

Chapter 10 - The Solid State

10-1: The halogenic atomic numbers are $Z = 9$ for flourine (F), $Z = 17$ for chlorine (Cl), $Z = 35$ for bromine (Br) and $Z = 53$ for iodine (I). The greater the atomic number of a halogen atom, the larger the atom is, hence the increase in the interatomic spacing with Z . The larger the ion spacing, the smaller the cohesive energy, hence the lower the melting point.

10-3: (a) The cohesive energy will be the negative of the Coulombic energy as given in Equation (10.1), minus the difference between the ionization energy of potassium and the electron affinity of chlorine;

$$\begin{aligned} & \alpha \frac{e^2}{4\pi \epsilon_0 r} - (E_i - E_a) \\ & = (1.748) \frac{(8.988 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2) (1.602 \times 10^{-19} \text{ C})^2}{(0.314 \times 10^{-9} \text{ m}) (1.602 \times 10^{-19} \text{ J/eV})} - (4.34 \text{ eV} - 3.61 \text{ eV}) \\ & = 7.29 \text{ eV}. \end{aligned}$$

(b) The difference between the observed binding energy and that found in part (a) must be due to the repulsive energy as given in Equation (10.1). From the observed binding energy, U_0 must be given by

$$-U_0 = 6.42 \text{ eV} + (4.34 \text{ eV} - 3.61 \text{ eV}) = 7.15 \text{ eV}.$$

The Coulombic energy, an intermediate calculation in part (a), is $U_{\text{coulomb}} = -8.0156 \text{ eV}$, and so solving Equation (10.5) for n ,

$$n = \left[1 - \frac{U_0}{U_{\text{coulomb}}} \right]^{-1} = \left[1 - \frac{7.15 \text{ eV}}{8.0156 \text{ eV}} \right]^{-1} = 9.26.$$

10-5: The heat lost by the expanding gas is equal to the work done against the attractive van der Waals forces between the gas molecules.

10-6: Van der Waals forces are too weak to hold inert gas atoms together against the forces exerted during collisions in the gaseous state.

10-7: (a) Van der Waals forces increase the cohesive energy because they are attractive, and the ions in the crystals tend more to cohere. (b) Zero-point oscillations decrease the cohesive energy because these oscillations represent a mode of energy that is present in a solid but not in individual atoms or ions.

10-9: The electrons that constitute the “gas” of freely moving electrons are only those that are loosely bound to the nuclei, specifically those electrons in the outer shells. As has been seen, the innermost electrons have binding energies that give rise to x-ray spectra, and will not be members of the free-electron gas.

10-11: The number density n is the mass density ρ_{Ag} divided by the mass m_{Ag} of each atom,

$$n = \frac{\rho_{\text{Ag}}}{m_{\text{Ag}}} = \frac{(10.5 \times 10^3 \text{ kg/m}^3)}{(108 \text{ u})(1.66054 \times 10^{-27} \text{ kg/u})} = 5.855 \times 10^{28} \text{ atoms/m}^3,$$

keeping an extra significant figure. With the assumption stated in the problem, this is the same as the electron density.

Using $\lambda = 200 d = 200 n^{-1/3}$ and $m v_F = \sqrt{2 m \epsilon_F}$ in Equation (10.16),

$$\begin{aligned} \rho &= \frac{m_e v_F}{n d^2 (200 n^{-1/3})} = \frac{\sqrt{2 m_e \epsilon_F}}{200 e^2 n^{2/3}} \\ &= \frac{\sqrt{2 (9.1095 \times 10^{-31} \text{ kg}) (5.51 \text{ eV}) (1.602 \times 10^{-19} \text{ J/eV})}}{(200) (1.602 \times 10^{-19} \text{ C})^2 (5.855 \times 10^{28} \text{ m}^{-3})^{2/3}} = 1.64 \times 10^{-8} \Omega \cdot \text{m} \end{aligned}$$

10-13: In both insulators and semiconductors, a forbidden band separates a filled valence band from the conduction band above it. In semiconductors, the band gap is smaller than in insulators, and the property of the gap that makes a semiconductor a semiconductor is that in the semiconductor some valence electrons have enough thermal energy to jump across the gap to the conduction band.

10-15: (a) Photons of visible light have energies of $\sim 1\text{-}3$ eV (see the back endpapers), which can be absorbed by free electrons in a metal without leaving the electrons’ valence band. Hence metals are opaque. The forbidden bands in insulators and semiconductors are too wide for valence electrons to jump across the gaps by absorbing 1-3 eV. Hence such solids are transparent.

(b) The minimum wavelengths are given by $\lambda = hc/E_g$, where E_g is the gap energy; light with shorter wavelengths would have energies larger than the gap energy. For silicon,

$$\lambda_{\min} = \frac{hc}{E_g} = \frac{1.240 \times 10^{-6} \text{ eV} \cdot \text{m}}{1.1 \text{ eV}} = 1.13 \times 10^{-6} \text{ m} = 1.13 \mu\text{m} = 1130 \text{ nm},$$

keeping an extra significant figure; elemental silicon is not transparent to visible light. For diamond,

$$\lambda_{\min} = \frac{hc}{E_g} = \frac{1.240 \times 10^{-6} \text{ eV} \cdot \text{m}}{6 \text{ eV}} = 2.07 \times 10^{-7} \text{ m} = 207 \text{ nm},$$

again keeping extra significant figures. Light with this wavelength is in the ultraviolet, but light with *longer* wavelengths, including the visible wavelengths, have less energy and cannot excite an electron to jump the band, and will pass through. Hence, diamond is transparent to visible light.

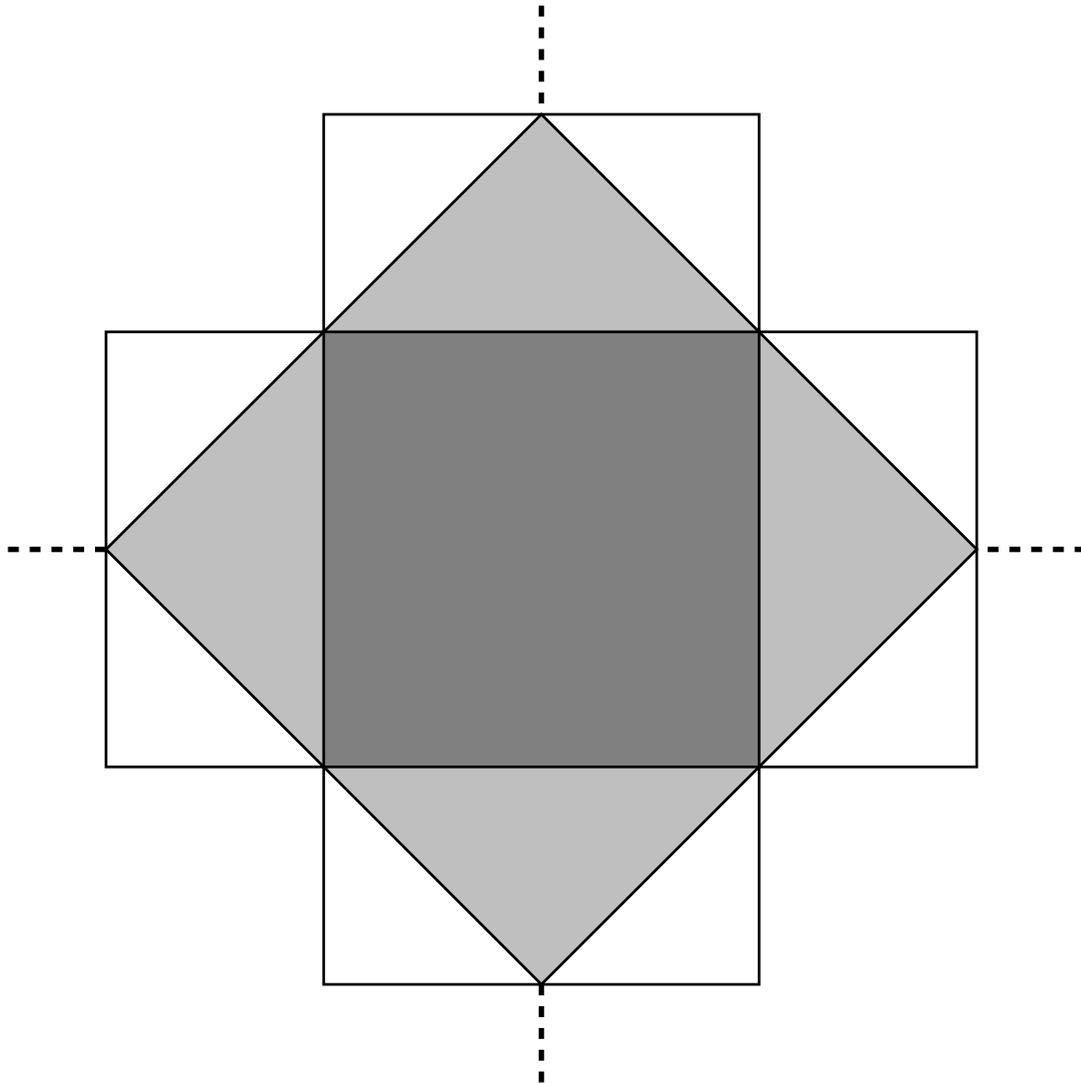
10-17: Aluminum atoms have 3 electrons in their outer shell, germanium atoms have 4 (see Table 7.4). Replacing a germanium atom with an aluminum atom leaves a hole, and the the result is a *p*-type semiconductor.

10-19: The figure on the next page shows the third Brillouin zone. The wavevectors in this zone will be those that do not fit into the first two zones but are not diffracted by the diagonal sets of atomic planes in Figure 10.40 that make angles of $\pm \arctan(1/2) = \pm 26.6^\circ$ or $\pm \arctan(2) = \pm 63.4^\circ$ with the *x*- or *y*-axes. These wavenumbers correspond to $|k_x| + |k_y| > \frac{2\pi}{a}$ (the condition that \mathbf{k} not be in the first or second zones) and

$$\left\{ \frac{\pi}{a} < |k_x| < \frac{2\pi}{a}, \quad 0 < |k_y| > \frac{\pi}{a} \right\} \quad \text{OR} \quad \left\{ 0 < |k_x| < \frac{\pi}{a}, \quad \frac{\pi}{a} < |k_y| > \frac{2\pi}{a} \right\}$$

Figure for Solution to Problem 9-19: More text on previous page

In the figure, the darker-shaded square is the first Brillouin zone and the lighter-shaded square is the second zone, as in Figure 10.41 (the axes and axis scales are not shown, but are the same as in the text figure). The unshaded area is the third Brillouin zone.



10-21: (a) The radius of a Bohr orbit for a given energy level n is proportional to the relative permittivity (the dielectric constant) and the mass, as given in Equation (4.13). The radius of the first Bohr orbit, in terms of a_0 , would then be

$$r_1 = a_0 \frac{(\epsilon/\epsilon_0)}{(m^*/m)} = (5.292 \times 10^{-11} \text{ m}) \frac{16}{0.17} = 5.0 \text{ nm}.$$

(b) The energy of an electron in a given Bohr orbit is proportional to the effective mass and inversely proportional to the square of the relative permittivity (see Equation (4.15)). The ionization energy is then

$$E = (-E_1) \frac{(m^*/m)}{(\epsilon/\epsilon_0)^2} = (13.6 \text{ eV}) \frac{(0.17)}{(16)^2} = 9.0 \times 10^{-3} \text{ eV}.$$

This is much smaller than the energy gap of 0.65 eV but comparable to the product $kT \approx 0.025 \text{ eV}$.

10-23: (a) In a uniform magnetic field of magnitude B , an electron moving with nonrelativistic speed v at right angles to the field will experience a force of magnitude evB . The inward acceleration of a particle moving with constant speed v in a circle of radius R has magnitude v^2/R . From $F = m^*a$,

$$evB = m^* \frac{v^2}{R}, \quad \text{or} \quad eB = m^* (2\pi\nu_c),$$

where $\nu_c = v/(2\pi R)$ has been used. Solving for the cyclotron frequency ν_c ,

$$\nu_c = \frac{eB}{2\pi m^*}.$$

Note that the cyclotron frequency is independent of the electron's speed, and hence its orbit radius.

(b) From the above expression for ν_c , solving for m^* gives

$$m^* = \frac{eB}{2\pi\nu_c} = \frac{(1.602 \times 10^{-19} \text{ C})(0.1 \text{ T})}{2\pi(1.4 \times 10^{10} \text{ Hz})} = 1.82 \times 10^{-31} \text{ kg} = 0.2 m_e.$$

(c) Solving for R as a function of v and ν ,

$$R = \frac{v}{2\pi\nu} = \frac{(3 \times 10^4 \text{ m/s})}{2\pi(1.4 \times 10^{10} \text{ Hz})} = 3.4 \times 10^{-7} \text{ m}.$$

10-25: From Equation (10.28), the frequency is

$$\nu = \frac{2(Ve)}{h} = \frac{2(5.0 \times 10^{-6} \text{ eV})}{4.136 \times 10^{-15} \text{ eV}\cdot\text{s}} = 2.4 \times 10^9 \text{ Hz} = 2.4 \text{ GHz}.$$