43 NUCLEAR PHYSICS

LEARNING GOALS

By studying this chapter, you will learn:

- Some key properties of atomic nuclei, including radii, densities, spins, and magnetic moments.
- How the binding energy of a nucleus depends on the numbers of protons and neutrons that it contains.
- The most important ways in which unstable nuclei undergo radioactive decay.
- How the decay rate of a radioactive substance depends on time.
- Some of the biological hazards and medical uses of radiation.
- How to analyze some important types of nuclear reactions.
- What happens in a nuclear fission chain reaction, and how it can be controlled.
- The sequence of nuclear reactions that allow the sun and stars to shine.

? These teeth belonged to the oldest known dental patient in the Americas, who had his upper teeth filed down 4500 years ago for ritual purposes. What physical principles make it possible to date biological specimens such as these?

During the past century, applications of nuclear physics have had enormous effects on humankind, some beneficial, some catastrophic. Many people have strong opinions about applications such as bombs and reactors. Ideally, those opinions should be based on understanding, not on prejudice or emotion, and we hope this chapter will help you to reach that ideal.

Every atom contains at its center an extremely dense, positively charged *nucleus,* which is much smaller than the overall size of the atom but contains most of its total mass. We will look at several important general properties of nuclei and of the nuclear force that holds them together. The stability or instability of a particular nucleus is determined by the competition between the attractive nuclear force among the protons and neutrons and the repulsive electrical interactions among the protons. Unstable nuclei *decay,* transforming themselves spontaneously into other structures by a variety of decay processes. Structure-altering nuclear reactions can also be induced by impact on a nucleus of a particle or another nucleus. Two classes of reactions of special interest are *fission* and *fusion.* We could not survive without the 3.90×10^{26}-watt output of one nearby fusion reactor, our sun.

43.1 Properties of Nuclei

As we described in Section 38.4, Rutherford found that the nucleus is tens of thousands of times smaller in radius than the atom itself. Since Rutherford's initial experiments, many additional scattering experiments have been performed, using high-energy protons, electrons, and neutrons as well as alpha particles (helium-4 nuclei). These experiments show that we can model a nucleus as a sphere with a radius R that depends on the total number of *nucleons* (neutrons

and protons) in the nucleus. This number is called the **nucleon number** A. The radii of most nuclei are represented quite well by the equation

$$R = R_0 A^{1/3} \qquad \text{(radius of a nucleus)} \qquad (43.1)$$

where R_0 is an experimentally determined constant:

$$R_0 = 1.2 \times 10^{-15}\ \text{m} = 1.2\ \text{fm}$$

The nucleon number A in Eq. (43.1) is also called the **mass number** because it is the nearest whole number to the mass of the nucleus measured in unified atomic mass units (u). (The proton mass and the neutron mass are both approximately 1 u.) The best current conversion factor is

$$1\ \text{u} = 1.66053886(28) \times 10^{-27}\ \text{kg}$$

In Section 43.2 we'll discuss the masses of nuclei in more detail. Note that when we speak of the masses of nuclei and particles, we mean their *rest* masses.

Nuclear Density

The volume V of a sphere is equal to $4\pi R^3/3$, so Eq. (43.1) shows that the *volume* of a nucleus is proportional to A. Dividing A (the approximate mass in u) by the volume gives us the approximate density and cancels out A. Thus *all nuclei have approximately the same density.* This fact is of crucial importance in understanding nuclear structure.

Example 43.1 Calculating nuclear properties

The most common kind of iron nucleus has a mass number of 56. Find the radius, approximate mass, and approximate density of the nucleus.

SOLUTION

IDENTIFY: We use two key ideas: The radius and mass of a nucleus depend on the mass number A and density is mass divided by volume.

SET UP: We use Eq. (43.1) to determine the radius of the nucleus. The mass of the nucleus in atomic mass units is approximately equal to the mass number.

EXECUTE: The radius is

$$R = R_0 A^{1/3} = (1.2 \times 10^{-15}\ \text{m})(56)^{1/3}$$
$$= 4.6 \times 10^{-15}\ \text{m} = 4.6\ \text{fm}$$

Since $A = 56$, the mass of the nucleus is approximately 56 u, or

$$m \approx (56)(1.66 \times 10^{-27}\ \text{kg}) = 9.3 \times 10^{-26}\ \text{kg}$$

The volume is

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi(4.6 \times 10^{-15}\ \text{m})^3 = 4.1 \times 10^{-43}\ \text{m}^3$$

and the density ρ is approximately

$$\rho = \frac{m}{V} \approx \frac{9.3 \times 10^{-26}\ \text{kg}}{4.1 \times 10^{-43}\ \text{m}^3} = 2.3 \times 10^{17}\ \text{kg/m}^3$$

EVALUATE: The density of solid iron is about $7000\ \text{kg/m}^3$, so we see that the iron nucleus is more than 10^{13} times as dense as the bulk material. Densities of this magnitude are also found in *neutron stars,* which are similar to gigantic nuclei made almost entirely of neutrons. A 1-cm cube of material with this density would have a mass of 2.3×10^{11} kg, or 230 million metric tons!

Nuclides and Isotopes

The basic building blocks of the nucleus are the proton and the neutron. In a neutral atom, the nucleus is surrounded by one electron for every proton in the nucleus. We introduced these particles in Section 21.1; we will recount the discovery of the neutron in Chapter 44. The masses of these particles are

Proton: $m_p = 1.007276\ \text{u} = 1.672622 \times 10^{-27}\ \text{kg}$

Neutron: $m_n = 1.008665\ \text{u} = 1.674927 \times 10^{-27}\ \text{kg}$

Electron: $m_e = 0.000548580\ \text{u} = 9.10938 \times 10^{-31}\ \text{kg}$

The number of protons in a nucleus is the **atomic number** Z. The number of neutrons is the **neutron number** N. The nucleon number or mass number A is the sum of the number of protons Z and the number of neutrons N:

$$A = Z + N \qquad (43.2)$$

A single nuclear species having specific values of both Z and N is called a **nuclide.** Table 43.1 lists values of A, Z, and N for a few nuclides. The electron structure of an atom, which is responsible for its chemical properties, is determined by the charge Ze of the nucleus. The table shows some nuclides that have the same Z but different N. These nuclides are called **isotopes** of that element; they have different masses because they have different numbers of neutrons in their nuclei. A familiar example is chlorine (Cl, $Z = 17$). About 76% of chlorine nuclei have $N = 18$; the other 24% have $N = 20$. Different isotopes of an element usually have slightly different physical properties such as melting and boiling temperatures and diffusion rates. The two common isotopes of uranium with $A = 235$ and 238 are usually separated industrially by taking advantage of the different diffusion rates of gaseous uranium hexafluoride (UF_6) containing the two isotopes.

Table 43.1 also shows the usual notation for individual nuclides: the symbol of the element, with a pre-subscript equal to Z and a pre-superscript equal to the mass number A. The general format for an element El is ${}^{A}_{Z}\mathrm{El}$. The isotopes of chlorine mentioned above, with $A = 35$ and 37, are written ${}^{35}_{17}\mathrm{Cl}$ and ${}^{37}_{17}\mathrm{Cl}$ and pronounced "chlorine-35" and "chlorine-37," respectively. This name of the element determines the atomic number Z, so the pre-subscript Z is sometimes omitted, as in ${}^{35}\mathrm{Cl}$.

Table 43.2 gives the masses of some common atoms, including their electrons. Note that this table gives masses of *neutral* atoms (with Z electrons) rather than masses of *bare* nuclei, because it is much more difficult to measure masses of bare nuclei with high precision. The mass of a neutral carbon-12 atom is exactly 12 u; that's how the unified atomic mass unit is defined. The masses of other atoms are *approximately* equal to A atomic mass units, as we stated earlier. In fact, the atomic masses are *less* than the sum of the masses of their parts (the Z protons, the Z electrons, and the N neutrons). We'll explain this very important mass difference in the next section.

Table 43.1 Compositions of Some Common Nuclides

Nucleus	Mass Number (Total Number of Nucleons), A	Atomic Number (Number of Protons), Z	Neutron Number, $N = A - Z$
${}^{1}_{1}\mathrm{H}$	1	1	0
${}^{2}_{1}\mathrm{D}$	2	1	1
${}^{4}_{2}\mathrm{He}$	4	2	2
${}^{6}_{3}\mathrm{Li}$	6	3	3
${}^{7}_{3}\mathrm{Li}$	7	3	4
${}^{9}_{4}\mathrm{Be}$	9	4	5
${}^{10}_{5}\mathrm{B}$	10	5	5
${}^{11}_{5}\mathrm{B}$	11	5	6
${}^{12}_{6}\mathrm{C}$	12	6	6
${}^{13}_{6}\mathrm{C}$	13	6	7
${}^{14}_{7}\mathrm{N}$	14	7	7
${}^{16}_{8}\mathrm{O}$	16	8	8
${}^{23}_{11}\mathrm{Na}$	23	11	12
${}^{65}_{29}\mathrm{Cu}$	65	29	36
${}^{200}_{80}\mathrm{Hg}$	200	80	120
${}^{235}_{92}\mathrm{U}$	235	92	143
${}^{238}_{92}\mathrm{U}$	238	92	146

Table 43.2 Neutral Atomic Masses for Some Light Nuclides

Element and Isotope	Atomic Number, Z	Neutron Number, N	Atomic Mass (u)	Mass Number, A
Hydrogen $({}^{1}_{1}\mathrm{H})$	1	0	1.007825	1
Deuterium $({}^{2}_{1}\mathrm{H})$	1	1	2.014102	2
Tritium $({}^{3}_{1}\mathrm{H})$	1	2	3.016049	3
Helium $({}^{3}_{2}\mathrm{He})$	2	1	3.016029	3
Helium $({}^{4}_{2}\mathrm{He})$	2	2	4.002603	4
Lithium $({}^{6}_{3}\mathrm{Li})$	3	3	6.015122	6
Lithium $({}^{7}_{3}\mathrm{Li})$	3	4	7.016004	7
Beryllium $({}^{9}_{4}\mathrm{Be})$	4	5	9.012182	9
Boron $({}^{10}_{5}\mathrm{B})$	5	5	10.012937	10
Boron $({}^{11}_{5}\mathrm{B})$	5	6	11.009305	11
Carbon $({}^{12}_{6}\mathrm{C})$	6	6	12.000000	12
Carbon $({}^{13}_{6}\mathrm{C})$	6	7	13.003355	13
Nitrogen $({}^{14}_{7}\mathrm{N})$	7	7	14.003074	14
Nitrogen $({}^{15}_{7}\mathrm{N})$	7	8	15.000109	15
Oxygen $({}^{16}_{8}\mathrm{O})$	8	8	15.994915	16
Oxygen $({}^{17}_{8}\mathrm{O})$	8	9	16.999132	17
Oxygen $({}^{18}_{8}\mathrm{O})$	8	10	17.999160	18

Source: A. H. Wapstra and G. Audi, *Nuclear Physics* **A595,** 4 (1995).

Nuclear Spins and Magnetic Moments

Like electrons, protons and neutrons are also spin-$\frac{1}{2}$ particles with spin angular momentum given by the same equations as in Section 41.3. The magnitude of the spin angular momentum $\vec{S}$ of a nucleon is

$$S = \sqrt{\frac{1}{2}\left(\frac{1}{2} + 1\right)}\hbar = \sqrt{\frac{3}{4}}\hbar \qquad (43.3)$$

and the z-component is

$$S_z = \pm\frac{1}{2}\hbar \qquad (43.4)$$

In addition to the spin angular momentum of the nucleons, there may be *orbital* angular momentum associated with their motions within the nucleus. The orbital angular momentum of the nucleons is quantized in the same way as that of electrons in atoms.

The *total* angular momentum $\vec{J}$ of the nucleus is the vector sum of the individual spin and orbital angular momenta of all the nucleons. It has magnitude

$$J = \sqrt{j(j + 1)}\hbar \qquad (43.5)$$

and z-component

$$J_z = m_j\hbar \qquad (m_j = -j, -j + 1, \ldots, j - 1, j) \qquad (43.6)$$

When the total number of nucleons A is *even, j* is an integer; when it is *odd, j* is a half-integer. All nuclides for which both Z and N are even have $J = 0$, which suggests that pairing of particles with opposite spin components may be an important consideration in nuclear structure. The total nuclear angular momentum quantum number j is usually called the *nuclear spin,* even though in general it refers to a combination of the orbital and spin angular momenta of the nucleons that make up the nucleus.

Associated with nuclear angular momentum is a *magnetic moment.* When we discussed *electron* magnetic moments in Section 41.2, we introduced the Bohr magneton $\mu_B = e\hbar/2m_e$ as a natural unit of magnetic moment. We found that the magnitude of the z-component of the electron-spin magnetic moment is almost exactly equal to μ_B; that is, $|\mu_{sz}|_{\text{electron}} \approx \mu_B$. In discussing *nuclear* magnetic moments, we can define an analogous quantity, the **nuclear magneton** μ_n:

$$\mu_n = \frac{e\hbar}{2m_p} = 5.05078 \times 10^{-27}\ \text{J/T} = 3.15245 \times 10^{-8}\ \text{eV/T} \qquad (43.7)$$

(nuclear magneton)

where m_p is the proton mass. Because the proton mass m_p is 1836 times larger than the electron mass m_e, the nuclear magneton μ_n is 1836 times smaller than the Bohr magneton μ_B.

We might expect the magnitude of the z-component of the spin magnetic moment of the proton to be approximately μ_n. Instead, it turns out to be

$$|\mu_{sz}|_{\text{proton}} = 2.7928\mu_n \qquad (43.8)$$

Even more surprising, the neutron, which has no charge, has a corresponding magnitude of

$$|\mu_{sz}|_{\text{neutron}} = 1.9130\mu_n \qquad (43.9)$$

The proton has a positive charge; as expected, its spin magnetic moment $\vec{\mu}$ is parallel to its spin angular momentum $\vec{S}$. However, $\vec{\mu}$ and $\vec{S}$ are opposite for a neutron, as would be expected for a negative charge distribution. These *anomalous* magnetic moments arise because the proton and neutron aren't really fundamental particles but are made of simpler particles called *quarks.* We'll discuss quarks in some detail in Chapter 44.

The magnetic moment of an entire nucleus is typically a few nuclear magnetons. When a nucleus is placed in an external magnetic field $\vec{B}$, there is an interaction energy $U = -\vec{\mu}\cdot\vec{B} = -\mu_z B$ just as with atomic magnetic moments. The components of the magnetic moment in the direction of the field μ_z are quantized, so a series of energy levels results from this interaction.

Example 43.2 Proton spin flips

Protons are placed in a magnetic field in the z-direction with magnitude 2.30 T. (a) What is the energy difference between a state with the z-component of proton spin angular momentum parallel to the field and one with the component antiparallel to the field? (b) A proton can make a transition from one of these states to the other by emitting or absorbing a photon with energy equal to the energy difference of the two states. Find the frequency and wavelength of such a photon.

SOLUTION

IDENTIFY: The proton is a spin-$\frac{1}{2}$ particle with a magnetic moment, so its energy depends on the orientation of its spin relative to an applied magnetic field.

SET UP: The magnetic field $\vec{B}$ is assumed to be in the positive z-direction, and the magnetic moment $\vec{\mu}$ of the proton is in the same direction as its spin. If the z-component of spin is aligned with $\vec{B}$, then μ_z is equal to the positive value given in Eq. (43.8); if the z-component of spin is opposite $\vec{B}$, then μ_z is the negative of this value. The interaction energy in either case is $U = -\mu_z B$, and the energy difference [our target variable in part (a)] is the difference between the values of U for the two spin orientations. We find the photon frequency and wavelength using the relationships $E = hf = hc/\lambda$.

EXECUTE: (a) When the z-component of $\vec{S}$ (and $\vec{\mu}$) is parallel to the field, the interaction energy is

$$U = -|\mu_z|B = -(2.7928)(3.152 \times 10^{-8}\ \text{eV/T})(2.30\ \text{T})$$
$$= -2.025 \times 10^{-7}\ \text{eV}$$

When the components are antiparallel to the field, the energy is $+2.025 \times 10^{-7}$ eV, and the energy *difference* between the two states is

$$\Delta E = 2(2.025 \times 10^{-7}\ \text{eV}) = 4.05 \times 10^{-7}\ \text{eV}$$

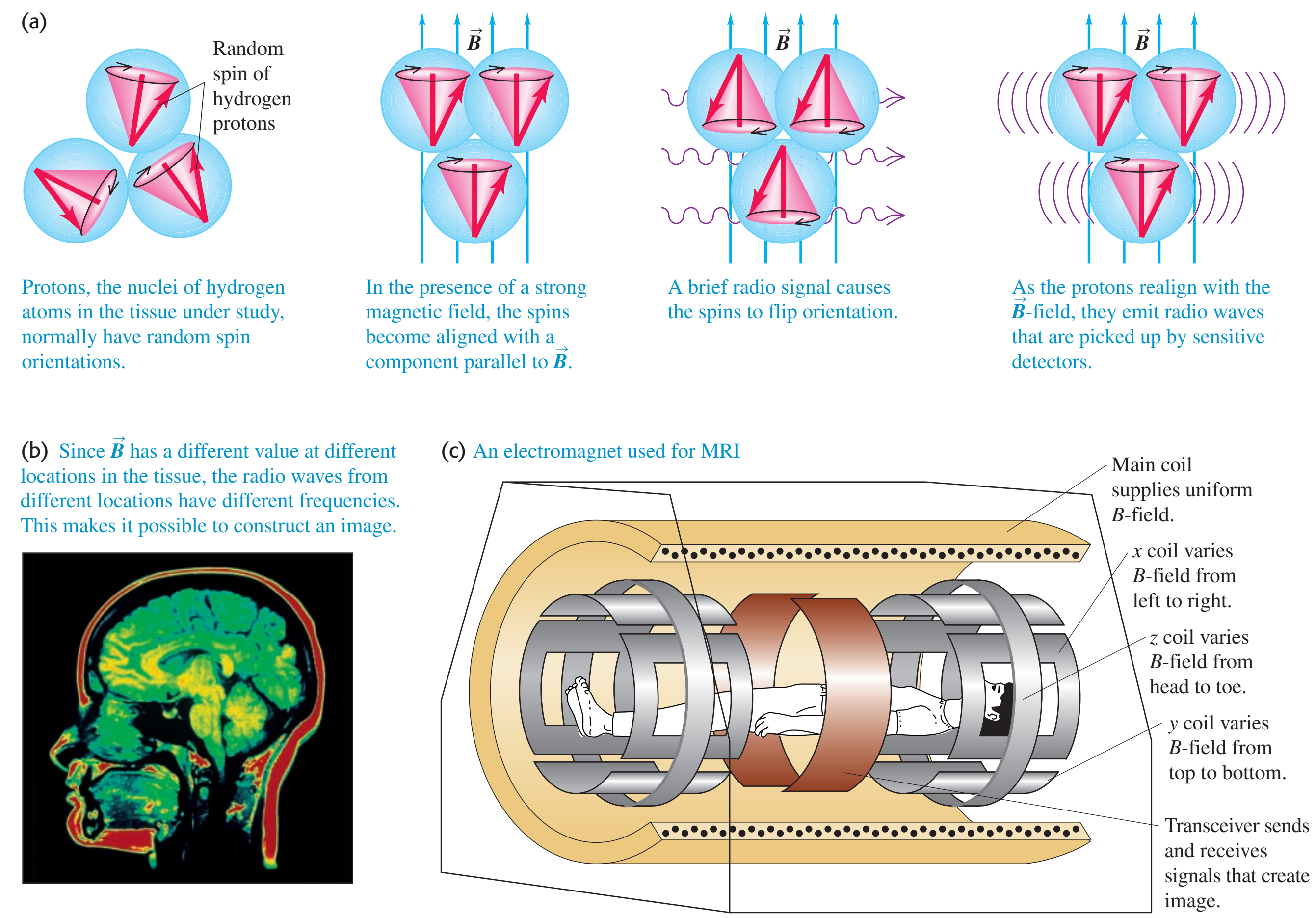
(b) The corresponding photon frequency and wavelength are

$$f = \frac{\Delta E}{h} = \frac{4.05 \times 10^{-7}\ \text{eV}}{4.136 \times 10^{-15}\ \text{eV}\cdot\text{s}} = 9.79 \times 10^{7}\ \text{Hz} = 97.9\ \text{MHz}$$

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^{8}\ \text{m/s}}{9.79 \times 10^{7}\ \text{s}^{-1}} = 3.06\ \text{m}$$

EVALUATE: This frequency is in the middle of the FM radio band. When a hydrogen specimen is placed in a 2.30-T magnetic field and then irradiated with radiation of this frequency, the proton *spin flips* can be detected by the absorption of energy from the radiation.

43.1 Magnetic resonance imaging (MRI).



Nuclear Magnetic Resonance and MRI

Spin-flip experiments of the sort referred to in Example 43.2 are called *nuclear magnetic resonance* (NMR). They have been carried out with many different nuclides. Frequencies and magnetic fields can be measured very precisely, so this technique permits precise measurements of nuclear magnetic moments. An elaboration of this basic idea leads to *magnetic resonance imaging* (MRI), a noninvasive imaging technique that discriminates among various body tissues on the basis of the differing environments of protons in the tissues. The principles of MRI are shown in Fig. 43.1.

The magnetic moment of a nucleus is also the *source* of a magnetic field. In an atom the interaction of an electron's magnetic moment with the field of the nucleus's magnetic moment causes additional splittings in atomic energy levels and spectra. We called this effect *hyperfine structure* in Section 41.3. Measurements of the hyperfine structure may be used to directly determine the nuclear spin.

Test Your Understanding of Section 43.1 (a) By what factor must the mass number of a nucleus increase to double its volume? (i) $\sqrt[3]{2}$; (ii) $\sqrt{2}$; (iii) 2; (iv) 4; (v) 8. (b) By what factor must the mass number increase to double the radius of the nucleus? (i) $\sqrt[3]{2}$; (ii) $\sqrt{2}$; (iii) 2; (iv) 4; (v) 8.

43.2 Nuclear Binding and Nuclear Structure

ActivPhysics Online 19.2 Nuclear Binding Energy

Because energy must be added to a nucleus to separate it into its individual protons and neutrons, the total rest energy E_0 of the separated nucleons is greater than the rest energy of the nucleus. The energy that must be added to separate the nucleons is called the **binding energy** E_B; it is the magnitude of the energy by which the nucleons are bound together. Thus the rest energy of the nucleus is $E_0 - E_B$. Using the equivalence of rest mass and energy (see Section 37.8), we see that the total mass of the nucleons is always greater than the mass of the nucleus by an amount E_B/c^2 called the *mass defect*. The binding energy for a nucleus containing Z protons and N neutrons is defined as

$$E_B = (ZM_H + Nm_n - {}^{A}_{Z}M)c^2 \quad \text{(nuclear binding energy)} \quad (43.10)$$

where ${}^{A}_{Z}M$ is the mass of the *neutral* atom containing the nucleus, the quantity in the parentheses is the mass defect, and $c^2 = 931.5\ \text{MeV/u}$. Note that Eq. (43.10) does not include Zm_p, the mass of Z protons. Rather, it contains ZM_H, the mass of Z protons and Z electrons combined as Z neutral ${}^{1}_{1}\text{H}$ atoms, to balance the Z electrons included in ${}^{A}_{Z}M$, the mass of the neutral atom.

The simplest nucleus is that of hydrogen, a single proton. Next comes the nucleus of ${}^{2}_{1}\text{H}$, the isotope of hydrogen with mass number 2, usually called *deuterium*. Its nucleus consists of a proton and a neutron bound together to form a particle called the *deuteron*. By using values from Table 43.2 in Eq. (43.10), the binding energy of the deuteron is

$$E_B = (1.007825\ \text{u} + 1.008665\ \text{u} - 2.014102\ \text{u})(931.5\ \text{MeV/u})$$
$$= 2.224\ \text{MeV}$$

This much energy would be required to pull the deuteron apart into a proton and a neutron. An important measure of how tightly a nucleus is bound is the *binding energy per nucleon*, E_B/A. At $(2.224\ \text{MeV})/(2\ \text{nucleons}) = 1.112\ \text{MeV}$ per nucleon, ${}^{2}_{1}\text{H}$ has the lowest binding energy per nucleon of all nuclides.

Problem-Solving Strategy 43.1 Nuclear Properties

IDENTIFY *the relevant concepts:* The key properties of any nucleus include the mass, radius, binding energy, mass defect, binding energy per nucleon, and angular momentum.

SET UP *the problem:* Once you have identified your target variables, collect the equations you need to solve the problem. A relatively small number of equations from this section and Section 43.1 are all you need, but make sure that you understand their meanings.

EXECUTE *the solution:* Solve the equations for the target variables. When doing energy calculations involving the binding energy and binding energy per nucleon, note that mass tables almost always list the masses of *neutral* atoms, including their full complement of electrons. To compensate for this, use the mass M_H of a ${}^{1}_{1}\text{H}$ atom in Eq. (43.10) rather than the mass of a bare proton. The binding energies of the electrons in the neutral atoms are much smaller and tend to cancel in the subtraction in Eq. (43.10), so we won't worry about them. As Eq. (43.10) shows, binding-energy calculations often involve subtracting two nearly equal quantities. To get enough precision in the difference, you often have to carry seven, eight, or nine significant figures, if that many are available. If not, you may have to be content with an approximate result.

EVALUATE *your answer:* Familiarity with some typical numerical magnitudes is very useful when checking your results. Many of these magnitudes are quite different in nuclear physics than in atomic physics. Protons and neutrons are about 1840 times as massive as electrons. The radius of a nucleus is of the order 10^{-15} m; the electric potential energy of two protons in a nucleus is of the order 10^{-13} J or 1 MeV. Thus typical nuclear interaction energies are of the order of a few MeV, rather than a few eV as with atoms. The typical binding energy per nucleon is about 1% of the rest energy of a nucleon. For comparison, the ionization energy of the hydrogen atom is only 0.003% of the electron's rest energy.

Angular momentum is of the same order of magnitude in both nuclei and atoms because it is determined by the value of $\hbar$. Magnetic moments of nuclei, however, are about 1000 times *smaller* than those of electrons in atoms because the nuclei are so much more massive than electrons.

Example 43.3 The most strongly bound nuclide

Because it has the highest binding energy per nucleon of all nuclides, ${}^{62}_{28}\text{Ni}$ may be described as the most strongly bound. Its neutral atomic mass is 61.928349 u. Find its mass defect, its total binding energy, and its binding energy per nucleon.

SOLUTION

IDENTIFY: The mass defect is the difference between the mass of the nucleus and the combined mass of its constituent nucleons. The binding energy E_B is this quantity multiplied by c^2, and the binding energy per nucleon is E_B divided by the mass number A.

SET UP: We use Eq. (43.10) to determine the binding energy.

EXECUTE: We use $Z = 28$, $M_H = 1.007825\ \text{u}$, $N = A - Z = 62 - 28 = 34$, $m_n = 1.008665\ \text{u}$, and ${}^{A}_{Z}M = 61.928349\ \text{u}$ in the parentheses of Eq. (43.10) to find a mass defect of 0.585361 u. Then

$$E_B = (0.585361\ \text{u})(931.5\ \text{MeV/u}) = 545.3\ \text{MeV}$$

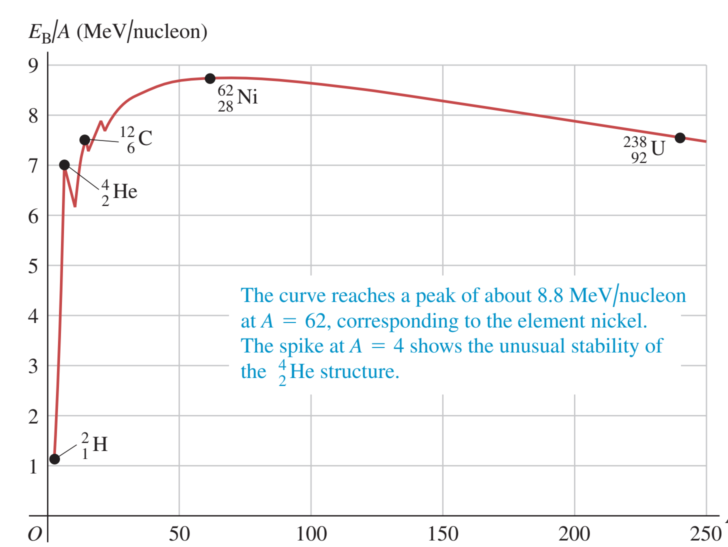
It would require a minimum of 545.3 MeV to pull a ${}^{62}_{28}\text{Ni}$ nucleus completely apart into 62 separate nucleons. The binding energy *per nucleon* is $\frac{1}{62}$ of this, or 8.795 MeV per nucleon.

EVALUATE: The mass defect of ${}^{62}_{28}\text{Ni}$ is about 1% of the mass of the atom: $(0.585361\ \text{u})/(61.928349\ \text{u}) = 0.00945223 = 0.945223\%$. Since the mass of the atom is almost the same as the mass of the nucleus, this means that the binding energy (mass defect times c^2) is about 1% of the rest energy of the nucleus, and the binding energy per nucleon is about 1% of the rest energy of a nucleon. This result is as expected. Note that the mass defect is more than half the mass of a neutron or a proton, which suggests how tightly bound nuclei are.

Nearly all stable nuclides, from the lightest to the most massive, have binding energies in the range of 7–9 MeV per nucleon. Figure 43.2 is a graph of binding energy per nucleon as a function of the mass number A. Note the spike at $A = 4$, showing the unusually large binding energy per nucleon of the ${}^{4}_{2}\text{He}$ nucleus (alpha particle) relative to its neighbors. To explain this curve, we must consider the interactions among the nucleons.

The Nuclear Force

The force that binds protons and neutrons together in the nucleus, despite the electrical repulsion of the protons, is an example of the *strong interaction* that we mentioned in Section 5.5. In the context of nuclear structure, this interaction is called the *nuclear force*. Here are some of its characteristics. First, it does not depend on charge; neutrons as well as protons are bound, and the binding is the same for both. Second, it has short range, of the order of nuclear dimensions—that is, 10^{-15} m. (Otherwise, the nucleus would grow by pulling in additional protons and neutrons.) But within its range, the nuclear force is much stronger than electrical forces; otherwise, the nucleus could never be stable. It would be nice if we could write a simple equation like Newton's law of gravitation or Coulomb's law for this force, but physicists have yet to fully determine its dependence on the separation r. Third, the nearly constant density of nuclear matter and the nearly constant binding energy per nucleon of larger nuclides show that a particular nucleon cannot interact simultaneously with *all* the other nucleons in a nucleus, but only with those few in its immediate vicinity. This is different from electrical forces; *every* proton in the nucleus repels every other one. This limited number of



43.2 Approximate binding energy per nucleon as a function of mass number A (the total number of nucleons) for stable nuclides.

interactions is called *saturation;* it is analogous to covalent bonding in molecules and solids. Finally, the nuclear force favors binding of *pairs* of protons or neutrons with opposite spins and of *pairs of pairs*—that is, a pair of protons and a pair of neutrons, each pair having opposite spins. Hence the alpha particle (two protons and two neutrons) is an exceptionally stable nucleus for its mass number. We'll see other evidence for pairing effects in nuclei in the next subsection. (In Section 42.8 we described an analogous pairing that binds opposite-spin electrons in Cooper pairs in the BCS theory of superconductivity.)

The analysis of nuclear structure is more complex than the analysis of many-electron atoms. Two different kinds of interactions are involved (electrical and nuclear), and the nuclear force is not yet completely understood. Even so, we can gain some insight into nuclear structure by the use of simple models. We'll discuss briefly two rather different but successful models, the *liquid-drop model* and the *shell model.*

The Liquid-Drop Model

The **liquid-drop model,** first proposed in 1928 by the Russian physicist George Gamow and later expanded on by Niels Bohr, is suggested by the observation that all nuclei have nearly the same density. The individual nucleons are analogous to molecules of a liquid, held together by short-range interactions and surface-tension effects. We can use this simple picture to derive a formula for the estimated total binding energy of a nucleus. We'll include five contributions:

1. We've remarked that nuclear forces show *saturation;* an individual nucleon interacts only with a few of its nearest neighbors. This effect gives a binding-energy term that is proportional to the number of nucleons. We write this term as C_1A, where C_1 is an experimentally determined constant.
2. The nucleons on the surface of the nucleus are less tightly bound than those in the interior because they have no neighbors outside the surface. This decrease in the binding energy gives a *negative* energy term proportional to the surface area $4\pi R^2$. Because R is proportional to $A^{1/3}$, this term is proportional to $A^{2/3}$; we write it as $-C_2A^{2/3}$, where C_2 is another constant.
3. Every one of the Z protons repels every one of the $(Z-1)$ other protons. The total repulsive electric potential energy is proportional to $Z(Z-1)$ and inversely proportional to the radius R and thus to $A^{1/3}$. This energy term is negative because the nucleons are less tightly bound than they would be without the electrical repulsion. We write this correction as $-C_3Z(Z-1)/A^{1/3}$.
4. To be in a stable, low-energy state, the nucleus must have a balance between the energies associated with the neutrons and with the protons. This means that N is close to Z for small A and N is greater than Z (but not too much greater) for larger A. We need a negative energy term corresponding to the difference $|N-Z|$. The best agreement with observed binding energies is obtained if this term is proportional to $(N-Z)^2/A$. If we use $N = A - Z$ to express this energy in terms of A and Z, this correction is $-C_4(A-2Z)^2/A$.
5. Finally, the nuclear force favors *pairing* of protons and of neutrons. This energy term is positive (more binding) if both Z and N are even, negative (less binding) if both Z and N are odd, and zero otherwise. The best fit to the data occurs with the form $\pm C_5A^{-4/3}$ for this term.

The total estimated binding energy E_B is the sum of these five terms:

$$E_B = C_1A - C_2A^{2/3} - C_3\frac{Z(Z-1)}{A^{1/3}} - C_4\frac{(A-2Z)^2}{A} \pm C_5A^{-4/3} \quad (43.11)$$

(nuclear binding energy)

The constants C_1, C_2, C_3, C_4, and C_5, chosen to make this formula best fit the observed binding energies of nuclides, are

$$C_1 = 15.75 \text{ MeV}$$
$$C_2 = 17.80 \text{ MeV}$$
$$C_3 = 0.7100 \text{ MeV}$$
$$C_4 = 23.69 \text{ MeV}$$
$$C_5 = 39 \text{ MeV}$$

The constant C_1 is the binding energy per nucleon due to the saturated nuclear force. This energy is almost 16 MeV per nucleon, about double the *total* binding energy per nucleon in most nuclides.

If we estimate the binding energy E_B using Eq. (43.11), we can solve Eq. (43.10) to use it to estimate the mass of any neutral atom:

$${}^{A}_{Z}M = ZM_H + Nm_n - \frac{E_B}{c^2} \quad \text{(semiempirical mass formula)} \quad (43.12)$$

Equation (43.12) is called the *semiempirical mass formula.* The name is apt; it is *empirical* in the sense that the C's have to be determined empirically (experimentally), yet it does have a sound theoretical basis.

Example 43.4 Estimating the binding energy and mass

Consider the nuclide ${}^{62}_{28}\text{Ni}$ of Example 43.3. (a) Calculate the five terms in the binding energy and the total estimated binding energy. (b) Find its neutral atomic mass using the semiempirical mass formula.

SOLUTION

IDENTIFY: We use the liquid-drop model of the nucleus and its five contributions to the binding energy.

SET UP: We use Eq. (43.11) to calculate the individual terms in the binding energy as well as the total binding energy, and we use Eq. (43.12) to find the neutral atomic mass.

EXECUTE: (a) If we substitute $Z = 28$, $A = 62$, and $N = 34$ into Eq. (43.11), the individual terms are

1. $C_1A = (15.75 \text{ MeV})(62) = 976.5 \text{ MeV}$
2. $-C_2A^{2/3} = -(17.80 \text{ MeV})(62)^{2/3} = -278.8 \text{ MeV}$
3. $-C_3\frac{Z(Z-1)}{A^{1/3}} = -(0.7100 \text{ MeV})\frac{(28)(27)}{(62)^{1/3}} = -135.6 \text{ MeV}$
4. $-C_4\frac{(A-2Z)^2}{A} = -(23.69 \text{ MeV})\frac{(62-56)^2}{62} = -13.8 \text{ MeV}$
5. $+C_5A^{-4/3} = (39 \text{ MeV})(62)^{-4/3} = 0.2 \text{ MeV}$

The pairing correction (term 5) is positive because both Z and N are even. Note that this is by far the smallest of all the terms. The total estimated binding energy is the sum of these five terms, or 548.5 MeV.

(b) Now we use the $E_B = 548.5$ MeV value in Eq. (43.12) to find

$$M = 28(1.007825 \text{ u}) + 34(1.008665 \text{ u}) - \frac{548.5 \text{ MeV}}{931.5 \text{ MeV/u}}$$
$$= 61.925 \text{ u}$$

EVALUATE: The binding energy of ${}^{62}_{28}\text{Ni}$ calculated in part (a) is only about 0.6% larger than the true value of 545.3 MeV found in Example 43.3, and the mass calculated in part (b) is only about 0.005% smaller than the measured value of 61.928349 u. These results show how accurate the semiempirical mass formula can be.

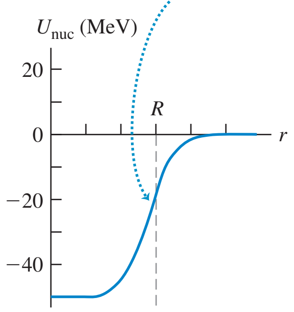
The liquid-drop model and the mass formula derived from it are quite successful in correlating nuclear masses, and we will see later that they are a great help in understanding decay processes of unstable nuclides. Some other aspects of nuclei, such as angular momentum and excited states, are better approached with different models.

The Shell Model

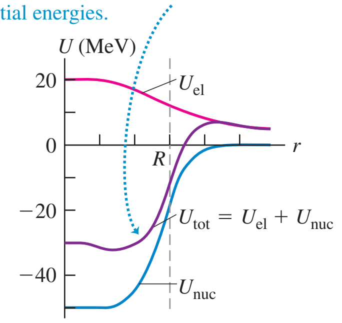
The **shell model** of nuclear structure is analogous to the central-field approximation in atomic physics (see Section 41.4). We picture each nucleon as moving in a potential that represents the averaged-out effect of all the other nucleons. This

43.3 Approximate potential-energy functions for a nucleon in a nucleus. The approximate nuclear radius is R.

(a) The potential energy U_{nuc} due to the nuclear force is the same for protons and neutrons. For neutrons, it is the *total* potential energy.



(b) For protons, the total potential energy U_{tot} is the sum of the nuclear (U_{nuc}) and electric (U_{el}) potential energies.



may not seem to be a very promising approach; the nuclear force is very strong, very short range, and therefore strongly distance dependent. However, in some respects, this model turns out to work fairly well.

The potential-energy function for the nuclear force is the same for protons as for neutrons. A reasonable assumption for the shape of this function is shown in Fig. 43.3a. This function is a three-dimensional version of the square well we discussed in Section 40.2. The corners are somewhat rounded because the nucleus doesn't have a sharply defined surface. For protons there is an additional potential energy associated with electrical repulsion. We consider each proton to interact with a sphere of uniform charge density, with radius R and total charge $(Z - 1)e$. Figure 43.3b shows the nuclear, electric, and total potential energies for a proton as functions of the distance r from the center of the nucleus.

In principle, we could solve the Schrödinger equation for a proton or neutron moving in such a potential. For any spherically symmetric potential energy, the angular-momentum states are the same as for the electrons in the central-field approximation in atomic physics. In particular, we can use the concept of *filled shells and subshells* and their relationship to stability. In atomic structure we found that the values $Z = 2$, 10, 18, 36, 54, and 86 (the atomic numbers of the noble gases) correspond to particularly stable electron arrangements.

A comparable effect occurs in nuclear structure. The numbers are different because the potential-energy function is different and the nuclear spin–orbit interaction is much stronger and of opposite sign than in atoms, so the subshells fill up in a different order from those for electrons in an atom. It is found that when the number of neutrons *or* the number of protons is 2, 8, 20, 28, 50, 82, or 126, the resulting structure is unusually stable—that is, has an unusually great binding energy. (Nuclides with $Z = 126$ have not been observed in nature.) These numbers are called *magic numbers.* Nuclides in which Z is a magic number tend to have an above-average number of stable isotopes. There are several *doubly magic* nuclides for which both Z and N are magic, including

$$^{4}_{2}\text{He} \qquad ^{16}_{8}\text{O} \qquad ^{40}_{20}\text{Ca} \qquad ^{48}_{20}\text{Ca} \qquad ^{208}_{82}\text{Pb}$$

All these nuclides have substantially higher binding energy per nucleon than do nuclides with neighboring values of N or Z. They also all have zero nuclear spin. The magic numbers correspond to filled-shell or -subshell configurations of nucleon energy levels with a relatively large jump in energy to the next allowed level.

Test Your Understanding of Section 43.2 Rank the following nuclei in order from largest to smallest value of the binding energy per nucleon. (i) $^{4}_{2}\text{He}$; (ii) $^{52}_{24}\text{Cr}$; (iii) $^{152}_{62}\text{Sm}$; (iv) $^{200}_{80}\text{Hg}$; (v) $^{252}_{92}\text{Cf}$.

43.3 Nuclear Stability and Radioactivity

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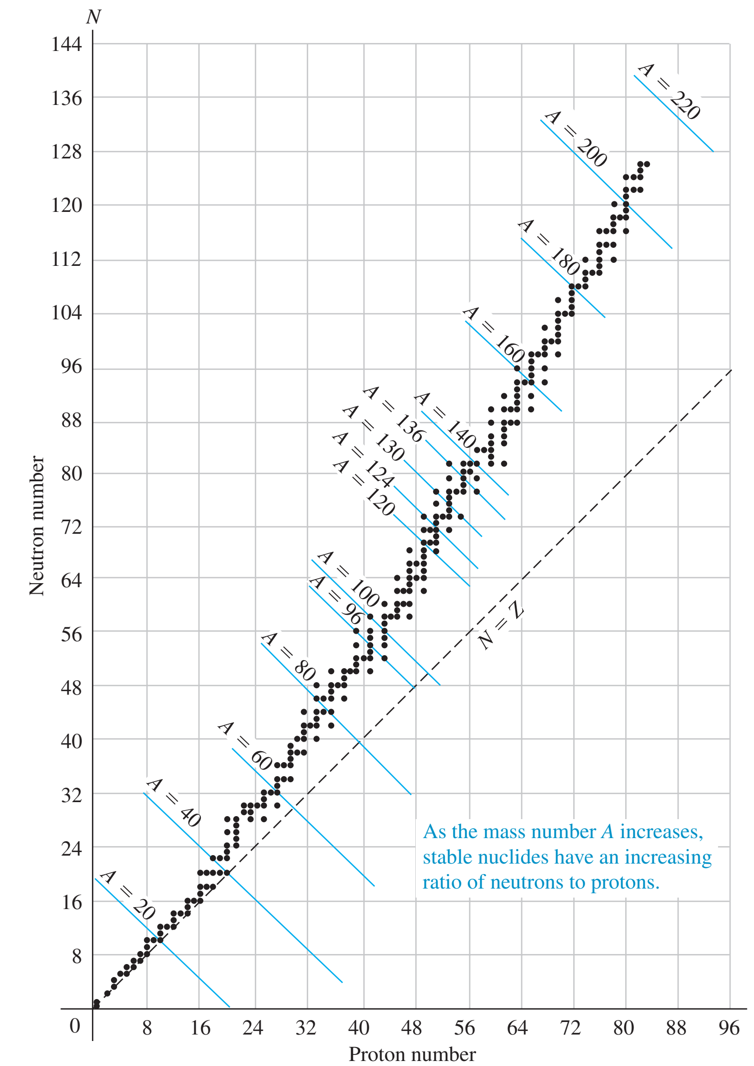
19.2 Nuclear Binding Energy
19.4 Radioactivity

Among about 2500 known nuclides, fewer than 300 are stable. The others are unstable structures that decay to form other nuclides by emitting particles and electromagnetic radiation, a process called **radioactivity.** The time scale of these decay processes ranges from a small fraction of a microsecond to billions of years. The *stable* nuclides are shown by dots on the graph in Fig. 43.4, where the neutron number N and proton number (or atomic number) Z for each nuclide are plotted. Such a chart is called a *Segrè chart,* after its inventor, the Italian-American physicist Emilio Segrè (1905–1989).

Each blue line perpendicular to the line $N = Z$ represents a specific value of the mass number $A = Z + N$. Most lines of constant A pass through only one or two stable nuclides; that is, there is usually a very narrow range of stability for a given mass number. The lines at $A = 20$, $A = 40$, $A = 60$, and $A = 80$ are



43.4 Segrè chart showing neutron number and proton number for stable nuclides.

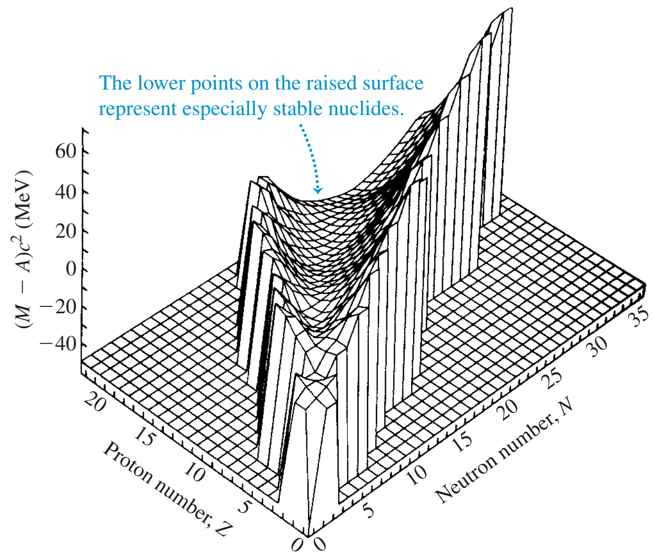
examples. In four cases these lines pass through *three* stable nuclides—namely, at $A = 96$, 124, 130, and 136.

Four stable nuclides have both odd Z and odd N:

$$^{2}_{1}\text{H} \qquad ^{6}_{3}\text{Li} \qquad ^{10}_{5}\text{B} \qquad ^{14}_{7}\text{N}$$

These are called *odd-odd nuclides.* The absence of other odd-odd nuclides shows the influence of pairing. Also, there is *no* stable nuclide with $A = 5$ or $A = 8$. The doubly magic $^{4}_{2}\text{He}$ nucleus, with a pair of protons and a pair of neutrons, has no interest in accepting a fifth particle into its structure. Collections of eight nucleons decay to smaller nuclides, with a $^{8}_{4}\text{Be}$ nucleus immediately splitting into two $^{4}_{2}\text{He}$ nuclei.

The points on the Segrè chart representing stable nuclides define a rather narrow stability region. For low mass numbers, the numbers of protons and neutrons are approximately equal, $N \approx Z$. The ratio N/Z increases gradually with A, up to about 1.6 at large mass numbers, because of the increasing influence of the electrical repulsion of the protons. Points to the right of the stability region represent nuclides that have too many protons relative to neutrons to be stable. In these cases, repulsion wins, and the nucleus comes apart. To the left are nuclides with too many neutrons relative to protons. In these cases the energy associated with the neutrons is out of balance with that associated with the protons, and the nuclides decay in a process that converts neutrons to protons. The graph also shows that no nuclide with $A > 209$ or $Z > 83$ is stable. A nucleus is unstable if



43.5 A three-dimensional Segrè chart for light nuclides up to $Z = 22$ (titanium). The quantity plotted on the third axis is $(M - A)c^2$, where M is the nuclide mass expressed in u. This quantity is related to the binding energy by a different constant for each nuclide.

it is too big. We also note that there is no stable nuclide with $Z = 43$ (technetium) or 61 (promethium). Figure 43.5, a three-dimensional version of the Segrè chart, shows the "valley of stability" for light nuclides (up to $Z = 22$).

Alpha Decay

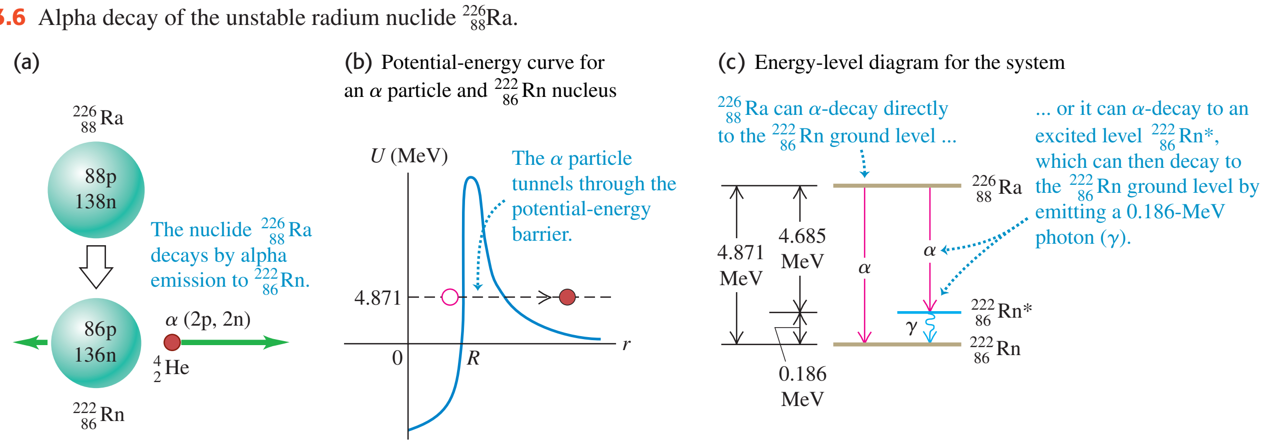
Nearly 90% of the 2500 known nuclides are *radioactive;* they are not stable but decay into other nuclides. When unstable nuclides decay into different nuclides, they usually emit alpha (α) or beta (β) particles. An **alpha particle** is a ^{4}He nucleus, two protons and two neutrons bound together, with total spin zero. Alpha emission occurs principally with nuclei that are too large to be stable. When a nucleus emits an alpha particle, its N and Z values each decrease by 2 and A decreases by 4, moving it closer to stable territory on the Segrè chart.

A familiar example of an alpha emitter is radium, $^{226}_{88}$Ra (Fig. 43.6a). The speed of the emitted alpha particle, determined from the curvature of its path in a transverse magnetic field, is about 1.52×10^7 m/s. This speed, although large, is only 5% of the speed of light, so we can use the nonrelativistic kinetic-energy expression $K = \frac{1}{2}mv^2$:

$$K = \frac{1}{2}(6.64 \times 10^{-27}\ \text{kg})(1.52 \times 10^7\ \text{m/s})^2 = 7.7 \times 10^{-13}\ \text{J} = 4.8\ \text{MeV}$$

Alpha particles are always emitted with definite kinetic energies, determined by conservation of momentum and energy. Because of their charge and mass, alpha

43.6 Alpha decay of the unstable radium nuclide $^{226}_{88}$Ra.



particles can travel only several centimeters in air, or a few tenths or hundredths of a millimeter through solids, before they are brought to rest by collisions.

Some nuclei can spontaneously decay by emission of α particles because energy is released in their alpha decay. You can use conservation of mass-energy to show that

alpha decay is possible whenever the mass of the original neutral atom is greater than the sum of the masses of the final neutral atom and the neutral helium-4 atom.

In alpha decay, the α particle tunnels through a potential-energy barrier, as shown in Fig. 43.6b. You may want to review the discussion of tunneling in Section 40.3.

Example 43.5 Alpha decay of radium

You are given the following neutral atomic masses:

$^{226}_{88}$Ra: 226.025403 u

$^{222}_{86}$Rn: 222.017571 u

Show that alpha emission is energetically possible and that the calculated kinetic energy of the emitted α particle agrees with the experimentally measured value of 4.78 MeV.

SOLUTION

IDENTIFY: Alpha emission is possible if the mass of the $^{226}_{88}$Ra atom is greater than the sum of the atomic masses of $^{222}_{86}$Rn and $^4_2$He.

SET UP: The mass difference between the initial radium atom and the final radon and helium atoms corresponds (through $E = mc^2$) to the energy E released in the decay. Because momentum is conserved as well as energy, *both* the alpha particle and the $^{222}_{86}$Rn atom are in motion after the decay; we will have to account for this fact in determining the kinetic energy of the alpha particle.

EXECUTE: From Table 43.2, the mass of the $^4_2$He atom is 4.002603 u. The difference in mass between the original nucleus and the decay products is

$$226.025403\ \text{u} - (222.017571\ \text{u} + 4.002603\ \text{u}) = +0.005229\ \text{u}$$

Since this is positive, alpha decay is energetically possible.

The energy equivalent of 0.005229 u is

$$E = (0.005229\ \text{u})(931.5\ \text{MeV/u}) = 4.871\ \text{MeV}$$

Thus we expect the decay products to emerge with total kinetic energy 4.871 MeV. Momentum is also conserved; if the parent nucleus is at rest, the daughter and the α particle have momenta of equal magnitude p but opposite direction. Kinetic energy is $K = p^2/2m$: Since p is the same for the two particles, the kinetic energy divides inversely as their masses. The α particle gets $222/(222 + 4)$ of the total, or 4.78 MeV, equal to the observed α-particle energy.

EVALUATE: An excellent way to check your results is to verify that the alpha particle and the $^{222}_{86}$Rn nucleus produced in the decay have the same magnitude of momentum $p = mv$. You can calculate the speed v of each of the decay products from its respective kinetic energy. You'll find that the alpha particle moves at a sprightly $0.0506c = 1.52 \times 10^7$ m/s; if momentum is conserved, you should find that the $^{222}_{86}$Rn nucleus moves 4/222 as fast. Does it?

Beta Decay

There are three different simple types of *beta decay: beta-minus, beta-plus,* and *electron capture.* A **beta-minus particle** (β^-) is an electron. It's not obvious how a nucleus can emit an electron if there aren't any electrons in the nucleus. Emission of a β^- involves *transformation* of a neutron into a proton, an electron, and a third particle called an *antineutrino.* In fact, if you freed a neutron from a nucleus, it would decay into a proton, an electron, and an antineutrino in an average time of about 15 minutes.

Beta particles can be identified and their speeds can be measured with techniques that are similar to the Thomson experiments we described in Section 27.5. The speeds of beta particles range up to 0.9995 of the speed of light, so their motion is highly relativistic. They are emitted with a continuous spectrum of energies. This would not be possible if the only two particles were the β^- and the recoiling nucleus, since energy and momentum conservation would then require a definite speed for the β^-. Thus there must be a *third* particle involved. From conservation of charge, it must be neutral, and from conservation of angular momentum, it must be a spin-$\frac{1}{2}$ particle.

This third particle is an antineutrino, the *antiparticle* of a **neutrino.** The symbol for a neutrino is ν_e (the Greek letter nu). Both the neutrino and the antineutrino have zero charge and zero (or very small) mass and therefore produce very little observable effect when passing through matter. Both evaded detection until 1953, when Frederick Reines and Clyde Cowan succeeded in observing the antineutrino directly. We now know that there are at least three varieties of neutrinos, each with its corresponding antineutrino; one is associated with beta decay and the other two are associated with the decay of two unstable particles, the muon and the tau particle. We'll discuss these particles in more detail in Chapter 44. The antineutrino that is emitted in β^- decay is denoted as $\bar{\nu}_e$. The basic process of β^- decay is

$$n \rightarrow p + \beta^- + \bar{\nu}_e \tag{43.13}$$

Beta-minus decay usually occurs with nuclides for which the neutron-to-proton ratio N/Z is too large for stability. In β^- decay, N decreases by 1, Z increases by 1 and A doesn't change. You can use conservation of mass-energy to show that

beta-minus decay can occur whenever the mass of the original neutral atom is larger than that of the final atom.

Example 43.6 Why cobalt-60 is a beta-minus emitter

The nuclide ${}^{60}_{27}\text{Co}$, an odd-odd unstable nucleus, is used in medical applications of radiation. Show that it is unstable relative to β^- decay. The following masses are given:

${}^{60}_{27}\text{Co}$:	59.933822 u
${}^{60}_{28}\text{Ni}$:	59.930791 u

SOLUTION

IDENTIFY: Beta-minus decay is possible if the mass of the original neutral atom is greater than that of the final atom.

SET UP: We must first decide which nuclide will result if ${}^{60}_{27}\text{Co}$ undergoes β^- decay and then compare its neutral atomic mass to that of ${}^{60}_{27}\text{Co}$.

EXECUTE: The original nuclide is ${}^{60}_{27}\text{Co}$. In β^- decay, Z increases by 1 from 27 to 28 and A remains at 60, so the final nuclide is ${}^{60}_{28}\text{Ni}$. Its mass is less than that of ${}^{60}_{27}\text{Co}$ by 0.003031 u, so β^- decay *can* occur.

EVALUATE: As with alpha decay, this beta-minus decay obeys the conservation laws for both momentum and energy. But with three decay products in β^- decay—the ${}^{60}_{28}\text{Ni}$ nucleus, the electron, and the antineutrino—the energy can be shared in many different ways that are consistent with the conservation laws. It's impossible to predict precisely how the energy will be shared for the decay of a particular ${}^{60}_{27}\text{Co}$ nucleus. By contrast, in alpha decay there are just two decay products, and their energies and momenta are determined uniquely (see Example 43.5).

We have noted that β^- decay occurs with nuclides that have too large a neutron-to-proton ratio N/Z. Nuclides for which N/Z is too *small* for stability can emit a *positron,* the electron's antiparticle, which is identical to the electron but with positive charge. (We mentioned the positron in connection with positronium in Section 38.5, and we will discuss it in more detail in Chapter 44.) The basic process, called *beta-plus decay* (β^+), is

$$p \rightarrow n + \beta^+ + \nu_e \tag{43.14}$$

where β^+ is a positron and ν_e is the electron neutrino.

Beta-plus decay can occur whenever the mass of the original neutral atom is at least two electron masses larger than that of the final atom;

you can show this using conservation of mass-energy.

The third type of beta decay is *electron capture.* There are a few nuclides for which β^+ emission is not energetically possible but in which an orbital electron (usually in the K shell) can combine with a proton in the nucleus to form a neutron and a neutrino. The neutron remains in the nucleus and the neutrino is emitted. The basic process is

$$p + \beta^- \rightarrow n + \nu_e \tag{43.15}$$

You can use conservation of mass-energy to show that

electron capture can occur whenever the mass of the original neutral atom is larger than that of the final atom.

In all types of beta decay, A remains constant. However, in beta-plus decay and electron capture, N increases by 1 and Z decreases by 1 as the neutron–proton ratio increases toward a more stable value. The reaction of Eq. (43.15) also helps to explain the formation of a neutron star, mentioned in Example 43.1.

CAUTION **Beta decay inside and outside nuclei** The beta-decay reactions given by Eqs. (43.13), (43.14), and (43.15) occur *within* a nucleus. Although the decay of a neutron outside the nucleus proceeds through the reaction of Eq. (43.13), the reaction of Eq. (43.14) is forbidden by conservation of mass-energy for a proton outside the nucleus. The reaction of Eq. (43.15) can occur outside the nucleus only with the addition of some extra energy, as in a collision.

Example 43.7 Why cobalt-57 is not a beta-plus emitter

The nuclide ${}^{57}_{27}\text{Co}$, an odd-even unstable nucleus, is often used as a source of radiation in a nuclear process called the *Mössbauer effect.* Show that this nuclide is stable relative to β^+ decay but can decay by electron capture. The following masses are given:

${}^{57}_{27}\text{Co}$:	56.936296 u
${}^{57}_{26}\text{Fe}$:	56.935399 u

SOLUTION

IDENTIFY: Beta-plus decay is possible if the mass of the original neutral atom is greater than that of the final atom plus two electron masses.

SET UP: We must first decide which nuclide will result if ${}^{57}_{27}\text{Co}$ undergoes β^+ decay and then compare its neutral atomic mass to that of ${}^{57}_{27}\text{Co}$.

EXECUTE: The original nuclide is ${}^{57}_{27}\text{Co}$. In β^+ decay and electron capture, Z decreases by 1 from 27 to 26, and A remains at 57. Thus the final nuclide is ${}^{57}_{26}\text{Fe}$. Its mass is less than that of ${}^{57}_{27}\text{Co}$ by 0.000897 u, a value smaller than 0.001097 u (two electron masses), so β^+ decay *cannot* occur. However, the mass of the original atom is greater than the mass of the final atom, so electron capture *can* occur. In Section 43.4 we'll see how to relate the probability that electron capture will occur to the *half-life* of this nuclide.

EVALUATE: In electron capture there are just two decay products: the final nucleus and the emitted neutrino. Unlike in β^- decay (Example 43.6) but like in alpha decay (Example 43.5), the decay products of electron capture have unique energies and momenta.

Gamma Decay

The energy of internal motion of a nucleus is quantized. A typical nucleus has a set of allowed energy levels, including a *ground state* (state of lowest energy) and several *excited states.* Because of the great strength of nuclear interactions, excitation energies of nuclei are typically of the order of 1 MeV, compared with a few eV for atomic energy levels. In ordinary physical and chemical transformations the nucleus always remains in its ground state. When a nucleus is placed in an excited state, either by bombardment with high-energy particles or by a radioactive transformation, it can decay to the ground state by emission of one or more photons called **gamma rays** or *gamma-ray photons,* with typical energies of 10 keV to 5 MeV. This process is called *gamma* (γ) *decay.* For example, alpha particles emitted from ${}^{226}\text{Ra}$ have two possible kinetic energies, either 4.784 MeV or 4.602 MeV. Including the recoil energy of the resulting ${}^{222}\text{Rn}$ nucleus, these correspond to a total released energy of 4.871 MeV or 4.685 MeV, respectively. When an alpha particle with the smaller energy is emitted, the ${}^{222}\text{Rn}$ nucleus is left in an excited state. It then decays to its ground state by emitting a gamma-ray photon with energy

$$(4.871 - 4.685)\ \text{MeV} = 0.186\ \text{MeV}$$

A photon with this energy is observed during this decay (Fig. 43.6c).

CAUTION **γ decay vs. α and β decay** In both α and β decay, the Z value of a nucleus changes and the nucleus of one element becomes the nucleus of a different element. In γ decay, the element does *not* change; the nucleus merely goes from an excited state to a less excited state.

Natural Radioactivity

Many radioactive elements occur in nature. For example, you are very slightly radioactive because of unstable nuclides such as carbon-14 and potassium-40 that are present throughout your body. The study of natural radioactivity began in 1896, one year after Röntgen discovered x rays. Henri Becquerel discovered a radiation from uranium salts that seemed similar to x rays. Intensive investigation in the following two decades by Marie and Pierre Curie, Ernest Rutherford, and many others revealed that the emissions consist of positively and negatively charged particles and neutral rays; they were given the names *alpha, beta,* and *gamma* because of their differing penetration characteristics.

The decaying nucleus is usually called the *parent nucleus;* the resulting nucleus is the *daughter nucleus.* When a radioactive nucleus decays, the daughter nucleus may also be unstable. In this case a *series* of successive decays occurs until a stable configuration is reached. Several such series are found in nature. The most abundant radioactive nuclide found on earth is the uranium isotope ^{238}U, which undergoes a series of 14 decays, including eight α emissions and six β^- emissions, terminating at a stable isotope of lead, ^{206}Pb (Fig. 43.7).

43.7 Earthquakes are caused in part by the radioactive decay of ^{238}U in the earth's interior. These decays release energy that helps to produce convection currents in the earth's interior. Such currents drive the motions of the earth's crust, including the sudden sharp motions that we call earthquakes (like the one that caused this damage).

Radioactive decay series can be represented on a Segrè chart, as in Fig. 43.8. The neutron number N is plotted vertically, and the atomic number Z is plotted horizontally. In alpha emission, both N and Z decrease by 2. In β^- emission, N decreases by 1 and Z increases by 1. The decays can also be represented in equation form; the first two decays in the series are written as

$$^{238}\text{U} \rightarrow {}^{234}\text{Th} + \alpha$$
$$^{234}\text{Th} \rightarrow {}^{234}\text{Pa} + \beta^- + \bar{\nu}_e$$

or more briefly as

$$^{238}\text{U} \xrightarrow{\alpha} {}^{234}\text{Th}$$
$$^{234}\text{Th} \xrightarrow{\beta^-} {}^{234}\text{Pa}$$

In the second process, the beta decay leaves the daughter nucleus ^{234}Pa in an excited state, from which it decays to the ground state by emitting a gamma-ray photon. An excited state is denoted by an asterisk, so we can represent the γ emission as

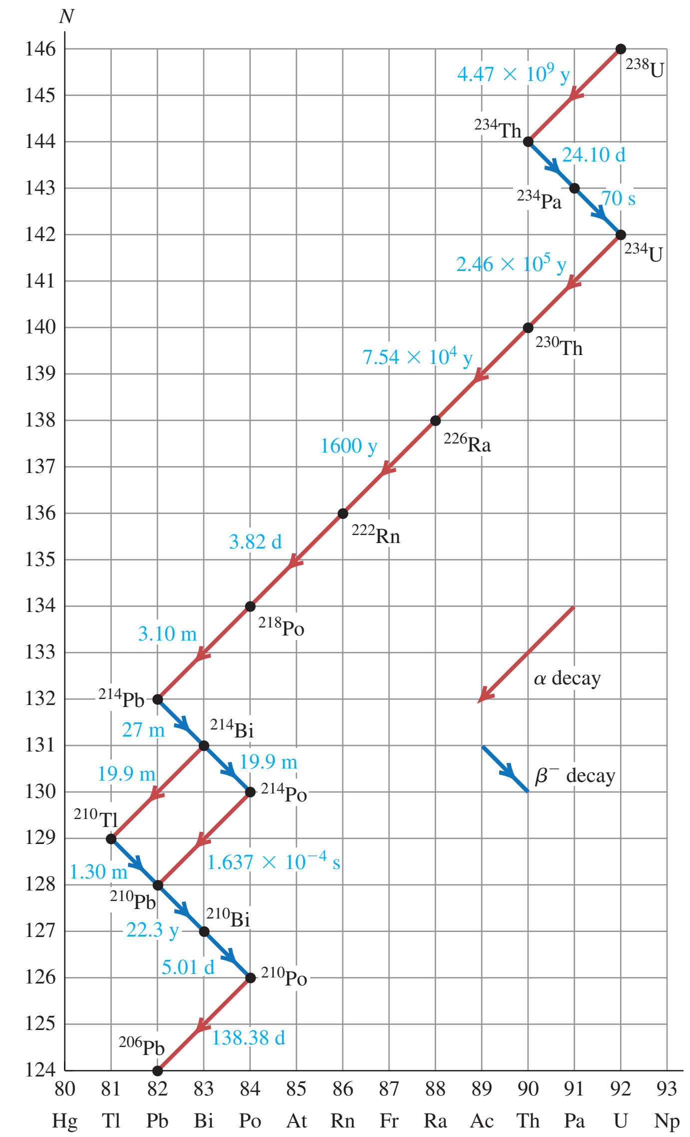
$$^{234}\text{Pa}^* \rightarrow {}^{234}\text{Pa} + \gamma$$

or

$$^{234}\text{Pa}^* \xrightarrow{\gamma} {}^{234}\text{Pa}$$

An interesting feature of the ^{238}U decay series is the branching that occurs at ^{214}Bi. This nuclide decays to ^{210}Pb by emission of an α and a β^-, which can occur in either order. We also note that the series includes unstable isotopes of several elements that also have stable isotopes, including thallium (Tl), lead (Pb), and bismuth (Bi). The unstable isotopes of these elements that occur in the ^{238}U series all have too many neutrons to be stable.

Many other decay series are known. Two of these occur in nature, one starting with the uncommon isotope ^{235}U and ending with ^{207}Pb, the other starting with thorium (^{232}Th) and ending with ^{208}Pb.



43.8 Segrè chart showing the uranium ^{238}U decay series, terminating with the stable nuclide ^{206}Pb. The times are half-lives (discussed in the next section), given in years (y), days (d), hours (h), minutes (m), or seconds (s).

Test Your Understanding of Section 43.3 A nucleus with atomic number Z and neutron number N undergoes two decay processes. The result is a nucleus with atomic number $Z - 3$ and neutron number $N - 1$. Which decay processes took place? (i) two β^- decays; (ii) two β^+ decays; (iii) two α decays; (iv) an α decay and a β^- decay; (v) an α decay and a β^+ decay.

43.4 Activities and Half-Lives

Suppose you need to dispose of some radioactive waste that contains a certain number of nuclei a particular radioactive nuclide. If no more are produced, that number decreases in a simple manner as the nuclei decay. This decrease is a statistical process; there is no way to predict when any individual nucleus will decay. No change in physical or chemical environment, such as chemical reactions or heating or cooling, greatly affects most decay rates. The rate varies over an extremely wide range for different nuclides.

Radioactive Decay Rates

Let $N(t)$ be the (very large) number of radioactive nuclei in a sample at time t, and let $dN(t)$ be the (negative) change in that number during a short time interval

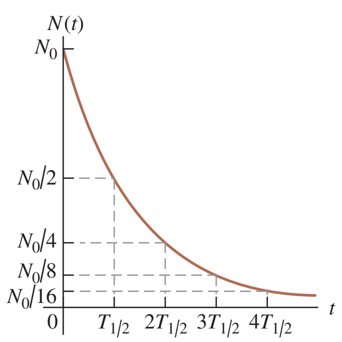
dt. (We'll use $N(t)$ to minimize confusion with the neutron number N.) The number of decays during the interval dt is $-dN(t)$. The rate of change of $N(t)$ is the negative quantity $dN(t)/dt$; thus $-dN(t)/dt$ is called the *decay rate* or the **activity** of the specimen. The larger the number of nuclei in the specimen, the more nuclei decay during any time interval. That is, the activity is directly proportional to $N(t)$; it equals a constant λ multiplied by $N(t)$:

$$-\frac{dN(t)}{dt} = \lambda N(t) \qquad (43.16)$$

The constant λ is called the **decay constant,** and it has different values for different nuclides. A large value of λ corresponds to rapid decay; a small value corresponds to slower decay. Solving Eq. (43.16) for λ shows us that λ is the ratio of the number of decays per time to the number of remaining radioactive nuclei; λ can then be interpreted as the *probability per unit time* that any individual nucleus will decay.

The situation is reminiscent of a discharging capacitor, which we studied in Section 26.4. Equation (43.16) has the same form as the negative of Eq. (26.15), with q and $1/RC$ replaced by $N(t)$ and λ. Then we can make the same substitutions in Eq. (26.16), with the initial number of nuclei $N(0) = N_0$, to find the exponential function:

$$N(t) = N_0 e^{-\lambda t} \qquad \text{(number of remaining nuclei)} \qquad (43.17)$$



43.9 The number of nuclei in a sample of a radioactive element as a function of time. The sample's activity has an exponential decay curve with the same shape.

Figure 43.9 is a graph of this function, showing the number of remaining nuclei $N(t)$ as a function of time.

The **half-life** $T_{1/2}$ is the time required for the number of radioactive nuclei to decrease to one-half the original number N_0. Then half of the remaining radioactive nuclei decay during a second interval $T_{1/2}$, and so on. The numbers remaining after successive half-lives are $N_0/2$, $N_0/4$, $N_0/8$, ...

To get the relationship between the half-life $T_{1/2}$ and the decay constant λ, we set $N(t)/N_0 = \frac{1}{2}$ and $t = T_{1/2}$ in Eq. (43.17), obtaining

$$\frac{1}{2} = e^{-\lambda T_{1/2}}$$

We take logarithms of both sides and solve for $T_{1/2}$:

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda} \qquad (43.18)$$

The mean lifetime T_{mean}, generally called the *lifetime,* of a nucleus or unstable particle is proportional to the half-life $T_{1/2}$:

$$T_{\text{mean}} = \frac{1}{\lambda} = \frac{T_{1/2}}{\ln 2} = \frac{T_{1/2}}{0.693} \qquad \text{(lifetime } T_{\text{mean}}\text{, decay constant } \lambda\text{, and half-life } T_{1/2}) \qquad (43.19)$$

In particle physics the life of an unstable particle is usually described by the lifetime, not the half-life.

Because the activity $-dN(t)/dt$ at any time equals $\lambda N(t)$, Eq. (43.17) tells us that the activity also depends on time as $e^{-\lambda t}$. Thus the graph of activity versus time has the same shape as Fig. 43.9. Also, after successive half-lives, the activity is one-half, one-fourth, one-eighth, and so on of the original activity.

CAUTION **A half-life may not be enough** It is sometimes implied that any radioactive sample will be safe after a half-life has passed. That's wrong. If your radioactive waste initially has ten times too much activity for safety, it is not safe after one half-life, when it still has five times too much. Even after three half-lives it still has 25% more activity than is safe. The number of radioactive nuclei and the activity approach zero only as t approaches infinity. ∎

A common unit of activity is the **curie,** abbreviated Ci, which is defined to be 3.70×10^{10} decays per second. This is approximately equal to the activity of one gram of radium. The SI unit of activity is the *becquerel,* abbreviated Bq. One becquerel is one decay per second, so

$$1 \text{ Ci} = 3.70 \times 10^{10} \text{ Bq} = 3.70 \times 10^{10} \text{ decays/s}$$

Example 43.8 Activity of ^{57}Co

The radioactive isotope ^{57}Co decays by electron capture with a half-life of 272 days. (a) Find the decay constant and the lifetime. (b) If you have a radiation source containing ^{57}Co, with activity 2.00 μCi, how many radioactive nuclei does it contain? (c) What will be the activity of your source after one year?

SOLUTION

IDENTIFY: This problem uses the relationships among decay constant λ, lifetime T_{mean}, and activity $-dN(t)/dt$.

SET UP: We determine the decay constant λ and lifetime T_{mean} from the half-life $T_{1/2}$ using Eq. (43.19). Once we have found λ, we calculate the number of nuclei $N(t)$ from the activity (which is the same as the decay rate $-dN(t)/dt$) using Eq. (43.16). We then use Eq. (43.17) to find the number of nuclei remaining after one year, and from this value find the activity after one year by using Eq. (43.16) once again.

EXECUTE: (a) To simplify the units, we convert the half-life to seconds:

$$T_{1/2} = (272 \text{ days})(86{,}400 \text{ s/day}) = 2.35 \times 10^7 \text{ s.}$$

From Eq. (43.19) the lifetime is

$$T_{\text{mean}} = \frac{T_{1/2}}{\ln 2} = \frac{2.35 \times 10^7 \text{ s}}{0.693} = 3.39 \times 10^7 \text{ s}$$

The decay constant is

$$\lambda = \frac{1}{T_{\text{mean}}} = 2.95 \times 10^{-8} \text{ s}^{-1}$$

(b) The activity $-dN(t)/dt$ is given as 2.00 μCi, so

$$-\frac{dN(t)}{dt} = 2.00\ \mu\text{Ci} = (2.00 \times 10^{-6})(3.70 \times 10^{10} \text{ s}^{-1})$$
$$= 7.40 \times 10^4 \text{ decays/s}$$

From Eq. (43.16) this is equal to $\lambda N(t)$, so we find

$$N(t) = -\frac{dN(t)/dt}{\lambda} = \frac{7.40 \times 10^4 \text{ s}^{-1}}{2.95 \times 10^{-8} \text{ s}^{-1}} = 2.51 \times 10^{12} \text{ nuclei}$$

If you feel we're being too cavalier about the "units" decays and nuclei, you can use decays/(nucleus · s) as the unit for λ.

(c) From Eq. (43.17) the number $N(t)$ of nuclei remaining after one year $(3.156 \times 10^7 \text{ s})$ is

$$N(t) = N_0 e^{-\lambda t} = N_0 e^{-(2.95\times10^{-8}\text{ s}^{-1})(3.156\times10^7\text{ s})} = 0.394 N_0$$

The number of nuclei has decreased to 0.394 of the original number. Equation (43.16) says that the activity is proportional to the number of nuclei, so the activity has decreased by this same factor to $(0.394)(2.00\ \mu\text{Ci}) = 0.788\ \mu\text{Ci}$.

EVALUATE: The number of nuclei found in part (b) is equivalent to 4.17×10^{-12} mol, with a mass of 2.38×10^{-10} g. This is a far smaller mass than even the most sensitive balance can measure.

After one 272-day half-life, the number of ^{57}Co nuclei has decreased to $N_0/2$; after $2(272 \text{ d}) = 544$ d, it has decreased to $N_0/2^2 = N_0/4$. This result agrees with our answer to part (c), which says that after 365 d the number of nuclei is between $N_0/2$ and $N_0/4$.

Radioactive Dating

?

An interesting application of radioactivity is the dating of archaeological and geological specimens by measuring the concentration of radioactive isotopes. The most familiar example is *carbon dating.* The unstable isotope ^{14}C, produced during nuclear reactions in the atmosphere that result from cosmic-ray bombardment, gives a small proportion of ^{14}C in the CO_2 in the atmosphere. Plants that obtain their carbon from this source contain the same proportion of ^{14}C as the atmosphere. When a plant dies, it stops taking in carbon, and its ^{14}C β^- decays to ^{14}N with a half-life of 5730 years. By measuring the proportion of ^{14}C in the remains, we can determine how long ago the organism died.

One difficulty with radiocarbon dating is that the ^{14}C concentration in the atmosphere changes over long time intervals. Corrections can be made on the basis of other data such as measurements of tree rings that show annual growth cycles. Similar radioactive techniques are used with other isotopes for dating geological specimens. Some rocks, for example, contain the unstable potassium isotope ^{40}K, a beta emitter that decays to the stable nuclide ^{40}Ar with a half-life of 2.4×10^8 y. The age of the rock can be determined by comparing the concentrations of ^{40}K and ^{40}Ar.

Example 43.9 Radiocarbon dating

Before 1900 the activity per unit mass of atmospheric carbon due to the presence of ^{14}C averaged about 0.255 Bq per gram of carbon. (a) What fraction of carbon atoms were ^{14}C? (b) In analyzing an archaeological specimen containing 500 mg of carbon, you observe 174 decays in one hour. What is the age of the specimen, assuming that its activity per unit mass of carbon when it died was that average value of the air?

SOLUTION

IDENTIFY: The key idea is that the present-day activity of a biological sample containing ^{14}C is related to both the elapsed time since it stopped taking in atmospheric carbon and its activity at that time.

SET UP: In part (a) we determine the number of ^{14}C atoms $N(t)$ from the activity $-dN(t)/dt$ using Eq. (43.16). We find the total number of carbon atoms in 500 mg by using the molar mass of carbon (12.011 g/mol, given in Appendix D), and we use the result to calculate the fraction of carbon atoms that are ^{14}C. The activity decays at the same rate as the number of ^{14}C nuclei; we use this fact in conjunction with Eq. (43.17) to solve for the age t of the specimen.

EXECUTE: (a) To use Eq. (43.16), we must first find λ from Eq. (43.18):

$$T_{1/2} = 5730\text{ y} = (5730\text{ y})(3.156 \times 10^7\text{ s/y}) = 1.808 \times 10^{11}\text{ s}$$

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{1.808 \times 10^{11}\text{ s}} = 3.83 \times 10^{-12}\text{ s}^{-1}$$

Alternatively,

$$\lambda = \frac{0.693}{5730\text{ y}} = 1.209 \times 10^{-4}\text{ y}^{-1}$$

Then, from Eq. (43.16),

$$N(t) = \frac{-dN/dt}{\lambda} = \frac{0.255\text{ s}^{-1}}{3.83 \times 10^{-12}\text{ s}^{-1}} = 6.66 \times 10^{10}\text{ atoms}$$

The *total* number of C atoms in one gram $(1/12.011\text{ mol})$ is $(1/12.011)(6.022 \times 10^{23}) = 5.01 \times 10^{22}$. The ratio of ^{14}C atoms to all C atoms is

$$\frac{6.66 \times 10^{10}}{5.01 \times 10^{22}} = 1.33 \times 10^{-12}$$

Only four carbon atoms in every 3 million million are ^{14}C.

(b) Assuming that the activity per gram of carbon in the specimen when it died was $0.255\text{ Bq/g} = (0.255\text{ s}^{-1}\cdot\text{g}^{-1})(3600\text{ s/h}) = 918\text{ h}^{-1}\cdot\text{g}^{-1}$, the activity of 500 mg of carbon then was $(0.500\text{ g}) \times (918\text{ h}^{-1}\cdot\text{g}^{-1}) = 459\text{ h}^{-1}$. The observed activity now, at time t later, is 174 h^{-1}. Since the activity is proportional to the number of radioactive nuclei, the activity ratio $174/459 = 0.379$ equals the number ratio $N(t)/N_0$.

Now we solve Eq. (43.17) for t and insert values for $N(t)/N_0$ and λ:

$$t = \frac{\ln(N(t)/N_0)}{-\lambda} = \frac{\ln 0.379}{-1.209 \times 10^{-4}\text{ y}^{-1}} = 8020\text{ y}$$

EVALUATE: After 8020 y the ^{14}C activity has decreased from 459 to 174 decays per hour. The specimen died and stopped taking CO_2 out of the air about 8000 years ago.

Radiation in the Home

A serious health hazard in some areas is the accumulation in houses of ^{222}Rn, an inert, colorless, odorless radioactive gas. Looking at the ^{238}U decay chain (Fig. 43.8), we see that the half-life of ^{222}Rn is 3.82 days. If so, why not just move out of the house for a while and let it decay away? The answer is that ^{222}Rn is continuously being *produced* by the decay of ^{226}Ra, which is found in minute quantities in the rocks and soil on which some houses are built. It's a dynamic equilibrium situation, in which the rate of production equals the rate of decay. The reason ^{222}Rn is a bigger hazard than the other elements in the ^{238}U decay series is that it's a gas. During its short half-life of 3.82 days it can migrate from the soil into your house. If a ^{222}Rn nucleus decays in your lungs, it emits a damaging α particle and its daughter nucleus ^{218}Po, which is *not* chemically inert and is likely to stay in your lungs until it decays, emits another damaging α particle and so on down the ^{238}U decay series.

How much of a hazard is radon? Although reports indicate values as high as 3500 pCi/L, the average activity per volume in the air inside American homes due to ^{222}Rn is about 1.5 pCi/L (over a thousand decays each second in an average-sized room). *If* your environment has this level of activity, it has been estimated that a lifetime exposure would reduce your life expectancy by about 40 days. For comparison, smoking one pack of cigarettes per day reduces life expectancy by 6 years, and it is estimated that the average emission from all the nuclear power plants in the world reduces life expectancy by anywhere from 0.01 day to 5 days. These figures include catastrophes such as the 1986 nuclear reactor disaster at Chernobyl, for which the *local* effect on life expectancy is much greater.

Test Your Understanding of Section 43.4 Which sample contains a greater number of nuclei: a 5.00-μCi sample of ^{240}Pu (half-life 6560 y) or a 4.45-μCi sample of ^{243}Am (half-life 7370 y)? (i) the ^{240}Pu sample; (ii) the ^{243}Am sample; (iii) both have the same number of nuclei.

43.5 Biological Effects of Radiation

The above discussion of radon introduced the interaction of radiation with living organisms, a topic of vital interest and importance. Under *radiation* we include radioactivity (alpha, beta, gamma, and neutrons) and electromagnetic radiation such as x rays. As these particles pass through matter, they lose energy, breaking molecular bonds and creating ions—hence the term *ionizing radiation.* Charged particles interact directly with the electrons in the material. X rays and γ rays interact by the photoelectric effect, in which an electron absorbs a photon and breaks loose from its site, or by Compton scattering (see Section 38.7). Neutrons cause ionization indirectly through collisions with nuclei or absorption by nuclei with subsequent radioactive decay of the resulting nuclei.

These interactions are extremely complex. It is well known that excessive exposure to radiation, including sunlight, x rays, and all the nuclear radiations, can destroy tissues. In mild cases it results in a burn, as with common sunburn. Greater exposure can cause very severe illness or death by a variety of mechanisms, including massive destruction of tissue cells, alterations of genetic material, and destruction of the components in bone marrow that produce red blood cells.

Calculating Radiation Doses

Radiation dosimetry is the quantitative description of the effect of radiation on living tissue. The *absorbed dose* of radiation is defined as the energy delivered to the tissue per unit mass. The SI unit of absorbed dose, the joule per kilogram, is called the *gray* (Gy); 1 Gy = 1 J/kg. Another unit, in more common use at present, is the *rad,* defined as 0.01 J/kg:

$$1\text{ rad} = 0.01\text{ J/kg} = 0.01\text{ Gy}$$

Absorbed dose by itself is not an adequate measure of biological effect because equal energies of different kinds of radiation cause different extents of biological effect. This variation is described by a numerical factor called the **relative biological effectiveness (RBE),** also called the *quality factor* (QF), of each specific radiation. X rays with 200 keV of energy are defined to have an RBE of unity, and the effects of other radiations can be compared experimentally. Table 43.3 shows approximate values of RBE for several radiations. All these values depend somewhat on the kind of tissue in which the radiation is absorbed and on the energy of the radiation.

Table 43.3 Relative Biological Effectiveness (RBE) for Several Types of Radiation

Radiation	RBE (Sv/Gy or rem/rad)
X rays and γ rays	1
Electrons	1.0–1.5
Slow neutrons	3–5
Protons	10
α particles	20
Heavy ions	20

The biological effect is described by the product of the absorbed dose and the RBE of the radiation; this quantity is called the *biologically equivalent dose,* or simply the equivalent dose. The SI unit of equivalent dose for humans is the sievert (Sv):

$$\text{Equivalent dose (Sv)} = \text{RBE} \times \text{Absorbed dose (Gy)} \qquad (43.20)$$

A more common unit, corresponding to the rad, is the rem (an abbreviation of *röntgen equivalent for man*):

$$\text{Equivalent dose (rem)} = \text{RBE} \times \text{Absorbed dose (rad)} \qquad (43.21)$$

Thus the unit of the RBE is 1 Sv/Gy or 1 rem/rad, and 1 rem = 0.01 Sv.

Example 43.10 A medical x-ray exam

During a diagnostic x-ray examination a 1.2-kg portion of a broken leg receives an equivalent dose of 0.40 mSv. (a) What is the equivalent dose in mrem? (b) What is the absorbed dose in mrad and mGy? (c) If the x-ray energy is 50 keV, how many x-ray photons are absorbed?

SOLUTION

IDENTIFY: We are asked to relate the equivalent dose (the biological effect of the radiation, measured in sieverts or rems) to the absorbed dose (the energy absorbed per mass, measured in grays or rads).

SET UP: In part (a) we use the conversion factor 1 rem = 0.01 Sv for equivalent dose. Table 43.3 gives the RBE for x rays; we use this value in part (b) to determine the absorbed dose using Eqs. (43.20) and (43.21). Finally, in part (c) we use the mass and the definition of absorbed dose to find the total energy absorbed and the total number of photons absorbed.

EXECUTE: (a) The equivalent dose in mrem is

$$\frac{0.40\ \text{mSv}}{0.01\ \text{Sv/rem}} = 40\ \text{mrem}$$

(b) For x rays, RBE = 1 rem/rad or 1 Sv/Gy, so the absorbed dose is

$$\frac{40\ \text{mrem}}{1\ \text{rem/rad}} = 40\ \text{mrad}$$

$$\frac{0.40\ \text{mSv}}{1\ \text{Sv/Gy}} = 0.40\ \text{mGy} = 4.0 \times 10^{-4}\ \text{J/kg}$$

(c) The total energy absorbed is

$$(4.0 \times 10^{-4}\ \text{J/kg})(1.2\ \text{kg}) = 4.8 \times 10^{-4}\ \text{J} = 3.0 \times 10^{15}\ \text{eV}$$

The number of x-ray photons is

$$\frac{3.0 \times 10^{15}\ \text{eV}}{5.0 \times 10^{4}\ \text{eV/photon}} = 6.0 \times 10^{10}\ \text{photons}$$

EVALUATE: The absorbed dose is relatively large because x rays have a low RBE. If the ionizing radiation had been a beam of α particles, for which RBE = 20, the absorbed dose needed for an equivalent dose of 0.40 mSv would be 0.020 mGy, corresponding to a total absorbed energy of 2.4×10^{-5} J.

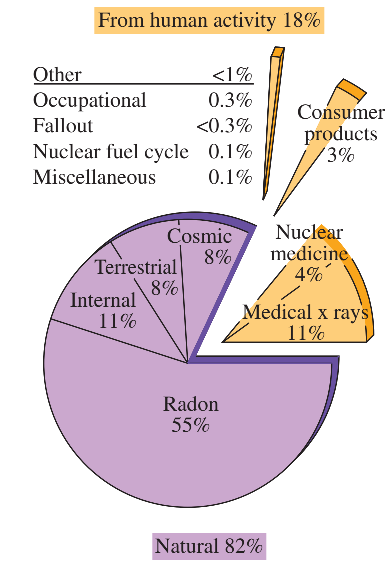
Radiation Hazards

Here are a few numbers for perspective. To convert from Sv to rem, simply multiply by 100. An ordinary chest x-ray exam delivers about 0.20–0.40 mSv to about 5 kg of tissue. Radiation exposure from cosmic rays and natural radioactivity in soil, building materials, and so on is of the order of 2–3 mSv per year at sea level and twice that at an elevation of 1500 m (5000 ft). A whole-body dose of up to about 0.20 Sv causes no immediately detectable effect. A short-term whole-body dose of 5 Sv or more usually causes death within a few days or weeks. A localized dose of 100 Sv causes complete destruction of the exposed tissues.

The long-term hazards of radiation exposure in causing various cancers and genetic defects have been widely publicized, and the question of whether there is any "safe" level of radiation exposure has been hotly debated. U.S. government regulations are based on a maximum *yearly* exposure, from all except natural resources, of 2 to 5 mSv. Workers with occupational exposure to radiation are permitted 50 mSv per year. Recent studies suggest that these limits are too high and that even extremely small exposures carry hazards, but it is very difficult to gather reliable statistics on the effects of low doses. It has become clear that any use of x rays for medical diagnosis should be preceded by a very careful estimation of the relationship of risk to possible benefit.

Another sharply debated question is that of radiation hazards from nuclear power plants. The radiation level from these plants is *not* negligible. However, to make a meaningful evaluation of hazards, we must compare these levels with the alternatives, such as coal-powered plants. The health hazards of coal smoke are serious and well documented, and the natural radioactivity in the smoke from a coal-fired power plant is believed to be roughly 100 times as great as that from a properly operating nuclear plant with equal capacity. But the comparison is not this simple; the possibility of a nuclear accident and the very serious problem of safe disposal of radioactive waste from nuclear plants must also be considered. It is clearly impossible to eliminate *all* hazards to health. Our goal should be to try to take a rational approach to the problem of *minimizing* the hazard from all sources. Figure 43.10 shows one estimate of the various sources of radiation exposure for the U.S. population. Ionizing radiation is a two-edged sword; it poses very serious health hazards, yet it also provides many benefits to humanity, including the diagnosis and treatments of disease and a wide variety of analytical techniques.

43.10 Contribution of various sources to the total average radiation exposure in the U.S. population, expressed as percentages of the total.



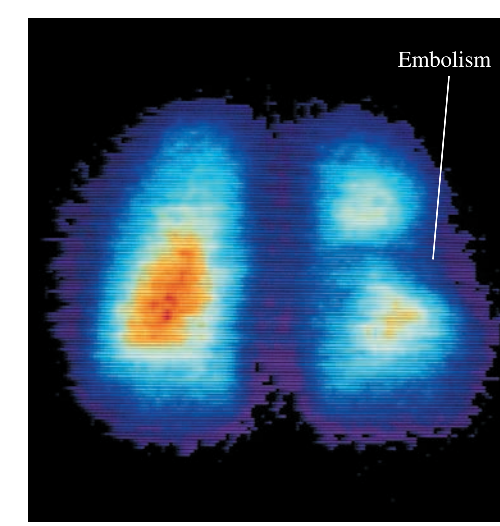
Beneficial Uses of Radiation

Radiation is widely used in medicine for intentional selective destruction of tissue such as tumors. The hazards are considerable, but if the disease would be fatal without treatment, any hazard may be preferable. Artificially produced isotopes are often used as radiation sources. Such isotopes have several advantages over naturally radioactive isotopes. They may have shorter half-lives and correspondingly greater activity. Isotopes can be chosen that emit the type and energy of radiation desired. Some artificial isotopes have been replaced by photon and electron beams from linear accelerators.

Nuclear medicine is an expanding field of application. Radioactive isotopes have virtually the same electron configurations and resulting chemical behavior as stable isotopes of the same element. But the location and concentration of radioactive isotopes can easily be detected by measurements of the radiation they emit. A familiar example is the use of radioactive iodine for thyroid studies. Nearly all the iodine ingested is either eliminated or stored in the thyroid, and the body's chemical reactions do not discriminate between the unstable isotope ^{131}I and the stable isotope ^{127}I. A minute quantity of ^{131}I is fed or injected into the patient, and the speed with which it becomes concentrated in the thyroid provides a measure of thyroid function. The half-life is 8.02 days, so there are no long-lasting radiation hazards. By use of more sophisticated scanning detectors, one can also obtain a "picture" of the thyroid, which shows enlargement and other abnormalities. This procedure, a type of *autoradiography,* is comparable to photographing the glowing filament of an incandescent light bulb by using the light emitted by the filament itself. If this process discovers cancerous thyroid nodules, they can be destroyed by much larger quantities of ^{131}I.

Another useful nuclide for nuclear medicine is technetium-99 (^{99}Tc), which is formed in an excited state by the β^- decay of molybdenum (^{99}Mo). The technetium then decays to its ground state by emitting a γ-ray photon with energy 143 keV. The half-life is 6.01 hours, unusually long for γ emission. (The ground state of ^{99}Tc is also unstable, with a half-life of 2.11×10^5 y; it decays by β^- emission to the stable ruthenium nuclide ^{99}Ru.) The chemistry of technetium is such that it can readily be attached to organic molecules that are taken up by various organs of the body. A small quantity of such technetium-bearing molecules is injected into a patient, and a scanning detector or *gamma camera* is used to produce an image, or *scintigram,* that reveals which parts of the body take up these γ-emitting molecules. This technique, in which ^{99}Tc acts as a radioactive *tracer,* plays an important role in locating cancers, embolisms, and other pathologies (Fig. 43.11).

43.11 This colored scintigram shows where a chemical containing radioactive ^{99}Tc was taken up by a patient's lungs. The orange color in the lung on the left indicates strong γ-ray emission by the ^{99}Tc, which shows that the chemical was able to pass into this lung through the bloodstream. The lung on the right shows weaker emission, indicating the presence of an embolism (a blood clot or other obstruction in an artery) that is restricting the flow of blood to this lung.



Tracer techniques have many other applications. Tritium (^3H), a radioactive hydrogen isotope, is used to tag molecules in complex organic reactions; radioactive tags on pesticide molecules, for example, can be used to trace their passage through food chains. In the world of machinery, radioactive iron can be used to study piston-ring wear. Laundry detergent manufacturers have even tested the effectiveness of their products using radioactive dirt.

Many direct effects of radiation are also useful, such as strengthening polymers by cross-linking, sterilizing surgical tools, dispersing of unwanted static electricity in the air, and intentional ionization of air in smoke detectors. Gamma rays are also being used to sterilize and preserve some food products.

Test Your Understanding of Section 43.5 Alpha particles have 20 times the relative biological effectiveness of 200-keV x rays. Which would be better to use to radiate tissue deep inside the body? (i) a beam of alpha particles; (ii) a beam of 200-keV x rays; (iii) both are equally effective.

43.6 Nuclear Reactions

In the preceding sections we studied the decay of unstable nuclei, especially spontaneous emission of an α or β particle, sometimes followed by γ emission. Nothing needs to be done to initiate this decay, and nothing can be done to control it. This section examines some *nuclear reactions,* rearrangements of nuclear components that result from a bombardment by a particle rather than a spontaneous natural process. Rutherford suggested in 1919 that a massive particle with sufficient kinetic energy might be able to penetrate a nucleus. The result would be either a new nucleus with greater atomic number and mass number or a decay of the original nucleus. Rutherford bombarded nitrogen (^{14}N) with α particles and obtained an oxygen (^{17}O) nucleus and a proton:

$$^4_2\text{He} + {}^{14}_{7}\text{N} \rightarrow {}^{17}_{8}\text{O} + {}^1_1\text{H} \qquad (43.22)$$

Rutherford used alpha particles from naturally radioactive sources. In Chapter 44 we'll describe some of the particle accelerators that are used nowadays to initiate nuclear reactions.

Nuclear reactions are subject to several *conservation laws.* The classical conservation principles for charge, momentum, angular momentum, and energy (including rest energies) are obeyed in all nuclear reactions. An additional conservation law, not anticipated by classical physics, is conservation of the total number of nucleons. The numbers of protons and neutrons need not be conserved separately; we have seen that in β decay, neutrons and protons change into one another. We'll study the basis of the conservation of nucleon number in Chapter 44.

When two nuclei interact, charge conservation requires that the sum of the initial atomic numbers must equal the sum of the final atomic numbers. Because of conservation of nucleon number, the sum of the initial mass numbers must also equal the sum of the final mass numbers. In general, these are *not* elastic collisions, and, correspondingly, the total initial mass does *not* equal the total final mass.

Reaction Energy

The difference between the masses before and after the reaction corresponds to the **reaction energy,** according to the mass-energy relationship $E = mc^2$. If initial particles A and B interact to produce final particles C and D, the reaction energy Q is defined as

$$Q = (M_A + M_B - M_C - M_D)c^2 \qquad \text{(reaction energy)} \qquad (43.23)$$

To balance the electrons, we use the neutral atomic masses in Eq. (43.23). That is, we use the mass of ${}^1_1\text{H}$ for a proton, ${}^2_1\text{H}$ for a deuteron, ${}^4_2\text{He}$ for an α particle, and so on. When Q is positive, the total mass decreases and the total kinetic energy increases. Such a reaction is called an *exoergic reaction.* When Q is negative, the mass increases and the kinetic energy decreases, and the reaction is called an *endoergic reaction.* The terms *exothermal* and *endothermal,* borrowed from chemistry, are also used. In an endoergic reaction the reaction cannot occur at all unless the initial kinetic energy in the center-of-mass reference frame is at

least as great as $|Q|$. That is, there is a **threshold energy,** the minimum kinetic energy to make an endoergic reaction go.

Example 43.11 Exoergic and endoergic reactions

(a) When lithium (^7Li) is bombarded by a proton, two alpha particles (^4He) are produced. Find the reaction energy. (b) Calculate the reaction energy for the nuclear reaction represented by Eq. (43.22).

SOLUTION

IDENTIFY: The reaction energy for any nuclear reaction is the total initial mass minus the total final mass, multiplied by c^2.

SET UP: In each case we determine the reaction energy using Eq. (43.23). The required masses are given in Table 43.2.

EXECUTE: (a) The reaction can be written

$$^1_1\text{H} + {}^7_3\text{Li} \rightarrow {}^4_2\text{He} + {}^4_2\text{He}$$

Here are the initial and final masses:

A: ${}^1_1\text{H}$	1.007825 u	C: ${}^4_2\text{He}$	4.002603 u
B: ${}^7_3\text{Li}$	7.016004 u	D: ${}^4_2\text{He}$	4.002603 u
	8.023829 u		8.005206 u

We see that

$$M_A + M_B - M_C - M_D = 0.018623\ \text{u}$$

Then Eq. (43.23) gives a reaction energy of

$$Q = (0.018623\ \text{u})(931.5\ \text{MeV/u}) = 17.35\ \text{MeV}$$

(b) The masses of the various particles are

A: ${}^4_2\text{He}$	4.002603 u	C: ${}^{17}_{8}\text{O}$	16.999132 u
B: ${}^{14}_{7}\text{N}$	14.003074 u	D: ${}^1_1\text{H}$1.	007825 u
	18.005677 u		18.006957 u

We see that the mass increases by 0.001280 u, and the corresponding reaction energy is

$$Q = (-0.001280\ \text{u})(931.5\ \text{MeV/u}) = -1.192\ \text{MeV}$$

EVALUATE: The reaction in (a) is *exoergic:* The final total kinetic energy of the two separating alpha particles is 17.35 MeV greater than the initial total kinetic energy of the proton and the lithium nucleus. By contrast, the reaction in (b) is *endoergic:* In the center-of-mass system—that is, in a head-on collision with zero total momentum—the minimum total initial kinetic energy for this reaction to occur is 1.192 MeV.

Ordinarily, the endoergic reaction of part (b) of Example 43.11 would be produced by bombarding stationary ^{14}N nuclei with alpha particles from an accelerator. In this case an alpha's kinetic energy must be *greater than* 1.192 MeV. If all the alpha's kinetic energy went solely to increasing the rest energy, the final kinetic energy would be zero, and momentum would not be conserved. When a particle with mass m and kinetic energy K collides with a stationary particle with mass M, the total kinetic energy K_{cm} in the center-of-mass coordinate system (the energy available to cause reactions) is

$$K_{\text{cm}} = \frac{M}{M + m}K \qquad (43.24)$$

This expression assumes that the kinetic energies of the particles and nuclei are much less than their rest energies. We leave the derivation of Eq. (43.24) to you (see Problem 43.75). In the present example, $K_{\text{cm}} = (14.00/18.01)K$, so K must be at least $(18.01/14.00) \times (1.192\ \text{MeV}) = 1.533\ \text{MeV}$.

For a charged particle such as a proton or an α particle to penetrate the nucleus of another atom and cause a reaction, it must usually have enough initial kinetic energy to overcome the potential-energy barrier caused by the repulsive electrostatic forces. In the reaction of part (a) of Example 43.11, if we treat the proton and the ^7Li nucleus as spherically symmetric charges with radii given by Eq. (43.1), their centers will be 3.5×10^{-15} m apart when they touch. The repulsive potential energy of the proton (charge $+e$) and the ^7Li nucleus (charge $+3e$) at this separation r is

$$U = \frac{1}{4\pi\epsilon_0}\frac{(e)(3e)}{r} = (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(3)(1.6 \times 10^{-19}\ \text{C})^2}{3.5 \times 10^{-15}\ \text{m}}$$
$$= 2.0 \times 10^{-13}\ \text{J} = 1.2\ \text{MeV}$$

Even though the reaction is exoergic, the proton must have a minimum kinetic energy of about 1.2 MeV for the reaction to occur, unless the proton *tunnels* through the barrier (see Section 40.3).

Neutron Absorption

Absorption of *neutrons* by nuclei forms an important class of nuclear reactions. Heavy nuclei bombarded by neutrons can undergo a series of neutron absorptions alternating with beta decays, in which the mass number A increases by as much as 25. Some of the *transuranic elements,* elements having Z larger than 92, are produced in this way. These elements have not been found in nature. Many transuranic elements, having Z possibly as high as 118, have been identified.

The analytical technique of *neutron activation analysis* uses similar reactions. When bombarded by neutrons, many stable nuclides absorb a neutron to become unstable and then undergo β^- decay. The energies of the β^- and associated γ emissions depend on the unstable nuclide and provide a means of identifying it and the original stable nuclide. Quantities of elements that are far too small for conventional chemical analysis can be detected in this way.

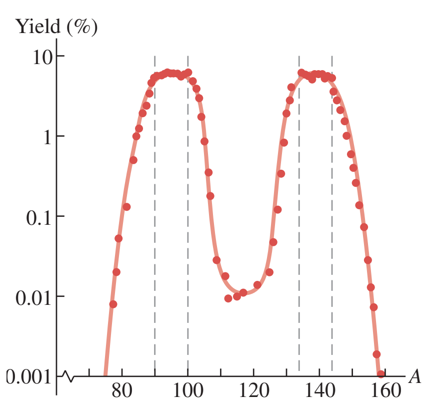
Test Your Understanding of Section 43.6 The reaction described in part (a) of Example 43.11 is exoergic. Can it happen naturally when a sample of solid lithium is placed in a flask of hydrogen gas?

43.7 Nuclear Fission

Nuclear fission is a decay process in which an unstable nucleus splits into two fragments of comparable mass. Fission was discovered in 1938 through the experiments of Otto Hahn and Fritz Strassman in Germany. Pursuing earlier work by Fermi, they bombarded uranium $(Z = 92)$ with neutrons. The resulting radiation did not coincide with that of any known radioactive nuclide. Urged on by their colleague Lise Meitner, they used meticulous chemical analysis to reach the astonishing but inescapable conclusion that they had found a radioactive isotope of barium $(Z = 56)$. Later, radioactive krypton $(Z = 36)$ was also found. Meitner and Otto Frisch correctly interpreted these results as showing that uranium nuclei were splitting into two massive fragments called *fission fragments.* Two or three free neutrons usually appear along with the fission fragments and, very occasionally, a light nuclide such as ^3H.

Both the common isotope (99.3%) ^{238}U and the uncommon isotope (0.7%) ^{235}U (as well as several other nuclides) can be easily split by neutron bombardment: ^{235}U by slow neutrons (kinetic energy less than 1 eV) but ^{238}U only by fast neutrons with a minimum of about 1 MeV of kinetic energy. Fission resulting from neutron absorption is called *induced fission.* Some nuclides can also undergo *spontaneous fission* without initial neutron absorption, but this is quite rare. When ^{235}U absorbs a neutron, the resulting nuclide $^{236}U^*$ is in a highly excited state and splits into two fragments almost instantaneously. Strictly speaking, it is $^{236}U^*$, not ^{235}U, that undergoes fission, but it's usual to speak of the fission of ^{235}U.

43.12 Mass distribution of fission fragments from the fission of $^{236}U^*$ (an excited state of ^{236}U), which is produced when ^{235}U absorbs a neutron. The vertical scale is logarithmic.



Over 100 different nuclides, representing more than 20 different elements, have been found among the fission products. Figure 43.12 shows the distribution of mass numbers for fission fragments from the fission of ^{235}U. Most of the fragments have mass numbers from 90 to 100 and from 135 to 145; fission into two fragments with nearly equal mass is unlikely.

Fission Reactions

You should check the following two typical fission reactions for conservation of nucleon number and charge:

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{236}_{92}U^* \rightarrow ^{144}_{56}Ba + ^{89}_{36}Kr + 3^{1}_{0}n$$

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{236}_{92}U^* \rightarrow ^{140}_{54}Xe + ^{94}_{38}Sr + 2^{1}_{0}n$$

The total kinetic energy of the fission fragments is enormous, about 200 MeV (compared to typical α and β energies of a few MeV). The reason for this is that nuclides at the high end of the mass spectrum (near $A = 240$) are less tightly bound than those nearer the middle ($A = 90$ to 145). Referring to Fig. 43.2, we see that the average binding energy per nucleon is about 7.6 MeV at $A = 240$ but about 8.5 MeV at $A = 120$. Therefore a rough estimate of the expected *increase* in binding energy during fission is about 8.5 MeV − 7.6 MeV = 0.9 MeV per nucleon, or a total of $(235)(0.9\text{ MeV}) \approx 200$ MeV.

CAUTION **Binding energy and rest energy** It may seem to be a violation of conservation of energy to have an increase in both the binding energy and the kinetic energy during a fission reaction. But relative to the total rest energy E_0 of the separated nucleons, the rest energy of the nucleus is E_0 *minus* E_B. Thus an *increase* in binding energy corresponds to a *decrease* in rest energy as rest energy is converted to the kinetic energy of the fission fragments.

Fission fragments always have too many neutrons to be stable. We noted in Section 43.3 that the neutron–proton ratio (N/Z) for stable nuclides is about 1 for light nuclides but almost 1.6 for the heaviest nuclides because of the increasing influence of the electrical repulsion of the protons. The N/Z value for stable nuclides is about 1.3 at $A = 100$ and 1.4 at $A = 150$. The fragments have about the same N/Z as ^{235}U, about 1.55. They usually respond to this surplus of neutrons by undergoing a series of β^- decays (each of which increases Z by 1 and decreases N by 1) until a stable value of N/Z is reached. A typical example is

$$^{140}_{54}Xe \xrightarrow{\beta^-} ^{140}_{55}Cs \xrightarrow{\beta^-} ^{140}_{56}Ba \xrightarrow{\beta^-} ^{140}_{57}La \xrightarrow{\beta^-} ^{140}_{58}Ce$$

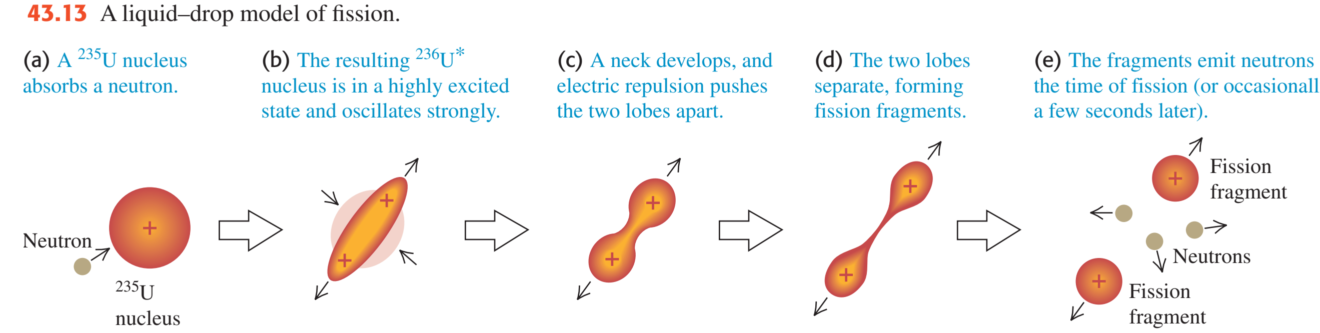
The nuclide ^{140}Ce is stable. This series of β^- decays produces, on average, about 15 MeV of additional kinetic energy. The neutron excess of fission fragments also explains why two or three free neutrons are released during the fission.

Fission appears to set an upper limit on the production of transuranic nuclei, mentioned in Section 43.6, that are relatively stable. There are theoretical reasons to expect that nuclei near $Z = 114$, $N = 184$ or 196, might be stable with respect to spontaneous fission. In the shell model (Section 43.2), these numbers correspond to filled shells and subshells in the nuclear energy-level structure. Such *superheavy nuclei* would still be unstable with respect to alpha emission, but they might live long enough to be identified. As of this writing, there is evidence that nuclei with $Z = 114$ have been produced in the laboratory.

Liquid-Drop Model

We can understand fission qualitatively on the basis of the liquid-drop model of the nucleus (Section 43.2). The process is shown in Fig. 46.13 in terms of an electrically charged liquid drop. These sketches shouldn't be taken too literally, but they may help to develop your intuition about fission. A ^{235}U nucleus absorbs a neutron (Fig. 43.13a), becoming a $^{236}U^*$ nucleus with excess energy (Fig. 43.13b). This excess energy causes violent oscillations, during which a neck between two

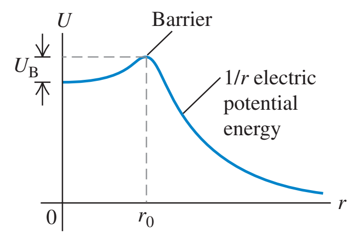
43.13 A liquid–drop model of fission.



lobes develops (Fig. 43.13c). The electrical repulsion of these two lobes stretches the neck farther (Fig. 43.13d), and finally two smaller fragments are formed (Fig. 43.13e) that move rapidly apart.

This qualitative picture has been developed into a more quantitative theory to explain why some nuclei undergo fission and others don't. Figure 43.14 shows a hypothetical potential-energy function for two possible fission fragments. If neutron absorption results in an excitation energy greater than the energy barrier height U_B, fission occurs immediately. Even when there isn't quite enough energy to surmount the barrier, fission can take place by quantum-mechanical *tunneling,* discussed in Section 40.3. In principle, many stable heavy nuclei can fission by tunneling. But the probability depends very critically on the height and width of the barrier. For most nuclei this process is so unlikely that it is never observed.

43.14 Hypothetical potential-energy function for two fission fragments in a fissionable nucleus. At distances r beyond the range of the nuclear force, the potential energy varies approximately as $1/r$. Fission occurs if there is an excitation energy greater than U_B or an appreciable probability for tunneling through the potential-energy barrier.



Chain Reactions

Fission of a uranium nucleus, triggered by neutron bombardment, releases other neutrons that can trigger more fissions, suggesting the possibility of a **chain reaction** (Fig. 43.15). The chain reaction may be made to proceed slowly and in a controlled manner in a nuclear reactor or explosively in a bomb. The energy release in a nuclear chain reaction is enormous, far greater than that in any chemical reaction. (In a sense, *fire* is a chemical chain reaction.) For example, when uranium is "burned" to uranium dioxide in the chemical reaction

$$U + O_2 \rightarrow UO_2$$

the heat of combustion is about 4500 J/g. Expressed as energy per atom, this is about 11 eV per atom. By contrast, fission liberates about 200 MeV per atom, nearly 20 million times as much energy.

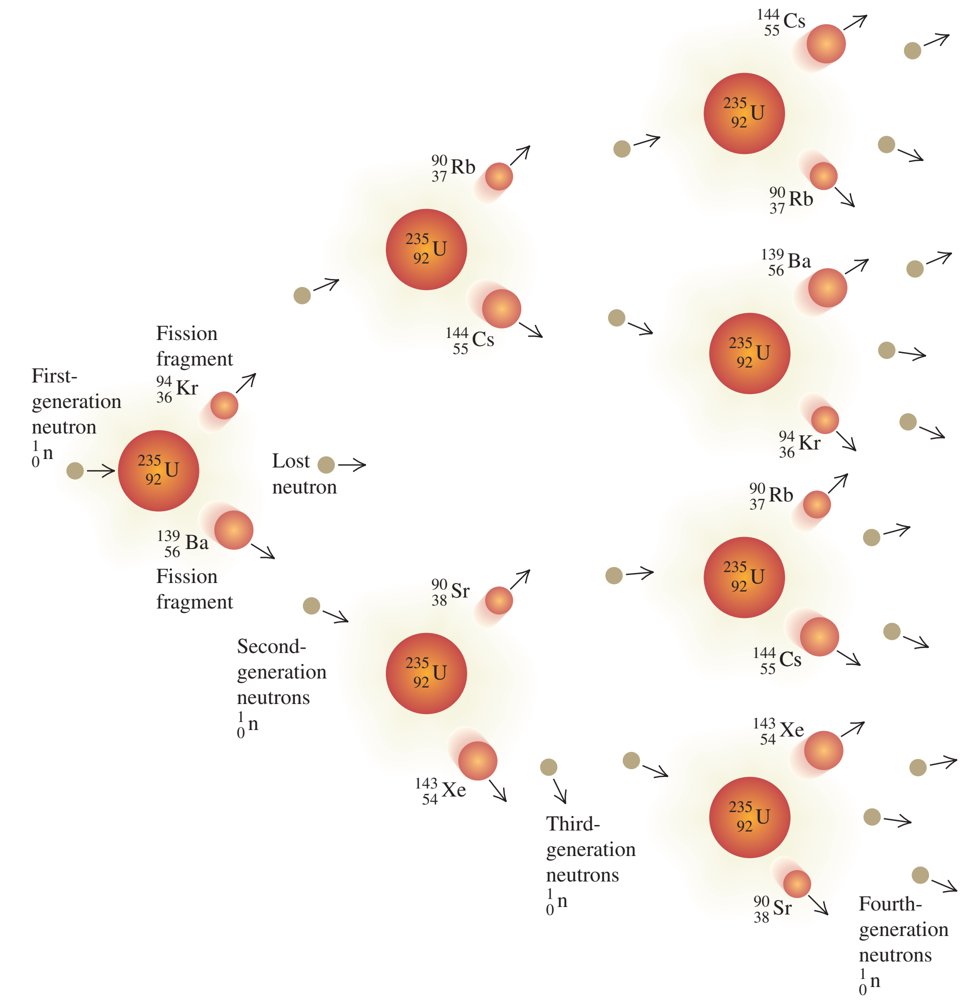
Nuclear Reactors

A *nuclear reactor* is a system in which a controlled nuclear chain reaction is used to liberate energy. In a nuclear power plant, this energy is used to generate steam, which operates a turbine and turns an electrical generator.

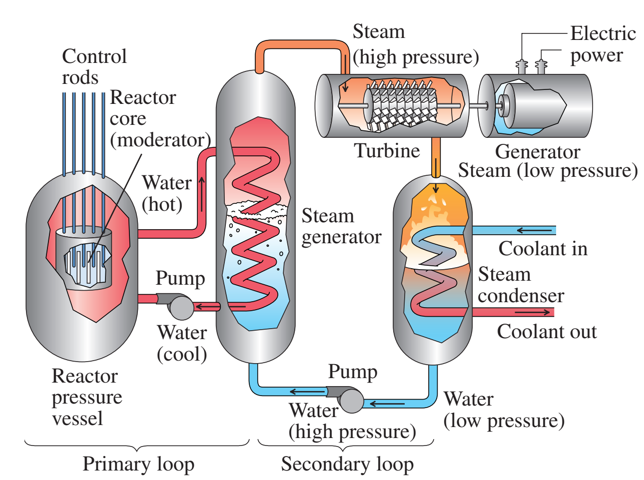
On average, each fission of a ^{235}U nucleus produces about 2.5 free neutrons, so 40% of the neutrons are needed to sustain a chain reaction. A ^{235}U nucleus is much more likely to absorb a low-energy neutron (less than 1 eV) than one of the higher-energy neutrons (1 MeV or so) that are liberated during fission. In a nuclear reactor the higher-energy neutrons are slowed down by collisions with nuclei in the surrounding material, called the *moderator,* so they are much more likely to cause further fissions. In nuclear power plants, the moderator is often water, occasionally graphite. The *rate* of the reaction is controlled by inserting or withdrawing *control rods* made of elements (such as boron or cadmium) whose nuclei *absorb* neutrons without undergoing any additional reaction. The isotope ^{238}U can also absorb neutrons, leading to $^{239}U^*$, but not with high enough probability for it to sustain a chain reaction by itself. Thus uranium that is used in reactors is often "enriched" by increasing the proportion of ^{235}U above the natural value of 0.7%, typically to 3% or so, by isotope-separation processing.

The most familiar application of nuclear reactors is for the generation of electric power. As was noted above, the fission energy appears as kinetic energy of the fission fragments, and its immediate result is to increase the internal energy of the fuel elements and the surrounding moderator. This increase in internal energy is transferred as heat to generate steam to drive turbines, which spin the electrical generators. Figure 43.16 is a schematic diagram of a nuclear power plant. The energetic fission fragments heat the water surrounding the reactor core. The steam generator is a heat exchanger that takes heat from this highly radioactive water and generates nonradioactive steam to run the turbines.

43.15 Schematic diagram of a nuclear fission chain reaction.



A typical nuclear plant has an electric-generating capacity of 1000 MW (or 10^9 W). The turbines are heat engines and are subject to the efficiency limitations imposed by the second law of thermodynamics, discussed in Chapter 20. In modern nuclear plants the overall efficiency is about one-third, so 3000 MW of thermal power from the fission reaction is needed to generate 1000 MW of electrical power.



43.16 Schematic diagram of a nuclear power plant.

Example 43.12 Uranium consumption in a nuclear reactor

What mass of ^{235}U has to undergo fission each day to provide 3000 MW of thermal power?

SOLUTION

IDENTIFY: The critical bit of information is that fission liberates about 200 MeV per atom.

SET UP: We use the energy released per atom along with the mass of the ^{235}U atom to determine the required amount of uranium.

EXECUTE: Each second, we need 3000 MJ or 3000×10^6 J. Each fission provides 200 MeV, which is

$$(200\ \text{MeV})(1.6 \times 10^{-13}\ \text{J/MeV}) = 3.2 \times 10^{-11}\ \text{J}$$

The number of fissions needed each second is

$$\frac{3000 \times 10^6\ \text{J}}{3.2 \times 10^{-11}\ \text{J}} = 9.4 \times 10^{19}$$

Each ^{235}U atom has a mass of $(235\ \text{u})(1.66 \times 10^{-27}\ \text{kg/u}) = 3.9 \times 10^{-25}$ kg, so the mass of ^{235}U needed each second is

$$(9.4 \times 10^{19})(3.9 \times 10^{-25}\ \text{kg}) = 3.7 \times 10^{-5}\ \text{kg} = 37\ \mu\text{g}$$

In one day (86,400 s), the total consumption of ^{235}U is

$$(3.7 \times 10^{-5}\ \text{kg/s})(86{,}400\ \text{s}) = 3.2\ \text{kg}$$

EVALUATE: For comparison, a 1000-MW coal-fired power plant burns 10,600 tons (about 10 million kg) of coal per day!

Nuclear fission reactors have many other practical uses. Among these are the production of artificial radioactive isotopes for medical and other research, production of high-intensity neutron beams for research in nuclear structure, and production of fissionable nuclides such as ^{239}Pu from the common isotope ^{238}U. The last is the function of *breeder reactors,* which can produce more fuel than they use.

We mentioned above that about 15 MeV of the energy released after fission of a ^{235}U nucleus comes from the β^- decays of the fission fragments. This fact poses a serious problem with respect to control and safety of reactors. Even after the chain reaction has been completely stopped by insertion of control rods into the core, heat continues to be evolved by the β^- decays, which cannot be stopped. For a 3000-MW reactor this heat power is initially very large, about 200 MW. In the event of total loss of cooling water, this power is more than enough to cause a catastrophic meltdown of the reactor core and possible penetration of the containment vessel. The difficulty in achieving a "cold shutdown" following an accident at the Three Mile Island nuclear power plant in Pennsylvania in March 1979 was a result of the continued evolution of heat due to β^- decays.

The catastrophe of April 26, 1986, at Chernobyl reactor No. 4 in Ukraine resulted from a combination of an inherently unstable design and several human errors committed during a test of the emergency core cooling system. Too many control rods were withdrawn to compensate for a decrease in power caused by a buildup of neutron absorbers such as ^{135}Xe. The power level rose from 1% of normal to 100 times normal in 4 seconds; a steam explosion ruptured pipes in the core cooling system and blew the heavy concrete cover off the reactor. The graphite moderator caught fire and burned for several days, and there was a meltdown of the core. The total activity of the radioactive material released into the atmosphere has been estimated as about 10^8 Ci.

Test Your Understanding of Section 43.7 The fission of ^{235}U can be triggered by the absorption of a slow neutron by a nucleus. Can a slow *proton* be used to trigger ^{235}U fission?

43.8 Nuclear Fusion

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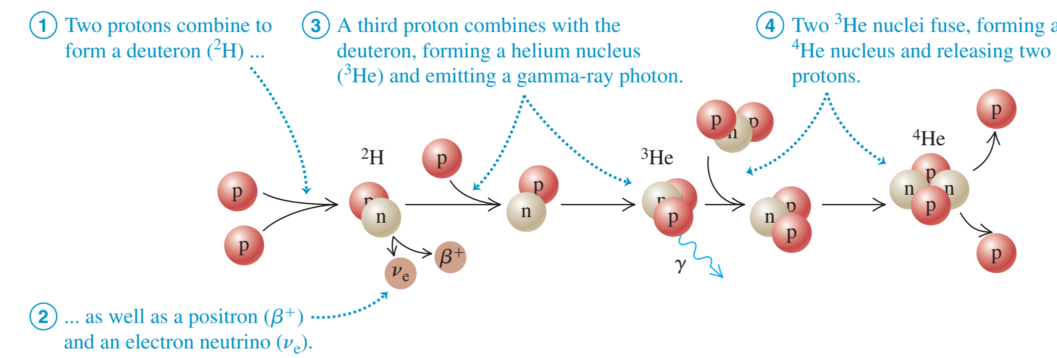
19.3 Fusion

In a **nuclear fusion** reaction, two or more small light nuclei come together, or *fuse,* to form a larger nucleus. Fusion reactions release energy for the same reason as fission reactions: The binding energy per nucleon after the reaction is greater than before. Referring to Fig. 43.2, we see that the binding energy per nucleon increases with A up to about $A = 60$, so fusion of nearly any two light nuclei to make a nucleus with A less than 60 is likely to be an exoergic reaction. In comparison to fission, we are moving toward the peak of this curve from the opposite side. Another way to express the energy relationships is that the total mass of the products is less than that of the initial particles.

43.17 The proton-proton chain.



Here are three examples of energy-liberating fusion reactions, written in terms of the neutral atoms:

$$^1_1\text{H} + {^1_1\text{H}} \rightarrow {^2_1\text{H}} + \beta^+ + \nu_e$$
$$^2_1\text{H} + {^1_1\text{H}} \rightarrow {^3_2\text{He}} + \gamma$$
$$^3_2\text{He} + {^3_2\text{He}} \rightarrow {^4_2\text{He}} + {^1_1\text{H}} + {^1_1\text{H}}$$

In the first reaction, two protons combine to form a deuteron (^2H), with the emission of a positron (β^+) and an electron neutrino. In the second, a proton and a deuteron combine to form the nucleus of the light isotope of helium, ^3He, with the emission of a gamma ray. Now double the first two reactions to provide the two ^3He nuclei that fuse in the third reaction to form an alpha particle (^4He) and two protons. Together the reactions make up the process called the *proton-proton chain* (Fig. 43.17).

The net effect of the chain is the conversion of four protons into one α particle, two positrons, two electron neutrinos, and two γ's. We can calculate the energy release from this part of the process: The mass of an α particle plus two positrons is the mass of neutral ^4He, the neutrinos have zero (or negligible) mass, and the gammas have zero mass.

Mass of four protons	4.029106 u
Mass of ^4He	4.002603 u
Mass difference and energy release	0.026503 u and 24.69 MeV

The two positrons that are produced during the first step of the proton-proton chain collide with two electrons; mutual annihilation of the four particles takes place, and their rest energy is converted into $4(0.511\ \text{MeV}) = 2.044$ MeV of gamma radiation. Thus the total energy release is $(24.69 + 2.044)\ \text{MeV} = 26.73$ MeV. The proton-proton chain takes place in the interior of the sun and other stars (Fig. 43.18). Each gram of the sun's mass contains about 4.5×10^{23} protons. If all of these protons were fused into helium, the energy released would be about 130,000 kWh. If the sun were to continue to radiate at its present rate, it would take about 75×10^9 years to exhaust its supply of protons. As we will see below, fusion reactions can occur only at extremely high temperatures; in the sun, these temperatures are found only deep within the interior. Hence the sun cannot fuse *all* of its protons, and can do so for a total of only about 10×10^9 years in total. The present age of the solar system (including the sun) is 4.6×10^9 years, so the sun is about halfway through its available store of protons.

43.18 The energy released as starlight comes from fusion reactions deep within a star's interior. When a star is first formed and for most of its life, it converts the hydrogen in its core into helium. As a star ages, the core temperature can become high enough for additional fusion reactions that convert helium into carbon, oxygen, and other elements.

Example 43.13 A fusion reaction

Two deuterons fuse to form a *triton* (a nucleus of tritium, or ^3H) and a proton. How much energy is liberated?

SOLUTION

IDENTIFY: This is a nuclear reaction of the type discussed in Section 43.6.

SET UP: We find the energy released using Eq. (43.23).

EXECUTE: Adding one electron to each particle makes each a neutral atom; we find their masses in Table 43.2. Substituting into Eq. (43.23), we find

$$Q = [2(2.014102\text{ u}) - 3.016049\text{ u} - 1.007825\text{ u}] \times (931.5\text{ MeV/u}) = 4.03\text{ MeV}$$

EVALUATE: Thus 4.03 MeV is released in the reaction; the triton and proton together have 4.03 MeV more kinetic energy than the two deuterons had together.

Achieving Fusion

For two nuclei to undergo fusion, they must come together to within the range of the nuclear force, typically of the order of 2×10^{-15} m. To do this, they must overcome the electrical repulsion of their positive charges. For two protons at this distance, the corresponding potential energy is about 1.2×10^{-13} J or 0.7 MeV; this represents the total initial *kinetic* energy that the fusion nuclei must have—for example, 0.6×10^{-13} J each in a head-on collision.

Atoms have this much energy only at extremely high temperatures. The discussion of Section 18.3 showed that the average translational kinetic energy of a gas molecule at temperature T is $\frac{3}{2}kT$, where k is Boltzmann's constant. The temperature at which this is equal to $E = 0.6 \times 10^{-13}$ J is determined by the relationship

$$E = \frac{3}{2}kT$$

$$T = \frac{2E}{3k} = \frac{2(0.6 \times 10^{-13}\text{ J})}{3(1.38 \times 10^{-23}\text{ J/K})} = 3 \times 10^9\text{ K}$$

Fusion reactions are possible at lower temperatures because the Maxwell-Boltzmann distribution function (see Section 18.5) gives a small fraction of protons with kinetic energies much higher than the average value. The proton-proton reaction occurs at "only" 1.5×10^7 K at the center of the sun, making it an extremely low-probability process; but that's why the sun is expected to last so long. At these temperatures the fusion reactions are called *thermonuclear* reactions.

Intensive efforts are under way to achieve controlled fusion reactions, which potentially represent an enormous new resource of energy (see Fig. 24.11). At the temperatures mentioned, light atoms are fully ionized, and the resulting state of matter is called a *plasma.* In one kind of experiment using *magnetic confinement,* a plasma is heated to extremely high temperature by an electrical discharge, while being contained by appropriately shaped magnetic fields. In another, using *inertial confinement,* pellets of the material to be fused are heated by a high-intensity laser beam (see Fig. 43.19). Some of the reactions being studied are

$$^2_1H + ^2_1H \rightarrow ^3_1H + ^1_1H + 4.0\text{ MeV} \quad (1)$$

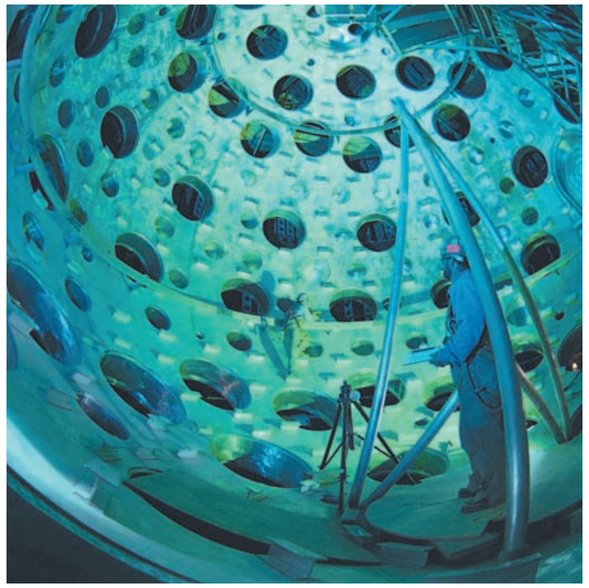
$$^3_1H + ^2_1H \rightarrow ^4_2He + ^1_0n + 17.6\text{ MeV} \quad (2)$$

$$^2_1H + ^2_1H \rightarrow ^3_2He + ^1_0n + 3.3\text{ MeV} \quad (3)$$

$$^3_2He + ^2_1H \rightarrow ^4_2He + ^1_1H + 18.3\text{ MeV} \quad (4)$$

We considered reaction (1) in Example 43.13; two deuterons fuse to form a triton and a proton. In reaction (2) a triton combines with another deuteron to form an alpha particle and a neutron. The result of both of these reactions together is the conversion of three deuterons into an alpha particle, a proton, and a neutron, with the liberation of 21.6 MeV of energy. Reactions (3) and (4) together achieve the same conversion. In a plasma that contains deuterons, the two pairs of reactions occur with roughly equal probability. As yet, no one has succeeded in producing these reactions under controlled conditions in such a way as to yield a net surplus of usable energy.

43.19 This target chamber at the National Ignition Facility in California has apertures for 192 powerful laser beams. When placed in operation in 2010, the lasers will deliver 5×10^{14} W of power for a few nanoseconds to a millimeter-sized pellet of deuterium and tritium at the center of the chamber, thus triggering thermonuclear fusion.



Methods of achieving fusion that don't require high temperatures are also being studied; these are called *cold fusion.* One scheme that does work uses an unusual hydrogen molecule ion. The usual H_2^+ ion consists of two protons bound by one shared electron; the nuclear spacing is about 0.1 nm. If the protons are replaced by a deuteron (^2H) and a triton (^3H) and the electron by a *muon,* which is 208 times as massive as the electron, the spacing is made smaller by a factor of 208. The probability then becomes appreciable for the two nuclei to tunnel through the narrow repulsive potential-energy barrier and fuse in reaction (2) above. The prospect of making this process, called *muon-catalyzed fusion,* into a practical energy source is still distant.

Test Your Understanding of Section 43.8 Are *all* fusion reactions exoergic?

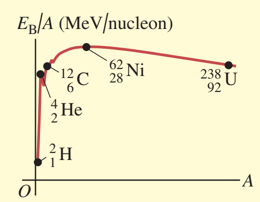
CHAPTER 43 SUMMARY

Nuclear properties: A nucleus is composed of A nucleons (Z protons and N neutrons). All nuclei have about the same density. The radius of a nucleus with mass number A is given approximately by Eq. (43.1). A single nuclear species of a given Z and N is called a nuclide. Isotopes are nuclides of the same element (same Z) that have different numbers of neutrons. Nuclear masses are measured in atomic mass units. Nucleons have angular momentum and a magnetic moment. (See Examples 43.1 and 43.2.)

$$R = R_0 A^{1/3} \quad (R_0 = 1.2 \times 10^{-15}\ \text{m}) \qquad (43.1)$$

Nuclear binding and structure: The mass of a nucleus is always less than the mass of the protons and neutrons within it. The mass difference multiplied by c^2 gives the binding energy E_B. The binding energy for a given nuclide is determined by the nuclear force, which is short range and favors pairs of particles, and by the electric repulsion between protons. A nucleus is unstable if A or Z is too large or if the ratio N/Z is wrong. Two widely used models of the nucleus are the liquid-drop model and the shell model; the latter is analogous to the central-field approximation for atomic structure. (See Examples 43.3 and 43.4.)

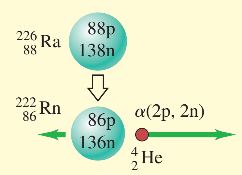
$$E_B = (ZM_H + Nm_n - {}^A_ZM)c^2 \qquad (43.10)$$



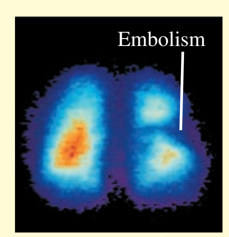
Radioactive decay: Unstable nuclides usually emit an alpha particle (a $^4_2$He nucleus) or a beta particle (an electron) in the process of changing to another nuclide, sometimes followed by a gamma-ray photon. The rate of decay of an unstable nucleus is described by the decay constant λ, the half-life $T_{1/2}$, or the lifetime T_{mean}. If the number of nuclei at time $t = 0$ is N_0 and no more are produced, the number at time t is given by Eq. (43.17). (See Examples 43.5–43.9.)

$$N(t) = N_0 e^{-\lambda t} \qquad (43.17)$$

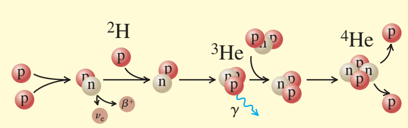
$$T_{mean} = \frac{1}{\lambda} = \frac{T_{1/2}}{\ln 2} = \frac{T_{1/2}}{0.693} \qquad (43.19)$$



Biological effects of radiation: The biological effect of any radiation depends on the product of the energy absorbed per unit mass and the relative biological effectiveness (RBE), which is different for different radiations. (See Example 43.10.)



Nuclear reactions: In a nuclear reaction, two nuclei or particles collide to produce two new nuclei or particles. Reactions can be exoergic or endoergic. Several conservation laws, including charge, energy, momentum, angular momentum, and nucleon number, are obeyed. Energy is released by the fission of a heavy nucleus into two lighter, always unstable, nuclei. Energy is also released by the fusion of two light nuclei into a heavier nucleus. (See Examples 43.11–43.13.)



Key Terms

nucleon number, *1469*
mass number, *1469*
atomic number, *1470*
neutron number, *1470*
nuclide, *1470*
isotope, *1470*
nuclear magneton, *1472*
binding energy, *1474*
liquid-drop model, *1476*
shell model, *1477*
radioactivity, *1478*
alpha particle, *1480*
beta-minus particle, *1481*
neutrino, *1482*
gamma ray, *1483*
activity, *1486*
decay constant, *1486*
half-life, *1486*
curie, *1487*
relative biological effectiveness (RBE), *1489*
reaction energy, *1492*
threshold energy, *1493*
nuclear fission, *1494*
chain reaction, *1496*
nuclear fusion, *1498*

Answer to Chapter Opening Question ?

When an organism dies, it stops taking in carbon from atmospheric CO_2. Some of this carbon is radioactive ^{14}C, which decays with a half-life of 5730 years. By measuring the proportion of ^{14}C that remains in the specimen, scientists can determine how long ago the organism died. (See Section 43.4.)

Answers to Test Your Understanding Questions

43.1 Answers: (a) (iii), (b) (v) The radius R is proportional to the cube root of the mass number A, while the volume is proportional to R^3 and hence to $(A^{1/3})^3 = A$. Therefore, doubling the volume requires increasing the mass number by a factor of 2; doubling the radius implies increasing both the volume and the mass number by a factor of $2^3 = 8$.

43.2 Answer: (ii), (iii), (iv), (v), (i) You can find the answers by inspecting Fig. 43.2. The binding energy per nucleon is lowest for very light nuclei such as $^4_2$He, is greatest around $A = 60$, and then decreases with increasing A.

43.3 Answer (v) Two protons and two neutrons are lost in an α decay, so Z and N each decrease by 2. A β^+ decay changes a proton to a neutron, so Z decreases by 1 and N increases by 1. The net result is that Z decreases by 3 and N decreases by 1.

43.4 Answer: (iii) The activity $-dN(t)/dt$ of a sample is the product of the number of nuclei in the sample $N(t)$ and the decay constant $\lambda = (\ln 2)/T_{1/2}$. Hence $N(t) = (-dN(t)/dt)\,T_{1/2}/(\ln 2)$. Taking the ratio of this expression for ^{240}Pu to this same expression for ^{243}Am, the factors of $\ln 2$ cancel and we get

$$\frac{N_{Pu}}{N_{Am}} = \frac{(-dN_{Pu}/dt)T_{1/2\text{-Pu}}}{(-dN_{Am}/dt)\,T_{1/2\text{-Am}}} = \frac{(5.00\mu\text{Ci})(6560\ \text{y})}{(4.45\mu\text{Ci})(7370\ \text{y})} = 1.00$$

The two samples contain *equal* numbers of nuclei. The ^{243}Am sample has a longer half-life and hence a slower decay rate, so it has a lower activity than the ^{240}Pu sample.

43.5 Answer: (ii) We saw in Section 43.3 that alpha particles can travel only a very short distance before they are stopped. By contrast, x-ray photons are very penetrating, so they can easily pass into the body.

43.6 Answer: no The reaction $^1_1\text{H} + ^7_3\text{Li} \rightarrow ^4_2\text{He} + ^4_2\text{He}$ is a *nuclear* reaction, which can take place only if a proton (a hydrogen nucleus) comes into contact with a lithium nucleus. If the hydrogen is in atomic form, the interaction between its electron cloud and the electron cloud of a lithium atom keeps the two nuclei from getting close to each other. Even if isolated protons are used, they must be fired at the lithium atoms with enough kinetic energy to overcome the electric repulsion between the protons and the lithium nuclei. The statement that the reaction is exoergic means that more energy is released by the reaction than had to be put in to make the reaction occur.

43.7 Answer: no Because the neutron has no electric charge, it experiences no electric repulsion from a ^{235}U nucleus. Hence a slow-moving neutron can approach and enter a ^{235}U nucleus, thereby providing the excitation needed to trigger fission. By contrast, a slow-moving *proton* (charge $+e$) feels a strong electric repulsion from a ^{235}U nucleus (charge $+92e$). It never gets close to the nucleus, so it cannot trigger fission.

43.8 Answer: no Fusion reactions between sufficiently light nuclei are exoergic because the binding energy per nucleon E_B/A increases. If the nuclei are too massive, however, E_B/A decreases and fusion is *endoergic* (i.e., it takes in energy rather than releasing it). As an example, imagine fusing together two nuclei of $A = 100$ to make a single nucleus with $A = 200$. From Fig. 43.2, E_B/A is more than 8.5 MeV for the $A = 100$ nuclei but is less than 8 MeV for the $A = 200$ nucleus. Such a fusion reaction is possible, but requires a substantial input of energy.

PROBLEMS

For instructor-assigned homework, go to www.masteringphysics.com

Discussion Questions

Q43.1. Neutrons have a magnetic dipole moment and can undergo spin flips by absorbing electromagnetic radiation. Why, then, are protons rather than neutrons used in MRI of body tissues? (See Fig. 43.1.)

Q43.2. In Eq. (43.11), as the total number of nucleons becomes larger, the importance of the second term in the equation decreases relative to that of the first term. Does this make physical sense? Explain.

Q43.3. Why aren't the masses of all nuclei integer multiples of the mass of a single nucleon?

Q43.4. Can you tell from the value of the mass number A whether to use a plus value, a minus value, or zero for the fifth term of Eq. (43.11)? Explain.

Q43.5. What are the six known elements for which Z is a magic number? Discuss what properties these elements have as a consequence of their special values of Z.

Q43.6. The binding energy per nucleon for most nuclides doesn't vary much (see Fig. 43.2). Is there similar consistency in the *atomic* energy of atoms, on an "energy per electron" basis? If so, why? If not, why not?

Q43.7. Heavy, unstable nuclei usually decay by emitting an α or β particle. Why don't they usually emit a single proton or neutron?

Q43.8. The only two stable nuclides with more protons than neutrons are ${}^{1}_{1}H$ and ${}^{3}_{2}He$. Why is $Z > N$ so uncommon?

Q43.9. Since lead is a stable element, why doesn't the ${}^{238}U$ decay series shown in Fig. 43.8 stop at lead, ${}^{214}Pb$?

Q43.10. In the ${}^{238}U$ decay series shown in Fig. 43.8, some nuclides in the series are found much more abundantly in nature than others, even though every ${}^{238}U$ nucleus goes through every step in the series before finally becoming ${}^{206}Pb$. Why don't the intermediate nuclides all have the same abundance?

Q43.11. Compared to α particles with the same energy, β particles can much more easily penetrate through matter. Why is this?

Q43.12. If ${}^{A}_{Z}El_i$ represents the initial nuclide, what is the decay process or processes if the final nuclide is (a) ${}_{Z+1}{}^{A}El_f$; (b) ${}^{A-4}_{Z-2}El_f$; (c) ${}_{Z-1}{}^{A}El_f$?

Q43.13. In a nuclear decay equation, why can we represent an electron as ${}_{-1}^{0}\beta^-$? What are the equivalent representations for a positron, a neutrino, and an antineutrino?

Q43.14. Why is the alpha, beta, or gamma decay of an unstable nucleus unaffected by the *chemical* situation of the atom, such as the nature of the molecule or solid in which it is bound? The chemical situation of the atom can, however, have an effect on the half-life in electron capture. Why is this?

Q43.15. In the process of *internal conversion,* a nucleus decays from an excited state to a ground state by giving the excitation energy directly to an atomic electron rather than emitting a gamma-ray photon. Why can this process also produce x-ray photons?

Q43.16. In Example 43.9 (Section 43.4), the activity of atmospheric carbon *before* 1900 was given. Discuss why this activity may have changed since 1900.

Q43.17. One problem in radiocarbon dating of biological samples, especially very old ones, is that they can easily be contaminated with modern biological material during the measurement process. What effect would such contamination have on the estimated age? Why is such contamination a more serious problem for samples of older material than for samples of younger material?

Q43.18. The most common radium isotope found on earth, ${}^{226}Ra$, has a half-life of about 1600 years. If the earth was formed well over 10^9 years ago, why is there any radium left now?

Q43.19. Fission reactions occur only for nuclei with large nucleon numbers, while exoergic fusion reactions occur only for nuclei with small nucleon numbers. Why is this?

Q43.20. When a large nucleus splits during nuclear fission, the daughter nuclei of the fission fly apart with enormous kinetic energy. Why does this happen?

Q43.21. As stars age, they use up their supply of hydrogen and eventually begin producing energy by a reaction that involves the fusion of three helium nuclei to form a carbon nucleus. Would you expect the interiors of these old stars to be hotter or cooler than the interiors of younger stars? Explain.

Exercises

Section 43.1 Properties of Nuclei

43.1. How many protons and how many neutrons are there in a nucleus of the most common isotope of (a) silicon, ${}^{28}_{14}Si$; (b) rubidium, ${}^{85}_{37}Rb$; (c) thallium, ${}^{205}_{81}Tl$?

43.2. Consider the three nuclei of Exercise 43.1. Estimate (a) the radius, (b) the surface area, and (c) the volume of each nucleus. Determine (d) the mass density (in kg/m^3) and (e) the nucleon density (in nucleons per cubic meter) for each nucleus. Assume that the mass of each nucleus is A atomic mass units.

43.3. Hydrogen atoms are placed in an external magnetic field. The protons can make transitions between states in which the nuclear spin component is parallel and antiparallel to the field by absorbing or emitting a photon. What magnetic-field magnitude is required for this transition to be induced by photons with frequency 22.7 MHz?

43.4. Neutrons are placed in a magnetic field with magnitude 2.30 T. (a) What is the energy difference between the states with the nuclear spin angular momentum components parallel and antiparallel to the field? Which state is lower in energy, the one with its spin component parallel to the field or the one with its spin component antiparallel to the field? How do your results compare with the energy states for a proton in the same field (see Example 43.2)? (b) The neutrons can make transitions from one of these states to the other by emitting or absorbing a photon with energy equal to the energy difference of the two states. Find the frequency and wavelength of such a photon.

43.5. Hydrogen atoms are placed in an external 1.65-T magnetic field. (a) The *protons* can make transitions between states where the nuclear spin component is parallel and antiparallel to the field by absorbing or emitting a photon. Which state has lower energy: the state with the nuclear spin component parallel or antiparallel to the field? What are the frequency and wavelength of the photon? In which region of the electromagnetic spectrum does it lie? (b) The *electrons* can make transitions between states where the electron spin component is parallel and antiparallel to the field by absorbing or emitting a photon. Which state has lower energy: the state with the electron spin component parallel or antiparallel to the field? What are the frequency and wavelength of the photon? In which region of the electromagnetic spectrum does it lie?

Section 43.2 Nuclear Binding and Nuclear Structure

43.6. The most common isotope of uranium, ${}^{238}_{92}U$, has atomic mass 238.050783 u. Calculate (a) the mass defect; (b) the binding energy (in MeV); (c) the binding energy per nucleon.

43.7. What is the maximum wavelength of a γ ray that could break a deuteron into a proton and a neutron? (This process is called photodisintegration.)

43.8. Calculate (a) the total binding energy and (b) the binding energy per nucleon of ${}^{12}C$ (c) What percent of the rest mass of this nucleus is its total binding energy?

43.9. A photon with a wavelength of 3.50×10^{-13} m strikes a deuteron, splitting it into a proton and a neutron. (a) Calculate the kinetic energy released in this interaction. (b) Assuming the two particles share the energy equally, and taking their masses to be 1.00 u, calculate their speeds after the photodisintegration.

43.10. Calculate the mass defect, the binding energy (in MeV), and the binding energy per nucleon of (a) the nitrogen nucleus, ${}^{14}_{7}N$ and (b) the helium nucleus, ${}^{4}_{2}He$. (c) How do the results of parts (a) and (b) compare?

43.11. The most common isotope of boron is ${}^{11}_{5}B$. (a) Determine the total binding energy of ${}^{11}_{5}B$ from Table 43.2 in Section 43.1. (b) Calculate this binding energy from Eq. (43.11). (Why is the fifth term zero?) Compare to the result you obtained in part (a). What is the percent difference? Compare the accuracy of Eq. (43.11) for ${}^{11}_{5}B$ to its accuracy for ${}^{62}_{28}Ni$ (see Example 43.4).

43.12. The most common isotope of copper is ${}^{63}_{29}Cu$. The measured mass of the neutral atom is 62.929601 u. (a) From the measured mass, determine the mass defect, and use it to find the total binding energy and the binding energy per nucleon. (b) Calculate the binding energy from Eq. (43.11). (Why is the fifth term zero?) Compare to the result you obtained in part (a). What is the percent difference? What do you conclude about the accuracy of Eq. (43.11)?

Section 43.3 Nuclear Stability and Radioactivity

43.13. What nuclide is produced in the following radioactive decays? (a) α decay of ${}^{239}_{4}Pu$; (b) β^- decay of ${}^{24}_{11}Na$; (c) β^+ decay of ${}^{15}_{8}O$.

43.14. (a) Is the decay $n \rightarrow p + \beta^- + \bar{\nu}_e$ energetically possible? If not, explain why not. If so, calculate the total energy released. (b) Is the decay $p \rightarrow n + \beta^+ + \nu_e$ energetically possible? If not, explain why not. If so, calculate the total energy released.

43.15. The α decay of ${}^{238}U$ is accompanied by a γ ray of measured wavelength 0.0248 nm. This decay is due to a transition of the nucleus between two energy levels. What is the difference in energy (in MeV) between these two levels?

43.16. ${}^{238}U$ decays spontaneously by α emission to ${}^{234}Th$. Calculate (a) the total energy released by this process and (b) the recoil velocity of the ${}^{234}Th$ nucleus. The atomic masses are 238.050788 u for ${}^{238}U$ and 234.043601 u for ${}^{234}Th$.

43.17. The atomic mass of ${}^{14}C$ is 14.003242 u. Show that the β^- decay of ${}^{14}C$ is energetically possible, and calculate the energy released in the decay.

43.18. What particle (α particle, electron, or positron) is emitted in the following radioactive decays? (a) ${}^{27}_{14}Si \rightarrow {}^{27}_{13}Al$; (b) ${}^{238}_{92}U \rightarrow {}^{234}_{90}Th$; (c) ${}^{74}_{33}As \rightarrow {}^{74}_{34}Se$.

43.19. (a) Calculate the energy released by the electron-capture decay of ${}^{57}_{27}Co$ (see Example 43.7). (b) A negligible amount of this energy goes to the resulting ${}^{57}_{26}Fe$ atom as kinetic energy. About 90% of the time, the ${}^{57}_{26}Fe$ nucleus emits two successive gamma-ray photons after the electron-capture process, of energies 0.122 MeV and 0.014 MeV, respectively, in decaying to its ground state. What is the energy of the neutrino emitted in this case?

Section 43.4 Activities and Half-Lives

43.20. The isotope ${}^{90}Sr$ undergoes β^- decay with a half-life of 28 years. (a) What nucleus is produced by this decay? (b) If a nuclear power plant is contaminated with ${}^{90}Sr$, how long will it take for the radiation level to decrease to 1.0% of its initial value?

43.21. If a 6.13-g sample of an isotope having a mass number of 124 decays at a rate of 0.350 Ci, what is its half-life?

43.22. Radioactive isotopes used in cancer therapy have a "shelf-life," like pharmaceuticals used in chemotherapy. Just after it has been manufactured in a nuclear reactor, the activity of a sample of ${}^{60}Co$ is 5000 Ci. When its activity falls below 3500 Ci, it is considered too weak a source to use in treatment. You work in the radiology department of a large hospital. One of these ${}^{60}Co$ sources in your inventory was manufactured on October 6, 2004. It is now April 6, 2007. Is the source still usable? The half-life of ${}^{60}Co$ is 5.271 years.

43.23. A 12.0-g sample of carbon from living matter decays at the rate of 180.0 decays/min due to the radioactive ${}^{14}C$ in it. What will be the decay rate of this sample in (a) 1000 years and (b) 50,000 years?

43.24. Radioactive Tracers. Radioactive isotopes are often introduced into the body through the bloodstream. Their spread through the body can then be monitored by detecting the appearance of radiation in different organs. ${}^{131}I$, a β^- emitter with a half-life of 8.0 d is one such tracer. Suppose a scientist introduces a sample with an activity of 375 Bq and watches it spread to the organs. (a) Assuming that the sample all went to the thyroid gland, what will be the decay rate in that gland 24 d (about $2\frac{1}{2}$ weeks) later? (b) If the decay rate in the thyroid 24 d later is actually measured to be 17.0 Bq, what percentage of the tracer went to that gland? (c) What isotope remains after the I-131 decays?

43.25. Tritium $({}^{3}H)$ undergoes β^- decay with a half-life of 12.3 years. It is also highly toxic to living things. (a) What nucleus is produced in the β^- decay of tritium? (b) Suppose some tritium gas is released into the atmosphere in a nuclear power plant accident. How long will it take for 90.0% of the tritium to become nonradioactive?

43.26. As a health physicist, you are being consulted about a spill in a radiochemistry lab. The isotope spilled was 500μCi of ${}^{131}Ba$, which has a half-life of 12 days. (a) What mass of ${}^{131}Ba$ was spilled? (b) Your recommendation is to clear the lab until the radiation level has fallen 1.00μCi. How long will the lab have to be closed?

43.27. Measurements on a certain isotope tell you that the decay rate decreases from 8318 decays/min to 3091 decays/min in 4.00 days. What is the half-life of this isotope?

43.28. The isotope ${}^{226}Ra$ undergoes α decay with a half-life of 1620 years. What is the activity of 1.00 g of ${}^{226}Ra$? Express your answer in Bq and in Ci.

43.29. The ratio of ${}^{14}C$ to ${}^{12}C$ in living matter is measured to be ${}^{14}C/{}^{12}C = 1.3 \times 10^{-12}$ at the present time. A 12.0-g sample of carbon produces 180 decays/min due to the small amount of ${}^{14}C$ in it. From this information, calculate the half-life of ${}^{14}C$.

43.30. If you are of average mass, about 360 million nuclei in your body undergo radioactive decay each day. Express your activity in curies.

43.31. The radioactive nuclide ${}^{199}Pt$ has a half-life of 30.8 minutes. A sample is prepared that has an initial activity of 7.56×10^{11} Bq. (a) How many ${}^{199}Pt$ nuclei are initially present in the sample? (b) How many are present after 30.8 minutes? What is the activity at this time? (c) Repeat part (b) for a time 92.4 minutes after the sample is first prepared.

43.32. Radiocarbon Dating. A sample from timbers at an archeological site containing 500 g of carbon provides 3070 decays/min. What is the age of the sample?

43.33. The unstable isotope ${}^{40}K$ is used for dating rock samples. Its half-life is 1.28×10^9 y. (a) How many decays occur per second in a sample containing 1.63×10^{-6} g of ${}^{40}K$? (b) What is the activity of the sample in curies?

Section 43.5 Biological Effects of Radiation

43.34. A person exposed to fast neutrons receives a radiation dose of 200 rem on part of his hand, affecting 25 g of tissue. The RBE of these neutrons is 10. (a) How many rad did he receive? (b) How many joules of energy did this person receive? (c) Suppose the person received the same rad dosage, but from beta rays with a RBE of 1.0 instead of neutrons. How many rem would he have received?

43.35. A nuclear chemist receives an accidental radiation dose of 5.0 Gy from slow neutrons $(\text{RBE} = 4.0)$. What does she receive in rad, rem, and J/kg?

43.36. To Scan or Not to Scan? It has become popular for some people to have yearly whole-body scans (CT scans, formerly called CAT scans) using x rays, just to see if they detect anything suspicious. A number of medical people have recently questioned the advisability of such scans, due in part to the radiation they impart. Typically, one such scan gives a dose of 12 mSv, applied to the *whole body*. By contrast, a chest x ray typically administers 0.20 mSv to only 5.0 kg of tissue. How many chest x rays would deliver the same *total* amount of energy to the body of 75-kg person as one whole-body scan?

43.37. Food Irradiation Food is often irradiated with either x rays or electron beams to help prevent spoilage. A low dose of 5–75 kilorads (krad) helps to reduce and kill inactive parasites, a medium dose of 100–400 krad kills microorganisms and pathogens such as salmonella, and a high dose of 2300-5700 krad sterilizes food so that it can be stored without refrigeration, (a) A dose of 175 krad kills spoilage microorganisms in fish. If x rays are used, what would be the dose in Gy, Sv, and rem, and how much energy would a 150-g portion of fish absorb? (See Table 43.3.) (b) Repeat part (a) if electrons of RBE 1.50 are used instead of x rays.

43.38. In an industrial accident a 65-kg person receives a lethal whole-body equivalent dose of 5.4 Sv from x rays. (a) What is the equivalent dose in rem? (b) What is the absorbed dose in rad? (c) What is the total energy absorbed by the person's body? How does this amount of energy compare to the amount of energy required to raise the temperature of 65 kg of water 0.010°C?

43.39. A 50-kg person accidentally ingests 0.35 Ci of tritium. (a) Assume that the tritium spreads uniformly throughout the body and that each decay leads on the average to the absorption of 5.0 keV of energy from the electrons emitted in the decay. The half-life of tritium is 12.3 y, and the RBE of the electrons is 1.0. Calculate the absorbed dose in rad and the equivalent dose in rem during one week. (b) The β^- decay of tritium releases more than 5.0 keV of energy. Why is the average energy absorbed less than the total energy released in the decay?

43.40. A person ingests an amount of a radioactive source with a very long lifetime and activity 0.72 μCi. The radioactive material lodges in the lungs, where all of the 4.0-MeV α particles emitted are absorbed within a 0.50-kg mass of tissue. Calculate the absorbed dose and the equivalent dose for one year.

Section 43.6 Nuclear Reactions, Section 43.7 Nuclear Fission, and Section 43.8 Nuclear Fusion

43.41. Consider the nuclear reaction

$$^{2}_{1}\mathrm{H} + {}^{14}_{7}\mathrm{N} \rightarrow \mathrm{X} + {}^{10}_{5}\mathrm{B}$$

where X is a nuclide. (a) What are Z and A for the nuclide X? (b) Calculate the reaction energy Q (in MeV). (c) If the $^{2}_{1}\mathrm{H}$ nucleus is incident on a stationary $^{14}_{7}\mathrm{N}$ nucleus, what minimum kinetic energy must it have for the reaction to occur?

43.42. Energy from Nuclear Fusion. Calculate the energy released in the fusion reaction

$$^{3}_{2}\mathrm{He} + {}^{2}_{1}\mathrm{H} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{1}_{1}\mathrm{H}$$

43.43. Consider the nuclear reaction

$$^{2}_{1}\mathrm{H} + {}^{9}_{4}\mathrm{Be} \rightarrow \mathrm{X} + {}^{4}_{2}\mathrm{He}$$

where X is a nuclide. (a) What are the values of Z and A for the nuclide X? (b) How much energy is liberated? (c) Estimate the threshold energy for this reaction.

43.44. The United States uses 1.0×10^{19} J of electrical energy per year. If all this energy came from the fission of ^{235}U, which releases 200 MeV per fission event, (a) how many kilograms of ^{235}U would be used per year and (b) how many kilograms of uranium would have to be mined per year to provide that much ^{235}U? (Recall that only 0.70% of naturally occurring uranium is ^{235}U.)

43.45. At the beginning of Section 43.7 the equation of a fission process is given in which ^{235}U is struck by a neutron and undergoes fission to produce ^{144}Ba, ^{89}Kr, and three neutrons. The measured masses of these isotopes are 235.043930 u (^{235}U), 143.922953 u (^{144}Ba), 88.917630 u (^{89}Kr), and 1.0086649 u (neutron). (a) Calculate the energy (in MeV) released by each fission reaction. (b) Calculate the energy released per gram of ^{235}U, in MeV/g.

43.46. Consider the nuclear reaction

$$^{28}_{14}\mathrm{Si} + \gamma \rightarrow {}^{24}_{12}\mathrm{Mg} + \mathrm{X}$$

where X is a nuclide. (a) What are Z and A for the nuclide X? (b) Ignoring the effects of recoil, what minimum energy must the photon have for this reaction to occur? The mass of a $^{28}_{14}$Si atom is 27.976927 u, and the mass of a $^{24}_{12}$Mg atom is 23.985042 u.

43.47. The second reaction in the proton–proton chain (see Fig. 43.17) produces a $^{3}_{2}$He nucleus. A $^{3}_{2}$He nucleus produced in this way can combine with a $^{4}_{2}$He nucleus:

$$^{3}_{2}\mathrm{He} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{7}_{4}\mathrm{Be} + \gamma$$

Calculate the energy liberated in this process. (This is shared between the energy of the photon and the recoil kinetic energy of the beryllium nucleus.) The mass of a $^{7}_{4}$Be atom is 7.016929 u.

43.48. Consider the nuclear reaction

$$^{4}_{2}\mathrm{He} + {}^{7}_{3}\mathrm{Li} \rightarrow \mathrm{X} + {}^{1}_{0}\mathrm{n}$$

where X is a nuclide. (a) What are Z and A for the nuclide X? (b) Is energy absorbed or liberated? How much?

Problems

43.49. Use conservation of mass-energy to show that the energy released in alpha decay is positive whenever the mass of the original neutral atom is greater than the sum of the masses of the final neutral atom and the neutral ^{4}He atom. (*Hint:* Let the parent nucleus have atomic number Z and nucleon number A. First write the reaction in terms of the nuclei and particles involved, and then add Z electron masses to both sides of the reaction and allot them as needed to arrive at neutral atoms.)

43.50. Use conservation of mass-energy to show that the energy released in β^- decay is positive whenever the neutral atomic mass of the original atom is greater than that of the final atom. (See the hint in Problem 43.49.)

43.51. Use conservation of mass-energy to show that the energy released in β^+ decay is positive whenever the neutral atomic mass of the original atom is at least two electron masses greater than that of the final atom. (See the hint in Problem 43.49.)

43.52. Comparison of Energy Released per Gram of Fuel. (a) When gasoline is burned, it releases 1.3×10^{8} J of energy per gallon (3.788 L). Given that the density of gasoline is 737 kg/m^3, express the quantity of energy released in J/g of fuel. (b) During fission, when a neutron is absorbed by a ^{235}U nucleus, about 200 MeV of energy is released for each nucleus that undergoes fission. Express this quantity in J/g of fuel. (c) In the proton-proton chain that takes place in stars like our sun, the overall fusion reaction can be summarized as six protons fusing to form one ^{4}He nucleus with two leftover protons and the liberation of 26.7 MeV of energy. The fuel is the six protons. Express the energy produced here in units of J/g of fuel. Notice the huge difference between the two forms of nuclear energy, on the one hand, and the chemical energy from gasoline, on the other (d) Our sun produces energy at a measured rate of 3.86×10^{26} W. If its mass of 1.99×10^{30} kg were all gasoline, how long could it last before consuming all its fuel? (*Historical note:* Before the discovery of nuclear fusion and the vast amounts of energy it releases, scientists were confused. They knew that the earth was at least many millions of years old, but could not explain how the sun could survive that long if its energy came from chemical burning.)

43.53. The experimentally determined mass of the neutral ^{24}Na atom is 23.990963 u. Calculate the mass from the semiempirical mass formula, Eq. (43.12). What is the percent error of the result compared to the experimental value? What percent error is made if the E_B term is ignored entirely?

43.54. Thorium $^{230}_{90}$Th decays to radium $^{226}_{88}$Ra by α emission. The masses of the neutral atoms are 230.033127 u for $^{230}_{90}$Th and 226.025403 u for $^{226}_{88}$Ra. If the parent thorium nucleus is at rest, what is the kinetic energy of the emitted α particle? (Be sure to account for the recoil of the daughter nucleus.)

43.55. The atomic mass of $^{25}_{12}$Mg is 24.985837 u, and the atomic mass of $^{25}_{13}$Al is 24.990429 u. (a) Which of these nuclei will decay into the other? (b) What type of decay will occur? Explain how you determined this. (c) How much energy (in MeV) is released in the decay?

43.56. The polonium isotope $^{210}_{84}$Po has atomic mass 209.982857 u. Other atomic masses are $^{206}_{82}$Pb, 205.974449 u; $^{209}_{83}$Bi, 208.980383 u; $^{210}_{83}$Bi, 209.984105 u; $^{209}_{84}$Po, 208.982416 u; and $^{210}_{85}$At, 209.987131 u. (a) Show that the alpha decay of $^{210}_{84}$Po is energetically possible, and find the energy of the emitted α particle. (b) Is $^{210}_{84}$Po energetically stable with respect to emission of a proton? Why or why not? (c) Is $^{210}_{84}$Po energetically stable with respect to emission of a neutron? Why or why not? (d) Is $^{210}_{84}$Po energetically stable with respect to β^- decay? Why or why not? e) Is $^{210}_{84}$Po energetically stable with respect to β^+ decay? Why or why not?

43.57. Irradiating Ourselves! The radiocarbon in our bodies is one of the naturally occurring sources of radiation. Let's see how large a dose we receive. ^{14}C decays via β^- emission, and 18% of our body's mass is carbon. (a) Write out the decay scheme of carbon-14 and show the end product. (A neutrino is also produced.) (b) Neglecting the effects of the neutrino, how much kinetic energy (in MeV) is released per decay? The atomic mass of C-14 is 14.003242 u. (See Table 43.2.) (c) How many grams of carbon are there in a 75-kg person? How many decays per second does this carbon produce? (d) Assuming that all the energy released in these decays is absorbed by the body, how many MeV/s and J/s does the C-14 release in this person's body? (e) Consult Table 43.3 and use the largest appropriate RBE for the particles involved. What radiation dose does the person give himself in a year, in Gy, rad, Sv, and rem?

43.58. Pion Radiation Therapy. A neutral pion (π^0) has a mass of 264 times the electron mass and decays with a lifetime of 8.4×10^{-17} s to two photons. Such pions are used in the radiation treatment of some cancers. (a) Find the energy and wavelength of these photons. In which part of the electromagnetic spectrum do they lie? What is the RBE for these photons? (b) if you want to deliver a dose of 200 rem (which is typical) in a single treatment to 25 g of tumor tissue, how many π^0 mesons are needed?

43.59. Gold, $^{198}_{79}$Au, undergoes β^- decay to an excited state of $^{198}_{80}$Hg. If the excited state decays by emission of a γ photon with energy 0.412 MeV, what is the maximum kinetic energy of the electron emitted in the decay? This maximum occurs when the antineutrino has negligible energy. (The recoil energy of the $^{198}_{80}$Hg nucleus can be ignored. The masses of the neutral atoms in their ground states are 197.968225 u for $^{198}_{79}$Au and 197.966752 u for $^{198}_{80}$Hg.)

43.60. Calculate the mass defect for the β^+ decay of $^{11}_{6}$C. Is this decay energetically possible? Why or why not? The atomic mass of $^{11}_{6}$C is 11.011434 u.

43.61. Calculate the mass defect for the β^+ decay of $^{13}_{7}$N. Is this decay energetically possible? Why or why not? The atomic mass of $^{13}_{7}$N is 13.005739 u.

43.62. The results of activity measurements on a radioactive sample are given in the table. (a) Find the half-life. (b) How many radioactive nuclei were present in the sample at $t = 0$? (c) How many were present after 7.0 h?

Time (h)	Decays/s
0	20,000
0.5	14,800
1.0	11,000
1.5	8,130
2.0	6,020
2.5	4,460
3.0	3,300
4.0	1,810
5.0	1,000
6.0	550
7.0	300

43.63. If $A(t)$ is the activity of a sample at some time t and A_0 is the activity at $t = 0$, show that $A(t) = A_0 e^{-\lambda t}$.

43.64. Show that Eq. (43.17) may be written as $N(t) = N_0\left(\frac{1}{2}\right)^n$, where $n = t/T_{1/2}$ is the number of half-lives that have elapsed since $t = 0$. (This expression is valid even if n is not a whole number.)

43.65. We Are Stardust. In 1952 spectral lines of the element technetium-99 (^{99}Tc) were discovered in a red giant star. Red giants are very old stars, often around 10 billion years old, and near the end of their lives. Technetium has *no* stable isotopes, and the half-life of ^{99}Tc is 200,000 years. (a) For how many half-lives has the ^{99}Tc been in the red-giant star if its age is 10 billion years? (b) What fraction of the original ^{99}Tc would be left at the end of that time? This discovery was extremely important because it provided convincing evidence for the theory (now essentially known to be true) that most of the atoms heavier than hydrogen and helium were made inside of stars by thermonuclear fusion and other nuclear processes. If the ^{99}Tc had been part of the star since it was born, the amount remaining after 10 billion years would have been so minute that it would not have been detectable. This knowledge is what led the late astronomer Carl Sagan to proclaim that "we are stardust."

43.66. Measuring Very Long Half-Lives. Some radioisotopes such as samarium (^{149}Sm) and gadolinium (^{152}Gd) have half-lives that are much longer than the age of the universe, so we can't measure their half-lives by watching their decay rate decrease. Luckily, there is another way of calculating the half-life, using Eq. (43.16). Suppose a 12.0-g sample of ^{149}Sm is observed to decay at a rate of 2.65 Bq. Calculate the half-life of the sample in years. (*Hint:* How many nuclei are there in the 12.0-g sample?)

43.67. Measurements indicate that 27.83% of all rubidium atoms currently on the earth are the radioactive ^{87}Rb isotope. The rest are the stable ^{85}Rb isotope. The half-life of ^{87}Rb is 4.75×10^{10} y. Assuming that no rubidium atoms have been formed since, what percentage of rubidium atoms were ^{87}Rb when our solar system was formed 4.6×10^{9} y ago?

43.68. A 70.0-kg person experiences a whole-body exposure to α radiation with energy 4.77 MeV. A total of 6.25×10^{12} α particles are absorbed. (a) What is the absorbed dose in rad? (b) What is the equivalent dose in rem? (c) If the source is 0.0320 g of ^{226}Ra (half-life 1600 y) somewhere in the body, what is the activity of this source? (d) If all the alpha particles produced are absorbed, what time is required for this dose to be delivered?

43.69. A ^{60}Co source with activity 2.6×10^{-4} Ci is embedded in a tumor that has mass 0.500 kg. The source emits γ photons with average energy 1.25 MeV. Half the photons are absorbed in the tumor, and half escape. (a) What energy is delivered to the tumor per second? (b) What absorbed dose (in rad) is delivered per second? (c) What equivalent dose (in rem) is delivered per second if the RBE for these γ rays is 0.70? (d) What exposure time is required for an equivalent dose of 200 rem?

43.70. The nucleus $^{15}_{8}$O has a half-life of 122.2 s; $^{19}_{8}$O has a half-life of 26.9 s. If at some time a sample contains equal amounts of $^{15}_{8}$O and $^{19}_{8}$O, what is the ratio of $^{15}_{8}$O to $^{19}_{8}$O (a) after 4.0 minutes and (b) after 15.0 minutes?

43.71. A bone fragment found in a cave believed to have been inhabited by early humans contains 0.21 times as much ^{14}C as an equal amount of carbon in the atmosphere when the organism containing the bone died. (See Example 43.9 in Section 43.4.) Find the approximate age of the fragment.

43.72. An Oceanographic Tracer. Nuclear weapons tests in the 1950s and 1960s released significant amounts of radioactive tritium ($^{3}_{1}$H, half-life 12.3 years) into the atmosphere. The tritium atoms were quickly bound into water molecules and rained out of the air, most of them ending up in the ocean. For any of this tritium-tagged water that sinks below the surface, the amount of time during which it has been isolated from the surface can be calculated by measuring the ratio of the decay product, $^{3}_{2}$He, to the remaining tritium in the water. For example, if the ratio of $^{3}_{1}$He to $^{3}_{1}$H in a sample of water is 1:1, the water has been below the surface for one half-life, or approximately 12 years. This method has provided oceanographers with a convenient way to trace the movements of subsurface currents in parts of the ocean. Suppose that in a particular sample of water, the ratio of $^{3}_{2}$He to $^{3}_{1}$H is 4.3 to 1.0. How many years ago did this water sink below the surface?

43.73. Consider the fusion reaction

$$^{2}_{1}\text{H} + ^{2}_{1}\text{H} \rightarrow ^{3}_{2}\text{He} + ^{1}_{0}\text{n}$$

(a) Estimate the barrier energy by calculating the repulsive electrostatic potential energy of the two $^{2}_{1}$H nuclei when they touch. (b) Compute the energy liberated in this reaction in MeV and in joules. (c) Compute the energy liberated *per mole* of deuterium, remembering that the gas is diatomic, and compare with the heat of combustion of hydrogen, about 2.9×10^{5} J/mol.

43.74. In the 1986 disaster at the Chernobyl reactor in the Soviet Union (now Ukraine), about $\frac{1}{8}$ of the ^{137}Cs present in the reactor was released. The isotope ^{137}Cs has a half-life for β decay of 30.07 y and decays with the emission of a total of 1.17 MeV of energy per decay. Of this, 0.51 MeV goes to the emitted electron and the remaining 0.66 MeV to a γ ray. The radioactive ^{137}Cs is absorbed by plants, which are eaten by livestock and humans. How many ^{137}Cs atoms would need to be present in each kilogram of body tissue if an equivalent dose for one week is 3.5 Sv? Assume that all of the energy from the decay is deposited in that 1.0 kg of tissue and that the RBE of the electrons is 1.5.

43.75. (a) Prove that when a particle with mass m and kinetic energy K collides with a stationary particle with mass M, the total kinetic energy K_{cm} in the center-of-mass coordinate system (the energy available to cause reactions) is

$$K_{cm} = \frac{M}{M + m}K$$

Assume that the kinetic energies of the particles and nuclei are much lower than their rest energies. (b) If K_{th} is the minimum, or threshold, kinetic energy to cause an endoergic reaction to occur in the situation of part (a), show that

$$K_{th} = -\frac{M + m}{M}Q$$

43.76. A $^{186}_{76}$Os nucleus at rest decays by the emission of a 2.76-MeV α particle. Calculate the atomic mass of the daughter nuclide produced by this decay, assuming that it is produced in its ground state. The atomic mass of $^{186}_{76}$Os is 185.953838 u.

43.77. Calculate the energy released in the fission reaction

$$^{235}_{92}\text{U} + ^{1}_{0}\text{n} \rightarrow ^{140}_{54}\text{Xe} + ^{94}_{38}\text{Sr} + 2^{1}_{0}\text{n}$$

You can ignore the initial kinetic energy of the absorbed neutron. The atomic masses are $^{235}_{92}$U, 235.043923 u; $^{140}_{54}$Xe, 139.921636 u; and $^{94}_{38}$Sr, 93.915360 u.

Challenge Problems

43.78. The results of activity measurements on a mixed sample of radioactive elements are given in the table. (a) How many different nuclides are present in the mixture? (b) What are their half-lives? (c) How many nuclei of each type are initially present in the sample? (d) How many of each type are present at $t = 5.0$ h?

Time (h)	Decays/s
0	7500
0.5	4120
1.0	2570
1.5	1790
2.0	1350
2.5	1070
3.0	872
4.0	596
5.0	414
6.0	288
7.0	201
8.0	140
9.0	98
10.0	68
12.0	33

43.79. In an experiment, the isotope ^{128}I is created by the irradiation of ^{127}I with a beam of neutrons that creates 1.5×10^{6} ^{128}I nuclei per second. Initially no ^{128}I nuclei are present. The half-life of ^{128}I is 25 minutes. (a) Graph the number of ^{128}I nuclei present as a function of time. (b) What is the activity of the sample 1, 10, 25, 50, 75, and 180 minutes after irradiation begins? (c) What is the maximum number of ^{128}I atoms that can be created in the sample after it is irradiated for a long time? (This steady-state situation is called *saturation.*) (d) What is the maximum activity that can be produced?

43.80. Industrial Radioactivity. Radioisotopes are used in a variety of manufacturing and testing techniques. Wear measurements can be made using the following method. An automobile engine is produced using piston rings with a total mass of 100 g, which includes 9.4 μCi of ^{59}Fe whose half-life is 45 days. The engine is test-run for 1000 hours, after which the oil is drained and its activity is measured. If the activity of the engine oil is 84 decays/s, how much mass was worn from the piston rings per hour of operation?