Bulletin on Comparison of Marine Drilling Riser Analyses

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API Bulletin on COMPARISON OF ANALYSES OF MARINE DRILLING RISERS

Foreword

a. Recognizing the need for a standard method of analyzing marine drilling risers, the API Committee on the Standardization of Offshore Structures began a program of comparing existing computer programs for such analyses. To make this comparison, a standard set of problems was defined and a Task Group formed by the Committee on Standardization of Offshore Structures obtained computer solutions of this set from various participants. The First Edition of API RP 2J was issued in January 1977. This Edition now under the jurisdiction of the Committee on Standardization of Drilling Well Control Systems and renumbered Bul 16J has added a number of deepwater cases along with some random wave cases. The number of participants has increased from nine to 14. The statistical summaries of these solutions are presented in this bulletin.

b. The purposes of issuing these results are: (1) to show the degree of agreement among a representative group of riser analysis computer programs, and (2) to present data which can be used to help validate other such programs.

c. In cases where the results of a participant differed significantly (some 25% or more) from the consistently clustered results of the majority of the participants, results were omitted. Numerous attempts were made by the committee members to assist participants in correcting input errors, problem definitions, and other likely sources of disagreement. The omissions were handled on a problem by problem basis, i.e., the number of solutions used to compile the results for each problem varies. The number of solutions included for each individual problem is indicated throughout the tables and figures in parentheses following the problem designation.

d. None of these results have been directly validated by measurements on equivalent real risers, although some of the programs have been partially validated in other applications. Comparisons such as those described here are not intended to replace or lessen the need for such validation.

e. American Petroleum Institute (API) Bulletins are published to provide information for which there is a broad industry need but which does not constitute either Specifications or Recommended Practices.

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Note:
This is the first edition of this bulletin. It was formerly numbered API Bul 2J.

SECTION 1
DESCRIPTION OF CASES

General

1.1 There are a total of forty cases as shown in Table 1, DESIGNATION OF CASES. All but two are for 500-, 1500-, and 3000-ft water depths.

1.2 Static cases are analyzed for three water depths, two current profiles and two top tensions. Also included are three disconnected cases, one at each depth, and a 6000-ft water depth case.

1.3 Periodic dynamic cases are analyzed for three water depths, two current profile/wave height combinations, and two top tensions. Also included are three disconnected cases, one at each depth, and a 6000-ft water depth case. There are also two cases, 1500-40-1-D2 and 1500-40-2-D2, for which the vessel surge phase angle is 180° greater than that of the other cases, i.e., the peak vessel surge leads the wave crest. These cases are intended to illustrate the importance of phase angle and to provide results that may be more useful for checking hydrodynamic force formulations.

1.4 Random dynamic cases are analyzed for three water depths, one current profile/wave height combination, and two top tensions.

1.5 The input data specifications for the 500-, 1500-, and 3000-ft water depth cases are given in Table 2. Table 3 shows input data specifications for the 6000-ft cases. Figure 1 shows typical riser configurations and Figure 2 shows a cross section of the riser. Figure 3 is the vessel's surge response amplitude operator (RAO) to be used for the random cases.

1.6 The 21-inch and 18⅝-inch risers are assumed to contain drilling mud only — with no drill pipe or other internal connections. For all cases, the choke and kill lines are assumed to be filled with seawater to the mean water level. The riser is attached at its lower end to a pin connection, the lower ball joint. At its top, it is restrained by another pin connection at the tensioner ring. Both joints are frictionless, and for simplicity, the telescopic joint is assumed to have the same physical properties as the rest of the riser. The riser has no upper, or intermediate, ball joint.

1.7 For disconnected riser cases, the riser is assumed to contain sea water, with the lower marine riser package (LMRP) twenty feet above the sea floor. Included in the LMRP is a pin connection, the lower ball joint, ten feet above its bottom (thirty feet above the sea floor).

1.8 Furthermore, it is assumed that all cases are for a semisubmersible drilling vessel with the upper end of the riser exposed to wave forces. Wave and current forces are assumed to act in the same direction as the positive offset. Choke and kill lines are broadside to waves and current, as shown in Figure 2, and are assumed to contribute no stiffness to the riser.

Static Cases

1.9 The static cases are for current plus offset only — no wave forces. The remaining input data, including current profiles, are given in Tables 2 and 3.

1.10 Note that the 500- and 1500-ft cases are with a 20-inch unbuoyed riser, the 3000-ft cases with a 21-inch buoyed riser, and the 6000-ft case with an 18⅝-inch buoyed riser.

Dynamic Cases

1.11 The dynamic cases are similar to the static cases, with the addition of time-varying wave forces. Dynamic cases are for combined wave and current. Wave propagation is in the direction of positive offset. Input data specifications for the two periodic waves and the random waves are included in Tables 2 and 3. The periodic wave model is not specified. Top tension is assumed to be constant throughout the wave cycle.

1.12 For the periodic wave cases, horizontal motion (surge) is assumed to be sinusoidal with the same period as the wave. For those dynamic cases with the designation "D," the surge motion lags the wave crest. For "D2" cases, the surge motion leads the crest. For the random wave cases, the vessel's surge response amplitude operator (RAO) is shown in Figure 3.

TABLE 1
DESIGNATION OF CASES

Water Depth (Feet)	Top Tension (Kips)	Current Profile A		Current Profile B		
		Static	Periodic Dynamic (Wave Height 20 Ft)	Static	Periodic Dynamic (Wave Height 40 Ft)	Random Dynamic (Significant Wave Height 21.5 Ft)
500	170	500-A-1-S	500-20-1-D	500-B-1-S	500-40-1-D	500-21.5-1-R
	240	500-A-2-S	500-20-2-D	500-B-2-S	500-40-2-D	500-21.5-2-R
	Disconnected			500-B-FREE-S	500-40-FREE-D	
1500	370	1500-A-1-S	1500-20-1-D	1500-B-1-S	1500-40-1-D	1500-21.5-1-R
					1500-40-1-D2	
	600	1500-A-2-S	1500-20-2-D	1500-B-2-S	1500-40-2-D	1500-21.5-2-R
					1500-40-2-D2	
	Disconnected			1500-B-FREE-S	1500-40-FREE-D	
3000	500	3000-A-1-S	3000-20-1-D	3000-B-1-S	3000-40-1-D	3000-21.5-1-R
	650	3000-A-2-S	3000-20-2-D	3000-B-2-S	3000-40-2-D	3000-21.5-2-R
	Disconnected			3000-B-FREE-S	3000-40-FREE-D	
				Current Profile C		
6000	700			6000-C-1-S	6000-40-1-D	

500-A-1-S

Water Depth:	500, 1500, 3000, or 6000 ft	
Current Profile:	A, B, or C	
Static:	Current plus offset only	
Top Tension:	1 -	Low tension
	2 -	High tension
	FREE -	Disconnected

500-20-1-D

Water Depth:	500, 1500, 3000, or 6000 ft	
Wave and Current:	20 -	20 ft Regular Wave (Current A)
	40 -	40 ft Regular Wave (Current C: 6000 ft; Current B: all others)
	21.5 -	21.5 ft Significant, Random Wave (Current B)
Dynamic:	D -	Periodic Dynamic, ϕ - -90
	D2 -	Periodic Dynamic, ϕ - +90
	R -	Random Dynamic
Top Tension:	1 -	Low Tension
	2	High Tension
	FREE -	Disconnected

TABLE 2
INPUT DATA SPECIFICATIONS
500, 1500, AND 3000-FT CASES

I. CONSTANT WITH WATER DEPTH

Vertical Distances, Feet

Mean water level to riser tensioner ring	50
Sea floor to lower ball joint	30

Riser Data[1]

Diameters, Inches

r_o, riser pipe outside diameter	21.0
r_i, riser pipe inside diameter	20.0
c_o & k_o, choke and kill line outside diameters	4.0
c_i & k_i, choke and kill line inside diameters	3.0
b_o, buoyant material outside diameter (3000-ft water depth cases only)	38.0
Choke and Kill Lines Offset (Centerline to Centerline of Riser), Inches	15.88
Modulus of Elasticity of Riser Pipe, E, psi x 10^6	30

Densities, pounds/ cubic foot

Seawater	64.0
Drilling mud[2]	89.8 (12 ppg)

Hydraulic Force Constants

C_d, drag coefficient	0.7
C_m, mass coefficient[3]	1.5
d_e, effective diameter for current and wave forces, inches - unbuoyed riser	29.0
buoyed riser	38.0

Weight, pounds, of 50-foot joint, complete with all associated lines, couplings, and buoyant material, if any

	Wa, in air	Wp, in seawater
Unbuoyed 21-in	8800	7660
Buoyed 21-in (3000-ft water depth cases)	14740	2950

Current Profile

A. Linear, 1/2 knot at mean water level, zero at lower ball joint.

B. Linear, 2 knot at mean water level, 0.4 knot at lower ball joint.

II. VARYING WITH WATER DEPTH

D, water depth, feet	500	1500	3000
L, riser length, feet	520	1520	3020
T, top tension, Kips[4]	170	370	500
	240	600	650
O_s, static offset, feet	15	45	90

III. DYNAMIC-PERIODIC REGULAR WAVE

H, wave height, peak to trough, feet	20	40
T, wave period, seconds	9	12.8
V, vessel surge amplitude (peak to peak), ft	4	26.7
ϕ, vessel surge phase angle, degrees[5]	-90	-90
Current profile	A	B

NOTE:
Cases 1500-40-1-D2 and 1500-40-2-D2 are for ϕ = +90. (Peak vessel surge leads wave crest.)

IV. DYNAMIC-RANDOM WAVE (Pierson-Moskowitz Spectrum)

H_s, significant wave height, feet	21.5
T_z, zero upcrossing period, seconds	12.8
V_s, vessel surge amplitude	From Figure 3
ϕ, vessel surge phase angle	From Figure 3
Current profile	B

V. DISCONNECT CASES

Riser Properties

Same as connected cases (Data in I. above)
Riser pinned at top, pin at LMRP ball joint
Contents: Seawater to mean water level
Static offset: 0
Dynamic analysis for 40-ft periodic regular wave only (Data in III. above)

Lower Marine Riser Package (LMRP) Data

LMRP length, feet	10
W_d, in air, pounds	150000
W_w, in seawater, pounds	120000
C_d, drag coefficient	0.7
C_m, mass coefficient[3]	1.5
d_e, effective diameter for current and wave forces, inches	36.0

Vertical Distances, Feet

Sea floor to bottom of LMRP (Ball joint at top of LMRP)	20

[1] Assume constant properties over entire length. Choke and kill lines are on opposite sides of riser. Riser is oriented so tubes are broadside to current/wave. Weight or other effects above tensioner ring are ignored.

[2] Riser is filled with drilling mud to top of riser. For simplicity, assume choke and kill lines filled with seawater to mean water level.

[3] This is the mass coefficient used in Morison's equation and defined in his original paper on the subject. Morison, J. R., O'Brien, M. P., Johnson, J. W., and Schaaf, S. A. (1950). "The Force Exerted by Surface Waves on Piles", Petroleum Transactions of AIME, Vol. 189, pages 149-154.

[4] Assume constant (with time) top tension.

[5] Negative phase angle is peak vessel surge after wave crest. (Assume surge has same period as wave.)

TABLE 3
INPUT DATA SPECIFICATIONS
6000-FT CASES

Vertical Distances, Feet

Mean water level to riser tensioner ring	50
Sea floor to lower ball joint	30

Riser Data[1]

Diameters, Inches

r_o, riser pipe outside diameter	18.63
r_i, riser pipe inside diameter	17.63
c_o & k_o, choke and kill line outside diameters	4.0
c_i, & k_i, choke and kill line inside diameters	3.0
b_o, buoyant material outside diameter	38.0
Choke and Kill Lines Offset (Centerline to Centerline of Riser), Inches	14.75
Modulus of Elasticity of Riser Pipe, E, psi x 10^6	30

Densities, Pounds/Cubic Foot

Seawater	64.0
Drilling Mud[2]	89.8 (12 ppg)

Hydraulic Force Constants

C_d, drag coefficient	0.7
C_m, mass coefficient[3]	1.5
d_e, effective diameter for current and wave forces, inches	38.0

Weight, Pounds, of 50-foot joint, complete with all associated lines, couplings, and buoyant material,

	Wa, in air	Wp, in seawater
18-5/8-in, buoyed	16350	1300

Current Profile

C. Linear, 2 knot at mean water level, 1 knot at 150 ft below MWL, 0.4 knot at lower ball joint.

Model

D, water depth feet	6000
L, riser length, feet	6020
T, top tension, Kips[4]	700
O_s, static offset, feet	180
H, wave height, peak to trough, feet	40
T, wave period, seconds	12.8
V, vessel surge amplitude (peak to peak), feet	26.7
ϕ, vessel surge phase angle, degrees[5]	-90
Current Profile	C

[1]Assume constant properties over entire length. Choke and kill lines are on opposite sides of riser. Riser is oriented so tubes are broadside to current/wave. Weight or other effects above tensioner ring are ignored.

[2]Riser is filled with drilling mud to top of riser. For simplicity, assume choke and kill lines filled with seawater to mean water level.

[3]This is the mass coefficient used in Morison's equation and defined in his original paper on the subject. Morison, J. R., O'Brien, M. P., Johnson, J. W., and Schaaf, S. A. (1950). "The Force Exerted by Surface Waves on Piles", Petroleum Transactions of AIME, Vol. 189, pages 149-154.

[4]Assume constant (with time) top tension.

[5]Negative phase angle is peak vessel surge after wave crest. (Assume surge has same period as wave.)

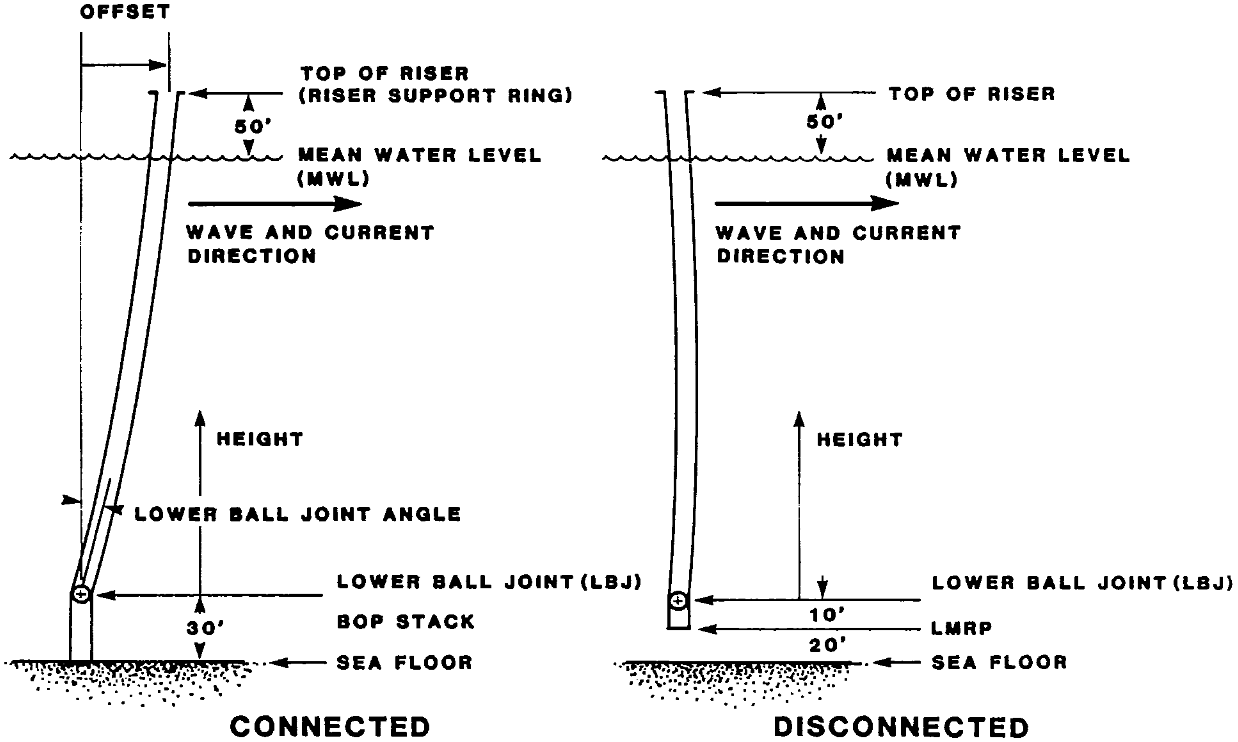


FIGURE 1
TYPICAL RISER CONFIGURATIONS

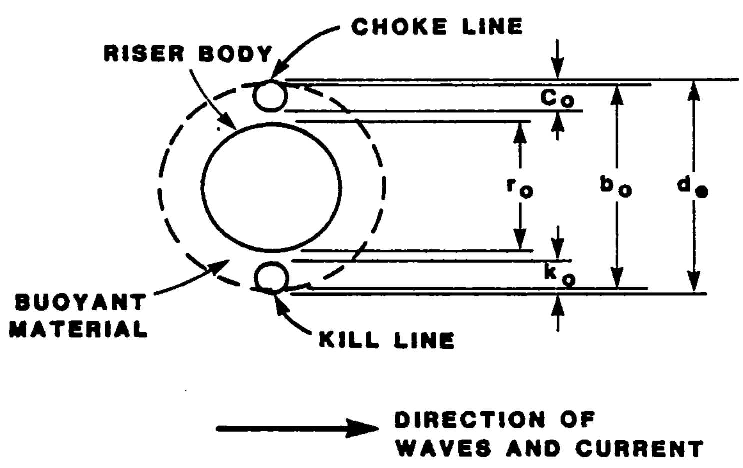
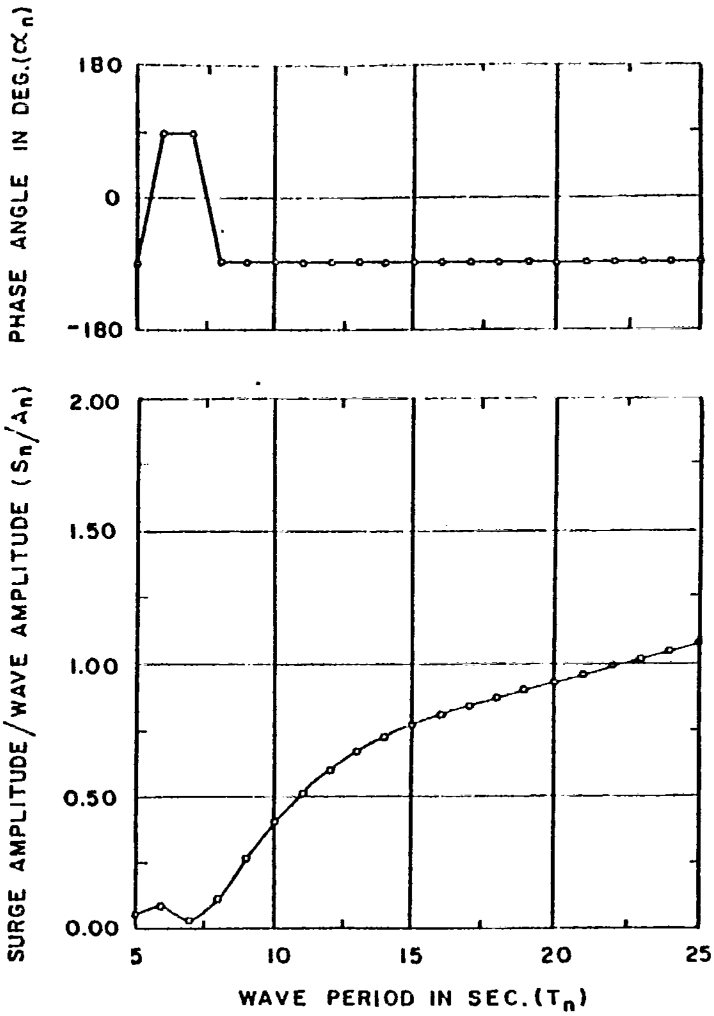


FIGURE 2
CROSS SECTION OF RISER



T_n (sec)	S_n/A_n	α_n (deg)
5	.06	-90
6	.09	+90
7	.04	+90
8	.12	-90
9	.26	-90
10	.40	-90
11	.51	-90
12	.60	-90
13	.67	-90
14	.72	-90
15	.76	-90
16	.80	-90
17	.84	-90
18	.87	-90
19	.91	-90
20	.94	-90
21	.96	-90
22	.99	-90
23	1.02	-90
24	1.06	-90
25	1.09	-90

FIGURE 3
VESSEL RESPONSE OPERATOR*

NOTE: Negative phase angles are for peak vessel surge after the wave crest.

* Sexton, R.H. and Agbezuge, L.K. (1979), Random Wave and Vessel Motion Effects on Drilling Riser Dynamics, Offshore Technology Conference, Paper No. OTC-2650.

SECTION 2
RESULTS

General

2.1 The computer programs used to obtain the results described here were developed by different organizations, using varying modeling and mathematical techniques. The programs are all based on the concept of a laterally loaded vertical beam with axial tension varying over its length. Some programs simulate static or steady state conditions only while others simulate dynamic or time-varying effects. The dynamic programs use either frequency or time domain solutions.

2.2 For dynamic cases with periodic waves, maximum and minimum deflection and bending stress were the parameters chosen for comparative purposes. For random waves, mean and RMS values were used. All dynamic cases compare top and bottom angles as well. For static cases, maximum bending stress, maximum total stress, and top and bottom riser angles were chosen. The premise was that agreement among these parameters would indicate the degree of similarity of the various solutions.

Static Analyses

2.3 Fourteen participants submitted solutions to the static cases. Since the general character of the deflection and stress profiles were basically similar for all solutions, results are conveniently presented in tabular form in Table 4.

2.4 Table 4 provides a statistical analysis of maximum bending stress, maximum total stress, and angles at the lower ball joint and the top of the riser. The statistical parameters are defined in Appendix A.

Periodic Dynamic Analyses

2.5 Fourteen participants submitted solutions to the periodic dynamic cases. Unlike the static solutions, the dynamic deflection and stress profiles varied too much for a simple tabular comparison to be meaningful. Therefore, the one-standard-deviation envelopes of the computed profiles have been plotted and are presented as Figures 4 through 39. The purpose of these plots is to show what is judged to be the correct solution along with the general level of agreement among the various solutions, rather than compare specific solutions.

2.6 The envelopes plotted bracket the parameter (deflection or stress) through one or more wave cycles. For example, in case 500-20-1-D, the top deflection is specified as 15 feet (static) with a dynamic oscillation of four feet (± two feet) about this static value. An examination of the deflection profile for this case (Figure 4) shows that, at the top of the riser, the deflection profiles do converge within the specified oscillation limits of ± 2 feet. Each solution consists of two envelopes which describe the left and right hand excursions of the parameter plotted.

2.7 The top and bottom riser angle extremes are tabulated in Table 5.

Random Dynamic Analyses

2.8 Six participants submitted solutions to the random dynamic cases. Profile envelopes are presented in Figures 40 through 51. Note, as pointed out in Section 1, that mean and RMS values have been substituted for the minimum and maximum values used in the periodic dynamic runs.

2.9 The top and bottom riser angle results are tabulated in Table 5.

TABLE 4
RESULTS — STATIC ANALYSES

CASE	MAX BENDING STRESS VALUE	MAX BENDING STRESS LOC	MAX TOTAL STRESS VALUE	MAX TOTAL STRESS LOC	ANGLE FROM VERTICAL LBJ	ANGLE FROM VERTICAL TOP
500-A-1-S (10)						
MEAN	2.05	127.40	5.69	444.90	2.51	1.00
STANDARD DEVIATION	0.09	6.22	0.15	27.22	0.03	0.04
COEF. OF VARIATION%	4.53	4.88	2.61	6.12	1.24	3.62
RANGE	0.36	21.00	0.52	90.00	0.10	0.12
500-A-2-S (11)						
MEAN	1.14	126.27	7.75	470.91	2.17	1.22
STANDARD DEVIATION	0.05	6.99	0.08	19.62	0.02	0.02
COEF. OF VARIATION%	4.77	5.53	1.02	4.17	0.71	1.45
RANGE	0.23	26.00	0.31	75.00	0.04	0.06
500-B-FREE-S (5)						
MEAN	1.59	391.20	7.19	413.20	-0.03	-1.04
STANDARD DEVIATION	0.03	14.04	0.08	21.53	0.02	0.03
COEF. OF VARIATION%	1.63	3.59	1.16	5.21	-55.90	-3.12
RANGE	0.07	34.00	0.22	54.00	0.04	0.09
500-B-1-S (9)						
MEAN	3.59	168.00	7.53	369.67	3.28	0.19
STANDARD DEVIATION	0.09	9.68	0.08	14.14	0.05	0.03
COEF. OF VARIATION%	2.44	5.76	1.01	3.83	1.42	17.15
RANGE	0.31	26.00	0.27	42.00	0.17	0.11
500-B-2-S (10)						
MEAN	2.17	352.80	8.92	420.00	2.62	0.67
STANDARD DEVIATION	0.06	38.23	0.14	13.94	0.02	0.02
COEF. OF VARIATION%	2.75	10.84	1.62	3.32	0.64	2.97
RANGE	0.18	132.00	0.50	44.00	0.06	0.07
1500-A-1-S (13)						
MEAN	3.91	122.45	11.49	1504.65	4.57	0.64
STANDARD DEVIATION	0.18	8.46	0.05	14.49	0.07	0.01
COEF. OF VARIATION%	4.58	6.91	0.47	0.96	1.64	1.45
RANGE	0.76	36.00	0.22	50.10	0.31	0.04

TABLE 4 (Continued)
RESULTS — STATIC ANALYSES

CASE	MAX BENDING STRESS VALUE	MAX BENDING STRESS LOC	MAX TOTAL STRESS VALUE	MAX TOTAL STRESS LOC	ANGLE FROM VERTICAL LBJ	ANGLE FROM VERTICAL TOP
1500-A-2-S (13)						
MEAN	0.71	118.15	18.66	1499.88	2.52	1.14
STANDARD DEVIATION	0.09	10.65	0.09	12.36	0.01	0.01
COEF. OF VARIATION%	12.46	9.01	0.51	0.82	0.51	0.49
RANGE	0.34	47.00	0.37	50.10	0.05	0.02
1500-B-FREE-S (10)						
MEAN	0.98	1342.60	11.04	1429.94	-0.02	-1.91
STANDARD DEVIATION	0.06	20.05	0.98	18.59	0.01	0.07
COEF. OF VARIATION%	5.66	1.49	8.89	1.30	-57.52	-3.64
RANGE	0.22	63.00	3.30	73.40	0.03	0.28
1500-B-1-S (12)						
MEAN	6.04	125.12	11.93	1430.87	6.63	-0.77
STANDARD DEVIATION	0.28	8.12	0.21	30.19	0.12	0.03
COEF. OF VARIATION%	4.64	6.49	1.72	2.11	1.75	-4.15
RANGE	1.21	32.00	0.62	123.50	0.45	0.12
1500-B-2-S (11)						
MEAN	1.00	121.35	18.95	1449.26	3.12	0.36
STANDARD DEVIATION	0.09	6.43	0.19	13.97	0.02	0.01
COEF. OF VARIATION%	9.23	5.30	1.02	0.96	0.58	4.09
RANGE	0.35	20.00	0.78	37.30	0.08	0.06
3000-A-1-S (9)						
MEAN	1.03	150.70	15.49	3021.67	3.45	0.80
STANDARD DEVIATION	0.09	18.35	0.08	36.74	0.06	0.02
COEF. OF VARIATION%	8.98	12.18	0.53	1.22	1.71	2.77
RANGE	0.31	50.80	0.24	140.00	0.21	0.08
3000-A-2-S (8)						
MEAN	0.44	140.72	20.15	3028.13	2.64	1.06
STANDARD DEVIATION	0.03	26.06	0.11	33.37	0.03	0.03
COEF. OF VARIATION%	6.07	18.52	0.55	1.10	1.10	2.47
RANGE	0.09	68.80	0.32	100.00	0.09	0.07

TABLE 4 (Continued)
RESULTS — STATIC ANALYSES

CASE	MAX BENDING STRESS VALUE	MAX BENDING STRESS LOC	MAX TOTAL STRESS VALUE	MAX TOTAL STRESS LOC	ANGLE FROM VERTICAL LBJ	ANGLE FROM VERTICAL TOP
3000-B-FREE-S (7)						
MEAN	1.73	2764.57	10.51	2813.00	-0.03	-5.87
STANDARD DEVIATION	0.21	73.25	0.21	105.40	0.01	0.10
COEF. OF VARIATION%	12.14	2.65	1.99	3.75	-44.10	-1.76
RANGE	0.61	170.00	0.68	236.00	0.03	0.29
3000-B-1-S (8)						
MEAN	2.11	158.41	15.87	2913.66	6.22	-1.85
STANDARD DEVIATION	0.18	14.42	0.24	70.48	0.10	0.03
COEF. OF VARIATION%	8.48	9.11	1.54	2.42	1.63	-1.58
RANGE	0.64	34.80	0.72	216.70	0.33	0.10
3000-B-2-S (8)						
MEAN	0.96	2839.91	20.36	2942.64	4.14	-0.87
STANDARD DEVIATION	0.03	32.22	0.24	50.30	0.06	0.06
COEF. OF VARIATION%	3.55	1.13	1.20	1.71	1.47	-6.56
RANGE	0.12	100.00	0.66	124.00	0.20	0.17
6000-C-1-S (7)						
MEAN	0.57	5948.14	25.11	5992.14	4.13	-0.18
STANDARD DEVIATION	0.07	22.73	1.55	35.57	0.03	0.03
COEF. OF VARIATION%	12.93	0.38	6.16	0.59	0.68	-18.06
RANGE	0.20	68.00	4.40	75.00	0.08	0.09

TABLE 5
RESULTS — DYNAMIC ANALYSES

	ANGLE FROM VERTICAL			
	LBJ		TOP	
CASE	MIN	MAX	MIN	MAX
500-20-1-D (8)				
MEAN	2.04	3.20	0.09	1.71
STANDARD DEVIATION	0.09	0.20	0.09	0.07
COEF. OF VARIATION%	4.48	6.30	98.14	4.30
RANGE	0.31	0.53	0.23	0.20
500-20-2-D (9)				
MEAN	1.45	3.01	0.52	1.77
STANDARD DEVIATION	0.15	0.19	0.09	0.14
COEF. OF VARIATION%	10.25	6.44	16.71	8.15
RANGE	0.49	0.60	0.23	0.39
500-21.5-1-R (3)				
MEAN	3.32	0.82	0.13	0.38
STANDARD DEVIATION	0.18	0.18	0.11	0.03
COEF. OF VARIATION%	5.35	21.33	83.63	6.57
RANGE	0.33	0.35	0.21	0.05
500-21.5-2-R (3)				
MEAN	2.64	0.71	0.63	0.30
STANDARD DEVIATION	0.10	0.12	0.09	0.07
COEF. OF VARIATION%	3.95	16.84	13.59	24.04
RANGE	0.20	0.24	0.16	0.14
500-40-FREE- D (4)				
MEAN	-2.62	2.31	-2.71	0.08
STANDARD DEVIATION	0.31	0.20	0.01	0.06
COEF. OF VARIATION%	-11.94	8.53	-0.43	81.18
RANGE	0.69	0.37	0.02	0.13
500-40-1-D (5)				
MEAN	1.17	7.07	-2.16	1.70
STANDARD DEVIATION	0.16	0.42	0.21	0.21
COEF. OF VARIATION%	13.35	5.92	-9.80	12.13
RANGE	0.34	0.90	0.53	0.53

TABLE 5 (Continued)
RESULTS — DYNAMIC ANALYSES

	ANGLE FROM VERTICAL			
	LBJ		TOP	
CASE	MIN	MAX	MIN	MAX
500-40-2-D (5)				
MEAN	0.31	5.87	-0.82	1.71
STANDARD DEVIATION	0.26	0.51	0.31	0.13
COEF. OF VARIATION%	82.91	8.74	-38.40	7.64
RANGE	0.64	1.25	0.76	0.33
1500-20-1-D (11)				
MEAN	3.89	5.37	0.02	1.14
STANDARD DEVIATION	0.13	0.12	0.05	0.10
COEF. OF VARIATION%	3.26	2.32	230.18	8.86
RANGE	0.45	0.43	0.17	0.27
1500-20-2-D (10)				
MEAN	2.00	3.16	0.55	1.60
STANDARD DEVIATION	0.05	0.08	0.06	0.07
COEF. OF VARIATION%	2.50	2.43	10.01	4.40
RANGE	0.18	0.22	0.15	0.20
1500-21.5-1-R (4)				
MEAN	6.69	0.31	-0.80	0.39
STANDARD DEVIATION	0.11	0.05	0.06	0.03
COEF. OF VARIATION%	1.58	15.88	-6.90	6.37
RANGE	0.24	0.09	0.13	0.06
1500-21.5-2-R (5)				
MEAN	3.18	0.26	0.32	0.36
STANDARD DEVIATION	0.04	0.05	0.03	0.04
COEF. OF VARIATION%	1.19	18.21	8.44	10.68
RANGE	0.10	0.13	0.07	0.10
1500-40-FREE-D (7)				
MEAN	-0.80	1.01	-4.03	-0.68
STANDARD DEVIATION	0.10	0.32	0.18	0.22
COEF. OF VARIATION%	-12.18	31.55	-4.40	-31.57
RANGE	0.30	0.81	0.52	0.64

TABLE 5 (Continued)
RESULTS — DYNAMIC ANALYSES

	ANGLE FROM VERTICAL			
	LBJ		TOP	
CASE	MIN	MAX	MIN	MAX
1500-40-1-D (8)				
MEAN	6.13	8.63	-2.64	0.45
STANDARD DEVIATION	0.07	0.36	0.27	0.15
COEF. OF VARIATION%	1.07	4.14	-10.11	33.05
RANGE	0.21	1.06	0.80	0.35
1500-40-1-D2 (8)				
MEAN	6.59	10.68	-9.57	2.12
STANDARD DEVIATION	0.23	0.24	0.63	0.46
COEF. OF VARIATION%	3.44	2.22	-6.56	21.58
RANGE	0.62	0.82	1.49	1.29
1500-40-2-D (9)				
MEAN	2.48	4.93	-1.39	1.45
STANDARD DEVIATION	0.15	0.23	0.10	0.09
COEF. OF VARIATION%	6.12	4.72	-7.42	5.93
RANGE	0.50	0.65	0.30	0.28
1500-40-2-D2 (8)				
MEAN	2.36	5.83	-6.36	2.76
STANDARD DEVIATION	0.20	0.21	0.50	0.17
COEF. OF VARIATION%	8.48	3.64	-7.93	6.33
RANGE	0.55	0.68	1.55	0.47
3000-20-1-D (7)				
MEAN	3.17	3.80	-0.12	1.40
STANDARD DEVIATION	0.11	0.12	0.08	0.10
COEF. OF VARIATION%	3.58	3.18	72.22	7.27
RANGE	0.33	0.35	0.25	0.24
3000-20-2-D (9)				
MEAN	2.39	3.10	0.44	1.56
STANDARD DEVIATION	0.11	0.21	0.12	0.10
COEF. OF VARIATION%	4.52	6.71	27.60	6.46
RANGE	0.36	0.56	0.43	0.27

TABLE 5 (Continued)
RESULTS — DYNAMIC ANALYSES

	ANGLE FROM VERTICAL			
	LBJ		TOP	
CASE	MIN	MAX	MIN	MAX
3000-21.5-1-R (3)				
MEAN	6.09	0.04	-1.86	0.43
STANDARD DEVIATION	0.23	0.01	0.07	0.07
COEF. OF VARIATION%	3.81	31.49	-3.51	16.21
RANGE	0.42	0.02	0.13	0.14
3000-21.5-2-R (3)				
MEAN	4.14	0.04	-0.94	0.38
STANDARD DEVIATION	0.09	0.01	0.01	0.12
COEF. OF VARIATION%	2.06	13.32	-1.37	32.12
RANGE	0.16	0.01	0.02	0.23
3000-40-FREE-D (5)				
MEAN	-0.05	-0.01	-8.13	-4.04
STANDARD DEVIATION	0.02	0.02	0.17	0.18
COEF. OF VARIATION%	-44.60	-345.61	-2.06	-4.52
RANGE	0.06	0.05	0.36	0.46
3000-40-1-D (9)				
MEAN	5.90	6.39	-3.90	-0.35
STANDARD DEVIATION	0.84	0.81	0.22	0.24
COEF. OF VARIATION%	14.22	12.62	-5.75	-69.21
RANGE	2.85	2.77	0.72	0.69
3000-40-2-D (7)				
MEAN	4.15	4.47	-2.84	0.52
STANDARD DEVIATION	0.22	0.16	0.15	0.18
COEF. OF VARIATION%	5.19	3.63	-5.43	34.24
RANGE	0.63	0.48	0.44	0.56
6000-40-1-D (6)				
MEAN	4.17	4.33	-2.31	1.04
STANDARD DEVIATION	0.09	0.15	0.29	0.25
COEF. OF VARIATION%	2.13	3.41	-12.77	23.60
RANGE	0.24	0.38	0.74	0.67

FIGURE 4
DEFLECTION 500-20-1-D (10)

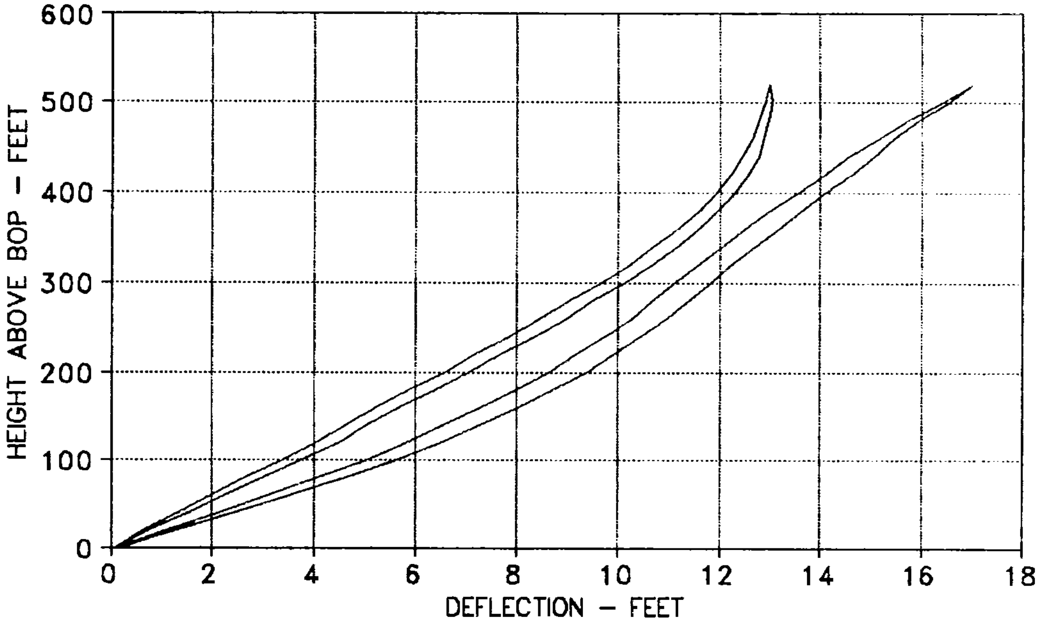


FIGURE 5
BENDING STRESS 500-20-1-D (5)

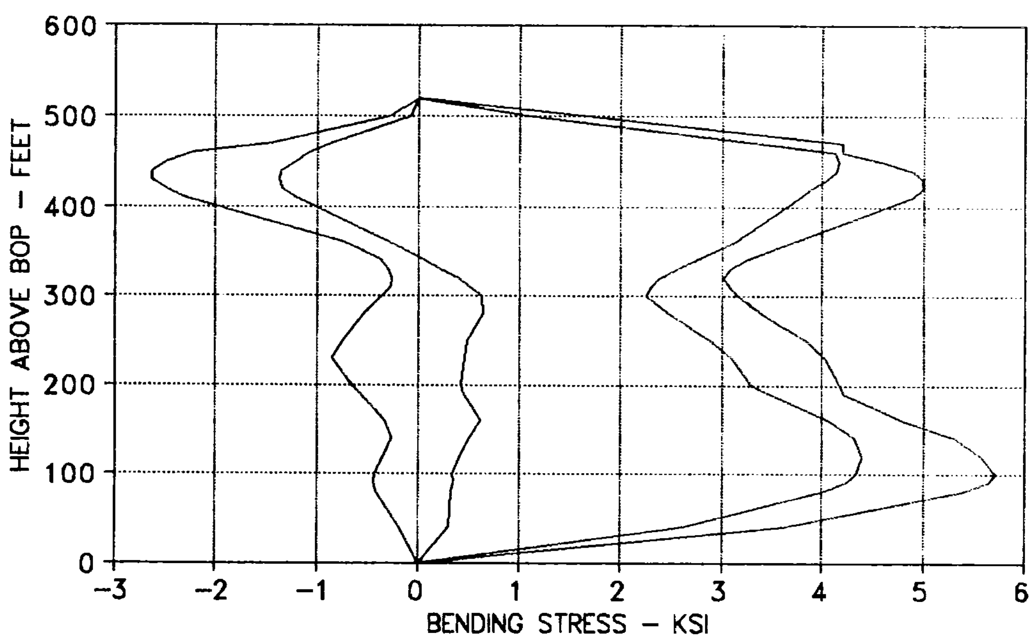


FIGURE 6
DEFLECTION 500-20-2-D (10)

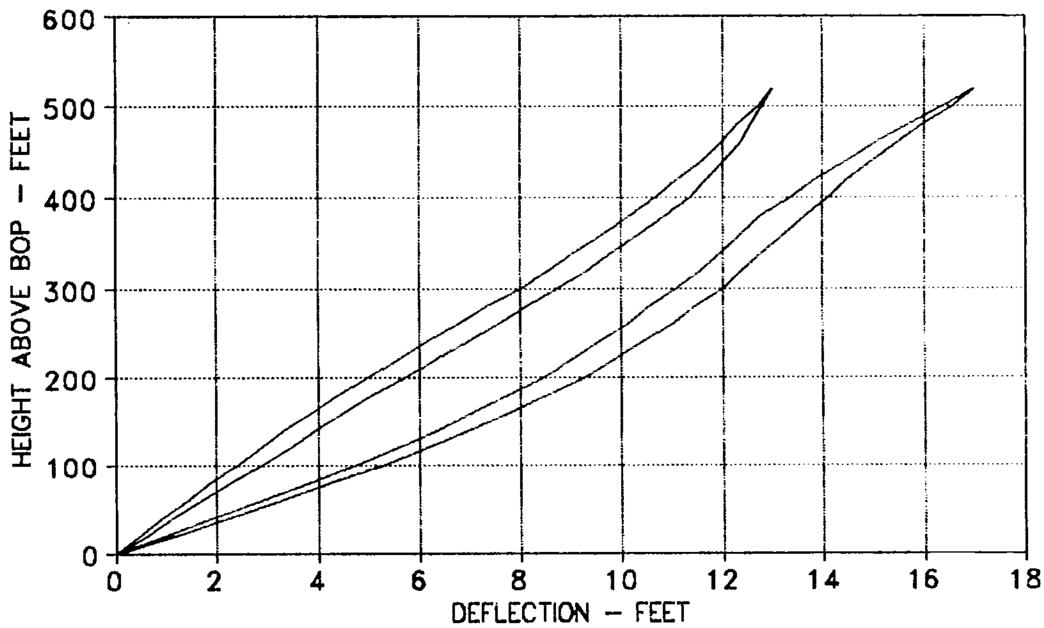


FIGURE 7
BENDING STRESS 500-20-2-D (5)

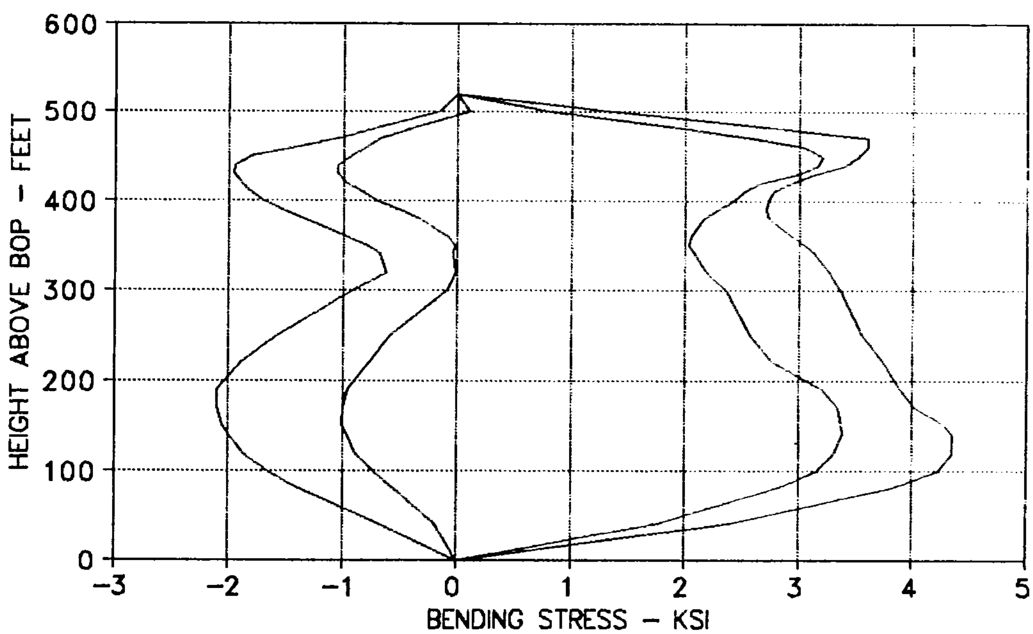


FIGURE 8
DEFLECTION 500-40-1-D (9)

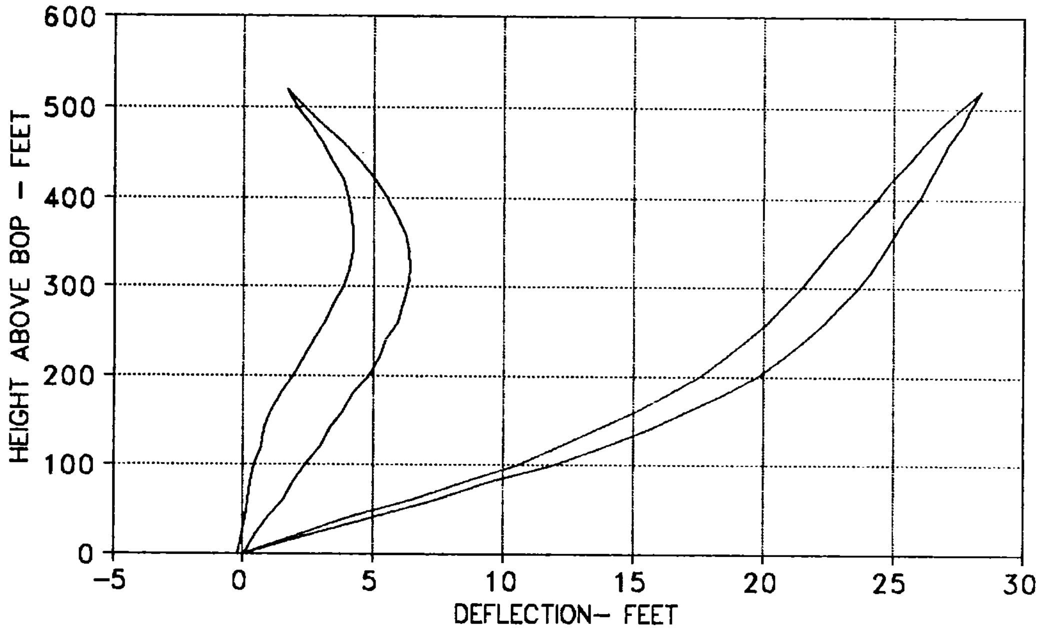


FIGURE 9
BENDING STRESS 500-40-1-D (5)

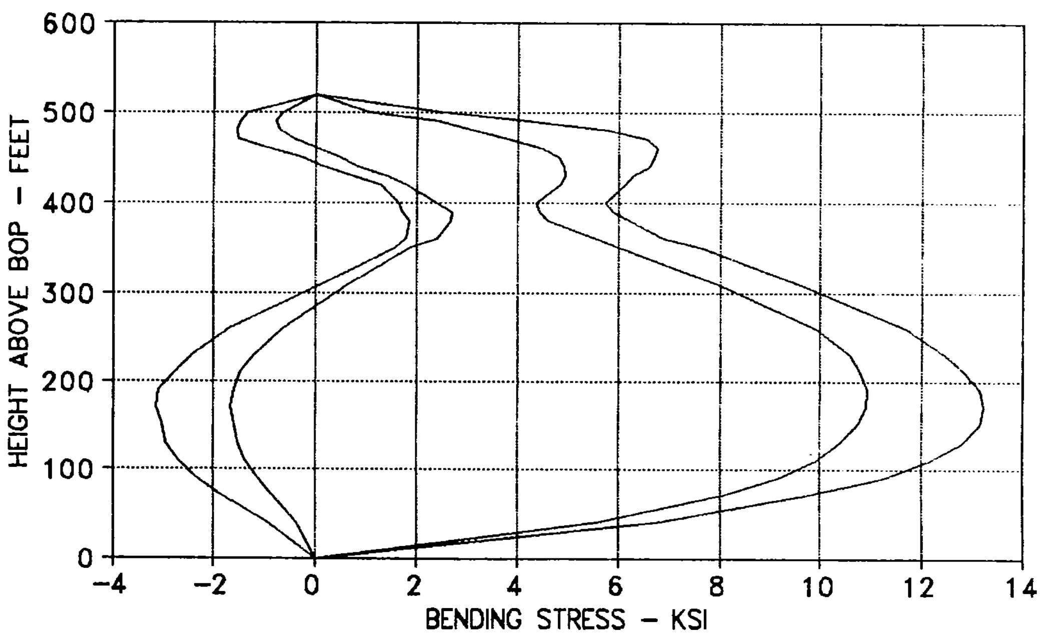


FIGURE 10
DEFLECTION 500-40-2-D (9)

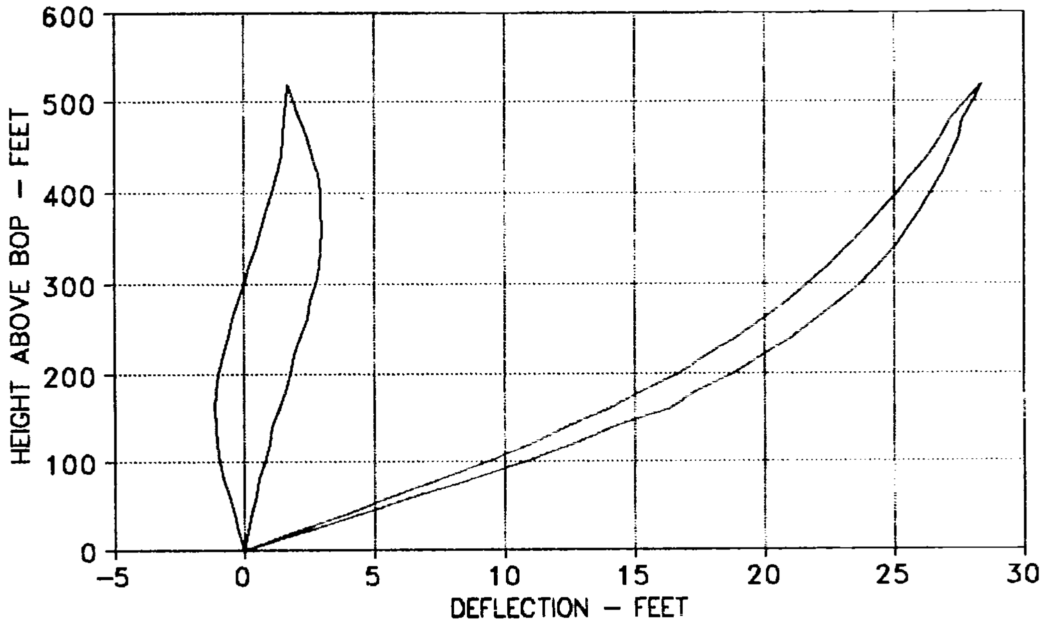


FIGURE 11
BENDING STRESS 500-40-2-D (5)

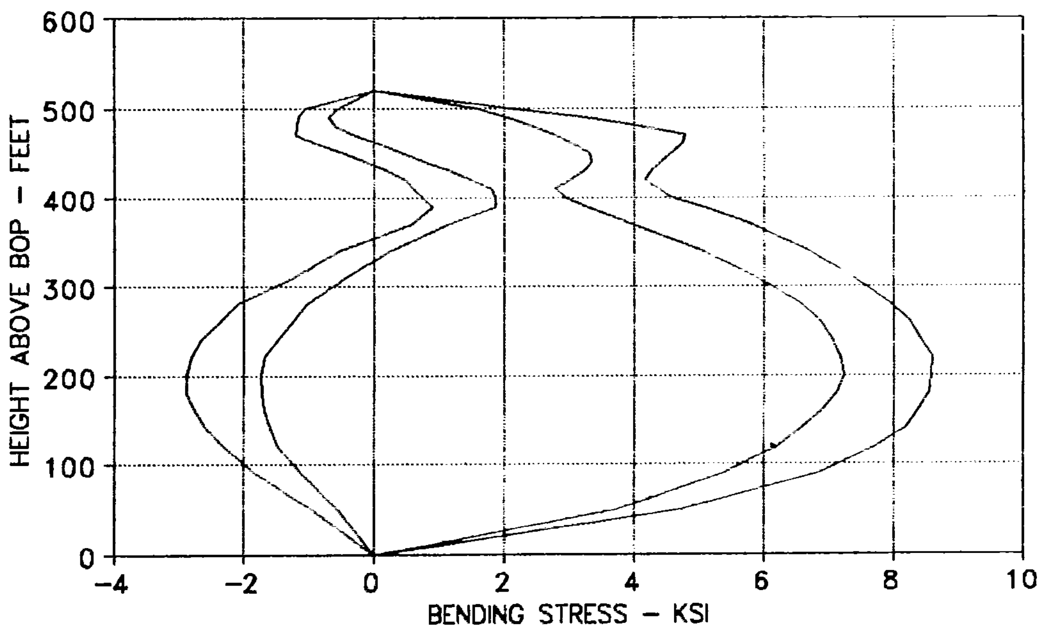


FIGURE 12
DEFLECTION 500-40-FREE-D (1)

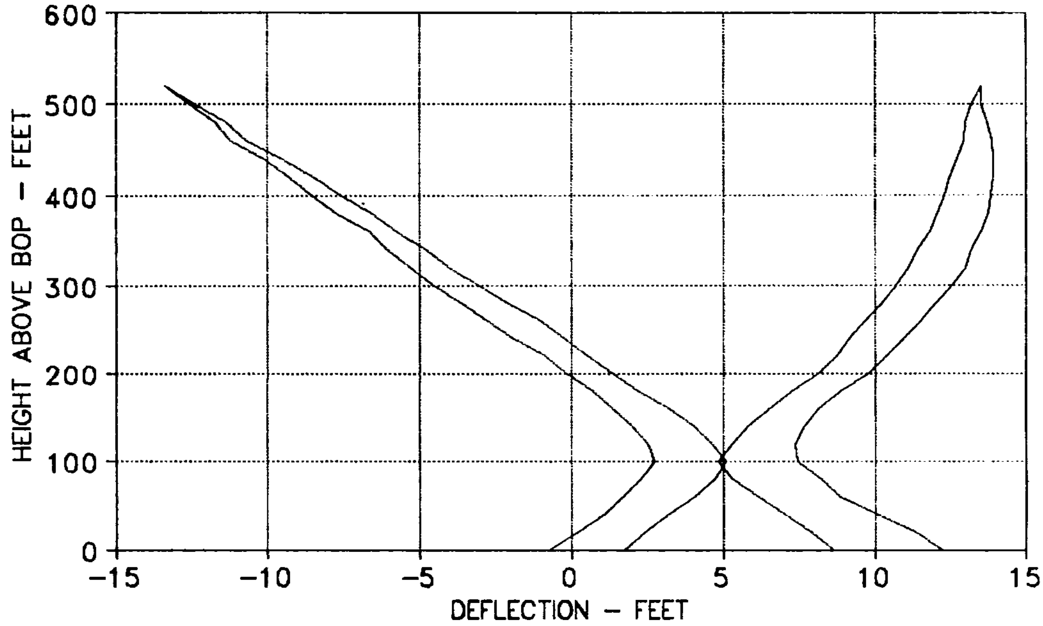


FIGURE 13
BENDING STRESS 500-40-FREE-D (5)

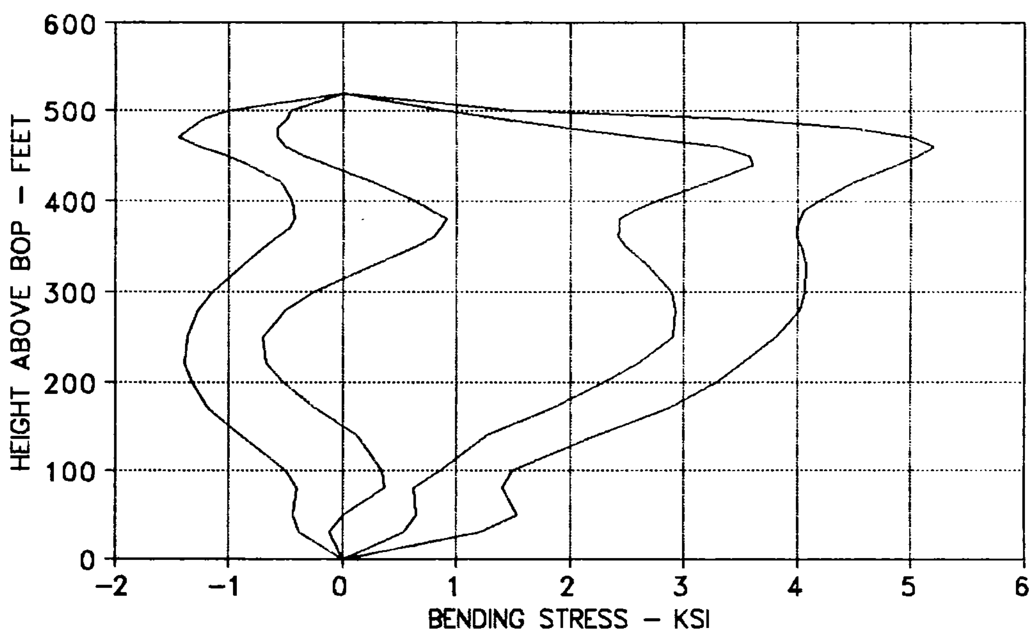


FIGURE 14
DEFLECTION 1500-20-1-D (12)

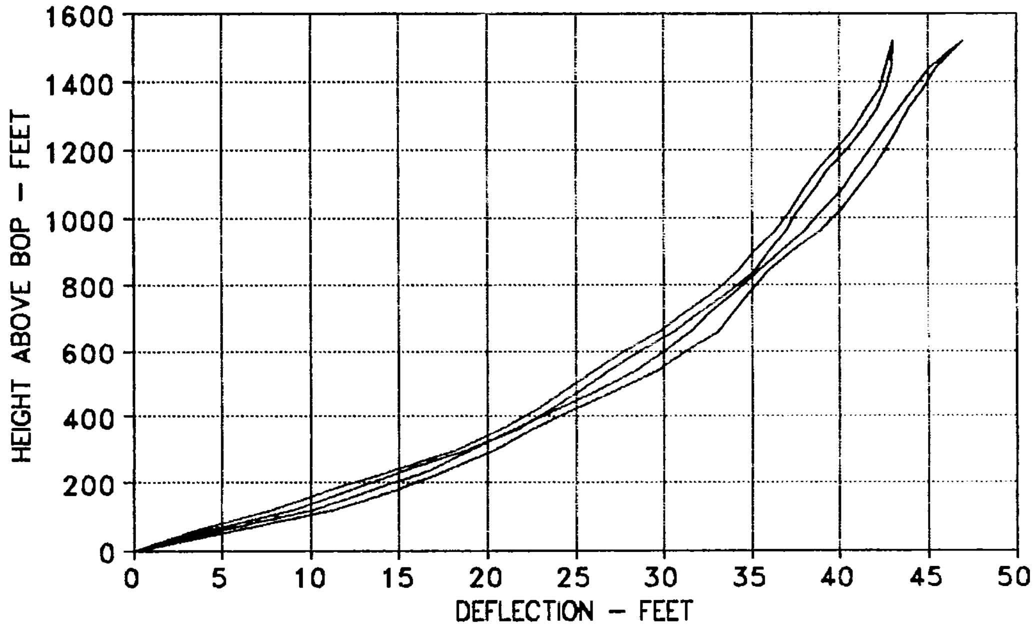


FIGURE 15
BENDING STRESS 1500-20-1-D (5)

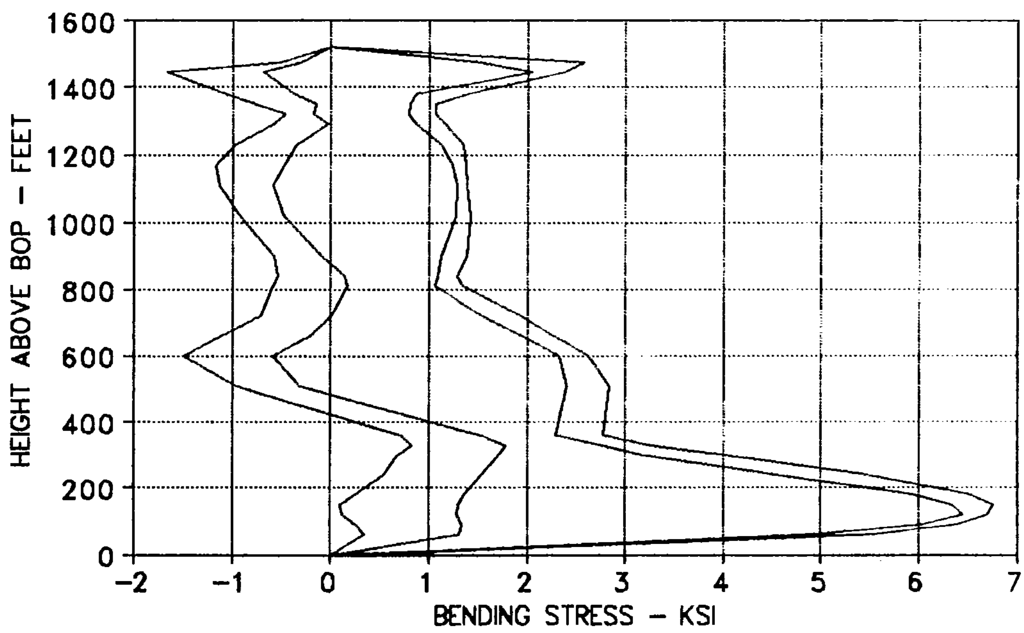


FIGURE 16
DEFLECTION 1500-20-2-D (12)

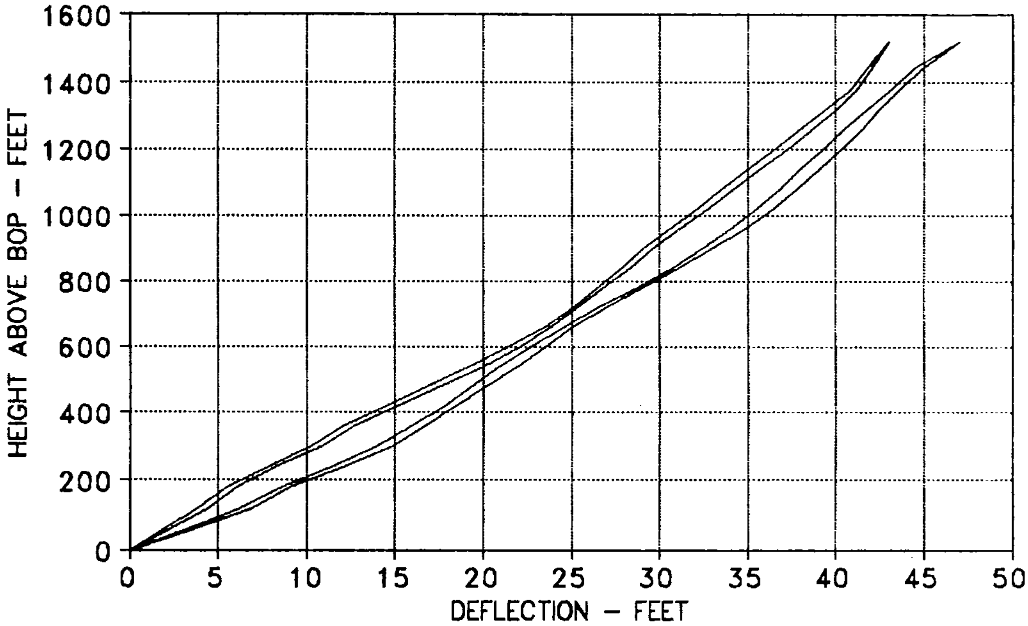


FIGURE 17
BENDING STRESS 1500-20-2-D (5)

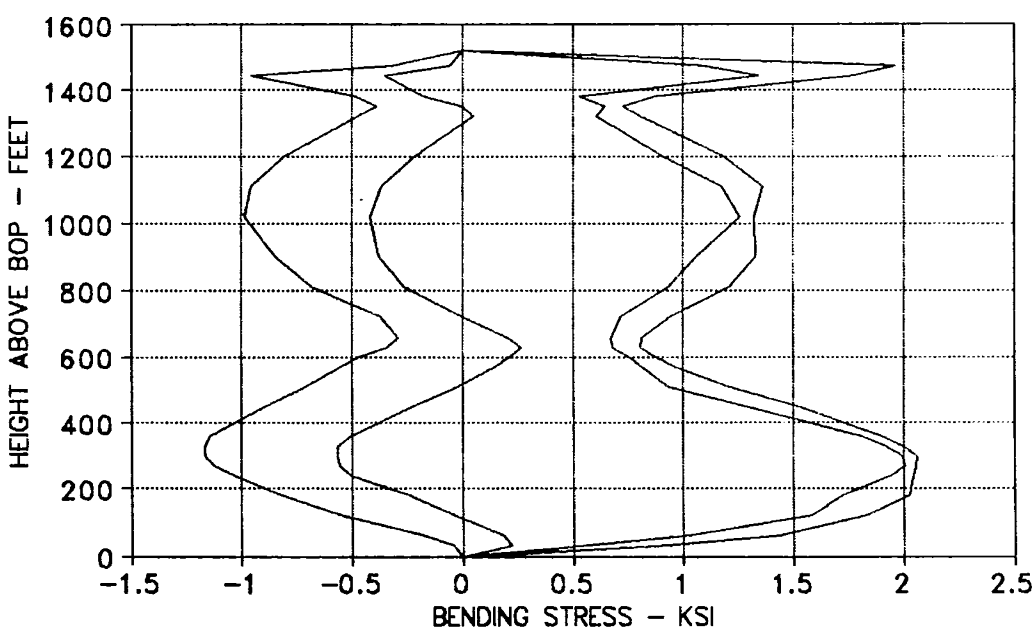


FIGURE 18
DEFLECTION 1500-40-1-D (12)

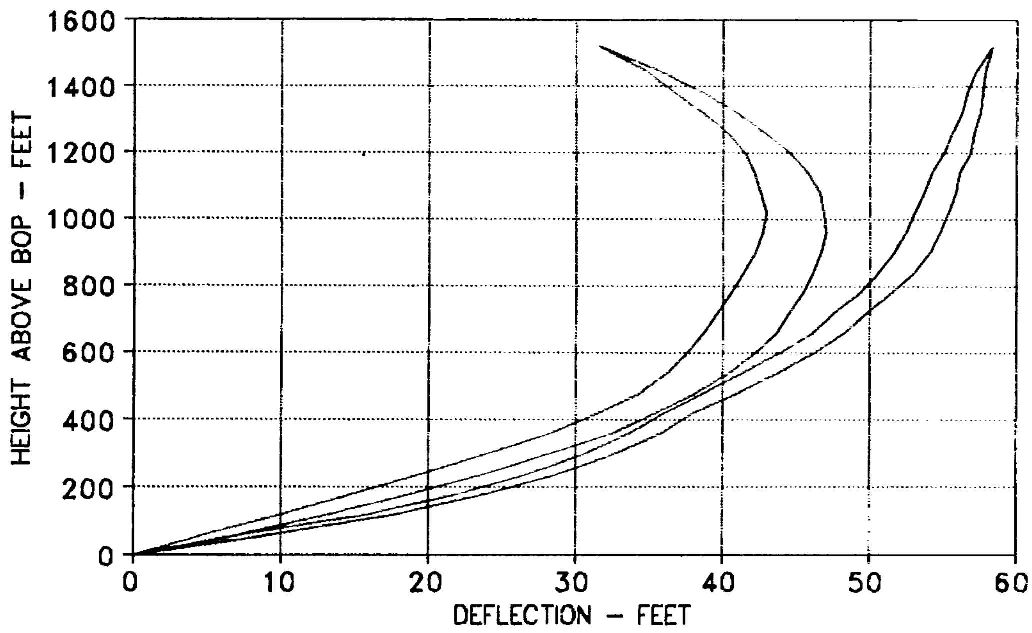


FIGURE 19
BENDING STRESS 1500-40-1-D (7)

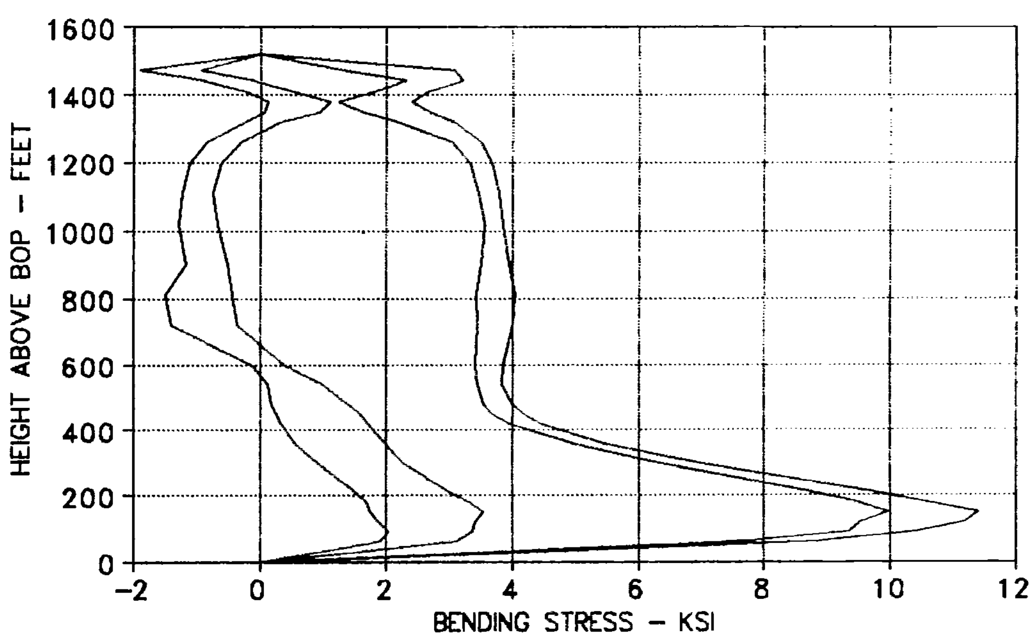


FIGURE 20
DEFLECTION 1500-40-1-D2 (12)

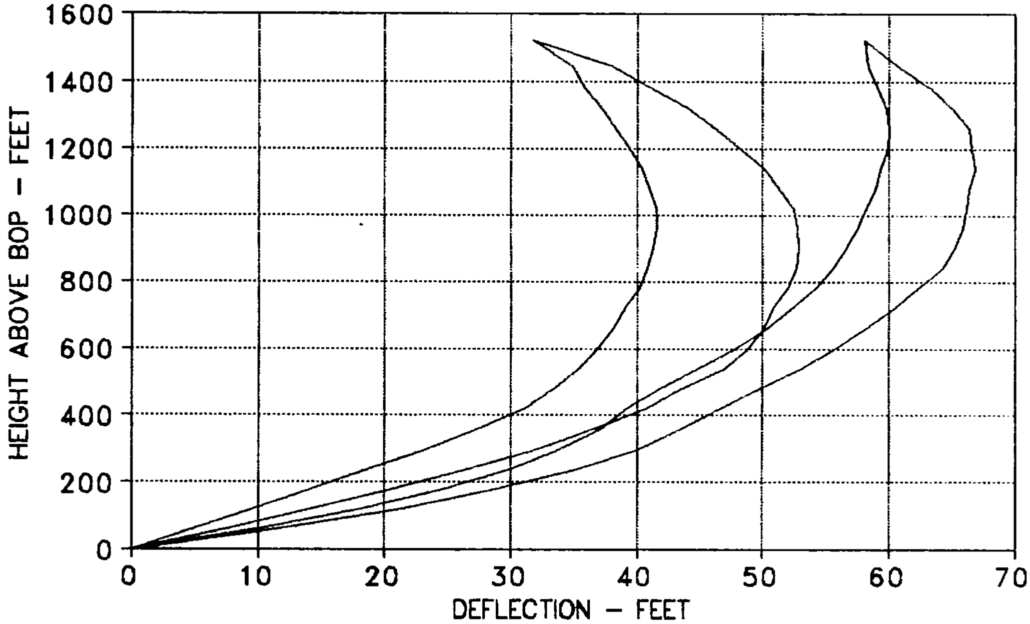


FIGURE 21
BENDING STRESS 1500-40-1-D2 (6)

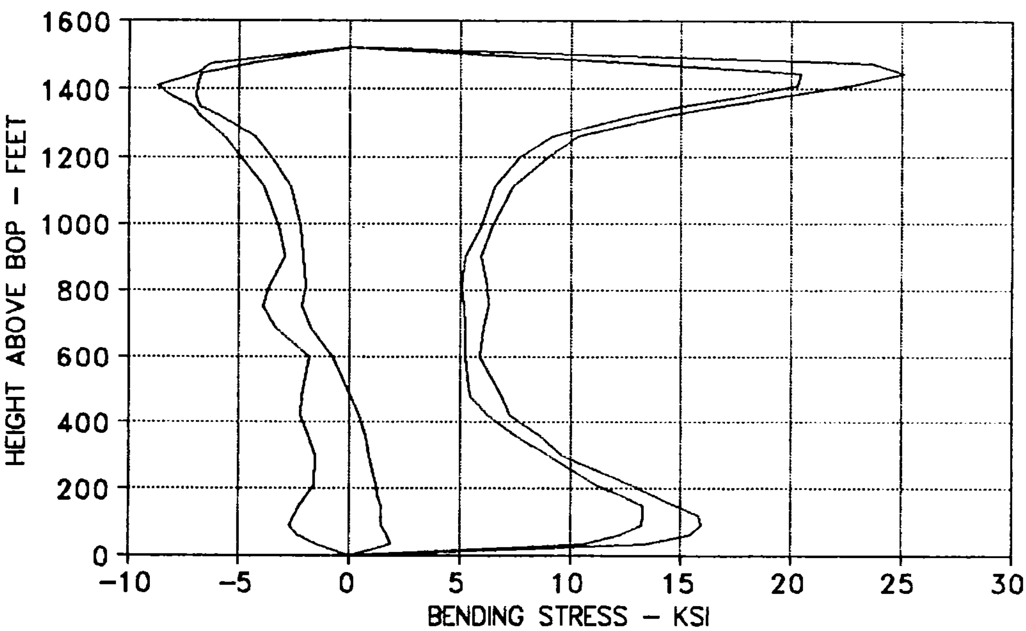


FIGURE 22
DEFLECTION 1500-40-2-D (12)

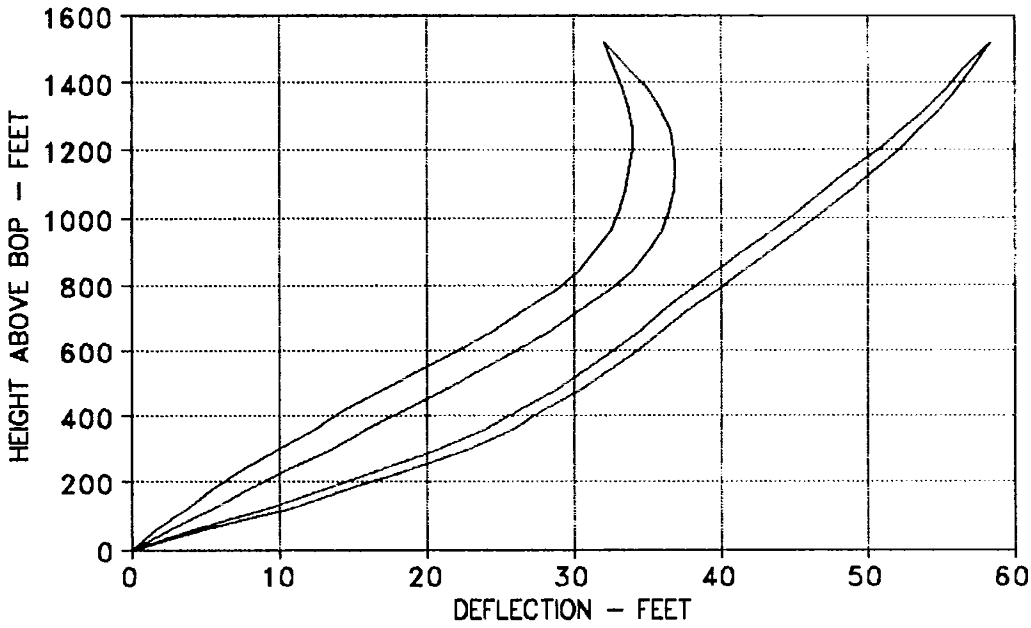


FIGURE 23
BENDING STRESS 1500-40-2-D (7)

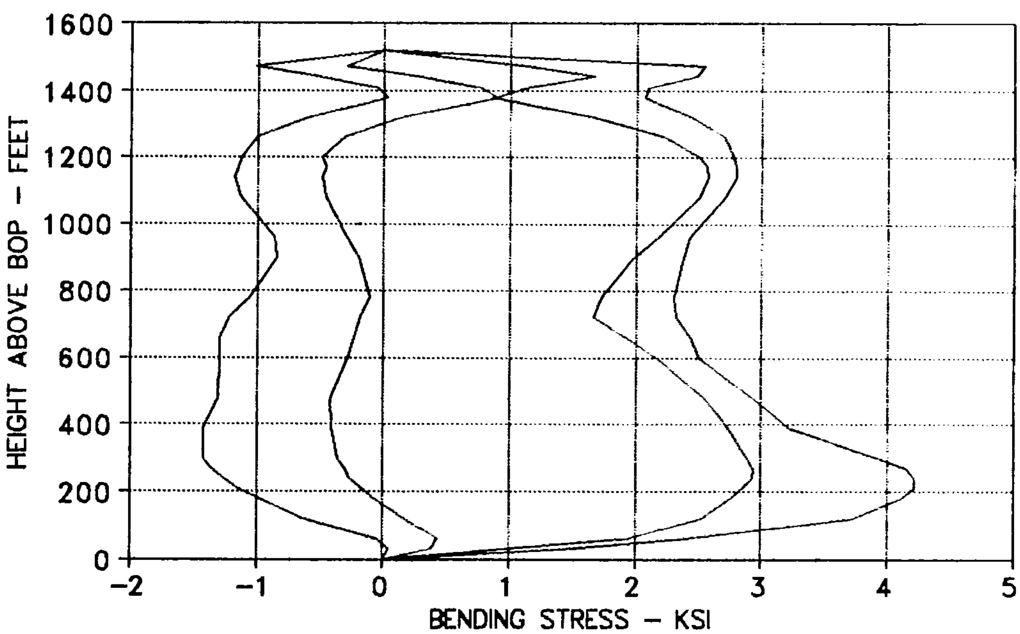


FIGURE 24
DEFLECTION 1500-40-2-D2 (12)

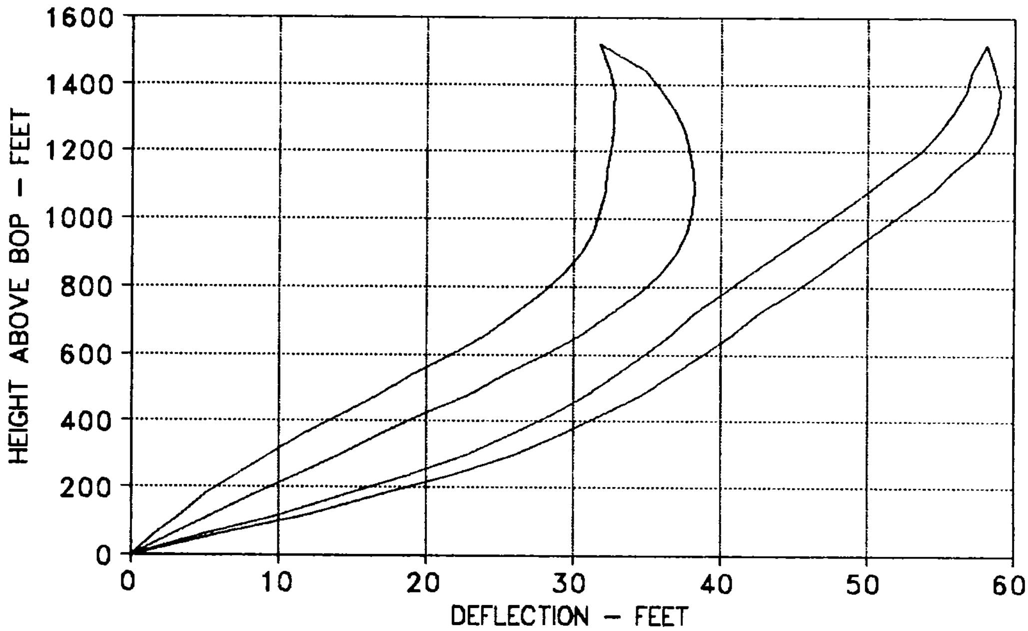


FIGURE 25
BENDING STRESS 1500-40-2-D2 (6)

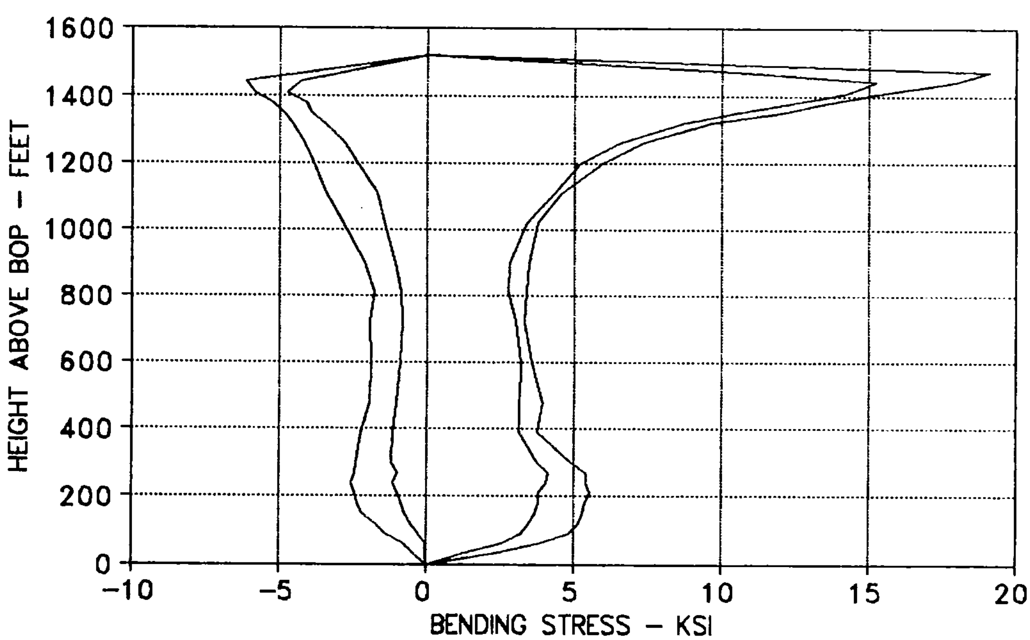


FIGURE 26
DEFLECTION 1500-40-FREE-D (11)

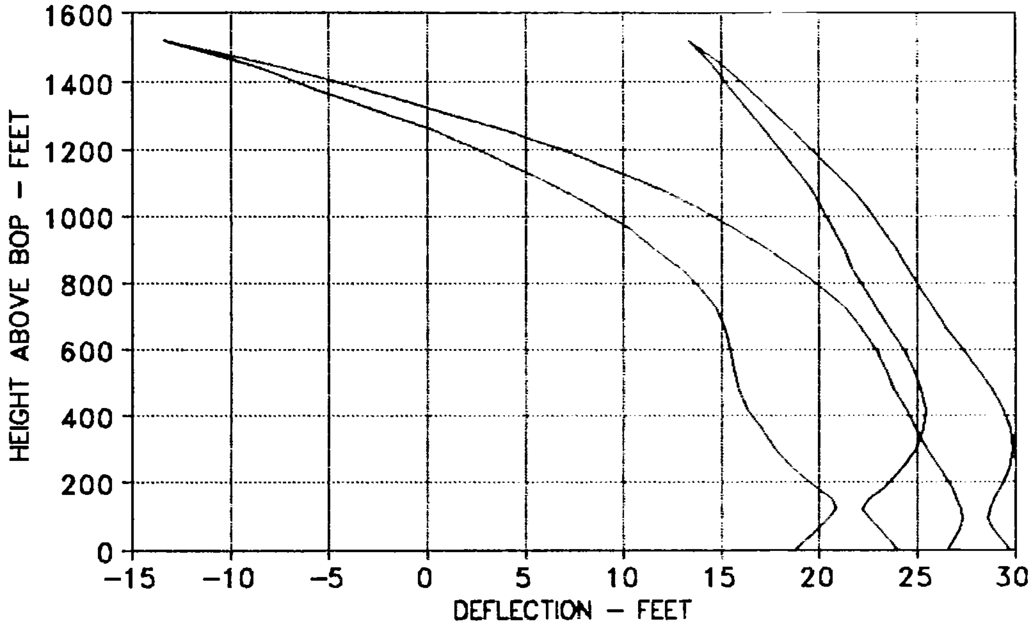


FIGURE 27
BENDING STRESS 1500-40-FREE-D (5)

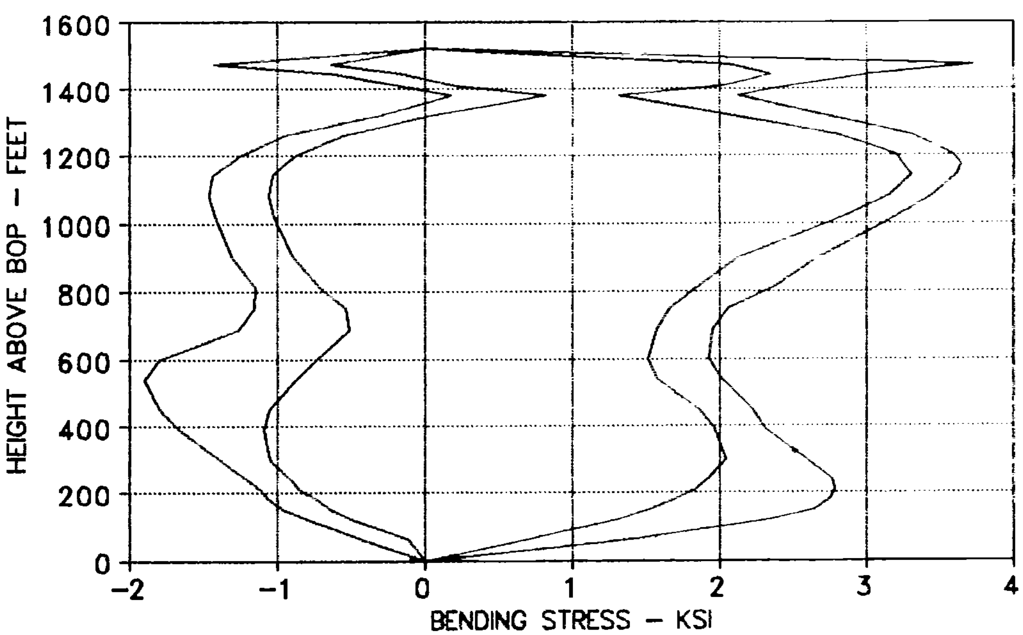


FIGURE 28
DEFLECTION 3000-20-1-D (10)

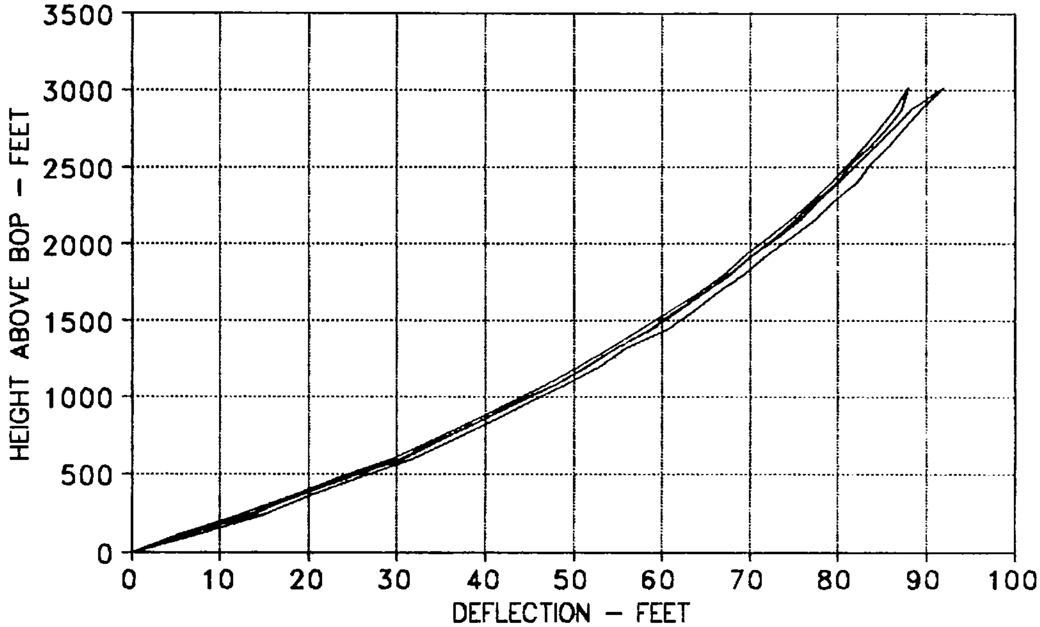


FIGURE 29
BENDING STRESS 3000-20-1-D (5)

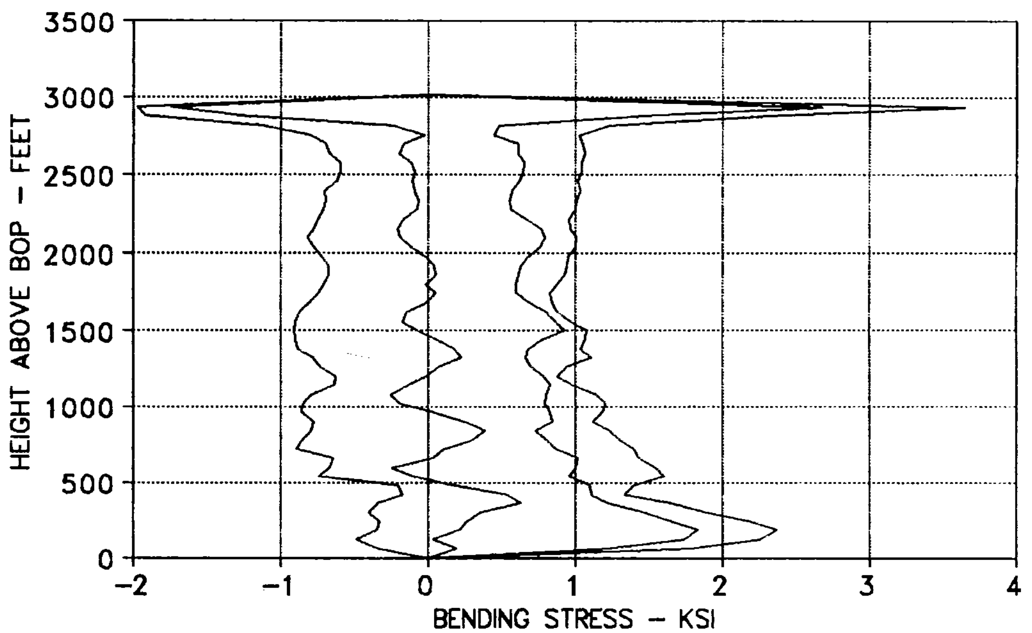


FIGURE 30
DEFLECTION 3000-20-2-D (10)

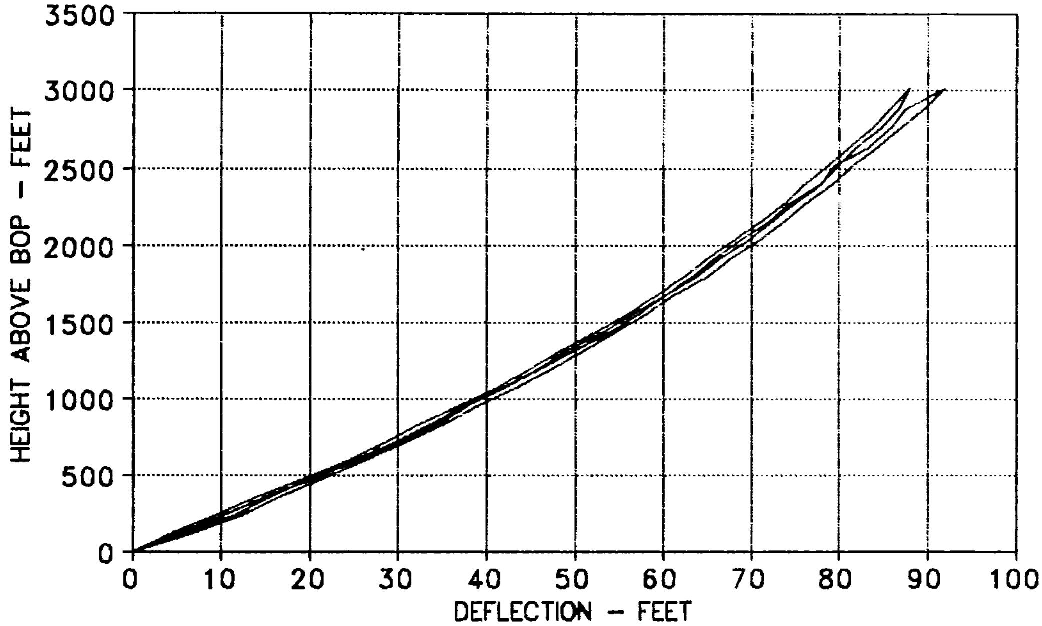


FIGURE 31
BENDING STRESS 3000-20-2-D (5)

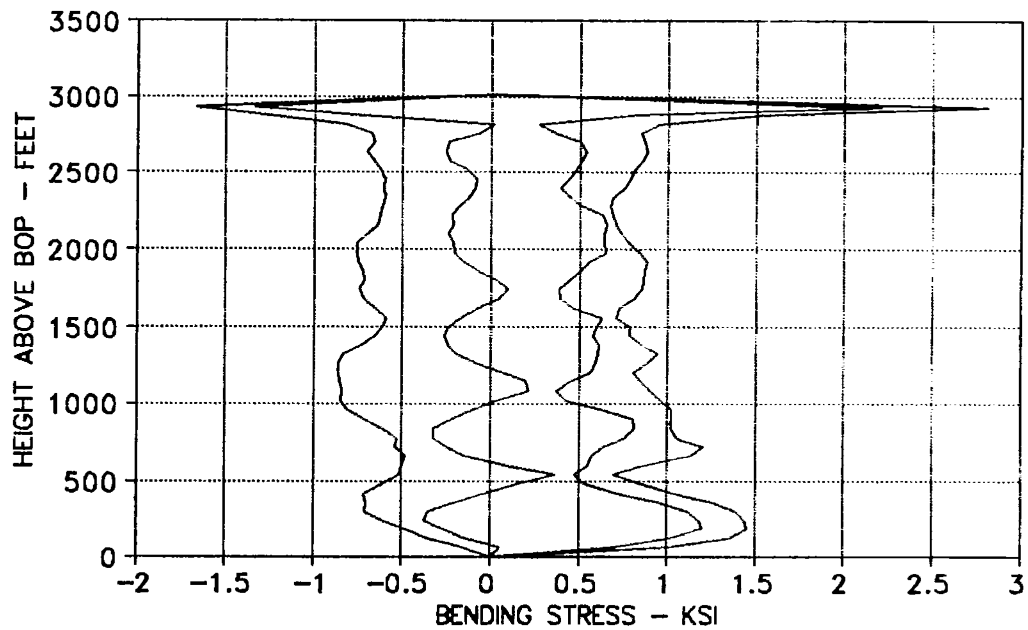


FIGURE 32
DEFLECTION 3000-40-1-D (10)

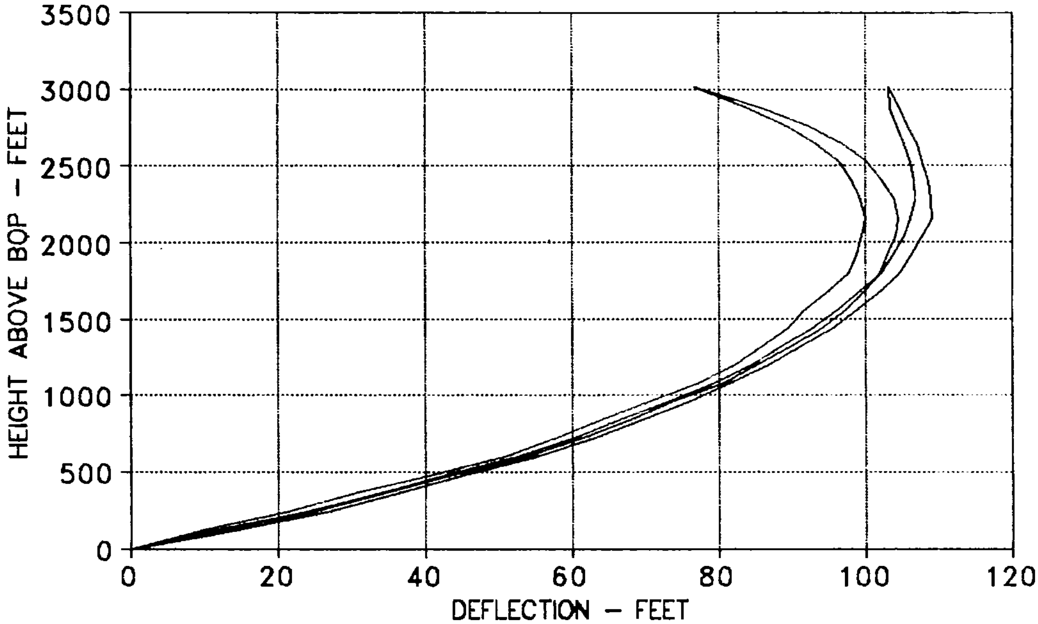


FIGURE 33
BENDING STRESS 3000-40-1-D (5)

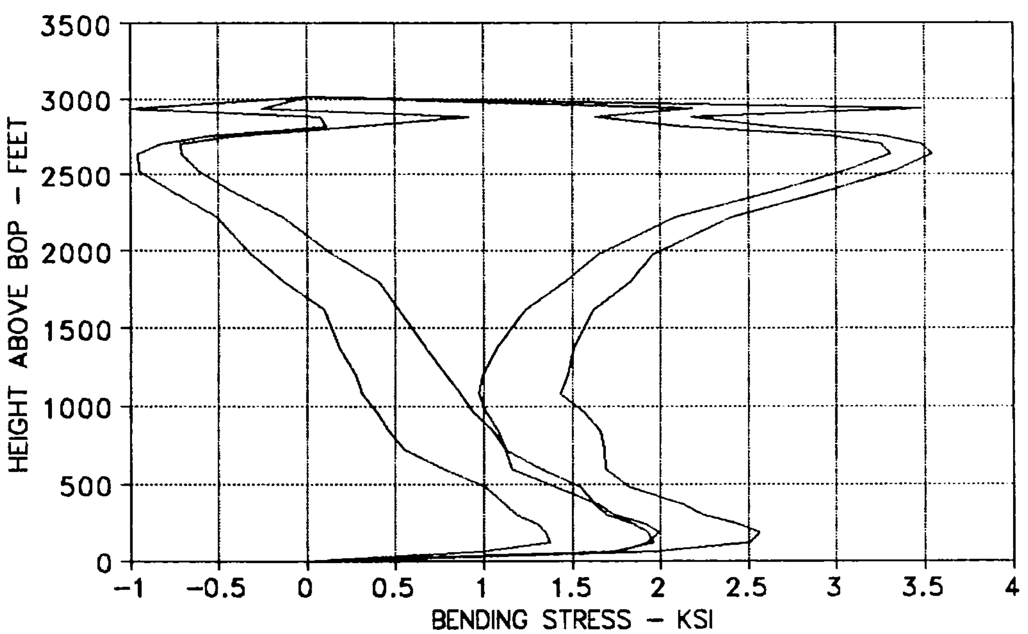


FIGURE 34
DEFLECTION 3000-40-2-D (10)

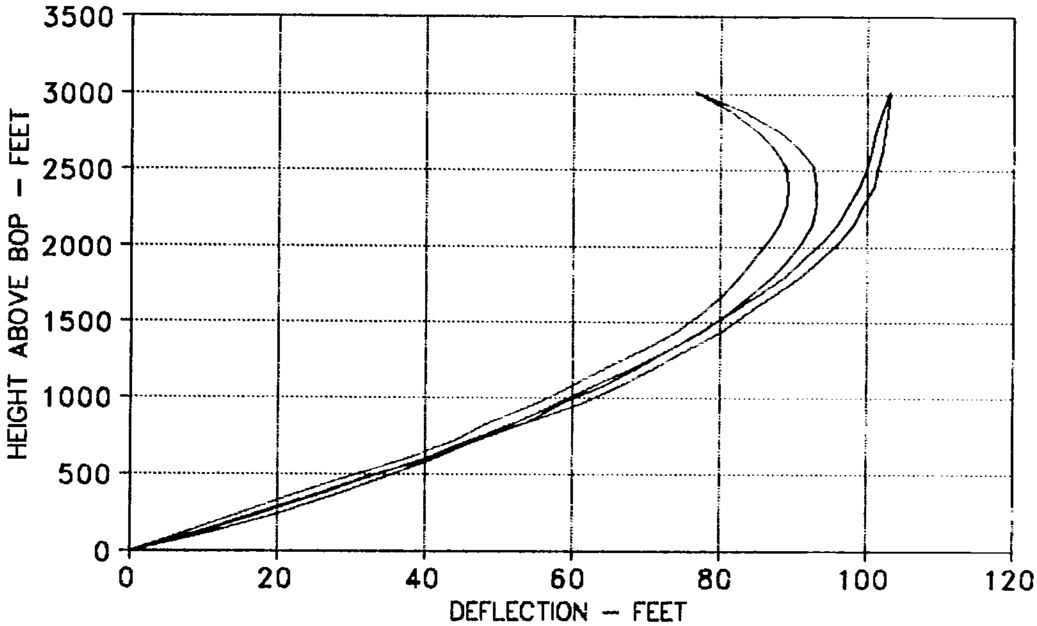


FIGURE 35
BENDING STRESS 3000-40-2-D (5)

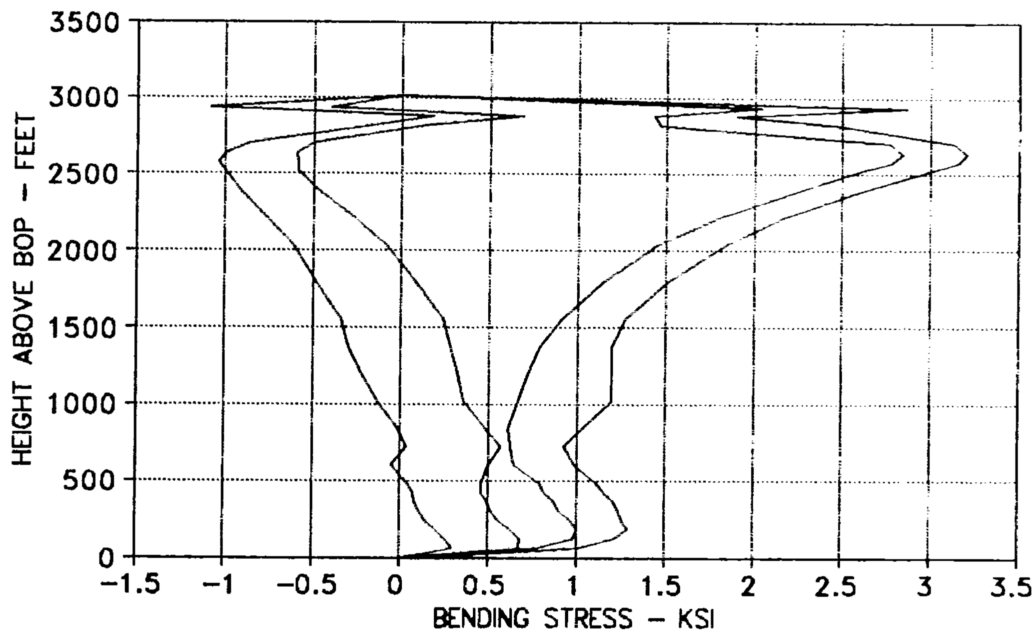


FIGURE 36
DEFLECTION 3000-40-FREE-D (9)

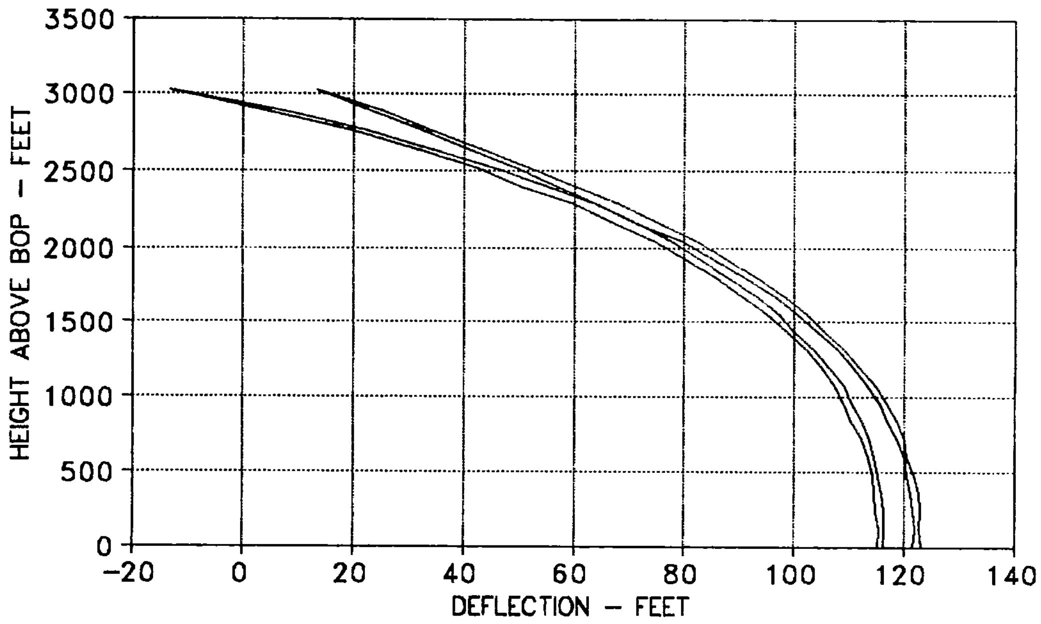


FIGURE 37
BENDING STRESS 3000-40-FREE-D (5)

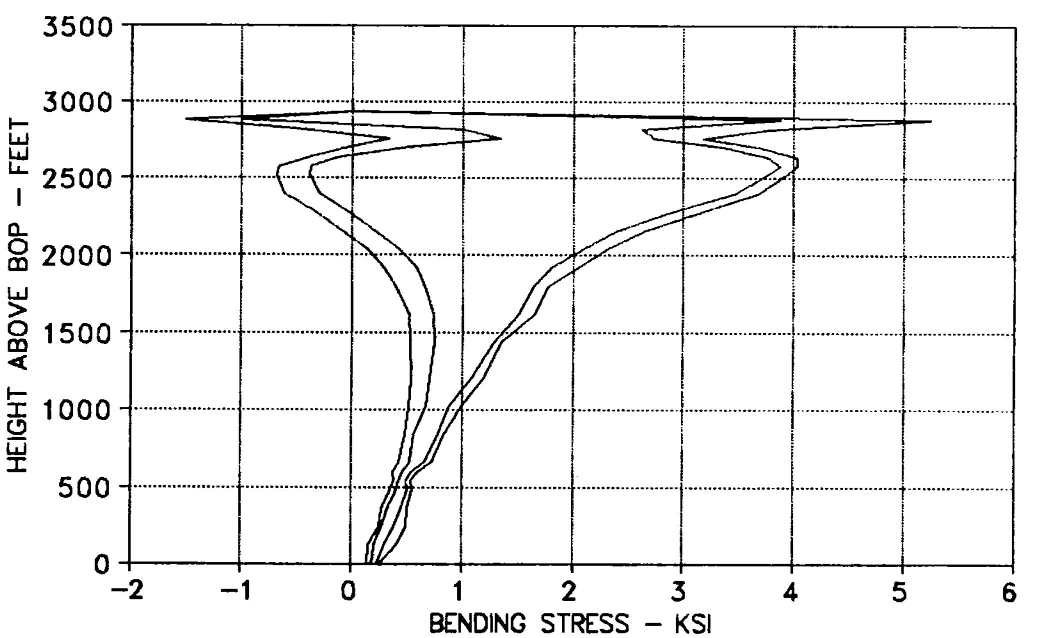


FIGURE 38
DEFLECTION 6000-40-1-D (8)

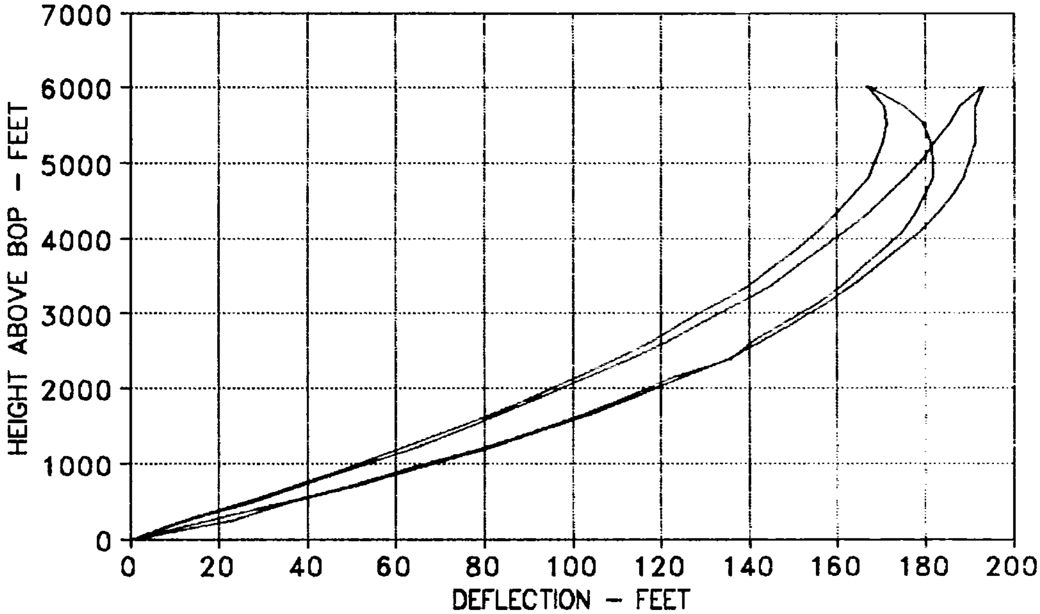


FIGURE 39
BENDING STRESS 6000-40-1-D (4)

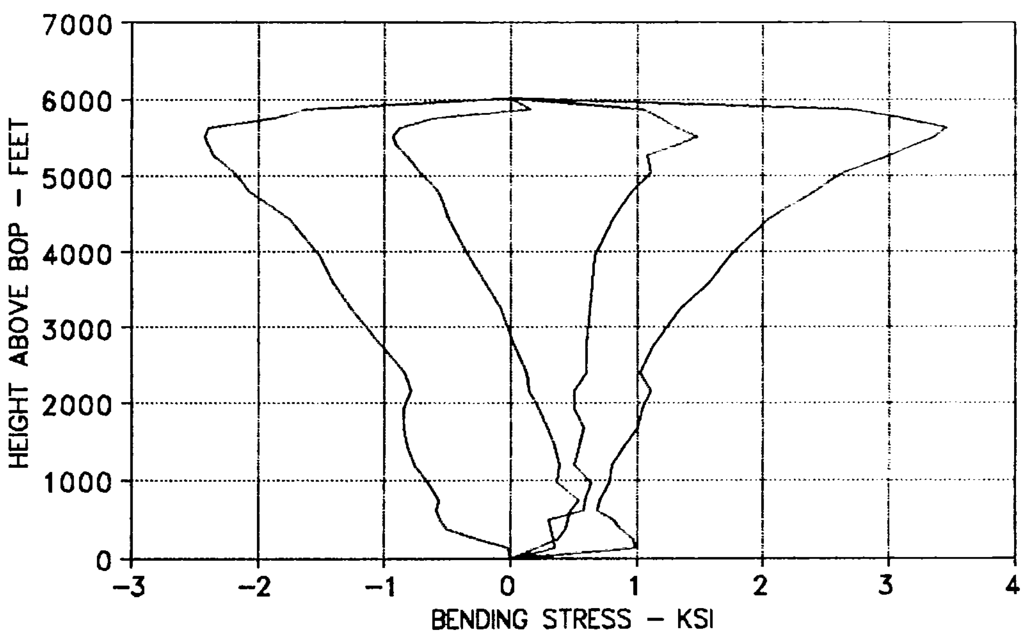


FIGURE 40
DEFLECTION 500-21.5-1-R (3)

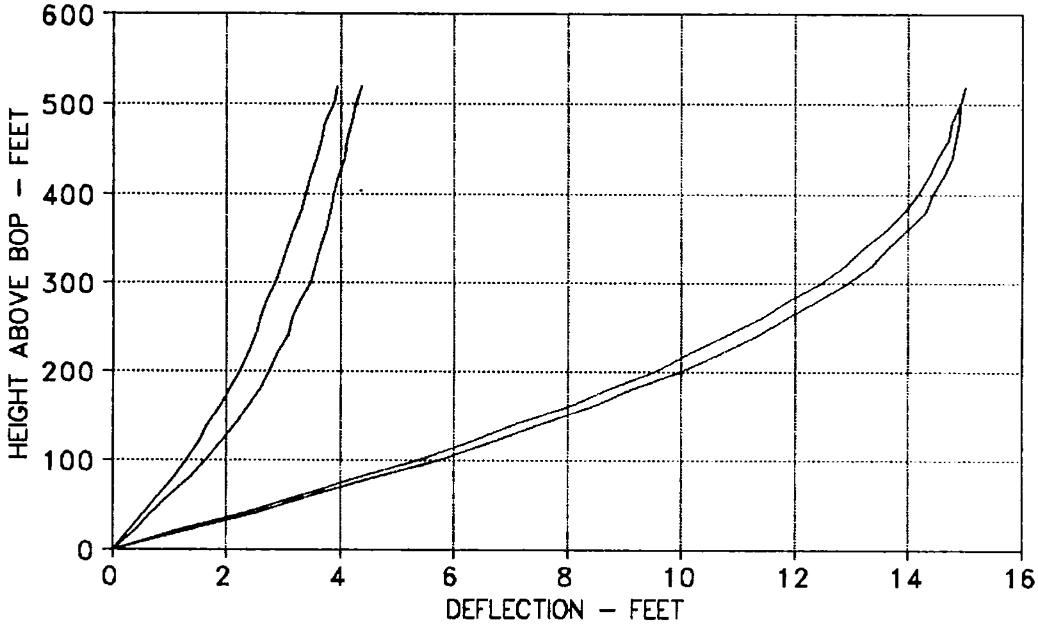


FIGURE 41
BENDING STRESS 500-21.5-1-R (3)

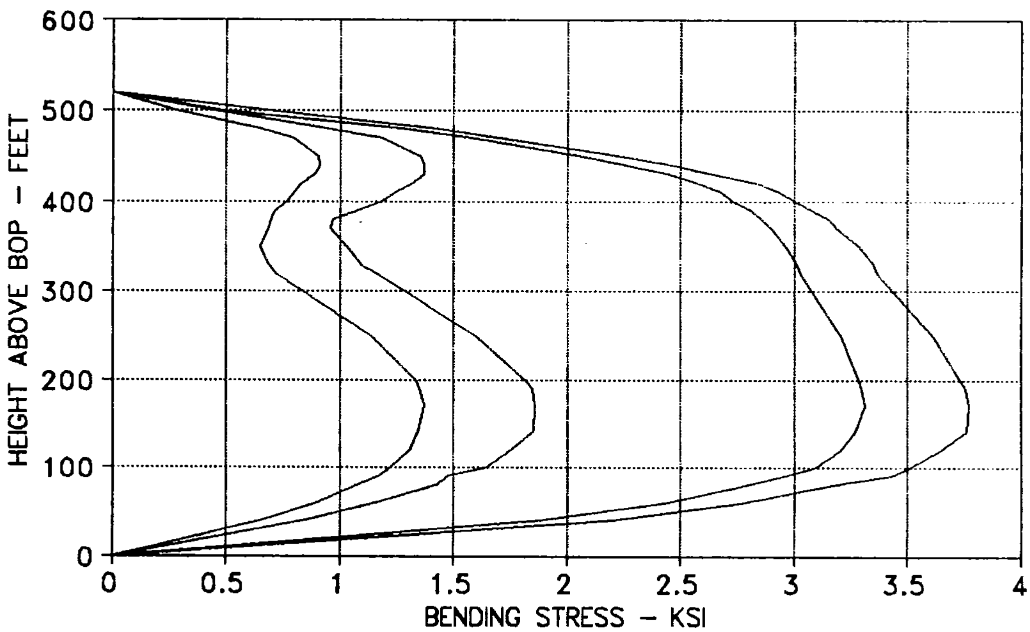


FIGURE 42
DEFLECTION 500-21.5-2-R (3)

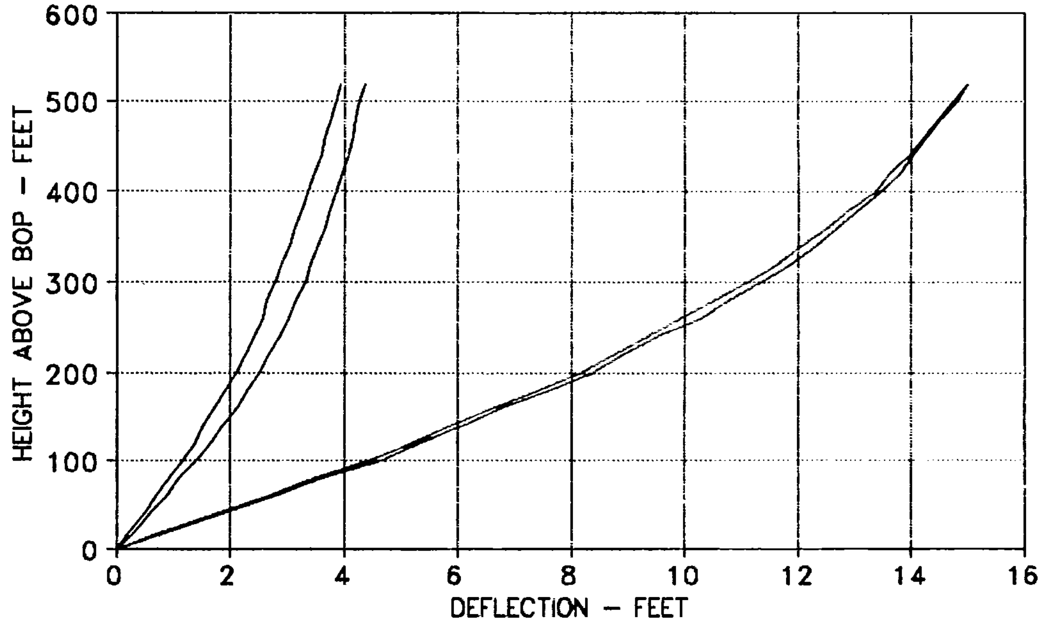


FIGURE 43
BENDING STRESS 500-21.5-2-R (3)

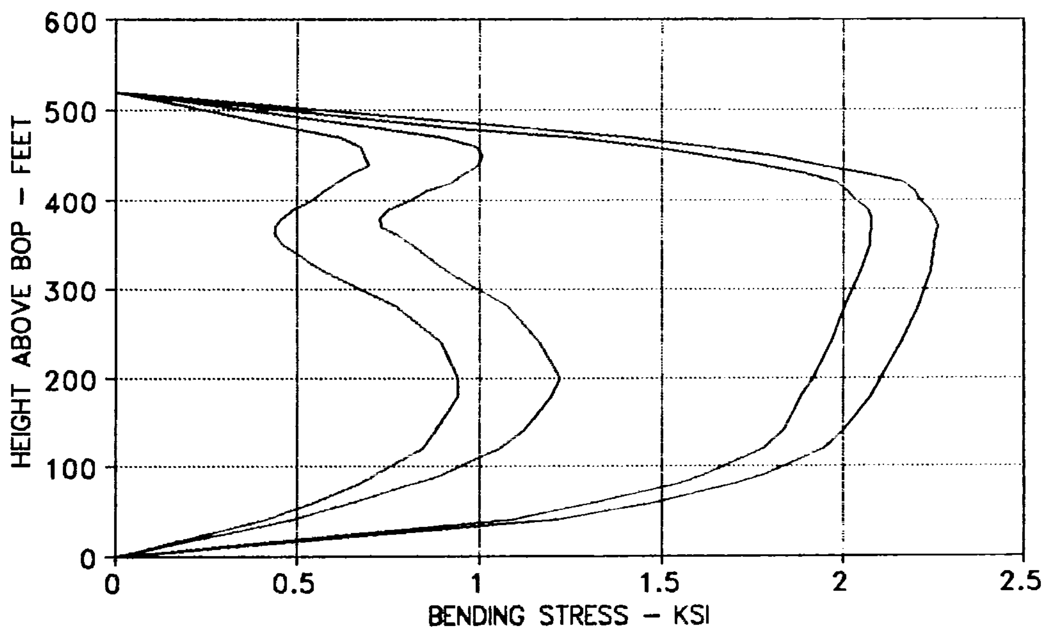


FIGURE 44
DEFLECTION 1500-21.5-1-R (6)

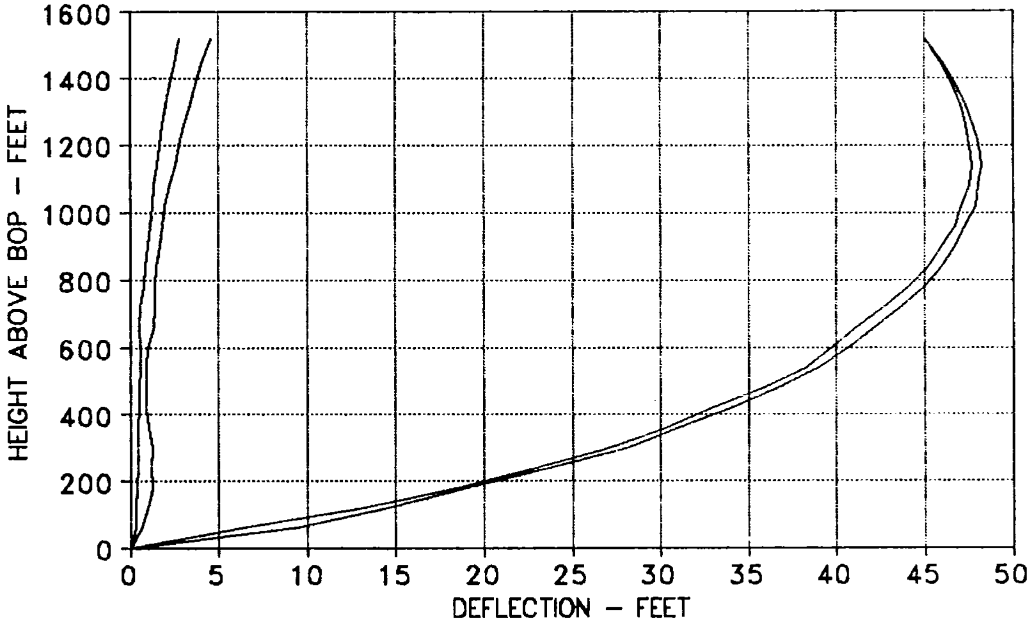


FIGURE 45
BENDING STRESS 1500-21.5-1-R (4)

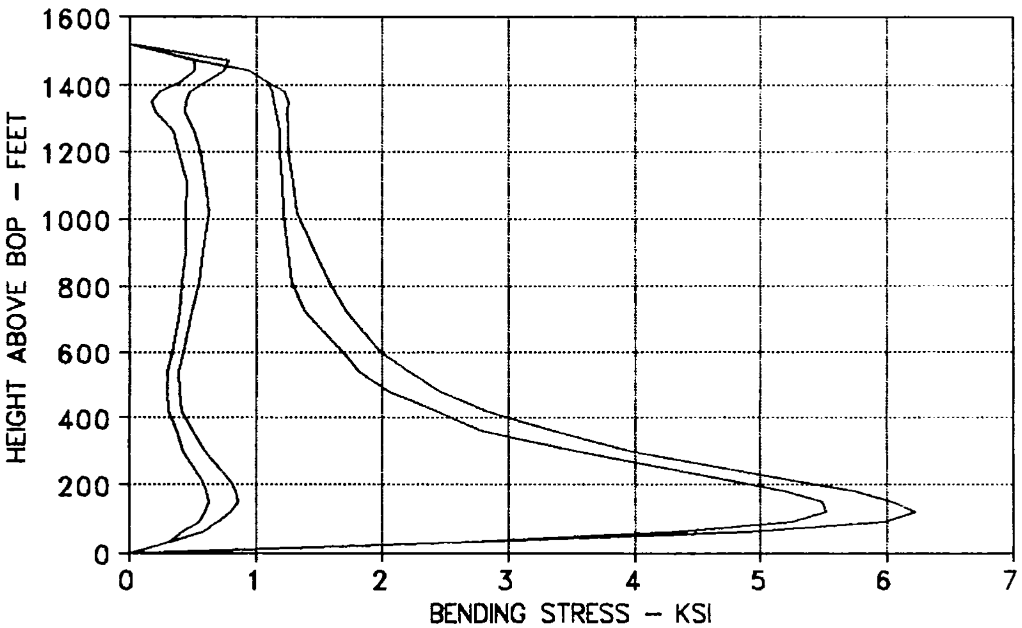


FIGURE 46
DEFLECTION 1500-21.5-2-R (6)

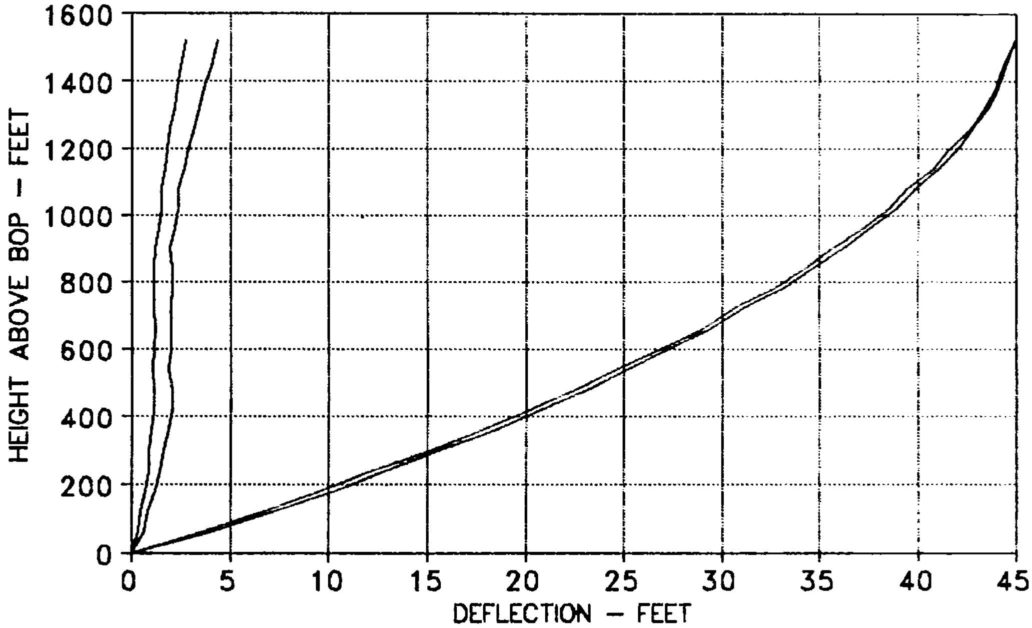


FIGURE 47
BENDING STRESS 1500-21.5-2-R (4)

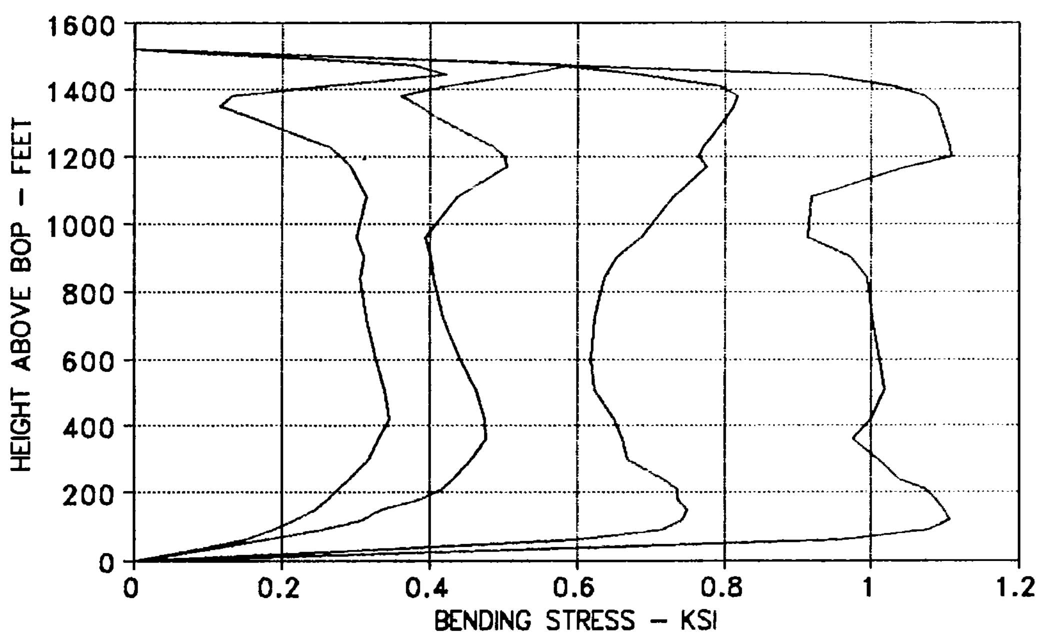


FIGURE 48
DEFLECTION 3000-21.5-1-R (3)

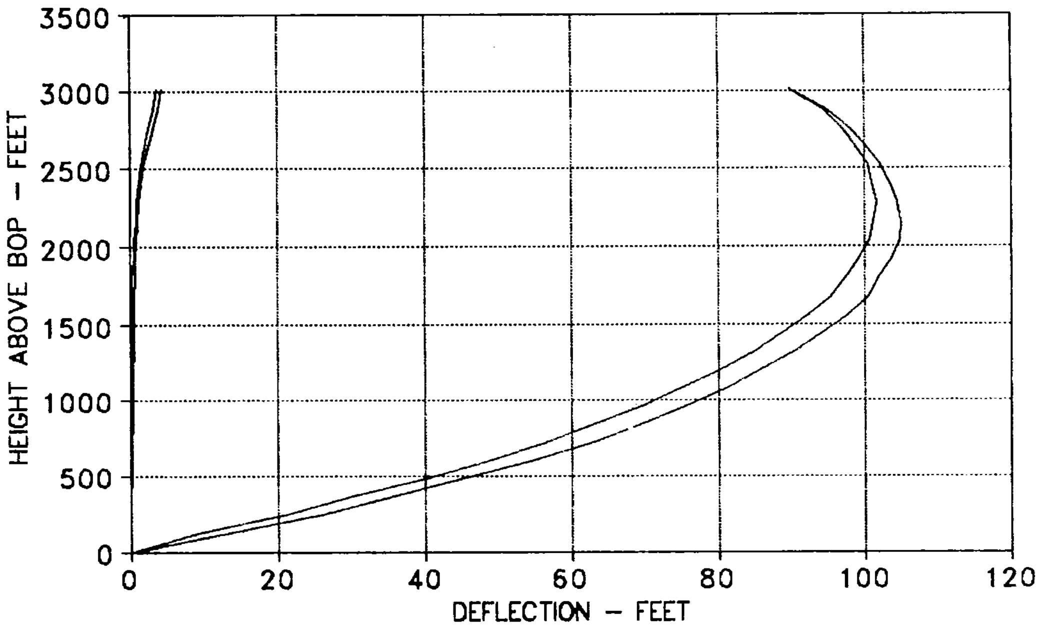


FIGURE 49
BENDING STRESS 3000-21.5-1-R (3)

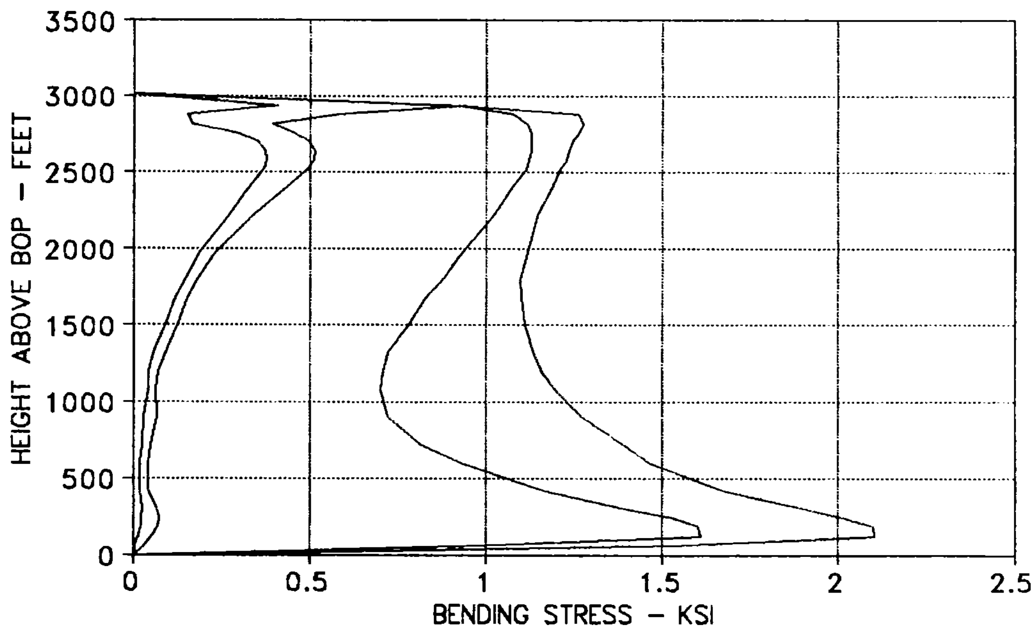


FIGURE 50
DEFLECTION 3000-21.5-2-R (2)

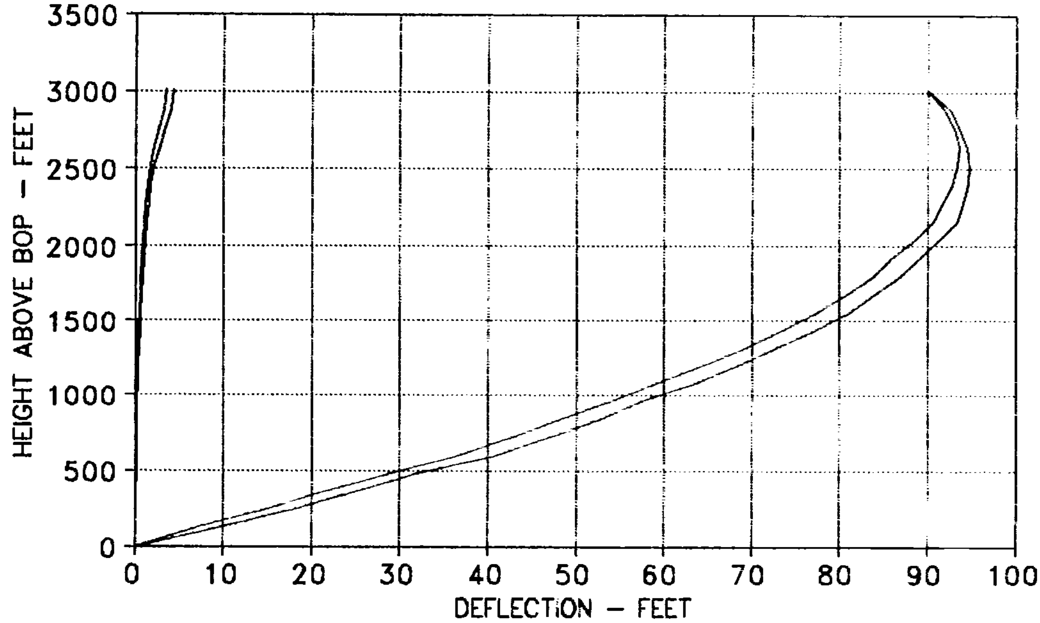
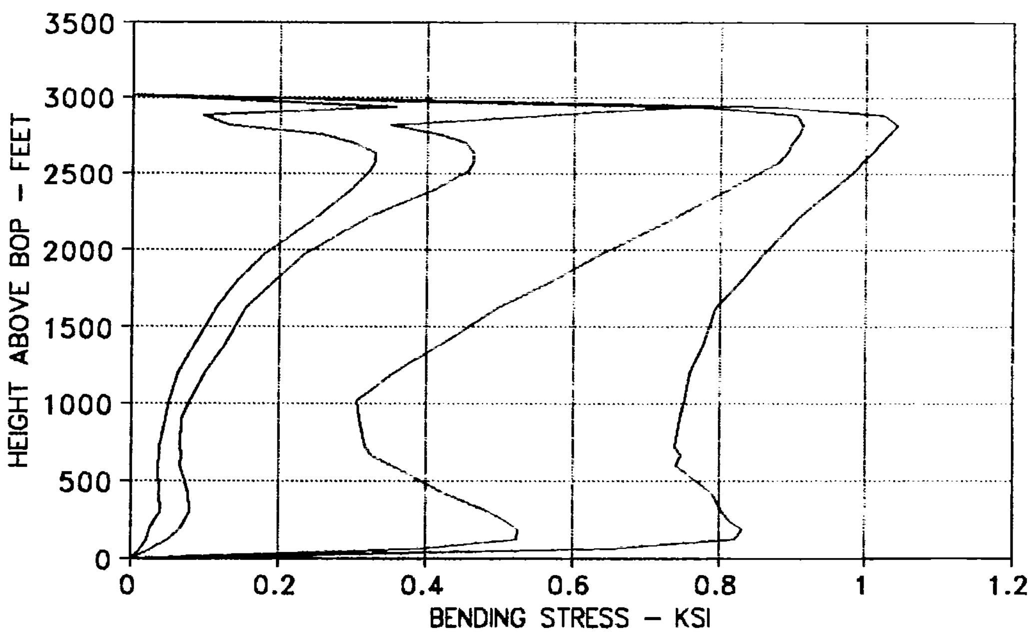


FIGURE 51
BENDING STRESS 3000-21.5-2-R (3)



APPENDIX A
DEFINITIONS OF STATISTICAL PARAMETERS

$$\text{mean} = \bar{x} = \frac{\sum_{i=1}^{N} X_i}{N}$$

$$\text{standard deviation} = s = \sqrt{\frac{\sum_{i=1}^{N} (X_i - \bar{x})^2}{N-1}}$$

$$\text{coefficient of variation, \%} = 100\,\frac{s}{x}$$

$$\text{range} = X_{max} - X_{min}$$

where:

$X_1, X_2, X_3 \ldots X_i \ldots X_n$ = series of independent values of x

N = total number of values

X_{max} = largest value of X found in series

X_{min} = smallest value of X found in series

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