ASME B89.3.4-2010

[Revision of ANSI/ASME B89.3.4M-1985 (R1992)]

# Axes of Rotation: Methods for Specifying and Testing

AN AMERICAN NATIONAL STANDARD

The American Society of Mechanical Engineers

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SETTING THE STANDARD

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Date of Issuance: May 12, 2010

This Standard will be revised when the Society approves the issuance of a new edition. There will be no addenda issued to this edition.

ASME issues written replies to inquiries concerning interpretations of technical aspects of this Standard. Periodically certain actions of the ASME B89 Committee may be published as Cases. Cases and interpretations are published on the ASME Web site under the Committee Pages at http://cstools.asme.org as they are issued.



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# FOREWORD

The testing of axes of rotation is at least as old as machine tools since most forms of machine tools incorporate such an axis. One of the more widely distributed European works on testing machine tools[1] devotes considerable attention to the problems encountered. Consideration of principles, equipment, and methods were included in the work.

Other European work[2] was carried forward and was published, in part, in 1959. As a result, a variety of terms came into use throughout the world to describe and explain the various phenomena found during testing and subsequent use of machine tool spindles.

In the United States, work published in 1967[3] represented a new viewpoint both in definitions and methods of testing. This work also underscored the lack of standardization of the entire subject of rotational axes. When the American National Standards Subcommittee B89.3, Geometry, was formed in February 1963, axes of rotation were not initially considered as a separate topic. This Standard, which was initiated by J. K. Emery in August 1968 as a part of the Geometry Subcommittee work, is the result of recognizing the need for uniform technology and methods of testing for axes of rotation.

The goal in preparing the 1985 Standard was to produce a comprehensive document for the description, specification, and testing of axes of rotation. Extensive advisory material is provided in the Appendices as an aid to the user. It is recommended that this material be studied before putting the Standard to use. While the examples of the Appendices involve machine tools and measuring machines, the terminology and the underlying concepts are applicable to any situation in which the performance of a rotary axis is of concern.

The 1985 edition was adopted as an American National Standard by the American National Standards Institute (ANSI) on May 17, 1985.

The 1985 Standard laid the modern foundation for understanding, specifying, and testing axes of rotation. The cornerstones of this foundation are the following: the concept of error motion as opposed to runout; recognition of the role of the structural loop; differentiation between fixed and rotating sensitive direction; classification of radial, axial, tilt, and face error motions; separation of thermal drift from error motion; and dividing total error motion into average and asynchronous components. These concepts are illuminated by appendices with examples of test procedures and equipment, including a method of separating error motion from out-of-roundness of the test ball.

This revision more fully describes the periodic nature of error motions in order to point out the nonrandom, deterministic behavior of bearings. The term "average error motion" is now called "synchronous error motion." The distinction between synchronous and asynchronous is described in terms of frequency analysis. Distinction is also emphasized between axis error motions, axis shifts (displacements due to changes in operating conditions), and structural motions.

The least squares circle is now preferred for determining the center when calculating most error motions. New definitions include stator, rotor, bearing, artifact, orientation angle, axis shift, spindle error motion, synchronous error motion, residual synchronous error motion, static error motion, stationary-point runout, setup hysteresis, frequency analysis, aliasing, and master axis. Manual evaluation of polar plots remains a valid method. A new appendix describes representative uncertainty evaluation procedures for error motion measurement.

ASME B89.3.4-2010 was approved by the American National Standards Institute on April 1, 2010.

[1] Schlesinger, G., Testing Machine Tools, Machinery Publishing Co.

[2] Tlusty, J., System and Methods of Testing Machine Tools, Microtechnic, 13, 162 (1959)

[3] Bryan, J. B., Clouser, R. W., and Holland, E., Spindle Accuracy, *American Machinist*, Dec. 4, 1967

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| | |
|---|---|
| Subject: | Cite the applicable paragraph number(s) and the topic of the inquiry. |
| Edition: | Cite the applicable edition of the Standard for which the interpretation is being requested. |
| Question: | Phrase the question as a request for an interpretation of a specific requirement suitable for general understanding and use, not as a request for an approval of a proprietary design or situation. The inquirer may also include any plans or drawings that are necessary to explain the question; however, they should not contain proprietary names or information. |

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# AXES OF ROTATION: METHODS FOR SPECIFYING AND TESTING

## 1 SCOPE

This Standard is primarily intended for, but not limited to, the standardization of methods for specifying and testing axes of rotation of spindles used in machine tools and measuring machines. Appendices provide advisory information for the interpretation and use of this Standard.

### 1.1 Properties Included in This Standard

(a) error motion
(b) structural motion
(c) compliance
(d) axis shifts

### 1.2 Properties Not Included in This Standard

(a) angular positioning accuracy
(b) accelerometer, velocity, or microphone based measurements
(c) dynamic compliance measurements
(d) torque measurements
(e) speed stability or load capacity

## 2 DEFINITIONS

### 2.1 General Concepts

The definitions in this Standard have been arranged to help the user develop an understanding of the terminology of axes of rotation.

#### 2.1.1 Axis of Rotation

*axis of rotation:* a line segment about which rotation occurs.

NOTE: In general, this line segment translates and tilts with respect to the reference coordinate axes, as shown in Fig. 1.

#### 2.1.2 Spindle

*spindle:* a device that provides an axis of rotation.

NOTE: Other-named devices such as rotary tables, trunnions, and live centers are included within this definition.

#### 2.1.3 Rotor

*rotor:* the rotating element of a spindle.

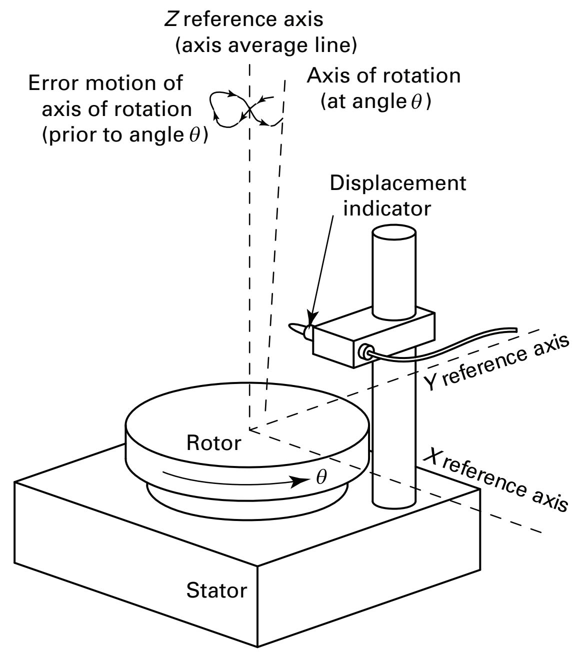
#### 2.1.4 Stator

*stator:* the nonrotating element of a spindle.

#### 2.1.5 Bearing

*bearing:* an element of a spindle that supports the rotor and allows rotation between the rotor and the stator.

**Fig. 1 Reference Coordinate Axes Directions, Axis of Rotation, and Error Motion of Spindle**




#### 2.1.6 Reference Coordinate Axes

*reference coordinate axes:* mutually perpendicular $X$, $Y$, and $Z$ axes, fixed with respect to a specified object.

NOTES:
(1) For simplicity, the $Z$ axis is chosen to lie along the axis average line, as in Fig. 1.
(2) The specified object may be fixed or rotating.

#### 2.1.7 Perfect Spindle

*perfect spindle:* a spindle having no motion of its axis of rotation relative to the reference coordinate axes.

#### 2.1.8 Perfect Workpiece

*perfect workpiece:* a rigid body having a perfect surface of revolution about a centerline.

#### 2.1.9 Axis Average Line

*axis average line:* a line segment passing through two axially separated radial error motion polar profile centers.

NOTES:
(1) If the centers are not specified, the least squares circle (LSC) center is to be assumed.

(2) The axis average line concept is used to define an unambiguous location of an axis of rotation for a given set of operating conditions.

### 2.1.10 Axis Shift

*axis shift:* a change in position of the axis of rotation caused by a change in operating conditions.

NOTES:
(1) Causes of axis shift include thermal drift, load changes, preload changes, and speed changes.
(2) An axis shift that occurs during an error motion measurement will affect the error motion values.
(3) Error motion specifications assume constant conditions unless specified otherwise.

### 2.1.11 Displacement Indicator

*displacement indicator:* a device that measures changes in distance between two objects.

NOTE: Examples include capacitive gages, linear variable differential transformers (LVDTs), eddy current probes, laser interferometers, and dial indicators.

### 2.1.12 Structural Loop

*structural loop:* the assembly of components that maintain the relative position between two specified objects.

NOTE: A typical pair of specified objects is the cutting tool and the workpiece; the structural loop would include the workpiece, chuck, spindle rotor, bearings, stator, headstock, the machine slideways and frame, the tool holder, and the cutting tool. (In this Standard, a displacement indicator qualifies as a tool.)

### 2.1.13 Structural Error Motion

*structural error motion:* error motion measured from the spindle stator to the tool, from the rotor to an object mounted to the rotor, or from any two specified objects outside the stator-to-rotor structural loop.

### 2.1.14 Sensitive Direction

*sensitive direction:* the direction normal to the surface of a perfect workpiece through the instantaneous point of machining or measurement (as shown in Fig. 2).

### 2.1.15 Nonsensitive Direction

*nonsensitive direction:* any direction perpendicular to the sensitive direction.

### 2.1.16 Fixed Sensitive Direction

*fixed sensitive direction:* the sensitive direction is fixed when the workpiece is rotated by the spindle and the point of machining or measurement is not rotating.

NOTES:
(1) A lathe has a fixed sensitive direction.
(2) The reference coordinates are fixed with respect to the stator.

### 2.1.17 Rotating Sensitive Direction

*rotating sensitive direction:* the sensitive direction is rotating when the workpiece is fixed and the point of machining or measurement rotates.

NOTES:
(1) A jig borer has a rotating sensitive direction; the point of machining or measurement rotates with the rotor (see Fig. A-3).
(2) The reference coordinate axes rotate with the rotor.
(3) For some measurements, a displacement indicator rotates with the spindle; in an equivalent arrangement two displacement indicators are arranged at 90 deg to each other (see para. A-7.5).

### 2.1.18 Orientation Angle

*orientation angle:* the angle between the circumferential position of a designated feature on the spindle stator or rotor and the point of machining or gaging.

NOTES:
(1) Specification of the orientation angle enables a spindle to be installed with the same orientation in which it was tested or specified.
(2) The orientation angle is specified with respect to a designated feature on the stator for fixed sensitive direction or on the rotor for rotating sensitive direction.

### 2.1.19 Direction Angle

*direction angle:* the angle of the sensitive direction with respect to the axis of rotation.

NOTES:
(1) Axial measurements have a direction angle of 0 deg and radial measurements have a direction angle of 90 deg.
(2) The direction angle must be specified if the measurement direction is at some angle other than in the radial or axial direction (see Fig. 2).

### 2.1.20 Runout

*runout:* the total displacement measured by an indicator sensing against a moving surface or moved with respect to a fixed surface.

NOTES:
(1) The term "TIR" (total indicator reading) is equivalent to runout.
(2) Surfaces have runout; axes of rotation have error motion.
(3) Runout includes errors due to centering and workpiece form errors and hence is not equivalent to error motion.

### 2.1.21 Stationary-Point Runout

*stationary-point runout:* the total displacement measured by sensing against a point on a surface that is not intended to move laterally with respect to the indicator.

NOTES:
(1) This term applies when two or more axes of a machine are simultaneously moved to keep a point stationary with respect to the indicator.
(2) Stationary-point runout also describes a variety of chase-the-point measurements such as rim-and-face measurements for alignment of two axes of rotation.

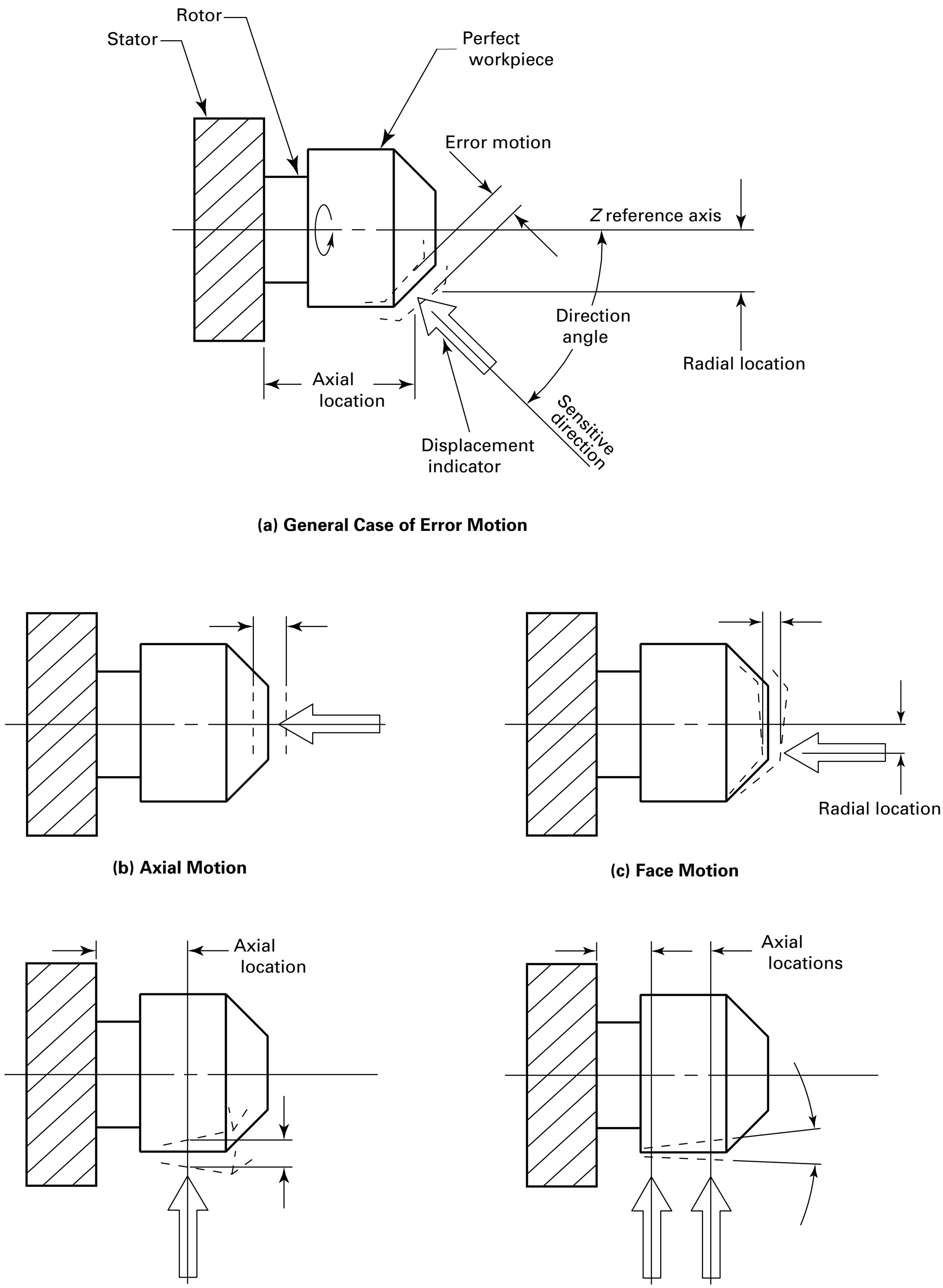
### 2.1.22 Master Axis

*master axis:* the axis of rotation of a precision spindle used to measure error motions of another spindle.

### 2.1.23 Artifact

*artifact:* a test ball, optical flat, test cylinder, or other target for error motion measurement.

**Fig. 2 Plan View of Spindle Showing General Case of Error Motion and Axial, Face, Radial, and Tilt Motions**




(a) General Case of Error Motion

(b) Axial Motion

(c) Face Motion

(d) Radial Motion

(e) Tilt Motion

#### 2.1.24 Squareness

*squareness:* a plane surface is square to an axis of rotation if coincident polar profile centers are obtained for an axial and a face error motion polar plot or for two face error motion polar plots at different radii.

NOTE: The term "perpendicularity" is equivalent to squareness.

#### 2.1.25 Play

*play:* a condition of low or zero stiffness over a limited range of displacement, due to clearance between elements of a structural loop.

#### 2.1.26 Setup Hysteresis

*setup hysteresis:* nonrepeatability of position between elements in a test setup, normally due to loose mechanical connections or friction.

#### 2.1.27 Pure Radial Motion

*pure radial motion:* the concept of radial motion in the absence of tilt motion.

### 2.2 Error Motions

#### 2.2.1 Error Motion

*error motion:* changes in position, relative to the reference coordinate axes, of the surface of a perfect workpiece, as a function of rotation angle, with the workpiece centerline coincident with the axis of rotation.

NOTES:
(1) Error motions are specified as to location and direction as shown in Fig. 2, illustration (a) and do not include motions due to changes in operating conditions unless specified otherwise.
(2) The concept of a perfect workpiece enables easier visualization of error motion. Also, in practice all measurements are made from solid surfaces rather than abstract lines.

#### 2.2.2 Spindle Error Motion

*spindle error motion:* generic term for any error motion associated with a spindle measured between the ends of a structural loop.

NOTES:
(1) This term only is used quantitatively when combined with a modifier defined in this Standard, as in spindle radial error motion.
(2) Using this term alone is nonpreferred due to ambiguity in common use concerning the specific error motion and the specific structural loop.

#### 2.2.3 Stator-to-Rotor Error Motion

*stator-to-rotor error motion:* generic term for any error motion associated with a spindle measured between the ends of a minimal structural loop.

NOTES:
(1) This term only is used quantitatively when combined with a modifier defined in this Standard, as in stator-to-rotor radial error motion.
(2) "Minimal" is understood to mean that the contribution of the structural error motion is reduced as much as practical so that direct stator-to-rotor error motions are being observed.
(3) Using this term alone is nonpreferred due to ambiguity in common use concerning the specific error motion and the specific structural loop.

#### 2.2.4 Error Motion Sources

*error motion sources:* the sources of error motion are as follows:

*(a)* bearing error motion, due to imperfect bearings and seats

*(b)* structural error motion, due to internal or external excitation and affected by elasticity, mass, and damping of the structural loop

#### 2.2.5 Total Error Motion

*total error motion:* the complete error motion as recorded.

#### 2.2.6 Synchronous Error Motion

*synchronous error motion:* the components of total error motion that occur at integer multiples of the rotation frequency.

NOTE: The term "average error motion" is equivalent but no longer preferred. The method of averaging is acceptable for the determination of synchronous error motion.

##### 2.2.6.1 Fundamental Error Motion

*fundamental error motion:* the sinusoidal component of synchronous error motion that occurs at the rotation frequency.

NOTES:
(1) Fundamental radial and fundamental tilt displacements are not error motions because they represent misalignment of the artifact, not a property of the axis of rotation.
(2) Fundamental axial and fundamental face motions are error motions and have important engineering consequences. (See para. A-7.9.)

##### 2.2.6.2 Residual Synchronous Error Motion

*residual synchronous error motion:* the components of synchronous error motion that occur at integer multiples of the rotation frequency other than the fundamental.

NOTE: This term applies only to axial and face error motions.

#### 2.2.7 Asynchronous Error Motion

*asynchronous error motion:* the portion of total error motion that occurs at frequencies other than integer multiples of the rotation frequency.

NOTES:
(1) Asynchronous error motion comprises those components of error motion that are
*(a)* not periodic
*(b)* periodic but occur at frequencies other than the rotational frequency and its integer multiples
*(c)* periodic at frequencies that are subharmonics of the rotational frequency
(2) The previously used term "random error motion" is now nonpreferred because of confusion with the statistical meaning of the word random and because most errors are deterministic.

### 2.2.8 Frequency Analysis

*frequency analysis:* the process of calculating the individual spectral components that superimpose to recreate the original signal.

### 2.2.9 Aliasing

*aliasing:* the incorrect assignment of a frequency component higher than one-half the sampling rate.

## 2.3 Error Motion Types

### 2.3.1 Radial Error Motion

*radial error motion:* error motion perpendicular to the $Z$ reference axis and at a specified axial location [see Fig. 2, illustration (d)].

NOTE: The term "radial runout" has an accepted meaning, which includes errors due to centering and workpiece out-of-roundness and hence is not equivalent to radial error motion.

### 2.3.2 Tilt Error Motion

*tilt error motion:* error motion in an angular direction relative to the $Z$ reference axis [see Fig. 2, illustration (e)].

NOTES:
(1) Coning, wobble, swash, tumbling, and towering errors are nonpreferred terms for tilt error motion.
(2) The term "tilt error motion" rather than "angular motion" is chosen to avoid confusion with rotation about the axis or with angular positioning error of devices such as rotary tables.

### 2.3.3 Axial Error Motion

*axial error motion:* error motion coaxial with the $Z$ reference axis [Fig. 2, illustration (b)].

NOTE: Axial slip, end-camming, pistoning, and drunkenness are nonpreferred terms for axial error motion.

### 2.3.4 Face Error Motion

*face error motion:* the sum of the axial error motion and the axial component of tilt motion at the specified radius [Fig. 2, illustration (c)].

NOTES:
(1) Face error motion is parallel to the $Z$ reference axis at a specified radial location.
(2) The term "face runout" has an accepted meaning analogous to radial runout and hence is not equivalent to face error motion.

## 2.4 Static Error Motion

*static error motion:* a special case in which error motion is sampled with the spindle at rest in a series of discrete rotational positions.

NOTE: This technique measures error motion exclusive of dynamic influences.

## 2.5 Polar Plots

### 2.5.1 Error Motion Polar Plot

*error motion polar plot:* a polar plot of error motion versus angle of rotation of the spindle (Fig. 3).

### 2.5.2 Total Error Motion Polar Plot

*total error motion polar plot:* the complete error motion polar plot as recorded.

### 2.5.3 Synchronous Error Motion Polar Plot

*synchronous error motion polar plot:* a polar plot of the error motion components having frequencies that are integer multiples of the rotation frequency.

NOTE: It is acceptable to create the synchronous error polar plot by averaging the total error motion polar plot.

### 2.5.4 Asynchronous Error Motion Polar Plot

*asynchronous error motion polar plot:* a polar plot of that portion of the total error motion that occurs at frequencies that are not integer multiples of the rotational frequency.

### 2.5.5 Axial Error Motion Polar Plot

*axial error motion polar plot:* a polar plot of the axial error motion, including the fundamental, residual synchronous, and asynchronous error motions.

### 2.5.6 Fundamental Error Motion Polar Plot

*fundamental error motion polar plot:* the best fit reference circle fitted to the axial or face synchronous error motion polar plot.

### 2.5.7 Residual Synchronous Error Motion Polar Plot

*residual synchronous error motion polar plot:* a polar plot of the portion of the axial or face synchronous error motion that occurs at frequencies other than the fundamental.

### 2.5.8 Inner Error Motion Polar Plot

*inner error motion polar plot:* the contour of the inner boundary of the total error motion polar plot.

### 2.5.9 Outer Error Motion Polar Plot

*outer error motion polar plot:* the contour of the outer boundary of the total error motion polar plot.

## 2.6 Error Motion Centers

The following centers are defined for the assessment of error motion polar plots (see Fig. 4).

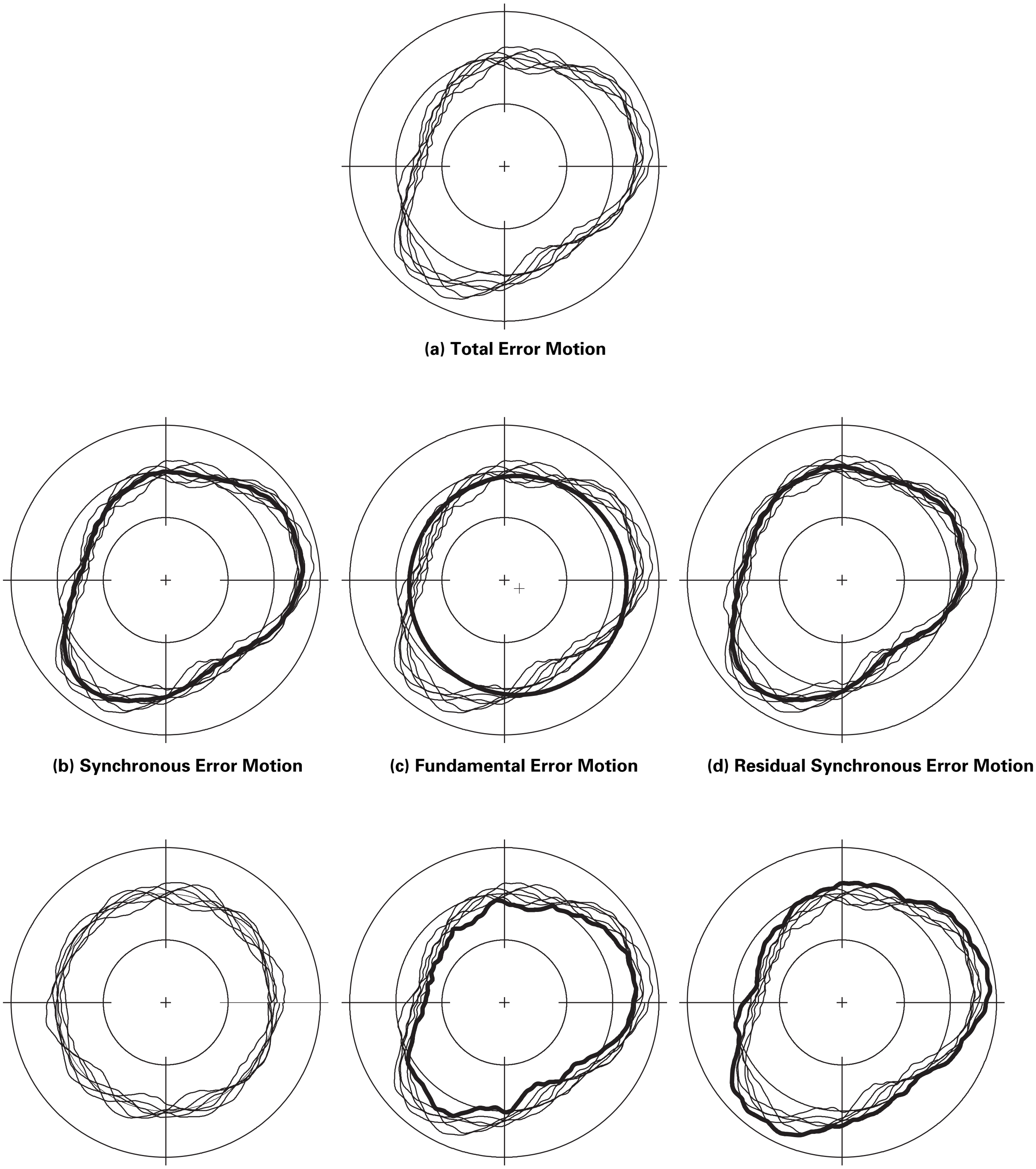
Table 1 gives the preferred centers for the assessment of error motion values. If the center is not specified, the preferred center is to be assumed.

### 2.6.1 Polar Chart (PC) Center

*polar chart (PC) center:* the center of the polar chart.

NOTES:
(1) Unless otherwise specified, the polar profile center is determined using the synchronous error motion polar plot.
(2) A workpiece is centered with zero centering error when the polar chart center coincides with the chosen polar profile center.

## Fig. 3 Polar Plots of Error Motion and Its Components



(a) Total Error Motion

(b) Synchronous Error Motion

(c) Fundamental Error Motion

(d) Residual Synchronous Error Motion

(e) Asynchronous Error Motion

(f) Inner Error Motion

(g) Outer Error Motion

**Fig. 4 Error Motion Polar Plot Showing PC Center and LSC Center and Error Motion Values About These Centers**

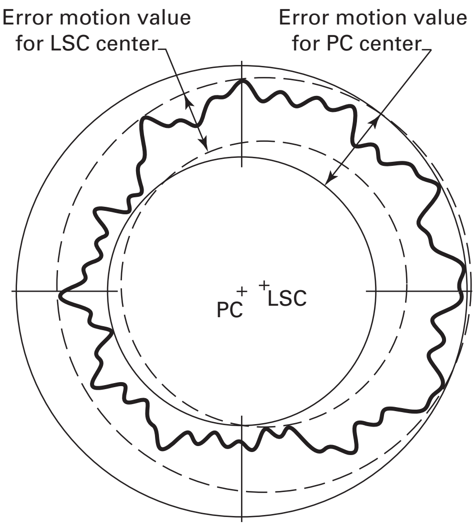



**Table 1 Error Motion Type and Preferred Center**

| Motion Type | Preferred Center |
|---|---|
| Radial | LSC |
| Tilt | LSC |
| Axial | PC |
| Face | PC |
| Residual synchronous axial | LSC |
| Residual synchronous face | LSC |

#### 2.6.2 Polar Profile Center

*polar profile center:* a center derived from the polar profile by a mathematical or graphical technique.

#### 2.6.3 Least Squares Circle (LSC) Center

*least squares circle (LSC) center:* the center of a circle that minimizes the sum of the squares of a sufficient number of equally spaced radial deviations measured from it to the error motion polar plot.

#### 2.6.4 Minimum Radial Separation (MRS) Center

*minimum radial separation (MRS) center:* the center that minimizes the radial difference required to contain the error motion polar plot between two concentric circles.

#### 2.6.5 Maximum Inscribed Circle (MIC) Center

*maximum inscribed circle (MIC) center:* the center of the largest circle that can be inscribed within the error motion polar plot.

#### 2.6.6 Minimum Circumscribed Circle (MCC) Center:

*minimum circumscribed circle (MCC) center:* the center of the smallest circle that will just contain the error motion polar plot.

### 2.7 Error Motion Value

*error motion value:* a magnitude assessment of error motion. The following definitions are presented in terms of polar plots to aid in understanding the phenomena and the computations. Mathematical analysis allows values to be calculated without constructing polar plots.

#### 2.7.1 Total Error Motion Value

*total error motion value:* the scaled difference in radii of two concentric circles from a specified error motion center just sufficient to contain the total error motion polar plot.

#### 2.7.2 Synchronous Error Motion Value

*synchronous error motion value:* the scaled difference in radii of two concentric circles from a specified error motion center just sufficient to contain the synchronous error motion polar plot.

#### 2.7.3 Asynchronous Error Motion Value

*asynchronous error motion value:* the maximum scaled width of the asynchronous error motion polar plot, measured along a radial line through the PC center.

NOTE: Methods that compute the asynchronous error motion value based on the range of data from the same angular position are equivalent to measurements from the PC center.

#### 2.7.4 Fundamental Error Motion Value

*fundamental error motion value:* twice the amplitude of the component of synchronous error motion that occurs at the rotation frequency.

NOTES:
(1) The fundamental error motion value is alternatively defined as twice the scaled distance between the PC center and a specified polar profile center of the synchronous error motion polar plot.
(2) The value is twice the amplitude because, in this case, amplitude represents the average-to-peak value rather than the peak-to-peak value.
(3) Fundamental axial value and fundamental face value are the same value.

#### 2.7.5 Residual Synchronous Error Motion Value

*residual synchronous error motion value:* the scaled difference in radii of two concentric circles from a specified error motion center just sufficient to contain the residual synchronous error motion polar plot.

#### 2.7.6 Inner Error Motion Value

*inner error motion value:* the scaled difference in radii of two concentric circles from a specified error motion center just sufficient to contain the inner error motion polar plot.

#### 2.7.7 Outer Error Motion Value

*outer error motion value:* the scaled difference in radii of two concentric circles from a specified error motion center just sufficient to contain the outer error motion polar plot.

#### 2.7.8 Compliance

*compliance:* the displacement per unit static force between two objects, specified as to the structural loop, the location and direction of the applied force, and the location and direction of the displacement of interest. The following special terms can be applied when the structural loop includes a spindle:

##### 2.7.8.1 Radial Compliance

*radial compliance:* applicable when the force and displacement directions are at 90 deg to the $Z$ reference axis.

##### 2.7.8.2 Axial Compliance

*axial compliance:* applicable when the force and displacement directions are colinear with the $Z$ reference axis.

##### 2.7.8.3 Tilt Compliance

*tilt compliance:* applicable for a pure moment and a tilt displacement in a plane containing the $Z$ reference axis.

##### 2.7.8.4 Face Compliance

*face compliance:* applicable when the force and displacement directions are parallel to the $Z$ reference axis and at a specified radial location.

#### 2.7.9 Stiffness

*stiffness:* the reciprocal of compliance.

#### 2.7.10 Thermal Drift

*thermal drift:* a changing distance or angle between two objects, associated with a changing temperature distribution within the structural loop.

NOTES:
(1) Thermal drift may be caused by internal or external sources.
(2) Thermal drift is specified as to axial and radial location and direction angle relative to the $Z$ reference axis. Unless otherwise noted, the direction is assumed to be the sensitive direction and error motion is not included. The following special terms can be applied when the structural loop includes a spindle.

##### 2.7.10.1 Radial Thermal Drift

*radial thermal drift:* applicable when the displacement is perpendicular to the $Z$ reference axis.

##### 2.7.10.2 Axial Thermal Drift

*axial thermal drift:* applicable when the displacement is colinear with the $Z$ reference axis.

##### 2.7.10.3 Tilt Thermal Drift

*tilt thermal drift:* applicable to a tilt displacement relative to the $Z$ reference axis.

##### 2.7.10.4 Face Thermal Drift

*face thermal drift:* applicable to a combination of axial and tilt displacement measured at a specified radial location.

##### 2.7.10.5 Thermal Drift Plot

*thermal drift plot:* a time-based record of thermal drift.

##### 2.7.10.6 Thermal Drift Value

*thermal drift value:* the difference between the maximum and minimum displacement over a specified time and at specified conditions.

#### 2.7.11 Axis Shift Caused by Other Changes in Operating Conditions

*axis shift caused by other changes in operating conditions:* a changing distance or angle between the axis of rotation and a specified object; associated with a change in operating conditions such as speed, preload, load variations, and direction of rotation.

## 3 SPECIFICATION OR DESCRIPTION OF AXIS OF ROTATION

### 3.1 Error Motion

The following information is necessary for specifying or describing error motion:

*(a)* the type of motion: total, synchronous, asynchronous, fundamental, or residual synchronous

*(b)* the axial and/or radial position(s) at which the measurement is made

*(c)* the direction angle of the sensitive direction, e.g., axial, radial, or intermediate angles, as appropriate

*(d)* orientation angle of the displacement indicator with respect to a specified mark on the spindle stator or rotor

*(e)* the error motion component and type of information (polar plot or value)

*(f)* the rotation speed and direction of the spindle

*(g)* the time duration in seconds or number of spindle revolutions

*(h)* model and serial numbers of all sensors, artifacts, targets, and fixtures

*(i)* the structural loop, including the specified object with respect to which the spindle axis and the reference coordinate axes are located and the components connecting these objects

*(j)* the magnitude, direction, and location of any external forces

*(k)* other operating conditions that may influence the error motion measurement such as setup hysteresis and temperature variation

### 3.2 Compliance

The following information is necessary for specifying or describing compliance:

*(a)* the axial and radial location and direction angle of the applied static force relative to the $Z$ reference axis

*(b)* the same information, but for the displacement measurement

*(c)* the structural loop connecting the two ends of the load measuring instrument

*(d)* the structural loop connecting the displacement indicator and the surface against which it reads

### 3.3 Thermal Drift

The following information is necessary for specifying or describing thermal drift:

*(a)* the axial and radial location and direction angle of the displacement measurement relative to the Z reference axis

*(b)* the structural loop connecting the displacement indicator and the surface against which it reads

*(c)* the rotational speed of the spindle

*(d)* the time duration of the measurement

*(e)* the ambient temperature conditions of the test environment

*(f)* other operating conditions that may influence the thermal drift

*(g)* the drift characteristics of the displacement measuring system due to temperature change with zero displacement input

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# NONMANDATORY APPENDIX A
# DISCUSSION OF GENERAL CONCEPTS

## A-1 INTRODUCTION

This Standard was developed in response to the need of industry to specify and measure the quality of axes of rotation found in machines used in the fabrication and inspection of manufactured products. While the terminology and concepts of this Standard may be applied in other areas, no attempt will be made to do so in this Nonmandatory Appendix.

For clarity, this Nonmandatory Appendix uses specific examples, such as the spindle of a lathe. However, it is emphasized that the concepts under discussion can be applied to all rotational axes found in machine tool or measuring machine components, such as rotary tables, trunnion bearings, and live centers.

## A-2 PERFECT AXIS OF ROTATION

It is helpful to begin by considering the requirements to be met by a perfect axis of rotation. While it may seem appropriate to use a simple phrase such as "capable of pure rotation of a workpiece about a line fixed in space," several important points should be noted to show that this description is inadequate.

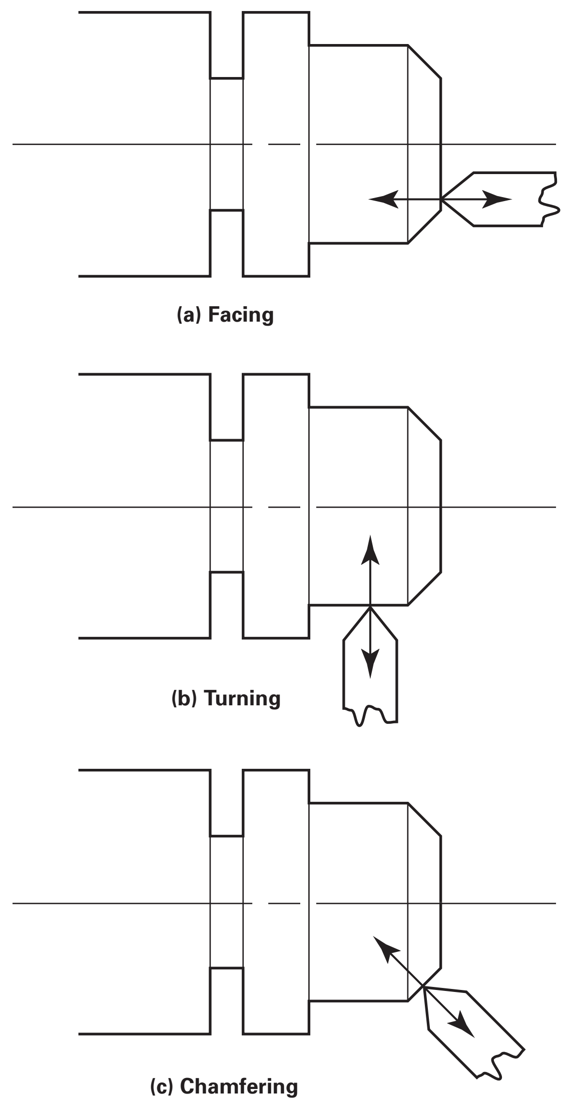
### A-2.1 Relative Motion

Consider a lathe mounted aboard a ship that is rolling in the ocean. The spindle axis can clearly undergo large motions in space without influencing the workpiece accuracy. What is important is relative motion between the workpiece and the cutting tool. This involves only the structural loop, a term that refers to the mechanical components that maintain the relative position between the workpiece and the tool (the chuck, spindle shaft, spindle bearings, stator, headstock, frame, slides, and tool post in the present example).

### A-2.2 Sensitive Direction

Assume that a flat facing cut is being made in a lathe. If imperfections in the spindle bearings cause small axial movements of the workpiece relative to the tool at the point of cutting, one-for-one errors will be cut into the workpiece, and hence the axial movement is in a sensitive direction. By contrast, small motions tangential to the face do not cause cutting errors since these motions are in a nonsensitive direction as shown in Fig. A-1. The sensitive direction is always along a line that is perpendicular to the surface of revolution being generated and through the point of machining. Any line perpendicular to the sensitive direction is a nonsensitive direction. Relative motion in the nonsensitive direction will cause some error when dealing with a curved surface such as the cylinder of Fig. A-2. In practice, this error is very small and can usually be neglected in return

**Fig. A-1 Illustration of Sensitive Direction in Facing, Turning, and Chamfering**

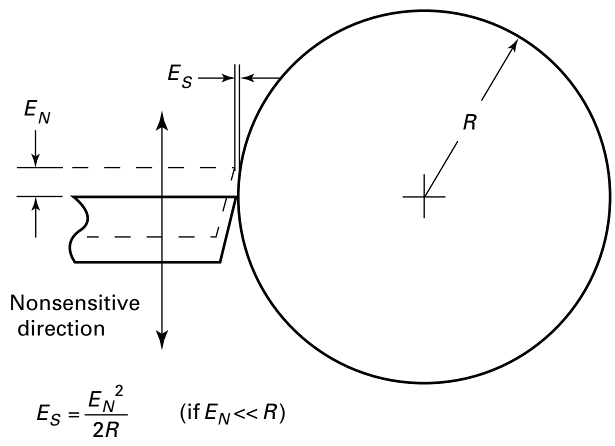


(a) Facing

(b) Turning

(c) Chamfering

**Fig. A-2 Second-Order Error Due to Relative Motion in Nonsensitive Direction Along Curved Surface**




for a substantial reduction in effort. The following formula is useful in estimating this error. Let

$E_N$ = motion in the nonsensitive direction
$E_S$ = error in the sensitive direction due to $E_N$
$R$ = part radius

then

$$E_S = \frac{(E_N)^2}{2R} \left(\text{if } E_N \text{ is small compared to } R\right) \qquad \text{(A-1)}$$

For example, let $E_N$ = 0.001 in. (0.025 mm) and $R$ = 0.5 in. (12.7 mm).

Then

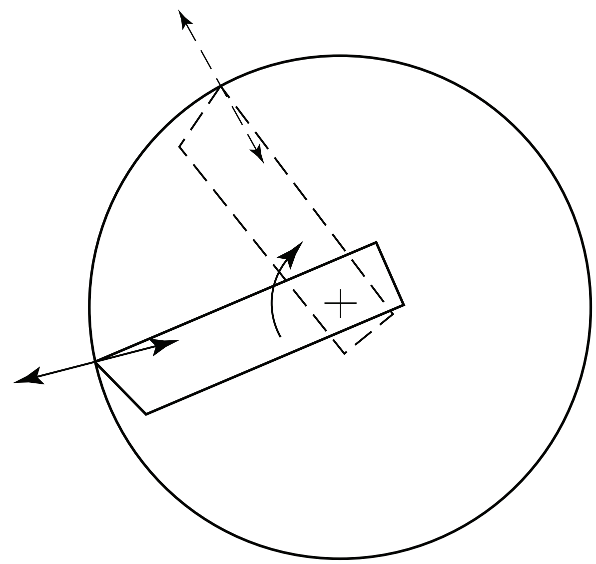
$$E_S = \frac{(0.001)^2}{2 \times 0.5} = 10^{-6} \text{ in.} = 1 \ \mu\text{in.} \ (0.025 \ \mu\text{m}) \qquad \text{(A-2)}$$

The 1 μin. (0.025 μm) error due to moving tangentially to the circle is $\frac{1}{1000}$ as large as if the same 0.001 in. (0.025 mm) motion had occurred in the sensitive directions, i.e., it is a second-order error. Ignoring motion in the nonsensitive direction is therefore justified if it is the same order of magnitude as the motion in the sensitive direction.

### A-2.3 Fixed Workpiece

In contrast to a machine such as a lathe, which rotates the workpiece, another type of machine exists in which the workpiece is fixed and the cutting tool rotates. An example of this type is a boring machine. Since the sensitive direction is always normal to the workpiece surface at the point of machining, the sensitive direction rotates with the tool (see Fig. A-3). Different test arrangements are used for axes of rotation depending on whether the sensitive direction is fixed or rotating with respect to the machine frame.

**Fig. A-3 Illustration of Rotating Sensitive Direction at Two Instants in Time in Jig-Boring Hole**



### A-2.4 Displacement Indicators Versus Tools

The previous examples have all referred to cutting tools. However, the concepts apply with equal validity to measuring machines, with a displacement indicator replacing the cutting tool.

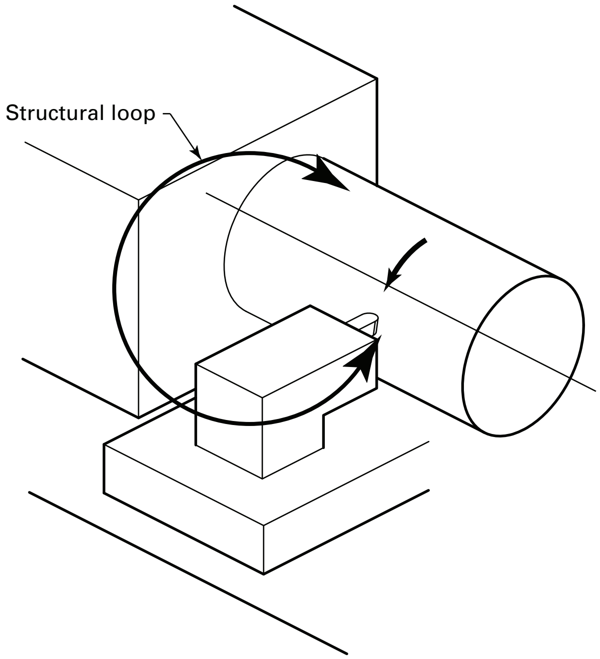
## A-3 IMPERFECT AXIS OF ROTATION

For a real axis of rotation, the general term "error motion" will be used to refer to relative displacements in the sensitive direction between the tool or displacement indicator and the workpiece. The physical causes of error motion can be classified as spindle error motion due to factors such as misaligned or out-of-round bearing components, and structural error motion due to internal or external sources of excitation and affected by the finite mass, compliance, and damping of the structural loop. The separation of error motion test data into these two categories is not always possible, although the recording of data on synchronized polar charts is useful in this regard.

## A-4 STRUCTURAL ERROR MOTION

The term "structural error motion" is used rather than "vibration" to emphasize the relationship of the structural loop (see Fig. A-4) as it applies to relative motion. It would be incorrect, for example, to measure the structural error motion by attaching an accelerometer to the tool post of a lathe and integrating the output twice, since this would yield the absolute motion. For a rigid structural loop, the entire loop could undergo virtually the same absolute vibratory motion, resulting in a negligible structural error motion.

Fig. A-4 Structural Loop Example




The decision to include structural error motion in an axis of rotation standard is deserving of some comment. Since only relative motion is important, the structural loop is as important to the functional use of an axis of rotation as the C-frame and anvil are to a hand micrometer. To attempt to include structural error motion due to noisy rolling-element bearings and to exclude that from drive gears or motors, or to include resonance in a spindle shaft but not a tool post, seems arbitrary and unrealistic. The approach taken in this Standard is to include structural error motion from all sources as a valid topic of discussion, but to leave to the user the choice of the structural loop best suited to his objectives. Thus the present Standard can be applied to testing a spindle as a stand-alone unit on a surface plate or as an integrated part of a complete machine. It follows that the user of this Standard should allow no ambiguity to arise regarding the structural loop associated with an error motion measurement or specification. It should also be noted that asymmetric compliance of the spindle or other part of the structural loop can cause changes of error motion as measured under various conditions of loading; load conditions of a test should therefore be specified where appropriate.

## A-5 THERMAL DRIFT

An additional cause of relative motion between the tool or displacement indicator and the workpiece is a changing temperature distribution within the structural loop. The relative motion in the sensitive direction due to the accompanying thermal expansion or contraction is referred to as thermal drift. Thermal drift represents a change in operating conditions and is therefore treated separately.

The situation regarding thermal drift is similar to that of structural motion, in that it can be caused by heat generated in the spindle bearings or from some external source. The choice of what to include or exclude is left open to the user of this Standard. Additional advisory material on thermal drift can be found in ANSI B89.6.2 (reference [2]).

## A-6 ERROR MOTION GEOMETRY

A workpiece has six degrees of freedom, consisting of three linear motions and three angular motions, as shown individually in Fig. A-5 for a given instant in time, $t$, corresponding to a spindle position, $\theta$. Spindle rotation, $\theta$, about the $Z$ reference axis, Fig. A-5, illustration (h), is the intended function of the axis of rotation. Which of the remaining five degrees of freedom contributes significantly to the error motion depends on the sensitive direction and the axial and radial location of the point of machining or gaging. For the lathe operations shown in Fig. A-1, it can be concluded that the sensitive direction always lies in the plane parallel to that of the tool travel. Examination of other rotating workpiece machine tools and measuring machines shows that in virtually all cases the sensitive direction is restricted to one plane. Calling this the $X$-$Z$ plane for convenience, it follows that the motions $Y(\theta)$ and $\beta(\theta)$ are always in a nonsensitive direction and can be ignored. In other words, the only motions of concern are the motions $X(\theta)$, $Z(\theta)$, and $\alpha(\theta)$ in the $X$-$Z$ plane. The following terms will be used.

### A-6.1 Pure Radial Motion

*pure radial motion:* motion $X(\theta)$ in Fig. A-5, illustration (c), in which the axis of rotation remains parallel to the $Z$ reference axis and moves perpendicular to it in the sensitive direction.

### A-6.2 Axial Motion

*axial motion:* motion $Z(\theta)$ in Fig. A-5, illustration (e), in which the axis of rotation remains coaxial with the $Z$ reference axis and moves parallel to it.

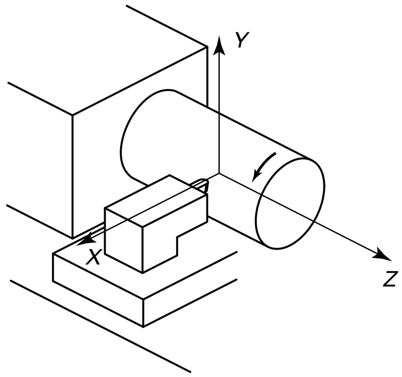
### A-6.3 Tilt Motion

*tilt motion:* motion $\alpha(\theta)$ in Fig. A-5, illustration (g), in which the axis of rotation moves angularly with respect to the $Z$ reference axis and in the plane of the axial error and pure radial error motions.

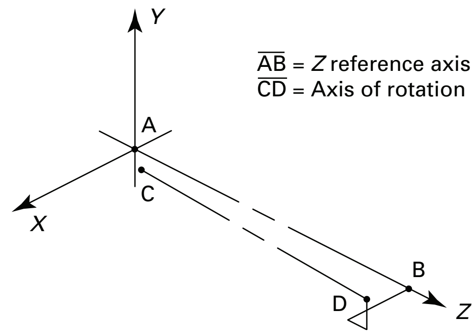
### A-6.4 Radial Motion

*radial motion:* in general, tilt error motion and pure radial error motion occur at the same time, and the sum at any particular axial position is referred to as radial error motion. A knowledge of radial error motion $r_0(\theta)$ at one axial position and tilt error motion $\alpha(\theta)$ allows the radial

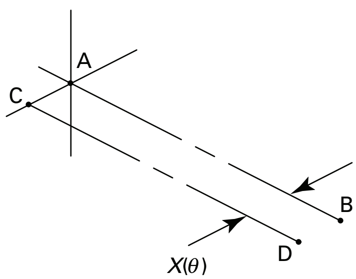
**Fig. A-5 Schematic Diagrams of Six Degrees of Freedom of Axis of Rotation**



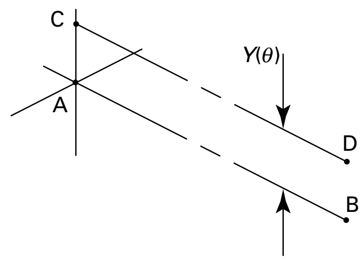

**(a) Lathe Example**



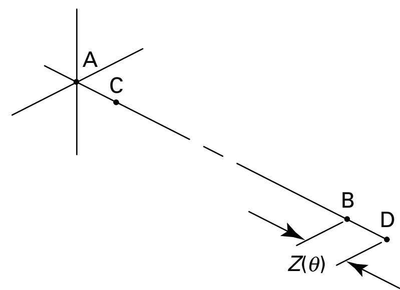

**(b) General Case**



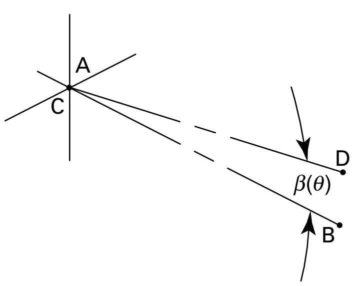

**(c) Radial, Sensitive**



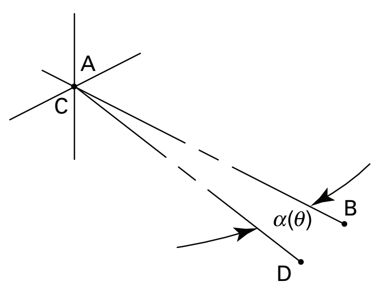

**(d) Radial, Nonsensitive**



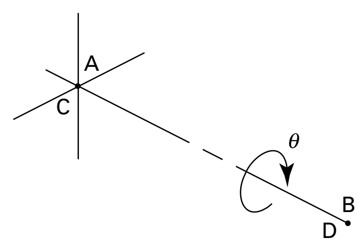

**(e) Axial**




**(f) Tilt, Nonsensitive**




**(g) Tilt, Sensitive**




**(h) Pure Rotation**

error motion $r(\theta)$ at another axial position to be predicted as shown in Fig. A-6, illustration (a),

$$r(\theta) = r_0(\theta) + L\alpha(\theta) \tag{A-3}$$

where $L$ is the distance between the two axial locations. Since radial error motion varies with axial position, it is necessary to specify the axial location of a radial error motion measurement.

### A-6.5 Face Motion (Face Error Motion)

*face motion (face error motion):* denotes error motion in the axial direction at a specified distance $R$ from the $Z$ reference axis, as shown in Fig. A-6, illustration (b). Face motion $f(\theta)$ is related to axial error and tilt error motion by

$$f(\theta) = Z(\theta) - R\alpha(\theta) \tag{A-4}$$

Since face motion varies with radial position, it is necessary to specify the radius of a face motion measurement.

### A-6.6 Error Motion: General Case

*error motion (general case):* the most general case of error motion involves the direction angle $\phi$ of the sensitive direction with respect to the $Z$ reference axis, as shown in Fig. A-7 for the spherical surface. The error motion depends on both the axial and radial locations, which must be specified together with $\phi$. The equation for error motion $e(\theta)$ in terms of axial error, radial error, and tilt error motion is

$$\begin{aligned} e(\theta) &= r(\theta)\sin\phi + f(\theta)\cos\phi \\ &= r_0(\theta)\sin\phi + Z(\theta)\cos\phi + \alpha(\theta) \\ &\quad (L\sin\phi - R\cos\phi) \end{aligned} \tag{A-5}$$

It can be seen from eqs. (A-3), (A-4), and (A-5) that error motion in general or for any of the special cases can be obtained from a knowledge of axial error motion $Z(\theta)$, tilt error motion $\alpha(\theta)$, and radial error motion $r_0(\theta)$ at a known axial error position, plus the location dimensions $L$, $R$, and $\phi$.

Figure A-8 shows schematic diagrams of two test arrangements that can be used to measure the necessary motions. It is assumed that the test pieces used in these arrangements have perfect geometry and are perfectly centered. In both cases, the radial error and axial error motions are measured directly. In Fig. A-8, illustration (a), tilt error motion is derived from face motion by use of eq. (A-4),

$$\alpha(\theta) = \frac{1}{R}\left[f(\theta) - Z(\theta)\right] \tag{A-6}$$

In Fig. A-8, illustration (b), a second radial error motion measurement is used to obtain tilt error motion from eq. (A-3),

$$\alpha(\theta) = \frac{1}{L}\left[r_2(\theta) - r_1(\theta)\right] \tag{A-7}$$

It should be noted that pure radial motion does not appear in any of these error motion equations. However, it is useful as a concept in understanding error motion geometry and is usually not a factor that needs to be measured in determining the behavior of an axis of rotation.

It should also be noted that the illustrations in this section have progressed from line segments in Fig. A-5 to solid workpieces in Figs. A-6, A-7, and A-8. These are equivalent in that the centerline of a perfectly centered workpiece having ideal geometry is coincident with the axis of rotation, and the latter can be viewed as a workpiece of zero diameter. It is easier to visualize the concepts of face error motion or error motion in the general case from solid workpieces rather than the lines of Fig. A-5, and of course in practice all measurements are made from solid surfaces rather than abstract lines. This is why the motion of an ideal solid object is the basis for defining error motion.

## A-7 ERROR MOTION POLAR PLOTS

A very useful form for displaying error motion measurements of an axis of rotation is a polar plot of the error motion versus the angular position of the axis. The following are advantages for this method:

*(a)* prediction of the part roundness and surface finish potential of a machine tool

*(b)* diagnosis of spindle bearing error motion and structural error motion

*(c)* reduction of the required accuracy of centering the master test ball

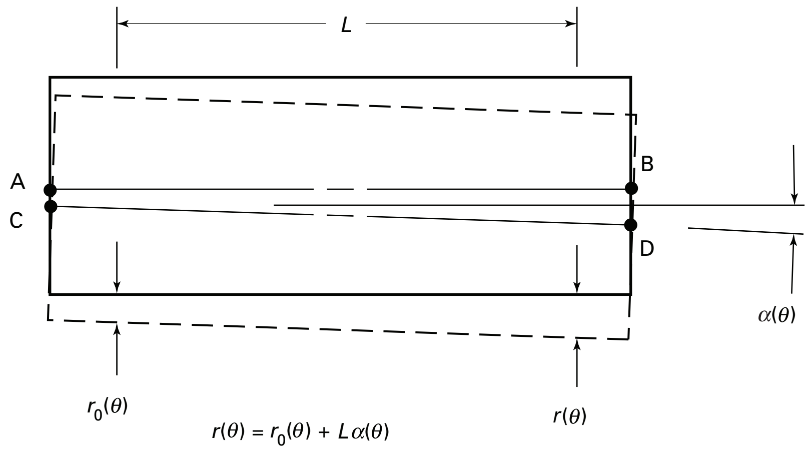
*(d)* assessment of the error motion value

### A-7.1 Specific Example: Radial Error Motion Polar Plot

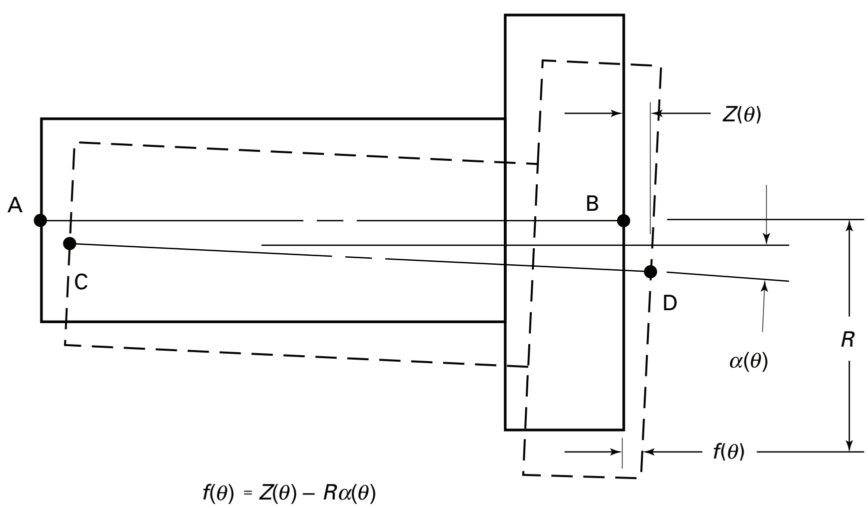
A specific example of an error motion polar plot will be used as a basis for discussion. Using radial error motion for illustration, Fig. A-9, illustration (a) shows a test arrangement involving a master test ball (assumed to be perfectly round) with a displacement indicator arranged to measure in the sensitive direction. Figure A-9, illustration (b) shows an enlarged view of the assumed path of the axis of rotation in the $X$-$Y$ plane relative to the displacement indicator. The assumed path consists of a repetitive figure-eight pattern that has been labeled with the angle of rotation at various points. Figure A-9, illustration (c) shows a rectilinear plot of the radial error motion measured by a displacement indicator versus angle of rotation as a result of the figure-eight pattern, with motion of the ball toward the displacement indicator being positive. Figure A-9, illustration (d) shows the same data as Fig. A-9, illustration (c), but in the form of a polar plot of radial error motion (which may, for example, be obtained by connecting the

**Fig. A-6 Geometry of Radial and Face Motion**

$\overline{AB}$ = $Z$ reference axis
$\overline{CD}$ = Axis of rotation

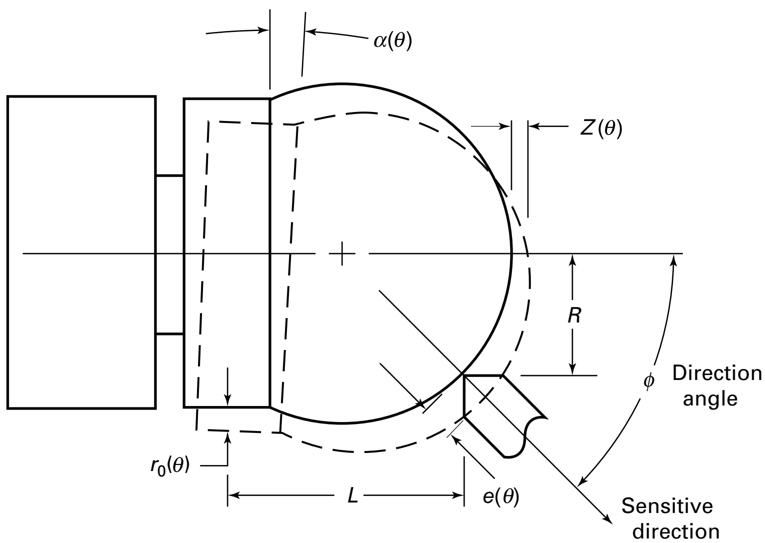



**(a) Radial Motion Variation With Axial Distance**



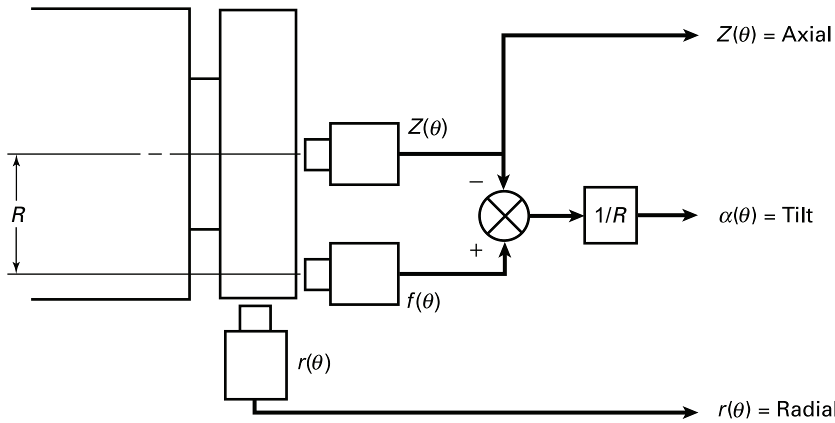

**(b) Face Motion Variation With Radial Distance**

## Fig. A-7 General Case of Error Motion

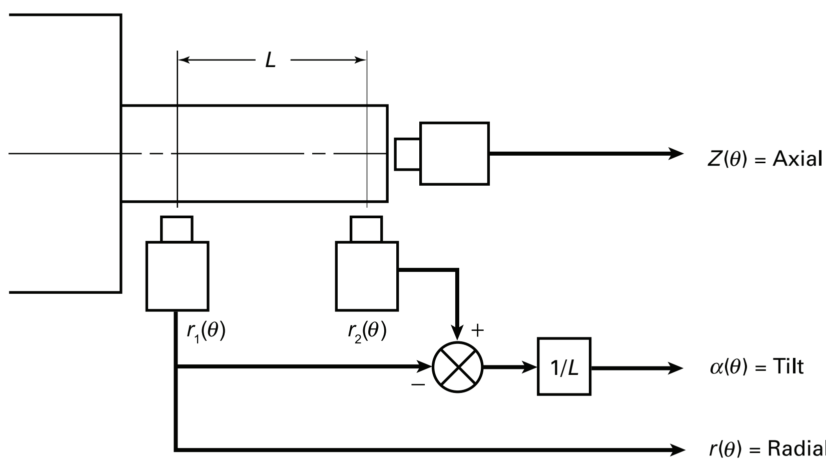



$$e(\theta) = Z(\theta)\cos\phi + r_0(\theta)\sin\phi + \alpha(\theta)\,(L\sin\phi - R\cos\phi)$$

**Fig. A-8 Schematic Test Arrangements for Radial, Axial, and Tilt Error Motion With Fixed Sensitive Direction**

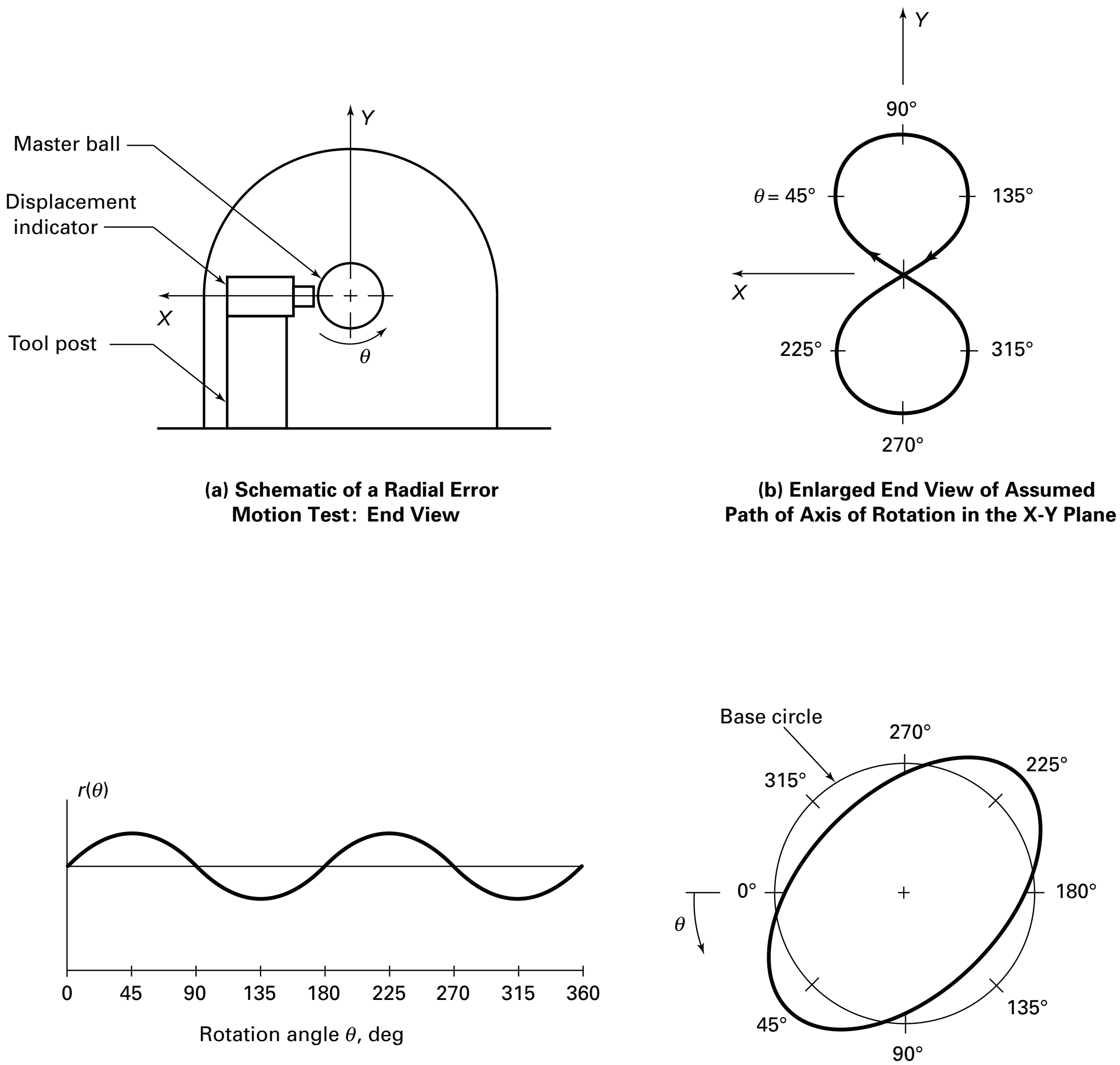



**(a) Basic Motions Derived From Radial, Face, and Axial Motion Measurements**




**(b) Basic Motions Derived From Two Radial and an Axial Motion Measurement**

**Fig. A-9 Hypothetical Example of Radial Error Motion Measurement and Plotting**




**(a) Schematic of a Radial Error Motion Test: End View**

**(b) Enlarged End View of Assumed Path of Axis of Rotation in the X-Y Plane**

**(c) Rectilinear Plot of Radial Error Motion [X-Component in Illustration (b) Versus Angle of Rotation]**

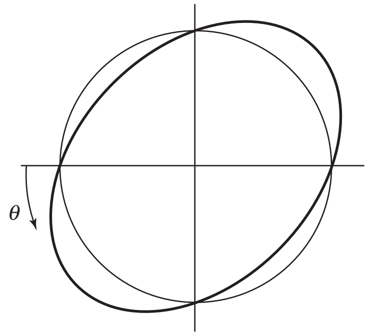
**(d) Radial Error Motion Polar Plot [Polar Plot of Illustration (c)]**

displacement indicator to a polar recorder synchronized to the spindle). Thus the figure-eight pattern results in a skewed elliptical radial error motion polar plot. It is not necessary to have a figure-eight pattern to produce an ellipse since other motions in the nonsensitive direction could occur without changing the radial error motion.

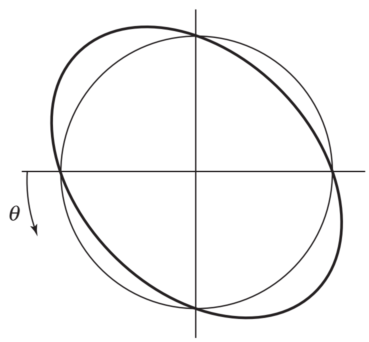
### A-7.2 Synchronous Radial Error Motion and Part Roundness

If the displacement indicator in Fig. A-9, illustration (a) was replaced by an ideal cutting tool, it is clear that the figure-eight motion would result in an out-of-round part. Since the part radius is influenced only by motion of the axis in the sensitive direction, it follows that a motion defined as positive in para. A-7.1 (toward the tool post) will lead to a smaller part radius and vice-versa. If the part is removed and placed in an error-free roundness measuring machine, the roundness chart will be identical to that of Fig. A-9, illustration (d), except for a reversal of sign (peaks and valleys interchanged with respect to the base circle), as shown in Fig. A-10. The out-of-roundness of the two charts is identical, and hence the radial error motion polar plot of a machine

**Fig. A-10 Relationship of Radial Error Motion to Part Roundness Using Example of Fig. A-9**




**(a) Radial Error Motion Polar Plot**




**(b) Part Roundness Profile Plot**

tool axis of rotation predicts the best workpiece roundness the machine is capable of producing under ideal cutting conditions. Other factors such as nonideal cutting (built-up edge, tool wear, variable tool deflection), feed marks, chucking distortion, thermal distortion, and residual stress relief can result in this capability not being realized.

If the above part was left in place after cutting and the cutting tool replaced by a displacement indicator, then under the present assumptions of ideal cutting and no asynchronous radial error motion, the radial runout of the part surface would be zero. The radial error motion and the part roundness errors cancel due to their equal magnitudes and opposite signs. This is one example of the difference between radial error motion and radial runout measurements; see sections A-13 and A-14 for a more extensive discussion.

The above example is idealized in that the motion of the axis of rotation was assumed to be exactly repetitive from revolution to revolution. Figure A-11, illustration (a) shows a more typical case of error motion that also includes an asynchronous component. Figure A-11, illustration (a) is known as a total error motion polar plot. Figure A-11, illustration (b) shows the average position over the number of revolutions recorded, known as the synchronous error motion polar plot. Figure A-11, illustration (c) shows an asynchronous error motion polar plot, which consists of the difference between the total and the synchronous error motion polar plots. It can be argued that the synchronous error motion polar plot is indicative of form error (such as roundness for radial error motion). This is true only to the extent that the shape of the total error motion polar plot for any single revolution is similar to the shape of the synchronous error motion polar plot.

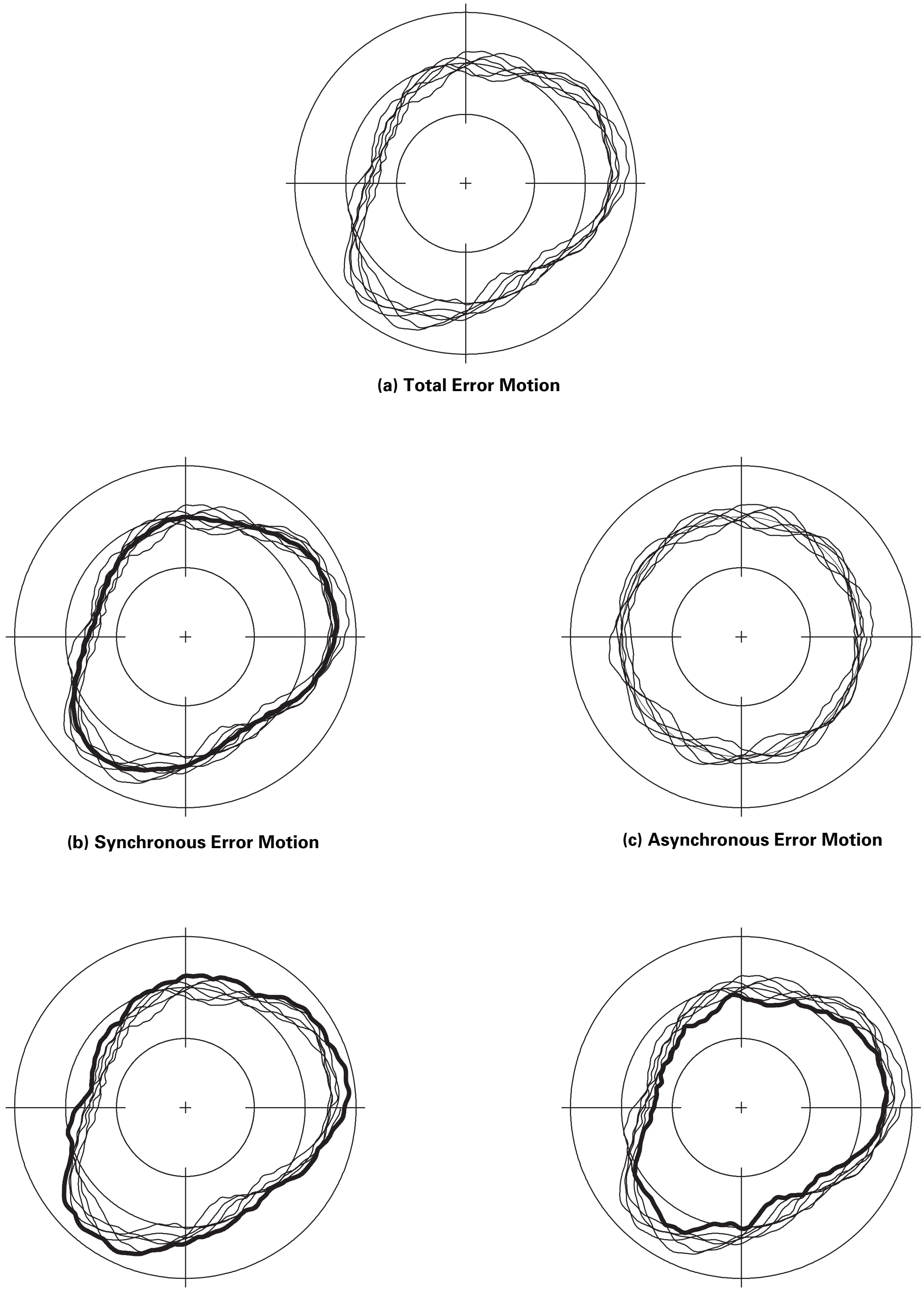
### A-7.3 Asynchronous Error Motion and Surface Roughness

It can be shown that the asynchronous error motion polar plot can be used to predict the surface roughness obtained under ideal cutting conditions. Recalling that surface roughness is ordinarily measured across the lay (i.e., parallel to the axis for a cylinder or radially on a flat face), it follows that the measurement corresponds to crossing a number of successive revolutions at one particular angle on the total error motion polar plot. If the asynchronous error motion were zero, the only irregularity present would be the scallop-marks associated with the tool radius as shown in Fig. A-12, illustration (a), which is referred to as the theoretical finish. The peak-to-valley height, $H$, of the theoretical finish associated with a tool radius, $R$, and the feed per revolution $F$ is

$$H = \frac{F^2}{8R} \text{ (if } F \text{ is small compared to } R\text{)} \qquad \text{(A-8)}$$

The value of $H$ can easily be made quite small, e.g., if $F = 0.001$ in. (0.025 mm)/rev and $R = \frac{1}{8}$ in. (3.1 mm), then $H$ is 1 μin. (0.025 μm). However, if asynchronous motion is present, then the surface is cut to varying levels on successive revolutions as in Fig. A-12, illustration (b). It is evident that a given asynchronous motion level is translated into an equal peak-to-valley surface roughness if the roughness cutoff width [usually 0.03 in. (0.8 mm)] is several times larger than the feed per revolution. The sum of the asynchronous error motion level and $H$ from eq. (A-8) represents the potential peak-to-valley surface roughness for the machine under ideal cutting conditions, with the arithmetic average (AA) value being approximately one-fourth as large. This potential can be realized for sharp diamond tools cutting certain nonferrous metals, but under most cutting conditions, the presence of a built-up edge on the tool leads to a larger surface roughness. In some situations the tool has repeated contact with the same point on the work over a large number of revolutions, as, for example, in turning with a flat-nosed tool, cylindrical grinding with a flat-faced wheel, or dwelling at zero feed rate with any tool. In such a case, it can be argued that material will be removed to the level of the maximum excursions of the work toward the tool, and hence (using the sign

**Fig. A-11 Total, Synchronous, Asynchronous, Outer, and Inner Error Motion Polar Plots**



(a) Total Error Motion

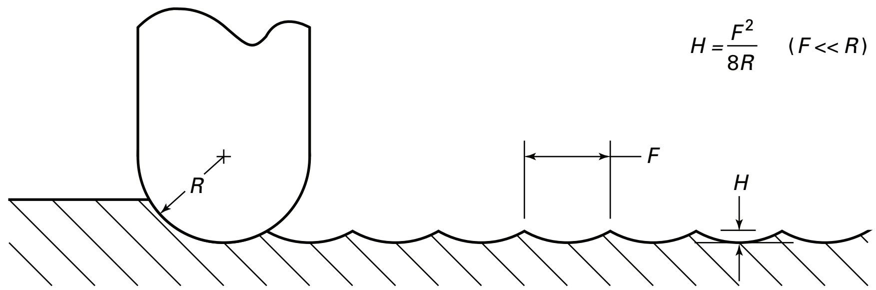
(b) Synchronous Error Motion

(c) Asynchronous Error Motion

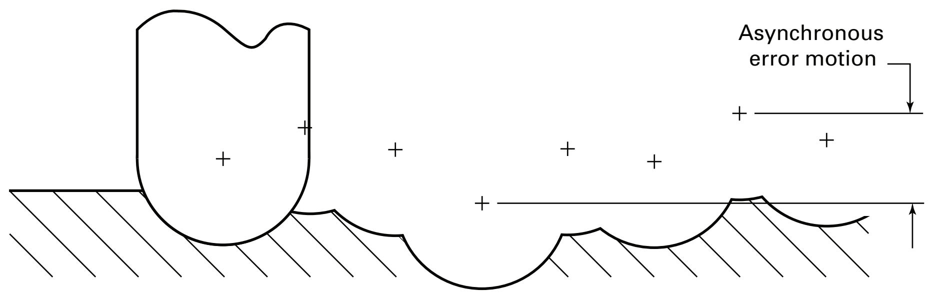
(d) Outer Error Motion

(e) Inner Error Motion

**Fig. A-12 Relationship of Surface Roughness to Asynchronous Error Motion**




**(a) Theoretical Finish for Ideal Cutting With Zero Asynchronous Error Motion**




**(b) Effect of Asynchronous Error Motion on Peak-to-Valley Roughness With Ideal Cutting**

convention of Fig. A-9) the potential part roundness can be predicted from the outer error motion polar plot, consisting of the contour of the total error motion polar plot as shown in Fig. A-11, illustration (d). For operations inside a cylindrical bore, the inner error motion polar plot has a similar significance [see Fig. A-11, illustration (e)]. The reliability of such a prediction is limited by the similarity of a succession of such plots as well as by nonideal cutting conditions. For an axis of rotation of a measuring machine, the asynchronous error motion represents the basic level to which the machine can repeat its readings on a given workpiece from revolution to revolution. The synchronous error motion represents the basic systematic error of the axis, which will still be present if there is no asynchronous error motion (perfect repeatability from revolution to revolution) or if the asynchronous error motion is averaged out over a number of revolutions.

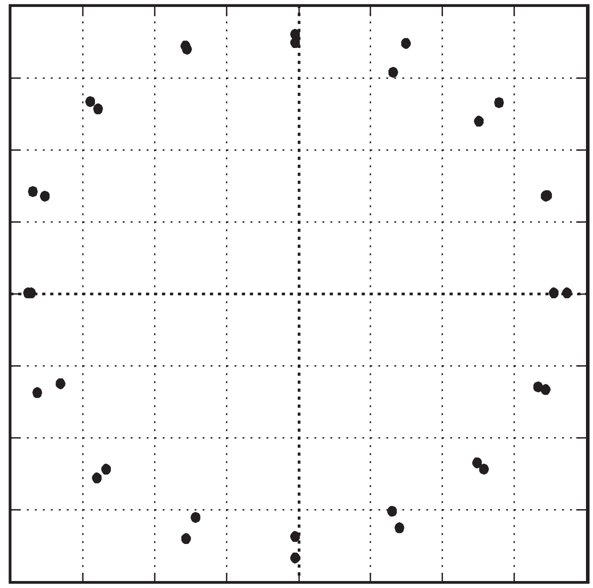
## A-7.4 Spindle and Structural Error Motions

In addition to being useful in predicting the performance of a machine, the polar plot can help in diagnosing the physical causes of the observed error motion. In this context, it is helpful to view the total error motion as an asynchronous error motion superimposed on a completely repetitive synchronous error motion profile. It can be shown mathematically that a repetitive profile can involve only those frequencies that are equal to or are whole number multiples of the axis rotational frequency. Thus the axis bearings and the axis drive system are the most likely sources of synchronous error motion. Fluid film bearings (hydrodynamic, hydrostatic, aerostatic) in particular show highly repetitive patterns.

Asynchronous error motion has, in the past, often been referred to as random error motion. This latter term is now nonpreferred since the physical causes of asynchronous error motion are frequently not random in the statistical sense. For example, asynchronous error motion is often due to deterministic sources such as motors or pumps operating at frequencies that are not integer multiples of the axis rotational frequency. These frequencies are best determined by spectral analysis.

The above discussion suggests that synchronous error motion can be equated with spindle bearing error motion and similarly for asynchronous and structural error motions. This is an oversimplification. Asynchronous error motion can originate in a rolling-element bearing due to imperfect balls and rollers. Ball and roller bearings sometimes exhibit a pattern that is repetitive only every other revolution and this behavior is associated with the rolling elements traveling at approximately one-half of the shaft speed. Plain bearings can have a similar behavior due to a hydrodynamic effect called half-speed whirl. In these cases the deviation between successive revolutions represents asynchronous error motion that is caused by the bearings. Synchronous error motion can also be caused by sources other than the spindle bearings, such as a drive component operating at a whole number multiple of the axis rotational frequency or by a piece of equipment unrelated to the axis but having a chance synchronization with the axis rotational frequency. A useful technique for locating the sources of error motion is to note changes as potential sources are turned on and off or varied in speed. An alternative approach is to vary the axis speed of rotation. At zero axis speed, the remaining cloud band thickness represents the structural motion that is asynchronous error motion due to sources other than the axis bearings and drive system. The synchronous error motion polar plot at zero axis speed can also be obtained from a static error motion polar plot, performed by placing the nonrotating axis in a succession of discrete angular positions. It should be noted that aliasing limits the resolvable cycles per revolution to one-half the number of sampled points per revolution.

**Fig. A-13 Representative Data From Static Error Motion Measurement Using Oscilloscope Camera**



### A-7.5 Static Error Motion Measurement

The purpose of this test is to isolate errors caused by the spindle from errors caused by structural motion. Structural error motion of the spindle support structure with respect to the tool can be as high as 95% of the error motion as measured at operational speed. A static error motion measurement is strongly recommended before a decision is made to change spindle bearings since experience has shown that it is a common mistake to assume that imperfect bearings are the root cause of the error motion.

For a milling machine, the test setup is identical to one used to measure rotating sensitive direction. Two displacement indicators, mounted 90 deg to each other, are set up in a nest supported from the table to read against a test ball or pin attached to the spindle rotor. It is important to note that the spindle error motion tests show the combined, functional effect of bearing error motion and structural error motion. After the spindle error motion tests are completed, the static error motion measurement procedure should be performed to determine the contribution of the bearings. The detailed procedure is as follows:

*Step 1:* Put the spindle drive into neutral. Belt tension should be removed so that the spindle is free of all external forces.

*Step 2:* Rotate the spindle, by hand, a minimum of two revolutions, stopping at a minimum of eight points per revolution.

*Step 3:* Release all hand forces and record the average indicator reading at each point.

*Step 4:* Averaging the readings eliminates the effect of structural error motion with the spindle stopped.

Play or loss of preload in the bearings can be identified by applying opposing forces to the spindle.

The data are analyzed for synchronous and asynchronous radial errors using methods previously described. If an oscilloscope is used, the recording can be done by photographing the screen at each rotational position. Figure A-13 is an example of data from this test. If a data acquisition system is used, the recording can be done by manual activation of the data recording system.

The static error motion measurement concept assumes that bearings that have acceptable performance at zero speed will continue to have acceptable performance at operational speeds. Experience with rolling-element bearing analyzers has shown that this assumption is generally true. The small differences that are observed when changing speed can be assigned to the effects of centrifugal forces that cause small changes in the line of contact of the balls or rollers with the races. The procedure works for hydrostatic and aerostatic bearings as well as rolling element bearings. However, it does not work for hydrodynamic bearings, which depend on rotational velocity for their load carrying and centering ability.

If there is doubt about the validity of the static error motion measurement, a structural error motion measurement, with the spindle running, can be performed as a cross-check. This measurement requires a special setup and careful attention to the design of the bracket that holds the indicator.

A significant increase in error motion with an increase in speed indicates a problem with the drive system. Worn belts and couplings, unbalance, and misalignment of the drive motors, pulleys, and gears, are typical drive system problems.

This Standard does not specify which sources of error motion are to be included in the assessment of an axis of rotation. For example, in the case of a roundness measuring machine subjected to a high level of building vibration, the machine's performance may be adversely affected by its environment. However, it is conceivable that the machine was purchased with special design features intended to deal with such an environment, in which case the error motion tests would intentionally include building vibration as a source.

### A-7.6 Structural Error Motion

Error motion measurements involve several structural elements, some of which may have displacements greater than the error motion of the axis of rotation.

Structural error motion is displacement of a nonspindle component of the structural loop, resulting in unintended motion between the tool and the work, in the sensitive direction. Structural error motion is categorized separately from spindle error motion; it may or may not affect the error motion of the axis of rotation. It does affect the relative position of the work and the tool. Structural error motion can cause the axis of rotation to move relative to the tool, or the tool to move relative to the axis of rotation, or the work to move relative to the axis of rotation.

Thermal drift is a special case of structural motion and is categorized separately as an axis shift caused by a change in operating conditions.

Diagnostic procedures include various environmental tests with the spindle stopped, static error motion tests, and push-pull tests for lost motion. Another test separates error sources by changing spindle speed and noting which structural elements are affected.

Spindle testing involves more than just the spindle. By understanding the role of the structural loop, the user will be in a better position to find the root cause of various problems.

### A-7.7 Variation in Sag Causes One-for-One Roundness Error (VISCOORE)

Variation in the gravitational sag of cantilevered workpieces causes a one-for-one roundness error when grinding or turning with stationary tools on horizontal-spindle machine tools. An acronym for the effect is VISCOORE, which stands for Variation In Sag Causes a One-for-One Roundness Error. The roundness error profile is elliptical. The effect is superimposed on the radial error motion of the spindle. Constant sag causes a second order variation in size as the tool moves above or below center as shown in Fig. A-2. There is no effect on roundness or eccentricity.

**Fig. A-14 Ellipse Generated by Nonuniform Sag of Rotating Workpiece**

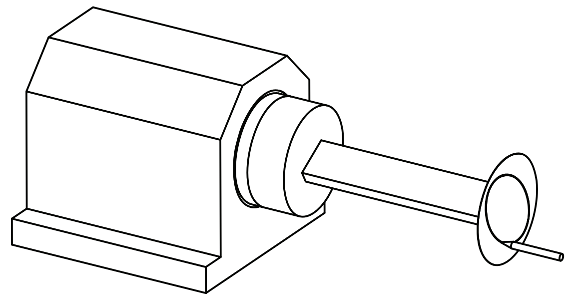
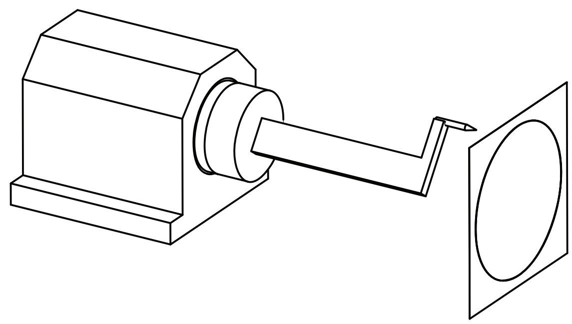


**Fig. A-15 Circle (Not Ellipse) Generated by Rotating Toolholder Having Large (But Irrelevant) Variation in Cross-Section**



GENERAL NOTE: The center of the circle is offset from the axis of rotation by an amount that depends upon the stiffness (in the sensitive direction) of the toolholder.

The problem can be visualized by imagining a rotating I-beam, as shown in Figs. A-14 and A-15. Because of its nonuniform stiffness, the sag is different when the web of the I-beam is horizontal or vertical. When the web is at 45 deg with respect to gravity, the centerline of the beam moves horizontally as well as downward. This horizontal component of gravitational sag in anisoelastic beams is nonintuitive. It causes out-of-roundness for fixed-sensitive-direction applications and eccentricity for rotating-sensitive-direction applications.

The horizontal and vertical radial movements of the workpiece with respect to the average position of the end of the beam are equal to half the total sag variation. The trajectory of the center of the beam with respect to its average is a circle. Therefore there is no roundness error with rotating tools, since the displacement is constant at all rotational angles.

If the cutting point of a rotating tool is in line with the soft direction, the workpiece will be round but eccentric with respect to the axis of rotation average line. If the cutting point is located in-line with the stiff direction, eccentricity will be minimized.

The sources of sag variation may be asymmetrical cross-sections in the work or variations in the stiffness of the fixtures that attach the work to the spindle. In the case of long, cantilevered workpieces, variations in the torquing of fixture attachment screws can cause roundness errors that are larger than those created by the radial error motion of the spindle.

Machine tools having a vertical axis of rotation are not subject to the VISCOORE effect, which is restricted to the consequences of gravitational sag. Sag is a type of structural error motion; the axis of rotation is unaffected.

An awareness of the VISCOORE problem can lead to some solutions, but they are not trivial. The simplest test for the existence of this problem is to take a light cut on the workpiece and then indicate the work from an effective point 90 deg away from the tool. Note that this effect is present to some degree in all horizontal applications. Usually the variation is insignificant, but for precision work, it requires consideration. Also note that regardless of the shape of the cross-section, the variation, if any, is always twice per revolution, since there is always one angle having maximum stiffness and the minimum-stiffness orientation is always at right angles (the Principal Axis Principle).

Reference [18] describes the errors that result from fixed and rotating radial forces in detail.

### A-7.8 Centering Error

Returning to the radial error motion example shown in Fig. A-9, it is assumed that the master test ball is perfectly centered on the axis of rotation (as well as being perfectly round). An off-center test ball adds an unwanted component to the radial motion, which can be examined most simply by assuming a perfect axis of rotation. As shown in Fig. A-16, the consequence of centering error is a once-per-revolution sinusoidal wave in rectilinear coordinates. In polar coordinates, Fig. A-16, illustration (c), the sinusoid is very nearly a circle having its center offset from the center of the polar chart. When radial error motion and centering error are both present, it is difficult to determine the sinusoidal component of the total waveform in rectilinear coordinates, as is evident in Fig. A-16, illustration (d). In contrast, the sinusoid can be eliminated from the polar plot of Fig. A-16, illustration (e) by choosing an offset best fit center. Since it is time consuming and often virtually impossible to reduce centering error to a negligible level in the mechanical mounting of a test ball, the ability to eliminate its effect represents another advantage for polar plots.

It should be noted that the amount of centering error must be limited when using polar charts. Strictly speaking, the polar plot of a pure sinusoid is a limaçon, which ranges from a circular shape to a heart shape as the centering error increases, as shown in Fig. A-17, illustration (b). This causes a fictitious synchronous radial error motion to appear on the polar chart of a perfect axis of rotation due to an off-center test ball. Figure A-18 shows the distortion that occurs for a given eccentricity of the best fit center and a given minimum radius of the polar plot. A conservative rule is to limit the offset of the best fit center on the recording chart to 0.1 in. (2.5 mm) plus 5% of the minimum polar plot radius, which limits the limaçon error to 0.01 in. (0.25 mm) (approximately the limit of graphical reading accuracy). Radial error motions and centering error may be readily separated using frequency analysis or least squares fit techniques. The centering error is the fundamental component of the radial motion data set.

### A-7.9 Fundamental and Residual Synchronous Error Motions

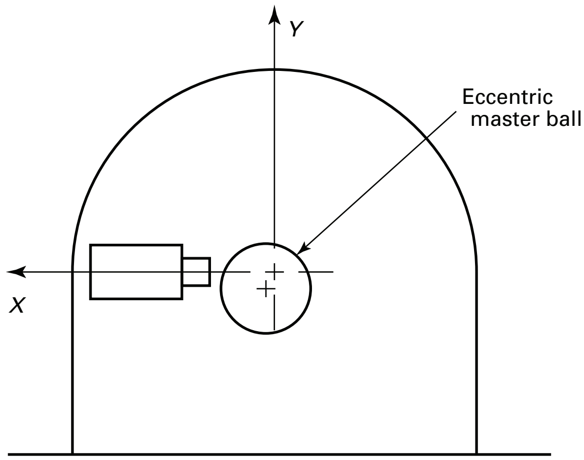
The term used to refer to the once-per-revolution sinusoidal component of an error motion polar plot is fundamental error motion. Since a test ball is perfectly centered when this component vanishes, it follows that fundamental radial error motion of an axis of rotation does not exist.

Similarly, fundamental tilt error motion does not exist. This can be understood by visualizing a perfect cylinder mounted on an axis of rotation. If the mounting is adjusted so that the cylinder has no centering error at either end, then there can be no once-per-revolution tilt error motion. Since familiar terms such as coning, wobble, and swash suggest a once-per-revolution component, their use is inappropriate to describe tilt error motion.

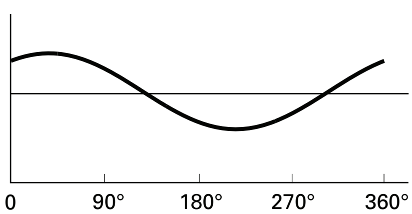
In contrast, fundamental axial error motion does exist and is not caused by a master ball mounting error as with centering error. It consists of a once-per-revolution axial sliding motion of the axis of rotation along the $Z$ reference axis and can arise, for example, from out-of-square thrust-bearing components. An excellent illustration of results of this motion, shown in exaggerated detail on a model, is given in Fig. A-17 of reference given in reference [5].

Regarding face motion, reference to eq. (A-4) shows that fundamental face error motion does exist and is equal to fundamental axial error motion. This can be understood by visualizing a perfectly flat disk mounted on a perfect axis of rotation. Mounting error can result in a once-per-revolution sinusoidal face motion (increasing in direct proportion to radius), but this will vanish if the disk is perfectly square to the axis of rotation. Assuming perfect squareness and then changing from a perfect axis to an axis having fundamental axial error motion, it follows that the same fundamental error motion will occur at all radii. Thus a perfectly flat disk is square to an imperfect axis of rotation if the fundamental face error motion is the same at all radii. It is possible to cancel the fundamental face error motion by mounting the disk out-of-square to the axis of rotation, but this cancelation can only occur at one radius. The out-of-squareness angle necessary for this cancelation becomes

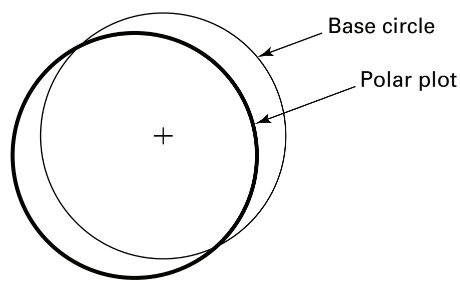
### Fig. A-16 Effect of Centering Error on Radial Error Motion Measurement



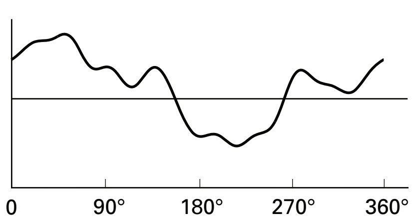

**(a) Radial Error Motion Test With Centering Error**



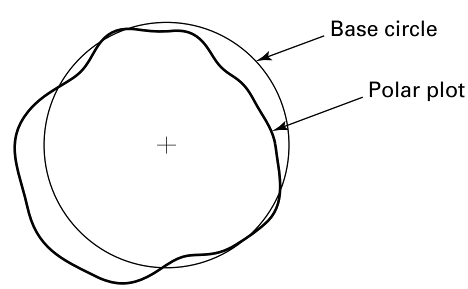

**(b) Effect of Centering Error for Perfect Axis: Rectilinear Plot**




**(c) Polar Plot of Illustration (b)**

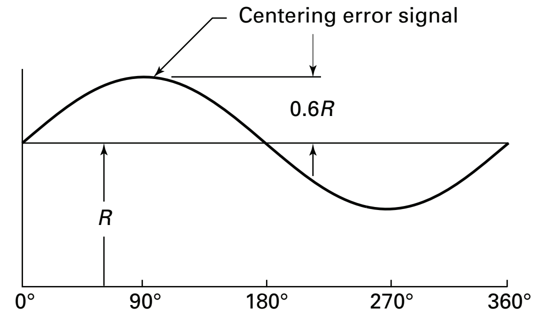



**(d) Radial Error Motion With Superimposed Centering Error**

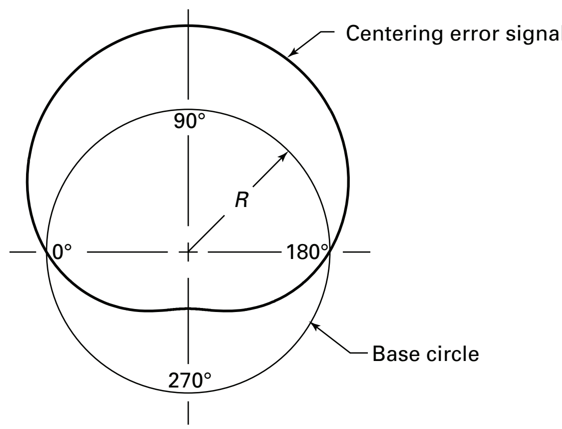



**(e) Polar Plot of Illustration (d)**

**Fig. A-17 Example of Polar Distortion for Centering Error Signal Amplitude of 60% of Base Circle Radius**

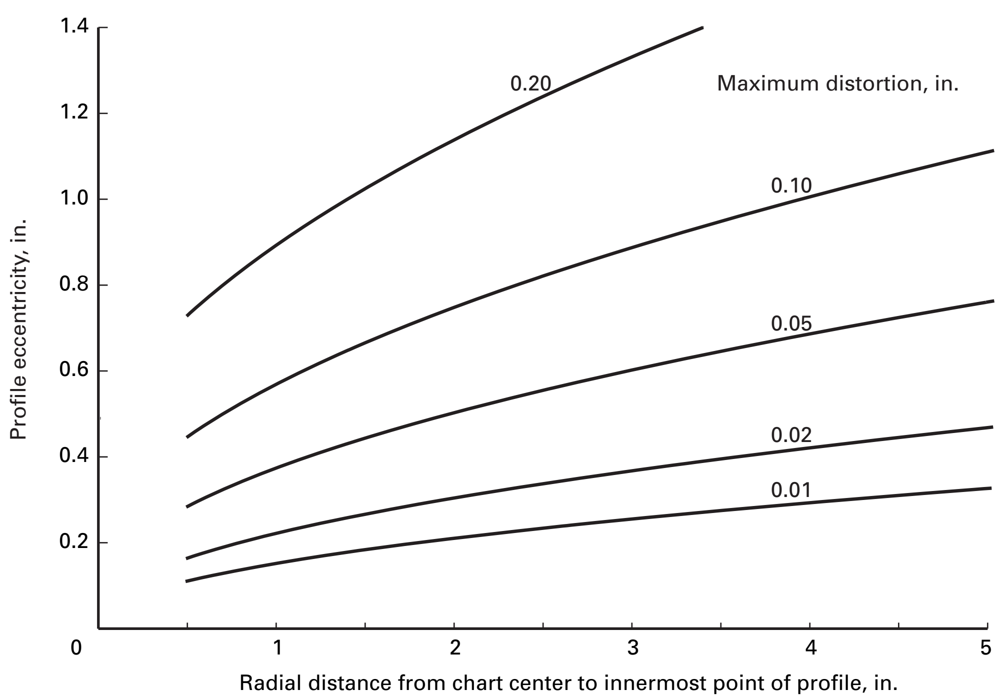



**(a) Rectilinear Plot of Centering Error Signal**




**(b) Polar Plot of Illustration (a), Showing Polar Distortion**

**Fig. A-18 Polar Distortion Due to Centering Error**



larger as the radius becomes smaller, reaching an impossible situation at zero radius.

The existence of fundamental face error motion has an interesting consequence in machining and measuring flat faces. If a flat disk is faced on an axis that is perfect except for the presence of fundamental axial error motion, then the part can be viewed as made up of many flat-faced thin rings, each of which is out-of-square with the axis of rotation by an amount that increases with decreasing radius. Such a part is not flat over its full area. However, if the part is mounted in a roundness-measuring machine with the displacement indicator sensing axially, then the part can be tilted so that no flatness error is sensed during a trace around a circular path concentric with the part center. Such a part is said to have circular flatness. Since it does not have area flatness, it follows that circular flatness measurements can be misleading if this effect is not properly understood.

Residual synchronous error motion is the general term applied to the difference between synchronous and fundamental error motion. The consequences of residual synchronous error motion are analogous to those of synchronous radial error motion. For example, residual synchronous face error motion during machining leads to errors in circular flatness in the same way that synchronous radial error motion leads to errors in roundness.

The fundamental error motion is proportional to cos $\phi$ times the fundamental axial error motion (see eq. A-5), where $\phi$ is the direction angle. Thus a 45 deg taper involves 70.7% as much fundamental error motion as a flat face.

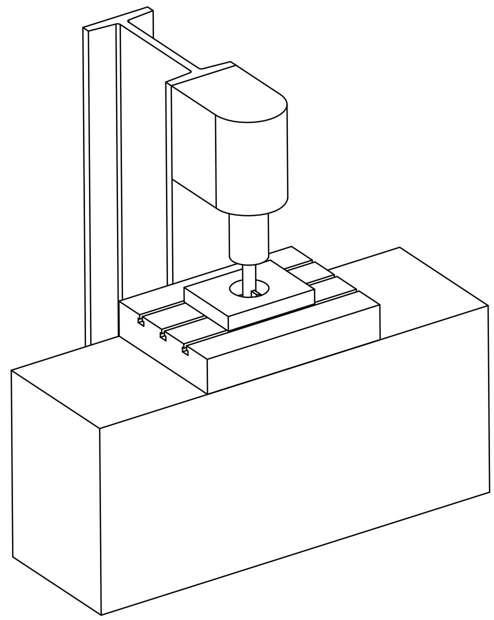
## A-8 EFFECT OF UNBALANCE

### A-8.1 Effect of Unbalance on Fixed Sensitive Directions

Unbalance of the rotating elements introduces a once-per-revolution sinusoidal force with maximum amplitude varying as the square of the spindle speed. When machining in a fixed sensitive direction, the consequence of this for an otherwise perfect axis is that although a perfectly round part can be machined at a given speed, it will exhibit a centering error at other speeds. If two cylindrical sections are machined on the same part at different speeds, their geometric centerlines will not be coaxial. If the shift of the axis average line involves a tilt as well as a radial component, then the centerlines of the above two cylinders will not be parallel. Shifts in tilt also change the parallelism or squareness of the axis of rotation to the machine slideways, causing cylinders to be machined with a taper and flat faces to be machined conical.

In this discussion, unbalance is assumed to cause a circular orbit of an initially centered test ball. If the structural loop has nonlinear and/or nonuniform compliance (see Fig. A-19), unbalance may excite higher harmonic motions that lead to roundness and flatness errors. Balancing of rotating elements can be as important for this reason as any other.

**Fig. A-19 Boring Machine With Nonuniform Compliance**



A simple method to measure the magnitude and phase of unbalance forces is to take a trial cut at high speed and measure the once-per-revolution runout at slow speed using a displacement indicator. Trial weights are then installed (or stock removed) and the test repeated to determine the calibration of the weight/runout relationship. An important advantage of this method of balancing is that it can be performed on a facing cut to measure the effect of angular unbalance. It can also be performed on any number of axial planes on a long shaft supported by a tailstock. Two-plane balance is achieved when radial and face runouts do not vary with speed.

An alternative to taking trial cuts is to measure the fundamental portion of the radial and face runout of an existing surface on the spindle (or workpiece) at two different rotational speeds. The change in position of the center in the fundamental is a measure of the unbalance amplitude and location (phase) of the unbalance center. A high frequency response displacement transducer is required. It may be a capacitive, optical, or eddy current transducer. The surface effects noise of eddy current transducers is of no consequence since it is filtered out in the process of separating the fundamental from the residual synchronous motion. Trial weights must be used in the usual way to determine the weights and locations of the final corrections for both static and dynamic unbalance.

In the case of surface finish it can be demonstrated that, in single-point turning, there is no relationship between surface finish and unbalance. This may be difficult to believe since the necessity of a smooth, quiet, vibration-free machine for achieving mirror finishes seems obvious. It is, in fact, absolutely essential for a cylindrical grinding machine. To understand why it is not necessary for a lathe requires insight into the difference between synchronous and asynchronous vibration. Unbalance introduces synchronous motion which, in single-point turning, does not affect finish since the relative position of the tool with respect to the axis of rotation at each complete revolution is the same. An otherwise perfect lathe with a large amount of unbalance will achieve theoretical finish.

Asynchronous motion, on the other hand, is motion of the tool with respect to the spindle at frequencies other than whole number multiples of the spindle frequency. It affects the position of the tool with respect to the axis of rotation at each complete revolution and therefore affects surface finish. The surface finish achieved by cylindrical grinders is influenced by unbalance because the wheel spindle rotates at a different speed than the work spindle and synchronous motion of the wheel spindle automatically becomes asynchronous motion with respect to the work spindle.

### A-8.2 Effect of Unbalance on Rotating Sensitive Direction Applications

Paragraph A-8.1 discusses the effect of unbalance for fixed sensitive direction applications. The effect of unbalance on rotating sensitive direction applications is different. Every structure has a stiff direction, and at 90 deg, it has a soft direction. This characteristic may cause an elliptical out-of-roundness of a bored workpiece. The nonroundness is related to

*(a)* orientation angle of the tool

*(b)* the orientation angle of the unbalance

*(c)* the difference in compliance in the two orthogonal directions

The orientation of the ellipse axes depends on the location of the tool with respect to the center of the unbalance. If the tool is on the same side as the center of mass, the ellipse will have its long axis in line with the soft direction, as shown in Case 1 in Fig. A-20.

Unbalance has no effect on the asynchronous error motion of a rotating sensitive direction. However, the surface finish may be affected by unbalance in milling with a multiple tooth cutter. In the case of a two flute cutter with no static runout it is possible that unbalance could cause tooth runout to exceed the depth of cut, in which case only one tooth would be cutting. The effective feed per revolution of the spindle would therefore be doubled. The theoretical finish would then be four times larger.

## A-9 COMPLIANCE TESTING

Performance under load is influenced by the compliance of the various structural elements affected by tool force. This section discusses general principles and gives an example of a procedure for evaluating specific components of the structural loop.

Machine compliance is the property of a structure to deflect when a force is applied. Compliance is meaured by applying a static load and measuring the corresponding displacement. The units of linear compliance are length/force (micrometers/newton or microinches/pound force) and the units of angular compliance are angle/torque (radians/newton-meter or microradians/pound-inch). Total compliance is the result of the contributions of the deflection of each component of the structural loop and may vary with slide positions.

Compliance is measured using a displacement indicator and a force gage. To avoid the cost and complexity of using an electronic load cell, a useful alternative is a handheld tension gage or scale.

A rapid assessment can be made by hand, but for improved accuracy it is desirable to construct a simple fixture for applying the force. The use of a second scale enables assessment of the important characteristic of response to a low initial force. In Fig. A-21, the right-hand scale establishes the preload and the left one applies incremental force through the region of zero net force.

By moving the applied force and the displacement indicator as shown in Fig. A-22, the compliance of each joint in the structural loop can be investigated. With this approach to compliance measurement, detection of the source of unwanted compliance is simplified (loose joints show up as a discontinuity in the compliance curve) so that improvement efforts can be directed where they will have the greatest impact. Refer to Figs. A-23 and A-24 for additional arrangements for measuring compliance.

## A-10 AXIS SHIFT SPECIFICATION AND TESTING

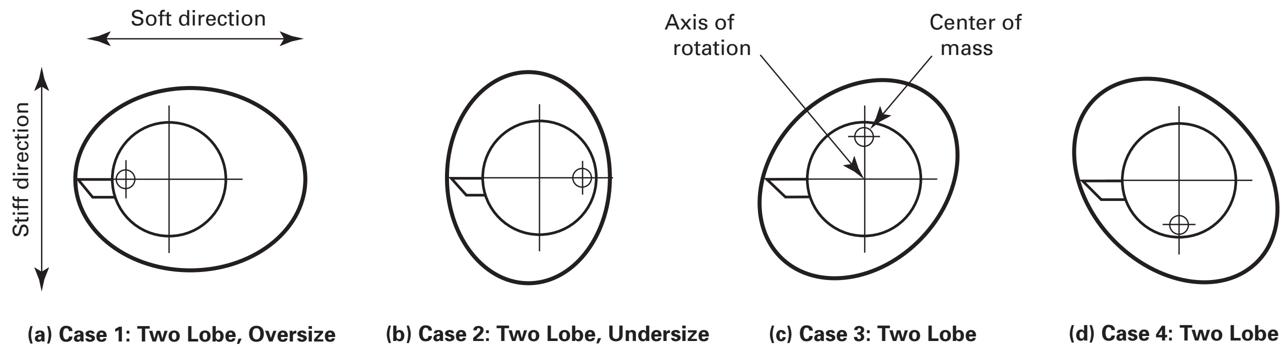
Axis shifts are displacements associated with changes in temperature, load, speed, direction of rotation, or other operating conditions.

Axis shift tests are similar to error motion tests except for the inclusion of changes in operating conditions. For instance, a thermal drift test measures the gradual shift of the axis average line as a spindle warms and cools.

Load-induced axis shifts occur when tool force changes, weight is added or removed, drive force changes, and as unbalance effects change with speed. If the force variation is known, displacement magnitudes can be predicted from compliance specifications.

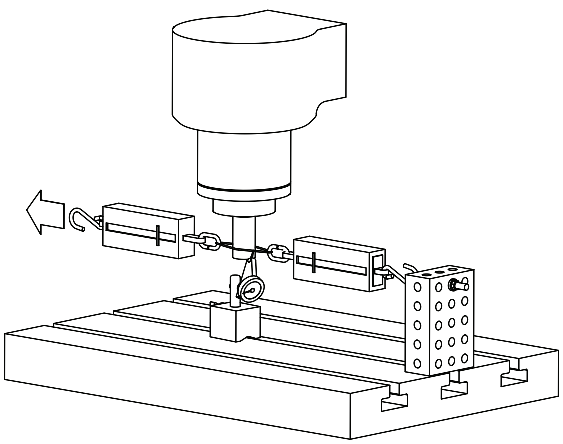
Speed-induced axis shifts typically have a prompt response due to dynamic effects, followed by a delayed shift as the operating temperature gradually readapts to the new equilibrium condition. Drive influences and unbalance are typical root causes of speed-related axis shift. Other causes include hydrodynamic yaw, which describes an axis shift due to speed-induced change in

## Fig. A-20 Effect of Unbalance on Hole Boring

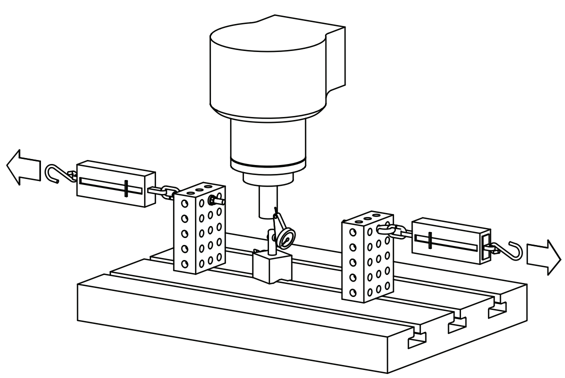



GENERAL NOTE: The difference in major and minor diameter is the same for all cases.

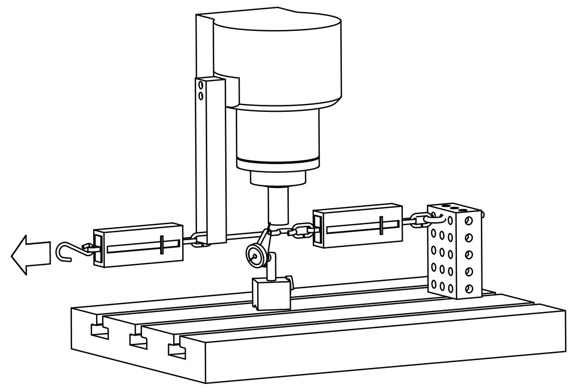
## Fig. A-21 Arrangement for Measuring Compliance From Tool to Frame of Machine



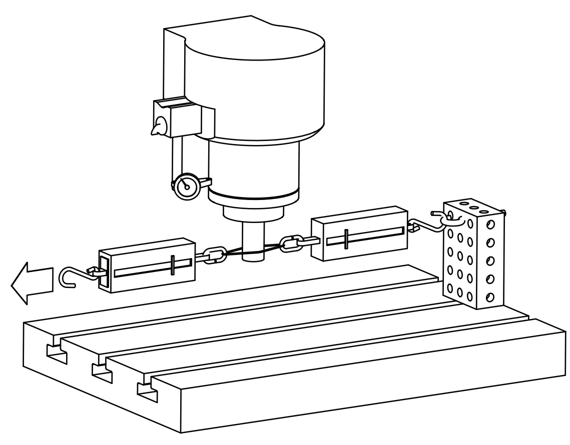
## Fig. A-22 Arrangement for Measuring Compliance of Table-Support Structure



## Fig. A-23 Arrangement for Measuring Compliance of Spindle-Support Structure



## Fig. A-24 Arrangement for Measuring Compliance of Sliding Joint Between Head and Quill



fluid-pressure distribution within a self-pressurized bearing.

Spindles with externally pressurized bearings are subject to axis shifts as the inlet pressure rises and falls. Some spindles experience an axis shift when the direction of rotation reverses, due to drive influence or bearing characteristics.

In practice, it can be expected that several operating conditions will vary during the time period of interest. By characterizing their effects individually, it is possible to identify root causes and to separate axis shifts from the error motion that would be observed under constant conditions.

The term axis shift enables an unambiguous specification to be written that includes responses to changes in operating conditions.

(a) Thermal Drift Test = ________ μin./°F/time period of interest

(b) Compliance Test= ________ μin./lb

(c) Speed Variation Test= ________ μin./rpm

(d) Inlet Pressure Variation Test = ________ μin./psi

(e) Reversal-of-Rotation Test= ________ μin.

(f) Temporal Test (All-inclusive) = ________ μin./time period of interest

Note that error motion specifications assume constant conditions — axis shifts are not included unless so stated. However, this does not preclude grouping them together under the heading of error motion, as long as the test condition variations are described.

## A-11 TEST ARTIFACT ERRORS

Thus far it has been assumed that a geometrically perfect test ball or equivalent was being used in the various error motion measurement examples. It is clear that the form errors in an artifact can cause misleading error motion measurements, and it cannot always be assumed that the artifact has negligible errors, since high quality axes of rotation may have error motions of the order of 1 μin. (0.025 μm). Nonmandatory Appendix B describes methods for separating the errors of the artifact from the synchronous error motion component of the axis of rotation.

## A-12 MASTER AXIS TECHNIQUE FOR SPINDLE TESTING

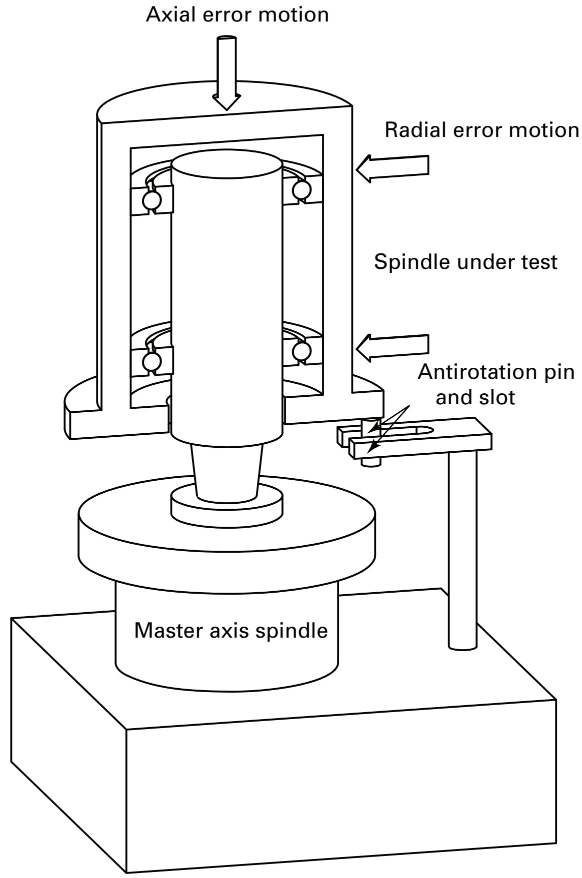
The axis of rotation of an air-bearing spindle can serve as the reference for measuring error motion of another spindle (reference [17]). As shown in Fig. A-25, the spindles are connected rotor-to-rotor with an antirotation arm also connecting the stators. Thus the stator of the spindle being tested is a target that does not rotate but is otherwise free to move in response to various error motions.

Displacement indicators are positioned as desired for radial, axial, tilt, and face motion measurement. The observed displacement is termed stationary-point runout. It comprises the misalignment of the two axes plus their combined error motion. Misalignment is compensated in the usual way, by removing the once-per-turn sinusoidal component of the displacement signal. But note that once-per-turn axial displacement is not an alignment error and must not be removed. (For axial measurements it is advantageous to measure from a flat, nonrotating target as in this case, since there is no concern that an imperfection of a test ball could be mistaken for fundamental axial error motion.)

Another advantage of a nonrotating target is that it is easy to connect an effective grounding strap, thereby reducing electrical noise in the displacement indicator amplifier circuit.

A further advantage is that the target surface does not need to be as smooth or as clean as that of a rotating artifact. Also, angular information can be obtained by attaching an encoder disc to the master axis rotor, thus improving the resolution compared to triggering from

**Fig. A-25 Roundness Tester Illustrating the Principle of the Axis of Rotation of a Precision Spindle Used as the Basis for Error Motion Measurements of Another Spindle**



an eccentric target. (This applies to a portable master axis spindle used to measure a machine tool spindle.)

It is convenient if an air bearing master axis spindle has negligible error motion relative to the tested spindle. If not, it is possible to use conventional error-separation techniques to construct individual error maps. Compliance during rotation can be measured by applying load to the master axis stator.

**CAUTION: Overloading can seize the air bearing, causing the air lines to whip around if the antirotation bar is not strong enough to stop the drive motor. Always use appropriate guards and restraints.**

## A-13 ERROR MOTION VERSUS RUNOUT OR TOTAL INDICATOR READING (TIR)

It should be noted that error motion measurements differ from measurements of runout or total indicator reading (TIR) in several respects. It is important to understand these differences, since runout tests have been used extensively in the past in assessing the accuracy of rotational axes. Runout is defined as the total displacement measured by an instrument sensing against a moving surface or moving with respect to a fixed surface. Under this definition, a radial runout measurement includes both the roundness error and the centering error of the surface that the displacement indicator senses against, and hence radial runout will be identical to radial error motion only if both of these errors are zero. As noted previously, neither of these conditions is easily accomplished. While centering error unavoidably makes the runout larger than the error motion, it is possible for roundness errors to make the runout either larger or smaller than the error motion. The latter situation can arise if the surface against which the displacement indicator is sensing was machined in place on the spindle bearings. Similar comments apply to face motion versus face runout; the latter measurement includes nonsquareness and circular flatness errors.

## A-14 RUNOUT OF REFERENCE SURFACES SUCH AS TAPERS, PILOT DIAMETERS, AND SHOULDERS

George Schlesinger's book Testing Machine Tools, published in 1927, used reference-surface runout as the only accuracy parameter of a spindle. Questions may therefore arise as to why this Standard does not discuss spindle-nose runout.

Although spindle reference-surface runout may be an important accuracy parameter for certain applications, it does not represent the accuracy of the axis of rotation. In fact, runout can be minimized by simply machining the spindle reference surfaces in situ, using the machine tool itself, while not affecting the axis of rotation at all. Most machine tool builders and some machine tool users apply this method to maintain low runout values for their spindle reference surfaces. In this way, the runout of these surfaces can be reduced to less than the synchronous error motion of the axis of rotation, but only at the orientation angle of the point of machining. This procedure has no effect on the form errors of subsequent workpieces, since the error motions of the axis of rotation were not improved, only concealed.

*(a)* However, there remain important reasons for in situ machining for low runout reference surfaces:

*(1)* The runout of workpieces held in collets depends upon the accuracy of the collet and the runout of the mating taper in the spindle.

*(2)* The runout of tapered-shank drills, dead centers, and milling cutters depends on the runout of the tapered surface that receives the shank.

*(3)* The radius of a rotating boring bar tool bit with respect to the axis of rotation determines the diameter of the hole that is bored. Some boring bar tool bits are nonadjustable and located with a taper.

*(b)* In other applications, the runout of spindle surfaces is irrelevant. Some examples are as follows:

*(1)* Runout of the spindle nose of a rotating-stylus roundness measuring machine has nothing to do with its roundness measuring accuracy.

*(2)* Runout of a single-tooth flycutter used for facing has no effect on the flatness of the work that is faced.

*(3)* Runout of the spindle nose has no effect on the roundness of a workpiece that is held in a four-jaw chuck and turned on a lathe.

## A-15 ERROR MOTION VALUES

In most cases, an error motion value is equal to the difference in radii of two concentric circles that will just enclose the corresponding error motion polar plot, and the value obtained depends upon the location of the common center of these two circles. The following four methods are recognized in this Standard for locating polar plot centers:

*(a)* least squares circle (LSC) center
*(b)* minimum radial separation (MRS) center
*(c)* maximum inscribed circle (MIC) center
*(d)* minimum circumscribed circle (MCC) center

In addition, a fifth center, the polar chart (PC) center, is used in establishing error motion values, which include fundamental axial error motion and asynchronous error.

### A-15.1 Least Squares Circle Center

The least squares circle (LSC) center has been chosen as the preferred polar plot center for error motion assessment in this Standard and is to be understood as the method to be used if no method is specified. Any of the other three methods can be used provided that the method is specified.

The LSC center is based on the mathematical approach of choosing a center that will minimize the sum of the squares of the polar plot deviations from a circle about that center. Since it can be defined mathematically, the LSC center is unique and can be found without trial and error methods. Figure A-26 is a description of the LSC method, abstracted from Section A3 of the British Standard 3730:1964. A proof of the formulae and guidance on the effect of the number of radial ordinates used can be found in the same reference.

### A-15.2 Minimum Radial Separation Center

The concept of the minimum radial separation (MRS) center is, as the name implies, that of a center chosen so as to make the difference in radii of the two concentric circles that contain the error motion polar plot an absolute minimum. By definition, the MRS center yields the smallest possible number for the error motion value. There is no direct method of locating this center, and some form of iterative trial and error must be used. In unusual cases, more than one such center may exist. Figure A-27 shows three successive trials using a bow compass; further reduction is still possible as the reader may wish to verify. In general, the minimum has not been found until the inner and outer circles both touch the polar profile at two points; in unusual cases more than two points per circle may occur. In the common case of two points per circle, the points must also alternate between the inner and outer circles. The time required for trial and error searching can be reduced by use of a transparent template having engraved concentric circles. Further reductions in time, together with improved accuracy, can be obtained with computer-aided systems using iterative algorithms.

### A-15.3 Maximum Inscribed and Minimum Circumscribed Circle Centers

These terms are self-explanatory. The error motion value is the radial distance to a second circle drawn about the same center so as to just contain the polar plot. These centers are sometimes used in roundness measurement and are included in this Standard for completeness in view of the relationship already discussed between radial error motion and part roundness. For example, assume a circular part has a machined profile in accordance with the total radial error motion polar plot and that it is placed in a perfectly round ring gage of the smallest diameter that will accept the part. Then the total radial error motion value about the MCC center represents the largest gap between the part and gage, and, if the part is rotated in the ring gage, it will rotate about the MCC center.

### A-15.4 Polar Chart Center

As noted previously, fundamental axial error motion is a real property of an axis of rotation rather than an error analogous to centering error. As a consequence, the total and synchronous axial error motion values are always measured using concentric circles drawn from the polar chart (PC) center. If it is desired to separate the synchronous axial error motion values into fundamental and residual synchronous axial error motion components, then the LSC method is preferred to find the center of the residual synchronous axial error motion polar plot. Twice the radial distance between this center and the PC center is the fundamental axial error motion value (also referred to as the axial error motion center offset). The LSC center is to be assumed unless another center is specified.

Since face error motion involves the same fundamental component as axial error motion, it follows that total and synchronous face error motion should also be based on the PC center. However, if face error motion is measured directly from a master flat mounted on the axis of rotation, then squareness error of the flat to the axis of rotation will introduce a second fundamental motion component. This component cannot be separated from that due to the axis unless the fundamental axial error motion value is known. If the latter is known, the simplest procedure is to assess the residual synchronous portion of the face motion polar plot using the appropriate polar plot center and then add the fundamental axial error motion value to obtain the desired result.

In the general case of error motion at a direction angle $\phi$ to the $Z$ reference axis, the fundamental error motion is equal to cos $\phi$ times the fundamental axial error motion. Therefore the addition procedure of the preceding paragraph can be used, except that the added quantity is the fundamental axial error motion times cos $\phi$.

Finally, for synchronous radial and tilt error motions, the nonexistence of fundamental error motion excludes the use of the PC center.

### A-15.5 Asynchronous Error Motion Value

The asynchronous motion value is found from the total error motion polar plot as the maximum radial width of the cloud band at any angular position around the circumference. It does not employ concentric circles, since it involves the radial variation at the particular angle where the maximum width occurs, rather than the radial variation around the full circumference.

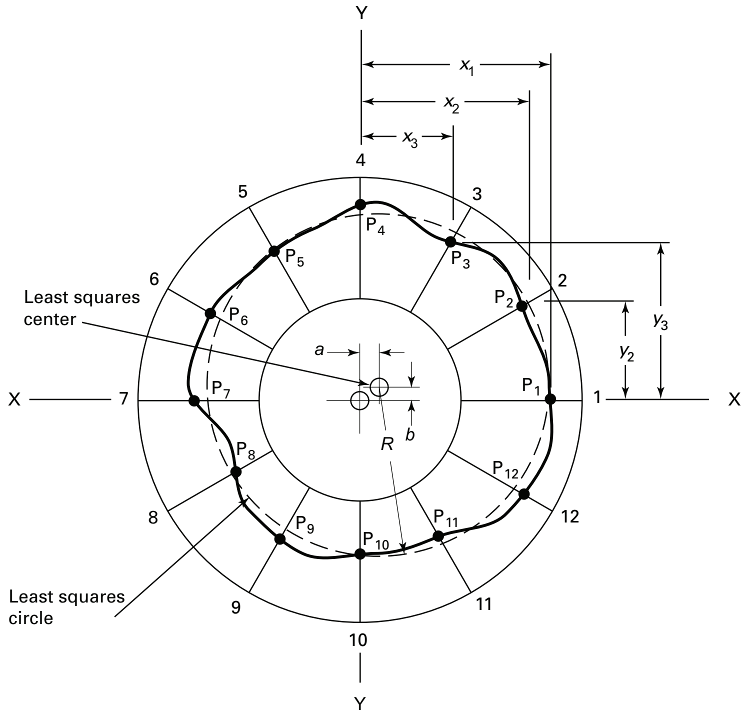
To be strictly correct, the asynchronous error motion value should be measured along a radial line from the polar chart (PC) center rather than from a best-fit center, even though this is contrary to what seems intuitively correct. Figure A-28 illustrates this point by means of a computer-generated plot of a high frequency sinusoid of uniform amplitude that is superimposed on a limaçon. The sinusoid amplitude is constant if it is measured radially from the PC center, as it should be, but is up to 8% smaller (in this case) if it is measured radially from the MRS center.

Methods that compute asynchronous error motion value based solely on the range of data (minimum to

## Fig. A-26 Determination of Least Squares Center and Circle

In determining the least squares center and circle, the position of the center of the least squares circle and the value of its radius can be calculated from simple approximate formulae.

Referring to the figure below, the practical procedure is as follows:




**Determination of Least Squares Center and Circle**

From the center of the chart, draw a sufficient even number of equally spaced radial ordinates. In the illustration they are shown numbered 1–12. Two of these at right angles are selected to provide a system of rectangular coordinates X–X and Y–Y.

The distances of the points of intersection of the polar graph with these radial ordinates, $P_1$ to $P_{12}$, are measured from the axes X–X and Y–Y, taking positive and negative signs into account.

The distances $a$ and $b$ of the least squares center from the center of the paper are calculated from the following approximate formulae:

$$a = \frac{2 \times \text{sum of } x \text{ values}}{\text{number of ordinates}} = \frac{2\sum x}{n} \qquad b = \frac{2 \times \text{sum of } y \text{ values}}{\text{number of ordinates}} = \frac{2\sum y}{n}$$

The radius $R$, of the least squares circle, if wanted, is calculated as the average radial distance of the points P from the center; that is:

$$R = \frac{\text{sum of radial values}}{\text{number of ordinates}} = \frac{\sum r}{n}$$

In practice, if it is required to know only the radial width of the zone enclosing the curve, there is no point in finding $R$, and it is sufficient to draw the inscribing and circumscribing circles from the least squares center.

GENERAL NOTE: Permission to reproduce extract from BS 3730:1964 is granted by BSI.

**Fig. A-27 Determination of Minimum Radial Separation**

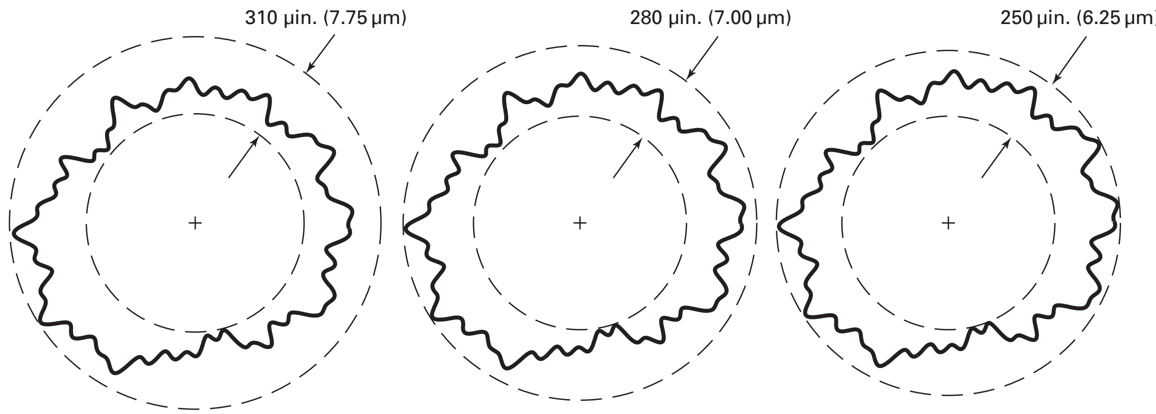
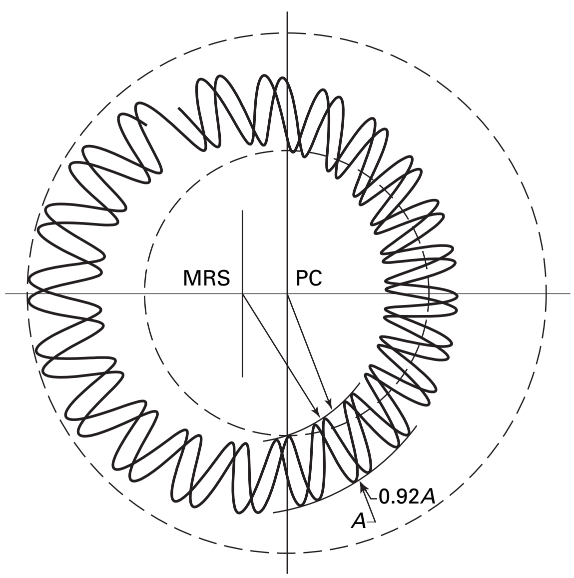



**Fig. A-28 Asynchronous Motion Assessment: Constant-Amplitude Sinusoid with Centering Error, Showing Proper Measurement Along Radius From PC Center**




maximum) from the same angular position are insensitive to the choice of plot center. Data acquisition schemes that use position-based triggers, such as a rotary encoder on a spindle (many points/revolution) or a grease pencil mark (single point/revolution) in the SPAM test are examples.

## A-15.6 Single-Point Asynchronous Motion (SPAM) Test

The single-point asynchronous motion test (formerly called the grease pencil test) is used to estimate asynchronous radial or face error motion of a spindle. There are several cases in which this test may be preferred or used in addition to asynchronous tests that require a spindle analyzer. It is quicker to perform, requires less test equipment, and does not require a precision artifact. This enables measurement at the point of interest even on large machines.

This test measures asynchronous error motion at a specific orientation angle of the spindle rotor, rather than the worst-case angular orientation, which is the default case when reporting asynchronous error. It is important to note that there can be large differences in asynchronous error motion at different angular orientations. It is possible to reduce this unknown factor by reporting the worst of several points at different angular orientations.

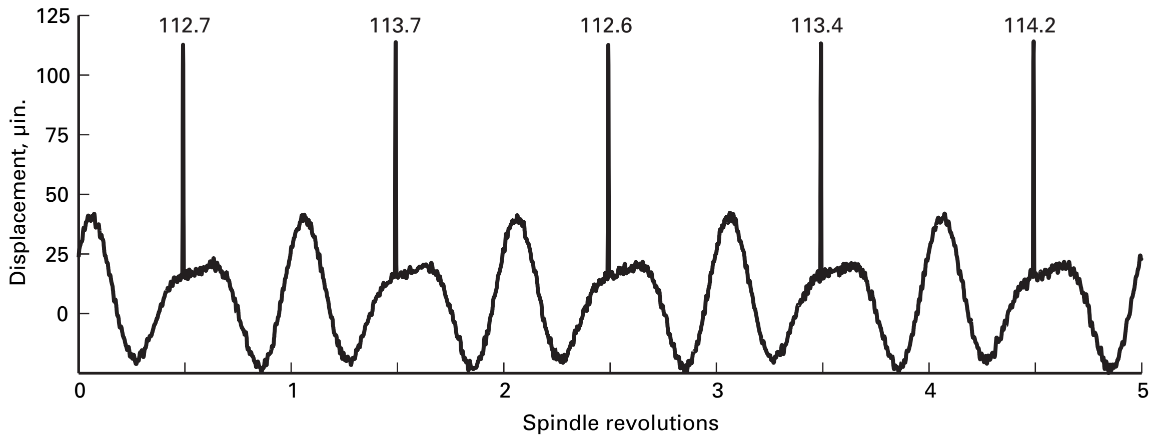
The test is performed by making a high- or low-point mark on a rotating element of the spindle. A high-point mark can be made with a grease pencil. The rotating element can be a workpiece or the nose of the spindle itself.

The high or low points are recorded on the oscilloscope as a series of narrow spikes over a specified number of revolutions. The variation in the amplitude of the spikes is an estimate of asynchronous error motion.

Care must be taken with digital data acquisition equipment to avoid aliasing the sampled signal. Furthermore, it is imperative that the sampling rate be set such that the fiducial mark contains at least ten samples. For example, if at a particular spindle speed the mark is observed to span 1 ms, then the minimum sampling rate should be 10 kHz. The only reliable way to prevent aliasing from frequency components above this frequency is by analog low-pass filtering. When reporting a SPAM result, the spindle speed, sampling rate, and low-pass filter cutoff frequency must be included.

There are a number of methods of collecting data in the spatial domain. They all involve using a trigger (e.g., encoder or fiducial mark) slaved to the rotational position of the spindle, to ensure that the sample is acquired at the same angular position for each revolution, at least

**Fig. A-29 Single-Point Asynchronous Error Motion Estimate From Encoder Sampled Estimate**




GENERAL NOTE: The SPAM result is 1.6 µin.

within acceptable angular position limits. In the case of an encoder, an external or high mark is unnecessary because the acquisition system may be triggered from the encoder at any orientation angle. Analysis of the signal magnitude for the same angular orientation for multiple revolutions for the encoder-triggered system is a measure of asynchronous error motion at that angular orientation. Commercially available spindle error analyzers enable measurement of single-point asynchronous error motion without a fiducial mark, through the use of encoder output synchronized to rotation. Data sampled as a function of the encoder pulse train for multiple revolutions enables data from the same angular orientation to be analyzed. Figure A-29 shows the single point asynchronous error motion estimate for five revolutions.

Asynchronous error motion measurement predicts the surface finish that may be achieved under perfect cutting conditions, i.e., perfect diamond tools on diamond-turnable materials at a specific tool radius and feed rate. It includes the effect of structural motion as well as spindle error motion. Note that air and hydrostatic bearings have a comparatively small level of asynchronous error motion, and that the source of the measured asynchronous error motion is largely due to structural motion caused by acoustic and mechanical vibration, the spindle drive system, and electrical noise in the displacement indicator.

## A-16 TEST METHODS

A computer provides the most convenient basis for a system to collect, manipulate, and present data from axis of rotation measurements. Vanherck and Peters (see reference [7]) first described a system in which the signal from a displacement indicator is sampled at closely spaced angular intervals, using a perforated disk mounted on the axis and a photoelectric trigger. Today, this approach is readily carried out using samples evenly spaced in time (assuming constant spindle speed) or with a rotary encoder to precisely synchronize the data acquisition system to the axis rotation. In either case, the samples are digitized and stored for subsequent calculation and display of the total, synchronous, and asynchronous error motion polar plots with elimination of the fundamental component as appropriate. Error motion values may also be calculated and displayed. In addition, a computer-based measurement system allows the analyst to

*(a)* remove the once-per-revolution component from radial measurements

*(b)* separately plot the fundamental and residual error motion in the axial direction

*(c)* remove very long time-scale variations to compensate for thermal drift

*(d)* perform an error separation technique to distinguish artifact form error from spindle error motion

*(e)* apply zero-phase-shift digital filtering to remove any frequency component or range of frequencies

Another advantage of the computer-based system is the ease with which it can deal with arbitrary orientation angles for fixed or rotating sensitive direction measurements. As shown in Fig. A-30, adding a second radially sensing displacement indicator at 90 deg to the first, similar to that shown in Fig. A-31, the radial (or face) error motion along a sensitive direction at any angle $\phi$ to the first displacement indicator can be calculated from the following equation:

$$r(\theta) = r_0 + \Delta X(\theta) \cos \theta + \Delta Y(\theta) \sin \theta \qquad \text{(A-9)}$$

where $\Delta X(\theta)$ and $\Delta Y(\theta)$ are the two displacement indicator signals and $r_0$ is an arbitrary radius of the plotted base circle. For a rotating sensitive direction having any angle $\phi$ relative to $\theta = 0$, the radial error motion using the same signals is given by

$$r(\theta) = r_0 + \Delta X(\theta) \cos(\theta + \phi) + \Delta Y(\theta) \sin(\theta + \phi) \qquad \text{(A-10)}$$

Fig. A-30 Vector Diagram for Rotating Sensitive Direction

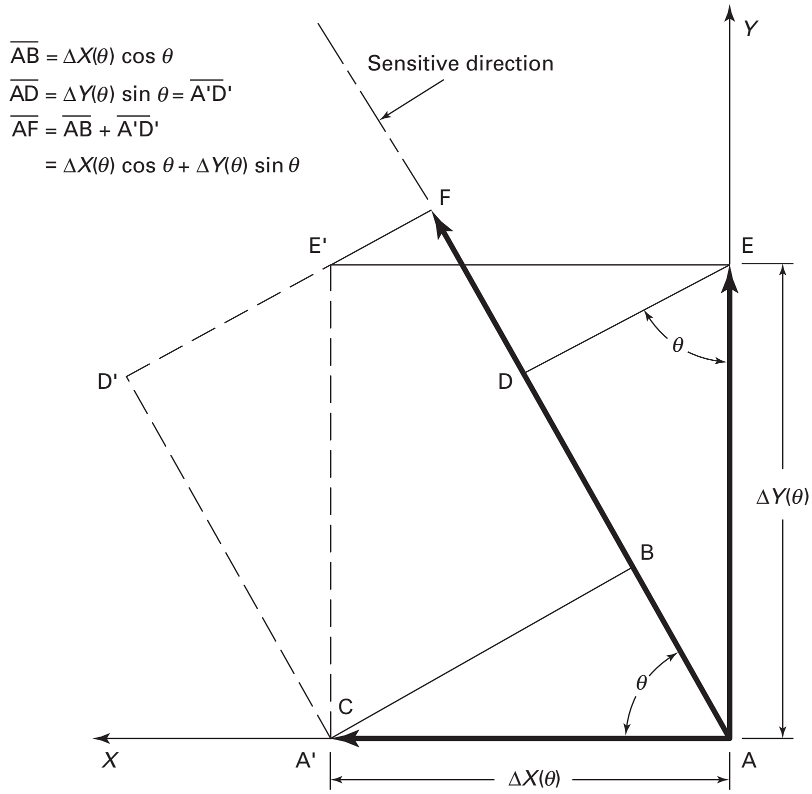



Fig. A-31 Test Method for Error Motion With Rotating Sensitive Direction

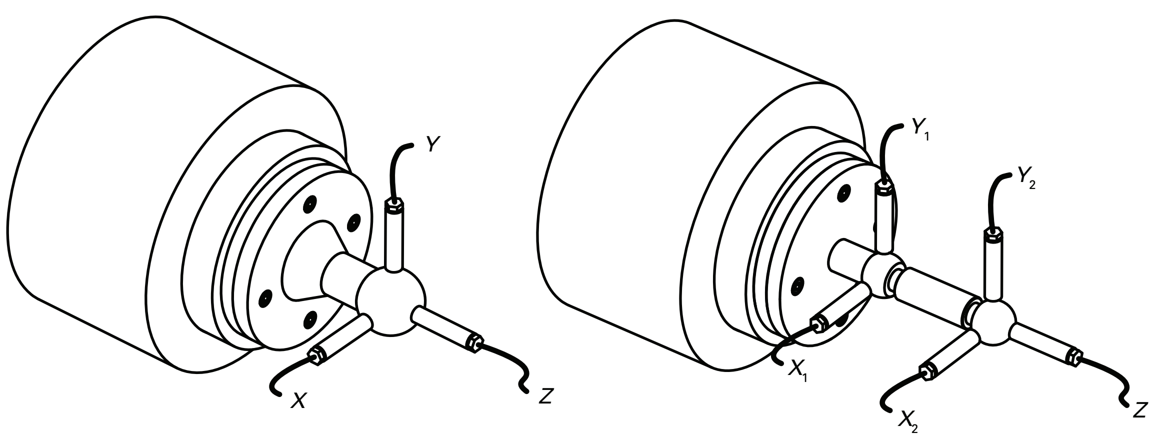
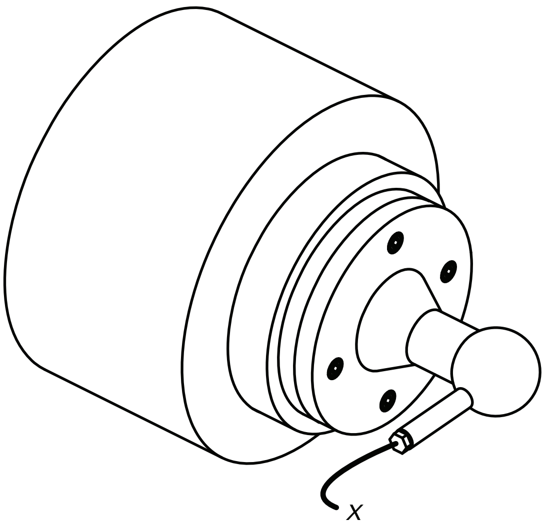


**Fig. A-32 Test Method for Radial Error Motion With Fixed Sensitive Direction**




### A-16.1 Rotating Sensitive Direction Measurements

For low speed rotary axes such as rotary tables, trunnions, etc., both fixed and rotating sensitive direction cases can be dealt with by use of a polar recorder whose angular drive is mechanically or electrically synchronized to the axis of rotation. For the rotating sensitive direction, the master test ball is supported by the machine frame and the displacement indicator is supported by the axis of rotation. For one or a few revolutions of the axis, it is usually possible to coil the displacement indicator cable around the axis in a noninfluencing manner; for continuous rotation, slip rings or their equivalent are necessary.

For rotating sensitive direction measurement of high speed rotary axes it is more appropriate to use a setup such as shown in Fig. A-31 with two (or four) displacement indicators sensing radially against a master artifact. Alternatively, the third and fourth indicators may be oriented to measure face motion at a known radius from the $Z$-axis. An additional displacement indicator may also be used to measure in the axial direction to completely specify the five degrees of spindle motion. In this case, the error motion can be calculated in any direction at any location (refer to Fig. A-7). An error separation technique to distinguish error motion from artifact form error may also be applied as needed (see Nonmandatory Appendix B).

The displacement indicator may be synchronized to the spindle rotation through the use of a rotary encoder or, if the spindle speed is sufficiently constant, the data acquisition may be evenly spaced in increments of time.

### A-16.2 Fixed Sensitive Direction Measurements

Figure A-32 shows hardware to measure the fixed sensitive direction motion. As with the rotating sensitive direction measurements, additional displacement indicators may be used to measure the axial and face motion.

## A-17 TWO-INDICATOR MEASUREMENTS FOR A FIXED SENSITIVE DIRECTION

It is natural to consider using rotating sensitive direction measurement data to plot the error motion for a fixed sensitive direction. However, if this substitution is made, the resulting radial error motion polar plot will not be representative of the potential part out-of-roundness. If $\phi = 0$ deg is the fixed sensitive direction, then the polar plot reflects radial error motion in this direction only in the vicinity of $\phi = 0$ deg and $\phi = 180$ deg. Moreover, if a given localized movement of the axis of rotation occurring at $\phi = 0$ deg appears as a peak on the polar plot, the same movement occurring at $\phi = 180$ deg will have an undesired sign reversal and will appear as a valley. At $\phi = 90$ deg and $\phi = 270$ deg the same movement will not register on the polar plot.

Despite the above observations, it still appears intuitively plausible that the radial error motion value should be roughly the same for both fixed and rotating sensitive directions, even if the details of the polar plot are different. This view appears reasonable if the factor of concern is asynchronous radial error motion. However, for synchronous radial error motion, Donaldson (see reference [8]) has noted a case giving precisely the opposite result, in which an axis that exhibits an elliptical pattern when tested in a fixed sensitive direction is free of radial error motion when tested in a rotating sensitive direction. The case occurs for the following motions:

$$\Delta X(\theta) = A \cos 2\theta \tag{A-11}$$

$$\Delta Y(\theta) = A \sin 2\theta \tag{A-12}$$

where the coordinate system is that of Fig. A-9, illustration (a). With a fixed sensitive direction along the $X$-axis, the radial error motion polar plot has the following equation:

$$r(\theta) = r_0 + A \cos 2\theta \tag{A-13}$$

where $r_0$ is the base circle radius. Equation (A-13) represents an elliptical shape, having a value $r_0 + A$ at $\theta = 0$ deg and $\theta = 180$ deg and a value of $r_0 - A$ at $\theta = 90$ deg and $\theta = 270$ deg. The radial error motion value based on any of the polar profile centers is $2A$. If the sensitive direction rotates with angle $\theta$, the radial error motion is given by the following equation:

$$r(\theta) = r_0 + \Delta X(\theta) \cos \theta + \Delta Y(\theta) \sin \theta \tag{A-14}$$

Figure A-30 shows the resolution of $\Delta X(\theta)$ and $\Delta Y(\theta)$ into components along the rotating sensitive direction that leads to eq. (A-14). Combining eqs. (A-11) and

(A-12) with eq. (A-14) and using the following trigonometric identities:

$$\cos \alpha \cos \beta = \frac{1}{2}\left[\cos(\alpha - \beta) + \cos(\alpha + \beta)\right] \quad \text{(A-15)}$$

$$\sin \alpha \sin \beta = \frac{1}{2}\left[\cos(\alpha - \beta) - \cos(\alpha + \beta)\right] \quad \text{(A-16)}$$

The result is

$$\begin{aligned} r(\theta) &= r_0 + \frac{A}{2}\left[\cos\theta + \cos 3\theta\right] + \frac{A}{2}\left[\cos\theta - \cos 3\theta\right] \\ &= r_0 + A\cos\theta \end{aligned} \quad \text{(A-17)}$$

Equation (A-17) is the equation of a circle that is offset from the origin by a distance $A$ (aside from a second order limaçon distortion), and hence the axis would be perfect if tested by the two-displacement indicator system.

The view might be taken that the above example is a mathematical oddity that is unlikely to occur in practice. In this regard it can be noted that radial error motion polar plots commonly exhibit an elliptical pattern, and that to the extent that the overall patterns in the $X$ and $Y$ directions contain components as given in eqs. (A-11) and (A-12), these components will not contribute to the measured radial error motion value.

## A-18 FREQUENCY ANALYSIS OF SPINDLE ERROR MOTION AND ROUNDNESS DATA

Frequency analysis is useful for identifying the root cause of error motions in rotating machinery. For example, a two-lobe shape is often associated with machine setup errors and asymmetrical stiffness in the structural loop. Also, lobing or chatter frequencies are often associated with vibrations in the machine and can generally be related to natural frequencies of the spindle, natural frequencies of structural machine elements, rolling-element bearing frequencies, or forcing frequencies from various machine drives. Therefore a brief outline of frequency analysis methods is included here.

### A-18.1 Definition of Discrete Fourier Transform

Any discrete data set, such as sampled spindle error motion, can be represented as the sum of a series of sine and cosine waves.

$$X_k = \frac{a_0}{2} + \sum_{j=1}^{\infty} a_j \cos\left(\frac{2\pi k}{N} j\right) + \sum_{j=1}^{\infty} b_j \sin\left(\frac{2\pi k}{N} j\right) \quad \text{(A-18)}$$

Here, $X_k$ represents the measured radial error motion for the $k^{\text{th}}$ sample of the measured data. The angle of rotation for the $k^{\text{th}}$ sample is $2\pi kj/N$, where $N$ is the number of data points and $j$ is the harmonic number. The subscripted terms $a_j$ and $b_j$ are the Fourier coefficients and are calculated using the equations of the Discrete Fourier Transform.

$$a_j = \frac{1}{N}\sum_{k=0}^{N} X_j \cos\left(\frac{2\pi k}{N} j\right) \qquad j = 0, 1, 2, \ldots \quad \text{(A-19)}$$

$$b_j = \frac{1}{N}\sum_{k=0}^{N} X_j \sin\left(\frac{2\pi k}{N} j\right) \qquad j = 0, 1, 2, \ldots \quad \text{(A-20)}$$

To avoid aliasing, it is important to sample the inspection data at a frequency that is at least two times greater than the highest expected frequency. In practice, this is achieved by analog low-pass filtering the data with a cut-off frequency that is less than one-half the sampling frequency.

### A-18.2 Example of Application of Method to Spindle Error Motion

Consider the spindle error motion data presented in polar plot form in Fig. A-33. The Discrete Fourier Transform equations may be applied to determine the frequency content of the error motion. These results are plotted in Fig. A-34. This plot shows the frequency or harmonic on the $X$-axis and the magnitude of that component on the $Y$-axis. Table A-1 offers sample frequency content. Two excellent references for Fourier methods are references [11] and [12]. These texts should be consulted for additional guidance in implementation of the method.

**Fig. A-33 Sample Spindle Error Motion Data**

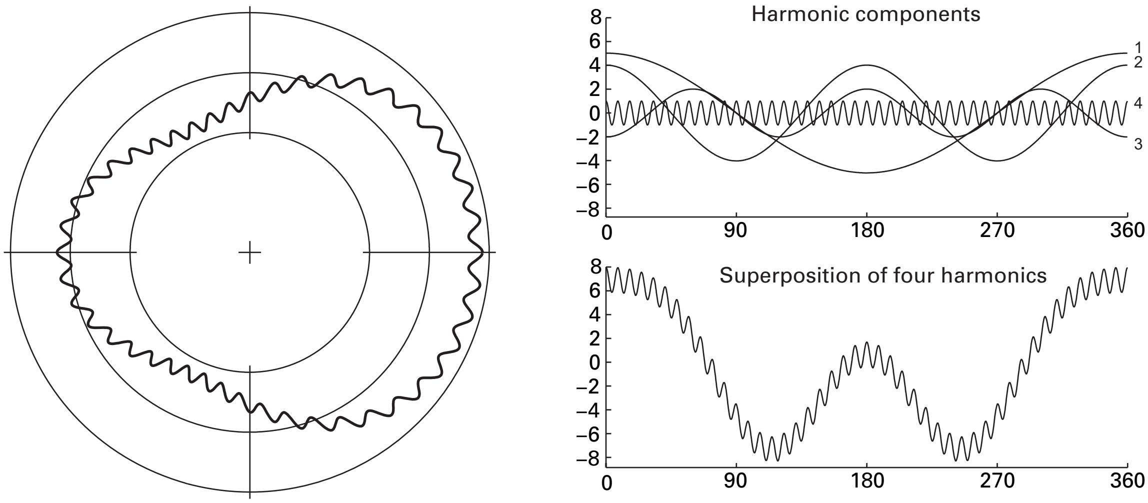



**Fig. A-34 Frequency Plot of Sample Data**

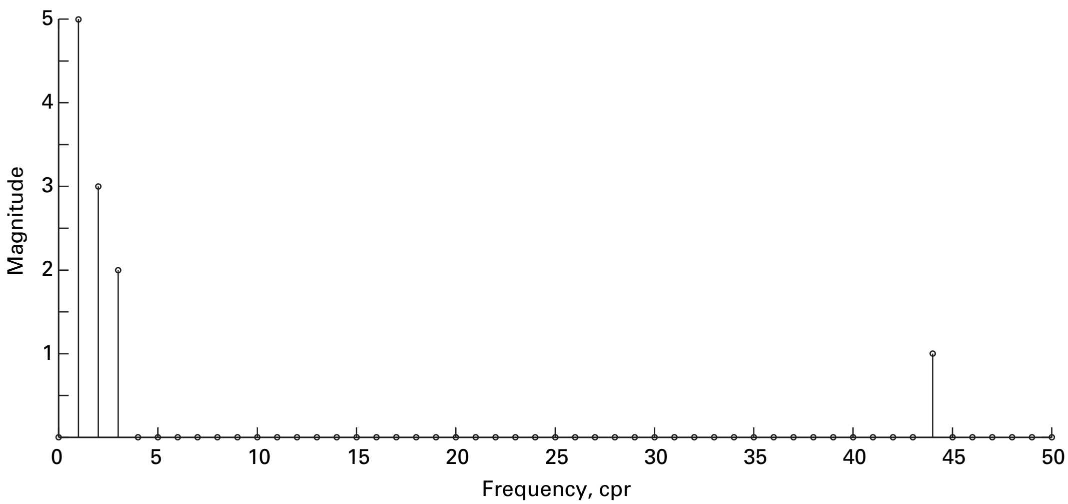



**Table A-1 Sample Frequency Content**

| Harmonic | Magnitude | Cause |
|---|---|---|
| 1 | 5 | Centering error |
| 2 | 4 | Two-lobe error |
| 3 | 2 | Three-lobe error |
| 44 | 1 | Chatter frequency |

# NONMANDATORY APPENDIX B
# ELIMINATION OF MASTER BALL ROUNDNESS ERROR

## B-1 INTRODUCTION

Measurements of radial error motion are directly influenced by the out-of-roundness of the test ball or other circular master against which the displacement indicator senses. This Nonmandatory Appendix describes methods for separating the out-of-roundness of the master from the radial error motion of the axis of rotation. These methods can be described as multiorientation error separation techniques since they involve taking two or more measurements with differing relative orientations between master and spindle. When correctly applied, they are a very effective means of accurately determining test ball errors. The two most widely used techniques, the reversal and multistep methods, are described here.

## B-2 THE REVERSAL METHOD

The reversal method has been described by Donaldson (reference [8]) and is also contained in para. D2.5 of ANSI B89.3.1 (reference [1]). The method requires two profile measurements to be taken; in the second measurement the orientations of both the part and the displacement indicator are reversed by 180 deg relative to the spindle. The relative position of part and displacement indicator is unchanged while the effect of the spindle radial error motion on the displacement indicator at any position is equal and opposite. Thus the part or master ball roundness error can be extracted as the mean of the two traces while the difference gives the spindle radial error motion.

The method is described in detail below. In the interest of consistency, the same notation is used here as in para. D2.5 of ANSI B89.3.1, with $P(\theta)$ (for part) representing the out of roundness of the master and $S(\theta)$ (for spindle) representing the radial error motion. It is assumed that the axis of rotation is free of asynchronous radial error motion; means of dealing with asynchronous motion will be discussed in section B-5. The method can be divided into the following two procedures:

*(a)* Procedure *P*, which yields the roundness error of the master

*(b)* Procedure *S*, which yields the radial error motion of the spindle

### B-2.1 Procedure *P*

Procedure *P* begins by recording an initial polar plot; the deviations from the base circle will be designated at $T_1(\theta)$. Figure B-1, illustration (a) shows a schematic diagram of the test arrangement with the arbitrary initial positions being marked as $\theta = 0$ deg by coincident marks on the master, displacement indicator, rotor, and stator of the axis of rotation. The recorded value of $T_1(\theta)$ is the sum of the master roundness profile $P(\theta)$ and the radial error motion $S(\theta)$.

$$T_1(\theta) = P(\theta) + S(\theta) \tag{B-1}$$

It is assumed that the sign convention for roundness measurement is used so that hills and valleys on the polar plot correspond to hills and valleys on the master. The second step of Procedure *P* is to make a second polar plot $T_{2P}(\theta)$ using the arrangement of Fig. B-l, illustration (b), in which the rotor and stator marks are coincident at $\theta = 0$ deg, but the master and displacement indicator positions are reversed (rotated 180 deg about the axis of rotation). The same sign convention must be used as for $T_1(\theta)$. Comparison between Fig. B-1, illustrations (a) and (b) shows that the out-of-roundness of the master is recorded in the same manner since the relative position of the displacement indicator and the master is unchanged. However, radial error motion is recorded with a reversed sign in Fig. B-1, illustration (b) since a movement of the spindle toward the displacement indicator in Fig. B-1, illustration (a) becomes a movement away from the displacement indicator in Fig. B-1, illustration (b). Expressed as an equation,

$$T_{2P}(\theta) = P(\theta) - S(\theta) \tag{B-2}$$

Adding eqs. (B-1) and (B-2) causes $S(\theta)$ to cancel. Solving for $P(\theta)$ gives

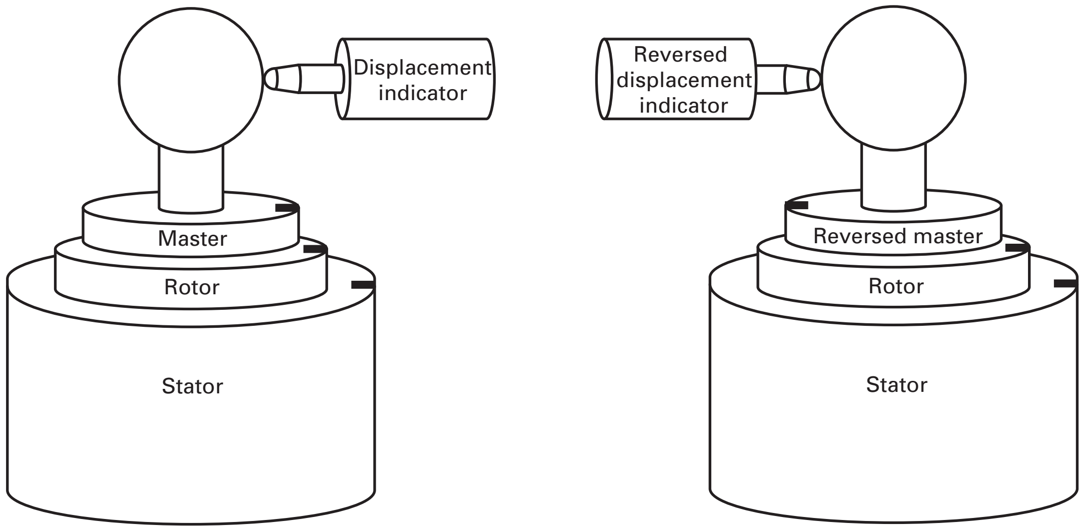
$$P(\theta) = \frac{T_1(\theta) + T_{2P}(\theta)}{2} \tag{B-3}$$

Equation (B-3) states that the out-of-roundness profile of the master, $P(\theta)$, is the average of the first and second polar plots. If $T_1(\theta)$ and $T_{2P}(\theta)$ are recorded on the same polar chart, $P(\theta)$ can be obtained by drawing a third polar plot halfway between the first two as shown in Fig. B-2, illustration (a).

### B-2.2 Procedure *S*

Procedure *S* begins by recording an initial profile $T_1(\theta)$ as in Procedure *P*. The second step of Procedure *S* is also identical to the second step of Procedure *P* except that

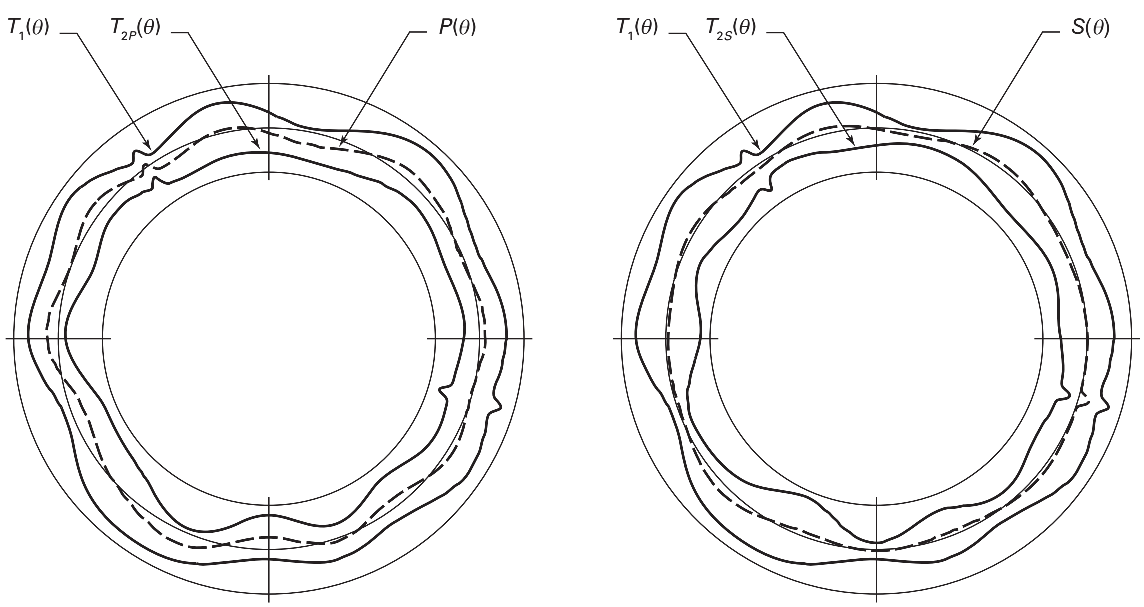
**Fig. B-1 Schematic Test Setups**




(a) For $T_1(\theta)$

(b) For $T_{2P}(\theta)$ and $T_{2S}(\theta)$

**Fig. B-2 Error Separation by Profile Averaging**



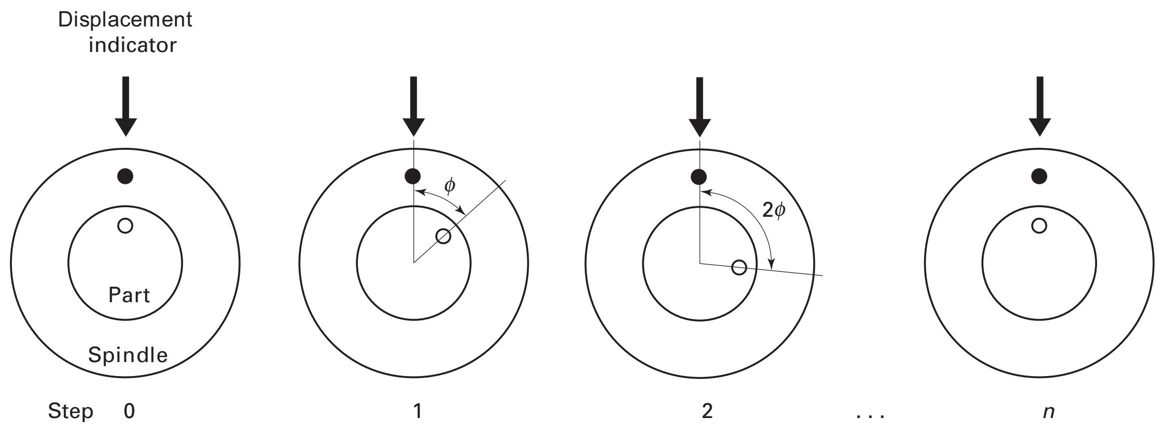

(a) Master Out-of-Roundness $P(\theta)$

(b) Radial Error Motion $S(\theta)$

**Table B-1 Procedure *P* and Procedure *S***

| Procedure | Reverse for Record 2 | Average |
|---|---|---|
| *P* | Master, displacement indicator | Master out-of-roundness |
| *S* | Master, displacement indicator, sign | Radial error motion |

**Fig. B-3 Multistep Method**




the sign convention must be reversed. Calling the second polar plot $T_{2S}(\theta)$ it follows that

$$T_{2S}(\theta) = -T_{2P}(\theta) = -P(\theta) + S(\theta) \quad \text{(B-4)}$$

If eqs. (B-1) and (B-4) are added, $P(\theta)$ cancels; solving for $S(\theta)$ gives

$$S(\theta) = \frac{T_1(\theta) + T_{2S}(\theta)}{2} \quad \text{(B-5)}$$

Equation (B-5) states that a third polar plot drawn halfway between $T_1(\theta)$ and $T_{2S}(\theta)$ will be the radial error motion polar plot $S(\theta)$.

Table B-1 summarizes these two procedures. Both procedures are equally valid with either a fixed or a rotating sensitive direction.

## B-3 THE MULTISTEP METHOD

The multistep method (reference [9]) entails taking a whole series of roundness profile measurements in each of the part that is stepped through equal angles relative to the spindle. The most effective way of implementing the method is to take *n* separate but equiangled orientations adding up to 360 deg as illustrated schematically in Fig. B-3. It is then possible to separate the part error, which rotates with each step, from the spindle error, which remains stationary. To obtain the part error, it is necessary to pick one angle of the spindle's rotation and to identify the changes in displacement indicator signal at this angle for all the different orientations in sequence. To obtain spindle errors, a fixed angle on the part has to be chosen instead. Before the error separation can be carried out, the profile sets from each orientation have to be normalized, i.e., they have to be adjusted so that the profile eccentricity and radius are always the same. Because of the large amount of data processing, the method is much more conveniently carried out using a computer. As in the case of the reversal method, the technique can, in principle, be applied to either rotating workpiece (fixed sensitive direction) or rotating stylus (rotating sensitive direction) roundness instruments.

## B-4 FACE ERROR MOTION REVERSAL

A new method for separating residual face error motion from artifact circular flatness form error has been proposed by Estler and documented by Salsbury in reference [16].

## B-5 PRACTICAL CONSIDERATIONS

Several practical considerations arise in obtaining accurate results with error separation techniques. A crucial assumption in these techniques is that the part and spindle error profiles are highly repeatable. In the case of the part profile, this involves ensuring that the plane of measurement remains constant in each orientation without axial shift or tilt of the measurement track. Sensitivity to track location can be tested by examining the repeatability of the measured profile as the track is shifted by small amounts in the first setup of each method.

In the presence of asynchronous radial motion, the spindle error must be interpreted as the synchronous radial error motion polar plot, and the resulting accuracy depends upon being able to obtain a repeatable synchronous radial motion in each orientation. This can be tested by successive recordings of the measured profile in the first setup. Repeatability over a single revolution is sometimes improved by turning the spindle backward to the same starting point, particularly with rolling-element bearings. With computer-aided measurement systems, averaging over several profiles in each orientation can be used to minimize the effects of asynchronous spindle motion.

Both the reversal and multistep methods have their respective advantages and disadvantages. The reversal method may require modification of some commercial instruments and only works for radial error motion measurements. Other error motions can be computed by measuring axial error motion and tilt error motion as well and then combining the three error profiles according to eq. (A-5), which is practical only with the aid of a computer. The multistep method is directly applicable to any error motion but requires a computer for effective implementation. It also suffers to some extent from harmonic distortions and is therefore limited to errors involving only low numbers of undulations per revolution. Both methods are, however, capable of giving excellent results. A comparison of the two methods, using computer aided roundness equipment (see reference [10], has given agreement to within 0.04 μin. (0.001 μm) standard deviation.

Once the test ball errors are known, it is a simple matter, in a digital system, to store them in memory. By subtracting them point-for-point from the measurement, the spindle radial error motion can be evaluated accurately in one step.

# NONMANDATORY APPENDIX C
# UNCERTAINTY EVALUATION PROCEDURE FOR AXES OF ROTATION

## C-1 INTRODUCTION

This Nonmandatory Appendix provides concepts, terminology, and procedures for constructing a valid numerical response to the question, "How much confidence do you have in the result of an error motion measurement?" The examples include narrative justifications for the decisions that were made. To minimize complexity, informal language is used, the topic is limited to error motion, and the math does not go beyond taking square roots. A closer study would show that each topic is subject to refinement. For a higher level treatment refer to ANSI/NCSL Z540.2-1997, U.S. Guide to the Expression of Uncertainty in Measurement, reference [13].

> Although this Guide provides a framework for assessing uncertainty, it cannot substitute for critical thinking, intellectual honesty, and professional skill. The evaluation of uncertainty is neither a routine task nor a purely mathematical one; it depends on detailed knowledge of the nature of the measurand and of the measurement. The quality and utility of the uncertainty quoted for the result of a measurement therefore ultimately depend on the understanding, critical analysis, and integrity of those who contribute to the assignment of its value. (reference [13])

### C-1.1 Procedure

The following steps illustrate a systematic approach to evaluating the uncertainty of axis of rotation measurement results.

*Step 1:* Define the measurand.
*Step 2:* List the significant influence quantities and create the input quantities.
*Step 3:* Assign best estimates and standard uncertainties to the input quantities.
*Step 4:* Construct the uncertainty budget.
*Step 5:* Evaluate the best estimate and combined standard uncertainty of the measurand.
*Step 6:* State a coverage factor and level of confidence (expanded uncertainty).
*Step 7:* Sign and date.

### C-1.2 Purpose

Uncertainty evaluation is a disciplined approach to defining the level of confidence that can be placed in the result of a measurement. It is particularly helpful for making decisions where the result of a meaurement is close to its specified value for acceptability. The procedure may also be used to identify areas where improvement would result in a significant reduction of uncertainty and thereby indicate practices that should be brought into common usage within a company. Whether to make an uncertainty evaluation is a business decision and not every measurement need be accompanied by an uncertainty evaluation.

## C-2 MEASURANDS

"A quantity or property subjected to a measurement process." — Microsoft glossary of terminology

> . . . a well-defined physical quantity — the measurand — that can be characterized by an essentially unique value. If the phenomenon of interest can be represented only as a distribution of values or is dependent on one or more parameters, such as time, then the measurands required for its description are the set of quantities describing that distribution or that dependence. (reference [13])
>
> The first step in making a measurement is to specify the measurand — the quantity to be measured; the measurand cannot be specified by a value but only by a description of a quantity. However, in principle, a measurand cannot be completely described without an infinite amount of information. Thus, to the extent that it leaves room for interpretation, incomplete definition of the measurand introduces into the uncertainty of the results of a measurement a component of uncertainty that may or may not be significant relative to the accuracy required of the measurement. (reference [13])

### C-2.1 Description of the Measurand

For error motion measurements, a suffcient description of the measurand requires a detailed description, including sensor location, filter settings, and operating conditions such as rpm, number of turns, load, temperature variation, and any other influence that may be significant.

### C-2.2 Types of Measurand

The term measurand has subtle gradations of meaning, and this is a problem for situations requiring exactitude because the word lends itself to various interpretations. Measurand means different things depending on the point of view of the designer, measurement planner, metrologist, and evaluator of the measurement result. These different points of view cause communication problems stemming from trying to make one word cover a range of meanings. A remedy is to add modifiers: specified measurand, planned measurand, realized measurand, and measurand under extended validity conditions.

### C-2.3 Specified Measurand

The specified measurand is that which is named on the specification sheet, for instance axial error motion. It is expected to be suffciently descriptive to convey design intent so that if the measurement result is in the conformance zone, functionality is assured. Note that the definition of axial error motion is complete and unambiguous; there is no uncertainty about the conformance zone, only about the ability to know if the measurement result does or does not conform. The specified numerical value is exact, with no uncertainty; it is that particular value and no other. Uncertainty arises to the extent that the specified measurand does not fully describe test conditions such as the number of turns, the speed, or the load, since ". . . in principle, a measurand cannot be completely described without an infinite amount of information." (reference [13])

### C-2.4 Planned Measurand

A measurement plan addresses the type of displacement indicator, means of providing angular data, structural loop, filters and other settings, test ball or other artifact, number of samples per turn, number of turns, the environment, and how the data are to be displayed and evaluated. These are the validity conditions used to construct the uncertainty budget.

Uncertainties enter in to the extent that the gage is not perfect, the filters have allowance for variation, the test ball will not be perfectly round, the number of points to be recorded is not infinite, the temperature variation is not known exactly, the circle-fitting algorithm choices may not be spelled out, and the question may be open as to how to deal with discrepancies and outliers.

Thus the planned measurand is more complex than the specified measurand, and any lack of detail in the definition of the planned measurand will introduce uncertainty into interpretation of the final result.

### C-2.5 Realized Measurand Quantity

The realized measurand refers to the result of a particular measurement as it was accomplished. This includes the test environment with all its complexity and imperfections such as electrical noise, drive influence, contamination, vibration, and the effects of inadvertent variations in speed, load, temperature, or other operating conditions. Also, the gage is not perfect; it will have finite resolution, inexact amplification, and imperfect alignment. The test ball will have imperfect form and finish and will be imperfectly held and located; all of which introduce deviations from the measurement plan and thus add uncertainties.

Operator influences can be important in several ways: test ball and probe mount installation, probe selection and alignment, judgment as to adequacy of soak time, and deciding if a measurement is to be rerun due to transient environmental imperfections.

### C-2.6 Measurand Under Extended Validity Conditions

For error motion measurements, constant operating conditions are assumed unless specified otherwise. For performance evaluation including known variations of conditions, refer to the principles described in section A-10.

Specified validity conditions are the conditions under which the results of the uncertainty statement are valid. Generally the validity conditions are either those specified in the definition of the measurand or are extended conditions.

Industrial measurements often involve extended validity conditions that may differ significantly from those specified in the definition of the measurand. For example, production testing of bearings or spindles may require compromises such as abbreviated warmup time. This extended validity condition involves stating the permitted range of values for the input. Uncertainty budgets associated with extended validity conditions typically have much larger expanded uncertainties than those associated with the conditions specified in the definition of the measurand, since the effects of the extended conditions must be included in the uncertainty evaluation (see ASME B89.7.3.2, Guidelines for the Evaluation of Dimensional Measurement Uncertainty).

For axis of rotation measurements, a typical specification assumes zero influence from the form errors of the test ball. The measurement plan may include use of an error map to attempt to fulfill this specified validity condition. Or, the validity conditions could be extended to include additional uncertainty associated with unknown form errors of an unmapped artifact.

## C-3 INFLUENCE QUANTITIES

The first step in constructing an uncertainty budget is to list all recognized influence quantities that could potentially be significant sources of error. The following influence quantities are typical of those encountered in axis of rotation error motion measurements:

*(a)* incomplete definition of the measurand
*(b)* thermal drift and other slow changes such as variation in air bearing inlet pressure
*(c)* indeterminate location of the polar plot center location
*(d)* number of turns in the test period
*(e)* test ball out-of-roundness, including texture
*(f)* operator influences
*(g)* indicator resolution, including electrical noise and lost motion due to loose joints
*(h)* inexact indicator amplification
*(i)* indicator periodic errors
*(j)* indicator misalignment
*(k)* frequency response of indicators, including structural limitations
*(l)* numerical and computational issues, digital sampling limits, and rounding effects
*(m)* drive influences, speed variations, and direction reversals
*(n)* angular resolution limitation
*(o)* inexact value for eccentricity removal or thermal drift compensation
*(p)* load variation, including unbalance effects

Note that some influence quantities affect synchronous error motion, some asynchronous error motion, and some affect both. Some influence quantities may be grouped together when forming input quantities included in the uncertainty budget. Others may have zero or near-zero amplitude and will not need further consideration. See Section 7 of reference [1] for a categorical list of inputs. The list should be evaluated to avoid counting the same input more than one time.

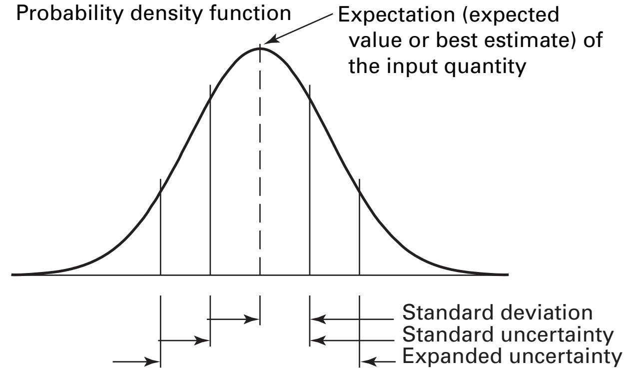
### C-3.1 Narrative Discussion

A narrative discussion supports the decision-making process. The state of knowledge of an input quantity can be modeled by a probability distribution. See Figs. C-1, C-2, C-3, and C-4. Such knowledge is summarized by giving a best estimate for the value, accompanied by an associated standard uncertainty. The best estimate and standard uncertainty are the expectation and standard deviation, respectively, of the probability distribution.

### C-3.2 Distribution Factors

Distribution factors are used to convert input quantity ranges, expressed in units of the measurand, into the associated standard uncertainty components. These uncertainty components are then combined in root-sum-of-squares fashion in order to calculate the combined standard uncertainty associated with the measurement result.

**Fig. C-1 Graphical Representation of Expectation, Standard Deviation, Estimate, and Standard Uncertainty**

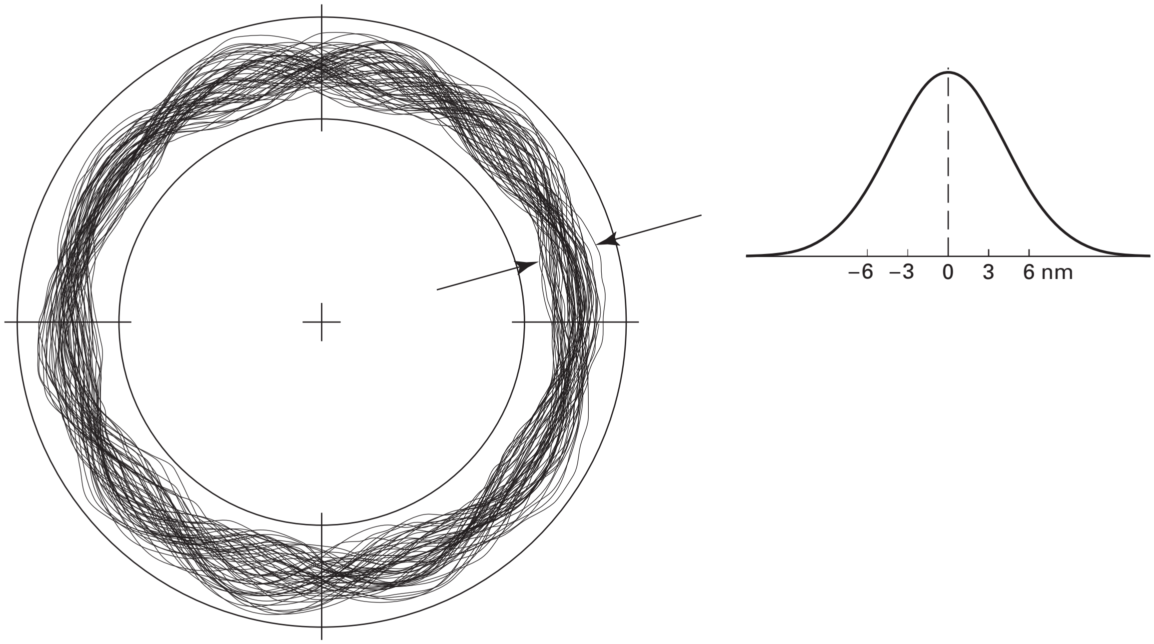



In the following examples, the rationale for selecting a particular distribution is described in sufficient detail so that the reader can judge its appropriateness. Note that the objective of the evaluation is to arrive at the most reasonable range, not some conservative, timid, or pessimistic extra-large estimation that would lead to zero probability of being wrong. A commonly accepted rule is 95% confidence in the end result, so there is a 5% probability that the actual value lies outside the stated range. In other words, do not select a nonnormal distribution just to be on the safe side. Refer to Annex E of reference [13] for further discussion. Distribution factors are selected according to the available information. In the absence of information the reasonable inference is that the potential values are normally distributed. For our purposes, normal applies to any shape that tends to be high in the middle, as opposed to high at the ends (U-shaped) or uniformly distributed (rectangular). It is not necessary to elaborate further than these three basic forms since these are assumptions, and it would be misleading to assign a higher degree of precision than circumstances warrant. Thus use 0.7 not 0.707 for thermal drift, regulator imperfections, speed variation effects, and other cyclical inputs. Use 0.6 not 0.577 for rectangular distributions such as digitization of analog data. And, of course, use 0.5 to convert from two standard deviations to one. An exception would be: if a significant input is known well enough to justify the extra significant figures and is large enough to affect the outcome. See Section 7.2.6 of reference [13] for further discussion of rounding.

### C-3.3 Incomplete Definition of the Measurand

To minimize uncertainty due to incomplete definition of the measurand, describe the setup in detail, including indicator locations and operating conditions such as rpm, number of turns, temperature variation, load, and any other influences deemed to be potentially significant (see Fig. C-5). Uncertainty enters in to the extent that

## Fig. C-2 Visualization of Data as Normal Distribution

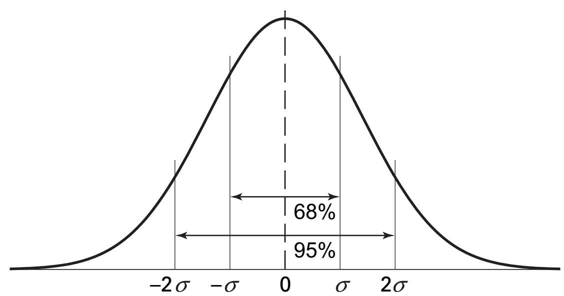



GENERAL NOTES:

(a) This is not random motion, it is the mathematical result of superimposing several noninteger multiples of the rotation frequency. Each turn is deterministic and therefore predictable.

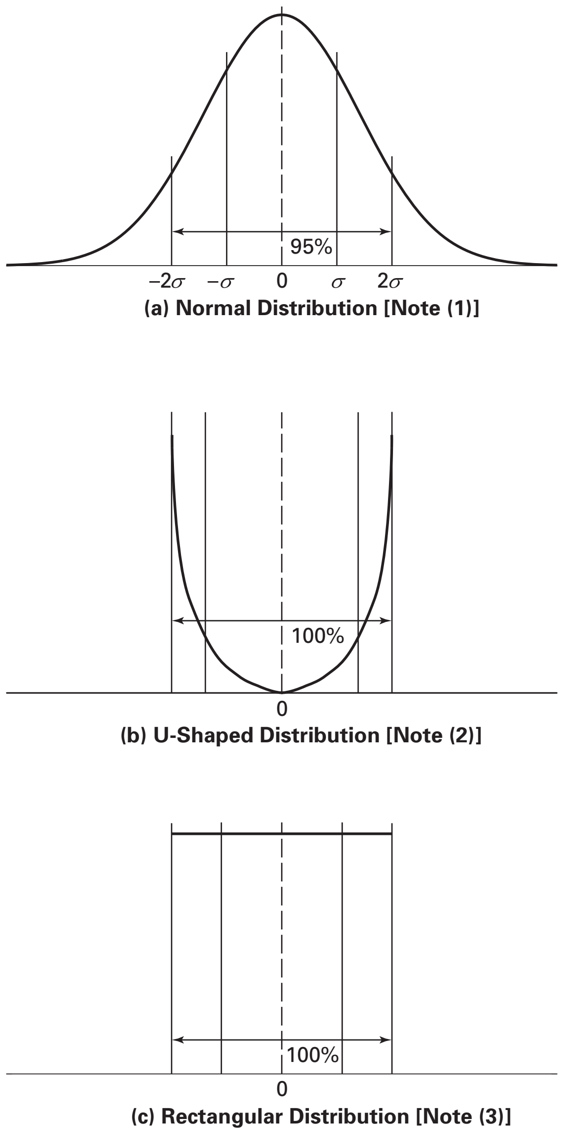
(b) At this orientation angle, the width of the cloud band is 36 nm. The data are normally distributed and 95% of the points fall within a 12 nm zone as shown in the histogram. By definition, this 95% zone is also plus/minus two standard deviations; the standard deviation is therefore 3 nm.

## Fig. C-3 Visualization of Standard Deviation




GENERAL NOTE: In order to combine uncertainty inputs, each is put in terms of one standard deviation. Note that the original 95% range spanned four standard deviations (±2$\sigma$), so that the conversion to one standard deviation is a four-fold reduction.

**Fig. C-4 Visualization of Distribution Factors**




NOTES:

(1) To find the value of one standard deviation, take the 95% range from the midpoint and multiply by 0.5.

(2) To find the value of one standard deviation, take the range from the midpoint and multiply by 0.7.

(3) To find the value of one standard deviation, take the range from the midpoint and multiply by 0.6.

none of these values can be known exactly. However, good measurement practices can reduce incompleteness to a manageable level.

The length of the list depends on the application. It may be exhaustive for troubleshooting complex setups with tight specifications where every possible input is listed in order to show that they have all been considered. Or the list may be short when it is known that the result will be dominated by a few inputs. As a general rule, if the magnitude of an uncertainty component is less than 10% of the largest component in the budget, its contribution to the final result can be assumed to be negligible. Keep in mind that the final value is based on expert judgment and is not inflexible.

### C-3.4 Thermal Drift

Temperature variation is often cyclical; it can be assumed that the temperature is always rising or falling so that it would graph as a time-varying wave, with more time spent near the reversal points and the least time passing though the center of the range. This leads to the assignment of a U-shaped distribution. Even when the test period is shorter than a complete cycle the distribution factor remains 0.7, due to the relatively strong effect of cyclical variations.

Sometimes the observation time is brief enough to reduce to insignificance the magnitude of thermal error, but this is not the case for slow-turning applications or when many revolutions are to be observed as when searching for precessional effects or outliers, or when measuring in nanometers. Depending upon the application low frequency displacements may be filtered out, but at the risk of inadvertent loss of data that is not thermal drift but is in the same range of near-zero frequency. (Precessional errors are typically in this range.) Operator influence may affect the time required to reach thermal equilibrium, especially if any part of the structure was recently handled without gloves. The decision to include or exclude thermal drift from the definition of the measurand should be explicit in the specification and the measurement plan (see section A-5).

### C-3.5 Indeterminate Location of the Polar Plot Center

Incompleteness of information limits the ability to unambiguously identify a central point from which to construct a polar plot and compute synchronous displacements, including eccentricity. This is not an input to asynchronous values; only synchronous and total error motion values are affected. This ambiguity is reduced as the number of turns increases.

The preferred least squares method of locating the polar plot center always produces a mathematically unique location irrespective of the number of data points used. The amount of data required depends principally on the amount of asynchronous error motion: axes with low asynchronous errors and minimal precession converge quickly while the opposite circumstance may require a large number of turns to converge to an acceptable level. Convergence can be thought of as the difference in location between the least squares solution using $N$ turns and the solution using $N + 1$ turns. When this difference is small, data from enough turns has been collected.

### C-3.6 Number of Turns

If the phenomenon of interest can be represented only as a distribution of values or is

## Fig. C-5 Error Motion Test Report Sample Format

Spindle model # __________ S/N __________ Machine model # __________ S/N ________________
Date ______________________ Time __________ Location ____________________ Tested by __________

RPM _____ / _____ Temp _____ / _____ Number of revolutions (or duration) __________
Run-in time before test __________ Warm-up time before data acquisition __________

Axis horizontal __________ or vertical __________
External force:
Magnitude __________
Location __________
Direction __________

**Fixed Sensitive Direction**: Show stator orientation
Structural motion __________

| | Total | Synchronous | Asynchronous |
|---|---|---|---|
| Radial #1 | | | |
| Radial #2 | | | |
| Axial | | / | |
| Face #1 | | | |
| Face #2 | | | |
| Tilt | | | |

**Rotating Sensitive Direction**: Show rotor orientation
Structural motion __________

| | Total | Synchronous | Asynchronous |
|---|---|---|---|
| Radial #1 | | | |
| Radial #2 | | | |
| Axial | | / | |
| Face #1 | | | |
| Face #2 | | | |
| Tilt | | | |

Target type, size, shape, model, and S/N __________________
Probe holder description and locations __________________

Displacement indicator(s) model and S/N:
#1 ____________________
#2 ____________________
#3 ____________________
#4 ____________________
#5 ____________________

Sampling rate ________ Cutoff frequency ________
Sensor resolution ________ Angular resolution ________

Hysteresis of structural loop: target side_____probe side _____
Environmental noise _________ / time period of interest_____

Remarks / Conclusions ______________________________
______________________________
______________________________
______________________________
______________________________

**Polar plot: fixed sensitive direction**

**Polar plot: rotating sensitive direction**

**Thermal drift plot**

**Frequency plot**

**Sketch of setup**

> dependent on one or more parameters, such as time, then the measurands required for its description are the set of quantities describing that distribution or that dependence. (reference [13])

The performance evaluation of an axis of rotation is affected by the number of turns sampled. For example, each successive turn increases the ability to separate synchronous from asynchronous displacements. As data accumulates, the component due to asynchronous motion can be reduced by averaging. Such averaging makes it easier to identify the underlying synchronous components. The improvement increases with the square root of the number of turns. Some effects such as slow precession require many revolutions to be separable from thermal drift. (Precession may be seen when two bearings have different diameters.) Also, occasional events occur that are outside the normal range and the choice of the number of turns to sample may be affected by the need for information about these outliers. Statistical processes such as averaging or other signal processing techniques like frequency analysis become increasing useful in separating asynchronous from synchronous motion as turns accumulate.

An axis of rotation is a dynamic entity, capable of showing variations even under apparently identical conditions. This opens the question of how closely a short test represents long-term operational performance. A way to manage this is to invoke the concept of extended validity conditions. For example, a production line may allow only eight revolutions to sort bearings for radial error motion so the uncertainty evaluation is valid only for tests of this duration, as specified in the measurement plan.

### C-3.7 Test Ball Out-of-Roundness

Test ball out-of-roundness can be directly measured by Donaldson Reversal, which potentially reduces uncertainty down to the repeatability of the setup. Alternatively, the ball manufacturer's specification can be used. Note that form errors of the ball constitute a bias that can add to or subtract from error motion of the axis of rotation.

Assume a normal distribution for the ball manufacturer's specification. If the specified maximum out-of-roundness is 20 nm, it is reasonable to assume that the most likely value is on the order of 10 nm, with a 95% level of confidence that the specific ball lies within a range of ±10 nm. Multiply by 0.5 to get a plus/minus one-sigma value of ±5 nm for the standard uncertainty associated with this input. (This is an example of simplification; the actual distribution shape is not known, but the best assumption is that it is not one of the other two models to which consideration is limited.) In the absence of a polar plot of the equator of the ball, there is no way of knowing if form error adds to or subtracts from the error motion of the axis of rotation; it is only known that it is a bias that influences the uncertainty of an error motion measurement.

In the following examples, the estimated form error of the ball (artifact) adds in a root-sum-square fashion after the uncertainty of the measurement result is obtained (reference [22]).

### C-3.8 Operator Influences

Operator influences include cleaning and lubrication techniques and the ability to recognize potential problems with loose joints, or nonuniform deflection under load. Adequacy of the mounting surfaces may also be an issue, especially for high-speed applications. Also, if the support structure does not provide uniform stiffness, the resulting structural error motion masquerades as two-lobed out-of-roundness of the ball (the VISCOORE effect; see para. A-7.7).

### C-3.9 Indicator Resolution and Noise

The smallest displacement that can be resolved is established by the background noise, which can be measured with a capped-probe test. This reveals the electronic noise of the amplifier as well as the influence of external stray currents and imperfect grounding. Note that random noise (the most common component of noise in well-designed electronic systems) is reduced by averaging or other signal processing techniques as data accumulates so that the least observable indicated displacement is reduced as the number of turns increases. This type of noise has a rectangular distribution. The value should be derived from a peak-to-average measurement of the noise component.

Structural vibration and lost motion from loose joints also limit resolution. Lost motion is checked by lightly pushing to-and-fro on the indicator bracket and on the test ball shank and noting if the original value is restored.

Whatever the source, noise is setup-dependent and is to be evaluated at the time of the test, making it a type A influence (the data distribution shape is directly observable).

Operator influence can be considerable; training, experience, skill, and motivation are required to minimize the influence of setup imperfections.

### C-3.10 Inexact Indicator Amplification

Displacement can be measured with various types of indicators, all of which are subject to this potential error source. Electronic indicators are designed to have a specific number of volts per unit of displacement, called sensitivity or scale factor. Uncertainty from this source is not apt to be a major contributor since it is easy to check with gage blocks. The most likely error is failure to set the correct range but this blunder is unquantifiable and therefore does not lend itself to uncertainty evaluation. Since axis of rotation measurements typically involve small displacements, a 1% or so deviation is probably insignificant. However, when $X$ and $Y$ indicators are used for rotating sensitive direction measurement, unequal sensitivity causes a spurious twice-per-revolution result. Equivalency can be determined by swapping probe positions. Operator influence consists of deciding if in-situ calibration with gage blocks is appropriate.

### C-3.11 Indicator Periodic Errors

An indicator with closely spaced periodic errors could meet the sensitivity specification per calibration with two gage blocks but would introduce errors into the record of continuous displacement during an axis of rotation test. This can be examined by testing at two different eccentricities. If after eccentricity is mathematically removed the polar plots are the same, periodic errors can be regarded as insubstantial contributors to uncertainty.

### C-3.12 Indicator Misalignment

Axis of rotation testing assumes that the indicators are properly aligned and usually this can be confirmed by $X$, $Y$, $Z$ motion of the machine slides. An exception is when eyeball alignment is used for axial error motion tests. The consequence of off-axis location is once per revolution axial displacement that is a combination of fundamental axial error motion and test ball eccentricity measured at a small radial distance. Note that these influences can add or cancel, depending upon the orientation angle of the eccentricity. (As discussed in para. A-7.9, a once-per-revolution axial displacement is classified as an error.)

### C-3.13 Indicator Frequency Response and Structural Limitations

As rotational speed increases, at some point the measurement setup sets a limit to the number of undulations per revolution that can be resolved. The limiting factor could be the bandwidth of the indicator, the natural frequency of the indicator holder, or some other element of the structural loop. This is evaluated on a case-by-case basis. As an example, an indicator bandwidth of 10,000 Hz implies the ability to measure 100 undulations per revolution at 100 revolutions per second (6,000 rpm). However, most setups are not nearly stiff enough to be noninfluencing above a few hundred Hz.

### C-3.14 Computational Influences

Computational influences include rounding, removal of eccentricity, and partitioning of synchronous and asynchronous errors.

### C-3.15 Drive Influences, Speed Variations, and Direction Reversals

The performance of an axis of rotation can be significantly affected by the drive, especially as the load changes or the speed varies. The specification and the measurement plan may call for testing at 1,000 rpm, but the speed during the test may vary, perhaps even within a single turn. Also, in some applications a reversal of direction of rotation can cause a momentary shift of the axis of rotation. Uncertainty enters in to the extent that these potential influences are not included in the definition of the specified measurand.

### C-3.16 Angular Resolution

Construction of a polar plot requires angular information linked to displacement data. Imperfect correlation of these two data streams will result in misidentification of synchronous data as asynchronous. This is most likely to occur when eccentricity is used to generate angular information. In an extreme example of low angular resolution, loops are sometimes seen on polar plots, leading to the impossible conclusion that rotation briefly reversed direction. This blunder would hopefully be recognized and corrected by the operator, since being unquantifiable it cannot be considered in an uncertainty evaluation. The problem is most acute when the eccentricity is not substantially greater than the radial error motion. The ideal solution is to couple an angular position sensor such as an incremental encoder to the axis under test and collect data using the encoder count pulse train as the sample clock. In either case, the angular sample resolution must be at least twice as high as the highest spatial frequency encountered in the sampled signal. This is required to comply with the Sampling Theorem in order to prevent aliasing of out-of-band spatial information into the data of interest.

### C-3.17 Closure Errors and Repeatability

A polar plot of a single turn can be expected to fail to close perfectly; the first and last data points are unlikely to have the exact same radial value. This is the first opportunity to observe asynchronous error motion. The root cause could be thermal drift, in which case continuing revolutions will plot as a spiral, and the decision has to be made to continue or to wait for thermal stability. Or, the cause could be a periodic error at a noninteger frequency, to be revealed in detail as turns accumulate. Best practice is to acquire data for a sufficient number of turns that the closure error is small compared to other errors.

A polar plot containing multiple traces gives an opportunity to assess repeatability. Good practice is to repeat the test and correct repeatability problems due to loose joints or environmental effects. Note that repeatability studies, while valuable, are not equivalent to uncertainty evaluations since it is possible to have large uncertainty but good repeatability.

### C-3.18 Items Not Included In Uncertainty Evaluations

**C-3.18.1 Blunders.** Blunders include transcription errors, which are unquantifiable.

**C-3.18.2 Dirt and Contamination.** A stray particle can be of any size; the big ones are easily identified and removed, but the concern is potential misidentification of a particle as an error motion. Since particle size is unpredictable, it must be assumed that the equipment is properly prepared and that the operator will recognize the presence of a particle and deal with it. In special cases it may be practical to study the effects of various cleaning procedures and make a business decision to accept a certain degree of uncertainty due to abbreviated cleaning, but for most applications it is better to assure adequate cleanliness by testing repeatability of each setup.

**C-3.18.3 Outliers.** An occasional reading may be observed that is so far from the expected value that it appears discrepant. Vibration and dirt are possible causes of these outliers. Or, the outlier may represent relevant displacement and should not be arbitrarily censored. Depending on circumstances the outlying data point can be removed, the series can be re-run, more data can be taken so that the effect of the outlier is swamped, or the measurement can be continued without special treatment. The best practice is to include the outlier treatment policy in a written procedure.

## C-4 CONSTRUCTING AN UNCERTAINTY BUDGET

### C-4.1 List of Significant Input Quantities

It is useful to list the largest contributors first and to limit the calculations to inputs with values no smaller than 10% of the largest input. Figure C-6 illustrates the insignificance of minor inputs. (If in doubt about the influence of a minor input, it can be tested by doubling the value and recalculating the result.) In the following examples, many separate influences have been combined into a single input. For a more rigorous treatment see reference [13].

### C-4.2 Calculating Combined Standard Uncertainty

The various uncertainty components are squared and added. The square root of this is the Combined Standard Uncertainty, $u$, which defines an interval about the stated value that would be expected to yield a 68% probability that the stated value is correct. However, the desired probability is 95%, so a Coverage Factor, $k = 2$, is applied to give the Expanded Uncertainty, $U$. See Fig. C-1.

Note the relative influence of the largest value. This procedure intentionally emphasizes the larger terms and reduces the impact of minor terms.

### C-4.3 Assessing the Uncertainty Statement

The combined standard uncertainty should encompass a range deemed reasonable by persons skilled in the specific case, keeping in mind that experts often disagree by 25% due to differing assumptions and data, see reference [15]. A level of confidence of 95% indicates the willingness of the evaluator to act as if the value of the measurand lies within $\pm U$ of the stated best estimate, accepting a 5% risk that this may be wrong. Such an action could be visualized as a wager at appropriate odds; in this case the odds for a fair bet would be 19:1. Arriving at this confidence level may require an iterative process of reevaluation of assumptions, estimations, and data until the result satisfies the well-considered expert judgment of the evaluator.

### C-4.4 Signature and Date

A signature on the evaluation report establishes credibility and indicates to the user that the author is willing to discuss the result and the process by which it was generated. At this point the information may become the basis for a business decision such as whether further testing is economically appropriate (see reference [15]).

## C-5 UNCERTAINTY EVALUATION EXAMPLES

Two hypothetical examples will be discussed: one for a universal milling machine and another for an ultraprecision lathe.

### C-5.1 Hypothetical Milling Machine Example

This example represents a test of a typical machine tool. Assume a rotating sensitive direction radial error motion test at 600 rpm for 32 turns after 1 hr warmup. The location of the indicators is to be recorded with a sketch so that the definition of the planned measurand will be sufficiently complete. Figure C-5 suggests a format for a test report, and Fig. A-8 suggests a format for a sketch. See Table C-1 for the uncertainty evaluation.

**C-5.1.1 Input Quantities.** For completeness, all reasonable input quantities appear in the budget to show that they have been considered. However, only a few will affect the result, the others are included in this example to show which distribution factor was deemed appropriate and to illustrate the insignificant contribution of minor inputs due to the effect of summing the squares. Some are even assigned zero value and are shown only as placeholders.

*(a) Test Pin Out-of-Roundness.* Assume a gage pin having no documentation for out-of-roundness. Also assume a 95% probability that it is between zero and 40 μin. out-of-round. To select the distribution factor, consider whether it is reasonable to assume that the true value, if it could be known, is about as likely to be near the boundaries as it is to be near the center; if not, this rules out the uniform and U-shaped distributions, leaving the normal distribution as the clear choice. Since the 95% probability represents plus/minus two standard deviations, multiply 20 μin. (half the range) by 0.5 to arrive at 10 μin. for the value of one standard deviation.

*(b) Drive Influence and Load Variations, Including Unbalance Effects.* If speed varies within a single turn, some

**Fig. C-6 Visualization of Bias Inherent in Contributors to Error Motion Measurement Results**

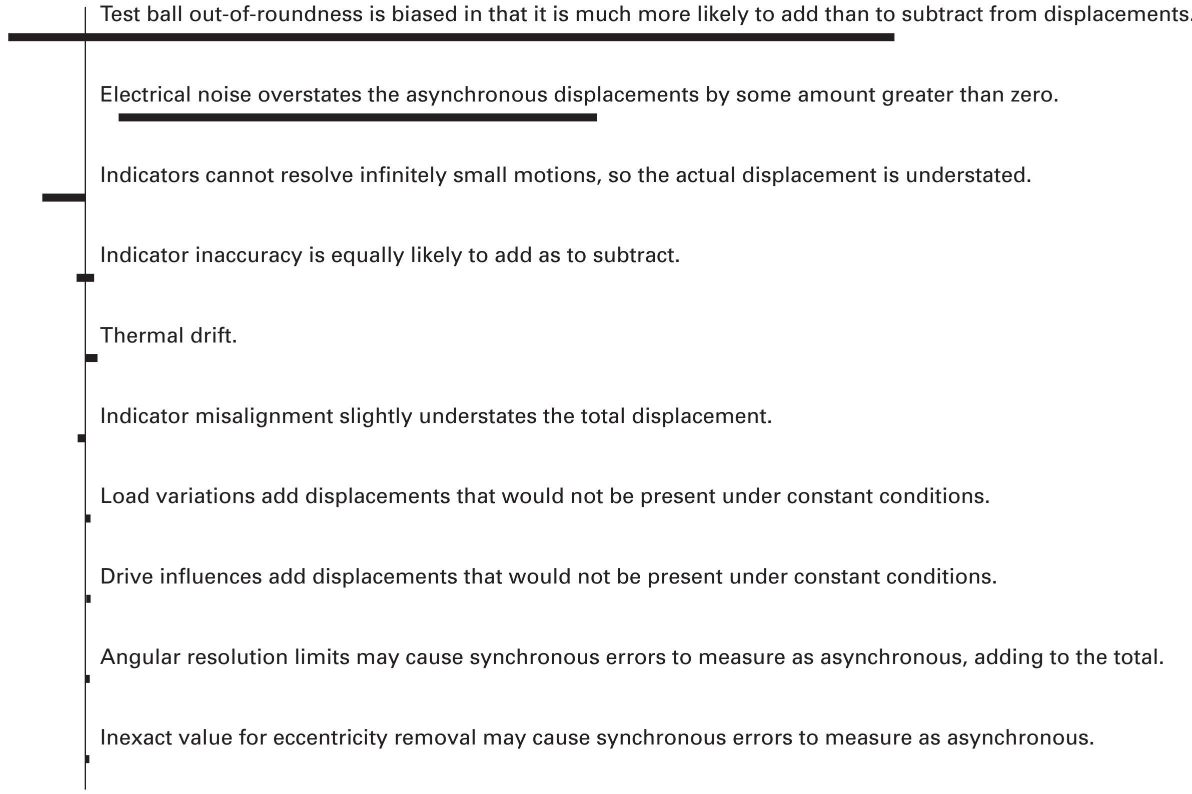



**Table C-1 Uncertainty Evaluation Example for Hypothetical Milling Machine**

| Name of Uncertainty Input | μin. | Distribution | Factor | Result | Squared |
|---|---|---|---|---|---|
| Drive influence, including speed variations | 6 | U-shaped | 0.7 | 4.2 | 17.64 |
| Angular resolution | 4 | Normal | 0.5 | 2 | 4 |
| Thermal drift and other slow changes | 3 | U-shaped | 0.7 | 2.1 | 4.41 |
| Indicator resolution and noise | 3 | Rectangular | 0.6 | 1.8 | 3.24 |
| Indeterminate location of polar plot center | 1 | U-shaped | 0.7 | 0.7 | 0.49 |
| Indicator sensitivity and periodic errors | 1 | Normal | 0.5 | 0.5 | 0.25 |
| Indicator misalignment | 0.5 | U-shaped | 0.7 | 0.35 | 0.1225 |
| Indicator frequency response limitations | 0.1 | Normal | 0.5 | 0.05 | 0.0025 |
| Sum of the squared values | | | | | 30.155 |
| Square root (68% confidence level) | | | | | 5.5 |
| $k = 2$ (for 95% confidence) | | | | | 11 |
| Expanded uncertainty of the measurement result | 1 | Normal | 0.5 | 5.5 | 30.25 |
| Gage pin out-of-roundness | 20 | Normal | 0.5 | 10 | 100 |
| Sum of the squared values | | | | | 130 |
| Square root (68% confidence level) | | | | | 11.4 |
| $k = 2$ (for 95% confidence) | | | | | 23 |

synchronous displacements will appear asynchronous due to low angular resolution resulting from pin eccentricity used here in lieu of an angular encoder. An arbitrary value of 12 μin. is assigned in this example. This is assumed to be 100% asynchronous, with 95% of the additional displacement contained in a zone ±6 μin. wide. This example assumes constant load but other cases may be affected by belt tension, unbalance, or tool force. Note that unbalance produces a rotating force that explores the rotational symmetry of the spindle support structure, producing a twice-per-revolution displacement as shown in Fig. A-19. Also note that the definition of the measurand must specify the structural loop so that spindle error motion can be distinguished from structural error motion (see paras. A-7.6 and A-8.2).

(c) *Thermal Drift and Other Slow Changes.* In this example the test period was only about 3 sec so thermal drift was assumed to be insignificant. However, the spindle has a potential for a precessional error that may take many turns to complete a cycle. Therefore the thermal drift compensation feature was not engaged. Thus the 3 μin. value was chosen as a likely value for the uncertainty arising from imperfect ability to partition an axis shift due to thermal drift from radial error motions in this frequency range (see sections A-5 and A-10).

(d) *Indicator Resolution, Including Electrical Noise and Lost Motion Due to Loose Joints.* In this example this is deemed to be not a significant factor since the probe holders were assumed to be robustly supported and the one-microinch nominal resolution of the indicator is an insignificant percentage the measured error motion. A hypothetical capped probe test showed 6 μin. increase in electrical noise with the motor running. This was treated as a normal distribution with ±3 μin. encompassing 95% of the signal.

(e) *Indeterminate Identification of the Center of the Polar Plot.* As the number of turns increases the calculated center point moves slightly, but at a diminishing rate of change. This is especially notable in the early turns and in the presence of low-frequency displacements such as cage rotation frequency or half-speed whirl (see Nonmandatory Appendix A, paras. A-7.4 and A-7.8). Uncertainty about the location of the center sets a limit to the ability to remove eccentricity and this in turn affects the value for total error motion. In this case, 32 turns was sufficient. Assume that the measurement plan designated which of the four recognized methods was to be used (see section A-15).

(f) *Incomplete Definition of the Measurand.* In this example the setup is well described and it is assumed that further refinement would not significantly affect the outcome. Note that the definition of the measurand was axis of rotation, not bearing error motion or spindle error motion; these involve different structural loops. See para. A-7.5, Static Error Motion Measurement.

(g) *Number of Turns in the Test Period.* The number of turns used in the computation of synchronous and asynchronous error motion affects how inputs such as thermal drift and other time-varying inputs appear in the final results. This is particularly so with ball-bearing spindles because as more turns are added to the data set, the synchronous error motion will tend to decrease while the asynchronous components will increase.

(h) *Angular Resolution.* In this example angular information was derived from the eccentricity of the pin. This technique has less resolution than an angular encoder but is deemed to be adequate in this case. The line item is included in the uncertainty budget to show that it has been considered.

(i) *Frequency Response of Indicators, Including Structural Limitations.* This was not a significant input since the capacitive gage is rated at 10,000 Hz, which would be 1,000 cycles per revolution at 600 rpm, which is beyond the range of interest here. The hypothetical indicator holders were suffciently robust to measure the frequency range of interest.

(j) *Indicator Sensitivity and Periodic Errors.* Two indicators are used for this test and they must be matched to avoid a twice-per-revolution spurious displacement reading. For noncontact gages, tip damage can cause mismatch. In this example, it is assumed that the operator was alert to this possibility and compared them offline or by swapping them in their holders so that this influence is deemed insignificant.

(k) *Indicator Misalignment.* The $X$ and $Y$ travel of the mill in this example were used to confirm that both indicators were positioned to intersect the axis of rotation; thereby this influence is deemed to be insignificant.

(l) *Numerical and Computational Issues, Digital Sampling Limits, Rounding Effects, Indeterminate Value for Eccentricity Removal, or Thermal Drift Compensation.* These influences are combined into one input that is deemed to be less than 10% of the amplitude of the largest input and is only shown here to show that it has been considered.

(m) *Operator Influence.* Operator influence is unquantified but assumed to minimize uncertainty to the extent that care was taken to make in-situ tests for obvious errors and shortcomings.

### C-5.2 Hypothetical Lathe Example

Assume a fixed sensitive direction radial error motion test at 6,000 rpm for 512 turns after 1 hr warmup, using a 1 in. diameter test ball and an angular encoder with 2,048 counts, with data taken 1,024 times per revolution. A sketch of the setup shows the axial location of the ball and the orientation of the indicator. Since this machine is to produce optical-quality surfaces, units are nanometers and close consideration is given to all potential influences. See Table C-2.

(a) *Remarks on the Lathe Uncertainty Budget.* Inputs were assigned reasonable values for the purpose of demonstration. The list includes small-magnitude inputs as illustrations, but in practice minor inputs may be left out since the larger contributors dominate the result.

**Table C-2 Uncertainty Evaluation Example for Hypothetical Ultraprecision Lathe**

| Name of Uncertainty Input | nm | Distribution | Factor | Result | Squared |
|---|---|---|---|---|---|
| Drive influence, including speed variations | 3 | U-shaped | 0.7 | 2.1 | 4.41 |
| Electrical noise | 2 | Rectangular | 0.6 | 1.2 | 1.44 |
| Thermal drift and other slow displacements | 2 | U-shaped | 0.7 | 1.4 | 1.96 |
| Load variations, including unbalance | 1.5 | U-shaped | 0.7 | 1.05 | 1.1025 |
| Indicator resolution | 0.15 | Rectangular | 0.6 | 0.09 | 0.0081 |
| Indicator misalignment | 0.10 | U-shaped | 0.7 | 0.07 | 0.0049 |
| Angular resolution | 0.05 | Normal | 0.5 | 0.025 | 0.000625 |
| Indicator frequency response limitations | 0.04 | Normal | 0.5 | 0.02 | 0.0004 |
| Indeterminate location of polar plot center | 0.03 | U-shaped | 0.7 | 0.021 | 0.000441 |
| Indicator sensitivity and periodic errors | 0.02 | Normal | 0.5 | 0.01 | 0.0001 |
| Computational limitations | 0.01 | Rectangular | 0.6 | 0.006 | 0.000036 |
| Sum of the squared values | | | | | 8.927102 |
| Square root (68% confidence level) | | | | | 3 |
| $k = 2$ (for 95% confidence) | | | | | 6 |
| Expanded uncertainty of the measurement result | 6 | Normal | 0.5 | 3 | 9 |
| Test ball out-of-roundness | 12 | Normal | 0.5 | 6 | 36 |
| Sum of the squared values | | | | | 45 |
| Square root (68% confidence level) | | | | | 6.7 |
| $k = 2$ (for 95% confidence) | | | | | 13.4 |

The example yields an expanded uncertainty of 12.6 nm. This is interpreted as a 95% probability that the true value, if it could be known, lies within a range of ±12.6 nm of the stated measurement result, assuming a normal distribution. However, as shown in Fig. C-6 there is an inherent bias toward overstating the displacements attributed to the error motion of the axis of rotation. Each case is different but for this example it is reasonable to assert that the most likely true value is substantially less that the stated measurement result since the primary input (test ball out-of-roundness) was unlikely to have a form error that compensated the radial error motion of the axis of rotation. However, as previously noted, a different set of conditions, such as a larger number of turns or a higher speed, will yield different values.

*(b) Test Ball Out-of-Roundness, Including Texture.* Assume a ball having no documentation for out-of-roundness. Also assume a 95% probability that it lies between zero and 25 nm, with the most likely value deemed to be 12 nm. To select the distribution factor the following is considered: whether it is reasonable to assume that the true value, if it could be known, is just as likely to be zero as it is to be 25 nm; if not, this rules out the uniform and U-shaped distributions, leaving the normal distribution as the clear choice. Since the 95% probability represents plus/minus two standard deviations, 12 nm (half the range) is multiplied by 0.5 to arrive at 6 nm for the value of one standard deviation. Note that form errors of the ball create an unknown bias that could add or subtract from the synchronous and total error motions of the axis of rotation but will only affect asynchronous values to the extent that a scratch will show as an asynchronous error if the angular readout varies.

*(c) Electrical Noise and Indicator Resolution.* A hypothetical capped probe test showed 12 nm increase in electrical noise with the motor running. This was treated as a normal distribution with ±6 nm encompassing 95% of the signal.

*(d) Indicator Resolution, Including Structural Limitations.* The capacitive gaging system has a sensitivity of 500 nm/V and the data acquisition system resolution is 305 μV per least significant bit so the gaging system resolution is 0.153 nm/LSB (least-significant bit). The distribution is rectangular. The indicator is robustly held in the toolpost, and the rest of the structural loop is stiff enough to be noninfluencing.

*(e) Indicator Sensitivity and Periodic Errors.* For noncontact gages, tip damage can cause a change in the voltage output for a given displacement. The radius of curvature of the target also has a potential influence. In this example it is assumed that the operator was alert to these possibilities and calibrated the gage off-line using gage blocks. Alternatively, an on-the-machine affirmation could be performed by commanding a specific radial motion of the machine slide and comparing the results, assuming that the lathe metrology system is traceable to the SI meter. In this example, a tiny value was assigned in order to show the selection of a normal distribution for the error associated with this input quantity. Note that if calibration had revealed a known bias, a correction would be applied, and this input would

apply to the uncertainty of this bias. Note that an inexact value for sensitivity would not affect the shape of the polar plot or the frequency proportions, only the numerical values would be affected. However, this does not apply to periodic sensitivity variations; these cyclical errors would show as synchronous error motions. An in-situ test can compare results at different eccentricities of the test ball; to the extent that synchronous errors are constant, the possibility of periodic errors can be discounted.

*(f) Thermal Drift and Other Slow Changes Such as Variation in Air Bearing Inlet Pressure.* The test period was only about 5 sec, and the machine was warmed up so thermal drift was slight but not zero. Thermal drift influence can be removed by filtering out low-frequency displacements but at the risk of inadvertently removing error motion contributors such as a precessional error that may take many turns to be revealed. In this hypothetical test, the frequency plot did not reveal any significant peaks in the smooth decline attributed to thermal drift. This justified the use of a thermal drift compensation filter. Thus the 0.1 nm value was chosen as a likely value for the uncertainty arising from imperfect ability to separate thermal drift from other low-frequency displacements.

Note that thermal drift, if unfiltered, adds to total error motion and perhaps to asynchronous error motion, but not significantly to synchronous error motion. It plots as a spiral on the polar display and as a smooth curve on the low end of the frequency plot. Pressure variations in fluid-film bearings may cycle like thermal drift and are best evaluated with explicit tests.

*(g) Indicator Misalignment.* A radial error motion test assumes that the indicator is perpendicular to the axis of rotation and aimed at the center of the test ball. This can be confirmed with measurements using the lathe slide travel.

Referring to Fig. A-2, note the consequences of measurement above or below center when the target diameter is small. For this example the ball diameter is large enough and the potential for misalignment is small enough that the consequences can be deemed to be insignificant. For the sake of illustration this example selects the U-shaped distribution on the basis that the influence is zero when alignment is perfect, and uncertainty increases with increasing potential for misalignment.

*(h) Load Variations, Including the Effect of Unbalance.* The measurement plan assumes constant conditions, but in this ultra-precision application even small variations may be significant. The effect can be visualized as half of a U-shaped distribution with a factor of 0.7, noting that the effect adds to but does not subtract from error motion. (In this example it is assumed that the rotating structural elements do not exhibit variation in gravitational sag as discussed in Nonmandatory Appendix A, section A-7.)

*(i) Drive Influence, Including the Effect of Speed Variations.* This can be significant since speed variations can shift the axis of rotation. Also, the drive may induce radial force variations.

*(j) Angular Resolution.* Angular data are produced by an encoder on the spindle rotor, enabling accurate measurements regardless of speed variations. Since the 2,048-count encoder was triggered 1,024 times per revolution, up to 512 undulations per revolution can be adequately measured. The 512-turn test period produced such a vast amount of data that the frequency peaks were very sharply delineated, so separating synchronous from asynchronous frequencies was not a problem. Nevertheless, a tiny value was assigned to illustrate the distribution. A normal distribution is assigned since there is no reason to assume otherwise, but note that for this example the initial value is so small that the choice of a different factor would have no effect on the final result.

*(k) Indeterminate Value for Removal of Eccentricity or Thermal Drift.* Eccentricity is removed by deducting the once-per-revolution frequency; accuracy depends upon sharp delineation of this frequency from the nearby peaks. For the sake of illustration this example selects the U-shaped distribution on the basis that the influence is zero when delineation is perfect and uncertainty increases to the extent that the peak suffers from spectral leakage.

*(l) Indicator Frequency Response Limitations, Including Structural Limitations.* This is not a significant factor since the capacitive gage is rated at 10,000 Hz, which is equivalent to 100 cycles per revolution at 6,000 rpm. A tiny value was assigned to illustrate the distribution. Indicator linearity over the range of interest is not significant due to the small range used. Table travel was used to affirm that the proper range was selected to avoid a ten-to-one blunder. A small value was assigned to illustrate the distribution.

*(m) Computational Limitations.* A rectangular distribution is appropriate because the true value has equal probability of lying anywhere between the bounds set by adjacent digital code transitions.

*(n) Incomplete Definition of the Measurand.* Incomplete definition of the measurand is not a problem since the setup is adequately described, including a sketch showing angular orientation marks on the stator, rotor, and test ball. It is included in the budget as a placeholder but is unquantifiable so no attempt is made to assign a range or a distribution factor. Note that the measurand in this case is not a prediction of results that may occur under any other conditions. It would be acceptable to define a measurand for other validity conditions but that is not what this example illustrates.

*(o) Number of Turns in the Test Period.* This influence affects several inputs such as thermal drift and the ability

to separate synchronous frequencies from nearby asynchronous frequencies. The choice of the number of turns to sample affects uncertainty in two ways: by increasing the resolution of a particular measurement and by increasing confidence that the sample represents typical performance over an extended duration. But note that this budget applies only to one specific measurement.

*(p) Operator Influence.* Operator influence was assumed to minimize uncertainty to the extent that care was taken to align the probe, clean the ball, and make in-situ sanity checks for blunders and obvious errors. It is shown as a line item but without an assigned value since it applies to multiple inputs.

*(q) Improvements.* For this example the best improvement would be to self-calibrate the ball, thus reducing the expanded uncertainty from 12 nm to 2.8 nm.

*(r) Evaluate the Result.* Note that normal distribution is assumed in the final result since this is standard practice for expressing the result of an uncertainty evaluation. Alternatively, a biased distribution may be selected to reflect the asymmetry inherent in the effects of most inputs, see Fig. C-6. See F.2.4.4 of reference [13].

The result is required to correspond to the informed opinion of the author of the evaluation. The desired result is a numerical value for the range that best represents a personal level of confidence of 95%. In other words, the expectation is that there is one chance in twenty that the true value of the measurand, if it could be known, would be outside the stated range (see para. C-4.4).

# APPENDIX D
# REFERENCES

In the case of ANSI Standards, refer to the most recent edition.

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*[2]* ANSI B89.6.2, Temperature and Humidity Environment for Dimensional Measurement

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*[4]* Tlusty, J., "System and methods of testing machine tools," *Microtechnic*, 13, 1959

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