Chapter 9 - Statistical Mechanics

9-1: As in Example 9.1, $g(\epsilon_2) = 8$ and $g(\epsilon_1) = 1$. Then,

$$\frac{n(\epsilon_2)}{n(\epsilon_1)} = \frac{1}{1000} = 4\,e^{-(\epsilon_2 - \epsilon_1)/kT} = 4\,e^{3\epsilon_1/4kT},$$

where $\epsilon_2 = \epsilon_1/4$. Using $\epsilon_1 = -13.6$ eV, and solving for T,

$$T = \left(\frac{1}{k}\right)\frac{(3/4)\,(-\epsilon_1)}{\ln 4000} = \frac{(3/4)(13.6\text{ eV})}{(8.617\times10^{-5}\text{ eV/K})\,(\ln 4000)} = 1.43\times10^4\text{ K}.$$

9-3: In this situation, the "multiplicity" that is the presuperscript in the term symbol is not the same as the number of states of a given energy. The number of states is $2\mathbf{L}+1 = 3$ for a P level and 1 for an S level. The ratio of the numbers of atoms in the states is then

$$(3)\exp\left(-\frac{(2.093\text{ eV})}{(8.617\times10^{-5}\text{ eV/K})\,(1200\text{ K})}\right) = 4.86\times10^{-9}.$$

9-5: (a) As in Example 9.2, there are $2J+1$ states with the same rotational energy for a given rotational quantum number J. The $J=0$ state has 0 energy, and so the populations relative to $J=0$ are

$$\begin{aligned}
&(2J+1)\exp\left(-\frac{J(J+1)\,\hbar^2}{2\,I\,k\,T}\right)\\
&= (2J+1)\left[\exp\left(-\frac{\hbar^2}{2\,I\,k\,T}\right)\right]^{J(J+1)}\\
&= (2J+1)\left[\exp\left(-\frac{\left(1.055\times10^{-34}\text{ J}\cdot\text{s}\right)^2}{2\,(4.64\times10^{-48}\text{ kg}\cdot\text{m}^2)\,(1.381\times10^{-23}\text{ J/K})\,(300\text{ K})}\right)\right]^{J(J+1)}\\
&= (2J+1)\,[0.74864]^{J(J+1)}.
\end{aligned}$$

Applying this expression to $J = 0$, 1, 2, 3 and 4 gives, respectively, 1 exactly, 1.68, 0.880, 0.217 and 0.275.

If more precise values for the constants $\hbar$ and k than those given in the endpapers are used, the answers might differ in the third figure. For instance, using $\hbar = 1.0545716\times10^{-34}$ J·s gives, for $J = 2$, 3 and 4 the values 0.882, 0.218 and 0.277.

(b) Introduce the dimensionless parameter $x \equiv e^{-\hbar^2/2\,I\,kT}$ (in part (a), with $T = 300$ K, $x = 0.74864$). Then, for the populations of the $J = 2$ and $J = 3$ states to be equal,

$$5\,x^6 = 7\,x^{12}, \qquad x^6 = \frac{5}{7} \qquad \text{and} \qquad 6\,\ln x = \ln\frac{5}{7}.$$

Using $\ln x = -\hbar^2/2\,I\,kT$ and $\ln\frac{5}{7} = -\ln\frac{7}{5}$, and solving for T,

$$\begin{aligned} T &= \frac{6\,\hbar^2}{2\,I\,k\,\ln(7/5)} \\ &= \frac{6\,\left(1.055 \times 10^{-34}\ \text{J}\cdot\text{s}\right)^2}{2\,(4.64 \times 10^{-48}\ \text{kg}\cdot\text{m}^2)\,(1.381 \times 10^{-23}\ \text{J/K})\,\ln(1.4)} = 1.55 \times 10^3\ \text{K}. \end{aligned}$$

9-7: The mean speed $\overline{v} = \frac{1}{2}(1.00\ \text{m/s} + 3.00\ \text{m/s}) = 2.00\ \text{m/s}$. The root-mean-square speed is

$$v_{\text{rms}} = \sqrt{\frac{1}{2}\left((1.00\ \text{m/s})^2 + (2.00\ \text{m/s})^2\right)} = 2.24\ \text{m/s}.$$

9-9: For monatomic hydrogen, the kinetic energy is all translational and $\overline{\text{KE}} = (3/2)\,kT$; solving for T with $\overline{\text{KE}} = -E_1$,

$$T = \frac{2}{3}\left(-\frac{E_1}{k}\right) = \frac{(2/3)\,(13.6\ \text{eV})}{(8.617 \times 10^{-5}\ \text{eV/K})} = 1.05 \times 10^5\ \text{K}.$$

9-11: For these nonrelativistic atoms, the shift in wavelengths will be between $+\lambda(v/c)$ and $-\lambda(v/c)$ and the width of the doppler-broadened line will be $2\lambda(v/c)$. Using the rms speed from $\overline{\text{KE}} = (3/2)\,kT = (1/2)\,mv^2$, $v = \sqrt{3kT/m}$, and

$$\begin{aligned} \Delta\lambda &= 2\lambda\,\frac{\sqrt{3\,kT/m}}{c} \\ &= 2\,(656.3 \times 10^{-9}\ \text{m})\,\frac{\sqrt{3\,(1.381 \times 10^{-23}\ \text{J/K})\,(500\ \text{K})/\,(1.6736 \times 10^{-27}\ \text{kg})}}{(2.998 \times 10^8\ \text{m/s})} \\ &= 1.54 \times 10^{-11}\ \text{m} = 15.4\ \text{pm}. \end{aligned}$$

9-13: The average value of $\frac{1}{v}$ is

$$\left\langle \frac{1}{v} \right\rangle = \frac{1}{N} \int_0^\infty \frac{1}{v}\, n(v)\, dv.$$

With $n(v)$ as given in Equation (9.14),

$$\begin{aligned} \left\langle \frac{1}{v} \right\rangle &= \frac{1}{N}\, 4\pi\, N \left(\frac{m}{2\pi\, kT} \right)^{3/2} \int_0^\infty v\, e^{-mv^2/2kT}\, dv \\ &= 4\pi \left(\frac{m}{2\pi\, kT} \right)^{3/2} \left(\frac{kT}{m} \right) = \sqrt{\frac{2\, m}{\pi\, kT}}, \end{aligned}$$

where the improper definite integral given in the problem,

$$\int_0^\infty v\, e^{-a\, v^2}\, dv = \frac{1}{2a}$$

has been used.

Note that
$$\left\langle \frac{1}{v} \right\rangle = 2\, \frac{1}{\langle v \rangle},$$

where the notation $\overline{v} = \langle v \rangle$ has been used.

9-15: See Figure 9.5. The curves are not normalized, in that when α is adjusted to give the same areas under the curves, the curves will interect at a finite energy. A fermion gas will exert the greatest pressure because the Fermi distribution has a larger proportion of high-energy particles than the other distributions (note that the *proportion* of high-energy particles is larger). A boson gas will exert the least pressure because the Bose distribution has a larger proportion of low-energy particles than the others.

9-17: **NOTE:** For convenience in this problem, the quantity $g(\lambda)$ is used as well as $g(\nu)$, even though they are different functions, with different arguments and, as will be seen, different functional forms.

The condition that $g(\lambda)$ must satisfy is

$$g(\nu)\, d\nu = g(\lambda)\, d\lambda, \qquad \text{so} \qquad g(\lambda) = g(\nu)\, \frac{d\nu}{d\lambda}.$$

The quantity $\frac{d\nu}{d\lambda}$ is negative, so it is convenient and conventional to use instead

$$g(\lambda) = g(\nu) \left| \frac{d\nu}{d\lambda} \right| = g(\nu)\, \frac{c}{\lambda^2} = \frac{8\pi\, L^3\, \nu^2}{c^2\, \lambda^2} = \frac{8\pi\, L^3}{\lambda^4},$$

where Equation (9.34) has been used for $g(\nu)$.

The number of waves between 9.5 mm and 10.5 mm is then

$$g(\lambda)\,\Delta\lambda = \frac{8\pi\,(1\ \mathrm{m})^3}{(10\ \mathrm{mm})^4}\,(1.00\ \mathrm{mm}) = 2.5\times 10^6.$$

(It should be noted that integrating $g(\lambda)\,d\lambda$ between the frequencies and including the variation of $g(\lambda)$ with λ, instead of using $g(\lambda) = g\left(\overline{\lambda}\right)$, as was done above, gives the same answer to the given precision.)

Similarly, the number of waves between 99.5 mm and 100.5 mm is 2.5×10^2, lower by a factor of 10^4.

9-19: The percentage difference is the percentage difference in the fourth powers of the Kelvin temperatures; specifically,

$$\frac{\sigma\,T1^4 - \sigma\,T_2^4}{\sigma\,T_1^4} = \frac{T_1^4 - T_2^4}{T_1^4} = 1 - \left(\frac{T_2}{T_1}\right)^4 = 1 - \left(\frac{307\ \mathrm{K}}{308\ \mathrm{K}}\right)^4 = 0.013 = 1.3\%.$$

For temperature variations this small, the fractional variation may be approximated by

$$\frac{\Delta\left(T^4\right)}{T^4} = \frac{3\,T^3\,\Delta T}{T^4} = 3\,\frac{\Delta T}{T} = 3\,\frac{1\ \mathrm{K}}{308\ \mathrm{K}} = 0.013$$

to the given precision.

9-21: See Example 9.7. Lowering the Kelvin temperature by a given fraction will lower the radiation by a factor equal to the fourth power of the ratio of the temperatures. Using 1.4 kW/m^2 as the rate at which the sun's energy arrives at the surface of the earth,

$$\left(1.4\ \mathrm{kW/m^2}\right)(0.90)^4 = 0.92\ \mathrm{kW/m^2}.$$

9-23: To radiate at twice the rate, the fourth power of the Kelvin temperature would need to double. The new temperature would be

$$((400+273)\ \mathrm{K})\ 2^{1/4} = 800\ \mathrm{K},$$

which is 527°C.

9-25: From Equation (9.41), with unit emissivity for the hole in the wall,

$$P = \sigma\,T^4 = \left(5.670\times 10^{-8}\ \mathrm{W}/\left(\mathrm{m^2{\cdot}K^4}\right)\right)(973\ \mathrm{K})^4\ \left(10\times 10^{-4}\ \mathrm{m^2}\right) = 51\ \mathrm{W}.$$

9-27: Using Equation (9.41) for the radiated power per unit area, the area of the blackbody (assuming unit emissivity) is

$$A = \frac{P}{R} = \frac{P}{e\,\sigma\,T^4} = \frac{(1.00 \times 10^3\ \text{W})}{(1)\,(5.670 \times 10^{-8}\ \text{W}/\,(\text{m}^2{\cdot}\text{K}^4))\,((500 + 273)\ \text{K})^4}$$
$$= 4.94 \times 10^{-2}\ \text{m}^2 = 494\ \text{cm}^2.$$

The radius of a sphere with this surface area is found from $A = 4\pi\,r^2$, or

$$r = \sqrt{\frac{A}{4\pi}} = \sqrt{\frac{494\ \text{cm}^2}{4\pi}} = 6.27\ \text{cm}.$$

9-29: Equation (9.38) is not integrable in terms of elementary functions; however, approximating $g(\nu)$ by $g\,(\overline{\nu})$, where $\overline{\nu}$ is the average frequency in the wavelength interval ($\overline{\nu}$ will be approximated by $c/\overline{\lambda}$), is valid. Before using Equation (9.38), it is convenient to note that because the proportion of the radiation in this wavelength interval is desired, division by $u = \int u(\nu)\,d\nu$ gives the fraction

$$\frac{\Delta u}{u} = 15\left(\frac{h\,\nu}{\pi\,kT}\right)^4 \frac{(\Delta\nu/\nu)}{e^{h\nu/kT} - 1}.$$

The quantity $\Delta\nu/\nu$ is approximated by $\Delta\lambda/\lambda$ (supressing the minus sign). The dimensionless quantity $h\nu/kT$ that appears in the above expression is evaluated at the average frequency, in terms of the average wavelength, as outlined above, so that

$$\frac{h\overline{\nu}}{kT} = \frac{hc}{kT\overline{\lambda}} = \frac{\left(1.240 \times 10^{-6}\ \text{eV}{\cdot}\text{m}\right)}{(8.617 \times 10^{-5}\ \text{eV/K})\,(6000\ \text{K})(580 \times 10^{-9}\ \text{m})} = 4.135,$$

keeping extra significant figures. The result is

$$\frac{\Delta u}{u} = 15\left(\frac{4.135}{\pi}\right)^4 \frac{(20\ \text{nm})/(580\ \text{nm})}{\exp(4.135) - 1} = 0.025 = 2.5\%.$$

To do the integral numerically, an example of a sequence of Maple commands that reproduce the above result, and allows similar calculations for arbitrary ranges of wavelengths and temperatures, is:

```
>u:=x^3/(exp(y*x)-1);
>y:=1.24E-6/8.167E-5/6E3;
>x1:=1/590E-9; x2:=1/570E-9;
>evalf(int(u,x=x1..x2))/int(u,x=0..infinity);
```

9-31: From the Wien displacement law (Equation (9.40)), the surface temperature of Sirius is

$$T = \frac{2.898 \times 10^{-3}\ \mathrm{m{\cdot}K}}{290 \times 10^{-9}\ \mathrm{m}} = 1.0 \times 10^{4}\ \mathrm{K}.$$

9-33: From the Wien displacement law (Equation (9.40)), the surface temperature of the cloud is

$$T = \frac{2.898 \times 10^{-3}\ \mathrm{m{\cdot}K}}{10 \times 10^{-6}\ \mathrm{m}} = 2.9 \times 10^{2}\ \mathrm{K}$$

(the form of the answer indicates that this result is valid to no more than two significant figures).

Assuming unit emissivity, the radiation rate is

$$R = \sigma\, T^4 = \frac{P}{A} = \frac{P}{\pi D^2},$$

where D is the cloud's diameter. Solving for D using the given power and the temperature found above,

$$D = \sqrt{\frac{P}{\pi\, \sigma\, T^4}} = 8.9 \times 10^{11}\ \mathrm{m},$$

roughly but slightly larger than the distance from the sun to Jupiter.

9-35: The total energy (denoted by uppercase U) is related to the energy density by $U = V\, u$, where V is the volume. In terms of the temperature,

$$U = V\, u = V\, a\, T^4 = V\, \frac{4\sigma}{c}\, T^4.$$

The specific heat at constant volume is then

$$\begin{aligned} c_V = \frac{\partial U}{\partial T} &= \frac{16\, \sigma}{c}\, T^3\, V \\ &= \frac{16\,\left(5.670 \times 10^{-8}\ \mathrm{W}/\left(\mathrm{m}^2{\cdot}\mathrm{K}^4\right)\right)}{(2.998 \times 10^{8}\ \mathrm{m/s})}\,(1000\ \mathrm{K})^3\left(1.00 \times 10^{-6}\ \mathrm{m}^3\right) \\ &= 3.03 \times 10^{-12}\ \mathrm{J/K}. \end{aligned}$$

9-37: At $T = 0$, all states with energy less than the Fermi energy ϵ_F are occupied, and all states with energy above the Fermi energy are empty. For $0 \le \epsilon \le \epsilon_F$, the electron energy distribution, given in Equation (9.58), is proportional to $\sqrt{\epsilon}$. The median energy is that energy for which there are as many occupied states below the median as there are above. The median energy ϵ_M is then the energy such that

$$\int_0^{\epsilon_M} \sqrt{\epsilon}\, d\epsilon = \frac{1}{2} \int_0^{\epsilon_F} \sqrt{\epsilon}\, d\epsilon.$$

Evaluating the integrals,

$$\frac{2}{3} (\epsilon_M)^{3/2} = \frac{1}{3} (\epsilon_F)^{3/2}, \qquad \text{or} \qquad \epsilon_M = \left(\frac{1}{2}\right)^{3/2} \epsilon_F = 0.630\, \epsilon_F.$$

9-39: (a) The average energy at $T = 0$, from Equation (9.59), is $(3/5)\,\epsilon_F = 3.31$ eV.

(b) Setting $(3/2)\,kT = (3/5)\,\epsilon_F$ and solving for T,

$$T = \frac{2}{5}\frac{\epsilon_F}{k} = \frac{2}{5}\, \frac{5.51 \text{ eV}}{8.617 \times 10^{-5} \text{ eV/K}} = 2.56 \times 10^4 \text{ K}.$$

(c) The speed in terms of the kinetic energy is

$$v = \sqrt{\frac{2\,\text{KE}}{m}} = \sqrt{\frac{6\,\epsilon_F}{5\,m}} = \sqrt{\frac{6\,(5.51 \text{ eV})\,(1.602 \times 10^{-19} \text{ J/eV})}{5\,(9.1095 \times 10^{-31} \text{ kg})}} = 1.08 \times 10^6 \text{ m/s}.$$

9-41: The denominator is not well-defined at $T = 0$, but the expression in Equation (9.29) may be evaluated by taking the limit as $T \to 0^+$. If $\epsilon > \epsilon_F$, the argument of the exponent is positive for positive T, and as $T \to 0^+$ the exponent becomes unboundedly large and $f_{FD}(\epsilon) \to 0$. If $\epsilon < \epsilon_F$, the argument of the exponent is always negative and the exponent goes to zero as $T \to 0^+$, so the denominator approaches 1 and $f_{FD} \to 1$. The interpretation of these results is that in the limit $T \to 0^+$, states with $\epsilon > \epsilon_F$ are unoccupied and states with $\epsilon < \epsilon_F$ are fully occupied.

9-43: Using Equation (9.29),

$$f_1 = f_{FD}\,(\epsilon_F + \Delta\epsilon) = \frac{1}{e^{\Delta\epsilon/kT} + 1}, \qquad \text{and}$$

$$f_2 = f_{FD}\,(\epsilon_F + \Delta\epsilon) = \frac{1}{e^{-\Delta\epsilon/kT} + 1}.$$

From these expressions,

$$\begin{aligned} f_1 + f_2 &= \frac{1}{e^{\Delta\epsilon/kT}+1} + \frac{1}{e^{-\Delta\epsilon/kT}+1} \\ &= \frac{1}{e^{\Delta\epsilon/kT}+1} + \frac{e^{\Delta\epsilon/kT}}{e^{\Delta\epsilon/kT}+1} \\ &= 1. \end{aligned}$$

9-45: In using Equation (9.56) to find the Fermi energy, the proper values for N/V, the number of free electrons per unit volume, and m^*, the effective electron mass, must be used. From Table 7.4, zinc in its ground state has two electrons in the $4s$ subshell and completely filled K, L and M shells. Thus, there are two free electrons per atom. The number of atoms per unit volume is the ratio of the mass density ρ_{Zn} to the mass per atom m_{Zn}. Combining in Equation (9.56),

$$\begin{aligned} \epsilon_F &= \frac{h^2}{2\,m^*}\left(\frac{3\,(2)\,\rho_{\mathrm{Zn}}}{8\pi\,m_{\mathrm{Zn}}}\right)^{2/3} \\ &= \left(\frac{\left(6.626\times10^{-34}\ \mathrm{J\cdot s}\right)^2}{2\,(0.85)\,(9.1095\times10^{-31}\ \mathrm{kg})}\right)\left(\frac{3\,(2)\,\left(7.13\times10^{3}\ \mathrm{kg/m^3}\right)}{8\pi\,(65.4\ \mathrm{u})\,(1.66054\times10^{-27}\ \mathrm{kg/u})}\right)^{2/3} \\ &= 1.78\times10^{-18}\ \mathrm{J} = 11\ \mathrm{eV} \end{aligned}$$

to two significant figures.

9-47: At $T = 0$, the electron distribution $n(\epsilon)$ as given in Equation (9.58) reduces to

$$n(\epsilon) = \frac{3\,N}{2}\,(\epsilon_F)^{-3/2}\,\sqrt{\epsilon},$$

as explained in the derivation of Equation (9.59) and in Problem 9-41.

At $\epsilon = (\epsilon_F)/2$,

$$n\left(\frac{\epsilon_F}{2}\right) = \frac{3}{\sqrt{8}}\,\frac{N}{\epsilon_F}.$$

The number of atoms is the mass divided by the mass per atom,

$$N = \frac{\left(1.00\times10^{-3}\ \mathrm{kg}\right)}{(63.55\ \mathrm{u})\,(1.66054\times10^{-27}\ \mathrm{kg/u})} = 9.48\times10^{21},$$

with the atomic mass of copper from the front endpapers and $\epsilon_F = 7.04$ eV is from Table 9.2 or Problem 9-40. The number of states per electronvolt is

$$n\left(\frac{\epsilon_F}{2}\right) = \frac{3}{\sqrt{8}}\,\frac{9.48\times10^{21}}{7.04\ \mathrm{eV}} = 1.43\times10^{21}\ \mathrm{states/eV},$$

and the distribution may certainly be considered to be continuous.

9-49: Using the approximation $f(\epsilon) = A\,e^{-\epsilon/kT}$, and a factor of 4 instead of 8 in Equation (9.47), Equation (9.57) becomes

$$n(\epsilon)\,d\epsilon = g(\epsilon)\,f(\epsilon)\,d\epsilon = A\,4\sqrt{2}\,\pi\,\frac{V\,m^{3/2}}{h^3}\,\sqrt{\epsilon}\,e^{-\epsilon/kT}\,d\epsilon.$$

Integrating over all energies,

$$N = \int_0^\infty n(\epsilon)\,d\epsilon = A\,4\sqrt{2}\,\pi\,\frac{V\,m^{3/2}}{h^3}\int_0^\infty \sqrt{\epsilon}\,e^{-\epsilon/kT}\,d\epsilon.$$

The integral is that given in the problem with $x = \epsilon$ and $a = kT$,

$$\int_0^\infty \sqrt{\epsilon}\,e^{-\epsilon/kT}\,d\epsilon = \frac{\sqrt{\pi\,kT}}{2\,(1/kT)} = \frac{\sqrt{\pi\,(kT)^3}}{2}, \qquad \text{so that}$$

$$N = A\,4\sqrt{2}\,\pi\,\frac{V\,m^{3/2}}{h^3}\,\frac{\sqrt{\pi\,(kT)^3}}{2} = A\,\frac{V}{h^3}\,(2\pi\,m\,kT)^{3/2}.$$

Solving for A,

$$A = \frac{N}{V}\,h^3\,(2\pi\,m\,kT)^{-3/2}.$$

Using the given numerical values,

$$\begin{aligned} A &= \left(\begin{array}{l} \dfrac{6.022\times10^{26}\ \text{kmol}^{-1}}{22.4\ \text{kg/kmol}}\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)^3\times \\ \quad\left[2\pi\,(4.00\ \text{u})\left(1.66054\times10^{-27}\ \text{kg/u}\right)\left(1.381\times10^{-23}\ \text{J/K}\right)(293\ \text{K})\right]^{-3/2} \end{array}\right) \\ &= 3.56\times10^{-6}, \end{aligned}$$

which is much less than one. In the above calculation, care must be taken in evaluating A; in SI units, the exponent of h^3 is greater than 100, and will cause difficulty if tried on some standard calculators. A possible method of evaluation is to find the last term first, multiply by h and then multiply by h^2.

9-51: See Problem 9-49. Here, the original factor of 8 must be retained, with the result that

$$\begin{aligned} A &= \frac{1}{2}\,\frac{N}{V}\,h^3\,(2\pi\,m_e\,kT)^{-3/2} \\ &= \left(\begin{array}{l} (1/2)\left(8.48\times10^{28}\ \text{m}^{-3}\right)\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)^3\times \\ \quad\left[2\pi\left(9.1095\times10^{-31}\ \text{kg}\right)\left(1.381\times10^{-23}\ \text{J/K}\right)(293\ \text{K})\right]^{-3/2} \end{array}\right) \\ &= 3.50\times10^{3}, \end{aligned}$$

which is much greater than one, and so the Fermi-Dirac distribution cannot be approximated by a Maxwell-Boltzmann distribution. (See the above note for Problem 9-49 regarding the difficulties involved in using h^3 numerically.)

9-53: The number density ($N_{\rm atom}/V$) for either gas is the ratio of the total mass and the mas of a single atom, divided by the volume (assumed spherical);

$$\begin{aligned}\frac{N}{V} &= \frac{M_{\rm star}}{m_{\rm C}}\,\frac{1}{(4\pi/3)\,R_{\rm star}^3}\\ &= \frac{\left(2.0\times10^{30}\ {\rm kg}\right)/2}{(12\ {\rm u})\,(1.66054\times10^{-27}\ {\rm kg/u})}\,\frac{1}{(4\pi/3)\,((0.010)\times7.0\times10^{8}\ {\rm m})^3}\\ &= 3.49\times10^{34}\ {\rm atoms/m^3}.\end{aligned}$$

(a) The Fermi energy of the carbon nucleus gas is found with the above value of ($N_{\rm atom}/V$), with, of course, one nucleus per atom;

$$\begin{aligned}\epsilon_F &= \frac{h^2}{2\,m_C}\left(\frac{3}{8\pi}\,\frac{N_{\rm atom}}{V}\right)^{2/3}\\ &= \frac{\left(6.626\times10^{-34}\ {\rm J\cdot s}\right)^2}{2\,(12\ {\rm u})\,(1.66054\times10^{-27}\ {\rm kg/u})}\left(\frac{3}{8\pi}\left(3.49\times10^{34}\ {\rm nuceli/m^3}\right)\right)^{2/3}\\ &= 2.85\times10^{-19}\ {\rm J} = 1.78\ {\rm eV}.\end{aligned}$$

Because there are six electrons per carbon atom, the Fermi energy of the electron gas is found with 6 times the above value of ($N_{\rm atom}/V$);

$$\begin{aligned}\epsilon_F &= \frac{h^2}{2\,m_e}\left(\frac{3}{8\pi}\,\frac{6\,N_{\rm atom}}{V}\right)^{2/3}\\ &= \frac{\left(6.626\times10^{-34}\ {\rm J\cdot s}\right)^2}{2\,(9.1095\times10^{-31}\ {\rm kg})}\left(\frac{18}{8\pi}\left(3.49\times10^{34}\ {\rm nuceli/m^3}\right)\right)^{2/3}\\ &= 2.06\times10^{-14}\ {\rm J} = 129\ {\rm keV}\end{aligned}$$

(use of less precise values for the constants, or roundoff in intermediate calculations, may lead to a result that differs in the last significant figure).

(b) For either gas, $kT = \left(8.617\times10^{-5}\ {\rm eV/K}\right)\left(10^7\ {\rm K}\right) = 862\ {\rm eV}$. The gas of nuclei is nondegenerate, and the gas of electrons will be mostly degenerate (the factor $kT/\epsilon_F \approx 10^{-2}$, and there will be a small fraction of the electrons above the Fermi energy).