Chapter 12 - Nuclear Transformations

In finding energy changes in nuclear transformations, accurate values of atomic masses are needed. The Appendix in the text, like most tables, gives the masses of *neutral* atoms; that is, the masses are the sum of the nuclear masses and the electrons, and include any binding energy of the electrons in their orbits. These electron binding energies are usually on the order of a few electronvolts and the precision of the masses given does not depend on the electron energies. As a specific example, the mass equivalent of the binding energy of an electron in the first Bohr orbit in a hydrogen atom is $(13.6\ \mathrm{eV})/\,(931.49\ \mathrm{MeV/u}) = 1.5\times10^{-8}\ \mathrm{u}$, a value too small to be reflected in the given atomic masses.

For four of the five possible nuclear transformation, it is sufficient to recognize that the initial and final states consist only of neutral atoms, and the electron masses need not be considered. For positron emission, the extra mass of the positron, which normally cannot exist in a netural atom, must be included.

Consider the reaction corresponding to positron emission (negative beta decay), represented as

$$\,^{A}_{Z}\mathrm{X} \qquad \longrightarrow \qquad \,^{A}_{Z-1}\mathrm{Y} + e^{+}.$$

The change in the nuclear masses is related to the change in the neutral atomic masses by

$$\left[m\left(\,^{A}_{Z}\mathrm{X}\right) - Z\,m_e\right] - \left[m\left(\,^{A}_{Z-1}\mathrm{Y}\right) - (Z-1)\,m_e\right] = \left[m\left(\,^{A}_{Z}\mathrm{X}\right) - m\left(\,^{A}_{Z-1}\mathrm{Y}\right)\right] + m_e,$$

where $m\left(\,^{A}_{Z}\mathrm{X}\right)$ and $m\left(\,^{A}_{Z-1}\mathrm{Y}\right)$ are the atomic masses as tabulated in the Appendix. However, the final state is *not* a neutral Y atom; there are still Z electrons and the emitted postitron. The atom is neutral, but it contains an extra electron-positron pair (for a short while), and this mass must be included in the final state. The result is that for positron emission, the energy released in the nuclear reaction is the energy of

$$m\left(\,^{A}_{Z}\mathrm{X}\right) - m\left(\,^{A}_{Z-1}\mathrm{Y}\right) - 2\,m_e,$$

m_e being the common mass of the electron and positron.

The above discussion relates to Problems 12-27 and 12-62, among others.

12-1: 25 y is twice the half-life, so the fraction of the sample remaining is $2^{-2} = 1/4$.

12-3: The time of 1.00 s is small compared to the half-life of 37.3 min = 2232 s, and so N may be taken as a constant during this time. The probability that a particular nucleus will undergo beta decay is then the product of the decay constant and the time,

$$P = \lambda\,\Delta t = \frac{\ln 2}{T_{1/2}}\Delta t = \frac{\ln 2}{2232\ \text{s}}\,1.00\ \text{s} = 3.1 \times 10^{-4}.$$

It should be noted that for calculators with sufficient precision, the above result is the same as

$$P = 1 - 2^{-\Delta t/T_{1/2}}.$$

12-5: The decay constant is $\lambda = \ln 2/T_{1/2}$; using this in Equation (12.5) and solving for the time t,

$$t = \frac{T_{1/2}}{\ln 2}\ \ln\left(N_0/N\right) = \frac{15.0\ \text{hr}}{\ln 2}\ \ln(5.0) = 34.8\ \text{hr}.$$

12-7: Using Equation (12.3), the half-life is related to the decay constant by

$$T_{1/2} = \frac{\ln 2}{\lambda} = \ln 2\,\frac{N}{R}.$$

The number N of nuclei is the mass of the sample divided by the atomic mass. Approximating the atomic mass by the atomic number (see the Appendix),

$$\begin{aligned} T_{1/2} &= \ln 2\,\frac{\left(1.00 \times 10^{-3}\ \text{kg}\right)}{(226\ \text{u})\,(1.66054 \times 10^{-27}\ \text{kg/u})}\,\frac{1}{(3.70 \times 10^{10}\ \text{Bq})} \\ &= 5.0 \times 10^{10}\ \text{s} = 1.6 \times 10^{3}\ \text{y}. \end{aligned}$$

12-9: From Equation (12.8), the activity is $R = \lambda N$ and the total number of atoms is the total mass divided by the mass of an atom, so

$$\begin{aligned} R = \lambda\,N &= \frac{\ln 2}{T_{1/2}}\,\frac{m}{m\left({}^{238}_{92}\text{U}\right)} \\ &= \frac{\ln 2}{(4.5 \times 10^{9}\ \text{yr})\,(3.156 \times 10^{7}\ \text{s/y})}\,\frac{\left(1.0 \times 10^{-3}\ \text{kg}\right)}{(238\ \text{u})\,(1.66054 \times 10^{-27}\ \text{kg/u})} \\ &= 1.23\ \text{Bq}, \end{aligned}$$

keeping an extra significant figure.

12-11: The mass needed is the number of nuclei times the mass of each nucleus. Using Equation (12.8),

$$\begin{aligned} m &= m\left({}^{210}_{84}\text{Po}\right)\frac{R}{\lambda} = m\left({}^{210}_{84}\text{Po}\right)\frac{R T_{1/2}}{\ln 2} \\ &= (210\ \text{u})\left(1.66054\times10^{-27}\ \text{kg/u}\right)\frac{\left(3.7\times10^{8}\ \text{Bq}\right)(138\ \text{d})(86,400\ \text{s/d})}{\ln 2} \\ &= 2.22\times10^{-9}\ \text{kg}. \end{aligned}$$

12-13: Solving Equation (12.2) for the product λt gives $\lambda t = -\ln(R/R_0)$. Taking the time of the first measurement to be $t = 0$, so that the first measurement is taken as R_0, the natural logarithms of the activites as fractions of the initial activity are

$$\ln\left(\frac{80.5}{80.5}\right) = 0, \qquad \ln\left(\frac{36.2}{80.5}\right) = -0.799, \qquad \ln\left(\frac{16.3}{80.5}\right) = -1.597,$$

$$\ln\left(\frac{7.3}{80.5}\right) = -2.400, \qquad \ln\left(\frac{3.3}{80.5}\right) = -3.194.$$

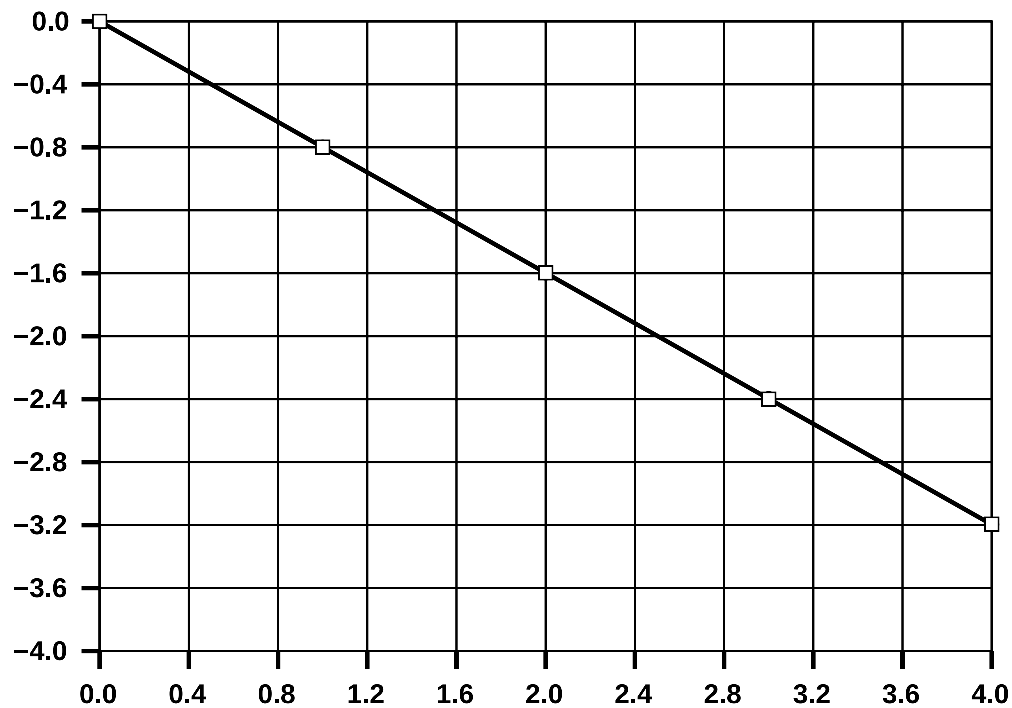
Four significant figures are not really warranted here, but are included for the intermediate calculations. Rounded to the hundredths place, the natural logarithms are 0, -0.80, -1.60, -2.40 and -3.20, and so $\ln(R/R_0)$ is proportional to the time since the measurements began. From these data, it should be clear that to two significant figures, $-\lambda\,(1.0\ \text{h}) = -0.80$. Using a plot to see the proportionality confirms this.

The plot for Problem 12-13 is shown on the next page.

The slope of the line that best fits the data is $-0.80\ \text{h}^{-1}$, and so the experimental value for λ is $0.80\ \text{h}^{-1}$ The plot was generated using a spreadsheet program that finds the slope of the best-fit line as $-0.799\ \text{hr}^{-1}$, the same to two significant figures. The half-life $T_{1/2}$ is

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{\left(0.80\ \text{h}^{-1}\right)(1\ \text{h}/60\ \text{min})} = 52\ \text{min}.$$

Plot for Problem 12-13



12-15: When the rock was formed, each nucleus that is currently a lead nucleus (^{206}Pb) was a uranium nulceus (^{238}U). The original mass M_0 of the sample would therefore have been

$$M_0 = M_{\rm U} + M_{\rm Pb}\,\frac{238}{206} = (4.00\ {\rm mg})\left(1 + \left(\frac{1.00\ {\rm mg}}{4.00\ {\rm mg}}\right)\left(\frac{238}{206}\right)\right) = (4.00\ {\rm mg})(1.289).$$

The ratio of the initial number of unranium nuclei to the current number is the same as the ratio of the masses. Solving Equation (12.5) for the time t,

$$t = \frac{\ln(N_0/N)}{\lambda} = \frac{\ln(M_0/M)}{(\ln 2/T_{1/2})} = \frac{T_{1/2}\,\ln(M_0/M)}{\ln 2}$$

$$= \frac{(4.47\times 10^9\ {\rm y})\,\ln(1.289)}{\ln 2} = 1.64\times 10^9\ {\rm y}.$$

12-17: See Example 12.5. The time since the wood was burned is

$$t = -\frac{1}{\lambda}\ln\left(\frac{R}{R_0}\right) = -\frac{T_{1/2}}{\ln 2}\ln\left(\frac{R}{R_0}\right) = -\frac{5760\ {\rm y}}{\ln 2}\ln(0.18) = 1.4\times 10^4\ {\rm y}.$$

12-19: With the assumption that equal amounts of the nuclides (deonted A and B) were created, the ratio of the relative abundances as a function of time is

$$\frac{N_A}{N_B} = \frac{e^{-\lambda_A t}}{e^{-\lambda_B t}} = e^{(\lambda_B - \lambda_A)\,t}.$$

Solving for t,

$$t = \frac{\ln{(N_A/N_B)}}{\lambda_A - \lambda_B} = \frac{\ln{(N_A/N_B)}}{\ln 2\left((1/T_{B,\,1/2}) - (1/T_{A,\,1/2})\right)}.$$

Using ^{238}U (the nuclide with the longer half-life, and hence the more abundant) for A and ^{235}U for B,

$$t = \frac{\ln(99.3/0.7)}{\ln 2\,((7.0\times10^{8}\ \mathrm{y})^{-1} - (4.5\times10^{9}\ \mathrm{y})^{-1})} = 5.9\times10^{9}\ \mathrm{y}.$$

12-21: Each alpha decay lowers the mass number by 4, so the mass number of the lead isotope is $238 - 8\,(4) = 206$. Each alpha decay lowers the atomic number by 2, and each negative beta decay (electron emission) increases the atomic number by 1, so the atomic number of the lead isotope is $92 - 8\,(2) + 6 = 82$ (the isotope is given to be lead, and so the atomic number must be 82). The symbol is ${}^{206}_{82}$Pb. The energy released is the equivalent of

$$\begin{aligned} &m\left({}^{238}_{92}\mathrm{U}\right) - m\left({}^{206}_{82}\mathrm{Pb}\right) - 8\,m\left({}^{4}_{2}\mathrm{He}\right) - 6\,m_e \\ &= \left[238.050786\ \mathrm{u} - 205.974455\ \mathrm{u} - 8\,(4.002603\ \mathrm{u}) - 6\left(5.84\times10^{-4}\ \mathrm{u}\right)\right] \\ &\quad\times(931.49\ \mathrm{MeV/u}) = 48.64\ \mathrm{MeV}. \end{aligned}$$

12-23: The kinetic energies of the alpha particle and the daughter nucleus are related by $\mathrm{KE}_\alpha + \mathrm{KE}_d = \mathcal{Q}$. The magnitude of the momenta of the alpha particle and the daughter nucleus must be the same, and from the nonrelativistic expression $p^2 = 2\,M\,\mathrm{KE}$,

$$2\,M_\alpha\,\mathrm{KE}_\alpha = 2\,M_d\,\mathrm{KE}_d, \qquad \text{or} \qquad \frac{M_\alpha}{M_d}\,\mathrm{KE}_\alpha = \mathrm{KE}_d.$$

Substituting this expression into the expression for $\mathcal{Q}$,

$$\mathrm{KE}_\alpha\left(1 + \frac{M_\alpha}{M_d}\right) = \mathrm{KE}_\alpha\left(1 + \frac{4}{A-4}\right) = \mathrm{KE}_\alpha\left(\frac{A}{A-4}\right), \qquad \text{and}$$

$$\mathrm{KE}_\alpha = \frac{A-4}{A}\,\mathcal{Q}.$$

12-25: An electron leaving a nucleus is attracted by the positive nuclear charge, which reduces the electron's energy. A positron leaving a nucleus is repelled by the nucleus and is accordingly accelerated outward, and so leaves the nucleus with greater energy.

12-27: See the remarks at the beginning of this chapter regarding positron emission.

The available energy when ^{7}Be decays to ^{7}Li is

$$m\left({}^7_4\text{Be}\right) - m\left({}^7_3\text{Li}\right) = 7.016930\ \text{u} - 7.016004\ \text{u} = 9.26 \times 10^{-4}\ \text{u},$$

which is less than $2\,m_e = 1.0972$ u.

12-29: See Problem 12-26 and the discussion at the beginning of this chapter regarding proper treatment of the electron masses.

In electron emission, ${}^{80}_{35}$Br becomes ${}^{80}_{36}$Kr. The difference between the masses of the neutral atoms is

$$79.918528\ \text{u} - 79.916375\ \text{u} = 2.153 \times 10^{-3}\ \text{u} > 0,$$

so the reaction can occur. The energy released is

$$\left(2.153 \times 10^{-3}\ \text{u}\right)(931.49\ \text{MeV/u}) = 2.01\ \text{MeV}.$$

In positron emission, ${}^{80}_{35}$Br becomes ${}^{80}_{34}$Se. The atomic mass of the neutral copper atom must exceed the mass of the neutral selenium atom by twice the electron mass;

$$79.918528\ \text{u} - 79.916520\ \text{u} - 2\left(5.486 \times 10^{-4}\ \text{u}\right) = 9.11 \times 10^{-4}\ \text{u} > 0,$$

and so the reaction can occur. The energy released is

$$\left(9.11 \times 10^{-4}\ \text{u}\right)(931.49\ \text{MeV/u}) = 0.85\ \text{MeV}.$$

In electron capture, ${}^{80}_{35}$Br becomes ${}^{80}_{34}$Se, as in positron emission. The difference between the masses of the neutral atoms is

$$63.929766\ \text{u} - 63.927968\ \text{u} = 2.008 \times 10^{-3}\ \text{u}.$$

The energy released is $\left(2.008 \times 10^{-3}\ \text{u}\right)(931.49\ \text{MeV/u}) = 1.87$ MeV. Note that this is larger than the energy released in positron emission by twice the rest energy of an electron.

12-31: The minimum antineutrino energy needed is the energy equivalent of the difference between the rest masses of the final neutron and electron and the initial proton. Using the energy equivalents directly,

$$(m_n + m_e - m_p)\, c^2 = 939.57 \text{ MeV} + 0.511 \text{ MeV} - 938.28 \text{ MeV} = 1.80 \text{ MeV}.$$

12-33: The thirty-ninth proton in ^{89}Y is normally in a $p_{1/2}$ state and the next higher state available to this proton is a $g_{9/2}$ state; hence a radiative transition between the states has a low probability.

12-35: The neutron cross section decreases with increasing energy because the likelihood that a neutron will be captured depends on how much time the neutron spends near a particular nucleus; this time is inversely proportional to the neutron speed. The proton cross section is smaller at smaller energies because of the repulsive force exerted by the positive nuclear charge. See Problem 4-3 for a quantitative consideration.

12-37: The number density n is the ratio of the mass density and the mass of each atom,

$$n = \frac{\left(8.9 \times 10^3 \text{ kg/m}^3\right)}{(59 \text{ u})\,(1.66054 \times 10^{-27} \text{ kg/u})} = 9.08 \times 10^{28} \text{ atoms/m}^3.$$

(a) The fraction that penetrates is given by Equation (12.20),

$$\begin{aligned}\frac{N}{N_0} &= \exp(-n\,\sigma\,x) \\ &= \exp\left(-\left(9.08 \times 10^{28} \text{ atoms/m}^3\right)\left(37 \times 10^{-28} \text{ m}^2\right)\left(1.0 \times 10^{-3} \text{ m}\right)\right) \\ &= 0.71 = 71\%.\end{aligned}$$

(b) From Equation (12.21), the mean free path is

$$\lambda = \frac{1}{n\,\sigma} = \frac{1}{(9.08 \times 10^{28} \text{ atoms/m}^3)\,(37 \times 10^{-28} \text{ m}^2)} = 3.0 \text{ mm}.$$

12-39: Using $N = (1 - 0.99)\,N_0 = (0.01)\,N_0$ is Equation (12.20) and solving for x,

$$x = -\frac{\ln(0.01)}{n\,\sigma} = \frac{\ln(100)}{n\,\sigma}.$$

The number density n is the mass density divided by the mass per atom,

$$n = \frac{\left(2.2 \times 10^3 \text{ kg/m}^3\right)}{(10 \text{ u})\,(1.66054 \times 10^{-27} \text{ kg/u})} = 1.32 \times 10^{29} \text{ m}^{-3}, \qquad \text{and so}$$

$$x = \frac{\ln(100)}{(1.32 \times 10^{29} \text{ m}^{-3})\,(4.0 \times 10^{-25} \text{ m}^2)} = 8.7 \times 10^{-5} \text{ m} = 0.087 \text{ mm}.$$

12-41: In this situation the exposure time $\Delta t = 10.0$ h is much less than the half-life. This means that any decays of ^{60}Co that occur *while* the sample is exposed may be neglected (the correction is on the order of 2×10^{-3} %). The number N_{60} of ^{60}Co atoms after the sample has been exposed is the product of the number N_{59} of the original ^{59}Co atoms, the neutron flux S, the cross section σ and the exposure time Δt. The original number of ^{59}Co atoms is the ratio of the total mass to the mass of an atom. Combining,

$$\begin{aligned} N_{60} &= N_{59}\, S\, \sigma\, \Delta t \\ &= \frac{(10.0 \times 10^{-3}\ \text{kg})}{(59\ \text{u})\,(1.66054 \times 10^{-27}\ \text{kg/u})}\,\left(5.00 \times 10^{17}\ \text{neutrons/(m·s)}\right) \\ &\quad \times \left(37 \times 10^{-28}\ \text{m}^2\right)\left(10.0\ \text{h} \times 3600\ \frac{\text{s}}{\text{h}}\right) \\ &= 6.80 \times 10^{18}. \end{aligned}$$

The activity of the sample after exposure is

$$\begin{aligned} R = \lambda\, N_{60} &= \frac{\ln 2}{T_{1/2}}\, N_{60} \\ &= \frac{\ln 2}{(5.27\ \text{y})\,(3.156 \times 10^{7}\ \text{s/y})}\,\left(6.80 \times 10^{18}\right) \\ &= 2.83 \times 10^{10}\ \text{Bq} = 0.77\ \text{Ci}. \end{aligned}$$

Note that in the above, the quantity λ is a decay constant, not a mean free path.

12-43: The mass number of the unknown constituent must be $7 + 1 - 6 = 2$ and the atomic number must be $4 + 0 - 3 = 1$, and so the unknown nuclide is ${}^2_1$H (a deuterium nucleus):

$${}^{6}_{3}\text{Li} + {}^{2}_{1}\text{H} \longrightarrow {}^{7}_{4}\text{Be} + {}^{1}_{0}n.$$

The mass number of the unknown constituent must be $32 + 4 - 35 = 1$ and the atomic number must be $16 + 2 - 17 = 1$, and so the unknown nuclide is ${}^1_1$H (a proton):

$${}^{35}_{17}\text{Li} + {}^{1}_{1}\text{H} \longrightarrow {}^{32}_{16}\text{Be} + {}^{4}_{2}\text{He}.$$

The mass number of the unknown constituent must be $9 + 4 - 3\,(4) = 1$ and the atomic number must be $4 + 2 - 3\,(2) = 0$, and so the unknown nuclide is ${}^1_0n$ (a neutron):

$${}^{9}_{4}\text{LBe} + {}^{4}_{2}\text{He} \longrightarrow 3\,{}^{4}_{2}\text{He} + {}^{1}_{0}n.$$

The mass number of the unknown constituent must be $79 + 2 - 2\,(1) = 79$ and the atomic number must be $35+1 = 36$, and so the unknown nuclide is ${}^{79}_{36}\mathrm{Kr}$ (a rare isotope of krypton):

$$\begin{matrix}79\\35\end{matrix}\mathrm{Be} + \begin{matrix}2\\1\end{matrix}\mathrm{H} \longrightarrow \begin{matrix}79\\36\end{matrix}\mathrm{Kr} + 2\begin{matrix}1\\0\end{matrix}n.$$

12-45: From Equation (12.26), with $m_A = m_p = 1$ u and $m_B = m_d = 2$ u,

$$\mathrm{KE}_{\,\mathrm{lab}} = \frac{3}{2}\,\mathrm{KE}_{\,\mathrm{cm}} = \frac{3}{2}\,(2.22\ \mathrm{MeV}) = 3.33\ \mathrm{MeV}.$$

12-47: Using Equation (12.24) with $m_A = 4$ u and $m_B = 16$ u, the speed of the center of mass of the system is (using a nonrelativistic calculation)

$$V = \frac{4}{20}\,v = \frac{1}{5}\sqrt{\frac{2\,\mathrm{KE}_{\,\mathrm{lab}}}{m}} = \frac{1}{5}\sqrt{\frac{2\,(5.0\ \mathrm{MeV})}{(4\ \mathrm{u})\,(931.49\ \mathrm{MeV}/u\,c^2)}} = 0.014\,c = 3.1{\times}10^6\ \mathrm{m/s}.$$

From Equation (12.16), the kinetic energy relative to the center of mass is

$$\mathrm{KE}_{\,\mathrm{cm}} = \frac{4}{5}\,\mathrm{KE}_{\,\mathrm{lab}} = 4\ \mathrm{MeV}.$$

12-49: There are many possible ways to approach this problem; two are given here. Both methods must assume nonrelativistic motion.

Method (I): Let the original direction of the alpha particle be the x-direction, and let the plane of the interaction be the x-y plane. The original alpha particle has initial speed v_0 and final speed v'. The target nucleus has mass M and final speed V.

Conservation of momentum in both the x- and y-directions gives

$$\begin{aligned} m_\alpha\, v_0 &= m_\alpha\, v' \cos 60^\circ + M\,V \cos 30^\circ \\ 0 &= m_\alpha\, v' \sin 60^\circ - M\,V \sin 30^\circ. \end{aligned}$$

Multiplying the first equation by $\sin 30^\circ$ and the second by $\cos 30^\circ$ and adding,

$$m_\alpha\, v_0 \sin 30^\circ = m_\alpha v' \left(\cos 60^\circ \sin 30^\circ + \sin 30^\circ \cos 60^\circ\right) = m_\alpha v' \sin 90^\circ = m_\alpha\, v',$$

and so $v' = v_0 \sin 30^\circ = v_0/2$.

Multiplying the first of the above equations by $\sin 60^\circ$ and the second by $\cos 60^\circ$ and subtracting,

$$m_\alpha\, v_0 \sin 60^\circ = M\,V \left(\cos 30^\circ \sin 60^\circ + \sin 30^\circ \cos^\circ\right) = M\,V \sin 90^\circ = M\,V,$$

and so $M\,V = m_\alpha\,v_0 \sin 60°$.

For an elastic collision, kinetic energy is conserved;

$$\frac{1}{2}\,m_\alpha\,v_0^2 = \frac{1}{2}\,m_\alpha\,v'^2 + \frac{1}{2}\,M\,V^2, \qquad \text{or}$$

$$m_\alpha\,v_0^2 = m_\alpha\,\left(v_0 \sin 30°\right)^2 + \frac{\left(m_\alpha\,v_0 \sin 60°\right)^2}{M} = m_\alpha\,v_0^2 \frac{1}{4} + m_\alpha\,v_0^2\,\frac{3}{4}\,\frac{m_\alpha}{M}.$$

This is solved for $M = m_\alpha$, and so the target has a mass number of 4.

In the above calculation, if the angle 60° is replaced by an arbitrary angle θ the result is

$$M = m_\alpha\,\frac{\sin^2\theta}{1 - \sin^2\left(90° - \theta\right)} = m_\alpha\,\frac{\sin^2\theta}{\cos^2\left(90° - \theta\right)} = m_\alpha,$$

suggesting perhaps an equivalent and more direct method of solution.

Method (II): Vector algebra may be used together with the fact that the particles move in perpendicular directions after the collision. Denote the initial momentum of the alpha particle by $\mathbf{p}_\alpha$, its final momentum by $\mathbf{p}'_\alpha$, and the final momentum of the target nucleus as $\mathbf{P}$. Then

$$\mathbf{p}_\alpha = \mathbf{p}'_\alpha + \mathbf{P}.$$

Taking the dot product of each side of the above equation with itself,

$$\mathbf{p}_\alpha \cdot \mathbf{p}_\alpha = \left(\mathbf{p}'_\alpha + \mathbf{P}\right) \cdot \left(\mathbf{p}'_\alpha + \mathbf{P}\right) = \mathbf{p}'_\alpha \cdot \mathbf{p}'_\alpha + \mathbf{P} \cdot \mathbf{P} + 2\,\mathbf{p}'_\alpha \cdot \mathbf{P}.$$

Because $\mathbf{p}'_\alpha$ and $\mathbf{P}$ are given as perpendicular, the last term on the right above vanishes, and so

$$p^2 = p'^2 + P^2.$$

For energy to be conserved,

$$\frac{p^2}{2\,m_\alpha} = \frac{p'^2}{2\,m_\alpha} + \frac{P^2}{2\,M},$$

and comparison with the expression obtained from conservation of momentum gives the result $M = m_\alpha$.

12-51: (a) The excitation energy will be the kinetic energy of particle A in the center of mass frame, plus the $\mathcal{Q}$ value,

$$E^* = \left(\frac{m_B}{m_A + m_B}\right) \mathrm{KE}_A + \mathcal{Q} = \left(\frac{m_C - m_A}{m_C}\right) \mathrm{KE}_A + \mathcal{Q} = \left(1 - \frac{m_A}{m_C}\right) \mathrm{KE}_A + \mathcal{Q}.$$

In this expression, the approximation $m_c \approx m_A + m_B$, valid when $\mathcal{Q} \ll m_C c^2$, has been made.

(b) The $\mathcal{Q}$ value for this reaction is

$$\begin{aligned}
&\left[m\left({}^{15}_{7}\mathrm{N}\right) + m_p - m\left({}^{16}_{8}\mathrm{O}\right)\right] c^2 \\
&= [15.000109\ \mathrm{u} + 1.007825\ \mathrm{u} - 15.994915\ \mathrm{u}]\,(931.49\ \mathrm{MeV/u}) \\
&= 12.13\ \mathrm{MeV}.
\end{aligned}$$

Solving the above expression for KE_A,

$$\mathrm{KE}_A = (E^* - Q)\,\frac{m_C}{m_C - m_A} = (16.2\ \mathrm{MeV} - 12.13\ \mathrm{MeV})\,\frac{16}{15} = 4.43\ \mathrm{MeV}.$$

12-53: The neutron to proton ratio required for stability decreeases with decreasing mass number A, hence there is an excess of neutrons when fission occurs. Some of the excess neutrons are released directly, and the others change to protons by beta decay in the fission fragments.

12-55: Using Equation (11.1) for the nuclear radii, the centers of the nuclei are separated by $R_0\left(A_1^{1/3} + A_2^{1/3}\right)$, and the electrostatic potential energy is

$$\begin{aligned}
U &= \frac{Q_1 Q_2}{4\pi\,\epsilon_0\,R_0\left(A_1^{1/3} + A_2^{1/3}\right)} \\
&= \left(8.988 \times 10^9\ \mathrm{N{\cdot}m^2/C^2}\right) \frac{(38)(54)\left(1.602 \times 10^{-19}\ \mathrm{C}\right)^2}{(1.2 \times 10^{-15}\ \mathrm{m})\left(94^{1/3} + 140^{1/3}\right)} \\
&= 4.05 \times 10^{-11}\ \mathrm{J} = 253\ \mathrm{MeV}.
\end{aligned}$$

12-57: The ${}^{1}_{1}\mathrm{H}$ nuclei in ordinary water are protons, which readily capture neutrons to form ${}^{2}_{1}\mathrm{H}$ (deuterium) nuclei. The neutrons cannot contribute to the chain recation in a reactor, so a reactor using ordinary water as a moderator needs enriched unranium with a greater content of the fissionable ${}^{235}\mathrm{U}$ isotope to function. Deuterium nuclei are less likely to capture neutrons than are protons; hence a reactor moderated with heavy water can operate with ordinary uranium as fuel.

12-59: Let the initial speed of the particle with mass m_1 be v_1 and the final speeds by v_1' and v_2'.

(a) Conservation of momentum gives

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2' \qquad \text{or} \qquad m_1 (v_1 - v_1') = m_2 v_2'$$

and conservation of kinetic energy gives

$$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2 \qquad \text{or} \qquad m_1 \left(v_1^2 - v_1'^2\right) = m_2 v_2'^2.$$

If $v_1' = v_1$, there is no collision. If there is a collision, dividing the equation obtained from conservation of kinetic energy by the equation obtained from conservation of momentum, and using $v_1^2 - v_1'^2 = (v_1 - v_1')(v_1 + v_1')$ gives

$$v_1 + v_1' = v_2', \qquad \text{or} \qquad v_1 = v_2' - v_1'.$$

This standard result from classical mechanics is often interpreted as the relative speeds of the particles being the same before and after the collision, a result that holds even if the second particle is moving initially; that is, the relative speed is independent of the frame of the observer if the observer and the particles are not moving relativistically.

The two equations

$$m_1 v_1 = m_1 v_1' + m_2 v_2' \qquad \text{and} \qquad v_1 = v_2' - v_1'$$

are solved for

$$v_2' = \frac{2 m_1}{m_1 + m_2} v_1.$$

The desired ratio of kinetic energies is

$$\frac{\text{KE}_2'}{\text{KE}_1} = \frac{(1/2) m_2 v_2'^2}{(1/2) m_1 v_1^2} = \frac{4 m_1 m_2}{(m_1 + m_2)^2} = 4 \frac{(m_2/m_1)}{(1 + (m_2/m_1))^2}.$$

(b) Virtually all of the neutron's kinetic energy will be transferred to the proton, as the masses are almost indentical (use of the actual masses gives $1 - 2 \times 10^{-7}$). For a collision with a deuteron, the ratio m_2/m_1 is essentially 2, and $4(2)/(3)^2 = 0.89 =$ 89%. For a collision with a ^{12}C nucleus, $m_2/m_1 = 12$ and $4(12)/(13)^2 = 0.28 =$ 28%. For a collision with a ^{238}U nucleus, $m_2/m_1 = 238$, and $4(238)/(239)^2 =$ $0.017 = 1.7\%$.

12-61: The minimum kinetic energy the proton must have is the electrostatic potential energy of the proton-nucleus combination when the proton is at the nuclear surface. Using Equation (1.11) to give the radius of the nucleus and using R_0 for the radius of the proton,

$$U = \frac{Z\,e^2}{4\pi\,\epsilon_0\,R_0\left(1 + A^{1/3}\right)} = \left(8.988 \times 10^9\ \mathrm{N{\cdot}m^2/C^2}\right)\frac{(6)\left(1.602 \times 10^{-19}\ \mathrm{C}\right)^2}{(1.2 \times 10^{-15}\ \mathrm{m})\left(1 + 12^{1/3}\right)}$$
$$= 3.51 \times 10^{-10}\ \mathrm{J} = 2.19\ \mathrm{MeV}.$$

12-63: (a) The electrostatic energy of the deuterons separated by the given distance is

$$U = \frac{e^2}{4\pi\,\epsilon_0 r} = \left(8.988 \times 10^9\ \mathrm{N{\cdot}m^2/C^2}\right)\frac{\left(1.602 \times 10^{-19}\ \mathrm{C}\right)^2}{(5 \times 10^{-15}\ \mathrm{m})}$$
$$= 4.6 \times 10^{-14}\ \mathrm{J} = 2.9 \times 10^5\ \mathrm{eV}.$$

For the average translational kinetic energy $(3/2)kT = U$,

$$T = \frac{2}{3}\frac{U}{k} = \frac{2}{3}\frac{2.9 \times 10^5\ \mathrm{eV}}{8.617 \times 10^{-5}\ \mathrm{eV/K}} = 2.2 \times 10^9\ \mathrm{K}.$$

(b) This temperature corresponds to the average deuteron energy, but many deuterons have considerably higher energies than the average. Also, quantum-mechanical tunneling through the potential barrier can occur, permitting deuterons to react despite having insufficient energy to come together classically.