Design of Flat Plate Structures

API BULLETIN 2V
THIRD EDITION, JUNE 2004

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FOREWORD

This Bulletin is under jurisdiction of the API Subcommittee on Offshore Structures.

This Bulletin provides guidance for the design of steel flat plate structures. Used in conjunction with API RP 2T or other applicable codes and standards, this Bulletin will be helpful to engineers involved in the design of offshore structures which include flat plate structural components.

The buckling formulations and design considerations contained herein are based on the latest available information. As experience with the use of the Bulletin develops, and additional research results become available, it is anticipated that the Bulletin will be updated periodically to reflect the latest technology.

Suggested revisions are invited and should be submitted to API, Standards Department, 1220 L Street, NW, Washington, DC 20005

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Section 1-Nomenclature and Glossary

1.1 Nomenclature

Note: The terms not defined here are uniquely defined in the sections in which they are used.

1.1.1 Material Properties

E	=	modulus of elasticity, [ksi].
G	=	shear modulus, [ksi].
v	=	Poisson's ratio.
F_y	=	minimum specified yield stress of material, [ksi].
τ_y	=	$F_y / \sqrt{3}$ yield stress in shear, [ksi].
F_p	=	proportional limit stress in compression, [ksi].
p_r	=	F_p / F_y stress ratio defining the beginning of nonlinear effects in compression.

1.1.2 Plate Geometry and Related Parameters

a	=	plate length or larger dimension, [in.]
b	=	plate width or shorter dimension, [in.]
D	=	$Et^3/[12 (1 - v^2)]$ plate flexural rigidity, [kips-in].
t	=	plate thickness, [in.]
α	=	$a/b \geq 1$ aspect ratio
β	=	$(b/t)\sqrt{F_y / E}$ slenderness ratio

1.1.3 Stiffener Geometry and Related Parameters

A	=	cross sectional area, [in.^{2}]
A_w	=	web area, [in.^{2}]
b	=	spacing between stiffeners, [in.]
b_e	=	effective width of attached plating, [in.]
b_f	=	flange total width, [in.]
C_w	=	warping constant (see formulas in Table 4.4-1), [in.^{6}]
d	=	web depth, [in].
I	=	minimum moment of inertia, [in.^{4}]
I_c	=	polar moment of inertia about centroid, [in.^{4}]
Is	=	polar moment of inertia about shear center, [in.^{4}]
I_1	=	moment of inertia of symmetric I-section in the plane of minimum stiffness, [in.^{4}]
I_2	=	moment of inertia of symmetric I-section in the plane of maximum stiffness, [in.^{4}]
J	=	torsion constant (see formulas in Table 4.4-1), [in.^{4}]
K	=	effective length ratio, normally taken as unity.
L	=	unsupported length, [in.]
L_b	=	bracing distance, [in.]

L_y = length at which there is a transition between elastic and plastic limit state moments for lateral buckling, [in.]
r = $\sqrt{I/A}$ radius of gyration, [in.]
S = section modulus for bending of symmetric I-section in the plane of maximum stiffness, [in.^{3}]
s = spacing between tripping brackets, [in.]
t = attached plate thickness, [in.]
t_f = flange thickness, [in.]
t_w = web thickness, [in.]
λ = $[KL/(r\pi)]\sqrt{F_y/E}$ stiffener slenderness ratio.

1.1.4 Stiffened Panel Geometry and Related Parameters

A = entire panel length, [in.]
A_2 = area of flange in stiffened plating (zero in the case of flat bar stiffeners), in.^{2}
A_s = stiffener area, [in.^{2}]
B = entire stiffened panel width in the case of a stiffened panel (see Figure 5.1-1), or distance between webs for effective flange breadth calculations (see Figure 5.2-1), [in.]
$2b$ = plate breadth, or distance between webs, [in.] (See Figure 5.6-1)
b_{ef} = effective breadth, [in.]
d = spacing between stiffeners = $2b$, [in.]
h = one half web depth, [in.]
I_s = moment of inertia of one stiffener about an axis parallel to the plate surface at the base of the stiffener, [in.^{4}]
L = length, [in.]
cL = distance between points of zero bending moment, [in.]
n = number of sub-panels (individual plates).
t = plate thickness, [in.]
t_f = flange thickness, [in.]
t_w = web thickness, [in.]
α = aspect ratio of whole panel
γ = $12(1-v^2)I_s/(t^3 d)$
δ = $A_s/(Bt)$
$\bar{\lambda}$ = $(B/t)\sqrt{F_y 12(1-v^2)/(E\pi^2 k)}$, modified slenderness ratio for uniaxially stiffened panels, where k is the buckling coefficient.
I_x, I_y = moment of inertia of the stiffeners with effective plating extending in the x- or y-direction,
respectively, [in.^{4}]
I_{px}, I_{py} = moment of inertia of the effective plating alone associated with stiffeners extending in the x- or y-direction, respectively, about the neutral axis of the entire section, [in.^{4}]
s_x, s_y = spacing of the stiffeners extending in the y- or x-direction, respectively, [in.]

t_x, t_y = equivalent thickness of the plate and the stiffeners (diffused) extending in the *x*-direction or *y*-direction, respectively, [in.]

M_x, M_y = moment per unit length that produces a stress f_x or f_y, respectively, [kips]

r_a, r_b = bending lever arm associated with f_x or f_y, respectively, i.e., distance from the neutral axis of the stiffener with the effective breadth of plate to the outer fiber of the flange (for the flange stress) or of the plate (for the plate field stress), [in.]

1.1.5 Deep Plate Girder Geometry and Related Parameters

A_f = flange cross-sectional area, [$in.^2$]

a = spacing between transverse web stiffeners, [in.]

a_h = web opening height, [in.]

B_f = width of unstiffened flange in a beam with only one web, or half the distance between successive longitudinal stiffeners or webs, together with any adjacent outstand, [in.] (See Fig. 6.1-4.)

b = spacing between longitudinal web stiffeners, [in.] (See Fig. 6.3-1.)

b_e = effective plate flange width attached to web stiffeners, [in.]

b_h = web opening length, [in.] (See Fig. 6.3-1)

d_s = spacing between web longitudinal stiffeners, [in.]

d_w = web depth, [in.]

R_f = flange radius of curvature, [in.]

s_h = clear distance along the longitudinal direction between web openings, [in.]

t_f = flange thickness, [in.]

t_w = web thickness, [in.]

θ = slope of web to horizontal.

1.1.6 Stresses

1.1.6.1 Normal Stresses:

f = normal stress, [ksi].

f_x , f_y = normal stress directed along the *x* and *y* axis, [ksi].

f_{xy} = in-plane shear stress, [ksi]

f_{se} = elastic serviceability limit state stress, [ksi].

f_{sp} = plastic serviceability limit state stress, [ksi].

f_u = ultimate limit state stress, [ksi].

f_{xse} = normal stress f_{se} when the plate is compressed in the *x* direction alone, [ksi]

f_{yse} = normal stress f_{se} when the plate is compressed in the *y* direction alone, [ksi].

f_{xyse} = edge shear stress f_{se} when the plate is loaded in pure shear, [ksi].

f_{xysp} = edge shear stress f_{sp} when the plate is loaded in pure shear, [ksi].

f_{xyu} = edge shear stress f_u when the plate is loaded in pure shear, [ksi].

f_{xl} = limit state normal stress in the x direction when the plate is compressed in the *x* direction, [ksi].

f_{yl} = limit state normal stress in the y direction when the plate is compressed in the *y* direction, [ksi].
f_{xyl} = limit state shear stress when the plate is loaded in pure shear, [ksi].

1.1.6.2 Shear Stresses:
f_{xy} = in-plane shear stress, [ksi].
f_{xyse} = elastic serviceability limit state stress, [ksi].
f_{xysp} = plastic serviceability limit state stress, [ksi].
f_{xyu} = ultimate limit state stress, [ksi].

1.1.7 Plate Lateral Deflections
W_a = maximum allowable deflection, [in.]
W_e = maximum elastic deflection, [in.]
W_p = plastic set (maximum permanent plastic deflection), [in.]

1.1.8 Plate Lateral Pressures
p = uniform lateral pressure, [ksi].
p_u = ultimate limit state pressure, [ksi].

1.1.9 Stiffener Axial Loads
P = applied axial force, [kips].
P_y = fully plastic axial force = A F_y , [kips].
P_{Ee} = column elastic ultimate state axial force, [kips].
P_{Ep} = column plastic ultimate state axial force, [kips].
P_{Te} = column torsional elastic ultimate state axial force, [kips].
P_{T_p} = column torsional plastic ultimate state axial force, [kips].
P_{TFe} = column torsional/flexural elastic ultimate state axial force, [kips].
P_{TF_p} = column torsional/flexural plastic ultimate state axial force, [kips].

1.1.10 Stiffener Lateral Distributed Loads
q = uniform lateral load per unit length, kips per [in.]
q_a = load *q* per unit length on stiffener of length *a*, kips per [in.]
q_b = load *q* per unit length on stiffener of length *b*, [kips per in.]
q_u = ultimate load, [kips per in.]

1.1.11 Stiffener Bending Moments
M = applied bending moment, [in-kips].
M_o = fully plastic bending moment, [in-kips].
M_1 = smaller end moment in the plane of bending, [in-kips].
M_2 = larger end moment in the plane of bending, [in-kips].
M_{fy} = moment at which the flanges are fully plastic, [in-kips].
M_y = moment at which yield first occurs in the flanges, [in-kips].
M_u = ultimate limit state *M*, [in-kips].
M_{ue} = elastic ultimate limit state *M*, [in-kips].
M_{up} = plastic ultimate limit state *M*, [in-kips].

1.1.12 SI Metric Conversion Factors

in x 25.4 = mm
ksi x 6.894757 = MPa

1.2 GLOSSARY

1.2.1 chord: Deep plate girder flange.

1.2.2 deep plate girder: Deep plate girder with the web stiffened in both the longitudinal and transverse directions and satisfying the requirements of 6.1.1. See also 6.1.2.

1.2.3 design variables: Quantities that define for the purpose of structural design or analysis a structural component and material, its state of stress, and the applied loads.

1.2.4 distortion energy theory: Failure theory defined by the following equation, where the applied stresses are positive for tension and negative for compression:

$$f_x^2 - f_x f_y + f_y^2 + 3 f_{xy}^{\ 2} = F_y^2$$

1.2.5 effective flange breadth: The reduced breadth of a plate subjected to bending and/or tensile load, which, with an assumed uniform stress distribution, produces the same effect on the behavior of a structural member as the actual breadth of the plate with its non-uniform stress distribution. While the effective flange width applies to a member under compression, the effective flange breadth applies to a member under bending and/or tensile loading, and is associated with shear lag effects. See 5.6.

1.2.6 effective flange width: The reduced width of a plate subjected to compressive load, which, with an assumed uniform stress distribution produces the same effect on the behavior of a structural member as the actual width of the plate with its non-uniform stress distribution. See 4.1.2.

1.2.7 panel: See stiffened panel.

1.2.8 plate: In Bulletin 2V this term refers to a flat thin rectangular plate, see 3.1.2.

1.2.9 global stresses: Stresses resulting from global deformation of the structure.

1.2.10 proportional limit stress (F_p): Stress above which the stress-strain curve is no longer linear and which represents the onset of plastic behavior. If no specific value for the steel being used is available F_p can be taken as 0.60 F_y , where F_y is the yield stress.

1.2.11 residual stresses: The stresses that remain in an unloaded member after it has been formed and installed in a structure. Some typical causes are forming, welding and corrections for misalignment during installation in the structure.

1.2.12 panel stresses: Stresses on stiffened panels resulting from local applied pressures or transverse loads.

1.2.13 serviceability limit state: Function of design variables which defines a condition at which a member no longer satisfies functional requirements, although it is still capable of carrying additional loads before reaching an ultimate limit state. See 2.4.3.

1.2.14 shear lag: Shear effects on beams that cause a non-uniform distribution of longitudinal bending stresses across the flange.

1.2.15 stiffened panel: Structural component comprising one or two sets of equally spaced uniform stiffeners of equal cross section supporting a thin plate. If there is only one set of stiffeners the panel is uniaxially stiffened, and if there are two the panel is orthogonally stiffened. See 5.1.2.

1.2.16 stiffener: Straight and slender thin-walled member of uniform cross which serves as a stiffening element for a flat plate structure. See 4.1.2.

1.2.17 plate stresses: Stresses on a thin rectangular plate resulting from lateral pressure.

1.2.18 tripping: Torsional buckling of stiffener.

1.2.19 ultimate limit state: Function of design variables that defines the resistance of a member to failure (i.e., its maximum load carrying capacity at failure), see 2.4.2.

1.2.20 yield stress: The yield stress of the material determined in accordance with ASTM A307.

Section 2-General

2.1 SCOPE

2.1.1 Bulletin 2V provides guidance for the design of steel flat plate structures. These often constitute main components of offshore structures. When applied to Tension Leg Platforms (TLPs) this Bulletin should be viewed as a complement to API RP 2T. The Bulletin combines good practice considerations with specific design guidelines and information on structural behavior. As such it provides a basis for taking a "design by analysis" approach to structural design of offshore structures.

2.1.2 Flat plate structures include thin plates, stiffened panels and deep plate girders, and they can constitute the main component of decks, bulkheads, web frames and flats. The external shell of pontoons or columns can also be made of flat stiffened panels if their cross section is, for example, square or rectangular, rather than circular.

2.1.3 Bulletin 2V is not a comprehensive document, and users have to recognize the need to exercise engineering judgment in actual applications, particularly in the areas that are not specifically covered.

2.1.4 Plates are discussed in Section 3, stiffeners in Section 4, stiffened panels in Section 5, and deep plate girders in Section 6. Limit states are given for each relevant load and load combination, and design requirements are also defined. Figure 2.1-1 summarizes the structural components and the limit states covered in Bulletin 2V.

2.2 REFERENCES

Background and references on the contents of Bulletin 2V are included in a Commentary given in the Appendix. Reference is made to API RP 2T, *Recommended Practice for Design of Tension Leg Platforms*, and API RP 2A, *Recommended Practice for Planning, Designing, and Constructing Fixed Offshore Platforms, American Petroleum Institute, and to the American Institute of Steel Construction, Specification for the Design, Fabrication and Erection of Structural Steel for Buildings*, latest edition.

2.3 RANGE OF VALIDITY AND LIMITATIONS

2.3.1 The formulations given apply only to members made of structural steel used for offshore structures, as defined in API RP 2T.

2.3.2 Structural components must comply with the dimensional tolerance limits defined in API RP 2T. Members not complying with these requirements should be given special consideration, given the potential negative impact dimensional imperfections can have on structural performance.

2.3.3 The formulations for the limit states given may be replaced by more refined analyses, or model tests, taking into account the real boundary conditions, the actual load distribution, geometrical imperfections, material properties, and residual stresses.

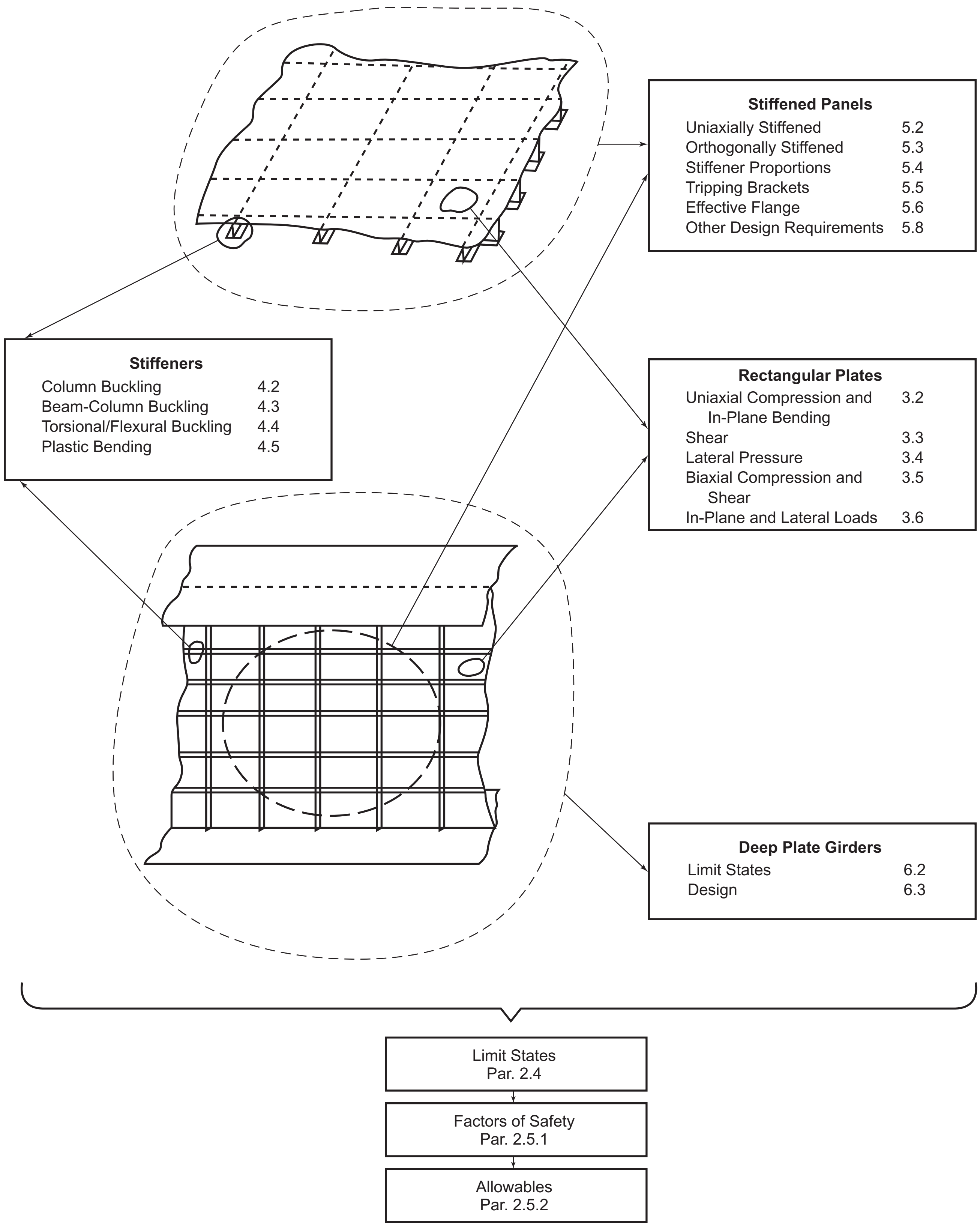


Figure 2.1-1—Structural Components and Limit States Covered in this Bulletin

2.3.4 Ultimate limit states associated with failure due to material fracture are not considered. Provisions have to be made to ensure that this type of failure is properly addressed in the design.

2.3.5 Ultimate limit states associated with accidental loads such as collisions, dropped objects, fire, explosion, or flooding are not considered. Design criteria for these loads have to be established, and provisions have to be made to ensure structural adequacy under such conditions.

2.4 LIMIT STATES

2.4.1 Working Stress Design

2.4.1.1 The design basis adopted in this Bulletin is the working stress design method, whereby stresses in all components of the structure cannot exceed specified allowable values. Allowable stresses are associated with two basic structural requirements: resistance to failure (ultimate limit states); and stiffness and strength criteria (serviceability limit states).

2.4.1.2 In addition to specifying allowable stress values, certain limits on non-dimensional parameters can be defined. Examples are upper limits on web depth to thickness ratio, or flange width to thickness ratio for I-section stiffening elements, which are in general defined to limit the possibility of buckling of the web or flange. These limits on cross sectional proportions are normally associated with good design practice.

2.4.2 Ultimate Limit States

2.4.2.1 Ultimate limit states correspond to the maximum load carrying capacity of a member at failure. Thus, if an ultimate limit state is reached, the structure collapses and loses its load carrying capacity. Failure may be due to:

1. Material plastic flow,
2. Material fracture,
3. Collapse due to local or general instability.

2.4.2.2 The ultimate limit states considered here include only failure due to material plasticity, and collapse due to local or general instability.

2.4.2.3 In identifying material plastic failure as an ultimate limit state it is necessary to distinguish those cases where the material yields, but there is no plastic mechanism and as such no collapse, and those cases where a plastic mechanism leads to structural instability. If material yielding does not lead to collapse, failure is not an ultimate limit state but a serviceability limit state. This distinction is important, since by designing for limited and controlled material yield a more weight efficient design can possibly be achieved. The designer must use critical judgment in identifying those areas and components where plastic design can be adopted.

2.4.2.4 Local instability refers to the type of failure whereby only a localized portion or subcomponent of the structure fails. In a rectangular panel stiffened by two sets of stiffeners intersecting at right angles, such as a transverse bulkhead or flat, the buckling of a single rectangular plate spanning between consecutive stiffeners is an example of local instability. The tripping of a single stiffener over a single span is another example of local instability. If the complete panel buckles as a whole, the mode of failure is general instability.

2.4.3 Serviceability Limit States

2.4.3.1 Serviceability limit states correspond to loads at which a member no longer satisfies functional requirements, although it is capable of carrying additional loads before reaching an ultimate limit state. Serviceability limit states include:

1. Material yield;
2. Local instability;
3. Deformation;
4. Vibration.

2.4.3.2 Material plastic flow should not adversely affect the structure's appearance or efficiency, and should not lead to excessive deformations. The same applies to local instability, such as the buckling of an individual plate, or the local tripping of a secondary stiffener in a stiffened panel.

2.4.3.3 The deformation of the structure or any of its parts resulting from the normal operating conditions or from damage should not adversely affect its appearance or efficiency, violate minimum specified clearances, or cause drainage difficulties. Damage occurring in specific parts of the structure which might entail excessive maintenance or lead to excessive deformation or corrosion, and hence adversely affect the structure's appearance or efficiency, should be limited.

2.4.3.4 Where there is a likelihood of the structure being subjected to vibration from causes such as wind forces, equipment or other transient loads, measures should be taken to prevent discomfort or alarm, or impairment of a proper function.

2.4.3.5 Serviceability limit states associated with local damage or vibration are not considered in Bulletin 2V. Provisions have to be made by the designer to ensure that these are properly accounted for in the design process.

2.5 VERIFICATION OF STRUCTURAL ADEQUACY

2.5.1 Factors of Safety

2.5.1.1 A design is considered satisfactory if the structure has an adequate margin against failure, or reserve strength, for all applicable limit states. The margin against failure to be adopted in the design is defined in terms of allowable values for the stresses, or other relevant design variables (e.g., pressure, axial load, etc.). The allowables are obtained by dividing limit state values by factors of safety, as described in more detail in 2.5.2. The factors of safety recommended for design are as follows:

F.S. = 1.67 for serviceability limit states

F.S. = 1.67ψ for ultimate limit states

2.5.1.2 The effects of imperfections are very significant in the elastic range but have little effect in the yield and strain hardening ranges of the material. Therefore, a partial factor of safety, ψ, dependent on the buckling stress is recommended for ultimate limit states. The value of ψ is 1.20 when the buckling stress is elastic, 1.00 when the buckling stress equals the yield stress and varies linearly between these limits.

2.5.1.3 A 1/3 increase in allowable stresses may be used where appropriate. The structure should be designed so that all components are proportioned for basic allowable stresses specified by API RP 2A, API RP 2T, or by the AISC *Specification for the Design, Fabrication and Erection of Structural Steel for Buildings*, latest edition. Where the structural element or type is not covered by the above, a rational analysis should be used to determine the basic allowable stresses, with factors of safety equivalent to those defined. Alternative methods for verifying structural adequacy may also be acceptable, as defined in 2.5.6.

2.5.1.4 In determining structural adequacy two types of load conditions have to be considered: a single load acting on the structure and multiple loads (or load combinations).

2.5.2 Single Load Limit States

2.5.2.1 Each limit is defined in terms of a design variable Qi. Depending on the particular limit state, this design variable can be, for example, a stress component, a pressure, or a deflection. When a limit state is satisfied:

$$Q_i = Q_i^u \qquad (2.5\text{-}1)$$

where

Q_i = actual value of the relevant design variable (stress, pressure, deflection, etc.),

Q_i^u = limit state value of Q_i, as defined by the formulas in this Bulletin.

2.5.2.2 Given a particular limit state, a design is considered satisfactory if the associated design variable does not exceed an allowable value given by:

$$\frac{Q_i^u}{F.S.}$$

where F.S. is the appropriate factor of safety.

2.5.3 Combined Load Limit States

When n loads $Q_1, \ldots, Q_n$ act on a structure a limit state is defined in this Bulletin in terms of an interaction equation:

$$\left(\frac{Q_1}{Q_1^u}\right)^{m_1} + \left(\frac{Q_2}{Q_2^u}\right)^{m_2} + \ldots + \left(\frac{Q_n}{Q_n^u}\right)^{m_u} = 1 \qquad (2.5\text{-}2)$$

where Q_i^u, is the limit state value of Q_i when Q_i is the only load acting on the structure.

Interaction equations are in most cases of an empirical nature, with the exponents m_i being determined on the basis of a best fit of experimental data.

2.5.3.1 Given a particular limit state, a design is considered satisfactory if the relevant design variables do not exceed allowable values given by Q_1^u /F.S., Q_2^u /F.S., … Q_n^u/F.S., where Q_1^u … Q_n^u are the limit state design variables satisfying the interaction equation above, and F.S. is the appropriate factor of safety.

2.5.3.2 The interaction equations and the formulations for the limit state values of the relevant design variables given in this Bulletin reflect serviceability and ultimate limit states. In using them for specific applications the designer must ensure that the appropriate factors of safety (F.S.'s) are adopted, as prescribed in 2.5.1, 2.5.2, and 2.5.3.

2.5.4 Governing Limit State

In general, both serviceability and ultimate limit states are defined for each mode of failure. Either of these limit states can govern the design by imposing a lower allowable value on the design variable Q_i. However, the allowable values for Q_i resulting from serviceability and ultimate limit state considerations should be close for an efficient design. A design is considered satisfactory if the design variables do not exceed their allowable values for all the applicable limit states.

Note: formulations given in this Bulletin for the ultimate limit state sometimes yield lower values than the serviceability limit state. This is a function of the plate geometry and material properties.

2.5.5 Other Limit States

To ensure that a structure is adequate, it is necessary to consider other modes of failure not treated in Bulletin 2V. These include failure due to material fracture or fatigue, and failure caused by accidental loads.

2.5.6 Alternative Methods for Verifying Structural Adequacy

2.5.6.a General. The formulations for the limit states included in Bulletin 2V may be replaced by more refined analyses, or model tests, taking into account the real boundary conditions, the actual load distribution, geometrical imperfections, material properties and residual stresses. In adopting these alternative methods it is necessary to ensure that the structure is correctly modeled, and that all relevant limit states are considered. In particular if weight savings and increased structural efficiency are necessary, more refined methods of analysis should be explored.

2.5.6.b Methods of Analysis. The methods of analysis that are adequate for considering the ultimate limit states include elastic methods, and plastic or yield-line methods. Elastic methods (in which P-delta effect is included and all failure modes are accounted for by appropriate stress limits, but plastic load redistribution does not occur) are acceptable as lower bound collapse solutions, and they will also lead to solutions less likely to violate serviceability criteria. Elastic methods imply that a valid yield criterion is adopted to ensure, together with equilibrium, the static admissibility of the solution.
Plastic or yield-line methods may be adopted when appropriate to the structural configuration. Plastic methods or other procedures for permitting redistribution of moments and shears may be used only when:

a. The structural configuration and the materials have an adequate plateau of resistance under the appropriate ultimate conditions, and are not prone to deterioration of strength due to shakedown under repeated loading;

b. The development of bending plasticity does not cause an indeterminate deterioration in shear, torsional or axial strength, when relevant;

c. The supports or supporting structures are capable of withstanding reactions calculated by elastic methods.

The methods of analysis that are adequate for considering the serviceability limit states are in general elastic methods. Linear methods may be used when changes in geometry do not significantly influence the structure's performance. Nonlinear methods may be adopted with appropriate allowances for loss of stiffness, and should be used where geometric changes significantly modify the structure's performance. The method used should at all times satisfy equilibrium requirements and compatibility of deformations.

The mathematical idealization of the structure should reflect the nature of its response. The boundaries assumed in such an idealization should either calculate accurately the stiffness of adjacent parts, or be sufficiently remote from the part under consideration, for the stresses to be insensitive to the boundary assumptions.

2.5.6.c Model Analysis and Testing. Model analysis and testing may be used either to define the load effects in a structure, or to verify a proposed theoretical analysis. The models used should be capable of simulating the response of the structure appropriately, and the interpretation of the results should be carried out by engineers having the relevant experience. Model tests are particularly important in those cases where the geometry being proposed is novel, or not proven for the specific application under consideration.

The reliability of the test results depends upon the accuracy or knowledge of several factors, such as:

a. Material properties (model and prototype);

b. Methods of measurement;
c. Methods used to derive load effects from measurements;
d. Loading and reactions.

In interpreting results, the load effects to be used in design should exceed those derived from the test data by a margin dependent upon:
e. Number of tests;
f. Method of testing;
g. An assessment of a., b., and c. above.

In all cases the interpreted results should satisfy equilibrium and compatibility.

Where prototype testing is adopted as a basis for proving the resistance of a component, the test loading should adequately reproduce the range of stress combinations to be sustained in service. A sufficient number of prototypes should be tested to enable a mean value and standard deviation of resistance to be calculated for each critical stress condition. A particular aspect of structural behavior that may not be modeled correctly in small scale testing is residual stresses. It is important that this factor be accounted for in interpreting results, and in extrapolating to full scale.

The material strengths to be specified for construction of the model should have mean values and coefficients of variation compatible with those in the prototypes. Tolerances and dimensions should be similarly prescribed so that the models are compatible with the prototypes.

2.6 STRUCTURAL COMPONENT LOADS AND LOAD COMBINATIONS

2.6.1 General

The loads and load combinations that are to serve as a basis of design are defined in appropriate documents such as API RP 2T, API RP2FPS, etc.

2.6.2 Primary Loads

2.6.2.1 Primary loads and load combinations for structural component design, such as stiffened panels or deep girders, result in general from global platform analysis, to be discussed in 2.7. These primary loads can typically be classified as follows:

- axial tension or compression;
- shear;
- bending;
- twisting;
- lateral loading (distributed or concentrated).

Typical load combinations that are relevant for design include, for example:

- axial compression and shear;
- axial compression and bending;
- biaxial bending;
- bending and torsion.

2.6.2.2 The most relevant loads and load combinations for structural component analysis are treated in Bulletin 2V. The structural components considered are thin rectangular plates, stiffeners, stiffened panels and deep plate girders. However, the treatment is not comprehensive, and the designer should use other methods to ensure structural adequacy for those loads or load combinations not treated in the Bulletin. In particular, no consideration is given to concentrated loads on plates.

2.6.3 Secondary Loads

2.6.3.1 For most commonly encountered load cases, secondary loads do not directly affect the limit states, but the designer should ensure that they are included, when appropriate.

2.6.3.2 Examples of secondary loads include:

- shrinkage forces due to welding;
- stresses due to construction tolerances;
- thermal loads.

2.6.3.3 In cases controlled by fire considerations, thermal loads should be treated as primary loads.

2.6.4 Accidental Loads

As indicated in 2.3, accidental loads, such as those caused by collisions, dropped objects, fire, explosion, or flooding, are not considered. Some of these loads can lead to the rapid loss of strength of the primary structure and bring about an ultimate limit state. The designer should use acceptable methods to assess the adequacy of the structure to withstand such loads.

2.7 GENERAL APPROACH TO STRUCTURAL ANALYSIS

2.7.1 General

General principles regarding analysis methods, modeling, stress analysis and fatigue analysis for structures are covered in API RP 2T.

2.7.2 Global, Panel, and Plate Stresses

2.7.2.1 The structural analysis of a stiffened plate structure requires the consideration of several models. Global behavior can be represented through the use of a 3-D finite element model describing the whole structure. A more precise definition of stress distribution requires the consideration of smaller models, representing main structural components, or more localized areas of the structure, such as stiffened panels. Finally, main structural components can be further subdivided into the most basic elements, which are thin plates and stiffeners.

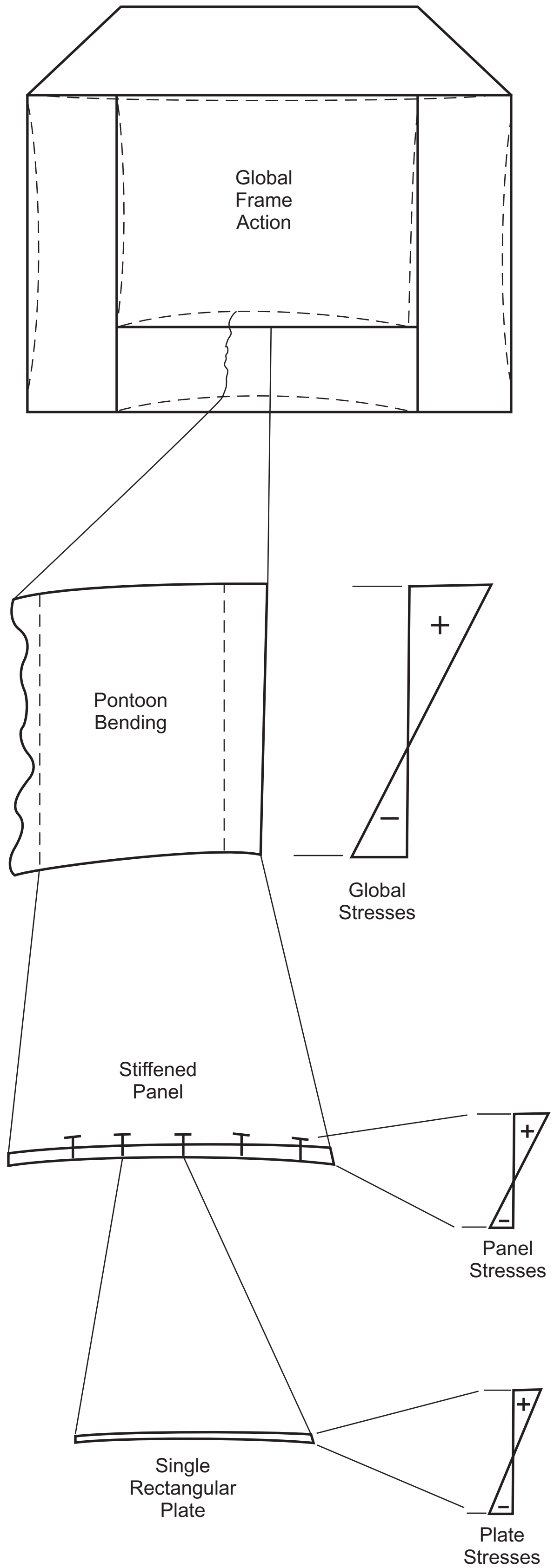


Figure 2.7-1—Global, Panel, and Plate Stresses

2.7.2.2 The 3-D finite elements model leads to stress distributions over gross cross sections of the structure, such as the columns or pontoons. These stresses resulting from deformation of the structure are global stresses. In the case of a pontoon of rectangular cross section, for example, the global stresses result from axial load, shear, biaxial bending and torsion. Assuming that the members in the space frame model are slender the global stresses can be obtained from simple beam theory, with corrections for shear, if necessary.

2.7.2.3 The next main structural component is the stiffened panel. The main stresses are generally due to bending and transverse shear, and are a result of local applied pressures or transverse loads. These stresses can be called panel stresses, and can be derived on the basis of orthotropic plate or grillage theory.

2.7.2.4 A single rectangular plate is the most basic component of flat plate structures. If the plate behavior between stiffeners under lateral pressure is considered, the resulting stresses are the plate stresses. These can be derived on the basis of thin plate theory.

2.7.2.5 Typical global longitudinal bending stress distributions for a pontoon cross section are sketched in Figure 2.7-1. They vary linearly across the depth of the cross section. Typical panel stresses for the pontoon bottom are also shown. They vary linearly across the depth of the stiffened panel, reaching maximum values at the extreme fiber of the stiffener flange, or at the shell plate. Plate bending stresses vary linearly across the plate thickness and are zero at its middle surface.

2.7.2.6 Given this breakdown of stresses into the three main categories, global, panel and plate, it becomes possible to use linear superposition to assess the resulting stress in different components of the structure, assuming elastic material properties and small deformations.

2.7.2.7 This classification of stresses is practical in those areas where the structure can easily be subdivided into global (space frame), panel (stiffened panel), and plate (plate) functions. In areas such as the nodes (where the columns and pontoons intersect), more refined stress analysis methods become necessary, such as the finite element method (Ref. APPENDIX B).

2.7.3 Dimensional Imperfections

Dimensional imperfections, such as out-of-straightness of stiffeners or out-of-flatness of plates, can have a strong impact on structural performance. Structural analysis has to account for dimensional imperfections in case these are beyond the tolerances established in 10.2.3 of API RP 2T. Numerical methods, such as the finite element method, are usually required to study the implications of imperfections on performance.

2.7.4 Residual Stresses and Weld Shrinkage Forces

2.7.4.1 Residual stresses can have some impact on structural performance. There are no simple analytical ways of determining how they affect the structure. Weld shrinkage forces can only be estimated on the basis of empirical equations, but they depend on many factors that cannot be controlled by the designer.

2.7.4.2 Examples of factors that can affect residual stresses and weld shrinkage forces are the assembly sequence, the welding procedure and the use of temporary bracing.

2.7.4.3 The designer should use engineering judgment in deciding how relevant residual stresses and weld shrinkage forces can be for a particular application.

2.8 GENERAL APPROACH TO STRUCTURAL DESIGN

2.8.1 General

Structural design is an iterative process through which the layout and scantlings for a structure are determined, such that it meets all the requirements of structural adequacy. The overall configuration results from a synthesis of all design requirements, which are in general dictated by non-structural considerations, such as volume and space requirements, global stability, safety, etc. Thus, structural design is assumed here to concentrate on the choice of an appropriate structural layout and scantlings, or cross-sectional dimensions, of structural components.

2.8.2 Major Structural Design Steps

2.8.2.1 There is no unique way of designing a structure, but in general terms the major steps that are involved can be summarized as follows:

a. Identify loads and load combinations acting on the structure as a whole, or on its main subcomponents.
b. Select initial structural layout and scantlings. In general this is based on past experience with similar structures. In those cases where some limits on proportions are specified, these should be respected in the initial configuration. Examples are stiffener proportions, such as maximum web depth to thickness ratio. Absolute minimum or maximum scantlings result in general from practical considerations related to constructability, weldability, etc.
c. Identify structure’s main components, and determine through structural analysis the loads and load combinations acting on each component. Structural analysis would normally start with a global space frame analysis and would then move into specific components, such as stiffened panels and single plates. For selected areas of the structure, global, panel and plate stresses can be computed and combined using linear superposition. In those areas where the structural arrangement is complex, a numerical method of analysis, such as the finite element method, may have to be adopted in order to obtain an accurate picture of the stress distributions.
d. Identify relevant limit states and associated factors of safety.
e. Check structural adequacy. If any limit state is violated, adjust scantlings and repeat the analysis and the structural checks. Perform the iterations required to converge to a structurally adequate design. Exercise engineering judgment in

those cases where the design is governed by serviceability, see 2.5.4. Investigate structural adequacy with alternative acceptable methods, in case limit state checks are perceived to lead to structural inefficiency.

f. Check other limit states, such as fatigue, which requires the selection of main structural detail configurations. Also check the adequacy of the design against accidental loads. If the structure is found to be inadequate, then new design iterations have to be conducted.
g. “Optimize” structural design. Once an adequate design has been achieved it is in general possible to “optimize” it for a given objective. The objective depends on the structure’s intended use, and can be, for example, the structural weight or the cost of fabrication and installation. Thus, once a new configuration and set of scantlings are derived, structural adequacy (Step e) has to be checked again, in an iterative fashion.

2.8.2.2 Structural “optimization” as a tool of structural design has to be considered with some caution, since proper balance between all desirable features, such as weight efficiency and cost, is in general very difficult to attain. However, it is important that the iterative nature of the design be recognized, and that possible and practical improvements be explored at the design stage. It is also important to note that special attention should be given to a weight engineering function.

2.8.3 Structural Details

2.8.3.1 The importance of good structural details must be emphasized. These have a great impact on structural efficiency and ensure that the structure will perform adequately.

2.8.3.2 The design of structural details requires a coordinated effort between designer, fabricator and installer to ensure constructability. Whenever possible, details should be made uniform, and advantage should be taken of repeatability.

2.8.3.3 Considerations regarding the design of structural details are not provided herein. However, the designer must ensure that good engineering practice is followed in designing details.

Section 3-Plates

3.1 GENERAL

3.1.1 Scope

3.1.1.1 Flat thin rectangular plates, where the thickness is very small as compared to the other plate dimensions, are considered. It is assumed that normal stress in the direction transverse to the plate surface can be disregarded.

3.1.1.2 The provisions in this Bulletin are not valid when the plate thickness is not small, in which case more refined analyses have to be conducted.

3.1.2 Definitions

3.1.2.1 Thin rectangular plates are the simplest component of flat stiffened plate structures. Each plate is usually supported around the four edges by stiffeners. When considering an individual rectangular plate the edge stiffeners are assumed to be sufficiently strong to remain essentially straight under loading.

3.1.2.2 If the plate slope at the edges is fixed, as happens with plating under uniform lateral pressure over continuous supports, the edges can be taken as perfectly clamped. If the edges rotate freely about the supports simply supported conditions govern the plate behavior. The plate edges should in general be assumed simply supported, unless it can be shown that other conditions apply. In particular partial fixity (degree of restraint between fully clamped and simply supported) should be examined, if engineering judgment indicates it is a better representation of the actual structural arrangement.

3.1.2.3 In the case of plate deflections that are not small in comparison with the thickness it is necessary to distinguish between immovable edges and edges free to move in the plane of the plate. This distinction can have a considerable impact on the magnitude of deflections and stresses. If the plate edges are fully prevented from moving in the plane of the plate, membrane effects can significantly affect its carrying capacity, and could be included provided the deflection limits are not exceeded.

3.1.2.4 The following nomenclature will be adopted here: The long plate dimension or length is parallel to the x-axis or longitudinal direction and is labeled a. The small plate dimension or width is parallel to the y-axis or transverse direction and is labeled b. Thus the plate's aspect ratio, = a/b, is always equal to or larger than unity. The plate thickness is t.

3.1.3 Loads and Load Combinations

3.1.3.1 A rectangular plate can be subjected to a variety of primary and secondary loads and load combinations.

3.1.3.2 The following loads can be classified as primary loads, as shown in Figure 3.1-1.

- In-plane longitudinal tension or compression;
- In-plane transverse tension or compression;
- In-plane longitudinal bending;
- In-plane transverse bending;
- In-plane shear;
- Twisting;
- Lateral pressure.

3.1.3.3 In addition to these primary loads the plate can also be subjected to secondary loads as follows:

- Shrinkage forces due to welding;
- Stresses due to construction tolerances;
- Loads due to thermal effects.

3.1.3.4 The following loads and load combinations are considered in Bulletin 2V:

- Uniaxial (longitudinal or transverse) compression;
- In-plane bending;
- In-plane edge shear;
- Uniform lateral pressure;
- Biaxial compression with or without edge shear;
- Uniform lateral pressure and in-plane biaxial loading.

3.1.3.5 If other load types or load combinations are known to be acting on the plate, special consideration will have to be given to their treatment, since they are not covered by the provisions in this Bulletin. This applies in particular to the case of concentrated loads.

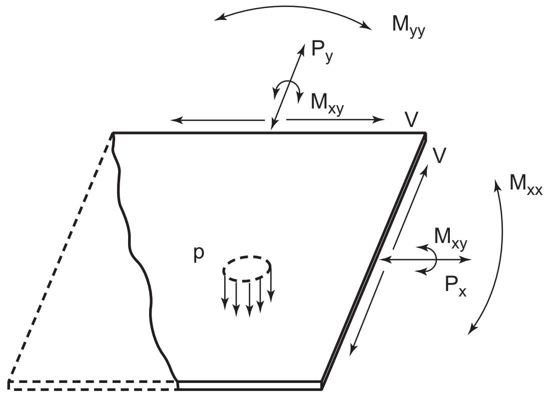
3.1.4 Stress Analysis

3.1.4.1 The stresses in a thin plate can be calculated on the assumption that plane sections remain plane, following the approach adopted in classical thin plate theory.

3.1.4.2 Finite element or other type of numerical analysis can be used in those cases where the applied loads and/or boundary conditions require a more refined treatment, or when the thin plate assumptions are no longer acceptable.

3.1.5 Stress Distributions

3.1.5.1 For an in-plane load P applied uniformly across the plate's edges the corresponding stress is $f = P/A_e$, where A_e. is the edge area. Similarly, for an in-plane shear load V the average shear stress is $f_{xy} = V/A_e$.



P_x, P_y: in-plane tension or compression
M_{xx}, M_{yy}: in-plane bending
M_{xy}: Twisting
V: in-plane shear
p: lateral pressure

Figure 3.1-1—Primary Loads Acting on a Rectangular Plate

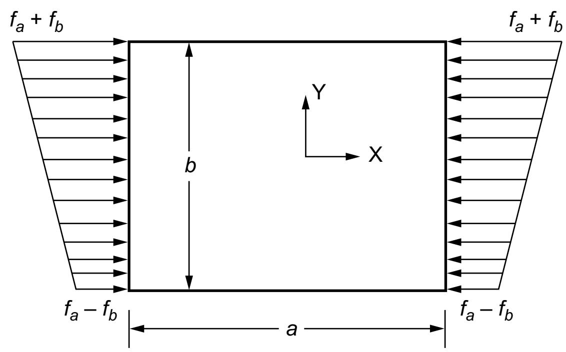


Figure 3.2-1—Long Rectangular Plate

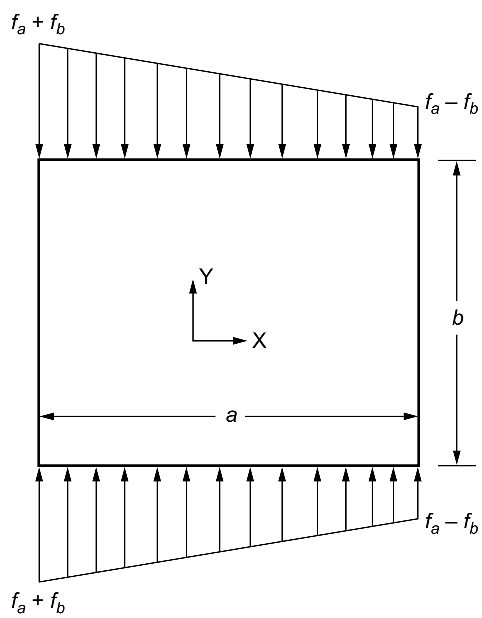


Figure 3.2-2—Wide Rectangular Plate

3.1.5.2 In the case of lateral loads the bending stresses are zero at the mid-surface and vary linearly across the thickness of the plate, with a maximum at the surface given by:

$$f_{x\max} = \frac{6M_x}{t^2} \qquad (3.1\text{-}1)$$

$$f_{y\max} = \frac{6M_y}{t^2} \qquad (3.1\text{-}2)$$

where M_x is the bending moment per unit length for bending about the *y* axis, and M_y is the bending moment per unit length for bending about the *x*-axis. The shear stress resulting from a twisting moment per unit length M_{xy} is also zero at the plate's mid-surface and varies linearly across the thickness, with a maximum at the surface given by:

$$f_{xy\max} = \frac{6M_{xy}}{t^2} \qquad (3.1\text{-}3)$$

3.1.5.3 The shear stresses f_{xz} and f_{yz} can be determined by assuming that they are distributed across the plate thickness according to a parabolic law, as in simple beam theory. Thus the maximum values are:

$$f_{xz\max} = \frac{3}{2}\frac{Q_x}{t} \qquad (3.1\text{-}4)$$

$$f_{yz\max} = \frac{3}{2}\frac{Q_y}{t} \qquad (3.1\text{-}5)$$

where Q_x and Q_y are the transverse shear force per unit length along the edges parallel to the *y* and *x* axis, respectively.

3.2 UNIAXIAL COMPRESSION AND IN-PLANE BENDING

3.2.1 Definitions

Two types of plates are considered. Figure 3.2-1 shows long plates under longitudinal compression stress (f_a) and in-plane bending stress (f_b), where the load is applied to the shorter edges. Figure 3.2-2 shows wide plates, or plates under transverse compression stress (f_a) and in-plane bending stress (f_b), where the load is applied to the larger edges.

The serviceability limit state is reached when the applied in-plane compressive stress, f, equals the appropriate limiting stress. The limit stress is f_{se} when f is in the elastic range, or f_{sp} when f is in the inelastic or plastic range. Specifically, elastic serviceability limit f_{xse} applies for long plates, and f_{yse} applies for wide plates. Likewise, the plastic serviceability limit f_{xsp} applies for long plates, and f_{ysp} for wide plates. The ultimate limit state is reached when *f* equals f_{xu} for long plates, or f_{yu} for wide plates, respectively. The allowable stress is obtained by dividing the limit state stress f_{se}, f_{sp}, or f_u by the appropriate factor of safety F.S. The wide plate formulas should be used for square plates.

3.2.2 Serviceability Limit State

a. **Long Plates** (Figure 3.2-1)

$$f_{xse} = k\frac{\pi^2 E}{12(1-v^2)}\left(\frac{t}{b}\right)^2 \qquad (3.2\text{-}1)$$

$$k = 8.4/(1.1+r),\ 0 \le r \le 1$$
$$= 7.6 - 6.4r + 10r^2, -1 \le r < 0$$

where

$$r = (f_a - f_b)/(f_a + f_b),\ f_b \ge 0$$

The expression above is based on the assumption that the plate edges are simply supported. If other boundary conditions apply the buckling coefficient k can be determined from Figure 3.2-3.

Elastic range ($f_{xse} < F_p$):

$$f_{xsp} = f_{xse} \qquad (3.2\text{-}2)$$

Plastic range ($f_{xse} \ge F_p$):

$$f_{xsp} = \frac{F_y f_{xse}{}^2}{F_p(F_y - F_p) + f_{xse}{}^2} \qquad (3.2\text{-}3)$$

b. **Wide Plates** (Figure 3.2-2)

$$f_{yse} = k\frac{\pi^2 E}{12(1-v^2)}\left(\frac{t}{b}\right)^2 \qquad (3.2\text{-}4)$$

$$k = (1+(b/a)^2)^2\, 2.1/(1.1+r) \quad 0 \le r \le 1$$

$$= (1+(b/a)^2)^2(2.1/1.1)(1+r) + 10r(1+r)(b/a)^2 - 24r(b/a)^2 \qquad -1 \le r < 0 \text{ and } a/b \le 1.5$$

$$= (1+(b/a)^2)^2(2.1/1.1)(1+r) + 10r(1+r)(b/a)^2 - r[2+16(b/a)^2 + 8(b/a)^4] \qquad -1 \le r < 0 \text{ and } a/b > 1.5$$

where

$$r = (f_a - f_b)/(f_a + f_b),\ f_b \ge 0$$

Elastic Range ($f_{yse} < F_p$):

$$f_{ysp} = f_{yse} \qquad (3.2\text{-}5)$$

Plastic Range ($f_{yse} \ge F_p$):

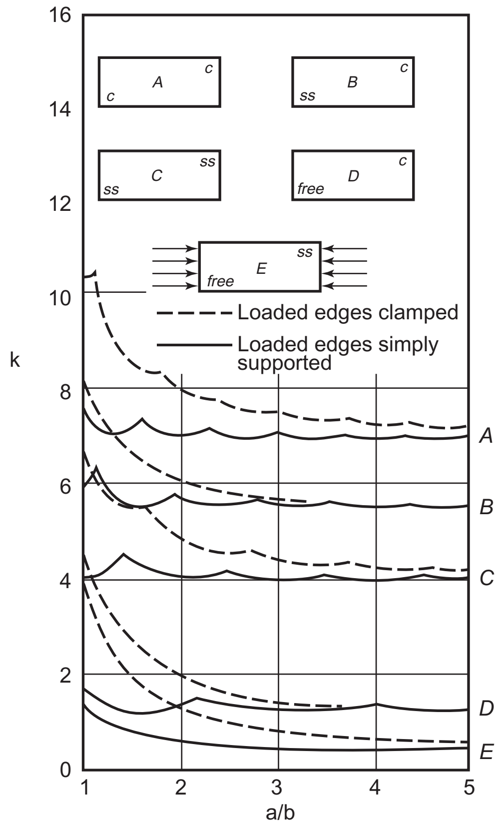


Figure 3.2-3—Buckling Coefficients for Plates in Uniaxial Compression[1]

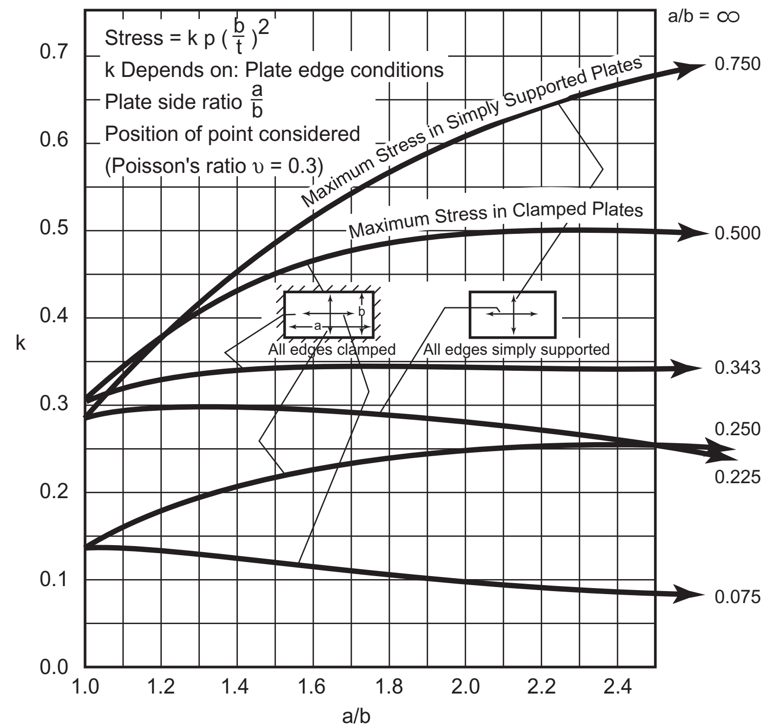


Figure 3.4-2—Stresses in Plates Under Uniform Lateral Pressure[3]

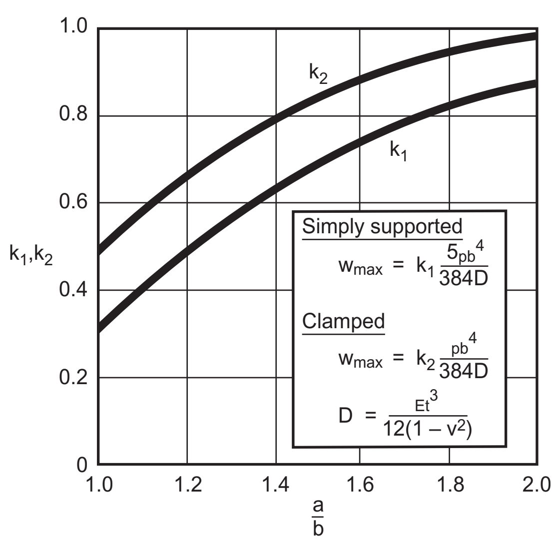


Figure 3.4-1—Coefficients for Computing Plate Deflections[2]

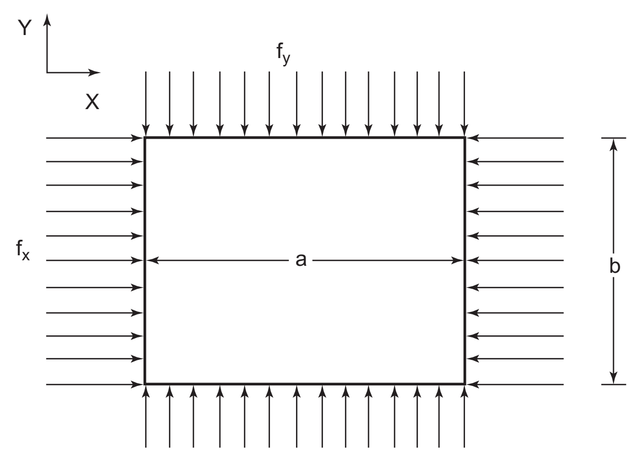


Figure 3.5-1—Rectangular Plate Under Biaxial Compression

[1]From D.O. Brush and B.O. Almroth, "Buckling of Bars, Plates and Shells," McGraw-Hill, 1975.

[2]From O. Hughes, "Ship Structural Design: A Rationally Based Computer-Aided, Optimization Approach," Wiley Interscience, 1983.

[3]From S.P. Timoshenko and S. Woinowski-Krieger, "Theory of Plates and Shells," McGraw-Hill, 1959.

$$f_{ysp} = F_y - \frac{F_p(F_y - F_p)}{f_{yse}} \tag{3.2-6}$$

3.2.3 Ultimate Limit State

a. **Long Plates** (Figure 3.2-1)

$$f_{xu} = F_y\left(\frac{2}{\beta} - \frac{1}{\beta^2}\right), \beta \geq 1 \tag{3.2-7}$$

$$f_{xu} = F_y, \beta < 1 \tag{3.2-8}$$

where

$$\beta = \frac{b}{t}\sqrt{\frac{F_y}{E}}$$

These apply when the plate edge stress reaches yield before the stiffener fails. Otherwise, the following formulas should be used:

$$f_{xu} = F_y \frac{1}{\beta}, \beta \geq 1 \tag{3.2-9}$$

$$f_{xu} = F_y, \beta < 1 \tag{3.2-10}$$

b. **Wide Plates** (Figure 3.2-2)

$$f_{yu} = F_y\left[\frac{1}{\alpha}C + 0.10\left(1 - \frac{1}{\alpha}\right)\left(1 + \frac{1}{\beta^2}\right)^2\right] \leq F_y \tag{3.2-11}$$

$$C = \frac{2}{\beta} - \frac{1}{\beta^2}, \beta \geq 1$$

$$C = 1, \beta < 1$$

where

$$\alpha = a/b \geq 1$$

3.3 EDGE SHEAR

3.3.1 Definitions

The serviceability limit state is reached when the applied edge shear stress f_{xy} equals f_{xyse} or f_{xysp}. The limit f_{xyse} applies in the elastic range, while f_{xysp} applies in the plastic range. The ultimate limit state is reached when f_{xy} equals f_{syu}. The allowable stress is obtained by dividing the limit state stress (f_{xyse}, f_{xysp} or f_{syu}) by the appropriate factor of safety F.S.

3.3.2 Serviceability Limit State

$$f_{xyse} = k\frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2 \qquad (3.3\text{-}1)$$

$$k = 5.34 + \frac{4}{\alpha^2}$$

The result given is based on the assumption that the plate edges are simply supported. If the plate edges can be considered clamped the buckling coefficient *k* takes the form:

$$k = 8.98 + \frac{5.60}{\alpha^2}$$

Elastic range ($f_{xyse} < F_p/\sqrt{3}$):

$$f_{xysp} = f_{xyse} \qquad (3.3\text{-}2)$$

Plastic range ($f_{xyse} \geq F_p/\sqrt{3}$):

$$f_{xysp} = \frac{3\tau_y f_{xyse}^2}{F_p(F_y - F_p) + 3f_{xyse}^2} \qquad (3.3\text{-}3)$$

3.3.3 Ultimate Limit State

$$f_{xyu} = f_{xysp} + \frac{\sqrt{3}}{2\sqrt{1+\alpha+\alpha^2}}\left(\tau_y - f_{xysp}\right) \qquad (3.3\text{-}4)$$

where f_{xysp} is the serviceability limit state shear stress defined in 3.3.2.

where $\tau_y = \frac{1}{\sqrt{3}}(F_y)$

3.4 UNIFORM LATERAL PRESSURE

3.4.1 Definitions

a. Serviceability Limit State. The serviceability limit state is based on a deflection criterion and a stress criterion.

b. Deflection Criterion. The deflection criterion is associated with a maximum allowable deflection W_a. Two cases have to be considered: (1) no permanent plastic deformations allowed, so that W_a is an elastic deflection; (2) permanent plastic deformation or plastic set allowed, so that W_a is a plastic deformation. No specific guidelines can be given on the allowable deflection, and whether it should remain purely elastic or become a permanent plastic set, since it depends on the type of service intended for the structure. In general the deflection should not be such as to adversely affect the structure's appearance or its performance requirements. In those cases where in-plane compressive loads are not present, and where specific operational requirements do not rule against it, a permanent plastic set W_p can be allowed. If as a result of a permanent set membrane effects are induced in the plate its capacity to carry in-plane tensile loads and structural efficiency are improved.

The designer has to use engineering judgment in establishing a maximum allowable deflection, and deciding if a permanent plastic set is acceptable.

If an absolute value cannot be specified, a criterion based on the maximum span and/or thickness can be adopted, such as the maximum of $W_a = C_1$ x (span) and $W_a = C_2$ x (thickness), where C_1 and C_2 are non-dimensional parameters (such as $C_1 = 1/360$ and $C_2 = 1$). If a permanent plastic set is allowed a criterion for determining its magnitude is given in 3.4.2.

Expressions for estimating the maximum elastic deflection in a rectangular plate subjected to uniform lateral pressure are given in 3.4.2.

c. Stress Criterion. The serviceability limit state stress criterion implies that the plate's material must remain in the elastic range, and it is expressed in the form of a yield criterion, defined in 3.4.2. In cases where a permanent plastic set is allowed this stress criterion does not apply.

d. Ultimate Limit State. The ultimate limit state is reached when the lateral pressure equals ρ_u, as defined in 3.4.3.

3.4.2 Serviceability Limit State

a. Deflection Criterion. If no permanent plastic set is allowed a maximum allowable elastic deflection W_a must be selected by the designer, given the particular application being considered (see discussion in 3.4.1). The computed maximum elastic deflection W_a must satisfy:

$$W_e \leq W_a \qquad (3.4\text{-}1)$$

W_e can be estimated from the following expressions:

$$W_e = k_1 \frac{5pb^4}{384D}, \text{ simply supported edges} \qquad (3.4\text{-}2)$$

$$W_e = k_2 \frac{pb^4}{384D}, \text{ clamped edges} \qquad (3.4\text{-}3)$$

where D is the plate's flexural rigidity

$$D = \frac{Et^3}{12(1-v^2)}$$

and the coefficients k_1 and k_2 can be found from the graphs in Figure 3.4-1.

If a permanent plastic set is allowed (again, the designer has to take into consideration all aspects of performance requirements, as discussed in 3.4.1), it should be limited to:

$$W_p \le 0.2b\sqrt{\frac{F_y}{E}} \qquad (3.4\text{-}4)$$

b. Stress Criterion. If no permanent plastic set is allowed the plate's material must remain in the elastic regime, so that the maximum stresses f_x and f_y must satisfy the following relation:

$$f_x^2 + f_y^2 - f_x f_y \le F_p^2 \qquad (3.4\text{-}5)$$

where tensile stresses are taken as positive and compressive stresses as negative.

The maximum stresses f_x and f_y can be estimated from the following expression:

$$f_x \text{ or } f_y = kp\left(\frac{b}{t}\right)^2 \qquad (3.4\text{-}6)$$

where the coefficient k can be found from the graphs in Figure 3.4-2 for simply supported and clamped edge conditions.

If a permanent plastic set is allowed the stress criterion is not applicable.

3.4.3 Ultimate Limit State

$$p_u = F_y\left(\frac{t}{b}\right)^2 \frac{6}{\sqrt{\alpha}}\left(1+\frac{2}{\alpha}\frac{W_p}{t}\right) \qquad (3.4\text{-}7)$$

where W_p is the permanent set (see 3.4.2). If no permanent set is allowed $W_p = 0$. These formulas are restricted to plates with aspect ratio $1 \le \alpha \le 5$. The allowable pressure is obtained by dividing the limit state pressure p_u by the appropriate factor of safety F.S.

3.5 BIAXIAL COMPRESSION WITH OR WITHOUT EDGE SHEAR

3.5.1 Definitions

The limit state (serviceability or ultimate) is reached if the combination of the applied compressive stresses due to axial compression only, in the x and y directions, or f_x and f_y respectively, Figure 3.5-1, and the edge shear stress f_{xy} are equal to the limit state stresses, f_{xl}, f_{yl} and f_{xyl}, respectively, that satisfy the interaction formulas defined in 3.5.2 and 3.5.3.

3.5.2 Serviceability Limit State

Elastic range:

$$\left[\left(f_{xl}/f_{xse}\right)^c + \left(f_{yl}/f_{yse}\right)^c + \left(f_{xyl}/f_{xyse}\right)^c\right]^{(2/c)} + \left(f_e/F_y\right)^2 = 1.0 \qquad (3.5\text{-}1)$$

where

c = $2 - 1/\alpha, \alpha \ge 1.0$

f_e = limit state von Mises stress

= $\left(f_{xl}^2 + f_{yl}^2 - f_{xl}f_{yl} + 3f_{xyl}^2\right)^{1/2}$

and f_{xse} is given in 3.2.2a considering axial compression only, f_{yse} is given in 3.2.2b considering axial compression only, and f_{xyse} is given in 3.3.2.

The allowable stresses are obtained by dividing these limit state stresses, f_{xl} , f_{yl} and f_{xyl} , by the appropriate factor of safety F.S.

3.5.3 Ultimate Limit State

$$\left(f_{xl} / f_{xu}\right)^A - \bar{\eta}\left(f_{xl} / f_{xu}\right)\left(f_{yl} / f_{yu}\right) + \left(f_{yl} / f_{yu}\right)^2 + \left(f_{xyl} / f_{xyu}\right)^2 = 1 \qquad (3.5\text{-}2)$$

where f_{xu} is given in Par. 3.2.3a, f_{yu} is given in Par. 3.2.3b, f_{xyu} is given in Par. 3.3.3 and
A = 1, $\bar{\eta}$ = 0.25, for $\alpha \geq 3$
A = 2, $\bar{\eta} = 3.2e^{-0.35\beta} - 2$, for $\alpha = 1$

For 1< α < 3 and for a given value of the ratio f_{yl} / f_{yu}, the corresponding values of f_{xl} / f_{xu} and f_{xyl} / f_{xyu} can be found by linear interpolation between the values of A and $\bar{\eta}$ obtained for α = 3 and for α = 1.

The allowable stresses are obtained by dividing these limit state stresses, f_{xl}, f_{yl}, and f_{xyl}, by the appropriate factor of safety F.S.

3.6 COMBINED IN-PLANE AND LATERAL LOADS

3.6.1 Definitions

The serviceability or ultimate limit state is reached if the combination of applied axial stresses in the *x* and y directions, or f_x and f_y, respectively, edge shear stress f_{xy}, and pressure *p*, satisfy the interaction formulas defined in 3.6.2 and 3.6.3.

3.6.2 Serviceability Limit State

The serviceability limit state shall be checked if a permanent set is not allowed.

- f_x compression, f_y compression:

$$\left(f_x / f_{xsp}\right)^2 + \left(f_y / f_{ysp}\right)^2 + \left(f_{xy} / f_{xysp}\right)^2 + p / p_s = 1 \qquad (3.6\text{-}1)$$

where *p* = applied pressure, p_s = collapse pressure calculated assuming zero permanent plastic set and f_{xsp}, f_{ysp}, and f_{xysp} are the serviceability limit state stresses defined in 3.2.2 and 3.3.2.

- f_x tension, f_y compression:

$$\left(f_x / F_y\right)^2 + \left(f_y / f_{ysp}\right)^2 + \left(f_{xy} / f_{xysp}\right)^2 + p / p_s = 1 \qquad (3.6\text{-}2)$$

- f_x tension, f_y,tension:

$$-\left[\left(f_x / F_y\right)^2 + \left(f_y / F_y\right)^2\right]^{1/2} + f_{xy} / f_{xysp} = 1 \qquad (3.6\text{-}3)$$

- f_x compression, f_y tension:

$$\left(f_x / f_{xsp}\right)^2 + \left(f_{xy} / f_{xysp}\right)^2 + p / p_s = 1 \qquad (3.6\text{-}4)$$

A von Mises based yield criterion is also applied in all quadrants but does not control for compression-compression:

$$\left(f_x / f_{xcr}\right)^2 - \left(f_x / f_{xcr}\right)\left(f_y / f_{ycr}\right) + \left(f_y / f_{ycr}\right)^2 + 3\left(f_{xy} / f_{xycr}\right)^2 = 1 \qquad (3.6\text{-}5)$$

3.6.3 Ultimate Limit State:

- f_x compression, f_y compression:

$$\left[\left(f_x / \sigma'_{xu}\right)^2 + \left(f_y / \sigma'_{yu}\right)^2\right]^{1/2} + \left(f_{xy} / f_{xyu}\right)^2 = 1 \qquad (3.6\text{-}6)$$

In this case, σ'_{xu} and σ'_{yu} are reduced from the API Bulletin 2V values due to the presence of lateral pressure:

$$\sigma'_u / F_y = \left(f_u / F_y\right)^{\left(0.8Q^2 + 0.84Q + 1\right)}$$

where $Q = pE / F_y{}^2$, p = applied pressure.

- f_x tension, f_y compression:

$$\left(f_x / F_y\right)^2 + \left(f_y / f_{yu}\right)^2 + \left(f_{xy} / f_{xyu}\right)^2 + p / p_u = 1 \qquad (3.6\text{-}7)$$

where p = applied pressure, p_u = ultimate pressure under pressure loading only.

- f_x tension, f_y tension:

$$-\left[\left(f_x / F_y\right)^2 + \left(f_y / F_y\right)^2\right]^{1/2} + f_{xy} / f_{xyu} = 1 \qquad (3.6\text{-}8)$$

- f_x compression, f_y tension:

$$\left(f_x / f_{xu}\right)^2 + \left(f_{xy} / f_{xyu}\right)^2 + p / p_u = 1 \qquad (3.6\text{-}9)$$

A von Mises based yield criterion is also applied in all quadrants but does not control for compression-compression:

$$\left(f_x / \sigma'_x\right)^2 - \left(f_x / \sigma'_x\right)\left(f_y / \sigma'_y\right) + \left(f_y / \sigma'_y\right)^2 + 3\left(f_{xy} / f_{xyu}\right)^2 = 1$$

where:

$$\sigma'_x = F_y\left(1 - \left(Q / Q_u\right)\bar{\lambda}\right)^{1/2}$$

$$\sigma'_y = F_y\left(1 - \left(Q / Q_u\right)\bar{\lambda}\lambda\right)^{1/2}$$

$\lambda = f_{yb} / f_{xb}$, bending stress ratio

$\bar{\lambda} = \left(1 - \lambda + \lambda^2\right)^{-1/2}$

$Q = pE / F_y^2$

$Q_u = p_u E / F_y^2$

3.7 DESIGN CONSIDERATIONS

When a thin rectangular plate of a given material is subjected to compressive stresses it can fail by instability, and the strength depends primarily on the type of loads and/or load combinations, the boundary conditions and the geometry (dimensions, aspect ratio).

The plate is in general part of a stiffened panel, such as in a deck or bulkhead, and it is supported by stiffeners. The stiffener spacing should be selected so as to limit the plate geometry and aspect ratio to dimensions and proportions that can provide the necessary strength. The designer must change the plate proportions and thickness until all applicable limit states are satisfied. If necessary, additional stiffeners might have to be introduced in the design. The minimum stiffener spacing should be based on fabrication considerations.

When the plate is primarily subjected to lateral loading, the tensile membrane effects substantially improve its carrying capacity. In designing the supports, full in-plane fixity should be provided whenever possible in order to take advantage of membrane effects.

Section 4-Stiffeners

4.1 GENERAL

4.1.1 Scope

Straight and slender thin-walled members of uniform cross section containing at least one plane of symmetry and thin-walled angles that serve as stiffening elements for flat plate structures are considered.

4.1.2 Definitions

Stiffeners are used to strengthen plates and to increase their load carrying capacity. In most cases they are made of a thin-walled web welded to the plate and a flange. Thus, when determining the cross sectional properties, account should be given to the attached plating acting with the stiffener as an effective flange. When the stiffeners are subjected to axial compressive loads the effective plate flange width b_e, when the maximum edge stress reaches the yield stress, is

$$b_e = b\frac{f_u}{F_y} \qquad (4.1\text{-}1)$$

where f_u (f_{xu} or f_{yu}) is determined from 3.2.3. When the stiffeners are subjected to lateral or tensile loading alone, the effective plate flange is governed by shear lag effects and should be determined from 5.6.

The following ultimate limit states will be considered:

- column buckling;
- beam-column buckling;
- torsional/flexural buckling;
- plastic bending.

4.1.3 Loads and Load Combinations

A plate stiffener can be subjected to a variety of primary and secondary loads and load combinations.

The following loads can be classified as primary loads:

- axial tension or compression;
- bending about the axis of maximum moment of inertia;
- bending about the axis of minimum moment of inertia;
- lateral distributed load;
- lateral concentrated loads.

In addition to these primary loads the plate can also be subjected to secondary loads as follows:

- shrinkage forces due to welding;
- stresses due to construction tolerances;
- loads due to thermal effects.

The following loads and load combinations are considered here:

- axial compression;
- axial compression and lateral load;
- lateral load.

If other load types or load combinations are known to be acting on the plate special consideration will have to be given to their treatment, since they are not covered by the provisions of this Bulletin.

4.1.4 Stress Analysis

The stresses in a slender thin-walled stiffener can be calculated on the assumption that plane sections remain plane, following the approach adopted in classical beam theory.

Finite element or other type of numerical analysis can be used in those cases where the applied loads and/or boundary conditions require a more refined treatment, or when the classical beam theory assumptions are no longer acceptable.

4.1.5 Stress Distributions

As a result of conventional beam theory, the longitudinal bending stresses in a stiffener vary linearly across the depth. If the section is subjected to both compression and bending, the stress distribution is given by:

$$f = \frac{P}{A} + \frac{M\bar{y}}{I_{ef}} \tag{4.1-2}$$

where P is the compressive load, A is the cross sectional area, M is the applied bending moment, $\bar{y}$ is the distance to the neutral axis and I_{ef} is the effective moment of inertia about the neutral axis. In computing I_{ef} the effective flange should be used, as prescribed in 4.1.2. If the stiffener is subjected to lateral or tensile load alone, the effective flange is governed by shear lag effects and should be determined from 5.6.

The shear stress distribution can be obtained from:

$$f_{xy} = \frac{VQ}{It} \tag{4.1-3}$$

where V is the shear force, Q is the moment of the area above the point where shear stress is being determined about the neutral axis, I is the moment of inertia about the neutral axis, and t is the thickness at the point under investigation.

For webs with constant thickness the shear stress can be approximated by:

$$f_{xy} = \frac{V}{A_w} \tag{4.1-4}$$

where A_w is the web area.

4.2 COLUMN BUCKLING

4.2.1 Definitions

The ultimate limit state is reached when the applied axial load P equals P_{Ee} or P_{Ep}. The limit P_{Ee} applies in the elastic range, while P_{Ep} applies in the plastic range. The allowable axial load is obtained by dividing the limit state axial load (P_{Ee} or P_{Ep}) by the appropriate factor of safety F.S.

4.2.2 Ultimate Limit State

$$P = \frac{P_y}{\lambda^2}$$

where $\lambda = \frac{1}{\pi}\left(\frac{KL}{r}\right)\sqrt{\frac{F_y}{E}}$ (4.2-1)

Elastic range ($P < p_r P_y$):

$$P_{Ee} = P \tag{4.2-2}$$

Plastic range ($P \geq p_r P_y$):

$$P_{Ep} = P_y\left[1 - \frac{p_r(1-p_r)}{P/P_y}\right]$$

where $\mathrm{p}_r = \frac{F_p}{F_y}$; $\mathrm{P}_y = AF_y$ (4.2-3)

4.3 BEAM-COLUMN BUCKLING

4.3.1 Definitions

The ultimate limit state is reached when the applied axial load P and bending moment M satisfy the interaction curve specified in 4.3.2.

4.3.2 Ultimate Limit State

$$\frac{P}{P_u} + B_1\frac{M}{M_u} = 1 \tag{4.3-1}$$

where P_u is equal to P_{Ee} or P_{Ep}, as given in 4.2.2, depending on whether the material is in the elastic or inelastic range, respectively, and M_u = M_{fy}, and where the amplification reduction factor C_m is defined in AISC.

$$B_1 = \frac{C_m}{1 - P/P_{Ee}} \geq 1.0$$

The allowable axial load and bending moment are obtained by dividing the limit state axial load *P* and bending moment *M* by the appropriate factor of safety F.S.

4.4 TORSIONAL/FLEXURAL BUCKLING

4.4.1 Definitions

The following properties of a cross section are particularly related to stiffener torsional/flexural buckling, as well as lateral buckling: the location of the shear center, the torsion constant J, and the warping constant C_w. Expressions for determining the value of these parameters for a number of thin-walled open cross sections are listed in Table 4.4-1.

Two cases have to be considered when dealing with stiffener torsional/flexural buckling. If the stiffener shear center and centroid coincide (as happens with doubly symmetric sections such as equal flanged I-sections), buckling by twisting, with the longitudinal axis through the centroid remaining straight, can occur. In such cases twisting and flexure are decoupled, and the ultimate limit state discussed here is determined by torsional buckling only. If the shear center and the centroid do not coincide (as happens with sections containing only one plane of symmetry such as unequal flanged I-sections), the ultimate limit state is governed by a combination of twisting and bending, since these two actions cannot be decoupled.

In the case of doubly symmetric sections the limit state is reached when the applied axial compressive load *P* equals P_{Te} or P_{T_p}. The limit P_{Te} applies in the elastic range, while P_{T_p} applies in the plastic range.

In the case of sections containing only one plane of symmetry the limit state is reached when the applied axial compressive load equals P_{TFe} or P_{TF_p}, which correspond to the elastic and inelastic ranges, respectively. The allowable axial compressive load is obtained by dividing the limit state load (P_{Te}, P_{T_p}, P_{TFe}, or P_{TFp}) by the appropriate factor of safety F.S.

Sections containing no plane of symmetry, such as angle stiffeners, shall meet the compact section criteria of 4.4.4a.

4.4.2 Ultimate Limit State for Doubly Symmetric Sections

$$P_T = \frac{A}{I_s}\left(GJ + \frac{\pi^2 E C_w}{L^2}\right) \qquad (4.4\text{-}1)$$

where I_s is the polar moment of inertia about shear center

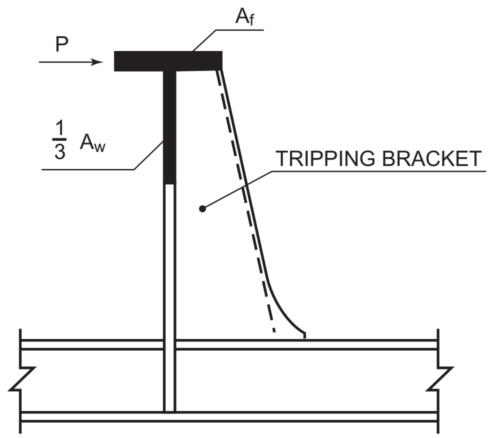
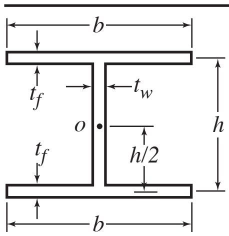
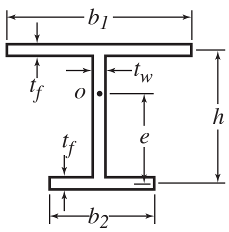
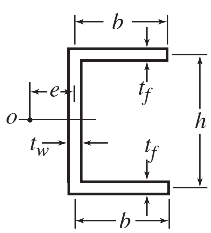
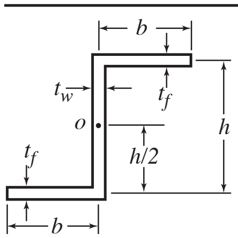
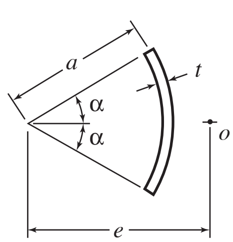


Figure 4.4-1—Design Lateral Load for Tripping Bracket

Table 4.4-1—Properties of Thin-Walled Open Cross Sections[1]

0 = shear center	J = torsion constant	Cw = warping constant
	$J = \frac{2bt_f^3 + ht_w^3}{3}$ $C_w = \frac{t_f h^2 b^3}{24}$	If $t_f = t_w = t$: $J = \frac{t^3}{3}(2b + h)$
	$e = h \frac{b_1^3}{b_1^3 + b_2^3}$ $J = \frac{(b_1 + b_2)\, t_f^3 + ht_w^3}{3}$ $C_w = \frac{t_f h^2}{12} \frac{b_1^3 b_2^3}{b_1^3 + b_2^3}$	If $t_f = t_w = t$: $J = \frac{t^3}{3}(b_1 + b_2 + h)$
	$e = \frac{3b^2 t_f}{6bt_f + ht_w}$ $J = \frac{2bt_f^3 + ht_w^3}{3}$ $C_w = \frac{t_f b^3 h^2}{12} \frac{3bt_f + 2ht_w}{6bt_f + ht_w}$	If $t_f = t_w = t$: $e = \frac{3b^2}{6b + h}$ $J = \frac{t^3}{3}(2b + h)$ $C_w = \frac{tb^3h^2}{12} \frac{3b + 2h}{6b + h}$
	$J = \frac{2bt_f^3 + ht_w^3}{3}$ $C_w = \frac{b^3h^2}{12\,(2b + h)^2}$ X $[2t_f(b^2 + bh + h^2) + 3t_w bh]$	If $t_f = t_w = t$: $J = \frac{t^3}{3}(2b + h)$ $C_w = \frac{tb^3h^2}{12} \frac{b + 2h}{2b + h}$
	$e = 2\alpha \frac{\sin\alpha - \alpha\cos\alpha}{\alpha - \sin\alpha\ \cos\alpha}$ $J = \frac{2a\alpha t^2}{3}$ $C_w = \frac{2ta^5}{3}$ X $\left[\alpha^3 - \frac{6\,(\sin\alpha - \alpha\cos\alpha)^2}{\alpha - \sin\alpha\ \cos\alpha}\right]$	If $2\alpha = \pi$: $e = \frac{4a}{\pi}$ $J = \frac{\pi\alpha t^3}{3}$ $C_w = \frac{2ta^5}{3}\left(\frac{\pi^3}{8} - \frac{12}{\pi}\right)$ $= 0.0374ta^5$

[1]From S.P. Timoshenko, "Theory of Elastic Stability," McGraw-Hill, 1961

Elastic range ($P_T < AF_p$):

$$P_{Te} = P_T \qquad (4.4\text{-}2)$$

Plastic range ($P_T \geq AF_p$):

$$P_{Tp} = P_y \left[1 - \frac{p_r(1 - p_r)}{P_T / P_y} \right] \qquad (4.4\text{-}3)$$

4.4.3 Ultimate Limit State for Sections With a Single Plane of Symmetry

P_{TF} is the smallest root of the following quadratic equation:

$$\frac{I_c}{I_s} P_{TF}^2 - P_{TF}\left(P_{Te} + P_{Ee}\right) + P_{Ee} P_{Te} = 0 \qquad (4.4\text{-}4)$$

where P_{Ee} is the buckling load for buckling normal to the plane of symmetry, as given in 4.2.2, and P_{Te} is the torsion buckling load given in 4.4.2.

Elastic range ($P_{TF} < AF_p$):

$$P_{TFe} = P_{TF} \qquad (4.4\text{-}5)$$

Plastic range ($P_{TF} \geq AF_p$):

$$P_{TFp} = p_y \left[1 - \frac{p_r(1 - p_r)}{P_{TF} / P_y} \right] \qquad (4.4\text{-}6)$$

4.4.4 Stiffener Proportions

In order to avoid the possibility of torsional/flexural buckling, the stiffener proportions should satisfy certain requirements, depending on whether the section is compact or non-compact. If the section is compact and homogeneous, local buckling will not occur before the full plastic moment is reached.

a. Compact Sections

1. The compression flange must be continuously connected to the beam web.
2. The width/thickness ratio of unstiffened elements of the compression flange must satisfy the following requirement:

$$\frac{b_f}{t_f} \leq 0.75 \sqrt{\frac{E}{F_y}} \qquad (4.4\text{-}7)$$

3. The width/thickness ratio of stiffened elements of the compression flange must satisfy the following requirement:

$$\frac{b_f}{t_f} \leq 1.10\sqrt{\frac{E}{F_y}} \quad (4.4\text{-}8)$$

4. The web depth/thickness ratio must satisfy the following requirements:

$$\frac{d}{t_w} \leq 3.70\sqrt{\frac{E}{F_y}}\left(1 - 3.74\frac{P}{P_y}\right), \frac{P}{P_y} \leq 0.16 \quad (4.4\text{-}9)$$

$$\frac{d}{t_w} \leq 1.48\sqrt{\frac{E}{F_y}}, \frac{P}{P_y} > 0.16 \quad (4.4\text{-}10)$$

where P is the computed axial load.

5. The laterally unsupported length of the compression flange of members other than circular or box members shall not exceed either of the following two distances:

$$L_1 = 0.44 b_f \sqrt{\frac{E}{F_y}} \quad (4.4\text{-}11)$$

$$L_2 = 1.05\frac{b_f t_f}{d}\frac{E}{F_y} \quad (4.4\text{-}12)$$

The compression flange must be adequately braced if the unsupported length does not meet the above criteria. The bracing distance L_b is the lesser of the two distances L_1 and L_2.

b. Non-Compact Sections.

Unstiffened elements subject to axial compression due to bending shall be considered as fully effective when the ratio of width to thickness is not greater than:

$$\frac{\frac{1}{2}b_f}{t_f} \leq 0.56\sqrt{\frac{E}{F_y}} \quad (4.4\text{-}13)$$

Stiffened elements subject to axial compression or to uniform compression due to bending shall be considered as fully effective when the ratio of width to thickness is not greater than:

$$\frac{b_f}{t_f} \leq 1.49\sqrt{\frac{E}{F_y}} \quad (4.4\text{-}14)$$

4.4.5 Tripping Brackets

The possibility of overall stiffener tripping can be minimized by means of tripping brackets. The spacing s between tripping brackets should not exceed:

$$\frac{s}{b_f} = 0.44\sqrt{\frac{E}{F_y}} \tag{4.4-15}$$

The design lateral load on the flange for tripping bracket sizing can be taken as the compressive stress in the flange, *f*, multiplied by 2% of the combined area of the flange plus one-third of the web area, see Figure 4.4-1 or:

$$P = 0.02\left(A_f + \frac{A_w}{3}\right)f \tag{4.4-16}$$

4.5 PLASTIC BENDING

4.5.1 Definitions

For a stiffener subjected to a uniform distributed lateral load, or line load q, the plastic limit state in bending is reached when q equals q_u.

When the lateral load on the stiffener is the result of a uniform pressure p acting on a plate stiffened by two orthogonal sets of stiffeners with uniform spacings a and b for each set, with $a > b$ the line load on each stiffener component can be found from

$$q_a = pb\left(1 - \frac{b}{2a}\right) \tag{4.5-1}$$

$$q_b = \frac{pb}{2} \tag{4.5-2}$$

4.5.2 Ultimate Limit State

$$q_u = \frac{16M'}{L^2} \tag{4.5-3}$$

where $M' = M_o$ when the stiffener is not subjected to axial load. When the stiffener is subjected to both bending and axial tension or compression P, M' is determined from:

$$\left(\frac{M'}{M_o}\right) + \left(\frac{P}{P_y}\right)^2 \frac{1}{2\frac{A_w}{A} - \left(\frac{A_w}{A}\right)^2} = 1, \frac{P}{P_y} \le \frac{A_w}{A} \tag{4.5-4}$$

$$\left(\frac{M'}{M_o}\right) + \left(1 - \frac{A_w}{2A}\right) + \frac{P}{P_y} = 1, \frac{P}{P_y} \ge \frac{A_w}{A} \tag{4.5-5}$$

The allowable axial load and bending moment are obtained by dividing the limit state axial load P and bending moment M' by the appropriate factor of safety F.S.

4.5.3 Stiffener Proportions

The ultimate limit state in 4.5.2 can be used as a basis for design if the stiffener proportions satisfy the following compactness requirements:

$$\frac{b_f}{t_f} < 0.56\sqrt{\frac{E}{F_y}} \qquad (4.5\text{-}6)$$

$$\frac{d}{t_w} < 2.38\sqrt{\frac{E}{F_y}}\left(1-1.4\frac{P}{P_y}\right), \frac{P}{P_y} \leq 0.27 \qquad (4.5\text{-}7)$$

$$\frac{d}{t_w} < 1.48\sqrt{\frac{E}{F_y}}, \frac{P}{P_y} > 0.27 \qquad (4.5\text{-}8)$$

4.6 DESIGN CONSIDERATIONS

4.6.1 Stiffener proportions satisfying the requirements in 4.4.4 should be selected. In case the design is based on plastic methods the proportions in 4.5.3 should govern the design. Cross section dimensions satisfying the proportion requirements should then be chosen to meet the required section modulus (or plastic modulus).

4.6.2 Normally the stiffener length is determined by functional requirements (such as main dimensions of a stiffened panel). Thus the main variables that can be selected by the designer are the cross sectional dimensions. These will have to be refined through several iterations until all the applicable limit states are satisfied. There are obviously many cross sections that can meet a given set of strength requirements, and the designer must make a selection that will contribute to the overall structure’s weight and cost efficiency.

Section 5-Stiffened Panels

5.1 GENERAL

5.1.1 Scope

Flat stiffened panels, comprising one or two sets of equally spaced uniform stiffeners of equal cross section, supporting a thin plate, are considered (see Figure 5.1-1). If there are two sets, they intersect each other at right angles.

5.1.2 Definitions

5.1.2.1 If there is only one set of stiffening elements the panel is uniaxially stiffened, while if there are two the panel is orthogonally stiffened. All the stiffeners in each set are slender, straight, and of uniform cross section, and they all have the same cross sectional dimensions. The entire panel length is A, and the entire panel width is B.

5.1.2.2 The ultimate limit state is defined for the case of uniaxially stiffened panels under end compression, and orthogonally stiffened panels under uniaxial compression, biaxial compression and uniform lateral load. The serviceability limit state is also defined for orthogonally stiffened panels under uniform lateral load. Requirements for avoiding stiffener local instabilities and stiffener tripping are included in 5.4 and 5.5. Design charts for determining the effective flange breadth are given in 5.6. The minimum stiffener inertia required for panels to reach their ultimate shear strength is given in 5.7. Requirements for avoiding the interaction of buckling modes in stiffened panels are included in 5.8.

5.1.2.3 In determining the cross sectional properties of stiffeners account should be taken of the attached plating acting with the stiffener as an effective flange, as defined in 4.1 and 5.6.

5.1.3 Loads and Load Combinations

5.1.3.1 A stiffened panel can be subjected to a variety of primary and secondary loads and load combinations. These can be classified in the same basic categories adopted for a thin rectangular plate, see 3.1.3.

5.1.3.2 The following loads and load combinations are considered in Bulletin 2V:

a. Uniaxially stiffened panels under end compression;
b. Orthogonally stiffened panels under uniaxial and biaxial compression, and uniform lateral pressure.

5.1.3.3 If other load types or load combinations are known to be acting on the plate, special consideration should be given to their treatment, since they are not covered by the provisions in this Bulletin.

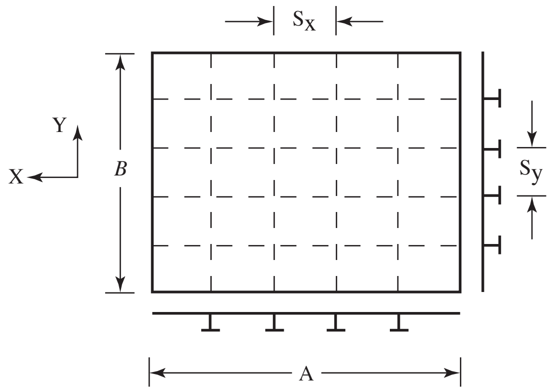


Figure 5.1-1—Flat Stiffened Panel

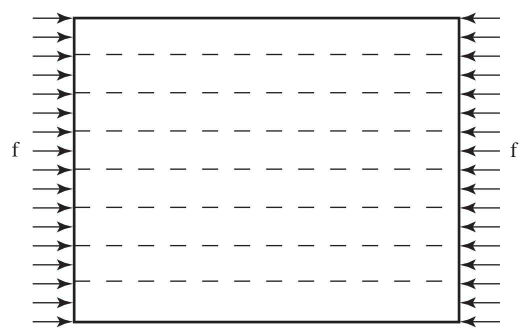


Figure 5.2-1—Uniaxially Stiffened Panel in End Compression

5.1.4 Stress Analysis

5.1.4.1 The stresses in a stiffened panel can be calculated on the assumption that plane sections remain plane. Individual stiffeners with attached effective breadth or width of plating can be analyzed on the basis of the principles established for stiffeners in 4.1.2 and 5.6. Single thin rectangular plates supported by stiffeners can be analyzed on the basis of thin plate theory, as indicated in 3.1.4.

5.1.4.2 A more refined approach to stiffened panel analysis, where the orthotropic nature of the structure is retained, can be provided by thin orthotropic plate theory. Grillage analysis can also be used. Neither one of these two methods is in general conducive to simple hand calculations, and in those cases where the applied loads and/or boundary conditions require a more refined treatment, numerical methods, such as the finite element method, might be preferred.

5.1.5 Stress Distributions

The stress distributions across the stiffener and thin rectangular plate cross sections can be derived on the basis of the same general methods proposed in 4.1.5 and 3.1.5, respectively.

5.2 UNIAXIALLY STIFFENED PANELS IN END COMPRESSION

5.2.1 Definitions

A uniaxially stiffened panel subjected to an applied in-plane compressive stress acting in the same direction as the stiffeners is considered here, see Figure 5.2-1. The ultimate limit state is reached when the applied in-plane compressive stress f equals f_u, as defined in 5.2.2. The allowable in-plane compressive stress is obtained by dividing the limit state stress f_u by the appropriate factor of safety F.S.

5.2.2 Ultimate Limit State

$$f_u = F_y, \bar{\lambda} \le 0.5 \qquad (5.2\text{-}1)$$

$$f_u = F_y\left(1.5 - \bar{\lambda}\right), 0.5 \le \bar{\lambda} \le 1.0 \qquad (5.2\text{-}2)$$

$$f_u = F_y\left(\frac{0.5}{\bar{\lambda}}\right), \bar{\lambda} > 1.0 \qquad (5.2\text{-}3)$$

where $\bar{\lambda} = \left(\frac{B}{t}\right)\frac{1}{\pi}\sqrt{\frac{F_y}{E}\frac{12(1-\nu^2)}{k}}$

$$k = \min.\left(k_R, k_F\right)$$

$$k_R = 4n^2$$

where n = number of sub-panels (individual plates)

$$k_F = \frac{\left(1+\alpha^2\right)^2 + n\gamma}{\alpha^2\left(1+n\delta\right)}, \quad \alpha \le \left(1+n\gamma\right)^{1/4}$$

where $\delta = \frac{A_s}{Bt}$

$$k_F = \frac{2\left(1+\sqrt{1+n\gamma}\right)}{1+n\gamma}, \alpha \geq (1+n\gamma)^{1/4}$$

where $\gamma = \frac{12(1-\nu^2)}{t^3}\left(\frac{I_s}{d}\right)$,

$\alpha =$ aspect ratio of whole panel,
$I_s =$ moment of inertia of one stiffener about the axis parallel to the plate surface at the base of the stiffener,
$t =$ plate thickness,
$d =$ spacing between stiffeners

5.3 ORTHOGONALLY STIFFENED PANELS

5.3.1 Definitions

5.3.1.1 Limit states for the entire stiffened panel including both longitudinal and transverse stiffeners are considered.

5.3.1.2 The serviceability limit state for a panel subjected to uniaxial compression is reached when the axial stress f reaches the value f_{se} or f_{sp} defined in 5.3.2. The limit f_{se} applies in the elastic range, while the limit f_{sp} applies in the plastic range. The serviceability limit state for a panel subjected to biaxial compression is reached when the equations in 5.3.3 are satisfied. The case of lateral pressure is defined in 5.3.4.

5.3.2 Uniaxial Compression

$$f_{se} = K\frac{\pi^2\left(D_x D_y\right)^{1/2}}{t_x B^2} \qquad (5.3\text{-}1)$$

where t_x = equivalent thickness of the plates and stiffeners (diffused), extending in the x direction.

For $A/B \geq 1: K = 4.0$

For $A/B < 1: K = \left(\frac{1}{\rho^2} + 2\eta + \rho^2\right)$

$$D_x = \frac{EI_x}{s_y\left(1-\nu^2\right)} \quad , \quad D_y = \frac{EI_y}{s_x\left(1-\nu^2\right)}$$

$$\rho = \frac{A}{B}\left(\frac{D_y}{D_x}\right)^{1/4} \quad , \quad \eta = \left(\frac{I_{px}}{I_x}\frac{I_{py}}{I_y}\right)^{1/2}$$

Elastic range ($f_{se} < F_p$):

$$f_{sp} = f_{se} \qquad (5.3\text{-}2)$$

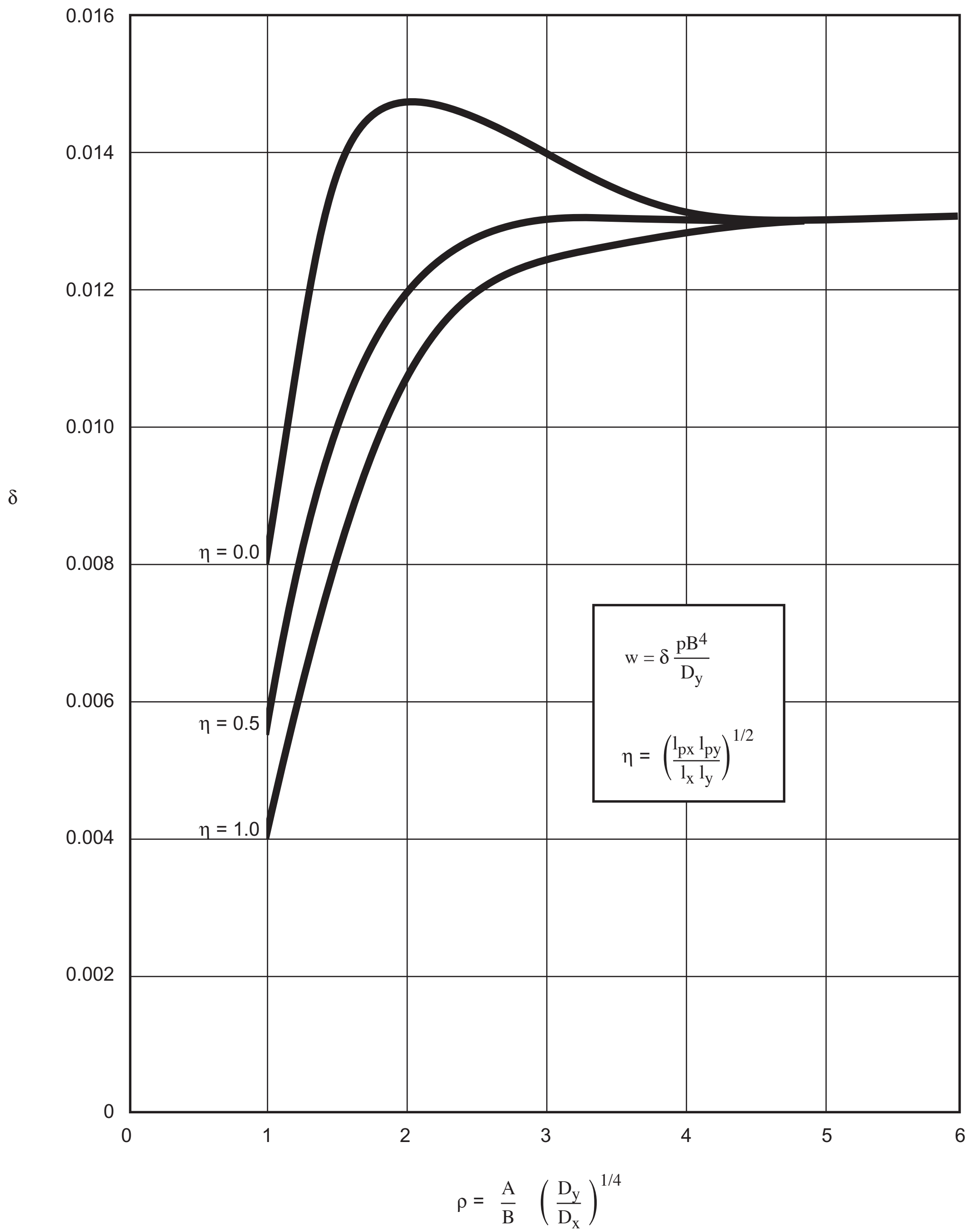


Figure 5.3-1—Deflection Coefficient for Orthogonally Stiffened Panels

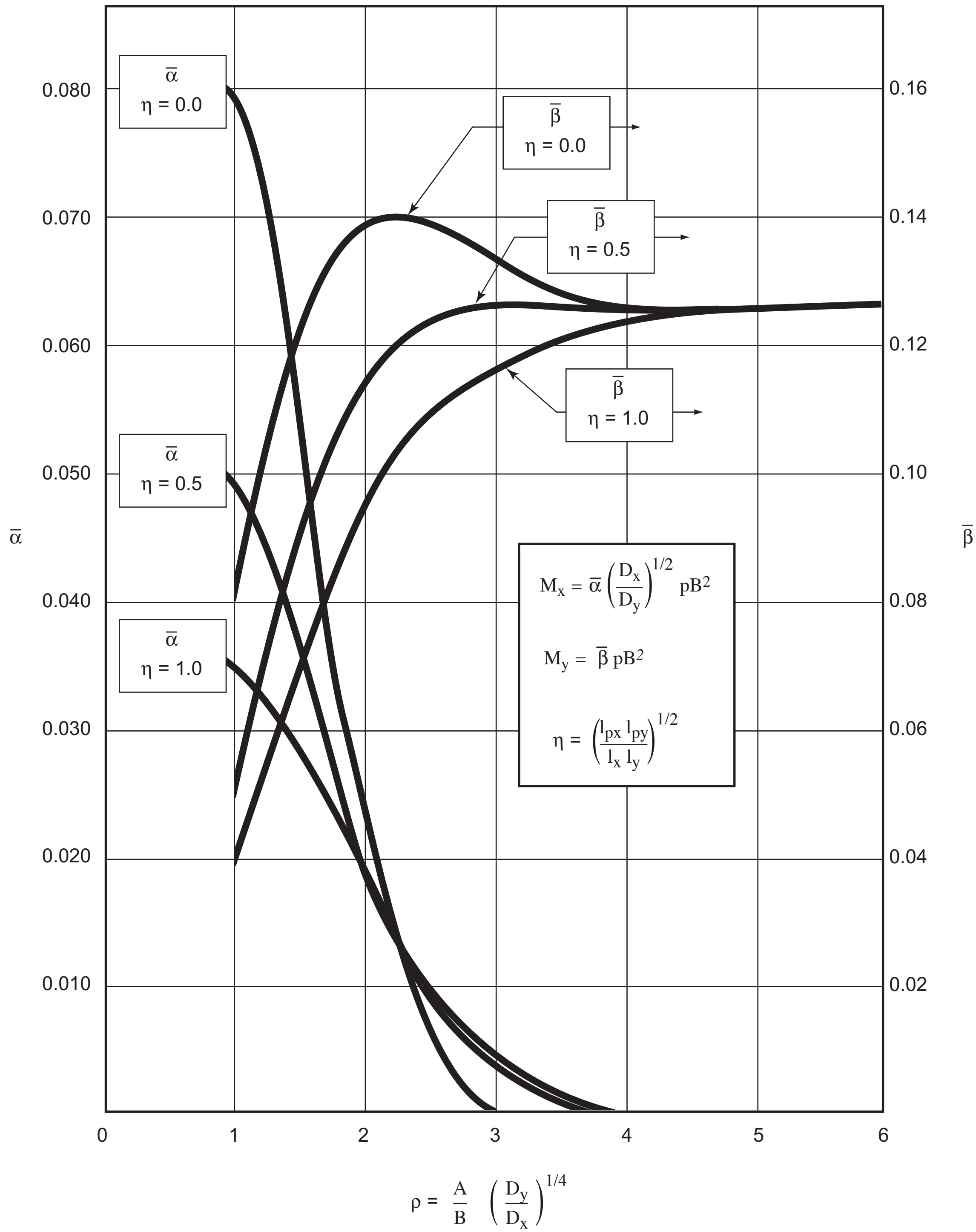


Figure 5.3-2—Coefficients for Computing Stresses for Orthogonally Stiffened Panels

Plastic range ($f_{se} \geq F_p$):

$$f_{sp} = \frac{C_e F_y}{C_e + 1}, \frac{A}{B} \geq 1 \qquad (5.3\text{-}3)$$

$$f_{sp} = F_y - \frac{1}{C_s}, \frac{A}{B} < 1 \qquad (5.3\text{-}4)$$

$$C_e = \frac{f_e^2}{F_p\left(F_y - F_p\right)}, C_s = \frac{f_s}{F_p\left(F_y - F_p\right)}, f_e = \frac{4\pi^2\left(D_x D_y\right)^{1/2}}{t_x B^2}$$

$$f_s = \left(\frac{1}{\rho^2} + 2\eta + \rho^2\right)\frac{f_e}{4}$$

The allowable in-plane compressive stress is obtained by dividing the limit state stress (f_{se} or f_{sp}) by the appropriate factor of safety F.S.

5.3.3 Biaxial Compression

$$\frac{f_{xl}}{f_{xl}^*} m^2 + \frac{f_{yl}}{f_{yl}^*} n^2 = \frac{m^4}{\rho^2} + 2\eta m^2 n^2 + \rho^2 n^4 \qquad (5.3\text{-}5)$$

$$f_{xl} \leq F_p$$

$$f_{xl}^* = \frac{\pi^2\left(D_x D_y\right)^{1/2}}{t_x B^2}$$

$$f_{yl}^* = \frac{\pi^2\left(D_x D_y\right)^{1/2}}{t_y A^2}$$

See 5.3.2 for definition of symbols.

A trial procedure can be used to determine the values of f_{xl}, f_{yl} , m and n (these represent the integer number of half waves in which the panel buckles in the x and y directions, respectively).

Elastic range: The serviceability limit state is elastic if the stresses f_{xl} and f_{yl} obtained from the expressions above satisfy the following criterion:

$$f_{xl}^2 - f_{xl} f_{yl} + f_{yl}^2 < F_p^2$$

If this criterion is satisfied the stresses f_{xl} and f_{yl} are the elastic serviceability limit state stresses f_{xse} and f_{yse}, respectively. The allowable stresses are obtained by dividing f_{xse} and f_{yse} by the appropriate factor of safety F.S.

Plastic range:

$$f_{xl}^2 - f_{xl} f_{yl} + f_{yl}^2 \geq F_p^2$$

$$\left(\frac{f_{xl}}{f_{xsp}}\right)^2 + \left(\frac{f_{yl}}{f_{ysp}}\right)^2 = 1 \qquad (5.3\text{-}6)$$

where f_{xsp} and f_{ysp} are given in 5.3.2. The allowable stresses are obtained by dividing the limit state stresses f_{xl} and f_{yl} by the appropriate factor of safety F.S.

5.3.4 Uniform Lateral Load

a. Serviceability Limit State

As with rectangular plates, shown in 3.4, a deflection criterion and a stress criterion can be defined. The deflection criterion is associated with a maximum allowable deflection, while the stress criterion implies that the panel must remain in the elastic range. Expressions for computing the maximum elastic deflection and the stresses are given below.

The maximum elastic deflection at the center of a simply supported cross stiffened plate can be calculated from:

$$w = \delta \frac{pB^4}{D_y} \qquad (5.3\text{-}7)$$

where the non-dimensional coefficient δ depends on the virtual aspect ratio ρ as shown in Figure 5.3-1.

In order to ensure that the panel will not suffer any plastic deformations the distribution of plate stresses f_x and f_y should be determined from linear elastic theory. Elastic behavior is ensured if the stresses satisfy the following relation:

$$f_x^2 - f_x f_y + f_y^2 \le F_p^{\;2}$$

The stresses should be checked at both a stiffener's free flange and in the plate field, and may be determined at the panel's center according to the following formulas:

Stiffener's free flange:

$$M_x = \overline{\alpha}\left(\frac{D_x}{D_y}\right)^{1/2} pB^2 \qquad (5.3\text{-}8)$$

$$M_y = \overline{\beta} pB^2 \qquad (5.3\text{-}9)$$

Plate field:

$$M_x = \overline{\alpha}'\left(\frac{D_x}{D_y}\right)^{1/2} pB^2 \qquad (5.3\text{-}10)$$

$$M_y = \overline{\beta}' pB^2 \qquad (5.3\text{-}11)$$

where $\overline{\alpha}' = \overline{\alpha} + 0.3\left(\dfrac{D_x}{D_y}\right)^{1/2} \overline{\beta}$

$$\overline{\beta}' = \overline{\beta} + 0.3\left(\frac{D_x}{D_y}\right)^{1/2} \overline{\alpha}$$

The stresses f_x and f_y are determined from:

$$f_x = \frac{M_x s_y r_a}{I_x} \qquad (5.3\text{-}12)$$

$$f_y = \frac{M_y s_x r_b}{I_y} \qquad (5.3\text{-}13)$$

where r_a, r_b = bending lever arm associated with f_x or f_y respectively, i.e. distance from the neutral axis of the stiffener with the effective breath of plate to the outer fiber of the flange (for the flange stress) or of the plate (for the plate field stress).

The non-dimensional coefficients $\overline{\alpha}$ and $\overline{\beta}$ depend on the virtual aspect ratio ρ and the torsional coefficient η as shown in Figure 5.3-2.

b. Ultimate Limit State

$$p_u = \frac{p_c(n+1)}{A} \qquad (5.3\text{-}14)$$

where

A = length of longitudinal stiffeners,

n = number of transverse stiffeners,

p_c = a parameter of dimension load/length to be determined according to the following equations:

$$p_c = \frac{8(m+1)^2}{m(m+2)B^2} M_t + \frac{m+1}{B} R_c$$

for m = even

or

$$p_c = \frac{8}{B^2} M_t + \frac{m+1}{B} R_c$$

for m = odd

The values of the interaction forces between the longitudinal and transverse stiffeners R_c are given by:

for n = even

$$R_c = \frac{8(n+1)}{n(n+2)A} M_l \qquad (5.3\text{-}15)$$

for n = odd

$$R_c = \frac{8}{(n+1)A} M_l \tag{5.3-16}$$

where

p_u = ultimate uniform pressure,
M_t = plastic moment of transverse stiffener at center,
M_l = plastic moment of longitudinal stiffener at center,
A = length of longitudinal stiffener,
B = length of transverse stiffener,
m = number of longitudinal stiffeners,
n = number of transverse stiffeners.

In determining M_l and M_t the effect of in-plane loads should be taken into account, as suggested in 4.5.2.

The allowable pressure is obtained by dividing the limit state pressure p_u by the appropriate factor of safety F.S.

5.4 STIFFENER PROPORTIONS

In order to limit the possibility of local instability such as torsional/flexural buckling, or lateral buckling, the stiffener proportions should satisfy the requirements in 4.4.4.

If the design is based on plastic methods, the stiffener proportions should satisfy the requirements in 4.5.3.

5.5 TRPPING BRACKETS

The overall tripping of stiffeners can be avoided by means of tripping brackets. These should satisfy the requirements of 4.4.5.

5.6 EFFECTIVE FLANGE

5.6.1 Definitions

5.6.1.1 Data for effective flange calculations in plate girders and box girders subjected primarily to bending type loads is given. The approach followed leads to the effective breadth ratio b_{ef}/b, where b_{ef} is the effective half flange breadth and b is the half flange breadth. Note that for this section only, the term b is defined as one-half of the flange breadth (see Figure 5.6-1).

5.6.1.2 Three cases are considered, as sketched in Figure 5.6-1:
Case I: Single web, symmetrical flange with free sides;
Case II: Double web, flange bounded by webs;
Case III: Multiple webs.

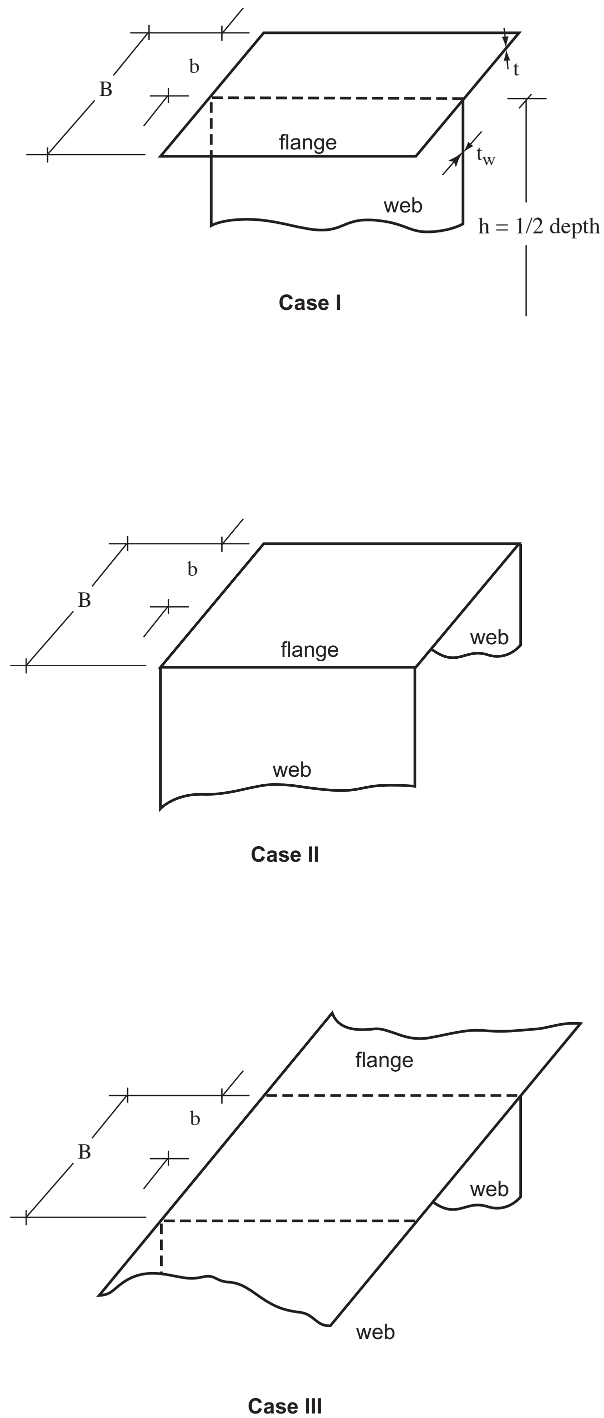


Figure 5.6-1—Cases for Effective Flange Calculations

Figures 5.6-2 through 5.6-4 give in a graphical form the effective breadth ratio $b_{ef}\ /b$ for the three cases described above, and for a number of load conditions.

The non-dimensional coefficient depends on the cross sectional shape. For identical lower and upper flanges:

$$\beta = \frac{1}{6}\frac{h}{b}\frac{t_w}{t} \tag{5.6-1}$$

and for stiffened plating:

$$\beta = \frac{1}{4}\frac{h}{b}\frac{t_w}{t}\frac{4A_2 + 2ht_w}{3A_2 + 2ht_w} \tag{5.6-2}$$

where t is the flange thickness, t_w is the web thickness, h is the half web depth, b is the half breadth and A_2 is the lower flange area (zero in the case of flat bar stiffeners). The remaining symbols in Figures 5.6-2 through 5.6-4 are defined as follows:

B = distance between webs,
cL = distance between points of zero bending moment.

5.6.2 Stress Distribution Across Flange

In computing the effective section modulus S_{ef} for the purpose of stress calculations, the effective flange breadth b_{ef} determined by the approach in 5.6.1 should be used (in place of the actual flange breadth b). Then the stress at the flange web junction f_{max} is given by:

$$f_{\max} = \frac{M}{S_{ef}} \tag{5.6-3}$$

where M is the bending moment acting on the cross section.

Across the flange breadth the actual stress distribution can be approximated by the following quartic equation:

$$f = f_{\max}\left\{\left(\frac{x}{b}\right)^4 + \left[\frac{5\left(b_{ef}\ /b\right)-1}{4}\right]\left[1-\left(\frac{x}{b}\right)^4\right]\right\} \tag{5.6-4}$$

where x is the distance measured across the flange breadth, as shown in Figure 5.6-5.

5.6.3 Calculation of Deflections

The effective flange breadth b_{ef} determined by the approach in 5.6.1 should be used to find the effective moment of inertia I_{ef} of the cross section. This effective moment of inertia I_{ef} multiplied by the modulus of elasticity E gives the bending rigidity EI_{ef}, which should be used in computing girder deflections.

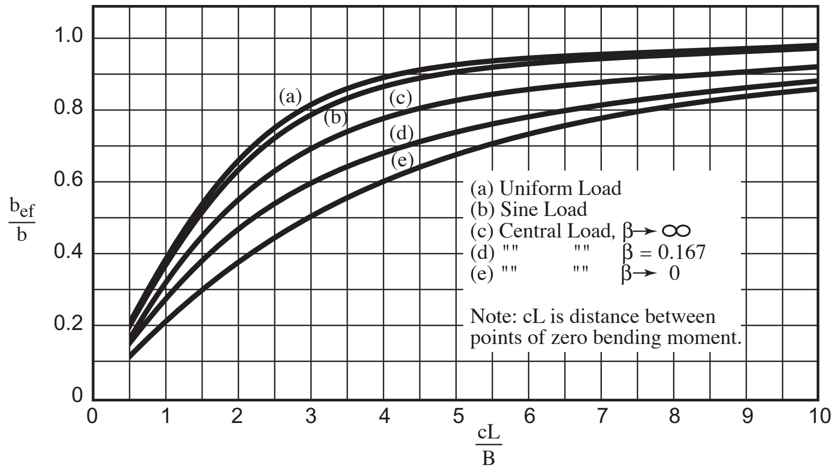


Figure 5.6-2—Effective Breadth Ratio for Case I (Single Web)*

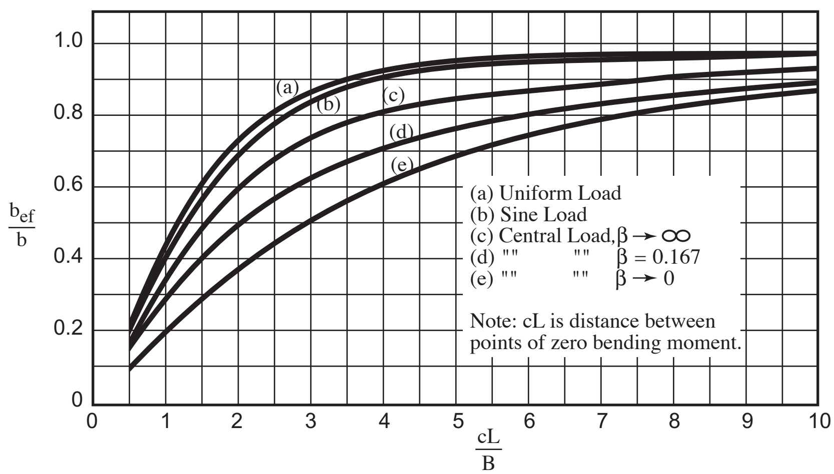


Figure 5.6-3—Effective Breadth Ratio for Case II (Double Web)*

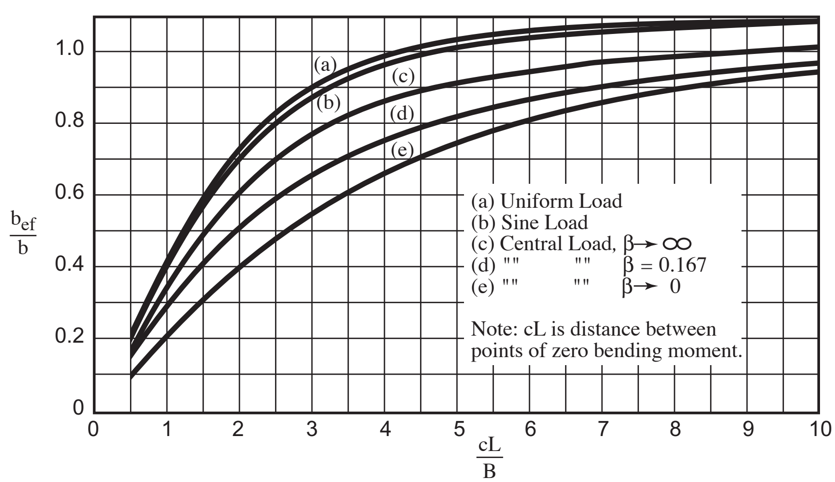


Figure 5.6-4—Effective Breadth Ratio for Case III (Multiple Webs)*

*From H.A. Schade, "The Effective Breadth of Stiffened Plating Under Bending Loads," *SNAME Transactions*, Vol. 59, 1951.

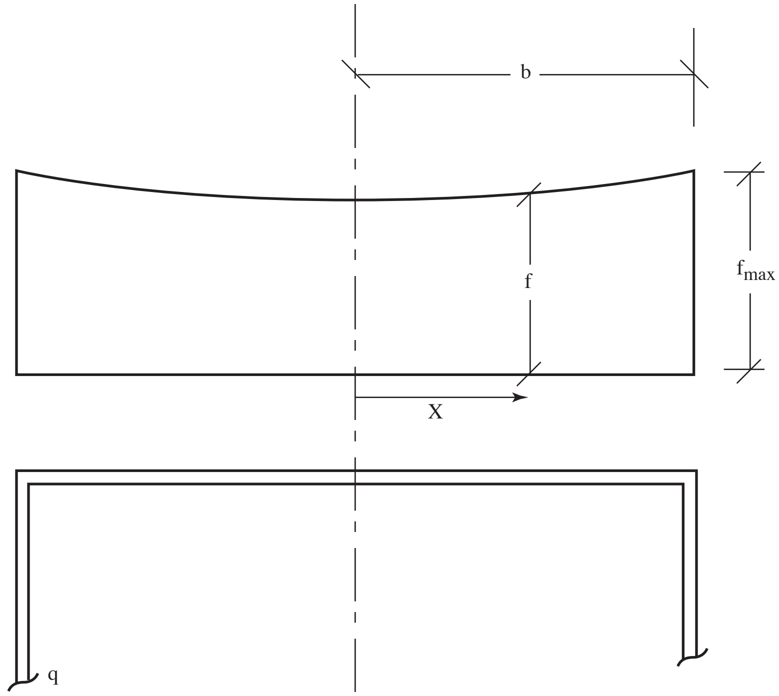


Figure 5.6-5—Stress Distribution Across Flange

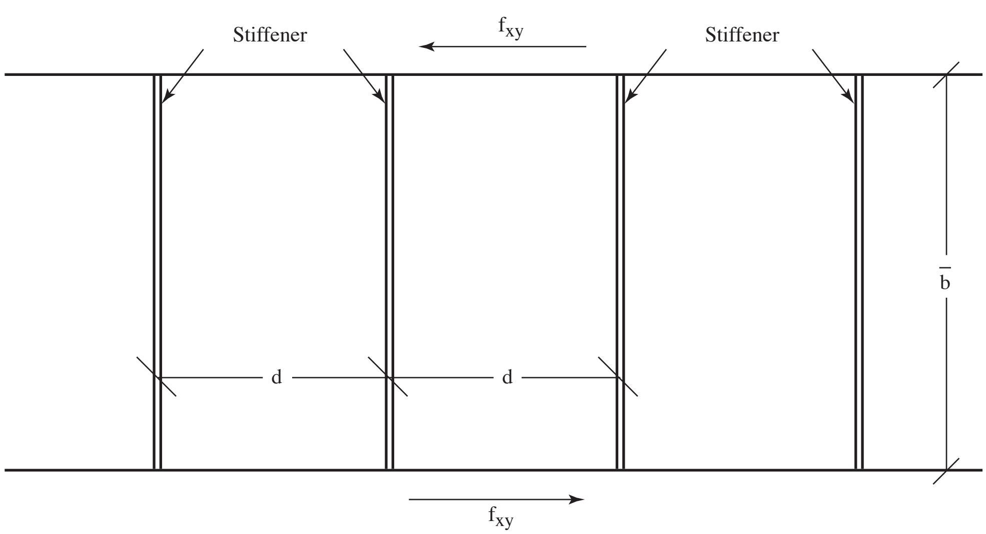


Figure 5.7-1—Geometry of Stiffened Panels Subjected to In-Plane Shear

5.7 STIFFENER REQUIREMENT FOR IN-PLANE SHEAR

The moment of inertia of stiffeners in panels subjected to edge shear should satisfy the following requirement:

$$I > 0.09 t^3 \bar{b} \gamma \qquad (5.7\text{-}1)$$

$$\gamma = 8 \frac{f_{xy}}{f_{xyu}}, \frac{\bar{b}}{d} \leq 1$$

$$\gamma = 10 \left(2.8 \frac{\bar{b}}{d} - 2 \frac{d}{\bar{b}} \right) \frac{f_{xy}}{f_{xyu}}, \frac{\bar{b}}{d} > 1$$

where f_{xy} is the design in-plane shear stress in the plate, f_{xyu} is given in 3.3.3, and the plate's geometry is shown in Figure 5.7-1. I is the moment of inertia of the stiffener's web plus flange about an axis coinciding with the surface of the plate at the plate/web intersection.

5.8 OTHER DESIGN REQUIREMENTS

Good design practice dictates a sufficient separation of local plate and stiffened panel buckling modes. Therefore, the stiffened panel design should ensure that the elastic buckling stresses in the panel longitudinal and transverse direction exceed the associated elastic buckling stresses for each plate panel by at least 20 percent. For uniaxially stiffened panels, only elastic buckling stresses in the direction of the stiffening must meet this recommendation.

Where the appropriate elastic stress is not specified in this bulletin, it may be determined from:

$$f_e = \left[\frac{F_p \left(F_y - F_p \right)}{\left(F_y / f_{cr} \right) - 1} \right]^{1/2} \qquad (5.8\text{-}1)$$

where f_{cr} is the determined stiffened panel limit state stress under consideration.
where $f_{cr} < F_y$

5.9 DESIGN CONSIDERATIONS

5.9.1 The most relevant step in the design of a stiffened panel involves a proper choice of the stiffening system, to provide an adequate overall strength, and to limit the plate dimensions and proportions to values that will prevent plate failure by instability. Many choices for the stiffening system are available, and no specific guidelines can be given, since the optimum configuration depends on dimensions, loads and boundary conditions.

5.9.2 Typically, a stiffened panel would be orthogonally stiffened, with the set of primary stiffeners or girders providing the main support structure for the whole panel, and the set of secondary stiffeners providing local plating support. The large number of stiffener intersections justifies a careful detail design of the stiffener crossings.

5.9.3 In general a weight efficient structure would make use of a high density of stiffeners in both directions, but the cost implications could be adverse. Since stiffened plates are important components of TLP and other floating structures, and contribute to a large share of the structural weight, the designer should perform several iterations with alternative stiffener arrangements in order to reach an efficient design.

5.9.4 Important aspects of stiffened panel design are the selection of stiffener cross sectional dimensions, and the rectangular plate aspect ratio, discussed in 4.6 and 3.7, respectively.

Section 6-Deep Plate Girders

6.1 GENERAL

6.1.1 Scope

Deep plate girders with the web stiffened in both the longitudinal and transverse directions are considered. The requirements given apply specifically to the case where the transverse stiffener spacing is not larger than 1.5 times the girder depth, and the ratio of the clear distance between flanges to the web thickness d_w / t_w exceeds

$$11.75\sqrt{\frac{E}{F_y}} \qquad (6.1\text{-}1)$$

When the girder web is not stiffened, or the depth to thickness ratio is smaller than the value above, the design should comply with the *AISC Specification for Structural Steel Buildings*.

6.1.2 Definitions

6.1.2.a Deep Plate Girders. Deep plate girders, sometimes also referred to as bulkhead girders, are in general orthogonally stiffened. They may form the main support structure for platform decks and they may be arranged as a grillage, hence dividing the deck structure into discrete compartments. A typical arrangement is shown in Figure 6.1-1. The orthogonal stiffening can be single or double sided.

The girders can span between points of support with continuous or intermittent lateral restraint for the compression flange. Flanges can be single or multiple, thick, unstiffened plates or thinner, stiffened plates, which can also function as a deck. See Figure 6.1-2. The girder webs will, in some circumstances, form part of a fire wall and/or boundary of a hazardous area. For some floating structures the deck girders may be utilized as part of the reserve buoyancy, and must also be designed for lateral pressure.

6.1.2.b Flanges. The girder flanges, sometimes also referred to as chords, are the upper and lower girder flanges of the plate girders. The primary function of the flanges is to provide sufficient area at the extremities of the girder to resist bending moment. During fabrication the flanges may act alone in resisting bending moment, but in service they are integrated with the deck plate, which contributes to the plate girder resisting moment.
Depending on the geometry and loading the girder compression flange may have to be longitudinally and/or transversely stiffened. The stiffening arrangement also provides adequate strength to resist local concentrated loads. Continuous or intermediate lateral restraint at, or remote from, the compression flange, might also be required.

6.1.2.c Girder Web. The girder web transmits the shear loads to the joints, with the plating in consequence carrying shear, axial and bending stresses. In some cases the web might also

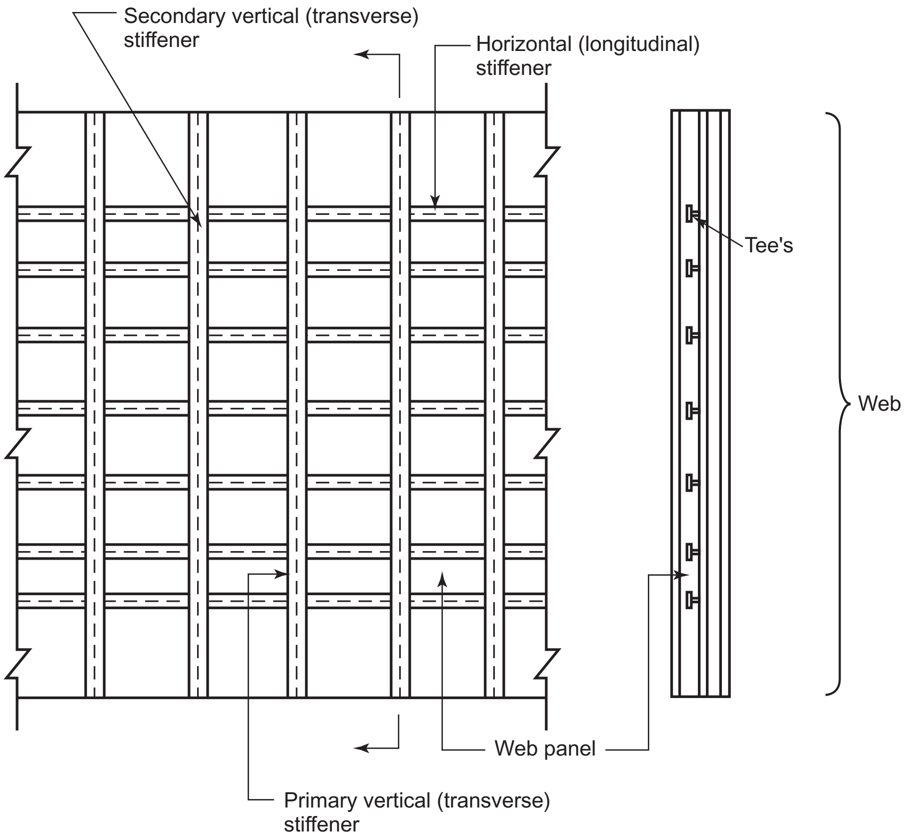


Figure 6.1-1—Typical Deep Plate Girder Structural Arrangement

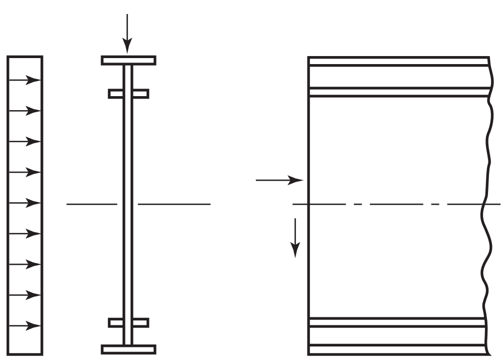


Figure 6.1-2—Primary Loads Acting on Plate Girder

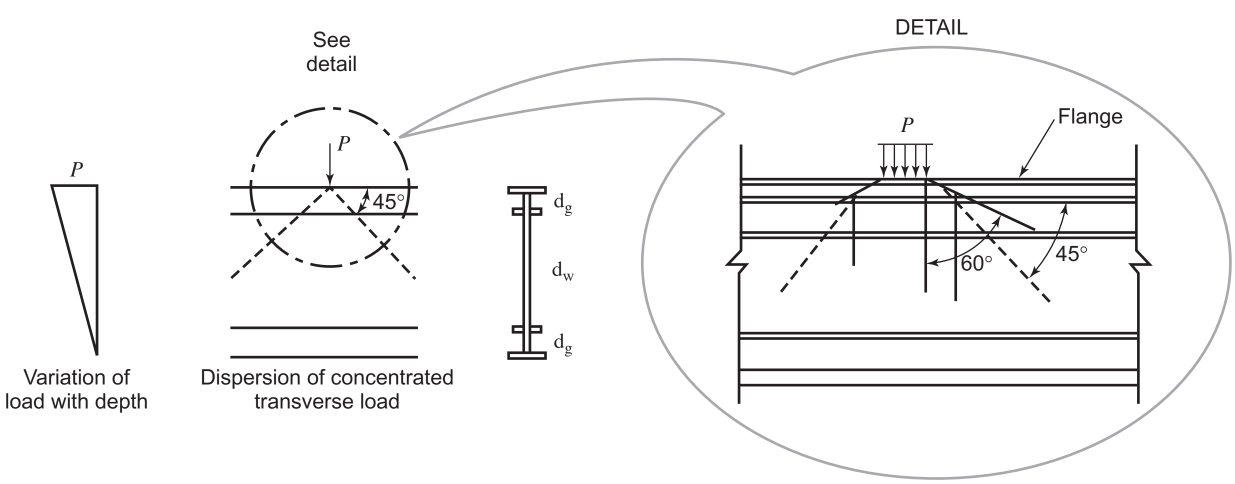


Figure 6.1-3—Stress Distribution Across Section Due to Concentrated Load Applied at the Flange Level

be subjected to directly applied lateral loads due to hydrostatic pressure. To prevent buckling, the webs are divided into panels by longitudinal (horizontal) and transverse (vertical) stiffeners. A web panel is defined as an area of web plate bounded on each edge by a stiffener, diaphragm, or girder flange. An 'outer panel' is a web panel adjacent to the girder flange.

The orthogonal stiffening arrangement for the girder web has to be selected to achieve maximum structural efficiency. This requires a balanced choice of panel aspect ratios and stiffener proportions. The design of the outer panel connection to the girder flange requires special consideration to ensure adequate shear transfer. Openings in the web may be required for operational reasons. If these cannot be completely avoided, web reinforcement may be required, and special consideration has to be given to the geometry of the openings.

6.1.2.d Primary Transverse Stiffeners. Primary transverse (or vertical) stiffeners support the flanges for tension field action, serve as stiffening elements for the girder web plate, and also connect with the deck girders. The connection between deck girders and stiffeners provides full continuity, and leads to frame action. Transverse stiffeners may also be required to support concentrated loads, such as transportation loads, deck/hull mating loads, etc.

6.1.2.e Secondary Transverse Stiffeners. Secondary transverse (or vertical) stiffeners span the full height of the girder, stiffen the web plate, and support the flanges for tension field action.

6.1.2.f Longitudinal Stiffeners. Longitudinal (or horizontal) stiffeners in the compression zone of the web increase the buckling resistance of the web plate between transverse stiffeners by limiting the unsupported panel sizes. Their spacing should be chosen to ensure continuity of stiffening between interconnecting girders. The longitudinal stiffeners also contribute to the girder bending resistance, and as such must be designed to carry axial loads due to applied bending/axial forces, and possibly also lateral loads due to hydrostatic pressure.

6.1.3 Loads and Load Combinations

6.1.3.a Primary Loads. Primary loads are obtained from three-dimensional space frame action. Five main types of primary loads act on girders, as shown in Figure 6.1-2:

- Longitudinal tension or compression;
- Transverse tension or compression;
- Bending;
- In-plane shear;
- Lateral load.

6.1.3.b Secondary Loads. Secondary loads consist of the following categories:

- Shrinkage forces due to welding;
- Stresses due to construction tolerances;
- Vertical load on the web due to a slope change in the undeformed flange;

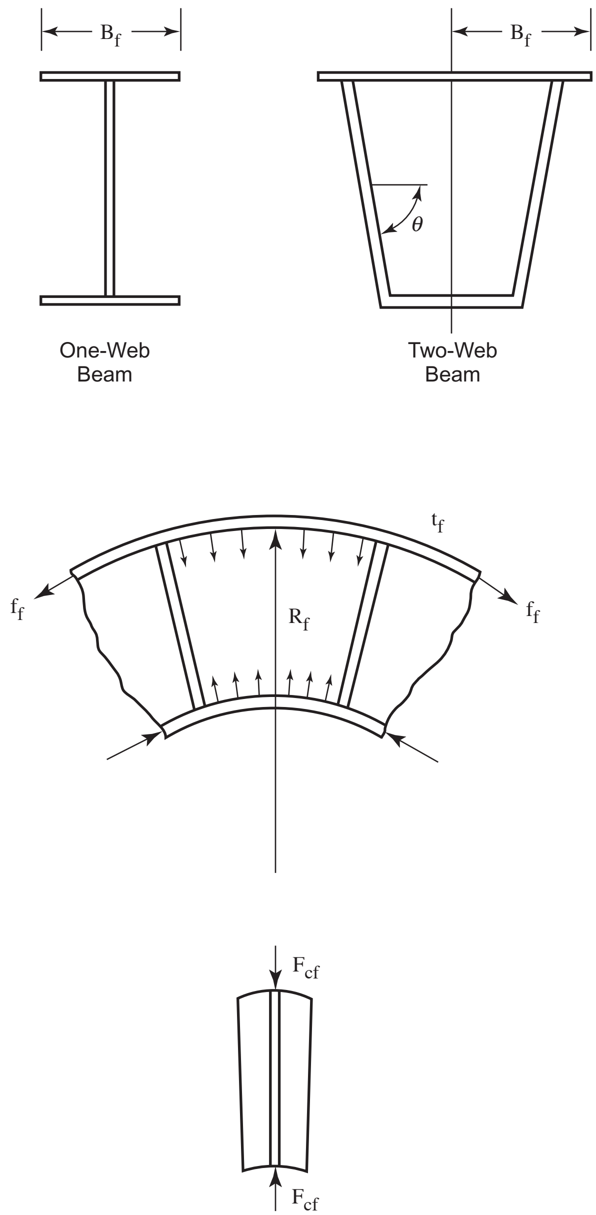


Figure 6.1-4—Transverse Stresses in Webs Due to
Flanges Curved in Elevation

- Vertical load on the web due to concentrated load applied to the upper and lower flange level;
- Secondary bending, redistribution of primary bending and shear in the vicinity of web openings;
- Local vertical forces on bearing surfaces;
- Loads due to deck girder connections;
- Thermal loads.

The combined effect of the above loads should be accounted for in designing the girder components.

6.1.4 Stress Analysis

6.1.4.1 The stresses in a plate girder can be calculated on the assumption that plane sections remain plane, provided the girder unsupported span to depth ratio is larger than 5. The cross sectional properties required to determine longitudinal bending stresses must take account of shear lag effects, as prescribed in 5.6. However, shear lag effects may be neglected when considering the ultimate limit state.

6.1.4.2 The transverse shear stresses can also be derived on the basis of simple beam theory, but allowance must be made for web openings when computing the shear stress distribution across the web.

6.1.4.3 Finite element or other type of numerical analysis can be used to obtain a more exact stress distribution, if required for the particular geometry and load conditions, or if simple beam theory is no longer valid (as for very short, stocky girders).

6.1.5 Stress Distribution

6.1.5.a Longitudinal Stress. If simple beam theory is applicable the longitudinal bending stress distribution across the girder depth is given by

$$f_x = \frac{M\bar{y}}{I_{ef}} \qquad (6.1\text{-}2)$$

where M is the applied bending moment, $\bar{y}$ is the distance to the neutral axis, and I_{ef} is the effective moment of inertia of the cross section about the neutral axis. In computing I_{ef} the effective flange should be used, as prescribed in 5.6.

The distribution of longitudinal bending stresses across the flange width can be obtained by following the approach described in 5.6.2.

6.1.5.b Shear Stresses. The shear stress distribution can be obtained from:

$$f_{xy} = \frac{VQ}{It} \qquad (6.1\text{-}3)$$

where V is the shear force, Q is the moment of the area above the point where shear stress is being determined about the neutral axis, I is the moment of inertia about the neutral axis, and t is the thickness at the point under investigation.

For webs with constant thickness the average shear stress can be approximated by

$$f_{xy} = \frac{V}{A_w} \qquad (6.1\text{-}4)$$

where A_w is the web area. When the web has openings the web area should be computed on the basis of $d_w - a_h$, where d_w is the depth of the web plate between flanges and a_h is the height of the opening.

6.1.5.c Transverse Stresses in Webs Due to Local Vertical Forces. The transverse stress in the plane of the web due to load applied to a flange may be calculated on the assumption that the load is dispersed uniformly. It can be assumed that the load decreases linearly from its point of application to zero at the extremity of the opposite flange. Also, it is assumed that the stress disperses inside the flange at a 60° angle and inside the web at a 45° angle, as shown in Figure 6.1-3.

6.1.5.d Transverse Stresses in Webs Due to Flanges Curved in Elevation. The edge of a web attached to a portion of a flange curved in elevation, Figure 6.1-4, should be considered to be subjected to a force per unit length F_{cf}, acting in the plane of the web, given by:

$$F_{cf} = \frac{f_f B_f t_f}{R_f \sin\theta} \qquad (6.1\text{-}5)$$

where f_f is the flange longitudinal stress, B_f is the width of an unstiffened flange in a beam having only one web (or half the distance between successive longitudinal stiffeners or webs, together with any adjacent outstand), θ is the slope of the web to the horizontal, t_f is the flange thickness in the panel being considered, and R_f is the radius of curvature of the flange.

6.2 LIMIT STATES

6.2.1 General

The serviceability and ultimate limit states governing deep girder structural performance are defined for each girder main component, namely the flanges, web plates, and their stiffeners.

6.2.2 Girder Flanges

6.2.2.1 If girder flanges are longitudinally and/or transversely stiffened, each individual rectangular plate should be considered. The serviceability and ultimate limit states that apply, for the appropriate loads and load combinations, are given in Section 3.

6.2.2.2 The serviceability and ultimate limit states that apply to the stiffeners are given in Section 4. In particular the stiffener proportions should follow the requirements in 4.4.4.

6.2.3 Girder Web

6.2.3.1 The serviceability and ultimate limit states that govern the strength of the individual rectangular web plates supported by stiffeners are given in Section 3.

6.2.3.2 The serviceability and ultimate limit states that apply to the longitudinal and transverse stiffeners are given in Section 4. In particular the stiffener proportions should follow the requirements in 4.4.4.

6.2.3.3 The girder web is also subjected to in-plane bending. In general, when treating this particular loading condition the guidelines described below can be followed, but the designer should exercise engineering judgment in applying them.

6.2.3.4 Under in-plane bending the longitudinal bending stress varies linearly across the plate transverse edge (of length b). For an individual plate if this variation is small the applied stress should be assumed uniform and equal to the average stress acting across the transverse edge. If the individual rectangular plate is close to the girder neutral axis and the average stress is very small, it should be assumed that it is subjected to a uniform compressive stress equal to the maximum stress acting across its edge.

6.3 DESIGN CONSIDERATIONS

6.3.1 Girder Flanges

6.3.1.1 The design of girder flanges should comply with the *AISC Specification for Structural Steel Buildings*. The thickness of outstanding parts of flanges should conform with the requirements of 4.4.4.

6.3.1.2 The effect of shear lag must be considered, as prescribed in 5.6.

6.3.1.3 Where possible compact sections should be used, thus allowing the whole section to be effective without requiring stiffening.

6.3.2 Web Panels

6.3.2.a General. The girder web is divided into rectangular plates by longitudinal and transverse stiffeners. In general these web plates are subjected to longitudinal and transverse loads, as well as in-plane shear. Lateral loads can also be present.

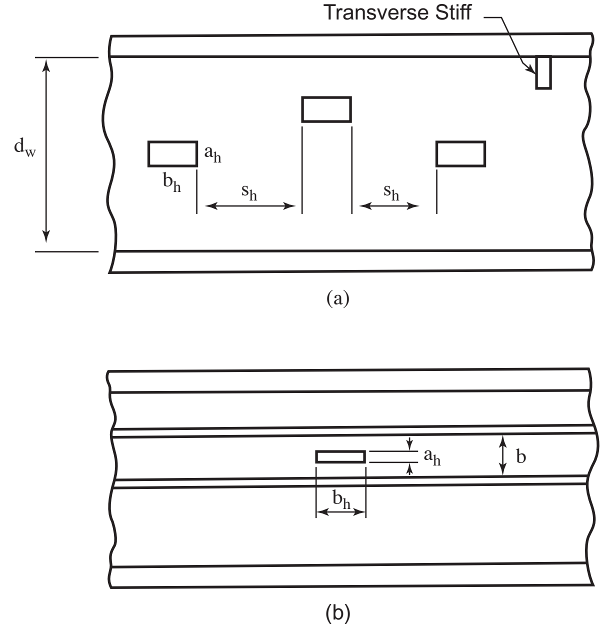
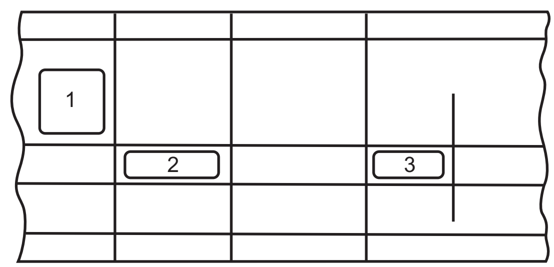


Figure 6.3-1—Web with Small Openings



1 & 2 = Diagonally Opposite Penetrations
2 & 3 = Penetrations in Line

Figure 6.3-2—Web with Large Openings

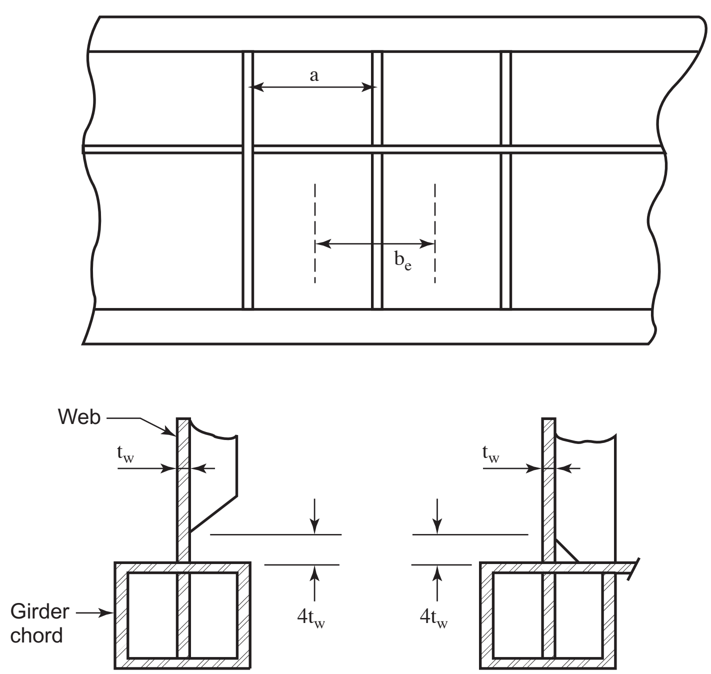


Figure 6.3-3—Vertical Stiffener Termination

In choosing the stiffener spacing, serviceability and ultimate limit states associated with rectangular plate instability, due to compressive longitudinal and transverse loads and in-plane shear, must be considered.

The longitudinal web stiffeners are designed to allow the adjacent web panels to reach their required load capacity, without premature stiffener failure by buckling or yielding. The stiffeners must have sufficient rigidity to enforce nodal lines on the web in conjunction with the transverse stiffeners. The longitudinal and transverse web stiffeners must be designed as beam-columns for flexural and axial loads due to web panel buckling and lateral pressure.

The vertical spacing of the longitudinal stiffeners should be such that the web panels between the compression flange and the first longitudinal stiffener are capable of reaching yield in shear, or combined compression and shear, before reaching the critical buckling load.

When $d_w > 180t_w$ (where d_w is the web depth and t_w is the web thickness), at least one longitudinal stiffener should be provided and placed between the neutral axis and compression flange. Additional longitudinal stiffeners should be provided to restrict the ratios of *b/t* of the web panels to values which can adequately prevent limit states associated with plate buckling. Adjacent to internal supports in continuous spans, where the lower part of the web can be overstressed due to concentration of shear stress, additional intermediate longitudinal stiffeners should be provided, for a distance of at least d_w on each side of the support. These stiffeners should terminate on transverse stiffeners. They should restrict the proportions of all the web plate panels to acceptable values. Longitudinal stiffeners should extend between and be attached to transverse stiffeners.

6.3.2.b Webs With Openings. In general, web openings are subject to special investigation for stress concentration, buckling around the opening perimeter, or fatigue. These considerations could be effectively satisfied if the design of openings complies with the following recommendations:

a. In the absence of special framing around the opening, its overall dimension should be limited to [see Figure 6.3-1(a)]:

$$a_h \text{ or } b_h \leq \frac{1}{10} d_w \tag{6.3-1}$$

b. The overall dimension of an opening in longitudinally stiffened webs should be limited to [see Figure 6.3-1(b)]:

$$a_h \text{ or } b_h \leq \frac{1}{3} b \tag{6.3-2}$$

c. Openings should be spaced horizontally with a clear distance between them of at least s_h [see Figure 6.3-1(a)]:

$$a_h \text{ or } b_h \leq \frac{1}{3} s_h \tag{6.3-3}$$

d. No more than one opening is recommended at any one web cross section between longitudinal stiffeners.

e. Cutouts in webs for the connection of transverse stiffeners should be welded over at least 1/3 of the opening perimeter.

f. Openings should be designed with adequate corner radius, or reinforcing, to avoid stress concentration.

In cases where the web opening dimensions are large and/or do not comply with the requirements in items a through f above, the following requirements apply:

g. Each opening should be reinforced by longitudinal and transverse stiffeners (Figure 6.3-2). Sufficient corner radius should be provided at each opening to reduce the stress concentration.

h. Diagonally opposite openings should be bounded by at least two common orthogonal stiffeners (Figure 6.3-2).

i. In line openings should be bounded by at least two parallel stiffeners.

j. The height of openings should be limited to a maximum of one-third (1/3) the web depth.

k. To the extent possible openings should be away from points of load concentration.

l The stiffeners adjacent to openings should have a minimum cross sectional area equal to the area of the opening in each direction. Furthermore, the stiffener should provide adequate strength to resist the primary, as well as secondary, axial loads and bending moment.

m. A detailed finite element analysis is recommended to obtain the load and stress distribution around an opening.

6.3.3 Longitudinal Web Stiffeners

Stiffener proportions should comply with the requirements in 4.4.4. Longitudinal stiffeners should extend between and be attached to transverse stiffeners.

The longitudinal stiffener bending stiffness necessary to ensure that a stiffened plate can reach the ultimate strength of the web panel between stiffeners is greater than that required to develop maximum local buckling stress. The moment of inertia I_s of the stiffener cross section about the neutral axis should be larger than the following value.

$$I_s = 4at_w^3 \tag{6.3-4}$$

where a is the spacing between transverse stiffeners and t_w is the web thickness.

6.3.4 Transverse Web Stiffeners

6.3.4.a General. Transverse (vertical) stiffeners provide adequate support for the web and longitudinal stiffeners. A transverse stiffener should be included at the junction with cross beams, and at sloping flange locations.

Transverse stiffeners should be shaped to allow space for weld material connecting the web to the flange, with a clearance not exceeding 4 t_w, as shown in Figure 6.3-3. The stiffener should extend over the whole remaining depth of the web.

A primary transverse stiffener should be fitted to the flange near each point of concentrated load application.

Where cutouts are provided in transverse stiffeners to allow passage of longitudinal stiffeners, at least 1/3 of the cutout perimeter should be welded to the longitudinal stiffeners.

6.3.4.b Effective Stiffener Section. The effective stiffener section should include the stiffener plus a portion of the web plate on each side of the stiffener, as shown in Figure 6.3-3. The effective plate flange width b_e is given in 4.1.2.

6.3.4.c Design Load for Transverse Stiffeners. The following loads should be considered when designing transverse stiffeners, as applicable:

- axial force due to tension field action, see 6.3.4d;
- axial force assumed in preventing web buckling, see 6.3.4e;
- axial force due to vertical distribution of load through a cross frame;
- axial force due to load applied at the girder chord level;
- axial force due to initial flange curvature, see 6.3.4.f;
- axial force due to change in chord girder slope;
- bending moment about an axis in or parallel to the plane of the web, arising from eccentricity of axial force, or from flexure of a cross-frame or deck.

6.3.4.d Axial Force Due to Tension Field Action. Tension field action should be assumed to occur in the web plate, and to act in the mid-plane of the web, when the average shear stress in the web plate, f_{xy}, is greater than τ_o to given by:

$$\tau_o = 3.6E\left[1+\left(\frac{b}{a}\right)^2\right]\left(\frac{t_w}{b}\right)^2\sqrt{1-\frac{f_1}{2.9E}\left(\frac{b}{t_w}\right)^2} \qquad (6.3\text{-}5)$$

$$f_1 < 2.9E\left(\frac{t_w}{b}\right)^2$$

$$\tau_o = 0,\ f_1 \geq 2.9E\left(\frac{t_w}{b}\right)^2 \qquad (6.3\text{-}6)$$

where a is the plate length or spacing between transverse stiffeners, b is the plate width or spacing between longitudinal stiffeners, t_w is the web thickness, and f_1 is the average longitudinal stress in the web panel, to be taken as positive when compressive. In computing f_1 it is assumed that the bending moment and/or axial force are not redistributed to the flanges.

The tension field action should be assumed to cause a compressive force F_{tw} in the adjacent transverse stiffener over its entire length equal to the smaller of the two values:

$$F_{tw} = \left(f_{xy} - \tau_o\right)t_w a \qquad (6.3\text{-}7)$$

$$F_{tw} = \left(f_{xy} - \tau_o\right)t_w l_s \qquad (6.3\text{-}8)$$

where l_s is the clear distance between the flanges of the girder.

When F_{tw} is different on the two sides of a transverse stiffener the average value may be taken. If there are longitudinal stiffeners, F_{tw} for one side of the transverse stiffener should be taken as the average of the two smallest values of F_{tw} occurring in the web panels, on that side of the transverse stiffener.

6.3.4.e Axial Force Assumed in Preventing Web Buckling. In order to resist buckling of the web plate the effective stiffener section should be assumed to carry, along its centroidal axis, a compressive force F_{wi} given by:

$$F_{wi} = \frac{l_s^2}{a_{\max}} t_w k_s f_R \qquad (6.3\text{-}9)$$

where l_s is the clear distance between the flanges of the girder, $a_{\max}$ is the maximum spacing of transverse stiffeners, t_w is the web thickness and k_s is a coefficient given in Figure 6.3-4. The coefficient k_s is a function of the slenderness parameter λ:

$$\lambda = 24 \frac{l_s}{r_{se}} \sqrt{\frac{F_y}{E}} \qquad (6.3\text{-}10)$$

where r_{se} is the radius of gyration of the effective stiffener section about the maximum moment of inertia axis through the centroid.

The stress f_R is defined by:

$$f_R = \tau_R + \left(1 + \frac{\sum A_s}{l_s t_w}\right)\left(f_1 + \frac{f_b}{6}\right) \qquad (6.3\text{-}11)$$

where τ_R is equal to f_{xy} or τ_o (as defined in 6.3.4d), whichever is less, $\sum A_s$ is the sum of the cross sectional areas of all the longitudinal stiffeners not including any adjacent web plate. f_1 is the average longitudinal stress in the web, taken as positive when compressive, calculated without any redistribution to the flanges (see Figure 6.3-5), and f_b is the maximum value of the stress in the web due to bending alone, calculated without any redistribution of moment to the flanges, and always taken as positive (see Figure 6.3-5).

For a longitudinally stiffened web, the force F_{wi} should be factored by n_s

where

$$n_s = \frac{1}{\left(1 + 0.4\left(\frac{\left(\sum I_s\right) l_s^3}{I a^3}\right)\right)}$$

$\sum I_s$ is the sum of the moments of inertia of the effective section of all the longitudinal web stiffeners in depth l_s,

I is the moment of inertia of the effective section of the transverse stiffener

6.3.4.f Axial Force Due to Initial Flange Curvature. The effective web included in the effective stiffener section should be considered to be subjected to an axial force, due to initial flange curvature, F_{cf}, given by

$$F_{cf} = \frac{f_f A_f b_e}{R_f \sin\theta} \qquad (6.3\text{-}12)$$

where

f_f	=	flange longitudinal stress,
R_f	=	flange radius of curvature,
A_f	=	flange cross-sectional area,
θ	=	slope of web to the horizontal,
b_e	=	effective web acting with stiffener =16t_w or a/2, whichever is less, unless a larger value is demonstrated by analysis.

6.3.4.g Axial Loading Distribution Within a Stiffener. The force in a stiffener due to load applied at the flange level, or due to curvature or change of slope of a stressed flange, or due to transfer of load through a cross frame, should be assumed to vary uniformly along the length of the stiffener, from the value at the point of application, to zero at the remote end of the stiffener.

The force due to tension field action or restraint of web buckling should be assumed constant over the length of the stiffener.

6.3.4.h Yielding of Vertical Stiffener. The maximum stress in the stiffener itself at every point along its length, due to all the relevant forces and moments listed in 6.3.4.c, except the axial force assumed in preventing web buckling should not exceed $F_{ys} = 0.66F_y$. A one-third increase is allowed for extreme load conditions. In areas where cutouts are provided an appropriate reduced section should be taken.

Where the end of a stiffener is fitted closely to the flange of a girder, the bearing stress over the area in contact should not exceed 1.33 F_{ys} . In calculating this stress, the effective bearing area should be taken to consist of only those portions of the area of the stiffener and web plate that satisfy all of the following:

a. In contact with the flange;

b. Clear of the weld or root fillet at the web flange junction;

c. Within the dispersal lines drawn at 60° from the line of application, at any local load through the thickness of a flange plate.

6.3.4.i Load Bearing Support Stiffeners. At each support position, or beneath concentrated loads carried by plate girder flanges, load bearing stiffeners are required.

The section of a bearing stiffener should be symmetrical about the mid-plane of the web. When this condition is not met, the effect of the resulting eccentricity should be taken into account.

The bearing stiffener ends should be adequately connected to both flanges, and particular attention should be given to the detail design of bearing stiffener intersections with longitudinal stiffeners.

Where cut-outs are provided in bearing stiffeners to allow the passage of longitudinal stiffeners, at least one side of the opening in the bearing stiffener should be cleated to the longitudinal stiffener by full perimeter welding of the cleat, or at least one-third of the perimeter of the cut-out should be connected to the longitudinal stiffener by welding.

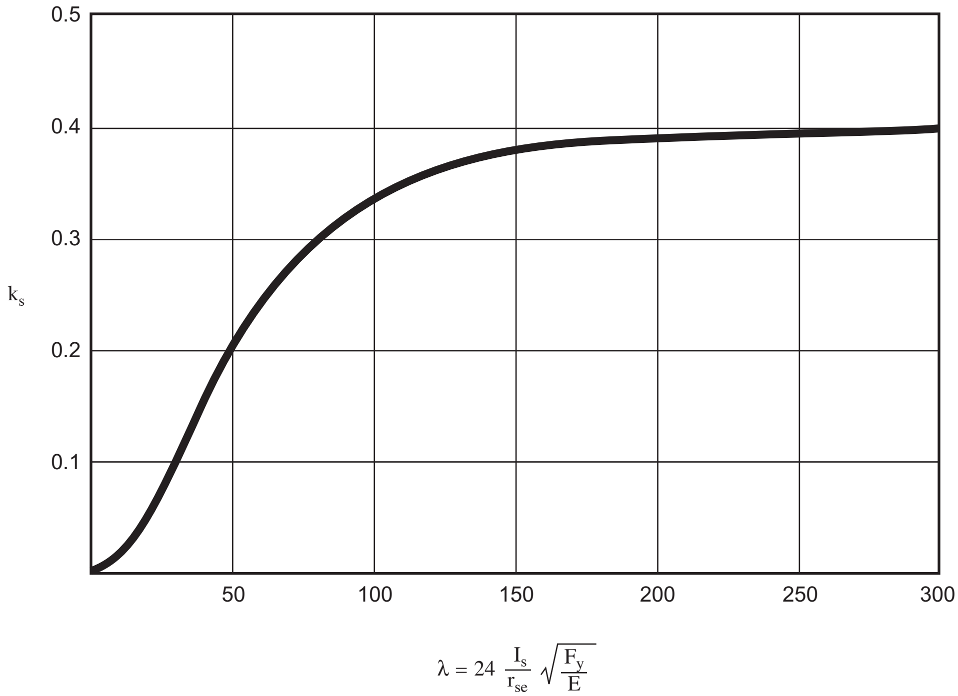


Figure 6.3-4—Coefficient for Computing Axial Force Assumed in Preventing Web Buckling

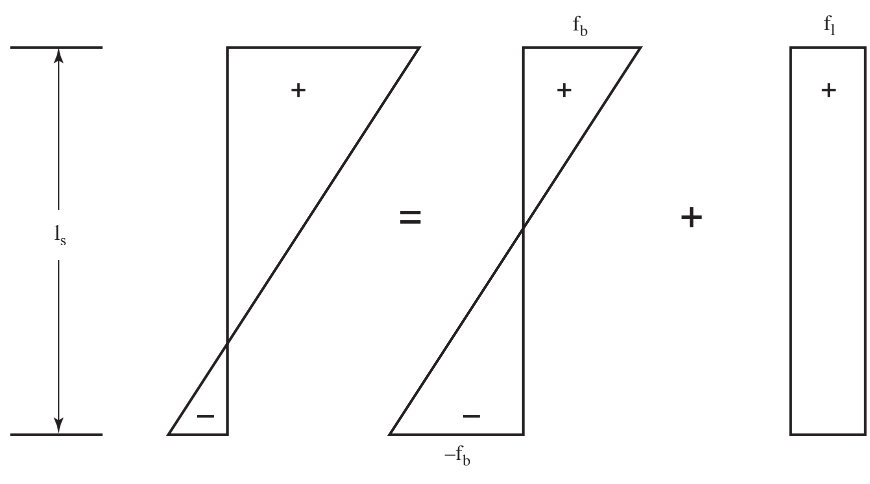


Figure 6.3-5—Longitudinal Stress in Webs with Transverse Stiffeners

APPENDIX A—COMMENTARY

Note: The section, figure and table numbers in this Appendix correspond directly with those found in the main body of the document (i.e., C1.2 provides commentary on section 1.2)

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C1 INTRODUCTION

This Commentary provides background information on the formulations and the design guidance given in Bulletin 2V. Whenever applicable, references are provided, and the rationale for the recommendations made is discussed in some detail.

The objective of this Commentary is to help the designer understand some of the fundamental principles of structural engineering that form the basis of the Bulletin. Rather than applying design formulas that are difficult to interpret, and as such mean very little, a broader understanding of their background can lead to a more efficient design process.

The Commentary follows the same format as the Bulletin. Paragraph numbers are the same as in the Bulletin but are preceded by the letter C. In most cases the same nomenclature is adopted, and where there are changes these are indicated in the text when new symbols are defined.

C2 GENERAL

The design basis adopted in Bulletin 2V is the working stress method, whereby stresses are not allowed to exceed specified values. Allowable stresses are associated with two basic structural requirements: resistance to failure (ultimate limit states); and stiffness and strength criteria (serviceability limit states). The distinction between ultimate and serviceability limit states is used by several codes of practice, such as the British Standard BS5400 *Steel, Concrete and Composite Bridges*, Reference 2.1.

The approach to design implied in Bulletin 2V is deterministic, and the uncertainties in loads and resistance or strength are not specifically addressed. Uncertainties are lumped into factors of safety defined in API RP 2T. The factors of safety depend on the design case, which is associated with the project phase, the system condition and the environment. Factors of safety also depend on the type of limit state.

The classification of relevant modes of failure into limit states gives the designer some more insight into structural behavior. Rather than defining a procedure and a set of ’blind’ formulas, the designer has a better understanding of the implications of each formulation, and is asked to exercise good engineering judgment in following what may be considered a ‘design by analysis’ approach. The definition of limit states also paves the way to an eventual adoption of probabilistic or reliability based methods, such as the load and resistance factor design (LRFD) method. The LRFD approach is already adopted in several codes of practice, see References 2.2 through 2.6. The LRFD approach requires a statistical description of the design variables which define loading and resistance. It is then possible to account for those uncertainties that have a stronger impact on performance, such as:

a. Sensitivity of the structural element resistance to residual stresses and initial geometric imperfections;
b. Uncertainty in procedures used to convert loads to load effects;
c. Unfavorable deviations of the loads from their calculated values allowing for unforeseen action;

d. Deviation in material strengths from those used to calculate resistance;
e. Reduced probability that components of loading combinations will act simultaneously at their full levels;
f. Accuracy of theory used to calculate the characteristic resistance of a section.

Extensive research has been conducted in the area of reliability based design, see References 2.7 through 2.10. A simple assessment of safety can be done on the basis of approximate Level II reliability methods, which provide a systematic way of deriving partial safety factors. Level II methods make use of the safety or reliability index, which is related to a notional probability of failure. Important steps in such a procedure include the definition of limit state functions, and the derivation of partial safety factors for a given target reliability level.

First-order second-moment methods are based on a first-order (linear) approximation of the failure variables, and the only required information regarding the probabilistic description of the random variables is their mean and variance, which makes these methods attractive from the design point of view. First-order second-moment methods are very simple to implement, see for example Reference 2.11, and the designer might wish to use them in checking the reliability level of the structure's main components.

If a reliability approach to design is adopted, partial safety factors reflecting uncertainties in different load and resistance design variables can be derived. For a limit state representing a combination of loads several partial safety factors would be used, rather than the single factor of safety F.S.

In future revisions and refinements of Bulletin 2V the adoption of a probabilistic approach to safety should be considered. This would contribute to a more efficient and balanced design, and would follow the path already established by existing and well-established codes of practice.

C2.5.1 Factors of safety

The first edition of API Bulletin 2V specifies a basic factor of safety of 1.67 for the serviceability limit state and 2.0 for the ultimate limit state. *DNV*'s working stress method is based on usage factors, the minimum being 0.6 for the serviceability limit state and 0.6 for the ultimate limit state. This corresponds to a basic factor of safety of 1.67 for the serviceability and ultimate limit states. Clearly, there is a major discrepancy in the basic premise behind the working stress design philosophy. API Bulletin 2V is clearly too conservative with respect to the ultimate limit state and needs revision and clarification.

It is unclear how the Bulletin 2V (first edition) factors of safety were originally developed. The first edition Commentary makes specific reference to API RP 2T but does not quantify the factors of safety. The quantification occurs in the body of RP 2T but its commentary makes no reference to this quantification either. The 2T Commentary does make reference to the safety factors used in API RP 2A, which are basically 1.67 to 2.0 for longitudinal stress and 2.0 for pressure which, in the context of Bulletin 2V, is not directly related to limit state.

Hence, among RPs 2A and 2T, Bulletins 2U and 2V and *DNVDNV*, there are a multitude of safety factor formulations, all of which are somewhat inconsistent with each other.

Factors of safety should be set to provide consistent reliability of all structure components considering analysis and design unknowns or variabilities. Since Bulletins 2U and 2V cover similar types of structure (i.e., orthogonally stiffened cylinders and flat plate structures), it is logical that both bulletins should have a similar basis for factors of safety. Bulletin 2U, with its partial factors of safety, is more in line with RP 2A, at least for longitudinal stresses.

For orthogonally stiffened structures, it is unclear why a factor of safety of 2.0 should always apply, especially in the inelastic range, considering that most of the applied pressure is static and well defined for floating structures, especially at depth (this applies to API RP 2A also). API RP 2A's higher factor of safety for external pressure is based on the sensitivity of cylindrical shells to geometric imperfections at D/t less than 300. For orthogonally stiffened cylindrical shells designed with a hierarchical order of buckling mode instability, geometrical imperfections have a negligible effect on critical buckling stresses. Relative to cylindrical plate panels, flat plate panels have increased post-buckling strength; hence, the higher safety factor of 2.0 may not be warranted for orthogonally stiffened flat plate configurations.

In order to maintain consistency with Bulletin 2U, it is recommended that the factor of safety for the ultimate limit state in Bulletin 2V be revised to 1.67 times a partial safety factor that varies from 1.2 at the proportional limit to 1.0 at the yield stress. The safety factor for the serviceability limit state should remain at 1.67. This revision brings the safety factor formulation in line with that of *DNV* for serviceability limit states and provides a more conservative design for ultimate limit states in the elastic range, a design range that is undesirable and inefficient.

C3 PLATES

C3.2 UNIAXIAL COMPRESSION AND IN-PLANE BENDING

C3.2.2 Serviceability Limit State

Elastic Behavior. The elastic buckling of simply supported rectangular plates uniformly compressed in one direction, Figure C3-1, is a classical structures problem first solved by Bryan in 1891, and well-documented in several textbooks, e.g. References 3.1, 3.2, 3.3. In this solution the common assumptions of perfect material and geometry are adopted, namely the material is linear elastic, isotropic and homogeneous, the plate is thin and perfectly flat, the load is applied on the mid-plane of the plate, the deformations are small, shear effects are disregarded, and the direct stresses normal to the plate's through thickness direction are zero.

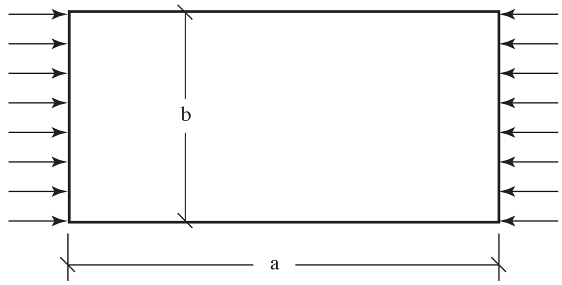

Figure C3-1—Rectangular Plate Under
Uniaxial Compression

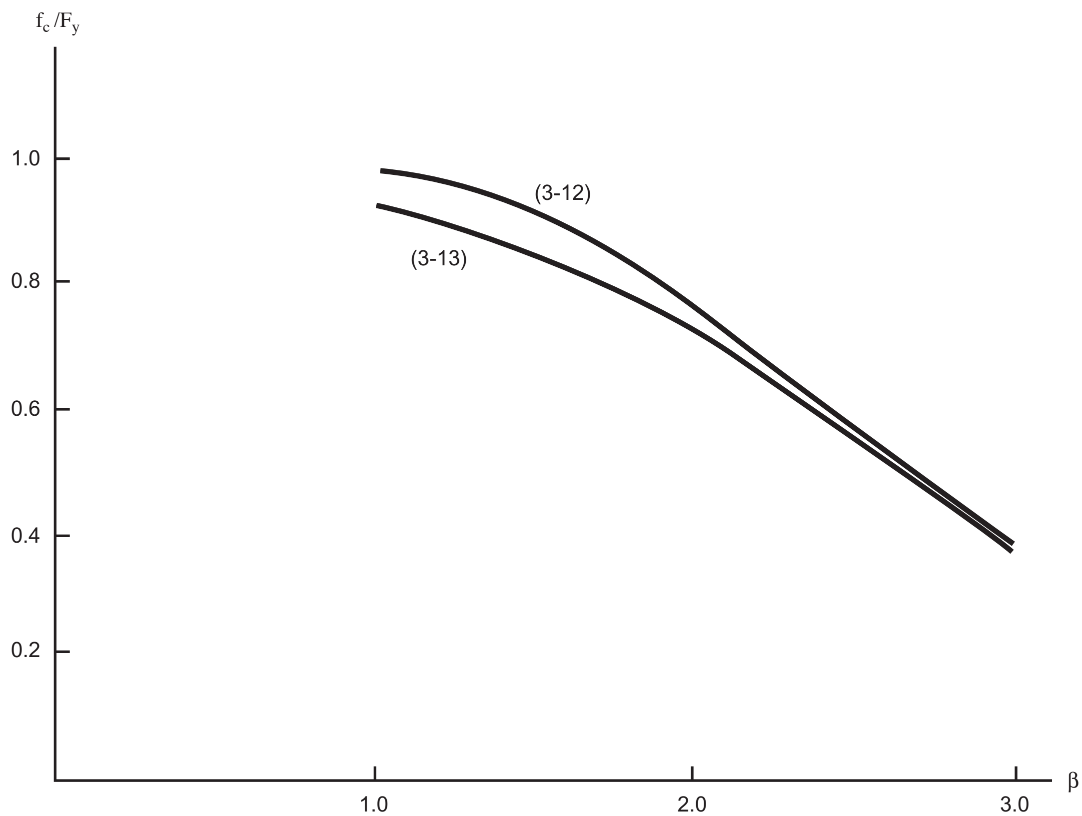

Figure C3-2—Comparison of Inelastic Buckling Formulations for
Rectangular Plates Under Uniaxial Compression

The critical stress f_{cr} is given by:

$$f_{cr} = \left(m\frac{b}{a}+\frac{1}{m}\frac{a}{b}\right)^2 \frac{\pi^2 E}{12\left(1-v^2\right)}\left(\frac{t}{b}\right)^2 \qquad \text{(C3-1)}$$

where m is the integer number of half-waves in which the plate buckles, a is the plate's length (its dimension along the direction of loading), b is the plate's width (the dimension of its loaded edges), t the plate's thickness, E the modulus of elasticity and v is Poisson's ratio.

For a given plate the critical stress can be determined from equation (C3-1) by choosing the value of m which makes f_{cr} a minimum, and this can easily be accomplished by using a graphical representation. As shown in Reference 3.1, the transition from m to m+1 half-waves occurs for the following value of the aspect ratio a/b:

$$\frac{a}{b} = \left[m(m+1)\right]^{1/2} \qquad \text{(C3-2)}$$

For long plates ($a / b \geq 1$) it can easily be shown that the following bound applies:

$$4.0 \leq \left(m\frac{b}{a}+\frac{1}{m}\frac{a}{b}\right)^2 \leq 4.49 \qquad \text{(C3-3)}$$

Thus for practical purposes when considering long plates it is reasonable to adopt the value 4.0 for the term in parenthesis in equation (C3-3), since it represents a lower bound to the exact critical value. For wide plates (a/b < 1) m = 1 always applies independently of the exact value of the aspect ratio a/b.

On the basis of the foregoing discussion the following expressions are proposed for computing the critical stress of simply supported rectangular plates uniformly compressed in one direction:

$$f_{cr} = k\frac{\pi^2 E}{12\left(1-v^2\right)}\left(\frac{t}{b}\right)^2 \qquad \text{(C3-4)}$$

$$k = 4.0 \text{ for } \frac{a}{b} \geq 1 \qquad \text{(C3-4a)}$$

$$k = \left(\frac{a}{b}+\frac{b}{a}\right)^2 \text{ for } \frac{a}{b} < 1 \qquad \text{(C3-4b)}$$

These equations are adopted in Bulletin 2V to define the serviceability limit state in the elastic range for uniform compression. Note that the expression for k for wide plates in Bulletin 2V is slightly different, because the aspect ratio is always assumed to be larger than unity, as in Figure C3-1.

For future reference it is convenient to express equation (C3-4) in terms of the plate slenderness or width factor β:

$$\beta = \frac{b}{t}\sqrt{\frac{F_y}{E}} \qquad \text{(C3-5)}$$

where F_y is the material yield stress. Then equation (C3-4) takes the form:

$$\frac{f_{cr}}{F_y} = k\,\frac{\pi^2}{12\left(1-\nu^2\right)}\frac{1}{\beta^2} \qquad \text{(C3-6)}$$

The above formulas apply in the linear elastic range, defined by the material proportional limit F_p. For $f_{cr} > F_p$ inelastic effects have to be taken into consideration.

The above solution applies to the case where the plate is simply supported around the four edges. It is obvious that in a real case the boundary conditions can significantly depart from this ideal situation, since in general surrounding stiffeners will give the plate a varying degree of rotational restraint. However, from the design point of view, assuming simple support conditions is reasonable, since the results lie on the conservative side. Also, in most practical situations the plate geometry is such that failure will be determined by plastic effects and imperfections, so that the exact form of the elastic buckling equation is not very relevant. It should be noted that the approach suggested here is the one proposed by *DNV*, equation (C2-1) in Reference 3.4.

In Bulletin 2V buckling coefficients k for boundary conditions other than simply supported are also given, to be used if other conditions are known to govern the design. Figure 3.2-3 in Bulletin 2V was adapted from Reference 3.3.

For the second edition, expressions for k were revised to include the effect of in-plane bending. Neglecting the effect of in-plane bending is unconservative while including the bending stress as uniform compression is unduly conservative. The revised expressions are based on classical solutions and follow those of *DNV* (Reference 3.30).

Inelastic Behavior. When the critical stress f_{cr} as given by equation (C3-4) exceeds the proportional limit, inelastic effects have to be taken into consideration. The approach which is normally suggested in the literature implies using the equations which apply in the elastic range, with the modulus of elasticity E replaced by the tangent modulus E_t, or a function of E_t, Reference 3.5.

There is no unique way of defining an appropriate value for E_t. Bleich, Reference 3.5, suggests a quadratic parabolic approximation which is often referred to as the Ostenfeld-Bleich quadratic parabola, Reference 3.6:

$$\frac{E_t}{E} = \frac{f_c\left(F_y - f_c\right)}{F_p\left(F_y - F_p\right)} \qquad \text{(C3-7)}$$

where f_c is the ultimate average stress. Equation 3-7 seems to be quite adequate for materials having a well defined yield plateau. In this approximation the stress strain curve is assumed to be a straight line up to the proportional limit F_p, and a quadratic parabola from the proportional limit F_p to the yield point.

For strain-hardening materials the Ramberg-Osgood three parameter stress-strain relation is usually adopted, Reference 3.6:

$$\frac{E_t}{E} = \left[1 + \frac{3n}{7}\left(\frac{f}{F_y}\right)^{n-1}\right]^{-1} \qquad \text{(C3-8)}$$

where n is an empirical constant derived from curve fitting.

In Reference 3.7, Bleich suggests the following expression for the inelastic critical stress for a rectangular plate under uniaxial compression:

$$f_c = k\frac{\pi^2 E\eta}{12\left(1-v^2\right)}\left(\frac{t}{b}\right)^2 \qquad \text{(C3-9)}$$

where all the parameters have been defined, except η, which is a modulus factor, or characteristic of the plate material, equal to unity when f_{cr} is equal to or below the proportional limit, and smaller than unity, varying with f_{cr}, when the critical stress exceeds the proportional limit.

For long plates ($a/b \geq 1$) Bleich, Reference 3.7, suggests the following value for η:

$$\eta = \sqrt{\frac{E_t}{E}} \qquad \text{(C3-10)}$$

with E_t / E given by equation (C3-7). Then, combining equations (C3-4), (C3-9), (C3-10), and (C3-7) the following expression for f_c can be obtained:

$$\frac{f_c}{F_y} = \frac{C}{1+C} \qquad \text{(C3-11)}$$

where

$$C = \frac{\left(f_{cr} / F_y\right)^2}{p_r\left(1-p_r\right)} \qquad \text{(C3-11a)}$$

$$p_r = \frac{F_p}{F_y} \qquad \text{(C3-11b)}$$

f_{cr} is the elastic buckling stress given by equation 3-4. The stress ratio p_r defines the beginning of inelastic effects in compression, and a typical value for welded ship panels is 0.5, Reference 3.6. For p_r = 0.5 the inelastic buckling stress becomes:

$$\frac{f_c}{F_y} = \frac{4\left(f_{cr} / F_y\right)^2}{1 + 4\left(f_{cr} / F_y\right)^2} \qquad \text{(C3-12)}$$

DNV proposes the following expression for f_c [using the notation adopted here, see Reference 3.4, equation (C2-1)]:

$$\frac{f_c}{F_y} = 1 - \frac{1}{4\left(f_{cr} / F_y\right)} \qquad \text{(C3-13)}$$

Equations (C3-12) and (C3-13) are plotted in Figure C3-2 as a function of the plate slenderness ratio β, for $v = 0.3$ and $k = 4$. It can be concluded that *DNV*'s formula is slightly more conservative than Bleich's equation (C3-12). In Bulletin 2V, equation (C3-11) is adopted to define the serviceability limit state for long plates in the plastic range.

For wide plates ($a/b < 1/2$) Bleich suggests that the collapse stress is again given by equation (C3-9) with η now defined as

$$\eta = \frac{E_t}{E} \qquad \text{(C3-14)}$$

with E_t /E again given by equation (C3-7). Combining equations (C3-4), (C3-9), (C3-14), and (C3-7) the following expression for f_c can be obtained:

$$\frac{f_c}{F_y} = 1 - \frac{p_r\left(1 - p_r\right)}{f_{cr} / F_y} \qquad \text{(C3-15)}$$

where f_{cr} is given by equation (C3-4) with k defined by equation (C3-4b). In the range $1/2 < a/b < 1$, Bleich suggests that the critical stress may be calculated from the interpolation formula.

$$\frac{f_c}{F_y} = 2\left(\frac{f_{c1}}{F_y} - \frac{f_{c2}}{F_y}\right)\frac{a}{b} + 2\frac{f_{c2}}{F_y} - \frac{f_{c1}}{F_y} \qquad \text{(C3-16)}$$

where f_{c1} is found from equation (C3-11) and f_{c2} from equation (C3-15). Comparing equations (C3-11) and (C3-15) for p_r = 0.5, it can easily be concluded that they give results which are quite close, with equation (C3-11) lying roughly less than 7% above equation (C3-15). Thus in practical applications there does not seem to be a need for using the linear interpolation scheme expressed by equation (C3-16).

In Bulletin 2V equation (C3-15) is adopted to define the serviceability limit state for wide plates in the plastic range.

It is interesting to note that *DNV*'s equation (C2-1), Reference 3.4, which is suggested for both long and wide plates, corresponds to equation (C3-15) with p_r = 0.5. This is a reasonable way of representing by a single expression the inelastic buckling of both long and wide plates.

C3.2.3 Ultimate Limit State

Long Plates. The concept of effective width is widely used in structural engineering to estimate the ultimate strength of rectangular plates. An extensive review of the subject is given in Reference 3.8. The effective span of plating required for computing section properties, as discussed in Reference 3.8, is given by:

$$b_e = \frac{f_a}{f_e} b \tag{C3-17}$$

where f_e is the edge stress in the plating, f_a the average stress, and b the plate element width over which uniform compression strain is applied. If it is postulated that the maximum post-buckling load the plate can sustain occurs when the edge stress f_e reaches the yield stress, then we have from equation (C3-17):

$$\frac{f_a}{F_y} = \frac{b_e}{b} \tag{C3-18}$$

In Reference 3.8, the following empirical formula for the effective width ratio of simply supported plates is proposed:

$$\frac{f_a}{F_y} = \frac{b_e}{b} = \frac{2}{\beta} - \frac{1}{\beta^2}, \beta \geq 1 \tag{C3-19}$$

This expression has been found to provide excellent agreement with strut-panel test data and with recent box-girder bridge reviews. In Bulletin 2V it is adopted to define the ultimate limit state for long plates. The generalized form of this equation for use in the restricted range 0.7 $F_y \leq f_e \leq F_y$ and $\beta \geq 1$ is:

$$\frac{f_a}{f_e} = \frac{b_e}{b} = \frac{2}{\beta}\sqrt{\frac{F_y}{f_e}} - \frac{1}{\beta^2}\left(\frac{F_y}{f_e}\right) \tag{C3-20}$$

In Reference 3.8, Faulkner also defines a reduced effective width b_e', or tangent width, which is intended to take account of a possible stiffener failure before the edge stress in the plate elements has reached the yield stress. In the case of simply supported plates the reduced effective width for $\beta \geq 1$ is given by:

$$\frac{b_e'}{b} = \frac{1}{\beta} \tag{C3-21}$$

and if the edge stress f_e is smaller than F_y we have

$$\frac{b_e'}{b} = \frac{1}{\beta}\sqrt{\frac{F_y}{f_e}} \tag{C3-22}$$

In the case where the longitudinal stiffeners in a panel are torsionally strong, or where the lateral pressure is sufficiently large (say larger than $F_y^2 / E\beta^2$), clamped boundary conditions for the plate might be more appropriate, Reference 3.8. The following expressions then apply for $\beta \geq 1.25$ and $0.7 F_y \leq f_e \leq F_y$:

$$\frac{b_e}{b} = \frac{2.5}{\beta} - \frac{1.5625}{\beta^2} \tag{C3-23}$$

$$\frac{b_e}{b} = \frac{2.5}{\beta}\sqrt{\frac{F_y}{f_e}} - \frac{1.5625}{\beta^2}\frac{F_y}{f_e} \tag{C3-24}$$

$$\frac{b'_e}{b} = \frac{1.25}{\beta} \tag{C3-25}$$

$$\frac{b'_e}{b} = \frac{1.25}{\beta}\sqrt{\frac{F_y}{f_e}} \tag{C3-26}$$

Wide Plates. Several formulations have been proposed for the ultimate strength of wide plates, and these will now be briefly discussed. For convenience a different notation will be used, as indicated in Figure C3-3. Now the length of the loaded edges is *a*, and *a /b* > 1.

In Reference 3.6 Faulkner refers to the following formula proposed by Bureau Veritas for simply supported wide plates:

$$\frac{f_u}{F_y} = \frac{0.9}{\beta^2} + \frac{1.9}{\alpha\beta}\left(1 - \frac{0.9}{\beta^2}\right) \tag{C3-27}$$

where the aspect ratio is $\alpha = a/b > 1$.

As reported also in Reference 3.8 Schnadel proposes the following formula for simply supported wide plates:

$$\frac{f_u}{F_y} = \frac{1}{3}\left(1 + 2\frac{f_E}{F_y}\right) \tag{C3-28}$$

where

$$\frac{f_E}{F_y} = \frac{\pi^2}{12\left(1 - v^2\right)}\frac{1}{\beta^2}\left(1 + \frac{1}{\alpha^2}\right)^2 \tag{C3-29}$$

In Reference 3-5 Bleich proposed the following formula resulting from an extension of Marguerre’s theory:

$$\frac{f_u}{F_y} = \frac{1+\alpha^4}{1+3\alpha^4} + \frac{2\alpha^4}{1+3\alpha^4}\left(\frac{f_E}{F_y}\right) \tag{C3-30}$$

where the ratio f_E/F_y is again given by equation (C3-29). As indicated in Reference 3.8, for large values of α the results given by equations (C3-28) and (C3-30) practically coincide.

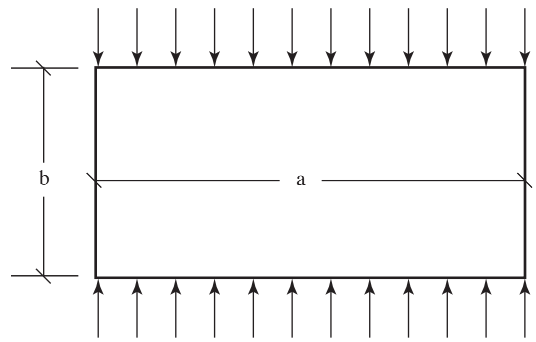

Figure C3-3—Wide Rectangular Plate

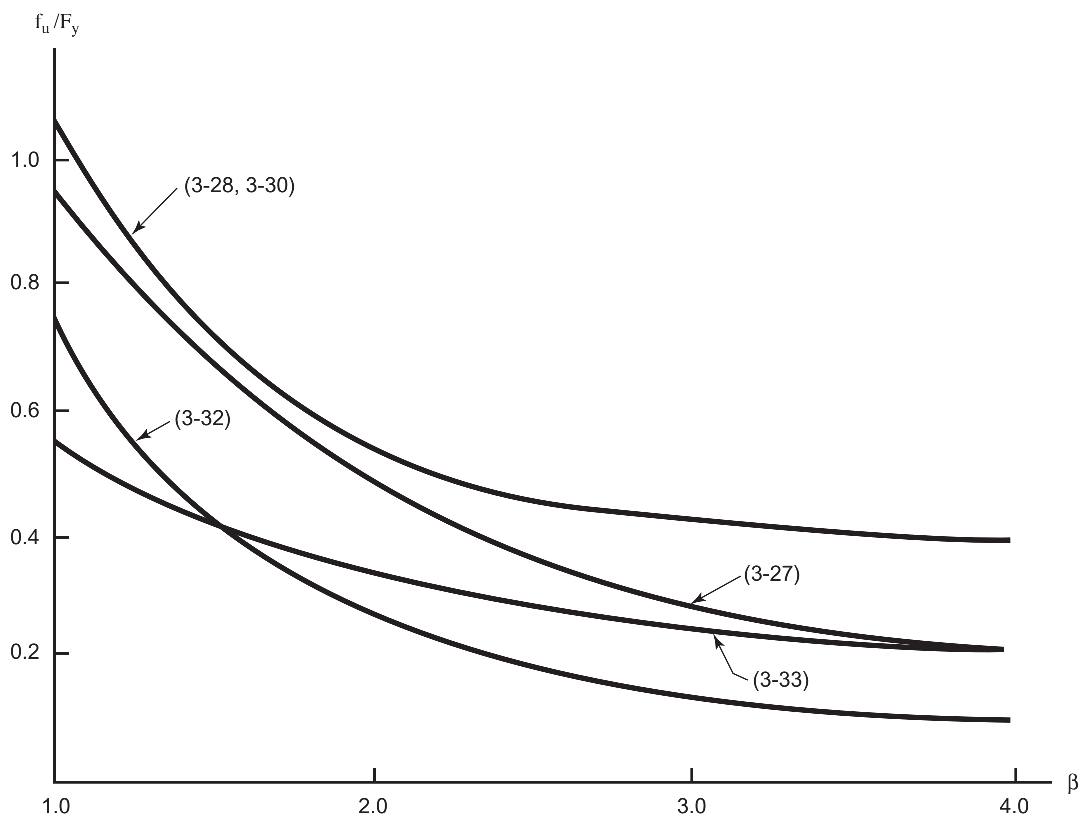

Figure C3-4—Comparison of Formulations for the Ultimate Strength of Wide Plates with *a*/*b* = 3

In Reference 3.9 Evans proposes the following wide plate ultimate strength formula:

$$f_u = 0.175\frac{\pi^2 E}{12(1-\nu^2)}\left(\alpha+\frac{1}{\alpha}\right)^{1.25}\left(\frac{t}{a}\right)^{1.5} \qquad \text{(C3-31)}$$

and in terms of the plate slenderness parameter β this can be recast in the form:

$$\frac{f_u}{F_y} = 0.175\frac{\pi^2}{12(1-\nu^2)}\left(\alpha+\frac{1}{\alpha}\right)^{1.25}\frac{1}{\alpha^{1.5}\beta^{1.5}}\left(\frac{E}{F_y}\right)^{1/4} \qquad \text{(C3-32)}$$

In Reference 3.10 Valsgard proposes the following formula for the transverse compression of simply supported and unrestrained plates:

$$\frac{f_u}{F_y} = \frac{1}{\alpha}C_{x1} + 0.08\left(1-\frac{1}{\alpha}\right)\left(1+\frac{1}{\beta^2}\right)^2 \le 1.0 \qquad \text{(C3-33)}$$

$$C_{x1} = \frac{2}{\beta}-\frac{1}{\beta^2}, \beta \ge 1 \qquad \text{(C3-33a)}$$

$$C_{x1} = 1.0, \beta < 1 \qquad \text{(C3-33b)}$$

Equations (C3-27), (C3-28), (C3-30), (C3-32), and (C3-33) are plotted in Figure C3-4 for α= 3. Equation (C3-32) provides the most conservative prediction, a fact which is discussed and explained in Reference 3.6. Valsgard's curve intersects equation (C3-32), but it lies below the remaining curves. This curve also appears in DNV Classification, Note 30.1 with the "0.08" coefficient revised to "0.10." Since this information is based on a study in which extensive numerical analysis and correlation with experimental data were performed, it is thought to be the most adequate, and it is adopted with the revised 0.10 coefficient in Bulletin 2V to represent the ultimate limit state.

C3.3 EDGE SHEAR

C3.3.2 Serviceability Limit State

Elastic Behavior. The elastic buckling of simply supported rectangular plates subjected to uniform edge shear is well documented in several textbooks, e.g., References 3.1, 3.2, 3.3. The critical stress can be written in the following form:

$$f_{xycr} = k\frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2 \qquad \text{(C3-34)}$$

where k can be approximated by:

$$k = 5.34 + 4\left(\frac{b}{a}\right)^2, \frac{b}{a} \le 1 \qquad \text{(C3-35)}$$

As a result of symmetry the above formula applies to both long and wide plates, and the sides should always be labeled a and b, such that $b/a \le 1$. Equation (C3-34) is adopted in the *DNV*

Rules, equation (C2-1), Reference 3.4. It is also adopted in Bulletin 2V to represent the serviceability limit state in the elastic range.

Plates built-in along all edges have been studied by Budiansky and Conner, Reference 3.11, who computed values of *k* by the Lagrange multiplier method. An approximate parabolic curve, fitting the results, is

$$k = 8.98 + 5.6\left(\frac{b}{a}\right)^2, \frac{b}{a} < 1 \qquad \text{(C3-36)}$$

This equation is also adopted in Bulletin 2V.

In terms of the slenderness factor β equation (C3-34) can be written in the following form:

$$\frac{\sqrt{3} f_{xycr}}{F_y} = k \frac{\sqrt{3}\pi^2}{12\left(1-\nu^2\right)} \frac{1}{\beta^2} \qquad \text{(C3-37)}$$

For future reference it is convenient to write equation (C3-34) in non-dimensional terms as follows:

$$\frac{\sqrt{3} f_{xycr}}{F_y} = \frac{1}{\lambda^2} \qquad \text{(C3-38)}$$

where the slenderness ratio λ is given by

$$\lambda = 0.8 \frac{b}{t} \sqrt{\frac{F_y}{kE}} \qquad \text{(C3-39)}$$

The foregoing results are valid when the applied stress remains in the elastic range, or

$$\frac{\sqrt{3} f_{xycr}}{F_y} \leq p_r \qquad \text{(C3-40)}$$

Inelastic Behavior. Following an approach similar to the one adopted for plates under uniaxial compression, as suggested by Bleich in Reference 3.15, the critical shear stress now takes the form:

$$f_{xyc} = \frac{f_i}{\sqrt{3}} \qquad \text{(C3-41)}$$

where

$$f_i = k \frac{\pi^2 E\sqrt{3}}{12\left(1-\nu^2\right)} \left(\frac{t}{b}\right)^2 \sqrt{\frac{E_t}{E}} \qquad \text{(C3-42)}$$

and *k* is the buckling coefficient for elastic buckling, given by equations (C3-35) or (C3-36), depending on the boundary conditions. Combining equations (C3-34), (C3-41), and (C3-42) we obtain the following expression for f_{xyc},

$$f_{xyc} = f_{xycr} \sqrt{\frac{E_t}{E}} \qquad \text{(C3-43)}$$

Again using the Ostenfeld-Bleich quadratic parabola, equation (C3-7), with $f_c = \sqrt{3} f_{xyc}$ (implying the von Mises yield criterion), the following expression for f_{xyc} can be obtained:

$$\frac{\sqrt{3} f_{xyc}}{F_y} = \frac{C}{1+C} \tag{C3-44}$$

where

$$C = \frac{\left(\sqrt{3} f_{xycr} / F_y\right)^2}{p_r (1 - p_r)} \tag{C3-45}$$

and where as defined earlier $p_r = F_p / F_y$. Equation (C3-44) is valid when the proportional limit of the material is exceeded, or when $\sqrt{3} f_{xycr} / F_y > p_r$. This equation is adopted in Bulletin 2V to represent the serviceability limit state in the plastic range.

DNV proposes the following expression for f_{xyc} (using the notation adopted here, equation (C2-1) in Reference 3.4):

$$\frac{\sqrt{3} f_{xyc}}{F_y} = 1 - \frac{1}{4\left(\sqrt{3} f_{xycr} / F_y\right)} \tag{C3-46}$$

It is interesting to note that equations (C3-44) and (C3-46) are formally identical to equations (C3-11) and (C3-13), respectively. Thus, as discussed earlier, for p_r = 0.5 these two formulations (the one due to Bleich and represented by equation (C3-44), and the one suggested by *DNV*, equation (C3-46), are quite similar.

It is useful, for comparison purposes to express equations (C3-44) and (C3-46) in terms of the slenderness ratio defined by equation (C3-39). Equation (C3-44) becomes:

$$\frac{\sqrt{3} f_{xyc}}{F_y} = \left[1 + \lambda^4 p_r (1 - p_r)\right]^{-1} \tag{C3-47}$$

and equation (C3-46) becomes:

$$\frac{\sqrt{3} f_{xyc}}{F_y} = 1 - \frac{\lambda^2}{4} \tag{C3-48}$$

In Reference 3.12 Ostapenko proposes the following expression for the collapse shear stress in the inelastic range:

$$\frac{\sqrt{3} f_{xyc}}{F_y} = 1, \lambda \le 0.58 = 1/\sqrt{3} \tag{C3-49}$$

$$\frac{\sqrt{3} f_{xyc}}{F_y} = 1 - 0.618(\lambda - 0.58)^{1.18}, \quad 0.58 \le \lambda < 1.41 \tag{C3-50}$$

Equations (C3-47), (C3-48), (C3-49), and (C3-50) are plotted in Figure C3-5 for p_r = 0.5, which indicates that Bleich's curve, equation (C3-47) is less conservative. It is felt, however, that it is quite adequate for design purposes, since it provides the possibility of adopting a proportional limit ratio p_r different from 0.5.

C3.3.3 Ultimate Limit State

The ultimate shear capacity of flat plates has been the subject of extensive research, particularly in the context of the design of plate girders loaded in shear.

The AISC Specification, Reference 3.13, uses Basler's approach, References 3.14 and 3.15, which leads to the following equation for the allowable shear stress:

$$f_{xyu} = 0.6\tau_y \left[C_v + \frac{1 - C_v}{1.15\sqrt{1 + (a/b)^2}} \right] \le 0.7\tau_y \qquad \text{(C3-51)}$$

where

$$\tau_y = \frac{F_y}{\sqrt{3}}$$

$$C_v = \frac{f_{xycr}}{\tau_y} \text{ when } C_v \le 0.8$$

$$C_v = 0.9\left(\frac{f_{xycr}}{\tau_y} \right)^{1/2} \text{ when } C_v > 0.8$$

In Reference 3.16 Basler's theory is discussed, and it is pointed out that equation (C3-51) does not actually represent the true resistance of the Basler model, which is correctly given by:

$$f_{xyu} = f_{xycr} + \frac{F_y}{2\sqrt{1 + \alpha + \alpha^2}} \left(1 - \frac{f_{xycr}}{\tau_y} \right) \qquad \text{(C3-52)}$$

where $\alpha \ge 1$ is the plate's aspect ratio. In non-dimensional form equation (C3-52) can be written as follows:

$$\frac{\sqrt{3} f_{xyu}}{F_y} = \frac{\sqrt{3} f_{xycr}}{F_y} + \frac{\sqrt{3}}{2\sqrt{1 + \alpha + \alpha^2}} \left(1 - \frac{\sqrt{3} f_{xycr}}{F_y} \right) \qquad \text{(C3-53)}$$

In Balser's solution it is assumed that the edge girder flanges have insufficient flexural rigidity to resist diagonal tension, which is consequently reacted by the transverse stiffeners. As a result the transverse stiffeners are subject to compressive loading.

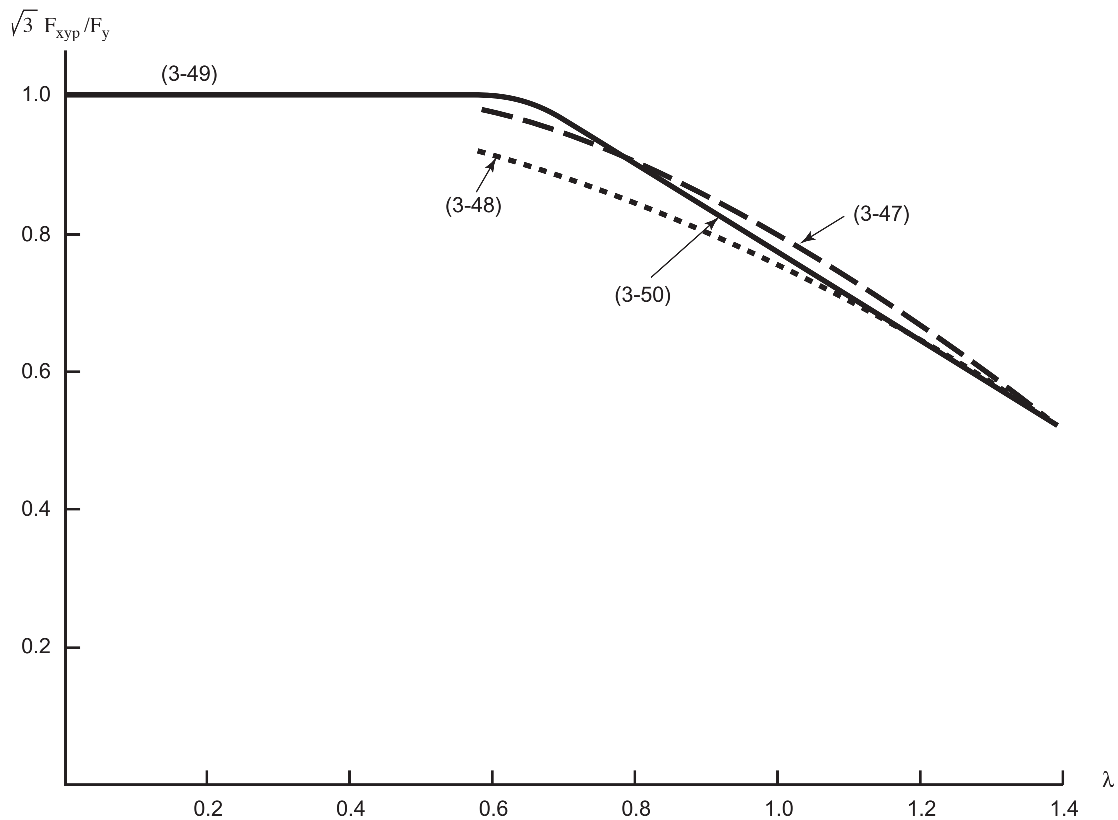

Figure C3-5—Comparison of Formulations for the Inelastic Buckling of Rectangular Plates Under Edge Shear

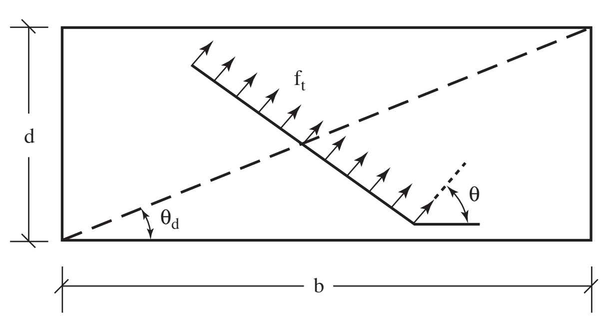

Figure C3-6—Model for the Ultimate Strength of Rectangular Plates in Shear

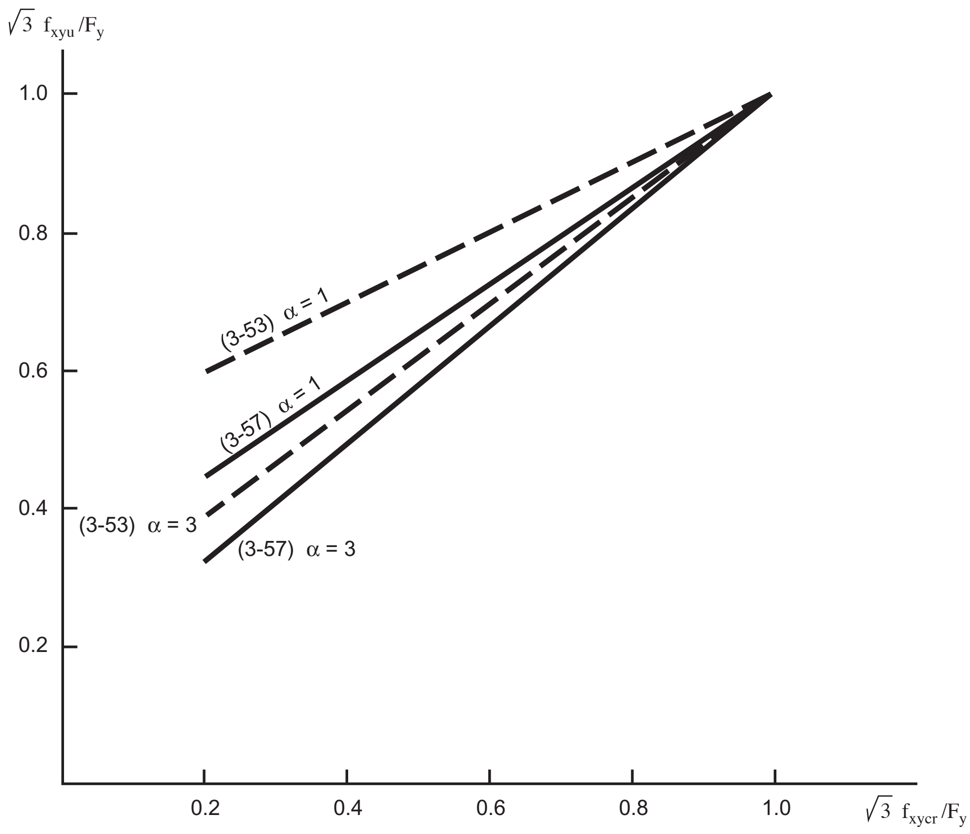

Figure C3-7—Comparison of Formulations for the Ultimate Strength of Rectangular Plates in Shear

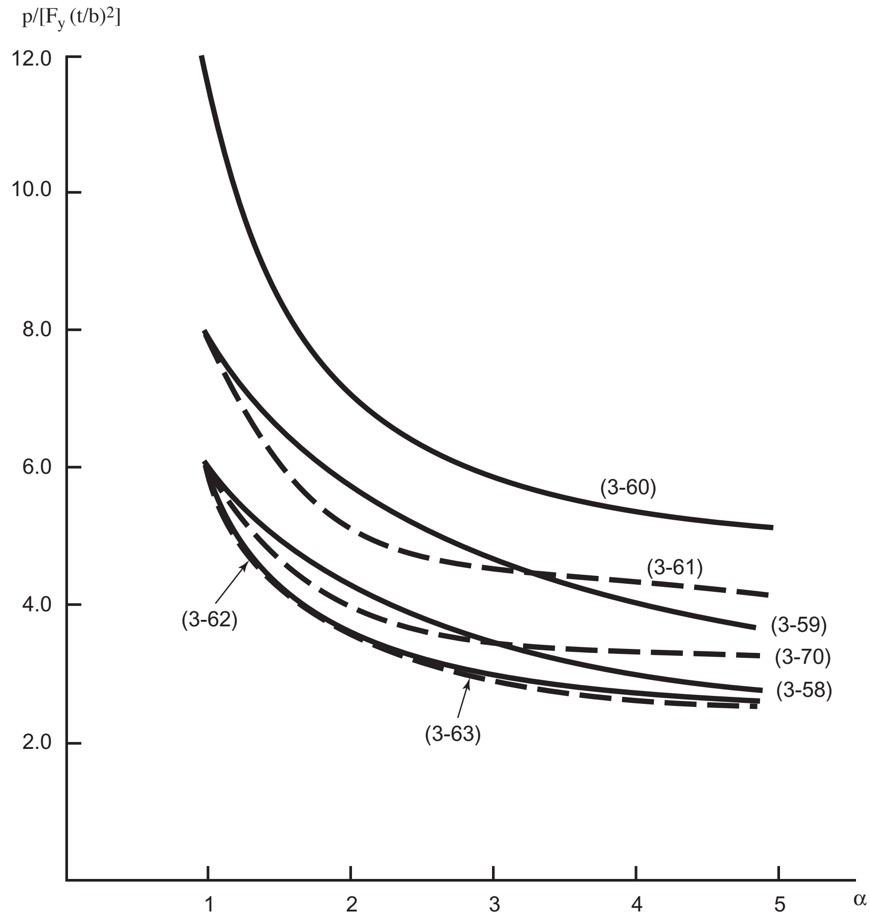

Figure C3-8—Comparison of Formulations for the Ultimate Strength of Rectangular Plates Under Lateral Pressure

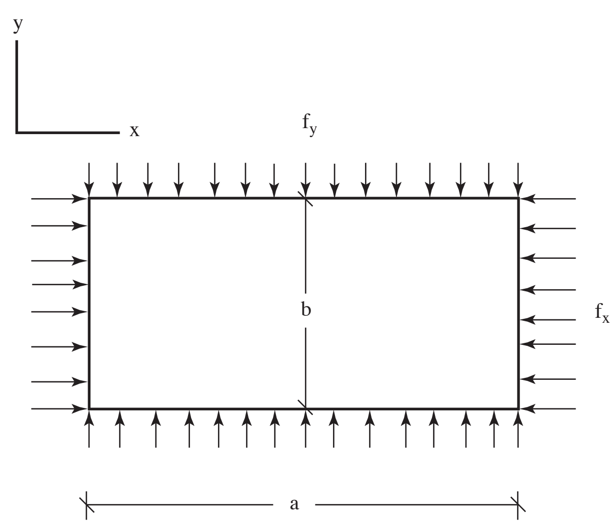

Figure C3-9—Rectangular Plate Under Biaxial Compression

In Reference 3.16 a model which takes into account the carrying capacity of the edge stiffeners is developed. It leads to the following equations:

$$f_{xyu} = 2\frac{c}{d} f_t \sin^2\theta + f_t \sin^2\theta(\cot\theta - \cot\theta_d) + f_{xycr} \qquad \text{(C3-54)}$$

$$\frac{c}{d} = \frac{2}{\sin\theta}\left(\frac{M_p}{f_t t d^2}\right)^{1/2}, 0 < c < b \qquad \text{(C3-55)}$$

$$f_t = -\frac{3}{2} f_{xycr}\sin\theta_d + \sqrt{F_y^2 + f_{xycr}^2\left[\left(\frac{3}{2}\sin 2\theta_d\right)^2 - 3\right]} \qquad \text{(C3-56)}$$

where M_p is the full plastic moment of the flange. The geometry related parameters θ_d, b, and d are defined in Figure C3-6. The angle θ is the inclination of the tensile membrane stress field f_t in the web. This angle is unknown and has to be found numerically, such that the maximum value of f_{xyu} is obtained. In most cases θ lies in the range $\theta_d/2 < \theta < \pi/4$.

Ostapenko, Reference 3.12, also suggests a model which follows along the lines of Basler's model, since it does not recognize the formation of the internal plastic hinges in the flanges. It leads to the following result:

$$\frac{\sqrt{3} f_{xyu}}{F_y} = \frac{\sqrt{3} f_{xycr}}{F_y} + \frac{1 - \sqrt{3} f_{xycr}/F_y}{2\sqrt{1.6 + \alpha^2}} \qquad \text{(C3-57)}$$

Figure C3-7 shows plots of equations (C3-53) and (C3-57), indicating that Ostapenko's model leads to more conservative results.

When designing stiffened plate structures it is obviously important to consider the strength of the stiffeners as well as the strength of the plate elements. If the stiffener's ultimate capacity is taken into account, it is reasonable to treat the plate independently, and it is recommended that the design be based on equation (C3-53). This equation is adopted in Bulletin 2V to represent the plate's ultimate limit state in shear.

C3.4 UNIFORM LATERAL PRESSURE

C3.4.2 Serviceability Limit State

The expression suggested in Bulletin 2V for estimating plate deflections in the elastic range can be derived from thin plate theory, as shown for example in Reference 3.17. The same applies to the maximum elastic stresses. The graphs in Bulletin 2V for computing elastic deflections and stresses (Figures 3.4-1 and 3.4-2) were adopted from Reference 3.18.

C3.4.3 Ultimate Limit State

There are many studies in the literature on the ultimate capacity of rectangular plates under uniform lateral pressure. In practice when considering rectangular plates which are supported

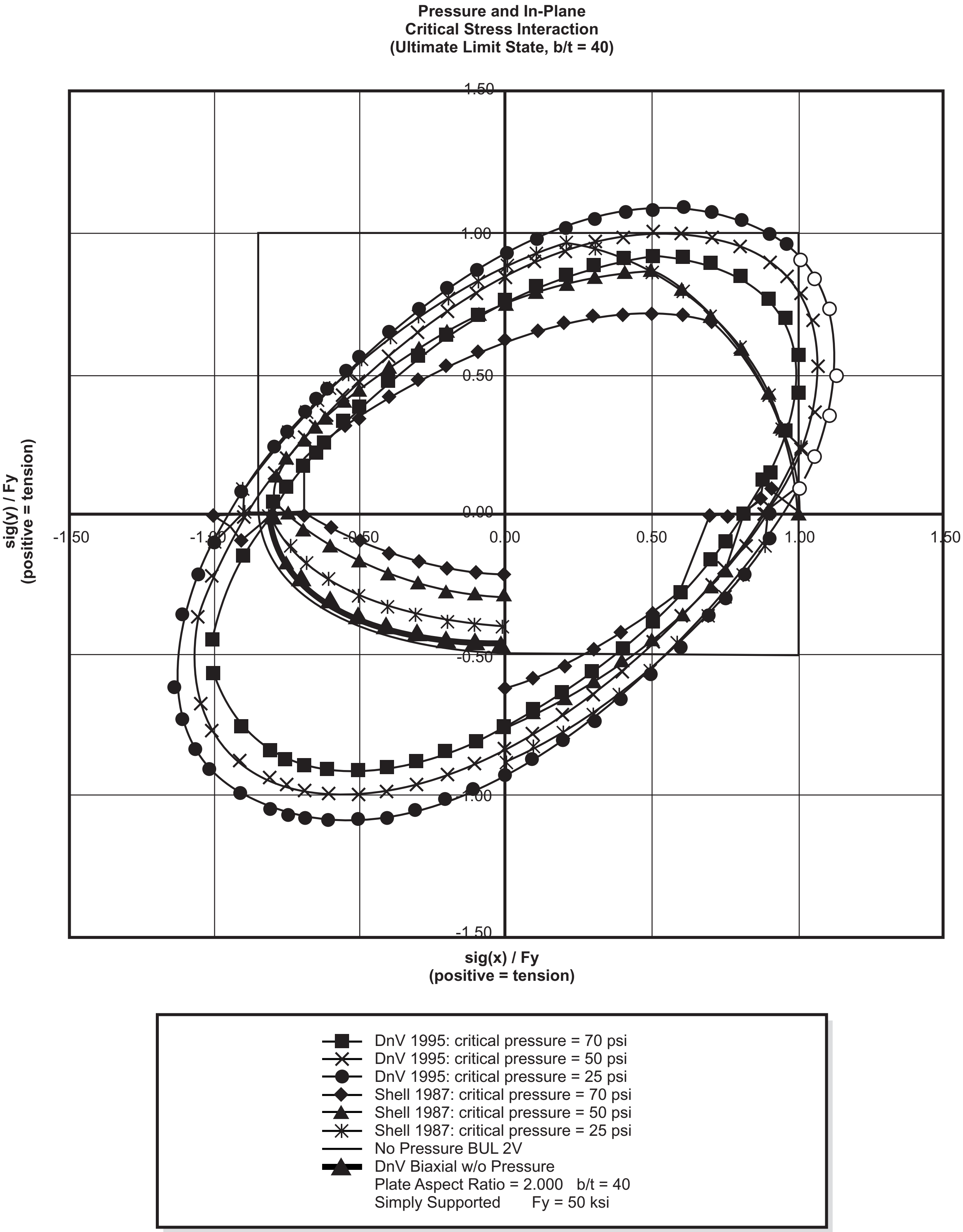

Figure C3-10—Combined In-Plane and Lateral Loads (*b*/*t* = 40)

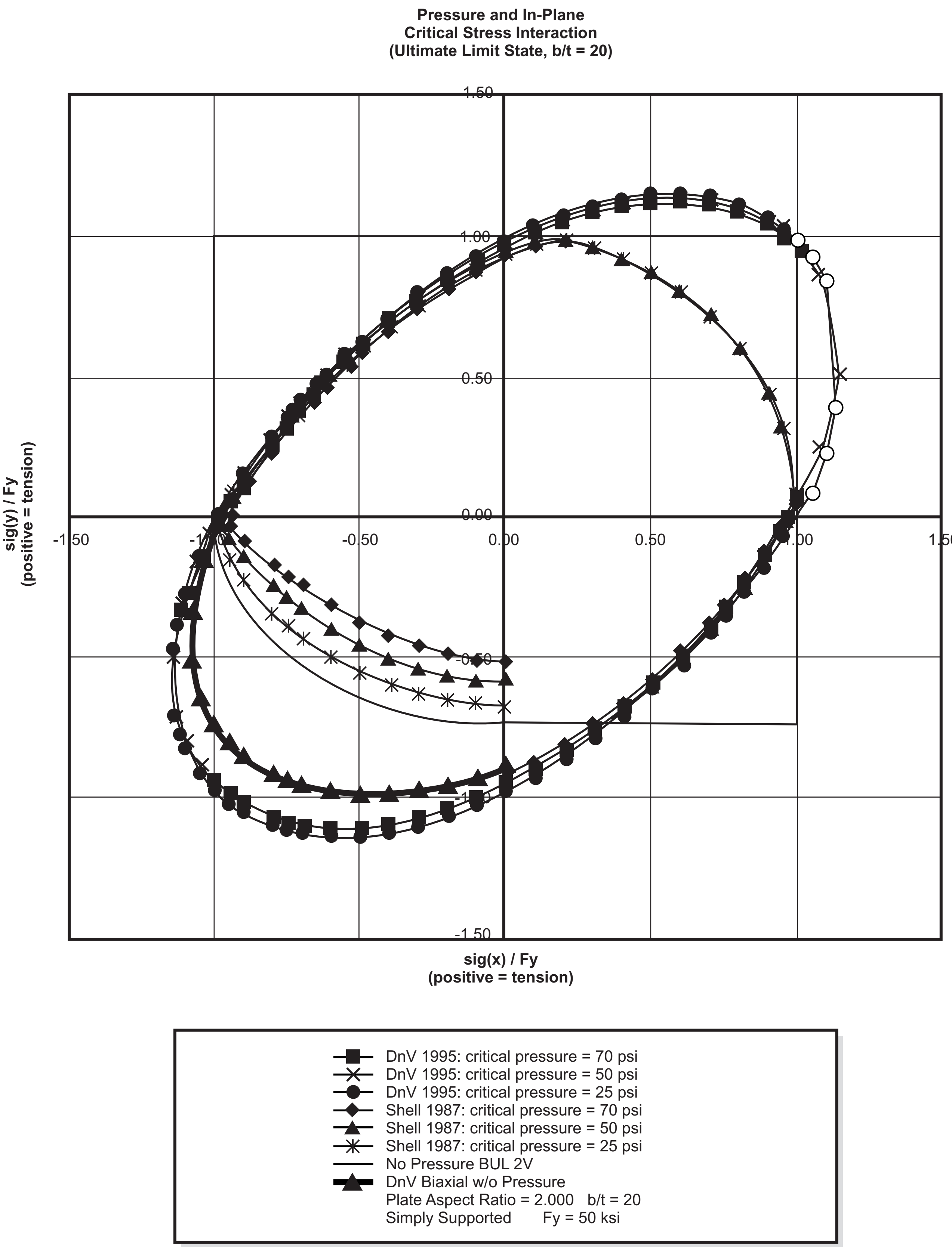

Figure C3-11—Combined In-Plane and Lateral Loads (*b*/*t* = 20) by orthogonal stiffening elements, the plate edges have a certain degree of in-plane axial and rotational restraint. If there is axial in-plane restraint the plate is able to resist lateral pressure by membrane action, and this provides a very large degree of reserve strength. The ultimate strength is in such cases primarily determined by fracture. Membrane action is to a large extent a function of lateral deflections, and as such it is reasonable to design plates on the basis of a certain allowable permanent set. A permanent set is desirable if lateral pressure acts alone, while it is in general undesirable if in-plane compression is also present, since then the buckling mode can easily be triggered.

In Reference 3.19 the following load/permanent set curves are suggested, based on a curve fitting study of Clarkson's experimental data in Reference 3.20:

$$p = \frac{6F_y(t/b)^2}{\sqrt{\alpha}}\left(1 + \frac{2W}{\alpha t}\right), \beta < 2.5 \qquad \text{(C3-58)}$$

$$p = \frac{6F_y(t/b)^2}{\sqrt{\alpha}}\left(\frac{4}{3} + \frac{2W}{\alpha t}\right), \beta \geq 2.5 \qquad \text{(C3-59)}$$

These expressions are restricted to rectangular steel plates with aspect ratios falling in the range 1 to 5. It should be noted that these two equations show a discontinuity for $\beta = 2.5$, with equation (C3-59) leading to larger values of p when the remaining parameters stay the same. No obvious explanation for this discontinuity is available.

An upper bound to the collapse pressure of clamped rectangular plates has been proposed by Johansen. It is an upper bound in the context of the Theorems of Limit Analysis, so that the plate's material is considered to be rigid-perfectly plastic, Reference 3.21. The collapse pressure is given by:

$$p = \frac{12F_y(t/b)^2\alpha^2}{\left(\sqrt{3\alpha^2+1}-1\right)^2} \qquad \text{(C3-60)}$$

A lower bound associated with the Johansen yield curve, as given in Reference 3.21, is

$$p = 4F_y\left(\frac{t}{b}\right)^2\left(1 + \frac{1}{\alpha^2}\right) \qquad \text{(C3-61)}$$

If the plate boundaries are simply supported the collapse pressure is one-half the value given by equation (C3-60), or:

$$p = \frac{6F_y(t/b)^2\alpha^2}{\left(\sqrt{3\alpha^2+1}-1\right)^2} \qquad \text{(C3-62)}$$

As discussed in Reference 3.21 a lower bound is in this case given by:

$$p = 2F_y\left(\frac{t}{b}\right)^2\left(1 + \frac{1}{\alpha} + \frac{1}{\alpha^2}\right) \qquad \text{(C3-63)}$$

As discussed earlier, membrane effects play an important role in the behavior of rectangular plates under uniform lateral pressure. Jones and Walters, Reference 3.22, developed expressions for the collapse pressure when finite plastic deflections are taken into account. These are based on the material rigid plastic assumption, and follow an approach similar to the upper bound method adopted in the derivation of equations (C3-60) and (C3-62). However, it should be noted that the results given by Jones and Walters cannot in a strict sense be considered as an upper bound, since the Theorems of Limit Analysis are only valid in the case of infinitesimal deflections.

For fully clamped boundaries the pressure versus deflection curve is given by:

$$p = \frac{12F_y(t/b)^2}{\zeta^2\alpha^2}\left\{1+\frac{1}{3}\left(\frac{W}{t}\right)^2\left[\frac{\zeta+(3-2\zeta)^2}{3-\zeta}\right]\right\}, \frac{W}{t} \le 1 \qquad \text{(C3-64)}$$

$$p = \frac{12F_y(t/b)^2}{\zeta^2\alpha^2}2\frac{W}{t}\left[1+\frac{\zeta(2-\zeta)}{3-\zeta}\left(\frac{1}{3}\frac{t^2}{W^2}-1\right)\right], \frac{W}{t} \ge 1 \qquad \text{(C3-65)}$$

where ζ is given by

$$\zeta = \frac{1}{\alpha^2}\left(\sqrt{3\alpha^2+1}-1\right) \qquad \text{(C3-66)}$$

The pressure versus deflection curves for simply supported boundaries are given by:

$$p = \frac{6F_y(t/b)^2}{\zeta^2\alpha^2}\left[1+\frac{4}{3}\left(\frac{W}{t}\right)^2\frac{\zeta+(3-2\zeta)^2}{3-\zeta}\right], \frac{W}{t} \le \frac{1}{2} \qquad \text{(C3-67)}$$

$$p = \frac{24F_y(t/b)^2}{\zeta^2\alpha^2}\frac{W}{t}\left[1+\frac{\zeta(\zeta-2)}{3-\zeta}\left(1-\frac{t^2}{12W^2}\right)\right], \frac{W}{t} \ge \frac{1}{2} \qquad \text{(C3-68)}$$

In the *DNV* rules [equation (C2-2)], Reference 3.4, there is a minimum thickness requirement for plates subjected to lateral pressure. In the absence of other loads and using the notation adopted here, this requirement takes the form:

$$t > \frac{b}{2}\left\{0.77\frac{F_y}{p}\left[1+\left(\frac{b}{a}\right)^2\right]\right\}^{-\frac{1}{2}} \qquad \text{(C3-69)}$$

where p is the design hydrostatic pressure. This requirement implies a collapse pressure which can be obtained from equation (C3-69) by solving for p:

$$p = F_y\left(\frac{t}{b}\right)^2 3.08\left(1+\frac{1}{\alpha^2}\right) \qquad \text{(C3-70)}$$

Equations (C3-58) through (C3-63) and (C3-70) are plotted in Figure C3-8. It can be concluded from this figure that equations (C3-58), (C3-62), (C3-63), and (C3-70) show a reasonable agreement, while equations (C3-59) and (C3-61) also show a reasonable agreement, lying above the previous ones. Equation (C3-60), corresponding to the collapse load of a perfectly clamped plate, gives a pressure which lies consistently above the values given by all the previous curves.

In a real case the support conditions lie somewhere between the extreme cases of simple supports and clamped supports. For continuous plating under uniform lateral pressure resting on stiffeners, support conditions might be close to clamped, since the edge slopes are not very different from zero. On the other hand the degree of axial restraint is difficult to predict, but it also has a great influence on the collapse pressure, particularly beyond initial collapse. Thus, referring to Figure C3-8, it is understandable that equation (C3-60) provides a nonconservative estimate, since it is based on the assumptions of fully clamped and axially restraining supports. The other rigid-plastic upper and lower bound solutions equations (C3-61), (C3-62), and (C3-63) are very attractive and are quite convenient for design purposes. However, their major limitation is that they are based on support conditions which are not realistic.

Since all the curves under consideration have their own limitations, it is suggested that equation (C3-58) be selected, since it is based on credible experimental evidence, it agrees reasonably well with equations (C3-62), (C3-63) and (C3-70), and it allows for the consideration of plastic set.

The permanent set or maximum plastic deformation suffered by a plate largely depends on the degree of in-plane axial restraint at the boundaries. Equations (C3-64) through (C3-68) assume that the edges do not move inwards, a situation which is not likely to be encountered in practice. In fact, as discussed for example in Reference 3.23, the pressures associated with even modest values of permanent set for plates with rigidly held edges are so high that if the plate element boundaries are supported by stiffeners, these would collapse at much lower pressures. At the other extreme, equations (C3-58) and (C3-59) are based on tests where the plate edges are free to slide inwards, which is a more reasonable and conservative assumption. It is convenient to specify in establishing design guidance a certain magnitude of permissible permanent set, in the absence of in-plane compression, since this can lead in general to a more weight efficient structure. For example, in Reference 3.24 and for naval ship design, the value $W/t = 0.25$ is suggested for bottom plating and strength deck, and the value $W/t = 0.5$ for other decks, bulkheads and remaining structure. In Reference 3.15 for example, in the design of icebreaker shell plating, the maximum acceptable permanent set is related to panel width, and the value of 0.3 percent of panel width seems to be acceptable. For a mild steel with $F_y = 36{,}000$ psi this corresponds to $W/t = 0.1\beta$, and for a higher strength steel with $F_y = 50{,}000$ psi this corresponds to $W/t = 0.07\beta$, and these are smaller than the values suggested in Reference 3.24. The value $W/t = 0.2\beta$ is suggested here as an acceptable maximum permanent set value.

C3.5 BIAXIAL COMPRESSION WITH OR WITHOUT EDGE SHEAR BIAXIAL COMPRESSION ALONE

C3.5.2 Serviceability Limit State

Elastic Behavior. The elastic buckling of simply supported rectangular plates compressed in two perpendicular directions, Figure C3-9, is treated by Timoshenko in Reference 3.1. The critical stresses satisfy the following relation:

$$f_x m^2 + f_y n^2 \left(\frac{a}{b}\right)^2 = f_e \left(m^2 + n^2 \frac{a^2}{b^2}\right)^2 \qquad \text{(C3-71)}$$

where *m* and *n* are the number of half waves in which the plate buckles in the x and y directions, respectively, and

$$f_e = \frac{\pi^2 E}{12\left(1-v^2\right)}\left(\frac{t}{a}\right)^2 \qquad \text{(C3-72)}$$

As suggested in Reference 3.2 it is convenient to recast equation (C3-71) in the following form:

$$f_{xcr} = \frac{\pi^2 E}{12\left(1-v^2\right)}\left(\frac{t}{b}\right)^2 \left[\left(\frac{mb}{a}\right)^2 + n^2\right]^2 \frac{1}{\left(\frac{mb}{a}\right)^2 + Rn^2} \qquad \text{(C3-73)}$$

where *R* is the load ratio:

$$R = \frac{f_y}{f_x} \qquad \text{(C3-74)}$$

For given values of the load ratio *R* and plate aspect ratio *a/b*, the values of *m* and *n* may be chosen by trial to give the smallest eigenvalue f_{xcr}. Alternatively equations (C3-71) and (C3-72) can be used, and this is the approach adopted in Bulletin 2V.

Equation (C3-73) can be recast in a non-dimensional form similar to equation (C3-6) as follows:

$$\frac{f_{xcr}}{F_y} = K_b \frac{\pi^2}{12\left(1-v^2\right)} \frac{1}{\beta^2} \qquad \text{(C3-75)}$$

$$K_b = \frac{\left[\left(\frac{mb}{a}\right)^2 + n^2\right]^2}{\left[\left(\frac{mb}{a}\right)^2 + Rn^2\right]}$$

Inelastic Behavior. When f_x and/or f_y exceed the proportional limit plasticity effects have to be considered. Note that strictly speaking, plasticity effects are governed by a combination of stresses, as given for example by the von Mises yield criterion, and not by individual stress components. Thus, rather than stating that plasticity effects must be considered when either f_x or f_y or both exceed the proportional limit, it is more rigorous to say that plasticity must be considered when the equivalent stress, defined in the context of the von Mises yield criterion, for example, exceeds the proportional limit.

Plasticity effects can conceptually be included for design purposes by adopting an approach similar to the one used for combined uniaxial compression and edge shear. However, there is really no theoretical justification for such a procedure in the case of biaxial compression, so this approach is not recommended here. It seems more reasonable to adopt an interaction type of relationship between f_x and f_y, and the form suggested in Reference 3.15 is reasonable:

$$\left(\frac{f_x}{f_{xcr}}\right)^2 + \left(\frac{f_y}{f_{ycr}}\right)^2 = 1 \qquad \text{(C3-76)}$$

This interaction equation was adopted in the first edition of Bulletin 2V, but was eliminated in the second edition as the formulation was assumed unconservative relative to the recommendations of Section 3.5 for biaxial compression and edge shear. An alternative form for this interaction has been proposed by Faulkner, Reference 3.6:

$$\frac{f_x}{f_{xcr}} + \left(\frac{f_y}{f_{ycr}}\right)^2 = 1 \qquad \text{(C3-77)}$$

C3.5.3 Ultimate Limit State

Valsgard, Reference 3.10, has recently conducted a detailed study on the ultimate strength of plates in biaxial in-plane compression. The interaction curves proposed in this study are recommended here and adopted in Bulletin 2V.

For plates with an aspect ratio equal to or larger than 3, Valsgard proposes the following interaction curve:

$$R_x^2 - 0.25 R_x R_y + R_y^2 = 1 \qquad \text{(C3-78)}$$

where $R_x = f_x / f_{xu}$ and $R_y = f_y / f_{yu}$ are the plate longitudinal and transverse strength ratios, respectively.

For square plates (aspect ratio equal to unity) the following interaction curve is suggested in Reference 3.10, based on a study conducted by Frieze et al. in Reference 3.26:

$$R_x^2 - \overline{\eta} R_x R_y + R_y^2 = 1 \qquad \text{(C3-79)}$$

where

$$\overline{\eta} = 3.2 e^{-0.35\beta} - 2 \qquad \text{(C3-80)}$$

and β is the plate slenderness parameter defined by equation (C3-5).

For plates with aspect ratios lying between 1 and 3, for a given value of R_y the corresponding value of R_x can be found by linear interpolation.

The set of equations (C3-78) and (C3-79) is adopted in Bulletin 2V to represent the ultimate limit state of plates in biaxial compression.

Biaxial Compression with Edge Shear. In the case of a rectangular plate under combined biaxial compression and edge shear, an exact treatment of the interaction problem becomes very difficult, if not impossible, as discussed in Reference 3.23. *DNV* (Reference 3.30) adopts a spherical interaction surface based on a combination of elastic and von Mises stresses. This formulation has been rewritten and recommended for Bulletin 2V for the serviceability limit state:

$$\left[\left(\frac{f_{xl}}{f_{xse}}\right)^c + \left(\frac{f_{yl}}{f_{yse}}\right)^c + \left(\frac{f_{xyl}}{f_{xyse}}\right)^c\right]^{2/c} + \left(\frac{f_e}{F_y}\right)^2 = 1.0 \qquad \text{(C3-81)}$$

where

c = $2 - \dfrac{1}{\alpha}, \alpha \geq 1.0$

f_e = limit state von Mises stress

= $\left(f_{xl}^2 + f_{yl}^2 - f_{xl}\left(f_{yl}\right) + 3 f_{xyl}^2\right)^{1/2}$

C3.6 COMBINED IN-PLANE AND LATERAL LOADS

The influence of lateral pressure on the behavior of plates subjected to in-plane loads is a complex problem, which has been studied by several authors (e.g., References 3.27, 3.28, and 3.29). At present a clear understanding of this problem is lacking, and additional testing seems necessary to clarify some of the aspects involved, Reference 3.23. In Reference 3.28, for example, experiments appear to have demonstrated negligible influence of normal pressure upon uniaxial longitudinal compressive strength. The same may be said of biaxial strength for *b/t* = 50 or less. However, for greater *b/t* ratios the pressure can have a negative impact on biaxial strength. Thus, in order to quantify the exact influence of lateral pressure on ultimate strength an extensive experimental program seems necessary. Attempting to postulate a linear interaction for wide plate collapse, as discussed in Reference 3.23, also seems premature, given the lack of data available on the subject.

Reference 3.30 provides an explicit formulation for combined in-plane and lateral loads based on yield-line theory, a reduced moment capacity along the yieldlines based on von Mises' equivalent stresses. This formulation may be appropriate when support conditions produce tensile membrane effects in the plate under combined in-plane and lateral loads. This formulation will probably have only a minor impact on design for typical offshore installations.

Another interaction formulation for this condition (Reference 3.31), developed for some Gulf of Mexico TLP designs, uses the API Bulletin 2V formulations for component critical stresses. This formulation may be appropriate when support conditions are unable to produce tensile membrane effects in the plate under combined inplane and lateral loads. At present, this buckling interaction takes the following form (shown for the ultimate limit state; the serviceability limit state is similar):

(σ_x compression, σ_y compression):

$$\left[(\sigma_x / \sigma'_{xu})^2 + (\sigma_y / \sigma'_{yu})^2\right]^{1/2} + (\tau / \tau_u)^2 = 1$$

In this case, σ'_{xu} and σ'_{yu} are reduced from the API Bulletin 2V values due to the presence of lateral pressure:

$$\sigma'_u / F_y = (\sigma_u / F_y)^{0.8*Q^\wedge 2+0.84*Q+1}$$

where

Q	=	pE / F_y^2,
p	=	applied pressure.

(σ_x tension, σ_y compression):

$$(\sigma_x / F_y)^2 + (\sigma_y / \sigma_{yu})^2 + (\tau / \tau_u)^2 + p / p_u = 1$$

where

p	=	applied pressure,
p_u	=	ultimate pressure under pressure loading only.

(σ_x tension, σ_y tension):

$$-\left[(\sigma_x / F_y)^2 + (\sigma_y / F_y)^2\right]^{1/2} + \tau / \tau_u = 1$$

(σ_x compression, σ_y tension):

$$(\sigma_x / \sigma_{xu})^2 + (\tau / \tau_u)^2 + p / p_u = 1$$

A von Mises based yield criterion is also applied in all quadrants but does not control for compression-compression:

$$(\sigma_x / \sigma'_x)^2 - (\sigma_x / \sigma'_x)(\sigma_y / \sigma_y) + (\sigma_y / \sigma'_y)^2 = 1$$

where

$$\sigma'_x = F_y\left(1 - (\sigma / \sigma_u)\overline{\lambda}\right)^{1/2}$$

$$\sigma'_y = F_y\left(1 - (\sigma / \sigma_u)\overline{\lambda}\lambda\right)^{1/2}$$

$\lambda = \sigma_{yb} / \sigma_{xb}$, bending stress ratio

$$\overline{\lambda} = \left(1 - \lambda + \lambda^2\right)^{-1/2}$$

$$Q = pE / F_{y^2}$$

$$Q_u = p_u E / F_{y^2}$$

The above formulations will have an impact on design relative to the present API Bulletin 2V and *DNV* recommendations, especially for compression-compression biaxial stress states.

The *DNV* 1995 and the above ultimate limit state formulations are compared on Figures C3-10 and C3-11 for a 50 ksi yield steel plate with aspect ratio of 2.0 and a breadth to thickness of 40 and 20, respectively. The effect of edge shear is eliminated for simplification. The above formulations are generally more conservative over the tension-tension and compression-compression quadrants of the interaction curve, as compared with the *DNV* pressure interaction formulations. This is especially true in the compression-compression range (the lower left-hand quadrant of the plots), which is a very typical biaxial stress state for floating structures. Three critical pressure ratios are plotted (25, 50 and 70 psi).

The figures also plot biaxial interaction without pressure per *DNV* and API Bulletin 2V formulations. Looking at the compression-compression quadrant of the figures, the *DNV* curve without pressure appears to control over the *DNV* curves with pressure, converging as the b/t ratio decreases. This implies that the *DNV* pressure interaction curve never controls design for biaxial compression-compression. For API Bulletin 2V, the opposite is true. Another observation is that *DNV* and Bulletin 2V will be closer in agreement as the configurations become more elastic since the *DNV* biaxial interaction is based on elastic buckling stress, whereas Bulletin 2V is based on critical buckling stress. However, the addition of pressure will always result in a more conservation design using API Bulletin 2V and the above formulations than using *DNV* formulations.

C4 STIFFENERS

C4.2 COLUMN BUCKLING

For a perfectly straight column made of a linear elastic material under an axial concentric load, elastic buckling is governed by the well-known Euler formula, Reference 4.1:

$$P_E = \frac{\pi^2 EI}{(KL)^2} \qquad \text{(C4-1)}$$

where L is the unsupported column’s span and I is the moment of inertia of the cross section. K is a coefficient which defines the effective length KL, and depends on the boundary conditions. For example, for a column perfectly clamped at both ends K = 0.5, while for a column pinned at both ends K = 1.0. K is normally taken in the range 0.7 to 1.0, but sometimes other values are suggested, see for example Reference 4.2.

It is convenient to rewrite equation (C4-1) in the following non-dimensional form:

$$\frac{f_E}{F_y} = \frac{1}{\lambda^2} \qquad \text{(C4-2)}$$

where the slenderness ratio λ is given by:

$$\lambda = \frac{1}{\pi}\frac{KL}{r}\sqrt{\frac{F_y}{E}} \qquad \text{(C4-3)}$$

In equation (C4-3) *r* is the radius of gyration of the cross section, or $r = (I/A)^{1/2}$ where *A* is the cross sectional area and *I* has already been defined.

Equation (C4-2) is valid in the linear elastic range, where $f_E / F_y \le p_r$, or $\lambda \ge 1/\sqrt{p_r}$. For $f_E / F_y > p_r$ plastic effects have to be considered, and the column buckling problem can be treated in a way similar to the approach used for plates under uniaxial compression, using the Ostenfeld-Bleich quadratic parabola, equation (C3-7). The following result can be obtained:

$$\frac{f_c}{F_y} = 1 - p_r(1 - p_r)\lambda^2, \lambda \le \frac{1}{\sqrt{p_r}} \qquad \text{(C4-4)}$$

With p_r = 0.5 we obtain from equations (C4-2) and (C4-4) the values of f_E / F_y adopted by AISC, Reference 4.3.

Since these formulas are to be applied to stiffener design, in determining the cross section properties the attached effective plating should be considered. The effective width of plating b_e can be determined by using the formulas presented in C3.2.3.

C4.3 BEAM-COLUMN BUCKLING

The behavior of beam-columns has been the subject of considerable research, as reviewed for example in Reference 4.4. In this reference the following design formula is proposed, which closely follows the expression given in Part 2 of the AISC Specification, Reference 4.3:

$$\frac{P}{P_u} + B_1\frac{M}{M_u} \le 1.0 \qquad \text{(C4-5)}$$

where

P	=	axial force,
M	=	end moment,
M_u	=	maximum moment that can be resisted by the member in the absence of axial loads. For minor axis bending, lateral torsional buckling does not exist, so that $M_u = M_p$, where M_p is the full plastic moment of the cross section,
P_u	=	ultimate axial load of the column as given by $f_c A$,
P_y	=	axial yield load = $F_y A$,
B_1	=	amplification factor given by

$$\frac{C_m}{1 - P/P_E} \geq 1.0 \qquad \text{(C4-6)}$$

P_E = Af_E with f_E given by equation (C4-2),

C_m = $0.6 + 0.4\dfrac{M_1}{M_2} \geq 0.4$ (C4-7)

M_1/M_2 = ratio of the smaller to larger end-moments in the plane of bending, positive for single curvature bending, and negative for double curvature bending.

The above beam-column interaction equation has been criticized by several researchers, Reference 4.4, and some nonlinear interaction equations have been suggested as being more adequate, e.g., Reference 4.5. Equation (C4-5) is recommended for Bulletin 2V, with the understanding that future studies will be required to determine a more suitable formulation.

C4.4 TORSIONAL/FLEXURAL BUCKLING

C4.4.2 Ultimate Limit State for Doubly Symmetric Sections

Torsional buckling refers to the case where a thin-walled bar subjected to uniform axial compression buckles torsionally, while its longitudinal axis remains straight.

The elastic torsional buckling stress (axial compressive stress) for a thin walled prismatic member is derived in various textbooks, e.g., References 4.1, 4.6, and 4.7. For a built-in section with the ends fixed against rotation and not free to warp, if the shear center and centroid coincide, the buckling stress is equal to:

$$f_{cr} = \frac{1}{I_o}\left[GJ + \frac{4\pi^2 EC_w}{L^2}\right] \qquad \text{(C4-8)}$$

where

I_o = polar moment of inertia about the shear center,

G = shear modulus,

J = torsional constant. For thin-walled open sections consisting of n flat elements of width b and thickness t,

$$J = \sum_i^n b_i t_i^3 / 3$$

C_w = warping constant. For uniform thickness I section, web width b, and moment of inertia I_y about an axis coincident with the middle line of the web, $C_w = b^2 I_y / 4$,

L = member length.

Formulas giving the constants J and C_w for a number of typical thin-walled open cross sections are included in Table 4.4-1, Section 4.4.1, Bulletin 2V.

In the *DNV* rules [equation (C2-16), Reference 4.8] equation (C4-8) is adopted for the torsional buckling of flanged profiles, with $C_w = h_s^{\,2} I_y / 4$, where h_s is the distance from the stiffener toe to the shear center.

If the calculated value of the elastic critical stress exceeds the elastic limit of the material, inelastic buckling will occur at a stress lower than the value predicted by equation (C4-8). Inelastic buckling can again be taken into consideration by using the concept of the tangent modulus. However, both *E* and *G* now affect the critical stress, and there is little information about the correct reduced modulus to be used in place of *G*. As discussed in Reference 4.6, the usual assumption is to take the reduced modulus for *G* as GE_t /E, and the approximation is accepted on the grounds that, in most torsional buckling problems, the shearing stresses play only a minor part. By following this approach the term E_t /E can be factored out, and if equation (C3-7) is used the following result can be obtained:

$$\frac{f_c}{F_y} = 1 - \frac{p_r\left(1 - p_r\right)}{\left(f_{cr} / F_y\right)} \qquad \text{(C4-9)}$$

where f_{cr} is given by equation (C4-8). Note that this equation as exactly the same form as equation (C3-15) which applies to rectangular plates under uniaxial compression.

A possible approach to design is to specify that the elastic buckling stress given by equation (C4-8) should be much higher than the yield stress. For example *DNV* (equation C2-16), Reference 4.8), specifies that f_{cr} should be 2.5 times larger than F_y .

Equations (C4-8) and (C4-9) are adopted in Bulletin 2V.

C4.4.3 Ultimate Limit State for Sections with a Single Plane of Symmetry

When the shear center and the centroid do not coincide the section can buckle by a combination of twisting and bending. This is the case of an I-section with unequal flanges, for which there is only one axis of symmetry.

As derived for example in Reference 4.6, the critical axial load *P* for a simply supported section with one axis of symmetry, with the ends free to warp but fixed against rotation, can be found from the following quadratic equation:

$$\frac{I_c}{I_o} P^2 - P\left(P_\theta + P_x\right) + P_x P_\theta = 0 \qquad \text{(C4-10)}$$

where

I_c = polar moment of inertia about the centroid,

P_x = $\pi^2 EI_x / L^2$ = Euler buckling load for buckling normal to the plane of symmetry,

P_θ = $\left(A / I_o\right)\left(GJ + EC_w \pi^2 / L^2\right)$ is the buckling load in pure torsion.

The quadratic equation (C4-10) gives two solutions for the critical load P, one of which is smaller than either P_x or P_θ, while the other is larger than either. The smaller of these roots, or the Euler load for buckling in the plane of symmetry, represents the critical load for the column.

As discussed in the case of torsional buckling, when the critical stress exceeds the elastic limit inelastic effects have to be taken into consideration. This implies that the inelastic buckling load becomes $P\ E_t\ /E$, where P is the elastic solution with the shear modulus G replaced by $GE_t\ /E$.

C4.4.4 Stiffener Proportions

The possibility of occurrence of different forms of local stiffener instability, such as torsional buckling or web crippling, can to a large extent be minimized if certain local slenderness ratios are respected. These usually involve the flange width/thickness ratio *d/t*.

The AISC Specification, Reference 4.3, gives the following requirements for local ratios of compact sections. If the section is compact local buckling will not occur before the full plastic moment is reached. As a result, the AISC Specification, Reference 4.3, increases the allowable bending stress for compact members from 0.60 F_y to 0.66 F_y.

For compact sections the width/thickness ratio of unstiffened projecting elements of a compression flange must satisfy (see Reference 4.3, 1.5.1.4.1):

$$\frac{b_f}{2t_f} \leq \frac{65}{\sqrt{F_y}} \tag{C4-11}$$

with F_y expressed in kips/in^2.

The depth/thickness ratio of the web must satisfy:

$$\frac{d}{t} \leq \frac{640}{\sqrt{F_y}}\left(1-3.74\frac{f_a}{F_y}\right), \frac{f_a}{F_y} \leq 0.16 \tag{C4-12}$$

$$\frac{d}{t} \leq \frac{257}{\sqrt{F_y}}, \frac{f_a}{F_y} > 0.16 \tag{C4-12a}$$

where f_a = computed axial stress.

The compression flange shall be supported laterally at intervals s satisfying:

$$s \leq 76\frac{b_f}{\sqrt{F_y}} \tag{C4-13}$$

$$s \leq \frac{20{,}000}{\frac{d}{A_f}F_y} \tag{C4-14}$$

where

d	=	depth of girder,
A_f	=	area of compression flange.

It is convenient to express the foregoing ratios in terms of $\sqrt{E/F_y}$. For $E = 30x10^6$ psi equations (C4-11) through (C4-13) can be rewritten as follows:

$$\frac{b_f}{t_f} \le 0.75\sqrt{\frac{E}{F_y}} \qquad \text{(C4-15)}$$

$$\frac{d}{t} \le 3.70\sqrt{\frac{E}{F_y}}\left(1-3.74\frac{f_a}{F_y}\right) \qquad \text{(C4-16)}$$

$$\frac{d}{t} \le 1.48\sqrt{\frac{E}{F_y}} \qquad \text{(C4-16a)}$$

$$\frac{s}{b} \le 0.44\sqrt{\frac{E}{F_y}} \qquad \text{(C4-17)}$$

For non-compact sections the AISC Specification, Reference 4.3, 1.9.1.2 and 1.9.2.2, gives some stiffener proportions that allow the design to proceed with no reduction in allowable stress, while preventing local buckling. In this case the maximum ratio for unstiffened compression elements is given by

$$\frac{b_f}{2t_f} \le 0.55\sqrt{\frac{E}{F_y}} \qquad \text{(C4-18)}$$

In the case of stiffened compression elements the following ratio applies:

$$\frac{b_f}{t_f} \le 1.46\sqrt{\frac{E}{F_y}} \qquad \text{(C4-19)}$$

DNV [equations (C2-15) and (C2-17), Reference 4.8] proposes the following limits:

$$\frac{b_f}{t_f} \le 0.8\sqrt{\frac{E}{F_y}} \qquad \text{(C4-20)}$$

$$\frac{d}{t} \le 1.4\sqrt{\frac{E}{F_y}} \qquad \text{(C4-21)}$$

Comparing equations (C4-15) and (C4-20), and (C4-16a) and (C4-21), it can be concluded that the AISC requirements for compact sections and the *DNV* requirements are similar.

In References 4.9 and 4.10 the following limit for the depth to thickness ratio of flat bars is proposed:

$$\frac{d}{t} \le 0.37\sqrt{\frac{E}{F_y}} \qquad \text{(C4-22)}$$

corresponding to the limit $f_{TE} > 2.5\ F_y$, where f_{TE} is the elastic buckling stress in torsion. This limit on d/t is comparable to the limit on b/t_f given by *DNV*, equation (C4-20).

It is recommended that the limits proposed by AISC be adopted in Bulletin 2V, since they agree reasonably well with the limits proposed by other sources.

C4.5 PLASTIC BENDING

The plastic collapse load for a fully clamped beam of span a, subjected to a uniform distributed load is given by Reference 4.11:

$$q = \frac{16M_o}{a^2} \tag{C4-23}$$

where M_o is the plastic moment of the cross section. Equation (C4-23) assumes the supports can withstand the full plastic moment M_o, and that no shear and axial effects influence the structural behavior.

As shown in Reference 4.12, for a symmetric I section the plastic moment M_o is given by:

$$M_o = \left[bt(h-t) + s\left(\frac{h}{2} - t\right)^2 \right] F_y \tag{C4-24}$$

where b is the flange width, t the flange thickness, h the depth and s the web thickness.

For thin-walled sections ($t << h$) the following approximation is acceptable:

$$M_o = \left(bth + \frac{sh^2}{4} \right) F_y \tag{C4-24a}$$

In the presence of axial force N the bending capacity of the cross section decreases. The moment/axial force interaction for an *I* section takes the following form, Reference 4.12:

$$\frac{M}{M_o} + \left(\frac{N}{N_o}\right)^2 \left[2\frac{A_w}{A} - \left(\frac{A_w}{A}\right)^2 \right]^{-1} = 1, \frac{N}{N_o} \le \frac{A_w}{A} \tag{C4-25}$$

$$\frac{M}{M_o}\left(1 - \frac{A_w}{2A}\right) + \frac{N}{N_o} = 1, \frac{N}{N_o} \ge \frac{A_w}{A} \tag{C4-26}$$

where $N_o = AF_y$ = plastic axial capacity, A is the total cross sectional area, and A_w = web area. These interaction equations are adopted in Bulletin 2V.

The *European Recommendations Steel Construction*, Reference 4.13, give two formulas for the moment/axial force interaction, for both strong axis bending and weak axis bending. For strong axis bending for I beams the suggested interaction curve is linear and compares well with the results of equations (C4-25) and (C4-26).

In the presence of shear force the bending capacity of beams also decreases. For thin-walled I beams loaded in the plane of the web it is reasonable to assume that the maximum carrying capacity in shear V_o is:

$$V_o = A_w \frac{F_y}{\sqrt{3}} \qquad \text{(C4-27)}$$

Shear influences bending in extreme cases, when the length/depth ratio for the beam is very small, and this is not likely to occur in practical situations of interest here. As suggested in Reference 4.14 a possible shear and bending moment interaction is:

$$\left(\frac{M}{M_o}\right)^2 + \left(\frac{V}{V_o}\right)^2 = 1 \qquad \text{(C4-28)}$$

For a plated structure consisting of a plate stiffened by orthogonally intersecting stiffeners, and subjected to a uniformly distributed lateral pressure p, the line loads on the stiffeners can be estimated from the following formulas suggested by Faulkner in Reference 4.15:

$$q_a \cong pb\left(1 - \frac{1}{2\alpha}\right) \qquad \text{(C4-29)}$$

$$q_b \cong \frac{pb}{2} \qquad \text{(C4-30)}$$

where q_a is the line load on the stiffener of length a, q_b is the line load on the stiffener of length b, and α is the plate aspect ratio. These two equations are adopted in Bulletin 2V.

C4.5.3 Stiffener Proportions

Requirements for stiffener proportions in members under lateral load and axial compression, to ensure that plastic hinges develop, are discussed here.

AISC, Reference 4.3, gives in a tabular form the maximum values for the ratio $b_f / 2t_f$ as a function of the yield stress (kips/in^2), as shown in the first two columns below:

F_y	$b_f / 2t_f$	$(b_f / t_f)/\sqrt{E/F_y}$
36	8.5	0.59
42	8.0	0.60
45	7.4	0.57
50	7.0	0.57
55	6.6	0.56
60	6.3	0.56
65	6.0	0.56

The third column contains the constant which when multiplied by $\sqrt{E/F_y}$ (for E = 30 x 10^6 psi) gives the ratio b_f/t_f. Thus if the smallest value of the constant is selected the requirement for b_f/t_f takes the following form:

$$\frac{b_f}{t_f} \le 0.56\sqrt{\frac{E}{F_y}} \qquad \text{(C4-31)}$$

Regarding the depth/thickness ratio of webs the following requirements are given in Reference 4.3:

$$\frac{d}{t} \le \frac{412}{\sqrt{F_y}}\left(1-1.4\frac{P}{P_y}\right), \frac{P}{P_y} \le 0.27 \qquad \text{(C4-32)}$$

$$\frac{d}{t} \le \frac{257}{\sqrt{F_y}}, \frac{P}{P_y} > 0.27 \qquad \text{(C4-33)}$$

where P_y = plastic axial load = F_yA

For E = 30 x 10^6 psi equations (C4-32) and (C4-33) take the form:

$$\frac{d}{t} \le 2.38\sqrt{\frac{E}{F_y}}\left(1-1.4\frac{P}{P_y}\right), \frac{P}{P_y} \le 0.27 \qquad \text{(C4-34)}$$

$$\frac{d}{t} \le 1.48\sqrt{\frac{E}{F_y}}, \frac{P}{P_y} > 0.27 \qquad \text{(C4-35)}$$

The equivalent requirements proposed by *DNV* (equations (C2-18) and (C2-20), Reference 4.8) are:

$$\frac{b_f}{t_f} \le 0.6\sqrt{\frac{E}{F_y}} \qquad \text{(C4-36)}$$

$$\frac{d}{t} < 1.15\sqrt{\frac{E}{F_y}} \qquad \text{(C4-37)}$$

The AISC and *DNV* requirements agree reasonably well. The AISC requirements are adopted in Bulletin 2V.

C5 STIFFENED PANELS

A very wide range of papers have been published in the literature on the structural behavior of stiffened panels, see for example References 5.1 and 5.2. Due to the large number of parameters required to fully define a stiffened panel it is difficult to develop simple design formulas. The *DNV* rules, Reference 5.3, give guidelines for plate, stiffener and girder design, but do not include any specific recommendations regarding the overall design of stiffened panels or grillages. Some possible ways of treating this problem and the difficulties involved will now be discussed.

C5.2 UNIAXIALLY STIFFENED PANELS IN END COMPRESSION

In Reference 5.12 an approach for deriving the average failure stress of uniaxially stiffened plate panels in end compression is given. In this approach interaction between adjacent stiffener fields is neglected, it is assumed that flexural failure is plate induced, and simple support conditions at the transverse edges are assumed. The average failure stress is given by:

$$\phi = \frac{f_w}{F_y} = f_e \frac{A_s + b_e t}{F_y (A_s + bt)} \tag{C5-1}$$

$$\frac{f_e}{F_y} = 1 - p_r (1 - p_r) \left(\frac{a}{\pi r_{ce}} \right)^2 \frac{F_y}{E}, \; f_e \geq p_r F_y \tag{C5-2}$$

$$r_{ce}^2 = \frac{I_e'}{A_s + b_e t} \tag{C5-3}$$

$$p_r F_y = F_p \tag{C5-4}$$

where

EI_e'	=	buckling flexural rigidity of the stiffener plus the effective width b_e of plating,
b_e	=	effective width,
A_s	=	stiffener area,
t	=	plate thickness,
F_p	=	proportional limit stress.

Both widths b_e and b_e' should be reduced by the product $R_r R_y R_{xy}$ for the effects of any other in-plane stresses f_r, f_y, and f_{xy}. The reduction factors R_r, R_y, and R_{xy} are given by:

$$R_r = 1 - \frac{f_r}{f_m} \frac{E_t}{E} \tag{C5-5}$$

$$\frac{f_r}{F_y} \cong \frac{2\eta}{b/t - 2\eta}, \eta \cong 4.5 \tag{C5-6}$$

$$\frac{E_t}{E} \cong \frac{1}{\beta} 2(\beta - 1), 1 \leq \beta \leq 2.5 \tag{C5-7}$$

$$\frac{E_t}{E} \cong 1, \beta > 2.5 \tag{C5-8}$$

$$\frac{E_t}{E} \cong 0, \beta < 1 \tag{C5-9}$$

$$R_y = 1 - \left(f_y / f_{ym} \right)^2 \tag{C5-10}$$

$$R_{xy} = \left[1 - \left(f_{xy} / \tau_y \right)^2 \right]^{1/2} \tag{C5-11}$$

The subscript m for f denotes the maximum or ultimate stress for the plate, as discussed in C3.2.3.

Since b_e and b_e' are both functions of the required plate edge stress f_e, an iterative procedure is needed in order to find ϕ. However, as indicated in Reference 5.4, experience shows that only a few iterations are required for convergence.

In Reference 5.5 some studies on the ultimate strength of simply supported uniaxially stiffened panels (equally spaced and sized stiffeners) under edge compression are given. A design criterion is discussed, which seems easy to apply in practical design situations, and which will now be described. For uniform compression a maximum plate width/thickness ratio is suggested as follows:

plane plate (no stiffener):

$$\frac{b}{t} \leq 1.33\sqrt{\frac{E}{F_y}} \tag{C5-12}$$

panel with one stiffener:

$$\frac{b}{t} \leq 2.66\sqrt{\frac{E}{F_y}} \tag{C5-13}$$

panel with more than two equally spaced stiffeners:

$$\frac{b}{t} \leq 1.33n\sqrt{\frac{E}{F_y}} \tag{C5-14}$$

where n = number of sub-panels (individual plates).

In case the compressive edge stress f_1 is less than the ultimate compressive stress f_u, the thickness can be reduced to $\sqrt{f_1 / f_u}$ times the values given above.

The moment of inertia of any type of stiffener shall not be less than

$$I = \frac{1}{11} bt^3\gamma \tag{C5-15}$$

where

I	=	required moment of inertia of stiffener,
b	=	entire plate width,
t	=	plate thickness,
γ	=	required flexural rigidity ratio given by:

$$\gamma = \frac{5}{k_{req} - k_o}\left[\frac{k_{req}(b/t)_s^2}{(b/t)_o^2} - k_o\right]\gamma_m \tag{C5-16}$$

The parameter $(b/t)_o$ is the maximum width/thickness ratio of the entire plate as specified by equations (C5-12) through (C5-14), and $(b/t)_s$ is the actual width/thickness ratio of the entire plate. The parameter γ_m is given by:

$$\gamma_m = 4\alpha^2 n(1+n\delta) - \frac{(\alpha^2+1)^2}{n}, \alpha \le \alpha_o \tag{C5-17}$$

$$\gamma_m = \frac{\alpha_o^4 - 1}{n}, \alpha > \alpha_o \tag{C5-18}$$

$$\alpha_o = \sqrt{2n^2(1+n\delta)} - 1 \tag{C5-19}$$

$$\delta = \frac{A_s}{bt}$$

where

A_s	=	area of stiffener,
α	=	aspect ratio of a whole panel,
k_o	=	buckling coefficient = 4,
k_{req}	=	n^2

Reference 5.5 also gives ultimate strength curves reflecting the influence of residual stresses, initial geometric imperfections, and inelastic behavior. The ultimate stress for multiple stiffened plates under pure compression takes the following form:

$$\frac{f_u}{F_y} = \begin{cases} 1.0, & \bar{\lambda} \le 0.5 \\ 1.5 - \bar{\lambda}, & 0.5 < \bar{\lambda} \le 1.0 \\ 0.5/\bar{\lambda}, & \bar{\lambda} > 1.0 \end{cases} \tag{C5-21}$$

where

$$\bar{\lambda} = \frac{b}{t}\sqrt{\frac{F_y 12(1-\nu^2)}{E\pi^2 k}} \tag{C5-22}$$

The buckling coefficient k is given by:

$$k = \min(k_R, k_F) \tag{C5-23}$$

$$k_R = 4n^2 \tag{C5-24}$$

$$k_F = \frac{(1+\alpha^2)^2 + n\gamma}{\alpha^2(1+n\delta)}, \quad \alpha \le (1+n\gamma)^{1/4} \tag{C5-25}$$

$$\delta = \frac{A_s}{Bt}$$

$$k_F = \frac{2\left[1+(1+n\gamma)^{1/2}\right]}{1+n\gamma} \quad \alpha > (1+n\gamma)^{1/4} \tag{C5-26}$$

$$\gamma = \frac{EI_s}{bD} \tag{C5-27}$$

where I_s is the moment of inertia of one stiffener about an axis parallel to the plate surface at the base of the stiffener, and D is the plate flexural rigidity.

C5.3 ORTHOGONALLY STIFFENED PANELS

Grillage and orthotropic plate theory have been suggested for the analysis of cross stiffened plates by several authors, e.g., References 5.4 through 5.7. *The Rules for the Classification and Construction of Seagoing Steel Ships* published by Germanisher Lloyd, Reference 5.7 for example, define the buckling strength of cross stiffened panels, in terms of orthotropic plate theory. There is no doubt that these methods provide a powerful tool for studying the strength of cross stiffened panels, as demonstrated for example in Reference 5.6. A possible shortcoming of these methods is the difficulty of giving an exact definition for the flexural rigidities of the orthotropic panel, or the rigidities of grillage elements. Also, in the case of a grillage it is difficult to establish how the applied loads should be distributed among the various grillage components.

The limit states given in Bulletin 2V for orthogonally stiffened panels under uniaxial and biaxial compression and lateral load are based on the work of Mansour, References 5.4 and 5.6. In particular the graphs in Figures 5.3-1 and 5.3-2 are adapted from Reference 5.4.

C5.4 STIFFENER PROPORTIONS

The torsional and lateral buckling of stiffeners has been covered in C4.4.3. Some additional guidelines can be given in order to prevent the possibility of tripping. The *DNV* rules, for example [equation (C2-24), Reference 5.3] state that overall tripping of the girder should be avoided by means of tripping brackets, and that the spacing between these should not exceed:

$$s = f \cdot b_f \sqrt{\frac{E}{F_y}} \qquad \text{(C5-28)}$$

where f = 0.4 for symmetric flanges, and f = 0.8 for one-sided flanges. This result compares reasonably well with the AISC requirement for lateral support of the compression flange, Reference 5.8, as expressed by equation (C4-17).

Requirements for tripping brackets and stiffener proportions are also suggested in References 5.9 and 5.10 on the basis of U.S. Navy practice. More recent work on the subject is given in Reference 5.11.

As mentioned in Reference 5.12, a possible basis for design includes two requirements:

i) the elastic torsional buckling stresses are kept well above yield;

ii) the stiffener outer-fiber stresses under compressive load, allowing for residual stresses and initial flexural and torsional deformations, are kept below yield by an appropriate margin.

The margin and deformations are not specified in Reference 5.12. In the *DNV* rules, for example (equation (C2-16) Reference 5.3), there is a requirement that for flanged profiles the elastic torsional buckling stress be larger than 2.5 times the yield stress, and this seems to be reasonable. For the outer fiber stress the *DNV* rules do not contain an explicit requirement, but they give a minimum value for the girder's moment of inertia [equation (C2-25)].

C5.6 EFFECTIVE FLANGE

The concept of effective breadth is normally adopted in the design of flange structures, such as box girders, in order to take the shear lag phenomenon into account. The problem has been studied by several authors, and is briefly reviewed in Reference 5.13. In naval architecture applications, Schade, Reference 5.14, performed the pioneer work in the area and his design curves are suggested here, until a more thorough review of more recent literature can be conducted, e.g., References 5.15, 5.16, and 5.17.

Schade's approach is based on simple plane-stress solutions, and thus, as mentioned in Reference 5.13, does not take into account the following effects:

a. normal deflections, such as initial distortion, or those caused by lateral load;
b. residual stresses;
c. plate buckling;

The *DNV* rules (section 5, in Reference 5.3) also contain design curves for effective breadth calculations, probably based on Reference 5.15, and further study should be conducted on comparing these with Schade's curves, and other approaches available in the literature.

The three Schade design curves, Reference 5.14, are reproduced here in Figures 5.6-2 to 5.6-4. The first figure applies to a single web, the second to double webs and the third to multiple webs. The following nomenclature is used:

B = plate breadth, or distance between webs,
b = half breadth,
L = length,
cL = distance between points of zero bending moment,
b_{ef} = effective breadth.

The parameter β is a non-dimensional coefficient to be computed as follows. For a box girder with identical lower and upper flanges:

$$\beta = \frac{1}{6}\frac{h}{b}\frac{t_w}{t} \qquad \text{(C5-29)}$$

where

h = one-half the depth of the web,
t = flange thickness,
t_w = web thickness.

For stiffened plating

$$\beta = \frac{1}{4}\frac{h}{b}\frac{t_w}{t}\frac{4A_2 + 2ht_w}{3A_2 + 2ht_w} \tag{C5-30}$$

where A_2 is the area of the lower flange (zero in the case of flat bar stiffeners). As suggested in Reference 5.18 the stress distribution across the breadth of a flange can be approximated by the following quartic equation:

$$f_x = f_{\max}\left\{\left(\frac{x}{b}\right)^4 + \left[\frac{5(b_e/b)-1}{4}\right]\left[1-\left(\frac{x}{b}\right)^4\right]\right\} \tag{C5-31}$$

where, as shown in Figure 5.6-5, f_{max} is the maximum stress occurring at the web intersection. Equation (C5-31) is adopted in the *DNV* rules (Section C5.5.1.1, Reference 5.3).

C5.7 STIFFENER REQUIREMENTS FOR IN-PLANE SHEAR

The ultimate strength of plates loaded in shear depends to a large extent on the rigidity of the surrounding stiffeners. In order to study this problem the non-dimensional parameter γ is usually defined:

$$\gamma = \frac{EI}{Dd} = \frac{12(1-v^2)I}{t^3 d} \tag{C5-32}$$

where d is the spacing between stiffeners, I is the stiffener cross section's moment of inertia about an axis coinciding with the surface of the plate, and t is the plate thickness.

In Reference 5.19 it is shown that γ does not need to be larger than the limiting ratio γ_o, in order to ensure that the shear stress reaches its maximum or critical value, and where γ_o is given by:

$$\gamma_o = 4(7\alpha^2 - 5) \tag{C5-33}$$

$\alpha = \bar{b}/d$ is the plate's aspect ratio. Combining equations (C5-32) and (C5-33) we get for I:

$$I = \frac{1}{12(1-v^2)}t^3 d\, 4(7\alpha^2 - 5) \tag{C5-34}$$

For $v = 0.3$ equation (C5-34) can be rewritten in the following form:

$$I = 0.092t^3\bar{b}\gamma \tag{C5-35}$$

$$\gamma = 10\left(2.8\frac{\bar{b}}{d} - 2\frac{d}{\bar{b}}\right) \tag{C5-36}$$

The *DNV* rules (equation (C2-22), Reference 5.3), give the following requirement for I, where I is the moment of inertia of the stiffener with full plate width (using the present notation):

$$I > 0.1t^3\bar{b}\gamma \tag{C5-37}$$

$$\gamma = 12.5\frac{\tau_d}{R_{\tau d}}, \frac{\bar{b}}{d} \le 1 \qquad \text{(C5-38)}$$

$$\gamma = 25\left(2.5\frac{\bar{b}}{d} - 2\frac{d}{b}\right)\frac{\tau_d}{R_{\tau d}}, \frac{\bar{b}}{d} > 1 \qquad \text{(C5-39)}$$

where τ_d is the design in-plane shear stress in the plate and $R_{\tau d}$ is the maximum shear resistance of the plate. Comparing equations (C5-36) and (C5-39) we can conclude that the *DNV* requirements seem conservative, which is partly due to a different definition for *I*. It should be noted that no inelastic effects are included in equation (C5-36), and this can be achieved in simple terms, as in the *DNV* approach, by multiplying γ by the ratio $\tau_d / R_{\tau d}$.

C5.8 OTHER DESIGN CONSIDERATIONS

It is possible, to develop a stiffened panel with relatively small stiffening that will meet the recommendations of Bulletin 2V with elastic stress ratios that are insufficient to ensure a reasonable hierarchy of failure modes. At loads close to the structure's ultimate capacity, local instability could trigger progressive collapse. Thus, an explicit hierarchy check is needed in the bulletin for stiffened panel design.

For uniaxially stiffened panels in end compression, the bulletin first edition formulations cover ultimate (critical) buckling stresses only. A similar problem occurs for orthogonally stiffened panels under biaxial compression when the resultant critical stress is in the material plastic range. Elastic buckling stresses are not directly computed and are not available for use in an elastic stress hierarchy check. This problem is overcome by back-calculating the equivalent elastic stress (a similar procedure is used in API Bulletin 2U(first edition), Section 4.5.2b). However, the back-calculation is complicated by the fact that that Bulletin 2V proposes various plasticity reduction factors for various buckling modes and the appropriate reduction factor for back-calculation of elastic stresses is not immediately clear. A single plasticity reduction factor formulation would be preferable and development of such a formulation should be considered for future work. Meanwhile, since the long plate plasticity reduction equation produces the lowest elastic stress for a give critical stress, this formulation is rewritten for elastic stress calculation and used in the hierarchy check.

C6 DEEP PLATE GIRDERS

C6.1.1 Scope

Plate girder design is covered in the AISC *Specification for the Design, Fabrication and Erection of Structural Steel for Buildings*, Reference 6.1. However, AISC limits the web depth to thickness ratio, and this excludes the very deep girders, or bulkhead girders, that can potentially be used in offshore structure decks. This limitation prompted the inclusion of this topic in Bulletin 2V, and the basic approach that has been adopted follows in some aspects the philosophy of BS5400, Reference 6.2. Thus Bulletin 2V recommends that the AISC

Specification be used, but in those cases where the web depth to thickness ratio exceeds the AISC limit, Bulletin 2V Section 6.0 should guide the design. It is worthwhile noting that other sources of relevant information on deep girder design are the *Specifications for Highway Bridges (AASHTO), Reference 6.3,* and the *Specifications for Steel Railway Bridges (AREA)*, Reference 6.4.

C6.1.5.d Transverse Stresses in Webs Due to Flanges Curved in Elevation. The transverse load transmitted to the web as a result of flange curvature can easily be obtained from equilibrium. The transverse loads on the web are simply the components of the flange force along the transverse direction. The formula in Bulletin 2V is identical to that in BS5400, Par. 9.5.7.2, Reference 6.2, except that the slope is referenced to the horizontal.

C6.2 LIMIT STATES

The web plate in a deep girder is normally subjected to a combination of longitudinal and transverse compression or tension, in-plane bending and shear. Lateral loads can also be present. Thus, ideally each single rectangular plate component should be examined for a combination of all these loads. However, limit states involving such a combination are not available. It, therefore, becomes necessary to assess structural performance in order to compare the relative importance of the several stress components. It can be assumed, for example, that the flexural stresses in the web are effectively shed to the girder flanges, and in this case the web plate can be designed for shear alone. For the deep girders being considered, where a number of longitudinal stiffeners is used, the in-plane bending stresses are almost uniform across each individual rectangular plate, so that a combination of shear and uniform edge compression can be adopted in the design. Where significant transverse stresses due to flange curvature or transversely applied loads are present, a load combination involving biaxial compression and shear would be adequate. However, as discussed in C3.5, there are no widely accepted methods to deal with the problem of combined biaxial compression and shear, and the same applies to the case where lateral pressure acts together with these loads. Engineering judgment must be used to address such cases.

C6.3 DESIGN CONSIDERATIONS

C6.3.2.b Webs With Openings. Openings can obviously affect the web strength, since its ability to carry shear is reduced as a result of the decreased web area. Also, stress concentrations occur around openings, particularly at the corners, and good detail design is required to ensure an adequate level of performance. Extensive surveys of ship structural details, as reported in References 6.5, 6.6, and 6.7, have shown that serious structural failures can occur if openings are not properly designed. If openings cannot be avoided in highly stressed areas, detailed analysis using, for example, the finite element method might be required.

The guidelines given in Bulletin 2V on the subject of webs with openings are of an empirical nature, and are associated with good design practice. The impact of plate openings has been extensively studied in the context of different applications. The impact on ship structures is discussed for example in Reference 6.8. Specific detail design guidelines on openings in ship hull structures are given in Reference 6.5. These include in particular the case of long openings or groups of long openings all in the same section, such as discussed in Bulletin 2V.

C6.3.3 Longitudinal Web Stiffeners

The minimum moment of inertia I_s of the stiffener cross section in Bulletin 2V is

$$I_s = 4at^3 \tag{C6-1}$$

The minimum value specified by AASHTO, Reference 6.3, is

$$I_s = bt^3\left[2.4\left(\frac{a}{b}\right)^2 - 0.13\right] \tag{C6-2}$$

This formula is valid for *a/b* smaller than unity, where *a* is the spacing between transverse web stiffeners, and *b* is the spacing between longitudinal web stiffeners.

In order to compare these two formulas it is convenient to normalize I_s with respect to the moment of inertia of the web plate about its own mid-surface, or $\left(1/12\right)bt^3$. This leads to the following non-dimensional parameter:

$$I_s' = 48\frac{a}{b} \tag{C6-3}$$

$$I_s' = 12\left[2.4\left(\frac{a}{b}\right)^2 - 0.13\right] \tag{C6-4}$$

These two formulas are compared in Figure C6-1, and it can be concluded that $I_s = 4at^3$ provides a more conservative requirement. An alternative expression for the minimum stiffener inertia, where the full attached plate width is included, is provided by *DNV*, Reference 6.9:

$$I > 0.25a^3\left(A_s + bt\right)F_y / E \tag{C6-5}$$

where A_s is the stiffener cross sectional area, excluding any attached plating. Comparisons of equations (C6-1), (C6-2), and (C6-5) for a typical girder arrangement indicate that they all lead to similar results.

C6.3.4.d Axial Force Due to Tension Field Action. The AISC Specification, Chapter G, G3, Reference 6.1, provides a formulation for deriving the axial force on transverse stiffeners due to tension field action. The AISC formulation was based on extensive tests for girders with multiple transverse stiffeners, as discussed in Reference 6.10. In the present application the webs are intended to be both longitudinally and transversely stiffened, and the treatment in BS5400, Reference 6.2, is preferred. The formulations in Par. 6.3.4.d of Bulletin 2V are identical to those in Par. 9.13.3.2 of BS5400.

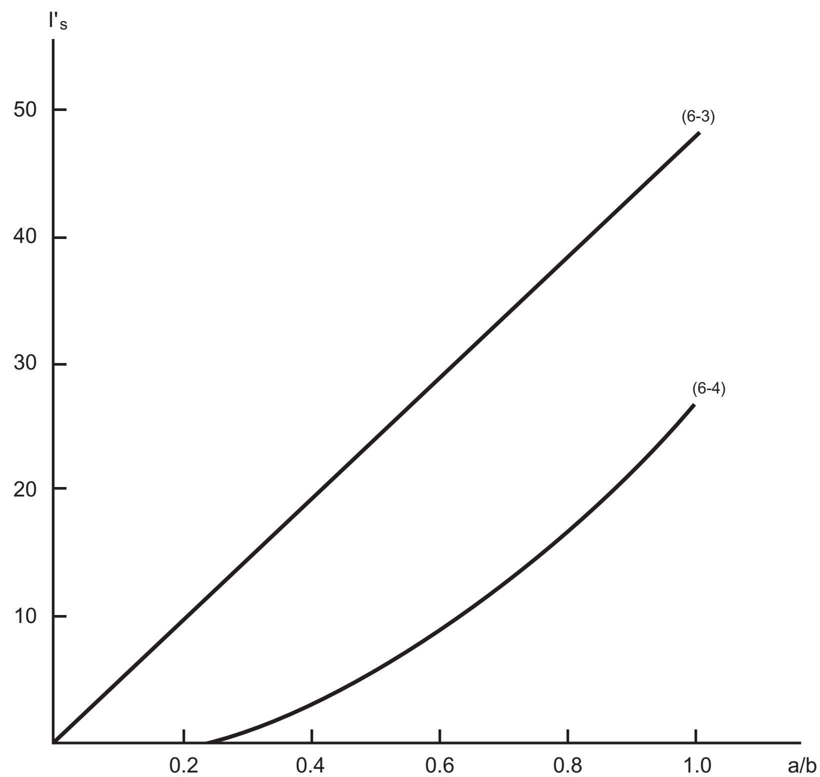

Figure C6-1—Comparison of Minimum Longitudinal Stiffener Stiffness Requirements

C6.3.4.e Axial Force Assumed in Preventing Web Buckling. The formulation in Bulletin 2V is identical to the one given in Par. 9.13.3.3 of BS5400, Reference 6.2. The coefficient k_s was redefined in a form that is consistent with the nomenclature adopted in Bulletin 2V.

C6.3.4.f Axial Force Due to Curvature. The formula given in 6.5.4.f is similar to the one in 6.1.5.d of Bulletin 2V. Some comments on its basis are given in C6.1.5.d.

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APPENDIX B—GUIDELINES FOR FINITE ELEMENT ANALYSIS USE

TABLE OF CONTENTS

B1 BACKGROUND

The first edition of API Bulletins 2U and 2V were published in 1987. At that time, most offshore structures were analyzed and designed based on three-dimensional space frame structural models. Thus, the applied load and stress formulations in the bulletins were written assuming load and stress results from these space frame models.

Since 1987, the use of partial or full finite element plate and/or shell modeling of offshore structures has increased dramatically. Determination of applied stresses from such models for use in bulletin formulations is presently left up to the analyst or designer. Actual values of these applied stresses are a function of model complexity and mesh definition, individual element capability, and interpretation of analysis results. Because of this and the additional expertise required to properly perform a finite element analysis of complex structures such as offshore platforms, a general guideline for the minimum requirements of such an analysis is needed to ensure that the bulletin formulations remain commensurate with the analysis results and the bulletin's intent.

In 1996, Basu et al (Ship Structure Committee Paper No. SSC-387) developed a systematic and practical methodology to assess the validity of FEA results based on the selected analysis procedure, type of elements, model size, boundary conditions, load application, etc. Models and analyses that meet their assessment should produce response results appropriate for use with API bulletin formulations. The more important aspects are extracted and summarized in the following, which may serve as guidance for the minimum requirements of a finite element model and analysis in determining the structural response for use with API bulletin formulations.

B2 BULLETIN INTENT

The major purpose of the API 2V and 2U bulletins is to provide guidance for the design of stiffened steel flat plate or cylindrical shell structures. The guidance takes the form of buckling formulations and design considerations with respect to strength and, in the case of Bulletin 2V, serviceability. Working stress design methods are assumed with sufficient factors of safety to ensure that the material remains in the linear range under design loads. The bulletin formulations also account for the normal fabrication residual stresses and geometric imperfections that need not be modeled in an analysis for the purposes of bulletin evaluation.

In order to implement the bulletin buckling formulations, average applied stresses need to be determined at or near the center of each plate panel, assuming a more or less uniform stress gradient across the plate panel. Likewise, yielding considerations require additional stress determination along the edges of each plate panel. Assuming a generally uniform stress gradient, this establishes the minimum number of locations for applied stress determination for a quadrilaterally shaped plate at nine (9), namely at the center, four corners and midspan at the four edges of the quadrilateral plate. Similarly, stiffener stresses should be determined at each support and at midspan at the associated extreme fibers of the stiffener. Of course, the stress gradient should be reviewed to determine if evaluation at additional locations is needed at specific plate panel locations.

B3 BULLETIN USE

Assuming an appropriate finite element model and analysis, the analysis results may be used to determine the applied stresses for use with the bulletin buckling stress formulations. Generally, this may be done by integrating the FEA stress results along the edges and centerlines of each plate panel. The in-plane directional axial and shear stresses are determined as the average stress along each line of integration and the in-plane bending stresses are determined from the variation of stress from its associated average normal stress. Out-of-plane stresses due to lateral pressure may also be determined from the element stresses assuming the element types that are used accurately predict the out-of-plane response.

Once these applied stresses are determined, they may be used directly in the bulletin buckling stress formulations and checked against the bulletin allowables. Plate buckling checks are performed for applied stresses at, or near, the center of the plate. Plate yielding checks are performed for applied stresses at all locations. This is most easily done by determining the von Mises equivalent stress at each location and comparing it against the specified limit criterion.

B4 FINITE ELEMENT ANALYSIS GUIDELINES

It is important that the finite element analysis accurately models the intended loading and structural response. This is accomplished by selecting a model size, element mesh, element types and boundary conditions that are commensurate with the area of interest. In most cases these parameters are inter-related and the proper selection for all these parameters requires an experienced analyst. Lack of experience should be supplemented by supervision and review by others with appropriate levels of finite element analysis experience with similar types of offshore structures of structural components.

B5 MODEL

Prior to modeling, it is useful to have a general idea of the anticipated behavior of the structure. This knowledge serves as a useful guide in several modeling decisions that need to be made in developing the model. For example, stiffened plate structure that is subject primarily to in-plane loads rather than transverse loads is better modeled using membrane elements rather than plate/shell elements. However, if the analysis of the stiffened plate structure is local in nature, or the loading is transverse, shear effects may be significant and certain element formulations may not account for shear, or such an option must be specifically selected by the analyst.

B6 MESH

Mesh design is one of the most critical tasks in finite element modeling and is often a difficult one. Mesh density, mesh transitions and the ratio of stiffness of adjacent elements must all be considered when developing a finite element mesh. As a general rule, a finer mesh is required in areas of higher stress gradient. Of course, a finer mesh could be used for the entire model but this approach sacrifices computational economy and increases the possibility of manipulation errors. For these reasons, variations in mesh density are often used.

The mesh density depends on the element type used, distribution of applied load and purpose of the analysis. In general, the mesh should be finest in regions of steepest stress gradients. Thus, where stresses show a sharp variation between adjacent elements, the mesh should be refined and the analysis rerun. Mesh density also depends on the type of analysis (i.e., linear, non-linear, or dynamic) and the number and type of element integration points.

B7 ELEMENT TYPE

At present, linear stress field elements are the most commonly used. This is due, in part, to the requirement that the order of the stress function should properly match the stress gradient, and this is easy to visualize for linear stress elements in a properly sized mesh. For most portions of structures, a mesh of linear stress elements can provide a good description of the stress state. Even in areas of discontinuities or in areas of non-linearity, linear elements in a relatively fine mesh can give excellent results. Thus, the use of properly meshed linear stress elements is appropriate for structure components covered by the bulletin formulations. The use of higher order stress fields may be appropriate for coarser meshes although free surface stress prediction can be in error.

B8 ELEMENT SHAPE

Element performance is affected by element shape, where element shape is a function of the element aspect ratio, element skewness and element warping.

A general rule of thumb is to limit the aspect ratio of membrane and bending elements to 3 for good stress results. The best shape for quadrilateral and triangular elements is square and equilateral, respectively. Thus, the use of square and/or equilateral elements is particularly desirable in areas of the highest stress gradients. However, higher order elements will be less sensitive to deviations from the ideal aspect ratio than lower order elements.

Element performance also degrades with element skewness. For quadrilateral elements, vertex angels greater than 135° or less than 45° are not recommended and the quadrilateral element will perform better if its shape is that of a parallelogram. For triangular elements, vertex angels should remain in the range of 45° to 90°.

Element warping occurs when the element nodes are not coplanar. The degradation in element performance depends on the element formulation. Triangular elements may be used in place of warped quadrilateral elements where curvature is high.

B9 STIFFENED PLATE STRUCTURE MODELING

Based on the above, the following guidance is provided for modeling typical stiffened plate structure for offshore structures. Minimum requirements are summarized on Table B-1.

Individual plate panels should be modeled with linear stress membrane elements, where transverse load effects are negligible, or bending elements where transverse load effects are important.

Since most plate panels are rectangular, or at least quadrilateral in shape, elements should be generally quadrilateral and as nearly square as possible. The minimum number of elements on any one side of a plate panel should be two if the element stress formulations adequately predict stresses at the element nodes. If element prediction is inadequate at the element nodes but acceptable at the element center, then the minimum number of elements modeling any one side of a plate panel should be three (figure B-1). In any case, the model should be developed such that accurate stress predictions are obtained at each corner, midspan along each edge and at the center of the plate. This may require acceptable node stress prediction from the elements unless an acceptable interpolation technique is developed to obtain the stresses at the edges of the plate.

Stiffener flanges and webs may be modeled similar to plate panels or as single beam elements with structural properties accounting for the associated plate effective width and offset of the stiffener. The first approach has the advantage of being easier to visualize, provides more local results that may be of interest, but suffers from an increase in computational time and increased volume of data to manipulate. The second approach is more common because of the inherent computational efficiency. Care should be taken that the stiffener plate effective width is not double counted in the model; software capabilities in this area vary with each program.

B10 APPLIED STRESSES FOR BULLETIN CODE CHECKS

The purpose of this section is to provide a minimum FEA guideline for determining the average applied stresses compatible with those locations shown on Figure B-3 and the critical stresses obtained from Bulletin 2V formulations. When a very fine mesh is use, peaked stress concentrations should not be used in conjunction with stresses computed from Bulletin 2V formulations.

The specific procedure for a rectangular plate or stiffened panel is as follows:

1) Assuming relatively constant stress gradients across the plat or panel spans, determine the FEA stresses at locations 1 through 9 as shown on Figure B-3.

Where stress gradients vary, determine FEA stresses at additional appropriate locations and adjust the remaining procedure accordingly.

2) Determine the applicable in-plane longitudinal axial average stress, f_{xa}, maximum bending stress, f_{xb}, and average in-plane shear stress, τ_{xy}, along the two short edges (lines 1-4-7 and 3-6-9) and the plate midspan (line 2-5-8). For example, along line 2-5-8:

$$f_{xa258} = 0.25 f_{x2} + 0.5 f_{x5} + 0.25 f_{x8}$$

$$f_{xb258} = \max\left[abs\left(f_{xa258} - f_{x2}\right), abs\left(f_{xa258} - f_{x8}\right)\right]$$

$$T_{xy258} = 0.25\tau_{xy2} + 0.50\tau_{xy5} + 0.25\tau_{xy8}$$

3) Determine the applicable in=plane longitudinal axial average stress, f_{xa}, maximum bending stress, f_{xb}, and average in-plan shear stress, T_{xy}, along the two long edges (lines 1-2-3 and 7-8-9) and the plate midspan (line 4-5-6). For example, along line 4-5-6:

$$f_{ya456} = 0.25 f_{y4} + 0.50 f_{y5} + 0.25 f_{y6}$$

$$f_{yb456} = \max\left[abs\left(f_{ya456} - f_{y4}\right), abs\left(f_{ya456} - f_{y6}\right)\right]$$

$$\tau_{xy456} = 0.25\tau_{xy4} + 0.50\tau_{xy5} + 0.25\tau_{xy6}$$

4) For Bulletin 2V only, if lateral pressure is present, the plate panel out-of-plane stress effects should be similarly determined from FEA element stresses, if available, or explicitly calculated based on the plate panel geometry, thickness and applied pressure.

5) Use the axial (f_{xa}, f_{ya}) and bending (f_{xb}, f_{yb}) stresses computed above in the appropriate Bulletin 2V code checking formulations, in accordance with Table B-2. The applied stresses f_{xa258}, f_{xb258}, f_{ya456}, f_{yb456}, and the absolute maximum of τ_{xy258} and τ_{xy456} should be used in the bulletin uniaxial and biaxial compression buckling checks, with or without additional effects due to lateral pressure. All locations (e.g., 1 through 9) should be checked against yield or the appropriate tension interaction equations.

Again, the above procedure assumes that the stress gradient is relatively constant. If this is not true, stresses at additional locations should be determined in a similar manner so that a more accurate stress state for the plate or panel may be determined.

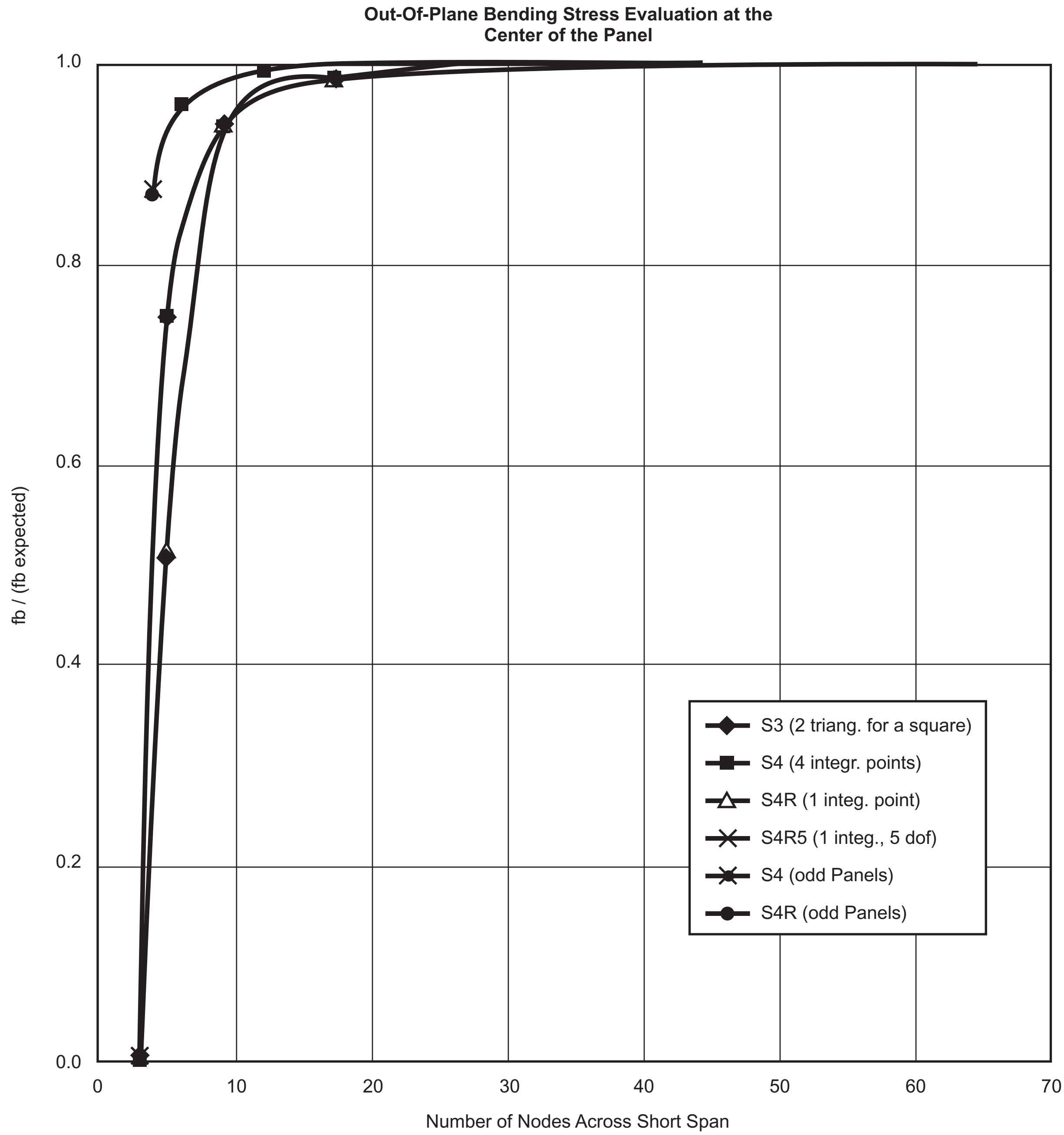


Figure B-1—Panel Weak Axis Bending Stress Evaluation at Center of Panel

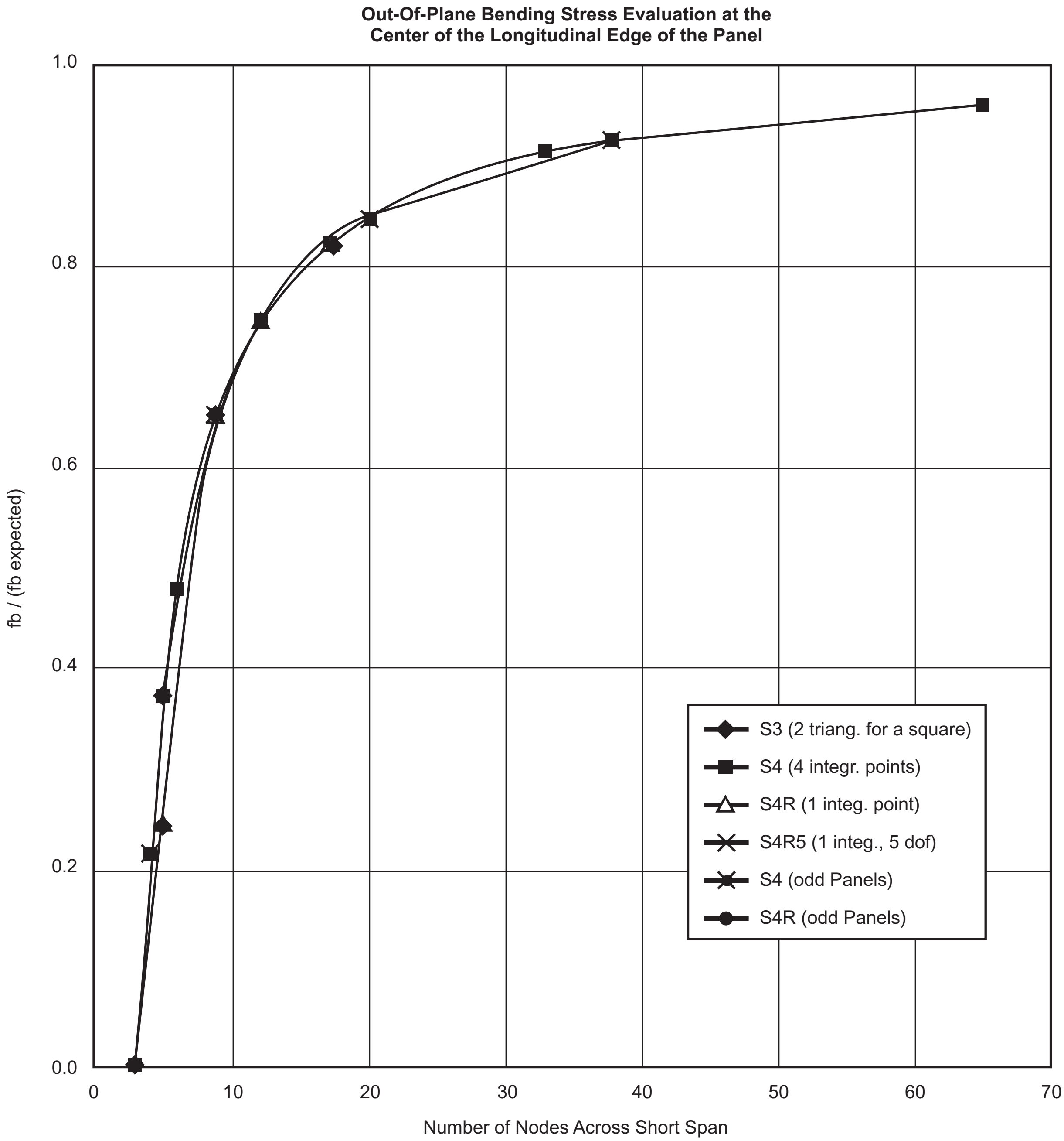
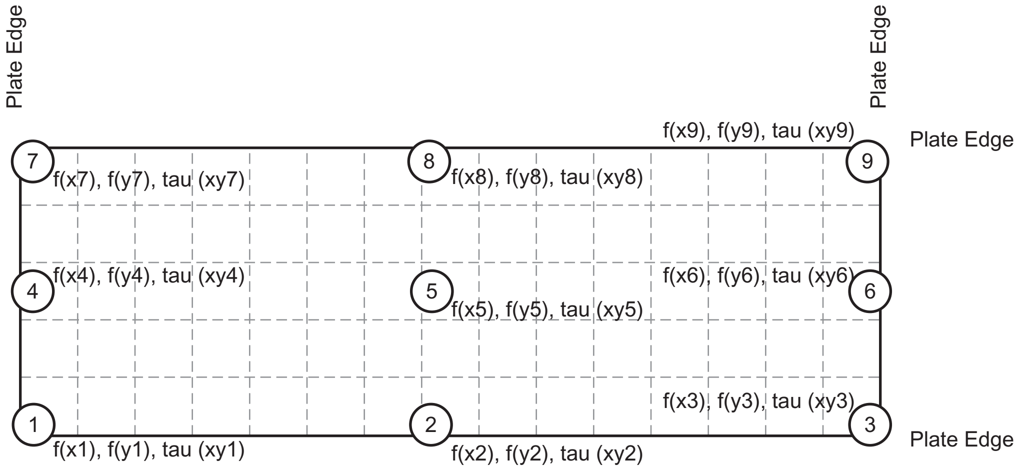
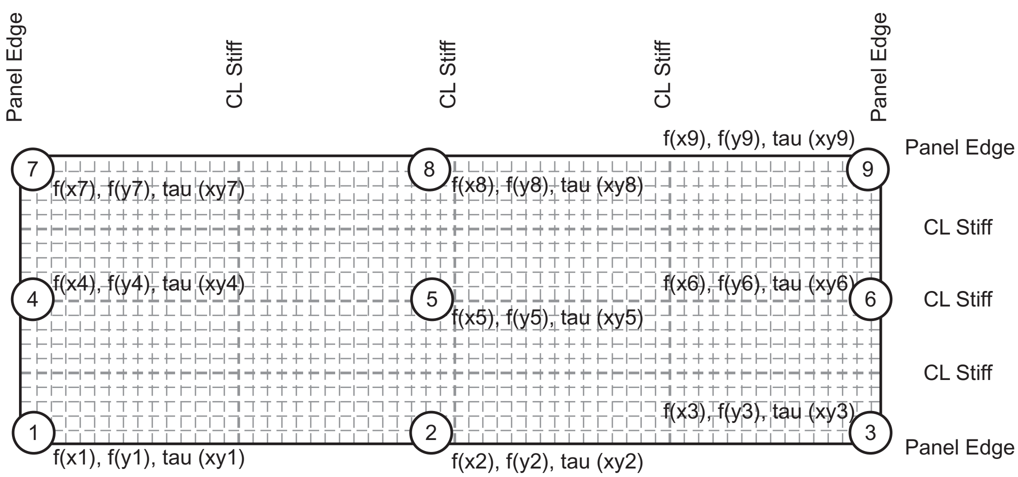


Figure B-2—Panel Weak Axis Bending Stress Evaluation at Center of Longitudinal Edge



FEA Stress Locations for Rectangular Plate



FEA Stress Locations for Rectangular Panel

Figure B-3—Design Guideline—Plate and Stiffened Panel Applied Stress Locations

MINIMUM FEA REQUIREMENTS FOR STIFFENED FLAT PLATE STRUCTURE		
Item	**Coarse Mesh**	**Fine Mesh**
Model Purpose	Strength Analysis Bulletin Code Check	Fatigue Analysis Stress Concentration
Element Model for Plate	In-Plane Load: Linear Stress Membrane Elements Transverse Load: Linear Stress Membrane Element w/ Shear lag Capability	
Element Mesh for Plate	Max. Aspect Ration = 3.0 Max. Element Dimension = 10t	Max. Aspect Ratio = 3.0 Max. Element Dimension = 2t
Element Shape for Plate	4-Node Quadrilateral, Vertices 45 to 135 deg, Square Optimal 3-Node Triangular, Vertices 45 to 90 deg, Equilateral Optimal	
Element Model for Stiffeners	Beam or Spar or Linear Stress Membrane Elements	
Element Mesh for Stiffeners	Same as Plate	
Element Shape for Stiffeners	2-Node Beam or Spar or Same as Plate	

MINIMUM FEA REQUIREMENTS FOR STIFFENED CYLINDRICAL PLATE STRUCTURE		
Item	**Coarse Mesh**	**Fine Mesh**
Model Purpose	Strength Analysis Bulletin Code Check	Fatigue Analysis Stress Concentration
Element Model for Plate	In-Plane Load: Linear Stress Membrane Elements Transverse Load: Linear Stress Shell Element w/ Shear lag Capability	
Element Mesh for Plate	Max. Aspect Ration = 3.0 Max. Element Dimension = 10t	Max. Aspect Ratio = 3.0 Max. Element Dimension = 2t
Element Shape for Plate	4-Node Quadrilateral, Vertices 45 to 135 deg, Square Optimal 3-Node Triangular, Vertices 45 to 90 deg, Equilateral Optimal	
Element Model for Stiffeners	Beam or Spar or Linear Stress Membrane Elements	
Element Mesh for Stiffeners	Same as Plate	
Element Shape for Stiffeners	2-Node Beam or Spar or Same as Plate	

Table B-1—Minimum FEA Requirements for Stiffened Plate Structure

FEA Stress Location	Applicable Applied Stresses from FEA for Code Check				Comment
	In-Plane f_{xa}, f_{xb}	In-Plane f_{ya}, f_{yb}	In-Plane T_{xy}	Out-of-Plane Due To Pressure	
1	f_{x1}	f_{y1}	T_{xy1}	None	Yield check
2	f_{x2} f_{xa258} f_{xb258}	f_{y2} f_{ya123} f_{yb123}	T_{xy2} T_{xy123} T_{xy258}	f_{zb2}	Buckling checks optional
3	f_{x3}	f_{y3}	T_{xy3}	None	Yield check
4	f_{x4} f_{xa147} f_{xb147}	f_{y4} f_{ya456} f_{yb456}	T_{xy4} T_{xy147} T_{xy456}	f_{zb4}	Buckling checks optional
5 (Center of plate or panel)	f_{x5} f_{xa258} f_{xb258}	f_{y5} f_{ya456} f_{yb456}	T_{xy5} T_{xy258} T_{xy456}	f_{xb258} f_{xb456}	Buckling checks required
6	f_{x6} f_{xa369} f_{xb369}	f_{y6} f_{ya456} f_{yb456}	T_{xy6} T_{xy369} T_{xy456}	f_{zb6}	Buckling checks optional
7	f_{x7}	f_{y7}	T_{xy7}	None	Yield check
8	f_{x8} f_{xa258} f_{xb258}	f_{y8} f_{ya789} f_{yb789}	T_{xy8} T_{xy258} T_{xy789}	f_{zb8}	Buckling checks optional
9	f_{x9}	f_{y9}	T_{xy9}	None	Yield check

Notes:
This table presents the minimum stress components for bulletin code checking at each stress location. Additional locations may be needed for plates or panels with varying stress gradients or large aspect ratios. See Figure B-3 for FEA stress locations.

Average stresses are used for uniaxial and biaxial interaction buckling checks. Point stresses are used for von Mises stress determination. Where more than one shear stress result is available, the largest value shall be used.

This table does not apply for locations of local stress concentration.

Table B-2—FEA Design Guideline for Applied Stresses

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