Bulletin on Stability Design of Cylindrical Shells

API BULLETIN 2U
THIRD EDITION, JUNE 2004

American Petroleum Institute

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Upstream Segment

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FOREWORD

This Bulletin is under jurisdiction of the API Subcommittee on Offshore Structures.

This Bulletin contains semi-empirical formulations for evaluating the buckling strength of stiffened and unstiffened cylindrical shells. Used in conjunction with API RP 2T or other applicable codes and standards, this Bulletin will be helpful to engineers involved in the design of offshore structures which include large diameter stiffened or unstiffened cylinders.

The buckling formulations and design considerations contained herein are based on classical buckling formulations, the latest available test data, and analytical studies. This third edition of the Bulletin provides buckling formulations and design considerations based on classical buckling solutions. It also incorporates user experience and feedback from users. It is intended for design and/or review of large diameter cylindrical shells, typically identified as those with D/t ratios greater than or equal to 300. Equations are provided for the prediction of stresses at which typical modes of buckling failures occur for unstiffened and stiffened cylindrical shells, from which the design of the shell plate and the stiffeners may be developed. Used in conjunction with API RP 2T or other applicable codes and standards, this Bulletin will be helpful to engineers involved in the design of offshore structures that include large diameter unstiffened and stiffened cylindrical shells.

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Suggested revisions are invited and should be submitted to API, Standards Department, 1220 L Street, NW, Washington, DC 20005

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Nomenclature

Note: The terms not defined here are uniquely defined in the sections in which they are used.

I = subscript denoting direction and load.

ϕ = longitudinal direction and any load combination

θ = hoop direction and any load combination

x = longitudinal direction and axial compression load only ($N_\theta = 0$).

h = hoop direction and hydrostatic external pressure ($N_\phi / N_\theta = 0.5$).

r = hoop direction and radical external pressure ($N_\phi = 0$).

j = subscript denoting buckling failure mode.

L = local shell buckling.

B = bay instability.

G = general instability.

C = column buckling.

$\bar{A}_r$ = $A_r/(L_r t)$.

A_r = cross-sectional area of one ring stiffener, [in.^{2}].

A_s = cross-sectional area of one stringer stiffener, [in.^{2}].

A_t = total cross-sectional area of cylinder, [in^2].

A_t = $2\pi Rt + N_s A_s$, [in^2].

b = stringer spacing as an arc dimension on the shell centerline, [in].

b_e = effective width of shell in at the shell centerline in the circumferential direction, [in].

B = mean bias factor.

C_m = end moment coefficient in interaction equation.

C_s = ratio of structural proportional limit to yield strength for a material.

C_x = elastic buckling coefficient = $\sigma_{xeL} R/E\ t$.

D_{max}, D_{min} = maximum and minimum shell diameters used to determine the out-of-roundness factor, γ, at a particular cross-section, in. The location giving the largest γ factor should be used.

D = centerline diameter of shell, [in].

D_o = outside diameter of shell, [in].

D_{nom} = nominal outside diameter of cylinder, [in].

E = modulus of elasticity, [ksi].

E_s = secant modulus

E_t = tangent modulus

f_a = applied (computed) axial stress, [ksi].

f_b = applied (computed) bending stress, [ksi].

f_θ = applied (computed) hoop stress, [ksi].

F_a = axial compressive stress permitted in the absence of bending moment, [ksi].

F_b = bending stress permitted in a cylinder in the absence of axial force, [ksi].

F_θ = hoop compressive stress permitted in a cylinder, [ksi].

F_{icj} = inelastic shell buckling stress for fabricated shell, [ksi].

F_{iej} = elastic shell buckling stress for fabricated shell, [ksi].

FS = factor of safety.

$\overline{F}_{iej}$ = F_{iej}/β.

$\overline{F}_{icj}$ = F_{icj}/β

F_y = minimum specified yield stress of material, [ksi].

F_{ys} = static yield stress of material (zero strain rate), [ksi].

G = shear modulus, $E/2\,(1 + \nu)$, [ksi].

g = $M_x M_\theta L_r t A_s / I_s$.

h_s = width (or depth) of stiffening element, [in].

I_{es}, I_{er} = moment of inertia of stringer and ring stiffener, respectively, plus effective width of shell about centroidal axis of combined section (see Fig. 2.2), [in.^{4}]

I_s, I_r = moment of inertia of stringer and ring stiffener, respectively, about its centroidal axis, [in.^{4}]

J_s, J_r = torsional stiffness constant of stringer and ring stiffener, respectively (for general non-circular shapes use $\Sigma\, h_s t_s^3/3$), [in.^{4}]

K = effective length factor for column buckling.

k = ratio of axial load to circumferential load (N_Φ / N_θ).

K_G = factor used in calculating ring stiffener stresses when a cylinder is subjected to external pressure.

K_L = factor used when calculating the shell stress at mid-bay, to account for the effects of a ring or end stiffeners, when a cylinder is subjected to external pressure.

K_p = effective pressure correction factor used in calculation of collapse pressure.

L = unsupported length of shell between rings, [in].

L_j = cylinder length for calculation of bay instability and general instability, [in].

L_e = effective width of shell in the longitudinal direction, [in].

L_b = length of cylinder between bulkheads or lines of support with sufficient stiffness to act as bulkheads. Lines of support which act as bulkheads include end ring stiffeners, [in].

L_r = ring spacing, [in].

L_t = unbraced length of cylinder, [in].

m = number of half waves into which the shell will buckle in the longitudinal direction.

M = applied bending moment.

M_1 = the smaller of the moments at the ends of the unbraced length of a beam-column, [in-kips].

M_2 = the larger of the moments at the ends of the unbraced length of a beam-column, [in-kips].

M_x = $L_r / \sqrt{Rt}$

M_θ = $b / \sqrt{Rt}$

n = number of waves into which the shell will buckle in the circumferential direction.

N_{iej} = theoretical elastic buckling load per unit length of shell (longitudinal or circumferential) for a perfect cylinder for both bay instability and general instability, kips per [in].

N_s	=	number of stringers.
N_ϕ	=	axial load per unit of circumference, [kips/in].
N_θ	=	circumferential load per unit of length, kips/in].
p	=	applied external pressure, [ksi].
p_{eB}	=	theoretical elastic failure pressure for bay instability mode, [ksi].
p_{eG}	=	theoretical elastic failure pressure for general instability mode, [ksi].
p_{eL}	=	theoretical failure pressure for local buckling mode, [ksi].
p_s	=	contribution of stringers to collapse pressure, [ksi].
p_{cG}	=	failure pressure for general instability mode, [ksi].
P	=	applied axial load, [ksi].
P_{cB}	=	inelastic axial compression bay instability load, kips.
p_{cB}	=	failure pressure for bay instability mode, [ksi].
p_{cL}	=	failure pressure for local buckling mode, [ksi].
r	=	radius of gyration, $r = (0.5R^2 + 0.125t^2)^{1/2} \approx 0.707R$, [in].
R	=	radius to centerline of shell, [in].
R_c	=	radius to centroidal axis of the combined ring stiffener and effective width of shell, [in].
R_o	=	radius to outside of shell, [in].
R_r	=	radius to centroid of ring stiffener, [in].
t	=	thickness of shell, [in].
t_w	=	thickness of web of ring stiffener, [in].
t_r, t_x	=	effective shell thickness, $t_r = (A_r + L_e t)/L_e$, $t_x = (A_s + b_e t)/b$, [in].
t_s	=	thickness of stiffening element, [in].
Z_r, Z_s	=	distance from centerline of shell to centroid of stiffener, for ring and stringer stiffeners, respectively (positive outward), [in].
α_{ij}	=	capacity reduction factor to account for the difference between classical theory and predicted instability stresses for fabricated shells.
β	=	a factor applied to the bay instability and general instability failure stresses to avoid interaction with the local buckling mode.
ρ_n , ρ_s	=	reduction factors used in computing collapse load for axial compression.

Δ_c, Δ_d = $F_{iej} / F_y, \overline{F}_{iej} / F_y$

η = plasticity reduction factor which accounts for the nonlinearity of material properties and the effects of residual stresses.

γ = $(D_{max} - D_{min})\ 100/D_{nom}$.

λ = $\pi R/L_r$.

λ_o, λ_e = slenderness parameters as defined in text and used for computing collapse load for axial compression.

λ_G = $\pi R/L_b$.

ν = Poisson's ratio.

σ_{iej} = theoretical elastic instability stress, [ksi].

ψ = partial safety factor.

SI Metric Conversion Factors

in. x 2.54 = mm

ksi x 6.894757= MPa

Glossary

amplification reduction factor (Cm): Coefficient applied to bending term in interaction equation for members subjected to combined bending and axial compression to account for overprediction of secondary moment given by the amplification factor $1/\left(1 - f_a / F_e'\right)$.

asymmetric buckling: The buckling of the shell plate between the circumferential (i.e., ring) stiffeners characterized by the formation of two or more lobes (waves) around the circumference.

axial direction: Longitudinal direction of the member.

axisymmetric collapse: The buckling of the shell plate between the circumferential stiffeners characterized by accordion-like pleats around the circumference.

bay: The section of cylinder between rings.

bay instability: Simultaneous lateral buckling of the shell and stringers with the rings remaining essentially round.

capacity reduction factor (α_{ij}): Coefficient which accounts for the effects of shape imperfections, nonlinear behavior and boundary conditions (other than classical simply supported) on the buckling capacity of the shell.

critical buckling stress: The stress level associated with initiation of buckling. Critical buckling stress is also referred to as the inelastic buckling stress.

distortion energy theory: Failure theory defined by the following equation where the applied stresses are positive for tension and negative for compression.

$$f_a^2 - f_a f_\theta + f_\theta^2 = f_y^2$$

effective length (*KL_t*): The equivalent length used in compression formulas and determined by a bifurcation analysis.

effective length factor (*K*): The ratio between the effective length and the unbraced length of the member.

effective section: Stiffener together with the effective width of shell acting with the stiffener.

effective width: The reduced width of shell or plate which, with an assumed uniform stress distribution, produces the same effect on the behavior of a structural member as the actual width of shell or plate with its nonuniform stress distribution.

elastic buckling stress: The buckling stress of a cylinder based upon elastic behavior.

general instability: Buckling of one or more circumferential (i.e., ring) stiffeners with the attached shell plate in ring-stiffened cylindrical shells. For a ring- and stringer-stiffened cylindrical shell general instability refers to the buckling of one or more rings and stringers with the attached shell plate.

hierarchical order of instability: Refers to a design method that will ensure development of a design with the most critical instability mode (i.e., general instability) having a higher critical buckling stress than the less critical instability mode (i.e., local instability).

hydrostatic pressure: Uniform external pressure on the sides and ends of a member.

inelastic buckling stress: The buckling stress of a cylinder which exceeds the elastic stress limit of the member material. The inelastic material properties are accounted for, including effects of residual stresses due to forming and welding.

interaction of instability modes: Critical buckling stress determined for one instability mode may be affected (i.e., reduced) by another instability mode. Elastic buckling stresses for two or more instability modes should be kept apart to preclude an interaction between instability modes.

local instability: Buckling of the shell plate between the stiffeners with the stiffeners (i.e., rings or rings and stringers) remaining intact.

membrane stresses: The in-plane stresses in the shell; longitudinal, circumferential or shear.

maximum shear stress theory: Failure theory defined by the following equation:

$$\sigma_1 - \sigma_2 = F_y$$

where σ_1 is the maximum principal stress and σ_2, is the minimum principal stress, with tension positive and compression negative.

orthogonally stiffened: A member with circumferential (ring) and longitudinal (stringer) stiffeners.

radial pressure: Uniform external pressure acting only on the sides of a member.

residual stresses: The stresses that remain in an unloaded member after it has been formed and installed in a structure. Some typical causes are forming, welding and corrections for misalignment during installation in the structure. The misalignment stresses are not accounted for by the plasticity reduction factor η.

ring stiffened: A member with circumferential stiffeners.

shell panel: That portion of a shell which is bounded by two adjacent rings in the longitudinal direction and two adjacent stringers in the circumferential direction.

slenderness ratio (KL_t/r): The ratio of the effective length of a member to the radius of gyration of the member.

stress relieved: The residual stresses are significantly reduced by post weld heat treatment.

stringer stiffened: A member with longitudinal stiffeners.

yield stress: The yield stress of the material determined in accordance with ASTM A307.

SECTION 1—General Provisions

1.1 SCOPE

1.1.1 This Bulletin provides stability criteria for determining the structural adequacy against buckling of large diameter circular cylindrical members when subjected to axial load, bending, shear and external pressure acting independently or in combination. The cylinders may be unstiffened, longitudinally stiffened, ring stiffened or stiffened with both longitudinal and ring stiffeners. Research and development work leading to the preparation and issue of all three editions of this Bulletin is documented in References 1 through 16 and the Commentary.

1.1.2 The buckling capacities of the cylinders are based on linear bifurcation (classical) analyses reduced by capacity reduction factors which account for the effects of imperfections and nonlinearity in geometry and boundary conditions and by plasticity reduction factors which account for nonlinearity in material properties. The reduction factors were determined from tests conducted on fabricated steel cylinders. The plasticity reduction factors include the effects of residual stresses resulting from the fabrication process.

1.1.3 Fabricated cylinders are produced by butt-welding together cold or hot formed plate materials. Long fabricated cylinders are generally made by butt-welding together a series of short sections, commonly referred to as cans, with the longitudinal welds rotated between the cans. Long fabricated cylinders generally have *D/t* ratios less than 300 and are covered by AP RP 2A.

1.2 LIMITATIONS

1.2.1 The criteria given are for stiffened cylinders with uniform thickness between ring stiffeners or for unstiffened cylinders of uniform thickness. All shell penetrations must be properly reinforced. The results of experimental studies on buckling of shells with reinforced openings and some design guidance are given in Ref. 2. The stability criteria of this bulletin may be used for cylinders with openings that are reinforced in accordance with the recommendations of Ref. 2 if the openings do not exceed 10% of the cylinder diameter or 80% of the ring spacing. Special consideration must be given to the effects of larger penetrations.

1.2.2 The stability criteria are applicable to shells with diameter-to-thickness (*D/t*) ratios equal to or greater than 300 but less than 1200 and shell thicknesses of 5 mm (3/16 in.) or greater. The deviations from true circular shape and straightness must satisfy the requirements stated in this bulletin, refer to section 10.

1.2.3 Special considerations should be given to the ends of members and other areas of load application where the stress distribution may be nonlinear and localized stresses may exceed those predicted by linear theory. When the localized stresses extend over a distance equal to one half wave length of the buckling mode, they should be considered as a uniform stress around the full circumference. Additional thickness or stiffening may be required.

1.2.4 Failure due to material fracture or fatigue and failure caused by dents resulting from accidental loads are not considered in the bulletin.

1.3 STRESS COMPONENTS FOR STABILITY ANALYSIS AND DESIGN

The internal stress field which controls the buckling of a cylindrical shell consists of the longitudinal membrane, circumferential membrane and in-plane shear stresses. The stresses resulting from a dynamic analysis should be treated as equivalent static stresses.

1.4 STRUCTURAL SHAPEAND PLATE SPECIFICATIONS

Unless otherwise specified by the designer, structural shapes and plates should conform to one of the specifications listed in Table 8.1.4-1/2 of API RP 2A, 20th edition, or Table 4 of API RP 2T.

1.5 HIERARCHICAL ORDER AND INTERATCTIONOF BUCKLING MODES

1.5.1 This Bulletin requires avoidance of failure in any mode, and recommends sizing of the cylindrical shell plate and the arrangement and sizing of the stiffeners to ensure that the buckling stress for the most critical general instability is higher than the less critical local instability buckling stress.

1.5.2 A hierarchical order of buckling stresses with adequate separation of general, bay and local instability stresses is also desirable for a cylindrical shell subjected to loading resulting in longitudinal and circumferential stresses to preclude any interaction of buckling modes. To prevent a reduction in buckling stress due to interaction of buckling modes, it is recommended that bay and general instability mode elastic buckling stresses remain at least 1.2 times the elastic buckling stress for local instability.

SECTION 2—Geometries, Failure Modes, and Loads

The maximum stresses corresponding to all of the failure modes will be referred to as buckling stresses. Buckling stress equations are given for the following geometries, failure modes and load conditions.

2.1 GEOMETRIES

a. Unstiffened.
b. Ring Stiffened.
c. Stringer Stiffened.
d. Ring and Stringer Stiffened.

The four cylinder geometries are illustrated in Figure 2.1 and the stiffener geometries in Figure 2.2.

2.2 FAILURE MODES

a. **Local Shell Buckling**—buckling of the shell plate between stiffeners. The stringers remain straight and the rings remain round.
b. **Bay Instability**—buckling of the stringers together with the attached shell plate between rings (or the ends of the cylinders for stringer stiffened cylinders). The rings and the ends of the cylinders remain round.
c. **General Instability**—buckling of one or more rings together with the attached shell (shell plus stringers for ring and stringer stiffened cylinders).
d. **Local Stiffener Buckling**—buckling of the stiffener elements.
e. **Column Buckling**—buckling of the cylinder as a column.

The first four failure modes are illustrated in Figure 2.3.

2.3 LOADS AND LOAD COMBINATIONS

a. **Determination of Applied Stresses Due to the Following Loads:**

1. Longitudinal stress due to axial compression/tension and overall bending.
2. Shear stress due to transverse shear and torsion.
3. Circumferential stress due to external pressure.
4. Combined (von Mises) stress due to combination of loads.

b. Determination of Utilization Ratios Based on Recommended Interaction Relationships for Combined Loads:

1. Longitudinal (axial) tension and circumferential (hoop) compression.
2. Longitudinal (axial) compression and circumferential compression.

Note: Stresses and stress combinations considered are for in-plane loads and do not account for secondary bending stresses due to out-of-plane pressure loading on shell plate.

Some of the external pressure on an orthogonally stiffened cylindrical shell will be directly transferred to the rings through the stringers and the resulting bending stresses in the stringers may be appreciable. Local, bay and general instability stresses compared against the applied axial and hoop stresses, whether obtained from a finite element analysis or determined based on equations in Section 11, may need to be supplemented by checking effective stringer column instability as an appropriate beam column element.

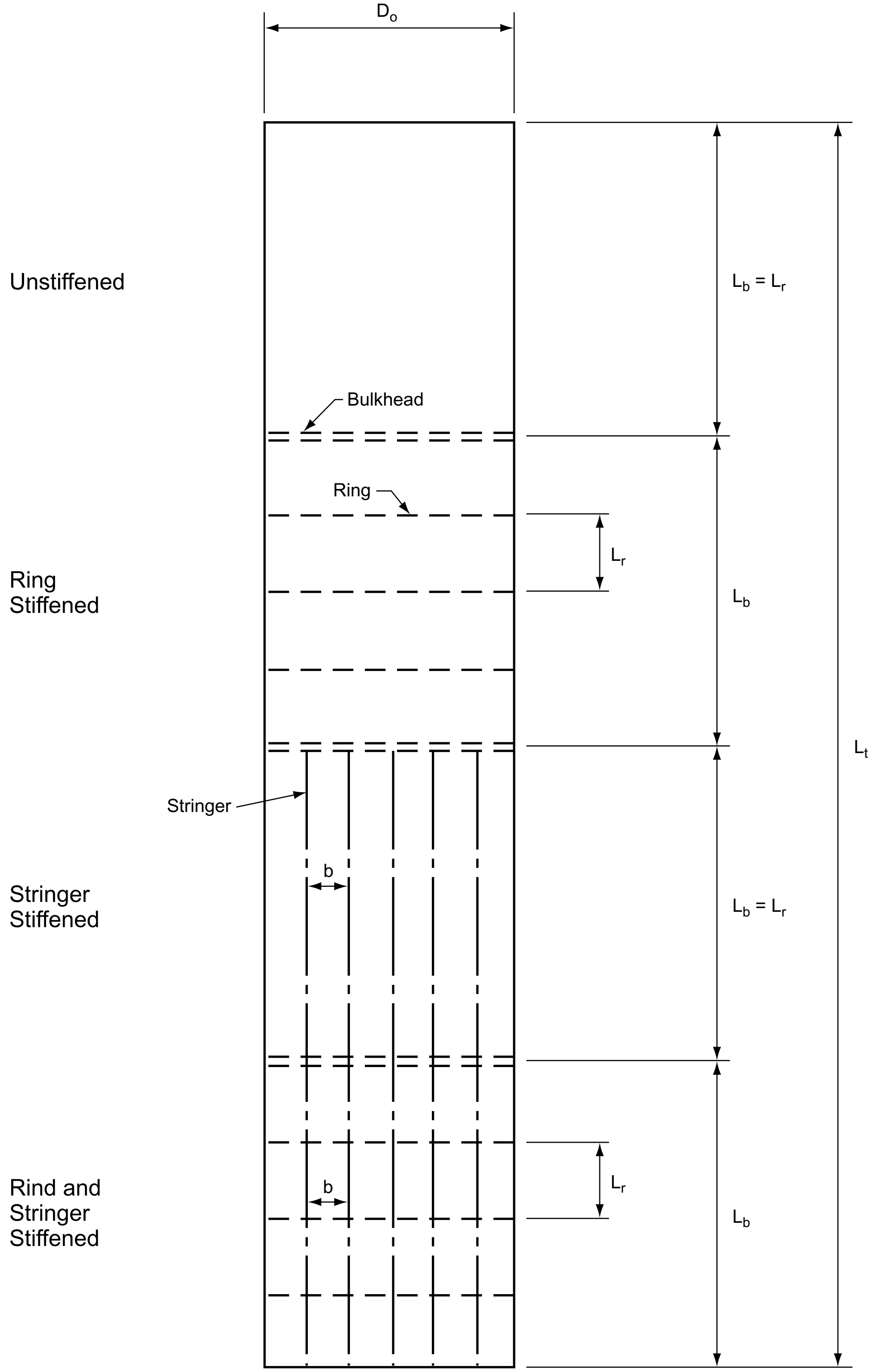
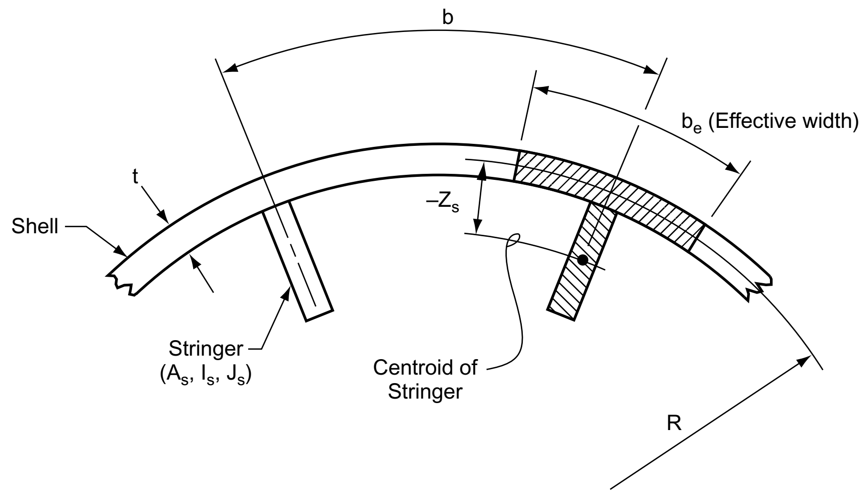
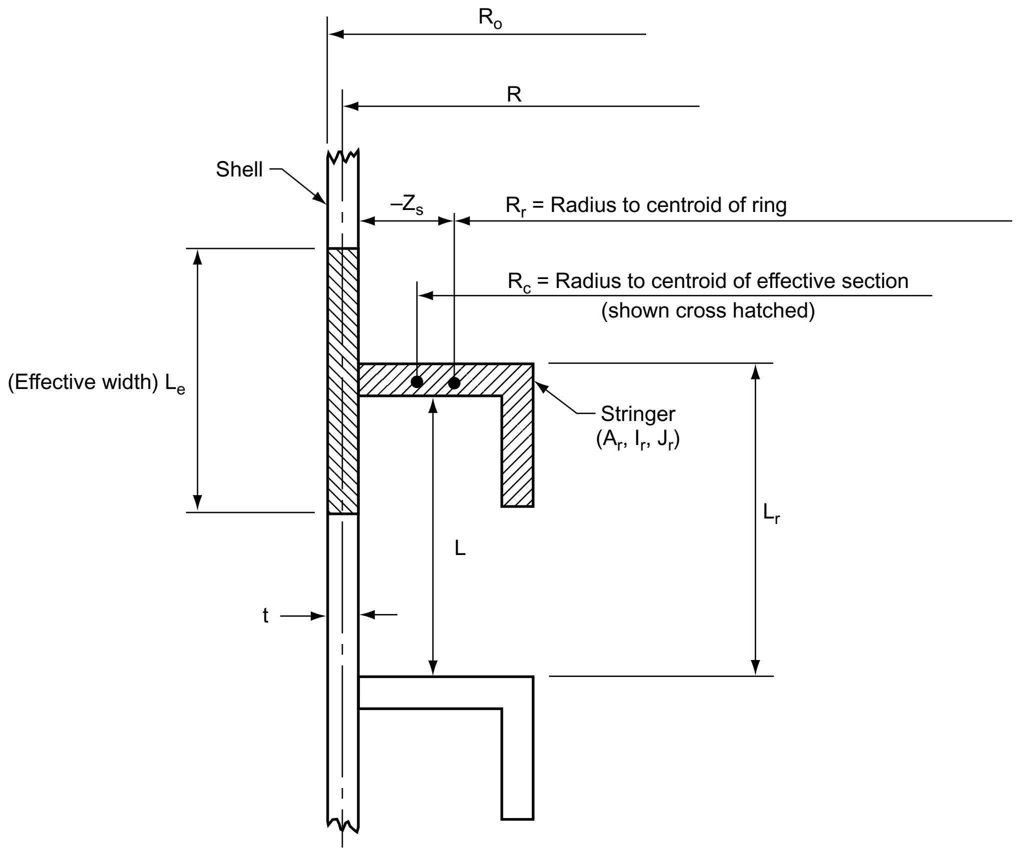


Figure 2.1--Geometry of Cylinder



Section Through Stringers



Section Through Rings

Figure 2.2--Geometry of Stiffeners

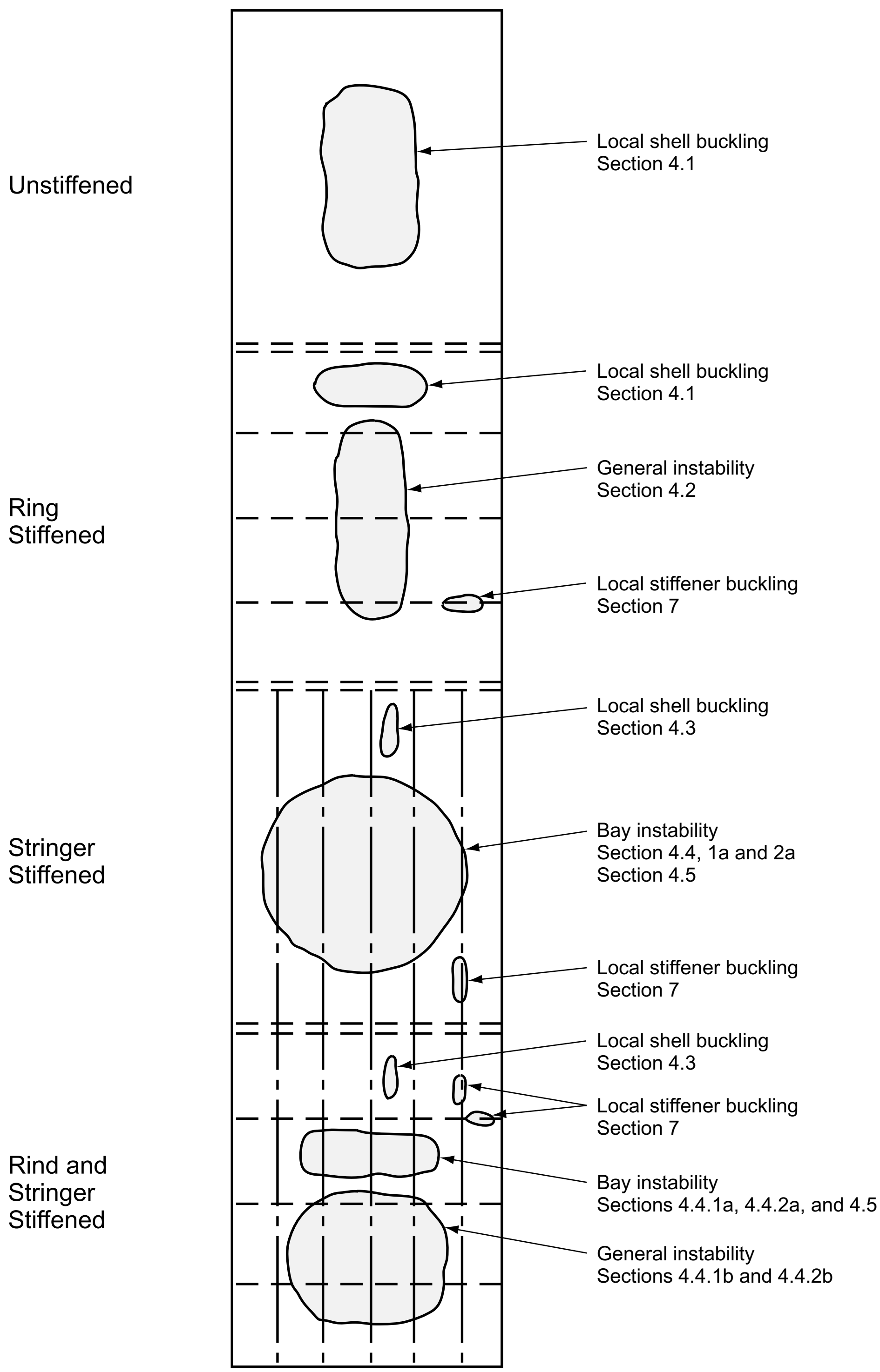


Figure 2.3--Shell Buckling Modes for Cylinders

SECTION 3—Buckling Design Method

3.1 The buckling strength formulations presented in this bulletin are based upon classical linear theory which is modified by reduction factors α_{ij} and η which account for the effects of imperfections, boundary conditions, nonlinearity of material properties and residual stresses. The reduction factors are determined from approximate lower bound values of test data of shells with initial imperfections representative of the specified tolerance limits given in Section 10.

3.2 The general equations for the predicted shell buckling stresses for fabricated steel cylinders subjected to the individual load cases of axial compression, bending and external pressure are given by Equations (3.2-1) and (3.2-2). The equations for α_{ij} and σ_{iej} are given in Section 4 and the equations for η are given in Section 5.

a. Elastic Shell Buckling Stress

$$F_{iej} = \alpha_{ij}\sigma_{iej} \tag{3.2-1}$$

b. Inelastic Shell Buckling Stress

$$F_{icj} = \eta F_{iej} \tag{3.2-2}$$

3.3 The bay instability stresses for cylinders with stringer stiffeners are given by orthotropic shell theory. This theory requires that the number of stringers must be greater than about three times the number of circumferential waves corresponding to the buckling mode. An alternate method is given for determining the bay instability stresses for cylinders which do not satisfy this requirement.

3.4 The buckling stress equations for cylinders under the individual load cases of axial compression, bending, radial external pressure ($N_\phi = 0$) and hydrostatic external pressure ($N_\phi/N_\theta = 0.5$) are given in Section 4. Interaction equations are given in Section 6 for cylinders subjected to combinations of axial load, bending and external pressure. The interaction between column buckling and shell buckling is considered in Section 8. The method for determining the size of stiffeners is given in Section 7.

3.5 A flow chart is given in Figure 3.1 for determining the allowable stresses. The equations for allowable stresses are given in Section 9 and equations for determining the stresses due to applied load are given in Section 11. A summary of the sections relating to the buckling modes for each of the different shell geometries is given on Table 3.1.

Table 3.1—Section Numbers Relating to Buckling Modes for Different Shell Geometries

	Geometry			
Buckling Mode	Unstiffened	Ring Stiff	Stringer Stiff	Ring and Stringer Stiff
Local Shell Buckling	4.1	4.1	4.3	4.3
Bay Instability			4.4 4.5	4.4 4.5
General Instability		4.2		4.4 (1b, 2b)
Local Stiffener Buckling		7.2	7.2	7.2
Column Buckling	8.0	8.0	8.0	8.0

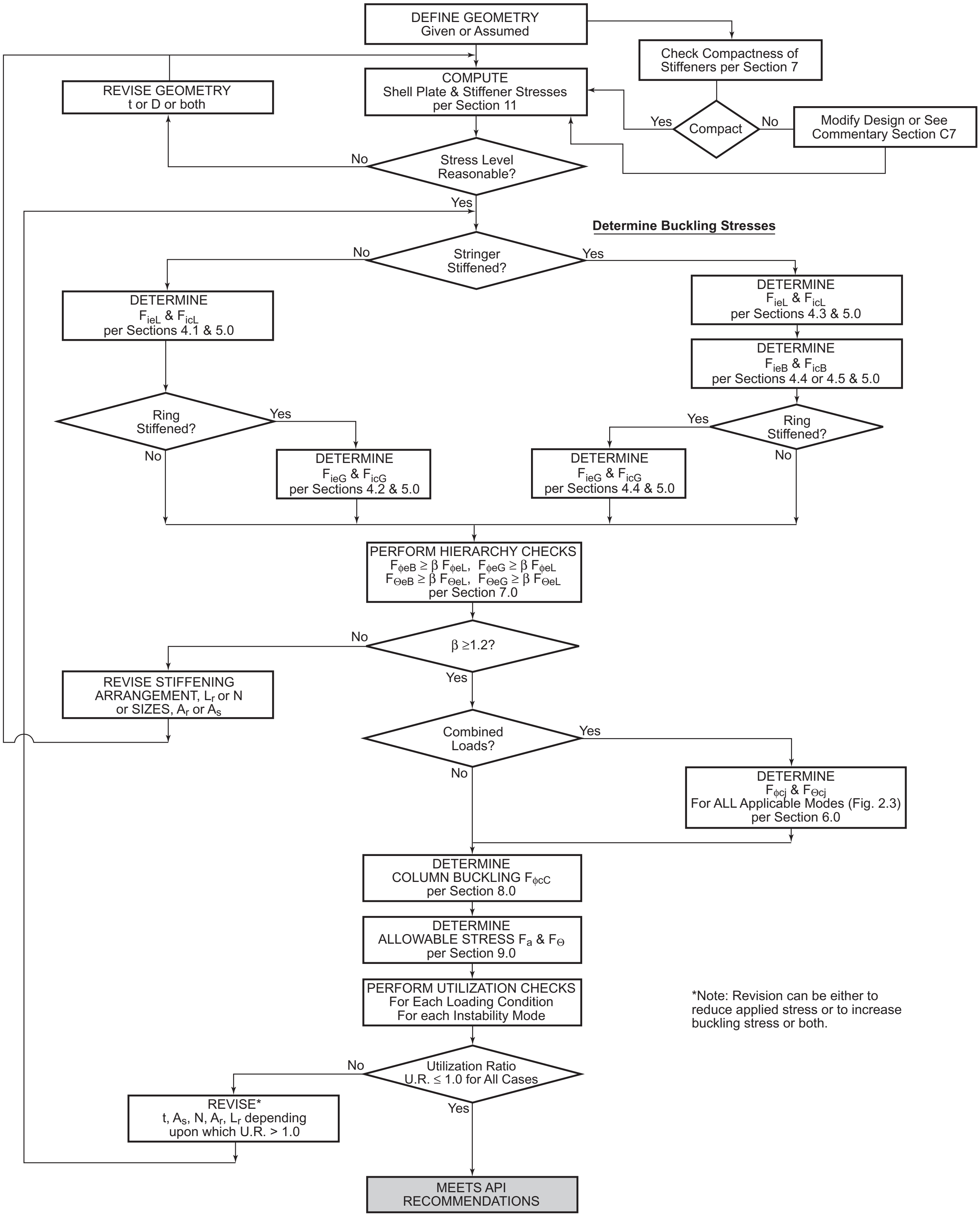


Figure 3.1--Flow Chart for Meeting API Recommendations

SECTION 4—Predicted Shell Buckling Stresses for Axial Load, Bending and External Pressure

This section gives equations for determining the shell buckling stresses for the load cases of axial compression, bending, radial external pressure (N_ϕ= 0) and hydrostatic external pressure (N_ϕ= 0.5 N_θ). The general equations for predicting the elastic and inelastic buckling stresses for fabricated cylinders are given by Equations 3.2-1 and 3.2-2. The equations for determining α_{ij} and σ_{iej} are given in the following section. The equations for determining the plasticity reduction factors, η, are given in Section 5. Equations given in this section are based on the behavior of large diameter cylindrical shells and permit determination of local, bay and general instability mode buckling stresses. As illustrated in the Commentary, predicted stresses include imperfection/correction factors and are compatible with test data. Predicted stresses are based on the assumption that the instability modes are separated and do not interact. To ensure the assumption remains valid, a hierarchy among the instability modes is required. As shown in Section 7, ring and stringer stiffener spacing and sizes should be modified, as necessary, to achieve the desirable hierarchy.

The values of M_x and M_θ appearing in the following equations are defined as:

$$M_x = L_r/(Rt)^{0.5} \text{ and } M_\theta = b/(Rt)^{0.5} \tag{4-1a}$$

$$Z_x = M_x^2\left(1-\nu^2\right)^{0.5} \text{ and } Z_\theta = M_\theta^2\left(1-\nu^2\right)^{0.5} \tag{4-1b}$$

Where the term Z represents the classical definition of geometric parameter.

4.1 LOCAL BUCKLING OF UNSTIFFENED OR RING STIFFENED CYLINDERS

4.1.1 Axial Compression or Bending (N_θ= 0)

The buckling stresses for cylinders subjected to axial compression or bending are assumed to be the same (see Commentary).

a. Elastic Buckling Stresses

$$F_{xeL} = C_{xL}\frac{\pi^2 E}{12(1-\nu^2)}(t/L_r)^2 \tag{4.1-1}$$

where the buckling coefficient, C_{xL}, is expressed in terms of geometric curvature parameter, M_x, the D/t ratio and the imperfection factor, α_{xL} :

$$C_{xL} = \left[1+\left(\frac{150}{(D/t)}\right)(\alpha_{xL})^2\left(M_x^4\right)\right]^{0.5} \tag{4.1-2}$$

where, the imperfection factor is defined in paragraph 4.1.1(b).

b. **Imperfection Factor, α_{xL}**

$$\alpha_{xL} = 9.0/(300 + D/t)^{0.4} \tag{4.1-3}$$

c. **Inelastic Buckling Stresses**: The buckling stress in the material elasto-plastic zone is determined following the empirical formulation given in Section 5.

$$F_{cL} = \eta F_{eL} \quad \text{for} \quad F_{eL} > 0.5F_y$$

(i.e., F_{eL} > material proportional limit)

$$F_{cL} = F_{eL} \text{ for } F_{eL} < 0.5F_y \tag{4.1-4}$$

(i.e., F_{eL} < material proportional limit)

4.1.2 External Pressure (N_ϕ/N_θ= 0 or 0.5)

a. **Elastic Buckling Stresses**

$$F_{heL} = F_{reL} = C_{\theta L} \frac{\pi^2 E}{12(1-\nu^2)}(t/L_r)^2 \tag{4.1-5}$$

where the buckling coefficient, $C_{\theta L}$, will have a different definition for each geometry as defined by its asymmetric buckling mode (i.e., number of lobes, n).

The buckling coefficient, $C_{\theta L}$, can be readily obtained by defining the geometric curvature parameter, M_x, and the following formula based on Batdorf-introduced simplifications to Donnell's equations. A simple iterative approach is necessary to determine the number of half-waves (i.e., lobes) "n". Since API provides for determination of instability modes higher than that of local instability mode, local instability is considered not to interact with other instability modes. This is achieved by implementing the hierarchical failure order as required by Section 7. For unstiffened and ring-stiffened cylindrical shells and imperfection factor is defined in Section 4.1.2b.

Assuming a single mode, m = 1, between the rings, Batdorf's equation permits determination of the number of buckling lobes, n, from the following equation:

$$\frac{\beta^2(1+\beta^2)^4}{2+3\beta^2} = Z_m \tag{4.1-6}$$

where the modified geometric curvature parameter, Z_m, and β can be expressed as:

$$Z_m = \frac{12Z_x^2}{\pi^4} = \frac{12\left[M_x^2(1-\nu^2)^{1/2}\right]^2}{\pi^4}$$

$Z_m = 0.112 M_x^4$ for Poisson's ration of 0.3

$\beta = L_r / (\pi R / n)$

The smallest "n" that causes that left and the right side of the equation (4.1-6) to be approximately equal defines the asymmetric buckling of the shell plate. Then the buckling coefficient, $C_{\theta L}$, can be directly computed from the following equation:

$$C_{\theta L} = \left[\frac{(1+\beta^2)^2}{(0.5+\beta^2)} + \frac{0.112 M_x^4}{(1+\beta^2)^2 (0.5+\beta^2)} \right] [\alpha_{\theta L}] \qquad (4.1\text{-}7)$$

b. Imperfection Factor, $\alpha_{\theta L}$

For cylindrical shells with D/t ratios greater than 300, test-to-predicted stress ratios indicate that the use of an imperfection factor equal to 0.8 is too conservative. It is recommended that:

$$\alpha_{\theta L} = 1.0 \quad \text{if } M_x < 5 \qquad (4.1\text{-}8)$$
$$\alpha_{\theta L} = 0.8 \quad \text{if } M_x > 5$$

c. Inelastic Buckling Stresses

Inelastic buckling stress definitions in terms of plasticity reduction factors to be applied on elastic buckling stresses are given in Section 5.

4.1.3 Transverse Shear

Panel instability due to transverse shear and torsion can be critical at interfaces. Critical buckling stress is affected not only by the shell thickness and the panel aspect ratio, but also by the boundary conditions.

As a minimum, it is necessary to incorporate the shear stress in a von Mises stress check to assess the overall effect of combined loads.

The local shear stress may become important when concentrated local load transfers occur due to attachments/appurtenances. Further discussion on this subject it presented in Section 4.3.3.

4.2 GENERAL INSTABILITY OF RING STIFFENED CYLINDERS

4.2.1 Axial Compression or Bending ($N_\theta = 0$)

a. Elastic Buckling Stresses

$$F_{xeG} = \alpha_{xG}\sigma_{xeG} = \alpha_{xG} 0.605E\frac{t}{R}\left(1+\overline{A}_r\right)^{1/2} \quad (4.2\text{-}1)$$

$$\overline{A}_r = \frac{A_r}{L_r t}$$

where $\overline{A}_r$ is the ring area and L_r is the ring spacing.

b. Imperfection Factors

$$\alpha_{xG} = \begin{cases} 0.72 & \text{if} \quad \overline{A}_r \geq 0.2 \\ (3.6-5.0\alpha_x)\overline{A}_r + \alpha_x & \text{if} \quad 0.06 < \overline{A}_r < 0.2 \\ \alpha_x & \text{if} \quad \overline{A}_r \leq 0.06 \end{cases} \quad (4.2\text{-}2)$$

where $\alpha_x = 0.85/[1+0.0025(D/t)]$ (4.2-3)

c. Inelastic Buckling Stresses

Inelastic buckling stress definitions in terms of plasticity reduction factors to be applied on elastic buckling stresses are given in Section 5.

4.2.2 External Pressure (N_ϕ/N_θ = 0 or 0.5)

a. Elastic Buckling Stresses With or Without End Pressure

$$F_{reG} \text{ or } F_{heG} = \alpha_{\theta G}\frac{p_{eG}R_o}{t}K_{\theta G} \quad (4.2\text{-}4)$$

where $K_{\theta G}$ is given by Equation 11.3-12a

$$p_{eG} = \frac{E(t/R)\lambda_G^4}{\left(n^2+k\lambda_G^2-1\right)\left(n^2+\lambda_G^2\right)^2} + \frac{EI_{er}\left(n^2-1\right)}{L_r R_c^2 R_o} \quad (4.2\text{-}5)$$

where $\lambda_G = \pi R/L_b$, $k = 0$ for radial pressure and 0.5 for hydrostatic pressure, R_c is the radius to the centroid of the effective section, R_o is the radius to the outside of the shell and I_{er} is the moment of inertia of the effective section given by the following equation:

$$I_{er} = I_r + A_r Z_r^2 \frac{L_e t}{A_r + L_e t} + \frac{L_e t^3}{12} \quad (4.2\text{-}6)$$

where Z_r is the distance from the centerline of the shell to the centroid of the stiffener ring (positive outward).

The value of L_e can be approximated by $1.1\sqrt{Dt}+t_w$ when M_x > 1.56 and L_r when $M_x \leq 1.56$. The correct value for n is the value which gives the minimum value of p_{eG} in Equation 4.2-5. The minimum value of n is 2 and the maximum value will be less than 10 for most shells of interest. The minimum pressure will correspond to a noninteger value of n. The pressure p_{eG} is determined by trial and error.

b. **Imperfection Factors**: For fabricated cylinders which meet the fabrication tolerances given in Section 10, a constant value of $\alpha_{\theta G} = 0.8$ is adequate.

c. **Inelastic Buckling Stresses**: Inelastic buckling stress definitions in terms of plasticity reduction factors to be applied on elastic buckling stresses are given in Section 5.

d. **Failure Pressures**

$$p_{cG} = \eta \alpha_{\theta G} p_{eG} \tag{4.2-7}$$

See a, b, and c above for determination of the terms in Equation 4.2-7.

4.3 LOCAL BUCKLING OF STRINGER STIFFENED OR RING AND STRINGER STIFFENED CYLINDERS

The following equations are based upon the assumption that the stringers satisfy the compact section requirements of Section 7. A method is presented in the Commentary for noncompact sections.

4.3.1 Axial Compression or Bending (N_θ= 0)

For the stringers to be effective in increasing the buckling stress, they must be spaced sufficiently close so that M_θ < 15 and $b < 2L_r$. For values of M_θ > 15 or $b > 2L_r$ the buckling stresses are computed as if the stringers were omitted. However, the stringers may be assumed to be effective in carrying part of the axial loading when computing the stresses due to applied loads.

a. **Elastic Buckling Stresses**

$$F_{xeL} = C_{xL} \frac{\pi^2 E}{12(1-\nu^2)} (t/b)^2 \tag{4.3-1}$$

where the buckling coefficient, C_{xL}, is expressed in terms of geometric curvature parameter, M_θ:

$$C_{xL} = 4.0 \qquad M_\theta \leq 2 \tag{4.3.2}$$

$$C_{xL} = 4.0\{1 + 0.038[M_\theta - 2]^3\}[\alpha_{xL}] \qquad M_\theta > 2$$

where the stringer spacing, b, is defined as $\pi D/N$, and the imperfection reduction factor, α_{xL}, is set equal to 1.0 for geometries meeting API-recommended tolerances.

b. **Inelastic Buckling Stresses**: Inelastic buckling stresses should be determined by applying plasticity reduction factors to elastic buckling stresses as recommended in Section 5.

4.3.2 External Pressure ($N_\phi / N_\theta = 0$ or 0.5)

The local buckling pressure of a stringer stiffened cylinder will be greater than a corresponding unstiffened or ring stiffened cylinder if $0.5N_s$ (N_s = Number of Stringers) is greater than the number of circumferential waves at buckling for the cylinder without stringers. This is based upon the assumption that one-half wave will form between stringers at buckling. For stringers with high torsional rigidity a full wave might form between stringers with a concurrent increase in the buckling pressure. This possible increase in buckling pressure is not considered.

a. **Elastic Buckling Stresses With or Without End Pressure**

$$F_{\theta eL} = C_{\theta L} \frac{\pi^2 E}{12\left(1-\nu^2\right)} \left(t / L_r\right)^2 \qquad (4.3\text{-}3)$$

where the buckling coefficient, $C_{\theta L}$, is defined by:

$$C_{\theta L} = \frac{\left(1+[L_r/b]^2\right)^2}{\left(L_r/b\right)^2} \left[1 + \frac{0.011 M_x^3}{0.5\left(1+[L_r/b]^2\right)^2}\right] \left[\alpha_{\theta L}\right] \qquad (4.3\text{-}4)$$

If the stringer spacing is large and the aspect ratio is small, the minimum number of buckling lobes, n, for an unstiffened cylindrical shell may yield a buckling coefficient larger than that obtained from above given buckling coefficient. In such instances, cylindrical shell should be treated as unstiffened and the buckling coefficient determined from equations in Section 4.1.2.

b. **Imperfection Factors:** The test results indicate that no imperfection reduction factor is needed for stringer stiffened cylinders. Therefore:

$\alpha_{\theta L} = 1.0$

c. **Inelastic Buckling Stresses:** Inelastic instability stresses are determined by applying plasticity reduction factors to elastic buckling stresses as recommended in Section 5.

4.4 BAY INSTIBALITY OF STRINGER STIFFENED OR RING AND STRINGER STIFFENED CYLINDERS, AND GENERAL INSTABILITY OF RING AND STRING STIFFENED CYLINDERS BASED UPON ORTHOTROPIC SHELL THEORY

The theoretical elastic buckling loads for both bay instability and general instability are given by the following orthotropic shell equation (Equation 4.4-1). The elastic buckling load per unit length of shell is denoted N_{iej} where i is the stress direction and j is the buckling mode with $j = B$ for bay instability and $j = G$ for general instability. The bay instability stress is determined by letting the cylinder length equal the ring spacing ($L_j = L_r$) and the general instability stress is determined by letting the cylinder length equal the distance between bulkheads or stiffener rings that are sufficiently sized to act as bulkheads ($L_j = L_b$).

When the rings and stringers are not sufficiently close together so that the shell plating is fully effective, the rigidity parameters (E_x, E_θ, D_x, D_θ, $D_{x\theta}$, $G_{x\theta}$) of Equation 4.4-1 are modified by the ratios of effective width to stiffener spacing. Equations are given for Le and b_e which are the effective widths of plate in the x and θ directions, respectively. When $L_e < L_r$ or $b_e < b$, set ν = 0; otherwise ν= 0.3. In all cases, use ν=0.3 when calculating G.

The values of m and n to use in the following equation are those which minimize N_{iej} where $m \geq 1$ and $n \geq 2$. For the following equation to be valid, the number of stringers must be greater than about $3n$ and the bay instability stress should be less than 1.5 times the local shell buckling stress. When these conditions are not met, the equations in Section 4.5 should be used for sizing the stringers. Section 4.2 should be used for sizing the rings.

$$N_{iej} = \left[A_{33} + \frac{A_{12}A_{23} - A_{13}A_{22}}{A_{11}A_{22} - A_{12}^2} A_{13} + \frac{A_{12}A_{13} - A_{11}A_{23}}{A_{11}A_{22} - A_{12}^2} A_{23} \right] / Y \tag{4.4-1}$$

where

$$A_{11} = E_x \left(\frac{m\pi}{L_j} \right)^2 + G_{x\theta} \left(\frac{n}{R} \right)^2$$

$$A_{22} = E_\theta \left(\frac{n}{R} \right)^2 + G_{x\theta} \left(\frac{m\pi}{L_j} \right)^2$$

$$A_{33} = D_x \left(\frac{m\pi}{L_j} \right)^4 + D_{x\theta} \left(\frac{m\pi}{L_j} \right)^2 \left(\frac{n}{R} \right)^2 + D_\theta \left(\frac{n}{R} \right)^4 + \frac{E_\theta}{R^2} + \frac{2C_\theta}{R} \left(\frac{n}{R} \right)^2$$

$$A_{12} = (E_{x\theta} + G_{x\theta}) \left(\frac{m\pi}{L_j} \right) \left(\frac{n}{R} \right)$$

$$A_{23} = \frac{E_\theta}{R} \left(\frac{n}{R} \right) + C_\theta \left(\frac{n}{R} \right)^3$$

$$A_{13} = \frac{E_{x\theta}}{R} \left(\frac{m\pi}{L_j} \right) + C_x \left(\frac{m\pi}{L_j} \right)^3$$

$$E_x = \frac{Et}{1-\nu^2} \left(\frac{b_e}{b} \right) + \frac{EA_s}{b}$$

$$E_{x\theta} = \frac{\nu Et}{1-\nu^2}$$

$$E_\theta = \frac{Et}{1-v^2}\left(\frac{L_e}{L_r}\right) + \frac{EA_r}{L_r}$$

$$G_{x\theta} = \frac{Gt}{2}\left(\frac{L_e}{L_r} + \frac{b_e}{b}\right)$$

$$D_x = \frac{Et^3}{12(1-v^2)}\left(\frac{b_e}{b}\right) + \frac{EI_s}{b} + \frac{EA_s Z_s^2}{b}$$

$$D_\theta = \frac{Et^3}{12(1-v^2)}\left(\frac{L_e}{L_r}\right) + \frac{EI_r}{L_r} + \frac{EA_r Z_r^2}{L_r}$$

$$D_{x\theta} = \frac{vEt^3}{6(1-v^2)} + \frac{Gt^3}{6}\left(\frac{L_e}{L_r} + \frac{b_e}{b}\right) + \frac{GJ_s}{b} + \frac{GJ_r}{L_r}$$

$$C_\theta = \frac{EA_r Z_r}{L_r}$$

$$C_x = \frac{EA_s Z_s}{b}$$

The term Y in Equation 4.4-1 is dependent upon the loading condition and is defined in the sections below.

4.4.1 Axial Compression or Bending ($N_\theta = 0$)

The elastic buckling stresses in the longitudinal direction for the bay instability and general instability modes of failure are given by Equations 4.4-3 and 4.4-5 with N_{xej} determined from Equation 4.4-1. The following relationships for Y, t_x, and L_e are to be used for both bay and general instability stresses. When $b_e < b$, the values for F_{xej} must be determined by iteration since the effective width is a function of the buckling stress.

$$Y = \left(\frac{m\pi}{L_j}\right)^2$$

$$t_x = \frac{A_s + b_e t}{b}$$

$$L_e = L_r$$

a. For Bay Instability

1. **Elastic Buckling Stresses**. Use the following relationships together with those above for Y, t_x, and L_e to determine the bay instability buckling stresses:

$$j = B,\ A_r = I_r = J_r = 0,\ L_j = L_r$$

$$b_e = 1.9t\sqrt{E / F_{xeB}} \leq b \tag{4.4-2}$$

$$F_{xeB} = \alpha_{xB} \frac{N_{xeB}}{t_x} \tag{4.4-3}$$

F_y is to be substituted for F_{xeB} when $F_{xeB} > F_y$ so that $b_e = 1.9t(E / Fy)^{1/2} \leq b$.

Equations 4.4-2 and 4.4-3 may require an iterative solution

2. Imperfection Factors

$$\alpha_{xB} = \begin{cases} 0.65 & \text{if} \quad \overline{A}_s \geq 0.06 \\ \alpha_{xL} & \text{if} \quad \overline{A}_s < 0.06 \end{cases}$$

$\overline{A}_s = \dfrac{A_s}{bt}$ and α_{xL} is given by Equation 4.1-3

3. Inelastic Buckling Stresses: Inelastic instability stresses are determined by applying plasticity reduction factors to elastic buckling stresses as recommended in Section 5.

b. For General Instability

1. Elastic Buckling Stresses. Use the following relationships together with those above for Y, t_x, and L_e to determine the general instability stresses. The local buckling stress, F_{xcL}, obtained from Equations 4.3-1 and 5-1 and the general instability stress, F_{xcG}, obtained from Equations 4.4-5 and 5-1 should be substituted into Equation 4.4-4.

$j = G, L_j = L_b$

$$b_e = b\sqrt{F_{xcL} / F_{xcG}} \leq b \tag{4.4-4}$$

$$F_{xeG} = \alpha_{xG} \frac{N_{xeG}}{t_x} \tag{4.4-5}$$

Equations 4.4-4 and 4.4-5 may require an iterative solution.

2. Imperfection Factors

α_{xG} is given by Equation 4.2-2.

3. Inelastic Buckling Stresses : Inelastic instability stresses are determined by applying plasticity reduction factors to elastic buckling stresses as recommended in Section 5.

4.4.2 External Pressure (N_ϕ $/N_\theta$ = 0 or 0.5)

The elastic buckling stresses in the hoop direction for the bay instability and general instability modes of failure are given by Equations 4.4-6 and 4.4-7 with $N_{\theta ej}$ determined from Equation 4.4-1. The following relationships for Y and t_r are to be used for both bay and general instability stresses.

$$Y = k\left(\frac{m\pi}{L_j}\right)^2 + \left(\frac{n}{R}\right)^2$$

$$t_r = \frac{A_r + L_e t}{L_e}$$

where k = 0 for radial pressure and k = 0.5 for hydrostatic pressure.

a. For Bay Instability

1. **Elastic Buckling Stresses**. Use the following relationships together with that given above for Y to determine the bay instability stresses.

$$j = B,\ A_r = I_r = J_r = 0,\ L_j = L_r$$

$$L_e = L_r\ ,\ b_e = b$$

$$F_{reB} \text{ or } F_{heB} = \alpha_{\theta B}\frac{N_{\theta eB}}{t}K_{\theta L} \qquad (4.4\text{-}6)$$

See Equation 11. 3-3b for $K_{\theta L}$.

2. **Imperfection Factors**

$$\alpha_{\theta B} = 1.0$$

3. **Inelastic Buckling Stresses**: Inelastic instability stresses are determined by applying plasticity reduction factors to elastic buckling stresses as recommended in Section 5.

b. For General Instability

1. **Elastic Buckling Stresses**. Use the following relationships together with that above for Y to determine the general instability stresses.

$$j = G,\ L_j = L_b$$

$$L_e = 1.56\sqrt{Rt} \le L_r, b_e = b$$

$$F_{reG} \text{ or } F_{heG} = \alpha_{\theta G} \frac{N_{\theta eG}}{t} K_{\theta G} \tag{4.4-7}$$

Use the larger of the $K_{\theta G}$ values obtained from Equations 11.3-12 and 11.3-16.

2. Imperfection Factors

$\alpha_{\theta G} = 0.8$

3. Inelastic Buckling Stresses: Inelastic instability stresses are determined by applying plasticity reduction factors to elastic buckling stresses as recommended in Section 5.

4.5 BAY INSTABILITY OF STRINGER STIFFENED AND RING AND STRINGER STIFFENEDE CYLINDERS- ALTERNATE METHOD

The method of determining the bay instability stresses for stringer stiffened and ring and stringer stiffened cylinders given in Section 4.4 is based upon a modified orthotropic shell equation. This equation is not valid if the minimum number of stringers is less than about three times the number of circumferential waves for the bay instability mode. The following equations can be used when these restrictions are not met. The rings are to be sized using the equations in Section 4.2 for ring stiffened cylinders.

4.5.1 Axial Compression or Bending ($N_\theta = 0$)

The following method for determining the bay instability loads and stresses for axial compression and bending is quite lengthy but gives the best correlation between test and predicted loads of those methods considered. The method is based on the procedure proposed by Faulkner, et al. in Ref. 3.

a. **Elastic Buckling Stresses**. The elastic bay instability stress F_{xeB} is approximated by summing the buckling stress of a shell panel and the column buckling stress of a stringer plus effective width of shell.

$$F_{xeB} = \frac{\alpha_{xL} C_x E 2t / D}{1 + A_s / bt} + \frac{\pi^2 E I'_{es}}{\left(b_{eu} t + A_s\right) L_r^2} \tag{4.5-1}$$

where $\alpha_{xL}\, C_x$ is obtained from Equations 4.5-12. The other parameters are defined as follows:

$$I'_{es} = I_s + A_s Z_s^2 \frac{b'_e t}{A_s + b'_e t} + \frac{b'_e t^3}{12} \tag{4.5-2}$$

$$b_{eu} = \begin{cases} b\left(\dfrac{1.05}{\lambda_0} - \dfrac{0.28}{\lambda_0^2}\right)R_r & \text{for} \quad \lambda_o > 0.53 \\ b & \text{for} \quad \lambda_o \le 0.53 \end{cases} \tag{4.5-3}$$

$$b'_e = \begin{cases} b\left(\dfrac{0.53}{\lambda_o}\right)R_r & \text{if} \quad \lambda_o \ge 0.53 \\ b & \text{if} \quad \lambda_o < 0.53 \end{cases} \tag{4.5-4}$$

$$\lambda_o = \sqrt{F_y / \sigma_e} \tag{4.5-5}$$

$$\sigma_e = B\rho_\eta \sigma_{xeL} \tag{4.5-6}$$

$$\sigma_{xeL} = \begin{cases} 0.605E2t/D & M_\theta \ge 3.46 \\ \left(\dfrac{3.62}{M_\theta^2} + 0.0253M_\theta^2\right)E2t/D & M_\theta < 3.46 \end{cases} \tag{4.5-7}$$

$$\rho_\eta = \begin{cases} 0.27 + \dfrac{1.57}{M_\theta^2} + \dfrac{29.6}{M_\theta^4} + 0.008\left(1 - \dfrac{D/t}{600}\right)M_\vartheta \\ 3.46 < M_\theta < 8.57 \\ 1.0 - 0.018M_\theta^{2.5} + 0.0023M_\theta^2\left(1 - \dfrac{D/t}{600}\right) \\ M_\theta \le 3.46 \end{cases} \tag{4.5-8}$$

$$B = \begin{cases} 1.15 & \text{for} \quad \lambda_\eta \ge 1.0 \\ 1 + 0.15\lambda_\eta & \text{for} \quad \lambda_\eta < 1.0 \end{cases} \tag{4.5-9}$$

$$\lambda_\eta = \sqrt{F_y / \left(\rho_\eta \sigma_{xcL}\right)} \tag{4.5-10}$$

$$R_r = \begin{cases} 1.0 & \text{if} \quad \lambda_\eta \le 0.53 \\ 1.0 - \dfrac{2c}{b/t - 2c}\left(\dfrac{\lambda_\eta^2}{1 + 0.25\lambda_\eta^4}\right)^2 \dfrac{\lambda_\eta^2}{1.05\lambda_\eta - 0.28} & \text{if} \quad \lambda_\eta > 0.53 \end{cases} \tag{4.5-11}$$

where $c = 4.5$ for continuous structural fillet welds.

The term $\alpha_{xL}\ C_x$ in Equation 4.5-1 can be computed from Equation 4.5-12:

at $M_\theta < 3$

$$\alpha_{xL} C_x = 0.33 + 160(M_x)^{-0.5} / [200 + 0.5(D/t)] \tag{4.5-12}$$

if $M_\theta = 15$

$$\alpha_{xL} C_x = 350(M_x)^{-0.5} / [200 + 0.5(D/t)]$$

For values of M_θ between 3 and 15, $\alpha_{xL}\ C_x$ can be obtained by interpolation.

b. **Inelastic Buckling Stresses**. Inelastic instability stresses are determined by applying plasticity reduction factors to elastic buckling stresses as recommended in Section 5.

c. **Failure Load**. The failure load is the product of the failure stress and the effective area. The effective shell width for determining the failure load or applied stresses (see Equations 11.1-2 and 11.2-2) is given by:

$$b_e = \begin{cases} b\left(\dfrac{1.05}{\lambda_e} - \dfrac{0.28}{\lambda_e^2}\right) R_r & \text{if} \quad \lambda_e \geq 0.53 \\ b & \text{if} \quad \lambda_e < 0.53 \end{cases} \tag{4.5-13}$$

where

$$\lambda_e = \lambda_o \sqrt{F_{xcB} / F_y}$$

See Equation 4.5-1 for F_{xeB}, Equation 4.5-5 for λ_o, Equation 4.5-11 for R_r, and the critical buckling stress, F_{xcB} should be obtained from equations in Section 5.

The failure load P_{cB} is computed from:

$$P_{cB} = N_s F_{xcB} \left(A_s + b_e t\right) \tag{4.5-14}$$

4.5.2 External Pressure

a. **Elastic Buckling Stresses**: Elastic instability stresses are determined from either inelastic instability or yield stresses as defined in Equation 4.5-15, Section 4.5.2b and the use of equations in Section 5.

b. **Inelastic Buckling Stresses**

Inelastic Bay Instability Stress, $F_{\theta cB}$, is determined from Equation 4.5-15:

$$F_{\tau cB} \text{ or } F_{hcB} = \frac{p_{cB} R_o}{t} K_{\theta L} \tag{4.5-15}$$

NOTE: $K_{\theta L}$ is defined in Section 11.

The failure pressure for bay instability, p_{cB} , is defined as the total capacity of shell plate and stringers in Equation (4.5-16):

$$p_{cB} = (p_{cL} + p_s)K_\rho \tag{4.5-16}$$

The local shell failure pressure, p_{cL} , is determined by Equation 4.5-17:

$$p_{cL} = F_{rcL} t / R_o \tag{4.5-17}$$

The local shell instability stress, F_{rcL}, is determined from Equation (4.1-5) and the equations in Section 5 by assuming that the cylinder is without stringer stiffeners and has a ring spacing equal to L_r.

The failure pressure associated with the development of plastic hinges in the stringers with effective shell plate, p_s , is determined from Equation 4.5-18:

$$p_s = \left(\frac{16}{bL_r^2}\right) A_s \mid Z_s \mid F_y \tag{4.5-18}$$

An effective pressure correction factor, K_p, is applied to the computed bay instability pressure to normalize the test data. Effective pressure correction factor is determined from Equation 4.5-19.

$$K_p = \begin{cases} 0.20 + 0.90(g / 500) & \text{for} \quad g < 500 \\ 1.10 & \text{for} \quad g > 500 \end{cases} \tag{4.5-19}$$

where, $g = M_x M_\theta L_r t A_s / I_s$

SECTION 5—Plasticity Reduction Factors

The predicted elastic buckling stresses given in Section 4 for local shell buckling, bay instability and general instability must be reduced by a plasticity factor when the elastic buckling stresses exceed $0.50F_y$. Inelastic buckling stress definitions given in Equations 5-1 and 5-2 should be used together with Equation 5-3 defining the plasticity reduction factor.

$$F_{icj} = \eta F_{iej} \qquad \text{for } F_{iej} > 0.5F_y \tag{5-1}$$

(i.e., F_{iej} > material proportional limit)

$$F_{icj} = F_{iej} \qquad \text{for } F_{iej} \le 0.5F_y \tag{5-2}$$

(i.e., F_{iej} < material proportional limit)

where

$$\eta = \left(F_y / F_{iej}\right)\left[\frac{1.0}{\left\{1.0 + 3.75\left(F_y / F_{iej}\right)^2\right\}}\right]^{1/4} \tag{5-3}$$

SECTION 6—Predicted Shell Buckling Stresses for Combined Loads

Interaction equations are given for determining the failure stresses for cylinders subjected to combined loads. The stresses due to bending moment are treated as equivalent axial stresses. The interaction equations are applicable to all modes of failure and to both elastic and inelastic buckling stresses. Each mode must be checked independently.

6.1 GENERAL LOAD CASES

In the following equations for N_ϕ and N_θ , P is the total axial load including any pressure load on the end of the cylinder, M is the bending moment and p is the external radial pressure.

a. Axial Compression and Hoop Compression

$$N_\phi = P/(2\pi R)$$

$$N_\theta = pR_o$$

b. Bending and Hoop Compression

$$N_\phi = M/(\pi R^2)$$

$$N_\theta = pR_o$$

c. Axial Compression, Bending and Hoop Compression

$$N_\phi = P/(2\pi R) + M/(\pi R^2)$$

$$N_\theta = pR_o$$

6.2 AXIAL TENSION, BENDING AND HOOP COMPRESSION

The failure stresses are the lower of the values determined from Equations 6.2-1 and 6.2-2. The longitudinal stress $F_{\phi cj}$ is the sum of the axial and bending stresses.

$$F_{\theta cj} = F_{rcj}\left(1 - 0.25\frac{F_{\phi cj}}{F_y}\right) \tag{6.2-1}$$

$$F_{\phi cj} + F_{\theta cj} = F_y \tag{6.2-2}$$

where $F_{\theta cj}$ is the failure stress in the hoop direction corresponding to $F_{\phi cj}$ which is the coexistent failure stress in the axial direction, and F_{rcj} is the predicted failure stress for radial pressure. F_{rcj} is given by Equation 4.5-15 and the equations in Section 5.

The values of $F_{\phi cj}$ and $F_{\theta cj}$ are determined from Equations 6.2-1 and 6.2-2 by letting $F_{\phi cj} = F_{\theta cj} k K_{\phi j} / K_{\theta j}$ where $k = N_{\phi} / N_{\theta}$ and then solving for $F_{\theta cj}$. See Table 6.2-1 for K_{ij}.

6.3 AXIAL COMPRESSION, BENDING AND HOOP COMPRESSION

The following equation is applicable to any combination of axial compression, bending and external pressure. The axial buckling stress, $F_{\phi cj}$, is the sum of the longitudinal stresses due to axial compression and bending. If the bending stress is greater than the axial stress, the failure stresses are determined from Section 6.2. $F_{\theta cj}$ is the failure stress in the hoop direction.

$$R_a^2 - cR_a R_h + R_h^2 = 1.0 \tag{6.3-1}$$

where

$$R_a = F_{\phi cj} / F_{xcj}$$

$$R_h = F_{\theta cj} / F_{rcj}$$

The equations for c are dependent upon the cylinder geometry.

a. Unstiffened and Ring Stiffened Cylinders (all buckling modes)

$$c = \frac{F_{xcj} + F_{rcj}}{F_y} - 1.0 \tag{6.3-2}$$

b. Stringer Stiffened and Ring and Stringer Stiffened Cylinders

1. Local Buckling *(j = L)*

$$c = \frac{0.4\left(F_{xcj} + F_{rcj}\right)}{F_y} - 0.8 \tag{6.3-3}$$

2. Bay Instability and General Instability *(j = B, j = G)*

$$c = \frac{1.5\left(F_{xcj} + F_{rcj}\right)}{F_y} - 2.0 \tag{6.3-4}$$

The buckling stresses for any combination of longitudinal compression and hoop compression, N_{ϕ} / N_{θ}, are determined by the following procedure for each of the buckling modes.

Step 1. Calculate F_{xcj} and F_{rcj} from the equations in Section 4.

Step 2. Solve for $F_{\theta cj}$ in Equation 6.3-1 by letting $F_{\phi cj} = F_{\theta cj} k K_{\phi j} / K_{\theta j}$.

where

$k = N_{\phi} / N_{\theta}$,

$K_{\phi j}$ = axial stress modifier (see Table 6.2-1),

$K_{\theta j}$ = hoop stress modifier (see Table 6.2-1).

Table 6.2-1: Stress Distribution Factors, K_{ij}

K_{ij}	Unstiffened	Ring Stiffened	Stringer Stiffened	Ring and Stringer Stiffened
$K_{\phi j}$	1.0	1.0	1.0	1.0
$K_{\phi B}$			t/t_x Section 4.4.1	t/t_x Section 4.4.1
$K_{\phi G}$		1.0		t/t_x Section 4.4.1
$K_{\theta L}$	1.0	Equation 11.3-3a	Equation 11.3-3b	Equation 11.3-3b
$K_{\theta B}$			Same as $K_{\theta L}$	Same as $K_{\theta L}$
$K_{\theta G}$		Equation 11.3-12a		Larger of Eqn. 11.3-12a or 11.3-16

SECTION 7—Stiffener Requirements

The flow chart in Figure 3.1 shows the procedure for determining the allowable stresses for all buckling modes. The sizes of the stringers are determined from the bay instability mode equations and the sizes of the rings are determined from the general instability mode equations.

The shell buckling stress equations were developed on the basis of no interaction between the buckling modes. The buckling stresses for local buckling, however, may be reduced if the predicted buckling stress for either bay instability or general instability is approximately equal to the predicted local buckling stress. Similarly, if the predicted general instability stress is approximately equal to the bay instability stress, the actual stress for either of these modes may be less than predicted.

Mode interaction can be avoided by applying a factor β to the strains corresponding to the buckling stresses. It is desirable to provide a hierarchy for failure with general instability preceded by bay instability and bay instability preceded by local shell buckling. A minimum factor of β = 1.2 is recommended for both the bay and general instability modes. The designer may elect to select a higher β value for the general instability mode.

7.1 HIERARCHY CHECKS

The buckling stresses which include the factor β are called the “design buckling stresses” and defined by the following equations:

$$\overline{F}_{iej} = F_{iej} / \beta \tag{7.1-1}$$

$$\overline{F}_{icj} = F_{icj} / \beta \tag{7.2-2}$$

where β= 1.0 for local buckling and at least 1.2 for bay instability and general instability. Critical buckling stresses five by Equation 7.1-2 are determined through the use of equations given in Section 5.

It should be noted that β factor is intended to be applied to the failure strain and not the failure stress. Thus, while the bay and general instability-to-local stress ratios will be equal to β in the material elastic range (i.e., below material proportional limit), these ratios will be less than β in the material elasto-plastic range (i.e., above material proportional limit).

Cylindrical shell design should meet the desired hierarchy checks (i.e., $F_{ieG} > F_{ieB} > 1.2F_{ieL}$). If the hierarchy is not achieved, the design should be modified by either raising F_{ieG} and F_{ieB} stresses or by lowering F_{ieL} stress. These objectives can be achieved by changing:

(1) ring and stringer stiffener spacing
(2) ring and stringer stiffener sizes
(3) shell plate thickness

7.2 STIFFENER STRESSES AND BUCKLING

Both the ring and the stringer are subjected to localized stresses that need to be combined with global stresses. The stiffeners also need to be checked against local web or flange buckling.

7.2.1 Local Stiffener Buckling

To preclude stiffener buckling prior to shell buckling, the local stiffener buckling stress must be greater than the shell buckling stress given by the foregoing equations. The local stiffener buckling stress can be assumed to be equal to the yield stress for stiffeners which satisfy the following compact section requirements. For stiffeners not meeting these requirements, the local stiffener buckling stress can be determined from Equation C7.2-1 in the Commentary.

a. Flat Bar Stiffener, Flange of a Tee Stiffener and Outstanding Leg of an Angle Stiffener

$$\frac{h_s}{t_s} \leq 0.375\sqrt{E / F_y} \tag{7.2-1}$$

where h_s is the full width of a flat bar stiffener or outstanding leg of an angle stiffener and one-half of the full width of the flange of a tee stiffener and t_s is the thickness of the bar, leg of angle or flange of tee.

b. Web of Tee Stiffener or Leg of Angle Stiffener Attached to Shell

$$\frac{h_s}{t_s} \leq 1.0\sqrt{E / F_y} \tag{7.2-2}$$

where h_s is the full depth of a tee section or full width of an angle leg and t_s is the thickness of the web or angle leg.

7.2.2 Stiffener Global and Local Stresses

Some of the external pressure on an orthogonally stiffened cylindrical shell will be directly transferred to the rings through the stringers and the resulting bending stresses in the stringers may be appreciable. In addition to meeting API requirements on bay instability, it is recommended that an effective stringer column instability check be performed for an appropriate beam column element subjected to combined global axial and local bending stresses.

7.2.3 Tripping Brackets

The ring stiffeners supporting the stringers may be susceptible to tripping and the ring tripping can be minimized by introducing tripping brackets. The spacing, s, between the tripping brackets should not exceed:

$$s < 0.44 b_t \left[E / F_y\right]^{0.5} \tag{7.2-3}$$

The design lateral load on the flange for tripping bracket sizing can be taken as the compressive stress in the flange multiplied by 2% of the combined area of the flange plus one-third of the web area, see Figure 7.2-1.

7.3 STIFFENER ARRANGEMENT AND SIZES

An optimum design provides a natural hierarchical order of failure modes, minimizes steel requirements and simplifies fabrication. Ring spacing and shell thickness are primarily controlled by external pressure and the stringer spacing and size are primarily controlled by axial and bending loads.

Stiffener arrangement and sizes should meet both the applied combined loads as discussed in Section 6 and the axial and bending loads and external pressure separately. For cylinders subjected to loads in one direction alone, $F_{\phi ej} = F_{xej}$ and $F_{\theta ej} = F_{hej}$. The ring and stringer stiffeners, together with the effective shell area, should yield general, bay and local instability stresses that meet the following requirements:

a. Stringers

$$\overline{F}_{\phi eB} = \overline{F}_{\phi eB} / 1.2 \geq \overline{F}_{\phi eL} \tag{7.3-1}$$

$$\overline{F}_{\theta eB} = \overline{F}_{\theta eB} / 1.2 \geq \overline{F}_{\theta eL} \tag{7.3-2}$$

b. Rings

$$\overline{F}_{\phi eG} = \overline{F}_{\phi eG} / 1.2 \geq \overline{F}_{\phi eL} \tag{7.3-3}$$

$$\overline{F}_{\theta eG} = \overline{F}_{\theta eG} / 1.2 \geq \overline{F}_{\theta eL} \tag{7.3-4}$$

A general procedure that can be used to meet both design safety factors and the hierarchical failure mode requirements is presented in Section C7.3 of the Commentary.

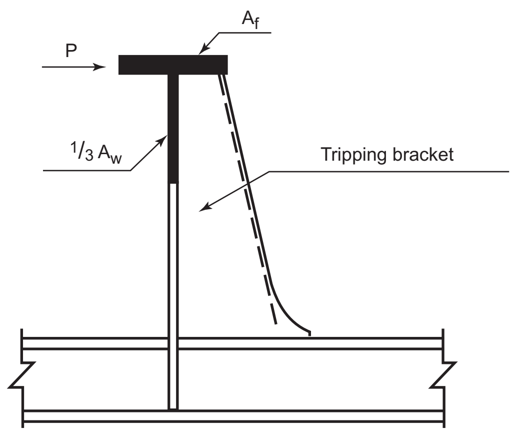


Figure 7.2-1--Design Lateral Load for Tripping Bracket

SECTION 8—Column Buckling

The shell buckling stresses determined based on Section 4.1.1 [Equation 4.1-1] and Section 5 [Equations 5-1 and 5-2] for cylinders subjected to axial compression only, and based on Section 6.3 [Equation 6.3-1] for cylinders subjected to combined axial and hoop compression. Equations given in Sections 4 and 6 do not include the effect of column buckling. Although column buckling phenomena is not likely to occur in large diameter cylindrical shells with small slenderness rations (i.e., *KL/r*), column buckling should be routinely checked.

The buckling stress of an unstiffened or ring-stiffened cylindrical shell is determined by substituting the shell buckling stress, $F_{\phi cL}$, for the yield stress in the column buckling equation. Without the external pressure, the shell buckling stress, $F_{\phi cL}$, is equal to uniaxial shell buckling stress in the member longitudinal axis, F_{xcj}.

The buckling stress of a tubular column is equal to the shell buckling stress for cylinders with $KL_t / r < 0.5\sqrt{E / F_{\phi cL}}$. For longer columns the buckling stresses are given by the following equations. These equations are based on the premise that the stiffeners for stiffened cylinders are sized in accordance with the bulletin and only the local shell buckling mode is considered to interact with column buckling.

8.1 ELASTIC COLUMN BUCKLING STRESSES

$$F_{\phi eC} = \alpha_{xC}\sigma_{xeC} = \alpha_{xC}\frac{\pi^2 E}{(KL_t / r)^2} \tag{8.1-1}$$

where

$$\alpha_{xC} = 0.87$$

8.2 INELASTIC COLUMN BUCKLING STRESSES

$$F_{\phi cC} = \begin{cases} F_{\phi eC} & \dfrac{KL_t}{r} \geq 3.56C_c \\ \left(0.48 + 0.37\sqrt{\dfrac{C_c}{(KL_t)/r}}F_{\phi cL}\,\dfrac{C_c}{2}\right) & < \dfrac{KL_t}{r} < 3.56C_c \\ F_{\phi cL} & \dfrac{KL_t}{r} \leq 0.5C_c \end{cases} \tag{8.2-1}$$

where

$$C_c = \sqrt{E / F_{\phi cL}}$$

For axial load only, $F_{\phi cL} = F_{xcL}$, where F_{xcL} is determined from Equations 4.1-1 and 5-1 and 5-2. For combined axial and hoop compression, $F_{\phi cL}$ is determined from Equation 6.3-1.

Column buckling check is not necessary for ring and stringer stiffened cylindrical shells.

SECTION 9—Allowable Stresses

The allowable stresses for short cylinders, $KL_r / r \le 0.5\sqrt{E / F_{\phi cj}}$, are determined by applying an appropriate factor of safety to the predicted buckling stresses given in Sections 4 and 6. Without external pressure $F_{\phi cj} = F_{xcj}$ .The effects of imperfections due to out-of-roundness and out-of-straightness on the shell buckling stresses are very significant in the elastic range but have little effect in the yield and strain hardening ranges. Therefore a partial factor of safety, ψ, that is dependent upon the buckling stress is recommended. The value of ψ is 1.2 when the buckling stress is elastic and 1.0 when the buckling stress equals the yield stress. A linear variation is recommended between these limits. The equation for ψ is given below. A value of ψ = 1.0 may be used for axial tension stresses and for column buckling mode stresses.

$$\psi = \begin{cases} 1.20 & F_{icj} \le 0.50F_y \\ 1.40 - 0.40F_{icj} / F_y & 0.50F_y < F_{icj} < F_y \\ 1.00 & F_{icj} = F_y \end{cases} \tag{9-1}$$

For longer cylinders, $KL_r / r > 0.5\sqrt{E / F_{\phi cj}}$, is subjected to axial compression, the cylinders will fail in the column buckling mode and the column buckling stresses are given by Equation 8-2-1. An interaction equation is given in this section for long cylinders subjected to bending in combination with axial compression. This same interaction equation can be used when the cylinder is also subjected to external pressure.

The allowable stresses F_a, F_b, and F_θ are to be taken as the lowest values given for all modes of failure. If the stiffeners are sized in accordance with the method given in Section 7, only the local shell buckling mode need be considered in the equations which follow. The allowable stresses must be greater than the applied stresses which can be calculated using the equations given in Section 11 or by more exact methods using computer codes. The factor of safety, *FS*, is provided by the design specifications. In general for normal design conditions:

$$FS = 1.67\psi$$

For extreme load conditions where a one-third increase in allowable stresses is appropriate:

$$FS = 1.25\psi$$

9.1 ALLOWABLE STRSSES FOR SHELL BUCKLING MODE

The following equations provide the allowable stresses for the local shell buckling mode. The same equations are applicable to other modes of failure by substituting the design inelastic buckling stresses for those modes in the equations. The following equations should be satisfied for all loads. See Section 11 for f_a, f_b, and f_q.

$$f_a + f_b < F_a \qquad\qquad f_\theta < F_\theta$$

9.1.1 Axial Tension

$$F_a = \frac{F_y}{FS} \qquad F_\theta = 0 \tag{9.1-1}$$

9.1.2 Axial Compression or Bending

$$F_a = F_b = \frac{F_{xcL}}{FS} \qquad F_\theta = 0 \tag{9.1-2}$$

See Equations 4.1-1, 4.3-1, and Section 5 for F_{xcL} . [See Equations 4.4-3, 4.5-1, and Section 5 for F_{xcB} ; and, See Equations 4.2-1, 4.4-5, and Section 5 for F_{xcG} .]

9.1.3 External Pressure

$$F_a = 0, F_\theta = \frac{F_{rcL}}{FS} \tag{9.1-3}$$

See Equations 4.1-5, 4.3-3, and Section 5 or $F_{\theta cL}$. [See Equations 4.4-6, 4.5-19, Section 5 for $F_{\theta cB}$; and, See Equations 4.2-4, 4.4-7 and Section 5 for $F_{\theta cG}$.]

9.1.4 Axial Tension and Hoop Compression and Axial Tension, Bending, and Hoop Compression

$$F_a = F_b = \frac{F_{\phi cL}}{FS}, F_\theta = \frac{F_{\theta cL}}{FS} \tag{9.1-4}$$

See Equations 6.2-1 and 6.2-2 for $F_{\phi cL}$ and $F_{\theta cL}$.

9.1.5 Axial Compression and Hoop Compression and Axial Compression, Bending, and Hoop Compression

$$F_a = F_b = \frac{F_{\phi cj}}{FS}, F_\theta = \frac{F_{\theta cj}}{FS} \tag{9.1-5}$$

See Equation 6.3-1 for $F_{\phi cj}$ and $F_{\theta cj}$.

9.1.6 Bending and Hoop Compression

$$F_b = \frac{F_{\phi cL}}{FS}, F_\theta = \frac{F_{\theta cL}}{FS} \tag{9.1-6}$$

See Equation 6.3-1 for $F_{\phi cL}$ and $F_{\theta cL}$.

9.2 ALLOWABLE STRESSES FOR COLUMN BUCKLING MODE

When $KL_t / r > 0.5\sqrt{E / F_{\phi cj}}$ the following equations as well as those in Section 9.1 must be satisfied:

9.2.1 Axial Compression

$$F_a = \frac{F_{\phi cC}}{FS} \tag{9.2-1}$$

See Equation 8.2-1 for $F_{\phi cC}$.

9.2.2 Axial Compression and Bending

Members subjected to both axial compression and bending stresses should satisfy Equations 9.2-2 and 9.2-3. See Equations 9.2-1 for F_a and 9.1-2 for F_b and the latest edition of API RP 2A for C_m and K. C_m must be greater than or equal to $(1 - f_a / F'_e)$.

a. For $f_a / F_a \leq 0.15$

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1.0 \tag{9.2-2}$$

b. For $f_a / F_a > 0.15$

$$\frac{f_a}{F_a} + \frac{f_b}{F_b}\left(\frac{C_m}{1 - f_a / F'_e}\right) \leq 1.0 \tag{9.2-3}$$

$$F'_e = \frac{\pi^2 E}{(KL_t / r)^2 FS}$$

9.2.3 Axial Compression, Bending, and Hoop Compression

Members subjected to combinations of axial compression, bending and hoop compression should satisfy Equations 9.2-2 and 9.2-3 with F_a determined from 9.2-1 and F_b from 9.1-5.

SECTION 10—Tolerances

The foregoing rules are based upon the assumption that the cylinders will be fabricated within the following tolerances. The Commentary provides additional information on the buckling strength of cylinders which do not meet these tolerances. The requirements for out of roundness are from the ASME *Pressure Vessel Code* (Ref. 17) and the requirement for straightness is from the ECCS rules (Ref. 18).

10.1 MAXIMUM DIFFERENCES IN CROSS-SECTIONAL DIAMETERS

The difference between the maximum and minimum diameters at any cross section should not exceed 1% of the nominal diameter at the cross section under consideration.

$$\frac{D_{\max} - D_{\min}}{0.01 D_{nom}} \le 1.0 \qquad (10.1\text{-}1)$$

10.2 LOCAL DEVIATION FROM STRAIGHT LINE ALONG A MERIDIAN

Cylinders designed for axial compression should meet the following tolerances. The local deviation from a straight line measured along a meridian over a gauge length L_x should not exceed the maximum permissible deviation e_x.

$$e_x = 0.01 L_x \qquad (10.1\text{-}2)$$

$L_x = 4\sqrt{Rt}$ but not greater than L_r

10.3 LOCAL DEVIATION FROM TRUE CIRCLE

Cylinders designed for external pressure should meet the following tolerances. The local deviation from a true circle should not exceed the maximum permissible deviation obtained from Figure 10.3-1. Measurements are to be made with a gauge or template with the arc length obtained from Figure 10.3-2.

Additionally the difference between the actual radius to the shell at any point and the theoretical radius should not exceed $0.005R$.

10.4 PLATE STIFFENERS

The lateral deviation of the free edge of a plate stiffener should not exceed 0.002 times the length of the stiffener. The length of a stringer stiffener is the distance between rings. The length of a ring stiffener is the distance between stringers when present, or $\pi R/n$ where n is determined from Equation 4.2-5. A conservative value for n is given by Equation 10.4-1.

$$n^2 = 1.875 \frac{R}{L_b} \sqrt{R/t} \ge 4 \qquad (10.4\text{-}1)$$

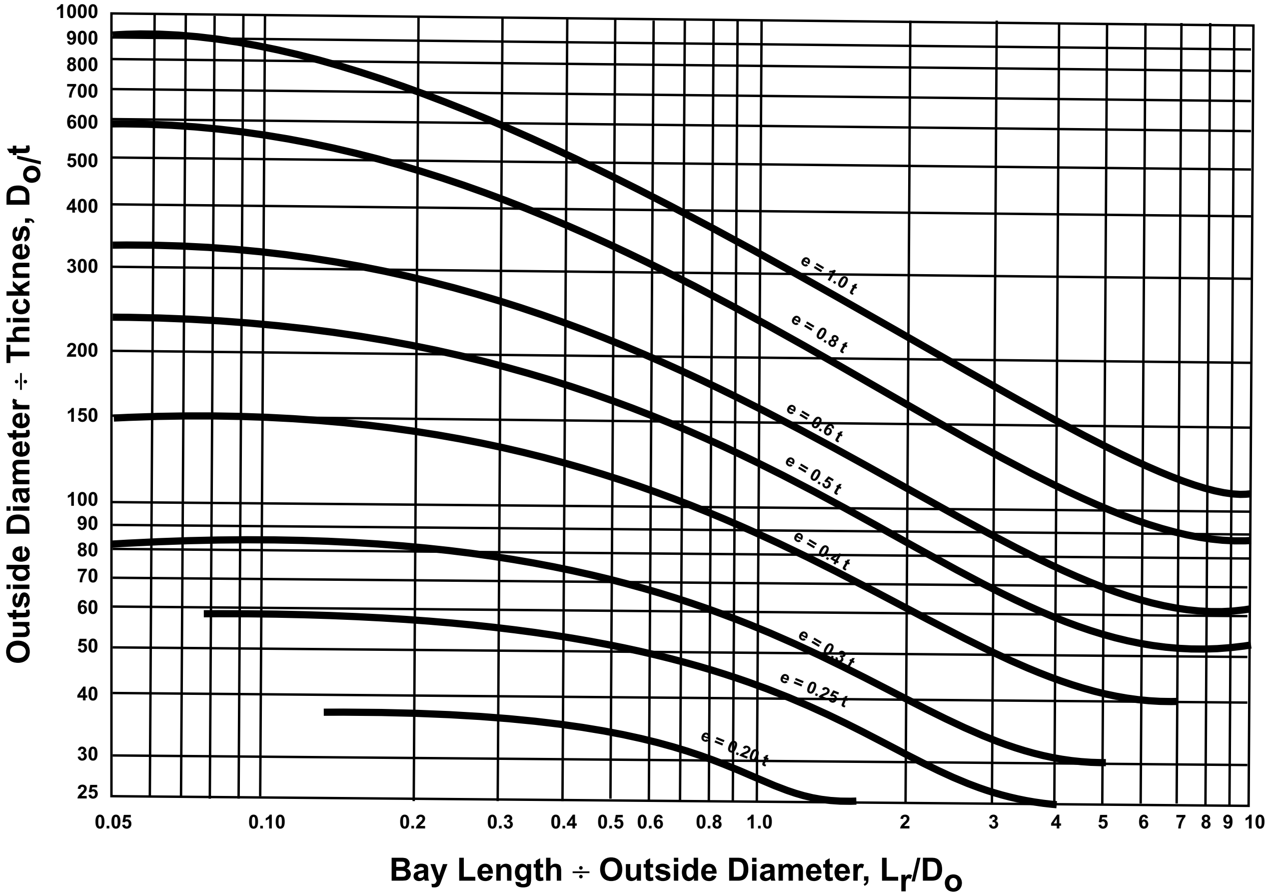


Figure 10.3-1--Maximum Possible Deviation *e* from a True Circular Form

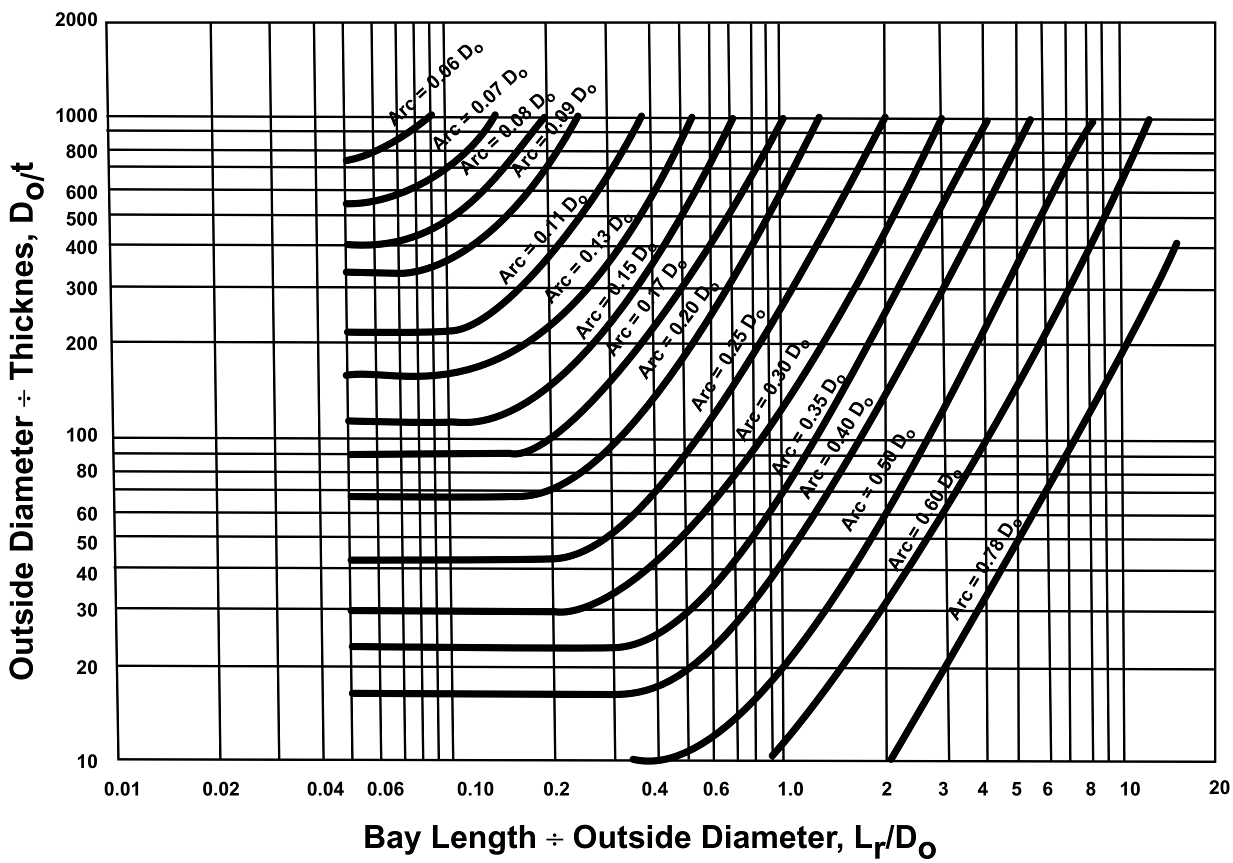


Figure 10.3-2--Maximum Arc Length for Determining Plus or Minus Deviation

SECTION 11—Stress Calculations

It is recommended that the applied stresses in the shell and the stiffeners are obtained from a finite element analysis. However, the following equations may also be used to determine the approximate average stress levels in the shell plate and the effective stiffener cross-sections.

11.1 AXIAL STRESSES

In Equations 11.1-1 and 11.1-2, P is the total axial load including any pressure load on the end of the cylinder.

a. Unstiffened and Ring Stiffened Cylinders

$$f_a = \frac{P}{2\pi Rt} \tag{11.1-1}$$

b. Cylinders With Longitudinal Stiffeners. When the stringers are not spaced sufficiently close to make the shell fully effective, the effective area is used to determine the axial and bending stresses. The factor Q_a is a ratio of the effective area to the actual area. Q_a = 1.0 for the local shell buckling mode. See Equations 4.4-2, 4.4-4, and 4.5-13 for b_e.

$$f_a = \frac{P}{Q_a A_t} \tag{11.1-2}$$

where

$$Q_a = \frac{A_s + b_e t}{A_s + bt}$$

$$A_t = 2\pi Rt + N_s A_s$$

11.2 BENDING STRESSES

In Equations 11.2-1 and 11.2-2, M is the bending moment at the cross section under consideration.

a. Unstiffened and Ring Stiffened Cylinders

$$f_b = \frac{M}{\pi R^2 t} \times K_b \tag{11.2-1}$$

where

$$K_b = \frac{1 + 0.5t/R}{1 + 0.25(t/R)^2}$$

The value of K_b is approximately 1.0.

b. Cylinders with Longitudinal Stiffeners

See Equation 11.1-2 for definition of Q_a.

$$f_b = \frac{M}{Q_a \pi R^2 t_e} \tag{11.2-2}$$

where

$$t_e = t + A_s / b$$

11.3 HOOP STRESSES

The presence of longitudinal stiffeners affect the distribution of hoop stresses between the shell plate and the rings. Equations given in this section were validated through the use of finite element analysis (References 13, 14, and 15) and further discussed in Section C11. The external pressure, p, is assumed to be uniform around the cylindrical shell.

a. Unstiffened and Stringer Stiffened Cylinder

$$f_\theta = \frac{pR_o}{t} \tag{11.3-1}$$

b. Ring-Stiffened Cylindrical Shells

The hoop stress in ring-stiffened cylindrical shell midway between ring spacing is in general greater than the stress at the ring and its magnitude depends primarily on external pressure, D/t ratio, shell thickness and the ring spacing, L_r .

1. Hoop stress in Shell Midway between Rings

The shell stress is expressed by:

$$f_{\theta S} = \frac{pR_o}{t} K_{\theta L} \tag{11.3-2}$$

where

$$K_{\theta L} = 1 - \psi \frac{p_\sigma}{p}\left(\frac{k_d}{k_t + k_d}\right) \tag{11.3-3a}$$

$$p_\sigma = p + \frac{\nu \sigma_{xa} t}{R_o} \le p \tag{11.3-4}$$

In which p is the externally applied pressure and σ_{xa} is the uniformly applied axial stress(axial tension is positive in sign in above equation)

$$k_t = 8\beta^3 D\left(\frac{Cosh\beta L_r - Cos\beta L_r}{Sinh\beta L_r + Sin\beta L_r}\right) \tag{11.3-5a}$$

$$k_d = \frac{Et_{ws}\left(R_o^2 - R_f^2\right)}{R_o\left[(1+\upsilon)R_o^2 + (1-\upsilon)R_f^2\right]} \tag{11.3-6}$$

$$t_{ws} = \frac{A_r}{h} \tag{11.3-7}$$

$$\psi = \frac{2\left(Sin\frac{\beta L_r}{2}Cosh\frac{\beta L_r}{2} + Cos\frac{\beta L_r}{2}Sinh\frac{\beta L_r}{2}\right)}{Sinh\beta L_r + Sin\beta L_r} \geq 0 \tag{11.3-8a}$$

$$\beta^4 = \frac{Et}{4R_o^2 D} \tag{11.3-9a}$$

$$D = \frac{Et^3}{12\left(1-\nu^2\right)} \tag{11.3-10a}$$

where, in the above equations R_t is the radius to the flange of ring and h is the ring web height.

2. Hoop Stress in the Shell at the Ring

The stress in the ring is expressed by:

$$f_{\theta R} = \frac{pR_o}{t}K_{\theta G} \tag{11.3-11}$$

where

$$K_{\theta G} = 1 - \frac{p_\sigma}{p}\left(\frac{k_d}{k_t + k_d}\right) \tag{11.3-12a}$$

c. Ring and Stringer Stiffened Cylindrical Shells

The addition of stringers to ring-stiffened cylindrical shell in general tends to decrease the stress midway between ring spacing while the stress at the ring increases. Thus, the stress midway between ring spacing and at the ring comes closer to each other. This effect is greater when the stringers are closely spaced.

1. **Hoop Stress in the Shell Midway Between Rings**

The hoop stress is expressed by Equation 11.3-2. To account for effect of stringers requires modification of $K_{\theta L}$, k_t, ψ , β , D defined in Equations 11.3-3a, 11.3-5a, 11.3-8a, 11.3-9a, and 11.3-10a), respectively,

$$K_{\theta L} = 1 - \psi_{ef}\frac{p_\sigma}{p}\left(\frac{k_d}{k_{tef} + k_d}\right) \tag{11.3-3b}$$

$$k_{tef} = 8\beta_{ef}^3 D_{ef}\left(\frac{Cosh\beta_{ef}L_r - Cos\beta_{ef}L_r}{Sinh\beta_{ef}L_r + Sin\beta_{ef}L_r}\right) \tag{11.3-5b}$$

$$\psi_{ef} = \delta \frac{2\left(Sin\frac{\beta_{ef} L_r}{2} Cosh\frac{\beta_{ef} L_r}{2} + Cos\frac{\beta_{ef} L_r}{2} Sinh\frac{\beta_{ef} L_r}{2} \right)}{Sinh\beta_{ef} L_r + Sin\beta_{ef} L_r} \geq 0 \qquad (11.3\text{-}8b)$$

$$\beta_{ef}^4 = \frac{Et_{ef}}{4R_o^2 D_{ef}} \qquad (11.3\text{-}9b)$$

$$D_{ef} = \frac{N_s EI_{ef}}{2\pi R_o} \qquad (11.3\text{-}10b)$$

where

$$t_{ef} = t\delta \frac{Sin\rho}{\rho} \qquad (11.3\text{-}13)$$

$$\delta = \frac{1}{\left[\left(1 + 12\left(\frac{R}{t} \right)^2 \right) \frac{2\rho + Sin2\rho}{4Sin\rho} - 12\left(\frac{R}{t} \right)^2 \frac{Sin\rho}{\rho} \right]} \qquad (11.3\text{-}14)$$

and ρ is the half angle between the stringer spacing:

$$\rho = \frac{\pi}{N_s} \qquad (11.3\text{-}15)$$

In Equation (11.3-10b), I_{ef} is the moment of inertia of stiffener inclusive of the plate acting as a flange. The effective plate breadth can be calculated using shear lag.

2. Hoop stress in the Shell at the Ring

The stress in the ring is expressed by Equation 11.3-11 and 11.3-12b, except for the definition of k_t , β, and *D*. Equations 11.3-5b, 11.3-9b and 11.3-10b should be used together with Equation 11.3-12b.

$$K_{\theta G} = 1 - \frac{p_\sigma}{p}\left(\frac{k_d}{k_{tef} + k_d} \right) \qquad (11.3\text{-}12b)$$

The equations given above provide the means to determine the ring stress accurately when the stringers are closely spaced. For cylindrical shell configurations with high *D/t* ratios and loosely spaced stringers computed hoop stresses are inaccurate. Thus, hoop stresses in the plate at the ring should be also checked with Equation 11.3-16 by assuming that even lightly stiffened shell behavior allows for substitution of ring spacing for effective shell width acting with the ring. The larger of the two $K_{\theta G}$ value obtained from Equation 11.3-12b and 11.3-16 should be used in defining ring hoop stress.

$$K_{\theta G} = (1 - 0.3k)\frac{L_e t}{A_r + L_e t} \qquad (11.3\text{-}16)$$

where

$k = N_\phi / N_\theta$

A_r = Ring flange and web area

L_e = Effective Length $= 1.56\sqrt{Rt}$

SECTION 12—References

1. Miller, C. D. Grove, R. B., and Vojta, J. F., "Design of Stiffened Cylinders for Offshore Structures," American Welding Society Welded Offshore Structures Conference, New Orleans, Louisiana, December 1983.

2. Miller, C. D., "Experimental Study of the Buckling of Cylindrical Shells with Reinforced Openings," ASME/ANS Nuclear Engineering Conference, Portland, Oregon, July 1982.

3. Faulkner, D., Chen, Y. N. and deOliveira, J. G., "Limit State Design Criteria for Stiffened Cylinders of Offshore Structures," ASME 4th National Congress of Pressure Vessels and Piping Technology, Portland, Oregon, June 1983.

4. Stephens, M., Kulak, G. and Montgomery, C., "Local Buckling of Thin-Walled Tubular Steel Members," Third International Colloquium on Stability of Metal Structures, Toronto, Canada, May 1983.

5. Sherman, D. R., "Flexural Tests of Fabricated Pipe Beams," Third International Colloquium on Stability of Metal Structures, Toronto, Canada, May 1983.

6. Miller, C. D., Kinra, R. K. and Marlow, R. S., "Tension and Collapse Tests of Fabricated Steel Cylinders," Paper OTC 4218, Offshore Technology Conference, May 1982.

7. Earl and Wright, "API Bulletin 2U Verification," Joint Industry Project(JIP) Supported by ABS, Arco, Chevron, Conoco, Marathon, Mobil, Pennzoil, Shell and Texaco, Final Report, San Francisco, CA June 1986.

8. Advanced Mechanics & Engineering, Ltd., "Evaluation of TLP Design Code API RP 2T," Prepared for Department of Energy, London, England, November 1989.

9. Miller, C.D. and Saliklis, E.P., "Analysis of Cylindrical Shell Database and Validation of Design Formulations for D/t Values > 300," API PRAC 92-56 Phase 2 Final Report, Prepared by CBI Technical Services Company for American Petroleum Institute, Plainfield, Illinois, April 1995.

10. I.D.E.A.S., Inc., "API Bulletin 2U and Other Recommended Formulations: Cross-Correlation Study," Joint Industry Project (JIP) on API Bulletins 2U and 2V, Supported by ABS, Amoco, BP, Chevron, Exxon, Mobil, Shell, and Texaco, Phase 1 Technical Memorandum No.1, 96201-TM-05 , November 1996.

11. I.D.E.A.S., Inc., "API Bulletin 2U Proposed Revision for Subcommittee Consideration," Joint Industry Project (JIP) on API Bulletins 2U an 2V, Supported by ABS, Amaco, BP, Chevron, Exxopn, Mobil, Shell and Texaco, Phase 1 Technical Memorandum No.5 96201-TM01, June 1999.

12. I.D.E.A.S., Inc., "API Bulletin 2U and 2V Phase 2 Work-Data Applicable to API Bulletin 2U, "API PRAC and TG on Bulletins 2U and 2V, Phase 2 Technical Memorandum No.1, 98210-TM01, June 1999.

13. I.D.E.A.S., Inc., "API Bulletins 2U and 2V Phase 2 Work-Proposed Revisions for Task Group Consideration," API PRAC and TG on Bulletins 2U and 2V, Phase 2 Technical Memorandum No. 3, 98210-TM-03, October 1999.

14. I.D.E.A.S., Inc., "API Bulletins 2U and 2V Phase 3 Work-Gathering, Review and Evaluation of Test Data Applicable to Cylindrical Shells Subjected to Axial Compressions, External Pressure and Combined Loading," API PRAC and TG on Bulletins 2U and 2V, Phase 3 Technical Memorandum No. 1, 99210-TM-01, April 2000.

15. Kamal, Rajiv, "Distribution of Stresses in Ring Stiffened and Ring and Stringer Stiffened Cylindrical Shells Subjected to Axial Loads and External Pressure," Submitted to API Task Group, Rev. 2, June 2001.

16. I.D.E.A.S., Inc., "API Bulletins 2U and 2V Phase 3 Worked-Proposed Mat-Ready Revisions to Bulletin 2U for Task Group Consideration," API PRAC and TG on Bulletins 2U and 2V, Phase 3 Technical Memorandum No. 5., 99210-TM-05, August 2001.

17. American Society of Mechanical Engineers, *Boiler and Pressure Vessel Code*, Section VII, Division 1 and 2 and Section III, Subsection NE, 1983.

18. European Convention for Constructional Steelwork, ***"European Recommendations for Steel Construction,"*** Section 4.6, ***Buckling of Shells***, Publication 29, Second Edition, 1983.

19. American Petroleum Institute, ***Recommended Practice for Planning, Designing and Construction Fixed Offshore Platforms,*** API RP 2A, Twentieth Edition, July 1993.

20. American Institute of Steel Construction, "Manual of Steel Construction," Latest Edition.

APPENDIX A—Commentary on Stability Design of Cylindrical Shells

Table of Contents

Note: The section, figure and table numbers in this Appendix correspond directly with those found in the main body of the document (i.e., C1.2 provides commentary on section 1.2)

INTRODUCTION

This third edition of Bulletin 2U differs from earlier editions (Ref. C.01 and C.02) in:

a) providing buckling equations that are easier to comprehend and implement so that the engineer can design more robust cost-effective structures.
b) taking advantage of more test data to develop less conservative buckling equations that predict buckling stresses close to test data.
c) offering new guidelines on the correct use of finite element analysis (FEA/modeling and a new set of equations to determine the applied stresses compatible with FEA and each instability mode.
d) providing sample calculations to illustrate application of equations and the sensitivity of key variables.

This commentary provides the designer with the basis for the design methods presented in Bulletin 2U. The design criteria are applicable to shells that are fabricated from steel plates where the plates are cold or hot formed and joined by welding. The stability criteria are based upon classical linear theory which has been reduced by capacity reduction factors and plasticity reduction factors which are determined from approximate lower-bound buckling values of test data of shells with initial imperfections which are representative of the tolerance limits given in Section 10 of the Bulletin.

Equations given in this bulletin are based on the behavior of large diameter cylindrical shells having D/t ratios of 300 or greater and define buckling stresses for local, bay and general instability modes. As illustrated in this Commentary, predicted stresses include imperfection/correction factors and are compatible with test data. Predicted stresses are based on the assumption that the instability modes are separated and do not interact. To ensure this assumption remains valid, a hierarchy among the instability modes is required. As shown in Section 7, ring and stringer stiffener spacing and sizes should be modified, as necessary, to achieve the desirable hierarchy.

Recommendations of API RP 2A (ref. C03) are applicable to unstiffened and ring stiffened cylinders with D/t ratios of less than 300.

C1 GENERAL PROVISIONS

C1.1 Scope

The present rules are limited to cylindrical shells.

C1.2 Limitations

The minimum thickness of 3/16 in. is quite arbitrary. Many tests have been performed on fabricated steel models with $t = 0.075$ in. These models required very closely controlled fabrication and welding procedures to obtain the desired tolerances. Also, the thinner models are much more sensitive to nonuniform distribution of loads. The limit of $D/t < 2{,}000$

corresponds to the largest D/t ratio for a fabricated model test. It should be noted that there are only few data points beyond D/t = 1,200.

C1.4 Material

The stability criteria are applicable to steels which have a well defined yield plateau such as those specified in API RP 2A or API RP 2T. Most of the materials used for model tests had minimum specified yield strengths of 36 or 50 ksi. A few additional models have been made from steels with 80 to 100 ksi yield strengths.

C2 Geometries, Failure Modes and Loads

The geometric proportions of a cylindrical shell member will vary widely depending on the application. The load carrying capacity is determined by the shell buckling strength for short members with $KL_t\ /r$ less than about 12. The column buckling mode is not an issue for typical large diameter cylindrical shells. However, some cylindrical shells in transition region (i.e., D/t ratio of close to 300), such as a ring stiffened crane pedestal, need to be check against column buckling.

The shell buckling strength is a function of both the geometry and the type of load or load combination. Unstiffened shells fail by local shell buckling. The local buckling stresses for unstiffened cylindrical shells are very low, susceptible to geometric imperfections and exhibit large reduction in post buckling strength. Ring and stringer stiffened cylindrical shells meet tighter tolerances and minimize the effect of geometric imperfections. Stiffeners, when arranged and sized adequately, greatly increase cylindrical shell local, bay and general instability stresses as discussed below.

C2.3.1 Axial Compression

The axial compression buckling stress can be increased by the addition of stringers (longitudinal stiffeners). The stringers carry part of the load as well as increase the local shell buckling stress. They must be placed less than about $10\sqrt{Rt}$ apart to be effective for axial compression. The stringer spacing must be less than one half the wave length determined for a shell without stringers to be effective in increasing the failure stress for external pressure. The use of stringers introduces two more possible modes of failure. The stringer elements must be compact sections (see Section 7) or local buckling of the stringers may occur. Another possible mode of failure is the buckling of the stringers and shell plating together. This mode of failure is termed bay instability and the failure stress is mainly a function of the moment of inertia of the stringers and attached shell. Waves form in both the longitudinal and circumferential directions for axial compression loads. A single half wave forms in the longitudinal direction and several waves form in the circumferential direction for external pressure. If the bay instability stress is greater than the local shell buckling stress, the cylinder will continue to carry load after local shell buckling occurs. If there is only a small difference, local shell buckling will probably initiate the bay instability mode. Bulletin 2U recommends that the shell be designed so that the bay instability stress is 1.2 times the local shell buckling stress.

A large diameter orthotropically stiffened cylindrical shell is not likely to fail in a column buckling mode. However, a cylindrical shell stiffened with rings only needs to be checked against column buckling by substituting the local shell buckling stress for yield stress in the column buckling equation as the local buckling can precipitate column buckling.

C2.3.2 External Pressure

Ring stiffeners are much more effective than stringers in increasing the buckling stress of a cylinder subjected to external pressure. The use of rings introduces two more possible modes of failure. One mode is local buckling of the ring elements which can be avoided by the use of compact sections. The other mode of failure is the buckling of one or more rings together with the shell (and stringers when used). This mode of failure is called general instability and the failure stress is a function of the moment of inertia of the ring together with an effective width of shell. This mode of failure should be avoided because it results in gross distortions of the shell. Local shell buckling may precipitate a general instability failure if there is only a small difference in the buckling stresses for the two modes. The bulletin recommends that the shell be designed so that the general instability stress is more than 1.2 times the local buckling stress for both ring stiffened cylinders and ring and stringer stiffened cylinders.

A stringer stiffened cylinder may continue to carry load after the shell has buckled locally between stringers until failure occurs by bay instability. This mode of failure when due to external pressure or external pressure combined with axial compression results in the postbuckling formation of a series of longitudinal plastic hinges between stringers at locations around the circumference where the circumferential waves are radially outward. The formation of hinges may also occur in the rings due to local buckling of the ring elements. Under axial compression load the mode of failure is a joint collapse of stringers and shell. The post buckling load may be as much as 80% of the collapse load for either axial compression or external pressure if the plastic hinges do not develop in the rings.

C3 Buckling Design Method

The design of cylindrical shells subjected to applied axial loading and external pressure is an interactive procedure. It requires an understanding of how to first determine and then to change, whenever necessary, both the buckling stresses and the applied stresses to meet the hierarchy and the safety factor/utilization ratio requirements for each load condition and load combination.

<u>Applied Stresses</u>
The design process differs from a design review process only in terms of defining cylindrical shell configuration, namely the diameter, thickness, stiffener arrangement and stiffener sizes. Whether the geometry is defined or assumed, the first step is to assess the adequacy of the configuration with respect to applied stress levels. The shell thickness should be compatible with the configuration so that adequate area is provided to maintain reasonable stress levels when the cylindrical shell is subjected to applied loads. Then, the applied shell and stiffener stresses are more accurately determined either from a finite element analysis or through the use of recommended equations in Section 11.

Elastic Buckling Stresses

The next step is to determine elastic buckling stresses for each instability mode and the load case in accordance with the recommendations of Section 4. To ensure that the assumed/given geometry will meet the hierarchy requirements, a check is performed in accordance with the recommendations of Section 7.1. If the hierarchy is not achieved, it may be necessary to revise the shell plate thickness or the stiffener spacing to raise the bay and general instability buckling stresses or perhaps to reduce the local instability stress.

Plasticity Reduction Factor

Having met the hierarchy requirements, computed elastic buckling stresses in the material elasto-plastic region (i.e., above the material proportional limit) are corrected by applying a plasticity reduction factor in accordance with the recommendations of Section 5.

Buckling Stresses for Combined Loading

Computed buckling stresses for uniaxial loading have to be downgraded when the buckling phenomena can be initiated due to multiaxial loading. Interaction equations recommended in Section 6 define a limiting buckling stress envelope that can be used in conjunction with any stress combination.

Stiffener Sizing

Buckling stress equations are based on the assumption that stiffeners meet compact section requirements and will not exhibit local buckling of the stiffener web/flange that may initiate bay or general instability. Stiffener shape, web/flange thickness/width may be revised, whenever necessary, either to meet the compact section requirement or to make subtle changes to bay or general instability stresses. In some instances it may be acceptable to utilize non-compact sections provided that the applicable bay or general instability stresses are corrected accordingly.

Allowable Stresses

Typically, column buckling is not an issue for an orthropically stiffened cylindrical shell. However, column buckling stresses are determined in accordance with Section 8 to ensure that all possible instability modes are checked. Determined buckling stresses are reduced by safety factors as recommended in Section 9 to determine allowable stresses. The applied-to-allowable stress ratio (i.e., utilization ratio) for each load case and load combination (i.e., per Section 6) for each instability mode should be less than unity.

Conclusion

A design or a design review that follows the steps shown on Figure 3.1 and discussed above should yield not only adequate utilization ratios for each instability mode but also yield general instability stresses that are higher than the bay instability stresses that are at least 20 percent larger than the local buckling stresses so that one instability mode will not initiate another.

The separation of the local buckling mode from the bay or general instability modes is accomplished with the factor β given in Section 7. The separation of the bay and general instability modes is left to the discretion of the designer.

C4 Predicted Shell Buckling Stresses for Axial Load, Bending and External Pressure

The theoretical elastic buckling stress equations given in the Bulletin are based upon classical theory with simple support boundary conditions and Poisson's ratio of 0.3. The differences between tests on fabricated cylindrical shells and the theoretical stresses are accounted for by the factor α_{ij}.. This factor is equivalent to the ratio of the strain in a fabricated cylinder under load to the strain in the tensile coupon from which the material properties are determined. The values of α_{ij} apply to cylinders with initial shapes which meet the fabrication tolerances of Section 10. Design guidance is also given in the commentary for cylinders which do not meet the tolerances of the Bulletin.

C4.1 Local Buckling of Unstiffened or Ring Stiffened Cylinders

The buckling strength of a section of shell between ring stiffeners is assumed to be the same as an unstiffened shell.

C4.1.1 Axial Compression and Bending

For a cylindrical shell that can locally fail, the elastic buckling stress was previously expressed in terms of its geometric characteristics by:

$$F_{xeL} = \alpha_{xL} 2C_x E(t/D) \qquad \text{(C4.1.1-1)}$$

This equation is determined from classic elastic theory (Ref. C04, p.465) by assuming the number of circumferential half-waves (i.e., lobes) being zero (n = 0) and the number of longitudinal half-waves being one (m = 1 in Equation (C.4.1-2). This is an axisymmetric (accordion-like) buckling mode.

$$\sigma_{xeL} = \frac{N_\phi}{t} = \left[\frac{(n^2+\lambda^2)^2}{12(1-\nu^2)\lambda^2}\left(\frac{t}{R}\right)^2 + \frac{\lambda^2}{(n^2+\lambda^2)^2} \right] E \qquad \text{(C4.1.1-2)}$$

where

$$\lambda = \frac{m\pi R}{L}$$

m = number of half waves in the longitudinal direction at buckling
n = number of circumferential waves at buckling

API RP 2A recommends the use of Equation C4.1.1-1 for determination of local buckling stresses in the material elastic zone (i.e., below material proportional limit). Inelastic buckling stress is defined to be equal to material yield stress for a D/t ratio equal to 60. For a D/t ratio in excess of 60, an empirical relationship (Equation C4.1.1-3) is used to determined inelastic buckling stress.

$$F_{xcL} = F_y \left[1.64 - 0.23(D/t)^{1/4}\right] < F_{xeL} \qquad \text{(C4.1.1-3)}$$

A comparison of test data (see Figure C3.2.2-2, Ref.C03) for cylinders with D/t ratios up to 340 indicates validity of API RP 2A recommendation.

API Bulletin 2U is applicable to D/t ratios greater than 300, namely large diameter cylindrical shells outside of the scope of API RP 2A. As the cylinder diameter increases and the curvature decreases, the buckling behavior of a cylindrical shell becomes less dependent on diameter and more dependent on the unsupported length of the shell plate. Thus, the failure mode of a cylinder with large curvature changes to essentially that of a flat plate when the shell plate curvature is small. API Bulletin 2U covers the transition from one type of behavior to the other.

Donnell's eighth-order partial differential equation (Ref. C05) is applicable to an axisymmetric buckling mode when the number of lobes (n) is not small (Equation C4.1.1-4).

$$\frac{Et^3}{12\left(1-v^2\right)}\nabla^8 w+\frac{Et}{r^2}\frac{\partial^4 w}{\partial x^4}+\nabla^4\left[N_x\frac{\partial^2 w}{\partial x^2}+2N_{xy}\frac{\partial^2 w}{r\partial x\partial\theta}+N_y\frac{\partial^2 w}{r^2\partial\theta^2}+p\right]=0 \qquad \text{(C.4.1.1-4)}$$

where

- E = modulus of elasticity
- t = cylinder shell plate thickness
- r = cylinder radius
- v = Poisson's ratio
- x = cylinder longitudinal axis
- θ = cylinder circumferential axis
- w = radical displacement
- N = applied line loads
- p = pressure

Donnell's equation was simplified by Batdorf (ref.C06) for curved panels with complex boundary conditions and gives the theoretical buckling stress as:

$$F_{ieL}=k_i\frac{\pi^2 E}{12\left(1-v^2\right)}(t/a)^2 \qquad \text{(C4.1.1-5)}$$

where

- k_i = buckling coefficient, C_{xL} , for local axial buckling
- a = ring spacing, L_r , for axial loading

The buckling coefficient is expressed in terms of the geometric curvature parameter, M_x , the *D/t* ratio and the imperfection factor in Equation 4.1-2.

$$C_{xL}=\left[1+\{150/(D/t)\}\{\alpha_{xL}\}^2\{M_x^4\}\right]^{0.5}$$

and the imperfection factor in the axial direction is expressed by Equation 4.1-3 as a function of the D/t ratio:

$$\alpha_{xL}=9.0/[300+D/t]^{0.4}$$

Tests conducted by Stephens, Kulak, et. al. (Ref. C07), Wilson and Newmark (Ref. C08), Akiyama, et.al. (Ref. C09), Chen, et.al. (Ref. C10), Dowling and Harding (Ref. C12), Galletley and Pemsing (Ref. C13), Miller (Ref. C14), and Odland (c15) were evaluated and presented in Reference C16.

These test data were normalized by taking the buckling stress-to-yield stress ratios (i.e., f_{xcL} / F_y) and comparing them against API-predicted elastic buckling stress-to-yield stress ratios (i.e., F_{xeL} / F_y). All the test data in the material elastic range (i.e., $F_{xeL} / F_y < 0.5$) are greater than API-predicted buckling stresses (see Figure C.4.1.1-1). In the material elasto-plastic zone some of the API-predicted inelastic buckling stress are slightly higher than the test data. This scatter is acceptable due to variations in geometric imperfections of small scale tests and is further discussed in Section C6.2. Figure C.4.1.1-2 shows that the test-to-API predicted inelastic (i.e., critical) stress ratios (f_{xcL} / F_{xcL}) are substantially above 1.0. Thus, the use of somewhat less conservative buckling coefficient definition in conjunction with an LRFD-based design could be considered.

C4.1.2 External Pressure

The theoretical elastic buckling stress can be determined based on cylindrical shell geometry, modulus of elasticity and Poisson's ratio. Von Mises (Ref. C17) and others have analyzed local buckling of cylindrical shells subjected to external pressure. Von Mises' equation (C.4.1.2-1) for external pressure is not exact for cylindrical shells with closely spaced rings where axisymmetric instability is more likelythan assymmetric buckling.

An empirical relationship based on Von Mises' solution using Donnell's equation (C.4.1.1-4) was developed by Batdorf (Ref. C06). This classical definition of elastic buckling stress ($F_{\theta eL}$ in Equation 4.1-5) and the buckling coefficient ($C_{\theta L}$ in Equation 4.1-7) neglect the bending effect on a cylindrical shell and render the predicted buckling stresses inaccurate for instability modes with a small number of circumferential lobes (i.e., n).

a) <u>Large Diameter Cylindrical Shells</u>

Defining the buckling coefficient based on Donnell's equation as:

$$C_{\theta L} = \frac{\left(m^2 + \beta^2\right)^2}{m^2/2 + \beta^2} + \frac{12M^4\left(1 - v^2\right)m^4}{\pi^2\left(m^2 + \beta^2\right)^2\left(m^2/2 + \beta^2\right)} \qquad \text{(C4.1.2-1)}$$

and minimizing it by $\partial C_{\theta L} / \partial m = 0$ and $\partial C_{\theta L} / \partial \beta = 0$, the following relationship is obtained for a simple span between rings (i.e., $m = 1$).

$$\frac{\beta^2\left(1+\beta^2\right)^4}{2+3\beta^2} = \frac{12M^4\left(1-v^2\right)}{\pi^4} \qquad \text{(C4.1.2-2)}$$

The smallest number "n" that causes the left and the right side of Equation C.4.1.2-2 to be approximately equal defines the asymmetric buckling mode of a cylindrical shell.

The term, β, is the ratio of ring-spacing-to-half wave buckle length $\left[\beta = L_r / \pi R / n\right]$.

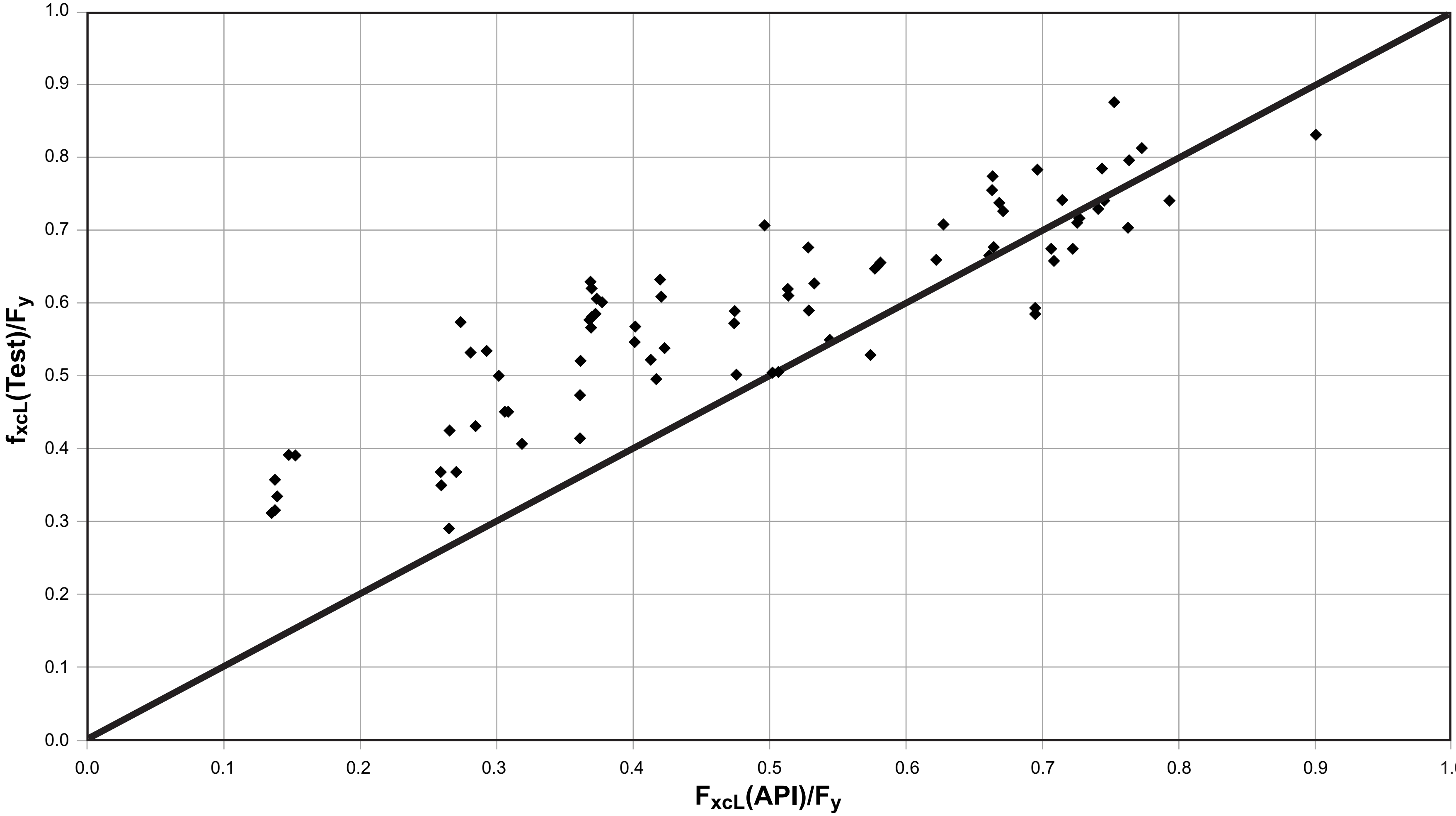


Figure C.4.1.1-1--Test $f_{\bar{x}cL}/F_y$ versus API F_{xcL}/F_y
Ring Stiffened Cylindrical Shells Under Axial Compression

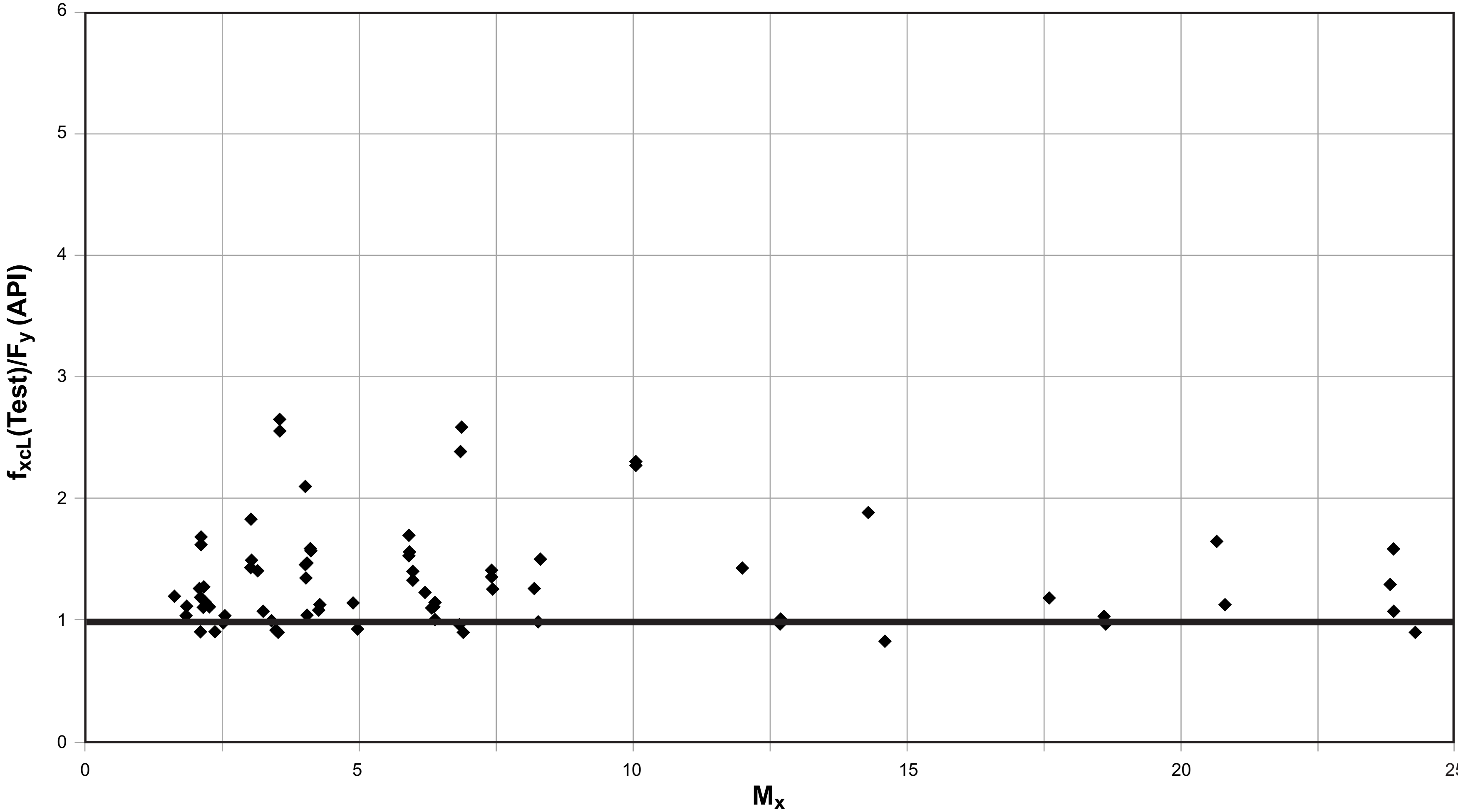


Figure C.4.1.1-2--Test f_{xcL}/API F_{xcL} Versus M_x
Ring Stiffened Cylindrical Shells Under Axial Compression

Tests conducted by Chen, et. al. (Ref. C11), Galletley and Pemsing (Ref. C13, Bannon (Ref. C18), and Miller (C14) were analyzed and the studies conducted by Miller (C19 and C20) were carefully reviewed. Analysis and review results are presented in Reference C16.

These test data were normalized by taking the buckling stress-to-yield stress ratios (i.e., $f_{\theta cL} / F_y$) and comparing them against API-predicted elastic buckling stress-to-yield stress ratios (i.e., $F_{\theta eL} / F_y$). The test data in the material elastic range (i.e., $F_{\theta eL} / F_y < 0.5$) are very close to the API-predicted buckling stresses (see Figure C.4.1.2-1). In the material elasto-plastic zone some of the API-predicted inelastic buckling stress are less than the test data. When additional test data becomes available, the buckling coefficient for buckling in this region can be modified to be less conservative. This topic is further discussed in Section C6.2.

b) Smaller Diameter Cylinders

The Von Mises equation (C.4.1.2-3) for external pressure does not accurately define the behavior of cylinders with closely spaced rings where axisymmetric collapse, rather than asymmetric buckling, is likely. However, it is a satisfactory method for estimating elastic buckling strength.

$$p_e = \frac{2E(t/D)}{n^2 + (\lambda^2/2) - 1}\left[\frac{(t/D)^2}{3(1-v^2)}\left\{(n^2+\lambda^2)^2 - 2n^2 + 1\right\} + \frac{\lambda^4}{(n^2+\lambda^2)^2}\right] \qquad \text{(C.4.1.2-3)}$$

If the instability mode is that of an ellipse (i.e., n = 2), the above equation reduces to:

$$p_e = \frac{2E(t/D)}{3 + (\lambda^2/2) - 1}\left[\frac{(t/D)^2}{3(1-v^2)}\left\{(4+\lambda^2)^2 - 7\right\} + \frac{\lambda^4}{(4+\lambda^2)^2}\right] \qquad \text{(C.4.1.2-4)}$$

Predicted stressed based on Batdorf's approach are applicable for a wide range of shell geometry parameter, *G*, defined as a function of ring spacing, L_r, diameter (*D*) and shell thickness (*t*) in Equation C.4.1.2-5.

For a shell geometry parameter, *G*, greater than 4(*D/t*):

$$G = 1.82 L_r (1/D)(t/D)^{0.5} > 4(D/t) \qquad \text{(C.4.1.2-5)}$$

Since the slenderness function, $\lambda = \pi D / n L_r$, becomes less significant, equation C.4.1.2-4 can be reduced to:

$$p_e = \frac{2E(t/D)^3}{(1-v^2)} \qquad \text{(C.4.1.2-6)}$$

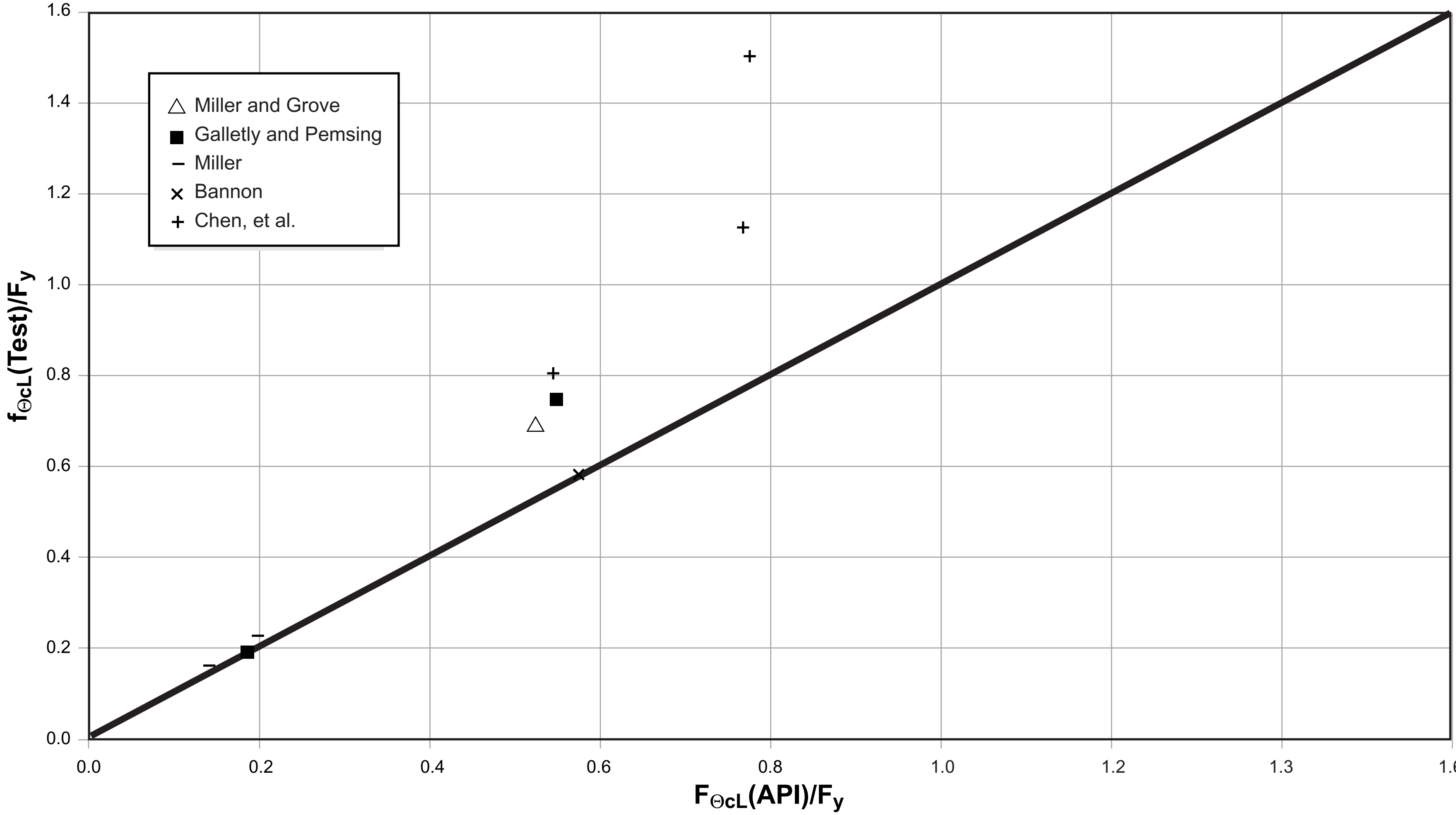


Figure C.4.1.2-1--Test $f_{\Theta cL}/F_y$ versus API $F_{\Theta cL}/F_y$
Ring Stiffened Cylindrical Shells Under External Pressure

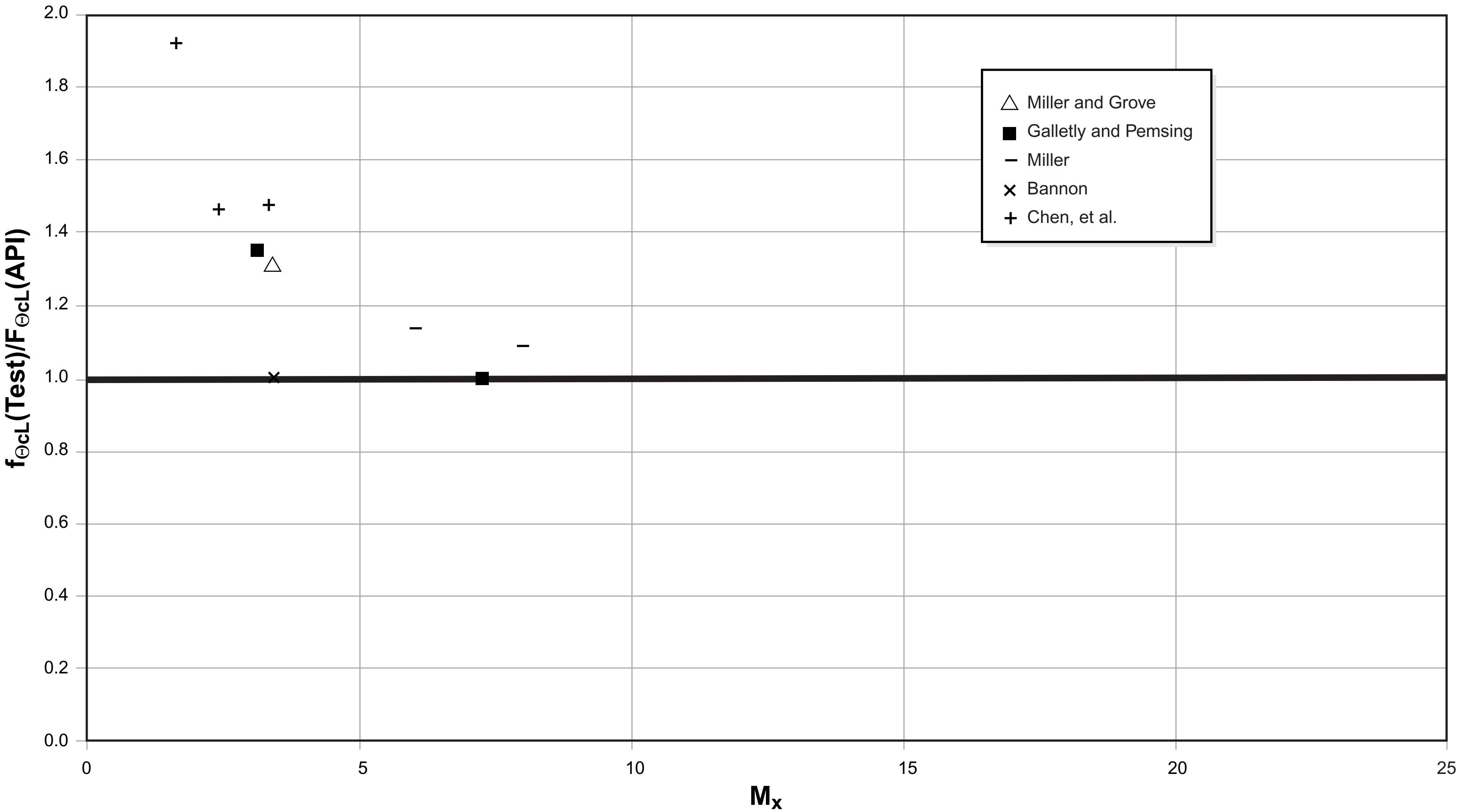


Figure C.4.1.2-2--Test $f_{\Theta cL}$/API $F_{\Theta cL}$ versus M_x
Ring Stiffened Cylindrical Shells Under External Pressure

which is the well known equation form used in unstiffened brace member analysis:

$$p_e = \frac{F_{\theta eL}t}{R} = \frac{2E(t/D)^2}{(1-\nu^2)}(t/2R)$$

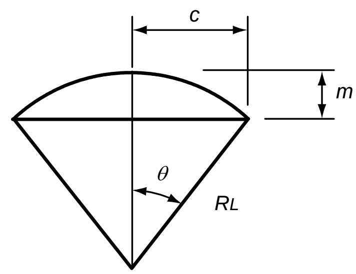
$$F_{\theta eL} = \frac{E}{(1-\nu^2)}(t/D)^2 \qquad \text{(C.4.1.2-7)}$$

Theoretical methods have been developed by several authors to account for the effects of imperfections. All of these methods are based upon the assumption that the initial out-of-roundness is similar in form to the assumed buckling mode shape. The bending stresses resulting from the initial out-of-roundness are combined with the membrane stresses. The buckling pressure is determined by equating the combined hoop stress to the yield stress or by the von Mises failure theory. Reference C21 gives a comparison of test pressures to those predicted by the methods of Timoshenko & Gere (Ref. C04), Galletly and Bart (Ref. C22), and Sturm (Ref.C23). The correlation between these theories and test results is very poor and the methods are much too conservative to be practical for use. The ratios of p_{Test} / p_{Theory} varied from 1.93 to 3.25 for the theory of Timoshenko and Gere and from 1.40 to 2.82 for the other two theories.

Miller and Grove (Ref. C14) have found an alternate method which provides excellent correlation for cylinders of all geometries. The theory behind the method is that a flat spot on the shell having a larger than nominal radius of curvature will buckle at a lower pressure. Also, it was noted from experimental results that an imperfect shell will buckle in the same or nearly the same number of waves as a shell without imperfections. To determine the local buckling pressure, the local radius measured over half of a theoretical wave length is substituted for the nominal radius in the theoretical shell equation. The buckling pressure is taken as the minimum pressure given by either n or n + 1 where n is the theoretical wave number for the shell without imperfections.

Since the local shell imperfections are measured over a half wave length, the gage angle, 2θ, is equal to π/n radians ($\theta = \pi/2n$). The local radius, R_L, can be computed by knowing the versine, m, and the half chord, c, corresponding to the versine.

$$R_L = \frac{m^2 + c^2}{2m}$$



A comparison of test results with the proposed method was made for 30 cylinders in Ref. C14 and the average of $\rho_{Test} / \rho_{Theory}$ was 1.007 with a convergence of 13.2%. Tests include elastic and elastic-plastic values with R/t ratios from 14 to 500 and yield stresses of 31.0 to 61.7 ksi.

C4.2 General Instability of Ring Stiffened Cylinders

C4.2.1 Axial Compression or Bending

Equation 4.2-1 was determined from Equation 29 of Ref. C24 for U = 0. An analysis of available test data is give in Ref. C25 and Equation 4.2-2 is based upon this study. The imperfection factor α_{xG} is a constant when the area of the stiffener exceeds 20% of the shell area. It is equal to an unstiffened cylinder when the stiffener area is zero. A straight line variation is assumed between these two limits. Additionally, it is recommended that the minimum area of the stiffener must equal 6% of the shell area for it to be effective.

C4.2.2 External Pressure

The equation for external pressure is based upon the split rigidity principle where the first term is the contribution of the shell of length between bulkheads and the second term is the contribution of the effective ring section. The shell contribution is taken from Equation C.4.1.2-1. Only the second term of this equation is used since the first term has little contribution if included. The second term of Equation 4.2-5 is the buckling pressure for a ring under uniform load which is given by Equation d, p. 289, of Ref. C04.

A comparison of test data with Equation 4.2-5 was made in Ref. C21. A constant value of $\alpha_{\theta G} = 0.8$ is recommended for cylinders which meet the fabrication tolerances of Section 10.

Theoretical methods have been developed for predicting the effect of out-of-roundness on the general instability pressure. The test results are compared with the methods proposed by Strum (Ref. C23), Kendrick (Ref. C26), Hom(Ref. C27) and Griemann (Ref. C28) in Reference C21. The closest correlation for instability is given by Griemann with the ratios of $\rho_{Test} / \rho_{Theory}$ ranging from 0.58 to 0.77 compared with 1.16 to 2.55 for Sturm. The theories of Kendrick and Hom gave ratios as high as 2.73 and 3.30. A later comparison was made with Kendrick's elasto-plastic theory (see Ref. C29, p. 637). This method gave ratios of 2.43 to 6.23.

Note that if axial load is being considered separately, use $k = 0$ in equation 4.2-5. If axial load is due to end cap pressure alone, then using $k = 0.5$ incorporates its effect, so a separate axial check at 4.2.1 is not required.

C4.3 Local Buckling of Stringer Stiffened or Ring and Stringer Stiffened Cylinders

C4.3.1 Axial Compression or Bending

A cylindrical shell, stiffened with reasonably sized rings and stringers that provide adequate rigidity, can be treated as a series of curved plates supported along all four edges with rings and stringers. Curved plates subjected to axial compression will buckle like flat plates when the curvature is small and buckle like cylinders when the curvature is large.

Equation 4.3-1 is the classical buckling equation for a curved plate supported at its edges with rings and stringers. Batdorf's solution (Ref. C06) for unstiffened cylinders yields a buckling coefficient, C_{xL}:

$$C_{xL} = 4[3]^{0.5} Z_\theta / \pi^2 = 0.702 Z_\theta \qquad \text{(C.4.3.1-1)}$$

Substituting this into Equation 4.3-1:

$$F_{xeL} = 0.702 \frac{b^2}{Rt}\left(1-\nu^2\right)\frac{\pi^2 E}{12\left(1-\nu^2\right)}\left(t/b\right)^2 \sim 0.6\left(Et/R\right) \qquad \text{(C.4.3.1-2)}$$

which is the classical buckling stress for long cylinders subjected to axial compression. The behavior of a large diameter cylindrical shell panel with a small curvature is close to that of a flat plate supported at four edges. If the ring spacing is assumed to be equal to or greater than the stringer spacing, the aspect ratio, $A = \left(L_r / b\right)$, can be set equal to 1.0 and the buckling coefficient is defined (Ref. C04) as:

$$C_{xL} = \left(A + 1/A\right)^2 = 4 \qquad \text{(C.4.3.1-3)}$$

Utilizing Kollbrunner's (Ref. C30) buckling coefficient equation for flat plates based on panel aspect ratios, the buckling coefficient is:

$$C_{xL} = \left[1 + \left(1/A\right)^2\right]^2 / \left(1/A\right)^2 = 4.0 \qquad \text{(C.4.3.1-4)}$$

Thus, the most conservative value for the buckling coefficient is 4.0.

Tests conducted by Miller (Ref. C14) and the analyses of these test results (References C20 and C31) validate API recommendations for local buckling.

These test data were normalized by taking the buckling stress-to-yield ratios (i.e., f_{xcL} / F_y) and comparing them against F_{xeL} / F_y). All of the test data in the material elastic range (i.e., $F_{xeL} / F_y < 0.5$) are greater than API-predicted buckling stresses (see Figure C.4.3.1-1). Although the number of test data is limited for the material elasto-plastic zone, the test data are higher than the API-predicted inelastic buckling stresses. This scatter is acceptable due to variations of geometric imperfections of small scale tests and the subject is further discussed in Section C6.2.

Figure C.4.3.1-2 shows that the test-to-API predicted inelastic (i.e., critical) stress ratios f_{xcL} / F_{xcL} as a function of the geometric curvature parameter. Although the stress ratios are greater than 1.0, it is difficult to justify a less conservative prediction of buckling stresses when the number of test data are limited and inadequate to accurately define the transition

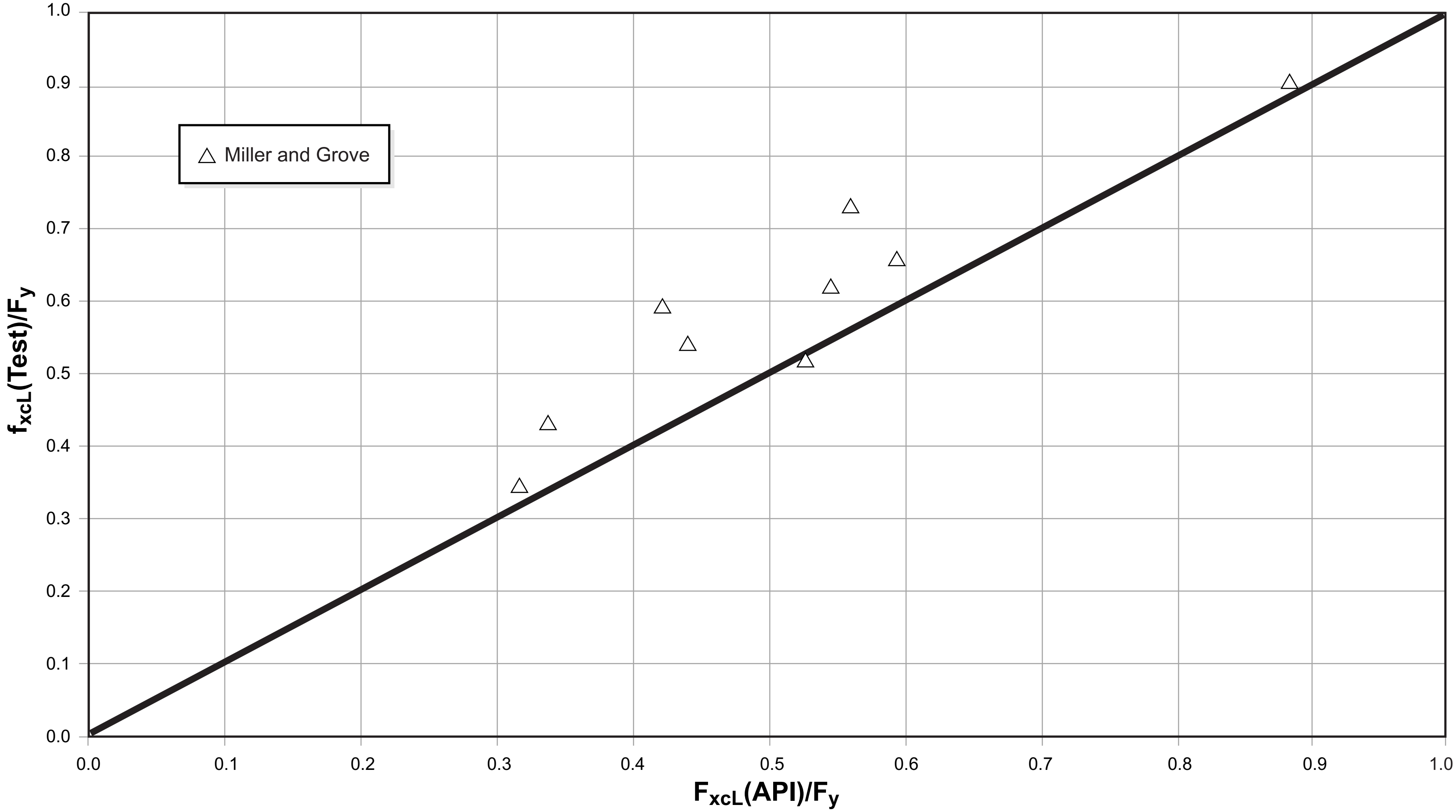


Figure C.4.3.1-1--Test $f_{\Theta cL}/F_y$ versus $F_{\Theta cL}/F_y$
Ring and Stringer Stiffened Cylindrical Shells Under Axial Compression

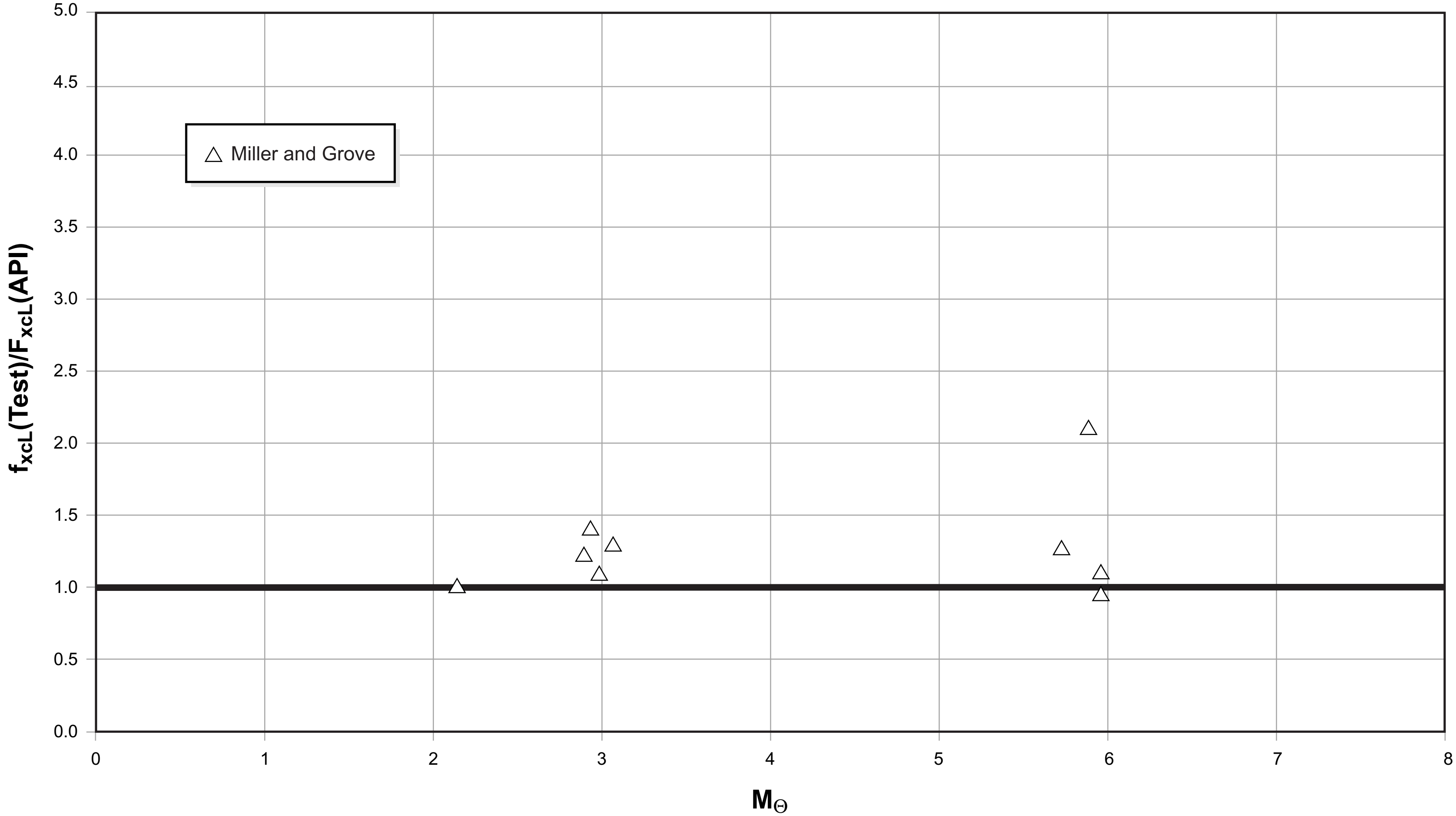


Figure C.4.3.1-2--Test f_{xcL}/API F_{xcL} versus M_{Θ}
Ring and Stringer Stiffened Cylindrical Shells Under Axial Compression

from a small curvature stiffened panel behavior (i.e., small M_θ) to a larger curvature unstiffened cylinder behavior (i.i., large M_θ).

C4.3.2 External Pressure

The behavior of a ring and stringer stiffened cylindrical shell differs from that of a ring stiffened cylindrical shell with the introduction of another instability mode, namely, bay instability. Since bay instability defines the stress level for the failure of shell plate with the stringer(s), the local instability mode is now defined as the instability of only the shell plate uniformly supported at its edges with rings and stringers.

The stringers will be effective only if they can force the number of buckle waves (n) to increase. The hoop buckling stress of an unstiffened shell plate is increased only when the distance between stringers is less than a one-half buckle wave length (i.e., $2N > n$).

Buckling stress equations given in Sections 4.1.2 and 4.3.2 utilize slightly different buckling coefficients. When the rings are reasonably far apart and an adequate number of stringers are provided (i.e., aspect ratio, $\beta = L_r / b > 1.5$), buckling stresses computed by setting $n = N/2$ in Equation 4.1-7 will be very close to those computed from Equation 4.3-4. It should be noted that:

- the exact equations given in Section 4.1.2, derived from Von Mises and neglecting the bending of shell plate, will yield conservative stresses when the rings are closely spread.
- the equations given in Section 4.3.2 can produce buckling stresses less than those predicted by Section 4.1.2 (i.e., no stringers) when the stringers are far apart.

Tests conducted (or sponsored) by Miller (Ref. C14), Bannon (Ref. C18) and Kinra (Ref. C21) were thoroughly analyzed (References C20 and C31). These test data correlate very well with predicted data based on equations in Sections 4.1.2 and 4.3.2. It should be noted that the test data cover only cylindrical shell geometries with a reasonable range of ring and stringer spacings and the results can not be extrapolated from cylindrical shells with inadequate number of stringers.

Available test data were normalized by taking the buckling stress-to-yield stress ratios (i.e., $f_{\theta cL} / F_y$) and comparing them against API-predicted elastic buckling stress-to-yield stress ratios (i.e., $F_{\theta cL} / F_y$). The test data in the material elastic range (i.e., $F_{\theta cL} < 0.5 F_y$) exhibit substantial scatter and remain consistently above the API-predicted buckling stresses (see Figure C.4.3.2-1). In the material elasto-plastic zone, only three data points exist and the API-predicted inelastic buckling stresses remain below the test data. When additional test data become available, the buckling coefficient can be modified to be less conservative. This topic is further discussed in Section C6.2.

Figure C.4.3.2-2 shows that the test-to-API predicted inelastic (i.e., critical) stress ratios ($f_{\theta cL} / F_{\theta cL}$) are substantially above 1.0. Thus, the use of somewhat less conservative buckling coefficient definition is appropriate.

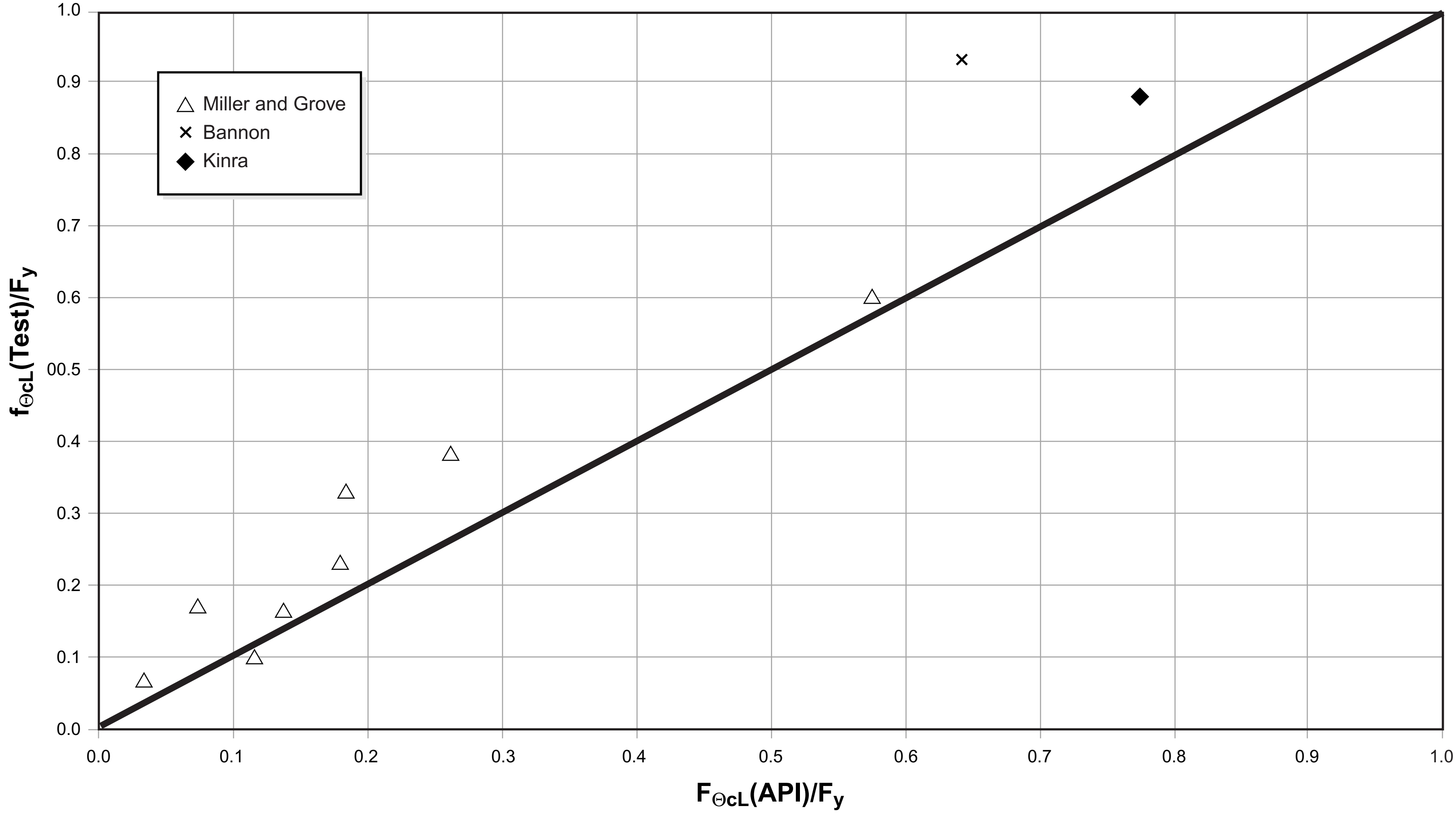


Figure C.4.3.2-1--Test $f_{\Theta cL}/F_y$ versus API $F_{\Theta cL}/F_y$
Ring and Stringer Stiffened Cylindrical Shells Under External Pressure

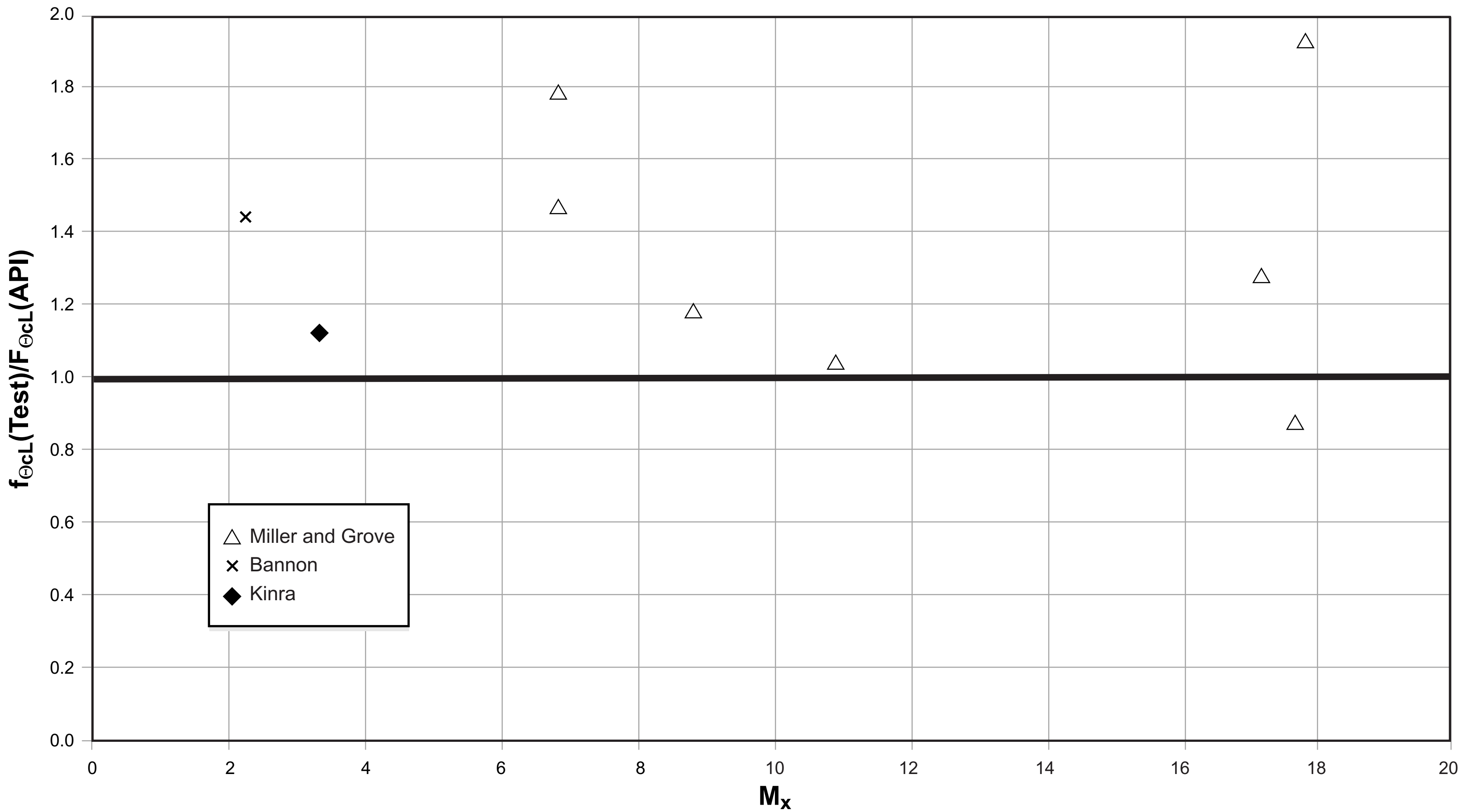


Figure C.4.3.2-2--Test $f_{\Theta cL}$/API $F_{\Theta cL}$ versus M_x
Ring and Stringer Stiffened Cylindrical Shells Under External Pressure

A comparison of test results with the theoretical predictions indicates that imperfections permitted by the Bulletin do not significantly affect the buckling capacities of stringer stiffened cylinders subjected to external pressure. The reason may be that the stringers essentially fix the nominal radius and the membrane stress is a function of the nominal radius, not the local radius. In comparison, the buckling pressures and stresses of ring stiffened cylinders are best predicted using the measured local radius. (See Reference C14)

For a shell without stringers, the shell is free to deflect and rotate at points of inflection of the buckle waves. This produces a buckle pattern with a uniform in-out pattern. Stringer stiffened cylinders provide restraint in the radial direction and some restraint against rotation, depending on the torsional stiffness of the stringers. The buckle wave may vary between a half and a full wave between stringers. The shell panels which buckle inward are much more pronounced than those that buckle outward.

C4.4 Bay Instability of Stringer Stiffened or Ring and Stringer Stiffened Cylinders and General Instability of Ring and Stringer Stiffened Cylinders Based Upon Orthotropic Shell Theory

Equation 4.4-1 is a modification of the equation given in Ref. C32 for simply supported orthotropic shells in which the effective membrane thickness in the longitudinal direction is equal to the area per unit length of shell and the bending rigidity is based upon the effective moment of inertia per unit length of shell. This equation has been modified so that it also applies to stiffened shells with stringers that are not spaced close enough to make the shell plate fully effective. This effect is accounted for in the rigidity parameters (E_x, E_θ, D_x, D_θ and $D_{x\theta}$) of Equation 4.4-1 by introducing the ratios of b_e / b and L_e / L_r. When both ratios equal 1.0 the rigidity factors are the same as those given in Ref. C33, p. 306, for a stiffened shell with the plate fully effective. When both ratios equal zero, the rigidity factors are the same as those given in Ref. C33, p. 303, for a gridwork shell. The equations for the rigidity parameters are given below. The first term in each equation is the Bulletin nomenclature, and the second term is Ref. C33 nomenclature.

a. Stiffened Shell-Plate Fully Effective

$$E_x = D_x = \frac{Et}{1-v^2} + \frac{EA_s}{b} \qquad E_{x\theta} = D_v = \frac{vEt}{1-v^2}$$

$$E_\theta = D_\phi = \frac{Et}{1-v^2} + \frac{EA_r}{L_r} \qquad G_{x\theta} = D_{x\phi} = \frac{Et}{2(1+v)} = Gt$$

$$D_x = K_x = \frac{Et^3}{12(1-v^2)} + \frac{EI_s}{b} + \frac{EA_sZ_s^2}{b}$$

$$D_\theta = K_\theta = \frac{Et^3}{12(1-v^2)} + \frac{EI_r}{L_r} + \frac{EA_rZ_r^2}{L_r}$$

$$D_{x\theta} = 2K_v + K_{x\phi} + K_{\phi x} = \frac{vEt^3}{6(1-v^2)} + \frac{Gt^3}{3} + \frac{GJ_s}{b} + \frac{GJ_r}{L_r}$$

$$C_\theta = S_\phi = \frac{EA_r Z_r}{L} \qquad\qquad C_x = S_x = \frac{EA_s Z_s}{b}$$

b. Gridwork Shell

$$E_x = D_x = \frac{EA_s}{b} \qquad\qquad E_\theta = D_\theta = \frac{EA_r}{L_r} \qquad\qquad E_{x\theta} = 0$$

$$G_{x\theta} = D_{x\phi} + \frac{3}{2L_r b}\left(\frac{b}{EI'_r} + \frac{L_r}{EI'_s}\right)^{-1}$$

(I'_r and I'_s are moment of inertia about the weak axis of the stiffeners.)

$$D_x = K_x = \frac{EI_s}{b} + \frac{EA_s Z_s^2}{b} \qquad\qquad D_\theta = K_\phi = \frac{EI_r}{L_r} + \frac{A_r Z_r^2}{L_r}$$

$$D_{x\theta} = K_{x\phi} + K_{\phi x} = \frac{GJ_s}{b} + \frac{GJ_r}{L_r}$$

$$C_x = S_x = \frac{EA_r Z_r}{L_r}$$

The bay instability mode is determined by letting the length of the cylinder equal the ring spacing. The general instability mode is determined by letting the length of cylinder equal the overall length. Local buckling of the stiffener elements is not accounted for in Equation 4.4-1. Although several methods for predicting the effects of local stiffener buckling have been investigated, further study is deemed necessary. The present recommendation is to substitute the buckling stress given by Equation C7-1 for the yield stress.

An analysis of approximately 300 tests from data published prior to 1977 is contained in Ref. C34. Local buckling of stiffeners occurred on only a few models. The applied loads were either axial compression or bending moment. A large test program (6) was conducted by CBI Industries in 1983 on ring and stringer stiffened cylinders subjected to combinations of axial compression and external pressure. The fabrication methods and materials used for the test models were representative of offshore structures. The *R/t* values were 190, 300 and 500 and the material was hot rolled steel sheets with yield stresses of 50 to 80 ksi. The stringer spacings were $b/\sqrt{Rt}$ of 2.2, 3, and 6. The test results are analyzed and compared with Equation 4.4-1 in Ref. C14.

Many of the models had stiffeners which did not satisfy the compact section requirements of Section 7.2. For the analysis of these models an effective yield stress was substituted for the actual yield stress. The effective yield stress is used for all failure modes. The effective yield stress was taken as the buckling stress of a bar stiffener determined from the AISI *Cold Formed Steel Design Manual* (Ref.C35). The buckling stress was assumed to be 1.67 times the allowable stress given in Equation 3.2-2 of Ref. C35 (see Equation C7-1).

C4.4.1 Axial Compression or Bending

When the study of Ref. C34 was made, a factor of 1.7 rather than 1.9 was used in Equation 4.4-2 for b_e and 0.9 rather than 1.0 in Equation 4.4-4. The correct mode of failure was predicted for almost all models. The higher factor of 0.9 is based upon the tests reported in Ref. C14.

a. Bay Instability

A majority of the test models in Ref. C34 were one bay long (stringers only) while all the models of Ref. C14 were 3 bays long with the end bays 0.7 times the length of the center bay. The one bay models failed at values of 0.8 to 2.5 times the predicted values. This wide range is attributed to the effects of end fixity. Reference C34 included a group of tests on models with multiple bays subjected to bending moments. The shells were not fully effective ($b_e < b$). The test stresses were 0.9 to 2.1 times the predicted stresses (with changes noted in Par. 1). Most of the tests in Ref. C14 were made on stress relieved models. Additional tests are now in progress on nonstress relieved models.

b. General Instability

None of the tests in Ref. C14 failed in the general instability mode. There were two groups of tests in Ref. C34 which failed by general instability. The cylinders subjected to axial load failed at stresses 1.0 to 1.3 times the values predicted values (with changes noted in Par. 1) and the cylinders subjected to bending moment failed at 0.8 to 1.3 times the predicted values.

C4.4.2 External Pressure

External pressure tests have been conducted on ring and stringer stiffened cylinders with pressure loadings corresponding to k = 0, 0.5 and 1.8 where $k = N_x/N_\theta$. The results of these tests were analyzed in Ref. *C14*.

a. Bay Instability

The bay instability stresses given by the equations in Section 4.4 require that the minimum number of stringers must be about 3 times the number of circumferential waves for this mode. Several of the test models did not satisfy this requirement. For these models the buckling stresses are predicted by the bay instability formulations of Section 4.5.

If the local shell buckling stress is significantly less than the bay instability stress the accuracy of Equation 4.4-1 decreases. This equation has been found to provide good correlation for stringer stiffened shells when the bay instability stresses do not exceed 1.5 times the local shell buckling stresses. A ratio of 1.2 is recommended. This corresponds to the value of b recommended for Equation 7-1.

b. General Instability

One test (Ref. C36) has been conducted where the cylinder failed by general instability when subjected to external pressure. The rules suggest that the general instability stresses should be 1.2 times the local shell buckling stresses (β = 1.2). This can be accomplished with little additional material in the rings because the general instability stress is a function of the moment of inertia of the effective ring section. The imperfection factor for ring stiffened cylinders is also recommended for ring and stringer stiffened cylinders.

C4.5 Bay Instability of Stringer Stiffened and Ring and Stringer Stiffened Cylinders—Alternate Method

The buckling stresses given by the equations in Section 4.4 are based on small deformation theory. The strains at which buckling occurs are typically less than the yield strain.

When stringer stiffeners are used on a cylinder, local buckling of the shell plate between stiffeners may occur without precipitating a collapse of the cylinder. If the local shell buckling stress is significantly less than the bay instability stress the modified orthotropic shell equation (Equation 4.4-1) becomes increasingly less accurate as the difference becomes greater. A ratio of bay instability stress to local shell buckling stress of 1.2 is recommended. Also, Equation 4.4-1 is not applicable to stiffened shells with less than about three stringers for each wave length in the bay instability mode.

An alternate method is given in Ref. C37 for those cases where the orthotropic shell equation should not be used. Equations were derived based upon the formation of a collapse mechanism. The equations of Section 4.5 for axial compression are taken from Ref. C37.

Although an error in the alternate method was corrected and the method predicts bay instability stresses that compare well with available test data, the method is less conservative than the approach take in Section 4.4 For some geometric configurations with low instability stresses, reducing the number of stringers will force the use of Section 4.5 rather than 4.4 and result in an increase in predicted instability stress. While bay instability stresses based on Section 4.4 will be low when the method is not applicable due to inadequate number of stiffeners, the number of stiffeners should not be intentionally reduced to take advantage of higher bay instability stresses based on Section 4.5. A reduction in the number of stiffeners (i.e., increase stiffener spacing) will reduce local instability stresses.

C4.5.1 Axial Compression or Bending

The failure load given by Equation 4.5-14 was developed by Faulkner, Chen and de Oliveira (Ref. C37). A discrete stiffener-shell approach was used for determining the elastic collapse load and the inelastic collapse load was then determined by using the Ostenfeld-Bleich equation. Specific values were assumed for factors such as the shell shape reduction and bias factors. The values for coefficient c in Equation 4.5-8 are subject to further review. The authors of Ref. C37 also suggest c = 3.0 for light fillets and c = 0 for stress relieved shells.

The method of Section 4.5.1, although highly empirical, provides the best correspondence between test and predicted loads of any of the methods that have been studied.

A much simpler alternate method has been developed by the ECCS Committee on Buckling of Shells for Ref. C38. This method is being studied as an alternative to the equations in Section 4.5.1.

C4.5.2 External Pressure

The formulations for bay instability under external pressure are taken from Ref. C39. The external pressure load for bay failure is assumed to be made up of two components similar to Equation 4.2-5 for ring stiffened cylinders. The first term in Equation 4.5-15 is the buckling capacity of a cylinder with the stringers removed and the length equal to the ring spacing. The second term is the pressure that will develop through the formation of plastic hinges in the composite stiffener and shell. This total is then modified by an effective pressure correction factor, K_p, determined from tests. Equation 4-60 is compared with test data in Fig. C4.5.2-1.

C5 Plasticity Reduction Factors

The elastic buckling stress of a fabricated cylinder, F_{iej}, is the product of the elastic buckling stress for a perfect shell and the capacity reduction factor α_{ij}, which accounts for the differences in geometry and boundary conditions between the fabricated shell and a perfect shell. The factor α_{ij} can also be considered to be the ratio of the strain in a tensile coupon used to determine the material properties and the strain in the fabricated cylinder under applied load. When F_{iej} exceeds the elastic limit of the shell material after fabrication, the buckling stress is given by F_{icj} which is the product of the elastic shell buckling stress and the plasticity reduction factor,η.

For axial compression, Gerard (Ref. 40) derived a plasticity reduction factor as a function of secant modulus, tangent modulus and a variable Poisson's ratio.

$$\eta = \frac{(E_s E_t)^{0.5}}{E}\left\{\frac{1-\nu_e}{1-\nu^2}\right\}^{0.5} \tag{C5-1}$$

where the variable Poisson's ratio is defined equal to (a) 0.3 in the material elastic zone, (b) 0.5 in the fully plastic zone, and is defined by:

$$\nu = \nu_p - (Es/E)(\nu_p - \nu_e) \tag{C5-1a}$$

Equation C5-1 does not compare well with available test data throughout the elasto-plastic zone. Another disadvantage of the equations give above is that they require knowledge of both E_s and E_t, name the stress-strain curve from tests or an assumption of the shape of the curve in material elasto-plastic range.

A more commonly used equation for plate buckling was recommended by Johnston (Ref. C41). This simpler relationship to determine the local instability stress in the material elasto-plastic zone is:

$$\eta = [E_t / E]^{0.5} \tag{C5-2}$$

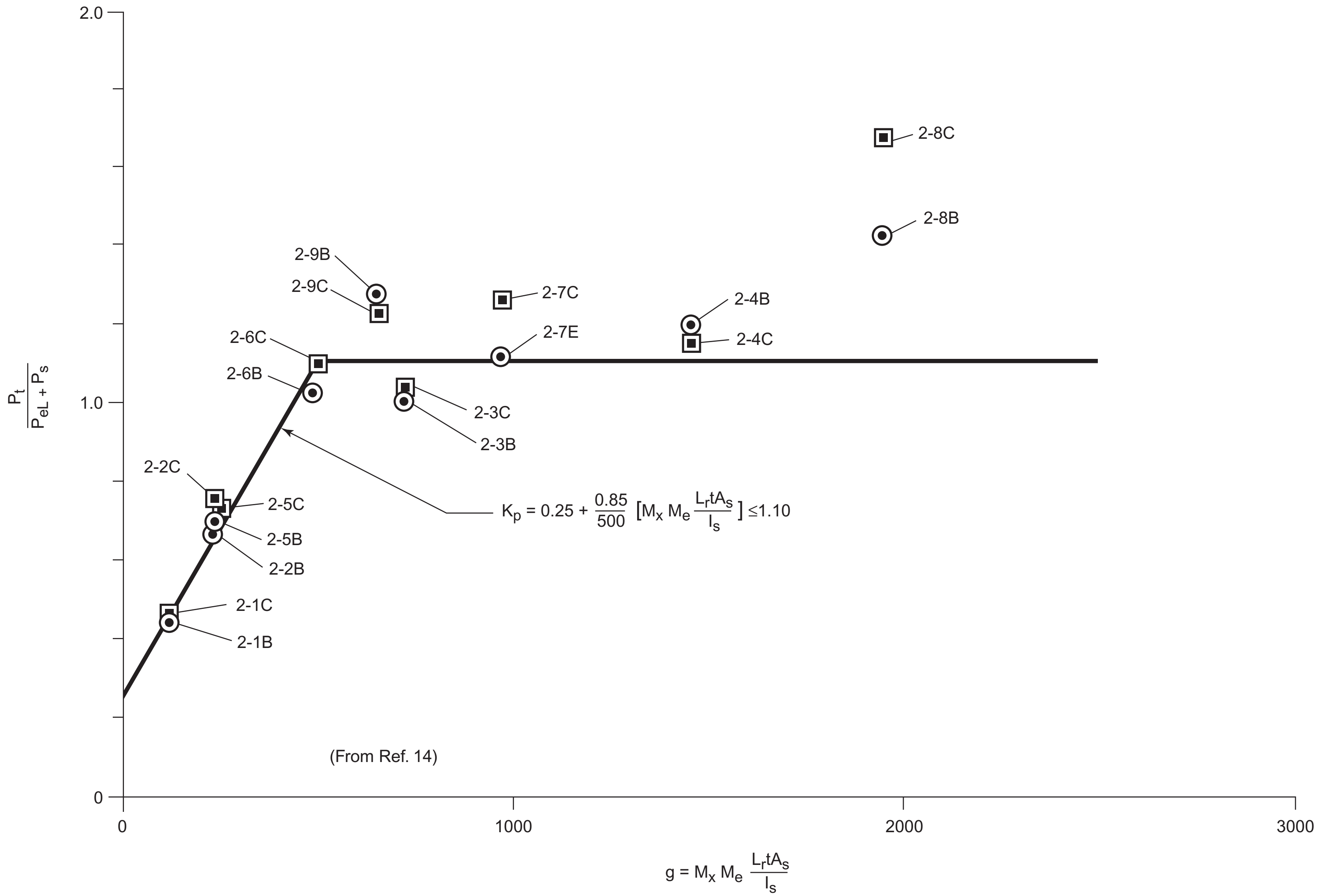


Figure C.4.5.2-1--Comparison of Test Pressures with Predicted Failure Pressures for Stringer Stiffened Cylinders

For shell plate instability due to hoop stresses, although somewhat conservative, Equation C5-2 is equally applicable as the ring-supported shell plate behaves like a panel.

The use of Equation C5-2 still requires that E_t be determined from a stress-strain curve. Although an added advantage exists in having residual stresses due to fabrication incorporated into the definition of E_t, such information may not be readily available.

Instability equations given in Section 4 accurately predict buckling stresses (i.e., F_{iej} and F_{icj}) in the material elastic zone. By reviewing test data in the material elasto-plastic region for uniaxial compression (i.e., either axial compression or hoop compression) an empirical relationship was derived, requiring the knowledge of only the elastic instability stress and the material yield stress. This formulation, Equation C5-3 compares quite well with other recommended plasticity reduction factor formulations and is illustrated on Figure C5-1.

$$\eta = \left(F_y / F_{iej}\right)\left[1.0 / \left\{1.0 + 3.75\left(F_y / F_{iej}\right)^2\right\}\right]^{1/4} \qquad \text{(C5-3)}$$

Applicable interaction relationships (See Section 6) that define behavior of a cylindrical shell due to combined loading typically define material elastic behavior. Few theoretical studies exist that define behavior of cylindrical shells in the material elasto-plastic zone when subjected to combined loading. Further discussion is provided in Section C6.

Figure C5-1 provides a comparison of several plasticity reduction factor equations.

C6 Predicted Shell Buckling Stresses for Combined Loads

The test data documented in Reference C07 indicate that the buckling stresses for cylinders subjected to bending are approximately the same as for cylinders under axial compression for R/t values greater than 150.

C6.1 Axial Tension, Bending and Hoop Compression

The theoretical elastic buckling equation for a cylinder subjected to combinations of axial tension and hoop compression indicates that it is safe to assume no interaction for elastic buckling (See Figure 11-22 of Ref. C04). However, Ref. C42 shows that interaction must be considered for buckling stresses in the elastic as well as the inelastic range. Equation 6.2-1 was shown to be a lower bound on test data for buckling stresses not limited by the stress intensity.

The stress intensity was found to be limited by the Hencky-von Mises distortion energy theory for all but a few tests. However, the more conservative maximum shear stress theory given by Equation 6.2-2 is recommended for design.

The failure stresses are the lower of the values determined from Equations 6.2-1 and 6.2-2. Test data is compared with the interaction curves in Figures C6.1-1 and C6.1-2. Also shown are the Hencky-von Mises curve labeled μ= 0.5 and a modification labeled μ = 0.75 which has merit as an alternate to Equation 6.2-2. The curve labeled API is the interaction curve given in API RP 2A (1) and is the same as Equation 6.3-1 with $C = 1.5, F_{xcj} = F_y$, and

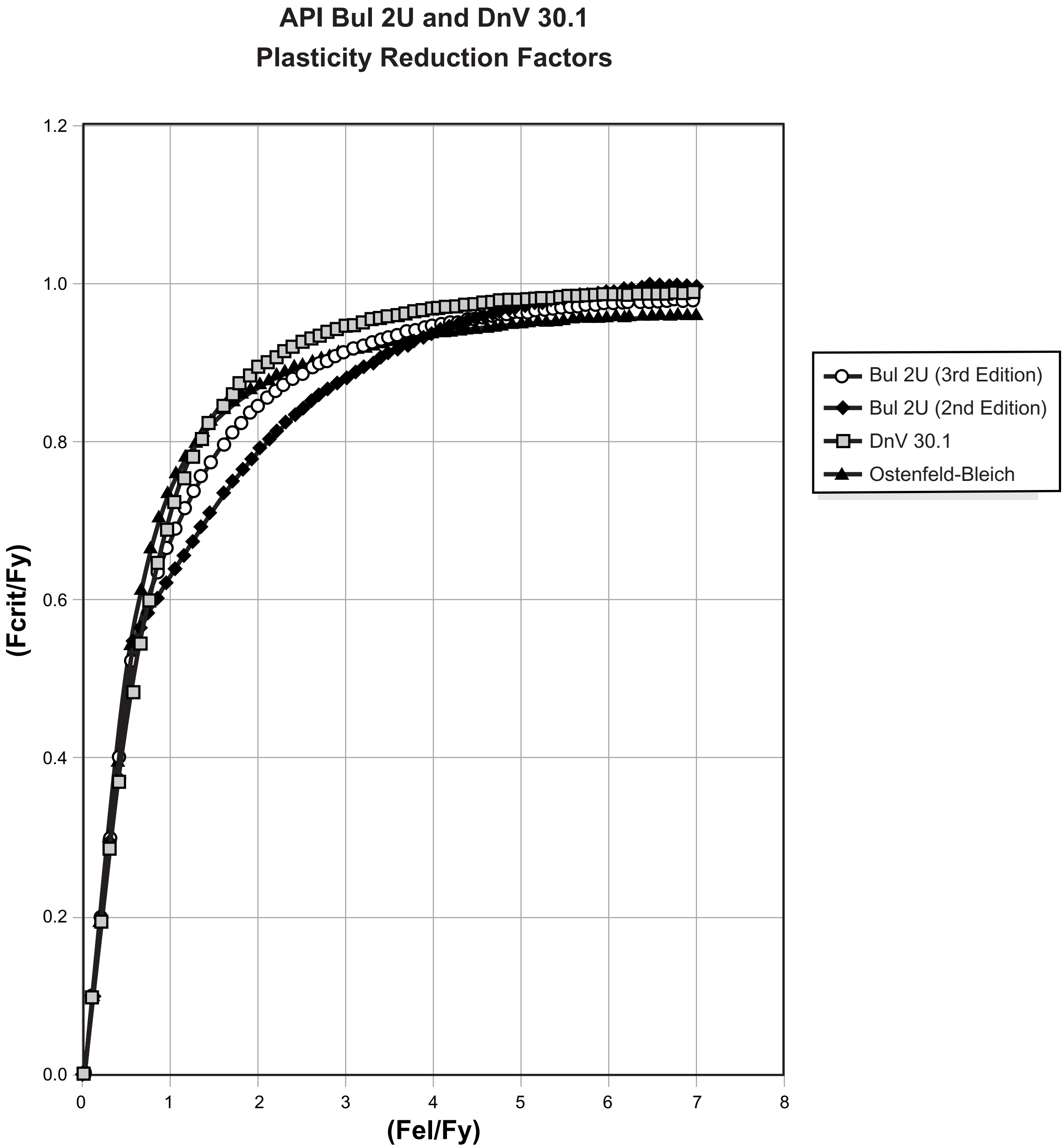


Figure C.5-1--Comparison of Plasticity Reduction Factor Equations

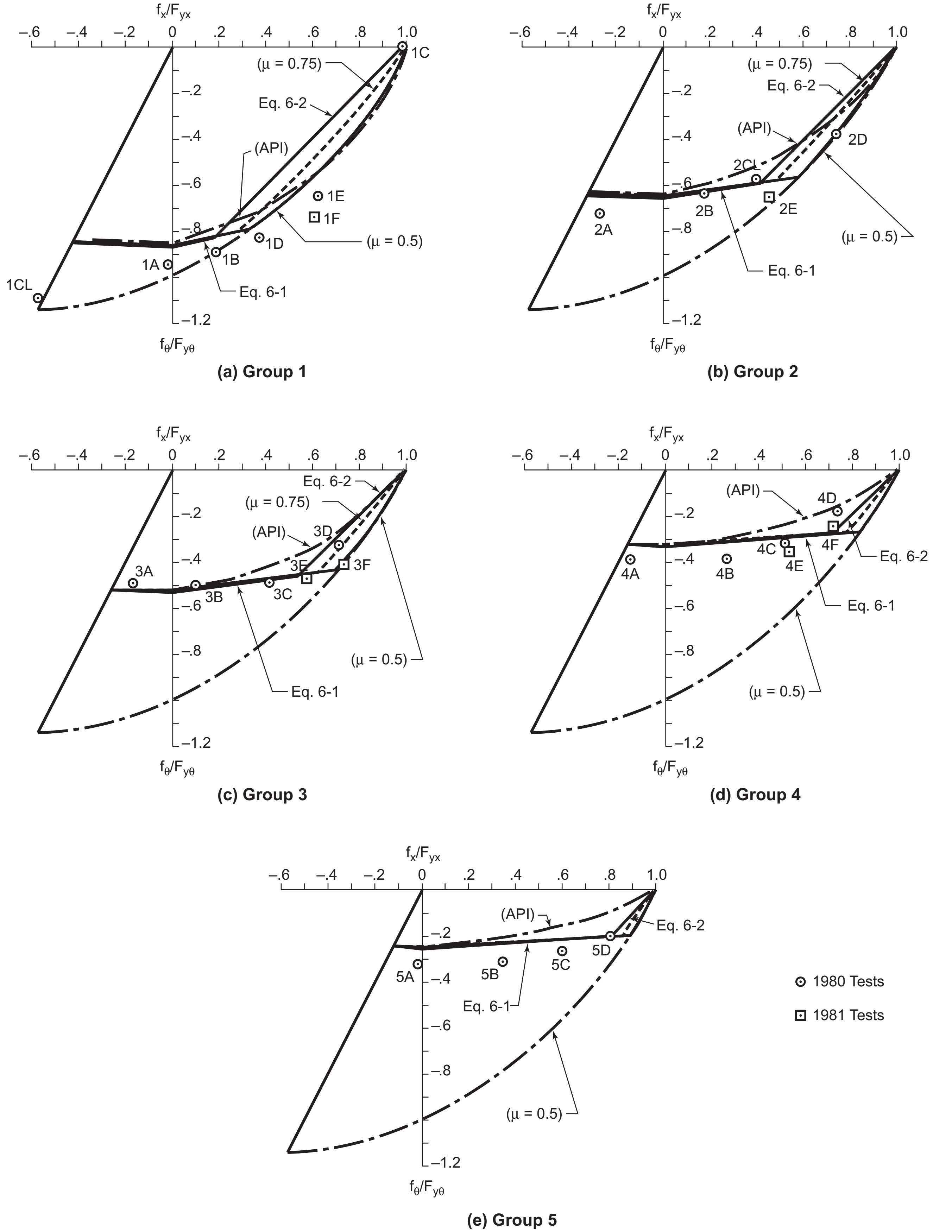


Figure C.6.1-1—Comparison of Test Data from Fabricated Cylinders Under Combined Axial Tension and Hoop Compression with Interaction Curves (F_y = 36 ksi)

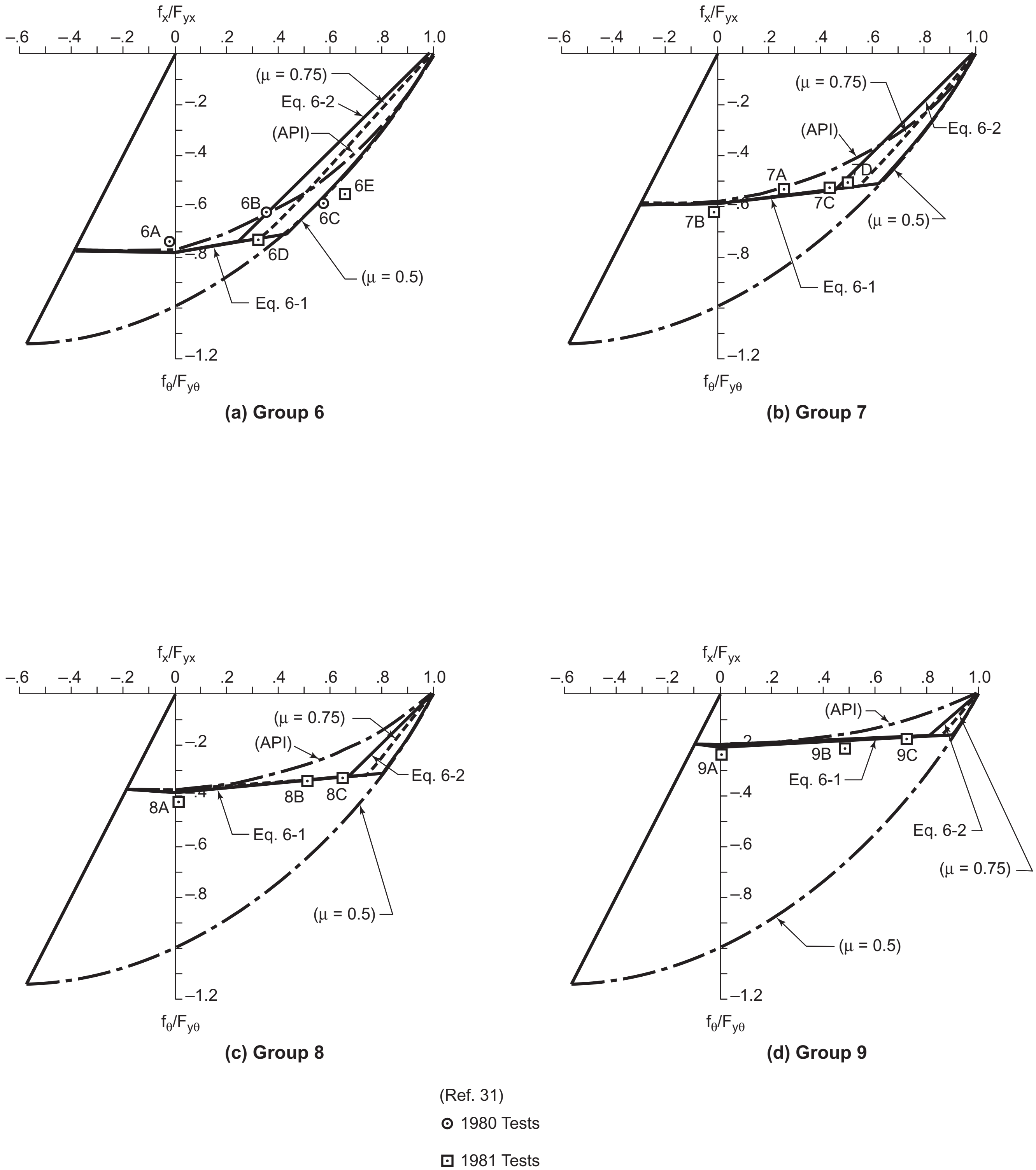


Figure C.6.1-2—Comparison of Test Data from Fabricated Cylinders Under Combined Axial Tension and Hoop Compression with Interaction Curves (F_y = 50 ksi)

$F_{rcj} = F_{hcj}$. This equation is less conservative than Hencky-von Mises for cylinders with values of F_{hcj} approaching F_y and values of f_x exceeding 0.5 F_y as shown in Fig. C6.1-1(a).

C6.2 Axial Compression, Bending and Hoop Compression

Probably the greatest differences in the various recommended rules for shell buckling are the interaction equations for cylinders subjected to combinations of axial compression and external pressure. Seven different recommendations are discussed in Ref. C43. Equation 6.3-1 is based upon a method proposed by Miller and Grove (Ref. C44).

The interaction equation for combinations of axial compression and hoop compression is a modification of the Hencky-von Mises failure theory. Equation 6.3-1 is identical to this theory when c = 1.0. Test data was found to conform quite closely to the interaction curves obtained by varying the value for c. The value of c was found to vary with F_{xcj} / F_y and F_{rcj} / F_y where F_{xcj} is the failure stress for axial compression only and F_{rcj} is the failure stress for hoop compression only. When both F_{xcj} and F_{rcj} equal F_y, c = 1.0. The values for c decrease with decreasing values of F_{xcj} and F_{rcj} and Equation 6.3-1 becomes a straight line for c = -2.0. The values for c were found to be less for stringer stiffened cylinders than for unstiffened and ring stiffened cylinders.

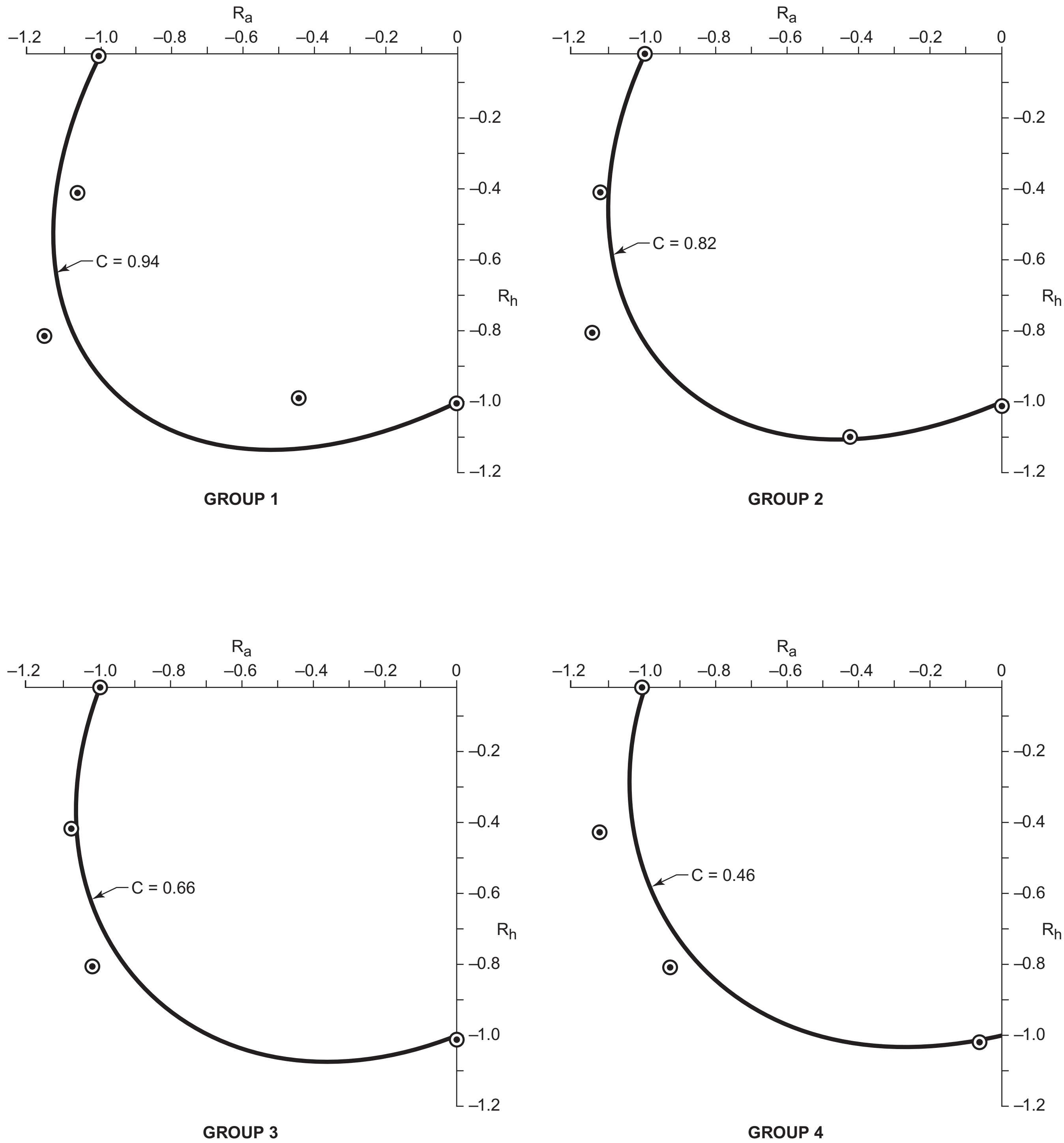
The equation for c for ring stiffened and unstiffened cylinders is given by Equation 6.3-2. This equation is similar to the equation in Ref. C44. Equations 6.3-3 and 6.3-4 were determined from test data for stringer stiffened cylinders. Comparisons of Equation 6-3 with test data are shown in Figures C6.2-1 to C6.2-7.

Comparisons of Equation 6.3-1 with test data are shown on Figures C6.2-1 through C6.2-7 to validate the interaction relationship. Further comparative assessment is provided to underscore substantial scatter in test data, the level of conservatism of predicted instability stresses and compatibility of API's interaction relationship with that of test data for a range of geometric configurations.

C6.2.1 Ring-Stiffened Cylindrical Shells

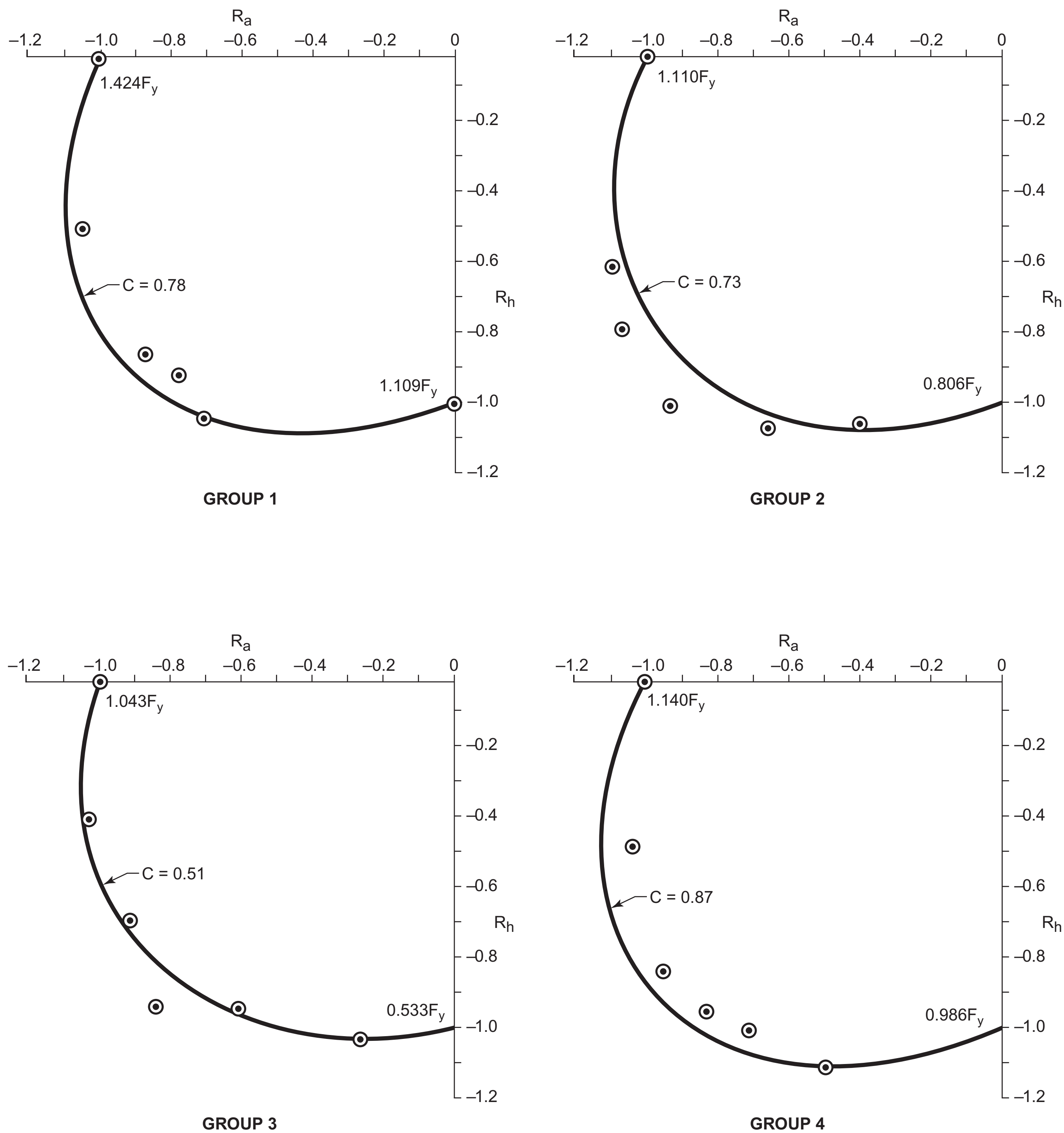
Figure C6.2-8 provides a good comparison of predicted local buckling stress and test data for a series of ring-stiffened cylindrical shells with the nominal D/t and L_r /t ratios of (300,30), (300,60), (600,30), respectively. API-predicted buckling stresses match very well with test data fro axial compression. API-predicted hoop buckling stresses are smaller than the test data and the difference is large with an increase in D/t and L_r /t ratios. It should be noted that:

> (a1) the test data for shell plate are computed based on axial load at failure and external pressure at failure and the use of FEA-validated analytical equations in Section 11.



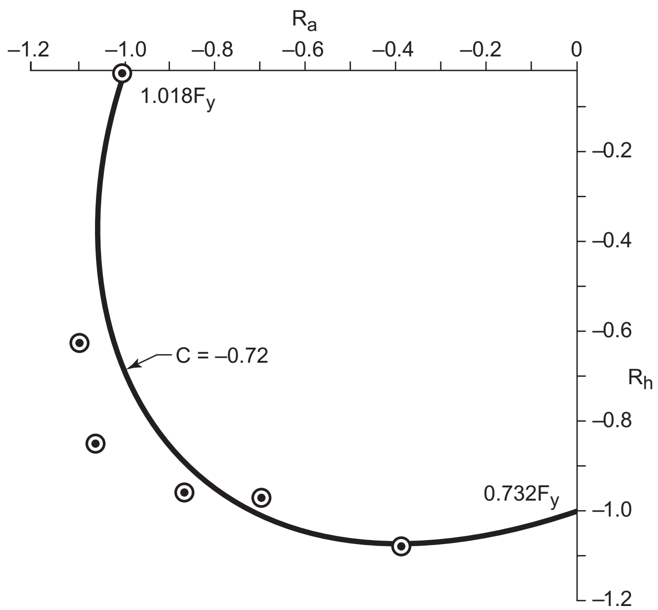
(Data Ref. 5)

Figure C.6.2-1--Comparison of Test Data with Interaction Equation for Unstiffened Cylinders Under Combined Axial Compression and Hoop Compression



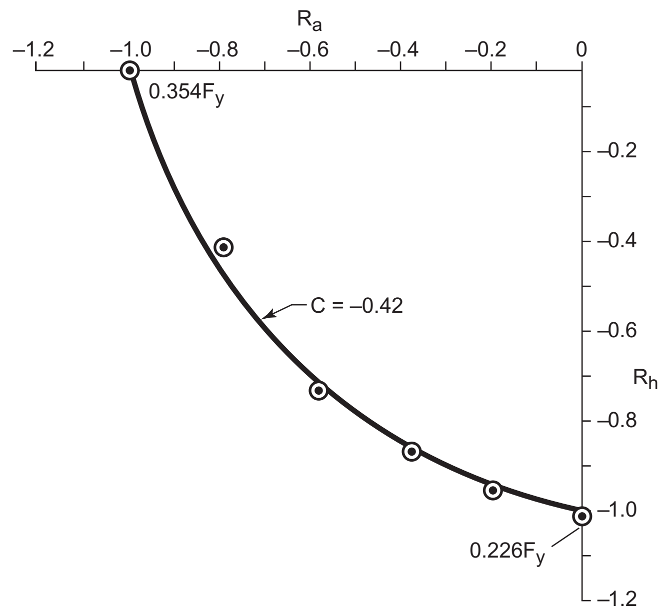
(Data Ref. 5)

Figure C.6.2-2--Comparison of Test Data with Interaction Equation for Ring Stiffened Cylinders Under Combined Axial Compression and Hoop Compression



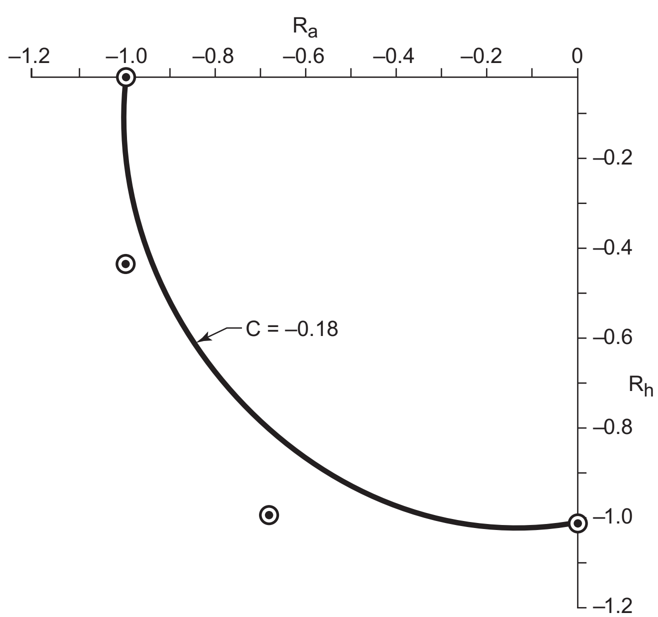
GROUP 5

(Ref. 5)



GROUP 6

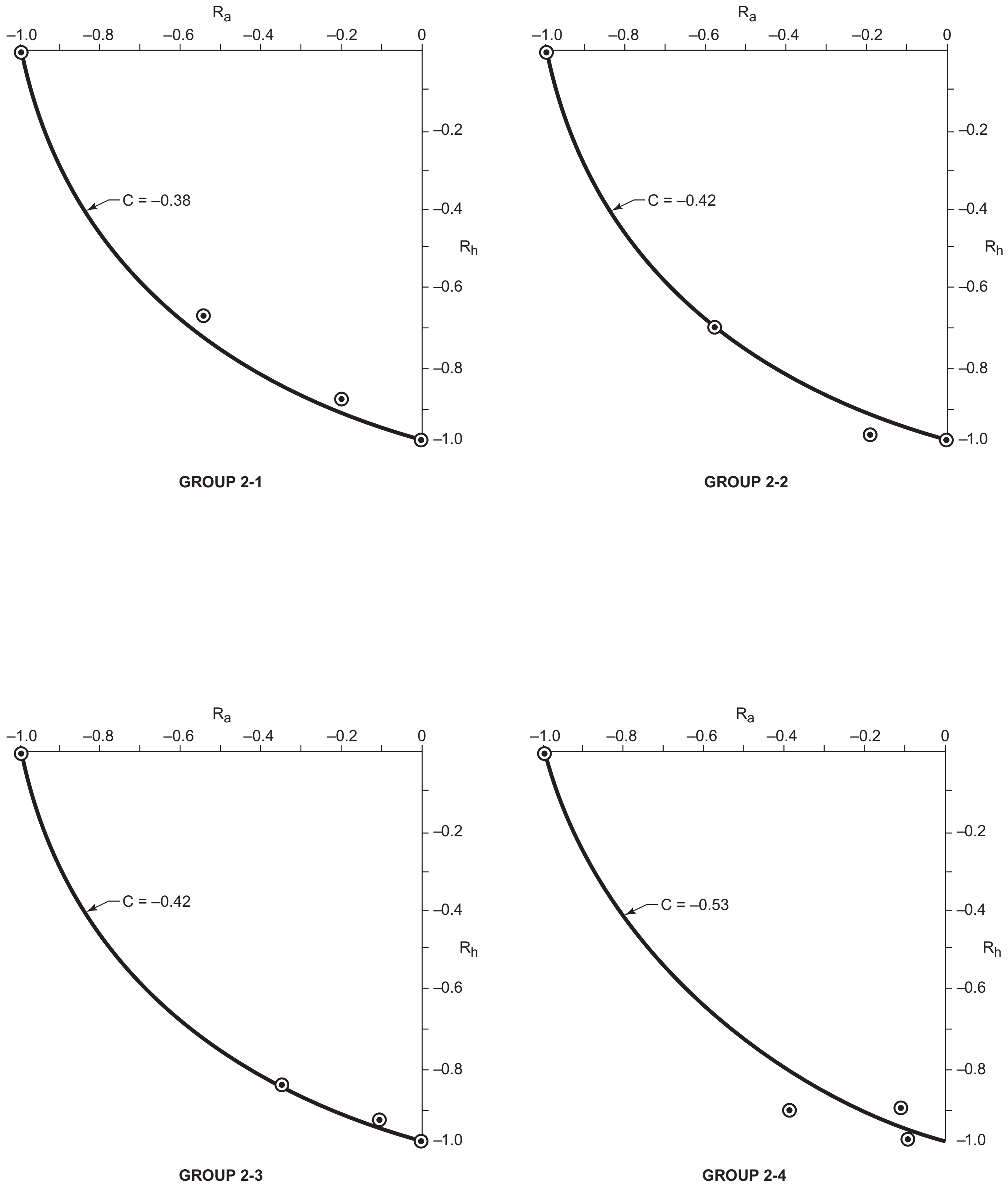
(Ref. 50)



GROUP 1-4

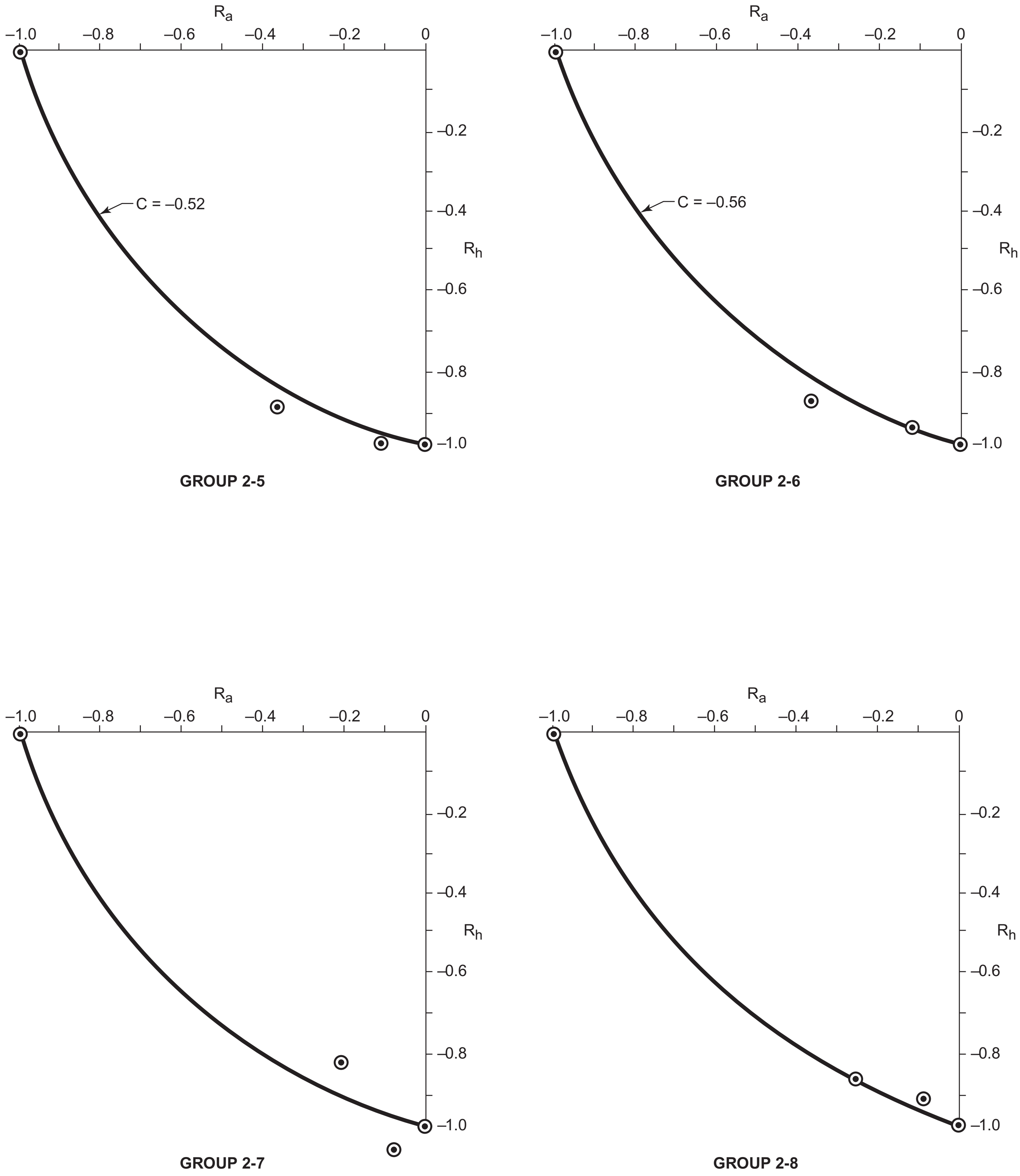
(Ref. 6)

Figure C.6.2-3--Comparison of Test Data with Interaction Equation for Ring Stiffened Cylinders Under Combined Axial Compression and Hoop Compression



(Data Ref. 6)

Figure C.6.2-4--Comparison of Test Data with Interaction Equation for Local Buckling of Ring and Stringer Stiffened Cylinders Under Combined Axial Compression and Hoop Compression



(Data Ref. 6)

Figure C.6.2-5--Comparison Test Data with Interaction Equation for Local Buckling of Ring and Stringer Stiffened Cylinders Under Combined Axial Compression and Hoop Compression

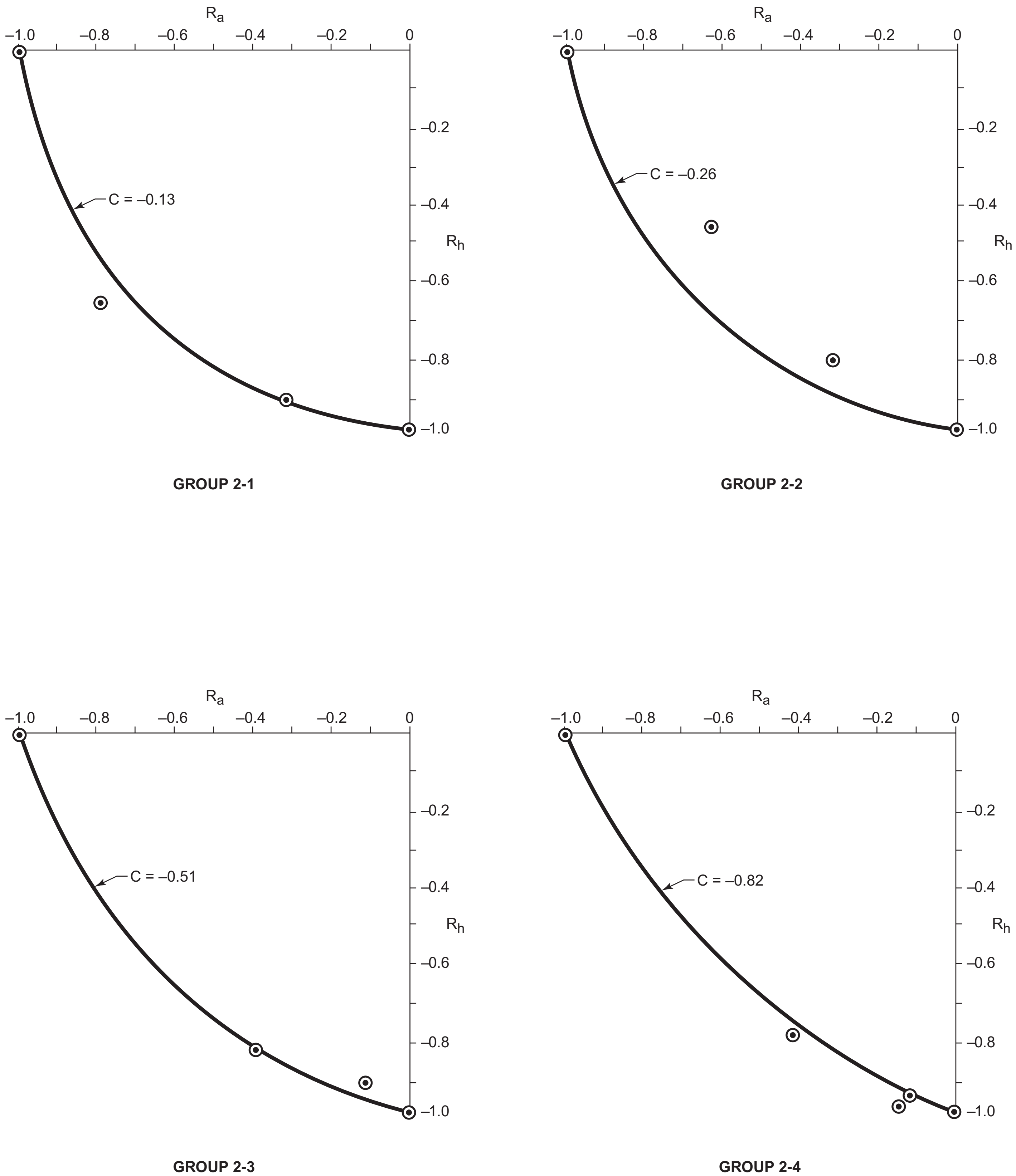


Figure C.6.2-6--Comparison of Test Data with Interaction Equation for Bay Instability of Ring and Stringer Stiffened Cylinders Under Combined Axial Compression and Hoop Compression

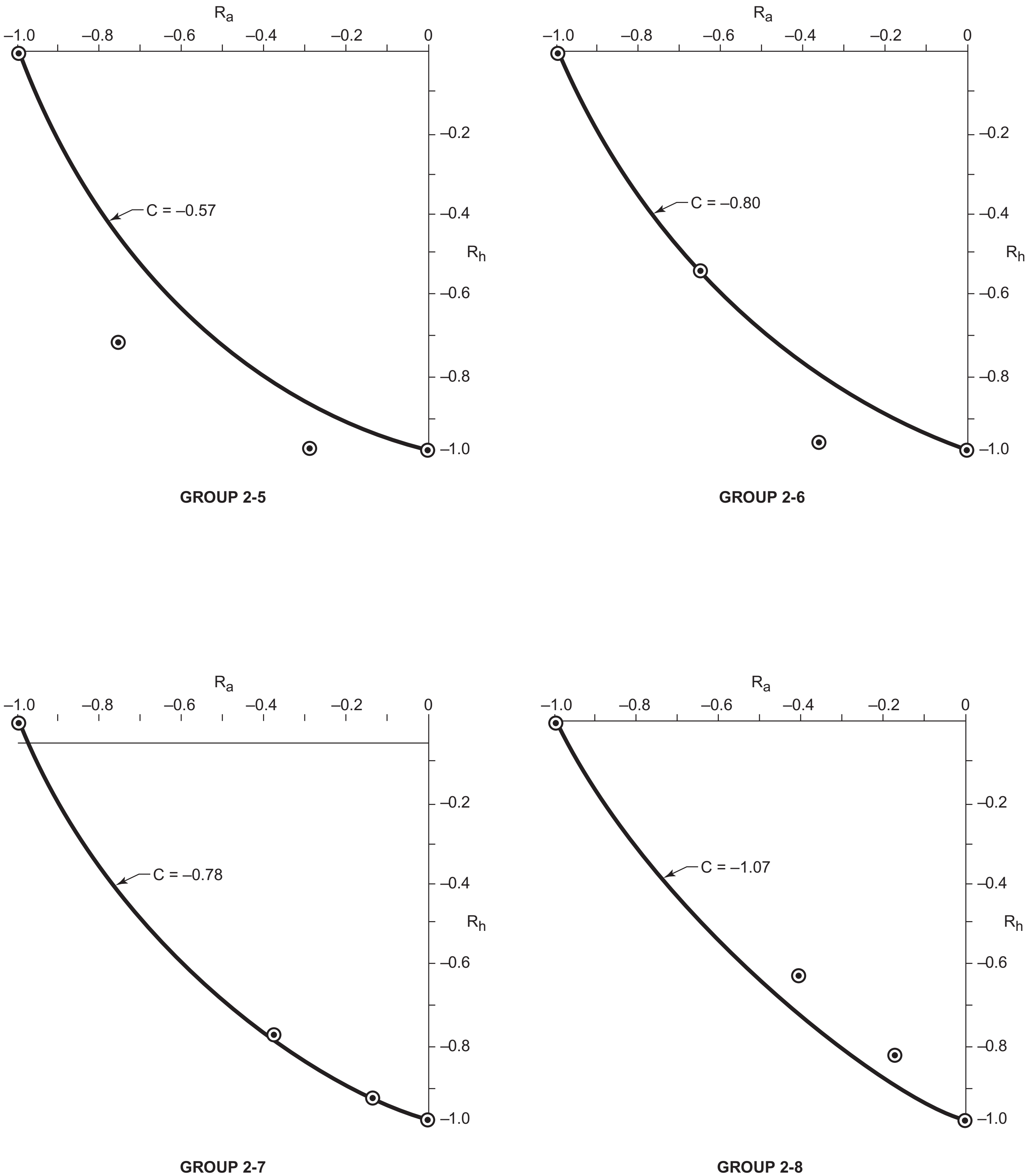


Figure C.6.2-7--Comparison of Test Data with Interaction Equation for Bay Instability of Ring and Stringer Stiffened Cylinders Under Combined Axial Compression and Hoop Compression

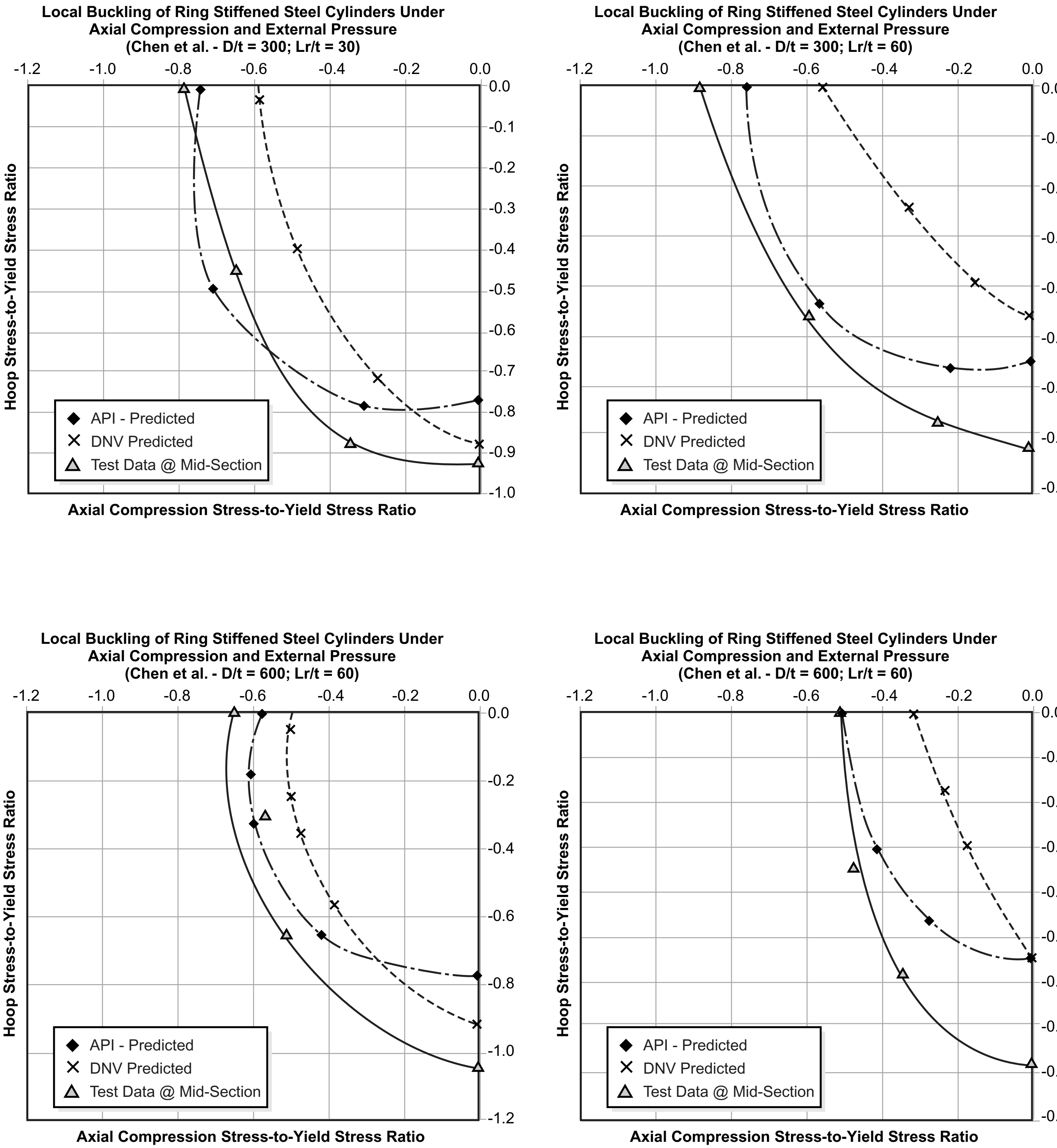


Figure C.6.2-8--Local Instability of Ring Stiffened Cylindrical Shells Subject to Combined Loading--
Four Series by Chen et al for
D/t & L_r/t at 300 & 30, 300 & 60, 600 & 30, and 600 & 60

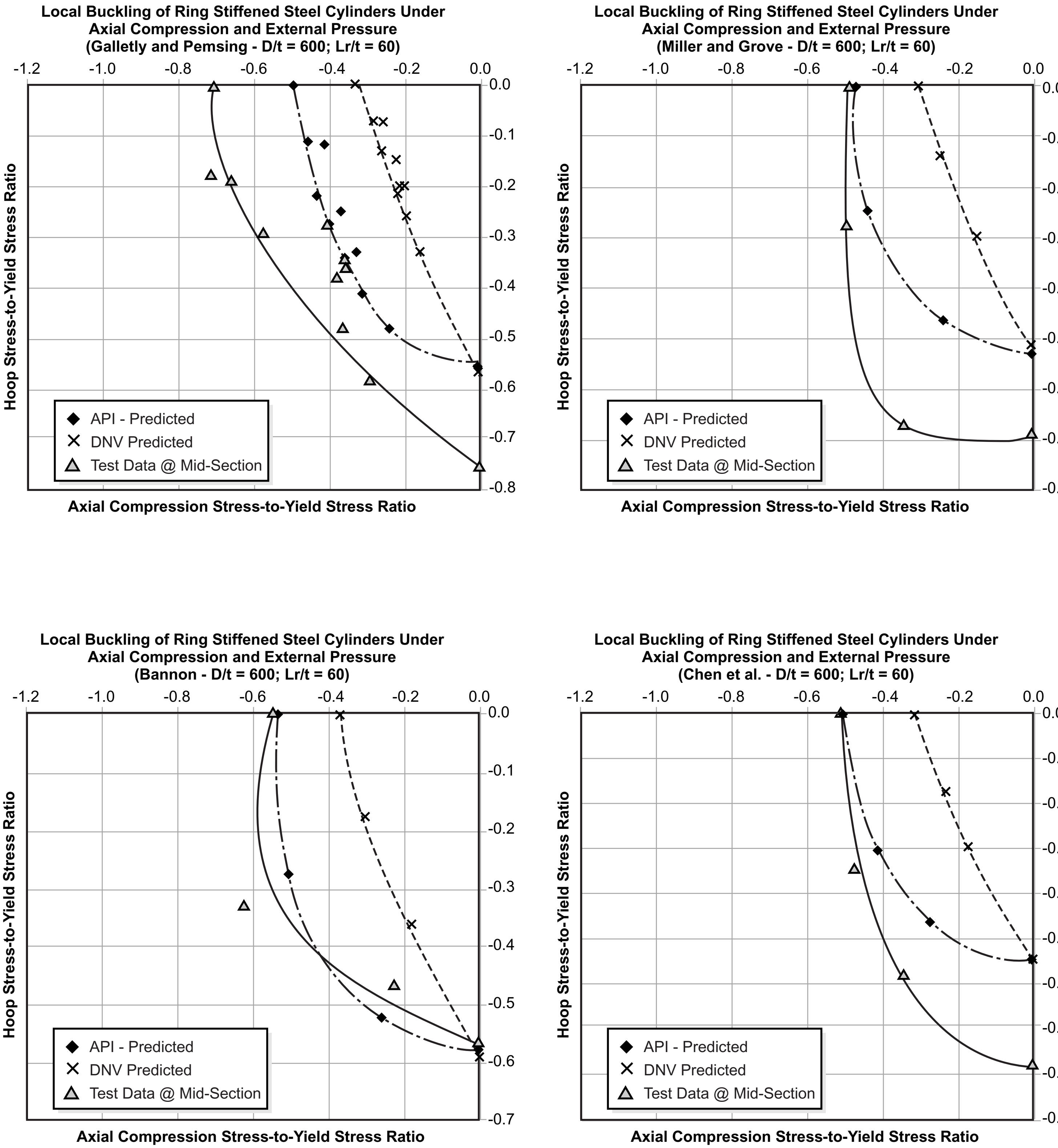


Figure C.6.2-9--Local Instability of Ring Stiffened Cylindrical Shells Subject to Combined Loading--
Four Series for D/t & L_r/t at 600 &60
from Galletly, Miller, Bannon and Chen

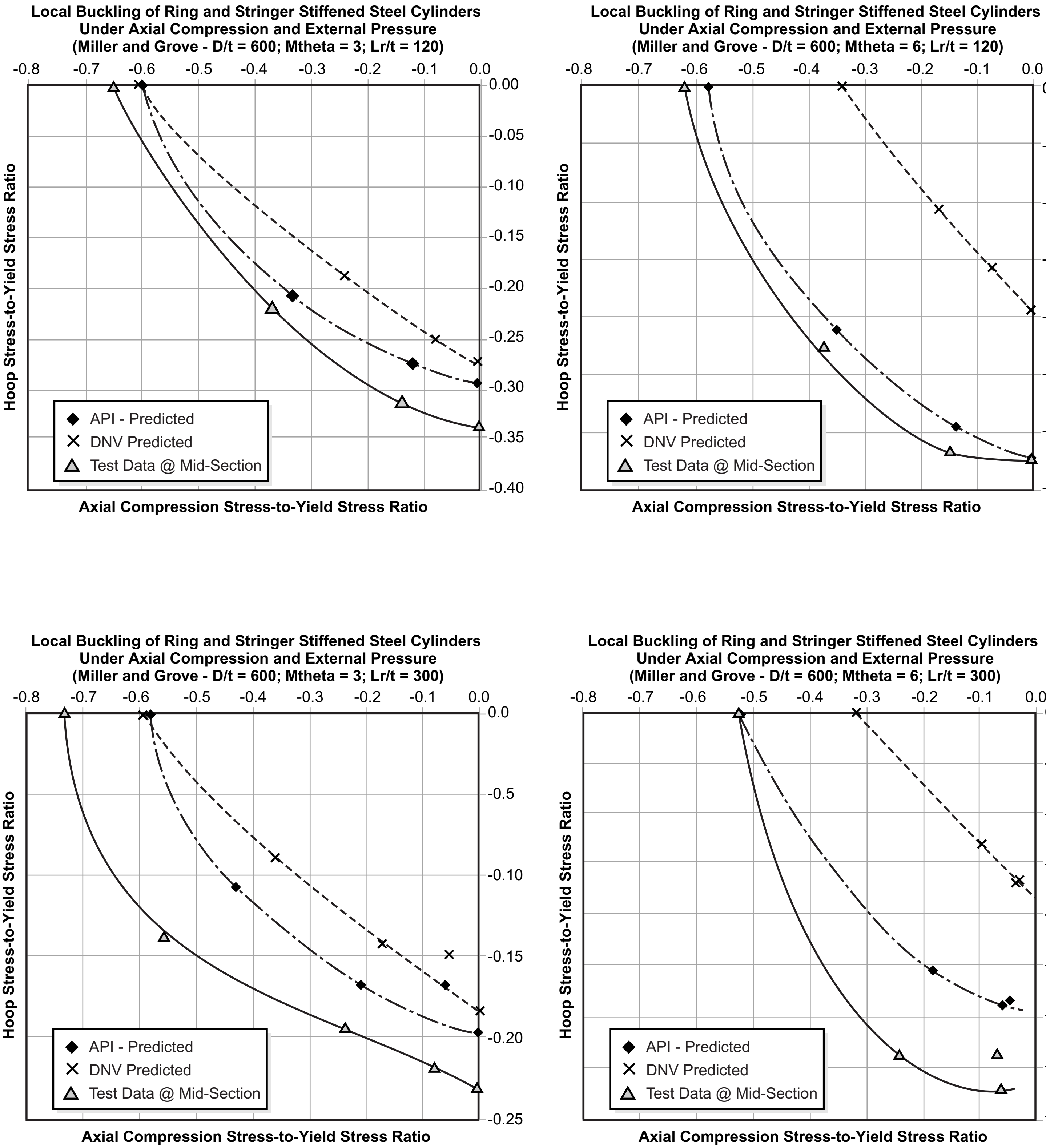
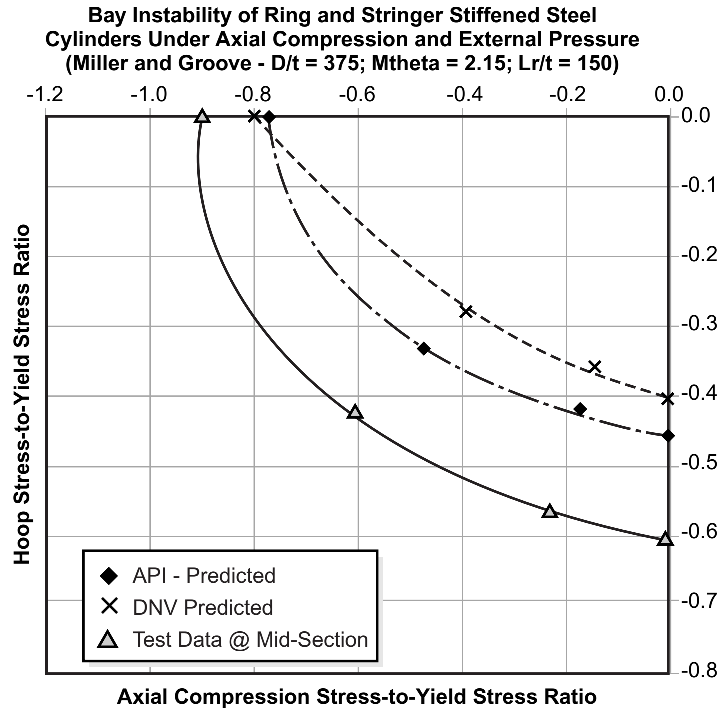


Figure C.6.2-10--Local Instability of Ring- and Stringer-Stiffened Cylindrical Shells Subject to Combined Loading--Four Series for D/t, L_r/t and M_Θ at 600, 120 & 3, 600, 120 & 6, 600, 300 &3, and 600, 300 & 6, respectively



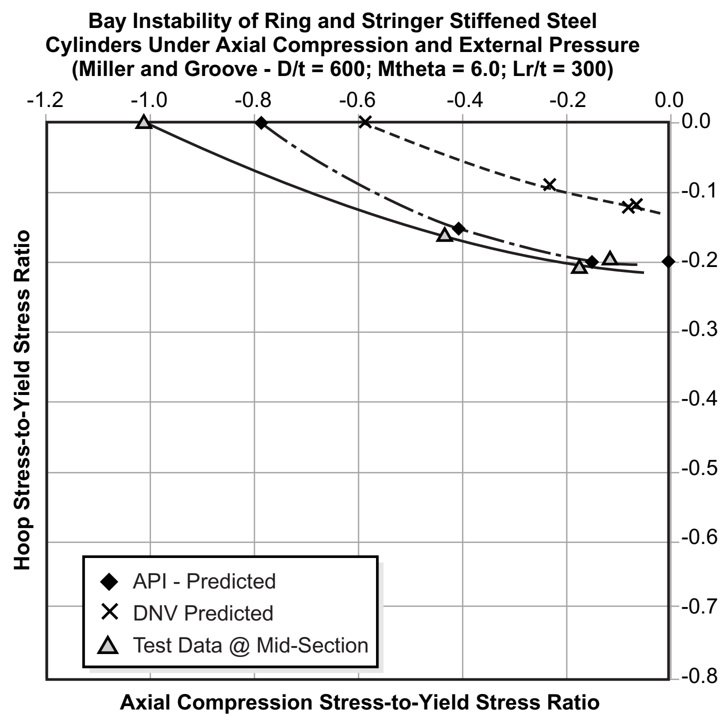
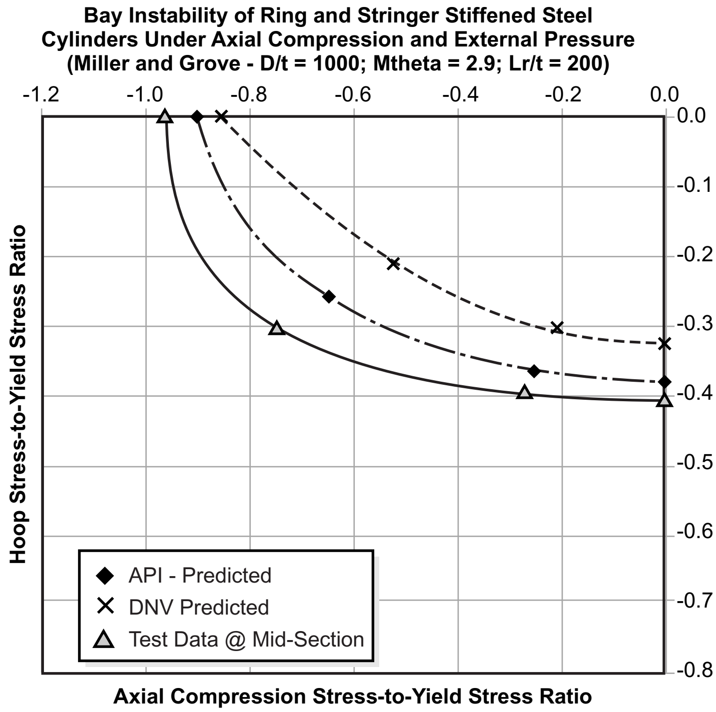
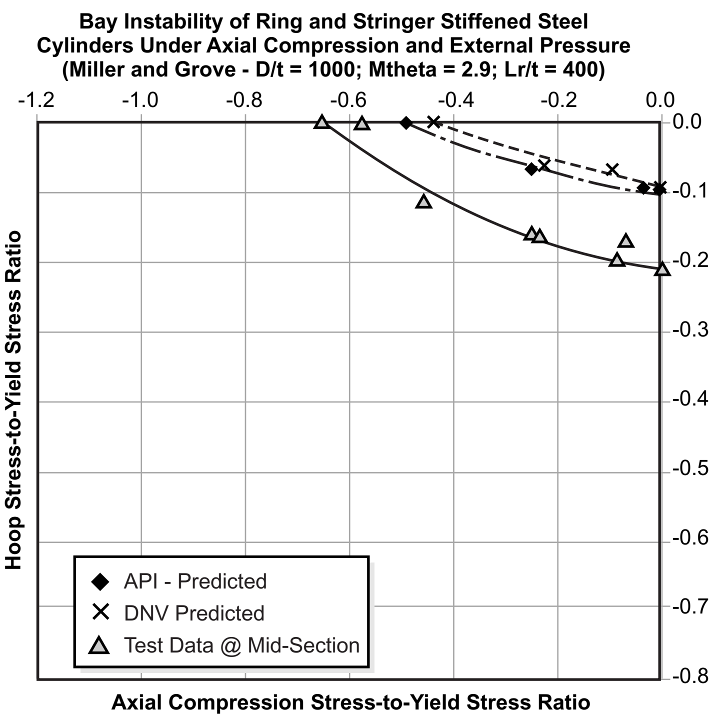


Figure C.6.2-11--Bay Instability of Ring Stiffened Cylindrical Shells Subject to Combined Loading--
For D/t = 375, L_r/t = 150 & M_Θ = 2.15, and
For D/t = 600, L_r/t = 300 & M_Θ = 6.0
From Miller and Grove





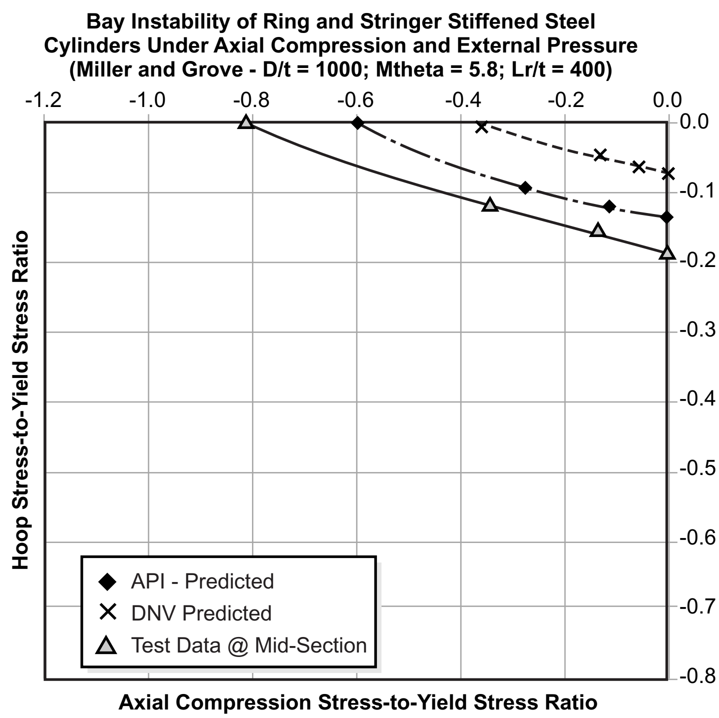


Figure C.6.2-12--Bay Instablility of Ring Stiffened Cylindrical Shells Subject to Combined Loading--
For D/t = 1000, L_r/t = 200 & 400 and M_Θ = 2.9 and 5.8
From Miller and Grove

(a2) the predicted instability stresses are determined through the use of equations in Sections 4, 5, and 6 for each cylindrical shell geometry with slightly different plate thicknesses and yield strengths.

Figure C6.2-9 provides an even better illustration that an equation defining the instability stress as a lower bound curve to test data is acceptable when few data points are available. A more appropriate equation defining the instability stress would be one that perhaps underpredicts 90 percent of test data. Tests conducted/sponsored by Galletley and Pemsing (Ref. C13), Miller and Grove (Ref. C14, Bannon (Ref. C18 and Chen, et. al. (Ref. C11) with ring stiffened cylindrical shells having D/t and L_r/t ratios of 600 and 60, respectively show large differences from one series of tests to another as follows:

(b1) API-predicted axial buckling stress-to-yield stress ratio is about 0.5, with slight differences due to plate thickness and yield stress differences. API-predicted and test-to-yield stress ratios compare very well with published data by Miller (Ref. C14), Bannon (Ref. C18), and Chen (ref. C11. However, Galletley and Pemsing reported test-to-yield stress ratios that are about 40% higher than those predicted by API.

(b2) API-predicted hoop buckling stress-to-yield stress ratio vary from about 0.52 to 0.57 due to slight plate thickeness and yield stress differences. API-predicted and test-to-yield stress ratios compare very well with published data by Bannon (Ref. C18). However, Galletley and Pemsing (Ref.C13), Miller (Ref. C14), and Chen (Ref. C11) reported test-to-yield stress ratios that are 35 to 40% higher than those predicted by API.

C6.2.2 Ring and Stringer Stiffened Cylindrical Shells

a. Local Instability

Figure C6.2-10 provides a good comparison of predicted local instability stresses and test data for a series of ring and stringer stiffened cylindrical shells with the nominal D/t, L_r/t, M_θ of 600, 120 & 3, 600, 120 & 6, 600, 300 & 6, respectively. API-predicted buckling stresses are consistently less than the test data and the predicted and test data exhibit very similar interaction relationship between axial and hoop compression.

It should be noted that the axial instability stresses for some of the tested specimens would have been higher had it not been for the premature failure of stiffener web (i.e., noted as “LS”) or the failure of shell plate together with the stringer (i.e., noted as “BS”).

b. Bay Instability

Figure C6.2-11 provides a good comparison of predicted bay instability stresses and test data for a series of ring-stiffened cylindrical shells with the nominall D/t, L_r/t, and M_θ of 375, 150 & 2.15 and 600, 300 & 6, respectively. API-predicted buckling st6resses are consistently less than the test data and the predicted and test data exhibit very similar interaction relationship between axial and hoop compression.

Figure C6.2-12 provides a good comparison of predicted by instability stresses and test data for three separate series of ring-stiffened cylindrical shells with the nominal D/t, L_r/t, and M_θ of: (1) 1000, 200, and 2.9, (2) 1000, 400, and 2.9, (3) 1000, 400, and 5.8, respectively. API-predicted instability stresses are very close to the test data and exhibit similar interaction relationship at L_r/t, and M_θ of 200 and 2.9, respectively. When the ring spacing is increased by a factor of two (i.e., L_r/t increased from 200 to 400), API-predicted instability stresses are substantially smaller than the test data due to relative conservativeness of orthotropic theory for lightly stiffened cylindrical shells.

When the number of stringers are reduced by a factor two (i.e., L_r/t, and M_θ of 400 and 5.8), API-predicted instability stresses in axial direction remain unchanged by the substantial improvement in the hoop direction. The reason for this is that the number of stringers are less than three times the number of lobes and the orthotropic method is no longer applicable, therefore the predicted hoop stress is now based on Section 4.5, rather than Section 4.4.

Although none of the predicted instability stresses are less than the test data, it would have been acceptable to occasionally overpredict the instability stresses due to the use of hierarchical order. Thus, to items (a1) and (a2), above, a third comment should be added:

(a3) the predicted instability stresses represent not the design stresses but the true failure stresses. This, even if few of the predicted instability stresses are greater than the test data, the design stresses would most likely be smaller than the test data.

C7 Stiffener Requirements

C7.1 Hierarchy Checks

A factor β has been added to the shell buckling stress equations to provide a convenient method for separating the local buckling mode from the bay and general instability modes of failure. The factor is applied to the failure strain rather than the failure stress. For elastic buckling the design shell buckling stresses are inversely proportional to β whereas for inelastic buckling the ratio of $F_{icj} / \overline{F}_{icj}$ is less than β.

C7.2 Local Stiffener Buckling

The recommended buckling criteria require that the stiffeners be adequately proportioned so that local instability of the stiffeners is prevented. The requirements of Equations 7.2-1 and 7.2-2 will preclude this mode of failure for the most generally used stiffener configurations. These requirements are the same as those specified by AISI (Ref. C35) for fully effective sections. For other configurations, the AISI or other guidelines must be consulted. Equation 7.2-1 was determined from AISI Equation 3.2-1 and Equation 7.2-2 from AISI Equation 2.3.1-1.

The buckling stresses of bar stiffeners which do not meet the compact section requirements are assumed to be 1.67 times the allowable stresses given by AISI Equation 3.2-2 which follows:

$$F_{xc} = (1.28 - 0.75\lambda_s)F_y \text{ for } 0.375 < \lambda_s < 0.846 \tag{C7.2-1}$$

$$\lambda_s = \frac{h_s}{t_s}\sqrt{\frac{F_y}{E}}$$

When stringer stiffeners are noncompact sections the failure stress from Equation C7.2-1 should be substituted for the yield stress when determining the local shell buckling stresses for axial compression or bending and the bay instability stresses for all load conditions. When ring stiffeners are noncompact sections the failure stress from Equation C7.2-1 should be substituted for the yield stress when determining the general instability stress for all load conditions.

C7.3 Stiffener Arrangement and Sizes

An optimum design provides a natural hierarchical order of failure modes, minimizes steel requirements and simplifies fabrication. Ring spacing and shell thickness are primarily controlled by external pressure and the stringer spacing and size are primarily controlled by axial and bending loads.

The following general procedure may be used to meet both the design safety factors and the hierarchical failure mode requirements:

1. Determine a shell thickness and a ring spacing that would yield a reasonable applied hoop stress in the shell plate.
2. Determine the local instability stress, $F_{\theta cL}$, divide it by the applied shell hoop stress and if the ratio is less than the required safety factor, increase the shell plate thickness until a desirable safety factor is achieved (i.e., local instability SF check in circumferential direction).
3. Determine the general instability stress, $F_{\theta eG}$, divide it by the applied hoop stress at the ring and if the ratio is less than the required safety factor, change the ring spacing or size until a desirable safety factor is achieved (i.e., general instability check in circumferential direction).
4. Divide the general instability stress, $F_{\theta eG}$, by a β factor, and apply a plasticity reduction factor. If the obtained $F_{\theta cG}$ is not equal to or greater than $F_{\theta cL}$, revise ring spacing or ring size to meet the requirements (i.e., general instability hierarchy check).
5. For the selected shell thickness, determine an appropriate stringer spacing and size that would yield a reasonable applied axial shell stress.

6. Determine the local shell instability stress, $F_{\phi cL}$, divide it by the applied axial shell stress and if the ratio is less than the required safety factor, increase the number of stringers or the stringer size until the desirable safety factor is achieved (i.e., local instability check in axial direction).
7. Determine the bay instability stress, $F_{\phi cB}$, divide it by the applied axial stringer stress and if the ratio is less than the required safety factor, revise the number of stringers or the stringer size until a desirable safety factor is achieved)i.e., bay instability check in axial direction).
8. Divide the bay instability stress, $F_{\phi cB}$, by a β factor, and apply a plasticity reduction factor. If the obtained $F_{\phi cB}$ is not equal to or greater that $F_{\phi cL}$, revise stringer spacing or size to meet the requirements (i.e., bay instability hierarchy check).
9. Perform an interaction check for combined loads. Utilization ratios for all instability modes should be less than 1.0. Repeat the appropriate steps to ensure that all utilization ratios remain under 1.0.

C8 Column Buckling

Column buckling is not likely to occur in large diameter cylindrical shells as they typically have small slenderness ratios (i.e., *KL/r*). However, tall unsupported columns with high curvatures (i.e., small D/t ratios) need to be checked for column buckling stresses.

The local shell buckling stress based on Section 4.1.1 [Equation 4.1-1] and Section 5 [Equation 5-1] and [s-2] should be substituted for material yield strength in determining the column buckling stress. The AISC (Ref. C45) and AISI (Ref. C35) specifications use CRC Column-Strength Curve which can be modified to account for column and shell buckling interaction by substituting the shell buckling stress, $F_{\phi cj}$, for the yield stress. The buckling stress, $F_{\phi cC}$ is given by:

$$F_{\phi cC} = \left(1 - \frac{F_{\phi cj}}{4F_{\phi ec}}\right)F_{\phi cj} \qquad \text{for } \frac{F_{\phi eC}}{F_{\phi cj}} > 2.0 \tag{C8-1}$$

$$F_{\phi cC} = F_{\phi eC} \qquad \text{for } \frac{F_{\phi eC}}{F_{\phi cj}} \le 2.0 \tag{C8-2}$$

The shell buckling stress $F_{\phi cj}$ should be taken as the lowest stress for all possible modes of failure. This will always be the local shell buckling stress when the hierarchy requirements are met. The local shell buckling stress based on Section 4.3.1 should not be substituted for material yield strength as local shell buckling can not initiate column buckling of a cylindrical shell with longitudinal (i.e., stringers) stiffening. The elastic column buckling stress, $F_{\phi eC}$, is given by the following equation:

$$F_{\phi eC} = \frac{\pi^2 E}{(KL_t / r)^2} \tag{C8-3}$$

where KL_t is the effective column length and r is the radius of gyration.

The test results on fabricated tubular columns from Refs. C46 and C47 were compared with Equation C8-1 in Ref. C48 and several test points were found to be less than the predicted value. Equation 8.2-1 of the Bulletin which was taken from Ref. C48 is based upon a lower bound of test data. The comparison of test data with Equation 8-2 is shown in Figure C8-1. There is no data from fabricated cylinders in the elastic region. A reduction factor of 0.87 is assumed. The AISC Specification (Ref. C45) gives an allowable stress of 0.522 times the Euler buckling stress. The factor 0.87 is equal to 1.67 x 0.522.

The differences between test results and the predicted values when $F_{\phi cC}$ is determined from Equation C8-1 are partially compensated for in the AISC specification by using a variable factor of safety whereas a constant factor of safety can be used with Equation 8.2-1. Also Equation 8.2-1 reflects that no reduction in buckling stress occurs due to overall length for short columns ($KL_t / r < 0.5\sqrt{E / F_{\phi cj}}$). Comparisons of the column buckling curves given by the Bulletin, CRC Column-Strength Curve, and AISC are shown in Figure C8-2. The buckling stress curve for AISC is equal to $1.667F_a$ where F_a is the allowable stress.

C9 Allowable Stresses

The allowable stresses for axial compression and bending are assumed to be equal for the shell buckling modes of failure. Equations 9.1-1 through 9.1-6 are obtained by applying factors of safety to the failure stresses given by the equations in Sections 4 and 6.

The allowable stress equations for the column buckling mode for members subjected to axial compression and bending stresses are the same as given in AISC (Ref. C45). Equations 9.2-1, 9.2-2 and 9.2-3 are simpler in form than the AISC equations because the properties of tubular members are identical in the X and Y directions. When external pressure is combined with axial compression and bending the stresses for F_a and F_b are determined from the shell buckling interaction equations.

C10 Tolerances

The tolerances for out-of-roundness are from the ASME Pressure Vessel Code (Ref. C49) and the requirement for straightness is from the ECCS rules (Ref. C38).

C10.1 Maximum Differences in Cross-Section Diameters

The equation for maximum differences provides a shell that appears reasonably round to the eye. One exception is for a shape conforming to n = 3. Provision is made for this case by the second paragraph of Section 10.3.

C10.2 Location Deviation from Straight Line Along a Meridian

The reference length $L_x = 4\sqrt{Rt}$ is related to the size of the potential buckles. There are no published papers which show a correlation between measured values of ex and the reduction in axial strength. In Ref. C38 when $e_x = 0.02L_x$ the values of α_{xL} are halved. When the ratio

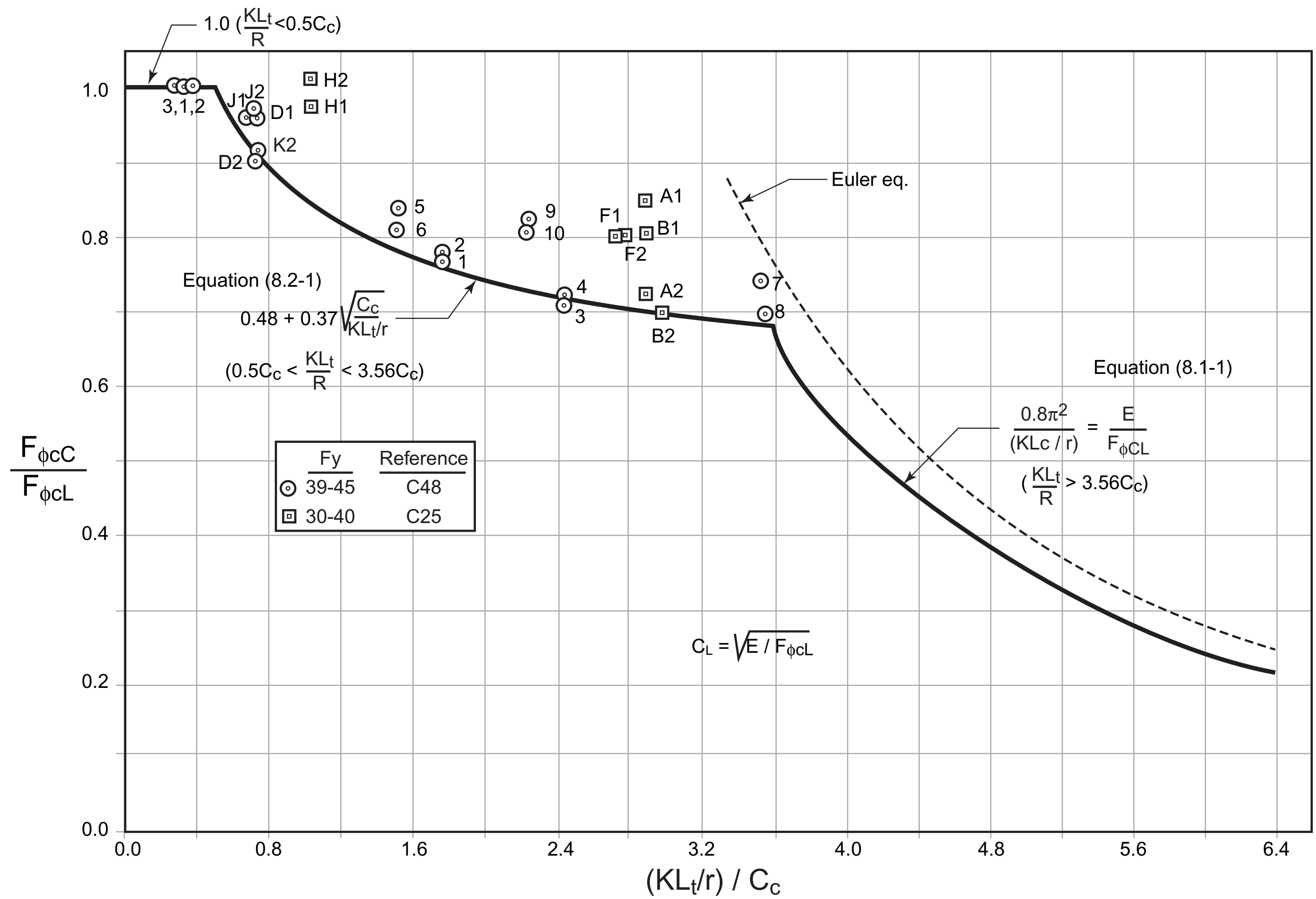


Figure C.8-1--Axial Compression of Fabricated Cylinders--Column Buckling

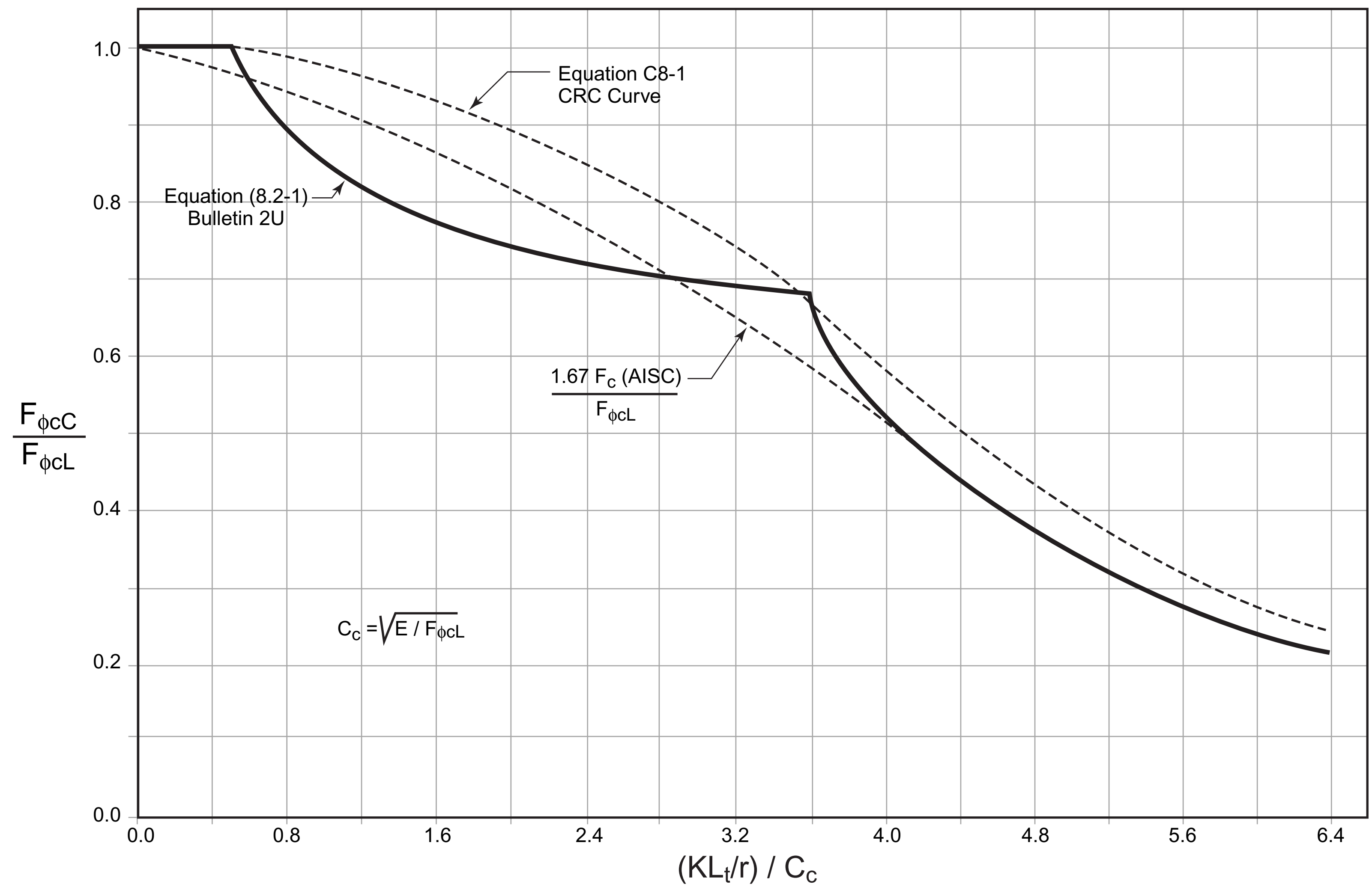


Figure C.8-2--Comparison of Column Buckling Equations is between 0.01 L_x and 0.02 L_x, linear interpolation between α_{xL} and $0.5\alpha_{xL}$ is recommended for the reduction factor.

C10.3 Local Deviation from True Circle

Figures 10.3-1 and 10.3-2 are based upon the following equation developed by Windenburg (Ref. C50).

$$\frac{e}{t} = \frac{0.018D/t}{n} + 0.015n \qquad \text{(C10.3-1)}$$

This equation is based upon the analogy between a pressure vessel and a column by considering the shell of the pressure vessel to be made up of a series of columns with length of one-half wave length. The eccentricity of a column corresponds to the out-of-roundness of a cylinder. The constants in Equation C10.3-1 were derived from available test data to provide tolerance limits which would reduce the collapsing strength by a maximum of 20% ($\alpha_{\theta L}$ = 0.8).

The value of n is the number of waves in the cylinder at collapse and e is the allowable deviation measured over one half wave length. The values of n were determined from the equation for hydrostatic pressure developed by Sturm (Ref. C23). Identical values for n can be determined from Equation 4.1-6. Noninteger values are selected for n and n corresponds to the lowest buckling pressure for the assumed geometry. The arc length in Figure 10.3-2 is given by Arc = $\pi D/2n$.

C10.4 Plate Stiffeners

The permissible lateral deviation of the free edge of a plate stiffener corresponds to the fabrication tolerances specified for the models reported in Ref. C51.

The value of n given by Equation 10.4-1 is based upon the assumption that the stiffening ring will buckle into one-half the number of waves of an unstiffened shell of length L_b. This equation can be safely used in lieu of Equation 4.2-5 because it will always predict a smaller value of n.

C11 Stress Calculations

It is recommended that the applied stresses in the shell and the stiffeners be obtained from an appropriate finite element analysis. The equations given in Section 11 are based on two independently modeled finite element analyses of cylindrical shells (Ref. C52 and C53). Studies conducted covered 50-ft diameter cylindrical shells with D/t ratios of 300,600, and 1200 for ring spacings of 40 and 80 inches. For the ring and stringer stiffened configurations, 36 and 72 stringers were considered.

Test data plotted on Figures in Sections C4 and C6 are based on failure axial loads and external pressure data. These load and pressures were used to compute axial and hoop stresses based on equations recommended in Section 11.

C11.2 Bending Stresses

The bending stress in a cylinder is given by the equation:

$$F_b = \frac{M}{S}$$

where

$$S = \pi R^2 t \frac{1 + 0.25\left(t / R^2\right)}{1 + \left(0.5t\right)/ R} = \pi\left(D_o^4 - D_i^4\right)/ 32D_o$$

C11.3 Hoop Stresses

Previous editions of this bulletin neglected the effect of the longitudinal stiffener on cylindrical shell and ring stiffener hoop stresses. These effects were they indirectly addressed in the equations for imperfection and plasticity reduction factors. This approach is no longer acceptable when the applied stresses are directly obtained from finite element analyses.

a. Ring Stiffened Cylindrical Shells

The magnitude of hoop stress on the cylindrical shell and ring stiffener depends on external pressure and the cylindrical shell configuration, namely the D/t ratio, shell plate thickness, ring spacing and, to a lesser extent, the ring size. In effect, relative rigidity between the shell plate and the ring determines the hoop stress levels in both.

Equations 11.3-2 and 11.3-11 modify the computed shell hoop stress for an unstiffened cylindrical shell with distribution factors, $K_{\theta L}$ and $K_{\theta G}$, for the shell plate (i.e. Local Instability) and the ring stiffener (i.e., General Instability) stress level, respectively. Equations 11.3-3a through 11.3-10a and Equation 11.3-12 quantify the stress distribution factors $K_{\theta L}$ and $K_{\theta G}$. Tables C11.3-1 and C11.3-2 present comparisons of shell plate and ring stiffener hoop stresses for a range of D/t ratios, shell plate thicknesses and ring stiffener spacings. The ratios of FEA-to-predicted hoop stresses are very good for the entire range of configurations considered for both the 2nd and 3rd editions of API Bulletin 2U.

b. Ring and Stringer Stiffened Cylindrical Shells

Cylindrical shells that are ring- and stringer-stiffened differ from ring stiffened cylindrical shells in transmitting some of the external pressure directly to the rings. Thus, existing recommendations that neglect the effect of stringers overpredict shell plate hoop stresses and underpredict ring hoop stresses. While the error can be tolerated in a *D/t* range of 300 to 500, the error is magnified as the *D/t* ratios reach 1200. The stress distribution between the ring and the shell is also affected by the stringer spacing.

1. **Hoop Stress in Shell Midway Between Rings**
   The hoop stress in the shell is determined from Equations 11.3-2 through 11.3-10a. To account for the effect of stringers, k_t, j, β, and D, defined in Equations 11.3-5a, 11.3-8a, 11.3-9a, and 11.3-10a, respectively are revised.

   Table C11.3-1 and the Figures C11.3-1 and C11.3-2 present FEA-to-predicted shell plate hoop stress ratios. The equations in the 3rd edition of Bulletin 2U accurately predict hoop stresses fro the entire range of *D/t* ratios and for reasonable ranges of ring and string stringer spacings.

2. **Hoop Stress in Shell at the Ring**
   The hoop stress in the ring is determined from Equations 11.3-11, 11.3-12b, and 11.3-16, except for the definition of k_t, β, and D. Equations 11.3-5b, 11.3-9b, and 11.3-10b should be used together with Equation 11.3-12b).

   As illustrated on Table C11.3-2 and the Figures C11.3-3 and C11.3-4, FEA-to-predicted ring hoop stress ratios indicate that the equations accurately predict the ring stress when the number of stringers is adequate. These formulations underpredict the ring hoop stress when the number of stringers is small. Thus, for lightly stiffened shells an alternate equation is provided where the effective shell width acting with the ring is used to compare the ring hoop stress distribution factor. The larger of the two stress distribution factors obtained from Equation 11.3-12 and 11.3-16 should be used in defining the ring hoop stress.

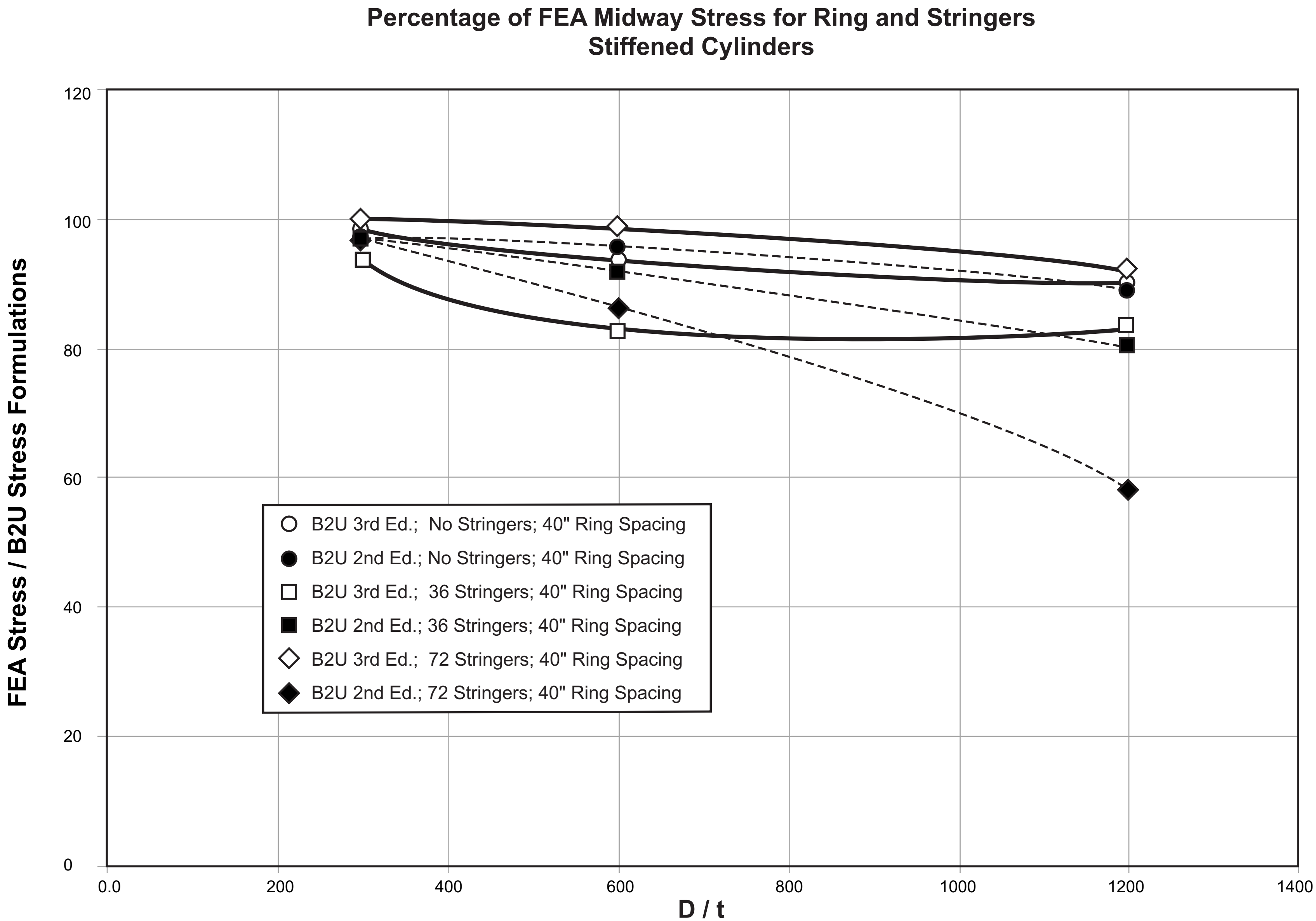


Figure C.11.3-1--Shell Hoop Stress Ratios at Mid Panel
for a Range of Cylindrical Shell Configurations at L_r = 40"

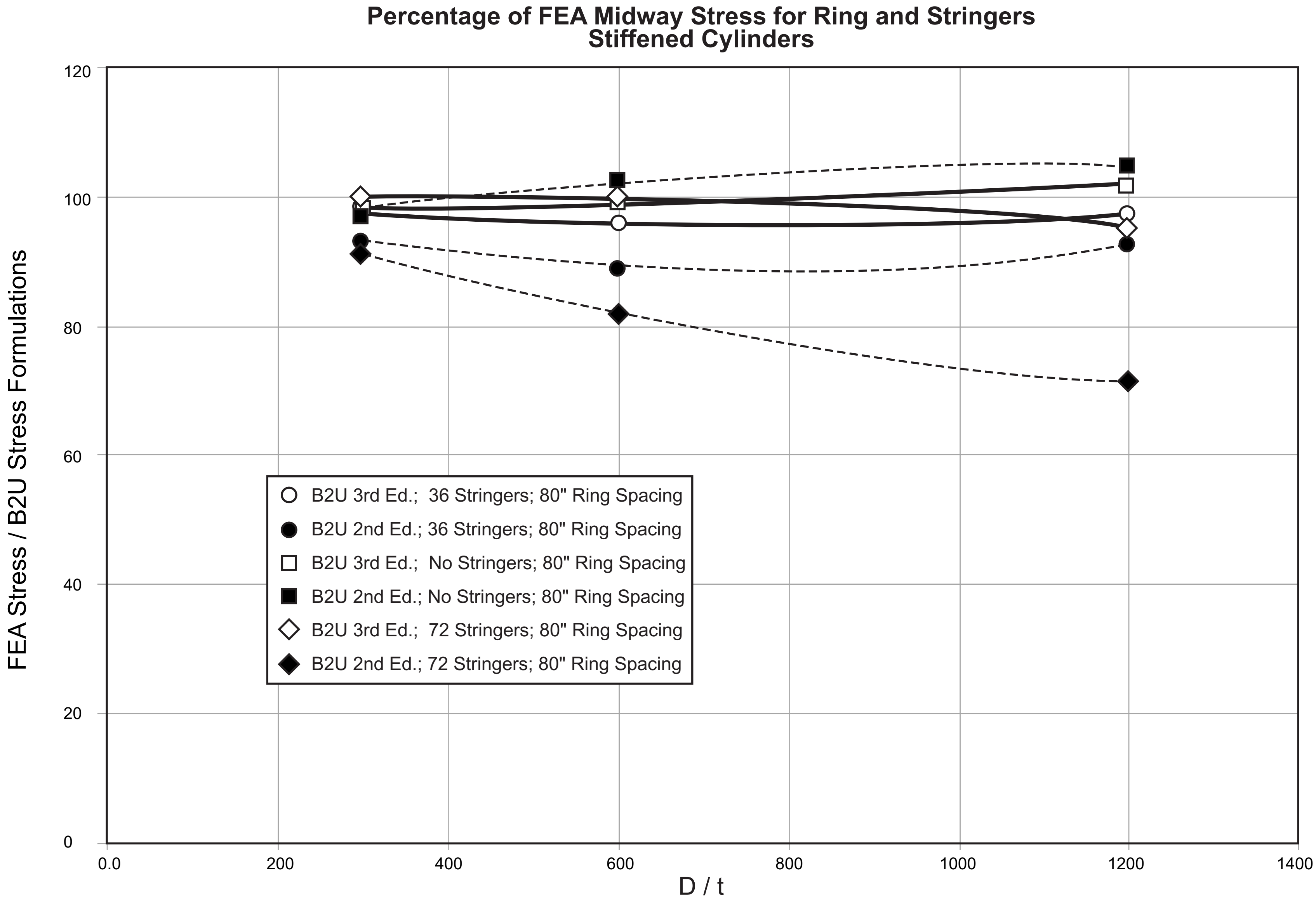


Figure C.11.3-2--Shell Hoop Stress Ratios at Mid Panel
for a Range of Cylindrical Shell Configurations at L_r = 80"

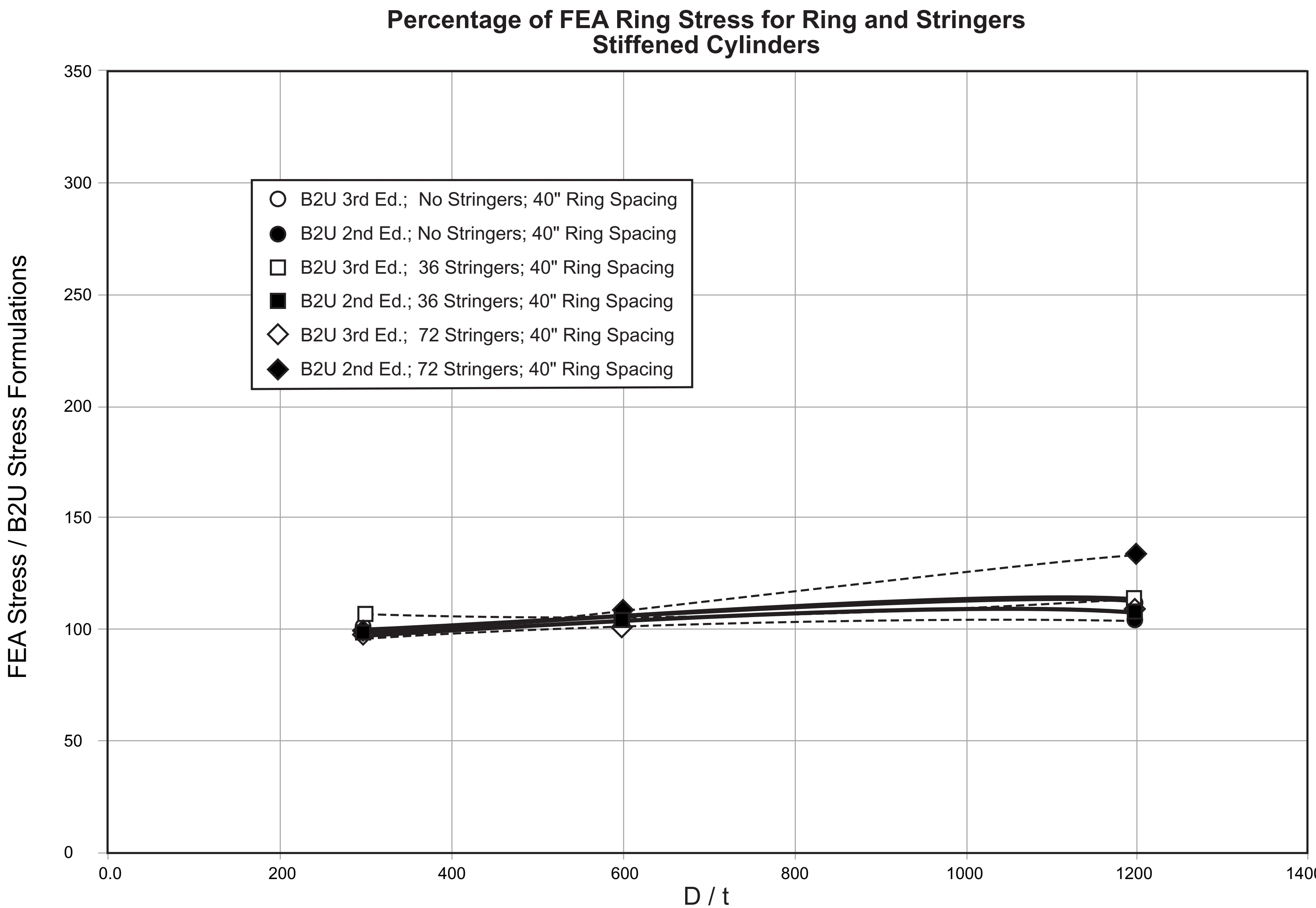


Figure C.11.3-3--Ring Hoop Stress Ratios for a Range of Cylindrical Shell Configurations at L_r = 40"

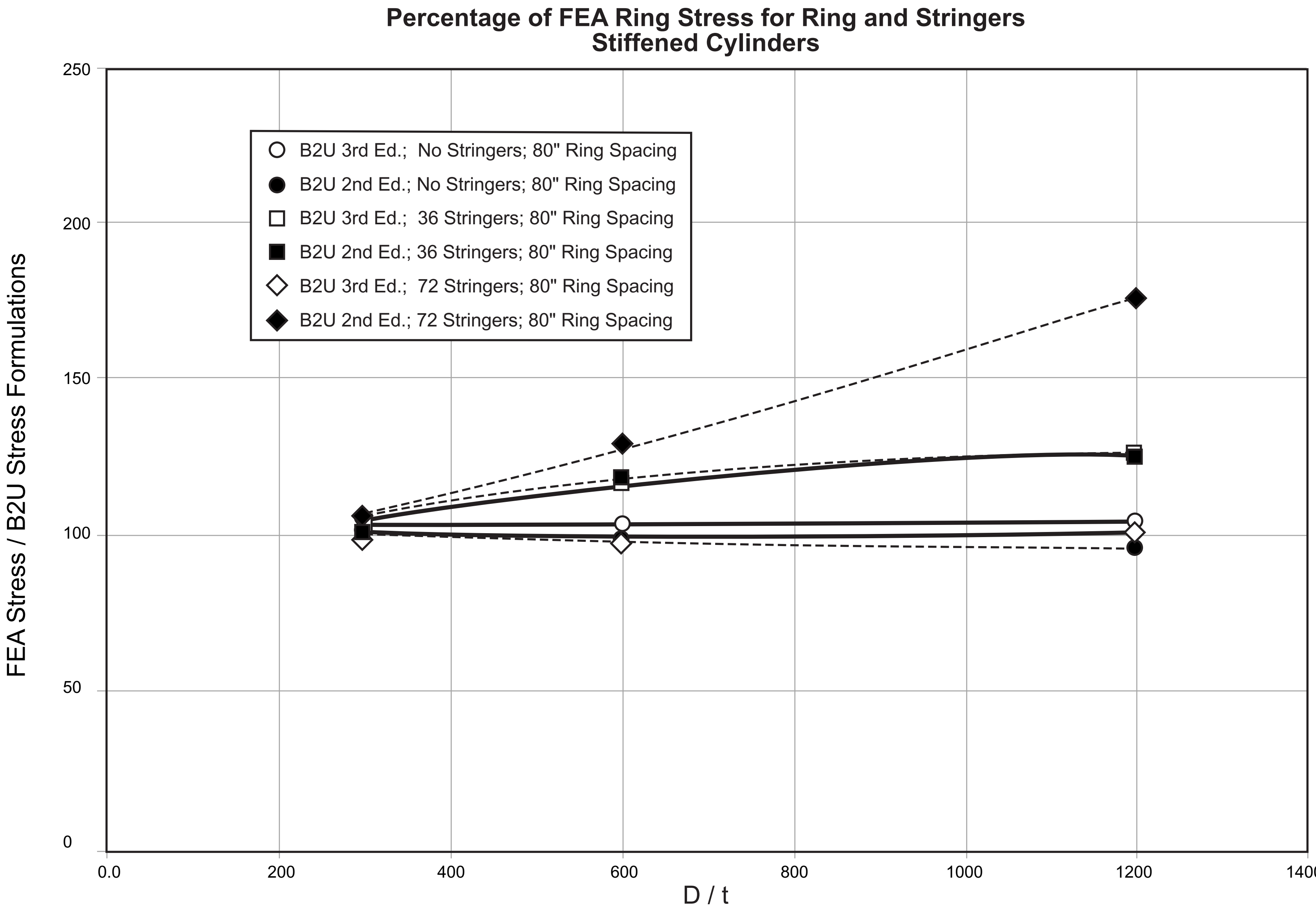


Figure C.11.3-4--Ring Hoop Stress Ratios for a Range of Cylindrical Shell Configurations at L_r = 80"

Ring and Stringer Stiffened Cylindrical Shells – Mid Panel Hoop Stress

D/t	1200	600	300	1200	600	300
Ring Spacing (in).	40.00	40.00	40.00	80.00	80.00	80.00
Number of stringers	0	0	0	0	0	0
Hoop Stress						
FEA	-18.11	-7.78	-4.21	-22.36	-10.99	-5.12
B2U- 3rd Ed.	-19.91	-8.30	-4.28	-21.95	-11.08	-5.22
B2U- 2nd Ed.	-20.19	-8.15	-4.32	-21.35	-10.68	-5.25
FEA/API						
3rd Ed.	90.97	93.70	98.44	101.87	99.18	98.04
2nd Ed.	89.71	95.49	97.41	104.73	102.81	97.37

D/t	1200	600	300	1200	600	300
Ring Spacing (in).	40.00	40.00	40.00	80.00	80.00	80.00
Number of stringers	36	36	36	36	36	36
Hoop Stress						
FEA	-16.39	-7.48	-4.20	-19.81	-9.53	-4.90
B2U- 3rd Ed.	-19.62	-9.07	-4.47	-20.34	-9.90	-4.98
B2U- 2nd Ed.	-20.19	-8.15	-4.32	-21.35	-10.68	-5.25
FEA/API						
3rd Ed.	83.53	82.48	93.76	97.36	96.31	98.43
2nd Ed.	81.20	91.84	97.09	92.76	89.20	93.25

D/t	1200	600	300	1200	600	300
Ring Spacing (in).	40.00	40.00	40.00	80.00	80.00	80.00
Number of stringers	72	72	72	72	72	72
Hoop Stress						
FEA	-11.90	-7.03	-4.20	-19.81	-9.53	-4.90
B2U- 3rd Ed.	-12.94	-7.13	-4.19	-16.09	-8.76	-4.79
B2U- 2nd Ed.	-20.19	-8.15	-4.32	-21.35	-10.68	-5.25
FEA/API						
3rd Ed.	91.98	98.59	99.83	95.23	99.85	100.33
2nd Ed.	58.97	86.22	96.69	71.78	81.84	91.39

Table C11.3-1: Shell Hoop Stresses and Stress Ratios at Mid Panel for a Range of Cylindrical Shell Configurations

Ring and Stringer Stiffened Cylindrical Shells – Ring Hoop Stress

D/t	1200	600	300	1200	600	300
Ring Spacing (in).	40.00	40.00	40.00	80.00	80.00	80.00
Number of stringers	0	0	0	0	0	0
Hoop Stress						
FEA	-6.87	-6.14	-4.06	-6.54	-5.93	-4.21
B2U- 3rd Ed.	-6.45	-5.98	-4.05	-6.26	-5.75	-4.14
B2U- 2nd Ed.	-6.74	-6.02	-4.19	-6.74	-6.02	-4.19
FEA/API						
3rd Ed.	106.62	102.59	100.12	104.46	102.97	101.74
2nd Ed.	101.92	101.94	96.73	96.96	98.39	100.41

D/t	1200	600	300	1200	600	300
Ring Spacing (in).	40.00	40.00	40.00	80.00	80.00	80.00
Number of stringers	36	36	36	36	36	36
Hoop Stress						
FEA	-7.50	-6.30	-4.06	-8.48	-7.11	-4.41
B2U- 3rd Ed.	-6.74	-6.02	-4.19	-6.74	-6.09	-4.31
B2U- 2nd Ed.	-6.74	-6.02	-4.19	-6.74	-6.02	-4.19
FEA/API						
3rd Ed.	111.16	104.62	96.84	125.66	116.83	102.45
2nd Ed.	111.16	104.62	96.84	125.66	118.11	105.26

D/t	1200	600	300	1200	600	300
Ring Spacing (in).	40.00	40.00	40.00	80.00	80.00	80.00
Number of stringers	72	72	72	72	72	72
Hoop Stress						
FEA	-8.96	-6.51	-4.06	-11.90	-7.76	-4.48
B2U- 3rd Ed.	-8.36	-6.55	-4.19	-11.74	-7.92	-4.56
B2U- 2nd Ed.	-6.74	-6.02	-4.19	-6.74	-6.02	-4.19
FEA/API						
3rd Ed.	107.14	99.37	96.89	101.36	97.97	98.42
2nd Ed.	132.78	108.11	96.89	176.43	128.93	106.90

Table C11.3-2: Ring Hoop Stresses and Stress Ratios for a Range of Cylindrical Shell Configurations

C12 References

C01. American Petroleum Institute, ***Bulletin on Stability Design of Cylindrical Shells***, API Bulletin 2U (BUL 2U), First Edition, May 1987.

C02. American Petroleum Institute, ***Bulletin on Stability Design of Cylindrical Shells,*** API Bulletin 2U (BUL 2U), Second Edition, October 2000.

C03. American Petroleum Institute, ***Recommended Practice for Planning, Designing and Constructing Fixed Offshore Plateforms***, API RP2A, Twentieth Edition, July 1993.

C04. Timoshenko, S. P. and Gere, J. M., ***Theory of Elastic Stability***, 2nd Edition, McGraw Hill, New York, 1961.

C05. Donnell, L.H., ***Stability of Thin-Walled Tubes in Torison***, NACA Report No. 479, 1933.

C06. Batdorf, S.B., ***A Simplified Method of Elastic-Stability Analysis for Thin Cylindrical Shells***, NACA Report No. 874, 1947.

C07. Stephens, M., Kulak, G., and Montgomery, C., ***"Local Buckling of Thin-Walled Tubular Members,"*** Proc. Of Structural Stability Research Council (SSRC), Toronto, Canada, 1983, pp. 489-508.

C08. Wilson, W. M. and Newmark, N. M., **"The Strength of Thin Cylindrical Shells as Columns,"** University of Illinois Engineering Experiment Station Bulletin, No. 255, 1933.

C09. Akiyama, H., Shimizu, D., Takahashi, T. and Yuhara, T., ***"Buckling of Steel Containment Vessels Under Earthquake Loadings,"*** Proceedings of 1986 Fall Meeting of Atomic Energy Society of Japan, 1986..

C10. Brockenbrough, R.L., ***"Determination of Critical Buckling Stress of Cylindrical Plates Having Low t/R Values,"*** PDM Company Report, Pittsburgh, Pennsylvania, 1960.

C11. Chen, Y., Simmer, R.A., DeOliveira, J.G., and Jan, H.Y., ***"Buckling and Ultimate Strength of Stiffened Cylinders: Model Experiments and Strength Formulations,"*** OTC Paper No. 4583, Proceedings of International Offshore Technology Conference, Houston, Texas, 1985.

C12. Dowling, P. J. and Harding, J. E., ***"Experimental Behavior of Ring and Stringer Stiffened Shells,"*** Buckling of Shells in Offshore Structures, Granada Publishing, pp. 73-107,1982.

C13. Galletley, G.D., and Pemsing, K., ***"Buckling of Cylinders and Combined External Pressure and Axial Compression,"*** IUTAM Symposium on Collapse, University College, London, England, 1982.

C14. Miller, C. D. and Grove, R. B., ***"Interpretive Report on the Conoco/ABS Test Program,"*** CBI Company Report, Plainfield, Illinois, 1983.

C15. Odland, J. ***"On the Strength of Welded Ring Stiffened Shells Primarily Subjected to Axial Compression,"*** Report No UR-81-15, University of Trondheim, Trondheim, Norway, 1981.

C16. I.D.E.A.S., Inc., ***"API Bulletins 2U and 2V Phase 2 Work-Data Applicable to API Bulletin 2U,"*** API PRAC and TG on Bulletins 2U and 2V, Phase 2 Technical Memorandum No. 1, 98210-TM-01, June 1999.

C17. Von Mises, R., ***"The Critical External Pressure of Cylindrical Tubes Under Uniform Radial and Axial Load,"*** (Der Kritische Aussendrück für Alseitis Belastete Zylindrische Röhre), Stodolas Festschrift, Zürich, 1931, pp. 418-430 [David Taylor Model Basin Translation #5].

C18. Bannon, R.J., ***"Buckling of Ring and String Stiffened Cylindrical Models Subjected to Combined Loads,"*** Final Report, CBI Contract No. U1851606, CBI Industries, Plainfield, Illinois, 1986.

C19. Miller, C. D., ***"Summary of Buckling Tests on Fabricate Steel Cylindrical Shells In USA,"*** Buckling of Shells in Offshore Structures, Granada, Publishing, pp. 429-472, 1982.

C20. Miller, C. D., and Saliklis, Edmund P., ***"Analysis of Cylindrical Shell Database and Validation of Design Formulations,"*** Phase 2: For *D/t* for Values >300, API PRAC Project 92-56, Final Report, 1996.

C21. Miller, C. D. and Kinra, R. K., "***External Pressure Tests of Ring Stiffened Fabricated Steel Cylinders,"*** Paper OTC 4107, Offshore Technology Conference, Houston, Texas, May 1981.

C22. Galletly, G. D. and Bart, R., "***Effects of Boundary Conditions and Initial Out-of-Roundness on the Strength of Thin-Walled Cylinders Subjected to External Hydrostatic Pressure,"*** David Taylor Model Basin (DTMB) Report 1505, May1962..

C23. Sturm, R. G., ***"A Study of the Collapsing Pressure of Thin-Walled Cylinders,"*** University of Illinois, Engineering Experiment Station Bulletin No. 329, 1941.

C24. Becker, H. and Gerard, G., ***"Elastic Stability of Orthotropic Shells,"*** Journal of the Aerospace Sciences, Vol. 29, No. 5, May 1962, pp. 505-512.

C25. Miller, C. D., ***"Buckling Stresses for Axially Compressed Cylinders,"*** Journal of the Structural Division, ASCE, Vol. 103, No. ST3, March 1977, pp. 695-721.

C26. Kendrick, S. B., ***"Collapse of Stiffened Cylinders Under External Pressure,"*** Paper No. C190/72, Proceedings of Institution of Mechanical Engineers Conference on Vessels Under Buckling Conditions, London, Dec. 1972, pp. 33-42.

C27. Hom, K., ***"Elastic Stress in Ring-Frames of Imperfectly Circular Cylindrical Shells Under External Pressure Loading,"*** David Taylor Model Basin (DTMB) Report 1066, Nov. 1957.

C28. Greimann, L. F., DeHart, R. C., and Franz, D. R., ***"Effect of Out-of-Roundness and Residual Stresses on Ring Reinforced Steel Shells,"*** Southwest Research Institute Report, Project No. 03-2435, AISI Project 155, Oct. 1969.

C29. Faulkner, D., Cowling, M. J., and Frieze, P. A., Editors, ***Integrity of Offshore Structures***, Applied Science Publishers, London and New Jersey, 1981.

C30. Kollburnner, C.F., and Meiseter, M., Ausbeulen, Springer, 1958.

C31. I.D.E.A.S., Inc., ***"API Bulletins 2U and 2V Phase 3 Work-Gathering Review and Evaluation of Test Data Applicable to Cylindrical Shells Subjected to Axial Compression, External Pressure and Combined Loading,"*** API PRAC and TG on Bulletins 2U and 2V, Phase 3 Technical Memorandum NO.1, 99210-Tm-01, April 2000.

C32. Block, D. L., Card, M. F., and Mikulas, M. M., ***"Buckling of Eccentrically Stiffened Orthotropic Cylinders,"*** NASA TN-2960, August 1965.

C33. Flügge, W., ***"Stresses in Shells,"*** Springer-Verlag, Belin/Gottinger/Heidelberg, 1960.

C34. Miller, C. D., ***"Buckling Stresses of Ring and Stringer Stiffened Cylindrical Shells under Axial Compressive Load,"*** Research Report, Chicago Bridge & Iron Co., April 1977.

C35. American Iron and Steel Institute, ***Cold-Formed Steel Design Manual***, 1983 Edition.

C36. Kinra, R. K., ***"Hydrostatic and Axial Collapse Tests of Stiffened Cylinders,"*** Paper OTC 2685, International Offshore Technology Conference, Houston, Texas, May 1976.

C37. Faulkner, D., Chen, Y. N., and de Oliveira, J. G., ***"Limit State Design Criteria for Stiffened Cylinders of Offshore Structures,"*** 4th ASME PV&P Conference, Portland, Oregon, June 1983.

C38. European Convention for Constructional Steelwork, ***"European Recommendations for Steel Construction,"*** Section 4.6 ***Buckling of Shells***, Publication 29, Second Edition, 1983.

C39. Miller, C. D., Grove, R. B., and Vojta, J. F., "***Design of Stiffened Cylinders for Offshore Structures,"*** American Welding Society Welded Offshore Structures Conference, New Orleans, Louisiana, December 1983.

C40. Gerard, G., ***Compressive and Torsional Buckling of Thin-Walled Cylinders in Yield Region***, NACA Technical Note, TN 3726, 1956.

C41. Johnston, B. G., ***"Guide to Design Criteria for Metal Compression Members,"*** Column Research Council, John Wiley and Sons, New York, 1966.

C42. Miller, C. D., Kinra, R. K., and Marlow, R. S., "***Tension and Collapse Tests of Fabricated Steel Cylinders,"*** Paper OTC 4218, Offshore Technology Conference, May 1982.

C43. Miller, C. D. and Grove, R. B., ***"Current Research Related to Buckling of Shells for Offshore Structures,"*** Paper OTC 4474, International Offshore Technology Conference, Houston, Texas, May 1983.

C44. Miller, C. D. and Grove, R. B., ***"Collapse Tests of Ring Stiffened Cylinders Under Combinations of Axial Compression and External Pressure,"*** ASME, PVP Conference, Chicago, Illinois, July 20-24, 1986.

C45. American Institute of Steel Construction, ***Manual of Steel Construction***, AISI, 8th Edition, 1981.

C46. Chen, W. F. and Ross, D. A., ***"The Strength of Axially Loaded Tubular Columns,"*** Fritz Engineering Lab Report No. 393.8, Lehigh University, Bethlehem, Pennsylvania, September 1978.

C47. Wilson, W. M., ***"Tests of Steel Columns,"*** University of Illinois Engineering Experiment Station Bulletin, No. 292, 1937.

C48. Miller, C. D., "**Axial Compression and Bending Strength of Unstiffened Steel Cylinders,"** Research Report, CBI Industries, October 1984.

C49. American Society of Mechanical Engineers, ***Boiler and Pressure Vessel Code*** , Section VIII, Division 1 and 2 and Section III, Subesection NE, 1983.

C50. Windenburg, D. F., ***"Vessels Under External Pressure,"*** Pressure Vessel and Piping Design, collected papers 1927-1959, ASME, 1960, pp. 625-632.

C51. Vojta, J. F. and Miller, C. D., ***"Buckling Tests on Ring and Stringer Stiffened Cylindrical Models Subject to Combined Loads,"*** Final Report, Contract No. 11896, Vol. I—Main Report and 3 Volume Appendix, CBI Industries, Inc., April 1983.

Appendix B – Example - Ring Stiffened Cylinders

In the following, the process of using API 2U to perform buckling checks using the third edition of API 2U Bulletin will be explained. Notice that while the terms were calculated exactly using all decimal places, they appear in the following text as rounded numbers.

Problem Definition

Material Data

Modulus of Elasticity, E 29,000[ksi]
Poisson's ratio, υ 0.3
Shear Modulus, G 11,154[ksi]
Yield Stress, F_y 50[ksi]
Water density, ρ_w 64[lb/ft^3]

Dimensions

Cylinder Length, L 150[ft]
Diameter, D 600[in]
Distance between bulkheads, L_b 50[ft]

Plate

Thickness, t 0.75[in]

Ring

Number of ring spacings, 10
Ring spacing, Lr 5[ft]
Web height, 14[in]
Web thickness, 5/8[in]
Flange width, 10[in]
Flange thickness, 1[in]

Loading

Pressure Head 60[ft]
Axial Loading (compression) 9000[kips]
Loading Condition Extreme

Ring Section property Calculations

Similar to calculations of section properties of stringers, the section properties of rings can be calculated as:

$$A_r = 18.75[in^2]$$

Check Ring Section Compactness per Section 7

Compactness of the ring web:

$$\frac{h_s}{t_s} = \frac{14}{0.625} = 22.4 \le \sqrt{\frac{29000}{50}} = 24.1[Okay]$$

Compactness of ring flange:

$$\frac{h_s}{t_s} = \frac{5}{1} = 5 \le 0.375\sqrt{\frac{29000}{50}} = 9.03[Okay]$$

Check Stress Level in Plate and Ring Per Section 11

Hoop stress in Shell Midway Between Rings

We have the stress in the plate midway between rings given by:

$$f_{\theta S} = \frac{pR_0}{t} K_{\theta L} \qquad (11.3\text{-}2)$$

in which:

$$K_{\theta L} = 1 - \psi_k \frac{p_\sigma}{p}\left(\frac{k_d}{k_d + k_t}\right) \qquad (11.3\text{-}3a)$$

$$p_\sigma = p + \frac{\upsilon \sigma_{xa} t}{R_0} \le p \qquad (11.3\text{-}4)$$

$$\sigma_{xa} = -f_a = -\frac{P}{2\pi R t} = -\frac{9000}{2\pi \times 299.625 \times 0.75}$$

$$= -6.37[ksi]$$

$$p = \frac{\rho_w}{144 \times 1000} \times 60 = \frac{64}{1000} \times 60 = 0.0267[ksi]$$

Thus we have:

$$p_\sigma = 0.027 - \frac{0.3 \times 6.37 \times 0.75}{300} = 0.022[ksi]$$

The terms required to evaluate k_t and k_d are given by:

$$D = \frac{Et^3}{12(1-\upsilon^2)} = \frac{29000 \times 0.75^3}{12(1-0.3^2)} = 1120.36[k-in]$$

$$\beta = \sqrt[4]{\frac{Et}{4R^2 D}} = \sqrt[4]{\frac{29000 \times 0.75}{4 \times 299.625^2 \times 1120.36}} = 0.0857$$

$$t_{ws} = \frac{A_R}{h} = \frac{18.75}{14} = 1.34[in]$$

$$R_0 = 300[in]$$

$$R_f = 300 - 14 = 286[in]$$

$$L_r = 60[in]$$

Using the above, we have k_t and k_d and ψ given by:

$$k_t = 5.67 \qquad (11.3\text{-}5a)$$

$$k_d = 6.10 \qquad (11.3\text{-}6)$$

$$\psi_k = 0 \qquad (11.3\text{-}8a)$$

Thus we get:

$$K_{\theta L} = 1$$

Hoop stress in shell midway between rings is thus given by:

$$f_{\theta S} = \frac{pR_0}{t} K_{\theta L} = \frac{0.0267 \times 300}{0.75} \times 1 = 10.67[ksi]$$

Hoop Stress in Ring

The hoop stress in the ring is given by:

$$f_{\theta R} = \frac{pR_0}{t} K_{\theta G} \qquad (11.3\text{-}11)$$

in which:

$$K_{\theta G} = 1 - \frac{p_\sigma}{p}\left(\frac{k_d}{k_d + k_t}\right) = 1 - \frac{0.022}{0.0267}\left(\frac{6.10}{6.10 + 5.67}\right) \qquad (11.3\text{-}12a)$$

$$K_{\theta G} = 0.5748$$

Notice that the external applied compressive load increases the hoop stress in the ring.

Hoop stress in shell at ring is given by:

$$f_{\theta R} = \frac{pR_0}{t} K_{\theta G} = \frac{0.0267 \times 300}{0.75} \times 0.5748 = 6.13[ksi]$$

4 Predicted Shell Buckling Stresses for Axial Load, Bending and External Pressure

The value of M_x is given by:

$$M_x = \frac{L_r}{\sqrt{Rt}} = \frac{60}{\sqrt{299.625 \times 0.75}} = 4.00 \qquad (4.1\text{ a})$$

4.1 Local Buckling of Unstiffened or Ring Stiffened Cylinders

4.1.1 Axial Compression or Bending

a. Elastic Buckling Stresses

The elastic buckling stresses is given by Eq. 4.1-1 as:

$$F_{xeL} = C_{xL} \frac{\pi^2 E}{12(1-\upsilon^2)} (t / L_r)^2 \qquad (4.1\text{-}1)$$

in which

$$C_{xL} = \sqrt{1 + \frac{150\alpha_{xL}^2 M_x^4}{D/t}} \qquad (4.1\text{-}2)$$

In the above equation, the imperfection factor is given by:

$$\alpha_{xL} = \frac{9}{(300 + D/t)^{0.4}} = \frac{9}{(300 + 799)^{0.4}} = 0.5468 \qquad (4.1\text{-}3)$$

Thus, we get C_{xL} as:

$$C_{xL} = \sqrt{1 + \frac{150 \times 0.5468^2 \times 4^4}{799}} = 3.925$$

Elastic buckling stress under axial compression becomes:

$$F_{xeL} = 3.925 \frac{\pi^2 \times 29000}{12(1-0.3^2)} (0.75/60)^2 = 16.07[ksi]$$

b. Inelastic Buckling Stress

The inelastic buckling stress is calculated as:

$$F_{xcL} = \eta F_{xeL}$$

in which, η is referred to as plasticity reduction factor calculated using Section 5 as:

$\eta = 1$ (5-3)

For local buckling under axial compression, $i = x$ and $j = L$; Thus $F_{iej} = F_{xeL}$. The inelastic buckling stress is given by:

$F_{xcL} = 1 \times 16.07 = 16.07[ksi]$

4.1.2 External Pressure

The elastic buckling stress is given by:

$$F_{reL} = C_{\theta L} \frac{\pi^2 E}{12(1-\upsilon^2)}(t / L_r)^2 \qquad (4.1\text{-}5)$$

In order to determine $C_{\theta L}$, the number of lobes n into which the shell buckles between rings has to be determined. The term Z_m is first calculated as:

$$Z_m = \frac{12 M_x^4 (1-\upsilon^2)}{\pi^4} = 28.77$$

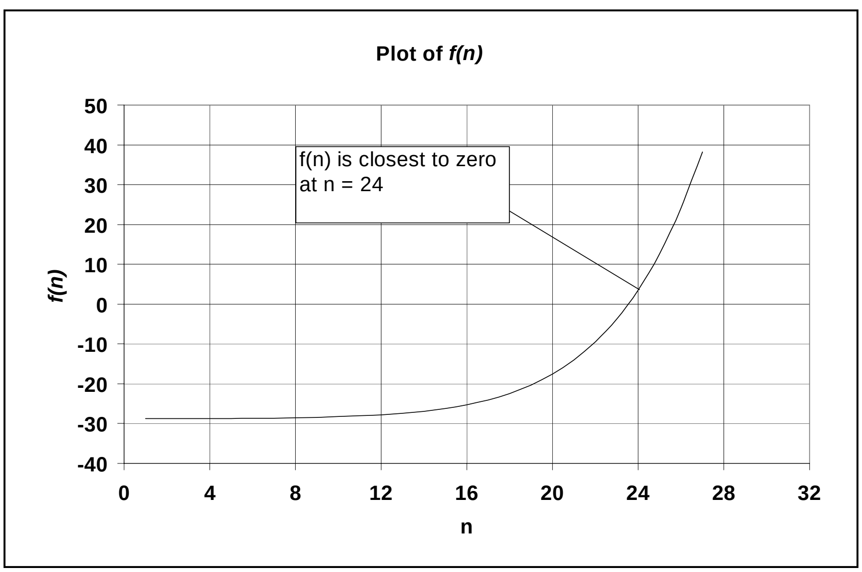
The number of lobes, n, is then found by equating the left hand side of Eq. 4.1-6 to the value of Z_m found above:

$$f(n) = \frac{\beta^2 (1+\beta^2)^4}{2+3\beta^2} - Z_m \cong 0$$

in which:

$$\beta = \frac{L_r}{(\pi R / n)}$$

There are several ways of solving for n. In the following the function *f(n)* is shown graphically:



As shown in the figure above, the function *f(n)* is closest to zero at *n=24*, hence, the shell would buckle in this example into 24 lobes under external pressure. Once *n* is determined, β and consequently $C_{\theta L}$ can be determined:

$\beta = 1.53$

$\alpha_{\theta L} = 1.0$ (4.1-8)

$C_{\theta L} = 4.84$ (4.1-7)

We get the elastic buckling stress as:

$$F_{reL} = 4.84 \frac{\pi^2 \times 29000}{12(1-0.3^2)} (0.75/60)^2 = 19.8[ksi]$$

The inelastic buckling stress is calculated using plasticity reduction factor in section 5, i.e.,

$$F_{rcL} = \eta F_{reL}$$

Since elastic buckling stress is less than half of yield stress, there is no plasticity reduction and thus the inelastic buckling stress is same as the elastic buckling stress:

$$F_{rcL} = 19.8[ksi]$$

4.2 General Instability of Ring Stiffened Cylinders

4.2.1 Axial Compression or Bending

a. Elastic Buckling Stresses

The elastic buckling stress is given by:

$$F_{xeG} = \alpha_{xG}\sigma_{xeG} = 0.605\alpha_{xG} \frac{Et}{R}(1+\overline{A}_r)^{1/2} \quad (4.2\text{-}1)$$

in which:

$$\overline{A}_r = A_r / L_r t = 18.75/60/0.75 = 0.4167$$

Thus $\alpha_{xG} = 0.72$ per Eq. 4.2-2, giving the elastic buckling stress as:

$$F_{xeG} = 0.605 \times 0.72 \times \frac{29000 \times 0.75}{300}(1+0.4167)^{1/2}$$

$$= 37.64[ksi]$$

The plasticity reduction factor is calculated using Section 5 as:

$$\eta = \frac{F_y}{F_{iej}}\left(\frac{1}{1+3.75(F_y/F_{iej})^2}\right)^{1/4} \quad (5\text{-}3)$$

$$\eta = \frac{50}{37.64}\left(\frac{1}{1+3.75(50/37.64)^2}\right)^{1/4} = 0.7996$$

The inelastic buckling stress is given by:

$$F_{xcG} = \eta F_{xeG} = 0.7996 \times 37.64 = 30.10[ksi] \quad (5\text{-}1)$$

4.2.2 External Pressure

a. Elastic Buckling Stress

The elastic buckling stress is given by the equation:

$$F_{reG} = \alpha_{\theta G} \frac{p_{eG} R_0}{t} K_{\theta G} \quad (4.2\text{-}4)$$

In which $K_{\theta G}$ is calculated in Section 11 as:

$$K_{\theta G} = 0.5748$$

The ring properties are calculated as:

$A_r = 18.75[in^2]$

$y_{na} = 11[in.]$

$\sum Ad^2 = 262.5[in^4]$

$I_0 = 143.75[in^4]$

$I_r = I_0 + \sum Ad^2 = 406.25[in^4]$

$L_e = 23.94[in]$

$t = 0.75[in]$

$Z_r = -11.375[in]$

$I_{er} = 1593.98[in^4]$

λ_G, R_c and R_0 are given by:

$\lambda_G = \pi R/L_b = 1.5688$

$R_c = 293.82[in]$

$R_0 = 300[in]$

The non-integer value of n that gives the minimum p_{eG} can be found by trial and error (method of bisection or Newton Raphson):

$n = 3.65$

$p_{eG} = 0.510[ksi]$ (4.2-5)

The imperfection factor is given by:

$\alpha_{\theta G} = 0.8$

Hence, the elastic buckling stress is given finally as:

$$F_{reG} = 0.8\frac{0.510\times 300}{0.75}0.57 = 93.77[ksi]$$

The plasticity reduction factor is calculated using Section 5 as:

$$\eta = \frac{50}{93.77}\left(\frac{1}{1+3.75(50/93.77)^2}\right)^{1/4} = 0.445 \qquad (5\text{-}3)$$

The inelastic buckling stress is given by:

$F_{rcG} = \eta F_{reG} = 0.445\times 93.77 = 41.70[ksi]$

Summary of Buckling Stresses

Buckling Mode	Elastic Stress (ksi)	Inelastic Stress (ksi)
Axial Compression		
Local Buckling	F_{xeL}=16.07	F_{xcL}=16.07
General Instability	F_{xeG}=37.64	F_{xcG}=30.10
External Pressure		
Local Buckling	F_{reL}=19.8	F_{rcL}=19.8
General Instability	F_{reG}=93.77	F_{rcG}=41.7

6.0 Predicted Shell Buckling Stresses for Combined Loads

6.1 General Load Cases

The values of N_ϕ and N_θ is given by:

$$N_\phi = \frac{P}{2\pi R} = \frac{9000}{2\pi \times 299.625} = 4.78[k/in]$$

$$N_\theta = pR_0 = 0.0267 \times 300 = 8.01[k/in]$$

6.3 Axial Compression Bending and Hoop Compression

Equation 6.3-1 is an interaction equation used to determine the combined buckling stresses. The use of interaction equation will be demonstrated separately for local and general instability modes.

Local Buckling

For local buckling, the interaction equation is given by:

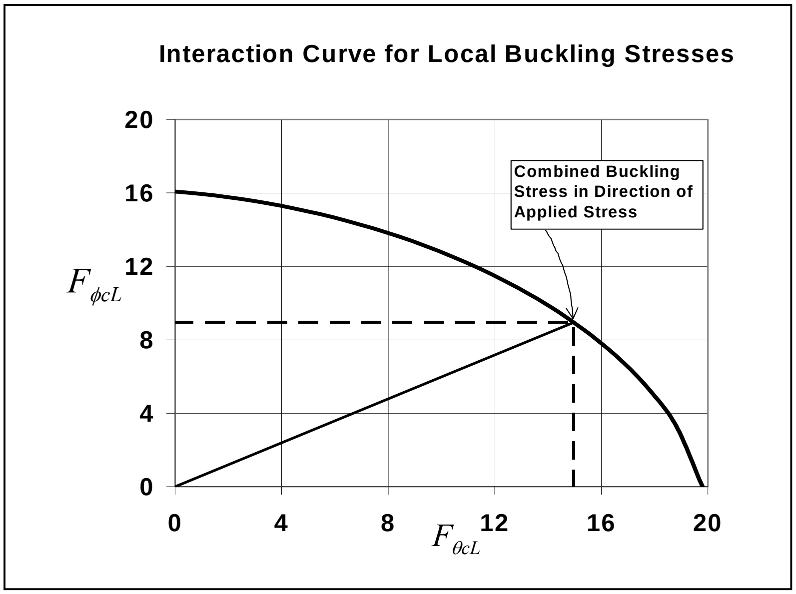
$$\left(\frac{F_{\phi cL}}{F_{xcL}}\right)^2 - c\left(\frac{F_{\phi cL}}{F_{xcL}}\right)\left(\frac{F_{\theta cL}}{F_{rcL}}\right) + \left(\frac{F_{\theta cL}}{F_{rcL}}\right)^2 = 1 \qquad (6.3\text{-}1)$$

The term c in the above equation is given by:

$$c = \frac{F_{xcL} + F_{rcL}}{F_y} - 1.0 = \frac{16.07 + 19.8}{50} - 1 = -0.28 \qquad (6.3\text{-}2)$$

The interaction equation becomes:

$$\left(\frac{F_{\phi cL}}{16.07}\right)^2 + 0.28\frac{F_{\phi cL}}{16.07}\frac{F_{\theta cL}}{19.8} + \left(\frac{F_{\theta cL}}{19.8}\right)^2 = 1$$



The figure above shows the interaction diagram. Points on the curve represent the pairs of combined inelastic buckling stresses. The combined buckling stress is determined in the direction of the applied stress by setting:

$$F_{\phi cL} = F_{\theta cL} k \frac{K_{\phi L}}{K_{\theta L}}$$

in which,

$$k = \frac{N_\phi}{N_\theta} = 0.6 \qquad K_{\phi L} = 1 \qquad K_{\theta L} = 1$$

Using the above, the interaction equation becomes:

$$\left(\frac{0.6F_{\theta cL}}{16.07}\right)^2 + 0.28\frac{0.6}{16.07}\frac{F_{\theta cL}^2}{19.8} + \left(\frac{F_{\theta cL}}{19.8}\right)^2 = 1$$

$$\Rightarrow F_{\theta cL} = 14.97[ksi]$$

Substituting value of $F_{\theta cL}$ back into the interaction diagram we get:

$$F_{\phi cL} = 8.95[ksi]$$

General Instability

For general instability, the term c in the above equation is given by:

$$c = \frac{F_{xcG} + F_{rcG}}{F_y} - 1.0 = \frac{30.10 + 41.7}{50} - 1 = 0.436$$

We have:

$$k\frac{K_{\phi G}}{K_{\theta G}} = k\frac{1}{K_{\theta G}} = 0.6\frac{1}{0.57} = 1.04$$

Following the same procedure as local instability, we get the combined general instability stresses as:

$$F_{\theta cG} = 26.66[ksi]$$

$$F_{\phi cG} = 27.71[ksi]$$

Summary of Combined Buckling Stresses

Buckling Mode		Combined Inelastic Stress (ksi)
Axial Load	Local Buckling	$F_{\phi cL} = 8.95$
	General Instability	$F_{\phi cG} = 27.71$
External Pressure	Local Buckling	$F_{\theta cL} = 14.97$
	General Instability	$F_{\theta cG} = 26.66$

9.0 Allowable Stresses

The factor of safety for extreme conditions is given by:

$$F.S = 1.25\psi$$

in which ψ is calculated using Eq. 9.1. Since we have axial compression and hoop compression, the allowable stresses are calculated using Eq. 9.1-5. The allowable axial load and external pressure for local and general instability modes are given by:

Summary of Allowable Stresses

Buckling Mode		Allowable Stresses (ksi)
Axial Load	Local Buckling	$\psi = 1.2 \quad F.S = 1.5$ $F_a = F_{\phi cL}/F.S = 5.96$
	General Instability	$\psi = 1.18 \quad F.S = 1.47$ $F_a = F_{\phi cG}/F.S = 18.82$
External Pressure	Local Buckling	$\psi = 1.2 \quad F.S = 1.5$ $F_\theta = F_{\theta cL}/F.S = 9.98$
	General Instability	$\psi = 1.19 \quad F.S = 1.48$ $F_\theta = F_{\theta cG}/F.S = 17.97$

We have the applied stresses given by:

$$f_a = \frac{P}{2\pi Rt} = 6.37[ksi]$$

$$f_\theta = 10.67[ksi]$$

Notice that the applied stresses are greater than allowable stresses for local buckling. The unity ratios are given by:

Summary of Unity Ratios

Buckling Mode		Unity Ratios
Axial Load	Local Buckling	1.07
	General Instability	0.34
External Pressure	Local Buckling	1.07
	General Instability	0.59

Based on these results, the designer would need to strengthen the structure to bring the local buckling unity check values to below 1.0.

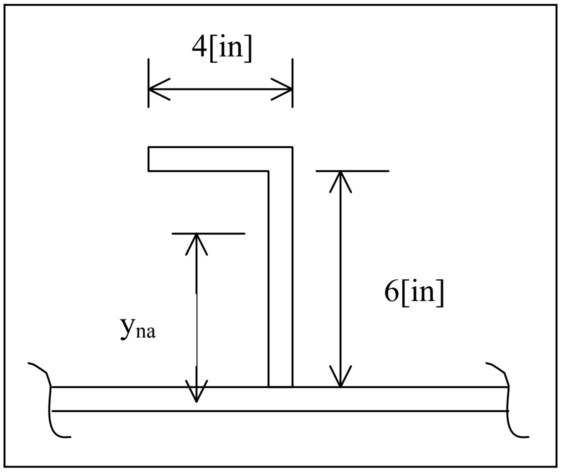
Appendix C – Example - Ring and Stringer Stiffened Cylinders

In the following, the process of using API 2U to perform buckling checks using the third edition of API 2U Bulletin will be explained. Notice while the terms were calculated exactly using all decimal places, they appear in the following text as rounded numbers.

Problem Definition

Material Data	
Modulus of Elasticity, E	29,000[ksi]
Poisson's ratio, υ	0.3
Shear Modulus, G	11,154[ksi]
Yield Stress, Fy	50[ksi]
Dimensions	
Cylinder Length, L	150[ft]
Diameter, D	600[in]
Distance between bulkheads, L_b	50[ft]
Plate	
Thickness, t	0.75[in]
Longitudinal Stiffeners	
Number of stiffener spacings,	64
Stiffener spacing,	2.45[ft]
Web height,	6[in]
Web thickness,	1/2[in]
Flange width,	4[in]
Flange thickness,	1/2[in]
Ring	
Number of ring spacings,	10
Ring spacing, L_r	5[ft]
Web height,	14[in]
Web thickness,	5/8[in]
Flange width,	10[in]
Flange thickness,	1[in]
Loading	
Pressure Head	60[ft]
Axial Loading (Compression)	9000[kip]
Loading Condition	Extreme

Stringer Section Properties Calculations



$$A_s = 6\times 0.5 + 4\times 0.5 = 5\ [in^2]$$

$$y_{na} = \frac{6\times 0.5\times 3 + 6.25\times 0.5\times 4}{A_s} = 4.3[in]$$

$$I_0 = \frac{1}{12}6^3\times 0.5 + \frac{1}{12}0.5^3\times 4 = 9.042[in^4]$$

$$\sum Ad^2 = 6\times 0.5\times(4.3-6/2)^2 + 4\times 0.5\times$$
$$\times(6.25-4.3)^2 = 12.675[in^4]$$

$$I_s = I_0 + \sum Ad^2 = 21.7167[in^4]$$

$$Z_s = -(y_{na}+t/2) = -(4.3+0.75/2) = -4.675[in]$$

Ring Section property Calculations

Similar to calculations of section properties of stringers, the section properties of rings can be calculated as:

$$A_r = 18.75[in^2]$$

Stringer Section Compactness per Section 7

Compactness of the stringer web:

$$\frac{h_s}{t_s} = \frac{6}{0.5} = 12 \le \sqrt{\frac{29000}{50}} = 24.1[Okay]$$

Compactness of stringer flange:

$$\frac{h_s}{t_s} = \frac{4}{0.5} = 8 \le 0.375\sqrt{\frac{29000}{50}} = 9.03[Okay]$$

Ring Section Compactness per Section 7

Compactness of the ring web:

$$\frac{h_s}{t_s} = \frac{14}{0.625} = 22.4 \le \sqrt{\frac{29000}{50}} = 24.1[Okay]$$

Compactness of ring flange:

$$\frac{h_s}{t_s} = \frac{5}{1} = 5 \le 0.375\sqrt{\frac{29000}{50}} = 9.03[Okay]$$

Check Stress Level in Plate and Ring Per Section 11

Hoop stress in Shell Midway Between Rings

We have the stress in the plate midway between rings given by:

$$f_{\theta S} = \frac{pR_0}{t} K_{\theta L} \qquad (11.3\text{-}2)$$

in which:

$$k_{\theta L} = 1 - \psi_{ef} \frac{p_\sigma}{p}\left(\frac{k_d}{k_d + k_{tef}}\right) \qquad (11.3\text{-}3b)$$

$$p_\sigma = p + \frac{\upsilon \sigma_{xa} t}{R_0} \le p \qquad (11.3\text{-}4)$$

$$\sigma_{xa} = -f_a = -\frac{P}{2\pi Rt + N_s A_s} =$$

$$= -\frac{9000}{2\pi \times 300 \times 0.75 + 64 \times 5} = -5.19[ksi]$$

Notice in the above that the axial stress is reduced when compared to ring stiffened shells, due to presence of stringers.

$$p = \frac{\rho_w}{144 \times 1000} \times 60 = \frac{64}{1000} \times 60 = 0.0267[ksi]$$

Thus we have:

$$p_\sigma = 0.027 - \frac{0.3 \times 5.91 \times 0.75}{300} = 0.023[ksi]$$

In equation 11.3-10b, the moment of inertia of stringer including the effective breadth is needed. Effective breadth is calculated using shear lag assuming stiffener to be supported at ring with fixed-fixed end conditions. This is calculated as:

$$b_e = 23.2[in.]$$

The stringer moment of inertia becomes:

$$I_{ef} = 126.23[in^4]$$

We have the terms required to evaluate k_{tef} and k_d given by:

$$D_{ef} = \frac{N_s EI_{ef}}{2\pi R_0} = 124689[k - in]$$

$$\rho = \frac{\pi}{N_s} = 0.0491$$

$$\delta = 0.8021$$

$$t_{ef} = t\delta \frac{Sin\rho}{\rho} = 0.6[in]$$

$$\beta_{ef} = \sqrt[4]{\frac{Et_{ef}}{4R^2 D_{ef}}} = 0.025$$

$$t_{ws} = \frac{A_R}{h} = \frac{18.75}{14} = 1.34[in]$$

$$R_0 = 300[in]$$

$$R_r = 300 - 14 = 286[in]$$

$$L_r = 60[in]$$

Using the above, we have k_{tef} and k_d and ψ_{ef} given by:

$$k_{tef} = 11.31$$

$$k_d = 6.10$$

$$\psi_{ef} = 0.762$$

Thus we get:

$$K_{\theta L} = 0.77$$

Hoop stress in shell midway between rings is given by:

$$f_{\theta S} = \frac{pR_0}{t} K_{\theta L} = \frac{0.0267 \times 300}{0.75} \times 0.77 = 8.24[ksi]$$

Hoop Stress in Ring

We have the stress in the ring given by:

$$f_{\theta R} = \frac{pR_0}{t} k_{\theta G} \qquad (11.3\text{-}11)$$

in which:

$$K_{\theta G} = 1 - \frac{p_\sigma}{p}\left(\frac{k_d}{k_d + k_{tef}}\right) = \qquad (11.3\text{-}12b)$$

$$= 1 - \frac{0.023}{0.0267}\left(\frac{6.10}{6.10 + 11.31}\right) = 0.70$$

Notice that the effect of external applied compressive load is to increase the hoop stress in the ring. The value of $K_{\theta G}$ calculated using Eq. 11.3-16 is given by:

$$K_{\theta G} = 0.44 \qquad (11.3\text{-}16)$$

Since the value of $K_{\theta G}$ evaluated using Eq. 11.3-12b is greater we use $K_{\theta G}$ =0.7.

Hoop stress in shell at ring is given by:

$$f_{\theta R} = \frac{pR_0}{t} K_{\theta G} = \frac{0.0267 \times 300}{0.75} \times 0.7 = 7.48[ksi]$$

Note: Comparison with stresses in the corresponding ring stiffened cylindrical shows that the hoop stress midway between ring spacing has decreased while the hoop stress at ring has increased.

4 Predicted Shell Buckling Stresses for Axial Load, Bending and External Pressure

The values of M_x and M_θ is given by:

$$M_x = \frac{L_r}{\sqrt{Rt}} = \frac{60}{\sqrt{299.625 \times 0.75}} = 4.00$$
$$M_\theta = \frac{b}{\sqrt{Rt}} = \frac{29.42}{\sqrt{299.625 \times 0.75}} = 1.96 \qquad (4.1\ a)$$

4.3 Local Buckling of Stringer Stiffened or Ring and Stringer Stiffened Cylinders

4.3.1 Axial Compression or Bending

a. Elastic Buckling Stress

The elastic buckling stress is given by:

$$F_{xeL} = C_{xL} \frac{\pi^2 E}{12(1-\upsilon^2)} (t/b)^2 \qquad (4.3\text{-}1)$$

in which

$$C_{xL} = 4 \qquad (4.3\text{-}2)$$

Thus:

$$F_{xeL} = 4 \frac{\pi^2 \times 29000}{12(1-0.3^2)} (0.75/29.42)^2$$
$$= 68.16[ksi]$$

b. Inelastic Buckling Stress

The inelastic buckling stress is calculated using plasticity reduction factor in section 5, i.e.,

$$F_{xcL} = \eta F_{xeL}$$

in which, η is referred to as plasticity reduction factor calculated using Section 5 as:

$$\eta = \frac{F_y}{F_{iej}} \left(\frac{1}{1 + 3.75(F_y / F_{iej})^2} \right)^{1/4} \qquad (5\text{-}3)$$

For local buckling under axial compression, $i = x$ and $j = L$; Thus $Fiej = FxeL$. We get:

$$\eta = \frac{50}{68.16} \left(\frac{1}{1 + 3.75(50/68.16)^2} \right)^{1/4} = 0.5566$$

Thus, the inelastic buckling stress is given by:

$$F_{xcL} = 0.5566 \times 68.16 = 37.93[ksi]$$

4.3.2 External Pressure

The local buckling pressure of a stringer stiffened cylinder will be greater than a corresponding unstiffened or ring stiffened cylinder if $0.5N_s$ is greater than the number of circumferential waves at buckling for the cylinder without stringers. The number of circumferential waves for stiffened cylinder without stringers is given by (calculated using Eq. 4.1-6 to Eq. 4.1-7):

$$n = 24$$

Stringers are effective if $N_s > 2n = 48$. Since the actual number of stringer spacings used is 64, we use Eqn. 4.3-3 to calculate elastic buckling stress for stringer-stiffened shells:

$$F_{\theta eL} = C_{\theta L} \frac{\pi^2 E}{12(1-\upsilon^2)}(t/L_r)^2 \qquad (4.3\text{-}3)$$

in which:

$$C_{\theta L} = \frac{\left(1+(60/29.42)^2\right)^2}{(60/29.42)^2}\left(1+\frac{0.011\times 4^3}{0.5\left(1+(60/29.42)^2\right)^2}\right)(1) = 6.74 \qquad (4.3\text{-}4)$$

Thus we get:

$$F_{\theta eL} = 6.74\frac{\pi^2 \times 29000}{12(1-0.3^2)}(0.75/60)^2 = 27.6[ksi]$$

The inelastic buckling stress is calculated using plasticity reduction factor in section 5, i.e.,

$$F_{\theta cL} = \eta F_{\theta eL}$$

in which, η is referred to as plasticity reduction factor calculated using Section 5. For local buckling under external pressure, $i = \theta$ and $j = L$; Thus $F_{iej} = F_{\theta eL}$. We get:

$$\eta = \frac{50}{27.6}\left(\frac{1}{1+3.75(50/27.6)^2}\right)^{1/4} = 0.9485 \qquad (5\text{-}3)$$

Thus, the inelastic buckling stress is given by:

$$F_{\theta cL} = 0.9485 \times 27.6 = 26.2[ksi]$$

4.4 Bay Instability of Stringer Stiffened or Ring and Stringer Stiffened Cylinders, and General Instability of Ring and Stringer Stiffened Cylinders Based Upon Orthotropic Shell Theory

4.4.1 Axial Compression or Bending

a. Bay Instability

1. Elastic Buckling Stresses

The calculations in this section will be shown for *m* and *n* pair that minimizes N_{xeB}. The table below shows the N_{xeB} values for various *m* and *n* pair:

n	m	NxeB
1	1	433.01
2	1	427.96
3	1	420.06
4	1	410.02
-	-	-
-	-	-
15	1	318.98
16	1	317.65
17	1	317.32
18	1	317.89
19	1	319.29
20	1	321.44
1	2	1,286.85
2	2	1,285.72
3	2	1,283.89
4	2	1,281.39
5	2	1,278.32

Minimum value reached

Greater when compared to m = 1

As seen in the table above, the minimum N_{xeB} is obtained for *n=17* and *m=1*. Now the process of calculating N_{xeB} will be explained for *n=17* and *m = 1*. The same process can be used to calculate N_{xeB} for any n and m pair. Notice that the value of effective width, b_e, depends on F_{xeB}, see Eq. 4.4-2. Thus, the process of determining N_{xeB} and consequently F_{xeB} is iterative. We start with $b_e = b = 29.42[in]$.

$$j = B \quad A_r = I_r = J_r = 0 \quad L_j = L_r = 60[in]$$

Using the above we get:

$$Y = \left(\frac{m\pi}{L_j}\right)^2 = \left(\frac{1\times\pi}{60}\right)^2 = 2.74\times10^{-3}$$

$$t_x = \frac{A_s + b_e t}{b} = \frac{5 + 29.42\times0.75}{29.42} = 0.92[in]$$

$$L_e = L_r = 60[in]$$

In the following, the terms of Eq. 4.4-1 are determined. The value of Poisson's ratio in Eq. 4.4-1 is determined by the following condition:

$$\upsilon = 0 \quad for \quad L_e < L_r \; or \quad b_e < b$$

$$\upsilon = 0.3 \qquad otherwise$$

Thus $b_e = b$, $\nu = 0.3$ is used in the terms below:

$$C_x = \frac{EA_s Z_s}{b} = \frac{29000 \times 5 \times (-4.675)}{29.4156} = -23044.7$$

$$C_\theta = 0; \quad D_{x\theta} = 2406.62; \quad D_\theta = 1120.36;$$

$$D_x = 130264.32; \quad G_{x\theta} = 8365.4; \quad E_\theta = 23901.1;$$

$$E_{x\theta} = 7170.33; \qquad E_x = 28830.45$$

$$A_{11} = 105.97; \quad A_{22} = 99.88; \quad A_{33} = 1.278;$$

$$A_{12} = 46.15; \quad A_{23} = 4.53; \quad A_{13} = -2.055 \qquad (4.4\text{-}1)$$

Using the above terms, N_{xeB} is obtained as:

$$N_{xeB} = 317.32[k/in]$$

Next, F_{xeB} is determined. We have:

$$\overline{A}_s = A_s/bt = 5/29.42/0.75 = 0.23$$

Thus, imperfection factor is given by:

$$\alpha_{xB} = 0.65$$

Thus, F_{xeB} is given by:

$$F_{xeB} = \alpha_{xB} \frac{N_{xeB}}{t_x} = 0.23 \frac{317.05}{0.92} = 224.2[ksi] \qquad (4.4\text{-}3)$$

Since, $F_{xeB} > F_y$ we have the effective width given by:

$$b_e = 1.9t\sqrt{\frac{E}{F_y}} = 1.9 \times 0.75 \times \sqrt{\frac{29000}{50}} = 34.32[in] \qquad (4.4\text{-}2)$$

Since, $b_e > b$ we have:

$$b_e = b = 29.42[in]$$

Since, the value of b_e at the end of iteration remains the same as the start of iteration, the calculation process of N_{xeB} converges in one iteration.

<u>3. Inelastic Buckling Stresses</u>

The plasticity reduction factor is calculated using Section 5 as:

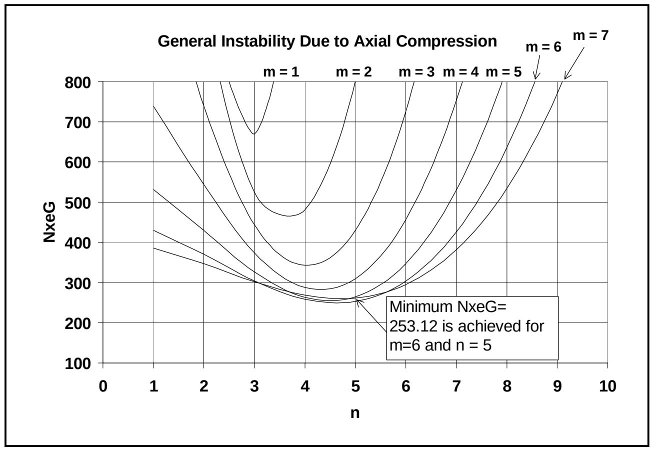
$$\eta = \frac{50}{224.2}\left(\frac{1}{1 + 3.75(50/224.2)^2}\right)^{1/4} = 0.2137 \qquad (5\text{-}3)$$

The inelastic buckling stress is given by:

$$F_{xcB} = \eta F_{xeB} = 0.2137 \times 224.2 = 47.907[ksi]$$

b. General Instability

Similar to Section 4.4.1.a, the calculations in this section will be shown for m and n pair that minimizes N_{xeG}. The figure below shows that minimum N_{xeG} is obtained for m=6 and n=5:



We start again with $b_e = b = 29.42[in]$:

$$L_j = L_b = 600[in]$$

$$Y = \left(\frac{m\pi}{L_j}\right)^2 = \left(\frac{6\times\pi}{600}\right)^2 = 9.87\times10^{-4}$$

$$t_x = \frac{A_s + b_e t}{b} = \frac{5 + 29.42\times0.75}{29.42} = 0.92[in]$$

$$L_e = L_r = 60[in]$$

In the following, the terms of Eq. 4.4-1 are determined. The value of Poisson's ratio in Eq. 4.4-1 are determined by the following condition:

$$\upsilon = 0 \quad for \quad L_e < L_r \; or \quad b_e < b$$

$$\upsilon = 0.3 \qquad otherwise$$

Since $b_e = b$, $\nu = 0.3$ is used in the terms below:

$$C_x = \frac{EA_s Z_s}{b} = \frac{29000\times5\times(-4.675)}{29.4156} = -23044.7$$

$$C_\theta = -103085.94; \quad D_{x\theta} = 3238.1; \quad D_\theta = 1370077.1;$$

$$D_x = 130264.32; \quad G_{x\theta} = 8365.4; \quad E_\theta = 32963.6;$$

$$E_{x\theta} = 7170.3; \qquad E_x = 28830.45$$

$$A_{11} = 30.78; \quad A_{22} = 17.44; \quad A_{33} = 0.41;$$

$$A_{12} = 8.15; \quad A_{23} = 1.36; \quad A_{13} = -0.037 \qquad (4.4\text{-}1)$$

Using the above terms, N_{xeG} is obtained as:

$$N_{xeG} = 294.65$$

Next, F_{xeG} is determined. We have

$$\overline{A}_r = A_r / L_r t = 18.75/60/0.75 = 0.4167$$

Thus, imperfection factor is given by:

$$\alpha_{xG} = 0.72 \qquad (4.2\text{-}2)$$

F_{xeG} is given by:

$$F_{xeG} = \alpha_{xG} \frac{N_{xeG}}{t_x} = 0.72 \frac{294.65}{0.92} = 230.6[ksi] \qquad (4.4\text{-}5)$$

The inelastic general instability stress is determined using plasticity reduction factor:

$$\eta = \frac{50}{230.6}\left(\frac{1}{1+3.75(50/230.6)^2}\right)^{1/4} = 0.208 \qquad (5\text{-}3)$$

Thus, the inelastic buckling stress is given by:

$$F_{xcG} = 0.208 \times 230.6 = 48.01[ksi]$$

The effective width is now determined using Eq. 4.4-4, used in this equation was determined in Section 4.3.1:

$$b_e = b\sqrt{\frac{F_{xcL}}{F_{xcG}}} = 29.42\sqrt{\frac{37.93}{48.01}} = 26.15[in] \qquad (4.4\text{-}4)$$

This completes the first iteration, at the end of which we have a new value of b_e which is not equal the value of b_e at the start of iteration. We start the second iteration with the new effective width b_e:

$$b_e = 26.15[in]$$

$$t_x = \frac{A_s + b_e t}{b} = \frac{5 + 26.15 \times 0.75}{29.42} = 0.84[in]$$

Since $b_e < b$, $\nu=0$ is used in the terms below:

$$\begin{aligned}
&C_x = -23044.7\\
&C_\theta = -103085.94; \quad D_{x\theta} = 2478.71; \quad D_\theta = 1369976.24;\\
&D_x = 130050.20; \quad G_{x\theta} = 7900.63; \quad E_\theta = 30812.50;\\
&E_{x\theta} = 0; \quad E_x = 24262.62\\
&A_{11} = 26.15; \quad A_{22} = 16.38; \quad A_{33} = 0.39;\\
&A_{12} = 4.14; \quad A_{23} = 1.24; \quad A_{13} = -0.72
\end{aligned} \qquad (4.4\text{-}1)$$

Using the above terms, N_{xeG} is obtained as:

$$N_{xeG} = 253.01[k/in]$$

F_{xeG} is given by:

$$F_{xeG} = \alpha_{xG} \frac{N_{xeG}}{t_x} = 0.72 \frac{253.01}{0.84} = 217.74[ksi] \qquad (4.4\text{-}5)$$

The inelastic general instability stress is determined using plasticity reduction factor:

$$\eta = \frac{50}{217.74}\left(\frac{1}{1+3.75(50/217.74)^2}\right)^{1/4} = 0.22 \qquad (5\text{-}3)$$

Thus, the inelastic buckling stress is given by:

$$F_{xcG} = 0.22 \times 217.74 = 47.79[ksi]$$

The effective width becomes:

$$b_e = b\sqrt{\frac{F_{xcL}}{F_{xcG}}} = 29.42\sqrt{\frac{37.93}{47.79}} = 26.21[in] \qquad (4.4\text{-}4)$$

This completes the second iteration, after which the effective width converges to first decimal place in the effective width. The table below shows the convergence up to four decimal places:

Iter. no	*be*	*NxeG*	
1	29.4156	294.6492	
2	26.1471	253.0114	
3	26.2062	253.1216	
4	26.2077	253.1244	
5	26.2077	253.1244	← Converged

Notice that N_{xeG} converges to fourth decimal places in five iterations. The final values of elastic and inelastic general instability stresses are given by:

$$F_{xeG} = 217.43[ksi]$$

$$F_{xcG} = 47.79[ksi]$$

4.4.2 External Pressure

a. Bay Instability

1. Elastic Buckling Stresses

Similar to Section 4.4.1.a, the calculations in this section will be shown for m and n pair that minimizes $N_{\theta eB}$. The table below shows the $N_{\theta eB}$ values for various m and n pair:

n	*m*	NθeB	
1	1	106,573.70	
2	1	26,332.57	
3	1	11,487.54	
4	1	6,307.20	
-	-		
-	-		
47	1	70.93	
48	1	70.69	
49	1	70.55	
50	1	70.52	← Minimum value reached
51	1	70.58	
52	1	70.73	
1	2	1,266,899.24	
2	2	316,447.30	
3	2	140,442.04	
4	2	78,845.21	
5	2	50,339.99	↓ Greater when compared to m = 1

As seen in the table above, the minimum $N_{\theta eB}$ is obtained for $n=50$ and $m=1$. Now the process of calculating $N_{\theta eB}$ will be explained for $n=50$ and $m = 1$. The same process can be used to calculate $N_{\theta eB}$ for any n and m pair. Notice that the value of effective width, b_e, remains constant. Hence, no iterations will be needed in the process of determining $N_{\theta eB}$.

$$j = B \quad A_r = I_r = J_r = 0 \quad L_j = L_r = 60[in]$$
$$L_e = L_r = 60[in] \qquad b_e = b = 29.42[in]$$

We get using the above:

$$k = 0$$
$$Y = k\left(\frac{m\pi}{L_j}\right)^2 + \left(\frac{n}{R}\right)^2 = \left(\frac{50}{299.625}\right)^2 = 2.785\times10^{-2}$$

In the following, the terms of Eq. 4.4-1 are determined. The value of Poisson's ratio in Eq. 4.4-1 are determined by the following condition:

$$\upsilon = 0 \quad for \quad L_e < L_r \; or \quad b_e < b$$
$$\upsilon = 0.3 \qquad otherwise$$

Since $b_e = b$, $\nu = 0.3$ is used in the terms below:

$$C_x = \frac{EA_s Z_s}{b} = \frac{29000\times5\times(-4.675)}{29.4156} = -23044.7$$
$$C_\theta = 0; \quad D_{x\theta} = 2406.62; \quad D_\theta = 1120.36;$$
$$D_x = 130264.32; \quad G_{x\theta} = 8365.4; \quad E_\theta = 23901.1;$$
$$E_{x\theta} = 7170.33; \qquad E_x = 28830.45$$
$$A_{11} = 311.99; \quad A_{22} = 688.52; \quad A_{33} = 2.298;$$
$$A_{12} = 135.74; \quad A_{23} = 13.31; \quad A_{13} = -2.055 \qquad (4.4\text{-}1)$$

Using the above terms, $N_{\theta eB}$ is obtained as:

$$N_{\theta eB} = 70.52[k/in]$$

Next, F_{reB} is determined. We have from Section 11:

$$K_{\theta L} = 0.77$$

Imperfection factor is given by:

$$\alpha_{xB} = 1$$

Thus, F_{xeB} is given by:

$$F_{reB} = \alpha_{\theta B}\frac{N_{\theta eB}}{t}K_{\theta L} = 1\frac{70.52}{0.75}0.77 = 72.61[ksi] \qquad (4.4\text{-}6)$$

3. Inelastic Buckling Stresses

The plasticity reduction factor is calculated using Section 5 as:

$$\eta = \frac{50}{72.61}\left(\frac{1}{1+3.75(50/72.61)^2}\right)^{1/4} = 0.53 \qquad (5\text{-}3)$$

The inelastic buckling stress is given by:

$$F_{rcB} = \eta F_{reB} = 0.53\times72.61 = 38.73[ksi]$$

b. General Instability

Similar to previous sections, the calculations in this section will be shown for *m* and *n* pair that minimizes $N_{\theta eG}$. The table below shows that minimum $N_{\theta eG}$ is obtained for m=1 and n=3:

n	m	NθeG	
1	1	7,195.05	
2	1	511.21	
3	1	136.95	Minimum value reached
4	1	145.87	
1	2	12,889.30	
2	2	1,619.47	
3	2	377.74	
4	2	202.51	
5	2	228.01	
1	3	15,344.40	Greater when compared to m = 1
2	3	2,503.52	
3	3	674.98	
4	3	301.73	
5	3	262.85	
6	3	325.72	

As with bay instability stress under external pressure, no iterations will be needed in the process of determining $N_{\theta eG.}$

$$j = B \quad A_r = I_r = J_r = 0 \quad L_j = L_r = 60[in]$$

$$L_e = 1.56\sqrt{Rt} = 23.39 \qquad b_e = b = 29.42[in]$$

We get using the above:

$$k = 0$$

$$Y = k\left(\frac{m\pi}{L_j}\right)^2 + \left(\frac{n}{R}\right)^2 = \left(\frac{3}{299.625}\right)^2 = 1.00\times10^{-4}$$

Since $b_e = b$, $\nu=0.3$ is used in the terms below:

$$C_x = \frac{EA_sZ_s}{b} = \frac{29000\times5\times(-4.675)}{29.4156} = -23044.7$$

$$C_\theta = -103085.94; \quad D_{x\theta} = 2565.86; \quad D_\theta = 1369976.24;$$

$$D_x = 130163.49; \quad G_{x\theta} = 8365.4; \quad E_\theta = 30812.5;$$

$$E_{x\theta} = 0; \qquad E_x = 26679.36$$

$$A_{11} = 1.314; \quad A_{22} = 1.918; \quad A_{33} = 0.14;$$

$$A_{12} = 0.305; \quad A_{23} = 0.483; \quad A_{13} = -3.308 \qquad (4.4\text{-}1)$$

Using the above terms, $N_{\theta eG}$ is obtained as:

$$N_{\theta eG} = 136.95[k/in]$$

Next, F_{reG} is determined. We have from Section 11:

$$k_{\theta G} = 0.701$$

Imperfection factor is given by:

$$\alpha_{\theta G} = 0.8$$

Thus, F_{reG} is given by:

$$F_{reG} = \alpha_{\theta G} \frac{N_{\theta eG}}{t} K_{\theta G} = \qquad (4.4\text{-}7)$$
$$= 0.8 \frac{136.95}{0.75} 0.701 = 102.4[ksi]$$

The inelastic general instability stress is determined using plasticity reduction factor:

$$\eta = \frac{50}{102.4}\left(\frac{1}{1+3.75(50/102.4)^2}\right)^{1/4} = 0.42 \qquad (5\text{-}3)$$

Thus, the inelastic buckling stress is given by:

$$F_{rcG} = 0.42 \times 102.4 = 42.62[ksi]$$

4.5 Bay Instability of Stringer Stiffened and Ring and Stringer Stiffened Cylinders - Alternate Method

This is section is used to size stringers when the number of stringers is less than about 3n and the bay instability stress is greater than 1.5 times the local shell buckling stress.

4.5.1 Axial Compression or Bending

The elastic bay instability stress is given by the equation 4.5-1:

$$F_{xeB} = \frac{\alpha_{xL} C_x E 2t/D}{1 + A_s/bt} + \frac{\pi^2 E I'_{es}}{(b_{eu}t + A_s)L_r^2}$$

The terms in the above equation are determined in the following sequence:

$\alpha_{xL} C_x = 0.46$ (4.5-12)

$\sigma_{xeL} = 75.32[ksi]$ (4.5-7)

$\rho_\eta = 0.90$ (4.5-8)

$\lambda_\eta = 0.86$ (4.5-10)

$B = 1.13$ (4.5-9)

$\sigma_e = 76.52$ (4.5-6)

$\lambda_0 = 0.81$ (4.5-5)

$R_r = 0.85$ (4.5-11)

$b'_e = 16.41$ (4.5-4)

$b_{eu} = 21.79$ (4.5-3)

$I'_{es} = 100.01$ (4.5-2)

The above terms give the elastic bay instability stress as:

$$F_{xeB} = 399.97[ksi]$$

The inelastic general instability stress is determined using plasticity reduction factor:

$$\eta = \frac{50}{399.97}\left(\frac{1}{1+3.75(50/399.97)^2}\right)^{1/4} = 0.12 \qquad (5\text{-}3)$$

Thus, the inelastic buckling stress is given by:

$$F_{xcB} = 0.12 \times 399.97 = 49.29[ksi]$$

We have the failure load calculated as:

$$P_{cB} = 67{,}516[kip] \qquad (4.5\text{-}14)$$

The effective shell width used in above equation is given by Eq. 4.5-13:

$$b_e = 21.87[in]$$

4.5.2 External Pressure

The inelastic bay instability stress is given by the equation 4.5-15:

$$F_{rcB} = \frac{p_{cB} R_0}{t} K_{\theta L}$$

Where, p_{cB} is given by:

$$p_{cB} = (p_{cL} + p_s) k_p$$

In the above equation, p_{cL} is found using inelastic local buckling stress of a ring-stiffened cylinder:

$$F_{rcL} = 19.8[ksi]:$$

See Eq. 4.5-1 in solved ring stiffened shell example problem. p_{cL} is given by:

$$p_{cL} = F_{rcL} t / R_0 = 0.0495 \qquad (4.5\text{-}17)$$

The term p_s and K_p are calculated as:

$$p_s = 0.18$$

$$K_p = 0.3465$$

Using the above we get:

$$p_{cL} = (0.0495 + 0.18) \times 0.3465 = 0.0783$$

The value of $k_{\square L}$ was calculated in Section 11 and is rewritten below as:

$$K_{\theta L} = 0.77$$

Using the terms given in foregoing, the elastic bay instability stress is determined as:

$$F_{rcB} = \frac{0.0783 \times 300}{0.75} 0.77 = 24.20[ksi]$$

Using Section 5, the elastic buckling stress can be back calculated. The equivalent of Eqs. 5.1 and 5.2 is given by:

$$F_{reB} = F_y \sqrt{\left[3.75 / \left(\left(F_y / F_{rcB} \right)^4 - 1 \right) \right]} \quad if \quad F_{rcB} > 0.5 F_y$$

$$F_{reB} = F_{rcB} \quad Otherwise$$

Using the above we get:

$$F_{reB} = 24.20[ksi]$$

<u>Summary of Buckling Stresses</u>

The buckling stresses for ring and stringer stiffened shells are now summarized in the table below:

	Buckling Mode	Elastic (ksi)	Inelastic (ksi)	Valid
Axial Compression	Local	68.16	37.93	
	Bay (Sec. 4.4)	224.2	47.9	Yes
	Bay (Sec. 4.5)	399.97	49.29	
	General (Sec. 4.4)	217.43	47.79	Yes
	General (Sec. 4.2)	37.64	30.10	
External Pressure	Local	27.60	26.18	
	Bay (Sec. 4.4)	72.61	38.73	No(1)
	Bay (Sec. 4.5)	24.20	24.20	
	General (Sec. 4.4)	102.4	42.62	Yes
	General (Sec. 4.2)	93.77	41.7	

(1)Note: Bay instability stress under <u>pressure</u> per Section 4.4 is invalid since number of stringer $N_s=64$ is smaller than $3n$ for $n=50$.

6.0 Predicted Shell Buckling Stresses for Combined Loads

6.1 General Load Cases

The values of N_ϕ and N_θ is given by:

$$N_\phi = \frac{P}{2\pi R} = \frac{9000}{2\pi \times 299.625} = 4.78[k/in]$$

$$N_\theta = pR_0 = 0.0267 \times 300 = 8.01[k/in]$$

Process of determining combined buckling stresses was explained in ring stiffened shell example. A similar process is used in ring and stringer stiffened shells:

<u>Summary of Combined Buckling Stresses</u>

<table>
<tr><td colspan="2">Buckling Mode</td><td>Combined Inelastic Stress (ksi)</td></tr>
<tr><td rowspan="3"><u>Axial Load</u></td><td>Local</td><td>16.89</td></tr>
<tr><td>Bay</td><td>14.90</td></tr>
<tr><td>General</td><td>32.87</td></tr>
<tr><td rowspan="3">External Pressure</td><td>Local</td><td>21.83</td></tr>
<tr><td>Bay</td><td>23.62</td></tr>
<tr><td>General</td><td>43.09</td></tr>
</table>

9.0 Allowable Stresses

The factor of safety for normal operating conditions is given by:

$$F.S = 1.25\psi$$

in which ψ is calculated using Eq. 9.1. Since we have axial compression and hoop compression, the allowable stresses are calculated using Eq. 9.1-5. The allowable axial load and external pressure for local and general instability modes are given by:

Summary of Allowable Stresses

Buckling Mode		Allowable Stresses (ksi)
Axial Load	Local	11.26
	Bay	9.93
	General	22.82
External Pressure	Local	14.55
	Bay	15.75
	General	32.48

We have the applied stresses given by:

$$f_a = \frac{P}{2\pi Rt + Q_a N_s A_s}$$

Since the effective width of plate attached to stringer is different for different buckling modes, the applied axial load is different for each mode.

Local Buckling

For local buckling, full width between stringers is used:

$$b_e = b$$

$$Q_a = 1$$

$$f_a = 5.2[ksi]$$

Bay Instability

For bay instability, since Section 4.4.1 is not valid, we pick effective width from Section 4.5.1, Eq. 4.5-13, thus:

$$b_e = 21.87[in]$$

$$Q_a = 0.79$$

$$f_a = 6.57[ksi]$$

General Instability

For general instability we use Section 4.4.1.b:

$$b_e = 26.21[in]$$

$$Q_a = 0.91$$

$$f_a = 5.7[ksi]$$

The stress midway between rings and at ring is given using Section 11 by:

$$f_{\theta S} = 8.24[ksi]$$

$$f_{\theta R} = 7.48[ksi]$$

Summary of Unity Ratios

The combined inelastic stresses, factor of safety, allowable stresses, applied stress and unity check ratios are summarized in table below:

Buckling Mode		Combined Inelastic Stresses	ψ	F.S	Allowable Stresses	Applied Stress	Unity Ratio
Axial Load	Local	16.89	1.2	1.5	11.26	5.2	0.46
	Bay	14.90	1.2	1.5	9.93	6.57	0.66
	General	32.87	1.14	1.42	23.13	5.7	0.25
External Pressure	Local	21.83	1.2	1.5	14.55	8.24	0.73
	Bay	23.62	1.2	1.5	15.75	8.24	0.52
	General	43.09	1.06	1.32	32.67	7.48	0.23

The above values indicate that the design is acceptable with regard to buckling resistance.

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