ASME B89.7.3.2-2007
(Technical Report)

Guidelines for the Evaluation of Dimensional Measurement Uncertainty

The American Society of Mechanical Engineers

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Three Park Avenue • New York, NY 10016

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FOREWORD

The ISO Guide to the Expression of Uncertainty in Measurement (GUM) is now the internationally accepted method of expressing measurement uncertainty [1]. The U.S. has adopted the GUM as a national standard [2]. The evaluation of measurement uncertainty has been applied for some time at national measurement institutes but more recently issues such as measurement traceability and laboratory accreditation are resulting in its widespread use in calibration laboratories.

Given the potential impact to business practices, national and international standards committees are working to publish new standards and technical reports that will facilitate the integration of the GUM approach and the consideration of measurement uncertainty. In support of this effort, ASME B89 Committee for Dimensional Metrology has formed Division 7 — Measurement Uncertainty.

Measurement uncertainty has important economic consequences for calibration and measurement activities. In calibration reports, the magnitude of the uncertainty is often taken as an indication of the quality of the laboratory, and smaller uncertainty values generally are of higher value and of higher cost. ASME B89.7.3.1, Guidelines for Decision Rules in Determining Conformance to Specifications [3], addresses the role of measurement uncertainty when accepting or rejecting products based on a measurement result and a product specification. This document, ASME B89.7.3.2, Guidelines for the Evaluation of Dimensional Measurement Uncertainty, provides a simplified approach (relative to the GUM) to the evaluation of dimensional measurement uncertainty. ASME B89.7.3.3, Guidelines for Assessing the Reliability of Dimensional Measurement Uncertainty Statements [4], examines how to resolve disagreements over the magnitude of the measurement uncertainty statement. Finally, ASME B89.7.4, Measurement Uncertainty and Conformance Testing: Risk Analysis [5], provides guidance on the risks involved in any product acceptance/rejection decision.

With the increasing number of laboratories that are accredited, more and more metrologists will need to develop skills in evaluating measurement uncertainty. This report provides guidance for both the novice and experienced metrologist in this endeavor. Additionally, this report may be used to understand the accuracy of measurements at a more comprehensive level than the variation captured by "Gage Repeatability and Reproducibility" (GR&R) studies. This will provide a higher level of confidence in the measurements and aid in determining if a measurement system is capable of meeting the expected capability as a percentage of specified tolerance. Emphasis is placed on simplified uncertainty evaluation appropriate for the reader who is experienced in measurement procedures but is new to uncertainty evaluation.

Comments and suggestions for improvement of this Technical Report are welcome. They should be addressed to The American Society of Mechanical Engineers, Secretary, B89 Main Committee, Three Park Avenue, New York, NY 10016-5990.

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GUIDELINES FOR THE EVALUATION OF DIMENSIONAL MEASUREMENT UNCERTAINTY

ABSTRACT

The primary purpose of this Technical Report is to provide introductory guidelines for assessing dimensional measurement uncertainty in a manner that is less complex than presented in the Guide to the Expression of Uncertainty in Measurement (GUM). These guidelines are fully consistent with the GUM methodology and philosophy. The technical simplifications include not assigning degrees of freedom to uncertainty sources, assuming uncorrelated uncertainty sources, and avoiding partial differentiation by always working with input quantities having units of the measurand. A detailed discussion is presented on measurement uncertainty concepts that should prove valuable to both the novice and experienced metrologist (Nonmandatory Appendices A and B). Potential influence quantities that can affect a measurement result are listed in Nonmandatory Appendix C. Worked examples, with an emphasis on thermal issues, are provided in Nonmandatory Appendix D. The bibliography is located in Nonmandatory Appendix E.

1 SCOPE

These guidelines address the evaluation of dimensional measurement uncertainty. Emphasis is placed on simplified methods appropriate for the industrial practitioner. The introductory methods presented are consistent with the Guide to the Expression of Uncertainty in Measurement (GUM), the nationally [2] and internationally [1] accepted method to quantify measurement uncertainty. The use of these guidelines does not preclude the use of more advanced methods in the uncertainty evaluation process.

2 SIMPLIFICATIONS IN THE EVALUATION OF MEASUREMENT UNCERTAINTY

To simplify and focus the uncertainty evaluation process in an industrial setting, issues associated with the effective degrees of freedom of the uncertainty statement and correlation between uncertainty sources are considered less important when compared to problems associated with underestimating or omitting uncertainty sources. The issue of effective degrees of freedom frequently confuses beginning uncertainty practitioners and at best represents a slight refinement of the uncertainty statement. Indeed, even in the determination of fundamental constants the practice of using degrees of freedom has been abandoned [6].

Correlations can exist between uncertainty sources; however, most uncertainty evaluations involve uncorrelated uncertainty sources. Consequently, correlation effects are omitted in this document, except for some guidelines to identify when they are present and hence more advanced methods (beyond the scope of this document) are needed.

Accordingly, this guideline has the following two assumptions:

(a) Uncertainty sources are not assigned any degrees of freedom (i.e., no attempt is made to evaluate the uncertainty of the uncertainty). Hence, it is assumed that the expanded ($k = 2$) uncertainty interval has a 95% probability of containing the true value of the measurand.

(b) All uncertainty sources are assumed to be uncorrelated. Finally, for simplicity, all input quantities of the uncertainty budget are packaged in quantities that have the unit of the measurand (i.e., length). This avoids the issue of sensitivity coefficients that typically involve partial differentiation.

3 BASIC CONCEPTS AND TERMINOLOGY OF UNCERTAINTY

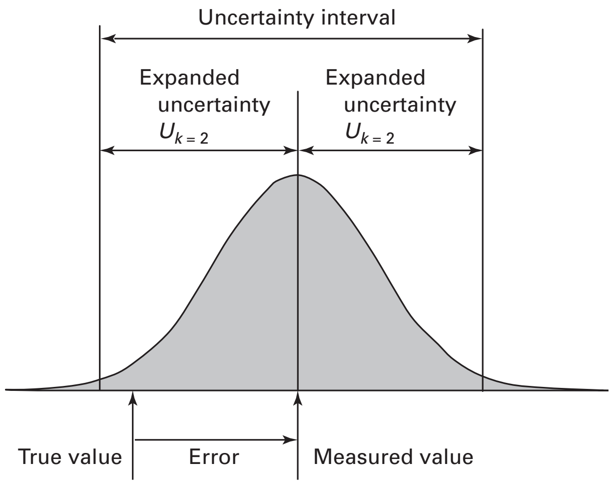
The formal definition of the term "uncertainty of measurement" in the current International Vocabulary of Basic and General Terms in Metrology (VIM) [7] (VIM entry 3.9) is as follows:

uncertainty (of measurement): parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand.

This can be interpreted as saying that measurement uncertainty is a number that describes an interval centered about the measurement result where we have reasonable confidence that it includes the "true value" of the quantity we are measuring.

expanded uncertainty (with a coverage factor of 2), U: a number that defines an interval around the measurement result, y, given by $y \pm U$, that has an approximate 95% level of confidence (i.e., probability) of including

Fig. 1 Measurement Uncertainty Quantities



GENERAL NOTE: Figure 1 illustrates the uncertainty interval of width $2U$ centered about the result of a measurement. There is a probability of about 95% that the true value of the measured quantity lies in this interval. The true value and hence the error are unknown; the error shown in the figure is among an infinite number of possible values. The subscript $k = 2$ indicates that U has been calculated with a coverage factor of two.

the true value of the quantity we are measuring. (In certain advanced applications of measurement uncertainty it may be necessary to have a different level of confidence or even an asymmetric uncertainty interval; these topics involve modifying the coverage factor and are beyond the scope of this document; refer to the GUM.) The expanded uncertainty is the end product of an uncertainty evaluation. In this document, unless otherwise stated, the term "measurement uncertainty" is considered to be the expanded uncertainty with a coverage factor of 2. (The issue of the coverage factor will be discussed later.) Several aspects of measurement uncertainty are described below.

(a) Measurement results have uncertainty; measurement instruments, gauges, and workpieces are sources of uncertainty. For example, measuring the diameter of a steel ball using a caliper will generally have smaller uncertainty than when measuring a foam rubber ball, even though it involves the same instrument.

(b) The expanded uncertainty, U, is always a positive number, and the uncertainty interval around a measurement result is of width $2U$. (See Fig. 1.)

(c) The expanded uncertainty (using the GUM procedures for evaluating uncertainty) is a statement of belief about the accuracy of a measurement result. When additional information becomes available the uncertainty is likely to be re-evaluated yielding a new value. Consequently, there is no "true" or "correct" uncertainty value, only a statement of belief that is based on the information available at the time the uncertainty is evaluated.

(d) The expanded uncertainty is a quantitative statement about our ignorance of the true value of the measurand.

influence quantity: any quantity, other than the quantity being measured, that affects the measurement result. Constructing the list of influence quantities is one of the first steps of an uncertainty evaluation. This list includes not only obvious sources of influence such as the uncertainty in the value of a reference standard, or the value of a force setting on an instrument, but also nuisance quantities such as environmental parameters or gauge contamination (dirt). (See Nonmandatory Appendix C.)

input quantity: a specific "line item" in the uncertainty budget that represents one or more influence quantities combined together into one quantity. That is, all significant influence quantities must be included (i.e., "packaged") in some input quantity. Different uncertainty budgets developed by different metrologists might use different input quantities, but all budgets include (in some input quantity) all the significant influence quantities. The selection of the input quantities is usually based on the type of the data available about the influence quantities. For example, if a long-term reproducibility study using a check standard has been conducted (e.g., measuring the same feature on a gauge once a week, for several years), then the effects of many influence quantities such as temperature, different operators, recalibration of the instrument, and other factors, are all combined in the observed variation of the check standard results. In this example, a very large number of influence quantities are combined into a single input quantity (i.e., the reproducibility of the check standard results).[1]

correlation: refers to a relationship between two input quantities. Correlation between two input quantities means that these two quantities are not completely independent. One way in which input quantities can be correlated is that the same influence quantity can appear in both input quantities. In this case the same influence quantity has the risk of being "double-counted." In advanced uncertainty budgets this issue is addressed by calculating correlation coefficients and then the effect of the double counting is subtracted. In this document a more modest approach is suggested, namely that input quantities should be constructed such that an influence quantity appears in only one input quantity.

EXAMPLE: Suppose that gauge blocks are calibrated using a set of master gauge blocks similar to the blocks under calibration. Suppose further that the laboratory's temperature slowly varies by ±1°C about 20°C and that no correction is made for the thermal expansion of either gauge block. A poor way to model the measurement is to employ a separate input quantity for the temperature,

[1] As will be described later, the variation captured by a reproducibility study can be quantitatively evaluated by a "Type A" evaluation.

T_m, of the master block and for the temperature, T_c, of the customer's block under calibration. These two input quantities are strongly correlated. This is easily shown by asking the question, "If I knew for sure that $T_m > 20°C$, would such knowledge tell me anything about T_c?" In this case the answer is affirmative (i.e., I would know that $T_c > 20°C$, because the blocks are similar and share the same thermal environment). Indeed $T_m \approx T_c$ and the two input quantities are fully correlated. This correlation can be completely removed by the observation that both blocks will have the same temperature. Thus there is only a single temperature, T, associated with both blocks, and all that is known is that $T = 20°C \pm 1°C$. Hence, the correlation is removed by eliminating a redundant uncertainty source.

measurand: the particular quantity subject to measurement. It is defined by a set of specifications (i.e., instructions) that specifies what we intend to measure; it is not a numerical value. It represents the quantity intended to be measured. It should specify, as generically as possible, exactly the quantity of interest, and avoid specifying details regarding experimental setups that might be used to measure the measurand. For example, measurands specified by ASME Y14.5 [8], such as the diameter of a feature of size or the concentricity of two bores, do not attempt to describe the measurement procedure in detail.[2] Ideally the measurand should be completely independent of experimental measurement details so that different measurement technologies can be used to measure the same measurand and get the same result. Indeed, the measurand is an idealized concept and it may be impossible to produce an actual gauge, artifact, or instrument exactly to the specifications of the measurand. Consequently, a well-specified measurand provides enough information, and is generic enough, to allow different techniques to be used to perform the measurement. The more completely defined the measurand, the less uncertainty will (potentially) be associated with its realization. A completely specified definition of the measurand has associated with it a unique value, and an incompletely specified measurand may have many values, each conforming to the (incompletely defined) measurand. The ambiguity associated with an incompletely defined measurand results in an uncertainty contributor that must be assessed during the measurement uncertainty evaluation.

As an example of the significance of the measurand, consider a bore that has a size tolerance specified by ASME Y14.5. An inspection of the workpiece involves a measurand defined as the diameter of the maximum inscribed cylinder that will just fit in the bore (i.e., this is the largest diameter cylinder that is constrained by the workpiece surface, regardless of any translations or rotations that may be applied).[3] Note the generic nature of this measurand, which avoids specifying any details about potential experimental measurement setups. Unless careful consideration is given to the measurand, different inspection techniques can lead to significantly different results. For example, when measuring a bore, a two-point diameter as measured with a micrometer,[4] a least squares fit diameter as measured with a coordinate measuring machine,[5] and a maximum inscribed diameter as found using a plug gauge, may each yield a different numerical value because each quantity realized by a particular measurement method measures a different measurand. No amount of improvement in the accuracy of these measurement methods will cause their results to converge as they are fundamentally measuring different quantities (two point, least squares, and maximum inscribed diameters). The metrologist must recognize that the two-point and least squares results are not the measurand under consideration in this inspection, and differences must be accounted for by applying appropriate corrections to the measurement result or, alternatively, to account for their difference by increasing the uncertainty associated with the measurement result. In the GUM, reference is made to the quantity "realized" by the measurement system; again this points out that many measurement systems do not yield a quantity fully consistent with the definition of the measurand and that corrections (or an increase in the uncertainty) are needed to bring the results of the measurement into alignment with the definition of the measurand.

A complete definition of the measurand will, in the general case, allow corrections to be applied for different measurement methods. For example, the calibration of a chrome-carbide gauge block using a gauge block comparator and a steel master requires the correction for the differential mechanical penetration of the probe tips since the length of the block is defined as the undeformed (i.e., unpenetrated) length.[6] The use of appropriate corrections will allow convergence of the results

[2] The nominal value that may be attributed to a measurand (e.g., the diameter of a feature of size) is not part of the measurand; rather, it is the desired result of a measurement of the measurand. Similarly, a tolerance associated with a feature is not part of the measurand, but rather describes a region within which a measurement result is considered to demonstrate the feature to be in conformance with the design intent.

[3] In this example, it is assumed that no additional control for orientation or location is specified for the bore and that the ASME Y14.5 "Rule #1" is in effect.

[4] A "two-point diameter" of a cross-section (defined as the "actual local size" in ASME Y14.5-1982) is an ambiguous measurand since it is a one-dimensional length and different cross-sections will in general yield different two-point diameters. For this reason, the 1994 revision of ASME Y14.5 and the associated ASME Y14.5.1 standard redefined this measurand to be a two-dimensional quantity.

[5] A measurand defined as a least squares diameter fit has an unambiguous value when computed from an infinite number of perfectly known points distributed around the workpiece surface. In practice the diameter will be measured with a finite number of imperfectly known points. The effects of finite sampling and errors in the sampled points are uncertainty sources associated with that particular measurement.

[6] The calibration of gauge blocks made of the same material as the master generally do not require a penetration correction since the deformation is the same on both blocks and hence cancels out.

from different measurement methods and bring them into accordance with the definition of the measurand.[7] Hence the methods divergence problem is actually a problem with an incompletely specified measurand or the failure to recognize that the measurement method is not measuring the intended measurand.

The definition of the measurand must also be sufficiently complete to avoid improper use of the calibrated artifact or instrument. For example, consider a handheld micrometer that is calibrated for measuring workpieces with flat and parallel surfaces by measuring several calibrated gauge blocks (with surfaces larger than the micrometer anvil size). Hence the micrometer is calibrated for this particular measurand; this means that a measurement result of this measurand has a known expanded uncertainty provided the measurement is performed within some set of conditions under which the uncertainty statement is valid.[8] This procedure does not calibrate the micrometer for measuring ball diameters because the flatness and parallelism of the anvils are unknown and are significant influence quantities for the (ball diameter) measurand.

Included in the definition of the measurand is a description of the set of conditions that specify the values of particular influence quantities relevant to the measurand. An obvious example is that the length of a gauge or workpiece is defined at 20°C (68°F); otherwise, an object would have multiple "true lengths" depending on the temperature at the time of measurement. Typically, the higher the accuracy requirements, the more extensive the list of specified influence quantities in order to have negligible uncertainty associated with the definition of the measurand. Note that definition of the measurand must address all significant conditions (i.e., influence quantities, not just environmental conditions).

error: defined in a measurement result is the measured value minus a true value of the measurand (VIM 3.10). (See Fig. 1.) Strictly speaking, the error of a measurement result is never exactly known since the value of the measurand is never exactly known. Just as there is a set of reasonably probable true values that could be attributed to the measurand after performing a measurement, there is also a set of reasonably probable errors that might have been made. Several aspects of an error are described below.

(a) An error is a quantitative statement about the difference between the measured value and the true value. (In contrast, an uncertainty statement expresses our ignorance about the true value.)

(b) Errors are typically measured only during the special case of a calibration when calibrated artifacts (representing the true value) are measured. [The measurement of an uncalibrated object (e.g., a workpiece) is described using a measurement uncertainty statement.]

(c) Errors may be either positive or negative (in contrast to uncertainty, which is always positive).

For a measurement corrected for all known significant systematic effects, the best estimate of the error is zero. The set of reasonably probable errors is contained in an interval centered on an error value of zero with its width equal to twice the expanded uncertainty.

Consider, for example, a measurement of the length, L, of a block that yields the result $L = 25$ mm ± 10 μm with a 95% level of confidence. It would be incorrect to say, for example, "The measurement error is equal to 10 μm," but one could say, "The measurement error is 0 ± 10 μm with a 95% level of confidence." The best terminology, however, would be to state that the best estimate of the value of the measurand, with all corrections applied, is 25 mm, with an expanded uncertainty of 10 μm.

In the special case of an instrument calibration, well-calibrated artifacts are measured and the difference between the measured and calibrated value is a good estimate of the error. For example, the calibration of a caliper using a series of gauge blocks uses the calibrated gauge block to represent a true value that is compared with the caliper reading to reveal the instrument's error. The measured error will have an associated (usually very small) uncertainty associated with the calibration of the gauge block (i.e., this small uncertainty is associated with the "true value" of the block's length).

systematic error: the (mathematical) expectation value of the error. In an instrument calibration, it can be estimated by the mean (i.e., average) error in the reading of the measuring instrument when repeatedly measuring a calibrated standard. Similar to the case of error, the systematic error is never exactly known because we never know the "true value" and we cannot perform an infinite number of measurements of a standard to produce the expectation value. The estimated systematic error may be determined from the mean of a series of repeated measurements or as a calculated value corresponding to a known systematic effect. If a systematic error is known to be present in the measurement result, then either a correction must be applied or it must be accounted for in the uncertainty budget. (A correction has the same magnitude as the systematic error but the opposite sign.)

[7] In some cases a metrologist will deliberately choose (for economy or convenience) to measure a related quantity that differs from the measurand (e.g., a least squares diameter instead of a maximum inscribed diameter). In this case an estimated systematic error may result; this must either be corrected or accounted for in the uncertainty statement of the measurement.

[8] The set of conditions under which an uncertainty statement is valid is known as the validity conditions. This may restrict such parameters as the measurement temperature range, the amount of force applied, the quality of the workpiece surfaces, the type of material measured (e.g., a dimensional measurement of a workpiece made of foam rubber may have a higher uncertainty than one of steel).

standard uncertainty: a quantitative value describing the magnitude of an uncertainty source. Each uncertainty source must be evaluated to yield its associated standard uncertainty. The standard uncertainty can be thought of as one standard deviation of potential variation associated with that uncertainty source. Each and every uncertainty source, as represented as an input quantity to the uncertainty budget, has an associated standard uncertainty that is combined with others in order to produce the final uncertainty statement.

combined standard uncertainty, u_c: the result of combining all of the standard uncertainties of the various uncertainty sources. The method of combining these sources will be discussed later, but u_c can be thought of as one standard deviation's worth of variation about the measurement result due to all of the uncertainty sources included in the uncertainty budget. In order to increase the level of confidence that the uncertainty includes all reasonable values of the measurand, the combined standard uncertainty is multiplied by a "coverage factor," which will always be equal to 2 in this document. Hence, the expanded uncertainty is just twice the combined standard uncertainty (i.e., $U = 2u_c$). If the uncertainty budget is well constructed such that it includes all relevant uncertainty sources and these sources are well evaluated, then generally there will be approximately a 95% level of confidence (or probability) that the value of the measurand is within the uncertainty interval of the measurement result. (See Fig. 1.)

Type A and Type B uncertainty evaluations: designations that refer to the method used to evaluate an uncertainty source. The GUM specifically avoids referring to uncertainty sources as "random" or "systematic." Such a classification is artificial since given more time and money an apparently random source can be tracked down to its systematic causes. Instead, the GUM focuses on the method of evaluation of the uncertainty source.

Type A evaluations assign the standard uncertainty of an uncertainty source to be equal to the standard deviation calculated based on repeated observations of experimental data. Calculating the standard deviation of experimental data is a simple and straightforward activity; see Nonmandatory Appendix A for details.

Type B evaluations are based on anything other than repeated observations (i.e., they involve more than just numerically evaluating experimental data). Often this information is from handbooks, manufacturer's specification sheets, or just educated guesses. Whatever the source, the information must be converted into a standard uncertainty. In some cases this conversion is trivial. For example, the standard uncertainty associated with the calibration value of a master artifact is easily obtained from the expanded uncertainty (that would be typically stated on a calibration report) by simply dividing by two (the coverage factor).

In other cases a Type B evaluated standard uncertainty involves deciding on the "probability density function" of the source; this is just a fancy way of stating how likely the uncertainty source might yield one value relative to another value. In this document several probability distributions will be used in the examples, including the uniform distribution, the normal (or Gaussian) distribution, and the triangular distribution. The only information needed to define a uniform distribution, for example, are the upper and lower bounds of the possible values of the uncertainty source. All other values that occur between the two limits are considered to be equally likely. Nonmandatory Appendix B presents some guidance on selecting the appropriate distribution as well as the details of calculating the standard uncertainty when using these distributions. While advanced uncertainty practitioners might employ many different distributions, generally the benefits of this additional fidelity are typically a small refinement to the numerical value of the expanded uncertainty.

specified validity conditions: the conditions under which the results of the uncertainty statement are valid. For example, an industrial metrologist may wish to use a calibrated artifact at nonstandard thermal conditions (e.g., 23°C), and desires an expanded uncertainty that includes these nonstandard conditions (i.e., the metrologist does not want to develop a new uncertainty statement for the nonstandard condition but rather wishes to refer directly to the calibration certificate to obtain the uncertainty relevant to the conditions of use). These conditions, which we will call the validity conditions,[9] include the values (or range of values) of all significant influence quantities for which the uncertainty results are valid. In the case of instruments, the validity conditions also include the number of measurements used to compute a result, because if repeated measurements by an instrument yield different results, then the mean (mathematical average) result will usually have a smaller uncertainty than a single result.

The validity conditions are either those specified in the definition of the measurand (i.e., the "measurand defining conditions," or are "extended conditions"). For stating the uncertainty of a calibration, which typically involves such artifacts as master gauges and reference standards, the validity conditions are often identical to the measurand defining conditions. For example, the results of a NIST gauge block calibration are valid only at exactly 20°C, which is the standard reference temperature for the length of a gauge block. Although the NIST laboratory cannot actually realize the measurand defining conditions during the calibration (i.e., the prevailing conditions during the calibration are never exactly 20°C), corrections for deviations from the measurand defining

[9] The term "validity conditions" is used in order to avoid confusion with the conditions that happen to prevail at the time of the measurement.

conditions (e.g., not exactly at 20°C) are applied and their associated uncertainties are included in the uncertainty budget of the calibration. Hence the calibration report states the best estimate of the block length and its uncertainty at exactly 20°C.

Subsequent use of these standards (e.g., in calibrating other artifacts) will generally not be at the validity conditions (i.e., not exactly at the measurand defining conditions). Hence the metrologist is obligated to develop an uncertainty budget that includes not only the uncertainty stated in the calibration report of the reference artifact, but also any failure to exactly realize the measurand defining conditions of the reference artifact during subsequent calibrations that use the reference artifact as the "master." Thus the uncertainty of each subsequent calibration in a traceability chain will be greater than the uncertainty of the previous calibration since the measurand defining conditions generally cannot be fully achieved. The benefit of having the validity conditions identical to those of the measurand defining conditions is that it can yield the smallest uncertainty statement. The disadvantage is that any subsequent use requires the evaluation of a new uncertainty budget. (See para. 6.1 for an example of developing an uncertainty statement for validity conditions constrained to be those specified in the measurand defining conditions.)

In contrast, industrial measurements often involve "extended validity conditions" that are appropriate for their particular needs. These conditions may differ significantly from those specified in the measurand defining conditions. For example, a factory floor worker using an instrument may not want to develop an uncertainty budget for every measurement performed. What may be desired is a calibration report that states an uncertainty under validity conditions that include the conditions of actual use. A common example is a voltmeter calibration that gives an uncertainty statement over a range of ambient temperatures. (See para. 6.2 for an example of developing an uncertainty statement for extended validity conditions.)

The calibration of an instrument or artifact under extended validity conditions must have its errors and uncertainties assessed over this range of conditions. As with the definition of the measurand, specifying the extended validity conditions involves stating the permitted values of any influence quantity that affects the measurement. Uncertainty budgets that are associated with extended validity conditions typically have much larger expanded uncertainties than those associated with measurand defining conditions, since the effects of the extended conditions must be included in the uncertainty evaluation.

Table 1 may further clarify the difference between the validity conditions of the uncertainty statement and the conditions that happen to prevail at the time of measurement. The columns of the table specify the applicability of the uncertainty statement. That is, it describes the set of conditions under which the uncertainty statement is valid; this is a critical piece of information for the user of the uncertainty statement. Note that the validity conditions of the uncertainty statement have no effect on the definition of the measurand; they merely restrict or extend the set of conditions under which the user may employ the results of the measurement with the stated uncertainty. Considering any particular measurement, the input quantities for an uncertainty evaluation with extended validity conditions will have larger standard uncertainties than those of a corresponding uncertainty statement applicable only for the measurand defining conditions. Hence, the expanded uncertainty having extended validity conditions will be larger than that of the corresponding case that is limited by the measurand defining conditions.

The rows of the table describe the conditions that prevail during the measurement. The special test scenario describes a specific measurement of a specific object and the associated evaluation of the measurement uncertainty. The multiple measurement scenario describes an ongoing measurement process where many measurements will be performed now and in the future and a single uncertainty statement is assigned to each of these measurement results. For example, consider a calibration lab that continuously calibrates gauge blocks all year long; an uncertainty budget is developed once and this uncertainty is attached to each of the future measurement results. In both the special test and multiple measurement scenarios, sources of uncertainty must be evaluated and corrections may need to be applied to bring the measurement results into alignment with the definition of the measurand. The difference between the two cases involves the magnitude of the uncertainty sources and corrections. In the multiple measurement scenario the range of variation of each influence quantity over the entire time of the measurement process must be considered. For example, if the uncertainty statement of a gauge block calibration process is to be applicable all year long, then variations that occur over that time period must be included. This will include, for example, seasonal temperature variations, multiple operators, drift in the measurement system, etc. In general, the standard uncertainties evaluated for a multiple measurement scenario will be larger than the corresponding standard uncertainties for a special test scenario.

4 COMBINING UNCERTAINTY SOURCES

Once all the influence quantities are known and included in some input quantity, the uncertainty components associated with each input quantity can be evaluated (in the units of the measurand, i.e., length). These uncertainty components are then combined to yield the combined standard uncertainty, u_c.

Table 1 Measurement and Validity Conditions

Prevailing Conditions at the Time of Measurement	Validity Conditions of the Uncertainty Statement	
	Measurand Defining Conditions	Extended Validity Conditions
Special test scenario: conditions limited to those at the time of the measurement	U_1: the smallest uncertainty for a particular measurement result	U_3: $U_3 > U_1$ typically $U_3 > U_2$
Multiple measurement scenario: conditions varying over the time of all measurements	U_2: typically $U_2 > U_1$	U_4: the largest uncertainty for a particular measurement result

In the common case where all input quantities are assumed to be uncorrelated, the method of combining uncertainty components is called a root sum of squares (RSS) calculation, given by the following simple expression:

$$u_c = \sqrt{u_1^2 + u_2^2 + \ldots + u_N^2} \quad (1)$$

where

u_c = the combined standard uncertainty

$u_1, u_2, \ldots, u_N$ = the standard uncertainty components associated with the N input quantities.[10]

The expanded uncertainty, U, is stated as just twice the combined standard uncertainty: $U = 2u_c$.

In actual practice, a metrologist may have determined that in order to have an acceptable level of risk[11] during conformance testing the expanded uncertainty must be no greater than a specific value. If the expanded uncertainty evaluated is greater than this value the metrologist must consider which input quantities can economically have their standard uncertainties reduced in magnitude. Due to the manner in which uncertainty sources combine, given in eq. (1), the largest uncertainty source is usually the target of this reduction since it will yield the greatest reduction in the expanded uncertainty. Generally, once the expanded uncertainty has been reduced to yield an acceptable amount of risk, it is economically unprofitable to make further efforts to lower the evaluated expanded uncertainty.

5 BASIC PROCEDURE FOR UNCERTAINTY EVALUATION

Having discussed the general concepts of measurement uncertainty the basic procedure for producing an uncertainty statement will now be discussed. For purposes of this document the evaluation will be considered in eight steps.

Step 1: Define the quantity to be measured (the measurand).
Step 2: State the desired validity conditions of the uncertainty statement.
Step 3: Define the measurement method, equipment, and environment.
Step 4: List the influence quantities.
Step 5: Determine the input quantities (select an uncertainty evaluation plan).
Step 6: Evaluate and rank the input quantities (determine the standard uncertainties).
Step 7: Combine the input quantities (combined standard uncertainty).
Step 8: State the expanded uncertainty and the coverage factor used.

NOTE: The use of the uncertainty statement in a decision rule regarding the acceptance or rejection of workpieces is discussed in [3]. The analysis of the risk (accepting a nonconforming workpiece or rejecting a conforming workpiece) associated with a decision rule is discussed in [5]. The evaluation of the reliability of uncertainty statements is discussed in [4].

6 EXAMPLES

6.1 Calibration of Gauge Blocks by Mechanical Comparison

A small calibration laboratory calibrates steel gauge blocks of lengths from 1 mm to 100 mm in an environment of (21°C ± 1°C), and the master and customer blocks are always within ±0.1°C of each other. An uncertainty statement for the length of the gauge block with validity conditions to be those specified by the measurand defining conditions is required. This is a multiple measurement scenario since the uncertainty statement will be applied to many future gauge block measurement results. (This example is intended to be a "crude" calibration; for a high accuracy example of mechanical comparison of gauge blocks, see [9].)

6.1.1 Measurand. The measurand is the length of the gauge block as defined by B89.1.9 [10]. This definition is based on an interferometric length measurement at 20°C; fortunately, National Measurement Institutes also provide gauge block calibrations based on mechanical comparison so the transfer of the interferometrically based definition to a mechanical (point-to-point) length

[10] Due to the RSS manner by which the uncertainty components are combined, the largest components dominate the combined standard uncertainty. For example, consider two components: u_1 = 1 μm and u_2 = 5 μm; u_2 will have 25 times more significance than u_1 in the combined standard uncertainty.

[11] The issue of risk analysis (i.e., the probability of rejecting a conforming workpiece or accepting a nonconforming workpiece during an inspection) is extensively discussed in [5].

is already included in the calibration report of the master blocks.

6.1.2 Uncertainty Validity Conditions. Since the customer intends to use the gauge blocks as their master artifacts, the validity conditions of the uncertainty will be the same as the measurand defining conditions defined by B89.1.9; this yields the smallest possible uncertainty statement given the conditions of the calibration.

6.1.3 Measurement Method and Environment. The measurement method will be by mechanical comparison to a master gauge block in an environment of 21°C ± 1°C (over all measurements) and is homogeneous in the measuring volume. The calibration includes "check standards" that are steel gauge blocks measured together with the customer's blocks as part of the quality assurance system. The check standard plays the role of a surrogate customer block for quality control purposes. Data on the check standards from many years is available and hence includes the long-term variation of many influence quantities. Since the master, check standard, and customer's blocks are all steel it is assumed that they have the similar temperatures, CTEs, and values of elastic modulus.

6.1.4 Influence Quantities. Referring to Nonmandatory Appendix C, the following influence quantities are relevant:

(a) master gauge block length uncertainty
(b) coefficient of thermal expansion (CTE) of the master block
(c) CTE of the customer's block
(d) temperature of the master block
(e) temperature of the customer's block
(f) thermal gradients between and within the blocks
(g) operator effects
(h) calibration of the indicator on the gauge block comparator
(i) comparator's length transfer ability
(j) master block geometry and modulus of elasticity
(k) customer's block geometry and modulus of elasticity
(l) check standard's block geometry and modulus of elasticity
(m) cleanliness/contamination
(n) resolution of the comparator

6.1.5 Input Quantities. Choosing the input quantities is guided by the availability of data and the measurement method.

(a) Reproducibility. Creating an input quantity called reproducibility, based on the check standard data, is advantageous as it include the effects of many (nuisance) influence quantities including cleanliness/contamination, multiple operator effects, thermal gradients between and within blocks (it is assumed that the thermal properties of the master, customer, and check standard blocks are all similar since they are all steel), block geometry effects, comparator's transfer ability, and scale calibration because the check standard data includes several scale recalibrations. (In this example, we assume the comparator is routinely recalibrated.)

(b) Master Block Length

(c) CTEs of the Master and Customer Blocks

6.1.6 Evaluate Input Quantities

(a) Reproducibility. Using historical data for each length of check standard block, the standard deviation of the data is computed for each check standard block. The standard deviations from each check standard are pooled as described in Nonmandatory Appendix A. The standard uncertainty associated with reproducibility is evaluated as the pooled standard deviation. Pooling the standard deviations has the advantage that it includes several different block geometrical effects (e.g., flatness), and these effects are quantified in the pooled standard deviation.

EXAMPLE: For brevity, only 20 measurements per block and only three check standard blocks will be considered. In a more complete example, more check standards might be used and the data could be segregated based on length (e.g., all check standards between 1 mm and 30 mm in length would be pooled, allowing possible different standard uncertainties associated with block length). Additionally, for quality control purposes, the check standards could be calibrated (or at least the current measurement value could be compared to the historical mean value) as a means of detecting blunders. (See [9] for further details.)

(a) Check Standard 1 Data (Deviations From Mean Value in Micrometers). 0.174467, 0.175691, 0.0852917, −0.0576673, −0.110542, 0.235682, 0.0954639, −0.283445, 0.0302104, −0.217087, 0.0156588, 0.248976, 0.134756, 0.315471, 0.0509718, 0.0263296, −0.0868594, 0.203901, 0.00287366, −0.287881

Standard deviation = 0.17 μm

(b) Check Standard 2 Data (Deviations From Mean Value in Micrometers). 0.103049, 0.246114, 0.0275914, 0.0778894, −0.0658711, 0.0374774, −0.0417286, 0.172826, 0.244662, 0.130969, 0.0290974, −0.172633, 0.0160234, 0.0659565, −0.200082, 0.173218, 0.111646, 0.172801, 0.0157569, 0.0378071

Standard deviation = 0.12 μm

(c) Check Standard 3 Data (Deviations From Mean Value in Micrometers). 0.0128391, 0.0884897, 0.348164, 0.0382762, 0.0250351, −0.170214, −0.0939044, −0.246856, −0.134731, −0.113879, 0.00409127, −0.104183, 0.139396, −0.243031, 0.152802, −0.127018, 0.212681, 0.307211, 0.0618147, −0.0564749

Standard deviation = 0.17 μm

(d) Pooled standard deviation =

$$\sqrt{\frac{19 \times (0.17)^2 + 19 \times (0.12)^2 + 19 \times (0.17)^2}{20 + 20 + 20 - 3}}\ \mu\text{m} = 0.16\ \mu\text{m}$$

NOTE: Although repeated measurements are used in the uncertainty evaluation, the resulting standard uncertainty (0.16 μm in this example) is applicable to a future measurement result based on a single measured value.

(b) Master Gauge Calibration. From the certificate the expanded ($k = 2$) uncertainty is 0.14 μm; hence, the standard uncertainty is 0.07 μm.

(c) Thermal Effects. From para. D-3 of Nonmandatory Appendix D, the standard uncertainty component due to thermal effects equals 0.17 μm.

6.1.7 Evaluate the Combined Standard Uncertainty. The combined standard uncertainty is

$$u_c = \sqrt{0.16^2 + 0.07^2 + 0.17^2}\ \mu\text{m} = 0.24\ \mu\text{m}$$

6.1.8 State the Expanded Uncertainty. The expanded uncertainty and associated coverage factor ($k = 2$) is

$$U = 2 \times 0.24\ \mu\text{m} = 0.48\ \mu\text{m}$$

NOTE: A more detailed uncertainty evaluation may result in an uncertainty statement that depends on the length of the block under measurement.

6.2 Uncertainty of Shop Floor Measurements Using a Caliper

A metrologist is interested in evaluating the uncertainty of measurements made on the shop floor using a steel caliper (i.e., the uncertainty statement shall have extended validity conditions including the conditions on the shop floor).[12]

NOTE: In this example, some of the validity conditions involve workpiece material properties.

The caliper has a calibration certificate stating that the maximum permissible error (MPE) is less than 10 μm over its full range when measuring at 20°C. The shop floor measurand of interest is the actual external size at a specified cross-section of the workpiece. The metal workpieces have a variety of rectangular and cylindrical shapes up to 100 mm in size, and consist of materials with CTEs between 1×10^{-6}/°C and 22×10^{-6}/°C. The shop temperature varies between 15°C and 25°C, and the temperatures of the caliper and the workpiece are assumed to be within 0.2°C of each other during a measurement.

It is desired to have an uncertainty statement for a single future measurement with extended validity conditions covering the shop environment without applying corrections. The form error on the workpiece surface is known to be small relative to the resolution of the caliper.

6.2.1 Measurand. The caliper does not realize the measurand of interest; rather it measures a point-to-point distance (on cylindrical workpieces) or a line-to-line distance (on rectangular workpieces). However, since the form error of the workpiece is known to be negligible, the difference between the measurand realized by the caliper and that of interest (actual external size) is also negligible. The size of an object is defined at 20°C, and with zero contact deformation (i.e., in its free state).

[12] This example is a special test scenario; however, since the extended validity conditions appropriate for the shop floor (including different caliper operators, different workpiece materials, broad and uncorrected thermal condition, etc.) result in significant uncertainty contributions, a multiple measurement scenario (calibrating many calipers for the shop floor) would likely have a similar uncertainty.

6.2.2 Uncertainty Validity Conditions

Caliper CTE:	$(11.5 \pm 1) \times 10^{-6}$/°C
Workpiece CTE range:	1×10^{-6}/°C to 22×10^{-6}/°C
Temperature range:	15°C to 25°C
Temperature difference:	−0.2°C to +0.2°C
Material hardness:	All common metallic engineering materials
Workpiece size:	0 mm to 100 mm continuous
Workpiece geometry:	All geometries (planar, cylindrical, etc.)
Workpiece form error:	Negligible

6.2.3 Measurement Method and Environment. Lengths are measured by hand using a calibrated caliper. Data consists of a single caliper reading. The shop temperature varies between 15°C and 25°C.

6.2.4 Influence Quantities

(a) calibration of caliper
(b) CTE of the caliper
(c) CTE of the workpieces
(d) temperature of caliper
(e) temperature of the workpieces
(f) thermal gradients
(g) operator effects
(h) geometry and modulus of elasticity of workpieces
(i) cleanliness/contamination
(j) resolution of the caliper
(k) parallelism and flatness of the caliper anvils

6.2.5 Input Quantities. The effects of workpiece form error and modulus, operator variability, and contamination are judged to be negligible compared to the effects of differential thermal expansion and caliper calibration at 20°C.

The input quantities then consist of the following:
(a) reading of the caliper with a resolution of 10 μm
(b) calibration of caliper
(c) differential thermal expansion
(d) anvil parallelism and flatness

6.2.6 Evaluate Input Quantities

(a) Caliper Resolution. The resolution (magnitude of the last digit in the display) of the caliper is 10 μm. Assigning a Type B uniform distribution of width 10 μm (±5 μm) yields a standard uncertainty of 2.9 μm.

(b) Caliper Calibration. Since the calibration report of the caliper at 20°C states that the maximum permissible error (MPE) is 10 μm over the full range of travel and there is no information about the actual calibration error in any particular measurement, the best estimate of this error is zero, with an uncertainty evaluated as follows.

Assigning a Type B uniform distribution of width 20 μm (±10 μm) yields a standard uncertainty of 5.8 μm.

NOTE: In this example, the calibration result was stated as a limiting value (i.e., an MPE). Had the calibration result been stated as an expanded uncertainty with $k = 2$, then the standard uncertainty would be one-half the stated expanded uncertainty.

(c) Differential Thermal Expansion. For any particular measurement, the best estimate of the differential expansion is zero since the average temperature on the shop floor is 20°C and the average workpiece CTE is equal to the CTE of the caliper. There is, however, a component of uncertainty evaluated as described in para. D-3 of Nonmandatory Appendix D.

Assuming a steel caliper with a CTE of $10.5 \times 10^{-6}/°C$ to $12.5 \times 10^{-6}/°C$, the maximum difference, $\Delta\alpha_{\max}$, between the caliper and workpiece CTEs is $11.5 \times 10^{-6}/°C$. The maximum temperature deviation, $\Delta T_{\max}$, is 5°C, and the maximum measured length, $L_{\max}$, is 100 mm. Then the possible values of the differential thermal expansion lie in the interval [see eq. (D-15)] $\pm L_{\max}\Delta\alpha_{\max}\Delta T_{\max} = \pm(0.1 \times 11.5 \times 10^{-6} \times 5)\text{m} = \pm 5.8\ \mu\text{m}$. Assigning a Type B triangular distribution to this possible error then yields a standard uncertainty of $5.8\ \mu\text{m}/\sqrt{6} = 2.3\ \mu\text{m}$.

The maximum error due to a possible temperature difference between the workpiece and the caliper can be evaluated following eq. (D-17). The maximum measured length, $L_{\max}$, is 100 mm. The maximum value of the caliper CTE is $12.5 \times 10^{-6}/°C$. The temperature difference, $\delta T = T_m - T$, between the caliper and the workpiece is assumed to lie in the interval ±0.2°C. Then error due to the temperature difference lies in the interval $\pm L_{\max}\alpha_{\max}\delta T_{\max} = \pm(0.1 \times 12.5 \times 10^{-6} \times 0.2)\text{m} = \pm 0.25\ \mu\text{m}$. Assigning a Type B triangular distribution to this possible error gives a standard uncertainty of $0.25\ \mu\text{m}/\sqrt{6} = 0.1\ \mu\text{m}$.

NOTE: When an uncertainty source is evaluated and is less than 10% of another uncertainty source typically it can be neglected. In this example we continue to include this small value for completeness of the example.

(d) Anvil Flatness and Parallelism. These effects are evaluated using a small gauge wire measured in multiple positions and orientations. Variation of the results leads to the assignment of a Type A standard uncertainty (see Nonmandatory Appendix A) equal to 4.5 μm. The best estimate of the correction for these effects in any particular workpiece measurement is zero due to lack of specific information about the position and orientation of the anvils with respect to the measured feature.

6.2.7 Evaluate the Combined Standard Uncertainty. The combined standard uncertainty is

$$u_c = \sqrt{2.9^2 + 5.8^2 + 2.3^2 + 0.1^2 + 4.5^2}\ \mu\text{m} = 8.2\ \mu\text{m}$$

6.2.8 State the Expanded Uncertainty. The expanded uncertainty is $U(k = 2) = 2 \times 8.2\ \mu\text{m} = 16.4\ \mu\text{m}$; valid for measurements of workpieces up to 100 mm in length, of any geometry, made of common metallic engineering materials with a CTE between $1 \times 10^{-6}/°C$ to $22 \times 10^{-6}/°C$, measured within a temperature range of 15°C to 25°C.

In this problem the temperature of the shop floor happened to be symmetrically distributed about 20°C (i.e., 15°C to 25°C). Had the temperature been biased away from 20°C (e.g., 18°C to 28°C) then, on average, we would expect the measured length to be longer than the 20°C value, resulting in a systematic error. In the case of 18°C to 28°C, the magnitude of the expected error is $L \times 11.5 \times 10^{-6}/°C \times 3°C$, where L is the length of the measured workpiece. It is recommended that the user apply this correction to the measurement results. However, in this example it is stated that the user does not want to apply any corrections, so the largest length (0.1 m) is assumed, yielding a systematic error of magnitude 3.5 μm. This value is then added in an arithmetic manner to the previous analysis to yield 16.4 μm + 3.5 μm = 19.9 μm as the expanded uncertainty. This manner of including the systematic error in an uncertainty statement assures that the containment probability is at least as large as that associated with the coverage factor (e.g., at least 95% at $k = 2$) [11]. However, the resulting value is no longer, strictly speaking, an uncertainty (since it contains a known systematic error), and, while this procedure may be useful for workpiece conformance decisions, it is not appropriate for the statement of calibration results where the uncertainty statement will be propagated into subsequent measurements.

NONMANDATORY APPENDIX A
TYPE A EVALUATION OF STANDARD UNCERTAINTY

A-1 MEASURES OF DISPERSION

Repeated measurements will always have some variation between them. The extent of this variation is known as the dispersion. There are several methods used to numerically characterize the dispersion. The simplest to use and understand is the range, which is the arithmetic difference between the largest and the smallest readings. A more commonly used estimate of the dispersion is the standard deviation.

A-1.1 Standard Deviation

By definition, a Type A evaluation of standard uncertainty involves the use of a statistical method on repeated observations of the same measurand. From these repeated observations the sample mean is defined by

$$\bar{x} = \frac{1}{n} \times \sum_{1}^{n} x_i \tag{A-1}$$

where

n = the number of observations
$x_1, x_2, \ldots, x_n$ = the individual readings

Given a set of n repeated observations of a quantity x, the Type A standard uncertainty associated with x is taken to be the sample standard deviation, defined by

$$s_x = \sqrt{\frac{\sum_{i=1}^{n} (\bar{x} - x_i)^2}{(n-1)}} \tag{A-2}$$

For a Type A evaluation, the standard uncertainty associated with a single observation of x is equal to the calculated standard deviation of eq. (A-2) (i.e., $u = s_x$). This standard uncertainty is frequently used when an uncertainty statement pertains to future measurements in a multiple measurement scenario. A measurement (in the future) is performed only once, and the standard uncertainty (equal to s_x) describes the dispersion of this value.

A-1.2 Pooled Standard Deviations

In some cases multiple sets of observations are available, but these sets of data do not have the same mean. For example, the reproducibility of gauge block measurements could be evaluated by examining repeated measurements on several different length gauge blocks. Data sets with different means cannot be directly combined to calculate the standard deviation but they can be pooled. To pool the data, the standard deviation of each data set is first determined and then the standard deviations are then combined, weighted by the number of observations in each set:

$$s = \sqrt{\frac{\sum_{i=1}^{N} (n_i - 1)s_i^2}{\sum_{i=1}^{N} n_i - N}} \tag{A-3}$$

where

N = the number of data sets
n_i = the number of observations in the ith data set
s = the pooled standard deviation
s_i = the standard deviation of the ith data set

A-1.3 Standard Deviation of the Mean

Frequently, the best estimate of a value is obtained by repeated observations and the calculation of the mean value, $\bar{x}$. [See eq. (A-1).] If the uncertainty statement pertains to the mean value (as opposed to a single value as described in para. A-1), then the standard uncertainty associated with the mean is equal to the standard deviation of the mean, computed by

$$u(\bar{x}) = s_{\bar{x}} = \frac{s_x}{\sqrt{n}} \tag{A-4}$$

where

s_x = the sample standard deviation given by eq. (A-1).

NOTE: A Type A evaluation can be performed on repeated observations regardless of whether the source of variation is from a systematic or random effect.

NONMANDATORY APPENDIX B
TYPE B EVALUATION OF STANDARD UNCERTAINTY

If repeated observations are available then the standard uncertainty of a quantity can be evaluated by a statistical Type A procedure, as described in Nonmandatory Appendix A. Otherwise a Type B evaluation is required. A Type B evaluation is based on available relevant information; this could be a handbook value, expert opinion based on engineering judgment, prior history of similar measurement systems, calibration certificate information, manufacturer's specification, etc.

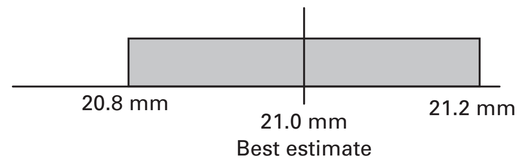
The essence of a Type B evaluation is assigning a probability distribution that describes the likelihood of the possible values of the quantity. The metrologist must estimate the likelihood of various values for a particular input quantity that is assigned a Type B evaluation. That is, are certain values of the input quantity more likely to occur than others? How rapidly does the likelihood of a value of an input quantity diminish as its value gets farther away from the most likely value? What is the maximum value that this input quantity may obtain? These questions are answered by the metrologist by assigning a probability distribution to the input quantity. This typically involves specifying the shape of the distribution and a measure of its width; once this is done the standard uncertainty associated with the distribution is readily evaluated.

B-1 THE UNIFORM DISTRIBUTION

The uniform distribution assigns equal probability for a value anywhere between two limits. Figure B-1 shows a uniform distribution for the possible values of the length of a measured workpiece; the uniform distribution is centered about the best estimate of the value. (In this document, the best estimate will always be at the center of the uniform distribution.)

To evaluate the standard uncertainty of a uniform distribution, take the half-width of the distribution and divide by the square root of 3 (equivalent to multiplying by 0.58). The standard uncertainty for the distribution in Fig. B-1 is computed as $u(L) = 0.58 \times 0.2$ mm $=$ 0.12 mm.

Fig. B-1 Uniform Probability Distribution

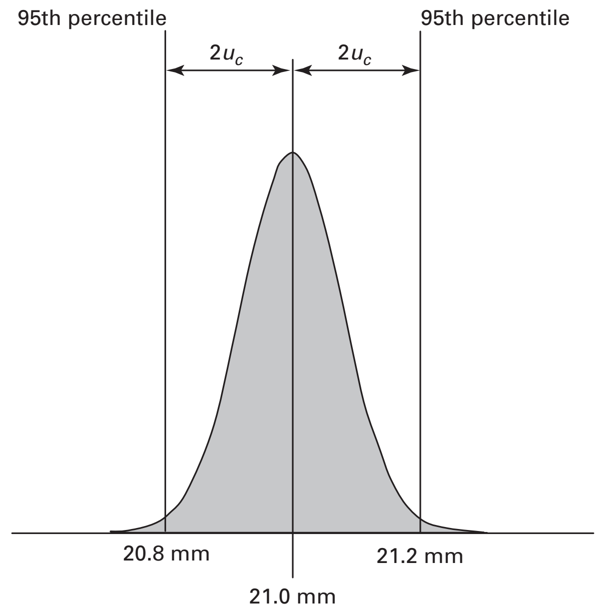


GENERAL NOTE: Figure B-1 illustrates a measured length of 21.0 mm, with an uncertainty characterized by a uniform distribution with limits 20.8 mm and 21.2 mm.

B-2 THE NORMAL DISTRIBUTION

The normal distribution assigns a higher probability around the best estimate of the value than does the uniform distribution. The assigned probability decreases as the difference between a possible value and the best estimate increases. Unlike the uniform distribution, the normal distribution does not have limits; rather it extends out to infinity, albeit with vanishingly small probability.

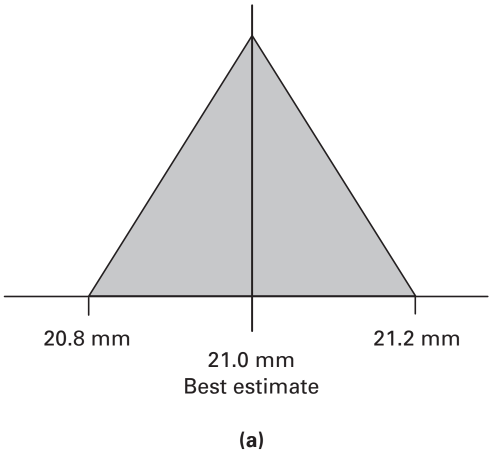
Fig. B-2 Normal Probability Distribution

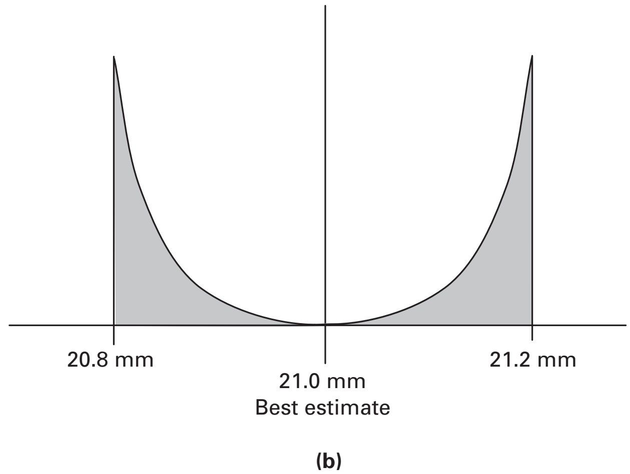


GENERAL NOTE: Figure B-2 displays the uncertainty in a measured value of 21.0 mm having with a normal distribution and an expanded ($k = 2$) uncertainty of 0.2 mm, yielding one standard uncertainty equal to 0.1 mm.

The normal distribution has the property that approximately 95% of its probability is contained within plus-minus two standard uncertainties of its mean (best estimate) value. (See Fig. B-2.) Consequently, if one believes that there is a probability of about 95% that the value of an unknown quantity lies between two limits (X1 and X2), and that it is most likely to lie midway between the two limits, then a normal distribution is a reasonable

Fig. B-3 Triangular and "U" Probability Distributions





GENERAL NOTE: Figure B-3 illustrates an example of a triangular distribution [illustration (a)] and a "U" distribution [illustration (b)]; for the values shown the one standard uncertainty is 0.08 mm and 0.14 mm, respectively.

Table B-1 Various Probability Distributions and Their Standard Uncertainty

Distribution	"Width" Specified By	Standard Uncertainty [Note (1)]	Example: From Figs. B-1, B-2, and B-3
Triangular	Upper minus lower bounding value	$0.41 \times \frac{1}{2}$ width	0.08 mm
Normal	Upper minus lower 95th percentile value	$0.50 \times \frac{1}{2}$ width	0.10 mm
Uniform	Upper minus lower bounding value	$0.58 \times \frac{1}{2}$ width	0.12 mm
"U"	Upper minus lower bounding value	$0.71 \times \frac{1}{2}$ width	0.14 mm

NOTE:
(1) For rigor in this report, two significant figures are shown for calculating the standard uncertainty of the various distributions. In practice, a single significant digit is usually sufficient (e.g., using 0.4 instead of 0.41 is sufficient).

assignment with a standard uncertainty of $0.5(X2 - X1)/2$.

The normal distribution is also commonly assigned to the distribution of reasonably probable values associated with a calibration report. The expanded uncertainty of a calibration result is usually (unless otherwise stated on the calibration report) two standard uncertainties associated with a normal distribution. Hence to obtain the standard uncertainty from a calibration report, just divide the expanded uncertainty by the coverage factor. A simple example is shown in Fig. B-2. Similarly, if a metrologist believes an input quantity has a normal distribution, to evaluate its standard uncertainty the 95th percentile limit of the distribution is estimated and then this value is divided by two to obtain the standard uncertainty.

B-3 OTHER DISTRIBUTIONS

For introductory uncertainty evaluations the uniform distribution, with its assignment of probability distributed broadly over the range of possible values, and the normal distribution, with its probability distribution peaked at the center of the range of possible values, are sufficient in most cases. Two other distributions are shown in Fig. B-3; the triangular distribution has the probability peaked at the center of the interval, while the "U" distribution has the probability peaked at its bounding values with low probability at the mid-value. [The "U" distribution typically occurs when an input quantity has a sinusoidal time dependence (e.g., some temperature cycles).] The triangular distribution is often used to describe the probability distribution that arises from the sum, product, or difference of two uniform distributions. Table B-1 summarizes the standard uncertainties of these probability distributions.

As seen in Table B-1, the normal and uniform distributions yield a standard uncertainty that is roughly midway between the two more extreme distributions. Since an uncertainty evaluation reduces all the information about a distribution to a single number (the standard uncertainty), the difference between using a normal or uniform versus a more extreme distribution is a factor of roughly 20% and is unlikely to greatly change the final combined standard uncertainty.

NOTE: A Type B evaluation can be performed on any uncertainty source regardless if the source of uncertainty is from a systematic or random effect.

NONMANDATORY APPENDIX C INFLUENCE QUANTITIES

A large number of potential influence quantities can affect a measurement result. Some potential quantities are listed in paras. C-1 through C-10.

C-1 ENVIRONMENT

(a) temperature: absolute, time variance, spatial variance
(b) vibrations/noise
(c) humidity
(d) contamination
(e) illumination
(f) ambient pressure
(g) air composition and flow
(h) EMI (electromagnetic interference)
(i) transients in power supply
(j) pressured air (e.g., air bearings)
(k) heat radiation
(l) gravity
(m) instrument thermal equilibrium

C-2 REFERENCE ELEMENT OF MEASUREMENT EQUIPMENT

(a) stability
(b) scale mark quality
(c) CTE, thermal time constant
(d) physical principle: line scale, optical digital scale, magnetic digital scale, spindle, rack and pinion, interferometer
(e) CCD-techniques
(f) uncertainty of the calibration
(g) resolution of the main scale (analog or digital)
(h) time since last calibration
(i) wavelength error

C-3 MEASURING EQUIPMENT

(a) interpretation system
(b) magnification stability
(c) wavelength error
(d) zero-point stability
(e) force stability/absolute force
(f) hysteresis
(g) guides/slideways
(h) stylus/probe configuration
(i) geometrical imperfections
(j) stiffness/rigidity
(k) sampling strategy
(l) probe/reading system
(m) contact geometry
(n) stiffness of probe system
(o) temperature, CTE, time constants
(p) temperature stability/sensitivity
(q) Abbe, Cosine errors
(r) time since last calibration
(s) response characteristic
(t) interpolation system
(u) interpolation resolution
(v) digitization

C-4 MEASURING SETUP

(a) Abbe, Cosine errors
(b) temperature sensitivity, warm up
(c) stiffness/rigidity/stability
(d) tip radius
(e) form deviation of tip
(f) stiffness of the probe system
(g) optical aperture
(h) interaction between workpiece and setup
(i) warming up

C-5 SOFTWARE CALIBRATIONS

(a) rounding/quantification
(b) algorithms and implementation
(c) significant digits in computation
(d) sampling, filtering
(e) validity/certification of algorithm
(f) interpolation/extrapolation
(g) outlier handling

C-6 METROLOGIST

(a) heat source (breath, radiation)
(b) physical ability
(c) experience, dedication
(d) education, training, knowledge
(e) personal equation, honesty

C-7 MEASURING OBJECT

(a) surface roughness
(b) form deviations
(c) elastic modulus (Young's modulus)
(d) Poisson ratio
(e) stiffness
(f) temperature, CTE
(g) thermal conductivity and diffusivity
(h) weight, shape, size
(i) magnetism
(j) hygroscopic characteristics
(k) temperature
(l) internal stress, stability
(m) creep characteristics
(n) workpiece distortion due to clamping
(o) aging
(p) cleanliness
(q) orientation

C-8 DEFINITIONS OF THE CHARACTERISTICS

(a) datum, reference system
(b) degrees of freedom
(c) toleranced feature
(d) distance
(e) angle
(f) reference conditions

C-9 MEASURING PROCEDURE

(a) conditioning
(b) number of measurements
(c) order of measurements
(d) duration of measurements
(e) choice of principle
(f) choice of reference
(g) choice of apparatus
(h) choice of metrologist
(i) alignment
(j) number of operators
(k) strategy
(l) clamping
(m) fixturing
(n) number of data points
(o) probing principle
(p) probing strategy
(q) alignment of probing system
(r) drift check
(s) reversal measurements
(t) multiple redundancies, error separation

C-10 PHYSICAL CONSTANTS AND CONVERSION FACTORS

knowledge of the correct physical values of, for example, material properties (workpiece, measuring instrument, ambient air, etc.)

NONMANDATORY APPENDIX D THERMAL EFFECTS IN DIMENSIONAL MEASUREMENTS

D-1 SYSTEMATIC THERMAL ERROR

The thermally related issues of a dimensional measurement are often a major source of uncertainty. In this Appendix the basic methods of addressing this issue are described. The effects of temperature in dimensional metrology are treated in more detail in ISO/TR 16015 [12] and ANSI/ASME B89.6.2 [13].

Consider a workpiece whose length, L, is measured at a nonstandard (other than 20°C) temperature. If the result of the measurement is to be compared with a tolerance requirement (specified on a drawing, for example) then the measured length must be corrected for thermal expansion. The length, $L^{20°C}$, of the workpiece at standard temperature is

$$L^{20°C} = L\,(1 - \alpha\Delta T) \tag{D-1}$$

where

L = the measured length at temperature, T
α = the coefficient of thermal expansion (CTE) of the workpiece material
ΔT = $T - 20°C$

The difference, $\Delta L = L - L^{20°C}$, between the measured length and the length at standard temperature is a thermally induced systematic error. From eq. (D-1) we see that

$$\Delta L = L\alpha\Delta T \tag{D-2}$$

In this Report it is assumed that corrections are always applied to eliminate known significant systematic errors.[1] Hence the thermally induced length error, ΔL, in eq. (D-2) is eliminated by a correction (having the opposite sign) added to the measured length. After correcting the measured length for the known systematic error, the uncertainty in the measured length, L, must be evaluated. The thermal aspects of the uncertainty evaluation are described below.

[1] If a significant systematic effect is known to exist but there is no way to know the magnitude or sign of the associated error, then the correction will be zero. The uncertainty budget, however, should include a component (usually a Type B assignment) that accounts for the possible magnitude of the unknown error. (See para. 6.2.)

D-2 THERMAL UNCERTAINTY EVALUATION

From eq. (D-1) we see that the measurand (the length of the workpiece at 20°C) depends on three input quantities: L, α, and ΔT. Uncertainties in each of these imperfectly known quantities will contribute to the uncertainty in $L^{20°C}$. In this Appendix we focus only on the thermal issues and hence consider only the uncertainty components due to CTE and temperature uncertainties. In a complete uncertainty evaluation, uncertainty components associated with the length measurement, such as reproducibility and calibration uncertainty, would also be included in the analysis.

The GUM procedure for uncertainty evaluation involves techniques from differential calculus that may be unfamiliar and seem overly complicated in an industrial environment. For simple thermal expansion problems an alternative procedure gives similar results. For the length measurement of eq. (D-1) the steps are as follows:

Step 1: Assign uniform probability distributions to α and T. Thus α is sure to lie between $\alpha_{\min}$ and $\alpha_{\max}$ with best estimate $\alpha = (\alpha_{\min} + \alpha_{\max})/2$, and T is sure to lie between $T_{\min}$ and $T_{\max}$ with best estimate $T = (T_{\min} + T_{\max})/2$.

Step 2: Evaluate the best estimate of the measurand using the best estimates of the input quantities. In this case $L^{20°C} = L(1 - \alpha\Delta T)$, according to eq. (D-1).

Step 3: Evaluate the change in length, L_α, when α is replaced by $\alpha_{\max}$. Using eq. (D-1) the result is

$$\Delta L_\alpha = L\Delta T(\alpha_{\max} - \alpha) \tag{D-3}$$

This is the maximum length error that could be caused by an error in the value of the CTE.

Step 4: Convert the error component, ΔL_α, to an uncertainty component, $u_\alpha(L)$, by dividing by $\sqrt{3}$ (assigning a Type B uniform distribution; see Nonmandatory Appendix B):

$$u_\alpha(L) = \frac{L\Delta T(\alpha_{\max} - \alpha_{\min})}{2\sqrt{3}} = \frac{L\Delta T(\alpha_{\max} - \alpha)}{\sqrt{3}} \tag{D-4}$$

Step 5: Evaluate the change in length, ΔL_T, when T is replaced by $T_{\max}$. Using eq. (D-1) the result is

$$\Delta L_T = L\alpha\frac{(T_{\max} - T_{\min})}{2} = L\alpha(T_{\max} - T) \tag{D-5}$$

This is the maximum length error that could be caused by an error in the temperature.

Step 6: Convert the error component, ΔL_T, to an uncertainty component, $u_T(L)$, by dividing by $\sqrt{3}$ (again assigning a uniform distribution):

$$u_T(L) = \frac{L\alpha(T_{\max} - T)}{\sqrt{3}} \qquad \text{(D-6)}$$

Step 7: Combine the uncertainty components in a root-sum-of-squares procedure. The result is the component of the combined standard uncertainty of the measured length due to thermal effects:

$$u_c^{th}(L) = \sqrt{u_\alpha^2(L) + u_T^2(L)} \qquad \text{(D-7)}$$

where the superscript *th* denotes the thermal component of uncertainty.

EXAMPLE: Suppose a laser interferometer is used to measure the distance between two points on an aluminum rod, and the measurement result is 2.000220 m. The laser is compensated for the wavelength of light in the measurement environment and hence its measurement scale is adjusted to 20°C. The temperature measurement of the rod is 25°C using a calibrated thermometer, with ±0.5°C assumed to be the maximum plausible error in the temperature measurement. The length of a material object is defined at 20°C, so a correction must be performed to account for the thermal expansion of the rod.

The expected thermal expansion is corrected for the systematic error due to thermal expansion using eq. (D-2) is $\Delta L = 2.0 \text{ m} \times 22 \times 10^{-6}/°\text{C} \times 5.0°\text{C} = 220\ \mu\text{m}$. Hence the best estimate of the rod is 2.000000 m. [Because the measured and corrected lengths of the rod are similar, a negligible error is committed by using only two significant digits for the rod length in eq. (D-2).]

Assuming the rod is isothermal, the uncertainty in the length measurement is shown below.

(a) CTE of the Rod. Since the type of aluminum is unknown, a uniform distribution is assigned to its CTE, with $\alpha_{\min} = 20 \times 10^{-6}$ / °C, $\alpha_{\max} = 24 \times 10^{-6}$ / °C, and $\alpha = (\alpha_{\min} + \alpha_{\max}) / 2 = 22 \times 10^{-6}$ / °C. The CTE-related component of length uncertainty [see eq. (D-4)] is

$$u_\alpha(L) = \frac{L\Delta T(\alpha_{\max} - \alpha)}{\sqrt{3}} \qquad \text{(D-8)}$$

$$= \frac{2 \text{ m} \times 5°\text{C} \times 2 \times 10^{-6}/°\text{C}}{\sqrt{3}} = 11.5\ \mu\text{m}$$

(b) Temperature of the Rod. Based on knowledge of the thermometer calibration, a uniform distribution is assigned to the temperature, with $T_{\min} = 24.5°\text{C}$, $T_{\max} = 25.5°\text{C}$, and $T = 25°\text{C}$. The temperature-related component of length uncertainty [see eq. (D-6)] is

$$u_T(L) = \frac{L\alpha(T_{\max} - T)}{\sqrt{3}} \qquad \text{(D-9)}$$

$$= \frac{2 \text{ m} \times 22 \times 10^{-6}/°\text{C} \times 0.5°\text{C}}{\sqrt{3}} = 12.1\ \mu\text{m}$$

The thermally related component of the combined standard uncertainty [see eq. (D-7)] is then

$$u_c^{\text{th}}(L) = \sqrt{11.5^2 + 12.1^2}\ \mu\text{m} = 16.7\ \mu\text{m}$$

The measurement result would then be stated $L = 2.000000$ m, $U(k = 2) = 33\ \mu$m

D-3 THERMAL SELF-COMPENSATION

In some length measurements no explicit temperature measurement is taken for the purpose of calculating the correction due to thermal expansion at a nonstandard temperature. Typically this occurs when the measurement is "self compensating" in the sense that both the instrument scale or master gauge and the artifact under measurement expand similarly. For example, when a master gauge block and customer gauge block of the same nominal length are both made of steel, and are in equilibrium in the same environment, it is reasonable to assume that they expand thermally by the same amount. In calibrating the customer's block by comparison with the master block, the resulting nominal differential expansion then "cancels out" and does not need an explicit correction. The uncertainty in the calibrated length of the customer's block will nevertheless have thermally related components due to uncertainties in the CTEs of the master and customer blocks and uncertainties in the temperatures of the blocks.

Although both the master and customer gauge blocks are in the same thermal environment, so that their temperatures are expected to be the same, there might exist a small temperature difference between the blocks, denoted by $\delta T = T_M - T$, where T_M and T are the temperatures of the master and customer blocks, respectively. The best estimate of δT is zero, but knowledge of the thermal environment would suggest a uniform distribution of possible values between $\delta T_{\min}$ and $\delta T_{\max}$.

Denote the lengths, at 20°C, of the master and customer blocks by L_m and L, respectively, and let d be the measured length difference. Then, using eq. (D-1),

$$d = L\,(1 + \alpha\Delta T) - L_M\,(1 + \alpha_M\Delta T_M) \qquad \text{(D-10)}$$

where α and α_M are the associated CTEs. The desired length, L, is then given by

$$L = \frac{d + L_M\,(1 + \alpha_M\Delta T_M)}{1 + \alpha\Delta T} \qquad \text{(D-11)}$$

Since $(1 + \alpha\Delta T)^{-1} \approx 1 - \alpha\Delta T$, and $\Delta T_M = \Delta T + \delta T$, eq. (D-11) can be simplified to

$$L \approx L_M + d + C_1 + C_2 \qquad \text{(D-12)}$$

where

$C_1 = L_M(\alpha_m - \alpha)\Delta T$

$C_2 = L_M\alpha_M\delta T$

The desired length, L, is thus equal to the length, L_M, of the master block plus the measured difference, d, with additive corrections, C_1 and C_2, that account for the effects of nominal differential expansion and the difference in temperatures between the blocks.

When the master and customer blocks are of the same material, then $\alpha_m = \alpha$ and the best estimate of correction, C_1, is equal to zero. Also, since the best estimate of the difference, δT, is zero, the best estimate of correction, C_2, is also equal to zero. Then the resulting best estimate, L_{est}, of the length of the customer block is

$$L_{est} = L_m + d \quad \text{(D-13)}$$

where

d = best estimate of the measured length difference
L_m = best estimate of the length of the master block

The associated standard uncertainty is then

$$u(L) = \sqrt{u^2(L_m) + u^2(d) + u^2(C_1) + u^2(C_2)} \quad \text{(D-14)}$$

The uncertainty component, $u(L_m)$, is evaluated based on the information supplied by the calibration certificate for the master block, and the component $u(d)$ is evaluated based on what is known about the comparator system used to measure the length difference. The thermally related components $u(C_1)$ and $u(C_2)$ may be evaluated as follows.

D-3.1 Thermally Related Uncertainty Evaluation

Uncertainty components associated with the correction terms C_1 and C_2 in eq. (D-12) are evaluated by estimating their maximum values and assigning appropriate probability distributions.

D-3.1.1 Uncertainty Component Due to Correction, C_1. Knowledge of the block CTEs and the temperatures are modeled by uniform distributions. Thus α and α_m are assumed to lie in the interval $[\alpha_{min}, \alpha_{max}]$,[2] with best estimates $\alpha_m = (\alpha_{min} + \alpha_{max})/2$ and $\alpha = (\alpha_{min} + \alpha_{max})/2$.

Let ΔT_{max} be the maximum possible value of the temperature deviation $\Delta T = T - 20°C$, regardless of sign. (For example, if the temperature is known to lie between 18°C and 21°C, $\Delta T_{max} = 2°C$.) Then the reasonably probable values of the correction, C_1, lie in the interval $\pm C_1^{max}$, where

$$C_1^{max} = L_m(\alpha_{max} - \alpha_{min})\Delta T_{max} = L_m \Delta\alpha_{max} \Delta T_{max} \quad \text{(D-15)}$$

[In this expression, the small uncertainty, $u(L_m)$, in the length of the master block has been ignored.] Since it is unlikely that $\Delta\alpha$ and ΔT will both be at their maximum limits, it is reasonable to assign a triangular distribution to the possible values of C_1. (See para. B-3 of Nonmandatory Appendix B.) Thus

$$u(C_1) = \frac{L_m \Delta\alpha_{max} \Delta T_{max}}{\sqrt{6}} \quad \text{(D-16)}$$

D-3.1.2 Uncertainty Component Due to Correction, C_2. Assume that knowledge of the temperature difference, δT, is characterized by a uniform distribution over the interval $\pm\delta T_{max}$. Then the reasonably probable values of C_2 lie in the interval $\pm C_2^{max}$, where

$$C_2^{max} = L_m \alpha_{max} \delta T_{max} \quad \text{(D-17)}$$

Since it is unlikely that the master block CTE and the temperature difference will both be at their maximum limits, a triangular distribution is assigned to the possible values of C_2, so that

$$u(C_2) = \frac{L_m \alpha_{max} \delta T_{max}}{\sqrt{6}} \quad \text{(D-18)}$$

EXAMPLE: Suppose a 100-mm steel gauge block is being calibrated by comparison to a 100-mm steel master gauge block. There is no temperature measurement (and associated thermal correction) performed, but it is known that the blocks are in an environment that varies between 20°C and 22°C. It is also known that the blocks are always within ±0.1°C of each other. Determine the thermally induced uncertainties.

For the steel blocks, assume $\alpha_{max} = 12.5 \times 10^{-6}/°C$ and $\alpha_{min} = 10.5 \times 10^{-6}/°C$. The maximum temperature deviation is $\Delta T_{max} = 2°C$, and the maximum temperature difference is $\delta T_{max} = 0.1°C$. Then setting $L_m \approx 100$ mm, the nominal length of the blocks, the desired uncertainty components are evaluated using eqs. (D-16) and (D-18).

$$u(C_1) = \frac{100 \text{ mm} \times 2 \times 10^{-6}/°C \times 2°C}{\sqrt{6}} \approx 0.16\ \mu m \quad \text{(D-19)}$$

$$u(C_2) = \frac{100 \text{ mm} \times 12.5 \times 10^{-6}/°C \times 0.1°C}{\sqrt{6}} \approx 0.05\ \mu m \quad \text{(D-20)}$$

The thermally related component of the combined standard uncertainty is then

$$u(L)_{thermal} = \sqrt{0.16^2 + 0.05^2}\ \mu m \approx 0.17\ \mu m \quad \text{(D-21)}$$

[2] Although the two CTEs are assigned the same probability distribution, knowledge of one of them would not change knowledge of the other, so that they are uncorrelated.

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