MOMENTUM, IMPULSE, AND COLLISIONS

8

?Which could potentially cause you the greater injury: being tackled by a lightweight, fast-moving football player, or being tackled by a player with double the mass but moving at half the speed?

Learning Goals

By studying this chapter, you will learn:

- the meaning of the momentum of a particle, and how the impulse of the net force acting on a particle causes its momentum to change.
- the conditions under which the total momentum of a system of particles is constant (conserved).
- how to solve problems in which two bodies collide with each other.
- the important distinction among elastic, inelastic, and completely inelastic collisions.
- the definition of the center of mass of a system, and what determines how the center of mass moves.
- how to analyze situations such as rocket propulsion in which the mass of a body changes as it moves.

There are many questions involving forces that cannot be answered by directly applying Newton's second law, $\sum \vec{F} = m\vec{a}$. For example, when an 18-wheeler collides head-on with a compact car, what determines which way the wreckage moves after the collision? In playing pool, how do you decide how to aim the cue ball in order to knock the eight ball into the pocket? And when a meteorite collides with the earth, how much of the meteorite's kinetic energy is released in the impact?

A common theme of all these questions is that they involve forces about which we know very little: the forces between the car and the 18-wheeler, between the two pool balls, or between the meteorite and the earth. Remarkably, we will find in this chapter that we don't have to know *anything* about these forces to answer questions of this kind!

Our approach uses two new concepts, *momentum* and *impulse,* and a new conservation law, *conservation of momentum.* This conservation law is every bit as important as that of conservation of energy. The law of conservation of momentum is valid even in situations in which Newton's laws are inadequate, such as bodies moving at very high speeds (near the speed of light) or objects on a very small scale (such as the constituents of atoms). Within the domain of Newtonian mechanics, conservation of momentum enables us to analyze many situations that would be very difficult if we tried to use Newton's laws directly. Among these are *collision* problems, in which two bodies collide and can exert very large forces on each other for a short time.

8.1 Momentum and Impulse

In Chapter 6 we re-expressed Newton's second law for a particle, $\sum \vec{F} = m\vec{a}$, in terms of the work–energy theorem. This theorem helped us tackle a great number of physics problems and led us to the law of conservation of energy. Let's now return to $\sum \vec{F} = m\vec{a}$ and see yet another useful way to restate this fundamental law.

Newton's Second Law in Terms of Momentum

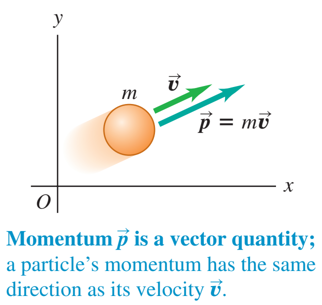
Consider a particle of constant mass m. (Later in this chapter we'll see how to deal with situations in which the mass of a body changes.) Because $\vec{a} = d\vec{v}/dt$, we can write Newton's second law for this particle as

$$\sum \vec{F} = m\frac{d\vec{v}}{dt} = \frac{d}{dt}(m\vec{v}) \tag{8.1}$$

We can take the mass m inside the derivative because it is constant. Thus Newton's second law says that the net force $\sum \vec{F}$ acting on a particle equals the time rate of change of the combination $m\vec{v}$, the product of the particle's mass and velocity. We'll call this combination the **momentum,** or **linear momentum,** of the particle. Using the symbol $\vec{p}$ for momentum, we have

$$\vec{p} = m\vec{v} \qquad \text{(definition of momentum)} \tag{8.2}$$

8.1 The velocity and momentum vectors of a particle.



The greater the mass m and speed v of a particle, the greater is its magnitude of momentum mv. Keep in mind, however, that momentum is a *vector* quantity with the same direction as the particle's velocity (Fig. 8.1). Hence a car driving north at 20 m/s and an identical car driving east at 20 m/s have the same *magnitude* of momentum (mv) but different momentum *vectors* $(m\vec{v})$ because their directions are different.

We often express the momentum of a particle in terms of its components. If the particle has velocity components v_x, v_y, and v_z, then its momentum components p_x, p_y, and p_z (which we also call the *x-momentum, y-momentum,* and *z-momentum*) are given by

$$p_x = mv_x \qquad p_y = mv_y \qquad p_z = mv_z \tag{8.3}$$

These three component equations are equivalent to Eq. (8.2).

The units of the magnitude of momentum are units of mass times speed; the SI units of momentum are kg · m/s. The plural of momentum is "momenta."

If we now substitute the definition of momentum, Eq. (8.2), into Eq. (8.1), we get

$$\sum \vec{F} = \frac{d\vec{p}}{dt} \qquad \text{(Newton's second law in terms of momentum)} \tag{8.4}$$

The net force (vector sum of all forces) acting on a particle equals the time rate of change of momentum of the particle. This, not $\sum \vec{F} = m\vec{a}$, is the form in which Newton originally stated his second law (although he called momentum the "quantity of motion"). This law is valid only in inertial frames of reference.

According to Eq. (8.4), a rapid change in momentum requires a large net force, while a gradual change in momentum requires less net force. This principle is used in the design of automobile safety devices such as air bags (Fig. 8.2).

8.2 If a fast-moving automobile stops suddenly in a collision, the driver's momentum (mass times velocity) changes from a large value to zero in a short time. An air bag causes the driver to lose momentum more gradually than would an abrupt collision with the steering wheel, reducing the force exerted on the driver as well as the possibility of injury.

The Impulse–Momentum Theorem

A particle's momentum $\vec{p} = m\vec{v}$ and its kinetic energy $K = \frac{1}{2}mv^2$ both depend on the mass and velocity of the particle. What is the fundamental difference between these two quantities? A purely mathematical answer is that momentum is a vector whose magnitude is proportional to speed, while kinetic energy is a scalar proportional to the speed squared. But to see the *physical* difference between momentum and kinetic energy, we must first define a quantity closely related to momentum called *impulse.*

Let's first consider a particle acted on by a *constant* net force $\sum \vec{F}$ during a time interval Δt from t_1 to t_2. (We'll look at the case of varying forces shortly.)

The **impulse** of the net force, denoted by $\vec{J}$, is defined to be the product of the net force and the time interval:

$$\vec{J} = \sum \vec{F}(t_2 - t_1) = \sum \vec{F}\,\Delta t \qquad \text{(assuming constant net force)} \tag{8.5}$$

Impulse is a vector quantity; its direction is the same as the net force $\sum \vec{F}$. Its magnitude is the product of the magnitude of the net force and the length of time that the net force acts. The SI unit of impulse is the newton-second $(\text{N} \cdot \text{s})$. Because 1 N = 1 kg · m/s^2, an alternative set of units for impulse is kg · m/s, the same as the units of momentum.

To see what impulse is good for, let's go back to Newton's second law as restated in terms of momentum, Eq. (8.4). If the net force $\sum \vec{F}$ is constant, then $d\vec{p}/dt$ is also constant. In that case, $d\vec{p}/dt$ is equal to the *total* change in momentum $\vec{p}_2 - \vec{p}_1$ during the time interval $t_2 - t_1$, divided by the interval:

$$\sum \vec{F} = \frac{\vec{p}_2 - \vec{p}_1}{t_2 - t_1}$$

Multiplying this equation by $(t_2 - t_1)$, we have

$$\sum \vec{F}(t_2 - t_1) = \vec{p}_2 - \vec{p}_1$$

Comparing with Eq. (8.5), we end up with a result called the **impulse–momentum theorem:**

$$\vec{J} = \vec{p}_2 - \vec{p}_1 \qquad \text{(impulse–momentum theorem)} \tag{8.6}$$

The change in momentum of a particle during a time interval equals the impulse of the net force that acts on the particle during that interval.

The impulse–momentum theorem also holds when forces are not constant. To see this, we integrate both sides of Newton's second law $\sum \vec{F} = d\vec{p}/dt$ over time between the limits t_1 and t_2:

$$\int_{t_1}^{t_2} \sum \vec{F}\,dt = \int_{t_1}^{t_2} \frac{d\vec{p}}{dt}\,dt = \int_{\vec{p}_1}^{\vec{p}_2} d\vec{p} = \vec{p}_2 - \vec{p}_1$$

The integral on the left is defined to be the impulse $\vec{J}$ of the net force $\sum \vec{F}$ during this interval:

$$\vec{J} = \int_{t_1}^{t_2} \sum \vec{F}\,dt \qquad \text{(general definition of impulse)} \tag{8.7}$$

With this definition, the impulse–momentum theorem $\vec{J} = \vec{p}_2 - \vec{p}_1$, Eq. (8.6), is valid even when the net force $\sum \vec{F}$ varies with time.

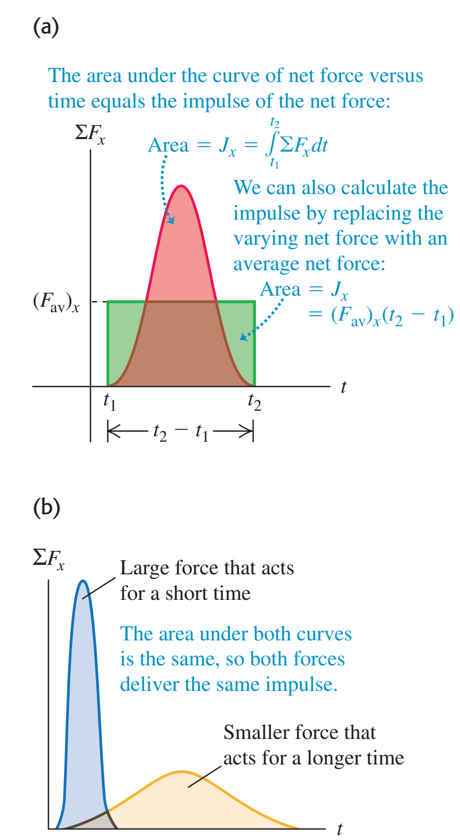
We can define an *average* net force $\vec{F}_{av}$ such that even when $\sum \vec{F}$ is not constant, the impulse $\vec{J}$ is given by

$$\vec{J} = \vec{F}_{av}(t_2 - t_1) \tag{8.8}$$

When $\sum \vec{F}$ is constant, $\sum \vec{F} = \vec{F}_{av}$ and Eq. (8.8) reduces to Eq. (8.5).

Figure 8.3a shows the x-component of net force $\sum F_x$ as a function of time during a collision. This might represent the force on a soccer ball that is in contact with a player's foot from time t_1 to t_2. The x-component of impulse during this interval is represented by the red area under the curve between t_1 and t_2. This area is equal to the green rectangular area bounded by t_1, t_2, and $(F_{av})_x$, so $(F_{av})_x(t_2 - t_1)$ is

8.3 The meaning of the area under a graph of $\sum F_x$ versus t.



equal to the impulse of the actual time-varying force during the same interval. Note that a large force acting for a short time can have the same impulse as a smaller force acting for a longer time if the areas under the force–time curves are the same (Fig. 8.3b). In this language, an automobile airbag (Fig. 8.2) provides the same impulse to the driver as would the steering wheel or the dashboard by applying a weaker and less injurious force for a longer time.

Impulse and momentum are both vector quantities, and Eqs. (8.5)–(8.8) are all vector equations. In specific problems, it is often easiest to use them in component form:

$$J_x = \int_{t_1}^{t_2} \Sigma F_x\, dt = (F_{\text{av}})_x(t_2 - t_1) = p_{2x} - p_{1x} = mv_{2x} - mv_{1x}$$
$$J_y = \int_{t_1}^{t_2} \Sigma F_y\, dt = (F_{\text{av}})_y(t_2 - t_1) = p_{2y} - p_{1y} = mv_{2y} - mv_{1y} \qquad (8.9)$$

and similarly for the z-component.

Momentum and Kinetic Energy Compared

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6.1 Momentum and Energy Change

We can now see the fundamental difference between momentum and kinetic energy. The impulse–momentum theorem $\vec{J} = \vec{p}_2 - \vec{p}_1$ says that changes in a particle's momentum are due to impulse, which depends on the *time* over which the net force acts. By contrast, the work–energy theorem $W_{\text{tot}} = K_2 - K_1$ tells us that kinetic energy changes when work is done on a particle; the total work depends on the *distance* over which the net force acts. Consider a particle that starts from rest at t_1 so that $\vec{v}_1 = \mathbf{0}$. Its initial momentum is $\vec{p}_1 = m\vec{v}_1 = \mathbf{0}$, and its initial kinetic energy is $K_1 = \frac{1}{2}mv_1^2 = 0$. Now let a constant net force equal to $\vec{F}$ act on that particle from time t_1 until time t_2. During this interval, the particle moves a distance s in the direction of the force. From Eq. (8.6), the particle's momentum at time t_2 is

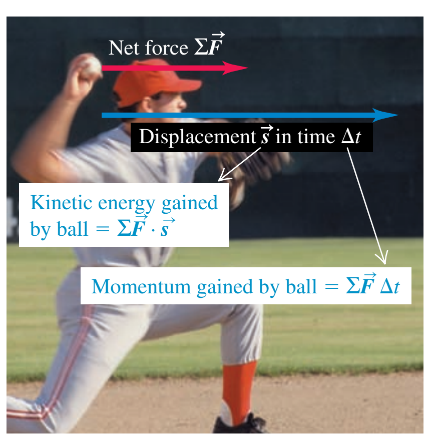
$$\vec{p}_2 = \vec{p}_1 + \vec{J} = \vec{J}$$

where $\vec{J} = \vec{F}(t_2 - t_1)$ is the impulse that acts on the particle. So *the momentum of a particle equals the impulse that accelerated it from rest to its present speed;* impulse is the product of the net force that accelerated the particle and the *time* required for the acceleration. By comparison, the kinetic energy of the particle at t_2 is $K_2 = W_{\text{tot}} = Fs$, the total *work* done on the particle to accelerate it from rest. The total work is the product of the net force and the *distance* required to accelerate the particle (Fig. 8.4).

Here's an application of the distinction between momentum and kinetic energy. Suppose you have a choice between catching a 0.50-kg ball moving at 4.0 m/s or a 0.10-kg ball moving at 20 m/s. Which will be easier to catch? Both balls have the same magnitude of momentum, $p = mv = (0.50\text{ kg})(4.0\text{ m/s}) = (0.10\text{ kg})(20\text{ m/s}) = 2.0\text{ kg}\cdot\text{m/s}$. However, the two balls have different values of kinetic energy $K = \frac{1}{2}mv^2$; the large, slow-moving ball has $K = 4.0$ J, while the small, fast-moving ball has $K = 20$ J. Since the momentum is the same for both balls, both require the same *impulse* to be brought to rest. But stopping the 0.10-kg ball with your hand requires five times more *work* than stopping the 0.50-kg ball because the smaller ball has five times more kinetic energy. For a given force that you exert with your hand, it takes the same amount of time (the duration of the catch) to stop either ball, but your hand and arm will be pushed back five times farther if you choose to catch the small, fast-moving ball. To minimize arm strain, you should choose to catch the 0.50-kg ball with its lower kinetic energy.

Both the impulse–momentum and work–energy theorems are relationships between force and motion, and both rest on the foundation of Newton's laws. They are *integral* principles, relating the motion at two different times separated by a finite interval. By contrast, Newton's second law itself (in either of the forms $\Sigma\vec{F} = m\vec{a}$ or $\Sigma\vec{F} = d\vec{p}/dt$) is a *differential* principle, relating the forces to the rate of change of velocity or momentum at each instant.

8.4 The *kinetic energy* of a pitched baseball is equal to the work the pitcher does on it (force multiplied by the distance the ball moves during the throw). The *momentum* of the ball is equal to the impulse the pitcher imparts to it (force multiplied by the time it took to bring the ball up to speed).



Conceptual Example 8.1 Momentum versus kinetic energy

Consider again the race described in Conceptual Example 6.5 (Section 6.2) between two iceboats on a frictionless frozen lake. The iceboats have masses m and $2m$, and the wind exerts the same constant horizontal force $\vec{F}$ on each iceboat (see Fig. 6.14). The two iceboats start from rest and cross the finish line a distance s away. Which iceboat crosses the finish line with greater momentum?

SOLUTION

In Conceptual Example 6.5 we asked how the *kinetic energies* of the iceboats compare when they cross the finish line. The way to answer this was not to use the formula $K = \frac{1}{2}mv^2$, but to remember that a body's kinetic energy equals the total work done to accelerate it from rest. Both iceboats started from rest, and the total work done between the starting and finish lines was the same for both iceboats (because the net force and displacement were the same for both). Hence both iceboats cross the finish line with the same kinetic energy.

Similarly, the best way to compare the *momenta* of the iceboats is *not* to use the formula $\vec{p} = m\vec{v}$. By itself this formula isn't enough to determine which iceboat has greater momentum at the finish line. The iceboat of mass $2m$ has greater mass, which suggests greater momentum, but this iceboat crosses the finish line going slower than the other one, which suggests less momentum.

Instead, we use the idea that the momentum of each iceboat equals the impulse that accelerated it from rest. For each iceboat the downward force of gravity and the upward normal force add to zero, so the net force equals the constant horizontal wind force $\vec{F}$. Let Δt be the time an iceboat takes to reach the finish line, so that the impulse on the iceboat during that time is $\vec{J} = \vec{F}\,\Delta t$. Since the iceboat starts from rest, this equals the iceboat's momentum $\vec{p}$ at the finish line:

$$\vec{p} = \vec{F}\,\Delta t$$

Both iceboats are subjected to the same force $\vec{F}$, but they take different amounts of time Δt to reach the finish line. The iceboat of mass $2m$ accelerates more slowly and takes a longer time to travel the distance s; thus there is a greater impulse on this iceboat between the starting and finish lines. So the iceboat of mass $2m$ crosses the finish line with a greater magnitude of momentum than the iceboat of mass m (but with the same kinetic energy). Can you show that the iceboat of mass $2m$ has $\sqrt{2}$ times as much momentum at the finish line as the iceboat of mass m?

Example 8.2 A ball hits a wall

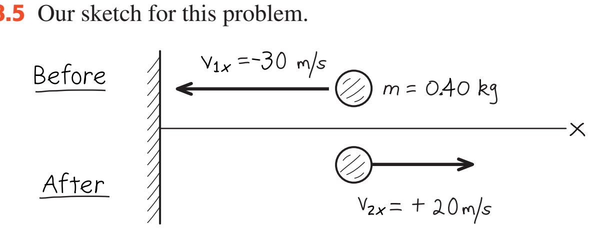
Suppose you throw a ball with a mass of 0.40 kg against a brick wall. It hits the wall moving horizontally to the left at 30 m/s and rebounds horizontally to the right at 20 m/s. (a) Find the impulse of the net force on the ball during its collision with the wall. (b) If the ball is in contact with the wall for 0.010 s, find the average horizontal force that the wall exerts on the ball during the impact.

SOLUTION

IDENTIFY: We're given enough information to determine the initial and final values of the ball's momentum, so we can use the impulse–momentum theorem to find the impulse. We'll then use the definition of impulse to determine the average force.

SET UP: Figure 8.5 shows our sketch. The motion is purely horizontal, so we need only a single axis. We'll take the x-axis to be horizontal and the positive direction to be to the right. Our target variable in part (a) is the x-component of impulse, J_x, which we'll find from the x-components of momentum before and after the impact, using Eqs. (8.9). In part (b), our target variable is the average x-component of force $(F_{\text{av}})_x$; once we know J_x, we can also find this force by using Eqs. (8.9).

8.5 Our sketch for this problem.



EXECUTE: (a) With our choice of x-axis, the initial and final x-components of momentum of the ball are

$$p_{1x} = mv_{1x} = (0.40\text{ kg})(-30\text{ m/s}) = -12\text{ kg}\cdot\text{m/s}$$
$$p_{2x} = mv_{2x} = (0.40\text{ kg})(+20\text{ m/s}) = +8.0\text{ kg}\cdot\text{m/s}$$

From the x-equation in Eqs. (8.9), the x-component of impulse equals the *change* in the x-momentum:

$$J_x = p_{2x} - p_{1x}$$
$$= 8.0\text{ kg}\cdot\text{m/s} - (-12\text{ kg}\cdot\text{m/s}) = 20\text{ kg}\cdot\text{m/s} = 20\text{ N}\cdot\text{s}$$

(b) The collision time is $t_2 - t_1 = \Delta t = 0.010$ s. From the x-equation in Eqs. (8.9), $J_x = (F_{\text{av}})_x(t_2 - t_1) = (F_{\text{av}})_x\,\Delta t$, so

$$(F_{\text{av}})_x = \frac{J_x}{\Delta t} = \frac{20\text{ N}\cdot\text{s}}{0.010\text{ s}} = 2000\text{ N}$$

EVALUATE: The x-component of impulse is positive—that is, to the right in Fig. 8.5. This is as it should be: The impulse represents the "kick" that the wall imparts to the ball, and this "kick" is certainly to the right.

CAUTION Momentum is a vector Because momentum is a vector, we had to include the negative sign in p_{1x}. Had we carelessly omitted it, we would have calculated the impulse to be $8.0\text{ kg}\cdot\text{m/s} - (12\text{ kg}\cdot\text{m/s}) = -4\text{ kg}\cdot\text{m/s}$. This incorrect answer would say that the wall had somehow given the ball a kick to the *left!* Make sure that you account for the *direction* of momentum in your calculations.

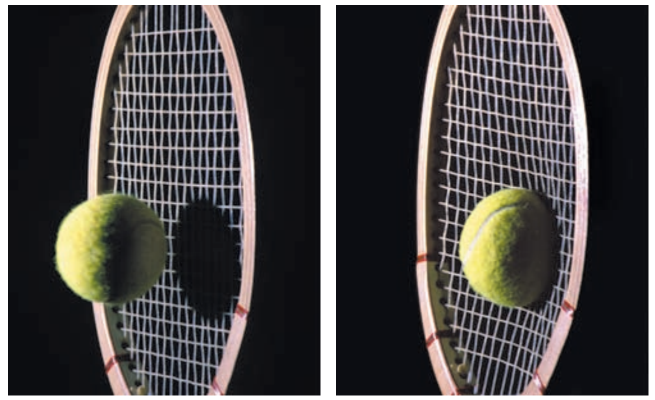
The force that the wall exerts on the ball has to have a large magnitude of 2000 N (equal to 450 lb, or the weight of a 200-kg

Continued

object) to change the ball's momentum in such a short time interval. Other forces that act on the ball during the collision are very weak by comparison; for instance, the gravitational force is only 3.9 N. Thus, during the brief time that the collision lasts, we can ignore all other forces on the ball to a very good approximation. Figure 8.6 is a photograph showing the impact of a tennis ball and racket.

Note that the 2000-N value we calculated is just the *average* horizontal force that the wall exerts on the ball during the impact. It corresponds to the horizontal line $(F_{av})_x$ in Fig. 8.3a. The horizontal force is zero before impact, rises to a maximum, and then decreases to zero when the ball loses contact with the wall. If the ball is relatively rigid, like a baseball or golf ball, the collision lasts a short time and the maximum force is large, as in the blue curve in Fig. 8.3b. If the ball is softer, like a tennis ball, the collision time is longer and the maximum force is less, as in the orange curve in Fig. 8.3b.

8.6 Typically, a tennis ball is in contact with the racket for approximately 0.01 s. The ball flattens noticeably due to the tremendous force exerted by the racket.



Example 8.3 Kicking a soccer ball

A soccer ball has a mass of 0.40 kg. Initially, it is moving to the left at 20 m/s, but then it is kicked and given a velocity at 45° upward and to the right, with a magnitude of 30 m/s (Fig. 8.7a). Find the impulse of the net force and the average net force, assuming a collision time $\Delta t = 0.010$ s.

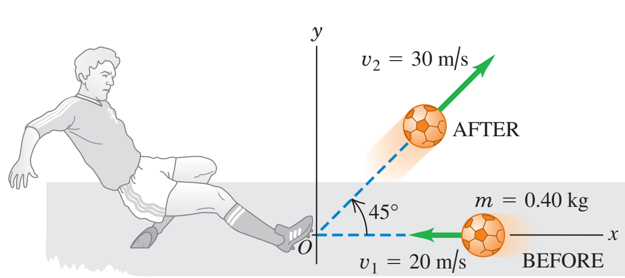
SOLUTION

IDENTIFY: This example uses the same principles as Example 8.2. The key difference is that the initial and final velocities are not along the same line, so we have to be careful to treat momentum and impulse as vector quantities, using their x- and y-components.

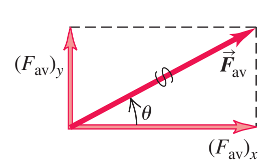
SET UP: We take the x-axis to be horizontally to the right and the y-axis to be vertically upward. Our target variables are the components of the net impulse on the ball, J_x and J_y, and the components of the average net force on the ball, $(F_{av})_x$ and $(F_{av})_y$. We'll find them using the x- and y-components of Eqs. (8.9).

8.7 (a) Kicking a soccer ball. (b) Finding the average force on the ball from its components.

(a) Before-and-after diagram



(b) Average force on the ball



EXECUTE: With our choice of axes, we find the ball's velocity components before (subscript 1) and after (subscript 2) it is kicked:

$$v_{1x} = -20\ \text{m/s} \qquad v_{1y} = 0$$

$$v_{2x} = v_{2y} = (30\ \text{m/s})(0.707) = 21.2\ \text{m/s}$$
$$(\text{since } \cos 45° = \sin 45° = 0.707)$$

The x-component of impulse is equal to the x-component of momentum change, and the same is true for the y-components:

$$J_x = p_{2x} - p_{1x} = m(v_{2x} - v_{1x})$$
$$= (0.40\ \text{kg})[21.2\ \text{m/s} - (-20\ \text{m/s})] = 16.5\ \text{kg}\cdot\text{m/s}$$
$$J_y = p_{2y} - p_{1y} = m(v_{2y} - v_{1y})$$
$$= (0.40\ \text{kg})(21.2\ \text{m/s} - 0) = 8.5\ \text{kg}\cdot\text{m/s}$$

The components of the average net force on the ball are

$$(F_{av})_x = \frac{J_x}{\Delta t} = 1650\ \text{N} \qquad (F_{av})_y = \frac{J_y}{\Delta t} = 850\ \text{N}$$

The magnitude and direction of the average force are

$$F_{av} = \sqrt{(1650\ \text{N})^2 + (850\ \text{N})^2} = 1.9 \times 10^3\ \text{N}$$

$$\theta = \arctan\frac{850\ \text{N}}{1650\ \text{N}} = 27°$$

where θ is measured upward from the $+x$-axis (Fig. 8.7b). Note that because the ball was not initially at rest, the ball's final velocity does *not* have the same direction as the average force that acted on it.

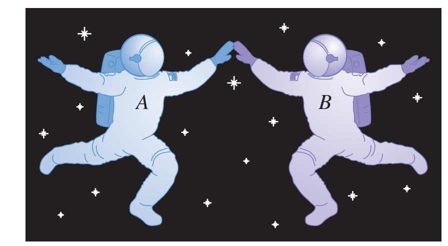
EVALUATE: The average net force $\vec{F}_{av}$ includes the effects of the force of gravity, although these are small; the weight of the ball is only 3.9 N. As in Example 8.2, the average force acting during the collision is exerted almost entirely by the object that the ball hit (in this case, the soccer player's foot).

Test Your Understanding of Section 8.1 Rank the following situations according to the magnitude of the impulse of the net force, from largest value to smallest value. In each situation a 1000-kg automobile is moving along a straight east–west road. (i) The automobile is initially moving east at 25 m/s and comes to a stop in 10 s. (ii) The automobile is initially moving east at 25 m/s and comes to a stop in 5 s. (iii) The automobile is initially at rest, and a 2000-N net force toward the east is applied to it for 10 s. (iv) The automobile is initially moving east at 25 m/s, and a 2000-N net force toward the west is applied to it for 10 s. (v) The automobile is initially moving east at 25 m/s. Over a 30-s period, the automobile reverses direction and ends up moving west at 25 m/s.

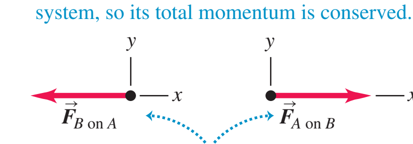
8.2 Conservation of Momentum

The concept of momentum is particularly important in situations in which we have two or more *interacting* bodies. To see why, let's consider first an idealized system consisting of two bodies that interact with each other but not with anything else—for example, two astronauts who touch each other as they float freely in the zero-gravity environment of outer space (Fig. 8.8). Think of the astronauts as particles. Each particle exerts a force on the other; according to Newton's third law, the two forces are always equal in magnitude and opposite in direction. Hence, the *impulses* that act on the two particles are equal and opposite, and the changes in momentum of the two particles are equal and opposite.

8.8 Two astronauts push each other as they float freely in the zero-gravity environment of space.



No external forces act on the two-astronaut system, so its total momentum is conserved.



The forces the astronauts exert on each other form an action–reaction pair.

Let's go over that again with some new terminology. For any system, the forces that the particles of the system exert on each other are called **internal forces.** Forces exerted on any part of the system by some object outside it are called **external forces.** For the system shown in Fig. 8.8, the internal forces are $\vec{F}_{B\ \text{on}\ A}$, exerted by particle B on particle A, and $\vec{F}_{A\ \text{on}\ B}$, exerted by particle A on particle B. There are *no* external forces; when this is the case, we have an **isolated system.**

The net force on particle A is $\vec{F}_{B\ \text{on}\ A}$ and the net force on particle B is $\vec{F}_{A\ \text{on}\ B}$, so from Eq. (8.4) the rates of change of the momenta of the two particles are

$$\vec{F}_{B\ \text{on}\ A} = \frac{d\vec{p}_A}{dt} \qquad \vec{F}_{A\ \text{on}\ B} = \frac{d\vec{p}_B}{dt} \tag{8.10}$$

The momentum of each particle changes, but these changes are related to each other by Newton's third law: The two forces $\vec{F}_{B\ \text{on}\ A}$ and $\vec{F}_{A\ \text{on}\ B}$ are always equal in magnitude and opposite in direction. That is, $\vec{F}_{B\ \text{on}\ A} = -\vec{F}_{A\ \text{on}\ B}$, so $\vec{F}_{B\ \text{on}\ A} + \vec{F}_{A\ \text{on}\ B} = \mathbf{0}$. Adding together the two equations in Eq. (8.10), we have

$$\vec{F}_{B\ \text{on}\ A} + \vec{F}_{A\ \text{on}\ B} = \frac{d\vec{p}_A}{dt} + \frac{d\vec{p}_B}{dt} = \frac{d(\vec{p}_A + \vec{p}_B)}{dt} = \mathbf{0} \tag{8.11}$$

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6.3 Momentum Conservation and Collisions
6.7 Explosion Problems
6.10 Pendulum Person-Projectile Bowling

The rates of change of the two momenta are equal and opposite, so the rate of change of the vector sum $\vec{p}_A + \vec{p}_B$ is zero. We now define the **total momentum** $\vec{P}$ of the system of two particles as the vector sum of the momenta of the individual particles; that is,

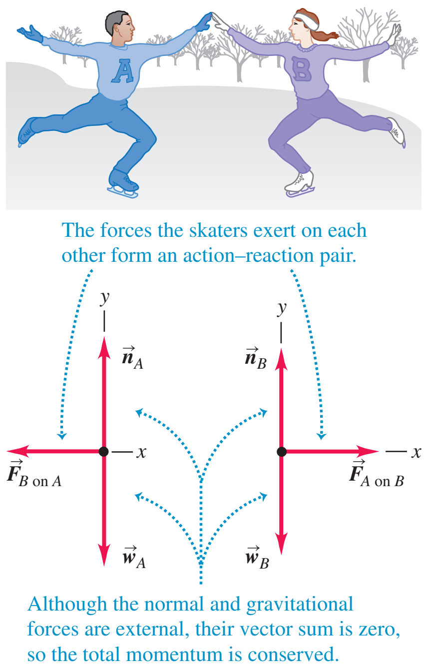
$$\vec{P} = \vec{p}_A + \vec{p}_B \tag{8.12}$$

Then Eq. (8.11) becomes, finally,

$$\vec{F}_{B\ \text{on}\ A} + \vec{F}_{A\ \text{on}\ B} = \frac{d\vec{P}}{dt} = \mathbf{0} \tag{8.13}$$

The time rate of change of the *total* momentum $\vec{P}$ is zero. Hence the total momentum of the system is constant, even though the individual momenta of the particles that make up the system can change.

8.9 Two ice skaters push each other as they skate on a frictionless, horizontal surface. (Compare to Fig. 8.8.)



If external forces are also present, they must be included on the left side of Eq. (8.13) along with the internal forces. Then the total momentum is, in general, not constant. But if the vector sum of the external forces is zero, as in Fig. 8.9, these forces don't contribute to the sum, and $d\vec{P}/dt$ is again zero. Thus we have the following general result:

If the vector sum of the external forces on a system is zero, the total momentum of the system is constant.

This is the simplest form of the **principle of conservation of momentum.** This principle is a direct consequence of Newton's third law. What makes this principle useful is that it doesn't depend on the detailed nature of the internal forces that act between members of the system. This means that we can apply conservation of momentum even if (as is often the case) we know very little about the internal forces. We have used Newton's second law to derive this principle, so we have to be careful to use it only in inertial frames of reference.

We can generalize this principle for a system that contains any number of particles A, B, C, . . . interacting only with each other. The total momentum of such a system is

$$\vec{P} = \vec{p}_A + \vec{p}_B + \cdots = m_A\vec{v}_A + m_B\vec{v}_B + \cdots \quad \text{(total momentum of a system of particles)} \tag{8.14}$$

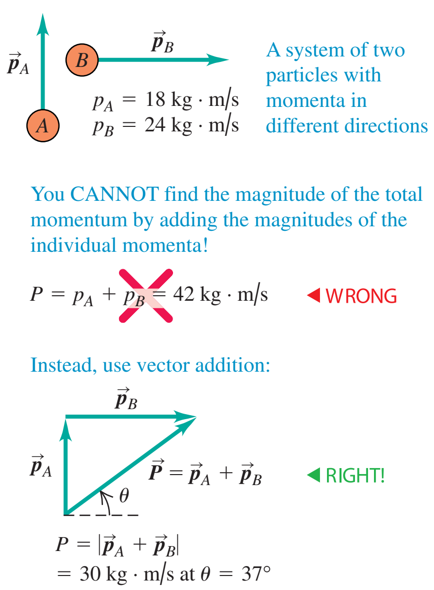
We make the same argument as before: The total rate of change of momentum of the system due to each action–reaction pair of internal forces is zero. Thus the total rate of change of momentum of the entire system is zero whenever the vector sum of the external forces acting on it is zero. The internal forces can change the momenta of individual particles in the system but not the *total* momentum of the system.

CAUTION **Conservation of momentum means conservation of its components** When you apply the conservation of momentum to a system, remember that momentum is a *vector* quantity. Hence you must use vector addition to compute the total momentum of a system (Fig. 8.10). Using components is usually the simplest method. If p_{Ax}, p_{Ay}, and p_{Az} are the components of momentum of particle A, and similarly for the other particles, then Eq. (8.14) is equivalent to the component equations

$$P_x = p_{Ax} + p_{Bx} + \cdots$$
$$P_y = p_{Ay} + p_{By} + \cdots \tag{8.15}$$
$$P_z = p_{Az} + p_{Bz} + \cdots$$

If the vector sum of the external forces on the system is zero, then P_x, P_y, and P_z are all constant.

8.10 When applying conservation of momentum, remember that momentum is a vector quantity!



In some ways the principle of conservation of momentum is more general than the principle of conservation of mechanical energy. For example, mechanical energy is conserved only when the internal forces are *conservative*—that is, when the forces allow two-way conversion between kinetic and potential energy—but conservation of momentum is valid even when the internal forces are *not* conservative. In this chapter we will analyze situations in which both momentum and mechanical energy are conserved, and others in which only momentum is conserved. These two principles play a fundamental role in all areas of physics, and we will encounter them throughout our study of physics.

Problem-Solving Strategy 8.1 Conservation of Momentum

IDENTIFY *the relevant concepts:* Before applying conservation of momentum to a problem, you must decide whether momentum *is* conserved! This will be true *only* if the vector sum of the external forces acting on the system of particles is zero. If this is not the case, you can't use conservation of momentum.

SET UP *the problem using the following steps:*

1. Define a coordinate system and show it in a sketch, including the positive direction for each axis. Often it is easiest to choose the x-axis in the direction of one of the initial velocities. Make sure you are using an inertial frame of reference. Most of the problems in this chapter deal with two-dimensional situations, in which the vectors have only x- and y-components, but this strategy can be generalized to include z-components when necessary.
2. Treat each body as a particle. Draw "before" and "after" sketches, and include vectors on each to represent all known velocities. Label the vectors with magnitudes, angles, components, or whatever information is given, and give each unknown magnitude, angle, or component an algebraic symbol. It's helpful to use the subscripts 1 and 2 for velocities before and after the interaction, respectively, and use letters (not numbers) to label each particle.
3. As always, identify the target variable(s) from among the unknowns.

EXECUTE *the solution as follows:*

1. Write an equation in symbols equating the total *initial* x-component of momentum (that is, before the interaction) to the total *final* x-component of momentum (that is, after the interaction), using $p_x = mv_x$ for each particle. Write another equation for the y-components, using $p_y = mv_y$ for each particle. (*Never* add the x- and y-components of velocity or momentum together in the same equation!) Even when all motions are along a line (such as the x-axis), the components of velocity along this line can be positive or negative; be careful with signs!
2. Solve these equations to determine whatever results are required. In some problems you will have to convert from the x- and y-components of a velocity to its magnitude and direction, or the reverse.
3. In some problems, energy considerations give additional relationships among the various velocities, as we will see later in this chapter.

EVALUATE *your answer:* Does your answer make physical sense? If your target variable is a certain body's momentum, check that the direction of the momentum is reasonable.

Example 8.4 Recoil of a rifle

A marksman holds a rifle of mass $m_R = 3.00$ kg loosely in his hands, so as to let it recoil freely when fired. He fires a bullet of mass $m_B = 5.00$ g horizontally with a velocity relative to the ground of $v_{Bx} = 300$ m/s. What is the recoil velocity v_{Rx} of the rifle? What are the final momentum and kinetic energy of the bullet? Of the rifle?

SOLUTION

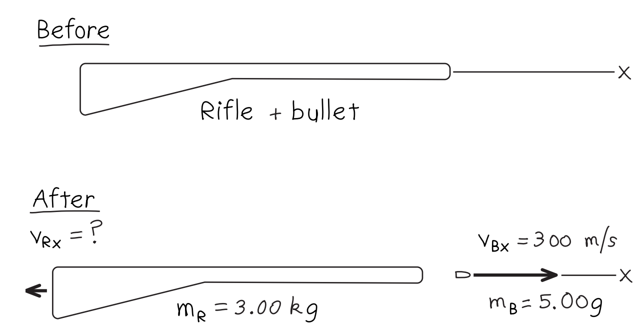
IDENTIFY: We consider an idealized model in which the horizontal forces the marksman exerts on the rifle are negligible. Then there is no net horizontal force on the system (the bullet and rifle) during the firing of the rifle, and so the total horizontal momentum of the system is the same before and after the rifle is fired (i.e., it is conserved).

SET UP: Figure 8.11 shows our sketch. We take the positive x-axis to be the direction the rifle is aimed. Initially, both the rifle and the bullet are at rest, so the initial x-component of total momentum is zero. After the shot is fired, the bullet's x-momentum is $p_{Bx} = m_B v_{Bx}$ and the rifle's x-momentum is $p_{Rx} = m_R v_{Rx}$. Our target variables are v_{Rx}, p_{Bx}, p_{Rx}, and $K_B = \frac{1}{2}m_B v_{Bx}^2$ and $K_R = \frac{1}{2}m_R v_{Rx}^2$ (the final kinetic energies of the bullet and rifle, respectively).

EXECUTE: Conservation of the x-component of total momentum gives

$$P_x = 0 = m_B v_{Bx} + m_R v_{Rx}$$
$$v_{Rx} = -\frac{m_B}{m_R}v_{Bx} = -\left(\frac{0.00500 \text{ kg}}{3.00 \text{ kg}}\right)(300 \text{ m/s}) = -0.500 \text{ m/s}$$

8.11 Our sketch for this problem.



The negative sign means that the recoil is in the direction opposite to that of the bullet. If the butt of a rifle hit your shoulder at this speed, you'd feel it. It's more comfortable to hold the rifle tightly against your shoulder when you fire it; then m_R is replaced by the sum of your mass and the rifle's mass, and the recoil speed is much less.

The final momentum and kinetic energy of the bullet are

$$p_{Bx} = m_B v_{Bx} = (0.00500 \text{ kg})(300 \text{ m/s}) = 1.50 \text{ kg} \cdot \text{m/s}$$
$$K_B = \tfrac{1}{2}m_B v_{Bx}^2 = \tfrac{1}{2}(0.00500 \text{ kg})(300 \text{ m/s})^2 = 225 \text{ J}$$

For the rifle, the final momentum and kinetic energy are

$$p_{Rx} = m_R v_{Rx} = (3.00 \text{ kg})(-0.500 \text{ m/s}) = -1.50 \text{ kg} \cdot \text{m/s}$$
$$K_R = \tfrac{1}{2}m_R v_{Rx}^2 = \tfrac{1}{2}(3.00 \text{ kg})(-0.500 \text{ m/s})^2 = 0.375 \text{ J}$$

Continued

EVALUATE: The bullet and the rifle have equal and opposite *momenta* after the interaction. That's because they were subjected to equal and opposite interaction forces for the same amount of *time* (i.e., equal and opposite impulses). But the bullet acquires much greater *kinetic energy* than the rifle because the bullet travels a much greater *distance* than the rifle during the interaction. Thus the force on the bullet does more work than the force on the rifle. The ratio of the two kinetic energies, 600 : 1, is equal to the inverse ratio of the masses; in fact, it can be shown that this always happens in recoil situations. We leave the proof as a problem (see Exercise 8.22).

Our calculation doesn't depend on the details of how the rifle works. In a real rifle, the bullet is propelled forward by an explosive charge; if instead the rifle used a very stiff spring, the answers would have been exactly the same.

Example 8.5 Collision along a straight line

Two gliders move toward each other on a frictionless linear air track (Fig. 8.12a). After they collide (Fig. 8.12b), glider B moves away with a final velocity of $+2.0\ \text{m/s}$ (Fig. 8.12c). What is the final velocity of glider A? How do the changes in momentum and in velocity compare for the two gliders?

SOLUTION

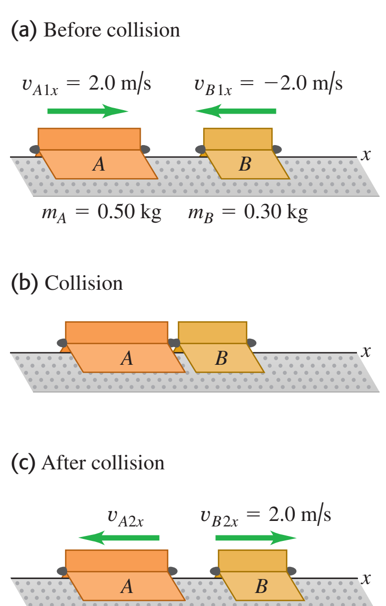
IDENTIFY: The total vertical force on each glider is zero; the net force on each glider is the horizontal force exerted on it by the other glider. The net *external* force on the two gliders together is zero, so the total momentum is conserved. (Compare Fig. 8.9.)

SET UP: We take the positive x-axis to be to the right, along the air track. We are given the masses and initial velocities of both gliders and the final velocity of glider B. Our target variables are v_{A2x}, the final x-component of velocity of glider A, and the changes in momentum and in velocity of the two gliders (the value after the collision minus the value before the collision).

EXECUTE: The x-component of total momentum before the collision is

$$\begin{aligned} P_x &= m_A v_{A1x} + m_B v_{B1x} \\ &= (0.50\ \text{kg})(2.0\ \text{m/s}) + (0.30\ \text{kg})(-2.0\ \text{m/s}) \\ &= 0.40\ \text{kg}\cdot\text{m/s} \end{aligned}$$

8.12 Two gliders colliding on an air track.



This is positive (to the right in Fig. 8.12) because glider A has a greater magnitude of momentum before the collision than does glider B. The x-component of total momentum has the same value after the collision, so

$$P_x = m_A v_{A2x} + m_B v_{B2x}$$

Solving this equation for v_{A2x}, the final x-velocity of A, we find

$$\begin{aligned} v_{A2x} &= \frac{P_x - m_B v_{B2x}}{m_A} = \frac{0.40\ \text{kg}\cdot\text{m/s} - (0.30\ \text{kg})(2.0\ \text{m/s})}{0.50\ \text{kg}} \\ &= -0.40\ \text{m/s} \end{aligned}$$

The change in x-momentum of glider A is

$$\begin{aligned} m_A v_{A2x} - m_A v_{A1x} &= (0.50\ \text{kg})(-0.40\ \text{m/s}) \\ &\quad - (0.50\ \text{kg})(2.0\ \text{m/s}) = -1.2\ \text{kg}\cdot\text{m/s} \end{aligned}$$

and the change in x-momentum of glider B is

$$\begin{aligned} m_B v_{B2x} - m_B v_{B1x} &= (0.30\ \text{kg})(2.0\ \text{m/s}) \\ &\quad - (0.30\ \text{kg})(-2.0\ \text{m/s}) = +1.2\ \text{kg}\cdot\text{m/s} \end{aligned}$$

The two interacting gliders undergo changes in momentum that are equal in magnitude and opposite in direction. The same is *not* true of their changes in velocity, however. For A, $v_{A2x} - v_{A1x} = (-0.40\ \text{m/s}) - 2.0\ \text{m/s} = -2.4\ \text{m/s}$; for B, $v_{B2x} - v_{B1x} = 2.0\ \text{m/s} - (-2.0\ \text{m/s}) = +4.0\ \text{m/s}$.

EVALUATE: Why do the momentum changes have the same magnitude for the two gliders, but the velocity changes do not? By Newton's third law, both gliders were acted on for equal amounts of time by an interaction force of the same magnitude. Hence both gliders experienced impulses of the same magnitude, and therefore equal-magnitude changes in momentum. But by Newton's second law, the less massive glider (B) had a greater magnitude of acceleration and hence a greater velocity change.

Here's an application of these ideas. When a large truck collides with a car of normal size, both vehicles undergo equal changes in momentum. The occupants of the car, however, are subjected to greater acceleration (and greater chance of injury) than the occupants of the truck. An even more extreme example is what happens when a truck collides with an insect: The truck driver won't notice the resulting acceleration at all, but the insect surely will!

Example 8.6 Collision in a horizontal plane

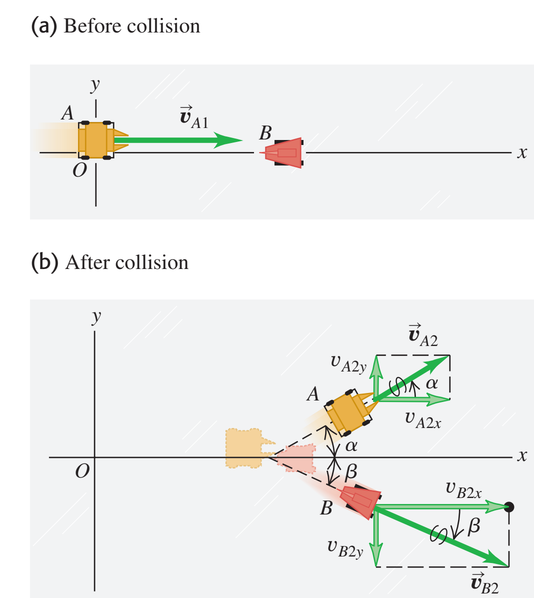
Figure 8.13a shows two battling robots sliding on a frictionless surface. Robot A, with mass 20 kg, initially moves at 2.0 m/s parallel to the x-axis. It collides with robot B, which has mass 12 kg and is initially at rest. After the collision, robot A is moving at 1.0 m/s in a direction that makes an angle $\alpha = 30°$ with its initial direction (Fig. 8.13b). What is the final velocity of robot B?

SOLUTION

IDENTIFY: There are no horizontal (x or y) external forces, so the x-component and the y-component of the total momentum of the system are both conserved in the collision.

SET UP: Figure 8.13 shows the coordinate axes. The velocities are not all along a single line, so we have to treat momentum as a vector quantity. Momentum conservation requires that the sum of the x-components of momentum *before* the collision (subscript 1) must equal the sum *after* the collision (subscript 2), and similarly for the sums of the y-components. We write a separate momentum conservation equation for each component. Our target variable is $\vec{v}_{B2}$, the final velocity of robot B.

8.13 Views from above of the velocities (a) before and (b) after the collision.



EXECUTE: Conservation of the x-component of total momentum says that

$$\begin{aligned} m_A v_{A1x} + m_B v_{B1x} &= m_A v_{A2x} + m_B v_{B2x} \\ v_{B2x} &= \frac{m_A v_{A1x} + m_B v_{B1x} - m_A v_{A2x}}{m_B} \\ &= \frac{\begin{bmatrix} (20\ \text{kg})(2.0\ \text{m/s}) + (12\ \text{kg})(0) \\ -(20\ \text{kg})(1.0\ \text{m/s})(\cos 30°) \end{bmatrix}}{12\ \text{kg}} \\ &= 1.89\ \text{m/s} \end{aligned}$$

Similarly, for the y-component of total momentum we have

$$\begin{aligned} m_A v_{A1y} + m_B v_{B1y} &= m_A v_{A2y} + m_B v_{B2y} \\ v_{B2y} &= \frac{m_A v_{A1y} + m_B v_{B1y} - m_A v_{A2y}}{m_B} \\ &= \frac{\begin{bmatrix} (20\ \text{kg})(0) + (12\ \text{kg})(0) \\ -(20\ \text{kg})(1.0\ \text{m/s})(\sin 30°) \end{bmatrix}}{12\ \text{kg}} \\ &= -0.83\ \text{m/s} \end{aligned}$$

After the collision, robot B moves in the positive x-direction and the negative y-direction (Fig. 8.13b). The magnitude of $\vec{v}_{B2}$ is

$$v_{B2} = \sqrt{(1.89\ \text{m/s})^2 + (-0.83\ \text{m/s})^2} = 2.1\ \text{m/s}$$

and the angle of its direction from the positive x-axis is

$$\beta = \arctan \frac{-0.83\ \text{m/s}}{1.89\ \text{m/s}} = -24°$$

EVALUATE: We can check our answer by looking at the values of momentum before and after the collision. Initially all of the momentum is in robot A, which has x-momentum $m_A v_{A1x} = (20\ \text{kg})(2.0\ \text{m/s}) = 40\ \text{kg}\cdot\text{m/s}$ and zero y-momentum. After the collision, robot A has x-momentum $m_A v_{A2x} = (20\ \text{kg})(1.0\ \text{m/s})(\cos 30°) = 17\ \text{kg}\cdot\text{m/s}$, while robot B has x-momentum $m_B v_{B2x} = (12\ \text{kg})(1.89\ \text{m/s}) = 23\ \text{kg}\cdot\text{m/s}$; the total x-momentum is $40\ \text{kg}\cdot\text{m/s}$, the same as before the collision (as it should be). In the y-direction, robot A acquires y-momentum $m_A v_{A2y} = (20\ \text{kg})(1.0\ \text{m/s})(\sin 30°) = 10\ \text{kg}\cdot\text{m/s}$, while robot B acquires y-momentum of the same magnitude but opposite direction: $m_B v_{B2y} = (12\ \text{kg})(-0.83\ \text{m/s}) = -10\ \text{kg}\cdot\text{m/s}$. Hence the *total* y-component of momentum after the collision has the same value (zero) as before the collision.

Test Your Understanding of Section 8.2 A spring-loaded toy sits at rest on a horizontal frictionless surface. When the spring releases, the toy breaks into three equal-mass pieces, A, B, and C, which slide along the surface. Piece A moves off in the negative x-direction, while piece B moves off in the negative y-direction. (a) What are the signs of the velocity components of piece C? (b) Which of the three pieces is moving the fastest?

MP

8.3 Momentum Conservation and Collisions

To most people the term *collision* is likely to mean some sort of automotive disaster. We'll use it in that sense, but we'll also broaden the meaning to include any strong interaction between bodies that lasts a relatively short time. So we include

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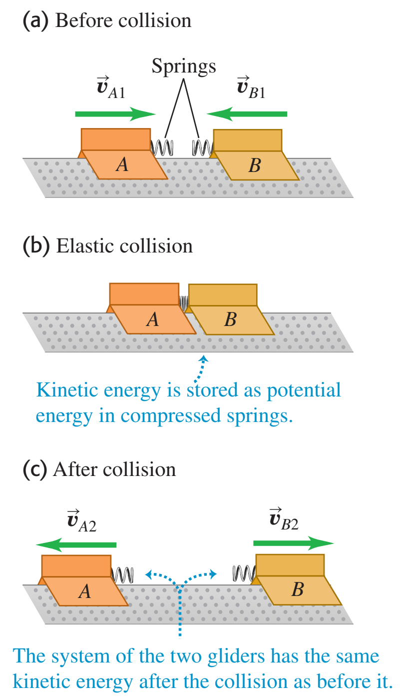
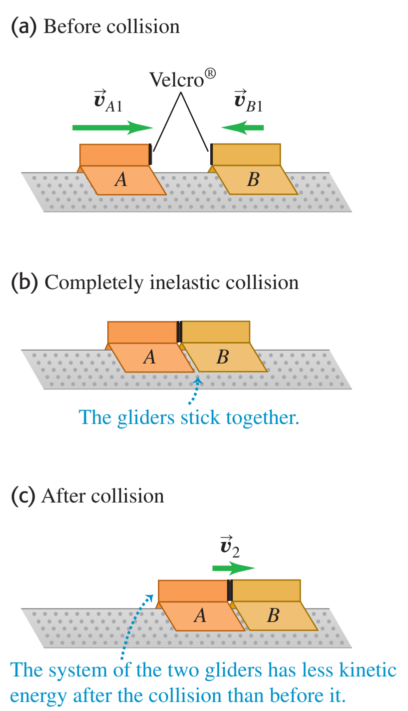
6.4 Collision Problems
6.8 Skier and Cart

8.14 Two gliders undergoing an elastic collision on a frictionless surface. Each glider has a steel spring bumper that exerts a conservative force on the other glider.



8.15 Two gliders undergoing a completely inelastic collision. The spring bumpers on the gliders are replaced by Velcro®, so the gliders stick together after collision.



not only car accidents but also balls colliding on a billiard table, neutrons hitting atomic nuclei in a nuclear reactor, the impact of a meteor on the Arizona desert, and a close encounter of a spacecraft with the planet Saturn.

If the forces between the bodies are much larger than any external forces, as is the case in most collisions, we can neglect the external forces entirely and treat the bodies as an *isolated* system. Then momentum is conserved and the total momentum of the system has the same value before and after the collision. Two cars colliding at an icy intersection provide a good example. Even two cars colliding on dry pavement can be treated as an isolated system during the collision if the forces between the cars are much larger than the friction forces of pavement against tires.

Elastic and Inelastic Collisions

If the forces between the bodies are also *conservative,* so that no mechanical energy is lost or gained in the collision, the total *kinetic* energy of the system is the same after the collision as before. Such a collision is called an **elastic collision.** A collision between two marbles or two billiard balls is almost completely elastic. Figure 8.14 shows a model for an elastic collision. When the gliders collide, their springs are momentarily compressed and some of the original kinetic energy is momentarily converted to elastic potential energy. Then the gliders bounce apart, the springs expand, and this potential energy is converted back to kinetic energy.

A collision in which the total kinetic energy after the collision is *less* than before the collision is called an **inelastic collision.** A meatball landing on a plate of spaghetti and a bullet embedding itself in a block of wood are examples of inelastic collisions. An inelastic collision in which the colliding bodies stick together and move as one body after the collision is often called a **completely inelastic collision.** Figure 8.15 shows an example; we have replaced the spring bumpers in Fig. 8.14 with Velcro®, which sticks the two bodies together.

CAUTION **An inelastic collision doesn't have to be *completely* inelastic** It's a common misconception that the *only* inelastic collisions are those in which the colliding bodies stick together. In fact, inelastic collisions include many situations in which the bodies do *not* stick. If two cars bounce off each other in a "fender bender," the work done to deform the fenders cannot be recovered as kinetic energy of the cars, so the collision is inelastic (Fig. 8.16).

Remember this rule: **In any collision in which external forces can be neglected, momentum is conserved and the total momentum before equals the total momentum after; in elastic collisions *only,* the total kinetic energy before equals the total kinetic energy after.**

Completely Inelastic Collisions

Let's look at what happens to momentum and kinetic energy in a *completely* inelastic collision of two bodies (A and B), as in Fig. 8.15. Because the two bodies stick together after the collision, they have the same final velocity $\vec{v}_2$:

$$\vec{v}_{A2} = \vec{v}_{B2} = \vec{v}_2$$

Conservation of momentum gives the relationship

$$m_A\vec{v}_{A1} + m_B\vec{v}_{B1} = (m_A + m_B)\vec{v}_2 \quad \text{(completely inelastic collision)} \quad (8.16)$$

If we know the masses and initial velocities, we can compute the common final velocity $\vec{v}_2$.

Suppose, for example, that a body with mass m_A and initial x-component of velocity v_{A1x} collides inelastically with a body with mass m_B that is initially at

rest $(v_{B1x} = 0)$. From Eq. (8.16) the common x-component of velocity v_{2x} of both bodies after the collision is

$$v_{2x} = \frac{m_A}{m_A + m_B}v_{A1x} \quad \text{(completely inelastic collision, } B \text{ initially at rest)} \quad (8.17)$$

Let's verify that the total kinetic energy after this completely inelastic collision is less than before the collision. The motion is purely along the x-axis, so the kinetic energies K_1 and K_2 before and after the collision, respectively, are

$$K_1 = \tfrac{1}{2}m_A v_{A1x}^2$$

$$K_2 = \tfrac{1}{2}(m_A + m_B)v_{2x}^2 = \tfrac{1}{2}(m_A + m_B)\left(\frac{m_A}{m_A + m_B}\right)^2 v_{A1x}^2$$

The ratio of final to initial kinetic energy is

$$\frac{K_2}{K_1} = \frac{m_A}{m_A + m_B} \quad \text{(completely inelastic collision, } B \text{ initially at rest)} \quad (8.18)$$

The right side is always less than unity because the denominator is always greater than the numerator. Even when the initial velocity of m_B is not zero, it is not hard to verify that the kinetic energy after a completely inelastic collision is always less than before.

Please note: We don't recommend memorizing Eqs. (8.17) or (8.18). We derived them only to prove that kinetic energy is always lost in a completely inelastic collision.

8.16 Automobile collisions are intended to be inelastic, so that the structure of the car absorbs as much of the energy of the collision as possible. This absorbed energy cannot be recovered, since it goes into a permanent deformation of the car.

Example 8.7 A completely inelastic collision

Suppose we repeat the collision described in Example 8.5 (Section 8.2), but this time equip the gliders so that they stick together instead of bouncing apart after they collide. Their masses and initial velocities are the same as in Example 8.5. Find the common final x-velocity v_{2x}, and compare the initial and final kinetic energies.

SOLUTION

IDENTIFY: There are no external forces in the x-direction, so the x-component of momentum is conserved.

SET UP: Figure 8.17 shows our sketch. As in Example 8.5, we take the positive x-axis to point to the right. Our target variables are the final x-velocity v_{2x} and the initial and final kinetic energies of the system.

EXECUTE: From conservation of the x-component of momentum,

$$m_A v_{A1x} + m_B v_{B1x} = (m_A + m_B)v_{2x}$$

$$v_{2x} = \frac{m_A v_{A1x} + m_B v_{B1x}}{m_A + m_B}$$

$$= \frac{(0.50\text{ kg})(2.0\text{ m/s}) + (0.30\text{ kg})(-2.0\text{ m/s})}{0.50\text{ kg} + 0.30\text{ kg}}$$

$$= 0.50\text{ m/s}$$

Because v_{2x} is positive, the gliders move together to the right (the $+x$-direction) after the collision. Before the collision, the kinetic energies of gliders A and B are

$$K_A = \tfrac{1}{2}m_A v_{A1x}^2 = \tfrac{1}{2}(0.50\text{ kg})(2.0\text{ m/s})^2 = 1.0\text{ J}$$

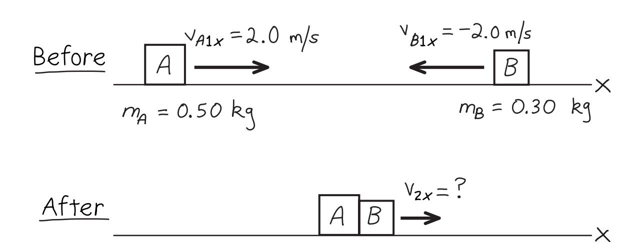
$$K_B = \tfrac{1}{2}m_B v_{B1x}^2 = \tfrac{1}{2}(0.30\text{ kg})(-2.0\text{ m/s})^2 = 0.60\text{ J}$$

(Note that the kinetic energy of glider B is positive, even though the x-components of its velocity v_{B1x} and momentum $m_B v_{B1x}$ are both negative.) The *total* kinetic energy before the collision is 1.6 J. The kinetic energy after the collision is

$$\tfrac{1}{2}(m_A + m_B)v_{2x}^2 = \tfrac{1}{2}(0.50\text{ kg} + 0.30\text{ kg})(0.50\text{ m/s})^2 = 0.10\text{ J}$$

EVALUATE: The final kinetic energy is only $\frac{1}{16}$ of the original; $\frac{15}{16}$ is converted from mechanical energy to various other forms. If there is a ball of chewing gum between the gliders, it squashes and becomes warmer. If there is a spring between the gliders that is compressed as they lock together, then the energy is stored as potential energy of the spring. In both of these cases the *total* energy of the system is conserved, although *kinetic* energy is not. However, in an isolated system, momentum is *always* conserved, whether the collision is elastic or not.

8.17 Our sketch for this problem.



Example 8.8 The ballistic pendulum

Figure 8.18 shows a ballistic pendulum, a system for measuring the speed of a bullet. The bullet, with mass m_B, is fired into a block of wood with mass m_W, suspended like a pendulum, and makes a completely inelastic collision with it. After the impact of the bullet, the block swings up to a maximum height y. Given the values of y, m_B, and m_W, what is the initial speed v_1 of the bullet?

SOLUTION

IDENTIFY: We'll analyze this event in two stages: (1) the embedding of the bullet in the block and (2) the subsequent swinging of the block on its strings.

During the first stage, the bullet embeds itself in the block so quickly that the block has no time to move appreciably. The supporting strings remain nearly vertical, so negligible external horizontal force acts on the system of bullet plus block, and the *horizontal component of momentum* is conserved. Mechanical energy is *not* conserved in this stage because a nonconservative force does work (the force of friction between bullet and block).

In the second stage, after the collision, the block and bullet move as a unit. The only forces acting on this unit are gravity (a conservative force) and the string tensions (which do no work). Thus, as the block swings upward and to the right, *mechanical energy* is conserved. Momentum is *not* conserved during this stage because there is a net external force (the forces of gravity and string tension don't cancel when the strings are inclined).

SET UP: We take the positive x-axis to be to the right and the positive y-axis to be upward in Fig. 8.18. Our target variable is v_1. Another unknown quantity is the speed v_2 of the block and bullet as a unit just after the collision (that is, just at the end of the first stage). We'll use momentum conservation in the first stage to relate v_1 to v_2, and we'll use energy conservation in the second stage to relate v_2 to the (given) maximum height y.

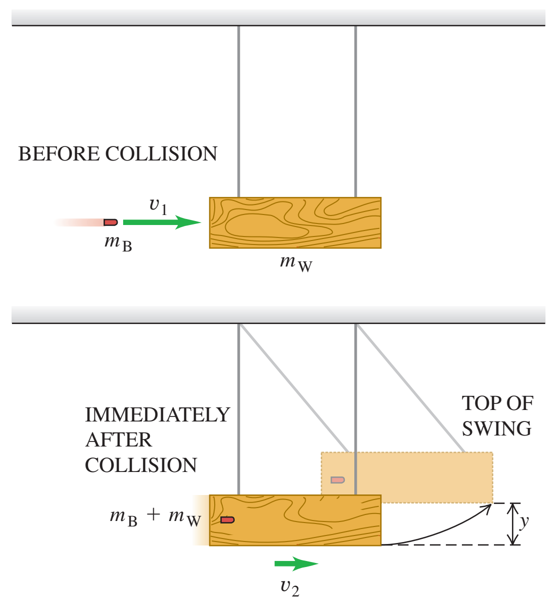
EXECUTE: In the first stage, the velocities are all in the positive x-direction. Momentum conservation gives

$$m_B v_1 = (m_B + m_W)v_2 \qquad v_1 = \frac{m_B + m_W}{m_B} v_2$$

At the beginning of the second stage, the block–bullet unit has kinetic energy $K = \frac{1}{2}(m_B + m_W)v_2^2$. [As in Eq. (8.18), this is less than the kinetic energy before the collision; the collision is inelastic!] The block–bullet unit swings up and comes to rest for an instant at a height y, where its kinetic energy is zero and the potential energy is $(m_B + m_W)gy$; it then swings back down. Energy conservation gives

$$\tfrac{1}{2}(m_B + m_W)v_2^2 = (m_B + m_W)gy \qquad v_2 = \sqrt{2gy}$$

8.18 A ballistic pendulum.



Now we substitute this expression into the momentum equation to find an expression for our target variable v_1:

$$v_1 = \frac{m_B + m_W}{m_B}\sqrt{2gy}$$

Hence measuring m_B, m_W, and y tells us the initial speed of the bullet.

EVALUATE: Let's check our answers by plugging in some realistic numbers. If $m_B = 5.00\ \text{g} = 0.00500\ \text{kg}$, $m_W = 2.00\ \text{kg}$, and $y = 3.00\ \text{cm} = 0.0300\ \text{m}$, the initial speed of the bullet is

$$v_1 = \frac{0.00500\ \text{kg} + 2.00\ \text{kg}}{0.00500\ \text{kg}}\sqrt{2(9.80\ \text{m/s}^2)(0.0300\ \text{m})}$$
$$= 307\ \text{m/s}$$

The speed v_2 of the block just after impact is

$$v_2 = \sqrt{2gy} = \sqrt{2(9.80\ \text{m/s}^2)(0.0300\ \text{m})}$$
$$= 0.767\ \text{m/s}$$

The kinetic energy of the bullet just before impact is $\frac{1}{2}(0.00500\ \text{kg})(307\ \text{m/s})^2 = 236\ \text{J}$. Just after impact the kinetic energy of the bullet and block is $\frac{1}{2}(2.005\ \text{kg})(0.767\ \text{m/s})^2 = 0.589\ \text{J}$. Nearly all the kinetic energy disappears as the wood splinters and the bullet and block become hotter.

Example 8.9 An automobile collision

A 1000-kg compact car is traveling north at 15 m/s when it collides with a 2000-kg truck traveling east at 10 m/s. All occupants are wearing seat belts and there are no injuries, but the two vehicles are thoroughly tangled and move away from the impact point as one mass. The insurance adjustor has asked you to find the velocity of the wreckage just after impact. What do you tell her?

SOLUTION

IDENTIFY: We'll assume that we can treat the cars as an isolated system during the collision. We can do so because the horizontal forces that the cars exert on each other during the collision have very large magnitudes, great enough to crumple the cars' metal

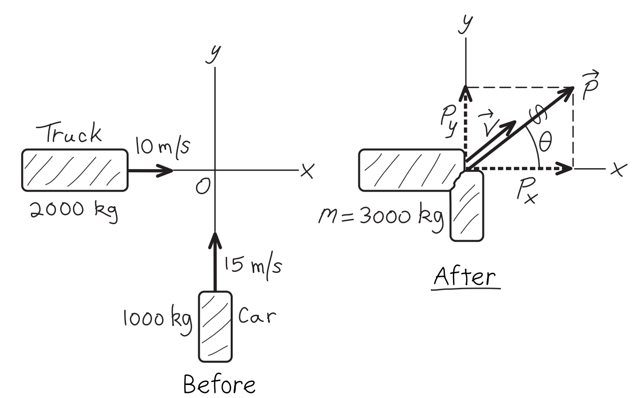
skins. Compared with these forces, we can neglect any external forces such as friction. (We'll justify this assumption later.) Hence the momentum of the system of two cars has the same value just before and just after the collision.

SET UP: Figure 8.19 shows our sketch. We can find the total momentum before the collision, $\vec{P}$, using Eqs. (8.15) and the coordinate axes shown in Fig. 8.19. The momentum has the same value just after the collision; hence, once we've found $\vec{P}$, we'll be able to find the velocity $\vec{V}$ just after the collision (our second target variable) using the relationship $\vec{P} = M\vec{V}$, where M is the combined mass of the wreckage. We'll use the subscripts C and T for the car and truck, respectively.

EXECUTE: From Eqs. (8.15) the components of the total momentum $\vec{P}$ are

$$P_x = p_{Cx} + p_{Tx} = m_C v_{Cx} + m_T v_{Tx}$$
$$= (1000\ \text{kg})(0) + (2000\ \text{kg})(10\ \text{m/s})$$
$$= 2.0 \times 10^4\ \text{kg}\cdot\text{m/s}$$
$$P_y = p_{Cy} + p_{Ty} = m_C v_{Cy} + m_T v_{Ty}$$
$$= (1000\ \text{kg})(15\ \text{m/s}) + (2000\ \text{kg})(0)$$
$$= 1.5 \times 10^4\ \text{kg}\cdot\text{m/s}$$

8.19 Our sketch for this problem.



The magnitude of $\vec{P}$ is

$$P = \sqrt{(2.0 \times 10^4\ \text{kg}\cdot\text{m/s})^2 + (1.5 \times 10^4\ \text{kg}\cdot\text{m/s})^2}$$
$$= 2.5 \times 10^4\ \text{kg}\cdot\text{m/s}$$

and its direction is given by the angle θ shown in Fig. 8.19, where

$$\tan\theta = \frac{P_y}{P_x} = \frac{1.5 \times 10^4\ \text{kg}\cdot\text{m/s}}{2.0 \times 10^4\ \text{kg}\cdot\text{m/s}} = 0.75 \qquad \theta = 37°$$

The total momentum just after the collision is the same as just before. Assuming that no parts fall off, the total mass of wreckage is $M = m_C + m_T = 3000\ \text{kg}$. From $\vec{P} = M\vec{V}$, the direction of the velocity $\vec{V}$ just after the collision is the same as that of the momentum, and its magnitude is

$$V = \frac{P}{M} = \frac{2.5 \times 10^4\ \text{kg}\cdot\text{m/s}}{3000\ \text{kg}} = 8.3\ \text{m/s}$$

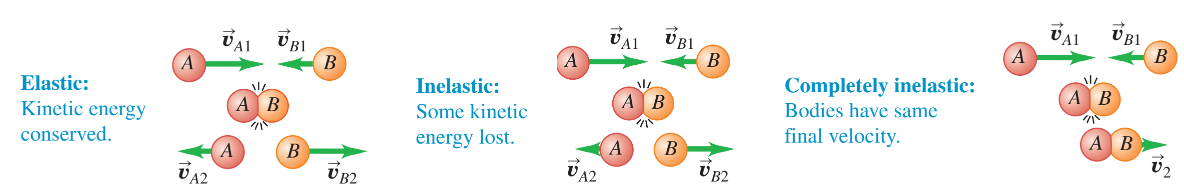
EVALUATE: This is an inelastic collision, so we expect the total kinetic energy to be less after the collision than before. Carry out the calculations yourself; you will find that the initial kinetic energy is 2.1×10^5 J and the final value is 1.0×10^5 J. More than half of the initial kinetic energy is converted to other forms.

We still need to justify our assumption that we can neglect the external forces on the vehicles during the collision. To do so, note that the mass of the truck is 2000 kg, its weight is about 20,000 N, and, if the coefficient of friction is about 0.5, the friction force when it slides across the pavement is about 10,000 N. The truck's kinetic energy just before the impact is $\frac{1}{2}(2000\ \text{kg})(10\ \text{m/s})^2 = 1.0 \times 10^5$ J. The car may crumple 0.2 m or so; to do -1.0×10^5 J of work on the car (required to stop it) in a distance of 0.2 m would require a force of 5.0×10^5 N, which is 50 times greater than the friction force. So it's reasonable to treat the external force of friction as negligible compared with the internal forces that the vehicles exert on each other.

Classifying Collisions

It's important to remember that we can classify collisions according to energy considerations (Fig. 8.20). A collision in which kinetic energy is conserved is called *elastic*. (We'll explore these in more depth in the next section.) A collision in which the total kinetic energy decreases is called *inelastic*. When the two bodies have a common final velocity, we say that the collision is *completely inelastic*. There are also cases in which the final kinetic energy is *greater* than the initial value. Rifle recoil, discussed in Example 8.4 (Section 8.2), is an example.

8.20 Collisions are classified according to energy considerations.



Finally, we emphasize again that we can sometimes use momentum conservation even when there are external forces acting on the system, if the net external force acting on the colliding bodies is small in comparison with the internal forces during the collision (as in Example 8.9)

Test Your Understanding of Section 8.3 For each situation, state whether the collision is elastic or inelastic. If it is inelastic, state whether it is completely inelastic. (a) You drop a ball from your hand. It collides with the floor and bounces back up so that it just reaches your hand. (b) You drop a different ball from your hand and let it collide with the ground. This ball bounces back up to half the height from which it was dropped. (c) You drop a ball of clay from your hand. When it collides with the ground, it stops.

8.4 Elastic Collisions

We saw in Section 8.3 that an *elastic collision* in an isolated system is one in which kinetic energy (as well as momentum) is conserved. Elastic collisions occur when the forces between the colliding bodies are *conservative.* When two billiard balls collide, they squash a little near the surface of contact, but then they spring back. Some of the kinetic energy is stored temporarily as elastic potential energy, but at the end it is reconverted to kinetic energy (Fig. 8.21).

ActivPhysics Online
6.2 Collisions and Elasticity
6.7 Car Collisions: Two Dimensions
6.9 Pendulum Bashes Box

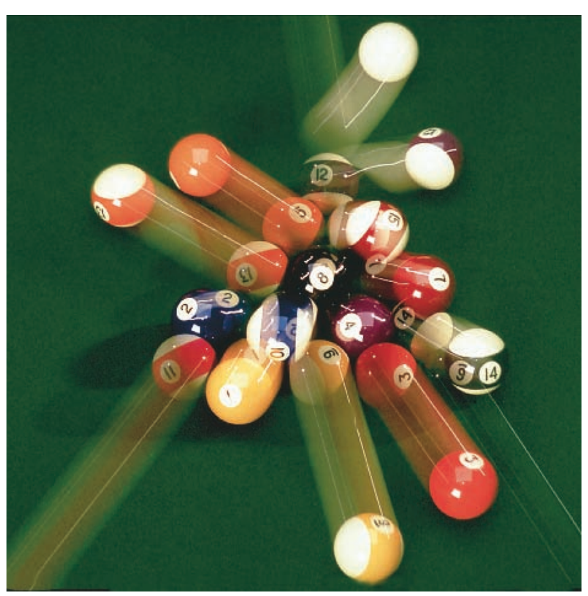
Let's look at an elastic collision between two bodies A and B. We start with a one-dimensional collision, in which all the velocities lie along the same line; we choose this line to be the x-axis. Each momentum and velocity then has only an x-component. We call the x-velocities before the collision v_{A1x} and v_{B1x}, and those after the collision v_{A2x} and v_{B2x}. From conservation of kinetic energy we have

$$\tfrac{1}{2}m_A v_{A1x}{}^2 + \tfrac{1}{2}m_B v_{B1x}{}^2 = \tfrac{1}{2}m_A v_{A2x}{}^2 + \tfrac{1}{2}m_B v_{B2x}{}^2$$

and conservation of momentum gives

$$m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$$

If the masses m_A and m_B and the initial velocities v_{A1x} and v_{B1x} are known, we can solve these two equations to find the two final velocities v_{A2x} and v_{B2x}.



8.21 Billiard balls deform very little when they collide, and they quickly spring back from any deformation they do undergo. Hence the force of interaction between the balls is almost perfectly conservative, and the collision is almost perfectly elastic.

Elastic Collisions, One Body Initially at Rest

The general solution to the above equations is a little complicated, so we will concentrate on the particular case in which body B is at rest before the collision (so $v_{B1x} = 0$). Think of body B as a target for body A to hit. Then the kinetic energy and momentum conservation equations are, respectively,

$$\tfrac{1}{2}m_A v_{A1x}{}^2 = \tfrac{1}{2}m_A v_{A2x}{}^2 + \tfrac{1}{2}m_B v_{B2x}{}^2 \tag{8.19}$$

$$m_A v_{A1x} = m_A v_{A2x} + m_B v_{B2x} \tag{8.20}$$

We can solve for v_{A2x} and v_{B2x} in terms of the masses and the initial velocity v_{A1x}. This involves some fairly strenuous algebra, but it's worth it. No pain, no gain! The simplest approach is somewhat indirect, but along the way it uncovers an additional interesting feature of elastic collisions.

First we rearrange Eqs. (8.19) and (8.20) as follows:

$$m_B v_{B2x}{}^2 = m_A(v_{A1x}{}^2 - v_{A2x}{}^2) = m_A(v_{A1x} - v_{A2x})(v_{A1x} + v_{A2x}) \tag{8.21}$$

$$m_B v_{B2x} = m_A(v_{A1x} - v_{A2x}) \tag{8.22}$$

Now we divide Eq. (8.21) by Eq. (8.22) to obtain

$$v_{B2x} = v_{A1x} + v_{A2x} \tag{8.23}$$

We substitute this expression back into Eq. (8.22) to eliminate v_{B2x} and then solve for v_{A2x}:

$$m_B(v_{A1x} + v_{A2x}) = m_A(v_{A1x} - v_{A2x})$$

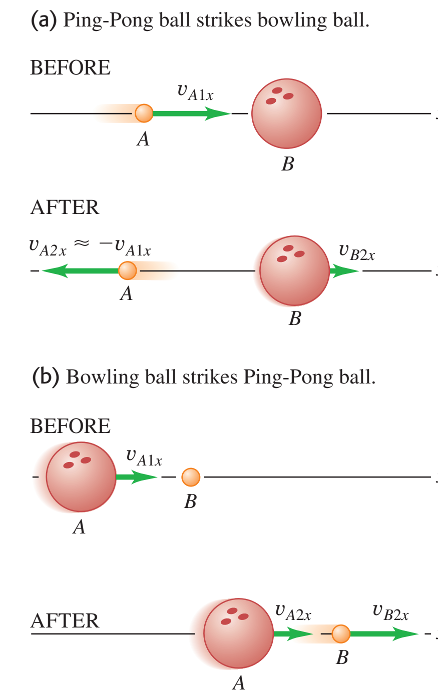
$$v_{A2x} = \frac{m_A - m_B}{m_A + m_B}v_{A1x} \tag{8.24}$$

Finally, we substitute this result back into Eq. (8.23) to obtain

$$v_{B2x} = \frac{2m_A}{m_A + m_B}v_{A1x} \tag{8.25}$$

Now we can interpret the results. Suppose body A is a Ping-Pong ball and body B is a bowling ball. Then we expect A to bounce off after the collision with a velocity nearly equal to its original value but in the opposite direction (Fig. 8.22a), and we expect B's velocity to be much less. That's just what the equations predict. When m_A is much smaller than m_B, the fraction in Eq. (8.24) is approximately equal to (-1), so v_{A2x} is approximately equal to $-v_{A1x}$. The fraction in Eq. (8.25) is much smaller than unity, so v_{B2x} is much less than v_{A1x}. Figure 8.22b shows the opposite case, in which A is the bowling ball and B the Ping-Pong ball and m_A is much larger than m_B. What do you expect to happen then? Check your predictions against Eqs. (8.24) and (8.25).

8.22 Collisions between (a) a moving Ping-Pong ball and an initially stationary bowling ball, and (b) a moving bowling ball and an initially stationary Ping-Pong ball.



Another interesting case occurs when the masses are equal (Fig. 8.23). If $m_A = m_B$, then Eqs. (8.24) and (8.25) give $v_{A2x} = 0$ and $v_{B2x} = v_{A1x}$. That is, the body that was moving stops dead; it gives all its momentum and kinetic energy to the body that was at rest. This behavior is familiar to all pool players.

Elastic Collisions and Relative Velocity

Let's return to the more general case in which A and B have different masses. Equation (8.23) can be rewritten as

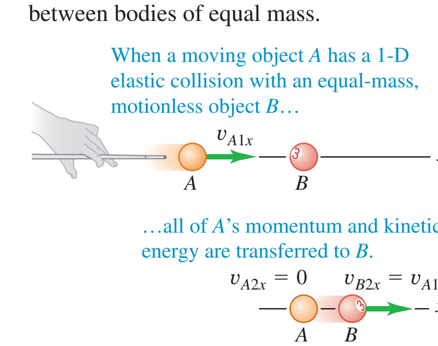
$$v_{A1x} = v_{B2x} - v_{A2x} \tag{8.26}$$

Here $v_{B2x} - v_{A2x}$ is the velocity of B relative to A *after* the collision; from Eq. (8.26), this equals v_{A1x}, which is the *negative* of the velocity of B relative to A *before* the collision. (We discussed relative velocity in Section 3.5.) The relative velocity has the same magnitude, but opposite sign, before and after the collision. The sign changes because A and B are approaching each other before the collision but moving apart after the collision. If we view this collision from a second coordinate system moving with constant velocity relative to the first, the velocities of the bodies are different but the *relative* velocities are the same. Hence our statement about relative velocities holds for *any* straight-line elastic collision, even when neither body is at rest initially. *In a straight-line elastic collision of two bodies, the relative velocities before and after the collision have the same magnitude but opposite sign.* This means that if B is moving before the collision, Eq. (8.26) becomes

$$v_{B2x} - v_{A2x} = -(v_{B1x} - v_{A1x}) \tag{8.27}$$

It turns out that a *vector* relationship similar to Eq. (8.27) is a general property of *all* elastic collisions, even when both bodies are moving initially and the velocities do not all lie along the same line. This result provides an alternative and equivalent definition of an elastic collision: *In an elastic collision, the relative velocity of the two bodies has the same magnitude before and after the collision.* Whenever this condition is satisfied, the total kinetic energy is also conserved.

8.23 A one-dimensional elastic collision between bodies of equal mass.



When an elastic two-body collision isn't head-on, the velocities don't all lie along a single line. If they all lie in a plane, then each final velocity has two unknown components, and there are four unknowns in all. Conservation of energy and conservation of the x- and y-components of momentum give only three equations. To determine the final velocities uniquely, we need additional information, such as the direction or magnitude of one of the final velocities.

Example 8.10 An elastic straight-line collision

We repeat the air-track experiment from Example 8.5 (Section 8.2), but now we add ideal spring bumpers to the gliders so that the collision is elastic. What are the velocities of A and B after the collision?

SOLUTION

IDENTIFY: As in Example 8.5, the net external force on the system of two gliders is zero, and the momentum of the system is conserved.

SET UP: Figure 8.24 shows our sketch. We again choose the positive x-axis to point to the right. We'll find our target variables, v_{A2x} and v_{B2x}, using Eq. (8.27) and the equation of momentum conservation.

EXECUTE: From conservation of momentum,

$$m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$$
$$(0.50\text{ kg})(2.0\text{ m/s}) + (0.30\text{ kg})(-2.0\text{ m/s}) = (0.50\text{ kg})v_{A2x} + (0.30\text{ kg})v_{B2x}$$
$$0.50v_{A2x} + 0.30v_{B2x} = 0.40\text{ m/s}$$

(In the last equation we divided through by the unit "kg.") From Eq. (8.27), the relative velocity relationship for an elastic collision, we have

$$v_{B2x} - v_{A2x} = -(v_{B1x} - v_{A1x}) = -(-2.0\text{ m/s} - 2.0\text{ m/s}) = 4.0\text{ m/s}$$

Before the collision, the velocity of B relative to A is to the left at 4.0 m/s; after the collision, the velocity of B relative to A is to the right at 4.0 m/s. Solving these equations simultaneously, we find

$$v_{A2x} = -1.0\text{ m/s} \qquad v_{B2x} = 3.0\text{ m/s}$$

EVALUATE: Both bodies reverse their directions of motion; A moves to the left at 1.0 m/s and B moves to the right at 3.0 m/s. This is different from the result of Example 8.5 because that collision was *not* elastic.

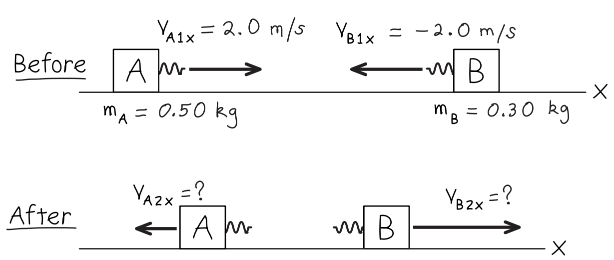
Note that unlike the situations shown in Fig. 8.22, the two gliders are *both* moving toward each other before the collision. Our results show that A (the more massive glider) moves slower after the collision than before the collision, and so loses kinetic energy. In contrast, B (the less massive glider) gains kinetic energy: It moves faster after the collision than before. The *total* kinetic energy after the elastic collision is

$$\tfrac{1}{2}(0.50\text{ kg})(-1.0\text{ m/s})^2 + \tfrac{1}{2}(0.30\text{ kg})(3.0\text{ m/s})^2 = 1.6\text{ J}$$

As expected, this equals the total kinetic energy *before* the collision (which we calculated in Example 8.7 in Section 8.3). Thus kinetic energy is transferred from A to B in the collision, with none of it lost in the process. Much the same happens when a baseball player swings a bat and hits an oncoming baseball. The collision is nearly elastic, and the more massive bat transfers kinetic energy to the less massive baseball. The baseball leaves the bat with a much greater speed—perhaps enough to make a home run.

CAUTION Be careful with the elastic collision equations You might have been tempted to solve this problem using Eqs. (8.24) and (8.25). These equations apply *only* to situations in which body B is initially at rest, which isn't the case here. When in doubt, always solve the problem at hand using equations that are applicable to a broad variety of cases.

8.24 Our sketch for this problem.



Example 8.11 Moderator in a nuclear reactor

The fission of uranium nuclei in a nuclear reactor produces high-speed neutrons. Before a neutron can trigger additional fissions, it has to be slowed down by collisions with nuclei in the *moderator* of the reactor. The first nuclear reactor (built in 1942 at the University of Chicago) and the reactor involved in the 1986 Chernobyl accident both used carbon (graphite) as the moderator material. Suppose a neutron (mass 1.0 u) traveling at 2.6×10^7 m/s undergoes a head-on elastic collision with a carbon nucleus (mass 12 u) initially at rest. The external forces during the collision are negligible. What are the velocities after the collision? (1 u is the *atomic mass unit,* equal to 1.66×10^{-27} kg.)

SOLUTION

IDENTIFY: We are given that the external forces can be neglected (so momentum is conserved in the collision) and that the collision is elastic (so kinetic energy is also conserved).

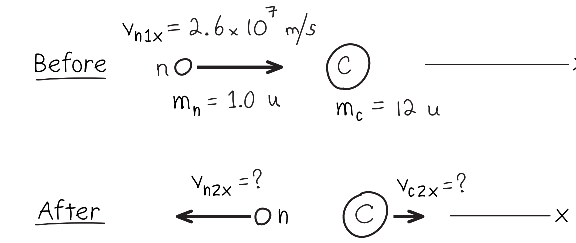
SET UP: Figure 8.25 shows our sketch. We take the x-axis to be in the direction in which the neutron is moving initially. Because the collision is head-on, both the neutron and the carbon nucleus move along this same axis after the collision. Furthermore, because one body is initially at rest, we can use Eqs. (8.24) and (8.25) with A replaced by n (for the neutron) and B replaced by C (for the carbon nucleus). We have $m_n = 1.0$ u, $m_C = 12$ u, and $v_{n1x} = 2.6 \times 10^7$ m/s, and we need to solve for the target variables v_{n2x} and v_{C2x} (the final velocities of the neutron and the carbon nucleus, respectively).

EXECUTE: We'll let you do the arithmetic; the results are

$$v_{n2x} = -2.2 \times 10^7\text{ m/s} \qquad v_{C2x} = 0.4 \times 10^7\text{ m/s}$$

EVALUATE: The neutron ends up with $\frac{11}{13}$ of its initial speed, and the speed of the recoiling carbon nucleus is $\frac{2}{13}$ of the neutron's initial speed. [These ratios are the factors $(m_n - m_C)/(m_n + m_C)$ and $2m_n/(m_n + m_C)$ that appear in Eqs. (8.24) and (8.25), with the subscripts revised for this problem.] Kinetic energy is proportional to speed squared, so the neutron's final kinetic energy is $(\frac{11}{13})^2$, or about 0.72 of its original value. If the neutron makes a second such collision, its kinetic energy is $(0.72)^2$, or about half its original value, and so on. After several collisions, the neutron will be moving quite slowly and will be able to trigger a fission reaction in a uranium nucleus.

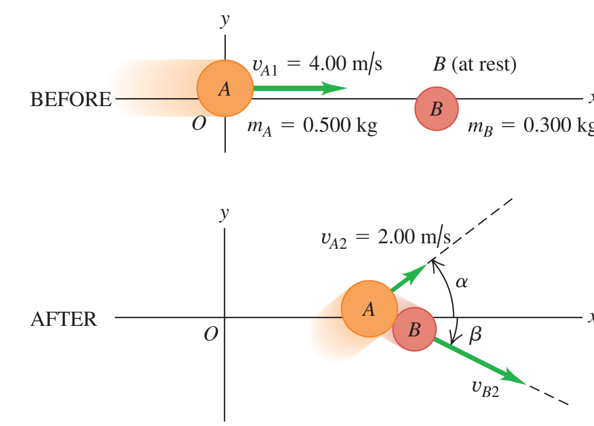
8.25 Our sketch for this problem.



Example 8.12 A two-dimensional elastic collision

Figure 8.26 shows an elastic collision of two pucks on a frictionless air-hockey table. Puck A has mass $m_A = 0.500$ kg and puck B has mass $m_B = 0.300$ kg. Puck A has an initial velocity of 4.00 m/s in the positive x-direction and a final velocity of 2.00 m/s in an unknown direction. Puck B is initially at rest. Find the final speed v_{B2} of puck B and the angles α and β in the figure.

8.26 An elastic collision that isn't head-on.



SOLUTION

IDENTIFY: Although the collision is elastic, it is *not* one-dimensional, so we can't use any of the one-dimensional formulas derived in this section. Instead, we'll use the equations for conservation of energy, conservation of x-momentum, and conservation of y-momentum.

SET UP: The target variables are given in the statement of the problem. We have three equations, which should be enough to solve for our three target variables.

EXECUTE: Because the collision is elastic, the initial and final kinetic energies are equal:

$$\tfrac{1}{2}m_A v_{A1}^2 = \tfrac{1}{2}m_A v_{A2}^2 + \tfrac{1}{2}m_B v_{B2}^2$$
$$v_{B2}^2 = \frac{m_A v_{A1}^2 - m_A v_{A2}^2}{m_B} = \frac{(0.500\text{ kg})(4.00\text{ m/s})^2 - (0.500\text{ kg})(2.00\text{ m/s})^2}{0.300\text{ kg}}$$
$$v_{B2} = 4.47\text{ m/s}$$

Conservation of the x-component of total momentum gives

$$m_A v_{A1x} = m_A v_{A2x} + m_B v_{B2x}$$
$$(0.500\text{ kg})(4.00\text{ m/s}) = (0.500\text{ kg})(2.00\text{ m/s})(\cos\alpha) + (0.300\text{ kg})(4.47\text{ m/s})(\cos\beta)$$

and conservation of the y-component gives

$$0 = m_A v_{A2y} + m_B v_{B2y}$$
$$0 = (0.500\text{ kg})(2.00\text{ m/s})(\sin\alpha) - (0.300\text{ kg})(4.47\text{ m/s})(\sin\beta)$$

These are two simultaneous equations for α and β. The simplest solution is to eliminate β as follows: We solve the first equation for $\cos\beta$ and the second for $\sin\beta$; we then square each equation and add. Since $\sin^2\beta + \cos^2\beta = 1$, this eliminates β and leaves an equation that we can solve for $\cos\alpha$ and hence for α. We can then substitute this value back into either of the two equations and solve the result for β. We leave the details for you to work out in Exercise 8.44; the results are

$$\alpha = 36.9° \qquad \beta = 26.6°$$

EVALUATE: A quick way to check the answers is to make sure that the y-momentum, which was zero before the collision, is still zero after the collision. The y-momenta of the pucks are

$$p_{A2y} = (0.500\text{ kg})(2.00\text{ m/s})(\sin 36.9°) = +0.600\text{ kg}\cdot\text{m/s}$$
$$p_{B2y} = -(0.300\text{ kg})(4.47\text{ m/s})(\sin 26.6°) = -0.600\text{ kg}\cdot\text{m/s}$$

The sum of these values is zero, as it should be.

Test Your Understanding of Section 8.4 Most present-day nuclear reactors use water as a moderator (see Example 8.11). Are water molecules (mass $m_w = 18.0$ u) a better or worse moderator than carbon atoms? (One advantage of water is that it also acts as a coolant for the reactor's radioactive core.)

8.5 Center of Mass

We can restate the principle of conservation of momentum in a useful way by using the concept of **center of mass.** Suppose we have several particles with masses m_1, m_2, and so on. Let the coordinates of m_1 be (x_1, y_1), those of m_2 be (x_2, y_2), and so on. We define the center of mass of the system as the point that has coordinates (x_{cm}, y_{cm}) given by

$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} = \frac{\sum_i m_ix_i}{\sum_i m_i}$$

$$y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} = \frac{\sum_i m_ix_i}{\sum_i m_i} \quad \text{(center of mass)} \quad (8.28)$$

The position vector $\vec{r}_{cm}$ of the center of mass can be expressed in terms of the position vectors $\vec{r}_1, \vec{r}_2, \ldots$ of the particles as

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} = \frac{\sum_i m_i\vec{r}_i}{\sum_i m_i} \quad \text{(center of mass)} \quad (8.29)$$

In statistical language, the center of mass is a *mass-weighted average* position of the particles.

Example 8.13 Center of mass of a water molecule

Figure 8.27 shows a simple model of the structure of a water molecule. The separation between atoms is $d = 9.57 \times 10^{-11}$ m. Each hydrogen atom has mass 1.0 u, and the oxygen atom has mass 16.0 u. Find the position of the center of mass.

SOLUTION

IDENTIFY: Nearly all the mass of each atom is concentrated in its nucleus, which is only about 10^{-5} times the overall radius of the atom. Hence we can safely represent each atom as a point particle.

SET UP: The coordinate system is shown in Fig. 8.27. We'll use Eqs. (8.28) to determine the coordinates x_{cm} and y_{cm}.

EXECUTE: The x-coordinate of each hydrogen atom is $d\cos(105°/2)$; the y-coordinates of the upper and lower hydrogen atoms are $+d\sin(105°/2)$ and $-d\sin(105°/2)$, respectively. The coordinates of the oxygen atom are $x = 0$, $y = 0$. From Eqs. (8.28) the x-coordinate of the center of mass is

$$x_{cm} = \frac{\left[\begin{array}{l}(1.0\text{ u})(d\cos 52.5°) + (1.0\text{ u})\\ \times (d\cos 52.5°) + (16.0\text{ u})(0)\end{array}\right]}{1.0\text{ u} + 1.0\text{ u} + 16.0\text{ u}} = 0.068d$$

and the y-coordinate is

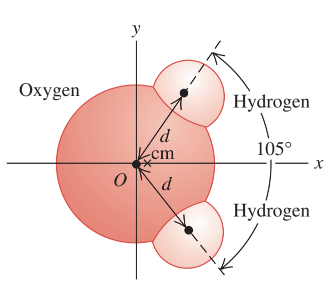
$$y_{cm} = \frac{\left[\begin{array}{l}(1.0\text{ u})(d\sin 52.5°) + (1.0\text{ u})\\ \times (-d\sin 52.5°) + (16.0\text{ u})(0)\end{array}\right]}{1.0\text{ u} + 1.0\text{ u} + 16.0\text{ u}} = 0$$

Substituting the value $d = 9.57 \times 10^{-11}$ m, we find

$$x_{cm} = (0.068)(9.57 \times 10^{-11}\text{ m}) = 6.5 \times 10^{-12}\text{ m}$$

EVALUATE: The center of mass is much closer to the oxygen atom than to either hydrogen atom because the oxygen atom is much more massive. Notice that the center of mass lies along the x-axis, which is the *axis of symmetry* of this molecule. If the molecule is rotated by 180° around this axis, it looks exactly the same as before. The position of the center of mass can't be affected by this rotation, so it must lie on the axis of symmetry.

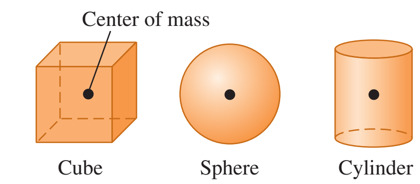
8.27 Where is the center of mass of a water molecule?



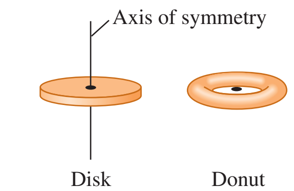
For solid bodies, in which we have (at least on a macroscopic level) a continuous distribution of matter, the sums in Eqs. (8.28) have to be replaced by integrals. The calculations can get quite involved, but we can say three general things about such problems (Fig. 8.28). First, whenever a homogeneous body has a geometric center, such as a billiard ball, a sugar cube, or a can of frozen orange juice, the center of mass is at the geometric center. Second, whenever a body has an axis of symmetry, such as a wheel or a pulley, the center of mass always lies on that axis. Third, there is no law that says the center of mass has to be within the body. For example, the center of mass of a donut is right in the middle of the hole.

We'll talk a little more about locating the center of mass in Chapter 11 in connection with the related concept of *center of gravity.*

8.28 Locating the center of mass of a symmetrical object.



If a homogeneous object has a geometric center, that is where the center of mass is located.



If an object has an axis of symmetry, the center of mass lies along it. As in the case of the donut, the center of mass may not be within the object.

Motion of the Center of Mass

To see the significance of the center of mass of a collection of particles, we must ask what happens to the center of mass when the particles move. The x- and y-components of velocity of the center of mass, $v_{cm\text{-}x}$ and $v_{cm\text{-}y}$, are the time derivatives of x_{cm} and y_{cm}. Also, dx_1/dt is the x-component of velocity of particle 1, and so on, so $dx_1/dt = v_{1x}$, and so on. Taking time derivatives of Eqs. (8.28), we get

$$v_{cm\text{-}x} = \frac{m_1v_{1x} + m_2v_{2x} + m_3v_{3x} + \cdots}{m_1 + m_2 + m_3 + \cdots}$$

$$v_{cm\text{-}y} = \frac{m_1v_{1y} + m_2v_{2y} + m_3v_{3y} + \cdots}{m_1 + m_2 + m_3 + \cdots} \quad (8.30)$$

These equations are equivalent to the single vector equation obtained by taking the time derivative of Eq. (8.29):

$$\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} \quad (8.31)$$

We denote the *total* mass $m_1 + m_2 + \cdots$ by M. We can then rewrite Eq. (8.31) as

$$M\vec{v}_{cm} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots = \vec{P} \quad (8.32)$$

The right side is simply the total momentum $\vec{P}$ of the system. Thus we have proved that *the total momentum is equal to the total mass times the velocity of the center of mass.* When you catch a baseball, you are really catching a collection of a very large number of molecules of masses $m_1, m_2, m_3, \ldots$. The impulse you feel is due to the total momentum of this entire collection. But this impulse is the same as if you were catching a single particle of mass $M = m_1 + m_2 + m_3 + \cdots$ moving with velocity $\vec{v}_{cm}$, the velocity of the collection's center of mass. So Eq. (8.32) helps to justify representing an extended body as a particle.

For a system of particles on which the net external force is zero, so that the total momentum $\vec{P}$ is constant, the velocity of the center of mass $\vec{v}_{cm} = \vec{P}/M$ is also constant. Suppose we mark the center of mass of a wrench and then slide the wrench with a spinning motion across a smooth, horizontal tabletop (Fig. 8.29). The overall motion appears complicated, but the center of mass follows a straight line, as though all the mass were concentrated at that point.

8.29 The center of mass of this wrench is marked with a white dot. The net external force acting on the wrench is almost zero. As the wrench spins on a smooth horizontal surface, the center of mass moves in a straight line with nearly constant velocity.

Example 8.14 A tug-of-war on the ice

James and Ramon are standing 20.0 m apart on the slippery surface of a frozen pond. Ramon has mass 60.0 kg and James has mass 90.0 kg. Midway between the two men a mug of their favorite beverage sits on the ice. They pull on the ends of a light rope that is stretched between them. When James has moved 6.0 m toward the mug, how far and in what direction has Ramon moved?

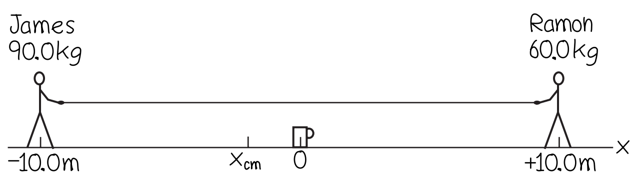
Continued

SOLUTION

IDENTIFY: The frozen surface is horizontal and essentially frictionless, so the net external force on the system of James, Ramon, and the rope is zero. Hence their total momentum is conserved. Initially there is no motion, so the total momentum is zero; thus the velocity of the center of mass is zero, and the center of mass remains at rest. We can use this to relate the positions of James and Ramon.

SET UP: Let's take the origin at the position of the mug, and let the $+x$-axis extend from the mug toward Ramon. Figure 8.30 shows our sketch. Since the rope is light, we can neglect its mass in calculating the position of the center of mass with Eq. (8.28).

8.30 Our sketch for this problem.



EXECUTE: The initial x-coordinates of James and Ramon are -10.0 m and $+10.0$ m, respectively, so the x-coordinate of the center of mass is

$$x_{cm} = \frac{(90.0\text{ kg})(-10.0\text{ m}) + (60.0\text{ kg})(10.0\text{ m})}{90.0\text{ kg} + 60.0\text{ kg}} = -2.0\text{ m}$$

When James moves 6.0 m toward the mug, his new x-coordinate is -4.0 m; we'll call Ramon's new x-coordinate x_2. The center of mass doesn't move, so

$$x_{cm} = \frac{(90.0\text{ kg})(-4.0\text{ m}) + (60.0\text{ kg})x_2}{90.0\text{ kg} + 60.0\text{ kg}} = -2.0\text{ m}$$

$$x_2 = 1.0\text{ m}$$

James has moved 6.0 m in the positive x-direction and is still 4.0 m from the mug, but Ramon has moved 9.0 m in the negative x-direction and is only 1.0 m from it.

EVALUATE: The ratio of how far each man moved, $(6.0\text{ m})/(9.0\text{ m}) = \frac{2}{3}$, equals the inverse ratio of their masses. Can you see why? If the two men keep moving (and if the surface is frictionless, they will!), Ramon will reach the mug first. This result is completely independent of how hard either person pulls; pulling harder just helps Ramon quench his thirst sooner.

External Forces and Center-of-Mass Motion

If the net external force on a system of particles is not zero, then total momentum is not conserved and the velocity of the center of mass changes. Let's look at the relationship between the motion of the center of mass and the forces acting on the system.

Equations (8.31) and (8.32) give the *velocity* of the center of mass in terms of the velocities of the individual particles. We take the time derivatives of these equations to show that the *accelerations* are related in the same way. Let $\vec{a}_{cm} = d\vec{v}_{cm}/dt$ be the acceleration of the center of mass; then we find

$$M\vec{a}_{cm} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + \cdots \qquad (8.33)$$

Now $m_1\vec{a}_1$ is equal to the vector sum of forces on the first particle, and so on, so the right side of Eq. (8.33) is equal to the vector sum $\sum\vec{F}$ of *all* the forces on *all* the particles. Just as we did in Section 8.2, we can classify each force as *external* or *internal.* The sum of all forces on all the particles is then

$$\sum\vec{F} = \sum\vec{F}_{ext} + \sum\vec{F}_{int} = M\vec{a}_{cm}$$

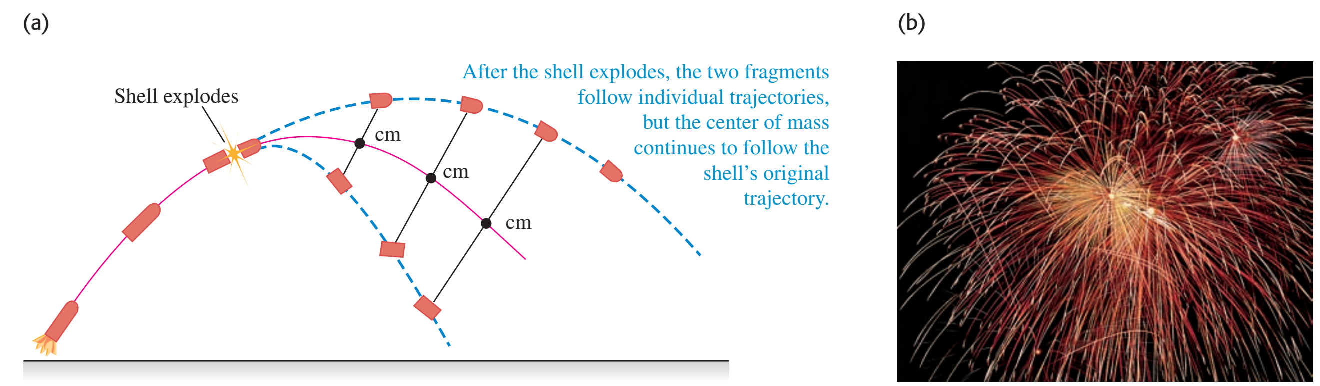
Because of Newton's third law, the internal forces all cancel in pairs, and $\sum\vec{F}_{int} = \mathbf{0}$. What survives on the left side is the sum of only the *external* forces:

$$\sum\vec{F}_{ext} = M\vec{a}_{cm} \qquad \text{(body or collection of particles)} \qquad (8.34)$$

When a body or a collection of particles is acted on by external forces, the center of mass moves just as though all the mass were concentrated at that point and it were acted on by a net force equal to the sum of the external forces on the system.

This result may not sound very impressive, but in fact it is central to the whole subject of mechanics. In fact, we've been using this result all along; without it, we would not be able to represent an extended body as a point particle when we apply Newton's laws. It explains why only *external* forces can affect the motion of an extended body. If you pull upward on your belt, your belt exerts an equal downward force on your hands; these are *internal* forces that cancel and have no effect on the overall motion of your body.

8.31 **(a)** A shell explodes into two fragments in flight. If air resistance is ignored, the center of mass continues on the same trajectory as the shell's path before exploding. **(b)** The same effect occurs with exploding fireworks.



Suppose a cannon shell traveling in a parabolic trajectory (neglecting air resistance) explodes in flight, splitting into two fragments with equal mass (Fig. 8.31a). The fragments follow new parabolic paths, but the center of mass continues on the original parabolic trajectory, just as though all the mass were still concentrated at that point. A skyrocket exploding in air (Fig. 8.31b) is a spectacular example of this effect.

This property of the center of mass is important when we analyze the motion of rigid bodies. We describe the motion of an extended body as a combination of translational motion of the center of mass and rotational motion about an axis through the center of mass. We will return to this topic in Chapter 10. This property also plays an important role in the motion of astronomical objects. It's not correct to say that the moon orbits the earth; rather, the earth and moon both move in orbits around their center of mass.

There's one more useful way to describe the motion of a system of particles. Using $\vec{a}_{cm} = d\vec{v}_{cm}/dt$, we can rewrite Eq. (8.33) as

$$M\vec{a}_{cm} = M\frac{d\vec{v}_{cm}}{dt} = \frac{d(M\vec{v}_{cm})}{dt} = \frac{d\vec{P}}{dt} \qquad (8.35)$$

The total system mass M is constant, so we're allowed to take it inside the derivative. Substituting Eq. (8.35) into Eq. (8.34), we find

$$\sum\vec{F}_{ext} = \frac{d\vec{P}}{dt} \qquad \text{(extended body or system of particles)} \qquad (8.36)$$

This equation looks like Eq. (8.4). The difference is that Eq. (8.36) describes a *system* of particles, such as an extended body, while Eq. (8.4) describes a single particle. The interactions between the particles that make up the system can change the individual momenta of the particles, but the *total* momentum $\vec{P}$ of the system can be changed only by external forces acting from outside the system.

Finally, we note that if the net external force is zero, Eq. (8.34) shows that the acceleration $\vec{a}_{cm}$ of the center of mass is zero. So the center-of-mass velocity $\vec{v}_{cm}$ is constant, as for the wrench in Fig. 8.29. From Eq. (8.36) the total momentum $\vec{P}$ is also constant. This reaffirms our statement in Section 8.3 of the principle of conservation of momentum.

Test Your Understanding of Section 8.5 Will the center of mass in Fig. 8.31a continue on the same parabolic trajectory even after one of the fragments hits the ground? Why or why not?

*8.6 Rocket Propulsion

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6.6 Saving an Astronaut

Momentum considerations are particularly useful for analyzing a system in which the masses of parts of the system change with time. In such cases we can't use Newton's second law $\sum \vec{F} = m\vec{a}$ directly because m changes. Rocket propulsion offers a typical and interesting example of this kind of analysis. A rocket is propelled forward by rearward ejection of burned fuel that initially was in the rocket (which is why rocket fuel is also called *propellant*). The forward force on the rocket is the reaction to the backward force on the ejected material. The total mass of the system is constant, but the mass of the rocket itself decreases as material is ejected.

As a simple example, consider a rocket fired in outer space, where there is no gravitational force and no air resistance. Let m denote the mass of the rocket, which will change as it expends fuel. We choose our x-axis to be along the rocket's direction of motion. Figure 8.32a shows the rocket at a time t, when its mass is m and its x-velocity relative to our coordinate system is v. (For simplicity, we will drop the subscript x in this discussion.) The x-component of total momentum at this instant is $P_1 = mv$. In a short time interval dt, the mass of the rocket changes by an amount dm. This is an inherently negative quantity because the rocket's mass m *decreases* with time. During dt, a *positive* mass $-dm$ of burned fuel is ejected from the rocket. Let v_{ex} be the exhaust *speed* of this material *relative to the rocket;* the burned fuel is ejected opposite the direction of motion, so its x-component of *velocity* relative to the rocket is $-v_{ex}$. The x-velocity v_{fuel} of the burned fuel relative to our coordinate system is then

$$v_{fuel} = v + (-v_{ex}) = v - v_{ex}$$

and the x-component of momentum of the ejected mass $(-dm)$ is

$$(-dm)v_{fuel} = (-dm)(v - v_{ex})$$

Figure 8.32b shows that at the end of the time interval dt, the x-velocity of the rocket and unburned fuel has increased to $v + dv$, and its mass has decreased to $m + dm$ (remember that dm is negative). The rocket's momentum at this time is

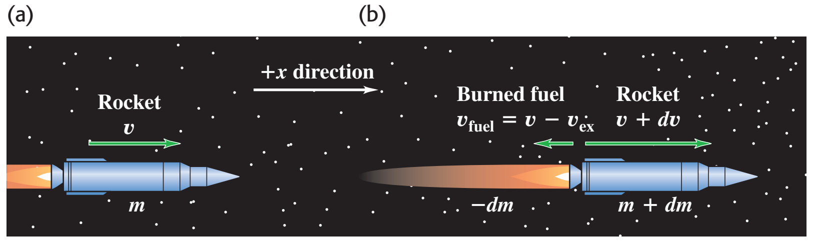
$$(m + dm)(v + dv)$$

Thus the *total* x-component of momentum P_2 of the rocket plus ejected fuel at time $t + dt$ is

$$P_2 = (m + dm)(v + dv) + (-dm)(v - v_{ex})$$

8.32 A rocket moving in gravity-free outer space at (a) time t and (b) time $t + dt$.

(a) (b)



At time t, the rocket has mass m and x-component of velocity v.

At time $t + dt$, the rocket has mass $m + dm$ (where dm is inherently *negative*) and x-component of velocity $v + dv$. The burned fuel has x-component of velocity $v_{fuel} = v - v_{ex}$ and mass $-dm$. (The minus sign is needed to make $-dm$ *positive* because dm is negative.)

According to our initial assumption, the rocket and fuel are an isolated system. Thus momentum is conserved, and the total x-component of momentum of the system must be the same at time t and at time $t + dt$: $P_1 = P_2$. Hence

$$mv = (m + dm)(v + dv) + (-dm)(v - v_{ex})$$

This can be simplified to

$$m\,dv = -dm\,v_{ex} - dm\,dv$$

We can neglect the term $(-dm\,dv)$ because it is a product of two small quantities and thus is much smaller than the other terms. Dropping this term, dividing by dt, and rearranging, we find

$$m\frac{dv}{dt} = -v_{ex}\frac{dm}{dt} \tag{8.37}$$

Now dv/dt is the acceleration of the rocket, so the left side of this equation (mass times acceleration) equals the net force F, or *thrust,* on the rocket,

$$F = -v_{ex}\frac{dm}{dt} \tag{8.38}$$

The thrust is proportional both to the relative speed v_{ex} of the ejected fuel and to the mass of fuel ejected per unit time, $-dm/dt$. (Remember that dm/dt is negative because it is the rate of change of the rocket's mass, so F is positive.)

The x-component of acceleration of the rocket is

$$a = \frac{dv}{dt} = -\frac{v_{ex}}{m}\frac{dm}{dt} \tag{8.39}$$

This is positive because v_{ex} is positive (remember, it's the exhaust *speed*) and dm/dt is negative. The rocket's mass m decreases continuously while the fuel is being consumed. If v_{ex} and dm/dt are constant, the acceleration increases until all the fuel is gone.

Equation (8.38) tells us that an effective rocket burns fuel at a rapid rate (large $-dm/dt$) and ejects the burned fuel at a high relative speed (large v_{ex}), as in Fig. 8.33. In the early days of rocket propulsion, people who didn't understand conservation of momentum thought that a rocket couldn't function in outer space because "it doesn't have anything to push against." On the contrary, rockets work *best* in outer space, where there is no air resistance! The launch vehicle in Fig. 8.33 is *not* "pushing against the ground" to get into the air.

If the exhaust speed v_{ex} is constant, we can integrate Eq. (8.39) to find a relationship between the velocity v at any time and the remaining mass m. At time $t = 0$, let the mass be m_0 and the velocity v_0. Then we rewrite Eq. (8.39) as

$$dv = -v_{ex}\frac{dm}{m}$$

We change the integration variables to v' and m', so we can use v and m as the upper limits (the final speed and mass). Then we integrate both sides, using limits v_0 to v and m_0 to m, and take the constant v_{ex} outside the integral:

$$\int_{v_0}^{v} dv' = -\int_{m_0}^{m} v_{ex}\frac{dm'}{m'} = -v_{ex}\int_{m_0}^{m}\frac{dm'}{m'}$$

$$v - v_0 = -v_{ex}\ln\frac{m}{m_0} = v_{ex}\ln\frac{m_0}{m} \tag{8.40}$$

The ratio m_0/m is the original mass divided by the mass after the fuel has been exhausted. In practical spacecraft this ratio is made as large as possible to maximize the speed gain, which means that the initial mass of the rocket is almost all fuel. The final velocity of the rocket will be greater in magnitude (and is often

8.33 To provide enough thrust to lift its payload into space, this Atlas V launch vehicle exhausts more than 1000 kg of burned fuel per second at speeds of nearly 4000 m/s.

much greater) than the relative speed v_{ex} if $\ln(m_0/m) > 1$—that is, if $m_0/m > e = 2.71828\ldots$

We've assumed throughout this analysis that the rocket is in gravity-free outer space. However, gravity must be taken into account when a rocket is launched from the surface of a planet, as in Fig. 8.33 (see Problem 8.110).

Example 8.15 Acceleration of a rocket

A rocket is in outer space, far from any planet, when the rocket engine is turned on. In the first second of firing, the rocket ejects $\frac{1}{120}$ of its mass with a relative speed of 2400 m/s. What is the rocket's initial acceleration?

SOLUTION

IDENTIFY: We are given the rocket's exhaust speed v_{ex}, but not its mass m or the rate of change of its mass dm/dt. However, we are told what fraction of the initial mass is lost during a given time interval, which should be enough.

SET UP: We'll use Eq. (8.39) to find the acceleration of the rocket.

EXECUTE: The initial rate of change of mass is

$$\frac{dm}{dt} = -\frac{m_0/120}{1\text{ s}} = -\frac{m_0}{120\text{ s}}$$

where m_0 is the initial $(t = 0)$ mass of the rocket. From Eq. (8.39) the initial acceleration is

$$a = -\frac{v_{ex}}{m_0}\frac{dm}{dt} = -\frac{2400\text{ m/s}}{m_0}\left(-\frac{m_0}{120\text{ s}}\right) = 20\text{ m/s}^2$$

EVALUATE: Note that the answer didn't depend on the value of m_0. If v_{ex} is the same, the initial acceleration is the same for a 120,000-kg spacecraft that ejects 1000 kg/s as for a 60-kg astronaut equipped with a small rocket that ejects 0.5 kg/s.

Example 8.16 Speed of a rocket

Suppose that $\frac{3}{4}$ of the initial mass m_0 of the rocket in Example 8.15 is fuel, so the final mass is $m = m_0/4$, and that the fuel is completely consumed at a constant rate in a total time $t = 90$ s. If the rocket starts from rest in our coordinate system, find its speed at the end of this time.

SOLUTION

IDENTIFY: We are given the initial velocity v_0 (equal to zero), the exhaust speed v_{ex}, and the final mass m in terms of the initial mass m_0.

SET UP: We'll use Eq. (8.40) directly to find the final speed v.

EXECUTE: We have $m_0/m = 4$, so from Eq. (8.40),

$$v = v_0 + v_{ex}\ln\frac{m_0}{m} = 0 + (2400\text{ m/s})(\ln 4) = 3327\text{ m/s}$$

EVALUATE: Let's examine what happens as the rocket gains speed. At the start of the flight, when the velocity of the rocket is zero, the ejected fuel is moving to the left, relative to our coordinate system, at 2400 m/s. At the end of the first second $(t = 1\text{ s})$, the rocket is moving at 20 m/s, and the fuel's speed relative to our system is 2380 m/s. During the next second the acceleration, given by Eq. (8.39), is a little greater. At $t = 2$ s, the rocket is moving a little faster than 40 m/s, and the fuel's speed is a little less than 2360 m/s. Detailed calculation shows that at about $t = 75.6$ s, the rocket's velocity v in our coordinate system equals 2400 m/s. The burned fuel ejected after this time moves *forward,* not backward, in our system. Since the final velocity of the rocket is 3327 m/s and the relative velocity is 2400 m/s, the last portion of the ejected fuel has a forward velocity (relative to our frame of reference) of $(3327 - 2400)$ m/s $= 927$ m/s. (To illustrate our point, we are using more figures than are significant.)

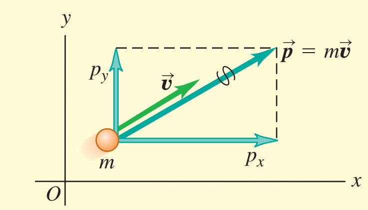
Test Your Understanding of Section 8.6 (a) If a rocket in gravity-free outer space has the same thrust at all times, is its acceleration constant, increasing, or decreasing? (b) If the rocket has the same acceleration at all times, is the thrust constant, increasing, or decreasing?

CHAPTER 8 SUMMARY

Momentum of a particle: The momentum $\vec{p}$ of a particle is a vector quantity equal to the product of the particle's mass m and velocity $\vec{v}$. Newton's second law says that the net force on a particle is equal to the rate of change of the particle's momentum.

$$\vec{p} = m\vec{v} \tag{8.2}$$

$$\Sigma\vec{F} = \frac{d\vec{p}}{dt} \tag{8.4}$$

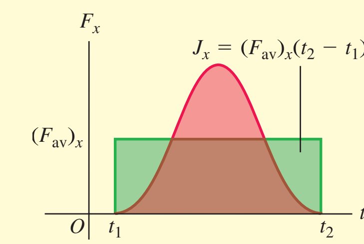


Impulse and momentum: If a constant net force $\Sigma\vec{F}$ acts on a particle for a time interval Δt from t_1 to t_2, the impulse $\vec{J}$ of the net force is the product of the net force and the time interval. If $\Sigma\vec{F}$ varies with time, $\vec{J}$ is the integral of the net force over the time interval. In any case, the change in a particle's momentum during a time interval equals the impulse of the net force that acted on the particle during that interval. The momentum of a particle equals the impulse that accelerated it from rest to its present speed. (See Examples 8.1–8.3.)

$$\vec{J} = \Sigma\vec{F}(t_2 - t_1) = \Sigma\vec{F}\,\Delta t \tag{8.5}$$

$$\vec{J} = \int_{t_1}^{t_2}\Sigma\vec{F}\,dt \tag{8.7}$$

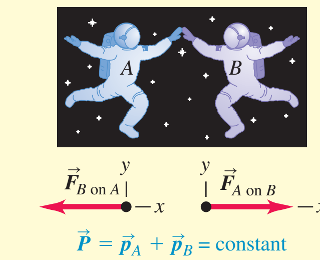
$$\vec{J} = \vec{p}_2 - \vec{p}_1 \tag{8.6}$$



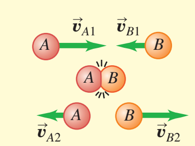
Conservation of momentum: An internal force is a force exerted by one part of a system on another. An external force is a force exerted on any part of a system by something outside the system. If the net external force on a system is zero, the total momentum of the system $\vec{P}$ (the vector sum of the momenta of the individual particles that make up the system) is constant, or conserved. Each component of total momentum is separately conserved. (See Examples 8.4–8.6)

$$\vec{P} = \vec{p}_A + \vec{p}_B + \cdots$$
$$= m_A\vec{v}_A + m_B\vec{v}_B + \cdots \tag{8.14}$$

If $\Sigma\vec{F} = \mathbf{0}$, then $\vec{P}$ = constant.



Collisions: In collisions of all kinds, the initial and final total momenta are equal. In an elastic collision between two bodies, the initial and final total kinetic energies are also equal, and the initial and final relative velocities have the same magnitude. In an inelastic two-body collision, the total kinetic energy is less after the collision than before. If the two bodies have the same final velocity, the collision is completely inelastic. (See Examples 8.7–8.12.)

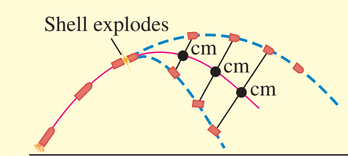


Center of mass: The position vector of the center of mass of a system of particles, $\vec{r}_{cm}$, is a weighted average of the positions $\vec{r}_1, \vec{r}_2, \ldots$ of the individual particles. The total momentum $\vec{P}$ of a system equals its total mass M multiplied by the velocity of its center of mass, $\vec{v}_{cm}$. The center of mass moves as though all the mass M were concentrated at that point. If the net external force on the system is zero, the center-of-mass velocity $\vec{v}_{cm}$ is constant. If the net external force is not zero, the center of mass accelerates as though it were a particle of mass M being acted on by the same net external force. (See Examples 8.13 and 8.14.)

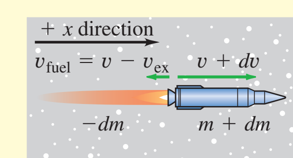
$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$$
$$= \frac{\Sigma_i m_i\vec{r}_i}{\Sigma_i m_i} \tag{8.29}$$

$$\vec{P} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots$$
$$= M\vec{v}_{cm} \tag{8.32}$$

$$\Sigma\vec{F}_{ext} = M\vec{a}_{cm} \tag{8.34}$$



Rocket propulsion: In rocket propulsion, the mass of a rocket changes as the fuel is used up and ejected from the rocket. Analysis of the motion of the rocket must include the momentum carried away by the spent fuel as well as the momentum of the rocket itself. (See Examples 8.15 and 8.16.)



Key Terms

Answer to Chapter Opening Question ?

The two players have the same magnitude of momentum $p = mv$ (the product of mass and speed), but the faster, lightweight player has twice as much kinetic energy $K = \frac{1}{2}mv^2$. Hence, the lightweight player can do twice as much work on you (and twice as much damage) in the process of coming to a halt (see Section 8.1).

Answers to Test Your Understanding Questions

8.1 Answer: (v), (i) and (ii) (tied for second place), (iii) and (iv) (tied for third place) We use two interpretations of the impulse of the net force: (1) the net force multiplied by the time that the net force acts, and (2) the change in momentum of the particle on which the net force acts. Which interpretation we use depends on what information we are given. We take the positive *x*-direction to be to the east. (i) The force is not given, so we use interpretation 2: $J_x = mv_{2x} - mv_{1x} = (1000\text{ kg})(0) - (1000\text{ kg})(25\text{ m/s}) = -25{,}000\text{ kg}\cdot\text{m/s}$, so the magnitude of the impulse is $25{,}000\text{ kg}\cdot\text{m/s} = 25{,}000\text{ N}\cdot\text{s}$. (ii) For the same reason as in (i), we use interpretation 2: $J_x = mv_{2x} - mv_{1x} = (1000\text{ kg})(0) - (1000\text{ kg})(25\text{ m/s}) = -25{,}000\text{ kg}\cdot\text{m/s}$, and the magnitude of the impulse is again $25{,}000\text{ kg}\cdot\text{m/s} = 25{,}000\text{ N}\cdot\text{s}$. (iii) The final velocity is not given, so we use interpretation 1: $J_x = (\sum F_x)_{\text{av}}(t_2 - t_1) = (2000\text{ N})(10\text{ s}) = 20{,}000\text{ N}\cdot\text{s}$, so the magnitude of the impulse is $20{,}000\text{ N}\cdot\text{s}$. (iv) For the same reason as in (iii), we use interpretation 1: $J_x = (\sum F_x)_{\text{av}}(t_2 - t_1) = (-2000\text{ N})(10\text{ s}) = -20{,}000\text{ N}\cdot\text{s}$, so the magnitude of the impulse is $20{,}000\text{ N}\cdot\text{s}$. (v) The force is not given, so we use interpretation 2: $J_x = mv_{2x} - mv_{1x} = (1000\text{ kg})(-25\text{ m/s}) - (1000\text{ kg})(25\text{ m/s}) = -50{,}000\text{ kg}\cdot\text{m/s}$, so the magnitude of the impulse is $50{,}000\text{ kg}\cdot\text{m/s} = 50{,}000\text{ N}\cdot\text{s}$.

8.2 Answers: (a) $v_{C2x} > 0$, $v_{C2y} > 0$, (b) piece *C* There are no external horizontal forces, so the *x*- and *y*-components of the total momentum of the system are both conserved. Both components of the total momentum are zero before the spring releases, so they must be zero after the spring releases. Hence

$$P_x = 0 = m_A v_{A2x} + m_B v_{B2x} + m_C v_{C2x}$$
$$P_y = 0 = m_A v_{A2y} + m_B v_{B2y} + m_C v_{C2y}$$

We are given that $m_A = m_B = m_C$, $v_{A2x} < 0$, $v_{A2y} = 0$, $v_{B2x} = 0$, and $v_{B2y} < 0$. You can solve the above equations to show that $v_{C2x} = -v_{A2x} > 0$ and $v_{C2y} = -v_{B2y} > 0$, so the velocity components of piece *C* are both positive. Piece *C* has speed $\sqrt{v_{C2x}{}^2 + v_{C2y}{}^2} = \sqrt{v_{A2x}{}^2 + v_{B2y}{}^2}$, which is greater than the speed of either piece *A* or piece *B*.

8.3 Answers: (a) inelastic, (b) elastic, (c) completely inelastic In each case gravitational potential energy is converted to kinetic energy as the ball falls, and the collision is between the ball and the ground. In (a) all of the initial energy is converted back to gravitational potential energy, so no kinetic energy is lost in the bounce and the collision is elastic. In (b) there is less gravitational potential energy at the end than at the beginning, so some kinetic energy was lost in the bounce. Hence the collision is inelastic. In (c) the ball loses all the kinetic energy it has to give, the ball and the ground stick together, and the collision is completely inelastic.

8.4 Answer: worse After a collision with a water molecule initially at rest, the speed of the neutron is $|(m_\text{n} - m_\text{w})/(m_\text{n} + m_\text{w})| = |(1.0\text{ u} - 18\text{ u})/(1.0\text{ u} + 18\text{ u})| = \frac{17}{19}$ of its initial speed, and its kinetic energy is $(\frac{17}{19})^2 = 0.80$ of the initial value. Hence a water molecule is a worse moderator than a carbon atom, for which the corresponding numbers are $\frac{11}{13}$ and $(\frac{11}{13})^2 = 0.72$.

8.5 Answer: no If gravity is the only force acting on the system of two fragments, the center of mass will follow the parabolic trajectory of a freely falling object. Once a fragment lands, however, the ground exerts a normal force on that fragment. Hence the net force on the system has changed, and the trajectory of the center of mass changes in response.

8.6 Answers: (a) increasing, (b) decreasing From Eqs. (8.37) and (8.38), the thrust F is equal to $m(dv/dt)$, where m is the rocket's mass and dv/dt is its acceleration. Because m decreases with time, if the thrust F is constant, then the acceleration must increase with time (the same force acts on a smaller mass); if the acceleration dv/dt is constant, then the thrust must decrease with time (a smaller force is all that's needed to accelerate a smaller mass).

PROBLEMS

Discussion Questions

Q8.1. In splitting logs with a hammer and wedge, is a heavy hammer more effective than a lighter hammer? Why?

Q8.2. Suppose you catch a baseball and then someone invites you to catch a bowling ball with either the same momentum or the same kinetic energy as the baseball. Which would you choose? Explain.

Q8.3. When rain falls from the sky, what happens to its momentum as it hits the ground? Is your answer also valid for Newton's famous apple?

Q8.4. A car has the same kinetic energy when it is traveling south at 30 m/s as when it is traveling northwest at 30 m/s. Is the momentum of the car the same in both cases? Explain.

Q8.5. A truck is accelerating as it speeds down the highway. One inertial frame of reference is attached to the ground with its origin at a fence post. A second frame of reference is attached to a police car that is traveling down the highway at constant velocity. Is the momentum of the truck the same in these two reference frames? Explain. Is the rate of change of the truck's momentum the same in these two frames? Explain.

Q8.6. When a large, heavy truck collides with a passenger car, the occupants of the car are more likely to be hurt than the truck driver. Why?

Q8.7. A woman holding a large rock stands on a frictionless, horizontal sheet of ice. She throws the rock with speed v_0 at an angle α above the horizontal. Consider the system consisting of the woman plus the rock. Is the momentum of the system conserved? Why or why not? Is any component of the momentum of the system conserved? Again, why or why not?

Q8.8. In Example 8.7 (Section 8.3), where the two gliders in Fig. 8.15 a stick together after the collision, the collision is inelastic because $K_2 < K_1$. In Example 8.5 (Section 8.2), is the collision inelastic? Explain.

Q8.9. In a completely inelastic collision between two objects, where the objects stick together after the collision, is it possible for the final kinetic energy of the system to be zero? If so, give an example in which this would occur. If the final kinetic energy is zero, what must the initial momentum of the system be? Is the initial kinetic energy of the system zero? Explain.

Q8.10. Since for a particle the kinetic energy is given by $K = \frac{1}{2}mv^2$ and the momentum by $\vec{p} = m\vec{v}$, it is easy to show that $K = p^2/2m$. How, then, is it possible to have an event during which the total momentum of the system is constant but the total kinetic energy changes?

Q8.11. In each of Examples 8.10, 8.11, and 8.12 (Section 8.4), verify that the relative velocity vector of the two bodies has the same magnitude before and after the collision. In each case what happens to the *direction* of the relative velocity vector?

Q8.12. A glass dropped on the floor is more likely to break if the floor is concrete than if it is wood. Why? (Refer to Fig. 8.3b.)

Q8.13. In Fig. 8.22b, the kinetic energy of the Ping-Pong ball is larger after its interaction with the bowling ball than before. From where does the extra energy come? Describe the event in terms of conservation of energy.

Q8.14. A machine gun is fired at a steel plate. Is the average force on the plate from the bullet impact greater if the bullets bounce off or if they are squashed and stick to the plate? Explain.

Q8.15. A net force of 4 N acts on an object initially at rest for 0.25 s and gives it a final speed of 5 m/s. How could a net force of 2 N produce the same final speed?

Q8.16. A net force with *x*-component $\sum F_x$ acts on an object from time t_1 to time t_2. The *x*-component of the momentum of the object is the same at t_1 as it is at t_2, but $\sum F_x$ is not zero at all times between t_1 and t_2. What can you say about the graph of $\sum F_x$ versus t?

Q8.17. A tennis player hits a tennis ball with a racket. Consider the system made up of the ball and the racket. Is the total momentum of the system the same just before and just after the hit? Is the total momentum just after the hit the same as 2 s later, when the ball is in midair at the high point of its trajectory? Explain any differences between the two cases.

Q8.18. In Example 8.4 (Section 8.2), consider the system consisting of the rifle plus the bullet. What is the speed of the system's center of mass after the rifle is fired? Explain.

Q8.19. An egg is released from rest from the roof of a building and falls to the ground. As the egg falls, what happens to the momentum of the system of the egg plus the earth?

Q8.20. A woman stands in the middle of a perfectly smooth, frictionless, frozen lake. She can set herself in motion by throwing things, but suppose she has nothing to throw. Can she propel herself to shore *without* throwing anything?

Q8.21. In a zero-gravity environment, can a rocket-propelled spaceship ever attain a speed greater than the relative speed with which the burnt fuel is exhausted?

Q8.22. When an object breaks into two pieces (explosion, radioactive decay, recoil, etc.), the lighter fragment gets more kinetic energy than the heavier one. This is a consequence of momentum conservation, but can you also explain it using Newton's laws of motion?

Q8.23. An apple falls from a tree and feels no air resistance. As it is falling, which of these statements about it are true? (a) Only its momentum is conserved; (b) only its mechanical energy is conserved, (c) both its momentum and its mechanical energy are conserved, (d) its kinetic energy is conserved.

Q8.24. Two pieces of clay collide and stick together. During the collision, which of these statements are true? (a) Only the momentum of the clay is conserved, (b) only the mechanical energy of the clay is conserved, (c) both the momentum and the mechanical energy of the clay are conserved, (d) the kinetic energy of the clay is conserved.

Q8.25. Two marbles are pressed together with a light ideal spring between them, but they are not attached to the spring in any way. They are then released on a frictionless horizontal table and soon move free of the spring. As the marbles are moving away from each other, which of these statements about them are true? (a) Only the momentum of the marbles is conserved, (b) only the mechanical energy of the marbles is conserved, (c) both the momentum and the mechanical energy of the marbles are conserved, (d) the kinetic energy of the marbles is conserved.

Q8.26. A very heavy SUV collides head-on with a very light compact car. Which of these statements about the collision are correct? (a) The amount of kinetic energy lost by the SUV is equal to the amount of kinetic energy gained by the compact, (b) the amount of momentum lost by the SUV is equal to the amount of momentum gained by the compact, (c) The compact feels a considerably greater force during the collision than the SUV does, (d) both cars lose the same amount of kinetic energy.

Exercises

Section 8.1 Momentum and Impulse

8.1. (a) What is the magnitude of the momentum of a 10,000-kg truck whose speed is 12.0 m/s? (b) What speed would a 2,000-kg SUV have to attain in order to have (i) the same momentum? (ii) the same kinetic energy?

8.2. In Conceptual Example 8.1 (Section 8.1), show that the iceboat with mass $2m$ has $\sqrt{2}$ times as much momentum at the finish line as does the iceboat with mass m.

8.3. (a) Show that the kinetic energy K and the momentum magnitude p of a particle with mass m are related by $K = p^2/2m$. (b) A 0.040-kg cardinal (*Richmondena cardinalis*) and a 0.145-kg baseball have the same kinetic energy. Which has the greater magnitude of momentum? What is the ratio of the cardinal's magnitude of momentum to the baseball's? (c) A 700-N man and a 450-N woman have the same momentum. Who has the greater kinetic energy? What is the ratio of the man's kinetic energy to that of the woman?

8.4. In a certain men's track and field event, the shotput has a mass of 7.30 kg and is released with a speed of 15.0 m/s at 40.0° above the horizontal over a man's straight left leg. What are the initial horizontal and vertical components of the momentum of this shotput?

8.5. One 110-kg football lineman is running to the right at 2.75 m/s while another 125-kg lineman is running directly toward him at

2.60 m/s. What are (a) the magnitude and direction of the net momentum of these two athletes, and (b) their total kinetic energy?

8.6. Two vehicles are approaching an intersection. One is a 2500-kg pickup traveling at 14.0 m/s from east to west (the $-x$-direction), and the other is a 1500-kg sedan going from south to north (the $+y$-direction at 23.0 m/s). (a) Find the x- and y-components of the net momentum of this system. (b) What are the magnitude and direction of the net momentum?

8.7. Force of a Golf Swing. A 0.0450-kg golf ball initially at rest is given a speed of 25.0 m/s when a club strikes. If the club and ball are in contact for 2.00 ms, what average force acts on the ball? Is the effect of the ball's weight during the time of contact significant? Why or why not?

8.8. Force of a Baseball Swing. A baseball has mass 0.145 kg. (a) If the velocity of a pitched ball has a magnitude of 45.0 m/s and the batted ball's velocity is 55.0 m/s in the opposite direction, find the magnitude of the change in momentum of the ball and of the impulse applied to it by the bat. (b) If the ball remains in contact with the bat for 2.00 ms, find the magnitude of the average force applied by the bat.

8.9. A 0.160-kg hockey puck is moving on an icy, frictionless, horizontal surface. At $t = 0$, the puck is moving to the right at 3.00 m/s. (a) Calculate the velocity of the puck (magnitude and direction) after a force of 25.0 N directed to the right has been applied for 0.050 s. (b) If, instead, a force of 12.0 N directed to the left is applied from $t = 0$ to $t = 0.050$ s, what is the final velocity of the puck?

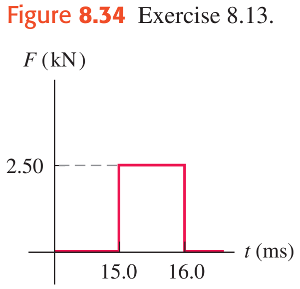
8.10. An engine of the orbital maneuvering system (OMS) on a space shuttle exerts a force of $(26{,}700\ \text{N})\hat{\jmath}$ for 3.90 s, exhausting a negligible mass of fuel relative to the 95,000-kg mass of the shuttle. (a) What is the impulse of the force for this 3.90 s? (b) What is the shuttle's change in momentum from this impulse? (c) What is the shuttle's change in velocity from this impulse? (d) Why can't we find the resulting change in the kinetic energy of the shuttle?

8.11. At time $t = 0$, a 2150-kg rocket in outer space fires an engine that exerts an increasing force on it in the $+x$-direction. This force obeys the equation $F_x = At^2$, where t is time, and has a magnitude of 781.25 N when $t = 1.25$ s. (a) Find the SI value of the constant A, including its units. (b) What impulse does the engine exert on the rocket during the 1.50-s interval starting 2.00 s after the engine is fired? (c) By how much does the rocket's velocity change during this interval?

8.12. A bat strikes a 0.145-kg baseball. Just before impact, the ball is traveling horizontally to the right at 50.0 m/s, and it leaves the bat traveling to the left at an angle of 30° above horizontal with a speed of 65.0 m/s. If the ball and bat are in contact for 1.75 ms, find the horizontal and vertical components of the average force on the ball.

8.13. A 2.00-kg stone is sliding to the right on a frictionless horizontal surface at 5.00 m/s when it is suddenly struck by an object that exerts a large horizontal force on it for a short period of time. The graph in Fig. 8.34 shows the magnitude of this force as a function of time. (a) What impulse does this force exert on the stone? (b) Just after the force stops acting, find the magnitude and direction of the stone's velocity if the force acts (i) to the right or (ii) to the left.

Figure 8.34 Exercise 8.13.



Section 8.2 Conservation of Momentum

8.14. A 68.5-kg astronaut is doing a repair in space on the orbiting space station. She throws a 2.25-kg tool away from her at 3.20 m/s relative to the space station. With what speed and in what direction will she begin to move?

8.15. Animal Propulsion. Squids and octopuses propel themselves by expelling water. They do this by keeping water in a cavity and then suddenly contracting the cavity to force out the water through an opening. A 6.50-kg squid (including the water in the cavity) at rest suddenly sees a dangerous predator. (a) If the squid has 1.75 kg of water in its cavity, at what speed must it expel this water to suddenly achieve a speed of 2.50 m/s to escape the predator? Neglect any drag effects of the surrounding water. (b) How much kinetic energy does the squid create by this maneuver?

8.16. You are standing on a sheet of ice that covers the football stadium parking lot in Buffalo; there is negligible friction between your feet and the ice. A friend throws you a 0.400-kg ball that is traveling horizontally at 10.0 m/s. Your mass is 70.0 kg. (a) If you catch the ball, with what speed do you and the ball move afterward? (b) If the ball hits you and bounces off your chest, so afterward it is moving horizontally at 8.0 m/s in the opposite direction, what is your speed after the collision?

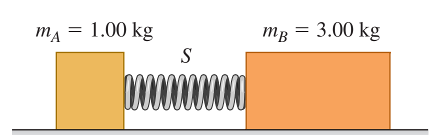
8.17. On a frictionless, horizontal air table, puck A (with mass 0.250 kg) is moving toward puck B (with mass 0.350 kg), which is initially at rest. After the collision, puck A has a velocity of 0.120 m/s to the left, and puck B has a velocity of 0.650 m/s to the right. (a) What was the speed of puck A before the collision? (b) Calculate the change in the total kinetic energy of the system that occurs during the collision.

8.18. When cars are equipped with flexible bumpers, they will bounce off each other during low-speed collisions, thus causing less damage. In one such accident, a 1750-kg car traveling to the right at 1.50 m/s collides with a 1450-kg car going to the left at 1.10 m/s. Measurements show that the heavier car's speed just after the collision was 0.250 m/s in its original direction. You can ignore any road friction during the collision. (a) What was the speed of the lighter car just after the collision? (b) Calculate the change in the combined kinetic energy of the two-car system during this collision.

8.19. The expanding gases that leave the muzzle of a rifle also contribute to the recoil. A .30-caliber bullet has mass 0.00720 kg and a speed of 601 m/s relative to the muzzle when fired from a rifle that has mass 2.80 kg. The loosely held rifle recoils at a speed of 1.85 m/s relative to the earth. Find the momentum of the propellant gases in a coordinate system attached to the earth as they leave the muzzle of the rifle.

8.20. Block A in Fig. 8.35 has mass 1.00 kg, and block B has mass 3.00 kg. The blocks are forced together, compressing a spring S between them; then the system is released from rest on a level, frictionless surface. The spring, which has negligible mass, is not fastened to either block and drops to the surface after it has expanded. Block B acquires a speed of 1.20 m/s. (a) What is the final speed of block A? (b) How much potential energy was stored in the compressed spring?

Figure 8.35 Exercise 8.20.



8.21. A hunter on a frozen, essentially frictionless pond uses a rifle that shoots 4.20-g bullets at 965 m/s. The mass of the hunter (including his gun) is 72.5 kg, and the hunter holds tight to the gun after firing it. Find the recoil velocity of the hunter if he fires the rifle (a) horizontally and (b) at 56.0° above the horizontal.

8.22. An atomic nucleus suddenly bursts apart (fissions) into two pieces. Piece A, of mass m_A, travels off to the left with speed v_A. Piece B, of mass m_B, travels off to the right with speed v_B. (a) Use conservation of momentum to solve for v_B in terms of m_A, m_B, and v_A. (b) Use the results of part (a) to show that $K_A/K_B = m_B/m_A$, where K_A and K_B are the kinetic energies of the two pieces.

8.23. The nucleus of ^{214}Po decays radioactively by emitting an alpha particle (mass 6.65×10^{-27} kg) with kinetic energy 1.23×10^{-12} J, as measured in the laboratory reference frame. Assuming that the Po was initially at rest in this frame, find the recoil velocity of the nucleus that remains after the decay.

8.24. You are standing on a large sheet of frictionless ice and holding a large rock. In order to get off the ice, you throw the rock so it has velocity 12.0 m/s relative to the earth at an angle of 35.0° above the horizontal. If your mass is 70.0 kg and the rock's mass is 15.0 kg, what is your speed after you throw the rock (see Discussion Question Q8.7)?

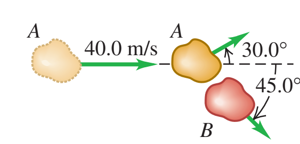
8.25. Two ice skaters, Daniel (mass 65.0 kg) and Rebecca (mass 45.0 kg), are practicing. Daniel stops to tie his shoelace and, while at rest, is struck by Rebecca, who is moving at 13.0 m/s before she collides with him. After the collision, Rebecca has a velocity of magnitude 8.00 m/s at an angle of 53.1° from her initial direction. Both skaters move on the frictionless, horizontal surface of the rink. (a) What are the magnitude and direction of Daniel's velocity after the collision? (b) What is the change in total kinetic energy of the two skaters as a result of the collision?

8.26. An astronaut in space cannot use a scale or balance to weigh objects because there is no gravity. But she does have devices to measure distance and time accurately. She knows her own mass is 78.4 kg, but she is unsure of the mass of a large gas canister in the airless rocket. When this canister is approaching her at 3.50 m/s, she pushes against it, which slows it down to 1.20 m/s (but does not reverse it) and gives her a speed of 2.40 m/s. What is the mass of this canister?

8.27. Changing Mass. An open-topped freight car with mass 24,000 kg is coasting without friction along a level track. It is raining very hard, and the rain is falling vertically downward. Originally, the car is empty and moving with a speed of 4.00 m/s. What is the speed of the car after it has collected 3000 kg of rainwater?

8.28. Asteroid Collision. Two asteroids of equal mass in the asteroid belt between Mars and Jupiter collide with a glancing blow. Asteroid A, which was initially traveling at 40.0 m/s, is deflected 30.0° from its original direction, while asteroid B travels at 45.0° to the original direction of A (Fig. 8.36). (a) Find the speed of each asteroid after the collision. (b) What fraction of the original kinetic energy of asteroid A dissipates during this collision?

Figure 8.36 Exercise 8.28.



Section 8.3 Momentum Conservation and Collisions

8.29. A 15.0-kg fish swimming at 1.10 m/s suddenly gobbles up a 4.50-kg fish that is initially stationary. Neglect any drag effects of the water. (a) Find the speed of the large fish just after it eats the small one. (b) How much mechanical energy was dissipated during this meal?

8.30. Two fun-loving otters are sliding toward each other on a muddy (and hence frictionless) horizontal surface. One of them, of mass 7.50 kg, is sliding to the left at 5.00 m/s, while the other, of mass 5.75 kg, is slipping to the right at 6.00 m/s. They hold fast to each other after they collide. (a) Find the magnitude and direction of the velocity of these free-spirited otters right after they collide. (b) How much mechanical energy dissipates during this play?

8.31. Deep Impact Mission. In July 2005, NASA's "Deep Impact" mission crashed a 372-kg probe directly onto the surface of the comet Tempel 1, hitting the surface at 37,000 km/h. The original speed of the comet at that time was about 40,000 km/h, and its mass was estimated to be in the range $(0.10\text{–}2.5) \times 10^{14}$ kg. Use the smallest value of the estimated mass. (a) What change in the comet's velocity did this collision produce? Would this change be noticeable? (b) Suppose this comet were to hit the earth and fuse with it. By how much would it change our planet's velocity? Would this change be noticeable? (The mass of the earth is 5.97×10^{24} kg.)

8.32. A 1050-kg sports car is moving westbound at 15.0 m/s on a level road when it collides with a 6320-kg truck driving east on the same road at 10.0 m/s. The two vehicles remain locked together after the collision. (a) What is the velocity (magnitude and direction) of the two vehicles just after the collision? (b) At what speed should the truck have been moving so that it and car are both stopped in the collision? (c) Find the change in kinetic energy of the system of two vehicles for the situations of part (a) and part (b). For which situation is the change in kinetic energy greater in magnitude?

8.33. On a very muddy football field, a 110-kg linebacker tackles an 85-kg halfback. Immediately before the collision, the linebacker is slipping with a velocity of 8.8 m/s north and the halfback is sliding with a velocity of 7.2 m/s east. What is the velocity (magnitude and direction) at which the two players move together immediately after the collision?

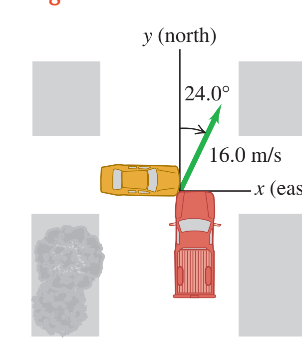
8.34. Two skaters collide and grab on to each other on frictionless ice. One of them, of mass 70.0 kg, is moving to the right at 2.00 m/s, while the other, of mass 65.0 kg, is moving to the left at 2.50 m/s. What are the magnitude and direction of the velocity of these skaters just after they collide?

8.35. Two cars, one a compact with mass 1200 kg and the other a large gas-guzzler with mass 3000 kg, collide head-on at typical freeway speeds. (a) Which car has a greater magnitude of momentum change? Which car has a greater velocity change? (b) If the larger car changes its velocity by Δv, calculate the change in the velocity of the small car in terms of Δv. (c) Which car's occupants would you expect to sustain greater injuries? Explain.

8.36. Bird Defense. To protect their young in the nest, peregrine falcons will fly into birds of prey (such as ravens) at high speed. In one such episode, a 600-g falcon flying at 20.0 m/s hit a 1.50-kg raven flying at 9.0 m/s. The falcon hit the raven at right angles to its original path and bounced back at 5.0 m/s. (These figures were estimated by the author as he watched this attack occur in northern New Mexico.) (a) By what angle did the falcon change the raven's direction of motion? (b) What was the raven's speed right after the collision?

8.37. At the intersection of Texas Avenue and University Drive, a yellow subcompact car with mass 950 kg traveling east on University collides with a red pickup truck with mass 1900 kg that is traveling north on Texas and ran a red light (Fig. 8.37). The two vehicles stick together as a result of the collision, and the wreckage slides at 16.0 m/s in the direction 24.0° east of north. Calculate the

Figure 8.37 Exercise 8.37.



speed of each vehicle before the collision. The collision occurs during a heavy rainstorm; you can ignore friction forces between the vehicles and the wet road.

8.38. A 5.00-g bullet is fired horizontally into a 1.20-kg wooden block resting on a horizontal surface. The coefficient of kinetic friction between block and surface is 0.20. The bullet remains embedded in the block, which is observed to slide 0.230 m along the surface before stopping. What was the initial speed of the bullet?

8.39. A Ballistic Pendulum. A 12.0-g rifle bullet is fired with a speed of 380 m/s into a ballistic pendulum with mass 6.00 kg, suspended from a cord 70.0 cm long (see Example 8.8 in Section 8.3). Compute (a) the vertical height through which the pendulum rises, (b) the initial kinetic energy of the bullet, and (c) the kinetic energy of the bullet and pendulum immediately after the bullet becomes embedded in the pendulum.

8.40. You and your friends are doing physics experiments on a frozen pond that serves as a frictionless, horizontal surface. Sam, with mass 80.0 kg, is given a push and slides eastward. Abigail, with mass 50.0 kg, is sent sliding northward. They collide, and after the collision Sam is moving at 37.0° north of east with a speed of 6.00 m/s and Abigail is moving at 23.0° south of east with a speed of 9.00 m/s. (a) What was the speed of each person before the collision? (b) By how much did the total kinetic energy of the two people decrease during the collision?

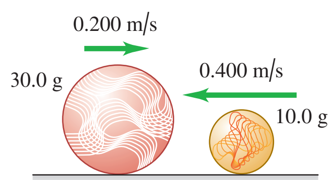
Section 8.4 Elastic Collisions

8.41. Blocks A (mass 2.00 kg) and B (mass 10.00 kg) move on a frictionless, horizontal surface. Initially, block B is at rest and block A is moving toward it at 2.00 m/s. The blocks are equipped with ideal spring bumpers, as in Example 8.10. The collision is head-on, so all motion before and after the collision is along a straight line. (a) Find the maximum energy stored in the spring bumpers and the velocity of each block at that time. (b) Find the velocity of each block after they have moved apart.

8.42. A 0.150-kg glider is moving to the right on a frictionless, horizontal air track with a speed of 0.80 m/s. It has a head-on collision with a 0.300-kg glider that is moving to the left with a speed of 2.20 m/s. Find the final velocity (magnitude and direction) of each glider if the collision is elastic.

8.43. A 10.0-g marble slides to the left with a velocity of magnitude 0.400 m/s on the frictionless, horizontal surface of an icy New York sidewalk and has a head-on, elastic collision with a larger 30.0-g marble sliding to the right with a velocity of magnitude 0.200 m/s (Fig. 8.38). (a) Find the velocity of each marble (magnitude and direction) after the collision. (Since the collision is head-on, all the motion is along a line.) (b) Calculate the *change in momentum* (that is, the momentum after the collision minus the momentum before the collision) for each marble. Compare the values you get for each marble. (c) Calculate the *change in kinetic energy* (that is, the kinetic energy after the collision minus the kinetic energy before the collision) for each marble. Compare the values you get for each marble.

Figure 8.38 Exercise 8.43.



8.44. Supply the details of the calculation of α and β in Example 8.12 (Section 8.4).

8.45. Moderators. Canadian nuclear reactors use *heavy water* moderators in which elastic collisions occur between the neutrons and deuterons of mass 2.0 u (see Example 8.11 in Section 8.4). (a) What is the speed of a neutron, expressed as a fraction of its original speed, after a head-on, elastic collision with a deuteron that is initially at rest? (b) What is its kinetic energy, expressed as a fraction of its original kinetic energy? (c) How many such successive collisions will reduce the speed of a neutron to 1/59,000 of its original value?

8.46. You are at the controls of a particle accelerator, sending a beam of 1.50×10^7 m/s protons (mass m) at a gas target of an unknown element. Your detector tells you that some protons bounce straight back after a collision with one of the nuclei of the unknown element. All such protons rebound with a speed of 1.20×10^7 m/s. Assume that the initial speed of the target nucleus is negligible and the collision is elastic. (a) Find the mass of one nucleus of the unknown element. Express your answer in terms of the proton mass m. (b) What is the speed of the unknown nucleus immediately after such a collision?

Section 8.5 Center of Mass

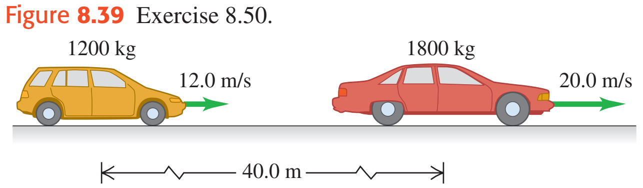
8.47. Three odd-shaped blocks of chocolate have the following masses and center-of-mass coordinates: (1) 0.300 kg, (0.200 m, 0.300 m); (2) 0.400 kg, (0.100 m, −0.400 m); (3) 0.200 kg, (−0.300 m, 0.600 m). Find the coordinates of the center of mass of the system of three chocolate blocks.

8.48. Find the position of the center of mass of the system of the sun and Jupiter. (Since Jupiter is more massive than the rest of the planets combined, this is essentially the position of the center of mass of the solar system.) Does the center of mass lie inside or outside the sun? Use the data in Appendix F.

8.49. Pluto and Charon. Pluto's diameter is approximately 2370 km, and the diameter of its satellite Charon is 1250 km. Although the distance varies, they are often about 19,700 km apart, center-to-center. Assuming that both Pluto and Charon have the same composition and hence the same average density, find the location of the center of mass of this system relative to the center of Pluto.

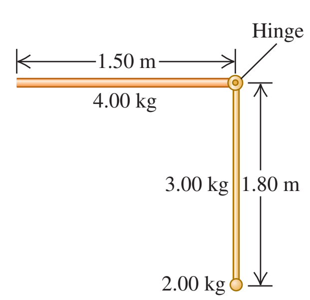
8.50. A 1200-kg station wagon is moving along a straight highway at 12.0 m/s. Another car, with mass 1800 kg and speed 20.0 m/s, has its center of mass 40.0 m ahead of the center of mass of the station wagon (Fig. 8.39). (a) Find the position of the center of mass of the system consisting of the two automobiles. (b) Find the magnitude of the total momentum of the system from the given data. (c) Find the speed of the center of mass of the system. (d) Find the total momentum of the system, using the speed of the center of mass. Compare your result with that of part (b).

Figure 8.39 Exercise 8.50.



8.51. A machine part consists of a thin, uniform 4.00-kg bar that is 1.50 m long, hinged perpendicular to a similar vertical bar of mass 3.00 kg and length 1.80 m. The longer bar has a small but dense 2.00-kg ball at one end (Fig. 8.40). By what distance will the center of mass of this part move horizontally and vertically if the vertical bar is pivoted counterclockwise through 90° to make the entire part horizontal?

Figure 8.40 Exercise 8.51.



8.52. At one instant, the center of mass of a system of two particles is located on the x-axis at $x = 2.0$ m and has a velocity of $(5.0\text{ m/s})\hat{\imath}$. One of the particles is at the origin. The other particle has a mass of 0.10 kg and is at rest on the x-axis at $x = 8.0$ m. (a) What is the mass of the particle at the origin? (b) Calculate the total momentum of this system. (c) What is the velocity of the particle at the origin?

8.53. In Example 8.14 (Section 8.5), Ramon pulls on the rope to give himself a speed of 0.70 m/s. What is James's speed?

8.54. A system consists of two particles. At $t = 0$ one particle is at the origin; the other, which has a mass of 0.50 kg, is on the y-axis at $y = 6.0$ m. At $t = 0$ the center of mass of the system is on the y-axis at $y = 2.4$ m. The velocity of the center of mass is given by $(0.75\text{ m/s}^3)t^2\hat{\imath}$. (a) Find the total mass of the system. (b) Find the acceleration of the center of mass at any time t. (c) Find the net external force acting on the system at $t = 3.0$ s.

8.55. A radio-controlled model airplane has a momentum given by $[(-0.75\text{ kg}\cdot\text{m/s}^3)t^2 + (3.0\text{ kg}\cdot\text{m/s})]\hat{\imath} + (0.25\text{ kg}\cdot\text{m/s}^2)t\hat{\jmath}$. What are the x-, y-, and z-components of the net force on the airplane?

*Section 8.6 Rocket Propulsion

***8.56.** A small rocket burns 0.0500 kg of fuel per second, ejecting it as a gas with a velocity relative to the rocket of magnitude 1600 m/s. (a) What is the thrust of the rocket? (b) Would the rocket operate in outer space where there is no atmosphere? If so, how would you steer it? Could you brake it?

***8.57.** A 70-kg astronaut floating in space in a 110-kg MMU (manned maneuvering unit) experiences an acceleration of 0.029 m/s^2 when he fires one of the MMU's thrusters. (a) If the speed of the escaping N_2 gas relative to the astronaut is 490 m/s, how much gas is used by the thruster in 5.0 s? (b) What is the thrust of the thruster?

***8.58.** A rocket is fired in deep space, where gravity is negligible. If the rocket has an initial mass of 6000 kg and ejects gas at a relative velocity of magnitude 2000 m/s, how much gas must it eject in the first second to have an initial acceleration of 25.0 m/s^2?

***8.59.** A rocket is fired in deep space, where gravity is negligible. In the first second it ejects $\frac{1}{160}$ of its mass as exhaust gas and has an acceleration of 15.0 m/s^2. What is the speed of the exhaust gas relative to the rocket?

***8.60.** A C6-5 model rocket engine has an impulse of 10.0 N · s for 1.70 s, while burning 0.0125 kg of propellant. It has a maximum thrust of 13.3 N. The initial mass of the engine plus propellant is 0.0258 kg. (a) What fraction of the maximum thrust is the average thrust? (b) Calculate the relative speed of the exhaust gases, assuming it is constant. (c) Assuming that the relative speed of the exhaust gases is constant, find the final speed of the engine if it was attached to a very light frame and fired from rest in gravity-free outer space.

***8.61.** A single-stage rocket is fired from rest from a deep-space platform, where gravity is negligible. If the rocket burns its fuel in 50.0 s and the relative speed of the exhaust gas is $v_{ex} = 2100$ m/s, what must the mass ratio m_0/m be for a final speed v of 8.00 km/s (about equal to the orbital speed of an earth satellite)?

***8.62.** Obviously, we can make rockets to go very fast, but what is a reasonable top speed? Assume that a rocket is fired from rest at a space station in deep space, where gravity is negligible. (a) If the rocket ejects gas at a relative speed of 2000 m/s and you want the rocket's speed eventually to be $1.00 \times 10^{-3}c$, where c is the speed of light, what fraction of the initial mass of the rocket and fuel is *not* fuel? (b) What is this fraction if the final speed is to be 3000 m/s?

Problems

8.63. A steel ball with mass 40.0 g is dropped from a height of 2.00 m onto a horizontal steel slab. The ball rebounds to a height of 1.60 m. (a) Calculate the impulse delivered to the ball during impact. (b) If the ball is in contact with the slab for 2.00 ms, find the average force on the ball during impact.

8.64. In a volcanic eruption, a 2400-kg boulder is thrown vertically upward into the air. At its highest point, it suddenly explodes (due to trapped gases) into two fragments, one being three times the mass of the other. The lighter fragment starts out with only horizontal velocity and lands 274 m directly north of the point of the explosion. Where will the other fragment land? Neglect any air resistance.

8.65. Just before it is struck by a racket, a tennis ball weighing 0.560 N has a velocity of $(20.0\text{ m/s})\hat{\imath} - (4.0\text{ m/s})\hat{\jmath}$. During the 3.00 ms that the racket and ball are in contact, the net force on the ball is constant and equal to $-(380\text{ N})\hat{\imath} + (110\text{ N})\hat{\jmath}$. (a) What are the x- and y-components of the impulse of the net force applied to the ball? (b) What are the x- and y-components of the final velocity of the ball?

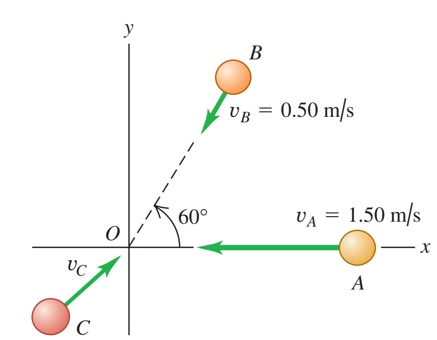
8.66. Three coupled railroad cars roll along and couple with a fourth car, which is initially at rest. These four cars roll along and couple with a fifth car initially at rest. This process continues until the speed of the final collection of railroad cars is one-fifth the speed of the initial three railroad cars. All the cars are identical. Ignoring friction, how many cars are in the final collection?

8.67. A 1500-kg blue convertible is traveling south, and a 2000-kg red SUV is traveling west. If the total momentum of the system consisting of the two cars is 8000 kg · m/s directed at 60.0° west of south, what is the speed of each vehicle?

8.68. Three identical pucks on a horizontal air table have repelling magnets. They are held together and then released simultaneously. Each has the same speed at any instant. One puck moves due west. What is the direction of the velocity of each of the other two pucks?

8.69. Spheres A (mass 0.020 kg), B (mass 0.030 kg), and C (mass 0.050 kg) are approaching the origin as they slide on a frictionless air table (Fig. 8.41). The initial velocities of A and B are given in the figure. All three spheres arrive at the origin at the same time and stick together. (a) What must the x- and y-components of the initial velocity of C be if all three objects are to end up moving at 0.50 m/s in the $+x$-direction after the collision? (b) If C has the velocity found in part (a), what is the change in the kinetic energy of the system of three spheres as a result of the collision?

Figure 8.41 Problem 8.69.



8.70. A railroad handcar is moving along straight, frictionless tracks with negligible air resistance. In the following cases, the car initially has a total mass (car and contents) of 200 kg and is traveling east with a velocity of magnitude 5.00 m/s. Find the *final* *velocity* of the car in each case, assuming that the handcar does not leave the tracks. (a) A 25.0-kg mass is thrown sideways out of the car with a velocity of magnitude 2.00 m/s relative to the car's initial velocity. (b) A 25.0-kg mass is thrown backward out of the car with a velocity of 5.00 m/s relative to the initial motion of the car. (c) A 25.0-kg mass is thrown into the car with a velocity of 6.00 m/s relative to the ground and opposite in direction to the initial velocity of the car.

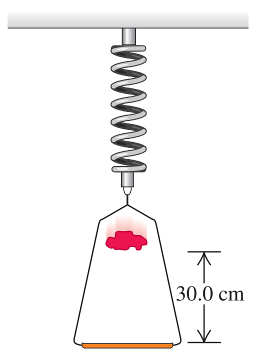
8.71. Changing Mass. A railroad hopper car filled with sand is rolling with an initial speed of 15.0 m/s on straight, horizontal tracks. You can ignore frictional forces on the railroad car. The total mass of the car plus sand is 85,000 kg. The hopper door is not fully closed so sand leaks out the bottom. After 20 min, 13,000 kg of sand has leaked out. Then what is the speed of the railroad car? (Compare your analysis with that used to solve Exercise 8.27.)

8.72. At a classic auto show, a 840-kg 1955 Nash Metropolitan motors by at 9.0 m/s, followed by a 1620-kg 1957 Packard Clipper purring past at 5.0 m/s. (a) Which car has the greater kinetic energy? What is the ratio of the kinetic energy of the Nash to that of the Packard? (b) Which car has the greater magnitude of momentum? What is the ratio of the magnitude of momentum of the Nash to that of the Packard? (c) Let F_N be the net force required to stop the Nash in time t, and let F_P be the net force required to stop the Packard in the same time. Which is larger: F_N or F_P? What is the ratio F_N/F_P of these two forces? (d) Now let F_N be the net force required to stop the Nash in a distance d, and let F_P be the net force required to stop the Packard in the same distance. Which is larger: F_N or F_P? What is the ratio F_N/F_P?

8.73. A soldier on a firing range fires an eight-shot burst from an assault weapon at a full automatic rate of 1000 rounds per minute. Each bullet has a mass of 7.45 g and a speed of 293 m/s relative to the ground as it leaves the barrel of the weapon. Calculate the average recoil force exerted on the weapon during that burst.

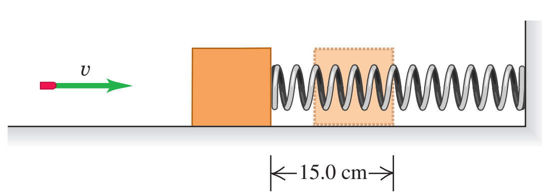
8.74. A 0.150-kg frame, when suspended from a coil spring, stretches the spring 0.050 m. A 0.200-kg lump of putty is dropped from rest onto the frame from a height of 30.0 cm (Fig. 8.42). Find the maximum distance the frame moves downward from its initial position.

Figure 8.42 Problem 8.74.



8.75. A rifle bullet with mass 8.00 g strikes and embeds itself in a block with mass 0.992 kg that rests on a frictionless, horizontal surface and is attached to a coil spring (Fig. 8.43). The impact compresses the spring 15.0 cm. Calibration of the spring shows that a force of 0.750 N is required to compress the spring 0.250 cm. (a) Find the magnitude of the block's velocity just after impact. (b) What was the initial speed of the bullet?

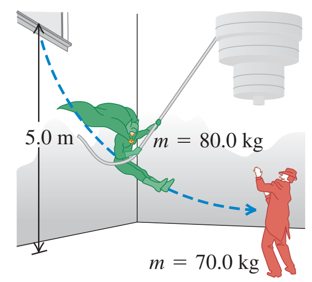
Figure 8.43 Problem 8.75.



8.76. A Ricocheting Bullet. 0.100-kg stone rests on a frictionless, horizontal surface. A bullet of mass 6.00 g, traveling horizontally at 350 m/s, strikes the stone and rebounds horizontally at right angles to its original direction with a speed of 250 m/s. (a) Compute the magnitude and direction of the velocity of the stone after it is struck. (b) Is the collision perfectly elastic?

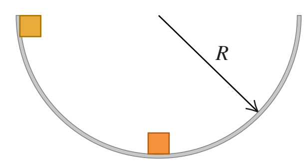
8.77. A movie stuntman (mass 80.0 kg) stands on a window ledge 5.0 m above the floor (Fig. 8.44). Grabbing a rope attached to a chandelier, he swings down to grapple with the movie's villain (mass 70.0 kg), who is standing directly under the chandelier. (Assume that the stuntman's center of mass moves downward 5.0 m. He releases the rope just as he reaches the villain.) (a) With what speed do the entwined foes start to slide across the floor? (b) If the coefficient of kinetic friction of their bodies with the floor is $\mu_k = 0.250$, how far do they slide?

Figure 8.44 Problem 8.77.



8.78. Two identical masses are released from rest in a smooth hemispherical bowl of radius R, from the positions shown in Fig. 8.45. You can ignore friction between the masses and the surface of the bowl. If they stick together when they collide, how high above the bottom of the bowl will the masses go after colliding?

Figure 8.45 Problem 8.78.



8.79. A ball with mass M, moving horizontally at 5.00 m/s, collides elastically with a block with mass $3M$ that is initially hanging at rest from the ceiling on the end of a 50.0-cm wire. Find the maximum angle through which the block swings after it is hit.

8.80. A 20.00-kg lead sphere is hanging from a hook by a thin wire 3.50 m long, and is free to swing in a complete circle. Suddenly it is struck horizontally by a 5.00-kg steel dart that embeds itself in the lead sphere. What must be the minimum initial speed of the dart so that the combination makes a complete circular loop after the collision?

8.81. An 8.00-kg ball, hanging from the ceiling by a light wire 135 cm long, is struck in an elastic collision by a 2.00-kg ball moving horizontally at 5.00 m/s just before the collision. Find the tension in the wire just after the collision.

8.82. A rubber ball of mass m is released from rest at height h above the floor. After its first bounce, it rises to 90% of its original height. What impulse (magnitude and direction) does the floor exert on this ball during its first bounce? Express your answer in terms of the variables m and h.

8.83. A 4.00-g bullet, traveling horizontally with a velocity of magnitude 400 m/s, is fired into a wooden block with mass 0.800 kg, initially at rest on a level surface. The bullet passes through the block and emerges with its speed reduced to 120 m/s. The block slides a distance of 45.0 cm along the surface from its initial position. (a) What is the coefficient of kinetic friction between block and surface? (b) What is the decrease in kinetic energy of the bullet? (c) What is the kinetic energy of the block at the instant after the bullet passes through it?

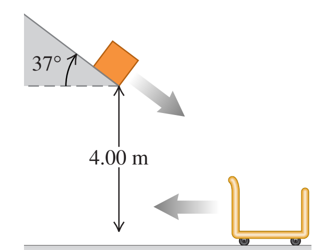
8.84. A 5.00-g bullet is shot *through* a 1.00-kg wood block suspended on a string 2.00 m long. The center of mass of the block rises a distance of 0.45 cm. Find the speed of the bullet as it emerges from the block if its initial speed is 450 m/s.

8.85. A neutron with mass m makes a head-on, elastic collision with a nucleus of mass M, which is initially at rest. (a) Show that if the neutron's initial kinetic energy is K_0, the kinetic energy that it loses during the collision is $4mMK_0/(M+m)^2$. (b) For what value of M does the incident neutron lose the most energy? (c) When M has the value calculated in part (b), what is the speed of the neutron after the collision?

8.86. Energy Sharing in Elastic Collisions. A stationary object with mass m_B is struck head-on by an object with mass m_A that is moving initially at speed v_0. (a) If the collision is elastic, what percentage of the original energy does each object have after the collision? (b) What does your answer in part (a) give for the special cases (i) $m_A = m_B$ and (ii) $m_A = 5m_B$? (c) For what values, if any, of the mass ratio m_A/m_B is the original kinetic energy shared equally by the two objects after the collision?

8.87. In a shipping company distribution center, an open cart of mass 50.0 kg is rolling to the left at a speed of 5.00 m/s (Fig. 8.46). You can ignore friction between the cart and the floor. A 15.0-kg package slides down a chute that is inclined at 37° from the horizontal and leaves the end of the chute with a speed of 3.00 m/s. The package lands in the cart and they roll off together. If the lower end of the chute is a vertical distance of 4.00 m above the bottom of the cart, what are (a) the speed of the package just before it lands in the cart and (b) the final speed of the cart?

Figure 8.46 Problem 8.87.



8.88. A blue puck with mass 0.0400 kg, sliding with a velocity of magnitude 0.200 m/s on a frictionless, horizontal air table, makes a perfectly elastic, head-on collision with a red puck with mass m, initially at rest. After the collision, the velocity of the blue puck is 0.050 m/s in the same direction as its initial velocity. Find (a) the velocity (magnitude and direction) of the red puck after the collision; and (b) the mass m of the red puck.

8.89. Two asteroids with masses m_A and m_B are moving with velocities $\vec{v}_A$ and $\vec{v}_B$ with respect to an astronomer in a space vehicle. (a) Show that the total kinetic energy as measured by the astronomer is

$$K = \tfrac{1}{2}Mv_{cm}^2 + \tfrac{1}{2}(m_A v_A'^2 + m_B v_B'^2)$$

with $\vec{v}_{cm}$ and M defined as in Section 8.5, $\vec{v}_A' = \vec{v}_A - \vec{v}_{cm}$, and $\vec{v}_B' = \vec{v}_B - \vec{v}_{cm}$. In this expression the total kinetic energy of the two asteroids is the energy associated with their center of mass plus the energy associated with the internal motion relative to the center of mass. (b) If the asteroids collide, what is the *minimum* possible kinetic energy they can have after the collision, as measured by the astronomer? Explain.

8.90. Suppose you hold a small ball in contact with, and directly over, the center of a large ball. If you then drop the small ball a short time after dropping the large ball, the small ball rebounds with surprising speed. To show the extreme case, ignore air resistance and suppose the large ball makes an elastic collision with the floor and then rebounds to make an elastic collision with the still-descending small ball. Just before the collision between the two balls, the large ball is moving upward with velocity $\vec{v}$ and the small ball has velocity $-\vec{v}$. (Do you see why?) Assume the large ball has a much greater mass than the small ball. (a) What is the velocity of the small ball immediately after its collision with the large ball? (b) From the answer to part (a), what is the ratio of the small ball's rebound distance to the distance it fell before the collision?

8.91. Jack and Jill are standing on a crate at rest on the frictionless, horizontal surface of a frozen pond. Jack has mass 75.0 kg, Jill has mass 45.0 kg, and the crate has mass 15.0 kg. They remember that they must fetch a pail of water, so each jumps horizontally from the top of the crate. Just after each jumps, that person is moving away from the crate with a speed of 4.00 m/s relative to the crate. (a) What is the final speed of the crate if both Jack and Jill jump simultaneously and in the same direction? (*Hint:* Use an inertial coordinate system attached to the ground.) (b) What is the final speed of the crate if Jack jumps first and then a few seconds later Jill jumps in the same direction? (c) What is the final speed of the crate if Jill jumps first and then Jack, again in the same direction?

8.92. Energy Sharing. An object with mass m, initially at rest, explodes into two fragments, one with mass m_A and the other with mass m_B, where $m_A + m_B = m$. (a) If energy Q is released in the explosion, how much kinetic energy does each fragment have immediately after the collision? (b) What percentage of the total energy released does each fragment get when one fragment has four times the mass of the other?

8.93. Neutron Decay. A neutron at rest decays (breaks up) to a proton and an electron. Energy is released in the decay and appears as kinetic energy of the proton and electron. The mass of a proton is 1836 times the mass of an electron. What fraction of the total energy released goes into the kinetic energy of the proton?

8.94. A ^{232}Th (thorium) nucleus at rest decays to a ^{228}Ra (radium) nucleus with the emission of an alpha particle. The total kinetic energy of the decay fragments is 6.54×10^{-13} J. An alpha particle has 1.76% of the mass of a ^{228}Ra nucleus. Calculate the kinetic energy of (a) the recoiling ^{228}Ra nucleus and (b) the alpha particle.

8.95. Antineutrino. In beta decay, a nucleus emits an electron. A ^{210}Bi (bismuth) nucleus at rest undergoes beta decay to ^{210}Po (polonium). Suppose the emitted electron moves to the right with a momentum of 5.60×10^{-22} kg·m/s. The ^{210}Po nucleus, with mass 3.50×10^{-25} kg, recoils to the left at a speed of 1.14×10^{-3} m/s. Momentum conservation requires that a second particle, called an antineutrino, must also be emitted. Calculate the magnitude and direction of the momentum of the antineutrino that is emitted in this decay.

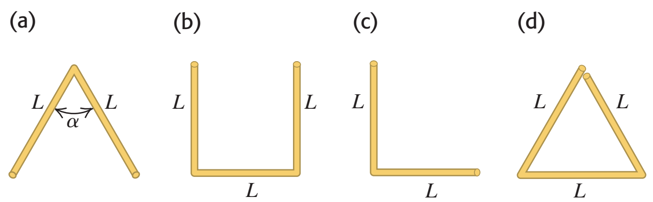
8.96. A proton moving with speed v_{A1} in the $+x$-direction makes an elastic, off-center collision with an identical proton originally at rest. After impact, the first proton moves with speed v_{A2} in the first quadrant at an angle α with the x-axis, and the second moves with speed v_{B2} in the fourth quadrant at an angle β with the x-axis (Fig. 8.13). (a) Write the equations expressing conservation of linear momentum in the x- and y-directions. (b) Square the equations from part (a) and add them. (c) Now introduce the fact that the collision is elastic. (d) Prove that $\alpha + \beta = \pi/2$. (You have shown that this equation is obeyed in any elastic, off-center collision between objects of equal mass when one object is initially at rest.)

8.97. Hockey puck B rests on a smooth ice surface and is struck by a second puck A, which has the same mass. Puck A is initially traveling at 15.0 m/s and is deflected 25.0° from its initial direction. Assume that the collision is perfectly elastic. Find the final speed of each puck and the direction of B's velocity after the collision. [*Hint:* Use the relationship derived in part (d) of Problem 8.96.]

8.98. Jonathan and Jane are sitting in a sleigh that is at rest on frictionless ice. Jonathan's weight is 800 N, Jane's weight is 600 N, and that of the sleigh is 1000 N. They see a poisonous spider on the floor of the sleigh and immediately jump off. Jonathan jumps to the left with a velocity of 5.00 m/s at 30.0° above the horizontal (relative to the ice), and Jane jumps to the right at 7.00 m/s at 36.9° above the horizontal (relative to the ice). Calculate the sleigh's horizontal velocity (magnitude and direction) after they jump out.

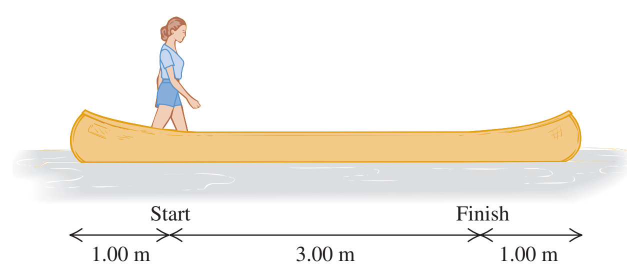
8.99. The objects in Fig. 8.47 are constructed of uniform wire bent into the shapes shown. Find the position of the center of mass of each.

Figure 8.47 Problem 8.99.



8.100. A 45.0-kg woman stands up in a 60.0-kg canoe 5.00 m long. She walks from a point 1.00 m from one end to a point 1.00 m from the other end (Fig. 8.48). If you ignore resistance to motion of the canoe in the water, how far does the canoe move during this process?

Figure 8.48 Problem 8.100.



8.101. You are standing on a concrete slab that in turn is resting on a frozen lake. Assume there is no friction between the slab and the ice. The slab has a weight five times your weight. If you begin walking forward at 2.00 m/s relative to the ice, with what speed, relative to the ice, does the slab move?

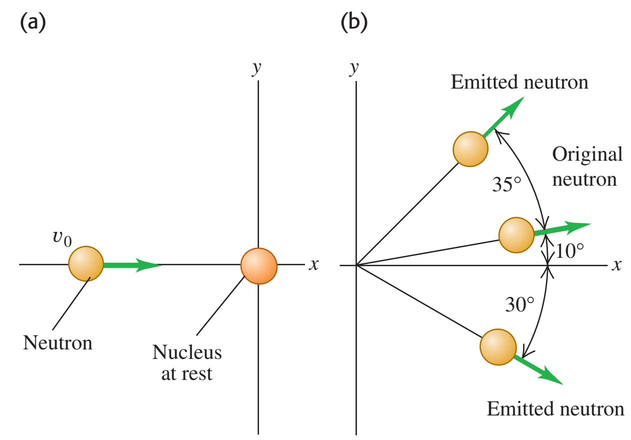
8.102. A 20.0-kg projectile is fired at an angle of 60.0° above the horizontal with a speed of 80.0 m/s. At the highest point of its trajectory, the projectile explodes into two fragments with equal mass, one of which falls vertically with zero initial speed. You can ignore air resistance. (a) How far from the point of firing does the other fragment strike if the terrain is level? (b) How much energy is released during the explosion?

8.103. A fireworks rocket is fired vertically upward. At its maximum height of 80.0 m, it explodes and breaks into two pieces, one with mass 1.40 kg and the other with mass 0.28 kg. In the explosion, 860 J of chemical energy is converted to kinetic energy of the two fragments. (a) What is the speed of each fragment just after the explosion? (b) It is observed that the two fragments hit the ground at the same time. What is the distance between the points on the ground where they land? Assume that the ground is level and air resistance can be ignored.

8.104. A 12.0-kg shell is launched at an angle of 55.0° above the horizontal with an initial speed of 150 m/s. When it is at its highest point, the shell exploded into two fragments, one three times heavier than the other. The two fragments reach the ground at the same time. Assume that air resistance can be ignored. If the heavier fragment lands back at the same point from which the shell was launched, where will the lighter fragment land and how much energy was released in the explosion?

8.105. A Nuclear Reaction. Fission, the process that supplies energy in nuclear power plants, occurs when a heavy nucleus is split into two medium-sized nuclei. One such reaction occurs when a neutron colliding with a ^{235}U (uranium) nucleus splits that nucleus into a ^{141}Ba (barium) nucleus and a ^{92}Kr (krypton) nucleus. In this reaction, two neutrons also are split off from the original ^{235}U. Before the collision, the arrangement is as shown in Fig. 8.49a. After the collision, the ^{141}Ba nucleus is moving in the $+z$-direction and the ^{92}Kr nucleus in the $-z$-direction. The three neutrons are moving in the xy-plane, as shown in Fig. 8.49b. If the incoming neutron has an initial velocity of magnitude 3.0×10^3 m/s and a final velocity of magnitude 2.0×10^3 m/s in the directions shown, what are the speeds of the other two neutrons, and what can you say about the speeds of the ^{141}Ba and ^{92}Kr nuclei? (The mass of the ^{141}Ba nucleus is approximately 2.3×10^{-25} kg, and the mass of ^{92}Kr is about 1.5×10^{-25} kg.)

Figure 8.49 Problem 8.105.



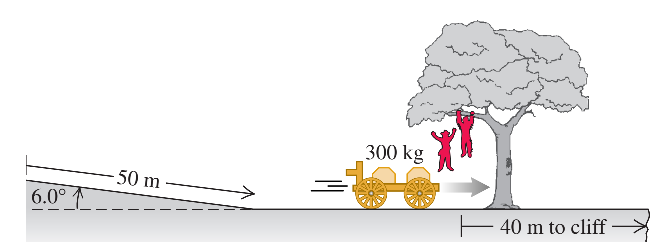
8.106. Center-of-Mass Coordinate System. Puck A (mass m_A) is moving on a frictionless, horizontal air table in the $+x$-direction with velocity $\vec{v}_{A1}$ and makes an elastic, head-on collision with puck B (mass m_B), which is initially at rest. After the collision, both pucks are moving along the x-axis. (a) Calculate the velocity of the center of mass of the two-puck system before the collision. (b) Consider a coordinate system whose origin is at the center of mass and moves with it. Is this an inertial reference frame? (c) What are the initial velocities $\vec{u}_{A1}$ and $\vec{u}_{B1}$ of the two pucks in this center-of-mass reference frame? What is the total momentum in this frame? (d) Use conservation of momentum and energy, applied in the center-of-mass reference frame, to relate the final momentum of each puck to its initial momentum and thus the final velocity of each puck to its initial velocity. Your results should show that a one-dimensional, elastic collision has a very simple description in center-of-mass coordinates. (e) Let $m_A = 0.400$ kg, $m_B = 0.200$ kg, and $v_{A1} = 6.00$ m/s. Find the center-of-mass velocities $\vec{u}_{A1}$ and $\vec{u}_{B1}$, apply the simple result found in part (d), and transform back to velocities in a stationary frame to find the final velocities of the pucks. Does your result agree with Eqs. (8.24) and (8.25)?

8.107. The *coefficient of restitution* ϵ for a head-on collision is defined as the ratio of the relative speed after the collision to the relative speed before. (a) What is ϵ for a completely inelastic collision? (b) What is ϵ for an elastic collision? (c) A ball is dropped from a height h onto a stationary surface and rebounds back to a height H_1. Show that $\epsilon = \sqrt{H_1/h}$. (d) A properly inflated basketball should have a coefficient of restitution of 0.85. When dropped from a height of 1.2 m above a solid wood floor, to what height should a properly inflated basketball bounce? (e) The height of the first bounce is H_1. If ϵ is constant, show that the height of the nth bounce is $H_n = \epsilon^{2n}h$. (f) If ϵ is constant, what is the height of the eighth bounce of a properly inflated basketball dropped from 1.2 m?

8.108. Binding Energy of the Hydrogen Molecule. When two hydrogen atoms of mass m combine to form a diatomic hydrogen molecule (H_2), the potential energy of the system after they combine is $-\Delta$, where Δ is a positive quantity called the *binding energy* of the molecule. (a) Show that in a collision that involves only two hydrogen atoms, it is *impossible* to form an H_2 molecule because momentum and energy cannot simultaneously be conserved. (*Hint:* If you can show this to be true in one frame of reference, then it is true in all frames of reference. Can you see why?) (b) An H_2 molecule can be formed in a collision that involves *three* hydrogen atoms. Suppose that before such a collision, each of the three atoms has speed 1.00×10^3 m/s, and they are approaching at 120° angles so that at any instant, the atoms lie at the corners of an equilateral triangle. Find the speeds of the H_2 molecule and of the single hydrogen atom that remains after the collision. The binding energy of H_2 is $\Delta = 7.23 \times 10^{-19}$ J, and the mass of the hydrogen atom is 1.67×10^{-27} kg.

8.109. A wagon with two boxes of gold, having total mass 300 kg, is cut loose from the horses by an outlaw when the wagon is at rest 50 m up a 6.0° slope (Fig. 8.50). The outlaw plans to have the wagon roll down the slope and across the level ground, and then fall into a canyon where his confederates wait. But in a tree 40 m from the canyon edge wait the Lone Ranger (mass 75.0 kg) and Tonto (mass 60.0 kg). They drop vertically into the wagon as it passes beneath them. (a) If they require 5.0 s to grab the gold and jump out, will they make it before the wagon goes over the edge? The wagon rolls with negligible friction. (b) When the two heroes drop into the wagon, is the kinetic energy of the system of the heroes plus the wagon conserved? If not, does it increase or decrease, and by how much?

Figure 8.50 Problem 8.109.



***8.110.** In Section 8.6, we considered a rocket fired in outer space where there is no air resistance and where gravity is negligible. Suppose instead that the rocket is accelerating vertically upward from rest on the earth's surface. Continue to ignore air resistance and consider only that part of the motion where the altitude of the rocket is small so that g may be assumed to be constant. (a) How is Eq. (8.37) modified by the presence of the gravity force? (b) Derive an expression for the acceleration a of the rocket, analogous to Eq. (8.39). (c) What is the acceleration of the rocket in Example 8.15 (Section 8.6) if it is near the earth's surface rather than in outer space? You can ignore air resistance. (d) Find the speed of the rocket in Example 8.16 (Section 8.6) after 90 s if the rocket is fired from the earth's surface rather than in outer space. You can ignore air resistance. How does your answer compare with the rocket speed calculated in Example 8.16?

***8.111. A Multistage Rocket.** Suppose the first stage of a two-stage rocket has total mass 12,000 kg, of which 9000 kg is fuel. The total mass of the second stage is 1000 kg, of which 700 kg is fuel. Assume that the relative speed v_{ex} of ejected material is constant, and ignore any effect of gravity. (The effect of gravity is small during the firing period if the rate of fuel consumption is large.) (a) Suppose the entire fuel supply carried by the two-stage rocket is utilized in a single-stage rocket with the same total mass of 13,000 kg. In terms of v_{ex}, what is the speed of the rocket, starting from rest, when its fuel is exhausted? (b) For the two-stage rocket, what is the speed when the fuel of the first stage is exhausted if the first stage carries the second stage with it to this point? This speed then becomes the initial speed of the second stage. At this point, the second stage separates from the first stage. (c) What is the final speed of the second stage? (d) What value of v_{ex} is required to give the second stage of the rocket a speed of 7.00 km/s?

***8.112.** For the rocket described in Examples 8.15 and 8.16 (Section 8.6), the mass of the rocket as a function of time is

$$m(t) = \begin{cases} m_0 & \text{for } t < 0 \\ m_0\left(1 - \dfrac{t}{120\text{ s}}\right) & \text{for } 0 \le t \le 90\text{ s} \\ m_0/4 & \text{for } t \ge 90\text{ s} \end{cases}$$

(a) Calculate and graph the velocity of the rocket as a function of time from $t = 0$ to $t = 100$ s. (b) Calculate and graph the acceleration of the rocket as a function of time from $t = 0$ to $t = 100$ s. (c) A 75-kg astronaut lies on a reclined chair during the firing of the rocket. What is the maximum net force exerted by the chair on the astronaut during the firing? How does your answer compare with her weight on earth?

Challenge Problems

8.113. In Section 8.5 we calculated the center of mass by considering objects composed of a *finite* number of point masses or objects that, by symmetry, could be represented by a finite number of point masses. For a solid object whose mass distribution does not allow for a simple determination of the center of mass by symmetry, the sums of Eqs. (8.28) must be generalized to integrals

$$x_{cm} = \frac{1}{M}\int x\,dm \qquad y_{cm} = \frac{1}{M}\int y\,dm$$

where x and y are the coordinates of the small piece of the object that has mass dm. The integration is over the whole of the object. Consider a thin rod of length L, mass M, and cross-sectional area A. Let the origin of the coordinates be at the left end of the rod and the positive x-axis lie along the rod. (a) If the density $\rho = M/V$ of the object is uniform, perform the integration described above to show that the x-coordinate of the center of mass of the rod is at its geometrical center. (b) If the density of the object varies linearly with x—that is, $\rho = \alpha x$, where α is a positive constant—calculate the x-coordinate of the rod's center of mass.

8.114. Use the methods of Challenge Problem 8.113 to calculate the x- and y-coordinates of the center of mass of a semicircular metal plate with uniform density ρ and thickness t. Let the radius of the plate be a. The mass of the plate is thus $M = \frac{1}{2}\rho\pi a^2 t$. Use the coordinate system indicated in Fig. 8.51.

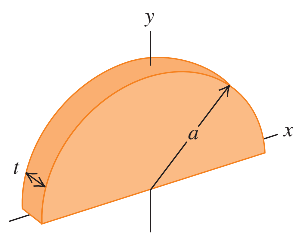


Figure 8.51 Challenge Problem 8.114.

8.115. One-fourth of a rope of length l is hanging down over the edge of a frictionless table. The rope has a uniform, linear density (mass per unit length) λ (Greek lambda), and the end already on the table is held by a person. How much work does the person do when she pulls on the rope to raise the rest of the rope slowly onto the table? Do the problem in two ways as follows. (a) Find the force that the person must exert to raise the rope and from this the work done. Note that this force is variable because at different times, different amounts of rope are hanging over the edge. (b) Suppose the segment of the rope initially hanging over the edge of the table has all of its mass concentrated at its center of mass. Find the work necessary to raise this to table height. You will probably find this approach simpler than that of part (a). How do the answers compare, and why is this so?

***8.116 A Variable-Mass Raindrop.** In a rocket-propulsion problem the mass is variable. Another such problem is a raindrop falling through a cloud of small water droplets. Some of these small droplets adhere to the raindrop, thereby *increasing* its mass as it falls. The force on the raindrop is

$$F_{\text{ext}} = \frac{dp}{dt} = m\frac{dv}{dt} + v\frac{dm}{dt}$$

Suppose the mass of the raindrop depends on the distance x that it has fallen. Then $m = kx$, where k is a constant, and $dm/dt = kv$. This gives, since $F_{\text{ext}} = mg$,

$$mg = m\frac{dv}{dt} + v(kv)$$

Or, dividing by k,

$$xg = x\frac{dv}{dt} + v^2$$

This is a differential equation that has a solution of the form $v = at$, where a is the acceleration and is constant. Take the initial velocity of the raindrop to be zero. (a) Using the proposed solution for v, find the acceleration a. (b) Find the distance the raindrop has fallen in $t = 3.00$ s. (c) Given that $k = 2.00$ g/m, find the mass of the raindrop at $t = 3.00$ s. For many more intriguing aspects of this problem, see K. S. Krane, *Amer. Jour. Phys.,* Vol. 49 (1981), pp. 113–117.