Manual of Petroleum Measurement Standards Chapter 13—Statistical Aspects of Measuring and Sampling

Section 2—Methods of Evaluating Meter Proving Data

FIRST EDITION, NOVEMBER 1994

REAFFIRMED, FEBRUARY 2011

ERRATA, OCTOBER 2015

AMERICAN PETROLEUM INSTITUTE

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Suggested revisions are invited and should be submitted to Measurement Coordination, Industry Services Department, American Petroleum Institute, 1220 L Street, N.W., Washington, D.C. 20005.

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ERRATA

Page 9, **Table 8**, row $n = 5$, 95% Confidence Level, *replace*

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Chapter 13—Statistical Aspects of Measuring and Sampling

SECTION 2—METHODS OF EVALUATING METER PROVING DATA

13.2.0 Introduction

Minimizing systematic and random errors, estimating remaining errors, and informing affected parties of errors are becoming increasingly important to the petroleum industry. A consistent basis of estimating the size and significance of errors is essential for communications between affected parties. A consistent basis of estimating and controlling errors can help to avoid disputes and dispel delusions on the accuracy of activities and equipment related to meter proving operations.

A wide range of designs, equipment, and service operating conditions is experienced in meter proving operations. Because of these variations, it is impractical to establish fixed procedures for maintenance, calibration, and proving activities for all installations.

Meter proving factors (meter factors) are normally monitored to detect trends or sudden deviations as indications of when to perform maintenance and calibration of measurement equipment.

The purpose of this chapter is to provide procedures for recording, analyzing, and controlling variations in meter factors so that random uncertainties are understood and consistent with the objectives of parties affected by the measurement operations. Limits on meter factor variations are left to the agreement of parties affected by the measurement operations.

13.2.1 Scope

Chapter 13.2 will address procedures for evaluating any meter's performance where meter proving factors are developed in accordance with Chapter 12.2. The data in examples used in this chapter are intended to be typical of custody transfer operations of low-vapor-pressure fluids using displacement or turbine meters in accordance with Chapters 4, 5, and 6 of the *Manual of Petroleum Measurement Standards*. However, the procedures in Chapter 13.2 can be used for non-custody transfer metering applications and for custody transfer metering of high-vapor-pressure and gaseous fluids where meter proving data are available.

Procedures and examples are given for analyzing the random uncertainties associated with meter proving data (see note). Procedures and examples are also given for evaluating and controlling trends in meter factors with control charts and control logs to ensure that meter factor variations exhibit a random nature that results in the propagation of a lower average uncertainty in measurements with time and throughput.

Note: Uncertainty computations are based on procedures given in Chapter 13.1 of the *Manual of Petroleum Measurement Standards*.

Since no single document can cover all of the statistical procedures and applications being practiced, procedures other than those appearing herein may be used. Alternate statistical procedures are not expected to duplicate the computational values provided by procedures in this chapter. However, alternative procedures should achieve the same purpose intended by the procedures given herein. When alternative statistical computational procedures are to be used and affect metered quantities, parties directly involved with the metering operations should be notified prior to their implementation.

Some of the procedures in Chapter 13.2 are suitable for hand calculations and graphs employed by field personnel. These procedures are discussed and illustrated in 13.2.5 of this chapter. However, the statistical evaluations and control charts are generally too rigorous for manual field computations, and computers should be employed. The statistical procedures in this document may serve as a guide for developing software for computer applications.

13.2.2 Definitions

The following definitions supplement the definitions appearing in Chapter 13.1.

Action limits are control limits applied to a control chart or log to indicate when action is necessary to inspect or calibrate equipment and possibly issue a correction ticket. Action limits are normally based on 95 percent to 99 percent confidence levels for statistical uncertainty analyses of the group of measurements.

Control chart is a graphical method for evaluating whether meter proving operations are in or out of a state of statistical control.

Control log is a tabular method for evaluating whether meter proving operations are in or out of a state of statistical control.

Control chart or *log, fixed limit* is a control chart or log whose control limits are based on adopted fixed values applicable to the statistical measurements displayed on the log or chart. Historically, fixed limits have been used to control the limits on meter factor changes.

Control chart or *log, no fixed limit* is a control chart or log whose control limits are based on the statistical variations of measurements displayed on the chart or log.

Control limits are limits applied to a control chart or log to indicate the need for action and/or whether or not data is in a state of statistical control. Several control limits can be applied to a single control chart or log to determine when various levels of action are warranted. Terms used to describe various control limits are "warning," "action," and "tolerance" limits.

Central line is a line on a control chart representing the average or standard value of the statistical measure being plotted and is the reference line or value from which control limits are determined and plotted.

Central value is the average or standard value of the statistical measure being logged and is the reference value from which control limits are determined.

Moving series is a cumulative group of data (meter factors) whose scope increases as more data become available.

Rational subgroup is a grouping of measurements chosen in such a way that variations within the group and subgroups are independent and random in nature.

A *run* is an uninterrupted, consecutive sequence of events that are measured cumulatively, such as meter pulses during a single trip or round-trip of a displacer in a volumetric meter prover.

Set is a series of runs made under similar and controlled conditions.

A *Shewhart control chart* consists of control chart procedures developed by Walter A. Shewhart. The control limits are based on empirical and economic considerations for the industrial processes evaluated by Shewhart. Control charts are generally based on the variation of individual measurements within ±3 times the average standard deviation for all measurements. These charts require rational subgroups. The limits do not relate directly to the 99.7 percent uncertainty bounds for measurements because average standard deviations are used. Refer to Appendix C for information on Shewhart control chart procedures.

Tolerance limits are control limits that define the conformance boundaries for meter factor variation. Tolerance boundaries indicate when an audit or technical review of the facility design operating variables and/or computations may be necessary to determine sources of errors and changes required to reduce variations in the meter factors. Tolerance limits are normally based on 99 percent or greater confidence levels.

Warning limits are control limits applied to a control chart or log to indicate when meter proving equipment, operating conditions, or computations should be checked because meter factors are outside preestablished limits. Warning limits are normally based on 90 to 95 percent confidence levels for assessing statistical uncertainty in the group of measurements.

13.2.3 Nomenclature

The following alphanumeric and algebraic symbols are used in Chapter 13.2:

a	Estimate of A.
D	Conversion factor (used to derive s from w).
G, H, I, J, K, L, M, N	Conversion factors for computing control limits in Shewhart control chart procedures. The sequence of nomenclature corresponds to A_2, A_3, B_3, B_4, D_3, D_4, E_2, and E_3, respectively, appearing in other published sources on Shewhart control charts.
k	number of sets in a series of measurements.
$\bar{k}$	Average of k sets of measurements.
MF	Meter factor.
$\overline{MF}$	Average meter factor.
n	Number of repeat measurements in a set.
$\bar{n}$	Average of n repeated measurements in a set.
s	Estimate of standard deviation.
t	Value of Student's t distribution.
$T(\%)$	Conversion factor used to derive uncertainty from the standard deviation.
$T(\%, n \text{ or } k)$	Conversion factor for deriving uncertainty of individual measurements from the standard deviation of a series of runs or sets.
$T(\%, \bar{n} \text{ or } \bar{k})$	Conversion factor for deriving uncertainty of the average from the standard deviation of a series of runs or sets.
w	Range of a set of data.
x	Observed value of a variable.
$\bar{x}$	Observed mean value of a set of data.
$\bar{\bar{x}}$	Overall average of the means of individual sets.
$Z(\%, n \text{ or } k)$	Conversion factor for deriving uncertainty of individual measurements from the average range of a series of runs or sets.
$Z(\%, \bar{n} \text{ or } \bar{k})$	Conversion factor for deriving uncertainty of the average from the average range of a series of runs or sets.
$(\%)$	Denotes the confidence level for conversion factors for calculating uncertainty from standard deviation or range data.
ϕ	Degrees of freedom, $n - 1$.

13.2.4 Referenced Publications

The most recent editions of the following standards, specifications, and publications are cited in this standard:

ANSI[1]/ASQC[2]

A1	*Definitions, Symbols, Formulas, and Tables for Control Charts*
B1, B2, and B3	*Guide for Quality Control Charts, Method of Analyzing Data, and Controlling Quality During Production*

[1]American National Standards Institute, 11 West 42nd Street, New York, New York 10036.

[2]American Society of Quality Control, P.O. Box 3005, Milwaukee, Wisconsin 53201-9402.

API

Manual of Petroleum Measurement Standards

Chapter 4, "Proving Systems"
Chapter 5, "Metering"
Chapter 6, "Metering Assemblies"
Chapter 12.2, "Calculation of Liquid Petroleum Quantities Measured by Turbine or Displacement Meters"
Chapter 13.1, "Statistical Concepts and Procedures in Measurement"

ASTM[3]

MNL 7 *Manual on Presentation of Data and Control Chart Analysis*

13.2.5 Meter Factor Logs and Graphs

A flow meter will vary in response to changes in flow rate, mechanical condition of the meter, changes in fluid properties, contaminants content of the flowing stream, and the amount of paraffin deposits. The control chart provides a method of determining whether an adjustment should be initiated, the meter should be repaired, or both.

Fluid properties that directly affect the flow meter's performance are viscosity, density (API gravity), and lubricity.

A flow meter's performance is represented by its meter factor. However, the meter factor also represents changes in the performance of the prover, interchange valves, detector switches, prover sphere, prover coating, pulse generators, and proving counters. Simply stated, the meter factor represents the performance of the flow meter and the proving system.

13.2.5.1 METER FACTOR LOG

A meter factor log is a table for recording in sequence each meter factor and data that affect the meter's operation. A simple example of a meter log is shown in Figure 1.

The meter factor log facilitates hand or computer computation of changes between consecutive meter factors and net accumulation of changes from a baseline meter factor. Meter factor logs are normally based on fixed values to determine when warning, action, or tolerance limits are exceeded. In the example in Figure 1, action limits of ±0.50 percent from the baseline factor are shown for illustration purposes.

13.2.5.2 METER FACTOR GRAPH

The data on the meter factor log can also be plotted on a graph as shown in Figure 2. Using graphs facilitates the detection of trends by field operating personnel. The meter factors plotted in Figure 2 exhibit a significant trend that could result in a systematic error in the custody measurement quantities.

[3]American Society for Testing and Materials, 1916 Race Street, Philadelphia, Pennsylvania 19103-1187.

13.2.6 Application of Statistics to Meter Proving Data

13.2.6.1 FIXED LIMITS

Fixed criteria are normally used for judging the acceptability of meter factor data. Industry practice is for custody transfer parties to reach agreement on a minimum number of proving runs that agree within a maximum range between high and low meter proving pulses or meter factors. Custody transfer parties also agree on a meter proving interval and deviation limit between consecutive meter factors. Cumulative meter factor change for a new or overhauled meter is usually given a fixed limit unless fixed time or throughput intervals are applied for inspection and maintenance. The deviation limits between consecutive meter provings and cumulative meter factor changes serve as warning and action limits to operating personnel in judging the reliability of the meter proving data and the physical condition of the meter and proving systems.

Fixed limits should be based on the experience of attainable performance of metering systems and the stability of operating conditions such as flow rate, pressure, temperature, and fluid properties. However, experience-based criteria will not always be appropriate for new equipment or operating conditions. Statistical methods should be used to develop meter proving limits for new installations and applications where the suitability of experience-based limits is questionable. Statistics can also be used to judge the suitability of fixed meter proving ranges and deviation limits. Conversely, fixed limits can also be used to judge the suitability of statistically based limits for metering systems that exhibit broad deviations in meter proving data.

13.2.6.2 EVALUATING TRENDS IN A SINGLE SET OF METER PROVING DATA

Operating changes during and between meter proving runs should be minimized so that variations in meter pulses or meter factors are primarily due to the performance of the meter and proving system. Meter factors or meter pulses for each run should be evaluated in sequence to determine if there is a time related trend due to changing operational parameters or malfunctioning equipment. The set of meter factors listed for the example in Table 1 illustrates a sequential bias and should not be used for correcting the meter's output unless it can be determined that the midpoint of the set represents the midpoint of operating conditions of the meter.

Trending analyses may not be practical or cost effective every time a meter factor is determined. It is presented for periodically analyzing meter proving data to determine that proving conditions are consistent with actual operations.

The set of meter factors shown in Table 1 are graphed in sequence in Figure 3. The meter factors show a trend that would be undetectable if only two meter proving runs were made and used for the meter factor.

XYZ Company

METER FACTOR LOGGING SHEET

Location: Anywhere | Meter Make/Style: Generic PD | Flow Range: Min: 85 Max: 850 BPH

Serial #: G123456 | Temp Comp: Yes / No ATG | Action Limit: ±0.50%

Product: Crude | ProverType: Bi-Dir Eall | Prover Serial #: C12345

Sequence No.	Date	Meter Factor	Proving Report #	Meter Temp.	Meter Press.	Gravity API @ 60°F	Flow Rate	% Deviation Last Proving	% Deviation Baseline	Remarks
1	1-2-87	1.0005	101	78.0	60	35.5	180	Initial	Initial	Baseline factor
2	2-3-87	1.0008	102	76.5	58	35.4	178	+0.03	+0.03	
3	3-5-87	1.0010	103	80.5	61	35.8	181	+0.02	+0.05	
4	4-3-87	1.0015	104	81.2	60	36.0	180	+0.05	+0.10	
5	5-5-87	1.0021	105	84.0	61	35.8	179	+0.06	+0.16	
6	6-4-87	1.0019	106	83.5	60	35.5	182	-0.02	+0.14	
7	7-7-87	1.0028	107	87.0	62	35.8	180	+0.09	+0.23	
8	8-4-87	1.0037	108	88.0	60	35.4	180	+0.09	+0.32	
9	9-6-87	1.0048	109	89.0	60	35.8	181	+0.11	+0.43	
10	10-3-87	1.0042	110	85.0	60	35.2	185	+0.06	+0.37	
11	11-4-87	1.0061	111	81.0	60	35.4	180	+0.19	+0.55	Repaired meter
12	11-5-87	1.0002	112	81.5	59	35.6	176	–	–	First factor
13	11-6-87	1.0010	113	83.5	61	36.1	181	–	–	Baseline factor
14	12-8-87	1.0002	114	78.5	62	35.7	175	-0.08	-0.08	
15	1-4-88	1.0009	115	77.0	61	36.1	172	+0.07	-0.01	
16	2-6-88	1.0018	116	74.5	63	36.6	170	+0.09	+0.08	
17	3-4-88	1.0015	117	76.0	61	36.1	175	-0.03	+0.05	
18	4-6-88	1.0028	118	81.5	60	36.0	180	+0.13	+0.18	
19	5-4-88	1.0020	119	82.5	59	35.9	181	-0.08	+0.10	

Figure 1—Example of a Meter Factor Log

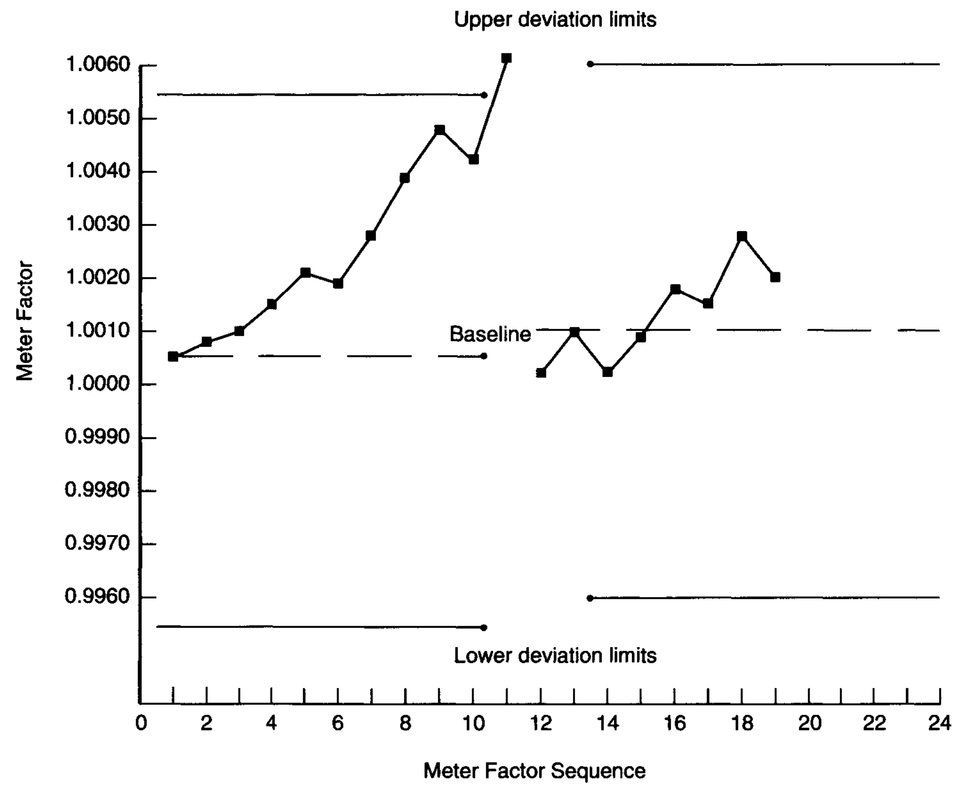


Figure 2—Example of a Meter Factor Graph

Further examination of the meter factor data yields the additional information in Table 2.

Historical data on operating flow rates for the meter in this example are summarized in Table 3 and illustrated in Figure 4.

The cumulative distribution of operating flow rates for the meter in question is shown in Figure 4. Since the meter operates between 280 and 320 barrels per hour 95 percent of the time, meter proving factors outside this range should normally be excluded unless performance curves and operating history establish the linearity of the meter for the metered fluids over the range of operating flow rates. In this example, meter factors one through seven should be excluded from the meter factor determination. The average meter factor for the set should be based on runs eight, nine, and ten if only three runs are required to meet the company's meter proving procedure. If more meter proving factors are required, additional provings should be made within the normal operating flow range of the meter. If three more meter provings are conducted and added to meter factors eight, nine, and ten, the modified meter proving set becomes that shown in Table 4.

Table 1—Example: Sequential Set of Meter Factors

Sequence	Meter Factor, *MF*
1	1.0006
2	1.0008
3	1.0012
4	1.0010
5	1.0011
6	1.0015
7	1.0014
8	1.0016
9	1.0021
10	1.0020

13.2.6.3 STATISTICAL ANALYSES OF A SINGLE SET OF METER PROVING DATA

Fixed limits are usually applied to determine the acceptability of meter proving data. Statistical procedures can be used to evaluate the uncertainty of various meter proving criteria consisting of varying deviation limits and time intervals. A statistically based uncertainty criterion can also be used to determine the acceptability of a set of meter proving data.

The computed average or arithmetic mean of a single set of values is calculated as follows:

$$\bar{x} = \frac{1}{n}\left(x_1 + x_2 + \dots . x_n\right) \quad (1)$$

For the set of meter proving runs in Table 4, the average meter factor is determined as follows:

Table 2— Example: Meter Proving Operating Parameters

Sequence	Meter Factor, *MF*	Flow Rate (bph)	Temperature (°F)
1	1.0006	200	46
2	1.0008	205	47
3	1.0012	215	49
4	1.0010	230	51
5	1.0011	250	52
6	1.0015	255	53
7	1.0014	275	57
8	1.0016	290	58
9	1.0021	305	59
10	1.0020	310	60

Note: bph = barrels per hour.

$$\overline{MF} = \frac{1}{6}(1.0016 + 1.0021 + 1,0020 + 1.0018 + 1.0021 + 1.0020)$$
$$= 1.0019$$

The standard deviation of a single set of meter proving runs is determined with the following equation:

$$s(x) = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2} \quad (2)$$

Table 5 shows the calculation of the standard deviation for the modified meter proving set in Table 4.

Another statistical quantity used to evaluate a set of meter proving runs is range, which is determined as follows:

$$w(x) = \textit{largest value} - \textit{smallest value} \quad (3)$$

For the data in Table 5, the range is calculated using Equation 3 as follows:

$$w(MF) = 1.0021 - 1.0016 = 0.0005$$

The standard deviation for a single set of meter proving runs can be estimated from the average range as follows:

Table 3—Example: Historical Statistical Data on Flow Rates

Flow Rate (bph)	Historical Occurrence (percent)	Cumulative Occurrence (percent)
220 to 230	.25	.25
230 to 240	.25	.50
240 to 250	0.5	1
250 to 260	1	2
260 to 270	1	3
270 to 280	2	5
280 to 290	10	15
290 to 300	40	55
300 to 310	30	85
310 to 320	15	100

Note: bph = barrels per hour.

$$s(x) = \frac{\overline{w}(x)}{D(n)} \text{ or } \frac{\overline{w}(x)}{D(k)} \quad (4)$$

Values of the range to standard deviation conversion factors are given in Table 6.

For the set of meter factors in Table 4, the standard deviation can be estimated from its range and Equation 4 as follows:

$$s(MF) = \frac{0.0005}{D(6)} = \frac{0.0005}{2.534} = 0.0002$$

The standard deviation of the average of a single set of meter proving runs is estimated as follows:

$$s(\bar{x}) = \frac{s(x)}{\sqrt{n}} \quad (5)$$

For the example in Table 5, the standard deviation of the average of a set of meter proving runs is estimated as follows:

$$s(\overline{MF}) = \frac{0.0002}{\sqrt{6}} = 0.0001$$

For the example given in Table 5, both the calculated and estimated standard deviations are the same within the number of significant digits for computations. This is a coincidence because range conversion factors in Table 6 are primarily for

Table 4—Example: Modified Meter Proving Set

Sequence	Meter Factor, *MF*	Flow Rate (bph)	Temperature (°F)
8	1.0016	290	58
9	1.0021	305	59
10	1.0020	310	60
11	1.0018	305	60
12	1.0021	310	61
13	1.0020	310	61

Note: bph = barrels per hour.

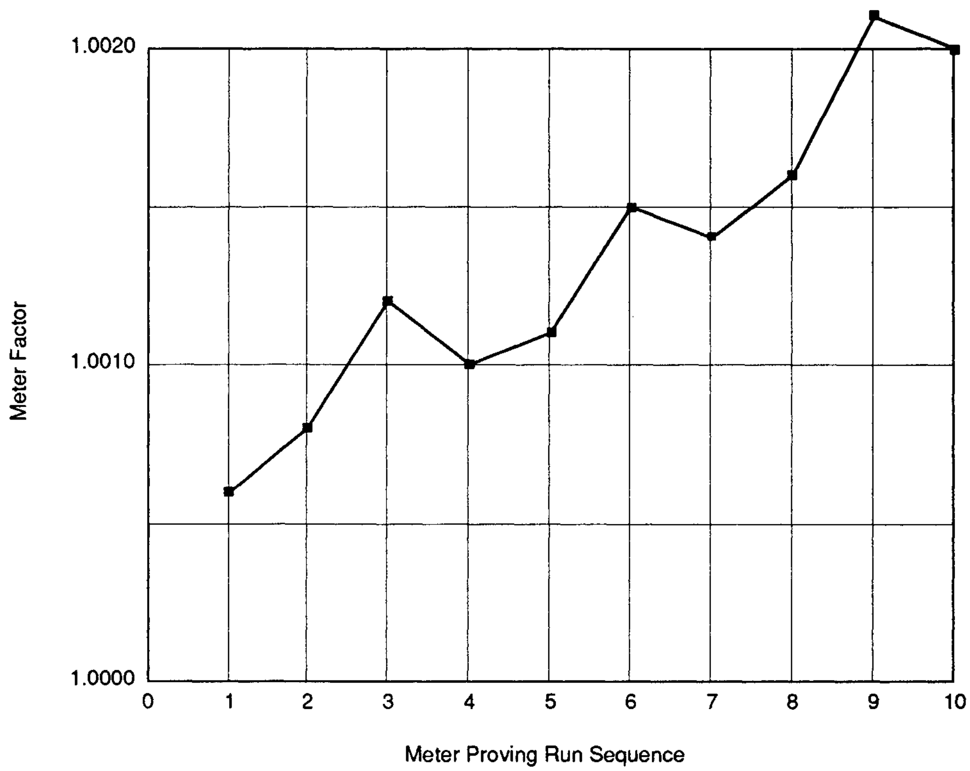


Figure 3—Meter Factors for Example

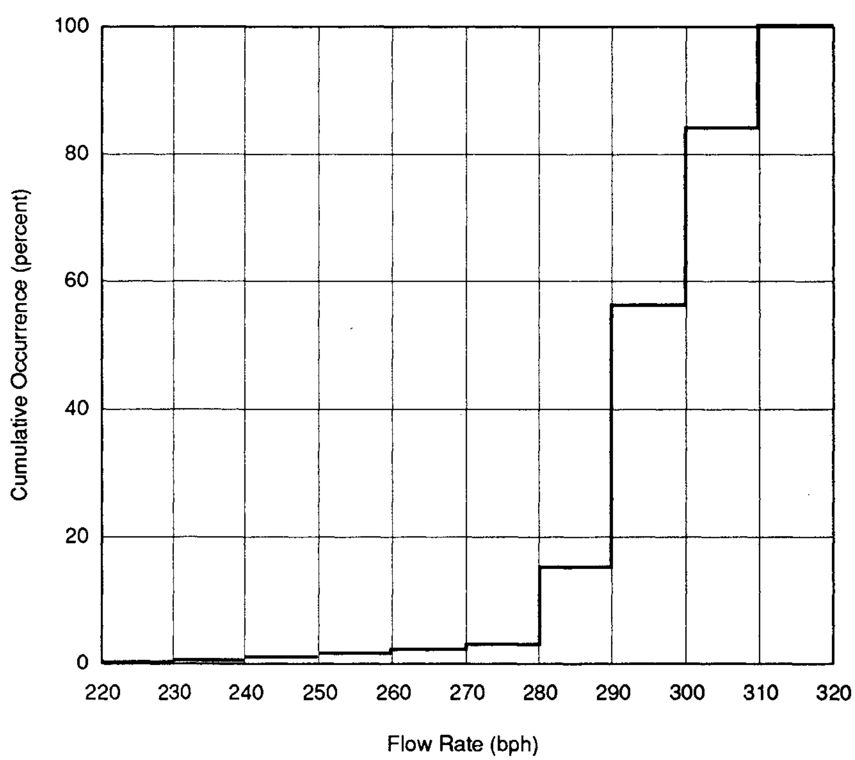


Figure 4—Cumulative Distribution of Flow Rates

Table 5—Standard Deviation Calculation

Sequence	Meter Factor, MF	$MF - \overline{MF}$	$(MF - \overline{MF})^2$
8	1.0016	–0.0003	0.00000009
9	1.0021	0.0002	0.00000004
10	1.0020	0.0001	0.00000001
11	1.0018	–0.0001	0.00000001
12	1.0021	0.0002	0.00000004
13	1.0020	0.0001	0.00000001
	$\overline{MF}$ = 1.0019		0.00000020

$$s(x) = \sqrt{\left(\frac{1}{6-1}\right)(0.00000020)} = 0.0002$$

use with average range ($\overline{w}$) from a series of measured values. In this example, only one range value is given.

In evaluating the statistical variation of a set of values, the range, w, is an easier quantity to use than the standard deviation calculated in accordance with Equation 2, especially for hand calculations. However, standard deviation is generally a more suitable indicator of the statistical variation of a set of values from its mean.

13.2.6.4 UNCERTAINTY ANALYSES OF A SINGLE SET OF METER PROVING DATA

The random uncertainty of individual measurements in a set of data in relation to the true mean is the product of the standard deviation and statistical factors to convert standard deviation to uncertainty. The uncertainty of the individual measurements in a set is expressed as follows:

$$a(x) = [T(\%)][s(x)] \qquad (6)$$

For large sets (greater than 25) of independent random measurements, statistical variations in the data usually follow a normal (Gaussian) distribution. Standard deviation to uncertainty conversion factors, $T(\%)$, versus confidence level are shown in Table 7.

For small sets (25 or less), t-distribution values should be applied to the standard deviation to estimate uncertainty. The t-distribution values to replace $T(\%)$ in Equation 6 are given in Table 8 for confidence levels of 90 percent, 95 percent, 99 percent, and 99.5 percent.

In this document, the degrees of freedom (ϕ) associated with the Student t functions are taken as $n - 1$ or $k - 1$. To avoid confusion between n or k and degrees of freedom, the following term is defined and used in place of the Student t functions:

$$T(\%,\ n \text{ or } k) = t(\%,\ \phi) \qquad (7)$$

The random uncertainty of individual measurements in a small set is estimated as follows:

$$a(x) = [T(\%,\ n)][s(x)] \qquad (8)$$

Table 6—Range to Standard Deviation Conversion Factors

Number of Values n or k	Range to Standard Deviation Conversion Factor $D(n)$ or $D(k)$
2	1.128
3	1.693
4	2.059
5	2.326
6	2.534
7	2.704
8	2.847
9	2.970
10	3.078
11	3.173
12	3.258
13	3.336
14	3.407
15	3.472
16	3.532
17	3.588
18	3.640
19	3.689
20	3.735
21	3.778
22	3.819
23	3.858
24	3.895
25	3.931

The random uncertainty of individual means in k sets of data is estimated as follows:

$$a(\bar{x}) = [T(\%,\ k)][s(\bar{x})] \qquad (9)$$

For applications of statistics to custody measurement, the 95 percent confidence level is traditionally used for analyzing and reporting uncertainties in measured values. However, other confidence levels can be used as appropriate for specific measurements.

For measurements that are known to be random and are frequently repeated, the average value can be propagated to a lower value of uncertainty. Confidence levels of 90 percent or lower are suitable for analyzing and reporting uncertainties of these individual measurements. For example, the

Table 7—Uncertainty Conversion Factors for Normal Distribution

Confidence Limit (%)[a]	Conversion Factor, T (%)
50	0.6745
68.3	1.00
90	1.645
95	1.960
95.5	2.00
99	2.576
99.7	3.00

[a]Probability that true value falls within limits stated for calculated value.

Table 8—*t*-Distribution Factors for Individual Measurements

Number of Sets or Measurements, *n* or *k*	*T* (%, *n* or *k*) Versus Confidence Level			
	90%	95%	99%	99.5%
2	6.314	12.706	63.657	127.320
3	2.920	4.303	9.925	14.089
4	2.353	3.182	5.841	7.453
5	2.132	2.770	4.604	5.598
6	2.015	2.571	4.032	4.773
7	1.943	2.447	3.707	4.317
8	1.895	2.365	3.499	4.029
9	1.860	2.306	3.355	3.833
10	1.833	2.262	3.250	3.690
11	1.812	2.228	3.169	3.581
12	1.796	2.201	3.106	3.497
13	1.782	2.179	3.055	3.428
14	1.771	2.160	3.012	3.372
15	1.761	2.145	2.977	3.326
16	1.753	2.131	2.947	3.286
17	1.746	2.120	2.921	3.252
18	1.740	2.110	2.898	3.222
19	1.734	2.101	2.878	3.197
20	1.729	2.093	2.861	3.174
21	1.725	2.086	2.845	3.153
22	1.721	2.080	2.831	3.135
23	1.717	2.074	2.819	3.119
24	1.714	2.069	2.807	3.104
25	1.711	2.064	2.797	3.091
∞	1.645	1.960	2.576	2.807

random uncertainty in a single reading of a thermometer graduated in and read to whole degree units is ±0.5°. If a single thermometer is used for five different measurements, the random error in the average of five measured temperatures would be propagated to the following:

$$a(\overline{scale}) = \frac{\pm 0.5°}{\sqrt{5}} = \pm 0.2°$$

If in the above example, the thermometer experienced a scale position error of –2°, this error would not be propagated to a lower average value with multiple measurements. Therefore, uncertainties regarding potential scale position errors should be evaluated at a higher confidence level than scale discrimination errors that are random in nature.

The random uncertainty of the calculated mean in a set of *n* measurements is estimated as follows:

$$a(\bar{x}) = \frac{a(x)}{\sqrt{n}} \quad (10)$$

The random uncertainty of the overall average of calculated means in *k* sets of measurements is estimated as follows:

$$a(\bar{\bar{x}}) = \frac{a(\bar{x})}{\sqrt{k}} = \frac{a(x)}{\sqrt{nk}} \quad (11)$$

The following term is defined for calculating uncertainties of averages using Student *t* functions and standard deviation.

$$T(\%, \bar{n} \text{ or } \bar{k}) = \frac{T(\%, n)}{\sqrt{n}} \text{ or } \frac{T(\%, k)}{\sqrt{k}} \quad (12)$$

Equations 8, 10, and 12 can be combined as follows to calculate uncertainty for the average in a set of *n* measurements.

$$a(\bar{x}) = [T(\%, \bar{n})][s(x)] \quad (13)$$

Equations 9, 11, and 12 can be combined as follows to calculate uncertainty for the overall average of the means of *k* sets of data:

$$a(\bar{\bar{x}}) = [T(\%, \bar{k})][s(\bar{x})] \quad (14)$$

Values of $T(\%, \bar{n}$ or $k)$ are given in Table 9.

For the meter proving example shown in Table 4 having an average meter factor ($\overline{MF}$) of 1.0019 and a standard deviation of 0.0002, the uncertainty in the calculated average meter factor at a 95 percent confidence level using Equation 13 is as follows:

$$a(\overline{MF}) = [T(95, \bar{6})][s(MF)]$$
$$a(1.0019) = (1.050)(0.0002) = \pm 0.0002$$

When ranges are used for estimating standard deviation, Equations 4 and 8 or 9 can be combined as follows and the following term is defined for range to estimated uncertainty conversion factors for individual measurements in a set:

Table 9—*t*-Distribution Factors for Averages

Number of Sets or Measurements, n or k	$T(\%, \bar{n} \text{ or } \bar{k})$ Versus Confidence Level			
	90%	95%	99%	99.5%
2	4.465	8.984	45.012	90.029
3	1.686	2.484	4.730	8.134
4	1.177	1.591	2.921	3.726
5	0.953	1.241	2.059	2.504
6	0.823	1.050	1.646	1.949
7	0.734	0.925	1.401	1.632
8	0.670	0.836	1.237	1.424
9	0.620	0.769	1.118	1.278
10	0.580	0.715	1.028	1.167
11	0.546	0.672	0.955	1.080
12	0.518	0.635	0.897	1.009
13	0.494	0.604	0.847	0.951
14	0.473	0.577	0.805	0.901
15	0.455	0.554	0.769	0.859
16	0.438	0.533	0.737	0.822
17	0.423	0.514	0.708	0.789
18	0.410	0.497	0.683	0.759
19	0.398	0.482	0.660	0.733
20	0.387	0.468	0.640	0.710
21	0.376	0.455	0.621	0.688
22	0.367	0.443	0.604	0.668
23	0.358	0.432	0.588	0.650
24	0.350	0.422	0.573	0.634
25	0.342	0.413	0.559	0.618

$$Z(\%,\ n \text{ or } k) = \frac{T(\%,\ n \text{ or } k)}{D(n)} \qquad (15)$$

$$a(x) = [Z(\%,\ n)][\bar{w}(x)] \qquad (16)$$

$$a(\bar{x}) = [Z(\%,\ k)][\bar{w}(\bar{x})] \qquad (17)$$

Range to estimated uncertainty conversion factors for individual values in a set or individual means in a series of data sets are given in Table 10.

The following term is defined for calculating uncertainties of averages using Student *t* functions, range, and number of measurements in a set, *n*, or number of sets, *k*.

$$Z(\%,\ \bar{n}) = \frac{Z(\%,\ n)}{\sqrt{n}} \text{ or} \qquad (18)$$

$$Z(\%,\ \bar{k}) = \frac{Z(\%,\ k)}{\sqrt{k}} \qquad (19)$$

The uncertainty in the average of a set of measurements from the range of the set of measurements can be estimated as follows:

$$a(\bar{x}) = [Z(\%,\ \bar{n})][\bar{w}(x)] \qquad (20)$$

$$a(\bar{\bar{x}}) = [Z(\%,\ \bar{k})][\bar{w}(\bar{x})] \qquad (21)$$

Range to estimated uncertainty conversion factors for the mean in a set of *n* measurements or for the overall average of the means of *k* sets of data are given in Table 11.

For the example in Table 4 having a range of 0.0005, the uncertainty of the average of the meter factor set at the 95 percent confidence level is calculated as follows:

$$a(\overline{MF}) = [Z(95,\ \bar{6})][w(MF)]$$

$$a(1.0019) = (0.420)(0.0005)$$

$$= \pm 0.0002$$

The average meter factor for the meter proving set in this example, along with the random computational uncertainty, is expressed as follows:

$\overline{MF} = 1.0019 \pm 0.0002$ (95 percent confidence level, 6 measurements)

or abbreviated as

$\overline{MF} = 1.0019 \pm 0.0002\ (95, \bar{6})$

13.2.6.5 UNCERTAINTY ANALYSES OF A SERIES OF METER PROVING FACTORS

The example in Table 12 will be used to illustrate the statistical analyses of the random uncertainty in a series of meter proving factors.

The following computations cover two methods of estimating the uncertainty in the overall average of the series of ten meter factors. One method uses standard deviations, and

Table 10—Range to Estimated Uncertainty Conversion Factors for Individual Measurements

Number of Sets or Measurements *n* or *k*	*Z* (%, *n* or *k*) Versus Confidence Level			
	90%	95%	99%	99.5%
2	5.598	11.264	56.433	112.870
3	1.725	2.542	5.863	8.322
4	1.143	1.545	2.836	3.620
5	0.917	1.193	1.871	2.407
6	0.795	1.015	1.538	1.884
7	0.719	0.905	1.339	1.597
8	0.666	0.831	1.209	1.415
9	0.626	0.776	1.115	1.291
10	0.596	0.735	1.045	1.199
11	0.571	0.702	0.991	1.129
12	0.551	0.676	0.948	1.073
13	0.534	0.653	0.911	1.028
14	0.520	0.634	0.880	0.990
15	0.507	0.618	0.855	0.958
16	0.496	0.603	0.831	0.930
17	0.487	0.591	0.812	0.906
18	0.478	0.580	0.795	0.885
19	0.470	0.570	0.779	0.864
20	0.463	0.560	0.764	0.850
21	0.457	0.553	0.753	0.835
22	0.451	0.546	0.742	0.821
23	0.445	0.538	0.730	0.808
24	0.440	0.532	0.721	0.797
25	0.435	0.525	0.710	0.786

Table 11—Range to Estimated Uncertainty Conversion Factors for Averages

Number of Sets or Measurements, *n* or *k*	*Z* (%, $\bar{n}$ or $\bar{k}$) Versus Confidence Level			
	90%	95%	99%	99.5%
2	3.958	7.965	39.904	79.811
3	0.996	1.467	3.385	4.805
4	0.572	0.780	1.419	1.810
5	0.410	0.540	0.885	1.076
6	0.325	0.420	0.650	0.769
7	0.271	0.340	0.518	0.604
8	0.235	0.290	0.434	0.500
9	0.209	0.260	0.376	0.430
10	0.188	0.230	0.334	0.379
11	0.172	0.212	0.301	0.340
12	0.159	0.195	0.275	0.310
13	0.148	0.181	0.254	0.285
14	0.139	0.169	0.236	0.265
15	0.131	0.160	0.221	0.247
16	0.124	0.151	0.209	0.232
17	0.118	0.143	0.197	0.220
18	0.113	0.137	0.188	0.209
19	0.108	0.131	0.179	0.198
20	0.104	0.125	0.171	0.190
21	0.100	0.120	0.164	0.182
22	0.096	0.116	0.158	0.175
23	0.093	0.112	0.152	0.168
24	0.090	0.108	0.147	0.163
25	0.087	0.105	0.142	0.157

Table 12—Example: Statistical Summary of Consecutive Series of Meter Factors

	Individual Runs		Individual Set Statistical Data		
Series Sequence Number	Set of Runs MF	Set Average $\overline{MF}$	Standard Deviations, $s(MF)$[a]	Ranges for Set, $w(MF)$[b]	Uncertainties of Set Averages, $a(\overline{MF})$[c]
1	0.9995				
	0.9994				
	0.9998	0.99962	0.00020	0.0005	±0.00025
	0.9995				
	0.9999				
2	1.0011				
	1.0010	1.00120	0.00018	0.0004	±0.00029
	1.0013				
	1.0014				
3	0.9992				
	0.9994				
	0.9990	0.99930	0.00020	0.0006	±0.00021
	0.9993				
	0.9993				
	0.9996				
4	1.0010				
	1.0009				
	1.0011	1.00092	0.00015	0.0004	±0.00016
	1.0007				
	1.0009				
5	1.0002				
	1.0007				
	1.0004	1.00048	0.00019	0.0005	±0.00024
	1.0005				
	1.0006				
6	0.9992				
	0.9989				
	0.9991	0.99902	0.00013	0.0003	±0.00016
	0.9989				
	0.9990				
7	1.0002				
	1.0006				
	1.0003	1.00042	0.00013	0.0003	±0.00016
	1.0003				
	1.0005				
8	1.0010				
	1.0016				
	1.0013	1.00128	0.00021	0.0006	±0.00023
	1.0014				
	1.0011				
	1.0013				
9	1.0002				
	1.0000				
	0.9998	1.00002	0.00018	0.0004	±0.00022
	0.9999				
	1.0002				
10	1.0018				
	1.0016				
	1.0020	1.00177	0.00019	0.0005	±0.00020
	1.0019				
	1.0015				
	1.0018				
Overall average, $\overline{\overline{MF}}$		1.00040			
Overall average standard deviation of sets, $\bar{s}(MF)$			0.00018		
Overall average range of sets, $\bar{w}(MF)$				0.00048	
Overall average of the uncertainties of individual set averages, $\bar{a}(95, \bar{n})$					±0.00020

[a]See Equation 2.
[b]See Equation 3.
[c]See Equation 10.

the other method uses ranges of the average meter factors, $(\overline{MF})$ of the sets.

a. The standard deviation for the series of ten consecutive meter factors (set averages) is 0.00092. The uncertainty in the overall average meter factor is as follows:

$$a(\overline{\overline{MF}}) = [T(95, \overline{10})][s(\overline{MF})]$$
$$a(1.00040) = (0.715)(0.00092)$$
$$= \pm 0.00066$$

b. The uncertainty in the overall average meter factor (series average) can also be estimated with the series range of 0.0028 of ten meter factors (set averages) as follows:

$$w(\overline{MF}) = \overline{MF}(10) - \overline{MF}(6)$$
$$= 1.00177 - 0.99902$$
$$= 0.00275$$
$$a(\overline{\overline{MF}}) = [Z(95, \overline{10})][w(\overline{MF})]$$
$$a(1.00040) = (0.230)(0.00275)$$
$$= \pm 0.00063$$

The following computations in Items c and d will cover two additional methods of estimating the uncertainty in the overall average of the series of ten meter factors using the average standard deviation and average range for the sets.

c. T.e uncertainty in the overall average of a series of meter proving factors can be based on the average standard deviation for the series of ten meter factors and approximate average of five runs in each set of meter factors as follows:

$$a(\overline{\overline{MF}}) = [T(95, \overline{5})][\bar{s}(MF)]$$
$$a(1.00040) = (1.241)(0.00018)$$
$$= \pm 0.00022$$

d. The estimated uncertainty in the average of a series of meter factor proving factors can be based on the average range for each of the ten sets of meter proving factors as follows:

$$a(\overline{\overline{MF}}) = [Z(95, \overline{5})][\bar{w}(MF)]$$
$$a(1.00040) = (0.54)(0.00048)$$
$$= \pm 0.00026$$

The computations of uncertainties in Items c and d are not as appropriate as the computations in Items a and b because those in c and d do not measure the statistical variations between sets of meter proving runs. The values in Items c and d approximate the overall average of the uncertainties of individual set averages of ±0.00020. The standard deviations and ranges within sets of meter proving runs are significantly less than the variations between sets. Therefore, the procedures in Items a and b are more appropriate for estimating the uncertainty in the overall average meter factor of 1.00040. The overall average meter factor should be stated as follows:

$$\overline{\overline{MF}} = 1.00040 \pm 0.00066 \text{ (95 percent confidence level, 10 measurements)}$$

or abbreviated as

$$\overline{\overline{MF}} = 1.00040 \pm 0.00066\ (95, \overline{10})$$

13.2.6.6 UNCERTAINTY ANALYSES OF MOVING AVERAGE IN A SERIES OF METER FACTORS

The example in 13.2.6.5 illustrates the analyses of statistical quantities after completion of a series of ten meter proving factors. However, it is often necessary to evaluate a moving series of meter factors when few measured values are available for statistical analysis shortly after startup of a new or overhauled meter as shown in the example in Table 13, which is based on the example in 13.2.6.5.

Figure 5 plots the individual meter factors and moving overall average meter factors given in Table 13. The moving average meter factor exhibits significantly less variation than individual meter factors, and this variation decreases as the number of values in the moving average increases.

Figure 6 plots the individual meter factors and random uncertainties in the individual meter factors given in Table 13. The uncertainty in the moving averages decreases significantly as the number of meter factors in the moving average increases.

Figure 7 shows the moving average of individual meter factors and the moving uncertainty of the moving average. The moving uncertainty of the moving average decreases rapidly for the initial four meter factors in the sequence as more data are added.

Additional examples on calculating uncertainties associated with meter proving activities are given in Appendix A.

13.2.7 Meter Factor Control Charts Based on Uncertainty Analyses

If plotted on a control chart, a series of meter factors obtained over a period of time under normal operating conditions will exhibit a band of scatter about an average value. The width of this band will depend on the mechanical condition of the meter and prover; the uniformity of the fluid; and the constancy of the flowing temperature, pressure, and flow. Variations in the meter factors are caused by a combination of random and systematic influences. In a meter factor, these variations are collectively lumped together, and meter factor variations are treated as random uncertainty to simplify statistical analyses.

A meter factor control chart clearly illustrates the meter factor variations and alerts the operator to changing conditions that otherwise may not be noticed.

Control charts are preferably prepared and kept by personnel responsible for proving the meter so that the charts are

Table 13—Example: Statistical Summary on Moving Series of Average Meter Factors

Series Sequence Number	Statistical Values of Individuals in Moving Sequence: $\overline{MF}$ [a]	Standard Deviation, $s(\overline{MF})$ [b]	Uncertainty, $a(\overline{MF})$ [c]	Statistical Values of Moving Overall Average: $\overline{\overline{MF}}$	Uncertainty, $a(\overline{\overline{MF}})$ [d]
1	0.9996				
2	1.0012	0.0011	±0.0140	1.0004	±0.0099
3	0.9993	0.0010	±0.0043	1.0000	±0.0025
4	1.0009	0.0009	±0.0029	1.0002	±0.0014
5	1.0005	0.0008	±0.0022	1.0003	±0.0010
6	0.9990	0.0009	±0.0023	1.0001	±0.0009
7	1.0004	0.0008	±0.0020	1.0001	±0.0007
8	1.0013	0.0009	±0.0021	1.0003	±0.0008
9	1.0000	0.0008	±0.0018	1.0002	±0.0006
10	1.0018	0.0009	±0.0020	1.0004	±0.0006

[a]Rounded values from Table 12
[b]Standard deviation of prior average meter factors, up to and with respect to current moving average.
[c]$a(\underline{1 \text{ to } k}) = [T(95, k)][s(1 \text{ to } k)]$.
[d]$a(\overline{1 \text{ to } k}) = [T(95, k)][s(1 \text{ to } k)]$.

available at the time of proving. Individual control charts for each meter shall be maintained since meters vary, as do their fluid properties and operating conditions.

Procedures for developing control charts based on the statistical concepts in 13.2.6.6 are covered in this section. Procedures for control charts based on Shewhart's procedures are given in Appendix B.

13.2.7.1 CONTROL LIMITS

A hierarchy of control limits can be established as a criterion for several levels of activities for responding to unusual changes in meter factors. The control limits may be based on the statistical variations of the individual meter or a group of meters or may be fixed based on experience and

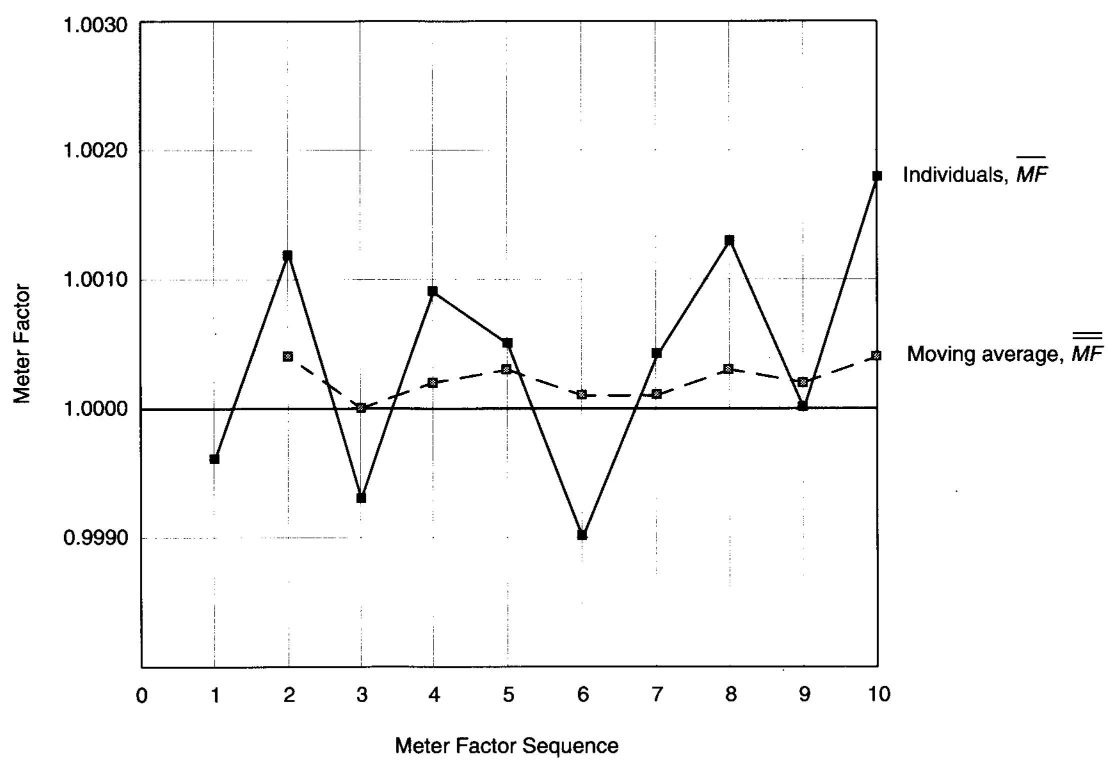


Figure 5—Individual and Moving Average Meter Factors for Example

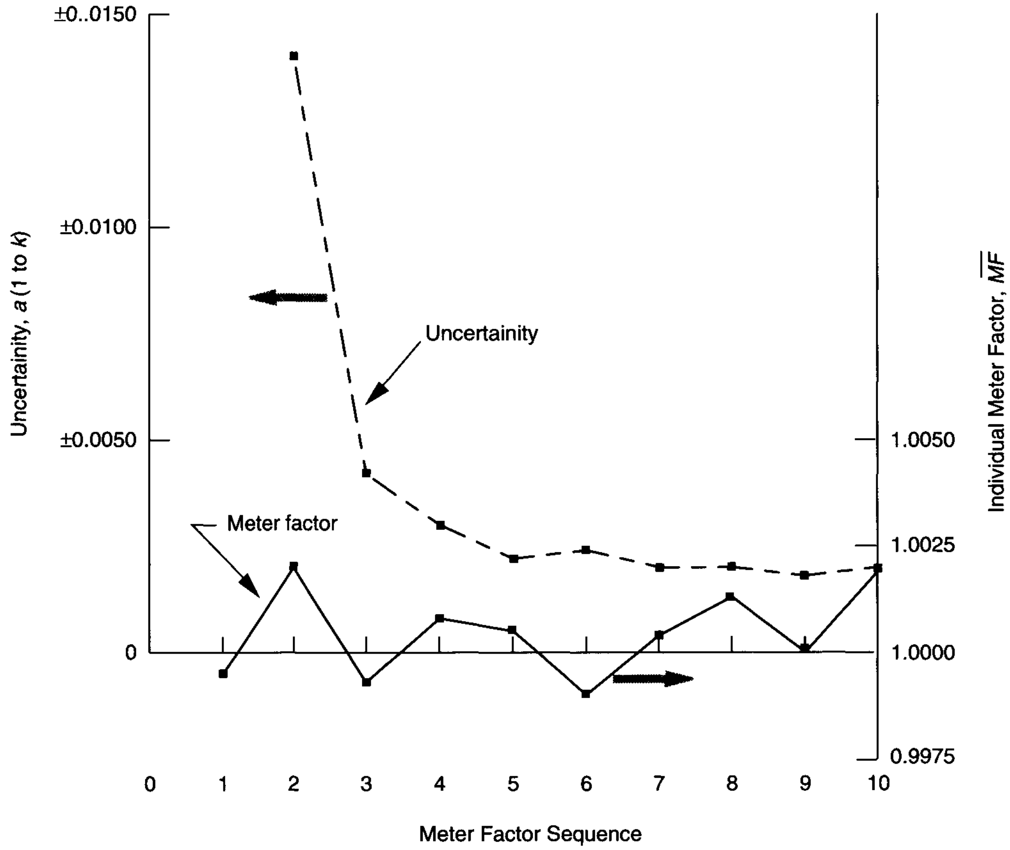


Figure 6—Uncertainty of Individual Meter Factors in Relation to Current Moving Average

mutual consent of the affected custody measurement parties. Control limits are computed on the assumption that meter factor variations are random or at least exhibit random characteristics. If meter factors deviate beyond control limits, it is normally assumed that the metering system is exhibiting a nonrandom or systematic change that results in a measurement bias.

Statistically based control limits would normally be based on the ranges of confidence levels listed in Table 14.

Warning limits can be used to indicate when it is appropriate to perform the following activities:

a. Check meter proving equipment.
b. Evaluate the stability of operating conditions.
c. Check for valve leakage.
d. Check computations.

Action limits can be used to indicate when to consider any or all of the following activities:

a. Recalibrating instrumentation.
b. Inspecting, adjusting, cleaning, and repairing mechanical equipment.
c. Issuing a correction ticket or tickets.

Tolerance limits can be used to indicate when to consider performing any or all of the following activities:

a. Conducting an intercompany audit or review of all equipment and computational procedures.
b. Reviewing the adequacy of the custody transfer facility for potential equipment changes.
c. Conducting laboratory analyses of metered fluid or fluids to verify properties used for computational purposes and to control operating conditions.

13.2.7.2 CONTROL CHART OR LOG—FIXED LIMITS GIVEN

Historically, fixed limits have been used for determining the acceptability of change between consecutive meter factors and cumulative change from a baseline meter factor. Table 15 shows an example of a meter factor control log that incorporates a fixed limit for changes between consecutive meter factors and cumulative change in meter factors.

Figure 8 is a graphical version of a control chart with fixed limits. Graphed control charts can also be combined with control chart logs.

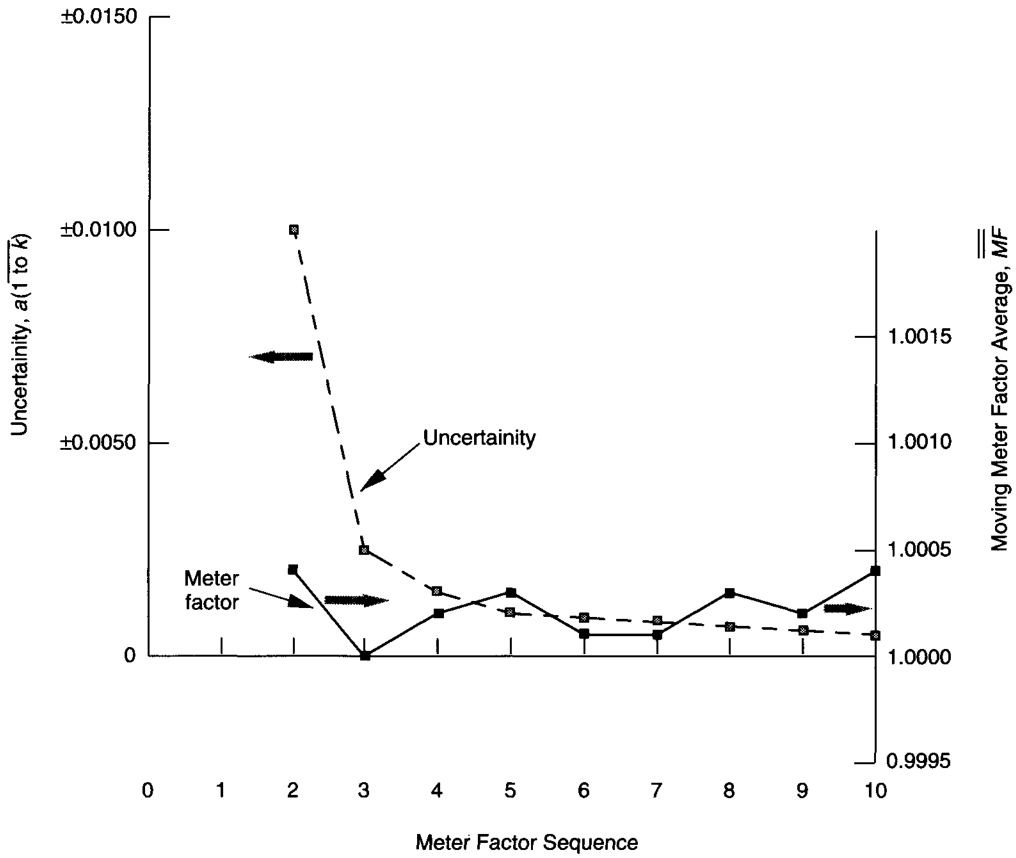


Figure 7—Moving Average of Individual Meter Factors and Uncertainties of Moving Average

13.2.7.3 STATISTICAL CONTROL CHARTS FOR MOVING SERIES—SINGLE METER, NO FIXED LIMIT

The control chart procedures in this section are based on uncertainties of a moving series of meter factors. The example in 13.2.6.5 and 13.2.6.6 (Table 12) will also be used to illustrate the development of a meter factor control chart. This example is repeated in Table 17. For illustration purposes, the control levels shown in Table 16 will be used for the control chart.

The uncertainties of the moving average for the warning, action, and tolerance control limits with confidence levels of 90 percent, 95 percent, and 99 percent, respectively, are given in Table 18. Uncertainty estimates of the moving average based on standard deviations are given in the left column for each control level. The right column under each control level gives uncertainty estimates of the moving average based on range values.

Table 14—Statistical Control Levels

Control Level	Confidence Level
Warning	90% to 95%
Action	95% to 99%
Tolerance	99% and greater

Control limits based on the uncertainty estimates using standard deviations of the moving average meter factor are plotted in Figure 9.

Uncertainties calculated from ranges are higher than those calculated from standard deviations for the first three meter factors. For meter factors four through ten, the uncertainty estimates of the moving average are similar and probably within the overall uncertainties to determine individual meter factors.

The uncertainties based on standard deviations for individual meter factors in Table 18 are plotted in Figure 9 for the three control levels with respect to the moving average meter factor, $\overline{\overline{MF}}$. Uncertainties for meter factors less than four are excluded from Figure 9 because the values are inflated due to the characteristics of the t-distribution for a small amount of data during the learning period for a new or overhauled meter.

As indicated in Table 18 and Figure 9, the uncertainties for individual meter factors in the moving series are relatively stable after five meter factors. The central line (baseline) and control limits for a control chart after five meter factors are shown in Table 19.

Figure 10 contains a control chart based on the limits given in Table 19. A log of individual meter factors and

Table 15—Example: Meter Factor Control Log

	Warning Limit	Action Limit
Consecutive *MF* change	none	±0.0025
Cumulative *MF* change	±0.0050	±0.0075

Sequence or Date	Meter Factor, *MF*	Meter Factor Change From Consecutive	Meter Factor Change From Baseline	Comments
1	0.9996			Baseline meter factor after repairs
2	1.0012	+0.0016	+0.0010	
3	0.9993	–0.0019	–0.0003	
4	0.9999	+0.0006	+0.0003	
5	1.0010	+0.0011	+0.0016	
6	1.0021	+0.0011	+0.0025	
7	1.0026	+0.0005	+0.0030	
8	1.0046	+0.0020	+0.0050	Warning limit met
9	1.0050	+0.0004	+0.0054	Warning limit exceeded
10	1.0000	—	—	Meter repaired, new baseline
11	1.0010	+0.0010	+0.0010	
12	1.0005	–0.0005	+0.0005	
13	1.0022	+0.0017	+0.0022	
14	1.0078	+0.0066	+0.0078	Action limit exceeded
15	1.0006	—	—	Meter repaired; first factor
16	1.0010	—	—	New baseline
17	1.0002	–0.0008	–0.0008	
18	0.9992	–0.0010	–0.0018	

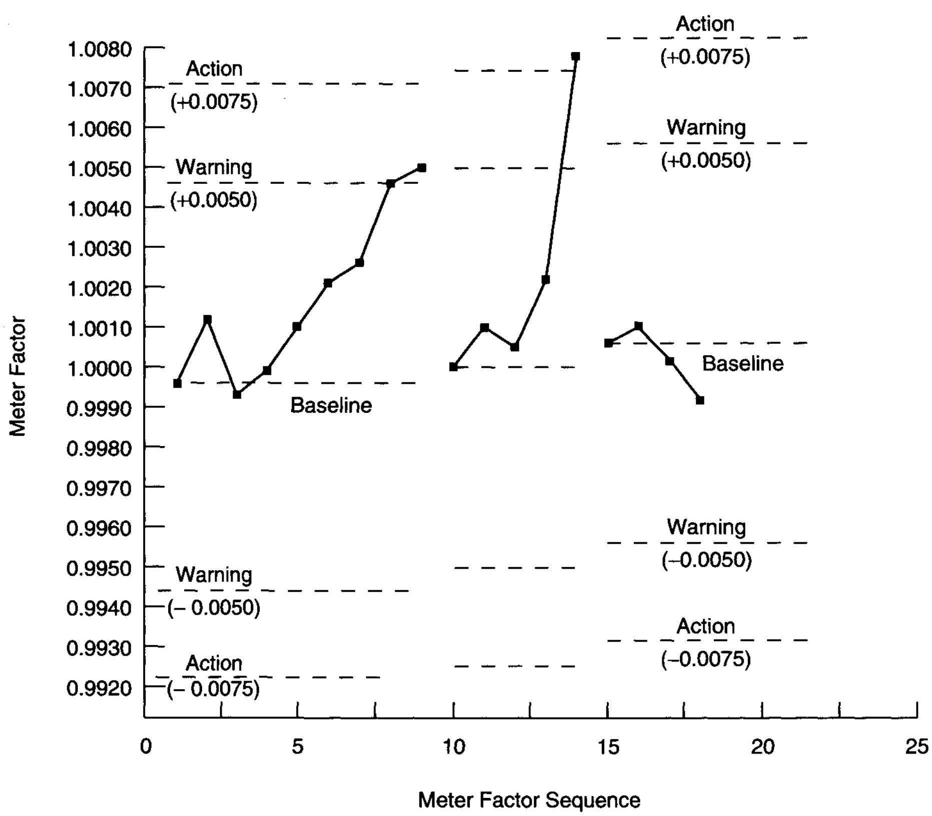


Figure 8—Meter Factor Control Chart With Fixed Control Limits

Table 16—Example on Meter Factor Control Levels

Control Level	Confidence Level
Warning	90%
Action	95%
Tolerance	99%

moving statistical values should also be maintained as part of the historical records. The control limits should be reviewed and revised when appropriate after more meter factors are developed. Control limits should also be compared to fixed limits so that broad limits based on erratic meter proving data do not prevail. Normally, the control limits can be reduced or tightened when a substantial number of meter factors are available. However, delays in establishing control limits will delay the use of the control chart in detecting outliers and systematic trends.

For trend analyses, control logs and/or charts can be established for the cumulative average meter factor of a moving series based on the statistical values given in Table 20.

Control lines for the cumulative average of the series of meter factors in Table 20 are shown in Table 21. The initial control lines are based on the cumulative average and uncertainties after five meter factors. Also note that the control lines can be revised after ten meter factors as the uncertainty estimate becomes lower.

The control chart for the example in Table 20 and control lines in Table 21 is shown in Figure 11. Also included for illustration purposes are additional meter factors beyond the example in Table 20.

Control charts for moving averages can also be weighted so that the most current data is given a higher weighted value than earlier data. This will cause the moving average to be more sensitive to recent changes in meter factors, and trends will be detected sooner. However, normal statistical variations in current data will also be amplified and control limits are more likely to be exceeded by both random and systematic changes.

Table 17—Example: Statistical Values for a Moving Series

Individual Set Data		Moving Statistical Values of Moving Series		
Sequence Number	Average Meter Factor, MF	Moving Average, $\overline{MF}$	Range, $w(\overline{MF})$	Standard Deviation, $s(\overline{MF})$[a]
1	0.9996			
2	1.0012	1.0004	0.0016	±0.00113
3	0.9993	1.0000	0.0019	±0.00102
4	1.0009	1.0002	0.0019	±0.00094
5	1.0005	1.0003	0.0019	±0.00082
6	0.9990	1.0001	0.0022	±0.00091
7	1.0004	1.0001	0.0022	±0.00084
8	1.0013	1.0003	0.0023	±0.00088
9	1.0000	1.0002	0.0023	±0.00083
10	1.0018	1.0004	0.0028	±0.00092

[a]Standard deviation of prior individual meter factors, $\overline{MF}$, up to and with respect to current moving average.

Table 18—Uncertainties of Individual Meter Factors in a Moving Series for Various Control Levels

	Uncertainties Versus Control Level					
	Warning Level		Action Level		Tolerance Level	
Set Sequence Number	Based on Range	Based on Standard Deviation	Based on Range	Based on Standard Deviation	Based on Range	Based on Standard Deviation
1	—	—	—	—	—	—
2	±0.0090	±0.0071	±0.0180	±0.0144	±0.0093	±0.0719
3	±0.0033	±0.0030	±0.0048	±0.0044	±0.0111	±0.0101
4	±0.0022	±0.0022	±0.0029	±0.0030	±0.0054	±0.0055
5	±0.0017	±0.0017	±0.0023	±0.0023	±0.0036	±0.0038
6	±0.0017	±0.0018	±0.0022	±0.0023	±0.0034	±0.0037
7	±0.0016	±0.0016	±0.0020	±0.0021	±0.0029	±0.0031
8	±0.0015	±0.0017	±0.0019	±0.0021	±0.0028	±0.0031
9	±0.0014	±0.0015	±0.0018	±0.0019	±0.0026	±0.0028
10	±0.0017	±0.0017	±0.0021	±0.0021	±0.0029	±0.0030

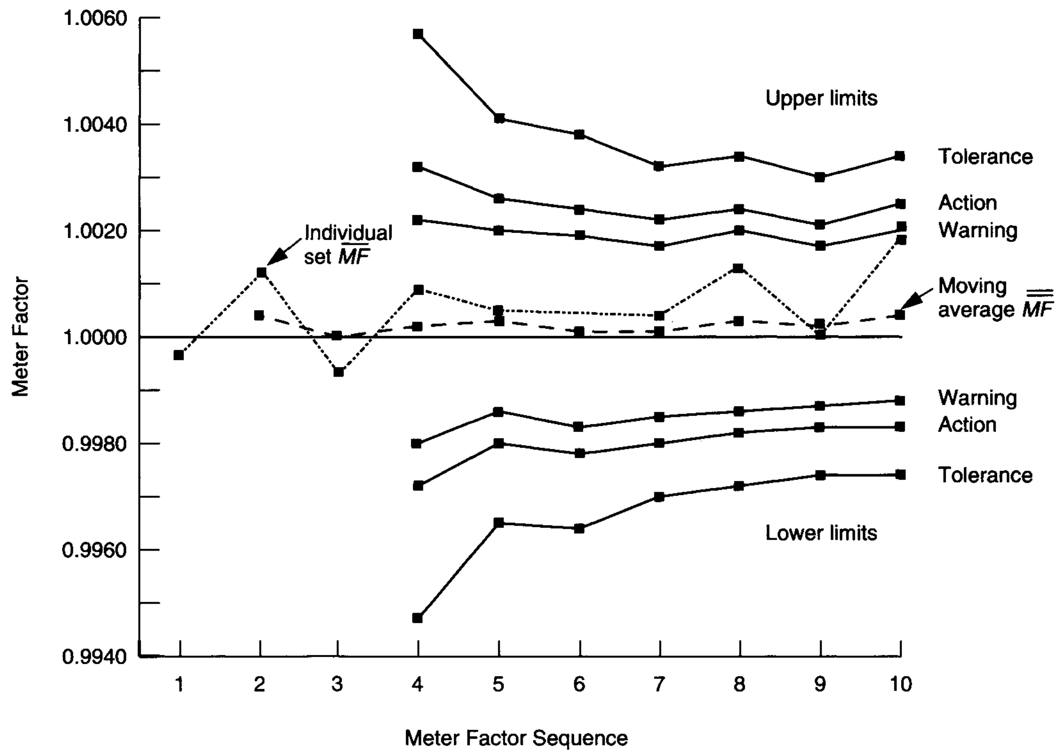


Figure 9—Statistics of Moving Series of Meter Factors

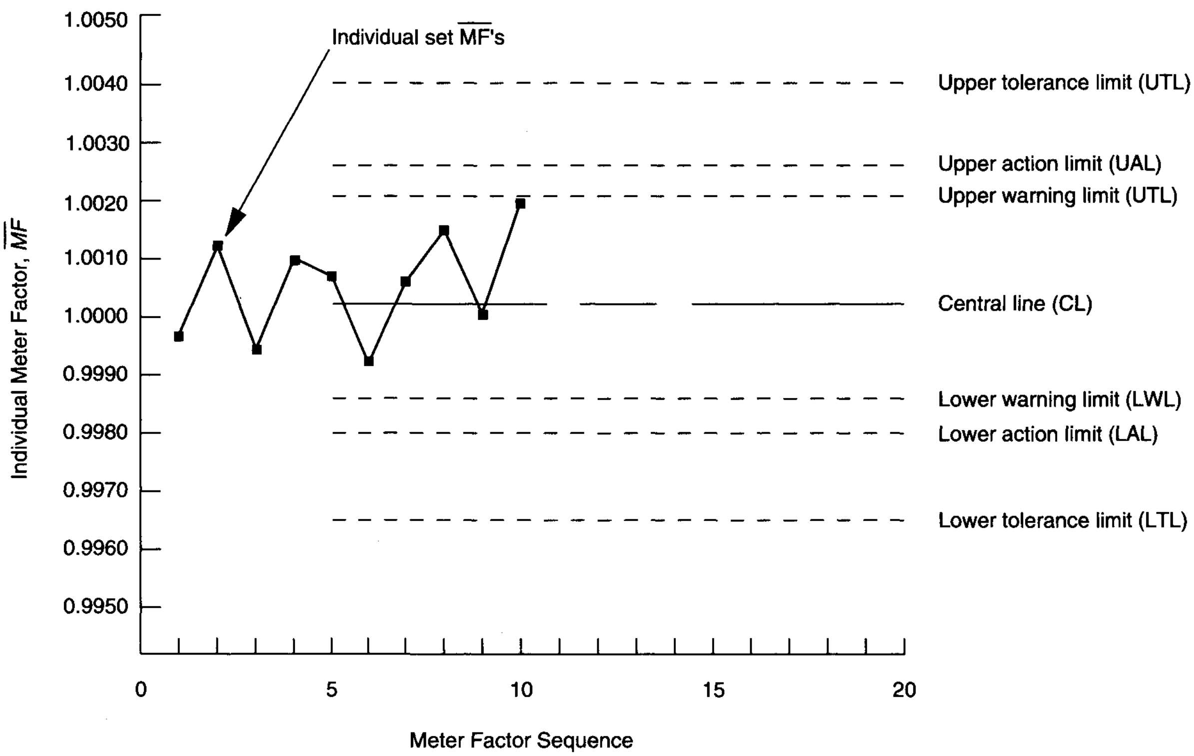


Figure 10—Control Chart for Individual Meter Factors

Table 19—Control Chart Lines for Individual Meter Factors in a Moving Series

Description of Control Chart Line	Meter Factor Value
Central line (baseline)	1.0003
Upper warning limit	1.0020
Upper action limit	1.0026
Upper tolerance limit	1.0041
Lower warning limit	0.9986
Lower action limit	0.9980
Lower tolerance limit	0.9965

13.2.7.4 CONTROL CHARTS FOR GROUPS OF METERS

Control charts can also be developed for groups of meters that experience similar fluids, design, and operating parameters. The example in Table 22 illustrates how control charts or logs can be used to identify meters and meter factors that are out of statistical control. These statistical procedures can also be used to evaluate fixed standards for meter factor acceptance.

Control chart analyses can be performed on a bank of similar meters experiencing similar operating conditions and metering similar fluids. This relative comparison can be helpful in identifying erratic performance of one or more meters in a group caused by poor mechanical conditions, inadequate flow control, or other facility deficiencies. Control charts covering groups of meters can also be helpful in evaluating maintenance and operating programs at different locations or between different operating parties.

Table 23 provides a summary of changes in consecutive meter factors for the five meters in Table 22. A control chart will be constructed for the changes that occur between consecutive meter factors.

Control charts (or logs) can be developed for the example in Table 23 to evaluate the following variations:

a. Average absolute change between consecutive meter factors of each meter with respect to the average absolute change between consecutive meter factors for all meters.
b. Absolute changes in consecutive meter factors changes of each meter with respect to its average absolute change.
c. Absolute change in consecutive meter factors of each meter with respect to the overall average of absolute changes in consecutive meter factors for a group of meters.

The control chart lines for the average absolute change in consecutive meter factors of each meter with respect to the overall average of absolute changes in consecutive meter factors of all meters are determined as follows:

a. Central line (CL):

$$CL = \overline{\overline{w}} \text{ (overall average change)}$$

$$\overline{\overline{w}} = \frac{(0.00084 + 0.00098 + 0.00080 + 0.00072 + 0.00132)}{5}$$

$$= 0.00093$$

b. Upper action limit (UAL):

$$UAL = \overline{\overline{w}} + [Z(95,\ 5)][w(\overline{w})] \text{ (see note)}$$

$$w(\overline{w}) = 0.00132 - 0.00072 = 0.00060$$

$$UAL = 0.00093 + (1.193)(0.00060)$$

$$= 0.00165$$

c. Lower action limit (LAL):

$$LAL = \overline{\overline{w}} - [Z(95,\ 5)][\overline{w}(w)] \text{ (see note)}$$

$$= 0.00093 - 0.00072$$

$$= 0.00021$$

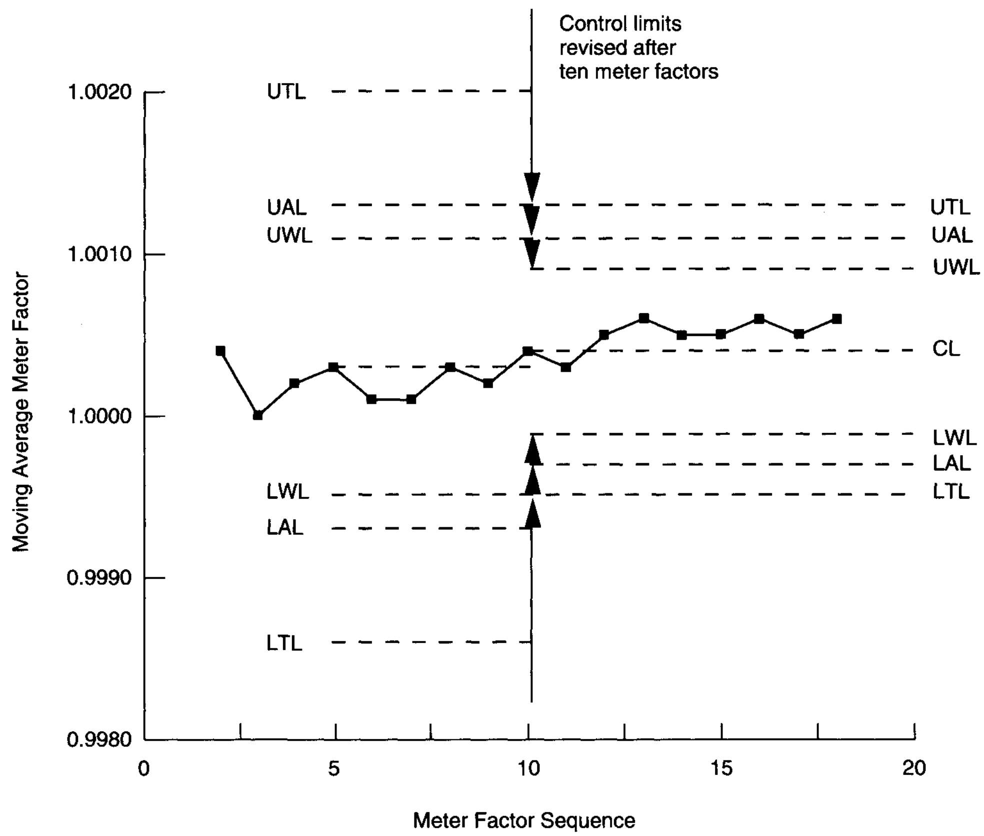
Note: See Table 10 for Z in Items b and c above.

Figure 12 shows the control chart with action control limits for the overall average absolute change for meters A, B, C, D, and E. Warning and tolerance control levels could also be included on the control chart but are excluded for simplification.

Table 20—Statistical Values for a Moving Average, $\overline{\overline{MF}}$

Sequence Number	Average Meter Factor, $\overline{MF}$	Moving Average, $\overline{\overline{MF}}$	Uncertainty Values for Control Levels[a]		
			Warning	Action	Tolerance
1	0.9996	—	—	—	—
2	1.0012	1.0004	±0.0050	±0.0102	±0.0508
3	0.9993	1.0000	±0.0017	±0.0025	±0.0058
4	1.0009	1.0002	±0.0011	±0.0015	±0.0028
5	1.0005	1.0003	±0.0008	±0.0010	±0.0017
6	0.9990	1.0001	±0.0007	±0.0009	±0.0015
7	1.0004	1.0001	±0.0006	±0.0008	±0.0012
8	1.0013	1.0003	±0.0006	±0.0007	±0.0011
9	1.0000	1.0002	±0.0005	±0.0006	±0.0009
10	1.0018	1.0004	±0.0005	±0.0007	±0.0009

[a]Based on dividing the calculated uncertainty values in Table 19 by the square root of the sequence number (number of meter factors).



Note: UTL = upper tolerance level; UAL = upper action level; UWL = upper warning level;
LWL = lower warning level; LAL = lower action level; LTL = lower tolerance level; and CL = central line.

Figure 11—Control Chart for Moving Cumulative Average of Meter Factors

Although all five meters are within the action limits shown in Figure 12, the average absolute change for meter E is substantially greater than that of the other meter. Inclusion of meter E causes substantially broader action limits than those of meters A, B, C, and D without meter E. If meter E is excluded from the database for the control limits, the revised control lines would be as follows:

Table 21—Control Chart Lines for the Moving Average of Series of Meter Factors

Description of Control	Meter Factor Values After 5 *MF*	After 10 *MF*
Central line (CL)	1.0003	1.0004
Upper warning limit (UWL)	1.0011	1.0009
Upper action limit (UAL)	1.0013	1.0011
Upper tolerance limit (UTL)	1.0020	1.0013
Lower warning limit (LWL)	0.9995	0.9999
Lower action limit (LAL)	0.9993	0.9997
Lower tolerance limit (LTL)	0.9986	0.9995

a. Central line (CL):

$$CL = \bar{\bar{w}}$$

$$\bar{\bar{w}} = \frac{(0.00084 + 0.00098 + 0.00080 + 0.00072)}{4}$$

$$= 0.00084$$

b. Upper action limit (UAL):

$$UAL = \bar{\bar{w}} + [Z(95,\ 4)][w(\bar{w})]$$

$$w(\bar{w}) = 0.00098 - 0.00072$$

$$= 0.00026$$

$$UAL = 0.00084 + (1.545)(0.00026)$$

$$= 0.00124$$

c. Lower action limit (LAL):

$$LAL = \bar{\bar{w}} - [Z(95,\ 4)][w(\bar{w})]$$

$$LAL = 0.00084 - 0.00040$$

$$= 0.00044$$

Table 22—Example: Meter Factors for Five Meters

Sequence Number	Meter Factors Versus Meter Designation				
	A	B	C	D	E
1	1.0002	0.9994	0.9980	1.0028	0.9997
2	1.0010	0.9982	0.9994	1.0021	0.9988
3	0.9995	1.0001	0.9990	1.0022	0.9991
4	0.9999	0.9992	0.9982	1.0030	0.9980
5	1.0005	0.9996	0.9996	1.0028	1.0000
6	0.9996	1.0010	0.9991	1.0039	0.9981
7	1.0006	1.0006	1.0000	1.0042	0.9996
8	1.0009	1.0018	0.9998	1.0031	1.0016
9	1.0018	1.0000	1.0010	1.0039	1.0004
10	1.0010	1.0002	1.0001	1.0044	1.0011
11	1.0011	0.9990	0.9993	1.0030	0.9990
12	0.9992	0.9992	0.9996	1.0021	0.9998

Table 23—Summary of Change Between Consecutive Meter Factors

Sequence Number	Absolute Change Between Consecutive Meter Factors, w				
	A	B	C	D	E
1	—	—	—	—	—
2	0.0008	0.0012	0.0014	0.0007	0.0009
3	0.0015	0.0019	0.0004	0.0001	0.0003
4	0.0004	0.0009	0.0008	0.0008	0.0011
5	0.0006	0.0004	0.0014	0.0002	0.0020
6	0.0009	0.0014	0.0005	0.0011	0.0019
7	0.0010	0.0004	0.0009	0.0003	0.0015
8	0.0003	0.0012	0.0002	0.0011	0.0020
9	0.0009	0.0018	0.0012	0.0008	0.0012
10	0.0008	0.0002	0.0009	0.0005	0.0007
11	0.0001	0.0012	0.0008	0.0014	0.0021
12	0.0019	0.0002	0.0003	0.0009	0.0008
Average absolute change ($\overline{w}$)	0.00084	0.00098	0.00080	0.00072	0.00132
Overall range in consecutive meter factor changes, $w(w)$	0.0018	0.0017	0.0012	0.0013	0.0017

Figure 13 shows a modified control chart with action limits based on meters A, B, C, and D. For development of attainable standard meter factor ranges, it would normally be preferable to delete meter E's ranges from the database since it is not typical of the other meters. Inclusion of meter E in the database results in considerably broader limits.

The control chart for the individual absolute changes in consecutive meter factors on all five meters would have the following lines:

a. Central line (CL):

$$CL = \overline{\overline{w}}$$

$$\overline{\overline{w}} = \frac{(0.00084 + 0.00098 + 0.00080 + 0.00072 + 0.00132)}{5}$$

$$= 0.00093$$

b. Upper action limit (UAL):

$$UAL = \overline{\overline{w}} + [Z(95,\ 11)]\left[\overline{w}(\overline{w})\right]$$

$$\overline{w}(\overline{w}) = \frac{(0.0018 + 0.0017 + 0.0012 + 0.0013 + 0.0017)}{5}$$

$$= 0.00154$$

$$UAL = 0.00093 + (0.702)(0.00154)$$

$$= 0.00201$$

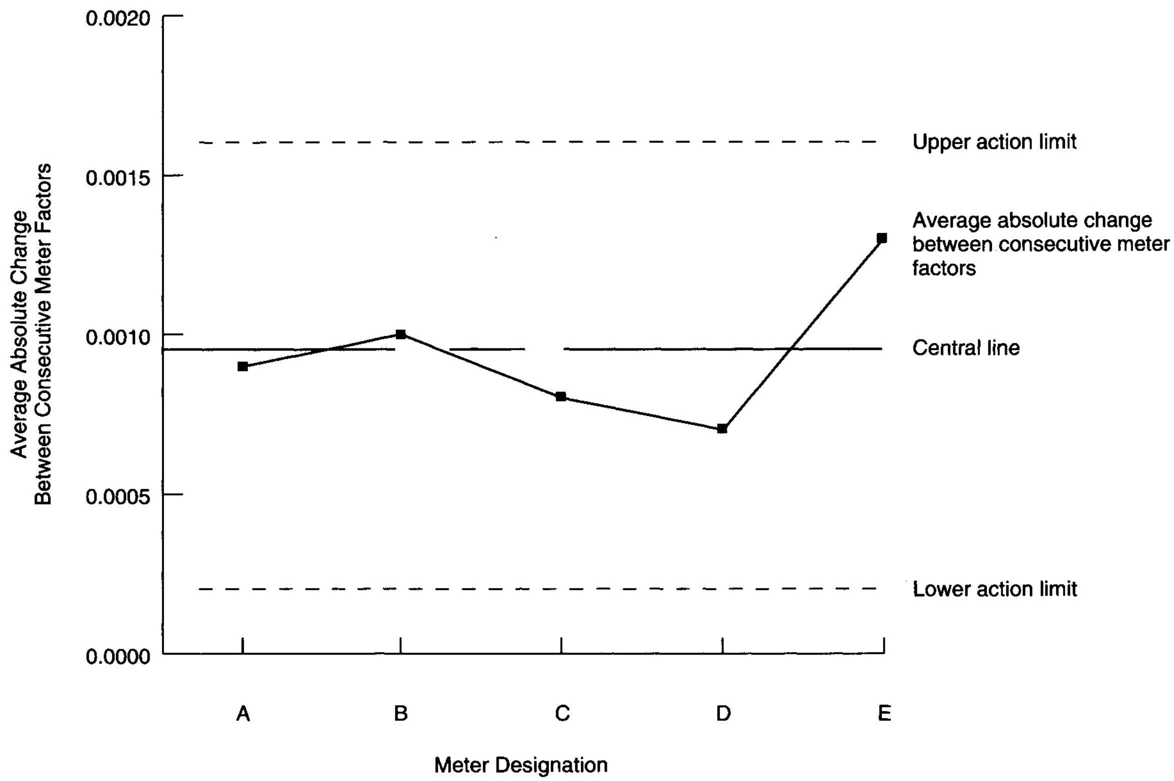
c. Lower action limit (LAL):

$$LAL = \overline{\overline{w}} - [Z(95,\ 11)]\left[\overline{w}(\overline{w})\right]$$

$$= 0.00093 - (0.702)(0.00154)$$

$$= -0.00015$$

$$LAL = 0, \text{ since } LAL \text{ cannot be less than zero}$$



Note: Upper action limit, central line, and lower action limit are based on five meters.

Figure 12—Control Chart for Average Absolute Change Between Consecutive Meter Factors

The control chart for individual absolute changes between consecutive meter factors based on all five meters is shown in Figure 14.

Absolute changes between consecutive meter factors for meter E are also plotted in Figure 14 for illustration purposes.

If the absolute changes in consecutive meter factors for meter E are excluded from the database, the control lines for individual ranges in consecutive meter factors would be as follows:

a. Central line (CL):

$$CL = \overline{\overline{w}}$$

$$\overline{\overline{w}} = 0.00084$$

b.. Upper action limit (UAL):

$$UAL = \overline{\overline{w}} + [Z(95,\ 11)]\left[\overline{w}(\overline{w})\right]$$

$$\overline{w}(\overline{w}) = \frac{0.0017 + 0.0017 + 0.0012 + 0.0013}{4}$$

$$= 0.0015$$

$$UAL = 0.00084 + (0.702)(0.0015)$$

$$= 0.00189$$

c. Lower action limit (LAL):

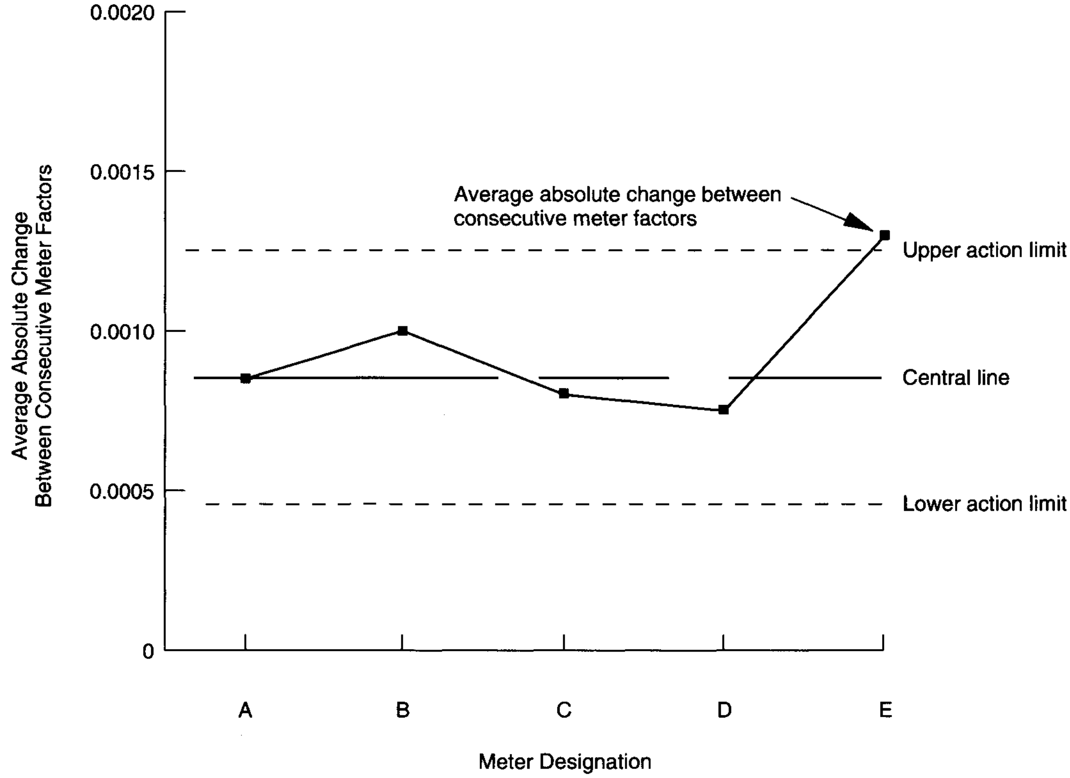
$$LAL = \overline{\overline{w}} - [Z(95,\ 11)]\left[\overline{w}(\overline{w})\right]$$

$$= 0.00084 - 0.00105$$

$$= -0.00021$$

$$LAL = 0,\ \text{since } -0.00021 \text{ is less than zero}$$

Figure 15 plots the absolute changes in consecutive meter factors for meter E with respect to the control lines based on meters A, B, C, and D. Three of meter E's absolute changes in consecutive meter factors fall outside the action limits shown in Figure 15.



Note: Upper action limit, centerline, and lower action limit are based on meters A, B, C, and D.

Figure 13—Revised Control Chart for Average Absolute Change Between Consecutive Meter Factors

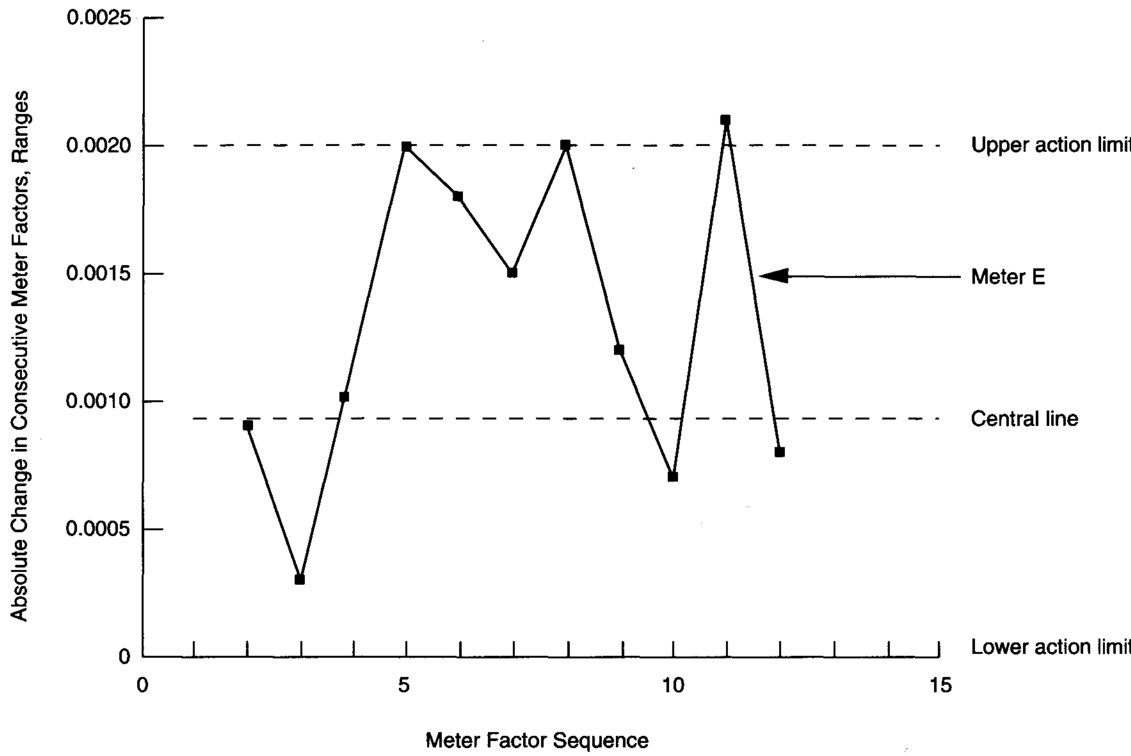


Figure 14—Control Chart for Absolute Consecutive Meter Factor Changes for Meter E (All Meters Used for Control Limits)

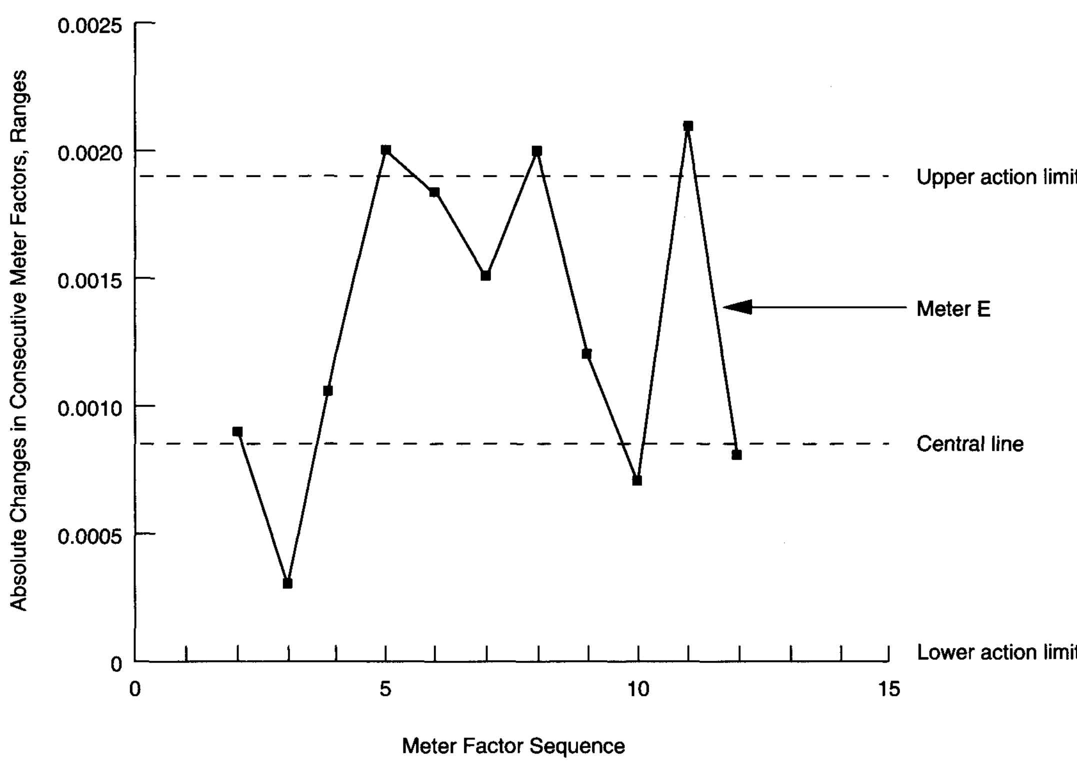


Figure 15—Control Chart for Absolute Changes in Consecutive Meter Factors (Meter E Excluded for Control Limits)

APPENDIX A—EXAMPLES OF METER PROVING UNCERTAINTY COMPUTATIONS

A.1 Example A-1: Uncertainties in a Moving Average

Estimate uncertainties in the moving average of the set of meter proving runs shown in Table A-1.

A.1.1 STEP 1

Calculate the moving range using Equation 3 and estimate the uncertainty of the moving average using Equation 20 for proving runs one and two in Table A-1 as follows:

$$w(MF) = 0.9990 - 0.9988$$
$$= 0.0002$$
$$a(0.99890) = \left[Z(95, \bar{2})\right]\left[w(MF)\right] \text{ (see Table 11)}$$
$$= (7.965)(0.0002)$$
$$= \pm 0.0016$$

A.1.2 STEP 2

Calculate the moving ranges and estimate the uncertainties of moving averages for proving runs 3 through 10 and summarize the results as shown in Table A-2.

A.2 Example A-2: Potential Annual Uncertainty in Meter Factor Acceptance Criteria

Estimate the potential average annual uncertainty at a 95 percent confidence level with the following meter proving acceptance criteria:

a. Minimum number of proving runs = three.
b. Range limits for three proving runs = 0.0005.

Table A-1—Example A-1: Meter Proving Set

Proving Runs	Average Meter Factors	
Sequence, n	Individual, $\overline{MF}$	Moving Average, $\overline{\overline{MF}}$
1	0.9990	—
2	0.9988	0.99890
3	0.9994	0.99907
4	0.9992	0.99910
5	0.9993	0.99914
6	0.9989	0.99910
7	0.9995	0.99916
8	0.9992	0.99916
9	0.9985	0.99909
10	0.9992	0.99910
11	0.9990	0.99909
12	0.9998	0.99915
13	0.9986	0.99911
14	0.9992	0.99911
15	0.9991	0.99911

c. Minimum proving period = monthly.
d. Deviation limit between consecutive meter factors = 0.0025.

A.2.1 STEP 1

Estimate the potential average annual uncertainty associated with meter run acceptance criteria as follows:

a. The estimated standard deviation of the average in a single set of three runs that agree within a range of 0.0005 is calculated as follows:

$$s\left(\overline{MF}\right) = \frac{w}{\sqrt{3}\,D(3)}$$
$$= \frac{0.0005}{(1.732)(1.693)}$$
$$= 0.00017$$

b. The potential average annual uncertainty associated with meter run acceptance criteria is as follows:

$$a\left(\overline{\overline{MF}}\right) = \left[T(95, \overline{12})\right]\left[s\left(\overline{MF}\right)\right] \text{ (see Table 9)}$$
$$a\left(\overline{\overline{MF}}\right) = (0.635)(\pm 0.00017)$$
$$= \pm 0.00011$$

The potential average annual uncertainty associated with the meter run acceptance criteria of three runs that agree within a range of 0.0005 propagates to an average annual value of ±0.00011 when repeated 12 times (monthly) over a year.

A.2.2 STEP 2

Estimate the potential average annual uncertainty associated with a deviation limit between consecutive meter factors of 0. 0025.

a. The standard deviation of the average in two consecutive meter factors that agree within a range of 0.0025 is as follows:

$$s\left(\overline{MF}\right) = \frac{w}{\sqrt{(2)}\,D(2)}$$
$$= \frac{0.00025}{(1.414)(1.128)}$$
$$= 0.00157$$

b. The potential average annual uncertainty associated with the consecutive meter factor deviation limit is as follows:

$$a\left(\overline{\overline{MF}}\right) = \left[T(95, \overline{12})\right]\left[s\left(\overline{MF}\right)\right] \text{ (see Table 9)}$$
$$a\left(\overline{\overline{MF}}\right) = (0.635)(0.00157)$$
$$= \pm 0.00100$$

Table A-2—Statistical Summary of the Moving Average of a Sequence of Meter Proving Factors

Proving Runs Data		Moving Statistical Values		
Sequence, n	Meter Factor	Average	Range	Uncertainty[a]
1	0.9990	—	—	—
2	0.9988	0.99890	0.0002	±0.0016
3	0.9994	0.99907	0.0006	±0.0009
4	0.9992	0.99910	0.0006	±0.0005
5	0.9993	0.99914	0.0006	±0.0003
6	0.9989	0.99910	0.0006	±0.0003
7	0.9995	0.99916	0.0007	±0.0002
8	0.9992	0.99916	0.0007	±0.0002
9	0.9985	0.99909	0.0010	±0.0003
10	0.9992	0.99910	0.0010	±0.0002
11	0.9990	0.99909	0.0010	±0.0002
12	0.9998	0.99915	0.0013	±0.0003
13	0.9986	0.99911	0.0013	±0.0002
14	0.9992	0.99911	0.0013	±0.0002
15	0.9991	0.99911	0.0013	±0.0002

[a]Of moving average, i.e., $a(\overline{\overline{MF}}) = Z(95, \bar{n})[w(n)]$.

The potential average annual uncertainty associated with the deviation limit between consecutive meter factors propagates to an average annual value of ±0.00100 when repeated 12 times over a year.

A.2.3 STEP 3

Combine the potential average annual uncertainties of the two meter proving acceptance requirements as follows:

$$a\left(\overline{\overline{MF}}\right) = \sqrt{(\pm 0.00011)^2 + (0.00100)^2}$$
$$= \pm 0.00101$$

In example A-2, the uncertainty associated with the meter run acceptance criteria has a negligible effect on the potential average annual uncertainty. This effect is primarily due to the relatively large deviation limit between consecutive meter factors, which is five times greater than the range limit for a set of three meter proving runs. The potential average annual uncertainty completed in example A-2 is a worst-case value because it is unlikely that the range for sets of meter proving runs and the deviation between consecutive meter factors will be equal to the limits every time a meter factor is determined.

A.3 Example A-3: Variable Range Meter Factor Acceptance Criteria

Calculate ranges for sets of 3 to 15 meter proving runs that equal the estimated uncertainty of the average of 5 runs that agree within a range of 0.0005. The confidence level for analyses is 95 percent.

A.3.1 STEP 1

Estimate the uncertainty of the average associated with five proving runs that agree within a range of 0.0005:

$$a\left(\overline{MF}\right) = \left[Z\left(95,\ \bar{5}\right)\right]\left[w(MF)\right] \text{ (see note)}$$
$$= (0.540)(0.0005)$$
$$= \pm 0.00027$$

A.3.2 STEP 2

Calculate moving ranges that equal the uncertainty of Step 1 above:

$$w(MF) = \frac{a\left(\overline{MF}\right)}{Z\left(95,\ \bar{n}\right)} \text{ (see note)}$$

A.3.3. STEP 3

For three runs, calculate:

$$w(MF) = \frac{\pm 0.00027}{1.467}$$
$$= 0.0002$$

A.3.4 STEP 4

For meter proving sets of 4 to 15 runs, calculate the variable ranges and summarize as shown in Table A-3.

A.4 Example A-4: Uncertainty Limit Meter Factor Acceptance Criteria

Calculate the uncertainty of a moving series of meter factors until the uncertainty of the average of the cumulated set equals ±0.00025 or less at 95 percent confidence level. The example in Table A-2 will be used for illustration purposes. Statistical values are summarized in Table A-4.

The uncertainty of the average in the moving set after six runs meets the uncertainty criteria for the meter proving set. Therefore, the meter proving set is accepted after six runs are completed.

Table A-3—Variable Ranges for ±0.00027 Uncertainty in Meter Factors

Number of Meter Factors	Moving Range
3	0.0002
4	0.0003
5	0.0005
6	0.0006
7	0.0008
8	0.0009
9	0.0010
10	0.0012
11	0.0013
12	0.0014
13	0.0015
14	0.0016
15	0.0017

Table A-4—Summary of Uncertainties in the Average in a Moving Series of Meter Proving Factors

Proving Data		Moving Statistical Values		
Sequence, *n*	Meter Factor	Average	Standard Deviation[a]	Uncertainty[b]
1	0.9990	—	—	—
2	0.9988	0.99890	±0.00014	±0.00127
3	0.9994	0.99907	±0.00031	±0.00076
4	0.9992	0.99910	±0.00026	±0.00041
5	0.9993	0.99914	±0.00024	±0.00030
6	0.9995	0.99910	±0.00024	±0.00025

[a]Standard deviation of the moving set up to the number of proving runs.
[b]Uncertainty in the average of the moving set of meter proving runs.

For illustration purposes, calculations of standard deviation and uncertainty after three meter factors are given below using Equations 3 and 14 and the data in Table A-4.

$$s(MF) = \sqrt{\frac{1}{3-1}\left[\left(MF_1-\overline{MF}\right)^2 + \left(MF_2-\overline{MF}\right)^2 + \left(MF_3-\overline{MF}\right)^2\right]}$$

$$\left(MF_1-\overline{MF}\right)^2 = (0.99900 - 0.99907)^2 = 0.5 \times 10^{-8}$$

$$\left(MF_2-\overline{MF}\right)^2 = (0.99880 - 0.99907)^2 = 7.3 \times 10^{-8}$$

$$\left(MF_3-\overline{MF}\right)^2 = (0.99940 - 0.99907)^2 = 10.9 \times 10^{-8}$$

$$\Sigma\left(MF-MF\right)^2 = 18.7 \times 10^{-8}$$

$$s(MF) = \sqrt{\frac{1}{3-1}\left(18.7 \times 10^{-8}\right)}$$

$$= 3.1 \times 10^{-4}$$

$$a\left(\overline{MF}\right) = \left[T(95,\ 3)\right]\left[s(MF)\right]$$

$$a(0.99907) = (2.484)(0.00031)$$

$$= \pm 0.00076$$

APPENDIX B—EXAMPLES OF OUTLIER TESTS

Since no single document can cover all of the statistical procedures and applications being practiced, outlier tests other than those appearing in Appendix B may be used. Alternate outlier tests are not expected to duplicate the exact results provided by these procedures; however, alternate computational methods should achieve the same purpose intended by the outlier tests in this appendix.

B.1 Example B-1—Dixon's Test for Outliers

Appendix B of Chapter 13.1 of the *Manual of Petroleum Measurement Standards* includes information on Dixon's test for outliers that can be applied to a meter proving set. Outliers can inflate the standard deviation, range, and uncertainty of a meter proving set.

Dixon's outlier test is not normally practical for hand calculations. However, increasing use of computers in custody measurement now permits the use of statistical outlier tests on sets of meter proving data. The example of a set of meter proving data shown in Table B-1 will be used to illustrate the use of Dixon's outlier test.

In the outlier test, the meter factors are arranged in ascending order of magnitude as shown in Table B-2.

The following steps from Appendix B of Chapter 13.1 illustrate the outlier test for high and low values in the meter proving set using the ascending order run sequence.

B.1.1 STEP 1

Test the lowest ascending order value as follows:

a. Calculate Dixon R ratio.

$$R_{low} = \frac{MF_3 - MF_1}{MF_{13} - MF_1} = \frac{1.0004 - 1.0000}{1.0008 - 1.0000} = 0.500$$

b. Look up Dixon R limits for $n = 15$ values.
1. $R(95)$ limit = 0.525.
2. $R(99)$ limit = 0.616.

c. Compare calculated Dixon R ratio against Dixon limits. Since 0.500 is less than 0.525, the lowest value is not excluded from the meter proving set.

B.1.2 STEP 2

Test the highest ascending order value as follows:

a. Calculate Dixon R ratio.

$$R_{high} = \frac{MF_{15} - MF_{13}}{MF_{15} - MF_3} = \frac{1.0015 - 1.0008}{1.0015 - 1.0004} = 0.636$$

b. Look up Dixon R limits for $n = 15$ values.
1. $R(95)$ limit = 0.525.
2. $R(99)$ limit = 0.616.

c. Compare calculated Dixon R ratio against Dixon limits. Since 0.636 is greater than 0.525 and 0.616, the highest ascending order value should be automatically excluded from the set.

d. Repeat test with remaining values.

Table B-1—Initial Meter Factor Sequence

Meter Factor Sequence	Calculated Meter Factor
1	1.0004
2	1.0006
3	1.0005
4	1.0007
5	1.0000
6	1.0004
7	1.0009
8	1.0005
9	1.0003
10	1.0008
11	1.0006
12	1.0007
13	1.0007
14	1.0015
15	1.0009

Table B-2—Ascending Order for Outlier Test

Ascending Order	Initial Run Sequence	Value
1	5	1.0000
2	9	1.0003
3	1	1.0004
4	6	1.0004
5	8	1.0005
6	3	1.0005
7	11	1.0005
8	15	1.0006
9	2	1.0006
10	12	1.0006
11	4	1.0007
12	13	1.0007
13	10	1.0008
14	7	1.0009
15	14	1.0015
		$\overline{MF}$ = 1.0006

B.1.3 STEP 3

Test the lowest value in the remaining ascending order set as follows:

a. Calculate Dixon R ratio:

$$R_{low} = \frac{MF_3 - MF_1}{MF_{12} - MF_1} = \frac{1.0004 - 1.0000}{1.0007 - 1.0000} = 0.571$$

b. Look up Dixon R limits for $n = 14$ values:
1. $R(95) = 0.546$.
2. $R(99) = 0.641$.

c. Compare Dixon R ratio against Dixon limits. Since R_{low}, 0.571, is greater than 0.546 but less than 0.641, ascending order run number one is not automatically excluded but could be rejected. Since exclusion of run number one will not appreciably reduce the number of meter factors, it should be eliminated from the data set.

B.1.4 STEP 4

Test the highest value in the remaining set as follows:

a. Calculate Dixon R ratio for $n = 14$ values.

$$R_{high} = \frac{MF_{14} - MF_{12}}{MF_{14} - MF_3} = \frac{1.0009 - 1.0007}{1.0009 - 1.0004} = 0.400$$

b. Look up Dixon R limits for $n = 14$ values.
1. $R(95) = 0.546$.
2. $R(99) = 0.641$.

c. Compare Dixon ratio against Dixon limits. Since R_{high} is less than 0.546 [$R(95)$] and 0.641 [$R(99)$], the high value in the set should not be excluded.

B.1.5 STEP 5

Test the lowest meter factor in the remaining ascending order set as follows:

a. Calculate Dixon R ratio.

$$R_{low} = \frac{MF_4 - MF_2}{MF_{13} - MF_2} = \frac{1.0004 - 1.0003}{1.0008 - 1.0003} = 0.200$$

b. Look up Dixon R limits for $n = 13$ values.
1. $R(95) = 0.521$.
2. $R(99) = 0.615$.

c. Compare Dixon R ratio against Dixon limits. Since R_{low} is less than 0.521 [$R(95)$] and 0.615 [$R(99)$], the remaining low value in the set should not be excluded.

The data set remaining after completion of the outlier test is given in Table B-3.

Although the average meter factor ($\overline{MF}$) in this example remained at 1.0006 after the outlier test as shown in Table B-3, the estimated uncertainty in the calculated value was reduced from ±0.0002 to ±0.0001 as shown with the following computations:

a. Estimated $a(\overline{MF})$ before outlier test:

$$a(\overline{MF}) = [Z(95, \overline{15})][w(MF)] = (0.160)(0.0015) = \pm 0.0002$$

b. Estimated $a(\overline{MF})$ after outlier test:

$$a(\overline{MF}) = [Z(95, \overline{13})][w(MF)] = (0.181)(0.0006) = \pm 0.0001$$

B-2 Example B-2—Uncertainty Minimization Outlier Test

The purpose of this example problem is to illustrate an alternate method of evaluating potential outliers in a set of data such as a set of meter proving runs. The procedure involves a series of uncertainty estimates on reducing ranges and using Student t functions until the uncertainty of the average in the set approaches a minimum value. The extreme values in the set that constitute the high and/or low values are then rejected so that the remaining values in the set result in a minimum value of uncertainty in the average of the remaining

Table B-3—Remaining Data Set After Outlier Test

Initial Sequence[a]	Meter Factor
1	1.0004
2	1.0006
3	1.0005
4	1.0007
6	1.0004
7	1.0009
8	1.0005
9	1.0003
10	1.0008
11	1.0006
12	1.0007
13	1.0007
15	1.0009
	$\overline{MF} = 1.0006$

[a]Original meter factors 5 and 14 are excluded from the final set.

data set. Statistical values for a 99 percent or higher confidence level are normally used for outlier tests. Example B-1 will be used to illustrate this procedure.

The estimated uncertainty of the average in the set of proving runs in Table B-2 is as follows:

$$a(\overline{MF}) = [Z(99, \overline{15})][w(MF)]$$

$$Z(99, \overline{15}) = 0.221$$

$$w(MF) = 1.0015 - 1.0000$$
$$= 0.0015$$

$$a(\overline{MF}) = (0.221)(0.0015)$$
$$= \pm 0.00033$$

If the first and last values in ascending order are eliminated, the resulting uncertainty in the average of the remaining 13 values would be as follows:

$$a(\overline{MF}) = [Z(99, \overline{13})][w(MF)]$$

$$Z(99, \overline{13}) = 0.254$$

$$w(MF) = 1.0009 - 1.0003$$
$$= 0.0004$$

$$a(\overline{MF}) = (0.254)(0.0004)$$
$$= \pm 0.00010$$

In this example, the uncertainty of the average in the set of runs is reduced from ±0.00033 to ±0.00010 (99 percent confidence level) by eliminating high and low values in the original data set. This process can be continued by evaluating the uncertainty of the remaining set with and without the remaining high and/or low values. Table B-4 summarizes the uncertainties in averages of sets that are reduced one at a time by the run that is the farthest from the average of the set.

As shown in Table B-4, the uncertainty of the average in the set reduces until only ten values remain in the set. Further elimination of high and/or low values from the set results in an increase in the estimated uncertainty of the average in the remaining set. Runs 1, 2, 13, 14, and 15 are excluded as outliers from the original data set.

Table B-4—Summary of Uncertainty Estimates

Number of Values	Eliminated Run Numbers[a]	Range	Uncertainty of Average at 99% Confidence
15	0	0.0015	±0.00033
14	15	0.0009	±0.00021
13	1	0.0006	±0.00015
11	2, 14	0.0004	±0.00012
10	13	0.0003	±0.00010
7	3, 4, 12	0.0002	±0.00013

[a]Ascending order runs from Table B-2.

APPENDIX C—SHEWHART CONTROL CHARTS

C.1 Introduction

The basic control chart used in most industrial applications was developed by Walter A. Shewhart. However, general use of Shewhart statistical procedures for control charts on meter factors is not recommended. Shewhart's procedures were developed to evaluate whether a process is or is not in a state of statistical control relative to itself or to some standard value. Statistics are derived from data collected within rational subgroups and compared to limits based on past experience, the data themselves, or standard values. Control charts can be developed for individual measurement sets, averages of measurement sets, ranges within sets, standard deviations within sets, and ranges between individual measurements.

Procedures by Shewhart are based on his concepts of appropriate empirical and economic considerations. Control limits are based on three multiples of the standard deviation of the data being evaluated for statistical control. However, the uncertainties associated with the control limits are not always equivalent to a 99.7 percent confidence level, which is implied with ±3 standard deviations, because averages of statistical quantities are used for computations of control limits.

Shewhart control chart procedures are generally appropriate for post manufacturing evaluation of relatively large samples where a manufacturing process is mature and somewhat predictable. However, some of the control chart procedures may not be appropriate during startup of a new facility where experience with the operation is limited and/or where the number of samples is low. The use of averaged statistical variations within the sets requires that statistical variations between sets must be minimal. This is seldom the case for meter factors unless proving intervals are relatively short. Therefore, Shewhart control charts are not normally appropriate for statistical control of meter proving data.

Extensive information on Shewhart control chart procedures may be found in ASTM MNL 7, ANSI/ASQC A1, and ANSI/ASQC B1, B2, and B3 (see 13.2.4 for complete information).

C.2 Shewhart Control Charts for Averages

Shewhart statistical control limits for the averages of sets of meter factors are determined by the following equations:

$$\overline{\overline{MF}} \pm \frac{3\left[\bar{s}\left(\overline{MF}\right)\right]}{k - 0.5}$$

or

$$\overline{\overline{MF}} \pm \frac{3\left[\bar{w}\left(\overline{MF}\right)\right]}{D(k)\,k}$$

Where:

$$\bar{s} = \frac{s_1 + s_2 + \ldots\ldots s_k}{k}$$

and

$$\bar{w} = \frac{w_1 + w_2 \ldots\ldots w_k}{k}$$

The following factors are provided in publications referenced in C.1, and tabulated values are given in Table C-1 for use in hand computations:

$$H = \frac{3}{k - 0.5}$$

$$\textit{Control limits} = \overline{\overline{MF}} \pm H\left[\bar{s}(MF)\right]$$

$$G = \frac{3}{(k)\left[D(k)\right]}$$

$$\textit{Control limits} = \overline{\overline{MF}} \pm G\left[\bar{w}\left(\overline{MF}\right)\right]$$

Equations for central lines and control limits for standard deviation and range control charts are given in Table C-1, and factors for computing control limits are shown in Table C-2.

Table C-3 contains an example illustrating the use of Shewhart methodology for developing a control chart. In the example shown in Table C-3, control charts can be developed for each set's average, standard deviation, and range. The control lines for the control charts are as follows:

Table C-1—Nomenclature for Control Chart Lines

Quantity to Be Evaluated	Central Line	Lower Control Limit	Upper Control Limit
Averages	$\overline{\overline{MF}}$	$\overline{\overline{MF}} - (H)(\bar{s})$	$\overline{\overline{MF}} + (H)(\bar{s})$
Standard Deviations	$\bar{s}$	$(I)(\bar{s})$	$(J)(\bar{s})$
Ranges	$\bar{w}$	$(K)(\bar{w})$	$(L)(\bar{w})$

Table C-2—Shewhart Control Chart Factors for Averages

Number of Measurements in Sets, k	Shewhart Conversion Factors					
	G A_2[a]	H A_3[a]	I B_3[a]	J B_4[a]	K D_3[a]	L D_4[a]
2	1.880	2.659	0	3.267	0	3.267
3	1.023	1.954	0	2.568	0	2.575
4	0.729	1.628	0	2.266	0	2.282
5	0.577	1.427	0	2.089	0	2.114
6	0.483	1.287	0.030	1.970	0	2.004
7	0.419	1.182	0.118	1.882	0.076	1.924
8	0.373	1.099	0.185	1.815	0.136	1.864
9	0.337	1.032	0.239	1.761	0.184	1.816
10	0.308	0.975	0.284	1.716	0.223	1.777
11	0.285	0.927	0.321	1.679	0.256	1.744
12	0.266	0.886	0.354	1.646	0.283	1.717
13	0.249	0.850	0.382	1.618	0.307	1.693
14	0.235	0.817	0.406	1.594	0.328	1.672
15	0.223	0.789	0.428	1.572	0.347	1.653
16	0.212	0.763	0.448	1.552	0.363	1.637
17	0.203	0.739	0.466	1.534	0.378	1.622
18	0.194	0.718	0.482	1.518	0.391	1.609
19	0.187	0.698	0.497	1.503	0.404	1.596
20	0.180	0.680	0.510	1.490	0.415	1.585
21	0.173	0.663	0.523	1.477	0.425	1.575
22	0.167	0.647	0.534	1.466	0.435	1.565
23	0.162	0.633	0.545	1.455	0.443	1.557
24	0.157	0.619	0.555	1.445	0.452	1.548
25	0.153	0.606	0.565	1.435	0.459	1.541

[a]Traditional nomenclature for Shewhart conversion factors.

Table C-3—Statistical Data for Shewhart Control Chart

Set Number	Set of Runs					Average Meter Factor, $\overline{MF}$	Standard Deviation, $s(MF)$	Range, $w(MF)$
	$MF1$	$MF2$	$MF3$	$MF4$	$MF5$			
1	0.9995	0.9994	0.9998	0.9995	0.9999	0.9996	0.00020	0.0005
2	1.0011	1.0010	1.0013	1.0014	1.0012	1.0012	0.00018	0.0004
3	0.9992	0.9994	0.9990	0.9993	0.9996	0.9993	0.00022	0.0006
4	1.0010	1.0009	1.0011	1.0007	1.0009	1.0009	0.00015	0.0004
5	1.0002	1.0007	1.0004	1.0005	1.0006	1.0005	0.00019	0.0005
6	0.9992	0.9989	0.9991	0.9989	0.9990	0.9990	0.00013	0.0003
7	1.0004	1.0006	1.0003	1.0003	1.0005	1.0004	0.00013	0.0003
8	1.0010	1.0016	1.0013	1.0014	1.0012	1.0013	0.00022	0.0006
9	1.0002	1.0000	0.9998	0.9999	1.0002	1.0000	0.00018	0.0004
10	1.0018	1.0016	1.0020	1.0019	1.0015	1.0018	0.00021	0.0005
						$\overline{\overline{MF}} = 1.0004$		
							$\bar{s}(MF) = 0.00018$	
								$\bar{w}(MF) = 0.00045$

a. For set averages:
1. Central line (CL):

$$CL = \overline{\overline{MF}} = 1.0004$$

2. Upper control limit (UCL):

$$UCL = \overline{\overline{MF}} + H[\bar{s}(MF)] = 1.0004 + (1.427)(0.00018) = 1.0007$$

3. Lower control limit (LCL):

$$LCL = \overline{\overline{MF}} - H[\bar{s}(MF)] = 1.0004 - (1.427)(0.00018) = 1.0001$$

b. Control limits for the standard deviations of each set are as follows:
1. Central line (CL):

$$CL = \bar{s}(MF) = 0.00018$$

2. Upper control limit (UCL):

$$UCL = J[\bar{s}(MF)] = (2.089)(0.00018) = 0.00038$$

3. Lower control limit (LCL):

$$LCL = I[\bar{s}(MF)] = (0)(0.00018) = 0$$

c. Control limits for ranges of each set are as follows:
1. Central line (CL):

$$CL = \bar{w} = 0.00045$$

2. Upper control limit (UCL):

$$UCL = L[\bar{w}(MF)] = (2.114)(0.00045) = 0.00095$$

3. Lower control limit (LCL):

$$LCL = K[\bar{w}(MF)] = (0)(0.00045) = 0$$

Control charts for average meter factors and standard deviations for each set are shown in Figure C-1. Most of the meter factors are outside of the Shewhart control limits. However, the standard deviations of sets are well within their control limits. This occurs because variations between the average meter factor in each set are substantially greater than the variations within each meter factor set. The control chart limits shown in Figure C-1 for set averages are not appropriate for this example.

C.3 Shewhart Control Charts for Individuals

Shewhart control charts can also be developed for all individual measurements in a database. Control limits for individuals are based on the overall average of standard deviations of individual measurements in each set. Control chart lines for individual meter factors are determined as follows:

a. Central line (CL):

$$CL = \overline{\overline{MF}}$$

b. Upper control limit (UCL):

$$UCL = \overline{\overline{MF}} + M[\bar{s}(MF)]$$

c. Lower control limit (LCL):

$$LCL = \overline{\overline{MF}} - M[\bar{s}(MF)]$$

Since ranges can be used for estimating standard deviations, the control chart lines can be:

a. Central line (CL):

$$CL = \overline{\overline{MF}}$$

b. Upper control limit (UCL):

$$UCL = \overline{\overline{MF}} + N[\bar{w}(MF)]$$

c. Lower control limit (LCL):

$$LCL = \overline{\overline{MF}} - N[\bar{w}(MF)]$$

The values for the Shewhart conversion factors, M and N, are given in Table C-4.

For the example in Table C-3, the control chart lines would be as follows:

a. Based on standard deviations:

$CL = 1.0004$
$UCL = 1.0004 + (3.192)\ (0.00018) = 1.0010$
$LCL = 1.0004 - (3.192)\ (0.00018) = 0.9998$

b. Based on ranges:

$CL = 1.0004$
$UCL = 1.0004 + (1.290)\ (0.00045) = 1.0010$
$LCL = 1.0004 - (1.290)\ (0.00045) = 0.9998$

The control chart for individuals is shown in Figure C-2. The control limits are broader than for the averages shown on Figure C-1. However, a substantial number of meter proving runs fall outside the control limits because the control limits only reflect the statistical variations within sets. The variations between sets are not reflected in the control limit computations.

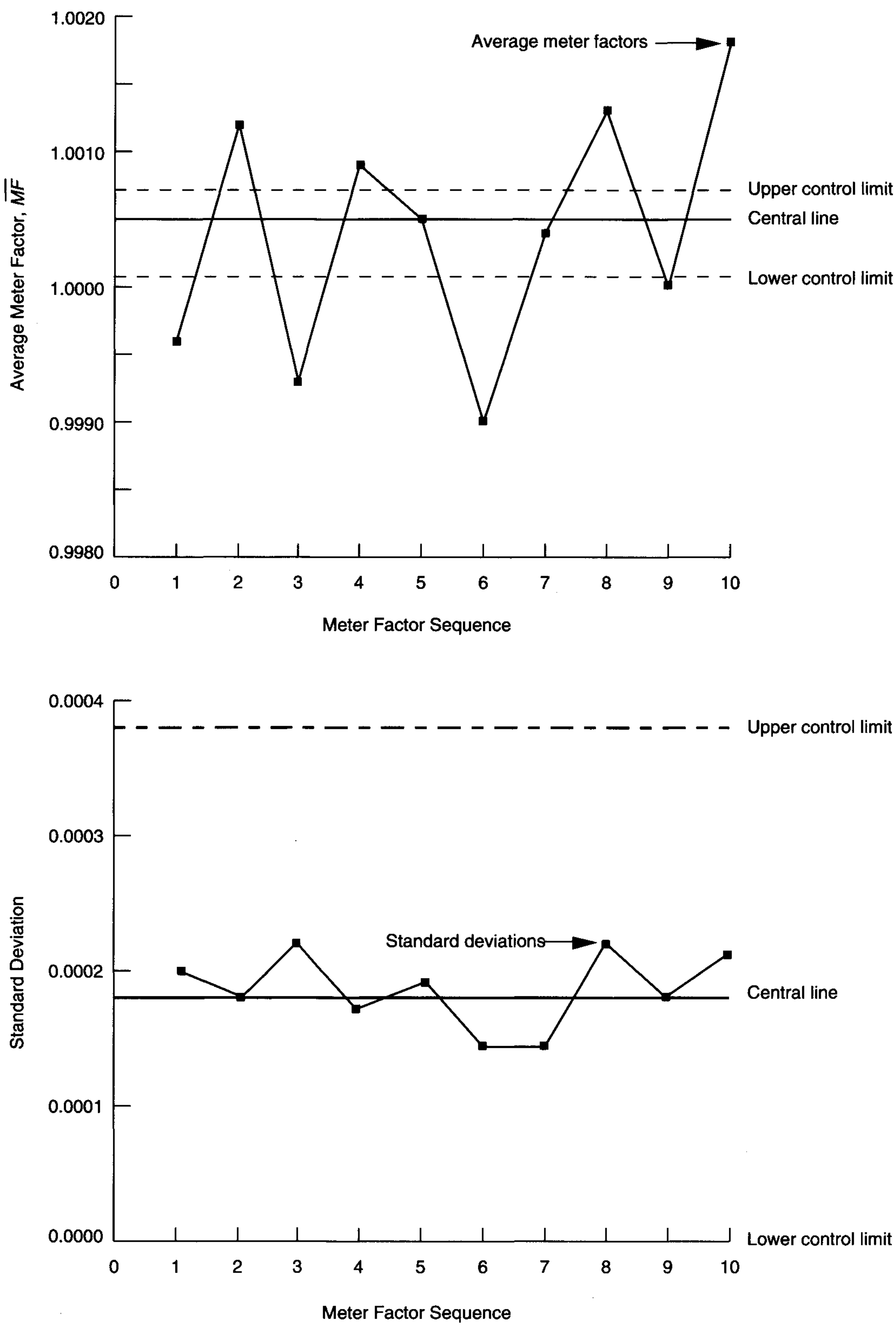


Figure C-1—Examples of Shewhart Control Charts

Control charts for individual meter factors can also be based on ranges between consecutive values such as the set average in Table C-3. Ranges for the example being analyzed are given in Table C-5.

Table C-4—Shewhart Control Chart Factors for Individuals

Number of Measurements in Set, n	Conversion Factors for Individuals[a] $M(n)$	$N(n)$
2	3.760	2.659
3	3.385	1.772
4	3.256	1.457
5	3.192	1.290
6	3.153	1.184
7	3.127	1.109
8	3.109	1.054
9	3.095	1.010
10	3.084	0.975
11	3.076	0.946
12	3.069	0.921
13	3.063	0.899
14	3.058	0.881
15	3.054	0.864
16	3.050	0.849
17	3.047	0.836
18	3.044	0.824
19	3.042	0.813
20	3.040	0.803
21	3.038	0.794
22	3.036	0.785
23	3.034	0.778
24	3.033	0.770
25	3.031	0.763
Over 25	3.000	$3/D(n)$

[a]E_2 and E_3 are traditional nomenclature for Shewhart conversion factors for N and M respectively.

The control chart lines based on the ranges between consecutive sets are as follows:

a. Upper control limit (UCL):

$$UCL = \overline{\overline{MF}} + [N(2)][\overline{w}(\overline{MF})]$$
$$= 1.00040 + (2.659)(0.00138)$$
$$= 1.0041$$

b. Lower control limit (LCL):

$$LCL = \overline{\overline{MF}} - [N(2)][\overline{w}(\overline{MF})]$$
$$= 1.00040 - (2.659)(0.00138)$$
$$= 0.9967$$

A control chart based on the above control limits is shown in Figure C-3. In this example, the control limits are broad. The standard deviation for the individual set averages ($\overline{MF}$) with respect to the overall average is 0.0009, and the overall range of the individual set averages is 0.0028. The control limits for set averages when considered as individual measurements are based on the following:

a. Standard deviation of the averages of a series of sets:

$$\textit{Control limits} = \overline{\overline{MF}} \pm [M(10)][s(\overline{MF})]$$
$$= 1.0004 \pm (3.084)(0.0009)$$
$$= 1.0004 \pm 0.0028$$
$$= 0.9976 \text{ and } 1.0032$$

b. Range of the averages of a series of sets:

$$\textit{Control limits} = \overline{\overline{MF}} \pm [N(9)][w(\overline{MF})]$$
$$= 1.0004 \pm (1.010)(0.0028)$$
$$= 1.0004 \pm 0.0028$$
$$= 0.9976 \text{ and } 1.0032$$

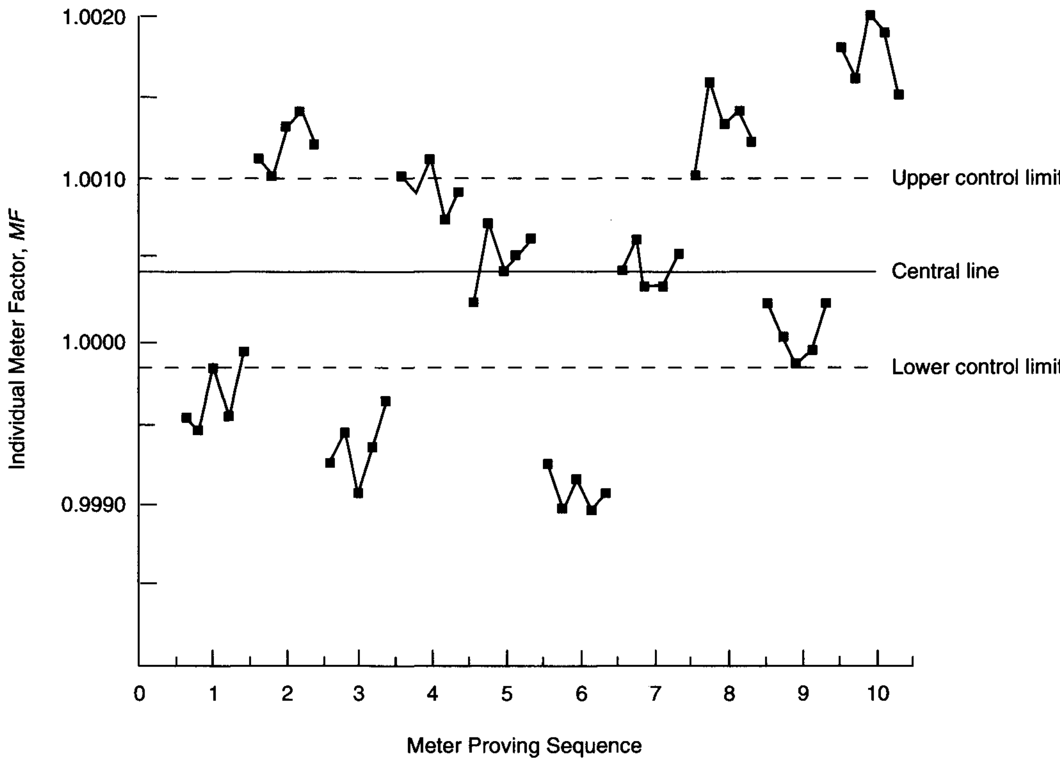


Figure C-2—Example of Shewhart Control Chart for Individuals

Table C-5—Moving Ranges of Set Averages for Example

Set Number	Set Averages, $\overline{MF}$	Ranges Between Consecutive Sets
1	0.9996	—
2	1.0012	0.0016
3	0.9993	0.0019
4	1.0009	0.0016
5	1.0005	0.0004
6	0.9990	0.0015
7	1.0004	0.0014
8	1.0013	0.0009
9	1.0000	0.0013
10	1.0018	0.0018
Average	1.00040	0.00138

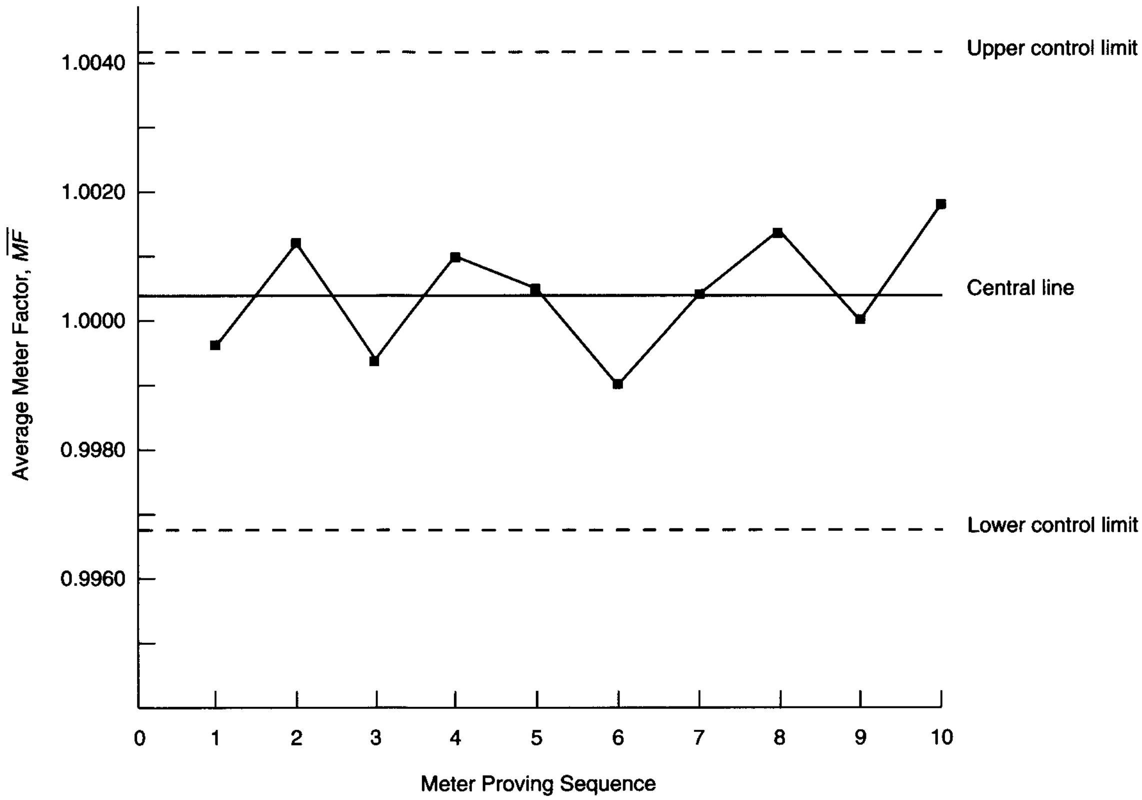


Figure C-3—Example of Shewhart Control Chart for Individuals Using Moving Ranges

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