Chapter 13 - Elementary Particles

13-1: (a) From the uncertainty principle, with $\Delta E = 2\,mc^2$, the uncertainty in the time that such an electron-positron pair may exist is

$$\Delta t \geq \frac{\hbar/2}{2\,mc^2} = \frac{\left(6.582 \times 10^{-16}\ \text{eV}\cdot\text{s}\right)}{4\left(0.511 \times 10^{6}\ \text{eV}\right)} = 3.22 \times 10^{-22}\ \text{s}.$$

(b) The strong electric field of the nucleus separates the electron and positron sufficiently so that they cannot recombine afterward to reconstitute the photon.

13-3: A relativistic calculation, including the recoil of the Λ^0 particle, is necessary. The Λ^0 particle and the photon will have momentum of magnitude

$$p_{\Lambda^0} = p_\gamma = \frac{E_\gamma}{c}.$$

This may be used to relate the total energy of the Λ^0 particle to the photon energy,

$$p_{\Lambda^0}^2 c^2 = E_{\Lambda^0}^2 - \left(m_{\Lambda^0} c^2\right)^2 = E_\gamma^2.$$

From conservation of energy,

$$m_{\Sigma^0} c^2 = E_{\Lambda^0} + E_\gamma, \qquad \text{and}$$

$$E_{\Lambda^0}^2 = \left(m_{\Sigma^0} c^2\right)^2 + E_\gamma^2 - 2\,m_{\Sigma^0} c^2\, E_\gamma.$$

Equating the two expressions for $E_{\Lambda^0}^2$, canceling the E_γ^2 term and solving for the photon energy,

$$E_\gamma = \frac{\left(m_{\Sigma^0} c^2\right)^2 - \left(m_{\Lambda^0} c^2\right)^2}{2\,m_{\Sigma^0} c^2} = \frac{(1193\ \text{MeV})^2 - (1116\ \text{MeV})^2}{2\,(1193\ \text{MeV})} = 74.5\ \text{MeV}.$$

The above expression for the photon energy may be expressed as

$$E_\gamma = \left(m_{\Sigma^0} c^2 - m_{\Lambda^0} c^2\right)\left(1 - \frac{m_{\Sigma^0} - m_{\Lambda^0}}{2\,m_{\Sigma^0}}\right),$$

which shows that in the nonrelativistic limit, $m_{\Sigma^0} \gg m_{\Sigma^0} - m_{\Lambda^0}$, the photon energy is just the difference between the rest mass energies of the particles.

13-5: The minimum photon energy would be for the situation where all three of the final electrons have the same momentum (no relative motion, and hence no motion in the center of mass frame). Denote this common momentum magnitude by p'. Assuming the initial electron to be at rest, the initial photon momentum would be $3\,p'$, and the initial photon energy is $E_\gamma = 3\,p'c$. From conservation of energy,

$$E_\gamma + m_e c^2 = 3\,E', \qquad \left(E_\gamma + m_e c^2\right)^2 = 9\,E'^2 = 9\left(\left(p'c\right)^2 + \left(m_e c^2\right)^2\right),$$

where the common final energy of each electron is E'. Squaring the binomial and using $9\left(p'c\right)^2 = E_\gamma^2$,

$$\begin{aligned} E_\gamma^2 + 2\,E_\gamma\,m_e c^2 + \left(m_e c^2\right)^2 &= E_\gamma^2 + 9\left(m_e c^2\right)^2 \\ 2\,E_\gamma\,m_e c^2 &= 8\left(m_e c^2\right)^2. \end{aligned}$$

from which $E_\gamma = 4\left(m_e c^2\right)^2$ follows.

As an equivalent alternative, consider the center of mass frame (more accurately, the center of momentum frame) in which the three electrons are created at rest, and hence with zero momentum. In this frame, the initial momentum of the photon and electron would have the same magnitude p_0, and the photon would have energy $E_0 = p_0 c$. From conservation of energy,

$$E_0 + \sqrt{m_e^2 c^4 + p_0^2 c^2} = 3\,m_e c^2, \qquad \sqrt{m_e^2 c^4 + E_0^2} = 3\,m_e c^2 - E_0.$$

Squaring both sides of the second relation and canceling E_0^2 gives

$$m_e^2 c^4 = 9\,m_e^2 c^4 - 6\,m_e c^2\,E_0, \qquad \text{and} \qquad E_0 = \frac{4}{3}\,m_e c^2, \qquad p_0 = \frac{4}{3}\,m_e c.$$

From Equation (1.16), then,

$$\frac{(v/c)}{\sqrt{1-(v/c)^2}} = \frac{4}{3},$$

which is solved for $(v/c) = (4/5)$, and this must be the speed of the center of mass frame relative to the frame where the electron was initially at rest (the lab frame). From Equation (1.8), the energy of the photon in the lab frame would be higher than E_0,

$$E_\gamma = E_0\sqrt{\frac{1+(v/c)}{1-(v/c)}} = \frac{4}{3}\,m_e c^2\sqrt{(9/5)/(1/5)} = 4\,m_e c^2.$$

13-7: Denote the initial pion momentum by p_π and the gamma-ray momentum magnitude by p_γ. From conservation of momentum, $p_\pi = 2\,p_\gamma \cos\theta$, where θ is the half-angle between the gamma-ray paths (θ is the angle that each gamma ray makes with respect to the initial direction of the pion's momentum). The inital energy of the pion in terms of its momentum is

$$\sqrt{p_\pi^2\,c^2 + m_\pi^2\,c^4} = 2\,m_\pi\,c^2, \qquad \text{so} \qquad p_\pi = \sqrt{3}\,m_\pi\,c.$$

From conservation of energy,

$$\mathrm{KE}_\pi + m_\pi\,c^2 = w\,m_\pi\,c^2 = 2\,p_\gamma\,c, \qquad \text{so} \qquad m_\pi c = p_\gamma \qquad \text{and} \qquad p_\pi = \sqrt{3}\,p_\gamma.$$

Equating the two expressions relating p_π and p_γ yields $\cos\theta = \dfrac{\sqrt{3}}{2}$, and so $\theta = 30°$ (giving θ in radians as $\pi/6$ might cause confusion). The angle between the two gamma rays is $2\,\theta = 60°$.

13-9: (**a**) does not conserve baryon number. (**b**) can occur. (**c**) does not conserve charge. (**d**) can occur.

13-11: The spontaneous appearance of neutron-antineutron pairs would violate conservation of energy.

13-13: The other particle must have charge $\mathcal{Q} = 0$ and muonic lepton number $L_\mu = +1$. The only such particle is ν_μ, a μ-neutrino.

13-15: The other particle must have charge $\mathcal{Q} = -e$, baryon number $B = +1$ and strangeness $S = -2$. (The original negative kaon had strangeness $S = +1$ and the final kaon has strangeness $S = -1$.) From Table 13.1, the only such particle is the Ξ^-, a negative xi particle.

13-17: Quarks are fermions, and if quarks with the same color combined to form a hadron, the exclusion principle would be violated. If the spins of quarks were integral instead of half-integral, the exclusion principle would not apply, and quarks of the same color could be the constituents of a hadron.

13-19: From Table 13.4, the sum of the charges of two u quarks and an s quark is $2\,\frac{2}{3}e - \frac{1}{3}e = +e$, and the particle is Σ^+ as given in Table 13.3

13-21: The combination uus has strangeness $S = -1$ and charge $\mathcal{Q} = 2\,\frac{2}{3}e - \frac{1}{3}e = +e$, and from Table 13.3 the particle is Σ^+.

13-23: Only the strong interaction, which affects only hadrons, can produce such rapid decays.

13-25: Because a positron and a neutrino are emitted, the weak interaction is involved; the weak interaction is so much feebler than the strong interaction that the initial reaction of the proton-proton cycle has a low probability of occuring even when the protons are energetic enough to overcome the Coulomb barrier.

13-27: (a) If the angular separation of two spots is the angle θ, in radians, then $s = r\,\theta$ and

$$\frac{ds}{dt} = \frac{dr}{dt}\,\theta = \frac{dr}{dt}\,\frac{s}{r} = \frac{1}{r}\,\frac{dr}{dt}\,s.$$

where the constancy of the angular separation is used to set $\frac{d\theta}{dt} = 0$. The radius r and the rate of change $\frac{dr}{dt}$ are the same for all points on the sphere at any time.

(b) The parameter H is then the factor multiplying s in the above expression for $\frac{ds}{dt}$;

$$H = \frac{1}{r}\,\frac{dr}{dt},$$

which is sometimes expressed as

$$H = \frac{d}{dt}\ln(r).$$

In this form, it is readily seen that if $\ln r = \ln r_0 + k\,t$, H is constant and equal to k, but if not, H is not constant. Thus, H will be constant if $r = r_0\,e^{k\,t}$. If this is the case with $k > 0$, the balloon is expanding at an every-increasing rate. This phenomenon would be like the proposed "inflationary universe" theory of the early universe. If $k < 0$, the balloon is shrinking, gradually approaching zero radius. If $k = 0$, $H = 0$ and the balloon is neither expanding or shrinking.