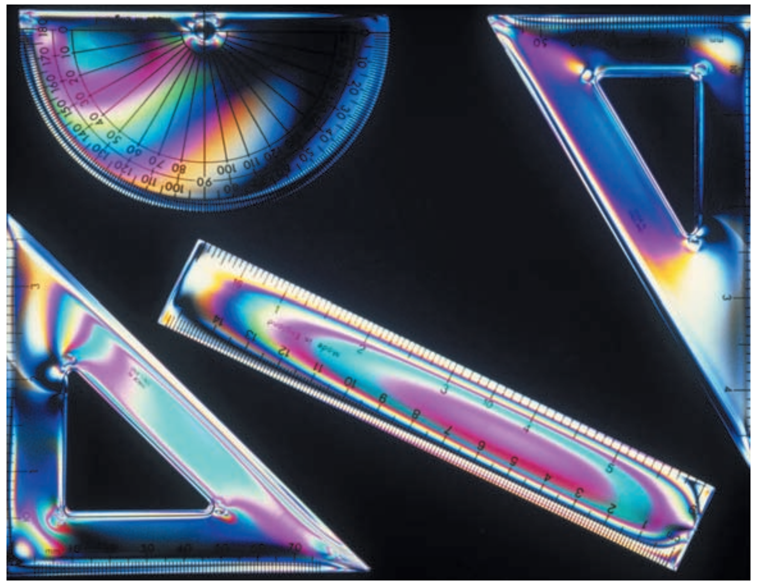
33 THE NATURE AND PROPAGATION OF LIGHT



? These drafting tools are made of clear plastic, but a rainbow of colors appears when they are placed between two special filters called polarizers. How does this cause the colors?

LEARNING GOALS

By studying this chapter, you will learn:

- What light rays are, and how they are related to wave fronts.
- The laws that govern the reflection and refraction of light.
- The circumstances under which light is totally reflected at an interface.
- How to make polarized light out of ordinary light.
- How Huygens's principle helps us analyze reflection and refraction.

Blue lakes, ochre deserts, green forests, and multicolored rainbows can be enjoyed by anyone who has eyes with which to see them. But by studying the branch of physics called **optics,** which deals with the behavior of light and other electromagnetic waves, we can reach a deeper appreciation of the visible world. A knowledge of the properties of light allows us to understand the blue color of the sky and the design of optical devices such as telescopes, microscopes, cameras, eyeglasses, and the human eye. The same basic principles of optics also lie at the heart of modern developments such as the laser, optical fibers, holograms, optical computers, and new techniques in medical imaging.

The importance of optics to physics, and to science and engineering in general, is so great that we will devote the next four chapters to its study. In this chapter we begin with a study of the laws of reflection and refraction and the concepts of dispersion, polarization, and scattering of light. Along the way we compare the various possible descriptions of light in terms of particles, rays, or waves, and we introduce Huygens's principle, an important link that connects the ray and wave viewpoints. In Chapter 34 we'll use the ray description of light to understand how mirrors and lenses work, and we'll see how mirrors and lenses are used in optical instruments such as cameras, microscopes, and telescopes. We'll explore the wave characteristics of light further in Chapters 35 and 36.

33.1 The Nature of Light

Until the time of Isaac Newton (1642–1727), most scientists thought that light consisted of streams of particles (called *corpuscles*) emitted by light sources. Galileo and others tried (unsuccessfully) to measure the speed of light. Around

1665, evidence of *wave* properties of light began to be discovered. By the early 19th century, evidence that light is a wave had grown very persuasive.

In 1873, James Clerk Maxwell predicted the existence of electromagnetic waves and calculated their speed of propagation, as we learned in Chapter 32. This development, along with the experimental work of Heinrich Hertz starting in 1887, showed conclusively that light is indeed an electromagnetic wave.

The Two Personalities of Light

The wave picture of light is not the whole story, however. Several effects associated with emission and absorption of light reveal a particle aspect, in that the energy carried by light waves is packaged in discrete bundles called *photons* or *quanta.* These apparently contradictory wave and particle properties have been reconciled since 1930 with the development of quantum electrodynamics, a comprehensive theory that includes *both* wave and particle properties. The *propagation* of light is best described by a wave model, but understanding emission and absorption requires a particle approach.

33.1 An electric heating element emits primarily infrared radiation. But if its temperature is high enough, it also emits a discernible amount of visible light.

The fundamental sources of all electromagnetic radiation are electric charges in accelerated motion. All bodies emit electromagnetic radiation as a result of thermal motion of their molecules; this radiation, called *thermal radiation,* is a mixture of different wavelengths. At sufficiently high temperatures, all matter emits enough visible light to be self-luminous; a very hot body appears "red-hot" (Fig. 33.1) or "white-hot." Thus hot matter in any form is a light source. Familiar examples are a candle flame, hot coals in a campfire, the coils in an electric room heater, and an incandescent lamp filament (which usually operates at a temperature of about 3000°C).

Light is also produced during electrical discharges through ionized gases. The bluish light of mercury-arc lamps, the orange-yellow of sodium-vapor lamps, and the various colors of "neon" signs are familiar. A variation of the mercury-arc lamp is the *fluorescent* lamp (see Fig. 30.7). This light source uses a material called a *phosphor* to convert the ultraviolet radiation from a mercury arc into visible light. This conversion makes fluorescent lamps more efficient than incandescent lamps in transforming electrical energy into light.

33.2 Ophthalmic surgeons use lasers for repairing detached retinas and for cauterizing blood vessels in retinopathy. Pulses of blue-green light from an argon laser are ideal for this purpose, since they pass harmlessly through the transparent part of the eye but are absorbed by red pigments in the retina.

A light source that has attained prominence in the last forty years is the *laser.* In most light sources, light is emitted independently by different atoms within the source; in a laser, by contrast, atoms are induced to emit light in a cooperative, coherent fashion. The result is a very narrow beam of radiation that can be enormously intense and that is much more nearly *monochromatic,* or single-frequency, than light from any other source. Lasers are used by physicians for microsurgery, in CD players and computers to scan the information encoded on a compact disc or CD-ROM, in industry to cut through steel and to fuse high-melting-point materials, and in many other applications (Fig. 33.2).

No matter what its source, electromagnetic radiation travels in vacuum at the same speed. As we saw in Sections 1.3 and 32.1, the speed of light in vacuum is defined to be

$$c = 2.99792458 \times 10^8 \text{ m/s}$$

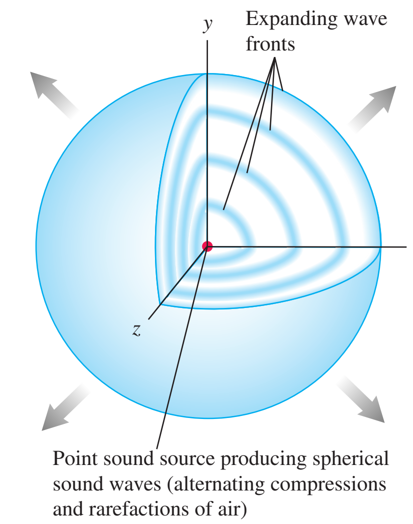
or 3.00×10^8 m/s to three significant figures. The duration of one second is defined by the cesium clock (see Section 1.3), so one meter is defined to be the distance that light travels in 1/299,792,458 s.

Waves, Wave Fronts, and Rays

We often use the concept of a **wave front** to describe wave propagation. We introduced this concept in Section 32.2 to describe the leading edge of a wave. More generally, we define a wave front as *the locus of all adjacent points at which the phase of vibration of a physical quantity associated with the wave is the same.* That is, at any instant, all points on a wave front are at the same part of the cycle of their variation.

When we drop a pebble into a calm pool, the expanding circles formed by the wave crests, as well as the circles formed by the wave troughs between them, are wave fronts. Similarly, when sound waves spread out in still air from a pointlike source, or when electromagnetic radiation spreads out from a pointlike emitter, any spherical surface that is concentric with the source is a wave front, as shown in Fig. 33.3. In diagrams of wave motion we usually draw only parts of a few wave fronts, often choosing consecutive wave fronts that have the same phase and thus are one wavelength apart, such as crests of water waves. Similarly, a diagram for sound waves might show only the "pressure crests," the surfaces over which the pressure is maximum, and a diagram for electromagnetic waves might show only the "crests" on which the electric or magnetic field is maximum.

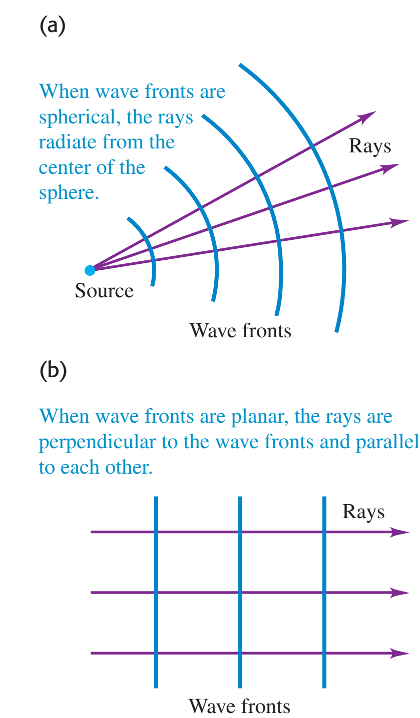
33.3 Spherical wave fronts of sound spread out uniformly in all directions from a point source in a motionless medium, such as still air, that has the same properties in all regions and in all directions. Electromagnetic waves in vacuum also spread out as shown here.



We will often use diagrams that show the shapes of the wave fronts or their cross sections in some reference plane. For example, when electromagnetic waves are radiated by a small light source, we can represent the wave fronts as spherical surfaces concentric with the source or, as in Fig. 33.4a, by the circular intersections of these surfaces with the plane of the diagram. Far away from the source, where the radii of the spheres have become very large, a section of a spherical surface can be considered as a plane, and we have a *plane* wave like those discussed in Sections 32.2 and 32.3 (Fig. 33.4b).

To describe the directions in which light propagates, it's often convenient to represent a light wave by **rays** rather than by wave fronts. Rays were used to describe light long before its wave nature was firmly established. In a particle theory of light, rays are the paths of the particles. From the wave viewpoint *a ray is an imaginary line along the direction of travel of the wave.* In Fig. 33.4a the rays are the radii of the spherical wave fronts, and in Fig. 33.4b they are straight lines perpendicular to the wave fronts. When waves travel in a homogeneous isotropic material (a material with the same properties in all regions and in all directions), the rays are always straight lines normal to the wave fronts. At a boundary surface between two materials, such as the surface of a glass plate in air, the wave speed and the direction of a ray may change, but the ray segments in the air and in the glass are straight lines.

33.4 Wave fronts (blue) and rays (purple).



The next several chapters will give you many opportunities to see the interplay of the ray, wave, and particle descriptions of light. The branch of optics for which the ray description is adequate is called **geometric optics;** the branch dealing specifically with wave behavior is called **physical optics.** This chapter and the following one are concerned mostly with geometric optics. In Chapters 35 and 36 we will study wave phenomena and physical optics.

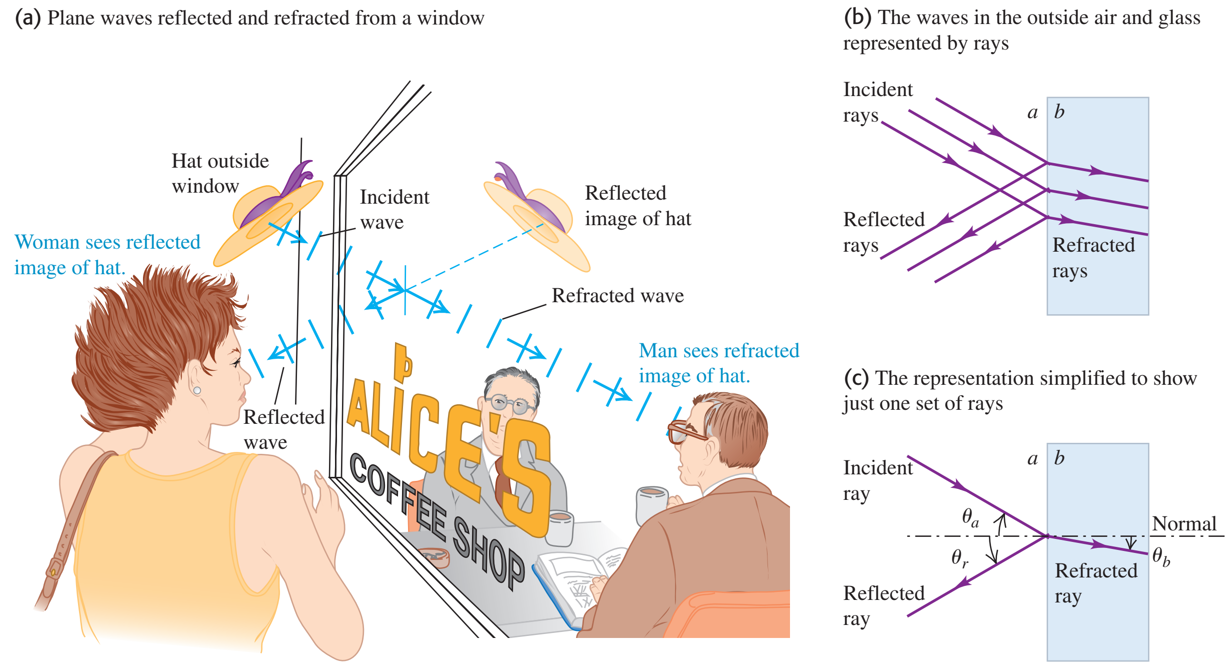
Test Your Understanding of Section 33.1 Some crystals are *not* isotropic: Light travels through the crystal at a higher speed in some directions than in others. In a crystal in which light travels at the same speed in the x- and z-directions but at a faster speed in the y-direction, what would be the shape of the wave fronts produced by a light source at the origin? (i) spherical, like those shown in Fig. 33.3; (ii) ellipsoidal, flattened along the y-axis; (iii) ellipsoidal, stretched out along the y-axis.

33.2 Reflection and Refraction

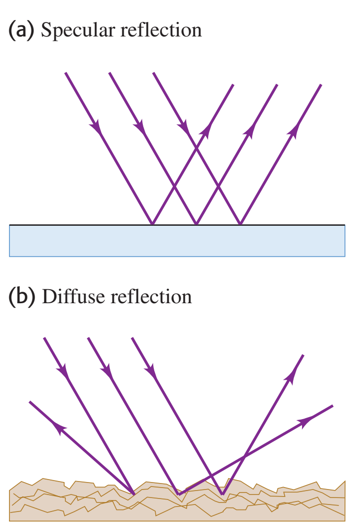
In this section we'll use the *ray* model of light to explore two of the most important aspects of light propagation: **reflection** and **refraction.** When a light wave strikes a smooth interface separating two transparent materials (such as air and glass or water and glass), the wave is in general partly *reflected* and partly *refracted* (transmitted) into the second material, as shown in Fig. 33.5a. For example, when you look into a restaurant window from the street, you see a reflection of the street scene, but a person inside the restaurant can look out through the window at the same scene as light reaches him by refraction.

15.1 Reflection and Refraction
15.3 Refraction Applications

33.5 (a) A plane wave is in part reflected and in part refracted at the boundary between two media (in this case, air and glass). The light that reaches the inside of the coffee shop is refracted twice, once entering the glass and once exiting the glass. (b), (c) How light behaves at the interface between the air outside the coffee shop (material a) and the glass (material b). For the case shown here, material b has a larger index of refraction that material a $(n_b > n_a)$ and the angle θ_b is smaller than θ_a.



33.6 Two types of reflection.



The segments of plane waves shown in Fig. 33.5a can be represented by bundles of rays forming *beams* of light (Fig. 33.5b). For simplicity we often draw only one ray in each beam (Fig. 33.5c). Representing these waves in terms of rays is the basis of geometric optics. We begin our study with the behavior of an individual ray.

We describe the directions of the incident, reflected, and refracted (transmitted) rays at a smooth interface between two optical materials in terms of the angles they make with the *normal* (perpendicular) to the surface at the point of incidence, as shown in Fig. 33.5c. If the interface is rough, both the transmitted light and the reflected light are scattered in various directions, and there is no single angle of transmission or reflection. Reflection at a definite angle from a very smooth surface is called **specular reflection** (from the Latin word for "mirror"); scattered reflection from a rough surface is called **diffuse reflection.** This distinction is shown in Fig. 33.6. Both kinds of reflection can occur with either transparent materials or *opaque* materials that do not transmit light. The vast majority of objects in your environment (including clothing, plants, other people, and this book) are visible to you because they reflect light in a diffuse manner from their surfaces. Our primary concern, however, will be with specular reflection from a very smooth surface such as highly polished glass, plastic, or metal. Unless stated otherwise, when referring to "reflection" we will always mean *specular* reflection.

The **index of refraction** of an optical material (also called the **refractive index**), denoted by n, plays a central role in geometric optics. It is the ratio of the speed of light c in vacuum to the speed v in the material:

$$n = \frac{c}{v} \qquad \text{(index of refraction)} \qquad (33.1)$$

Light always travels *more slowly* in a material than in vacuum, so the value of n in anything other than vacuum is always greater than unity. For vacuum, $n = 1$.

Since n is a ratio of two speeds, it is a pure number without units. (The relationship of the value of n to the electric and magnetic properties of a material is described in Section 32.3.)

CAUTION **Wave speed and index of refraction** Keep in mind that the wave speed v is *inversely* proportional to the index of refraction n. The greater the index of refraction in a material, the *slower* the wave speed in that material. Failure to remember this point can lead to serious confusion!

The Laws of Reflection and Refraction

Experimental studies of the directions of the incident, reflected, and refracted rays at a smooth interface between two optical materials lead to the following conclusions (Fig. 33.7):

1. **The incident, reflected, and refracted rays and the normal to the surface all lie in the same plane.** The plane of the three rays is perpendicular to the plane of the boundary surface between the two materials. We always draw ray diagrams so that the incident, reflected, and refracted rays are in the plane of the diagram.
2. **The angle of reflection θ_r is equal to the angle of incidence θ_a for all wavelengths and for any pair of materials.** That is, in Fig. 33.5c,

$$\theta_r = \theta_a \qquad \text{(law of reflection)} \qquad (33.2)$$

This relationship, together with the observation that the incident and reflected rays and the normal all lie in the same plane, is called the **law of reflection.**

3. For monochromatic light and for a given pair of materials, a and b, on opposite sides of the interface, **the ratio of the sines of the angles θ_a and θ_b, where both angles are measured from the normal to the surface, is equal to the inverse ratio of the two indexes of refraction:**

$$\frac{\sin\theta_a}{\sin\theta_b} = \frac{n_b}{n_a} \qquad (33.3)$$

or

$$n_a \sin\theta_a = n_b \sin\theta_b \qquad \text{(law of refraction)} \qquad (33.4)$$

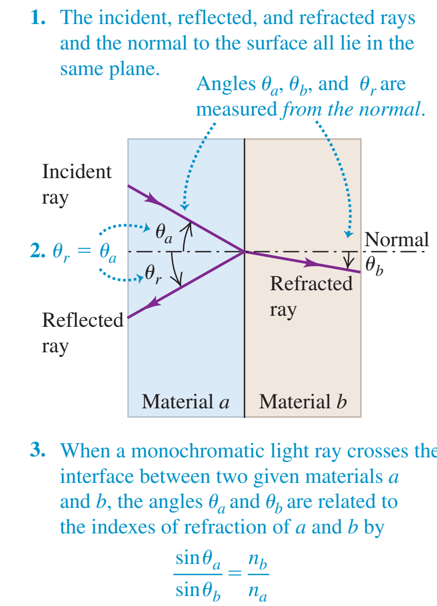
This experimental result, together with the observation that the incident and refracted rays and the normal all lie in the same plane, is called the **law of refraction** or **Snell's law,** after the Dutch scientist Willebrord Snell (1591–1626). There is some doubt that Snell actually discovered it. The discovery that $n = c/v$ came much later.

While these results were first observed experimentally, they can be derived theoretically from a wave description of light. We do this in Section 33.7.

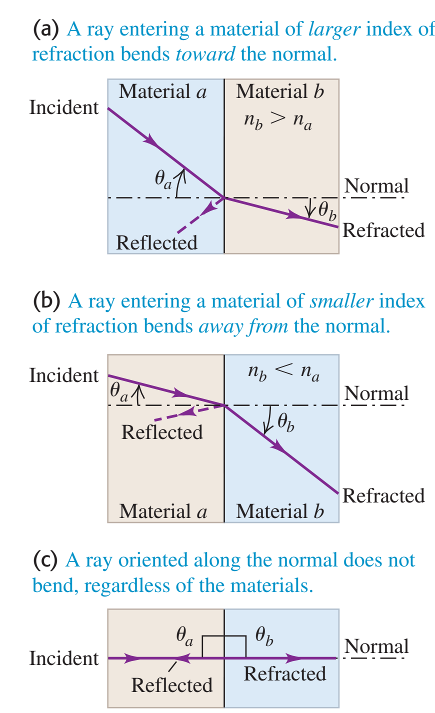
Equations (33.3) and (33.4) show that when a ray passes from one material (a) into another material (b) having a larger index of refraction $(n_b > n_a)$ and hence a slower wave speed, the angle θ_b with the normal is *smaller* in the second material than the angle θ_a in the first; hence the ray is bent *toward* the normal (Fig. 33.8a). When the second material has a *smaller* index of refraction than the first material $(n_b < n_a)$ and hence a faster wave speed, the ray is bent *away from* the normal (Fig. 33.8b).

No matter what the materials on either side of the interface, in the case of *normal* incidence the transmitted ray is not bent at all (Fig. 33.8c). In this case $\theta_a = 0$ and $\sin\theta_a = 0$, so from Eq. (33.4) θ_b is also equal to zero, so the transmitted ray is

33.7 The laws of reflection and refraction.



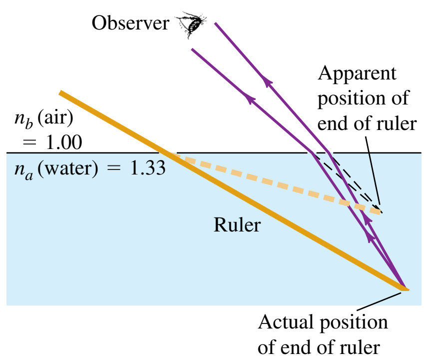
33.8 Refraction and reflection in three cases. (a) Material b has a larger index of refraction than material a. (b) Material b has a smaller index of refraction than material a. (c) The incident light ray is normal to the interface between the materials.



33.9 (a) This ruler is actually straight, but it appears to bend at the surface of the water. (b) Light rays from any submerged object bend away from the normal when they emerge into the air. As seen by an observer above the surface of the water, the object appears to be much closer to the surface than it actually is.

(a) A straight ruler half-immersed in water

(b) Why the ruler appears bent



also normal to the interface. Equation (33.2) shows that θ_r, too, is equal to zero, so the reflected ray travels back along the same path as the incident ray.

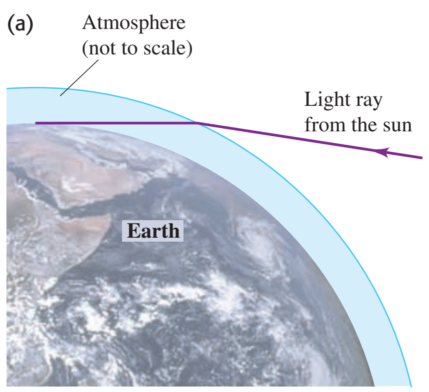
The law of refraction explains why a partially submerged ruler or drinking straw appears bent; light rays coming from below the surface change in direction at the air–water interface, so the rays appear to be coming from a position above their actual point of origin (Fig. 33.9). A similar effect explains the appearance of the setting sun (Fig. 33.10).

An important special case is refraction that occurs at an interface between vacuum, for which the index of refraction is unity by definition, and a material. When a ray passes from vacuum into a material (b), so that $n_a = 1$ and $n_b > 1$, the ray is always bent *toward* the normal. When a ray passes from a material into vacuum, so that $n_a > 1$ and $n_b = 1$, the ray is always bent *away from* the normal.

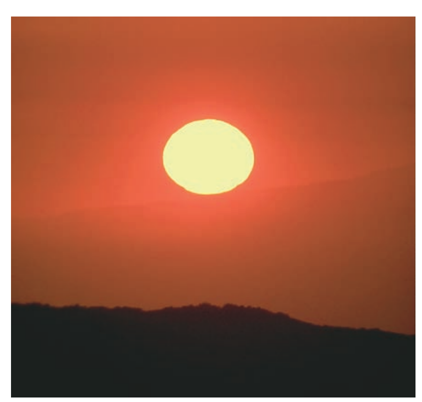
The laws of reflection and refraction apply regardless of which side of the interface the incident ray comes from. If a ray of light approaches the interface in Fig. 33.8a or 33.8b from the right rather than from the left, there are again reflected and refracted rays; these two rays, the incident ray, and the normal to the surface again lie in the same plane. Furthermore, the path of a refracted ray is *reversible;* it follows the same path when going from b to a as when going from a to b. [You can verify this using Eq. (33.4).] Since reflected and incident rays make the same angle with the normal, the path of a reflected ray is also reversible. That's why when you see someone's eyes in a mirror, they can also see you.

33.10 (a) The index of refraction of air is slightly greater than 1, so light rays from the setting sun bend downward when they enter our atmosphere. (The effect is exaggerated in this figure.) (b) Stronger refraction occurs for light coming from the lower limb of the sun (the part that appears closest to the horizon), which passes through denser air in the lower atmosphere. As a result, the setting sun appears flattened vertically. (See Problem 33.55.)

(a)



(b)



The *intensities* of the reflected and refracted rays depend on the angle of incidence, the two indexes of refraction, and the polarization (that is, the direction of the electric-field vector) of the incident ray. The fraction reflected is smallest at normal incidence $(\theta_a = 0°)$, where it is about 4% for an air–glass interface. This fraction increases with increasing angle of incidence to 100% at grazing incidence, when $\theta_a = 90°$.

It's possible to use Maxwell's equations to predict the amplitude, intensity, phase, and polarization states of the reflected and refracted waves. Such an analysis is beyond our scope, however.

The index of refraction depends not only on the substance but also on the wavelength of the light. The dependence on wavelength is called *dispersion;* we will consider it in Section 33.4. Indexes of refraction for several solids and liquids are given in Table 33.1 for a particular wavelength of yellow light.

The index of refraction of air at standard temperature and pressure is about 1.0003, and we will usually take it to be exactly unity. The index of refraction of a gas increases as its density increases. Most glasses used in optical instruments have indexes of refraction between about 1.5 and 2.0. A few substances have larger indexes; one example is diamond, with 2.417.

Table 33.1 Index of Refraction for Yellow Sodium Light $\lambda_0 = 589$ nm

Substance	Index of Refraction, n
Solids	
Ice (H_2O)	1.309
Fluorite (CaF_2)	1.434
Polystyrene	1.49
Rock salt (NaCl)	1.544
Quartz (SiO_2)	1.544
Zircon $(ZrO_2 \cdot SiO_2)$	1.923
Diamond (C)	2.417
Fabulite $(SrTiO_3)$	2.409
Rutile (TiO_2)	2.62
Glasses (typical values)	
Crown	1.52
Light flint	1.58
Medium flint	1.62
Dense flint	1.66
Lanthanum flint	1.80
Liquids at 20°C	
Methanol (CH_3OH)	1.329
Water (H_2O)	1.333
Ethanol (C_2H_5OH)	1.36
Carbon tetrachloride (CCl_4)	1.460
Turpentine	1.472
Glycerine	1.473
Benzene	1.501
Carbon disulfide (CS_2)	1.628

Index of Refraction and the Wave Aspects of Light

We have discussed how the direction of a light ray changes when it passes from one material to another material with a different index of refraction. It's also important to see what happens to the *wave* characteristics of the light when this happens.

First, the frequency f of the wave does not change when passing from one material to another. That is, the number of wave cycles arriving per unit time must equal the number leaving per unit time; this is a statement that the boundary surface cannot create or destroy waves.

Second, the wavelength λ of the wave *is* different in general in different materials. This is because in any material, $v = \lambda f$; since f is the same in any material as in vacuum and v is always less than the wave speed c in vacuum, λ is also correspondingly reduced. Thus the wavelength λ of light in a material is *less than* the wavelength λ_0 of the same light in vacuum. From the above discussion, $f = c/\lambda_0 = v/\lambda$. Combining this with Eq. (33.1), $n = c/v$, we find

$$\lambda = \frac{\lambda_0}{n} \quad \text{(wavelength of light in a material)} \qquad (33.5)$$

When a wave passes from one material into a second material with larger index of refraction, so that $n_b > n_a$, the wave speed decreases. The wavelength $\lambda_b = \lambda_0/n_b$ in the second material is then shorter than the wavelength $\lambda_a = \lambda_0/n_a$ in the first material. If instead the second material has a smaller index of refraction than the first material, so that $n_b < n_a$, then the wave speed increases. Then the wavelength λ_b in the second material is longer than the wavelength λ_a in the first material. This makes intuitive sense; the waves get "squeezed" (the wavelength gets shorter) if the wave speed decreases and get "stretched" (the wavelength gets longer) if the wave speed increases.

Problem-Solving Strategy 33.1 Reflection and Refraction

IDENTIFY *the relevant concepts:* You need to use the ideas of this section, called *geometric optics,* whenever light encounters a boundary between two different materials. In general, part of the light is reflected back into the first material and part is refracted into the second material. These ideas apply to electromagnetic radiation of all frequencies and wavelengths, not just visible light.

SET UP *the problem* using the following steps:
1. In geometric optics problems involving rays and angles, *always* start by drawing a large, neat diagram. Label all known angles and indexes of refraction.
2. Determine the target variables.

EXECUTE *the solution* as follows:
1. Apply the laws of reflection, Eq. (33.2), and refraction, Eq. (33.4). Remember to always measure the angles of incidence, reflection, and refraction from the *normal* to the surface where the reflection and refraction occur, *never* from the surface itself.
2. You will often have to use some simple geometry or trigonometry in working out angular relationships. The sum of the interior angles in a triangle is 180°, an angle and its complement differ by 180°, and so on. Ask yourself, "What information am I given?", "What do I need to know in order to find this angle?", or "What other angles or other quantities can I compute using the information given in the problem?"
3. Remember that the frequency of the light does not change when it moves from one material to another, but the wavelength changes in accordance with Eq. (33.5).

EVALUATE *your answer:* In problems that involve refraction, check that the direction of refraction makes sense. If the second material has a higher index of refraction than the first material, the refracted ray bends toward the normal and the refracted angle is smaller than the incident angle. If the first material has the higher index of refraction, the refracted ray bends away from the normal and the refracted angle is larger than the incident angle. Do your results agree with these rules?

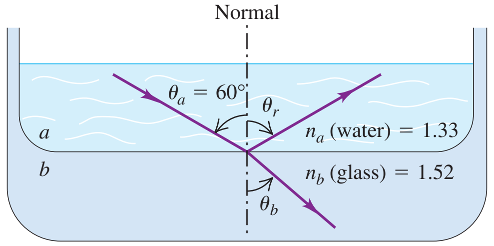
Example 33.1 Reflection and refraction

In Fig. 33.11, material a is water and material b is a glass with index of refraction 1.52. If the incident ray makes an angle of 60° with the normal, find the directions of the reflected and refracted rays.

SOLUTION

IDENTIFY: This is a problem in geometric optics. We are given the incident angle and the index of refraction of each material, and we need to find the reflected and refracted angles.

33.11 Reflection and refraction of light passing from water to glass.



SET UP: Figure 33.11 shows the rays and angles for this situation. The target variables are the reflected angle θ_r and the refracted angle θ_b. Since n_b is greater than n_a, the refracted angle must be smaller than the incident angle θ_a; this is shown in the figure.

EXECUTE: According to Eq. (33.2), the angle the reflected ray makes with the normal is the same as that of the incident ray, so $\theta_r = \theta_a = 60.0°$.

To find the direction of the refracted ray, we use Snell's law, Eq.(33.4), with $n_a = 1.33$, $n_b = 1.52$, and $\theta_a = 60.0°$. We find

$$n_a \sin\theta_a = n_b \sin\theta_b$$

$$\sin\theta_b = \frac{n_a}{n_b}\sin\theta_a = \frac{1.33}{1.52}\sin 60.0° = 0.758$$

$$\theta_b = 49.3°$$

EVALUATE: The second material has a larger refractive index than the first, just like the situation shown in Fig. 33.8a. Hence, the refracted ray is bent toward the normal as the wave slows down upon entering the second material, and $\theta_b < \theta_a$.

Example 33.2 Index of refraction in the eye

The wavelength of the red light from a helium-neon laser is 633 nm in air but 474 nm in the aqueous humor inside your eyeball. Calculate the index of refraction of the aqueous humor and the speed and frequency of the light in this substance.

SOLUTION

IDENTIFY: The key ideas here are the relationship between index of refraction n and wave speed v and the relationship between index of refraction and wavelength λ.

SET UP: We use the definition of index of refraction given by Eq. (33.1), $n = c/v$, as well as Eq. (33.5), $\lambda = \lambda_0/n$. It will also be helpful to use the relationship $v = \lambda f$ among wave speed, wavelength, and frequency.

EXECUTE: The index of refraction of air is very close to unity, so we assume that the wavelengths in air and vacuum are the same. Then the wavelength λ in the material is given by Eq. (33.5) with $\lambda_0 = 633$ nm:

$$\lambda = \frac{\lambda_0}{n} \qquad n = \frac{\lambda_0}{\lambda} = \frac{633\text{ nm}}{474\text{ nm}} = 1.34$$

This is about the same index of refraction as for water. Then $n = c/v$ gives

$$v = \frac{c}{n} = \frac{3.00 \times 10^8\text{ m/s}}{1.34} = 2.25 \times 10^8\text{ m/s}$$

Finally, from $v = \lambda f$,

$$f = \frac{v}{\lambda} = \frac{2.25 \times 10^8\text{ m/s}}{474 \times 10^{-9}\text{ m}} = 4.74 \times 10^{14}\text{ Hz}$$

EVALUATE: Note that while the speed and wavelength have different values in air and in the aqueous humor, the *frequency* in air, f_0, is the same as the frequency f in the aqueous humor:

$$f_0 = \frac{c}{\lambda_0} = \frac{3.00 \times 10^8\text{ m/s}}{633 \times 10^{-9}\text{ m}} = 4.74 \times 10^{14}\text{ Hz}$$

This illustrates the general rule that when a light wave passes from one material into another, the wave frequency is unchanged.

Example 33.3 A twice-reflected ray

Two mirrors are perpendicular to each other. A ray traveling in a plane perpendicular to both mirrors is reflected from one mirror, then the other, as shown in Fig. 33.12. What is the ray's final direction relative to its original direction?

SOLUTION

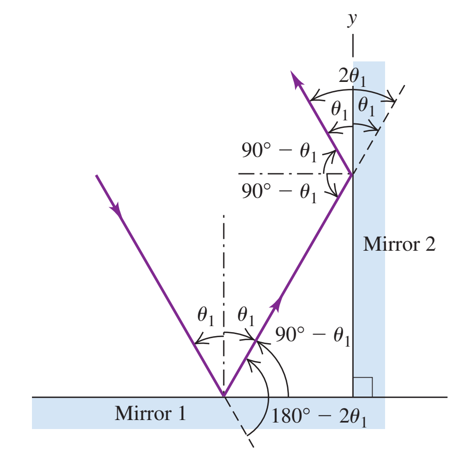
IDENTIFY: This problem involves only the law of reflection.

SET UP: There are two reflections in this situation, so we must apply the law of reflection twice.

EXECUTE: For mirror 1 the angle of incidence is θ_1, and this equals the angle of reflection. The sum of interior angles in the triangle shown in the figure is 180°, so we see that the angles of incidence and reflection for mirror 2 are both $90° - \theta_1$. The total change in direction of the ray after both reflections is therefore $2(90° - \theta_1) + 2\theta_1 = 180°$. That is, the ray's final direction is opposite to its original direction.

EVALUATE: An alternative viewpoint is that specular reflection reverses the sign of the component of light velocity perpendicular to the surface but leaves the other components unchanged. We invite you to verify this in detail. You should also be able to use this result to show that when a ray of light is successively reflected by three mirrors forming a corner of a cube (a "corner reflector"), its final direction is again opposite to its original direction. This principle is widely used in tail-light lenses and bicycle reflectors to improve their night-time visibility. Apollo astronauts placed arrays of corner reflectors on the moon. By use of laser beams reflected from these arrays, the earth–moon distance has been measured to within 0.15 m.

33.12 A ray moving in the xy-plane. The first reflection changes the sign of the y-component of its velocity, and the second reflection changes the sign of the x-component. For a different ray with a z-component of velocity, a third mirror (perpendicular to the two shown) could be used to change the sign of that component.



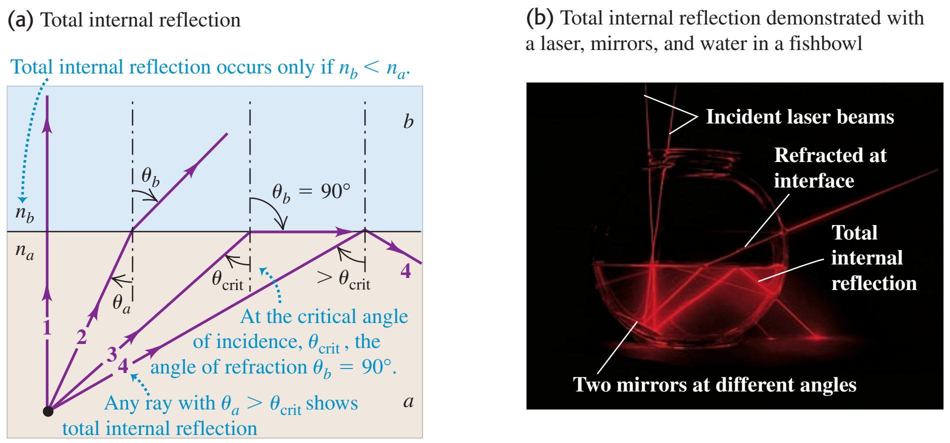
Test Your Understanding of Section 33.2 You are standing on the shore of a lake. You spot a tasty fish swimming some distance below the lake surface. (a) If you want to spear the fish, should you aim the spear (i) above, (ii) below, or (iii) directly at the apparent position of the fish? (b) If instead you use a high-power laser to simultaneously kill and cook the fish, should you aim the laser (i) above, (ii) below, or (iii) directly at the apparent position of the fish?

33.3 Total Internal Reflection

We have described how light is partially reflected and partially transmitted at an interface between two materials with different indexes of refraction. Under certain circumstances, however, *all* of the light can be reflected back from the interface, with none of it being transmitted, even though the second material is transparent. Figure 33.13a shows how this can occur. Several rays are shown radiating from a point source in material a with index of refraction n_a. The rays strike the surface of a second material b with index n_b, where $n_a > n_b$. (For

ActivPhysics Online
15.2 Total Internal Reflection

33.13 (a) Total internal reflection. The angle of incidence for which the angle of refraction is 90° is called the critical angle: this is the case for ray 3. The reflected portions of rays 1, 2, and 3 are omitted for clarity. (b) Rays of laser light enter the water in the fishbowl from above; they are reflected at the bottom by mirrors tilted at slightly different angles. One ray undergoes total internal reflection at the air–water interface.



instance, materials a and b could be water and air, respectively.) From Snell's law of refraction,

$$\sin\theta_b = \frac{n_a}{n_b}\sin\theta_a$$

Because n_a/n_b is greater than unity, $\sin\theta_b$ is larger than $\sin\theta_a$; the ray is bent *away from* the normal. Thus there must be some value of θ_a *less than* 90° for which Snell's law gives $\sin\theta_b = 1$ and $\theta_b = 90°$. This is shown by ray 3 in the diagram, which emerges just grazing the surface at an angle of refraction of 90°. Compare the diagram in Fig. 33.13a to the photograph of light rays in Fig. 33.13b.

The angle of incidence for which the refracted ray emerges tangent to the surface is called the **critical angle,** denoted by θ_{crit}. (A more detailed analysis using Maxwell's equations shows that as the incident angle approaches the critical angle, the transmitted intensity approaches zero.) If the angle of incidence is *larger* than the critical angle, the sine of the angle of refraction, as computed by Snell's law, would have to be greater than unity, which is impossible. Beyond the critical angle, the ray *cannot* pass into the upper material; it is trapped in the lower material and is completely reflected at the boundary surface. This situation, called **total internal reflection,** occurs only when a ray is incident on the interface with a second material whose index of refraction is *smaller* than that of the material in which the ray is traveling.

We can find the critical angle for two given materials by setting $\theta_b = 90°\,(\sin\theta_b = 1)$ in Snell's law. We then have

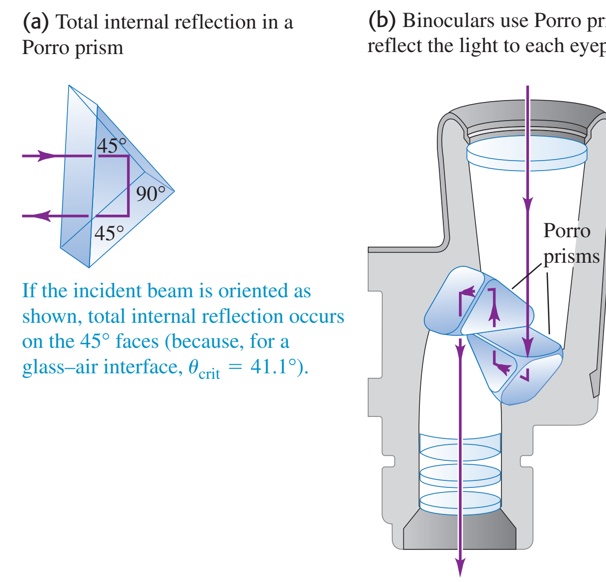
$$\sin\theta_{crit} = \frac{n_b}{n_a} \qquad \text{(critical angle for total internal reflection)} \qquad (33.6)$$

Total internal reflection will occur if the angle of incidence θ_a is larger than or equal to θ_{crit}.

Applications of Total Internal Reflection

Total internal reflection finds numerous uses in optical technology. As an example, consider glass with index of refraction $n = 1.52$. If light propagating within this glass encounters a glass–air interface, the critical angle is

$$\sin\theta_{crit} = \frac{1}{1.52} = 0.658 \qquad \theta_{crit} = 41.1°$$



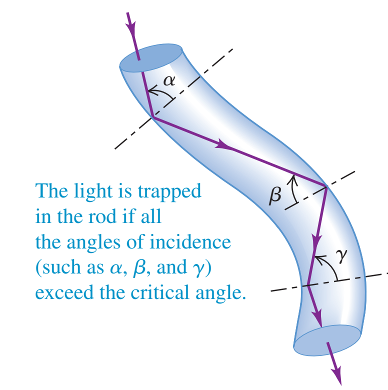
33.14 (a) Total internal reflection in a Porro prism. (b) A combination of two Porro prisms in binoculars.

The light will be *totally reflected* if it strikes the glass–air surface at an angle of 41.1° or larger. Because the critical angle is slightly smaller than 45°, it is possible to use a prism with angles of 45°−45°−90° as a totally reflecting surface. As reflectors, totally reflecting prisms have some advantages over metallic surfaces such as ordinary coated-glass mirrors. While no metallic surface reflects 100% of the light incident on it, light can be *totally* reflected by a prism. The reflecting properties of a prism have the additional advantages of being permanent and unaffected by tarnishing.

A 45°–45°–90° prism, used as in Fig. 33.14a, is called a *Porro* prism. Light enters and leaves at right angles to the hypotenuse and is totally reflected at each of the shorter faces. The total change of direction of the rays is 180°. Binoculars often use combinations of two Porro prisms, as in Fig. 33.14b.

When a beam of light enters at one end of a transparent rod (Fig. 33.15), the light can be totally reflected internally if the index of refraction of the rod is greater than that of the surrounding material. The light is "trapped" within the rod even if the rod is curved, provided that the curvature is not too great. Such a rod is sometimes called a *light pipe.* A bundle of fine glass or plastic fibers behaves in the same way and has the advantage of being flexible. A bundle may consist of thousands of individual fibers, each of the order of 0.002 to 0.01 mm in diameter. If the fibers are assembled in the bundle so that the relative positions of the ends are the same (or mirror images) at both ends, the bundle can transmit an image, as shown in Fig. 33.16.

33.15 A transparent rod with refractive index greater than that of the surrounding material.



33.16 Image transmission by a bundle of optical fibers.

Fiber-optic devices have found a wide range of medical applications in instruments called *endoscopes,* which can be inserted directly into the bronchial tubes, the bladder, the colon, and so on, for direct visual examination. A bundle of fibers can be enclosed in a hypodermic needle for study of tissues and blood vessels far beneath the skin.

Fiber optics also have applications in communication systems, in which they are used to transmit a modulated laser beam. The rate at which information can be transmitted by a wave (light, radio, or whatever) is proportional to the frequency. To see qualitatively why this is so, consider modulating (modifying) the wave by chopping off some of the wave crests. Suppose each crest represents a binary digit, with a chopped-off crest representing a zero and an unmodified crest representing a one. The number of binary digits we can transmit per unit time is thus proportional to the frequency of the wave. Infrared and visible-light waves have much higher frequency than do radio waves, so a modulated laser beam can transmit an enormous amount of information through a single fiber-optic cable.

33.17 To maximize their brilliance, diamonds are cut so that there is total internal reflection on their back surfaces.

Another advantage of optical fibers is that they can be made thinner than conventional copper wire, so more fibers can be bundled together in a cable of a given diameter. Hence more distinct signals (for instance, different phone lines) can be sent over the same cable. Because fiber-optic cables are electrical insulators, they are immune to electrical interference from lightning and other sources, and they don't allow unwanted currents between source and receiver. For these and other reasons, fiber-optic cables are playing an increasingly important role in long-distance telephone, television, and Internet communication.

Total internal reflection also plays an important role in the design of jewelry. The brilliance of diamond is due in large measure to its very high index of refraction $(n = 2.417)$ and correspondingly small critical angle. Light entering a cut diamond is totally internally reflected from facets on its back surface, and then emerges from its front surface (Fig. 33.17). "Imitation diamond" gems, such as cubic zirconia, are made from less expensive crystalline materials with comparable indexes of refraction.

Conceptual Example 33.4 A leaky periscope

A periscope for a submarine uses two totally reflecting $45°-45°-90°$ prisms with total internal reflection on the sides adjacent to the $45°$ angles. It springs a leak, and the bottom prism is covered with water. Explain why the periscope no longer works.

SOLUTION

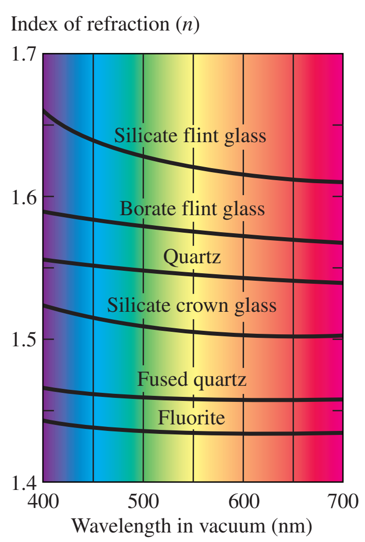
The critical angle for water $(n_b = 1.33)$ on glass $(n_a = 1.52)$ is

$$\theta_{crit} = \arcsin\frac{1.33}{1.52} = 61.0°$$

The $45°$ angle of incidence for a totally reflecting prism is *smaller than* the $61°$ critical angle, so total internal reflection does not occur at the glass–water boundary. Most of the light is transmitted into the water, and very little is reflected back into the prism.

Test Your Understanding of Section 33.3 In which of the following situations is there total internal reflection? (i) Light propagating in water $(n = 1.33)$ strikes a water–air interface at an incident angle of 70°; (ii) light propagating in glass $(n = 1.52)$ strikes a glass–water interface at an incident angle of 70°; (iii) light propagating in water strikes a water–glass interface at an incident angle of 70°.

33.18 Variation of index of refraction n with wavelength for different transparent materials. The horizontal axis shows the wavelength λ_0 of the light *in vacuum;* the wavelength in the material is equal to $\lambda = \lambda_0/n$.

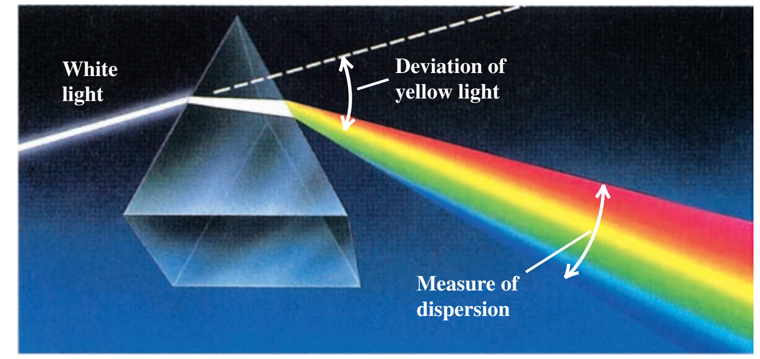


*33.4 Dispersion

Ordinary white light is a superposition of waves with wavelengths extending throughout the visible spectrum. The speed of light *in vacuum* is the same for all wavelengths, but the speed in a material substance is different for different wavelengths. Therefore the index of refraction of a material depends on wavelength. The dependence of wave speed and index of refraction on wavelength is called **dispersion.**

Figure 33.18 shows the variation of index of refraction n with wavelength for a few common optical materials. Note that the horizontal axis of this figure is the wavelength of the light *in vacuum,* λ_0; the wavelength in the material is given by Eq. (33.5), $\lambda = \lambda_0/n$. In most materials the value of n *decreases* with increasing wavelength and decreasing frequency, and thus n *increases* with decreasing wavelength and increasing frequency. In such a material, light of longer wavelength has greater speed than light of shorter wavelength.

Figure 33.19 shows a ray of white light incident on a prism. The deviation (change of direction) produced by the prism increases with increasing index of refraction and frequency and decreasing wavelength. Violet light is deviated most, and red is deviated least; other colors are in intermediate positions. When it comes out of the prism, the light is spread out into a fan-shaped beam, as shown.



33.19 Dispersion of light by a prism. The band of colors is called a spectrum.

The light is said to be *dispersed* into a spectrum. The amount of dispersion depends on the *difference* between the refractive indexes for violet light and for red light. From Fig. 33.18 we can see that for a substance such as fluorite, the difference between the indexes for red and violet is small, and the dispersion will also be small. A better choice of material for a prism whose purpose is to produce a spectrum would be silicate flint glass, for which there is a larger difference in the value of n between red and violet.

As we mentioned in Section 33.3, the brilliance of diamond is due in part to its unusually large refractive index; another important factor is its large dispersion, which causes white light entering a diamond to emerge as a multicolored spectrum. Crystals of rutile and of strontium titanate, which can be produced synthetically, have about eight times the dispersion of diamond.

Rainbows

When you experience the beauty of a rainbow, as in Fig. 33.20a, you are seeing the combined effects of dispersion, refraction, and reflection. Sunlight comes from behind you, enters a water droplet, is (partially) reflected from the back surface of the droplet, and is refracted again upon exiting the droplet (Fig. 33.20b). A light ray that enters the middle of the raindrop is reflected straight back. All other rays exit the raindrop within an angle Δ of that middle ray, with many rays "piling up" at the angle Δ. What you see is a disk of light of angular radius Δ centered on the down-sun point (the point in the sky opposite the sun); due to the "piling up" of light rays, the disk is brightest around its rim, which we see as a rainbow (Fig. 33.20c). Because no light reaches your eye from angles larger than Δ, the sky looks dark outside the rainbow (see Fig. 33.20a). The value of the angle Δ depends on the index of refraction of the water that makes up the raindrops, which in turn depends on the wavelength (Fig. 33.20d). The bright disk of red light is slightly larger than that for orange light, which in turn is slightly larger than that for yellow light, and so on. As a result, you see the rainbow as a band of colors.

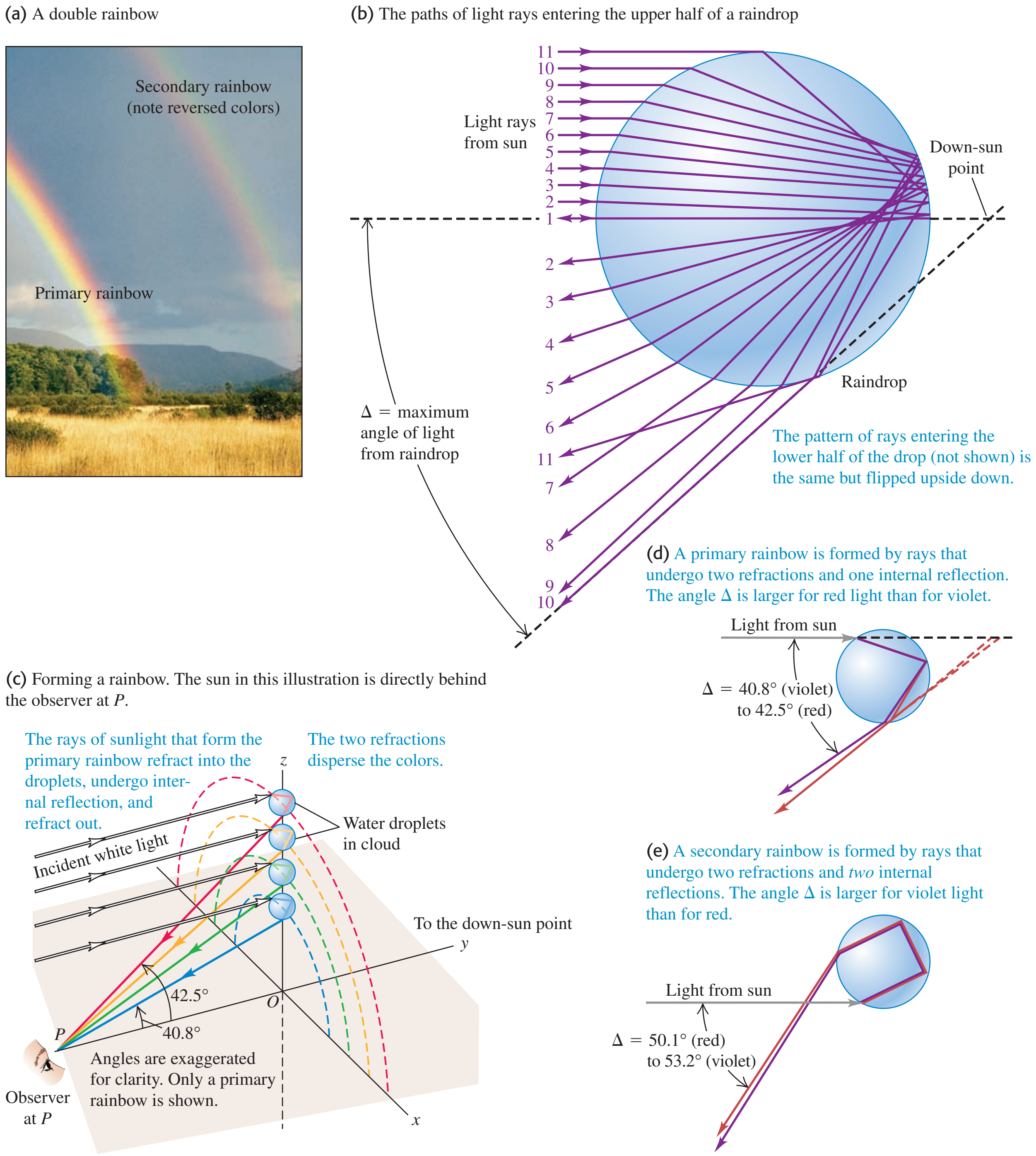
In many cases you can see a second, larger rainbow. It is the result of dispersion, refraction, and *two* reflections from the back surface of the droplet (Fig. 33.20e). Each time a light ray hits the back surface, part of the light is refracted out of the drop (not shown in Fig. 33.20); after two such hits, relatively little light is left inside the drop, which is why the secondary rainbow is noticeably fainter than the primary rainbow. Just as a mirror held up to a book reverses the printed letters, so the second reflection reverses the sequence of colors in the secondary rainbow. You can see this effect in Fig 33.20a.

33.5 Polarization

Polarization is a characteristic of all transverse waves. This chapter is about light, but to introduce some basic polarization concepts, let's go back to the transverse waves on a string that we studied in Chapter 15. For a string that in

ActivPhysics Online 16.9 Physical Optics: Polarization

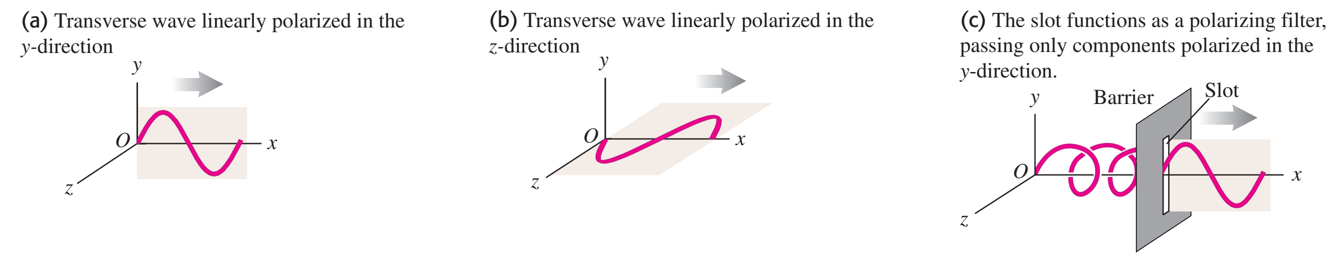
33.20 How rainbows form.



equilibrium lies along the x-axis, the displacements may be along the y-direction, as in Fig. 33.21a. In this case the string always lies in the xy-plane. But the displacements might instead be along the z-axis, as in Fig. 33.21b; then the string always lies in the xz-plane.

When a wave has only y-displacements, we say that it is **linearly polarized** in the y-direction; a wave with only z-displacements is linearly polarized in the z-direction. For mechanical waves we can build a **polarizing filter,** or **polarizer,** that permits only waves with a certain polarization direction to pass. In Fig. 33.21c the string can slide vertically in the slot without friction, but no horizontal motion is possible. This filter passes waves that are polarized in the y-direction but blocks those that are polarized in the z-direction.

33.21 (a), (b) Polarized waves on a string. (c) Making a polarized wave on a string from an unpolarized one using a polarizing filter.



This same language can be applied to electromagnetic waves, which also have polarization. As we learned in Chapter 32, an electromagnetic wave is a *transverse* wave; the fluctuating electric and magnetic fields are perpendicular to each other and to the direction of propagation. We always define the direction of polarization of an electromagnetic wave to be the direction of the *electric*-field vector $\vec{E}$, not the magnetic field, because many common electromagnetic-wave detectors respond to the electric forces on electrons in materials, not the magnetic forces. Thus the electromagnetic wave described by Eq. (32.17),

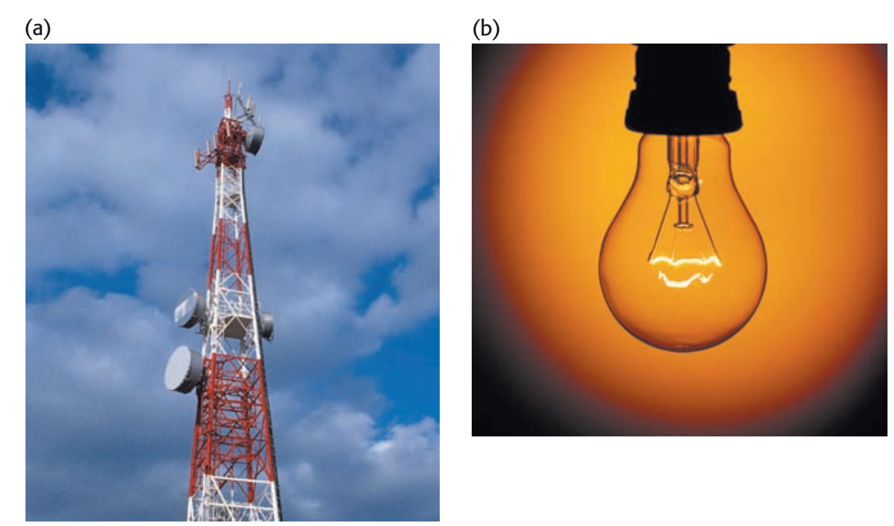
$$\vec{E}(x,t) = \hat{\jmath}E_{max}\cos(kx - \omega t)$$
$$\vec{B}(x,t) = \hat{k}B_{max}\cos(kx - \omega t)$$

is said to be polarized in the y-direction because the electric field has only a y-component.

CAUTION **The meaning of "polarization"** It's unfortunate that the same word "polarization" that is used to describe the direction of $\vec{E}$ in an electromagnetic wave is also used to describe the shifting of electric charge within a body, such as in response to a nearby charged body; we described this latter kind of polarization in Section 21.2 (see Fig. 21.7). You should remember that while these two concepts have the same name, they do not describe the same phenomenon.

Polarizing Filters

Waves emitted by a radio transmitter are usually linearly polarized. The vertical antennas that are used for radio broadcasting emit waves that, in a horizontal plane around the antenna, are polarized in the vertical direction (parallel to the antenna) (Fig. 33.22a). Rooftop TV antennas have horizontal elements in the United States and vertical elements in Great Britain because the transmitted waves have different polarizations.

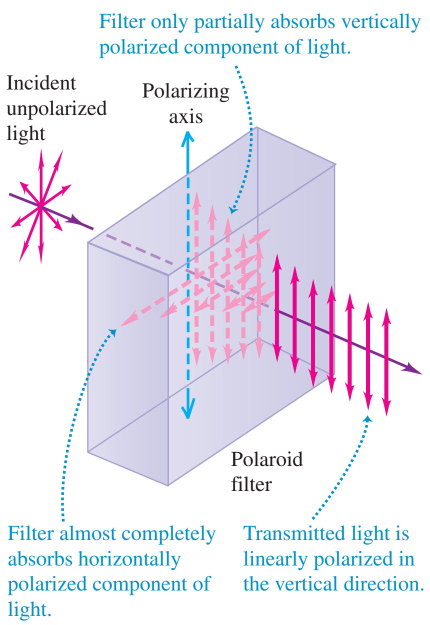


33.22 (a) Electrons in the red and white broadcast antenna oscillate vertically, producing vertically polarized electromagnetic waves that propagate away from the antenna in the horizontal direction. (The small gray antennas are for relaying cellular phone signals.) (b) No matter how this light bulb is oriented, the random motion of electrons in the filament produces unpolarized light waves.

The situation is different for visible light. Light from ordinary sources, such as incandescent light bulbs and fluorescent light fixtures, is *not* polarized (Fig. 33.22b). The "antennas" that radiate light waves are the molecules that make up the sources. The waves emitted by any one molecule may be linearly polarized, like those from a radio antenna. But any actual light source contains a tremendous number of molecules with random orientations, so the emitted light is a random mixture of waves linearly polarized in all possible transverse directions. Such light is called **unpolarized light** or **natural light.** To create polarized light from unpolarized natural light requires a filter that is analogous to the slot for mechanical waves in Fig. 33.21c.

Polarizing filters for electromagnetic waves have different details of construction, depending on the wavelength. For microwaves with a wavelength of a few centimeters, a good polarizer is an array of closely spaced, parallel conducting wires that are insulated from each other. (Think of a barbecue grill with the outer metal ring replaced by an insulating one.) Electrons are free to move along the length of the conducting wires and will do so in response to a wave whose $\vec{E}$ field is parallel to the wires. The resulting currents in the wires dissipate energy by I^2R heating; the dissipated energy comes from the wave, so whatever wave passes through the grid is greatly reduced in amplitude. Waves with $\vec{E}$ oriented perpendicular to the wires pass through almost unaffected, since electrons cannot move through the air between the wires. Hence a wave that passes through such a filter will be predominantly polarized in the direction perpendicular to the wires.

33.23 A Polaroid filter is illuminated by unpolarized natural light (shown by $\vec{E}$ vectors that point in all directions perpendicular to the direction of propagation). The transmitted light is linearly polarized along the polarizing axis (shown by $\vec{E}$ vectors along the polarization direction only).

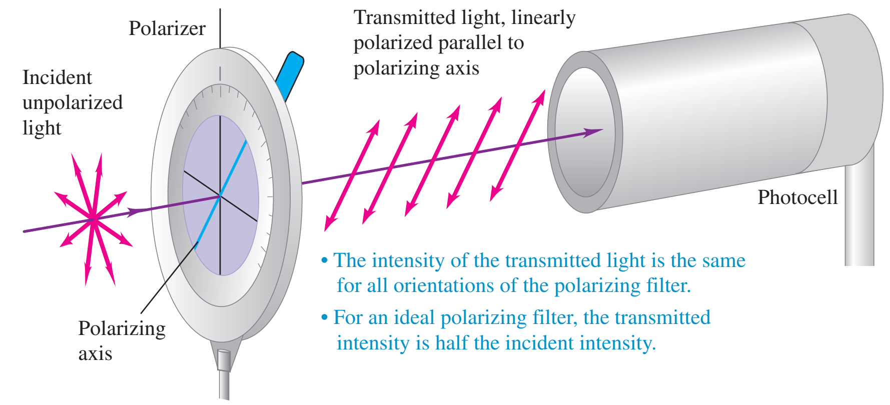


The most common polarizing filter for visible light is a material known by the trade name Polaroid, widely used for sunglasses and polarizing filters for camera lenses. Developed originally by the American scientist Edwin H. Land, this material incorporates substances that have **dichroism,** a selective absorption in which one of the polarized components is absorbed much more strongly than the other (Fig. 33.23). A Polaroid filter transmits 80% or more of the intensity of a wave that is polarized parallel to a certain axis in the material, called the **polarizing axis,** but only 1% or less for waves that are polarized perpendicular to this axis. In one type of Polaroid filter, long-chain molecules within the filter are oriented with their axis perpendicular to the polarizing axis; these molecules preferentially absorb light that is polarized along their length, much like the conducting wires in a polarizing filter for microwaves.

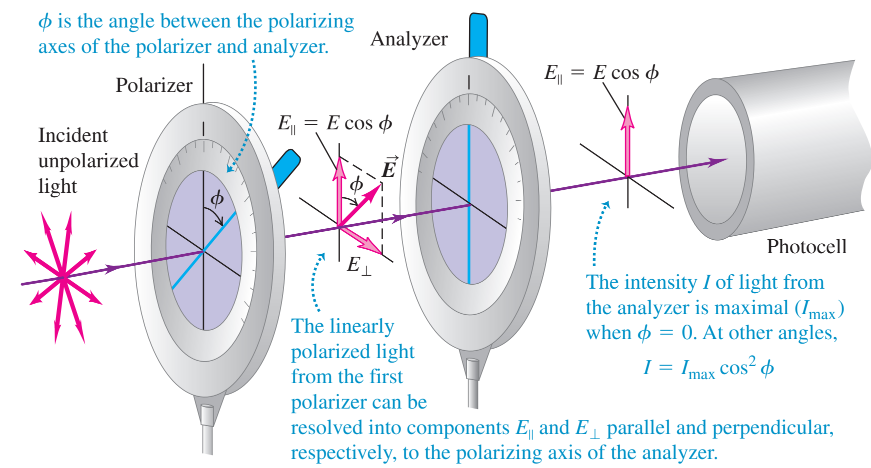
Using Polarizing Filters

An *ideal* polarizing filter (polarizer) passes 100% of the incident light that is polarized in the direction of the filter's polarizing axis but completely blocks all light that is polarized perpendicular to this axis. Such a device is an unattainable idealization, but the concept is useful in clarifying the basic ideas. In the following discussion we will assume that all polarizing filters are ideal. In Fig. 33.24 unpolarized light is incident on a flat polarizing filter. The polarizing axis is represented by the blue line. The $\vec{E}$ vector of the incident wave can be represented in terms of components parallel and perpendicular to the polarizer axis; only the component of $\vec{E}$ parallel to the polarizing axis is transmitted. Hence the light emerging from the polarizer is linearly polarized parallel to the polarizing axis.

When unpolarized light is incident on an ideal polarizer as in Fig. 33.24, the intensity of the transmitted light is *exactly half* that of the incident unpolarized light, no matter how the polarizing axis is oriented. Here's why: We can resolve the $\vec{E}$ field of the incident wave into a component parallel to the polarizing axis and a component perpendicular to it. Because the incident light is a random mixture of all states of polarization, these two components are, on average, equal. The ideal polarizer transmits only the component that is parallel to the polarizing axis, so half the incident intensity is transmitted.

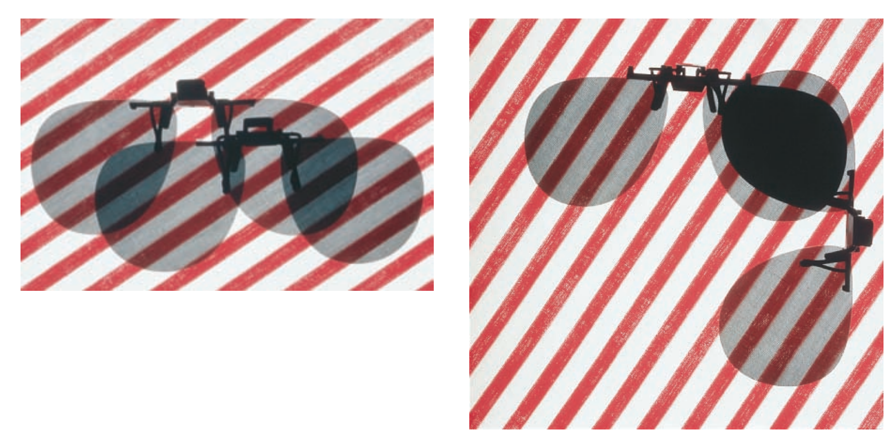


33.24 Unpolarized natural light is incident on the polarizing filter. The photocell measures the intensity of the transmitted linearly polarized light.



33.25 An ideal analyzer transmits only the electric field component parallel to its transmission direction (that is, its polarizing axis).

What happens when the linearly polarized light emerging from a polarizer passes through a second polarizer, as in Fig. 33.25? Consider the general case in which the polarizing axis of the second polarizer, or *analyzer,* makes an angle ϕ with the polarizing axis of the first polarizer. We can resolve the linearly polarized light that is transmitted by the first polarizer into two components, as shown in Fig. 33.25, one parallel and the other perpendicular to the axis of the analyzer. Only the parallel component, with amplitude $E\cos\phi$, is transmitted by the analyzer. The transmitted intensity is greatest when $\phi = 0$, and it is zero when polarizer and analyzer are *crossed* so that $\phi = 90°$ (Fig. 33.26). To determine the direction of polarization of the light transmitted by the first polarizer, rotate the analyzer until the photocell in Fig. 33.25 measures zero intensity; the polarization axis of the first polarizer is then perpendicular to that of the analyzer.



33.26 These photos show the view through Polaroid sunglasses whose polarizing axes are (left) aligned ($\phi = 0$) and (right) perpendicular ($\phi = 90°$). The transmitted intensity is greatest when the axes are aligned; it is zero when the axes are perpendicular.

To find the transmitted intensity at intermediate values of the angle ϕ, we recall from our energy discussion in Section 32.4 that the intensity of an electromagnetic wave is proportional to the *square* of the amplitude of the wave [see Eq.(32.29)]. The ratio of transmitted to incident *amplitude* is $\cos\phi$, so the ratio of transmitted to incident *intensity* is $\cos^2\phi$. Thus the intensity of the light transmitted through the analyzer is

$$I = I_{max}\cos^2\phi \quad \text{(Malus's law, polarized light passing through an analyzer)} \tag{33.7}$$

where I_{max} is the maximum intensity of light transmitted (at $\phi = 0$) and I is the amount transmitted at angle ϕ. This relationship, discovered experimentally by Etienne Louis Malus in 1809, is called **Malus's law.** Malus's law applies *only* if the incident light passing through the analyzer is already linearly polarized.

Problem-Solving Strategy 33.2 Linear Polarization

MP

IDENTIFY *the relevant concepts:* Remember that in all electromagnetic waves, including light waves, the direction of the $\vec{E}$ field is the direction of polarization and is perpendicular to the propagation direction. When working with polarizers, you are really dealing with components of $\vec{E}$ parallel and perpendicular to the polarizing axis. Everything you know about components of vectors is applicable here.

SET UP *the problem* using the following steps:

1. Just as for problems in geometric optics, you should *always* start by drawing a large, neat diagram. Label all known angles, including the angles of any and all polarizing axes.
2. Determine the target variables.

EXECUTE *the solution* as follows:

1. Remember that a polarizer lets pass only electric-field components parallel to its polarizing axis.
2. If the incident light is linearly polarized and has amplitude E and intensity I_{max}, the light that passes through an ideal polarizer has amplitude $E\cos\phi$ and intensity $I_{max}\cos^2\phi$, where ϕ is the angle between the incident polarization direction and the filter's polarizing axis.
3. Unpolarized light is a random mixture of all possible polarization states, so on the average it has equal components in any two perpendicular directions. When passed through an ideal polarizer, unpolarized light becomes linearly polarized light with half the incident intensity. Partially linearly polarized light is a superposition of linearly polarized and unpolarized light.
4. The intensity (average power per unit area) of a wave is proportional to the *square* of its amplitude. If you find that two waves differ in amplitude by a certain factor, their intensities differ by the square of that factor.

EVALUATE *your answer:* Check your answer for any obvious errors. If your results say that light emerging from a polarizer has greater intensity than the incident light, something's wrong: a polarizer can't add energy to a light wave.

Example 33.5 Two polarizers in combination

In Fig. 33.25 the incident unpolarized light has intensity I_0. Find the intensities transmitted by the first and second polarizers if the angle between the axes of the two filters is 30°.

SOLUTION

IDENTIFY: This problem involves a polarizer (a polarizing filter on which unpolarized light shines, producing polarized light) and an analyzer (a second polarizing filter on which the polarized light shines).

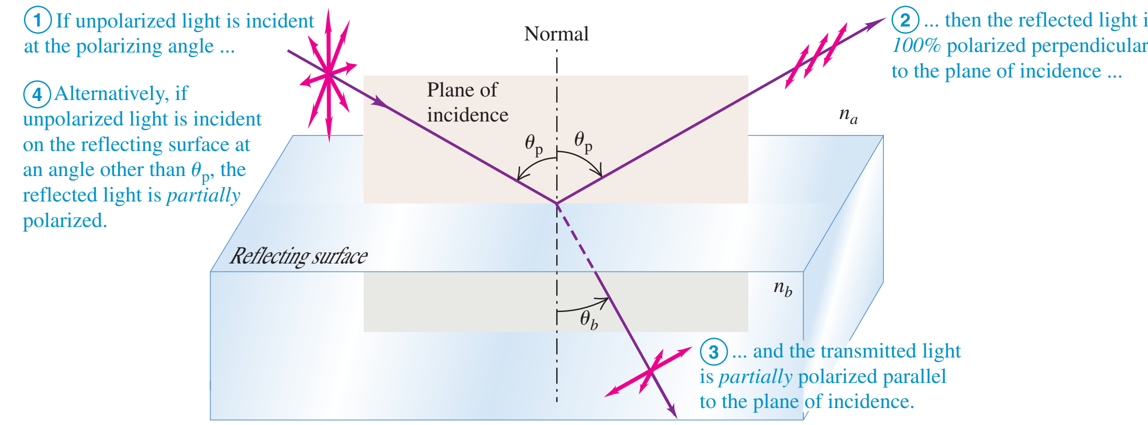
SET UP: The diagram has already been drawn for us in Fig. 33.25. We are given the intensity I_0 of the incident natural light and the angle $\phi = 30°$ between the polarizing axes. Our target variables are the intensities of the light emerging from the first polarizer and of the light emerging from the second polarizer.

EXECUTE: As we explained above, the intensity of the linearly polarized light transmitted by the first filter is $I_0/2$. According to Eq. (33.7) with $\phi = 30°$, the second filter reduces the intensity by a factor of $\cos^2 30° = \frac{3}{4}$. Thus the intensity transmitted by the second polarizer is

$$\left(\frac{I_0}{2}\right)\left(\frac{3}{4}\right) = \frac{3}{8}I_0$$

EVALUATE: Note that the intensity decreases after each passage through a polarizer. The only situation in which the transmitted intensity does *not* decrease is if the polarizer is ideal (so it absorbs none of the light that passes through it) and if the incident light is linearly polarized along the polarizing axis, so $\phi = 0$.

33.27 When light is incident on a reflecting surface at the polarizing angle, the reflected light is linearly polarized.



Polarization by Reflection

Unpolarized light can be polarized, either partially or totally, by *reflection.* In Fig. 33.27, unpolarized natural light is incident on a reflecting surface between two transparent optical materials; the plane containing the incident and reflected rays and the normal to the surface is called the **plane of incidence.** For most angles of incidence, waves for which the electric-field vector $\vec{E}$ is perpendicular to the plane of incidence (that is, parallel to the reflecting surface) are reflected more strongly than those for which $\vec{E}$ lies in this plane. In this case the reflected light is *partially polarized* in the direction perpendicular to the plane of incidence.

But at one particular angle of incidence, called the **polarizing angle** θ_p, the light for which $\vec{E}$ lies in the plane of incidence is *not reflected at all* but is completely refracted. At this same angle of incidence the light for which $\vec{E}$ is perpendicular to the plane of incidence is partially reflected and partially refracted. The *reflected* light is therefore *completely* polarized perpendicular to the plane of incidence, as shown in Fig. 33.27. The *refracted* (transmitted) light is *partially* polarized parallel to this plane; the refracted light is a mixture of the component parallel to the plane of incidence, all of which is refracted, and the remainder of the perpendicular component.

In 1812 the British scientist Sir David Brewster discovered that when the angle of incidence is equal to the polarizing angle θ_p, the reflected ray and the refracted ray are perpendicular to each other (Fig. 33.28). In this case the angle of refraction θ_b becomes the complement of θ_p, so $\theta_b = 90° - \theta_p$. From the law of refraction,

$$n_a \sin\theta_p = n_b \sin\theta_b$$

so we find

$$n_a \sin\theta_p = n_b \sin(90° - \theta_p) = n_b \cos\theta_p$$

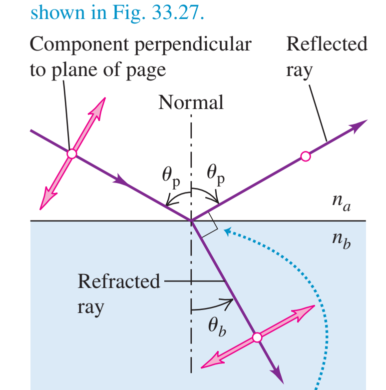
$$\tan\theta_p = \frac{n_b}{n_a} \quad \text{(Brewster's law for the polarizing angle)} \tag{33.8}$$

This relationship is known as **Brewster's law.** Although discovered experimentally, it can also be *derived* from a wave model using Maxwell's equations.

Polarization by reflection is the reason polarizing filters are widely used in sunglasses (Fig. 33.26). When sunlight is reflected from a horizontal surface, the plane of incidence is vertical, and the reflected light contains a preponderance of

33.28 The significance of the polarizing angle. The open circles represent a component of $\vec{E}$ that is perpendicular to the plane of the figure (the plane of incidence) and parallel to the surface between the two materials.

Note: This is a side view of the situation shown in Fig. 33.27.



When light strikes a surface at the polarizing angle, the reflected and refracted rays are perpendicular to each other and $\tan\theta_p = \frac{n_b}{n_a}$

light that is polarized in the horizontal direction. When the reflection occurs at a smooth asphalt road surface or the surface of a lake, it causes unwanted glare. Vision can be improved by eliminating this glare. The manufacturer makes the polarizing axis of the lens material vertical, so very little of the horizontally polarized light reflected from the road is transmitted to the eyes. The glasses also reduce the overall intensity of the transmitted light to somewhat less than 50% of the intensity of the unpolarized incident light.

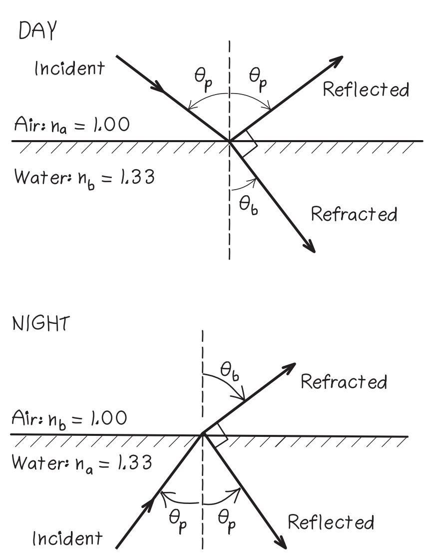
Example 33.6 Reflection from a swimming pool's surface

Sunlight reflects off the smooth surface of an unoccupied swimming pool. (a) At what angle of reflection is the light completely polarized? (b) What is the corresponding angle of refraction for the light that is transmitted (refracted) into the water? (c) At night an underwater floodlight is turned on in the pool. Repeat parts (a) and (b) for rays from the floodlight that strike the smooth surface from below.

SOLUTION

IDENTIFY: This problem involves polarization by reflection at an air–water interface in parts (a) and (b) and at a water–air interface in part (c).

SET UP: Figure 33.29 shows our sketches of the light rays for the situation during the day [parts (a) and (b)] and at night [part (c)]. In part (a) we're looking for the polarizing angle for light that is first in air, then in water; we find this with Brewster's law, Eq. (33.8). In part (b) we want the angle of the refracted light for this situation. In part (c) we again want the polarizing angle, but for light that is first in water, then in air. Again we use Eq. (33.8) to determine this angle.



33.29 Our sketches for this problem.

EXECUTE: (a) The top part of Fig. 33.29 shows the situation during the day. Since the light moves from air toward water, we have $n_a = 1.00$ (air) and $n_b = 1.33$ (water). From Eq. (33.8),

$$\theta_p = \arctan\frac{n_b}{n_a} = \arctan\frac{1.33}{1.00} = 53.1°$$

(b) The incident light is at the polarizing angle, so the reflected and refracted rays are perpendicular; hence

$$\theta_p + \theta_b = 90°$$
$$\theta_b = 90° - 53.1° = 36.9°$$

(c) The situation at night is shown in the bottom part of Fig. 33.29. Now the light is *first* in the water, then in the air, so $n_a = 1.33$ and $n_b = 1.00$. Again using Eq. (33.8), we have

$$\theta_p = \arctan\frac{1.00}{1.33} = 36.9°$$
$$\theta_b = 90° - 36.9° = 53.1°$$

EVALUATE: We can check our answer in part (b) using Snell's law, $n_a \sin\theta_a = n_b \sin\theta_b$, or

$$\sin\theta_b = \frac{n_a \sin\theta_p}{n_b} = \frac{1.00 \sin 53.1°}{1.33} = 0.600$$
$$\theta_b = 36.9°$$

Note that the two polarizing angles found in parts (a) and (c) add to 90°. This is *not* an accident; can you see why?

Circular and Elliptical Polarization

Light and other electromagnetic radiation can also have *circular* or *elliptical* polarization. To introduce these concepts, let's return once more to mechanical waves on a stretched string. In Fig. 33.21, suppose the two linearly polarized waves in parts (a) and (b) are in phase and have equal amplitude. When they are superposed, each point in the string has simultaneous y- and z-displacements of equal magnitude. A little thought shows that the resultant wave lies in a plane oriented at 45° to the y- and z-axes (i.e., in a plane making a 45° angle with the

xy- and xz-planes). The amplitude of the resultant wave is larger by a factor of $\sqrt{2}$ than that of either component wave, and the resultant wave is linearly polarized.

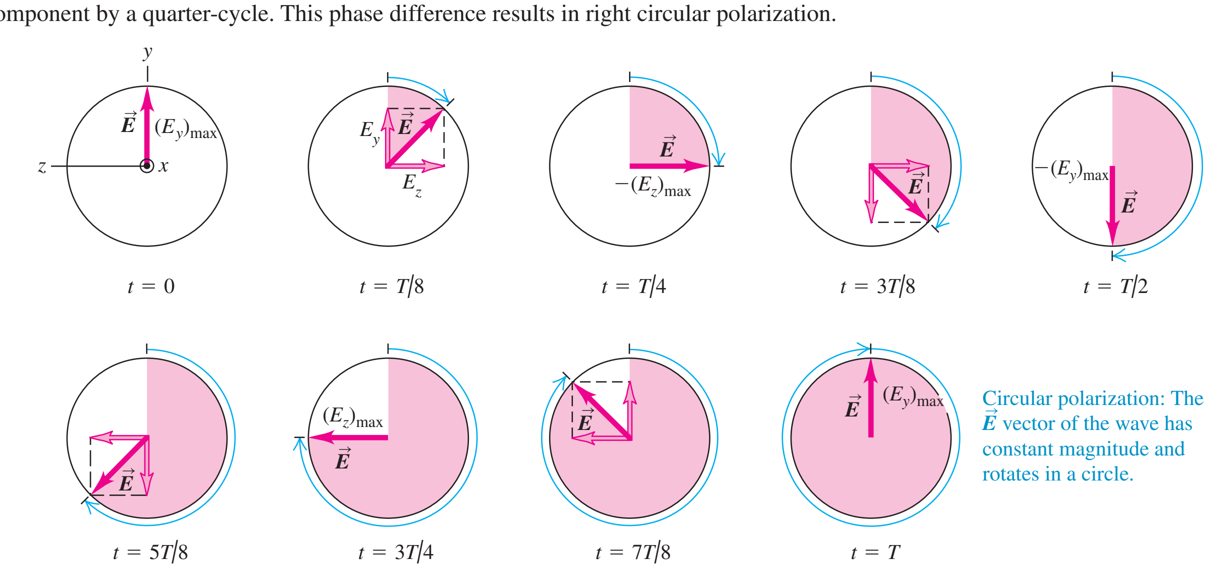
But now suppose the two equal-amplitude waves differ in phase by a quarter-cycle. Then the resultant motion of each point corresponds to a superposition of two simple harmonic motions at right angles, with a quarter-cycle phase difference. The y-displacement at a point is greatest at times when the z-displacement is zero, and vice versa. The motion of the string as a whole then no longer takes place in a single plane. It can be shown that each point on the rope moves in a *circle* in a plane parallel to the yz-plane. Successive points on the rope have successive phase differences, and the overall motion of the string has the appearance of a rotating helix. This is shown to the left of the polarizing filter in Fig. 33.21c. This particular superposition of two linearly polarized waves is called **circular polarization.** By convention, the wave is said to be *right circularly polarized* when the sense of motion of a particle of the string, to an observer looking *backward* along the direction of propagation, is *clockwise;* the wave is said to be *left circularly polarized* if the sense of motion is the reverse.

Figure 33.30 shows the analogous situation for an electromagnetic wave. Two sinusoidal waves of equal amplitude, polarized in the y- and z-directions and with a quarter-cycle phase difference, are superposed. The result is a wave in which the $\vec{E}$ vector at each point has a constant magnitude but *rotates* around the direction of propagation. The wave in Fig. 33.30 is propagating toward you and the $\vec{E}$ vector appears to be rotating clockwise, so it is called a *right circularly polarized* electromagnetic wave. If instead the $\vec{E}$ vector of a wave coming toward you appears to be rotating counterclockwise, it is called a *left circularly polarized* electromagnetic wave.

If the phase difference between the two component waves is something other than a quarter-cycle, or if the two component waves have different amplitudes, then each point on the string traces out not a circle but an *ellipse.* The resulting wave is said to be **elliptically polarized.**

For electromagnetic waves with radio frequencies, circular or elliptical polarization can be produced by using two antennas at right angles, fed from the same transmitter but with a phase-shifting network that introduces the appropriate phase difference. For light, the phase shift can be introduced by use of a material that exhibits *birefringence*—that is, has different indexes of refraction for different directions of polarization. A common example is calcite $(CaCO_3)$. When a

33.30 Circular polarization of an electromagnetic wave moving toward you parallel to the x-axis. The y-component of $\vec{E}$ lags the z-component by a quarter-cycle. This phase difference results in right circular polarization.



calcite crystal is oriented appropriately in a beam of unpolarized light, its refractive index (for a wavelength in vacuum of 589 nm) is 1.658 for one direction of polarization and 1.486 for the perpendicular direction. When two waves with equal amplitude and with perpendicular directions of polarization enter such a material, they travel with different speeds. If they are in phase when they enter the material, then in general they are no longer in phase when they emerge. If the crystal is just thick enough to introduce a quarter-cycle phase difference, then the crystal converts linearly polarized light to circularly polarized light. Such a crystal is called a *quarter-wave plate.* Such a plate also converts circularly polarized light to linearly polarized light. Can you prove this? (See Problem 33.43.)

33.31 Photoelastic stress analysis of a model of a cross section of a Gothic cathedral. The masonry construction that was used for this kind of building had great strength in compression but very little in tension (see Section 11.4). Inadequate buttressing and high winds sometimes caused tensile stresses in normally compressed structural elements, leading to some spectacular collapses.

Photoelasticity

Some optical materials that are not normally birefringent become so when they are subjected to mechanical stress. This is the basis of the science of *photoelasticity.* Stresses in girders, boiler plates, gear teeth, and cathedral pillars can be analyzed by constructing a transparent model of the object, usually of a plastic material, subjecting it to stress, and examining it between a polarizer and an analyzer in the crossed position. Very complicated stress distributions can be studied by these optical methods.

Figure 33.31 is a photograph of a photoelastic model under stress. The polarized light that enters the model can be thought of as having a component along each of the two directions of the birefringent plastic. Since these two components travel through the plastic at different speeds, the light that emerges from the other side of the model can have a different overall direction of polarization. Hence some of this transmitted light will be able to pass through the analyzer even though its polarization axis is at a 90° angle to the polarizer's axis, and the stressed areas in the plastic will appear as bright spots. The amount of birefringence is different for different wavelengths and hence different colors of light; the color that appears at each location in Fig. 33.31 is that for which the transmitted light is most nearly polarized along the analyzer's polarization axis.

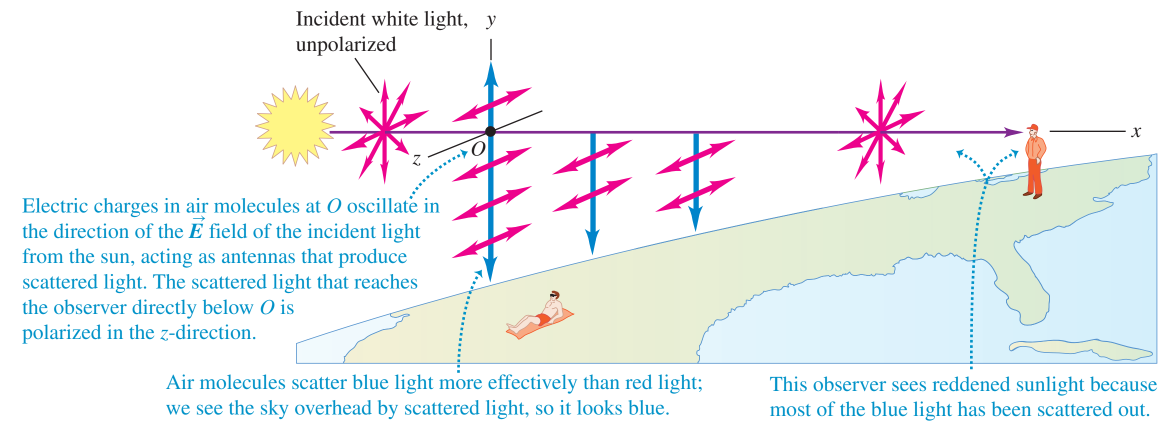
Test Your Understanding of Section 33.5 You are taking a photograph of a sunlit high-rise office building. In order to minimize the reflections from the building's windows, you place a polarizing filter on the camera lens. How should you orient the filter? (i) with the polarizing axis vertical; (ii) with the polarizing axis horizontal; (iii) either orientation will minimize the reflections just as well; (iv) neither orientation will have any effect.

*33.6 Scattering of Light

The sky is blue. Sunsets are red. Skylight is partially polarized; that's why the sky looks darker from some angles than from others when it is viewed through Polaroid sunglasses. As we will see, a single phenomenon is responsible for all of these effects.

When you look at the daytime sky, the light that you see is sunlight that has been absorbed and then re-radiated in a variety of directions. This process is called **scattering.** (If the earth had no atmosphere, the sky would appear as black in the daytime as it does at night, just as it does to an astronaut in space or on the moon; you would see the sun's light only if you looked directly at it, and the stars would be visible in the daytime.) Figure 33.32 shows some of the details of the scattering process. Sunlight, which is unpolarized, comes from the left along the x-axis and passes over an observer looking vertically upward along the y-axis. (We are viewing the situation from the side.) Consider the molecules of the earth's atmosphere located at point O. The electric field in the beam of sunlight sets the electric charges in these molecules into vibration.

33.32 When the sunbathing observer on the left looks up, he sees blue, polarized sunlight that has been scattered by air molecules. The observer on the right sees reddened, unpolarized light when he looks at the sun.



Since light is a transverse wave, the direction of the electric field in any component of the sunlight lies in the yz-plane, and the motion of the charges takes place in this plane. There is no field, and hence no motion of charges, in the direction of the x-axis.

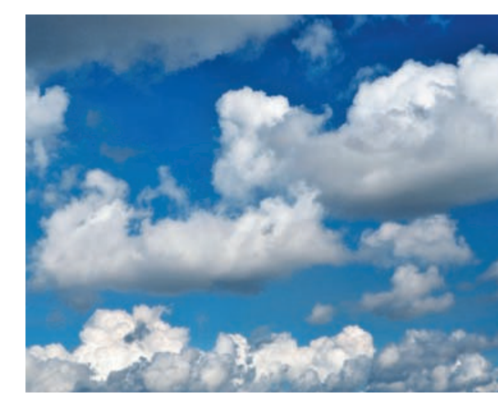
An incident light wave sets the electric charges in the molecules at point O vibrating along the line of $\vec{E}$. We can resolve this vibration into two components, one along the y-axis and the other along the z-axis. Each component in the incident light produces the equivalent of two molecular "antennas," oscillating with the same frequency as the incident light and lying along the y- and z-axes.

We mentioned in Chapter 32 that an oscillating charge, like those in an antenna, does not radiate in the direction of its oscillation. (See Fig. 32.3 in Section 32.1.) Thus the "antenna" along the y-axis does not send any light to the observer directly below it, although it does emit light in other directions. Therefore the only light reaching this observer comes from the other molecular "antenna," corresponding to the oscillation of charge along the z-axis. This light is linearly polarized, with its electric field along the z-axis (parallel to the "antenna"). The red vectors on the y-axis below point O in Fig. 33.32 show the direction of polarization of the light reaching the observer.

As the original beam of sunlight passes though the atmosphere, its intensity decreases as its energy goes into the scattered light. Detailed analysis of the scattering process shows that the intensity of the light scattered from air molecules increases in proportion to the fourth power of the frequency (inversely to the fourth power of the wavelength). Thus the intensity ratio for the two ends of the visible spectrum is $(700\text{ nm}/400\text{ nm})^4 = 9.4$. Roughly speaking, scattered light contains nine times as much blue light as red, and that's why the sky is blue.

Clouds contain a high concentration of water droplets or ice crystals, which also scatter light. Because of this high concentration, light passing through the cloud has many more opportunities for scattering than does light passing through a clear sky. Thus light of *all* wavelengths is eventually scattered out of the cloud, so the cloud looks white (Fig. 33.33). Milk looks white for the same reason; the scattering is due to fat globules in the milk. If you dilute milk by mixing it with enough water, the concentration of fat globules will be so low that only blue light will be substantially scattered; the dilute solution will look blue, not white. (Nonfat milk, which also has a very low concentration of globules, looks somewhat bluish for this same reason.)

33.33 Clouds are white because they efficiently scatter sunlight of all wavelengths.



Near sunset, when sunlight has to travel a long distance through the earth's atmosphere, a substantial fraction of the blue light is removed by scattering. White light minus blue light looks yellow or red. This explains the yellow or red hue that we so often see from the setting sun (and that is seen by the observer at the far right of Fig. 33.32).

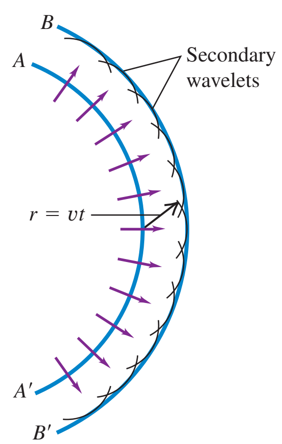
Because skylight is partially polarized, polarizers are useful in photography. The sky in a photograph can be darkened by orienting the polarizer axis to be perpendicular to the predominant direction of polarization of the scattered light. The most strongly polarized light comes from parts of the sky that are 90° away from the sun—for example, from directly overhead when the sun is on the horizon at sunrise or sunset.

33.7 Huygens's Principle

The laws of reflection and refraction of light rays that we introduced in Section 33.2 were discovered experimentally long before the wave nature of light was firmly established. However, we can *derive* these laws from wave considerations and show that they are consistent with the wave nature of light. The same kind of analysis that we use here will be of central importance in Chapters 35 and 36 in our discussion of physical optics.

We begin with a principle called **Huygens's principle.** This principle, stated originally by the Dutch scientist Christiaan Huygens in 1678, is a geometrical method for finding, from the known shape of a wave front at some instant, the shape of the wave front at some later time. Huygens assumed that **every point of a wave front may be considered the source of secondary wavelets that spread out in all directions with a speed equal to the speed of propagation of the wave.** The new wave front at a later time is then found by constructing a surface *tangent* to the secondary wavelets or, as it is called, the *envelope* of the wavelets. All the results that we obtain from Huygens's principle can also be obtained from Maxwell's equations. Thus it is not an independent principle, but it is often very convenient for calculations with wave phenomena.

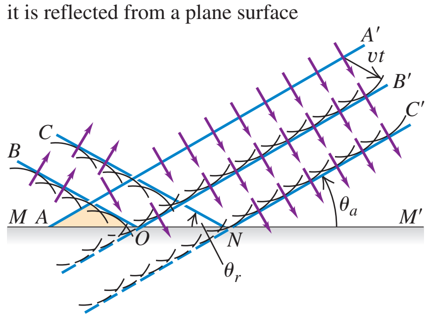
33.34 Applying Huygens's principle to wave front AA' to construct a new wave front BB'.



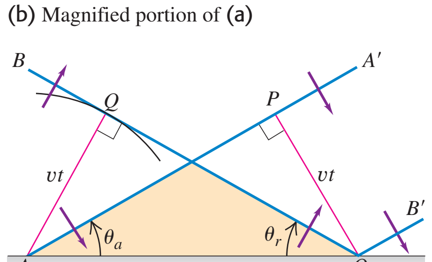
Huygens's principle is shown in Fig. 33.34. The original wave front AA' is traveling outward from a source, as indicated by the arrows. We want to find the shape of the wave front after a time interval t. Let v be the speed of propagation of the wave; then in time t it travels a distance vt. We construct several circles (traces of spherical wavelets) with radius $r = vt$, centered at points along AA'. The trace of the envelope of these wavelets, which is the new wave front, is the curve BB'. We are assuming that the speed v is the same at all points and in all directions.

33.35 Using Huygens's principle to derive the law of reflection.

(a) Successive positions of a plane wave AA' as it is reflected from a plane surface



(b) Magnified portion of (a)



Reflection and Huygens's Principle

To derive the law of reflection from Huygens's principle, we consider a plane wave approaching a plane reflecting surface. In Fig. 33.35a the lines AA', OB', and NC' represent successive positions of a wave front approaching the surface MM'. Point A on the wave front AA' has just arrived at the reflecting surface. We can use Huygens's principle to find the position of the wave front after a time interval t. With points on AA' as centers, we draw several secondary wavelets with radius vt. The wavelets that originate near the upper end of AA' spread out unhindered, and their envelope gives the portion OB' of the new wave front. If the reflecting surface were not there, the wavelets originating near the lower end of AA' would similarly reach the positions shown by the broken circular arcs. Instead, these wavelets strike the reflecting surface.

The effect of the reflecting surface is to *change the direction* of travel of those wavelets that strike it, so the part of a wavelet that would have penetrated the surface actually lies to the left of it, as shown by the full lines. The first such wavelet is centered at point A; the envelope of all such reflected wavelets is the portion OB of the wave front. The trace of the entire wave front at this instant is the bent line BOB'. A similar construction gives the line CNC' for the wave front after another interval t.

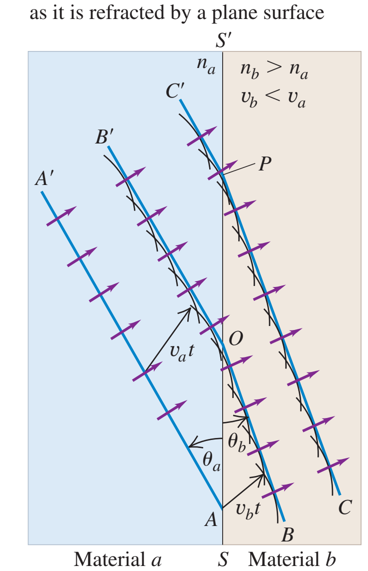
From plane geometry the angle θ_a between the incident *wave front* and the *surface* is the same as that between the incident *ray* and the *normal* to the surface and is therefore the angle of incidence. Similarly, θ_r is the angle of reflection. To find the relationship between these angles, we consider Fig. 33.35b. From O we draw $OP = vt$, perpendicular to AA'. Now OB, by construction, is tangent to a circle of radius vt with center at A. If we draw AQ from A to the point of tangency, the triangles APO and OQA are congruent because they are right triangles with the side AO in common and with $AQ = OP = vt$. The angle θ_a therefore equals the angle θ_r, and we have the law of reflection.

Refraction and Huygens's Principle

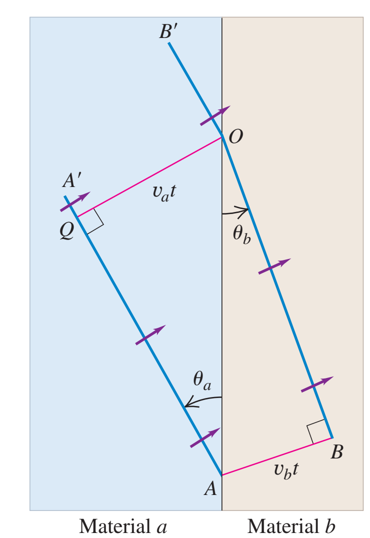
We can derive the law of *refraction* by a similar procedure. In Fig. 33.36a we consider a wave front, represented by line AA', for which point A has just arrived at the boundary surface SS' between two transparent materials a and b, with indexes of refraction n_a and n_b and wave speeds v_a and v_b. (The *reflected* waves are not shown in the figure; they proceed as in Fig. 33.35.) We can apply Huygens's principle to find the position of the refracted wave fronts after a time t.

33.36 Using Huygens's principle to derive the law of refraction. The case $v_b < v_a$ $(n_b > n_a)$ is shown.

(a) Successive positions of a plane wave AA' as it is refracted by a plane surface



(b) Magnified portion of (a)



With points on AA' as centers, we draw several secondary wavelets. Those originating near the upper end of AA' travel with speed v_a and, after a time interval t, are spherical surfaces of radius v_at. The wavelet originating at point A, however, is traveling in the second material b with speed v_b and at time t is a spherical surface of radius v_bt. The envelope of the wavelets from the original wave front is the plane whose trace is the bent line BOB'. A similar construction leads to the trace CPC' after a second interval t.

The angles θ_a and θ_b between the surface and the incident and refracted wave fronts are the angle of incidence and the angle of refraction, respectively. To find the relationship between these angles, refer to Fig. 33.36b. We draw $OQ = v_at$, perpendicular to AQ, and we draw $AB = v_bt$, perpendicular to BO. From the right triangle AOQ,

$$\sin\theta_a = \frac{v_at}{AO}$$

and from the right triangle AOB,

$$\sin\theta_b = \frac{v_bt}{AO}$$

Combining these, we find

$$\frac{\sin\theta_a}{\sin\theta_b} = \frac{v_a}{v_b} \qquad (33.9)$$

We have defined the index of refraction n of a material as the ratio of the speed of light c in vacuum to its speed v in the material: $n_a = c/v_a$ and $n_b = c/v_b$. Thus

$$\frac{n_b}{n_a} = \frac{c/v_b}{c/v_a} = \frac{v_a}{v_b}$$

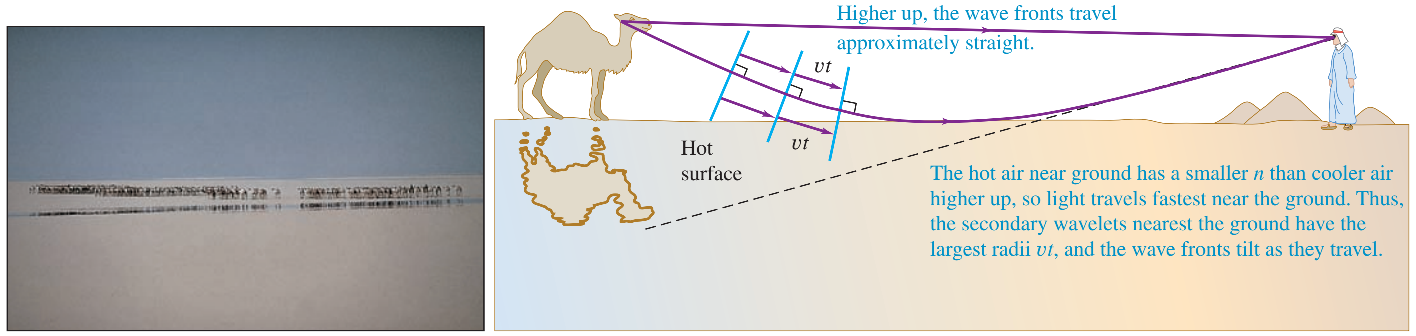
and we can rewrite Eq. (33.9) as

$$\frac{\sin\theta_a}{\sin\theta_b} = \frac{n_b}{n_a} \quad \text{or}$$

$$n_a\sin\theta_a = n_b\sin\theta_b$$

which we recognize as Snell's law, Eq. (33.4). So we have derived Snell's law from a wave theory. Alternatively, we may choose to regard Snell's law as an experimental result that defines the index of refraction of a material; in that case

33.37 How mirages are formed.



this analysis helps to confirm the relationship $v = c/n$ for the speed of light in a material.

Mirages offer an interesting example of Huygens's principle in action. When the surface of pavement or desert sand is heated intensely by the sun, a hot, less dense, smaller-n layer of air forms near the surface. The speed of light is slightly greater in the hotter air near the ground, the Huygens wavelets have slightly larger radii, the wave fronts tilt slightly, and rays that were headed toward the surface with a large angle of incidence (near 90°) can be bent up as shown in Fig. 33.37. Light farther from the ground is bent less and travels nearly in a straight line. The observer sees the object in its natural position, with an inverted image below it, as though seen in a horizontal reflecting surface. Even when the turbulence of the heated air prevents a clear inverted image from being formed, the mind of the thirsty traveler can interpret the apparent reflecting surface as a sheet of water.

It is important to keep in mind that Maxwell's equations are the fundamental relationships for electromagnetic wave propagation. But it is a remarkable fact that Huygens's principle anticipated Maxwell's analysis by two centuries. Maxwell provided the theoretical underpinning for Huygens's principle. Every point in an electromagnetic wave, with its time-varying electric and magnetic fields, acts as a source of the continuing wave, as predicted by Ampere's and Faraday's laws.

Test Your Understanding of Section 33.7 Sound travels faster in warm air than in cold air. Imagine a weather front that runs north-south, with warm air to the west of the front and cold air to the east. A sound wave traveling in a northeast direction in the warm air encounters this front. How will the direction of this sound wave change when it passes into the cold air? (i) The wave direction will deflect toward the north; (ii) the wave direction will deflect toward the east; (iii) the wave direction will be unchanged.

CHAPTER 33 SUMMARY

Light and its properties: Light is an electromagnetic wave. When emitted or absorbed, it also shows particle properties. It is emitted by accelerated electric charges. The speed of light is a fundamental physical constant.

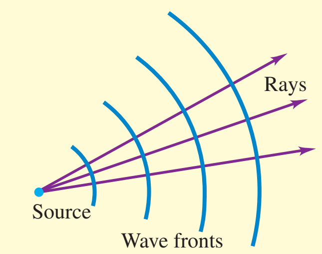
A wave front is a surface of constant phase; wave fronts move with a speed equal to the propagation speed of the wave. A ray is a line along the direction of propagation, perpendicular to the wave fronts. Representation of light by rays is the basis of geometric optics.

When light is transmitted from one material to another, the frequency of the light is unchanged, but the wavelength and wave speed can change. The index of refraction n of a material is the ratio of the speed of light in vacuum c to the speed v in the material. If λ_0 is the wavelength in vacuum, the same wave has a shorter wavelength λ in a medium with index of refraction n. (See Example 33.2.)

The variation of index of refraction n with wavelength λ is called dispersion. Usually n decreases with increasing λ.

$$n = \frac{c}{v} \quad (33.1)$$

$$\lambda = \frac{\lambda_0}{n} \quad (33.5)$$



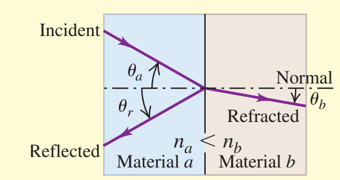
Reflection and refraction: At a smooth interface between two optical materials, the incident, reflected, and refracted rays and the normal to the interface all lie in a single plane called the plane of incidence. The law of reflection states that the angles of incidence and reflection are equal. The law of refraction relates the angles of incidence and refraction to the indexes of refraction of the materials. Angles of incidence, reflection, and refraction are always measured from the normal to the surface. (See Examples 33.1 and 33.3.)

$$\theta_r = \theta_a \quad (33.2)$$

(law of reflection)

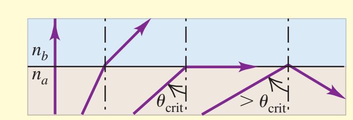
$$n_a \sin\theta_a = n_b \sin\theta_b \quad (33.4)$$

(law of refraction)



Total internal reflection: When a ray travels in a material of greater index of refraction n_a toward a material of smaller index n_b, total internal reflection occurs at the interface when the angle of incidence exceeds a critical angle θ_{crit}. (See Example 33.4.)

$$\sin\theta_{crit} = \frac{n_b}{n_a} \quad (33.6)$$

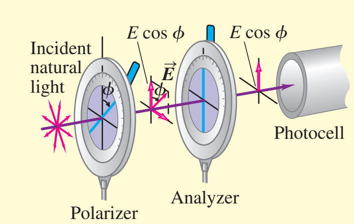


Polarization of light: The direction of polarization of a linearly polarized electromagnetic wave is the direction of the $\vec{E}$ field. A polarizing filter passes waves that are linearly polarized along its polarizing axis and blocks waves polarized perpendicularly to that axis. When polarized light of intensity I_{max} is incident on a polarizing filter used as an analyzer, the intensity I of the light transmitted through the analyzer depends on the angle ϕ between the polarization direction of the incident light and the polarizing axis of the analyzer. (See Example 33.5.)When two linearly polarized waves with a phase difference are superposed, the result is circularly or elliptically polarized light. In this case the $\vec{E}$ vector is not confined to a plane containing the direction of propagation, but rather describes circles or ellipses in planes perpendicular to the propagation direction.

Light is scattered by air molecules. The scattered light is partially polarized.

$$I = I_{max}\cos^2\phi \quad (33.7)$$

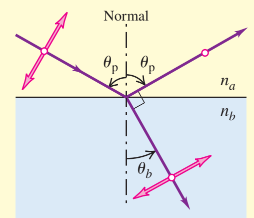
(Malus's law)



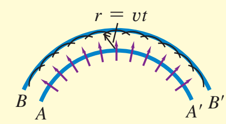
Polarization by reflection: When unpolarized light strikes an interface between two materials, Brewster's law states that the reflected light is completely polarized perpendicular to the plane of incidence (parallel to the interface) if the angle of incidence equals the polarizing angle θ_p. (See Example 33.6.)

$$\tan\theta_p = \frac{n_b}{n_a} \quad (33.8)$$

(Brewster's law)



Huygens's principle: Huygens's principle states that if the position of a wave front at one instant is known, then the position of the front at a later time can be constructed by imagining the front as a source of secondary wavelets. Huygens's principle can be used to derive the laws of reflection and refraction.



Key Terms

optics, *1121*
wave front, *1122*
ray, *1123*
geometric optics, *1123*
physical optics, *1123*
reflection, *1123*
refraction, *1123*
specular reflection, *1124*
diffuse reflection, *1124*
index of refraction (refractive index), *1124*
law of reflection, *1125*
law of refraction (Snell's law), *1125*
critical angle, *1130*
total internal reflection, *1130*
dispersion, *1132*
linear polarization, *1134*
polarizing filter (polarizer), *1134*
unpolarized light (natural light), *1135*
dichroism, *1136*
polarizing axis, *1136*
Malus's law, *1138*
plane of incidence, *1139*
polarizing angle, *1139*
Brewster's law, *1139*
circular polarization, *1141*
elliptical polarization, *1141*
scattering, *1142*
Huygens's principle, *1144*

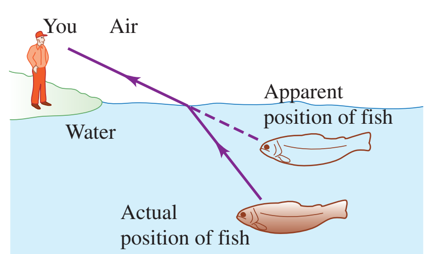
Answer to Chapter Opening Question ?

This is the same effect as shown in Fig. 33.31. The drafting tools are placed between two polarizing filters whose polarizing axes are perpendicular. In places where the clear plastic is under stress, the plastic becomes birefringent; that is, light travels through it at a speed that depends on its polarization. The result is that the light that emerges from the plastic has a different polarization than the light that enters. A spot on the plastic appears bright if the emerging light has the same polarization as the second polarizing filter. The amount of birefringence depends on the wavelength of the light as well as the amount of stress on the plastic, so different colors are seen at different locations on the plastic.

Answers to Test Your Understanding Questions

33.1 Answer: (iii) The waves go farther in the y-direction in a given amount of time than in the other directions, so the wave fronts are elongated in the y-direction.

33.2 Answers: (a) (ii), (b) (iii) As shown in the figure, light rays coming from the fish bend away from the normal when they pass from the water $(n = 1.33)$ into the air $(n = 1.00)$. As a result, the fish appears to be higher in the water than it actually is. Hence you should aim a spear *below* the apparent position of the fish. If you use a laser beam, you should aim *at* the apparent position of the fish: The beam of laser light takes the same path from you to the fish as ordinary light takes from the fish to you (though in the opposite direction).



33.3 Answers: (i), (ii) Total internal reflection can occur only if two conditions are met: n_b must be less than n_a, and the critical angle θ_{crit} (where $\sin\theta_{crit} = n_b/n_a$) must be smaller than the angle of incidence θ_a. In the first two cases both conditions are met: The critical angles are (i) $\theta_{crit} = \sin^{-1}(1/1.33) = 48.8°$ and (ii) $\theta_{crit} = \sin^{-1}(1.33/1.52) = 61.0°$, both of which are smaller than $\theta_a = 70°$. In the third case $n_b = 1.52$ is greater than $n_a = 1.33$, so total internal reflection cannot occur for any incident angle.

33.5 Answer: (ii) The sunlight reflected from the windows of the high-rise building is partially polarized in the vertical direction, since each window lies in a vertical plane. The Polaroid filter in front of the lens is oriented with its polarizing axis perpendicular to the dominant direction of polarization of the reflected light.

33.7 Answer: (ii) Huygens's principle applies to waves of all kinds, including sound waves. Hence this situation is exactly like that shown in Fig. 33.36, with material a representing the warm air, material b representing the cold air in which the waves travel more slowly, and the interface between the materials representing the weather front. North is toward the top of the figure and east is toward the right, so Fig. 33.36 shows that the rays (which indicate the direction of propagation) deflect toward the east.

PROBLEMS

Discussion Questions

Q33.1. Light requires about 8 minutes to travel from the sun to the earth. Is it delayed appreciably by the earth's atmosphere? Explain.

Q33.2. Sunlight or starlight passing through the earth's atmosphere is always bent toward the vertical. Why? Does this mean that a star is not really where it appears to be? Explain.

Q33.3. A beam of light goes from one material into another. On *physical* grounds, explain *why* the wavelength changes but the frequency and period do not.

Q33.4. A student claimed that, because of atmospheric refraction (see Discussion Question Q33.2), the sun can be seen after it has set and that the day is therefore longer than it would be if the earth had no atmosphere. First, what does she mean by saying that the sun can be seen after it has set? Second, comment on the validity of her conclusion.

Q33.5. When hot air rises from a radiator or heating duct, objects behind it appear to shimmer or waver. What causes this?

Q33.6. Devise straightforward experiments to measure the speed of light in a given glass using (a) Snell's law; (b) total internal reflection; (c) Brewster's law.

Q33.7. Sometimes when looking at a window, you see two reflected images slightly displaced from each other. What causes this?

Q33.8. If you look up from underneath toward the surface of the water in your aquarium, you may see an upside-down reflection of your pet fish in the surface of the water. Explain how this can happen.

Q33.9. A ray of light in air strikes a glass surface. Is there a range of angles for which total reflection occurs? Explain.

Q33.10. When light is incident on an interface between two materials, the angle of the refracted ray depends on the wavelength, but the angle of the reflected ray does not. Why should this be?

Q33.11. A salesperson at a bargain counter claims that a certain pair of sunglasses has Polaroid filters; you suspect that the glasses are just tinted plastic. How could you find out for sure?

Q33.12. Does it make sense to talk about the polarization of a *longitudinal* wave, such as a sound wave? Why or why not?

Q33.13. How can you determine the direction of the polarizing axis of a single polarizer?

Q33.14. It has been proposed that automobile windshields and headlights should have polarizing filters to reduce the glare of oncoming lights during night driving. Would this work? How should the polarizing axes be arranged? What advantages would this scheme have? What disadvantages?

Q33.15. When a sheet of plastic food wrap is placed between two crossed polarizers, no light is transmitted. When the sheet is stretched in one direction, some light passes through the crossed polarizers. What is happening?

Q33.16. If you sit on the beach and look at the ocean through Polaroid sunglasses, the glasses help to reduce the glare from sunlight reflecting off the water. But if you lie on your side on the beach, there is little reduction in the glare. Explain why there is a difference.

Q33.17. When unpolarized light is incident on two crossed polarizers, no light is transmitted. A student asserted that if a third polarizer is inserted between the other two, some transmission will occur. Does this make sense? How can adding a third filter *increase* transmission?

Q33.18. For the old "rabbit-ear" style TV antennas, it's possible to alter the quality of reception considerably simply by changing the orientation of the antenna. Why?

Q33.19. In Fig. 33.32, since the light that is scattered out of the incident beam is polarized, why is the transmitted beam not also partially polarized?

Q33.20. You are sunbathing in the late afternoon when the sun is relatively low in the western sky. You are lying flat on your back, looking straight up through Polaroid sunglasses. To minimize the amount of sky light reaching your eyes, how should you lie: with your feet pointing north, east, south, west, or in some other direction? Explain your reasoning.

Q33.21. Light scattered from blue sky is strongly polarized because of the nature of the scattering process described in Section 33.6. But light scattered from white clouds is usually *not* polarized. Why not?

Q33.22. Atmospheric haze is due to water droplets or smoke particles ("smog"). Such haze reduces visibility by scattering light, so that the light from distant objects becomes randomized and images become indistinct. Explain why visibility through haze can be improved by wearing red-tinted sunglasses, which filter out blue light.

Q33.23. The explanation given in Section 33.6 for the color of the setting sun should apply equally well to the *rising* sun, since sunlight travels the same distance through the atmosphere to reach your eyes at either sunrise or sunset. Typically, however, sunsets are redder than sunrises. Why? (*Hint:* Particles of all kinds in the atmosphere contribute to scattering.)

Q33.24. Huygens's principle also applies to sound waves. During the day, the temperature of the atmosphere decreases with increasing altitude above the ground. But at night, when the ground cools, there is a layer of air just above the surface in which the temperature *increases* with altitude. Use this to explain why sound waves from distant sources can be heard more clearly at night than in the daytime. (*Hint:* The speed of sound increases with increasing temperature. Use the ideas displayed in Fig. 33.37 for light.)

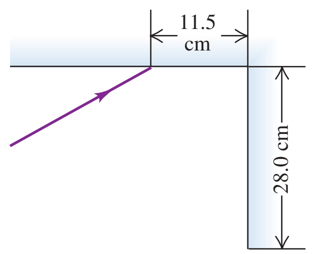
Q33.25. Can water waves be reflected and refracted? Give examples. Does Huygens's principle apply to water waves? Explain.

Exercises

Section 33.2 Reflection and Refraction

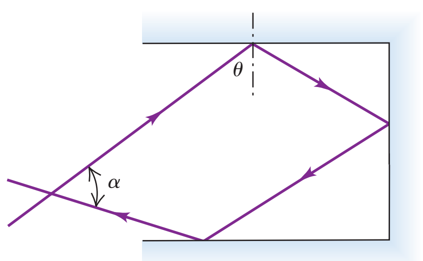
33.1. Two plane mirrors intersect at right angles. A laser beam strikes the first of them at a point 11.5 cm from their point of intersection, as shown in Fig. 33.38 For what angle of incidence at the first mirror will this ray strike the midpoint of the second mirror (which is 28.0 cm long) after reflecting from the first mirror?

Figure 33.38 Exercise 33.1.



33.2. Three plane mirrors intersect at right angles. A beam of laser light strikes the first of them at an angle θ with respect to the normal (Fig. 33.39). (a) Show that when this ray is reflected off of the other two mirrors and crosses the original ray, the angle α between these two rays will be $\alpha = 180° - 2\theta$. (b) For what angle θ will the two rays be perpendicular when they cross?

Figure 33.39 Exercise 33.2



33.3. A beam of light has a wavelength of 650 nm in vacuum. (a) What is the speed of this light in a liquid whose index of refraction at this wavelength is 1.47? (b) What is the wavelength of these waves in the liquid?

33.4. Light with a frequency of 5.80×10^{14} Hz travels in a block of glass that has an index of refraction of 1.52. What is the wavelength of the light (a) in vacuum and (b) in the glass?

33.5. A light beam travels at 1.94×10^{8} m/s in quartz. The wavelength of the light in quartz is 355 nm. (a) What is the index of refraction of quartz at this wavelength? (b) If this same light travels through air, what is its wavelength there?

33.6. Light of a certain frequency has a wavelength of 438 nm in water. What is the wavelength of this light in benzene?

33.7. A parallel beam of light in air makes an angle of 47.5° with the surface of a glass plate having a refractive index of 1.66. (a) What is the angle between the reflected part of the beam and the surface of the glass? (b) What is the angle between the refracted beam and the surface of the glass?

33.8. Using a fast-pulsed laser and electronic timing circuitry, you find that light travels 2.50 m within a plastic rod in 11.5 ns. What is the refractive index of the plastic?

33.9. Light traveling in air is incident on the surface of a block of plastic at an angle of 62.7° to the normal and is bent so that it makes a 48.1° angle with the normal in the plastic. Find the speed of light in the plastic.

33.10. (a) A tank containing methanol has walls 2.50 cm thick made of glass of refractive index 1.550. Light from the outside air strikes the glass at a 41.3° angle with the normal to the glass. Find the angle the light makes with the normal in the methanol. (b) The tank is emptied and refilled with an unknown liquid. If light incident at the same angle as in part (a) enters the liquid in the tank at an angle of 20.2° from the normal, what is the refractive index of the unknown liquid?

33.11. (a) Light passes through three parallel slabs of different thicknesses and refractive indexes. The light is incident in the first slab and finally refracts into the third slab. Show that the middle slab has no effect on the final direction of the light. That is, show that the direction of the light in the third slab is the same as if the light had passed directly from the first slab into the third slab. (b) Generalize this result to a stack of N slabs. What determines the final direction of the light in the last slab?

33.12. A horizontal, parallel-sided plate of glass having a refractive index of 1.52 is in contact with the surface of water in a tank. A ray coming from above in air makes an angle of incidence of 35.0° with the normal to the top surface of the glass. (a) What angle does the ray refracted into the water make with the normal to the surface? (b) What is the dependence of this angle on the refractive index of the glass?

33.13. In a material having an index of refraction n, a light ray has frequency f, wavelength λ, and speed v. What are the frequency, wavelength, and speed of this light (a) in vacuum and (b) in a material having refractive index n'? In each case, express your answers in terms of *only* f, λ, v, n, and n'.

33.14. Prove that a ray of light reflected from a plane mirror rotates through an angle of 2θ when the mirror rotates through an angle θ about an axis perpendicular to the plane of incidence.

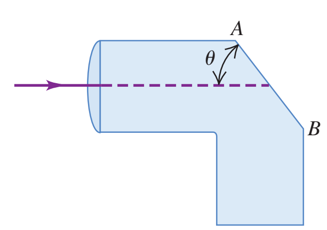
33.15. A ray of light is incident on a plane surface separating two sheets of glass with refractive indexes 1.70 and 1.58. The angle of incidence is 62.0°, and the ray originates in the glass with $n = 1.70$. Compute the angle of refraction.

33.16. In Example 33.1 the water–glass interface is horizontal. If instead this interface were tilted 15.0° above the horizontal, with the right side higher than the left side, what would be the angle from the vertical of the ray in the glass? (The ray in the water still makes an angle of 60.0° with the vertical.)

Section 33.3 Total Internal Reflection

33.17. Light Pipe. Light enters a solid pipe made of plastic having an index of refraction of 1.60. The light travels parallel to the upper part of the pipe (Fig. 33.40). You want to cut the face AB so that all the light will reflect back into the pipe after it first strikes that face. (a) What is the largest that θ can be if the pipe is in air? (b) If the pipe is immersed in water of refractive index 1.33, what is the largest that θ can be?

Figure 33.40 Exercise 33.17



33.18. A beam of light is traveling inside a solid glass cube having index of refraction 1.53. It strikes the surface of the cube from the inside. (a) If the cube is in air, at what minimum angle with the normal inside the glass will this light *not* enter the air at this surface? (b) What would be the minimum angle in part (a) if the cube were immersed in water?

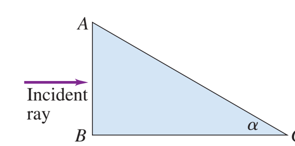
33.19. The critical angle for total internal reflection at a liquid–air interface is 42.5°. (a) If a ray of light traveling in the liquid has an angle of incidence at the interface of 35.0°, what angle does the refracted ray in the air make with the normal? (b) If a ray of light traveling in air has an angle of incidence at the interface of 35.0°, what angle does the refracted ray in the liquid make with the normal?

33.20. At the very end of Wagner's series of operas *Ring of the Nibelung,* Brünnhilde takes the golden ring from the finger of the dead Siegfried and throws it into the Rhine, where it sinks to the bottom of the river. Assuming that the ring is small enough compared to the depth of the river to be treated as a point and that the Rhine is 10.0 m deep where the ring goes in, what is the area of the largest circle at the surface of the water over which light from the ring could escape from the water?

33.21. A ray of light is traveling in a glass cube that is totally immersed in water. You find that if the ray is incident on the glass–water interface at an angle to the normal larger than 48.7°, no light is refracted into the water. What is the refractive index of the glass?

33.22. Light is incident along the normal on face AB of a glass prism of refractive index 1.52, as shown in Fig. 33.41. Find the largest value the angle α can have without any light refracted out of the prism at face AC if (a) the prism is immersed in air and (b) the prism is immersed in water.

Figure 33.41 Exercise 33.22.



33.23. A ray of light in diamond (index of refraction 2.42) is incident on an interface with air. What is the *largest* angle the ray can make with the normal and not be totally reflected back into the diamond?

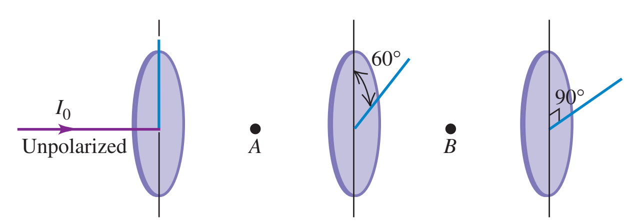
Section 33.4 Dispersion

33.24. A beam of light strikes a sheet of glass at an angle of 57.0° with the normal in air. You observe that red light makes an angle of 38.1° with the normal in the glass, while violet light makes a 36.7° angle. (a) What are the indexes of refraction of this glass for these colors of light? (b) What are the speeds of red and violet light in the glass?

Section 33.5 Polarization

33.25. A beam of unpolarized light of intensity I_0 passes through a series of ideal polarizing filters with their polarizing directions turned to various angles as shown in Fig. 33.42. (a) What is the light intensity (in terms of I_0) at points A, B, and C? (b) If we remove the middle filter, what will be the light intensity at point C?

Figure 33.42 Exercise 33.25.

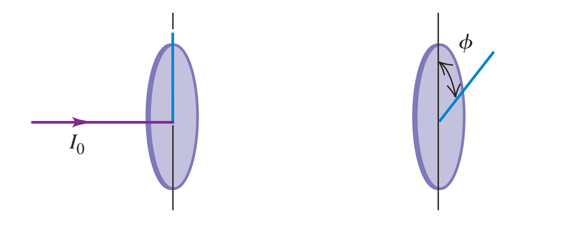


33.26. Light traveling in water strikes a glass plate at an angle of incidence of 53.0°; part of the beam is reflected and part is refracted. If the reflected and refracted portions make an angle of 90.0° with each other, what is the index of refraction of the glass?

33.27. A parallel beam of unpolarized light in air is incident at an angle of 54.5° (with respect to the normal) on a plane glass surface. The reflected beam is completely linearly polarized. (a) What is the refractive index of the glass? (b) What is the angle of refraction of the transmitted beam?

33.28. Light of original intensity I_0 passes through two ideal polarizing filters having their polarizing axes oriented as shown in Fig. 33.43. You want to adjust the angle ϕ so that the intensity at point P is equal to $I_0/10$. (a) If the original light is unpolarized, what should ϕ be? (b) If the original light is linearly polarized in the same direction as the polarizing axis of the first polarizer the light reaches, what should ϕ be?

Figure 33.43 Exercise 33.28.



33.29. A beam of polarized light passes through a polarizing filter. When the angle between the polarizing axis of the filter and the direction of polarization of the light is θ, the intensity of the emerging beam is I. If you now want the intensity to be $I/2$, what should be the angle (in terms of θ) between the polarizing angle of the filter and the original direction of polarization of the light?

33.30. The refractive index of a certain glass is 1.66. For what incident angle is light reflected from the surface of this glass completely polarized if the glass is immersed in (a) air and (b) water?

33.31. Unpolarized light of intensity 20.0 W/cm^2 is incident on two polarizing filters. The axis of the first filter is at an angle of 25.0° counterclockwise from the vertical (viewed in the direction the light is traveling), and the axis of the second filter is at 62.0° counterclockwise from the vertical. What is the intensity of the light after it has passed through the second polarizer?

33.32. A polarizer and an analyzer are oriented so that the maximum amount of light is transmitted. To what fraction of its maximum value is the intensity of the transmitted light reduced when the analyzer is rotated through (a) 22.5°; (b) 45.0°; (c) 67.5°?

33.33. Three Polarizing Filters. Three polarizing filters are stacked with the polarizing axes of the second and third at 45.0° and 90.0°, respectively, with that of the first. (a) If unpolarized light of intensity I_0 is incident on the stack, find the intensity and state of polarization of light emerging from each filter. (b) If the second filter is removed, what is the intensity of the light emerging from each remaining filter?

33.34. Three polarizing filters are stacked, with the polarizing axis of the second and third filters at 23.0° and 62.0°, respectively, to that of the first. If unpolarized light is incident on the stack, the light has intensity 75.0 W/cm^2 after it passes through the stack. If the incident intensity is kept constant, what is the intensity of the light after it has passed through the stack if the second polarizer is removed?

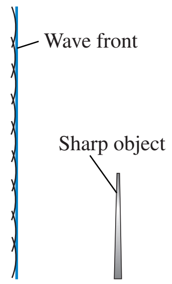
*Section 33.6 Scattering of Light

33.35. A beam of white light passes through a uniform thickness of air. If the intensity of the scattered light in the middle of the green part of the visible spectrum is I, find the intensity (in terms of I) of scattered light in the middle of (a) the red part of the spectrum and (b) the violet part of the spectrum. Consult Table 32.1.

Section 33.7 Huygens's Principle

33.36. Bending Around Corners. Traveling particles do not bend around corners, but waves do. To see why, suppose that a plane wave front strikes the edge of a sharp object traveling perpendicular to the surface (Fig. 33.44). Use Huygens's principle to show that this wave will bend around the upper edge of the object. (*Note:* This effect, called *diffraction,* can easily be seen for water waves, but it also occurs for light, as you will see in Chapters 35 and 36. However due to the very short wavelength of visible light, it is not so apparent in daily life.)

Figure 33.44 Exercise 33.36.



Problems

33.37. The Corner Reflector. An inside corner of a cube is lined with mirrors to make a corner reflector (see Example 33.3 in Section 33.2). A ray of light is reflected successively from each of three mutually perpendicular mirrors; show that its final direction is always exactly opposite to its initial direction.

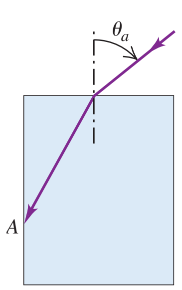
33.38. A light beam is directed parallel to the axis of a hollow cylindrical tube. When the tube contains only air, it takes the light 8.72 ns to travel the length of the tube, but when the tube is filled with a transparent jelly, it takes the light 2.04 ns longer to travel its length. What is the refractive index of this jelly?

33.39. Light traveling in a material of refractive index n_1 is incident at angle θ_1 with respect to the normal at the interface with a slab of material that has parallel faces and refractive index n_2. After the light passes through this material, it is refracted into a material with refractive index n_3 and in this third material it makes an angle of θ_3 with the normal. (a) Find θ_3 in terms of θ_1 and the refractive indexes of the materials. (b) The ray in the third material is now reversed, so that it is incident on the n_3-to-n_2 interface with the angle θ_3 found in part (a). Show that when the light refracts into the material with refractive index n_1, the angle it makes with the normal is angle θ_1. This shows that the refracted ray is reversible. (c) Are reflected rays also reversible? Explain.

33.40. In a physics lab, light with wavelength 490 nm travels in air from a laser to a photocell in 17.0 ns. When a slab of glass 0.840 m thick is placed in the light beam, with the beam incident along the normal to the parallel faces of the slab, it takes the light 21.2 ns to travel from the laser to the photocell. What is the wavelength of the light in the glass?

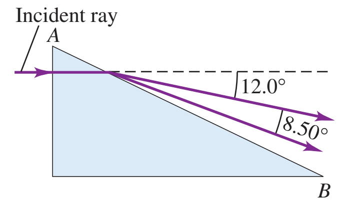
33.41. A ray of light is incident in air on a block of a transparent solid whose index of refraction is n. If $n = 1.38$, what is the *largest* angle of incidence θ_a for which total internal reflection will occur at the vertical face (point A shown in Fig. 33.45)?

Figure 33.45 Problem 33.41.



33.42. A light ray in air strikes the right-angle prism shown in Fig. 33.46. This ray consists of two different wavelengths. When it emerges at face AB, it has been split into two different rays that diverge from each other by 8.50°. Find the index of refraction of the prism for each of the two wavelengths.

Figure 33.46 Problem 33.42.



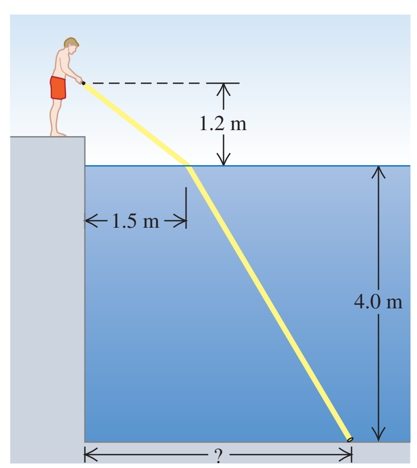
33.43. A quarter-wave plate converts linearly polarized light to circularly polarized light. Prove that a quarter-wave plate also converts circularly polarized light to linearly polarized light.

33.44. A glass plate 2.50 mm thick, with an index of refraction of 1.40, is placed between a point source of light with wavelength 540 nm (in vacuum) and a screen. The distance from source to screen is 1.80 cm. How many wavelengths are there between the source and the screen?

33.45. Old photographic plates were made of glass with a light-sensitive emulsion on the front surface. This emulsion was somewhat transparent. When a bright point source is focused on the front of the plate, the developed photograph will show a halo around the image of the spot. If the glass plate is 3.10 mm thick and the halos have an inner radius of 5.34 mm, what is the index of refraction of the glass? (*Hint:* Light from the spot on the front surface is scattered in all directions by the emulsion. Some of it is then totally reflected at the back surface of the plate and returns to the front surface.)

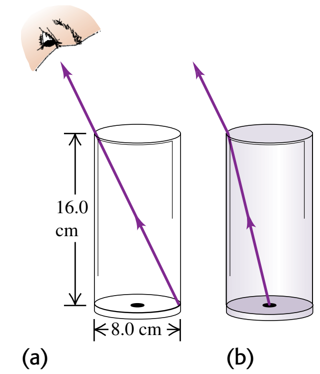
33.46. After a long day of driving you take a late-night swim in a motel swimming pool. When you go to your room, you realize that you have lost your room key in the pool. You borrow a powerful flashlight and walk around the pool, shining the light into it. The light shines on the key, which is lying on the bottom of the pool, when the flashlight is held 1.2 m above the water surface and is directed at the surface a horizontal distance of 1.5 m from the edge (Fig. 33.47). If the water here is 4.0 m deep, how far is the key from the edge of the pool?

Figure 33.47 Problem 33.46.



33.47. You sight along the rim of a glass with vertical sides so that the top rim is lined up with the opposite edge of the bottom (Fig. 33.48a). The glass is a thin-walled, hollow cylinder 16.0 cm high with a top and bottom of the glass diameter of 8.0 cm. While you keep your eye in the same position, a friend fills the glass with a transparent liquid, and you then see a dime that is lying at the center of the bottom of the glass (Fig. 33.48b). What is the index of refraction of the liquid?

Figure 33.48 Problem 33.47.



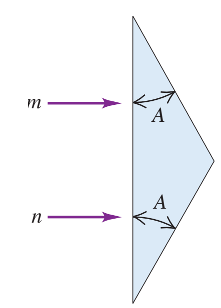
33.48. A beaker with a mirrored bottom is filled with a liquid whose index of refraction is 1.63. A light beam strikes the top surface of the liquid at an angle of 42.5° from the normal. At what angle from the normal will the beam exit from the liquid after traveling down through the liquid, reflecting from the mirrored bottom, and returning to the surface?

33.49. A thin layer of ice $(n = 1.309)$ floats on the surface of water $(n = 1.333)$ in a bucket. A ray of light from the bottom of the bucket travels upward through the water. (a) What is the largest angle with respect to the normal that the ray can make at the ice–water interface and still pass out into the air above the ice? (b) What is this angle after the ice melts?

33.50. A 45°−45°−90° prism is immersed in water. A ray of light is incident normally on one of its shorter faces. What is the minimum index of refraction that the prism must have if this ray is to be totally reflected within the glass at the long face of the prism?

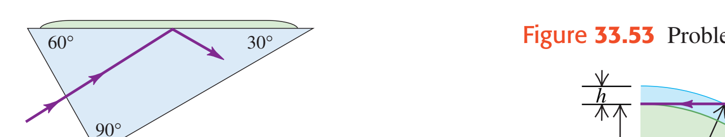
33.51. The prism shown in Fig. 33.49 has a refractive index of 1.66, and the angles A are 25.0°. Two light rays m and n are parallel as they enter the prism. What is the angle between them after they emerge?

Figure 33.49 Problem 33.51.



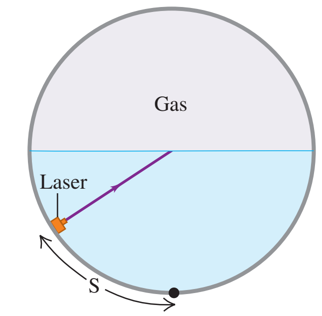
33.52. Light is incident normally on the short face of a 30°−60°−90° prism (Fig. 33.50). A drop of liquid is placed on the hypotenuse of the prism. If the index of the prism is 1.62, find the maximum index that the liquid may have if the light is to be totally reflected.

Figure 33.50 Problem 33.52.



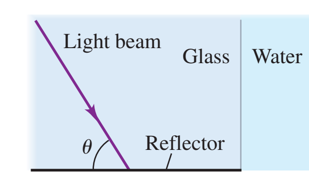
33.53. A horizontal cylindrical tank 2.20 m in diameter is half full of water. The space above the water is filled with a pressurized gas of unknown refractive index. A small laser can move along the curved bottom of the water and aims a light beam toward the center of the water surface (Fig. 33.51). You observe that when the laser has moved a distance $S = 1.09$ m or more (measured along the curved surface) from the lowest point in the water, no light enters the gas. (a) What is the index of refraction of the gas? (b) How long does it take the light beam to travel from the laser to the rim of the tank when (i) $S > 1.09$ m and (ii) $S < 1.09$ m?

Figure 33.51 Problem 33.53.



33.54. A large cube of glass has a metal reflector on one face and water on an adjoining face (Fig. 33.52). A light beam strikes the reflector, as shown. You observe that as you gradually increase the angle of the light beam, if $\theta \geq 59.2°$ no light enters the water. What is the speed of light in this glass?

Figure 33.52 Problem 33.54.

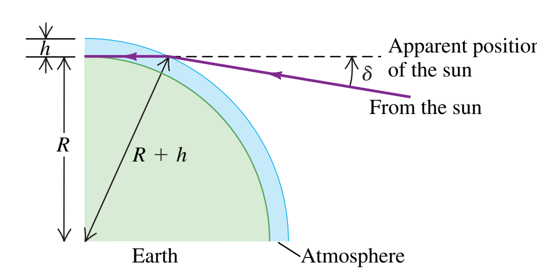


33.55. When the sun is either rising or setting and appears to be just on the horizon, it is in fact *below* the horizon. The explanation for this seeming paradox is that light from the sun bends slightly when entering the earth's atmosphere, as shown in Fig. 33.53. Since our perception is based on the idea that light travels in straight lines, we perceive the light to be coming from an apparent position that is an angle δ above the sun's true position. (a) Make the simplifying assumptions that the atmosphere has uniform density, and hence uniform index of refraction n, and extends to a height h above the earth's surface, at which point it abruptly stops. Show that the angle δ is given by

$$\delta = \arcsin\left(\frac{nR}{R + h}\right) - \arcsin\left(\frac{R}{R + h}\right)$$

where $R = 6378$ km is the radius of the earth. (b) Calculate δ using $n = 1.0003$ and $h = 20$ km. How does this compare to the

Figure 33.53 Problem 33.55.



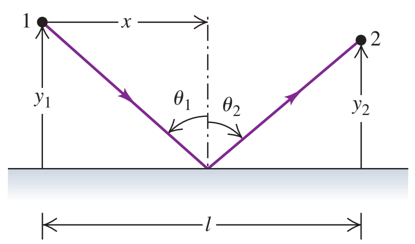
angular radius of the sun, which is about one quarter of a degree? (In actuality a light ray from the sun bends gradually, not abruptly, since the density and refractive index of the atmosphere change gradually with altitude.)

33.56. Fermat's Principle of Least Time. A ray of light traveling with speed c leaves point 1 shown in Fig. 33.54 and is reflected to point 2. The ray strikes the reflecting surface a horizontal distance x from point 1. (a) Show that the time t required for the light to travel from 1 to 2 is

$$t = \frac{\sqrt{y_1^2 + x^2} + \sqrt{y_2^2 + (l - x)^2}}{c}$$

(b) Take the derivative of t with respect to x. Set the derivative equal to zero to show that this time reaches its *minimum* value when $\theta_1 = \theta_2$, which is the law of reflection and corresponds to the actual path taken by the light. This is an example of Fermat's *principle of least time,* which states that among all possible paths between two points, the one actually taken by a ray of light is that for which the time of travel is a *minimum.* (In fact, there are some cases in which the time is a maximum rather than a minimum.)

Figure 33.54 Problem 33.56.

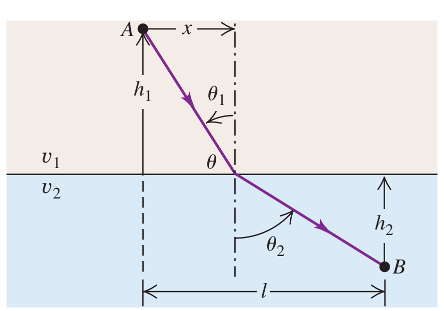


33.57. A ray of light goes from point A in a medium in which the speed of light is v_1 to point B in a medium in which the speed is v_2 (Fig. 33.55). The ray strikes the interface a horizontal distance x to the right of point A. (a) Show that the time required for the light to go from A to B is

$$t = \frac{\sqrt{h_1^2 + x^2}}{v_1} + \frac{\sqrt{h_2^2 + (l - x)^2}}{v_2}$$

(b) Take the derivative of t with respect to x. Set this derivative equal to zero to show that this time reaches its *minimum* value when $n_1 \sin\theta_1 = n_2 \sin\theta_2$. This is Snell's law, and corresponds to the actual path taken by the light. This is another example of Fermat's principle of least time (see Problem 33.56).

Figure 33.55 Problem 33.57.

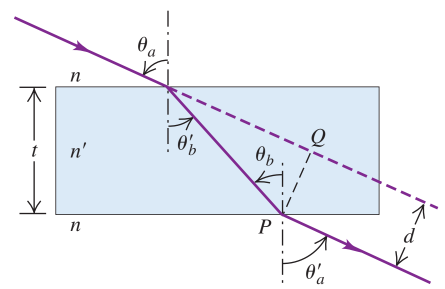


33.58. Light is incident in air at an angle θ_a (Fig. 33.56) on the upper surface of a transparent plate, the surfaces of the plate being plane and parallel to each other. (a) Prove that $\theta_a = \theta'_a$. (b) Show that this is true for any number of different parallel plates. (c) Prove that the lateral displacement d of the emergent beam is given by the relationship

$$d = t\frac{\sin(\theta_a - \theta'_b)}{\cos\theta'_b}$$

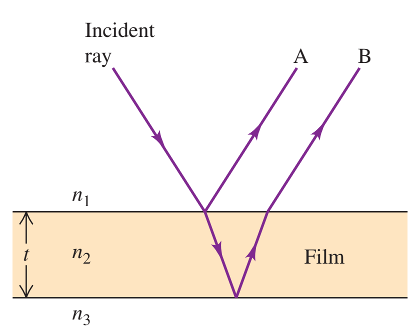
where t is the thickness of the plate. (d) A ray of light is incident at an angle of 66.0° on one surface of a glass plate 2.40 cm thick with an index of refraction 1.80. The medium on either side of the plate is air. Find the lateral displacement between the incident and emergent rays.

Figure 33.56 Problem 33.58.



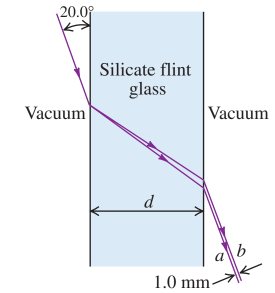
33.59. Light traveling downward is incident on a horizontal film of thickness t, as shown in Fig. 33.57. The incident ray splits into two rays, A and B. Ray A reflects from the top of the film. Ray B reflects from the bottom of the film and then refracts back into the material that is above the film. If the film has parallel faces, show that rays A and B end up parallel to each other.

Figure 33.57 Problem 33.59.



33.60. A thin beam of white light is directed at a flat sheet of silicate flint glass at an angle of 20.0° to the surface of the sheet. Due to dispersion in the glass, the beam is spread out as shown in a spectrum in Fig. 33.58. The refractive index of silicate flint glass versus wavelength is graphed in Fig. 33.18. (a) The rays a and b shown in Fig. 33.58 correspond to the extremes of the visible spectrum. Which corresponds to red and which to violet? Explain your reasoning. (b) For what thickness d of the glass sheet will the spectrum be 1.0 mm wide, as shown (see Problem 33.58)?

Figure 33.58 Problem 33.60.

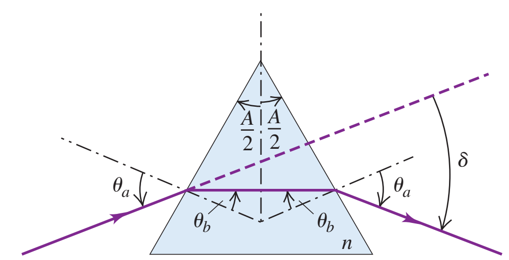


33.61. Angle of Deviation. The incident angle θ_a shown in Fig. 33.59 is chosen so that the light passes symmetrically through the prism, which has refractive index n and apex angle A. (a) Show that the angle of deviation δ (the angle between the initial and final directions of the ray) is given by

$$\sin\frac{A + \delta}{2} = n\sin\frac{A}{2}$$

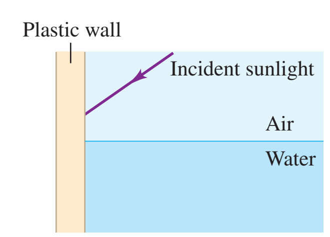
(When the light passes through symmetrically, as shown, the angle of deviation is a minimum.) (b) Use the result of part (a) to find the angle of deviation for a ray of light passing symmetrically through a prism having three equal angles $(A = 60.0°)$ and $n = 1.52$. (c) A certain glass has a refractive index of 1.61 for red light (700 nm) and 1.66 for violet light (400 nm). If both colors pass through symmetrically, as described in part (a), and if $A = 60.0°$, find the difference between the angles of deviation for the two colors.

Figure 33.59 Problem 33.61.



33.62. A beam of unpolarized sunlight strikes the vertical plastic wall of a water tank at an unknown angle. Some of the light reflects from the wall and enters the water (Fig. 33.60). The refractive index of the plastic wall is 1.61. If the light that has been reflected from the wall into the water is observed to be completely polarized, what angle does this beam make with the normal inside the water?

Figure 33.60 Problem 33.62.



33.63. A beam of light traveling horizontally is made of an unpolarized component with intensity I_0 and a polarized component with intensity I_p. The plane of polarization of the polarized component is oriented at an angle of θ with respect to the vertical. The data in the table give the intensity measured through a polarizer with an orientation of ϕ with respect to the vertical. (a) What is the orientation of the polarized component? (That is, what is the angle θ?) (b) What are the values of I_0 and I_p?

ϕ (°)	I_{total} (W/m²)	ϕ (°)	I_{total} (W/m²)
0	18.4	100	8.6
10	21.4	110	6.3
20	23.7	120	5.2
30	24.8	130	5.2
40	24.8	140	6.3
50	23.7	150	8.6
60	21.4	160	11.6
70	18.4	170	15.0
80	15.0	180	18.4
90	11.6		

33.64. A certain birefringent material has indexes of refraction n_1 and n_2 for the two perpendicular components of linearly polarized light passing through it. The corresponding wavelengths are $\lambda_1 = \lambda_0/n_1$ and λ_0/n_2, where λ_0 is the wavelength in vacuum. (a) If the crystal is to function as a quarter-wave plate, the number of wavelengths of each component within the material must differ by $\frac{1}{4}$. Show that the minimum thickness for a quarter-wave plate is

$$d = \frac{\lambda_0}{4(n_1 - n_2)}$$

(b) Find the minimum thickness of a quarter-wave plate made of siderite $(FeO \cdot CO_2)$ if the indexes of refraction are $n_1 = 1.875$ and $n_2 = 1.635$ and the wavelength in vacuum is $\lambda_0 = 589$ nm.

Challenge Problems

33.65. Consider two vibrations of equal amplitude and frequency but differing in phase, one along the x-axis,

$$x = a\sin(\omega t - \alpha)$$

and the other along the y-axis,

$$y = a\sin(\omega t - \beta)$$

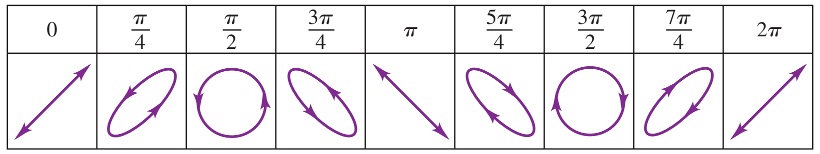
These can be written as follows:

$$\frac{x}{a} = \sin\omega t\cos\alpha - \cos\omega t\sin\alpha \qquad (1)$$

$$\frac{y}{a} = \sin\omega t\cos\beta - \cos\omega t\sin\beta \qquad (2)$$

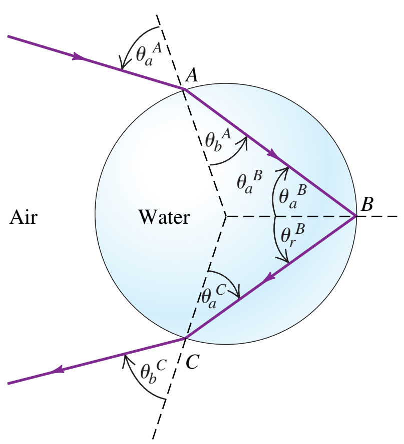
(a) Multiply Eq. (1) by $\sin\beta$ and Eq. (2) by $\sin\alpha$, and then subtract the resulting equations. (b) Multiply Eq. (1) by $\cos\beta$ and Eq. (2) by $\cos\alpha$, and then subtract the resulting equations. (c) Square and add the results of parts (a) and (b). (d) Derive the equation $x^2 + y^2 - 2xy\cos\delta = a^2\sin^2\delta$, where $\delta = \alpha - \beta$. (e) Use the above result to justify each of the diagrams in Fig. 33.61 (next page). In the figure, the angle given is the phase difference between two simple harmonic motions of the same frequency and amplitude, one horizontal (along the x-axis) and the other vertical (along the y-axis). The figure thus shows the resultant motion from the superposition of the two perpendicular harmonic motions.

Figure 33.61 Challenge Problem 33.65.



33.66. A rainbow is produced by the reflection of sunlight by spherical drops of water in the air. Figure 33.62 shows a ray that refracts into a drop at point A, is reflected from the back surface of the drop at point B, and refracts back into the air at point C. The angles of incidence and refraction, θ_a and θ_b, are shown at points A and C, and the angles of incidence and reflection, θ_a and θ_r, are shown at point B. (a) Show that $\theta_a^B = \theta_b^A$, $\theta_a^C = \theta_b^A$, and $\theta_b^C = \theta_a^A$. (b) Show that the angle in radians between the ray before it enters the drop at A and after it exits at C (the total angular deflection of the ray) is $\Delta = 2\theta_a^A - 4\theta_b^A + \pi$. (*Hint:* Find the angular deflections that occur at A, B, and C, and add them to get Δ.) (c) Use Snell's law to write Δ in terms of θ_a^A and n, the refractive index of the water in the drop. (d) A rainbow will form when the angular deflection Δ is *stationary* in the incident angle θ_a^A—that is, when $d\Delta/d\theta_a^A = 0$. If this condition is satisfied, all the rays with incident angles close to θ_a^A will be sent back in the same direction, producing a bright zone in the sky. Let θ_1 be the value of θ_a^A for which this occurs. Show that $\cos^2\theta_1 = \frac{1}{3}(n^2 - 1)$. (*Hint:* You may find the derivative formula $d(\arcsin u(x))/dx = (1 - u^2)^{-1/2}(du/dx)$ helpful.) (e) The index of refraction in water is 1.342 for violet light and 1.330 for red light. Use the results of parts (c) and (d) to find θ_1 and Δ for violet and red light. Do your results agree with the angles shown in Fig. 33.20d? When you view the rainbow, which color, red or violet, is higher above the horizon?

Figure 33.62 Challenge Problem 33.66.



33.67. A *secondary rainbow* is formed when the incident light undergoes two internal reflections in a spherical drop of water as shown in Fig. 33.20e. (See Challenge Problem 33.66.) (a) In terms of the incident angle θ_a^A and the refractive index n of the drop, what is the angular deflection Δ of the ray? That is, what is the angle between the ray before it enters the drop and after it exits? (b) What is the incident angle θ_2 for which the derivative of Δ with respect to the incident angle θ_a^A is zero? (c) The indexes of refraction for red and violet light in water are given in part (e) of Challenge Problem 33.66. Use the results of parts (a) and (b) to find θ_2 and Δ for violet and red light. Do your results agree with the angles shown in Fig. 33.20e? When you view a secondary rainbow, is red or violet higher above the horizon? Explain.