

Design Calculations for Sucker Rod Pumping Systems (Conventional Units)

API TECHNICAL REPORT 11L
FIFTH EDITION, JUNE 2008



Design Calculations for Sucker Rod Pumping Systems (Conventional Units)

Upstream Segment

API TECHNICAL REPORT 11L
FIFTH EDITION, JUNE 2008



Special Notes

API publications necessarily address problems of a general nature. With respect to particular circumstances, local, state, and federal laws and regulations should be reviewed.

API is not undertaking to meet the duties of employers, manufacturers, or suppliers to warn and properly train and equip their employees, and others exposed, concerning health and safety risks and precautions, nor undertaking their obligations under local, state, or federal laws.

Information concerning safety and health risks and proper precautions with respect to particular materials and conditions should be obtained from the employer, the manufacturer or supplier of that material, or the material safety data sheet.

Neither API nor any of API's employees, subcontractors, consultants, committees, or other assignees make any warranty or representation, either express or implied, with respect to the accuracy, completeness, or usefulness of the information contained herein, or assume any liability or responsibility for any use, or the results of such use, of any information or process disclosed in this publication. Neither API nor any of API's employees, subcontractors, consultants, or other assignees represent that use of this publication would not infringe upon privately owned rights.

Classified areas may vary depending on the location, conditions, equipment, and substances involved in any given situation. Users of this technical report should consult with the appropriate authorities having jurisdiction.

Users of this technical report should not rely exclusively on the information contained in this document. Sound business, scientific, engineering, and safety judgement should be used in employing the information contained herein.

API publications may be used by anyone desiring to do so. Every effort has been made by the Institute to assure the accuracy and reliability of the data contained in them; however, the Institute makes no representation, warranty, or guarantee in connection with this publication and hereby expressly disclaims any liability or responsibility for loss or damage resulting from its use or for the violation of any authorities having jurisdiction with which this publication may conflict.

Any manufacturer marking equipment or materials in conformance with the marking requirements of an API standard is solely responsible for complying with all the applicable requirements of that standard. API does not represent, warrant, or guarantee that such products do in fact conform to the applicable API standard.

API publications are published to facilitate the broad availability of proven, sound engineering and operating practices. These publications are not intended to obviate the need for applying sound engineering judgment regarding when and where these publications should be utilized. The formulation and publication of API publications is not intended in any way to inhibit anyone from using any other practices.

Nothing contained in any API publication is to be construed as granting any right, by implication or otherwise, for the manufacture, sale, or use of any method, apparatus, or product covered by letters patent. Neither should anything contained in the publication be construed as insuring anyone against liability for infringement of letters patent.

This document was produced under API standardization procedures that ensure appropriate notification and participation in the developmental process and is designated as an API standard. Questions concerning the interpretation of the content of this standard or comments and questions concerning the procedures under which this standard was developed should be directed in writing to the director/general manager of the Upstream Segment, American Petroleum Institute, 1220 L Street, N.W., Washington, D.C. 20005. Requests for permission to reproduce or translate all or any part of the material published herein should also be addressed to the director. A catalog of API publications and materials is published annually and updated quarterly by API, 1220 L Street, N.W., Washington, D.C. 20005.

All rights reserved. No part of this work may be reproduced, stored in a retrieval system, or transmitted by any means, electronic, mechanical, photocopying, recording, or otherwise, without prior written permission from the publisher. Contact the Publisher, API Publishing Services, 1220 L Street, N.W., Washington, D.C. 20005.

Copyright © 2008 American Petroleum Institute

Foreword

This technical report is under the jurisdiction of the API Standards Subcommittee on Field Operating Equipment (API SC11). This technical report replaces Recommended Practice 11L, *Design Calculations for Sucker Rod Pumping Systems*, 4th Edition. This technical report shall become effective on the date printed on the cover but may be used voluntarily from the date of distribution.

Nothing contained in any API publication is to be construed as granting any right, by implication or otherwise, for the manufacture, sale, or use of any method, apparatus, or product covered by letters patent. Neither should anything contained in the publication be construed as insuring anyone against liability for infringement of letters patent.

Suggested revisions are invited and should be submitted to the Standards Department, API, 1220 L Street, NW, Washington, D.C. 20005.

Contents

	Page
1 Introduction	1
2 Validity of Calculations	1
3 Symbols and Formulas	2
3.1 Symbols, with Units Where Applicable, Used in the Technical Report.	2
3.2 Formulas.	5
4 Design Procedure	6
Annex A Discussion of Non-dimensional Parameters.	21
Bibliography	31
Figure	
3.1 Basic Dynagraph Card	5
4.1 S_p/S Plunger Stroke Factor	15
4.2 F_1/Sk_r Peak Polished Rod Load	16
4.3 F_2/Sk_r Minimum Polished Rod Load	17
4.4 $2T/S^2K_r$ Peak Torque	18
4.5 F_3/Sk_r Polished Rod Horse Power.	19
4.6 T_a , Adjustment for Peak Torque for Values of W_{rf}/Sk_r Other Than 0.3	20
A.1 Percentage Increase in Fundamental Frequency 1 $1/8$ in., 1 in., and $7/8$ in. Three-way Taper String.	24
A.2 Percentage Increase in Fundamental Frequency 1 in., $7/8$ in., and $3/4$ in. Three-way Taper String	25
A.3 Percentage Increase in Fundamental Frequency $7/8$ in., $3/4$ in., and $5/8$ in. Three-way Taper String	26
A.4 Percentage Increase in Fundamental Frequency $3/4$ in., $5/8$ in., and $1/2$ in. Three-way Taper String	27
A.5 Four-way Taper Strings Percent Increase in Fundamental Frequency	28
A.6 Percentage Increase in Fundamental Frequency Specific Rod Combinations	29
Tables	
4.1 Rod and Pump Data (See 4.5)	10
4.2 Tubing Data	13
4.3 Sucker Rod Data	13
4.4 Pump Constants	14

Design Calculations for Sucker Rod Pumping Systems (Conventional Units)

1 Introduction

1.1 In 1954, a group of users and manufacturers of sucker rod pumping equipment undertook a study in depth of the many complex problems associated with this means of lifting fluid from a well. To control and direct the effort, Sucker Rod Pumping Research, Incorporated, a non-profit organization was created. The services of Midwest Research Institute at Kansas City were retained to perform the work necessary to achieve the objectives of the organization.

1.2 The design calculations are based on correlations of the test data that were obtained during the research phase of the project. Sucker Rod Pumping Research, Inc., before its dissolution, released these correlated test results to the American Petroleum Institute for publication. This technical report for the design calculations of sucker rod pumping systems using conventional pumping units is based on these correlations.

1.3 Three discussions included in the final reports of test results by Midwest Research Institute have been published for permanent reference in API *Drilling and Production Practice* (1968). These discussions include the following topics:

- a) vibration characteristics of sucker-rod strings;
- b) physical characteristics of sucker rods;
- c) dimensional analysis of sucker-rod pumping systems.

1.4 A catalog of over 1100 dynamometer cards derived from the electronic analog computer for many combinations of the independent non-dimensional parameters F_o/Sk_r and N/N_o was included in the material released to API by Sucker Rod Pumping Research, Inc. This catalog has been printed as API 11L2, *Catalog of Analog Computer Dynamometer Cards*.

1.5 Two computer programs have been developed from the data in API 11L. One program developed tabular material calculated for depths of 2000 ft to 12,000 ft in increments of 500 ft and for production rates of 100 barrels/day to over 1500 barrels/day in varying increments. Rod and pump size combinations as listed in Table 4.1 were used, except for the elimination of rod no. 88 and rod no. 99. All API stroke lengths are covered. This material is printed as API 11L3, *Sucker Rod Pumping System Design Book*.

1.6 The other program developed a series of curves for selecting beam pumping units for depths of 1600 ft to 9900 ft and various rates of production and combinations of rod sizes, pump sizes, and speeds. Generally, the limiting factor on the curve is the peak torque rating of the unit. This material was printed as API 11L4, *Curves for Selecting Beam Pumping Units* (withdrawn from publication in 2008).

2 Validity of Calculations

2.1 In a majority of cases, it has been found that the values calculated by the following method have been in reasonably close agreement with measured values. Several groups conducting independent surveys have found this design method to give better results than other methods formerly used. However, since this method is based on the best interpretations of average values, the actual conditions found in individual cases may not always yield valid predictions of pumping system performance.

2.2 The designer must realize that there are a number of unusual conditions which may be present in a well that could cause misleading conclusions from these design calculations. Some of these unusual conditions are:

- a) slanted or crooked holes;
- b) very viscous fluid;
- c) excessive sand production;
- d) excessive gas production through the pump; and
- e) well flowing-off.

2.3 The research work was limited to simulated problems in which the tubing was assumed as being anchored at the pump. Therefore, the test results reflected only this condition. However, because of the many known cases in which tubing is unanchored, a formula is included which, experience indicates, will give a very close approximation of relative plunger travel with respect to the pump. This value is identified with the symbol S_p . Examination of the formula will reveal that the contraction of the tubing caused by the transfer of the fluid load from the standing valve to the traveling valve is subtracted from the calculated plunger stroke. It is realized that this formula is highly simplified and not mathematically correct, but it is close enough for practical application.

2.4 These design calculations may be used with confidence when applied to the broad category of average, normal pumping wells fitting the assumed conditions outlined in Annex A. Unusual, conditions not fitting the assumptions will cause deviations from calculated performance. The designer must recognize this fact even though he cannot calculate quantitative values for this deviation.

3 Symbols and Formulas

3.1 Symbols, with Units Where Applicable, Used in the Technical Report

3.1.1

CBE

counterweight required, lb

3.1.2

D

plunger diameter, in.

3.1.3

E_r

elastic constant—rods, in./lb-ft (Table 4.1, Column 4).

NOTE E_r represents the inches of elongation caused by the application of a load of 1 lb to a rod 1 ft in length

3.1.4

E_t

elastic constant—tubing, in./lb ft (see Table 4.2, Column 5)

NOTE E_t represents the inches of elongation caused by application of a load of 1 lb to a section of tubing 1 ft in length.

3.1.5

F_1

PPRL factor (see Figure 3.1)

3.1.6 F_2

MPRL factor (see Figure 3.1)

3.1.7 F_c

frequency factor (see Table 4.1, Column 5)

3.1.8 F_o

differential fluid load on full plunger area, lb (see Figure 3.1)

3.1.9 F_3

PRHP factor

3.1.10 G

specific gravity of produced fluid

3.1.11 H

net lift, ft

3.1.12 L

pump depth, ft

3.1.13 $MPRL$

minimum polished rod load, lb (see Figure 3.1)

3.1.14 N pumping speed, SPM **3.1.15** N_o natural frequency of straight rod string, SPM **3.1.16** N'_o natural frequency of tapered rod string, SPM **3.1.17** PD

pump displacement, barrels/day

3.1.18 $PPRL$

peak polished rod load, lb (see Figure 3.1)

3.1.19*PRHP*

polished rod horsepower

3.1.20*PT*

peak crank torque, lb-in.

3.1.21*S*

polished rod stroke, in. (see Figure 3.1)

3.1.22*Sk_r*

lb of load necessary to stretch the total rod string an amount equal to the polished rod stroke, S

3.1.23*S_p*

bottom hole pump stroke, in.

3.1.24*SPM*

strokes/minute

3.1.25*T*

crank torque, lb-in.

3.1.26*T_a*torque adjustment constant for values of W_{rf}/Sk_r other than 0.3**3.1.27***W*

total weight of rods in air, lb

3.1.28*W_r*

average unit weight of rods in air, lb/ft (see Table 4.1, Column 3)

3.1.29*W_{rf}*

total weight of rods in fluid, lb (see Figure 3.1)

3.1.30 $1/k_r$

elastic constant—total rod string, in./lb

NOTE k_r is the spring constant of the total rod string and represents the load in lb required to stretch the total rod string 1 in.

3.1.31 $1/k_t$

elastic constant—unanchored portion of tubing string, in./lb

NOTE k_t is the spring constant of the unanchored tubing and represents the load in pounds required to stretch the unanchored portion of the tubing, between the anchor and the pump, 1 in.

3.2 Formulas

An understanding of the formulas utilized for the solution of sucker rod pumping problems will be gained by referring to Figure 3.1. The variables F_0 , F_1 , F_2 , W_{rf} , and S are illustrated with this figure.

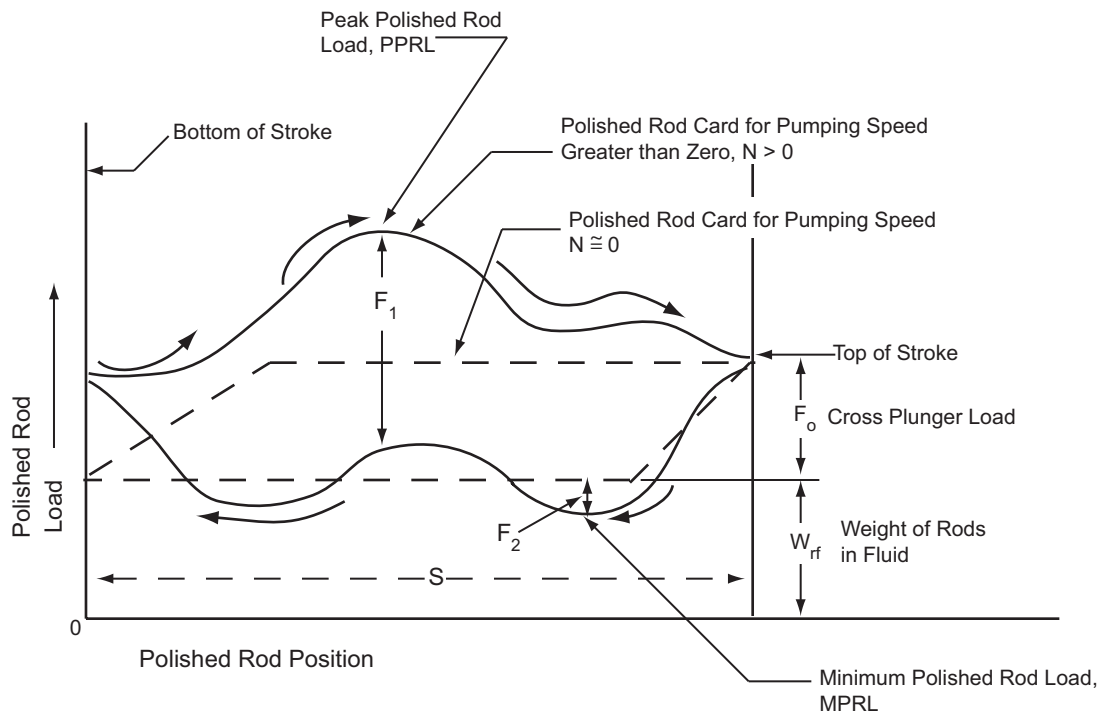


Figure 3.1—Basic Dynagraph Card

a) At pumping speed, $N \approx 0$

$$\begin{aligned} \text{peak polished rod load,} & \quad PPRL = W_{rf} + F_0 \\ \text{minimum polished rod load,} & \quad MPRL = W_{rf} \end{aligned}$$

b) For pumping speed, $N > 0$

$$\begin{aligned} \text{peak polished rod load,} & \quad PPRL = W_{rf} + F_1 \\ \text{minimum polished rod load,} & \quad MPRL = W_{rf} - F_2 \end{aligned}$$

3.2.1 The problem is generalized by using parameters of variables that are non-dimensional.

a) The independent non-dimensional variables are:

$$N/N_o \text{ (Dimensionally } = SPM/SPM = 1), \text{ and}$$

$$F_o/S_{kr} \text{ (Dimensionally } = \frac{\text{lb}}{\text{in.} \times \text{lb/in.}} = 1)$$

where

N is *SPM*
 N_o is *SPM* at natural frequency of rod string
 S is surface stroke
 k_r is spring constant of rod string

b) The dependent non-dimensional variables are:

peak polished rod load, *PPRL*: F_1/Sk_r
 minimum polished rod load, *MPRL*: F_2/Sk_r
 peak torque, *PT*: $2TS^2k_r$
 polished rod horsepower, *PRHP*: F_3/Sk_r
 plunger stroke, S_p : S_p/S

3.2.2 In the research project the sucker rod pumping system was simulated by an electronic analog computer. Computer runs were made for many combinations of N/N_o and F_o/Sk_r with the dependent non-dimensional variables being measured on each test. Test results were correlated by plotting the families of curves shown in Figures 4.1 through 4.5. From these curves, values for the various non-dimensional variables may be determined for substitution in the following design calculation formulas:

plunger stroke,

$$S_p = [(S_p/S) \times S] - [F_o \times 1/k_t]$$

NOTE When tubing is anchored, the value of $1/k_t$ equals zero, therefore the formula for S_p with anchored tubing becomes $(S_p/S) \times S$.

pump displacement,

$$PD = 0.1166 \times S_p \times N \times D^2$$

peak polished rod load,

$$PPRL = W_{rf} + [(F_1/Sk_r) \times Sk_r]$$

minimum polished rod load,

$$MPRL = W_{rf} - [(F_2/Sk_r) \times Sk_r]$$

peak torque,

$$PT = (2T/S^2k_r) \times Sk_r \times S/2 \times T_a$$

polished rod horsepower,

$$PRHP = (F_3/Sk_r) \times Sk_r \times S \times N \times 2.53 \times 10^{-6}$$

counterweight required,

$$CBE = 1.06 (W_{rf} + 1/2 F_o)$$

4 Design Procedure

4.1 The final solution to this design problem is reached through trial and error methods. Generally, three steps are required in designing an installation.

- a) A preliminary selection of components for the installation must be made.
- b) The operating characteristics of the preliminary selection are calculated by use of the formulas, tables, and figures presented herein.
- c) The calculated pump displacement and loads are compared with the volumes, load ratings, stresses, and other limitations of the preliminary selection.

It will usually be found necessary to make more than one calculation to bring the limitations of the various components of the installation into agreement.

4.2 The minimum amount of information which must be known (or assumed) for a particular sucker rod pumping unit installation design calculation must include:

- fluid level— H , the net lift in ft;
- pump depth— L , ft;
- pumping speed— N , SPM ;
- length of surface stroke— S , in.;
- pump, plunger diameter— D , in.;
- specific gravity of the fluid— G ;
- the nominal tubing diameter and whether it is anchored or hanging free;
- sucker rod size and design.

4.3 With these factors, the designer will be able to calculate the following:

- plunger stroke— S_p , in.;
- pump displacement— PD , barrels/day;
- peak polished rod load— $PPRL$, lb;
- minimum polished rod load— $MPRL$, lb;
- peak crank torque— PT , lb-in.;
- polished rod horsepower— $PRHP$;
- counterweight required— CBE , lb.

4.4 Accumulate the known (or assumed) factors on a data sheet or similar document. An example of a completed design calculations form is included in this document (see Data Sheet 11L).

4.5 Refer to Table 4.1, use the sucker rod string designation in Column 1 and the plunger diameter in Column 2 as guides, read and record the values for W_r , E_r , and F_c found in Columns 3, 4, and 5 respectively. Table 4.2, Column 5 will give the value of E_t . This factor becomes significant only when working with an unanchored tubing string. If the tubing is anchored, E_t need not be recorded.

NOTE The values of rod percentages, rod weights, elastic constants, and frequency factors shown in Table 4.1 differ from those in previous editions of this document and those shown in Tables 1 and 2, API 11L3, First Edition, May 1970. Values in current Table 4.1 were adopted at the June 1976 Standardization Conference, based on the article by A. B. Neely. Changed rod percentages have negligible effect upon values calculated in API 11L3 except for the weight of rods in fluid (W_{rf}).

4.6 Perform the indicated mathematical operations indicated through step 11 (per example Data Sheet 11L). If the tubing is anchored, $1/k_t$ (step 11) is equal to zero and need not be calculated. The values are now available with which the bottom hole pump stroke, S_p , and the pump displacement, PD , may be calculated.

4.7 With the calculated values of F_o/Sk_r and NN'_o , record the value of S_p/S from figure 4.1 and solve for S_p and PD in steps 13 and 14 (per example Data Sheet 11L). Pump displacement is the first test being made to see if the preliminary selection of components for the installation is satisfactory. If the pump displacement calculated in step 14 fails to satisfy known or anticipated requirements, appropriate adjustments must be made in the assumed data and steps 1 through 14 repeated. When the calculated pump displacement is acceptable, proceed with the design calculations by performing steps 15, 16 and 17.

4.8 By using the calculated values of F_o/Sk_r and NN'_o , the values of F_1/Sk_r (see Figure 4.2), F_2/Sk_r (see Figure 4.3), $2T/S^2k_r$ (see Figure 4.4), and F_3/Sk_r (see Figure 4.5) are read from the curves and recorded. When referring to Figures 4.1 and 4.6 to determine S_p/S and T_a , the value of NN'_o must be used. Record the value of T_a .

4.9 Substitution of the appropriate values in the various formulas and performance of the indicated mathematics in steps 23 through 27 of the example Data Sheet 11L will yield the various loads to be expected from the preliminary selection of equipment. It is now necessary to compare these calculated loads with limitations imposed by the preliminary selection. Calculate the stress in the sucker rods to determine if it is within acceptable limits.

4.10 Generally, more than one selection of equipment and calculation of operating conditions is necessary before the optimum selection can be made..

EXAMPLE DESIGN CALCULATIONS CONVENTIONAL SUCKER ROD PUMPING SYSTEM

Object: To solve for—Sp, PD, PPRL, MPRL, PT, PRHP, and CBE

Known or Assumed Data:

Fluid Level, H = 4,500 ft. Pumping Speed, N = 16 SPM Plunger Diameter, D = 1.50 in.
 Pump Depth, L = 5,000 ft. Length of Stroke, S = 54 in. Spec. Grav. of Fluid, G = 0.9
 Tubing Size 2 in. Is it anchored? Yes, (No) Sucker Rods 33.8% - 7/8" & 66.2% - 3/4"

Record Factors from Tables 4.1 & 4.2:

1. $W_r = \underline{1.833}$ (Table 4.1, Column 3) 3. $F_c = \underline{1.082}$ (Table 4.1, Column 5)
 2. $E_r = \underline{.804 \times 10^{-6}}$ (Table 4.1, Column 4) 4. $E_t = \underline{.307 \times 10^{-6}}$ (Table 4.2, Column 5)

Calculate Non-Dimensional Variables:

5. $F_o = .340 \times G \times D^2 \times H = .340 \times \underline{0.9} \times \underline{2.25} \times \underline{4,500} = \underline{3,098}$ lbs.
 6. $1/k_r = E_r \times L = \underline{.804 \times 10^{-6}} \times \underline{5,000} = \underline{4.020 \times 10^{-3}}$ in/lb. 9. $N/N_o = NL / 245,000 = \underline{16} \times \underline{5,000} / 245,000 = \underline{.326}$
 7. $Sk_r = S / 1/k_r = \underline{54} / \underline{4.020 \times 10^{-3}} = \underline{13,433}$ lbs. 10. $N/N_o' = N/N_o \div F_c = \underline{.326} \div \underline{1.082} = \underline{.301}$
 $F_o/Sk_r = \underline{3,098} / \underline{13,433} = \underline{.231}$ 11. $1/k_t = E_t \times L = \underline{.307 \times 10^{-6}} \times \underline{5,000} = \underline{1.535 \times 10^{-3}}$ in/lb.

Solve for Sp and PD:

12. $S_p/S = \underline{.86}$ (Figure 4.1)
 13. $S_p = [(S_p/S) \times S] - [F_o \times 1/k_t] = [\underline{.86} \times \underline{54}] - [\underline{3,098} \times \underline{1.535 \times 10^{-3}}] = \underline{41.7}$ in.
 14. $PD = 0.1166 \times S_p \times N \times D^2 = 0.1166 \times \underline{41.7} \times \underline{16} \times \underline{2.25} = \underline{175}$ barrels per day

If the calculated pump displacement fails to satisfy known or anticipated requirements, appropriate adjustments must be made in the assumed data and steps 1 through 14 repeated. When the calculated pump displacement is acceptable, proceed with the Design Calculation.

Determine Non-Dimensional Parameters:

15. $W = W_r \times L = \underline{1.833} \times \underline{5,000} = \underline{9,165}$ lbs. 17. $W_{rt}/Sk_r = \underline{8,110} / \underline{13,433} = \underline{.604}$
 16. $W_{rt} = W[1 - (.128G)] = \underline{9,165} [1 - (.128 \times \underline{.9})] = \underline{8,110}$ lbs.

Record Non-Dimensional Factors from Figures 4.2 through 4.6:

18. $F_1/Sk_r = \underline{.465}$ (Figure 4.2) 20. $2T/S^2k_r = \underline{.37}$ (Figure 4.4)
 19. $F_2/Sk_r = \underline{.213}$ (Figure 4.3) 21. $F_3/Sk_r = \underline{.29}$ (Figure 4.5) 22. $T_a = \underline{.997}$ (Figure 4.6)

Solve for Operating Characteristics:

23. $PPRL = W_{rt} + [(F_1/Sk_r) \times Sk_r] = \underline{8,110} + [\underline{.465} \times \underline{13,433}] = \underline{14,356}$ lbs.
 24. $MPRL = W_{rt} - [(F_2/Sk_r) \times Sk_r] = \underline{8,110} - [\underline{.22} \times \underline{13,433}] = \underline{5,249}$ lbs.
 25. $PT = (2T/S^2k_r) \times Sk_r \times S / 2 \times T_a = \underline{.37} \times \underline{13,433} \times \underline{27} \times \underline{.997} = \underline{133,793}$ lb inches
 26. $PRHP = (F_3/Sk_r) \times Sk_r \times S \times N \times 2.53 \times 10^{-6} = \underline{.29} \times \underline{13,433} \times \underline{54} \times \underline{16} \times 2.53 \times 10^{-6} = \underline{8.5}$
 27. $CBE = 1.06(W_{rt} + 1/2 F_o) = 1.06 \times (\underline{8,110} + \underline{1,549}) = \underline{10,239}$ lbs.

Data Sheet 11L—Example Design Calculation*

* The technical report calculations are merely examples for illustration purposes only (each company should develop its own approach). They are not to be considered exclusive or exhaustive in nature. API makes no warranties, express or implied for reliance on or any omissions from the information contained in this document.

* Users of these technical report instructions should not rely exclusively on the information contained in this document. Sound business, scientific, engineering, and safety judgement should be used in employing the information contained herein

Table 4.1—Rod and Pump Data (See 4.5)

1	2	3	4	5	6	7	8	9		
Rod No.	Plunger Diameter in. D	Rod Weight lb/ft W_r	Elastic Constant in./lb-ft E_r	Frequency Factor F_c	Rod String, % of each size					
					1 1/8	1	7/8	3/4	5/8	1/2
44	All	0.726	1.990×10^{-6}	1.000	—	—	—	—	—	100.0
54	1.06	0.908	1.668×10^{-6}	1.138	—	—	—	—	44.6	55.4
54	1.25	0.929	1.633×10^{-6}	1.140	—	—	—	—	49.5	50.5
54	1.50	0.957	1.584×10^{-6}	1.137	—	—	—	—	56.4	43.6
54	1.75	0.990	1.525×10^{-6}	1.122	—	—	—	—	64.6	35.4
54	2.00	1.027	1.460×10^{-6}	1.095	—	—	—	—	73.7	26.3
54	2.25	1.067	1.391×10^{-6}	1.061	—	—	—	—	83.4	16.6
54	2.50	1.108	1.318×10^{-6}	1.023	—	—	—	—	93.5	6.5
55	All	1.135	1.270×10^{-6}	1.000	—	—	—	—	100.0	—
64	1.06	1.164	1.382×10^{-6}	1.229	—	—	—	33.3	33.1	33.5
64	1.25	1.211	1.319×10^{-6}	1.215	—	—	—	37.2	35.9	26.9
64	1.50	1.275	1.232×10^{-6}	1.184	—	—	—	42.3	40.4	17.3
64	1.75	1.341	1.141×10^{-6}	1.145	—	—	—	47.4	45.2	7.4
65	1.06	1.307	1.138×10^{-6}	1.098	—	—	—	34.4	65.6	—
65	1.25	1.321	1.127×10^{-6}	1.104	—	—	—	37.3	62.7	—
65	1.50	1.343	1.110×10^{-6}	1.110	—	—	—	41.8	58.2	—
65	1.75	1.369	1.090×10^{-6}	1.114	—	—	—	46.9	53.1	—
65	2.00	1.394	1.070×10^{-6}	1.114	—	—	—	52.0	48.0	—
65	2.25	1.426	1.045×10^{-6}	1.110	—	—	—	58.4	41.6	—
65	2.50	1.460	1.018×10^{-6}	1.099	—	—	—	65.2	34.8	—
65	2.75	1.497	0.990×10^{-6}	1.082	—	—	—	72.5	27.5	—
65	3.25	1.574	0.930×10^{-6}	1.037	—	—	—	88.1	11.9	—
66	All	1.634	0.883×10^{-6}	1.000	—	—	—	100.0	—	—
75	1.06	1.566	0.997×10^{-6}	1.191	—	—	27.0	27.4	45.6	—
75	1.25	1.604	0.973×10^{-6}	1.193	—	—	29.4	29.8	40.8	—
75	1.50	1.664	0.935×10^{-6}	1.189	—	—	33.3	33.3	33.3	—
75	1.75	1.732	0.892×10^{-6}	1.174	—	—	37.8	37.0	25.1	—
75	2.00	1.803	0.847×10^{-6}	1.151	—	—	42.4	41.3	16.3	—
75	2.25	1.875	0.801×10^{-6}	1.121	—	—	46.9	45.8	7.2	—
76	1.06	1.802	0.816×10^{-6}	1.072	—	—	28.5	71.5	—	—
76	1.25	1.814	0.812×10^{-6}	1.077	—	—	30.6	69.4	—	—
76	1.50	1.833	0.804×10^{-6}	1.082	—	—	33.8	66.2	—	—
76	1.75	1.855	0.795×10^{-6}	1.088	—	—	37.5	62.5	—	—
76	2.00	1.880	0.785×10^{-6}	1.093	—	—	41.7	58.3	—	—
76	2.25	1.908	0.774×10^{-6}	1.096	—	—	46.5	53.5	—	—
76	2.50	1.934	0.764×10^{-6}	1.097	—	—	50.8	49.2	—	—
76	2.75	1.967	0.751×10^{-6}	1.094	—	—	56.5	43.5	—	—
76	3.25	2.039	0.722×10^{-6}	1.078	—	—	68.7	31.3	—	—

Table 4.1—Rod and Pump Data (See 4.5) (Continued)

1	2	3	4	5	6	7	8	9		
Rod No.	Plunger Diameter in. D	Rod Weight lb/ft W_r	Elastic Constant in./lb-ft E_r	Frequency Factor F_c	Rod String, % of each size					
					1 1/8	1	7/8	3/4	5/8	1/2
76	3.75	2.119	0.690×10^{-6}	1.047	—	—	82.3	17.7	—	—
77	All	2.224	0.649×10^{-6}	1.000	—	—	100.0	—	—	—
85	1.06	1.883	0.873×10^{-6}	1.261	—	22.2	22.4	22.4	33.0	—
85	1.25	1.943	0.841×10^{-6}	1.253	—	23.9	24.2	24.3	27.6	—
85	1.50	2.039	0.791×10^{-6}	1.232	—	26.7	27.4	26.8	19.2	—
85	1.75	2.138	0.738×10^{-6}	1.201	—	29.6	30.4	29.5	10.5	—
86	1.06	2.058	0.742×10^{-6}	1.151	—	22.6	23.0	54.3	—	—
86	1.25	2.087	0.732×10^{-6}	1.156	—	24.3	24.5	51.2	—	—
86	1.50	2.133	0.717×10^{-6}	1.162	—	26.8	27.0	46.3	—	—
86	1.75	2.185	0.699×10^{-6}	1.164	—	29.4	30.0	40.6	—	—
86	2.00	2.247	0.679×10^{-6}	1.161	—	32.8	33.2	33.9	—	—
86	2.25	2.315	0.656×10^{-6}	1.153	—	36.9	36.0	27.1	—	—
86	2.50	2.385	0.633×10^{-6}	1.138	—	40.6	39.7	19.7	—	—
86	2.75	2.455	0.610×10^{-6}	1.119	—	44.5	43.3	12.2	—	—
87	1.06	2.390	0.612×10^{-6}	1.055	—	24.3	75.7	—	—	—
87	1.25	2.399	0.610×10^{-6}	1.058	—	25.7	74.3	—	—	—
87	1.50	2.413	0.607×10^{-6}	1.062	—	27.7	72.3	—	—	—
87	1.75	2.430	0.603×10^{-6}	1.066	—	30.3	69.7	—	—	—
87	2.00	2.450	0.598×10^{-6}	1.071	—	33.2	66.8	—	—	—
87	2.25	2.472	0.594×10^{-6}	1.075	—	36.4	63.6	—	—	—
87	2.50	2.496	0.588×10^{-6}	1.079	—	39.9	60.1	—	—	—
87	2.75	2.523	0.582×10^{-6}	1.082	—	43.9	56.1	—	—	—
87	3.25	2.575	0.570×10^{-6}	1.084	—	51.6	48.4	—	—	—
87	3.75	2.641	0.556×10^{-6}	1.078	—	61.2	38.8	—	—	—
87	4.75	2.793	0.522×10^{-6}	1.038	—	83.6	16.4	—	—	—
88	All	2.904	0.497×10^{-6}	1.000	—	100.0	—	—	—	—
96	1.06	2.382	0.670×10^{-6}	1.222	19.1	19.2	19.5	42.3	—	—
96	1.25	2.435	0.655×10^{-6}	1.224	20.5	20.5	20.7	38.3	—	—
96	1.50	2.511	0.633×10^{-6}	1.223	22.4	22.5	22.8	32.3	—	—
96	1.75	2.607	0.606×10^{-6}	1.213	24.8	25.1	25.1	25.1	—	—
96	2.00	2.703	0.578×10^{-6}	1.196	27.1	27.9	27.4	17.6	—	—
96	2.25	2.806	0.549×10^{-6}	1.172	29.6	30.7	29.8	9.8	—	—
97	1.06	2.645	0.568×10^{-6}	1.120	19.6	20.0	60.3	—	—	—
97	1.25	2.670	0.563×10^{-6}	1.124	20.8	21.2	58.0	—	—	—
97	1.50	2.707	0.556×10^{-6}	1.131	22.5	23.0	54.5	—	—	—
97	1.75	2.751	0.548×10^{-6}	1.137	24.5	25.0	50.4	—	—	—
97	2.00	2.801	0.538×10^{-6}	1.141	26.8	27.4	45.7	—	—	—
97	2.25	2.856	0.528×10^{-6}	1.143	29.4	30.2	40.4	—	—	—

Table 4.1—Rod and Pump Data (See 4.5) (Continued)

1	2	3	4	5	6	7	8	9		
Rod No.	Plunger Diameter in. D	Rod Weight lb/ft W_r	Elastic Constant in./lb-ft E_r	Frequency Factor F_c	Rod String, % of each size					
					1 1/8	1	7/8	3/4	5/8	1/2
97	2.50	2.921	0.515×10^{-6}	1.141	32.5	33.1	34.4	—	—	—
97	2.75	2.989	0.503×10^{-6}	1.135	36.1	35.3	28.6	—	—	—
97	3.25	3.132	0.475×10^{-6}	1.111	42.9	41.9	15.2	—	—	—
98	1.06	3.068	0.475×10^{-6}	1.043	21.2	78.8	—	—	—	—
98	1.25	3.076	0.474×10^{-6}	1.045	22.2	77.8	—	—	—	—
98	1.50	3.089	0.472×10^{-6}	1.048	23.8	76.2	—	—	—	—
98	1.75	3.103	0.470×10^{-6}	1.051	25.7	74.3	—	—	—	—
98	2.00	3.118	0.468×10^{-6}	1.055	27.7	72.3	—	—	—	—
98	2.25	3.137	0.465×10^{-6}	1.058	30.1	69.9	—	—	—	—
98	2.50	3.157	0.463×10^{-6}	1.062	32.7	67.3	—	—	—	—
98	2.75	3.180	0.460×10^{-6}	1.066	35.6	64.4	—	—	—	—
98	3.25	3.231	0.453×10^{-6}	1.071	42.2	57.8	—	—	—	—
98	3.75	3.289	0.445×10^{-6}	1.074	49.7	50.3	—	—	—	—
98	4.75	3.412	0.428×10^{-6}	1.064	65.7	34.3	—	—	—	—
99	All	3.676	0.393×10^{-6}	1.000	100.0	—	—	—	—	—
107	1.06	2.977	0.524×10^{-6}	1.184	16.9	16.8	17.1	49.1	—	—
107	1.25	3.019	0.517×10^{-6}	1.189	17.9	17.8	18.0	46.3	—	—
107	1.50	3.085	0.506×10^{-6}	1.195	19.4	19.2	19.5	41.9	—	—
107	1.75	3.158	0.494×10^{-6}	1.197	21.0	21.0	21.2	36.9	—	—
107	2.00	3.238	0.480×10^{-6}	1.195	22.7	22.8	23.1	31.4	—	—
107	2.25	3.336	0.464×10^{-6}	1.187	25.0	25.0	25.0	25.0	—	—
107	2.50	3.435	0.447×10^{-6}	1.174	26.9	27.7	27.1	18.2	—	—
107	2.75	3.537	0.430×10^{-6}	1.156	29.1	30.2	29.3	11.3	—	—
108	1.06	3.325	0.447×10^{-6}	1.097	17.3	17.8	64.9	—	—	—
108	1.25	3.345	0.445×10^{-6}	1.101	18.1	18.6	63.2	—	—	—
108	1.50	3.376	0.441×10^{-6}	1.106	19.4	19.9	60.7	—	—	—
108	1.75	3.411	0.437×10^{-6}	1.111	20.9	21.4	57.7	—	—	—
108	2.00	3.452	0.432×10^{-6}	1.117	22.6	23.0	54.3	—	—	—
108	2.25	3.498	0.427×10^{-6}	1.121	24.5	25.0	50.5	—	—	—
108	2.50	3.548	0.421×10^{-6}	1.124	26.5	27.2	46.3	—	—	—
108	2.75	3.603	0.415×10^{-6}	1.126	28.7	29.6	41.6	—	—	—
108	3.25	3.731	0.400×10^{-6}	1.123	34.6	33.9	31.6	—	—	—
108	3.75	3.873	0.383×10^{-6}	1.108	40.6	39.5	19.9	—	—	—
109	1.06	3.839	0.378×10^{-6}	1.035	18.9	81.1	—	—	—	—
109	1.25	3.845	0.378×10^{-6}	1.036	19.6	80.4	—	—	—	—
109	1.50	3.855	0.377×10^{-6}	1.038	20.7	79.3	—	—	—	—
109	1.75	3.867	0.376×10^{-6}	1.040	22.1	77.9	—	—	—	—
109	2.00	3.880	0.375×10^{-6}	1.043	23.7	76.3	—	—	—	—

Table 4.1—Rod and Pump Data (See 4.5) (Continued)

1	2	3	4	5	6	7	8	9		
Rod No.	Plunger Diameter in. D	Rod Weight lb/ft W_r	Elastic Constant in./lb-ft E_r	Frequency Factor F_c	Rod String, % of each size					
					1 1/8	1	7/8	3/4	5/8	1/2
109	2.25	3.896	0.374×10^{-6}	1.046	25.4	74.6	—	—	—	—
109	2.50	3.911	0.372×10^{-6}	1.048	27.2	72.8	—	—	—	—
109	2.75	3.930	0.371×10^{-6}	1.051	29.4	70.6	—	—	—	—
109	3.25	3.971	0.367×10^{-6}	1.057	34.2	65.8	—	—	—	—
109	3.75	4.020	0.363×10^{-6}	1.063	39.9	60.1	—	—	—	—
109	4.75	4.120	0.354×10^{-6}	1.066	51.5	48.5	—	—	—	—
1010	All	4.538	0.318×10^{-6}	1.000	100.00	—	—	—	—	—

* Rod No. shown in first column refers to the largest and smallest rod size in eighths of an inch. For example, Rod No. 76 is a two-way taper of 7/8 and 6/8 rods. Rod No. 85 is a four-way taper of 8/8, 7/8, 6/8 and 5/8 rods. Rod No. 109 is a two-way taper of 1 1/4 and 1 1/8 rods. Rod No. 77 is a straight string of 7/8 rods, etc.

Table 4.2—Tubing Data

1	2	3	4	5
Tubing Size	Outside Diameter, in.	Inside Diameter, in.	Metal Area, sq. in.	Elastic Constant in./lb-ft E_t
1.900	1.900	1.610	0.800	0.500×10^{-6}
2 3/8	2.375	1.995	1.304	0.307×10^{-6}
2 7/8	2.875	2.441	1.812	0.221×10^{-6}
3 1/2	3.500	2.992	2.590	0.154×10^{-6}
4	4.000	3.476	3.077	0.130×10^{-6}
4 1/2	4.500	3.958	3.601	0.111×10^{-6}

Table 4.3—Sucker Rod Data

1	2	3	4
Rod Size	Metal Area Sq. in.	Rod Weight in air, lb/ft W_r	Elastic Constant, in./lb-ft E_r
1/2	0.196	0.72	1.990×10^{-6}
5/8	0.307	1.13	1.270×10^{-6}
3/4	0.442	1.63	0.883×10^{-6}
7/8	0.601	2.22	0.649×10^{-6}
1	0.785	2.90	0.497×10^{-6}
1 1/8	0.994	3.67	0.393×10^{-6}

Table 4.4—Pump Constants

1	2	3	4
Plunger Diameter, in. D	Plgr. Diam, Squared Sq. in. D^2	Fluid Factor lb/ft $(.340 \times D^2)$	Load Pump Factor, $(.1166 \times D^2)$
1 ¹ / ₁₆	1.1289	0.384	0.132
1 ¹ / ₄	1.5625	0.531	0.182
1 ¹ / ₂	2.2500	0.765	0.262
1 ³ / ₄	3.0625	1.041	0.357
2	4.0000	1.360	0.466
2 ¹ / ₄	5.0625	1.721	0.590
2 ¹ / ₂	6.2500	2.125	0.728
2 ³ / ₄	7.5625	2.571	0.881
3 ³ / ₄	14.0625	4.781	1.640
4 ³ / ₄	22.5625	7.671	2.630

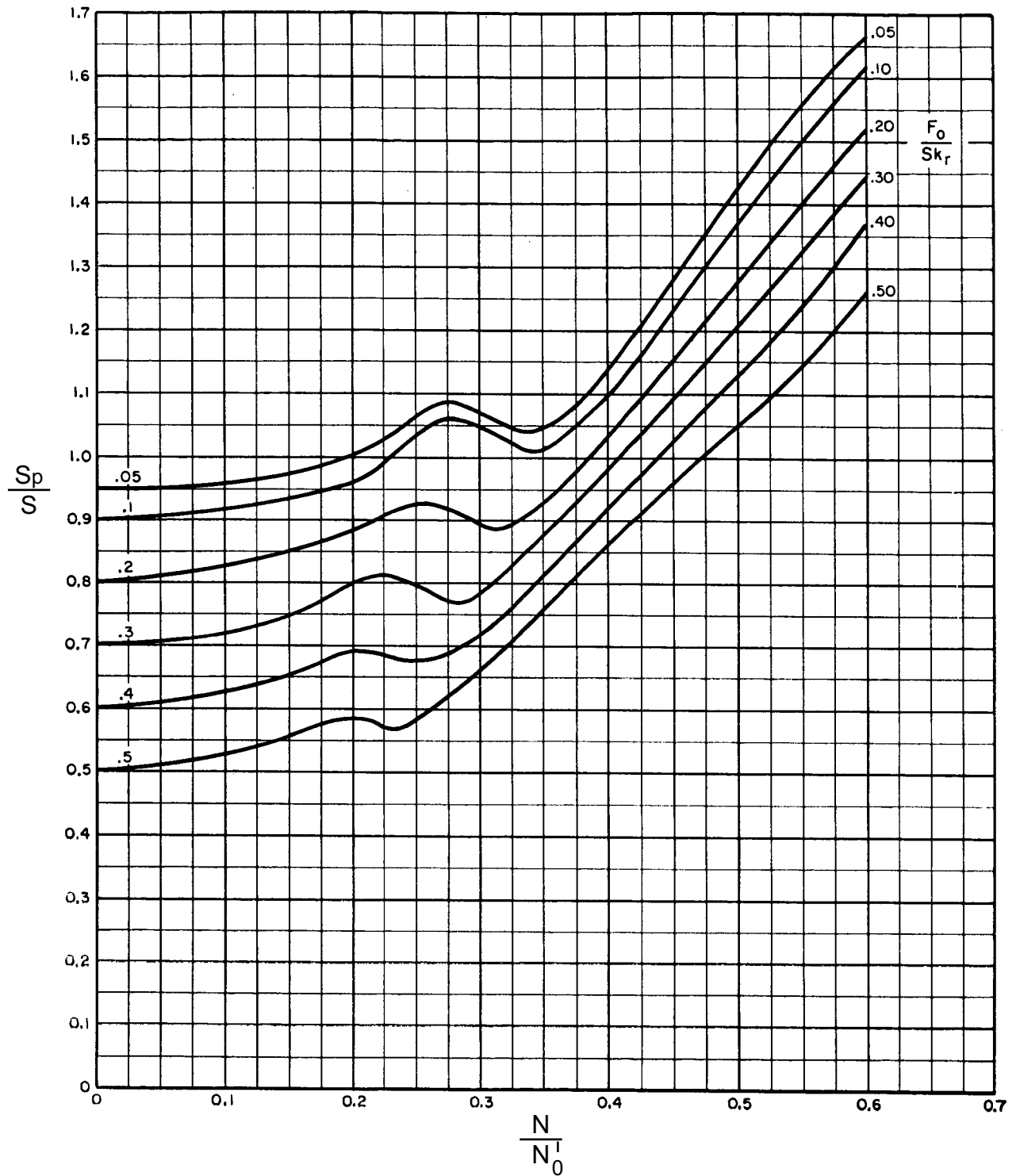


Figure 4.1— $\frac{S_p}{S}$, Plunger Stroke Factor

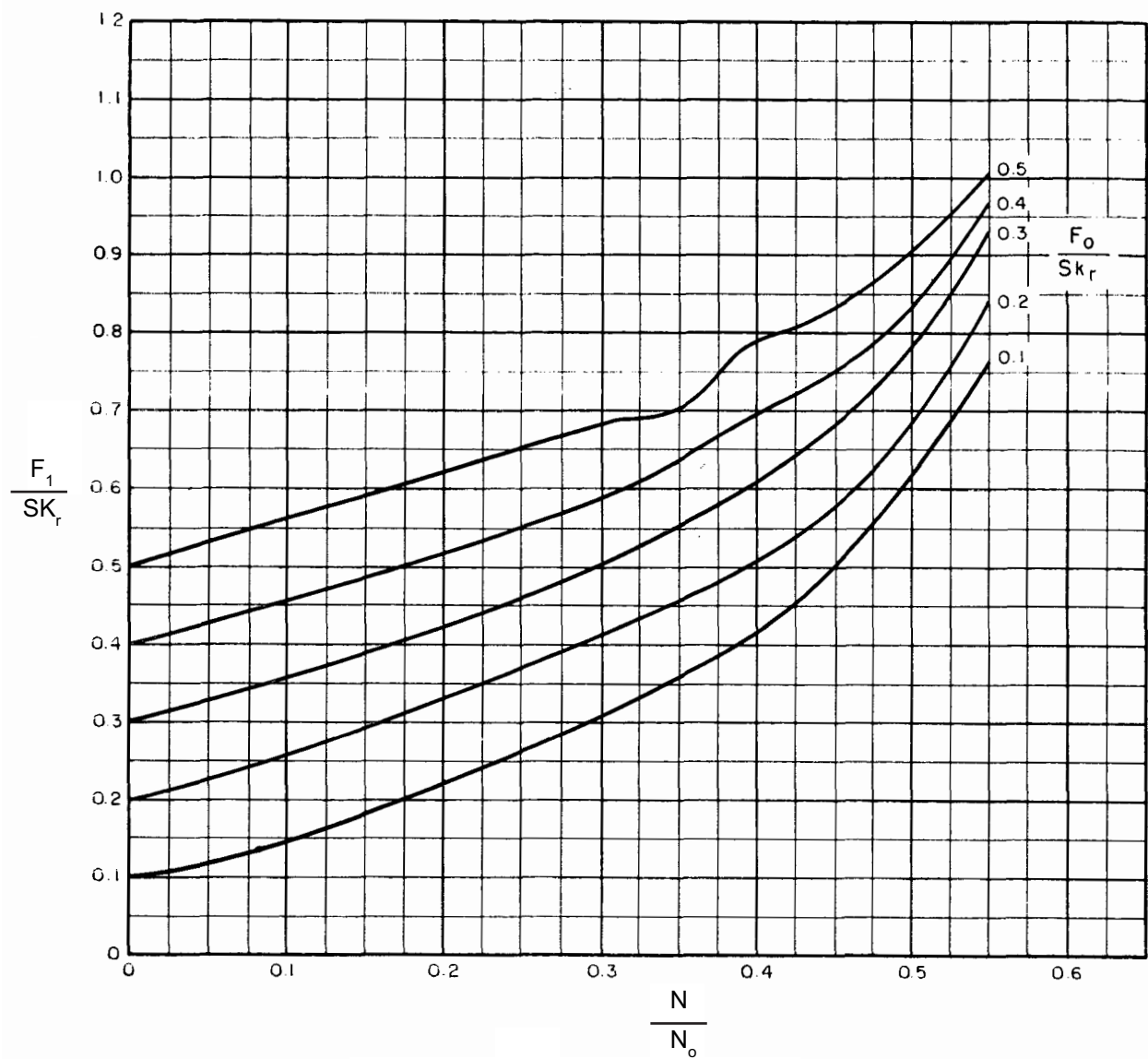


Figure 4.2— $\frac{F_1}{Sk_r}$, Peak Polished Rod Load

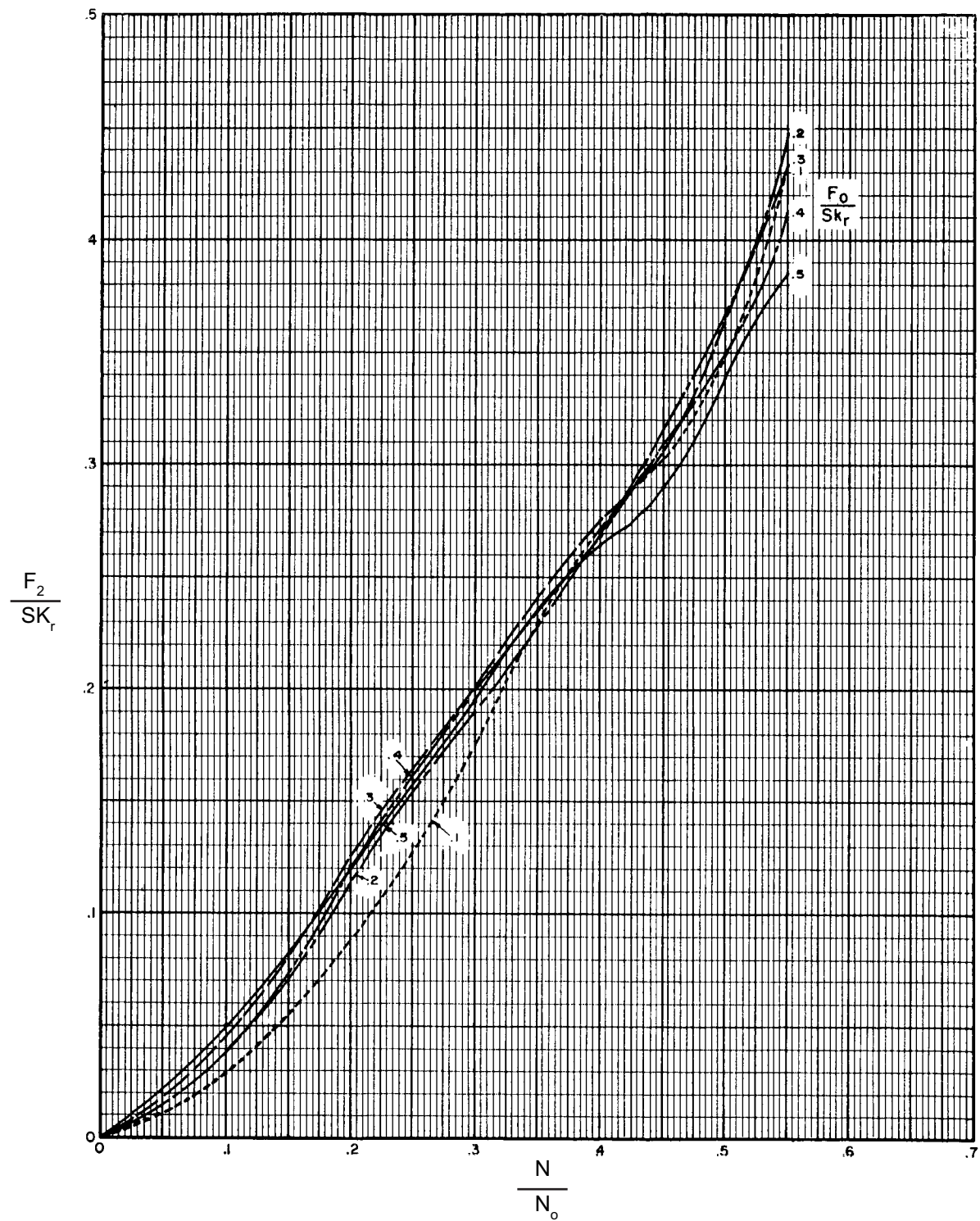
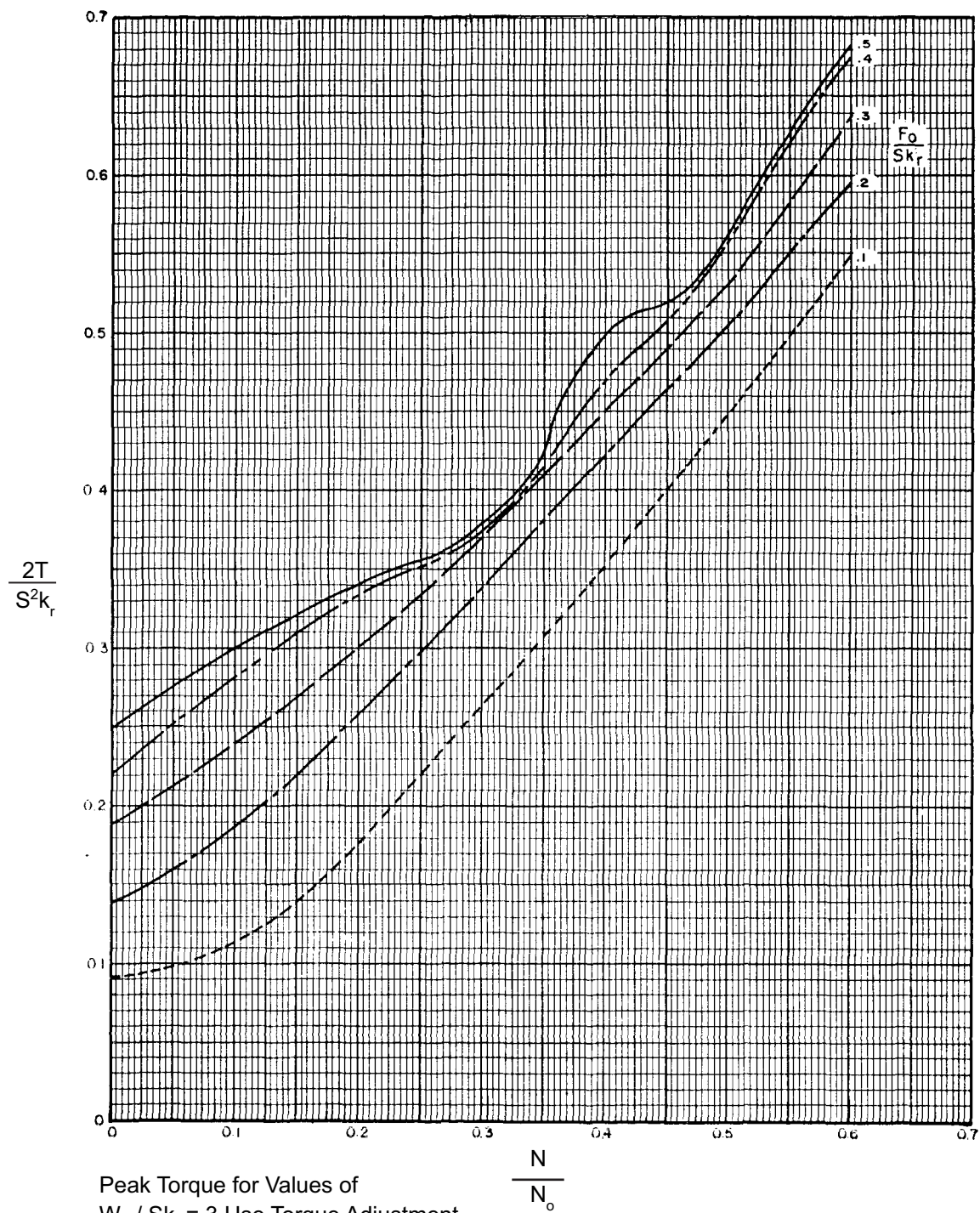


Figure 4.3— $\frac{F_2}{Sk_r}$, Minimum Polished Rod Load



Peak Torque for Values of
 $\frac{W_{rf}}{S k_r} = 3$ Use Torque Adjustment
 For Other Values of $\frac{W_{rf}}{S k_r}$.

Figure 4.4— $\frac{2T}{S^2 K_r}$, Peak Torque

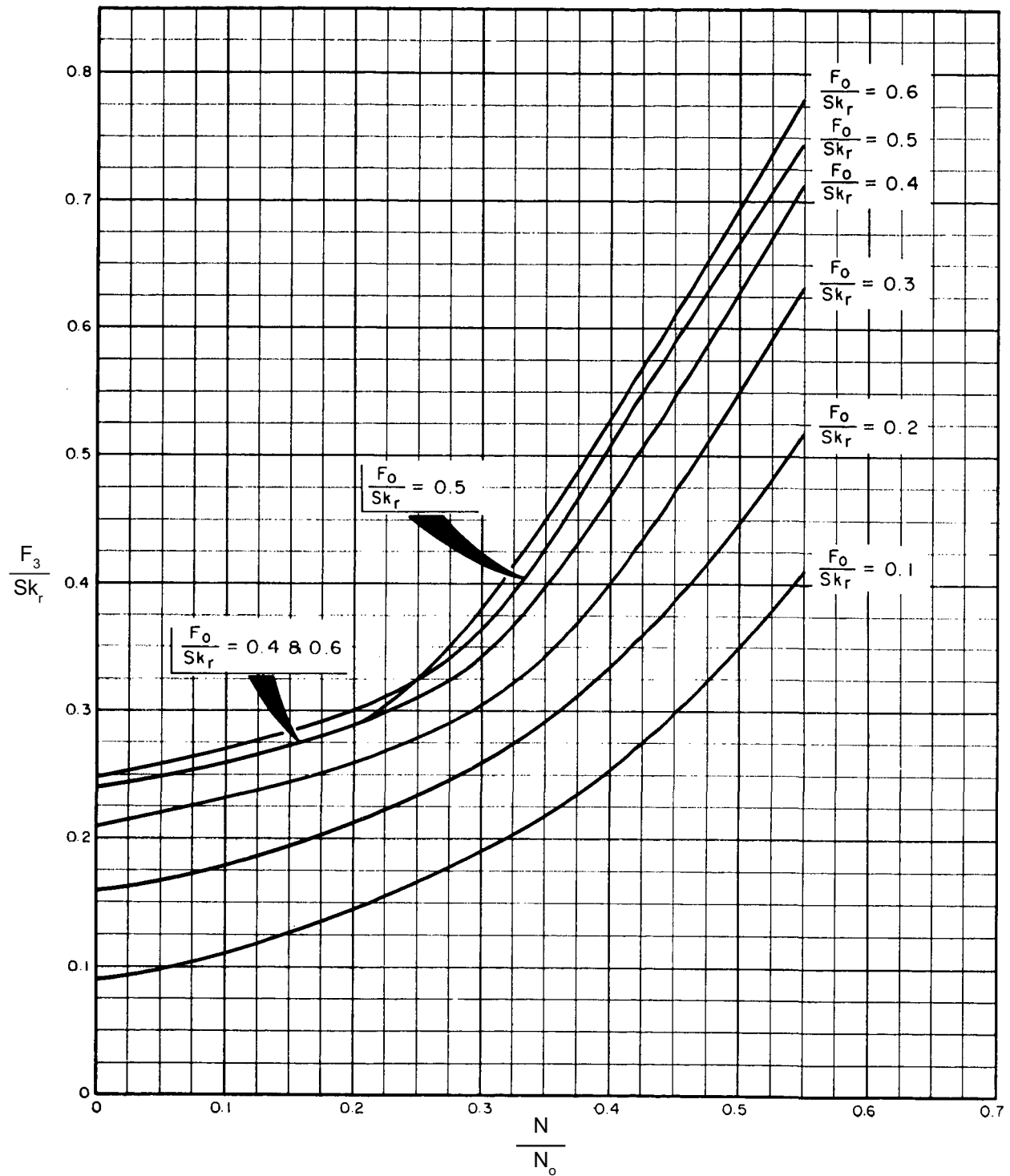
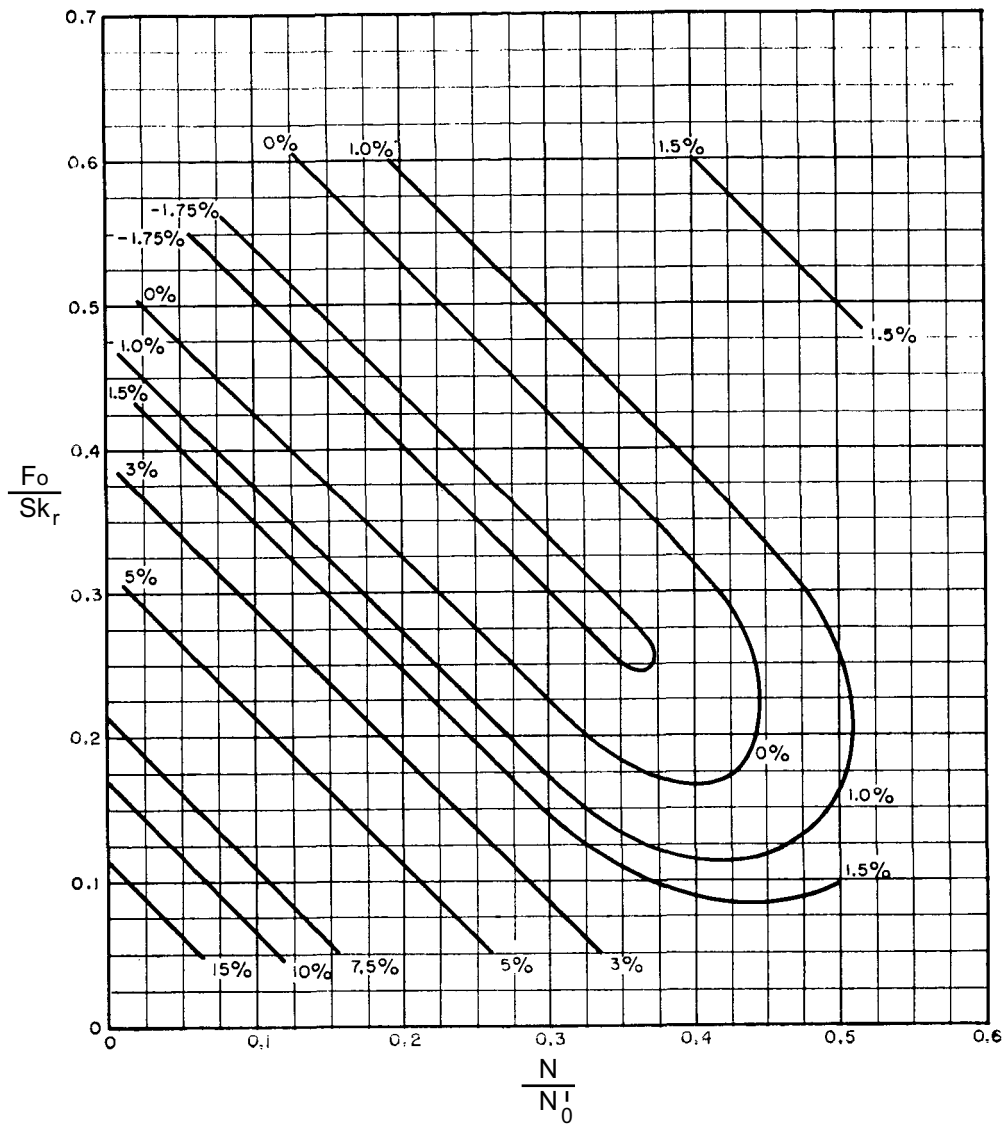


Figure 4.5— $\frac{F_3}{Sk_r}$, Polished Rod Horse Power



To Use: Multiply % Indicated on Curve by $\frac{(W_{rf} - 0.3)}{0.1}$

For example: $W_{rf} = 0.600$

$$\frac{N}{N_o} = 0.200 \quad \frac{F_o}{S_{k_r}} = 0.188$$

Adjustment = 3 % for each 0.1 Increase in $\frac{W_{rf}}{S_{k_r}}$ above 0.3

Total Adjustment = $3 \times 3\% = 9\%$

$$T_a = 1.00 + 0.09 = 1.09$$

Note: If $\frac{W_{rf}}{S_{k_r}}$ is less than 0.3, Adjustment becomes negative.

Figure 4.6— T_a , Adjustment for Peak Torque for Values of $\frac{W_{rf}}{S_{k_r}}$ Other Than 0.3

Annex A

Discussion of Non-dimensional Parameters

A.1 A mathematical evaluation of sucker-rod pumping, for practical purposes, revolves around the development of equations to express the motion and the state of stress of sucker rods. The development of representative formulas is mathematically complicated because of the difficulty of expressing boundary conditions which suitably reflect the opening and closing of the valves of the pump when these are themselves a function of the motion of the system and the fluid-flow conditions in the pump.

A.2 The criteria generally used for the design of sucker rod installations are based on greatly oversimplified concepts of pumping system mechanics. Consequently, the formulas derived from these viewpoints at best represent average conditions occurring in pumping systems and may not yield valid predictions of system performance, especially in deep wells and at high pumping speeds.

A.3 The tendency to oversimplify the methods for predicting system performance is not due to a limited knowledge concerning the operation of sucker rod equipment. Quite the contrary, the basic operating characteristics of sucker rod systems have been reasonably well understood for many years. Simplifications are used when computer programs are not available to help calculate the designs. There are computer programs available that can accurately describe the performance of pumping systems, including the impact of system masses, when design is properly modeled.

A.4 The simplifications in the mathematical model of sucker rod pumping can be grouped into two broad categories:

- a) inadequate representation of the rod string;
- b) inadequate treatment of the downhole pump action.

A.5 Many of the current methods have been developed on the basis that the mass of the sucker rod is concentrated at a point. This assumption makes possible a mathematical simplification wherein the spring equation (an ordinary differential equation) is used to simulate the characteristics of the sucker rod. Although this assumption is mathematically convenient, it destroys the analogy between the actual sucker rod and the mathematical model used to represent the sucker rod. In reality, the mass of the sucker rod is distributed along its length, and this fact must be incorporated into any realistic mathematical model of the rod. Without this provision, the analysis does not include the effect of force waves traveling within the sucker rod, which is an important characteristic of real systems.

A.6 Some of the later techniques for predicting rod performance have used an adequate representation of the sucker rod. Such a representation requires that the wave equation (a partial differential equation) be used to describe the behavior of the sucker rod. However, use of the wave equation introduces a new mathematical difficulty, in that partial differential equations are inherently more difficult to solve. Much of the difficulty arises in formulating the boundary conditions that describe the loading and motion at the extremities of the rod. The boundary condition which describes the behavior of the downhole pump presents particular difficulties, and investigators in the past have made many specialized assumptions in its regard.

A.7 The apparent necessity for making specialized assumptions about pump action has been brought about by peculiarities in the sucker rod problem. The sucker rod problem presents a particular difficulty, in that the pump boundary condition depends upon the behavior of the sucker rod string itself, which is the very thing to be established by solution of the problem. This apparent impasse has led to the special assumptions mentioned heretofore, which, in effect, amount to idealized guesses about pump operation. Consequently, the methods based on the special assumptions yield usable results only to the extent to which the idealizations approximate pump action.

A.8 In the work at Midwest Research Institute, sucker rod pumping systems were simulated by an electronic analog computer. Numerous runs were made representing a wide range of conditions. These runs were correlated on the basis

of two non-dimensional parameters. These parameters are N/N'_o , the dimensionless pumping speed, and F_o/Sk_r , the dimensionless rod stretch. The use of these non-dimensional parameters allows a complete suite of sucker rod pumping conditions to be correlated without having to run an infinite number of cases.

A.9 The dimensionless pumping speed, N/N'_o , is a highly important index of the rod strings' behavior. This parameter is the ratio of the forcing frequency, N (the pumping speed), to the undamped natural frequency of the rod string, N'_o . The undamped natural frequency is shown in vibration theory to be inversely proportional to the time required for a force wave to make four traversals along the sucker rod. Thus, the undamped natural frequency is given by:

$$N'_o = \frac{F_c a}{4L}$$

where F_c is a constant of proportionality which depends on the rod design and " a " is the speed of sound in steel. For untapered rod strings, F_c equals 1. From theoretical considerations, it can be shown that the natural frequency of a tapered rod string is greater than that of a uniform string of equal length. Thus, for tapered strings, F_c has a value greater than unity. Values for F_c in tapered rod strings can be found in Table 4.1. The formula for dimensionless pumping speed N/N'_o immediately follows as:

$$\frac{N}{N'_o} = \frac{4NL}{F_c a}$$

Figures A.1 through A.6 contain charts from which most of the frequency factors, F_c , in Table 4.1 were obtained. These charts may be used to determine F_c for any arrangement of rod sizes for some of the most commonly used tapered strings. Figure A.6 plots a number of rod combinations on a single curve as a function of pump diameter.

A.10 In determining the natural frequency, the velocity of force propagation " a " plays a key role. The theoretical value for this velocity is about 17,000 ft/second. In practice, however, it has been found that lower frequencies normally occur. The speed of sound in long narrow rods is usually somewhat lower than in normal size vessels. Also, the effect of the rod coupling can cause an apparent increase in density resulting in a decrease in propagation velocity. In practice, it has been found that " a " equals about 16,300 ft/second. With this value, the non-dimensional pumping speed can be calculated by:

$$\frac{N}{N'_o} = \frac{NL}{245,000F_o}$$

A.11 Another dimensionless parameter of importance in describing the behavior of the sucker rod string is the dimensionless rod stretch, F_o/Sk_r . In this parameter, the spring constant, k_r , is involved. For untapered rod strings, it is evaluated by the relation:

$$k_r = \frac{EA}{L}$$

For tapered rod strings, the spring constant is computed from the familiar reciprocal formula:

$$\frac{1}{k_r} = \frac{1}{E} \left[\frac{L_1}{A_1} + \frac{L_2}{A_2} + \dots \right]$$

The complete term, F_o/Sk_r , gives the rod stretch caused by static application of the fluid load as a percentage of the polished rod stroke. For example,

$$\frac{F_o}{Sk_r} = 0.1$$

means that the rod stretch is 10 % of the polished rod stroke when the fluid load is statically applied. At very low speeds where static conditions are approached, the dimensionless rod stretch and pump stroke are related as follows:

$$\frac{S_p}{S} = 1 - \frac{F_o}{Sk_r}$$

At higher speeds, this relation breaks down as dynamic effects become more important.

A.12 The charts are used to determine the dependent parameters as defined in the report. Three forces of particular importance are determined from the charts. F_1 is the fluid load plus the maximum dynamic effect on the up stroke. F_2 is the dynamic effects during the down stroke taken away from the load on the polished rod. F_3 is a force which will give horsepower when applied to the full stroke length at the speed of the pumping unit.

A.13 The surface dynamometer card which is generated in the analog computer is independent of the weight of the rod string involved. The shape of the card will be the same and a different rod load will simply shift the card up or down in relation to the zero line. In calculating all parameters except torque, this can be handled very easily by calculating fluid and dynamic loads independent of the rod load and then adding in the rod load. In the case of torque, the amount of rod load is important in determining the torque involved in the unit. All torque values were calculated using a rod load of:

$$\frac{W_{rf}}{Sk_r} = 0.3$$

If a rod load different from this is used, a correction must be made as shown in Figure 4.6.

A.14 In the analog model, it has been assumed that the tubing is anchored and no tubing motion occurs during the pumping stroke. If the tubing is unanchored, then a correction must be made for the shortening of pump stroke which will occur. This happens because the tubing shortens when the load is transferred to the rod string and then lengthens during down stroke when the fluid load is transferred back to the tubing. A correction in pump stroke is made by simply calculating the amount of tubing stretch which will occur with the fluid load used in the design and subtracting this amount of stroke from the net plunger stroke at the bottom of the hole. This should give a reasonable correction. It will not be absolutely correct due to the fact that dynamic effects occur in the tubing string as well as in the rod string and these dynamic effects are not taken into account.

A.15 Average conventional pumping unit geometry has been used in the simulation. This is a conventional unit with the counterweights in phase with the crank and the tail bearing being over the slow speed shaft at midpoint of the stroke. For units with drastically different geometry from that assumed, the simulation will not be accurate and the values calculated will be more approximate. It is believed, however, that the values for maximum and minimum loads and for pump stroke will be reasonably good. The calculated value for torque will not be even approximately right and a torque calculation must be made by some other method.

A.16 Pumping unit motion was assumed based on a medium slip motor. Generally speaking, a higher slip prime mover results in slight decreases in the maximum load and a slight increase in the minimum load but also tends to reduce the subsurface pump stroke; therefore, some error may be introduced in the calculations if a prime mover with a considerably different slip characteristic than a medium slip electric motor is used. Also, the assumption is made that no friction occurs in the stuffing box or in the pump itself. This, of course, is an unreal assumption although the values for friction in this case should be minor and of negligible importance in the design.

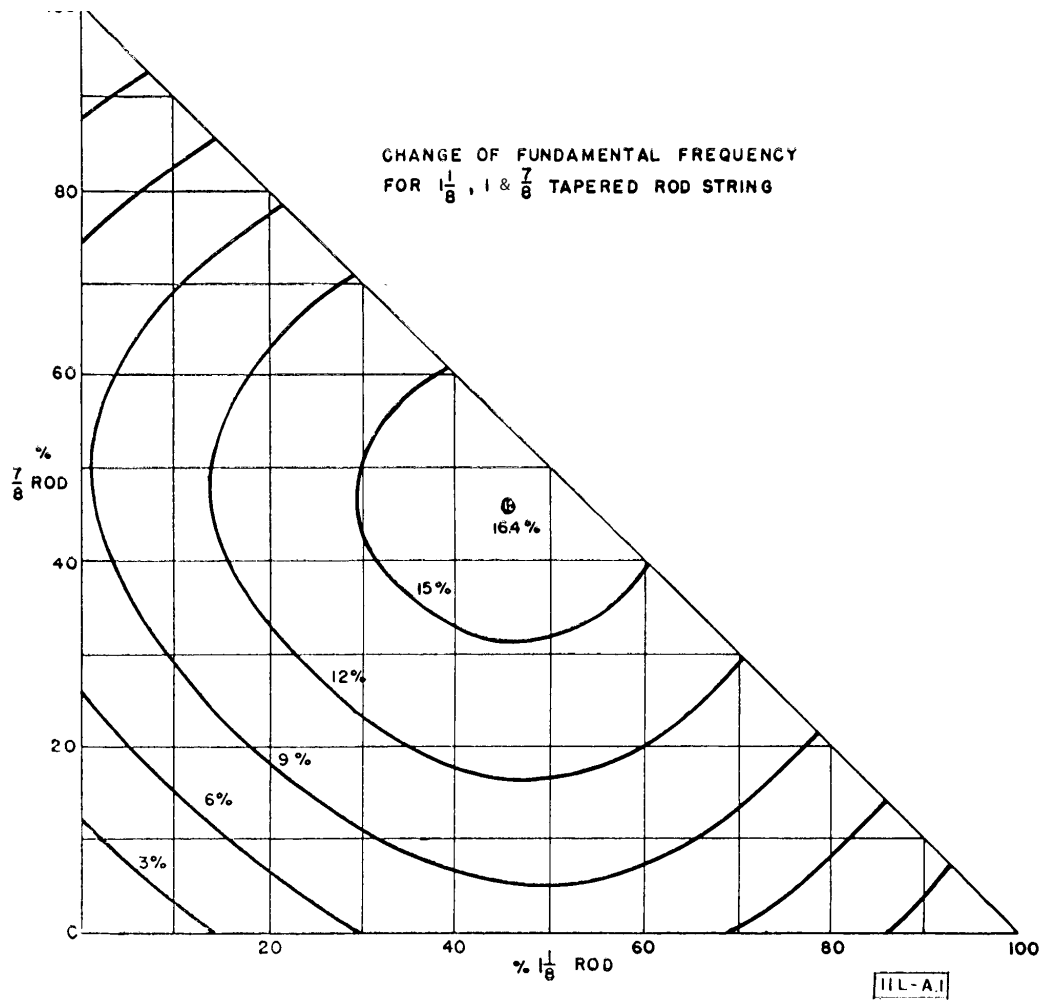
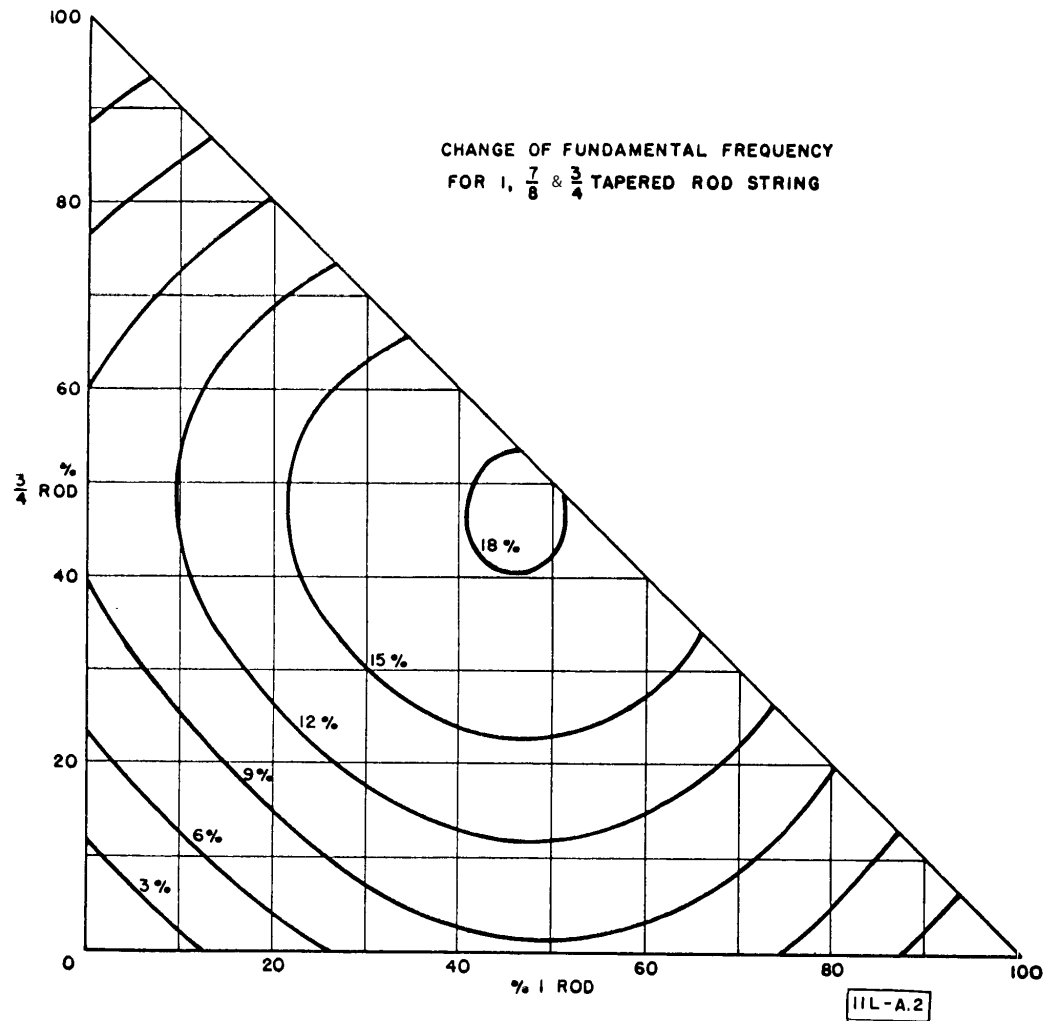


Figure A.1—Percentage Increase in Fundamental Frequency
 $1\frac{1}{8}$ in., 1 in., and $\frac{7}{8}$ in. Three-way Taper String



**Figure A.2—Percentage Increase in Fundamental Frequency
1 in., $\frac{7}{8}$ in., and $\frac{3}{4}$ in. Three-way Taper String**

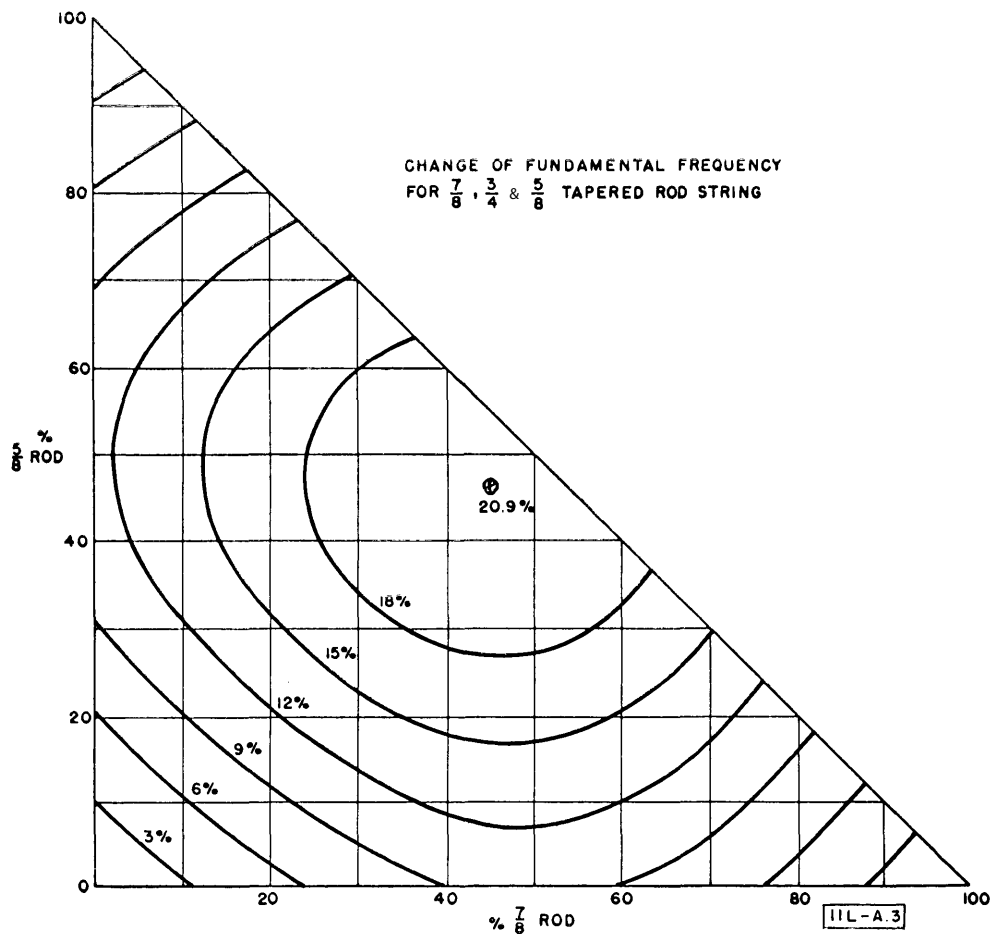
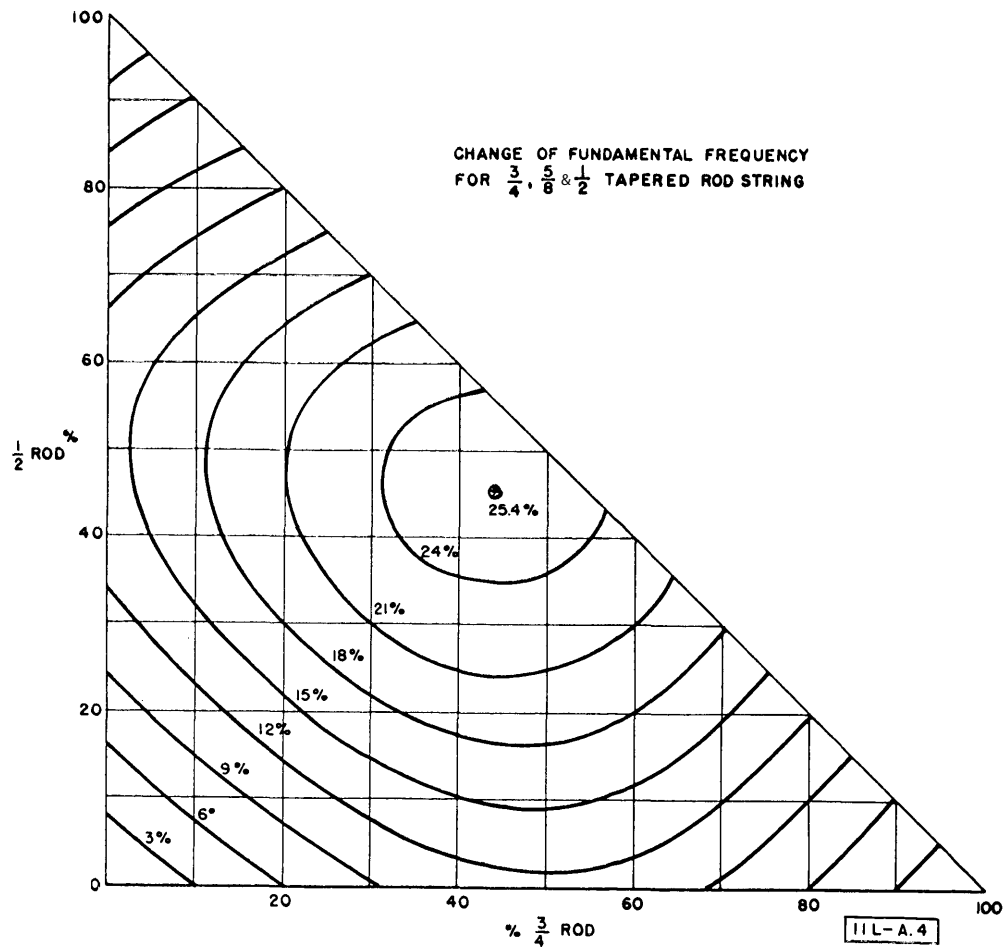


Figure A.3—Percentage Increase in Fundamental Frequency
 $\frac{7}{8}$ in., $\frac{3}{4}$ in., and $\frac{5}{8}$ in. Three-way Taper String



**Figure A.4—Percentage Increase in Fundamental Frequency
 $\frac{3}{4}$ in., $\frac{5}{8}$ in., and $\frac{1}{2}$ in. Three-way Taper String**

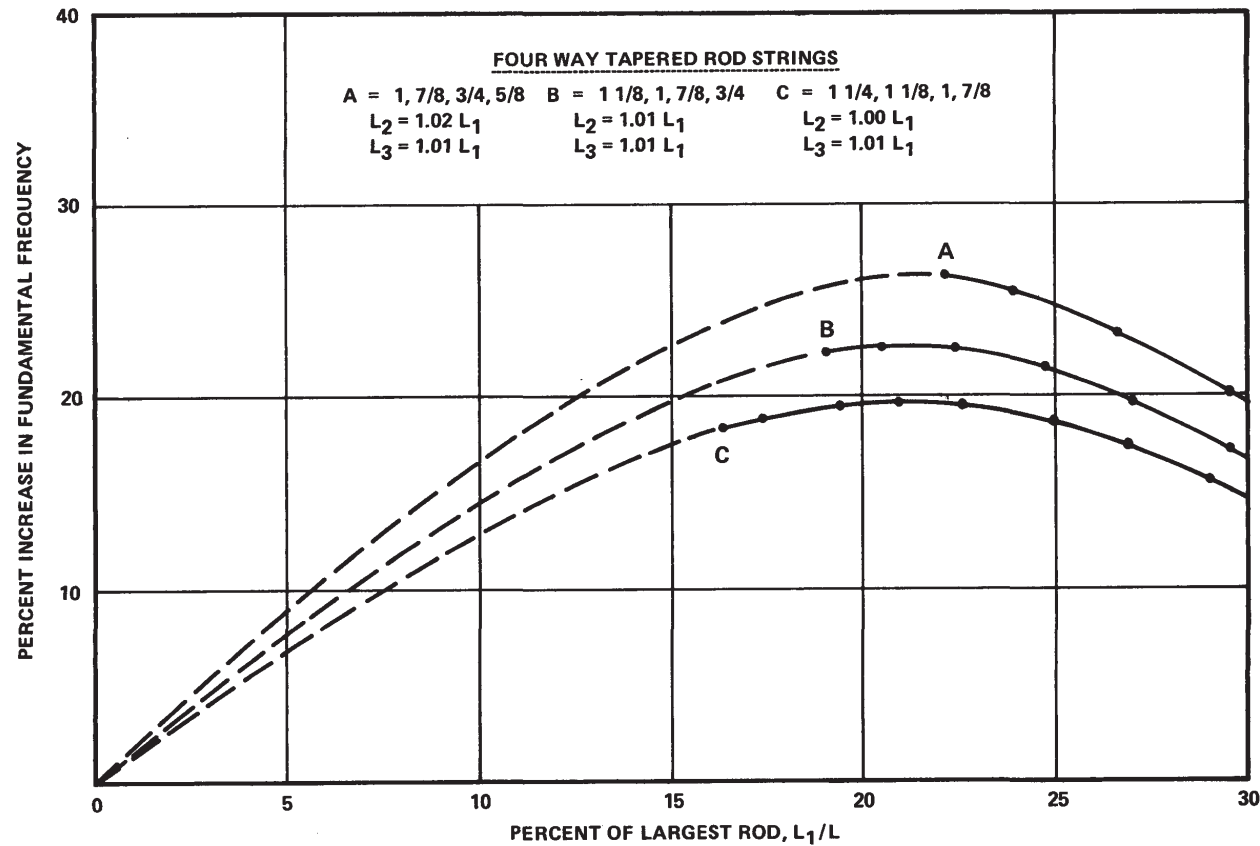


Figure A.5—Four-way Taper Strings
Percent Increase in Fundamental Frequency

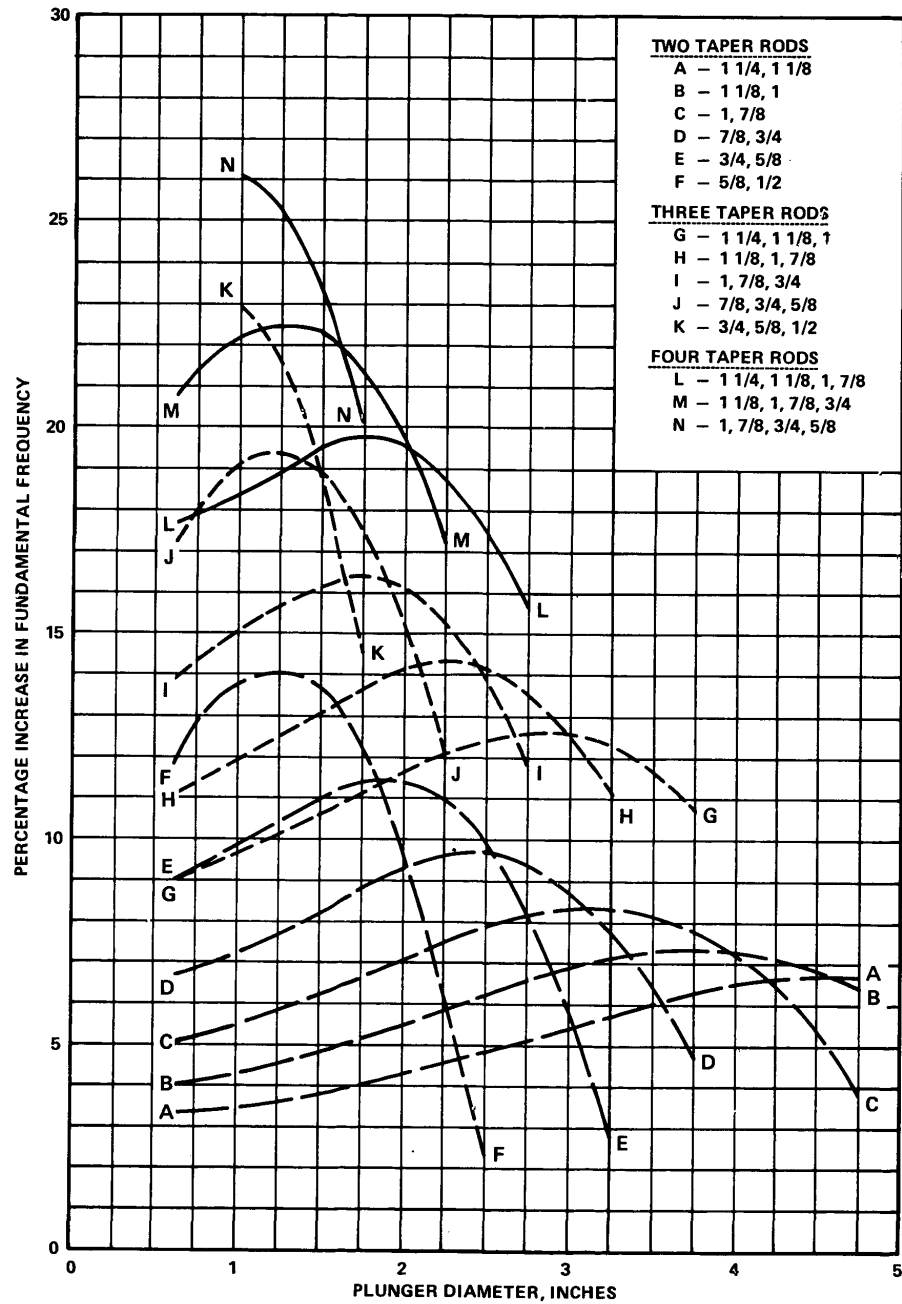


Figure A.6—Percentage Increase in Fundamental Frequency
Specific Rod Combinations

Bibliography

- [1] API Recommended Practice 11L, *Design Calculations for Sucker Rod Pumping Systems*
- [2] API Bulletin 11L2, *Catalog of Analog Computer Dynamometer Cards*
- [3] API Bulletin 11L3, *Sucker Rod Pumping System Design Book*
- [4] API Bulletin 11L4, *Curves for Selecting Beam Pumping Units*, withdrawn 2008
- [5] Midwest Research Institute, "Electric Analog Study of Sucker-rod Pumping Systems," American Petroleum Institute's Drilling and Production Practice, 1968, p. 232.
- [6] Neely, A.B, "Sucker Rod String Design," *Petroleum Engineer*, March 1976.



2008

Publications

Order Form

Effective January 1, 2008.

API Members receive a 30% discount where applicable.

The member discount does not apply to purchases made for the purpose of resale or for incorporation into commercial products, training courses, workshops, or other commercial enterprises.

Available through IHS:

Phone Orders: **1-800-854-7179** (Toll-free in the U.S. and Canada)
303-397-7956 (Local and International)
Fax Orders: **303-397-2740**
Online Orders: **global.ihs.com**

Date: _____

☐ **API Member** (Check if Yes)

Invoice To (☐ Check here if same as "Ship To")

Name: _____
Title: _____
Company: _____
Department: _____
Address: _____

City: _____ State/Province: _____
Zip/Postal Code: _____ Country: _____
Telephone: _____
Fax: _____
Email: _____

Ship To (UPS will not deliver to a P.O. Box)

Name: _____
Title: _____
Company: _____
Department: _____
Address: _____

City: _____ State/Province: _____
Zip/Postal Code: _____ Country: _____
Telephone: _____
Fax: _____
Email: _____

Quantity	Title	SO★	Unit Price	Total

☐ Payment Enclosed ☐ P.O. No. (Enclose Copy) _____

☐ Charge My IHS Account No. _____

☐ VISA ☐ MasterCard ☐ American Express
☐ Diners Club ☐ Discover

Credit Card No.: _____

Print Name (As It Appears on Card): _____

Expiration Date: _____

Signature: _____

Subtotal

Applicable Sales Tax (see below)

Rush Shipping Fee (see below)

Shipping and Handling (see below)

Total (in U.S. Dollars)

★ To be placed on Standing Order for future editions of this publication, place a check mark in the SO column and sign here:

Pricing and availability subject to change without notice.

Mail Orders - Payment by check or money order in U.S. dollars is required except for established accounts. State and local taxes, \$10 processing fee, and 5% shipping must be added. Send mail orders to: **API Publications, IHS, 15 Inverness Way East, c/o Retail Sales, Englewood, CO 80112-5776, USA.**

Purchase Orders - Purchase orders are accepted from established accounts. Invoice will include actual freight cost, a \$10 processing fee, plus state and local taxes.

Telephone Orders - If ordering by telephone, a \$10 processing fee and actual freight costs will be added to the order.

Sales Tax - All U.S. purchases must include applicable state and local sales tax. Customers claiming tax-exempt status must provide IHS with a copy of their exemption certificate.

Shipping (U.S. Orders) - Orders shipped within the U.S. are sent via traceable means. Most orders are shipped the same day. Subscription updates are sent by First-Class Mail. Other options, including next-day service, air service, and fax transmission are available at additional cost. Call 1-800-854-7179 for more information.

Shipping (International Orders) - Standard international shipping is by air express courier service. Subscription updates are sent by World Mail. Normal delivery is 3-4 days from shipping date.

Rush Shipping Fee - Next Day Delivery orders charge is \$20 in addition to the carrier charges. Next Day Delivery orders must be placed by 2:00 p.m. MST to ensure overnight delivery.

Returns - All returns must be pre-approved by calling the IHS Customer Service Department at 1-800-624-3974 for information and assistance. There may be a 15% restocking fee. Special order items, electronic documents, and age-dated materials are non-returnable.

THERE'S MORE WHERE THIS CAME FROM.

API provides additional resources and programs to the oil and natural gas industry which are based on API Standards. For more information, contact:

API MONOGRAM® LICENSING PROGRAM

Phone: 202-962-4791
Fax: 202-682-8070
Email: certification@api.org

API QUALITY REGISTRAR (APIQR®)

- > ISO 9001 Registration
- > ISO/TS 29001 Registration
- > ISO 14001 Registration
- > API Spec Q1® Registration

Phone: 202-962-4791
Fax: 202-682-8070
Email: certification@api.org

API PERFORATOR DESIGN REGISTRATION PROGRAM

Phone: 202-682-8490
Fax: 202-682-8070
Email: perfdesign@api.org

API TRAINING PROVIDER CERTIFICATION PROGRAM (API TPCP™)

Phone: 202-682-8490
Fax: 202-682-8070
Email: tpcp@api.org

API INDIVIDUAL CERTIFICATION PROGRAMS (ICP®)

Phone: 202-682-8064
Fax: 202-682-8348
Email: icp@api.org

API ENGINE OIL LICENSING AND CERTIFICATION SYSTEM (EOLCS)

Phone: 202-682-8516
Fax: 202-962-4739
Email: eolcs@api.org

API PETROTEAM (TRAINING, EDUCATION AND MEETINGS)

Phone: 202-682-8195
Fax: 202-682-8222
Email: petroteam@api.org

API UNIVERSITY™

Phone: 202-682-8195
Fax: 202-682-8222
Email: training@api.org

Check out the API Publications, Programs, and Services Catalog online at www.api.org.



Copyright 2008 – API, all rights reserved. API, API monogram, APIQR, API Spec Q1, API TPCP, ICP, API University and the API logo are either trademarks or registered trademarks of API in the United States and/or other countries.



1220 L Street, NW
Washington, DC 20005-4070
USA

202.682.8000

Additional copies are available through IHS

Phone Orders: 1-800-854-7179 (Toll-free in the U.S. and Canada)
303-397-7956 (Local and International)

Fax Orders: 303-397-2740

Online Orders: global.ihs.com

Information about API Publications, Programs and Services
is available on the web at www.api.org

Product No. G1105