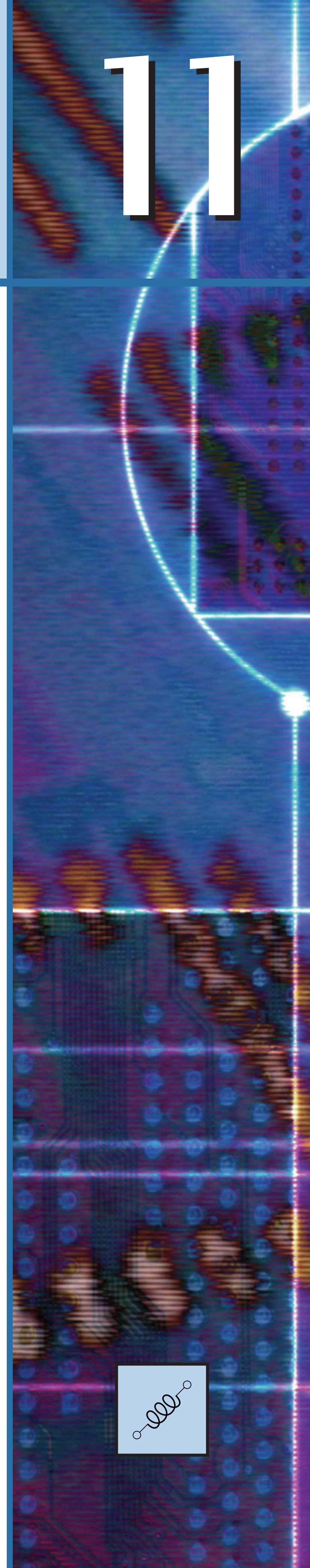
Inductors



Objectives

- ***Become familiar with the basic construction of an inductor, the factors that affect the strength of the magnetic field established by the element, and how to read the nameplate data.***
- ***Be able to determine the transient (time-varying) response of an inductive network and plot the resulting voltages and currents.***
- ***Understand the impact of combining inductors in series or parallel.***
- ***Develop some familiarity with the use of PSpice or Multisim to analyze networks with inductive elements.***

11.1 INTRODUCTION

Three basic components appear in the majority of electrical/electronic systems in use today. They include the *resistor* and the *capacitor,* which have already been introduced, and the **inductor,** to be examined in detail in this chapter. In many ways, the inductor is the dual of the capacitor; that is, the voltage of one is applicable to the current of the other, and vice versa. In fact, some sections in this chapter parallel those in Chapter 10 on the capacitor. Like the capacitor, *the inductor exhibits its true characteristics only when a change in voltage or current is made in the network.*

Recall from Chapter 10 that a capacitor can be replaced by an open-circuit equivalent under steady-state conditions. You will see in this chapter that an inductor can be replaced by a short-circuit equivalent under steady-state conditions. Finally, you will learn that while resistors dissipate the power delivered to them in the form of heat, ideal capacitors store the energy delivered to them in the form of an electric field. Inductors, in the ideal sense, are like capacitors in that they also store the energy delivered to them—but in the form of a magnetic field.

11.2 MAGNETIC FIELD

Magnetism plays an integral part in almost every electrical device used today in industry, research, or the home. Generators, motors, transformers, circuit breakers, televisions, computers, tape recorders, and telephones all employ magnetic effects to perform a variety of important tasks.

The compass, used by Chinese sailors as early as the second century A.D., relies on a **permanent magnet** for indicating direction. A permanent magnet is made of a material, such as steel or iron, that remains magnetized for long periods of time without the need for an external source of energy.

In 1820, the Danish physicist Hans Christian Oersted discovered that the needle of a compass deflects if brought near a current-carrying conductor. This was the first demonstration that electricity and magnetism were related. In the same year, the French physicist André-Marie Ampère performed experiments in this area and developed what is presently known as **Ampère's circuital law.** In subsequent years, others such as Michael Faraday, Karl Friedrich

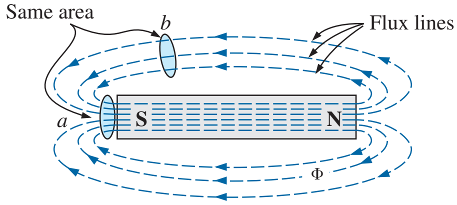


FIG. 11.1
Flux distribution for a permanent magnet.

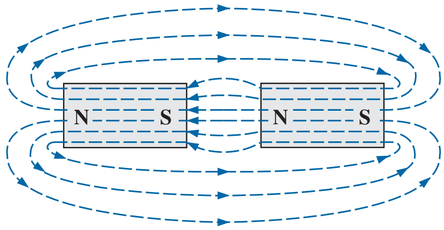


FIG. 11.2
Flux distribution for two adjacent, opposite poles.

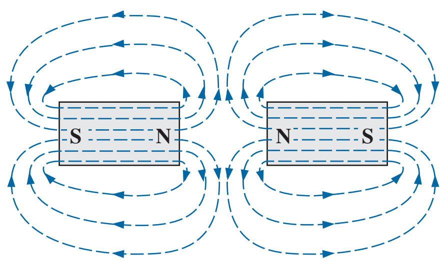


FIG. 11.3
Flux distribution for two adjacent, like poles.

Gauss, and James Clerk Maxwell continued to experiment in this area and developed many of the basic concepts of **electromagnetism**—magnetic effects induced by the flow of charge, or current.

A magnetic field exists in the region surrounding a permanent magnet, which can be represented by **magnetic flux lines** similar to electric flux lines. Magnetic flux lines, however, do not have origins or terminating points as do electric flux lines but exist in *continuous loops,* as shown in Fig. 11.1.

The magnetic flux lines radiate from the north pole to the south pole, returning to the north pole through the metallic bar. Note the equal spacing between the flux lines within the core and the symmetric distribution outside the magnetic material. These are additional properties of magnetic flux lines in homogeneous materials (that is, materials having uniform structure or composition throughout). It is also important to realize that the continuous magnetic flux line will strive to occupy as small an area as possible. This results in magnetic flux lines of minimum length between the unlike poles, as shown in Fig. 11.2. The strength of a magnetic field in a particular region is directly related to the density of flux lines in that region. In Fig. 11.1, for example, the magnetic field strength at point *a* is twice that at point *b* since twice as many magnetic flux lines are associated with the perpendicular plane at point *a* than at point *b.* Recall from childhood experiments that the strength of permanent magnets is always stronger near the poles.

If unlike poles of two permanent magnets are brought together, the magnets attract, and the flux distribution is as shown in Fig. 11.2. If like poles are brought together, the magnets repel, and the flux distribution is as shown in Fig. 11.3.

If a nonmagnetic material, such as glass or copper, is placed in the flux paths surrounding a permanent magnet, an almost unnoticeable change occurs in the flux distribution (Fig. 11.4). However, if a magnetic material, such as soft iron, is placed in the flux path, the flux lines pass through the soft iron rather than the surrounding air because flux lines pass with greater ease through magnetic materials than through air. This principle is used in shielding sensitive electrical elements and instruments that can be affected by stray magnetic fields (Fig. 11.5).

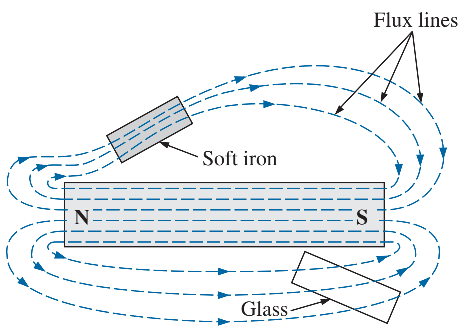


FIG. 11.4
Effect of a ferromagnetic sample on the flux distribution of a permanent magnet.

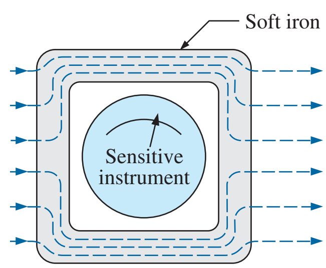


FIG. 11.5
Effect of a magnetic shield on the flux distribution.

A magnetic field (represented by concentric magnetic flux lines, as in Fig. 11.6) is present around every wire that carries an electric current. The direction of the magnetic flux lines can be found simply by placing the thumb of the *right* hand in the direction of *conventional* current flow and noting the direction of the fingers. (This method is commonly called the *right-hand rule.*) If the conductor is wound in a

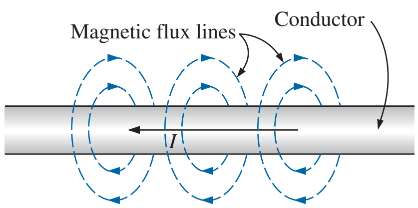


FIG. 11.6
Magnetic flux lines around a current-carrying conductor.

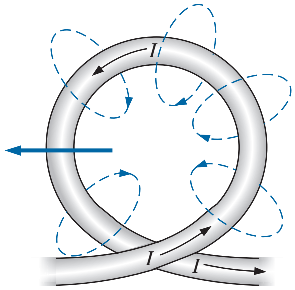


FIG. 11.7
Flux distribution of a single-turn coil.

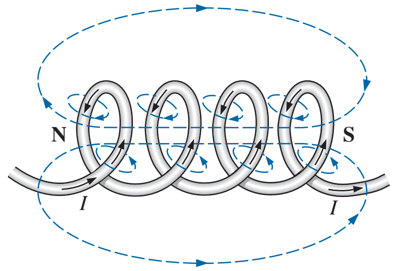


FIG. 11.8
Flux distribution of a current-carrying coil.

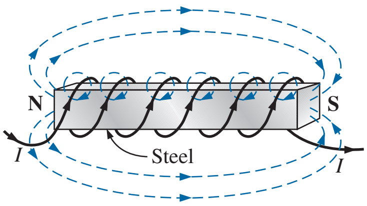


FIG. 11.9
Electromagnet.

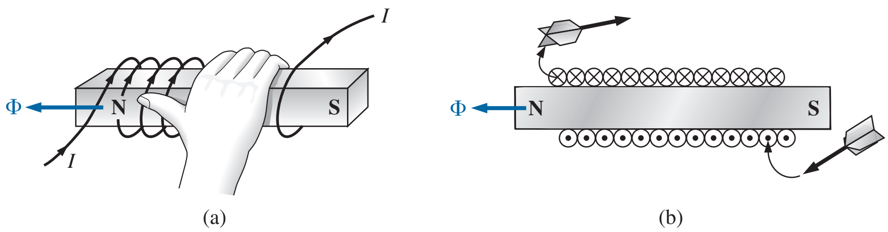


FIG. 11.10
Determining the direction of flux for an electromagnet: (a) method; (b) notation.

single-turn coil (Fig. 11.7), the resulting flux flows in a common direction through the center of the coil. A coil of more than one turn produces a magnetic field that exists in a continuous path through and around the coil (Fig. 11.8).

The flux distribution of the coil is quite similar to that of the permanent magnet. The flux lines leaving the coil from the left and entering to the right simulate a north and a south pole, respectively. The principal difference between the two flux distributions is that the flux lines are more concentrated for the permanent magnet than for the coil. Also, since *the strength of a magnetic field is determined by the density of the flux lines,* the coil has a weaker field strength. The field strength of the coil can be effectively increased by placing certain materials, such as iron, steel, or cobalt, within the coil to increase the flux density within the coil. By increasing the field strength with the addition of the core, we have devised an *electromagnet* (Fig. 11.9) that not only has all the properties of a permanent magnet but also has a field strength that can be varied by changing one of the component values (current, turns, and so on). Of course, current must pass through the coil of the electromagnet for magnetic flux to be developed, whereas there is no need for the coil or current in the permanent magnet. The direction of flux lines can be determined for the electromagnet (or in any core with a wrapping of turns) by placing the fingers of your right hand in the direction of current flow around the core. Your thumb then points in the direction of the north pole of the induced magnetic flux, as demonstrated in Fig. 11.10(a). A cross section of the same electromagnet is in Fig. 11.10(b) to introduce the convention for directions perpendicular to the page. The cross and the dot refer to the tail and the head of the arrow, respectively.

In the SI system of units, magnetic flux is measured in **webers (Wb)** as derived from the surname of Wilhelm Eduard Weber (Fig. 11.11). The

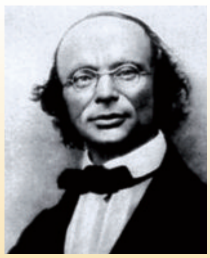


FIG. 11.11
Wilhelm Eduard Weber.
Courtesy of the Smithsonian Institution, Photo No. 52,604.

German (Wittenberg, Göttingen)
(1804–91)
Physicist
Professor of Physics, University of Göttingen

An important contributor to the establishment of a system of *absolute units* for the electrical sciences, which was beginning to become a very active area of research and development. Established a definition of electric current in an electromagnetic system based on the magnetic field produced by the current. He was politically active and, in fact, was dismissed from the faculty of the University of Göttingen for protesting the suppression of the constitution by the King of Hanover in 1837. However, he found other faculty positions and eventually returned to Göttingen as director of the astronomical observatory. He received honors from England, France, and Germany, including the Copley Medal of the Royal Society of London.

applied symbol is the capital Greek letter *phi,* Φ. The number of flux lines per unit area, called the **flux density,** is denoted by the capital letter B and is measured in **teslas (T)** to honor the efforts of Nikola Tesla, a scientist of the late 1800s (Fig. 11.12).

In equation form:

$$\boxed{B = \frac{\Phi}{A}} \qquad \begin{aligned} B &= \text{Wb/m}^2 = \text{teslas (T)} \\ \Phi &= \text{webers (Wb)} \\ A &= \text{m}^2 \end{aligned} \tag{11.1}$$

where Φ is the number of flux lines passing through area A in Fig. 11.13. The flux density at point a in Fig. 11.1 is twice that at point b because twice as many flux lines pass through the same area.

In Eq. (11.1), the equivalence is given by

$$\boxed{1 \text{ tesla} = 1 \text{ T} = 1 \text{ Wb/m}^2} \tag{11.2}$$

which states in words that if 1 weber of magnetic flux passes through an area of 1 square meter, the flux density is 1 tesla.

For the CGS system, magnetic flux is measured in maxwells and the flux density in gauss. For the English system, magnetic flux is measured in lines and the flux density in lines per square inch. The relationship between such systems is defined in Appendix F.

The flux density of an electromagnet is directly related to the number of turns of, and current through, the coil. The product of the two, called the **magnetomotive force,** is measured in **ampere-turns (At)** as defined by

$$\boxed{\mathscr{F} = NI} \qquad \text{(ampere-turns, At)} \tag{11.3}$$

In other words, if you increase the number of turns around a core and/or increase the current through the coil, the magnetic field strength also increases. In many ways, the magnetomotive force for magnetic circuits is similar to the applied voltage in an electric circuit. Increasing either one results in an increase in the desired effect: magnetic flux for magnetic circuits and current for electric circuits.

For the CGS system, the magnetomotive force is measured in gilberts, while for the English system, it is measured in ampere-turns.

Another factor that affects the magnetic field strength is the type of core used. Materials in which magnetic flux lines can readily be set up are said to be **magnetic** and to have a high **permeability.** Again, note the similarity with the word "permit" used to describe permittivity for the dielectrics of capacitors. Similarly, the permeability (represented by the Greek letter *mu,* μ) of a material is a measure of the ease with which magnetic flux lines can be established in the material.

Just as there is a specific value for the permittivity of air, there is a specific number associated with the permeability of air:

$$\boxed{\mu_o = 4\pi \times 10^{-7} \text{ Wb/A}\cdot\text{m}} \tag{11.4}$$

Practically speaking, the permeability of all nonmagnetic materials, such as copper, aluminum, wood, glass, and air, is the same as that for free space. Materials that have permeabilities slightly less than that of free space are said to be **diamagnetic,** and those with permeabilities slightly greater than that of free space are said to be **paramagnetic.** Magnetic materials,

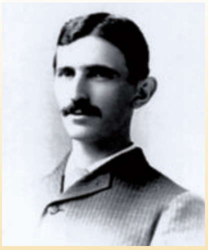


FIG. 11.12
Nikola Tesla.
Courtesy of the Smithsonian Institution, Photo No. 52,223.

Croatian-American (Smiljan, Paris, Colorado Springs, New York City)
(1856–1943)
Electrical Engineer and Inventor Recipient of the Edison Medal in 1917

Often regarded as one of the most innovative and inventive individuals in the history of the sciences. He was the first to introduce the *alternating-current machine,* removing the need for commutator bars of dc machines. After emigrating to the United States in 1884, he sold a number of his patents on *ac machines, transformers,* and *induction coils* (including the Tesla coil as we know it today) to the Westinghouse Electric Company. Some say that his most important discovery was made at his laboratory in Colorado Springs, where in 1900 he discovered *terrestrial stationary waves.* The range of his discoveries and inventions is too extensive to list here but extends from lighting systems to *polyphase power systems* to a *wireless world broadcasting system.*

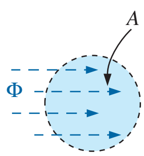


FIG. 11.13
Defining the flux density B.

such as iron, nickel, steel, cobalt, and alloys of these metals, have permeabilities hundreds and even thousands of times that of free space. Materials with these very high permeabilities are referred to as **ferromagnetic.**

The ratio of the permeability of a material to that of free space is called its **relative permeability;** that is,

$$\mu_r - \frac{\mu}{\mu_o} \tag{11.5}$$

In general, for ferromagnetic materials, $\mu_r \geq 100$, and for nonmagnetic materials, $\mu_r = 1$.

A table of values for μ to match the provided table for permittivity levels of specific dielectrics would be helpful. Unfortunately, such a table cannot be provided because *relative permeability is a function of the operating conditions.* If you change the magnetomotive force applied, the level of μ can vary between extreme limits. At one level of magnetomotive force, the permeability of a material can be 10 times that at another level.

An instrument designed to measure flux density in gauss (CGS system) appears in Fig. 11.14. Appendix F reveals that 1 T = 10^4 gauss. The magnitude of the reading appearing on the face of the meter in Fig. 11.14 is therefore

$$20.2159 \text{ gauss}\left(\frac{1\text{ T}}{10^4 \text{ gauss}}\right) = 2.02 \times 10^{-3}\text{ T}$$

Although our emphasis in this chapter is to introduce the parameters that affect the nameplate data of an inductor, the use of magnetics has widespread application in the electrical/electronics industry, as shown by a few areas of application in Fig. 11.15.

FIG. 11.14
Digital display gaussmeter.
(Courtesy of Walker LDJ Scientific Inc.
www.walkerldjscientific.com)

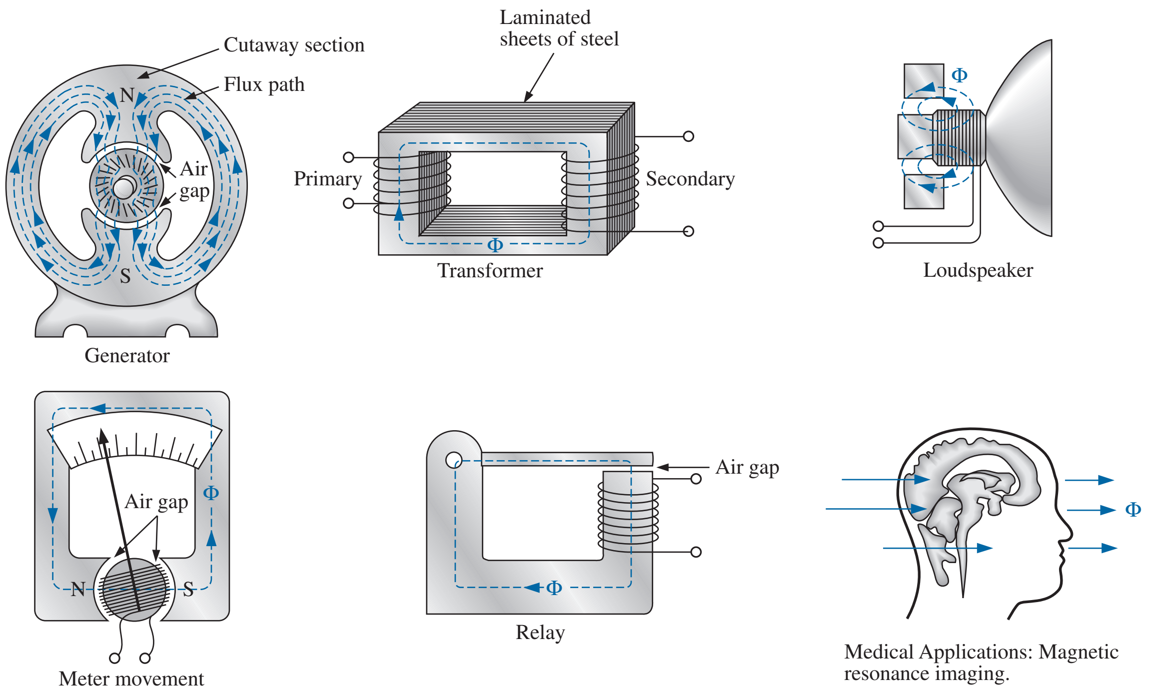


FIG. 11.15
Some areas of application of magnetic effects.

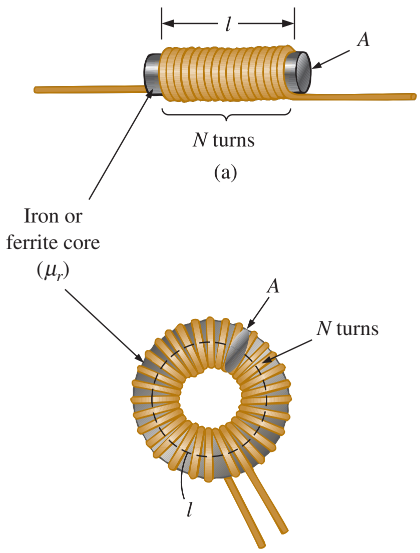


FIG. 11.16
Defining the parameters for Eq. (11.6).

11.3 INDUCTANCE

In the previous section, we learned that sending a current through a coil of wire, with or without a core, establishes a magnetic field through and surrounding the unit. This component, of rather simple construction (see Fig. 11.16), is called an **inductor** (often referred to as a **coil**). Its **inductance** level determines the strength of the magnetic field around the coil due to an applied current. The higher the inductance level, the greater the strength of the magnetic field. In total, therefore,

inductors are designed to set up a strong magnetic field linking the unit, whereas capacitors are designed to set up a strong electric field between the plates.

Inductance is measured in **henries (H),** after the American physicist Joseph Henry (Fig. 11.17). However, just as the farad is too large a unit for most applications, most inductors are of the millihenry (mH) or microhenry (μH) range.

In Chapter 10, 1 farad was defined as a capacitance level that would result in 1 coulomb of charge on the plates due to the application of 1 volt across the plates. For inductors,

1 henry is the inductance level that will establish a voltage of 1 volt across the coil due to a change in current of 1 A/s through the coil.

FIG. 11.17
Joseph Henry.
Courtesy of the Smithsonian Institutions, Photo No. 59,054.

American (Albany, NY; Princeton, NJ)
(1797–1878)
Physicist and Mathematician
Professor of Natural Philosophy,
Princeton University

In the early 1800s the title Professor of Natural Philosophy was applied to educators in the sciences. As a student and teacher at the Albany Academy, Henry performed extensive research in the area of electromagnetism. He improved the design of *electromagnets* by insulating the coil wire to permit a tighter wrap on the core. One of his earlier designs was capable of lifting 3600 pounds. In 1832 he discovered and delivered a paper on *self-induction.* This was followed by the construction of an effective *electric telegraph transmitter and receiver* and extensive research on the oscillatory nature of lightning and discharges from a *Leyden jar.* In 1845 he was appointed the first Secretary of the Smithsonian.

Inductor Construction

In Chapter 10, we found that capacitance is sensitive to the area of the plates, the distance between the plates, and the dielectric employed. The level of inductance has similar construction sensitivities in that it is dependent on the area within the coil, the length of the unit, and the permeability of the core material. It is also sensitive to the number of turns of wire in the coil as dictated by Eq. (11.6) and defined in Fig. 11.16 for two of the most popular shapes:

$$L = \frac{\mu N^2 A}{l} \qquad \textbf{(11.6)}$$

μ = permeability (Wb/A · m)
N = number of turns (t)
A = m^2
l = m
L = henries (H)

First note that since the turns are squared in the equation, the number of turns is a big factor. However, also keep in mind that the more turns, the bigger the unit. If the wire is made too thin to get more windings on the core, the rated current of the inductor is limited. Since higher levels of permeability result in higher levels of magnetic flux, permeability should, and does, appear in the numerator of the equation. Increasing the area of the core or decreasing the length also increases the inductance level.

Substituting $\mu = \mu_r \mu_o$ for the permeability results in Eq. (11.7), which is very similar to the equation for the capacitance of a capacitor:

$$L = \frac{\mu_r \mu_o N^2 A}{l}$$

or

$$L = 4\pi \times 10^{-7} \frac{\mu_r N^2 A}{l} \quad \text{(henries, H)} \qquad \textbf{(11.7)}$$

If we break out the relative permeability as follows:

$$L = \mu_r\left(\frac{\mu_o N^2 A}{l}\right)$$

we obtain the following useful equation:

$$\boxed{L = \mu_r L_o} \qquad \textbf{(11.8)}$$

which is very similar to the equation $C = \epsilon_r C_o$. Eq. (11.8) states the following:

The inductance of an inductor with a ferromagnetic core is μ_r times the inductance obtained with an air core.

Although Eq. (11.6) is approximate at best, the equations for the inductance of a wide variety of coils can be found in reference handbooks. Most of the equations are mathematically more complex than Eq. (11.6), but the impact of each factor is the same in each equation.

EXAMPLE 11.1 For the air-core coil in Fig. 11.18:

a. Find the inductance.
b. Find the inductance if a metallic core with $\mu_r = 2000$ is inserted in the coil.

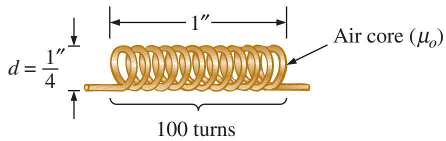


FIG. 11.18
Air-core coil for Example 11.1.

Solutions:

a. $$d = \frac{1}{4}\text{ in.}\left(\frac{1\text{ m}}{39.37\text{ in.}}\right) = 6.35\text{ mm}$$

$$A = \frac{\pi d^2}{4} = \frac{\pi(6.35\text{ mm})^2}{4} = 31.7\ \mu\text{m}^2$$

$$l = 1\text{ in.}\left(\frac{1\text{ m}}{39.37\text{ in.}}\right) = 25.4\text{ mm}$$

$$L = 4\pi \times 10^{-7}\frac{\mu_r N^2 A}{l}$$

$$= 4\pi \times 10^{-7}\frac{(1)(100\text{ t})^2(31.7\ \mu\text{m}^2)}{25.4\text{ mm}} = \mathbf{15.68\ \mu H}$$

b. Eq. (11.8): $L = \mu_r L_o = (2000)(15.68\ \mu\text{H}) = \mathbf{31.36\ mH}$

EXAMPLE 11.2 In Fig. 11.19, if each inductor in the left column is changed to the type appearing in the right column, find the new inductance level. For each change, assume that the other factors remain the same.

Solutions:

a. The only change was the number of turns, but it is a squared factor, resulting in

$$L = (2)^2 L_o = (4)(20\ \mu\text{H}) = \mathbf{80\ \mu H}$$

b. In this case, the area is three times the original size, and the number of turns is 1/2. Since the area is in the numerator, it increases

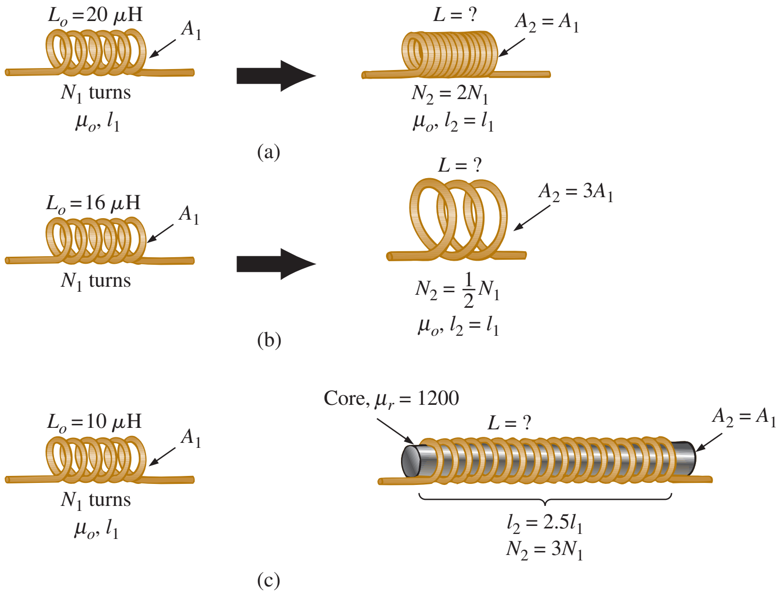


FIG. 11.19
Inductors for Example 11.2.

the inductance by a factor of three. The drop in the number of turns reduces the inductance by a factor of $(1/2)^2 = 1/4$. Therefore,

$$L = (3)\left(\frac{1}{4}\right)L_o = \frac{3}{4}(16\ \mu\text{H}) = \mathbf{12\ \mu H}$$

c. Both μ and the number of turns have increased, although the increase in the number of turns is squared. The increased length reduces the inductance. Therefore,

$$L = \frac{(3)^2(1200)}{2.5}L_o = (4.32 \times 10^3)(10\ \mu\text{H}) = \mathbf{43.2\ mH}$$

Types of Inductors

Inductors, like capacitors and resistors, can be categorized under the general headings **fixed** or **variable.** The symbol for a fixed air-core inductor is provided in Fig. 11.20(a), for an inductor with a ferromagnetic core in Fig. 11.20(b), for a tapped coil in Fig. 11.20(c), and for a variable inductor in Fig. 11.20(d).

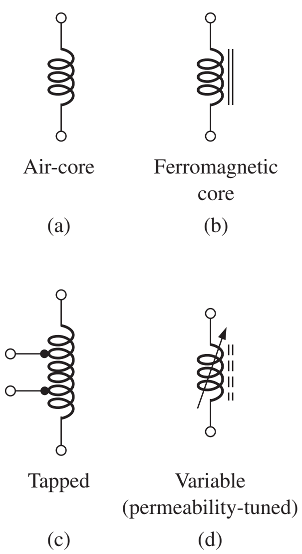


FIG. 11.20
Inductor (coil) symbols.

Fixed Fixed-type inductors come in all shapes and sizes. However,

in general, the size of an inductor is determined primarily by the type of construction, the core used, or the current rating.

In Fig. 11.21(a), the 10 μH and 1 mH coils are about the same size because a thinner wire was used for the 1 mH coil to permit more turns in the same space. The result, however, is a drop in rated current from 10 A to only 1.3 A. If the wire of the 10 μH coil had been used to make the 1 mH coil, the resulting coil would have been many times the size of the

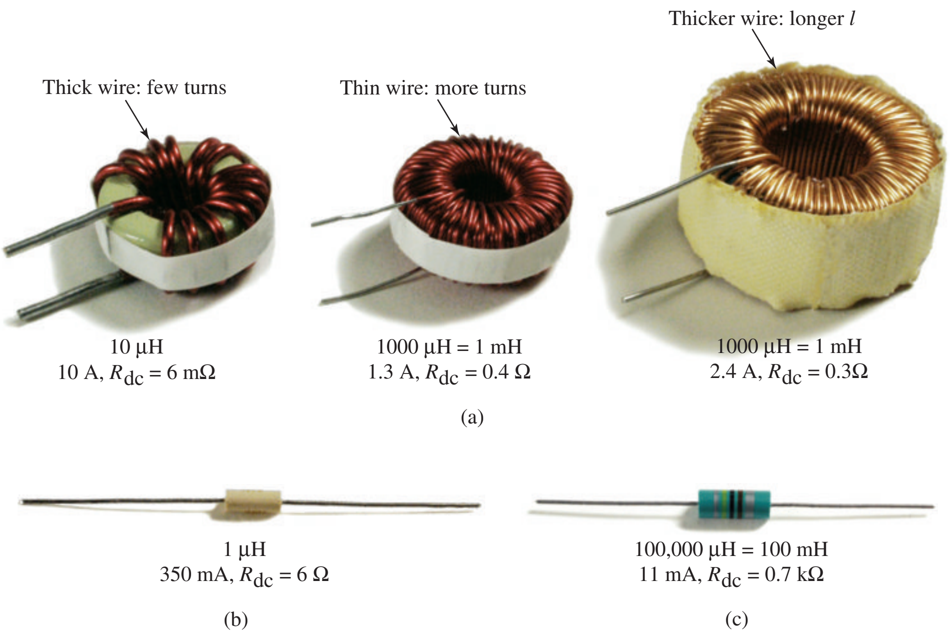


FIG. 11.21
Relative sizes of different types of inductors: (a) toroid, high-current; (b) phenolic (resin or plastic core); (c) ferrite core.

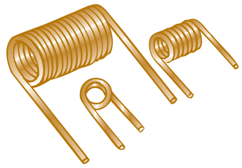
10 μH coil. The impact of the wire thickness is clearly revealed by the 1 mH coil at the far right in Fig. 11.21(a), where a thicker wire was used to raise the rated current level from 1.3 A to 2.4 A. Even though the inductance level is the same, the size of the toroid is 4 or 5 times greater.

The phenolic inductor (using a nonferromagnetic core of resin or plastic) in Fig. 11.21(b) is quite small for its level of inductance. We must assume that it has a high number of turns of very thin wire. Note, however, that the use of a very thin wire has resulted in a relatively low current rating of only 350 mA (0.35 A). The use of a ferrite (ferromagnetic) core in the inductor in Fig. 11.21(c) has resulted in an amazingly high level of inductance for its size. However, the wire is so thin that the current rating is only 11 mA $= 0.011$ A. Note that for all the inductors, the dc resistance of the inductor increases with a decrease in the thickness of the wire. The 10 μH toroid has a dc resistance of only 6 mΩ, whereas the dc resistance of the 100 mH ferrite inductor is 700 Ω—a price to be paid for the smaller size and high inductance level.

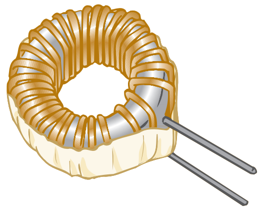
Different types of fixed inductive elements are displayed in Fig. 11.22 on the next page, including their typical range of values and common areas of application. Based on the earlier discussion of inductor construction, it is fairly easy to identify an inductive element. The shape of a molded film resistor is similar to that of an inductor. However, careful examination of the typical shapes of each reveals some differences, such as the ridges at each end of a resistor that do not appear on most inductors.

Variable A number of variable inductors are depicted in Fig. 11.23 on the next page. In each case, the inductance is changed by turning the slot at the end of the core to move it in and out of the unit. The farther in the core is, the more the ferromagnetic material is part of the magnetic circuit, and the higher the magnetic field strength and the inductance level.

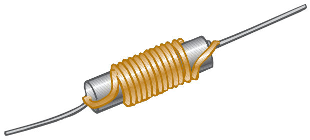
Type: Air-core inductors (1–32 turns)
Typical values: 2.5 nH–1 μH
Applications: High-frequency applications



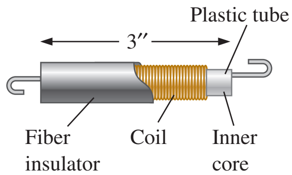
Type: Toroid coil
Typical values: 10 μH–30 mH
Applications: Used as a choke in ac power line circuits to filter transient and reduce EMI interference. This coil is found in many electronic appliances.



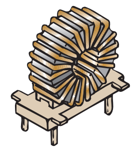
Type: Hash choke coil
Typical values: 3 μH–1 mH
Applications: Used in ac supply lines that deliver high currents.



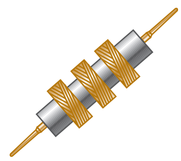
Type: Delay line coil
Typical values: 10 μH–50 μH
Applications: Used in color televisions to correct for timing differences between the color signal and the black-and-white signal.



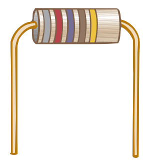
Type: Common-mode choke coil
Typical values: 0.6 mH–50 mH
Applications: Used in ac line filters, switching power supplies, battery chargers, and other electronic equipment.



Type: RF chokes
Typical values: 10 μH–470 mH
Applications: Used in radio, television, and communication circuits. Found in AM, FM, and UHF circuits.



Type: Molded coils
Typical values: 0.1 μH–100 mH
Applications: Used in a wide variety of circuits such as oscillators, filters, pass-band filters, and others.



Type: Surface-mount inductors
Typical values: 0.01 μH–250 μH
Applications: Found in many electronic circuits that require miniature components on multilayered PCBs (printed circuit boards).

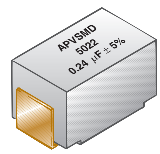


FIG. 11.22
Typical areas of application for inductive elements.

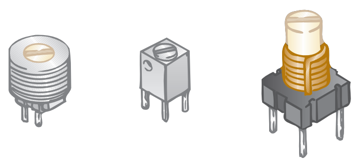


FIG. 11.23
Variable inductors with a typical range of values from 1 μH to 100 μH; commonly used in oscillators and various RF circuits such as CB transceivers, televisions, and radios.

Practical Equivalent Inductors

Inductors, like capacitors, are not ideal. Associated with every inductor is a resistance determined by the resistance of the turns of wire (the thinner the wire, the greater the resistance for the same material) and by the core losses (radiation and skin effect, eddy current and hysteresis losses—all discussed in more advanced texts). There is also some stray capacitance due to the capacitance between the current-carrying turns of wire of the coil. Recall that capacitance appears whenever there are two conducting surfaces separated by an insulator, such as air, and when those wrappings are fairly tight and are parallel. Both elements are included in the equivalent circuit in Fig. 11.24. For most applications in this text, the capacitance can be ignored, resulting in the equivalent model in Fig. 11.25. The resistance R_l plays an im-

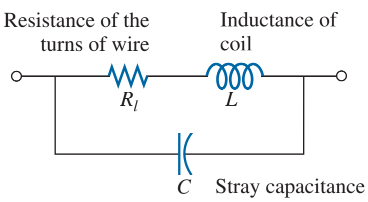


FIG. 11.24
Complete equivalent model for an inductor.

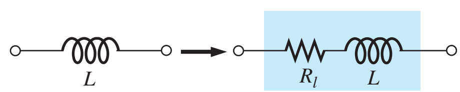


FIG. 11.25
Practical equivalent model for an inductor.

portant part in some areas (such as resonance, discussed in Chapter 20), because the resistance can extend from a few ohms to a few hundred ohms depending on the construction. For this chapter, the inductor is considered an ideal element and the series resistance is dropped from Fig. 11.25.

Inductor Labeling

Because some inductors are larger in size, their nameplate value can often be printed on the body of the element. However, on smaller units, there may not be enough room to print the actual value, so an abbreviation is used that is fairly easy to understand. First, realize that the *microhenry* (μH) is the fundamental unit of measurement for this marking. Most manuals list the inductance value in μH even if the value must be reported as 470,000 μH rather than as 470 mH. If the label reads 223K, the third number (3) is the power to be applied to the first two. The K is not from *kilo,* representing a power of three, but is used to denote a tolerance of ±10% as described for capacitors. The resulting number of 22,000 is, therefore, in μH so the 223K unit is a 22,000 μH or 22 mH inductor. The letters J and M indicate a tolerance of ±5% and ±20%, respectively.

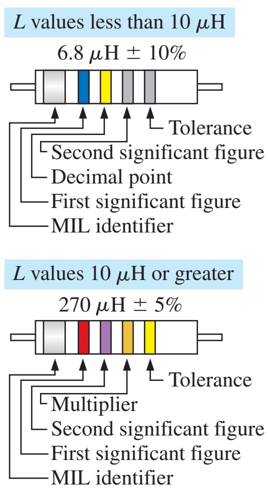
For molded inductors, a color-coding system very similar to that used for resistors is used. The major difference is that *the resulting value is always in μH,* and a wide band at the beginning of the labeling is an MIL ("meets military standards") indicator. Always read the colors in sequence, starting with the band closest to one end as shown in Fig. 11.26.

The standard values for inductors employ the same numerical values and multipliers used with resistors and capacitors. In general, therefore,

Color Code Table

Color[1]	Significant Figure	Multiplier[2]	Inductance Tolerance (%)
Black	0	1	
Brown	1	10	
Red	2	100	
Orange	3	1000	
Yellow	4		
Green	5		
Blue	6		
Violet	7		
Gray	8		
White	9		
None[2]			±20
Silver			±10
Gold	Decimal point		±5

[1] Indicates body color.
[2] The multiplier is the factor by which the two significant figures are multiplied to yield the nominal inductance value.



Cylindrical molded choke coils are marked with five colored bands. A wide silver band, located at one end of the coil, identifies military radio-frequency coils. The next three bands indicate the inductance in microhenries, and the fourth band is the tolerance.

Color coding is in accordance with the color code table, shown on the left. If the first or second band is gold, it represents the decimal point for inductance values less than 10. Then the following two bands are significant figures. For inductance values of 10 or more, the first two bands represent significant figures, and the third is the multiplier.

FIG. 11.26
Molded inductor color coding.

expect to find inductors with the following multipliers: 1 μH, 1.5 μH, 2.2 μH, 3.3 μH, 4.7 μH, 6.8 μH, 10 μH, and so on.

Measurement and Testing of Inductors

The inductance of an inductor can be read directly using a meter such as the Universal LCR Meter (Fig. 11.27), also discussed in Chapter 10 on capacitors. Set the meter to *L* for inductance, and the meter automatically chooses the most appropriate unit of measurement for the element, that is, H, mH, μH, or pH.

An inductance meter is the best choice, but an ohmmeter can also be used to check whether a short has developed between the windings or whether an open circuit has developed. The open-circuit possibility is easy to check because a reading of infinite ohms or very high resistance results. The short-circuit condition is harder to check because the resistance of many good inductors is relatively small, and the shorting of a few windings may not adversely affect the total resistance. Of course, if you are aware of the typical resistance of the coil, you can compare it to the measured value. A short between the windings and the core can be checked by simply placing one lead of the ohmmeter on one wire (perhaps a terminal) and the other on the core itself. A reading of zero ohms reveals a short between the two that may be due to a breakdown in the insulation jacket around the wire resulting from excessive currents, environmental conditions, or simply old age and cracking.

FIG. 11.27
Digital reading inductance meter.
(Image provided by B & K Precision Corporation.)

11.4 INDUCED VOLTAGE v_L

Before analyzing the response of inductive elements to an applied dc voltage, we must introduce a number of laws and equations that affect the transient response.

The first, referred to as **Faraday's law of electromagnetic induction,** is one of the most important in this field because it enables us to establish ac and dc voltages with a generator. If we move a conductor (any material with conductor characteristics as defined in Chapter 2) through a magnetic field so that it cuts magnetic lines of flux as shown in Fig. 11.28, a voltage is induced across the conductor that can be measured with a sensitive voltmeter. That's all it takes, and, in fact, the faster you move the conductor through the magnetic flux, the greater the induced voltage. The same effect can be produced if you hold the conductor still and move the magnetic field across the conductor. Note that the direction in which you move the conductor through the field determines the polarity of the induced voltage. Also, if you move the conductor through the field at right angles to the magnetic flux, you generate the maximum induced voltage. Moving the conductor parallel with the magnetic flux lines results in an induced voltage of zero volts since magnetic lines of flux are not crossed.

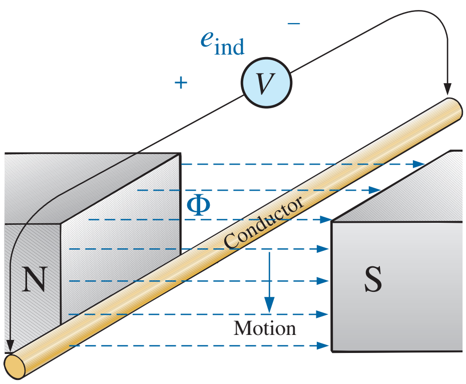


FIG. 11.28
Generating an induced voltage by moving a conductor through a magnetic field.

If we now go a step further and move a coil of *N* turns through the magnetic field as shown in Fig. 11.29, a voltage will be induced across the coil as determined by **Faraday's law:**

$$e = N\frac{d\phi}{dt} \qquad \text{(volts, V)} \qquad \textbf{(11.9)}$$

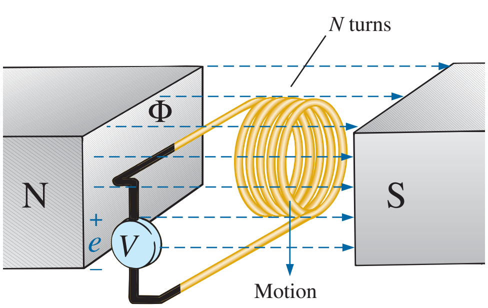


FIG. 11.29
Demonstrating Faraday's law.

The greater the number of turns or the faster the coil is moved through the magnetic flux pattern, the greater the induced voltage. The term $d\phi/dt$

is the differential change in magnetic flux through the coil at a particular instant in time. If the magnetic flux passing through a coil remains constant—no matter how strong the magnetic field—the term will be zero, and the induced voltage zero volts. It doesn't matter whether the changing flux is due to moving the magnetic field or moving the coil in the vicinity of a magnetic field: The only requirement is that the flux linking (passing through) the coil changes with time. Before the coil passes through the magnetic poles, the induced voltage is zero because there are no magnetic flux lines passing through the coil. As the coil enters the flux pattern, the number of flux lines cut per instant of time increases until it peaks at the center of the poles. The induced voltage then decreases with time as it leaves the magnetic field.

This important phenomenon can now be applied to the inductor in Fig. 11.30, which is simply an extended version of the coil in Fig. 11.29. In Section 11.2, we found that the magnetic flux linking the coil of N turns with a current I has the distribution shown in Fig. 11.30. If the current through the coil increases in magnitude, the flux linking the coil also increases. We just learned through Faraday's law, however, that a coil in the vicinity of a changing magnetic flux will have a voltage induced across it. The result is that a voltage is induced across the coil in Fig. 11.30 due to the *change in current through the coil.*

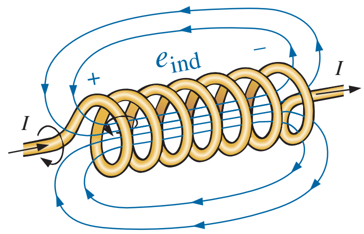


FIG. 11.30
Demonstrating the effect of Lenz's law.

It is very important to note in Fig. 11.30 that the polarity of the induced voltage across the coil is such that it opposes the increasing level of current in the coil. In other words, the changing current through the coil induces a voltage across the coil that is opposing the applied voltage that establishes the increase in current in the first place. The quicker the change in current through the coil, the greater the opposing induced voltage to squelch the attempt of the current to increase in magnitude. The "choking" action of the coil is the reason inductors or coils are often referred to as **chokes.** This effect is a result of an important law referred to as **Lenz's law,** which states that

an induced effect is always such as to oppose the cause that produced it.

The inductance of a coil is also a measure of the change in flux linking the coil due to a change in current through the coil. That is,

$$L = N\frac{d\phi}{di_L} \qquad \text{(henries, H)} \qquad \textbf{(11.10)}$$

The equation reveals that the greater the number of turns or the greater the change in flux linking the coil due to a particular change in current, the greater the level of inductance. In other words, coils with smaller levels of inductance generate smaller changes in flux linking the coil for the same change in current through the coil. If the inductance level is very small, there will be almost no change in flux linking the coil, and the induced voltage across the coil will be very small. In fact, if we now write Eq. (11.9) in the following form:

$$e = N\frac{d\phi}{dt} = \left(N\frac{d\phi}{di_L}\right)\left(\frac{di_L}{dt}\right)$$

and substitute Eq. (11.10), we obtain

$$e_L = L\frac{di_L}{dt} \qquad \text{(volts, V)} \qquad \textbf{(11.11)}$$

which relates the voltage across a coil to the number of turns of the coil and the change in current through the coil.

When induced effects are used in the generation of voltages such as those from dc or ac generators, the symbol e is applied to the induced voltage. However, in network analysis, the voltage induced across an inductor will always have a polarity that opposes the applied voltage (like the voltage across a resistor). Therefore, the following notation is used for the induced voltage across an inductor:

$$\boxed{v_L = L\frac{di_L}{dt}} \qquad \text{(volts, V)} \qquad \textbf{(11.12)}$$

The equation clearly states that

the larger the inductance and/or the more rapid the change in current through a coil, the larger will be the induced voltage across the coil.

If the current through the coil fails to change with time, the induced voltage across the coil will be zero. We will find in the next section that for dc applications, when the transient phase has passed, $di_L/dt = 0$, and the induced voltage across the coil is

$$v_L = L\frac{di_L}{dt} = L(0) = 0\text{ V}$$

The duality that exists between inductive and capacitive elements is now abundantly clear. Simply interchange the voltages and currents of Eq. (11.12), and interchange the inductance and capacitance. The following equation for the current of a capacitor results:

$$v_L = L\frac{di_L}{dt}$$
$$\downarrow \quad \downarrow \quad \downarrow$$
$$i_C = C\frac{dv_C}{dt}$$

We are now at a point where we have all the background relationships necessary to investigate the transient behavior of inductive elements.

11.5 *R-L* TRANSIENTS: THE STORAGE PHASE

A great number of similarities exist between the analyses of inductive and capacitive networks. That is, what is true for the voltage of a capacitor is also true for the current of an inductor, and what is true for the current of a capacitor can be matched in many ways by the voltage of an inductor. The storage waveforms have the same shape, and time constants are defined for each configuration. Because these concepts are so similar (refer to Section 10.5 on the charging of a capacitor), you have an opportunity to reinforce concepts introduced earlier and still learn more about the behavior of inductive elements.

The circuit in Fig. 11.31 is used to describe the storage phase. Note that it is the same circuit used to describe the charging phase of capacitors, with a simple replacement of the capacitor by an ideal inductor. Throughout the analysis, it is important to remember that energy is stored in the form of an electric field between the plates of a capacitor. For inductors, on the other hand, energy is stored in the form of a magnetic field linking the coil.

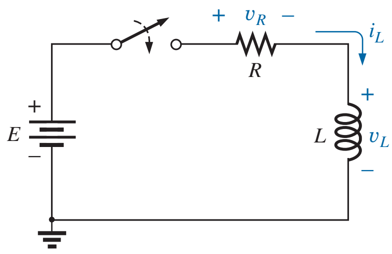


FIG. 11.31
Basic R-L transient network.

At the instant the switch is closed, the choking action of the coil prevents an instantaneous change in current through the coil, resulting in $i_L = 0$ A as shown in Fig. 11.32(a). The absence of a current through the coil and circuit at the instant the switch is closed results in zero volts across the resistor as determined by $\upsilon_R = i_R R = i_L R = (0 \text{ A})R = 0$ V, as shown in Fig. 11.32(c). Applying Kirchhoff's voltage law around the closed loop results in E volts across the coil at the instant the switch is closed, as shown in Fig. 11.32(b).

Initially, the current increases very rapidly as shown in Fig. 11.32(a) and then at a much slower rate as it approaches its steady-state value determined by the parameters of the network (E/R). The voltage across the resistor rises at the same rate because $\upsilon_R = i_R R = i_L R$. Since the voltage across the coil is sensitive to the rate of change of current through the coil, the voltage will be at or near its maximum value early in the storage phase. Finally, when the current reaches its steady-state value of E/R amperes, the current through the coil ceases to change, and the voltage across the coil drops to zero volts. At any instant of time, the voltage

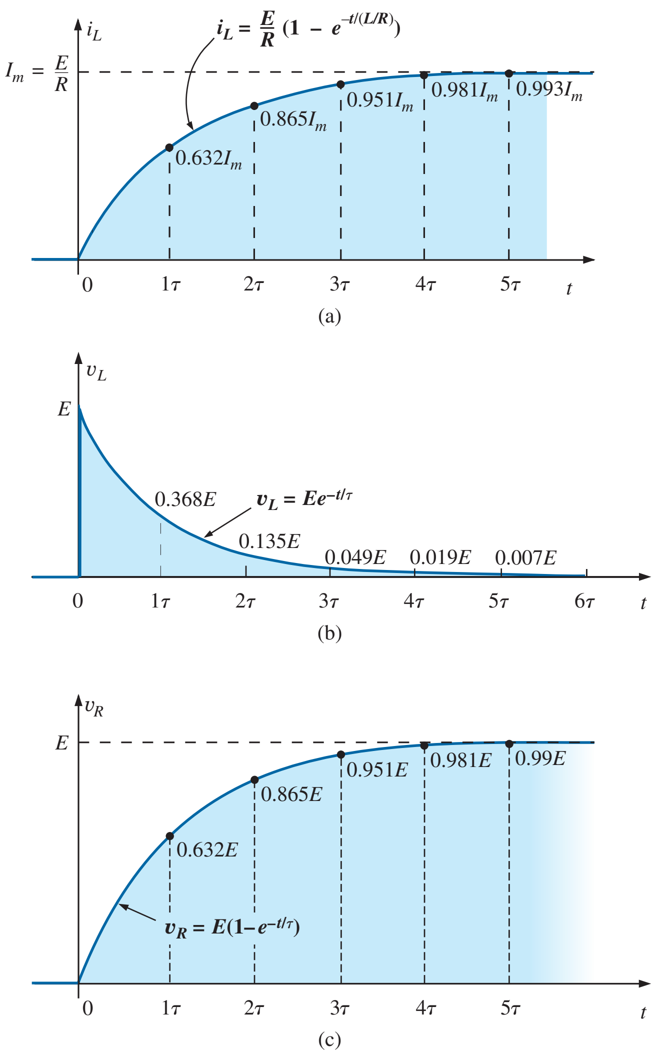


FIG. 11.32

i_L, υ_L, and υ_R for the circuit in Fig. 11.31 following the closing of the switch.

across the coil can be determined using Kirchhoff's voltage law in the following manner: $\upsilon_L = E - \upsilon_R$.

Because the waveforms for the inductor have the same shape as obtained for capacitive networks, we are familiar with the mathematical format and can feel comfortable calculating the quantities of interest using a calculator or computer.

The equation for the transient response of the current through an inductor is the following:

$$i_L = \frac{E}{R}(1 - e^{-t/\tau}) \quad \text{(amperes, A)} \tag{11.13}$$

with the time constant now defined by

$$\tau = \frac{L}{R} \quad \text{(seconds, s)} \tag{11.14}$$

Note that Eq. (11.14) is a ratio of parameters rather than a product as used for capacitive networks, yet the units used are still seconds (for time).

Our experience with the factor $(1 - e^{-t/\tau})$ verifies the level of 63.2% for the inductor current after one time constant, 86.5% after two time constants, and so on. If we keep R constant and increase L, the ratio L/R increases, and the rise time of 5τ increases as shown in Fig. 11.33 for increasing levels of L. The change in transient response is expected because the higher the inductance level, the greater the choking action on the changing current level, and the longer it will take to reach steady-state conditions.

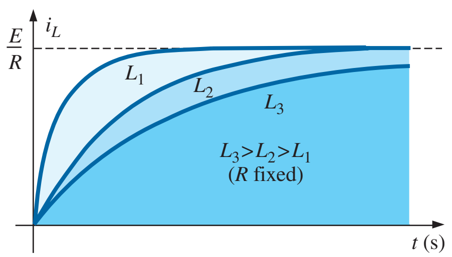


FIG. 11.33
Effect of L on the shape of the i_L storage waveform.

The equation for the voltage across the coil is the following:

$$\upsilon_L = Ee^{-t/\tau} \quad \text{(volts, V)} \tag{11.15}$$

and for the voltage across the resistor:

$$\upsilon_R = E(1 - e^{-t/\tau}) \quad \text{(volts, V)} \tag{11.16}$$

As mentioned earlier, the shape of the response curve for the voltage across the resistor must match that of the current i_L since $\upsilon_R = i_R R = i_L R$.

Since the waveforms are similar to those obtained for capacitive networks, we will assume that

the storage phase has passed and steady-state conditions have been established once a period of time equal to five time constants has occurred.

In addition, since $\tau = L/R$ will always have some numerical value, even though it may be very small at times, the transient period of 5τ will always have some numerical value. Therefore,

the current cannot change instantaneously in an inductive network.

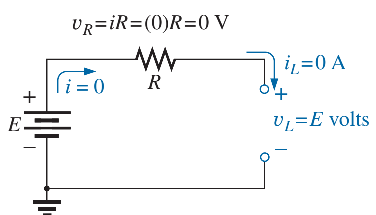


FIG. 11.34
Circuit in Figure 11.31 the instant the switch is closed.

If we examine the conditions that exist at the instant the switch is closed, we find that the voltage across the coil is E volts, although the current is zero amperes as shown in Fig. 11.34. In essence, therefore,

the inductor takes on the characteristics of an open circuit at the instant the switch is closed.

However, if we consider the conditions that exist when steady-state conditions have been established, we find that the voltage across the coil is zero volts and the current is a maximum value of E/R amperes as shown in Fig. 11.35. In essence, therefore,

the inductor takes on the characteristics of a short circuit when steady-state conditions have been established.

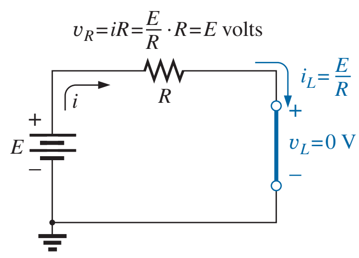


FIG. 11.35
Circuit in Fig. 11.31 under steady-state conditions.

EXAMPLE 11.3 Find the mathematical expressions for the transient behavior of i_L and v_L for the circuit in Fig. 11.36 if the switch is closed at $t = 0$ s. Sketch the resulting curves.

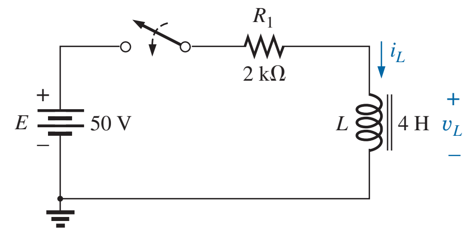


FIG. 11.36
Series R-L circuit for Example 11.3.

Solution: First, the time constant is determined:

$$\tau = \frac{L}{R_1} = \frac{4\text{ H}}{2\text{ k}\Omega} = 2\text{ ms}$$

Then the maximum or steady-state current is

$$I_m = \frac{E}{R_1} = \frac{50\text{ V}}{2\text{ k}\Omega} = 25 \times 10^{-3}\text{A} = 25\text{ mA}$$

Substituting into Eq. (11.13):

$$i_L = \mathbf{25\ mA\ (1 - e^{-t/2ms})}$$

Using Eq. (11.15):

$$v_L = \mathbf{50\ Ve^{-t/2ms}}$$

The resulting waveforms appear in Fig. 11.37.

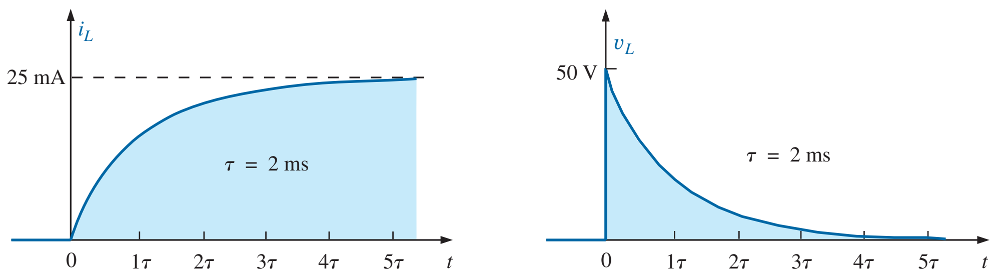


FIG. 11.37
i_L and v_L for the network in Fig. 11.36.

11.6 INITIAL CONDITIONS

This section parallels Section 10.7 ("Initial Conditions") on the effect of initial values on the transient phase. Since the current through a coil cannot change instantaneously, the current through a coil begins the transient phase at the initial value established by the network (note Fig. 11.38) before the switch was closed. It then passes through the transient phase until it reaches the steady-state (or final) level after about five time constants. The steady-state level of the inductor current can be found by substituting its short-circuit equivalent (or R_l for the practical equivalent) and finding the resulting current through the element.

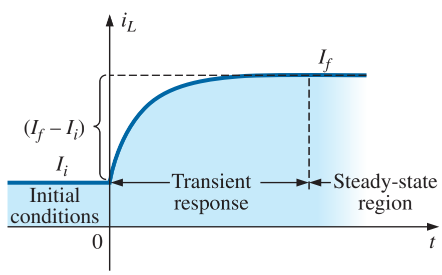


FIG. 11.38
Defining the three phases of a transient waveform.

Using the transient equation developed in the previous section, an equation for the current i_L can be written for the entire time interval in Fig. 11.38; that is,

$$i_L = I_i + (I_f - I_i)(1 - e^{-t/\tau})$$

with $(I_f - I_i)$ representing the total change during the transient phase. However, by multiplying through and rearranging terms:

$$\begin{aligned} i_L &= I_i + I_f - I_f e^{-t/\tau} - I_i + I_i e^{-t/\tau} \\ &= I_f - I_f e^{-t/\tau} + I_i e^{-t/\tau} \end{aligned}$$

we find

$$\boxed{i_L = I_f + (I_i - I_f)e^{-t/\tau}} \qquad \textbf{(11.17)}$$

If you are required to draw the waveform for the current i_L from initial value to final value, start by drawing a line at the initial value and steady-state levels, and then add the transient response (sensitive to the time constant) between the two levels. The following example will clarify the procedure.

EXAMPLE 11.4 The inductor in Fig. 11.39 has an initial current level of 4 mA in the direction shown. (Specific methods to establish the initial current are presented in the sections and problems to follow.)

a. Find the mathematical expression for the current through the coil once the switch is closed.
b. Find the mathematical expression for the voltage across the coil during the same transient period.
c. Sketch the waveform for each from initial value to final value.

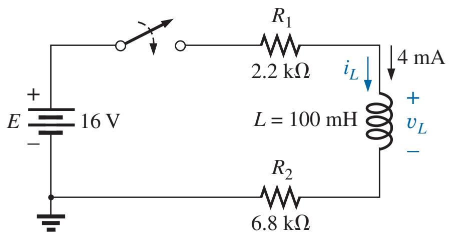


FIG. 11.39
Example 11.4.

Solutions:

a. Substituting the short-circuit equivalent for the inductor results in a final or steady-state current determined by Ohm's law:

$$I_f = \frac{E}{R_1 + R_2} = \frac{16\text{ V}}{2.2\text{ k}\Omega + 6.8\text{ k}\Omega} = \frac{16\text{ V}}{9\text{ k}\Omega} = 1.78\text{ mA}$$

The time constant is determined by

$$\tau = \frac{L}{R_T} = \frac{100\text{ mH}}{2.2\text{ k}\Omega + 6.8\text{ k}\Omega} = \frac{100\text{ mH}}{9\text{ k}\Omega} = 11.11\ \mu\text{s}$$

Applying Eq. (11.17):

$$\begin{aligned} i_L &= I_f + (I_i - I_f)\, e^{-t/\tau} = 1.78\text{ mA} + (4\text{ mA} - 1.78\text{ mA})e^{-t/11.11\mu\text{s}} \\ &= \mathbf{1.78\ mA + 2.22\ mA}\boldsymbol{e^{-t/11.11\mu s}} \end{aligned}$$

b. Since the current through the inductor is constant at 4 mA prior to the closing of the switch, the voltage (whose level is sensitive only to changes in current through the coil) must have an initial value of 0 V. At the instant the switch is closed, the current through the coil cannot change instantaneously, so the current through the resistive elements is 4 mA. The resulting peak voltage at $t = 0$ s can then be found using Kirchhoff's voltage law as follows:

$$\begin{aligned} V_m &= E - V_{R_1} - V_{R_2} = 16\text{ V} - (4\text{ mA})(2.2\text{ k}\Omega) - (4\text{ mA})(6.8\text{ k}\Omega) \\ &= 16\text{ V} - 8.8\text{ V} - 27.2\text{ V} = 16\text{ V} - 36\text{ V} = -20\text{ V} \end{aligned}$$

Note the minus sign to indicate that the polarity of the voltage v_L is opposite to the defined polarity of Fig. 11.39.

The voltage then decays (with the same time constant as the current i_L) to zero because the inductor is approaching its short-circuit equivalence.

The equation for v_L is therefore

$$v_L = -20\text{V}e^{-t/11.11\mu s}$$

c. See Fig. 11.40. The initial and final values of the current were drawn first, and then the transient response was included between these levels. For the voltage, the waveform begins and ends at zero, with the peak value having a sign sensitive to the defined polarity of v_L in Fig. 11.39.

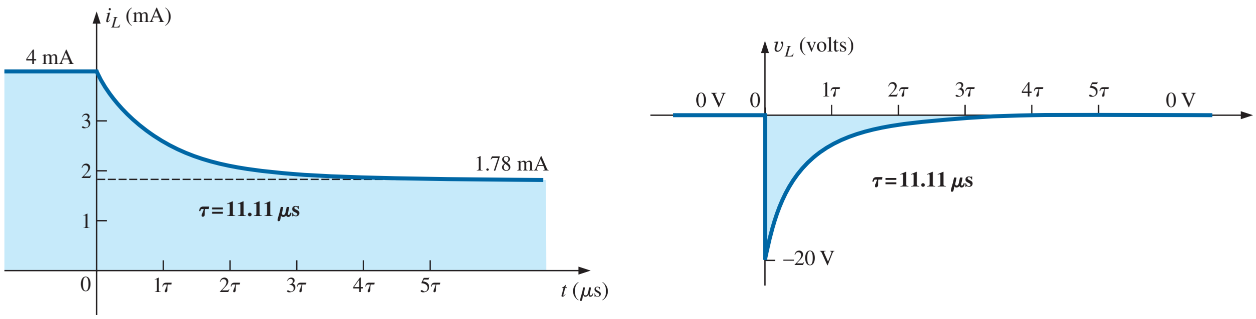


FIG. 11.40
i_L and v_L for the network in Fig. 11.39.

Let us now test the validity of the equation for i_L by substituting $t = 0$ s to reflect the instant the switch is closed.

$$e^{-t/\tau} = e^{-0} = 1$$

and $\quad i_L = 1.78 \text{ mA} + 2.22 \text{ mA}e^{-t/\tau} = 1.78 \text{ mA} + 2.22 \text{ mA} = 4 \text{ mA}$

When $t > 5\tau$, $\qquad e^{-t/\tau} \cong 0$

and $\qquad i_L = 1.78 \text{ mA} + 2.22 \text{ mA}e^{-t/\tau} = 1.78 \text{ mA}$

11.7 *R-L* TRANSIENTS: THE RELEASE PHASE

In the analysis of *R-C* circuits, we found that the capacitor could hold its charge and store energy in the form of an electric field for a period of time determined by the leakage factors. In *R-L* circuits, the energy is stored in the form of a magnetic field established by the current through the coil. Unlike the capacitor, however, an isolated inductor cannot continue to store energy, because the absence of a closed path causes the current to drop to zero, releasing the energy stored in the form of a magnetic field. If the series *R-L* circuit in Fig. 11.41 reaches steady-state conditions and the switch is quickly opened, a spark will occur across the contacts due to the rapid change in current from a maximum of *E/R* to zero amperes. The change in current *di/dt* of the equation $v_L = L(di/dt)$ establishes a high voltage v_L across the coil that, in conjunction with the applied voltage *E*, appears across the points of the switch. This is the same mechanism used in the ignition system of a car to ignite the fuel in the cylinder. Some 25,000 V are

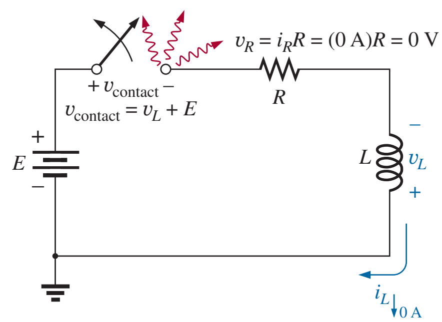


FIG. 11.41
Demonstrating the effect of opening a switch in series with an inductor with a steady-state current.

generated by the rapid decrease in ignition coil current that occurs when the switch in the system is opened. (In older systems, the "points" in the distributor served as the switch.) This inductive reaction is significant when you consider that the only independent source in a car is a 12 V battery.

If opening the switch to move it to another position causes such a rapid discharge in stored energy, how can the decay phase of an *R-L* circuit be analyzed in much the same manner as for the *R-C* circuit? The solution is to use a network like that in Fig. 11.42(a). When the switch is closed, the voltage across resistor R_2 is E volts, and the *R-L* branch responds in the same manner as described above, with the same waveforms and levels. A Thévenin network of E in parallel with R_2 results in the source as shown in Fig. 11.42(b), since R_2 will be shorted out by the short-circuit replacement of the voltage source E when the Thévenin resistance is determined.

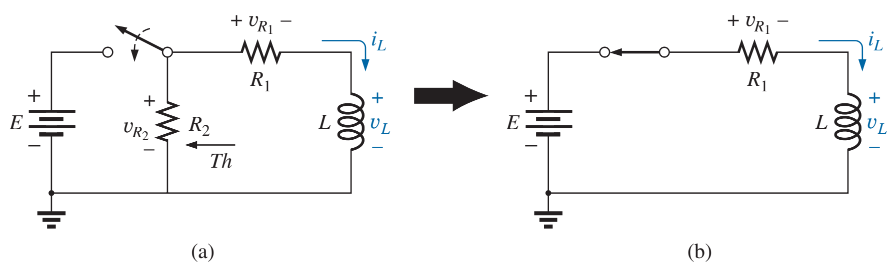


FIG. 11.42
Initiating the storage phase for an inductor by closing the switch.

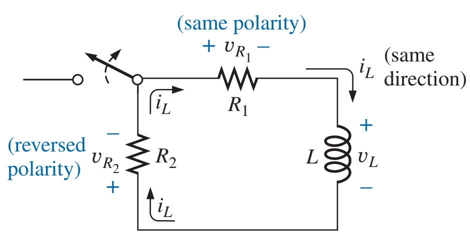


FIG. 11.43
Network in Fig. 11.42 the instant the switch is opened.

After the storage phase has passed and steady-state conditions are established, the switch can be opened without the sparking effect or rapid discharge due to resistor R_2, which provides a complete path for the current i_L. In fact, for clarity the discharge path is isolated in Fig. 11.43. The voltage v_L across the inductor reverses polarity and has a magnitude determined by

$$v_L = -(v_{R_1} + v_{R_2}) \qquad \textbf{(11.18)}$$

Recall that the voltage across an inductor can change instantaneously but the current cannot. The result is that the current i_L must maintain the same direction and magnitude as shown in Fig. 11.43. Therefore, the instant after the switch is opened, i_L is still $I_m = E/R_1$, and

$$\begin{aligned} v_L &= -(v_{R_1} + v_{R_2}) = -(i_1R_1 + i_2R_2) \\ &= -i_L(R_1 + R_2) = -\frac{E}{R_1}(R_1 + R_2) = -\left(\frac{R_1}{R_1} + \frac{R_2}{R_1}\right)E \end{aligned}$$

and

$$v_L = -\left(1 + \frac{R_2}{R_1}\right)E \qquad \text{switch opened} \qquad \textbf{(11.19)}$$

which is bigger than E volts by the ratio R_2/R_1. In other words, when the switch is opened, the voltage across the inductor reverses polarity and drops instantaneously from E to $-[1 + (R_2/R_1)]E$ volts.

As an inductor releases its stored energy, the voltage across the coil decays to zero in the following manner:

$$\boxed{\upsilon_L = -V_i e^{-t/\tau'}} \tag{11.20}$$

with $$V_i = \left(1 + \frac{R_2}{R_1}\right)E$$

and $$\tau' = \frac{L}{R_T} = \frac{L}{R_1 + R_2}$$

The current decays from a maximum of $I_m = E/R_1$ to zero.

Using Eq. (11.17):

$$I_i = \frac{E}{R_1} \quad \text{and} \quad I_f = 0\text{ A}$$

so that $$i_L = I_f + (I_i - I_f)e^{-t/\tau'} = 0\text{ A} + \left(\frac{E}{R_1} - 0\text{ A}\right)e^{-t/\tau'}$$

and $$\boxed{i_L = \frac{E}{R_1}e^{-t/\tau'}} \tag{11.21}$$

with $$\tau' = \frac{L}{R_1 + R_2}$$

The mathematical expression for the voltage across either resistor can then be determined using Ohm's law:

$$\upsilon_{R_1} = i_{R_1}R_1 = i_L R_1 = \frac{E}{R_1}R_1 e^{-t/\tau'}$$

and $$\boxed{\upsilon_{R_1} = Ee^{-t/\tau'}} \tag{11.22}$$

The voltage υ_{R_1} has the same polarity as during the storage phase since the current i_L has the same direction. The voltage υ_{R_2} is expressed as follows using the defined polarity of Fig. 11.42:

$$\upsilon_{R_2} = -i_{R_2}R_2 = -i_L R_2 = -\frac{E}{R_1}R_2 e^{-t/\tau'}$$

and $$\boxed{\upsilon_{R_2} = -\frac{R_2}{R_1}Ee^{-t/\tau'}} \tag{11.23}$$

EXAMPLE 11.5 Resistor R_2 was added to the network in Fig. 11.36 as shown in Fig. 11.44.

a. Find the mathematical expressions for i_L, υ_L, υ_{R_1}, and υ_{R_2} for five time constants of the storage phase.
b. Find the mathematical expressions for i_L, υ_L, υ_{R_1}, and υ_{R_2} if the switch is opened after five time constants of the storage phase.
c. Sketch the waveforms for each voltage and current for both phases covered by this example. Use the defined polarities in Fig. 11.43.

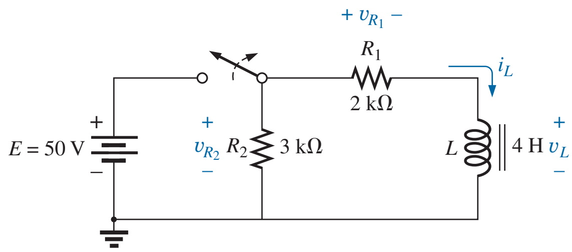


FIG. 11.44

Defined polarities for v_{R_1}, v_{R_2}, v_L, and current direction for i_L for Example 11.5.

Solutions:

a. From Example 11.3:

$$i_L = \mathbf{25\ mA(1 - e^{-t/2ms})}$$
$$v_L = \mathbf{50\ Ve^{-t/2ms}}$$
$$v_{R_1} = i_{R_1}R_1 = i_L R_1$$
$$= \left[\frac{E}{R_1}(1 - e^{-t/\tau})\right]R_1$$
$$= E(1 - e^{-t/\tau})$$

and
$$v_{R_1} = \mathbf{50\ V}(1 \mathbf{-} \mathbf{e^{-t/2ms}})$$
$$v_{R_2} = E = \mathbf{50\ V}$$

b. $$\tau' = \frac{L}{R_1 + R_2} = \frac{4\text{ H}}{2\text{ k}\Omega + 3\text{ k}\Omega} = \frac{4\text{ H}}{5 \times 10^3\ \Omega}$$
$$= 0.8 \times 10^{-3}\text{ s} = 0.8\text{ ms}$$

By Eqs. (11.19) and (11.20):

$$V_i = \left(1 + \frac{R_2}{R_1}\right)E = \left(1 + \frac{3\text{ k}\Omega}{2\text{ k}\Omega}\right)(50\text{ V}) = 125\text{ V}$$

and
$$v_L = -V_i e^{-t/\tau'} = \mathbf{-125\ Ve^{-t/-0.8ms}}$$

By Eq. (11.21):

$$I_m = \frac{E}{R_1} = \frac{50\text{ V}}{2\text{ k}\Omega} = 25\text{ mA}$$

and
$$i_L = I_m e^{-t/\tau'} = \mathbf{25\ mAe^{-t/0.8ms}}$$

By Eq. (11.22):

$$v_{R_1} = Ee^{-t/\tau'} = \mathbf{50\ Ve^{-t/0.8ms}}$$

By Eq. (11.23):

$$v_{R_2} = -\frac{R_2}{R_1}Ee^{-t/\tau'} = -\frac{3\text{ k}\Omega}{2\text{ k}\Omega}(50\text{ V})e^{-t/\tau'} = \mathbf{-75\ Ve^{-t/0.8\ ms}}$$

c. See Fig. 11.45:

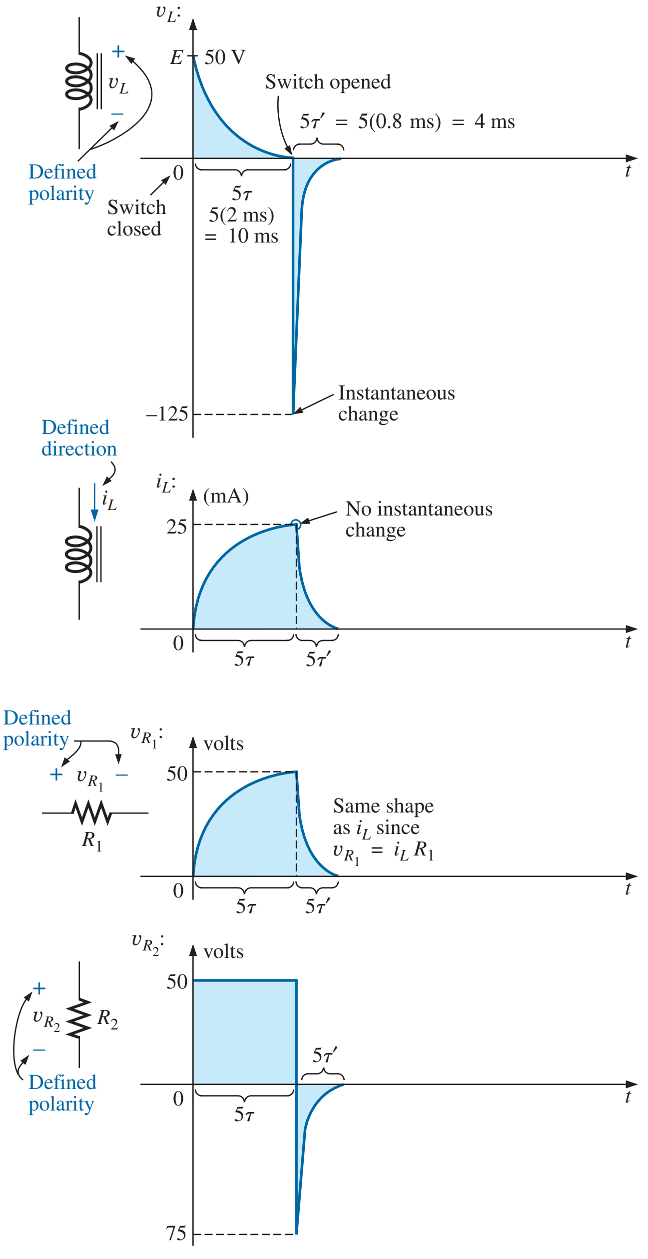


FIG. 11.45

The various voltages and the current for the network in Fig. 11.44.

In the preceding analysis, it was assumed that steady-state conditions were established during the charging phase and $I_m = E/R_1$, with $v_L = 0$ V. However, if the switch in Fig. 11.42 is opened before i_L reaches its maximum value, the equation for the decaying current of Fig. 11.42 must change to

$$i_L = I_i e^{-t/\tau'} \qquad \textbf{(11.24)}$$

where I_i is the starting or initial current. The voltage across the coil is defined by the following:

$$\upsilon_L = -V_i e^{-t/\tau'} \tag{11.25}$$

with $$V_i = I_i(R_1 + R_2)$$

11.8 THÉVENIN EQUIVALENT: $\tau = L/R_{Th}$

In Chapter 10 on capacitors, we found that a circuit does not always have the basic form in Fig. 11.31. The solution is to find the Thévenin equivalent circuit before proceeding in the manner described in this chapter. Consider the following example.

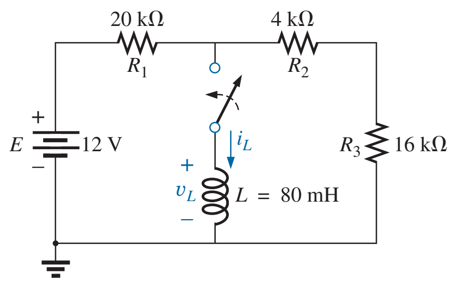


FIG. 11.46
Example 11.6.

EXAMPLE 11.6 For the network in Fig. 11.46.

a. Find the mathematical expression for the transient behavior of the current i_L and the voltage υ_L after the closing of the switch ($I_i = 0$ mA).
b. Draw the resultant waveform for each.

Solutions:

a. Applying Thévenin's theorem to the 80 mH inductor (Fig. 11.47) yields

$$R_{Th} = \frac{R}{N} = \frac{20\text{ k}\Omega}{2} = 10\text{ k}\Omega$$

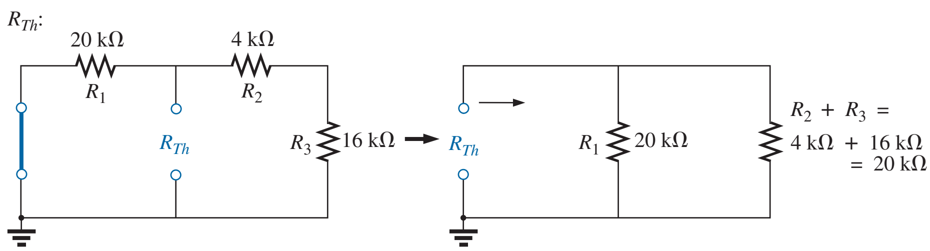


FIG. 11.47
Determining R_{Th} for the network in Fig. 11.46.

Applying the voltage divider rule (Fig. 11.48), we obtain

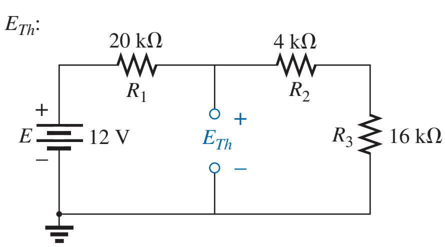


FIG. 11.48
Determining E_{Th} for the network in Fig. 11.46.

$$E_{Th} = \frac{(R_2 + R_3)E}{R_1 + R_2 + R_3}$$
$$= \frac{(4\text{ k}\Omega + 16\text{ k}\Omega)(12\text{ V})}{20\text{ k}\Omega + 4\text{ k}\Omega + 16\text{ k}\Omega} = \frac{(20\text{ k}\Omega)(12\text{ V})}{40\text{ k}\Omega} = 6\text{ V}$$

The Thévenin equivalent circuit is shown in Fig. 11.49. Using Eq. (11.13):

$$i_L = \frac{E_{Th}}{R}(1 - e^{-t/\tau})$$
$$\tau = \frac{L}{R_{Th}} = \frac{80 \times 10^{-3}\text{ H}}{10 \times 10^3\ \Omega} = 8 \times 10^{-6}\text{ s} = 8\ \mu\text{s}$$
$$I_m = \frac{E_{Th}}{R_{Th}} = \frac{6\text{ V}}{10 \times 10^3\ \Omega} = 0.6 \times 10^{-3}\text{ A} = 0.6\text{ mA}$$

and $\mathbf{i_L = 0.6\text{ mA}\ (1 - e^{-t/8\mu s})}$

Using Eq. (11.15):

$$\upsilon_L = E_{Th}e^{-t/\tau}$$

so that $\mathbf{\upsilon_L = 6\text{ V}e^{-t/8\mu s}}$

b. See Fig. 11.50.

Thévenin equivalent circuit:

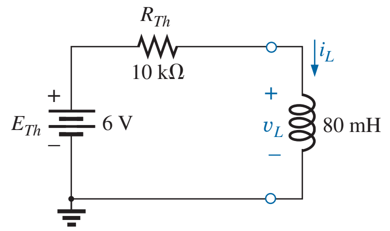


FIG. 11.49
The resulting Thévenin equivalent circuit for the network in Fig. 11.46.

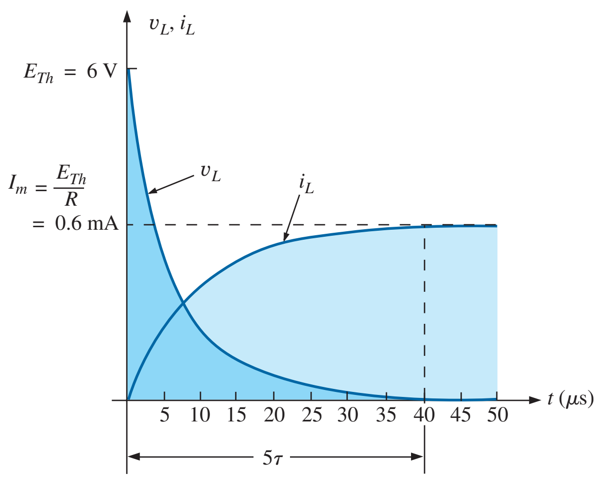


FIG. 11.50
The resulting waveforms for i_L and υ_L for the network in Fig. 11.46.

EXAMPLE 11.7 Switch S_1 in Fig. 11.51 has been closed for a long time. At $t = 0$ s, S_1 is opened at the same instant that S_2 is closed to avoid an interruption in current through the coil.

a. Find the initial current through the coil. Pay particular attention to its direction.
b. Find the mathematical expression for the current i_L following the closing of switch S_2.
c. Sketch the waveform for i_L.

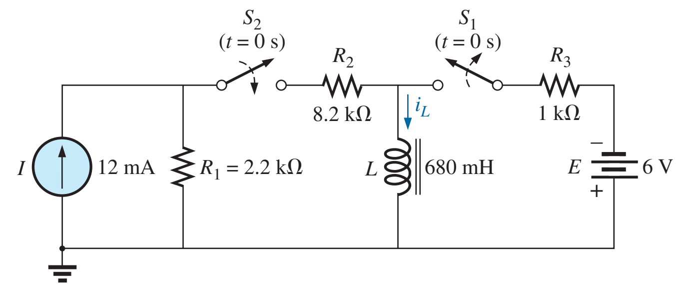


FIG. 11.51
Example 11.7.

Solutions:

a. Using Ohm's law, the initial current through the coil is determined by

$$I_i = -\frac{E}{R_3} = -\frac{6\text{ V}}{1\text{ k}\Omega} = -6\text{ mA}$$

b. Applying Thévenin's theorem:

$$R_{Th} = R_1 + R_2 = 2.2\text{ k}\Omega + 8.2\text{ k}\Omega = 10.4\text{ k}\Omega$$
$$E_{Th} = IR_1 = (12\text{ mA})\,(2.2\text{ k}\Omega) = 26.4\text{ V}$$

The Thévenin equivalent network appears in Fig. 11.52.

The steady-state current can then be determined by substituting the short-circuit equivalent for the inductor:

$$I_f = \frac{E}{R_{Th}} = \frac{26.4\text{ V}}{10.4\text{ k}\Omega} = 2.54\text{ mA}$$

The time constant is

$$\tau = \frac{L}{R_{Th}} = \frac{680\text{ mH}}{10.4\text{ k}\Omega} = 65.39\ \mu\text{s}$$

Applying Eq. (11.17):

$$\begin{aligned} i_L &= I_f + (I_i - I_f)e^{-t/\tau} \\ &= 2.54\text{ mA} + (-6\text{ mA} - 2.54\text{ mA})e^{-t/65.39\mu\text{s}} \\ &= \mathbf{2.54\ mA - 8.54\ mA}\boldsymbol{e^{-t/65.39\mu s}} \end{aligned}$$

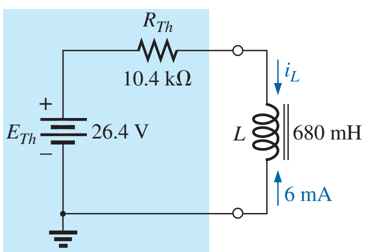


FIG. 11.52
Thévenin equivalent circuit for the network in Fig. 11.51 for $t \geq 0$ s.

c. Note Fig. 11.53.

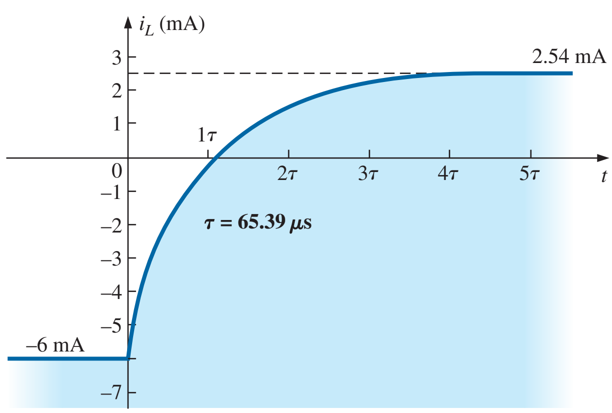


FIG. 11.53
The current i_L for the network in Fig. 11.51.

11.9 INSTANTANEOUS VALUES

The development presented in Section 10.8 for capacitive networks can also be applied to *R-L* networks to determine instantaneous voltages, currents, and time. The instantaneous values of any voltage or current can be determined by simply inserting *t* into the equation and using a calculator or table to determine the magnitude of the exponential term.

The similarity between the equations

$$\upsilon_C = V_f + (V_i - V_f)e^{-t/\tau}$$

and

$$i_L = I_f + (I_i - I_f)e^{-t/\tau}$$

results in a derivation of the following for *t* that is identical to that used to obtain Eq. (10.22):

$$t = \tau \log_e \frac{(I_i - I_f)}{(i_L - I_f)} \qquad \text{(seconds, s)} \qquad \textbf{(11.26)}$$

For the other form, the equation $\upsilon_C = Ee^{-t/\tau}$ is a close match with $\upsilon_L = Ee^{-t/\tau} = V_i e^{-t/\tau}$, permitting a derivation similar to that employed for Eq. (10.23):

$$t = \tau \log_e \frac{V_i}{\upsilon_L} \qquad \text{(seconds, s)} \qquad \textbf{(11.27)}$$

For the voltage υ_R, $V_i = 0$ V and $V_f = E$ V since $\upsilon_R = E(1 - e^{-t/\tau})$. Solving for *t* yields

$$t = \tau \log_e \left(\frac{E}{E - \upsilon_R}\right)$$

or

$$t = \tau \log_e \left(\frac{V_f}{V_f - \upsilon_R}\right) \qquad \text{(seconds, s)} \qquad \textbf{(11.28)}$$

11.10 AVERAGE INDUCED VOLTAGE: $\upsilon_{L_{av}}$

In an effort to develop some feeling for the impact of the derivative in an equation, the average value was defined for capacitors in Section 10.10, and a number of plots for the current were developed for an applied voltage. For inductors, a similar relationship exists between the induced voltage across a coil and the current through the coil. For inductors, the average induced voltage is defined by

$$\upsilon_{L_{av}} = L\frac{\Delta i_L}{\Delta t} \qquad \text{(volts, V)} \qquad \textbf{(11.29)}$$

where Δ indicates a finite (measurable) change in current or time. Eq. (11.12) for the instantaneous voltage across a coil can be derived from Eq. (11.29) by letting V_L become vanishingly small. That is,

$$\upsilon_{L_{inst}} = \lim_{\Delta t \to 0} L\frac{\Delta i_L}{\Delta t} = L\frac{di_L}{dt}$$

In the following example, the change in current Δi_L is considered for each slope of the current waveform. *If the current increases with time, the average current is the change in current divided by the change in time,* *with a positive sign. If the current decreases with time, a negative sign is applied.* Note in the example that the faster the current changes with time, the greater the induced voltage across the coil. When making the necessary calculations, do not forget to multiply by the inductance of the coil. Larger inductances result in increased levels of induced voltage for the same change in current through the coil.

EXAMPLE 11.8 Find the waveform for the average voltage across the coil if the current through a 4 mH coil is as shown in Fig. 11.54.

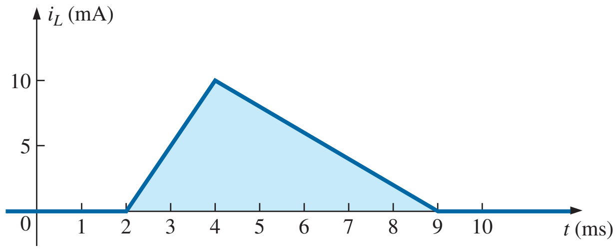


FIG. 11.54
Current i_L to be applied to a 4 mH coil in Example 11.8.

Solutions:

a. *0 to 2 ms:* Since there is no change in current through the coil, there is no voltage induced across the coil. That is,

$$v_L = L\frac{\Delta i}{\Delta t} = L\frac{0}{\Delta t} = \mathbf{0\ V}$$

b. *2 ms to 4 ms:*

$$v_L = L\frac{\Delta i}{\Delta t} = (4 \times 10^{-3}\,\text{H})\left(\frac{10 \times 10^{-3}\,\text{A}}{2 \times 10^{-3}\,\text{s}}\right) = 20 \times 10^{-3}\,\text{V} = \mathbf{20\ mV}$$

c. *4 ms to 9 ms:*

$$v_L = L\frac{\Delta i}{\Delta t} = (-4 \times 10^{-3}\,\text{H})\left(\frac{10 \times 10^{-3}\,\text{A}}{5 \times 10^{-3}\,\text{s}}\right) = -8 \times 10^{-3}\,\text{V} = \mathbf{-8\ mV}$$

d. *9 ms to ∞:*

$$v_L = L\frac{\Delta i}{\Delta t} = L\frac{0}{\Delta t} = \mathbf{0\ V}$$

The waveform for the average voltage across the coil is shown in Fig. 11.55. Note from the curve that

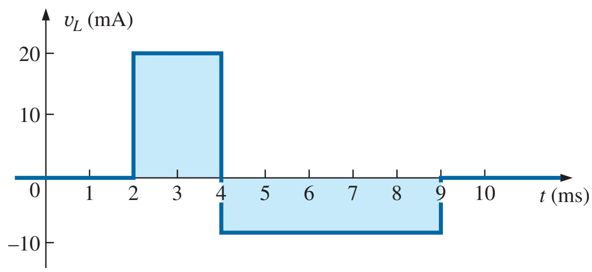


FIG. 11.55
Voltage across a 4 mH coil due to the current in Fig. 11.54.

the voltage across the coil is not determined solely by the magnitude of the change in current through the coil (Δi), but by the rate of change of current through the coil ($\Delta i/\Delta t$).

A similar statement was made for the current of a capacitor due to change in voltage across the capacitor.

A careful examination of Fig. 11.55 also reveals that the area under the positive pulse from 2 ms to 4 ms equals the area under the negative pulse from 4 ms to 9 ms. In Section 11.13, we will find that the area under the curves represents the energy stored or released by the inductor. From 2 ms to 4 ms, the inductor is storing energy, whereas from 4 ms to 9 ms, the inductor is releasing the energy stored. For the full period zero to 10 ms, energy has been stored and released; there has been no dissipation as experienced for the resistive elements. Over a full cycle, both the ideal capacitor and inductor do not consume energy but store and release it in their respective forms.

11.11 INDUCTORS IN SERIES AND IN PARALLEL

Inductors, like resistors and capacitors, can be placed in series or in parallel. Increasing levels of inductance can be obtained by placing inductors in series, while decreasing levels can be obtained by placing inductors in parallel.

For inductors in series, the total inductance is found in the same manner as the total resistance of resistors in series (Fig. 11.56):

$$L_T = L_1 + L_2 + L_3 + \ldots + L_N \tag{11.30}$$

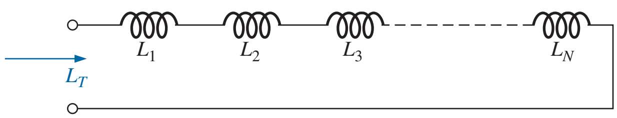


FIG. 11.56
Inductors in series.

For inductors in parallel, the total inductance is found in the same manner as the total resistance of resistors in parallel (Fig. 11.57):

$$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_N} \tag{11.31}$$

For two inductors in parallel,

$$L_T = \frac{L_1 L_2}{L_1 + L_2} \tag{11.32}$$

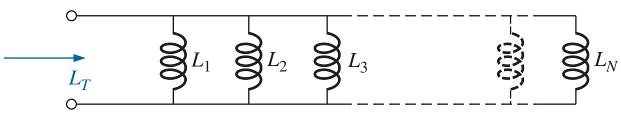


FIG. 11.57
Inductors in parallel.

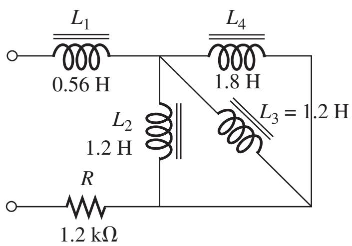


FIG. 11.58
Example 11.9.

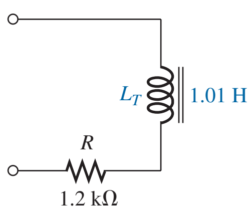


FIG. 11.59
Terminal equivalent of the network in Fig. 11.58.

EXAMPLE 11.9 Reduce the network in Fig. 11.58 to its simplest form.

Solution: Inductors L_2 and L_3 are equal in value and they are in parallel, resulting in an equivalent parallel value of

$$L'_T = \frac{L}{N} = \frac{1.2\text{ H}}{2} = 0.6\text{ H}$$

The resulting 0.6 H is then in parallel with the 1.8 H inductor, and

$$L''_T = \frac{(L'_T)(L_4)}{L'_T + L_4} = \frac{(0.6\text{ H})(1.8\text{ H})}{0.6\text{ H} + 1.8\text{ H}} = 0.45\text{ H}$$

Inductor L_1 is then in series with the equivalent parallel value, and

$$L_T = L_1 + L''_T = 0.56\text{ H} + 0.45\text{ H} = \mathbf{1.01\text{ H}}$$

The reduced equivalent network appears in Fig. 11.59.

11.12 STEADY-STATE CONDITIONS

We found in Section 11.5 that, for all practical purposes, an ideal (ignoring internal resistance and stray capacitances) inductor can be replaced by a short-circuit equivalent once steady-state conditions have been established. Recall that the term *steady state* implies that the voltage and current levels have reached their final resting value and will no longer change unless a change is made in the applied voltage or circuit configuration. For all practical purposes, our assumption is that steady-state conditions have been established after five time constants of the storage or release phase have passed.

For the circuit in Fig. 11.60(a), for example, if we assume that steady-state conditions have been established, the inductor can be removed and replaced by a short-circuit equivalent as shown in Fig. 11.60(b). The short-circuit equivalent shorts out the 3 Ω resistor, and current I_1 is determined by

$$I_1 = \frac{E}{R_1} = \frac{10\text{ V}}{2\ \Omega} = \mathbf{5\text{ A}}$$

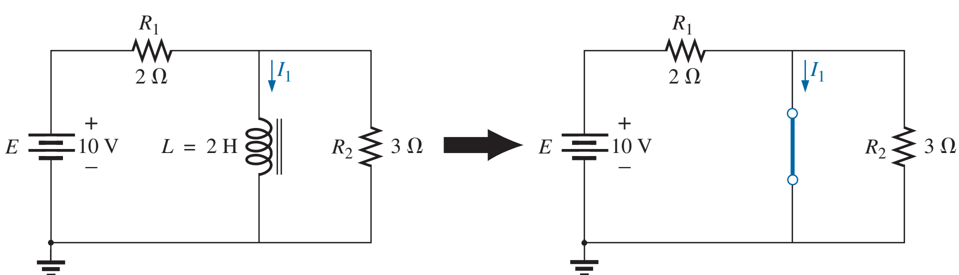


FIG. 11.60
Substituting the short-circuit equivalent for the inductor for $t > 5\tau$.

For the circuit in Fig. 11.61(a), the steady-state equivalent is as shown in Fig. 11.61(b). This time, resistor R_1 is shorted out, and resistors R_2 and R_3 now appear in parallel. The result is

$$I = \frac{E}{R_2 \parallel R_3} = \frac{21\text{ V}}{2\ \Omega} = \mathbf{10.5\text{ A}}$$

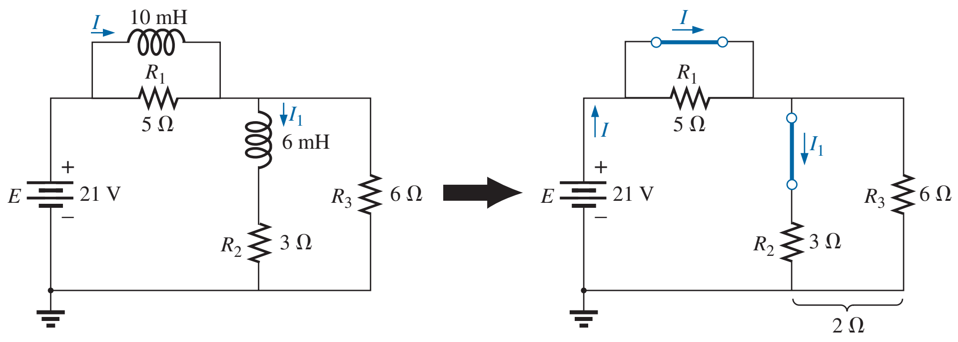


FIG. 11.61
Establishing the equivalent network for $t > 5\tau$.

Applying the current divider rule yields

$$I_1 = \frac{R_3 I}{R_3 + R_2} = \frac{(6\ \Omega)(10.5\ \text{A})}{6\ \Omega + 3\ \Omega} = \frac{63}{9}\text{A} = \mathbf{7\ A}$$

In the examples to follow, it is assumed that steady-state conditions have been established.

EXAMPLE 11.10 Find the current I_L and the voltage V_C for the network in Fig. 11.62.

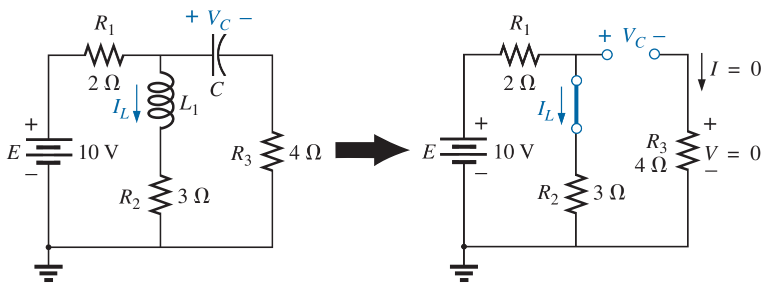


FIG. 11.62
Example 11.10.

Solution:

$$I_L = \frac{E}{R_1 + R_2} = \frac{10\ \text{V}}{5\ \Omega} = \mathbf{2\ A}$$

$$V_C = \frac{R_2 E}{R_2 + R_1} = \frac{(3\ \Omega)(10\ \text{V})}{3\ \Omega + 2\ \Omega} = \mathbf{6\ V}$$

EXAMPLE 11.11 Find currents I_1 and I_2 and voltages V_1 and V_2 for the network in Fig. 11.63.

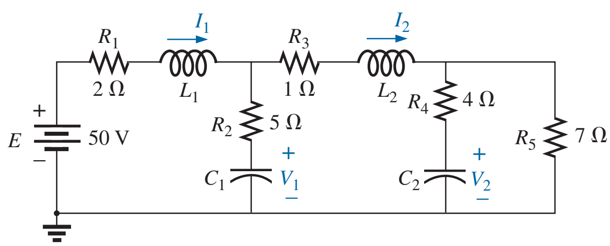


FIG. 11.63
Example 11.11.

Solution: Note Fig. 11.64.

$$I_1 = I_2$$
$$= \frac{E}{R_1 + R_3 + R_5} = \frac{50\text{ V}}{2\ \Omega + 1\ \Omega + 7\ \Omega} = \frac{50\text{ V}}{10\ \Omega} = \mathbf{5\ A}$$
$$V_2 = I_2R_5 = (5\text{ A})(7\ \Omega) = \mathbf{35\ V}$$

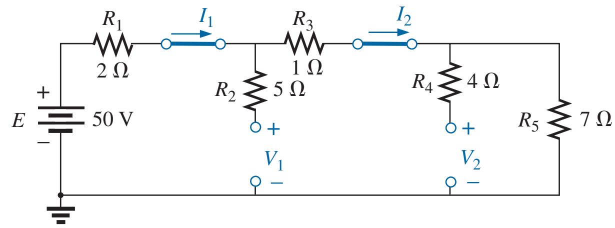


FIG. 11.64
Substituting the short-circuit equivalents for the inductors and the open-circuit equivalents for the capacitor for $t > 5\tau$.

Applying the voltage divider rule yields

$$V_1 = \frac{(R_3 + R_5)E}{R_1 + R_3 + R_5} = \frac{(1\ \Omega + 7\ \Omega)(50\text{ V})}{2\ \Omega + 1\ \Omega + 7\ \Omega} = \frac{(8\ \Omega)(50\text{ V})}{10\ \Omega} = \mathbf{40\ V}$$

11.13 ENERGY STORED BY AN INDUCTOR

The ideal inductor, like the ideal capacitor, does not dissipate the electrical energy supplied to it. It stores the energy in the form of a magnetic field. A plot of the voltage, current, and power to an inductor is shown in Fig. 11.65 during the buildup of the magnetic field surrounding the inductor. The energy stored is represented by the shaded area under the power curve. Using calculus, we can show that the evaluation of the area under the curve yields

$$W_{\text{stored}} = \frac{1}{2}LI_m^2 \qquad \text{(joules, J)} \qquad \textbf{(11.33)}$$

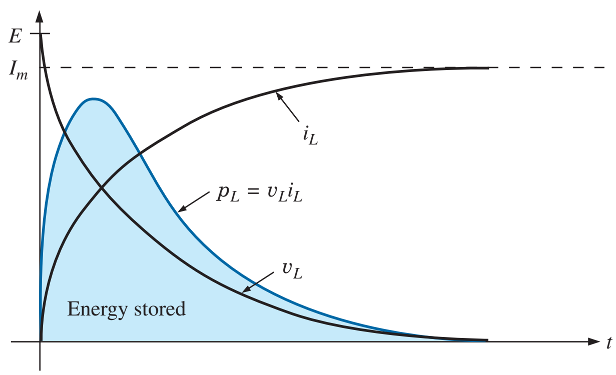


FIG. 11.65
The power curve for an inductive element under transient conditions.

EXAMPLE 11.12 Find the energy stored by the inductor in the circuit in Fig. 11.66 when the current through it has reached its final value.

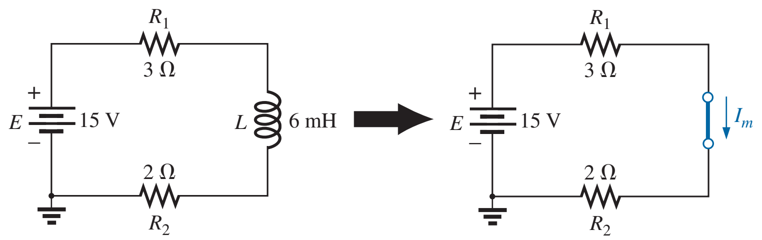


FIG. 11.66
Example 11.12.

Solution:

$$I_m = \frac{E}{R_1 + R_2} = \frac{15\text{ V}}{3\ \Omega + 2\ \Omega} = \frac{15\text{ V}}{5\ \Omega} = 3\text{ A}$$

$$W_{\text{stored}} = \frac{1}{2}LI_m^2 = \frac{1}{2}(6 \times 10^{-3}\text{ H})(3\text{ A})^2 = \frac{54}{2} \times 10^{-3}\text{ J} = \mathbf{27\ mJ}$$

11.14 APPLICATIONS

Camera Flash Lamp

The inductor played an important role in the camera flash lamp circuitry described in the Application section of Chapter 10 on capacitors. For the camera, the inductor was the important component that resulted in the high spike voltage across the trigger coil, which was then magnified by the autotransformer action of the secondary to generate the 4000 V necessary to ignite the flash lamp. Recall that the capacitor in parallel with the trigger coil charged up to 300 V using the low-resistance path provided by the SCR (silicon-controlled rectifier). However, once the capacitor was fully charged, the short-circuit path to ground provided by the SCR was removed, and the capacitor immediately started to discharge through the trigger coil. Since the only resistance in the time constant for the inductive network is the relatively low resistance of the coil itself, the current through the coil grew at a very rapid rate. A significant voltage was then developed across the coil as defined by Eq. (11.12): $v_L = L\,(di_L/dt)$. This voltage was in turn increased by transformer action to the secondary coil of the autotransformer, and the flash lamp was ignited. That high voltage generated across the trigger coil also appears directly across the capacitor of the trigger network. The result is that it begins to charge up again until the generated voltage across the coil drops to zero volts. However, when it does drop, the capacitor again discharges through the coil, establishes another charging current through the coil, and again develops a voltage across the coil. The high-frequency exchange of energy between the coil and capacitor is called *flyback* because of the "flying back" of energy from one storage element to the other. It begins to decay with time because of the resistive elements in the loop. The more resistance, the more quickly it dies out. If the capacitor-inductor pairing is isolated and "tickled" along the way with the application of a dc voltage, the high frequency-generated voltage across the coil can be maintained and put to good use. In fact, it is this flyback effect that is used to generate a steady dc voltage (using rectification to convert the oscillating waveform to one of a steady dc nature) that is commonly used in TVs.

Household Dimmer Switch

Inductors can be found in a wide variety of common electronic circuits in the home. The typical household dimmer uses an inductor to protect the other components and the applied load from "rush" currents—currents that increase at very high rates and often to excessively high levels. This feature is particularly important for dimmers since they are most commonly used to control the light intensity of an incandescent lamp. When a lamp is turned on, the resistance is typically very low, and relatively high currents may flow for short periods of time until the filament of the bulb heats up. The inductor is also effective in blocking high-frequency noise (RFI) generated by the switching action of the triac in the dimmer. A capacitor is also normally included from line to neutral to prevent any voltage spikes from affecting the operation of the dimmer and the applied load (lamp, etc.) and to assist with the suppression of RFI disturbances.

A photograph of one of the most common dimmers is provided in Fig. 11.67(a), with an internal view shown in Fig. 11.67(b). The basic compo-

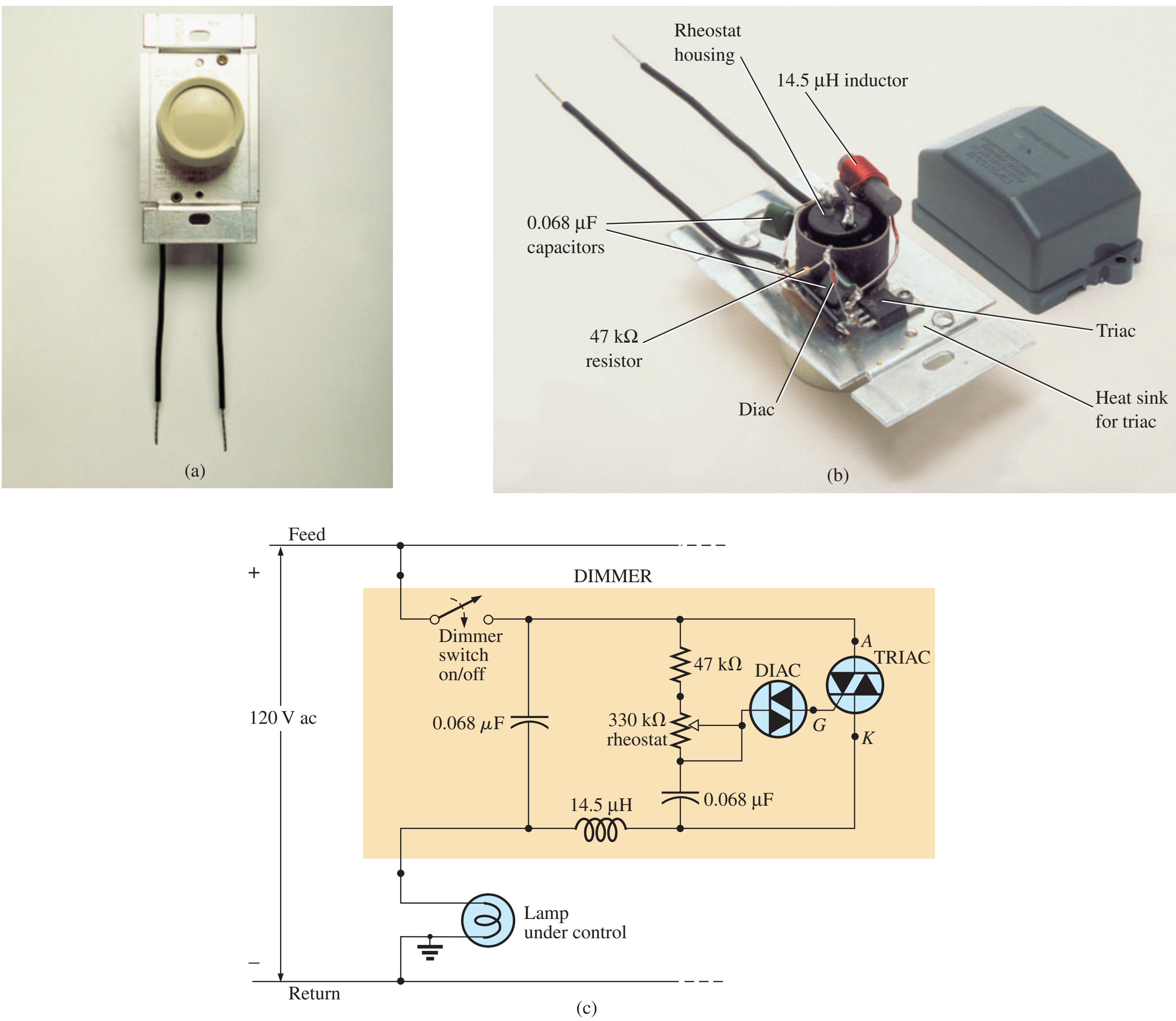


FIG. 11.67

Dimmer control: (a) external appearance; (b) internal construction; (c) schematic.

nents of most commercially available dimmers appear in the schematic in Fig. 11.67(c). In this design, a 14.5 μH inductor is used in the choking capacity described above, with a 0.068 μF capacitor for the "bypass" operation. Note the size of the inductor with its heavy wire and large ferromagnetic core and the relatively large size of the two 0.068 μF capacitors. Both suggest that they are designed to absorb high-energy disturbances.

The general operation of a dimmer is shown in Fig. 11.68. The controlling network is in series with the lamp and essentially acts as an impedance (like resistance—to be introduced in Chapter 15) that can vary between very low and very high levels. Very low impedance levels resemble a short circuit so that the majority of the applied voltage appears across the lamp [Fig. 11.68(a)] and very high impedances approach an open circuit where very little voltage appears across the lamp [Fig. 11.68(b)]. Intermediate levels of impedance control the terminal voltage of the bulb accordingly. For instance, if the controlling network has a very high impedance (open-circuit equivalent) through half the cycle, as shown in Fig. 11.68(c), the brightness of the bulb will be less than full voltage but not 50% due to the nonlinear relationship between the brightness of a bulb and the applied voltage. A lagging effect is also present in the actual operation of the dimmer, which we will learn about when leading and lagging networks are examined in the ac chapters.

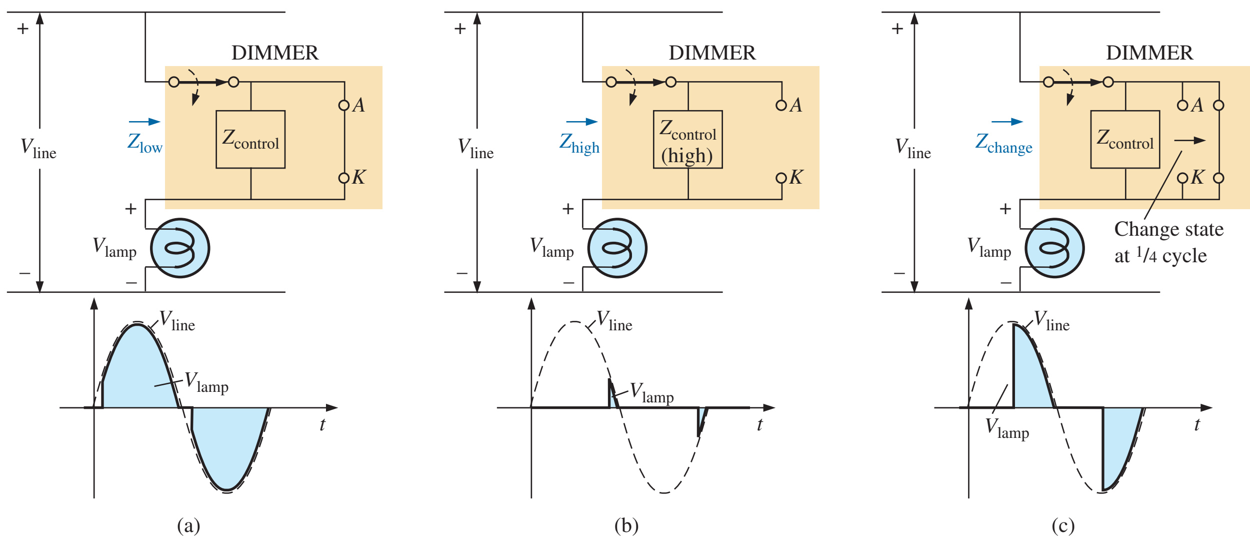


FIG. 11.68

Basic operation of the dimmer in Fig. 11.67: (a) full voltage to the lamp; (b) approaching the cutoff point for the bulb; (c) reduced illumination of the lamp.

The controlling knob, slide, or whatever other method is used on the face of the switch to control the light intensity is connected directly to the rheostat in the branch parallel to the triac. Its setting determines when the voltage across the capacitor reaches a sufficiently high level to turn on the diac (a bidirectional diode) and establish a voltage at the gate (G) of the triac to turn it on. When it does, it establishes a very low resistance path from the anode (A) to the cathode (K), and the applied voltage appears directly across the lamp. When the SCR is off, its terminal resistance between anode and cathode is very high and can be approximated by an open circuit. During this period, the applied voltage does not reach the load (lamp). At this time, the impedance of the parallel branch containing the rheostat, fixed resistor, and capacitor is sufficiently high compared to the load that

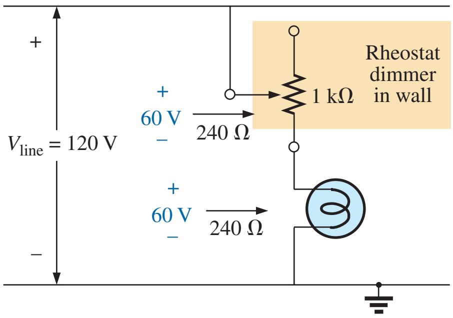


FIG. 11.69
Direct rheostat control of the brightness of a 60 W bulb.

it can also be ignored, completing the open-circuit equivalent in series with the load. Note the placement of the elements in the photograph in Fig. 11.67(b) and that the metal plate to which the triac is connected is actually a heat sink for the device. The on/off switch is in the same housing as the rheostat. The total design is certainly well planned to maintain a relatively small size for the dimmer.

Since the effort here is to control the amount of power getting to the load, the question is often asked, Why don't we just use a rheostat in series with the lamp? The question is best answered by examining Fig. 11.69, which shows a rather simple network with a rheostat in series with the lamp. At full wattage, a 60 W bulb on a 120 V line theoretically has an internal resistance of $R = V^2/P$ (from the equation $P = V^2/R$) $= (120\text{ V})^2/60\text{ W} = 240\ \Omega$. Although the resistance is sensitive to the applied voltage, we will assume this level for the following calculations.

If we consider the case where the rheostat is set for the same level as the bulb, as shown in Fig. 11.69, there will be 60 V across the rheostat and the bulb. The power to each element is then $P = V^2/R = (60\text{ V})^2/240\ \Omega = 15\text{ W}$. The bulb is certainly quite dim, but the rheostat inside the dimmer switch is dissipating 15 W of power on a continuous basis. When you consider the size of a 2 W potentiometer in your laboratory, you can imagine the size rheostat you would need for 15 W, not to mention the purchase cost, although the biggest concern would probably be all the heat developed in the walls of the house. You would be paying for electric power that was not performing a useful function. Also, if you had four dimmers set at the same level, you would actually be wasting sufficient power to fully light another 60 W bulb.

On occasion, especially when the lights are set very low by the dimmer, a faint "singing" can sometimes be heard from the light bulb. This effect sometimes occurs when the conduction period of the dimmer is very small. The short, repetitive voltage pulse applied to the bulb sets the bulb into a condition similar to a resonance state (Chapter 20). The short pulses are just enough to heat up the filament and its supporting structures, and then the pulses are removed to allow the filament to cool down again for a longer period of time. This repetitive heating and cooling cycle can set the filament in motion, and the "singing" can be heard in a quiet environment. Incidentally, the longer the filament, the louder the "singing." A further condition for this effect is that the filament must be in the shape of a coil and not a straight wire so that the "slinky" effect can develop.

TV or PC Monitor Yolk

Inductors and capacitors play a multitude of roles in the operation of a TV or PC monitor. However, the most obvious use of the coil is in the yolk assembly wrapped around the neck of the tube as shown in Fig. 11.70. The tube itself, in addition to providing the screen for viewing, is actually a large capacitor which plays an integral part in establishing the high dc voltage for the proper operation of the monitor.

A photograph of the yolk assembly of a cathode-ray tube is provided in Fig. 11.71(a). It is constructed of four 28 mH coils with two sets of two coils connected at one point [Fig 11.71(b)] so that they share the same current and establish the same magnetic field. The purpose of the yolk assembly is to control the direction of the electron beam from the cathode to the screen of the tube. When the cathode is heated to a very temperature by a filament internal to the structure, electrons are emitted into the surrounding media. The placement of a very high positive potential (10 kV to 25 kV

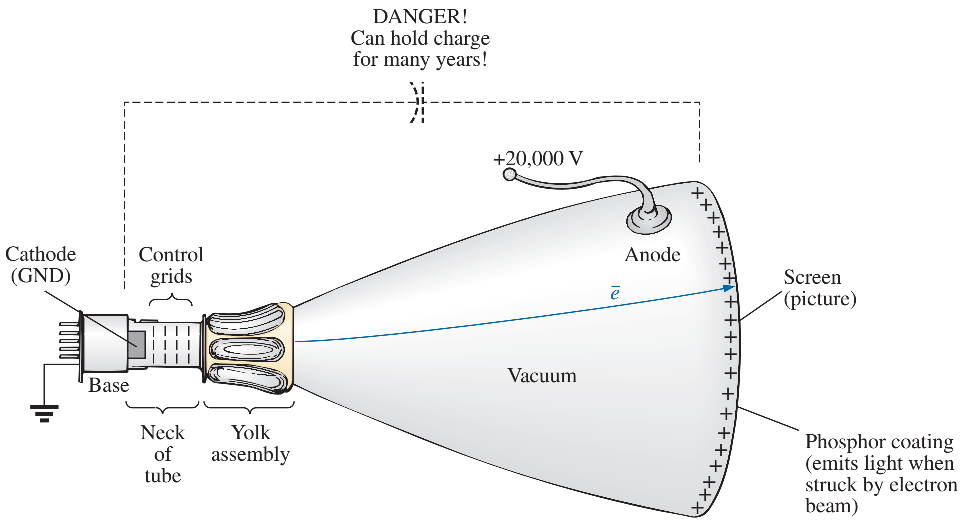


FIG. 11.70
Yolk assembly for a TV or PC tube.

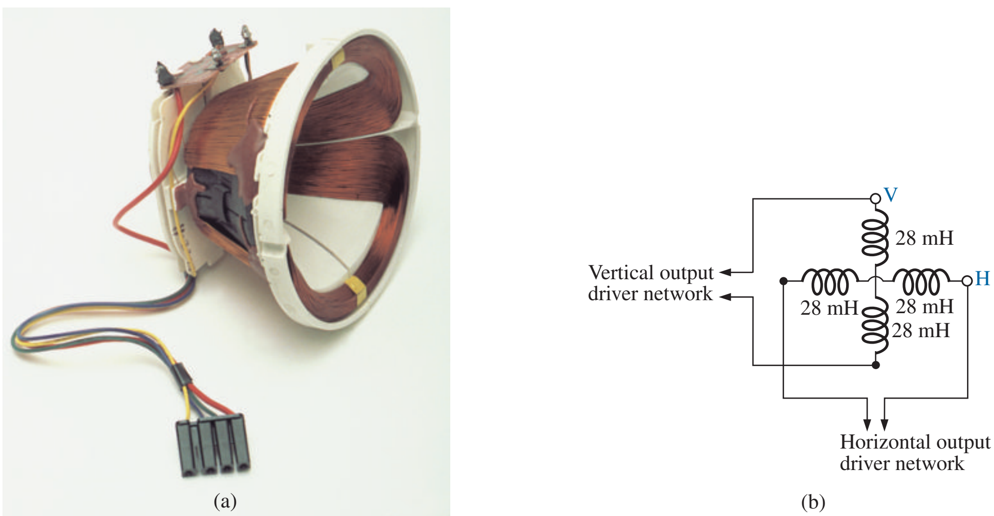


FIG. 11.71
(a) Black-and-white TV yolk assembly; (b) schematic representation.

dc) to the conductive coating on the face of the tube attracts the emitted electrons at a very high speed and therefore at a high level of kinetic energy. When the electrons hit the phosphorescent coating (usually white, green, or amber) on the screen, light is emitted which can be seen by someone facing the monitor. The beam characteristics (such as intensity, focus, and shape) are controlled by a series of grids placed relatively close to the cathode in the neck of the tube. The grid is such that the negatively charged electrons can easily pass through, but the number and speed with which they pass can be controlled by a negative potential applied to the grid. The grids cannot have a positive potential because the negatively charged electrons would be attracted to the grid structure and would eventually disintegrate from the high rate of conduction. Negative potentials on the grids control the flow of electrons by repulsion and by masking the attraction for the large positive potential applied to the face of the tube.

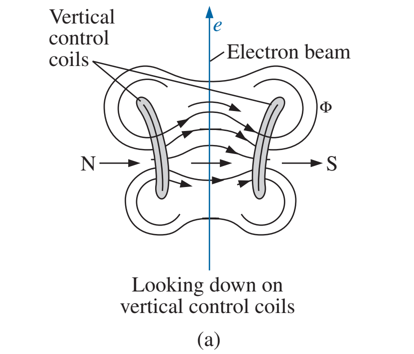


FIG. 11.72
Deflection coils: (a) vertical control; (b) right-hand-rule (RHR) for electron flow.

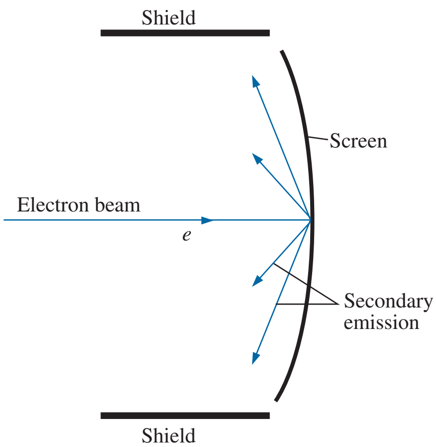


FIG. 11.73
Secondary emission from and protective measures for a TV or PC monitor.

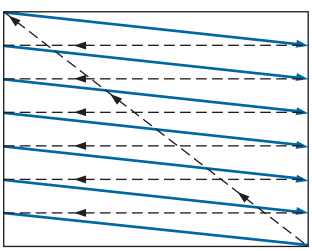


FIG. 11.74
Pattern generation.

Once the beam has been established with the desired intensity and shape, it must be directed to a particular location on the screen using the yolk assembly. For vertical control, the two coils on the side establish a magnetic flux pattern as shown in Fig. 11.72(a). The resulting direction of the magnetic field is from left to right as shown in Figs. 11.72(a) and 11.72(b). Using your right hand, with the index finger pointing in the direction of the magnetic field and the middle finger (at right angles to the index finger) in the direction of electron flow, results in the thumb (also at right angles to the index finger) pointing in the direction of the force on the electron beam. The result is a bending of the beam as shown in Fig. 11.70. The stronger the magnetic field of the coils as determined by the current through the coils, the greater the deflection of the beam.

Before continuing, it is important to realize that when the electron beam hits the phosphorescent screen as shown in Fig. 11.73, it is moving with sufficient velocity to cause a secondary emission of X-rays that scatter to all sides of the monitor. Even though the X-rays die off exponentially with distance from the source, there is some concern about safety, and monitors today have shields all around the outside surface of the tube as shown in Fig. 11.73. Actually, it is not direct viewing that is of some concern but rather viewing by individuals to the side, above, or below the screen. Monitors are currently limited to 25 kV at the anode because the application of voltages in excess of 25 kV can result in a direct emission of X-rays. Internally, all monitors currently have a safety shutoff to ensure that this level is never attained in the operating system.

A detailed discussion of the full operation of a monitor is not possible here, but there are some facts about its operation that reveal the sophistication of the design. When an image is generated on a screen, it is done one *pixel* at a time along one horizontal line at a time. A pixel is one point on the screen. Pixels are black (no signal) or white (with signal) for black-and-white (monochromic) TVs or black and white or some color for color TVs. VGA monitors are 640 pixels wide and 480 pixels high. Obviously the more pixels in the same area, the sharper the image. A typical scan rate is 31.5 kHz which means that 31,500 lines can be drawn in 1 s, or one line of 640 pixels can be drawn in about 31.7 μs.

Patterns on the screen are developed by the sequence of lines appearing in Fig. 11.74. Starting at the top left, the image moves across the screen down to the next line until it ends at the bottom right of the screen, at which point there is a rapid retrace (invisible) back to the starting point. Typical scanning rates (full image generated) extend from 60 frames per second to 80 frames per second. The slower the rate, the higher the possibility of flickering in the images. At 60 frames per second, one entire frame is generated every 1/60 = 16.67 ms = 0.017 s.

Color monitors are particularly interesting because all colors on the screen are generated by the colors red, blue, and green. The reason is that the human eye responds to the wavelengths and energy levels of the various colors. The absence of any color is black, and the result of full energy to each of the three colors is white. The color yellow is a combination of red and green with no blue, and pink is primarily red energy with smaller amounts of blue and green. An in-depth description of this "additive" type of color generation must be left for another course.

The fact that three colors define the resulting color requires that there be three cathodes in a color monitor to generate three electron beams. However, the three beams must sweep the screen in the same relative positions. Each pixel is now made up of three color dots in the same relative position for each pixel, as shown in Fig. 11.75. Each dot has a phosphorescent material that generates the desired color when hit with an electron beam. For situations where the desired color has no green, the electron beam associated with the color green is turned off. In fact, between each pixel, each beam is shut down to provide definition between the color pixels. The dots within the pixel are so close that the human eye cannot pick up the individual colors but simply the color that results from the "additive" process.

During the entire "on" time of a monitor, a full 10 kV to 25 kV are applied to the conductor on the screen to attract electrons. Over time there is naturally an accumulation of negative charge on the screen which remains after the power is turned off—a typical capacitive storage charge. For a brief period of time, it sits with 25 kV across the plates which drop as the "capacitor" begins to discharge. However, the lack of a low-resistance path often results in a storage of the charge for a fairly long period of time. This stored charge and the associated voltage across the plates are sufficiently high to cause severe damage. It is therefore paramount that TVs and monitors be repaired or investigated only by someone who is well versed in how to discharge the tube. One commonly applied procedure is to attach a long lead from the metal shaft of a flat-edge screwdriver to a good ground connection. Then leave the anode connection to the tube in place, and insert the screwdriver under the cap until it touches the metal clip of the cap. You will probably hear a loud snap when discharge occurs. Because of the enormous amount of residual charge, it is recommended that the above procedure be repeated two or three times. Even then, treat the tube with a great deal of respect. In short, until you become familiar with the discharge procedure, leave the investigation of TVs and monitors to someone with the necessary experience. Very high pulse voltages are also generated in an operating system. Be aware that they are of a magnitude that could destroy standard test equipment.

The capacitive effect of the tube is an integral part of developing the high dc anode potential. Its filtering action smooths out the repetitive, high-voltage pulses generated by the flyback action of the TV. Otherwise, the screen would simply be a flickering pattern as the anode potential switched on and off with the pulsating signal.

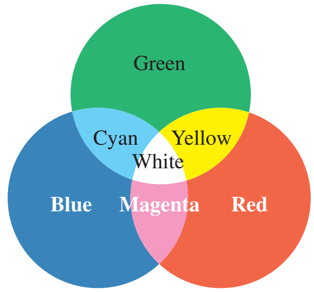


FIG. 11.75
Color television pixels.

11.15 COMPUTER ANALYSIS

PSpice

Transient *RL* Response The computer analysis begins with a transient analysis of the network of parallel inductive elements in Fig. 11.76. The inductors are picked up from the **ANALOG** library in the **Place Part** dialog box. As noted in Fig. 11.76, the inductor is displayed with its terminal identification which is helpful for identifying nodes when calling for specific output plots and values. In general, when an element is first placed on a schematic, the number 1 is assigned to the left end on a horizontal display and to the top on a vertical display. Similarly, the number 2 is assigned to the right end of an element in the horizontal position and to the bottom in the vertical position. Be aware, however, that the option **Rotate** rotates the element in the CCW direction, so taking a horizontal resistor to the vertical position requires three rotations to get the number 1 to the top again. In previous chapters, you may have noted that a number of the outputs were taken off terminal 2 because a single rotation placed this terminal at the top of the vertical display.

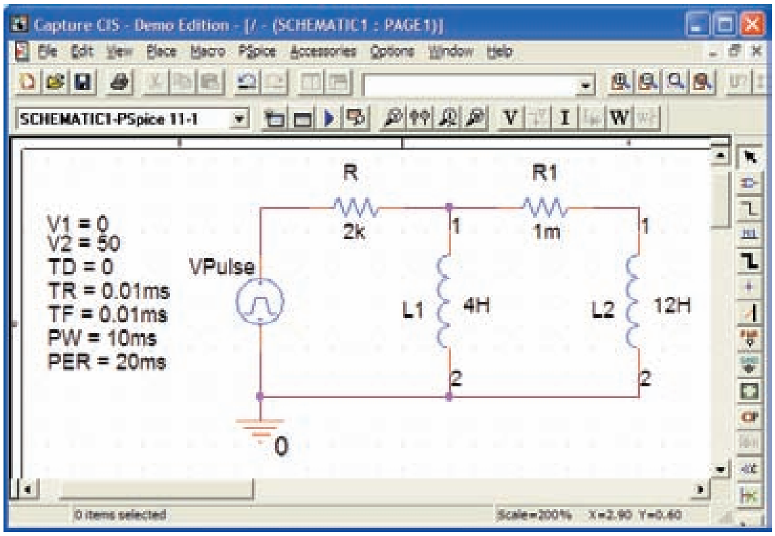


FIG. 11.76
Using PSpice to obtain the transient response of a parallel inductive network due to an applied pulse of 50 V.

Also note in Fig. 11.76 the need for a series resistor R_1 within the parallel loop of inductors. In PSpice, inductors must have a series resistor to reflect real-world conditions. The chosen value of 1 mΩ is so small, however, that it will not affect the response of the system. For **VPulse** (obtained from the SOURCE Library), the rise and fall times were selected as 0.01 ms, and the pulse width was chosen as 10 ms because the time constant of the network is $\tau = L_T/R = (4\text{ H} \parallel 12\text{ H})/2\text{ k}\Omega = 1.5\text{ ms}$ and $5\tau = 7.5$ ms.

The simulation is the same as applied when obtaining the transient response of capacitive networks. In condensed form, the sequence to obtain a plot of the voltage across the coils versus time is as follows: **New SimulationProfile** key-**PSpice 11-1-Create-TimeDomain(Transient)-Run to time:**10ms-**Start saving data after:**0s and **Maximum step size:**5μs-**OK-Run PSpice** key-**Add Trace** key-**V1(L2)-OK.** The resulting trace appears in the bottom of Fig. 11.77. A maximum step size of 5 μs was chosen to ensure that it was less than the rise or fall times of 10 μs. Note that the voltage across the coil jumps to the 50 V level almost immediately; then it decays to 0 V in about 8 ms. A plot of the total current through the parallel coils can be obtained through **Plot-Plot to Window-Add Trace** key-**I(R)-OK,** resulting in the trace appearing at the top of Fig. 11.77. When the trace first appeared, the vertical scale extended from 0 A to 40 mA even though the maximum value of i_R was 25 mA. To bring the maximum value to the top of the graph, **Plot** was selected followed by **Axis Settings-Y Axis-User Defined-0A to 25mA-OK.**

For values, the voltage plot was selected, **SEL>>,** followed by the **Toggle cursor** key and a click on the screen to establish the crosshairs. The left-click cursor was set on one time constant to reveal a value of 18.461 V for **A1** (about 36.8% of the maximum as defined by the exponential waveform). The right-click cursor was set at 7.5 ms or five time constants, resulting in a relatively low 0.338 V for **A2.**

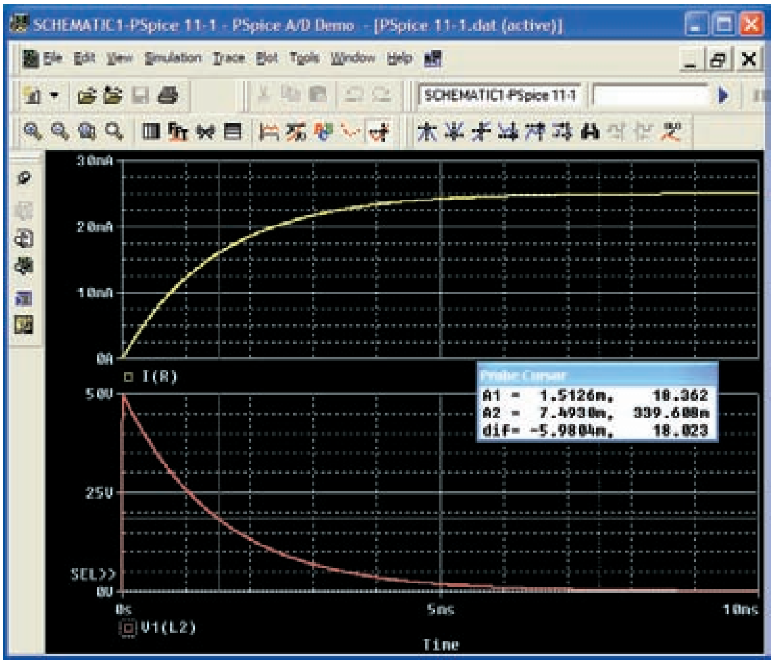


FIG. 11.77

The transient response of v_L and i_R for the network in Fig. 11.76.

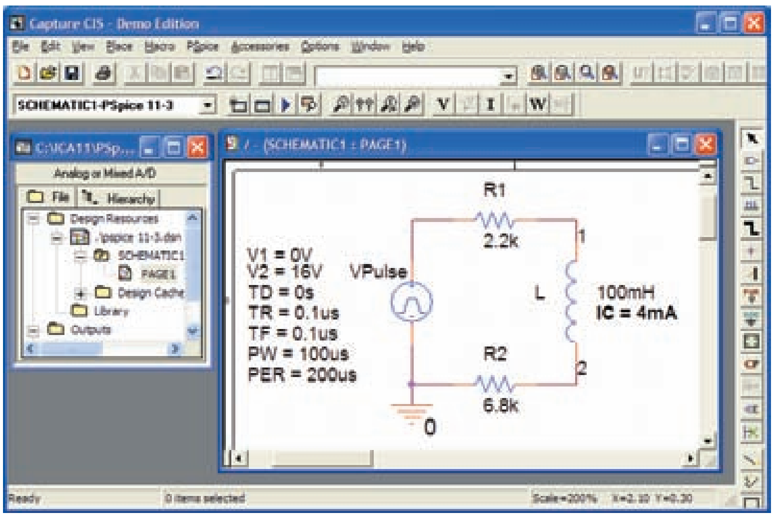


FIG. 11.78

Using PSpice to determine the transient response for a circuit in which the inductive element has an initial condition.

Transient Response with Initial Conditions The next application verifies the results of Example 11.4 which has an initial condition associated with the inductive element. **VPULSE** is again employed with the parameters appearing in Fig. 11.78. Since $\tau = L/R = 100\text{ mH}/(2.2\text{ k}\Omega + 6.8\text{ k}\Omega) = 100\text{ mH}/9\text{ k}\Omega = 11.11\ \mu\text{s}$ and $5\tau = 55.55\ \mu\text{s}$, the pulse width (**PW**) was set to $100\ \mu\text{s}$. The rise and fall times were set at $100\ \mu\text{s}/1000 = 0.1\ \mu\text{s}$. Note again that the labels 1 and 2 appear with the inductive element.

Setting the initial conditions for the inductor requires a procedure that has not been described as yet. First double-click on the inductor symbol to obtain the **Property Editor** dialog box. Then select **Parts** at the bottom of the dialog box, and select **New Column** to obtain the **Add New Column** dialog box. Under **Name,** enter **IC** (an abbreviation for "initial condition"—not "capacitive current") followed by the initial condition of 4 mA under **Value;** then click **OK.** The **Property Editor** dialog box appears again, but now the initial condition appears as a **New Column** in the horizontal listing dedicated to the inductive element. Now select **Display** to obtain the **Display Properties** dialog box, and under **Display Format** choose **Name and Value** so that both **IC** and **4mA** appear. Click **OK** to return to the **Property Editor** dialog box. Finally, click on **Apply** and exit the dialog box (**X**). The result is the display in Fig. 11.78 for the inductive element.

Now for the simulation. First select the **New Simulation Profile** key, insert the name **PSpice 11-3,** and follow up with **Create.** Then in the **Simulation Settings** dialog box, select **Time Domain(Transient)** for the **Analysis type** and **General Settings** for the **Options.** The **Run to time** should be 200 μs so that you can see the full effect of the pulse source on the transient response. The **Start saving data after** should remain at 0 s, and the **Maximum step size** should be 200 μs/1000 = 200 ns. Click **OK** and then select the **Run PSpice** key. The result is a screen with an *x*-axis extending from 0 to 200 μs. Selecting **Trace** to get to the **Add Traces** dialog box and then selecting **I(L)** followed by **OK** results in the display in Fig. 11.79. The plot for **I(L)** clearly starts at the initial value of 4 mA and then decays to 1.78 mA as defined by the left-click cursor. The right-click cursor reveals that the current has dropped to 0.222 μA (essentially 0 A) after the pulse source has dropped to 0 V for 100 μs. The **VPulse** source was placed in the same figure through **Plot-Add Plot to Window-Trace-Add Trace-V(VPulse:+)-OK** to permit a comparison between the applied voltage and the resulting inductor current.

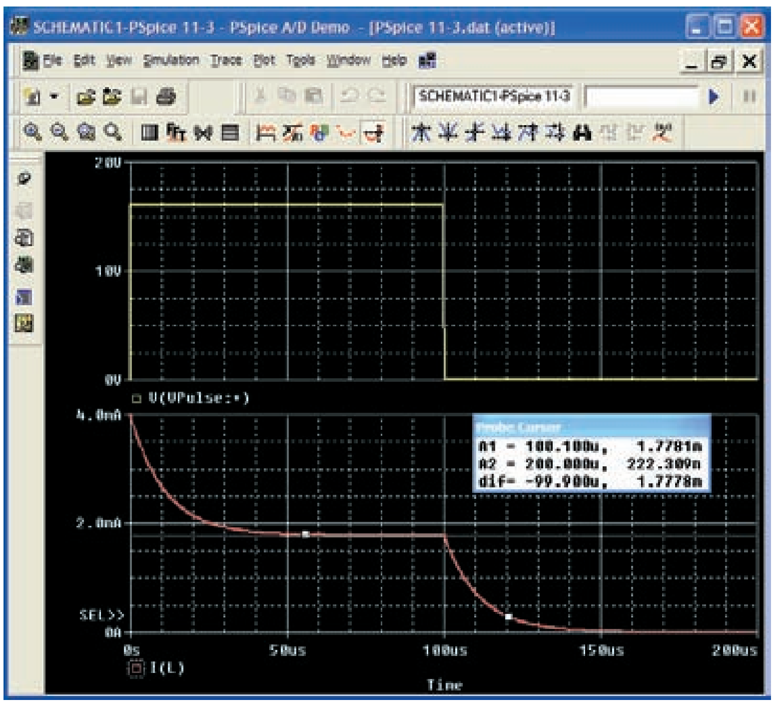


FIG. 11.79

A plot of the applied pulse and resulting current for the circuit in Fig. 11.78.

Multisim

The transient response of an *R-L* network can also be obtained using Multisim. The circuit to be examined appears in Fig. 11.80 with a pulse voltage source to simulate the closing of a switch at $t = 0$ s. The source, **PULSE_VOLTAGE,** is found under **SIGNAL_VOLTAGE** Source Family. When selected, it appears with a label, an initial voltage, a step voltage, and the time period for each level. All can be changed by double-clicking on the source symbol to obtain the dialog box. As shown in Fig. 11.80, the **Pulsed Value** will be set at 20 V, and the **Delay Time** to 0 s. The **Rise Time** and **Fall Time** will both remain at the default levels of 1 ns. For our analysis we want a **Pulse Width** that is at least twice the 5τ transient period of the circuit. For the chosen values of R and L, $\tau = L/R = 10\text{ mH}/100\ \Omega = 0.1\text{ ms} = 100\ \mu\text{s}$. The transient period of 5τ is therefore 500 μs or 0.5 ms. Thus, a **Pulse Width** of 1 ms would seem appropriate with a **Period** of 2 ms. The result is a frequency of $f = I/T = 1/2\text{ ms} = 500$ Hz. When all have been set and selected, the parameters of the pulse source appear as shown in Fig. 11.80. Next the resistor, inductor, and ground are placed on the screen to complete the circuit.

The simulation process is initiated by the following sequence: **Simulate-Analyses-Transient Analysis.** The result is the **Transient Analysis** dialog box in which **Analysis Parameters** is chosen first. Under **Parameters,** use 0 s as the **Start time** and 4 ms as the **End time** so that we get two full cycles of the applied voltage. After enabling the **Maximum time step settings(TMAX),** set the **Minimum number of time points** at 1000 to get a reasonably good plot during the rapidly changing transient period. Next, select the **Output variables** section and tell the program which voltage and current levels you are interested in. On the left side of the dialog box is a list of **Variables** that have been defined for the circuit. On the right is a list of **Selected variables for analysis.** In between you see **Add** or **Remove.** To move a variable from the left to the right column, select it in the left column and choose **Add.** It then appears in the right column. To plot both the applied voltage and the voltage across the coil, move **1** and **2** to the right column. Then select **Simulate.** A window titled **Grapher View** appears with the selected plots as shown in Fig. 11.80. Click on the **Show/Hide Grid** key (a red grid on a black axis), and the grid lines appear. Selecting the **Show/Hide**

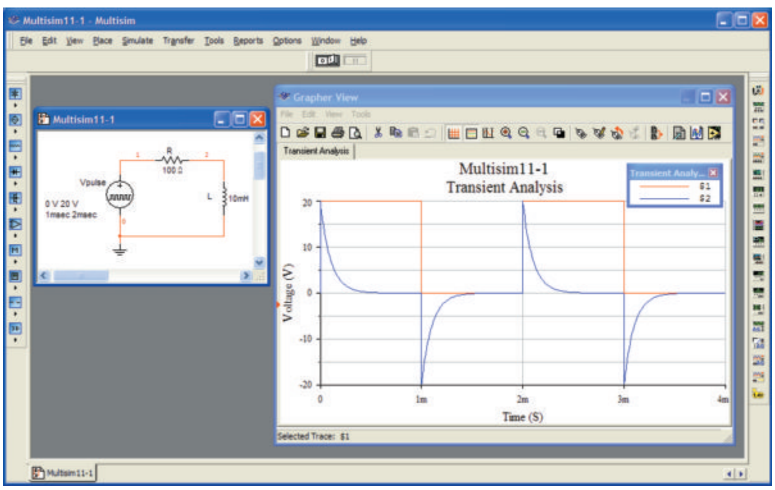


FIG. 11.80
Using Multisim to obtain the transient response for an inductive circuit.

Legend key on the immediate right results in the small **Transient Analysis** dialog box that identifies the color that goes with each nodal voltage. In this case, red is the color of the applied voltage, and blue is the color of the voltage across the coil.

The source voltage appears as expected with its transition to 20 V, 50% duty cycle, and the period of 2 ms. The voltage across the coil jumped immediately to the 20 V level and then began its decay to 0 V in about 0.5 ms as predicted. When the source voltage dropped to zero, the voltage across the coil reversed polarity to maintain the same direction of current in the inductive circuit. Remember that for a coil, the voltage can change instantaneously, but the inductor "chokes" any instantaneous change in current. By reversing its polarity, the voltage across the coil ensures the same polarity of voltage across the resistor and therefore the same direction of current through the coil and circuit.

PROBLEMS

SECTION 11.2 Magnetic Field

1. For the electromagnet in Fig. 11.81:
   a. Find the flux density in Wb/m^2.
   b. What is the flux density in teslas?
   c. What is the applied magnetomotive force?
   d. What would the reading of the meter in Fig. 11.14 read in gauss?

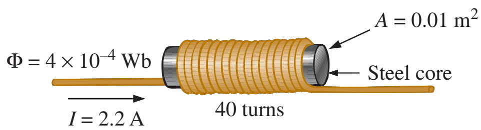


FIG. 11.81
Problem 1.

SECTION 11.3 Inductance

2. For the inductor in Fig. 11.82, find the inductance L in henries.

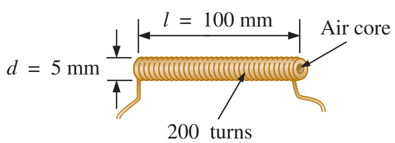


FIG. 11.82
Problems 2 and 3.

3. Repeat Problem 2 with $l = 1.6$ in., $d = 0.2$ in., and a ferromagnetic core with $\mu_r = 500$.
4. For the inductor in Fig. 11.83, find the inductance L in henries.
5. An air-core inductor has a total inductance of 5 mH.
   a. What is the inductance if the only change is to increase the number of turns by a factor of three?

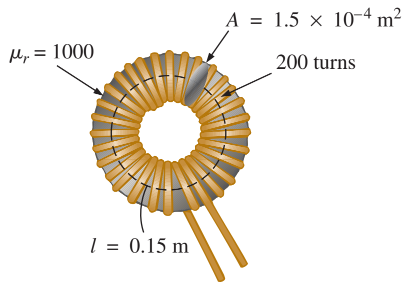


FIG. 11.83
Problem 4.

   b. What is the inductance if the only change is to increase the length by a factor of three?
   c. What is the inductance if the area is doubled, the length cut in half, and the number of turns doubled?
   d. What is the inductance if the area, length, and number of turns are cut in half and a ferromagnetic core with a μ_r of 1500 is inserted?
6. What are the inductance and the range of expected values for an inductor with the following label?
   a. 123J
   b. 47K

SECTION 11.4 Induced Voltage v_L

7. If the flux linking a coil of 50 turns changes at a rate of 120 mW/s, what is the induced voltage across the coil?
8. Determine the rate of change of flux linking a coil if 20 V are induced across a coil of 200 turns.
9. How many turns does a coil have if 42 mV are induced across the coil by a change in flux of 3 mW/s?
10. Find the voltage induced across a coil of 5 H if the rate of change of current through the coil is:
    a. 1 A/s
    b. 60 mA/s
    c. 0.5 A/ms
11. Find the induced voltage across a 50 mH inductor if the current through the coil changes at a rate of 0.1 mA/μs.

SECTION 11.5 *R-L* Transients: The Storage Phase

12. For the circuit in Fig. 11.84:
- **a.** Determine the time constant.
- **b.** Write the mathematical expression for the current i_L after the switch is closed.
- **c.** Repeat part (b) for v_L and v_R.
- **d.** Determine i_L and v_L at one, three, and five time constants.
- **e.** Sketch the waveforms of i_L, v_L, and v_R.

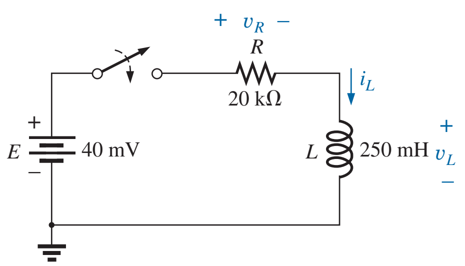


FIG. 11.84
Problem 12.

13. For the circuit in Fig. 11.85:
- **a.** Determine τ.
- **b.** Write the mathematical expression for the current i_L after the switch is closed at $t = 0$ s.
- **c.** Write the mathematical expression for v_L and v_R after the switch is closed at $t = 0$ s.
- **d.** Determine i_L and v_L at $t = 1\tau$, 3τ, and 5τ.
- **e.** Sketch the waveforms of i_L, v_L, and v_R for the storage phase.

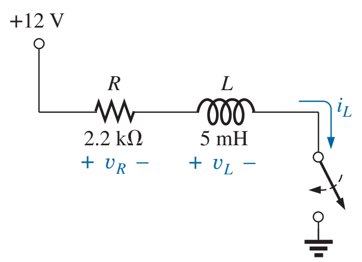


FIG. 11.85
Problem 13.

***14.** For the circuit in Fig. 11.86:
- **a.** Determine the time constant.
- **b.** Write the mathematical expression for the voltage v_L and the current i_L using the defined polarities and direction.
- **c.** Sketch the waveforms of v_L and i_L.

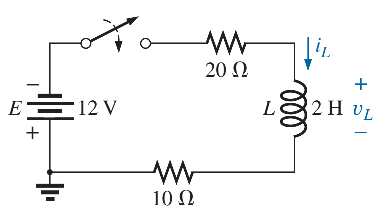


FIG. 11.86
Problem 14.

SECTION 11.6 Initial Conditions

15. For the circuit in Fig. 11.87:
- **a.** Write the mathematical expressions for the current i_L and the voltage v_L following the closing of the switch. Note the magnitude and the direction of the initial current.
- **b.** Sketch the waveform of i_L and v_L for the entire period from initial value to steady-state level.

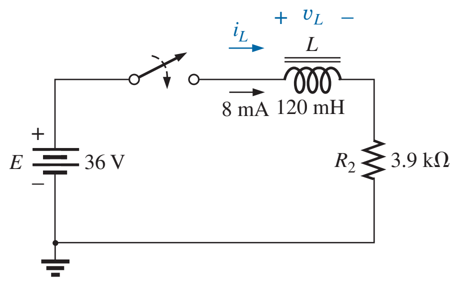


FIG. 11.87
Problems 15 and 47.

16. In this problem, the effect of reversing the initial current is investigated. The circuit in Fig. 11.88 is the same as that appearing in Fig. 11.87, with the only change being the direction of the initial current.
- **a.** Write the mathematical expressions for the current i_L and the voltage v_L following the closing of the switch. Take careful note of the defined polarity for v_L and the direction for i_L.
- **b.** Sketch the waveform of i_L and v_L for the entire period from initial value to steady-state level.
- **c.** Compare the results with those of Problem 15.

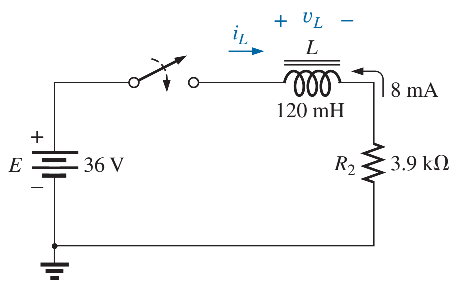


FIG. 11.88
Problem 16.

17. For the network in Fig. 11.89:

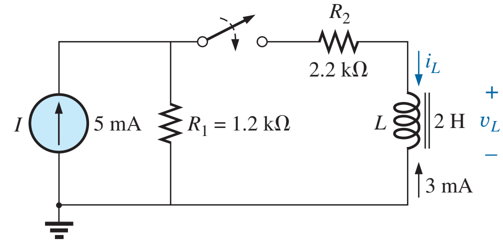


FIG. 11.89
Problem 17.

a. Write the mathematical expressions for the current i_L and the voltage v_L following the closing of the switch. Note the magnitude and the direction of the initial current.

b. Sketch the waveform of i_L and v_L for the entire period from initial value to steady-state level.

*__18.__ For the network in Fig. 11.90:

a. Write the mathematical expressions for the current i_L and the voltage v_L following the closing of the switch. Note the magnitude and direction of the initial current.

b. Sketch the waveform of i_L and v_L for the entire period from initial value to steady-state level.

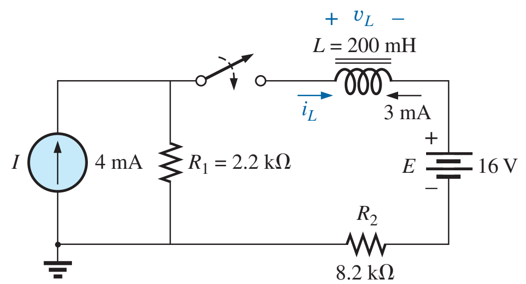


FIG. 11.90
Problem 18.

SECTION 11.7 *R-L* Transients: The Release Phase

19. For the network in Fig. 11.91:

a. Determine the mathematical expressions for the current i_L and the voltage v_L when the switch is closed.

b. Repeat part (a) if the switch is opened after a period of five time constants has passed.

c. Sketch the waveforms of parts (a) and (b) on the same set of axes.

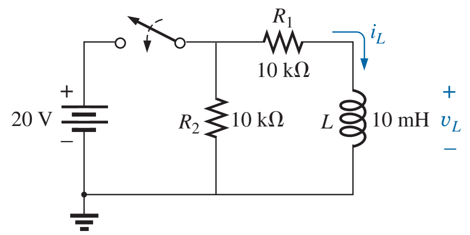


FIG. 11.91
Problem 19.

*__20.__ For the network in Fig. 11.92:

a. Determine the mathematical expressions for the current i_L and the voltage v_L following the closing of the switch.

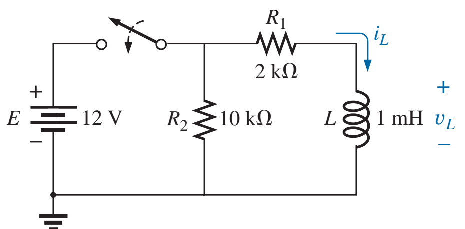


FIG. 11.92
Problem 20.

b. Repeat part (a) if the switch is opened at $t = 1\ \mu s$.

c. Sketch the waveforms of parts (a) and (b) on the same set of axes.

*__21.__ For the network in Fig. 11.93:

a. Write the mathematical expression for the current i_L and the voltage v_L following the closing of the switch.

b. Determine the mathematical expressions for i_L and v_L if the switch is opened after a period of five time constants has passed.

c. Sketch the waveforms of i_L and v_L for the time periods defined by parts (a) and (b).

d. Sketch the waveform for the voltage across R_2 for the same period of time encompassed by i_L and v_L. Take careful note of the defined polarities and directions in Fig. 11.93.

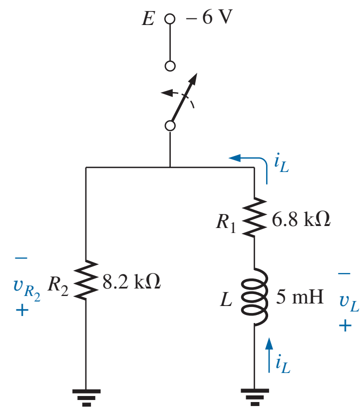


FIG. 11.93
Problem 21.

SECTION 11.8 Thévenin Equivalent: $\tau = L/R_{Th}$

22. For Fig. 11.94:

a. Determine the mathematical expressions for i_L and v_L following the closing of the switch.

b. Determine i_L and v_L after one time constant.

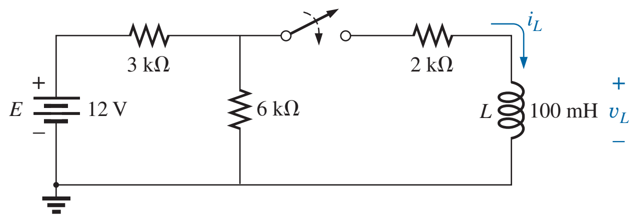


FIG. 11.94
Problems 22 and 48.

23. For Fig. 11.95:

a. Determine the mathematical expressions for i_L and v_L following the closing of the switch.

b. Determine i_L and v_L at $t = 100$ ns.

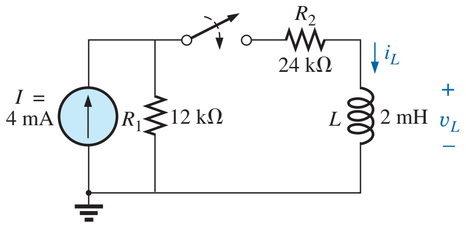


FIG. 11.95
Problem 23.

*__24.__ For Fig. 11.96:

a. Determine the mathematical expressions for i_L and v_L following the closing of the switch.
b. Calculate i_L and v_L at $t = 10\ \mu s$.
c. Write the mathematical expressions for the current i_L and the voltage v_L if the switch is opened at $t = 10\ \mu s$.
d. Sketch the waveforms of i_L and v_L for parts (a) and (c).

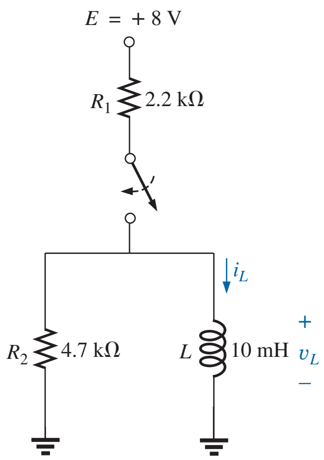


FIG. 11.96
Problem 24.

*__25.__ For the network in Fig. 11.97, the switch is closed at $t = 0$ s.

a. Determine v_L at $t = 25$ ms.
b. Find v_L at $t = 1$ ms.
c. Calculate v_{R_1} at $t = 1\tau$.
d. Find the time required for the current i_L to reach 100 mA.

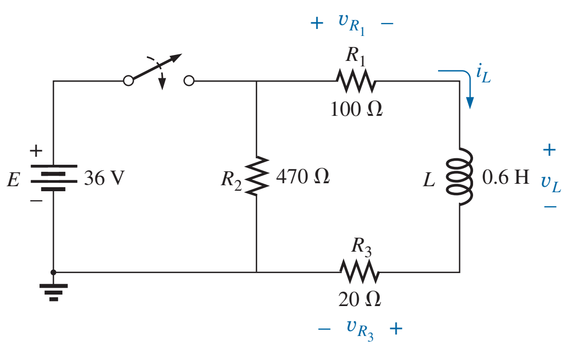


FIG. 11.97
Problem 25.

*__26.__ The switch in Fig. 11.98 has been open for a long time. It is then closed at $t = 0$ s.

a. Write the mathematical expression for the current i_L and the voltage v_L after the switch is closed.
b. Sketch the waveform of i_L and v_L from the initial value to the steady-state level.

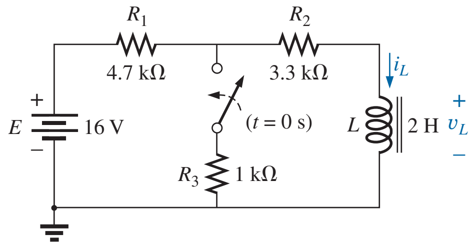


FIG. 11.98
Problem 26.

*__27.__ **a.** Determine the mathematical expressions for i_L and v_L following the closing of the switch in Fig. 11.99.
b. Determine i_L and v_L after two time constants of the storage phase.
c. Write the mathematical expressions for the current i_L and the voltage v_L if the switch is opened at the instant defined by part (b).
d. Sketch the waveforms of i_L and v_L for parts (a) and (c).

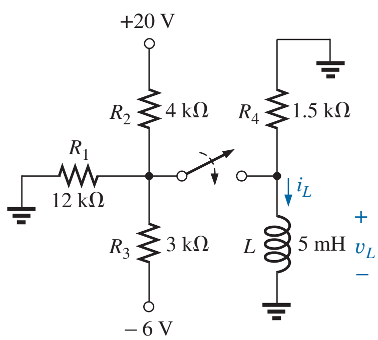


FIG. 11.99
Problem 27.

*__28.__ The switch for the network in Fig. 11.100 has been closed for about 1 h. It is then opened at the time defined as $t = 0$ s.

a. Determine the time required for the current i_R to drop to 1 mA.

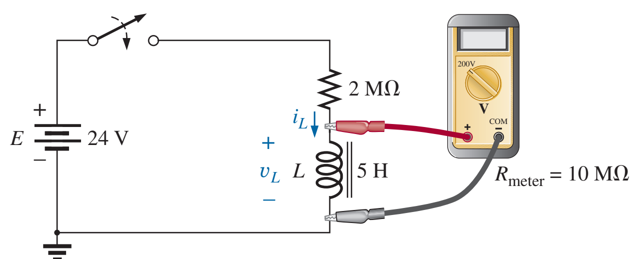


FIG. 11.100
Problem 28.

b. Find the voltage v_L at $t = 1$ ms.
c. Calculate v_R at $t = 5\tau$.

***29.** The switch in Fig. 11.101 has been closed for a long time. It is then opened at $t = 0$ s.
a. Write the mathematical expression for the current i_L and the voltage v_L after the switch is opened.
b. Sketch the waveform of i_L and v_L from initial value to the steady-state level.

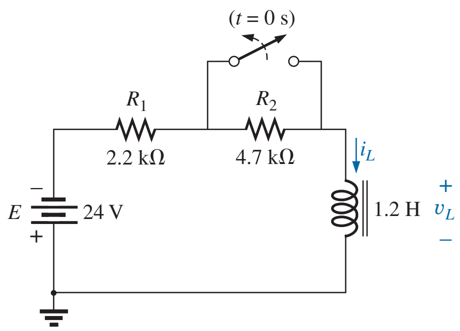


FIG. 11.101
Problem 29.

SECTION 11.9 Instantaneous Values

30. Given $i_L = 100 \text{ mA} (1 - e^{-t/20\text{ms}})$:
a. Determine i_L at $t = 1$ ms.
b. Determine i_L at $t = 100$ ms.
c. Find the time t when i_L will equal 50 mA.
d. Find the time t when i_L will equal 99 mA.

31. The network in Fig. 11.102 employs a DMM with an internal resistance of 10 MΩ in the voltmeter mode. The switch is closed at $t = 0$ s.
a. Find the voltage across the coil the instant after the switch is closed.
b. What is the final value of the current i_L?
c. How much time must pass before i_L reaches 10 μA?
d. What is the voltmeter reading at $t = 12$ μs?

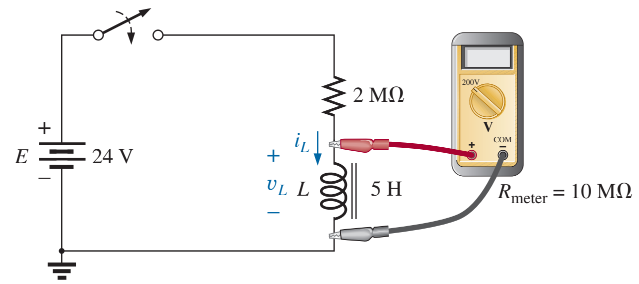


FIG. 11.102
Problem 31.

SECTION 11.10 Average Induced Voltage: $v_{L_{av}}$

32. Find the waveform for the voltage induced across a 200 mH coil if the current through the coil is as shown in Fig. 11.103.

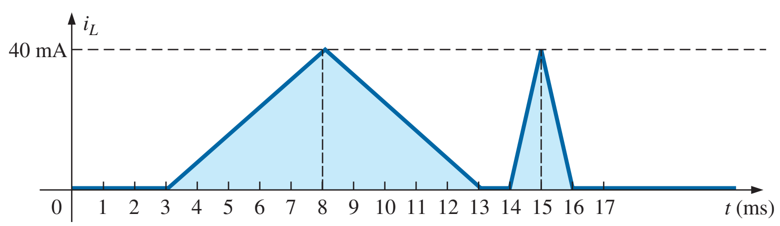


FIG. 11.103
Problem 32.

33. Find the waveform for the voltage induced across a 5 mH coil if the current through the coil is as shown in Fig. 11.104.

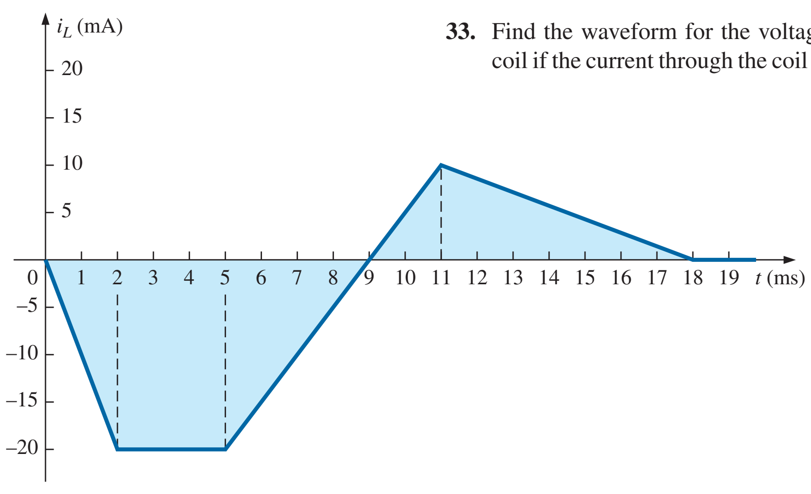


FIG. 11.104
Problem 33.

*34. Find the waveform for the current of a 10 mH coil if the voltage across the coil follows the pattern in Fig. 11.105. The current i_L is 4 mA at $t = 0$ s.

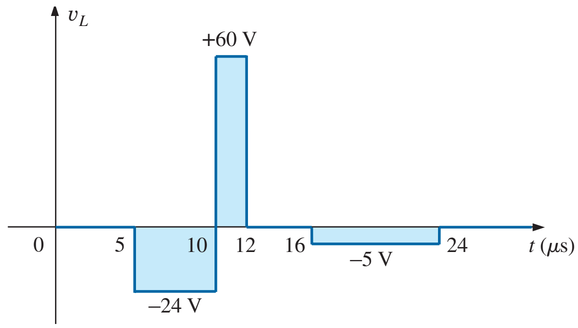


FIG. 11.105
Problem 34.

SECTION 11.11 Inductors in Series and in Parallel

35. Find the total inductance of the circuits in Fig. 11.106.

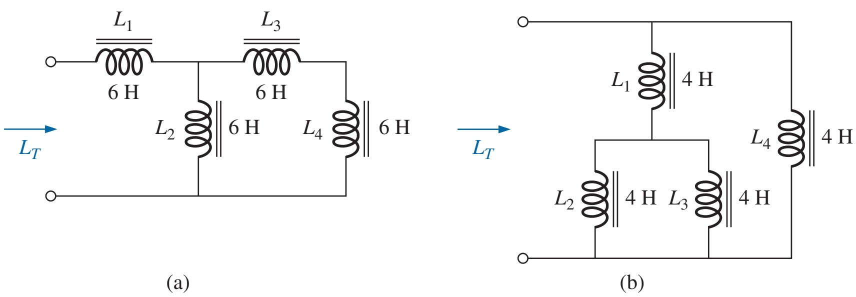


FIG. 11.106
Problem 35.

36. Reduce the network in Fig. 11.107 to the fewest number of components.

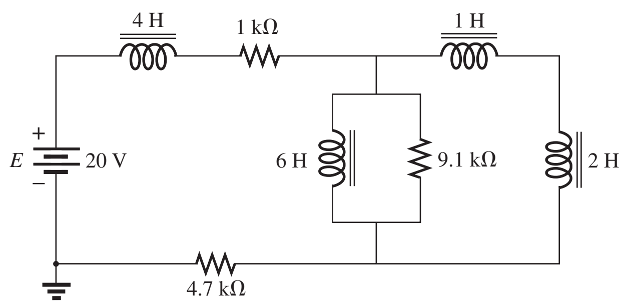


FIG. 11.107
Problem 36.

37. Reduce the network in Fig. 11.108 to the fewest elements.

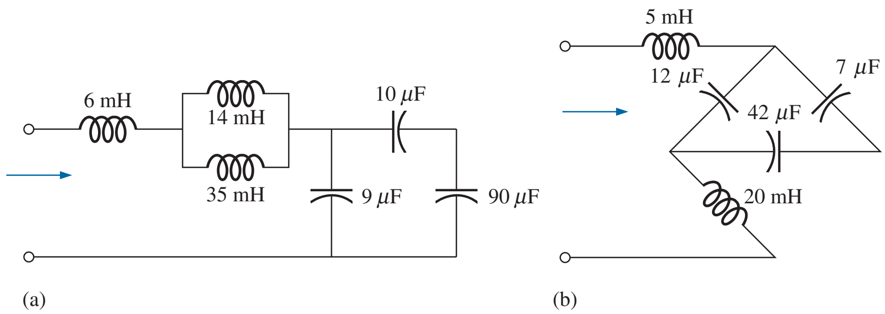


FIG. 11.108
Problem 37.

*__38.__ For the network in Fig. 11.109:

a. Write the mathematical expressions for the voltages v_L and v_R and the current i_L if the switch is closed at $t = 0$ s.

b. Sketch the waveforms of v_L, v_R, and i_L.

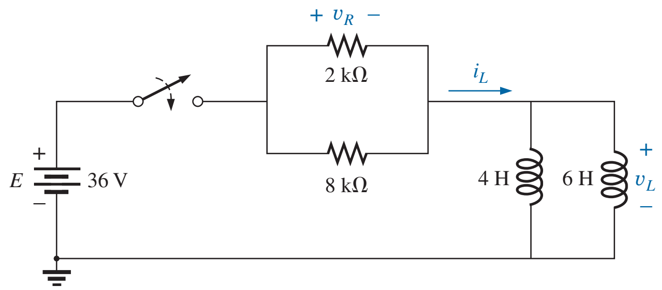


FIG. 11.109
Problem 38.

*__39.__ For the network in Fig. 11.110:

a. Write the mathematical expressions for the voltage v_L and the current i_L if the switch is closed at $t = 0$ s.

b. Sketch the waveforms of v_L and i_L.

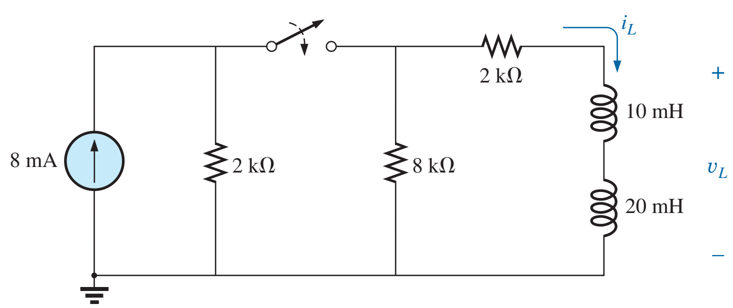


FIG. 11.110
Problem 39.

*40. For the network in Fig. 11.111:
  a. Find the mathematical expressions for the voltage v_L and the current i_L following the closing of the switch.
  b. Sketch the waveforms of v_L and i_L obtained in part (a).
  c. Determine the mathematical expression for the voltage v_{L_3} following the closing of the switch, and sketch the waveform.

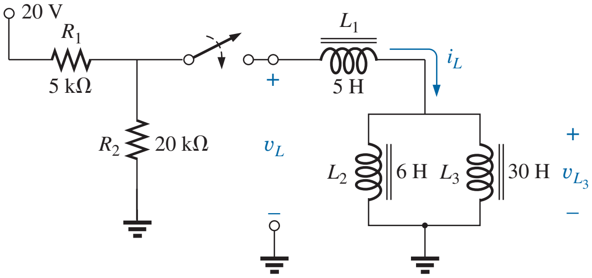


FIG. 11.111
Problem 40.

SECTION 11.12 Steady-State Conditions

41. Find the steady-state currents I_1 and I_2 for the network in Fig. 11.112.

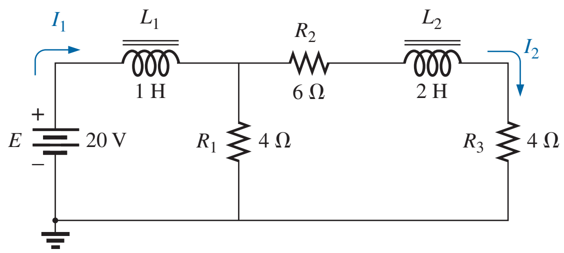


FIG. 11.112
Problem 41.

42. Find the steady-state currents and voltages for the network in Fig. 11.113.

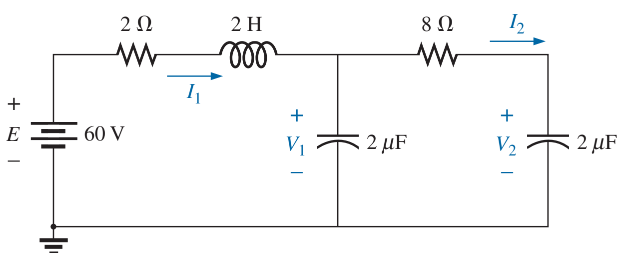


FIG. 11.113
Problem 42.

43. Find the steady-state currents and voltages for the network in Fig. 11.114.

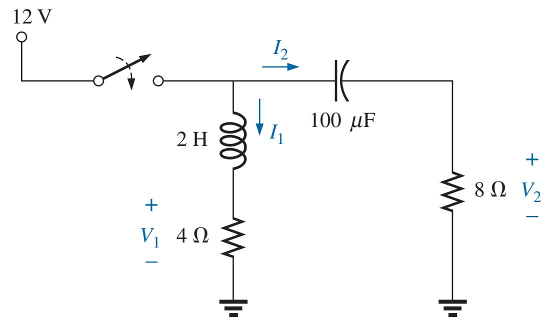


FIG. 11.114
Problem 43.

44. Find the steady-state currents and voltages for the network in Fig. 11.115.

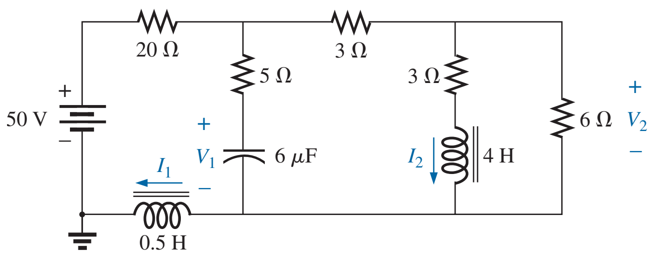


FIG. 11.115
Problem 44.

SECTION 11.15 Computer Analysis

45. Using PSpice or Multisim, verify the results of Example 11.3.
46. Using PSpice or Multisim, verify the results of Example 11.4.
47. Using PSpice or Multisim, find the solution to Problem 15.
48. Using PSpice or Multisim, find the solution to Problem 22.
49. Using PSpice or Multisim, verify the results of Example 11.8.

GLOSSARY

Ampère's circuital law A law establishing the fact that the algebraic sum of the rises and drops of the magnetomotive force (mmf) around a closed loop of a magnetic circuit is equal to zero.

Choke A term often applied to an inductor, due to the ability of an inductor to resist a change in current through it.

Diamagnetic materials Materials that have permeabilities slightly less than that of free space.

Electromagnetism Magnetic effects introduced by the flow of charge, or current.

Faraday's law A law stating the relationship between the voltage induced across a coil and the number of turns in the coil and the rate at which the flux linking the coil is changing.

Ferromagnetic materials Materials having permeabilities hundreds and thousands of times greater than that of free space.

Flux density (*B*) A measure of the flux per unit area perpendicular to a magnetic flux path. It is measured in teslas (T) or webers per square meter (Wb/m^2).

Inductance (*L*) A measure of the ability of a coil to oppose any change in current through the coil and to store energy in the form of a magnetic field in the region surrounding the coil.

Inductor (coil) A fundamental element of electrical systems constructed of numerous turns of wire around a ferromagnetic core or an air core.

Lenz's law A law stating that an induced effect is always such as to oppose the cause that produced it.

Magnetic flux lines Lines of a continuous nature that reveal the strength and direction of a magnetic field.

Magnetomotive force (mmf) ($\mathscr{F}$) The "pressure" required to establish magnetic flux in a ferromagnetic material. It is measured in ampere-turns (At).

Paramagnetic materials Materials that have permeabilities slightly greater than that of free space.

Permanent magnet A material such as steel or iron that will remain magnetized for long periods of time without the aid of external means.

Permeability (μ) A measure of the ease with which magnetic flux can be established in a material. It is measured in $Wb/A \cdot m$.

Relative permeability (μ_r) The ratio of the permeability of a material to that of free space.