11.025 - 0) = 42.0 \ ksi$$

$$\sigma_{max} = \max \left[ \left( {}^{m} \sigma_{m}^{e} + {}^{m} \sigma_{b}^{e} \right), \left( {}^{n} \sigma_{m}^{e} + {}^{n} \sigma_{b}^{e} \right) \right]$$

$$\sigma_{max} = \max \left[ (11.025 + 42.0), (0 + 0) \right] = 53.025 \ ksi$$

$$\sigma_{min} = \min \left[ \left( {}^{m} \sigma_{m}^{e} + {}^{m} \sigma_{b}^{e} \right), \left( {}^{n} \sigma_{m}^{e} + {}^{n} \sigma_{b}^{e} \right) \right]$$

$$\sigma_{min} = \min \left[ (11.025 + 42.0), (0 + 0) \right] = 0 \ ksi$$

$$\sigma_{mean} = \frac{\sigma_{max}^{e} + \sigma_{min}^{e}}{2} = \frac{53.025 + 0}{2} = 26.512 \ ksi$$

Paragraph B1.5.3, STEP 4 – Determine the elastically calculated structural stress range,  $\Delta \sigma^e$ , for the cycle using Equation (B1.51)

$$\Delta \sigma^{e} = \Delta \sigma_{m}^{e} + \Delta \sigma_{b}^{e} = 11.025 + 42.0 = 53.025 \ ksi$$

Paragraph B1.5.3, STEP 5 – Determine the elastically calculated structural strain,  $\Delta \varepsilon^e$ , from the elastically calculated structural stress range,  $\Delta \sigma^e$ , using Equation (B1.52) and the elastic modulus for the material at the average temperature of 335° F.

$$\Delta \varepsilon^{e} = \frac{\Delta \sigma^{e}}{E_{ya}} = \frac{53.025}{28.0(10)^{3}} = 1.8940(10)^{-3}$$

Determine the stress range,  $\Delta\sigma$ , and strain range,  $\Delta\varepsilon$ , by correcting the elastically computed values of  $\Delta\sigma^e$  and  $\Delta\varepsilon^e$  by solving Equations (B1.53) and (B1.54), these equations are shown below, simultaneously (this is an application of Neuber's Rule).

$$\Delta \sigma \cdot \Delta \varepsilon = \Delta \sigma^{e} \cdot \Delta \varepsilon^{e}$$
$$\Delta \varepsilon = \frac{\Delta \sigma}{E_{ya}} + 2 \left(\frac{\Delta \sigma}{K_{css}}\right)^{\frac{1}{n_{css}}}$$

The parameters  $K_{css}$  and  $n_{css}$  are determined from Table F.8 in Annex F for the average temperature during the cycle. The values for Carbon Steel (0.75 in. – weld metal) are:

$$T = 70^{\circ} F \qquad n_{css} = 0.110 \qquad K_{css} = 100.8$$
  
$$T = 390^{\circ} F \qquad n_{css} = 0.118 \qquad K_{css} = 99.6$$

The average temperature of the cycle is 335° F. Therefore, the values of  $n_{css}$  and  $K_{css}$  can be conservatively set at the value given for 390° F, or:

$$n_{css} = 0.118$$
$$K_{css} = 99.6$$

Substituting the above values into the simultaneous equations previously shown (i.e. Neuber's Rule), results in the following simultaneous equations.

$$\Delta \sigma \cdot \Delta \varepsilon = \Delta \sigma^{e} \cdot \Delta \varepsilon^{e} = (55.305) (1.97581 (10)^{-3}) = 1.004 (10)^{-1}$$
$$\Delta \varepsilon = \frac{\Delta \sigma}{28.0E + 03} + 2 \left(\frac{\Delta \sigma}{99.6}\right)^{\frac{1}{0.118}}$$

The solution of these equations is:

$$\Delta \sigma = 40.6896 \ ksi$$
$$\Delta \varepsilon = 2.46785 (10)^{-3}$$

Modify the value of  $\Delta\sigma$  is modified or low-cycle fatigue using Equation (B1.55),

$$\Delta \sigma = \left(\frac{E_{ya}}{1 - v^2}\right) \Delta \varepsilon = \left(\frac{28.0E + 03}{1 - (0.3)^2}\right) \left(2.46785(10)^{-3}\right) = 75.9340 \ ksi$$

Paragraph B1.5.3, STEP 6 – Compute the equivalent structural stress range  $\Delta S_{ess}$  using Equation (B1.56) where the input parameters are as follows:

$$\begin{split} \Delta \sigma &= 75.9340 \ ksi \\ t_{ess} &= 0.625 \ in \qquad \left( since \ the \ component \ thickness \ is \ t_c = 0.437 \le \ 0.625 \ in \right) \\ R_b &= \frac{\left| \Delta \sigma_b \right|}{\left| \Delta \sigma_m \right| + \left| \Delta \sigma_b \right|} = \frac{42.0}{11.025 + 42.0} = 0.7921 \\ I^{\frac{1}{m_{ss}}} &= \frac{1.23 - 0.364R_b - 0.17R_b^2}{1.007 - 0.306R_b - 0.178R_b^2} \\ I^{\frac{1}{m_{ss}}} &= \frac{1.23 - 0.364\left(0.7921\right) - 0.17\left(0.7921\right)^2}{1.007 - 0.306\left(0.7921\right) - 0.178\left(0.7921\right)^2} = 1.2789 \\ R &= \frac{\sigma_{\min}}{\sigma_{\max}} = \frac{0}{53.025} = 0 \\ f_M &= 1.0 \qquad \left( since \ R = 0 \le 0; \ see \ Equations \ (B1.63) \ and \ (B1.64) \right) \end{split}$$

 $m_{ss} = 3.6$ 

$$\Delta S_{ess} = \frac{\Delta \sigma}{t_{ess}^{\left(\frac{2-m_{ss}}{2\cdot m_{ss}}\right)} \cdot I^{\frac{1}{m_{ss}}} \cdot f_{M}}} = \frac{75.9340}{(0.625)^{\left(\frac{2-3.6}{2\cdot 3.6}\right)} \cdot (1.2789) \cdot (1)} = 53.4882 \ ksi$$

Paragraph B1.5.3, STEP 7 – Determine the permitted number of cycles, N, using the value of  $\Delta S_{ess}$  from STEP 6 and the welded component fatigue curves in Annex F. The welded component fatigue curves are represented in Annex F by Equation (F.218):

$$N = \frac{f_1}{f_E} \left( \frac{f_{MT} \cdot C}{\Delta S_{ess}} \right)^{\frac{1}{h}}$$

The adjustment factors set as follows.

$$f_{1} = 1.0 \qquad (fatigue improvement techniques have not been used)$$

$$f_{E} = 4.0 \qquad (the process fluid is considered mildly aggressive)$$

$$f_{MT} = \frac{E_{ACS}}{E_{T}} = \frac{2.94E + 04}{2.80E + 04} = 1.0500$$

From Table F.29, for a lower 99% prediction interval  $(-3\sigma)$ , the values of C and h for low alloy steel are, C = 818.3 and h = 0.3195, respectively. The number of cycles is computed as follows.

$$N = \frac{f_1}{f_E} \left(\frac{f_{MT} \cdot C}{\Delta S_{ess}}\right)^{\frac{1}{h}} = \frac{1.0}{4.0} \left(\frac{(1.0500)(818.3)}{53.4882}\right)^{\frac{1}{0.3195}} = 1486 \ cycles$$

Paragraph B1.5.3, STEP 8 – Evaluate the component by comparing the number of permitted cycles to the number of desired cycles:

$$\{N = 1486\} \ge \{Specified \ Design \ Cycles = 1200\}$$
 (True)

The component is acceptable for cyclic operation at the specified design conditions.

In summary, the fatigue life is satisfied by both Method 1 and Method 3, or

Method 1:

$$\{N = 3303\} \ge \{Specified \ Design \ Cycles = 1200\}$$
 (True)

Method 3:

$$\{N = 1486\} \ge \{Specified \ Design \ Cycles = 1200\}$$
 (True)

## 4.14.3 Example E4.14.3 – Local Thin Area

For the vessel in Example Problem 1, an arch strike was removed during fabrication by blend grinding that has resulted in a region of local metal loss. If the region of local metal loss is made to conform to the requirements of Paragraph 4.14.2.2 by blend grinding, determine whether the local thin area is acceptable using Part 5 of API 579-1/ASME FFS-1.

## Vessel Data

•	Material	=	SA-106, Grade B
•	Pipe Outside Diameter	=	30 in
•	Wall Thickness	=	0.5 in
•	Design Conditions	=	325 psig@600 °F
•	Joint Efficiency	=	100 %
•	Future Corrosion Allowance	=	0.063 in
•	Supplemental Loads	=	Negligible

• The vessel is not in cyclic service (subject to less than 150 cycles)

## Examination Data

Based on inspection data, the initial thickness profile in the longitudinal direction has a length s of 8.0 in and a uniform measured thickness of 0.36 in. The critical thickness profile in the circumferential direction has a length c of 10.0 in with the same uniform thickness. The region of local metal loss is located 45 in away from the nearest structural discontinuity and is the only region of local metal loss found in the vessel during inspection.

The user has agreed to permit the assessment procedures in API 579-1/ASME FFS-1 to be used to qualify the design. When API 579-1/ASME FFS-1 is used in the assessment, a Remaining Strength Factor of 0.98 shall be used in the calculations unless another value is agreed to by the user.

The assessment procedure for evaluation of a local thin area is provided in API 579-1/ASME FFS-1, Part 5, paragraph 5.4.2.2. The step-by-step procedure is shown below.

- a) STEP 1 Determine the CTP (Critical Thickness Profiles) (See Inspection Data above).
- b) STEP 2 Determine the wall thickness to be used in the assessment using equation (5.3).

$$t_{nom} = 0.5 \text{ in}$$
  
 $LOSS = 0.0 \text{ in}$   
 $FCA = 0.063 \text{ in}$   
 $t_c = t_{nom} - LOSS - FCA = 0.5 - 0.0 - 0.063 = 0.437 \text{ in}$ 

c) STEP 3 – Determine the minimum measured thickness,  $t_{mm}$ , and the dimension, s, for the longitudinal CTP. There is only one LTA in the vessel; therefore, the spacing criteria in Part 4, paragraph 4.3.3.3.f.3 of API 579-1/ASME FFS-1 do not apply.

$$t_{mm} = 0.36 in$$
  
 $s = 8.0 in$ 

d) STEP 4 – Determine the remaining thickness ratio and the longitudinal flaw length parameter,  $\lambda$  using equations (5.5) and (5.6) of API 579-1/ASME FFS-1.

$$R_{t} = \frac{t_{mm} - FCA}{t_{c}} = \frac{0.36 - 0.063}{0.437} = 0.6796$$
  

$$D = 30 - 2t_{nom} + 2(LOSS + FCA) = 30 - 2(0.5) + 2(0.0 + 0.063) = 29.176 \text{ in}$$
  

$$\lambda = \frac{1.285s}{\sqrt{Dt_{c}}} = \frac{1.285(8)}{\sqrt{29.176(0.437)}} = 2.881$$

e) STEP 5 – Check the limiting flaw size criteria for a Level 1 Assessment using equations (5.7), (5.8), and (5.9).

$$\{R_t = 0.6796\} \ge 0.20$$
 True

$$\{t_{mm} - FCA = 0.36 - 0.063 = 0.297 \ in\} \ge 0.10 \ in$$
 True

$$L_{msd} = 45 \ in\} \ge \left\{ 1.8\sqrt{Dt_c} = 1.8\sqrt{29.126(0.437)} = 6.422 \ in \right\}$$
True

- f) STEP 6 Check the criteria for a groove-like flaw. This step is not applicable because the region of localized metal loss is categorized as an *LTA*.
- g) STEP 7 Determine the *MAWP* for the component (see A.3.4) using equations (A.10), (A.16), and (A.22) of API 579-1/ASME FFS-1.

$$R = \frac{D}{2} = \frac{29.176}{2} = 14.563 \text{ in}$$

$$MAWP^{C} = \frac{SEt_{c}}{R+0.6t_{c}} = \frac{(17900)(1.0)(0.437)}{(14.563)+0.6(0.437)} = 527.64 \text{ psi}$$

$$MAWP^{L} = \frac{2SE(t_{c}-t_{sl})}{R-0.4(t_{c}-t_{sl})} = \frac{(2)(17900)(1.0)(0.437-0.0)}{(14.563)-0.4(0.437-0.0)} = 1087.32 \text{ psi}$$

$$MAWP = \min[527.64, 1087.32] = 527.64 \text{ psi}$$

h) STEP 8 – Evaluate the longitudinal extent of the flaw.

From Figure 5.6 with  $\begin{cases} \lambda = 2.881 \\ R_t = 0.6796 \end{cases}$ , the longitudinal extent of the flaw is acceptable. Using Table 5.2 and equation (5.11) of API 579-1/ASME FFS-1:

$$M_{t} = 1.857 + \frac{(2.881 - 2.5)}{(3.0 - 2.5)} (2.103 - 1.857) = 2.044$$
$$\left(RSF = \frac{R_{t}}{1 - \frac{1}{M_{t}} (1 - R_{t})} = \frac{0.6796}{1 - \frac{1}{2.044} (1 - 0.6796)} = 0.8059\right) < (RSF_{a} = 0.9)$$

Since  $RSF < RSF_a$ , the reduced MAWP can be calculated using equation (2.2)

$$MAWP_r = MAWP \frac{RSF}{RSF_a} = (527.64) \left(\frac{0.8059}{0.98}\right) = 433.9 \ psi$$

 $\{MAWP_r = 433.9 \ psi\} > \{P_{Design} = 325 \ psi\}$ The longitudinal extent of the flaw is acceptable.

STEP 9 – Evaluate circumferential extent of the flaw.

i)

1) STEP 9.1 – From the circumferential CTP, determine  $\lambda_c$  using equation (5.12).

$$c = 10.0 \text{ in}$$
$$\lambda_c = \frac{1.285c}{\sqrt{Dt_c}} = \frac{1.285(10.0)}{\sqrt{(29.126)(0.437)}} = 3.6018$$

2) STEP 9.2 – Check the following conditions (equations (5.13) to (5.17)).

$$\{\lambda_c = 3.6018\} \le 9 \qquad \qquad True$$

$$\left\{\frac{D}{t_c} = \frac{29.176}{0.437} = 66.650\right\} \ge 20$$
 True

$$0.7 \le \{RSF = 0.8059\} \le 1.0$$
 True

$$0.7 \le \{E_L = 1\} \le 1.0$$
 True

$$0.7 \le \{E_C = 1\} \le 1.0$$
 True

3) STEP 9.3 – Calculate tensile strength factor using equation (5.18),

$$TSF = \frac{E_C}{2 \cdot RSF} \left( 1 + \frac{\sqrt{4 - 3E_L^2}}{E_L} \right) = \frac{1}{2(0.8059)} \left( 1 + \frac{\sqrt{4 - 3(1)^2}}{1} \right) = 1.2408$$
  
From Figure 5.8 with  $\begin{cases} \lambda_C = 3.6018\\ R_i = 0.6796 \end{cases}$ , the circumferential extent of the flaw is acceptable

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with a TSF = 1.2408.

The Level 1 Assessment Criteria are satisfied. Therefore, the vessel is acceptable for operation at the specified design conditions.

## 4.15 Supports and Attachments

## 4.15.1 Example E4.15.1 – Horizontal Vessel, Zick's Analysis

Determine if the stresses in the horizontal vessel induced by the proposed saddle supports are with acceptable limits. The vessel is supported by two symmetric equally spaced saddles welded to the vessel, without reinforcing plates or stiffening rings. All Category A and B joints are Type 1 butt welds and have been 100% radiographically examined. See Figure E4.15.1

## Vessel Data:

•	Material	=	SA-516, Grade 70
•	Design Conditions	=	2074 psig@175°F
•	Outside Diameter	=	66.0 in
•	Thickness	=	3.0 <i>in</i>
•	Corrosion Allowance	=	0.125 in
•	Formed Head Type	=	2:1 Elliptical
•	Head Height (Based on OD)	=	16.5 in
•	Allowable Stress	=	23500 psi
•	Weld Joint Efficiency	=	1.0
•	Shell Tangent to Tangent Length	=	292.0 in
<u>Sa</u>	ddle Data:		
•	Material	=	SA-516, Grade 70
•	Saddle Center Line to Head Tangent Line	=	41.0 <i>in</i>
•	Saddle Contact Angle	=	123.0 <i>deg</i>
•	Width of Saddles	=	8.0 <i>in</i>
•	Vessel Load per Saddle	=	50459.0 lbs

Adjust variables for corrosion and calculate the mean shell radius.

$$ID = 60.0 + 2(Corrosion \ Allowance) = 60.0 + 2(0.125) = 60.25 \ in$$
$$t = 3.0 - Corrosion \ Allowance = 3.0 - 0.125 = 2.875 \ in$$
$$R_m = \frac{OD + ID}{4} = \frac{66.0 + 60.25}{4} = 31.5625 \ in$$

Use the procedure described in paragraph 4.15.3.

Paragraph 4.15.3.1,

a) The stress calculation method is based on linear elastic mechanics and covers modes of failure by excessive deformation and elastic instability.

b) Saddle supports for horizontal vessels shall be configured to provide continuous support for at least one-third of the shell circumference, or  $\theta = 120.0 \ deg$ . Since the contact angle specified is  $123.0 \ deg$ , the geometry is acceptable.

Paragraph 4.15.3.2,

c) The vessel is composed of a cylindrical shell with formed heads at each end that is supported by two equally spaced saddle supports. The moment at the saddle,  $M_1$ , the moment at the center of the vessel,  $M_2$ , and the shear force at the saddle, T, may be computed if the distance between the saddle centerline and head tangent line satisfies the following limit.

$$\{a = 41.0\} \le \{0.25L = 0.25(292.0) = 73.0\}$$
 Satisfied

Bending Moment at the Saddle

$$M_{1} = -Qa \left( 1 - \frac{1 - \left(\frac{a}{L}\right) + \frac{R_{m}^{2} - h_{2}^{2}}{2aL}}{1 + \frac{4h_{2}}{3L}} \right)$$
$$M_{1} = -(50459.0)(41.0) \left( 1 - \frac{1 - \left(\frac{41.0}{292.0}\right) + \frac{(31.5625)^{2} - (16.5)^{2}}{2(41.0)(292.0)}}{1 + \frac{4(16.5)}{3(292.0)}} \right) = -356913.7 \text{ in - lbs}$$

Bending Moment at the Center of the Vessel

$$M_{2} = \frac{QL}{4} \left( \frac{1 + \frac{2(R_{m}^{2} - h_{2}^{2})}{L^{2}}}{1 + \frac{4h_{2}}{3L}} - \frac{4a}{L} \right)$$
$$M_{2} = \frac{50459.0(292.0)}{4} \left( \frac{1 + \frac{2[(31.5625)^{2} - (16.5)^{2}]}{(292.0)^{2}}}{1 + \frac{4(16.5)}{3(292.0)}} - \frac{4(41.0)}{292.0} \right) = 1414775.7 \text{ in - lbs}$$

Shear Force at the Saddle

$$T = \frac{Q(L-2a)}{L+\frac{4h_2}{3}} = \frac{50459.0[292.0-2(41.0)]}{292.0+\frac{4(16.5)}{3}} = 33746.5 \ lbs$$

Paragraph 4.15.3.3,

d) The longitudinal membrane plus bending stresses in the cylindrical shell between the supports are given by the following equations.

At the top of shell:

$$\sigma_1 = \frac{PR_m}{2t} - \frac{M_2}{\pi R_m^2 t} = \frac{2074(31.5625)}{2(2.875)} - \frac{1414775.7}{\pi (31.5625)^2 (2.875)} = 11227.2 \text{ psi}$$

Note: A load combination that includes zero internal pressure and the vessel full of contents would provide the largest compressive stress at the top of the shell, and should be checked as part of the design.

At the bottom of the shell:

$$\sigma_2 = \frac{PR_m}{2t} + \frac{M_2}{\pi R_m^2 t} = \frac{2074(31.5625)}{2(2.875)} + \frac{1414775.7}{\pi (31.5625)^2 (2.875)} = 11541.7 \text{ psi}$$

e) The longitudinal stresses in the cylindrical shell at the support location are given by the following equations. The values of these stresses depend on the rigidity of the shell at the saddle support. The cylindrical shell may be considered as suitably stiffened if it incorporates stiffening rings at, or on both sides of the saddle support, or if the support is sufficiently close defined as  $a \le 0.5R_m$  to the elliptical head.

$$\{a = 41.0\} > \{0.5R_m = 0.5(31.5625) = 15.7813\}$$
 Not Satisfied

Therefore, for an unstiffened shell, calculate the maximum values of longitudinal membrane plus bending stresses at the saddle support as follows.

At points A and B in Figure 4.15.5:

$$\sigma_3^* = \frac{PR_m}{2t} - \frac{M_1}{K_1 \pi R_m^2 t} = \frac{2074(31.5625)}{2(2.875)} - \frac{-356913.7}{0.0682(\pi)(31.5625)^2(2.875)} = 11740.5 \text{ psi}$$

Where the coefficient  $K_1$  is found in Table 4.15.1,

$$K_{1} = \frac{\Delta + \sin \Delta \cdot \cos \Delta - \frac{2 \sin^{2} \Delta}{\Delta}}{\pi \left(\frac{\sin \Delta}{\Delta} - \cos \Delta\right)} = \frac{1.4181 + \sin \left[1.4181\right] \cdot \cos \left[1.4181\right] - \frac{2 \sin^{2} \left[1.4181\right]}{1.4181}}{\pi \left(\frac{\sin \left[1.4181\right]}{1.4181} - \cos \left[1.4181\right]\right)}$$

$$K_{1} = 0.1114$$

$$\Delta = \frac{\pi}{6} + \frac{5\theta}{12} = \frac{\pi}{6} + \frac{5 \left\lfloor (123.0) \left(\frac{\pi}{180}\right) \right\rfloor}{12} = 1.4181 \ rad$$

At the bottom of the shell:

$$\sigma_4^* = \frac{PR_m}{2t} + \frac{M_1}{K_1^* \pi R_m^2 t} = \frac{2074(31.5625)}{2(2.875)} + \frac{-356913.7}{0.2003(\pi)(31.5625)^2(2.875)}$$
  
$$\sigma_4^* = 11186.4 \ psi$$

Where the coefficient  $K_1^*$  is found in Table 4.15.1,

$$K_{1}^{*} = \frac{\Delta + \sin\Delta \cdot \cos\Delta - \frac{2\sin^{2}\Delta}{\Delta}}{\pi \left(1 - \frac{\sin\Delta}{\Delta}\right)} = \frac{1.4181 + \sin\left[1.4181\right] \cdot \cos\left[1.4181\right] - \frac{2\sin^{2}\left[1.4181\right]}{1.4181}}{\pi \left(1 - \frac{\sin\left[1.4181\right]}{1.4181}\right)}$$

 $K_1^* = 0.2003$ 

f) Acceptance Criteria:

$$\begin{split} |\sigma_1| &\leq SE \to \{ |11227.2| \ psi \} \leq \{ 23500(1.0) = 23500 \ psi \} \\ |\sigma_2| &\leq SE \to \{ |11541.7| \ psi \} \leq \{ 23500(1.0) = 23500 \ psi \} \\ |\sigma_3^*| &\leq SE \to \{ |11740.5| \ psi \} \leq \{ 23500(1.0) = 23500 \ psi \} \\ |\sigma_4^*| &\leq SE \to \{ |1186.4| \ psi \} \leq \{ 23500(1.0) = 23500 \ psi \} \\ \end{split}$$

Since all calculated stresses are positive (tensile), the compressive stress check per paragraph 4.15.3.3.c.2 is not required.

Paragraph 4.15.3.4,

The shear stress in the cylindrical shell without stiffening ring(s) that is not stiffened by a formed head,  $\{a = 41.0 \ in\} > \{0.5R_m = 0.5(31.5625) = 15.7813 \ in\}$ , is calculated as follows.

$$\tau_2 = \frac{K_2 T}{R_m t} = \frac{1.1229(33746.5)}{31.5625(2.875)} = 417.6 \ psi$$

Where the coefficient  $K_2$  is found in Table 4.15.1,

$$K_{2} = \frac{\sin \alpha}{\pi - \alpha + \sin \alpha \cdot \cos \alpha} = \frac{\sin[1.9648]}{\pi - (1.9648) + \sin[1.9648] \cdot \cos[1.9648]} = 1.1229$$
$$\alpha = 0.95 \left(\pi - \frac{\theta}{2}\right) = 0.95 \left(\pi - \frac{123.0 \left(\frac{\pi}{180}\right)}{2}\right) = 1.9648 \ rad$$

Acceptance Criteria, where C = 0.8 for ferritic materials:

$$|\tau_2| \le CS \rightarrow \{ |417.6| \ psi \} \le \{ 0.8(23500) = 18800 \ psi \}$$
 True

Paragraph 4.15.3.5,

g) Maximum circumferential bending moment - the distribution of the circumferential bending moment at the saddle support is dependent on the use of stiffeners at the saddle location. For a cylindrical shell without a stiffening ring, the maximum circumferential bending moment is shown in Figure 4.15.6 Sketch (a) and is calculated as follows.

$$M_{\beta} = K_7 Q R_m = (0.0504)(50459.0)(31.5625) = 80267.7 \text{ in} - lbs$$

Where the coefficient  $K_7$  is found in Table 4.15.1,

$$\begin{aligned} & \text{When } \frac{a}{R_m} \ge 1.0, \ K_7 = K_6 \\ & \left\{ \frac{a}{R_m} = \frac{41.0}{31.5625} = 1.2990 \right\} \ge 1.0 \rightarrow K_7 = K_6 = 0.0504 \\ & K_6 = \left[ \frac{\frac{3\cos\beta}{4} \left(\frac{\sin\beta}{\beta}\right)^2 - \frac{5\sin\beta\cos^2\beta}{4\beta} + \frac{\cos^3\beta}{2} - \frac{\sin\beta}{4\beta} + \frac{\cos^2\beta}{2} - \frac{\sin\beta}{4\beta} + \frac{\cos^2\beta}{4\beta} - \beta\sin\beta \left[ \left(\frac{\sin\beta}{\beta}\right)^2 - \frac{1}{2} - \frac{\sin2\beta}{4\beta} \right] \right] \\ & 2\pi \left[ \left(\frac{\sin\beta}{\beta}\right)^2 - \frac{1}{2} - \frac{\sin2\beta}{4\beta} \right] \\ & 2\pi \left[ \left(\frac{\sin\beta}{\beta}\right)^2 - \frac{1}{2} - \frac{\sin2\beta}{4\beta} \right] \\ & \left( \frac{\cos^3[2.0682]}{4} - \frac{\sin[2.0682]}{2} - \frac{\sin[2.0682]}{4(2.0682)} + \frac{\cos[2.0682]}{4(2.0682)} + \frac{\cos[2.0682]}{4(2.0682)} \right] \\ & K_6 = \left( \frac{(2.0682)\sin[2.0682] \left[ \left(\frac{\sin[2.0682]}{2.0682}\right)^2 - \frac{1}{2} - \left(\frac{\sin[2(2.0682)]}{4(2.0682)}\right) \right] \\ & \left( \frac{\sin[2.0682]}{2} - \frac{1}{2} - \left(\frac{\sin[2(2.0682)]}{4(2.0682)}\right) \right] \\ & = 0.0504 \end{aligned} \right) \end{aligned}$$
$$\beta = \pi - \frac{\theta}{2} = \pi - \frac{123.0\left(\frac{\pi}{180}\right)}{2} = 2.0682 \ rad$$

h) Width of cylindrical shell - the width of the cylindrical shell that contributes to the strength of the cylindrical shell at the saddle location shall be determined as follows.

$$x_1, x_2 \le 0.78\sqrt{R_m t} = 0.78\sqrt{31.5625(2.875)} = 7.4302$$
 in

If the width  $(0.5b + x_1)$  extends beyond the limit of *a*, as shown in Figure 4.15.2, then the width  $x_1$  shall be reduced such as not to exceed *a*.

$$\{(0.5b+x_1)=0.5(8.0)+7.4302=11.4302 \text{ in}\} \le \{a=41.0 \text{ in}\}$$

- i) Circumferential stresses in the cylindrical shell without stiffening ring(s).
  - 1) The maximum compressive circumferential membrane stress in the cylindrical shell at the base of the saddle support shall be calculated as follows.

Satisfied

$$\sigma_6 = \frac{-K_5Qk}{t(b+x_1+x_2)} = \frac{-0.7492(50459.0)(0.1)}{2.875(8.0+7.4302+7.4302)} = -57.5 \ psi$$

Where the coefficient  $K_5$  is found in Table 4.15.1,

$$K_5 = \frac{1 + \cos \alpha}{\pi - \alpha + \sin \alpha \cdot \cos \alpha} = \frac{1 + \cos[1.9648]}{\pi - (1.9648) + \sin[1.9648] \cdot \cos[1.9648]} = 0.7492$$

k = 0.1 when the vessel is welded to the saddle support

- The circumferential compressive membrane plus bending stress at Points G and H of Figure 4.15.6 Sketch (a) is determined as follows.
  - iii) If  $L \ge 8R_m$ , then the circumferential compressive membrane plus bending stress shall be computed using Equation (4.15.24).

Since  $\{L = 292.0 \ in\} \ge \{8R_m = 8(31.5625) = 252.5 \ in\}$ , the criterion is satisfied.

$$\sigma_{7} = \frac{-Q}{4t(b+x_{1}+x_{2})} - \frac{3K_{7}Q}{2t^{2}}$$
  
$$\sigma_{7} = \frac{-(50459.0)}{4(2.875)(8+7.4302+7.4302)} - \frac{3(0.0504)(50459.0)}{2(2.875)^{2}} = -653.4 \text{ psi}$$

3) The stresses at  $\sigma_6$  and  $\sigma_7$  may be reduced by adding a reinforcement or wear plate at the saddle location that is welded to the cylindrical shell.

A wear plate was not specified in this example.

j) Circumferential stress in the cylindrical shell with a stiffening ring along the plane of the saddle support.

Stiffeners were not specified in the example.

 k) Circumferential stress in the cylindrical shell with stiffening rings on both sides of the saddle support.

Stiffeners were not specified in the example.

Acceptance Criteria:

$$\{ | \sigma_6 |= |57.5| \ psi \} \le \{ S = 23500 \ psi \}$$
 True  
 
$$\{ | \sigma_7 |= |653.4| \ psi \} \le \{ 1.25S = 1.25(23500) = 29375 \ psi \}$$
 True

Paragraph 4.15.3.6,

The horizontal force at the minimum section at the low point of the saddle is given by Equation (4.15.42). The saddle shall be designed to resist this force.

$$F_{h} = Q \left( \frac{1 + \cos \beta - 0.5 \sin^{2} \beta}{\pi - \beta + \sin \beta \cdot \cos \beta} \right)$$
  

$$F_{h} = (50459.0) \left( \frac{1 + \cos[2.0682] - 0.5 \sin^{2}[2.0682]}{\pi - (2.0682) + \sin[2.0682] \cdot \cos[2.0682]} \right) = 10545.1 \ lbs$$

Note: The horizontal splitting force is equal to the sum of all of the horizontal reactions at the saddle due to the weight loading of the vessel. The splitting force is used to calculate tension stress and bending stress in the web of the saddle. The following provides one possible method of calculating the tension and bending stress in the web and its acceptance criteria. However, other methods may also be deemed acceptable by the Manufacturer and accepted by the Authorized Inspector.

The membrane stress is given by,

$$\left\{\sigma_t = \frac{F_h}{A_s}\right\} \leq \left\{0.6S_y\right\}$$

where  $A_s$  is the cross-sectional area of the web at the low point of the saddle with units of  $in^2$ , and  $S_v$  is the yield stress of the saddle material with units of psi.

The bending stress is given by,

$$\left\{\sigma_{b} = \frac{F_{h} \cdot d \cdot c}{I}\right\} \leq \left\{0.66S_{y}\right\}$$

where d is the moment arm of the horizontal splitting force, measured from the center of gravity of the saddle arc to the bottom of the saddle baseplate with units of in, c is the distance from the centroid of the saddle composite section to the extreme fiber with units of in, I is the moment of inertia of the composite section of the saddle with units of  $in^4$ , and  $S_y$  is the yield stress of the saddle material with units of psi.



Figure E4.15.1 - Saddle Details

## 4.15.2 Example E4.15.2 – Vertical Vessel, Skirt Design

Determine if the proposed cylindrical vessel skirt is adequately designed considering the following loading conditions.

<u>Skirt Data:</u>

• Mate	erial	=	SA-516, Grade 70
• Des	ign Temperature	=	300°F
• Skir	t Inside Diameter	=	150.0 in
• Thic	kness	=	0.625 in
• Len	gth of Skirt	=	147.0 in
• Allo	wable Stress at Design Temperature	=	22400 psi
• Mod	lulus of Elasticity at Design Temperature	=	$28.3E + 06 \ psi$
• Yiel	d Strength at Design Temperature	=	33600 <i>psi</i>
<ul> <li>Axia</li> </ul>	al Force, Weight	=	-363500 <i>lbs</i>
<ul> <li>Axia</li> </ul>	al Force, Appurtenance Live Loading	=	-85700 <i>lbs</i>
• Ben	ding Moment, Appurtenance Loading	=	90580 in – lbs
• Ben	ding Moment, Earthquake Loading	=	18550000 <i>in–lbs</i>
• Ben	ding Moment, Wind Loading	=	29110000 in – lbs

Adjust variable for corrosion and determine outside dimensions.

 $D = 150.0 + 2(Corrosion \ Allowance) = 150.0 + 2(0.0) = 150.0 \ in$   $R = 0.5D = 0.5(150.0) = 75.0 \ in$   $t = 0.625 - Corosion \ Allowance = 0.625 - 0.0 = 0.625 \ in$   $D_o = 150.0 + 2(Uncorroded \ Thickness) = 150.0 + 2(0.625) = 151.25 \ in$  $R_o = 0.5D_o = 0.5(151.25) = 75.625 \ in$ 

Evaluate per paragraph 4.15.4 with reference to paragraph 4.3.10.

The loads transmitted to the base of the skirt are given in the Table E4.15.2.2. Note that this table is given in terms of the load parameters and load combinations shown in Table 4.1.1 and Table 4.1.2. (Table E4.15.2.1 of this example). As shown in Table E4.15.2.1, the acceptance criteria is that the general primary membrane stress for each load case must be less than or equal to the allowable stress at the specified design condition.

In accordance with paragraph 4.3.10.2, the following procedure shall be used to design cylindrical, spherical, and conical shells subjected to internal pressure plus supplemental loads of applied net section axial force, bending moment, and torsional moment. By inspection of the results shown in Table E4.15.2.3, Load Case 5 is determined to be a potential governing load case. The pressure, net section axial force, and bending moment at the location of interest for Load Case 5 are:

 $0.9P + P_s = 0.0 \ psi$   $F_5 = -363500 \ lbs$   $M_5 = 17466000 \ in - lbs$  $M_{15} = 0.0 \ in - lbs$ 

Determine applicability of the rules of paragraph 4.3.10 based on satisfaction of the following requirements.

The section of interest is at least  $2.5\sqrt{Rt}$  away from any major structural discontinuity.

$$2.5\sqrt{Rt} = 2.5\sqrt{(75.0)(0.625)} = 17.1163 \text{ in}$$

Shear force is not applicable.

The shell R/t ratio is greater than 3.0, or:

$$\left\{ R / t = \frac{75.0}{0.625} = 120.0 \right\} > 3.0$$
 True

a) STEP 1 – Calculate the membrane stress. For the skirt, weld joint efficiency is set as E = 1.0. Note, that the maximum bending stress occurs at  $\theta = 0.0 \ deg$ .

$$\sigma_{\theta m} = \frac{P}{E(D_o - D)} = \frac{P}{E(151.25 - 150.0)} = 0.0 \ psi$$
  
$$\sigma_{sm} = \frac{1}{E} \left( \frac{PD^2}{D_o^2 - D^2} + \frac{4F}{\pi (D_o^2 - D^2)} \pm \frac{32MD_o \cos[\theta]}{\pi (D_o^4 - D^4)} \right)$$
  
$$\sigma_{sm} = \frac{1}{1.0} \left( 0.0 + \frac{4(-363500)}{\pi ((151.25)^2 - (150.0)^2)} \pm \frac{32(17466000)(151.25)\cos[0.0]}{\pi ((151.25)^4 - (150.0)^4)} \right)$$
  
$$\sigma_{sm} = \begin{cases} -1229.0724 + 1574.7814 = 345.7090 \ psi \\ -1229.0724 - 1574.7814 = -2803.8538 \ psi \end{cases}$$
  
$$\tau = \frac{16M_t D_o}{\pi (D_o^4 - D^4)} = 0.0 \ psi$$

b) STEP 2 – Calculate the principal stresses.

$$\sigma_{1} = 0.5 \left( \sigma_{\theta m} + \sigma_{sm} + \sqrt{\left( \sigma_{\theta m} - \sigma_{sm} \right)^{2} + 4\left( \tau \right)^{2}} \right)$$
  
$$\sigma_{1} = \begin{cases} 0.5 \left( 0.0 + 345.7090 + \sqrt{\left( 0.0 - 345.7090 \right)^{2} + 4\left( 0.0 \right)^{2}} \right) = 345.7090 \ psi \\ 0.5 \left( 0.0 + \left( -2803.8538 \right) + \sqrt{\left( 0.0 - \left( -2803.8538 \right) \right)^{2} + 4\left( 0.0 \right)^{2}} \right)} = 0.0 \ psi \end{cases}$$

$$\sigma_{2} = 0.5 \left( \sigma_{\theta m} + \sigma_{sm} - \sqrt{(\sigma_{\theta m} - \sigma_{sm})^{2} + 4(\tau)^{2}} \right)$$
  
$$\sigma_{2} = \begin{cases} 0.5 \left( 0.0 + 345.7090 - \sqrt{(0.0 - 345.7090)^{2} + 4(0.0)^{2}} \right) = 0.0 \ psi \\ 0.5 \left( 0.0 + (-2803.8538) - \sqrt{(0.0 - (-2803.8538))^{2} + 4(0.0)^{2}} \right) = -2803.8538 \ psi \end{cases}$$

 $\sigma_3 = \sigma_r = -0.5P = 0.0 \ psi$ 

c) STEP 3 – Check the allowable stress acceptance criteria.

$$\sigma_{e} = \frac{1}{\sqrt{2}} \left[ \left( \sigma_{1} - \sigma_{2} \right)^{2} + \left( \sigma_{2} - \sigma_{3} \right)^{2} + \left( \sigma_{3} - \sigma_{1} \right)^{2} \right]^{0.5} \le SE$$

$$\sigma_{e} = \begin{cases} \frac{1}{\sqrt{2}} \left[ \left( 345.7090 - 0.0 \right)^{2} + \left( 0.0 - 345.7090 \right)^{2} \right]^{0.5} = 345.7090 \ psi \\ \left( 0.0 - \left( 0.0 \right)^{2} + \left( 0.0 - 345.7090 \right)^{2} \right]^{0.5} = 345.7090 \ psi \\ \frac{1}{\sqrt{2}} \left[ \left( 0.0 - \left( -2803.8538 \right) \right)^{2} + \left( \left( -2803.8538 \right) - 0.0 \right)^{2} + \left( 0.0 - 0.0 \right)^{2} \right]^{0.5} = 2803.8538 \ psi \\ \left\{ \sigma_{e} = 345.7 \ psi \\ \sigma_{e} = 2803.9 \ psi \\ \right\} \le \{SE = 22400 \ psi \}$$

Since the equivalent stress is less than the acceptance criteria, the shell section is adequately designed considering Load Case 5.

d) STEP 4 – For cylindrical and conical shells, if the axial membrane stress,  $\sigma_{sm}$  is compressive, then Equation (4.3.45) shall be satisfied where  $F_{xa}$  is evaluated using paragraph 4.4.12.2 with  $\lambda = 0.15$ .

$$\sigma_{sm} \leq F_{xa}$$

Since  $\sigma_{sm}$  is compressive,  $\{\sigma_{sm} = -2803.8538 \ psi < 0\}$ , a buckling check is required.

VIII-2, paragraph 4.4.12.2.b - Axial Compressive Stress Acting Alone.

In accordance with paragraph 4.4.12.2.b, the value of  $F_{\rm xa}$  is calculated as follows, with  $\lambda=0.15$  .

The design factor *FS* used in paragraph 4.4.12.2.b is dependent on the predicted buckling stress  $F_{ic}$  and the material's yield strength,  $S_y$  as shown in paragraph 4.4.2. An initial calculation is required to determine the value of  $F_{xa}$  by setting FS = 1.0, with  $F_{ic} = F_{xa}$ . The initial value of  $F_{ic}$  is then compared to  $S_y$  as shown in paragraph 4.4.2 and the value of *FS* is determined. This computed value of *FS* is then used in paragraph 4.4.12.2.b.

For  $\lambda_c=0.15$  , (Local Buckling)

$$F_{xa} = \min[F_{xa1}, F_{xa2}]$$

$$\frac{D_o}{t} = \frac{151.25}{0.625} = 242.0$$

$$M_x = \frac{L}{\sqrt{R_o t}} = \frac{147.0}{\sqrt{75.625(0.625)}} = 21.3818$$

Since  $135 < \frac{D_o}{t} \le 600$ , calculate  $F_{xa1}$  as follows with an initial value of FS = 1.0.

$$F_{xa1} = \frac{466S_y}{FS\left(331 + \frac{D_o}{t}\right)} = \frac{466(33600)}{1.0\left(331 + \frac{151.25}{0.625}\right)} = 27325.6545 \ psi$$

The value of  ${\it F}_{\rm xa2}$  is calculated as follows with an initial value of  ${\it FS}\,{=}\,1.0$  .

$$F_{xa2} = \frac{F_{xe}}{FS}$$
$$F_{xe} = \frac{C_x E_y t}{D_o}$$

Since  $\frac{D_o}{t} \le 1247$ , calculate  $C_x$  as follows:

$$C_{x} = \min\left[\frac{409\overline{c}}{\left(389 + \frac{D_{o}}{t}\right)}, \ 0.9\right]$$

Since  $M_{x} \ge 15$ , calculate  $\overline{c}$  as follows:

$$\overline{c} = 1.0$$

$$C_x = \min\left[\frac{409(1.0)}{389 + \frac{151.25}{0.625}}, \ 0.9\right] = 0.6482$$

Therefore,

$$F_{xe} = \frac{0.6482(28.3E + 06)(0.625)}{151.25} = 75801.9008 \ psi$$
$$F_{xa2} = \frac{F_{xe}}{FS} = \frac{75801.9008}{1.0} = 75801.9008 \ psi$$
$$F_{xa} = \min[27325.6545, 75801.9008] = 27325.6545 \ psi$$

With a value of  $F_{ic} = F_{xa} = 27325.6545$ , in accordance with paragraph 4.4.2, the value of *FS* is determined as follows.

Since 
$$\{0.55S_y = 0.55(33600) = 18480\} \le \{F_{ic} = 27325.6545\} \le \{S_y = 33600\}$$
  
 $FS = 2.407 - 0.741 \left(\frac{F_{ic}}{S_y}\right) = 2.407 - 0.741 \left(\frac{27325.6545}{33600}\right) = 1.8044$ 

Using this computed value of FS = 1.8044 in paragraph 4.4.12.2.b,  $F_{xa}$  is calculated as follows.

$$F_{xa1} = \frac{466S_y}{FS\left(331 + \frac{D_o}{t}\right)} = \frac{466(33600)}{1.8044\left(331 + \frac{151.25}{0.625}\right)} = 15143.9007 \ psi$$
$$F_{xa2} = \frac{F_{xe}}{FS} = \frac{75801.9008}{1.8044} = 42009.4773 \ psi$$
$$F_{xa} = \min[15143.9007, 42009.4773] = 15143.9007 \ psi$$

Compare the calculated axial compressive membrane stress,  $\sigma_{sm}$  to the allowable axial compressive membrane stress,  $F_{xa}$  per following criteria

$$\{\sigma_{sm} = 2803.9 \ psi\} \le \{F_{xa} = 15143.9 \ psi\}$$
 True

Therefore, local buckling due to axial compressive membrane stress is not a concern.

Table 4.1.1 – Design Loads				
Design Load Parameter	Description			
Р	Internal or External Specified Design Pressure (see paragraph 4.1.5.2.a)			
$P_s$	Static head from liquid or bulk materials (e.g. catalyst)			
D	<ul> <li>Dead weight of the vessel, contents, and appurtenances at the location of interest, including the following:</li> <li>Weight of vessel including internals, supports (e.g. skirts, lugs, saddles, and legs), and appurtenances (e.g. platforms, ladders, etc.)</li> <li>Weight of vessel contents under operating and test conditions</li> <li>Refractory linings, insulation</li> <li>Static reactions from the weight of attached equipment, such as motors, machinery, other vessels, and piping</li> <li>Transportation Loads (The static forces obtained as equivalent to the dynamic loads experienced during normal operation of a transport vessel – see paragraph 1.2.1.2.b)</li> </ul>			
L	<ul> <li>Appurtenance Live loading</li> <li>Effects of fluid flow, steady state or transient</li> <li>Loads resulting from wave action</li> </ul>			
Ε	Earthquake loads (see ASCE 7 for the specific definition of the earthquake load, as applicable)			
W	Wind Loads (See 4.1.5.3.b)			
S	Snow Loads			
F	Loads due to Deflagration			

Table E4.15.2.1 -	Design Loads a	nd Load Combinatio	ons from VIII-2

Table 4.1.2 – Design Load Combinations		
Design Load Combination (1)	General Primary Membrane Allowable Stress (2)	
$P + P_s + D$	S	
$P+P_s+D+L$	S	
$P+P_s+D+S$	S	
$0.9P + P_s + D + 0.75L + 0.75S$	S	
$0.9P + P_s + D + (0.6W \text{ or } 0.7E)$	S	
$0.9P + P_s + D + 0.75(0.6W \text{ or } 0.7E) + 0.75L + 0.75S$	S	
0.6D + (0.6W  or  0.7E) (3)	S	
$P_s + D + F$	See Annex 4.D	

1) The parameters used in the Design Load Combination column are defined in Table 4.1.1.

2) S is the allowable stress for the load case combination (see paragraph 4.1.5.3.c)

3) This load combination addresses an overturning condition for foundation design. It does not apply to design of anchorage (if any) to the foundation. Refer to ASCE/SEI 7-10, 2.4.1 Exception 2 for an additional reduction to W that may be applicable.

Design Load Parameter	Description	Magnitude of Pressure, Force and Moment	
Р	Internal or External Specified Design Pressure (see paragraph 4.1.5.2.a)	P = 0.0	
$P_s$	Static head from liquid or bulk materials (e.g. catalyst)	$P_{s} = 0.0$	
D	The dead weight of the vessel including skirt, contents, and appurtenances at the location of interest	$D_F = -363500 \ lbs$ $D_M = 0.0 \ in - lbs$	
L	Appurtenance live loading and effects of fluid flow	$L_F = -85700 \ lbs$ $L_M = 90580 \ in - lbs$	
E	Earthquake loads	$E_F = 0.0 \ lbs$ $E_M = 18550000 \ in - lbs$	
W	Wind Loads	$W_F = 0.0 \ lbs$ $W_M = 29110000 \ in - lbs$	
S	Snow Loads	$S_F = 0.0 \ lbs$ $S_M = 0.0 \ in - lbs$	
F	Loads due to Deflagration	$F_F = 0.0 \ lbs$ $F_M = 0.0 \ in - lbs$	

# Table E4.15.2.2 - Design Loads (Net-Section Axial Force and Bending Moment) at the Location of Interest

Based on these loads, the shell is required to be designed for the load case combinations shown in Table E4.15.2.3. Note that this table is given in terms of the load combinations shown in Table 4.1.2 (Table E4.15.2.1 of this example).

Load Case	Design Load Combination	Magnitude of Pressure, Force and Moment	General Primary Membrane Allowable Stress
1	$P + P_s + D$	$P + P_s = 0.0 \ psi$ $F_1 = -363500 \ lbs$ $M_1 = 0.0 \ in - lbs$	S
2	$P + P_s + D + L$	$P + P_s = 0.0 \ psi$ $F_2 = -449200 \ lbs$ $M_2 = 90580 \ in - lbs$	S
3	$P + P_s + D + S$	$P + P_s = 0.0 \ psi$ $F_3 = -363500 \ lbs$ $M_3 = 0.0 \ in - lbs$	S
4	$0.9P + P_s + D + 0.75L + 0.75S$	$0.9P + P_s = 0.0 \ psi$ $F_4 = -427775 \ lbs$ $M_4 = 67935 \ in - lbs$	S
5	$0.9P + P_s + D + (0.6W \text{ or } 0.7E)$	$0.9P + P_s = 0.0 \ psi$ $F_5 = -363500 \ lbs$ $M_5 = 17466000 \ in - lbs$	S
6	$\begin{pmatrix} 0.9P + P_s + D + 0.75(0.6W \text{ or } 0.7E) + \\ 0.75L + 0.75S \end{pmatrix}$	$0.9P + P_s = 0.0 \ psi$ $F_6 = -427775 \ lbs$ $M_6 = 13167435 \ in - lbs$	S
7	0.6D + (0.6W  or  0.7E) Anchorage is included in the design. Therefore, consideration of this load combination is not required.	$F_7 = -218100 \ lbs$ $M_7 = 17466000 \ in - lbs$	S
8	$P_s + D + F$	$P_{s} = 0.0 \ psi$ $F_{8} = -363500 \ lbs$ $M_{8} = 0.0 \ in - lbs$	See Annex 4.D

 Table E4.15.2.3 - Load Case Combination at the Location of Interest

## 4.16 Flanged Joints

#### 4.16.1 Example E4.16.1 – Integral Type

Determine if the stresses in the heat exchanger girth flange are with acceptable limits, considering the following design conditions. The flange is of an integral type and is attached to a cylindrical shell with a Category C, Type 1 butt weld and has been 100% radiographically examined. See Figure E4.16.1.

## General Data:

•	Cylinder Material	=	<i>SA</i> –516, <i>Grade</i> 70
•	Design Conditions	=	135 psig@650°F
•	Allowable Stress at Design Temperature	=	18800 <i>psi</i>
•	Allowable Stress at Ambient Temperature	=	25300 psi
•	Corrosion Allowance	=	0.125 in
Fla	ange Data		
•	Material	=	SA-105
•	Allowable Stress at Design Temperature	=	17800 <i>psi</i>
•	Allowable Stress at Ambient Temperature	=	24000 psi
•	Modulus of Elasticity at Design Temperature	=	$26.0E + 06 \ psi$
•	Modulus of Elasticity at Ambient Temperature	=	$29.4E + 06 \ psi$
Bo	It Data		
•	Material	=	SA-193, Grade B7
•	Allowable Stress at Design Temperature	=	25000 psi
•	Allowable Stress at Ambient Temperature	=	25000 psi
•	Diameter	=	0.75 in
•	Number of Bolts	=	44
•	Root area	=	$0.302 in^2$
<u>Ga</u>	isket Data		
•	Material	=	Flat Metal Jacketed (Iron/Soft Steel)
•	Gasket Factor	=	3.75
•	Seating Stress	=	7600 <i>psi</i>
•	Inside Diameter	=	29.0 in
•	Outside Diameter	=	30.0 <i>in</i>

Evaluate the girth flange in accordance with paragraph 4.16.

Paragraph 4.16.6, Design Bolt Loads. The procedure to determine the bolt loads for the operating and gasket seating conditions is shown below.

a) STEP 1 – Determine the design pressure and temperature of the flanged joint

$$P = 135 psig at 650^{\circ}F$$

b) STEP 2 – Select a gasket and determine the gasket factors m and y from Table 4.16.1.

$$m = 3.75$$
  
 $y = 7600 \ psi$ 

c) STEP 3 – Determine the width of the gasket, N, basic gasket seating width,  $b_o$ , the effective gasket seating width, b, and the location of the gasket reaction, G.

$$N = 0.5(GOD - GID) = 0.5(30.0 - 29.0) = 0.500$$
 in

From Table 4.16.3, Facing Sketch Detail 2, Column II,

$$b_o = \frac{w+3N}{8} = \frac{0.125 + 3(0.500)}{8} = 0.2031$$
 in

Where,

w = raised nubbin width = 0.125 in

For 
$$\{b_o = 0.2031 \ in\} \leq \{0.25 \ in\}$$
 ,

$$b = b_o = 0.2031$$
 in  
 $G = mean \ diameter \ of \ the \ gasket \ contact \ face$   
 $G = 0.5(30.0 + 29.0) = 29.5$  in

d) STEP 4 – Determine the design bolt load for the operating condition.

$$W_{o} = \frac{\pi}{4}G^{2}P + 2b\pi GmP \qquad for \ non-self-energized \ gaskets$$
$$W_{o} = \frac{\pi}{4}(29.5)^{2}(135) + 2(0.2031)\pi(29.5)(3.75)(135) = 111329.5 \ lbs$$

e) STEP 5 – Determine the design bolt load for the gasket seating condition.

$$W_g = \left(\frac{A_m + A_b}{2}\right) S_{bg} = \left(\frac{5.7221 + 13.2880}{2}\right) 25000 = 237626.3 \ lbs$$

Where,

$$A_b = (Number of bolts)(Root area of one bolt) = 44(0.302) = 13.2880 in^2$$

$$A_{m} = \max\left[\left(\frac{W_{o} + F_{A} + \frac{4M_{E}}{G}}{S_{bo}}\right), \left(\frac{W_{gs}}{S_{bg}}\right)\right] = \max\left[\left(\frac{111329.5 + 0.0 + 0.0}{25000}\right), \left(\frac{143052.5}{25000}\right)\right]$$
$$A_{m} = \max\left[4.4532, 5.7221\right] = 5.7221 \text{ in}^{2}$$

And,

$$W_{gs} = \pi b G(C_{us}y)$$
 for non-self-energized gaskets  
 $W_{gs} = \pi (0.2031)(29.5)(1.0(7600)) = 143052.5 \ lbs$   
 $F_A = 0$  and  $M_E = 0$  since there are no externally applied net-section axial forces or bending moments.

Paragraph 4.16.7, Flange Design Procedure. The procedure in this paragraph can be used to design circular integral, loose or reverse flanges, subject to internal or external pressure, and external loadings. The procedure incorporates both a strength check and a rigidity check for flange rotation.

a) STEP 1 – Determine the design pressure and temperature of the flanged joint and the external net-section axial force,  $F_A$ , and bending moment,  $M_E$ .

$$P = 135 \ psig \ at \ 650^{\circ}F$$
  
 $F_A = 0$   
 $M_E = 0$ 

b) STEP 2 – Determine the design bolt loads for operating condition  $W_o$ , and the gasket seating condition  $W_g$ , and the corresponding actual bolt load area  $A_b$ , from paragraph 4.16.6.

$$W_o = 111329.5 \ lbs$$
  
 $W_g = 2237626.3 \ lbs$   
 $A_b = 13.288 \ in^2$ 

- c) STEP 3 Determine an initial flange geometry, see Figure E4.16.1, in addition to the information required to determine the bolt load, the following geometric parameters are required.
  - 1) Flange bore

$$B = [26.0 + 2(Corrosion \ Allowance)] = [26.0 + 2(0.125)] = 26.25 \ in$$

2) Bolt circle diameter

C = 31.25 in

3) Outside diameter of the flange

A = 32.875 in

- 4) Flange thickness
  - t = 1.625 0.1875 = 1.4375 in

5) Thickness of the hub at the large end

 $g_1 = (0.5(Hub \ OD \ at \ Back \ of \ Flange - Uncorroded \ Bore) - Corrosion \ Allowance)$  $g_1 = (0.5(27.625 - 26.0) - 0.125) = 0.6875 \ in$ 

6) Thickness of the hub at the small end

$$g_0 = (Hub Thickness at Cylinder Attachment - Corrosion Allowance)$$
  
 $g_0 = (0.4375 - Corrosion Allowance) = (0.4375 - 0.125) = 0.3125 in$ 

7) Hub length

h = 2.125 in

d) STEP 4 – Determine the flange stress factors using the equations in Table 4.16.4 and Table 4.16.5.

$$\begin{split} K &= \frac{A}{B} = \frac{32.875}{26.25} = 1.2524 \\ Y &= \frac{1}{K-1} \Biggl[ 0.66845 + 5.71690 \Biggl( \frac{K^2 \log_{10} K}{K^2 - 1} \Biggr) \Biggr] \\ Y &= \frac{1}{1.2524 - 1} \Biggl[ 0.66845 + 5.71690 \frac{(1.2524)^2 \log_{10} [1.2524]}{(1.2524)^2 - 1} \Biggr] = 8.7565 \\ T &= \frac{K^2 (1 + 8.55246 \log_{10} K) - 1}{(1.04720 + 1.9448K^2)(K - 1)} = \frac{(1.2524)^2 (1 + 8.55246 \log_{10} [1.2524]) - 1}{(1.04720 + 1.9448(1.2524)^2)(1.2524 - 1)} = 1.8175 \\ U &= \frac{K^2 (1 + 8.55246 \log_{10} K) - 1}{1.36136 (K^2 - 1)(K - 1)} = \frac{(1.2524)^2 (1 + 8.55246 \log_{10} [1.2524]) - 1}{1.36136 ((1.2524)^2 - 1)(1.2524 - 1)} = 9.6225 \\ Z &= \frac{(K^2 + 1)}{(K^2 - 1)} = \frac{((1.2524)^2 + 1)}{((1.2524)^2 - 1)} = 4.5180 \\ h_o &= \sqrt{Bg_0} = \sqrt{(26.25)(0.3125)} = 2.8641 in \\ X_g &= \frac{g_1}{g_0} = \frac{0.6875}{0.3125} = 2.2000 \\ X_h &= \frac{h}{h_o} = \frac{2.125}{2.8641} = 0.7419 \end{split}$$

$$F = \begin{pmatrix} 0.897697 - 0.297012 \ln [X_g] + 9.5257 (10^{-3}) \ln [X_h] + \\ 0.123586 (\ln [X_g])^2 + 0.0358580 (\ln [X_h])^2 - 0.194422 (\ln [X_g]) (\ln [X_h]) - \\ 0.0181259 (\ln [X_g])^3 + 0.0129360 (\ln [X_h])^3 - \\ 0.0377693 (\ln [X_g]) (\ln [X_h])^2 + 0.0273791 (\ln [X_g])^2 (\ln [X_h]) \end{pmatrix}$$

$$F = \begin{pmatrix} 0.897697 - 0.297012 \ln [2.20] + 9.5257 (10^{-3}) \ln [0.7419] + 0.123586 (\ln [2.20])^2 + \\ 0.0358580 (\ln [0.7419])^2 - 0.194422 (\ln [2.20]) (\ln [0.7419]) - \\ 0.0181259 (\ln [2.20])^3 + 0.0129360 (\ln [0.7419])^3 - \\ 0.0377693 (\ln [2.20]) (\ln [0.7419])^2 + 0.0273791 (\ln [2.20])^2 (\ln [0.7419]) \end{pmatrix}$$

$$F = \begin{bmatrix} 0.0181259 (\ln[X_g])^3 + 0.0129360 (\ln[X_h])^3 - \\ 0.0377693 (\ln[X_g]) (\ln[X_h])^2 + 0.0273791 (\ln[X_g])^2 (\ln[X_h]) \end{bmatrix} \\ F = \begin{bmatrix} 0.897697 - 0.297012 \ln[2.20] + 9.5257 (10^{-3}) \ln[0.7419] + 0.123586 (\ln[2.20])^2 + \\ 0.0358580 (\ln[0.7419])^2 - 0.194422 (\ln[2.20]) (\ln[0.7419]) - \\ 0.0181259 (\ln[2.20])^3 + 0.0129360 (\ln[0.7419])^3 - \\ 0.0377693 (\ln[2.20]) (\ln[0.7419])^2 + 0.0273791 (\ln[2.20])^2 (\ln[0.7419]) \end{bmatrix}$$

$$F = 0.7695$$
  
For  $0.5 \le X_{\rm r} \le 2.0$ ,

$$V = \begin{pmatrix} 0.0144868 - \frac{0.135977}{X_g} - \frac{0.0461919}{X_h} + \frac{0.560718}{X_g^2} + \frac{0.0529829}{X_h^2} + \\ \frac{0.244313}{X_g X_h} + \frac{0.113929}{X_g^3} - \frac{0.00929265}{X_h^3} - \frac{0.0266293}{X_g X_h^2} - \frac{0.217008}{X_g^2 X_h} \end{pmatrix}$$

$$V = \begin{pmatrix} 0.0144868 - \frac{0.135977}{2.20} - \frac{0.0461919}{0.7419} + \frac{0.560718}{(2.20)^2} + \frac{0.0529829}{(0.7419)^2} + \\ \frac{0.244313}{(2.20)(0.7419)} + \frac{0.113929}{(2.20)^3} - \frac{0.00929265}{(0.7419)^3} - \frac{0.0266293}{(2.20)(0.7419)^2} - \frac{0.217008}{(2.20)^2(0.7419)} \end{pmatrix}$$

$$V = 0.1577$$

$$f = \max \left[ 1.0, \left( \frac{0.0927779 - 0.0336633X_g + 0.964176X_g^2 +}{0.0566286X_h + 0.347074X_h^2 - 4.18699X_h^3}{1 - 5.96093(10^{-3})X_g + 1.62904X_h +} \right) \right]$$

$$f = \max \left[ 1.0, \left( \frac{0.0927779 - 0.0336633(2.20) + 0.964176(2.20)^2 +}{0.0566286(0.7419) + 0.347074(0.7419)^2 - 4.18699(0.7419)^3}{1 - 5.96093(10^{-3})(2.20) + 1.62904(0.7419) +} \right) \right]$$

$$f = 1.0$$

$$d = \frac{Ug_0^2 h_o}{V} = \frac{(9.6225)(0.3125)^2 (2.8641)}{0.1577} = 17.0665 \text{ in}$$
$$e = \frac{F}{h_o} = \frac{0.7695}{2.8641} = 0.2687$$
$$L = \frac{te+1}{T} + \frac{t^3}{d} = \frac{1.4375(0.2687) + 1}{1.8175} + \frac{(1.4375)^3}{17.0665} = 0.9368$$

e) STEP 5 – Determine the flange forces.

$$H_{D} = \frac{\pi}{4}B^{2}P = \frac{\pi}{4}(26.25)^{2}(135) = 73060.4 \ lbs$$
$$H = \frac{\pi}{4}G^{2}P = \frac{\pi}{4}(29.5)^{2}(135) = 92271.5 \ lbs$$
$$H_{T} = H - H_{D} = 92271.5 - 73060.4 = 19211.1 \ lbs$$
$$H_{G} = W_{o} - H = 111329.5 - 92227.5 = 19058.0 \ lbs$$

f) STEP 6 – Determine the flange moment for the operating condition using Equation (4.16.14) or Equation (4.16.15), as applicable. When specified by the user or his designated agent, the maximum bolt spacing  $(B_{smax})$  and the bolt spacing correction factor  $(B_{sc})$  shall be applied in calculating the flange moment for internal pressure using the equations in Table 4.16.11. The flange moment  $M_o$  for the operating condition and the flange moment  $M_g$  for the gasket seating condition without correction for bolt spacing  $B_{sc} = 1$  is used for the calculation of the rigidity index in Step 10. In these equations,  $h_D$ ,  $h_T$ , and  $h_G$  are determined from Table 4.16.6. For integral and loose type flanges, the moment  $M_{oe}$  is calculated using Equation (4.16.16) where and I and  $I_p$  in this equation are determined from Table 4.16.7.

For internal pressure,

$$M_{o} = abs \left[ \left( (H_{D}h_{D} + H_{T}h_{T} + H_{G}h_{G})B_{sc} + M_{oe} \right)F_{s} \right]$$
  

$$M_{o} = abs \left[ \left( (73060.4(2.1563) + 19211.1(1.6875) + 19058.0(0.875))1.0 + 0.0)1.0 \right]$$
  

$$M_{o} = 206634.6 \text{ in} - lbs$$

From Table 4.16.11, the maximum bolt spacing and the bolt spacing correction factor are calculated as follows.

$$B_{smax} = 2a + \frac{6t}{m+0.5} = 2(0.75) + \frac{6(1.4375)}{3.75+0.5} = 3.5294 \text{ in}$$
$$B_{sc} = \sqrt{\frac{B_s}{2a+t}} = \sqrt{\frac{0.7102}{2(0.75)+1.4375}} = 0.4917$$

The actual bolt spacing is determined using the following equation.

$$B_s = \frac{C}{No. of bolts} = \frac{31.25}{44} = 0.7102 in$$

Since  $\{B_s = 0.7102 \ in\} \le \{B_{smax} = 3.5294 \ in\}$ , the value of  $B_{sc} = 1.0$ .

From Table 4.16.6,

$$h_{D} = \frac{C - B - g_{1}}{2} = \frac{31.25 - 26.25 - 0.6875}{2} = 2.1563 \text{ in}$$

$$h_{G} = \frac{C - G}{2} = \frac{31.25 - 29.5}{2} = 0.875 \text{ in}$$

$$h_{T} = \frac{1}{2} \left[ \frac{C - B}{2} + h_{G} \right] = \frac{1}{2} \left[ \frac{31.25 - 26.25}{2} + 0.875 \right] = 1.6875 \text{ in}$$

Since  $F_A = 0$  and  $M_E = 0$ , the flange cross-section bending moment of inertia, I, and polar moment of inertia,  $I_p$ , need not be calculated; and the flange design moment calculation for net-section bending moment and axial force supplemental loads,  $M_{oe} = 0$ . Additionally,  $F_s = 1.0$  for non-split rings.

g) STEP 7 – Determine the flange moment for the gasket seating condition using Equation (4.16.17) or Equation (4.16.18), as applicable.

For internal pressure,

$$M_g = \frac{W_g (C-G)F_s}{2} = \frac{(237626.3)(31.25-29.5)(1.0)}{2} = 207923.0 \text{ in - lbs}$$

 STEP 8 – Determine the flange stresses for the operating and gasket seating conditions using the equations in Table 4.16.8.

Note: As provided in paragraph 4.16.12 for the definition of B, if  $B < 20g_1$ , the designer may substitute the value of  $B_1$  for B in the equation for  $S_H$ , where,

For integral flanges when  $f \ge 1.0$ ,

 $B_1 = B + g_o$ 

Since  $\{B = 26.25 \ in\} \ge \{20g_1 = 20(0.6875) = 13.75 \ in\}$ , the value of  $B = 26.25 \ in$  will be used in the equation for  $S_H$ .

**Operating Condition** 

$$S_{H} = \frac{fM_{o}}{Lg_{1}^{2}B} = \frac{(1.0)(206634.6)}{(0.9368)(0.6875)^{2}(26.25)} = 17777.9 \text{ psi}$$

$$S_{R} = \frac{(1.33te+1)M_{o}}{Lt^{2}B} = \frac{\left[(1.33)(1.4375)(0.2687)+1\right](206634.6)}{(0.9368)(1.4375)^{2}(26.25)} = 6155.4 \text{ psi}$$
$$S_{T} = \frac{YM_{o}}{t^{2}B} - ZS_{R} = \frac{(8.7565)(206634.6)}{(1.4375)^{2}(26.25)} - 4.5180(6155.4) = 5547.0 \text{ psi}$$

**Gasket Seating Condition** 

$$S_{H} = \frac{fM_{g}}{Lg_{1}^{2}B} = \frac{(1.0)(207923.0)}{(0.9368)(0.6875)^{2}(26.25)} = 17888.8 \ psi$$

$$S_{R} = \frac{(1.33te+1)M_{g}}{Lt^{2}B} = \frac{\left[(1.33)(1.4375)(0.2687)+1\right](207923.0)}{(0.9368)(1.4375)^{2}(26.25)} = 6193.8 \ psi$$

$$S_{T} = \frac{YM_{g}}{t^{2}B} - ZS_{R} = \frac{(8.7565)(207923.0)}{(1.4375)^{2}(26.25)} - 4.5180(6193.8) = 5581.5 \ psi$$

STEP 9 – Check the flange stress acceptance criteria. The criteria below shall be evaluated. If the stress criteria are satisfied, go to STEP 10. If the stress criteria are not satisfied, rei) proportion the flange dimensions and go to STEP 4.

Allowable normal stress – The criteria to evaluate the normal stresses for the operating and gasket seating conditions are shown in Table 4.16.9, (for integral type flanges).

**Operating Condition** 

$$S_{H} \leq \min \left[ 1.5S_{fo}, 2.5S_{no} \right]$$
  
$$\{S_{H} = 17777.9 \ psi\} \leq \left\{ \min \left[ 1.5(17800), 2.5(18800) \right] = 26700 \ psi \right\} \qquad True$$

$$\{S_{R} = 6155.4 \ psi\} \le \{S_{fo} = 17800 \ psi\}$$
True

$$\{S_T = 5547.0 \ psi\} \le \{S_{fo} = 17800 \ psi\}$$
 True

$$\left\{\frac{\left(S_{H}+S_{R}\right)}{2}=\frac{\left(17777.9+6155.4\right)}{2}=11966.7 \ psi\right\} \le \left\{S_{fo}=17800 \ psi\right\} \qquad True$$

$$\left\{\frac{\left(S_{H}+S_{T}\right)}{2}=\frac{\left(17777.9+5547.0\right)}{2}=11662.5 \ psi\right\} \le \left\{S_{fo}=17800 \ psi\right\} \qquad True$$

**Gasket Seating Condition** 

$$S_{H} \leq \min \left[ 1.5S_{fg}, 2.5S_{ng} \right]$$

$$\{S_{H} = 17888.8 \ psi\} \leq \left\{ \min \left[ 1.5(24000), 2.5(25300) \right] = 36000 \ psi \right\} \qquad True$$

$$\{S_{R} = 6193.8 \ psi\} \leq \left\{ S_{fg} = 24000 \ psi \right\} \qquad True$$

$$S_R = 6193.8 \ psi\} \le \{S_{fg} = 24000 \ psi\}$$
 True

$$\{S_T = 5581.5 \ psi\} \le \{S_{fg} = 24000 \ psi\}$$
 True

$$\left\{\frac{\left(S_{H}+S_{R}\right)}{2}=\frac{\left(17888.8+6193.8\right)}{2}=12041.3 \ psi\right\} \le \left\{S_{fg}=24000 \ psi\right\} \qquad True$$

$$\left\{ \frac{\left(S_{H} + S_{T}\right)}{2} = \frac{\left(17888.8 + 5581.5\right)}{2} = 11735.2 \ psi \right\} \le \left\{S_{fg} = 24000 \ psi\right\} \qquad True$$

j) STEP 10 – Check the flange rigidity criterion in Table 4.16.10. If the flange rigidity criterion is satisfied, then the design is complete. If the flange rigidity criterion is not satisfied, then reproportion the flange dimensions and go to STEP 3.

**Operating Condition** 

$$J = \frac{52.14VM_o}{LE_{yo}g_0^2 K_R h_o} \le 1.0$$

$$\left\{ J = \frac{52.14(0.1577)(206634.6)}{(0.9368)(26.0E + 06)(0.3125)^2(0.3)(2.8641)} = 0.8313 \right\} \le 1.0 \qquad True$$

Where,  $K_R = 0.3$  for integral flanges

**Gasket Seating Condition** 

$$J = \frac{52.14VM_g}{LE_{yg}g_o^2 K_R h_o} \le 1.0$$

$$\left\{ J = \frac{52.14(0.1577)(207923.0)}{(0.9368)(29.4E + 06)(0.3125)^2(0.3)(2.8641)} = 0.7398 \right\} \le 1.0 \qquad True$$

Where,  $K_R = 0.3$  for integral flanges

Since the acceptance criteria are satisfied, the design is complete.



# Figure E4.16.1 - Flanged Joints

Note: The blind flange bolted to the heat exchanger girth flange is evaluated in Example Problem E4.6.1.

# 4.16.2 Example E4.16.2 – Loose Type

Determine if the stresses in the ASME B16.5, Class 300, NPS 20 Slip–on Flange are with acceptable limits, considering the following design conditions. The flange is of a loose type with hub and is attached to a cylindrical shell with Category C, Type 10 fillet welds. See Table 4.2.9, Detail 1.

#### General Data:

•	Cylinder Material	=	SA-516, Grade 70
•	Design Conditions	=	450 psig@650°F
•	Allowable Stress at Design Temperature	=	18800 <i>psi</i>
•	Allowable Stress at Ambient Temperature	=	25300 psi
•	Corrosion Allowance	=	0.0 <i>in</i>
Fla	ange Data		
•	Material	=	SA-105
•	Allowable Stress at Design Temperature	=	17800 <i>psi</i>
•	Allowable Stress at Ambient Temperature	=	24000 psi
•	Modulus of Elasticity at Design Temperature	=	$26.0E + 06 \ psi$
•	Modulus of Elasticity at Ambient Temperature	=	$29.4E + 06 \ psi$
<u>Bc</u>	It Data		
•	Material	=	SA-193, Grade B7
•	Allowable Stress at Design Temperature	=	25000 psi
•	Allowable Stress at Ambient Temperature	=	25000 psi
•	Diameter	=	1.25 in
•	Number of Bolts	=	24
•	Root area	=	$0.929 in^2$
<u>Ga</u>	asket Data		
•	Material	=	Kammprofile
•	Gasket Factor	=	2.0
•	Seating Stress	=	2500 psi
•	Inside Diameter	=	20.875 in
•	Outside Diameter	=	22.875 in

Evaluate the flange in accordance with paragraph 4.16.

Paragraph 4.16.6, Design Bolt Loads. The procedure to determine the bolt loads for the operating and gasket seating conditions is shown below.

a) STEP 1 – Determine the design pressure and temperature of the flanged joint

 $P = 450 psig at 650^{\circ}F$ 

b) STEP 2 – Select a gasket and determine the gasket factors m and y from Table 4.16.1.

m = 2.0 $y = 2500 \ psi$ 

Note: Table 4.16.1 provides a list of many commonly used gasket materials and contact facings with suggested design values of m and y that have generally proved satisfactory in actual service when using effective seating width b given in Table 4.16.3. The design values and other details given in this table are suggested only and are not mandatory.

For this example, gasket manufacturer's suggested m and y values were used.

c) STEP 3 – Determine the width of the gasket, N, basic gasket seating width,  $b_o$ , the effective gasket seating width, b, and the location of the gasket reaction, G.

$$N = 0.5(GOD - GID) = 0.5(22.875 - 20.875) = 1.0 in$$

From Table 4.16.3, Facing Sketch Detail 1a, Column II,

$$b_o = \frac{N}{2} = \frac{1.0}{2} = 0.500$$
 in

For  $\{b_o = 0.500 \ in\} > \{0.25 \ in\}$ ,

$$b = 0.5C_{ul}\sqrt{\frac{b_o}{C_{ul}}} = 0.5(1.0)\sqrt{\frac{0.500}{1.0}} = 0.3536 \text{ in}$$
$$G = G_c - 2b = 22.875 - 2(0.3536) = 22.1678 \text{ in}$$

Where,

$$C_{ul} = 1.0$$
, for US Custumary Units  
 $G_C = \min[Gasket \ OD, \ Flange \ Face \ OD] = \min[22.875, 23.0] = 22.875 \ in$ 

d) STEP 4 – Determine the design bolt load for the operating condition.

$$W_{o} = \frac{\pi}{4}G^{2}P + 2b\pi GmP \qquad for \ non-self-energized \ gaskets$$
$$W_{o} = \frac{\pi}{4}(22.1678)^{2}(450) + 2(0.3536)\pi(22.1678)(2.0)(450) = 218005.0 \ lbs$$

e) STEP 5 – Determine the design bolt load for the gasket seating condition.

$$W_g = \left(\frac{A_m + A_b}{2}\right) S_{bg} = \left(\frac{8.7202 + 22.2960}{2}\right) 25000 = 387702.5 \ lbs$$

Where,

$$A_b = (Number of bolts)(Root area of one bolt) = 24(0.929) = 22.2960 in^2$$

$$A_{m} = \max\left[\left(\frac{W_{o} + F_{A} + \frac{4M_{E}}{G}}{S_{bo}}\right), \left(\frac{W_{gs}}{S_{bg}}\right)\right] = \max\left[\left(\frac{220219.1 + 0.0 + 0.0}{25000}\right), \left(\frac{61563.7}{25000}\right)\right]$$
$$A_{m} = \max\left[8.7202, 2.4625\right] = 8.7202 \ in^{2}$$

And,

$$W_{gs} = \pi b G(C_{us}y)$$
 for non-self-energized gaskets  
 $W_{gs} = \pi (0.3536)(22.1678)(1.0(2500)) = 61563.7 \ lbs$   
 $F_A = 0$  and  $M_E = 0$  since there are no externally applied net-section axial forces  
bending moments.

or

Paragraph 4.16.7, Flange Design Procedure. The procedure in this paragraph can be used to design circular integral, loose or reverse flanges, subject to internal or external pressure, and external loadings. The procedure incorporates both a strength check and a rigidity check for flange rotation.

a) STEP 1 – Determine the design pressure and temperature of the flanged joint and the external net-section axial force,  $F_A$ , and bending moment,  $M_E$ .

$$P = 450 \text{ psig at } 650^{\circ}F$$
$$F_A = 0$$
$$M_E = 0$$

b) STEP 2 – Determine the design bolt loads for operating condition  $W_o$ , and the gasket seating condition  $W_g$ , and the corresponding actual bolt load area  $A_b$ , from paragraph 4.16.6.

 $W_o = 218005.0 \ lbs$  $W_g = 387702.5 \ lbs$  $A_b = 22.2960 \ in^2$ 

- c) STEP 3 Determine an initial flange geometry, in addition to the information required to determine the bolt load, the following geometric parameters are required. The flange is an ASME B16.5, Class 300, NPS 20 Slip–on Flange.
  - 1) Flange bore

B = 20.20 in

2) Bolt circle diameter

C = 27.0 in

3) Outside diameter of the flange

A = 30.5 in

4) Flange thickness

t = 2.44 in

5) Thickness of the hub at the large end

$$g_1 = 1.460$$
 in

6) Thickness of the hub at the small end

$$g_0 = 1.460$$
 in

7) Hub length

h = 1.25 in

d) STEP 4 – Determine the flange stress factors using the equations in Table 4.16.4 and Table 4.16.5.

$$\begin{split} &K = \frac{A}{B} = \frac{30.5}{20.20} = 1.5099 \\ &Y = \frac{1}{K-1} \Biggl[ 0.66845 + 5.71690 \Biggl( \frac{K^2 \log_{10} K}{K^2 - 1} \Biggr) \Biggr] \\ &Y = \frac{1}{1.5099 - 1} \Biggl[ 0.66845 + 5.71690 \frac{(1.5099)^2 \log_{10} [1.5099]}{(1.5099)^2 - 1} \Biggr] = 4.8850 \\ &T = \frac{K^2 (1 + 8.55246 \log_{10} K) - 1}{(1.04720 + 1.9448K^2)(K - 1)} = \frac{(1.5099)^2 (1 + 8.55246 \log_{10} [1.5099]) - 1}{(1.04720 + 1.9448(1.5099)^2)(1.5099 - 1)} = 1.7064 \\ &U = \frac{K^2 (1 + 8.55246 \log_{10} K) - 1}{1.36136 (K^2 - 1)(K - 1)} = \frac{(1.5099)^2 (1 + 8.55246 \log_{10} [1.5099]) - 1}{1.36136 ((1.5099)^2 - 1)(1.5099 - 1)} = 5.3681 \\ &Z = \frac{(K^2 + 1)}{(K^2 - 1)} = \frac{((1.5099)^2 + 1)}{((1.5099)^2 - 1)} = 2.5627 \\ &h_o = \sqrt{Bg_0} = \sqrt{(20.20)(1.46)} = 5.4307 \ in \\ &X_g = \frac{g_1}{g_0} = \frac{1.460}{1.460} = 1.0 \\ &X_h = \frac{h}{h_o} = \frac{1.25}{5.4307} = 0.2302 \end{split}$$

$$F_{L} = \begin{cases} \left\{ \begin{array}{l} 0.941074 + 0.176139 \left( \ln \left[ X_{g} \right] \right) - 0.188556 \left( \ln \left[ X_{h} \right] \right) + \\ 0.0689847 \left( \ln \left[ X_{g} \right] \right)^{2} + 0.523798 \left( \ln \left[ X_{h} \right] \right)^{2} - \\ 0.513894 \left( \ln \left[ X_{g} \right] \right) \left( \ln \left[ X_{h} \right] \right) \\ \hline \left\{ \begin{array}{l} 1 + 0.379392 \left( \ln \left[ X_{g} \right] \right) + 0.184520 \left( \ln \left[ X_{h} \right] \right) - \\ 0.00605208 \left( \ln \left[ X_{g} \right] \right)^{2} - 0.00358934 \left( \ln \left[ X_{h} \right] \right)^{2} + \\ 0.110179 \left( \ln \left[ X_{g} \right] \right) \left( \ln \left[ X_{h} \right] \right) \\ \hline \left\{ \begin{array}{l} 0.941074 + 0.176139 \left( \ln \left[ 1.0 \right] \right) - 0.188556 \left( \ln \left[ 0.2302 \right] \right) + \\ 0.0689847 \left( \ln \left[ 1.0 \right] \right)^{2} + 0.523798 \left( \ln \left[ 0.2302 \right] \right)^{2} - \\ 0.513894 \left( \ln \left[ 1.0 \right] \right) \left( \ln \left[ 0.2302 \right] \right) \\ \hline \left\{ \begin{array}{l} 1 + 0.379392 \left( \ln \left[ 1.0 \right] \right) + 0.184520 \left( \ln \left[ 0.2302 \right] \right)^{2} - \\ 0.00605208 \left( \ln \left[ 1.0 \right] \right)^{2} - 0.00358934 \left( \ln \left[ 0.2302 \right] \right)^{2} + \\ 0.110179 \left( \ln \left[ 1.0 \right] \right) \left( \ln \left[ 0.2302 \right] \right) \\ \hline \\ F_{L} = 3.2556 \\ For \ 0.1 \le X_{h} \le 0.25, \end{cases} \right\}$$

$$\ln[V_{L}] = \begin{pmatrix} 6.57683 - 0.115516X_{g} + 1.39499\sqrt{X_{g}} \left( \ln[X_{g}] \right) + \\ 0.307340 \left( \ln[X_{g}] \right)^{2} - 8.30849\sqrt{X_{g}} + 2.62307 \left( \ln[X_{g}] \right) + \\ 0.239498X_{h} \left( \ln[X_{h}] \right) - 2.96125 \left( \ln[X_{h}] \right) + \frac{7.035052 \left( 10^{-4} \right)}{X_{h}} \end{pmatrix} \\ \ln[V_{L}] = \begin{pmatrix} 6.57683 - 0.115516 (1.0) + 1.39499\sqrt{1.0} \left( \ln 1.0 \right) + \\ 0.307340 \left( \ln[1.0] \right)^{2} - 8.30849\sqrt{1.0} + 2.62307 \left( \ln[1.0] \right) + \\ 0.239498 (0.2302) \left( \ln[0.2302] \right) - 2.96125 \left( \ln[0.2302] \right) + \frac{7.035052 \left( 10^{-4} \right)}{0.2302} \right) \\ \ln[V_{L}] = 2.4244 \\ V_{L} = \exp[2.4244] = 11.2955 \\ f = 1.0 \end{cases}$$

$$d = \frac{Ug_0^2 h_o}{V_L} = \frac{(5.3681)(1.460)^2 (5.4307)}{11.2955} = 5.5014 \text{ in}$$
$$e = \frac{F_L}{h_o} = \frac{3.2556}{5.4307} = 0.5995$$
$$L = \frac{te+1}{h_o} + \frac{t^3}{5.4307} = \frac{2.44(0.5995)+1}{11.2955} + \frac{(2.44)^3}{11.2955} = 4.0838$$

$$L = \frac{1}{T} + \frac{1}{d} = \frac{1}{1.7064} + \frac{1}{5.5014} = 1$$

e) STEP 5 – Determine the flange forces.

$$H_{D} = \frac{\pi}{4}B^{2}P = \frac{\pi}{4}(20.20)^{2}(450) = 144213.2 \ lbs$$
$$H = \frac{\pi}{4}G^{2}P = \frac{\pi}{4}(22.1678)^{2}(450) = 173679.1 \ lbs$$
$$H_{T} = H - H_{D} = 173679.1 - 144213.2 = 29465.9 \ lbs$$
$$H_{G} = W_{o} - H = 218005.0 - 173679.1 = 44325.9 \ lbs$$

f) STEP 6 – Determine the flange moment for the operating condition using Equation (4.16.14) or Equation (4.16.15), as applicable. When specified by the user or his designated agent, the maximum bolt spacing  $(B_{smax})$  and the bolt spacing correction factor  $(B_{sc})$  shall be applied in calculating the flange moment for internal pressure using the equations in Table 4.16.11. The flange moment  $M_o$  for the operating condition and the flange moment  $M_g$  for the gasket seating condition without correction for bolt spacing  $B_{sc} = 1$  is used for the calculation of the rigidity index in Step 10. In these equations,  $h_D$ ,  $h_T$ , and  $h_G$  are determined from Table 4.16.6. For integral and loose type flanges, the moment  $M_{oe}$  is calculated using Equation (4.16.16) where and I and  $I_p$  in this equation are determined from Table 4.16.7.

For internal pressure,

$$M_{o} = abs \left[ \left( \left( H_{D}h_{D} + H_{T}h_{T} + H_{G}h_{G} \right) B_{sc} + M_{oe} \right) F_{s} \right]$$
  

$$M_{o} = abs \left[ \left( \left( 144213.2(3.40) + 29465.9(2.9081) + 44325.9(2.4161) \right) 1.0 + 0.0 \right) 1.0 \right]$$
  

$$M_{o} = 683110.5 \ in - lbs$$

From Table 4.16.11, the maximum bolt spacing and the bolt spacing correction factor are calculated as follows.

$$B_{smax} = 2a + \frac{6t}{m+0.5} = 2(1.25) + \frac{6(2.44)}{2.0+0.5} = 8.3560 \text{ in}$$
$$B_{sc} = \sqrt{\frac{B_s}{2a+t}} = \sqrt{\frac{1.125}{2(1.25)+2.44}} = 0.4772$$

The actual bolt spacing is determined using the following equation.

$$B_s = \frac{C}{No. \ of \ bolts} = \frac{27.0}{24} = 1.125 \ in$$

Since  $\{B_s = 1.125 \ in\} \le \{B_{smax} = 8.3560 \ in\}$ , the value of  $B_{sc} = 1.0$ .

From Table 4.16.6,

$$h_{D} = \frac{C - B}{2} = \frac{27.0 - 20.20}{2} = 3.40 \text{ in}$$

$$h_{G} = \frac{C - G}{2} = \frac{27.0 - 22.1678}{2} = 2.4161 \text{ in}$$

$$h_{T} = \frac{h_{D} + h_{G}}{2} = \frac{3.40 + 2.4161}{2} = 2.9081 \text{ in}$$

Since  $F_A = 0$  and  $M_E = 0$ , the flange cross-section bending moment of inertia, I, and polar moment of inertia,  $I_p$ , need not be calculated; and the flange design moment calculation for net-section bending moment and axial force supplemental loads,  $M_{oe} = 0$ . Additionally,  $F_s = 1.0$  for non-split rings.

g) STEP 7 – Determine the flange moment for the gasket seating condition using Equation (4.16.17) or Equation (4.16.18), as applicable.

For internal pressure,

$$M_g = \frac{W_g (C-G)F_s}{2} = \frac{(387702.5)(27.0 - 22.1678)(1.0)}{2} = 936728.0 \text{ in - lbs}$$

 h) STEP 8 – Determine the flange stresses for the operating and gasket seating conditions using the equations in Table 4.16.8.

Note: As provided in paragraph 4.16.12 for the definition of B, if  $B < 20g_1$ , the designer may substitute the value of  $B_1$  for B in the equation for  $S_H$ , where,

For integral flanges when f < 1.0 and for loose type flanges,

$$B_1 = B + g_1 = 20.20 + 1.460 = 21.66$$
 in

Since  $\{B = 20.20 \ in\} < \{20g_1 = 20(1.460) = 29.2 \ in\}$ , the value of  $B_1 = 21.66 \ in$  will be used in the equation for  $S_H$ .

**Operating Condition** 

$$S_{H} = \frac{fM_{o}}{Lg_{1}^{2}B} = \frac{(1.0)(683110.5)}{(4.0838)(1.460)^{2}(21.66)} = 3622.9 \text{ psi}$$

$$S_{R} = \frac{(1.33te+1)M_{o}}{Lt^{2}B} = \frac{\left[(1.33)(2.44)(0.5995)+1\right](683110.5)}{(4.0838)(2.44)^{2}(20.20)} = 4096.9 \ psi$$
$$S_{T} = \frac{YM_{o}}{t^{2}B} - ZS_{R} = \frac{(4.8850)(683110.5)}{(2.44)^{2}(20.20)} - 2.5627(4096.9) = 17248.4 \ psi$$

**Gasket Seating Condition** 

$$S_{H} = \frac{fM_{g}}{Lg_{1}^{2}B} = \frac{(1.0)(936728.0)}{(4.0838)(1.460)^{2}(21.66)} = 4968.0 \ psi$$
$$S_{R} = \frac{(1.33te+1)M_{g}}{Lt^{2}B} = \frac{\left[(1.33)(2.44)(0.5995)+1\right](936728.0)}{(4.0838)(2.44)^{2}(20.20)} = 5617.9 \ psi$$
$$S_{T} = \frac{YM_{g}}{t^{2}B} - ZS_{R} = \frac{(4.8850)(936728.0)}{(2.44)^{2}(20.20)} - 2.5627(5617.9) = 23652.3 \ psi$$

 STEP 9 – Check the flange stress acceptance criteria. The criteria below shall be evaluated. If the stress criteria are satisfied, go to STEP 10. If the stress criteria are not satisfied, reproportion the flange dimensions and go to STEP 4.

Allowable normal stress – The criteria to evaluate the normal stresses for the operating and gasket seating conditions are shown in Table 4.16.9, (for loose type flanges with a hub).

**Operating Condition** 

$$\{S_H = 3622.9 \ psi\} \le \{\min[1.5(17800), 2.5(18800)] = 26700 \ psi\}$$
 True

$$\{S_R = 4096.9 \ psi\} \le \{S_{fo} = 17800 \ psi\}$$
 True

$$\{S_T = 17248.4 \ psi\} \le \{S_{fo} = 17800 \ psi\}$$
 True

$$\left\{\frac{\left(S_{H}+S_{R}\right)}{2}=\frac{\left(3622.9+4096.9\right)}{2}=3859.9 \ psi\right\} \leq \left\{S_{fo}=17800 \ psi\right\}$$
True

$$\left\{\frac{\left(S_{H}+S_{T}\right)}{2}=\frac{\left(3622.9+17248.4\right)}{2}=10435.7 \ psi\right\} \le \left\{S_{fo}=17800 \ psi\right\} \qquad True$$

Gasket Seating Condition

$$\{S_H = 4968.0 \ psi\} \le \{\min[1.5(24000), 2.5(25300)] = 36000 \ psi\}$$
 True

$$\{S_R = 5617.9 \ psi\} \le \{S_{fg} = 24000 \ psi\}$$
 True

$$\{S_T = 23652.3 \ psi\} \le \{S_{fg} = 24000 \ psi\}$$
 True

$$\begin{cases} \frac{(S_H + S_R)}{2} = \frac{(4968.0 + 5617.9)}{2} = 5293.0 \ psi \\ \end{cases} \le \begin{cases} S_{fg} = 24000 \ psi \end{cases} \qquad True \\ \begin{cases} \frac{(S_H + S_T)}{2} = \frac{(4968.0 + 23652.3)}{2} = 14310.2 \ psi \\ \end{cases} \le \begin{cases} S_{fg} = 24000 \ psi \end{cases} \qquad True \end{cases}$$

STEP 10 - Check the flange rigidity criterion in Table 4.16.10. If the flange rigidity criterion is j) satisfied, then the design is complete. If the flange rigidity criterion is not satisfied, then reproportion the flange dimensions and go to STEP 3.

2

**Operating Condition** 

2

$$J = \frac{52.14V_L M_o}{LE_{yo} g_0^2 K_R h_o} \le 1.0$$

$$\left\{ J = \frac{52.14(11.2955)(683110.5)}{(4.0838)(26.0E+06)(1.460)^2 (0.2)(5.4307)} = 1.6366 \right\} \le 1.0 \text{ Not Satisfied}$$

Where,  $K_{R} = 0.2$  for loose type flanges.

Gasket Seating Condition

$$J = \frac{52.14V_L M_g}{LE_{yg} g_o^2 K_R h_o} \le 1.0$$

$$\left\{ J = \frac{52.14(11.2955)(936728.0)}{(4.0838)(29.4E+06)(1.460)^2 (0.2)(5.4307)} = 1.9847 \right\} \le 1.0 \text{ Not Satisfied}$$

Where,  $K_{R} = 0.2$  for loose type flanges.

Since the flange rigidity criterion is not satisfied for either the operating condition or the gasket seating condition, the flange dimensions should be re-proportioned and the design procedure shall be performed beginning with STEP 3.

NOTE: Although the proposed ASME B16.5 slip-on flange is shown not to satisfy the flange rigidity acceptance criteria of VIII-2 paragraph 4.16 Design Rules for Flanged Joints, Table 4.16.10, the ASME B16.5–2009, permits the use of an ASME Class 300 flange to operate at a pressure of 550 psi for a coincident temperature of 650°F.

# 4.17 Clamped Connections

# 4.17.1 Example E4.17.1 – Flange and Clamp Design Procedure

Using the data shown below, determine if the clamp design meets the design requirements of Section VIII, Division 2.

# Data (Refer to Figure E4.17.1)

•	Design Conditions	=	3000 psi@200°F
•	Corrosion Allowance	=	0.0 <i>in</i>
<u>Cla</u>	amp		
•	Material	=	SA-216, Grade WCB
•	Inside Diameter	=	43.75 in
•	Thickness	=	7.625 in
•	Width	=	28.0 <i>in</i>
•	Gap	=	14.0 in
•	Lug height	=	15.0 in
•	Lug Width	=	28.0 in
٠	Lip Length	=	2.75 in
٠	Radial Distance from Connection Centerline to Bolts	=	32.25 in
٠	Distance from W to the point where the clamp lug	=	3.7 <i>in</i>
	joins the clamp body		
•	Allowable Stress @ Design Temperature	=	22000 <i>psi</i>
•	Allowable Stress @ Ambient Temperature	=	24000 <i>psi</i>
<u>Hu</u>	<u>Ib</u>		-
•	Material	=	SA-105
•	Inside Diameter	=	18.0 <i>in</i>
•	Pipe End Neck Thickness	=	12.75 in
•	Shoulder End Neck Thickness	=	12.75 in
•	Shoulder Thickness	=	7.321 in
•	Shoulder Height	=	2.75 in
•	Friction Angle	=	5 deg
•	Shoulder Transition Angle	=	10 <i>deg</i>
•	Allowable Stress @ Design Temperature	=	22000 psi
•	Allowable Stress @ ambient Temperature	=	24000 psi
Bo	It Data		
•	Material	=	SA-193, Grade B7
•	Allowable Stress @ Design Temperature	=	23000 psi
•	Allowable Stress @ Gasket Temperature	=	23000 psi
•	Diameter	=	1.75 in
•	Number of Bolts	=	2
•	Root area	=	$1.980 in^2$

#### Gasket Data

•	Material Gasket Reaction Location	=	Self Energizing O-ring Type 19.0 <i>in</i>
•	Gasket Factor	=	0
•	Seating Stress	=	0 psi

#### Determine the Design Bolt Loads

The procedure to determine the bolt loads for the operating and gasket seating conditions is given in Part 4, paragraph 4.17.4.2, and is shown below.

a) STEP 1 – Determine the design pressure and temperature of the flanged joint

$$P = 3000 psig at 200^{\circ}F$$

b) STEP 2 – Select a gasket and determine the gasket factors m and y from Table 4.16.1.

$$m = 0.0$$
 for self – energized gaskets  
 $y = 0.0$ 

c) STEP 3 – Determine the width of the gasket, N, basic gasket seating width,  $b_o$ , the effective gasket seating width, b, and the location of the gasket reaction, G.

$$N = 0.0$$
 for self – energized gaskets per paragraph 4.16.12

From Table 4.16.3, Effective Gasket Width (not required because gasket is self-energized)

$$b_o = \frac{N}{2} = \frac{0.0}{2} = 0.0$$
 in

For  $b_o \leq 0.25$  in ,

$$b = b_0 = 0.0$$
 in

Therefore, from paragraph 4.16.6, the location of the gasket reaction is calculated as follows.

 $G = mean \ diameter \ of \ the \ gasket \ contact \ face$  $G = 19.0 \ in$ 

d) STEP 4 - Determine the flange forces for the bolt load calculation.

$$H = 0.785G^{2}P = 0.785(19.0)^{2}(3000) = 850155.0 \ lbs$$
  

$$H_{p} = 0.0 \qquad (for \ self - energized \ gaskets)$$
  

$$H_{m} = 0.0 \qquad (for \ self - energized \ gaskets)$$

e) STEP 5 – Determine the design bolt load for the operating condition.

$$W_o = \frac{2}{\pi} (H + H_p) \tan[\phi - \mu] = \frac{2}{\pi} (850155.0 + 0) \cdot \tan[10 - 5] = 47351.0941 \ lbs$$

STEP 6 – Determine the minimum required total bolt load for the gasket seating and assembly conditions.

$$W_{g1} = \frac{2}{\pi} H_m \tan\left[\phi + \mu\right] = \frac{2}{\pi} (0.0) \tan\left[10 + 5\right] = 0.0 \ lbs$$
$$W_{g2} = \frac{2}{\pi} (H + H_p) \tan\left[\phi + \mu\right] = \frac{2}{\pi} (850155 + 0) \tan\left[10 + 5\right] = 145020.9308 \ lbs$$

g) STEP 7 - Determine the design bolt load for the gasket seating and assembly conditions.

$$W_g = (A_m + A_b)S_{bg} = (3.1526 + 3.96)23000 = 163589.8 \ lbs$$

Where,

$$A_{m} = \max\left[\frac{W_{o}}{2S_{bo}}, \frac{W_{g1}}{2S_{bg}}, \frac{W_{g2}}{2S_{bg}}\right]$$
$$A_{m} = \max\left[\frac{47531.0941}{2(23000)}, \frac{0}{2(23000)}, \frac{145020.9308}{2(23000)}\right] = 3.1526 \ in^{2}$$

The actual bolt area is calculated as follows (using two 1.75 in diameter bolts).

$$A_{b} = (Number of bolts)(Root area of one bolt) = 2(1.980) = 3.96 in^{2}$$

Alternatively, if controlled bolting (e.g., bolt tensioning or torque control) techniques are used to assemble the clamp, assembly design bolt load may be calculated as follows.

$$W = 2A_m \cdot S_a = 2(3.1526)23000 = 145019.6 \ lbs$$

Note: This calculation is shown for informational purposes only and will not be used in the example problem.

Flange Design Procedure:

Refer to paragraph 4.17.5

a) STEP 1 – Determine the design pressure and temperature of the flange joint.

See above data.

- b) STEP 2 Determine an initial flange and clamp geometry see Figures 4.17.1(c) and 4.17.2(a), and Figure E4.17.1 of this example.
- c) STEP 3 Determine the design bolt loads for operating condition,  $W_o$ , and the gasket seating and assembly condition,  $W_o$ , from paragraph 4.17.4.2.

 $W_o = 47351.0941 \ lbs$  $W_g = 163589.8 \ lbs$  d) STEP 4 – Determine the flange forces, H,  $H_p$ , and  $H_m$  from paragraph 4.17.4.2, and:

$$H = 850155.0 \ lbs$$

$$H_p = 0.0 \ lbs$$

$$H_m = 0.0 \ lbs$$

$$H_D = 0.785B^2P = 0.785(18.0)^2 (3000) = 763020.0 \ lbs$$

$$H_G = \frac{1.571W_o}{\tan[\phi + \mu]} - (H + H_p) = \frac{1.571(47531.0941)}{\tan[10 + 5]} - (850155.0 + 0.0) = -571477.7323 \ lbs$$

$$H_T = H - H_D = 850155.0 - 763020.0 = 87135.0 \ lbs$$

e) STEP 5 – Determine the flange moment for the operating condition.

$$\begin{split} M_o &= M_D + M_G + M_T + M_F + M_P + M_R \\ M_o &= 5961093.75 + 0 + 1214444.063 + 0 + 25957.3484 - 254998.8042 \\ M_o &= 6946496.357 \ in - lbs \end{split}$$

Where,

$$\begin{split} M_{D} &= H_{D} \left[ \frac{C - (B + g_{1})}{2} \right] = 763020.0 \left[ \frac{46.375 - (18.0 + 12.75)}{2} \right] = 5961093.75 \ lbs \\ M_{G} &= H_{G}h_{G} = -571477.7323(0.0) = 0.0 \ lbs \\ M_{T} &= H_{T} \left[ \frac{C}{2} - \frac{(B + G)}{4} \right] = 87135.0 \left[ \frac{46.375}{2} - \frac{(18.0 + 19.0)}{4} \right] = 1214444.063 \ in - lbs \\ M_{F} &= H_{D} \left( \frac{g_{1} - g_{0}}{2} \right) = 763020.0 \left( \frac{12.75 - 12.75}{2} \right) = 0.0 \ in - lbs \\ M_{P} &= PBT\pi \left( \frac{T}{2} - \overline{h} \right) = 3000(18)(7.321)\pi \left( \frac{7.321}{2} - 3.6396 \right) = 25957.3484 \ lbs \\ M_{R} &= 1.571W_{o} \left( \overline{h} - T + \frac{(C - N)\tan[\phi]}{2} \right) \\ M_{R} &= 1.571(47351.0941) \left( 3.6396 - 7.321 + \frac{(46.375 - 43.5)\tan[10]}{2} \right) \\ M_{R} &= -254998.8042 \ lbs \end{split}$$

And,

$$A = B + 2(g_1 + g_2) = 18.0 + 2(12.75 + 2.75) = 49.0$$
 in

$$N = B + 2g_1 = 18 + 2(12.75) = 43.5 \text{ in}$$

$$C = \frac{(A + C_i)}{2} = \frac{(49 + 43.75)}{2} = 46.375 \text{ in}$$

$$\overline{h} = \frac{T^2g_1 + h_2^2g_2}{2(Tg_1 + h_2g_2)} = \frac{(7.321)^2 12.75 + (7.0785)^2 2.75}{2(7.321(12.75) + 7.0785(2.75))} = 3.6396 \text{ in}$$

$$h_2 = T - \frac{g_2 \tan[\phi]}{2} = 7.321 - \frac{2.75 \tan[10]}{2} = 7.0786 \text{ in}$$

f) STEP 6 – Determine the flange moment for the gasket seating condition

$$M_g = \frac{0.785W_g (C-G)}{\tan[\phi + \mu]} = \frac{0.785(163589.8)(46.375 - 19.0)}{\tan[10 + 5]} = 13119810.24 \text{ in-lbs}$$

g) STEP 7 – Determine the hub factors

$$\begin{split} F_{H} &= 1 + \frac{1.818}{\sqrt{Bg_{1}}} \left[ T - \bar{h} + \frac{3.305I_{h}}{g_{1}^{2} (0.5B + \bar{g})} \right] \\ F_{H} &= 1 + \frac{1.818}{\sqrt{18(12.75)}} \left[ 7.321 - 3.6396 + \frac{3.305(498.4148)}{(12.75)^{2} (0.5(18.0) + 7.7123)} \right] \\ F_{H} &= 1.5146 \\ I_{h} &= \frac{g_{1}T^{3}}{3} + \frac{g_{2}h_{2}^{3}}{3} - (g_{2}h_{2} + g_{1}T)\bar{h}^{2} \\ I_{h} &= \frac{12.75(7.321)^{3}}{3} + \frac{2.75(7.0785)^{3}}{3} - (2.75(7.0786) + 12.75(7.321))(3.6396)^{2} \\ I_{h} &= 498.4148 \ in^{4} \\ \bar{g} &= \frac{Tg_{1}^{2} + h_{2}g_{2}(2g_{1} + g_{2})}{2(Tg_{1} + h_{2}g_{2})} = \frac{7.321(12.75)^{2} + 7.0786(2.75)(2(12.75) + 2.75)}{2(7.321(12.75) + 7.0786(2.75))} \\ \bar{g} &= 7.7123 \ in \end{split}$$

h) STEP 8 – Determine the reaction shear force at the hub neck for the operating condition.

$$Q_o = \frac{1.818M_o}{F_H \sqrt{Bg_1}} = \frac{1.818(6946496.357)}{1.5146\sqrt{(18.0)(12.75)}} = 550389.8215 \ lbs$$

i) STEP 9 – Determine the reaction shear force at the hub neck for the gasket seating condition.

$$Q_g = \frac{1.818M_g}{F_H \sqrt{Bg_1}} = \frac{1.818(13119810.24)}{1.5146\sqrt{(18.0)(12.75)}} = 1039518.29 \ lbs$$

j) STEP 10 – Determine the clamp factors.

$$e_{b} = B_{c} - \frac{C_{i}}{2} - l_{c} - X = 32.25 - \frac{43.75}{2} - 2.75 - 2.7009 = 4.9241 \text{ in}$$
$$X = \frac{\left(\frac{C_{w}}{2} - \frac{C_{t}}{3}\right)C_{t}^{2} - 0.5\left(C_{w} - C_{g}\right)l_{c}^{2}}{A_{c}} = \frac{\left(\frac{28.0}{2} - \frac{7.625}{3}\right)(7.625)^{2} - 0.5\left(28.0 - 14.0\right)(2.75)^{2}}{227.0577}$$

$$X = 2.7009$$
 in

$$A_c = A_1 + A_2 + A_3 = 97.2188 + 91.3389 + 38.5 = 227.0577 in^2$$

$$I_{c} = \left(\frac{A_{1}}{3} + \frac{A_{2}}{4}\right)C_{t}^{2} + \frac{A_{3}l_{c}^{2}}{3} - A_{c}X^{2}$$

$$I_{c} = \left(\frac{97.2188}{3} + \frac{91.3389}{4}\right)(7.625)^{2} + \frac{38.5(2.75)^{2}}{3} - 227.0577(2.7009)^{2}$$

$$I_{c} = 1652.4435 \ in^{4}$$

$$A_{1} = (C_{w} - 2C_{t})C_{t} = (28.0 - 2(7.625))7.625 = 97.2188 \ in^{2}$$

$$A_{2} = 1.571C_{t}^{2} = 1.571(7.625)^{2} = 91.3389 \ in^{2}$$

$$A_{3} = (C_{w} - C_{g})l_{c} = (28.0 - 14.0)2.75 = 38.5 \ in^{2}$$

k) STEP 11 – Determine the hub stress correction factor, f, based on  $g_1$ ,  $g_0$ , h, and B using the equations in Table 4.16.4 and 4.16.5 and  $l_m$  using the following equation.

$$X_{g} = \frac{g_{1}}{g_{0}} = \frac{12.75}{12.75} = 1$$
$$X_{h} = \frac{h}{h_{o}} = 0$$
$$f = \max \left[ 1.0, \left[ \begin{array}{c} 0.0927779 - 0.0336633X_g + 0.964176X_g^2 + \\ 0.0566286X_h + 0.347074X_h^2 - 4.18699X_h^3 \\ \hline 1 - 5.96093(10)^{-3}X_g + 1.62904X_h + \\ 3.49329X_h^2 + 1.39052X_h^3 \end{array} \right] \right]$$

$$f = \max \left[ 1.0, \left[ \begin{array}{c} 0.0927779 - 0.0336633(1) + 0.964176(1)^2 + \\ 0.0566286(1) + 0.347074(0) - 4.18699(0) \\ \hline 1 - 5.96093(10)^{-3}1 + 1.62904(0) + \\ 3.49329(0) + 1.39052(0) \end{array} \right] \right]$$

$$f = \max \left[ 1.0, 1.0294 \right] = 1.0294$$

$$l_m = l_c - 0.5(C - C_i) = 2.75 - 0.5(46.375 - 43.75) = 1.4375 \text{ in}$$

I) STEP 12 – Determine the flange and clamp stresses for the operating and gasket seating conditions using the equations in Table 4.17.1.

Operating Condition – Location: Flange

Longitudinal Stress:

$$S_{1o} = f \left[ \frac{PB^2}{4g_1(B+g_1)} + \frac{1.91M_o}{g_1^2(B+g_1)F_H} \right]$$
  
$$S_{1o} = 1.0294 \left[ \frac{3000(18.0)^2}{4(12.75)(18.0+12.75)} + \frac{1.91(6946496.357)}{(12.75)^2(18.0+12.75)1.5146} \right] = 2442.0 \ psi$$

Lame Hoop Stress:

$$S_{2o} = P\left(\frac{N^2 + B^2}{N^2 - B^2}\right) = 3000 \left(\frac{\left(43.5\right)^2 + \left(18.0\right)^2}{\left(13.5\right)^2 - \left(18.0\right)^2}\right) = 4239.6 \ psi$$

Axial Shear Stress:

$$S_{3o} = \frac{0.75W_o}{T(B+2g_1)\tan[\phi-\mu]} = \frac{0.75(47351.0941)}{7.321(18.0+2(12.75))\tan[10-5]} = 1274.6 \text{ psi}$$

Radial Shear Stress:

$$S_{4o} = \frac{0.477Q_o}{g_1(B+g_1)} = \frac{0.477(550389.8215)}{12.75(18+12.75)} = 669.6 \text{ psi}$$

Operating Condition - Location: Clamp

Longitudinal Stress:

$$S_{5o} = \frac{W_o}{2C \tan\left[\phi - \mu\right]} \left[ \frac{1}{C_t} + \frac{3(C_t + 2l_m)}{C_t^2} \right]$$
$$S_{5o} = \frac{47351.0941}{2(46.375) \tan\left[10 - 5\right]} \left[ \frac{1}{7.625} + \frac{3(7.625 + 2(1.4375))}{(7.625)^2} \right] = 3926.8 \ psi$$

**Tangential Stress:** 

$$S_{6o} = \frac{W_o}{2} \left[ \frac{1}{A_c} + \frac{|e_b| \cdot (C_t - X)}{I_c} \right]$$
$$S_{6o} = \frac{47351.0941}{2} \left[ \frac{1}{227.0577} + \frac{|4.9241| \cdot (7.625 - 2.7009)}{1652.4435} \right] = 451.7 \text{ psi}$$

Lip Shear Stress:

$$S_{7o} = \frac{1.5W_o}{\left(C_w - C_g\right)C\tan\left[\phi - \mu\right]} = \frac{1.5(47351.0941)}{\left(28.0 - 14.0\right)\left(46.375\right)\tan\left[10 - 5\right]} = 1250.4 \text{ psi}$$

Lug Bending Stress:

$$S_{8o} = \frac{3W_oL_a}{L_wL_h^2} = \frac{3(47351.0941)(3.7)}{28.0(15.0)^2} = 83.4 \text{ psi}$$

Bearing Stress:

$$S_{9o} = \frac{W_o}{(A - C_i)C\tan[\phi - \mu]} = \frac{47351.0941}{(49.0 - 43.75)(46.375)\tan[10 - 5]} = 2223.0 \text{ psi}$$

Gasket Seating/Assembly Condition - Location: Flange

Longitudinal Stress:

$$S_{1g} = f\left[\frac{1.91M_g}{g_1^2(B+g_1)F_H}\right] = 1.0294 \left\lfloor\frac{1.91(13119810.24)}{(12.75)^2(18.0+12.75)(1.5146)}\right\rfloor = 3407.1 \text{ psi}$$

Lame Hoop Stress:

$$S_{2g} = 0.0$$

Axial Shear Stress:

$$S_{3g} = \frac{0.75W_g}{T(B+2g_1)\tan[\phi+\mu]} = \frac{0.75(163589.8)}{7.321(18.0+2(12.75))\tan[10+5]} = 1437.8 \text{ psi}$$

Radial Shear Stress:

$$S_{4g} = \frac{0.477Q_g}{g_1(B+g_1)} = \frac{0.477(1039518.29)}{12.75(18.0+12.75)} = 1264.7 \ psi$$

Gasket Seating/Assembly Condition-Location: Clamp

Longitudinal Stress:

$$S_{5g} = \frac{W_g}{2C \tan\left[\phi + \mu\right]} \left[ \frac{1}{C_t} + \frac{3(C_t + 2l_m)}{C_t^2} \right]$$
$$S_{5g} = \frac{163589.8}{2(46.375) \tan\left[10 + 5\right]} \left[ \frac{1}{7.625} + \frac{3(7.625 + 2(1.4375))}{7.625^2} \right] = 4429.6 \ psi$$

Tangential Stress:

$$S_{6g} = \frac{W_g}{2} \left[ \frac{1}{A_c} + \frac{|e_b| \cdot (C_t - X)}{I_c} \right]$$
  
$$S_{6g} = \frac{163589.8}{2} \left[ \frac{1}{227.0577} + \frac{|4.9241| \cdot (7.625 - 2.7009)}{1652.4435} \right] = 1560.4 \text{ psi}$$

Lip Shear Stress:

$$S_{7g} = \frac{1.5W_g}{\left(C_w - C_g\right)C\tan\left[\phi + \mu\right]} = \frac{1.5(163589.8)}{\left(28.0 - 14.0\right)\left(46.375\right)\tan\left[10 + 5\right]} = 1410.5 \ psi$$

Lug Bending Stress:

$$S_{8g} = \frac{3W_g L_a}{L_w L_h^2} = \frac{3(163589.8)(3.7)}{28.0(15.0)^2} = 288.2 \ psi$$

Bearing Stress:

$$S_{9g} = \frac{W_g}{(A - C_i)C\tan[\phi + \mu]} = \frac{163589.8}{(49.0 - 43.75)(46.375)\tan[10 + 5]} = 2507.6 \text{ psi}$$

m) STEP 13 – Check the flange stress acceptance criteria for the operating and gasket seating conditions are shown in Table 4.17.2.

Operating Condition – Location: Flange

$$\{S_{1o} = 2442.0 \ psi\} \le \{1.5S_{ho} = 33000 \ psi\}$$
True

$$\{S_{2o} = 4239.6 \ psi\} \le \{S_{ho} = 22000 \ psi\}$$
True

$$\{S_{3o} = 1274.6 \ psi\} < \{0.8S_{ho} = 17600 \ psi\}$$
 True

### PTB-3-2013

$$\{S_{4o} = 669.6 \ psi\} \le \{0.8S_{ho} = 17600 \ psi\}$$
True

Operating Condition – Location: Clamp

$$\{S_{5o} = 3926.8 \ psi\} \le \{1.5S_{co} = 33000 \ psi\}$$
True

$$\left\{ S_{6o} = 451.7 \ psi \right\} \leq \left\{ 1.5S_{co} = 33000 \ psi \right\}$$
 True

$$\{S_{7o} = 1250.4 \ psi\} < \{0.8S_{co} = 17600 \ psi\}$$
 True

$$\{S_{8o} = 83.4 \ psi\} \le \{S_{co} = 22000 \ psi\}$$
True

$$\{S_{9o} = 2223.0 \ psi\} \le \{1.6 \min[S_{ho}, S_{co}] = 35200 \ psi\}$$
 True

Gasket Seating/Assembly Condition – Location: Flange

$$\left\{S_{1g} = 3407.1 \ psi\right\} \le \left\{1.5S_{hg} = 33000 \ psi\right\}$$
True

$$\{S_{2g} = 0.0 \ psi\} \le \{S_{hg} = 22000 \ psi\}$$
 True

$$\left\{S_{3g} = 1437.8 \ psi\right\} \le \left\{0.8S_{hg} = 17600 \ psi\right\}$$
True

$$\left\{S_{4g} = 1264.7 \ psi\right\} \le \left\{0.8S_{hg} = 17600 \ psi\right\}$$
True

Gasket Seating/Assembly Condition – Location: Clamp

$$\left\{S_{5g} = 4429.6 \ psi\right\} \le \left\{1.5S_{cg} = 33000 \ psi\right\}$$
True

$$\{S_{6g} = 1560.4 \ psi\} \le \{1.5S_{cg} = 33000 \ psi\}$$
 True

$$\{S_{7g} = 1410.5 \ psi\} \le \{0.8S_{cg} = 17600 \ psi\}$$
 True

$${S_{8g} = 288.2 \ psi} < {S_{cg} = 22000 \ psi}$$
 True

$$\{S_{9g} = 2507.6 \ psi\} < \{1.6 \min[S_{hg}, S_{cg}] = 35200 \ psi\}$$
 True

The proposed hub/clamp assembly is acceptable for the specified design conditions.







Notes:

1) See Figure 4.17.2 for sction B-B

2) Clamp may have spherical depressions at bolt holes to facilitate the use of spherical nuts



Section B-B

Figure E4.17.1 - Typical Hub and Clamp Configuration

# 4.18 Tubesheets in Shell and Tube Heat Exchangers

### 4.18.1 Example E4.18.1 – U-Tube Tubesheet Integral with Shell and Channel

A U-tube heat exchanger is to be designed with the tubesheet construction in accordance with configuration a as shown in Figure 4.18.4.

- The shell side design conditions are -10 and 60 psig at 500°F.
- The tube side design conditions are -15 and 140 psig at 500°F.
- The tube material is SA-249 S31600 (Stainless Steel 316). The tubes are 0.75 in. outside diameter and 0.065 in. thick and are to be full-strength welded with no credit taken for expansion.
- The tubesheet material is SA-240 S31600 (Stainless Steel 316) with no corrosion allowance on the tube side and no pass partition grooves. The tubesheet outside diameter is 12.939 in. The tubesheet has 76 tube holes on a 1.0 in. square pattern with one centerline pass lane. The largest center-to-center distance between adjacent tube rows is 2.25 in., and the radius to the outermost tube hole center is 5.438 in.
- The shell material is SA-312 S31600 (Stainless Steel 316) welded pipe. The shell inside diameter is 12.39 in. and the shell thickness is 0.18 in.
- The channel material is SA-240 S31600 (Stainless Steel 316). The channel inside diameter is 12.313 in. and the channel thickness is 0.313 in.

#### Data Summary

The data summary consists of those variables from the nomenclature (see paragraph 4.18.15) that are applicable to this configuration.

The data for paragraph 4.18.15 is:

$$c_t = 0$$
 in.  
 $d_t = 0.75$  in.  
 $E = 25.8E10^6$  psi from Table TM-1 of Section II, Part D at 500°F

 $E_t = 25.8E10^6$  psi from Table TM-1 of Section II, Part D at 500°F

 $h_{\sigma} = 0$  in.

p = 1.0 in.

 $r_{o} = 5.438 in.$ 

S = 18,000 psi from Table 5A of Section II, Part D at 500°F

 $S_t = 18,000 \text{ psi}$  from Table 5A of Section II, Part D at 500°F (for seamless tube, SA-213)

 $t_t = 0.065$  in.

$$U_{L1} = 2.25 in.$$

 $\rho = 0$  for no tube expansion

*A* = 12.939 *in*.

 $D_c = 12.313$  in.

 $D_s = 12.39$  in.

 $E = 25.8E10^{6} psi \text{ from Table TM-1 of Section II, Part D at 500° F}$   $E_{c} = 25.8E10^{6} psi \text{ from Table TM-1 of Section II, Part D at 500° F}$   $E_{s} = 25.8E10^{6} psi \text{ from Table TM-1 of Section II, Part D at 500° F}$   $P_{sd,max} = 60 psig$   $P_{sd,max} = 140 psig$   $P_{td,max} = 140 psig$  S = 18,000 psi from Table 5A of Section II, Part D at 500° F  $S_{c} = 18,000 psi \text{ from Table 5A of Section II, Part D at 500° F}$   $S_{s} = 18,000 psi \text{ from Table 5A of Section II, Part D at 500° F}$   $S_{s} = 18,000 psi \text{ from Table 5A of Section II, Part D at 500° F}$   $S_{c} = 18,000 psi \text{ from Table 5A of Section II, Part D at 500° F}$   $V_{c} = 0.313 in.$   $V_{c} = 0.3$   $V_{s} = 0.3$ 

Calculation Procedure

The calculation procedure for a U-tube heat exchanger tubesheet is given in paragraph 4.18.7. The calculation results are shown for loading case 1 where  $P_s = P_{sd,min} = -10 \, psig$  and  $P_t = P_{td,max} = 140 \, psig$ , since this case yields the greatest value of  $\sigma$ .

a) STEP 1 – Calculate  $D_o$ ,  $\mu$ ,  $\mu$ \*, and  $h'_g$  from paragraph 4.18.6.4.a.

$$D_o = 11.626 in.$$
  

$$L_{L1} = 11.6 in.$$
  

$$A_L = 26.2 in.^2$$
  

$$\mu = 0.25$$
  

$$d* = 0.75 in.$$
  

$$p* = 1.15 in.$$
  

$$\mu* = 0.349$$
  

$$h'_g = 0 in.$$

b) STEP 2 – Calculate  $\rho_{s}, \rho_{c}$ , and  $M_{\rm TS}$  for configuration a.

$$\rho_s = 1.07$$
  
 $\rho_c = 1.06$ 
  
 $M_{TS} = -160 \text{ in.} - lb/in.$ 

c) STEP 3 – Assume a value for the tubesheet thickness, h, and calculate h/p. Determine  $E^*/E$  and  $v^*$  from paragraph 4.18.6.4.b and calculate  $E^*$ .

$$h = 0.521 in.$$
  
 $h/p = 0.521$   
 $E*/E = 0.445$   
 $v* = 0.254$   
 $E* = 11.5E10^6 psi$ 

d) STEP 4 – For configuration a, calculate  $\beta_s$ ,  $k_s$ ,  $\lambda_s$ ,  $\delta_s$ , and  $\omega_s$  for the shell and  $\beta_c$ ,  $k_c$ ,  $\lambda_c$ ,  $\delta_c$ , and  $\omega_c$  for the channel.

$$\beta_{s} = 1.21 in.^{-1}$$

$$k_{s} = 33,300 \, lb.$$

$$\lambda_{s} = 32.0 \times 10^{6} \, psi$$

$$\delta_{s} = 7.02 \times 10^{-6} \, in.^{3} / lb$$

$$\omega_{s} = 0.491 in.^{2}$$

$$\beta_{c} = 0.914 \, in.^{-1}$$

$$k_{c} = 132,000 \, lb$$

$$\lambda_{c} = 110 \times 10^{6} \, psi$$

$$\delta_{c} = 3.99 \times 10^{-6} \, in.^{3} / lb$$

$$\omega_{c} = 0.756 \, in.^{2}$$

e) STEP 5 – Calculate K and F for configuration a.

$$K = 1.11$$
  
 $F = 9.41$ 

f) STEP 6 – Calculate  $M^*$  for configuration a.

 $M^* = -49.4$  in. -lb/in.

g) STEP 7 – Calculate  $M_p$ ,  $M_o$ , and M.

$$M_p = 568 \text{ in.} -lb/in.$$
  
 $M_o = -463 \text{ in.} -lb/in.$ 

$$M = 568 in. -lb/in.$$

h) STEP 8 – Calculate  $\sigma$  and check the acceptance criterion.

 $\sigma = 36,000 \ psi \le 2S = 36,000 \ psi$ 

i) STEP 9 – Calculate the average shear stress in the tubesheet at the outer edge of the perforated region, if required. Compare to the allowable.

If 
$$|P_s - P_t| \le \frac{3.2S\mu h}{D_o}$$
, then the shear stress is not required to be calculated

Load Case 1:

$$\left\{ \left| P_{s} - P_{t} \right| = 150 \ psi \right\} \le \left\{ \frac{3.2S \,\mu h}{D_{o}} = 645 \ psi \right\}$$

Calculate the average shear stress, au .

Not required for this example.

j)

STEP 10 – For configuration a, calculate  $\sigma_{s,m}$ ,  $\sigma_{s,b}$ , and  $\sigma_s$  for the shell and  $\sigma_{c,m}$ ,  $\sigma_{c,b}$ , and  $\sigma_c$  for the channel, and check the acceptance criterion. The shell thickness shall be 0.18 in. for a minimum length of 2.69 in. adjacent to the tubesheet and the channel thickness shall be 0.313 in. for a minimum length of 3.53 in. adjacent to the tubesheet.

$$\sigma_{s,m} = -170 \ psi$$
  
 $\sigma_{s,b} = -17,600 \ psi$   
 $\sigma_s = 17,700 \ psi \le 1.5S_s = 27,000 \ psi$   
 $\sigma_{c,m} = 1,340 \ psi$   
 $\sigma_{c,b} = 25,300 \ psi$   
 $\sigma_c = 26,600 \ psi \le 1.5S_c = 27,000 \ psi$ 

The assumed value for the tubesheet thickness, h, is acceptable and the shell and channel stresses are within the allowable stresses; therefore, the calculation procedure is complete.

# 4.18.2 Example E4.18.2 – U-Tube Tubesheet Gasketed With Shell and Channel

A U-tube heat exchanger is to be designed with the tubesheet construction in accordance with configuration d as shown in Figure 4.18.4.

- The shell side design conditions are -15 and 10 psig at 300°F.
- The tube side design condition is 135 psig at 300°F.
- The tube material is SB-111 C44300 (Admiralty). The tubes are 0.625 in. outside diameter and 0.065 in. thick and are to be expanded for the full thickness of the tubesheet.
- The tubesheet material is SA-285, Grade C (K02801) with a 0.125 in. corrosion allowance on the tube side and no pass partition grooves. The tubesheet outside diameter is 20.0 in. The tubesheet has 386 tube holes on a 0.75 in. equilateral triangular pattern with one centerline pass lane. The largest center-to-center distance between adjacent tube rows is 1.75 in., and the radius to the outermost tube hole center is 8.094 in.
- The diameter of the shell gasket load reaction is 19.0 in. and the shell flange design bolt load is 147,000 lb.
- The diameter of the channel gasket load reaction is 19.0 in. and the channel flange design bolt load is 162,000 lb.

#### Data Summary

The data summary consists of those variables from the nomenclature (see paragraph 4.18.15) that are applicable to this configuration.

The data for paragraph 4.18.15 is:

 $c_t = 0.125 in.$ 

 $d_t = 0.625 in.$ 

 $E = 28.3E10^6$  psi from Table TM-1 of Section II, Part D at 300°F

 $E_t = 15.4E10^6 \text{ psi}$  from Table TM-3 of Section II, Part D at 300°F

 $h_g = 0$  in.

$$p = 0.75 in.$$

 $r_o = 8.094 in.$ 

S = 17,700 psi from Table 5A of Section II, Part D at  $300^{\circ}$ F

 $S_t = 10,000 \text{ psi}$  from Table 5B of Section II, Part D at  $300^{\circ}$ F

 $t_t = 0.065 in.$ 

$$U_{L1} = 1.75 in.$$

 $\rho = 1.0$  for a full length tube expansion

$$A = 20.0 in.$$
  

$$E = 28.3E10^{6} psi \text{ from Table TM-1 of Section II, Part D at 300°F}$$
  

$$G_{c} = 19.0 in.$$
  

$$G_{s} = 19.0 in.$$
  

$$P_{sd,max} = 10 psig$$
  

$$P_{sd,max} = -15 psig$$
  

$$P_{td,max} = 135 psig$$
  

$$P_{td,min} = 0 psig$$
  

$$S = 17,700 psi \text{ per Table 5A of Section II, Part D at 300°F}$$
  

$$W^{*} = 162,000 lb \text{ from Table 4.18.6}$$

#### **Calculation Procedure**

The calculation procedure for a U-tube heat exchanger tubesheet is given in paragraph 4.18.7. The calculation results are shown for loading case 1 where  $P_s = P_{sd,min} = -15 \, psig$  and  $P_t = P_{td,max} = 135 \, psig$  since this case yields the greatest value of  $\sigma$ .

a) STEP 1 – Calculate  $D_o, \mu, \mu^*$ , and  $h'_g$  from paragraph 4.18.6.4.a.

$$D_o = 16.813 in.$$
  

$$L_{L1} = 16.8 in.$$
  

$$A_L = 29.4 in.^2$$
  

$$\mu = 0.167$$
  

$$d* = 0.580 in.$$
  

$$p* = 0.805 in.$$
  

$$\mu* = 0.280$$
  

$$h'_g = 0 in.$$

b) STEP 2 – Calculate  $\rho_s, \rho_c$ , and  $M_{\rm TS}$  for configuration d.

$$\rho_s = 1.13$$
  
 $\rho_c = 1.13$ 
  
 $M_{TS} = -785 \text{ in.} -lb/in.$ 

c) STEP 3 – Assume a value for the tubesheet thickness, h, and calculate h/p. Determine  $E^*/E$  and  $v^*$  from paragraph 4.18.6.4.b and calculate  $E^*$ .

$$h = 1.28 in.$$
  
 $h/p = 1.71$   
 $E */E = 0.265$   
 $v * = 0.358$   
 $E * = 7.50E10^6 psi$ 

- d) STEP 4 For configuration d, skip STEP 4 and proceed to STEP 5.
- e) STEP 5 Calculate K and F for configuration d.

$$K = 1.19$$
  
 $F = 0.420$ 

f) STEP 6 – Calculate  $M^*$  for configuration d.

 $M^* = -785 in. -lb/in.$ 

g) STEP 7 – Calculate  $M_p, M_q$ , and M.

 $M_p = -160 \text{ in.} -lb/in.$  $M_o = -2,380 \text{ in.} -lb/in.$ M = 2,380 in. -lb/in.

h) STEP 8 – Calculate  $\sigma$  and check the acceptance criterion.

 $\sigma = 31,200 \ psi \le 2S = 35,400 \ psi$ 

i) STEP 9 – Calculate the average shear stress in the tubesheet at the outer edge of the perforated region, if required. Compare to the allowable.

If 
$$|P_s - P_t| \le \frac{3.2S \mu h}{D_o}$$
, then the shear stress is not required to be calculated

Load Case 1:

$$\left\{ \left| P_s - P_t \right| = 150 \ psi \right\} \le \left\{ \frac{3.2S\,\mu h}{D_o} = 639 \ psi \right\}$$

Calculate the average shear stress, au .

Not required for this example.

The assumed value for the tubesheet thickness, h, is acceptable and the calculation procedure is complete.

### PTB-3-2013

### 4.18.3 Example E4.18.3 – U-Tube Tubesheet Gasketed With Shell and Channel

A U-tube heat exchanger is to be designed with the tubesheet construction in accordance with configuration d as shown in Figure 4.18.4.

- The shell side design condition is 375 psig at 500°F.
- The tube side design condition is 75 psig at 500°F.
- The tube material is SB-111 C70600 (90/10 copper-nickel). The tubes are 0.75 in. outside diameter and 0.049 in. thick and are to be expanded for one-half of the tubesheet thickness.
- The tubesheet material is SA-516, Grade 70 (K02700) with a 0.125 in. corrosion allowance on the tube side and a 0.1875 in. deep pass partition groove. The tubesheet outside diameter is 48.88 in. The tubesheet has 1,534 tube holes on a 0.9375 in. equilateral triangular pattern with one centerline pass lane. The largest center-to-center distance between adjacent tube rows is 2.25 in., and the radius to the outermost tube hole center is 20.5 in.
- The diameter of the shell gasket load reaction is 43.5 in. and the shell flange design bolt load is 675,000 lb.
- The diameter of the channel gasket load reaction is 44.88 in. and the channel flange design bolt load is 584,000 lb.
- The tubesheet shall be designed for the differential design pressure.

#### Data Summary

The data summary consists of those variables from the nomenclature (see paragraph 4.18.15) that are applicable to this configuration.

The data for paragraph 4.18.15 is:

$$c_{t} = 0.125 in.$$

$$d_{t} = 0.75 in.$$

$$E = 27.1E10^{6} psi \text{ from Table TM-1 of Section II, Part D at 500° F}$$

$$E_{t} = 16.6E10^{6} psi \text{ from Table TM-3 of Section II, Part D at 500° F}$$

$$h_{g} = 0.1875 in.$$

$$p = 0.9375 in.$$

$$r_{o} = 20.5 in.$$

$$S = 20,600 psi \text{ from Table 5A of Section II, Part D at 500° F}$$

$$S_{t} = 8,000 psi \text{ from Table 5B of Section II, Part D at 500° F}$$

$$t_{t} = 0.049 in.$$

$$U_{L1} = 2.25 in.$$

 $\rho = 0.5$  for tubes expanded for one-half the tubesheet thickness

A = 48.88 in.  $E = 27.1E10^{6} psi \text{ from Table TM-1 of Section II, Part D at 500° F}$   $G_{c} = 44.88 in.$   $G_{s} = 43.5 in.$   $P_{sd,max} = 375 psig$   $P_{td,max} = 75 psig$  S = 20,600 psi from Table 5A of Section II, Part D at 500° F $W^{*} = 675,000 lb \text{ from Table UHX-8.1}$ 

#### Calculation Procedure

The calculation procedure for a U-tube heat exchanger tubesheet is given in paragraph 4.18.7. Since differential pressure design is specified, the calculation results are shown for loading case 3.

a) STEP 1 – Calculate  $D_o, \mu, \mu^*$ , and  $h'_g$  from paragraph 4.18.6.4.a.

$$D_{o} = 41.75 \text{ in.}$$

$$L_{L1} = 41.8 \text{ in.}$$

$$A_{L} = 93.9 \text{ in.}^{2}$$

$$\mu = 0.2$$

$$d* = 0.738 \text{ in.}$$

$$p* = 0.971 \text{ in.}$$

$$\mu* = 0.240$$

$$h'_{g} = 0.0625 \text{ in.}$$

b) STEP 2 – Calculate  $\rho_s, \rho_c$ , and  $M_{\rm TS}$  for configuration d.

$$\rho_s = 1.04$$
  
 $\rho_c = 1.07$ 
  
 $M_{TS} = 2,250 \, in. -lb/in.$ 

c) STEP 3 – Assume a value for the tubesheet thickness, h, and calculate h/p. Determine  $E^*/E$  and  $v^*$  from paragraph 4.18.6.4.b and calculate  $E^*$ .

$$h = 4.15 in.$$
  
 $h/p = 4.43$   
 $E*/E = 0.204$   
 $v* = 0.407$   
 $E* = 5.54E10^6 psi$ 

d) STEP 4 – For configuration d, skip STEP 4 and proceed to STEP 5.

e) STEP 5 – Calculate K and F for configuration d.

$$K = 1.17$$

$$F = 0.458$$

f) STEP 6 – Calculate M \* for configuration d.

 $M^* = 5800 \text{ in.} -lb/in.$ 

g) STEP 7 – Calculate  $M_p, M_o, and M$ .

 $M_p = -1150 \text{ in.} -lb/in.$  $M_o = 26,700 \text{ in.} -lb/in.$ M = 26,700 in. -lb/in.

h) STEP 8 – Calculate  $\sigma$  and check the acceptance criterion.

$$\sigma = 39,900 \ psi \le 2S = 41,200 \ psi$$

i) STEP 9 – Calculate the average shear stress in the tubesheet at the outer edge of the perforated region, if required. Compare to the allowable.

If 
$$|P_s - P_t| \le \frac{3.2S\mu h}{D_o}$$
, then the shear stress is not required to be calculated

Load Case 3:

$$\left\{ \left| P_s - P_t \right| = 300 \ psi \right\} \le \left\{ \frac{3.2S \,\mu h}{D_o} = 1311 \ psi \right\}$$

Calculate the average shear stress, au .

Not required for this example.

The assumed value for the tubesheet thickness, h, is acceptable and the calculation procedure is complete.

# 4.18.4 Example E4.18.4 – U-Tube Tubesheet Gasketed With Shell and Integral with Channel, Extended as a Flange

A U-tube heat exchanger is to be designed with the tubesheet construction in accordance with configuration e as shown in Figure 4.18.4.

- The shell side design condition is 650 psig at 400°F.
- The tube side design condition is 650 psig at 400°F.
- The tube material is SA-179 (K01200). The tubes are 0.75 in. outside diameter and 0.085 in. thick and are to be expanded for the full thickness of the tubesheet.
- The tubesheet material is SA-516, Grade 70 (K02700) with a 0.125 in. corrosion allowance on the tube side and no pass partition grooves. The tubesheet outside diameter is 37.25 in. The tubesheet has 496 tube holes on a 1.0 in. square pattern with one centerline pass lane. The largest center-to-center distance between adjacent tube rows is 1.375 in., and the radius to the outermost tube hole center is 12.75 in.
- The diameter of the shell gasket load reaction is 32.375 in., the shell flange bolt circle is 35 in., and the shell flange design bolt load is 656,000 lb.
- The channel material is SA-516, Grade 70, (K02700). The channel inside diameter is 31 in. and the channel thickness is 0.625 in.

# Data Summary

The data summary consists of those variables from the nomenclature (see paragraph 4.18.15) that are applicable to this configuration.

The data for paragraph 4.18.15 is:

$$c_{t} = 0.125 in.$$

$$d_{t} = 0.75 in.$$

$$E = 27.7E10^{6} psi \text{ from Table TM-1 of Section II, Part D at 400°F}$$

$$E_{t} = 27.7E10^{6} psi \text{ from Table TM-1 of Section II, Part D at 400°F}$$

$$h_{g} = 0 in.$$

$$p = 1.0 in.$$

$$r_{o} = 12.75 in.$$

$$S = 21,600 psi \text{ from Table 5A of Section II, Part D at 400°F}$$

$$S_{t} = 13,400 psi \text{ from Table 1A of Section II, Part D at 400°F}$$

$$t_{t} = 0.085 in.$$

$$U_{L1} = 1.375 in.$$

$$\rho = 1.0 \text{ for full length tube expansion}$$

$$\begin{split} &A = 37.25 \, in. \\ &C = 35 \, in. \\ &D_c = 31 \, in. \\ &E = 27.7 E10^6 \, psi \, \text{from Table TM-1 of Section II, Part D at 400°F} \\ &E_c = 27.7 E10^6 \, psi \, \text{from Table TM-1 of Section II, Part D at 400°F} \\ &G_s = 32.375 \, in. \\ &P_{sd,\text{max}} = 650 \, psig \\ &P_{sd,\text{min}} = 0 \, psig \\ &P_{ud,\text{max}} = 650 \, psig \\ &P_{ud,\text{max}} = 650 \, psig \\ &S = 21,600 \, psi \, \text{from Table 5A of Section II, Part D at 400°F} \\ &S_c = 21,600 \, psi \, \text{from Table 5A of Section II, Part D at 400°F} \\ &S_{y,c} = 32,500 \, psi \, \text{from Table 5A of Section II, Part D at 400°F} \\ &S_{PS,c} = 65,000 \, psi \, (\text{MYS/UTS} < 0.7; \, \text{therefore use } 2S_{y,c}) \\ &t_c = 0.625 \, in. \\ &W^* = 656,000 \, lb \\ &V_c = 0.3 \end{split}$$

#### **Calculation Procedure**

The calculation procedure for a U-tube heat exchanger tubesheet is given in paragraph 4.18.7. The calculation results are shown for loading case 2 where  $P_s = P_{sd,max} = 650 \ psig$  and  $P_t = P_{td,min} = 0 \ psig$ , since this case yields the greatest value of  $\sigma$ .

a) STEP 1 – Calculate  $D_o, \mu, \mu * \text{ and } h'_g$  from paragraph 4.18.6.4.a.

$$\begin{split} D_o &= 26.25 \, in. \\ L_{L1} &= 26.3 \, in. \\ A_L &= 36.1 \, in.^2 \\ \mu &= 0.25 \\ d^* &= 0.636 \, in. \\ p^* &= 1.04 \, in. \\ \mu^* &= 0.385 \\ h_g' &= 0 \, in. \end{split}$$

b) STEP 2 – Calculate  $\rho_s, \rho_c$ , and  $M_{\rm TS}$  for configuration e.

$$\rho_s = 1.23$$
  
 $\rho_c = 1.18$ 
  
 $M_{TS} = 16,500 \text{ in.} -lb/in.$ 

c) STEP 3 – Assume a value for the tubesheet thickness, h, and calculate h/p. Determine  $E^*/E$  and  $v^*$  from paragraph 4.18.6.4.b and calculate  $E^*$ .

$$h = 3.50 in.$$
  
 $h/p = 3.50$   
 $E */E = 0.441$   
 $v* = 0.318$   
 $E* = 12.2E10^6 psi$ 

d) STEP 4 – For configuration e, calculate  $\beta_c$ ,  $k_c$ ,  $\lambda_c$ ,  $\delta_c$ , and  $\omega_c$  for the channel.

$$\beta_{c} = 0.409 in.^{-1}$$

$$k_{c} = 506,000 lb$$

$$\lambda_{c} = 7.59 \times 10^{6} psi$$

$$\delta_{c} = 1.18 \times 10^{-5} in.^{3} / lb$$

$$\omega_{c} = 7.01 in.^{2}$$

e) STEP 5 – Calculate K and F for configuration e.

$$K = 1.42$$
  
 $F = 0.964$ 

f) STEP 6 – Calculate  $M^*$  for configuration e.

 $M^* = 26,900 \text{ in.} -lb/in.$ 

g) STEP 7 – Calculate  $M_p, M_o, and M$ .

$$M_p = 6830 \text{ in.} -lb/in.$$
  
 $M_o = 30,000 \text{ in.} -lb/in.$   
 $M = 30,000 \text{ in.} -lb/in.$ 

h) STEP 8 – Calculate  $\sigma$  and check the acceptance criterion.

 $\sigma = 38,200 \ psi \le 2S = 43,200 \ psi$ 

i) STEP 9 – For each loading case, calculate the average shear stress in the tubesheet at the outer edge of the perforated region, if required. Compare to the allowable.

If 
$$|P_s - P_t| \le \frac{3.2S\mu h}{D_o}$$
, then the shear stress is not required to be calculated.

Load Case 2:

$$\left\{ \left| P_s - P_t \right| = 650 \ psi \right\} \le \left\{ \frac{3.2S\mu h}{D_o} = 2133 \ psi \right\}$$

Calculate the average shear stress, au .

Not required for this example.

j) STEP 10 – For configuration e, calculate  $\sigma_{c,m}, \sigma_{c,b}$ , and  $\sigma_c$  for the channel, and check the acceptance criterion. The channel thickness shall be 0.625 in. for a minimum length of 7.92 in. adjacent to the tubesheet.

$$\sigma_{c,m} = 0 \ psi$$
  
 $\sigma_{c,b} = -57,000 \ psi$   
 $\sigma_c = 57,000 \ psi > 1.5S_c = 32,400 \ psi$ 

- k) STEP 11 Since the channel stress exceeds the allowable stress, the design must be reconsidered using one of three options.
  - Option 1 requires that the tubesheet thickness be increased until the channel stresses calculated in STEP 9 are within the allowable stress for each loading case.
  - Option 2 requires that the shell and/or channel thickness be increased until their respective stresses calculated in STEP 9 are within the allowable stress for each loading case.
  - Option 3 permits one elastic-plastic calculation for each design. If the tubesheet stress is still within the allowable stress given in STEP 8, the design is acceptable and the calculation procedure is complete. If the tubesheet stress is greater than the allowable stress, the design shall be reconsidered by using Option 1 or 2.

Choose Option 3, configuration e. Since  $\sigma_c \leq S_{PS,c} = 65,000 \ psi$  for all loading cases, this option may be used. The calculations for this option are only required for each loading case where  $\sigma_c > 1.5S_c = 32,400 \ psi$ .

Calculate  $E_c^*$  for each loading case where  $\sigma_c > 32,400 \text{ psi}$ . For this example,  $E_c^*$  and the calculations for loading case 2 are shown.

 $E_c^* = 20.1E10^6 \ psi$ 

Recalculate  $k_c$  and  $\lambda_c$  given in STEP 4 using the applicable reduced effective modulus  $E_c$ .

$$k_c = 368,000 \, lb$$
$$\lambda_c = 5.51 E 10^6 \, psi$$

Recalculate F given in STEP 5.

F = 0.848

Recalculate  $M_n, M_a$ , and M given in STEP 7.

$$M_p = 8,130 \text{ in.} -lb/in.$$
  
 $M_o = 31,400 \text{ in.} -lb/in.$   
 $M = 31,400 \text{ in.} -lb/in.$ 

Recalculate  $\sigma$  given in STEP 8.

 $\sigma = 39,800 \ psi \le 2S = 43,200 \ psi$ 

The assumed value for the tubesheet thickness, h, is acceptable and the calculation procedure is complete.

# 4.18.5 Example E4.18.5 – Fixed Tubesheet Exchanger, Configuration b, Tubesheet Integral with Shell, Extended as a Flange and Gasketed on the Channel Side

A fixed tubesheet heat exchanger is to be designed with the tubesheet construction in accordance with configuration b as shown in VIII-1, Figure UHX-13.1.

- For the Design Condition, the shell side design pressure is 150 psig at 700°F, and the tube side design pressure is 400 psig at 700°F.
- There is one operating condition. For Operating Condition 1, the shell side design pressure is 150 psig at 700°F, the tube side design pressure is 400 psig at 700°F, the shell mean metal temperature is 550°F, and the tube mean metal temperature is 510°F. For this example, the operating pressures and operating metal temperatures are assumed to be the same as the design values.
- The tube material is SA-214 welded (K01807). The tubes are 1 in. outside diameter, 0.083 in. thick and are to be expanded to 95% of the tubesheet thickness.
- The tubesheet material is SA-516, Grade 70 (K02700). The tubesheet outside diameter is 40.5 in. There are 649 tube holes on a 1.25 in. triangular pattern. There is no pass partition lane, and the outermost tube radius from the tubesheet center is 16.625 in. The distance between the outer tubesheet faces is 168 in. There is no corrosion allowance on the tubesheet.
- The shell material is SA-516, Grade 70 (K02700). The shell inside diameter is 34.75 in. and the thickness is 0.1875 in. There is no corrosion allowance on the shell. The shell contains an expansion joint that has an inside diameter of 38.5 in. and an axial rigidity of 11,388 lb/in. The efficiency of the shell circumferential welded joint (Category B) is 1.0.
- The diameter of the channel flange gasket load reaction is 36.8125 in., the bolt circle diameter is 38.875 in., the design bolt load is 512,937 lb, and the operating condition bolt load is 512,473 lb.

### Data Summary - Tubesheet

Tube Layout: Triangular h = 3.0625 in. $h_g = 0$  in.  $c_t = 0$  in. A = 40.5 in. $r_o = 16.625 in.$  $A_{L} = 0.0 in.^{2}$  $N_t = 649$  $L_t = 168 in.$ p = 1.2500 in.  $T = 700^{\circ} \mathrm{F}$  $T_a = 70^{\circ} \mathrm{F}$ S = 18,100 psi at T from Table 5A of Section II, Part D  $S_y = 27,200 \ psi$  at T  $S_{PS} = 54,400 \ psi$  at T E = 25.5E6 psi at T from TM-1 of Section II, Part D v = 0.3

#### Data Summary - Tubes

 $P_{td, \max} = 400 \, psig$  $P_{td,\min} = 0 psig$  $P_{to1} = 400 \ psig$  $\ell_{tr} = 2.909 in.$ k = 1 $\ell = 59 in.$  $t_{t} = 0.083 in.$  $d_t = 1$  in.  $T_{t} = 700^{\circ} \text{F}$  $T_{tm} = 510^{\circ} \text{F}$  $S_t = 10,500 \text{ psi}$  at  $T_t$  from Table 1A of Section II, Part D  $S_{v,t} = 18,600 \text{ psi} \text{ at } T_t \text{ from Y-lof Section II, Part D}$  $S_{tT} = 10,500 \text{ psi} \text{ at } T \text{ from Table 1A of Section II, Part D}$  $\alpha_{tm} = 7.3E - 06 in. / in. ^{\circ}F$  at  $T_{tm}$  $E_t = 25,500,000 \text{ psi}$  at  $T_t$  from TM-1 of Section II, Part D  $E_{T} = 25,500,000 \text{ psi}$  at T from TM-1 of Section II, Part D  $v_{t} = 0.3$ 

Note: Since the tubes are welded (SA-214), the tube allowable stresses  $S_t$  and  $S_{tT}$  can be divided by 0.85 per paragraph 4.18.15.d. This results in adjusted values of  $S_t = 12,353 psi$  and  $S_{tT} = 12,353 psi$ . Data Summary - Shell

 $P_{sd \max} = 150 psig$  $P_{sd,min} = 0 psig$  $P_{sol} = 150 psig$  $t_{s} = 0.1875$  in.  $D_{\rm s} = 34.75$  in.  $D_i = 38.5 in.$  $K_i = 11,388 lb/in.$  $T_{\circ} = 700^{\circ} \,\mathrm{F}$  $T_{s.m} = 550^{\circ} \mathrm{F}$  $S_s = 18,100 \text{ psi}$  at  $T_s$  from Table 5A of Section II, Part D  $E_{sw} = 1.0$  $S_{v,s} = 27,200 \text{ psi}$  from Table Y-1 of Section II, Part D  $S_{PSs} = 54,400 \text{ psi}$  at T see paragraph 5.5.6.1.d  $E_s = 25,500,000 \text{ psi}$  from TM-1 of Section II, Part D  $\alpha_{sm} = 7.3E - 06 \text{ in.} / \text{ in.} /^{\circ} \text{ F} \text{ at } T_{sm}$  $v_{\rm c} = 0.3$ 

Data Summary - Channel Flange

Gasket I.D. = 36.3125 in. Gasket O.D. = 37.3125 in. Mean Gasket Diameter,  $G = G_c = 36.8125 in$ . Gasket, m, Factor = 3.75Gasket, y, Factor = 7,600 psiFlange Outside Diameter = 40.5 in. Bolt Circle, C = 38.875 in. Bolting Data = 68 bolts, 0.75 in.diameter Bolting Material = SA-193 B7 Bolt Load,  $W_g = 512,937 lb$  per VIII – 2 paragraph 4.16 Bolt Load,  $W_o = 512,473 lb$  per VIII – 2 paragraph 4.16 W<sup>\*</sup> from Table 4.18.6 (see Summary Table for Step 5) Gasket Monument Arm,  $h_g = (C - G_c)/2 = 1.03125 in$ .

#### Calculation Procedure

The tubesheet is extended as a flange. The calculation procedure for a tubesheet flanged extension is given in paragraph 4.18.5.4.

The tubesheet flanged extension required thickness for the operating condition (S = 18,100 psi at T) is:

 $h_r = 1.228 in.$ 

The tubesheet flanged extension required thickness for the gasket seating condition  $(S = 25,300 \text{ psi at } T_a)$  is:

$$h_r = 1.039$$
 in.

The calculation procedure for a Fixed Tubesheet heat exchanger is given in paragraph 4.18.8. The following results are for the design and operating loading cases required to be analyzed (see paragraph 4.18.8.3). This example illustrates the calculation of both the elastic and elastic-plastic solutions.

a) STEP 1 – Calculate  $D_o$ ,  $\mu$ ,  $\mu *$  and  $h'_g$  from paragraph 4.18.6.4.a.

$$L = 161.875 in.$$
  

$$D_o = 34.25 in.$$
  

$$a_o = 17.125 in.$$
  

$$\rho = 0.95$$
  

$$d^* = 0.8924 in.$$
  

$$\mu = 0.2000$$
  

$$\rho^* = 1.2500 in.$$
  

$$\mu^* = 0.2861$$
  

$$\rho_s = 1.014598$$
  

$$\rho_c = 1.074818$$
  

$$x_s = 0.4467$$
  

$$x_t = 0.6152$$

b) STEP 2 – Calculate the shell axial stiffness,  $K_s$ , tube axial stiffness,  $K_t$ , stiffness factors,  $K_{s,t}$  and J.

 $K_s = 3,241,928 lb/in.$   $K_t = 37,666 lb/in.$   $K_{s,t} = 0.13262$ J = 0.0035 Calculate the shell coefficients  $\beta_s$ ,  $k_s$ ,  $\lambda_s$ , and  $\delta_s$ .

$$\beta_s = 0.7102 \text{ in.}^{-1}$$
  
 $k_s = 21,866 lb$   
 $\lambda_s = 879,437 \text{ psi}$   
 $\delta_s = 0.0000536694 \text{ in.}^3 / lb$ 

Calculate the channel coefficients  $\beta_c, k_c, \lambda_c$ , and  $\delta_c$ .

$$\beta_c = 0 in.^{-1}$$

$$k_c = 0 lb.$$

$$\lambda_c = 0 psi$$

$$\delta_c = 0 in.^3 / lb$$

c) STEP 3 – Calculate h/p. Determine  $E^*/E$  and  $v^*$  from paragraph 4.18.6.4.b and calculate  $E^*$ .

$$h/p = 2.45$$
  
 $E*/E = 0.262993$   
 $v* = 0.363967$   
 $E* = 6,706,322 \ psi$ 

Calculate,  $\,X_{\scriptscriptstyle a}^{}$  , and the parameters from Table 4.18.3.

$$X_{a} = 3.9630$$

$$Z_{d} = 0.024609$$

$$Z_{v} = 0.064259$$

$$Z_{m} = 0.371462$$

$$Z_{a} = 6.54740$$

$$Z_{m} = 0.064259$$

d) STEP 4 – Calculate the diameter ratio, K , the coefficient F , and the associated parameters.

$$K = 1.1825$$
  
 $F = 0.4888$ 

Calculate  $\Phi$ ,  $Q_1$ ,  $Q_{z1}$ ,  $Q_{z2}$  and U.

 $\Phi = 0.6667$   $Q_1 = -0.022635$   $Q_{Z1} = 2.8556$   $Q_{Z2} = 6.888$  U = 13.776

Summary Table for Step 5 – Design Condition									
Loading Case	P <sub>s</sub> (psi)	$P_t$ (psi)	γ	W*					
1	0	400	0	512473					
2	150	0	0	0					
3	150	400	0	512473					

e) STEP 5 – Calculate  $\gamma$ ,  $\omega_s$ ,  $\omega_s^*$ ,  $\omega_c$ ,  $\omega_c^*$ , and  $\gamma_b$ 

Summary Table for Step 5 – Operating Condition 1								
Loading Case	P <sub>s</sub> (psi)	P <sub>t</sub> (psi)	γ	W*				
1	0	400	-0.047	512937				
2	150	0	-0.047	512937				
3	150	400	-0.047	512937				
4	0	0	-0.047	512937				

 $\omega_{s} = 2.685 in.^{2}$   $\omega_{s}^{*} = -2.6536 in.^{2}$   $\omega_{c}^{*} = 0 in.^{2}$   $\omega_{c}^{*} = 9.6816 in.^{2}$   $\gamma_{b}^{*} = -0.06022$ 

f) STEP 6 – For each loading case, calculate  $P'_{s}$ ,  $P'_{t}$ ,  $P_{\gamma}$ ,  $P_{\omega}$ ,  $P_{W}$ ,  $P_{rim}$  and effective pressure  $P_{e}$ .

	Summary Table for STEP 6 – Design Condition									
Loading Case	P's (psi)	$P'_t$ (psi)	$P_{\gamma}$ (psi)	$P_{\omega}$ (psi)	$P_W$ (psi)	P <sub>rim</sub> (psi)	$P_e$ (psi)			
1	0	862,002	0	0	230.7	181.9	-399.4			
2	-46,387	0	0	0	0	18.7	-21.5			
3	-46,387	862,002	0	0	230.7	200.6	-420.9			

Summary Table for STEP 6 – Operating Condition 1									
Loading	$P'_s$	$P'_t$	$P_t'$ $P_{\gamma}$ $P_{\omega}$ $P_W$ $P_{rim}$						
Case	(psi)	(psi)	(psi)	(psi)	(psi)	(psi)	(psi)		
1	0	862,002	-1,254	0	230.9	181.9	-400		
2	-46,387	0	-1,254	0	230.9	18.7	-22		
3	-46,387	862,002	-1,254	0	230.9	200.6	-421.5		
4	0	0	-1,254	0	230.9	0	-0.5		

	Summary Table for STEP 7 – Design Condition									
Loading Case	$Q_2$ (lbs)	$Q_3$	$F_m$	$h_{g}^{\prime}$ (in)	$h-h_g'$ (in)	$\left  \sigma_{_{elastic}}  ight $ (psi)	1.5 <i>S</i> (psi)	$S_{_{PS}}$ (psi)		
1	-7,040.7	0.0976	0.0975	0	3.0625	25,540	27,150			
2	-319	0.0786	0.0901	0	3.0625	-1,269	27,150			
3	-7,359.7	0.0966	0.0971	0	3.0625	26,809	27,150			

g) STEP 7 – Elastic Iteration, calculate  $Q_2$  and  $Q_3$ , the tubesheet bending stress, and the allowable tubesheet bending stress.

	Summary Table for STEP 7 – Operating Condition 1										
Loading Case	$Q_2$ (lbs)	$Q_3$	$F_m$	$h_g'$ (in)	$h-h'_g$ (in)	$ert \sigma_{_{elastic}} ert$ (psi)	1.5 <i>S</i> (psi)	$S_{_{PS}}$ (psi)			
1	-7,044.2	0.09746	0.09749	0	3.0625	25,569		54,400			
2	-4,259.3	1.299	0.67047	0	3.0625	9,658		54,400			
3	-7,363.3	0.09650	0.09711	0	3.0625	26,839		54,400			
4	-3,940.3	56.627	28.409	0	3.0625	8,838		54,400			

For Design Loading Cases 1-3  $|\sigma_{elastic}| < 1.5S$ , and for Operating Cases 1-4  $|\sigma_{elastic}| < S_{PS}$ . The bending stress criterion for the tubesheet is satisfied.

h) STEP 8 – For each loading case, calculate the average shear stress in the tubesheet at the outer edge of the perforated region, if required. Compare to the allowable.

If  $|P_e| \leq \frac{1.6S \mu h}{a_o}$ , then the shear stress is not required to be calculated.

Design Condition 1, Load Case 1:

$$\left\{ \left| P_e \right| = 399.4 \ psi \right\} \le \left\{ \frac{1.6S \,\mu h}{a_o} = 1036 \ psi \right\}$$

Therefore, shear stress is not required to be calculated for design condition, case 1. Similarly, the shear stress is not required to be calculated for design conditions, case 2 and case 3, and operating condition cases 1-4.

Summary Table for STEP 8 – Design Condition							
Loading Case	0.8 <i>S</i> (psi)						
1	Not required	14,480					
2	Not required	14,480					
3	Not required	14,480					

Summary Table for STEP 8 – Operating Condition 1							
Loading Case	τ  (psi)	0.8 <i>S</i> (psi)					
1	Not required	14,480					
2	Not required	14,480					
3	Not required	14,480					
4	Not required	14,480					

For all Loading Cases, the shear stress criterion is not required.

i) STEP 9 – For each load case, calculate the axial tube stress and the allowable axial tube stress based on tube buckling.

 $r_t = 0.3255 \ in.$  $F_t = 181.24 \ in.$ 

 $C_{t}^{\prime} = 164.5$ 

Summary Table for STEP 9 – Design Condition								
Loading Case	$F_t$ , min	$\sigma_{\scriptscriptstyle t,1}$ (psi)	$F_t$ , max	$\sigma_{\scriptscriptstyle t,2}$ (psi)				
1	-1.081	-4024	3.809	7,570				
2	-1.011	269	3.658	865				
3	-1.077	-3,755	3.801	8,435				

Summary Table for STEP 9 – Operating Condition 1								
Loading Case	$F_t$ , min	$\sigma_{\scriptscriptstyle t,1}$ (psi)	$F_t$ , max	$\sigma_{\scriptscriptstyle t,2}$ (psi)				
1	-1.081	-4,028.8	3.807	7,580.9				
2	-5.520	-322.2	13.334	2,137				
3	-1.078	-3,760	3.8	8,445.5				
4	-213.188	-600.4	451.8	1,272.4				

# PTB-3-2013

Summary Table for STEP 9 – Design Condition (continued)									
Loading Case	$\sigma_{_{t,\mathrm{max}}}$ (psi)	$S_t$ (psi)	$\left  \sigma_{_{t,\min}}  ight $ (psi)	$F_s$	$S_{tb}$ (psi)				
1	7,570	12,353	4,024	1.346	5,693.9				
2	865	12,353	0	0	0				
3	8,435	12,353	3,755	1.349	5,677				

Summary Table for STEP 9 – Operating Condition 1 (continued)									
Loading Case	$\sigma_{_{t,\mathrm{max}}}$ (psi)	$S_t$ (psi)	$\left  \sigma_{_{t,\min}}  ight $ (psi)	$F_s$	$S_{tb}$ (psi)				
1	7,580.9	24,706	4,028.8	1.346	5,690.9				
2	2,137.0	24,706	322.2	1.250	6,129.4				
3	8,445.5	24,706	3,760	1.350	5,674.9				
4	1,272.4	24,706	600.4	1.250	6,129.4				

For all Loading Cases  $|\sigma_{t,\max}| < S_t$ . The axial tension stress criterion for the tube is satisfied. For all Loading Cases  $|\sigma_{t,\min}| < S_{tb}$ . The buckling criterion for the tube is satisfied.

Summary Table for STEP 10 – Design Condition					
Loading Case	$\sigma_{_{s,m}}$ (psi)	$S_s E_{s,w}$ (psi)	$S_{_{PS,s}}$ (psi)	<i>S</i> <sub><i>s,b</i></sub> (psi)	
1	26.1	18,100			
2	-760	18,100		8,505	
3	-738.7	18,100		8,508	

j) STEP 10 – For each loading case, calculate the axial membrane stress in each shell section and determine the maximum allowable longitudinal compressive stress.

Summary Table for STEP 10 – Operating Condition 1				
Loading Case	$\sigma_{_{s,m}}$ (psi)	$S_s E_{s,w}$ (psi)	$S_{_{PS,s}}$ (psi)	$S_{s,b}$ (psi)
1	0.0579		36,200	
2	-786.1		36,200	8,505
3	-764.8		36,200	8,505
4	-21.2		36,200	8,505

k) STEP 11 – For each loading case, calculate the stresses in the shell and/or channel when integral with the tubesheet.

Summary Table for STEP 11 – Design Condition					
Loading	$\sigma_{_{s,m}}$	$\sigma_{_{s,b}}$	$\sigma_{_s}$	$1.5S_{s}$	$S_{_{PS,s}}$
Case	(psi)	(psi)	(psi)	(psi)	(psi)
1	26.1	-42,440	42,466	27,150	
2	-760	19,214	19,978	27,150	
3	-738.7	-23,227	23,966	27,150	

Summary Table for STEP 11 – Operating Condition 1					
Loading Case	$\sigma_{\scriptscriptstyle s,m}$ (psi)	$\sigma_{\scriptscriptstyle s,b}$ (psi)	$\sigma_{_s}$ (psi)	1.5 <i>S</i> <sub>s</sub> (psi)	$S_{_{PS,s}}$ (psi)
1	0.0579	-42,484	42,484		54,400
2	-786.1	8,633	9,419		54,400
3	-764.8	-23,271	24,035		54,400
4	-21.2	-10,581	10,602		54,400

For Design Loading Cases 1 and 3  $|\sigma_s| > 1.5S_s$ , and for Operating Loading Cases 1-4  $|\sigma_s| < S_{_{PS,s}}$ . The stress criterion for the shell is not satisfied.

- I) STEP 12 The design shall be reconsidered by using one or a combination of the following options.
  - Option 1 Increase the tubesheet thickness and return to STEP 1.
  - Option 2 Increase the integral shell and/or channel thickness and return to STEP 1
  - Option 3 Perform the elastic-plastic calculation procedures as defined in paragraph 4.18.8.6.

Choose Option 3. Since the total axial stress in the shell  $\sigma_s$  is between  $1.5S_s$  and  $S_{PS,s}$  for Design Condition Loading Case 1 and 3, the procedure of paragraph 4.18.8.6 may be performed to determine if the tubesheet stresses are acceptable when the plasticity of the shell occurs.

Summary Results for STEP 12, Elastic Plastic Iteration Results per paragraph 4 18 8 6				
Design Condition Loading Case				
$S^*_{s}$ , psi	27,200			
fact <sub>s</sub>	0.776			
$E^*{}_s, psi$	19,785,000			
$k_s, lb$	16,965			
$\lambda_s$	0.682E+06			
F	0.470			
φ	0.641			
$Q_1$	-0.0215			
$Q_{Z1}$	2.865			
$Q_{Z2}$	6.941			
U	13.882			
$P_W, psi$	232.5			
$P_{\rm rim}$ , psi	183.309			
$P_e, psi$	-399.4			
$Q_2, lb$	-7,095			
Q <sub>3</sub>	0.100			
F <sub>m</sub>	0.098			
$ \sigma , psi$	25,752			

The final calculated tubesheet bending stress of 25,752 psi (Design Loading Case 1) is less than the allowable tubesheet bending stress of 27,150 psi. As such, this geometry meets the requirements of paragraph 4.18.8.6. The intermediate results for the elastic-plastic calculation are shown above.

# 4.18.6 Example E4.18.6 – Fixed Tubesheet Exchanger, Configuration b, Tubesheet Integral with Shell, Extended as a Flange and Gasketed on the Channel Side

A fixed tubesheet heat exchanger is to be designed with the tubesheet construction in accordance with configuration b as shown in VIII-1, Figure UHX-13.1.

- For the Design Condition, the shell side design pressure is 335 psig at 675°F, and the tube side design pressure is 1040 psig at 650°F.
- There is one operating condition. For Operating Condition 1, the shell side design pressure is 335 psig at 675°F, the tube side design pressure is 1040 psig at 650°F, the shell mean metal temperature is 550°F, and the tube mean metal temperature is 490°F. For this example, the operating pressures and operating metal temperatures are assumed to be the same as the design values.
- The tube material is welded SA-214 (K01807). The tubes are 0.75 in. outside diameter, are 0.083 in. thick, and are to be expanded for a length of 4.374 in.
- The tubesheet material is SA-516, Grade 70 (K02700). The tubesheet outside diameter is 32.875 in. There are 434 tube holes on a 0.9375 in. triangular pattern. There is no pass partition lane and the outermost tube radius from the tubesheet center is 10.406 in. The distance between the outer tubesheet faces is 144.375 in. There is a 0.125 in. corrosion allowance on both sides of the tubesheet.
- The shell material is SA-516, Grade 70 (K02700). The shell outside diameter is 24 in. and the thickness is 0.5 in. There is a 0.125 in. corrosion allowance on the shell. There is also a shell band 1.25 in. thick, 9.75 in. long with a 0.125 in. corrosion allowance. The shell and shell band materials are the same. The shell contains an expansion joint that has an inside diameter of 29.46 in. and an axial rigidity of 14,759 lb/in. The efficiency of shell circumferential welded joint (Category B) is 0.85.
- The diameter of the channel flange gasket load reaction is 25.625 in., the bolt circle diameter is 30.125 in., the design bolt load is 804,478 lb, and the operating condition bolt load is 804,456 lb.

Data Summary - Tubesheet

Tube Layout: Triangular h = 4.75 in. -0.125 in. -0.125 in. = 4.5 in. $h_{\sigma} = 0$  in.  $c_t = 0.125 in.$ A = 32.875 in.  $r_{o} = 10.406 in.$  $A_L = 0.0 in.^2$  $N_t = 434$  $L_t = 144.375$  in. p = 0.9375 in.  $T = 675^{\circ} F$  $T_a = 70^{\circ} F$ S = 18,450 psi at T from Table 5A of Section II, Part D  $S_v = 27,700 \text{ psi}$  at T from Table Y-1 of Section II, Part D  $S_{PS} = 55,400 \ psi$  at T E = 25,575,000 psi at T from TM-1 of Section II, Part D v = 0.3

#### Data Summary - Tubes

 $P_{td,max} = 1040 \ psig$  $P_{td,\min} = 0 psig$  $P_{to1} = 1040 \ psig$  $\ell_{tx} = 4.374 in.$ k = 1 $\ell = 34$  in.  $t_t = 0.083$  in.  $d_t = 0.75 in.$  $T_{t} = 675^{\circ} F$  $T_{tm} = 490^{\circ} F$  $S_t = 10,700 psi$  from Table 1A of Section II, Part D  $S_{v,t} = 18,950 \text{ psi}$  from Y-1 of Section II, Part D  $S_{iT} = 10,700 \text{ psi}$  at T from Table 1A of Section II, Part D  $\alpha_{tm} = 7.28E - 06 in. / in. /^{\circ} F$  at  $T_{tm}$  $E_t = 25,750,000 \text{ psi at } T_t \text{ from TM-1 of Section II, Part D}$  $E_{tT} = 25,750,000 \text{ psi} \text{ at } T$  from TM-1 of Section II, Part D  $v_{t} = 0.3$ 

Since the tubes are welded (SA-214), the tube allowable stresses  $S_t$  and  $S_{tT}$  can be delivered by 0.85 per paragraph 4.18.15.d. This results in adjusted values of  $S_t = 12,588 psi$  and  $S_{tT} = 12,588 psi$ .

```
Data Summary - Shell
      P_{sd,max} = 335 \, psig
      P_{sd \min} = 0 psig
      P_{so1} = 335 \, psig
      t_s = 0.5 in. - 0.125 in. = 0.375 in.
      D_{\rm s} = 23 \, in. + 2(0.125) \, in. = 23.25 \, in.
      D_i = 29.46 in.
      K_i = 14,759 \, lb \, / \, in.
      T_{s} = 675^{\circ} F
      T_{sm} = 550^{\circ} F
      E_s = 25,750,000 \text{ psi} from TM-1 of Section II, Part D
      \alpha_{sm} = 7.3E - 06 \text{ in.} / \text{in.} /^{\circ} F at T_{sm}
      v_{s} = 0.3
      t_{s1} = 1.25 in. -0.125 in. = 1.125 in.
      \ell_1 = 9.75 in. + 0.125 in. = 9.875 in.
      \ell_1' = 9.75 in. + 0.125 in. = 9.875 in.
      S_{s1} = 18,450 \text{ psi} at T_s from Table 5A of Section II, Part D
      S_{v,s,1} = 27,700 \text{ psi} at T_s from Table Y-1 of Section II, Part D
      S_{PS s1} = 55,400 \text{ psi}; \text{ see paragraph } 5.5.6.1.d
      E_{s1} = 25,750,000 \text{ psi} from TM-1 of Section II, Part D
      E_{sw} = 0.85
      \alpha_{sm1} = 7.3E - 06 in. / in. /^{\circ} F at T_{sm}
```

Data Summary - Channel Flange

Gasket I.D. = 25.125 in. Gasket O.D. = 26.125 in. Mean Gasket Diameter,  $G = G_c = 25.625$  in. Gasket, m, Factor = 6.5 Gasket, y, Factor = 26,000 psi Flange Outside Diameter = 32.875 in. Bolt Circle, C = 30.125 in. Bolting Data = 28 bolts, 1.375 in.diameter, SA – 193 B7 Bolt Load,  $W_g = 808,478$  lb per VIII – 2 paragraph 4.16 Bolt Load,  $W_o = 808,456$  lb per VIII – 2 paragraph 4.16 W<sup>\*</sup> from Table 4.18.6 (see Summary Table for Step 5) Gasket Moment Arm,  $h_c = (C - G)/2 = 2.25$  in.

#### Calculation Procedure

The tubesheet is extended as a flange. The calculation procedure for a tubesheet flanged extension is given in paragraph 4.18.5.4.

The tubesheet flanged extension required thickness for the operating condition (S = 18,450 psi at T) is:

 $h_r = 2.704 in.$ 

The tubesheet flanged extension required thickness for the gasket seating condition  $(S = 25,300 \text{ psi at } T_a)$  is:

 $h_{r} = 2.309$  in.

The calculation procedure for a Fixed Tubesheet heat exchanger is given in paragraph 4.18.8. The following results are for the design and operating loading cases required to be analyzed (see paragraph 4.18.8.3). This example illustrates the calculation of both the elastic and elastic-plastic solutions.
a) STEP 1 – Calculate the parameters from paragraph 4.18.6.4.a.

$$\begin{split} L &= 134.875 \, in. + 0.125 \, in. + 0.125 \, in. = 135.125 \, in. \\ D_o &= 21.562 \, in. \\ a_o &= 10.781 \, in. \\ \rho &= 0.972 \\ d^* &= 0.6392 \, in. \\ \mu &= 0.2 \\ p^* &= 0.9375 \, in. \\ \mu^* &= 0.3182 \\ \rho_s &= 1.078286 \\ \rho_c &= 1.188433 \\ x_s &= 0.4749 \\ x_t &= 0.6816 \end{split}$$

b) STEP 2 – Calculate the shell axial stiffness,  $K_s$ , tube axial stiffness,  $K_t$ , stiffness factors,  $K_{s,t}$  and J.

$$K_{s}^{*} = 5,876,500 \, lb \, / \, in.$$
  
 $K_{t} = 33,143 \, lb \, / \, in.$   
 $K_{st} = 0.40854$   
 $J = 0.0025063$ 

Calculate the shell coefficients  $\beta_s$ ,  $k_s$ ,  $\lambda_s$ , and  $\delta_s$ .

$$\beta_{s} = 0.3471 in.^{-1}$$

$$k_{s} = 2,331,037 \, lb$$

$$\lambda_{s} = 13,497,065 \, psi$$

$$\delta_{s} = 0.0000039653 in.^{3} / lb$$

Calculate the channel coefficients  $\beta_c$ ,  $k_c$ ,  $\lambda_c$ , and  $\delta_c$ .

$$\beta_c = 0 in.^{-1}$$

$$k_c = 0 lb$$

$$\lambda_c = 0 psi$$

$$\delta_c = 0 in.^3 / lb$$

c) STEP 3 – Calculate h/p. Determine  $E^*/E$  and  $v^*$  from paragraph 4.18.6.4.b and calculate  $E^*$ .

h/p = 4.80 E\*/E = 0.305132 v\* = 0.342304E\* = 7,803,761 psi

Calculate,  $\,X_{\scriptscriptstyle a}$  , and the parameters from Table 4.18.3.

$$\begin{split} X_a &= 1.9955\\ Z_d &= 0.174495\\ Z_m &= 0.667867\\ Z_v &= 0.160532\\ Z_a &= 0.809161\\ Z_w &= 0.160532 \end{split}$$

d) STEP 4 – Calculate the diameter ratio, K , the coefficient F , and the associated parameters.

$$K = 1.5247$$
  
 $F = 2.0466$ 

Calculate  $\Phi$ ,  $Q_1$ ,  $Q_{z1}$ ,  $Q_{z2}$  and U.

$$\Phi = 2.747$$

$$Q_1 = -0.128$$

$$Q_{Z1} = 1.2206$$

$$Q_{Z2} = 0.5952$$

$$U = 1.1904$$

Summary Table for Step 5 – Design Condition								
Loading Case $P_s$ (psi) $P_t$ (psi) $\gamma$ $W^*$								
1	0	1,040	0	808456				
2	335	0	0	0				
3	335	1040	0	808456				

e) STEP 5 – Calculate  $\gamma$ ,  $\omega_s$ ,  $\omega_s^*$ ,  $\omega_c$ ,  $\omega_c^*$ , and  $\gamma_b$ . The following results are those for the corroded condition, elastic solution.

Summary Table for Step 5 – Operating Condition 1								
Loading Case $P_s$ (psi) $P_t$ (psi) $\gamma$								
1	0	1040	-0.060	808478				
2	335	0	-0.060	808478				
3	335	1040	-0.060	808478				
4	0	0	-0.060	808478				

 $\omega_s = 8.8648 in.^2 \qquad \qquad \omega_c = 0 in.^2$ 

$$\omega_{s}^{*} = -8.4947 in.^{2}$$
  $\omega_{c}^{*} = 8.6591 in.^{2}$   
 $\gamma_{b} = -0.2087$ 

STEP 6 – For each loading case, calculate  $P'_s$ ,  $P'_t$ ,  $P_\gamma$ ,  $P_\omega$ ,  $P_W$ ,  $P_{rim}$  and effective pressure  $P_e$ . f)

	Summary Table for STEP 6 – Design Condition									
Loading	$P'_s$	$P'_t$	$P_{\gamma}$	$P_{\omega}$	$P_{W}$	$P_{rim}$	$P_e$			
Case	(psi)	(psi)	(psi)	(psi)	(psi)	(psi)	(psi)			
1	0	1,017,041	0	0	275	92.2	-1,039.2			
2	-167,351	0	0	0	0	29.1	-170.7			
3	-167,351	1,017,041	0	0	275	121.4	-1,210.2			

	Summary Table for STEP 6 – Operating Condition 1									
Loading	$P'_s$	$P'_t$	$P_{\gamma}$	$P_{\omega}$	$P_{W}$	$P_{rim}$	$P_e$			
Case	(psi)	(psi)	(psi)	(psi)	(psi)	(psi)	(psi)			
1	0	1,017,041	-2,376	0	275	92.2	-1,041.6			
2	-167,351	0	-2,376	0	275	29.1	-173.2			
3	-167,351	1,017,041	-2,376	0	275	121.4	-1,212.7			
4	0	0	-2,376	0	275	0	-2.1			

	Summary Table for STEP 7 – Design Condition									
Loading Case	$Q_2$ (lbs)	$Q_3$	$F_m$	$h_{g}^{\prime}$ (in)	$h-h_g'$ (in)	$\left  \sigma_{_{elastic}}  ight $ (psi)	1.5 <i>S</i> (psi)	$S_{_{PS}}$ (psi)		
1	-12,650	0.0815	0.19861	0	4.5	22,335.8	27,675			
2	-1003.9	-0.027	0.1574	0	4.5	2913	27,675			
3	-13,654	0.06617	0.1927	0	4.5	25,249.4	27,675			

g) STEP 7 – Elastic Iteration, calculate  $Q_2$  and  $Q_3$ , the tubesheet bending stress and the allowable tubesheet bending stress.

	Summary Table for STEP 7 – Operating Condition 1									
Loading Case	$Q_2$ (lbs)	$Q_3$	$F_m$	$h_g'$ (in)	$h-h_g'$ (in)	$\left  \sigma_{_{elastic}}  ight $ (psi)	1.5 <i>S</i> (psi)	$S_{_{PS}}$ (psi)		
1	-12,650	0.08101	0.1984	0	4.5	22,367.1		55,400		
2	-10,477	0.91305	0.5333	0	4.5	9,994.5		55,400		
3	-13,654	0.06578	0.19264	0	4.5	25,280.7		55,400		
4	-9,473	75.77	37.935	0	4.5	8,817.0		55,400		

For Design Loading Cases 1-3  $|\sigma_{elastic}| < 1.5S$ , and for Operating Cases 1-4  $|\sigma_{elastic}| < S_{PS}$ . The bending stress criterion for the tubesheet is satisfied.

h) STEP 8 – For each loading case, calculate the average shear stress in the tubesheet at the outer edge of the perforated region, if required. Compare to the allowable.

If  $|P_e| \leq \frac{1.6S \,\mu h}{a_o}$ , then the shear stress is not required to be calculated.

Design Condition 1, Load Case 1:

$$\left\{ \left| P_e \right| = 1039.2 \ psi \right\} \le \left\{ \frac{1.6S\mu h}{a_o} = 2464 \ psi \right\}$$

Therefore, shear stress is not required to be calculated for design condition, case 1. Similarly, the shear stress is not required to be calculated for design conditions, case 2 and case 3, and operating condition cases 1-4.

Summary Table for STEP 8 – Design Condition						
Loading Case $ \tau $ 0.8S (psi) (psi)						
1	Not required	14,760				
2	Not required	14,760				
3	Not required	14,760				

Summary Table for STEP 8 – Operating Condition 1							
Loading Case	τ  (psi)	0.8 <i>S</i> (psi)					
1	Not required	14,760					
2	Not required	14,760					
3	Not required	14,760					
4	Not required	14,760					

For all Loading Cases, the shear stress criterion is not required.

i) STEP 9 – For each load case, calculate the axial tube stress and the allowable axial tube stress based on tube buckling.

$$r_t = 0.2376 in.$$

$$F_t = 143.07$$

$$C_t = 163.7755$$

Summary Table for STEP 9 – Design Condition								
Loading Case $F_t$ , min $\sigma_{t,1}$ (psi) $F_t$ , max $\sigma_{t,2}$ (psi)								
1	0.459	-1,120.1	1.487	4,046.9				
2	0.59	1258	1.349	1886				
3	0.478	137.7	1.468	5,932.7				

Summary Table for STEP 9 – Operating Condition 1								
Loading Case	$F_t$ , min	$\sigma_{\scriptscriptstyle t,1}$ (psi)	$F_t$ , max	$\sigma_{\scriptscriptstyle t,2}$ (psi)				
1	0.460	-1,111.8	1.487	4,061.2				
2	-0.543	-314.9	2.545	2,902.1				
3	0.478	146	1.467	5,947				
4	-90.755	-942.9	97.817	1,016.3				

Summary Table for STEP 9 – Design Condition (continued)								
Loading Case	$\begin{array}{c c c c c c c c c c c c c c c c c c c $							
1	4,046	12,588.2	1,120.1	2	5,336.3			
2	1886	12,588.2						
3	5,932	12,588.2						

Summary Table for STEP 9 – Operating Condition 1 (continued)								
Loading Case	$\sigma_{_{t,\mathrm{max}}}$ (psi)	$S_t$ (psi)	$ert \sigma_{\iota, \min} ert$ (psi)	$F_s$	S <sub>tb</sub> (psi)			
1	4,061	25,176.5	1,111.8	2	5,336.3			
2	2,902	25,176.5						
3	5,947	25,176.5						
4	1,016	25,176.5	942.9	1.25	8,538.1			

For all Loading Cases  $|\sigma_{_{t,\max}}| < S_{_t}$ . The axial tension stress criterion for the tube is satisfied.

For all Loading Cases  $\left|\sigma_{t,\min}\right| < S_{tb}$ . The buckling criterion for the tube is satisfied.

j) STEP 10 – For each loading case, calculate the axial membrane stress in each shell section and determine the maximum allowable longitudinal compressive stress.

Summary Table for STEP 10 – Main Shell – Design Condition								
Loading Case	pading Case $\sigma_{s,m}$ $S_s E_{s,w}$ $S_{PS,s}$ (psi) (psi) (psi)							
1	10.4	15,682.5						
2	-1,525.1	15,682.5		10,802				
3	-1,518.3	15,682.5		10,802				

Summary Table for STEP 10 – Main Shell – Operating Condition 1								
Loading Case	$\sigma_{_{s,m}}$ (psi)	$S_s E_{s,w}$ (psi)	$S_{PS,s}$ $S_{s,b}$ (psi) (psi)					
1	-21.4		36,900	10,802				
2	-1,556.9		36,900	10,802				
3	-1,550.2		36,900	10,802				
4	-28.2		36,900	10,802				

## PTB-3-2013

Summary Table for STEP 10 – Shell Band – Design Condition								
Loading Case	ise $\sigma_{s,m}$ $S_s E_{s,w}$ $S_{PS,s}$ $S_{s,b}$ (psi) (psi) (psi) (psi)							
1	3.4	15,682.5		-41040				
2	-492.7	15,682.5		618				
3	-490.5	15,682.5		-40422				

Summary Table for STEP 10 – Shell Band – Operating Condition 1								
Loading Case	$\sigma_{_{s,m}}$ (psi)	$S_s E_{s,w}$ (psi)	$S_{PS,s}$ (psi)	$S_{s,b}$ (psi)				
1	-6.9		36,900	-41074				
2	-503		36,900	-19412				
3	-500.8		36,900	-40456				
4	-9.1		36,900	-20030				

k) STEP 11 – For each loading case, calculate the stresses in the shell and/or channel when integral with the tubesheet.

Summary Table for STEP 11 – Design Condition								
Loading Case	$\sigma_{s,m}$	$\sigma_{s,b}$	$\sigma_s$	$S_s$ (psi)	$S_{_{PS,s}}$			
1	3.4	-41,040	41,043	27,675				
2	-492.7	618	1112	27,675				
3	-490.5	-40,422	40,912	27,675				
	Summary	Table for STEP 1	11 – Operating C	ondition 1				
Loading Case	$\sigma_{_{s,m}}$ (psi)	$\sigma_{\scriptscriptstyle s,b}$ (psi)	$\sigma_{s}$ (psi)	S <sub>s</sub> (psi)	$S_{_{PS,s}}$ (psi)			
1	-6.9	-41,074	41,081		55,400			
2	-503	-19,412	19,915		55,400			
3	-500.8	-40,456	40,957		55,400			
4	-9.1	-20,030	20,039		55,400			

For Design Loading Cases 1 and 3  $|\sigma_s| > 1.5S_s$ , and for Operating Loading Cases 1-4  $|\sigma_s| < S_{_{PS,s}}$ . The stress criterion for the shell is not satisfied.

- I) STEP 12 The design shall be reconsidered by using one or a combination of the following options.
  - Option 1 Increase the tubesheet thickness and return to STEP 1.
  - Option 2 Increase the integral shell and/or channel thickness and return to STEP 1
  - Option 3 Perform the elastic-plastic calculation procedures as defined in paragraph 4.18.8.6.

Since the total axial stress in the shell  $\sigma_s$  is between  $1.5S_{s,1}$  and  $S_{PS,s,1}$  for Design Condition Loading Cases 1 and 3, the procedure of paragraph 4.18.8.6 may be performed to determine if the tubesheet stresses are acceptable when the plasticity of the shell occurs.

Summary Results for STEP 12,									
Elastic Plastic Iteration Results per paragraph 4.18.8.6									
Loading Case	1	3							
$S^*_{s}$ , psi	27,700	27,700							
<i>fact</i> <sub>s</sub>	0.807	0.820							
$E^*_{s}$ , psi	20.789E6	21.1E6							
$k_s, lb$	1.88E6	1.91E6							
$\lambda_s$	0.109E8	0.111E8							
F	1.827	1.842							
φ	2.453	2.472							
$Q_1$	-0.1196	-0.1202							
$Q_{Z1}$	1.231	1.230							
$Q_{Z2}$	0.640	0.636							
U	1.279	1.273							
$P_W, psi$	295.5	294.1							
$P_{\rm rim}, psi$	99.099	129.78							
$P_e, psi$	-1,039.2	-1,210.2							
$Q_2, lb$	-13,592	-14,599							
Q3	0.105	0.087							
$F_m$	0.208	0.201							
$ \sigma , psi$	23,358	26,304							

The final calculated tubesheet bending stresses of 23,358 psi (Loading Case 1) and 26,304 psi (Loading Case 3) are less than the allowable tubesheet bending stress of 27,675 psi. As such, this geometry meets the requirements of paragraph 4.18.8.6. The intermediate results for the elastic-plastic calculation are shown above.

## 4.18.7 Example E4.18.7 – Fixed Tubesheet Exchanger, Configuration a

A fixed tubesheet heat exchanger with the tubesheet construction in accordance with configuration a as shown in VIII-1, Figure UHX-13.1.

- For the Design Condition, the shell side design pressure is 325 psig at 400°F, and the tube side design pressure is 200 psig at 300°F.
- There is one operating condition. For Operating Condition 1, the shell side design pressure is 325 psig at 400°F, the tube side design pressure is 200 psig at 300°F, the shell mean metal temperature is 151°F, and the tube mean metal temperature is 113°F. For this example, the operating pressures and operating metal temperatures are assumed to be the same as the design values.
- The tube material is SA-249, Type 304L (S30403). The tubes are 1 in. outside diameter and are 0.049 in. thick.
- The tubesheet material is SA-240, Type 304L (S30403). The tubesheet outside diameter is 43.125 in. There are 955 tube holes on a 1.25 in. triangular pattern. There is no pass partition lane and the outermost tube radius from the tubesheet center is 20.125 in. The distance between the outer tubesheet faces is 240 in. The option for the effect of differential radial expansion is not required. There is no corrosion allowance on the tubesheet.
- The shell material is SA-240, Type 304L (S30403). The shell inside diameter is 42 in. and the thickness is 0.5625 in. There is no corrosion allowance on the shell and no expansion joint in the shell. The efficiency of shell circumferential welded joint (Category B) is 0.85.
- The channel material is SA-516, Grade 70 (K02700). The inside diameter of the channel is 42.125 in. and the channel is 0.375 in. thick. There is no corrosion allowance on the channel.

## Data Summary - Tubesheet

Tube Layout: Triangular

$$\begin{split} h &= 1.375 \, in. \\ h_g &= 0 \, in. \\ c_i &= 0 \, in. \\ A &= 43.125 \, in. \\ r_o &= 20.125 \, in. \\ A_L &= 0.0 \, in.^2 \\ N_t &= 955 \\ L_t &= 240 \, in. \\ p &= 1.25 \, in. \\ T &= 400^\circ F \\ T_a &= 70^\circ F \\ S &= 15,800 \, psi \, \text{at } T \text{ from Table 5A of Section II, Part D} \\ S_y &= 17,500 \, psi \, \text{at } T \\ S_{PS} &= 47,400 \, psi \, \text{at } T \\ E &= 26,400,000 \, psi \, \text{at } T \\ v &= 0.3 \end{split}$$

```
Data Summary - Tubes
```

 $P_{td,max} = 200 \, psig$  $P_{td,\min} = 0 psig$  $P_{to1} = 200 \ psig$  $\ell_{tx} = 1.25$ k = 1 $\ell = 48 in.$  $t_t = 0.049 in.$  $d_t = 1$  in.  $T_{t} = 300^{\circ} F$  $T_{t,m} = 113^{\circ}F$  $S_t = 16,700 \text{ psi}$  at  $T_t$  from Table 5A of Section II, Part D  $S_{vt} = 19,200 \text{ psi}$  at  $T_t$  from Y-1 of Section II, Part D  $S_{tT} = 15,800 \text{ psi}$  at T from Table 5A of Section II, Part D  $\alpha_{t.m} = 8.65E - 06 in. / in. /^{\circ} F$  at  $T_{t.m}$  $E_t = 27,000,000 \text{ psi}$  at  $T_t$  from TM-1 of Section II, Part D  $E_{T} = 26,400,000 \text{ psi}$  at T from TM-1 of Section II, Part D  $v_t = 0.3$ 

# Data Summary - Shell

Since there is no expansion joint in the shell, J = 1 and  $D_j$  and  $K_j$  need not be defined.

$$P_{sd,max} = 325 \ psig$$

$$P_{sd,min} = 0 \ psig$$

$$P_{so1} = 325 \ psig$$

$$t_s = 0.5625 \ in.$$

$$D_s = 42 \ in.$$

$$T_s = 400^{\circ} F$$

$$T_{s,m} = 151^{\circ} F$$

$$S_s = 15,800 \ psi \ at \ T_s \ from \ Table \ 5A \ of \ Section \ II, \ Part \ D$$

$$E_{s,w} = 0.85$$

$$S_{PS,s} = 47,400 \ psi \ at \ T_s$$

$$S_{y,s} = 17,500 \ psi \ at \ T_s$$

$$E_s = 26,400,000 \ psi \ from \ TM-1 \ of \ Section \ II, \ Part \ D$$

$$\alpha_{s,m} = 8.802E - 06 \ in./ \ in./^{\circ} \ F \ at \ T_{s,m}$$

$$\nu = 0.3$$

Data Summary - Channel

$$t_c = 0.375 in.$$
  

$$D_c = 42.125 in.$$
  

$$T_c = 300^{\circ} F$$
  

$$S_c = 22,400 psi \text{ at } T_c \text{ from Table 5A of Section II, Part D}$$
  

$$S_{y,c} = 33,600 psi \text{ at } T_c$$
  

$$S_{PS,c} = 67,200 psi \text{ at } T_c$$
  

$$E_c = 28,300,000 psi \text{ at } T_c \text{ from TM-1 of Section II, Part D}$$
  

$$v_c = 0.3$$

### **Calculation Procedure**

The calculation procedure for a Fixed Tubesheet heat exchanger is given in paragraph 4.18.8. The following results are for the design and operating loading cases required to be analyzed (see paragraph 4.18.8.3). This example illustrates the calculation of both the elastic and elastic-plastic solutions.

a) STEP 1 – Calculate the parameters from paragraph 4.18.6.4.a.

L = 237.25 in.  $D_o = 41.25 in.$   $a_o = 20.625 in.$   $\rho = 0.9091$   $d^* = 0.9111 in.$   $\mu = 0.2$   $p^* = 1.25 in.$   $\mu^* = 0.2711$   $\rho_s = 1.0182$   $\rho_c = 1.0212$   $x_s = 0.4388$  $x_t = 0.5434$ 

b) STEP 2 – Calculate the shell axial stiffness,  $K_s$ , tube axial stiffness,  $K_t$ , stiffness factors,  $K_{s,t}$  and J.

$$K_s = 8,369,456 \ lb / in.$$
  
 $K_t = 16,660 \ lb / in.$   
 $K_{st} = 0.526$   
 $J = 1$ 

Calculate the shell coefficients  $\beta_s, k_s, \lambda_s$ , and  $\delta_s$ 

$$\beta_s = 0.3715 \text{ in.}^{-1}$$
  
 $k_s = 319,712 \text{ lb}$   
 $\lambda_s = 50,867,972 \text{ psi}$   
 $\delta_s = 25.24E - 6 \text{ in.}^3 / \text{lb}$ 

Calculate the channel coefficients  $\beta_c, k_c, \lambda_c$ , and  $\delta_c$ .

$$\beta_{c} = 0.4554 \text{ in.}^{-1}$$

$$k_{c} = 124,461 \text{ lb}$$

$$\lambda_{c} = 22,049,112 \text{ psi}$$

$$\delta_{c} = 35.532E - 6 \text{ in.}^{3} / \text{lb}$$

c) STEP 3 – Calculate h/p. Determine  $E^*/E$  and  $v^*$  from paragraph 4.18.6.4.b and calculate  $E^*$ .

h/p = 1.10000 E\*/E = 0.274948 v\* = 0.340361E\* = 7.26E6 psi

Calculate,  $\,X_{\rm a}$  , and the parameters from Table 4.18.3.

 $X_a = 7.0155$   $Z_d = 0.00433$   $Z_v = 0.02064$   $Z_m = 0.2067$   $Z_a = 295.63$  $Z_w = 0.02064$ 

d) STEP 4 – Calculate the diameter ratio, K, the coefficient F, and the associated parameters.

$$K = 1.0455$$
  
 $F = 6.7322$ 

Calculate  $\Phi$ ,  $Q_1$ ,  $Q_{z1}$ ,  $Q_{z2}$  and U.

$$\Phi = 9.0236$$

$$Q_1 = -0.058647$$

$$Q_{Z1} = 3.7782$$

$$Q_{Z2} = 10.3124$$

$$U = 20.6248$$

e) STEP 5 – Calculate  $\gamma$ ,  $\omega_s$ ,  $\omega_s^*$ ,  $\omega_c$ ,  $\omega_c^*$ , and  $\gamma_b$ . The following results are those for the corroded condition, elastic solution

Summary Table for Step 5 – Design Condition								
Loading Case	$P_{s}$ (psi)	$P_t$ (psi)	γ					
1	0	200	0					
2	325	0	0					
3	325	200	0					

Summary Table for Step 5 – Operating Condition 1								
Loading Case	P <sub>s</sub> (psi)	$P_t$ (psi)	γ					
1	0	200	-0.0809					
2	325	0	-0.0809					
3	325	200	-0.0809					
4	0	0	-0.0809					

 $\omega_s = 4.6123 \text{ in.}^2$   $\omega_c = 3.344 \text{ in.}^2$   $\omega_s^* = -4.5413 \text{ in.}^2$   $\omega_c^* = -2.6027 \text{ in.}^2$  $\gamma_b = 0$ 

f) STEP 6 – For each loading case, calculate  $P'_{s}$ ,  $P'_{t}$ ,  $P_{\gamma}$ ,  $P_{\omega}$ ,  $P_{W}$ ,  $P_{rim}$  and effective pressure  $P_{e}$ .

Summary Table for STEP 6 – Design Condition									
Loading	$P'_s$	$P'_t$	$P_{\gamma}$	$P_{\omega}$	$P_{W}$	$P_{rim}$	$P_e$		
Case	(psi)	(psi)	(psi)	(psi)	(psi)	(psi)	(psi)		
1	0	543.7	0	0	0	-25.2	-97		
2	613.7	0	0	0	0	71.6	116.8		
3	613.7	543.7	0	0	0	46.3	19.8		

Summary Table for STEP 6 – Operating Condition 1								
Loading	$P'_s$	$P'_t$	$P_{\gamma}$	$P_{\omega}$	$P_{\scriptscriptstyle W}$	$P_{rim}$	$P_e$	
Case	(psi)	(psi)	(psi)	(psi)	(psi)	(psi)	(psi)	
1	0	543.7	-963	0	0	-25.2	-261.1	
2	613.7	0	-963	0	0	71.6	-47.3	
3	613.7	543.7	-963	0	0	46.3	-144.3	
4	0	0	-963	0	0	0	-164.1	

	Summary Table for STEP 7 – Design Condition									
Loading Case	$Q_2$ (lbs)	$Q_3$	$F_m$	$h_{g}^{\prime}$ (in)	$h-h_g'$ (in)	$ \sigma_{_{elastic}} $ (psi)	1.5 <i>S</i> (psi)	$S_{_{PS}}$ (psi)		
1	181.7	-0.0675	0.03373	0	1.375	16,286	23,700			
2	-515.1	-0.0794	0.03969	0	1.375	23,084	23,700			
3	-333.4	-0.138	0.06886	0	1.375	6,798	23,700			

g) STEP 7 – Elastic Iteration, calculate  $Q_2$  and  $Q_3$ , the tubesheet bending stress and the allowable tubesheet bending stress.

Summary Table for STEP 7 – Operating Condition 1										
Loading Case	$Q_2$ (lbs)	$Q_3$	$F_m$	$h_{g}^{\prime}$ (in)	$h-h_g'$ (in)	$ \sigma_{_{elastic}} $ (psi)	1.5 <i>S</i> (psi)	$S_{_{PS}}$ (psi)		
1	181.7	-0.0619	0.03096	0	1.375	40,253		47,400		
2	-515.1	-0.00749	0.03210	0	1.375	7,566		47,400		
3	-333.4	-0.0478	0.02389	0	1.375	17,169		47,400		
4	0	-0.0587	0.02932	0	1.375	23,967		47,400		

For Design Loading Cases 1-3  $|\sigma_{elastic}| < 1.5S$ , and for Operating Cases 1-4  $|\sigma_{elastic}| < S_{PS}$ . The bending stress criterion for the tubesheet is satisfied.

h) STEP 8 – For each loading case, calculate the average shear stress in the tubesheet at the outer edge of the perforated region, if required. Compare to the allowable.

If  $|P_e| \leq \frac{1.6S \mu h}{a_o}$ , then the shear stress is not required to be calculated.

Design Condition 1, Load Case 1:

$$\left\{ \left| P_e \right| = 97 \ psi \right\} \le \left\{ \frac{1.6S \mu h}{a_o} = 337 \ psi \right\}$$

Therefore, shear stress is not required to be calculated for design condition, case 1. Similarly, the shear stress is not required to be calculated for design conditions, case 2 and case 3, and operating condition cases 1-4.

Summary Table for STEP 8 – Design Condition					
Loading Case $\begin{vmatrix}  \tau  & 0.8S \\ (psi) & (psi) \end{vmatrix}$					
1	Not required	12,640			
2	Not required	12,640			
3	Not required	12,640			

Summary Table for STEP 8 – Operating Condition 1					
Loading Case	0.8 <i>S</i> (psi)				
1	Not required	12,640			
2	Not required	12,640			
3	Not required	12,640			
4	Not required	12,640			

For all Loading Cases the shear stress criterion is not required.

i) STEP 9 – For each load case, calculate the axial tube stress and the allowable axial tube stress based on tube buckling.

$$r_t = 0.3367 in.$$
  
 $F_t = 142.57$ 

$$C_t = 166.6$$

Summary Table for STEP 9 – Design Condition						
Loading Case $F_t$ , min $\sigma_{t,1}$ $\sigma_{t,2}$ (psi) $\sigma_{t,2}$ (psi)						
1	-0.270	-1,289.5	3.558	2,259.3		
2	-0.243	1,634.3	3.260	-2,276.6		
3	-0.191	360.4	2.123	-78.2		

Summary Table for STEP 9 – Operating Condition 1						
Loading Case	$F_t$ , min	$\sigma_{\scriptscriptstyle t,1}$ (psi)	$F_t$ , max	$\sigma_{\scriptscriptstyle t,2}$ (psi)		
1	-0.285	-1,751.2	3.696	8,187.4		
2	-0.490	1,141.5	5.057	3,651.5		
3	-0.329	-129.1	4.050	5,910.8		
4	-0.295	-462.4	3.778	5,928.1		

# PTB-3-2013

Summary Table for STEP 9 – Design Condition (continued)						
Loading Case	$\sigma_{_{t,\mathrm{max}}}$ (psi)	S <sub>t</sub> (psi)	$\left  \sigma_{_{t,\min}}  ight $ (psi)	$F_s$	$S_{tb}$ (psi)	
1	2,259	16,700	1,289.5	1.471	7,468	
2	-2,277	16,700	2,276.6	1.620	6,781	
3	360.4	16,700	78.2	2	5,493	

Summary Table for STEP 9 – Operating Condition 1 (continued)						
Loading Case	$\sigma_{_{t,\mathrm{max}}}$ (psi)	$S_t$ (psi)	$ert \sigma_{\iota, \min} ert$ (psi)	$F_s$	$S_{tb}$ (psi)	
1	8,187	33,400	1,751.2	1.402	7,836.5	
2	3,652	33,400				
3	5,911	33,400	129	1.25	8788	
4	5,928	33,400	462.4	1.361	8,072	

For all Loading Cases  $|\sigma_{t,\max}| < S_t$ . The axial tension stress criterion for the tube is satisfied.

For all Loading Cases  $\left|\sigma_{\!_{t,\min}}\right|\!<\!S_{\!_{tb}}$  . The buckling criterion for the tube is satisfied.

j) STEP 10 – For each loading case, calculate the axial membrane stress in each shell section and determine the maximum allowable longitudinal compressive stress.

Summary Table for STEP 10 – Main Shell – Design Condition						
Loading Case	$\sigma_{s,m}$ $S_s E_{s,w}$ $S_{PS,s}$ $S_{s,b}$ (psi) (psi)					
1	1,830.6	13,430				
2	2,287.2	13,430				
3	4,117.8	13,430				

Summary Table for STEP 10 – Main Shell – Operating Condition 1						
Loading Case	$\sigma_{_{s,m}}$ (psi)	$1.5S_s$ (psi)	$S_{PS,s}$ (psi)	$S_{s,b}$ (psi)		
1	-1,085.9		47,400	6,730		
2	-629.3		47,400	6,730		
3	1,201.3		47,400			
4	-2,916.5		47,400	6,730		

Summary Table for STEP 11, Shell Results – Design Condition						
Loading Case	$\sigma_{_{s,m}}$ (psi)	$\sigma_{_{s,b}}$ (psi)	$\sigma_{s}$ (psi)	$1.5S_s$ (psi)	$S_{_{PS,s}}$ (psi)	
1	1,830.6	-12,184	14,015	23,700		
2	2,287.2	27,748	30,036	23,700		
3	4,117.8	15,564	19,682	23,700		

k) STEP 11 – For each loading case, calculate the stresses in the shell and/or channel when integral with the tubesheet.

Summary Table for STEP 11, Shell Results – Operating Condition 1						
Loading Case	$\sigma_{s,m}$	$\sigma_{s,b}$	$\sigma_s$	$1.5S_s$	$S_{_{PS,s}}$	
1	-1.085.9	-38.418	(psi) 39.504	(psi) 	47.400	
2	-629.3	1,514	2,144		47,400	
3	1,201.3	-10,670	11,871		47,400	
4	-2,916.5	-26,234	29,150		47,400	

For Design Loading Case 2  $|\sigma_s| > 1.5S_s$ , and for Operating Cases 1-4  $|\sigma_s| < S_{PS,s}$ . The stress criterion for the shell is not satisfied.

Summary Table for STEP 11, Channel Results – Design Condition						
Loading Case	$\sigma_{_{c,m}}$ (psi)	$\sigma_{_{c,b}}$ (psi)	$\sigma_{_c}$ (psi)	1.5 <i>S<sub>c</sub></i> (psi)	$S_{_{PS,c}}$ (psi)	
1	5,567	28,346	33,913	33,600		
2	0	-8,492	8,492	33,600		
3	5,567	19,854	25,420	33,600		

Summary Table for STEP 11, Channel Results – Operating Condition 1						
Loading	$\sigma_{\scriptscriptstyle c,m}$	$\sigma_{_{c,b}}$	$\sigma_{c}$	$1.5S_{c}$	$S_{_{PS,c}}$	
Case	(psi)	(psi)	(psi)	(psi)	(psi)	
1	5,567	52,380	57,946		67,200	
2	0	15,541	15,541		67,200	
3	5,567	43,887	49,453		67,200	
4	0	24,033	24,033		67,200	

For Design Loading Case 1  $|\sigma_c| > 1.5S_c$ , and for Operating Cases 1-4  $|\sigma_c| < S_{PS,c}$ . The stress criterion for the channel is not satisfied.

- I) STEP 12 The design shall be reconsidered by using one or a combination of the following options.
  - Option 1 Increase the tubesheet thickness and return to STEP 1.
  - Option 2 Increase the integral shell and/or channel thickness and return to STEP 1
  - Option 3 Perform the elastic-plastic calculation procedures as defined in paragraph 4.18.8.6.

Since the total axial stress in the shell  $\sigma_s$  is between  $1.5S_s$  and  $S_{PS,s}$  for Design Condition Loading Case 2, the procedure of paragraph 4.18.8.6 may be performed to determine if the tubesheet stresses are acceptable when the plasticity of the shell occurs.

Since the total axial stress in the channel  $\sigma_c$  is between  $1.5S_c$  and  $S_{PS,c}$  for Design Condition Loading Case 1, the procedure of paragraph 4.18.8.6 may be performed to determine if the tubesheet stresses are acceptable when the plasticity of the channel occurs. The results are not presented for Design Condition Loading Case 1, because the calculated values of *facts* and *factc* do not exceed 1.0 for this case and further plasticity calculations are not required.

Summary Results for STEP 12, Elastic Plastic Iteration Results per paragraph 4.18.8.6		
Design Condition Loading Case	1	
$S^*{}_s, psi$	17,500	
$S^*_{c}$ , psi	33,600	
fact <sub>s</sub>	0.766	
fact <sub>c</sub>	1.000	
$E^{*}_{s}$ , psi	20.2E6	
$E^{*}_{c}, psi$	28.3E6	
$k_s, lb$	24.48E4	
$\lambda_s$	0.390E+08	
F	5.65	
φ	7.572	
$Q_1$	-0.0538	
$Q_{Z1}$	3.898	
$Q_{z_2}$	11.518	
U	23.037	
$P_W$ , psi	0	
$P_{\rm rim}, psi$	79.9	
$P_e, psi$	115.4	
$Q_2, lb$	-575	
$Q_3$	-0.0773	
$F_m$	0.0386	
$ \sigma , psi$	22,204	

The final calculated tubesheet bending stress is 22,204 psi, which is less than the Code allowable of 23,700 psi. As such, this geometry meets the requirement of paragraph 4.18.8.6. The intermediate results for the elastic-plastic iteration are shown above.

## PTB-3-2013

# 4.18.8 Example E4.18.8 – Stationary Tubesheet Gasketed With Shell and Channel; Floating Tubesheet Gasketed, Not Extended as a Flange

A floating tubesheet exchanger with an immersed floating head is to be designed with configuration a as shown in Figure 4.18.10. The stationary tubesheet is gasketed with the shell and channel in accordance with configuration d as shown in Figure 4.18.11. The floating tubesheet is not extended as a flange in accordance with configuration C as shown in Figure 4.18.12. There is no allowance for corrosion.

Data Summary - Data Common to Both Tubesheets

 $P_{sd,max} = 250 \ psig$  $P_{sd,min} = 0 \ psig$  $P_{td,max} = 150 \ psig$  $P_{td,min} = 0 \ psig$ 

Data Summary – Tubesheet

The tube layout pattern is triangular with one centerline pass lane

$$N_{t} = 466$$
  

$$p = 1in.$$
  

$$r_{o} = 12.5 in.$$
  

$$\rho = 0.8$$
  

$$U_{L1} = 2.5 in.$$
  

$$c_{t} = 0 in.$$
  

$$v = 0.31$$
  

$$E = 27.0E10^{6} psi$$
  

$$S = 19,000 psi$$

Data Summary – Tubes

$$d_{t} = 0.75 in.$$
  

$$t_{t} = 0.083 in.$$
  

$$L_{t} = 256 in.$$
  

$$\ell_{t} = 15.375 in.$$
  

$$v_{t} = 0.31$$
  

$$E_{t} = 27.0 \times 10_{6} psi$$
  

$$S_{t} = 13,350 psi$$
  

$$S_{y,t} = 20,550 psi$$

Data Summary – Stationary Tubesheet Data Summary

 $W^* = 211,426\,lb$  from Table 4.18.6  $A = 33.071\,in.$   $h = 1.75\,in.$   $G_s = 29.375\,in.$   $a_s = 14.6875\,in.$   $G_c = 29.375\,in.$   $a_c = 14.6875\,in.$   $C = 31.417\,in.$  $h_g = 0.197\,in.$ 

Data Summary – Floating Tubesheet Data Summary

$$W^* = 26,225 lb$$
 from Table 4.18.6  
 $A = 26.89 in.$   
 $h = 1.75 in.$   
 $G_1 = 26.496 in.$   
 $G_c = 26.496 in.$   
 $a_c = 13.248 in.$   
 $a_s = 13.248 in.$   
 $C = 27.992 in.$   
 $h_g = 0 in.$ 

# Calculation Procedure – Stationary Tubesheet

The following results are for the 3 load cases required to be analyzed (see paragraph 4.18.9.3).

a) STEP 1 – Calculate the parameters from paragraph 4.18.6.4.a.

 $D_o = 25.75 \text{ in.}$   $L_{L1} = 25.8 \text{ in.}$   $A_L = 64.4 \text{ in.}^2$   $d^* = 0.6567 \text{ in.}$   $\mu = 0.250$   $\mu^* = 0.385$   $h'_g = 0.197 \text{ in.}$   $a_o = 12.875 \text{ in.}$   $\rho_s = 1.14$   $\rho_c = 1.14$   $x_s = 0.605$   $x_t = 0.760$ 

b) STEP 2 – Calculate the shell coefficients  $\beta_s$ ,  $k_s$ ,  $\lambda_s$ , and  $\delta_s$  and the channel coefficients  $\beta_c$ ,  $k_c$ ,  $\lambda_c$ , and  $\delta_c$ .

$$\beta_{s} = 0 \text{ in.}^{-1}$$

$$k_{s} = 0 \text{ lb}$$

$$\lambda_{s} = 0 \text{ psi}$$

$$\delta_{s} = 0 \text{ in.}^{3} / \text{ lb}$$

$$\beta_{c} = 0 \text{ in.}^{-1}$$

$$k_{c} = 0 \text{ lb}$$

$$\lambda_{c} = 0 \text{ psi}$$

$$\delta_{c} = 0 \text{ in.}^{3} / \text{ lb}$$

c) STEP 3 – Calculate h/p. Determine  $E^*/E$  and  $v^*$  from paragraph 4.18.6.4.b and calculate  $E^*$ .

$$h/p = 1.75$$
  
 $E*/E = 0.404$   
 $v* = 0.308$   
 $E* = 10.91E6 \ psi$ 

Calculate,  $X_a$ , and the parameters from Table 4.18.3.

$$\begin{split} X_a &= 3.61 \\ Z_d &= 0.0328 \\ Z_v &= 0.0787 \\ Z_m &= 0.421 \\ Z_w &= 0.0787 \end{split}$$

d) STEP 4 – Calculate the diameter ratio, K , the coefficient F , and the associated parameters.

$$K = 1.28$$
  
 $F = 0.429$ 

Calculate  $\Phi$  and  $Q_1$ .

$$\Phi = 0.561$$
  
 $Q_1 = 0.0782$ 

e) STEP 5 – Calculate  $\omega_s$ ,  $\omega_s^*$ ,  $\omega_c$ ,  $\omega_c^*$ ,  $\gamma_b$ ,  $P_s^*$  and  $P_c^*$ .

$$\omega_s = 0 \text{ in.}^2$$
  

$$\omega_s^* = 1.758 \text{ in.}^2$$
  

$$\omega_c = 0 \text{ in.}^2$$
  

$$\omega_c^* = 1.758 \text{ in.}^2$$
  

$$\gamma_b = 0$$
  

$$P_s^* = 0 \text{ psi}$$
  

$$P_c^* = 0 \text{ psi}$$

f) STEP 6 – Calculate

Summary Table for STEP 6		
Loading Case	$P_e$ (psi)	
1	-150	
2	250	
3	100	

Summary Table for STEP 7					
Loading Case	Q <sub>2</sub> (in-lb/in)	$Q_3$	$F_m$	σ  (psi)	1.5 <i>S</i> (psi)
1	-213	0.0953	0.102	16,400	28,500
2	356	0.0953	0.102	27,400	28,500
3	142	0.0953	0.102	10,900	28,500

g) STEP 7 – Calculate the tubesheet bending stress and the allowable tubesheet bending stress.

For all loading cases  $|\sigma| \le 1.5S$ . The bending stress criterion for the tubesheet is satisfied.

h) STEP 8 – For each loading case, calculate the average shear stress in the tubesheet at the outer edge of the perforated region, if required. Compare to the allowable.

If 
$$|P_e| \leq \frac{1.6S \mu h}{a_o}$$
, then the shear stress is not required to be calculated.

Load Case 1:

$$\left\{ \left| P_{e} \right| = 150 \ psi \right\} \le \left\{ \frac{1.6S \,\mu h}{a_{o}} = 1033 \ psi \right\}$$

Therefore, shear stress is not required to be calculated for design load case 1. Similarly, the shear stress is not required to be calculated for load case 2 and case 3.

Summary Table for STEP 8			
Loading Case	τ  (psi)	0.8 <i>S</i> (psi)	
1	Not required	15200	
2	Not required	15200	
3	Not required	15200	

For all loading cases, the shear stress criterion is not required.

i) STEP 9 – For each load case, calculate the axial tube stress and the allowable axial tube stress based on tube buckling.

$$r_t = 0.238 in.$$
  
 $F_t = 64.7$   
 $C_t = 161$ 

Summary Table for STEP 9					
Loading Case	$F_s$	$\sigma_{_{t,1}}$ (psi)	$\sigma_{\scriptscriptstyle t,2}$ (psi)	$S_{tb}$ (psi)	S <sub>t</sub> (psi)
1	1.54	-1,716	2,564	10,700	13350
2	1.54	2,609	-4,524	10,700	13350
3	1.54	894	-1,959	10,700	13350

Determine  $\sigma_{t,\max} = \max\left[|\sigma_{t,1}|, |\sigma_{t,2}|\right]$  and  $\sigma_{t,\min} = \min\left[\sigma_{t,1}, \sigma_{t,2}\right]$ 

For all loading cases  $\sigma_{\rm r,max}$  < $S_{\rm r}$ . The axial tension stress criterion for the tube is satisfied.

For all loading cases  $\mid\!\sigma_{\rm r,min}\mid<\!S_{\rm tb}$  . The buckling criterion for the tube is satisfied.

### Calculation Procedure – Floating Tubesheet

The following results are for the 3 load cases required to be analyzed (see paragraph 4.18.9.3).

a) STEP 1 – Calculate the parameters from paragraph 4.18.6.4.a.

$$D_o = 25.75 in.$$

$$L_{L1} = 25.8 in.$$

$$A_L = 64.4 in.^2$$

$$d^* = 0.6567 in.$$

$$\mu = 0.250$$

$$\mu^* = 0.385$$

$$h'_g = 0 in.$$

$$a_o = 12.875 in.$$

$$\rho_s = 1.03$$

$$\rho_c = 1.03$$

$$x_s = 0.605$$

$$x_t = 0.760$$

b) STEP 2 – Calculate the shell coefficients  $\beta_s$ ,  $k_s$ ,  $\lambda_s$ , and  $\delta_s$  and the channel coefficients  $\beta_c$ ,  $k_c$ ,  $\lambda_c$ , and  $\delta_c$ .

$$\beta_{s} = 0 in.^{-1}$$

$$k_{s} = 0 lb$$

$$\lambda_{s} = 0 psi$$

$$\delta_{s} = 0 in.^{3} / lb$$

$$\beta_{c} = 0 in.^{-1}$$

$$k_{c} = 0 lb$$

$$\lambda_{c} = 0 psi$$

$$\delta_{c} = 0 in.^{3} / lb$$

c) STEP 3 – Calculate h/p. Determine E \*/E and  $\nu$  from paragraph 4.18.6.4.b and calculate E \*.

$$h/p = 1.75$$
  
 $E*/E = 0.404$   
 $v* = 0.308$   
 $E* = 10.91E6 \ psi$ 

Calculate,  $X_a$ , and the parameters from Table 4.18.3.

$$X_a = 3.61$$
  
 $Z_d = 0.0328$   
 $Z_v = 0.0787$   
 $Z_m = 0.421$   
 $Z_w = 0.0787$ 

d) STEP 4 – Calculate the diameter ratio, K, the coefficient F, and the associated parameters.

K = 1.04F = 0.0742

Calculate  $\Phi$  and  $Q_1$ .

$$\Phi = 0.0971$$
  
 $Q_1 = 0.0205$ 

e) STEP 5 – Calculate  $\omega_s$ ,  $\omega_s^*$ ,  $\omega_c$ ,  $\omega_c^*$ ,  $\gamma_b$ ,  $P_s^*$  and  $P_c^*$ .

f) STEP 6 – Calculate  $P_e$ .

Summary Table for STEP 6		
Loading Case	P <sub>e</sub> (psi)	
1	-150	
2	250	
3	100	

g) STEP 7 – Calculate the tubesheet bending stress and the allowable tubesheet bending stress.

Summary Table for STEP 7					
Loading Case	Q <sub>2</sub> (in-lb/in)	$\mathcal{Q}_3$	$F_m$	σ  (psi)	1.5 <i>S</i> (psi)
1	-10.2	0.0213	0.0751	9,500	28,500
2	16.9	0.0213	0.0751	15,800	28,500
3	6.78	0.0213	0.0751	6,330	28,500

For all loading cases  $|\sigma| < 1.5S$  . The bending stress criterion for the tubesheet is satisfied.

The calculation procedure is complete and the unit geometry is acceptable for the given design conditions.

## PTB-3-2013

# 4.18.9 Example E4.18.9 – Stationary Tubesheet Gasketed With Shell and Channel; Floating Tubesheet Integral

A floating tubesheet exchanger with an externally sealed (packed) floating head is to be designed in accordance with configuration b as shown in Figure 4.18.10. The stationary tubesheet is gasketed with the shell and channel in accordance with configuration d as shown in Figure 4.18.11. The floating tubesheet is integral with the head in accordance with configuration A as shown in Figure 4.18.12. There is no allowance for corrosion.

Data Summary - Data Common to Both Tubesheets

 $P_{sd,max} = 150 \ psig$  $P_{sd,min} = 0 \ psig$  $P_{td,max} = 30 \ psig$  $P_{td,max} = 0 \ psig$ 

Data Summary – Tubesheet

The tube layout pattern is triangular with no pass lanes

$$N_t = 1189$$
  
 $p = 1.25 in.$   
 $r_o = 22.605 in.$   
 $\rho = 0.958$   
 $h_g = 0 in.$   
 $c_t = 0 in.$   
 $v = 0.32$   
 $E = 14.8E10^6 psi$   
 $S = 11,300 psi$   
 $S_{ps} = 33,900 psi$  (MYS/UTS > 0.7; therefore use 3S)

Data Summary – Tubes

$$d_t = 1.0 in.$$
  
 $t_t = 0.049 in.$   
 $L_t = 144 in.$   
 $\ell_t = 16 in.$   
 $v_t = 0.32$   
 $E_t = 14.8E10^6 psi$   
 $S_t = 11,300 psi$   
 $S_{y,t} = 31,600 psi$ 

Data Summary – Stationary Tubesheet

 $W^* = 288,910 lb$  from Table 4.18.6 A = 51 in. h = 1.375 in.  $G_s = 49.71 in.$   $a_s = 24.9 in.$   $G_c = 49.616 in.$   $a_c = 24.8 in.$ C = 49.5 in.

Data Summary – Floating Tubesheet

$$P_{sol} = 150 \text{ psig}$$

$$P_{tol} = 30 \text{ psig}$$

$$W^* = 0 \text{ lb from Table 4.18.6}$$

$$T' = 200^{\circ} F$$

$$T'_{c} = 235^{\circ} F$$

$$A = 47.625 \text{ in.}$$

$$h = 1.375 \text{ in.}$$

$$\alpha' = 4.8E - 6 \text{ in.} / \text{ in.} / ^{\circ} F$$

$$D_{c} = 47 \text{ in.}$$

$$a_{c} = 23.5 \text{ in.}$$

$$t_{c} = 0.3125 \text{ in.}$$

$$v_{c} = 0.32$$

$$E_{c} = 14.8E6 \text{ psi}$$

$$S_{c} = 11,300 \text{ psi}$$

$$S_{ps,c} = 33,900 \text{ psi (MYS/UTS > 0.7; therefore use 3S)}$$

$$\alpha'_{c} = 4.8E - 6 \text{ in.} / \text{ in.} / ^{\circ} F$$

## PTB-3-2013

# Calculation Procedure – Stationary Tubesheet

The following results are for the 3 load cases required to be analyzed (see paragraph 4.18.9.3).

a) STEP 1 – Calculate the parameters from paragraph 4.18.6.4.a.

$$D_{o} = 46.21 in.$$

$$A_{L} = 0 in.^{2}$$

$$\mu = 0.200$$

$$\mu^{*} = 0.275$$

$$h'_{g} = 0 in.$$

$$a_{o} = 23.105 in.$$

$$\rho_{s} = 1.08$$

$$\rho_{c} = 1.07$$

$$x_{s} = 0.443$$

$$x_{t} = 0.547$$

b) STEP 2 – Calculate the shell coefficients  $\beta_s$ ,  $k_s$ ,  $\lambda_s$ , and  $\delta_s$  and the channel coefficients  $\beta_c$ ,  $k_c$ ,  $\lambda_c$ , and  $\delta_c$ .

$$\beta_{s} = 0 in.^{-1}$$

$$k_{s} = 0 lb$$

$$\lambda_{s} = 0 psi$$

$$\delta_{s} = 0 in.^{3} / lb$$

$$\beta_{c} = 0 in.^{-1}$$

$$k_{c} = 0 lb$$

$$\lambda_{c} = 0 psi$$

$$\delta_{c} = 0 in.^{3} / lb$$

c) STEP 3 – Calculate h/p. Determine  $E^*/E$  and  $v^*$  from paragraph 4.18.6.4.b and calculate  $E^*$ .

$$h/p = 1.10$$
  
 $E*/E = 0.280$   
 $v* = 0.337$   
 $E* = 4.149E6 \ psi$ 

Calculate,  $X_a$ , and the parameters from Table 4.18.3.

 $X_a = 8.84$   $Z_a = 3161.6$   $Z_d = 0.00214$   $Z_v = 0.0130$   $Z_m = 0.163$  $Z_w = 0.013$ 

d) STEP 4 – Calculate the diameter ratio, K, the coefficient F, and the associated parameters.

K = 1.10F = 0.233

Calculate  $\Phi$  and  $Q_1$ .

$$\Phi = 0.312$$
  
 $Q_1 = 0.0682$ 

e) STEP 5 – Calculate  $\omega_s$ ,  $\omega_s^*$ ,  $\omega_c$ ,  $\omega_c^*$ ,  $\gamma_b$ ,  $P_s^*$  and  $P_c^*$ .

$$\begin{split} \omega_{s} &= 0 in.^{2} & \omega_{c} &= 0 in.^{2} \\ \omega_{s}^{*} &= 1.59 in.^{2} & \omega_{c}^{*} &= 0.961 in.^{2} \\ \gamma_{b} &= -2.03 \times 10^{-3} \\ P_{s}^{*} &= 0 psi \\ P_{c}^{*} &= 0 psi \end{split}$$

f) STEP 6 – Calculate  $P_e$ .

Summary Table for STEP 6		
Loading Case	P <sub>e</sub> (psi)	
1	-30	
2	-23.6	
3	-53.6	

	Summary Table for STEP 7				
Loading Case	$Q_2$ (in-lb/in)	$Q_3$	$F_m$	σ  (psi)	1.5 <i>S</i> (psi)
1	-116	0.0828	0.0594	11,000	16950
2	138	0.0463	0.0442	6,420	16950
3	110	0.0605	0.0499	16,500	16950

g) STEP 7 – Calculate the tubesheet bending stress and the allowable tubesheet bending stress.

For all loading cases  $|\sigma| \le 1.5S$ . The bending stress criterion for the tubesheet is satisfied.

h) STEP 8 – For each loading case, calculate the average shear stress in the tubesheet at the outer edge of the perforated region, if required. Compare to the allowable.

If 
$$|P_e| \leq \frac{1.6S \mu h}{a_o}$$
, then the shear stress is not required to be calculated.

Load Case 1:

$$\left\{ \left| P_{e} \right| = 30 \ psi \right\} \le \left\{ \frac{1.6S \mu h}{a_{o}} = 215 \ psi \right\}$$

Therefore, shear stress is not required to be calculated for design load case 1. Similarly, the shear stress is not required to be calculated for load case 2 and case 3.

Summary Table for STEP 8			
Loading Case	ert art art (psi)	0.8 <i>S</i> (psi)	
1	Not required	9040	
2	Not required	9040	
3	Not required	9040	

For all loading cases the shear stress criterion is not required.

i) STEP 9 – For each load case, calculate the axial tube stress and the allowable axial tube stress based on tube buckling.

 $r_t = 0.3367 in.$  $F_t = 47.5235$  $C_t = 96.1507$ 

Summary Table for STEP 9					
Loading Case	$F_s$	$\sigma_{_{t,1}}$ (psi)	$\sigma_{_{t,2}}$ (psi)	$S_{tb}$ (psi)	$S_t$ (psi)
1	1.25	-525	2,685	11300	11300
2		424	2,546	11300	11300
3	1.25	-72	5,104	11300	11300

Determine  $\sigma_{t,\max} = \max[|\sigma_{t,1}|, |\sigma_{t,2}|]$  and  $\sigma_{t,\min} = \min[\sigma_{t,1}, \sigma_{t,2}]$ 

For all loading cases  $\sigma_{t,\max} < S_t$ . The axial tension stress criterion for the tube is satisfied.

For all loading cases  $\mid\!\sigma_{\rm r,min}\!\mid<\!S_{\rm tb}$  . The buckling criterion for the tube is satisfied.

### Calculation Procedure – Floating Tubesheet

The following results are for the design and operating loading cases required to be analyzed (see paragraph 4.18.9.3).

a) STEP 1 – Calculate the parameters from paragraph 4.18.6.4.a.

$$D_{o} = 46.2 in.$$

$$A_{L} = 0 in.^{2}$$

$$\mu = 0.200$$

$$\mu^{*} = 0.275$$

$$h'_{g} = 0 in.$$

$$a_{o} = 23.1 in.$$

$$\rho_{s} = 1.02$$

$$\rho_{c} = 1.02$$

$$x_{s} = 0.443$$

$$x_{t} = 0.547$$

b) STEP 2 – Calculate the shell coefficients  $\beta_s$ ,  $k_s$ ,  $\lambda_s$ , and  $\delta_s$  and the channel coefficients  $\beta_c$ ,  $k_c$ ,  $\lambda_c$ , and  $\delta_c$ .

$$\beta_{s} = 0 in.^{-1}$$

$$k_{s} = 0 lb$$

$$\lambda_{s} = 0 psi$$

$$\delta_{s} = 0 in.^{3} / lb$$

$$\beta_{c} = 0.471 in.^{-1}$$

$$k_{c} = 39,500 lb$$

$$\lambda_{c} = 7.96 \times 10^{6} psi$$

$$\delta_{c} = 1.00 \times 10^{-4} in.^{3} / lb$$

c) STEP 3 – Calculate h/p. Determine E \*/E and  $\nu$  from paragraph 4.18.6.4.b and calculate E \*.

$$h/p = 1.10$$
  
 $E*/E = 0.280$   
 $v* = 0.337$   
 $E* = 4.149E6 \ psi$ 

Calculate,  $X_a$ , and the parameters from Table 4.18.3.

$X_a = 8.84$	$Z_{\nu} = 0.0130$
$Z_a = 3161.6$	$Z_m = 0.163$
$Z_d = 0.00214$	$Z_w = 0.013$

d) STEP 4 – Calculate the diameter ratio, K, the coefficient F, and the associated parameters.

$$K = 1.03$$
  
 $F = 1.34$ 

Calculate  $\Phi$  and  $Q_1$ .

$$\Phi = 1.80$$
  
$$Q_1 = -4.83 \times 10^{-3}$$

e) STEP 5 – Calculate  $\omega_s$ ,  $\omega_s^*$ ,  $\omega_c$ ,  $\omega_c^*$ ,  $\gamma_b$ ,  $T_r$ ,  $T_c^*$ ,  $P_s^*$  and  $P_c^*$ .

$$\omega_{s} = 0 in.^{2} \qquad \omega_{c} = 3.13 in.^{2}$$
  

$$\omega_{s}^{*} = 7.87 \times 10^{-2} in.^{2} \qquad \omega_{c}^{*} = -3.05 in.^{2}$$
  

$$\gamma_{b} = 0$$
  

$$T_{r} = 217.5^{\circ} F$$
  

$$T_{c}^{*} = 226.25^{\circ} F$$
  

$$P_{s}^{*} = 0 psi$$
  

$$P_{c}^{*} = 8.27 psi$$

Summary Table for STEP 6 – Design Condition			
Loading Case	P <sub>e</sub> (psi)		
1	-30		
2	-5.17		
3	-35.2		

f)	STEP 6 – Calculate	$P_{e}$ .
• /		- e

Summary Table for STEP 6 – Operating Condition			
Loading Case	P <sub>e</sub> (psi)		
1	-30		
2	-5.17		
3	-35.2		
4	0		

g)	STEP 7 - Calculate the	tubesheet bending stress	and the allowable tubeshee	t bending stress.
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Summary Table for STEP 7 – Design Condition						
Loading Case	$Q_2$ (in-lb/in)	$Q_3$	$F_m$	$ \sigma $ (psi)	1.5 <i>S</i> (psi)	$S_{_{PS}}$ (psi)
1	70.8	-0.0137	0.0228	4,210	16950	
2	9.12	-0.0114	0.0235	748	16950	
3	79.9	-0.0133	0.0229	4,950	16950	

Summary Table for STEP 7 – Operating Condition						
Loading Case	$Q_2$ (in-lb/in)	$Q_3$	$F_m$	$ \sigma $ (psi)	1.5 <i>S</i> (psi)	$S_{_{PS}}$ (psi)
1	90.8	-0.0162	0.0220	4,070		33900
2	29.1	-0.0259	0.0193	615		33900
3	99.9	-0.0155	0.0222	4,810		33900
4	20.0			231		33900

For Design Loading Cases 1-3  $|\sigma| \le 1.5S$  and for Operating Cases 1-4  $|\sigma| \le S_{PS}$ . The bending stress criterion for the tubesheet is satisfied.

h) STEPS 8 and 9 – For configuration A, skip STEPS 8 and 9 and proceed to STEP 10.
Summary Table for STEP 10 – Design Condition						
Loading Case	$\sigma_{_{c,m}}$ (psi)	$\sigma_{\scriptscriptstyle c,b}$ (psi)	$\sigma_{_c}$ (psi)	1.5 <i>S</i> <sub>c</sub> (psi)	S <sub>PS c</sub> (psi)	
1	1,110	9,750	10,900	16950		
2	0	1,120	1,120	16950		
3	1,110	10,900	12,000	16950		

i) STEP 10 – Calculate the stresses in the shell and/or integral channel with the tubesheet.

Summary Table for STEP 10 – Operating Condition					
Loading Case	$\sigma_{_{c,m}}$ (psi)	$\sigma_{\scriptscriptstyle c,b}$ (psi)	$\sigma_{_c}$ (psi)	1.5 <i>S<sub>c</sub></i> (psi)	$S_{_{PS_c}}$ (psi)
1	1,110	10,600	11,800		33900
2	0	2,010	2,010		33900
3	1,110	11,800	12,900		33900
4	0	890	890		33900

For Design Loading Cases 1-3  $\sigma_c \leq 1.5S_c$  and for Operating Cases 1-4  $\sigma_c \leq S_{PS,c}$ . The stress criterion for the shell and/or integral channel with tubesheet is satisfied.

The calculation procedure is complete and the unit geometry is acceptable for the given design conditions.

# 4.18.10 Example E4.18.10 – Stationary Tubesheet Gasketed With Shell and Channel; Floating Tubesheet Internally Sealed

A floating tubesheet exchanger with an internally sealed floating head is to be designed in accordance with sketch (c) as shown in Figure 4.18.10. The stationary tubesheet is gasketed with the shell and channel in accordance with configuration d as shown in Figure 4.18.11. The floating tubesheet is packed and sealed on its edge in accordance with configuration D as shown in Figure 4.18.12. There is no allowance for corrosion.

Data Summary - Data Common to Both Tubesheets

$$P_{sd,max} = 150 \ psig$$
$$P_{sd,min} = 0 \ psig$$
$$P_{td,max} = 175 \ psig$$
$$P_{td,max} = 0 \ psig$$

Data Summary – Tubesheets

The tube layout pattern is triangular with no pass lanes

$$N_t = 1066$$
  
 $p = 0.9375 in.$   
 $r_o = 15.563 in.$   
 $\rho = 0.88$   
 $h_g = 0 in.$   
 $c_t = 0 in.$   
 $v = 0.31$   
 $E = 26.5E6 psi$   
 $S = 15,800 psi$ 

Data Summary - Tubes

$$d_{t} = 0.75 in.$$
  

$$t_{t} = 0.065 in.$$
  

$$L_{t} = 155.875 in.$$
  

$$\ell_{t} = 20.75 in.$$
  

$$v_{t} = 0.31$$
  

$$E_{t} = 26.5E6 psi$$
  

$$S_{t} = 15,800 psi$$
  

$$S_{y,t} = 17,500 psi$$

# Data Summary - Stationary Tubesheet

 $W^* = 290,720\,lb$  from Table 4.18.6  $A = 39.875\,in.$   $h = 1.188\,in.$   $G_s = 39.441\,in.$   $a_s = 19.7\,in.$   $G_c = 39.441\,in.$   $a_c = 19.7\,in.$  $C = 41.625\,in.$ 

Data Summary - Floating Tubesheet

 $W^* = 0 lb$  from Table 4.18.6 A = 36.875 in.  $a_c = 18.4375 in.$   $a_s = 18.4375 in.$ h = 1.188 in.

Calculation Procedure – Stationary Tubesheet

The following results are for the 3 load cases required to be analyzed (see paragraph 4.18.9.3).

a) STEP 1 – Calculate the parameters from paragraph 4.18.6.4.a.

$$D_o = 31.876 in.$$
  

$$A_L = 0 in.^2$$
  

$$\mu = 0.200$$
  

$$\mu^* = 0.322$$
  

$$h'_g = 0 in.$$
  

$$a_o = 15.938 in.$$
  

$$\rho_s = 1.24$$
  

$$\rho_c = 1.24$$
  

$$x_s = 0.410$$
  

$$x_t = 0.597$$

b) STEP 2 – Calculate the shell coefficients  $\beta_s$ ,  $k_s$ ,  $\lambda_s$ , and  $\delta_s$  and the channel coefficients  $\beta_c$ ,  $k_c$ ,  $\lambda_c$ , and  $\delta_c$ .

$$\beta_{s} = 0 \text{ in.}^{-1}$$

$$k_{s} = 0 \text{ lb}$$

$$\lambda_{s} = 0 \text{ psi}$$

$$\delta_{s} = 0 \text{ in.}^{3} / \text{ lb}$$

$$\beta_{c} = 0 \text{ in.}^{-1}$$

$$k_{c} = 0 \text{ lb}$$

$$\lambda_{c} = 0 \text{ psi}$$

$$\delta_{c} = 0 \text{ in.}^{3} / \text{ lb}$$

c) STEP 3 – Calculate h/p. Determine  $E^*/E$  and  $v^*$  from paragraph 4.18.6.4.b and calculate  $E^*$ .

$$h/p = 1.27$$
  
 $E*/E = 0.338$   
 $v* = 0.316$   
 $E* = 8.947E6 \ psi$ 

Calculate,  $X_a$ , and the parameters from Table 4.18.3.

$$X_a = 7.40$$
  
 $Z_a = 482.2$   
 $Z_d = 0.00369$   
 $Z_v = 0.0186$   
 $Z_m = 0.197$   
 $Z_w = 0.0186$ 

d) STEP 4 – Calculate the diameter ratio, K, the coefficient F, and the associated parameters.

$$K = 1.25$$
  
 $F = 0.454$ 

Calculate  $\Phi$  and  $Q_1$ .

$$\Phi = 0.597$$
  
 $Q_1 = 0.202$ 

e) STEP 5 – Calculate  $\omega_s$ ,  $\omega_s^*$ ,  $\omega_c$ ,  $\omega_c^*$ ,  $\gamma_b$ ,  $P_s^*$  and  $P_c^*$ .

$$\omega_{s} = 0 in.^{2} \qquad \omega_{c} = 0 in.^{2}$$
$$\omega_{s}^{*} = 8.00 in.^{2} \qquad \omega_{c}^{*} = 8.00 in.^{2}$$
$$\gamma_{b} = 0$$
$$P_{s}^{*} = 0 psi$$
$$P_{c}^{*} = 0 psi$$

f) STEP 6 – Calculate  $P_e$ .

Summary Table for STEP 6				
Loading Case	P <sub>e</sub> (psi)			
1	92.9			
2	-79.6			
3	13.3			

g) STEP 7 – Calculate the tubesheet bending stress and the allowable tubesheet bending stress.

Summary Table for STEP 7					
Loading Case	Q <sub>2</sub> (in-lb/in)	$\mathcal{Q}_3$	$F_m$	σ  (psi)	1.5 <i>S</i> (psi)
1	-1,250	0.0962	0.0702	21,900	23700
2	1,070	0.0962	0.0702	18,800	23700
3	-179	0.0962	0.0702	3,130	23700

For all loading cases  $|\sigma| \le 1.5S$ . The bending stress criterion for the tubesheet is satisfied.

h) STEP 8 – For all loading cases, calculate the average shear stress in the tubesheet at the outer edge of the perforated region, if required. Compare to the allowable.

If 
$$|P_e| \leq \frac{1.6S \mu h}{a_o}$$
, then the shear stress is not required to be calculated.

Load Case 1:

$$\left\{ \left| P_e \right| = 92.9 \ psi \right\} \le \left\{ \frac{1.6S \mu h}{a_o} = 377 \ psi \right\}$$

Therefore, shear stress is not required to be calculated for design load case 1. Similarly, the shear stress is not required to be calculated for load case 2 and case 3.

Summary Table for STEP 8					
Loading Case	τ  (psi)	0.8 <i>S</i> (psi)			
1	Not required	12640			
2	Not required	12640			
3	Not required	12640			

For all loading cases the shear stress criterion is not required.

i) STEP 9 – For each load case, calculate the axial tube stress and the allowable axial tube stress based on tube buckling.

$$r_t = 0.243 in.$$
  
 $F_t = 85.3$   
 $C_t = 173$ 

Summary Table for STEP 9					
Loading Case	$F_s$	$\sigma_{\scriptscriptstyle t,1}$ (psi)	$\sigma_{_{t,2}}$ (psi)	$S_{tb}$ (psi)	$S_t$ (psi)
1	1.25	2	-4,647	10,550	15800
2	1.25	-152	3,833	10,550	15800
3	1.25	-150	-814	10,550	15800

Determine  $\sigma_{t,\max} = \max\left[|\sigma_{t,1}|, |\sigma_{t,2}|\right]$  and  $\sigma_{t,\min} = \min\left[\sigma_{t,1}, \sigma_{t,2}\right]$ .

For all loading cases  $\sigma_{t,\max} < S_t$ . The axial tension stress criterion for the tube is satisfied.

For all loading cases  $\mid\!\sigma_{\!\scriptscriptstyle t,\min}\!\mid\!<\!S_{\!\scriptscriptstyle t\!b}$  . The buckling criterion for the tube is satisfied.

#### Calculation Procedure – Floating Tubesheet

The following results are for the 3 load cases required to be analyzed (see paragraphs 4.18.9.3).

a) STEP 1 – Calculate the parameters from paragraph 4.18.6.4.a.

$$D_o = 31.876 in.$$
  
 $A_L = 0 in.^2$   
 $\mu = 0.200$   
 $\mu^* = 0.322$   
 $h'_g = 0 in.$   
 $a_o = 15.938 in.$ 

- $\rho_s = 1.16$   $\rho_c = 1.16$   $x_s = 0.410$   $x_t = 0.597$
- b) STEP 2 Calculate the shell coefficients  $\beta_s$ ,  $k_s$ ,  $\lambda_s$ , and  $\delta_s$  and the channel coefficients  $\beta_c$ ,  $k_c$ ,  $\lambda_c$ , and  $\delta_c$ .
  - $\beta_{s} = 0 in.^{-1}$   $k_{s} = 0 lb$   $\lambda_{s} = 0 psi$   $\delta_{s} = 0 in.^{3} / lb$   $\beta_{c} = 0 in.^{-1}$   $k_{c} = 0 lb$   $\lambda_{c} = 0 psi$   $\delta_{c} = 0 in.^{3} / lb$
- c) STEP 3 Calculate h/p. Determine  $E^*/E$  and  $v^*$  from paragraph 4.18.6.4.b and calculate  $E^*$ .

$$h/p = 1.27$$
  
 $E*/E = 0.338$   
 $v* = 0.316$   
 $E* = 8.947E6 \ psi$ 

Calculate,  $X_a$ , and the parameters from Table 4.18.3.

$$X_a = 7.40$$
  
 $Z_a = 482.2$   
 $Z_d = 0.00369$   
 $Z_v = 0.0186$   
 $Z_m = 0.197$   
 $Z_w = 0.0186$ 

- d) STEP 4 Calculate the diameter ratio, K , the coefficient F , and the associated parameters.
  - K = 1.16F = 0.295

Calculate  $\Phi$  and  $Q_1$ .

$$\Phi = 0.388$$
  
 $Q_1 = 0.139$ 

e) STEP 5 – Calculate  $\omega_s$ ,  $\omega_s^*$ ,  $\omega_c$ ,  $\omega_c^*$ ,  $\gamma_b$ ,  $T_r$ ,  $T_c^*$ ,  $P_s^*$  and  $P_c^*$ .

$$\omega_{s} = 0 in.^{2} \qquad \omega_{c} = 0 in.^{2}$$
$$\omega_{s}^{*} = 3.37 in.^{2} \qquad \omega_{c}^{*} = 3.37 in.^{2}$$
$$\gamma_{b} = 0$$
$$P_{s}^{*} = 0 psi$$
$$P_{c}^{*} = 0 psi$$

f) STEP 6 – Calculate  $P_e$ .

Summary Table for STEP 6				
Loading Case	P <sub>e</sub> (psi)			
1	59.2			
2	-50.7			
3	8.46			

g) STEP 7 – Calculate the tubesheet bending stress and the allowable tubesheet bending stress.

Summary Table for STEP 7					
Loading Case	Q <sub>2</sub> (in-lb/in)	$\mathcal{Q}_3$	$F_m$	$ \sigma $ (psi)	1.5 <i>S</i> (psi)
1	-548	0.0661	0.0575	11,400	23700
2	469	0.0661	0.0575	9,780	23700
3	-78.2	0.0661	0.0575	1,630	23700

For loading cases  $|\sigma| \le 1.5S$ . The bending stress criterion for the tubesheet is satisfied.

The calculation procedure is complete and the unit geometry is acceptable for the given design conditions.

### 4.19 Bellows Expansion Joints

# 4.19.1 Example E4.19.1 – U-Shaped Un-reinforced Bellows Expansion Joint and Fatigue Evaluation

Check the acceptability of a U-shaped unreinforced bellows expansion joint for the given design conditions.

De	sign Conditions:		
•	Pressure (Internal)	=	50 psig@650°F
•	Axial Movements in Compression and Extension	=	Independent
•	Axial Movement (Compression)	=	4.5 in
•	Axial Movement (Extension)	=	0.375 in
•	Angular Deflection	=	None
•	Lateral Deflection	=	None
•	Number of Cycles Required	=	1000
Be	llows:		
•	Material	=	<i>SA</i> –240, <i>Type</i> 321
•	Allowable Stress	=	17900 <i>psi</i>
•	Yield Strength	=	19800 <i>psi</i>
•	Modulus of Elasticity at Design Temperature	=	$25.04E + 06 \ psi$
•	Modulus of Elasticity at Room Temperature	=	$28.26E + 06 \ psi$
•	Inside Diameter of Convolution	=	48.0 in
•	Outside Diameter of Convolution	=	52.0 in
•	Number of Convolutions	=	12
•	Number of Plies	=	1
•	Nominal Ply Thickness	=	0.048 in
•	Convolution Pitch	=	1.0 <i>in</i>
•	Mean Radius of Convolution	=	0.25 in
•	Crest Convolution Inside Radius	=	0.226 in
•	Root Convolution Inside Radius	=	0.226 in
•	End Tangent Length	=	1.25 in
•	Installed without Cold Spring	=	Yes
•	Circumferential welds	=	No

The bellows was formed with a mandrel from a cylinder with an inside diameter of 48.0 *in* and preformed 100% to the outside of the cylinder. The bellows is in as-formed condition.

# Collar:

Collar

= None

Cylindrical shell on which the bellows is attached:

•	Inside Diameter of Shell	=	47.25 in
•	Thickness of Shell	=	0.375 in
•	Minimum Length of Shell on each Side of the Bellows	=	10.5 in

Evaluate per paragraph 4.19.

- a) STEP 1 Check applicability of design rules per paragraph 4.19.2.
  - 1) Bellows length must satisfy  $Nq \leq 3D_b$ .

$$\{Nq = 12(1.0) = 12\} \le \{3D_b = 3(48.0) = 144\}$$
 (*True*)

2) Bellows thickness must satisfy  $nt \le 0.2$  in.

$${nt = 1(0.048) = 0.048} \le {0.2}$$
 (*True*)

3) Number of plies must satisfy  $n \le 5$ .

$$\{n=1\} \le \{5\} \tag{True}$$

4) Displacement shall be essentially axial.

No angular or lateral deflection is specified, so the condition is satisfied.

5) Material allowable stress must not be governed by time-dependent properties.

The material is SA-240, Type 321 austenitic stainless, the design temperature is  $650^{\circ}F$  which is less than the time-dependent value of  $800^{\circ}F$ ; therefore, the condition is satisfied.

6) The length of the cylindrical shell on each side of the bellows shall not be less than  $1.8\sqrt{D_s t_s}$ .

$$\{10.5 \ in\} \ge \left\{1.8\sqrt{D_s t_s} = 1.8\sqrt{(47.25)(0.375)} = 7.577 \ in\right\}$$
(*True*)

- b) STEP 2 Check the applicability of paragraph 4.19.5.2.
  - 1) Check that the following condition is satisfied.

$$0.9r_{ir} = 0.9(0.226) = 0.203 \} \le \{r_{ic} = 0.226\} \le \{1.1r_{ir} = 1.1(0.226) = 0.249\}$$
(True)

2) Torus radius shall satisfy  $r_i \ge 3t$ .

$$\left\{r_i = \frac{r_{ic} + r_{ir}}{2} = \frac{0.226 + 0.226}{2} = 0.226 \text{ in}\right\} \le \left\{3t = 3(0.048) = 0.144 \text{ in}\right\}$$
(True)

3) Sidewall offset angle shall meet  $-15^\circ \le \alpha \le 15^\circ$ .

$$\alpha = \operatorname{atan}\left[\left(\frac{q}{2} - 2r_{m}\right) / (w - 2r_{m})\right] = \operatorname{atan}\left[\left(\frac{(1.0)}{2} - 2(0.25)\right) / (2.0 - 2(0.25))\right] = 0 \ rad$$
  
-15° ≤ {\$\alpha\$ = 0} ≤ 15° (True)

4) Convolution height shall meet  $w \le D_b / 3$ .

$$\left\{w = \left(\frac{D_o}{2} - \frac{D_b}{2}\right) = \left(\frac{52.0}{2} - \frac{48.0}{2}\right) = 2.0 \text{ in}\right\} \le \left\{\frac{D_b}{3} = \frac{48.0}{3} = 16.0 \text{ in}\right\}$$
(True)

c) STEP 3 – Check stresses in bellows at Design Conditions per paragraph 4.19.5.3. Since the bellows are subject to internal pressure, calculations and acceptability criteria are per Table 4.19.1. The following values are calculated.

$$D_{m} = D_{b} + w + rt = 48.0 + 2.0 + 1(0.048) = 50.048 \text{ in}$$

$$t_{p} = t \sqrt{\frac{D_{b}}{D_{m}}} = 0.048 \sqrt{\frac{48.0}{50.048}} = 0.047 \text{ in}$$

$$K = \min\left[\left(\frac{L_{t}}{1.5\sqrt{D_{b}t}}\right), 10\right] = \min\left[\left(\frac{1.25}{1.5\sqrt{48.0(0.048)}}\right), 1.0\right] = 0.549$$

$$A = \left[2\pi r_{m} + 2\sqrt{\left\{\frac{q}{2} - 2r_{m}\right\}^{2} + \left\{w - 2r_{m}\right\}^{2}}\right] n t_{p}$$

$$A = \left[2\pi (0.25) + 2\sqrt{\left\{\frac{(1.0)}{2} - 2(0.25)\right\}^{2} + \left\{(2.0) - 2(0.25)\right\}^{2}}\right] (1)(0.047) = 0.215 \text{ in}^{2}$$

Table 4.19.2 is used to determine  $C_p$ . The following values are calculated.

$$\begin{split} C_1 &= \frac{2r_m}{w} = \frac{2(0.25)}{2.0} = 0.250 \\ \text{with } 0.0 \le C_1 \le 1.0 \\ C_2 &= \frac{1.82 r_m}{\sqrt{D_m t_p}} = \frac{1.82(0.25)}{\sqrt{(50.048)(0.047)}} = 0.297 \\ \text{with } 0.2 \le C_2 \le 4.0 \end{split}$$

The coefficients,  $\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$  are interpolated.

$$\begin{aligned} \alpha_{0} &= 1.000 & \alpha_{3} = 0.711 \\ \alpha_{1} &= -0.587 & \alpha_{4} = 0.662 \\ \alpha_{2} &= -0.589 & \alpha_{5} = -0.646 \\ C_{p} &= \alpha_{0} + \alpha_{1}C_{1} + \alpha_{2}C_{1}^{2} + \alpha_{3}C_{1}^{3} + \alpha_{4}C_{1}^{4} + \alpha_{5}C_{1}^{5} \\ C_{p} &= \begin{cases} 1.000 + (-0.587)(0.25) + (-0.589)(0.25)^{2} + \\ (0.711)(0.25)^{3} + (0.662)(0.25)^{4} + (-0.646)(0.25)^{5} \end{cases} = 0.830 \end{aligned}$$

#### **Calculate Stresses**

Circumferential Membrane stress in bellows tangent, due to pressure  $\left(S_{1}
ight)$  .

$$S_{1} = \frac{P(D_{b} + nt)^{2} L_{t}E_{b}k}{2\left[nt(D_{b} + nt)L_{t}E_{b} + t_{c}D_{c}L_{c}E_{c}k\right]}$$

$$S_{1} = \frac{50(48.0 + 1(0.048))^{2}(1.25)(25.04E + 06)(0.549)}{2\left[1(0.048)(48.0 + 1(0.048))(1.25)(25.04E + 06) + 0\right]} = 13738.7 \text{ psi}$$

Circumferential Membrane stress in bellows end convolutions, due to pressure  $\left(S_{2,E}\right)$ .

$$S_{2,E} = \frac{P[qD_m + L_t(D_b + nt)]}{2(A + nt_pL_t + t_cL_c)} = \frac{50[1.0(50.048) + 1.25(48.0 + 1(0.048))]}{2(0.215 + 1(0.047)(1.25) + 0)} = 10055.5 \text{ psi}$$

Circumferential Membrane stress in bellows intermediate convolutions, due to pressure  $(S_{2,I})$ .

$$S_{2,I} = \frac{P_q D_m}{2A} = \frac{50(1.0)(50.048)}{2(0.215)} = 5819.5 \ psi$$

Meridional Membrane stress in bellows due to pressure  $(S_3)$ .

$$S_3 = \frac{P_W}{2nt_p} = \frac{50(2.0)}{2(1(0.047))} = 1063.8 \ psi$$

Meridional Bending stress in bellows due to pressure  $\left(S_{4}
ight).$ 

$$S_4 = \frac{PC_p}{2n} \left(\frac{w}{t_p}\right)^2 = \frac{50(0.830)}{2(1)} \left(\frac{2.0}{0.047}\right)^2 = 37573.6 \text{ psi}$$

Acceptance Checks

$$\{S_1 = 13738.7 \ psi\} \le \{S = 17900 \ psi\}$$
(*True*)

$$\{S_{2,E} = 10055.5 \ psi\} \le \{S = 17900 \ psi\}$$
(*True*)

$$\{S_{2,I} = 5819.5 \ psi\} \le \{S = 17900 \ psi\}$$
(*True*)

$$\left\{ \left( S_3 + S_4 \right) = \left( 1063.8 + 37573.6 \right) = 38637.4 \ psi \right\} \le \left\{ K_m S = 3 \left( 17960 \right) = 53700 \ psi \right\} \quad (True)$$

Where,

Factor  $K_m$  is calculated by:

$$K_m = 1.5Y_{sm} = 1.5(2.0) = 3.0 \qquad (for \ As - Formed \ Bellows)$$

The forming stain  $\mathcal{E}_{f}$  for bellows formed 100% to the outside of the initial cylinder is:

$$\varepsilon_{f} = \sqrt{\left[\ln\left(1 + \frac{2w}{D_{b}}\right)\right]^{2} + \left[\ln\left(1 + \frac{nt_{p}}{2r_{m}}\right)\right]^{2}}$$
$$\varepsilon_{f} = \sqrt{\left[\ln\left(1 + \frac{2(2.0)}{(48.0)}\right)\right]^{2} + \left[\ln\left(1 + \frac{1(0.047)}{2(0.25)}\right)\right]^{2}} = 0.120$$

The forming method factor  $K_{f}$  for forming with expanding mandrel is:

$$K_{f} = 1.0$$

Since material SA-240, Type 321 is an austenitic stainless steel, the yield strength multiplier  $Y_{sm}$  is:

$$Y_{sm} = 1 + 9.94 (K_f \varepsilon_f) - 7.59 (K_f \varepsilon_f)^2 - 2.4 (K_f \varepsilon_f)^3 + 2.21 (K_f \varepsilon_f)^4$$
$$Y_{sm} = \begin{cases} 1 + 9.94 ((1.0)(0.120)) - 7.59 ((1.0)(0.120))^2 - \\ 2.4 ((1.0)(0.120))^3 + 2.21 ((1.0)(0.120))^4 \end{cases} = 2.083$$

If  $Y_{sm}$  is greater than 2.0, then  $Y_{sm} = 2.0$ .

The bellows meet internal pressure acceptance criteria at design conditions.

d) STEP 4 – Check column instability due to internal pressure per paragraph 4.19.5.4.

$$P_{sc} = \frac{0.34\pi K_b}{N_q} = \frac{0.34\pi (1647.5)}{12(1)} = 146.7 \ psi$$

The axial stiffness,  $K_b$ , is calculated using Equation 4.19.17.

$$K_{b} = \frac{\pi E_{b} D_{m}}{2(1-v_{b}^{2})C_{f}} \left(\frac{n}{N}\right) \left(\frac{t_{p}}{w}\right)^{3} = \frac{\pi (25.04E+06)(50.048)}{2(1-0.3^{2})(1.419)} \left(\frac{1}{12}\right) \left(\frac{0.047}{2.0}\right)^{3}$$
$$K_{b} = 1648.7 \frac{lbs}{in}$$

 $C_{f}$  is calculated using the method described in Table 4.19.3. With  $C_{2} = 0.297$ , interpolate for the coefficients  $\alpha_{0}, \alpha_{1}, \alpha_{2}, \alpha_{3}, \alpha_{4}, \alpha_{5}$ .

$$\begin{aligned} \alpha_{0} &= 1.006 & \alpha_{3} = 5.719 \\ \alpha_{1} &= 2.106 & \alpha_{4} = -5.501 \\ \alpha_{2} &= -2.930 & \alpha_{5} = 2.067 \\ C_{f} &= \alpha_{0} + \alpha_{1}C_{1} + \alpha_{2}C_{1}^{2} + \alpha_{3}C_{1}^{3} + \alpha_{4}C_{1}^{4} + \alpha_{5}C_{1}^{5} \\ C_{f} &= \begin{cases} 1.006 + (2.106)(0.25) + (-2.930)(0.25)^{2} + \\ (5.719)(0.25)^{3} + (-5.501)(0.25)^{4} + (2.067)(0.25)^{5} \end{cases} = 1.419 \\ \{P = 50 \ psi\} \leq \{P_{sc} = 146.7 \ psi\} \end{aligned}$$

$$(True)$$

The bellows meet columns instability criteria at design conditions.

e) STEP 5 – Check in-plane instability due to internal pressure per paragraph4.19.5.5.

$$P_{si} = \frac{AS_{y}^{*}(\pi - 2)}{D_{m}q \left[1 + 2\delta^{2} + (1 - 2\delta^{2} + 4\delta^{4})^{0.5}\right]^{0.5}}$$

$$P_{si} = \left(\frac{0.215(45540)(\pi - 2)}{50.048(1.0)\left[1 - 2(2.15)^{2} + (1 + 2(2.15)^{2} + 4(2.15)^{4})^{0.5}\right]^{0.5}}\right) = 51.2 \text{ psi}$$

$$S_{y}^{*} = 2.3S_{y} = 2.3(19800) = 45540 \text{ psi} \qquad (for \text{ As - Formed Bellows})$$

$$\delta = \frac{S_{4}}{3S_{2,I}} = \frac{37573.6}{3(5819.5)} = 2.15$$

$$\{P = 50 \text{ psi}\} \le \{P_{si} = 51.2 \text{ psi}\} \qquad (True)$$

The bellows meet in-plane instability criteria at design conditions.

f) STEP 6 – Perform a fatigue evaluation per paragraph 4.19.5.7

#### Calculate the equivalent axial displacement range

The following values are calculated.  $\Delta q$  is calculated using the procedure shown in paragraph 4.19.8.

$$\begin{cases} x_e = 0.375 \text{ in} \\ x_c = -4.5 \text{ in} \end{cases} \rightarrow \text{See design data}$$
$$\Delta q_{x,e} = \frac{x_e}{N} = \frac{0.375}{12} = 0.031 \text{ in}$$
$$\Delta q_{x,c} = \frac{x_c}{N} = \frac{4.5}{12} = -0.375 \text{ in}$$

Since no lateral or angular movement:

$$\Delta q_{e,1} = \Delta q_{x,e} = 0.031 \text{ in}$$
$$\Delta q_{c,1} = \Delta q_{x,c} = -0.375 \text{ in}$$

Since the bellows was installed without cold spring and the axial movements in extension and compression are independent (as opposed to concurrent):

Initial PositionFinal Position
$$\Delta q_{e,0} = 0.0$$
 in $\Delta q_{e,1} = +0.031$  in $\Delta q_{c,0} = 0.0$  in $\Delta q_{c,1} = -0.375$  in $\Delta q = \max \left[ |\Delta q_{e,1}|, |\Delta q_{c,1}| \right] = \max \left[ |0.031|, |-0.375| \right] = 0.375$  in

 $C_d$  is calculated using the method described in Table 4.19.4. With  $C_2 = 0.297$ , interpolate for the coefficients  $\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$ .

$$\alpha_{0} = 1.000 \qquad \alpha_{3} = -3.441$$

$$\alpha_{1} = 1.228 \qquad \alpha_{4} = 3.453$$

$$\alpha_{2} = 1.309 \qquad \alpha_{5} = -1.190$$

$$C_{d} = \alpha_{0} + \alpha_{1}C_{1} + \alpha_{2}C_{1}^{2} + \alpha_{3}C_{1}^{3} + \alpha_{4}C_{1}^{4} + \alpha_{5}C_{1}^{5}$$

$$C_{d} = \begin{cases} 1.000 + (1.228)(0.25) + (1.309)(0.25)^{2} + \\ (-3.441)(0.25)^{3} + (3.453)(0.25)^{4} + (-1.190)(0.25)^{5} \end{cases} = 1.347$$

Calculate stresses due to equivalent axial displacement range:

Meridional membrane  $(S_5)$ .

$$S_5 = \frac{E_b t_p^2 \Delta q}{2w^3 C_f} = \frac{(25.04E + 06)(0.047)^2 (0.375)}{2(2.0)^3 (1.419)} = 913.6 \text{ psi}$$

Meridional bending  $(S_6)$ .

$$S_6 = \frac{5E_b t_p \Delta q}{3w^2 C_d} = \frac{5(25.04E + 06)(0.047)(0.375)}{3(2.0)^2 (1.347)} = 136516.3 \text{ psi}$$

Total stress range due to cyclic displacement  $\left(S_{t}
ight)$ 

$$S_{t} = 0.7[S_{3} + S_{4}] + [S_{5} + S_{6}]$$
  

$$S_{t} = 0.7(1063.8 + 37573.6) + (913.6 + 136516.3) = 164476.1 \text{ psi}$$

Calculate allowable number of cycles,  $\,N_{\it alw}$  , using the equations from Table 4.19.5.

$$\begin{split} K_g \left(\frac{E_o}{E_b}\right) S_t &= \left\{ 1.0 \left(\frac{28.26E + 06}{25.04E + 06}\right) (164476.1) = 185626.78 \right\} \ge 65000 \\ N_{alw} &= \left(\frac{5.2 + 06}{K_g \left(\frac{E_0}{E_b}\right) S_t - 38300}\right)^2 = \left(\frac{5.2E + 06}{185626.78 - 38300}\right)^2 = 1246 \ cycles \\ \left\{ N_{alw} = 1246 \ cycles \right\} \ge \left\{ N_{spe} = 1000 \ cycles \right\} \end{split}$$

The bellows meets fatigue design criteria at design conditions.

The bellows meets all of the design requirements of paragraph 4.19 at design conditions.

## 4.19.2 Example E4.19.2 – Toroidal Bellows Expansion Joint and Fatigue Evaluation

Check the acceptability of a toroidal bellows for the given design conditions.

Design Conditions:		
Pressure (Internal)	=	400 psig@650°F
Axial Movements in Compression and Extension	=	Independent
Axial Displacement (Compression)	=	0.25 in
Axial Displacement (Extension)	=	0.745 in
Angular Deflection	=	None
Lateral Deflection	=	None
Number of Cycles Required	=	1000
Bellows:		
Material	=	<i>SA</i> – 240, <i>Type</i> 321
Allowable Stress	=	17900 <i>psi</i>
Modulus of Elasticity at Design Temperature	=	$25.04E + 06 \ psi$
Modulus of Elasticity at Room Temperature	=	$28.26E + 06 \ psi$
Inside Diameter of Bellows	=	36.0 in
Mean Diameter of Bellows	=	40.0 in
Number of Convolutions	=	2
Convolution Pitch	=	4.000 in
Mean Radius of Convolutions	=	1.5 in
Number of Plies	=	1
Ply Thickness	=	0.078 in
Installed without Cold Spring	=	Yes
Circumferential welds	=	No
The bellows is attached to the shell externally on both sides.		
Reinforcing and Tangent Collars:		
Material	=	<i>SA</i> – 240, <i>Type</i> 304
Allowable Stress	=	16200 <i>psi</i>
Modulus of Elasticity at Design Temperature	=	$25.04E + 06 \ psi$
Tangent Collars:		
Tangent Collar Joint Efficiency	=	1.0
Tangent Collar Thickness	=	0.75 in
Cross Sectional Metal Area of one Tangent Collar	=	1.034 <i>in</i> <sup>2</sup>
Length from Attachment Weld to the Center of the		
First Convolution	=	2.000 in

Reinforcing Collars:

Reinforcing Collar Joint Efficiency	=	1.0
Reinforcing Collar Thickness	=	0.75 in
Overall Length of one Reinforcing Collar	=	3.094 in
<ul> <li>Cross Sectional Metal Area of one Reinforcing Collar based on Overall Length</li> </ul>	=	2.068 in <sup>2</sup>
Cylindrical shell on which the bellows is attached:		
Inside Diameter of Shell	=	35.0 in
Thickness of Shell	=	0.50 in

• Minimum Length of Shell on each Side of the Bellows = 10.5 in

Evaluate per paragraph 4.19.

- a) STEP 1 Check applicability of design rules per paragraph 4.19.2.
  - 1) Bellows length must satisfy  $Nq \leq 3D_b$ .

$$\{(N-1)q+2L_d=(2-1)(4.0)+2(2.0)=8 \text{ in}\} \le \{3D_b=3(36)=108 \text{ in}\} \quad (True)$$

2) Bellows thickness must satisfy  $nt \le 0.2$  in.

$$\{nt = 1(0.078) = 0.078\} \le \{0.2\}$$
 (*True*)

3) Number of plies must satisfy  $n \le 5$ .

$$\{n=1\} \le \{5\} \tag{True}$$

4) Displacement shall be essentially axial.

No angular or lateral deflection is specified, so the condition is satisfied.

5) Material allowable stress must not be governed by time dependent properties.

The material is SA-240, Type 321 austenitic stainless, the design temperature is  $650^{\circ}F$  which is less than the time-dependent value of  $800^{\circ}F$ ; therefore, the condition is satisfied.

6) The length of the cylindrical shell on each side of the bellows shall not be less than  $1.8\sqrt{D_s t_s}$ .

$$\{10.5 \ in\} \ge \left\{1.8\sqrt{D_s t_s} = 1.8\sqrt{(35.0)(0.50)} = 7.530 \ in\right\}$$
(*True*)

- b) STEP 2 Check the applicability of paragraph 4.19.7.2.
  - The type of attachment to the shell shall be the same on both sides. The bellows is attached to the shell externally on both sides, so the condition is satisfied.
  - 2) Distance  $L_g$  shall be less than 0.75r in the maximum extended position.

The distance across the inside opening in the neutral position is calculated as follows,

$$L_{g0} = q - (L_{rt} + 2nt) = (4.0) - ((3.094) + 2(1)(0.078)) = 0.750$$
 in

The only movement is in the axial direction. The maximum opening corresponds to the maximum extension.

$$L_{g} = L_{g0} + \Delta q_{x,e} = L_{g0} + \frac{x_{e}}{N} = (0.75) + \left(\frac{0.745}{2}\right) = 1.1225 \text{ in}$$
  
$$\{L_{g} = 1.1225 \text{ in}\} < \{0.75r = 0.75(1.5) = 1.1250 \text{ in}\}$$
(True)

4) For internally attached bellows, the length of the shell on each side of the bellows having thickness  $t_s$  shall be at least equal to  $L_{sm} = \sqrt{D_s t_s}$ .

Not applicable

c) STEP 3 – Check stresses in bellows at design conditions per paragraph 4.19.7.3. Since the bellows are subject to internal pressure, calculations and acceptability criteria are per Table 4.19.8.

$$D_{c} = D_{b} + 2nt + t_{c} = 36.0 + 2(1)(0.078) + 0.75 = 36.906 in$$
  

$$D_{r} = D_{b} + 2nt + t_{r} = 36.0 + 2(1)(0.078) + 0.75 = 36.906 in$$
  

$$t_{p} = t\sqrt{\frac{D_{b}}{D_{m}}} = 0.078\sqrt{\frac{36.0}{40.0}} = 0.074 in$$

Calculate stresses

Circumferential Membrane stress in end tangent due to internal pressure  $(S_1)$ .

$$S_{1} = \frac{P(D_{b} + nt)^{2} L_{d}E_{b}}{2D_{c}E_{c}A_{tc}}$$

$$S_{1} = \frac{(400)(36.0 + (1)(0.078))^{2}(2.0)(25.04E + 06)}{2(36.906)(25.04E + 06)(1.034)} = 13643.6 \text{ psi}$$

Circumferential Membrane stress in collar due to internal pressure  $(S_1)$ .

$$S_1' = \frac{PD_cL_d}{2A_{tc}} = \frac{(400)(36.906)(2.0)}{2(1.034)} = 14277.0 \ psi$$

Circumferential Membrane stress in bellows due to internal pressure  $(S_2)$ .

$$S_2 = \frac{Pr}{2nt_p} = \frac{400(1.5)}{2(1)(0.074)} = 4054.1 \text{ psi}$$

Circumferential Membrane stress in reinforcing collar due to internal pressure  $(S'_2)$ .

Since 
$$\{L_{rt} = 3.094 \ in\} \le \left\{\frac{2}{3}\sqrt{D_r t_r} = \frac{2}{3}\sqrt{(36.906)(0.75)} = 3.507 \ in\right\}$$

$$S'_{2} = \frac{D_{r} (L_{rt} + L_{g} + 2nt)}{2 A_{rt}} P$$
  

$$S'_{2} = \frac{(36.906) (3.094 + 1.1225 + 2(1)(0.078))}{2(2.068)} \cdot (400) = 15606.6 \text{ psi}$$

Meridional Membrane stress in bellows due to internal pressure  $\left(S_3
ight).$ 

$$S_{3} = \frac{Pr}{nt_{p}} \left( \frac{D_{m} - r}{D_{m} - 2r} \right) = \frac{400(1.5)}{(1)(0.074)} \left( \frac{40.0 - 1.5}{40.0 - 2(1.5)} \right) = 8436.8 \text{ psi}$$

Acceptance Checks

$$\{S_1 = 13643.6 \ psi\} \le \{S = 17900 \ psi\}$$
(*True*)

$$\{S'_1 = 14277.0 \ psi\} \le \{C_{wc}S_c = (1)(16200) = 16200 \ psi\}$$
(*True*)

$$\{S_2 = 4054.1 \ psi\} \le \{S = 17900 \ psi\}$$
(*True*)

$$\left\{S_{2}^{'}=15606.6 \ psi\right\} \leq \left\{C_{wr}S_{r}=(1)(16200)=16200 \ psi\right\}$$
(*True*)

$$\{S_3 = 8436.8 \ psi\} \le \{S = 17900 \ psi\}$$
(*True*)

Therefore, bellows meets internal pressure stress acceptance criteria at design conditions.

d) STEP 4 – Check column instability due to internal pressure per paragraph 4.19.7.4.

The following values are calculated using the procedure shown in Table 4.19.9.

$$C_{3} = \frac{6.61r^{2}}{D_{m}t_{p}} = \frac{6.61(1.5)^{2}}{40.0(0.074)} = 5.024$$

$$B_{3} = \frac{0.99916 - 0.091665C_{3} + 0.040635C_{3}^{2} - 0.0038483C_{3}^{3} + 0.00013392C_{3}^{4}}{1 - 0.1527C_{3} + 0.013446C_{3}^{2} - 0.00062724C_{3}^{3} + 1.4374(10)^{-5}C_{3}^{4}}$$

$$B_{3} = \frac{\begin{bmatrix} 0.99916 - 0.091665(5.024) + 0.040635(5.024)^{2} - \\ 0.0038483(5.024)^{3} + 0.00013392(5.024)^{4} \end{bmatrix}}{\begin{bmatrix} 1 - 0.1527(5.024) + 0.013446((5.024)^{2}) - \\ 0.00062724(5.024)^{3} + 1.4374(10)^{-5}(5.024)^{4} \end{bmatrix}} = 2.315$$

The axial stiffness,  $K_b$ , is calculated using Equation 4.19.28.

$$K_{b} = \frac{E_{b}D_{m}B_{3}}{12(1-v_{b}^{2})} \left(\frac{n}{N}\right) \left(\frac{t_{p}}{r}\right)^{3} = \frac{(25.04E+06)(40.0)(2.315)}{12(1-(0.3)^{2})} \left(\frac{1}{2}\right) \left(\frac{0.074}{1.5}\right)^{3}$$
$$K_{b} = 12747.2 \ \frac{lb}{in}$$

Calculate allowable internal pressure:

$$P_{sc} = \frac{0.15\pi K_b}{Nr} = \frac{0.15\pi (12747.2)}{2(1.5)} = 2002 \ psi$$

Acceptance criteria

$$\{P = 400 \ psi\} \le \{P_{sc} = 2002 \ psi\}$$
 (True)

The bellows meets column instability criteria at design conditions.

e) STEP 5 – Perform a fatigue evaluation per paragraph 4.19.7.7.

The axial displacement range,  $\Delta q$  is calculated using the procedure shown in paragraph 4.19.8.

$$\begin{cases} x_e = 0.745 \text{ in} \\ x_c = -0.25 \text{ in} \end{cases} \rightarrow \qquad \text{See design data}$$
$$\Delta q_{x,e} = \frac{x_e}{N} = \frac{0.745}{2} = 0.3725 \text{ in}$$
$$\Delta q_{x,c} = \frac{x_c}{N} = \frac{-0.25}{2} = -0.125 \text{ in}$$

Since no angular or lateral displacement:

$$\Delta q_{e,1} = \Delta q_{x,e} = +0.3725 \text{ in}$$
  
$$\Delta q_{e,1} = \Delta q_{x,e} = -0.125 \text{ in}$$

Since the bellow was installed without cold spring and the axial movements in extension and compression are independent (as opposed to concurrent):

$$\Delta q = \max\left[\left|\Delta q_{e,1}\right|, \left|\Delta q_{c,1}\right|\right] = \max\left[\left|0.3725\right|, \left|-0.125\right|\right] = 0.3725 \text{ in}$$

Calculate coefficient  $B_1$ ,  $B_2$  from Table 4.19.9:

$$B_{1} = \frac{\left(1.00404 + 0.028725C_{3} + 0.18961C_{3}^{2} - 0.00058626C_{3}^{3}\right)}{\left(1 + 0.14069C_{3} - 0.0052319C_{3}^{2} + 0.00029867C_{3}^{3} - 6.2088(10)^{-6}C_{3}^{4}\right)}$$

$$B_{1} = \frac{\left(1.00404 + 0.028725(5.024) + 0.18961(5.024)^{2} - 0.00058626(5.024)^{3}\right)}{\left(1 + 0.14069(5.024) - 0.0052319(5.024)^{2} + 0.00029867(5.024)^{3} - 6.2088(10)^{-6}(5.024)^{4}\right)} = 3.643$$

$$B_{2} = \frac{\left(0.049198 - 0.77774C_{3} - 0.13013C_{3}^{2} + 0.080371C_{3}^{3}\right)}{\left(1 - 2.81257C_{3} + 0.63815C_{3}^{2} + 0.0006405C_{3}^{3}\right)}$$

$$B_{2} = \frac{\left(0.049198 - 0.77774(5.024) - 0.13013(5.024)^{2} + 0.080371(5.024)^{3}\right)}{\left(1 - 2.81257(5.024) + 0.63815(5.024)^{2} + 0.0006405(5.024)^{3}\right)} = 0.997$$

Calculate meridional stresses due to axial displacement range:

$$S_5(membrane) = \frac{E_b t_p^2 B_1 \Delta q}{34.3r^3} = \frac{(25.04E + 06)(0.074)^2 (3.643)(0.3725)}{34.3(1.5)^3} = 1607.4 \text{ psi}$$

$$S_6(bending) = \frac{E_b t_p B_2 \Delta q}{5.72r^2} = \frac{(25.04E + 06)(0.074)^2 (0.997)(0.3725)}{5.72(1.5)^2} = 53469.9 \ psi$$

Calculate total cycle stress range due to displacement:

$$S_t = 3S_3 + S_5 + S_6 = 3(8436.8) + 1607.4 + 53469.9 = 80387.7 \ psi$$

Calculate the allowable number of cycles,  $N_{\it alw}$  , per Table 4.19.10.

$$\begin{cases} K_g \left(\frac{E_o}{E_b}\right) S_t = 1.0 \left(\frac{28.26E + 06}{25.04E + 06}\right) (80387.7) = 90725.1 \ psi \\ \ge \{65000 \ psi \} \\ N_{alw} = \left(\frac{5.2E + 06}{\left(K_g \left(\frac{E_o}{E_b}\right) S_t - 38300\right)}\right)^2 = \left(\frac{5.2E + 06}{(90725.1 - 38300)}\right)^2 = 9838 \ cycles \\ \{N_{alw} = 9838 \ cycles\} \ge \{N_{spe} = 1000 \ cycles\} \end{cases}$$

The bellows meets fatigue design criteria at design conditions.

The bellows meets all of the design requirements of paragraph 4.19 at design conditions.

# PART 5

# DESIGN BY ANALYSIS REQUIREMENTS PART CONTENTS

#### 5.1 General Requirements

### 5.2 Protection Against Plastic Collapse

### 5.2.1 Example E5.2.1 – Elastic Stress Analysis

Evaluate the vessel top head and shell region for compliance with respect to the elastic stress analysis criteria for plastic collapse provided in paragraph 5.2.2. Do not include the standard flanges or NPS 6 piping in the assessment for compliance to allowable stresses. Internal pressure is the only load that is to be considered. Relevant design data and geometry are provided below and in Figures E5.2.1-1 and E5.2.1-2.

#### Vessel Data

•	Material – Shell and Heads	=	SA-516, Grade 70, Normalized
•	Material – Forgings	=	SA-105
•	Design Conditions	=	420 psig at 125°F
•	Corrosion Allowance	=	0.125 inches
•	PWHT	=	Yes



Figure E5.2.1-1 - Vessel Configuration



Figure E5.2.1-2 - Vessel Top Inlet Nozzle Geometry

a) STEP 1 – Determine the types of loads acting on the component. In general, separate load cases are analyzed to evaluate "load-controlled" loads such as pressure and externally applied reactions due to weight effects and "strain-controlled" loads resulting from thermal gradients and imposed displacements. The loads to be considered in the design shall include, but not be limited to, those given in Table 5.1. The load combinations that shall be considered for each loading condition shall include, but not be limited to those given in Table 5.3.

The only load to be considered is internal pressure of 420 psig

- 1) Develop the finite element model.
- 2) Due to symmetry in geometry and loading, an axisymmetric solid model is generated. To capture proper membrane behavior in the model the shell was extended a distance of  $5\sqrt{P_t}$

 $5\sqrt{Rt}$  below the head to shell transition. The FE model is illustrated in Figure E5.2.1-3. The model was generated with the ABAQUS commercial FEA program, version 6.9-1. The geometry was constructed based on the corroded dimensions.



Figure E5.2.1-3 - Axisymmetric FE Model

3) Generate mesh. ABAQUS 8-noded reduced integration elements (CAX8R) are specified for the analysis.



Figure E5.2.1-4 - Mesh of Flange and Nozzle to Head Junction



Figure E5.2.1-5 - Mesh of Head to Shell Transition

4) Apply the material properties given below to the appropriate components of the model. Material assignments are depicted in Figure E5.2.1-6. Note that the weld region was assigned the properties of SA-105 material. The difference in modulus between the two materials is negligible, so a single modulus value for the entire model could have been chosen as well.

Component	Material	Modulus of Elasticity (psi)	Poisson Ratio	
Weld Neck Flange & Nozzle	SA-105	2.91E+07	0.3	
Head and Shell	SA-516-70N	2.88E+07	0.3	



Figure E5.2.1-6 - Material Assignments

5) Apply the internal pressure load to the pressure boundaries of the vessel and an appropriate pressure thrust load to the flange face. Apply the appropriate boundary conditions to the shell edge as per the figure below.



Figure E5.2.1-7 - Load and Boundary Conditions for the FE Model

6) Run analysis and review results. Evaluate displacements and compare calculated reaction force values to hand calculated values.



Figure E5.2.1-8 - Results of Elastic Analysis, von Mises Stress in both Deformed and Nondeformed States

Reaction Force (y-direction):

Calculated (ABAQUS) 2.341E+06 lbf

Hand Calculations 2.341E+06 lbf

Note: Results for Steps 4, 5, and 6 were calculated automatically by analysis routines contained in the ABAQUS commercial FEA software, version 6.9-1. Through wall stress linearization was conducted at critical areas around the pressure boundary to provide data for the routines. The resultant von Mises stresses for  $P_m$ ,  $P_L$ , and  $P_b$  stress categories are summarized in Table E5.2.1-1.

Note that per Annex 5.A, bending stresses are calculated only for the local hoop and meridional (normal) component stresses, and not for the local component stress parallel to the SCL or in-plane shear stress.

- b) STEP 2 At the point on the vessel that is being investigated, calculate the stress tensor (six unique components of stress) for each type of load. Assign each of the computed stress tensors to one or to a group of the categories defined below. Assistance in assigning each stress tensor to an appropriate category for a component can be obtained by using Figure 5.1 and Table 5.6. Note that the equivalent stresses Q and F do not need to be determined to evaluate protection against plastic collapse. However, these components are needed for fatigue and ratcheting evaluations that are based on elastic stress analysis (see paragraphs 5.5.3 and 5.5.6, respectively).
  - 1) General primary membrane equivalent stress  $P_m$
  - 2) Local primary membrane equivalent stress  $P_{I}$
  - 3) Primary bending equivalent stress  $-P_{h}$
  - 4) Secondary equivalent stress Q
  - 5) Additional equivalent stress produced by a stress concentration or a thermal stress over and above the nominal (P+Q) stress level *F*



Figure E5.2.1-9 - Critical Locations Through the Vessel Requiring Stress Evaluation



Figure E5.2.1-10 - Stress Classification Lines (SCLs) in the Nozzle to Shell Region



Figure E5.2.1-11 - Stress Classification Lines (SCLs) in the Head to Shell Region and Main Body region

- c) STEP 3 Sum the stress tensors (stresses are added on a component basis) assigned to each equivalent stress category. The final result is a stress tensor representing the effects of all the loads assigned to each equivalent stress category. A detailed stress analysis performed using a numerical method such as finite element analysis typically provides a combination of  $P_L + P_b$  and  $P_L + P_b + Q + F$  directly.
  - 1) If a load case is analyzed that includes only "load-controlled" loads (e.g. pressure and weight effects), the computed equivalent stresses shall be used to directly represent the  $P_m$ ,  $P_L + P_b$ , or  $P_L + P_b + Q$ . For example, for a vessel subject to internal pressure with an elliptical head;  $P_m$  equivalent stresses occur away from the head to shell junction, and  $P_L$  and  $P_L + P_b + Q$  equivalent stresses occur at the junction.
  - 2) If a load case is analyzed that includes only "strain-controlled" loads (e.g. thermal gradients), the computed equivalent stresses represent Q alone; the combination  $P_L + P_b + Q$  shall be derived from load cases developed from both "load-controlled" and "strain-controlled" loads.
  - 3) If the stress in category F is produced by a stress concentration or thermal stress, the quantity F is the additional stress produced by the stress concentration in excess of the nominal membrane plus bending stress. For example, if a plate has a nominal primary membrane equivalent stress of  $S_e$ , and has a fatigue strength reduction characterized by a

factor  $K_j$ , then:  $P_m = S_e$ ,  $P_b = 0$ , Q = 0, and  $F = P_m (K_f - 1)$ . The total equivalent stress is  $P_m + F$ .

- d) STEP 4 Determine the principal stresses of the sum of the stress tensors assigned to the equivalent stress categories, and compute the equivalent stress using Equation (5.1).
- e) STEP 5 To evaluate protection against plastic collapse, compare the computed equivalent stress to their corresponding allowable values (see paragraph 5.2.2.4). See Table E5.2.1-1 for evaluation results.

Note: All bending stresses exhibited in this example fall under the category of secondary stress, Q; therefore  $P_b$  is zero for all cases in this example.

$$P_m \le S$$
$$P_L \le 1.5S$$
$$\left(P_L + P_b\right) \le 1.5S$$

SCL	Location	2013 Div 2		Equivalent Linearized Stresses		Stress Evaluation			
No.		Material	$S_m$	$P_m$	$P_{L}$	$P_b$	$P_m \leq S_m$	$P_{L} \leq 1.5S_{m}$	$P_L + P_b \le 1.5S_m$
1	Flange/nozzle connection	A 105	23300	N/A	6476	N/A	N/A	Pass	Pass
2	Upper nozzle transition	A 105	23300	N/A	1118	N/A	N/A	Pass	Pass
3	Nozzle to shell junction	A 105	23300	N/A	16077	N/A	N/A	Pass	Pass
4	Shell to nozzle junction	A 516- 70N	24550	N/A	22232	N/A	N/A	Pass	Pass
5	Elliptical head knuckle	A 516- 70N	24550	N/A	18025	N/A	N/A	Pass	Pass
6	Head tangent line	A 516- 70N	24550	N/A	9613	N/A	N/A	Pass	Pass
7	Head to shell transition	A 516- 70N	24550	N/A	10048	N/A	N/A	Pass	Pass
8	Shell to head transition	A 516- 70N	24550	N/A	10985	N/A	N/A	Pass	Pass
9	Shell (away from discontinuities)	A 516- 70N	24550	19028	N/A	N/A	Pass	N/A	N/A

# Table E5.2.1-1 - Results of the Elastic Analysis Using Criterion from Step 5 of Paragraph5.2.2.4 of the 2013 Section VIII, Div 2, Part 5 ASME Code

### 5.2.2 Example E5.2.2 – Limit Load Analysis

Evaluate the vessel top head and shell region given in Example Problem E5.2.1 for compliance with respect to the limit load analysis criteria for plastic collapse provided in paragraph 5.2.3.

a) STEP 1 – Develop a numerical model of the component including all relevant geometry characteristics. The model used for the analysis shall be selected to accurately represent the component geometry, boundary conditions, and applied loads.

The same model was used as in Example Problem E5.2.1, see Figure E5.2.1-3.

b) STEP 2 – Define all relevant loads and applicable load cases. The loads to be considered in the analysis shall include, but not be limited to, those given in Table 5.1.

The only load to be considered is internal pressure factored according to Table 5.4, 1.5(P) = 630 psig. Associated thrust load was factored accordingly.

c) STEP 3 – An elastic-perfectly plastic material model with small displacement theory was used in the analysis. The von Mises yield function and associated flow rule was utilized with a yield strength defining the plastic limit shall equal 1.5S.

Material and loading keywords used in the ABAQUS input file are shown in Figure E5.2.2-1.

**
** MATERIALS
**
*Material.name=SA105
*Flastic
$2.91_{P+07}$ 0.3
*nlastic
34950 0
*Material name=S451670N
*Elastic
$2.878_{P}+07.0.3$
*nlastic
36825 0
**
**
** STEP. Internal pressure 420 psig
**
*Step, name="Internal pressure 420 psig", nlgeom=no
*Static
0.1, 1., 1e-05, 1.
**
** BOUNDARY CONDITIONS
**
** Name: BC-1 Type: Symmetry/Antisymmetry/Encastre
*Boundary
_PickedSet4, YSYMM
**
** LOADS
**
** Name: Load-1 Type: Pressure
*Dsload
_PickedSurf5, P, 630.
** Name: end thrust Type: Pressure
*Dsload
_PickedSurf6, P, -1473.45
**

Figure E5.2.2-1 - Material Properties and Loading Conditions from ABAQUS Input File

d) STEP 4 – Determine the load case combinations to be used in the analysis using the information from STEP 2 in conjunction with Table 5.4. Each of the indicated load cases shall be evaluated. The effects of one or more loads not acting shall be investigated. Additional load cases for special conditions not included in Table 5.4 shall be considered, as applicable.

No additional load cases are applicable in this example.

e) STEP 5 – Perform a limit-load analysis for each of the load case combinations defined in STEP
 4. If convergence is achieved, the component is stable under the applied loads for this load case. Otherwise, the component configuration (i.e. thickness) shall be modified or applied loads reduced and the analysis repeated. Note that if the applied loading results in a compressive stress field within the component, buckling failure should be addressed (see paragraph 5.4).

The results of the analysis are shown in Figure E5.2.2-2, convergence was achieved at the factored load condition; therefore the vessel passes limit load analysis for this load case. Pressure versus displacement for this case is shown in Figure E5.2.2-3.



Figure E5.2.2-2 - Results of the Limit load Analysis at Factored Load of 630 psi



Figure E5.2.2-3 - Pressure vs. Displacement for Limit Load Case
### 5.2.3 Example E5.2.3 – Elastic-Plastic Analysis

Evaluate the vessel top head and shell region given in Example Problem E5.2.1 for compliance with respect to the elastic-plastic analysis criteria for plastic collapse provided in paragraph 5.2.4.

a) STEP 1 – Develop a numerical model of the component including all relevant geometry characteristics. The model used for the analysis shall be selected to accurately represent the component geometry, boundary conditions, and applied loads. In addition, refinement of the model around areas of stress and strain concentrations shall be provided. The analysis of one or more numerical models may be required to ensure that an accurate description of the stress and strains in the component is achieved.

The same model was used as in Example Problem E5.2.1, see Figure E5.2.1-3.

b) STEP 2 – Define all relevant loads and applicable load cases. The loads to be considered in the design shall include, but not be limited to, those given in Table 5.1.

The only load to be considered is internal pressure factored according to Table 5.5 for the global criterion, 2.4(P) = 1008 psig. Associated thrust load was factored accordingly.

c) STEP 3 – An elastic-plastic material model shall be used in the analysis. The von Mises yield function and associated flow rule was utilized. A material model that includes hardening or softening, or an elastic-perfectly plastic model may be utilized. A true stress-strain curve model that includes temperature dependent hardening behavior is provided in Annex 3.D. When using this material model, the hardening behavior shall be included up to the true ultimate stress and perfect plasticity behavior (i.e. the slope of the stress-strain curves is zero) beyond this limit. The effects of non-linear geometry shall be considered in the analysis.

The true stress-strain curve from Annex 3.D was used for the analysis. The material and loading keywords used in the ABAQUS input file are shown below.

```
** MATERIALS
**
*Material, name=SA105
*Elastic
2.91e+07, 0.3
*PLASTIC
**
**
   Prager Stress-Strain Curve - Units: (psi), (in/in), (F)
**
                            Temperature = 125 (F)
                           Yield stress = 34404.214 (psi)
**
              Ultimate Tensile Strength = 69836.572 (psi)
**
                    YS Reduction Factor = 1.0000
**
                   UTS Reduction Factor = 1.0000
                        Young's Modulus = 2.910E+07 (psi)
**
**
                 0.0000,
                            125.0000
 34404.214,
 37576.981,
              2.035E-03,
                           125.0000
  40749.748,
              5.551E-03,
                            125.0000
                          125.0000
  43922.515,
                 0.0118,
  47095.282,
                 0.0213,
                          125.0000
 50268.049,
                 0.0324,
                          125.0000
 53440.816,
                 0.0432,
                          125.0000
 56613.583,
                 0.0539,
                            125.0000
                 0.0653,
                          125.0000
 59786.350,
  62959.117,
                 0.0778,
                          125.0000
  66131.884,
                 0.0917,
                            125.0000
```

```
0.1073, 125.0000
 69304.651,
 72477.418,
              0.1245, 125.0000
                       125.0000
 75650.185,
               0.1436,
 78822.952,
               0.1647, 125.0000
 81995.719,
               0.1877, 125.0000
              0.2129, 125.0000
 85168.486,
                       125.0000
              0.2404,
 88341.253,
               0.2702, 125.0000
 91514.020,
 94686.787,
               0.3024, 125.0000
**
*Material, name=SA51670N
*Elastic
2.878e+07, 0.3
*PLASTIC
**
** Prager Stress-Strain Curve - Units: (psi), (in/in), (F)
**
                         Temperature = 125 (F)
**
                         Yield stress = 36315.560 (psi)
**
             Ultimate Tensile Strength = 69836.572 (psi)
**
                  YS Reduction Factor = 1,0000
**
                 UTS Reduction Factor = 1.0000
**
                      Young's Modulus = 2.878E+07 (psi)
**
 36315.560,
              0.0000, 125.0000
                       125.0000
 39306.562, 1.945E-03,
 42297.564, 5.227E-03, 125.0000
 45288.567,
              0.0110, 125.0000
 48279.569,
              0.0197, 125.0000
              0.0302, 125.0000
0.0407, 125.0000
 51270.571,
 54261.574,
 57252.576,
               0.0509, 125.0000
 60243.578,
              0.0615, 125.0000
 63234.581,
              0.0732, 125.0000
               0.0862, 125.0000
 66225.583,
               0.1008, 125.0000
 69216.586,
               0.1170, 125.0000
 72207.588,
 75198.590,
              0.1350, 125.0000
              0.1549, 125.0000
 78189.593,
                       125.0000
 81180.595,
               0.1767,
                       125.0000
               0.2006,
 84171.597,
              0.2267, 125.0000
 87162.600,
 90153.602,
              0.2551, 125.0000
 93144.604,
              0.2860,
                        125.0000
** _____
**
** STEP: Internal pressure factored load of 1008 psig
**
*Step, name="Internal pressure 1008 psig", nlgeom
*Static
0.05, 1., 1e-05, 1.
**
** BOUNDARY CONDITIONS
**
** Name: BC-1 Type: Symmetry/Antisymmetry/Encastre
*Boundary
```

```
_PickedSet4, YSYMM
**
** LOADS
**
*Dsload
_PickedSurf5, P, 1008.
** Name: end thrust Type: Pressure
*Dsload
_PickedSurf6, P, -2357.52
**
```

d) STEP 4 – Determine the load case combinations to be used in the analysis using the information from STEP 2 in conjunction with Table 5.5.

No additional load cases are applicable in this example.

e) STEP 5 – Perform an elastic-plastic analysis for each of the load cases defined in STEP 4. If convergence is achieved, the component is stable under the applied loads for this load case. Otherwise, the component configuration (i.e. thickness) shall be modified or applied loads reduced and the analysis repeated. Note that if the applied loading results in a compressive stress field within the component, buckling may occur, and an evaluation in accordance with paragraph 5.4 may be required.

The results of the analysis are shown in Figure E5.2.3-2, convergence was achieved therefore vessel passes elastic-plastic analysis for this load case. Pressure vs. radial displacement for this case is shown in Figure E5.2.3-3.



Figure E5.2.3-2 - Results of the Elastic-Plastic Analysis at Factored Load of 1008 psi; vonMises Stress and Equivalent Plastic Strain



Figure E5.2.3-3 - Pressure vs. Displacement for Elastic-Plastic Case

# 5.3 Protection Against Local Failure

# 5.3.1 Overview

Evaluate the vessel top head and shell region given in Example Problem E5.2.1 for compliance with respect to the elastic and elastic-plastic local failure criteria provided in paragraphs 5.3.2 and 5.3.3.

The same model and material conditions were used as in Example Problem E5.2.1.

# 5.3.2 Example E5.3.2 – Elastic Analysis

In addition to demonstrating protection against plastic collapse, the following elastic analysis criterion shall be satisfied for each point in the component. The sum of the local primary membrane plus bending principal stresses shall be used for checking this criterion.

$$(\sigma_1 + \sigma_2 + \sigma_3) \le 4S$$
$$4S = 93.2 \ ksi$$

The results for Example Problem E5.2.1 (elastic analysis) were evaluated using the above criterion. This analysis revealed that the model satisfies the elastic local failure criterion, see Figure E5.3.2-1. Note that the total (memb+bend+peak) principal stress summation as shown on the contour plot satisfied the allowable limit. Therefore, linearization of the elastic stresses was not necessary to satisfy the elastic local failure check. The elastic-plastic local failure criterion is evaluated in 5.3.3 for illustration since the component was shown to be acceptable based upon the elastic local failure results.



Figure E5.3.2-1 - Local Failure Elastic Analysis Results @ 420 psig

### 5.3.3 Example E5.3.3 – Elastic-Plastic Analysis

The following procedure shall be used to evaluate protection against local failure for a sequence of applied loads.

a) STEP 1 – Perform an elastic-plastic stress analysis based on the load case combinations for the local criterion given in Table 5.5. The effects of non-linear geometry shall be considered in the analysis.

The same model and material conditions were used as Example Problem E5.2.3. The only load to be considered is internal pressure factored according to Table 5.5 for the local criterion,  $1.7(P) = 714 \ psig$ . Associated thrust load was factored accordingly.

b) STEP 2 – For a location in the component subject to evaluation, determine the principal stresses,  $\sigma_1$ ,  $\sigma_2$ ,  $\sigma_3$ , the equivalent stress,  $\sigma_e$ , using Equation (5.1) and the total equivalent plastic strain,  $\varepsilon_{nea}$ .

Values for the principle stresses, equivalent stress and total equivalent plastic strain for each point in the model were extracted from the ABAQUS .odb file. The following example calculation is for one integration point in the model. The full model (all integration points) will be evaluated using the "user defined variable" option available in ABAQUS to create contour plot variables of the strain limit and strain limit ratio.

The principal stresses to be evaluated are shown below.

 $\sigma_1 = 45095 \ psi$   $\sigma_2 = 34603 \ psi$   $\sigma_3 = 1118 \ psi$  $\sigma_e = 39783 \ psi$ 

c) STEP 3 – Determine the limiting triaxial strain,  $\varepsilon_L$ , using Equation (5.6) where  $\varepsilon_{Lu}$ ,  $m_2$ , and  $\alpha_{sl}$  are determined from Table 5.7.

$$\varepsilon_{L} = \varepsilon_{Lu} \cdot \exp\left[-\left(\frac{\alpha_{sl}}{1+m_{2}}\right)\left(\left\{\frac{\left(\sigma_{1}+\sigma_{2}+\sigma_{3}\right)}{3\sigma_{e}}\right\}-\frac{1}{3}\right)\right]$$

The strain limit parameters are shown below.

$$R = 0.5429$$
$$\alpha_{sl} = 2.2$$
$$m_2 = 0.2743$$
$$\varepsilon_{Lu} = m_2$$

The computed limit strain is:

 $\varepsilon_{L} = 0.1515$ 

d) STEP 4 – Determine the forming strain  $\mathcal{E}_{cf}$  based on the material and fabrication method in accordance with Part 6. If heat treatment is performed in accordance with Part 6, the forming strain may be assumed to be zero. The forming strain is:

ne forming strain

 $\varepsilon_{cf} = 0$ 

e) STEP 5– Determine if the strain limit is satisfied. The location in the component is acceptable for the specified load case if Equation (5.7) is satisfied.

 $\varepsilon_{peq} + \varepsilon_{cf} \leq \varepsilon_L$ 

The total equivalent plastic strain is:

 $\varepsilon_{peq} = 0.002468$ 

Since  $\varepsilon_{peq} + \varepsilon_{cf}$  is less than  $\varepsilon_L$ , the strain at this integration point passes the Elastic-Plastic criterion.

A full model contour plot of the strain limit is shown in Figure E5.3.3-1.



Figure E5.3.3-1 - Contour plot of the Strain Limit,  $\varepsilon_L$ 

A full model contour plot of the equivalent plastic strain is shown in Figure E5.3.3-2



Figure E5.3.3-2 - Contour plot of Equivalent Plastic Strain,  $\mathcal{E}_{peq}$ 



Full model evaluation of the Elastic-Plastic criterion is shown in Figure 5.3.3-3.

Figure E5.3.3-3 - Elastic-Plastic Local Failure Analysis Results @ 714 psi

### PTB-3-2013

Full model evaluation indicates that all integration points meet the criterion of  $\varepsilon_{peq} + \varepsilon_{cf} \leq \varepsilon_L$ . The maximum strain limit ratio  $\left[\left(\varepsilon_{peq} + \varepsilon_{cf}\right)/\varepsilon_L\right]$  for this model is 0.018, as indicated in Figure E5.3.3-3. Since this value is less than 1.0 the model passes the elastic-plastic local strain analysis.

# 5.4 Example E5.4 – Protection Against Collapse from Buckling

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Evaluate the following tower, Figure E5.4-1, for compliance with respect to the Type-1 buckling criteria provided in paragraph 5.4.1.2.

- Material Shell and Heads
- SA-516, Grade 70, Normalized

• Design Conditions

- -14.7 psig at 300°F 0.125 inches
- Corrosion Allowance =



Figure E5.4-1 - Tower Configuration

### PTB-3-2013

In addition to evaluating protection against plastic collapse as defined in paragraph 5.2, a design factor for protection against collapse from buckling shall be satisfied to avoid buckling of components with a compressive stress field under applied design loads.

The design factor to be used in a structural stability assessment is based on the type of buckling analysis performed. The following design factor shall be the minimum value for use with shell components when the buckling loads are determined using a numerical solution (i.e. bifurcation buckling analysis or elastic-plastic collapse analysis).

<u>Type 1</u> – If a bifurcation buckling analysis is performed using an elastic stress analysis <u>without</u> <u>geometric nonlinearities</u> in the solution to determine the pre-stress in the component, a minimum design factor of  $\Phi_B = 2 / \beta_{cr}$  shall be used (see paragraph 5.4.1.3). In this analysis, the pre-stress in the component is established based on the loading combinations in Table 5.3.

For unstiffened and ring stiffened cylinders and cones under external pressure

$$\beta_{cr} = 0.80$$
  
 $\Phi_{B} = \frac{2}{\beta_{cr}} = \frac{2}{0.8} = 2.5$ 

a) STEP 1 – Define all relevant loads and applicable load cases. The loads to be considered in the design shall include, but not be limited to, those given in Table 5.3.

The load case considered in this example includes external pressure of 14.7 psig

- b) STEP 2 Create a finite element model of the tower and apply all relevant loads and boundary conditions.
  - 1) Create a finite element model of the tower using shell elements, see Figure E5.4-2. Apply appropriate material properties and thicknesses to the components of the tower.

Note: All possible buckling mode shapes shall be considered in determining the minimum buckling load for the component. Care should be taken to ensure that simplification of the model does not result in exclusion of a critical buckling mode shape.



Figure E5.4-2 - Cross-Sectional View of Finite Shell Element Model

2) Generate finite element mesh. For this example, 4-node reduced integration shell elements (ABAQUS - S4R elements) were used, see Figure E5.4-3.



Figure E5.4-3 - Mesh of Tower Components

3) Create a static preload step and apply an external pressure load to the pressure boundary of the tower to be active during this step. Apply a displacement boundary condition to the bottom of the skirt  $(U_x = U_y = U_z = 0)$ 

The preload pressure applied=-14.7 psig

4) Create a buckling step and request a minimum of 3 eigenvalues to be solved. Apply a perturbation load to the pressure boundary of the tower to be active in this step, see Figure E5.4-4.

The perturbation pressure applied=-14.7 psig





5) Run analysis and review results.

Evaluate mode shapes for proper displacement direction and record calculated eigenvalues, see Figure E5.4-5 for first mode shape and Table E5.4-1 for calculated eigenvalues.



Figure E5.4-5 - First Mode Shape

Table E5.4-1 - Eigenvalue Results

Mode	Eigenvalues		
1	7.939		
2	7.940		
3	14.351		

6) Calculate buckling load or maximum allowable vacuum pressure (MAVP) using design factors and the following equation:

 $Buckling \ Load = \frac{("Dead"Loads) + Eigenvalue*("Live"Loads)}{Design \ Factor}$ 

"Dead" Loads = Total Load (Preload) before \*BUCKLE STEP

"Live" Loads = Incremental (Perturbation) Load in \*BUCKLE STEP

Buckling Load = 
$$\frac{-14.7 + 7.939(-14.7)}{2.5} = -52.6 \text{ psig}$$

7) Since the maximum allowable external pressure of -52.6 psig is less than the external design pressure of -14.7 psig, the structure is acceptable with respect to buckling under the design conditions.

# 5.5 Protection Against Failure from Cyclic Loading

### 5.5.1 Overview

A Fatigue evaluation shall be performed if a component is subject to cyclic operation.

### 5.5.2 Example E5.5.2 – Fatigue Screening

Evaluate the Vessel top head and shell region given in Example E5.2.1 in accordance with the fatigue screening methodology provided in paragraph 5.5.2.4. The cyclic loading design requirements given in the Users' Design Specification are provided below:

•	Operating pressure	=	380 psig at 125°F
•	Corrosion Allowance	=	0.125 inches
•	Cyclic Life Requirement	=	20000 full pressure cycles
•	Number of Shutdowns/Startups	=	20

a) STEP 1 - Determine a load history for vessel.

Per the User's Design Specification, a full internal pressure cycle is the only applicable event to be considered. The vessel internal pressure is expected to cycle 20000 times between 0 psig and the operating pressure, 380 psig. This cycling occurs at a constant temperature of 125°F.

b) STEP 2 - Determine the fatigue screening factors,  $C_1$  and  $C_2$ .

Per Table 5.10 of the Code, the applicable factors for this assessment are determined for a vessel of Integral Construction. As there are no nozzles or attachments in the knuckle region of a formed head, the factors are:

$$C_1 = 3$$
  
 $C_2 = 2$ 

c) STEP 3 - Determine the number of full range pressure cycles,  $N_{\Delta FP}$ , and check initial screening criteria.

 $N_{\rm \Delta FP}$  = Number of Full Range Pressure Cycles + Number of Shutdowns

$$N_{\Delta FP} = 20000 + 20 = 20020 \ Cycles$$

To check the initial screening criteria,  $N_{\Delta FP} \leq N(C_1S)$ , the number of allowable cycles for a pressure range of  $C_1S$  must be obtained from Annex 3.F, where S is the allowable stress at the design temperature and  $C_1$  is from Step 2. Based on this equation, the screening criteria will be more restrictive for the shell/head material as the higher allowable stress will yield a lower number of cycles. For information, the calculations will be shown for both materials.

The allowable number of cycles for each component is calculated using Equation (3.F.1):

 $N = (10)^X$ 

Where,

$$\begin{split} X &= \frac{C_1 + C_3 Y + C_5 Y^2 + C_7 Y^3 + C_9 Y^4 + C_{11} Y^5}{1 + C_2 Y + C_4 y^2 + C_6 Y^3 + C_8 Y^4 + C_{10} Y^5} \\ Y &= \left(\frac{S_{alt,k}}{C_{us}}\right) \left(\frac{E_{FC}}{E_T}\right) \end{split}$$

 $E_{\rm T}$  = the material modulus of elasticity at the cycle temperature

 $E_{\rm FC}$  = the modulus of elasticity used to establish the design fatigue curve

 $S_a$  = the stress amplitude, which is equal to  $C_1S_1$  for the screening criteria as determined above.

 $C_{us} = 1$  (units of stress are in *ksi*).

For the vessel materials of construction, the coefficients  $C_i$  and the modulus  $E_{FC}$  are for the smooth bar fatigue curve for carbon steel with cycle temperature below  $700^{\circ}F$  and  $\sigma_{uts} \leq 80 \ ksi$  and are listed in Table 3.F.1. The calculated allowable number of cycles is shown in Table E5.5.2-1 for each vessel region.

Component	Material	$E_{T}$ (ksi)	$E_{_{FC}}$ (ksi)	S (ksi)	$S_a$ (ksi) $C_1S$	Х	$N(C_1S)$
Nozzle	SA-105	2.910E+04	2.83E+04	23.30	69.90	3.234	1713
Head/Shell	SA-516-70N	2.878E+04	2.83E+04	24.55	73.65	3.156	1431

Table E5.5.2-1 - Allowable Number of Cycles,  $N(C_1S)$ 

Check to see if the criteria of  $N_{\Delta FP} \leq N(C_1S)$  is satisfied:

 $20020 \ cycles \le 1431 \ cycles$  False

The criterion is not satisfied, and therefore, a detailed fatigue analysis of the vessel is required. The subsequent screening steps for paragraph 5.5.2.4 of the code need not be evaluated.

#### 5.5.3 Example E5.5.3 – Elastic Stress Analysis, and Equivalent Stresses

Evaluate the vessel top head and shell base metal regions given in Example E5.5.2 in accordance with the fatigue methodology provided in paragraph 5.5.3. Note that the nozzle to head weld is machined and subjected to full volumetric examination and both ID and OD surfaces receive MT/PT and VT. The shell to head weld is in the as-welded condition and the OD surface receive the same inspection as above. The ID surface receives only full volumetric examination.

a) STEP 1 - Determine a load history for vessel.

Per the User's Design Specification (see Example E5.5.2), a full internal pressure cycle is the only applicable event to be considered. The vessel internal pressure will cycle between 0 psig and the operating pressure, 380 psig.

b) STEP 2 - Determine the individual stress-strain cycles and cyclic stress ranges.

Since the only event under consideration is a full internal pressure cycle, the applicable cyclic stress range is between the stress in the vessel at 0 psig internal pressure and the stress in the vessel at 380 psig.

c) STEP 3 - Determine the equivalent stress range for the cycle determined in STEP 2.

As thermal loads are not applicable,  $\Delta \sigma_{ij,k}^{LT} = 0$  and  $\Delta S_{LT,k} = 0$ . Thus, the equations for the component stress range and equivalent stress range reduce to the form of the equations below from paragraph 5.5.3:

$$\Delta \sigma_{ij,k} = {}^{m} \sigma_{ij,k} - {}^{n} \sigma_{ij,k}$$
$$\Delta S_{P,k} = \frac{1}{\sqrt{2}} \begin{bmatrix} \left( \Delta \sigma_{11,k} - \Delta \sigma_{22,k} \right)^{2} + \left( \Delta \sigma_{11,k} - \Delta \sigma_{33,k} \right)^{2} + \\ \left( \Delta \sigma_{22,k} - \Delta \sigma_{33,k} \right)^{2} + 6 \left( \Delta \sigma_{12,k}^{2} + \Delta \sigma_{13,k}^{2} + \Delta \sigma_{23,k}^{2} \right) \end{bmatrix}^{0.5}$$

Further, since the start point in the loading cycle is 0 psig internal pressure, the initial stress,  ${}^{m}\sigma_{ij,k}$ , is 0, and the component stress range equation reduces to  $\Delta\sigma_{ij,k} = -{}^{n}\sigma_{ij,k}$ , and the equivalent stress range equation becomes:

$$\Delta S_{P,k} = \frac{1}{\sqrt{2}} \begin{bmatrix} \binom{n}{\sigma_{22,k}} - \binom{n}{\sigma_{11,k}}^2 + \binom{n}{\sigma_{33,k}} - \binom{n}{\sigma_{11,k}}^2 + \\ \binom{n}{\sigma_{33,k}} - \binom{n}{\sigma_{22,k}}^2 + 6\binom{n}{\sigma_{12,k}} + \binom{n}{\sigma_{13,k}} + \binom{n}{\sigma_{23,k}} \end{bmatrix}^{0.5}$$

The above equation is the von Mises equivalent stress at the end point of the cycle. This stress can be calculated using an elastic finite element analysis of the vessel, applying the loads at the end point of the cycle ( $380 \ psig$  internal pressure).

If the effect of a weld is not accounted for in the numerical model, a fatigue strength reduction factor,  $K_f$  shall be included as per the equation below, and applied to all of the linearized membrane plus bending stress components, unless more detailed test data is available. Recommended values for  $K_f$  are provided in Table 5.11.

For the machined nozzle to head weld a  $K_j$  of 1.0 is applicable. For the as-welded head to shell weld,  $K_j$  values of 3.0 and 1.2 are applicable for the ID and OD surfaces, respectively.

$$\Delta \sigma_{ij,k} = K_f \left( {}^{m} \sigma_{ij,k}^{MB} - {}^{n} \sigma_{ij,k}^{MB} \right)$$

#### Finite Element Model:

The axisymmetric model was taken from Example E5.2.1 (see Figures 5.2.1-3 – 5.2.1-6). The pressure load was modified to the  $380 \ psig$  operating pressure and the nozzle thrust load was adjusted accordingly. The component geometry was modeled in the corroded condition.

#### Elastic Analysis Results:

A plot of the equivalent stress range is shown in Figure E5.5.3-1. The maximum stress locations are shown in the figure.



Figure E5.5.3-1 - Equivalent Stress Range (380 psig internal pressure)

The total stress at each of the base metal locations was read directly from the finite element output and is taken as the equivalent stress range  $\Delta S_{P,k}$  for the component. Linearized membrane plus bending equivalent stresses were determined to evaluate the need for a fatigue penalty factor and for application of an appropriate fatigue strength reduction factor (FSRF) for weld locations.

In order to determine  $K_{e,k}$ , the fatigue penalty factor,  $S_{PS}$  and  $\Delta S_{n,k}$  must first be calculated using the method of Paragraph 5.5.6.1.  $\Delta S_{n,k}$  is also used for the application of the FSRF, where  $\Delta S_{n,k}$  is the equivalent stress range of the linearized primary membrane, primary

#### PTB-3-2013

bending, and secondary stresses ( $P_L + P_B + Q$ ) for each location under evaluation. The linearization of stresses for calculation of these stresses was performed using the guidance of Annex 5.A. The Stress Classification Lines used are shown in Figure E5.5.3-2. Results are shown in Table E5.5.3-1.



Figure E5.5.3-2 - Stress Classification Lines for Calculation of  $\Delta S_{n,k}$ 

Component	SCL	Location	$\Delta S_{n,k}$ (ksi)
Nozzle	1	Nozzle ring at inside corner radius	27.060
Nozzle	2	Nozzle transition at OD blend radius	29.877
Head	3	Head knuckle ID	26.821
Shell	4	Head to shell weld ID	7.637
Shell	4	Head to shell weld OD	17.193
Shell	5	Base metal	19.938

Table E5.5.3-1 - Calculated Values for Primary plus	s Secondary Equivalent Stress Range
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#### PTB-3-2013

 STEP 4 - Determine the effective alternating equivalent stress amplitude for the cycle, using the stresses calculated in STEP 3.

As noted in STEP 3, there are no thermal effects (  $\Delta S_{LT,k} = 0$  ). Therefore, the alternating stress is calculated as follows:

$$S_{alt,k} = \frac{K_{e,k} \cdot \Delta S_{P,k}}{2}$$

 $S_{PS}$  is defined as  $\max[3S, 2S_y]$ , where *S* is the material allowable stress at the cycle temperature and  $S_y$  is the material yield strength at the cycle temperature (125°*F*). Table E5.5.3-2 lists the values for  $S_{PS}$  for each component.

Component	S(ksi)	$S_{y}(ksi)$	$S_{_{PS}}(ksi)$
Nozzle	23.30	34.90	69.90
Head	24.55	36.85	73.70
Shell	24.55	36.85	73.70

Table E5.5.3-2 - Calculated Values for  $S_{\rm PS}$ 

Comparing  $\Delta S_{n,k}$  to  $S_{PS}$  shows that  $\Delta S_{n,k} \leq S_{PS}$  for all components, and therefore  $K_{e,k} = 1.0$  in each case. The effective alternating equivalent stress amplitudes can then be calculated and are listed in Table E5.5.3-3. Note that the total stress is reported in Table E5.5.3-3 for locations that do not have a fatigue penalty factor applied.

Component	Location	$K_{f}$	$K_{e,k}$	$\Delta S_{P,k}$ (ksi)	$S_{alt,k}$ (ksi)
Nozzle	Nozzle ring at inside corner radius base metal	1.0	1.0	27.263	13.632
Nozzle	Nozzle transition at OD blend radius machined weld	1.0	1.0	31.799	15.900
Head	Head knuckle ID base metal	1.0	1.0	27.356	13.678
Shell	Head to shell weld ID as-welded	3.0	1.0	7.637	11.456
Shell	Head to shell weld OD as-welded	1.2	1.0	17.193	10.316
Shell	base metal	1.0	1.0	19.727	9.863

Table E5.5.3-3 - Component Effective Alternating Equivalent Stress Amplitude	s $S_{alt,k}$
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e) STEP 5 - Determine the permissible number of cycles,  $N_k$ , for the alternating equivalent stress computed in STEP 4, using the fatigue curves provided in Annex 3.F.

The allowable number of cycles for each component is calculated using Equation (3.F.1):

$$N = (10)^{\lambda}$$

Where,

$$X = \frac{C_1 + C_3 Y + C_5 Y^2 + C_7 Y^3 + C_9 Y^4 + C_{11} Y^5}{1 + C_2 Y + C_4 y^2 + C_6 Y^3 + C_8 Y^4 + C_{10} Y^5}$$
$$Y = \left(\frac{S_{alt,k}}{C_{us}}\right) \left(\frac{E_{FC}}{E_T}\right)$$

 $E_{\rm T}$  = the material modulus of elasticity at the cycle temperature

 $E_{\rm FC}$  = the modulus of elasticity used to establish the design fatigue curve

 $C_{us} = 1$  (units of stress are in ksi).

#### PTB-3-2013

For the vessel materials of construction, the coefficients  $C_i$  and the modulus  $E_{FC}$  are for the smooth bar fatigue curve for carbon steel with cycle temperature below  $700^{\circ}F$  and  $\sigma_{uts} \leq 80 \ ksi$  and are listed in Table 3.F.1. The calculated allowable number of cycles is shown in Table E5.5.3-4 for each component.

Component	Location	$E_T$ (ksi)	$E_{FC}$ (ksi)	$S_{alt,k}$ (ksi)	X	$N_k$ (cycles)
Nozzle	Nozzle ring at inside corner radius base metal	2.91E+04	2.83E+04	13.632	5.742	5.522E+05
Nozzle	Nozzle transition at OD blend radius machined weld	2.88E+04	2.83E+04	15.900	5.437	2.734E+05
Head	Head knuckle ID base metal	2.88E+04	2.83E+04	13.678	5.708	5.101E+05
Shell	Head to shell weld ID as-welded	2.88E+04	2.83E+04	11.456	6.877	7.527E+06
Shell	Head to shell weld OD as-welded	2.88E+04	2.83E+04	10.316	7.776	5.972E+07
Shell	base metal	2.88E+04	2.83E+04	9.863	8.156	1.433E+08

Table E5.5.3-4 - Allowable Number of Cycles,  $N_k$ 

f) STEP 6 – Determine the fatigue damage for the  $k^{th}$  cycle.

The actual number of repetitions of the  $k^{th}$  cycle  $(n_k)$  is set to the cyclic life requirement provided in the User's Design Specification, 20020 cycles. The fatigue damage for the  $k^{th}$  cycle is then calculated by:

$$D_{f,k} = \frac{n_k}{N_k}$$

This results in calculated fatigue damage for the limiting region (nozzle outside radius) of 0.073. Similarly, a fatigue damage of 0.039 is calculated for the head knuckle.

g) STEP 7 – Repeat STEPs 3 through 6 for each different stress range identified.

The only stress range identified was the full pressure cycle, so no additional ranges need be evaluated.

h) STEP 8 – Compute the accumulated damage using the following equation:

$$D_f = \sum_{k=1}^M D_{f,k} \le 1.0$$

There is only one stress range,  $D_f = D_{f,k}$  for each region evaluated and  $D_f \le 1.0$  for each region. Therefore, all locations meet the cyclic life requirement of the User's Design Specification for the vessel operating conditions.

### 5.5.4 Example E5.5.4 – Elastic-Plastic Stress Analysis, and Equivalent Strains

Evaluate the vessel top head and shell base metal regions given in Example E5.5.2 in accordance with the fatigue methodology provided in paragraph 5.5.4, using an internal pressure of 1000 psig.

a) STEP 1 - Determine a load history for vessel.

Per the User's Design Specification (see Example E5.5.2), a full internal pressure cycle is the only applicable event to be considered. To better illustrate the method and generate plastic strain in the model, the operating pressure will be taken as1000 psig for this example.

b) STEP 2 - Determine the individual stress-strain cycles.

The only event under consideration is a full internal pressure cycle, from 0 psig to the operating pressure of 1000 psig.

c) STEP 3 – Determine the loadings at the start and end point of the cycle determined in STEP 2.

At the start point of the cycle, no loads are applied. At the end point of the cycle, a load of  $1000 \ psig$  internal pressure is applied. The loading range for internal pressure is therefore  $1000 \ psi$ .

d) STEP 4 – Perform elastic-plastic stress analysis for the cycle determined in STEP 2.

The Twice-Yield Method (Paragraph 5.5.4.1(b)) will be used for this example. The loading at the start point of the cycle is taken as zero pressure, and the loading at the end point is the loading range determined in STEP 3; in this case 1000 psig internal pressure.

#### Finite Element Model:

The axisymmetric model was taken from Example E5.2.1 (see Figures E5.2.1-3 – E5.2.1-6). The pressure load was modified to 1000 psig (load at the cycle end point) and the nozzle thrust load was adjusted accordingly.

#### Material Properties:

The material properties in the model were also modified to include plastic behavior. Per paragraph 5.5.4.1(b), a stabilized cyclic stress-strain range curve was used to model the plasticity.

This form of the curve in Annex 3.D, however, does not match up to the typical input form for a finite-element program, which requires a specific yield stress to separate the elastic and elastic-plastic regions of material behavior. To approximate this yield stress and modify the form of the curve, an offset of plastic strain,  $\mathcal{E}_{offset}$ , is assumed and a line is drawn along the

elastic slope. The intersection of this line and the cyclic stress-strain curve is taken as the yield stress. This method is described by A. Kalnins in ASME PVP2008-61397, 2008. This point can also be calculated using the equation:

$$\sigma_{vield} = K_{css} (\varepsilon_{offset})^{n_{css}}$$

The cyclic stress-strain curve (strain amplitude versus stress amplitude) then takes the form:

$$\varepsilon_{ta} = \frac{\sigma_a}{E_{ya}} \text{ for } \sigma_a \le \sigma_{yield}$$
$$\varepsilon_{ta} = \frac{\sigma_a}{E_{ya}} + \left[\frac{\sigma_a}{K_{css}}\right]^{\frac{1}{n_{css}}} - \varepsilon_{offset} \text{ for } \sigma_a > \sigma_{yield}$$

For the *SA*-516, *Grade* 70, *Normalized* head and shell, the coefficients were obtained from Table 3.D.2 for Carbon Steel (0.75 in - base metal) since this matches the thickness most closely. Linear interpolation was performed between the values for  $70^{\circ}F$  and  $390^{\circ}F$  to approximate the values for  $125^{\circ}F$ . The values used were:

$$n_{CSS} = 0.1290$$
  
 $K_{CSS} = 109.0781$ 

Using an  $\mathcal{E}_{offset}$  of 2E-5 (fitting parameter per Annex 3.D), the yield stress for the SA-5-6-70N head and shell is calculated as 27.004 ksi.

The coefficients for the SA-105 nozzle were obtained in a similar fashion, but using the values for Carbon Steel (2.0 inch base metal) since it better matches the nozzle thickness. The values used were:

$$n_{CSS} = 0.1238$$
  
 $K_{CSS} = 99.0734$ 

Using an  $\mathcal{E}_{offset}$  of 2E-5, the yield stress for the nozzle material is calculated as 25.965 ksi.

For the Twice-Yield Method, the curves are then converted to the hysteresis loop stressstrain curve form (strain range versus stress range). The plastic strain range is related to the stress range by the following equation:

$$\varepsilon_{pr} = 2 \left[ \frac{\sigma_r}{2K_{css}} \right]^{\frac{1}{n_{css}}} - 2\varepsilon_{offset}$$

The stress range and plastic strain range used in the analysis are shown in Table E5.5.4-1 for the head and shell, and Table E5.5.4-2 for the nozzle.

Stress Range $(\mathit{psi})$	Plastic Strain Range
54007.5	0
60000.0	0.0000504
65000.0	0.0001281
70000.0	0.0002586
75000.0	0.0004697
80000.0	0.0008004
85000.0	0.0013045
90000.0	0.0020538
95000.0	0.0031436
100000.0	0.0046976
105000.0	0.0068748
110000.0	0.0098764

Table E5.5.4-1 - Stress range versus plastic strain range SA-516-70N Head and Shell

Table E5.5.4-2 - Stress range versus plastic strain range SA-105 Nozzle

Stress Range $(\mathit{psi})$	Plastic Strain Range
51929.8	0
60000.0	0.0000885
65000.0	0.0002054
70000.0	0.0004065
75000.0	0.0007398
80000.0	0.0012735
85000.0	0.0021037
90000.0	0.0033619
95000.0	0.0052256
100000.0	0.0079296
105000.0	0.0117806
110000.0	0.0171739

Plots of the Mises stress and equivalent plastic strain in the overall model are shown in Figure E5.5.4-1.



Figure E5.5.4-1 - Mises Stress (psi) and Equivalent Plastic Strain

e) STEP 5 – Calculate the Effective Strain Range for the cycle.

The effective strain range is calculated using Equation (5.41):

$$\Delta \varepsilon_{eff,k} = \frac{\Delta S_{P,k}}{E_{ya,k}} + \Delta \varepsilon_{peq,k}$$

As noted in Paragraph 5.5.4.2(e), because the range of loading is applied in a single step, the equivalent plastic strain range  $\Delta \varepsilon_{peq,k}$  and equivalent stress  $\Delta S_{P,k}$  can both be obtained directly from the analysis results ("PEEQ" and "S, Mises", variables, respectively, in ABAQUS). Using the elastic-plastic analysis results, the highest stress and strain regions were identified for each component being evaluated. Plots of the equivalent stress and equivalent plastic strain for these three areas (nozzle inside radius, head knuckle, and shell) are shown in Figure E5.5.4-2, Figure E5.5.4-3, and Figure E5.5.4-4. The effective strain range was calculated for multiple points in each area to ensure that the maximum (limiting) strain range was analyzed. Table E5.5.4-3 shows the limiting effective strain range for each component.



Figure E5.5.4-2 - Mises Stress (psi) and Equivalent Plastic Strain at Nozzle Inner Radius



Figure E5.5.4-3 - Mises Stress (psi) and Equivalent Plastic Strain at Head Knuckle



Figure E5.5.4-4 - Mises Stress (psi) and Equivalent Plastic Strain in Shell

Component	Location	$E_{ya,k}$ (ksi)	$\Delta S_{P,k}$ (ksi)	$\Delta m{arepsilon}_{peq,k}$	$\Delta arepsilon_{e\!f\!f,k}$
Nozzle	Inside Radius	2.91(10) <sup>4</sup>	65.938	2.431(10) <sup>-4</sup>	2.509(10) <sup>-3</sup>
Head	Head knuckle (at ID)	2.88(10) <sup>4</sup>	67.811	1.857(10) <sup>-4</sup>	2.540(10) <sup>-3</sup>
Shell	OD below head transition	2.88(10) <sup>4</sup>	51.982	0.0	1.805(10) <sup>-3</sup>

Table E5.5.4-3 - Component Limiting Effective Strain Ranges

f) STEP 6 – Determine the effective alternating equivalent stress for the cycle.

The effective alternating equivalent stress is calculated using Equation (5.43):

$$S_{alt,k} = \frac{E_{yf} \cdot \Delta \varepsilon_{eff,k}}{2}$$

For the vessel materials of construction, the modulus  $E_{yf}$  is for the smooth bar fatigue curve for carbon steel with cycle temperature below 700°*F* and  $\sigma_{uts} \leq 80 \ ksi$  and is given in Annex 3.F, Table 3.F.1. Table E5.5.4-4 shows the limiting effective alternating equivalent stress for each component.

Component	Location	$E_{yf}$ (ksi)	$\Delta arepsilon_{e\!f\!f,k}$	$S_{alt,k}$ (ksi)
Nozzle	Inside Radius	2.91(10) <sup>4</sup>	2.509(10) <sup>-3</sup>	36.506
Head	Head knuckle (at ID)	2.88(10) <sup>4</sup>	2.540(10) <sup>-3</sup>	36.576
Shell	OD below head transition	2.88(10) <sup>4</sup>	1.805(10) <sup>-3</sup>	25.991

Table E5.5.4-4 - Component Limiting Effective Alternating Equivalent Stress

g) STEP 7 - Determine the permissible number of cycles,  $N_k$ , for the alternating equivalent stress computed in STEP 6, using the fatigue curves provided in Annex 3.F.

The allowable number of cycles for each component is calculated using Equation (3.F.1):

The allowable number of cycles for each component is calculated using Equation (3.F.1):

$$N = (10)^{\lambda}$$

Where,

$$X = \frac{C_1 + C_3 Y + C_5 Y^2 + C_7 Y^3 + C_9 Y^4 + C_{11} Y^5}{1 + C_2 Y + C_4 y^2 + C_6 Y^3 + C_8 Y^4 + C_{10} Y^5}$$
$$Y = \left(\frac{S_{alt,k}}{C_{us}}\right) \left(\frac{E_{FC}}{E_T}\right)$$

 $E_{\rm T}$  = the material modulus of elasticity at the cycle temperature

 $E_{\rm FC}$  = the modulus of elasticity used to establish the design fatigue curve

 $C_{us} = 1$  (units of stress are in ksi).

For the vessel materials of construction, the coefficients  $C_i$  and the modulus  $E_{FC}$  are for the smooth bar fatigue curve for carbon steel with cycle temperature below  $700^{\circ}F$  and  $\sigma_{uts} \leq 80 \ ksi$  and are listed in Table 3.F.1. The calculated allowable number of cycles is shown in Table E5.5.4-5 for each component.

Component	Location	$E_T(ksi)$	$E_{FC}$ (ksi)	$S_a(ksi)$	X	$N_k$ (cycles)
Nozzle	Inside Radius	2.91(10) <sup>4</sup>	2.83(10) <sup>4</sup>	36.506	4.104	12712
Head	Head knuckle (at ID)	2.88(10) <sup>4</sup>	2.83(10) <sup>4</sup>	36.576	4.086	12181
Shell	OD below head transition	2.88(10) <sup>4</sup>	2.83(10) <sup>4</sup>	25.991	4.557	34658

Table E5.5.4-5 - Allowable Number of Cycles,  $N_k$ 

# h) STEP 8 – Determine the fatigue damage for the $k^{th}$ cycle.

The actual number of repetitions of the  $k^{th}$  cycle  $(n_k)$  is set to the cyclic life requirement provided in the User's Design Specification, 20020 cycles. The fatigue damage for the  $k^{th}$  cycle is then calculated by:

$$D_{f,k} = \frac{n_k}{N_k}$$

This results in the calculated fatigue damage for the nozzle inside radius of 1.575. Similarly, a fatigue damage of 1.644 is calculated for the head knuckle and a fatigue damage of 0.577 for the shell.

i) STEP 9 – Repeat STEPs 3 through 8 for each different stress range identified.

The only stress range identified was the full pressure cycle, so no additional ranges need be evaluated.

j) STEP 10 – Compute the accumulated damage using the following equation:

$$D_f = \sum_{k=1}^{M} D_{f,k} \le 1.0$$

Since there is only one stress range,  $D_f = D_{f,k}$  for each region evaluated. As  $D_f > 1.0$  in the nozzle and head, these components do not meet the specified design cycle life for a 1000 *psig* operating pressure assumed for this example. For a 1000 *psig* operating pressure, the design cycle life for the base metal would be limited by the knuckle at 12181 cycles.

### 5.5.5 Example E5.5.5 – Elastic Stress Analysis, and Structural Stress

Evaluate the vessel top head and shell weld regions gien in Example E5.5.2 in accordance with the fatigue methodology provided in paragraph 5.5.5. Note that the vessel is in non-corrosive service with respect to environmental effects upon the fatigue behavior. However, the analysis was conservatively based on the corroded dimensions as specified in the design requirements. Additionally, the operating pressure is conservatively assumed equal to the design pressure in this example.

a) STEP 1 - Determine a load history for vessel.

Per the User's Design Specification (see Example E5.5.2), a full internal pressure cycle is the only applicable event to be considered. The vessel internal pressure is expected to cycle 20020 times between  $0 \ psig$  and the operating pressure,  $420 \ psig$ . This includes shutdown and startup conditions.

b) STEP 2 - Determine the individual stress-strain cycles.

Since the only event under consideration is a full internal pressure cycle, the applicable cyclic stress range is between the stress in the vessel at 0 psig internal pressure and the stress in the vessel at 420 psig.

c) STEP 3 - Determine the elastically calculated membrane and bending stress normal to the assumed hypothetical crack plane at the start and end points for the cycle determined in Step 2. Using this data, calculate the membrane and bending stress ranges, and the maximum, minimum and mean stress.

Assume the ending time point, assume  ${}^{n}t$ , for the cycle under consideration is at 0 *psig* internal pressure; the equations 5.46 through 5.49 reduce as follows:

$$\Delta \sigma_{m,k}^{e} = {}^{m} \sigma_{m,k}^{e} - 0$$
  

$$\Delta \sigma_{b,k}^{e} = {}^{m} \sigma_{b,k}^{e} - 0$$
  

$$\sigma_{\max,k} = \left( {}^{m} \sigma_{m,k}^{e} + {}^{m} \sigma_{b,k}^{e} \right)$$
  

$$\sigma_{\min,k} = 0$$
  

$$\sigma_{mean,k} = \frac{\sigma_{\max,k} + \sigma_{\min,k}}{2}$$

Finite Element Model:

The axisymmetric model was taken from Example E5.5.1 (see Figures 5. E5.2.1-3 – E5.2.1-6). The pressure load for this assessment was  $420 \ psig$ . The model also included nozzle thrust load. This is also the same model used for the Example E5.5.4. It may be useful to compare the fatigue results for each example problem.

#### Elastic Analysis Results:

A plot of the stress (von Mises) for the model is shown in Figure E5.5.5-1. The three weld locations to be analyzed are indicated on the figure (shell to head, head to top nozzle, nozzle to flange weld). It is assumed, per this analysis method that the hypothetical crack planes initiate at the toe of the welds.



Figure E5.5.5-1 - Stress in Model (420 psig internal pressure)

For each of the theoretical crack locations, the through-wall linearized stress is obtained. From the linearized stress results the membrane and bending stress normal to the crack plane is read directly from the output. For each of the 4 locations, the membrane and bending stress normal to the crack plane are summarized in Table E5.5.5-1 below. Note that the minimum state of stress comprising the stress range is for a zero loading case since the component cycles pressure only.

Location	Membrane Stress (ksi)	Bending Stress (ksi)		
А	0.913	6.694		
В	13.925	20.973		
С	5.199	13.002		
D	10.777	11.019		

|--|

For each of these locations, calculate the maximum, minimum, and mean stress. The results are summarized in Table E5.5.5-2.

Location	Maximum (ksi)	Mean (ksi)	Minimum (ksi)
А	7.607	3.804	0
В	34.898	17.449	0
С	18.201	9.101	0
D	21.796	10.898	0

Table E5.5.5-2 - Maximum, Minimum, and Mean Stress Results

d) STEP 4 – Determine the elastically calculated structural stress range using equation 5.51.

Because the stress level at time  ${}^{n}t$  is zero, the stress range for each location is simply the sum of the membrane and bending stress as the factors  ${}^{n}\sigma_{m,k}^{e}$  and  ${}^{n}\sigma_{b,k}^{e}$  are equal to zero in equations 5.46 and 5.47. The stress range for each location is summarized in Table E5.5.5-3 using the following:

$$\Delta \sigma_k^e = \Delta \sigma_{m,k}^e + \Delta \sigma_{b,k}^e$$

e) STEP 5 – Determine the elastically calculated structural strain,  $\Delta \varepsilon_k^e$ , from the elastically calculated structural stress obtained in Step 4.

The elastic structural strain is calculated using equation 5.52:

$$\Delta \varepsilon_k^e = \frac{\Delta \sigma_k^e}{E_{ya,k}}$$

Where  $E_{ya,k} = 28.8E6 \, psi$  (modulus of elasticity for the material at  $125^{\circ}F$ ) is obtained from ASME Sec. II, Part D.

The corresponding local nonlinear structural stress and strain ranges,  $\Delta \sigma_k$  and  $\Delta \varepsilon_k$ , respectively, are determined by simultaneously solving Neuber's Rule, Equation (5.53), and a model for the material hysteresis loop stress-strain curve given by Equation (5.54).

$$\Delta \varepsilon_k \cdot \Delta \sigma_k = \Delta \varepsilon_k^e \cdot \Delta \sigma_k^e$$
$$\Delta \varepsilon_k = \frac{\Delta \sigma_k}{E_{ya,k}} + 2 \left(\frac{\Delta \sigma_k}{2K_{css}}\right)^{\frac{1}{n_{css}}}$$

#### PTB-3-2013

The values for  $K_{css}$  and  $n_{css}$  are obtained from Table 3.D.2 in Annex 3.D. These values used were interpolated from the table data based on temperature. The values used are listed in Table E5.5.5-3 below.

	Elastic Ranges		Hysteresis Loop Stress-Strain Factors		Neuber Corrected Stress Range	
Location	$\Delta \sigma^e_k$ (ksi)	$\Delta arepsilon_k^e$	K <sub>css</sub> (ksi)	n <sub>ess</sub>	$\Delta \sigma_{\scriptscriptstyle k}$ (ksi)	
А	7.6070	2.641E-04	109.078	0.1290	7.6070	
В	34.8980	1.212E-03	109.078	0.1290	34.878	
С	18.2010	6.320E-04	109.078	0.1290	18.2010	
D	21.7960	7.568E-04	109.078	0.1290	21.7950	

Table E5.5.5-3 - Stress Range Summary

f) STEP 6 – Compute the equivalent structural stress range parameter for each location A through F.

The out-of-plane shear stresses for these cases are zero; therefore the modified procedure for this step, outlined in 5.5.5.3, need not be employed.

$$\Delta S_{ess,k} = \frac{\Delta \sigma_k}{t_{ess}^{\left(\frac{2-m_{ss}}{2m_{ss}}\right)} \cdot I^{\frac{1}{m_{ss}}} \cdot f_{M,k}}$$

Where,

$$m_{ss} = 3.6$$
  

$$t_{ess} = 16 mm (0.625 in.) for$$
  

$$t_{ess} = t for$$
  

$$t_{ess} = 150 mm (6 in.) for$$
  

$$I^{\frac{1}{m_{ss}}} = \frac{1.23 - 0.364R_{b,k} - 0.17R_{b,k}^2}{1.007 - 0.306R_{b,k} - 0.178R_{b,k}^2}$$

 $t \le 16 mm (0.625 in.)$  16 mm (0.625 in.) < t < 150 mm (6 in.) $t \ge 150 mm (6 in.)$


The results for this step are summarized in Table E5.5.5-4 for each location. As the minimum stress for the cycle is base on zero *psig* applied pressure;  $\sigma_{\min,k}$  is equal to zero, and therefore  $R_k$  is also equal to zero for all locations. Similarly,  $f_{M,k}$  is equal to 1.0 based on the  $R_k$  value. These values will not be shown in the results table.

Location	t <sub>ess</sub> (in)	$R_{bk}$	$I^{\frac{1}{m_{ss}}}$	$\Delta S_{ess,k}$ (ksi)
A	2.000	0.880	1.297	6.842
В	0.938	0.601	1.252	27.470
С	0.625	0.714	1.266	12.950
D	0.813	0.506	1.243	16.751

Table E5.5.5-4 - Results for Step 6

g) STEP 7 – Determine the permissible number of cycles,  $N_k$ , based on the equivalent structural stress range parameter computed in Step 6.

The welded joint fatigue curves can be developed using Annex 3.F, paragraph 3.F.2. In this example analysis, the carbon steel material of construction has curves limited to temperatures below  $700^{\circ}F$ . As our design temperature is  $125^{\circ}F$ , this criteria is met and the curves developed by this Annex may be used.

The design number of allowable design cycles, N, can be computed from the following equations. For this example calculation, it is assume that burr grinding in accordance with Part 6, Figure 6.2 is employed. For these equations  $\Delta S_{range} = \Delta S_{ess,k}$  from above.

$$N = \frac{f_I}{f_E} \left( \frac{f_{MT} \cdot C}{\Delta S_{ess,k}} \right)^{\frac{1}{h}}$$

Where,

• The improvement factor is:

$$f_{I} = 1.0 + 2.5(10)^{q} \qquad (for \ burr \ grinding)$$
$$q = -0.0016 \left(\frac{\Delta S_{ess,k}}{C_{us}}\right)^{1.6}$$

• The environmental factor is define as per 3.F.2.2 c); Since this is non-corrosive service, the default value of 4.0 is not applicable.

$$f_{E} = 1.0$$

• The material correction factor is calculated; In this case of this analysis, the average cycle temperature is very close to  $70^{\circ}F$ , therefore  $E_T$  will be equal to  $E_{ACS}$ , therefore:

$$f_{MT} = \frac{E_T}{E_{ACS}}$$
$$f_{MT} = 1$$

• The factors, *C* and *h*, are obtained from Table 3.F.11 for the  $-3\sigma$  curve.

C = 818.3h = 0.31950

For each of the locations, the allowable design cycles are calculated and summarized in Table E5.5.5-5.

Location	$\Delta S_{ess,k}$ $(ksi)$	q	$f_I$	N Cycles	$D_{f,k}$
A	6.842	-0.0347	3.3080	10535490	0.0019
В	27.470	-0.3208	2.1943	90141	0.2221
С	12.950	N/A	N/A	432377	0.0463
D	16.751	N/A	N/A	193177	0.1036

Table E5.5.5-5 - Results for Step 7

h) STEP 8 – Determine the fatigue damage for the cycle history.

The cycle history specified by the user design specification from Step 1 above, had a requirement for 20020 full pressure cycles (therefore  $n_k = 20020$ ). The fatigue damage factor can be calculated using the following equation.

$$D_{f,k} = \frac{n_k}{N_k}$$

By inspection, it is evident that location B is the controlling welded location, for those analyzed, as the allowable cycles for the stress range calculated is 90141 cycles. The damage factor is summarized for each location in Table E5.5.5-5 above.

i) STEPS 9-11 - Assessment of steps 9-11 are not required as there are no other stress ranges other than the 0 to 420 *psig* operational cycle.

#### 5.5.6 Example E5.5.6 – Protection Against Ratcheting Using Elastic Stress Analysis

Evaluate the vessel top head and shell region given in Example Problem E5.5.2 for compliance with respect to the elastic ratcheting criteria provided in paragraph 5.5.6.

a) STEP 1 – Evaluate the primary plus secondary equivalent stress range,  $\Delta S_{n,k}$ , for each component.

Note: The finite element model and elastic analysis for the non-weld locations are taken from Example 5.5.3 (See Figure 5.5.3-1 for analysis stress results). For the welded locations, the stress results are taken from Example 5.5.5.

 $\Delta S_{nk}$  is the equivalent stress range of the linearized combination of primary membrane, primary

bending, and secondary stresses ( $P_L + P_B + Q$ ) for each location under evaluation. The stress linearization is done as per Annex 5.A of the Code. The Stress Classification Lines used to calculate these values for the base metal locations are shown in Figure E5.5.6-1, and for the weld locations in Figure E5.5.6-2. Results are shown in Table E5.5.6-1.



Figure E5.5.6-1 - Stress Classification Lines for Calculation of  $\Delta S_{n,k}$  (Base Metal Locations)



Figure E5.5.6-2 - Stress Classification Lines for Calculation of  $\Delta S_{n,k}$  (Weld Locations)

Component Location		Туре	$\Delta S_{n,k}$ (ksi)
Nozzle	Inside Radius	Base Metal	27.060
Head	Head knuckle (at <i>ID</i> )	Base Metal	26.821
Shell	<i>OD</i> below head transition	Base Metal	19.938
Nozzle	A	Weld	16.41
Head	В	Weld	29.80
Nozzle	С	Weld	8.40
Nozzle	D	Weld	10.47
Shell	E	Weld	16.94
Shell	F	Weld	18.38

Table E5.5.6-1 - Stress Range Comparison

b) STEP 2 – Evaluate the limit for the equivalent stress range,  $\,S_{\rm PS}\,.$ 

 $S_{PS}$  is defined as  $\max[3S, 2S_y]$ , where *S* is the material allowable stress at the cycle temperature and  $S_y$  is the material yield strength at the cycle temperature  $(125^{\circ}F)$ . Table E5.5.6-2 lists the values for  $S_{PS}$  for each component.

Component	Location	Туре	S (ksi)	$S_{y}(ksi)$	$S_{PS}$ (ksi)
Nozzle	Inside Radius	Base Metal	23.3000	34.9000	69.9000
Head	Head knuckle (at $ID$ )	Base Metal	24.5500	36.8500	73.7000
Shell	<i>OD</i> below head transition	Base Metal	24.5500	36.8500	73.7000
Nozzle	A	Weld	23.3000	34.9000	69.9000
Head	В	Weld	24.5500	36.8500	73.7000
Nozzle	С	Weld	23.3000	34.9000	69.9000
Nozzle	D	Weld	23.3000	34.9000	69.9000
Shell	E	Weld	24.5500	36.8500	73.7000
Shell	F	Weld	24.5500	36.8500	73.7000

Table E5.5.6-2 - Calculated Values for  $S_{PS}$ 

c) STEP 3 – Compare equivalent stress ranges calculated in STEP 1 to the limits calculated in STEP 2.

Results of the comparison are shown in E5.5.6-3. Since  $\Delta S_{n,k} \leq S_{PS}$  for all locations, the elastic criteria for protection against racheting is satisfied.

Component	Location	Туре	$\Delta S_{n,k}$ (ksi)	$S_{_{PS}}(ksi)$	$\Delta S_{n,k} \leq S_{PS}$
Nozzle	Inside Radius	Base Metal	27.060	69.9000	Yes
Head	Head knuckle (at ID)	Base Metal	26.821	73.7000	Yes
Shell	OD below head transition	Base Metal	19.938	73.7000	Yes
Nozzle	A	Weld	16.41	69.9000	Yes
Head	В	Weld	29.80	73.7000	Yes
Nozzle	С	Weld	8.40	69.9000	Yes
Nozzle	D	Weld	10.47	69.9000	Yes
Shell	E	Weld	16.94	73.7000	Yes
Shell	F	Weld	18.38	73.7000	Yes

# Table E5.5.6-3 - Stress Range Comparison

### 5.5.7 Example E5.5.7 – Protection Against Ratcheting Using Elastic-Plastic Stress Analysis

Evaluate the vessel top head and shell region given in Example Problem E5.5.2 for compliance with respect to the elastic-plastic ratcheting criteria provided in paragraph 5.5.7.

a) STEP 1 – Develop a numerical model of the vessel components.

The axisymmetric finite element model geometry was taken from Example E5.2.1 (see Figures E5.2.1-3-5.2.1-6).

b) STEP 2 – Define all relevant loads and applicable load cases.

Per the User's Design Specification (see Example E5.5.2), a full internal pressure cycle is the only applicable event to be considered. The vessel internal pressure will cycle between 0 psig and the operating pressure, 380 psig.

The pressure load in the finite element model was modified to the 380 *psig* operating pressure and the nozzle thrust load was adjusted accordingly.

c) STEP 3 – Modify the material model to elastic-perfectly plastic behavior. The effects of nonlinear geometry shall be considered.

For each material, the yield strength defining the plastic limit was taken as the yield strength at temperature per Annex 3.D. For SA-105 (nozzle) and SA-516-70N (head, shell) the yield strengths were taken as  $34.90 \ ksi$  and  $36.85 \ ksi$ , respectively. The material section of the ABAQUS input deck is shown below.

```
** MATERIALS
**
*Material, name=SA105
*Elastic
2.91e+07, 0.3
*Plastic
34900.0,0.
*Material, name=SA516-70N
*Elastic
2.878e+07, 0.3
*Plastic
36850.0,0.0
**
```

The effects of nonlinear geometry were included in the analysis by setting ABAQUS keyword nlgeom=yes.

d) STEP 4 – Perform an elastic-plastic analysis using the applicable loading from STEP 2.

The elastic-plastic analysis was performed using the 380 *psig* internal pressure load from STEP 2 and the elastic-perfectly plastic material model from STEP 3. A plot of the Mises stress and equivalent plastic strain under load are shown in Figure E5.5.7-1 below.

S, Mises (Avg: 75%) + 3.253e+04 + 2.2982e+04 + 4.2.40e+04 + 4.2.40e+04 + 1.627e+04 + 1.627e+04 + 4.1369e+04 + 8.136e+00 + 8.138e+03 + 2.719e+03 + 9.084e+00	PEMAG (Avg: 75%) +0.000e+00 +0.000e+00 +0.000e+00 +0.000e+00 +0.000e+00 +0.000e+00 +0.000e+00 +0.000e+00 +0.000e+00 +0.000e+00 +0.000e+00 +0.000e+00
ODB: Part_S_EL-PL_Racheting.odb Abaqus/Standard Version 6.7-1 Thu Sep 04 14-0:39 Eastern Daylight Tim	ODB: Part_5_EL-PL_Racheting.odb Abaqus/Standard Version 6.7-1 Thu Sep 04 14:0.39 Eastern Daylight Tim
Y Step: Cyde_l_Load	Y Step: Cycle_1_Load
Incernent 1: Step Time = 1.000	Increment :: Step Time = 1.000
2X Deformed Var: U Deformation Scale Factor: +1.000e+00	2X Deformed Var: U Deformation Scale Factor: +1.000e+00

Figure E5.5.7-1 - Stress and Plastic Strain in Model (380 *psig* internal pressure, 1st cycle)

e) STEP 5 – Continue the elastic-plastic analysis for a minimum of three complete cycles and evaluate using the ratcheting criteria in paragraph 5.5.7.2(e).

Additional loading and unloading steps were added to the finite element analysis until three complete cycles were modeled. Figure E5.5.7-2 shows the Mises stress and plastic strain in the model following the completion of the third cycle.



Figure E5.5.7-2 - Stress and Plastic Strain in Model (End of 3<sup>rd</sup> Cycle)

It can be seen from Figure E5.5.7-2 that zero plastic strains have been incurred in the nozzle, head, or shell. Thus, these components meet the condition detailed in 5.5.7.2(e)(1) and the ratcheting criteria are satisfied.

The vessel components, therefore, are acceptable per the elastic-plastic ratcheting criteria for an operating pressure cycle between  $0 \ psig$  and  $380 \ psig$ .

# PART 6

# **FABRICATION REQUIREMENTS**

# **PART CONTENTS**

### 6.1 Example E6.1 – Postweld Heat Treatment of a Pressure Vessel

Establish the postweld heat treatment (PWHT) requirements for a process tower considering the following design conditions. All Category A and B joints are Type 1 butt welds and have been 100% radiographically examined.

Vessel Data:

•	Material	=	SA-537, Class 1
•	Design Conditions	=	1650 <i>psig</i> @600°F
•	Liquid Head	=	60 <i>ft</i>
•	Liquid Specific Gravity	=	0.89
•	Inside Diameter	=	96.0 in
•	Corrosion Allowance	=	0.125 in
•	Allowable Stress	=	25700 psi
•	P Number and Group	=	P-No. 1, Group 2
•	Weld Joint Efficiency	=	1.0
•	Tangent-to-Tangent Vessel Length	=	80 <i>ft</i>
•	Top and Bottom Heads	=	Hemispherical

Evaluate the requirements of PWHT per paragraph 6.4.

The design pressure used to establish the wall thickness for the bottom head and cylindrical shell section must be adjusted for the liquid head in accordance with Part 4, paragraph 4.1.5.2.a.

Adjusted pressure for the cylindrical shell:

Design Pressure = Specified Design Pressure + 
$$\gamma gh$$
  
Design Pressure =  $1650 + \frac{0.89(62.4)(56)}{144} = 1671.597 psig$ 

Adjusted pressure for the bottom hemispherical head:

Design Pressure = Specified Design Pressure +  $\gamma gh$ Design Pressure =  $1650 + \frac{0.89(62.4)(60)}{144} = 1673.140 psig$ 

In accordance with Part 4, paragraph 4.3.5, determine the required thickness of the top head.

$$D = 96.0 + 2(Corrosion \ Allowance) = 96.0 + 2(0.125) = 96.25 \ in$$
$$t = \frac{D}{2} \left( \exp\left[\frac{0.5P}{SE}\right] - 1 \right) = \frac{96.25}{2} \left( \exp\left[\frac{0.5(1650)}{25700(1.0)}\right] - 1 \right) = 1.5699 \ in$$
$$t = 1.5699 + Corrosion \ Allowance = 1.5699 + 0.125 = 1.6949 \ in$$

The required thickness of the top head is 1.6949 in.

In accordance with Part 4, paragraph 4.3.5, determine the required thickness of the bottom head, including the liquid head static pressure.

$$D = 96.0 + 2(Corrosion \ Allowance) = 96.0 + 2(0.125) = 96.25 \ in$$
$$t = \frac{D}{2} \left( \exp\left[\frac{0.5P}{SE}\right] - 1 \right) = \frac{96.25}{2} \left( \exp\left[\frac{0.5(1673.140)}{25700(1.0)}\right] - 1 \right) = 1.5923 \ in$$
$$t = 1.5923 + Corrosion \ Allowance = 1.5923 + 0.125 = 1.7173 \ in$$

The required thickness of the bottom head is 1.7173 in.

In accordance with Part 4, paragraph 4.3.3, determine the required thickness of the cylindrical shell, including the liquid head static pressure.

$$D = 96.0 + 2(Corrosion \ Allowance) = 96.0 + 2(0.125) = 96.25 \ in$$
$$t = \frac{D}{2} \left( \exp\left[\frac{P}{SE}\right] - 1 \right) = \frac{96.25}{2} \left( \exp\left[\frac{1671.597}{25700(1.0)}\right] - 1 \right) = 3.2342 \ in$$
$$t = 3.2373 + Corrosion \ Allowance = 3.2342 + 0.125 = 3.3592 \ in$$

The required thickness of the cylindrical shell is 3.3592 in.

Required Thickness Summary:

Top Head = 1.6949 in

Bottom Head = 1.7173 in

Cylindrical Shell = 3.3592 in

The requirements for postweld heat treatment are found in paragraph 6.4.2. Material specification SA-537 Class 1 is a P-No. 1, Group No. 2, material. Therefore, in accordance with paragraph 6.4.2.2.e, the PWHT requirements are provided in Table 6.8. The definition of nominal thickness governing PWHT is provided in paragraph 6.4.2.7. For pressure vessels or parts of pressure vessels being postweld heat treated in a furnace charge, the nominal thickness is the greatest weld thickness in any vessel or vessel part which has not previously been postweld heat treated. Therefore, the governing nominal thickness is that of the cylindrical shell, 3.3592 in.

The procedures for postweld heat treatment are found in paragraph 6.4.3. PWHT of the vessel will be performed by heating the vessel as a whole in a closed furnace in accordance with paragraph 6.4.3.2.

Based on Table 6.8, see Table E6.1 of this example, PWHT is mandatory due to the governing nominal thickness of 3.3592 in. The holding temperature and time based on the nominal thickness within the range of  $2 in < t_n < 5 in$  is 1100°F for 2 hours plus 15 minutes for each additional inch over 2 inches, respectively. For the vessel in question, the holding time is calculated as follows:

Holding time = 120 min + 
$$\left(\frac{15 \text{ min}}{\text{in}}\right)$$
 (3.3592 in - 2 in) = 140 min

The requirements for operation of PWHT are provided in paragraph 6.4.4. The operation of postweld heat treatment shall be carried out by one of the procedures given in paragraph 6.4.3 in accordance with the following requirements.

- a) When post weld heat treatment is performed in a furnace (see paragraph 6.4.3.2), the temperature of the furnace shall not exceed 800°F at the time the vessel or part is placed in it.
- b) Above 800°F, the rate of heating shall be not more than 400°F/hr divided by the maximum metal thickness of the shell or head plate in inches, but in no case more than 400°F/hr, and in no case need it be less than 100°F/hr. During the heating period there shall not be a greater variation in temperature throughout the portion of the vessel being heated than 250°F within any 15 ft interval of length.

*Maximum Heating Rate* = 
$$\frac{400^{\circ}F/hr}{3.3592 \text{ in}} = 119^{\circ}F/hr$$

- c) The vessel or vessel part shall be held at or above the temperature specified in paragraph 6.4.2 for the period of time specified in this paragraph. During the holding period, there shall not be a difference greater than 150°F between the highest and lowest temperatures throughout the portion of the vessel being heated, except where the range is further limited in paragraph 6.4.2.
- d) When post weld heat treatment is performed in a furnace (see paragraph 6.4.3.2), during the heating and holding periods, the furnace atmosphere shall be so controlled as to avoid excessive oxidation of the surface of the vessel. The furnace shall be of such design as to prevent direct impingement of the flame on the vessel.
- e) Above 800°F, cooling shall be done at a rate not greater than 500°F/hr divided by the maximum metal thickness of the shell or head plate in inches, but in no case need it be less than 100°F/hr. From 800°F, the vessel may be cooled in still air.

Maximum Cooling Rate = 
$$\frac{500^{\circ}F/hr}{3.3592 \text{ in}} = 149^{\circ}F/hr$$

	PWHT Requirements	Holding Temperature and Time Based On The Nominal Thickness		
a)	PWHT is mandatory for the following conditions:	<u>SI Units</u>		
	1) For welded joints over 38mm (1 1/2 in.) nominal thickness.	• For $t_n \le 50 \text{ mm}$ : 595°C, 0.04 hr/mm		
	2) For welded joints over 32 mm (1 1/4 in.) through 38 mm (1 1/2 in.)	15 minutes minimum		
	nominal thickness unless a 95°C (200°F) minimum preheat is applied during welding	• For: 50 $mm < t_n \le 125 mm 595^{\circ}$ C,		
b)	When it is impractical to perform PWHT at the temperatures specified in this table, it is permissible to carry out PWHT at lower temperatures	hr plus 0.6 minutes for e additional mm over 50 mm		
	for longer periods of time in accordance with Table 6.16.	• For $t_n > 125 \ mm$ : 595°C, 2 hr plu		
		0.6minutes for each additional m over 50 mm		
		US Customary Units		
		• For $t_n \le 2$ in : 1100°F, 1 hr/in, 1 minutes minimum		
		• For $2 in < t_n \le 5 in : 1100^{\circ}F$ , 2		
		plus 15 minutes for each addition inch over 2 in.		
		• For $t_n > 5$ in : 1100°F, 2 hr plus 1		
		minutes for each additional inch over		

# Table E6.1 - Design Loads and Load Combinations from VIII-2

#### 6.2 Example E6.2 – Out-of-Roundness of a Cylindrical Forged Vessel

A vessel is being fabricated using forged cylindrical shell segments. During fabrication, tolerances where checked and it was noted that out-of-roundness of one of the cylindrical shell segments is present that exceed tolerance limits specified in paragraph 6.1.2.7. In order to establish a plan of action, it was decided to use the provisions in Part 6 that permit a reduced permissible operating pressure be determined for cylindrical shells with general out-of-roundness characterized by a major and minor diameter. Establish the reduced permissible operating pressure requirements considering the following design conditions.

Vessel Data:

•	Material	=	SA-372, Grade C
•	Design Conditions	=	2800 psig@400°F
•	Inside Diameter	=	112.0 in
•	Corrosion Allowance	=	0.125 in
•	Allowable Stress	=	27600 psi
•	Weld Joint Efficiency	=	1.0
•	Modulus of Elasticity	=	27.9E + 06  psi

Evaluate the special requirements for forged fabrication per paragraph 6.7.4

In accordance with Part 4, paragraph 4.3.3, determine the required thickness of the cylindrical shell.

$$D = 96.0 + 2(Corrosion \ Allowance) = 112.0 + 2(0.125) = 112.25 \ in$$
$$t = \frac{D}{2} \left( \exp\left[\frac{P}{SE}\right] - 1 \right) = \frac{112.25}{2} \left( \exp\left[\frac{2800}{27600(1.0)}\right] - 1 \right) = 5.9927 \ in$$
$$t = 5.9927 + Corrosion \ Allowance = 5.9927 + 0.125 = 6.1177 \ in$$

The required thickness of the cylindrical shell is 6.1177 in; therefore a forging with a wall thickness of 6.25 in will be used.

During fabrication of a section of the cylindrical shell with a nominal inside diameter of 112.0 *in*, the following tolerance readings were taken.

Maximum Inside Diameter = 113.0 in

Minimum Inside Diameter = 110.0 in

The shell tolerance limits provided in paragraph 6.1.2.7.a and paragraph 4.3.2, state that the difference between the maximum and minimum inside diameters at any cross section shall not exceed 1% of the nominal diameter at the cross section under consideration.

$$\frac{Max \ Diameter - Min \ Diameter}{Nominal \ Diameter} = \frac{113.0 - 110.0}{112.0} = 2.7\%$$

In accordance with paragraph 6.7.4.2.b, if the out-of-roundness exceeds the limit in paragraph 6.1.2.7.a and the condition cannot be corrected, then the forging shall be rejected, except that if the out-of-roundness does not exceed 3%, the forging may be certified for a reduced pressure,  $P^*$ , calculated using Equations (6.1) and (6.2). The measurements used in these equations shall be corrected for the specified corrosion allowance.

With,

$$\{S_b = 14961.4 \ psi\} \ge \{0.25S = 0.25(27600) = 6900 \ psi\}$$
$$P^* = P\left(\frac{1.25}{S_b/S + 1}\right) = 2800\left(\frac{1.25}{(14961.4/27600) + 1}\right) = 2269.7 \ psi$$

Where,

$$S_{b} = \frac{1.5PR_{1}t(D_{1}-D_{2})}{t^{3}+3\left(\frac{P}{E_{y}}\right)(R_{1}R_{a}^{2})} = \frac{1.5(2800)(55.875)(6.125)(113.25-110.25)}{(6.125)^{3}+3\left(\frac{2800}{27.9E+06}\right)(55.875)(58.9375)^{2}}$$
  
$$S_{b} = 14961.4 \ psi$$

And,

$$\begin{split} D_1 &= 113.0 + 2 (Corrosion \ Allowance) = 113.0 + 2 (0.125) = 113.25 \ in \\ D_2 &= 110.0 + 2 (Corrosion \ Allowance) = 110.0 + 2 (0.125) = 110.25 \ in \\ t &= 6.25 - Corrosion \ Allowance = 6.25 - 0.125 = 6.125 \ in \\ R_1 &= \frac{D_1 + D_2}{4} = \frac{113.25 + 110.25}{4} = 55.875 \ in \\ R_a &= R_1 + \frac{t}{2} = 55.875 + \frac{6.125}{2} = 58.9375 \ in \end{split}$$

Therefore, with the current out-of-roundness in place, the maximum operating pressure of the vessel would be limited to  $2269.7 \ psi$ , which represents a 19% reduction in pressure. This is unacceptable for the planned operation of the vessel. It is determined that the condition cannot be corrected; therefore, the forging shall be rejected and a new cylindrical forging shall be fabricated.

# PART 7

# INSPECTION AND EXAMINATION REQUIREMENTS

# **PART CONTENTS**

The subject of Examination Groups is covered in Part 7, paragraph 7.4.2. The assignment of a welded joint to a particular Examination Group is dependent on the manufacturing complexity of the material, the maximum thickness, the welding process, and the selected weld joint efficiency. The Examination Groups are defined in Table 7.1. There are three Examination Groups defined within VIII-2, which are then further subdivided in sub-groups "a" and "b" to reflect the crack sensitivity of the material. The required method and extent of nondestructive examination is defined Table 7.2 based on the Examination Group, the joint category, and the corresponding permissible joint type.

Also introduced in Part 7, paragraph 7.4.2 is the concept of the governing welded joint, which is defined as that welded joint within a given vessel section (such as a shell course or vessel head) that, as a result of the selected joint efficiency, determines the thickness of that vessel section. For example, in a given shell course, the longitudinal weld seam would control the thickness of that shell course in most cases and would be the governing welded joint. However, if the component was subject to significant longitudinal stress from wind, seismic, or other external loadings such that the circumferential seam dictated the thickness of the shell course, then it would be the governing welded joint.

Since it is possible for a pressure vessel to have more than one governing welded joint, it is also possible to have a pressure vessel with multiple Examination Groups. The requirements for the case of a single vessel containing a combination of Examination Groups are covered in Part 7, paragraph 7.4.2.2.b. In each vessel section, the Examination Group of the governing welded joint shall be applied to all welds within that vessel section, including any nozzle attachment welds. A weld that joins two welded vessel sections assigned to different Examination Groups shall be assigned to that Group that requires the greater level of examination. Finally, a weld that joins a welded section to a seamless section, or a weld connecting two seamless sections, is assigned to an Examination Group based on the available thickness (the available thickness is defined as the thickness at the weld, less tolerances and corrosion allowance). If the ratio of available thickness to the minimum required thickness in a given vessel section is greater than 1.18, then Examination Group 3 may be used for that section. Otherwise, the Examination Group is assigned in accordance with the criteria in Table 7.1. The significance of the 1.18 value is that it represents a ratio of 1/0.85, the ratio of the joint efficiencies between Examination Groups 2 and 3.

#### 7.1 Example E7.1 – NDE Requirements: Vessel with One Examination Group Designation

A plant engineer is tasked with developing a design specification for a new pressure vessel that is to be constructed in accordance with VIII-2. Based on the anticipated design data, materials of construction, and welding process, the engineer selects Examination Group 1a to set the joint efficiency and NDE requirements for the entire vessel. A sketch of the vessel showing nozzle sizes, orientation, and weld seams is shown in Figure E7.1.

To assist with fabrication and inspection of the vessel, the engineer developed a table to summarize the NDE requirements applicable to each welded joint of the vessel based on the Examination Group selected. Table E7.1 is a sub-set of the original table and only addresses the weld joint identifiers referenced on the vessel sketch in Figure E7.1.



Figure E7.1 - Vessel Sketch

Examination G	Group		1a				
P Number/Gro	oup Number		P-1,Gr-2	Governing We	ld Joint Thickne	SS	2.75 in
Welding Process		Unrestricted	Weld Joint Efficiency		1.0		
			Non Destructive Examination				
Joint Identifier	Joint Category	Joint Type	Table - Detail	Volur	netric	Sur	face
			2010	Туре	Extent	Туре	Extent
1	A	1	4.2.4 - 1	RT	100%	PT	10%
2	В	1	4.2.4 - 1	RT	100%	PT	10%
3	В	1	4.2.4 - 1	RT	100%	PT	10%
4	A	1	4.2.4 - 1	UT	100%	PT	10%
5	А	1	4.2.4 - 1	UT	100%	PT	10%
6	А	1	4.2.4 - 1	RT	100%	PT	10%
7	А	1	4.2.4 - 1	RT	100%	МТ	10%
8	В	1	4.2.4 - 1	RT	100%	МТ	10%
9	В	1	4.2.4 - 1	RT	100%	МТ	10%
10	D	7	4.2.10 - 4	UT	100%	PT	10%
11	С	3	4.2.9 - 6	RT	100%	PT	10%
12	D	7	4.2.10 - 4	UT	100%	PT	10%
13	D	7	4.2.10 - 4			МТ	100%
14	С	3	4.2.9 - 6			MT	10%
15	E	10		UT	25%	PT	100%

### COMMENTARY:

Tables 7.1, 7.2, and 7.3 from Part 7 of VIII-2, and Figure 4.1 and Tables 4.1 and 4.2 from Part 4 of VIII-2 have been included at the end of the examples for reference.

- a) Based on Table 7.1, Examination Groups 1a and 1b are the only groups that have no restrictions on maximum thickness governing weld joints. All other Examination Groups have limitations on welded joint thickness.
- b) Other Examination Groups could have been specified for the elliptical and torispherical heads or the boot (communicating chamber), due to the different governing weld joint thicknesses. If multiple Examination Groups were specified for a single vessel, Table E7.1 would need to be repeated for each individual Examination Group.
- c) If one or multiple vessel sections were assigned alternate Examination Groups, requirements of the common welded joint between Examination Groups must be followed as noted in paragraph 7.4.2.2.b.

- d) All Examination Groups require 100% visual examination to the maximum extent possible.
- e) The type of volumetric NDE used in fabrication is a function of the shell thickness and is shown in Table 7.3. For the governing thickness of this Examination Group (1a), the volumetric NDE can either be RT or UT with the same 100% extent of examination. The extent of examination is a percentage of the total length of the welded joint under consideration. The selection of RT or UT in this example (see Table E7.1) was for illustration only.
- f) The type of surface NDE can either be MT or PT. The extent of examination varies based on the Joint Category under consideration. The percentage of surface examination refers to the percentage of length of the welds both on the inside and outside surfaces. The selection of MT or PT in this example (see Table E7.1) was for illustration only.
- g) Joint Categories and Type of Weld Joints:
  - 1) Joint Category A:
    - i) All welds shall be full penetration butt joints of Type 1 as defined in Table 4.2.2.
    - ii) All welds require volumetric (RT or UT) and surface examination (MT or PT).
  - 2) Joint Category B:
    - i) Welds shall be full penetration butt joints of Type 1, 2, or 3; or angle joints of Type 8 as defined in Table 4.2.2.
    - ii) All welds in the main shell require volumetric (RT or UT) and surface examination (MT or PT).
    - iii) Welds in nozzles may require volumetric (RT or UT) and surface examination (MT or PT) or only surface examination, depending on diameter and thickness.
  - 3) Joint Category C:
    - i) For the assembly of flat heads or tubesheets with cylindrical shells or the assembly of a flange or collar with a shell:
      - 1. Welds can be full penetration butt joints of Type 1, 2, or 3; or corner joints of Type 7 as defined in Table 4.2.2. These types of welds require volumetric (UT) and surface examination (MT or PT).
      - 2. Welds can be partial penetration joints of Type 9 or fillet welds of Type 10 as defined in Table 4.2.2, only when Examination Group 3a or 3b is specified. These types of welds require volumetric (UT) and surface examination (MT or PT).
    - ii) For the assembly of a flange or a collar with a nozzle:
      - 1. Welds can be full penetration butt joints of Type 1, 2, or 3; or corner joints of Type 7 as defined in Table 4.2.2. These welds require volumetric (RT or UT) and surface examination (MT or PT).
      - 2. Welds can be partial penetration joints of Type 9 or fillet welds of Type 10 as defined in Table 4.2.2, only when Examination Group 3a or 3b is specified. These types of welds only require surface examination (MT or PT).
      - 3. For full or partial penetration welds with a diameter less than or equal to NPS 6 and a thickness less than or equal to 0.625 in, only surface examination (MT or PT) is required.
  - 4) Joint Category D:
    - i) Welds can be full penetration butt joints of Type 1, 2, or 3; or corner joints of Type 7 as defined in Table 4.2.2.
      - 1. For welds with a nozzle diameter greater than or equal to NPS 6 or a thickness greater than or equal to 0.625 in, volumetric (RT or UT) and surface examination (MT or PT) is required.
      - 2. For welds with a nozzle diameter less than or equal to NPS 6 and a thickness greater than or equal to 0.625 in., only surface examination (MT or PT) is required.

- ii) Welds can be partial penetration joints of Type 9 or fillet welds of Type 10 as defined in Table 4.2.2., provided:
  - 1. For welds with any nozzle diameter and a thickness greater than or equal to 0.625 in., volumetric (UT) and surface examination (MT or PT) is required.
  - 2. For welds with nozzle diameters greater than NPS 6 and a thickness less than or equal to 0.625 in., with Examination Group 3a or 3b specified, only surface examination (MT or PT) is required.
  - 3. For welds with nozzle diameters less than NPS 6 and a thickness less than or equal to 0.625 in., only surface examination (MT or PT) is required.
- iii) For tube-to-tubesheet welds in accordance with Figure 4.18.13 and Table 4.C.1, only surface examination (MT or PT) is required.
- 5) Joint Category E:
  - Welds can be full penetration butt joints of Type 1 or Type 7 as defined in Table 4.2.2. These types of welds requires volumetric (RT or UT) and surface examination (MT or PT). The RT designation is only applicable to the Type 1 weld joint.
  - Welds can be partial penetration joints of Type 9 or fillet welds of Type 10 as defined in Table 4.2.2. These types of welds require volumetric (UT) and surface examination (MT or PT).

### 7.2 Example E7.2 – NDE Requirements: Vessel with Two Examination Group Designations

A plant engineer is tasked with developing a design specification for a new pressure vessel that is to be constructed in accordance with VIII-2. Based on the anticipated design data, materials of construction, and welding process, the engineer selects Examination Group 1a to set the joint efficiency and NDE requirements for the main vessel, but selects Examination Group 3b to set the joint efficiency and NDE requirements for the boot (communicating chamber). Refer to Figure E7.1.

Two important limitations require comment for this example problem.

- 1) The Category D weld joint attaching the boot (communicating chamber) to the main cylindrical shell (Joint Identifier 12) falls under the Examination Group 1a requirements, see paragraph 7.4.2.2.b.
- 2) All components assigned to Examination Group 3b are required to be designed in accordance with Part 4, Design-by-Rule. If any components in the boot (communicating chamber) required a design-by-analysis in accordance with Part 5, Examination Group 3b would not be permitted.

Similar to Example E7.1, the engineer develops a table to summarize the NDE requirements to assist with fabrication and inspection of the vessel. However, Table E7.1 is reduced by eliminating Joint Identifiers 7, 8, 9, 13, and 14 from Examination Group 1a, as shown in Table E7.2.1; and an additional table is developed to summarize the NDE requirements applicable to each welded joint of the boot (communicating chamber), based on Examination Group 3b, as shown in Table E7.2.2.

Examination Group		1a					
P Number/Gro	oup Number		P-1,Gr-2	Governing We	Governing Weld Joint Thickness		
Welding Process			Unrestricted	Weld Joint Eff	Weld Joint Efficiency		
			Tabla		Non Destructiv	e Examination	
Joint Identifier	Joint Category	Joint Type	Detail	Volur	netric	Sur	face
	0,			Туре	Extent	Туре	Extent
1	А	1	4.2.4 - 1	RT	100%	PT	10%
2	В	1	4.2.4 - 1	RT	100%	PT	10%
3	В	1	4.2.4 - 1	RT	100%	PT	10%
4	А	1	4.2.4 - 1	UT	100%	PT	10%
5	А	1	4.2.4 - 1	UT	100%	PT	10%
6	А	1	4.2.4 - 1	RT	100%	PT	10%
10	D	7	4.2.10 - 4	UT	100%	PT	10%
11	С	3	4.2.9 - 6	RT	100%	PT	10%
12	D	7	4.2.10 - 4	UT	100%	PT	10%
15	E	10		UT	25%	PT	100%

 Table E7.2.1 - Weld Joint Requirements

Examination G	Group		3b				
P Number/Gro	up Number		P-1,Gr-2	Governing We	SS	1.125 in	
Welding Proce	SS		Unrestricted	Weld Joint Effi	0.85		
Joint Identifier	Joint Category	Joint Type	l able - Detail	Volur	netric	Sur	ace
			Dotai	Туре	Extent	Туре	Extent
7	А	1	4.2.4 - 1	RT	10%	МТ	10%
8	В	1	4.2.4 - 1	RT	10%	МТ	10%
9	В	1	4.2.4 - 1	RT	10%	МТ	10%
13	D	7	4.2.10 - 4			МТ	10%
14	С	3	4.2.9 - 6			МТ	10%

#### Table E7.2.2 - Weld Joint Requirements

#### COMMENTARY:

Tables 7.1, 7.2, and 7.3 from Part 7 of VIII-2, and Figure 4.1 and Tables 4.1 and 4.2 from Part 4 of VIII-2 have been included at the end of the examples for reference.

- a) The extent of volumetric examination per Examination Group 3b for the Category A and B full penetration weld seams, Joint Identifiers 7, 8, and 9, is only 10% compared to the Examination Group 1a requirement of 100%.
- b) The extent of surface examination per Examination Group 3b for the Category D full penetration corner joint, Joint Identifier 13, is only 10% compared to the Examination Group 1a requirement of 100%. Due to limitations on diameter and thickness of the nozzle, volumetric examination was not required.
- c) Similar to Examination Group 1a, the extent of surface examination per Examination Group 3b for the Category C full penetration circumferential joint, Joint Identifier 14, is only 10%. Due to limitations on diameter and thickness of the nozzle, volumetric examination was not required.

	· · · ·	Examinatio	Table 7.1 n Groups For I	Pressure Vesse	2 <b>.</b>		
Demonstra			Exami	nation Group(1)			
Parameter	1a	1b	2a	2b	3a	3b	
Permitted Material (1)(2)	All Materials in Annex 3-A	P-No.1 Gr 1 and 2, P-No. 8 Gr 1	P-No. 8 Gr 2 P-No 9A Gr 1 P-No 9B Gr 1 P-No 11A Gr 1 P-No. 11A Gr 2 P-No. 10H Gr 1	P-No.1 Gr 1 and 2, P-No. 8 Gr 1	P-No. 8 Gr 2, P-No 9A Gr 1, P-No 9B Gr 1, P-No. 10H Gr 1	P-No.1 Gr 1 and 2 P-No. 8 Gr 1	
Maximum thickness of governing welded joints	Unlim	ited(4)	30 mm (1- 3/16 in.) for P-No 9A Gr 1 and P-No 9B Gr 1; 16 mm (5/8 in.) for P-No. 8,Gr 2(5) P-No. 11A Gr 1 P- No. 11A Gr 2 P- No. 10H Gr 1	50 mm (2 in.) for P-No.1 Gr 1 and P-No. 8 Gr 1; 30 mm (1- 3/16 in.) for P-No.1 Gr 2	30 mm (1-3/16 in.) for P-No. 9A Gr 1 and P-No. 9B Gr 1; 16 mm (5/8 in.) for P-No.8, Gr 2(5) P-No. 10H Gr 1	50 mm (2 in.) for P-No.1 Gr 1 and P-No. 8 Gr 1; 30 mm (1- 3/16 in.) for P-No.1 Gr 2	
Welding process Unrestricted(4)			Mechanized V	/elding Only(3)	Unrestricted(4)		

NOTES:

(1) All Examination Groups require 100% visual examination to the maximum extent possible.

(1) All Examination Groups require 100% visual examination to the maximum extent possible.
 (2) See Part 3 for permitted material.
 (3) Mechanized means machine and/or automatic welding methods.
 (4) Unrestricted with respect to weld application modes as set forth in this Table.
 (5) See Table 7.2 for NDE, joint category, and permissible weld joint details that differ between Examination Groups 1a and 1b.
 (6) The design basis is the analysis method used to establish the wall thickness.

uctive Examination	1a 1b 2a	P-No. 8 ( 2 P-No. 9A	All P-No. 1 Gr P-No. 9B Materials 1 & 2 1	in Annex 3- P-No. 8 Gr P-No. 11 A f181 1 Gr 1	P-No. 11	Gr 2 P-No. 10	1.0 1.0 1.0	Type of Extent of N VDE.(2)	KT or UT 100% 100% 100%	AT or PT 10% 10%(4) 10%	RT or UT 100% 100% 100% 100%	T of UT NA 100% NA	4T or PT NA 10% NA	XT or UT 100% 100% 100% 100% 100% 100%	T or UT NA 100% NA	IT OF T NA 10% NA	1T or PT 100% 10% 10%	XT or UT 100% 100% 100%	AT or PT 10% 10%(4) 10%	T or UT 100% 100% 100% 100%	AT or PT 10% 10%(4) 10%	VT or UT   100%   100%   100%
Nondestru	Examination Group		Permitted Materials				Weld Joint Efficiency	Type of Weld.(1)					2,3 M	1 R1	enetration RT	NUT Weld Z,3 MT	1				- T	

	3b			P-NO. I GF I	* 2 P-No. 8 Gr 1				0.85		10%	10%(4)	10%(3)	10%(4)	25%	10%	10%(3)	10%(4)	25%	10%	10%	10%	10%(4)	10%	10%(4)	10%	10%(4)
	3a	P-No. 8 Gr	P-No. 9A Gr	1	P-No. 9B Gr	P-No. 10H	Gr 1		0.85	(	25%	10%	10%	10%	NA	NA	10%	10%	NA	NA	10%	25%	10%	10%	10%	25%	100%
	2b		P-No. 1 Gr	1 & 2	P-No. 8 Gr	H :			1.0	(10)(11)(12	100%	10%(4)	100%	10%(4)	25%	10%	100%	10%(4)	25%	10%	10%	100%	10%(4)	100%	10%(4)	100%	10%[4]
	2a	P-No. 8 Gr 2 P-No. 9A Gr	P-No. 9B Gr		P-No. 11A	47 1 P-No. 11A	Gr 2	P-No. 10H Gr 1	1.0	xtent of NDE	100%	10%	100%	10%	NA	NA	100%	10%	NA	NA	100%	100%	10%	100%	10%	100%	100%
uo	1b		P-No. 1 Gr	1&2	P-No. 8 Gr	7			1.0	ш	100%	10%(4)	100%	10%(4)	100%	10%	100%	10%(4)	100%	10%	10%	100%	10%(4)	100%	10%(4)	100%	1.006(4)
.z Examinati	<b>1</b> a		All	Materials	in Annex 3-	(81) A			1.0		100%	10%	100%	10%	NA	NA	100%	10%	NA	NA	100%	100%	10%	100%	10%	100%	100%
structive I										Type of NDE.(2)	RT or UT	MT or PT	RT or UT	MT or PT	MT or PT	RT or UT	MT or PT	RT or UT	MT or PT	RT or UT	MT or DT						
Nondestr	iation Group			d Mataniale		•			nt Efficiency	Weld.(1)		1				1		Ŧ		1	:	Ξ		:		:	
	Examina			Downitto					Weld Join	Type of	•	T	*	T		Cr,2	•	1		Cr/7	1	+	-	+	-	•	ά
							:											:	Full penetration	חחון אכוע							
										Joint ategory		A		9	6	•	٥	۹.	٥	9	В	4	¥	۵	<u>a</u>		'n

	3b		P-No. 1 Gr 1 & 2 P-No. 8 Gr 1		0.85		10% 10%(4)	10%	10% 10%(4)	10%	10%	10%	10%(4) 10%(4)	10001
	3a	P-No. 8 Gr 2	P-No. 9A Gr 1 P-No. 9B Gr 1 P-No. 10H Gr 1		0.85		25% 10%	10%	25% 10%	10%	10%	25%	10% 100%	1000
	2b		P-No. 1 Gr 1 & 2 P-No. 8 Gr 1		1.0	(10)(11)(12	100% 10%(4)	10%	100% 10%(4)	NA	10%	100%	10%(4) 10%	10007
	Za	P-No. 8 Gr 2 P-No. 9A Gr	P-No. 9B Gr 1 P-No. 11A Gr 1 P-No. 11A Gr 2	P-No. 10H Gr 1	1.0	xtent of NDE	100% 10%	100%	100% 10%	NA	100%	100%	10% 100%	10001
Cont'd)	1b		P-Na. 1 Gr 1 & 2 P-No. 8 Gr 1		1.0		100% 10%(4)	10%	100% 10%(4)	NA	10%	100%	10%(4) 10%	1000
nination (	<b>1</b> a		All Materials in Annex 3- A (18)		1.0		100% 10%	100%	100% 10%	NA	100%	100%	25%(7) 100%	10001
tive Exan						Type of NDE.(2)	RT or UT MT or PT	MT or PT	UT MT or PT	MT ar PT	MT or PT	MT or PT	RT or UT MT or PT	
Nondestruc	ation Group		ed Materials		nt Efficiency	f Weld.(1)	With full penetration d> 150 mm (6 in.) or t> 16 mm (5/8 in.)	With full penetration <i>d</i> ≤ 150 mm (6 in.) and t ≲ 16 mm (5/8 in.)	With partial penetration for any d a>16 mm (5/8 in.) (17)	With partial penetration d > 150  mm  (6  in.) $a \le 16 \text{ mm } (5/8 \text{ in.})(17)$	With partial penetration $d \le 150 \text{ mm} (6 \text{ in.})$ $a \le 16 \text{ mm} (5/8 \text{ in.})$		With full penetration or partial penetration(15)	
	Examina		Permitte		Wełd Joi	Type of	1, 2, 3, 7	1, 2, 3, 7	9, 10	9, 10	9, 10		1, 7, 9, 10	MA
									Nozzle or branch (5)			Tube-to- Tubesheet Welds	Permanent attachments (6)	Pressure retaining
		•		-		Joint Category	đ	Q	Ē	Q	đ	Q	<u></u>	;

	3b		1 Gr P-No. 1 Gr 1 8 2 1 Gr P No. 0 Cr 1	H-100 8 GT 1	0.85		(13)	100%	100%	100%	is are required to
	3a	P-No. 8	r P-No. 94 r P-No. 9E	1 P-No. 1 Gr 1	0.85	12)	(13)	100%	100%	100%	nination.
	2b		P-No. 1 G 1 & 2 P-No. 8 G	-	1.0	(11)(11)(	(13)	100%	100%	100%	tetrant Exar No. 8 Gr 1 No. 8 Gr 1 tiside. di surface e di surface e 6.2.7.3.
	2a	P-No. 8 Gr 2 P-No. 9A Gr	1 P-No. 9B Gr 1 P-No. 11A	Gr 1 P-No. 11A Gr 2 P-No. 10H	1.0	Extent of NDI	(13)	100%	100%	100%	= Liquid Per Gr 1 and P- Gr 1 ie and the ou volumetric ar extent possil
Cont'd)	1b		P-No. 1 Gr 1 & 2 P-No. 8 Gr	<b>H</b>	1.0	-	(13)	100%	100%	100%	mination, PT eel of P-No. 1 h 7.4.3.5(b). 2, P-No. 10H h on the insic sthods. Both 1e maximum 1e maximum
nination (	<b>1</b> a		All Materials in Annex 3-	A (18)	1.0		(13)	100%	100%	100%	: Particle Exa udinal, for st see paragrap -No. 11A Gr -No. 10A Gr
uctive Exan						Type of NDE.(2)	RT or UT	MT or PT	RT or UT	MT or PT	MT = Magnetic ration as longit all the nozzles, no. 11A Gr 1, F No. 11A Gr 1, F e of length of th are surface ex 10% visual exar and the original
Nondestri	tion Group		d Materials		nt Efficiency	Weld.(1)		1			T = Ultrasonic Examination, ame weld procedure specifi if $t \le 30 \text{ mm}$ (1-3/16 in.) if $s \le 30 \text{ mm}$ (1-3/16 in.) e aggregate weld length of thicknesses $\le 16 \text{ mm}$ (5/8 i thicknesses $\le 16 \text{ mm}$ (5/8 i). ), 9A Gr 1, P-No. 9B Gr 1, P- aragraph 4.2. ition refers to the percentag tion methods, and MT and P1 ition methods, and MT and M
	Examina		Permitte		Weld Joir	Type of		:		1	uination, U in, and sc 6 in.), 0% effers to th 1 for weld Gr 2, P-No Gr 2, P-No Gr 2, P-No or 2 examinat c examinat c examinat c examinat c examinat c for 1 ful Type 1, ful in Figure 7
							Cladding by	welding	Dansine (14)	(TT) Slibdan	ragraph 4.2 ragraph 4.2 tadiographic Exan $\varepsilon \leq 30 mm (1-3/16$ ft > 30 mm (1-3/16 ht is not required UT is not required or steel of P-No. 8 mty not used.) mity not used.) if the extert sh ans "not applicab ragraph 7.4.8.1 fo ragraph 7.4.8.1 fo
						Joint Category	0	ı		:	NOTES: NOTES: (1) See pa (2) RT = I (2) RT = I (3) 2% if (4) 10% if (5) Percer (5) Percer (6) RT or (7) 10% f (6) RT or (7) 10% f (10) The pa (11) RT and be app (11) RT and (12) NA mé (13) See pa (13) See pa (13) The pe (13) The pe (13) The pe (15) The pe (16) The pe (17) The pe (16) The pe (16) The pe (17) The pe (16) The pe (16) The pe (16) The pe (16) The pe (17) The pe (16) The pe (16) The pe (17) The pe (16) The pe

Selection of I	Table 7.3 Nondestructive Testing Method F	or Full Penetration Joints							
Type of Joint	Shell thickness – t								
	t < 13 mm (1/2 in.)	$t \ge 13 \ mm (1/2 \ in.)$							
1, 2, 3	RT	RT or UT per 7.5.5							
7 9	N/A	IIT per 754 or 755							



	Table 4.2.1 Definition Of Weld Categories
Weld Category	Description
A	<ul> <li>Longitudinal and spiral welded joints within the main shell, communicating chambers (1), transitions in diameter, or nozzles</li> <li>Any welded joint within a sphere, within a formed or flat head, or within the side plates (2) of a flat-sided vessel</li> <li>Circumferential welded joints connecting hemispherical heads to main shells, to transitions in diameter, to nozzles, or to communicating chambers.</li> </ul>
В	<ul> <li>Circumferential welded joints within the main shell, communicating chambers (1), nozzles or transitions in diameter including joints between the transition and a cylinder at either the large or small end</li> <li>Circumferential welded joints connecting formed heads other than hemispherical to main shells, to transitions in diameter, to nozzles, or to communicating chambers.</li> </ul>
С	<ul> <li>Welded joints connecting flanges, Van Stone laps, tubesheets or flat heads to main shell, to formed heads, to transitions in diameter, to nozzles, or to communicating chambers (1)</li> <li>Any welded joint connecting one side plate (2) to another side plate of a flat-sided vessel.</li> </ul>
D	<ul> <li>Welded joints connecting communicating chambers (1) or nozzles to main shells, to spheres, to transitions in diameter, to heads, or to flat-sided vessels</li> <li>Welded joints connecting nozzles to communicating chambers (1) (for nozzles at the small end of a transition in diameter see Category B).</li> </ul>
E	Welded joints attaching nonpressure parts and stiffeners
NOTES:	

(1) Communicating chambers are defined as appurtenances to the vessel that intersect the shell or heads of a vessel and form an integral part of the pressure containing enclosure, e.g., sumps.
 (2) Side plates of a flat-sided vessel are defined as any of the flat plates forming an integral part of the pressure containing enclosure.

	Table 4.2.2Definition Of Weld Joint Types
Weld Joint Type	Description
1	Butt joints and angle joints where the cone half-apex angle is less than or equal to 30 deg produced by double welding or by other means which produce the same quality of deposited weld metal on both inside and outside weld surfaces. Welds using backing strips which remain in place do not qualify as Type No.1 butt joints.
2	Butt joints produced by welding from one side with a backing strip that remains in place.
3	Butt joints produced by welding from one side without a backing strip.
7	Corner joints made with full penetration welds with or without cover fillet welds
· 8	Angle joints made with a full penetration weld where the cone half-apex angle is greater than 30 deg
9	Corner joints made with partial penetration welds with or without cover fillet welds
10	Fillet welds

# **PART 8**

# PRESSURE TESTING REQUIREMENTS

# **PART CONTENTS**

## 8.1 Example E8.1 – Determination of a Hydrostatic Test Pressure

Establish the hydrostatic test pressure for a process tower considering the following design conditions. All Category A and B joints are Type 1 butt welds and have been 100% radiographically examined.

### Vessel Data:

•	Material	=	SA-537, Class 1
•	Design Conditions	=	1650 <i>psig</i> @600°F
•	Liquid Head	=	60 <i>ft</i>
•	Liquid Specific Gravity	=	0.89
•	Inside Diameter	=	96.0 in
•	Corrosion Allowance	= 0.125	5 in
•	Allowable Stress	=	25700 psi
•	Allowable Stress at Ambient Conditions	=	29200 psi
•	Yield Stress at Ambient Conditions	=	50000 <i>psi</i>
•	Weld Joint Efficiency	=	1.0
•	Tangent-to-Tangent Vessel Length	=	80 <i>ft</i>
•	Top and Bottom Heads	=	Hemispherical

Evaluate the requirements of hydrostatic testing per paragraph 8.2.

The design pressure used to establish the wall thickness for the bottom head and cylindrical shell section must be adjusted for the liquid head in accordance with Part 4, paragraph 4.1.5.2.a.

Adjusted pressure for the cylindrical shell:

Design Pressure = Specified Design Pressure +  $\gamma gh$ Design Pressure =  $1650 + \frac{0.89(62.4)(56)}{144} = 1671.597 psig$ 

Adjusted pressure for the bottom hemispherical head:

Design Pressure = Specified Design Pressure + 
$$\gamma gh$$
  
Design Pressure =  $1650 + \frac{0.89(62.4)(60)}{144} = 1673.140 psig$ 

In accordance with Part 4, paragraph 4.3.5, determine the required thickness of the top head.

$$D = 96.0 + 2(Corrosion \ Allowance) = 96.0 + 2(0.125) = 96.25 \ in$$
$$t = \frac{D}{2} \left( \exp\left[\frac{0.5P}{SE}\right] - 1 \right) = \frac{96.25}{2} \left( \exp\left[\frac{0.5(1650)}{25700(1.0)}\right] - 1 \right) = 1.5699 \ in$$

 $t = 1.5699 + Corrosion \ Allowance = 1.5699 + 0.125 = 1.6949 \ in$ 

The required thickness of the top head is 1.6949 in.

In accordance with Part 4, paragraph 4.3.5, determine the required thickness of the bottom head, including the liquid head static pressure.

$$D = 96.0 + 2(Corrosion \ Allowance) = 96.0 + 2(0.125) = 96.25 \ in$$
$$t = \frac{D}{2} \left( \exp\left[\frac{0.5P}{SE}\right] - 1 \right) = \frac{96.25}{2} \left( \exp\left[\frac{0.5(1673.140)}{25700(1.0)}\right] - 1 \right) = 1.5923 \ in$$
$$t = 1.5923 + Corrosion \ Allowance = 1.5923 + 0.125 = 1.7173 \ in$$

The required thickness of the bottom head is 1.7173 in.

In accordance with Part 4, paragraph 4.3.3, determine the required thickness of the cylindrical shell, including the liquid head static pressure.

$$D = 96.0 + 2(Corrosion \ Allowance) = 96.0 + 2(0.125) = 96.25 \ in$$
$$t = \frac{D}{2} \left( \exp\left[\frac{P}{SE}\right] - 1 \right) = \frac{96.25}{2} \left( \exp\left[\frac{1671.597}{25700(1.0)}\right] - 1 \right) = 3.2342 \ in$$
$$t = 3.2373 + Corrosion \ Allowance = 3.2342 + 0.125 = 3.3592 \ in$$

The required thickness of the cylindrical shell is 3.3592 in.

Required Thickness Summary:

Top Head = 1.6949 in Bottom Head = 1.7173 in Cylindrical Shell = 3.3592 in

Per paragraph 8.2.1,

a) The minimum hydrostatic test pressure shall be the greater of the following. In this example problem, the MAWP is taken as the design pressure and the ratio of  $S_T/S$  is based on the shell material.

$$P_{T} = \max\left[\left\{1.43 \cdot MAWP\right\}, \left\{1.25 \cdot MAWP \cdot \left(\frac{S_{T}}{S}\right)\right\}\right]$$
$$P_{T} = \max\left[\left\{1.43(1650) = 2359.5\right\}, \left\{1.25(1650)\left(\frac{29200}{25700}\right) = 2343.4\right\}\right] = 2360 \ psi$$

- b) The ratio  $S_T/S$  shall be the lowest ratio for the pressure-boundary materials, excluding bolting materials, of which the vessel is constructed.
- c) The test pressure is the pressure to be applied at the top of the vessel during the test. The vessel is to be pressure tested in the horizontal position; therefore, the additional pressure from hydrostatic head is negligible. This pressure is used in the applicable design equations to check the vessel under test conditions, see Part 4, paragraph 4.1.6.2.a.

Per paragraph 4.1.6.2.a, when a hydrostatic test is performed in accordance with Part 8, the hydrostatic test pressure of a completed vessel shall not exceed that value which results in the following equivalent stress limit.

$$P_m \leq 0.95 S_y$$

The general primary membrane stress in the hemispherical heads is computed using the ASME membrane equations from VIII-1. For the hemispherical head, note that it is only necessary to check the thinner head.

$$R = D_i/2 = 96.0/2 = 48.0 \text{ in}$$

$$P_m = \frac{P}{2E} \left(\frac{R}{t} + 0.2\right) = \frac{2360}{2(1.0)} \left(\frac{48.0}{1.6949} + 0.2\right) = 34163 \text{ psi}$$

$$\{P_m = 34163 \text{ psi}\} \le \{0.95S_y = 0.95(50000) = 47500 \text{ psi}\}$$

The general primary membrane stress in the cylindrical shell is computed using the ASME membrane equations from VIII-1.

$$R = D_i/2 = 96.0/2 = 48.0 \text{ in}$$

$$P_m = \frac{P}{E} \left(\frac{R}{t} + 0.6\right) = \frac{2360}{1.0} \left(\frac{48.0}{3.3592} + 0.6\right) = 35138 \text{ psi}$$

$$\{P_m = 35138 \text{ psi}\} \le \{0.95S_v = 0.95(50000) = 47500 \text{ psi}\}$$

Therefore, the proposed design is acceptable for a hydrostatic test pressure of 2360 psig.

- d) The requirement of paragraph 8.2.1.a represents the minimum required hydrostatic test pressure. The upper limit of the test pressure shall be determined using the method in paragraph 4.1.6.2.a. Any intermediate value or pressure may be used.
- A hydrostatic test based on a calculated pressure may be used by agreement between the user and the Manufacturer.

## 8.2 Example E8.2 – Determination of a Pneumatic Test Pressure

Establish the pneumatic test pressure for a vessel considering the following design conditions. All Category A and B joints are Type 1 butt welds and have been 100% radiographically examined.

### Vessel Data:

•	Material	=	SA-516, Grade 70
•	Design Conditions	=	150 <i>psig</i> @300°F
•	Inside Diameter	=	240.0 in
•	Corrosion Allowance	=	0.125 in
•	Allowable Stress	=	22400 psi
•	Allowable Stress at Ambient Conditions	=	25300 psi
•	Yield Stress at Ambient Conditions	=	38000 <i>psi</i>
•	Weld Joint Efficiency	=	1.0
•	Tangent-to-Tangent Vessel Length	=	80 <i>ft</i>
•	Top and Bottom Heads	=	Hemispherical

Evaluate the requirements of pneumatic testing per paragraph 8.3.

In accordance with Part 4, paragraph 4.3.5, determine the required thickness of the top and bottom hemispherical heads.

$$D = 240.0 + 2(Corrosion \ Allowance) = 240.0 + 2(0.125) = 240.25 \ in$$
$$t = \frac{D}{2} \left( \exp\left[\frac{0.5P}{SE}\right] - 1 \right) = \frac{240.25}{2} \left( \exp\left[\frac{0.5(150)}{22400(1.0)}\right] - 1 \right) = 0.4029 \ in$$
$$t = 0.4029 + Corrosion \ Allowance = 0.4029 + 0.125 = 0.5279 \ in$$

The required thickness of the heads is 0.5279 in.

In accordance with Part 4, paragraph 4.3.3, determine the required thickness of the cylindrical shell.

$$D = 240.0 + 2(Corrosion \ Allowance) = 240.0 + 2(0.125) = 240.25 \ in$$
$$t = \frac{D}{2} \left( \exp\left[\frac{P}{SE}\right] - 1 \right) = \frac{240.25}{2} \left( \exp\left[\frac{150}{22400(1.0)}\right] - 1 \right) = 0.8071 \ in$$
$$t = 0.8071 + Corrosion \ Allowance = 0.8071 + 0.125 = 0.9321 \ in$$

The required thickness of the cylindrical shell is 0.9321 in.

Required Thickness Summary:

Top/Bottom Head = 0.5279 inBottom Cylindrical Shell = 0.9321 in Per paragraph 8.2.1,

a) The minimum pneumatic test pressure shall be computed as follows. In this example problem, the MAWP is taken as the design pressure and the ratio of  $S_T/S$  is based on the shell material.

$$P_{T} = 1.15 \cdot MAWP \cdot \left(\frac{S_{T}}{S}\right)$$
$$P_{T} = 1.15(150.0) \left(\frac{25300}{22400}\right) = 195 \ psi$$

- b) The ratio  $S_T/S$  shall be the lowest ratio for the pressure-boundary materials, excluding bolting materials, of which the vessel is constructed.
- c) The requirement of paragraph 8.3.1.a represents the minimum required pneumatic test pressure. The upper limit of the test pressure shall be determined using the method in paragraph 4.1.6.2.b. Any intermediate value or pressure may be used.
- d) The test pressure is the pressure to be applied at the top of the vessel during the test. This pressure is used in the applicable design equations to check the vessel under test conditions, see Part 4, paragraph 4.1.6.2.b.

Per paragraph 4.1.6.2.b, when a pneumatic test is performed in accordance with Part 8, the pneumatic test pressure of a completed vessel shall not exceed that value which results in the following equivalent stress limit.

 $P_m \leq 0.80 S_v$ 

The general primary membrane stress in the hemispherical heads is computed using the ASME membrane equations from VIII-1.

$$R = D_i/2 = 240.0/2 = 120.0 \text{ in}$$

$$P_m = \frac{P}{2E} \left(\frac{R}{t} + 0.2\right) = \frac{195}{2(1.0)} \left(\frac{120.0}{0.5279} + 0.2\right) = 22183 \text{ psi}$$

$$\{P_m = 22183 \text{ psi}\} \le \{0.8S_y = 0.8(38000) = 30400 \text{ psi}\}$$

The general primary membrane stress in the cylindrical shell is computed using the ASME membrane equations from VIII-1.

$$R = D_i/2 = 240.0/2 = 120.0 \text{ in}$$

$$P_m = \frac{P}{E} \left(\frac{R}{t} + 0.6\right) = \frac{195}{1.0} \left(\frac{120.0}{0.9321} + 0.6\right) = 25222 \text{ psi}$$

$$\{P_m = 25222 \text{ psi}\} \le \{0.8S_y = 0.8(38000) = 30400 \text{ psi}\}$$

Therefore, the proposed design is acceptable for a pneumatic test pressure of 195 psi.
## **ASME PTB-3-2013**

