ASME MFC-26–2011

Measurement of Gas Flow by Bellmouth Inlet Flowmeters

AN AMERICAN NATIONAL STANDARD

The American Society of Mechanical Engineers

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Three Park Avenue • New York, NY • 10016 USA

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This Standard will be revised when the Society approves the issuance of a new edition. There will be no addenda issued to this edition.

ASME issues written replies to inquiries concerning interpretations of technical aspects of this Standard. Periodically certain actions of the ASME MFC Committee may be published as Cases. Cases and interpretations are published on the ASME Web site under the Committee Pages at http://cstools.asme.org as they are issued.

The American Society of Mechanical Engineers
Three Park Avenue, New York, NY 10016-5990

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FOREWORD

The bellmouth is a common device for flow conditioning and flow measurement in the aerospace industry. Specifically, the bellmouth is attached to the front end of a turbofan gas turbine engine. Turboshaft engine applications also use the bellmouth but typically for flow conditioning and less frequently for flow measurement. The automotive industry also uses the bellmouth in some test applications. This Standard was prepared by Subcommittee 26, Bellmouth Inlet Flowmeters, of the ASME Standards Committee on Measurement of Fluids in Closed Conduits (MFC).

This is the initial release of this Standard.

This Standard provides information in both SI (metric) units and U.S. Customary units.

Suggestions for improvement of this Standard are welcome. They should be sent to The American Society of Mechanical Engineers; Secretary, MFC Standards Committee; Three Park Avenue; New York, NY 10016-5990.

This Standard was approved by the American National Standards Institute on March 30, 2011.

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MEASUREMENT OF FLUID FLOW IN CLOSED CONDUITS

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MEASUREMENT OF GAS FLOW BY BELLMOUTH INLET FLOWMETERS

1 GENERAL

1.1 Scope

This Standard applies only to the steady flow of single-phase gases and gas mixtures and applies only to bellmouth inlet flowmeters in which the flow remains subsonic throughout the measuring section and the flow is steady or varies only slowly with time. It also addresses procedures by which calibration of the device can be made to allow for application with consistent conclusions and within known limits of uncertainty.

Bellmouth inlet flowmeters should be used only within the limits for which a given unit is tested, or if additional uncertainty can be tolerated, over a range within which extrapolation is reliable.

This Standard outlines the general geometry and method of use of bellmouth inlet flowmeters to determine the mass or volumetric flow rate of the gas or gas mixture flowing through the device. It also gives necessary information for calculating the flow rate and its associated uncertainty.

A bellmouth inlet flowmeter is a device that provides flow conditioning and flow measurement whose inlet is located or positioned in a large reservoir or supply source. The reservoir can be outside ambient, room, or plenum conditions depending on the application. The bellmouth inlet flowmeter is also referred to as an airbell, nozzle with zero beta ratio, borda tube, etc. Typical geometry consists of a convergent inlet followed by a constant throat area. This flowmeter is a differential pressure type device that allows determination of the flow rate from the differential pressure between the total pressure and static pressure at a single specified axial location in the constant area throat of the bellmouth.

1.2 Purpose

The purpose of this Standard is to provide guidance and recommendations for fluid flow measurement of gaseous applications using the bellmouth inlet flowmeter.

This Standard addresses the following:

(a) principle of operation
(b) design parameters and considerations
(c) calibration methods and procedures
(d) instrumentation and calculation methods
(e) installation requirements and considerations
(f) measurement uncertainty

1.3 Field of Application

The bellmouth inlet flowmeter is a common device that both conditions the flow and measures its rate and is widely used in the aerospace industry. Specifically, the discharge of the bellmouth flowmeter is often attached to the front end of a test article such as a turbofan gas turbine engine. Turboshaft engine applications also use the bellmouth but typically for flow conditioning and less frequently for flow measurement. The automotive industry also uses the bellmouth in some test applications.

2 REFERENCES

The following documents form a part of this Standard to the extent specified herein. Unless otherwise specified, the latest edition shall apply.

ASME Fluid Meters, 6th Edition, 1971

ASME MFC-1M, Glossary of Terms Used in the Measurement of Fluid Flow in Pipes

ASME MFC-3M, Measurement of Fluid Flow in Pipes Using Orifice, Nozzle, and Venturi

ASME PTC 19.5, Flow Measurement

Publisher: The American Society of Mechanical Engineers (ASME), Three Park Avenue, New York, NY 10016-5990; Order Department: 22 Law Drive, P.O. Box 2900, Fairfield, NJ 07007-2900 (www.asme.org)

ISO 5167, Measurement of fluid flow by means of pressure differential devices inserted in circular cross-section conduits running full

Publisher: International Organization for Standardization (ISO), Central Secretariat, 1, ch. de la Voie-Creuse, Case postale 56, CH-1211 Geneva 20, Switzerland (www.iso.org)

3 DEFINITIONS AND SYMBOLS

This Standard is written to serve the flow measurement community in general. Throughout this Standard flow measurement nomenclature will be given first, with aerospace-industry–specific nomenclature provided as ancillary. Similar treatment will be made for equations.

3.1 Definitions From ASME MFC-1M

base flow rate: the flow rate calculated from flowing conditions to base conditions of pressure and temperature.

calibration: the experimental determination of the relationship between the quantity being measured and the device that measures it, usually by comparison with a standard. Also, the act of adjusting the output of a device to bring it to a desired value, within a specified tolerance, for a particular value of the input.

differential pressure (of a Pitot tube): difference between the pressures measured at the total pressure tap and the static pressure tap.

flow rate: the quantity of fluid flowing through a cross-section of a pipe per unit of time.

ISA 1932 nozzle: a nozzle that consists of an upstream face that is perpendicular to the throat axis, a convergent section defined by two arcs, a cylindrical throat, and a recess. ISA 1932 nozzles always have corner tappings.

long radius nozzle: a nozzle that consists of an upstream face that is perpendicular to the throat axis, a convergent section whose shape is a quarter ellipse, a cylindrical throat, and a recess or a bevel.

Mach number: the ratio of the fluid velocity to the velocity of sound in the fluid at the same temperature and pressure.

mass flow rate: the rate of flow of fluid mass through a cross-section of a pipe.

nozzle: convergent device having a curved profile with no discontinuities leading to a throat.

Pitot tube: tubular device consisting of a cylindrical head attached perpendicularly to a stem. It is provided with one or more pressure tap holes, and it is inserted into a flowing fluid, thus giving the stagnation or static pressure.

Pitot-static tube: a Pitot tube provided with static pressure tap holes drilled at specific positions on the circumference of the cylinder that is oriented parallel to the flow direction. These holes can be drilled at one or more cross-sections. The total pressure tap faces the flow direction at the tip of the axisymmetric nose or head of the cylinder.

NOTE: When there is no possibility of confusion, the expression Pitot tube without further explanation may be used to designate a Pitot-static tube.

pressure taps (piezometric taps): a hole or annular slot in a flange, fitting or the wall of a pipe, or throat of a primary device that is flush with the inside surface.

steady flow: flow in which the flow rate in a measuring section is constant with the measurement uncertainty and over the time period of interest, aside from variations related to natural turbulence generated.

NOTE: The steady flows observed are, in practice, flows in which quantities such as velocity, pressure, mass, density, and temperature vary in time about mean values that are independent of time; these are actually statistically steady flows.

Taylor series: a power series to calculate the value of a function at a point in the neighborhood of some reference point. The series expresses the difference or differential between the new point and the reference point in terms of the successive derivatives of the function.

NOTE: The function is not listed as it is not referenced in this Standard.

total pressure Pitot tube: a Pitot tube with only a total pressure tap hole.

NOTE: A total pressure Pitot tube is generally associated with a separate static pressure tap located on the pipe wall.

traceability: property of a result of measurement whereby it can be related to appropriate standards, generally international or national standards, through an unbroken chain of comparisons. In the United States, the unbroken chain of comparison is with the standards at the NIST or at the state agency of weights and measures.

NOTE: Measurements have traceability if and only if scientifically rigorous evidence is produced on a continuing basis to show that the measurement process is producing measurement results (i.e., data) for which the total measurement uncertainty is quantified.

uncertainty (of measurement): range within which the true value of the measured quantity can be expected to lie with a specified probability and confidence level.

volume flow rate: the rate of flow of fluid volume through a cross-section of a pipe.

wall taps: annular or circular hole drilled in the wall of the pipe in such a way that its edge is flush with the internal surface of the pipe, the tap being such that the pressure within the hole is the static pressure at that point in the pipe.

3.2 Definitions Specific for This Standard

base conditions: specified conditions, base pressure, and base temperature to which the measured mass of a fluid is converted to the volume of the fluid.

base pressure: a specified reference pressure to which a fluid volume at flowing conditions is reduced.

base temperature: a specified reference temperature to which a fluid volume at flowing conditions is reduced.

bellmouth inlet flowmeter: a device that provides flow conditioning and flow measurement whose inlet is in a large reservoir or supply source. The reservoir can be outside ambient, room, or plenum conditions depending on the application. The bellmouth inlet flowmeter is also referred to as a bellmouth, airbell, nozzle with zero beta ratio, or borda tube.

boundary layer: influence of viscosity at high Reynolds numbers is confined to a very thin layer in the immediate neighborhood of the solid wall. In that thin layer the velocity of the fluid increases from zero at the wall (no slip) to its full value, which corresponds to external frictionless flow. This layer is called the *boundary layer.*

discharge coefficient [flow calibration factor(s)]: calibration factor(s) or coefficient(s) associated with the discharge coefficient used in determining mass flow rate. This coefficient is defined as the ratio of actual flow rate as determined by a calibration standard, and the theoretical flow rate of the flowmeter device assuming an initial unity discharge coefficient.

error: difference between a measured value and the "true" value of a measurand.

NOTE: The "true" value cannot usually be determined. In practice, a conventional recognized "standard" or "reference" value is typically used instead.

flowmeter: device for measuring the quantity or rate of flow of a moving fluid.

foreign object debris (FOD) screen: mechanism by which to ensure no foreign objects are sucked into the bellmouth inlet; also known as a *debris guard*.

installation effect: any difference in performance of a component or the measuring system arising between the calibration under ideal conditions and actual conditions of use. This difference may be caused by different flow conditions due to velocity profile, perturbations, or by different working regimes (pulsation, intermittent flow, alternating flow, vibrations, etc.).

instrumentation plane: all sensing points that are located in the same axial plane.

pressure loss: there are two pressure loss quantities referred to in this Standard.

(a) The first pressure loss of interest is the total difference between the reservoir pressure and the pressure downstream of the meter after pressure recovery.

(b) The second pressure loss of interest is that which is generated solely by use of an optional FOD screen (foreign object debris).

pressure differential: pressure differential between the total or stagnated pressure and the static pressure.

probe recovery error: correction for total pressure or total temperature probe errors as a function of Mach number resulting from probe design.

reference: a verifiable artifact or test facility that is traceable to a recognized national or international measurement standard.

static pressure: the pressure of a fluid that is independent of the kinetic energy of the fluid.

static temperature: actual temperature of the flowing gas.

total pressure (stagnation pressure): sum of the static pressure and the dynamic pressure. It characterizes the state of the fluid when its kinetic energy is completely transformed into potential energy.

total temperature (stagnation temperature): temperature that would exist in the gas if the flowing gas stream were brought to rest by an adiabatic process.

velocity distribution (flow profile): graphic representation of the velocity distribution. The velocities at relative radial physical positions across any axially defined plane (of specific interest is the axially defined instrumentation plane).

3.3 Symbols

See Table 3.3-1. Users of this Standard are cautioned to become familiar with units of all input and resultant calculated parameters. The desired units systems, measured parameter units, constants' units, and calculated parameter units should be thoroughly understood and included when generating calculation routines and reporting data.

4 PRINCIPLE OF MEASUREMENT AND METHOD OF COMPUTATION

The principle of operation of these devices is based on physical laws defined by Bernoulli's law and the mass continuity equation. The flow through these devices is typically achieved by drawing the gas from the space surrounding the inlet of the device. The flow rate calculation is typically achieved by monitoring the pressure difference between the total pressure and static pressure at a specified axial location in the flow cross-section of the device. The specified location is defined as an axial plane of the cross-section where a variety of instrumentation techniques can be employed to determine the free-stream and boundary layer velocities.

Velocity is calculated from the difference between the total and static pressures in the axial plane of the instrumentation located in the throat of the meter. This velocity is then multiplied by the cross-sectional area of the bellmouth throat in the same plane to produce the volumetric flow rate. Line pressure and temperature are also measured, which provide density. Mass flow rate can then be determined from density and volumetric flow rate.

4.1 General Description

Figures 4.1-1 and 4.1-2 show a side view and front view respectively, to introduce the fundamental description.

Table 3.3-1 Symbols

"Flow" [Note (1)]	"Aero" [Note (2)]	Description (First Use)	Dimensions [Note (3)]	SI Units	U.S. Customary Units
A	A	Area of the bellmouth determined at constant throat diameter and axial plane of measurement [eq. (C-1)]	L^2	m^2	in.^{2}
C	C_d	Discharge coefficient, defined as the ratio between the theoretical mass flow rate of the bellmouth with respect to the actual mass flow rate as determined from a recognized standard [eq. (4.2-1)]	Dimensionless	...	...
d	d_a	Diameter of constant area "throat" section of the bellmouth device, under operating conditions [eq. (4.2-1)]	L	m	in.
g_c	g_c	Conversion constant [eq. (4.2-1)]	$LM\Theta^{-2}(L^{-1}M^{-1}\Theta^2)$	m-kg/(s^2-N)	ft-lbm/(s^2-lbf)
M_{air}	MW_{air}	Molecular weight of air, adjusted for relative humidity [eq. (4.2-1)]	M	kg/kg·mol	lbm/lb-mol
Ma	M_n	Mach number (para. 4.4)	Dimensionless	...	...
P	P	Pressure (section 4)	$ML^{-1}T^{-2}$	Pa	psi
P_s	P_s	Pressure, static [eq. (4.2-1)]	$ML^{-1}T^{-2}$	Pa	psi
P_t	P_t	Pressure, total [eq. (4.2-1)]	$ML^{-1}T^{-2}$	Pa	psi
ΔP	DP	Pressure, differential [eq. (4.2-1)]	$ML^{-1}T^{-2}$	Pa, bar	psi
q_m	W_a	Mass flow rate [eq. (4.2-1)]	MT^{-1}	kg/s	lbm/sec, lbm/min
$q_{m,BL}$	$W_{a,BL}$	Mass flow rate, boundary layer (para. 7.3)	MT^{-1}	kg/s	lbm/sec, lbm/min
$q_{m,U}$	$W_{a,U}$	Mass flow rate, free-stream (para. 7.3)	MT^{-1}	kg/s	lbm/sec, lbm/min
$q_{m,ACT}$	$W_{a,ACT}$	Mass flow rate, actual (para. 7.3)	MT^{-1}	kg/s	lbm/sec, lbm/min
$q_{m,TH}$	$W_{a,TH}$	Mass flow rate, theoretical (para. 7.3)	MT^{-1}	kg/s	lbm/sec, lbm/min
R	RU	Universal gas constant [eq. (4.2-1)]	$L2T - 2\Theta - 1$	J/(kg·mol·K)	lbf-ft/(lb-mol-°R)
R_a	ε	Surface roughness of the bellmouth internal finish (para. 6.2.2)	L	m	in.
Re_d	Re_d	Reynolds number (para. 4.3)	Dimensionless	...	...
T	T	Temperature [eq. (4.2-1)]	K, °R	°C	°F
$u(q_m)$	$u(W_a)$	Uncertainty in mass flow rate (section 8)	MT^{-1}	kg/s, kg/min	lbm/sec, lbm/min
$u(x_i)$	$u(x_i)$	Uncertainty in x_i (section 8)	Same as x_i	Same as x_i	Same as x_i
$u(y)$	$u(y)$	Uncertainty in y (section 8)	Same as y	Same as y	Same as y
U	U	Expanded total uncertainty	Same as x_i, y_i	Same as x_i, y_i	Same as x_i, y_i
V	V	Velocity of flow (Nonmandatory Appendix C)	LT^{-1}	m/s	ft/sec
V_s	V_s	Sonic velocity (Nonmandatory Appendix C)	LT^{-1}	m/s	ft/sec
Z	Z	Compressibility	Dimensionless	...	...
$\%RH$	$\%RH$	Percent relative humidity (Nonmandatory Appendix A)	...	...	...
$\partial y/\partial x_i$	$\partial y/\partial x_i$	Sensitivity of y as a function of input, x_i, (Nonmandatory Appendix B)	Partial derivative of y with respect to x_i	Partial derivative of y with respect to x_i	Partial derivative of y with respect to x_i

Table 3.3-1 Symbols (Cont'd)

"Flow" [Note (1)]	"Aero" [Note (2)]	Description (First Use)	Dimensions [Note (3)]	SI Units	U.S. Customary Units
Δx_i	Δx_i	Change in x_i	Function of x_i	Function of x_i	Function of x_i
Δy	Δy	Change in y	Function of y	Function of y	Function of y
$\Delta y/\Delta x_i$	$\Delta y/\Delta x_i$	Ratio of the change in y as a function of an introduced change in x_i	Change in y with respect to change in x_i	Change in y with respect to change in x_i	Change in y with respect to change in x_i
δ	δ	Boundary layer thickness (Fig. 5.2-1)	L	m	in.
κ	γ	Specific heats ratio, isentropic exponent [eq. (4.2-1)]	Dimensionless	...	...
π	π	Universal constant [eq. (4.2-1)]	Dimensionless	...	...
ρ	ρ	Density	ML^{-3}	kg/m^3	lbm/ft^3
μ	μ	Dynamic viscosity of the fluid	$ML^{-1}T^{-1}$	Pa-s	lbm/in.-sec

NOTES:
(1) "Flow" indicates symbols used by the flow measurement community.
(2) "Aero" indicates symbols used by the aerospace community.
(3) Dimensions: M = mass, L = length, T = time, Θ = temperature

The throat section is usually connected directly or with a piping section to the process system.

(a) "Flow" indicates direction and is shown in the side view of Fig. 4.1-1, axially from left to right.

(b) Orientation is defined by "axial" (side view) "radial" and "circumferential" (both in front view).

(c) The "inlet screen" is optional and is often used to prevent foreign object destruction (FOD) to a downstream test article. This device is also referred to as a "FOD screen" or "debris guard."

(d) The "flared inlet" provides the flow conditioning of the air and can be of a variety of shapes; discussed further in para. 5.3.

(e) The "throat" is the axial portion of the bellmouth of constant diameter (area). The object is to have parallel lines of flow direction.

(f) The "measurement plane" is the axial plane used for a variety of instrumentation techniques; discussed further in para. 4.3.

4.2 Computation

The principle of this method of measurement is based on pressure and temperature at the throat of the bellmouth. Computation is achieved with any algebraic variation of eq. (4.2-1) or eq. (4.2-2). The total pressure and static pressure in the measurement plane can be measured directly or inferred by a variety of instrumentation techniques.

Users of this Standard are cautioned to become familiar with units of all input and resultant calculated parameters. The desired units systems, measured parameter units, constants' units, and calculated parameter units should be thoroughly understood and included when generating calculation routines and reporting data.

Mass flow rate is calculated from

$$q_m = C\left(\frac{\pi}{4}d^2\right)P_t\sqrt{\left(\frac{2\kappa g_c M_{\text{air}}}{TR(\kappa-1)}\right)\left(1-\frac{\Delta P}{P_t}\right)^{\left(\frac{2}{\kappa}\right)}\left[1-\left(1-\frac{\Delta P}{P_t}\right)^{\left(\frac{\kappa-1}{\kappa}\right)}\right]} \tag{4.2-1}$$

where

C = discharge coefficient, calibration coefficients (dimensionless)
d = diameter of the throat (constant area section) of the bellmouth, in.
g_c = gravitational constant, ft-lbm/lbf-sec^2
M_{air} = molecular weight of dry air corrected for relative humidity, lbm/lbmol
P_t = total absolute pressure in the axial plane of measurement, lbf/in.^{2}
q_m = mass flow rate of air, lbm/sec
R = universal gas constant, lbf-ft/lb-mol-°R
T = total absolute temperature in the axial plane of measurement, °R
ΔP = differential pressure between representative absolute total and static pressures in the axial plane of measurement, lbf/in.^{2}
κ = specific heats ratio (dimensionless)

This Standard also presents equations in a format that is independent of units' systems. U.S. Customary units are presented for illustration only. Equation (4.2-2)

Fig. 4.1-1 Bellmouth Inlet Flow Nozzle — General Description (Side View)

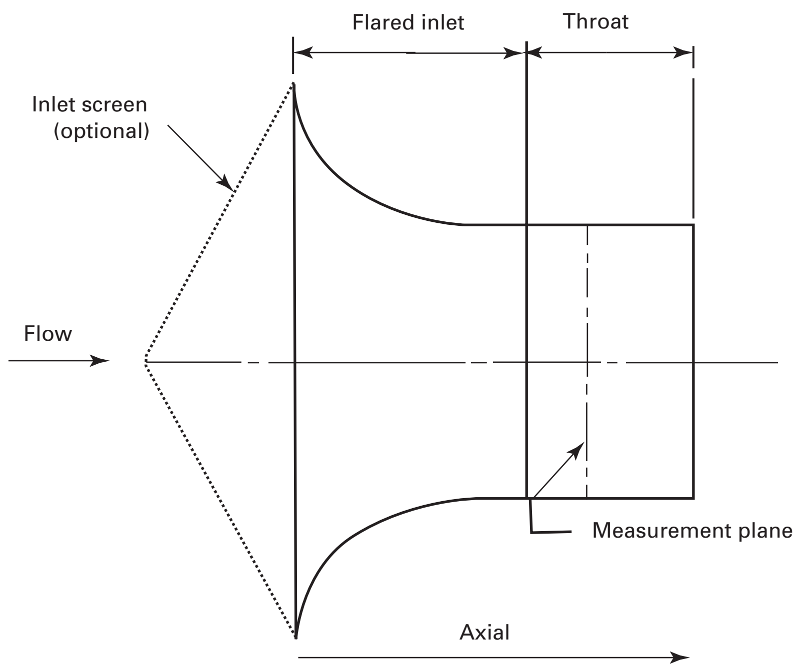
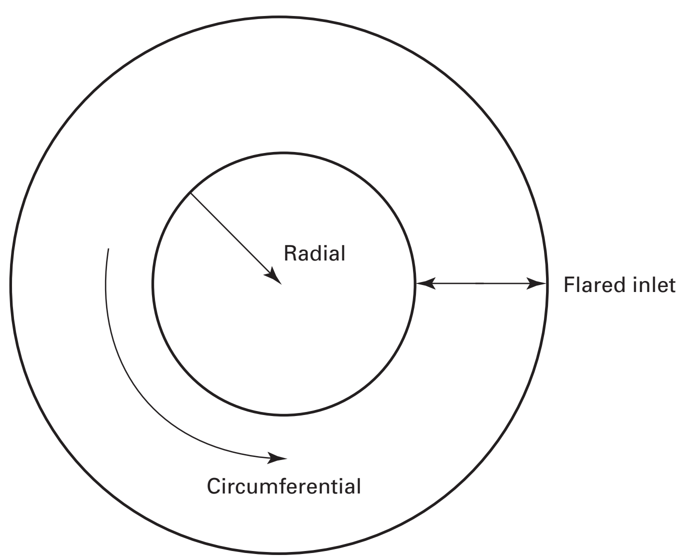


Fig. 4.1-2 Bellmouth Inlet Flow Nozzle — General Description (Front View)



presents use of aerospace nomenclature using the symbols prevalent in the aerospace industry. Mass flow (aerospace industry symbols) rate is calculated from

$$W_a = C_d\left(\frac{\pi}{4}d_a^{\ 2}\right)P_t\sqrt{\frac{\left(\frac{2\gamma g_c MW_{\text{air}}}{T_{\circ R}RU(\gamma-1)}\right)\left(1-\frac{DP}{P_t}\right)^{\left(\frac{2}{\gamma}\right)}}{\left[1-\left(1-\frac{DP}{P_t}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}\right]}} \tag{4.2-2}$$

where

C_d = discharge coefficient, calibration coefficients (dimensionless)
d_a = diameter of the throat (constant area section) of the bellmouth, in.
DP = pressure differential between absolute total and static pressures in the axial plane of measurement, lbf/in.^{2}
g_c = gravitational constant, ft-lbm/lbf-sec^2
MW_{air} = molecular weight of dry air corrected for relative humidity, lbm/lb-mol
P_t = total absolute pressure in the axial plane of measurement (lbf/in.^{2})
RU = universal gas constant, lbf-ft/lb-mol-°R
$T_{\circ R}$ = total absolute temperature in the axial plane of measurement, (°R)
W_a = mass flow rate of air, lbm/sec
γ = specific heats ratio (dimensionless)

4.3 Instrumentation Techniques

There are a variety of instrumentation techniques available. These include combining pressures: static, total, and/or differential, with temperature; all assumed, directly measured, inferred, or corrected to represent fluid conditions at the bellmouth throat.

The throat free-stream conditions are directly measured axially at the throat, and radially such that sensing points are not in the boundary layer. Alternatively, static pressure can be measured perpendicular to the flow via wall static taps. The sensing points of inserted probes used with wall static pressures should be such that

(a) all sensing points are located in the same axial plane that defines the instrumentation plane. The instrumentation plane shall be located at a distance greater than 0.5*d* downstream of where the flared inlet ends (i.e., the start of constant area throat inlet), although 1*d* or greater is recommended.

(b) insertion probes, if used, are circumferentially positioned such that there is no interference or disturbance to wall static taps. There are several considerations that influence relative circumferential positioning of insertion probes and wall static taps to minimize disturbance on the wall static taps:

(1) The size and shape of the insertion probes will impact the potential for disturbance. Probes should be designed with minimum flow disturbance, which achieves the necessary structural requirements (i.e., length for probe depth and ability to withstand flow forces at maximum Mach number).

(2) The quantity of probes will impact the positioning. For large diameter bellmouths, more probes are desired. Typically, four probes is an adequate number, equally circumferentially spaced. Two probes may be adequate for smaller bellmouths. Larger bellmouths may require up to eight insertion probes (see para. 5.3 for discussion on relative diameter sizes of bellmouths). Where possible, the wall static taps should be circumferentially spaced equally between the insertion probes.

A general rule is that any insertion probes should be a minimum of ±10 deg rotated from any wall static tap.

Larger diameter bellmouths are less likely to experience disturbance from relative circumferential positioning of insertion probes and wall static taps.

NOTE: Similar caution in positioning temporarily installed boundary layer probes during in situ calibration is required.

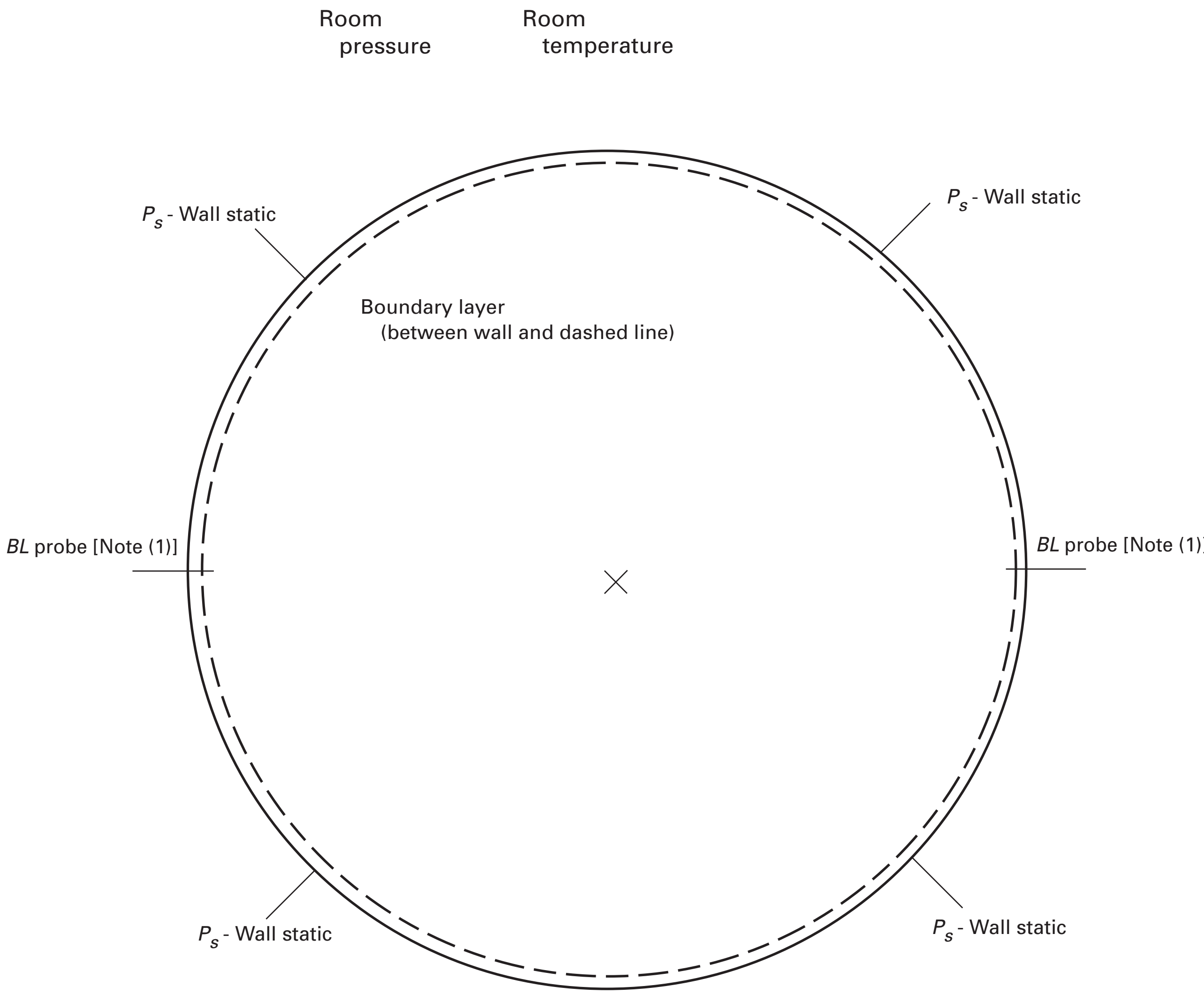
The following are three popular instrumentation techniques in industry, provided for demonstration. Within each of these, further options are provided by differential or absolute measurements of the various pressure sensing locations.

Figures 4.3-1, 4.3-2, and 4.3-3 provide examples for three sample instrumentation techniques demonstrating circumferential positioning of various measurements. Further suboptions are available in that any combination of two out of three of the following representative measurements, P_t, P_s, and ΔP, may be used with simple algebra. This is true for all three instrumentation technique examples presented below. Boundary layer probes are typically installed during in situ calibration only.

The assumption that the circumferential and radial profile of static pressure is constant must be assessed. This assumes the bellmouth flowmeter is positioned symmetrically horizontally and vertically in the "inlet room." In addition, the bypass ratio of air flow (entrainment-to-core flow ratio) must be low. If the bellmouth is located in an outside application, crosswind effects must be accounted for or minimized if low flow uncertainty is desired. It is imperative that the static pressure profile at the measurement plane be evaluated. If the profile is assumed to be constant in the functions of mass flow rate, velocity, and/or Reynolds number, then engineering rationalization must be presented; otherwise, computational fluid dynamics (CFD) or instrumentation for direct measurement is required.

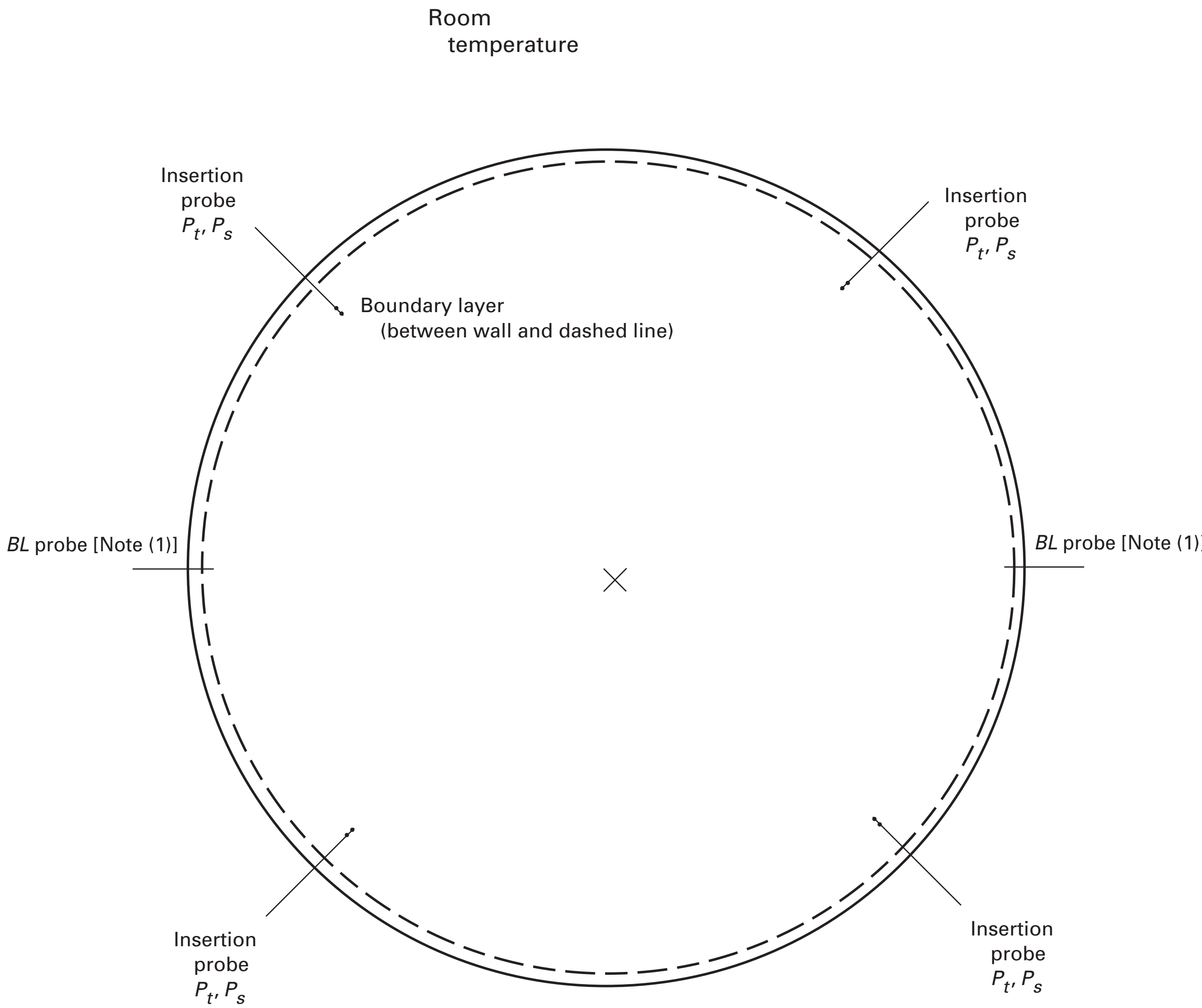
Caution must also be used specifically when using the technique described in para. 4.3.1, shown in Fig. 4.3-1, assuming the pseudostatic pressure in the "inlet room" for total pressure in the measurement plane. The use of an optional inlet screen (i.e., FOD screen or debris guard) will create a small unrecoverable total pressure

Fig. 4.3-1 Clean Inlet Method



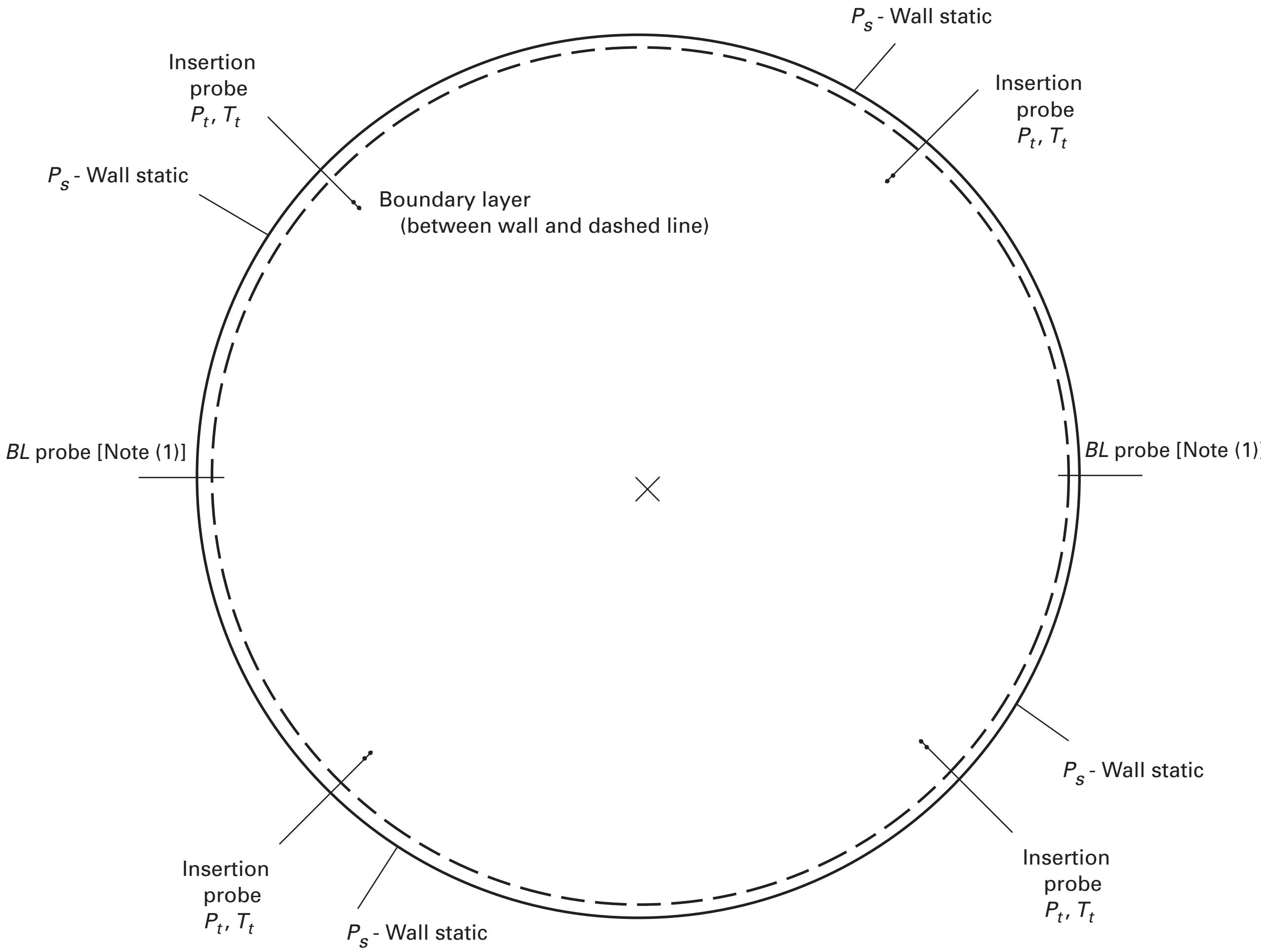
NOTE:
(1) Boundary Layer *(BL)* probe is monitored only during calibration.

Fig. 4.3-2 Combo Probe: P_t and P_s Method



NOTE:
(1) Boundary Layer *(BL)* probe is monitored only during calibration.

Fig. 4.3-3 Combo Probe: P_t and T_t Method



NOTE:
(1) Boundary Layer *(BL)* probe is monitored only during calibration.

loss that must be accounted for by engineering judgment or by in situ calibration. The generic C_d equation presented in para. 7.1 must be modified if the FOD screen is installed.

4.3.1 Clean Inlet Method. In the case where a "clean inlet" is desired for reduction of FOD risk, a pseudostatic pressure may be measured in the "inlet room" (reservoir or ambient surroundings) to infer total pressure at the throat. This is done assuming the air velocity is very low, such that the pseudostatic pressure very nearly equals total pressure, P_t, in the "inlet room," and very nearly equal to the total pressure in the instrumentation plane in the throat of the bellmouth. This is used in combination with wall static tap pressure, P_s. Total temperature, T_t, is also measured in the inlet room or reservoir.

Figure 4.3-1 shows the instrumentation technique of using a room pressure and throat wall static pressures. Temperature is measured in the inlet reservoir (room, plenum, etc.).

4.3.2 Combo Probe: P_t and P_s Method. Combination insertion probes may be used that consist of free-stream total pressure and free-stream static pressure. Temperature can be measured at the bellmouth inlet via attachment to optional inlet screen, fixed position in the "inlet room" (ambient surroundings), or other representative, consistent location.

Figure 4.3-2 shows the instrumentation technique of using an insertion combination probe consisting of free-stream total and static pressures.

4.3.3 Combo Probe: P_t and T_t Method. Combination insertion probes may be used that consist of free-stream total pressure, P_t, and free-stream total temperature, T_t. This is used in combination with wall static taps, P_s.

Figure 4.3-3 shows the instrumentation technique of using an insertion combo probe consisting of free-stream total pressure and total temperature.

4.4 Method of Sizing (Mach Number, Reynolds Number, Etc.)

Frequently, the size of the constant area throat section is dictated by the geometry definition of the application. However, in some cases it is desired to achieve higher Mach numbers at the throat instrumentation plane to improve pressure and temperature measurements. In this latter case, a throat diameter less than the application geometric diameter may require a divergent section downstream of the instrumentation plane.

Direct measurement of the bellmouth-generated pressure differential typically results in lower total system uncertainty than calculating a differential between two absolute measurements. Where possible the bellmouth diameter should be sized such that the application requirement for maximum mass flow rate occurs just below the full range of a commercially available differential pressure transducer.

The size and shape of the flow conditioning flare is discussed in section 5.

4.5 Fluid Properties That Affect the Flow Rate Measurement

For most applications, the fluid properties of the inlet room, plenum, or outside reservoir provide ambient conditions. However, in some testing applications, a plenum or room with conditioned inlet is desired. Careful consideration must be given to proper determination of fluid properties.

4.5.1 Density. Often mass flow rate is the desired result. These applications require the determination of density. The total pressure measurement used in the calculation as representative of the instrumentation plane is to be used. Total system uncertainty for mass flow rate as a function of density has relatively low sensitivity to temperature as measured in degrees Fahrenheit or centigrade; therefore, the various instrumentation techniques of para. 4.3 for temperature are equally adequate.

4.5.2 Viscosity. Reynolds number is used in the calculation and is a function of absolute viscosity. Auditable traceability is complemented by using a recognized standard for reporting the data for this fluid property. A curve fit of data for typical applications has negligible impact on the uncertainty of calculated result.

4.5.3 Isentropic Exponent. Paragraph 4.5.2 equally applies for specific heats ratio.

4.5.4 Thermal Expansion. The thermal expansion of the bellmouth material as a function of temperature will have an impact on the flow rate. This should be accounted for, and the value of the thermal expansion should be obtained from a reliable source. That value and its source should be documented.

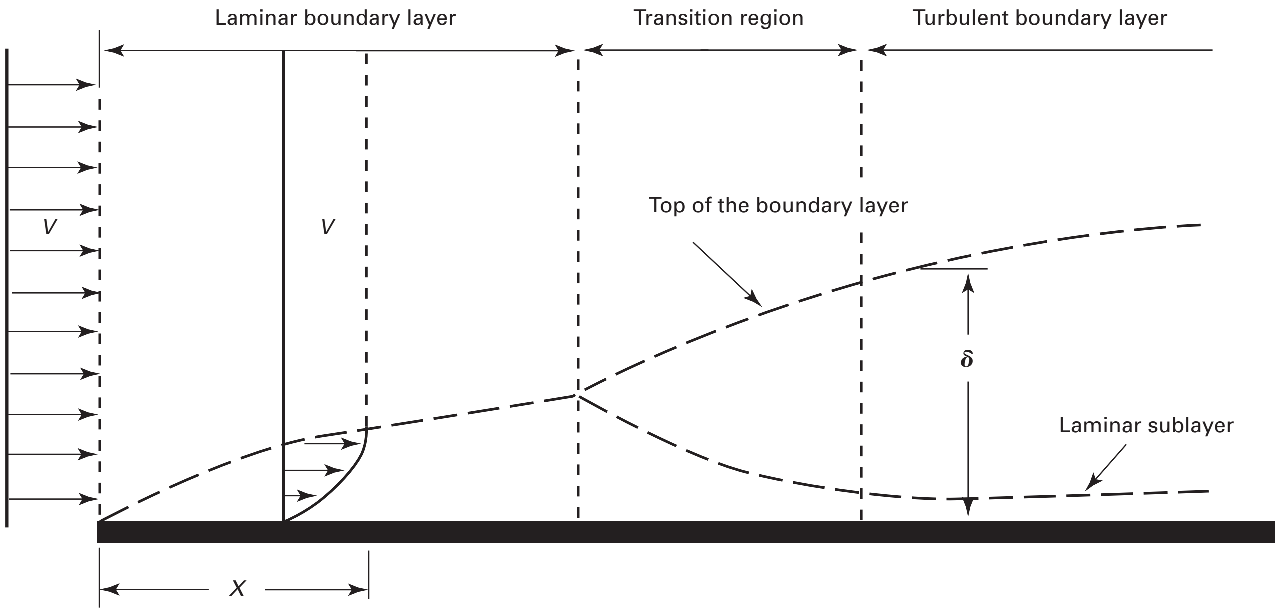
NOTE: Determination of an expansibility factor, often used with other differential producing flow measurement devices, is not required for the bellmouth inlet flowmeter.

5 FLOW CONDITIONING

5.1 Installation Effects (the Inlet Room, Reservoir)

The reservoir can be outside ambient, room, or plenum conditions depending on the application. The bellmouth differs from other differential producing devices in that the flow conditioning is close-coupled to the primary device. The calibration of the device is significantly impacted as a function of installation effects.

Fig. 5.2-1 Typical Boundary Layer Growth, Flat Plate Model



5.2 Boundary Layer

The influence of viscosity at high Reynolds numbers is confined to a very thin layer in the immediate neighborhood of the solid wall. In that thin layer the velocity of the fluid increases from zero at the wall (no slip) to its full value, which corresponds to external frictionless flow. This layer is called the boundary layer. The axial development of the boundary layer is a significant contributor to the deviation from theoretical flow rate as defined (corrected to actual) by the discharge coefficient.

Flat plate models, illustrated in Fig. 5.2-1, are often used for the determination of boundary layer growth as an acceptable method for design of boundary layer probes. The throat boundary layer is the primary influencing factor on the discharge coefficient.

5.3 Inlet Profiles

The design of the inlet flare for flow conditioning is frequently treated as proprietary. However, general guidelines can be presented. Considerations for the design include the flow conditioning, weight, and cost. Desired flow velocities for optimum instrumentation may also impact design.

The following guidelines are recommendations based on various experience in aerospace applications. The tradeoff is between flow conditioning required and the weight of the device. A larger or longer flare inlet will provide better flow conditioning than a smaller or shorter flare inlet. Yet the larger flare will present greater weight that must be supported compared to a shorter flare. In general, smaller diameter applications require more flow conditioning and can tolerate the weight of larger flares.

In all applications, flow conditioning requirements can be mitigated by two factors.

(a) sufficiently large facility when compared to engine diameter

(b) vertical and horizontal axisymmetric positioning

(1) For bellmouth throat diameters less than 20 in. ($d < 20$ in.), more flow conditioning is needed. Examples of appropriate flares for smaller applications include the long radius or ISA 1932 nozzle (see MFC-3M).

(2) Next, medium sized bellmouth throat diameters ($20 < d < 50$ in.) that require considerable support to handle the weight of the larger bell often require use of a shorter flare.

(3) For very large diameter applications ($d > 50$ in.) an even shorter flare may be required. Additional stability of the boundary layer can be achieved by installing boundary layer "trips" in the flare. This practice is often treated as proprietary.

6 GENERAL REQUIREMENTS

6.1 Manufacturing Materials

Bellmouth-inlet flow measuring devices should be made of corrosion-resistant material. For high-temperature service and many other services, stainless steel should be used. Aluminum and composite materials, such as fiberglass, are also recommended where operating temperatures and pressures do not exceed material limits.

6.2 Weight, Envelope, and Other Physical Constraints

The primary device is often hung from the front of a test article such as a gas turbine engine. Considerations of weight and subsequently size, shape, and material are frequently driven by the need to minimize moment forces on the test article.

Bellmouth flowmeters are somewhat similar to a pipe and orifice plate system, both of which are differential devices. It is desired that the measurement be taken at a point where the flow is swirl-free and fully developed. As stated above, the shape and size are often dictated by the moment forces caused by the bellmouth flowmeter; therefore, bellmouth flowmeters feature a straight section behind a curved leading section that conditions the flow. This curved section allows for a shorter upstream section and no need for a long pipe as in the traditional pipe and orifice plate setup. This design decreases weight while still achieving the desired flow measurement. A straight section is defined as not deviating more than 0.4% from a straight line over a pipe's length (see ASME MFC-3M). Flanges are commonly used to connect the straight sections and curved flow conditioning portion. Once again, the ISO Standard requires the flanges to be aligned so that it does not exceed 0.4% deviation from a straight line.

When constructing a bellmouth flowmeter, a seam may be used; however, the internal weld or connection must be parallel to the axis of the bellmouth along its entire length. The ISO dictates that the weld or connection cannot exceed the allowed step in diameter and notes that a weld or connection may not be placed within 30 deg of any pressure tap (see ASME MFC-3M). Annular slots (also used to secure instrumentation) do not require the weld to be placed in any specific location. Spirally wound pipe is permitted, but must be machined smooth along its internal surface.

The throat roughness of a bellmouth flowmeter is measured in terms of R_a, arithmetical mean deviation of the roughness profile. Again, ISO dictates that the surface roughness should be measured at the same location as the internal pipe diameter was measured (see para. 6.2 or para. 7.1.5 of ASME MFC-3M part 1); this should be done at a minimum of four places with an electronic averaging type surface roughness instrument. It is important to note films and deposits can form on bellmouths and instrumentation; this can change surface roughness and thus measured values. Bellmouths and instrumentation should be cleaned, and new surface roughness measurements should be taken and documented after cleaning.

ISO 5167 and ASME MFC-3M state acceptable construction and design are achieved when flow across a bellmouth is capable of swirl-free, fully developed flow over all Reynolds numbers desired for an individual test. Swirl-free is defined as a swirl angle at all points within the primary unit of less than 2 deg.

6.2.1 Cylindrical Throat Specifications. The following requirements for the bellmouth throat are in accordance with ASME Fluid Meters:

(a) The throat of a flow nozzle (bellmouth) should be as nearly cylindrical as possible. Any taper should not exceed the following negative amounts:
  (1) -0.0010 in. for d (in.) ≤ 3.00
  (2) -0.0015 in. for $3.01 \leq d$ (in.) ≤ 6.00
  (3) -0.0020 in. for d (in.) ≥ 6.01

That is, any taper should be such that the throat diameter decreases toward the outlet end.

(b) Any out-of-roundness of the nozzle throat should not exceed the following:
  (1) ± 0.002 in. for d (in.) ≤ 3.00
  (2) ± 0.003 in. for $3.01 \leq d$ (in.) ≤ 6.00
  (3) ± 0.004 in. for d (in.) ≥ 6.01

The actual diameter of the bellmouth throat should be determined by careful measurements after all machining and finishing has been completed. Measurements should be made on four or more diameters and at two or more axial planes. Care must be taken not to scratch the surface while making these measurements.

6.2.2 Surface Roughness Requirements. To ensure fully developed flow with low fluid wall friction throughout the bellmouth flowmeter, the following wall surface finish, R_a, requirement should be followed for all new designs:

$$R_a \leq 10^{-5} \times d$$

6.3 Pressure Taps and Instrumentation

Industry practice reveals a variety of instrumentation techniques. The flow rate through the bellmouth flowmeter is determined by the pressure differential developed by increasing flow velocity into the throat of the bellmouth. Total and static pressure measurements are required along with the inlet temperature.

One common instrumentation technique uses reservoir pressure as the total pressure. Depending on the application, the reservoir conditions can be considered as the outside ambient, room, or plenum conditions. In this technique, total pressure loss must be accounted for during calibration. A major factor in the total pressure loss of an engine test setup is an optional FOD screen (foreign object debris, also called a debris guard). It is used to prevent debris from entering the test article (engine) and is used in many test situations. One disadvantage is its contribution to total permanent pressure loss. Another factor to consider in a test setup is the velocity within the reservoir. If a debris guard is not used and the velocity or Mach number is low ($M_n < 0.1$), the flow conditioning of the bellmouth typically renders the total pressure loss negligible. Another commonly used instrumentation technique directly measures the total pressure at the axial plane where instrumentation is located within the throat of the bellmouth. In this approach it is critical to position instrumentation rakes such that required static pressure measurements in the same plane are unaffected. It is important to note that these techniques do not interfere with the flow within the bellmouth and are therefore valid.

Static pressure must be measured in the axial plane of instrumentation in the throat of the bellmouth. A primary consideration when locating the axial plane of static pressure instrumentation is to ensure static pressure measurement with no total pressure component or velocity. Size, shape, and integrity of the static pressure taps are critical. Notes on the general condition of wall static taps (e.g., burrs and shapes), and the impact on uncertainty can be found in the literature referenced in Nonmandatory Appendix D.

A temperature measurement must be made to calculate density when mass flow rate is the desired result and for any thermal expansion considerations. Temperature may be measured at the throat or in the reservoir, but careful consideration must be given to account for Mach number and probe recovery error. See also para. 4.3.

7 DISCHARGE COEFFICIENT DEVELOPMENT

Like all other differential pressure devices, for best measurement practice, a traceable calibration is required for the airbell. The discharge coefficient, C, given in the function of Reynolds number, Re_d, is the typical result of such a calibration effort. The discharge coefficient by definition is the ratio of actual flow rate to ideal or theoretical flow rate, where the actual flow rate includes the effects of nonideal velocity profiles (e.g., boundary layer growth, etc.), surface finish of the device, viscous drag, etc.

The following are three options for obtaining the C vs. Re_d relationship for differential pressure, ΔP, devices.

7.1 Generic or Standardized From Database

Nationally or internationally recognized standards organizations [e.g., The American Society of Mechanical Engineers (ASME), British Standard (BS), International Organization for Standardization (ISO), etc.] may publish an empirical relationship from results of extensive laboratory testing. At the time of developing this Standard there are no nationally or internationally published tables characterizing the bellmouth, airbells, or “nozzle with zero beta ratio (d/D).” Some industry users have utilized standards for nozzles, although these C vs. Re_d relationships are for nozzles in closed conduit. Although there is no standard table or equation for this device, users may use relationships presented herein with caution. The set of equations below is presented as having reported success, which also maintains representative fundamental relationships of laminar and turbulent boundary layer conditions. Associated uncertainties with cautionary note for use of this equation set are presented in para. 8.3. The discharge coefficient values (with aerospace industry symbols) defined are as follows:

(a) For $Re_d < 10^6$,

$$C = 0.99822 - 6.59298 \times Re_d^{-0.5} \tag{7.1-1}$$

(b) For $Re_d > 10^6$,

$$C = 0.99822 - 0.10449 \times Re_d^{-0.2} \tag{7.1-2}$$

NOTE: The current industry practice is to use eqs. (7.1-1) and (7.1-2). These equations are cited as one set among many available as approximating ASME flow nozzle data.[1] There is a modified later version of the same equation that may also be used by the industry.[2]

7.2 Third-Party Lab Calibration

The device is flow calibrated at a facility traceable to national or international standards (e.g., NIST in the U.S.) or state-recognized standards (state organization for weights and measures) to develop the discharge coefficient values as a function of the Reynolds number for the device. This method of developing the discharge coefficient value for the device is often used by industry, but with limited success. There are few facilities that have the capability to calibrate all sizes of the airbell meter, the calibration costs are often prohibitive, and there are still concerns regarding the effect of the installation specifics (i.e., the inlet size and relative position of the walls, space and available volume of the room, crosswinds) on the C vs. Re_d relationship.

7.3 In Situ Calibration

The preferred industry practice is to heavily instrument the bellmouth flowmeter and calibrate the device in situ, developing the discharge coefficient curve for the device as a function of Reynolds number. Boundary layer probes in the same axial plane as the “production” or “normal” instrumentation suite is a preferred method. This method used alone requires careful consideration of the static pressure profile. Additional measurements can be made to ensure thorough flow profile determination, including multiple total and static pressures in the free-stream.

The following summarizes key steps involved in this activity:

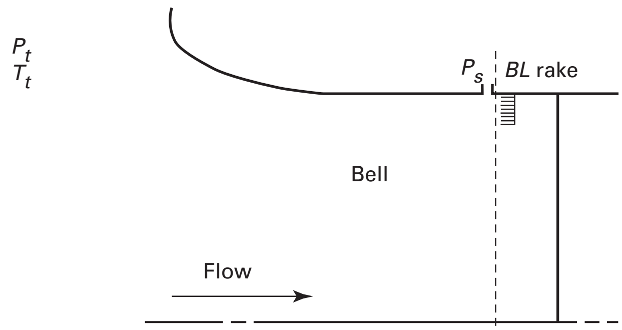
(a) Boundary layer (BL) predictions are conducted.

(b) Boundary layer rake(s) are selected or designed, etc. (See ASME PTC 19.5, section 9 for additional information on ASME-approved pressure probes and recommended traverse installations.)

[1] Robert P. Benedict, Fundamentals of Temperature, Pressure, and Flow. Third edition. New York: Wiley & Sons, 1984, pp. 440–441, equations 22.21 and 22.22.

[2] J. W. Murdock and D. Keyser. Apr. 1991. "Theoretical Basis for Extrapolation of Calibration Data of PTC 6 Throat Tap Nozzles," ASME J. Eng. Gas Turbines and Power 113:228–232; J. W. Murdock and D. Keyser. Apr. 1991. "A Method for the Extrapolation of Calibration Data of PTC 6 Throat Tap Nozzles," ASME J. Eng. Gas Turbines and Power 113: 233–241.

Fig. 7.3-1 Inlet Instrumentation (Technique From Para. 4.3.1)



GENERAL NOTE: This figure is shown without separate, larger free-stream rakes.

(c) Bellmouth flowmeter is designed for appropriate mounting of all rakes.

(d) Calibration plan is devised to coordinate other initial test configuration checkouts.

(e) Collection of data and initial review are conducted.

(f) Process fits are generated for the boundary layer data and total pressure as a function of radial distance from bellmouth wall.

(g) Actual airflow in the boundary layer, $q_{m,BL}$, is found using Pitot-static calculation.

(h) Actual airflow in the free-stream, $q_{m,U}$, is found by setting $C = 1$, resizing diameter to the free-stream boundary value (imaginary effective area).

(i) Theoretical airflow, $q_{m,TH}$, is found using $C = 1$ with "production" instrumentation.

(j) Actual airflow, $q_{m,ACT}$, equals $q_{m,BL} + q_{m,U}$.

(k) $C = q_{m,ACT} / q_{m,TH}$

The "production" or "normal" configuration for measuring airflow includes the pressure differential, ΔP, between static and total pressure for calculating velocity, and the total pressure, P_t, and total temperature, T_t, for calculating density, thus facilitating mass flow rate determination. By calibrating in situ, not only are the installation and boundary condition effects being accounted for, the "normal" configuration instrumentation technique can also be simplified. The parameters, P_t and T_t, can be obtained from inlet room pressures and temperatures instead of having rakes in the bellmouth at all times (see para. 4.3.1).

The "calibration" configuration can often consist of simply adding boundary layer rakes. A boundary layer rake consists of multiple total pressure sensing elements of significant quantity to properly map the total pressure profile radially from the wall to the free-stream (Fig. 7.3-1). If a review of the installation implies low distortion to the flow profile, a reduced number of boundary layer rakes, designed with a limited number of elements always in the free-stream, may suffice. If any suspicion exists regarding the inlet flow profile, additional rakes for P_t, P_s (free-stream), and T_t must be added.

Another in situ method is to use a transfer standard (either heavily instrumented or carefully controlled, i.e., length, calibration duct) in line with the bellmouth.

A practical approach used to develop the C vs. Re_d relationship is based on defining an imaginary "effective area" that allows maximum use of the boundary layer rake information.

In designing the boundary layer rake, it is recommended that at least two elements always reside in the free-stream throughout the flow range of interest. These elements can be used to assess the P_t measurement of the "normal" instrumentation. A significant number of elements of total pressure in the boundary layer should be used. A minimum of four elements are recommended to achieve reasonable representation. More is better, but since typically a low order rational polynomial curve fit of this data typically will be used to calculate the mass flow rate in the boundary layer by means of area averaging, large numbers of elements are generally unnecessary.

For $q_{m,BL}$ determination, a curve fit (process fit) is developed for the boundary layer (BL) rake total pressures and used with the "normal" configuration instrumentation, P_t and T_t. For mass flow rate in the free-stream determination, $q_{m,U}$, a Pitot-static calculation using the "normal" configuration instrumentation is used for airflow inside the free-stream. The actual flow rate, $q_{m,ACT}$, is simply the sum of $q_{m,BL}$ and $q_{m,U}$. This represents the calibration standard. The theoretical flow rate, $q_{m,TH}$, uses the "normal" configuration instrumentation and a standard differential producer calculation routine with $C = 1$.

If all secondary measurements are part of an established metrology quality system that is traceable to national standards and with known uncertainties, the in situ calibration results are traceable to national standards and allow uncertainty analysis of the test data and performance of the device.

CAUTION: It is imperative that the static pressure profile at the measurement plane be evaluated. If the profile is assumed to be constant, in the functions of mass flow, velocity, and/or Reynolds number, the engineering rationalization must be presented;

otherwise suitable modeling, such as computational fluid dynamics (CFD) modeling, or instrumentation for direct measurement is required (para. 4.3).

8 UNCERTAINTIES IN THE MEASUREMENT OF FLOW RATE

General guidelines and discussions are presented herein for determining the uncertainty of calculated airflow with known uncertainties of ancillary measurements. Users are directed to references for additional reading listed in Nonmandatory Appendix D for more detailed treatment.

8.1 Definition of Uncertainty

The expanded total uncertainty of a measurement is usually composed of numerous components, or elemental sources, of uncertainty. For consistency, all uncertainties, both total and elemental, shall be stated presuming 95% confidence intervals. The calculation of mass flow rate comes from para. 4.2. The uncertainty of the calculated parameter must be determined from the uncertainties of the "raw" or direct measured parameters.

The direct measurements on the right side of eq. (4.2-1) are not all independent. However, for practical application, assuming independence as a first order approximation proves reasonable. This is due to the fact that the only correlated terms that may present themselves among the direct-measured parameters are associated with calibration of the pressure measurements. These are typically small contributors to each parameter's total uncertainty and will be treated as negligible here.

Sensitivities can be determined analytically via partial derivatives, but are often found empirically by perturbation of the complete equation set (Nonmandatory Appendix B). Empirical determination proves practical in the case of mass airflow since the desired flow rate result is found by simultaneous and iterative solution of q_m, C, and Re_d, making analytical derivation of partial derivatives difficult.

The direct measured parameters' uncertainties are propagated into the calculation by way of Taylor series expansion, ignoring second order (and higher) covariant terms based on the assumption of independence as mentioned above. The direct measured parameters are then listed as C, d, P_t, $T_{\circ F}$, ΔP (relative humidity, $\%RH$, should also be included as it impacts the molecular weight of air, M_{air}).

The end-to-end expanded uncertainties of these direct measured parameters must be determined. For the discharge coefficient uncertainty, caution must be exercised as delineated throughout this Standard regarding determination method and installation effects. The simplified first-order Taylor series expansion appears as eq. (8.1-1).

$$U(q_m) = \sqrt{\begin{aligned} &\left[\left(\frac{\partial q_m}{\partial C}\right)\times(u(\mathrm{C}))\right]^2 + \left[\left(\frac{\partial q_m}{\partial d}\right)\times(u(d))\right]^2 + \\ &\left[\left(\frac{\partial q_m}{\partial P_t}\right)\times(u(P_t))\right]^2 + \left[\left(\frac{\partial q_m}{\partial T_{\circ F}}\right)\times(u(T_{\circ \mathrm{F}}))\right]^2 + \\ &\left[\left(\frac{\partial q_m}{\partial \Delta P}\right)\times(u(\Delta \mathrm{P}))\right]^2 + \left[\left(\frac{\partial q_m}{\partial \%RH}\right)\times(u(\%RH))\right]^2 \end{aligned}} \tag{8.1-1}$$

8.2 General Steps for Uncertainty Determination

The following steps are taken to develop uncertainties of calculated parameters:

(a) Define resultant calculated parameters of interest and all associated intermediate calculated parameters.

(b) Define the instrumentation list adequate to achieve all desired calculated parameters.

(c) Define typical values for all calculated parameters of interest, associated intermediate calculated parameters, and "raw" direct measurements to be used for the analysis (nominal values of interest).

NOTE: By generating a matrix of "input" or "raw" parameters that fully captures the operating envelope, full factorial calculation will not be necessary.

(d) Determine sensitivities of all intermediate and end calculations, at the typical values, such that "raw" direct-measured parameters' uncertainties are all that are needed to complete the analysis. Sensitivities may be obtained via partial derivatives or empirically derived by perturbation of the equations against each "raw" direct-measured input parameter. A simplified demonstration is provided in Nonmandatory Appendix B.

(e) Individual "raw" or direct-measured parameters' uncertainties are reported. A typical uncertainty analysis is done using values guaranteed achievable for 95% of all reported data for some specified calibration interval. This will often lead to conservative estimates.

(f) Uncertainties and sensitivities are combined via Taylor series, in a root-sum-square (RSS) method, which is statistically aligned with 95% confidence level (normal or Gaussian distribution coverage).

(g) Resulting uncertainties are presented both in engineering units and as percent of the nominal or typical values of interest.

Frequently it is convenient to evaluate each of the direct-measured parameters at four or five points that cover the full capability of the device. This then facilitates curve fitting of uncertainty as a function of each measured parameter's engineering values. This approach allows flexible updates in evaluating the calculated uncertainty for different operating conditions. The end result can also be treated in this way with appropriate caution.

8.3 Uncertainty for Reported "*C* Equations"

This paragraph provides discussion regarding the uncertainty of the discharge coefficient as reported in para. 7.1, eqs. (7.1-1) and (7.1-2).

The uncertainty reported herein is simply derived from the data scatter of the ASME nozzle data used to derive the curve fit. As mentioned throughout this Standard, the bellmouth device is highly susceptible to installation effects. Caution must be taken to ensure that installation effects do not invalidate the use of this or any other standardized equation for discharge coefficient. This caution includes, but is not limited to, all the items delineated in this Standard regarding limits of use, manufacture specifications of the primary device, symmetrical positioning of the bellmouth in surrounding reservoir, and the bypass ratio of surrounding air velocity. For $Re_d > 2 \times 10^4$,

$$u(C) = \pm 0.5\% \text{ of reading}$$

This set of equations is chosen from the reference in Nonmandatory Appendix D and is presented as having reported success. This set is also chosen in that it maintains representative fundamental relationships of laminar and turbulent boundary layer conditions. The variable transition point from laminar to turbulent boundary layer is typically approximated as $Re_d = 5 \times 10^5$ in the literature. This equation set better approximates the referenced author's empirical relationship in the regime of $5 \times 10^5 < Re_d < 1 \times 10^6$, using the $Re_d = 10^6$ transition approximation.

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NONMANDATORY APPENDIX A
SAMPLE CALCULATION EQUATION SET

The following equation set calculating mass flow rate, q_m, is for demonstration purposes. The fluid is air.

(a) Inputs (all units are critical)
  (1) System characteristics: d, α_d
  (2) Known constants: R, M_{air}, π, g_c
  (3) Measured parameters: P_t, $T_{°F}$, ΔP, $\%RH$
(b) Calculations

$$P_s = P_t - DP \tag{A-1}$$

$$T_{°R} = T_{°F} + 459.67 \tag{A-2}$$

$$d_a = d \times \left[1 + \alpha_d \times (T_{°R} - 529.67)\right] \tag{A-3}$$

NOTE: The use of d in subsequent equations assumes prerequisite correction for thermal expansion as described in eq. (A-3).

The following are examples of curve fits for air from traceable data for specific heats ratio and absolute viscosity[1]:

$$\gamma = 1.3930336 - 6.81374 \times 10^{-5} \times T_{°R} - 1.11831 \times 10^{-7} \times T_{°R}{}^2 + 3.16776 \times 10^{-11} \times T_{°R}{}^3 \tag{A-4}$$

$$\mu = \left[\frac{145.8 \times 10^{-7}\left(\frac{T_{°R}}{1.8}\right)^{1.5}}{\left(\frac{T_{°R}}{1.8}\right) + 110.4}\right] \times 0.0056 \tag{A-5}$$

[1] NBS Circular 564, Tables of Thermal Properties of Gases.

Iterate on q_m, C, and Re_d using initialized parameters (approximating the application).

$q_m = 400$ lbm/sec

$C = 0.99$

$Re_d = 100{,}000$

Three equations iterated and solved simultaneously.

$$q_m = C\left(\frac{\pi}{4}d^2\right)P_t\sqrt{\left(\frac{2\kappa g_c M_{air}}{TR(\kappa - 1)}\right)\left(1 - \frac{\Delta P}{P_t}\right)^{\left(\frac{2}{\kappa}\right)}\left[1 - \left(1 - \frac{\Delta P}{P_t}\right)^{\left(\frac{\kappa - 1}{\kappa}\right)}\right]} \tag{A-6}$$

$$Re_d = \frac{4 \times q_m}{\pi \times \mu \times d} \tag{A-7}$$

For $Re_d < 10^6$,

$$C = 0.99822 - 6.59298 \times Re_d{}^{-0.5} \tag{A-8}$$

For $Re_d > 10^6$,

$$C = 0.99822 - 0.10449 \times Re_d{}^{-0.2} \tag{A-9}$$

This published discharge coefficient can be replaced by the resultant C of an in situ calibration. The calibration result is typically a curve fit of discharge coefficient, C, as a function of bellmouth throat Reynolds number, Re_d.

This ends the calculation routine.

NONMANDATORY APPENDIX B
EXAMPLE OF EMPIRICALLY DERIVED SENSITIVITY

Sensitivities for conducting propagated uncertainty analyses for this calculation can be determined empirically by perturbating each direct measured parameter one at a time. It is recommended that small perturbations be used, such as 0.1% of the original nominal input value; this practice generally avoids potential discontinuities. The following example uses a perturbation of 0.001%. The sensitivities are then the quotient of the delta of calculated airflow over the delta of each input parameter.

Keep all other parameters constant.

$X1$ = input
Y = result

$$X1_p = 1.001\% \times X1 \tag{B-1}$$

$$Y_p = \text{perturbated result} \tag{B-2}$$

Sensitivity (partial derivative)

$$\frac{\partial Y}{\partial X1} = \frac{\Delta Y}{\Delta X} = \frac{(Y_p - Y)}{(X1_p - X1)} \tag{B-3}$$

NONMANDATORY APPENDIX C
DERIVATION OF SAMPLE EQUATION

This Nonmandatory Appendix provides a partial derivation of the flow equation found in para. 4.2. The equation is an algebraic derivation from several famous laws and equations derived over the centuries. Laws of motion, fluid dynamics, energy, etc., are all building blocks that provide practical solutions to modern industry, including flow measurement. In addition, there are many assumptions and simplifications used. The practitioner does not require a complete understanding of all these, but the following guidance is provided for background. See equation (4.2-1) for reference.

Mass flow rate is determined by

$$q_m = \rho AV \tag{C-1}$$

It is important to note that the desired density in eq. (C-1) is the flowing density that requires absolute P_s and T_s in the axial instrumentation plane of the bellmouth throat.

From compressible flow functions,

$$\frac{P_t}{P_s} = \left[1+\left(\frac{\kappa-1}{2}\right)Ma^2\right]^{\left(\frac{\kappa}{\kappa-1}\right)} \tag{C-2}$$

$$\frac{T_t}{T_s} = \left[1+\left(\frac{\kappa-1}{2}\right)Ma^2\right] \tag{C-3}$$

Velocity can be found from the Mach number and the sonic velocity at the fluid conditions. Do not confuse Mach number, Ma, with the molecular weight of air, M_{air}.

$$V = Ma \times V_s \tag{C-4}$$

$$V_s = \sqrt{g_c \kappa \left(\frac{R}{M_{\text{air}}}\right) T_s} \tag{C-5}$$

Rearrange eq. (C-2), solving for Mach number, Ma. Raise both sides to the exponent, $\left(\frac{\kappa-1}{\kappa}\right)$.

$$\left[\frac{P_t}{P_s}\right]^{\left(\frac{\kappa-1}{\kappa}\right)} = \left[\left[1+\left(\frac{\kappa-1}{2}\right)Ma^2\right]^{\left(\frac{\kappa}{\kappa-1}\right)}\right]^{\left(\frac{\kappa-1}{\kappa}\right)} \tag{C-6}$$

$$\left[\frac{P_t}{P_s}\right]^{\left(\frac{\kappa-1}{\kappa}\right)} = \left[1+\left(\frac{\kappa-1}{2}\right)Ma^2\right]^{\left(\frac{\kappa}{\kappa-1}\right)\times\left(\frac{\kappa-1}{\kappa}\right)} \tag{C-7}$$

$$\left[\frac{P_t}{P_s}\right]^{\left(\frac{\kappa-1}{\kappa}\right)} = \left[1+\left(\frac{\kappa-1}{2}\right)Ma^2\right] \tag{C-8}$$

$$\left[\frac{P_t}{P_s}\right]^{\left(\frac{\kappa-1}{\kappa}\right)} = 1+\left(\frac{\kappa-1}{2}\right)Ma^2 \tag{C-9}$$

Subtract 1 from both sides.

$$\left[\left[\frac{P_t}{P_s}\right]^{\left(\frac{\kappa-1}{\kappa}\right)} - 1\right] = 1-1+\left(\frac{\kappa-1}{2}\right)Ma^2 \tag{C-10}$$

Multiply by $\left(\frac{2}{\kappa-1}\right)$.

$$\left[\left[\frac{P_t}{P_s}\right]^{\left(\frac{\kappa-1}{\kappa}\right)} - 1\right]\times\left(\frac{2}{\kappa-1}\right) = \left(\frac{\kappa-1}{2}\right)\left(\frac{2}{\kappa-1}\right)Ma^2 \tag{C-11}$$

Swap sides of the equation, and take the square root to solve for Mach number, Ma.

$$Ma = \sqrt{\left[\left(\frac{P_t}{P_s}\right)^{\left(\frac{\kappa-1}{\kappa}\right)} - 1\right]\left(\frac{2}{\kappa-1}\right)} \tag{C-12}$$

Substitute Mach number, Ma, from eq. (C-12) into eq. (C-3), to solve for T_s as a function of total pressure, P_t, static pressure, P_s, and total temperature, T_t. Recall that flowing density requires absolute static temperature and absolute static pressure. Direct measurements being made in the bellmouth application include P_t, P_s, usually from P_t and ΔP, and T_t; all must be in absolute units. Static temperature, T_s, is not directly measured. The objective is to have a practical equation for calculating the air mass flow rate with all direct-measured input parameters. T_s needs to be defined in terms of direct-measured parameters. See eq. (C-3) for reference.

$$\frac{T_t}{T_s} = \left[1+\left(\frac{\kappa-1}{2}\right)\left[\sqrt{\left[\left(\frac{P_t}{P_s}\right)^{\left(\frac{\kappa-1}{\kappa}\right)} - 1\right]\left(\frac{2}{\kappa-1}\right)}\right]^2\right] \tag{C-13}$$

$$\frac{T_t}{T_s} = \left[1 + \left(\frac{\kappa - 1}{2}\right)\left[\left(\frac{P_t}{P_s}\right)^{\left(\frac{\kappa-1}{\kappa}\right)} - 1\right]\left(\frac{2}{\kappa - 1}\right)\right] \qquad \text{(C-14)}$$

$$\frac{T_t}{T_s} = \left[1 + \left[\left(\frac{P_t}{P_s}\right)^{\left(\frac{\kappa-1}{\kappa}\right)} - 1\right]\right] \qquad \text{(C-15)}$$

$$\frac{T_t}{T_s} = \left(\frac{P_t}{P_s}\right)^{\left(\frac{\kappa-1}{\kappa}\right)} \qquad \text{(C-16)}$$

Rearrange, or solve, for T_s (steps shown). Multiply both sides by T_s:

$$\frac{T_t}{T_s} \times T_s = \left(\frac{P_t}{P_s}\right)^{\left(\frac{\kappa-1}{\kappa}\right)} \times T_s \qquad \text{(C-17)}$$

$$T_t = \left(\frac{P_t}{P_s}\right)^{\left(\frac{\kappa-1}{\kappa}\right)} T_s \qquad \text{(C-18)}$$

Multiply both sides by $\left(\frac{P_t}{P_s}\right)^{\left(-\left(\frac{\kappa-1}{\kappa}\right)\right)}$.

$$T_t \times \left(\frac{P_t}{P_s}\right)^{\left(-\left(\frac{\kappa-1}{\kappa}\right)\right)} = \left[\left(\frac{P_t}{P_s}\right)^{\left(\frac{\kappa-1}{\kappa}\right)} \times \left(\frac{P_t}{P_s}\right)^{\left(-\left(\frac{\kappa-1}{\kappa}\right)\right)}\right] T_s \qquad \text{(C-19)}$$

$$T_s = T_t \left(\frac{P_t}{P_s}\right)^{\left(-\left(\frac{\kappa-1}{\kappa}\right)\right)} \qquad \text{(C-20)}$$

Substitute eq. (C-20) into eq. (C-5) to obtain sonic velocity, *Ma*, as a function of direct measured parameters,

$$V_s = \sqrt{g_c \kappa \left(\frac{R}{M_{air}}\right)\left[T_t \left(\frac{P_t}{P_s}\right)^{\left(-\left(\frac{\kappa-1}{\kappa}\right)\right)}\right]} \qquad \text{(C-21)}$$

Substituting eqs. (C-12) and (C-21) into eq. (C-4) provides the following equation for velocity.

$$V = \sqrt{\left[\left(\frac{P_t}{P_s}\right)^{\left(\frac{\kappa-1}{\kappa}\right)} - 1\right]\left(\frac{2}{\kappa - 1}\right)} \times \sqrt{g_c \kappa \left(\frac{R}{M_{\text{air}}}\right)\left[T_t \left(\frac{P_t}{P_s}\right)^{\left(-\left(\frac{\kappa-1}{\kappa}\right)\right)}\right]} \qquad \text{(C-22)}$$

Combine and reduce to simplify eq. (C-22).

$$V = \sqrt{\left[\frac{2\kappa g_c R T_t}{M_{air}(\kappa - 1)}\right]\left[\left(\frac{P_t}{P_s}\right)^{\left(\frac{\kappa-1}{\kappa}\right)} - 1\right]\left[\left(\frac{P_t}{P_s}\right)^{\left(-\left(\frac{\kappa-1}{\kappa}\right)\right)}\right]} \qquad \text{(C-23)}$$

As an intermediate step, combine the two pressure ratio terms under the radical.

$$\left[\left(\frac{P_t}{P_s}\right)^{\left(\frac{\kappa-1}{\kappa}\right)} - 1\right]\left[\left(\frac{P_t}{P_s}\right)^{\left(-\left(\frac{\kappa-1}{\kappa}\right)\right)}\right] \qquad \text{(C-24)}$$

$$\left[\left(\frac{P_t}{P_s}\right)^{\left(\frac{\kappa-1}{\kappa}\right)}\left(\frac{P_t}{P_s}\right)^{\left(-\left(\frac{\kappa-1}{\kappa}\right)\right)} - 1\left(\frac{P_t}{P_s}\right)^{\left(-\left(\frac{\kappa-1}{\kappa}\right)\right)}\right] \qquad \text{(C-25)}$$

$$\left[\left(\frac{P_t}{P_s}\right)^{\left(\frac{\kappa-1}{\kappa}\right)+\left(-\left(\frac{\kappa-1}{\kappa}\right)\right)} - 1\left(\frac{P_t}{P_s}\right)^{\left(-\left(\frac{\kappa-1}{\kappa}\right)\right)}\right] \qquad \text{(C-26)}$$

Where the first term's exponent can be simplified by

$$\left(\frac{\kappa - 1}{\kappa}\right) + \left(-\left(\frac{\kappa - 1}{\kappa}\right)\right) = \frac{\kappa - 1 - \kappa + 1}{\kappa} = 0$$

$$\left[\left(\frac{P_t}{P_s}\right)^{0} - 1\left(\frac{P_t}{P_s}\right)^{\left(-\left(\frac{\kappa-1}{\kappa}\right)\right)}\right] \qquad \text{(C-27)}$$

$$\left[1 - \left(\frac{P_t}{P_s}\right)^{\left(-\left(\frac{\kappa-1}{\kappa}\right)\right)}\right] \qquad \text{(C-28)}$$

Invert such that P_s is in the numerator.

$$\left[1 - \left(\frac{P_s}{P_t}\right)^{\left(\frac{\kappa-1}{\kappa}\right)}\right] \qquad \text{(C-29)}$$

Replace P_s with $P_s = P_t - \Delta P$.

$$\left[1 - \left(\frac{P_t - \Delta P}{P_t}\right)^{\left(\frac{\kappa-1}{\kappa}\right)}\right] \qquad \text{(C-30)}$$

$$\left[1 - \left(\frac{P_t}{P_t} - \frac{\Delta P}{P_t}\right)^{\left(\frac{\kappa-1}{\kappa}\right)}\right] \qquad \text{(C-31)}$$

$$\left[1 - \left(1 - \frac{\Delta P}{P_t}\right)^{\left(\frac{\kappa-1}{\kappa}\right)}\right] \qquad \text{(C-32)}$$

This eq. (C-32) can now be put into eq. (C-23) to represent the simplest form of the velocity equation, as a function of direct-measured parameters.

$$V = \sqrt{\left[\frac{2\kappa g_c R T_t}{M_{\text{air}}(\kappa - 1)}\right]\left[1 - \left(1 - \frac{\Delta P}{P_t}\right)^{\left(\frac{\kappa-1}{\kappa}\right)}\right]} \qquad \text{(C-33)}$$

The next parameter of interest is the flowing density.

$$\rho_s = \frac{(P_s \times M_{\text{air}})}{(T_s \times R)} \quad \text{(C-34)}$$

As previously mentioned, static pressure, P_s, is directly measured, but static temperature, T_s, is not. Substitute eq. (C-20) for T_s into eq. (C-34),

$$\rho_s = \frac{(P_s M_{\text{air}})}{R \times \left[T_t \left(\frac{P_t}{P_s} \right)^{\left(-\left(\frac{\kappa-1}{\kappa}\right)\right)} \right]} \quad \text{(C-35)}$$

Simplify with $\dfrac{1}{\left(\frac{P_t}{P_s}\right)^{\left(-\left(\frac{\kappa-1}{\kappa}\right)\right)}} = \left(\frac{P_t}{P_s}\right)^{\left(\frac{\kappa-1}{\kappa}\right)}$

$$\rho_s = \frac{(P_s M_{\text{air}})}{RT_t} \times \left(\frac{P_t}{P_s} \right)^{\left(\frac{\kappa-1}{\kappa}\right)} \quad \text{(C-36)}$$

The final parameter needed for the mass continuity eq. (C-1), is bellmouth throat area. The diameter of the bellmouth throat is presented, corrected for thermal expansion, as d.

$$A = \frac{\pi}{4} d^2 \quad \text{(C-37)}$$

Substitute eqs. (C-36), (C-37), and (C-33), into eq. (C-1).

$$q_m = \left[\frac{(P_s M_{\text{air}})}{RT_t} \left(\frac{P_t}{P_s} \right)^{\left(\frac{\kappa-1}{\kappa}\right)} \right] \left[\frac{\pi}{4} d^2 \right] \left[\sqrt{ \left[\frac{2\kappa g_c R T_t}{M_{\text{air}} (\kappa - 1)} \right] \left[1 - \left(1 - \frac{\Delta P}{P_t} \right)^{\left(\frac{\kappa-1}{\kappa}\right)} \right] } \right] \quad \text{(C-38)}$$

Square-root of the square allows combining like terms under the radical.

$$\left[\frac{P_s M_{\text{air}}}{RT_t} \left(\frac{P_t}{P_s} \right)^{\left(\frac{\kappa-1}{\kappa}\right)} \right] = \sqrt{ \left[\frac{P_s M_{\text{air}}}{RT_t} \left(\frac{P_t}{P_s} \right)^{\left(\frac{\kappa-1}{\kappa}\right)} \right]^2 } \quad \text{(C-39)}$$

Rearrange, with square root of square included under the radical.

$$q_m = \left[\frac{\pi}{4} d^2 \right] \left[\sqrt{ \left[\frac{P_s^2 M_{\text{air}}^2}{R^2 T_t^2} \left[\left(\frac{P_t}{P_s} \right)^{\left(\frac{\kappa-1}{\kappa}\right)} \right]^2 \right] \left[\frac{2\kappa g_c R T_t}{M_{\text{air}} (\kappa-1)} \right] \left[1 - \left(1 - \frac{\Delta P}{P_t} \right)^{\left(\frac{\kappa-1}{\kappa}\right)} \right] } \right] \quad \text{(C-40)}$$

Combine terms, rearrange, and simplify.

$$q_m = \left[\frac{\pi}{4} d^2 \right] \left[\sqrt{ P_s^2 \left(\frac{2\kappa g_c M_{\text{air}}}{T_t R (\kappa-1)} \right) \left[\left(\frac{P_t}{P_s} \right)^{\left(\frac{\kappa-1}{\kappa}\right)} \right]^2 \left[1 - \left(1 - \frac{\Delta P}{P_t} \right)^{\left(\frac{\kappa-1}{\kappa}\right)} \right] } \right] \quad \text{(C-41)}$$

Under the radical, there remains a pressure ratio term to simplify. Combine the P_s^2 term with this pressure ratio term,

$$P_s^2 \left[\left(\frac{P_t}{P_s} \right)^{\left(\frac{\kappa-1}{\kappa}\right)} \right]^2 \quad \text{(C-42)}$$

$$P_s^2 \left[\left(\frac{P_t}{P_s} \right)^{\left(\frac{2\kappa-2}{\kappa}\right)} \right] \quad \text{(C-43)}$$

$$P_s^2 \left[\left(\frac{P_t}{P_s} \right)^{\left(\frac{2\kappa}{\kappa}\right)} \left(\frac{P_t}{P_s} \right)^{\left(\frac{-2}{\kappa}\right)} \right] \quad \text{(C-44)}$$

$$P_s^2 \left[\left(\frac{P_t}{P_s} \right)^2 \left(\frac{P_t}{P_s} \right)^{\left(\frac{-2}{\kappa}\right)} \right] \quad \text{(C-45)}$$

$$P_s^2 \left(\frac{P_t^2}{P_s^2} \right) \left(\frac{P_t}{P_s} \right)^{\left(\frac{-2}{\kappa}\right)} \quad \text{(C-46)}$$

$$P_t^2 \left(\frac{P_t}{P_s} \right)^{\left(\frac{-2}{\kappa}\right)} \quad \text{(C-47)}$$

P_t^2 will be removed from the radical, leaving,

$$\left(\frac{P_t}{P_s} \right)^{\left(\frac{-2}{\kappa}\right)} \quad \text{(C-48)}$$

Invert such that P_s is in the numerator.

$$\left(\frac{P_s}{P_t} \right)^{\left(\frac{2}{\kappa}\right)} \quad \text{(C-49)}$$

Replace P_s with $P_s = P_t - \Delta P$.

$$\left(\frac{P_t - \Delta P}{P_t} \right)^{\left(\frac{2}{\kappa}\right)} \quad \text{(C-50)}$$

$$\left(\frac{P_t}{P_t} - \frac{\Delta P}{P_t} \right)^{\left(\frac{2}{\kappa}\right)} \quad \text{(C-51)}$$

$$\left(1 - \frac{\Delta P}{P_t} \right)^{\left(\frac{2}{\kappa}\right)} \quad \text{(C-52)}$$

This equation, eq. (C-52), represents the simplest form of this pressure ratio term as a function of direct-measured parameters.

Equation (C-41) now becomes

$$q_m = \left[\frac{\pi}{4}d^2\right]P_t\left[\sqrt{\left(\frac{2\kappa g_c M_{air}}{T_t R(\kappa - 1)}\right)\left[\left(1-\frac{\Delta P}{P_t}\right)^{\left(\frac{2}{\kappa}\right)}\right]\left[1-\left(1-\frac{\Delta P}{P_t}\right)^{\left(\frac{\kappa-1}{\kappa}\right)}\right]}\right] \tag{C-53}$$

Add in the calibration coefficient, or discharge coefficient, which converts the theoretical flow rate to actual flowrate, and eq. (C-53) becomes the now familiar equation, eq. (4.2-1).

This ends the derivation. Any algebraic equivalent of this equation is acceptable.

Users of this Standard are cautioned to become familiar with units of all input and resultant calculated parameters. The desired units systems, measured parameter units, constants' units, and calculated parameter units should be thoroughly understood and included when generating calculation routines and reporting data.

NONMANDATORY APPENDIX D
REFERENCES FOR ADDITIONAL READING

ASME MFC-2M, Measurement Uncertainty for Fluid Flow in Closed Conduits

ASME PTC 19.1, Test Uncertainty

Publisher: The American Society of Mechanical Engineers (ASME), Three Park Avenue, New York, NY 10016-5990; Order Department: 22 Law Drive, P.O. Box 2900, Fairfield, NJ 07007-2900 (www.asme.org)

Bryer, D.W., Pankhurst, R.C., Pressure-Probe Methods for Determining Wind Speed and Flow Direction, London, Her Majesty's Stationery Office, 1971

BS 1042, Measurement of Fluid Flow in Closed Conduits

Publisher: British Standards Institution (BSI), 12110 Sunset Hills Road, Reston, VA 20190 (www.bsigroup.com)

ISO Guide to the expression of uncertainty in measurement

Publisher: International Organization for Standardization (ISO) Central Secretariat, 1, ch. de la Voie-Creuse, Case postale 56, CH-1211 Geneva 20, Switzerland (www.iso.org)

NIST TN 1297, Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results

Publisher: National Institute of Standards and Technology (NIST), 100 Bureau Drive, Stop 1070, Gaithersburg, MD 20899 (www.nist.gov)

Ower, E., Pankhurst, R.C., The Measurement of Airflow, Fifth Edition, New York, Pergamon Press Inc., 1977

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