ASME MFC-7–2016

[Revision and Redesignation of ASME/ANSI MFC-7M–1987 (R2014)]

Measurement of Gas Flow by Means of Critical Flow Venturis and Critical Flow Nozzles

AN AMERICAN NATIONAL STANDARD

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Two Park Avenue • New York, NY • 10016 USA

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The American Society of Mechanical Engineers
Two Park Avenue, New York, NY 10016-5990

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FOREWORD

This Standard was prepared by Subcommittee 7 (SC 7) of the ASME Standards Committee on Measurement of Fluid Flow in Closed Conduits; it has been revised from ASME MFC-7M–1987 in its entirety. During the preparation, reference was made to older ASME standards and documents, including ASME MFC-3M–2004 and ASME PTC 19.5-2004, and to international standards including ISO 9300:2005 and ISO/IEC Guide 98-3:2008. In addition, information was gathered from many published papers and from the experience of the Subcommittee members and other knowledgeable engineers. This standard is a blend of the available technical information and best practices, and it is intended to be a practical guide to the proper use of critical flow venturis (CFV) and critical flow nozzles (CFN).

Changes made during the revision of this Standard are summarized as follows:

(a) The Scope and Field of Application was revised to clarify usage of the terms "critical flow venturi" and "critical flow nozzle."

(b) A few symbols and definitions have been added, and many have been clarified and updated.

(c) Manufacturing tolerances have been updated to be more verifiable and to accommodate smaller CFVs.

(d) The discharge coefficient equations have been brought into alignment with extensive research results and ISO 9300.

(e) Recommendations for the calculation of thermophysical properties have been directed almost entirely toward the NIST Reference Fluid Thermodynamic and Transport Properties Database (REFPROP), which is maintained by the National Institute of Standards and Technology (NIST).

(f) Uncertainty calculation methods have been extensively modified to be consistent with more modern methods and ISO/IEC Guide 98-3:2008. A statement of uncertainty is now required in order to be compliant with this Standard.

(g) The Nonmandatory Appendices have been modified to provide two new comprehensive examples, including uncertainty calculation, and to derive and clarify the mass flow equation, the real gas critical flow function, other gas property calculations, and humid air considerations.

(h) An "unchoking test procedure" is provided in a Nonmandatory Appendix.

Critical flow venturis are especially suited as transfer standards and reference flowmeters for calibration and testing and for precise flow control applications. CFVs provide a stable flow of compressible fluids, and per this Standard can and should be associated with a precise statement of uncertainty for the measured flow. Although this Standard is a complete guide that provides specific requirements and methods for the proper use of CFVs and CFNs, some latitude and variations in application are allowed if necessary tests are performed and proper judgment is applied.

Suggestions for improvement of this Standard will be welcomed. They should be sent to The American Society of Mechanical Engineers; Attn: Secretary, MFC Main Committee; Two Park Avenue; New York, NY 10016-5990.

This revision was approved as an American National Standard on January 6, 2016.

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Measurement of Fluid Flow in Closed Conduits

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Proposing Revisions. Revisions are made periodically to the Standard to incorporate changes that appear necessary or desirable, as demonstrated by the experience gained from the application of the Standard. Approved revisions will be published periodically.

The Committee welcomes proposals for revisions to this Standard. Such proposals should be as specific as possible, citing the paragraph number(s), the proposed wording, and a detailed description of the reasons for the proposal, including any pertinent documentation.

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Subject:	Cite the applicable paragraph number(s) and the topic of the inquiry in one or two words.
Edition:	Cite the applicable edition of the Standard for which the interpretation is being requested.
Question:	Phrase the question as a request for an interpretation of a specific requirement suitable for general understanding and use, not as a request for an approval of a proprietary design or situation. Please provide a condensed and precise question, composed in such a way that a "yes" or "no" reply is acceptable.
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MEASUREMENT OF GAS FLOW BY MEANS OF CRITICAL FLOW VENTURIS AND CRITICAL FLOW NOZZLES

1 SCOPE AND FIELD OF APPLICATION

This Standard applies only to the steady flow of single-phase gases through critical flow venturis (CFV) of shapes specified herein [also sometimes referred to as critical flow nozzles (CFN), sonic nozzles, or critical flow venturi nozzles]. This Standard applies to CFVs with diverging sections on the downstream side of the throat. When a CFN (no diverging section) is discussed, it is explicitly noted. This Standard specifies the method of use (installation and operating conditions) of CFVs. This Standard also gives information necessary for calculating the mass flow of the gas and its associated uncertainty.

This Standard applies only to CFVs and CFNs in which the flow is critical. Critical flow exists when the mass flow through the CFV is the maximum possible for the existing upstream conditions. At critical flow or choked conditions, the average gas velocity at the CFV throat closely approximates the local sonic velocity.

This Standard specifically applies to cases in which

(a) it can be assumed that there is a large volume upstream of the CFV or upstream of a set of CFVs mounted in a parallel flow arrangement (in a common plenum), thereby achieving higher flow; or

(b) the pipeline upstream of the CFV is of circular cross section with throat to pipe diameter ratio equal to or less than 0.25

2 REFERENCES

The following publications are referenced in this Standard. The latest edition of ASME publications should be used.

ASME MFC-3M, Measurement of Fluid Flow in Pipes Using Orifice, Nozzle, and Venturi
ASME PTC 19.5, Flow Measurement
Publisher: The American Society of Mechanical Engineers (ASME), Two Park Avenue, New York, NY 10016-5990 (www.asme.org)

ISO 9300:2005, Measurement of gas flow by means of critical flow Venturi nozzles
ISO/IEC Guide 98-3:2008, Uncertainty of measurement—Part 3: Guide to the expression of uncertainty in measurement
Publisher: International Organization for Standardization (ISO) Central Secretariat, Chemin de Blandonnet 8, Case Postale 401, 1214 Vernier, Geneva, Switzerland (www.iso.org)

NIST Standard Reference Database 23, NIST Reference Fluid Thermodynamic and Transport Properties Database (REFPROP): Version 9.1
Publisher: National Institute of Standards and Technology (NIST), 100 Bureau Drive, Stop 1070, Gaithersburg, MD 20899 (www.nist.gov)

3 SYMBOLS AND DEFINITIONS

3.1 Symbols and Nomenclature

See Table 3.1-1.

3.2 Definitions

3.2.1 Temperature Measurement

measured gas temperature: temperature of the gas after being irreversibly brought to rest against the temperature probe.

Table 3.1-1 Nomenclature Used in This Standard

Symbol	Name	Dimensions	SI Unit	U.S. Customary Unit
A^*	Area of CFV throat	L^2	m^2	ft^2
A_2	Area of CFV exit	L^2	m^2	ft^2
b_0, b_1, n	Coefficients for empirical C_d equation	Dimensionless	Dimensionless	Dimensionless
c	Sound speed	LT^{-1}	m/s	ft/sec
C_d	Discharge coefficient	Dimensionless	Dimensionless	Dimensionless
C_i^*	Ideal gas critical flow function	Dimensionless	Dimensionless	Dimensionless
C_P^*	Polytropic gas critical flow function	Dimensionless	Dimensionless	Dimensionless
C_R^*	Real gas critical flow function	Dimensionless	Dimensionless	Dimensionless
c_p	Constant pressure specific heat	$L^2T^{-2}\theta^{-1}$	kJ/kg K	Btu/lbm °R
c_v	Constant volume specific heat	$L^2T^{-2}\theta^{-1}$	kJ/kg K	Btu/lbm °R
D	Diameter of upstream conduit	L	m	ft
d	Diameter of CFV throat	L	m	ft
h	Specific enthalpy	L^2T^{-2}	J/kg	Btu/lbm
h_t	Total specific enthalpy	L^2T^{-2}	J/kg	Btu/lbm
k	Coverage factor	Dimensionless	Dimensionless	Dimensionless
M	Molar mass	$MM^{-1}mole^{-1}$	kg/kg mole	lbm/lbm mole
$\dot{m}$	Mass flow	MT^{-1}	kg/s	lbm/sec
Ma	Mach number: ratio of gas velocity to sound speed	Dimensionless	Dimensionless	Dimensionless
$\dot{m}_{th}$	Theoretical mass flow for one-dimensional isentropic flow of a real gas	MT^{-1}	kg/s	lbm/sec
P^*	Absolute static pressure of the gas at CFV throat	$ML^{-1}T^{-2}$	Pa	lbf/in.^{2}
P^*/P_0	Critical pressure ratio: ratio of throat pressure to inlet stagnation pressure	Dimensionless	Dimensionless	Dimensionless
P_0	Absolute stagnation (or total) pressure of the gas at CFV inlet	$ML^{-1}T^{-2}$	Pa	lbf/in.^{2}
P_1	Absolute static pressure of the gas in the upstream conduit	$ML^{-1}T^{-2}$	Pa	lbf/in.^{2}
P_2	Absolute static pressure of the gas at CFV exit	$ML^{-1}T^{-2}$	Pa	lbf/in.^{2}
P_2/P_0	Back pressure ratio: ratio of CFV exit static pressure to inlet stagnation pressure	Dimensionless	Dimensionless	Dimensionless
r_c	Radius of curvature of CFV inlet	L	m	ft
Re_d	CFV throat Reynolds number	Dimensionless	Dimensionless	Dimensionless
R_f	Recovery factor or temperature probe constant	Dimensionless	Dimensionless	Dimensionless
R_u	Universal gas constant: 8 314.4598 J/(mol · K) [1,545.3467 ft · lbf/(mol · °R)]	$ML^2T^{-2}\theta^{-1}$	J/(mol · K)	ft · lbf/(mol · °R)
s	Specific entropy of the gas	$L^2\theta^1T^{-2}$	J/(kg K)	Btu/(lbm °R)
T^*	Absolute static temperature at CFV throat	θ	K	°R
T_0	Absolute stagnation (or total) temperature of the gas	θ	K	°R
T_1	Absolute static temperature of the gas at CFV inlet	θ	K	°R
T_{m1}	Measured temperature	θ	K	°R
U	Expanded uncertainty (with specified coverage factor, k)	. . .	. . .	. . .
u	Standard uncertainty ($k = 1$)	. . .	. . .	. . .
u_c	Combined standard uncertainty ($k = 1$)	. . .	. . .	. . .
V	One-dimensional gas velocity	LT^{-1}	m/s	ft/sec
V^*	Velocity of gas at the throat equal to the sonic velocity	LT^{-1}	m/s	ft/sec
Z	Compressibility factor	Dimensionless	Dimensionless	Dimensionless

Table 3.1-1 Nomenclature Used in This Standard (Cont'd)

Symbol	Name	Dimensions	SI Unit	U.S. Customary Unit
β	Beta ratio: d/D	Dimensionless	Dimensionless	Dimensionless
γ	Ratio of specific heats	Dimensionless	Dimensionless	Dimensionless
κ	Isentropic (or polytropic) exponent	Dimensionless	Dimensionless	Dimensionless
μ_0	Dynamic viscosity of the gas at stagnation conditions	$ML^{-1}T^{-1}$	kg/m s	lbm/ft sec
ν_{eff}	Effective degrees of freedom	Dimensionless	Dimensionless	Dimensionless
ρ^*	Gas density at CFV throat	ML^{-3}	kg/m^3	lbm/ft^3
Superscript				
*	Value at the CFV throat based on one-dimensional, choked flow	. . .	. . .	. . .
Subscripts				
0	Stagnation property	. . .	. . .	. . .
1	Inlet conduit or upstream piping	. . .	. . .	. . .
2	CFV exit	. . .	. . .	. . .
i	Ideal gas	. . .	. . .	. . .
max	Maximum	. . .	. . .	. . .

recovery factor: parameter used to correct the temperature of the gas after being irreversibly brought to rest against the temperature probe.

$$R_f = \frac{T_{m1} - T_1}{T_0 - T_1} \tag{3-1}$$

stagnation (or total) temperature of a gas: temperature that would exist in the gas if the flowing gas stream were brought to rest by an isentropic process.

static temperature of a gas: actual bulk temperature of the flowing gas.

3.2.2 Critical Flow Venturis

CFV exit plane: surface at the exit of the divergent section.

CFV inlet plane: surface at the entrance of the convergent section.

critical flow venturi: a flowmeter having a geometrical configuration with a constant curvature convergent section to a minimum cross-sectional area (i.e., throat) at which sonic conditions exist followed by a conical divergent section.

critical (or choked) flow: maximum flow for a particular venturi that can exist for the given upstream conditions; the flow that exists when the ratio of the downstream static pressure, P_2, to the upstream absolute pressures, P_0, is such that the fluid velocity reaches sonic conditions at the throat. This condition is termed "choked" flow, and the flow is proportional to the inlet stagnation pressure.

critical pressure ratio: the ratio of the absolute static pressure at the CFV throat to the absolute stagnation pressure for which gas mass flow through the CFV is a maximum.

maximum back pressure ratio: the ratio of the highest absolute CFV exit static pressure to the absolute CFV upstream stagnation pressure at which the flow becomes critical.

throat: the cross section of the CFV with minimum diameter.

3.2.3 Pressure Measurement

stagnation (or total) pressure of a gas: pressure that would exist in the gas if the flowing gas stream were brought to rest by an isentropic process. Only the value of the absolute stagnation pressure is used in this Standard.

static pressure of a gas: the pressure of the flowing gas, which can be measured by connecting a pressure gauge to a wall pressure tap. Only the value of the absolute static pressure is used in this Standard.

wall pressure tap: a hole drilled in the wall of a conduit, the inside edge of which is flush with the inside surface of the conduit.

3.2.4 Flow

mass flow: the mass of gas per unit time passing through the CFV. In this Standard, mass flow is always the steady-state or equilibrium mass flow.

steady state: the conditions under which the inlet and other measured pressures and temperatures at a CFV do not change in a transient or periodic manner by more than two times the resolution of the transducers or two times the standard uncertainty of the measurement during the period of testing or measurement.

3.2.5 Thermodynamic Properties

entropy: a property related to the disorder of a thermodynamic system, often assumed to have a constant value in theoretical analysis of flow through a CFV, i.e., stagnation entropy equals the throat enthalpy, $s_0 = s^*$.

isentropic exponent (κ): a thermodynamic variable defined by

$$\kappa \equiv \frac{\rho}{p}\left(\frac{\partial P}{\partial \rho}\right)_s = \frac{\rho\, c^2}{P} \tag{3-2}$$

NOTES:
(1) For a perfect gas, the isentropic exponent equals the specific heat ratio, $\kappa = \gamma$.
(2) For a polytropic process, the value of κ remains fixed so that the static pressure divided by the density raised to the polytropic exponent is constant (P/ρ^κ = constant). This expression is derived by integrating $\kappa = \rho/P\ (\delta P/\delta\rho)_s$ for an isentropic process with a constant value of the polytropic exponent.
(3) In real gases, the forces exerted between molecules, as well as the volume occupied by the molecules, have a significant effect on gas behavior. In a perfect gas, intermolecular forces and the volume occupied by the molecules are neglected.

real gas critical flow function: a dimensionless coefficient used to correct the CFV mass flow for real gas effects, defined by the equation

$$C_R^* = \frac{\rho^* c^* \sqrt{R_u T_o}}{P_0 \sqrt{M}} \tag{3-3}$$

where
c^* = the sound velocity at the throat
M = molar (or molecular) weight
P_0 = the stagnation pressure
R_u = the universal gas constant
T_0 = the stagnation temperature
ρ^* = the gas density at the nozzle throat

NOTES:
(1) The throat density, ρ^*, and sound speed, c^*, are computed using a real gas equation of state for a one-dimensional, isentropic, isoenergetic flow model (see Nonmandatory Appendix C). For an isentropic process the entropy is constant (s = constant) while for an isoenergetic process the total enthalpy is constant ($h + V^2/2$ = constant).
(2) If the gas flowing through the CFV behaves ideally (i.e., real gas effects are negligible) and γ remains constant as the gas expands from the CFV inlet to the throat, C_R^* simplifies to the ideal gas critical flow function (C_i^*) and is a function only of the specific heat ratio.

$$C_i^* = \sqrt{\gamma\left(\frac{2}{1+\gamma}\right)^{\frac{\gamma+1}{\gamma-1}}} \tag{3-4}$$

The specific heat ratio should be determined using a low-uncertainty thermodynamic property database. It is evaluated as a function of the gas composition, and the measured upstream temperature and pressure.
(3) If the CFV flow can be accurately modeled by a polytropic process, C_R^* simplifies to the polytropic gas critical flow function $\left(C_p^*\right)$ and is a function of the polytropic exponent and the compressibility factor evaluated at the stagnation conditions.

$$C_p^* = \sqrt{\frac{\kappa}{Z_0}\left(\frac{2}{1+\kappa}\right)^{\frac{\kappa+1}{\kappa-1}}} \tag{3-5}$$

The polytropic exponent and compressibility factor should be determined using a low-uncertainty thermodynamic property database. The polytropic exponent is evaluated as a function of gas composition, and the measured upstream temperature and pressure.
(4) In practice, sometimes C_R^* is estimated by either C_i^* or C_p^*. If the gas behavior is not ideal (i.e., $Z \neq 1$) or if the CFV flow process is not polytropic (i.e., $\kappa \neq$ constant), neither of these idealized critical flow functions equals C_R^*. Consequently, mass flow calculations using either C_i^* or C_p^* will have higher uncertainty than corresponding calculations using C_R^*. For the lowest uncertainty, mass flow calculations should be based on C_R^*.

specific heat ratio: a thermodynamic variable defined by the ratio of the constant pressure, c_p, to constant volume, c_v, specific heats

$$\gamma = \frac{c_p}{c_v} \tag{3-6}$$

speed of sound: the speed that a sound wave travels

$$c = \sqrt{\left(\frac{\partial P}{\partial \rho}\right)_s} \tag{3-7}$$

where P and ρ are the absolute static pressure and density, respectively, and s refers to constant entropy.

stagnation enthalpy: thermodynamic variable equal to the specific enthalpy evaluated at the stagnation pressure and temperature, $h_0 = h(T_0,P_0)$.

total specific enthalpy: the sum of the specific enthalpy and the kinetic energy per unit mass, $h_t = h + V^2/2$.

NOTES:
(1) The total specific enthalpy equals the specific stagnation enthalpy for a steady, isentropic flow ($h_t = h_0$). This result is commonly referred to as the "isoenergetic condition."
(2) The total specific enthalpy is a thermodynamic property at the CFV throat since the fluid velocity equals the sound speed, $h_t^* = h^* + \frac{c^{*2}}{2}$.

3.2.6 Dimensionless Quantities

discharge coefficient: the dimensionless ratio of the actual flow to the theoretical flow

$$C_d = \frac{\dot{m}}{\dot{m}_{th}} \tag{3-8}$$

This coefficient corrects for viscous effects in the boundary layer and sonic line curvature effects in the far field (i.e., in the irrotational flow outside of the boundary layer region). For the CFV design and installation conditions specified in this Standard, it is a function of the throat Reynolds number.

Mach number: the ratio of the fluid velocity to the velocity of sound in the fluid at the same temperature and pressure

$$Ma = V/c \tag{3-9}$$

NOTE: For a one-dimensional isentropic flow the Mach number equals unity at the CFV throat, such that the fluid velocity equals the sound speed ($V^* = c^*$).

throat Reynolds number: dimensionless parameter calculated from the mass flow, $\dot{m}$, and the dynamic viscosity, μ_0, evaluated at the CFV inlet stagnation conditions using the throat diameter, d, as the length scale

$$R_e = \frac{4\dot{m}}{\pi d \mu_0} \tag{3-10}$$

4 BASIC EQUATIONS

4.1 State Equation

The behavior of a real gas can be described by

$$p = \frac{\rho R_U T Z}{M} \tag{4-1}$$

4.2 CFV Mass Flow Equations

Assuming that the flow is one-dimensional and isentropic (i.e., frictionless and adiabatic), the value of the theoretical mass flow through a CFV is

$$\dot{m}_{th} = \frac{A^* C_R^* P_0}{\sqrt{(R_u/M)T_0}} \tag{4-2}$$

A discharge coefficient C_d is needed to correct for the fact that the flow is neither entirely one-dimensional nor isentropic. Equation (4-2) becomes

$$\dot{m} = C_d \dot{m}_{th} = \frac{C_d A^* C_R^* P_0}{\sqrt{(R_u/M)T_0}} \tag{4-3}$$

If the throat diameter is known perfectly, C_d will be less than unity because the mass flow, $\dot{m}$, will be less than ideal due to curvature of the throat sonic line as well as subsonic velocities through the viscous boundary layer adjacent to the CFV wall.

NOTE: Paragraph 8.1 provides information for computing C_d if it is not obtained by calibration, and para. 8.2 provides information for computing C_R^*.

5 APPLICATIONS FOR WHICH THE METHOD IS SUITABLE

Each application should be evaluated to determine whether a CFV is suitable for the conditions and requirements. An important advantage of a CFV is that the flow through it is independent of the downstream pressure as long as the pressure conditions up- and downstream from the CFV lead to critical flow at the throat. The following are some other considerations:

(a) To calculate flow through a CFV, the only measurements required are the gas composition, and the pressure and temperature upstream of the CFV. These measurements enable the throat conditions to be calculated from thermodynamic considerations. A low-uncertainty measurement of the throat diameter is also required if C_d values are determined using the empirical equations in this Standard. In contrast, if the CFV is flow calibrated, only a nominal value of the diameter is necessary (see examples in Nonmandatory Appendix B). CFVs are applicable when there is no phase change as the gas accelerates from the inlet to the throat and the flow is not a function of the downstream pressure (i.e., the CFV is choked). Care must be taken when using an equation of state at or near the dew point of the gas so that correct gas phase properties are determined. Studies have shown that condensation rates in the presence of favorable pressure gradients and rapidly falling temperatures are much slower than the transit time of the fluid from the CFV entrance to the CFV throat. Therefore, the CFV will operate correctly and yield the correct flow, provided that the calculations for the speed of sound and density at the throat are correct.

(b) The velocity in the CFV throat is the maximum possible for the given upstream stagnation conditions; therefore, the sensitivity to installation effects is minimized, except for swirl, which must not exist in the inlet plane of the CFV.

(c) Unlike the subsonic differential pressure device, CFV flow is proportional to the inlet stagnation pressure and not to the square root of a measured differential pressure.

(d) The maximum flow range that can be obtained for a given CFV is limited to the range of inlet pressures that are available above the inlet pressure at which the flow becomes critical.

(e) The most common applications for CFVs are the calibration of other meters (working or reference standards) and verification or comparison of primary flow standards (check or transfer standards) in flow control applications and in product testing.

6 STANDARD CRITICAL FLOW VENTURIS

6.1 General Requirements

6.1.1 Discharge Coefficient. The discharge coefficient, C_d, for a CFV may be obtained by either the empirical method (using empirical or theoretical equations of C_d versus Reynolds number) or the calibration method (calibration of the particular CFV in a flow laboratory). When using empirical C_d values, the specifications for size, shape, and surface conditions are pertinent to obtaining the performance specified in this Standard. In these cases the CFV should be inspected to determine conformance to construction specifications of this Standard. In the case of laboratory-determined C_d values, compliance with the following construction specifications is less pertinent. When it is not practical to manufacture CFVs to the surface finish and curvature specifications herein, CFV performance must be demonstrated through calibration against a flow reference.

6.1.2 Materials. CFVs should be manufactured from material suitable for the intended application, such as the following:

(a) The material should be capable of being finished to the surface condition specified in this Standard. Some materials are unsuitable because of pits, voids, and other nonhomogeneities.

(b) The material, together with any surface treatment used, should not be subject to corrosion in the intended service. Experience has shown that 300 series stainless steel is often a suitable material.

(c) The material should be dimensionally stable and should have known and repeatable thermal expansion characteristics (if it is to be used at a temperature other than that at which the throat diameter has been measured), so throat diameter corrections and uncertainty estimates can be made. A period of time is generally required to achieve steady-state temperature conditions, and the flow reported by the CFV will change gradually as equilibrium is approached. The amount that the flow changes as steady state is approached depends on flow conditions, CFV geometry, ambient temperature conditions, gas type, and response time of the instrumentation. Generally, the time necessary to achieve steady state should be determined experimentally.

6.1.3 Surface Finish. The throat and toroidal inlet up to the conical divergent section of the CFV should be smoothly finished. Where it can be measured, the arithmetic average roughness height should not exceed $15 \times 10^{-6}d$. If the roughness cannot be measured the CFV should be flow calibrated. The throat and toroidal inlet up to the conical divergent section should be free from dirt, films, and other contamination. The form of the conical divergent portion of the CFV should be controlled such that any steps, discontinuities, irregularities, and lack of concentricity do not exceed 1% of the local diameter. If there is a diameter discontinuity in the divergent portion of the CFV, then the diameter should increase (not decrease) in the direction of flow. The arithmetic average roughness of the conical divergent section should not exceed $10^{-4}d$.

6.2 Standard CFV Geometries

Two different designs are possible for standard CFVs: a toroidal throat design and a cylindrical throat design. The toroidal throat design is the most widely used and is the primary focus of this Standard. However, for completeness, guidance is also given for the cylindrical throat design.

NOTE: Critical nozzles (i.e., CFNs with no divergent section) are not a recommended design (although they are allowed) due to poor pressure recovery and the greater possibility of flow performance being affected by downstream disturbances (i.e., flow pulsations). However, the same flow equations and discharge coefficients apply to CFNs as to CFVs.

6.2.1 Toroidal Throat CFVs. A toroidal throat CFV should meet the following requirements:

(a) It should conform to Fig. 6.2.1-1.

(b) For purposes of locating other elements of the CFV critical flow metering system, the inlet plane of the CFV is defined as the plane perpendicular to the axis of symmetry that intersects the inlet at a diameter equal to $2.5d \pm 0.1d$.

(c) The convergent part of the CFV (inlet) is a portion of a torus that extends through the minimum area section (throat). The curvature of this surface continues to become tangent to the divergent section. The contour of the inlet upstream of a diameter equal to $2.5d$ is not specified, except that the surface at each axial location has a diameter equal to or greater than the extension of the toroidal contour.

(d) The inlet toroidal surface of the CFV beginning at a diameter of $2.5d$ perpendicular to the axis of symmetry (see Fig. 6.2.1-1) and extending to the point of tangency should not deviate from the shape of a torus by more than $0.001d$. The radius of curvature of this toroidal surface in the plane of symmetry should be $1.8d$ to $2.2d$.

(e) The divergent portion of the CFV downstream of the point of tangency with the torus should form a frustum of a cone with a half-angle of 2.5 deg to 6 deg. The length of the conical section should not be less than one throat diameter.

(f) If these manufacturing tolerances cannot be achieved or verified by inspection, then flow calibration is recommended.

6.2.2 Cylindrical Throat CFVs. A cylindrical throat CFV should meet the following requirements:

(a) It should conform to Fig. 6.2.2-1.

(b) The inlet plane is defined as the plane tangent to the inlet contour of the CFV and perpendicular to the CFV centerline.

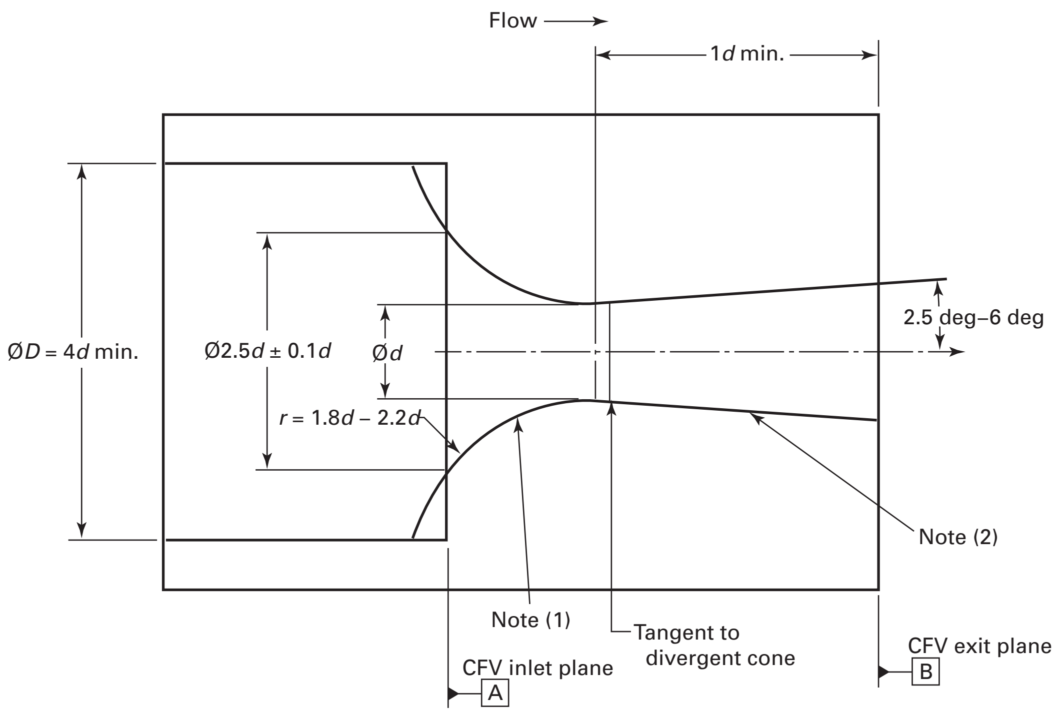
(c) The convergent part of the CFV (inlet) is a quarter of a torus tangent to the inlet plane and to the cylindrical throat. The radius of curvature of the convergent part of the CFV and the throat length is equal to the throat diameter, d. The length of the throat should equal the throat diameter within $0.05d$.

(d) The inlet toroidal surface of the CFV should not deviate from the shape of a torus by more than $0.001d$.

(e) The throat diameter should be the mean of at least four diameters measured at approximately equal angles to each other at the exit plane of the cylindrical throat. These diameters should not vary from the mean by more than $0.001d$.

(f) The transition between the convergent section and the throat should be inspected visually, and no defect should be observed. Where it can be measured, the whole inlet surface must be properly polished so that the arithmetic average roughness height does not exceed $15 \times 10^{-6}d$. If the roughness cannot be measured, the CFV should be flow calibrated. The transition between the cylindrical throat and the conical divergent section should

Fig. 6.2.1-1 Toroidal Throat CFV Geometry



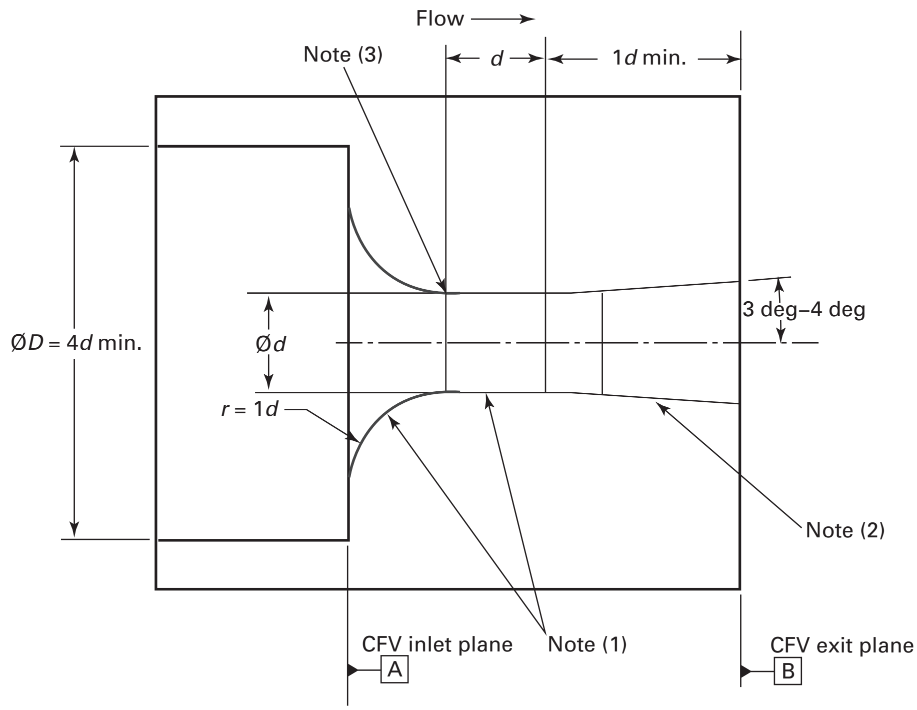
GENERAL NOTE: When it is not practical to manufacture CFVs to the surface finish and curvature specifications herein, CFV performance shall be demonstrated through calibration.

NOTES:

(1) In this region the surface shall not exceed $15 \times 10^{-6}d$ arithmetic average roughness, and the contour shall not deviate from toroidal form by more than $0.001d$.

(2) In this region the surface shall not exceed $10^{-4}d$ arithmetic average roughness.

Fig. 6.2.2-1 Cylindrical Throat CFV Geometry



GENERAL NOTE: When it is not practical to manufacture CFVs to the surface finish and curvature specifications herein, CFV performance shall be demonstrated through calibration.

NOTES:

(1) In this region the surface shall not exceed $15 \times 10^{-6}d$ arithmetic average roughness, and the contour shall not deviate from toroidal and cylindrical form by more than $0.001d$.

(2) In this region the surface shall not exceed $10^{-4}d$ arithmetic average roughness.

(3) For the transition region, see para. 6.2.2(f).

also be visually inspected, and no defect should be observed. When a defect of transition is observed, it must be checked that the local radius of curvature is never lower than $0.5d$ all along the inlet surface (convergent section and cylindrical throat).

(g) The divergent section of the CFV should be a frustum of a cone with a half-angle of 3.5 deg ± 0.5 deg. The length of the divergent section should not be less than the throat diameter.

(h) If these manufacturing tolerances cannot be achieved or verified by inspection, then flow calibration is recommended.

7 INSTALLATION REQUIREMENTS

7.1 General

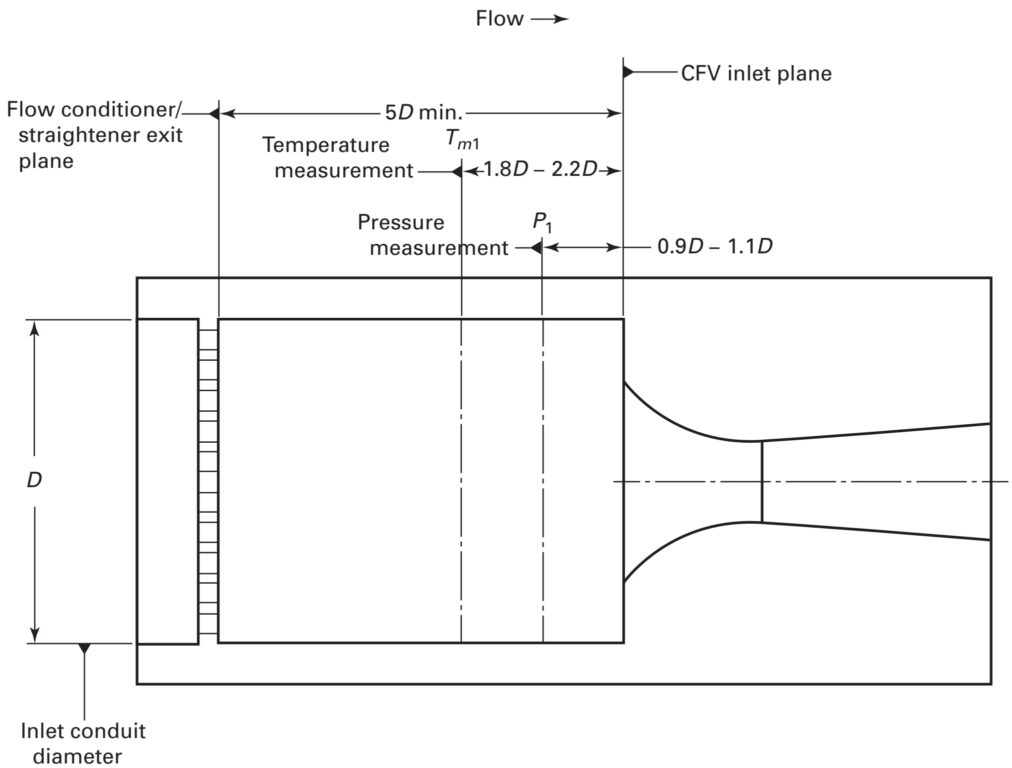
This Standard covers installation when either

(a) the pipeline upstream of the CFV is of circular cross section with $\beta \leq 0.25$ or

(b) there is a large volume (plenum) upstream of the CFV such that β is effectively zero

For the case in (a), the CFV should be installed in a system meeting the requirements of para. 7.2. For the case in (b), the CFV should be installed in a system meeting the requirements of para. 7.3. In both cases swirl must not exist upstream of the CFV. Where a pipeline is used upstream of the CFV, swirl-free conditions can be ensured by installing a flow straightener as shown in Fig. 7.1-1 at a distance $>5D$ upstream of the CFV inlet plane or any type of other flow conditioners of recognized type having equivalent or better performance (see ASME MFC-3M).

Fig. 7.1-1 Inlet Conduit Schematic



7.2 Upstream Pipeline

The CFV should be installed in a straight circular conduit that is concentric within 0.02D with the centerline of the CFV. The inlet conduit up to 3D upstream of the CFV should not deviate from circularity by more than 0.01D and should have an arithmetic roughness height that does not exceed $10^{-4}D$. In order to meet the coefficient specifications of this Standard, the diameter of the inlet conduit should be a minimum of 4d. It should be noted that the use of β ratios larger than 0.25 increases the effect of upstream disturbances, and corrections are necessary to the measured pressure and temperature (see para. 8.3).

In cases where upstream installation constraints are such that this requirement cannot be met, specific tests are recommended to investigate the influence of the installation conditions on the uncertainty of the flow measurement or the determination of C_d.

In most cases, the mass flow calculations used in this Standard will apply for $\beta > 0.25$; however, when real gas corrections are significant, the ideal calculations of the stagnation pressure and temperature (see para. 8.3) will no longer apply and corrections need to be made.

7.3 Large Upstream Volume (Plenum)

It can be assumed that there is a large volume upstream of the CFV if there is no wall closer than 5d to the axis of the CFV or to the inlet plane of the CFV (as defined in para. 3.2.2).

When multiple CFVs are used in parallel, testing should be done to ensure that performance is not degraded by interference between CFVs.[1]

NOTE: When determining the spacing requirements for CFVs mounted in parallel, the distance to the wall should also be considered.

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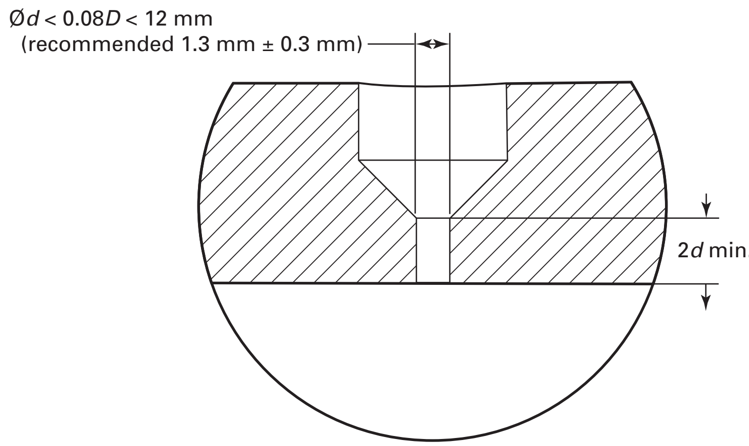
[1] For guidance for CFV spacing, see the following references:

Choi, Y. M., Park, K.-A., Park, J. T., Choi, H. M., and Park, S. O., Interference Effects of Three Sonic Nozzles of Different Throat Diameters in the Same Meter Tube, *Flow Meas. Instrum.*, 10, pp. 175–181, 1999.

Johnson, A. N., Li, C. H., Wright, J. D., Kline, G. M., and Crowley, C. J., Critical Flow Venturi Manifold Improves Gas Flow Calibrations, *Eighth International Symposium for Fluid Flow Measurement*, Colorado Springs, Colorado, USA, 2012.

Stevens, R. L., Development and Calibration of the Boeing 18 kg/sec Airflow Calibration Transfer Standard, *International Symposium on Fluid Flow Measurement*, Arlington, Virginia, USA, pp. 80–96, 1986.

Fig. 7.5-1 Pressure Tap Schematic



7.4 Downstream Requirements

No requirements are imposed on the outlet conduit except that it shall not restrict the flow so as to prevent critical flow in the CFV.

7.5 Pressure Measurement

When a circular conduit is used upstream of the primary device, the upstream static pressure should be measured at wall pressure taps located 0.9D to 1.1D from the inlet plane of the CFV (see Fig. 7.1-1). The pressure tap may be located upstream or downstream of this position, provided it has been demonstrated that the measured pressure can be used to reliably give the CFV inlet stagnation pressure.

When it can be assumed that there is a large volume upstream of the CFV, the upstream pressure tap should be located in a wall perpendicular to the inlet face of the CFV and within a distance of $10d \pm 1d$ from that plane. The pressure tap may be located upstream or downstream of this position, provided it has been demonstrated that the pressure measured can be used to reliably give the CFV inlet stagnation pressure.

For the pressure taps mentioned in the preceding paragraph, the centerline of the circular pressure tap should meet the centerline of the primary device and be at right angles to it. The edges should be free from burrs and be square or lightly rounded to a radius not exceeding 0.1 diameter of the pressure tap. Conformity of the pressure taps, with the two foregoing descriptions, is to be judged by visual inspection. When an upstream pipeline is used, the diameters of pressure taps should be 1.3 mm ± 0.3 mm (0.05 in. ± 0.02 in.) but no more than 0.08D or 12.7 mm (0.5 in.), whichever is less. The pressure tap should be cylindrical for a minimum length of two tap diameters (see Fig. 7.5-1).

The downstream pressure will be measured by a conduit wall tap to ensure that critical flow is maintained. The recommended location for the wall tap is within 0.5 conduit diameter downstream from the exit plane of the CFV so that the back pressure ratios specified in para. 8.4 are valid. However, locations further downstream can also be used, provided there is no substantial pressure change. Other tap locations may be used to check for critical flow conditions if an unchoking test is performed using that tap location.

In some applications, the outlet pressure can be determined without a pressure tap. For example, the CFV may discharge directly into the atmosphere or other region of known pressure. In these applications, the outlet pressure need not be measured.

7.6 Drain Holes

During measurement, flow must be single-phase upstream and in the throat with no condensation, and all surfaces must retain their cleanliness and surface finish. If this cannot be guaranteed, the measurement shall not be claimed to conform to this Standard.

If there is a possibility for condensate, the conduit may be provided with drain holes for the removal of condensate or other foreign substances that may collect in some applications. There must be no flow through these drain holes while the flow measurement is in progress. If drain holes are required, they should be located upstream of the CFV upstream pressure tap. The size of the drain holes is dependent on the viscosity of the fluid to be drained, but diameters are typically 6 mm to 12 mm. The axial distance from the drain hole to the plane of the upstream pressure

Table 8.1-1 Coefficients for Calculating Empirical C_d Values

CFV Type	Re_d Range	b_0	b_1	n
Toroidal	2.1×10^4 to 3.2×10^7	0.9959	2.720	0.5
Cylindrical	3.5×10^5 to 1.1×10^7	0.9976	0.1388	0.2

tap should be greater than 1*D*, and the hole should be located at the bottom or low in the upstream pipe and in a different plane from the pressure tap.

7.7 Temperature Measurement

The inlet temperature shall be measured using one or more sensors located upstream of the CFV. When an upstream pipeline is used, the recommended location of these sensors is 1.8*D* to 2.2*D* upstream of the inlet plane of the CFV. The diameter of the sensing element shall be not larger than 0.04*D*, and the element shall not be aligned with a wall pressure tap in the flow direction. If it is impractical to use a sensing element of diameter less than 0.04*D*, the sensing element shall be located so that it can be demonstrated that it does not affect the pressure measurement. The sensor may be located further upstream, provided that it has been demonstrated that the measured temperature can be used reliably to give the bulk gas temperature averaged over the CFV inlet.

Particular care should be exercised to ensure reliable temperature measurements considering such effects attributed to temperature sampling errors, time response of the temperature sensor, stem conduction, self-heating (for resistance temperature sensors), radiation effects, and heat transfer due to temperature gradient effects. Appropriate care, such as using insulation and selecting appropriate sensors, should be taken to minimize these effects.

8 CALCULATION METHODS

The mass flow through a CFV should be computed using eq. (4-3). Example calculations can be found in Nonmandatory Appendix B. In paras. 8.1 through 8.4, methods for calculation of C_d, C_R^*, P_0, T_0, and the maximum back pressure ratio are given.

8.1 Discharge Coefficient

The discharge coefficient, C_d, corrects the theoretical flow, $\dot{m}_{th}$, calculated from 1-D isentropic theory [eq. (4-2)] to give actual mass flow through a CFV. The discharge coefficient is less than unity so that the actual mass flow is lower than theoretical mass flow. Physically, $C_d < 1$ because there is a boundary layer along the CFV wall and there are momentum effects in the convergent section. Excluding the transition regime where the boundary layer transitions from laminar to turbulent flow, the discharge coefficient decreases with decreasing Re_d (decreasing mass flow or CFV diameter). CFV flows with either a laminar or a turbulent boundary layer have been successfully characterized by correlating C_d as a function of Re_d^{-n}.

The discharge coefficient can be obtained either by direct calibration against a flow reference standard (calibration method) or by using an equation based on prior research and theoretical methods (empirical method).

NOTE: For the empirical method, the throat diameter or cross-sectional area (A^*) of the CFV must be measured to achieve good flow uncertainty, whereas in the calibration method a nominal value for A^* is sufficient as long as the same value is used for both calibration and application. For small throat diameter CFVs, A^* is difficult to measure with low uncertainty, and flow calibration is generally preferred. If the nominal value of A^* is an underestimate, it is possible to obtain C_d values > 1 from the calibration method. However, if the same nominal value for the throat area is used during both the calibration and application phases, the CFV mass flow will be unaffected by the error in the nominal value of A^*. See section 9 and the example in Nonmandatory Appendix B for uncertainty effects due to errors in throat cross section area measurements.

For the empirical method, experimental discharge coefficient values for toroidal and cylindrical CFVs have been fitted to the following equation (see Table 8.1-1):[2]

$$C_d = b_0 - b_1 Re_d^{\ -n} \tag{8-1}$$

The uncertainty at a 95% confidence level for the discharge coefficients obtained from eq. (8-1) is 0.3%. Discharge coefficients are given in Tables A-1 and A-2 in Nonmandatory Appendix A.

[2] Arnberg, B. T., and Ishibashi, M., Discharge Coefficient Equations for Critical-Flow Toroidal-Throat Venturi Nozzles, *FEDSM2001-18030, Proceedings of the ASME Fluids Engineering Summer Meeting*, New Orleans, Louisiana, USA, May 2001.

NOTES:
(1) The uncertainties in the fitted equations for C_d are as large as 0.3% in part because the boundary layer transition from laminar to turbulent flow in the CFV causes a discontinuity in the discharge coefficient versus Reynolds number curve. The Reynolds number at which transition occurs is approximately 1×10^6 but depends on several factors (i.e., local curvature at the CFV throat, small geometric defects near the throat) and therefore is not known for a particular CFV without calibration against a flow reference. CFV flow uncertainty as low as 0.1% can be obtained by calibrating the CFV against a flow reference standard.
(2) The empirical method should not be used when measuring the flow of gases with significant vibrational relaxation effects (e.g., CO_2 and SF_6). Gases with energy in the vibrational modes can have C_d values that differ from the empirical method by 2% or more.[3]

CFVs that were precisely machined to match the geometry prescribed in this Standard have been manufactured and calibrated to show agreement with analytical C_d values within 0.04%.[4,5] C_d values calculated by analytical means also agree with experimental results for less precisely machined CFVs within 0.1%.[6,7] This makes CFVs an economical way to measure large gas flows with uncertainty of 0.3% or better as long as transition is avoided.

8.2 Computation of Real Gas Critical Flow Function

The critical flow function is a dimensionless thermodynamic property that accounts for real gas effects on CFV mass flow. It is defined by

$$C_R^* = \frac{\rho^* c^* \sqrt{R_u T_o}}{P_0 \sqrt{M}} \tag{8-2}$$

For specified stagnation conditions P_0 and T_0, the throat density, ρ^*, and sound speed, c^*, are calculated based on a one-dimensional, isentropic, isoenergetic flow model. Thermodynamic properties are calculated using a database that accounts for real gas effects. The isentropic condition requires that the throat entropy equals the stagnation entropy while the isoenergetic condition stipulates that the total enthalpy at the throat equals the stagnation enthalpy. Both the throat entropy, s^*, and total enthalpy, h_t^*, are independent thermodynamic variables, and therefore they determine the thermodynamic state at the CFV throat. The real gas critical flow function, C_R^*, is determined by finding the unique throat density and sound speed corresponding to the thermodynamic state at the CFV throat (see Fig. 8.2-1). An iterative procedure is generally required to determine ρ^* and c^* from the known entropy and total enthalpy (see Nonmandatory Appendix C for details).

Computations for C_R^* require a low-uncertainty thermodynamic database. In this Standard, C_R^* is computed using the REFPROP thermodynamic database.[8] This database documents the uncertainty of experimentally measured properties, includes numerous gases (e.g., O_2, CO_2, N_2, H_2, Ar, C_2H_4, and He), provides flexibility to create user-defined gas mixtures (e.g., natural gas, dry air, and humid air; see Nonmandatory Appendices C and D), and is maintained and kept up to date by the National Institute of Standards and Technology (NIST). In addition, the database internally solves the one-dimensional, isentropic, isoenergetic flow model for C_R^*. In this way C_R^* values are calculated as a function of gas composition, the stagnation pressure, and the stagnation temperature. Nonmandatory Appendix C shows values of C_R^* for selected gases and stagnation conditions.

If the flow measurement application does not require the lowest uncertainty then the ideal gas critical flow function (C_i^*) or the polytropic gas critical flow function (C_P^*) can be used instead of C_R^* (see Fig. 8.2-2).

[3] Johnson, A. N., Merkle, C. L., Moldover, M. R., and Wright, J. D., Relaxation Effects in Small Critical Nozzles, *ASME J. of Fluids Engrg.*, 128, pp. 170–176, 2006.

[4] Ishibashi, M., and Takamoto, M., Discharge Coefficient of Superaccurate Critical Nozzle Accompanied With the Boundary Layer Transition Measured by Reference Superaccurate Critical Nozzles Connected in Series, *FEDSM2001-18036, Proceedings of the ASME Fluids Engineering Summer Meeting*, New Orleans, Louisiana, USA, May 2001.

[5] Ishibashi, M., and Takamoto, M., Theoretical Discharge Coefficient of a Critical Circular-Arc Nozzle With Laminar Boundary Layer and Its Verification by Measurements Using Super-Accurate Nozzles, *Flow Meas. Instrum.*, 11, pp. 305–314, 2000.

[6] Johnson, A. N., and Wright, J. D., Comparison Between Theoretical CFV Models and NIST's Primary Flow Data in the Laminar, Turbulent, and Transition Flow Regimes, *ASME J. of Fluids Engrg.*, 130, 2008.

[7] Mickan, B., Kramer, R., Dopheide, D., Johnson, A., Wright, J., Hotze, H.-J., Hinze, H.-M., and Vallet, J.-P., Comparisons by PTB, NIST, and LNE-LADG in Air and Natural Gas with Critical Venturi Nozzles Agree Within 0.05%, *Sixth International Symposium for Fluid Flow Measurement*, Queretaro, Mexico, A.4.4, 2006.

[8] Lemmon, E. W., Huber, M. L., and McLinden, M. O., *NIST Standard Reference Database 23: Reference Fluid Thermodynamic and Transport Properties — REFPROP*, Version 9.1, National Institute of Standards and Technology, Standard Reference Data Program, Gaithersburg, Maryland, USA, 2013.

Fig. 8.2-1 Percent Difference Between the Ideal Gas Critical Flow Function, C_i^*, and the Real Gas Critical Flow Function, C_R^*, at $T_0 = 295$ K

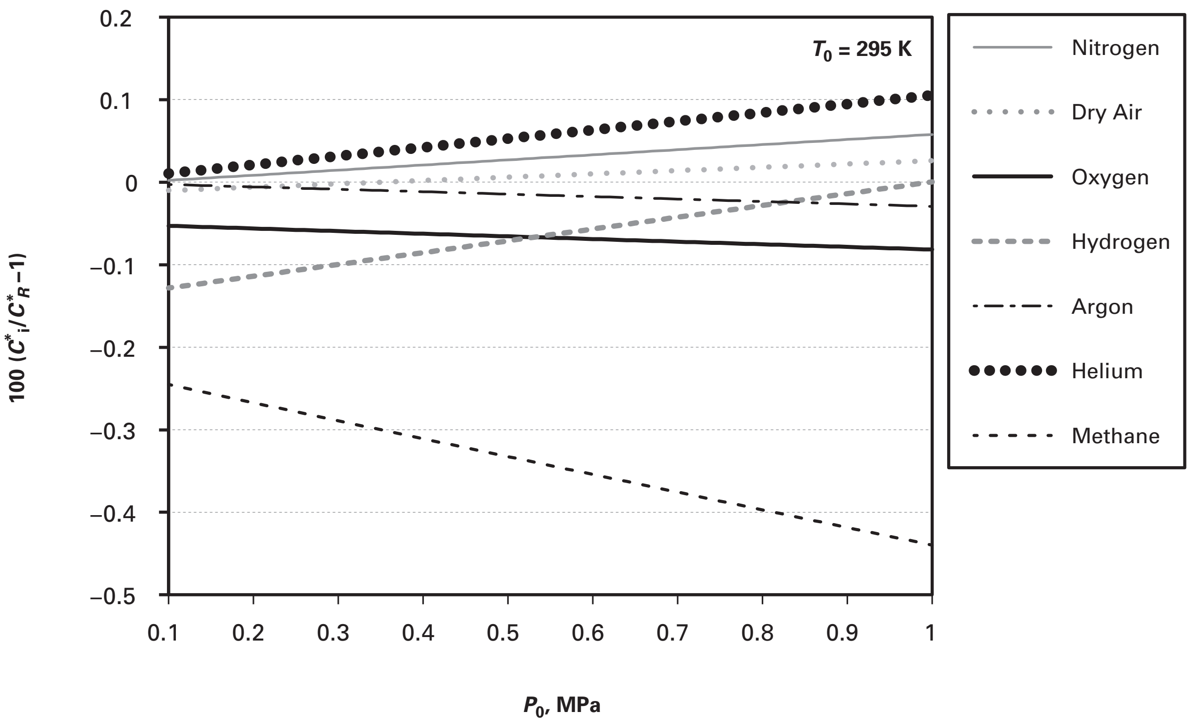
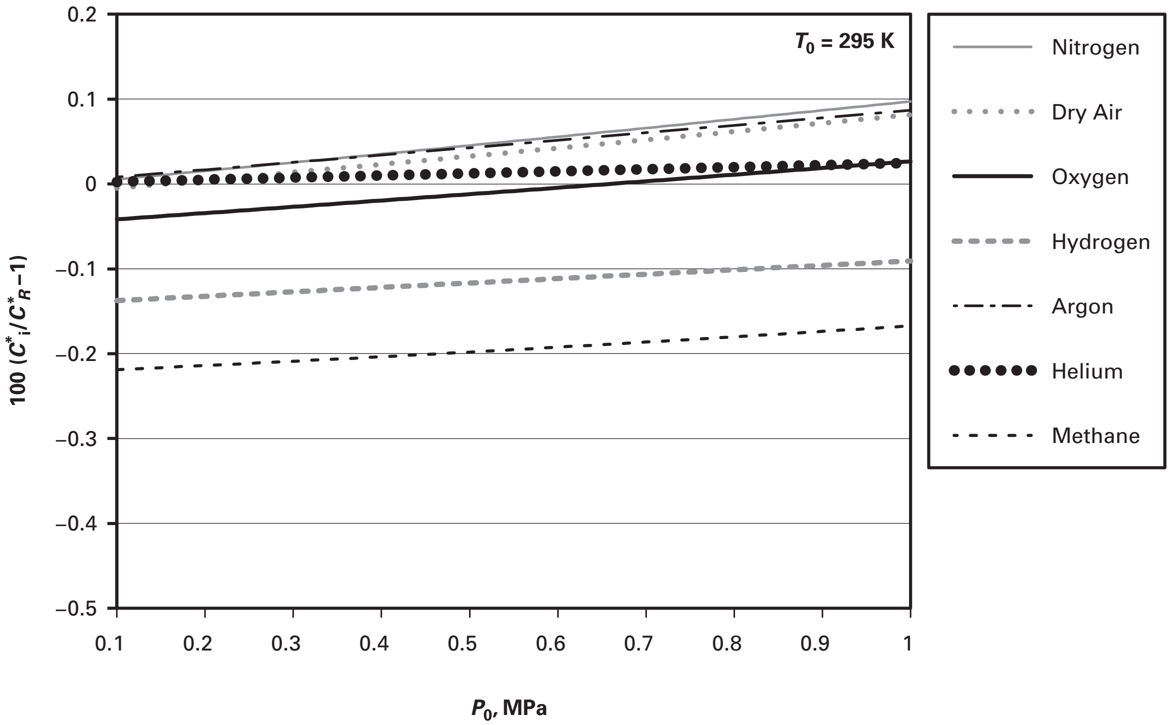


Fig. 8.2-2 Percent Difference Between the Polytropic Gas Critical Flow Function, C_p^*, and the Real Gas Critical Flow Function, C_R^*, at $T_0 = 295$ K



NOTES:
(1) If the discharge coefficient of a CFV is determined by flow calibration against a reference standard, and then used in the same gas at the same stagnation temperature and pressure, then the critical flow function should be computed using the same method for both the calibration and the application. In this case, any error introduced from using an approximate value of the critical flow factor during the calibration phase will identically cancel when the same value of the critical flow function is used in the application phase.
(2) If the discharge coefficient of a CFV is determined by flow calibration against a reference standard, and then used in a different gas at the stagnation conditions that yield the same Reynolds number, then the real gas critical flow function, C_R^*, should be used in both the calibration and the application in order to achieve the best uncertainty.

8.3 Conversion of Measured Pressure and Temperature to Stagnation Conditions

If the upstream plenum can be assumed to be infinitely large (i.e., $\beta = 0$), then the inlet Mach number can be taken to be zero ($Ma_1 = 0$). However, if flow is directed into the CFV through upstream piping with an internal diameter such that $\beta \leq 0.25$, then the Mach number in the pipe section can be calculated by

$$Ma_1 = \frac{1}{\beta^2}\left(\frac{2}{\kappa + 1}\right)^{(\kappa-3)/(2\kappa-2)}\left[1 - \sqrt{1 - 2\,\beta^4\left(\frac{2}{\kappa+1}\right)^{2/(\kappa-1)}}\right] \tag{8-3}$$

The inlet stagnation pressure can be determined from the relationship

$$P_0 = P_1\left(1 + \frac{\kappa - 1}{2}\,Ma_1^2\right)^{\kappa/(\kappa-1)} \tag{8-4}$$

and the inlet stagnation temperature may be determined from

$$T_0 = T_{m1}\left[1 + \frac{\kappa - 1}{2}\,Ma_1^2\,(1 - R_f)\right] \tag{8-5}$$

NOTES
(1) For CFV installations with $\beta \leq 0.25$, the magnitude of the difference between the measured pressure and the computed stagnation pressure is less than 0.1% as shown in Fig. 8.3-1. Therefore the Mach number, stagnation pressure, and stagnation temperature can be calculated with comparable uncertainty if the specific heat ratio, γ, is substituted for the polytropic exponent, κ. For example, the stagnation temperature could be computed with $T_0 = T_{m1}\,[1 + 0.5(\gamma - 1)\,Ma_1{}^2\,(1 - r_f)]$. When calculations of the ideal gas critical flow function, C_i^*, or when $\beta > 0.25$, low-uncertainty values of γ are required. When $\beta \leq 0.25$, estimated values of γ are adequate for calculating the Mach number, stagnation pressure, and stagnation temperature. For example, $\gamma = 5/3$ for ideal monatomic gases, $7/5$ for ideal diatomic gases, and $9/7$ for ideal triatomic gases can be used for the calculations.
(2) Temperature measurements are made by inserting a probe into the flow line. The temperature measured by the probe, T_{m1} in the equation, will be a function of the probe design, the fluid properties, the flow field at the probe, and the wall temperature. The measured temperature, T_{m1}, will be between the actual flowing static temperature, T_1, and the stagnation temperature, T_0. For CFVs with $\beta \leq 0.25$ the difference between T_1, T_{m1}, and T_0 will be small. However, if $\beta > 0.25$, the stagnation temperature and pressure corrections can be significant and should be made.
(3) In cases where a significant temperature correction is required ($\beta > 0.25$) the recovery factor, R_f, which is known, tested, or estimated, can be used to correct the measured temperature. The recovery factor is approximately constant for a given probe design and flow situation. The value of the recovery factor can range from 0.5 to 0.99. This Standard recommends a value of 0.75, a common value for many temperature probes.

8.4 Maximum Permissible Downstream Pressure (Maximum Back Pressure Ratio)

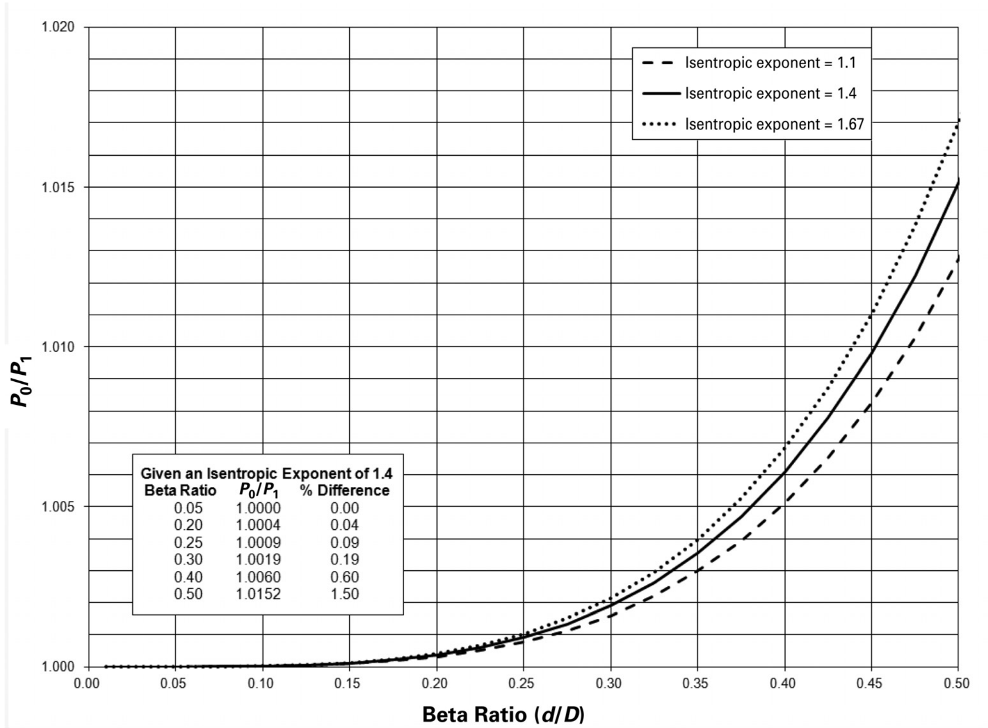
For CFVs operating at throat Reynolds numbers greater than 2×10^5 and with diffusers longer than $1d$, the maximum permissible downstream pressure can be estimated by the relationship

$$(P_2/P_0)_{\max} = 0.8\,[(P_2/P_0)_i - r^*] + r^* \tag{8-6}$$

where

$$r^* = \left(\frac{2}{\kappa + 1}\right)^{\kappa/(\kappa-1)} \tag{8-7}$$

Fig. 8.3-1 Difference Between Static and Stagnation Pressure for Various Beta Ratios and Isentropic Exponent Values



and

$$(P_2/P_0)_i = \left(1 + \frac{\kappa - 1}{2} Ma_2{}^2\right)^{-\kappa/(\kappa-1)} \tag{8-8}$$

and

$$Ma_2 = (A_2/A^*)\left(\frac{2}{\kappa + 1}\right)^{(\kappa-3)/(2\kappa-2)}\left\{1 - \sqrt{1 - 2\left[\frac{1}{(A_2/A^*)}\right]^2\left(\frac{2}{\kappa + 1}\right)^{2/(\kappa-1)}}\right\} \tag{8-9}$$

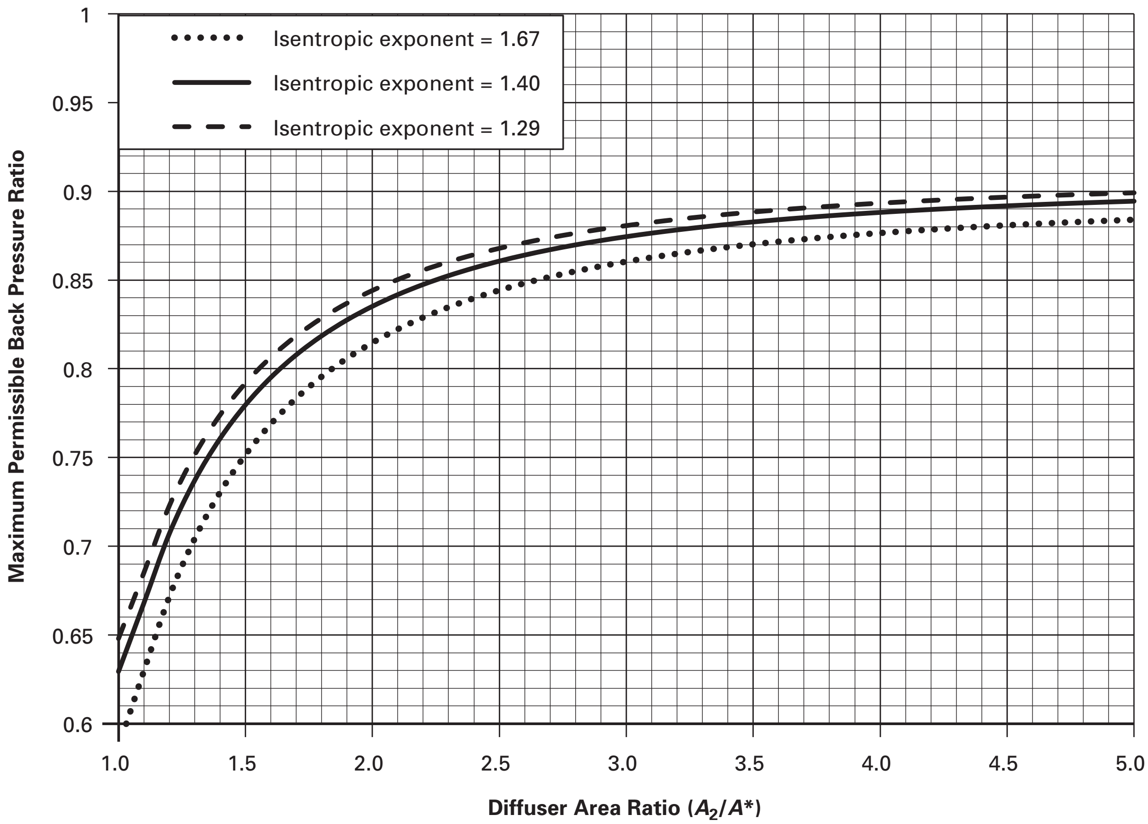
The value of $(P_2/P_0)_i$ is determined from the one-dimensional compressible flow of an ideal gas relationship as a function of area ratio (A_2/A^*) of the divergent section. Sample values of $(P_2/P_0)_{max}$ may be found in Fig. 8.4-1.

Higher back pressure ratios than shown can be used, provided it can be verified that the flow is critical. For Reynolds numbers greater than 2×10^5, the pressure ratio P_2/P_0 is not significantly affected by extending the cone length such that the exit area is greater than four times the throat area, i.e., beyond seven diameters for a cone half-angle of 4 deg.

For CFVs operating at throat Reynolds numbers from 5×10^4 to 2×10^5, it is recommended that users maintain a minimum back pressure ratio equal to the critical pressure ratio, r^*, or perform an unchoking test on their CFVs.

For CFVs operating at throat Reynolds numbers below 5×10^4, it is recommended that users maintain a back pressure ratio of $(P_2/P_0)_{max} = 0.30$ or perform an unchoking test on their CFVs. For some CFV diffuser geometries operated at these low Reynolds numbers, a decrease in C_d is observed from a back pressure ratio of approximately 0.35 to 0.50. This diffuser performance inversion, sometimes referred to as "premature unchoking" can be minimized by using 3-deg to 4-deg half-angles and diffuser lengths of 10d or greater.

Fig. 8.4-1 Recommended Maximum Back Pressure Ratio Versus Diffuser Area Ratio for Various Isentropic Exponent Values



GENERAL NOTE: Fig. 8.4-1 applies only for throat Reynolds numbers greater than 2×10^5.

Pressure ratios as high as 0.95 can be obtained for some CFVs at high Reynolds numbers. An unchoking test should be conducted to determine the $(P_2/P_0)_{max}$.

The procedure for performing an unchoking test can be found in Nonmandatory Appendix E.[9]

9 UNCERTAINTY OF CFV FLOW MEASUREMENTS

9.1 General Considerations

The uncertainty associated with each measurement of mass flow is an essential consideration and shall be calculated and reported whenever a measurement is claimed to conform to this Standard. The uncertainty for a mass flow measurement may be expressed in relative terms as a percentage, in relative (dimensionless) terms, or in absolute terms with the same units as the given mass flow. Uncertainty may be expressed as a standard uncertainty, u (at a confidence level of 68%), or as a combined standard uncertainty u_c, or as an expanded uncertainty U, which is usually the final result with a 95% confidence level. The uncertainty for mass flow as determined using eq. (4-3) is most simply evaluated using relative uncertainties expressed as a percentage. The quantities and notation herein refer to relative uncertainties expressed as percentages of the average value. Uncertainty calculations should conform to the procedures given in the following paragraphs of this Standard.[10]

[9] Maximum back pressure guidelines for CFVs with specific diffuser geometry operated with dry air in the Re range from 12,000 to 250,000 can be found in Carter, M., Sims, B., Britton, C., and McKee, R., Choking Pressure Ratio Guidelines for Small Critical Flow Venturis and the Effects of Diffuser Geometry, *16th International Flow Measurement Conference*, Paris, France, 2013.

[10] ISO/IEC Guide 98–3:2008.

In general, the expanded uncertainty for a measurand, U(mran), can be calculated from the combined standard uncertainty, u_c(mran), comprised of the relative Type A uncertainty, u_A(mran), which is obtained using statistics, and the relative Type B uncertainty, u_B(mran), which is obtained using methods other than statistics. These are combined by root sum of squares (RSS) as follows:

$$U(\mathrm{mran}) = k \times u_c(\mathrm{mran}) = k\sqrt{u_A(\mathrm{mran})^2 + u_B(\mathrm{mran})^2} \tag{9-1}$$

where k is the coverage factor, and $k = 1$ indicates a 68% confidence level and $k = 2$ is generally appropriate to calculate a 95% confidence level uncertainty. The Type A uncertainty, $u_A(\dot{m})$, is the standard deviation, at 68% confidence level, of the replicated measurements of mass flow from the repeatability and reproducibility test results. The relative Type B uncertainty, $u_B(\dot{m})$, is obtained from evaluations of the uncertainty of the components in the equation used to calculate the mass flow. It is common and usually more convenient to perform uncertainty calculations in relative uncertainty terms as shown in the practical computation methods that follow. Using relative uncertainty terms allows the use of normalized sensitivity coefficients, which are the exponents of the respective factors in the governing equation for the quantity being assessed.

Using a coverage factor $k = 2$ to obtain a 95% confidence level in uncertainty is appropriate when the system has many degrees of freedom, as is normally assumed for Type B uncertainty components. The degrees of freedom for the Type A component, calculated from the standard deviation of n repeated measurements, is $n - 1$, and this can lead to larger values of k when the number of replicates is approximately 20 or less. The Welch-Satterthwaite formula[10] allows calculation of effective degrees of freedom and, when the repeatability (Type A component) is small relative to the Type B components, leads to a coverage factor of approximately 2 for 95% confidence level. The Welch-Satterthwaite formula is applied in the examples in Nonmandatory Appendix B. There and in most applications, $k = 2$ is an acceptable value to obtain 95% confidence level uncertainties.

9.2 Practical Computation of Uncertainty

Equation (4-3), the governing equation for mass flow, $\dot{m}$, through a CFV, is

$$\dot{m} = \frac{C_d A^* C_R^* P_0}{\sqrt{(R_u/M)\, T_0}}$$

The uncertainty of a flow measurement should be calculated from the standard deviation of the available flow measurement data plus evaluations of uncertainty of the individual quantities in the governing equation. In most cases, uncertainty components can be assumed to be independent and uncorrelated, but there are certain cases where correlated uncertainties are an issue (see para. 9.3).

Because eq. (4-3) is comprised of products, it is simplest to use normalized sensitivity coefficients and relative uncertainties expressed as percentages (the dimensional uncertainty divided by the average value of the respective component). In this way, the sensitivity coefficients for a function like eq. (4-3) are the exponents of the respective components. In the case where the uncertainty components are assumed to be independent and uncorrelated or when the respective correlated effects are negligible, a practical working formula for calculating the relative combined standard uncertainty by RSS is

$$u_c(\dot{m}) = \sqrt{[u_A(\dot{m})]^2 + [u_B(C_D)]^2 + [u_B(A^*)]^2 + [u_B(C_R^*)]^2 + [u_B(P_0)]^2 + \frac{1}{4}\,[u_B(R_u)]^2 + \frac{1}{4}\,[u_B(M)]^2 + \frac{1}{4}\,[u_B(T_0)]^2} \tag{9-2}$$

where the squares of the square-bracketed terms are variances.

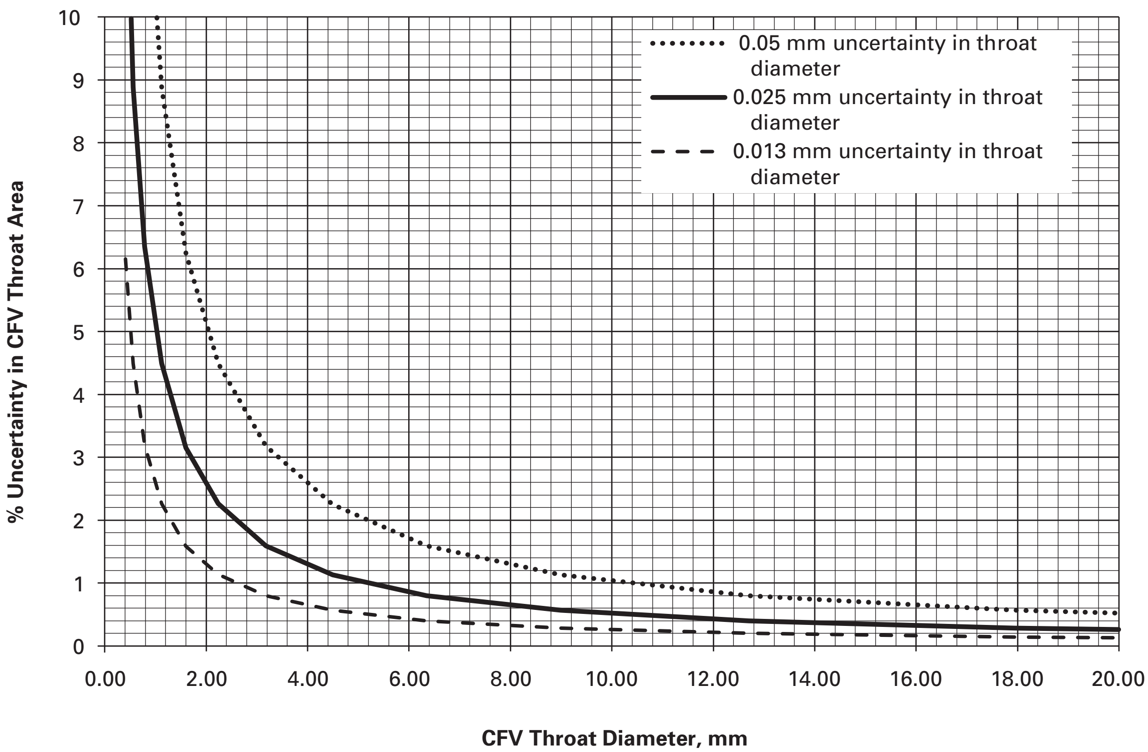
The Type A relative uncertainty term, $u_A(\dot{m})$, can be calculated from the standard deviation of the available replicated measurement data.

The Type B relative uncertainty terms in eq. (9-2) can be calculated from the uncertainty of each factor with the absolute uncertainties of each component being divided by the magnitude of that component to determine the relative uncertainties. Then the relative uncertainty terms are squared and combined by the RSS relationship.

Each uncertainty component has its own subset of uncertainty sources. Some of the uncertainty components that should be considered are

(a) long-term reproducibility (drift) of the discharge coefficient
(b) pressure and temperature sensor calibrations
(c) drift between periodic calibrations
(d) temperature effects on the CFV mass flow (e.g., stem conduction)
(e) sampling errors

Fig. 9.2-1 Percent Uncertainty in CFV Throat Area due to Uncertainty in Throat Diameter Measurement



(f) thermal expansion of the throat
(g) interference effects between CFVs in a plenum
(h) species effects (calibration in one gas, usage in another)
(i) leaks
(j) contamination of CFV surfaces with dirt
(k) pressure effects because real gas effects are not perfectly captured (e.g., errors in C_R^*)

An example of a subcomponent study is presented in Fig. 9.2-1, which shows the relationship between diameter uncertainties (of various magnitudes) and area uncertainties.

9.3 Correlated Uncertainty Components

In some measurement situations the components are not fully or predominately independent and the correlation of variables must be considered.

For the measurement cases where the terms in the governing equation cannot be assumed to be independent and the degree of correlation is significant, the computations become somewhat more complex because the respective relative correlation terms should be included. The correlated variable terms are computed or evaluated from data for the respective interacting terms.

For example, both C_R^* and M depend on the gas composition. When these terms are included, the combined relative uncertainty equation becomes

$$u_c(\dot{m}) = \sqrt{[u_A(\dot{m})]^2 + [u_B(C_D)]^2 + [u_B(A^*)]^2 + [u_B(C_R^*)]^2 + [u_B(P_0)]^2 + \frac{1}{4}[u_B(R_u)]^2 + \frac{1}{4}[u_B(M)]^2 + \frac{1}{4}[u_B(T_0)]^2 + \frac{1}{2}CV(M,gc)} \tag{9-3}$$

where *CV(M,gc)* is the covariance of the molecular weight and gas composition, as evaluated using data, and other correlated uncertainty terms have been left out because they are either zero or negligible. For example, the universal gas constant, R_u, is known to such low uncertainty, or if the same value of R_u is used during calibration and usage, its uncertainty can be neglected.

For some gases at high pressure, the normalized sensitivity coefficients for the stagnation pressure and temperature can deviate from unity due to contributions from the critical flow function. For example, for natural gas at room temperature and 9 MPa (1,300 lbf/in.^{2}), the sensitivity coefficient for pressure increases from 1 to 1.03.[11] In such cases, the upper value should be used to bound the effect.

CFVs used in a plenum also have significant correlated uncertainty in pressure, temperature, and C_d calibration.

Tabulating uncertainty assessments provides clear understanding of both the whole process and the quantitative results. Flow calculation examples and corresponding uncertainty assessments are given in Nonmandatory Appendix B, along with a suggested tabular format. Uncertainties are presented for two examples

(a) where C_d is determined using the empirical method (see para. 8.1)

(b) where C_d is from the calibration method

[11] Johnson, A. N., Natural Gas Flow Calibration Service, SP250-1081, NIST.

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NONMANDATORY APPENDIX A
CFV DISCHARGE COEFFICIENTS

See Table A-1 for toroidal throat CFV discharge coefficients.
See Table A-2 for cylindrical throat CFV discharge coefficients.

Table A-1 Toroidal Throat CFV Discharge Coefficient

Reynolds Number, Re_d	Discharge Coefficient, C_d
3×10^4	0.9802
4×10^4	0.9823
5×10^4	0.9837
6×10^4	0.9848
7×10^4	0.9856
8×10^4	0.9863
9×10^4	0.9868
1×10^5	0.9873
2×10^5	0.9898
3×10^5	0.9909
4×10^5	0.9916
5×10^5	0.9921
6×10^5	0.9924
7×10^5	0.9926
8×10^5	0.9929
9×10^5	0.9930
1×10^6	0.9932
2×10^6	0.9940
3×10^6	0.9943
4×10^6	0.9945
5×10^6	0.9947
6×10^6	0.9948
7×10^6	0.9949
8×10^6	0.9949
9×10^6	0.9950
1×10^7	0.9950
2×10^7	0.9953
3×10^7	0.9954

Table A-2 Cylindrical Throat CFV Discharge Coefficient

Reynolds Number, Re_d	Discharge Coefficient, C_d
4×10^5	0.9871
5×10^5	0.9875
6×10^5	0.9879
7×10^5	0.9882
8×10^5	0.9884
9×10^5	0.9887
1×10^6	0.9888
2×10^6	0.9900
3×10^6	0.9906
4×10^6	0.9910
5×10^6	0.9913
6×10^6	0.9915
7×10^6	0.9917
8×10^6	0.9918
9×10^6	0.9920
1×10^7	0.9921

NONMANDATORY APPENDIX B
EXAMPLE FLOW AND UNCERTAINTY CALCULATIONS

B-1 INTRODUCTION

This Appendix presents two examples of how the mass flow through a toroidal throat CFV installed in a circular conduit is calculated. Spreadsheet calls for use with the REFPROP gas properties database are provided for reference.

B-2 CALCULATIONS

B-2.1 C_d From Correlations

In this example, the discharge coefficient for the CFV is calculated using eq. (8-1). The CFV throat diameter was measured at 15.56°C (60.00°F).

Given:

diameter of CFV throat, $d = 0.1600$ cm (0.06300 in.)

diameter of upstream conduit, $D = 2.540$ cm (1.000 in.)

absolute inlet static pressure, $P_1 = 0.3447$ MPa (50.00 lbf/in.^{2})

inlet static temperature, $T_1 = 21.11$°C (70.00°F)

gas constant, $R = 8\,314$ J/kg × mole × K (1,545 ft × lbf/lbm × mole × °R)

dry air, where $N_2 = 0.7809$, $O_2 = 0.2094$, $Ar = 0.009332$, $CO_2 = 3.85 \times 10^{-4}$, and $He = 5 \times 10^{-6}$

REFPROP call to define dry air composition (A1 represents a cell in a spreadsheet):
A1=CONCATENATE ("Nitrogen",";",0.7809,";",Oxygen,";",0.2094,";","Argon",";",0.009332,";",Carbon Dioxide",";",3.85E-4,";",Helium,";",5E-6)

Calculation:
From eq. (4-3)

$$\dot{m} = \frac{C_d A^* C_R^* P_0}{\sqrt{(R_u/M)T_0}}$$

where
$A^* = \pi(d/2)^2 = 0.02011$ cm^2 (0.003117 in.^{2})
The measured temperature and pressure must be converted to stagnation conditions.
First calculate the Mach number in the upstream piping, using the value $\beta = d/D = 0.06300$.

REFPROP call for polytropic exponent:
=IsentropicExpansionCoef (Composition!A1,"TP","SI with C",21.11,0.3447)]
[=IsentropicExpansionCoef (Composition!A1,"TP","E",50.00,70.00)]

$$Ma_1 = \frac{1}{\beta^2}\left(\frac{2}{\kappa+1}\right)^{(\kappa-3)/(2\kappa-2)}\left[1-\sqrt{1-2\beta^4\left(\frac{2}{\kappa+1}\right)^{2/(\kappa-1)}}\right] \quad \text{(B-1)}$$

$$Ma_1 = \frac{1}{0.06300^2}\left(\frac{2}{1.405+1}\right)^{(1.405-3)/(2\times1.405-2)}\left\{1-\sqrt{1-2\times0.06300^4[2/(1.405+1)]^{2/(1.405-1)}}\right\} = 0.002296$$

Using the upstream Mach number and polytropic exponent, calculate the stagnation pressure.

$$\kappa = 1.405$$

$$P_0 = P_1 \left(1 + \frac{\kappa - 1}{2} Ma^2\right)^{\kappa/(\kappa-1)} \quad \text{(B-2)}$$

(SI Units)

$$P_0 = 0.3447 \left(1 + \frac{1.405 - 1}{2} 0.002296^2\right)^{\frac{1.405}{1.405-1}} = 0.3447 \text{ MPa}$$

(U.S. Customary Units)

$$P_0 = 50.00 \left(1 + \frac{1.405 - 1}{2} 0.002296^2\right)^{\frac{1.405}{1.405-1}} = 50.00 \text{ lbf/in.}^2$$

Using the upstream Mach number and polytropic exponent, calculate the stagnation temperature.

$$T_0 = T_1 \left[1 + \frac{\kappa - 1}{2} Ma^2 (1 - R_f)\right]$$

(SI Units)

$$T_0 = (21.1 + 273.2) \left[1 + \frac{1.405 - 1}{2} 0.002296^2(1 - 0.75)\right] = 294.3 \text{ K}$$

(U.S. Customary Units)

$$T_0 = (70 + 459.7) \left[1 + \frac{1.405 - 1}{2} 0.002296^2(1 - 0.75)\right] = 529.7°\text{R}$$

REFPROP call for real gas critical flow function:
= Cstar(Composition!A1,“TP”,“SI with C”,21.11,0.3447)
[= Cstar(Composition!A1,“TP”,“E”,70.00,50.00)]

$$C_R^* = 0.6858$$

REFPROP call for molar mass:
=MolarMass(Composition!A1,"TP","SI with C",21.11,0.3447
[= MolarMass(Composition!A1,"TP","E",70.00,50.00])

$$M = 28.97 \frac{\text{kg}}{\text{kg} \times \text{mole}} \quad \left(28.97 \frac{\text{lbm}}{\text{lbm} \times \text{mole}}\right)$$

To start the iteration assume the discharge coefficient, C_d, equals 1.0 and calculate the mass flow [from eq. (4-3)]. This will result in

(SI Units)

$$\dot{m} = \frac{1.0 \times [0.02011/(100^2)](0.6858) \times 0.3447 \times 10^6}{\sqrt{\frac{(8314)}{28.97} \times (294.3)}} = 0.001636 \text{ kg/s}$$

(U.S. Customary Units)

$$\dot{m} = \frac{1.0 \times (0.003117) \times (0.6858) \times 50 \times \sqrt{32.17}}{\sqrt{\frac{(1{,}545)}{28.97} \times 529.7}} = 0.003606 \text{ lbm/sec}$$

Using this mass flow, calculate the throat Reynolds number. From eq. (3-10)

$$Re_d = \frac{4\dot{m}}{\pi d \mu_0}$$

REFPROP call for dynamic viscosity:
=viscosity(Composition!A1,"TP","SI with C",21.11,0.3447)
[=viscosity(Composition! A1,"TP","E",70.00,50.00)]

$$\mu_0 = 18.34 \ \frac{\mu Pa}{s} \qquad \left(1.232 \times 10^{-5} \ \frac{\text{lbm}}{\text{ft} \cdot \text{sec}}\right)$$

(SI Units)

$$Re_d = \frac{400 \times 0.001636}{\pi \times 0.1600 \times 18.34 \times 10^{-6}} = 70\,990$$

(U.S. Customary Units)

$$Re_d = \frac{48 \times 0.003606}{\pi \times 0.06300 \times 1.232 \times 10^{-5}} = 70{,}980$$

From para. 8.1

$$C_d = 0.9959 - \frac{2.720}{\sqrt{Re_d}}$$
$$= 0.9857$$

Recalculate the mass flow with the new C_d and continue the Re_d, C_d, and $\dot{m}$ iteration until the solution converges. Final values after the solution has converged will be

$$Re_d = 69\,950 \ (69{,}970)$$

$$C_d = 0.9856$$

$$\dot{m} = 0.001612 \text{ kg/s } (0.003554 \text{ lbm/sec})$$

Uncertainty, u, in Calculated Mass Flow:
From the governing equation for $\dot{m}$

$$\dot{m} = \frac{C_d A^* C_R^* P_0}{\sqrt{(R_u/M)T_0}}$$

The relative differentials equation for $\dot{m}$ is

$$\frac{\Delta \dot{m}}{\overline{\dot{m}}} = \frac{\Delta C_d}{\overline{C}} + \frac{\Delta A^*}{\overline{A^*}} + \frac{\Delta C_R^*}{\overline{C^*_R}} + \frac{\Delta P_0}{\overline{P}_0} - \frac{1}{2}\frac{\Delta R_u}{\overline{R}_u} + \frac{1}{2}\frac{\Delta M}{\overline{M}} - \frac{1}{2}\frac{\Delta T_0}{\overline{T}_0} \tag{B-3}$$

where the overbars denote averages of respective quantities. Squaring eq. (B-3) and forming (relative) variances and covariances for the terms that are correlated gives the relative variance for $\dot{m}$

$$\begin{aligned}[u_c(\dot{m})^2] = {} & [u_A(\dot{m})^2] + [u_B(A^*)]^2 + [u_B(C_d)]^2 + [u_B(C_R^*)]^2 + [u_B(P_0)]^2 + \frac{1}{4}[u_B(R_u)]^2 + \frac{1}{4}[u_B(M)]^2 \\ & + \frac{1}{4}[u_B(T_0)]^2 + 2CV(C_R^*, P_0) + 2CV(C_R^*, T_0) + \frac{1}{2}CV(M, gc)\end{aligned} \tag{B-4}$$

resulting in the equation used to assess the relative uncertainty for $\dot{m}$ with correlation effects

$$u_c(\dot{m}) = \sqrt{\begin{array}{l}[u_A(\dot{m})]^2 + [u_B(C_d)]^2 + [u_B(A^*)]^2 + [u_B(C_R^*)]^2 + [u_B(P_0)]^2 + \frac{1}{4}[u_B(R_u)]^2 + \frac{1}{4}[u_B(M)]^2 \\ \quad + \frac{1}{4}[u_B(T_0)]^2 + 2r_{C^*_{R,P_0}} u_B(C_R^*)u_B(P_0) + 2r_{C^*_{R,T_0}} u_B(C_R^*)u_B(T_0) \\ \quad\quad + \frac{1}{2} r_{M,gc} u_B(M) u_B(gc)\end{array}} \tag{B-5}$$

where the correlation coefficients are defined as

$$r_{x,y} = \frac{CV(x,y)}{u(x)u(y)}$$

Where the correlated uncertainties are negligible, the remaining significant components of uncertainty are the C_d obtained for the correlation eq. (8-1), the pressure measurement, and the temperature measurement. However, uncertainties that are not obvious from the mass flow equation should be considered as well, e.g., the stability of the C_d over time (these being best quantified using subsequent calibrations) and any varying conditions of usage (such as different gas composition, environmental conditions, or P_0 or T_0). In a similar way, the uncertainties for temperature and pressure should include uncertainties from sensor calibration reports as well as other relevant uncertainty sources, e.g., spatial nonuniformity in the approach pipe, drift over time, and time to reach steady-state conditions. Experimental measurements are often necessary to quantify these uncertainties. In this example an uncertainty associated with the thermal expansion of the CFV is included.

The relative combined uncertainty, assuming no correlation effects, can then be calculated with the following equation:

$$u_c(\dot{m}) = \sqrt{[u_A(\dot{m})]^2 + [u_B(C_d)]^2 + [u_B(A^*)]^2 + [u_B(C_R^*)]^2 + [u_B(P_0)]^2 + \frac{1}{4}[u_B(R_u)]^2 + \frac{1}{4}[u_B(M)]^2 + \frac{1}{4}[u_B(T_0)]^2 + [u_B(T_{ex})]^2} \tag{B-6}$$

where the individual terms are

$u_A(\dot{m})$ = 0.1% [68% confidence level (CL)]: value from the standard deviation of 10 replicated tests
$u_B(A^*)$ = 3.15% (100% CL): value established by using Go/No Go gauge pins with diameter increments of 0.0025 cm (0.001 in.) to measure CFV throat diameter
$u_B(C_d)$ = 0.30% (95% CL): value specified in para. 8.1.1
$u_B(C_R{}^*)$ = 0.025% (68% CL): value from uncertainty in $C_R{}^*$ output from REFPROP
$u_B(P_0)$ = 0.01% (68% CL): value from pressure transducer manufacturer's specification
$u_B(R_u)$ = 0.00% (68% CL): value is known to very low uncertainty and can be neglected
$u_B(M)$ = 0.0025% (68% CL): value from uncertainty in M output from REFPROP
$u_B(T_0)$ = 0.05% (68% CL): value from temperature sensor manufacturer's specification, sensor drift between calibrations, sampling errors, and thermal equilibrium
$u_B(T_{ex})$ = 0.01% (68% CL): value from two alpha models of thermal expansion from temperature where throat diameter was measured to operational temperature

$$u_c(\dot{m}) = \sqrt{\begin{array}{l}[u_A(0.1)]^2 + [u_B(0.30/2)]^2 + [u_B(3.15/1.732)]^2 + [u_B(0.025)]^2 + u_B(0.01)]^2 \\ + \frac{1}{4}[u_B(0.0025)]^2 + \frac{1}{4}[u_B(0.05)]^2 + [u_B(0.01)]^2\end{array}} = 1.83 \tag{B-7}$$

The uncertainty calculations and results are displayed in Table B-2.1-1.

The resulting expanded uncertainty is

$$U_c(\dot{m}) = 3.66\% \text{ (95\% CL)}$$

The complete specification of the measurement result is $\dot{m} = 0.001612$ kg/s (0.003554 lbm/sec) with an expanded uncertainty of 3.66% ($k = 2$, at 95% CL and with 1.0×10^5 effective degrees of freedom).

Table B-2.1-1 Uncertainty Calculation and Results

Uncertainty Characteristics				Processing Uncertainties				Uncertainty Results		
Quantity	Source of Uncertainty	u(Quantity), %	Confidence Level, %	Type of Uncertainty	Probability Distribution	Divisor	Relative Sensitivity Coefficient	u(Quantity), %	Degrees of Freedom	Combined, %
Mass flow replications	Replicated measurements	0.1	68	A	Normal	1	1	0.1000	9 [Note (1)]	0.3
Throat area	Instrument specification	3.15 [Note (2)]	100 [Note (2)]	B	Rectangular	1.732	1	1.8187 [Note (2)]	Infinite	99.0
Discharge coefficient	Empirical equation	0.3	95	B	Normal	2	1	0.1500	Infinite [Note (3)]	0.7
Critical flow function	Data tabulation	0.025 [Note (4)]	68	B	Normal	1	1	0.0250 [Note (4)]	Infinite	0.0
Stagnation pressure	Instrument specification	0.01 [Note (5)]	68 [Note (5)]	B [Note (5)]	Normal [Note (5)]	1 [Note (5)]	1 [Note (5)]	0.0100 [Note (5)]	Infinite [Note (5)]	0.0
Universal gas constant	Data tabulation	0 [Note (6)]	68 [Note (6)]	B [Note (6)]	Normal [Note (6)]	1 [Note (6)]	0.5 [Note (6)]	0.0000 [Note (6)]	Infinite [Note (6)]	0.0
Molecular weight	Data tabulation	0.0025 [Note (7)]	68 [Note (7)]	B [Note (7)]	Normal [Note (7)]	1 [Note (7)]	0.5 [Note (7)]	0.0013 [Note (7)]	Infinite [Note (7)]	0.0
Stagnation temperature	Instrument specification	0.05 [Note (8)]	68 [Note (8)]	B [Note (8)]	Normal [Note (8)]	1 [Note (8)]	0.5 [Note (8)]	0.0250 [Note (8)]	Infinite [Note (8)]	0.0
Thermal expansion	2 alpha model	0.01 [Note (9)]	68 [Note (9)]	B [Note (9)]	Rectangular [Note (9)]	1.732 [Note (9)]	1 [Note (9)]	0.0058 [Note (9)]	Infinite [Note (9)]	0.0
Mass flow relative combined uncertainty, % →							RSS	1.828 [Note (10)]	1.0E+05 [Note (10)]	...
Mass flow relative expanded uncertainty, % →							2 × RSS	3.66 [Note (11)]	1.0E+05 [Note (11)]	...

NOTES:

(1) The degrees of freedom are based on the 10 replications taken for the measurement.

(2) Entries are based on the uncertainty of the throat diameter measurement.

(3) Entry is based on the correlation equation for the calculation of the discharge equation.

(4) Entries are based on the uncertainty in the REFPROP thermodynamic database.

(5) Entries are based on the instrument manufacturer's specification.

(6) Entries are based on the fact that the universal gas constant is known to a very low uncertainty.

(7) Entries are based on the uncertainty in the composition of the actual test gas.

(8) Entries are based on the instrument manufacturer's specification.

(9) Entries are based on a 2 alpha model of the change in throat area due to thermal expansion.

(10) RSS is the result from the values in the previous rows. The effective degrees of freedom are computed with the Welsh-Satterthwaite equation

$$\upsilon_{\text{eff}} = \frac{[u_c(\dot{m})]^4}{\sum_{i=1}^{N} \frac{[u(x_i)]^4}{v_i}}$$

(11) Entries give the uncertainty as 3.66% at 95% confidence level for 1.0E+05 effective degrees of freedom.

B-2.2 C_d Determined Through Flow Calibration

In this example, the discharge coefficient for the CFV is calculated with a 0.25% (95% CL) laboratory C_dA calibration that was performed 10 times at 21.11°C (70.00°F).

Given:

diameter of CFV throat, $d = 0.1600$ cm (0.06300 in.)

CFV laboratory calibration, $C_d = 0.9737 - 3.730/\sqrt{Re_d}$

diameter of upstream conduit, $D = 0.5334$ cm (0.210 in.)

absolute inlet static pressure, $P_1 = 0.3447$ MPa (50.00 lbf/in.^{2})

inlet static temperature, $T_1 = 21.11$°C (70.00°F)

gas constant, $R = 8\,314$ J/kg × mole × K (1,545 ft × lbf/lbm × mole × °R)

dry air, where $N_2 = 0.7809$, $O_2 = 0.2094$, Ar $= 0.009332$, $CO_2 = 3.85 \times 10^{-4}$, and He $= 5 \times 10^{-6}$

REFPROP call to define dry air composition:
A1=CONCATENATE ("Nitrogen",";",0.7809,";",Oxygen,";",0.2094,";","Argon",";",0.009332,";",Carbon Dioxide",";",3.85E-4,";",Helium,";",5E-6)

Calculation:
From eq. (4-3)

$$\dot{m} = \frac{C_d A^* C_R^* P_0}{\sqrt{(R_u/M)\; T_0}} \tag{B-8}$$

where

$$A^* = \pi(d/2)^2 = 0.02011\ \text{cm}^2\ (0.003117\ \text{in.}^2)$$

The measured temperature and pressure must be converted to stagnation conditions.
First calculate the Mach number in the upstream piping, using the value $\beta = d/D = 0.3000$.

$$Ma_1 = \frac{1}{\beta^2}\left(\frac{2}{\kappa + 1}\right)^{(\kappa-3)/(2\kappa-2)}\left[1 - \sqrt{1 - 2\,\beta^4\left(\frac{2}{\kappa + 1}\right)^{2/(\kappa-1)}}\right] \tag{B-9}$$

REFPROP call for polytropic exponent:
=IsentropicExpansionCoef (Composition!A1,"TP","SI with C", 21.11,0.3447)
=IsentropicExpansionCoef (Composition!A1, "TP","E",50.00,70.00)]

$$\kappa = 1.405$$

$$Ma_1 = \frac{1}{0.3000^2}\left(\frac{2}{1.405 + 1}\right)^{(1.405-3)/(2\times 1.405-2)}\left[1 - \sqrt{1 - 2 \times 0.3000^4\ (2/(1.405 + 1)^{2/(1.405-1)}}\right] = 0.05214$$

Using the upstream Mach number and polytropic exponent, calculate the stagnation pressure.

$$P_0 = P_1\left(1 + \frac{\kappa - 1}{2}\,Ma^2\right)^{\kappa/(\kappa-1)} \tag{B-10}$$

(SI Units)

$$P_0 = 0.3447\left(1 + \frac{1.405 - 1}{2}\,0.05214^2\right)^{\frac{1.405}{1.405-1}} = 0.3454\ \text{MPa}$$

(U.S. Customary Units)

$$P_0 = 50\left(1 + \frac{1.405 - 1}{2}\,0.05214^2\right)^{\frac{1.405}{1.405-1}} = 50.10\ \text{lbf/in.}^2$$

Using the upstream Mach number and polytropic exponent, calculate the stagnation temperature.

$$T_0 = T_1\left[1 + \frac{\kappa - 1}{2} Ma^2(1 - R_f)\right] \tag{B-11}$$

(SI Units)

$$T_0 = (21.11 + 273.2)\left[1 + \frac{1.405 - 1}{2} 0.05214^2 (1 - 0.75)\right] = 294.3 \text{ K}$$

(U.S. Customary Units)

$$T_0 = (70.00 + 459.7)\left[1 + \frac{1.405 - 1}{2} 0.05214^2 (1 - 0.75)\right] = 529.7°R$$

REFPROP call for real gas critical flow function:
=Cstar(Composition!A1,"TP","SI with C",(294.3-273.2),0.3454)
[=Cstar(Composition!A1,"TP","E",(529.7-459.7),50.10)]

$$C_R^* = 0.6858$$

REFPROP call for molar mass:
=MolarMass(Composition!A1,"TP","SI with C",(294.3-273.2),0.3454)
=MolarMass(Composition!A1,"TP","E",(529.7-459.7),50.10)

$$M = 28.97 \frac{\text{kg}}{\text{kg} \times \text{mole}} \qquad \left(28.97 \frac{\text{lbm}}{\text{lbm} \times \text{mole}}\right)$$

To start the iteration assume the discharge coefficient, C_d, equals 1.0 and calculate the mass flow [from eq. (4-3)].

(SI Units)

$$\dot{m} = \frac{1.0 \times [0.02011/(100^2)](0.6858) \times 0.3454 \times 10^6}{\sqrt{\frac{(8314)}{28.97} \times (294.3)}} = 0.001639 \text{ kg/s}$$

(U.S. Customary Units)

$$\dot{m} = \frac{1.0 \times (0.003117) \times (0.6858) \times 50.10 \times \sqrt{32.17}}{\sqrt{\frac{(1{,}545)}{28.97} \times (529.7)}} = 0.003614 \text{ lbm/sec}$$

Using this mass flow, the throat Reynolds number can be calculated from eq. (3-10).

$$Re_d = \frac{4\dot{m}}{\pi d \mu_0}$$

REFPROP call for dynamic viscosity:
=viscosity(Composition!A1,"TP","SI with C",(294.3-273.2),0.3454)
[=viscosity(Composition!A1,"TP","E",(529.7-459.7),50.10)]

$$\mu_0 = 18.34 \frac{\mu\text{Pa}}{\text{s}} \qquad \left(1.232 \times 10^{-5} \frac{\text{lbm}}{\text{ft} \times \text{sec}}\right)$$

$$Re_d = \frac{400 \times 0.001639}{\pi \times 0.160 \times 18.34 \times 10^{-6}} = 71\ 120$$

$$\left(\frac{48 \times 0.00361}{\pi \times 0.063 \times 1.232 \times 10^{-5}} = 70{,}110\right)$$

From CFV calibration

$$C_d = 0.9737 - \frac{3.730}{\sqrt{Re_d}} = 0.9596$$

Recalculate mass flow and continue the C_d, Re_d, and $\dot{m}$ iteration until the solution has converged. The final values after the solution has converged will be

$$Re_d = 68\ 230\ (68{,}250)$$
$$C_d = 0.9594$$
$$\dot{m} = 0.001573 \text{ kg/s } (0.003467 \text{ lbm/sec})$$

Uncertainty, u, in Calculated Mass Flow:

From the governing equation for $\dot{m}$

$$\dot{m} = \frac{C_d A^* C_R^* P_0}{\sqrt{(R_u/M)T_0}}$$

The relative differentials equation for $\dot{m}$ is

$$\frac{\Delta \dot{m}}{\bar{\dot{m}}} = \frac{\Delta C_d}{\bar{C}} + \frac{\Delta A^*}{\bar{A^*}} + \frac{\Delta C_R^*}{\bar{C_R^*}} + \frac{\Delta P_0}{\bar{P_0}} - \frac{1}{2}\frac{\Delta R_u}{\bar{R_u}} + \frac{1}{2}\frac{\Delta M}{\bar{M}} - \frac{1}{2}\frac{\Delta T_0}{\bar{T_0}} \tag{B-12}$$

where the overbars denote averages of respective quantities. Squaring eq. (B-12) and forming (relative) variances and covariances for the terms that are correlated gives the relative variance for $\dot{m}$

$$[u_c(\dot{m})^2] = [u_A(\dot{m})]^2 + [u_B(A^*)]^2 + [u_B(C_d)]^2 + [u_B(C_R^*)]^2 + [u_B(P_0)]^2 + \frac{1}{4}[u_B(R_u)]^2 + \frac{1}{4}[u_B(M)]^2 + \frac{1}{4}[u_B(T_0)]^2 + 2CV(C_R^*,P_0) + 2CV(C_R^*,T_0) + \frac{1}{2}CV(M, gc) \tag{B-13}$$

Taking the square root of both sides of eq. (B-13) gives the equation used to assess the relative uncertainty for $\dot{m}$ with correlation effects

$$u_c(\dot{m}) = \sqrt{\begin{array}{l}[u_A(\dot{m})]^2 + [u_B(C_d)]^2 + [u_B(A^*)]^2 + [u_B(C_R^*)]^2 + [u_B(P_0)]^2 + \frac{1}{4}[u_B(R_u)]^2 + \frac{1}{4}[u_B(M)]^2 \\ + \frac{1}{4}[u_B(T_0)]^2 + 2r_{C^*_R, P_0}u_B(C_R^*)u_B(P_0) + 2r_{C^*_R, T_0}u_B(C_R^*)u_B(T_0) + \frac{1}{2}r_{M, gc}u_B(M)u_B(gc)\end{array}} \tag{B-14}$$

where the correlation coefficients are defined as

$$r_{x,y} = \frac{CV(x,y)}{u(x)u(y)}$$

In this case we assume that the same values for the throat area, the critical flow function, the universal gas constant, and the molar mass are used during CFV calibration and subsequently for flow measurement. This assumes that the same gas species, nominally the same operating conditions, and the same equation for the critical flow function are used during both processes. For example, if an erroneous value for the throat area is used during the calibration process, it will not increase the uncertainty of the mass flow measurement made later via the CFV as long as the same erroneous value is used again. Through the flow calibration process, the discharge coefficient not only corrects for the nonideal behavior of the flow, but it also corrects for errors in the measurement of the throat area.

Where the correlated uncertainties are negligible, the remaining significant components of uncertainty are the C_d obtained during the CFV calibration, the pressure measurement, and the temperature measurement. However, uncertainties that are not obvious from the mass flow equation should be considered as well, e.g., the stability of the C_d over time (these being best quantified using subsequent calibrations) and any varying conditions of usage (such as different gas composition, different environmental conditions, or different P_0 or T_0). In a similar way, the uncertainties for temperature and pressure should include uncertainties from sensor calibration reports as well as other relevant uncertainty sources, e.g., spatial nonuniformity in the approach pipe, drift over time, and time to reach steady-state conditions. Experimental measurements are often necessary to quantify these uncertainties.

The relative combined uncertainty, assuming no correlation effects, can then be calculated with the following equation:

$$u_c(\dot{m}) = \sqrt{[u_A(\dot{m})]^2 + [u_B(C_d)]^2 + u_B(A^*)]^2 + [u_B(C_R^*)]^2 + [u_B(P_0)]^2 + \frac{1}{4}[u_B(R_u)]^2 + \frac{1}{4}[u_B(M)]^2 + \frac{1}{4}[u_B(T_0)]^2} \tag{B-15}$$

where the individual terms are

$u_A(\dot{m})$ = 0.1% (68% CL): value from the standard deviation of 10 replicated tests at steady state

$u_B(A^*)$ = 0.00%: the same value for A^* that was used during calibration is used during operation, and the value is fully correlated

$u_B(C_d)$ = 0.25% (95% CL): value specified in the laboratory calibration certificate

$u_B(C_R{}^*)$ = 0.00% (68% CL): REFPROP is used during calibration and usage at the same pressure and temperature conditions yielding the same value for $C_R{}^*$, and the value is fully correlated

$u_B(P_0)$ = 0.01% (68% CL): value from the pressure transducer manufacturer's specification

$u_B(R_u)$ = 0.00% (68% CL): value is known to very low uncertainty and can be neglected

$u_B(M)$ = 0.00% (68% CL): the same gas composition that was used during calibration is used during operation, and the value is fully correlated

$u_B(T_0)$ = 0.05% (68% CL): value from the temperature sensor manufacturer's specification

$$u_c(\dot{m}) = \sqrt{[u_A(0.1)]^2 + [u_B(0.25/2)]^2 + [u_B(0.01)]^2 + \frac{1}{4}[u_B(0.05)]^2} = 0.162 \tag{B-16}$$

The uncertainty calculations and results are displayed in Table B-2.2-1.

The resulting expanded uncertainty is in

$$U_c(\dot{m}) = 0.32\% \text{ (95\% CL, } k = 2)$$

The complete specification of the measurement is ($\dot{m}$) = 0.001573 kg/s (0.003467 lbm/sec) with an uncertainty of 0.32% (k = 2, at 95% CL and with 18.2 effective degrees of freedom).

Table B-2.2-1 Uncertainty Calculation and Results

Initial Uncertainty Characteristics				Processing Uncertainties				Uncertainty Results		
Quantity	**Source of Uncertainty**	**u(Quantity), %**	**Confidence Level, %**	**Type of Uncertainty**	**Probability Distribution**	**Divisor**	**Relative Sensitivity Coefficient**	**u(Quantity), %**	**Degrees of Freedom**	**Combined, %**
Mass flow replications	Replicated measurements	0.1	68	A	Normal	1	1	0.1000	9 [Note (1)]	38
Throat area	Instrument specification	0 [Note (2)]	100 [Note (2)]	B	Rectangular	1.732	1	0.0000 [Note (2)]	Infinite	0.0
Discharge coefficient	Calibration	0.25	95	B	Normal	2	1	0.1250	9 [Note (3)]	59.3
Critical flow function	Data tabulation	0 [Note (4)]	68	B	Normal	1	1	0.0000 [Note (4)]	Infinite	0.0
Stagnation pressure	Instrument specification	0.01 [Note (5)]	68 [Note (5)]	B [Note (5)]	Normal [Note (5)]	1 [Note (5)]	1 [Note (5)]	0.0100 [Note (5)]	Infinite [Note (5)]	0.4
Universal gas constant	Data tabulation	0 [Note (6)]	68 [Note (6)]	B [Note (6)]	Normal [Note (6)]	1 [Note (6)]	0.5 [Note (6)]	0.0000 [Note (6)]	Infinite [Note (6)]	0.0
Molecular weight	Data tabulation	0 [Note (7)]	68 [Note (7)]	B [Note (7)]	Normal [Note (7)]	1 [Note (7)]	0.5 [Note (7)]	0.0000 [Note (7)]	Infinite [Note (7)]	0.0
Stagnation temperature	Instrument specification	0.05 [Note (8)]	68 [Note (8)]	B [Note (8)]	Normal [Note (8)]	1 [Note (8)]	0.5 [Note (8)]	0.0250 [Note (8)]	Infinite [Note (8)]	2.4
Mass flow relative combined uncertainty, % →						RSS (Previous rows) =		0.162 [Note (9)]	18.2 [Note (9)]	100
Mass flow relative expanded uncertainty, % →						2 × RSS in preceding row =		0.32 [Note (10)]	18.2 [Note (10)]	...

NOTES:

(1) The degrees of freedom are based on the 10 replications taken for the measurement.

(2) Entries are due to the same value being used in both calibration and usage.

(3) Entry is based on the 10 degrees of freedom given in the calibration report for the meter.

(4) Entries are based on the fact that the same gas is used in the calibration and usage situations.

(5) Entries are based on the instrument manufacturer's specification.

(6) Entries are based on the fact that the same gas and same source for the gas constant are used in the calibration and when the meter is used.

(7) Entries are based on the fact that the same gas and same source are used for the molecular weight.

(8) Entries are based on the instrument manufacturer's specification.

(9) Entry is the RSS result from the values in the preceding rows. The effective degrees of freedom are computed with the Welsh-Satterthwaite equation

$$\upsilon_{\text{eff}} = \frac{[u_c(\dot{m})]^4}{\sum_{i=1}^{N} \frac{[u(x_i)]^4}{v_i}}$$

(10) Entry gives the uncertainty as 0.32% at 95% confidence level for 18.2 effective degrees of freedom.

NONMANDATORY APPENDIX C
CFV MASS FLOW EQUATION AND REAL GAS CRITICAL FLOW FUNCTION

C-1 GENERAL FLOW EQUATIONS

C-1.1 Formulation of the CFV Mass Flow Equation

The expression for steady mass flow through a choked CFV [see eq. (4-3)] has two correction factors, the discharge coefficient, C_d, and the real gas critical flow function, C_R*. The discharge coefficient scales primarily with Reynolds number and corrects for nonideal flow processes while the real gas critical flow function corrects for real gas effects. In this way corrections related to the flow process have been isolated from corrections corresponding to real gas effects. Equation (4-3) inherently assumes that these two phenomena can be treated separately. Herein eq. (4-3) is derived, and in the process the underlying assumption used to separate real gas corrections from flow-related corrections is explained and the limitations are discussed.

The first step in deriving eq. (4-3) is to determine the mass flow through a CFV for the special case when both the gas and the flow process are ideal. The ideal flow process confines the flow to be inviscid and one-dimensional while the ideal gas assumption requires that the compressibility factor is unity and the heat capacities are constant. Under these special conditions several researchers[1,2] have solved the mass, momentum, and energy conservation equations and determined that the ideal mass flow is[3]

$$\dot{m}_{\text{ideal}} = \frac{A^* C_i^* P_0}{\sqrt{\left(\dfrac{R_u}{M}\right) T_0}} \tag{C-1}$$

where C_i* is the ideal gas critical flow function defined by

$$C_i^* = \sqrt{\gamma \left(\frac{2}{\gamma - 1}\right)^{\frac{\gamma+1}{\gamma-1}}} \tag{C-2}$$

and $\gamma = \gamma(T_{1m}, P_1)$ is the specific heat ratio evaluated at the measured temperature and absolute inlet static pressure.

The difference between $\dot{m}_{\text{ideal}}$ and the actual mass flow is attributed to the following three phenomena:

(a) Viscous Boundary Layer Effects. The inviscid assumption used in the ideal flow model is not valid in the boundary layer adjacent to the CFV wall. In this region, viscous effects retard the fluid motion, thereby reducing the gas velocity below the sonic velocity. Simultaneously, shear between adjacent fluid layers heat the gas, leading to higher temperatures, and subsequently lower densities than the fluid density in the inviscid core beyond the boundary layer. Together, the lower velocity and lower density lead to the decreased mass flow through the boundary layer region than would be predicted by the ideal model.

(b) Curvature of the Sonic Line. The one-dimensional assumption used in the ideal flow model predicts a flat sonic line at the CFV throat. However, the flow outside of the boundary layer region or core flow is multidimensional so that the profile of the sonic line (i.e., locus of points where the Mach number is unity) is nearly parabolic instead of the flat profile predicted by the one-dimensional ideal flow model. The effect of the curved sonic line is to reduce the mass flow in the core region below the ideal flow model.

(c) Virial or Real Gas Effects. Real gas effects alter both the sound speed and the density, causing them to differ from the values predicted for an ideal gas. Virial effects can either increase or decrease the CFV mass flow depending on the upstream stagnation conditions and gas species.

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1 Anderson, J. D., *Modern Compressible Flow With Historical Perspective*, McGraw-Hill Series in Mechanical Engineering, New York, New York, USA, 1982.

2 John, J. E., *Gas Dynamics*, 2nd ed., Allyn and Bacon, Boston, Massachusetts, USA, 1984.

3 Note that the ideal mass flow ($\dot{m}_{\text{ideal}}$) does *not* equal the theoretical mass flow $\dot{m}_{th}$ defined in eq. (4-2). The difference between the two is that $\dot{m}_{\text{ideal}}$ is based on C_i* while $\dot{m}_{th}$ is defined using C_R*.

In general $\dot{m}_{\text{ideal}}$ predicts the actual mass flow to much better than 5%. Consequently, corrections to the ideal gas and flow processes due to (a), (b), and (c) are small and can be taken to be uncoupled from each other. For example, the correction for the viscous boundary layer development on the CFV wall can be determined independent of corrections for both one-dimensional core flow and real gas effects. In this way the reduction in mass flow due to the boundary layer, $\Delta\dot{m}_1$, can be determined assuming the core flow is one-dimensional and the gas is ideal. The reduction in mass flow attributed to sonic line curvature, $\Delta\dot{m}_2$, is determined for an axisymmetric (or three-dimensional) flow but assumes that the flow is inviscid and the gas is ideal. Analogously, the mass flow reduction (or increase) attributed to real gas effects, $\Delta\dot{m}_3$, incorporates real gas behavior in the model but assumes the flow is inviscid and one-dimensional.

The mass flow reduction relative to the ideal mass flow for any of the nonideal mechanisms alone is

$$\Delta\dot{m}_n = \dot{m}_{\text{ideal}} - \dot{m}_n \tag{C-3}$$

where $n = 1, 2$, or 3 and $\dot{m}_n$ is the mass flow attributed to the nth nonideal effect. The flow physics of each distinct nonideal effect is characterized by its discharge coefficient defined by

$$C_{d,n} = \frac{\dot{m}_n}{\dot{m}_{\text{ideal}}} \tag{C-4}$$

Each of the discharge coefficients corresponding to $n = 1, 2$, and 3 has been researched extensively. The first two discharge coefficients, $C_{d,1}$ and $C_{d,2}$, correct the ideal flow model for boundary layer effects and sonic line curvature, respectively. Accordingly, these are named the "viscous discharge coefficient," $C_{d,1}$, and the "inviscid discharge coefficient," $C_{d,2}$. Analytical expressions for $C_{d,1}$ and $C_{d,2}$ have been developed using boundary layer theory[4,5,6] and compressible potential flow theory,[7,8] respectively. In contrast, the virial discharge coefficient ($C_{d,3}$) has not been solved analytically, but instead numerical techniques have been used. Numerical solutions of $\dot{m}_3$ have traditionally been expressed in a form analogous to eq. (C-1) with C_i^* replaced with C_R^* so that all real gas effects are lumped into this parameter. Following this tradition, the virial discharge coefficient is equal to the ratio of the real gas critical flow function and the ideal gas critical flow function. Equation (C-5) shows this relationship and gives the functional dependence of the viscous discharge coefficient and the inviscid discharge coefficient

Viscous discharge coefficient

$$C_{d,1} = f_1(Re_d, \Omega, \gamma) \tag{C-5a}$$

Inviscid discharge coefficient

$$C_{d,2} = f_2(\Omega, \gamma) \tag{C-5b}$$

Virial discharge coefficient

$$C_{d,3} = C_R^*/C_i^* \tag{C-5c}$$

where the dimensionless geometric parameter, $\Omega = 2r_c/d$, is the ratio of the radius of curvature to the throat radius.

The actual CFV mass flow under real conditions is the ideal mass flow with linear corrections for each non-ideal effect

$$\dot{m} = \dot{m}_{\text{ideal}} - \Delta\dot{m}_1 - \Delta\dot{m}_2 - \Delta\dot{m}_3$$

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[4] Tang, S., Discharge Coefficients for Critical Flow Nozzles and Their Dependence on Reynolds Numbers, Ph.D. Thesis, Princeton University, Princeton, New Jersey, USA, 1969.

[5] Geropp, D., Laminare Grenzschichten in Ebenen und Rotationssymmetrischen Lavalduesen, Deutsche Luft-Und Raumfart, Forschungsbericht, pp. 71–90, 1971.

[6] Stratford, B. S., The Calculation of the Discharge Coefficient of Profiled Choked Nozzles and the Optimum Profile for Absolute Air Flow Measurement, *J. R. Aeronaut. Soc.*, 68, pp. 237–245, 1964.

[7] Hall, I. M., Transonic Flow in Two-Dimensional and Axially-Symmetric Nozzles, *Q. J. Mech. Appl. Math.*, 15, pp. 487–508, 1962.

[8] Kliegel, J. R., and Levine, J. N., Transonic Flow in Small Throat Radius of Curvature Nozzles, *AIAA J.*, 7, pp. 1375–1378, 1969.

or

$$\dot{m} = \dot{m}_{\text{ideal}}\left(1 - \frac{\Delta\dot{m}_1}{\dot{m}_{\text{ideal}}} - \frac{\Delta\dot{m}_2}{\dot{m}_{\text{ideal}}} - \frac{\Delta\dot{m}_3}{\dot{m}_{\text{ideal}}}\right) \tag{C-6}$$

By convention these corrections are subtracted from $\dot{m}_{\text{ideal}}$ to indicate that they reduce the CFV mass flow. However, only corrections due to eqs. (C-5a) and (C-5b) reduce the mass flow in all cases; corrections for eq. (C-5c) can either decrease or increase the mass flow depending on operating conditions and gas species. Higher-order terms such as $\Delta\dot{m}_1\,\Delta\dot{m}_2$, $\Delta\dot{m}_2\,\Delta\dot{m}_3$, and $\Delta\dot{m}_1\,\Delta\dot{m}_3$ are omitted since the nonideal effects are taken to be fully uncoupled. Physically, these higher-order terms account for weak coupling between the viscous effects, sonic line curvature, and virial effects. However, since each correction is already small, products of these corrections (i.e., higher-order terms) can generally be neglected.

Equation (C-6) can be approximately factored so that the mass flow equals the ideal mass flow multiplied by the three correction factors

$$\dot{m} = \dot{m}_{\text{ideal}}\left(1 - \frac{\Delta\dot{m}_1}{\dot{m}_{\text{ideal}}}\right)\left(1 - \frac{\Delta\dot{m}_2}{\dot{m}_{\text{ideal}}}\right)\left(1 - \frac{\Delta\dot{m}_3}{\dot{m}_{\text{ideal}}}\right) \tag{C-7}$$

The additional higher-order terms associated with this factorization are taken to be negligible, which is consistent with the composite linear theory used to formulate eq. (C-6). In cases where the higher-order terms $\Delta\dot{m}_1\,\Delta\dot{m}_3$ or $\Delta\dot{m}_2\,\Delta\dot{m}_3$ are significant, eq. (C-6) is not adequate due to coupling effects between real gas phenomena and flow processes. Consequently, the separation of flow-related corrections into C_d and real gas corrections into $C_R{}^*$ as implicitly done in eq. (C-7) is not justified. In this case an additional uncertainty commensurate with the magnitude of $\Delta\dot{m}_1\,\Delta\dot{m}_3$ or $\Delta\dot{m}_2\,\Delta\dot{m}_3$ should be included in the mass flow uncertainty budget.[9]

Substituting the correction terms ($\Delta\dot{m}_n$) given in eq. (C-3) into eq. (C-7) the mass flow is

$$\dot{m} = \dot{m}_{\text{ideal}}\left(\frac{\dot{m}_1}{\dot{m}_{\text{ideal}}}\right)\left(\frac{\dot{m}_2}{\dot{m}_{\text{ideal}}}\right)\left(\frac{\dot{m}_3}{\dot{m}_{\text{ideal}}}\right) \tag{C-8}$$

which can be further simplified by substituting $C_{d,n} = \dot{m}_n/\dot{m}_{\text{ideal}}$ for $n = 1$, 2, and 3 from eq. (C-4). The result is that the mass flow equals the product of the ideal mass flow multiplied by the viscous, inviscid, and virial discharge coefficients

$$\dot{m} = \dot{m}_{\text{ideal}} C_{d,1} C_{d,2} C_{d,3} \tag{C-9}$$

Finally, by substituting eqs. (C-1) and (C-5c) into eq. (C-9), we obtain the desired expression for the CFV mass flow

$$\dot{m} = \frac{C_d A^* C_R^* P_0}{\sqrt{(R_u/M)T_0}} \tag{C-10}$$

where the product of the viscous and inviscid discharge coefficients has been combined into a single discharge coefficient, $C_d = C_{d,1}C_{d,2}$. In this way C_d corrects for both boundary layer development and curvature of the sonic line. Researchers have shown that C_d can be successfully computed using the analytical expressions developed for $C_{d,1}$ and $C_{d,2}$.[10,11] However, in most flow measurement applications C_d is determined either by flow calibration or by the empirical correlations provided in this Standard. Nevertheless, the analytical models provide important insight on how C_d scales with Reynolds number in the laminar and turbulent regimes, the sensitivity of C_d on the CFV throat curvature, and an estimate of the C_d sensitivity on gas species.

[9] The additional uncertainty component need only be considered if the CFV is flow calibrated in one gas but applied in a different gas. If the same gas is used in the flow calibration and the application, the uncertainty is fully correlated and cancels.

[10] Johnson, A. N., and Wright, J. D., Comparison Between Theoretical CFV Models and NIST's Primary Flow Data in the Laminar, Turbulent, and Transition Flow Regimes, *ASME J. of Fluids Engrg.*, 130, July 2008.

[11] Mickan, B., Kramer, R., Dopheide, D., Johnson, A., Wright, J., Hotze, H.-J., Hinze, H.-M., and Vallet, J.-P., Comparisons by PTB, NIST, and LNE-LADG in Air and Natural Gas with Critical Venturi Nozzles Agree Within 0.05%, *Sixth International Symposium for Fluid Flow Measurement*, Queretaro, Mexico, A.4.4, 2006.

C-1.2 Formulation of the Real Gas Critical Flow Function

The real gas critical flow function, C_R*, corrects for departures from ideal gas behavior or real gas effects. As discussed in C-1.1, C_R* is determined for a one-dimensional, inviscid flow process. For this ideal flow process $C_d \rightarrow 1$ so that eq. (C-10) becomes

$$\dot{m}_3 = \frac{A^* C_R^* P_0}{\sqrt{(R_u/M)T_0}} \tag{C-11}$$

where the subscript 3 indicates that the mass flow is attributed solely to virial effects. Since the ideal flow process is one-dimensional, the mass flow at the CFV throat is

$$\dot{m}_3 = \rho^* c^* A^* \tag{C-12}$$

where $\rho*$ and $c*$ are the real gas throat density and sound speed, respectively. Substituting eq. (C-11) into (C-12) gives

$$C_R^* \equiv \frac{\rho^* c^* \sqrt{R_u T_0}}{P_0 \sqrt{M}} \tag{C-13}$$

which is the definition of the real gas critical flow function.

C-2 GOVERNING EQUATIONS TO COMPUTE THE REAL GAS CRITICAL FLOW FUNCTION

To compute the real gas critical flow function one must determine the density, $\rho*$, and the sound speed, $c*$, at the CFV throat for specified T_0, P_0, and gas species. The real gas throat properties $\rho*$ and $a*$ are determined for a one-dimensional inviscid flow process as discussed in C-1. The gas accelerates from the CFV inlet to the throat in accordance with the laws of conservation of mass, momentum, and energy. As the gas accelerates toward the CFV throat its temperature and pressure decrease. For these ideal flow conditions the conservation laws require that the decreasing temperature and pressure follow an isentropic and isoenergetic process.[6] For an isentropic process the entropy is constant ($s =$ constant), while for an isoenergetic process the total enthalpy is constant ($h + \nu^2/2 =$ constant). The isentropic and isoenergetic processes are used to relate the unknown throat conditions to the known stagnation conditions.[12] In particular, the entropy at the CFV throat, $s*$, equals the stagnation entropy, s_0, and the total enthalpy at the CFV throat ($h_t* = h* + a^{*2}/2$) equals the stagnation enthalpy, h_0, as expressed by

$$s_0 = s^* \tag{C-14}$$

$$h_0 = h^* + \frac{c^{*2}}{2} \tag{C-15}$$

Based on these two equations, the throat conditions are not arbitrary but are uniquely specified by the gas composition and stagnation conditions. The real gas critical flow function, C_R*, is determined by finding the unique throat density and sound speed consistent with eqs. (C-14) and (C-15). An iterative procedure is generally required to determine $\rho*$ and $c*$ from the known entropy and total enthalpy at the CFV throat. The REFPROP database internally performs the necessary iterations so that end users can conveniently compute C_R* as a function of T_0, P_0, and gas type. Nevertheless, we outline the numerical approach used in REFPROP to compute C_R*.

C-2.1 Procedure to Calculate the Real Gas Critical Flow Function

C-2.1.1 Introduction. Equations (C-14) and (C-15) must be solved using a low-uncertainty thermodynamic database to ensure high-fidelity real gas corrections. This Standard recommends the REFPROP thermodynamic database as it is maintained and updated by NIST and it specifies the uncertainty of the underlying measurement data used in the database. The REFPROP program also computes the required thermodynamic properties in eqs. (C-14) and (C-15) (i.e., entropy, enthalpy, and speed of sound) as a function of any two independent properties. For example, the enthalpy at the CFV throat can be computed as a function of the throat pressure and temperature, $h* = h(T*,P*)$, or equivalently using the dependent variables of temperature and entropy, $h* = h(T*,s*)$.

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[12] The stagnation values for the entropy and enthalpy are evaluated at the stagnation conditions so that the entropy and enthalpy are given by $s_0 = s(T_0,P_0)$ and $h_0 = h(T_0,P_0)$, respectively.

Equations (C-14) and (C-15) can be solved for any two independent thermodynamic properties. The choice of independent properties defines the dependent variables for the throat entropy, enthalpy, and speed of sound in eqs. (C-14) and (C-15). For example, if the two independent properties are the throat temperature and pressure, these equations are expressed as

$$s_0 = s(T^*,P^*) \tag{C-16}$$

$$h_0 = h(T^*,P^*) + c(T^*,P^*)^2/2 \tag{C-17}$$

where the throat entropy, enthalpy, and sound speed are explicit functions of T^* and P^*. This system of two equations can be iteratively solved for the unknowns T^* and P^* using any suitable numerical technique. If the thermodynamic database only evaluates properties as a function of temperature and pressure this approach should be used.

For thermodynamic databases, such as REFPROP, that compute thermodynamic properties as a function of any two independent properties, the solution procedure can be significantly simplified by a prudent choice of dependent variables. In particular, the dependent variables T^* and s^* reduce the system of two equations to a single equation. Equation (C-16) is identically satisfied since the throat entropy is known from the stagnation value. By replacing the unknown dependent variable P^* in eq. (C-17) with the known throat entropy, s^*, the equation becomes

$$h_0 = h(T^*,s^*) + c(T^*,s^*)^2/2 \tag{C-18}$$

where T^* is the only unknown. Since T^* is an implicit function of both the enthalpy and the square of the sound speed, eq. (C-18) must be solved numerically. Herein we outline how the secant method[13] can be used to determine the unknown throat temperature T^*, and how the result is used to compute C_R^*.

C-2.1.2 Outline of Numerical Algorithm That Determines C_R^* Using the Secant Method. The secant method is a numerical algorithm used to find the roots of a single equation or system of equations. Here this method is used to determine T^* in eq. (C-18), and subsequently to determine C_R^*.

Step 1: Determine the stagnation entropy and enthalpy, and define the number of iterations and logical parameters to prepare for iterations.

(a) Use the thermodynamic database to calculate the stagnation entropy and enthalpy as a function of the known stagnation temperature and pressure, $s_0 = s(T_0,P_0)$ and $h_0 = h(T_0,P_0)$.

(b) Set the throat entropy equal to the stagnation value, $s^* = s_0$.

(c) Define an acceptable convergence tolerance: $tol = 0.00000001$.

(d) Define the convergence flag: nFLAG = −1 (if nFLAG = −1 the solution is not converged, while if nFLAG = 1 the solution is converged within tolerance specifications).

(e) Define the maximum number of iterations: $N_{\max} = 100$ (generally a converged solution is obtained in 10 or fewer iterations).

Step 2: Begin iteration loop: $n = 1$ to $N_{\max}$

(a) If $n = 1$ then guess an initial throat temperature T_1^*

- $T_1^* = 2T_0/(\gamma + 1)$ where $\gamma = \gamma(T_m,P)$, T_m is the measured temperature, and P is the measured inlet static pressure

(b) If $n = 2$ then guess a second throat temperature T_2^*

- $T_2^* = (T_1^* + T_0)/2$ (here T_2^* is between T_1^* and T_0)

(c) Define a percent difference error function

$$\epsilon_n = 100\left[\frac{h(T_n^*,s^*) + a(T_n^*,s^*)^2/2}{h_0} - 1\right] \tag{C-19}$$

equal to the percent difference between throat total enthalpy and the stagnation enthalpy. By definition, if $\epsilon_n = 0$, then $T_n^* = T^*$ and eq. (C-18) is identically satisfied.

(d) If $|\epsilon_n| < tol$ then

- set nFLAG = 1
- exit loop and go to Step 3

[13] Press, W. H., Flannery, B. P., Teukolsky, S. A., and Vetterling, W. T., *Numerical Recipes*, Cambridge University Press, New York, 1989.

Table C-2.2-1 Sample Values of C_R^* Calculated with REFPROP

Gas	Stagnation Pressure, P_0, kPa (psia)	Stagnation Temperature, T_0, K (°F)	Critical Flow Function, C_R^*
Nitrogen	1 000 (145.038)	295 (71.33)	0.68725
Argon	1 000 (145.038)	295 (71.33)	0.73063
Helium	1 000 (145.038)	295 (71.33)	0.72528
Methane	1 000 (145.038)	295 (71.33)	0.67610
Hydrogen	1 000 (145.038)	295 (71.33)	0.68596
Dry air	1 000 (145.038)	295 (71.33)	0.68762
Natural gas (Ekofisk)	1 000 (145.038)	295 (71.33)	0.67284
Natural gas (Amarillo)	1 000 (145.038)	295 (71.33)	0.67459
Natural gas (Gulf Coast)	1 000 (145.038)	295 (71.33)	0.67512

(e) If $n \geq 2$ then estimate the derivative of the error function in eq. (C-19)

$$\left.\frac{d\epsilon}{dT^*}\right|_n = \frac{\epsilon_n - \epsilon_{n-1}}{T_n^* - T_{n-1}^*} \tag{C-20}$$

(f) If $n \geq 2$ then use the secant method to update the throat temperature

$$T_{n+1}^* = T_n^* - \frac{\epsilon_n}{(d\epsilon/dT^*)_n} \tag{C-21}$$

where the derivative term $(d\epsilon/dT^*)_n$ is computed using eq. (C-20).

(g) If $n < N_{\max}$ then go to Step 2 and increment n by 1.

(h) If $n = N_{\max}$ then go to Step 3.

Step 3: If nFLAG = 1 then

(a) Use the throat entropy at the throat, s^*, and the converged throat temperature, T^*, to compute both the throat density and sound speed, $\rho^* = \rho(T^*,s^*)$ and $c^* = c(T^*,s^*)$, respectively.

(b) Compute C_R^* by substituting ρ^* and c^* from Step 3(a) into eq. (C-3).

Step 4: If nFLAG = −1 then throat temperature is not converged to the specified tolerance level.

C-2.2 Example Calculations of the Real Gas Critical Flow Function

Table C-2.2-1 shows sample C_R^* calculations using the REFPROP thermodynamic database. Calculations are made for various pure gases, a mixture of dry air,[14] and various natural gas mixtures.[15]

[14] Wright, J. D., Properties for Accurate Gas Flow Measurements, *15th International Flow Measurement Conference*, Taipei, Taiwan, 2010.

[15] Starling, K. E., and Savidge, J. L., *Compressibility Factors of Natural Gas and Other Related Hydrocarbon Gases*, American Gas Association, Transmission Measurement Committee Report No. 8, 2nd ed., 1992.

NONMANDATORY APPENDIX D HUMID AIR COMPOSITION[1]

D-1 DETERMINING HUMID AIR COMPOSITION USING DEW POINT TEMPERATURE

Using dew point temperature, T_{dp} (K), as the input, the following method can be used to determine the humid air composition using measured absolute pressure, P_1 (Pa), and temperature, T_1 (K). The nomenclature in this Nonmandatory Appendix is independent from section 3 of this Standard.

Step 1: Calculate the saturation vapor pressure.

(a) For 273.15 K < T_1 < 373.15 K calculate the saturation vapor pressure, P_{ws}, over water at the measured dew point temperature, T_{dp}, using eq. (D-1)

$$\ln P_{ws} = \sum_{i=0}^{6} g_i T_{dp}^{i-2} + g_7 \ln T_{dp} \tag{D-1}$$

where

$g_0 = -2.8365744 \times 10^3$
$g_1 = -6.028076559 \times 10^3$
$g_2 = 1.954263612 \times 10^1$
$g_3 = -2.737830188 \times 10^{-2}$
$g_4 = 1.6261698 \times 10^{-5}$
$g_5 = 7.0229056 \times 10^{-10}$
$g_6 = -1.8680009 \times 10^{-13}$
$g_7 = 2.7150305$

(b) For 173.15 K < T_1 < 273.15 K calculate the saturation vapor pressure, P_{ws}, over ice at the measured dew point temperature, T_{dp}, using eq. (D-2)

$$\ln P_{ws} = \sum_{i=0}^{4} k_i T_{dp}^{\,i-1} + k_5 \ln T_{dp} \tag{D-2}$$

where

$k_0 = -5.8666426 \times 10^3$
$k_1 = 2.232870244 \times 10^1$
$k_2 = 1.39387003 \times 10^{-2}$
$k_3 = -3.4262402 \times 10^{-5}$
$k_4 = 2.7040955 \times 10^{-8}$
$k_5 = 6.7063522 \times 10^{-1}$

Step 2: Calculate the enhancement factor, f, at the measured temperature T_1 and pressure P_1 using eq. (D-3)

$$f = \exp\left[\alpha\left(1 - \frac{P_{ws}}{P_1}\right) + \beta\left(\frac{P_1}{P_{ws}} - 1\right)\right] \tag{D-3}$$

where

$$\alpha = \Sigma_{i=0}^{3} A_i T_1^i$$

$$\ln \beta = \sum_{i=0}^{3} B_i T_1^i$$

[1] The methods for calculating humid air composition in this Nonmandatory Appendix are sourced from Hardy, R., ITS-90 Formulations for Vapor Pressure, Frostpoint Temperature, Dewpoint Temperature, and Enhancement Factors in the Range −100 to +100 C, *Third International Symposium on Humidity and Moisture*, Teddington, London, England, April 1998.

(a) For 173.15 K < T_1 < 273.15 K

$$A_0 = -6.0190570 \times 10^{-2}$$
$$A_1 = 7.3984060 \times 10^{-4}$$
$$A_2 = -3.0897838 \times 10^{-6}$$
$$A_3 = 4.3669918 \times 10^{-9}$$
$$B_0 = -9.4868712 \times 10^{1}$$
$$B_1 = 7.2392075 \times 10^{-1}$$
$$B_2 = -2.1963437 \times 10^{-3}$$
$$B_3 = 2.4668279 \times 10^{-6}$$

(b) For 273.15 K < T_1 < 373.15 K

$$A_0 = -1.6302041 \times 10^{-1}$$
$$A_1 = 1.8071570 \times 10^{-3}$$
$$A_2 = -6.7703064 \times 10^{-6)}$$
$$A_3 = 8.5813609 \times 10^{-9}$$
$$B_0 = -5.9890467 \times 10^{1}$$
$$B_1 = 3.4378043 \times 10^{-1}$$
$$B_2 = -7.7326396 \times 10^{-4}$$
$$B_3 = 6.3405286 \times 10^{-7}$$

Step 3: Calculate the mole fraction of H_2O vapor in the humid air

$$(H_2O)_{mf} = f \times P_{ws} / P_1$$

Step 4: Define the dry air composition by volume. The following is a recommended dry air composition:[2]

(N_2)dry	78.0868%
(O_2)dry	20.9409%
(Ar)dry	0.9332%
(CO_2)dry	0.0385%
(He)dry	0.0005%

Step 5: Renormalize the dry air composition to include the mole fraction of H_2O vapor calculated

$$(N_2)\text{humid} = (N_2)\text{dry}/[(N_2)\text{dry} + (O_2)\text{dry} + (Ar)\text{dry} + (CO_2)\text{dry} + (He)\text{dry} + (H_2O)_{mf}]$$
$$(O_2)\text{humid} = (O_2)\text{dry}/[(N_2)\text{dry} + (O2)\text{dry} + (Ar)\text{dry} + (CO_2)\text{dry} + (He)\text{dry} + (H_2O)_{mf}]$$
$$(Ar)\text{humid} = (Ar)\text{dry}/[(N_2)\text{dry} + (O_2)\text{dry} + (Ar)\text{dry} + (CO_2)\text{dry} + (He)\text{dry} + (H_2O)_{mf}]$$
$$(CO_2)\text{humid} = (CO_2)\text{dry}/[(N_2)\text{dry} + (O_2)\text{dry} + (Ar)\text{dry} + (CO_2)\text{dry} + (He)\text{dry} + (H_2O)_{mf}]$$
$$(He)\text{humid} = (He)\text{dry}/[(N_2)\text{dry} + (O_2)\text{dry} + (Ar)\text{dry} + (CO_2)\text{dry} + (He)\text{dry} + (H_2O)_{mf}]$$
$$(H_2O)\text{humid} = (H_2O)\text{dry}/[(N_2)\text{dry} + (O_2)\text{dry} + (Ar)\text{dry} + (CO_2)\text{dry} + (He)\text{dry} + (H_2O)_{mf}]$$
$$(N_2)\text{humid} = (N_2)\text{dry}/[(N_2)\text{dry} + (O_2)\text{dry} + (Ar)\text{dry} + (CO_2)\text{dry} + (He)\text{dry} + (H_2O)_{mf}]$$

Step 6: Verify that the sum of the humid air constituents equals 100%. Tables D-1-1 and D-1-2 provide sample values attained with this method.

D-2 DETERMINING HUMID AIR COMPOSITION USING RELATIVE HUMIDITY

Using relative humidity, RH (%), as the input, the following method can be used to determine the humid air composition using measured absolute pressure, P_1 (Pa), and temperature, T_1 (K).

Step 1: Calculate the saturation vapor pressure.

(a) For 273.15 K < T_1 < 373.15 K, calculate the saturation vapor pressure, P_{ws}, over water at the measured temperature T_1 using eq. (D-4)

$$\ln P_{ws} = \sum_{i=0}^{6} g_i\, T_1^{i-2} + g_7 \ln T_1 \tag{D-4}$$

[2] Wright, J. D., Properties for Accurate Gas Flow Measurements, *15th International Flow Measurement Conference*, Taipei, Taiwan, 2010.

Table D-1-1 Results for Humid Air Calculations Using Dew Point Temperature of 277.05 K (39°F)

Preliminary Calculations (See Steps 1, 2, and 3) [Note (1)]

Parameter	Dry Air	Humid Air
Saturation vapor pressure	. . .	0.0807187 kPa (0.117073 psia)
Enhancement factor	. . .	1.0238593
Water vapor content	. . .	0.0011957 mole fraction

Composition of Air (See Steps 4 and 5)

Component	Dry Air, Mole Fraction	Humid Air, Mole Fraction
Nitrogen, N_2	0.7808685	0.7799359
Oxygen, O_2	0.2094101	0.2091600
Argon, Ar	0.0093317	0.0093206
Carbon dioxide, CO_2	0.0003845	0.0003840
Helium, He	0.0000052	0.0000052
Water, H_2O	. . .	0.0011942

Molecular Weight

	Dry Air, g/mol	Humid Air, g/mol	Δ Between Humid Air and Dry Air, %
Aggregate molecular weight	28.96546	28.95251	−0.045

Properties of Air

Property	Dry Air	Humid Air	Δ Between Humid Air and Dry Air, %
Critical flow factor	0.6867926	0.686778	−0.002
Compressibility factor, Z	0.99768	0.99765	−0.003
Density	8.1816 kg/m^3 (0.51076 lbm/ft^3)	8.1782 kg/m^3 (0.51055 lbm/ft^3)	−0.042
Isentropic exponent	1.4096	1.4095	−0.008
Absolute viscosity	18.39 μPa/s [1.236E−05 lbm/(ft sec)]	18.39 μPa/s [1.236E−05 lbm/(ft sec)]	−0.028
Sound speed	344.7 m/s (1,130.76 ft/sec)	344.7 m/s (1,130.95 ft/sec)	−0.017

GENERAL NOTE: Values in U.S. Customary units are shown in parentheses.

NOTE:

(1) Inputs for the calculations were as follows:
pressure = 689.5 kPa (100.0 psia)
dew point = 277.05 K (39°F)
temperature = 294.25 K (70.0°F)

Table D-1-2 Results for Humid Air Calculations Using Dew Point Temperature of −313.15 K (−40°F)

Preliminary Calculations (See Steps 1, 2, and 3) [Note (1)]

Parameter	Dry Air	Humid Air
Saturation vapor pressure, P_{ws}	. . .	0.012837 kPa (0.001862 psia)
Enhancement factor, f	. . .	1.0364175
Water vapor content	. . .	0.0000190 mole fraction

Composition of Air (See Steps 4 and 5)

Component	Dry Air, Mole Fraction	Humid Air, Mole Fraction
Nitrogen, N_2	0.7808685	0.7808536
Oxygen, O_2	0.2094101	0.2094061
Argon, Ar	0.0093317	0.0093315
Carbon dioxide, CO_2	0.0003845	0.0003845
Helium, He	0.0000052	0.0000052
Water, H_2O	. . .	0.0000190

Molecular Weight

	Dry Air, g/mol	Humid Air, g/mol	Δ Between Humid Air and Dry Air, %
Aggregate molecular weight	28.96546	28.96538	0.000

Properties of Air

Property	Dry Air	Humid Air	Δ Between Humid Air and Dry Air, %
Critical flow factor	0.6867926	0.686792	0.000
Compressibility factor, Z	0.99768	0.99768	0.000
Density	8.1816 kg/m^3 (0.51076 lbm/ft^3)	8.1816 kg/m^3 (0.51076 lbm/ft^3)	0.000
Isentropic exponent	1.4096	1.4096	0.000
Absolute viscosity	18.39 μPa/s [1.236E−05 lbm/(ft sec)]	18.39 μPa/s [1.236E−05 lbm/(ft sec)]	−0.001
Sound speed	344.7 m/s (1,130.76 ft/sec)	344.7 m/s (1,130.76 ft/sec)	0.000

GENERAL NOTE: Values in U.S. Customary units are shown in parentheses.

NOTE:

(1) Inputs for the calculations were as follows:

dew point = −313.15 K (−40°F)
pressure = 689.5 kPa (100.0 psia)
temperature = 294.25 K (70.0°F)

where
$g_0 = -2.8365744 \times 10^3$
$g_1 = -6.028076559 \times 10^3$
$g_2 = 1.954263612 \times 10^1$
$g_3 = -2.737830188 \times 10^{-2}$
$g_4 = 1.6261698 \times 10^{-5}$
$g_5 = 7.0229056 \times 10^{-10}$
$g_6 = -1.8680009 \times 10^{-13}$
$g_7 = 2.7150305$

(b) For 173.15 K < T_1 < 273.15 K, calculate the saturation vapor pressure, P_{ws}, over ice at the measured temperature T_1 using eq. (D-5)

$$\ln P_{ws} = \sum_{i=0}^{4} k_i T_1^{i-1} + k_5 \ln T_1 \quad \text{(D-5)}$$

where
$k_0 = -5.8666426 \times 10^3$
$k_1 = 2.232870244 \times 10^1$
$k_2 = 1.39387003 \times 10^{-2}$
$k_3 = -3.4262402 \times 10^{-5}$
$k_4 = 2.7040955 \times 10^{-8}$
$k_5 = 6.7063522 \times 10^{-1}$

Step 2: Calculate the enhancement factor, f, at the measured temperature T_1 and pressure P_1 using eq. (D-6)

$$f = \exp\left[\alpha\left(1 - \frac{P_{ws}}{P_1}\right) + \beta\left(\frac{P_1}{P_{ws}} - 1\right)\right] \quad \text{(D-6)}$$

where

$$\alpha = \Sigma_{i=0}^{3} A_i T_1^i$$

$$\ln \beta = \sum_{i=0}^{3} B_i T_1^i$$

(a) For 173.15 K < T_1 < 273.15 K
$A_0 = -6.0190570 \times 10^{-2}$
$A_1 = 7.3984060 \times 10^{-4}$
$A_2 = -3.0897838 \times 10^{-6}$
$A_3 = 4.3669918 \times 10^{-9}$
$B_0 = -9.4868712 \times 10^{-1}$
$B_1 = 7.2392075 \times 10^{-1}$
$B_2 = -2.1963437 \times 10^{-3}$
$B_3 = 2.4668279 \times 10^{-6}$

(b) For 273.15 K < T_1 < 373.15 K
$A_0 = -1.6302041 \times 10^{-1}$
$A_1 = 1.8071570 \times 10^{-3}$
$A_2 = -6.7703064 \times 10^{-6}$
$A_3 = 8.5813609 \times 10^{-9}$
$B_0 = -5.9890467 \times 10^1$
$B_1 = 3.4378043 \times 10^{-1}$
$B_2 = -7.7326396 \times 10^{-4}$
$B_3 = 6.3405286 \times 10^{-7}$

Step 3: Calculate the mole fraction of H_2O vapor in the humid air

$$(H_2O)_{mf} = (RH \times f \times P_{ws})/(P_1 \times 100)$$

Step 4: Define the dry air composition by volume. A recommended dry air composition is the following:[2]

(N_2)dry	78.0868%
(O_2)dry	20.9409%
(Ar)dry	0.9332%

$(CO_2)dry$ 0.0385%
$(He)dry$ 0.0005%

Step 5: Renormalize the dry air composition to include the mole fraction of H_2O vapor.

$$(N_2)humid = (N_2)dry/[(N_2)dry + (O_2)dry + (Ar)dry+(CO_2)dry + (He)dry+(H_2O)_{mf}]$$
$$(O_2)humid = (O_2)dry/[(N_2)dry + (O_2)dry + (Ar)dry + (CO_2)dry + (He)dry + (H_2O)_{mf}]$$
$$(Ar)humid = (Ar)dry/[(N_2)dry + (O_2)dry + (Ar)dry + (CO_2)dry + (He)dry + (H_2O)_{mf}]$$
$$(CO_2)humid = (CO_2)dry/[(N_2)dry + (O_2)dry + (Ar)dry + (CO_2)dry + (He)dry + (H_2O)_{mf}]$$
$$(He)humid = (He)dry/[(N_2)dry + (O_2)dry + (Ar)dry + (CO_2)dry + (He)dry + (H_2O)_{mf}]$$
$$(H_2O)humid = (H_2O)dry/[(N_2)dry + (O_2)dry + (Ar)dry + (CO_2)dry + (He)dry + (H_2O)_{mf}]$$
$$(N_2)humid = (N_2)dry/[(N_2)dry + (O_2)dry + (Ar)dry + (CO_2)dry + (He)dry + (H_2O)_{mf}]$$

Step 6: Verify that the sum of the "humid" air constituents equals 100%. Table D-2-1 provides sample values attained with this method.

Table D-2-1 Results for Humid Air Calculations Using Relative Humidity

Preliminary Calculations (See Steps 1, 2, and 3) [Note (1)]

Parameter	Dry Air	Humid Air
Saturation vapor pressure, P_{ws}	. . .	2.505220 kPa (0.363350 psia)
Enhancement factor, f	. . .	1.0039750
Water vapor content	. . .	0.0090546 mole fraction

Composition of Air (See Steps 4 and 5)

Component	Dry Air, Mole Fraction	Humid Air, Mole Fraction
Nitrogen, N_2	0.7808685	0.7738615
Oxygen, O_2	0.2094101	0.2075310
Argon, Ar	0.0093317	0.0092480
Carbon dioxide, CO_2	0.0003845	0.0003811
Helium, He	0.0000052	0.0000052
Water, H_2O	. . .	0.0089734

Molecular Weight

	Dry Air, g/mol	Humid Air, g/mol	Δ Between Humid Air and Dry Air, %
Aggregate molecular weight	28.96546	28.86720	–0.339%

Properties of Air

Property	Dry Air	Humid Air	Δ Between Humid Air and Dry Air, %
Critical flow factor	0.685118	0.684956	–0.024
Compressibility factor, Z	0.99965	0.99962	–0.003
Density	1.1843 kg/m^3 (0.07393 lbm/ft^3)	1.1803 kg/m^3 (0.07369 lbm/ft^3)	–0.336
Isentropic exponent	1.4014	1.4006	–0.058
Absolute viscosity	18.30 μPa/s [1.230E–05 lbm/(ft sec)]	18.26 μPa/s [1.227E–05 lbm/(ft sec)]	–0.228
Sound speed	344.0 m/s (1,128.59 ft/sec)	344.5 m/s (1,130.76 ft/sec)	0.140

GENERAL NOTE: Values in U.S. Customary units are shown in parentheses.

NOTE:
(1) Inputs for the calculations were as follows:
pressure = 100 kPa (14.5 psia)
relative humidity = 36%
temperature = 294.25 K (70.0°F)

NONMANDATORY APPENDIX E
CFV UNCHOKING TEST PROCEDURE

An unchoking test can be performed to determine the maximum back pressure ratio at which a CFV may be operated and maintain critical flow at the nozzle throat. If a CFV will be operated over a range of Re_d then it is recommended that the unchoking test be performed at the lowest operational Re_d.

Figure E-1 shows the preferred setup for performing an unchoking test. An upstream standard CFV (sized correctly to be choked when located upstream of the test CFV) is used to provide a stable mass flow into the test CFV, or unit under test (UUT). A pressure regulator located downstream of the UUT is used to control to the desired back pressure ratio.

The following steps can be followed to perform the unchoking test:

Step 1: Using the upstream regulator, control to a constant inlet pressure at the standard CFV, providing a stable, known mass flow through the UUT.

Step 2: Open the downstream pressure regulator to minimize P_{exit} pressure.

Step 3: Once conditions have stabilized, record the standard CFV mass flow and the test CFV inlet temperature and pressure. Calculate C_d for the UUT.

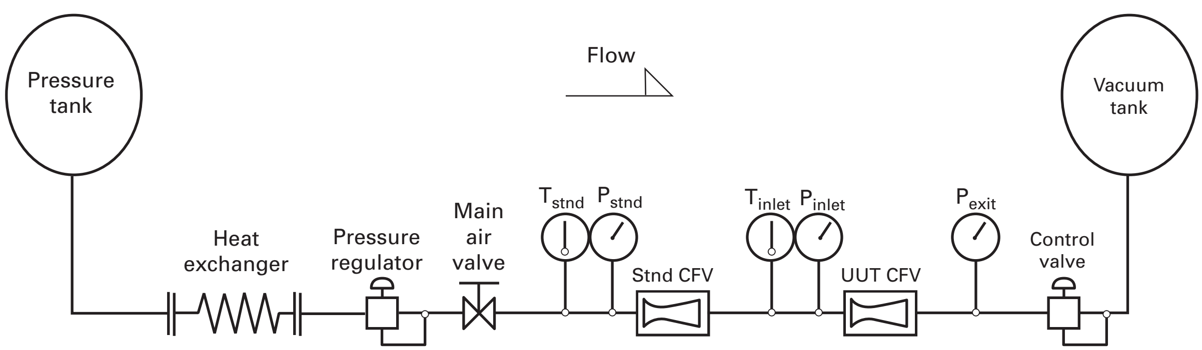
Step 4: Maintaining the same mass flow, use the downstream pressure regulator to increase P_{exit} to a higher back pressure ratio and wait for conditions to stabilize. Record the mass flow, temperature, and pressure data, and then recalculate C_d.

Step 5: Repeat Step 4 multiple times, slowly increasing through the back pressure ratio range.

Step 6: Plot the C_d versus back pressure ratio results. The CFV is choked for the linear (slope = 0) portion of this plot. A C_d decrease that is significant compared to the C_d uncertainty indicates that the CFV is unchoked.

Values for maximum back pressure ratios range from 0.528 to 0.98. CFVs, especially when the diffuser length is less than $7d$, where throat Re_d are below 50,000, may exhibit an undesired drop in C_d from a back pressure ratio of 0.35 to 0.5, and it is advised that multiple data points be collected in this region to verify performance.[1]

Fig. E-1 CFV Unchoking Test Setup



[1] Additional information on this effect is available in Carter, M., Sims, B., Britton, C., and McKee, R., Choking Pressure Ratio Guidelines for Small Critical Flow Venturis and the Effects of Diffuser Geometry, *16th International Flow Measurement Conference*, Paris, France, 2013.

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