CAPACITANCE AND DIELECTRICS

24

?The energy used in a camera's flash unit is stored in a capacitor, which consists of two closely spaced conductors that carry opposite charges. If the amount of charge on the conductors is doubled, by what factor does the stored energy increase?

LEARNING GOALS

By studying this chapter, you will learn:

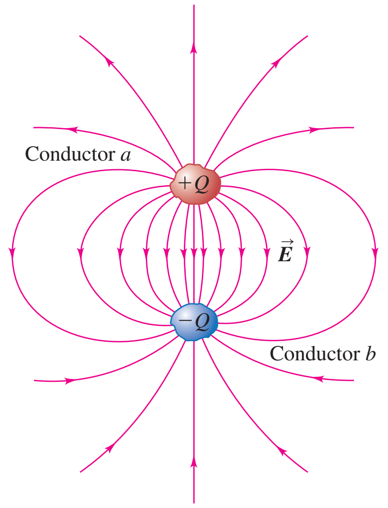
- The nature of capacitors, and how to calculate a quantity that measures their ability to store charge.
- How to analyze capacitors connected in a network.
- How to calculate the amount of energy stored in a capacitor.
- What dielectrics are, and how they make capacitors more effective.

When you set an old-fashioned spring mousetrap or pull back the string of an archer's bow, you are storing mechanical energy as elastic potential energy. A capacitor is a device that stores *electric* potential energy and electric charge. To make a capacitor, just insulate two conductors from each other. To store energy in this device, transfer charge from one conductor to the other so that one has a negative charge and the other has an equal amount of positive charge. Work must be done to move the charges through the resulting potential difference between the conductors, and the work done is stored as electric potential energy.

Capacitors have a tremendous number of practical applications in devices such as electronic flash units for photography, pulsed lasers, air bag sensors for cars, and radio and television receivers. We'll encounter many of these applications in later chapters (particularly Chapter 31, in which we'll see the crucial role played by capacitors in the alternating-current circuits that pervade our technological society). In this chapter, however, our emphasis is on the fundamental properties of capacitors. For a particular capacitor, the ratio of the charge on each conductor to the potential difference between the conductors is a constant, called the *capacitance.* The capacitance depends on the sizes and shapes of the conductors and on the insulating material (if any) between them. Compared to the case in which there is only vacuum between the conductors, the capacitance increases when an insulating material (a *dielectric*) is present. This happens because a redistribution of charge, called *polarization,* takes place within the insulating material. Studying polarization will give us added insight into the electrical properties of matter.

Capacitors also give us a new way to think about electric potential energy. The energy stored in a charged capacitor is related to the electric field in the space between the conductors. We will see that electric potential energy can be regarded as being stored *in the field itself.* The idea that the electric field is itself a storehouse of energy is at the heart of the theory of electromagnetic waves and our modern understanding of the nature of light, to be discussed in Chapter 32.

24.1 Capacitors and Capacitance



24.1 Any two conductors a and b insulated from each another form a capacitor.

Any two conductors separated by an insulator (or a vacuum) form a **capacitor** (Fig. 24.1). In most practical applications, each conductor initially has zero net charge and electrons are transferred from one conductor to the other; this is called *charging* the capacitor. Then the two conductors have charges with equal magnitude and opposite sign, and the *net* charge on the capacitor as a whole remains zero. We will assume throughout this chapter that this is the case. When we say that a capacitor has charge Q, or that a charge Q is *stored* on the capacitor, we mean that the conductor at higher potential has charge $+Q$ and the conductor at lower potential has charge $-Q$ (assuming that Q is positive). Keep this in mind in the following discussion and examples.

In circuit diagrams a capacitor is represented by either of these symbols:

In either symbol the vertical lines (straight or curved) represent the conductors and the horizontal lines represent wires connected to either conductor. One common way to charge a capacitor is to connect these two wires to opposite terminals of a battery. Once the charges Q and $-Q$ are established on the conductors, the battery is disconnected. This gives a fixed *potential difference* V_{ab} between the conductors (that is, the potential of the positively charged conductor a with respect to the negatively charged conductor b) that is just equal to the voltage of the battery.

The electric field at any point in the region between the conductors is proportional to the magnitude Q of charge on each conductor. It follows that the potential difference V_{ab} between the conductors is also proportional to Q. If we double the magnitude of charge on each conductor, the charge density at each point doubles, the electric field at each point doubles, and the potential difference between conductors doubles; however, the *ratio* of charge to potential difference does not change. This ratio is called the **capacitance** C of the capacitor:

$$C = \frac{Q}{V_{ab}} \quad \text{(definition of capacitance)} \tag{24.1}$$

The SI unit of capacitance is called one **farad** (1 F), in honor of the 19th-century English physicist Michael Faraday. From Eq. (24.1), one farad is equal to one *coulomb per volt* $(1\ \text{C/V})$:

$$1\ \text{F} = 1\ \text{farad} = 1\ \text{C/V} = 1\ \text{coulomb/volt}$$

CAUTION **Capacitance vs. coulombs** Don't confuse the symbol C for capacitance (which is always in italics) with the abbreviation C for coulombs (which is never italicized).

The greater the capacitance C of a capacitor, the greater the magnitude Q of charge on either conductor for a given potential difference V_{ab} and hence the greater the amount of stored energy. (Remember that potential is potential energy per unit charge.) Thus *capacitance is a measure of the ability of a capacitor to store energy.* We will see that the value of the capacitance depends only on the shapes and sizes of the conductors and on the nature of the insulating material between them. (The above remarks about capacitance being independent of Q and V_{ab} do not apply to certain special types of insulating materials. We won't discuss these materials in this book, however.)

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11.11.6 Electric Potential: Qualitative Introduction

11.12.1 and 11.12.3 Electric Potential, Field and, Force

Calculating Capacitance: Capacitors in Vacuum

We can calculate the capacitance C of a given capacitor by finding the potential difference V_{ab} between the conductors for a given magnitude of charge Q and then using Eq. (24.1). For now we'll consider only *capacitors in vacuum;* that is, we'll assume that the conductors that make up the capacitor are separated by empty space.

The simplest form of capacitor consists of two parallel conducting plates, each with area A, separated by a distance d that is small in comparison with their dimensions (Fig. 24.2a). When the plates are charged, the electric field is almost completely localized in the region between the plates (Fig. 24.2b). As we discussed in Example 22.8 (Section 22.4), the field between such plates is essentially *uniform,* and the charges on the plates are uniformly distributed over their opposing surfaces. We call this arrangement a **parallel-plate capacitor.**

We worked out the electric-field magnitude E for this arrangement in Example 21.13 (Section 21.5) using the principle of superposition of electric fields and again in Example 22.8 (Section 22.4) using Gauss's law. It would be a good idea to review those examples. We found that $E = \sigma/\epsilon_0$, where σ is the magnitude (absolute value) of the surface charge density on each plate. This is equal to the magnitude of the total charge Q on each plate divided by the area A of the plate, or $\sigma = Q/A$, so the field magnitude E can be expressed as

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

The field is uniform and the distance between the plates is d, so the potential difference (voltage) between the two plates is

$$V_{ab} = Ed = \frac{1}{\epsilon_0}\frac{Qd}{A}$$

From this we see that the capacitance C of a parallel-plate capacitor in vacuum is

$$C = \frac{Q}{V_{ab}} = \epsilon_0 \frac{A}{d} \quad \text{(capacitance of a parallel-plate capacitor in vacuum)} \tag{24.2}$$

The capacitance depends only on the geometry of the capacitor; it is directly proportional to the area A of each plate and inversely proportional to their separation d. The quantities A and d are constants for a given capacitor, and ϵ_0 is a universal constant. Thus in vacuum the capacitance C is a constant independent of the charge on the capacitor or the potential difference between the plates. If one of the capacitor plates is flexible, the capacitance C changes as the plate separation d changes. This is the operating principle of a condenser microphone (Fig. 24.3).

When matter is present between the plates, its properties affect the capacitance. We will return to this topic in Section 24.4. Meanwhile, we remark that if the space contains air at atmospheric pressure instead of vacuum, the capacitance differs from the prediction of Eq. (24.2) by less than 0.06%.

In Eq. (24.2), if A is in square meters and d in meters, C is in farads. The units of ϵ_0 are $\text{C}^2/\text{N}\cdot\text{m}^2$, so we see that

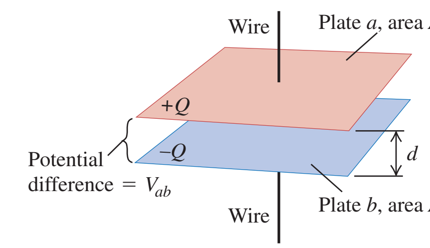
$$1\ \text{F} = 1\ \text{C}^2/\text{N}\cdot\text{m} = 1\ \text{C}^2/\text{J}$$

Because $1\ \text{V} = 1\ \text{J/C}$ (energy per unit charge), this is consistent with our definition $1\ \text{F} = 1\ \text{C/V}$. Finally, the units of ϵ_0 can be expressed as $1\ \text{C}^2/\text{N}\cdot\text{m}^2 = 1\ \text{F/m}$, so

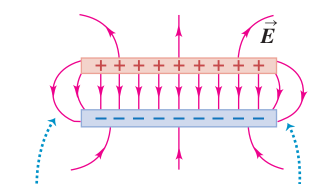
$$\epsilon_0 = 8.85 \times 10^{-12}\ \text{F/m}$$

24.2 A charged parallel-plate capacitor.

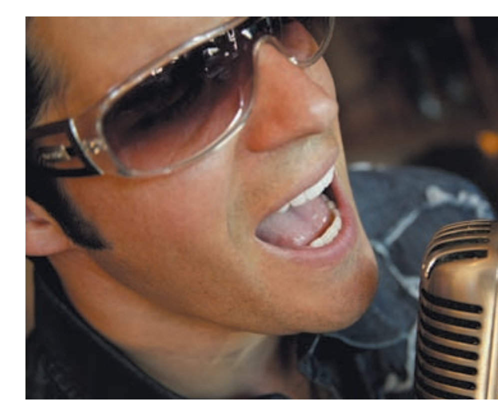
(a) Arrangement of the capacitor plates

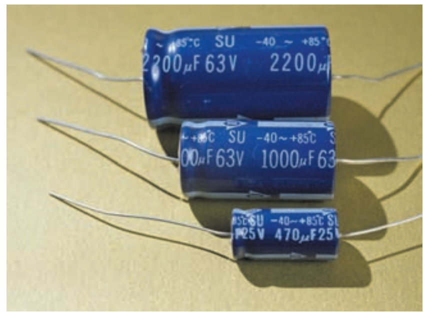


(b) Side view of the electric field $\vec{E}$



24.3 Inside a condenser microphone is a capacitor with one rigid plate and one flexible plate. The two plates are kept at a constant potential difference V_{ab}. Sound waves cause the flexible plate to move back and forth, varying the capacitance C and causing charge to flow to and from the capacitor in accordance with the relationship $C = Q/V_{ab}$. Thus a sound wave is converted to a charge flow that can be amplified and recorded digitally.





24.4 A commercial capacitor is labeled with the value of its capacitance. For these capacitors, $C = 2200\ \mu\text{F}$, $1000\ \mu\text{F}$, and $470\ \mu\text{F}$.

This relationship is useful in capacitance calculations, and it also helps us to verify that Eq. (24.2) is dimensionally consistent.

One farad is a very large capacitance, as the following example shows. In many applications the most convenient units of capacitance are the *microfarad* $(1\ \mu\text{F} = 10^{-6}\ \text{F})$ and the *picofarad* $(1\ \text{pF} = 10^{-12}\ \text{F})$. For example, the flash unit in a point-and-shoot camera uses a capacitor of a few hundred microfarads (Fig. 24.4), while capacitances in a radio tuning circuit are typically from 10 to 100 picofarads.

For *any* capacitor in vacuum, the capacitance C depends only on the shapes, dimensions, and separation of the conductors that make up the capacitor. If the conductor shapes are more complex than those of the parallel-plate capacitor, the expression for capacitance is more complicated than in Eq. (24.2). In the following examples we show how to calculate C for two other conductor geometries.

Example 24.1 Size of a 1-F capacitor

A parallel-plate capacitor has a capacitance of 1.0 F. If the plates are 1.0 mm apart, what is the area of the plates?

SOLUTION

IDENTIFY: This problem uses the relationship among the capacitance, plate separation, and plate area (our target variable) for a parallel-plate capacitor.

SET UP: We are given the values of C and d for a parallel-plate capacitor, so we use Eq. (24.2) and solve for the target variable A.

EXECUTE: From Eq. (24.2), the area A is

$$A = \frac{Cd}{\epsilon_0} = \frac{(1.0\ \text{F})(1.0 \times 10^{-3}\ \text{m})}{8.85 \times 10^{-12}\ \text{F/m}}$$
$$= 1.1 \times 10^{8}\ \text{m}^2$$

EVALUATE: This corresponds to a square about 10 km (about 6 miles) on a side! This area is about a third larger than Manhattan Island. Clearly this is not a very practical design for a capacitor.

In fact, it's now possible to make 1-F capacitors a few centimeters on a side. The trick is to have an appropriate substance between the plates rather than a vacuum. We'll explore this further in Section 24.4.

Example 24.2 Properties of a parallel-plate capacitor

The plates of a parallel-plate capacitor in vacuum are 5.00 mm apart and $2.00\ \text{m}^2$ in area. A potential difference of 10,000 V (10.0 kV) is applied across the capacitor. Compute (a) the capacitance; (b) the charge on each plate; and (c) the magnitude of the electric field in the space between them.

SOLUTION

IDENTIFY: We are given the plate area A, the plate spacing d, and the potential difference V_{ab} for this parallel-plate capacitor. Our target variables are the capacitance C, charge Q, and electric-field magnitude E.

SET UP: We use Eq. (24.2) to calculate C and then find the charge Q on each plate using the given potential difference V_{ab} and Eq. (24.1). Once we have Q, we find the electric field between the plates using the relationship $E = Q/\epsilon_0 A$.

EXECUTE: (a) From Eq. (24.2),

$$C = \epsilon_0 \frac{A}{d} = \frac{(8.85 \times 10^{-12}\ \text{F/m})(2.00\ \text{m}^2)}{5.00 \times 10^{-3}\ \text{m}}$$
$$= 3.54 \times 10^{-9}\ \text{F} = 0.00354\ \mu\text{F}$$

(b) The charge on the capacitor is

$$Q = CV_{ab} = (3.54 \times 10^{-9}\ \text{C/V})(1.00 \times 10^{4}\ \text{V})$$
$$= 3.54 \times 10^{-5}\ \text{C} = 35.4\ \mu\text{C}$$

The plate at higher potential has charge $+35.4\ \mu\text{C}$ and the other plate has charge $-35.4\ \mu\text{C}$.

(c) The electric-field magnitude is

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A} = \frac{3.54 \times 10^{-5}\ \text{C}}{(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(2.00\ \text{m}^2)}$$
$$= 2.00 \times 10^{6}\ \text{N/C}$$

EVALUATE: An alternative way to get the result in part (c) is to recall that the electric field is equal in magnitude to the potential gradient [Eq. (23.22)]. Since the field between the plates is uniform,

$$E = \frac{V_{ab}}{d} = \frac{1.00 \times 10^{4}\ \text{V}}{5.00 \times 10^{-3}\ \text{m}} = 2.00 \times 10^{6}\ \text{V/m}$$

(Remember that the newton per coulomb and the volt per meter are equivalent units.)

Example 24.3 A spherical capacitor

Two concentric spherical conducting shells are separated by vacuum. The inner shell has total charge $+Q$ and outer radius r_a, and the outer shell has charge $-Q$ and inner radius r_b (Fig. 24.5). (The inner shell is attached to the outer shell by thin insulating rods that have negligible effect on the capacitance.) Find the capacitance of this spherical capacitor.

SOLUTION

IDENTIFY: This isn't a parallel-plate capacitor, so we can't use the relationships developed for that particular geometry. Instead, we'll go back to the fundamental definition of capacitance: the magnitude of the charge on either conductor divided by the potential difference between the conductors.

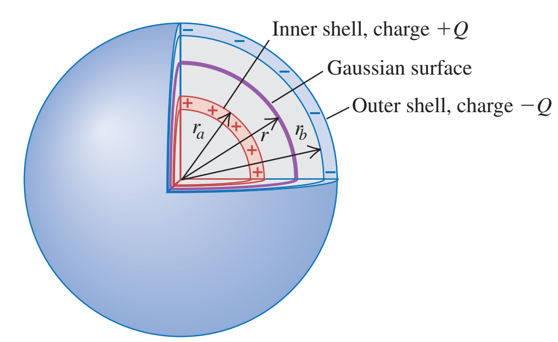
SET UP: We use Gauss's law to find the electric field between the spherical conductors. From this value we determine the potential difference V_{ab} between the two conductors; we then use Eq. (24.1) to find the capacitance $C = Q/V_{ab}$.

EXECUTE: Using the same procedure as in Example 22.5 (Section 22.4), we take as our Gaussian surface a sphere with radius r between the two spheres and concentric with them. Gauss's law, Eq. (22.8), states that the electric flux through this surface is equal to the total charge enclosed within the surface, divided by ϵ_0:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0}$$

By symmetry, $\vec{E}$ is constant in magnitude and parallel to $d\vec{A}$ at every point on this surface, so the integral in Gauss's law is equal

24.5 A spherical capacitor.



to $(E)(4\pi r^2)$. The total charge enclosed is $Q_{\text{encl}} = Q$, so we have

$$(E)(4\pi r^2) = \frac{Q}{\epsilon_0}$$
$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

The electric field between the spheres is just that due to the charge on the inner sphere; the outer sphere has no effect. We found in Example 22.5 that the charge on a conducting sphere produces zero field *inside* the sphere, which also tells us that the outer conductor makes no contribution to the field between the conductors.

The above expression for E is the same as that for a point charge Q, so the expression for the potential can also be taken to be the same as for a point charge, $V = Q/4\pi\epsilon_0 r$. Hence the potential of the inner (positive) conductor at $r = r_a$ with respect to that of the outer (negative) conductor at $r = r_b$ is

$$V_{ab} = V_a - V_b = \frac{Q}{4\pi\epsilon_0 r_a} - \frac{Q}{4\pi\epsilon_0 r_b}$$
$$= \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right) = \frac{Q}{4\pi\epsilon_0}\frac{r_b - r_a}{r_a r_b}$$

Finally, the capacitance is

$$C = \frac{Q}{V_{ab}} = 4\pi\epsilon_0 \frac{r_a r_b}{r_b - r_a}$$

As an example, if $r_a = 9.5$ cm and $r_b = 10.5$ cm,

$$C = 4\pi(8.85 \times 10^{-12}\ \text{F/m})\frac{(0.095\ \text{m})(0.105\ \text{m})}{0.010\ \text{m}}$$
$$= 1.1 \times 10^{-10}\ \text{F} = 110\ \text{pF}$$

EVALUATE: We can relate this result to the capacitance of a parallel-plate capacitor. The quantity $4\pi r_a r_b$ is intermediate between the areas $4\pi r_a^2$ and $4\pi r_b^2$ of the two spheres; in fact, it's the *geometric mean* of these two areas, which we can denote by A_{gm}. The distance between spheres is $d = r_b - r_a$, so we can rewrite the above result as $C = \epsilon_0 A_{\text{gm}}/d$. This is exactly the same form as for parallel plates: $C = \epsilon_0 A/d$. The point is that if the distance between spheres is very small in comparison to their radii, they behave like parallel plates with the same area and spacing.

Example 24.4 A cylindrical capacitor

A long cylindrical conductor has a radius r_a and a linear charge density $+\lambda$. It is surrounded by a coaxial cylindrical conducting shell with inner radius r_b and linear charge density $-\lambda$ (Fig. 24.6). Calculate the capacitance per unit length for this capacitor, assuming that there is vacuum in the space between cylinders.

SOLUTION

IDENTIFY: As in Example 24.3, we use the fundamental definition of capacitance.

SET UP: We first find expressions for the potential difference V_{ab} between the cylinders and the charge Q in a length L of the cylinders; we then find the capacitance of a length L using Eq. (24.1). Our target variable is this capacitance divided by L.

EXECUTE: To find the potential difference between the cylinders, we use a result that we worked out in Example 23.10 (Section 23.3). There we found that at a point outside a charged

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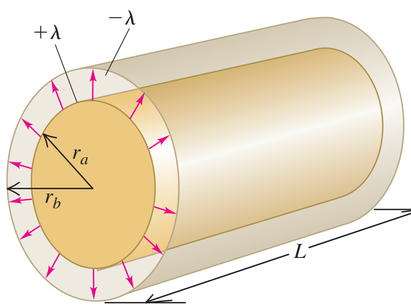
24.6 A long cylindrical capacitor. The linear charge density λ is assumed to be positive in this figure. The magnitude of charge in a length L of either cylinder is λL.



cylinder a distance r from the axis, the potential due to the cylinder is

$$V = \frac{\lambda}{2\pi\epsilon_0}\ln\frac{r_0}{r}$$

where r_0 is the (arbitrary) radius at which $V = 0$. We can use this same result for the potential *between* the cylinders in the present problem because, according to Gauss's law, the charge on the outer cylinder doesn't contribute to the field between cylinders (see Example 24.3). In our case, we take the radius r_0 to be r_b, the radius of the inner surface of the outer cylinder, so that the outer conducting cylinder is at $V = 0$. Then the potential at the outer surface of the inner cylinder (where $r = r_a$) is just equal to the potential V_{ab} of the inner (positive) cylinder a with respect to the outer (negative) cylinder b, or

$$V_{ab} = \frac{\lambda}{2\pi\epsilon_0}\ln\frac{r_b}{r_a}$$

This potential difference is positive (assuming that λ is positive, as in Fig. 24.6) because the inner cylinder is at higher potential than the outer.

The total charge Q in a length L is $Q = \lambda L$, so from Eq. (24.1) the capacitance C of a length L is

$$C = \frac{Q}{V_{ab}} = \frac{\lambda L}{\dfrac{\lambda}{2\pi\epsilon_0}\ln\dfrac{r_b}{r_a}} = \frac{2\pi\epsilon_0 L}{\ln(r_b/r_a)}$$

The capacitance per unit length is

$$\frac{C}{L} = \frac{2\pi\epsilon_0}{\ln(r_b/r_a)}$$

Substituting $\epsilon_0 = 8.85 \times 10^{-12}\ \text{F/m} = 8.85\ \text{pF/m}$, we get

$$\frac{C}{L} = \frac{55.6\ \text{pF/m}}{\ln(r_b/r_a)}$$

EVALUATE: We see that the capacitance of the coaxial cylinders is determined entirely by the dimensions, just as for the parallel-plate case. Ordinary coaxial cables are made like this but with an insulating material instead of vacuum between the inner and outer conductors. A typical cable for TV antennas and VCR connections has a capacitance per unit length of 69 pF/m.

Test Your Understanding of Section 24.1 A capacitor has vacuum in the space between the conductors. If you double the amount of charge on each conductor, what happens to the capacitance? (i) It increases; (ii) it decreases; (iii) it remains the same; (iv) the answer depends on the size or shape of the conductors.

24.2 Capacitors in Series and Parallel

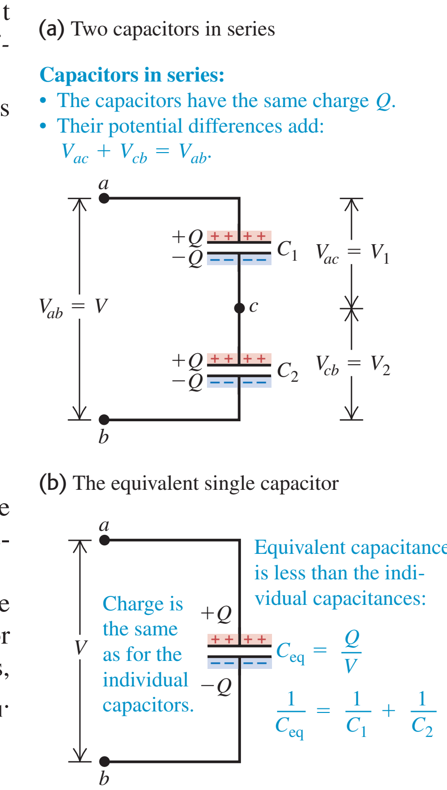
24.7 An assortment of commercially available capacitors.

Capacitors are manufactured with certain standard capacitances and working voltages (Fig. 24.7). However, these standard values may not be the ones you actually need in a particular application. You can obtain the values you need by combining capacitors; many combinations are possible, but the simplest combinations are a series connection and a parallel connection.

Capacitors in Series

Figure 24.8a is a schematic diagram of a **series connection.** Two capacitors are connected in series (one after the other) by conducting wires between points a and b. Both capacitors are initially uncharged. When a constant positive potential difference V_{ab} is applied between points a and b, the capacitors become charged; the figure shows that the charge on *all* conducting plates has the same magnitude. To see why, note first that the top plate of C_1 acquires a positive charge Q. The electric field of this positive charge pulls negative charge up to the bottom plate of C_1 until all of the field lines that begin on the top plate end on the bottom plate. This requires that the bottom plate have charge $-Q$. These negative charges had to come from the top plate of C_2, which becomes positively charged with charge $+Q$. This positive charge then pulls negative charge $-Q$ from the connection at point b onto the bottom plate of C_2. The total charge on the lower plate of C_1 and the upper plate of C_2 together must always be zero because these plates aren't connected to anything except each other. Thus *in a series connection the magnitude of charge on all plates is the same.*

24.8 A series connection of two capacitors.



Referring to Fig. 24.8a, we can write the potential differences between points a and c, c and b, and a and b as

$$V_{ac} = V_1 = \frac{Q}{C_1} \qquad V_{cb} = V_2 = \frac{Q}{C_2}$$

$$V_{ab} = V = V_1 + V_2 = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

and so

$$\frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2} \tag{24.3}$$

Following a common convention, we use the symbols V_1, V_2, and V to denote the potential *differences* V_{ac} (across the first capacitor), V_{cb} (across the second capacitor), and V_{ab} (across the entire combination of capacitors), respectively.

The **equivalent capacitance** C_{eq} of the series combination is defined as the capacitance of a *single* capacitor for which the charge Q is the same as for the combination, when the potential difference V is the same. In other words, the combination can be replaced by an *equivalent capacitor* of capacitance C_{eq}. For such a capacitor, shown in Fig. 24.8b,

$$C_{eq} = \frac{Q}{V} \quad \text{or} \quad \frac{1}{C_{eq}} = \frac{V}{Q} \tag{24.4}$$

Combining Eqs. (24.3) and (24.4), we find

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

We can extend this analysis to any number of capacitors in series. We find the following result for the *reciprocal* of the equivalent capacitance:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \quad \text{(capacitors in series)} \tag{24.5}$$

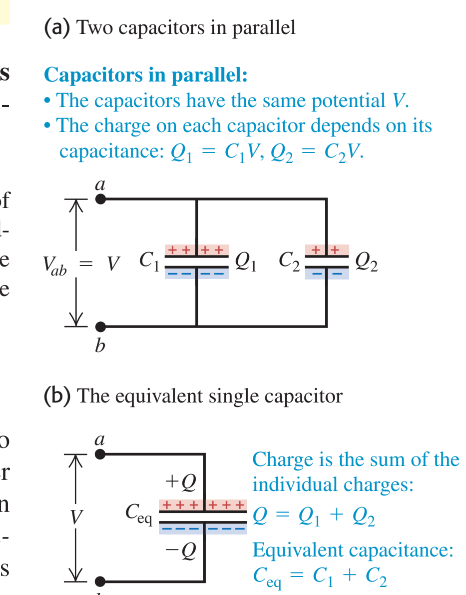
The reciprocal of the equivalent capacitance of a series combination equals the sum of the reciprocals of the individual capacitances. In a series connection the equivalent capacitance is always *less than* any individual capacitance.

CAUTION **Capacitors in series** The magnitude of charge is the same on all plates of all the capacitors in a series combination; however, the potential differences of the individual capacitors are *not* the same unless their individual capacitances are the same. The potential differences of the individual capacitors add to give the total potential difference across the series combination: $V_{total} = V_1 + V_2 + V_3 + \cdots$.

Capacitors in Parallel

The arrangement shown in Fig. 24.9a is called a **parallel connection.** Two capacitors are connected in parallel between points a and b. In this case the upper plates of the two capacitors are connected by conducting wires to form an equipotential surface, and the lower plates form another. Hence *in a parallel connection the potential difference for all individual capacitors is the same* and is equal to $V_{ab} = V$. The charges Q_1 and Q_2 are not necessarily equal, however,

24.9 A parallel connection of two capacitors.



since charges can reach each capacitor independently from the source (such as a battery) of the voltage V_{ab}. The charges are

$$Q_1 = C_1 V \quad \text{and} \quad Q_2 = C_2 V$$

The *total* charge Q of the combination, and thus the total charge on the equivalent capacitor, is

$$Q = Q_1 + Q_2 = (C_1 + C_2)V$$

so

$$\frac{Q}{V} = C_1 + C_2 \tag{24.6}$$

The parallel combination is equivalent to a single capacitor with the same total charge $Q = Q_1 + Q_2$ and potential difference V as the combination (Fig. 24.9b). The equivalent capacitance of the combination, C_{eq}, is the same as the capacitance Q/V of this single equivalent capacitor. So from Eq. (24.6),

$$C_{eq} = C_1 + C_2$$

In the same way we can show that for any number of capacitors in parallel,

$$C_{eq} = C_1 + C_2 + C_3 + \cdots \quad \text{(capacitors in parallel)} \tag{24.7}$$

The equivalent capacitance of a parallel combination equals the *sum* of the individual capacitances. In a parallel connection the equivalent capacitance is always *greater than* any individual capacitance.

CAUTION Capacitors in parallel The potential differences are the same for all the capacitors in a parallel combination; however, the charges on individual capacitors are *not* the same unless their individual capacitances are the same. The charges on the individual capacitors add to give the total charge on the parallel combination: $Q_{total} = Q_1 + Q_2 + Q_3 + \cdots$. [Compare these statements to those in the "Caution" paragraph following Eq. (24.5).]

Problem-Solving Strategy 24.1 Equivalent Capacitance

IDENTIFY *the relevant concepts:* The concept of equivalent capacitance is useful whenever two or more capacitors are connected.

SET UP *the problem* using the following steps:

1. Make a drawing of the capacitor arrangement.
2. Identify whether the capacitors are connected in series or in parallel. With more complicated combinations, you can sometimes identify parts that are simple series or parallel connections.
3. Keep in mind that when we say a capacitor has charge Q, we always mean that the plate at higher potential has charge $+Q$ and the other plate has charge $-Q$.

EXECUTE *the solution* as follows:

1. When capacitors are connected in series, as in Fig. 24.8a, they always have the same charge, assuming that they were uncharged before they were connected. The potential differences are *not* equal unless the capacitances are equal. The total potential difference across the combination is the sum of the individual potential differences.
2. When capacitors are connected in parallel, as in Fig. 24.9a, the potential difference V is always the same for all of the individual capacitors. The charges on the individual capacitors are *not* equal unless the capacitances are equal. The total charge on the combination is the sum of the individual charges.
3. For more complicated combinations, find the parts that are simple series or parallel connections and replace them with their equivalent capacitances, in a step-by-step reduction. If you then need to find the charge or potential difference for an individual capacitor, you may have to retrace your path to the original capacitors.

EVALUATE *your answer:* Check whether your result makes sense. If the capacitors are connected in series, the equivalent capacitance C_{eq} must be *smaller* than any of the individual capacitances. By contrast, if the capacitors are connected in parallel, C_{eq} must be *greater* than any of the individual capacitances.

Example 24.5 Capacitors in series and in parallel

In Figs. 24.8 and 24.9, let $C_1 = 6.0\ \mu\text{F}$, $C_2 = 3.0\ \mu\text{F}$, and $V_{ab} = 18$ V. Find the equivalent capacitance, and find the charge and potential difference for each capacitor when the two capacitors are connected (a) in series and (b) in parallel.

SOLUTION

IDENTIFY: This problem uses the ideas discussed in this section about capacitor connections.

SET UP: In both parts, one of the target variables is the equivalent capacitance C_{eq}. For the series combination in part (a), it is given by Eq. (24.5); for the parallel combination in part (b), C_{eq} is given by Eq. (24.6). In each part we find the charge and potential difference using the definition of capacitance, Eq. (24.1), and the rules outlined in the Problem-Solving Strategy 24.1.

EXECUTE: (a) Using Eq. (24.5) for the equivalent capacitance of the series combination (Fig. 24.8a), we find

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{6.0\ \mu\text{F}} + \frac{1}{3.0\ \mu\text{F}} \qquad C_{eq} = 2.0\ \mu\text{F}$$

The charge Q on each capacitor in series is the same as the charge on the equivalent capacitor:

$$Q = C_{eq}V = (2.0\ \mu\text{F})(18\ \text{V}) = 36\ \mu\text{C}$$

The potential difference across each capacitor is inversely proportional to its capacitance:

$$V_{ac} = V_1 = \frac{Q}{C_1} = \frac{36\ \mu\text{C}}{6.0\ \mu\text{F}} = 6.0\ \text{V}$$

$$V_{cb} = V_2 = \frac{Q}{C_2} = \frac{36\ \mu\text{C}}{3.0\ \mu\text{F}} = 12.0\ \text{V}$$

(b) To find the equivalent capacitance of the parallel combination (Fig. 24.9a), we use Eq. (24.6):

$$C_{eq} = C_1 + C_2 = 6.0\ \mu\text{F} + 3.0\ \mu\text{F} = 9.0\ \mu\text{F}$$

The potential difference across each of the two capacitors in parallel is the same as that across the equivalent capacitor, 18 V. The charges Q_1 and Q_2 are directly proportional to the capacitances C_1 and C_2, respectively:

$$Q_1 = C_1V = (6.0\ \mu\text{F})(18\ \text{V}) = 108\ \mu\text{C}$$

$$Q_2 = C_2V = (3.0\ \mu\text{F})(18\ \text{V}) = 54\ \mu\text{C}$$

EVALUATE: Note that the equivalent capacitance C_{eq} for the series combination in part (a) is indeed less than either C_1 or C_2, while for the parallel combination in part (b) the equivalent capacitance is indeed greater than either C_1 or C_2.

It's instructive to compare the potential differences and charges in each part of the example. For two capacitors in series, as in part (a), the charge is the same on either capacitor and the *larger* potential difference appears across the capacitor with the *smaller* capacitance. Furthermore, $V_{ac} + V_{cb} = V_{ab} = 18$ V, as it must. By contrast, for two capacitors in parallel, as in part (b), each capacitor has the same potential difference and the *larger* charge appears on the capacitor with the *larger* capacitance. Can you show that the total charge $Q_1 + Q_2$ on the parallel combination is equal to the charge $Q = C_{eq}V$ on the equivalent capacitor?

Example 24.6 A capacitor network

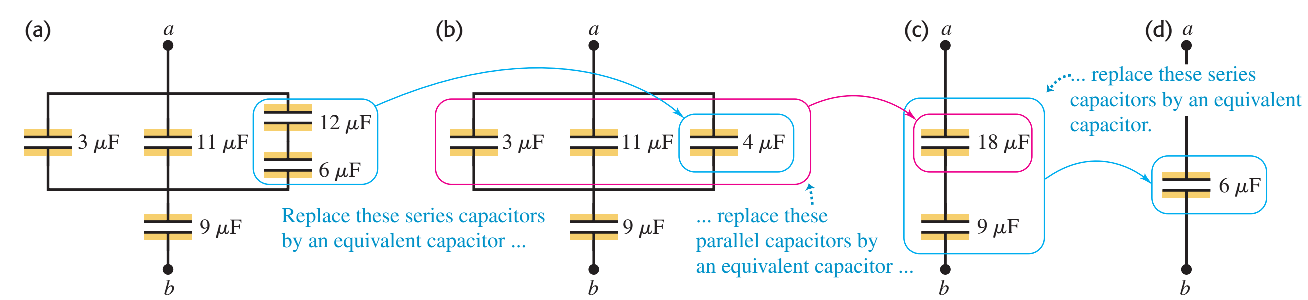
Find the equivalent capacitance of the combination shown in Fig. 24.10a.

SOLUTION

IDENTIFY: The five capacitors in Fig. 24.10a are neither all in series nor all in parallel. We can, however, identify portions of the arrangement that *are* either in series or parallel, which we combine to find the net equivalent capacitance.

SET UP: We use Eq. (24.5) to analyze portions of the network that are series connections and Eq. (24.7) to analyze portions that are parallel connections.

24.10 (a) A capacitor network between points a and b. (b) The 12-μF and 6-μF capacitors in series in (a) are replaced by an equivalent 4-μF capacitor. (c) The 3-μF, 11-μF, and 4-μF capacitors in parallel in (b) are replaced by an equivalent 18-μF capacitor. (d) Finally, the 18-μF and 9-μF capacitors in series in (c) are replaced by an equivalent 6-μF capacitor.



Continued

EXECUTE: We first replace the 12-μF and 6-μF series combination by its equivalent capacitance; calling that C', we use Eq. (24.5):

$$\frac{1}{C'} = \frac{1}{12\,\mu\text{F}} + \frac{1}{6\,\mu\text{F}} \qquad C' = 4\,\mu\text{F}$$

This gives us the equivalent combination shown in Fig. 24.10b. Next we find the equivalent capacitance of the three capacitors in parallel, using Eq. (24.7). Calling their equivalent capacitance C'', we have

$$C'' = 3\,\mu\text{F} + 11\,\mu\text{F} + 4\,\mu\text{F} = 18\,\mu\text{F}$$

This gives us the simpler equivalent combination shown in Fig. 24.10c. Finally, we find the equivalent capacitance C_{eq} of these two capacitors in series (Fig. 24.10d):

$$\frac{1}{C_{\text{eq}}} = \frac{1}{18\,\mu\text{F}} + \frac{1}{9\,\mu\text{F}} \qquad C_{\text{eq}} = 6\,\mu\text{F}$$

EVALUATE: The equivalent capacitance of the network is 6 μF; that is, if a potential difference V_{ab} is applied across the terminals of the network, the net charge on the network is 6 μF times V_{ab}. How is this net charge related to the charges on the individual capacitors in Fig. 24.10a?

Test Your Understanding of Section 24.2 You want to connect a 4-μF capacitor and an 8-μF capacitor. (a) With which type of connection will the 4-μF capacitor have a greater *potential difference* across it than the 8-μF capacitor? (i) series; (ii) parallel; (iii) either series or parallel; (iv) neither series nor parallel. (b) With which type of connection will the 4-μF capacitor have a greater *charge* than the 8-μF capacitor? (i) series; (ii) parallel; (iii) either series or parallel; (iv) neither series nor parallel.

24.3 Energy Storage in Capacitors and Electric-Field Energy

Many of the most important applications of capacitors depend on their ability to store energy. The electric potential energy stored in a charged capacitor is just equal to the amount of work required to charge it—that is, to separate opposite charges and place them on different conductors. When the capacitor is discharged, this stored energy is recovered as work done by electrical forces.

We can calculate the potential energy U of a charged capacitor by calculating the work W required to charge it. Suppose that when we are done charging the capacitor, the final charge is Q and the final potential difference is V. From Eq. (24.1) these quantities are related by

$$V = \frac{Q}{C}$$

Let q and v be the charge and potential difference, respectively, at an intermediate stage during the charging process; then $v = q/C$. At this stage the work dW required to transfer an additional element of charge dq is

$$dW = v\,dq = \frac{q\,dq}{C}$$

The total work W needed to increase the capacitor charge q from zero to a final value Q is

$$W = \int_0^W dW = \frac{1}{C}\int_0^Q q\,dq = \frac{Q^2}{2C} \qquad \text{(work to charge a capacitor)} \qquad (24.8)$$

This is also equal to the total work done by the electric field on the charge when the capacitor discharges. Then q *decreases* from an initial value Q to zero as the elements of charge dq "fall" through potential differences v that vary from V down to zero.

If we define the potential energy of an *uncharged* capacitor to be zero, then W in Eq. (24.8) is equal to the potential energy U of the charged capacitor. The final stored charge is $Q = CV$, so we can express U (which is equal to W) as

$$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV \qquad \text{(potential energy stored in a capacitor)} \qquad (24.9)$$

When Q is in coulombs, C in farads (coulombs per volt), and V in volts (joules per coulomb), U is in joules.

The last form of Eq. (24.9), $U = \frac{1}{2}QV$, shows that the total work W required to charge the capacitor is equal to the total charge Q multiplied by the *average* potential difference $\frac{1}{2}V$ during the charging process.

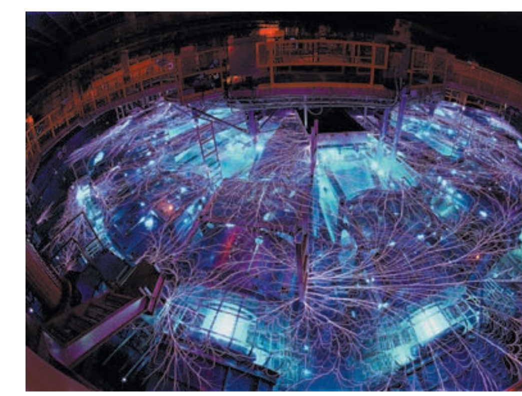
The expression $U = \frac{1}{2}(Q^2/C)$ in Eq. (24.9) shows that a charged capacitor is the electrical analog of a stretched spring with elastic potential energy $U = \frac{1}{2}kx^2$. The charge Q is analogous to the elongation x, and the *reciprocal* of the capacitance, $1/C$, is analogous to the force constant k. The energy supplied to a capacitor in the charging process is analogous to the work we do on a spring when we stretch it.

Equations (24.8) and (24.9) tell us that capacitance measures the ability of a capacitor to store both energy and charge. If a capacitor is charged by connecting it to a battery or other source that provides a fixed potential difference V, then increasing the value of C gives a greater charge $Q = CV$ and a greater amount of stored energy $U = \frac{1}{2}CV^2$. If instead the goal is to transfer a given quantity of charge Q from one conductor to another, Eq. (24.8) shows that the work W required is inversely proportional to C; the greater the capacitance, the easier it is to give a capacitor a fixed amount of charge.

Applications of Capacitors: Energy Storage

Most practical applications of capacitors take advantage of their ability to store and release energy. In electronic flash units used by photographers, the energy stored in a capacitor (see Fig. 24.4) is released by depressing the camera's shutter button. This provides a conducting path from one capacitor plate to the other through the flash tube. Once this path is established, the stored energy is rapidly converted into a brief but intense flash of light. An extreme example of the same principle is the Z machine at Sandia National Laboratories in New Mexico, which is used in experiments in controlled nuclear fusion (Fig. 24.11). A bank of charged capacitors releases more than a million joules of energy in just a few billionths of a second. For that brief space of time, the power output of the Z machine is 2.9×10^{14} W, or about 80 times the electric output of all the electric power plants on earth combined!

24.11 The Z machine uses a large number of capacitors in parallel to give a tremendous equivalent capacitance C (see Section 24.2). Hence a large amount of energy $U = \frac{1}{2}CV^2$ can be stored with even a modest potential difference V. The arcs shown here are produced when the capacitors discharge their energy into a target, which is no larger than a spool of thread. This heats the target to a temperature higher than 2×10^9 K.



In other applications, the energy is released more slowly. Springs in the suspension of an automobile, help smooth out the ride by absorbing the energy from sudden jolts and releasing that energy gradually; in an analogous way, a capacitor in an electronic circuit can smooth out unwanted variations in voltage due to power surges. And just as the presence of a spring gives a mechanical system a natural frequency at which it responds most strongly to an applied periodic force, so the presence of a capacitor gives an electric circuit a natural frequency for current oscillations. This idea is used in tuned circuits such as those in radio and television receivers, which respond to broadcast signals at one particular frequency and ignore signals at other frequencies. We'll discuss these circuits in detail in Chapter 31.

The energy-storage properties of capacitors also have some undesirable practical effects. Adjacent pins on the underside of a computer chip act like a capacitor, and the property that makes capacitors useful for smoothing out voltage variations acts to retard the rate at which the potentials of the chip's pins can be changed. This tendency limits how rapidly the chip can perform computations, an effect that becomes more important as computer chips become smaller and are pushed to operate at faster speeds.

Electric-Field Energy

We can charge a capacitor by moving electrons directly from one plate to another. This requires doing work against the electric field between the plates. Thus we can think of the energy as being stored *in the field* in the region between the plates. To develop this relationship, let's find the energy *per unit volume* in the space between the plates of a parallel-plate capacitor with plate area A and separation d. We call this the **energy density,** denoted by u. From Eq. (24.9) the total stored potential energy is $\frac{1}{2}CV^2$ and the volume between the plates is just Ad; hence the energy density is

$$u = \text{Energy density} = \frac{\frac{1}{2}CV^2}{Ad} \tag{24.10}$$

From Eq. (24.2) the capacitance C is given by $C = \epsilon_0 A/d$. The potential difference V is related to the electric field magnitude E by $V = Ed$. If we use these expressions in Eq. (24.10), the geometric factors A and d cancel, and we find

$$u = \frac{1}{2}\epsilon_0 E^2 \qquad \text{(electric energy density in a vacuum)} \tag{24.11}$$

Although we have derived this relationship only for a parallel-plate capacitor, it turns out to be valid for any capacitor in vacuum and indeed *for any electric field configuration in vacuum.* This result has an interesting implication. We think of vacuum as space with no matter in it, but vacuum can nevertheless have electric fields and therefore energy. Thus "empty" space need not be truly empty after all. We will use this idea and Eq. (24.11) in Chapter 32 in connection with the energy transported by electromagnetic waves.

CAUTION **Electrical-field energy is electric potential energy** It's a common misconception that electric-field energy is a new kind of energy, different from the electric potential energy described before. This is *not* the case; it is simply a different way of interpreting electric potential energy. We can regard the energy of a given system of charges as being a shared property of all the charges, or we can think of the energy as being a property of the electric field that the charges create. Either interpretation leads to the same value of the potential energy.

Example 24.7 Transferring charge and energy between capacitors

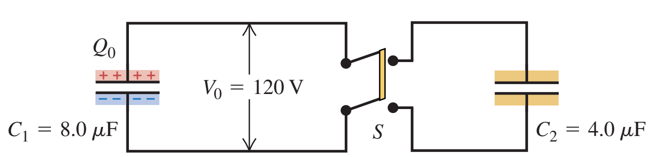
In Fig. 24.12 we charge a capacitor of capacitance $C_1 = 8.0\ \mu\text{F}$ by connecting it to a source of potential difference $V_0 = 120$ V (not shown in the figure). The switch S is initially open. Once C_1 is charged, the source of potential difference is disconnected. (a) What is the charge Q_0 on C_1 if switch S is left open? (b) What is the energy stored in C_1 if switch S is left open? (c) The capacitor of capacitance $C_2 = 4.0\ \mu\text{F}$ is initially uncharged. After we close switch S, what is the potential difference across each capacitor, and what is the charge on each capacitor? (d) What is the total energy of the system after we close switch S?

SOLUTION

IDENTIFY: Initially we have a single capacitor with a given potential difference between its plates. After the switch is closed, one wire connects the upper plates of the two capacitors and another wire connects the lower plates; in other words, the capacitors are connected in parallel.

SET UP: In parts (a) and (b) we find the charge and stored energy for capacitor C_1 using Eqs. (24.1) and (24.9), respectively. In part (c) we use the character of the parallel connection to determine how the charge Q_0 is shared between the two capacitors. In part (d) we again use Eq. (24.9) to find the energy stored in capacitors C_1 and C_2; the total energy is the sum of these values.

24.12 When the switch S is closed, the charged capacitor C_1 is connected to an uncharged capacitor C_2. The center part of the switch is an insulating handle; charge can flow only between the two upper terminals and between the two lower terminals.



EXECUTE: (a) The charge Q_0 on C_1 is

$$Q_0 = C_1V_0 = (8.0\ \mu\text{F})(120\ \text{V}) = 960\ \mu\text{C}$$

(b) The energy initially stored in the capacitor is

$$U_{\text{initial}} = \frac{1}{2}Q_0V_0 = \frac{1}{2}(960 \times 10^{-6}\ \text{C})(120\ \text{V}) = 0.058\ \text{J}$$

(c) When the switch is closed, the positive charge Q_0 becomes distributed over the upper plates of both capacitors and the negative charge $-Q_0$ is distributed over the lower plates of both capacitors. Let Q_1 and Q_2 be the magnitudes of the final charges on the two capacitors. From conservation of charge,

$$Q_1 + Q_2 = Q_0$$

In the final state, when the charges are no longer moving, both upper plates are at the same potential; they are connected by a conducting wire and so form a single equipotential surface. Both lower plates are also at the same potential, different from that of the upper plates. The final potential difference V between the plates is therefore the same for both capacitors, as we would expect for a parallel connection. The capacitor charges are

$$Q_1 = C_1V \qquad Q_2 = C_2V$$

When we combine these with the preceding equation for conservation of charge, we find

$$V = \frac{Q_0}{C_1 + C_2} = \frac{960\ \mu\text{C}}{8.0\ \mu\text{F} + 4.0\ \mu\text{F}} = 80\ \text{V}$$

$$Q_1 = 640\ \mu\text{C} \qquad Q_2 = 320\ \mu\text{C}$$

(d) The final energy of the system is the sum of the energies stored in each capacitor:

$$U_{\text{final}} = \frac{1}{2}Q_1V + \frac{1}{2}Q_2V = \frac{1}{2}Q_0V$$

$$= \frac{1}{2}(960 \times 10^{-6}\ \text{C})(80\ \text{V}) = 0.038\ \text{J}$$

EVALUATE: The final energy is less than the original energy $U_{\text{initial}} = 0.058$ J; the difference has been converted to energy of some other form. The conductors become a little warmer because of their resistance, and some energy is radiated as electromagnetic waves. We'll study the circuit behavior of capacitors in detail in Chapters 26 and 31.

Example 24.8 Electric-field energy

Suppose you want to store 1.00 J of electric potential energy in a volume of 1.00 m^3 in vacuum. (a) What is the magnitude of the required electric field? (b) If the field magnitude is 10 times larger, how much energy is stored per cubic meter?

SOLUTION

IDENTIFY: We use the relationship between the electric-field magnitude E and the energy density u, which equals the electric-field energy divided by the volume occupied by the field.

SET UP: In part (a) we use the given information to find u, then we use Eq. (24.11) to find the required value of E. This same equation gives us the relationship between changes in E and the corresponding changes in u.

EXECUTE: (a) The desired energy density is $u = 1.00\ \text{J/m}^3$. We solve Eq. (24.11) for E:

$$E = \sqrt{\frac{2u}{\epsilon_0}} = \sqrt{\frac{2(1.00\ \text{J/m}^3)}{8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2}}$$

$$= 4.75 \times 10^5\ \text{N/C} = 4.75 \times 10^5\ \text{V/m}$$

(b) Equation (24.11) shows that u is proportional to E^2. If E increases by a factor of 10, u increases by a factor of $10^2 = 100$, and the energy density is $100\ \text{J/m}^3$.

EVALUATE: The value of E found in part (a) is sizable, corresponding to a potential difference of nearly a half million volts over a distance of 1 meter. We will see in Section 24.4 that the field magnitudes in practical insulators can be as great as this or even larger.

Example 24.9 Two ways to calculate energy stored in a capacitor

The spherical capacitor described in Example 24.3 (Section 24.1) has charges $+Q$ and $-Q$ on its inner and outer conductors. Find the electric potential energy stored in the capacitor (a) by using the capacitance C found in Example 24.3 and (b) by integrating the electric-field energy density.

SOLUTION

IDENTIFY: This problem asks us to think about the energy stored in a capacitor, U, in two different ways: in terms of the work done to put the charges on the two conductors, $U = Q^2/2C$, and in terms of the energy in the electric field between the two conductors. Both descriptions are equivalent, so both must give us the same answer for U.

SET UP: In Example 24.3 we found the capacitance C and the field magnitude E between the conductors. We find the stored energy U in part (a) using the expression for C in Eq. (24.9). In part (b) we use the expression for E in Eq. (24.11) to find the electric-field energy density u between the conductors. The field magnitude depends on the distance r from the center of the capacitor, so u also depends on r. Hence we cannot find U by simply multiplying u by the volume between the conductors; instead, we must integrate u over this volume.

EXECUTE: (a) From Example 24.3, the spherical capacitor has capacitance

$$C = 4\pi\epsilon_0 \frac{r_a r_b}{r_b - r_a}$$

where r_a and r_b are the radii of the inner and outer conducting spheres. From Eq. (24.9) the energy stored in this capacitor is

$$U = \frac{Q^2}{2C} = \frac{Q^2}{8\pi\epsilon_0}\frac{r_b - r_a}{r_a r_b}$$

(b) The electric field in the volume between the two conducting spheres has magnitude $E = Q/4\pi\epsilon_0 r^2$. The electric field is zero inside the inner sphere and is also zero outside the inner surface of the outer sphere, because a Gaussian surface with radius $r < r_a$ or $r > r_b$ encloses zero net charge. Hence the energy density is nonzero only in the space between the spheres $(r_a < r < r_b)$. In this region,

$$u = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\epsilon_0\left(\frac{Q}{4\pi\epsilon_0 r^2}\right)^2 = \frac{Q^2}{32\pi^2\epsilon_0 r^4}$$

The energy density is *not* uniform; it decreases rapidly with increasing distance from the center of the capacitor. To find the

Continued

total electric-field energy, we integrate u (the energy per unit volume) over the volume between the inner and outer conducting spheres. Dividing this volume up into spherical shells of radius r, surface area $4\pi r^2$, thickness dr, and volume $dV = 4\pi r^2\,dr$, we have

$$U = \int u\,dV = \int_{r_a}^{r_b}\left(\frac{Q^2}{32\pi^2\epsilon_0 r^4}\right)4\pi r^2\,dr$$
$$= \frac{Q^2}{8\pi\epsilon_0}\int_{r_a}^{r_b}\frac{dr}{r^2} = \frac{Q^2}{8\pi\epsilon_0}\left(-\frac{1}{r_b}+\frac{1}{r_a}\right)$$
$$= \frac{Q^2}{8\pi\epsilon_0}\,\frac{r_b - r_a}{r_a r_b}$$

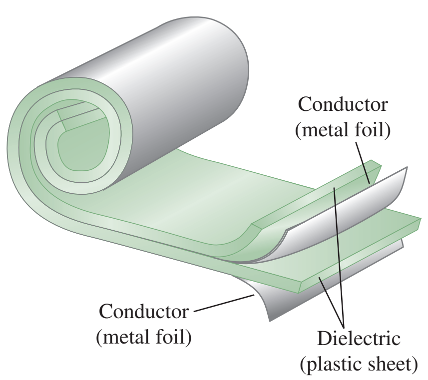
EVALUATE: We obtain the same result for U with either approach, as we must. We emphasize that electric potential energy can be regarded as being associated with either the *charges,* as in part (a), or the *field,* as in part (b); regardless of which viewpoint you choose, the amount of stored energy is the same.

Test Your Understanding of Section 24.3 You want to connect a 4-μF capacitor and an 8-μF capacitor. With which type of connection will the 4-μF capacitor have a greater amount of *stored energy* than the 8-μF capacitor? (i) series; (ii) parallel; (iii) either series or parallel; (iv) neither series nor parallel.

24.4 Dielectrics

Most capacitors have a nonconducting material, or **dielectric,** between their conducting plates. A common type of capacitor uses long strips of metal foil for the plates, separated by strips of plastic sheet such as Mylar. A sandwich of these materials is rolled up, forming a unit that can provide a capacitance of several microfarads in a compact package (Fig. 24.13).

24.13 A common type of capacitor uses dielectric sheets to separate the conductors.



Placing a solid dielectric between the plates of a capacitor serves three functions. First, it solves the mechanical problem of maintaining two large metal sheets at a very small separation without actual contact.

Second, using a dielectric increases the maximum possible potential difference between the capacitor plates. As we described in Section 23.3, any insulating material, when subjected to a sufficiently large electric field, experiences a partial ionization that permits conduction through it. This is called **dielectric breakdown.** Many dielectric materials can tolerate stronger electric fields without breakdown than can air. Thus using a dielectric allows a capacitor to sustain a higher potential difference V and so store greater amounts of charge and energy.

Third, the capacitance of a capacitor of given dimensions is *greater* when there is a dielectric material between the plates than when there is vacuum. We can demonstrate this effect with the aid of a sensitive *electrometer,* a device that measures the potential difference between two conductors without letting any appreciable charge flow from one to the other. Figure 24.14a shows an electrometer connected across a charged capacitor, with magnitude of charge Q on each plate and potential difference V_0. When we insert an uncharged sheet of dielectric, such as glass, paraffin, or polystyrene, between the plates, experiment shows that the potential difference *decreases* to a smaller value V (Fig. 24.14b). When we remove the dielectric, the potential difference returns to its original value V_0, showing that the original charges on the plates have not changed.

The original capacitance C_0 is given by $C_0 = Q/V_0$, and the capacitance C with the dielectric present is $C = Q/V$. The charge Q is the same in both cases, and V is less than V_0, so we conclude that the capacitance C with the dielectric present is *greater* than C_0. When the space between plates is completely filled by the dielectric, the ratio of C to C_0 (equal to the ratio of V_0 to V) is called the **dielectric constant** of the material, K:

$$K = \frac{C}{C_0} \quad \text{(definition of dielectric constant)} \tag{24.12}$$

When the charge is constant, $Q = C_0V_0 = CV$ and $C/C_0 = V_0/V$. In this case, Eq. (24.12) can be rewritten as

$$V = \frac{V_0}{K} \quad \text{(when } Q \text{ is constant)} \tag{24.13}$$

With the dielectric present, the potential difference for a given charge Q is *reduced* by a factor K.

The dielectric constant K is a pure number. Because C is always greater than C_0, K is always greater than unity. Some representative values of K are given in Table 24.1. For vacuum, $K = 1$ by definition. For air at ordinary temperatures and pressures, K is about 1.0006; this is so nearly equal to 1 that for most purposes an air capacitor is equivalent to one in vacuum. Note that while water has a very large value of K, it is usually not a very practical dielectric for use in capacitors. The reason is that while pure water is a very poor conductor, it is also an excellent ionic solvent. Any ions that are dissolved in the water will cause charge to flow between the capacitor plates, so the capacitor discharges.

24.14 Effect of a dielectric between the plates of a parallel-plate capacitor. (a) With a given charge, the potential difference is V_0. (b) With the same charge but with a dielectric between the plates, the potential difference V is smaller than V_0.

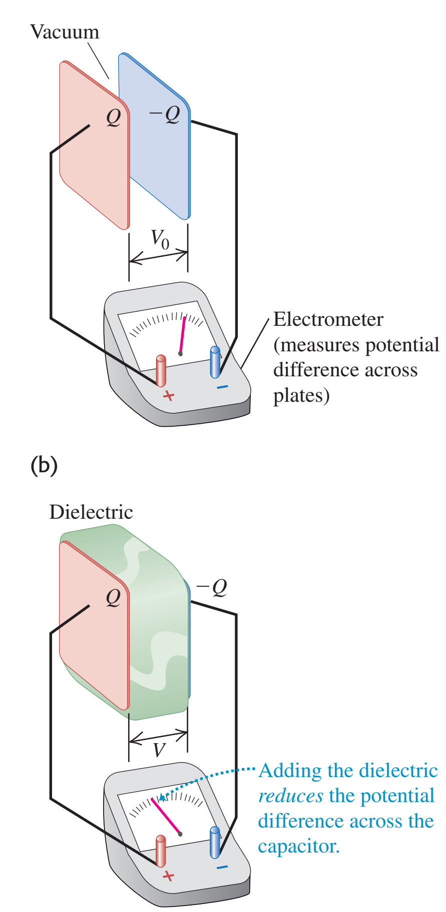


Table 24.1 Values of Dielectric Constant K at 20°C

Material	K	Material	K
Vacuum	1	Polyvinyl chloride	3.18
Air (1 atm)	1.00059	Plexiglas	3.40
Air (100 atm)	1.0548	Glass	5–10
Teflon	2.1	Neoprene	6.70
Polyethylene	2.25	Germanium	16
Benzene	2.28	Glycerin	42.5
Mica	3–6	Water	80.4
Mylar	3.1	Strontium titanate	310

No real dielectric is a perfect insulator. Hence there is always some *leakage current* between the charged plates of a capacitor with a dielectric. We tacitly ignored this effect in Section 24.2 when we derived expressions for the equivalent capacitances of capacitors in series, Eq. (24.5), and in parallel, Eq. (24.7). But if a leakage current flows for a long enough time to substantially change the charges from the values we used to derive Eqs. (24.5) and (24.7), those equations may no longer be accurate.

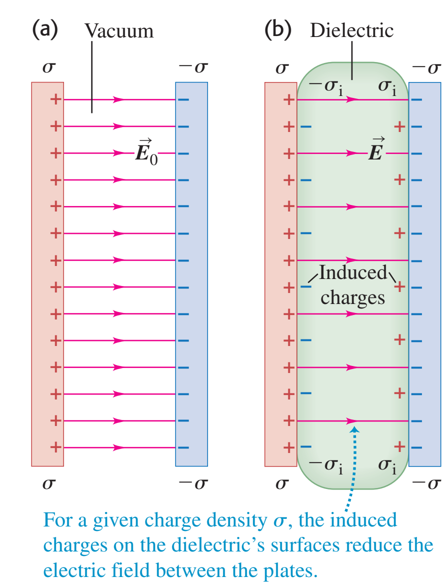
Induced Charge and Polarization

When a dielectric material is inserted between the plates while the charge is kept constant, the potential difference between the plates decreases by a factor K. Therefore the electric field between the plates must decrease by the same factor. If E_0 is the vacuum value and E is the value with the dielectric, then

$$E = \frac{E_0}{K} \quad \text{(when } Q \text{ is constant)} \tag{24.14}$$

Since the electric-field magnitude is smaller when the dielectric is present, the surface charge density (which causes the field) must be smaller as well. The surface charge on the conducting plates does not change, but an *induced* charge of the opposite sign appears on each surface of the dielectric (Fig. 24.15). The dielectric was originally electrically neutral and is still neutral; the induced surface charges arise as a result of *redistribution* of positive and negative charge within the dielectric material, a phenomenon called **polarization.** We first encountered polarization in Section 21.2, and we suggest that you reread the discussion of Fig. 21.8. We will assume that the induced surface charge is *directly proportional* to the electric-field magnitude E in the material; this is indeed the case for many common dielectrics. (This direct proportionality is analogous to

24.15 Electric field lines with (a) vacuum between the plates and (b) dielectric between the plates.



Hooke's law for a spring.) In that case, K is a constant for any particular material. When the electric field is very strong or if the dielectric is made of certain crystalline materials, the relationship between induced charge and the electric field can be more complex; we won't consider such cases here.

We can derive a relationship between this induced surface charge and the charge on the plates. Let's denote the magnitude of the charge per unit area induced on the surfaces of the dielectric (the induced surface charge density) by σ_i. The magnitude of the surface charge density on the capacitor plates is σ, as usual. Then the *net* surface charge on each side of the capacitor has magnitude $(\sigma - \sigma_i)$, as shown in Fig. 24.15b. As we found in Example 21.13 (Section 21.5) and in Example 22.8 (Section 22.4), the field between the plates is related to the net surface charge density by $E = \sigma_{net}/\epsilon_0$. Without and with the dielectric, respectively, we have

$$E_0 = \frac{\sigma}{\epsilon_0} \qquad E = \frac{\sigma - \sigma_i}{\epsilon_0} \tag{24.15}$$

Using these expressions in Eq. (24.14) and rearranging the result, we find

$$\sigma_i = \sigma\left(1 - \frac{1}{K}\right) \quad \text{(induced surface charge density)} \tag{24.16}$$

This equation shows that when K is very large, σ_i is nearly as large as σ. In this case, σ_i nearly cancels σ, and the field and potential difference are much smaller than their values in vacuum.

The product $K\epsilon_0$ is called the **permittivity** of the dielectric, denoted by ϵ:

$$\epsilon = K\epsilon_0 \quad \text{(definition of permittivity)} \tag{24.17}$$

In terms of ϵ we can express the electric field within the dielectric as

$$E = \frac{\sigma}{\epsilon} \tag{24.18}$$

The capacitance when the dielectric is present is given by

$$C = KC_0 = K\epsilon_0\frac{A}{d} = \epsilon\frac{A}{d} \quad \text{(parallel-plate capacitor, dielectric between plates)} \tag{24.19}$$

We can repeat the derivation of Eq. (24.11) for the energy density u in an electric field for the case in which a dielectric is present. The result is

$$u = \frac{1}{2}K\epsilon_0 E^2 = \frac{1}{2}\epsilon E^2 \quad \text{(electric energy density in a dielectric)} \tag{24.20}$$

In empty space, where $K = 1$, $\epsilon = \epsilon_0$ and Eqs. (24.19) and (24.20) reduce to Eqs. (24.2) and (24.11), respectively, for a parallel-plate capacitor in vacuum. For this reason, ϵ_0 is sometimes called the "permittivity of free space" or the "permittivity of vacuum." Because K is a pure number, ϵ and ϵ_0 have the same units, $C^2/N \cdot m^2$ or F/m.

Equation (24.19) shows that extremely high capacitances can be obtained with plates that have a large surface area A and are separated by a small distance d by a dielectric with a large value of K. In an *electrolytic double-layer capacitor,* tiny carbon granules adhere to each plate: The value of A is the combined surface area of the granules, which can be tremendous. The plates with granules attached are separated by a very thin dielectric sheet. A capacitor of this kind can have a capacitance of 5000 farads yet fit in the palm of your hand (compare Example 24.1 in Section 24.1).

Several practical devices make use of the way in which a capacitor responds to a change in dielectric constant. One example is an electric stud finder, used by home repair workers to locate metal studs hidden behind a wall's surface. It consists of a metal plate with associated circuitry. The plate acts as one half of a capacitor, with the wall acting as the other half. If the stud finder moves over a metal stud, the effective dielectric constant for the capacitor changes, changing the capacitance and triggering a signal.

Problem-Solving Strategy 24.2 Dielectrics

IDENTIFY *the relevant concepts:* The relationships in this section are useful whenever there is an electric field in a dielectric, such as a dielectric between charged capacitor plates. Typically you will be asked to relate the potential difference between the plates, the electric field in the capacitor, the charge density on the capacitor plates, and the induced charge density on the surfaces of the capacitor.

SET UP *the problem* using the following steps:
1. Make a drawing of the situation.
2. Identify the target variables, and choose which of the key equations of this section will help you find those variables.

EXECUTE *the solution* as follows:
1. In problems such as the next example, it is easy to get lost in a blizzard of formulas. Ask yourself at each step what kind of quantity each symbol represents. For example, distinguish clearly between charges and charge densities, and between electric fields and electric potential differences.
2. As you calculate, continually check for consistency of units. This effort is a bit more complex with electrical quantities than it was in mechanics. Distances must always be in meters. Remember that a microfarad is 10^{-6} farad, and so on. Don't confuse the numerical value of ϵ_0 with the value of $1/4\pi\epsilon_0$. There are several alternative sets of units for electric-field magnitude, including N/C and V/m. The units of ϵ_0 are $C^2/N \cdot m^2$ or F/m.

EVALUATE *your answer:* When you check numerical values, remember that with a dielectric present, (a) the capacitance is always greater than without a dielectric; (b) for a given amount of charge on the capacitor, the electric field and potential difference are less than without a dielectric; and (c) the induced surface charge density σ_i on the dielectric is always less in magnitude than the charge density σ on the capacitor plates.

Example 24.10 A capacitor with and without a dielectric

Suppose the parallel plates in Fig. 24.15 each have an area of $2000\ cm^2$ $(2.00 \times 10^{-1}\ m^2)$ and are 1.00 cm $(1.00 \times 10^{-2}\ m)$ apart. The capacitor is connected to a power supply and charged to a potential difference $V_0 = 3000\ V = 3.00\ kV$. It is then disconnected from the power supply, and a sheet of insulating plastic material is inserted between the plates, completely filling the space between them. We find that the potential difference decreases to 1000 V while the charge on each capacitor plate remains constant. Compute (a) the original capacitance C_0; (b) the magnitude of charge Q on each plate; (c) the capacitance C after the dielectric is inserted; (d) the dielectric constant K of the dielectric; (e) the permittivity ϵ of the dielectric; (f) the magnitude of the induced charge Q_i on each face of the dielectric; (g) the original electric field E_0 between the plates; and (h) the electric field E after the dielectric is inserted.

SOLUTION

IDENTIFY: This problem uses most of the relationships we have discussed for capacitors and dielectrics.

SET UP: Most of the target variables can be obtained in several different ways. The methods used below are a representative sample; we encourage you to think of others and compare your results.

EXECUTE: (a) With vacuum between the plates, we use Eq. (24.19) with $K = 1$:

$$C_0 = \epsilon_0\frac{A}{d} = (8.85 \times 10^{-12}\ F/m)\frac{2.00 \times 10^{-1}\ m^2}{1.00 \times 10^{-2}\ m} = 1.77 \times 10^{-10}\ F = 177\ pF$$

(b) Using the definition of capacitance, Eq. (24.1),

$$Q = C_0V_0 = (1.77 \times 10^{-10}\ F)(3.00 \times 10^3\ V) = 5.31 \times 10^{-7}\ C = 0.531\ \mu C$$

(c) When the dielectric is inserted, the charge remains the same but the potential decreases to $V = 1000$ V. Hence from Eq. (24.1), the new capacitance is

$$C = \frac{Q}{V} = \frac{5.31 \times 10^{-7}\ C}{1.00 \times 10^3\ V} = 5.31 \times 10^{-10}\ F = 531\ pF$$

(d) From Eq. (24.12), the dielectric constant is

$$K = \frac{C}{C_0} = \frac{5.31 \times 10^{-10}\ F}{1.77 \times 10^{-10}\ F} = \frac{531\ pF}{177\ pF} = 3.00$$

Alternatively, from Eq. (24.13),

$$K = \frac{V_0}{V} = \frac{3000\ V}{1000\ V} = 3.00$$

(e) Using K from part (d) in Eq. (24.17), the permittivity is

$$\epsilon = K\epsilon_0 = (3.00)(8.85 \times 10^{-12}\ C^2/N \cdot m^2) = 2.66 \times 10^{-11}\ C^2/N \cdot m^2$$

(f) Multiplying Eq. (24.15) by the area of each plate gives the induced charge $Q_i = \sigma_i A$ in terms of the charge $Q = \sigma A$ on each plate:

$$Q_i = Q\left(1 - \frac{1}{K}\right) = (5.31 \times 10^{-7}\ C)\left(1 - \frac{1}{3.00}\right) = 3.54 \times 10^{-7}\ C$$

Continued

(g) Since the electric field between the plates is uniform, its magnitude is the potential difference divided by the plate separation:

$$E_0 = \frac{V_0}{d} = \frac{3000\ \text{V}}{1.00 \times 10^{-2}\ \text{m}} = 3.00 \times 10^5\ \text{V/m}$$

(h) With the new potential difference after the dielectric is inserted,

$$E = \frac{V}{d} = \frac{1000\ \text{V}}{1.00 \times 10^{-2}\ \text{m}} = 1.00 \times 10^5\ \text{V/m}$$

or, from Eq. (24.17),

$$E = \frac{\sigma}{\epsilon} = \frac{Q}{\epsilon A} = \frac{5.31 \times 10^{-7}\ \text{C}}{(2.66 \times 10^{-11}\ \text{C}^2/\text{N}\cdot\text{m}^2)(2.00 \times 10^{-1}\ \text{m}^2)}$$
$$= 1.00 \times 10^5\ \text{V/m}$$

or, from Eq. (24.15),

$$E = \frac{\sigma - \sigma_\text{i}}{\epsilon_0} = \frac{Q - Q_\text{i}}{\epsilon_0 A}$$
$$= \frac{(5.31 - 3.54) \times 10^{-7}\ \text{C}}{(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(2.00 \times 10^{-1}\ \text{m}^2)}$$
$$= 1.00 \times 10^5\ \text{V/m}$$

or, from Eq. (24.14),

$$E = \frac{E_0}{K} = \frac{3.00 \times 10^5\ \text{V/m}}{3.00} = 1.00 \times 10^5\ \text{V/m}$$

EVALUATE: It's always useful to check the results by finding them in more than one way, as we did in parts (d) and (h). Our results show that inserting the dielectric increased the capacitance by a factor of $K = 3.00$ and reduced the electric field between the plates by a factor of $1/K = 1/3.00$. It did so by developing induced charges on the faces of the dielectric of magnitude $Q(1 - 1/K) = Q(1 - 1/3.00) = 0.667Q$.

Example 24.11 Energy storage with and without a dielectric

Find the total energy stored in the electric field of the capacitor in Example 24.10 and the energy density, both before and after the dielectric sheet is inserted.

SOLUTION

IDENTIFY: In this problem we have to extend the analysis of Example 24.10 to include the ideas of energy stored in a capacitor and electric-field energy.

SET UP: We use Eq. (24.9) to find the stored energy before and after the dielectric is inserted, and Eq. (24.20) to find the energy density.

EXECUTE: Let the original energy be U_0 and let the energy with the dielectric in place be U. From Eq. (24.9),

$$U_0 = \frac{1}{2}C_0V_0^2 = \frac{1}{2}(1.77 \times 10^{-10}\ \text{F})(3000\ \text{V})^2 = 7.97 \times 10^{-4}\ \text{J}$$

$$U = \frac{1}{2}CV^2 = \frac{1}{2}(5.31 \times 10^{-10}\ \text{F})(1000\ \text{V})^2 = 2.66 \times 10^{-4}\ \text{J}$$

The final energy is one-third of the original energy.

The energy density without the dielectric is given by Eq. (24.20) with $K = 1$:

$$u_0 = \frac{1}{2}\epsilon_0E_0^2 = \frac{1}{2}(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.0 \times 10^5\ \text{N/C})^2$$
$$= 0.398\ \text{J/m}^3$$

With the dielectric in place,

$$u = \frac{1}{2}\epsilon E^2 = \frac{1}{2}(2.66 \times 10^{-11}\ \text{C}^2/\text{N}\cdot\text{m}^2)(1.00 \times 10^5\ \text{N/C})^2$$
$$= 0.133\ \text{J/m}^3$$

The energy density with the dielectric is one-third of the original energy density.

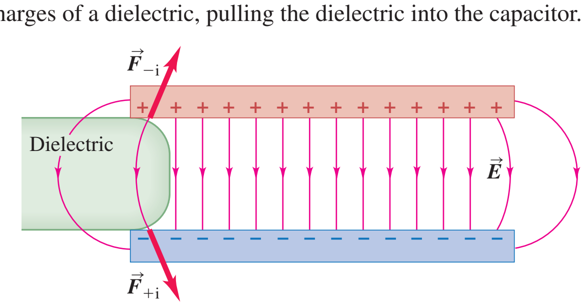
EVALUATE: We can check our answer for u_0 by noting that the volume between the plates is $V = (0.200\ \text{m})^2(0.0100\ \text{m}) = 0.00200\ \text{m}^3$. Since the electric field is uniform between the plates, u_0 is uniform as well and the energy density is just the stored energy divided by the volume:

$$u_0 = \frac{U_0}{V} = \frac{7.97 \times 10^{-4}\ \text{J}}{0.00200\ \text{m}^3} = 0.398\ \text{J/m}^3$$

which agrees with our earlier answer. You should use the same approach to check the value for U, the energy density with the dielectric.

We can generalize the results of this example. When a dielectric is inserted into a capacitor while the charge on each plate remains the same, the permittivity ϵ increases by a factor of K (the dielectric constant), the electric field decreases by a factor of $1/K$, and the energy density $u = \frac{1}{2}\epsilon E^2$ decreases by a factor of $1/K$. Where did the energy go? The answer lies in the fringing field at the edges of a real parallel-plate capacitor. As Fig. 24.16 shows, that field tends to pull the dielectric into the space between the plates, doing work on it as it does so. We could attach a spring to the left end of the dielectric in Fig. 24.16 and use this force to stretch the spring. Because work is done by the field, the field energy density decreases.

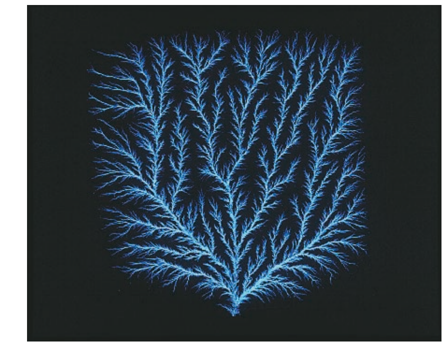
24.16 The fringing field at the edges of the capacitor exerts forces $\vec{F}_{-\text{i}}$ and $\vec{F}_{+\text{i}}$ on the negative and positive induced surface charges of a dielectric, pulling the dielectric into the capacitor.



Dielectric Breakdown

We mentioned earlier that when any dielectric material is subjected to a sufficiently strong electric field, *dielectric breakdown* takes place and the dielectric becomes a conductor (Fig. 24.17). This occurs when the electric field is so strong that electrons are ripped loose from their molecules and crash into other molecules, liberating even more electrons. This avalanche of moving charge, forming a spark or arc discharge, often starts quite suddenly.

24.17 A very strong electric field caused dielectric breakdown in a block of Plexiglas. The resulting flow of charge etched this pattern into the block.



Because of dielectric breakdown, capacitors always have maximum voltage ratings. When a capacitor is subjected to excessive voltage, an arc may form through a layer of dielectric, burning or melting a hole in it. This arc creates a conducting path (a short circuit) between the conductors. If a conducting path remains after the arc is extinguished, the device is rendered permanently useless as a capacitor.

The maximum electric-field magnitude that a material can withstand without the occurrence of breakdown is called its **dielectric strength.** This quantity is affected significantly by temperature, trace impurities, small irregularities in the metal electrodes, and other factors that are difficult to control. For this reason we can give only approximate figures for dielectric strengths. The dielectric strength of dry air is about $3 \times 10^6\ \text{V/m}$. Values of dielectric strength for a few common insulating materials are shown in Table 24.2. Note that the values are all substantially greater than the value for air. For example, a layer of polycarbonate 0.01 mm thick (about the smallest practical thickness) has 10 times the dielectric strength of air and can withstand a maximum voltage of about $(3 \times 10^7\ \text{V/m})(1 \times 10^{-5}\ \text{m}) = 300\ \text{V}$.

Table 24.2 Dielectric Constant and Dielectric Strength of Some Insulating Materials

Material	Dielectric Constant, K	Dielectric Strength, E_m (V/m)
Polycarbonate	2.8	3×10^7
Polyester	3.3	6×10^7
Polypropylene	2.2	7×10^7
Polystyrene	2.6	2×10^7
Pyrex glass	4.7	1×10^7

Test Your Understanding of Section 24.4 The space between the plates of an isolated parallel-plate capacitor is filled by a slab of dielectric with dielectric constant K. The two plates of the capacitor have charges Q and $-Q$. You pull out the dielectric slab. If the charges do not change, how does the energy in the capacitor change when you remove the slab? (i) It increases; (ii) it decreases; (iii) it remains the same.

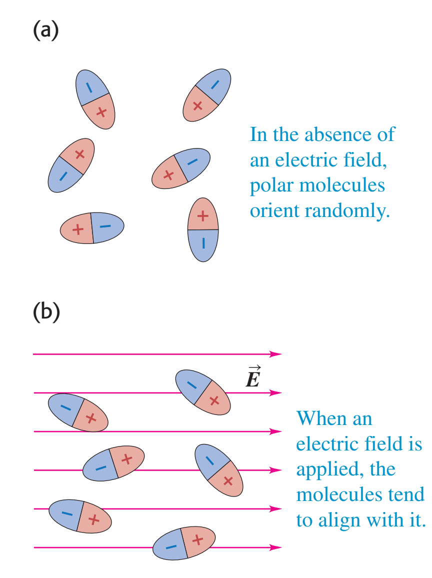
MP

*24.5 Molecular Model of Induced Charge

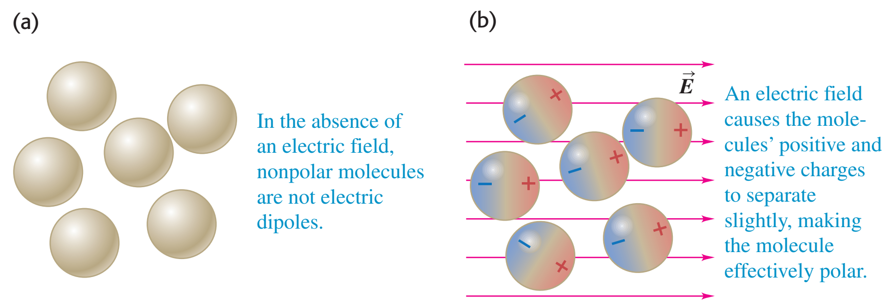
In Section 24.4 we discussed induced surface charges on a dielectric in an electric field. Now let's look at how these surface charges can arise. If the material were a *conductor,* the answer would be simple. Conductors contain charge that is free to move, and when an electric field is present, some of the charge redistributes itself on the surface so that there is no electric field inside the conductor. But an ideal dielectric has *no* charges that are free to move, so how can a surface charge occur?

To understand this, we have to look again at rearrangement of charge at the *molecular* level. Some molecules, such as H_2O and N_2O, have equal amounts of positive and negative charges but a lopsided distribution, with excess positive charge concentrated on one side of the molecule and negative charge on the other. As we described in Section 21.7, such an arrangement is called an *electric dipole,* and the molecule is called a *polar molecule.* When no electric field is present in a gas or liquid with polar molecules, the molecules are oriented randomly (Fig. 24.18a). When they are placed in an electric field, however, they tend

24.18 Polar molecules (a) without and (b) with an applied electric field $\vec{E}$.



24.19 Nonpolar molecules (a) without and (b) with an applied electric field $\vec{E}$.



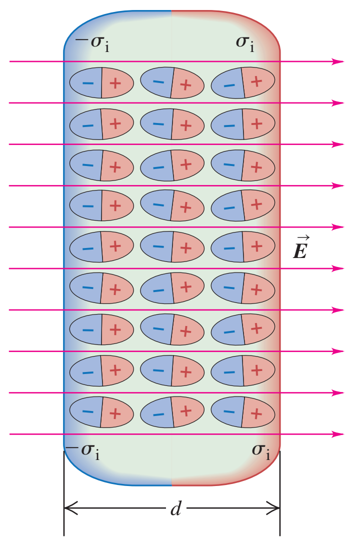
to orient themselves as in Fig. 24.18b, as a result of the electric-field torques described in Section 21.7. Because of thermal agitation, the alignment of the molecules with $\vec{E}$ is not perfect.

Even a molecule that is *not* ordinarily polar *becomes* a dipole when it is placed in an electric field because the field pushes the positive charges in the molecules in the direction of the field and pushes the negative charges in the opposite direction. This causes a redistribution of charge within the molecule (Fig. 24.19). Such dipoles are called *induced* dipoles.

With either polar or nonpolar molecules, the redistribution of charge caused by the field leads to the formation of a layer of charge on each surface of the dielectric material (Fig. 24.20). These layers are the surface charges described in Section 24.4; their surface charge density is denoted by σ_i. The charges are *not* free to move indefinitely, as they would be in a conductor, because each charge is bound to a molecule. They are in fact called **bound charges** to distinguish them from the **free charges** that are added to and removed from the conducting capacitor plates. In the interior of the material the net charge per unit volume remains zero. As we have seen, this redistribution of charge is called *polarization,* and we say that the material is *polarized.*

The four parts of Fig. 24.21 show the behavior of a slab of dielectric when it is inserted in the field between a pair of oppositely charged capacitor plates. Figure 24.21a shows the original field. Figure 24.21b is the situation after the dielectric has been inserted but before any rearrangement of charges has occurred.

24.20 Polarization of a dielectric in an electric field $\vec{E}$ gives rise to thin layers of bound charges on the surfaces, creating surface charge densities σ_i and $-\sigma_i$. The sizes of the molecules are greatly exaggerated for clarity.



24.21 (a) Electric field of magnitude E_0 between two charged plates. (b) Introduction of a dielectric of dielectric constant K. (c) The induced surface charges and their field. (d) Resultant field of magnitude E_0/K.

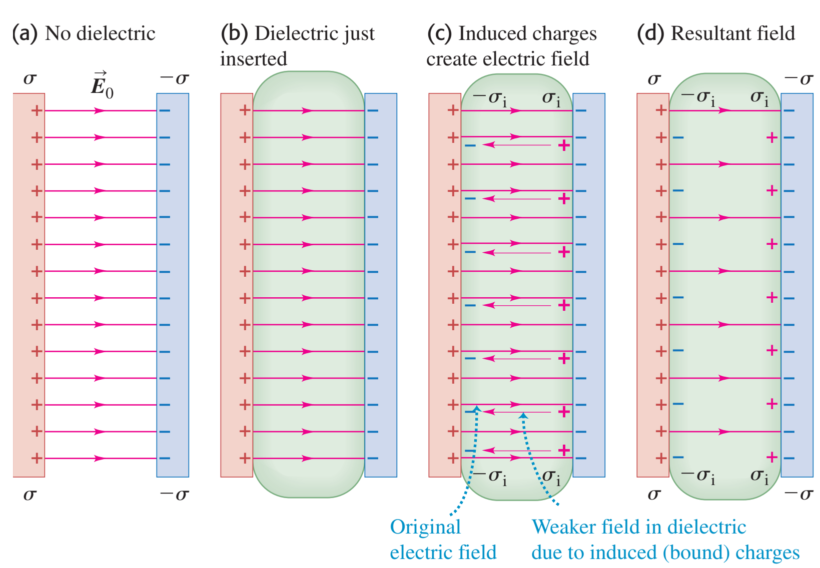
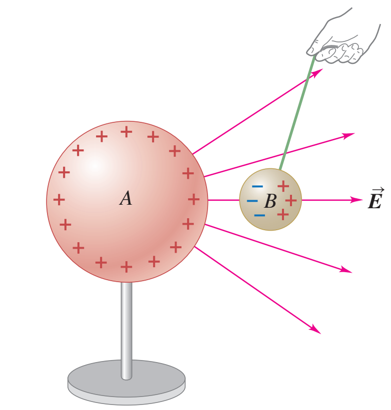


Figure 24.21c shows by thinner arrows the additional field set up in the dielectric by its induced surface charges. This field is *opposite* to the original field, but it is not great enough to cancel the original field completely because the charges in the dielectric are not free to move indefinitely. The resultant field in the dielectric, shown in Fig. 24.21d, is therefore decreased in magnitude. In the field-line representation, some of the field lines leaving the positive plate go through the dielectric, while others terminate on the induced charges on the faces of the dielectric.

As we discussed in Section 21.2, polarization is also the reason a charged body, such as an electrified plastic rod, can exert a force on an *uncharged* body such as a bit of paper or a pith ball. Figure 24.22 shows an uncharged dielectric sphere B in the radial field of a positively charged body A. The induced positive charges on B experience a force toward the right, while the force on the induced negative charges is toward the left. The negative charges are closer to A, and thus are in a stronger field, than are the positive charges. The force toward the left is stronger than that toward the right, and B is attracted toward A, even though its net charge is zero. The attraction occurs whether the sign of A's charge is positive or negative (see Fig. 21.8). Furthermore, the effect is not limited to dielectrics; an uncharged conducting body would be attracted in the same way.

24.22 A neutral sphere B in the radial electric field of a positively charged sphere A is attracted to the charge because of polarization.



Test Your Understanding of Section 24.5 A parallel-plate capacitor has charges Q and $-Q$ on its two plates. A dielectric slab with $K = 3$ is then inserted into the space between the plates as shown in Fig. 24.21. Rank the following electric-field magnitudes in order from largest to smallest. (i) the field before the slab is inserted; (ii) the resultant field after the slab is inserted; (iii) the field due to the bound charges.

*24.6 Gauss's Law in Dielectrics

We can extend the analysis of Section 24.4 to reformulate Gauss's law in a form that is particularly useful for dielectrics. Figure 24.23 is a close-up view of the left capacitor plate and left surface of the dielectric in Fig. 24.15b. Let's apply Gauss's law to the rectangular box shown in cross section by the purple line; the surface area of the left and right sides is A. The left side is embedded in the conductor that forms the left capacitor plate, and so the electric field everywhere on that surface is zero. The right side is embedded in the dielectric, where the electric field has magnitude E, and $E_\perp = 0$ everywhere on the other four sides. The total charge enclosed, including both the charge on the capacitor plate and the induced charge on the dielectric surface, is $Q_{encl} = (\sigma - \sigma_i)A$, so Gauss's law gives

$$EA = \frac{(\sigma - \sigma_i)A}{\epsilon_0} \qquad (24.21)$$

This equation is not very illuminating as it stands because it relates two unknown quantities: E inside the dielectric and the induced surface charge density σ_i. But now we can use Eq. (24.16), developed for this same situation, to simplify this equation by eliminating σ_i. Equation (24.16) is

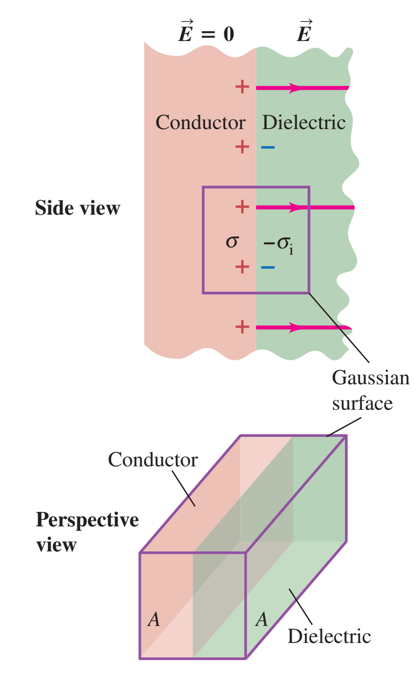
$$\sigma_i = \sigma\left(1 - \frac{1}{K}\right) \quad \text{or} \quad \sigma - \sigma_i = \frac{\sigma}{K}$$

Combining this with Eq. (24.21), we get

$$EA = \frac{\sigma A}{K\epsilon_0} \quad \text{or} \quad KEA = \frac{\sigma A}{\epsilon_0} \qquad (24.22)$$

Equation (24.22) says that the flux of $K\vec{E}$, not $\vec{E}$, through the Gaussian surface in Fig. 24.23 is equal to the enclosed *free* charge σA divided by ϵ_0. It turns out

24.23 Gauss's law with a dielectric. This figure shows a close-up of the left-hand capacitor plate in Fig. 24.15b. The Gaussian surface is a rectangular box that lies half in the conductor and half in the dielectric.



that for *any* Gaussian surface, whenever the induced charge is proportional to the electric field in the material, we can rewrite Gauss's law as

$$\oint K\vec{E}\cdot d\vec{A} = \frac{Q_{\text{encl-free}}}{\epsilon_0} \quad \text{(Gauss's law in a dielectric)} \quad (24.23)$$

where $Q_{\text{encl-free}}$ is the total *free* charge (not bound charge) enclosed by the Gaussian surface. The significance of these results is that the right sides contain only the *free* charge on the conductor, not the bound (induced) charge. In fact, although we have not proved it, Eq. (24.23) remains valid even when different parts of the Gaussian surface are embedded in dielectrics having different values of K, provided that the value of K in each dielectric is independent of the electric field (usually the case for electric fields that are not too strong) and that we use the appropriate value of K for each point on the Gaussian surface.

Example 24.12 A spherical capacitor with dielectric

In the spherical capacitor of Example 24.3 (Section 24.1), the volume between the concentric spherical conducting shells is filled with an insulating oil with dielectric constant K. Use Gauss's law to find the capacitance.

SOLUTION

IDENTIFY: This is essentially the same problem as Example 24.3. The only difference is the presence of the dielectric.

SET UP: As we did in Example 24.3, we use a spherical Gaussian surface of radius r between the two spheres. Since a dielectric is present, we use Gauss's law in the form of Eq. (24.23).

EXECUTE: The spherical symmetry of the problem is not changed by the presence of the dielectric, so we have

$$\oint K\vec{E}\cdot d\vec{A} = \oint KE\,dA = KE\oint dA = (KE)(4\pi r^2) = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{4\pi K\epsilon_0 r^2} = \frac{Q}{4\pi\epsilon r^2}$$

where $\epsilon = K\epsilon_0$ is the permittivity of the dielectric (introduced in Section 24.4). Compared to the case in which there is vacuum between the conducting shells, the electric field is reduced by a factor of $1/K$. The potential difference V_{ab} between the shells is likewise reduced by a factor of $1/K$, and so the capacitance $C = Q/V_{ab}$ is *increased* by a factor of K, just as for a parallel-plate capacitor when a dielectric is inserted. Using the result for the vacuum case in Example 24.3, we find that the capacitance with the dielectric is

$$C = \frac{4\pi K\epsilon_0 r_a r_b}{r_b - r_a} = \frac{4\pi\epsilon r_a r_b}{r_b - r_a}$$

EVALUATE: In this case the dielectric completely fills the volume between the two conductors, so the capacitance is just K times the value with no dielectric. The result is more complicated if the dielectric only partially fills this volume (see Challenge Problem 24.76).

Test Your Understanding of Section 24.6 A single point charge q is imbedded in a dielectric of dielectric constant K. At a point inside the dielectric a distance r from the point charge, what is the magnitude of the electric field? (i) $q/4\pi\epsilon_0 r^2$; (ii) $Kq/4\pi\epsilon_0 r^2$; (iii) $q/4\pi K\epsilon_0 r^2$; (iv) none of these.

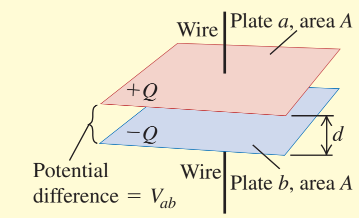
CHAPTER 24 SUMMARY

Capacitors and capacitance: A capacitor is any pair of conductors separated by an insulating material. When the capacitor is charged, there are charges of equal magnitude Q and opposite sign on the two conductors, and the potential V_{ab} of the positively charged conductor with respect to the negatively charged conductor is proportional to Q. The capacitance C is defined as the ratio of Q to V_{ab}. The SI unit of capacitance is the farad (F): $1\text{ F} = 1\text{ C/V}$.

A parallel-plate capacitor consists of two parallel conducting plates, each with area A, separated by a distance d. If they are separated by vacuum, the capacitance depends only on A and d. For other geometries, the capacitance can be found by using the definition $C = Q/V_{ab}$. (See Examples 24.1–24.4.)

$$C = \frac{Q}{V_{ab}} \quad (24.1)$$

$$C = \frac{Q}{V_{ab}} = \epsilon_0\frac{A}{d} \quad (24.2)$$



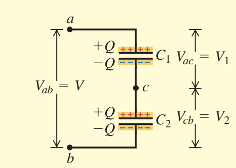
Capacitors in series and parallel: When capacitors with capacitances $C_1, C_2, C_3, \ldots$ are connected in series, the reciprocal of the equivalent capacitance C_{eq} equals the sum of the reciprocals of the individual capacitances. When capacitors are connected in parallel, the equivalent capacitance C_{eq} equals the sum of the individual capacitances. (See Examples 24.5 and 24.6.)

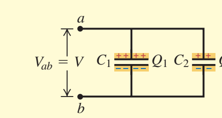
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \quad (24.5)$$

(capacitors in series)

$$C_{eq} = C_1 + C_2 + C_3 + \cdots \quad (24.7)$$

(capacitors in parallel)

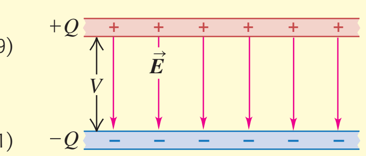




Energy in a capacitor: The energy U required to charge a capacitor C to a potential difference V and a charge Q is equal to the energy stored in the capacitor. This energy can be thought of as residing in the electric field between the conductors; the energy density u (energy per unit volume) is proportional to the square of the electric-field magnitude. (See Examples 24.7–24.9.)

$$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV \quad (24.9)$$

$$u = \frac{1}{2}\epsilon_0 E^2 \quad (24.11)$$



Dielectrics: When the space between the conductors is filled with a dielectric material, the capacitance increases by a factor K, called the dielectric constant of the material. The quantity $\epsilon = K\epsilon_0$ is called the permittivity of the dielectric. For a fixed amount of charge on the capacitor plates, induced charges on the surface of the dielectric decrease the electric field and potential difference between the plates by the same factor K. The surface charge results from polarization, a microscopic rearrangement of charge in the dielectric. (See Example 24.10.)

Under sufficiently strong fields, dielectrics become conductors, a situation called dielectric breakdown. The maximum field that a material can withstand without breakdown is called its dielectric strength.

In a dielectric, the expression for the energy density is the same as in vacuum but with ϵ_0 replaced by $\epsilon = K\epsilon$. (See Example 24.11.)

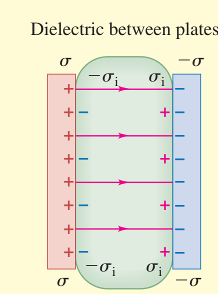
Gauss's law in a dielectric has almost the same form as in vacuum, with two key differences: $\vec{E}$ is replaced by $K\vec{E}$ and Q_{encl} is replaced by $Q_{\text{encl-free}}$, which includes only the free charge (not bound charge) enclosed by the Gaussian surface. (See Example 24.12.)

$$C = KC_0 = K\epsilon_0\frac{A}{d} = \epsilon\frac{A}{d} \quad (24.19)$$

(parallel-plate capacitor filled with dielectric)

$$u = \frac{1}{2}K\epsilon_0 E^2 = \frac{1}{2}\epsilon E^2 \quad (24.20)$$

$$\oint K\vec{E}\cdot d\vec{A} = \frac{Q_{\text{encl-free}}}{\epsilon_0} \quad (24.23)$$



Key Terms

capacitor, *816*
capacitance, *816*
farad, *816*
parallel-plate capacitor, *817*
series connection, *820*
equivalent capacitance, *821*
parallel connection, *821*
energy density, *826*
dielectric, *828*
dielectric breakdown, *828*
dielectric constant, *828*
polarization, *829*
permittivity, *830*
dielectric strength, *833*
bound charge, *834*
free charge, *834*

Answer to Chapter Opening Question ?

Equation (24.9) shows that the energy stored in a capacitor with capacitance C and charge Q is $U = Q^2/2C$. If the charge Q is doubled, the stored energy increases by a factor of $2^2 = 4$. Note that if the value of Q is too great, the electric-field magnitude inside the capacitor will exceed the dielectric strength of the material between the plates and dielectric breakdown will occur (see Section 24.4). This puts a practical limit on the amount of energy that can be stored.

Answers to Test Your Understanding Questions

24.1 Answer: (iii) The capacitance does not depend on the value of the charge Q. Doubling the value of Q causes the potential difference V_{ab} to double, so the capacitance $C = Q/V_{ab}$ remains the same. These statements are true no matter what the geometry of the capacitor.

24.2 Answers: (a) (i), (b) (iv) In a series connection the two capacitors carry the same charge Q but have different potential differences $V_{ab} = Q/C$; the capacitor with the smaller capacitance C has the greater potential difference. In a parallel connection the two capacitors have the same potential difference V_{ab} but carry different charges $Q = CV_{ab}$; the capacitor with the larger capacitance C has the greater charge. Hence a 4-μF capacitor will have a greater potential difference than an 8-μF capacitor if the two are connected in series. The 4-μF capacitor cannot carry more charge than the 8-μF capacitor no matter how they are connected: In a series connection they will carry the same charge, and in a parallel connection the 8-μF capacitor will carry more charge.

24.3 Answer: (i) Capacitors connected in series carry the same charge Q. To compare the amount of energy stored, we use the expression $U = Q^2/2C$ from Eq. (24.9); it shows that the capacitor with the *smaller* capacitance $(C = 4\ \mu\text{F})$ has more stored energy in a series combination. By contrast, capacitors in parallel have the same potential difference V, so to compare them we use $U = \frac{1}{2}CV^2$ from Eq. (24.9). It shows that in a parallel combination, the capacitor with the *larger* capacitance $(C = 8\ \mu\text{F})$ has more stored energy. (If we had instead used $U = \frac{1}{2}CV^2$ to analyze the series combination, we would have to account for the different potential differences across the two capacitors. Likewise, using $U = Q^2/2C$ to study the parallel combination would require us to account for the different charges on the capacitors.)

24.4 Answer: (i) Here Q remains the same, so we use $U = Q^2/2C$ from Eq. (24.9) for the stored energy. Removing the dielectric lowers the capacitance by a factor of $1/K$; since U is inversely proportional to C, the stored energy *increases* by a factor of K. It takes work to pull the dielectric slab out of the capacitor because the fringing field tries to pull the slab back in (Fig. 24.16). The work that you do goes into the energy stored in the capacitor.

24.5 Answer: (i), (iii), (ii) Equation (24.14) says that if E_0 is the initial electric-field magnitude (before the dielectric slab is inserted), then the resultant field magnitude after the slab is inserted is $E_0/K = E_0/3$. The magnitude of the resultant field equals the difference between the initial field magnitude and the magnitude E_i of the field due to the bound charges (see Fig. 24.21). Hence $E_0 - E_i = E_0/3$ and $E_i = 2E_0/3$.

24.6 Answer: (iii) Equation (24.23) shows that this situation is the same as an isolated point charge in vacuum but with $\vec{E}$ replaced by $K\vec{E}$. Hence KE at the point of interest is equal to $q/4\pi\epsilon_0 r^2$, and so $E = q/4\pi K\epsilon_0 r^2$. As in Example 24.12, filling the space with a dielectric reduces the electric field by a factor of $1/K$.

PROBLEMS

Discussion Questions

Q24.1. Equation (24.2) shows that the capacitance of a parallel-plate capacitor becomes larger as the plate separation d decreases. However, there is a practical limit to how small d can be made, which places limits on how large C can be. Explain what sets the limit on d. (*Hint:* What happens to the magnitude of the electric field as $d \to 0$?)

Q24.2. Suppose several different parallel-plate capacitors are charged up by a constant-voltage source. Thinking of the actual movement and position of the charges on an atomic level, why does it make sense that the capacitances are proportional to the surface areas of the plates? Why does it make sense that the capacitances are *inversely* proportional to the distance between the plates?

Q24.3. Suppose the two plates of a capacitor have different areas. When the capacitor is charged by connecting it to a battery, do the charges on the two plates have equal magnitude, or may they be different? Explain your reasoning.

Q24.4. At the Fermi National Accelerator Laboratory (Fermilab) in Illinois, protons are accelerated around a ring 2 km in radius to speeds that approach that of light. The energy for this is stored in capacitors the size of a house. When these capacitors are being charged, they make a very loud creaking sound. What is the origin of this sound?

Q24.5. In the parallel-plate capacitor of Fig. 24.2, suppose the plates are pulled apart so that the separation d is much larger than

the size of the plates. (a) Is it still accurate to say that the electric field between the plates is uniform? Why or why not? (b) In the situation shown in Fig. 24.2, the potential difference between the plates is $V_{ab} = Qd/\epsilon_0 A$. If the plates are pulled apart as described above, is V_{ab} more or less than this formula would indicate? Explain your reasoning. (c) With the plates pulled apart as described above, is the capacitance more than, less than, or the same as that given by Eq. (24.2)? Explain your reasoning.

Q24.6. A parallel-plate capacitor is charged by being connected to a battery and is kept connected to the battery. The separation between the plates is then doubled. How does the electric field change? The charge on the plates? The total energy? Explain your reasoning.

Q24.7. A parallel-plate capacitor is charged by being connected to a battery and is then disconnected from the battery. The separation between the plates is then doubled. How does the electric field change? The potential difference? The total energy? Explain your reasoning.

Q24.8. Two parallel-plate capacitors, identical except that one has twice the plate separation of the other, are charged by the same voltage source. Which capacitor has a stronger electric field between the plates? Which capacitor has a greater charge? Which has greater energy density? Explain your reasoning.

Q24.9. The charged plates of a capacitor attract each other, so to pull the plates farther apart requires work by some external force. What becomes of the energy added by this work? Explain your reasoning.

Q24.10. The two plates of a capacitor are given charges $\pm Q$. The capacitor is then disconnected from the charging device so that the charges on the plates can't change, and the capacitor is immersed in a tank of oil. Does the electric field between the plates increase, decrease, or stay the same? Explain your reasoning. How can this field be measured?

Q24.11. As shown in Table 24.1, water has a very large dielectric constant $K = 80.4$. Why do you think water is not commonly used as a dielectric in capacitors?

Q24.12. Is dielectric strength the same thing as dielectric constant? Explain any differences between the two quantities. Is there a simple relationship between dielectric strength and dielectric constant (see Table 24.2)?

Q24.13. A capacitor made of aluminum foil strips separated by Mylar film was subjected to excessive voltage, and the resulting dielectric breakdown melted holes in the Mylar. After this, the capacitance was found to be about the same as before, but the breakdown voltage was much less. Why?

Q24.14. Suppose you bring a slab of dielectric close to the gap between the plates of a charged capacitor, preparing to slide it between the plates. What force will you feel? What does this force tell you about the energy stored between the plates once the dielectric is in place, compared to before the dielectric is in place?

Q24.15. The freshness of fish can be measured by placing a fish between the plates of a capacitor and measuring the capacitance. How does this work? (*Hint:* As time passes, the fish dries out. See Table 24.1.)

Q24.16. *Electrolytic* capacitors use as their dielectric an extremely thin layer of nonconducting oxide between a metal plate and a conducting solution. Discuss the advantage of such a capacitor over one constructed using a solid dielectric between the metal plates.

Q24.17. In terms of the dielectric constant K, what happens to the electric flux through the Gaussian surface shown in Fig. 24.23 when the dielectric is inserted into the previously empty space between the plates? Explain.

Q24.18. A parallel-plate capacitor is connected to a power supply that maintains a fixed potential difference between the plates. (a) If a sheet of dielectric is then slid between the plates, what happens to (i) the electric field between the plates, (ii) the magnitude of charge on each plate, and (iii) the energy stored in the capacitor? (b) Now suppose that before the dielectric is inserted, the charged capacitor is disconnected from the power supply. In this case, what happens to (i) the electric field between the plates, (ii) the magnitude of charge on each plate, (iii) the energy stored in the capacitor? Explain any differences between the two situations.

Q24.19. Liquid dielectrics that have polar molecules (such as water) always have dielectric constants that decrease with increasing temperature. Why?

Q24.20. A conductor is an extreme case of a dielectric, since if an electric field is applied to a conductor, charges are free to move within the conductor to set up "induced charges." What is the dielectric constant of a perfect conductor? Is it $K = 0$, $K \to \infty$, or something in between? Explain your reasoning.

Exercises

Section 24.1 Capacitors and Capacitance

24.1. A capacitor has a capacitance of 7.28 μF. What amount of charge must be placed on each of its plates to make the potential difference between its plates equal to 25.0 V?

24.2. The plates of a parallel-plate capacitor are 3.28 mm apart, and each has an area of 12.2 cm^2. Each plate carries a charge of magnitude 4.35×10^{-8} C. The plates are in vacuum. (a) What is the capacitance? (b) What is the potential difference between the plates? (c) What is the magnitude of the electric field between the plates?

24.3. A parallel-plate air capacitor of capacitance 245 pF has a charge of magnitude 0.148 μC on each plate. The plates are 0.328 mm apart. (a) What is the potential difference between the plates? (b) What is the area of each plate? (c) What is the electric-field magnitude between the plates? (d) What is the surface charge density on each plate?

24.4. Capacitance of an Oscilloscope. Oscilloscopes have parallel metal plates inside them to deflect the electron beam. These plates are called the *deflecting plates.* Typically, they are squares 3.0 cm on a side and separated by 5.0 mm, with vacuum in between. What is the capacitance of these deflecting plates and hence of the oscilloscope? (*Note:* This capacitance can sometimes have an effect on the circuit you are trying to study and must be taken into consideration in your calculations.)

24.5. A 10.0-μF parallel-plate capacitor with circular plates is connected to a 12.0-V battery. (a) What is the charge on each plate? (b) How much charge would be on the plates if their separation were doubled while the capacitor remained connected to the battery? (c) How much charge would be on the plates if the capacitor were connected to the 12.0-V battery after the radius of each plate was doubled without changing their separation?

24.6. A 10.0-μF parallel-plate capacitor is connected to a 12.0-V battery. After the capacitor is fully charged, the battery is disconnected without loss of any of the charge on the plates. (a) A voltmeter is connected across the two plates without discharging them. What does it read? (b) What would the voltmeter read if (i) the plate separation were doubled; (ii) the radius of each plate were doubled and, but their separation was unchanged?

24.7. How far apart would parallel pennies have to be to make a 1.00-pF capacitor? Does your answer suggest that you are justified in treating these pennies as infinite sheets? Explain.

24.8. A 5.00-pF, parallel-plate, air-filled capacitor with circular plates is to be used in a circuit in which it will be subjected to potentials of up to 1.00×10^2 V. The electric field between the plates is to be no greater than 1.00×10^4 N/C. As a budding electrical engineer for Live-Wire Electronics, your tasks are to (a) design the capacitor by finding what its physical dimensions and separation must be; (b) find the maximum charge these plates can hold.

24.9. A capacitor is made from two hollow, coaxial, iron cylinders, one inside the other. The inner cylinder is negatively charged and the outer is positively charged; the magnitude of the charge on each is 10.0 pC. The inner cylinder has radius 0.50 mm, the outer one has radius 5.00 mm, and the length of each cylinder is 18.0 cm. (a) What is the capacitance? (b) What applied potential difference is necessary to produce these charges on the cylinders?

24.10. A cylindrical capacitor consists of a solid inner conducting core with radius 0.250 cm, surrounded by an outer hollow conducting tube. The two conductors are separated by air, and the length of the cylinder is 12.0 cm. The capacitance is 36.7 pF. (a) Calculate the inner radius of the hollow tube. (b) When the capacitor is charged to 125 V, what is the charge per unit length λ on the capacitor?

24.11. A cylindrical capacitor has an inner conductor of radius 1.5 mm and an outer conductor of radius 3.5 mm. The two conductors are separated by vacuum, and the entire capacitor is 2.8 m long. (a) What is the capacitance per unit length? (b) The potential of the inner conductor is 350 mV higher than that of the outer conductor. Find the charge (magnitude and sign) on both conductors.

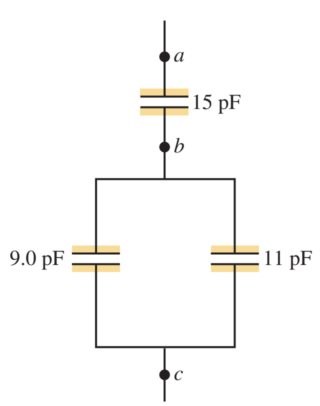
24.12. A spherical capacitor is formed from two concentric, spherical, conducting shells separated by vacuum. The inner sphere has radius 15.0 cm and the capacitance is 116 pF. (a) What is the radius of the outer sphere? (b) If the potential difference between the two spheres is 220 V, what is the magnitude of charge on each sphere?

24.13. A spherical capacitor contains a charge of 3.30 nC when connected to a potential difference of 220 V. If its plates are separated by vacuum and the inner radius of the outer shell is 4.00 cm, calculate: (a) the capacitance; (b) the radius of the inner sphere; (c) the electric field just outside the surface of the inner sphere.

Section 24.2 Capacitors in Series and Parallel

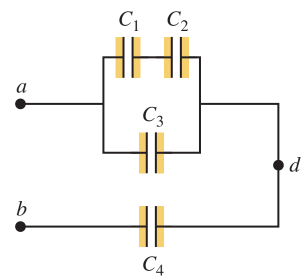
24.14. For the system of capacitors shown in Fig. 24.24, find the equivalent capacitance (a) between b and c, and (b) between a and c.

Figure 24.24 Exercise 24.14.



24.15. In Fig. 24.25, each capacitor has $C = 4.00\ \mu\text{F}$ and $V_{ab} = +28.0$ V. Calculate (a) the charge on each capacitor; (b) the potential difference across each capacitor; (c) the potential difference between points a and d.

Figure 24.25 Exercise 24.15.

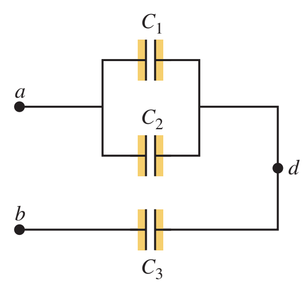


24.16. In Fig. 24.8a, let $C_1 = 3.00\ \mu\text{F}$, $C_2 = 5.00\ \mu\text{F}$, and $V_{ab} = +52.0$ V. Calculate (a) the charge on each capacitor and (b) the potential difference across each capacitor.

24.17. In Fig. 24.9a, let $C_1 = 3.00\ \mu\text{F}$, $C_2 = 5.00\ \mu\text{F}$, and $V_{ab} = +52.0$ V. Calculate (a) the charge on each capacitor and (b) the potential difference across each capacitor.

24.18. In Fig. 24.26, $C_1 = 6.00\ \mu\text{F}$, $C_2 = 3.00\ \mu\text{F}$, and $C_3 = 5.00\ \mu\text{F}$. The capacitor network is connected to an applied potential V_{ab}. After the charges on the capacitors have reached their final values, the charge on C_2 is 40.0 μC. (a) What are the charges on capacitors C_1 and C_3? (b) What is the applied voltage V_{ab}?

Figure 24.26 Exercises 24.18 and 24.19.



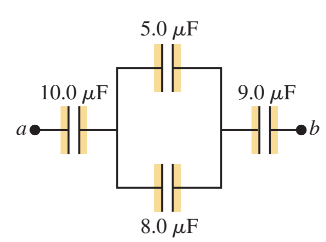
24.19. In Fig. 24.26, $C_1 = 3.00\ \mu\text{F}$ and $V_{ab} = 120$ V. The charge on capacitor C_1 is 150 μC. Calculate the voltage across the other two capacitors.

24.20. Two parallel-plate vacuum capacitors have plate spacings d_1 and d_2 and equal plate areas A. Show that when the capacitors are connected in series, the equivalent capacitance is the same as for a single capacitor with plate area A and spacing $d_1 + d_2$.

24.21. Two parallel-plate vacuum capacitors have areas A_1 and A_2 and equal plate spacings d. Show that when the capacitors are connected in parallel, the equivalent capacitance is the same as for a single capacitor with plate area $A_1 + A_2$ and spacing d.

24.22. Figure 24.27 shows a system of four capacitors, where the potential difference across ab is 50.0 V. (a) Find the equivalent capacitance of this system between a and b. (b) How much charge is stored by this combination of capacitors? (c) How much charge is stored in each of the 10.0-μF and the 9.0-μF capacitors?

Figure 24.27 Exercise 24.22.



24.23. Suppose the 3-μF capacitor in Fig. 24.10a were removed and replaced by a different one, and that this changed the equivalent capacitance between points a and b to 8 μF. What would be the capacitance of the replacement capacitor?

Section 24.3 Energy Storage in Capacitors and Electric-Field Energy

24.24. A parallel-plate air capacitor has a capacitance of 920 pF. The charge on each plate is 2.55 μC. (a) What is the potential difference between the plates? (b) If the charge is kept constant, what will be the potential difference between the plates if the separation is doubled? (c) How much work is required to double the separation?

24.25. A 5.80-μF, parallel-plate, air capacitor has a plate separation of 5.00 mm and is charged to a potential difference of 400 V. Calculate the energy density in the region between the plates, in units of J/m^3.

24.26. An air capacitor is made from two flat parallel plates 1.50 mm apart. The magnitude of charge on each plate is 0.0180 μC when the potential difference is 200 V. (a) What is the capacitance? (b) What is the area of each plate? (c) What maximum voltage can be applied without dielectric breakdown? (Dielectric breakdown for air occurs at an electric-field strength of 3.0×10^6 V/m.) (d) When the charge is 0.0180 μC, what total energy is stored?

24.27. A 450-μF capacitor is charged to 295 V. Then a wire is connected between the plates. How many joules of thermal energy are produced as the capacitor discharges if all of the energy that was stored goes into heating the wire?

24.28. A capacitor of capacitance C is charged to a potential difference V_0. The terminals of the charged capacitor are then connected to those of an uncharged capacitor of capacitance $C/2$. Compute (a) the original charge of the system; (b) the final potential difference across each capacitor; (c) the final energy of the system; (d) the decrease in energy when the capacitors are connected. (e) Where did the "lost" energy go?

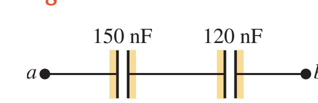
24.29. A parallel-plate vacuum capacitor with plate area A and separation x has charges $+Q$ and $-Q$ on its plates. The capacitor is disconnected from the source of charge, so the charge on each plate remains fixed. (a) What is the total energy stored in the capacitor? (b) The plates are pulled apart an additional distance dx. What is the change in the stored energy? (c) If F is the force with which the plates attract each other, then the change in the stored energy must equal the work $dW = F dx$ done in pulling the plates apart. Find an expression for F. (d) Explain why F is *not* equal to QE, where E is the electric field between the plates.

24.30. A parallel-plate vacuum capacitor has 8.38 J of energy stored in it. The separation between the plates is 2.30 mm. If the separation is decreased to 1.15 mm, what is the energy stored (a) if the capacitor is disconnected from the potential source so the charge on the plates remains constant, and (b) if the capacitor remains connected to the potential source so the potential difference between the plates remains constant?

24.31. (a) How much charge does a battery have to supply to a 5.0-μF capacitor to create a potential difference of 1.5 V across its plates? How much energy is stored in the capacitor in this case? (b) How much charge would the battery have to supply to store 1.0 J of energy in the capacitor? What would be the potential across the capacitor in that case?

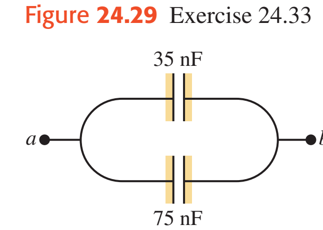
24.32. For the capacitor network shown in Fig. 24.28, the potential difference across ab is 36 V. Find (a) the total charge stored in this network; (b) the charge on each capacitor; (c) the total energy stored in the network; (d) the energy stored in each capacitor; (e) the potential differences across each capacitor.

Figure 24.28 Exercise 24.32.



24.33. For the capacitor network shown in Fig. 24.29, the potential difference across ab is 220 V. Find (a) the total charge stored in this network; (b) the charge on each capacitor; (c) the total energy stored in the network; (d) the energy stored in each capacitor; (e) the potential difference across each capacitor.

Figure 24.29 Exercise 24.33



24.34. A 0.350-m-long cylindrical capacitor consists of a solid conducting core with a radius of 1.20 mm and an outer hollow conducting tube with an inner radius of 2.00 mm. The two conductors are separated by air and charged to a potential difference of 6.00 V. Calculate (a) the charge per length for the capacitor; (b) the total charge on the capacitor; (c) the capacitance; (d) the energy stored in the capacitor when fully charged.

24.35. A cylindrical air capacitor of length 15.0 m stores 3.20×10^{-9} J of energy when the potential difference between the two conductors is 4.00 V. (a) Calculate the magnitude of the charge on each conductor. (b) Calculate the ratio of the radii of the inner and outer conductors.

24.36. A capacitor is formed from two concentric spherical conducting shells separated by vacuum. The inner sphere has radius 12.5 cm, and the outer sphere has radius 14.8 cm. A potential difference of 120 V is applied to the capacitor. (a) What is the energy density at $r = 12.6$ cm, just outside the inner sphere? (b) What is the energy density at $r = 14.7$ cm, just inside the outer sphere? (c) For a parallel-plate capacitor the energy density is uniform in the region between the plates, except near the edges of the plates. Is this also true for a spherical capacitor?

24.37. You have two identical capacitors and an external potential source. (a) Compare the total energy stored in the capacitors when they are connected to the applied potential in series and in parallel. (b) Compare the maximum amount of charge stored in each case. (c) Energy storage in a capacitor can be limited by the maximum electric field between the plates. What is the ratio of the electric field for the series and parallel combinations?

Section 24.4 Dielectrics

24.38. A parallel-plate capacitor has capacitance $C_0 = 5.00$ pF when there is air between the plates. The separation between the plates is 1.50 mm. (a) What is the maximum magnitude of charge Q that can be placed on each plate if the electric field in the region between the plates is not to exceed 3.00×10^4 V/m? (b) A dielectric with $K = 2.70$ is inserted between the plates of the capacitor, completely filling the volume between the plates. Now what is the maximum magnitude of charge on each plate if the electric field between the plates is not to exceed 3.00×10^4 V/m?

24.39. Two parallel plates have equal and opposite charges. When the space between the plates is evacuated, the electric field is $E = 3.20 \times 10^5$ V/m. When the space is filled with dielectric, the electric field is $E = 2.50 \times 10^5$ V/m. (a) What is the charge density on each surface of the dielectric? (b) What is the dielectric constant?

24.40. A budding electronics hobbyist wants to make a simple 1.0-nF capacitor for tuning her crystal radio, using two sheets of aluminum foil as plates, with a few sheets of paper between them as a dielectric. The paper has a dielectric constant of 3.0, and the thickness of one sheet of it is 0.20 mm. (a) If the sheets of paper measure 22 × 28 cm and she cuts the aluminum foil to the same dimensions, how many sheets of paper should she use between her plates to get the proper capacitance? (b) Suppose for convenience she wants to use a single sheet of posterboard, with the same dielectric constant but a thickness of 12.0 mm, instead of the paper. What area of aluminum foil will she need for her plates to get her 1.0 nF of capacitance? (c) Suppose she goes high-tech and finds a sheet of Teflon of the same thickness as the posterboard to use as a dielectric. Will she need a larger or smaller area of Teflon than of posterboard? Explain.

24.41. The dielectric to be used in a parallel-plate capacitor has a dielectric constant of 3.60 and a dielectric strength of 1.60×10^7 V/m. The capacitor is to have a capacitance of 1.25×10^{-9} F and must be able to withstand a maximum potential difference of 5500 V. What is the minimum area the plates of the capacitor may have?

24.42. Show that Eq. (24.20) holds for a parallel-plate capacitor with a dielectric material between the plates. Use a derivation analogous to that used for Eq. (24.11).

24.43. A capacitor has parallel plates of area 12 cm^2 separated by 2.0 mm. The space between the plates is filled with polystyrene (see Table 24.2). (a) Find the permittivity of polystyrene. (b) Find the maximum permissible voltage across the capacitor to avoid dielectric breakdown. (c) When the voltage equals the value found in part (b), find the surface charge density on each plate and the induced surface-charge density on the surface of the dielectric.

24.44. A constant potential difference of 12 V is maintained between the terminals of a 0.25-μF, parallel-plate, air capacitor. (a) A sheet of Mylar is inserted between the plates of the capacitor, completely filling the space between the plates. When this is done, how much additional charge flows onto the positive plate of the capacitor (see Table 24.1)? (b) What is the total induced charge on either face of the Mylar sheet? (c) What effect does the Mylar sheet have on the electric field between the plates? Explain how you can reconcile this with the increase in charge on the plates, which acts to *increase* the electric field.

24.45. When a 360-nF air capacitor $(1 \text{ nF} = 10^{-9} \text{ F})$ is connected to a power supply, the energy stored in the capacitor is 1.85×10^{-5} J. While the capacitor is kept connected to the power supply, a slab of dielectric is inserted that completely fills the space between the plates. This increases the stored energy by 2.32×10^{-5} J. (a) What is the potential difference between the capacitor plates? (b) What is the dielectric constant of the slab?

24.46. A parallel-plate capacitor has capacitance $C = 12.5$ pF when the volume between the plates is filled with air. The plates are circular, with radius 3.00 cm. The capacitor is connected to a battery and a charge of magnitude 25.0 pC goes onto each plate. With the capacitor still connected to the battery, a slab of dielectric is inserted between the plates, completely filling the space between the plates. After the dielectric has been inserted, the charge on each plate has magnitude 45.0 pC. (a) What is the dielectric constant K of the dielectric? (b) What is the potential difference between the plates before and after the dielectric has been inserted? (c) What is the electric field at a point midway between the plates before and after the dielectric has been inserted?

24.47. A 12.5-μF capacitor is connected to a power supply that keeps a constant potential difference of 24.0 V across the plates. A piece of material having a dielectric constant of 3.75 is placed between the plates, completely filling the space between them. (a) How much energy is stored in the capacitor before and after the dielectric is inserted? (b) By how much did the energy change during the insertion? Did it increase or decrease?

*Section 24.6 Gauss's Law in Dielectrics

***24.48.** A parallel-plate capacitor has plates with area 0.0225 m^2 separated by 1.00 mm of Teflon. (a) Calculate the charge on the plates when they are charged to a potential difference of 12.0 V. (b) Use Gauss's law (Eq. 24.23) to calculate the electric field inside the Teflon. (c) Use Gauss's law to calculate the electric field if the voltage source is disconnected and the Teflon is removed.

***24.49.** A parallel-plate capacitor has the volume between its plates filled with plastic with dielectric constant K. The magnitude of the charge on each plate is Q. Each plate has area A, and the distance between the plates is d. (a) Use Gauss's law as stated in Eq. (24.23) to calculate the magnitude of the electric field in the dielectric. (b) Use the electric field determined in part (a) to calculate the potential difference between the two plates. (c) Use the result of part (b) to determine the capacitance of the capacitor. Compare your result to Eq. (24.12).

Problems

24.50. A parallel-plate air capacitor is made by using two plates 16 cm square, spaced 4.7 mm apart. It is connected to a 12-V battery. (a) What is the capacitance? (b) What is the charge on each plate? (c) What is the electric field between the plates? (d) What is the energy stored in the capacitor? (e) If the battery is disconnected and then the plates are pulled apart to a separation of 9.4 mm, what are the answers to parts (a)–(d)?

24.51. Suppose the battery in Problem 24.50 remains connected while the plates are pulled apart. What are the answers then to parts (a)–(d) after the plates have been pulled apart?

24.52. Cell Membranes. Cell membranes (the walled enclosure around a cell) are typically about 7.5 nm thick. They are partially permeable to allow charged material to pass in and out, as needed. Equal but opposite charge densities build up on the inside and outside faces of such a membrane, and these charges prevent additional charges from passing through the cell wall. We can model a cell membrane as a parallel-plate capacitor, with the membrane itself containing proteins embedded in an organic material to give the membrane a dielectric constant of about 10. (See Fig. 24.30.) (a) What is the capacitance per square centimeter of such a cell wall? (b) In its normal resting state, a cell has a potential difference of 85 mV across its membrane. What is the electric field inside this membrane?

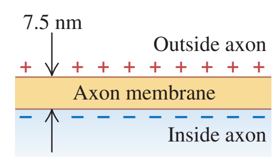


Figure 24.30 Problem 24.52.

24.53. Electronic flash units for cameras contain a capacitor for storing the energy used to produce the flash. In one such unit, the flash lasts for $\frac{1}{675}$ s with an average light power output of 2.70×10^5 W. (a) If the conversion of electrical energy to light is 95% efficient (the rest of the energy goes to thermal energy), how much energy must be stored in the capacitor for one flash? (b) The capacitor has a potential difference between its plates of 125 V when the stored energy equals the value calculated in part (a). What is the capacitance?

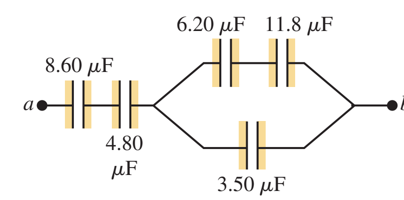
24.54. In one type of computer keyboard, each key holds a small metal plate that serves as one plate of a parallel-plate, air-filled capacitor. When the key is depressed, the plate separation decreases and the capacitance increases. Electronic circuitry detects the change in capacitance and thus detects that the key has been pressed. In one particular keyboard, the area of each metal plate is 42.0 mm^2, and the separation between the plates is 0.700 mm before the key is depressed. (a) Calculate the capacitance before the key is depressed. (b) If the circuitry can detect a change in capacitance of 0.250 pF, how far must the key be depressed before the circuitry detects its depression?

24.55. Consider a cylindrical capacitor like that shown in Fig. 24.6. Let $d = r_b - r_a$ be the spacing between the inner and outer conductors. (a) Let the radii of the two conductors be only slightly different, so that $d \ll r_a$. Show that the result derived in Example 24.4 (Section 24.1) for the capacitance of a cylindrical capacitor then reduces to Eq. (24.2), the equation for the capacitance of a parallel-plate capacitor, with A being the surface area of each cylinder. Use the result that $\ln(1 + z) \cong z$ for $|z| \ll 1$. (b) Even though the earth is essentially spherical, its surface appears flat to us because its radius is so large. Use this idea to explain why the result of part (a) makes sense from a purely geometrical standpoint.

24.56. In Fig. 24.9a, let $C_1 = 9.0\ \mu$F, $C_2 = 4.0\ \mu$F, and $V_{ab} = 28$ V. Suppose the charged capacitors are disconnected from the source and from each other, and then reconnected to each other with plates of *opposite* sign together. By how much does the energy of the system decrease?

24.57. For the capacitor network shown in Fig. 24.31, the potential difference across ab is 12.0 V. Find (a) the total energy stored in this network and (b) the energy stored in the 4.80-μF capacitor.

Figure 24.31 Problem 24.57.



24.58. Several 0.25-μF capacitors are available. The voltage across each is not to exceed 600 V. You need to make a capacitor with capacitance 0.25 μF to be connected across a potential difference of 960 V. (a) Show in a diagram how an equivalent capacitor with the desired properties can be obtained. (b) No dielectric is a perfect insulator that would not permit the flow of any charge through its volume. Suppose that the dielectric in one of the capacitors in your diagram is a moderately good conductor. What will happen in this case when your combination of capacitors is connected across the 960-V potential difference?

24.59. In Fig. 24.32, $C_1 = C_5 = 8.4\ \mu$F and $C_2 = C_3 = C_4 = 4.2\ \mu$F. The applied potential is $V_{ab} = 220$ V. (a) What is the equivalent capacitance of the network between points a and b? (b) Calculate the charge on each capacitor and the potential difference across each capacitor.

Figure 24.32 Problem 24.59.

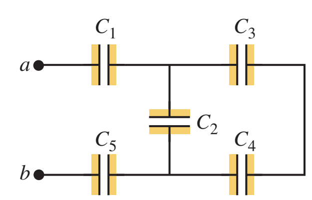
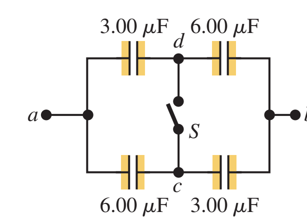


Figure 24.33 Problem 24.60.



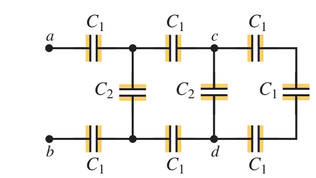
24.60. The capacitors in Fig. 24.33 are initially uncharged and are connected, as in the diagram, with switch S open. The applied potential difference is $V_{ab} = +210$ V. (a) What is the potential difference V_{cd}? (b) What is the potential difference across each capacitor after switch S is closed? (c) How much charge flowed through the switch when it was closed?

24.61. Three capacitors having capacitances of 8.4, 8.4, and 4.2 μF are connected in series across a 36-V potential difference. (a) What is the charge on the 4.2-μF capacitor? (b) What is the total energy stored in all three capacitors? (c) The capacitors are disconnected from the potential difference without allowing them to discharge. They are then reconnected in parallel with each other, with the positively charged plates connected together. What is the voltage across each capacitor in the parallel combination? (d) What is the total energy now stored in the capacitors?

24.62. Capacitance of a Thundercloud. The charge center of a thundercloud, drifting 3.0 km above the earth's surface, contains 20 C of negative charge. Assuming the charge center has a radius of 1.0 km, and modeling the charge center and the earth's surface as parallel plates, calculate: (a) the capacitance of the system; (b) the potential difference between charge center and ground; (c) the average strength of the electric field between cloud and ground; (d) the electrical energy stored in the system.

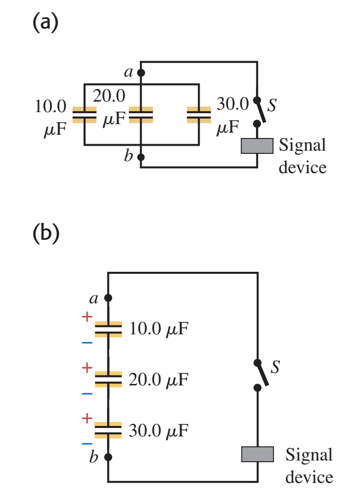
24.63. In Fig. 24.34, each capacitance C_1 is 6.9 μF, and each capacitance C_2 is 4.6 μF. (a) Compute the equivalent capacitance of the network between points a and b. (b) Compute the charge on each of the three capacitors nearest a and b when $V_{ab} = 420$ V. (c) With 420 V across a and b, compute V_{cd}.

Figure 24.34 Problem 24.63.



24.64. Each combination of capacitors between points a and b in Fig. 24.35 is first connected across a 120-V battery, charging the combination to 120 V. These combinations are then connected to make the circuits shown. When the switch S is thrown, a surge of charge for the discharging capacitors flows to trigger the signal device. How much charge flows through the signal device?

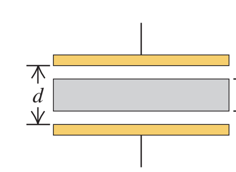
Figure 24.35 Problem 24.64.



24.65. A parallel-plate capacitor with only air between the plates is charged by connecting it to a battery. The capacitor is then disconnected from the battery, without any of the charge leaving the plates. (a) A voltmeter reads 45.0 V when placed across the capacitor. When a dielectric is inserted between the plates, completely filling the space, the voltmeter reads 11.5 V. What is the dielectric constant of this material? (b) What will the voltmeter read if the dielectric is now pulled partway out so it fills only one-third of the space between the plates?

24.66. An air capacitor is made by using two flat plates, each with area A, separated by a distance d. Then a metal slab having thickness a (less than d) and the same shape and size as the plates is inserted between them, parallel to the plates and not touching either plate (Fig. 24.36). (a) What is the capacitance of this arrangement? (b) Express the capacitance as a multiple of the capacitance C_0 when the metal slab is not present. (c) Discuss what happens to the capacitance in the limits $a \to 0$ and $a \to d$.

Figure 24.36 Problem 24.66.



24.67. Capacitance of the Earth. (a) Discuss how the concept of capacitance can also be applied to a *single* conductor. (*Hint:* In the relationship $C = Q/V_{ab}$, think of the second conductor as being located at infinity.) (b) Use Eq. (24.1) to show that $C = 4\pi\epsilon_0 R$ for a solid conducting sphere of radius R. (c) Use your result in part (b) to calculate the capacitance of the earth, which is a good conductor of radius 6380 km. Compare to typical capacitors used in electronic circuits that have capacitances ranging from 10 pF to 100 μF.

24.68. A solid conducting sphere of radius R carries a charge Q. Calculate the electric-field energy density at a point a distance r from the center of the sphere for (a) $r < R$ and (b) $r > R$. (c) Calculate the total electric-field energy associated with the charged sphere. (*Hint:* Consider a spherical shell of radius r and thickness dr that has volume $dV = 4\pi r^2\,dr$, and find the energy stored in this volume. Then integrate from $r = 0$ to $r \rightarrow \infty$.) (d) Explain why the result of part (c) can be interpreted as the amount of work required to assemble the charge Q on the sphere. (e) By using Eq. (24.9) and the result of part (c), show that the capacitance of the sphere is as given in Problem 24.67.

24.69. Earth-Ionosphere Capacitance. The earth can be considered as a single-conductor capacitor (see Problem 24.67). It can also be considered in combination with a charged layer of the atmosphere, the ionosphere, as a spherical capacitor with two plates, the surface of the earth being the negative plate. The ionosphere is at a level of about 70 km, and the potential difference between earth and ionosphere is about 350,000 V. Calculate: (a) the capacitance of this system; (b) the total charge on the capacitor; (c) the energy stored in the system.

24.70. The inner cylinder of a long, cylindrical capacitor has radius r_a and linear charge density $+\lambda$. It is surrounded by a coaxial cylindrical conducting shell with inner radius r_b and linear charge density $-\lambda$ (see Fig. 24.6). (a) What is the energy density in the region between the conductors at a distance r from the axis? (b) Integrate the energy density calculated in part (a) over the volume between the conductors in a length L of the capacitor to obtain the total electric-field energy per unit length. (c) Use Eq. (24.9) and the capacitance per unit length calculated in Example 24.4 (Section 24.1) to calculate U/L. Does your result agree with that obtained in part (b)?

24.71. A parallel-plate capacitor has the space between the plates filled with two slabs of dielectric, one with constant K_1 and one with constant K_2 (Fig. 24.37). Each slab has thickness $d/2$, where d is the plate separation. Show that the capacitance is

$$C = \frac{2\epsilon_0 A}{d}\left(\frac{K_1 K_2}{K_1 + K_2}\right)$$

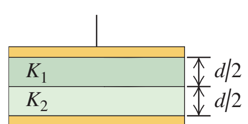


Figure 24.37 Problem 24.71.

24.72. A parallel-plate capacitor has the space between the plates filled with two slabs of dielectric, one with constant K_1 and one with constant K_2 (Fig. 24.38). The thickness of each slab is the same as the plate separation d, and each slab fills half of the volume between the plates. Show that the capacitance is

$$C = \frac{\epsilon_0 A(K_1 + K_2)}{2d}$$

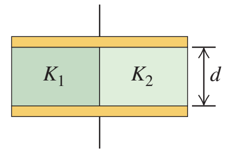


Figure 24.38 Problem 24.72.

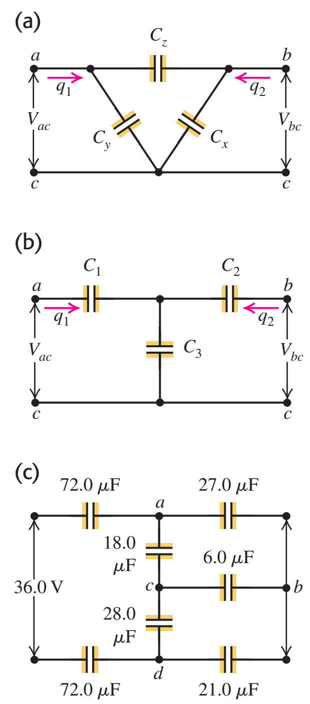
Challenge Problems

24.73. Capacitors in networks cannot always be grouped into simple series or parallel combinations. As an example, Fig. 24.39a shows three capacitors C_x, C_y, and C_z in a *delta network*, so called because of its triangular shape. This network has *three* terminals a, b, and c and hence cannot be transformed into a single equivalent capacitor. It can be shown that as far as any effect on the external circuit is concerned, a delta network is equivalent to what is called a *Y network*. For example, the delta network of Fig. 24.39a can be replaced by the Y network of Fig. 24.39b. (The name "Y network" also refers to the shape of the network.) (a) Show that the transformation equations that give C_1, C_2, and C_3 in terms of C_x, and C_y, and C_z are

$$C_1 = (C_xC_y + C_yC_z + C_zC_x)/C_x$$
$$C_2 = (C_xC_y + C_yC_z + C_zC_x)/C_y$$
$$C_3 = (C_xC_y + C_yC_z + C_zC_x)/C_z$$

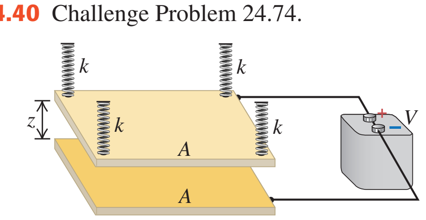
(*Hint:* The potential difference V_{ac} must be the same in both circuits, as V_{bc} must be. Also, the charge q_1 that flows from point a along the wire as indicated must be the same in both circuits, as must q_2. Obtain a relationship for V_{ac} as a function of q_1 and q_2 and the capacitances for each network, and obtain a separate relationship for V_{bc} as a function of the charges for each network. The coefficients of corresponding charges in corresponding equations must be the same for both networks.) (b) For the network shown in Fig. 24.39c, determine the equivalent capacitance between the terminals at the left end of the network. (*Hint:* Use the delta-Y transformation derived in part (a). Use points a, b, and c to form the delta, and transform the delta into a Y. The capacitors can then be combined using the relationships for series and parallel combinations of capacitors.) (c) Determine the charges of, and the potential differences across, each capacitor in Fig. 24.39c.

Figure 24.39 Challenge Problem 24.73.



24.74. The parallel-plate air capacitor in Fig. 24.40 consists of two horizontal conducting plates of equal area A. The bottom plate rests on a fixed support, and the top plate is suspended by four springs with spring constant k, positioned at each of the four corners of the top plate as shown in the figure. When uncharged, the plates are separated by a distance z_0. A battery is connected to the plates and produces a potential difference V between them. This causes the plate separation to decrease to z. Neglect any fringing effects. (a) Show that the electrostatic force between the charged plates has a magnitude $\epsilon_0 AV^2/2z^2$. (*Hint:* See Exercise 24.29.) (b) Obtain an expression that relates the plate separation z to the potential difference V. The resulting equation will be cubic in z. (c) Given the values $A = 0.300\text{ m}^2$, $z_0 = 1.20$ mm, $k = 25.0$ N/m, and $V = 120$ V, find the two values of z for which the top plate will be in equilibrium. (*Hint:* You can solve the cubic equation by plugging a trial value of z into the equation and then adjusting your guess until the equation is satisfied to three significant figures. Locating the roots of the cubic equation graphically can help you pick starting values of z for this trial-and-error procedure. One root of the cubic equation has a nonphysical negative value.) (d) For each of the two values of z found in part (c), is the equilibrium stable or unstable? For stable equilibrium a small displacement of the object will give rise to a net force tending to return the object to the equilibrium position. For unstable equilibrium a small displacement gives rise to a net force that takes the object farther away from equilibrium.

Figure 24.40 Challenge Problem 24.74.



24.75. Two square conducting plates with sides of length L are separated by a distance D. A dielectric slab with constant K with dimensions $L \times L \times D$ is inserted a distance x into the space between the plates, as shown in Fig. 24.41. (a) Find the capacitance C of this system (see Problem 24.72). (b) Suppose that the capacitor is connected to a battery that maintains a constant potential difference V between the plates. If the dielectric slab is inserted an additional distance dx into the space between the plates, show that the change in stored energy is

$$dU = +\frac{(K-1)\epsilon_0 V^2 L}{2D}dx$$

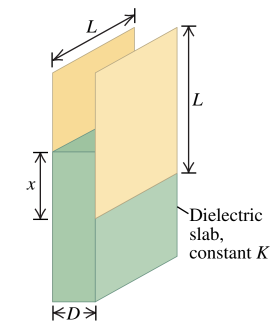
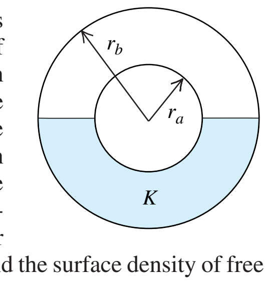


Figure 24.41 Challenge Problem 24.75.

(c) Suppose that before the slab is moved by dx, the plates are disconnected from the battery, so that the charges on the plates remain constant. Determine the magnitude of the charge on each plate, and then show that when the slab is moved dx farther into the space between the plates, the stored energy changes by an amount that is the *negative* of the expression for dU given in part (b). (d) If F is the force exerted on the slab by the charges on the plates, then dU should equal the work done *against* this force to move the slab a distance dx. Thus $dU = -F\,dx$. Show that applying this expression to the result of part (b) suggests that the electric force on the slab pushes it *out* of the capacitor, while the result of part (c) suggests that the force pulls the slab *into* the capacitor. (e) Figure 24.16 shows that the force in fact pulls the slab into the capacitor. Explain why the result of part (b) gives an incorrect answer for the direction of this force, and calculate the magnitude of the force. (This method does not require knowledge of the nature of the fringing field.)

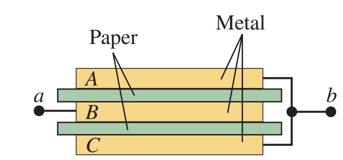
24.76. An isolated spherical capacitor has charge $+Q$ on its inner conductor (radius r_a) and charge $-Q$ on its outer conductor (radius r_b). Half of the volume between the two conductors is then filled with a liquid dielectric of constant K, as shown in cross section in Fig. 24.42. (a) Find the capacitance of the half-filled capacitor. (b) Find the magnitude of $\vec{E}$ in the volume between the two conductors as a function of the distance r from the center of the capacitor. Give answers for both the upper and lower halves of this volume. (c) Find the surface density of free charge on the upper and lower halves of the inner and outer conductors. (d) Find the surface density of bound charge on the inner $(r = r_a)$ and outer $(r = r_b)$ surfaces of the dielectric. (e) What is the surface density of bound charge on the flat surface of the dielectric? Explain.

Figure 24.42 Challenge Problem 24.76.



24.77. Three square metal plates A, B, and C, each 12.0 cm on a side and 1.50 mm thick, are arranged as in Fig. 24.43. The plates are separated by sheets of paper 0.45 mm thick and with dielectric constant 4.2. The outer plates are connected together and connected to point b. The inner plate is connected to point a. (a) Copy the diagram and show by plus and minus signs the charge distribution on the plates when point a is maintained at a positive potential relative to point b. (b) What is the capacitance between points a and b?

Figure 24.43 Challenge Problem 24.77.



24.78. A fuel gauge uses a capacitor to determine the height of the fuel in a tank. The effective dielectric constant K_{eff} changes from a value of 1 when the tank is empty to a value of K, the dielectric constant of the fuel, when the tank is full. The appropriate electronic circuitry can determine the effective dielectric constant of the combined air and fuel between the capacitor plates. Each of the two rectangular plates has a width w and a length L (Fig. 24.44). The height of the fuel between the plates is h. You can ignore any fringing effects. (a) Derive an expression for K_{eff} as a function of h. (b) What is the effective dielectric constant for a tank $\frac{1}{4}$ full, $\frac{1}{2}$ full, and $\frac{3}{4}$ full if the fuel is gasoline $(K = 1.95)$? (c) Repeat part (b) for methanol $(K = 33.0)$. (d) For which fuel is this fuel gauge more practical?

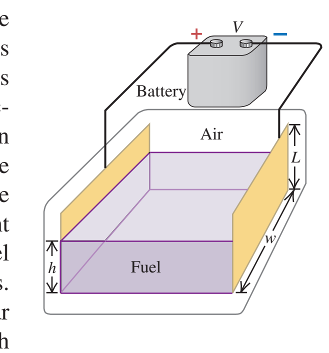


Figure 24.44 Challenge Problem 24.78.