# 7

# Consolidation of Soil and Settlement of Structures

## 7–1 INTRODUCTION

Structures built on soil are subject to settlement. Some settlement is often inevitable, and, depending on the circumstances, some settlement is tolerable. For example, small uniform settlement of a building throughout the floor area might be tolerable, whereas nonuniform settlement of the same building might not be. Or, settlement of a garage or warehouse building might be tolerable, whereas the same settlement (especially differential settlement) of a luxury hotel building would not be because of damage to walls, ceilings, and so on. In any event, a knowledge of the causes of settlement and a means of computing (or predicting) settlement quantitatively are important to the geotechnical engineer.

Although there are several possible causes of settlement (e.g., dynamic forces, changes in the groundwater table, adjacent excavation, etc.), probably the major cause is compressive deformation of soil beneath a structure. Compressive deformation generally results from reduction in void volume, accompanied by rearrangement of soil grains and compression of the material in the voids. If soil is dry, its voids are filled with air; and because air is compressible, rearrangement of soil grains can occur rapidly. If soil is saturated, its voids are filled with incompressible water, which must be extruded from the soil mass before soil grains can rearrange themselves. In soils of high permeability (i.e., coarse-grained soils), this process requires a short time interval for completion, and almost all settlement occurs by the time construction is complete. However, in soils of low permeability (i.e., fine-grained soils), the process requires a long time interval for completion. The result is that the strain occurs very slowly; thus, settlement takes place slowly and continues over a long period of time. The latter case (fine-grained soil) is of more concern because of long-term uncertainty.

Settlement of structures on fine-grained soil generally consists of three phases. The first phase is known as *immediate settlement*, or *volume distortion settlement*. As suggested by the expression, immediate settlement occurs rapidly after load is applied. Caused by soil volume distortion, immediate settlement is typically completed quickly and constitutes a relatively small amount of total settlement in fine-grained soils. Subsequent to immediate settlement, *primary consolidation settlement* occurs, the result of *primary consolidation.* (These are commonly referred to simply as *consolidation settlement* and *consolidation,* respectively.) Primary consolidation occurs due to extrusion of water from the voids as a result of increased loading. Primary consolidation settlement is very slow and continues over a long period of time. After primary settlement has ended, soil compression and additional associated settlement continue at a very slow rate, the result of plastic readjustment of soil grains due to new, changed stresses in the soil and progressive breaking of clayey particles and their interparticle bonds. This phenomenon is known as *secondary compression,* and associated settlement is called *secondary compression settlement.* Figure 7–1 illustrates the three phases of settlement as a function of time.

In analyzing clayey soil for consolidation settlement, one must differentiate between two types of clay—*normally consolidated clay* and *overconsolidated clay.* In the case of normally consolidated clay, the clay formation has never been subjected to

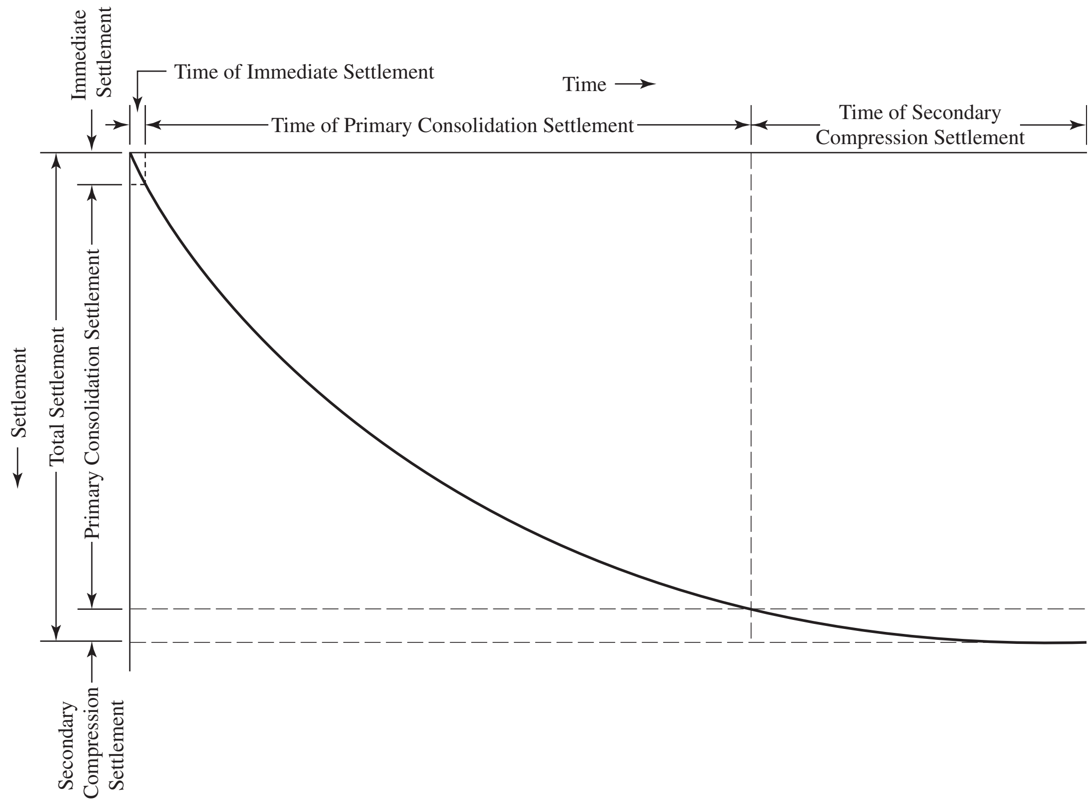



**FIGURE 7–1** Three phases of settlement for fine-grained soils as a function of time.

any loading larger than the present effective overburden pressure. This is the case whenever the height of soil above the clay formation (and therefore the weight of the soil above, which causes the pressure) has been more or less constant through time. With overconsolidated clay, the clay formation has been subjected at some time to a loading greater than the present effective overburden pressure. This occurs whenever the present height of soil above the clay formation is less than it was at some time in the past. Such a situation could exist if significant erosion has occurred at the ground surface. (Because of the erosion, the present height of soil above the clay formation is less than it was prior to the erosion.) It might be noted that overconsolidated clay is generally less compressible. As is related in this chapter, the analysis of clays for consolidation settlement differs somewhat depending on whether the clay is normally consolidated or overconsolidated.

This chapter deals primarily with the determination of settlement of structures. Sections 7–2 through 7–8 deal with settlement on clay. Section 7–2 deals with immediate settlement. Section 7–3 covers the laboratory testing required for analyzing consolidation settlement. Section 7–4 shows how laboratory data are analyzed to determine if the clay is normally consolidated, and Section 7–5 shows how they are analyzed to determine if the clay is overconsolidated. Section 7–6 demonstrates the development of a *field consolidation line,* which in turn is used to calculate consolidation settlement on clay (Section 7–7). Section 7–9 covers secondary compression and associated secondary compression settlement. Section 7–10 deals with settlement on sand.

## 7–2 IMMEDIATE SETTLEMENT OF LOADS ON CLAY

As noted previously, immediate settlement occurs rapidly, perhaps within hours or days after load is applied. It is caused by soil volume distortion, and it usually constitutes only a small amount of total settlement in fine-grained soils. Immediate settlement may be estimated based on the linear theory of elasticity. Equation (7–1) is applicable.*

$$S_i = C_s q B \frac{1 - \mu^2}{E_u} \tag{7–1}$$

where
$S_i$ = immediate settlement
$C_s$ = shape and foundation rigidity factor (see Tables 7–1 and 7–2)
$q$ = magnitude of evenly distributed load acting on the foundation area (total load/foundation area)
$B$ = width or diameter of foundation
$\mu$ = Poisson's ratio for the applied stress range (assume 0.5 for saturated clays, slightly less for partially saturated)
$E_u$ = undrained elastic modulus of clay (Young's Modulus or modulus of elasticity)

*From D. F. McCarthy, *Essentials of Soil Mechanics and Foundations,* 6th ed., 2002. Reprinted by permission of Pearson Education, Upper Saddle River, NJ.

**TABLE 7–1**
**Values of $C_s$ for Foundations on Clay Soil of Infinite Depth[a]**

| Shape | Center | Corner | Edge at Middle of Long Side | Average |
|---|---|---|---|---|
| Flexible foundation: | | | | |
| Circular | 1.00 | — | 0.64 | 0.85 |
| Square | 1.12 | 0.56 | 0.76 | 0.95 |
| Rectangular: | | | | |
| $L/B = 2$ | 1.53 | 0.76 | 1.12 | 1.30 |
| $L/B = 5$ | 2.10 | 1.05 | 1.68 | 1.82 |
| $L/B = 10$ | 2.56 | 1.28 | 2.10 | 2.24 |
| Rigid foundation: | | | | |
| Circular | 0.79 | — | 0.79 | 0.79 |
| Square | 0.82 | 0.82 | 0.82 | 0.82 |
| Rectangular: | | | | |
| $L/B = 2$ | 1.12 | 1.12 | 1.12 | 1.12 |
| $L/B = 5$ | 1.60 | 1.60 | 1.60 | 1.60 |
| $L/B = 10$ | 2.00 | 2.00 | 2.00 | 2.00 |

[a]Soil depth extends greater than $10B$.

*Source:* D. F. McCarthy, *Essentials of Soil Mechanics and Foundations,* 6th ed., 2002. Reprinted by permission of Pearson Education, Upper Saddle River, NJ.

**TABLE 7–2**
**Values of $C_s$ for Foundation on Clay Soil of Limited Depth (D) above a Rigid Substratum (Rock)**

| $C_s$ Under Center of Rigid Circular Foundation | Depth to Width Ratio ($D/B$) | $C_s$, Under Corner of Flexible Rectangular Foundation[a] | | | | |
|---|---|---|---|---|---|---|
| | | $L/B = 1$ | $L/B = 2$ | $L/B = 5$ | $L/B = 10$ | $L/B = \infty$ |
| 0.35 | 1 | 0.15 | 0.12 | 0.10 | 0.04 | 0.04 |
| 0.54 | 2 | 0.29 | 0.29 | 0.27 | 0.26 | 0.26 |
| 0.69 | 5 | 0.44 | 0.52 | 0.55 | 0.54 | 0.52 |
| 0.74 | 10 | 0.48 | 0.64 | 0.76 | 0.77 | 0.73 |
| 0.79 | ∞ | 0.56 | 0.76 | 1.05 | 1.28 | — |

[a]To determine $C_s$ for center of a foundation area, divide foundation shape into four equal subrectangles, then assign the $B$ dimension based on the size of one of the subrectangles. Multiply the selected value of $C_s$ by 4 to use with Eq. (7–1).

*Source:* D. F. McCarthy, *Essentials of Soil Mechanics and Foundations,* 6th ed., 2002. Reprinted by permission of Pearson Education, Upper Saddle River, NJ.

$E_u$ may be evaluated using the results of undrained triaxial compression tests (Chapter 8) performed on undisturbed soil samples. Values of $E_u$ frequently lie in the range $500c_u$ to $1{,}500c_u$, where $c_u$ is the soil cohesion shear strength as determined from the undrained tests. The lower range is for clays of high plasticity and where foundation loads are large, while the higher range is for clays of low plasticity and where foundation loading is low. If the results of undrained triaxial compression tests are not available, Table 7–3 can be used to estimate appropriate values of $E_u$.

**TABLE 7–3**
**Values of $E_u$ Related to Soil Consistency**

| Clay Consistency | Cohesion (Shear Strength) | Values for $E_u$ *MPa* | Values for $E_u$ *ksf* |
|---|---|---|---|
| Soft | <24 kPa (<500 psf) | 2.5–15 | 50–300 |
| Medium to stiff | 25–100 kPa (500–2000 psf) | 15–50 | 300–1000 |
| Very stiff to hard | >100 kPa (>2000 psf) | 50–200 | 1000–4000 |

*Source:* D. F. McCarthy, *Essentials of Soil Mechanics and Foundations,* 6th ed., 2002. Reprinted by permission of Pearson Education, Upper Saddle River, NJ.

## EXAMPLE 7–1

*Given*

1. A square, 3-m by 3-m rigid footing is resting on a deep clay deposit.
2. The footing is to carry a concentrated load of 1800 kN.
3. The undrained elastic modulus of clay ($E_u$) is estimated to be 40 MPa.
4. Assume Poisson's ratio of the clay is 0.5.

*Required*

The expected immediate settlement beneath the center of the footing.

**Solution**
From Eq. (7–1),

$$S_i = C_s qB\left(\frac{1 - \mu^2}{E_u}\right) \tag{7–1}$$

From Table 7–1, $C_s$ is 0.82.

$$S_i = (0.82)\left(\frac{1800 \text{ kN}}{(3 \text{ m})(3 \text{ m})}\right)(3 \text{ m})\left(\frac{1 - 0.5^2}{40 \times 10^3 \text{ kN/m}^2}\right)$$

$$S_i = 0.0092 \text{ m, or } 9.2 \text{ mm}$$

## 7–3 CONSOLIDATION TEST

As a means of estimating both the amount and time of consolidation and resulting settlement, consolidation tests are run in a laboratory. For complete and detailed instructions for conducting a consolidation test, the reader is referred to *Soil Properties: Testing, Measurement, and Evaluation*, 5th edition, by Liu and Evett (2003) or ASTM D 2435. A generalized discussion is given here.

To begin with, an undisturbed soil sample is placed in a metal ring. One porous disk is placed above the sample, and another is placed beneath the sample. The purpose of the disks is to allow water to flow vertically into and out of the soil sample. This assembly is immersed in water. As a load is applied to the upper disk, the sample is compressed, and deformation is measured by a dial gauge (see Figure 7–2).

To begin a particular test, a specific pressure (e.g., 500 lb/ft$^2$) is applied to the soil sample, and dial readings (reflecting deformation) and corresponding time observations are made and recorded until deformation has nearly ceased. Normally, this is done over a 24-hour period. Then, a graph is prepared using these data, with time along the abscissa on a logarithmic scale and dial readings along the ordinate on an arithmetic scale. An example of such a graph is given in Figure 7–3.

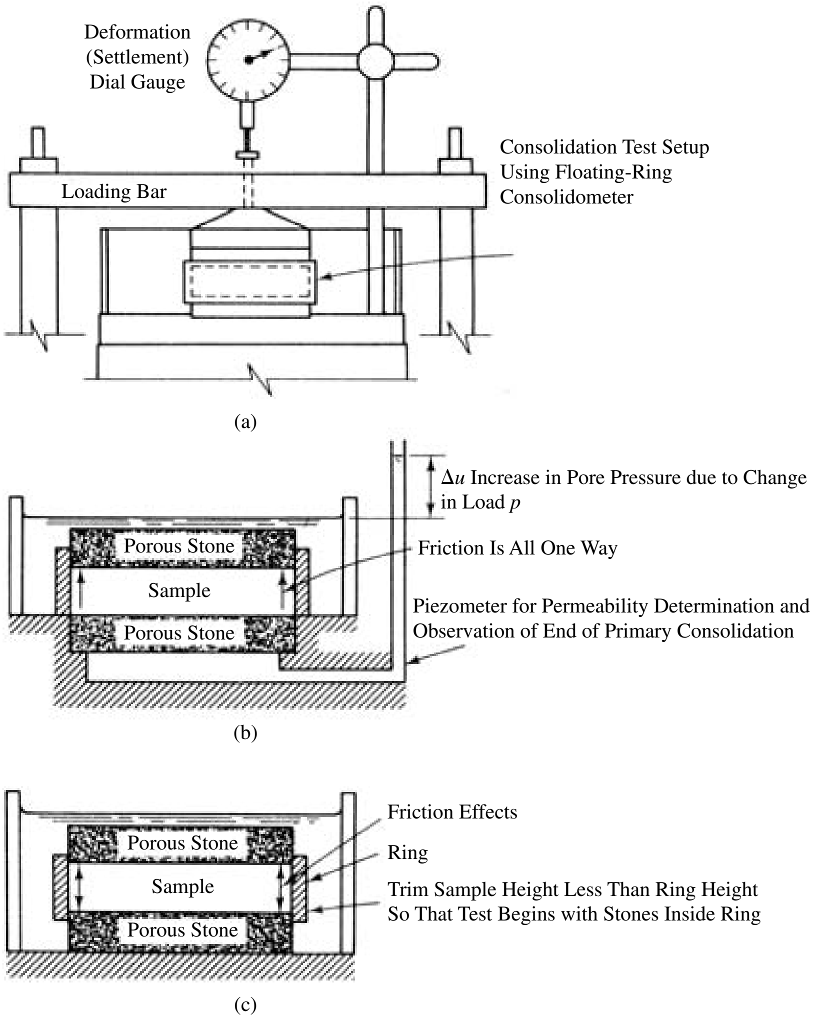



**FIGURE 7–2** (a) Consolidometer; (b) fixed-ring consolidometer, which may be used to obtain information during a consolidation test if a piezometer is installed; (c) floating-ring consolidometer.

*Source:* D. F. McCarthy, *Essentials of Soil Mechanics and Foundations,* 6th ed., 2002. Reprinted by permission of Pearson Education, Upper Saddle River, NJ.

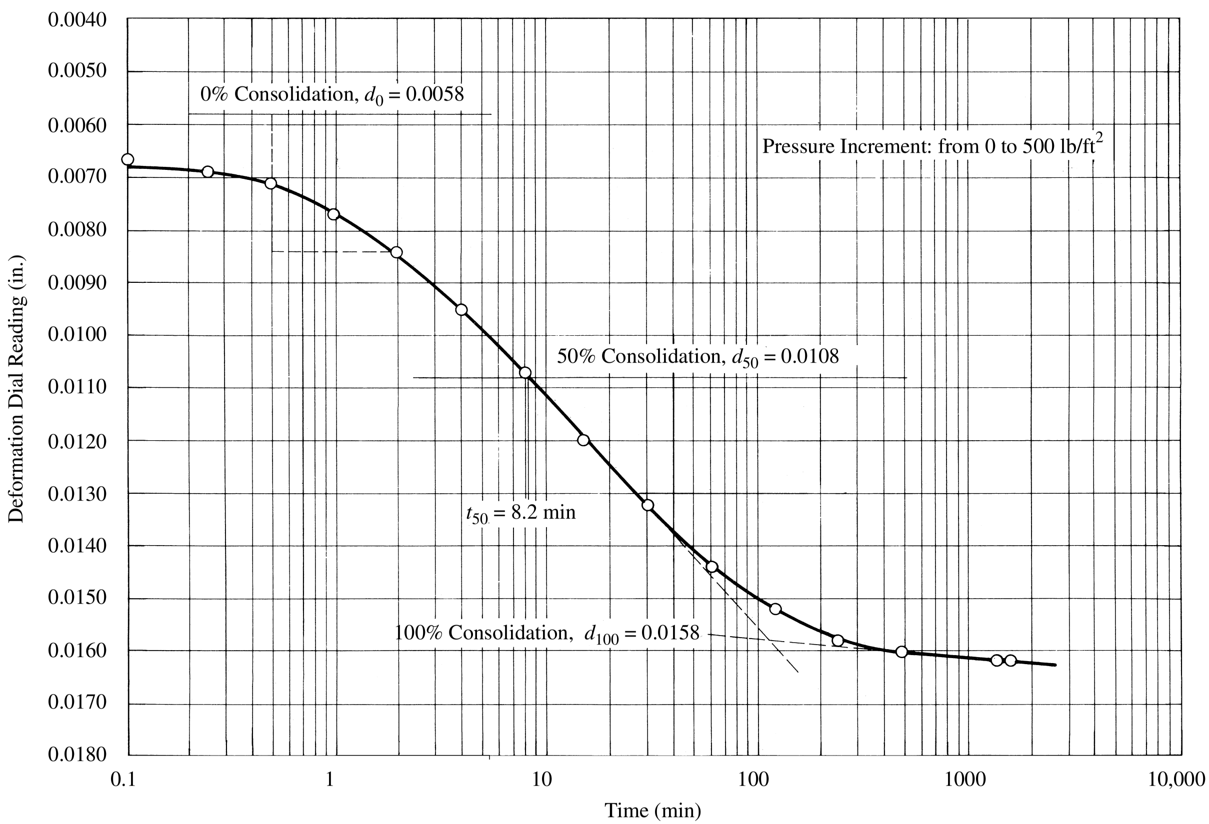



**FIGURE 7–3** Dial readings versus log time curve.
*Source:* J. E. Bowles, *Engineering Properties of Soils and Their Measurement,* 2nd ed., McGraw-Hill Book Company. New York, 1978. Reprinted by permission.

The procedure is repeated after the applied pressure is doubled, giving another graph of time versus dial readings corresponding to the new pressure. The procedure is repeated for additional doublings of applied pressure until the applied pressure is in excess of the total pressure to which the clay formation is expected to be subjected when the proposed structure is built. [The total pressure includes effective overburden pressure and net additional pressure (or consolidation pressure) due to the structure.]

From each graph of time versus dial readings, the void ratio ($e$) and coefficient of consolidation ($c_v$) that correspond to the specific applied pressure ($p$) for that graph are determined using the following steps.

1. Find the deformation representing 100% consolidation for each load increment. First, draw a straight line through the points representing the final readings that exhibit a straight-line trend and a flat slope. Draw a second straight line tangent to the steepest part of the deformation versus log time curve. The intersection of the two lines represents the deformation corresponding to 100% consolidation. Compression that occurs subsequent to 100% consolidation is defined as secondary compression (Figure 7–3).
2. Find the deformation representing 0% consolidation by selecting the deformations at any two times that have a ratio of 1:4. The deformation corresponding to the larger of the two times should be greater than one-fourth but less than one-half of the total change in deformation for the load increment. The deformation corresponding to 0% consolidation is equal to the deformation corresponding to the smaller time interval less the difference in the deformations for the two selected times (Figure 7–3).
3. The deformation corresponding to 50% consolidation for each load increment is equal to the average of the deformations corresponding to the 0 and 100% deformations. The time required for 50% consolidation under any load increment may be found graphically from the deformation versus log time curve for that load increment by observing the time that corresponds to 50% consolidation (ASTM, 2002) (Figure 7–3).
4. To obtain the change in thickness of the specimen, subtract the initial dial reading at the beginning of the first loading from the dial reading corresponding to 100% consolidation for the given loading. Find the change in void ratio ($\Delta e$) for the given loading by dividing the change in thickness of the specimen by the height of solid in the specimen. Determine the void ratio ($e$) for this loading by subtracting the change in void ratio ($\Delta e$) from the initial void ratio ($e_0$).
5. Compute the coefficient of consolidation ($c_v$) for this loading using the following equation:

$$c_v = \frac{0.196H^2}{t_{50}} \tag{7–2}$$

where $H$ = thickness of test specimen at 50% consolidation (i.e., initial height of specimen at beginning of test minus deformation dial reading at 50% consolidation). Use half the thickness if the specimen is drained on both top and bottom during the test.

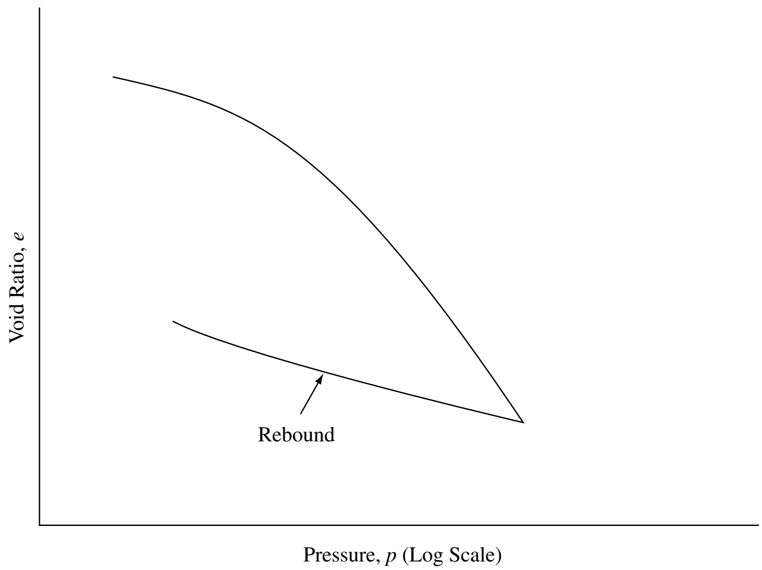
$t_{50}$ = time to 50% consolidation

By using values of $e$ and $c_v$ determined from the various graphs of time versus dial readings corresponding to the different test loadings, one can prepare two graphs—one of void ratio versus pressure ($e$–log $p$ curve), with pressure along the abscissa on a logarithmic scale and void ratio along the ordinate on an arithmetic scale, and another of consolidation coefficient versus pressure ($c_v$–log $p$ curve), with pressure along the abscissa on a logarithmic scale and coefficient of consolidation along the ordinate on an arithmetic scale. An example of an $e$–log $p$ curve is given in Figure 7–4, and an example of a $c_v$–log $p$ curve is given in Figure 7–5. As is related subsequently, the $e$–log $p$ curve is used to determine the amount of consolidation settlement, and the $c_v$–log $p$ curve is used to determine the timing of the consolidation settlement.

In Figure 7–4, the upper curve exhibits the relationship between void ratio and pressure as the pressure is increased. As is shown in Section 7–6, in the case of overconsolidated clay, it is necessary to have a "rebound curve." Exhibited by the lower curve in Figure 7–4, the rebound curve is obtained by unloading the soil sample during the consolidation test after the maximum pressure has been reached. As the sample is unloaded, the soil tends to swell, causing movement and associated dial readings to reverse direction.

The primary results of a laboratory consolidation test are (1) the $e$–log $p$ curve, (2) the $c_v$–log $p$ curve, and (3) the initial void ratio of the soil *in situ* ($e_0$).

**FIGURE 7–4** Void ratio versus logarithm of pressure.



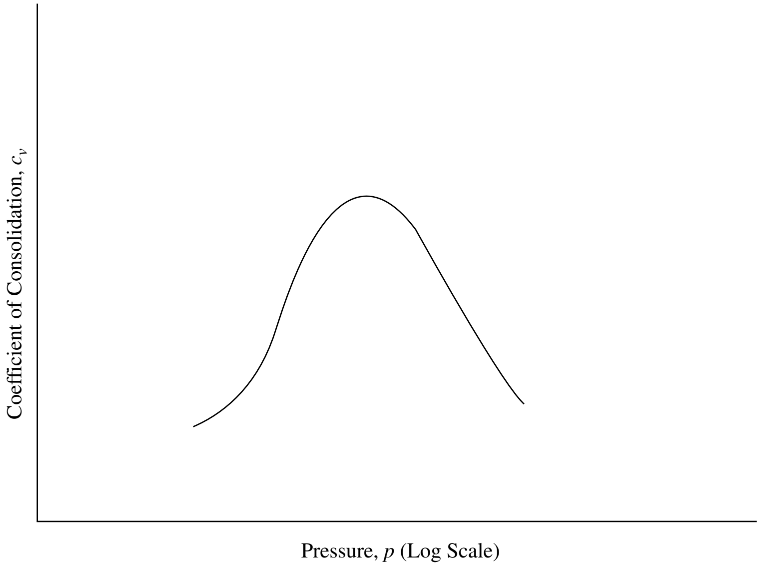



**FIGURE 7–5** Coefficient of consolidation versus logarithm of pressure.

## EXAMPLE 7–2

*Given*

A clayey soil obtained from the field was subjected to a laboratory consolidation test. The test results are as follows:

1. Diameter of test specimen = 2.50 in.
2. Initial height of specimen = 0.780 in.
3. Specific gravity of solids = 2.72.
4. Dry mass of specimen = 75.91 g.
5. Pressure versus deformation dial readings are as given in Table 7–4.

*Required*

1. Initial void ratio.
2. The $e$–log $p$ curve.

### Solution

1. Volume of solid in specimen $(V_s) = \dfrac{\text{Dry mass of solid}}{\text{Unit mass of solid}}$

$$= \frac{\text{Dry mass of solid}}{(\text{Specific gravity of solids})(\text{Unit mass of water})}$$

$$= \frac{75.91\text{ g}}{(2.72)(1.0\text{ g/cm}^3)} = 27.91\text{ cm}^3$$

**TABLE 7–4**
**Pressure Versus Deformation Dial Readings for Example 7–2**

| Pressure, $p$ (lb/ft$^2$) | Initial Deformation Dial Reading at Beginning of First Loading (in.) | Deformation Dial Reading Representing 100% Primary Consolidation (in.) |
|---|---|---|
| 0 | 0 | 0 |
| 500 | 0 | 0.0158 |
| 1000 | 0 | 0.0284 |
| 2000 | 0 | 0.0490 |
| 4000 | 0 | 0.0761 |
| 8000 | 0 | 0.1145 |
| 16,000 | 0 | 0.1580 |

$$\text{Initial volume of specimen } (V) = \frac{(0.780\text{ in.})(\pi)(2.50\text{ in.})^2}{4}$$
$$= 3.829\text{ in.}^3\text{, or } 62.74\text{ cm}^3$$
$$\text{Initial volume of void in specimen } (V_v) = 62.74\text{ cm}^3 - 27.91\text{ cm}^3$$
$$= 34.83\text{ cm}^3$$
$$\text{Initial void ratio } (e_0) = \frac{V_v}{V_s} = \frac{34.83\text{ cm}^3}{27.91\text{ cm}^3} = 1.248$$

2. To develop the $e$–log $p$ curve, one must determine the height of solid in the specimen.

$$\text{Height of solid in specimen } (H_s) = \frac{V_s}{\text{Area of specimen}}$$
$$= \frac{27.91\text{ cm}^3}{(\pi)[(2.50\text{ in.})(2.54\text{ cm/in.})]^2/4}$$
$$= 0.881\text{ cm}$$

The change in thickness of the specimen ($\Delta H$) can be found by subtracting the initial deformation dial reading from the deformation dial reading representing 100% primary consolidation. For the 500 lb/ft$^2$ pressure,

$$\Delta H = (0.0158\text{ in.} - 0\text{ in.})(2.54\text{ cm/in.}) = 0.0401\text{ cm}$$

The change in void ratio ($\Delta e$) can be determined by dividing $\Delta H$ by $H_s$. For the 500 lb/ft$^2$ pressure,

$$\Delta e = \frac{0.0401\text{ cm}}{0.881\text{ cm}} = 0.046$$

Finally, the void ratio ($e$) can be computed by subtracting $\Delta e$ from $e_0$. For the 500 lb/ft$^2$ pressure,

$$e = 1.248 - 0.046 = 1.202$$

Similar computations can be made for the remaining pressures and are presented in Table 7–5.

The $e$–log $p$ curve is prepared by plotting void ratio versus pressure, with the latter on a logarithmic scale. This curve is given in Figure 7–6.

## EXAMPLE 7–3

*Given*

Additional test results from the consolidation test on the clayey soil presented in Example 7–2 are as given in Table 7–6. In addition, the specimen was drained on both top and bottom during the test.

*Required*

The $c_v$–log $p$ curve.

**Solution**

The coefficient of consolidation is computed using Eq. (7–2).

$$c_v = \frac{0.196H^2}{t_{50}} \quad \textbf{(7–2)}$$

**TABLE 7–5**
**Computed Void Ratio–Pressure Relation for Example 7–2**

| Pressure, $p$ (lb/ft$^2$) | Initial Deformation Dial Reading at Beginning of First Loading (in.) | Deformation Dial Reading Representing 100% Primary Consolidation (in.) | Change in Thickness of Specimen, $\Delta H$ (cm) | Change in Void Ratio, $\Delta e$ $\left[\Delta e = \frac{\Delta H}{H_s}\right]$ | Void Ratio, $e$ $[e = e_0 - \Delta e]$ |
|---|---|---|---|---|---|
| *(1)* | *(2)* | *(3)* | *(4) = [(3) − (2)] × 2.54* | $(5) = \frac{(4)}{0.881}$ | *(6) = 1.248 − (5)* |
| 0 | 0 | 0 | 0 | 0 | 1.248 |
| 500 | 0 | 0.0158 | 0.0401 | 0.046 | 1.202 |
| 1000 | 0 | 0.0284 | 0.0721 | 0.082 | 1.166 |
| 2000 | 0 | 0.0490 | 0.1245 | 0.141 | 1.107 |
| 4000 | 0 | 0.0761 | 0.1933 | 0.219 | 1.029 |
| 8000 | 0 | 0.1145 | 0.2908 | 0.330 | 0.918 |
| 16,000 | 0 | 0.1580 | 0.4013 | 0.456 | 0.792 |

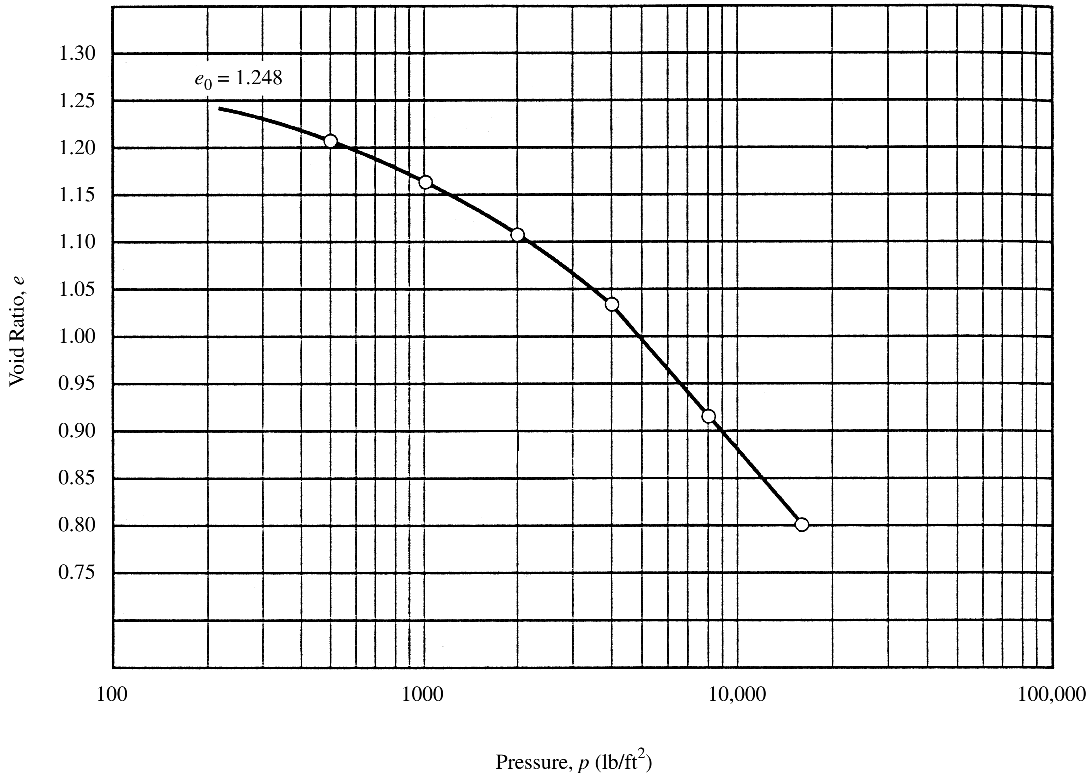



**FIGURE 7–6** Void ratio versus logarithm of pressure for Example 7–2.

**TABLE 7–6**
**Pressure Versus Deformation Dial Readings and Time for 50% Consolidation for Example 7–3**

| Pressure, $p$ (lb/ft$^2$) | Initial Height of Specimen at Beginning of Test, $H_0$ (in.) | Deformation Dial Reading at 50% Consolidation (in.) | Time for 50% Consolidation (min) |
|---|---|---|---|
| 0 | 0.780 | — | — |
| 500 | 0.780 | 0.0108 | 8.2 |
| 1000 | 0.780 | 0.0233 | 6.4 |
| 2000 | 0.780 | 0.0398 | 4.0 |
| 4000 | 0.780 | 0.0644 | 3.4 |
| 8000 | 0.780 | 0.0982 | 3.5 |
| 16,000 | 0.780 | 0.1387 | 4.0 |

**TABLE 7–7**
**Computed Coefficient of Consolidation–Pressure Relation for Example 7–3**

| Pressure, $p$ (lb/ft$^2$) | Initial Height of Specimen at Beginning of Test, $H_0$ (in.) | Deformation Dial Reading at 50% Consolidation (in.) | Thickness of Specimen at 50% Consolidation (in.) | Half-Thickness of Specimen at 50% Consolidation (in.) | Time for 50% Consolidation (min) | Coefficient of Consolidation (in.$^2$/min) |
|---|---|---|---|---|---|---|
| *(1)* | *(2) [from Example 7–2]* | *(3) [from dial readings versus log of time curves]* | $(4) = (2) - (3)$ | $(5) = \frac{(4)}{2}$ | *(6) [from dial readings versus log of time curves]* | $(7) = \frac{0.196 \times (5)^2}{(6)}$ |
| 0 | 0.780 | — | — | — | — | — |
| 500 | 0.780 | 0.0108 | 0.769 | 0.385 | 8.2 | $3.54 \times 10^{-3}$ |
| 1000 | 0.780 | 0.0233 | 0.757 | 0.378 | 6.4 | $4.38 \times 10^{-3}$ |
| 2000 | 0.780 | 0.0398 | 0.740 | 0.370 | 4.0 | $6.71 \times 10^{-3}$ |
| 4000 | 0.780 | 0.0644 | 0.716 | 0.358 | 3.4 | $7.39 \times 10^{-3}$ |
| 8000 | 0.780 | 0.0982 | 0.682 | 0.341 | 3.5 | $6.51 \times 10^{-3}$ |
| 16,000 | 0.780 | 0.1387 | 0.641 | 0.320 | 4.0 | $5.02 \times 10^{-3}$ |

The thickness of the specimen at 50% consolidation can be determined by subtracting the deformation dial reading at 50% consolidation from the initial height of the specimen (0.780 in., from Example 7–2). For the 500 lb/ft$^2$ pressure,

$$\text{Thickness of specimen} = 0.780 \text{ in.} - 0.0108 \text{ in.} = 0.769 \text{ in.}$$

Because the specimen was drained on both top and bottom, half the thickness of the specimen (0.385 in.) must be used for $H$ in Eq. (7–2). For the 500 lb/ft$^2$ pressure, the value of $t_{50}$ is given to be 8.2 min. Substituting these values into Eq. (7–2) gives the following:

$$c_v = \frac{(0.196)(0.385 \text{ in.})^2}{8.2 \text{ min}} = 3.54 \times 10^{-3} \text{ in.}^2/\text{min, or } 3.81 \times 10^{-4} \text{ cm}^2/\text{sec}$$

Similar computations can be made for the remaining pressures, and the results of $c_v$ at various values of $p$ are presented in Table 7–7.

The $c_v$–log $p$ curve is prepared by plotting the coefficient of consolidation versus pressure, with the latter on a logarithmic scale. This curve is given in Figure 7–7.

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Figure 7–8 gives a relationship from which approximate values of the coefficient of consolidation as a function of liquid limit for different soil types can be noted.

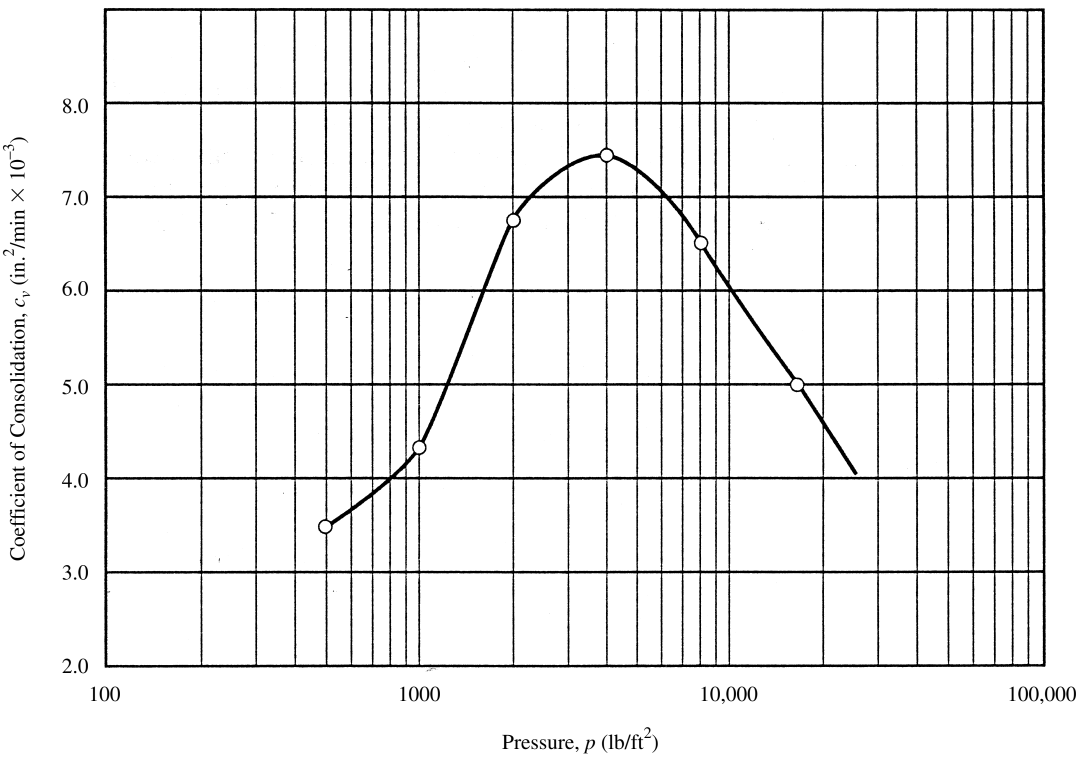



**FIGURE 7–7** Coefficient of consolidation versus logarithm of pressure for Example 7–3.

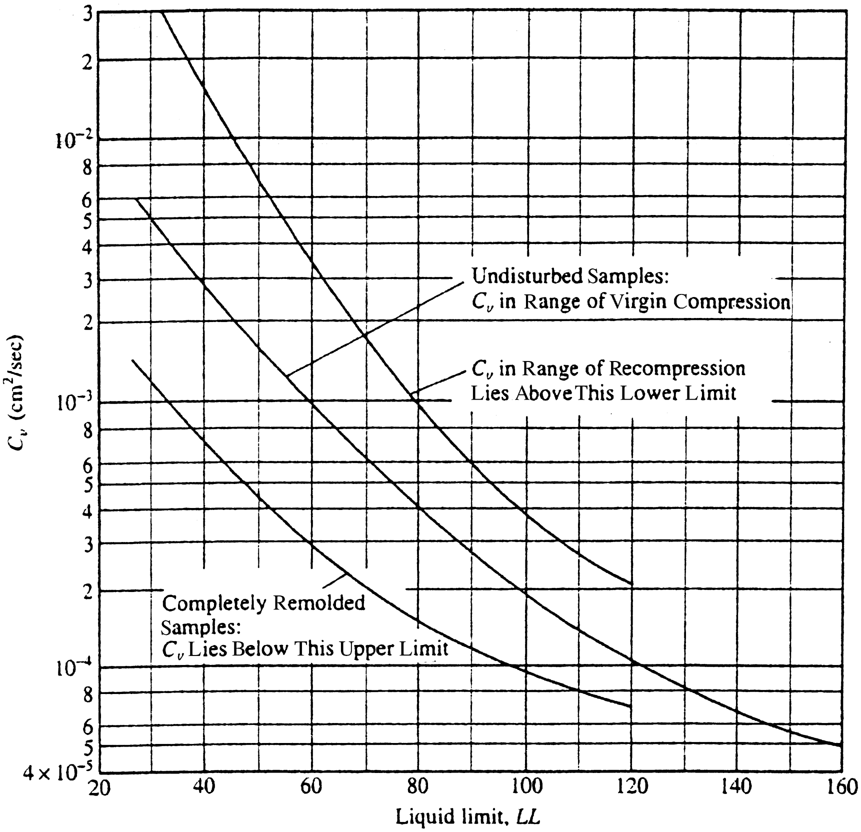



**FIGURE 7–8** Range of coefficient of consolidation (after U.S. Department of the Navy, 1971).

## 7–4 NORMALLY CONSOLIDATED CLAY

As indicated in Section 7–1, in the case of a clayey soil it is necessary to determine whether the clay is normally consolidated or overconsolidated. This section shows how to determine if a given clayey soil is normally consolidated.

It is first necessary, however, to determine the present effective overburden pressure ($p_0$). This pressure is the result of the (effective) weight of soil above midheight of the consolidating clay layer. Although the reader probably knows how to calculate the effective overburden pressure, the procedure is illustrated in Example 7–4.

### EXAMPLE 7–4

*Given*

The soil profile shown in Figure 7–9.

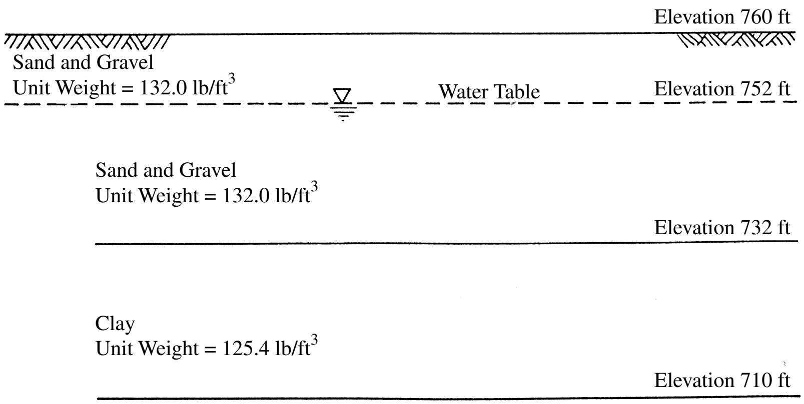



**FIGURE 7–9**

*Required*

Present effective overburden pressure ($p_0$) at midheight of the compressible clay layer.

**Solution**

The elevation of the midheight of the clay layer = (732 ft + 710 ft)/2 = 721 ft.

$$p_0 = (132.0\ \text{lb/ft}^3)(760\ \text{ft} - 752\ \text{ft}) + (132.0\ \text{lb/ft}^3 - 62.4\ \text{lb/ft}^3)$$

$$\times\ (752\ \text{ft} - 732\ \text{ft}) + (125.4\ \text{lb/ft}^3 - 62.4\ \text{lb/ft}^3)(732\ \text{ft} - 721\ \text{ft})$$

$$p_0 = 3141\ \text{lb/ft}^2 = 1.57\ \text{tons/ft}^2$$

---

The first step in determining if a given clayey soil is normally consolidated is to locate the point designated by a pressure of $p_0$ (distance along the abscissa) and void ratio of $e_0$ (distance along the ordinate). ($p_0$ is the present effective overburden pressure at midheight of the compressible clay layer, and $e_0$ is the initial void ratio of the soil *in situ*.) This point is labeled *a* in Figure 7–10. The next step is to project the lower right straight-line portion of the *e*–log *p* curve in a straight line upward and to the left. This is the dashed line in Figure 7–10; it will intersect a horizontal line drawn at $e = e_0$. The point of intersection of these two lines is labeled *b* in Figure 7–10. If point *b* is to the left of point *a* (as in Figure 7–10), the soil is normally consolidated clay (Peck et al., 1974).

## 7–5 OVERCONSOLIDATED CLAY

The procedure for determining if a given clay is overconsolidated is essentially the same as that for determining if it is normally consolidated. The point designated by a pressure of $p_0$ and void ratio of $e_0$ is located and labeled *a*. The lower right portion of the *e*–log *p* curve is projected in a straight line upward and to the left until it intersects

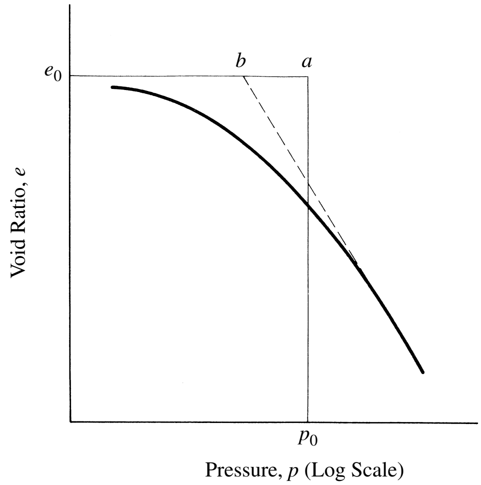



**FIGURE 7–10** Typical $e$–log $p$ curve for normally consolidated clay (Peck et al., 1974).

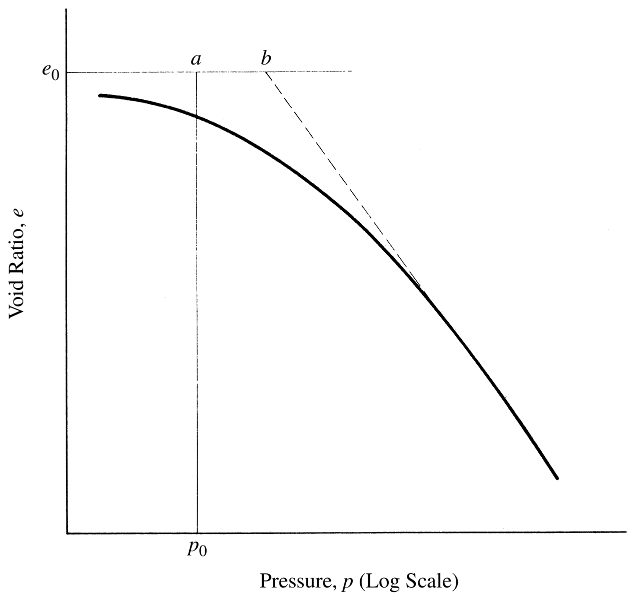



**FIGURE 7–11** Typical $e$–log $p$ curve for overconsolidated clay.
*Source:* R. B. Peck, W. E. Hansen, T. H. Thornburn, *Foundation Engineering,* 2nd ed., John Wiley & Sons, Inc. New York, 1974. Copyright © 1974 by John Wiley & Sons, Inc. Reprinted by permission of John Wiley & Sons, Inc.

a horizontal line drawn at $e = e_0$, with the point of intersection labeled $b$. If point $b$ is to the right of point $a$ (as in Figure 7–11), the soil is overconsolidated clay (Peck et al., 1974).

If the given clay is found to be overconsolidated, it is necessary to determine (for subsequent analysis of consolidation settlement) the maximum overburden pressure at the consolidated clay layer ($p_0'$). The following procedure, developed by

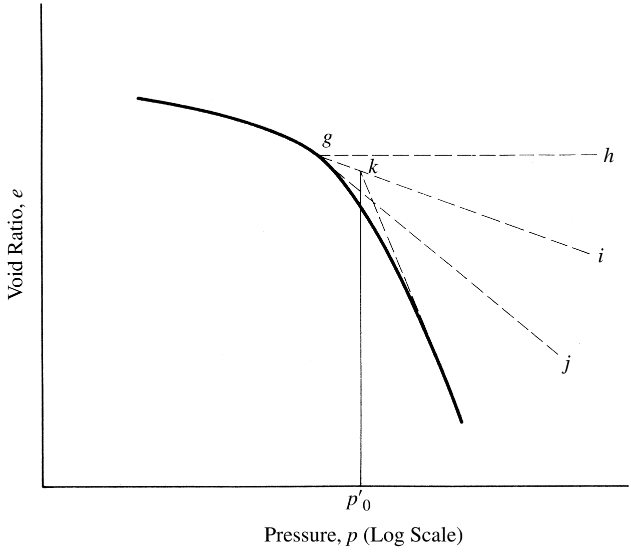



**FIGURE 7–12** Graphic construction for estimating maximum overburden pressure, $p_0'$ from $e$ –log $p$ curve.
*Source:* R. B. Peck, W. E. Hansen, T. H. Thornburn, *Foundation Engineering*, 2nd ed., John Wiley & Sons, Inc. New York, 1974. Copyright © 1974 by John Wiley & Sons, Inc. Reprinted by permission of John Wiley & Sons, Inc.

Casagrande (1936), can be used to determine $p_0'$. The first step is to locate the point on the $e$–log $p$ curve where the curvature is greatest (where the radius of curvature is smallest). This is indicated by point $g$ in Figure 7–12. From this point, two straight lines are drawn—one horizontal line (line $gh$ in Figure 7–12) and one tangent to the $e$–log $p$ curve (line $gj$ in Figure 7–12). The next step is to draw a line that bisects the angle between lines $gh$ and $gj$ (line $gi$ in Figure 7–12). The final step is to project the lower right straight-line portion of the $e$–log $p$ curve in a straight line upward and to the left. This projected line will intersect line $gi$ at a point such as $k$ in Figure 7–12. The value of $p$ corresponding to point $k$ ($p$ coordinate of point $k$ along the abscissa) is taken as $p_0'$ (Peck et al., 1974).

The overconsolidation ratio (OCR) can now be defined as the ratio of overconsolidation pressure $p_0'$ to present overburden pressure ($p_0$). Hence,

$$\text{OCR} = \frac{p_0'}{p_0} \quad \text{(7–3)}$$

The OCR is used to indicate the degree of overconsolidation.

## 7–6 FIELD CONSOLIDATION LINE

The $e$–log $p$ curves considered in previous sections give, of course, the relationship between void ratio and pressure for a given soil. Such a relationship is used in calculating consolidation settlement. The $e$–log $p$ curves of Figure 7–4 and Figures 7–10 through 7–12 reflect, however, the relationship between void ratio and pressure for the soil sample in the laboratory. Although an "undisturbed sample" is used in the laboratory test, it is not generally possible to duplicate soil in the laboratory exactly as it exists in the field. Thus, the $e$–log $p$ curves developed from laboratory consolidation

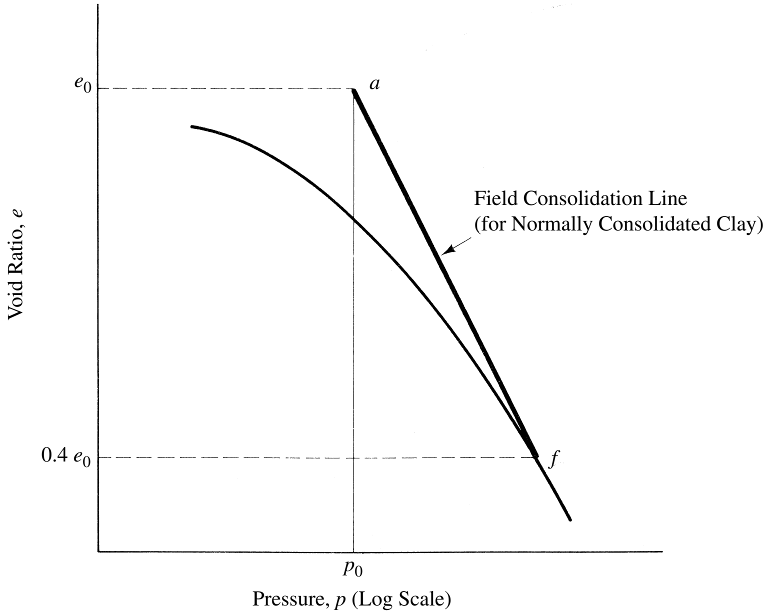



**FIGURE 7–13** Field consolidation line for normally consolidated clay.
*Source:* A. Casagrande, "The Determination of the Pre-consolidation Load and Its Practical Significance," *Proc. First Inst. Conf. Soil Mech.*, Cambridge, MA, **3**, 60–64 (1936).

tests are modified to give an $e$–log $p$ curve that is presumed to reflect actual field conditions. This modified $e$–log $p$ curve is called the *field consolidation line.* Two methods for determining the field consolidation line follow—one for normally consolidated clay, the other for overconsolidated clay.

In the case of normally consolidated clay, determination of the field consolidation line is fairly simple. With the given $e$–log $p$ curve developed from the laboratory test (Figure 7–13), the point on the $e$–log $p$ curve corresponding to $0.4e_0$ is determined (point $f$ in Figure 7–13). A straight line connecting points $a$ and $f$ gives the field consolidation line for the normally consolidated clay (Schmertmann, 1955). (The reader will recall that, as related in Section 7–4 and Figure 7–10, point $a$ is the point designated by a pressure of $p_0$ and void ratio of $e_0$.)

For overconsolidated clay, finding the field consolidation line is somewhat more difficult. With the given $e$–log $p$ curve developed from the laboratory test (Figure 7–14), the point on the $e$–log $p$ curve corresponding to $0.4e_0$ is determined (point $f$ in Figure 7–14). Point $a$ (the point designated by a pressure of $p_0$ and void ratio of $e_0$) is located, and a line is drawn through point $a$ parallel to the rebound line. This line through point $a$ parallel to the rebound line is shown as a dashed line in Figure 7–14; it will intersect a vertical line drawn at $p = p_0'$. (The procedure for evaluating $p_0'$ was given in Section 7–5.) This point of intersection is designated by $m$ in Figure 7–14. Points $m$ and $f$ are connected by a straight line, and points $a$ and $m$ are

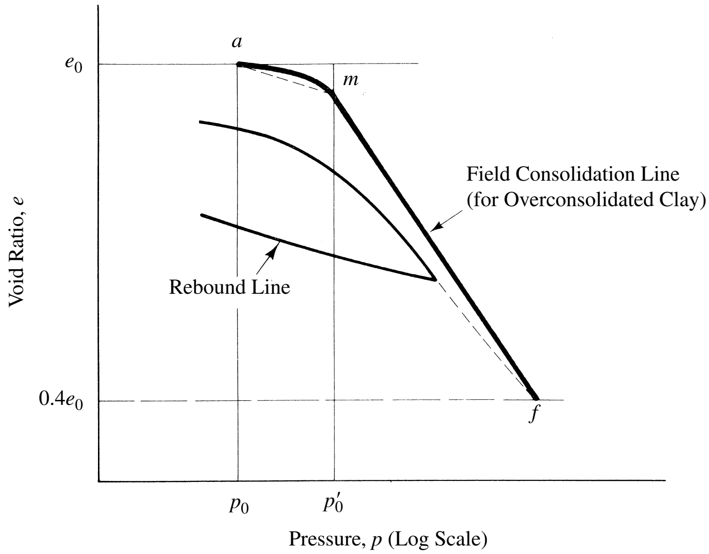



**FIGURE 7–14** Field consolidation line for overconsolidated clay.
*Source:* J. H. Schmertmann, "The Undisturbed Consolidation Behavior of Clay," *Trans. ASCE,* **120,** 1201–1227 (1955). Reprinted by permission.

connected by a curved line that follows the same general shape of the $e$–log $p$ curve. This curved line from $a$ to $m$ and the straight line from $m$ to $f$ give the field consolidation line for the overconsolidated clay (Schmertmann, 1953).

It is the field consolidation line—the dark line in Figure 7–13 (normally consolidated clay) and Figure 7–14 (overconsolidated clay)—that is used in calculating consolidation settlement. The other curves (dial readings versus time and $e$–log $p$ curve) are required only as a means of determining the field consolidation line. Once the field consolidation line is established, these other curves are no longer used in determining the amount of consolidation settlement.

The *slope* of the field consolidation line is called the *compression index* ($C_c$); it may be evaluated by finding coordinates of any two points on the field consolidation line [$(p_1, e_1)$ and $(p_2, e_2)$] and substituting these values into the following equation:

$$C_c = \frac{e_1 - e_2}{\log p_2 - \log p_1} = \frac{e_1 - e_2}{\log (p_2/p_1)} \tag{7–4}$$

Skempton (1944) has shown that the compression index for normally consolidated clays can be approximated in terms of the liquid limit ($LL$, in percent) by the following equation:

$$C_c = 0.009(LL - 10) \tag{7–5}$$

It should be emphasized that the value of $C_c$ computed from Eq. (7–4) is obtained from the field consolidation line, which is based on the results of a consolidation test, whereas that computed from Eq. (7–5) is based solely on the liquid limit. The consolidation test is much more lengthy, difficult, and expensive to perform than the test to determine the liquid limit. Also, calculation of $C_c$ using results of a consolidation test is much more involved than calculation using the liquid limit. However, calculation of $C_c$ using the liquid limit [Eq. (7–5)] is merely an approximation and should be used only when very rough values of settlement are acceptable (such as in a preliminary design).

## EXAMPLE 7–5

*Given*

When the total pressure acting at midheight of a consolidating clay layer is 200 kN/m$^2$, the corresponding void ratio of the clay is 0.98. When the total pressure acting at the same location is 500 kN/m$^2$, the corresponding void ratio decreases to 0.81.

*Required*

The void ratio of the clay if the total pressure acting at midheight of the consolidating clay layer is 1000 kN/m$^2$.

**Solution**

From Eq. (7–4),

$$C_c = \frac{e_1 - e_2}{\log (p_2/p_1)} \tag{7–4}$$

$$e_1 = 0.98$$

$$e_2 = 0.81$$

$$p_2 = 500 \text{ kN/m}^2$$

$$p_1 = 200 \text{ kN/m}^2$$

$$C_c = \frac{0.98 - 0.81}{\log\left(\dfrac{500 \text{ kN/m}^2}{200 \text{ kN/m}^2}\right)} = 0.427$$

Substituting the computed value of $C_c$ and the same values of $e_1$ and $p_1$ into Eq. (7–4) gives

$$0.427 = \frac{0.98 - e_2}{\log\left(\dfrac{1000 \text{ kN/m}^2}{200 \text{ kN/m}^2}\right)}$$

$$e_2 = 0.68$$

**EXAMPLE 7–6**

*Given*

A normally consolidated clay has a liquid limit of 51%.

*Required*

Estimate the compression index ($C_c$).

**Solution**

From Eq. (7–5),

$$C_c = 0.009(LL - 10) \quad \textbf{(7–5)}$$

$$C_c = 0.009(51 - 10) = 0.369$$

In the case of overconsolidated clays for present effective overburden pressure at midheight of the consolidating clay layer ($p_0$) plus net additional pressure at midheight of the consolidating clay layer due to structure load ($\Delta p$) less than the overconsolidation pressure ($p_0'$) [i.e., $p_0 + \Delta p < p_0'$], field $e$–log $p$ variations will be along the line $a$–$m$ in Figure 7–14. The slope of this line will be approximately the same as that of the laboratory rebound line (see Figure 7–14). The slope of the rebound line is known as the *swell index* and denoted $C_s$; it is an indication of volume increase of a soil as a result of pressure removal. Values of $C_s$ can be determined from laboratory consolidation tests. It is noted that the swell index is a considerably smaller value than that of the compression index (on the order of $C_s = 0.20C_c$).

## 7–7 SETTLEMENT OF LOADS ON CLAY DUE TO PRIMARY CONSOLIDATION

Once the field consolidation line has been defined for a given clayey soil, the total expected primary consolidation settlement of the load on the clay can be determined. Consider Figure 7–15, where a mass of soil is depicted before (Figure 7–15a) and after (Figure 7–15b) consolidation settlement has occurred in a clay layer of initial thickness $H$. From Figure 7–15,

$$\frac{\Delta H}{H} = \frac{\Delta H}{H_s + (H_v)_0} \quad \textbf{(7–6)}$$

where $\Delta H$ represents the amount of clay settlement and $H_s$ and $(H_v)_0$ denote the height of solids and initial height of voids, respectively. By definition, the initial void ratio, $e_0$ in Figure 7–15b, is given by the following:

$$e_0 = \frac{(V_v)_0}{V_s} \quad \textbf{(7–7)}$$

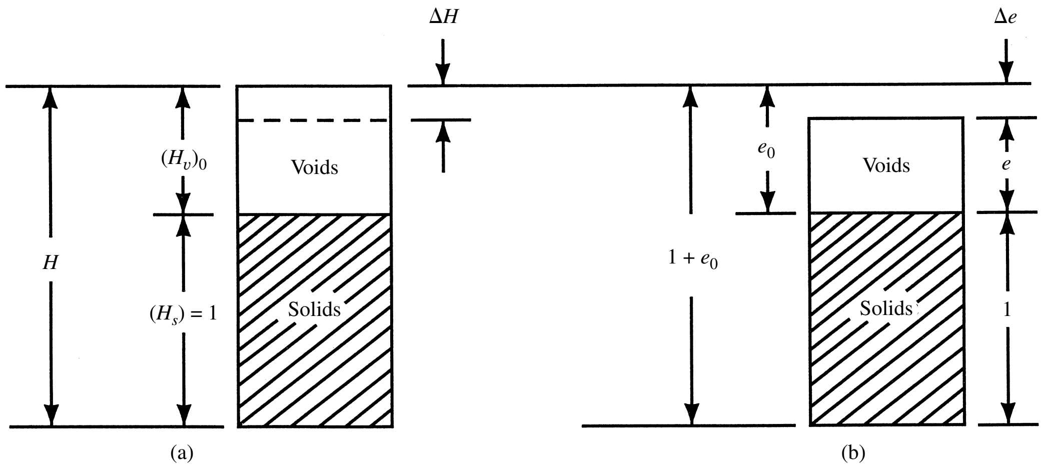



**FIGURE 7–15** Settlement of a mass of soil: (a) before consolidation settlement; (b) after consolidation settlement.
*Source:* J. H. Schmertmann, "Estimating the True Consolidation Behavior of Clay from Laboratory Test Results," *Proc. ASCE*, 79, Separate 311 (1953) 26 pp. Reprinted by permission.

where $(V_v)_0$ and $V_s$ represent the original volume of voids and the volume of solids, respectively. Because each volume can be replaced by the soil's cross-sectional area $(A)$ times the height of the soil, this equation can be modified as follows:

$$e_0 = \frac{(A)(H_v)_0}{(A)(H_s)} = \frac{(H_v)_0}{H_s} \tag{7–8}$$

Also,

$$\Delta e = \frac{\Delta H}{H_s} \tag{7–9}$$

where $\Delta e$ represents the change in void ratio as a result of consolidation settlement. If we let the height of solids $(H_s)$ equal unity, then Eqs. (7–6), (7–8), and (7–9) become

$$\frac{\Delta H}{H} = \frac{\Delta H}{1 + (H_v)_0} \tag{7–10}$$

$$e_0 = (H_v)_0 \tag{7–11}$$

$$\Delta e = \Delta H \tag{7–12}$$

Substituting Eqs. (7–11) and (7–12) into Eq. (7–10) yields

$$\frac{\Delta H}{H} = \frac{\Delta e}{1 + e_0} \tag{7–13}$$

or

$$\Delta H = \frac{\Delta e}{1 + e_0}(H) \tag{7–14}$$

Because $\Delta e = e_0 - e$ and $\Delta H =$ settlement $S_c$,

$$S_c = \frac{e_0 - e}{1 + e_0}(H) \tag{7–15}$$

where $S_c$ = settlement due to primary consolidation
$e_0$ = initial void ratio of the soil *in situ*
$e$ = void ratio of the soil corresponding to the total pressure ($p$) acting at midheight of the consolidating clay layer
$H$ = thickness of the consolidating clay layer

In practice, the value of $e_0$ is obtained from the laboratory consolidation test, and the value of $e$ is obtained from the field consolidation line based on total pressure (i.e., effective overburden pressure plus net additional pressure due to the structure—both at midheight of the consolidating clay layer). The value of $H$ is obtained from soil exploration (Chapter 3).

For normally consolidated clays, the expected primary consolidation settlement using the compression index (i.e., slope of the field consolidation line) can be derived by recalling Eq. (7–4):

$$C_c = \frac{e_1 - e_2}{\log (p_2/p_1)} \tag{7–4}$$

Because ($p_1$, $e_1$) and ($p_2$, $e_2$) can be the coordinates of any two points on the field consolidation line, let

$p_1$ = present effective overburden pressure at midheight of the consolidating clay layer (i.e., $p_0$)
$e_1$ = initial void ratio of the soil *in situ* [i.e., $e_0$ in Eq. (7–15)]
$p_2$ = total pressure acting at midheight of the consolidating clay layer [$p_0 + \Delta p$ (i.e., $p$)]
$e_2$ = void ratio of the soil corresponding to the total pressure ($p$) acting at midheight of the consolidating clay layer [i.e., $e$ in Eq. (7–15)]

Making these substitutions in Eq. (7–4) gives the following:

$$C_c = \frac{e_0 - e}{\log (p/p_0)} \tag{7–16}$$

Rearranging this equation results in

$$e_0 - e = C_c[\log (p/p_0)] \tag{7–17}$$

Substituting Eq. (7–17) into Eq. (7–15) yields

$$S_c = \frac{C_c[\log (p/p_0)]}{1 + e_0}(H) \tag{7–18}$$

or

$$S_c = C_c\left(\frac{H}{1 + e_0}\right) \log \frac{p}{p_0} \tag{7–19}$$

where $C_c$ = slope of the field consolidation line (compression index)
$p$ = total pressure acting at midheight of the consolidating clay layer ($= p_0 + \Delta p$)
$p_0$ = present effective overburden pressure at midheight of the consolidating clay layer
$\Delta p$ = net additional pressure at midheight of the consolidating clay layer due to the structure

The value of $C_c$ can be determined by evaluating the slope of the field consolidation line [Eq. (7–4)] or approximated based on the liquid limit [Eq. (7–5)]. If the latter method is used, computed settlement should be considered as a rough approximation.

For overconsolidated clays, as related in Section 7–6, the part of the field consolidation line between $p_0$ and $p_0'$ (line *a–m* in Figure 7–14) is a recompression curve. According to laboratory tests, it is approximately parallel to the laboratory rebound line, the slope of which is the swell index ($C_s$). For $p \leq p_0'$,

$$e_0 - e = C_s \log\frac{p}{p_0} \tag{7–20}$$

Substituting this equation into Eq. (7–15) gives

$$S_c = C_s\left(\frac{H}{1 + e_0}\right) \log\frac{p}{p_0} \tag{7–21}$$

Because $C_s$ is usually considerably less than $C_c$, settlements when $p \leq p_0'$ will usually be considerably less than they would be for normally consolidated clays. For $p > p_0'$, from Eqs. (7–19) and (7–21)

$$S_c = C_s\left(\frac{H}{1 + e_0}\right) \log \frac{p_0'}{p_0} + C_c\left(\frac{H}{1 + e_0}\right) \log \frac{p}{p_0'} \tag{7–22}$$

where $C_s$ = slope of the laboratory rebound line (swell index)
$p$ = total pressure acting at midheight of the consolidating clay layer ($= p_0 + \Delta p$)
$p_0'$ = overconsolidation pressure
$p_0$ = present effective overburden pressure at midheight of the consolidating clay layer
$\Delta p$ = net additional pressure at midheight of the consolidating clay layer due to the structure

The value of $C_s$ can be determined by evaluating the slope of the laboratory rebound line and is equal to roughly 20% of $C_c$.

For settlement calculations in thick clay layers, the thick clay deposit can be separated into several smaller sublayers. Then the settlement can be calculated individually for each sublayer, and the settlement for the entire thick deposit will be the sum of the individual settlements for each sublayer. In calculating the settlement for each sublayer, the overburden pressure and the stress increase caused by the structure are determined at the midheight of each sublayer.

## EXAMPLE 7–7

*Given*

A compressible normally consolidated clay layer is 7.40 m thick and has an initial void ratio *in situ* of 0.988. Consolidation tests and subsequent computations indicate that the void ratio of the clay layer corresponding to the total pressure acting at midheight of the consolidating clay layer after construction of a building is 0.942.

*Required*

Total expected primary consolidation settlement.

**Solution**

From Eq. (7–15),

$$S_c = \frac{e_0 - e}{1 + e_0}(H) \tag{7–15}$$

$$e_0 = 0.988$$

$$e = 0.942$$

$$H = 7.40 \text{ m}$$

$$S_c = \frac{0.988 - 0.942}{1 + 0.988}(7.40 \text{ m}) = 0.171 \text{ m}$$

## EXAMPLE 7–8

*Given*

1. A sample of normally consolidated clay was obtained by a Shelby tube sampler from the midheight of a compressible normally consolidated clay layer (see Figure 7–16).
2. A consolidation test was conducted on a portion of this sample. Results of the consolidation test are as follows:
   a. Natural (or initial) void ratio of the clay existing in the field ($e_0$) is 1.65.

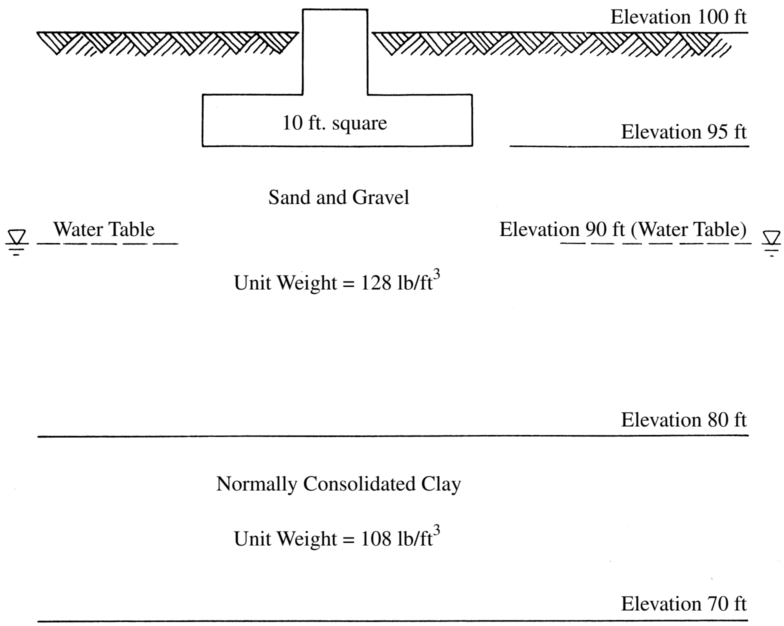



**FIGURE 7–16**

**b.** Pressure–void ratio relations are as follows:

| $p$ (tons/ft$^2$) | $e$ |
|---|---|
| 0.8 | 1.50 |
| 1.6 | 1.42 |
| 3.2 | 1.30 |
| 6.4 | 1.12 |
| 12.8 | 0.94 |

3. A footing is to be located 5 ft below ground level, as shown in Figure 7–16. The base of the square footing is 10 ft by 10 ft, and it exerts a total load of 250 tons, which includes column load, weight of footing, and weight of soil surcharge on the footing.

*Required*

1. From the given results of the consolidation test, prepare an $e$–log $p$ curve and construct a field consolidation line, assuming that point $f$ is located at $0.4e_0$.
2. Compute the total expected primary consolidation settlement for the clay layer.

**Solution**

1. Present effective overburden pressure ($p_0$) at midheight of clay layer = (128 lb/ft$^3$)(100 ft − 90 ft) + (128 lb/ft$^3$ − 62.4 lb/ft$^3$) × (90 ft − 80 ft) + (108 lb/ft$^3$ − 62.4 lb/ft$^3$) $\left(\frac{80\text{ ft} - 70\text{ ft}}{2}\right)$

$$p_0 = 2164\text{ lb/ft}^2\text{, or }1.08\text{ tons/ft}^2$$

$$e_0 = 1.65\text{ (given)}$$

$$0.4e_0 = (0.4)(1.65) = 0.66$$

The $e$–log $p$ curve is shown in Figure 7–17 together with the field consolidation line.

2. Effective weight of excavation = (128 lb/ft$^3$)(5 ft) = 640 lb/ft$^2$, or 0.32 ton/ft$^2$

$$\text{Net consolidation pressure at base of footing} = \frac{250\text{ tons}}{(10\text{ ft})(10\text{ ft})} - 0.32\text{ ton/ft}^2$$

$$= 2.18\text{ tons/ft}^2$$

To determine net consolidation pressure at midheight of the clay layer under the center of the footing, one must divide the base of the footing into four equal 5-ft by 5-ft square areas. Because each of these square areas has a common corner at the footing's center, the desired net consolidation pressure at midheight of the clay layer can be calculated upon determining an influence coefficient by using either Table 6–2 or Figure 6–8. Referring to Figure 6–8, we see that

$$mz = 5\text{ ft} \qquad z = 95\text{ ft} - \frac{80\text{ ft} + 70\text{ ft}}{2} = 20\text{ ft}$$

$$m = \frac{5\text{ ft}}{20\text{ ft}} = 0.25$$

$$nz = 5\text{ ft} \qquad z = 20\text{ ft} \qquad n = \frac{5\text{ ft}}{20\text{ ft}} = 0.25$$

From Figure 6–8, the influence coefficient = 0.027. Net consolidation pressure at midheight of the clay layer under the center of the footing is as follows:

$$\Delta p = (4)(0.027)(2.18\text{ tons/ft}^2) = 0.24\text{ ton/ft}^2$$

Thus, the final pressure at midheight of the clay layer is as follows:

$$p = p_0 + \Delta p = 1.08\text{ tons/ft}^2 + 0.24\text{ ton/ft}^2 = 1.32\text{ tons/ft}^2$$

Locate $p$ = 1.32 tons/ft$^2$ along the abscissa of the $e$–log $p$ curve (Figure 7–17) and move upward vertically until the field consolidation line is intersected. Then turn

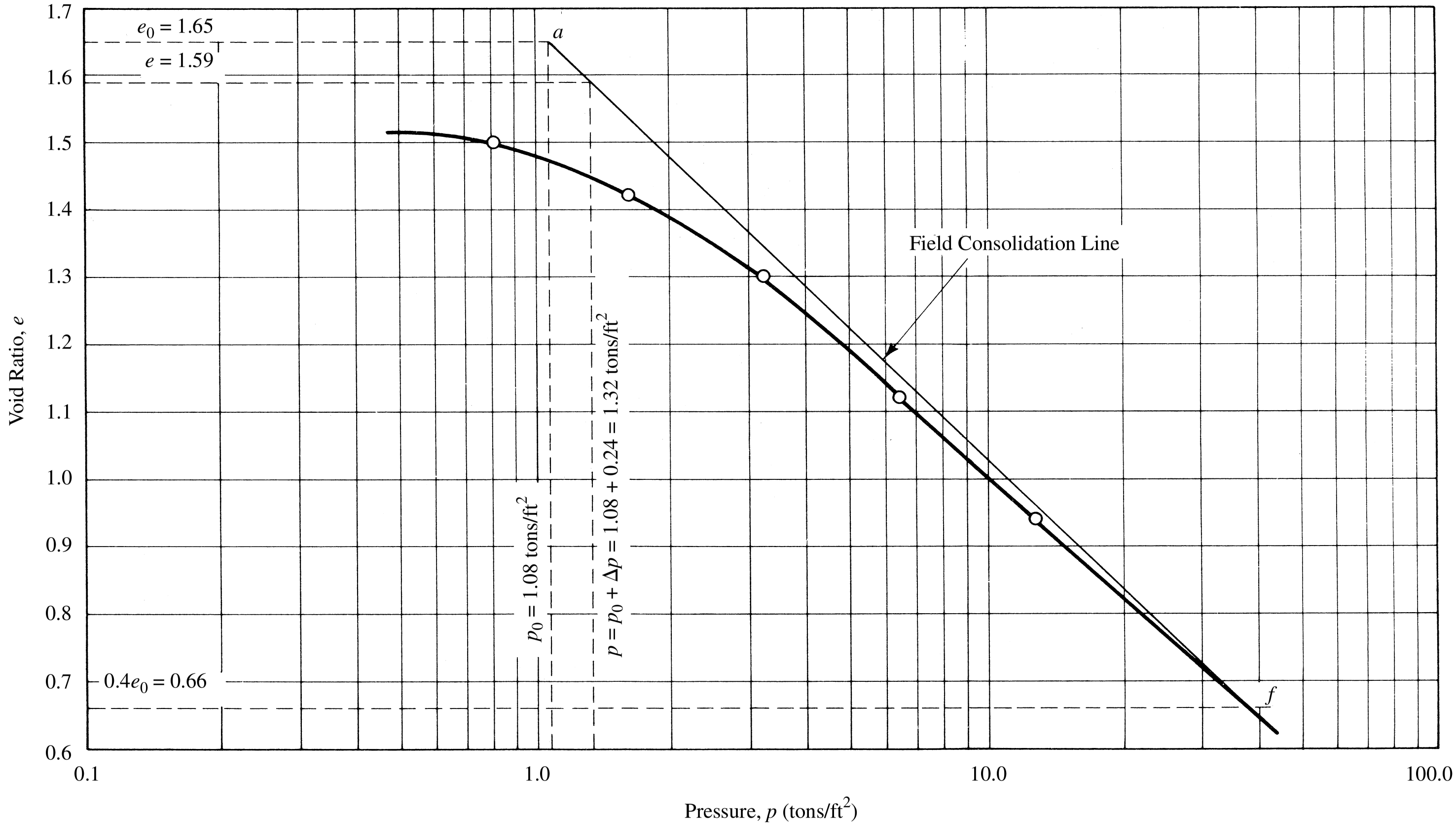



**FIGURE 7–17** $e$–log $p$ curve for Example 7–8.

left and move horizontally to read a void ratio $e$ of 1.59 on the ordinate of the $e$–log $p$ curve. With

$$e_0 = 1.65$$

$$e = 1.59$$

$$H = 10 \text{ ft} = 120 \text{ in.}$$

substitute into Eq. (7–15):

$$S_c = \frac{e_0 - e}{1 + e_0}(H) \qquad \textbf{(7–15)}$$

$$S_c = \frac{1.65 - 1.59}{1 + 1.65}(120 \text{ in.}) = 2.72 \text{ in., or } 6.91 \text{ cm}$$

The total expected primary consolidation settlement is 2.72 in., or 6.91 cm.

## EXAMPLE 7–9

*Given*

1. A foundation is to be constructed at a site where the soil profile is as shown in Figure 7–18.
2. A sample of overconsolidated clay was obtained by a Shelby tube sampler from the midheight of the clay layer (see Figure 7–18).
3. The initial void ratio *in situ* ($e_0$) of the overconsolidated clay layer is 0.72.
4. The compression index ($C_c$) of the clay layer was 0.28, and the swell index ($C_s$) was 0.054.
5. The net consolidation pressure at midheight of the clay layer under the center of the foundation ($\Delta p$) was calculated to be 65.4 kN/m$^2$.
6. The overconsolidated pressure ($p_0'$) is 128.6 kN/m$^2$.

*Required*

Expected primary consolidation settlement for the clay layer.

### Solution

Present effective overburden pressure ($p_0$) at midheight of clay layer = (16.18 kN/m$^3$)(2 m) + (16.18 kN/m$^3$ − 9.81 kN/m$^3$)(6 m−2 m) + (19.78 kN/m$^3$ − 9.81 kN/m$^3$)(5/2 m)

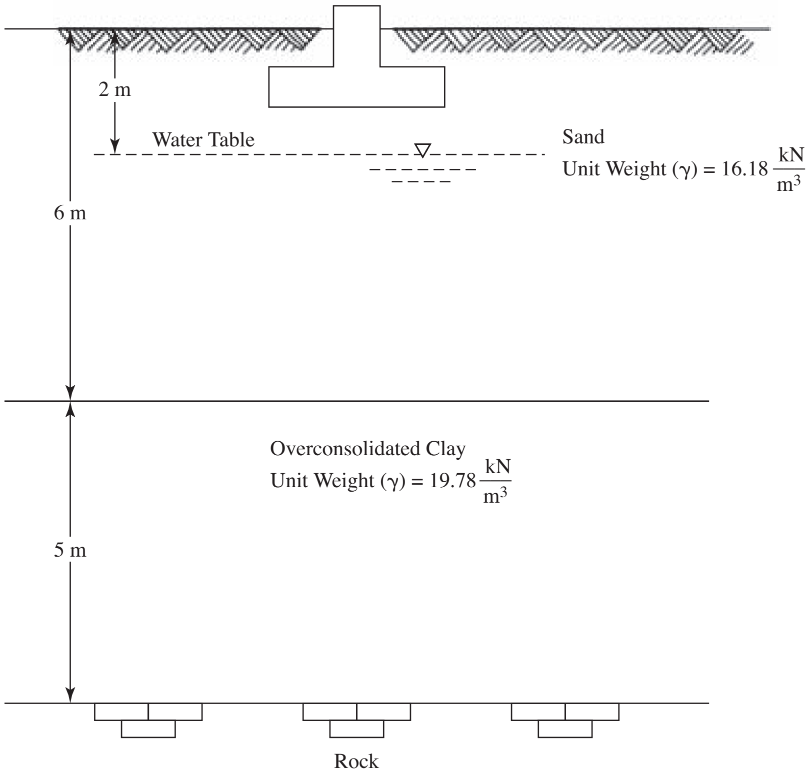
$$p_0 = 82.8 \text{ kN/m}^2$$

$$\Delta p = 65.4 \text{ kN/m}^2 \text{ (given)}$$

$$p = p_0 + \Delta p = 82.8 \text{ kN/m}^2 + 65.4 \text{ kN/m}^2 = 148.2 \text{ kN/m}^2$$

Therefore, $p = 148.2 \text{ kN/m}^2 > p_0' = 128.6 \text{ kN/m}^2$, so we need to to use Eq. (7–22).

**FIGURE 7–18**




$$S_c = C_s\left(\frac{H}{1+e_0}\right)\log\frac{p_0'}{p_0} + C_c\left(\frac{H}{1+e_0}\right)\log\frac{p}{p_0'} \tag{7–22}$$

$$S_c = (0.054)\left(\frac{5\text{ m}}{1+0.72}\right)\log\frac{128.6\text{ kN/m}^2}{82.8\text{ kN/m}^2} + (0.28)\left(\frac{5\text{ m}}{1+0.72}\right)\log\frac{148.2\text{ kN/m}^2}{128.6\text{ kN/m}^2}$$

$$S_c = 0.080\text{ m, or } 8.0\text{ cm}$$

## 7–8 TIME RATE OF SETTLEMENT DUE TO PRIMARY CONSOLIDATION

In addition to knowing the amount of settlement, it is also important to know the time rate of settlement. For a stratum of clay soil, the time rate of settlement depends in part on a number of factors including but not limited to the soil's compression properties, *in situ* void ratio, and permeability. The effect of all such factors may be combined into one parameter called the *coefficient of consolidation* ($c_v$). The coefficient of consolidation indicates how rapidly (or slowly) the process of consolidation takes place. This consolidation property can also be expressed as follows (Terzaghi et al., 1996):*

$$c_v = \frac{k}{\gamma_w m_v} \tag{7–23}$$

where $k$ = coefficient of permeability
$\gamma_w$ = unit weight of water
$m_v$ = coefficient of volume compressibility

The last term can be determined from the following:

$$m_v = \frac{a_v}{1 + e} \tag{7–24}$$

where $a_v$ = coefficient of compressibility
$e$ = void ratio

Substituting Eq. (7–24) into Eq. (7–23) gives the following:

$$c_v = \frac{k(1 + e)}{a_v \gamma_w} \tag{7–25}$$

For small strains, change in the void ratio ($\Delta e$) may be taken as directly proportional to change in the effective pressure ($\Delta p$). Therefore, $a_v$ in Eq. (7–25) can be expressed in equation form as follows:

$$a_v = \frac{\Delta e}{\Delta p} \tag{7–26}$$

Values of $k$, $e$, and $a_v$ can be evaluated separately and substituted into Eq. (7–25) to find $c_v$. However, it is common practice to evaluate $c_v$ directly from the results of a laboratory consolidation test (see Section 7–3 and Example 7–3).

As noted in Section 7–1, primary consolidation occurs due to extrusion of water from the voids as a result of increased loading. When a load increment (e.g., a structure) is applied to a saturated clayey soil stratum, the load is borne at first by the water in the soil's pores because the water is essentially incompressible compared to the soil makeup. The pressure in the water resulting from the applied load is known as *hydrostatic excess pressure*. Over time, as the water is extruded from the voids, the load shifts to the soil grains. As the water drains and the load shifts, soil volume decreases (the soil is compressed) by an amount equal to the volume of water drained, resulting in settlement of the overlying structure. This is the process known as *primary consolidation*.

---

*From K. Terzaghi, R. B. Peck, and G. Mesri, *Soil Mechanics in Engineering Practice,* 3rd ed., John Wiley & Sons, Inc., New York, 1996. 

The primary consolidation process is relatively slow, extending a very long time. When all the applied load is carried by the soil grains, the hydrostatic excess pressure will be zero, and the end of the consolidation process will have been reached. This is known as *full primary consolidation* or *100% primary consolidation*. For anything less than 100% primary consolidation, soil compression and settlement of the overlying structure are still progressing.

The average percent of consolidation (some refer to this as the average degree of consolidation) throughout the thickness of a compressing soil layer is related to the hydrostatic excess pressure by the expression

$$U\% = \left(\frac{U_i - U_t}{U_i}\right)(100\%) = (1 - U_t/U_i)(100\%) \qquad \textbf{(7–27)}$$

where $U\%$ = average percent of consolidation
$U_t$ = average hydrostatic excess pressure in the consolidating soil layer, corresponding to the time when the percent consolidation is being determined
$U_i$ = initial hydrostatic excess pressure

The value of $U\%$ can also be defined as

$$U\% = S_t/S_c(100\%) \qquad \textbf{(7–28)}$$

where $S_t$ = settlement of clay layer corresponding to the time when the percent of consolidation is being determined
$S_c$ = total settlement of clay layer due to primary consolidation

A time factor known as $T_v$ can be used to relate the rate at which hydrostatic excess pressure decreases to the time period required for an average percent of consolidation to occur. Taylor (1948) developed the following approximations for the variation of time factor ($T_v$) with average percent of consolidation ($U\%$).
For $U\% < 60\%$,

$$T_v = (\pi/4)(U\%/100\%)^2 \qquad \textbf{(7–29)}$$

For $U\% > 60\%$,

$$T_v = 1.781 - 0.933 \log(100 - U\%) \qquad \textbf{(7–30)}$$

The variation of time factor ($T_v$) with percent of consolidation ($U$) is also illustrated in Figure 7–19.

The time rate of settlement due to primary consolidation can be computed from the following equation (Terzaghi and Peck, 1967):

$$t = \frac{T_v}{c_v}H^2 \qquad \textbf{(7–31)}$$

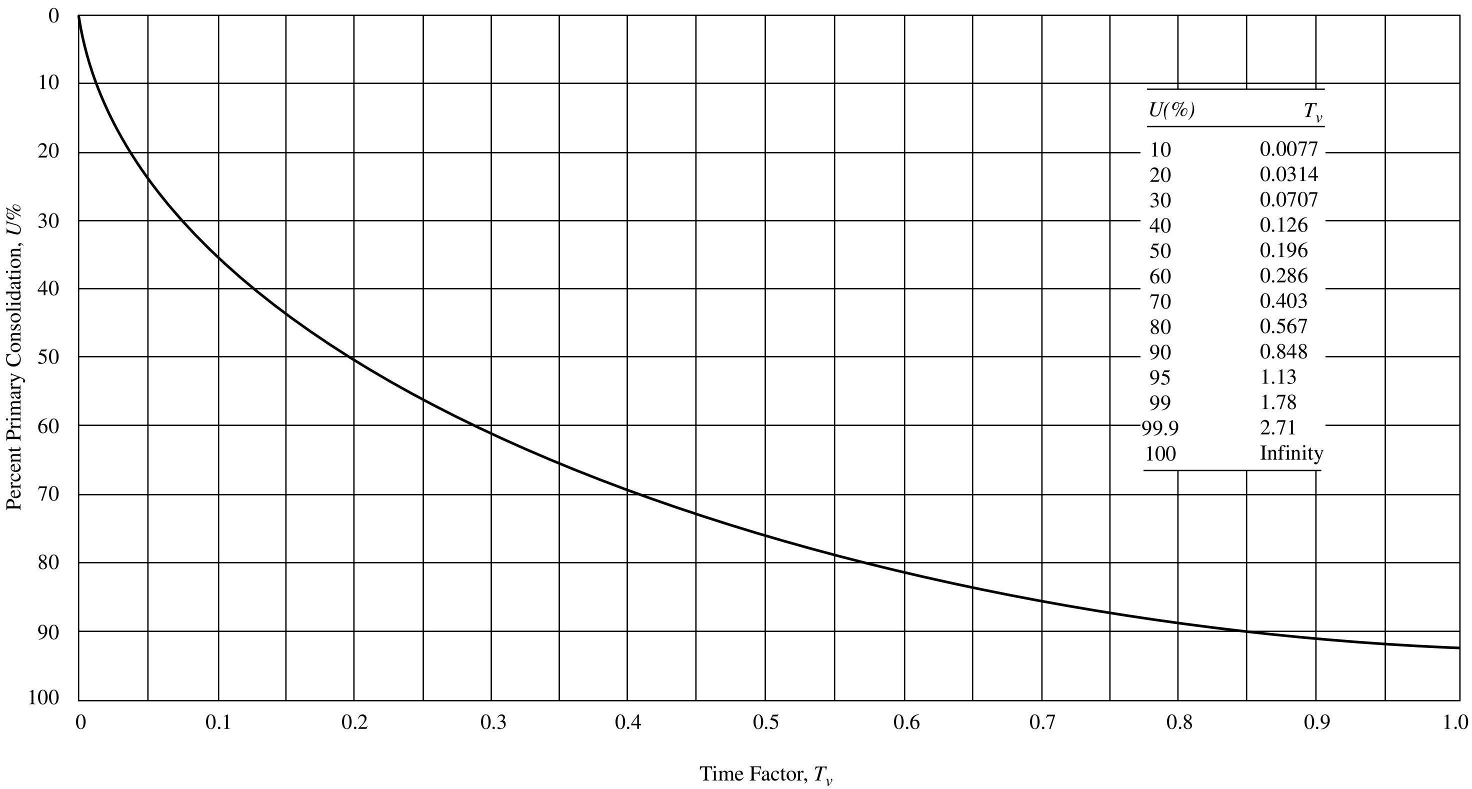



**FIGURE 7–19** Time factor as a function of percentage of consolidation (Teng, 1962).

where $t$ = time to reach a particular percent of consolidation; percent of consolidation is defined as the ratio of the amount of settlement at a certain time during the process of consolidation to the total settlement due to consolidation

$T_v$ = time factor, a coefficient depending on the particular percent of consolidation

$c_v$ = coefficient of consolidation corresponding to the total pressure ($p = p_0 + \Delta p$) acting at midheight of the clay layer

$H$ = thickness of the consolidating clay layer [however, if the clay layer *in situ* is drained on both top and bottom, half the thickness of the layer should be substituted for $H$ in Eq. (7–31)]

In practice, the value of $T_v$ is determined from Figure 7–19, based on the desired percent of consolidation ($U$%), and the value of $c_v$ is determined from the $c_v$–log $p$ curve (e.g., Figure 7–5) based on the total pressure acting at midheight of the clay layer. It will be recalled that the $c_v$–log $p$ curve is a product of the laboratory consolidation test.

To summarize the means of finding settlement of loads on clay due to primary consolidation, one can use either Eq. (7–15) or Eq. (7–19) to compute total settlement; then Eq. (7–31) can be used to find the time required to reach a particular percentage of that consolidation settlement. For example, if total settlement due to consolidation is computed to be 3.0 in., the time required for the structure to settle 1.5 in. could be determined from Eq. (7–31) by substituting a value of $T_v$ of 0.196 (along with applicable values of $c_v$ and $H$). The value of 0.196 is obtained from Figure 7–19 for a value of $U$ of 50%. $U$ is 50% because the particular settlement being considered (1.5 in.) is 50% of total settlement (3.0 in.).

## EXAMPLE 7–10

*Given*

1. Same as Example 7–8; total consolidation settlement = 2.72 in., or 6.91 cm.
2. Results of the laboratory consolidation test also indicated that the coefficient of consolidation ($c_v$) for the clay sample is $3.28 \times 10^{-3}$ in.$^2$/min for the pressure increment from 0.8 to 1.6 tons/ft$^2$.

*Required*

Time of primary consolidation settlement if the clay layer is underlain by

1. Permeable sand and gravel (double drainage).
2. Impermeable bedrock (single drainage).

Take $U$ at 10% increments and plot these values on a settlement–log time curve.

**Solution**

1. *Clay layer is underlain by permeable sand and gravel (double drainage).* Use Eq. (7–31):

$$t = \frac{T_v}{c_v} H^2 \tag{7–31}$$

where $c_v = 3.28 \times 10^{-3}$ in.$^2$/min
$H = 10$ ft$/2 = 5$ ft $= 60$ in. (double drainage)

**a.** When $U = 10\%$ (i.e., 10% of total settlement, $S_{10} = 2.72$ in. $\times$ $0.10 = 0.27$ in., or 0.69 cm),

$$T_v = 0.0077 \text{ (from Figure 7–19)}$$

$$t_{10} = \frac{(0.0077)(60 \text{ in.})^2}{3.28 \times 10^{-3} \text{ in.}^2/\text{min}} = 8451 \text{ min} = 0.016 \text{ yr}$$

This indicates that the footing will settle approximately 0.27 in., or 0.69 cm in 0.016 yr.

**b.** When $U = 20\%$ (i.e., 20% of total settlement, $S_{20} = 0.54$ in., or 1.37 cm),

$$T_v = 0.0314 \quad \text{(from Figure 7–19)}$$

$$t_{20} = \frac{(0.0314)(60 \text{ in.})^2}{3.28 \times 10^{-3} \text{ in.}^2/\text{min}} = 34{,}463 \text{ min} = 0.066 \text{ yr}$$

This indicates that the footing will settle approximately 0.54 in., or 1.37 cm in 0.066 yr.

**c.** When $U = 30\%$ (i.e., 30% of total settlement),

$$T_v = 0.0707 \quad \text{(from Figure 7–19)}$$

$$t_{30} = \frac{(0.0707)(60 \text{ in.})^2}{3.28 \times 10^{-3} \text{ in.}^2/\text{min}} = 77{,}598 \text{ min} = 0.15 \text{ yr}$$

**d.** When $U = 40\%$ (i.e., 40% of total settlement),

$$T_v = 0.126 \quad \text{(from Figure 7–19)}$$

$$t_{40} = \frac{(0.126)(60 \text{ in.})^2}{3.28 \times 10^{-3} \text{ in.}^2/\text{min}} = 1.383 \times 10^5 \text{ min} = 0.26 \text{ yr}$$

**e.** When $U = 50\%$ (i.e., 50% of total settlement),

$$T_v = 0.196 \quad \text{(from Figure 7–19)}$$

$$t_{50} = \frac{(0.196)(60 \text{ in.})^2}{3.28 \times 10^{-3} \text{ in.}^2/\text{min}} = 2.151 \times 10^5 \text{ min} = 0.41 \text{ yr}$$

**f.** When $U = 60\%$ (i.e., 60% of total settlement),

$$T_v = 0.286 \quad \text{(from Figure 7–19)}$$

$$t_{60} = \frac{(0.286)(60 \text{ in.})^2}{3.28 \times 10^{-3} \text{ in.}^2/\text{min}} = 3.139 \times 10^5 \text{ min} = 0.60 \text{ yr}$$

**g.** When $U = 70\%$ (i.e., 70% of total settlement),

$$T_v = 0.403 \quad \text{(from Figure 7–19)}$$

$$t_{70} = \frac{(0.403)(60\text{ in.})^2}{3.28 \times 10^{-3}\text{ in.}^2/\text{min}} = 4.423 \times 10^5\text{ min} = 0.84\text{ yr}$$

**h.** When $U = 80\%$ (i.e., 80% of total settlement),

$$T_v = 0.567 \quad \text{(from Figure 7–19)}$$

$$t_{80} = \frac{(0.567)(60\text{ in.})^2}{3.28 \times 10^{-3}\text{ in.}^2/\text{min}} = 6.223 \times 10^5\text{ min} = 1.18\text{ yr}$$

**i.** When $U = 90\%$ (i.e., 90% of total settlement),

$$T_v = 0.848 \quad \text{(from Figure 7–19)}$$

$$t_{90} = \frac{(0.848)(60\text{ in.})^2}{3.28 \times 10^{-3}\text{ in.}^2/\text{min}} = 9.307 \times 10^5\text{ min} = 1.77\text{ yr}$$

**2.** *Clay layer is underlain by impermeable bedrock (single drainage).* Eq. (7–31) is still applicable.

$$t = \frac{T_v}{c_v} H^2 \qquad \textbf{(7–31)}$$

where $c_v = 3.28 \times 10^{-3}\text{ in.}^2/\text{min}$

$H = 10\text{ ft} = 120\text{ in.}$ (single drainage)

**a.** When $U = 10\%$,

$$T_v = 0.0077$$

$$t_{10} = \frac{(0.0077)(120\text{ in.})^2}{3.28 \times 10^{-3}\text{ in.}^2/\text{min}} = 33{,}805\text{ min} = 0.064\text{ yr}$$

**b.** When $U = 20\%$,

$$T_v = 0.0314$$

$$t_{20} = \frac{(0.0314)(120\text{ in.})^2}{3.28 \times 10^{-3}\text{ in.}^2/\text{min}} = 1.379 \times 10^5\text{ min} = 0.26\text{ yr}$$

**c.** When $U = 30\%$,

$$T_v = 0.0707$$

$$t_{30} = \frac{(0.0707)(120\text{ in.})^2}{3.28 \times 10^{-3}\text{ in.}^2/\text{min.}} = 3.104 \times 10^5\text{ min} = 0.59\text{ yr}$$

**d.** When $U = 40\%$,

$$T_v = 0.126$$

$$t_{40} = \frac{(0.126)(120 \text{ in.})^2}{3.28 \times 10^{-3} \text{ in.}^2/\text{min}} = 5.532 \times 10^5 \text{ min} = 1.05 \text{ yr}$$

**e.** When $U = 50\%$,

$$T_v = 0.196$$

$$t_{50} = \frac{(0.196)(120 \text{ in.})^2}{3.28 \times 10^{-3} \text{ in.}^2/\text{min}} = 8.605 \times 10^5 \text{ min} = 1.64 \text{ yr}$$

**f.** When $U = 60\%$,

$$T_v = 0.286$$

$$t_{60} = \frac{(0.286)(120 \text{ in.})^2}{3.28 \times 10^{-3} \text{ in.}^2/\text{min}} = 1.256 \times 10^6 \text{ min} = 2.39 \text{ yr}$$

**g.** When $U = 70\%$,

$$T_v = 0.403$$

$$t_{70} = \frac{(0.403)(120 \text{ in.})^2}{3.28 \times 10^{-3} \text{ in.}^2/\text{min}} = 1.769 \times 10^6 \text{ min} = 3.37 \text{ yr}$$

**h.** When $U = 80\%$,

$$T_v = 0.567$$

$$t_{80} = \frac{(0.567)(120 \text{ in.})^2}{3.28 \times 10^{-3} \text{ in.}^2/\text{min}} = 2.489 \times 10^6 \text{ min} = 4.74 \text{ yr}$$

**i.** When $U = 90\%$,

$$T_v = 0.848$$

$$t_{90} = \frac{(0.848)(120 \text{ in.})^2}{3.28 \times 10^{-3} \text{ in.}^2/\text{min}} = 3.723 \times 10^6 \text{ min} = 7.08 \text{ yr}$$

The results of these computations are tabulated in Table 7–8 and are shown graphically by a settlement–log time curve in Figure 7–20.

## EXAMPLE 7–11

*Given*

1. An 8-ft clay layer beneath a building is overlain by a stratum of permeable sand and gravel and is underlain by impermeable bedrock.
2. The total expected primary consolidation settlement for the clay layer due to the footing load is 2.50 in.
3. The coefficient of consolidation ($c_v$) is $2.68 \times 10^{-3}$ in.$^2$/min.

**TABLE 7–8**
**Computed Time–Settlement Relation for Example 7–10**

| Fraction of Total Consolidation Settlement, *U* (%) | Consolidation Settlement (in.) | Time (yr) | |
|---|---|---|---|
| | | *Double Drainage* | *Single Drainage* |
| 10 | 0.27 | 0.016 | 0.064 |
| 20 | 0.54 | 0.066 | 0.26 |
| 30 | 0.82 | 0.15 | 0.59 |
| 40 | 1.09 | 0.26 | 1.05 |
| 50 | 1.36 | 0.41 | 1.64 |
| 60 | 1.63 | 0.60 | 2.39 |
| 70 | 1.90 | 0.84 | 3.37 |
| 80 | 2.18 | 1.18 | 4.74 |
| 90 | 2.45 | 1.77 | 7.08 |
| 100 | 2.72 | ∞ | ∞ |

*Required*

1. How many years will it take for 90% of the total expected primary consolidation settlement to take place?
2. Compute the amount of primary consolidation settlement that will occur in 1 yr.
3. How many years will it take for primary consolidation settlement of 1 in. to take place?

**Solution**

1. From Eq. (7–31),

$$t = \frac{T_v}{c_v} H^2 \qquad \textbf{(7–31)}$$

$$T_v = 0.848 \text{ (for } U = 90\%\text{; see Figure 7–19)}$$

$$c_v = 2.68 \times 10^{-3} \text{ in.}^2/\text{min (given)}$$

$$H = 8 \text{ ft} = 96 \text{ in. (single drainage)}$$

$$t_{90} = \frac{(0.848)(96 \text{ in.})^2}{2.68 \times 10^{-3} \text{ in.}^2/\text{min}} = 2.916 \times 10^6 \text{ min} = 5.55 \text{ yr}$$

2. From Eq. (7–31),

$$t = \frac{T_v}{c_v} H^2 \qquad \textbf{(7–31)}$$

$$t = 1 \text{ yr}$$

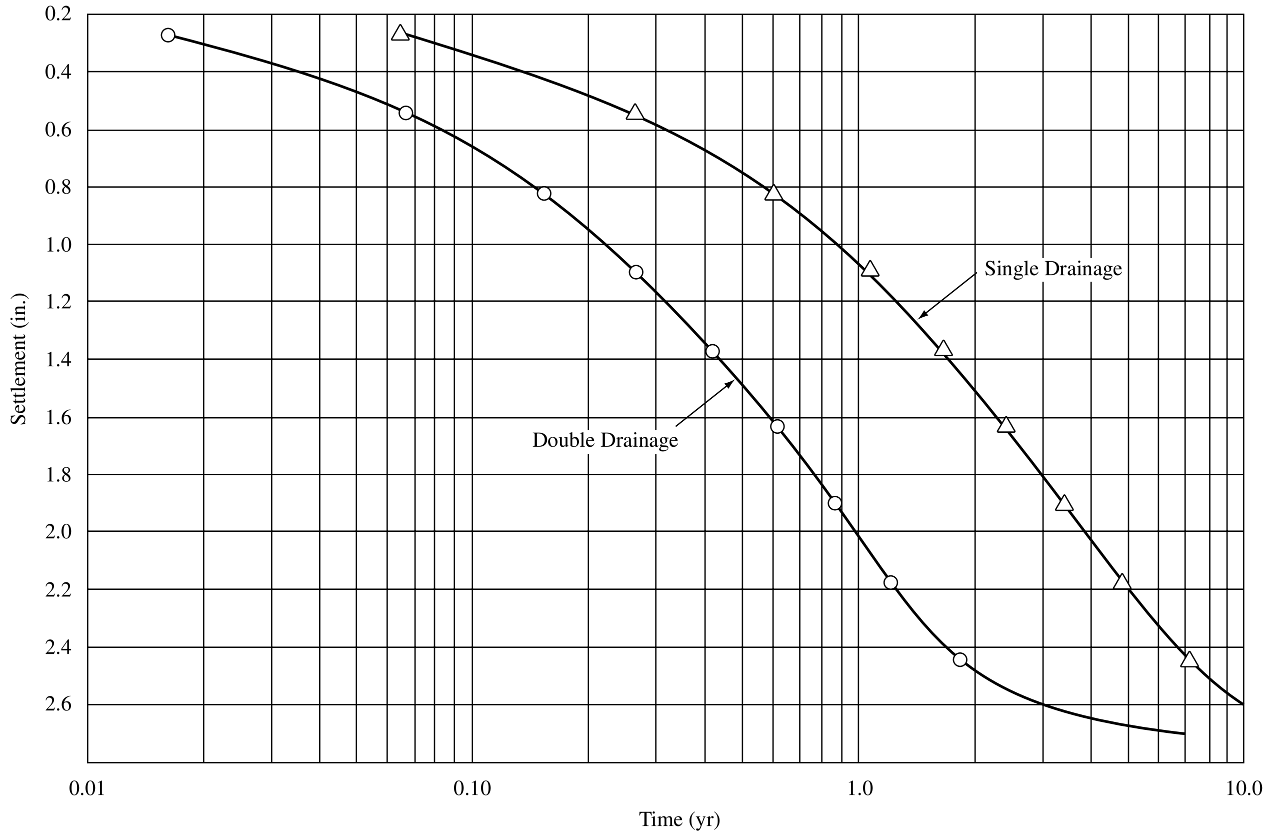



**FIGURE 7–20** Settlement–log time curves for Example 7–10.

$$c_v = 2.68 \times 10^{-3} \text{ in.}^2/\text{min}$$

$$H = 8 \text{ ft} = 96 \text{ in.}$$

$$1 \text{ yr} = \frac{T_v}{2.68 \times 10^{-3} \text{ in.}^2/\text{min}}(96 \text{ in.})^2 \times \frac{1}{(60 \text{ min/hr})(24 \text{ hr/day})(365 \text{ days/yr})}$$

$$T_v = 0.15$$

From Figure 7–19, with $T_v = 0.15$, $U = 43\%$.
Amount of primary consolidation settlement that will occur in 1 yr

$$= \text{Total primary consolidation settlement} \times U\%$$

$$= (2.50 \text{ in.})(0.43) = 1.08 \text{ in., or } 2.74 \text{ cm}$$

3. $U\%$ = Fraction of total primary consolidation settlement

$$U = \frac{1 \text{ in.}}{2.50 \text{ in.}} \times 100 = 40\%$$

From Figure 7–19, with $U = 40\%$, $T_v = 0.126$. From Eq. (7–31),

$$t = \frac{T_v}{c_v}H^2 \qquad \textbf{(7–31)}$$

$$t = \frac{(0.126)(96 \text{ in.})^2}{2.68 \times 10^{-3} \text{ in.}^2/\text{min}} = 4.333 \times 10^5 \text{ min} = 0.82 \text{ yr}$$

## EXAMPLE 7–12

*Given*

1. A foundation is to be constructed at a site where the soil profile is as shown in Figure 7–21.
2. The base of the foundation is 3 m by 6 m, and it exerts a total load of 5400 kN, which includes the weight of the structure, foundation, and soil surcharge on the foundation.
3. The initial void ratio *in situ* ($e_0$) of the compressible normally consolidated clay layer is 1.38.
4. The compression index ($C_c$) of the clay layer is 0.68.

*Required*

Expected primary consolidation settlement of the clay layer.

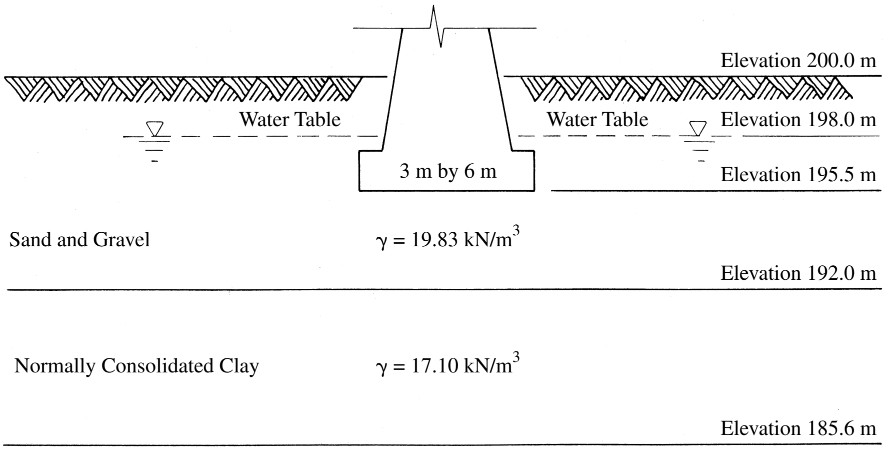



**FIGURE 7–21**

## Solution

Present effective overburden pressure ($p_0$) at midheight of clay layer

$$= (19.83\ \text{kN/m}^3)(200.0\ \text{m} - 198.0\ \text{m}) + (19.83\ \text{kN/m}^3 - 9.81\ \text{kN/m}^3) \times (198.0\ \text{m} - 192.0\ \text{m}) + (17.10\ \text{kN/m}^3 - 9.81\ \text{kN/m}^3) \times \left(\frac{192.0\ \text{m} - 185.6\ \text{m}}{2}\right) = 123.1\ \text{kN/m}^2$$

$$\text{Effective weight of excavation} = (19.83\ \text{kN/m}^3)(200.0\ \text{m} - 198.0\ \text{m}) + (19.83\ \text{kN/m}^3 - 9.81\ \text{kN/m}^3)(198.0\ \text{m} - 195.5\ \text{m}) = 64.7\ \text{kN/m}^2$$

Net consolidation pressure at the foundation's base

$$= \frac{5400\ \text{kN}}{(3\ \text{m})(6\ \text{m})} - 64.7\ \text{kN/m}^2 = 235.3\ \text{kN/m}^2$$

To determine the net consolidation pressure at midheight of the clay layer under the center of the foundation, one must divide the foundation's base into four equal 1.5-m by 3.0-m rectangular areas. Because each of these areas has a common corner at the foundation's center, the desired net consolidation pressure at midheight of the clay layer can be calculated by determining an influence coefficient using either Table 6–2 or Figure 6–8.

$$mz = 1.5\ \text{m} \qquad nz = 3.0\ \text{m}$$

$$z = 195.5\ \text{m} - \frac{192.0\ \text{m} + 185.6\ \text{m}}{2} = 6.7\ \text{m}$$

$$m = \frac{1.5\ \text{m}}{6.7\ \text{m}} = 0.224 \qquad n = \frac{3.0\ \text{m}}{6.7\ \text{m}} = 0.448$$

From Figure 6–8, the influence coefficient is 0.04. Therefore,

Net consolidation pressure at midheight of clay layer under center of foundation

$$(\Delta p) = (4)(0.04)(235.3\ \text{kN/m}^2) = 37.6\ \text{kN/m}^2$$

Final pressure at midheight of clay layer $(p) = p_0 + \Delta p$

$$= 123.1\ \text{kN/m}^2 + 37.6\ \text{kN/m}^2 = 160.7\ \text{kN/m}^2$$

From Eq. (7–19),

$$S_c = C_c\left(\frac{H}{1 + e_0}\right)\log\frac{p}{p_0} \qquad \textbf{(7–19)}$$

$C_c = 0.68$ (given)

$H = 192.0\ \text{m} - 185.6\ \text{m} = 6.4\ \text{m}$

$e_0 = 1.38$ (given)

$p = 160.7\ \text{kN/m}^2$

$p_0 = 123.1\ \text{kN/m}^2$

$$S_c = (0.68)\left(\frac{6.4\ \text{m}}{1 + 1.38}\right)\log\left(\frac{160.7\ \text{kN/m}^2}{123.1\ \text{kN/m}^2}\right) = 0.212\ \text{m}$$

## EXAMPLE 7–13

*Given*

1. Same data as for Example 7–12, including the computed primary consolidation settlement of 0.212 m.
2. Coefficient of consolidation $(c_v)$ is $4.96 \times 10^{-6}\ \text{m}^2/\text{min}$.

*Required*

How long will it take for half the expected consolidation settlement to take place if the clay layer is underlain by

1. Permeable sand and gravel?
2. Impermeable bedrock?

**Solution**

1. *Clay layer underlain by permeable sand and gravel.* From Eq. (7–31),

$$t = \frac{T_v}{c_v}H^2 \qquad \textbf{(7–31)}$$

From Figure 7–19, for $U = 50\%$, $T_v = 0.196$.

$$H = \frac{192.0\text{ m} - 185.6\text{ m}}{2} = 3.2\text{ m}$$

$$t_{50} = \left(\frac{0.196}{4.96 \times 10^{-6}\text{ m}^2/\text{min}}\right)(3.2\text{ m})^2 = 404{,}645\text{ min, or } 0.77\text{ yr}$$

2. *Clay layer underlain by impermeable bedrock.* Equation (7–31) is still applicable with $T_v = 0.196$ and $c_v = 4.96 \times 10^{-6}$ m$^2$/min, but with $H = 192.0$ m $-$ 185.6 m $= 6.4$ m.

$$t_{50} = \left(\frac{0.196}{4.96 \times 10^{-6}\text{ m}^2/\text{min}}\right)(6.4\text{ m})^2 = 1{,}618{,}581\text{ min, or } 3.08\text{ yr}$$

## 7–9 SETTLEMENT OF LOADS ON CLAY DUE TO SECONDARY COMPRESSION

After primary consolidation has ended (i.e., all water has been extruded from the voids in a fine-grained soil) and all primary consolidation settlement has occurred, soil compression (and additional associated settlement) continues very slowly at a decreasing rate. This phenomenon is known as *secondary compression* and perhaps results from plastic readjustment of soil grains due to new stresses in the soil and progressive breaking of clayey particles and their interparticle bonds.

Figure 7–22 gives a plot of void ratio as a function of the logarithm of time. Clearly, as the void ratio decreases, settlement increases. Secondary compression begins immediately after primary consolidation ends; it appears in Figure 7–22 as a straight line with a relatively flat slope. The void ratio corresponding to the end of primary consolidation (or the beginning of secondary compression) can be determined graphically

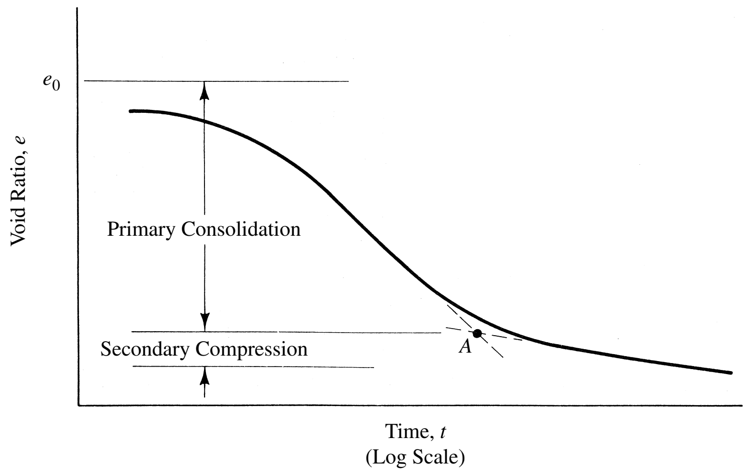



**FIGURE 7–22** Sketch showing primary consolidation and secondary compression.

as the point of intersection of the secondary compression line extended backward and a line tangent to the primary consolidation curve (i.e., point *A* in Figure 7–22).

Secondary compression settlement can be computed from the following equation (U.S. Department of the Navy, 1971):

$$S_s = C_\alpha H \log \frac{t_s}{t_p} \tag{7–32}$$

where $S_s$ = secondary compression settlement
$C_\alpha$ = coefficient of secondary compression
$H$ = (initial) thickness of the clay layer
$t_s$ = life of the structure (or time for which settlement is required)
$t_p$ = time to completion of primary consolidation

The coefficient of secondary compression ($C_\alpha$) varies with the clay layer's natural water content and can be determined from Figure 7–23.

The amount of secondary compression settlement may be quite significant for highly compressible clays, highly micaceous soils, and organic materials. On the other hand, it is largely insignificant for inorganic clay with moderate compressibility.

### EXAMPLE 7–14

*Given*

1. A foundation is to be built on a sand deposit underlain by a highly compressible clay layer 5.0 m thick.
2. The clay layer's natural water content is 80%.
3. Primary consolidation is estimated to be complete in 10 yr.

*Required*

Secondary compression settlement expected to occur from 10 to 50 yr after construction of the foundation.

**Solution**

From Eq. (7–32),

$$S_s = C_\alpha H \log \frac{t_s}{t_p} \tag{7–32}$$

$$C_\alpha = 0.015$$

(from Figure 7–23, with a natural water content of 80%)

$$H = 5.0 \text{ m}$$
$$t_s = 50 \text{ yr}$$
$$t_p = 10 \text{ yr}$$

$$S_s = (0.015)(5.0 \text{ m})\left(\log \frac{50 \text{ yr}}{10 \text{ yr}}\right) = 0.052 \text{ m}$$

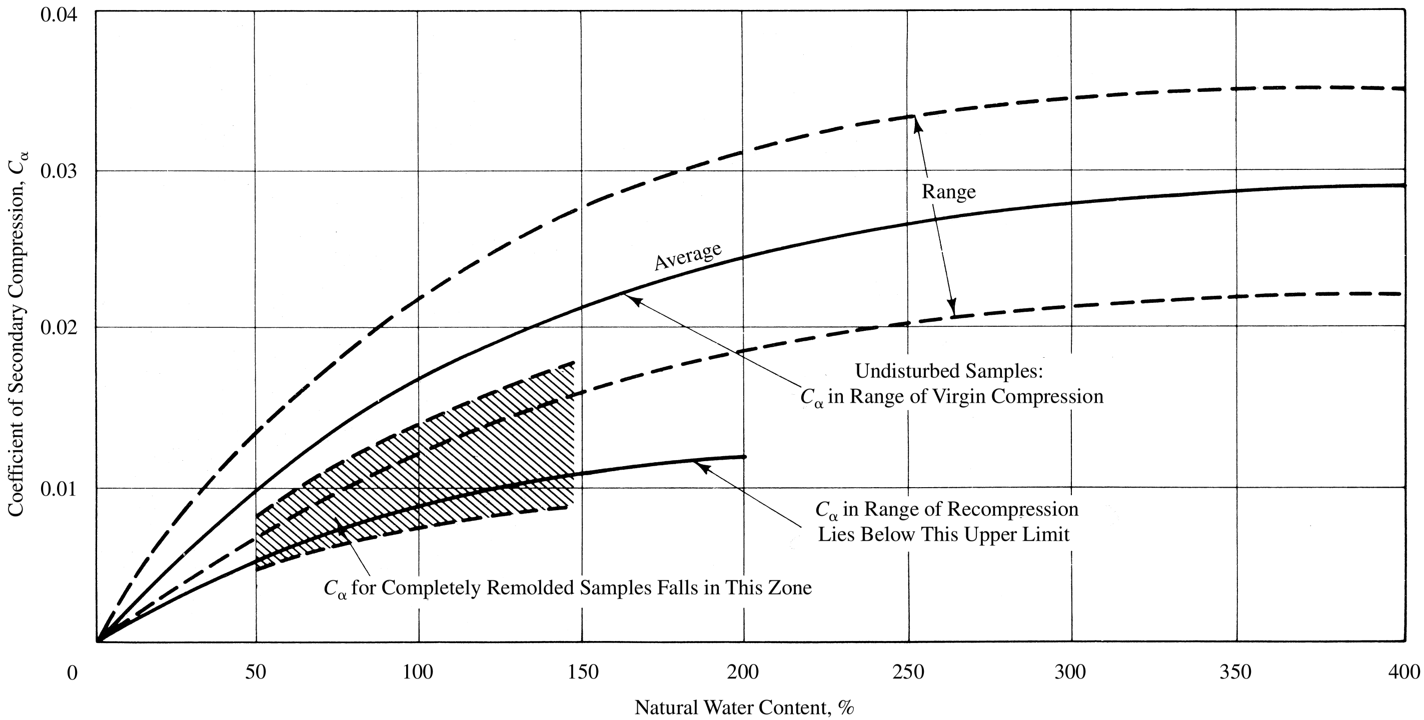



**FIGURE 7–23** Coefficient of secondary compression; $C_\alpha =$ ratio of decrease in sample height to initial sample height for one cycle of time on a logarithmic scale following completion of primary consolidation.
*Source:* U.S. Department of the Navy (1971).

**EXAMPLE 7–15**

*Given*

1. Same data as for Example 7–12.
2. Assume that primary consolidation will be complete in 15 yr.
3. Natural water content of the clay layer is 50%.

*Required*

Estimated secondary compression settlement 50 yr after construction.

**Solution**
From Eq. (7–32),

$$S_s = C_\alpha H \log \frac{t_s}{t_p} \tag{7–32}$$

From Figure 7–22, with a 50% natural water content of the clay layer,

$$C_\alpha = 0.010$$

$$H = 192.0 \text{ m} - 185.6 \text{ m} = 6.4 \text{ m}$$

$$t_s = 50 \text{ yr}$$

$$t_p = 15 \text{ yr}$$

$$S_s = (0.010)(6.4 \text{ m}) \log\left(\frac{50 \text{ yr}}{15 \text{ yr}}\right) = 0.033 \text{ m}$$

## 7–10 SETTLEMENT OF LOADS ON SAND

Most of the settlement of loads on sand has occurred by the time construction is complete. Thus, the time rate of settlement is not a factor as it is with clay. Settlement criteria rather than ultimate bearing capacity (see Chapter 9) commonly govern allowable bearing capacity for footings on sand; furthermore, settlement on sand is not amenable to solution based on laboratory consolidation tests. Indeed, settlement on sand is generally calculated by empirical means. Three methods for calculating settlement on sand follow—Bazaraa method, Burland and Burbidge method, and Schmertmann method.

### Bazaraa Method

One empirical method is based on the standard penetration test (SPT), which was discussed in Section 3–5. To determine settlement on sand, one makes SPT determinations at various depths at the test site, normally at depth intervals of $2\frac{1}{2}$ ft (0.76 m), beginning at a depth corresponding to the proposed footing's base. The SPT *N*-values must be corrected for overburden pressure (see Chapter 3). The next step is to compute the average corrected *N*-value for each boring for the sand between the footing's base and a depth $B$ below the base, where $B$ is the footing's width. The lowest of the average corrected $N$-values for all borings at the site is noted and designated $N_{\text{lowest}}$. Maximum settlement can then be computed from the following equation (Bazaraa, 1967):

$$s_{\max} = \frac{2q}{N_{\text{lowest}}}\left[\frac{2B}{1+B}\right]^2 \qquad \textbf{(7–33)}$$

where $s_{\max}$ = maximum settlement on dry sand, in.
$q$ = applied pressure, tons/ft$^2$
$B$ = width of footing, ft

Equation (7–33) is applicable to settlement on dry sand. If the groundwater table is located at a depth below the base of the footing less than half the footing's width, the settlement computed from Eq. (7–33) should be corrected by multiplying it by $x_B$, where (Bazaraa, 1967)

$$x_B = \frac{p_d}{p_w} \qquad \textbf{(7–34)}$$

where $p_d$ = effective overburden pressure at depth $B/2$ below the footing's base, assuming that the groundwater table is not present
$p_w$ = effective overburden pressure at the same depth with the groundwater table present

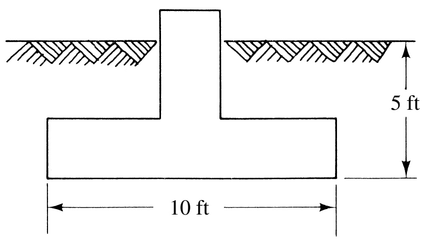
Examples 7–16 through 7–19 demonstrate the calculation of settlement on sand.

## EXAMPLE 7–16

*Given*

1. A 10-ft by 10-ft footing carrying a total load of 280 tons is to be constructed on sand as shown in Figure 7–24.

**FIGURE 7–24**



2. Standard penetration tests were conducted on the site. Test results were corrected for overburden pressure (see Chapter 3), and the corrected *N*-values are listed next.

| Depth (ft) | Corrected *N*-Values |
|---|---|
| 5.0 | 31 |
| 7.5 | 36 |
| 10.0 | 30 |
| 12.5 | 28 |
| 15.0 | 35 |
| 17.5 | 33 |
| 20.0 | 31 |

*Required*

Maximum expected settlement of this footing.

**Solution**

*Average Corrected* N-*Values*

The average corrected *N*-value is determined for each boring for the soil located between the level of the footing's base and a depth *B* below this level, where *B* is the footing's width. In this example, appropriate depths for calculating average corrected *N*-values are 5 to 15 ft. The average corrected *N*-value is a cumulative average down to the depth indicated.

For a depth of 5 ft,

$$\text{Average corrected } N\text{-value} = 31$$

For a depth of 7.5 ft,

$$\text{Average corrected } N\text{-value} = \frac{31 + 36}{2} = 33$$

For a depth of 10.0 ft,

$$\text{Average corrected } N\text{-value} = \frac{31 + 36 + 30}{3} = 32$$

For a depth of 12.5 ft,

$$\text{Average corrected } N\text{-value} = \frac{31 + 36 + 30 + 28}{4} = 31$$

For a depth of 15.0 ft,

$$\text{Average corrected } N\text{-value} = \frac{31 + 36 + 30 + 28 + 35}{5} = 32$$

*Lowest Average Corrected* N-*Value for Design*

Subsurface soil conditions generally vary somewhat at most construction sites. The *N*-value selected for design is usually the lowest average corrected *N*-value, which in this example is 31 (at depth 12.5 ft). From Eq. (7–33),

$$s_{\max} = \frac{2q}{N_{\text{lowest}}}\left[\frac{2B}{1 + B}\right]^2 \tag{7–33}$$

$$q = \frac{280\text{ tons}}{(10\text{ ft})(10\text{ ft})} = 2.8\text{ tons/ft}^2$$

$$N_{\text{lowest}} = 31$$

$$B = 10\text{ ft}$$

$$s_{\max} = \frac{(2)(2.8\text{ tons/ft}^2)}{31}\left[\frac{2 \times 10\text{ ft}}{1 + 10\text{ ft}}\right]^2 = 0.60\text{ in. on dry sand}$$

**EXAMPLE 7–17**

---

*Given*

Same conditions as in Example 7–16, except that the groundwater table is located 7 ft below ground level (see Figure 7–25).

*Required*

Maximum expected settlement of the footing.

**Solution**

From Example 7–16,

$$s_{\max} = 0.60\text{ in. on dry sand}$$

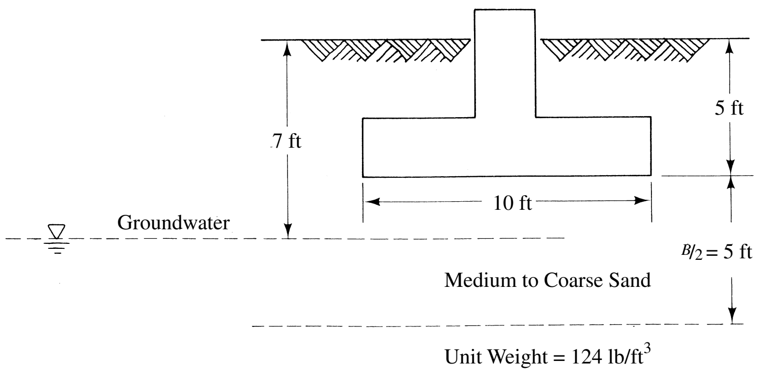



**FIGURE 7–25**

From Eq. (7–34),

$$x_B = \frac{p_d}{p_w} \tag{7–34}$$

$$p_d = (124\ \text{lb/ft}^3)\left(5\ \text{ft} + \frac{10\ \text{ft}}{2}\right) = 1240\ \text{lb/ft}^2$$

$$p_w = (124\ \text{lb/ft}^3)(7\ \text{ft}) + (124\ \text{lb/ft}^3 - 62.4\ \text{lb/ft}^3)\left(5\ \text{ft} + \frac{10\ \text{ft}}{2} - 7\ \text{ft}\right)$$

$$= 1053\ \text{lb/ft}^2$$

$$x_B = \frac{1240\ \text{lb/ft}^2}{1053\ \text{lb/ft}^2} = 1.178$$

$$s_{\text{max}} = (0.60\ \text{in.})(1.178) = 0.71\ \text{in. on wet sand}$$

## EXAMPLE 7–18

*Given*

1. A square footing 8 ft by 8 ft located 5 ft below ground level is to be constructed on sand.
2. Standard penetration tests were conducted on the site. Test results were corrected for overburden pressures, and the lowest average corrected $N$-value was determined to be 41.
3. Groundwater was not encountered.

*Required*

Allowable soil pressure for a maximum settlement of 1 in.

**Solution**

From Eq. (7–33),

$$s_{\text{max}} = \frac{2q}{N_{\text{lowest}}}\left[\frac{2B}{1 + B}\right]^2 \tag{7–33}$$

$$s_{\text{max}} = 1\ \text{in.}$$

$$N_{\text{lowest}} = 41$$

$$B = 8\ \text{ft}$$

$$1\ \text{in.} = \frac{2q}{41}\left[\frac{2 \times 8\ \text{ft}}{1 + 8\ \text{ft}}\right]^2$$

$$q = 6.49\ \text{tons/ft}^2$$

## EXAMPLE 7–19

*Given*

Same conditions as in Example 7–18, except that the groundwater table is located 6 ft below ground level and the sand's unit weight is 128 lb/ft$^3$ (see Figure 7–26).

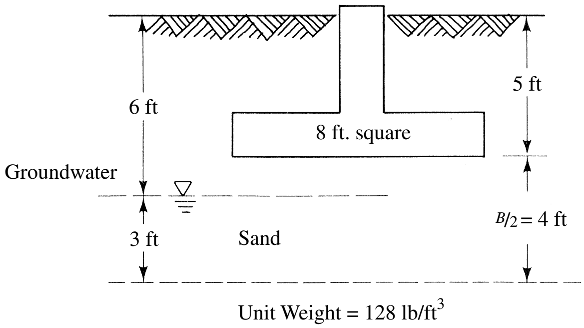



**FIGURE 7–26**

*Required*

Allowable soil pressure for a maximum settlement of 1 in.

**Solution**

From Eq. (7–34),

$$x_B = \frac{p_d}{p_w} \tag{7–34}$$

$$p_d = (128\ \text{lb/ft}^3)\left(5\ \text{ft} + \frac{8\ \text{ft}}{2}\right) = 1152\ \text{lb/ft}^2$$

$$p_w = (128\ \text{lb/ft}^3)(6\ \text{ft}) + (128\ \text{lb/ft}^3 - 62.4\ \text{lb/ft}^3)(3\ \text{ft}) = 964.8\ \text{lb/ft}^2$$

$$x_B = \frac{1152\ \text{lb/ft}^2}{964.8\ \text{lb/ft}^2} = 1.194$$

From Example 7–18, allowable soil pressure ($q$) is 6.49 tons/ft$^2$ for a settlement of 1 in. when no groundwater is encountered. When the groundwater table is at a depth below the base of the footing less than $B/2$, $s_{\text{max}}$ computed from Eq. (7–33) should be multiplied by $x_B$. Therefore, in this example an allowable soil pressure of 6.49 tons/ft$^2$ will produce a settlement of 1.194 × 1 in., or 1.194 in. Because settlement varies directly with bearing pressure,

$$\frac{6.49\ \text{tons/ft}^2}{1.194\ \text{in.}} = \frac{\text{Allowable soil pressure for a settlement of 1 in.}}{1\ \text{in.}}$$

Allowable soil pressure for a settlement of 1 in. = 5.44 tons/ft$^2$.

## Burland and Burbidge Method

Another empirical method for estimating foundation settlement on sand, which also uses SPT *N*-values, was developed by Burland and Burbidge (1985). A footing to be placed in a sandy soil at some depth below ground surface requires removal of sand above the level of the base of the foundation. As a result of such sand removal, sand located below the level of the base of the foundation becomes precompressed. Recompression, however, is assumed for bearing pressure up to the preconstruction effective vertical pressure at the foundation's base.

For sands normally compressed with respect to the original ground surface and for values of foundation contact pressure ($q$) greater than the preconstruction effective overburden pressure ($p_0$) at the foundation's base (Terzaghi et al., 1996)*,

$$S = B^{0.75}\frac{1.7}{\overline{N}^{1.4}}(q - 2p_0/3) \qquad \textbf{(7–35)}$$

where $S$ = settlement at end of construction and application of permanent live load (mm)
$B$ = width of footing (m)
$\overline{N}$ = arithmetic mean of SPT $N$-values measured within the zone of influence (i.e., $B^{0.75}$ m below the foundation's base)
$q$ = bearing (contact) pressure over the foundation's base (kN/m$^2$)
$p_0$ = *in situ* effective overburden pressure at base of foundation (kN/m$^2$)

For values of foundation contact pressure less than the preconstruction effective overburden pressure at the foundation's base (Terzaghi et al., 1996)*

$$S = \frac{1}{3}B^{0.75}\frac{1.7}{\overline{N}^{1.4}}q \qquad \textbf{(7–36)}$$

where the terms are the same as those in Eq. (7–35).

Equations (7–35) and (7–36) apply to foundations having a length-to-breadth ratio of unity ($L/B = 1$). Burland and Burbidge (1985) presented the following empirical relationship between settlement of foundations with $L/B > 1$ and $L/B = 1$:

$$S\,(L/B > 1) = S\,(L/B = 1)\left[\frac{1.25(L/B)}{(L/B) + 0.25}\right]^2 \qquad \textbf{(7–37)}$$

[Equation (7–37) gives the value of $S$ for $L/B > 1$ by multiplying the value of $S$ for $L/B = 1$ by the square of the value in brackets.] In the case of strip loading, $L/B$ becomes very large, and the square of the value in brackets in Eq. (7–37) approaches 1.56. Equations (7–35) and (7–36) also apply only to foundations on sands with a factor of safety against bearing capacity failure of at least 3; otherwise, excessive settlements associated with an approaching bearing capacity failure may develop.

In addition, Eqs. (7–35) and (7–36) are applicable only when the groundwater table is below the zone of influence (i.e., $B^{0.75}$ m below the foundation's base). If the groundwater table lies within the zone of influence, settlement will be

*From K. Terzaghi, R. B. Peck, and G. Mesri, *Soil Mechanics in Engineering Practice,* 3rd ed., John Wiley & Sons, Inc., New York, 1996. Copyright © 1996, by John Wiley & Sons, Inc. Reprinted by permission of John Wiley & Sons, Inc.

increased because the effective confining pressure is reduced. On the other hand, however, the reduced confining pressure results in a decrease in SPT $N$-values. Burland and Burbidge found that these two opposite effects more or less cancel each other; hence, the location of the groundwater table within the zone of influence can generally be neglected when one applies this method [i.e., Eqs. (7–35) and (7–36)] and no corrections are applied. If, however, the groundwater table rose into the zone of influence after the SPTs were performed, actual settlement could be considerably greater than (as great as twice as much) that computed from Eqs. (7–35) and (7–36) neglecting the groundwater table.

For saturated very dense fine or silty sand, measured SPT $N$-values should be reduced according to the following (Terzaghi et al., 1996* and Burland and Burbidge, 1985):

$$N' = 15 + \frac{(N - 15)}{2} \tag{7–38}$$

Ordinarily, foundations are designed to limit maximum settlement of any footing supporting a building to some acceptable value, such as 1 in. (25 mm). Because of the variability of sandy-soil deposits, settlements of equally loaded footings of any given size can vary from the mean by a factor of 1.6, or perhaps as large as 2.0. Therefore, to be reasonably sure that the largest footing will not settle more than about 1 in. (25 mm), one should strive for a value of $S$ of 25/1.6, or 16 mm, when applying Eqs. (7–35), (7–36), and (7–37).

Figure 7–27 may be used to facilitate computations involving Eqs. (7–35) and (7–36). Let

$$Q = \frac{\overline{N}^{1.4}}{1.7B^{0.75}} \tag{7–39}$$

Then, from Eqs. (7–35) and (7–36), with a value of 16 mm for $S$ for $q > p_0$,

$$q = 16Q + 2p_0/3 \tag{7–40}$$

for $q < p_0$,

$$q = 3 \times 16Q \tag{7–41}$$

To use Figure 7–27, locate a given width of footing and mean SPT $N$-value and find the corresponding value of $Q$. Using this value of $Q$ and, if needed, the given value of $p_0$, compute $q$ from Eq. (7–40) or (7–41). This value of $q$ is the bearing pressure corresponding to a maximum settlement of approximately 1 in. (25 mm) at the end of construction.

The relationship of Figure 7–27 is for square footings of side $B$. For rectangular footings, the value of $q$ should be reduced in accord with Eq. (7–37).

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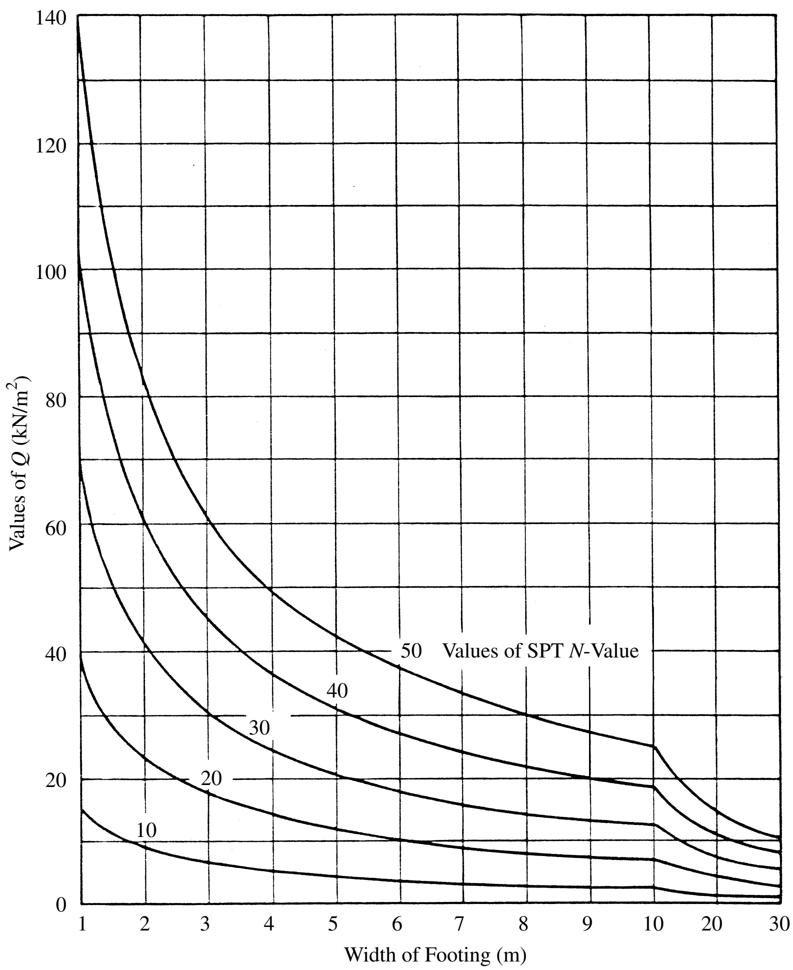


**FIGURE 7–27** Chart for estimating allowable soil pressure for footing on sand on the basis of results of the standard penetration test.
*Source:* K. Terzaghi, R. B. Peck, G. Mesri, *Soil Mechanics in Engineering Practice,* 3rd ed., John Wiley & Sons, Inc. New York, 1996. Copyright © 1996 by John Wiley & Sons, Inc. Reprinted by permission of John Wiley & Sons, Inc.

## EXAMPLE 7–20

*Given*

A square footing 3 m by 3 m located 1.5 m below ground level is to be constructed on sand having a unit weight of 18.30 kN/m$^3$. The arithmetic mean of the SPT $N$-values measured within the zone of influence is 30.

*Required*

Allowable soil pressure for a settlement of 25 mm.

**Solution**

Assume that the foundation contact pressure ($q$) is greater than the preconstruction effective overburden pressure ($p_0$) at the foundation's base, in which case Eq. (7–35) applies.

$$S = B^{0.75}\frac{1.7}{\overline{N}^{1.4}}(q - 2p_0/3) \quad \textbf{(7–35)}$$

Although settlement of 25 mm is required in this problem, as previously explained, it is recommended that a value of $S$ of 25/1.6, or 16 mm, be used in Eq. (7–35). Hence,

$$S = 16 \text{ mm}$$

$$B = 3 \text{ m}$$

$$\overline{N} = 30$$

$$p_0 = (1.5 \text{ m})(18.30 \text{ kN/m}^3) = 27.45 \text{ kN/m}^2$$

$$16 \text{ mm} = (3 \text{ m})^{0.75}\frac{1.7}{30^{1.4}}[q - (2)(27.45 \text{ kN/m}^2)/3]$$

$$q = 501 \text{ kN/m}^2$$

Check the assumption that $q > p_0$.
Because $[q = 501 \text{ kN/m}^2] > [p_0 = 27.45 \text{ kN/m}^2]$, use of Eq. (7–35) is correct.

As an alternative solution, using Figure 7–27, with $B = 3.0$ m and $\overline{N} = 30$, obtain (from Figure 7–27) $Q = 30 \text{ kN/m}^2$. Because $q > p_0$, use Eq. (7–40):

$$q = 16Q + 2p_0/3 \quad \textbf{(7–40)}$$

$$q = (16)(30 \text{ kN/m}^2) + (2)(27.45 \text{ kN/m}^2)/3 = 498 \text{ kN/m}^2$$

## Schmertmann Method

A third method for estimating foundation settlement on sand and gravel was developed by Schmertmann (1970). As noted previously in this section, settlement on sand is not amenable to solution based on laboratory tests, largely because of the problem of obtaining undisturbed soil samples for sandy soils for laboratory testing. Instead, settlement on sand is generally calculated by empirical means (Bazaraa method and Burkand and Burbidge method). The Schmertmann method evaluates settlement on sand using a semiempirical strain influence factor.

According to Schmertmann's investigation, settlement on sand can be calculated by the equation

$$S_t = C_1 C_2 \Delta p \sum_{0}^{2B} (I_z/E_s)\Delta z \quad (\text{for } L/B = 1)$$
$$S_t = C_1 C_2 \Delta p \sum_{0}^{4B} (I_z/E_s)\Delta z \quad (\text{for } L/B > 10) \tag{7–42}$$

where $S_t$ = settlement of foundation $t$ years after construction
$C_1$ = correction factor for effect of depth of foundation embedment
$C_1 = 1 - 0.5(p_0/\Delta p)$

where $p_0$ = soil overburden pressure at base of foundation (kN/m$^2$)
$\Delta p$ = net foundation pressure imposed onto soil at base of foundation (kN/m$^2$)

$C_2$ = correction factor for effect of a creep-type phenomenon and other factors over time
$C_2 = 1 + 0.2(\log 10t)$
where $t$ = elapsed time in years
$\Delta p$ = net foundation pressure imposed onto soil at base of foundation (kN/m$^2$)
$I_z$ = strain influence factor for soil zone $z$ depth below foundation (dimensionless) (see Figure 7–28)
$E_s$ = modulus of elasticity of sand (kN/m$^2$) (see Table 7–9)
$z$ = sand's layer thickness (m)

The variation of the strain influence factor ($I_z$) with depth below the foundation is shown Figure 7–28. The relationship of Figure 7–28 was developed on the basis of theory and model studies for vertical strain in sands below foundations as a function of depth. It can be noted from the figure that for square or circular foundations,

at $z = 0$ $\quad I_z = 0.1$
at $z = 0.5B$ $\quad I_z = 0.5$
at $z = 2B$ $\quad I_z = 0$

For foundations with $L/B > 10$,

at $z = 0$ $\quad I_z = 0.2$
at $z = B$ $\quad I_z = 0.5$
at $z = 4B$ $\quad I_z = 0$

[$B$ = width of foundation and $L$ = length of foundation. Values of $L/B$ between 1 and 10 can be interpolated.] The modulus of elasticity of sand ($E_s$) can be

**TABLE 7–9**
**Values of $E_s$ Related to Soil Type (McCarthy, 2002)**

| Soil Type | Approximate Value for $E_s$ (kgf/cm², 0.1 MPa, ton/ft²) In terms of $N$ | Approximate Value for $E_s$ (kgf/cm², 0.1 MPa, ton/ft²) In terms of $q_c$ |
|---|---|---|
| Sand–silt mixture | $4N$ | $1.5q_c$ |
| Fine-to-medium sands, fine–medium–coarse sands | $7N-10N$ (relating to density and compactness) | $2q_c-3q_c$ (relating to density and compactness) |
| Sand–gravel mixtures | $12N$ | $4q_c$ |

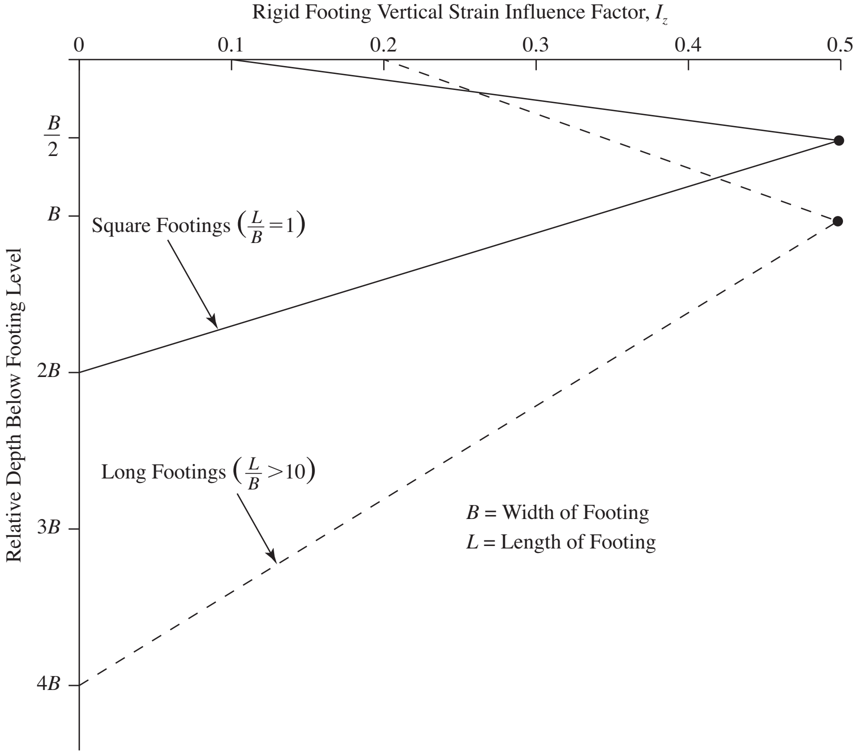


**FIGURE 7–28** Strain influence factors.
*Source:* J. H. Schmertmann, J. P. Hartman, and P. R. Brown, "Improved Strain Influence Factor Diagrams," *J. Geotech Eng. Div. ASCE*, **104**(GT8), 1131–1135 (1978). With permission from ASCE.

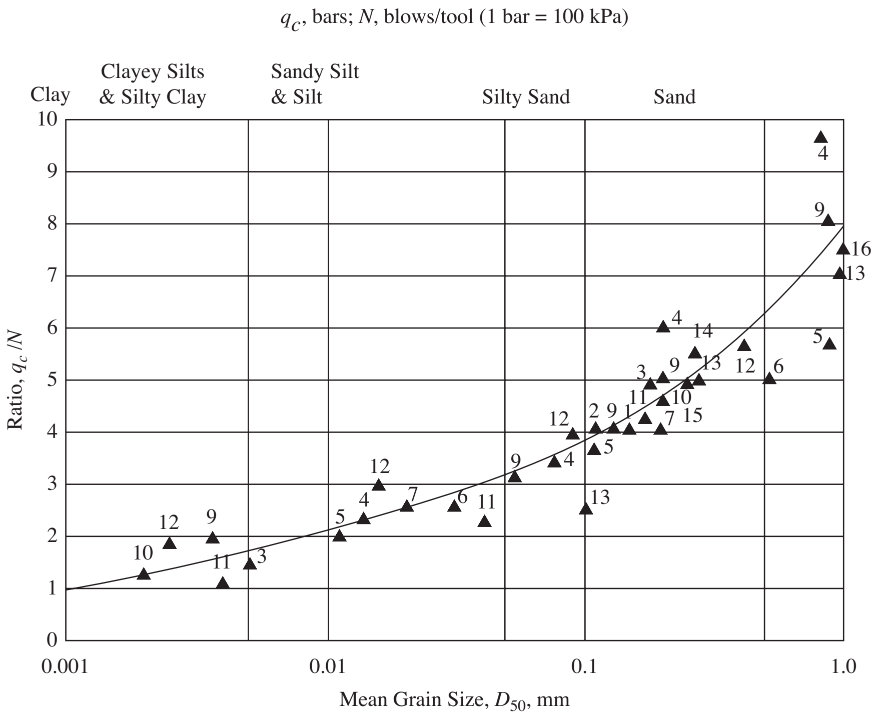


**FIGURE 7–29** Variation of $q_c/N$ ratio with mean grain size.
*Source:* P. K. Robertson, R. G. Campanella, and A. Whitman, "SPT–CPT Correlations," *J. Geotech. Eng. Div. ASCE*, **109** (GT11), 1449–1459 (1983). With permission from ASCE.

determined from cone penetration resistance ($q_c$) (see Chapter 3). If cone penetration resistances are not available, crude correlations between the SPT $N$-value and the cone penetration resistance can be used to estimate values of $E_s$ for use in Eq. (7–42). $q_c/N$ ratios as a function of mean grain size ($D_{50}$) are shown in Figure 7–29. Most of the data shown in Figure 7–29 were obtained using the standard donut-type hammer with rope and cathead system. A summary of the relationship between $q_c$ and SPT $N$-value and $E_s$ is given in Table 7–9.

To use Eq. (7–42) to calculate settlement, soil can be divided into several layers below the foundation's base, and each layer's settlement can be estimated using Eq. (7–42). The foundation's total settlement ($S_t$) is obtained by summing the individual settlements of all the layers. Example 7–21 illustrates this process.

## EXAMPLE 7–21

*Given*

A square footing 3 m by 3 m is resting on a sand deposit (see Figure 7–30). Assume the modulus of elasticity of the sand is $2.5q_c$.

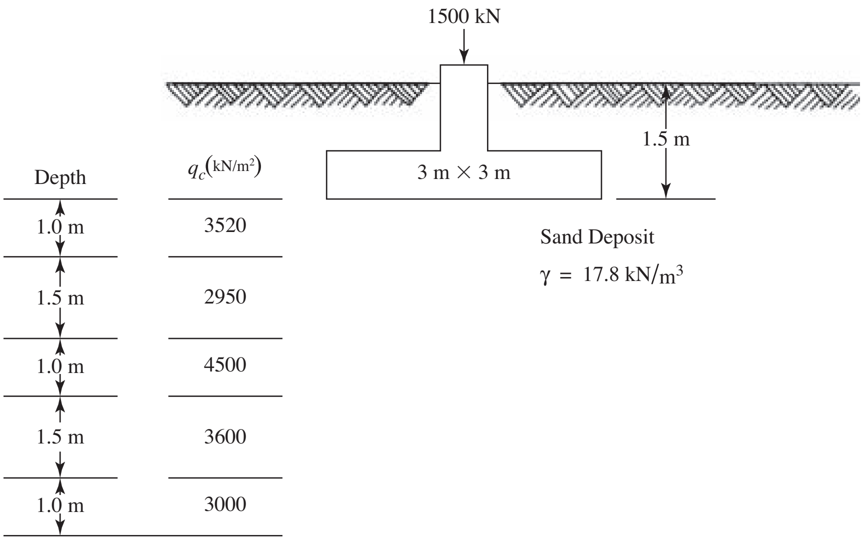


**FIGURE 7–30**

*Required*

Settlement of the footing five years after construction using the Schmertmann method.

**Solution**

From Eq. (7–42),

$$S_t = C_1 C_2 \Delta p \sum_{0}^{2B} (I_z/E_s)\Delta z \tag{7–42}$$

where $C_1 = 1 - 0.5(p_0/\Delta p)$

$$p_0 = (17.8 \text{ kN/m}^3)(1.5 \text{ m}) = 26.70 \text{ kN/m}^2$$

$$\Delta p = \frac{1500 \text{ kN}}{(3 \text{ m})(3 \text{ m})} - (17.8 \text{ kN/m}^3)(1.5 \text{ m}) = 139.97 \text{ kN/m}^2$$

$$C_1 = 1 - 0.5\left(\frac{26.70 \text{ kN/m}^2}{139.97 \text{ kN/m}^2}\right) = 0.905$$

$$C_2 = 1 + 0.2(\log 10t) = 1 + 0.2 \log[(10)(5 \text{ yr})] = 1.34$$

| Layer Number | Layer Thickness $\Delta z$ (m) | Depth from Base of Footing to Center of Layer (m) | $q_c$ (kN/m$^2$) | $E_s$ (kN/m$^2$) [2.5$q_c$] | $I_z$* | $(I_z/E_s)\Delta z$ (m$^3$/kN) |
|---|---|---|---|---|---|---|
| 1 | 1.0 | 0.50 | 3520 | 8800 | 0.233 | $2.65 \times 10^{-5}$ |
| 2 | 1.5 | 1.75 | 2950 | 7375 | 0.472 | $9.60 \times 10^{-5}$ |
| 3 | 1.0 | 3.00 | 4500 | 11,250 | 0.333 | $2.96 \times 10^{-5}$ |
| 4 | 1.5 | 4.25 | 3600 | 9000 | 0.194 | $3.23 \times 10^{-5}$ |
| 5 | 1.0 | 5.50 | 3000 | 7500 | 0.056 | $0.75 \times 10^{-5}$ |
| | | | | | | $\sum_{0}^{2B=6\text{ m}} (I_z/E_s)\Delta z = 19.19 \times 10^{-5}$ |

*Values of $I_z$ are obtained from Figure 7–31.

Substituting into Eq. (7–42) gives

$$S_{5\text{ yr}} = (0.905)(1.34)(139.97\text{ kN/m}^2)(19.19 \times 10^{-5}\text{m}^3/\text{kN}) = 0.033\text{ m, or 33 mm}$$

Maximum permissible settlement depends primarily on the nature of the superstructure. Some suggested maximum permissible settlement values are given in Table 7–10.

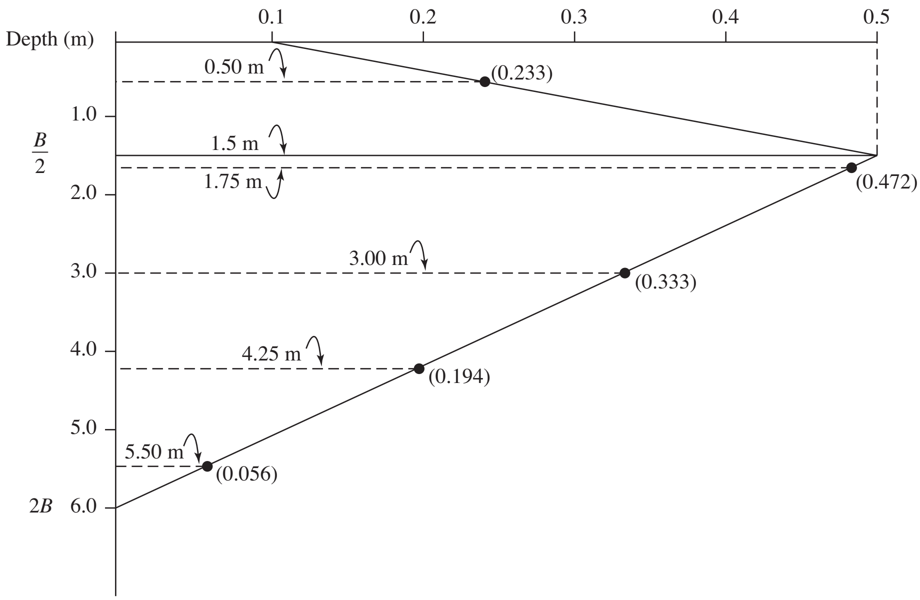


**FIGURE 7–31** Strain influence factor.

**TABLE 7–10**
**Maximum Permissible Settlement**

| Limiting Factor or Type of Structure | Maximum Permissible Settlement: *Differential*[1] | Maximum Permissible Settlement: *Total (in.)* |
|---|---|---|
| Drainage of floors | 0.01–0.02$L$ | 6–12 |
| Stacking, warehouse lift trucks | 0.01$L$ | 6 |
| Tilting of smokestacks, silos | 0.004$B$ | 3–12 |
| Framed structure, simple | 0.005$L$ | 2–4 |
| Framed structure, continuous | 0.002$L$ | 1–2 |
| Framed structure with diagonals | 0.0015$L$ | 1–2 |
| Reinforced concrete structure | 0.002–0.004$L$ | 1–3 |
| Brick walls, one-story | 0.001–0.002$L$ | 1–2 |
| Brick walls, high | 0.0005–0.001$L$ | 1 |
| Cracking of panel walls | 0.003$L$ | 1–2 |
| Cracking of plaster | 0.001$L$ | 1 |
| Machine operation, noncritical | 0.003$L$ | 1–2 |
| Crane rails | 0.003$L$ | |
| Machines, critical | 0.002$L$ | |

[1] L is the distance between adjacent columns; $B$ is the width of the base.

*Source*: *Soil Engineering*, 3rd ed., p. 587, by M. G. Spangler and R. L. Handy. Copyright © 1951, 1960, 1973. Reprinted by permission of Pearson Education, Upper Saddle River, NJ.

## 7–11 PROBLEMS

**7–1.** A laboratory consolidation test was performed on a clayey soil specimen, which was drained on both top and bottom. The time for 50% consolidation was 6.2 min, and the specimen's thickness at 50% consolidation was 0.740 in. Two points on the field consolidation line have coordinates ($p_1$, $e_1$) and ($p_2$, $e_2$) of (1000 lb/ft$^2$, 1.167) and (2000 lb/ft$^2$, 1.108), respectively. Find the coefficient of permeability of the clay for the given loading range.

**7–2.** Determine the present effective overburden pressure at midheight of the compressible clay layer in the soil profile shown in Figure 7–32.

**7–3.** When the total pressure acting at midheight of a compressible clay layer is 100 kN/m$^2$, the corresponding void ratio is 1.09. When the total pressure increases to 400 kN/m$^2$, the corresponding void ratio decreases to 0.89. What would be the void ratio for a total pressure of 800 kN/m$^2$?

**7–4.** A compressible clay layer 10.0 m thick has an initial void ratio *in situ* of 1.026. Tests and computations show that the final void ratio of the clay layer after construction of a structure is 0.978. Determine the estimated primary consolidation settlement of the structure.

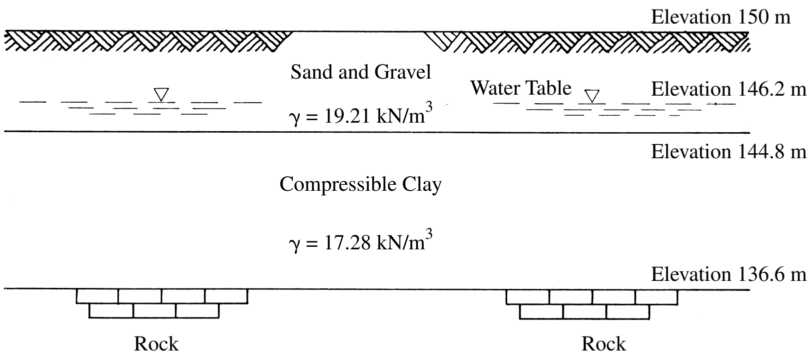


**FIGURE 7–32**

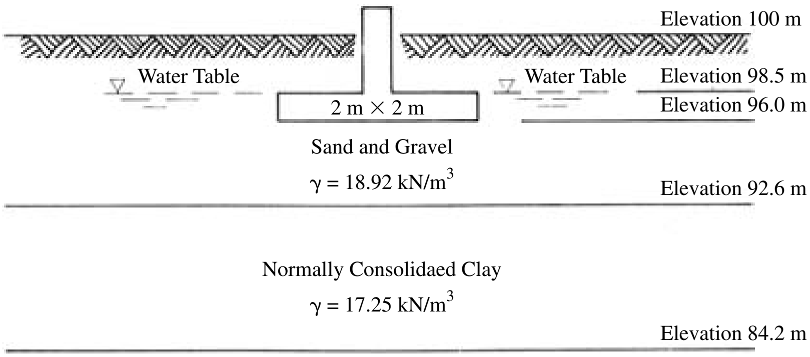


**FIGURE 7–33**

**7–5.** A foundation is to be constructed at a site where the soil profile is as shown in Figure 7–33. The base of the foundation, which is 2 m square, exerts a total load (weight of structure, foundation, and soil surcharge on the foundation) of 1000 kN. The initial void ratio *in situ* of the normally consolidated clay layer is 1.058, and its compression index is 0.60. Find the estimated primary consolidation settlement for the clay layer.

**7–6.** Continuing Problem 7–5, tests and computations indicate that the coefficient of consolidation is $6.98 \times 10^{-6}$ m$^2$/min. Compute the time required for 90% of the expected primary consolidation settlement to take place if the clay layer is underlain by (a) permeable sand and gravel, and (b) impermeable bedrock.

**7–7.** A sample of normally consolidated clay was obtained by a Shelby tube sampler from the midheight of a normally consolidated clay layer (see Figure 7–34). A consolidation test was conducted on a portion of this sample, the results of which are given as follows:

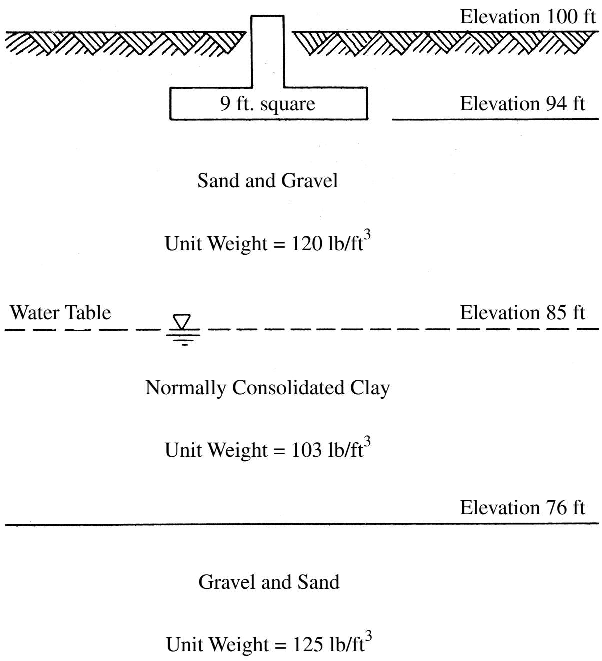


**FIGURE 7–34**

1. Natural (initial) void ratio of the clay existing in the field ($e_0$) = 1.80.
2. Pressure–void ratio relationships are as follows:

| $p$ (tons/ft$^2$) | $e$ |
|---|---|
| 0.250 | 1.72 |
| 0.500 | 1.70 |
| 1.00 | 1.64 |
| 2.00 | 1.51 |
| 4.00 | 1.34 |
| 8.00 | 1.15 |
| 16.00 | 0.95 |

A footing is to be constructed 6 ft below ground surface, as shown in Figure 7–34. The base of the footing is 9 ft by 9 ft, and it carries a total load of 200 tons, which includes the column load, weight of footing, and weight of soil surcharge on the footing.

a. From consolidation test results, prepare an $e$–log $p$ curve and construct a field consolidation line, assuming that point $f$ is located at $0.4e_0$.

b. Compute the total expected primary consolidation settlement of the compressible clay layer.

**7–8.** Estimate the primary consolidation settlement for a foundation on an overconsolidated clay layer for the following conditions.

1. Thickness of overconsolidated clay layer = 3.8 m.
2. Present effective overburden pressure ($p_0$) = 108 kN/m$^2$.
3. Overconsolidated pressure ($p_0{}'$) = 125 kN/m$^2$.
4. Initial void ratio of the clay layer existing in the field ($e_0$) = 0.70.
5. Compression index of clay layer ($C_c$) = 0.30.
6. Swell index of the clay layer ($C_s$) = 0.06.
7. Net consolidation pressure at midheight of clay layer under center of foundation ($\Delta p$) = 52 kN/m$^2$.

**7–9.** Continuing Problem 7–7, test results also indicated that the coefficient of consolidation ($c_v$) of the clay is $2.18 \times 10^{-3}$ in.$^2$/min for the pressure increment from 1 to 2 tons/ft$^2$. Compute the time of primary consolidation settlement. Take $U$ at 10% increments and plot these values on a settlement–log time curve.

**7–10.** For the information given in Problem 7–8, assume the overconsolidated pressure of the clay layer is 185 kN/m$^2$. Estimate the primary consolidation settlement.

**7–11.** A compressible 12-ft clay layer beneath a building is overlain by a stratum of sand and gravel and underlain by impermeable bedrock. The total expected primary consolidation settlement of the compressible clay layer due to the building load is 4.60 in. The coefficient of consolidation ($c_v$) is $9.04 \times 10^{-4}$ in.$^2$/min.

1. How long will it take for 90% of the expected total primary consolidation settlement to take place?
2. Compute the amount of primary consolidation settlement that will occur in 1 yr.
3. How long will it take for primary consolidation settlement of 1 in. to take place?

**7–12.** Continuing Problem 7–5, assume that 100% primary consolidation will be complete in 14 yr. If the clay layer's natural water content is 35%, compute the estimated secondary compression settlement that would occur from 14 to 40 yr after construction.

**7–13.** A 9-ft by 9-ft square footing to carry a total load of 300 tons is to be installed 6 ft below ground surface on a sand stratum. Standard penetration tests were conducted on the site. Test results were corrected for overburden pressures, and the corrected $N$-values are listed as follows:

| Depth (ft) | Corrected *N*-Values |
|---|---|
| 2.5 | 25 |
| 5.0 | 28 |
| 7.5 | 27 |
| 10.0 | 30 |
| 12.5 | 28 |
| 15.0 | 23 |
| 17.5 | 24 |
| 20.0 | 28 |

No groundwater was encountered during subsurface exploration. Estimate the maximum expected settlement of the footing. Use Bazaraa's method.

**7–14.** Assume the same conditions as in Problem 7–13, except that the groundwater table is located 8 ft below ground level and the sand's unit weight is 130 lb/ft$^3$. Estimate the maximum expected settlement of the footing.

**7–15.** A square footing 6 ft by 6 ft is to be installed 6 ft below ground level on a sand stratum. Standard penetration tests were conducted on the construction site. Test results were corrected for overburden pressures, and the lowest average corrected *N*-value was determined to be 18. Assuming that groundwater was not encountered, determine the allowable soil pressure for a maximum settlement of 1 in. Use Bazaraa's method.

**7–16.** Assume the same conditions as in Problem 7–15, except that the groundwater table is located 8 ft below ground level and the sand's unit weight is 118 lb/ft$^3$. Determine the allowable soil pressure for a maximum settlement of 1 in.

**7–17.** A rectangular footing 3 m by 4 m located 2 m below ground level is to be constructed on sand having a unit weight of 18.8 kN/m$^3$. The footing is designed to take a total load of 6000 kN. If the arithmetic mean of SPT *N*-values measured within the zone of influence is 36, compute the settlement of the footing. Use Burkand and Burbidge's method.

**7–18.** A proposed square footing 2 m by 2 m carrying a total load of 800 kN is to be constructed on a sand deposit. The depth of the footing will be 1 m below the ground surface, and the unit weight of the sand is 17.5 kN/m$^3$. The modulus of elasticity of the sand was determined to be 10,000 kN/m$^3$ throughout. Using Schmertmann's method, estimate the settlement of the footing four years after construction, using four 1-m-thick sublayers.