CHAPTER 1

Section 1.1

Algebra Aerobics 1.1

1.

Fraction	Decimal	Percent
$\frac{7}{12}$	0.583	58.3%
$\frac{1}{40}$	0.025	2.5%
$\frac{1}{50}$	0.02	2%
$\frac{1}{200}$	0.005	0.5%
$\frac{7}{20}$	0.35	35%
$\frac{1}{125}$	0.008	0.8%

2. a. 500 people

b. 38.94 or 39 students

c. 37.5%

3. mean = \$25,040; median = \$20,000

4. mean GPA ≈ 2.06; median GPA = 2.0

5. a. The numbers (in millions) for the Hispanic population.

b. 2000: $\frac{35.3}{0.125} = 282.4$ million; 2005: $\frac{42.7}{0.144} \approx 296.5$ million

6. a. Frequency Count (FC) for Age 1–20 interval is 38% of total: 38% of 137 = 0.38(137) ≈ 52.

FC for Age 61–80 interval is total FC minus all the others: 137 − (52 + 35 + 28) = 137 − 115 = 22.

Relative Frequency (RF) for each interval is its FC divided by total:

RF of (21–40) interval is: $\frac{35}{137} \approx 0.255 \approx 26\%$

RF of (41–60) interval is: $\frac{28}{137} \approx 0.204 \approx 20\%$

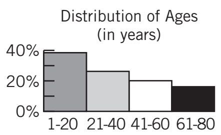
RF of (61–80) interval is: $\frac{22}{137} \approx 0.161 \approx 16\%$

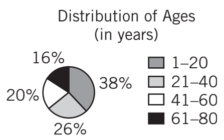
Age	Frequency Count	Relative Frequency (%)
1–20	52	38
21–40	35	26
41–60	28	20
61–80	22	16
Total	137	100

Table 1.3

b. 20% + 16% = 36%

7.





8. Table for the histogram in Figure 1.6

Age	Relative Frequency (%)	Frequency Count
1–20	20	(0.20)(1352) ≈ 270
21–40	35	(0.35)(1352) ≈ 473
41–60	30	(0.30)(1352) ≈ 406
61–80	15	(0.15)(1352) ≈ 203
Total	100	1352

9. a. sum = \$8750, so mean = \$8750/9 ≈ \$972.22; median = \$300

b. sum = 4.7, so mean = 4.7 ÷ 8 ≈ 0.59; median = (0.4 + 0.5)/2 = 0.45

10. One of the values (\$6,000) is much higher than the others which forces a high value for the mean. In cases like this, the median is generally a better choice for measuring central tendency.

Exercises for Section 1.1

1. a. 34.7% of 13- to 17-year-old females spent at least 3 hours per day watching TV in 2006.

b. 39.9%

c. The total numbers of male and female teenagers in that age range.

3. a. Paying off bills/debts. The other categories add up to 62%, not 66%.

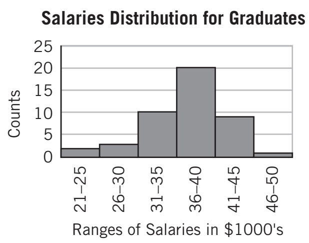
b. They do not add up to 100%. Also, the pie chart is 3D and tipped. Thus the front slices are disproportionately larger than the other slices.

5. a. 45

b. Quantitative data

c. $\frac{3}{45} \approx 6.7\%$

d.



7. a. In housing; 36.16%

b. 0.1438 · \$35,000 = \$5033

c. Answers will vary. Americans spend over 36% of their income for housing.

9. a. Mean = 386/7 = 55.14; median = 46.

b. Changing any entry in the list that is greater than the median to something still higher will not change the median of the list but will increase the mean. The same effect can be had if an entry less than the median is increased to a value that is still less than or equal to the median.

11. The mean annual salary is $24,700 and the median annual salary is $18,000. The mean is heavily weighted by the two high salaries. The mean salary is more attractive but is not likely to be an accurate indicator.

13. No answer is given here. (In general, when answers from students can vary quite a bit, either a typical answer or none is given.)

15. The mean age in the United States is slightly higher than the median age since there are a lot of older Americans (including the baby boomers), which pulls the mean age higher than the median. In developing countries, the mean age will be less than the median age since there are a lot of younger people and this pulls the mean lower than the median. Answers will vary by State and will depend on how the ages are distributed.

17. He is correct, provided the person leaving state A has an IQ that is below the average IQ of people in state A and above the average IQ of the people in state B.

19. The mean net worth of a group, e.g., American families, is heavily biased upward by the very high incomes of a relatively small subset of the group. The median net worth of a group such as this is not as biased. The two measures would be the same if net worths were distributed symmetrically about the mean.

21. **a.** $\left(\sum_{i=1}^{5} x_i\right)\Big/5$ **b.** $\left(\sum_{i=1}^{n} t_i\right)\Big/n$ **c.** $\sum_{k=1}^{5} 2k = 30$

23. Here is a progressive table of the work needed. The mean age of 36.6 years was obtained by dividing total people-years by the total number of persons. Total people-years was obtained by adding up the products of midpoint ages by the population counts in each age group.

Age (years)	Population (thousands)	Midpoint Age	Product of Midpoint Age and Population (people-years)
Under 10	39,677	4.5	178,546
10 to 19	41,875	14.5	607,187
20 to 29	40,532	24.5	993,034
30 to 39	41,523	34.5	1,432,543
40 to 49	45,179	44.5	2,010,465
50 to 59	35,986	54.5	1,961,237
60 to 74	31,052	67	2,080,484
75 to 84	12,971	79.5	1,031,194
85 and over	4,860	92.5	449,550
Total	293,655	n.a.	10,744,242
		Mean age	36.6

25. If we assume an estimated allowance of $5 for all students with an allowance in the $0–9 category, an estimate of $15 for all students with an allowance in the $10–19 category, etc., then the mean ≈ ($5 · 8 + $15 · 6 + $25 · 12 + $35 · 14 + $45 · 9)/49 = 1325/49 ≈ $27. The median lies somewhere between $20 and $29, probably closer to $29.

27. Answer is omitted. Student answers will vary.

29. Some factors to note are given below. You may find others. It is important in giving your answer to cite some population numbers and specific age brackets when making specific comparisons.

1) In Ghana there is a steady decline in the number in each age group from 5 to 9 up to over 80 (from approx. 2.7 million to 0.1 million), whereas in the United States the number of people in each age category is generally the same or more than the number of people 0 to 4 years of age (approx. 19 million) up to the ages 45 to 49 (approx. 23 million).
2) At almost all age levels there seems to be an even distribution of males and females in Ghana; it is somewhat the same in the United States except that from age 60 onward there is a dominance of females in each age category.
3) There is a small bulge in population in the teens as compared to those younger and those just older in the United States (by about 2 million) as compared to a steady decline in Ghana in the population in all age groups once past the 5 to 9 age bracket.

Section 1.2

Algebra Aerobics 1.2a

1. **a.** The median net worth of households, after decreasing from 1988 to 1993, increased from 1993 to 2000. It reached a low of about $43,600 in 1993 and increased to $55,000 in 2000.

b. Many factors would be useful, including size of households and number of wage earners in household.

2. **a.** 1975

b. 2030

c. 55 years

3. **a.** approximately 11 billion

b. approximately 1 billion

c. approximately 11 − 1 = 10 billion

d. The total world population increased rapidly from 1950 to 2000, and is projected to continue to increase but at a decreasing rate, reaching approximately 11 billion in the year 2150.

4. **a.** $\frac{\text{pop } 2000}{\text{pop } 1900} \approx \frac{6}{1.5} = 4$. The population of the world in year 2000 was approximately 4 times greater than in 1900. The difference ≈ 4.5 billion.

b. $\frac{\text{pop } 2100}{\text{pop } 2000} \approx \frac{10.5}{6} = 1.75$, so the population in 2100 was approximately 1.75 times greater than in 2000. The difference ≈ 4.5 billion.

c. The population from 2000 to 2100 is expected to grow by 4.5 billion people, which is the same as the 4.5 billion increase from 1900 to 2000. The world population in 2000 was approximately 4 times greater than in 1900 and it is projected to be about 1.75 times greater in 2100 than in 2000. While the population continues to increase, the rate of increase is slowing down.

Algebra Aerobics 1.2b

1. a. Square the value of x, then multiply that result by 3, then subtract the value of x and add $+1$.

b. (0, 1) is the only one of those ordered pairs that is a solution.

c.

x	−3	−2	−1	0	1	2	3
y	31	15	5	1	3	11	25

2. a. Subtract 1 from the value of x, then square the result.

b. (0, 1) and (1, 0) are the only ones of those ordered pairs that are solutions.

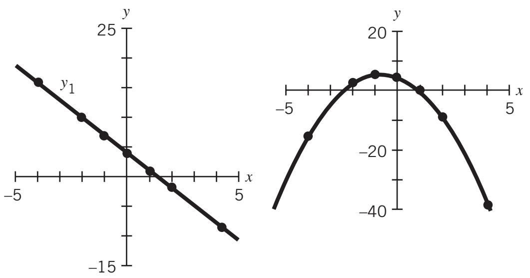
c.

x	−3	−2	−1	0	1	2	3
y	16	9	4	1	0	1	4

3.

x	−4	−2	−1	0	1	2	4
y_1	16	10	7	4	1	−2	−8
y_2	−15	3	6	5	0	−9	−39

a. & b.

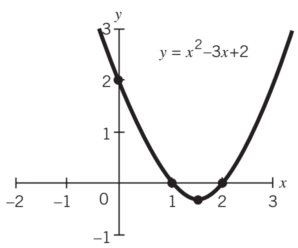


c. Yes for y_1; no for y_2.

d. No for y_1; yes for y_2.

e. No, since $4 - 3(-3) = 13$, $-2(-3)^2 - 3(-3) + 5 = -4$

4.



a. $y = -1/4$

b. Many answers: (0, 0) and (−2, 5) are examples

Exercises for Section 1.2

1. a. Answers may vary.

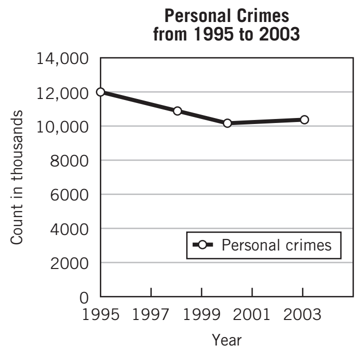
1) AIDS cases in the United States have nearly halved from 1993 to 2004, since they dropped from an all-time high of 79,879 in 1993 to 42,514 in 2004.

2) AIDS cases increased slightly from 2001 to 2004.

3) AIDS cases reached a low of about 39,200 in 2001.

b. Student answers will vary. Possible topic sentences: AIDS cases in the United States went down from 1993 to 2001 but then started to rise very slowly. The three points mentioned in part (a) could form the rest of the paragraph. A concluding sentence might be: "It is worrisome that the number of cases has been on the rise from 2001 to 2004. The number of cases has increased by a little over 3000 from 2001 to 2004."

3. a.

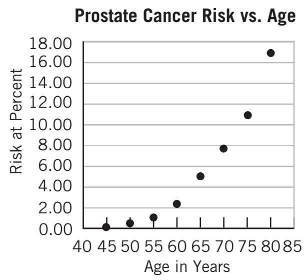


b. In 1995 there were approximately 6.7 times more property crimes than violent crimes. In 2003 there were approximately 7.6 times more property crimes than violent crimes.

c. Answers may vary. Some observations could be the following. Property crime, from 1995 to 2003, was over six times as great as personal crime. Both property crimes and personal crimes decreased from 1995 to 2003, but from 1995 to 2003 the ratio of property to personal crimes actually increased.

5. a. As men grow older, the risk of cancer goes up dramatically—for example, from 1 in 25,000 at age 45, to 1 in 25 at age 65, to 1 in 6 at age 80.

b.



c. The ratio of risks for a 50- vs. a 45-year-old man is $0.21\%/0.004\% = 52.5$. This means that a 50-year-old man is 52.5 times more likely to have had prostate cancer than a 45-year-old man. The ratio of risks for a 55- vs. 50-year-old man is $0.83\%/0.21\% \approx 4$. So a 55-year-old man is only 4 times as likely to have had prostate cancer than a 50-year-old man. At first this may seem contradictory. But the biggest incremental risk occurs between 45 and 50. So while the absolute risk continues to rise (note that the percent risk is cumulative), the ratios of the percent risk decrease over subsequent 5-year age intervals.

d. The medical profession has recommended this test for all men from age 40 up. The insurance companies think that this is too expensive. Your answer may be different.

7. a. Except for a dip in the count for 12- to 19-year-olds in the 1976 to 1980 period there has been a dramatic rise in the percentage of children who are overweight.

b. It increased in going from every time interval to the next, except in going from the first (1963 to 1970) to the second (1971 to 1974), when the percentage stayed the same.

c. From the period of 1963 to 1970 to the period of 1971 to 1974.

d. During the 1976–1980 period.

e. The goal is reasonable, but the data suggest that the trend in obesity will keep rising.

9. a. True; B is the newer car and it costs more than A.

b. True; A is the slower car in cruising speed and it has the larger size.

c. False; A is the larger car but it is older than B.

d. True; A carries more passengers and it is less expensive.

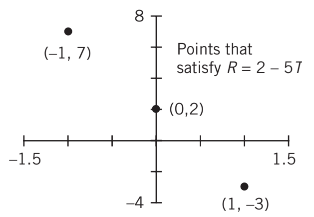
e. Your answers will vary. You may mention that the larger range car also has the larger passenger capacity.

f. Your answers will vary. Much depends on what features you value. There are many trade-offs.

11. a. Only $(1, -3)$ satisfies the formula.

b. There are many answers: e.g., $(0, 2)$, $(-1, 7)$. In general, pick a value of T and plug it into the formula to compute the corresponding value of R.

c. Here is the scatter plot for the three points given:



d. The plot suggests that solutions could be found on the straight line through these three points by eyeballing the line. Another source would be using the formula with other values of T to generate the corresponding values of R.

13. a. Add 1 to the value of x; divide the result by what one gets by subtracting 1 from the value of x.

b. $(5, 1.5)$

c. $(2, 3)$

d. No, the formula is not defined if $x = 1$.

15. a. $x = 0$ implies $y = 0$

b. If $x > 0$ then $y < 0$.

c. If $x < 0$ then $y < 0$.

d. No

17. a. Only $(-1, 3)$ satisfies $y = 2x + 5$.

b. $(1, 0)$ and $(2, 3)$ satisfy $y = x^2 - 1$.

c. $(-1, 3)$ and $(2, 3)$ satisfy $y = x^2 - x + 1$

d. Only $(1, 2)$ satisfies $y = 4/(x + 1)$.

19. a. There are far more people now than in 1971, and those who are most susceptible, the elderly, are now a greater proportion of the population than in 1971. Thus, it is not necessarily the case that proportionately twice as many now as then will have cancer. Also, better cancer detection tools could mean more frequent prevention of death. Lastly, not all cancers are fatal.

b. Most of the observations given by William M. London are cited in part (a) but his arguments make claims that are not buttressed clearly enough by data. We need to have data for cancer incidence, cancer treatment, cancer conquest, and cancer fatality, for all age groups by sex and type of cancer before we can make any solid overall claims either way.

c. The statements given above in parts (a) and (b) can be used to forge a paragraph.

Section 1.3

Algebra Aerobics 1.3

1. Table A represents a function. For each input value there is one and only one output value.

Table B does not represent a function. For the input of 2 there are two outputs, 7 and 8.

2. There are two output values, 5 and 7, for the input of 1. For the table to represent a function, there can be only one output value for each input. If you change 5 to 7 *or* change 7 to 5, then there will be only one output value for the input of 1.

3. Yes, it passes the vertical line test.

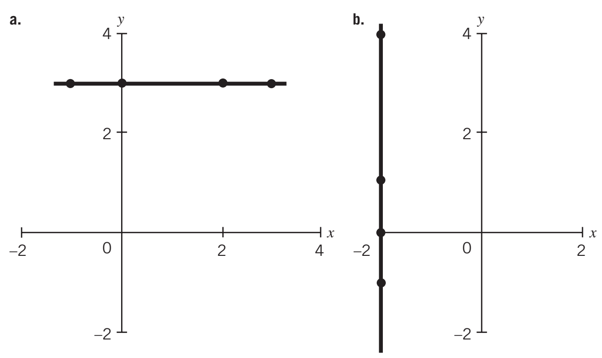
4. Graph B represents a function, while graphs A and C do not represent functions since they fail the vertical line test.

5. Neither is a function of the other. The graph fails the vertical line test, so weight is not a function of height. At height 51 inches there are two weights (115 and 120 pounds) and at height 56 inches there are two weights (135 and 140 pounds). If we reverse the axes, so that weight is on the horizontal axis and height is on the vertical axis, that graph will also fail the vertical line test, since the weight of 140 pounds has two corresponding heights of 56 inches and 58 inches.

6. a. D is a function of Y, since each value of Y determines a unique value of D.

b. Y is not a function of D, since one value of D, \$2.70, yields two values for Y, 1993 and 1997.

7.



a. The line containing the points is the graph of a function since for each value of x we have a unique value of y. The line is horizontal. Its equation can be written as $y = 3$.

b. The line containing the points is not the graph of a function since for one value of x we have infinitely many values of y. The line is vertical. Its equation can be written as $x = -2$.

8. a. Tip = 15% of the cost of meal, so $T = 0.15M$. Independent variable: M (meal price); Dependent variable: T (tip).

The equation is a function since to each value of M there corresponds a unique value of T.

b. $T = (0.15)(\$8) = \$1.20$

c. $T = (0.15)(\$26.42) = \$3.96$. One would probably round that up to $4.00.

Exercises for Section 1.3

1. a. Yes, each date has one and only one temperature.

b. No, the temperature 27°C goes with two different dates.

3. a. function (all input values are different and thus each input has one and only one output)

b. function [same reason as in part (a)]

c. not a function [same input values have different output values]

d. not a function [same reason as in part (c)]

5. a. Not a function: fails the vertical line test, e.g., look at the y-axis.

b. Function: passes the vertical line test.

c. Not a function: fails the vertical line test, e.g., look at the y-axis

7. a. The formulas are: $y = x + 5$; $y = x^2 + 1$; $y = 3$

b. All three represent y as a function of x; each input of x has only one output y.

9. a. $S_1 = 0.90 \cdot P$; $90

b. $S_2 = (0.90)^2 \cdot P$; $81

c. $S_3 = (0.90)^3 \cdot P$; $72.90

d. $S_5 = (0.90)^5 \cdot P$; $59.05; 40.95%

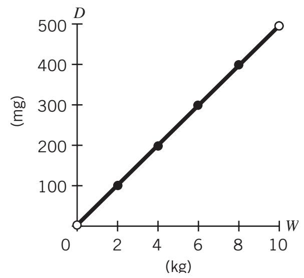
11. y is a function of x in parts (a), (b), and (c) but not in (d). In (d), for example, if $x = 1$, then $y = \pm 1$.

13. a. Since the dosage depends on the weight, the logical choice for the independent variable is W (expressed in kilograms) and for the dependent variable is D (expressed in milligrams).

b. In this formula, each value of W determines a unique dosage D, so D is a function of W.

c. The following table and graph are representations of the function. Since the points (0, 0) and (10, 500) are not included in the model, these points are represented with a hollow circle on the graph.

W (kg)	D (mg)
0	0
2	100
4	200
6	300
8	400



d. The domain and range are now all the real numbers. The table can now include values of W that are less than or equal to zero and values greater than or equal to 10. The graph will continue indefinitely in both directions.

Section 1.4

Algebra Aerobics 1.4a

1. $g(0) = 0$
$g(-1) = -3$
$g(1) = 3$
$g(20) = 60$
$g(100) = 300$

2. $f(0) = (0)^2 - 5(0) + 6 = 6$, so $f(0) = 6$
$f(1) = (1)^2 - 5(1) + 6 = 2$, so $f(1) = 2$
$f(-3) = (-3)^2 - 5(-3) + 6 = 30$, so $f(-3) = 30$

3. $f(0) = \frac{2}{0-1} = -2$, so $f(0) = -2$;
$f(-1) = \frac{2}{(-1)-1} = -1$, so $f(-1) = -1$;
$f(-3) = \frac{2}{(-3)-1} = -\frac{1}{2}$, so $f(-3) = -1/2$

4. $5 - 2t = 3$, so $t = 1$; $3t - 9 = 3$, so $t = 4$; $5t - 12 = 3$, so $t = 3$.

5. $2(x - 1) - 3(y + 5) = 10 \Rightarrow 2x - 2 - 3y - 15 = 10 \Rightarrow$
$2x - 17 - 3y = 10;\quad 2x - 27 = 3y \Rightarrow y = \frac{2x - 27}{3}$
So y is a function of x. $f(x) = \frac{2x - 27}{3}$.

6. $x^2 + 2x - y + 4 = 0 \Rightarrow x^2 + 2x + 4 = y$.
So y is a function of x. $f(x) = x^2 + 2x + 4$

7. $7x - 2y = 5 \Rightarrow -2y = -7x + 5 \Rightarrow y = (-7x + 5)/(-2) \Rightarrow$
$y = \frac{7x - 5}{2}$
So y is a function of x. $f(x) = \frac{7x - 5}{2}$

8. $f(-4) = 2$; $f(-1) = -1$; $f(0) = -2$; $f(3) = 1$. When $x = -2$ or $x = 2$, then $f(x) = 0$.

9. $f(0) = 20$, $f(20) = 0$, $f(x) = 10$ when $x = 10$ and $x = 30$. It is a function because for every value of x, there is one and only one value of $f(x)$.

Algebra Aerobics 1.4b

1. a. $(2, \infty)$

b. $[4, 20)$

c. $(-\infty, 0] \cup (500, +\infty)$

2. a. $-3 \le x < 10$

b. $-2.5 < x \le 6.8$

c. $x \le 5$ or $x \ge 12$

3. a. [2.5, 3.6]

b. [0.333, 1.000] (*Note:* The highest possible batting average is 1.000, meaning that the batter had a hit every time he has been at bat. This is commonly known as "batting a thousand.")

c. [35000, 50000]

4. $2(x + 1) + 3y = 5 \Rightarrow 2x + 2 + 3y = 5 \Rightarrow$

$$3y = -2x + 3 \Rightarrow$$
$$y = \frac{-2x + 3}{3}$$

So y is a function of x. The domain and range are all real numbers. $f(x) = \frac{-2x + 3}{3}$

5. $x + 2y = 3x - 4 \Rightarrow 2y = 2x - 4 \Rightarrow y = x - 2$

So y is a function of x. The domain and range are all real numbers. $f(x) = x - 2$

6. $y = \sqrt{x}$

So y is a function of x. The domain and range are all real numbers ≥ 0. $f(x) = \sqrt{x}$

7. $2xy = 6 \Rightarrow y = \frac{6}{2x} = \frac{3}{x}$. So y is a function of x. The domain is all real numbers except 0. The range is all real numbers except 0. $f(x) = \frac{3}{x}$

8. $6\left(\frac{x}{2} + \frac{y}{3}\right) = 6(1) \Rightarrow 3x + 2y = 6 \Rightarrow$

$2y = 6 - 3x \Rightarrow y = \frac{6 - 3x}{2}$

So y is a function of x. The domain and range are all real numbers. $f(x) = \frac{6 - 3x}{2}$

9. $f(x)$ is undefined at $x = -5$; domain: all real numbers except -5; range: all real numbers except 0.

$g(x)$ is undefined at $x = -1$; domain: all real numbers except -1; range: all real numbers except 0.

$h(x)$ is undefined at $x < 10$; domain: $[10, +\infty)$; range: $[0, +\infty)$.

Exercises for Section 1.4

1. a. $T(0) = 2$, $T(-1) = 6$, $T(1) = 0$, $T(-5) = 42$

3. a. Tax = 0.16 · Income

b. Income is the independent variable and Tax is the dependent variable.

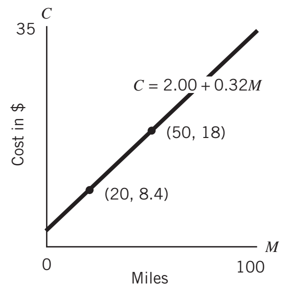
c. Yes, the formula represents a function: for each input there is only one output.

d. As the Income gets closer and closer to \$20,000, the Tax gets closer and closer to \$3200. In fact the Tax, at some point will round off to \$3200.00, even though the Income is not quite \$20,000. Its domain is [0, 20000] and its range is [0, 3200].

5. The equation is $C = 2.00 + 0.32M$. It represents a function. The independent variable is M measured in miles. The dependent variable is C measured in dollars. Here is a table of values.

Miles	Cost (\$)	Miles	Cost (\$)
0	2.00	30	11.60
10	5.20	40	14.80
20	8.40	50	18.00

The graph is in the accompanying diagram. Some of the table values are marked.



7. a. $f(2) = 4$

b. $f(-1) = 4$

c. $f(0) = 2$

d. $f(-5) = 32.$

9. a. $p(-4) = 0.063$, $p(5) = 32$ and $p(1) = 2$.

b. $n = 1$ only

11. a. $f(-2) = 5$, $f(-1) = 0$, $f(0) = -3$, and $f(1) = -4$.

b. $f(x) = -3$ if and only if $x = 0$ or 2.

c. The range of f is from -4 to $+\infty$ since we may assume that its arms extend out indefinitely.

13. $f(0) = 1$, $f(1) = 1$, and $f(-2) = 25$

15. a. $f(0) = 1$, $g(0) = 1$

b. $f(-2) = 2$, $g(-3) = 10$

c. $f(2) = 0$, $f(1) = 0.5$

d. $f(3) = -0.5$, $g(3) = 10$

17. a. $x = -2$ and 2 **b.** $x = 2$ **c.** $x = 0$ and 2

Section 1.5

Algebra Aerobics 1.5

1. *Possible titles:*

a. From 2004 to 2006 Prices Decreased 50% for LCD TVs

b. iTunes Sales Increase Tenfold From December 2003 to January 2005

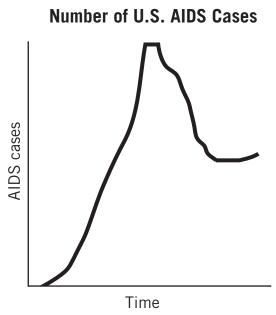
2. a. maximum value: approx. \$46,000 in 1999; (1999, \$46000)

b. minimum value: approx. \$40,000 in 1993; (1993, \$40000)

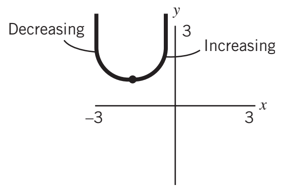
c. Median household income decreased from 1990 to 1993, reaching a low point of about \$40,000 in 1993. From 1993 to 1999 it increased steadily, reaching a high point of about \$46,000 in 1999, which was followed by a decrease until 2004.

3. Graph B is the best match for the situation. It is the only graph that represents the child stopping at the top of the slide with a speed of zero for a few minutes.

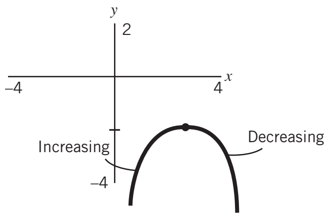
4.



5. a. Graph that is concave up with minimum at $(-2, 1)$



b. Graph that is concave down with maximum at $(3, -2)$



Exercises for Section 1.5

1. a. Graph B
b. Graph A
c. Graph A
d. Graph B

3. The price of Rockwell Collins hovered under \$30 at the beginning of the week and then rose to \$32 on Wednesday and then slowly climbed for the rest of the week. The price of Transkaryotic, by contrast, rose from \$16.50 on Friday to \$17 on Monday and then dropped to \$13 on Tuesday and hovered near \$13 for the rest of the week.

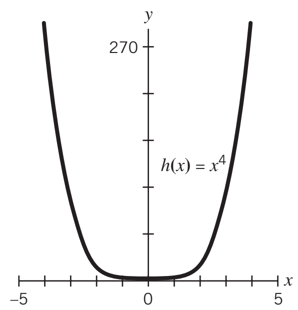
5. a. Positive over $(-5, 0)$ and $(5, +\infty)$
b. Negative over $(0, 5)$ and $(-\infty, -5)$
c. Decreasing over $(-3, 3)$
d. Increasing over $(-\infty, -3)$ and $(3, +\infty)$
e. There is no minimum.
f. There is no maximum.

7. a. A, domain $= (-\infty, +\infty)$ and range $= (-\infty, 2)$
B, domain $= [0, +\infty)$ and range $= [0, +\infty)$
C, domain $= (-\infty, +\infty)$ and range $= (0, +\infty)$
D, domain $= (-\infty, +\infty)$ and range $= [2, +\infty)$
b. A, $(-5, -1)$, B, $(0, +\infty)$, C, $(-\infty, +\infty)$, D, $(-\infty, +\infty)$
c. A, $(-\infty, -5)$ and $(-1, +\infty)$ B, nowhere
C, nowhere D, nowhere

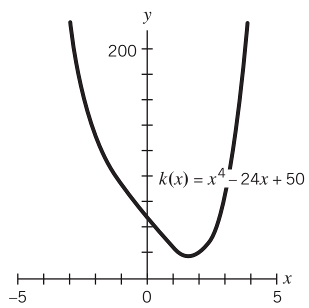
9. a. $[-6, -3)$ and $(5, 11)$
b. $(-3, 5)$ and $(11, 12]$
c. $(-6, 2)$ and $(8, 12)$
d. $(2, 8)$
e. Concave down approximately over interval $(0, 5)$; concave up approximately over interval $(5, 8)$.
f. $f(x) = 4$ when $x = 1$ and 3.
g. $f(-8)$ is not defined.

11. a. Graph B seems best. It indicates several stops, rises and falls in speed and, most importantly, it is the only one that ends with a stop.
b. Graph E is suitable. The horizontal parts on the graph indicate a time at which the bus stopped. Graphs D and F seem out of place. They would indicate that the bus went backwards.

13. a. Graph of $h(x) = x^4$



b. Graph of $k(x) = x^4 - 24x + 50$



The graphs of $h(x)$ and $k(x)$ are everywhere concave up.

15. a. 1996 to 1997, 1998 to 1999, 2000 to 2001.
b. 1995 to 1996, 1997 to 1998, 1999 to 2000, 2001 to 2003.
c. 17 billion dollars in 1999.
d. 10 billion dollars in 2003.

17. a. Johnsonville's population goes from $2.4 \cdot 100{,}000 = 240{,}000$ to $5.8 \cdot 100{,}000 = 580{,}000$. Palm City's population ranges from $1.8 \cdot 100{,}000 = 180{,}000$ to a high of $3.8 \cdot 100{,}000 = 380{,}000$. (This notation adheres to what is found in the graph.)
b. The population of Palm City increased from 1900 to 1930.
c. The population of Palm City decreased from 1930 to 1990.
d. The two populations were equal sometime around 1940.

19. a. Yes, P is a function of Y since the inputs are all distinct.
b. The domain is the set of years from 1990 to 1995 inclusive; the range is the set of corresponding values, namely $\{-0.5, 0, 1.2, 1.4, 2.3\}$.
c. The maximum P value is 2.3; it occurs when $Y = 1991$.
d. P is increasing from 1990 to 1991 and from 1993 to 1994. It is decreasing from 1991 to 1993 and 1994 to 1995.
e. Y is not a function of P, since the two inputs of 1.4 have different outputs.

21. **a.** From 1750 to 2100.

b. Yes, from 2100 to 2200.

c. Student answers will vary; it is hoped that something is said about the dramatic increase and the slowing down noted in parts (a) and (b).

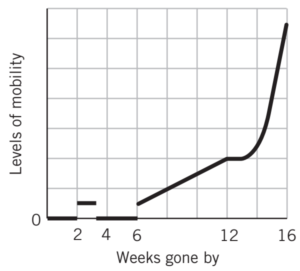
23. **a.** Male and female enlisted reserve personnel reached their respective maxima in 1992.

b. The maximum for males was approximately 1350 (thousand) and for women it was approximately 215 (thousand).

c. For men, there was a steady rise from 1990 to 1992 and then a gradual decline, reaching a low of 800,000 in 2004. For women there was a sharp rise from 1990 to 1992; then a steady decline to 175,000 until 1998; and then a slight rise from 1998 to 2001, when it climbed to 175,000 and a gradual decline since then to 2004, when it reached 170,000.

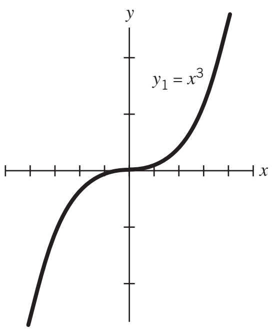
d. Answers will vary.

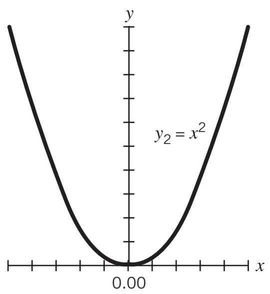
25.

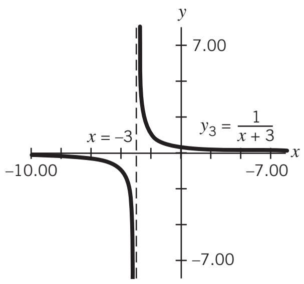


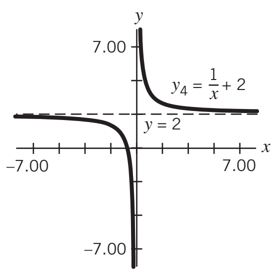
Here is one graph. Phrases like "after a while" and "for a while" were loosely interpreted. As for "levels of mobility," the idea was to give a good picture by means of a graph without being held to numerical values. Time intervals, when explicitly given, were respected. There are discontinuities in the going-to-crutches phases.

27. Here are the graphs asked for:









a. As $x \to +\infty$

y_1 approaches $+\infty$, y_2 approaches $+\infty$,

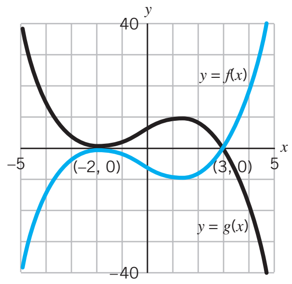
y_3 approaches 0; y_4 approaches 2

b. As $x \to -\infty$

y_1 approaches $-\infty$, y_2 approaches $+\infty$,

y_3 approaches 0 y_4 approaches 2.

29. Clearly one cannot simulate the session here. But one can give the graphs of f and g (see the accompanying diagram) and note that the graph of g is the mirror image of the graph of f with respect to the x-axis.



Ch. 1: Check Your Understanding

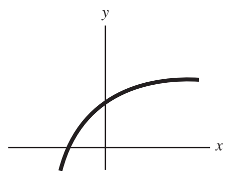
1. True	**8.** False	**15.** True	**22.** True
2. False	**9.** True	**16.** False	**23.** True
3. True	**10.** False	**17.** True	**24.** False
4. True	**11.** True	**18.** False	**25.** True
5. True	**12.** False	**19.** False	**26.** True
6. True	**13.** False	**20.** True	**27.** True
7. False	**14.** True	**21.** True	

28. Possible answer: $z^4 = 2w + 3$

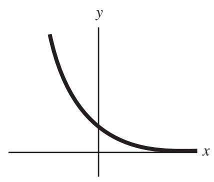
29. Possible answer:

z	1	2	3	4
w	10	−3	9	−3

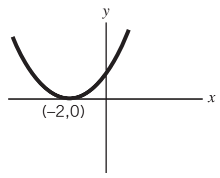
30. Possible answer:



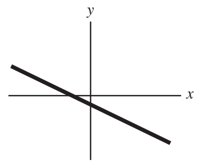
31. Possible answer:



32. Possible answer:



33. Possible answer:



34. Possible answer: The number of wolf pups born and the number that have survived in Yellowstone National Park have both increased significantly since 1995.

35. False **37.** False **39.** False **41.** True

36. False **38.** True **40.** False **42.** True

Ch. 1 Review: Putting It All Together

1. a. 25%

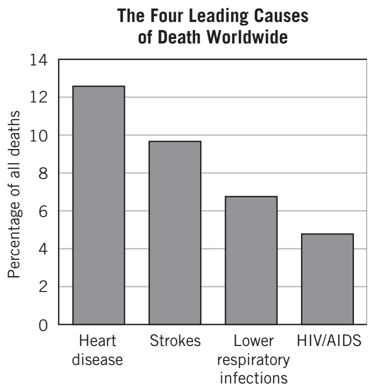
b. 20%

c. 40 lb is a smaller percentage of 200 lb (his weight at age 60) than it is of 160 lb (his weight at age 20).

3. This could be explained by changes in the distribution of incomes. For example, some incomes (originally below the median) could have increased to more than the previous median (to raise the median), while other incomes decreased on either side of the median (to lower the mean). To simultaneously decrease the mean while raising the median, the decrease in incomes would need to be greater than the increase in incomes. For example, for a small set of numbers such as {5, 15, 20, 50, 110}, the median is 20 and a mean is 40. If we change the set such that 5 increases to 25 and 110 decreases to 80, then the new set of {15, 20, 25, 50, 80} would have a median of 25 and a mean of 38.

5. a. Heart disease

b.



c. Sixty-six percent of the deaths were not accounted for. For example, tuberculosis, malaria, lung cancer, war, and car accidents are not included. No other cause can account for more than 5% of deaths, or else it would supersede HIV/AIDS and be one of the four leading causes.

7. $E = 0.025P$

9. Answers will vary. Possible title and summary: "The Popularity of the Name Emma over Time." Emma was a very popular name in 1880, when it was given to about 9,000 out of every million babies. Its use fell rapidly until it bottomed out in the 1970s, when it was given to about 200 out of every million babies. It has been rising in popularity since the 1980s, although it was only about half as popular in 2004 as it had been in 1880.

11. a. i. Function

ii. Not a function

iii. Function

iv. Not a function

b, c. Answers will vary.

13. a. Approx. 51%.

b. In November 2004.

c. There was a major increase between September and November of 2004; other lesser increases occurred between June and August of 2004 and between February 2005 and April 2005.

d. In June 2005 there was a difference of about 30% (approx. 63% minus 33%).

15. a.

d	1	2	3	4	10	20	100
y	8	16	24	32	80	160	800

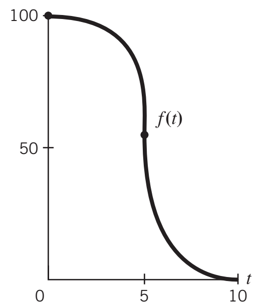
b. $y = 8d$ (d independent, y dependent)

c.

y	8	12	16	20	50	80	100
d	1	1.5	2	2.5	6.25	10	12.5

d. $d = (1/8)y$ (y independent, d dependant)

17.



19. a. Yes, it passes the vertical line test. Domain: [1970, 2010]; approx. range: [41, 63].

b. Approx. 1990; approx. 63 years of age.

c. Approx. 2003 (projected); approx. 41 years of age. AIDS is a likely candidate.

d. Increasing from approx. 1970 to 1990 and projected to increase from 2003 to 2010; decreasing from 1990 to 2003.

e. Concave down over interval (1970, 1997); concave up over interval (1997, 2010).

f. In 1970 the life expectancy in Botswana was about 52 years, as opposed to about 57 years worldwide. Over the next 20 years this gap gradually closed. In 1990 the life expectancy in Botswana was about 63, and 64 was the worldwide average. However, after 1990 the life expectancy in Botswana began to rapidly decline; this decline was expected to continue until about 2003, at which time it was projected to begin to increase slowly.

21. a. It means that 3% of the worldwide out-of-school population is in Latin America or the Caribbean.

b. It does not. It tells us only what percentage of the worldwide out-of-school population is in sub-Saharan Africa.

c. This graph does not show the number of children out of school worldwide or within each region. Nor does it show the percentage of children within each region who are out of school. The number of children out of school in a particular region may be relatively small compared to the total worldwide out-of-school population, but quite large compared to the total population of the region. This could be confusing.

d. Sub-Saharan Africa has the largest percentage of the world's out-of-school population. Seventy-three percent of the world's out-of-school population reside in sub-Saharan Africa or South Asia. Industrialized nations represent the smallest percentage.

e. Answers will vary. One idea is that UNICEF should consider focusing on sub-Saharan Africa and South Asia. This is where almost three-quarters of the children who are out of school can be found. Evidence from the graph should be cited.

23. a. $f(0) = -\frac{1}{2}$, $f(-1) = -\frac{1}{3}$, $f(2)$ is undefined

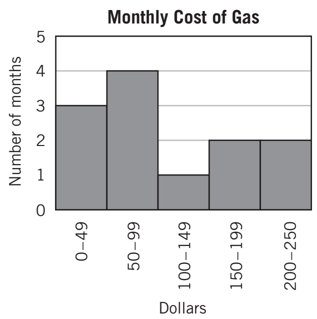
b. Domain: all real numbers, $x \neq 2$

Range: all real numbers, $f(x) \neq 0$.

25. a. February has the maximum cost and July the lowest. This makes sense, as heating costs would be less in the warm months and more in the cold ones.

b. Rounded off: mean = \$104.67 and median = \$86. Since the mean is higher than the median, together the mean and median suggest that there are a few expensive months, which account for most of the cost of gas for the year.

c.



d. Answers will vary. Some patterns: For one-third of the year, the cost of gas is between \$150 and \$250. Gas costs peak in February and then start to decrease throughout the spring and summer. In August they begin to increase, continuing throughout the fall and early winter.

CHAPTER 2

Section 2.1

Algebra Aerobics 2.1

1. $\frac{(143 - 135)\text{ lb}}{5\text{ yr}} = 1.6\,\frac{\text{lb}}{\text{yr}}$

2. a. i. $\frac{\$(820.2 - 537.2)\text{billion}}{(2006 - 1990)} = \frac{\$283\text{ billion}}{16\text{ yr}} \approx \$17.69\text{ billion/yr}$

ii. $\frac{\$(1{,}273.2 - 618.4)\text{billion}}{(2006 - 1990)} = \frac{\$654.8\text{ billion}}{16\text{ yr}} \approx \$40.93\text{ billion/yr}$

iii. $\frac{\$(-453.0 - (-81.2))\text{billion}}{(2006 - 1990)} = \frac{-\$371.8\text{ billion}}{16\text{ yr}}$

$\approx -\$23.24\text{ billion/yr}$

b. While both exports and imports were increasing, imports were increasing at a faster rate, and hence, the trade balance was decreasing. The trade deficit increased by over \$23 billion/year from 1990 to 2006.

3. a. $\frac{(41.8 - 52.1)\text{ thousand deaths}}{20\text{ yrs}} \approx \frac{-0.52\text{ thousand deaths}}{\text{yr}}$,

or a decrease of 520 deaths per year.

b. $\frac{(42.6 - 41.8)\text{ thousand deaths}}{4\text{ yrs}} = \frac{0.2\text{ thousand deaths}}{\text{yr}}$,

or an increase of 200 deaths per year.

4. 60 mph/5 sec = 12 mph per second

5. $\frac{(\$867 - \$689)}{6\text{ yrs}} \approx \$29.67/\text{yr}$

6. (978 − 1056)/4 = −78/4 = −19.5. On the average, his performance was declining by 19.5 yards per year.

7. (30,000 − 150,000)/10 = −12,000 elephants per year. On the average, the African elephant population is decreasing by 12,000 elephants per year.

Exercises for Section 2.1

1. a. inches/pound **b.** minutes/inch **c.** pounds/inch

3. 212 miles/10.8 gal. ≈ 19.6 miles per gallon

5. $\frac{7.5 - 9.4}{2000 - 1960} = \frac{-1.9}{40} = -0.0475$ pounds per year

7. a. Math: $\frac{504 - 498}{2005 - 2000} = \frac{6}{5} = 1.2$ points per year.

b. Verbal: $\frac{505 - 504}{2005 - 2000} = \frac{1}{5} = 0.2$ point per year.

9. a. $\frac{13{,}600{,}000 - 630{,}000}{2005 - 1985} = \frac{12{,}970{,}000}{20} = 648{,}500$ computers per year.

b. $\frac{4.0 - 84.1}{2005 - 1985} = \frac{-80.1}{20} = -4.005 \approx -4$ students per computer per year, or a decrease of 4 students per computer each year.

11. a. $\frac{500{,}000 - 4{,}000{,}000}{2000 - 1930} = \frac{-3{,}500{,}000}{70} = -50{,}000$;

i.e., on average there were 50,000 fewer elephants per year.

b. In 1980 the rate was twice as large as the rate computed in part (a) for 1930 to 2000. That means that either before or after the 1980s, the average rate of decline must have been much smaller.

13. a. For whites: $\frac{85.8 - 26.1}{2004 - 1940} = \frac{59.7}{64} \approx 0.9$ percentage points per year.

For blacks: $\frac{80.6 - 7.3}{2004 - 1940} = \frac{73.3}{64} \approx 1.1$ percentage points per year.

For Asian/Pacific Islanders: $\frac{85.0 - 22.6}{2004 - 1940} = \frac{62.4}{64} \approx 1.0$ percentage point per year.

For all: $\frac{85.2 - 24.5}{2004 - 1940} = \frac{60.7}{64} \approx 0.9$ percentage point per year.

b. In 2007:

For whites: $85.8 + 3 \cdot 0.9 = 88.5\%$ will have completed 4 years of high school.

For blacks: $80.6 + 3 \cdot 1.1 = 83.9\%$.

CH. 2 Algebra Aerobics Solutions for Section 2.2

For Asian/Pacific Islanders: $85.0 + 3 \cdot 1.0 = 88.0\%$.

For all: $85.2 + 3 \cdot 0.9 = 87.9\%$.

c. There was a major increase in the percentage of those who completed 4 years of high school or more in all four categories. Over the 64-year period the increases ranged from 59.7 percentage points for whites to 73.3 percentage points for blacks. Blacks showed the highest average gain: 1.14 percentage points per year. Other comments could be made.

d. For x years since 1940:

Whites: $100 = 26.1 + 0.9x \Rightarrow x \approx 82.1$ years, or in early 2022.

Blacks: $100 = 7.3 + 1.1x \Rightarrow x \approx 84.3$ years, or in early 2024.

Asian Pacific Islanders: $100 = 22.6 + 1.0x \Rightarrow x = 77.4$ years, or in mid-2017.

All: $100 = 24.5 + 0.9x \Rightarrow x = 83.9$, or in late 2023.

These 100% predictions may not make much sense since the time is too far into the future to allow a reliable prediction.

15. a. White females had the highest life expectancy in 1900 and in 2005. Black males had the lowest life expectancy in 1900 and in 2005.

b. For white males: $\frac{75.4 - 46.6}{2005 - 1900} = \frac{28.8}{105} \approx 0.27$ years per year.

For white females: $\frac{81.1 - 48.7}{2005 - 1900} = \frac{32.4}{105} \approx 0.31$ years per year.

For black males: $\frac{69.9 - 32.5}{2005 - 1900} = \frac{37.4}{105} \approx 0.36$ years per year.

For black females: $\frac{76.8 - 33.5}{2005 - 1900} = \frac{43.3}{105} \approx 0.41$ years per year.

Thus black females had the largest average rate of change in life expectancy between 1900 and 2005.

c. In all four groups the average rate of change of life expectancy rose over the 105 years. White males had the smallest increase, 0.27 years per year, and black females had the largest increase, 0.41 years per year.

Section 2.2

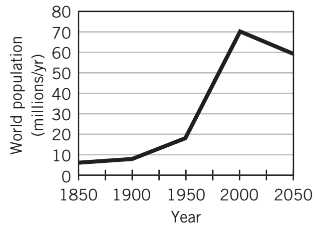
Algebra Aerobics 2.2

1. a.

World Population

Year	Total Population (in millions)	Average Rate of Change over Prior 50 yrs.
1800	980	n.a.
1850	1260	$\frac{(1260 - 980)\text{ million}}{(1850 - 1800)} = \frac{280\text{ million}}{50\text{ yrs}} = 5.6$ million/yr
1900	1650	$\frac{(1650 - 1260)\text{ million}}{(1990 - 1850)} = \frac{390\text{ million}}{50\text{ yrs}} = 7.8$ million/yr
1950	2520	$\frac{(2520 - 1650)\text{ million}}{(1950 - 1900)} = \frac{870\text{ million}}{50\text{ yrs}} = 17.4$ million/yr
2000	6090	$\frac{(6090 - 2520)\text{ million}}{(2000 - 1950)} = \frac{3570\text{ million}}{50\text{ yrs}} = 71.4$ million/yr
2050	9076	$\frac{(9076 - 6090)\text{ million}}{(2050 - 2000)} = \frac{2986\text{ million}}{50\text{ yrs}} = 59.72$ million/yr

b. Graph of average rate of change over prior 50 yrs.



c. During 1950–2000, average annual rate of change = 71.4 million/yr was the greatest.

d. The average rate of change increased in the time interval 1800 to 2000, yet despite a projected increase in world population from 2000 to 2050, the rate of change is projected to decrease to 59.72 million/yr.

2. From 2003 to 2005 the graph would rise, indicating that profits increased. From 2005 to 2006, the graph would be flat, so there would be no change in profits. From 2006 to 2007, the graph would fall, indicating that profits decreased. (Note, however, that a lower profit does not imply a loss.)

3. a. **High School Completers**

Year	Number (thousands)	Average Rate of Change (thousands/yr)
1960	1679	n.a.
1970	2757	107.8
1980	3089	33.2
1990	2355	−73.4
2000	2756	40.1
2004	2752	−1.0

Table 2.7

b. The number of individuals completing high school each year rose between 1960 and 1980, when it peaked at 3,089,000. The 1960s had the greatest rate of change, with an annual increase of 107.8 (thousand) per year. The rate slowed during the 1970s.

In the decade from 1980 to 1990, the number of high school completers each year showed a drastic decline on average of 73,400 per year. The trend reversed in the next decade (1990 to 2000), increasing on average by 40,100 per year. Between 2000 and 2004 the numbers remained almost the same, with only a slight decrease of 4000 over the 4 years.

c. If the average rate of change is positive, there is an increase in the number of high school completers. An example would be from 1970 to 1980, where the number of high school completers increased from 2,757,000 to 3,089,000.

d. If the average rate of change is negative, there is a decrease in the number of high school completers. An example would be from 1980 to 1990, where the number of high school completers decreased from 3,089,000 to 2,355,000.

e. The growth is slowing down.

Exercises for Section 2.2

1.

x	$f(x)$	Avg. Rate of Change
0	0	n.a.
1	1	1
2	8	7
3	27	19
4	64	37
5	125	61

a. The function is increasing.

b. The average rate of change is also increasing.

3. a.

Year	Registered Motor Vehicles (millions)	Average Annual Rate of Change (over prior decade)
1960	74	n.a.
1970	108	3.4
1980	156	4.8
1990	189	3.3
2000	218	2.9

b. From 1990 to 2000.

c. From 1970 to 1980.

d. Student paragraphs might include the observation that from 1960 to 2000 there was a continuous rise in the number of registered motor vehicles in the United States (from 74 million to 218 million). The largest average rate of change (4.8 million per year) was in the decade from 1970 to 1980 and the lowest average rate of change (2.9 million per year) was from 1990 to 2000.

5.

I.

x	$f(x)$	Average Rate of Change
0	5	n.a.
10	25	2
20	45	2
30	65	2
40	85	2
50	105	2

a. $f(x)$ is increasing.

b. The average rate of change is constant.

II.

x	$g(x)$	Average Rate of Change
0	270	n.a.
10	240	−3
20	210	−3
30	180	−3
40	150	−3
50	120	−3

a. $g(x)$ is decreasing.

b. The average rate of change is constant.

7. a. Graph *B* **b.** Graph *A* **c.** Graph *C*

9. a.

Table A

x	y	Average Rate of Change
0	2	n.a.
1	5	3
2	8	3
3	11	3
4	14	3
5	17	3
6	20	3

b. The average rate of change is constant.

c. Its graph is a straight line.

Table B

x	y	Average Rate of Change
0	0	n.a.
1	0.5	0.5
2	2	1.5
3	4.5	2.5
4	8	3.5
5	12.5	4.5
6	18	5.5

b. The average rate of change is increasing.

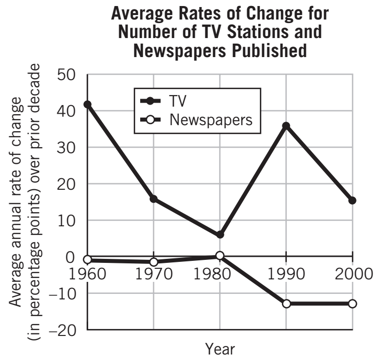
c. Its graph is concave up.

11. a. In 1920 there were $\frac{27{,}791{,}000}{106{,}000{,}000} = 0.3$ copy of newspapers printed per person (about one-third of a copy per person, or equivalently, roughly one copy for every three people).

In 2000 there were $\frac{55{,}800{,}000}{281{,}400{,}000} = 0.2$ copy of newspapers printed per person (about one-fifth of a copy per person, or equivalently, about one copy for every five people).

b.

Year	No. of TV Stations	Avg. Annual Rate of Change (over prior decade)	No. of Newspapers Published	Avg. Annual Rate of Change (over prior decade)
1950	98	n.a.	1772	n.a.
1960	515	41.7	1763	−0.9
1970	677	16.2	1748	−1.5
1980	734	5.7	1745	−0.3
1990	1092	35.8	1611	−13.4
2000	1248	15.6	1480	−13.1



c. From the table in part (b), the average rate of change from 1990 to 2000 was 15.6 stations per year. At this rate, in 2010 there will be $1248 + 15.6 \cdot 10 = 1404$ stations. This could be off, given that the avg. rate per year for each decade vacillates a fair amount.

d. It seems that news is being more disseminated through TV stations than through newspapers. But there is not enough information to really give an answer here. A great increase in TVs does not necessarily indicate a great increase in getting the news via TV. Many other programs besides the news are watched on TV.

Section 2.3

Algebra Aerobics 2.3

1. a. i. $m = (11 - 1)/(8 - 4) = 10/4 = 2\frac{1}{2}$

ii. $m = (6 - 6)/[2 - (-3)] = 0$

iii. $m = [-3 - (-1)]/[0 - (-5)] = -\frac{2}{5}$

b. i. $m = (1 - 11)/(4 - 8) = (-10)/(-4) = 2\frac{1}{2}$

ii. $m = (6 - 6)/(-3 - 2) = 0$

iii. $m = [-1 - (-3)]/(-5 - 0) = 2/(-5) = -\frac{2}{5}$

2. Between 1999 and 2002 the slope is approximately zero.

3. positive: 2001–2002, 2004–2005
negative: 1998–2000, 2003–2004
zero: 2000–2001, 2002–2003

4. $\frac{\Delta y}{\Delta x} = 4 \Rightarrow \frac{y - (-2)}{5 - 3} = 4 \Rightarrow \frac{y + 2}{2} = 4 \Rightarrow y + 2 = 8 \Rightarrow y = 6.$

5. $\frac{\Delta y}{\Delta x} = \frac{(9 + 2h) - 9}{(2 + h) - 2} = \frac{2h}{h} = 2$

6. a. Points lie on the graph $\Rightarrow$ the coordinates (x, y) satisfy the equation $y = x^2$ since

$$0 = 0^2, \quad 1 = 1^2, \quad 4 = 2^2, \quad 9 = 3^2.$$

b. P_1 and P_2: $m = \frac{1 - 0}{1 - 0} = 1$; P_2 and P_3: $m = \frac{4 - 1}{2 - 1} = 3$; P_3 and P_4: $m = \frac{9 - 4}{3 - 2} = 5$.

c. The positive slopes suggest that the function increases between $x = 0$ and $x = 3$; because the slopes increase in size as we move further to the right, the graph rises at an increasing rate.

Exercises for Section 2.3

1. a. $m = \frac{3 - (-6)}{2 - (-5)} = \frac{9}{7}$ **b.** $m = \frac{-3 - 6}{2 - (-5)} = \frac{-9}{7}$

3. a. Graph A crosses the y-axis at $(0, -4)$ and goes through $(1, -4)$. So the slope is $\frac{(-4) - (-4)}{1 - 0} = 0$.

b. Graph B crosses the x-axis at (2, 0) and the y-axis at $(0, -8)$. So the slope is $\frac{-8 - 0}{0 - 2} = 4$.

5. a. slope of segment $A = [2 - (-6)]/[-4 - (-8)] = 8/4 = 2$
slope of segment $B = [-8 - 2]/[0 - (-4)] = -10/4 = -2.5$
slope of segment $C = [-8 - (-8)]/[2 - 0] = 0/2 = 0$
slope of segment $D = [-6 - (-8)]/[6 - 2] = 2/4 = 0.5$
slope of segment $E = [6 - (-6)]/[10 - 6] = 12/4 = 3$
slope of segment $F = [-16 - 6]/[12 - 10] = -22/2 = -11$

b. The slope of segment F is steepest.

c. Segment C has a slope equal to 0.

7. a. $m = 75/10 = 7.5$

b. $70 - y = -4 \Rightarrow y = 74$

c. $32 = 4(28 - x) \Rightarrow 32 = 112 - 4x \Rightarrow x = 20$

d. $6 = 0.6(x - 10) \Rightarrow 12 = 0.6x \Rightarrow x = 20$

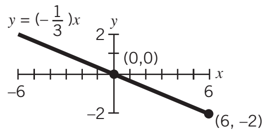
9. a. $-4 = (1 - t)/(-2 - 3) \Rightarrow 20 = 1 - t \Rightarrow t = -19$

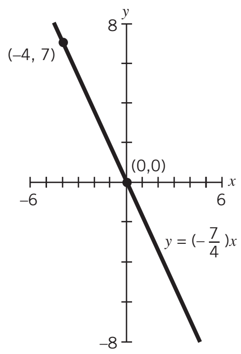
b. $2/3 = (9 - 6)/(t - 5) \Rightarrow 2(t - 5) = 9 \Rightarrow t - 5 = 4.5 \Rightarrow t = 9.5$

11. a. $m_1 = (7 - 3)/(4 - 2) = 4/2 = 2$; $m_2 = (15 - 7)/(8 - 4) = 2$; collinear

b. $m_1 = (4 - 1)/(2 + 3) = 3/5$; $m_2 = (8 - 1)/(7 + 3) = 0.7$; not collinear

13. a. $m = \frac{-2 - 0}{6 - 0} = -\frac{1}{3}$ **b.** $m = \frac{7 - 0}{-4 - 0} = -\frac{7}{4}$





15. a. $\frac{0 - \sqrt{2}}{\sqrt{2} - 0} = -1$

b. $\frac{0 + 3/2}{-3/2 - 0} = -1$

c. $\frac{0 - b}{b - 0} = -\frac{b}{b} = -1$

d. All slopes are -1, since each pair of points is of the form (0, a) and (a, 0).

17. The points in parts (a) and (c).

19. a. 1/10

b. 1/12

c. old: $\frac{3}{\text{run}} = \frac{1}{10} \Rightarrow$ run = 30 ft.; new: $\frac{3}{\text{run}} = \frac{1}{12} \Rightarrow$ run = 36 ft.

21. Student answers will vary.

Section 2.4

Algebra Aerobics 2.4

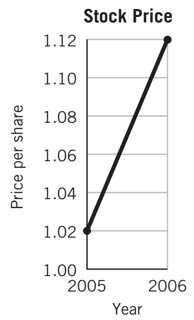
1. a. (1960, 22.2) and (2000, 11.3)

b. (1970, 12.6) and (2005, 12.6)

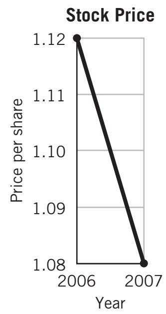
c. (2000, 11.3) and (2005, 12.6)

2. a. Between 2005 and 2006 the stock price surged from \$1.02 to \$1.12 per share, a 9.8% increase or, equivalently, an increase of \$0.10 per share. See Graph A.

b. Between 2006 and 2007 the stock price dropped drastically from \$1.12 to \$1.08 per share, about a 3.6% decrease or, equivalently, a decrease of \$0.04 per share. See Graph *B*.



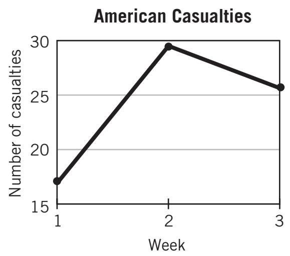
Graph *A*



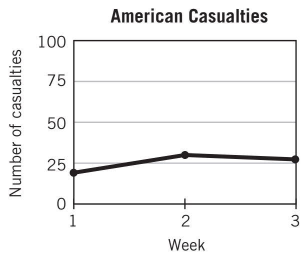
Graph *B*

3. a. You could draw a graph that is cropped and stretched vertically, as shown in Graph *A*.

b. You could show a graph that is not too steep (stretched horizontally or compressed vertically) and emphasize the decline in the number of casualties in week three, as in Graph *B*.



Graph *A*



Graph *B*

4. Some of the strategies are: use of dramatic language (the title "Gold Explodes" in big bold type, followed by "Experts Predict \$1,500.00 an Ounce"; cropped vertical axis on graph (it starts at \$300, not \$0, making the graph look steeper); use of powerful graphics (the arrow that increases in size), suggesting dynamic growth.

Exercises for Section 2.4

1. Graph *A*: "30-Year Mortgage Rates Steadily Climb"

Graph *B*: "30-Year Mortgage Rates Rise Sharply"

Graph *C*: "30-Year Mortgage Rates Show Little Gain"

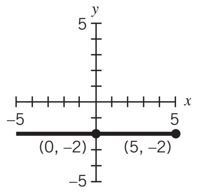
3. a. Graph *B* appears to have the steeper slope.

b. The slope of Graph *A* is -6 and the slope of Graph *B* is -4. So Graph *A* actually has the steeper slope.

5. a. From 1984 to 2005 the federal appropriations for the most part rose, but the reading scores stayed the same.

b. There is no relationship between the scale on the left (for federal funding) and the scale on the right (for NAEP scores). The NAEP scale was arbitrarily started at 185. Had it started at 0, the NAEP scores would still remain constant, but would lie much higher on the graph.

7. a.



b, c. The task is impossible. The slope of the line is 0, so it is not possible to make a graph through the given points appear to have a large positive slope.

9. "Precipitous," "dire," and "catastrophic" come to mind. Students will probably think of others.

11. a. The number of persons with AIDS who are still alive has been steadily growing since 1995 by approximately $\frac{425{,}000 - 200{,}000}{2004 - 1995} = 25{,}000$ people per year. Also, deaths from AIDS have declined since 1993 from 55,000 to less than 20,000 (using left scale).

b. Despite the progress in treating AIDS and education about it, there has been an increase in cases diagnosed since 2000 of about 1000 per year.

c. AIDS cases increased until 1993, when new treatments, public awareness, education, and testing for the disease may have caused a decrease in the number of new cases.

13. Graph *A* gives the appearance that the percentage has declined quite a bit, while Graph *B* gives the impression that the percentage has not declined much at all. The difference is in the vertical scale: it is greatly magnified in Graph *A*.

Section 2.5

Algebra Aerobics 2.5

1. The weight of a 4.5-month-old baby girl appears to be ≈ 14 lb. From the equation, the exact weight is

$W = 7.0 + 1.5(4.5) = 13.75$ lb. Our estimate is within 0.25 lb of the exact weight.

2. The age appears to be ≈ 2.5 months. Solve $11 = 7 + 1.5A \Rightarrow 11 - 7 = 1.5A \Rightarrow 4 = 1.5A \Rightarrow A = 2.67$ months.

3. a. The units for 15 are dollars/person. The units for 10 are dollars.

b. dollars = (dollars/person)(persons) + dollars

4. a. The units for 1200 are dollars and for 50 are dollars/month.

b. dollars = dollars + $\left(\frac{\text{dollars}}{\text{month}}\right)$months

5. a. 0.8 million dollars per year is the slope or average rate of change in the sales per year.

b. 19 million dollars represents the sales this year.

c. In 3 years $S = 0.8(3) + 19 = 21.4$ million dollars

d. dollars = $\left(\frac{\text{dollars}}{\text{year}}\right)$years + dollars, where all values for dollars are in millions.

6. a. The average cost to operate a car is \$0.45 per mile, and the units are dollars per mile.

b. $0.45(25{,}000) = \$11{,}250$

c. dollars = $\left(\frac{\text{dollars}}{\text{mile}}\right)$miles

7. a. \$840

b. The beginning mortgage is \$302,400.

c. After 10 years, B = \$302,400 − 840(12)(10) = \$201,600;
After 20 years, B = \$302,400 − 840(12)(20) = \$100,800;
After 30 years, B = \$302,400 − 840(12)(30) = \$0.

8. a. $m = 5, b = 3$

b. $m = 3, b = 5$

c. $m = 5, b = 0$

d. $m = 0, b = 3$

e. $m = -1, b = 7.0$

f. $m = -11, b = 10$

g. $m = -2/3, b = 1$

h. $m = 5, b = -3$ since $2y = 10x - 6 \Rightarrow y = 5x - 3$

9. a. $f(x)$ is a linear function because it is represented by an equation of the form $y = mx + b$, where here $b = 50$ and $m = -25$.

b. $f(0) = 50 - 25(0) = 50 - 0 = 50$
$f(2) = 50 - 25(2) = 50 - 50 = 0$

c. Since the line passes through (0, 50) and (2, 0), the slope m is: $m = \frac{50-0}{0-2} = -25$.

10. a. linear: $m = 3, b = 5$

b. linear: $m = 1, b = 0$

c. not linear

d. linear: $m = -2/3, b = 4$

11. a. $y = 3x + 4$

b. $y = -x$

c. $y = -3$

12. Graph A: $y = 4x + 3$
Graph B: $y = -2x + 0 = -2x$
Graph C: $y = 0x + 3 = 3$
Graph D: $y = x + 0 = x$

Exercises for Section 2.5

1. a. $E = 5000$ when $n = 0$, $E = 5100$ when $n = 1$, and $E = 7000$ when $n = 20$

b. (0, 5000), (1, 5100) and (20, 7000)

3. a. (5000, 0) is not a solution to either equation.

b. (15, 24000) is a solution to the second equation but not the first.

c. (35, 40000) is a solution to the second equation but not the first.

5. a. D = 3.40, 3.51, 3.62, 3.73, and 3.84, respectively

b. 0.11 is the slope; it represents the average rate of change of the average consumer debt per year; it is measured in thousands of dollars per year.

c. 3.40 represents the average consumer debt when $n = 0$ years. It is measured in thousands of dollars.

7. dollars = dollars + $\frac{\text{dollars}}{\text{year}}$ · years

9. a. $C(0)$ = \$11.00; $C(5) = 11 + 10.50 \cdot 5$ = \$63.50;
$C(10) = 11 + 10.50 \cdot 10$ = \$116.00

b. \$11.00

c. \$10.50 for every thousand cubic feet.

d. $C(96)$ = \$1019

11. a. 0; 4 **b.** 0; π **c.** 0; 2π **d.** −17.78; 5/9

13. a. matches **f**, since \$42.50 seems the most likely cost for producing one text.

b. matches **g**, since \$0.30 seems the most likely cost to produce one CD.

c. matches **e**, since \$800 seems the most likely cost of producing one computer.

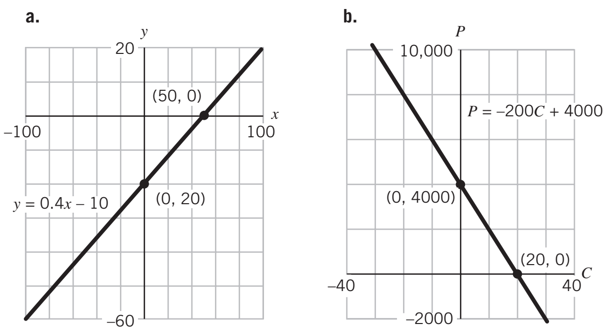
15. a. $C(p) = 1.06 \cdot p$

b. $C(9.50) = 1.06 \cdot 9.50 = 10.07$;
$C(115.25) = 1.06 \cdot 115.25 = 122.17$ (rounded up);
$C(1899) = 1.06 \cdot 1899 = 2012.94$. All function inputs and outputs are measured in dollars.

17. a. hours **b.** miles/gallon **c.** calories/gram of fat

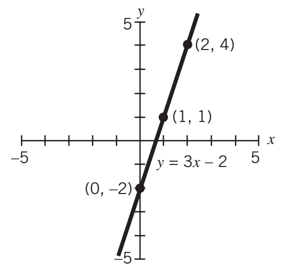
19. a. Slope = 0.4, vertical intercept = −20

b. Slope = −200, vertical intercept = 4000



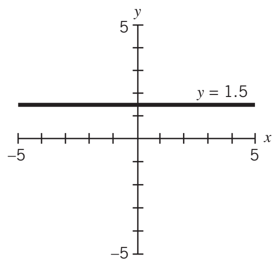
21. The equation is: $y = 3x - 2$

x	y
0	−2
1	1
2	4



23. The equation is $y = 0 \cdot x + 1.5 = 1.5$.

x	y
−5	1.5
0	1.5
5	1.5



Section 2.6

Algebra Aerobics 2.6

1. 0, $|-1|$, $|-3|$, 4, $|-7|$, 9, $|-12|$

2. a. The graph of $y = 6x$ goes through the origin and is two units above the graph of $y = 6x - 2$.

b. The graph of $y = 2 + 6x$ intersects the y-axis at (0, 2). The lines are parallel, so $y = 2 + 6x$ is four units above the graph of $y = 6x - 2$.

c. The graph of $y = -2 + 3x$ has the same y-intercept $(0, -2)$ but is less steep than the line $y = 6x - 2$.

d. The graph of $y = -2 - 2x$ has the same y-intercept $(0, -2)$ but is falling left to right, whereas $y = 6x - 2$ is rising and has a steeper slope.

3. For (a), $m = -2$, so $|m| = |-2| = 2$

For (b), $m = -1$, so $|m| = |-1| = 1$

For (c), $m = -3$, so $|m| = |-3| = 3$

For (d), $m = -5$, so $|m| = |-5| = 5$

Line d is the steepest. In order from least steep to steepest, we have b, a, c, d.

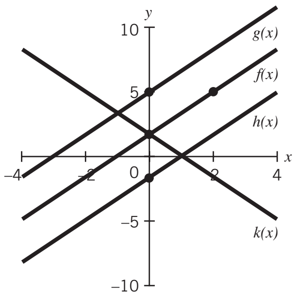
4. Answers will vary depending on the original $f(x)$. For example, if $f(x) = 2 + 2x$, then we could have:

a. $g(x) = 5 + 2x$

b. $h(x) = -2 + 2x$

c. $k(x) = 2 - 2x$

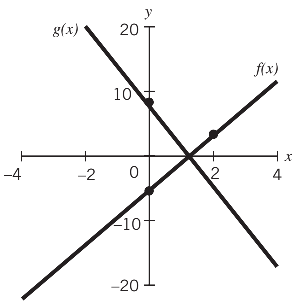
See graph below



5. For $f(x) = 3x - 5$, $m = 3$.

For $g(x) = 7 - 8x$, $m = -8$.

$g(x)$ has a steeper slope than $f(x)$ since $|-8| > |3|$



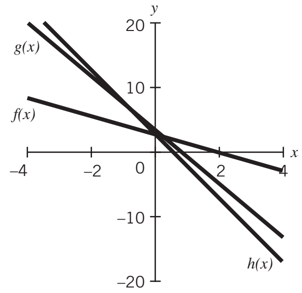
6. Answers will vary. For example,

$f(x) = 4 - 2x$

$g(x) = 4 - 5x$

$h(x) = 4 - 7x$

$h(x)$ has the steepest slope since $|-7| > |-5| > |-2|$



7. $f(x)$ matches Graph B, $g(x)$ matches Graph D, $h(x)$ matches Graph A, $k(x)$ matches Graph C.

Exercises for Section 2.6

1. a. Graph B

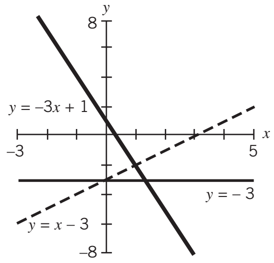
b. Graphs A, B, and D

c. Graph C

d. Graphs A and D

3. Answers may vary for (b) and (c).

a. $y = -3$ **b.** $y = x - 3$ **c.** $y = -3x + 1$



5. a. matches Graph B

b. matches Graph D

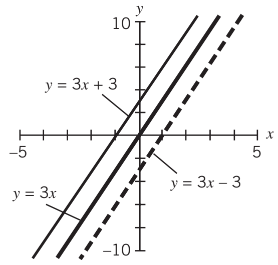
c. matches Graph C

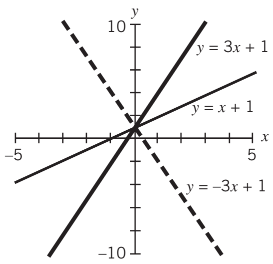
d. matches Graph A

7. Graphs may vary.

a. Same slope

b. Same vertical intercept





9. a. $R(t) = 5 + 13 - 5t = 18 - 5t$

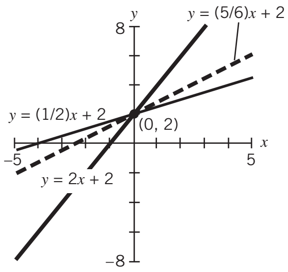
b. $S(t) = -3 + 13 - 5t = 10 - 5t$

c. $T(t) = 13 + 2t$ (slope may vary)

d. $U(t) = 12 - 5t$ (vertical intercept may vary)

e. $V(t) = 15 + 5t$ (vertical intercept may vary)

11.



13. a. *C*

b. *B*

c. $m_3 < m_2 < m_1$ (Note: m_3 is negative, while m_2 and m_1 are both positive and $m_2 < m_1$.)

d. The steepness is the absolute value of the slope. So although m_3 is negative, its absolute value is positive. Compared to the absolute values of the other slopes, we have $|m_2| < |m_1| < |m_3|$.

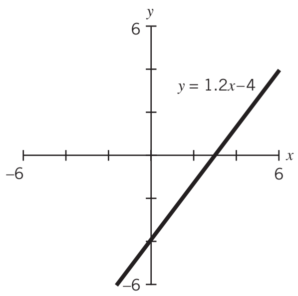
Section 2.7

Algebra Aerobics 2.7

1. a. $y = 1.2x - 4$

b.

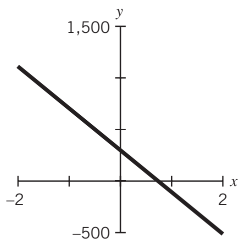
x	y
−3	−7.6
0	−4.0
3	−0.4



2. a. $y = 300 - 400x$

b.

x	y
−2	1100
0	300
2	−500

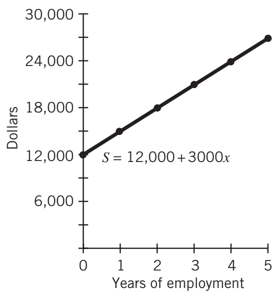


3. The vertical intercept is at (0, 1), so $b = 1$. The line passes through (0, 1) and (3, −5), so $m = \frac{1 - (-5)}{0 - 3} = -2$. The equation is: $y = -2x + 1$.

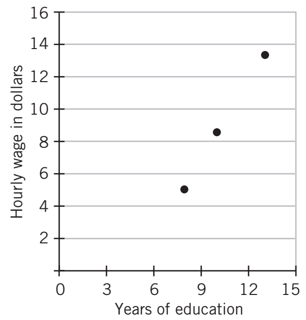
4. a. $S = \$12{,}000 + \$3000x$

b.

Year	$ Salary
0	12,000
1	15,000
3	21,000
5	27,000



5. a.



b. Yes. $\frac{8.50 - 5.30}{10 - 8} = \frac{3.20}{2} = 1.6$, $\frac{13.30 - 8.50}{13 - 10} = \frac{4.80}{3} = 1.6$, $\frac{13.30 - 5.30}{13 - 8} = \frac{8}{5} = 1.6$. Since the rate of change is 1.6, the same over all intervals, the points lie on the same line with slope 1.6.

c. If we choose the point $(8, 5.30) \Rightarrow x = 8$, $y = 5.30$. From part (b) we have $m = 1.6$.

Solve: $5.3 = (1.6)8 + b \Rightarrow 5.3 = 12.8 + b \Rightarrow -7.5 = b$, so the equation is $y = 1.6x - 7.5$.

6. 3

7. a. 4; (1, 4)

b. 4; (−1, 12)

8. a. $B = \$10{,}800 - 300P$

b. $300 per month

c. $B = \$10{,}800 - \$300(24) = \$3600$

d. $0 = \$10{,}800 - \$300P \Rightarrow \$300P = \$10{,}800 \Rightarrow P = 36$ or 36 months.

9. a. $R = 7.5T$

b. $7.50 per ticket

c. $R = \$7.50(120) = \$900.00$

10. Graph *A*: $y = x$

Graph *B*: $Q = 3t + 2$

Graph *C*: $y = 6 - 2x$

11. a. $R = 15{,}000 + 12T$

b. $R = 15{,}000 + 12(40{,}000) = \$495{,}000$

12. Any equation of the form $y = 6 + mx$ with three different values for m; some examples are $y = 6 + 2x$, $y = 6 - 5x$, and $y = 6 + 11x$.

13. Any equation of the form $y = -3x + b$ with three different values for b; some examples are $y = -3x + 5$, $y = -3x$, $y = -3x - 12$.

14. a. $y = 4x + 9$

b. $y = -\frac{2}{3}x + 7$

c. $y = -10x - 7$

d. $y = 2x + 2.1$

15. a. $m = \frac{6}{2} = 3 \Rightarrow y = 3x - 1$

b. $m = -\frac{1}{9} \Rightarrow y = -\frac{1}{9}x + \frac{5}{3}$

c. $m = \frac{-6}{-6} = 1 \Rightarrow y = x - 5$

16. a. $y = -\frac{3}{4}x - 3$

b. $y = 7x - 5$

c. $y = -\frac{1}{4}x + \frac{1}{8}$

d. $y = \frac{1}{2}x$

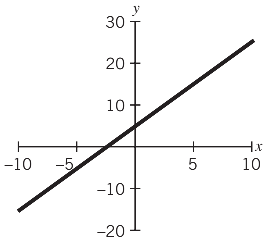
e. $y = 3x + 5$

f. $y = -5x + 6$

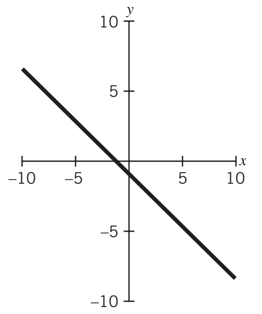
CH. 2 Exercises Solutions for Section 2.7

17. Note the difference in scales on all graphs.

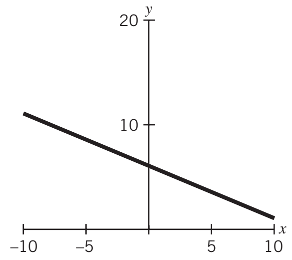
a. Graph of $y = 2x + 5$



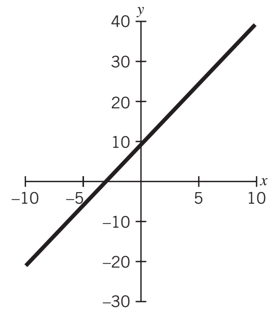
c. Graph of $y = -\left(\frac{3}{4}\right)x - 1$



b. Graph of $y = -\left(\frac{1}{2}\right)x + 6$



d. Graph of $y + 3x + 9$



Exercises for Section 2.7

1. a. Slope $= 5$, so equation is of the form $y = 5x + b$. The line passes through $(-2, 3)$,

$\Rightarrow 3 = 5(-2) + b$

$\Rightarrow$ b $= 13$. So $y = 5x + 13$

b. Slope $= -\frac{3}{4}$, so equation is of the form $y = -\frac{3}{4}x + b$. The line passes through $(-2, 3)$,

$\Rightarrow 3 = -\frac{3}{4}(-2) + b$

$\Rightarrow$ b $= 1.5$. So $y = -0.75x + 1.5$

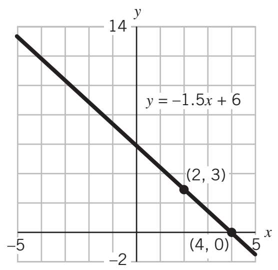
c. Slope $= 0$, so equation is of the form $y = b$. The line passes through $(-2, 3)$,

$\Rightarrow 3 = b$. So the equation is the horizontal line $y = 3$

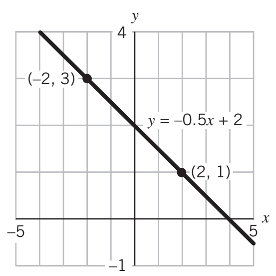
3. a. $m = (5.1 - 7.6)/(4 - 2) = -2.5/2 = -1.25$ and $y - 7.6 = -1.25(x - 2)$ or $y = -1.25x + 10.1$

b. $m = (16 - 12)/(7 - 5) = 4/2 = 2$ and $W - 12 = 2(A - 5)$ or $W = 2A + 2$

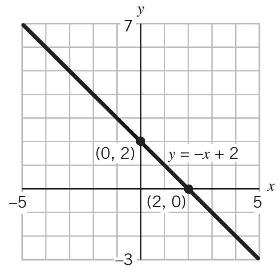
5. a. $y = -1.5x + 6$



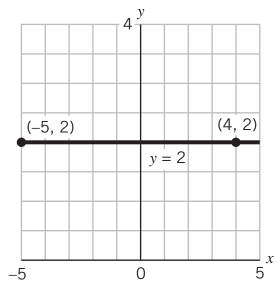
b. $y = -0.5x + 2$



c. $y = -x + 2$



d. $y = 2$



7. a. $y = \frac{2}{3}x - 2$; the slope is 2/3 and the y-intercept is -2.

b. $y = -\frac{3}{2}x + 3$; the slope is $-3/2$ and the y-intercept is 3.

c. $y = -\frac{2}{3}x + 12$; the slope is $-2/3$ and the y-intercept is 12.

d. $y = \frac{3}{2}x$; the slope is 3/2 and the y-intercept is 0.

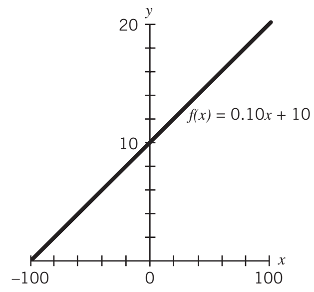
e. $y = \frac{3}{2}x$; the slope is 3/2 and the y-intercept is 0. So the equation is equivalent to the one in part (d).

f. $y = \frac{3}{4}x + \frac{1}{4}$; the slope is 3/4 and the y-intercept is 1/4.

9. a.

x	$f(x) = 0.10x + 10$
-100	0
0	10
100	20

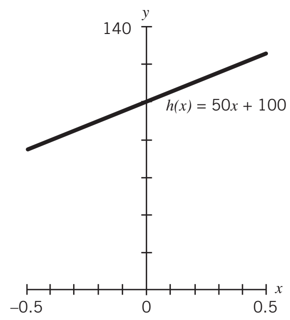
Graph of $f(x)$:



b.

x	$h(x) = 50x + 100$
-0.5	75
0	100
0.5	125

Graph of $h(x)$:



11. $C(n) = 150 + 120n$, where $C(n)$ is the cost of n credits, and 150 is measured in dollars and 120 in dollars per credit.

13. $C(n) = 2.50 + 0.10n$, where $C(n)$ is the cost of cashing n checks that month.

15. a. Annual increase = (32,000 − 26,000)/4 = $1500

b. $S(n) = 26{,}000 + 1500n$, where $S(n)$ is measured in dollars and n in years from the start of employment.

c. Here $0 \leq n \leq 20$ since the contract is for 20 years.

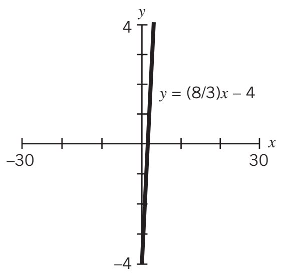
17. a. $P = 285 - 15t$

b. It will take approximately 16.33 years to make the water safe for swimming since $40 = 285 - 15t$ implies that $15t = 245$ or $t \approx 16.33$ years.

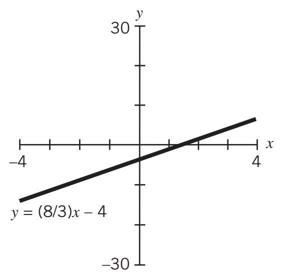
19. a. $V(t) \approx 70{,}000 + 26{,}000t$, where t = years since 1977.

b. $V(33)$ = $928,000.

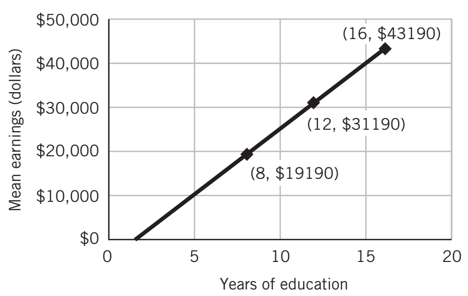
21. The equation of the line illustrated is $y = (8/3)x - 4$. The graph in the text can be altered to appear much steeper if the graph is stretched vertically—as in the accompanying graph—where the tick marks on the x-axis are 10 units apart.



If, however, we reversed the situation and vertically compressed the graph (as in the one below, where the tick marks on the y-axis are now 10 units apart), the graph would seem less steep than the one in the text.



23. a. The three data points are plotted in the accompanying graph.



b. The relationship is linear. The slope between any two points is $3000 per year of education.

c. The linear equation is: $M = 3000E - 4810$, where M = mean earning measured in dollars and E = years of education.

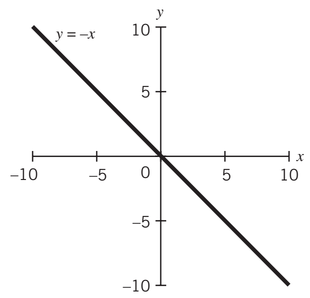
25. The entries in the table argue that the relationship is linear. The average rate of change in salinity per degree Celsius is a constant: −0.054. Since the freezing point, P, for 0 salinity is 0 degrees Celsius, we have that $P = -0.054S$, where S is the salinity measured in ppt and P, the freezing point, is measured in degrees Celsius.

Section 2.8

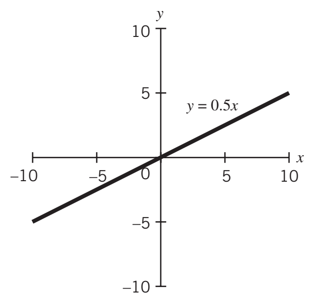
Algebra Aerobics 2.8a

1. If (0, 0) is on each line, then $b = 0$ in $y = mx + b$.

a. $y = -x$.



b. $y = 0.5x$.



2. a. The variables x and y are directly proportional; the equation is $y = -3x$.

b. The variables x and y are not directly proportional; the equation is $y = 3x + 5$.

3. E = euros; D = U.S. dollars

a. $E = 0.79D$

b. $E = 0.79D + 2.50$.

c. Only (a) because it is of the form $y = mx$ for some constant m.

4. $d = 60t$. This represents direct proportionality. If the value of t doubles, the value for d also doubles. If the value for t triples, then so does the value for d.

5. Since C and N are directly proportional, $C = kN$ for some constant k. Since $50 = k \cdot 2$, then k = $25, the cost per ticket. So the cost for 10 tickets is $C = 25 \cdot 10$ = $250.

CH. 2 Algebra Aerobics Solutions for Section 2.8

6. a. $4 = k(12) \Rightarrow k = \frac{1}{3} \Rightarrow y = \frac{1}{3}x$
b. $300 = k(50) \Rightarrow k = 6 \Rightarrow d = 6t$

7. a. $d = kC$ **b.** $T = kI$ **c.** $t = kc$

8. a. $a = kb$; $10 = k(15) \Rightarrow \frac{2}{3} = k$; $a = \frac{2}{3}(6) = 4$
b. $a = kb$; $4 = \frac{2}{3}b \Rightarrow b = 6$

Algebra Aerobics 2.8b

1. a. $y = -5$ **b.** $y = -3$ **c.** $y = 5$

2. a. $x = 3$ **b.** $x = 5$ **c.** $x = -3$

3. a. $y = -7$ **b.** $x = -4.3$

4. Slope is -1, y-intercept is 0, so $m = -1, b = 0 \Rightarrow y = -x$.

5. $m = 360$, $C = 4$, $W = 1000$ in $W = 360C + b$. To solve for b, use the point (4, 1000), so $1000 = (360)(4) + b \Rightarrow 1000 = 1440 + b \Rightarrow b = -440$. So, the equation is: $W = 360C - 440$.

6. Let m = slope of given line;
M = slope of perpendicular line. So $M = -\frac{1}{m}$.
a. $m = -3 \Rightarrow M = -\frac{1}{-3} = \frac{1}{3}$
b. $m = 1 \Rightarrow M = -\frac{1}{1} = -1$
c. $m = 3.1 \Rightarrow M = -\frac{1}{3.1} \approx -0.32$
b. $m = -\frac{3}{5} \Rightarrow M = -\frac{1}{-3/5} = \frac{5}{3}$

7. a. Slope of $y = 2x - 4$ is 2, so line perpendicular to it has slope $-1/2$ or -0.5. Since it passes through $(3, -5)$, $x = 3$ when $y = -5$. So, $y = mx + b$ is: $-5 = (-0.5)(3) + b$. Solve it for b. $-5 = -1.5 + b \Rightarrow b = -3.5$. So, equation is: $y = -0.5x - 3.5$.
b. Any line parallel to (but distinct from) $y = -0.5x - 3.5$ will be perpendicular to $y = 2x - 4$, but will not pass through $(3, -5)$. They have same slope m, (-0.5), but different values of b, in $y = mx + b$. Two examples are $y = -0.5x$ and $y = -0.5x - 7.5$.
c. The three lines are parallel.

8. $Ax + By = C \Rightarrow By = C - Ax \Rightarrow y = (C - Ax)/B \Rightarrow y = \frac{C}{B} - \frac{A}{B}x$, so $m = -\frac{A}{B}$

9. a. $2x + 3y = 5 \Rightarrow A = 2, B = 3 \Rightarrow m = -\frac{2}{3}$
b. $3x - 4y = 12 \Rightarrow A = 3, B = -4 \Rightarrow m = -\frac{3}{-4} = \frac{3}{4}$
c. $2x - y = 4 \Rightarrow A = 2, B = -1 \Rightarrow m = -\frac{2}{-1} = 2$
d. $x = -5 \Rightarrow A = 1, B = 0 \Rightarrow m = -\frac{1}{0}$, which is undefined; this line is vertical
e. $x - 3y = 5 \Rightarrow A = 1, B = -3 \Rightarrow m = -\frac{1}{-3} = \frac{1}{3}$
f. $y = 4 \Rightarrow A = 0, B = 1 \Rightarrow m = -\frac{0}{1} = 0$ (this line is horizontal)

10. $2x + 3y = 5 \Rightarrow 3y = -2x + 5 \Rightarrow y = -\frac{2}{3}x + \frac{5}{3} \Rightarrow m = -\frac{2}{3}$. Parallel lines have slopes that are equal, so $m = -\frac{2}{3}$. If the line passes through the point (0, 4) then $x = 0$, $y = 4$, the vertical intercept $b = 4 \Rightarrow y = -\frac{2}{3}x + 4$.

11. $3x + 4y = -7 \Rightarrow 4y = -7 - 3x \Rightarrow y = -\frac{7}{4} - \left(\frac{3}{4}\right)x \Rightarrow m = -\frac{3}{4}$. Because perpendicular lines have slopes that are negative reciprocals of each other, the slope of any line perpendicular to the given line is $m = -\frac{1}{-3/4} = \frac{4}{3}$. If the line passes through the point (0, 3) then $x = 0$, $y = 3$, the vertical intercept $b = 3$. So the equation is $y = \frac{4}{3}x + 3$.

12. $4x - y = 6 \Rightarrow y = 4x - 6$. The slope of the given line is 4; therefore the slope of any line perpendicular to it is $-1/4$. If the line passes through $(2, -3) \Rightarrow x = 2$, $y = -3$. So $y = -\left(\frac{1}{4}\right)x + b$, and $(-3) = \frac{-1}{4}(2) + b \Rightarrow$
$-3 = -\frac{1}{2} + b \Rightarrow b = \frac{-5}{2} \Rightarrow y = -\left(\frac{1}{4}\right)x - \frac{5}{2}$

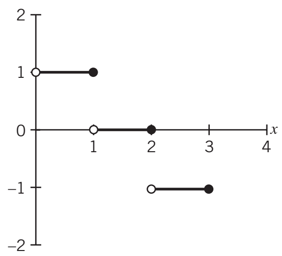
13. a. vertical line
b. neither $\left(m = -\frac{2}{3}\right)$
c. horizontal line

14. Slope of given line $= -\frac{2}{3}$, so slope of a perpendicular line $= \frac{3}{2}$. So:
a. $y = \frac{3}{2}x + 5$
b. Subsituting $(-6, 1)$, we have
$1 = \frac{3}{2}(-6) + b \Rightarrow b = 10 \Rightarrow y = \frac{3}{2}x + 10$

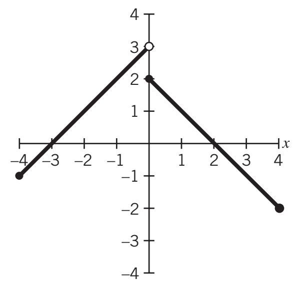
15. Slope = 2 for the given line and for lines parallel to it. So:
a. $y = 2x + 9$
b. Subsituting (4, 3), we have
$3 = 2(4) + b \Rightarrow b = -5 \Rightarrow y = 2x - 5$

Algebra Aerobics 2.8c

1. a. Graph of $f(x)$



b. Graph of $g(x)$



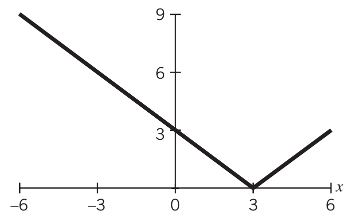
2. $Q(t) = \begin{cases} t - 2 & \text{for } -5 < t \le 0 \\ 2 - t & \text{for } \quad 0 < t \le 5 \end{cases}$

$C(r) = \begin{cases} r + 1 & \text{for } -3 < r \le 1 \\ 5 & \text{for } 1 \quad < r \le 4 \end{cases}$

3. a. 2 **b.** 6 **c.** 2 **d.** -2 **d.** -15

4. a. $g(-3) = |-6| = 6$; $g(0) = |-3| = 3$;
$g(3) = |0| = 0$; $g(6) = |3| = 3$

b.



Graph of $g(x) = |x - 3|$ for $-6 \le x \le 6$

c. The graph of g is the graph of f shifted three units to the right.

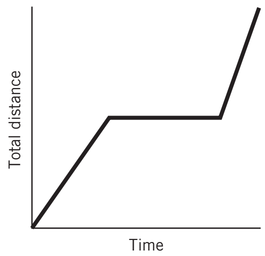
d. $g(x) = \begin{cases} x - 3 & \text{if } x \ge 3 \\ 3 - x & \text{if } x < 3 \end{cases}$

5. a. $3 \le t \le 7$, which means that the values of t lie between (and include) 3 and 7.

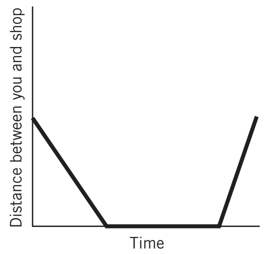
b. $69 < Q < 81$, which means that the values of Q lie between (but exclude) 69 and 81.

6. $|T - 55°| \le 20°$, or equivalently $35° \le T \le 75°$.

7. a.



b.



8. a. 4.25% in week 4; 5.25% in week 52; the longest period in which the rate remained the same was week 26 through week 52.

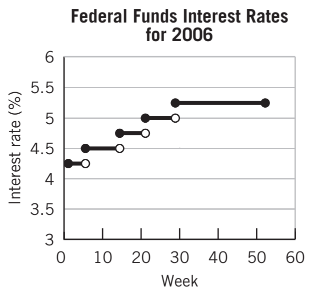
b. Typical rate increase = 0.25%. The increases are not at regular intervals.

c. Curbing inflation.

d. If $R(w)$ = the federal funds rate in week w of 2006, then we can write R(w) as:

$$R(w) = \begin{cases} 4.25\% & \text{for } 1 \le w < 5 \\ 4.50\% & \text{for } 5 \le w < 13 \\ 4.75\% & \text{for } 13 \le w < 19 \\ 5.00\% & \text{for } 19 \le w < 26 \\ 5.25\% & \text{for } 26 \le w \le 52 \end{cases}$$

e.



Exercises for Section 2.8

1. a. $2 = m \cdot 10$ means $m = 0.2$

b. $0.1 = m \cdot 0.2$ means $m = 0.5$

c. $1 = m \cdot 1/4$ means $m = 4$

3. a. The slope $m = 56.92 - 42.69 = 14.23$ and since y is directly proportional to x, then $y = 14.23x$ is the equation. Thus, if $x = 5$, then $5 \cdot 14.23 = 71.15$ is the missing value.

b. The coefficient of x is the cost of a single CD.

5. a. The independent variable is the price P; the dependent variable is the sales tax T. The equation is $T = 0.065P$.

b. Independent variable is amount of sunlight S received; dependent variable is the height of the tree H. The equation is $H = kS$, where k is a constant.

c. Time t in years since 1985 is the independent variable, and salary S in dollars is the dependent variable. The equation is $S = 25{,}000 + 1300t$.

7. $d = 5t$; yes, d is directly proportional to t; it is more likely to be the person jogging since the rate is only 5 mph.

9. a. $m = 0$; $y = 3$

b. $m = 0$; $y = -7$

c. slope is undefined; $x = -3$

d. slope is undefined, $x = 2$

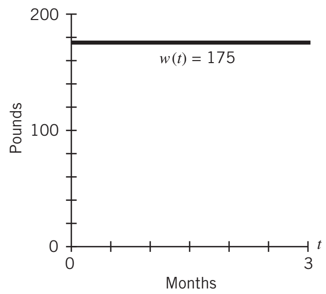
11. The horizontal line $N = 2300$, where N is measured in millions of books. (Student answers may vary.)

13. a. horizontal: $y = -4$; vertical: $x = 1$; line with slope 2: $-4 = 2 \cdot 1 + b \Rightarrow b = -6 \Rightarrow y = 2x - 6$.

b. horizontal: $y = 0$; vertical: $x = 2$; line with slope 2: $0 = 2 \cdot 2 + b \Rightarrow b = -4 \Rightarrow y = 2x - 4$.

15. a. The average rate of change is 10 lb per month.

b. $w(t) = 175$, where t = number of months after end of spring training and 175 is measured in pounds. The graph of this function is a horizontal line.



17. a. $7 = b - 3 \Rightarrow b = 10$ and therefore $y = 10 - x$.

b. $7 = b + 3 \Rightarrow b = 4$ and therefore $y = 4 + x$.

19. The lines described by: **a.** $x = 0$ **b.** $y = 0$ **c.** $y = x$.

21. a. Intercepts of one line are (0, 4) and (1, 0), and the intercepts of the other line are (0, 4) and (−1, 0). Thus the slope of the first line is −4 and the slope of the second line is 4, and thus the lines are not perpendicular.

b. Intercepts of the first line are (0, 2) and (1, 0) and the intercepts of the second line are (0, 2) and (−4, 0). So the slopes are now −2 and 1/2, and thus the lines are perpendicular.

23. For Graph *A*: both slopes are positive; same *y*-intercept.

For Graph *B*: one slope is positive, one negative; same *y*-intercept.

For Graph *C*: the lines are parallel; different *y*-intercepts.

For Graph *D*: one slope is positive, one negative; different *y*-intercepts.

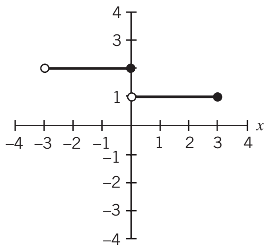
25. a. $y = (-A/B)x + (C/B),\ B \neq 0$

b. The slope is $-A/B,\ B \neq 0$

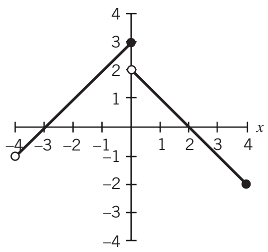
c. The slope is $-A/B,\ B \neq 0$

d. The slope is $B/A,\ A \neq 0$

27. a. Graph of $f(x)$



b. Graph of $g(x)$



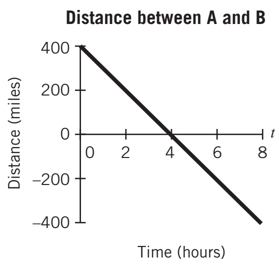
29. a. $g(x) = \begin{cases} -2 - x & \text{for } x < -2 \\ 2 + x & \text{for } x \geq -2 \end{cases}$

b. $g(x) = |2 + x|$

c. The graph of g is the same as the graph of f shifted horizontally two units to the left.

31. a. $d_A(t) = 60t, \quad d_B(t) = 40t$

b. $D_{AB}(t) = 400 - d_A(t) - d_B(t) = 400 - 60t - 40t$
$= 400 - 100t$

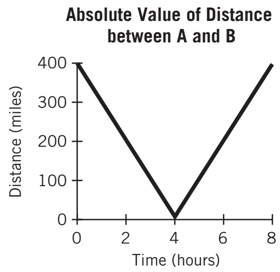


c. They will meet when the distance between them

$D_{AB}(t) = 0 \text{ miles} \Rightarrow 400 - 100t = 0 \Rightarrow 400 = 100t$
$\Rightarrow t = 4$ hours. A will have traveled 4 hr · 60 miles/hr = 240 miles. B will have traveled 4 hr · 40 miles/hr = 160 miles. (*Note:* Together they will have traveled 240 + 160 = 400 miles.)

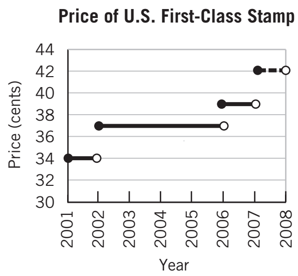
d. One hour before they meet (3 hours into the trip), $D_{AB}(3) = 400 - 100 \cdot 3 = +100$ miles, which means that they are 100 miles apart and traveling *toward* each other. One hour after they meet (at 5 hours), $D_{AB}(5) = 400 - 100 \cdot 5 = -100$ miles, which means that they are 100 miles apart and traveling *away* from each other.

e. $D(t) = |400 - 100t| = \begin{cases} 400 - 100t & \text{for } 0 \leq t \leq 4 \\ 100t - 400 & \text{for } t > 4 \end{cases}$



33. a. $S(x) = \begin{cases} 34 & \text{for } 2001 \leq x < 2002 \\ 37 & \text{for } 2002 \leq x < 2006 \\ 39 & \text{for } 2006 \leq x < 2007 \end{cases}$

b. Note that the top dotted line segment is from part (c).



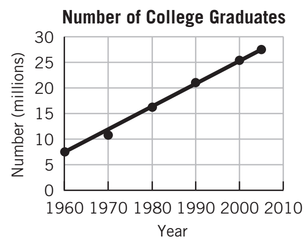
c. The domain would be $2001 \leq x < 2008$, and the top (dotted) line segment would be added to the graph.

Section 2.9

Algebra Aerobics 2.9

1. a. In 1960 there were about 7.5 million college graduates, in 2005 about 27.5 million.

b.



c. Two estimated points on the line are (1960, 7.5) and (2005, 27.5).

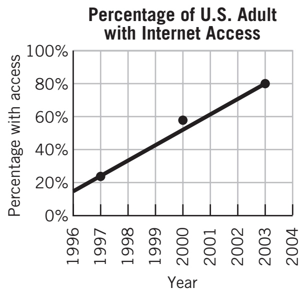
Slope $= \frac{(27.5 - 7.5)}{(2005 - 1960)} = \frac{20}{45} \approx 0.44$ million/yr.

d. (0, 7.5) and (45, 27.5)

e. $y = 7.5 + 0.44x$

f. The number of graduates was about 7.5 million in 1960 and has been steadily increasing since—by about 0.44 million, or 440,000, persons each year.

2. a.



b. Letting x = years from 1996, two estimated points on the line are (0, 17) and (7, 80). The slope = $(80-17)/(7-0)$ = 63/7 = 9 percentage points per year. This tells you that the adults with access to the Internet are increasing by 9 percentage points per year.

(*Note:* You *add* 9% each year to the previous year's % value, you are not calculating a percentage.)

c. (0, 17)

d. If we let $I(x)$ = percent of adult population with access to the Internet x years from 1996, then $I(x) = 17 + 9x$.

e. In 1998, we have $x = 2$. So $I(2) = 17 + (9 \cdot 2) = 35\%$. In 2002, we have $x = 6$. So $I(6) = 17 + (9 \cdot 6) = 71\%$.

f. As the percentage of adults with Internet access comes close to the maximum of 100%, we would expect the rate of change to slow down. In 2003 the percent is already close to 80%, so we should not expect the linear model to continue to be appropriate.

Exercises for Section 2.9

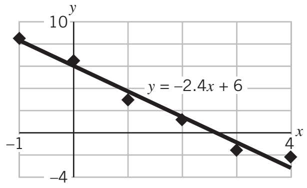
1. Equation **a** matches with table **C.**

Equation **b** matches with table **A.**

Equation **c** matches with table **B.**

3. a. Exactly linear; $y = 1.5x - 3.5$

b. An estimated best-fit line has intercepts at (0, 6) and (2.5, 0), so the slope = $(6 - 0)/(0 - 2.5) = -2.4$. The equation is then $y = -2.4x + 6$. (Note that the data point (0, 6.5) is not on the best-fit line.) Your answer may be somewhat different.

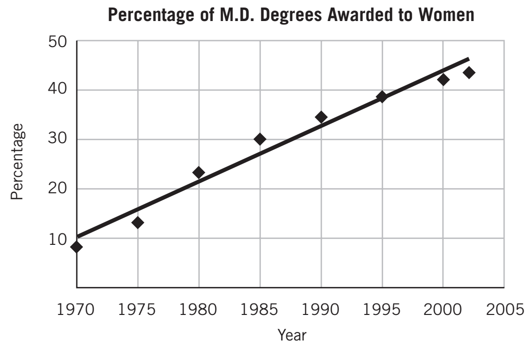


c. Exactly linear; $y = 3x + 35$

5. a. Exactly linear; $y = 0.07$ quadrillion Btus of solar units

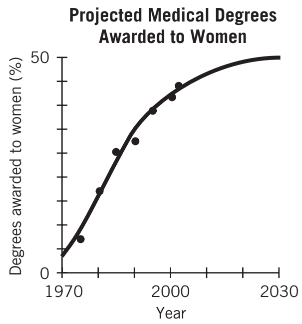
b. Approximately linear; farm output = 220 billion dollars

7. a. The accompanying graph shows a hand-drawn best-fit line. We can estimate two points on the line as (1970, 10) and (2000, 44). So the slope of the line is $(44 - 10)/(2000 - 1970) = 34/30 \approx 1.13$. So the average rate of change in the percentage of female M.D. degrees between 1970 and 2002 was about 1.13 percentage points per year.

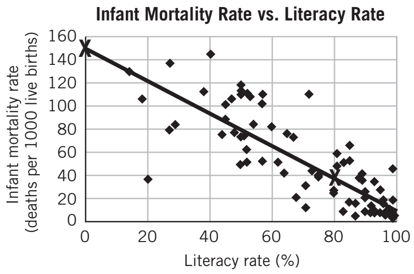


b. To extrapolate to 2010, it helps to generate the equation of the line. If we let x = years since 1970, the slope remains the same at 1.13 percentage points per year. If $x = 0$ (at year 1970), the vertical intercept is at 10. So the equation is $y = 10 + 1.13x$ where x = years since 1970, and y = percentage of M.D. degrees awarded to women. If $y = 100\%$, we would have $100 = 10 + 1.13x \Rightarrow x = 90/1.13 \approx 80$ years since 1970 or in 2050. So if we extrapolated the model to 2050, it would predict that in that year 100% of all medical degrees would be awarded to women.

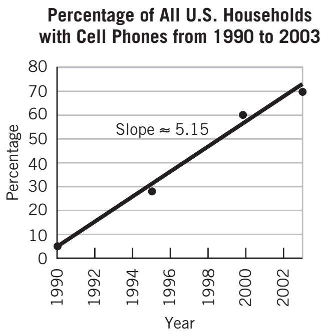
c. It is highly unlikely that this would happen. The most likely scenario is that the graph (and correspondingly the percentage of female M.D. degrees awarded) will taper off to some maximum percentage value, say at 50%. The accompanying graph fits that description. (Student answers may differ somewhat.)



9. The graph on the next page gives an eyeballed best-approximation line that goes through two estimated coordinates of (0, 150) and (80, 40). The y-intercept is 150. The slope is $\frac{40 - 150}{80 - 0} = \frac{-110}{80} = -1.375$. Thus the equation of the graph can be estimated as $y = 150 - 1.375x$, where y stands for infant mortality rate (deaths per 1000 live births) and x stands for literacy rate (%).This means that on average for each 1% increase in literacy rate, the number of infant deaths drops by 1.375 per thousand.



11. a. The hand-drawn best-fit model for the years 1990 to 2003 is in the graph below.



An eyeball estimate of two points on the best-fit line that is drawn are (1990, 5) and (2003, 72). The slope of the line between those two points is approximately $\frac{72 - 5}{2003 - 1990} = \frac{67}{13} \approx 5.15$ percentage points per year. This value indicates that on average the percent of U.S. households with cell phones grew approximately 5.15 percentage points each year between 1990 and 2003. Answers may vary slightly.

b. The percent of households having cell phones appears to be leveling off. Perhaps the saturation point has been reached.

c. Letting x = years since 1990, an estimated piecewise best-fit linear function for the combined time periods would be:

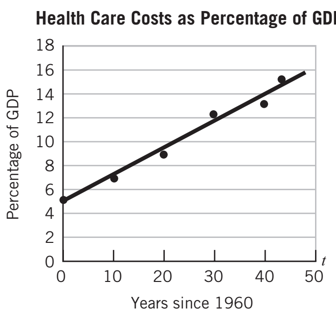
$$P(x) = \begin{cases} 5 + 5.15x & \text{for } 0 \le x < 13 \\ 70 & \text{for } 13 \le x \le 15 \end{cases}$$

where $P(x)$ = % of all households with cell phones.

13. a. Below is the graph of the data along with an estimated best-fit line. The two points (0, 5) and (40, 14) lie on the line ⇒ slope $= \frac{14 - 5}{40 - 0} = \frac{9}{40} = 0.225$

The equation of the line is $H(t) = 5 + 0.225t$

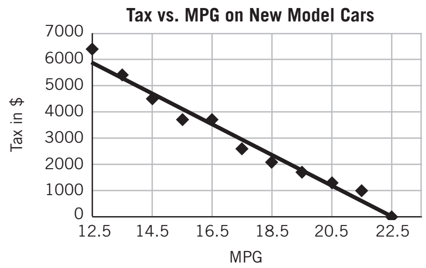
where t = years since 1960 and $H(t)$ = health care costs as % of GDP.



b. 2010 corresponds to $t = 50$ years and $H(50) = 5 + 0.225 \cdot 50 = 16.25\%$ of GDP.

c. One possible reason is that the GDP has grown very large and thus the percentage for health care has not grown as much as the health care costs themselves.

15. a. Below are the graph of the data and a hand-drawn best-fit line.



b. The approximate coordinates of two points on this line are (17.5, 3000) and (22.5, 0) ⇒ the slope of this line is $\frac{0 - 3000}{22.5 - 17.5} = \frac{-3000}{5} = -600$ dollars per mpg. The equation of this line is $T(x) = -600x + 13{,}500$, where x is in mpg and $T(x)$ = tax (in dollars). The vertical intercept is very large because its value is what one would get if mpg takes the value of 0. (This value, of course, represents an impossible situation.)

c. The average rate of change for the given line is −600 dollars per mpg. As the mpg rating increases, the tax paid goes down by $600 per mpg.

Ch. 2: Check Your Understanding

1. False	**9.** True	**17.** True	**25.** False
2. True	**10.** True	**18.** False	**26.** False
3. False	**11.** False	**19.** False	**27.** False
4. False	**12.** True	**20.** True	**28.** False
5. True	**13.** True	**21.** True	**29.** True
6. True	**14.** False	**22.** False	**30.** True
7. True	**15.** True	**23.** False	
8. False	**16.** True	**24.** False	

31. Possible answer: $y = -2x + 5$, y dependent variable, x independent variable.

32. Possible answer: $D = 0$, D dependent variable, p independent variable.

33. Possible answer: $2x - 3y = 6$, y dependent variable, x independent variable.

34. Possible answer: $3x + 5y = -15$, y dependent variable, x independent variable.

35. Possible answer: $T = 37l$, T dependent variable in minutes, l independent variable in laps.

36. Possible answer: $V = 19.25 + 0.25q$, V dependent variable in dollars, q independent variable in number of quarters from now.

37. Possible answer: $C = 2T + 3$ and $C = 2(T - 2) + 3$ (or $2T - 1$), C dependent variable, T independent variable.

38. Possible answer: $y = 2x + 4$, $y = 5x + 4$, $y = 4 - 2x$, $y = 4$, y dependent variable, x independent variable.

39. Possible answer: For $m = -1/4$ (or -0.25), $-1/m = 4$, $d = -0.25t + 3$ and $d = 4t - 1$, d dependent variable, t independent variable.

40. Possible answer: (1990, \$100), (1993, \$133), (1995, \$150), (2000, \$175), (2003, \$185).

41. True

42. False

43. False

44. False

45. True

46. True

47. True

48. False

49. False

50. False

51. False (it can be 0)

52. True

53. False (not a function)

Ch. 2 Review: Putting It All Together

1. a. \$217,300 − \$147,300 = \$70,000

b. \$70,000/\$147,300 ≈ 0.475, or about a 47.5% increase

c. (\$217,300 − \$147,300)/(2006 − 2000) = \$70,000/6 ≈ \$11,670/year

3. a. The function is positive between points B and E excluding the endpoints; negative between A and B (excluding point B) and between F and G (excluding point F); zero at point B and between points E and F.

b. The slope is positive between A and C; negative between D and E and between F and G; zero between C and D and between E and F.

5. a. They are both linear functions of U.S. sizes since the average rates of change are constant for both.

b. The British size equals the U.S. size plus 6.

c. If U.S. sizes are denoted by U, and French sizes by F, then $F = U + 34$.

7. a. 34% of 296.4 = 0.34 · 296.4 ≈ 100.78 million Americans (about one-third) were without a cell phone in 2005.

b. Average rate of change =

(207.9 − 109.4) million/(2005 – 2000) =

(98.5 million)/(5 years) = 19.7 million cell phone subscriptions per year.

c. $C(t) = 109.4 + 19.7\,t$, where t = years since 2000.

d. In 2010 we would have $t = 10$, and

$C(10) = 109.4 + (19.7 \cdot 10) = 306.4$ million cell phone subscriptions. The U.S. population did exceed 300 million in fall 2006, but this extrapolation seems somewhat excessive.

9. a. i and ii

b. i and ii

c. iii

d. both ii (which is decreasing) and iv (which is increasing).

11. a. Average rate of change between weeks 1 and 31 = (3.04 − 2.22)/(31 − 1) = 0.82/30 ≈ 0.03, which means that the price of gasoline was increasing by roughly 3 cents per week.

Average rate of change between weeks 31 and 43 = (2.22 − 3.04)/(43 − 31) = −0.82/12 ≈ −0.07, which means that the price of gas was decreasing by about 7 cents per week.

Average rate of change between weeks 1 and 43 = (2.22 − 2.22)/(43 − 1) = 0/42 = 0, which means, of course, that if we look only at weeks 1 and 43, we would have the impression that gas prices had not changed at all.

b. During the summer of 2006 gasoline prices soared to an all-time high. In January the price per gallon was relatively low. By August the price per gallon had increased on average by 3 cents each week to reach a cost of \$3.04 per gallon. In total the cost had increased by \$0.82 per gallon or by about 37%. By October gas prices fell back to the January rate, decreasing at a rate of 7 cents per week from the August high.

13. Possible answers are:

a. $Q = 3t - 2$ or $Q = 3t$

b. $Q = -2t + 3$ or $Q = -2t$

c. $Q = 3$ or $Q = -2$

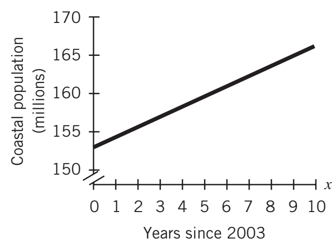
d. $Q = -2t$ or $Q = 3t$

e. $Q = 3t - 2$ or $Q = 3t$

f. $Q = -2t$ or $Q = -2t + 3$

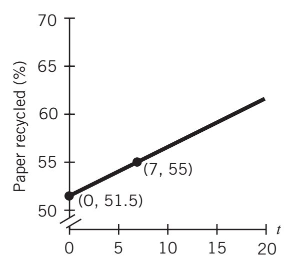
15. a. (3600 people/day) · (365 days/year) = 1,314,000 people/year or, equivalently, an increase of 1.314 million people per year living in the coastal regions of the United States.

b. The coastal population $P(x)$ (in millions) can be modeled by $P(x) = 153 + 1.314x$, where x is the number of years since 2003.



d. In 2008 $x = 5$, so $P(5) = 153 + 1.314(5) \approx 160$ million people living in coastal regions.

17. a.



The slope = (55 − 51.5)/7 = 0.5 percentage points per year.

b. $R(t) = 51.5 + 0.5t$, where $R(t)$ is the percentage of paper recycled in t years from 2005.

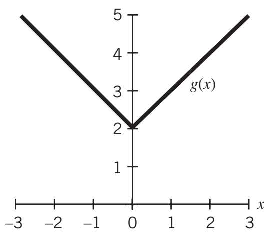
c. $R(0) = 51.5\%$; $R(5) = 54\%$; $R(20) = 61.5\%$ would mean that in the year 2025, 61.5% of all paper is recycled.

19. Graph A: $y = 5$; Graph B: $x = -2$: Graph C: $y = 2x + 1$; Graph D; $y = 2x + 4$; Graph E: $y = -(1/2)x + 6$

21. a.

x	−3	−2	−1	0	1	2	3
$g(x)$	5	4	3	2	3	4	5

b.

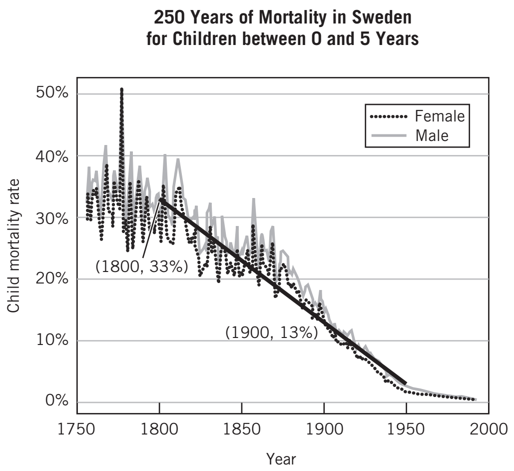


c. The graph of $g(x) = |x| + 2$ is the graph of the absolute value function $f(x) = |x|$ raised up two units.

23. Student answers will vary.

25. a. Over the last 250 years in Sweden the probability of a young child dying has steadily decreased. The child mortality rate has declined from about 40% in 1750 to less than 1% in 2000. The death rate is very similar for female and male children, though the male rates are consistently somewhat higher.

b.



Two estimated points on the line are (1800, 33%) and (1900, 13%). So the slope of the line is $\frac{13 - 33}{1900 - 1800} = -20/100 = -0.2\%$/year. This means that on average the probability of a child between the ages of 0 to 5 dying was decreasing by two-tenths of a percentage point each year, or equivalently, there were 2 fewer children dying per thousand.

c. If we let t = years since 1800, then (0, 33%) becomes the vertical intercept. The slope remains the same. So the linear model is $P(t) = 33 - 0.2t$, where $P(t)$ gives the female child mortality rate (as a percentage) at t years after 1800.

d. After 1950 the mortality rates are very low and still declining, almost approaching 0% per year. (In fact, Sweden currently has one of the lowest child mortality rates in the world.)

27. a.

Point	x-Coordinate of Point	y-Coordinate of Point	Average Rate of Change between Two Adjacent Points
A	0	4	n.a.
B	1	1	−3
C	2	0	−1
D	3	1	1
E	4	4	3

b. The function is increasing over the interval (2, 4). The average rates of change (or slopes) are increasing on that interval, which also suggests that the function is increasing.

c. The function is decreasing over the interval (0, 2). The absolute value of the average rate of change (or slope) is decreasing over that interval, which also suggests that the function is decreasing.

d. The function is concave up throughout, independent of whether the function is increasing or decreasing. The steepness of the curve (as approximated by the absolute values of the average rates of change) first decreases and then increases, also suggesting that the curve is concave up.

Exercises for EE on Education and Earnings

1. a. 0.65, 0.68, 0.07, 0.70

b. $|-0.07|$, $|0.65|$, $|-0.68|$, and $|0.70|$

3. a. The slope = 6139 dollars per year of education past grade 8; vertical intercept = −2105 dollars; cc = 0.72

b. On average, for each increase of a year of education past grade 8 the median personal earnings increase by \$6139.

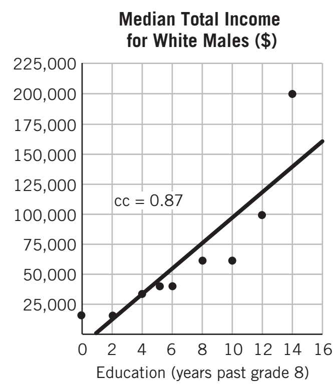
c. \$6139; \$61,390

5. a. The rate of change is 10,733 dollars per year of education past grade 8.

b. Three points are (4, 34242), (8, 77334), and (12, 10426). A sample computation: $\frac{77{,}334 - 34{,}242}{8 - 4} = \frac{43{,}092}{4} = 10{,}773$ dollars per year of education above grade 8.

c. For each extra year of education past grade 8 the median personal total income of white males on average rises \$10,733.

d. From the FAM 1000 data:



Median total income for white males = −8850 + 10,773 · yrs of educ. past grade 8

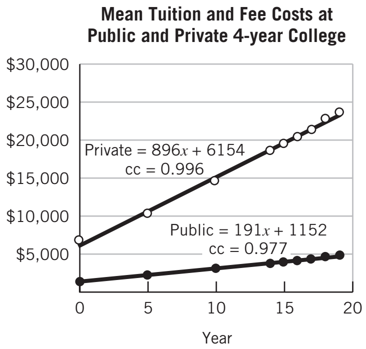
7. a. 0.516 is the slope of the regression line, and it indicates that, on average, each increase of an inch in the mean height of the fathers means there is an increase of 0.516 inch in the mean height of the sons.

b. If $F = 64$, then $S = 33.73 + 0.516 \cdot 64 = 66.75$ in.
If $F = 73$, then $S = 33.73 + 0.516 \cdot 73 = 71.40$ in.

c. If $F = S$, then $F = 33.73 + 0.516F$ or $0.484F = 33.73$ or $S = F = 69.69$ in.

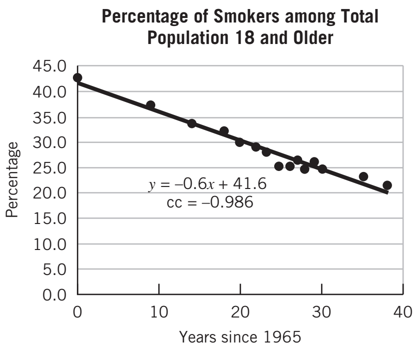
d. For each of the data points the S value represents the mean height of all the sons whose father has the given mean height of F. There are 17 mean heights listed (from 57 to 75 inches) for the 1000 fathers.

9. a, b. Here is a graph that gives the Excel-generated regression lines, their equations, and the cc values for public and private 4-year colleges over the time period given in the table. Student estimates of the regression lines for each will vary.



c. In 2010 the cost at a public college will be $191 \cdot 25 + 1152 = \$5927$ and in private colleges the cost will be $896 \cdot 25 + 6154 = \$28{,}554$.

11. a.



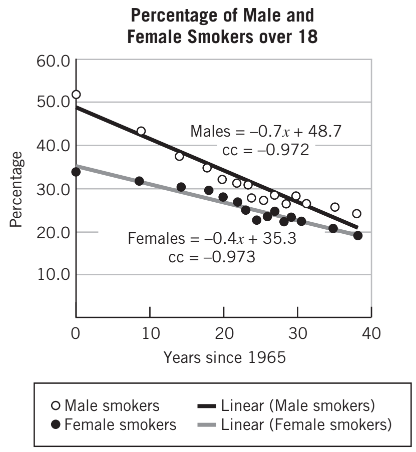
i. $\frac{20.9 - 42.4}{2005 - 1965} = \frac{-22}{40} \approx -0.55$ percentage point per year for all smokers from 1965 to 2005.

ii. $\frac{20.9 - 25.5}{2005 - 1990} = \frac{-4.6}{15} \approx -0.31$ percentage point per year for all smokers from 1990 to 2005.

b. Student estimates of the regression line will vary. The Excel-generated regression line is shown in the diagram in part (a), and the equation is Percentage $= -0.6x + 41.6$, where x = years since 1965. The average rate of change is -0.6 percentage point per year.

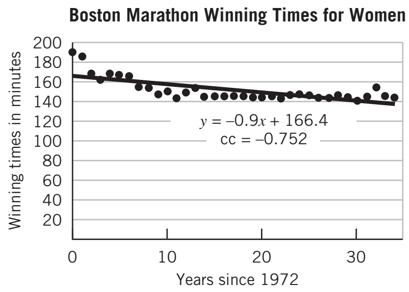
c. Student estimates of the regression line will vary. The line drawn in the diagram is the Excel-generated regression line. The cc is quite good: -0.986.

d. The regression line equation for males is: Percentage $= -0.7x + 48.7$, and the regression line equation for females is: Percentage $= -0.4x + 35.3$, where x = years since 1965 for both. The cc for males is -0.972 and for females it is -0.973, both quite good.



e. Student answers will vary. The more notable factors are the downward trend overall for both males and females and the fact that the downward trend is more rapid among males and that the percentage of smokers is always higher among males than among females. Students, in citing these factors, should also be citing actual figures.

13. a. The regression line and equation and the cc are given in the accompanying diagram.

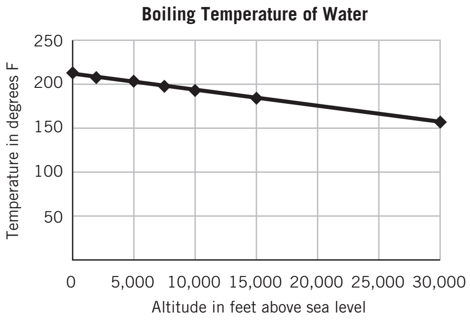


b. In 2010 $x = 38$ and time predicted $= -0.9 \cdot 38 + 166.4 \approx 132.2$ minutes. This does not seem to be a reasonable winning time. It is 10 minutes less than the recent winning times.

c. Using an eyeball estimate, one gets a horizontal line that goes through approximately $y = 144$. Yes, it is more realistic than the estimate in part (b).

d. Student answers will differ, but all should mention that the data seem to fall along a horizontal line in the constricted time span, indicating a leveling at times just above 140 minutes.

15. a. The plot of the data and the regression line are given in the accompanying diagram. The formula relating boiling temperatures in °F to altitude is $F = 211.80 - 0.0018H$, or when suitably rounded off, $F = 212 - 0.002H$, where H is feet above sea level. The correlation coefficient is -0.9999. The answer to the second part depends on where the student lives. Other factors could be something put into the water, such as salt, or variations in the air pressure.



b. On Mt. McKinley water boils at $212 - 0.002 \cdot 20320 = 171.36$ °F; in Death Valley water boils at $212 - 0.002 \cdot (-285) = 212.57$ °F.

c. $32 = 212 - 0.002H$ or $-180 = -0.002H$ or $H = 90{,}000$ feet or approximately 17 miles. But this seems unreasonable since then the water would be outside earth's atmosphere layer (which goes to 9 miles above the earth).

17. a. By computer or calculator one gets that the regression line equation as:

$y = 3.5x + 26.7$, where x = years since 1945 and y is measured in millions of registrations. [The cc. is 0.998].

b. On average the number of registrations goes up approximately 3.5 million per year since 1945.

c. 2004 is 59 years after 1945, and thus $y = 3.5 \cdot 59 + 26.7 = 233.2$ million. It is about 4.9 million too high.

d. 2010 is 65 years from 1945. Thus $y = 3.5 \cdot 65 + 26.7 = 254.2$ million. This seems to be too high since the entire U.S. population in 2007 was just over 300 million.

19. a. Of itself high correlation does not mean that there is causation involved. More studies would have to be done and in fact were done. The research leaves no doubt that cigarette smoking is indeed a cause of lung cancer. The high correlation coefficient was nevertheless an important factor.

CHAPTER 3

Section 3.1

Algebra Aerobics 3.1

1. Gas is the cheapest system from approximately 17.5 years of operation to approximately 32.5 years of operation. Solar becomes the cheapest system after approximately 32.5 years of operation.

2. a. $(3, -1)$ is a solution since: $4(3) + 3(-1) = 9$ and $5(3) + 2(-1) = 13$. It is a solution of both equations.

b. $(1, 4)$ is not a solution since: $5(1) + 2(4) = 13$ but $4(1) + 3(4) = 16$, not 9. So it is not a solution of $4x + 3 = 9$.

3. a. $12x - 9y = 18$ is equivalent to $4x = 6 + 3y$ because

$$\frac{1}{3}(12x - 9y) = \frac{1}{3}(18) \Rightarrow 4x - 3y = 6$$
$$\Rightarrow 4x = 6 + 3y.$$

So these equations represent the same line.

b. There is an infinite number of solutions to the system of equations in part (a) since every solution to the equation (every point on the line) is a solution to the system.

4. Graph A: (0, 2.5)
Graph B: $(4, -4)$

Exercises for Section 3.1

1. a. $4(5) - 3(-10) = 50$ and $2(5) + 2(-10) = -10$, and thus $(5, -10)$ does not solve the given system.

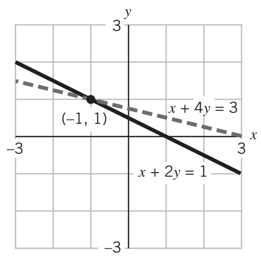
b. The coordinates must satisfy both equations.

3. A solution to a system of equations is a number (or set of numbers) that satisfies all of the equations in the system.

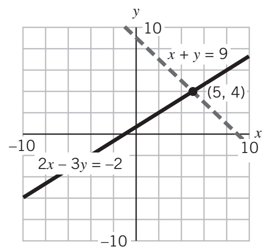
5. a. The coordinates are approximately (1995, \$380,000)

b. To the left of the intersection point the population of Pittsburgh is larger than that of Las Vegas, and to the right the population of Las Vegas is greater than that of Pittsburgh.

7. a.



b.



9. $y = -x - 2$ and $y = 2x - 8$ is the system, and the solution is $(2, -4)$. *Check:* $-2 - 2 = -4$ and $2 \cdot 2 - 8 = -4$, and thus the claimed solution works.

Section 3.2

Algebra Aerobics 3.2a

1. a. $y = 7 - 2x$

b. $y = \frac{6 - 3x}{5} = \frac{6}{5} - \frac{3}{5}x$

c. $x = 2y - 1$

2. a. no solution, because the lines have the same slope but different y-intercepts, so they are parallel.

b. one solution, because the lines have different slopes.

3. a. Set $y = y \Rightarrow x + 4 = -2x + 7 \Rightarrow 3x = 3 \Rightarrow x = 1$; $y = -2(1) + 7 = 5$. *Check*: $y = (1) + 4 \Rightarrow y = 5 \Rightarrow$ solution (x, y) is (1, 5).

b. Set $y = y \Rightarrow -1700 + 2100x = 4700 + 1300x \Rightarrow 800x = 6400 \Rightarrow x = 8$; $y = 4700 + 1300(8) = 15{,}100 \Rightarrow$ solution (x, y) is (8, 15100).

CH. 3 Algebra Aerobics Solutions for Section 3.2

c. Set $F = F \Rightarrow C = 32 + \frac{9}{5}C \Rightarrow 5C = 32(5) + 9C \Rightarrow -4C = 160 \Rightarrow C = -40$

$F = C \Rightarrow F = -40 \Rightarrow$ solution (C, F) is $(-40, -40)$.

4. a. Substitute $y = x + 3$ into $5y - 2x = 21 \Rightarrow 5(x + 3) - 2x = 21 \Rightarrow 5x + 15 - 2x = 21 \Rightarrow 3x = 6; x = 2$; so, $y = (2) + 3 = 5 \Rightarrow y = 5$. Solution (x, y) is $(2, 5)$.

b. Substitute $z = 3w + 1$ into $9w + 4z = 11 \Rightarrow 9w + 4(3w + 1) = 11 \Rightarrow 9w + 12w + 4 = 11 \Rightarrow 21w = 7 \Rightarrow w = 1/3$; so $z = 3\left(\frac{1}{3}\right) + 1 = 2 \Rightarrow z = 2$. Solution (w, z) is $\left(\frac{1}{3}, 2\right)$

c. Substitute $x = 2y - 5$ into $4y - 3x = 9 \Rightarrow 4y - 3(2y - 5) = 9 \Rightarrow 4y - 6y + 15 = 9 \Rightarrow -2y = -6 \Rightarrow y = 3$; so $x = 2(3) - 5 = 1 \Rightarrow x = 1$. Solution (x, y) is $(1, 3)$.

d. Solve: $r - 2s = 5$ for r, and substitute the resulting expression for r into $3r - 10s = 13$. $r = 2s + 5 \Rightarrow 3(2s + 5) - 10s = 13 \Rightarrow 6s + 15 - 10s = 13 \Rightarrow -4s = -2 \Rightarrow s = \frac{1}{2}$; so $r = 2\left(\frac{1}{2}\right) + 5 = 6 \Rightarrow r = 6$. Solution (r, s) is $\left(6, \frac{1}{2}\right)$.

Algebra Aerobics 3.2b

1. a. By the elimination method, add equations: $2y - 5x = -1$ and $3y + 5x = 11 \Rightarrow 5y = 10 \Rightarrow y = 2$; so $3(2) + 5x = 11 \Rightarrow 5x = 5 \Rightarrow x = 1$. Solution (x, y) is $(1, 2)$.

b. Multiply the equation $(3x + 2y = 16)$ by 3 and the equation $(2x - 3y = -11)$ by $2 \Rightarrow 9x + 6y = 48$ and $4x - 6y = -22$. By the elimination method, add these equations $\Rightarrow 13x = 26 \Rightarrow x = 2$, so $3(2) + 2y = 16 \Rightarrow 2y = 10 \Rightarrow y = 5$. Solution (x, y) is $(2, 5)$.

c. By substitution of $t = 3r - 4$ into $4t + 6 = 7r \Rightarrow 4(3r - 4) + 6 = 7r \Rightarrow 12r - 16 + 6 = 7r \Rightarrow -10 + 5r = 0 \Rightarrow 5r = 10 \Rightarrow r = 2$, so $t = 3(2) - 4 = 2 \Rightarrow t = 2$. Solution (r, t) is $(2, 2)$.

d. Substitute $z = 2000 + 0.4(x - 10{,}000)$ into $z = 800 + 0.2x \Rightarrow 2000 + 0.4(x - 10{,}000) = 800 + 0.2x \Rightarrow 2000 + 0.4x - 4000 = 800 + 0.2x \Rightarrow 0.2x = 2800 \Rightarrow x = 14{,}000$; so, $z = 800 + 0.2(14{,}000) = 3600 \Rightarrow z = 3600$. Solution (x, z) is $(14000, 3600)$.

2. a. By substitution: $2x + 4 = -x + 4 \Rightarrow 3x = 0 \Rightarrow x = 0$, so $y = -(0) + 4 = 4 \Rightarrow y = 4$. So solution (x, y) is $(0, 4)$.

b. By substitution of $(y = -6x + 4)$ into $(5y + 30x = 20) \Rightarrow 5(-6x + 4) + 30x = 20 \Rightarrow -30x + 20 + 30x = 20 \Rightarrow 20 = 20$. So both equations must be equivalent. There are infinitely many solutions since both equations describe the same line.

c. $2y = 700x + 3500 \Rightarrow y = 350x + 1750$. The slopes of the lines of both the equations are 350, but the y-intercepts are different (1500 and 1750), so the lines are parallel. There is no solution.

3. In order for a system of equations to have no solutions, they must produce parallel lines with the same slope, but different y-intercepts. One example is: $y = 5x + 10$; $y = 5x + 3$.

4. a. $2x + 5y = 7 \Rightarrow y = \frac{-2x + 7}{5}$; $3x - 8y = -1 \Rightarrow y = \frac{3x + 1}{8} \Rightarrow$ one solution since the lines have unequal slopes of $-\frac{2}{5}$ and $\frac{3}{8}$.

b. $3x + y = 6 \Rightarrow y = 6 - 3x$; $6x + 2y = 5 \Rightarrow y = \frac{5 - 6x}{2}$ or $y = \frac{5}{2} - 3x \Rightarrow$ no solution since the lines have the same slope of -3 and different y-intercepts of 6 and $\frac{5}{2}$.

c. $2x + 3y = 1 \Rightarrow y = \frac{1}{3} - \frac{2}{3}x$; $4x + 6y = 2 \Rightarrow y = \frac{1}{3} - \frac{2}{3}x \Rightarrow$ equivalent equations and an infinite number of solutions, since the lines have same slopes and same y-intercepts.

d. $3x + y = 8 \Rightarrow y = 8 - 3x$; $3x + 2y = 8 \Rightarrow y = 4 - \frac{3}{2}x \Rightarrow$ one solution since the lines have unequal slopes of -3 and $-\frac{3}{2}$.

5. a. $6\left(\frac{x}{2} + \frac{y}{3}\right) = 6(3) \Rightarrow 3x + 2y = 18$. Substitute $y = x + 4 \Rightarrow 3x + 2(x + 4) = 18 \Rightarrow 5x + 8 = 18 \Rightarrow 5x = 10 \Rightarrow x = 2$; so, $y = (2) + 4 = 6 \Rightarrow y = 6$. Solution (x, y) is $(2, 6)$.

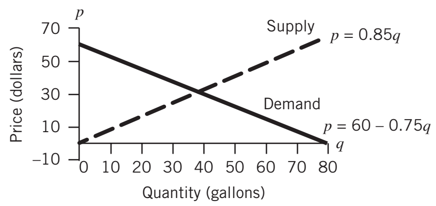
b. $-60(0.5x + 0.7y) = -60(10) \Rightarrow -30x - 42y = -600$. Add to $30x + 50y = 1000 \Rightarrow 8y = 400 \Rightarrow y = 50$, so $30x + 50(50) = 1000 \Rightarrow 30x + 2500 = 1000 \Rightarrow 30x = -1500 \Rightarrow x = -50$, so the solution (x, y) is $(-50, 50)$.

6. a. $4(39) + 3q = 240 \Rightarrow 3q = 84 \Rightarrow q = 28$ gals

b. $4p + 3(20) = 240 \Rightarrow 4p = 180 \Rightarrow p = \$45$ per gal

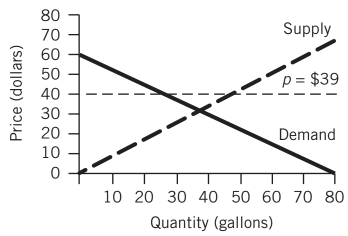
c, d. Solve for p: $4p = 240 - 3q \Rightarrow p = 60 - 3/4q \Rightarrow p = 60 - 0.75q$.

q	p
0	60
20	45
40	30



e. Solve by substitution of $p = 0.85q$ into $4p + 3q = 240 \Rightarrow 4(0.85q) + 3q = 240 \Rightarrow q = 37.5$; $p = 0.85(37.5) = 31.9$. So the equilibrium point is ~(38, \$32), which means that when the price is around \$32, the demand will be around 38 gallons.

f. There is a surplus of supply because where the line $p = 39$ crosses the supply curve, it is above the demand curve, so the supply is greater than the demand.



7. For this system to have an infinite number of solutions, the two linear equations should represent the same line. The slope of the line of the first equation is 2, and the y-intercept is 4. Solving the second equation for y, we get $y = -??/2x + 4$. Thus, ??, the coefficient for which we are solving, must be -4, so the slope of that line is also 2.

Exercises for Section 3.2

1. a. After approximately 9 months.

b. Approximately \$8.30 per hour.

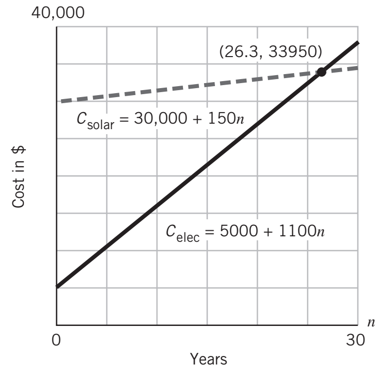
c. $W_A(m) = 7.00 + 0.15m$ and $W_B(m) = 7.45 + 0.10m$

d. The exact place where they meet is $m = 9$, and $W_A(9) = W_B(9) = \$8.35$ per hour.

e. The exact common hourly wage is \$8.35 per hour.

f. Before 9 months, the monthly wage rates at company B are higher; after 9 months the monthly wage rates at company A are higher. If one had more information about the number of hours worked at each place, one could judge the companies on accumulated wages instead of hourly rates. But no such information is given.

3. a. The graph of the linear system is given with the intersection point marked.



b. 1100 and 150 are the slopes of the heating cost lines; 1100 represents the rate of change of the total cost in dollars for electric heating per year since installation; 150 is the rate of change in the total cost for solar heat in the same units.

c. 5000 is the initial cost of installing the electric heat in dollars; 30,000 is the initial cost in dollars of installing solar heating. It cost a lot more initially to install solar heating than to install electrical heating.

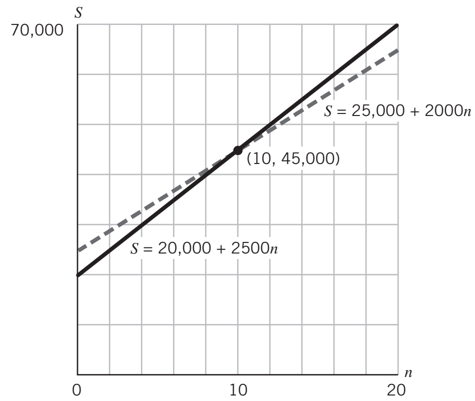
d. The point of intersection is approximately where $n = 26$ and $C = 34{,}000$.

e. $n \approx 26.32$, $C \approx 33{,}947.37$ is a more precise answer; the values have been rounded off to two decimal places; they were obtained by setting the equations equal to each other.

f. Assuming simultaneous installation of both heating systems, the total cost of solar heat was higher than the total cost of electric heat up to year 26 (plus nearly 4 months); after that the total cost of electric heat will be greater than that of solar heat.

5. a. Setting the two equations equal gives $20{,}000 + 2500n = 25{,}000 + 2000n$ or $500n = 5000$ or $n = 10$. Plugging in that n value gives $S = 20{,}000 + 2500 \cdot 10 = 45{,}000$

b. The graphs of the two linear equations are given in the diagram below. From inspecting the graphs it seems that the intersection occurs when $n = 10$ and $S = 45{,}000$.



7. a. The method that is easiest is often a judgment by the person solving the problem.

i. Either is easy	**iv.** Substitution
ii. Elimination	**v.** Either is easy
iii. Substitution	**vi.** Substitution

b. i. Setting the y values equal to each other gives $6 = -4$. Thus there is no solution.

ii. Letting $y = 2x - 5$ in the second equation gives $5x + 2(2x - 5) = 8$ or $9x - 10 = 8$ or $9x = 18$ or $x = 2$. Thus $y = 2 \cdot 2 - 5 = -1$ and therefore the solution is $(2, -1)$. *Check:* $2(2) - (-1) = 5$; and $5 \cdot 2 + 2 \cdot (-1) = 8$.

iii. Substituting $x = 7y - 30$ into the first equation gives $3 \cdot (7y - 30) + 2y = 2$ or $21y - 90 + 2y = 2$ or $23y = 92$ or $y = 4$ and then $x = 28 - 30 = -2$. Thus the solution is $(-2, 4)$. *Check:* $3 \cdot (-2) + 2 \cdot 4 = -6 + 8 = 2$ and $7 \cdot 4 - 30 = -2$.

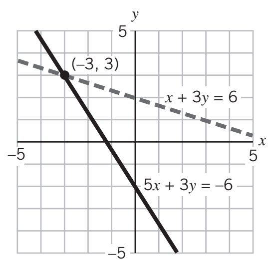
iv. Substituting $y = 2x - 3$ into second equation gives: $4(2x - 3) - 8x = -12$ or $8x - 12 - 8x = -12$ or $0 = 0$; thus the two equations have the same line as their graph. Thus all points on the line $y = 2x - 3$ are solutions.

v. Elimination yields $0 = -6$ and thus there is no solution.

vi. Substituting $y = 3$ into the second equation gives $x + 2 \cdot 3 = 11$ or $x = 5$. Thus the solution is $(5, 3)$. *Check:* $3 \cdot 3 = 9$ and $5 + 2 \cdot 3 = 11$.

9. a. Subtracting the first equation from the second yields $4x = -12$ or $x = -3$. Putting this value into the first equation gives $-3 + 3y = 6$ and thus $3y = 9$ and $y = 3$. Putting this value of x into the second equation gives $5(-3) + 3(3) = -6$ and thus the solution is $(-3, 3)$.

b. The graphs of the two equations and the coordinates of the intersection point are shown in the accompanying figure.



11. Let x = amount to be invested at 4% and let y = amount to be invested at 8%. Then the system of equations to be solved is $x + y = 2000$ and $0.04x + 0.08y = 100$. Substituting $y = 2000 - x$ into the second equation gives, after simplification, $x = \$1500$ and thus $y = \$500$. [*Check:* $1500 + 500 = 2000$ and $0.04 \cdot 1500 + 0.08 \cdot 500 = 100$.]

13. a. Letting $y = x - 4/3$ from the second equation and substituting this value in the first equation, we get $\frac{x}{3} + \frac{x - 4/3}{2} = 1$. Multiplying both sides by 6 gives $2x + 3(x - 4/3) = 6$ or $5x - 4 = 6$ or $x = 2$ and thus $y = 2 - 4/3 = 2/3$. [*Check:* $2 - 2/3 = 4/3$ and $2/3 + 1/3 = 1$.]

b. Substituting $y = x/2$ from the second equation into the first equation we get $x/4 + x/2 = 9$ or $(3/4)x = 9$ or $x = 12$. Then $y = 12/2 = 6$. [*Check:* $12/4 + 6 = 9$.]

15. a. The two equations in m and b are: $-2 = 2m + b$ and $13 = -3m + b$ and the solution is $m = -3$ and $b = 4$. [*Check:* $2(-3) + 4 = -2$ and $-3(-3) + 4 = 13$.]

b. The two equations in m and b are: $38 = 10m + b$ and $-4.5 = 1.5m + b$. The solution is $m = 5$ and $b = -12$. [*Check:* $38 = 5 \cdot 10 - 12$ and $-4.5 = 5 \cdot 1.5 - 12$.]

17. a. 12.5 is the production cost per shirt in dollars.

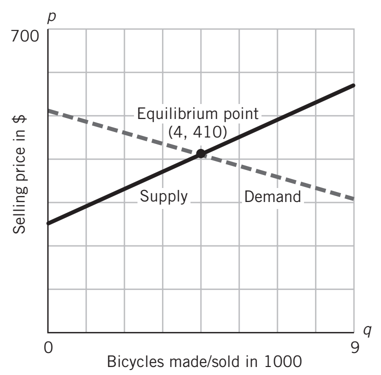
b. 15.5 is the selling price of a shirt in dollars.

c. $15.5x = 12.5x + 360$ or $3x = 360$ or $x = 120$ and $y = 15.5 \cdot 120 = \$1860$.

d. When $x = 120$, then $C = 12.5 \cdot 120 + 360 = \$1860$ and $R = 15.5 \cdot 120 = \$1860$.

19. a. The graphs of the supply and demand equations are in the accompanying diagram.

b. The equilibrium point is shown in the diagram. It is the spot where supply meets the demand; i.e., if the company charges \$410 for a bike it will sell exactly 4000 of them and have none left over.



21. Two equations are equivalent if their graphs are the same, i.e., they have the same sets of solutions. An example is the system $2x + y = 1$ and $4x + 2y = 2$.

23. If we make the origin the spot on the diagram where the height (in feet) or H-axis meets the ground and let d be the distance (in feet) from the origin, then the equation of the ramp is $H = 3 - \frac{1}{12}d$. The equation to describe the rising ground is $H = \frac{1}{20}d$. Setting them equal to each other gives $3 - \frac{1}{12}d = \frac{1}{20}d$ or $\frac{36 - d}{12} = \frac{d}{20}$ or $720 - 20d = 12d$ or $720 = 32d$ or $d = 22.5$ ft and $H = 22.5/20 = 1.125$ ft. Thus the point of meeting is where $d = 22.5$ ft from the platform and $H = 1.125$ ft above ground level.

25. a. Eliminate z; $11x + 7y = 68$ (4)

b. Eliminate z; $9x + 7y = 62$ (5)

c. $x = 3$ and $y = 5$ satisfy (4) and (5)

d. Thus $z = 2 \cdot 3 + 3 \cdot 5 - 11 = 10$

e. Thus the solution is $x = 3$, $y = 5$, $z = 10$ and the check is below:

(1) $2 \cdot 3 + 3 \cdot 5 - 10 = 11$

(2) $5 \cdot 3 - 2 \cdot 5 + 3 \cdot 10 = 35$

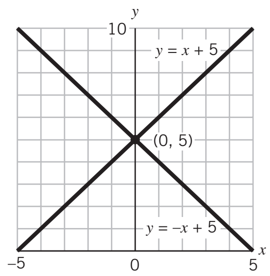
(3) $1 \cdot 3 - 5 \cdot 5 + 4 \cdot 10 = 18$

27. Answers will vary.

a. The system $y = x + 5$ and $y = x + 6$ has no solution.

b. The system $y = x + 5$ and $y = -x + 5$ has exactly one solution.

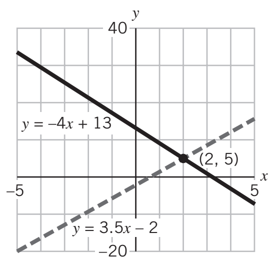
c. Algebraically: setting $x + 5 = -x + 5$ gives $2x = 0$ or $x = 0$ and thus $y = 5$; alternatively, adding the two equations together gives $2y = 10$ or $y = 5$ and thus $x = 0$. The graphs of the two lines intersecting at the point claimed is in the accompanying diagram. The answers agree.



29. The system of equations has no solution if the graphs of the two equations are parallel and distinct lines. This occurs when $m_1 = m_2$ (parallel means same slope) and $b_1 \neq b_2$ (different vertical intercepts).

31. a. The equations of this pair of lines are: $y - 5 = -4(x - 2)$ and $y - 5 = 3.5(x - 2)$, or in simplified form: $y = -4x + 13$ and $y = 3.5x - 2$.

b. The graphs of the two equations are given in the accompanying diagram. [*Check:* $-4 \cdot 2 + 13 = 5$ and $3.5 \cdot 2 - 2 = 5$.]



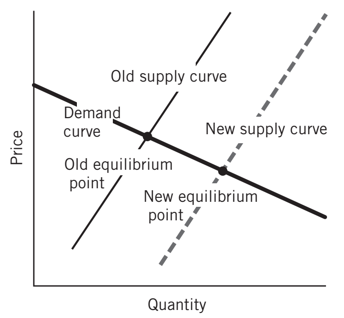
33. a. $y_B = 30$, $y_A = 0.625x$.

b. The common point in space that both planes will eventually occupy is where $x = 48$ and $y_B = 30$. (It is the intersection point of the graphs of $y = 30$ and $y_A = 0.625x$. These are equations for constant altitude of the flight paths of the two planes.)

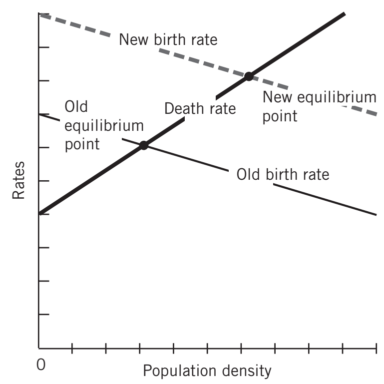
c. B, in going from $(-30, 30)$ to $(48, 30)$, travels a distance of 78 miles, and this takes B 13 minutes to do (since it is traveling at 6 miles/minute). A, in traveling from $(80, 50)$ to $(48, 30)$, covers $\sqrt{(30 - 50)^2 + (48 - 80)^2} = \sqrt{400 + 1024} = \sqrt{1424} \approx 37.7$ miles, and this will take approximately 18.9 minutes (since plane A is traveling at 2 miles per minute). Thus plane A will arrive at this point nearly 6 minutes after plane B. It is a safe situation.

35. a. For a given price the new supply curve shows more items being made.

b. The requested graph is given in the diagram. In going from the old equilibrium point to the new one, the price goes down and the quantity made/sold goes up at the equilibrium point.

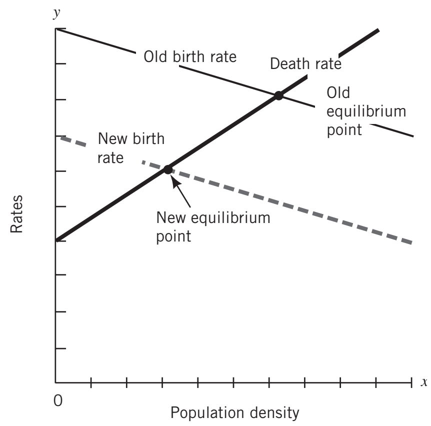


37. a. Higher birth rate:



If the birth rate increases (and the death rate stays the same) the equilibrium point moves to the right and up. This means that the equilibrium point will occur at a greater population density and a higher birth rate.

b. Lower birth rate:

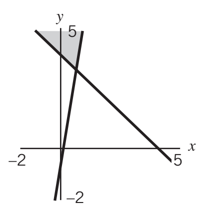


If the birth rate decreases (and the death rate stays the same) the equilibrium moves to the left and down. This means the equilibrium point will occur at a lesser population density and lower birth rate.

Section 3.3

Algebra Aerobics 3.3

1. a.



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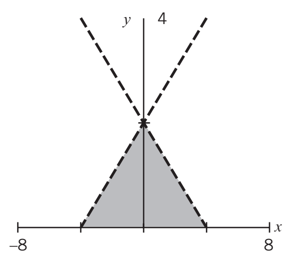
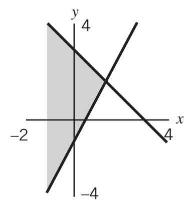
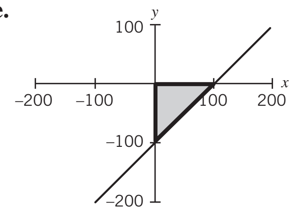


b.



e.



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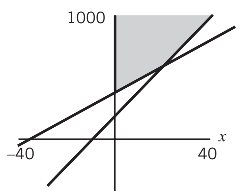
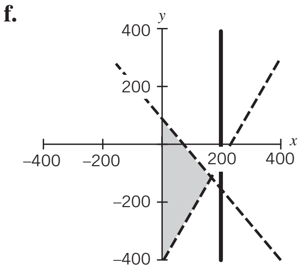


f.



2. a. $(2, 3)$ is a solution because: $3 > 2(2) - 3 \Rightarrow 3 > 1$ and $3 \le 3(2) + 8 \Rightarrow 3 \le 14$ are true.

b. $(-4, 7)$ is not a solution because: $7 \le 3(-4) + 8 \Rightarrow 7 \le -4$ is not true.

c. $(0, 8)$ is a solution because: $8 > 2(0) - 3 \Rightarrow 8 > -3$ and $(8) \le 3(0) + 8 \Rightarrow 8 \le 8$ are true.

d. $(-4, -6)$ is a solution because: $-6 > 2(-4) - 3 \Rightarrow -6 > -11$ and $-6 \le 3(-4) + 8 \Rightarrow -6 \le -4$ are true.

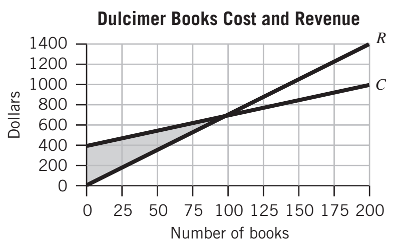
e. (20, −8) is not a solution because: $-8 > 2(20) - 3 \Rightarrow -8 > 37$ is not true.

f. (1, −1) is not a solution because: $-1 > 2(1) - 3 \Rightarrow -1 > -1$ is not true.

3. A. $y \le 2 - x$ **B.** $y > 1 + 2x$ **C.** $y \ge -3$ **D.** $x > 4$

4. a. Approximately (100, \$700). For sales of 100 books, the cost is equal to the revenue, which is \$700.

b. The region between the two graphs to the left of the breakeven point.



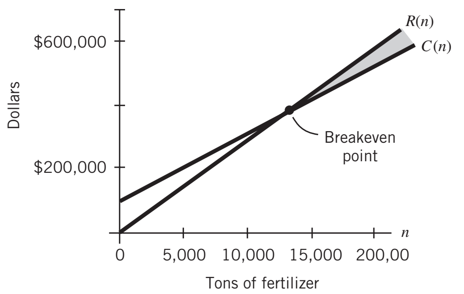
c. \$400 because that is the cost for selling 0 books (vertical intercept of the cost equation).

d. Assuming fixed costs at \$400, $C_1 \ge 3x + 400$, $R_1 \le 7x$.

5. a. $C(n) = \$50{,}000 + 235n$; $R(n) = 270n$.

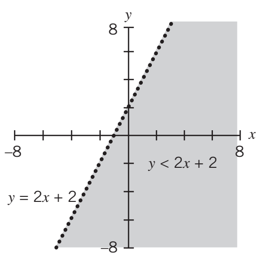
b. $C(n) = R(n)$ at breakeven point $\Rightarrow 50{,}000 + 235n = 270n \Rightarrow n \approx 1429$ tons. Selling about 1429 tons will yield a profit of \$0 since cost = revenue at the breakeven point.

c.

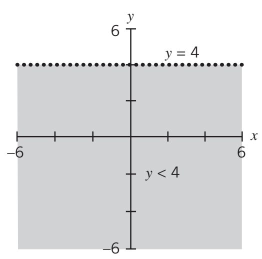


Exercises for Section 3.3

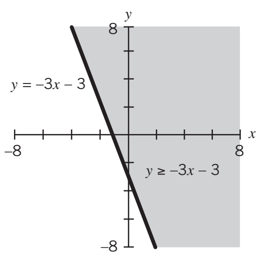
1. a.



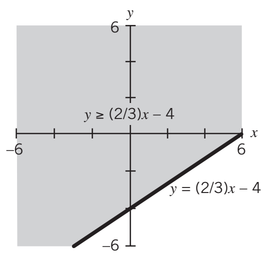
c.



b.

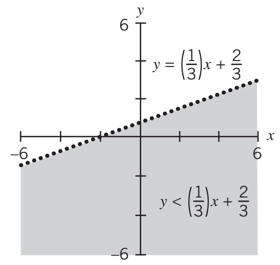


d.

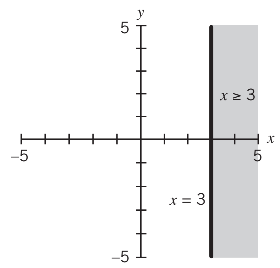


3. a. $y < \frac{2}{3}x + 2$ **b.** $y \ge -\frac{3}{2}x + 3$

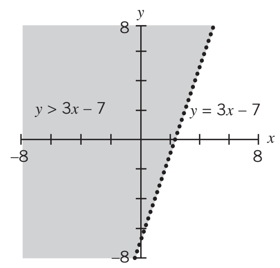
5. a. Yes, (0, 0) satisfies the inequality.



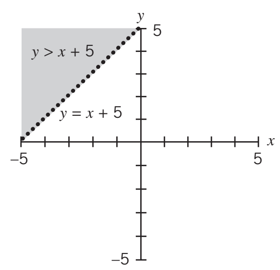
b. No, (0, 0) does not satisfy the inequality.



c. Yes, (0, 0) satisfies the inequality.



d. No, (0, 0) does not satisfy the inequality.



7. One looks to see if (0, 0) satisfies the inequality; it does here and thus the region is that half of the plane that contains (0, 0). In this case the shaded region is above the line.

9. a. goes with **g.** **c.** goes with **j.** **e.** goes with **h.**
b. goes with **i.** **d.** goes with **f.**

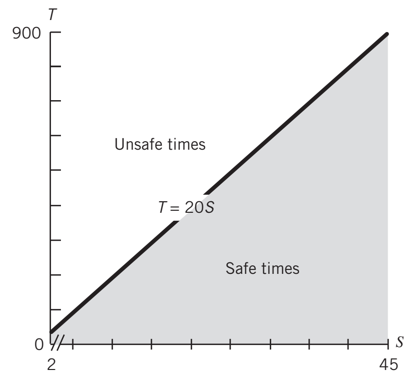
CH. 3 Exercises Solutions for Section 3.3

Ch. 3

11. a. l_1 has the equation $y = 1 + 0.25x$ and l_2 has the equation $y = 3 - 1.5x$.

b. $3 - 1.5x \leq y \leq 1 + 0.2x$

13.



a. $T = 20S$

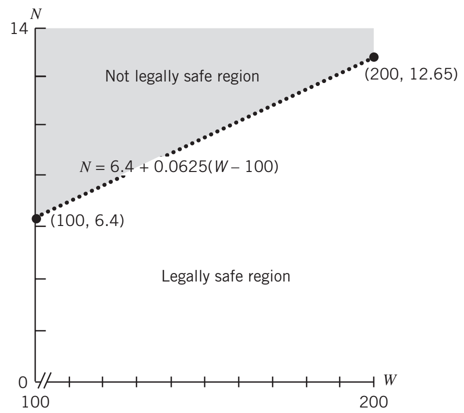
b. The graph is in the diagram. A suitable domain is $2 \leq S \leq 45$.

c. $T > 20S$ denotes unsafe times.

d. The shading and labels are found in the diagram.

e. The equation would be $T = 40S$ and its slope would be steeper.

15.



a. $N > 6.4 + 0.0625(W - 100)$, where W is measured in pounds and N measures the number of ounces of beer that gets one to the legal limit for safe driving.

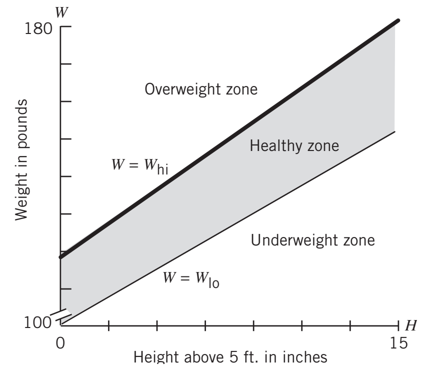
b. The sketch of the shaded areas is found in the diagram for $100 \leq W \leq 200$.

c. If $W = 100$ lb, then $N = 6.4$ oz. Thus one may legally drink 6.4 oz or less; for $W = 150$ lb we have $N = 9.525$ oz and for $W = 200$ lb we have $N = 12.65$ oz.

d. $N = 0.0625W + 0.15$

e. The given rule of thumb translates into the formula $N = 6 + 0.05(W - 100) = 0.050W + 1$. Thus it starts out higher and grows more slowly than the legal one. But its graph is lower from $W = 100$ to $W = 200$. It is a safe rule.

17.



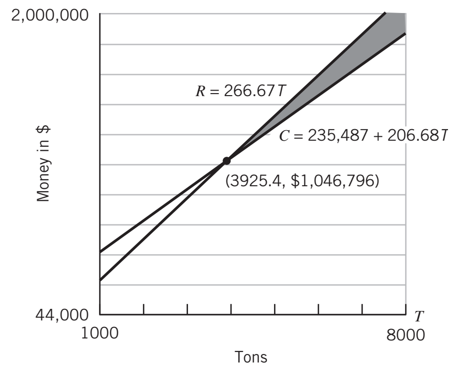
a. The two formulae and the three zones are graphed in the diagram.

b. $100 + 3.5H \leq W \leq 118.2 + 4.2H$ lb for $0 \leq H \leq 15$ in above 5 ft.

c. $W_{lo}(2) = 107$ lb and $W_{hi}(2) = 126.6$ lb. Thus the shorter woman is overweight. For the taller woman: $W_{lo}(5) = 117.5$ and $W_{hi}(5) = 139.2$. Thus the taller woman is in the healthy range.

d. $W_{hi}(4) = 135$ and $(165 - 135)/1.5 = 30/1.5 = 20$. Thus it would take 20 weeks for this woman to reach the top of the healthy range.

19.



a. $C = 235{,}487 + 206.68T$ gives the cost in dollars when T is measured in tons of fertilizer produced. $R = 266.67T$ gives the revenue in dollars from selling T tons of fertilizer.

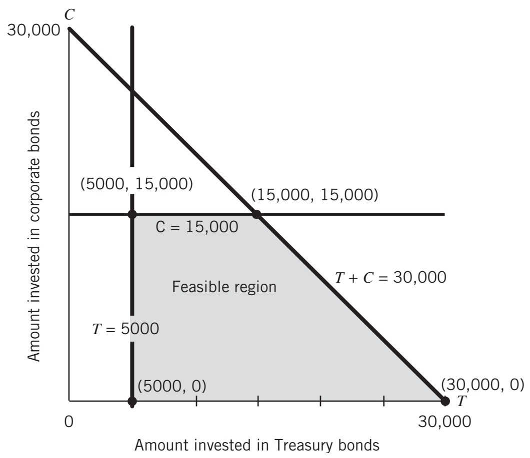
b. The graph is found in the accompanying diagram and the breakeven point is marked on the graph. It is where $T \approx 3925.4$ tons and $M \approx \$1{,}046{,}800$ dollars

c. The inequality $R - C > 0$ describes the profit region, and this occurs when $T > 3925.4$. It is shaded in the accompanying graph.

21. For Graph A: $x \geq 0$, $y \geq 0$, and $y < -1.5x + 3$

For Graph B: $x + 1 \leq y < 2x + 2$.

23.



a. $0 \le T + C \le 30{,}000$, $0 \le C \le 15{,}000$ and $5000 \le T \le 30{,}000$

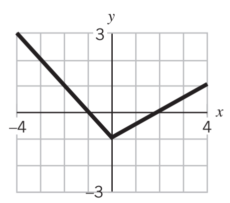
b. The feasible region is the shaded area of the graph.

c. Intersection points and interpretations: (5000, 0) is where \$5000 is invested in T bonds; (30000, 0) is where all \$30,000 is in T bonds; (5000, 15000) is where \$5000 is in T bonds and \$15,000 is in C bonds; and (15000, 15000) is where \$15,000 is in each kind.

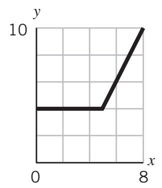
Section 3.4

Algebra Aerobics 3.4

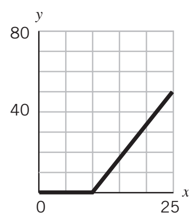
1. a.



b.



c.



2. Graph A: $f(x) = \begin{cases} x + 3 & \text{for } x \le 3 \\ -2x + 12 & \text{for } x > 3 \end{cases}$

Graph B: $f(x) = \begin{cases} -2 & \text{for } x \le 3 \\ 2x - 8 & \text{for } x > 3 \end{cases}$

3. a. $P(-5) = 3$, $P(0) = 3$, $P(2) = -3$, $P(10) = -19$

b. $W(-5) = -9$, $W(0) = -4$, $W(2) = 6$, $W(10) = 14$.

4. a. $g(i) = \begin{cases} 0.05i & \text{for } 0 \le i \le \$50{,}000 \\ 2500 + 0.08(i - 50{,}000) & \text{for } i > \$50{,}000 \end{cases}$

b. $g(i) = \begin{cases} 0.06i & \text{for } 0 \le i \le \$30{,}000 \\ 1800 + 0.09(i - 30{,}000) & \text{for } i > \$30{,}000 \end{cases}$

5. a. $f(30{,}000) = 0.0595(30{,}000) = \$1785$; $g(30{,}000) = 0.055(30{,}000) = \$1650$

Flat tax is \$135 higher for \$30,000 income.

b. $f(60{,}000) = 0.0595(60{,}000) = \$3570$

$g(60{,}000) = 2761 + 0.088(60{,}000 - 50{,}200)$
$= 2761 + 0.088(9800)$
$= 2761 + 862.40$
$= \$3623.40$

Graduated tax is \$53.40 higher for \$60,000 income.

c. $f(120{,}000) = 0.0595(120{,}000) = \$7{,}140$

$g(120.000) = 6263 + 0.098(120{,}000 - 90{,}000)$
$= 6263 + 2940$
$= \$9{,}203$

Graduated tax is \$2063 higher for \$120,000 income.

6. $y = \begin{cases} 100 + 1.5x & \text{for } 0 \le x < 200 \\ 400 & \text{for } 200 \le x \le 500 \end{cases}$

Exercises for Section 3.4

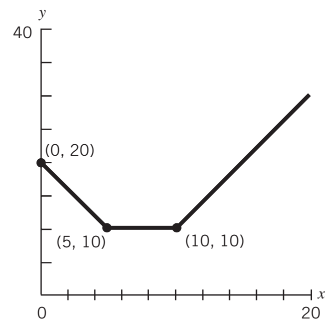
1. $f(-10) = 2 \cdot (-10) + 1 = -19$
$f(-2) = 2(-2) + 1 = -3$
$f(0) = 2 \cdot 0 + 1 = 1$
$f(2) = 3 \cdot 2 = 6$ and
$f(4) = 3 \cdot 4 = 12$

3. a. goes with Graph *B*. **b.** goes with Graph *A*.

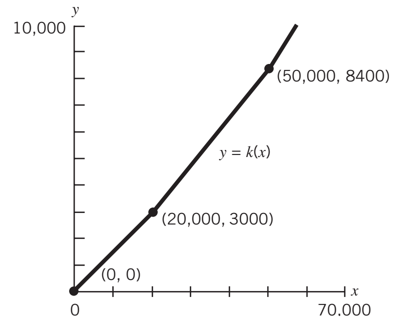
5. a. $y = \begin{cases} 1 & \text{for } 0 \le x \le 1 \\ x & \text{for } x > 1 \end{cases}$

b. $y = \begin{cases} 1 - x & \text{for } 0 \le x \le 1 \\ 1.5x - 1.5 & \text{for } x > 1 \end{cases}$

7. a. Graph of $y = h(x)$



b. Graph of $y = k(x)$



9. Answers will vary from state to state. Check student answers against the local tax form itself.

11. **a.** 45 mph is the speed limit.

b.

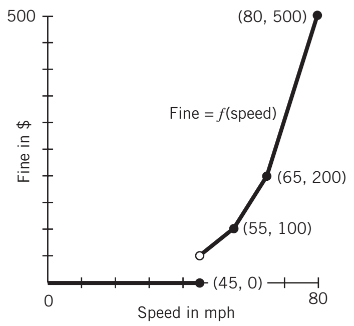
Speed (mph)	Fine ($)
40	0
45	0
50	75
55	100
60	150
65	200
70	300
75	400
80	500

c. Check the range category in which the clocked speed is found and then apply the formula for that range category.

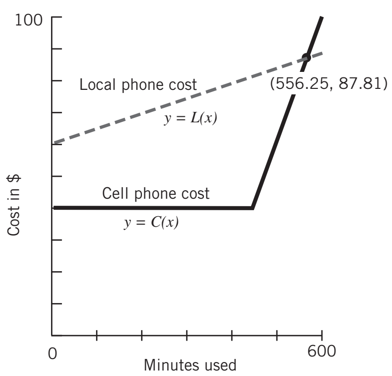
d. In each category this number represents how much more a person is fined for each increase of 1 mph in speed.

e. $F(30) = 0$; $F(57) = 100 + 10 \cdot 2 = \$120$; and $F(67) = 200 + 20 \cdot 2 = \$240$.

f.



13. The graph is given here to help one see the answers:



a. $C(x) = \begin{cases} 40 & \text{if } 0 \le x \le 450 \\ 40 + 0.45(x - 450) & \text{if } x > 450 \end{cases}$

$L(x) = 60 + 0.05x \quad \text{if } x \ge 0$

where x measures minutes used for long distance and $C(x)$ and $L(x)$ are measured in dollars.

The two cost functions are graphed in the diagram. They meet at $x = 556.25$ minutes and $C(x) = L(x) \approx \$87.81$. Thus the two plans cost the same at the point where one uses 556.25 minutes for long distance.

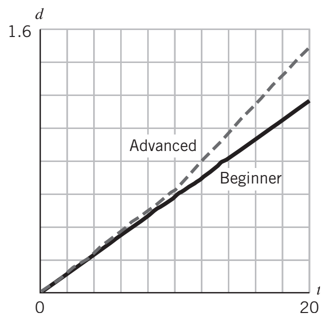
c. It would be more advantageous to use the cell phone for $0 \le x < 556.25$ minutes

d. It would be more advantageous to use the local company plan if $x > 556.25$ minutes.

15. The graphs for parts (a) and (b) are shown in the accompanying diagram.

a. For $0 \le T \le 20$: $D_{\text{beginner}} = (3.5/60)T$ or $0.0583T$ since there is 1/60 of an hour in a minute; note that T is measured in minutes and D_{beginner} is measured in miles.

b. For $0 \le T \le 10$: $D_{\text{advanced}} = (3.75/60)T$ or $0.0625T$ and for $10 < T \le 20$ we have $D_{\text{advanced}} = 0.625 + (5.25/60)(T - 10) = 0.0875T - 0.25$.



T	D_{advanced}	D_{beginner}
0	0.0000	0.0000
5	0.3125	0.2915
10	0.6250	0.5830
15	1.0630	0.8745
20	1.5000	1.1660

c. The graphs intersect only at $T = 0$.

Ch. 3: Check Your Understanding

1. False
2. True
3. True
4. False
5. False
6. False
7. False
8. True
9. True
10. True
11. False
12. True
13. False
14. False
15. True
16. True
17. True
18. False
19. False
20. True
21. False
22. True
23. True
24. True
25. False
26. True
27. False
28. True

29. True

30. Possible answer: $\begin{cases} 2x + 3y = 6 \\ 4x + 6y = 10 \end{cases}$

31. Possible answer: $\begin{cases} 2x + y = 7 \\ -6x - 3y = -21 \end{cases}$

32. Possible answer: $\begin{cases} y > 2x + 1 \\ y < 2x - 5 \end{cases}$

33. Possible answer: $\begin{cases} c = r + 1 \\ c = -r - 1 \end{cases}$

34. Possible answer: $\begin{cases} C = 25q + 2500 \\ R = 50q \end{cases}$

35. Possible answer: $p = 100 - 3q$

36. $\begin{cases} x > 0 \\ y < 0 \end{cases}$

37. False **38.** False **39.** True **40.** True

Ch. 3 Review: Putting It All Together

1. a. Maximum: approx. 1650 MMT in 2004; minimum: approx. 1420 MMT in 2002.

b. The three intersection points show when production equaled consumption.

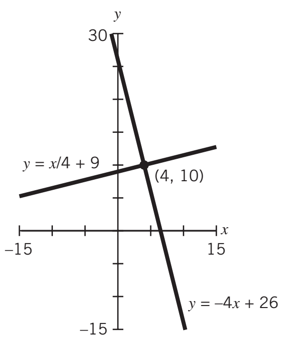
c. In 2002 there was a deficit of about 100 MMT.

d. Original title for the graph: Grain Consumption Outstrips Production Again. (Answers will vary.)

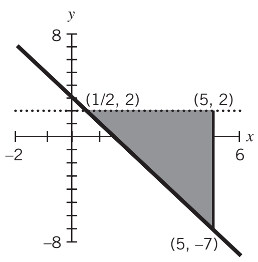
3. a. Recall that if two lines are perpendicular to each other (and neither is horizontal) then their slopes are negative reciprocals of each other.

$y = -4x + 26$ and $y = x/4 + 9$

b.



5.



7. a. $C = 10{,}000 + 7x$; $R = 12x$, where x is the number of CDs

b. $C = R \Rightarrow 10{,}000 + 7x = 12x \Rightarrow x = 2000$ CDs. Breakeven point is (2000, $24000).

c. If p is the new price per CD, then $P(1600) = 10{,}000 + 7(1600) = \$21{,}200 \Rightarrow p = \$13.25$ per CD. She would need to raise the price to $13.25 for each CD.

d. If c = new fixed cost, then $c + 7(1600) = 12(1600) \Rightarrow c = \$8000$. She would need to reduce fixed costs by $2000.

9. a. Estimates: production = 2900 thousand barrels/day and consumption = 2300 thousand barrels/day. The net difference is 600 thousand barrels per day. In 1990 China was producing more oil than it was consuming. It may have exported or stored this difference.

b. 1993. The amount of oil consumed is the same as the amount produced.

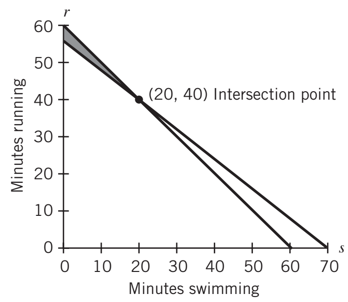
c. Estimates for 2006: production = 3800 thousand barrels/day and consumption = 7400 thousand barrels/day. The net difference is approximately 3600 thousand barrels per day. In the year 2006 China consumed almost double the amount of oil it produced. China needed to use its oil reserves or import this difference.

11. a. i. $x = -5$ and $y = 5/7$; **ii.** $a = 3$ and $b = 0.5$

b. Answers will vary. A system of two equations whose graphs are two distinct parallel lines will not have a solution.

13. a. $s + r \le 60$ minutes: $8s + 10r \ge 560$ calories; $s \ge 0$ and $r \ge 0$.

b.



c. There are many answers, for example: $s = 10$, $r = 50$ minutes is in the solution set and $s = 10$, $r = 40$ minutes is not in the solution set.

d. $r + s \le 70$ minutes; $10r + 8s \ge 560$ calories; $s \ge 0$ and $r \ge 0$. The intersection point of the boundary lines changes and the shaded area representing the solution set increases.

15.

a.

$$A(m) = \begin{cases} 39.99 & \text{for } 0 \le m \le 450 \\ 39.99 + 0.45(m - 450) & \text{for } 450 < m \le 2500 \end{cases}$$

b.

$$B(m) = \begin{cases} 59.99 & \text{for } 0 \le m \le 900 \\ 59.99 + 0.40(m - 900) & \text{for } 900 < m \le 2500 \end{cases}$$

c.

$$C(m) = \begin{cases} 79.99 & \text{for } 0 \le m \le 1350 \\ 79.99 + 0.35(m - 1350) & \text{for } 1350 < m \le 2500 \end{cases}$$

d.

Number of Minutes Used/Month	Cost		
	Plan A	Plan B	Plan C
500	39.99 + 0.45(50) = \$62.49	59.99	79.99
800	39.99 + 0.45(350) = \$197.49	59.99	79.99
1000	39.99 + 0.45(550) = \$287.49	59.99 + 0.40(100) = \$99.99	79.99

17. There have been wide swings in the real (adjusted for inflation) price of crude oil since the early 1860s. Dramatic increases occurred in the early 1860s and in the 1970s, with the maximum occurring in 1864 at over \$100 a barrel. In the early 1860s the price of crude oil increased over tenfold, and in 1974 the price was over three times the price in 1972. Even with the sharp rise in oil prices from 2004 to 2006, using real dollars adjusted for inflation, the price of oil in 2006 was only about two-thirds the price of crude oil in 1860s.

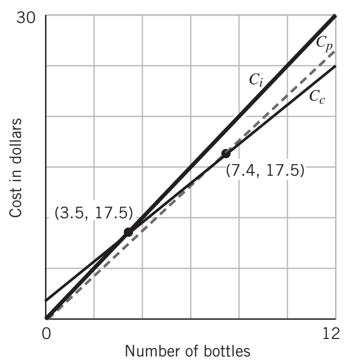
The nominal price or price actually paid for crude oil showed less variation than the price adjusted for inflation. It showed a huge surge during the Civil War, then remained fairly constant until the 1970s, after which the pattern more closely followed that of the price adjusted for inflation.

19. a. $C_p = 4.39N$

$C_c = 3.85N + 4.00$

$$C_i = \begin{cases} 4.99N & \text{for } 0 < N < 10 \\ 4.79N + 2.50 & \text{for } N \geq 10 \end{cases}$$

b. From the graphs of the three formulae given below, it can be seen that if one orders less than 7 bottles, then the C_p formula gives the best buy but if one orders 7 bottles or more, then the C_c formula gives the best buy.

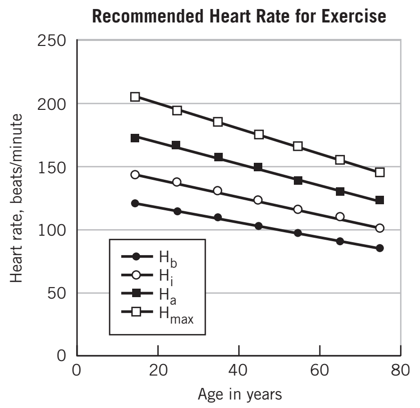


21. a. $H_b = 132 - 0.60A$

b. $H_i = 154 - 0.70A$

c. $H_a = 187 - 0.85A$

d.



Athletes are recommended to work in the zone on and between the top two lines, H_a and H_{max}.

e. $H_b = 120$, $H_i = 140$, $H_a = 170$, $H_{max} = 200$

f. 65-year-old: $I \approx 86\%$. She is just below her $H_{max} = 200 - 65 = 135$ beats per minute.

45-year-old: $I \approx 77\%$

25-year-old: $I \approx 69\%$

CHAPTER 4

Section 4.1

Algebra Aerobics 4.1

1. a. 10^{10}: to express 10 billion as a power of 10, start with 1.0, then count the ten place values the decimal must be moved to the right, in order to produce 10 billion.

b. 10^{-14}: the decimal point in 1.0 must be moved 14 place values to the left to produce 0.000 000 000 000 01.

c. 10^5

d. 10^{-5}

2. a. 0.000 000 01

b. 10,000,000,000,000

c. 0.000 1

d. 10,000,000

3. a. 10^{-9} or 0.000 000 001 sec

b. 10^3 or 1000 m

c. 10^9 or 1,000,000,000 bytes (a byte is a term used to describe a unit of computer memory).

4. a. $7 \text{ cm} \cdot \dfrac{1 \text{ m}}{100 \text{ cm}} = 7 \cdot \dfrac{1}{10^2} \text{ m} = 7 \cdot 10^{-2}$ or 0.07 m

b. $9 \text{ mm} \cdot \dfrac{1 \text{ m}}{1000 \text{ mm}} = 9 \cdot \dfrac{1}{10^3} \text{ m} = 9 \cdot 10^{-3}$ or 0.009 m

c. $5 \text{ km} \cdot \dfrac{1000 \text{ m}}{1 \text{ km}} = 5 \cdot 10^3$ or 5000 m

5. 602,000,000,000,000,000,000,000

6. $3.84 \cdot 10^8$ m

7. $1 \cdot 10^{-8}$ cm

8. 0.000 000 002 m

9. a. $-705{,}000{,}000$ **c.** 5,320,000
b. $-0.000\,040\,3$ **d.** 0.000 000 102 1

10. a. $-4.3 \cdot 10^{7}$ **c.** $5.83 \cdot 10^{3}$
b. $-8.3 \cdot 10^{-6}$ **d.** $2.41 \cdot 10^{-8}$

11. a. $\dfrac{1}{100{,}000} = \dfrac{1}{10^5} = 10^{-5}$

b. $\dfrac{1}{1{,}000{,}000{,}000} = \dfrac{1}{10^9} = 10^{-9}$

Exercises for Section 4.1

1. a. 10^{6} **d.** 10^{-3}
b. 10^{-5} **e.** 10^{13}
c. 10^{9} **f.** 10^{-8}

3. a. $1 \cdot 10^{-1}$ m **c.** $3 \cdot 10^{12}$ m
b. $4 \cdot 10^{3}$ m **d.** $6 \cdot 10^{-9}$ m

5. gigabyte $= 10^9$ bytes; terabyte $= 10^{12}$ bytes.

7. a. $2.9 \cdot 10^{-4}$ **d.** 10^{-11} **g.** $-4.9 \cdot 10^{-3}$
b. $6.54456 \cdot 10^{2}$ **e.** $2.45 \cdot 10^{-6}$
c. $7.2 \cdot 10^{5}$ **f.** $-1.98 \cdot 10^{6}$

9. a. 723,000 **c.** 0.001 **e.** 0.000188
b. 0.000526 **d.** 1,500,000 **f.** 67,800,000

11. a. False; $7.56 \cdot 10^{-3}$ **d.** False; $1.596 \cdot 10^{9}$
b. True **e.** True
c. False; $4.9 \cdot 10^{7}$ watts **f.** False; $6 \cdot 10^{-12}$ second

13. a. 9 **b.** 9 **c.** 1000 **d.** -1000

15. a. True **b.** False **c.** False **d.** True

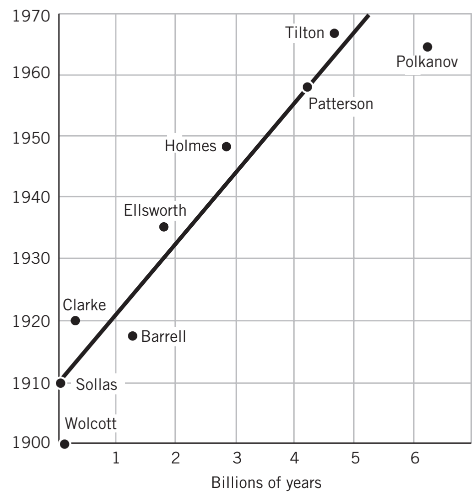
17. a. $|x - 1| < 5$ if $x = 5$; $|x - 1| > 5$ if $x = -5$
b. $2|3 - x| < 10$ if $x = 5$; $2|3 - x| > 10$ if $x = -5$.
c. $|x - 1| > 0$ whether $x = 5$ or -5
d. $|-x| > 4$ whether $x = 5$ or -5
e. $|2x - 1| < 11$ if $x = 5$; $|2x - 1| = 11$ if $x = -5$
f. $|-x| < 6$ whether $x = 5$ or -5

19. Note: Since the coordinates given below come from eyeball estimates, student answers may vary from those given here.
a. Wolcott (0.1, 1900); Sollas (0.2, 1908); and Clarke (0.3, 1921).
b. Barrell did it in 1918; the coordinates are approximately (1.3, 1918).
c. Estimating two points on a hand-drawn line could result in coordinates such as (0, 1910) and (4, 1956). Then the slope, m, of the line is

$$m = \frac{1956 - 1910}{4 - 0} = \frac{46}{4} = 11.5$$

A plot of the various estimates and the graph with this slope and y-intercept are given at the top of the next column.

d. One meaning for the slope is that for each increase of a billion years in the estimated age of Earth, time advances, on average, 11.5 years. However, it would make more sense to say that on average for every 11.5 years, estimates of Earth's age increased by a billion years.



Section 4.2

Algebra Aerobics 4.2a

1. a. $10^5 \cdot 10^7 = 10^{5+7} = 10^{12}$
b. $8^6 \cdot 8^{14} = 8^{6+14} = 8^{20}$
c. $z^5 \cdot z^4 = z^{5+4} = z^9$
d. Cannot be simplified because bases, 5 and 6, are different.
e. $7^3 + 7^3 = 7^3(1 + 1) = 2 \cdot 7^3$
f. $5 \cdot 5^6 = 5^1 \cdot 5^6 = 5^7$
g. $3^4 + 7 \cdot 3^4 = 3^4(1 + 7) = 8 \cdot 3^4$ or $2^3 \cdot 3^4$
h. $2^3 + 2^4 = 2^3 + 2^3 \cdot 2^1 = 2^3(1 + 2^1) = 3 \cdot 2^3$
i. Cannot be simplified because bases, 2 and 5, are different.

2. a. $\dfrac{10^{15}}{10^7} = 10^{15-7} = 10^8$

b. $\dfrac{8^6}{8^4} = 8^{6-4} = 8^2$

c. $\dfrac{3^5}{3^4} = 3^{5-4} = 3^1$ or 3

d. Cannot be simplified because bases, 5 and 6, are different.

e. $\dfrac{5^1}{5^6} = 5^{-5}$ **f.** $\dfrac{3^4}{3^1} = 3^3$

g. $\dfrac{2^3 \cdot 3^4}{2^1 \cdot 3^2} = \dfrac{2^3}{2^1} \cdot \dfrac{3^4}{3^2} = 2^{3-1} \cdot 3^{4-2} = 2^2 \cdot 3^2$

h. $\dfrac{6}{2^4} = \dfrac{2 \cdot 3}{2^4 \cdot 1} = \dfrac{2^1}{2^4} \cdot \dfrac{3}{1} = 2^{1-4} \cdot 3 = 3 \cdot 2^{-3}$

3. a. $10^5 \cdot 10^6 = 10^{5+6} = 10^{11}$
b. $10^3 \cdot 10^{-6} = 10^{3+(-6)} = 10^{-3}$
c. $10^{-11} \cdot 10^{-5} = 10^{-11+(-5)} = 10^{-16}$
d. $10^9 \cdot 10^{-4} = 10^{9+(-4)} = 10^5$

e. $10^6 \cdot 10^{-(-3)} = 10^{6+3} = 10^9$

f. $10^{-5} \cdot 10^{-(-4)} = 10^{-5+4} = 10^{-1}$

g. $10^{-6} \cdot 10^{-4} = 10^{-6+(-4)} = 10^{-10}$

4. a. $10^{4(5)} = 10^{20}$

b. $7^{2(3)} = 7^6$

c. $x^{4(5)} = x^{20}$

d. $(2x)^4 = 2^4x^4$ or $16x^4$

e. $(2a^4)^3 = 2^3(a^4)^3 = 2^3a^{12} = 8a^{12}$

f. $(-2a)^3 = (-2)^3a^3 = -8a^3$

g. $(-3x^2)^3 = (-3)^3(x^2)^3 = -27x^6$

h. $((x^3)^2)^4 = (x^{3\cdot 2})^4 = (x^6)^4 = x^{6\cdot 4} = x^{24}$

i. $(-5y^2)^3 = (-5)^3(y^2)^3 = -125y^6$

5. a. $\dfrac{(-2x)^3}{(4y)^3} = \dfrac{(-2)^3 \cdot x^3}{4^3 \cdot y^3} = \dfrac{-8x^3}{64y^3} = \dfrac{-x^3}{8y^3}$

b. $(-5)^2 = (-5)(-5) = 25$

c. $-5^2 = -(5)(5) = -25$

d. $-3(yz^2)^4 = -3(y)^4(z^2)^4 = -3y^4z^8$

e. $(-3yz^2)^4 = (-3)^4(y)^4(z^2)^4 = 81y^4z^8$

f. $(-3yz^2)^3 = (-3)^3(y)^3(z^2)^3 = -27y^3z^6$

6. $\dfrac{\text{(capacity of hard drive)}}{\text{(capacity of disk)}} = \text{number of disks}$

$\dfrac{4.0 \cdot 10^{10}}{7.37 \cdot 10^8} \approx 0.54 \cdot 10^2 \approx 54$ disks

7. a. $(3 + 5)^3 = 8^3 = 512$

b. $3^3 + 5^3 = 27 + 125 = 152$

c. $3 \cdot 5^2 = 3 \cdot 25 = 75$

d. $-3 \cdot 5^2 = -3 \cdot 25 = -75$

Algebra Aerobics 4.2b

1. a. $(0.000\,297\,6)(43{,}990{,}000) \approx (0.000\,3)(40{,}000{,}000)$

$= 3 \cdot 10^{-4} \cdot 4 \cdot 10^7 = 12 \cdot 10^3 = 12{,}000$

b. $\dfrac{453{,}897 \cdot 2{,}390{,}702}{0.004\,38} \approx \dfrac{500{,}000 \cdot 2{,}000{,}000}{0.004}$

$= \dfrac{(5 \cdot 10^5)(2 \cdot 10^6)}{4 \cdot 10^{-3}} = \dfrac{10}{4} \cdot \dfrac{10^{11}}{10^{-3}}$

$= 2.5 \cdot 10^{14} \approx 3 \cdot 10^{14}$

c. $\dfrac{0.000\,000\,319}{162{,}000} \approx \dfrac{0.000\,000\,3}{200{,}000} = \dfrac{3 \cdot 10^{-7}}{2 \cdot 10^5}$

$= 1.5 \cdot 10^{-12} \approx 2 \cdot 10^{-12}$

d. $28{,}000{,}000 \cdot 7629 \approx 30{,}000{,}000 \cdot 8000$

$= 3 \cdot 10^7 \cdot 8 \cdot 10^3 = 24 \cdot 10^{10}$

$= 2.4 \cdot 10^{11} \approx 2 \cdot 10^{11}$

e. $0.000\,021 \cdot 391{,}000{,}000 \approx 0.000\,02 \cdot 400{,}000{,}000$

$= 2 \cdot 10^{-5} \cdot 4 \cdot 10^8 = 8 \cdot 10^3 = 8{,}000$

2. a. $(3.0 \cdot 10^3)(4.0 \cdot 10^2) = 12 \cdot 10^5$

$= 1.2 \cdot 10^6 = 1{,}200{,}000$

b. $\dfrac{(5.0 \cdot 10^2)^2}{2.5 \cdot 10^3} = \dfrac{25 \cdot 10^4}{25 \cdot 10^2} = 1.0 \cdot 10^2 = 100$

c. $\dfrac{2.0 \cdot 10^5}{5.0 \cdot 10^3} = \dfrac{20 \cdot 10^4}{5.0 \cdot 10^3} = 4 \cdot 10^1 = 40$

d. $(4.0 \cdot 10^2)^3(2.0 \cdot 10^3)^2 = (4^3 \cdot 10^6)(4 \cdot 10^6)$

$= 4^4 \cdot 10^{12} = 256 \cdot 10^{12} = 2.56 \cdot 10^{14}$

$= 256{,}000{,}000{,}000{,}000$

3. If we use 3.14 to approximate π:

a. Surface area of Jupiter $= 4\pi r^2 \approx 4\pi(7.14 \cdot 10^4\text{ km})^2$

$= 4\pi(7.14)^2(10^4)^2\text{ km}^2$

$\approx 4(3.14)(50.98)10^8\text{ km}^2$

$\approx 640 \cdot 10^8\text{ km}^2$

$\approx 6.4 \cdot 10^2 \cdot 10^8\text{ km}^2$

$\approx 6.4 \cdot 10^{10}\text{ km}^2$

b. Volume of Jupiter $= \frac{4}{3}\pi r^3$

$\approx (1.3) \cdot (3.14)(7.14 \cdot 10^4\text{ km})^3$

$\approx (4.08)(7.14)^3(10^4)^3\text{ km}^3$

$\approx (4.08)(364)10^{12}\text{ km}^3$

$\approx 1486 \cdot 10^{12}\text{ km}^3$

$\approx 1.486 \cdot 10^3 \cdot 10^{12}\text{ km}^3$

$\approx 1.486 \cdot 10^{15}\text{ km}^3$

4. If only 3/7 of the farmable land is used, the people/sq. mi. of used farmland is:

$\dfrac{6.6 \cdot 10^9\text{ people}}{(3/7)12 \cdot 10^6\text{ sq.mi.}} = \left(\dfrac{7}{3}\right)\dfrac{6.6 \cdot 10^9\text{ people}}{12 \cdot 10^6\text{ sq.mi.}}$

$\approx 1.283 \cdot 10^3$ or 1283 people/sq.mi.

For fractions > 0, if the denominator is decreased, the value of that fraction is increased. So, one expects this ratio to be larger than the ratio of people to farmable land.

Exercises for Section 4.2

1. a. 10^7

b. $1.1 \cdot 10^4$

c. $2 \cdot 10^3$

d. x^{15}

e. x^{50}

f. $4^7 + 5^2$—this expression cannot be simplified without multiplying out the values and adding them together.

g. z^5

h. 1 or as is

i. 3^{-1} or 1/3

j. 4^{11}

3. a. $16a^4$

b. $-2a^4$

c. $-x^{15}$

d. $-8a^3b^6$

e. $32x^{20}$

f. $18x^6$

g. $2500a^{20}$

h. as is—nothing is simpler

5. a. $-\left(\frac{5}{8}\right)^2 = -\frac{25}{64}$

b. $\left(\frac{3x^3}{5y^2}\right)^3 = \frac{3^3x^9}{5^3y^6} = \frac{27x^9}{125y^6}$

c. $\left(\frac{-10x^5}{2b^2}\right)^4 = \frac{10^4x^{20}}{4^2b^8} = \frac{10{,}000x^{20}}{16b^8} = 625\frac{x^{20}}{b^8}$

d. $\left(\frac{-x^5}{x^2}\right)^3 = -x^9$

7. a. $(2 \cdot 10^6) \cdot (4 \cdot 10^3) = 8 \cdot 10^9$

b. $(1.4 \cdot 10^6) \div (7 \cdot 10^3) = 0.2 \cdot 10^3 = 2 \cdot 10^2$

c. $(5 \cdot 10^{10}) \cdot (6 \cdot 10^{13}) = 30 \cdot 10^{23} = 3 \cdot 10^{24}$

d. $(2.5 \cdot 10^{12}) \div (5 \cdot 10^5) = 0.5 \cdot 10^7 = 5 \cdot 10^6$

9. a. $x^{13}y^4$ **c.** $-8x^9y^9$ **e.** $81x^8y^{20}$
b. $5x^4y$ **d.** $16x^{10}y^8$ **f.** $\frac{9}{25}x^4$

11. a. $10^9/10^6 = 10^3 = 1000$
b. $1000/10 = 10^2 = 100$
c. $1000/0.001 = 10^6 = 1{,}000{,}000$
d. $10^{-6}/10^{-9} = 10^3 = 1000$

13. a. Japan's population density $= (1.275 \cdot 10^8$ people)/$(1.525 \cdot 10^5$ miles$^2) \approx 836$ people/mile^2.
b. The U.S. population density $= (3.0 \cdot 10^8$ people)/$(3.62 \cdot 10^6$ miles$^2) \approx 83$ people/mile^2.
c. Japan's population density is $(836)/(83) = 10$ times larger, or one order of magnitude larger than that of the United States.

15. $(1.5 \cdot 10^4$ beverages/sec)$(8.64 \cdot 10^4$ sec/day) $= 1.296 \cdot 10^9$ beverages/day, or over a billion Coca-Cola beverages were consumed each day worldwide in 2005.

17. a. If a is positive, then $-a$ is negative. If n is even, then $(-a)^n$ is positive; but if n is odd, then $(-a)^n$ is negative. If a is negative, then $-a$ is positive, then $(-a)^n$ is positive whether n is even or odd.
b. This is answered in part (a).

19. a. $\left(\frac{m^2n^3}{mn}\right)^2 = (mn^2)^2 = m^2n^4$ and $\left(\frac{m^2n^3}{mn}\right)^2 = \frac{m^4n^6}{m^2n^2} = m^2n^4$

b. $\left(\frac{2a^2b^3}{ab^2}\right)^4 = (2ab)^4 = 16a^4b^4$ and

$\left(\frac{2a^2b^3}{ab^2}\right)^4 = \frac{16a^8b^{12}}{a^4b^8} = 16a^4b^4$

21. $\left(\frac{2a^3}{5b^2}\right)^4 = \frac{2^4a^{12}}{5^4b^8} = \frac{16a^{12}}{625b^8}$

23. Two cases are distinguished:
If $n = 0$, then $(ab)^0 = 1$ and $a^0 \cdot b^0 = 1 \cdot 1 = 1$
If $n > 0$, then $a^n = a \cdot \cdots \cdot a$ (n factors) and $b^n = b \cdot \cdots \cdot b$ (n factors) and thus $a^n \cdot b^n = (a \cdot \cdots \cdot a) \cdot (b \cdot \cdots \cdot b) = (ab) \cdot \cdots \cdot (ab)$ (n factors), after rearrangement, and this product is what is meant by $(ab)^n$ when $n > 0$.

25. a. Generated in the United Kingdom:
(81 terawatt-hours)/$(60.6 \cdot 10^6$ persons) $\approx 1.34 \cdot 10^{-6}$ terawatt-hours/person.
(81 terawatt-hours)/(94,525 miles^2) $\approx 8.57 \cdot 10^{-4}$ terawatt-hours/mile^2.
b. Generated in the United States:
(780 terawatt-hours)/$(3.0 \cdot 10^8$ persons) $\approx 2.60 \cdot 10^{-6}$ terawatt-hours/person.
(780 terawatt-hours)/(3,675,031 miles^2) $\approx 2.12 \cdot 10^{-4}$ terawatt-hours/mile^2.
c. The United Kingdom produces about four times more terawatt-hours per square mile than the United States, because UK/US $= (8.57 \cdot 10^{-4}$ terawatt-hours/mile^2)/$(2.12 \cdot 10^{-4}$ terawatt-hours/mile$^2) \approx 4.04$.
d. Answers will vary. The United States produces (uses) $(2.60 \cdot 10^{-6}$ terawatt-hours/person)/$(1.34 \cdot 10^{-6}$ terawatt-hours/person) $\approx$ 1.9 times more nuclear energy per person than the United Kingdom, but the United Kingdom produces 4 times more nuclear energy than the United States per square mile.

Section 4.3

Algebra Aerobics 4.3

1. a. $10^{5-7} = 10^{-2} = \frac{1}{10^2}$
b. $11^{6-(-4)} = 11^{6+4} = 11^{10}$
c. $3^{-5-(-4)} = 3^{-5+4} = 3^{-1} = 1/3$
d. Cannot be simplified: different bases, 5 and 6.
e. $7^{3-3} = 7^0 = 1$
f. $a^{-2+(-3)} = a^{-5} = \frac{1}{a^5}$
g. $3^4 \cdot 3^3 = 3^7$
h. $(2^2 \cdot 3) \cdot (2^6) \cdot (2^4 \cdot 3) = 2^2 \cdot 2^6 \cdot 2^4 \cdot 3 \cdot 3 = 2^{12} \cdot 3^2$

2. Time for a TV signal to travel across the United States = (time to travel 1 kilometer) $\cdot$ (number of kilometers)
$= (3.3 \cdot 10^{-6})$ sec/km $\cdot$ $(4.3 \cdot 10^3)$ km
$= (3.3 \cdot 4.3) \cdot (10^{-6} \cdot 10^3)$ sec
$= 14 \cdot 10^{-3}$ sec
$= 1.4 \cdot 10 \cdot 10^{-3}$ sec
$= 1.4 \cdot 10^{-2}$ sec or 0.014 sec
So it would take less than two-hundredths of a second for the signal to cross the United States.

3. a. $x^{-2}(x^5 + x^{-6}) = x^{-2}(x^5) + x^{-2}(x^{-6})$
$= x^3 + x^{-8}$
$= x^3 + \frac{1}{x^8}$
b. $-a^2(b^2 - 3ab + 5a^2)$
$= b^2(-a^2) - 3ab(-a^2) + 5a^2(-a^2)$
$= -a^2b^2 + 3a^{1+2}b - 5a^{2+2}$
$= -a^2b^2 + 3a^3b - 5a^4$

4. a. $10^{(4)(-5)} = 10^{-20} = \frac{1}{10^{20}}$
b. $7^{(-2)(-3)} = 7^6$
c. $\frac{1}{(2a^3)^2} = \frac{1}{4a^6}$
d. $\left(\frac{8}{x}\right)^{-2} = \left(\frac{x}{8}\right)^2 = \frac{x^2}{64}$
e. $2^{-1}x^2 = \frac{x^2}{2}$
f. $2x^2$
g. $\left(\frac{3}{2y^2}\right)^{-4} = \left(\frac{2y^2}{3}\right)^4 = \frac{2^4y^8}{3^4} = \frac{16y^8}{81}$
h. $3 \cdot (2y^2)^4 = 3 \cdot 2^4y^8 = 48y^8$

5. a. $\frac{t^{-3}(1)}{t^{-12}} = t^{-3-(-12)} = t^9$
b. $\frac{v^{-3}w^7}{v^{-6}w^{-10}} = v^{-3-(-6)}w^{7-(-10)} = v^3w^{17}$
c. $\frac{7^{-8}x^{-1}y^2}{7^{-5}x^1y^3} = 7^{(-8)-(-5)}x^{(-1)-1}y^{2-3}$
$= 7^{-3}x^{-2}y^{-1} = \frac{1}{7^3x^2y}$
d. $\frac{a(5b^{-1}c^3)^2}{5ab^2c^{-6}} = \frac{a^1 \cdot 5^2b^{-2}c^6}{5^1a^1b^2c^{-6}} = 5^{2-1}a^{1-1}b^{(-2)-2}c^{6-(-6)}$
$= 5b^{-4}c^{12} = \frac{5c^{12}}{b^4}$

Exercises for Section 4.3

1. **a.** 10^1 **b.** $10^{-5} = \frac{1}{10^5}$ **c.** $10^{-6} = \frac{1}{10^6}$ **d.** 10^5

3. **a.** $\frac{1}{2^2x^4} = \frac{1}{4x^4}$ **b.** xy^{12} **c.** $\frac{1}{x^3y^2}$ **d.** $(x + y)^{11}$ **e.** $\frac{ab^7}{c^6}$

5. **a.** $4.6 \cdot 10^{10}$ **b.** $4.07 \cdot 10^3$ **c.** $1.525 \cdot 10^{11}$ **d.** $5.1669 \cdot 10^{-8}$ **e.** $2.3833 \cdot 10^{158}$ **f.** $2.601 \cdot 10^{-21}$

7. **a.** $1 \cdot 10^{-1} \cdot 10^{-5} = 1 \cdot 10^{-6}$
b. $5 \cdot 10^{-5}/(5 \cdot 10^4) = 1 \cdot 10^{-9}$
c. $3/(6 \cdot 10^{-3}) = 0.5 \cdot 10^3 = 5 \cdot 10^2$
d. $8 \cdot 10^3/(8 \cdot 10^{-4}) = 1 \cdot 10^7$
e. $6.4 \cdot 10^{-3}/(8 \cdot 10^3) = 0.8 \cdot 10^{-6} = 8 \cdot 10^{-7}$
f. $5 \cdot 10^6 \cdot 4 \cdot 10^4 = 20 \cdot 10^{10} = 2 \cdot 10^{11}$

9. **a.** -6 **b.** -6 **c.** 4 **d.** -5

11. **a.** $-\frac{y^3}{2^3x^9}$ **b.** $\frac{1}{2^2x^4}$ **c.** $\frac{5xy^9}{3}$ **d.** $\frac{3^6z^{24}}{x^6}$

13. 200 times longer or $1.6 \cdot 10^{-2}$ second

15. **a.** $\left(\frac{1}{x^2} - \frac{1}{y}\right) \cdot (xy^2) = \left(\frac{y - x^2}{x^2y}\right) \cdot (xy^2) = (y - x^2) \cdot \left(\frac{y}{x}\right)$

$$= \frac{y^2 - x^2y}{x}$$

(This one is difficult because you need to find a common denominator.)

b. $\frac{x^4y^6}{5^2} = \frac{x^4y^6}{25}$

17. $5.23 \cdot 10^{-3}$, 0.00523 and 5.23/1000

19. $\frac{1}{x^p} = \left(\frac{1}{x}\right)^p$ for any p. Here we have $p = -n$; thus

$$\frac{1}{x^{-n}} = \left(\frac{1}{x}\right)^{-n} = (x^{-1})^{-n} = x^{(-1)\cdot(-n)} = x^n$$

21. $(9 \cdot 10^9)/(35 \cdot 10^6) \approx$ 257 kernels per pound.

Section 4.4

Algebra Aerobics 4.4

1. $2\,\cancel{l} \cdot \frac{1\ \cancel{qt}}{0.946\,\cancel{l}} \cdot \frac{32\text{ oz}}{1\ \cancel{qt}} \approx 67.65$ oz

2. $120\ \cancel{cm} \cdot \frac{1\text{ in}}{2.54\ \cancel{cm}} \approx 47.24$ in.

3. **a.** $12\ \cancel{in.} \cdot \frac{2.54\text{ cm}}{1\ \cancel{in.}} = 30.48$ cm

b. $100\ \cancel{yd} \cdot \frac{0.914\text{ m}}{1\ \cancel{yd}} = 91.4$ m

c. $20\ \cancel{kg} \cdot \frac{1\text{ lb}}{0.4536\ \cancel{kg}} \approx 44.09$ lb

d. $\frac{\$40{,}000}{1\ \cancel{year}} \cdot \frac{1\ \cancel{year}}{52\ \cancel{weeks}} \cdot \frac{1\ \cancel{workweek}}{40\text{ hr}} \approx \$19.23$ per hour

e. $\frac{24\ \cancel{hours}}{1\text{ day}} \cdot \frac{60\ \cancel{min}}{1\ \cancel{hour}} \cdot \frac{60\text{ sec}}{1\ \cancel{min}} = 86{,}400$ sec/day

f. $1\ \cancel{gal} \cdot \frac{4\ \cancel{qt}}{1\ \cancel{gal}} \cdot \frac{946\text{ ml}}{1\ \cancel{qt}} = 3784$ ml

g. $\frac{1\ \cancel{mile}}{1\ \cancel{hour}} \cdot \frac{1\ \cancel{hour}}{3600\text{ sec}} \cdot \frac{5280\text{ ft}}{1\ \cancel{mile}} \approx 1.47$ ft/sec

4. 1 km = 1000 m, so the conversion factor from meters to kilometers is $\frac{1\text{ km}}{10^3\text{ m}}$. Hence,

$$3.84 \cdot 10^8\ \cancel{m} \cdot \frac{1\text{ km}}{10^3\ \cancel{m}} = 3.84 \cdot 10^5\text{ km}$$

5. 1 km = 1000 m, so the conversion factor from kilometers to meters is $\frac{10^3\text{ m}}{1\text{ km}}$. Hence

$$7.8 \cdot 10^8\ \cancel{km} \cdot \frac{10^3\text{ m}}{1\ \cancel{km}} = 7.8 \cdot 10^8 10^3 = 7.8 \cdot 10^{11}\text{ m}$$

6. 1 km = 0.62 mi, so the conversion factor to convert from km to mi is $\frac{0.62\text{ mi}}{1\text{ km}}$. Hence

$$9.46 \cdot 10^{12}\ \cancel{km} \cdot \frac{0.62\text{ mi}}{1\ \cancel{km}} = 5.87 \cdot 10^{12}\text{ mi}$$

which is close to $5.88 \cdot 10^{12}$ mi.

7. 1 m = 100 cm, so the conversion factor for converting from cm to m is $\frac{1\text{ m}}{100\text{ cm}}$.

Hence 1 Å = $10^{-8}\ \cancel{cm} \cdot \frac{1\text{ m}}{10^2\ \cancel{cm}} = 10^{-10}$ m.

8. 1 km = 0.62 mi, so the conversion factor for converting from km to miles is $\frac{0.62\text{ mi}}{1\text{ km}}$.

Hence, 218 km = $218\ \cancel{km} \cdot \frac{0.62\text{ mi}}{1\ \cancel{km}}$ = 135.16 mi or ~135 mi.

9. If a dollar bill is 6 in long, then two dollar bills/12 in = 2 dollars/ft. The number of dollars needed to reach from Earth to the sun is:

$93{,}000{,}000\ \cancel{mi} \cdot 5{,}280 \frac{\cancel{ft}}{\cancel{mi}} \cdot \frac{2\text{ dollars}}{\cancel{ft}}$

$\approx 9.3 \cdot 5.3 \cdot 2 \cdot 10^{7+3}$ dollars

$\approx 98.6 \cdot 10^{10} \approx 9.9 \cdot 10^{11}$

or 990,000,000,000 dollar bills, almost a trillion dollars.

10. $\frac{2{,}560\ \cancel{mi}}{4.2\text{ hrs}} \cdot \frac{1.6\text{ km}}{1\ \cancel{mi}} = \frac{4096\text{ km}}{4.2\text{ hrs}} = \frac{4096\text{ km}}{4.2\ \cancel{hrs}} \cdot \frac{1\ \cancel{hr}}{60\text{ min}}$

$= \frac{4096\text{ km}}{252\text{ min}}$ (or approximately 16.25 km/min)

11. $5\ \mu\text{m} \cdot \frac{1\text{ m}}{10^6\ \mu\text{m}} \cdot \frac{1000\text{ mm}}{1\text{ m}} = 5 \cdot 10^{-3}\text{ mm} \Rightarrow$ 1 spore is $5 \cdot 10^{-3}$ mm in diameter. To compare the 1 mm diameter of a pencil to the diameter of a spore, we divide: $\frac{1\text{ mm}}{5 \cdot 10^{-3}\text{ mm}} = 200$. The diameter of a pencil is approximately 200 times larger than the diameter of a spore.

12. **a.** $4.3\ \cancel{light\text{-}year} \cdot \frac{9.46 \cdot 10^{12}\text{ km}}{1\ \cancel{light\text{-}year}} \approx 4.07 \cdot 10^{13}$ km;

$4.3\ \cancel{light\text{-}year} \cdot \frac{5.88 \cdot 10^{12}\text{ miles}}{1\ \cancel{light\text{-}year}} \approx 2.53 \cdot 10^{13}$ miles

b. $10^8\ \cancel{light\text{-}years} \cdot \frac{9.46 \cdot 10^{12}\ \cancel{km}}{1\ \cancel{light\text{-}year}} \cdot \frac{1000\text{ m}}{1\ \cancel{km}}$

$= 9.46 \cdot 10^{23}$ m

c. $1.6 \cdot 10^3\ \cancel{light\text{-}year} \cdot \frac{5.88 \cdot 10^{12}\ \cancel{miles}}{1\ \cancel{light\text{-}year}} \cdot \frac{5.28 \cdot 10^3\text{ ft}}{1\ \cancel{mile}}$

$\approx 49.67 \cdot 10^{18} = 4.97 \cdot 10^{19}$ ft

13. **a.** $0.5\ \cancel{Å} \cdot \frac{10^{-10}\text{ m}}{1\ \cancel{Å}} = 5 \cdot 10^{-1} \cdot 10^{-10} = 5.0 \cdot 10^{-11}$ m

b. $10^5\,\text{Å} \cdot \frac{10^{-10}\,\text{m}}{1\,\text{Å}} = 10^{-5}$ m

c. $10^{-5}\,\text{Å} \cdot \frac{10^{-10}\,\text{m}}{1\,\text{Å}} = 10^{-15}$ m

14. $364\ \text{Smoot} \cdot \frac{5.6\ \text{ft}}{1\ \text{Smoot}} = 2038.4$ ft

Exercises for Section 4.4

1. a. $(50\ \text{miles}) \cdot \frac{1.609\ \text{km}}{1\ \text{mile}} = 80.45$ km

b. $(3\ \text{ft}) \cdot \frac{0.305\ \text{m}}{1\ \text{ft}} \approx 0.92$ m

c. $(5\ \text{lb}) \cdot \frac{0.4536\ \text{kg}}{1\ \text{lb}} \approx 2.27$ kg

d. $(12\ \text{in.}) \cdot \frac{2.54\ \text{cm}}{1\ \text{in.}} = 30.48$ cm

e. $(60\ \text{ft}) \cdot \frac{0.305\ \text{m}}{1\ \text{ft}} = 18.3$ m

f. $(4\ \text{qt}) \cdot \frac{0.946\ \text{liters}}{1\ \text{qt}} \approx 3.78$ liters

3. a. Student estimates will vary. There are approximately 30.5 cm per foot.

b. From conversation table, 1 ft = 0.305 m or approximately 30% of a meter.

5. Converting the units to decimeters we get: $(100\ \text{km}) \cdot (250\ \text{m}) \cdot (25\ \text{m}) = 10^6 \cdot (2.5 \cdot 10^3) \cdot (2.5 \cdot 10^2)$ cubic decimeters $= 6.25 \cdot 10^{11}$ cubic decimeters $= (6.25 \cdot 10^{11}\ \text{liters}) \cdot (10^3\ \text{droplets/liter}) = 6.25 \cdot 10^{14}$ droplets

7. 1 m ≈ 3.28 ft and thus 9.8 m/sec ≈ 9.8 · 3.28 = 32.144 ft/sec

9. a. 186,000 miles per second or one hundred and eighty-six thousand miles per second.

b. $\frac{1.86 \cdot 10^5\ \text{mi}}{\text{sec}} \cdot \frac{1609\ \text{m}}{\text{mi}} \cdot \frac{60\ \text{sec}}{\text{min}} \cdot \frac{60\ \text{min}}{\text{hr}} \cdot \frac{24\ \text{hr}}{\text{day}} \cdot \frac{365\ \text{days}}{\text{yr}}$

$\approx 9.438 \cdot 10^{15}$ m/yr

11. 500 seconds; 500 seconds or 8 minutes and 20 seconds from now.

13. U.S. barrel of oil = 42 gal = 42 · 4 qt = 168 qt ≈ 168 · 0.946 liter = 158.928 liters. Thus the British barrel of oil is larger than the U.S. barrel of oil by approximately 4.727 liters.

15. a. 1 acre = 43,560 ft^2 and thus is $\sqrt{43{,}560} \approx 208.71$ ft on each side.

b. $150x = 1.5 \cdot 43{,}560$ or $x = 435.6$ ft.; perimeter = 2 · (150 + 435.6) = 1171.2 feet or 390.4 yards.

c. 1 hectometer = 100 meters; 1 square hectometer = 10,000 square meters. 1 meter ≈ 3.28 ft. Thus 1 sq. meter ≈ 10.7584 square ft. Thus 1 acre = 43,560 sq. ft = 43,560/10.7584 ≈ 4048.929 square meters ≈ 0.4049 square hectometer.

d. 1 hectare = 100 acres = $4{,}356{,}000\ \text{ft}^2/[(5280)^2\ \text{ft}^2/\text{mi}^2]$ = 0.15625 sq. mi. Thus 1 acre = 0.0015625 sq. mile. Thus in a square mile there are 1/0.0015625 = 640 acres. This is 6.4 hectares.

17. Light travels 186,000 · 5280 ft/sec and thus it travels $186000 \cdot 5280/10^9 \approx 0.98208$ ft per nanosecond.

19. $\frac{15{,}000\ \text{bev}}{\text{second}} \cdot \frac{60\ \text{sec}}{1\ \text{min}} \cdot \frac{60\ \text{min}}{1\ \text{hr}} \cdot \frac{24\ \text{hr}}{1\ \text{day}} \cdot \frac{365\ \text{day}}{1\ \text{year}} \cdot \frac{1}{6.45 \cdot 10^9\ \text{persons}}$

≈ 73.34 beverages/year/person

21. a. Volume = $\pi \cdot 9^2 \cdot 4$ cu. ft = 324π cu. ft ≈ 1018 cu. ft, and 1018 cu. ft · 1728 (cu. in.)/(cu. ft) = 1,759,104 cu. in., and since there are 231 gal per cubic inch, there are 1,759,104/231 ≈ 7615 gal.

b. 7615 · 24 gal/(2500 gal/hr) ≈ 3.05 hr

c. 7615 · 24 gal/(10,000 gal/lb) ≈ 0.761 lb

23. a. Bush's BMI $\approx \frac{(194\ \text{lb})/(2.2\ \text{lb/kg})}{72\ \text{in.}/39.37\ \text{in.}/\text{m}^2} \approx \frac{88.18\ \text{kg}}{1.83^2\text{m}^2} \approx 26.33\ \text{kg/m}^2$.

$\approx 26.33\ \text{kg/m}^2$.

He is slightly overweight.

b. BMI in pounds and inches is BMI = $(\text{pounds}/2.2)/(\text{inches}/39.37)^2 = [(39.7)^2/2.2](\text{lb/in}^2)$, or approximately 704.5 lb/in^2. Thus for Bush we get $704.5 \cdot 194/72^2 \approx 26.36$, which is about the same.

c. Answers will vary, but note that 0.45 ≈ 1/2.2 and 0.254 ≈ 1/39.37.

d. A kilogram, more precisely, is approximately 2.2046 lb, and $39.37^2/2.2046 \approx 703.07$. Thus Kigner is correct.

25. a. 10^{-35} m $= 10^{-35} \cdot 10^{-3} = 10^{-38}$ km

b. 10^{-35} m $\approx 10^{-35} \cdot 0.00062 = 6.2 \cdot 10^{-39}$ mile

c. $x = 10^{-35}/3 \cdot 10^8 \approx 3.33 \cdot 10^{-44}$ sec

Section 4.5

Algebra Aerobics 4.5a

1. a. $\sqrt{81} = 9$

b. $\sqrt{144} = 12$

c. $\sqrt{36} = 6$

d. $-\sqrt{49} = -7$

e. not a real number

2. a. $\sqrt{9x} = 3x^{1/2}$

b. $\sqrt{\frac{x^2}{25}} = \frac{\sqrt{x^2}}{\sqrt{25}} = \frac{x}{5}$

c. $\sqrt{36x^2} = \sqrt{36}\sqrt{x^2} = 6x$

d. $\sqrt{\frac{9y^2}{25x^4}} = \frac{\sqrt{9y^2}}{\sqrt{25x^4}} = \frac{3y}{5x^2}$

e. $\sqrt{\frac{49}{x^2}} = \frac{\sqrt{49}}{\sqrt{x^2}} = \frac{7}{x}$

f. $\sqrt{\frac{4a}{169}} = \frac{\sqrt{4a}}{\sqrt{169}} = \frac{2\sqrt{a}}{13} = \frac{2a^{1/2}}{13}$

3. a. $S = \sqrt{30 \cdot 60} = \sqrt{1800} \approx 42$ mph

b. $S = \sqrt{30 \cdot 200} = \sqrt{6000} \approx 77$ mph

4. a. $\sqrt{25} < \sqrt{29} < \sqrt{36} \Rightarrow 5 < \sqrt{29} < 6$; 5 and 6

b. $\sqrt{81} < \sqrt{92} < \sqrt{100} \Rightarrow 9 < \sqrt{92} < 10$; 9 and 10

c. $\sqrt{100} < \sqrt{117} < \sqrt{121} \Rightarrow 10 < \sqrt{117} < 11$; 10 and 11.

d. $\sqrt{64} < \sqrt{79} < \sqrt{81} \Rightarrow 8 < \sqrt{79} < 9$; 8 and 9.

e. $\sqrt{36} < \sqrt{39} < \sqrt{49} \Rightarrow 6 < \sqrt{39} < 7$; 6 and 7.

5. a. $\sqrt[3]{27} = 3$

b. $\sqrt[4]{16} = 2$

c. $\frac{1}{\sqrt[3]{8}} = \frac{1}{2}$

d. $\sqrt[5]{32} = 2$

e. $\frac{1}{27^{1/3}} = \frac{1}{\sqrt[3]{27}} = \frac{1}{3}$

f. $\frac{1}{25^{1/2}} = \frac{1}{\sqrt{25}} = \frac{1}{5}$

g. $\left(\frac{8}{27}\right)^{-1/3} = \left(\frac{27}{8}\right)^{1/3}$

$= \sqrt[3]{\frac{27}{8}} = \frac{\sqrt[3]{27}}{\sqrt[3]{8}} = \frac{3}{2}$

h. $\sqrt{\frac{1}{16}} = \frac{1}{4}$

6. a. $(-27)^{1/3} = -3$ since $(-3)^3 = -27$

b. There is no real number solution to the fourth root of a negative number, since a negative or positive number raised to the fourth power is always positive.

c. $(-1000)^{1/3} = -10$ since $(-10)^3 = -1000$.

d. $-\sqrt[4]{16} = -2$, since $2^4 = 16 \Rightarrow -\sqrt[4]{2^4} = -2$.

e. $\sqrt[3]{-8} = -2$, since $(-2)^3 = -8$.

f. $\sqrt{2500} = 50$, since $50^2 = 2500$.

7. a. $V = \frac{4}{3}\pi r^3 \Rightarrow \frac{3}{4}V = \pi r^3 \Rightarrow \frac{3V}{4\pi} = r^3 \Rightarrow$

$r = \sqrt[3]{\frac{3 \cdot 2 \text{ feet}^3}{4\pi}} = \sqrt[3]{\frac{3 \text{ feet}^3}{2\pi}} \approx \sqrt[3]{0.478 \text{ feet}^3}$

≈ 0.78 feet

We can express that with a more meaningful figure if we convert it into inches. Since 1 ft = 12 in., we can use a conversion factor of 1 = (12 in.)/(1 foot). The radius of the balloon is:

$$(0.78 \text{ ft})\left(\frac{12 \text{ in.}}{1 \text{ ft}}\right) = 9.36 \text{ in. or } \approx 9.4 \text{ in.}$$

8. a. $\sqrt{25} = 5$

b. $-\sqrt{49} = -7$

c. -5

d. $\sqrt{45} - 3\sqrt{125} = \sqrt{9 \cdot 5} - 3\sqrt{25 \cdot 5}$

$= 3\sqrt{5} - 15\sqrt{5} = -12\sqrt{5}$

9. a. $\sqrt{36} = (6^2)^{1/2} = 6^{2/2} = 6$

b. $\sqrt[3]{27x^6} = (3^3 x^6)^{1/3} = 3^{3/3} x^{6/3} = 3x^2$

c. $\sqrt[4]{81a^4 b^{12}} = (3^4 a^4 b^{12})^{1/4} = 3^{4/4} a^{4/4} b^{12/4} = 3ab^3$

10. a. $r^2 = \frac{V}{\pi h} \Rightarrow r = \sqrt{\frac{V}{\pi h}}$

b. $r^2 = \frac{3V}{\pi h} \Rightarrow r = \sqrt{\frac{3V}{\pi h}}$

c. $S = \sqrt[3]{V}$

d. $a = \sqrt{c^2 - b^2}$

e. $x = \sqrt{\frac{S}{6}}$

Algebra Aerobics 4.5b

1. a. $2^{1/2} \cdot 2^{1/3} = 2^{1/2+1/3} = 2^{5/6}$

b. $5^{1/2} \cdot 5^{1/4} = 5^{1/2+1/4} = 5^{3/4}$

c. $3^{1/2} \cdot 9^{1/3} = 3^{1/2} \cdot (3^2)^{1/3} = 3^{1/2} \cdot 3^{2/3} = 3^{7/6}$

d. $x^{1/4} \cdot x^{1/3} = x^{7/12}$

e. $x^{3/4} \cdot x^{1/2} = x^{5/4}$

f. $x^{1/3} \cdot y^{2/3} \cdot x^{1/2} \cdot y^{1/2} = x^{5/6} \cdot y^{7/6}$

2. a. $\frac{2^{1/2}}{2^{1/3}} = 2^{1/2-1/3} = 2^{1/6}$

b. $\frac{2^1}{2^{1/4}} = 2^{1-1/4} = 2^{3/4}$

c. $\frac{5^{1/4}}{5^{1/3}} = 5^{1/4-1/3} = 5^{-1/12}$ or $\frac{1}{5^{1/12}}$

d. $\frac{x^{1/2}}{x^{3/4}} = x^{-\frac{1}{4}}$ or $\frac{1}{x^{1/4}}$

e. $\frac{x^{1/3} \cdot y^{2/3}}{x^{1/2} \cdot y^{1/2}} = x^{-1/6} \cdot y^{1/6}$ or $\frac{y^{1/6}}{x^{1/6}}$

3. a. $c = 17.1(0.25)^{3/8}$ cm

$c \approx 17.1(0.59)$ cm $\Rightarrow c \approx 10.2$ cm

b. $c = 17.1(25)^{3/8}$ cm

$c \approx 17.1(3.34)$ cm $\Rightarrow c \approx 57.2$ cm

4. a. $\sqrt{20x^2} = \sqrt{2^2 \cdot 5 \cdot x^2} = 2x\sqrt{5}$

b. $\sqrt{75a^3} = \sqrt{5^2 \cdot 3 \cdot a^2 \cdot a} = 5a\sqrt{3a}$

c. $\sqrt[3]{16x^3 y^4} = \sqrt[3]{2^3 \cdot 2 \cdot x^3 \cdot y^3 \cdot y} = 2xy\sqrt[3]{2y}$

d. $\frac{\sqrt[4]{32x^4y^6}}{\sqrt[4]{81x^8y^5}} = \frac{\sqrt[4]{2^4 \cdot 2 \cdot x^4 \cdot y^4 \cdot y^2}}{\sqrt[4]{3^4 \cdot (x^2)^4 \cdot y^4 \cdot y}}$

$= \frac{2xy \cdot \sqrt[4]{2y^2}}{3x^2y \cdot \sqrt[4]{y}} = \frac{2}{3x}\sqrt[4]{\frac{2y^2}{y}} = \frac{2}{3x}\sqrt[4]{2y}$

5. a. $\sqrt{4a^2 b^6} = (2^2 a^2 b^6)^{1/2} = 2^{2/2}a^{2/2}b^{6/2} = 2ab^3$

b. $\sqrt[4]{16x^4 y^6} = (2^4 x^4 y^6)^{1/4} = 2^{4/4} x^{4/4} y^{6/4}$

$= 2xy^{3/2}$

c. $\sqrt[3]{8.0 \cdot 10^{-9}} = (2^3 \cdot 10^{-9})^{1/3} = 2^{3/3} \cdot 10^{-9/3}$

$= 2 \cdot 10^{-3} = 0.002$

d. $\sqrt{8a^{-4}} = (2^3a^{-4})^{1/2} = 2^{3/2}a^{-2}$

Exercises for Section 4.5

1. a. 10 **b.** -10 **c.** $1/10 = 0.1$ **d.** $-1/10 = -0.1$ **e.** -10 **f.** -10

3. a. $\sqrt{\frac{a^2b^4}{c^6}} = \frac{ab^2}{c^3}$

b. $\sqrt{36x^4y} = 6x^2\sqrt{y}$

c. $\sqrt{\frac{49x}{y^6}} = \frac{7\sqrt{x}}{y^3}$

d. $\sqrt{\frac{x^4y^2}{100z^6}} = \frac{x^2y}{10z^3}$

5. a. $5\sqrt{5a}$ **b.** $\frac{1}{2xy^3}$ **c.** $2xy\sqrt{2x}$ **d.** $8x^2y^2\sqrt{y}$

7. a. $6 \cdot 10^3$ **b.** $2 \cdot 10^3$ **c.** $5 \cdot 10^5$ **d.** $1.0 \cdot 10^{-2}$

9. a. 0.1 **b.** 1/5 **c.** 4/3 **d.** 0.1

11. a. 4 **b.** -4 **c.** $\sqrt{2} \approx 1.414$ **d.** not defined **e.** $2\sqrt{2} \approx 2.8284$ **f.** 1

13. a. $\sqrt{3} + \sqrt{7} > \sqrt{3 + 7}$

b. $\sqrt{3^2 + 2^2} < 5$

c. $\sqrt{5^2 - 4^2} > 2$

15. a. 9 **b.** $\frac{1}{8}$ **c.** $\frac{1}{125}$ **d.** $\frac{1}{27}$

17. $V = (4/3)\pi r^3$ and thus if $V = 4$, then $r^3 = 3/\pi$. If one uses 3 as a crude estimate of π, then r is approximately 1 foot. (A more precise estimate, from using a calculator, is 0.985 ft.)

19. a. height $= 4\sqrt{3}$ cm ≈ 6.9 cm

b. area $= 16\sqrt{3}$ cm$^2 \approx 27.7$ cm^2

21. a. If $L \approx 2.67$ ft, then

$T = 2\pi\sqrt{\frac{2.67}{32}} \approx 0.578\pi$ sec ≈ 1.816 sec.

b. Solving for L, we get: $L = 8T^2/(\pi^2)$. So if T is 2 seconds, then $L = 8(2)^2/(\pi^2) = 32/\pi^2 \approx 3.24$ feet.

23. a. 3673 lb **b.** 6345 lb

Section 4.6

Algebra Aerobics 4.6

1. Since $\frac{\text{magnitude 1988 Armenia earthquake}}{\text{magnitude 1987 LA earthquake}}$

$\Rightarrow \frac{10^{6.9}}{10^{5.9}} = 10^1$, the Armenian earthquake had tremors about one order of magnitude larger than those in Los Angeles.

2. Since $\frac{\text{magnitude Chile earthquake}}{\text{magnitude 2003 Little Rock earthquake}}$

$\Rightarrow \frac{10^{9.5}}{10^{6.5}} = 10^3$, the maximum tremor size of the Little Rock earthquake of 2003 was 1000 times smaller (or three orders of magnitude smaller) than the maximum tremor size of the Chile earthquake.

3. Since $\frac{\text{your salary}}{\text{my salary of \$100,000}} = 10^1 \Rightarrow$

your salary is $1,000,000;

$\frac{\text{Henry's salary}}{\text{my salary \$100,000}} = 10^{-2} \Rightarrow$

$\frac{\text{Henry's salary}}{\text{my salary \$100,000}} = \frac{1}{10^2} \Rightarrow$ Henry's salary is $1000.

4. a. Since $\frac{\text{radius of the Milky Way}}{\text{radius of the sun}} = \frac{10^{21}}{10^9} = 10^{21-9} = 10^{12}$, the radius of the Milky Way is 12 orders of magnitude larger than the radius of the sun, or equivalently, the radius of the sun is 12 orders of magnitude smaller than the radius of the Milky Way.

b. Since $\frac{\text{radius of a proton}}{\text{radius of the hydrogen atom}} = \frac{10^{-15}}{10^{-11}} = 10^{-15-(-11)} = 10^{-4}$, the radius of the proton is four orders of magnitude smaller than the radius of the hydrogen atom, or equivalently, the radius of the hydrogen atom is four orders of magnitude larger than the radius of a proton.

5. a. $4 million since $400,000 · 10^1 = $4 million

b. $65,000 since $650,000 · 10^{-1} = $65,000

6. 1 km = 1000 m = 10^3 meters, so three orders of magnitude;

$1\ \cancel{\text{km}} \cdot \frac{1000\ \cancel{\text{m}}}{1\ \cancel{\text{km}}} \cdot \frac{1000\ \text{mm}}{1\ \cancel{\text{m}}} = 10^6$ mm, so six orders of magnitude

7. a. 1,000,000,000 m = 10^9 m $\Rightarrow$ plot at 10^9 m

b. 0.000 000 7 m = $7 \cdot 10^{-7}$ m $\approx 10 \cdot 10^{-7}$ m = 10^{-6} m $\Rightarrow$ plot at 10^{-6} m

c. Plot at $10^7 \cdot 10^{-2} = 10^5$ m

Exercises for Section 4.6

1. a. 12 **b.** 5 **c.** 7 **d.** 6

3. a. 52.61 is one order of magnitude larger than 5.261.

b. 5.261 is three orders of magnitude smaller than 5261.

c. 526.1 is four orders of magnitude smaller than $5.261 \cdot 10^6$.

5. a. $(50 \cdot 10^3) \cdot (3 \cdot 10^8) \cdot (365 \cdot 24 \cdot 60^2) \approx 4.7304 \cdot 10^{20}$ = number of meters in 50,000 light-years. Hence, the Milky Way is $(4.7302 \cdot 10^{20})/(0.5 \cdot 10^{-4}) = 9.4608 \cdot 10^{24}$ times larger than the first life form. Thus the Milky Way is nearly 25 orders of magnitude larger than the first living organism on Earth.

b. $(100 \cdot 10^6)/(100 \cdot 10^3) = 10^3$; thus Pleiades is three orders of magnitude older than *Homo sapiens*.

7. The raindrop is 10^{24} times heavier and this is an order of magnitude of 24.

9. a. $2.0 \cdot 10^{-5}$ in.

b. $2.0 \cdot 10^{-5}/39 \approx 5.128 \cdot 10^{-7}$ m

c. The name is a bit off — by two orders of magnitude (looking at meters).

d. The tweezers would have to be made able to grasp things two orders of magnitude smaller than they can grasp now.

11. a. i. Radius of the moon = 1,758,288.293 meters, or about $1.76 \cdot 10^6$ meters.

ii. Radius of Earth = 6,400,000 meters, or about $6.4 \cdot 10^6$ meters.

iii. Radius of the sun = 695,414,634.100 meters, or about $6.95 \cdot 10^8$ meters.

b. i. The surface area of a sphere is $4\pi r^2$, and thus the ratio of the surface area of Earth to that of the moon is the same as the ratio of the squares of their radii, or $[6.4 \cdot 10^6/(1.76 \cdot 10^6)]^2 \approx 13.22$. Thus the surface area of Earth is one order of magnitude bigger.

ii. The volume of a sphere is $(4/3)\pi r^3$, and thus the ratio of the volume of the sun to that of moon is the same as the ratio of the cubes of their radii, or $[6.95 \cdot 10^8/(1.76 \cdot 10^6)]^3 \approx 6.16 \cdot 10^7$. Thus the volume of the sun is seven orders of magnitude bigger than the volume of the moon.

13. Scale (a) is additive because the distances are equally spaced, and scale (b) is multiplicative or logarithmic because the distances are spaced like the logarithms of numbers.

15. a. Being 5 orders of magnitude larger than the first atoms means that it is 10^{-5} m and thus would appear at −5 on the log scale.

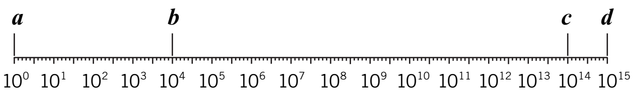
b. Being 20 orders of magnitude smaller than the radius of the sun means that it is 10^{-11} m and thus would appear at −11 on the log scale.

17. a. 1 watt = 10^0 watts

b. 10 kilowatts = 10^4 watts

c. 100 billion kilowatts = $10^2 \cdot 10^9 \cdot 10^3 = 10^{14}$ watts

d. 1000 terawatts = 10^{15} watts



Ch. 4

Section 4.7

Algebra Aerobics 4.7a

1. a. Since 10,000,000 = 10^7, log 10,000,000 = 7.

b. Since 0.000 000 1 = 10^{-7}, log 0.000 000 1 = −7.

c. Since 10,000 = 10^4, log 10,000 = 4.

d. Since 0.0001 = 10^{-4}, log 0.0001 = −4.

e. Since 1000 = 10^3, log 1000 = 3.

f. Since 0.001 = 10^{-3}, log 0.001 = −3.

g. Since 1 = 10^0, log 1 = 0.

2. a. 100,000 = 10^5 **c.** 10 = 10^1

b. 0.000 000 01 = 10^{-8} **d.** 0.01 = 10^{-2}

3. a. $\log N = 3 \Rightarrow N = 10^3 = 1000$

b. $\log N = -1 \Rightarrow N = 10^{-1} = 0.1$

c. $\log N = 6 \Rightarrow N = 10^6 = 1{,}000{,}000$

d. $\log N = 0 \Rightarrow N = 10^0 = 1$

e. $\log N = -2 \Rightarrow N = 10^{-2} = 0.01$

4. a. log 1000 = c, $c = 3$

b. log 0.001 = c, $c = -3$

c. log 100,000 = c, $c = 5$

d. log 0.000 01 = c, $c = -5$

e. log 1,000,000 = c, $c = 6$

f. log 0.000 001 = c, $c = -6$

5. a. $x - 3 = 2$, so $x = 5$

b. $2x - 1 = 4$, so $x = 5/2$

c. $10^1 = x - 2$, so $x = 12$

d. $10^{-1} = 5x$, so $x = \frac{0.1}{5} = 0.02$

Algebra Aerobics 4.7b

1. a. log 3 is the number c such that $10^c = 3$. With a calculator you can find that: $10^{0.4} \approx 2.512$ and $10^{0.5} \approx 3.162 \Rightarrow 0.4 < \log 3 < 0.5$. A calculator gives log 3 ≈ 0.477.

b. log 6 is the number c such that $10^c = 6$. With a calculator you can find that: $10^{0.7} \approx 5.012$ and $10^{0.8} \approx 6.310 \Rightarrow 0.7 < \log 6 < 0.8$. A calculator gives log 6 ≈ 0.778.

c. log 6.37 is the number c such that $10^c = 6.37$. Since $10^{0.8} \approx 6.310$ and $10^{0.9} \approx 7.943$, then $0.8 < \log 6.37 < 0.9$. A calculator gives log 6.37 ≈ 0.804.

2. a. Write 3,000,000 in scientific notation: $3{,}000{,}000 = 3 \cdot 10^6$

and substitute $10^{0.48}$ for 3: $3{,}000{,}000 \approx 10^{0.48} \cdot 10^6$

then combine powers: $3{,}000{,}000 \approx 10^{6.48}$

and rewrite as a logarithm: log 3,000,000 ≈ 6.48

A calculator gives log 3,000,000 ≈ 6.477121255.

b. Write 0.006 in scientific notation: $0.006 = 6 \cdot 10^{-3}$

and substitute $10^{0.78}$ for 6: $0.006 \approx 10^{0.78} \cdot 10^{-3}$

then combine powers: $0.006 \approx 10^{0.78-3} \approx 10^{-2.22}$

and rewrite as a logarithm: log 0.006 = −2.22

A calculator gives log 0.006 ≈ −2.22184875.

3. a. $0.000\,000\,7 = 10^{\log 0.0000007} \approx 10^{-6.1549}$

b. $780{,}000{,}000 = 10^{\log 780{,}000{,}000} \approx 10^{8.892}$

c. $0.0042 = 10^{\log 0.0042} \approx 10^{-2.3768}$

d. $5{,}400{,}000{,}000 = 10^{\log(5{,}400{,}000{,}000)} = 10^{9.732}$

4. a. You want to estimate a number x such that $10^{4.125} = x$. Since $10^4 = 10{,}000$, an estimate for x is 12,000. With a calculator, $x \approx 13{,}335$.

b. You want to estimate a number x such that $10^{5.125} = x$. Since $10^5 = 100{,}000$, an estimate for x is 120,000. With a calculator, $x \approx 133{,}352$.

c. You want to estimate a number x such that $10^{2.125} = x$. Since $10^2 = 100$, an estimate for x is 120. With a calculator, $x \approx 133$.

5. a. You want to estimate a value x such that $\log 250 = x$. $10^2 < 250 < 10^3 \Rightarrow 2 < x < 3$. With a calculator $x \approx 2.398$.

b. You want to estimate a value x such that $\log 250{,}000 = x$. $10^5 < 250{,}000 < 10^6 \Rightarrow 5 < x < 6$. With a calculator, $x \approx 5.398$.

c. You want to estimate a value x such that $\log 0.075 = x$. $10^{-2} < 0.075 < 10^{-1} \Rightarrow -2 < x < -1$. With a calculator, $x \approx -1.125$.

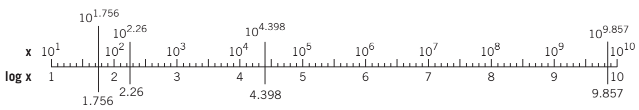
d. You want to estimate a value x such that $\log 0.000\,075 = x$. $10^{-5} < 0.000\,075 < 10^{-4} \Rightarrow -5 < x < -4$. With a calculator, $x \approx -4.125$.

6. a. $\log 57 \approx 1.756 \Rightarrow 57 \approx 10^{1.756}$

b. $\log 182 \approx 2.26 \Rightarrow 182 \approx 10^{2.26}$

c. $\log 25{,}000 \approx 4.398 \Rightarrow 25{,}000 \approx 10^{4.398}$

d. $\log 7.2 \cdot 10^9 \approx 9.857 \Rightarrow 7.2 \cdot 10^9 \approx 10^{9.857}$



Exercises for Section 4.7

1. a. log(10,000) = 4 **c.** log(1) = 0

b. log(0.01) = −2 **d.** log(0.00001) = −5

3. Since log(375) ≈ 2.574, we have that $375 \approx 10^{2.574}$.

5. a. $\log_{10}(100) = 2$

b. $\log_{10}(10{,}000{,}000) = 7$

c. $\log_{10}(0.001) = -3$
d. $10^1 = 10$
e. $10^4 = 10{,}000$
f. $10^{-4} = 0.0001$

7. **a.** $x = 10^{1.255} \approx 17.9887 \approx 18$
b. $x = 10^{3.51} \approx 3235.94 \approx 3236$
c. $x = 10^{4.23} \approx 16{,}982.44 \approx 16{,}982$
d. $10^{7.65} \approx 44{,}668{,}359.22 \approx 44{,}668{,}359$

9. **a.** $x = \log(12{,}500) \approx 4.097$
b. $x = \log(3{,}526{,}000) \approx 6.547$
c. $x = \log(597) \approx 2.776$
d. $x = \log(756{,}821) \approx 5.879$

11. **a.** $x = 10^{0.82} \approx 6.607$
b. $x = \log(0.012) \approx -1.921$
c. $x = 10^{0.33} \approx 2.138$
d. $x = \log(0.25) \approx -0.602$

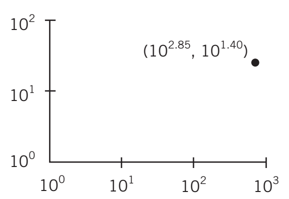
13. **a.** $x - 5 = 3$ or $x = 8$
b. $2x + 10 = 100$ or $2x = 90$ or $x = 45$
c. $3x - 1 = -4$ or $3x = -3$ or $x = -1$
d. $500 - 25x = 1000$ or $-25x = 500$ or $x = -20$

15. **a.** $1 < \log(11) < 2$ since $1 < 11 < 100$ and $\log(11) \approx 1.041$.
b. $4 < \log(12{,}000) < 5$ since $10{,}000 < 12{,}000 < 100{,}000$ and $\log(12{,}000) \approx 4.079$.
c. $-1 < \log(0.125) < 0$ since $10^{-1} = 0.1 < 0.125 < 1 = 10^0$ and $\log(0.125) \approx -0.903$.

17. **a.** Multiplying by 10^{-3}
b. Multiplying by $\sqrt{10}$
c. Multiplying by 10^2
d. Multiplying by 10^{10}

19. **a.** $\text{pH} = -\log(10^{-7}) = 7$
b. $\text{pH} = -\log(1.4 \cdot 10^{-3}) = 3 - \log(1.4) \approx 2.85$
c. $11.5 = -\log([\text{H}^+])$; thus $[\text{H}^+] = 10^{-11.5} \approx 3.16 \cdot 10^{-12}$
d. A higher pH means a lower hydrogen ion concentration, and one can see this because in plotting pH values one uses the numbers on the top of the given scale.
e. Pure water is neutral, orange juice is acidic, and ammonia is basic. In plotting, one uses the top numbers to find the right spots. Thus water would be placed at the 7 mark, orange juice 85% of the way between the 2 and 3 marks, and ammonia halfway between the 11 and 12 marks.

21.



23. We measure all in seconds, the smallest time unit.
a. 1 heartbeat ≈ 100 sec
b. 10 minutes $= 600$ sec $\approx 10^{2.8}$ sec
c. 7 days $= 7 \cdot 24 \cdot 3600$ sec $\approx 10^{5.8}$ sec
d. 1 year $= 365 \cdot 24 \cdot 3600$ sec $\approx 10^{7.5}$ sec
e. 38,000 years $= 38{,}000 \cdot 365 \cdot 24 \cdot 3600 \approx 10^{12.1}$ sec
f. $2.2 \cdot 10^6 \cdot 365 \cdot 24 \cdot 3600$ sec $\approx 10^{13.8}$ sec

Ch. 4: Check Your Understanding

1. True	**8.** True	**15.** True	**22.** True
2. False	**9.** False	**16.** True	**23.** True
3. False	**10.** False	**17.** False	**24.** False
4. False	**11.** False	**18.** False	**25.** False
5. True	**12.** False	**19.** False	**26.** True
6. True	**13.** False	**20.** False	
7. False	**14.** True	**21.** False	

27. Possible answer: population of city A $= 583{,}240$ and population of city B $= 3615$.

28. Possible answer: $x = 150{,}000{,}000$.

29. Possible answer: $x = 0.45$.

30. Possible answer: b such that $b = \frac{1}{4}$.

31. $b = 1$.

32. Possible answer: $b = -3$.

33. False	**36.** True	**39.** False	**42.** True
34. False	**37.** False	**40.** False	
35. True	**38.** False	**41.** False	

Ch. 4 Review: Putting It All Together

1. **a.** 4200 **b.** -40 **c.** -16 **d.** $\frac{1}{10}$ **e.** 27

3. When x is an odd integer the statement is true.

5. $\frac{20\text{ ft}}{3\text{ sec}} \cdot \frac{60\text{ sec}}{1\text{ min}} \cdot \frac{60\text{ min}}{1\text{ hr}} \cdot \frac{1\text{ mile}}{5280\text{ ft}} \approx 4.55$ miles/hour

7. Tiger Woods makes $\$111.9 \cdot 10^6$/yr. $= \$1.119 \cdot 10^8$/yr. The worker makes

$$\frac{\$6.40}{\text{hr}} \cdot \frac{40\text{ hr}}{\text{wk}} \cdot \frac{52\text{ wk}}{\text{yr}} = \frac{\$13{,}312}{\text{yr}} = \$1.3312 \cdot 10^4/\text{yr}.$$

Tiger Wood makes 4 orders of magnitude or about 10,000 times more than the minimum-wage worker.

The worker must work $\frac{\$1.119 \cdot 10^8}{\$1.3312 \cdot 10^4} \approx 0.84 \cdot 10^4$ years or about 8,400 years to make Tiger Wood's annual pay.

Item	Value	Value in Scientific Notation
Mass-energy of electron	0.000 000 000 000 051 J	$5.1 \cdot 10^{-14}$ J
The kinetic energy of a flying mosquito	0.000 000 160 2 J	$1.602 \cdot 10^{-7}$
An average person swinging a baseball bat	80 J	$8 \cdot 10^1$ J
Energy received from the sun at the Earth's orbit by one square meter in one second	1360 J	$1.360 \cdot 10^3$ J
Energy released by one gram of TNT	4184 J	$4.184 \cdot 10^3$ J
Energy released by metabolism of one gram of fat	38,000 J	$3.8 \cdot 10^4$ J
Approximate annual power usage of a standard clothes dryer	320,000,000 J	$3.2 \cdot 10^8$ J

9. Convert to same unit of measure—for example, km².

Country	Area	Scientific Notation
Russia	17,075,200 km²	$1.70752 \cdot 10^7$ km²
Chile	$(290{,}125 \text{ mi}^2)\left(\frac{1.609 \text{ km}}{1 \text{ mi}}\right)^2 \approx 751{,}099$ km²	$7.51099 \cdot 10^5$ km²
Canada	$(3{,}830{,}840 \text{ mi}^2)\left(\frac{1.609 \text{ km}}{1 \text{ mi}}\right)^2 \approx 9{,}917{,}589$ km²	$9.917589 \cdot 10^6$ km²
South Africa	1,184,825 km²	$1.184825 \cdot 10^6$ km²
Norway	323,895 km²	$3.23895 \cdot 10^5$ km²
Monaco	$0.5 \text{ mi}^2 \left(\frac{1.609 \text{ km}}{1 \text{ mi}}\right)^2 \approx 1.29$ km²	$1.29 \cdot 10^0$ km²

a. Russia is the largest in area. Monaco is the smallest in area.
b. Russia, Canada, South Africa, Chile, Norway, Monaco
c. Russia is seven orders of magnitude larger than Monaco.

11. False. An increase in one order of magnitude is the same as multiplying by 10. A 100% increase would only double the original amount.

13. a. $\frac{1 \text{ baby}}{7 \text{ sec}} \cdot \frac{60 \text{ sec}}{1 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \cdot \frac{24 \text{ hr}}{1 \text{ day}} = 12{,}342.9 \frac{\text{babies}}{\text{day}} \approx$ 12,343 babies/day

b. $\frac{1 \text{ immigrant}}{31 \text{ sec}} \cdot \frac{60 \text{ sec}}{1 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \cdot \frac{24 \text{ hr}}{1 \text{ day}} = 2787.1 \frac{\text{immigrants}}{\text{day}} \approx$ 2787 immigrants/day

c. $\frac{1 \text{ death}}{13 \text{ sec}} \cdot \frac{60 \text{ sec}}{1 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \cdot \frac{24 \text{ hr}}{1 \text{ day}} = 6646.15 \frac{\text{deaths}}{\text{day}} \approx$ 6646 deaths/day

15. Volume of a sphere $= \frac{4}{3}\pi r^3$, so the volume of Earth =

$$\frac{4}{3}\pi(6.3 \cdot 10^6 \text{ m})^3 = \left(\frac{4}{3}\pi(6.3)^3\right) \cdot 10^{18}\text{m}^3 \approx 1047.4 \cdot 10^{18}\text{ m}^3 \approx 1.0474 \cdot 10^{21}\text{ m}^3$$

$$\text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{5.97 \cdot 10^{24} \text{ kg}}{1.0474.10^{21} \text{ m}^3} \approx 5.7 \cdot 10^3 \text{ kg/m}^3$$

17. The patient's weight in kilograms is:

$$130 \text{ lb} \cdot \frac{0.4536 \text{ kg}}{1 \text{ lb}} \approx 59 \text{ kg}.$$

$$\text{daily dosage} = \frac{5 \text{ mg}}{\text{kg}} \cdot 59 \text{ kg} \approx 295 \text{ mg}$$

Since each tablet is 100 mg, the patient should take

$$\frac{295 \text{ mg}}{100 \text{ mg/tablet}} = 2.95, \text{ or 3 tablets each day.}$$

19. a. $\log 1 = 0$
b. $\log 1{,}000{,}000{,}000 = \log 10^9 = 9$
c. $\log 0.000\,001 = \log 10^{-6} = -6$

21. a. See chart at top of page for solution.
b. $\frac{3.2 \cdot 10^8 \text{ J}}{8 \cdot 10^1 \text{ J}} = 0.4 \cdot 10^7 = 4 \cdot 10^6$, or 4 million times more energy.
c. $\frac{3.8 \cdot 10^4 \text{ J}}{1.602 \cdot 10^{-7} \text{ J}} \approx 2.372 \cdot 10^{11}$, or approximately 200 billion times more energy.

23. (Requires scientific calculator.) Converting height and weight to kilograms and centimeters, respectively:

$W = 180 \text{ lb} \cdot \frac{0.4536 \text{ kg}}{1 \text{ lb}} \approx 81.6 \text{ kg}$ and

$H = 6 \text{ ft} \cdot \frac{30.5 \text{ cm}}{1 \text{ ft}} = 183 \text{ cm}$

$\text{BSA} = 71.84 \cdot 81.6^{0.425} \cdot 183^{0.725} \approx 20{,}376 \text{ cm}^2$

Converting cm² to m²:

$20{,}376 \text{ cm}^2 \cdot \frac{1 \text{ m}^2}{(100 \text{ cm})^2} = 2.04 \text{ m}^2$

$\text{daily dosage} = 15\frac{\text{mg}}{\text{m}^2} \cdot 2.04 \text{ m}^2 = 30.6 \text{ mg}$

CHAPTER 5

Section 5.1

Algebra Aerobics 5.1

1. a. Initial value = 350
Growth factor = 5

b. Initial value = 25,000
Growth factor = 1.5

c. Initial value = 7000
Growth factor = 4

d. Initial value = 5000
Growth factor = 1.025

2. a. $P = 3000 \cdot 3^t$
b. $P = (4 \cdot 10^7)(1.3)^t$
c. $P = 75 \cdot 4^t$
d. $P = \$30{,}000(1.12)^t$

3. a. $P = 28 \cdot 1.065^0 = 28$ million or 28,000,000 people; the initial population
b. $P = 28 \cdot 1.065^{10} \approx 52.6$ million or 52,600,000 people;
$P = 28 \cdot 1.065^{20} \approx 98.7$ million or 98,700,000;
$P = 28 \cdot 1.065^{30} \approx 185.2$ million or 185,200,000
c. $t \approx 11$ years for P to double or reach 56,000,000.

4.

t	0	1	10	15	20
P	80	84.80	143.27	191.72	256.57

a. 80
b. 84.8
c. The amount doubles (reaches 160) between $t = 10$ and $t = 15$
d. $t \approx 12$ time periods.

Exercises for Section 5.1

1. **a.** 275; 3 **b.** 15,000; 1.04 **c.** $6 \cdot 10^8$; 5 **d.** 25; 1.18 **e.** 8000; 2.718 **f.** $4 \cdot 10^5$; 2.5

3.

Initial Value C	Growth Factor a	Exponential Function $y = Ca^x$
1600	1.05	$y = 1600 \cdot 1.05^x$
$6.2 \cdot 10^5$	2.07	$y = 6.2 \cdot 10^5 \cdot (2.07)^x$
1400	3.25	$y = 1400 \cdot (3.25)^x$

5. **a.** $f(x) = 6 \cdot 1.2^x$ **b.** $f(x) = 10 \cdot 2.5^x$

7. **a.** 1.17 **b.** 63 cells per ml.

c.

d	C
0	63.0
1	73.7
2	86.2
3	100.9
4	118.1
5	138.1
6	161.6
7	189.1
8	221.2
9	258.8
10	302.8

d. C doubles somewhere between $d = 4$ and $d = 5$ days.

9. Expenditures $= 2016 \cdot (1.076)^5 \approx 2907.7$ billion dollars.

11. **a.** $G(t) = 5 \cdot 10^3 \cdot 1.185^t$ cells/ml, where t is measured in hours.
b. $G(8) = 5 \cdot 10^3 \cdot 1.185^8 = 19{,}440.92$ cells/ml

13. **a.** 10 fish **b.** It doubled in 5 months. **c.** 80 fish **d.** approx. 35 months

Section 5.2

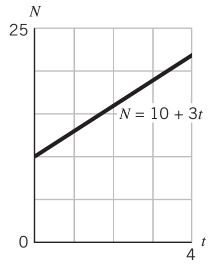
Algebra Aerobics 5.2

1.

t	$N = 10 + 3t$	$N = 10 \cdot 3^t$
0	10	10
1	13	30
2	16	90
3	19	270
4	22	810

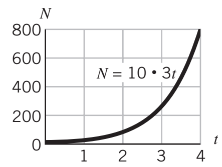
$N = 10 + 3t$

For every unit increase in t, N increases by 3, so the graph is linear with a slope of 3. The vertical intercept is 10.



$N = 10 \cdot 3^t$

The rate of change here is not constant as in the previous problem. For example, the average rate of change between 0 and 1 is $(30 - 10)/1 = 20$ and between 1 and 2 is $(90 - 30)/1 = 60$. But the vertical intercept for the graphs of both functions is the same, 10.

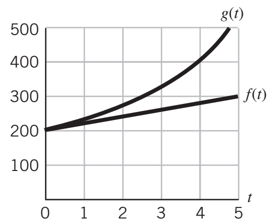


2. **a.**

t	$f(t) = 200 + 20t$
0	200
1	220
2	240
3	260
4	280
5	300

t	$g(t) = 200(1.20)^t$
0	200
1	240
2	288
3	345.6
4	414.72
5	497.66

b.



c. The function f is linear and the function g is exponential. While both functions have the same initial value, g is growing faster than f.

3. f is linear because the rate of change is constant; that is, the slope is 6.

g is linear because the rate of change is constant; that is, the slope is 0.

h is exponential because the ratio of any two consecutive terms is constant; that is, there is a constant growth factor of 1.5 or 150%.

p is linear because the rate of change is constant; that is, the slope is 90.

r is exponential because there is a constant growth factor of 1.04 or 104%

CH. 5 Exercises Solutions for Section 5.2

4. a. Linear: $m = \frac{620 - 500}{1 - 0} = 120$; $(0, 500) \Rightarrow b = 500 \Rightarrow y = 500 + 120t$

Exponential: growth factor $= \frac{620}{500} = 1.24$; $(0, 500) \Rightarrow$ initial amount $= 500 \Rightarrow y = 500(1.24)^t$

b. Linear: $m = \frac{3.2 - 3}{1 - 0} = 0.2$; $(0, 3) \Rightarrow b = 3 \Rightarrow y = 3 + 0.2t$

Exponential: growth factor $= \frac{3.2}{3} \approx 1.067$; $(0, 3) \Rightarrow$ initial amount $= 3 \Rightarrow y = 3(1.067)^t$

5. a. growth factor $= \frac{3750}{1500} = 2.50 \Rightarrow P = 1500 \cdot 2.50^t$

b. growth factor $= \frac{82{,}300}{80{,}000} = 1.029 \Rightarrow A = 80{,}000 \cdot 1.029^t$

c. growth factor $= \frac{32.7}{30} = 1.09 \Rightarrow Q = 30 \cdot 1.09^t$

6. a. 2

b.

t (years)	Q
0	10
2	20
4	40
6	80
8	160

c. Let a = growth factor. The 2-yr. growth factor $= 2 \Rightarrow a^2 = 2 \Rightarrow a = \sqrt{2} \approx 1.41$. The annual growth factor $\approx$ 1.41.

d. $Q = 10 \cdot 1.41^t$

Exercises for Section 5.2

1. a. linear; 5 **c.** neither; 3 **e.** exponential; 7

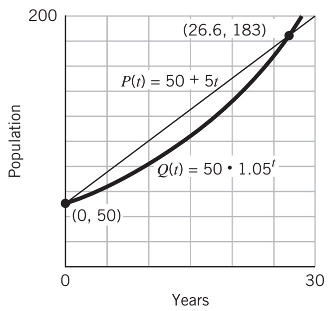
b. exponential; 3 **d.** exponential; 6 **f.** linear; 0

3. a. $P(t) = 50 + 5t$

b. $Q(t) = 50 \cdot 1.05^t$

c. The graph of the two functions is in the accompanying diagram.

d. Graphing software gives that the two are equal at (0, 50) and at (26.6, 183). Thus the populations were both 50 people at the start and were both approximately 183 persons after 26.6 years. (Student eyeball estimates may differ.)



5. linear: $y = 3x + 6$; exponential: $y = 6 \cdot 1.5^x$

7. a. linear: $y = 10x + 10$; exponential: $y = 10 \cdot (\sqrt{3})^x$

b. linear: $y = 100x + 100$; exponential $y = 100 \cdot (\sqrt[3]{4})^x$

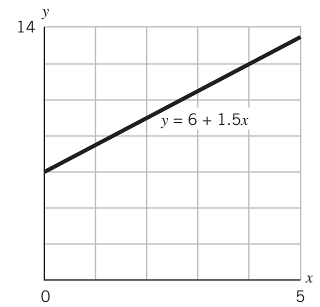
9. a. exponential; $f(x) = 7 \cdot 2.5^x$

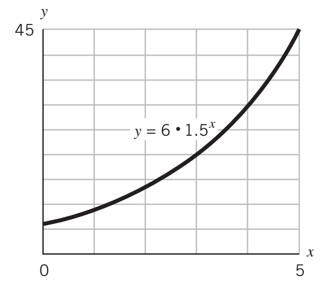
b. linear: $g(x) = 0.2x + 0.5$

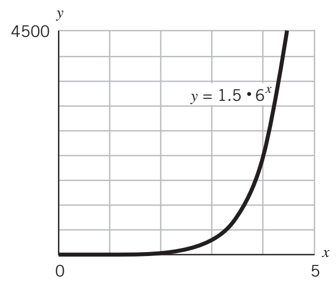
11.

x	$6 + 1.5x$	$6 \cdot 1.5^x$	$1.5 \cdot 6^x$
0	6.0	6	1.5
1	7.5	9	9
2	9.0	13.5	54
3	10.5	20.25	324
4	12.0	30.375	1,944
5	13.5	45.5625	11,664

The graphs are found in the accompanying diagrams.







13. a. For both populations we are looking for an exponential growth factor, i.e., for the value of a, where $A(t) = C \cdot a^t$ and where t measures years from 1986.

For the entire Atlantic coast: we have $C = 5800$ and $A(16) = 14{,}313$ and thus $14{,}313 = 5800 \cdot a^{16}$ or $a = \left(\frac{14{,}313}{5800}\right)^{1/16} \approx 1.058$. Thus the growth factor is about 1.058. For Massachusetts: we have $C = 585$ and $A(16) = 2939$ and thus $2939 = 585 \cdot a^{16}$ or $a = \left(\frac{2939}{585}\right)^{1/16} \approx 1.106$.

Thus 1.106 is the approximate growth factor. The growth factor in Massachusetts is somewhat larger than that of the entire Atlantic coast.

b. For the Atlantic coast, the average rate of change is the slope of the line between (0, 5800) and (16, 14313). This is: $\frac{14{,}313 - 5800}{16} = \frac{8513}{16} \approx 532$ swans/year.

For Massachusetts, the average rate of change is the slope of the line between (0, 585) and (16, 2939). This is $\frac{2939 - 585}{16} = \frac{2354}{16} \approx 147$ swans/year.

The average rate of change for the Atlantic coast is more than three times as large as the Massachusetts rate.

c. The linear model for Massachusetts since 1986 is $A(t) = 147t + 585$; the exponential model since 1986 for Massachusetts is $A(t) = 585 \cdot 1.106^t$.

d. Now 2010 is 24 years from 1986; thus $S(24) = 147 \cdot 24 + 585 = 4113$ swans and $A(24) = 585 \cdot 1.106^{24} \approx 6566$ swans. So the predictions are quite different for the two models.

15. a.

Years after 2004	Plan A	Plan B
0	\$7.00	\$7.00
1	\$7.35	\$7.50
2	\$7.72	\$8.00
3	\$8.10	\$8.50
4	\$8.51	\$9.00

b. Plan A: $F_A(t) = 7 \cdot 1.05^t$ and plan B: $F_B(t) = 7 + 0.50t$

c. $F_A(25) = 7 \cdot (1.05)^{21} \approx \$19.50$ and $F_B(25) = 7 + 0.50 \cdot 21 = \$17.50$

d. Student answers will vary, but one should note that plan B is more expensive until year 15; from then on plan A would be more expensive. Going for plan A for the short term seems like a better option.

Section 5.3

Algebra Aerobics 5.3

1. a. growth **c.** decay **e.** decay
b. decay **d.** growth **f.** growth

2. a. $y = 2300(1/3)^t$ **c.** $y = (375)(0.1)^t$
b. $y = (3 \cdot 10^9)(0.35)^t$

3. $y = 12(5)^{-x}$
$= 12(5^{-1})^x$
$= 12(1/5)^x$ so it represents decay.

4. a. $y = 23 \cdot 2.4^{-x} \Rightarrow y = 23(2.4^{-1})^x \Rightarrow y = 23\left(\frac{1}{2.4}\right)^x \Rightarrow y = 23(0.42)^x$; decay

b. $f(x) = 8000 \cdot (0.5^{-1})^x \Rightarrow f(x) = 8000\left(\frac{1}{0.5}\right)^x \Rightarrow f(x) = 8000 \cdot (2)^x$; growth

c. $P = 52{,}000 \cdot 1.075^{-t} \Rightarrow P = 52{,}000(1.075^{-1})^x \Rightarrow P = 52{,}000\left(\frac{1}{1.075}\right)^x \Rightarrow P = 52{,}000(0.93)^x$; decay

5. a. $g(x)$ decreases more rapidly.

b. For $g(x)$, 70% of the previous amount remains each time; that is, there is 30% less each time period. For $f(x)$, 90% of the previous amount remains each time; that is, there is 10% less each time period.

Exercises for Section 5.3

1. a. 0.43 **b.** 0.95 **c.** 1/3

3. a. $f(t) = 10{,}000 \cdot 0.4^t$ **c.** $h(x) = 219 \cdot 0.1^x$
b. $g(T) = 2.7 \cdot 10^{13} \cdot 0.27^T$

5. a. This is not exponential since the base is the variable.
b. This is exponential; the decay factor is 0.5; the vertical intercept is 100.
c. This is exponential; the decay factor is 0.999; the vertical intercept is 1000.

7.

Initial Value, C	Decay Factor, a	Exponential Function $y = Ca^x$
500	0.95	$y = 500 \cdot 0.95^x$
$1.72 \cdot 10^6$	0.75	$y = 1.72 \cdot 10^6 \cdot (0.75)^x$
1600	0.25	$y = 1600 \cdot (0.25)^x$

9. a. matches graph *B*. **b.** matches graph *A*.

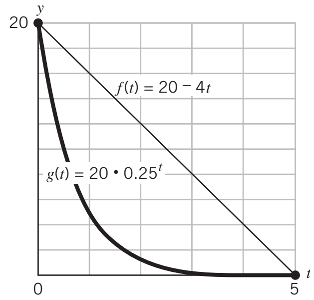
11. a.

Food	Formula	Amount in 2000
Beef	$B(t) = 72.1 \cdot 0.994^t$	63.9 lb
Chicken	$C(t) = 32.7 \cdot 1.024^t$	52.5 lb
Pork	$P(t) = 52.1 \cdot 0.996^t$	48.1 lb
Fish	$F(t) = 12.4 \cdot 1.01^t$	15.1 lb

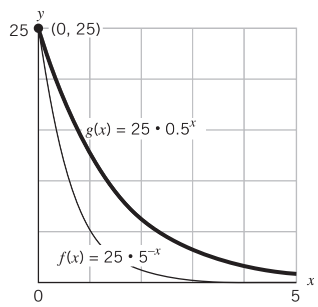
b. Growth: chicken and fish; decay: beef and pork.

c. Answers will vary with students. It is expected that the growth rates and per-capita consumption figures will be cited.

13. a. $f(t) = -4t + 20$
b. $g(t) = 20 \cdot 0.25^t$
c.



15. The accompanying diagram contains the graphs of $f(x) = 25 \cdot 5^{-x} = 25(0.2)^x$ and $g(x) = 25 \cdot 0.5^x$. It is clear that the graph of f has the faster rate of decline. Note that the decay factor for f is 0.20 and the decay factor for g is 0.50.



Ch. 5

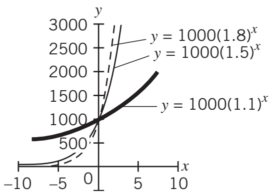
17. a. $P(t) = 100 \cdot b^t$; $99.2 = 100 \cdot b$ or $b = 0.992$ and therefore $P(t) = 100 \cdot 0.992^t$

b. $P(50) = 100 \cdot 0.992^{50} \approx 66.9$ grams; $P(500) = 100 \cdot 0.992^{500} \approx 1.8$ grams

Section 5.4

Algebra Aerobics 5.4

1. a.

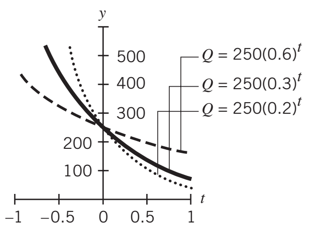


b. Yes, they intersect at (0, 1000).

c. In the first quadrant, $y = 1000(1.8)^x$ is on top, $y = 1000(1.5)^x$ is in the middle, and $y = 1000(1.1)^x$ is on the bottom.

d. In the second quadrant, $y = 1000(1.1)^x$ is on top, $y = 1000(1.5)^x$ is in the middle, and $y = 1000(1.8)^x$ is on the bottom.

2.

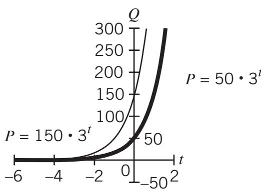


b. Yes, they intersect at (0, 250).

c. In the first quadrant: $Q = 250(0.6)^t$ is on top, $Q = 250(0.3)^t$ is in the middle, and $Q = 250(0.2)^t$ is on the bottom.

d. In the second quadrant: $Q = 250(0.2)^t$ is on top, $Q = 250(0.3)^t$ is in the middle, and $Q = 250(0.6)^t$ is on the bottom.

3. a.



b. No, the curves do not intersect.

c. $P = 150 \cdot 3^t$ is always above $P = 50 \cdot 3^t$.

4. a. The horizontal axis is the horizontal asymptote.

b. The horizontal axis is the horizontal asymptote.

c. No horizontal asymptotes. The graph of this function is a line.

5. Comparing the growth factors: $1.23 > 1.092 > 1.06$, so the function h has the most rapid growth. Comparing the decay factors: $0.89 < 0.956$, so the function g has the most rapid decay.

6. a. The function g has the larger initial value since it crosses the vertical axis above the function f.

b. The function f has the larger growth factor since it is steeper, growing at a faster rate.

c. The function f approaches zero more rapidly as $x \to -\infty$.

d. The graphs intersect at approximately (3, 35); f is greater than g after the point of intersection.

Exercises for Section 5.4

1. Let $y_1 = 2^x$, $y_2 = 5^x$, and $y_3 = 10^x$.

a. $C = 1$ for each case; $a = 2$ for y_1, $a = 5$ for y_2, and $a = 10$ for y_3.

b. Each represents growth; y_1's value doubles, y_2's value is multiplied by 5, and y_3's value is multiplied by 10.

c. All three graphs intersect at (0, 1).

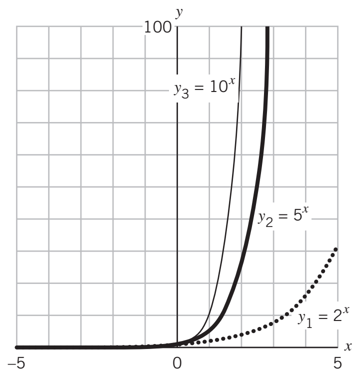
d. In the first quadrant the graph of y_3 is on top, the graph of y_2 is in the middle, and the graph of y_1 is on the bottom.

e. All have the graph of $y = 0$ (or the x-axis) as their horizontal asymptote.

f. Small table for each:

x	y_1	y_2	y_3
0	1	1	1
1	2	5	10
2	4	25	100

g. The graphs of the three functions are given in the accompanying diagram. They indeed confirm the answers given to the questions asked in parts (a) through (f).



3. Let $y_1 = 3^x$, $y_2 = (1/3)^x$, and $y_3 = 3 \cdot (1/3)^x$.

a. $C = 1$ for y_1 and y_2 and $C = 3$ for y_3.

b. y_1 represents growth; y_2 and y_3 represent decay.

c. y_1 and y_3 intersect at approx. (0.5, 1.7); y_1 and y_2 intersect at (0, 1).

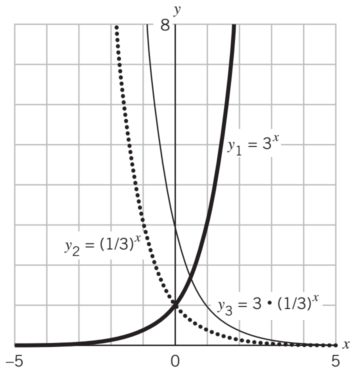
d. For $x > 0.5$, the graph of y_1 is on top, the graph of y_3 is in the middle, and the graph of y_2 is on the bottom. For $x < 0$, the graph of y_3 is on top, that of y_2 is in the middle, and that of y_1 is on the bottom.

e. All the graphs have the x-axis as their horizontal asymptote.

f. A small table for each:

x	y_1	y_2	y_3
0	1	1	3
1	3	1/3	1
2	9	1/9	1/3

g. The graphs of these three functions are given in the accompanying diagram, and they confirm the answers given to the questions in parts (a) through (f).



5. The function P goes with Graph C, the function Q goes with Graph A, the function R goes with Graph B, and the function S goes with Graph D.

7. a. The smaller the decay factor, the faster the descent; thus h has the smallest, then g, and finally f.

b. The point (0, 5) and no other point.

c. It will approach 0 more slowly than the other two functions.

d. h is on top and will stay on top for $x < 0$.

9. The accompanying diagram contains the graphs of $f(x) = 30 + 5x$ and $g(x) = 3 \cdot 1.6^x$ with the points of intersection marked.

a. $f(0) = 30$; $g(0) = 3$; $f(0) > g(0)$

b. $f(6) = 60$; $g(6) \approx 50.33$; $f(6) > g(6)$

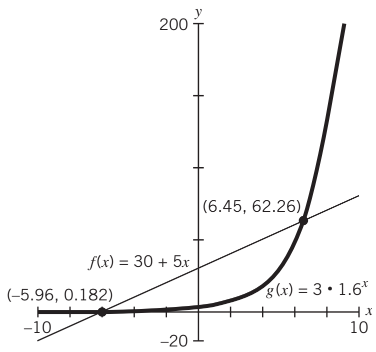
c. $f(7) = 65$; $g(7) \approx 80.53$; $f(7) < g(7)$

d. $f(-5) = 5$; $g(-5) \approx 0.286$; $f(-5) > g(-5)$

e. $f(-6) = 0$; $g(-6) \approx 0.179$; $f(-6) < g(-6)$

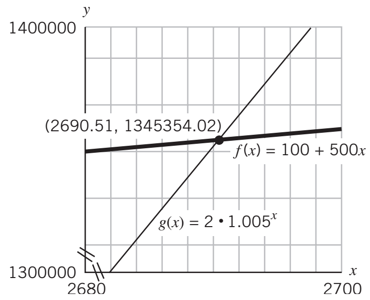
f. f and g go to infinity; $f(x) < g(x)$ for all $x > 6.5$, approximately.

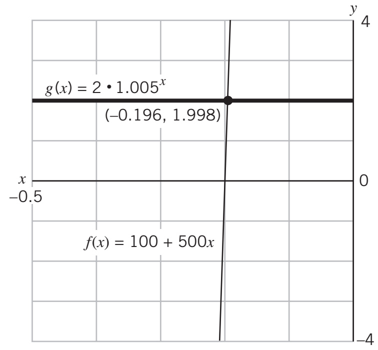
g. f goes to $-\infty$ and $g(x)$ goes to 0; thus $f(x) < g(x)$ for all $x < -6$, approximately.



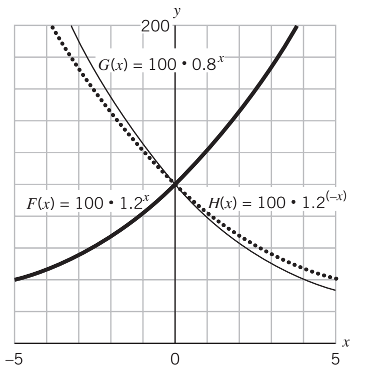
11. a. As $x \to +\infty$, $g(x)$ will dominate over $f(x)$, i.e., $g(x) > f(x)$

b. There is no one coordinate window that will display the graphs of f and g to help one see the answer to his question. Thus two displays are given: one over $2680 < x < 2700$ and another over $-0.5 < x < 0$. From these one can see that $f(x) > g(x)$ if $-0.196 < x < 2690.51$; otherwise $g(x) > f(x)$.





13. The graphs of the three functions are given in the accompanying diagram. The graphs of F and H are mirror images of each other and are equally steep when the absolute values of average rates of change are considered. $H(x) = 100(1/1.2)^x \approx 100(0.83)^x$ and $G(x) = 100 \cdot 0.8^x$, so $G(x)$ has a smaller decay factor and thus decays faster than $H(x)$ and is steeper than $H(x)$. Thus, the graph of G is steeper than the other two, again when considering the absolute values of the average rates of change.



15. The graphs are given in the accompanying diagram.

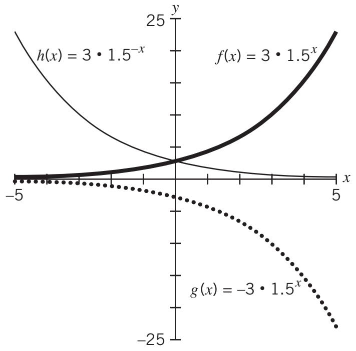
a. It is clear that the graphs of f and h are mirror images of each other with respect to the y-axis.

b. The graphs of f and g are mirror images of each other with respect to the x-axis.

c. The graphs of g and h are mirror images of each other with respect to the origin.

d. The graphs of these functions are mirror images of each other with respect to the x-axis.

e. These functions are mirror images of each other with respect to the y-axis.



Section 5.5

Algebra Aerobics 5.5

1. a. 22%

b. 106.7% or 1.067

c. $1 - 0.972 = 0.028$ or 2.8%

d. $1 - 0.123 = 0.877$

2.

Exponential Function	Initial Value	Growth or Decay?	Growth or Decay Factor	Growth or Decay Rate
$A = 4(1.03)^t$	4	growth	1.03	0.03 or 3%
$A = 10(0.98)^t$	10	decay	0.98	0.02 or 2%
$A = 1000(1.005)^t$	1000	growth	1.005	0.005 or 0.5%
$A = 30(0.96)^t$	30	decay	0.96	0.04 or 4%
$A = 50{,}000(1.0705)^t$	$50,000	growth	1.0705	0.0705 or 7.05%
$A = 200(0.51)^t$	200 g	decay	0.51	0.49 or 49%

3. a. growth factor = 1.06 = 106% growth rate = 6%

b. decay factor = 0.89 = 89% decay rate = 11%

c. growth factor = 1.23 = 123% growth rate = 23%

d. decay factor = 0.956 = 95.6% decay rate = 4.4%

e. growth factor = 1.092 = 109.2% growth rate = 9.2%

4. a. $\frac{\$6.00}{1.65} \approx 3.64$, so 3.64 (the growth factor) = 1 + growth rate. The growth rate = 2.64 or a 264% increase in 30 years.

b. $\frac{0.167}{0.299} \approx 0.56$, so 0.56 (the decay factor) = 1 − decay rate. The decay rate = 0.44 or a 44% decrease in one season.

c. $\frac{23}{17} \approx 1.35$, so 1.35 = 1 + growth rate. The growth rate = 0.35 or a 35% increase in one month.

Exercises for Section 5.5

1. a. $Q = 1000 \cdot 3^T$ **c.** $Q = 1000 \cdot 1.03^T$

b. $Q = 1000 \cdot 1.3^T$ **d.** $Q = 1000 \cdot 1.003^T$

3. a. 5% **c.** 55% **e.** 0.4%

b. 18% **d.** 34.5% **f.** 27.5%

5.

	Initial Value	Growth or Decay?	Growth or Decay Factor	Growth or Decay Rate	Exponential Function
a.	600	growth	2.06	106%	$N(t) = 600 \cdot 2.06^t$
b.	1200	growth	3.00	200%	$N(t) = 1200 \cdot 3^t$
c.	6000	decay	0.25	75%	$N(t) = 6000 \cdot 0.25^t$
d.	$1.5 \cdot 10^6$	decay	0.75	25%	$N(t) = 1.5 \cdot 10^6 \cdot 0.75^t$
e.	$1.5 \cdot 10^6$	growth	1.25	25%	$N(t) = 1.5 \cdot 10^6 \cdot 1.25^t$
f.	7	growth	4.35	335%	$N(t) = 7 \cdot 4.35^t$
g.	60	decay	0.35	65%	$N(t) = 60 \cdot 0.35^t$

7. a. $A(T) = 12{,}000 \cdot 1.12^T$ and $B(T) = 12{,}000 \cdot 0.88^T$, where T = decades since 1990.

b. If t = years since 1990, then $t = 10T$ or $T = t/10$, and thus $a(t) = 12{,}000 \cdot [1.12]^{t/10} = 12{,}000 \cdot [1.12^{0.1}]^t = 12{,}000 \cdot 1.0114^t$ people and $b(t) = 12{,}000 \cdot [0.88]^{t/10} = 12{,}000 \cdot [0.88^{0.1}]^t = 12000 \cdot 0.9873^t$ people where t is the number of years from 1990.

c. $A(2) \approx 15{,}053$; $a(20) \approx 15{,}053$; $B(2) \approx 9293$ and $b(20) \approx 9293$. $A(2)$ and $B(2)$ represent the population after two decades, whereas $a(20)$ and $b(2)$ represent the population after 20 years. Note that $A(2)$ should equal $a(20)$ and $B(2)$ should equal $b(20)$. Any small differences are because of round-off error.

9. $A(x) = 500 \cdot 0.85^x$; decay rate of 15%

11. $A(x) = 225 \cdot 1.015^x$; growth rate of 1.5%

13. a. $P(t) = 150 \cdot 3^t$ **c.** $P(t) = 150 \cdot 0.93^t$

b. $P(t) = 150 - 12t$ **d.** $P(t) = 150 + t$

15. In 2030 the population size will be approximately $200 \cdot 1.4^2 = 392$ million, and in 2060 it will be approximately $200 \cdot 1.4^3 = 548.8$ million. Using a graphing calculator to estimate the intersection point of the graphs of $y_1 = 200 \cdot 1.4^t$ and $y_2 = 1000$ (where y_1 and y_2 are measured in millions and t is measured in 30-year periods) gives $t \approx 4.78$. Thus, according to this model, sometime in the first part of 2113, the U.S. population will reach approximately 1 billion.

17. a. $A(n) = A_0 \cdot 0.75^n$, where n measures the number of years from the original dumping of the pollutant and A_0 represents the original amount of pollutant.

b. We are solving $0.1 \cdot A_0 = A_0 \cdot 0.75^n$ for n and we first divide out by A_0. Then we graph $y = 0.01$ and $y = 0.75^n$ and estimate their intersection. This gives $n \approx 16$ years.

19. a. $1.007^{12} \approx 1.087$ and thus the inflation rate is about 8.7% per year.

b. $(1 + r)^{12} = 1.05$ means that $r = (1.05)^{1/12} - 1 \approx 0.0041$. Thus the rate is about 0.4% per month.

21. a. They are equivalent, since $1.2^{0.1} = 1.0184$ to four decimal places.

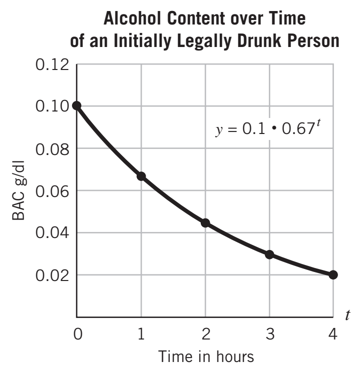
b.

x	0	5	10	15	20	25
$f(x)$	15,000.0	16431.7	18,000	19,718.0	21,600.0	23,661.6
$g(x)$	15,000.0	16431.7	18,000.1	19,718.2	21,600.3	23,662.0

c. If x is the number of years, then $f(x) = 15{,}000(1.2)^{x/10}$ represents a 20% growth factor over a decade, and $g(x) = 15{,}000(1.0184^x)$ represents a 1.84% annual growth factor.

CH. 5 Algebra Aerobics Solutions for Section 5.6

23. a. A graph with the data plotted and the best-fit exponential is given in the diagram.



b. If one looks at the exponential graph it is easy to see that it fits the data quite well. Moreover, the ratios of successive g/dl values are nearly a constant 0.67. Another equation, easier to handle, is $y = 0.1 \cdot 0.67^t$, where t is measured in hours and y in g/dl.

c. In this model, the g/dl decreases each hour by about 33%.

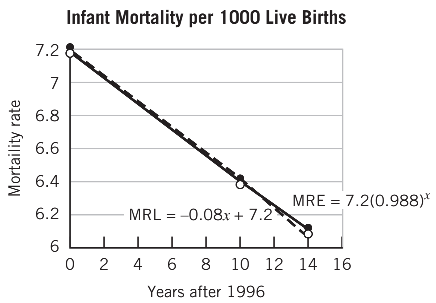
d. A reasonable domain could be 0 to 10 hours and the corresponding range would be 0.002 to 0.100 g/dl.

e. $0.005 = 0.1 \cdot 0.67^t$ implies that t is between 7 and 8 hours. This can be obtained by computing g/dl values for successive t values using the formula given in part (b).

25. a. If the model is linear, then the slope is $\frac{(6.4 - 7.2)}{(2006 - 1996)} = -\frac{0.8}{10} = -0.08$ and the equation is $MRL(y) = -0.08y + 7.2$, where y is the number of years since 1996 and MRL is the number of deaths per 1000 live births.

b. If the model is exponential, then the decay factor is $\left(\frac{6.4}{7.2}\right)^{1/10} \approx 0.988$ and thus the equation is $MRE(y) = 7.2 \cdot 0.988^y$, and the units are the same as in part (a).

c. The graphs requested are in the accompanying diagram.



d. In 2010, $MRL(14) = 6.08$ and $MRE(14) \approx 6.11$; both are measured in deaths per 1000 live births.

Section 5.6

Algebra Aerobics 5.6a

1. a. Exponential growth; doubling period is 30 days since $f(30) = 300 \cdot 2^{30/30} = 600$; Initial $= 300$;
one day later: $f(1) = 300 \cdot 2^{1/30} \approx 307.01$;
one month later: $f(30) = 300 \cdot 2^{30/30} = 600$;
one year later: $f(360) = 300 \cdot 2^{360/30} = 1{,}228{,}800$

b. Exponential decay; half-life period is 2 days since $g(2) = 32(0.5)^{2/2} = 16$; Initial $= 32$;
one day later: $g(1) = 32 \cdot 0.5^{1/2} \approx 22.6$;
one month later: $g(30) = 32 \cdot 0.5^{30/2} = 0.00098$;
one year later: $g(360) = 32 \cdot 0.5^{360/2} =$ trace amount (almost zero)

c. Exponential decay; half-life period is one day since $P = 32{,}000 \cdot 0.5^1 = 16{,}000$; Initial $= 32{,}000$;
one day later: $P = 32{,}000 \cdot 0.5^1 = 16{,}000$;
one month later: $P = 32{,}000 \cdot 0.5^{30} \approx 0.000\,03$;
one year later: $P = 32{,}000 \cdot 0.5^{360} =$ trace amount (almost zero)

d. Exponential growth; doubling period is 360 days since $h(360) = 40{,}000 \cdot 2^{360/360} = 80{,}000$; Initial $= 40{,}000$;
one day later, $h(1) = 40{,}000 \cdot 2^{1/360} \approx 40{,}077$;
one month later, $h(30) = 40{,}000 \cdot 2^{30/360} \approx 42{,}379$;
one year later, $h(360) = 40{,}000 \cdot 2^{360/360} = 80{,}000$

Ch. 5

2. a. $70/2 = 35$ yr

b. $70/0.5 = 140$ months

c. $70/8.1 = 8.64$ yr ≈ 9 yr

d. growth factor $= 1.065 \Rightarrow$ growth rate $= 6.5\%$
$70/6.5 = 10.77 \approx 11$ yr

3. a. $\frac{70}{10} = 7$; $R \approx 7\%$ per year

b. $70/5 = 14$; $R \approx 14\%$ per minute

c. $70/25 = 2.8$; $R \approx 2.8\%$ per second

4. a. Since $a = 0.95$, $r = 0.05$, so this is decay of $R = 5\%$, and half-life is $70/5 = 14$ months.

b. Since $a = 0.75$, $r = 0.25$, so this is decay of $R = 25\%$, and half-life is $70/25 = 2.8$ sec.

c. Half-life is $70/35 = 2$ yr.

Algebra Aerobics 5.6b

1. a. $70/3 \approx 23.3$ yr **b.** $70/5 = 14$ yr **c.** $70/7 = 10$ yr

2. a. $70/5 = 14$; 14% **b.** $70/10 = 7$; 7% **c.** $70/7 = 10$; 10%

3. a. $P = \$1000(1.04)^n$ **b.** $P = \$1000(1.11)^n$
c. $P = \$1000(2.10)^n$

4. If inflation is 10%/month, then what cost 1 cruzeiro this month would cost 1.10 cruzeiros next month. We have 1 cruzeiro $\approx 91\%$ of 1.10 cruzeiros (since $1/1.1 \approx 0.91$), so a month later 1 cruzeiro would only be worth 0.91 cruzeiros or 91% of its original value. Thus, the decay factor is 0.91. So the exponential decay function $Q = 100(0.91)^n$ gives the purchasing power of 100 of today's cruzeiros at n months in the future.

When $n = 3$ months, then Q, the value of 100 of today's cruzeiros, will be $100(0.91)^3 \approx 100(0.75) = 75$ cruzeiros. When $n = 6$ months, then Q, the value of 100 of today's cruzeiros, will be $100(0.91)^6 \approx 100(0.57) = 57$ cruzeiros. When $n = 12$ months or 1 yr, then Q, the value of 100 of today's cruzeiros, will be $100(0.91)^{12} \approx 100(0.32) = 32$ cruzeiros.

With a 10% monthly inflation rate, the value of a cruzeiro will shrink by more than two-thirds by the end of a year.

5. a. Initial investment is \$10,000; growth factor $= 1.065$; growth rate $= 6.5\%$; doubling time: using the rule of 70, $70/6.5 \approx 10.8$ years

b. Initial investment is \$25; growth factor = 1.08; growth rate = 8%; doubling time: using the rule of 70, $70/8 \approx 8.8$ years

c. Initial investment is \$300,000; growth factor = 1.11; growth rate = 11%; doubling time: using the rule of 70, $70/11 \approx 6.4$ years

d. Initial investment is \$200; growth factor = 1.092; growth rate = 9.2%; doubling time: using the rule of 70, $70/9.2 \approx 7.6$ years

6.

Function	Initial Investment	Growth Factor	Growth Rate	Amount 1 Year Later	Doubling Time (approx.)
$A = 50000 \cdot 1.072^t$	\$50,000	1.072	7.2%	\$53,600	9.7 years
$A = 100000 \cdot 1.067^t$	\$100,000	1.067	6.7%	\$106,700	10.4 years
$A = 49622 \cdot 1.058^t$	\$49,622	1.058	5.8%	\$52,500	12 years
$A = 3000 \cdot 1.13^t$	\$3000	1.13	13%	\$3390	5.4 years

Algebra Aerobics 5.6c

1. a. $x = 5$ since $2^5 = 32$

b. $x = 8$ since $2^8 = 256$

c. $x = 10$ since $2^{10} = 1024$

d. $x = 1$ since $2^1 = 2$

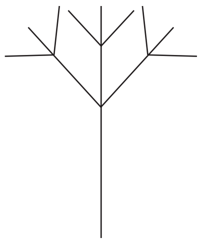
e. $x = 0$ since $2^0 = 1$

f. $x = -1$ since $2^{-1} = \frac{1}{2}$

g. $x = -3$ since $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

h. $x = 1/2$ since $2^{1/2} = \sqrt{2}$

2. a.



b. $N = 3^L$

3.

Level	# of People Called	Total Called
0	3^0	1
1	3^1	4
2	$3^2 = 9$	13
3	$3^3 = 27$	40
4	$3^4 = 81$	121
5	$3^5 = 243$	364
6	$3^6 = 729$	1093
7	$3^7 = 2187$	3280
8	$3^8 = 6561$	9841

So it will take eight levels to reach 8000 people.

Exercises for Section 5.6

1. a. Has a fixed doubling time

b. Has neither

c. Has a fixed half-life

d. Has a fixed doubling time

e. Has neither

f. Has a fixed half-life

3. b. $f(x) = 1000 \cdot 2^{t/7}$; $2^{1/7} = 1.1041$ per year; 10.41% per year

c. $f(x) = 4 \cdot 2^{t/25}$; $2^{1/25} = 1.0281$ per minute; 2.81% per minute

d. $f(x) = 5000 \cdot 2^{t/18}$; $2^{1/18} = 1.0393$ per month; 3.93% per month

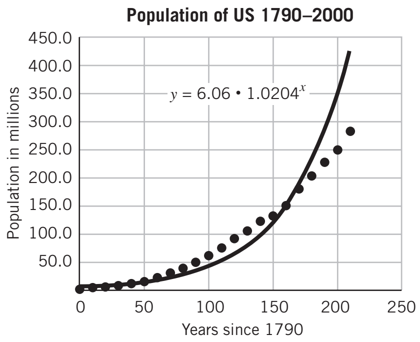
5. a. $3 \cdot 2^{x/5} < 3(1.225)^x$, if $x > 0$; the inequality is reversed if $x < 0$.

b. $50 \cdot (1/2)^{x/20} \approx 50 \cdot 0.9659^x$.

c. $200 \cdot 2^{x/8} \approx 200 \cdot 1.0905^x$.

d. $750 \cdot (1/2)^{x/165} > 750 \cdot 0.911^x$ if $x > 0$, and the inequality is reversed if $x < 0$.

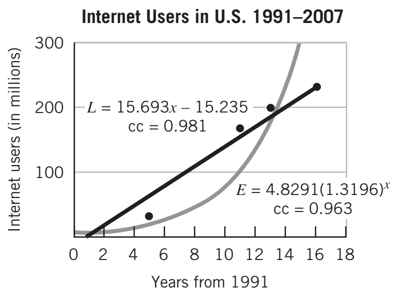
7.



a, b. The accompanying diagram contains the graph of both the data and the best-fit exponential function, via Excel. There y = population in millions and x = years since 1790. The annual growth factor is 1.0204; the annual growth rate is 2.04%; the estimated initial population is 6.06 million. One notes that the best-fit model is lower than the plotted data from 1850 to 1930. Its values surpass those of the data from 1940 on. It is a good model for the first 160 years but is not that good thereafter.

c. The model predicts the population to be 516.2 million in 2010 and 698.8 million in 2025. These seem very high indeed.

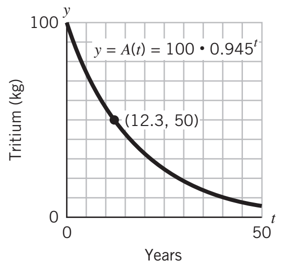
9. a. The accompanying diagram contains the graphs of the best-fit linear and exponential models for the given data. The linear and exponential formulae are also given there. The linear model seems to fit the data better because of a slightly higher correlation coefficient.



b. 2010 is 19 years after 1991. The linear model predicts $L(19) = 15.693 \cdot 19 - 15.235 \approx 283$ million users, and the exponential model predicts $E(19) = 4.8291(1.3196)^{19} \approx 938$ million users.

c. Student answers will vary.

11. The graph in the accompanying diagram goes through (0, 100) and (12.3, 50), where the first coordinate is measured in years and the second is measured in grams. Using the graphing utility gives: $A(t) = 100 \cdot 0.945^t$. Using algebra we get $A(t) = 100 \cdot (0.5)^{t/12.3}$, which gives the same formula as the best-fitting exponential formula. The graph of this function is given in the accompanying diagram.



13. a. $0.5^5 = 1/32 = 0.03125$ and thus 3.125% of the original dosage is left.

b. i. Approximately 2 hours.

ii. $A(t) = 100 \cdot 0.5^{t/2}$ where t = hours after taking the drug and $A(t)$ is in mg.

iii. $5 \cdot 2 = 10$ hours and $100 \cdot (1/2)^5 = 100/32 = 3.125$ milligrams. Also $A(10) = 100 \cdot 0.5^{10/2} = 3.125$.

iv. Student answers will vary. They should mention the half-life and present a graph to make the drug's behavior clear to any prospective buyer.

15. a. $R = 70/5730 \approx 0.012\%$ per year

b. $R = 70/11{,}460 \approx 0.0061\%$ per year

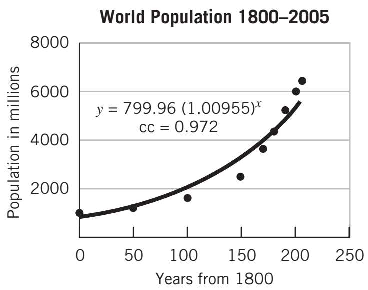
c. $R = 70/5 = 14\%$ per second.

d. $R = 70/10 = 7\%$ per second.

17. a.

Years from 1800	World Population (in millions)
0	980
50	1260
100	1650
150	2520
170	3700
180	4440
190	5270
200	6080
205	6480

b. $P(t) = 799.96 \cdot 1.00955^t$ is the best-fit exponential function formula, where t is measured in years since 1800 and $P(t)$ is measured in millions. Its graph is given in the accompanying diagram.



c. The initial value is 799.96 or approximately 800 for the world's population in millions in 1800 (when $t = 0$). (Note that it is approximately 180 million smaller than the actual size.) The growth factor is 1.00955, which gives a growth rate of 0.955% per year. The domain of the function P technically is any real number, but concretely, its values should not go much farther back than 1800 or much past 2005. The range in that domain is about 800 million to about 5600 million.

d. 1.00955 represents a growth rate of 0.955% per year.

e. i. Using a calculator gives the $P(t)$ values in the accompanying table. Eyeball estimates from students should be close to these.

Year	t	$P(t)$
1750	−50	498.6
1920	120	2487.9
2025	225	6714.1
2050	250	8504.5

ii. Using a calculator or the graph of the model one can see that 1 billion was reached during 1823 (or about 23.6 years after 1800); 4 billion was reached in the first half of 1970 (or about 170.2 years from 1800); and 8 billion will be reached during 2043 (about 243.5 years from 1800).

f. As can be seen from the answers in (ii) it takes approximately 73 years for the population to double.

19. a. 10 half-lives gives a $0.5^{10} = 0.001$ or 0.1% of the original or a 99.9% reduction.

b. $A(2) = 0.25A_0$, $A(3) = 0.125A_0$, $A(4) = 0.0625A_0$. After n half-lives, the amount left is $A(n) = 0.5^n \cdot A_0$.

21. a.

t, Time (months)	M, mass (g)
0	10
3	20
6	40
9	80
12	160

In general $M = 10(2^{1/3})^t = 10 \cdot 1.2599^t$ after t months.

b. Using a calculator or a graph of the model, when $M = 2000$, then $t \approx 23$ months or nearly 2 years.

c. $2000/10 = 200$ and thus at 2000 grams, it is 20,000% of its original size.

23. a. i. $A(n) = 5000 \cdot (1.035)^n$

ii. $B(n) = 5000 \cdot (1.0675)^n$

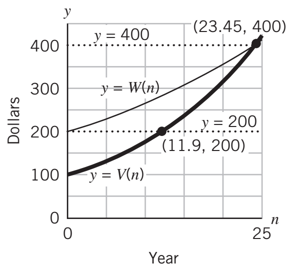
iii. $C(n) = 5000 \cdot (1.125)^n$

b. $A(40) = 19{,}796.30$ $\quad B(40) = 68{,}184.45$
$C(40) = 555{,}995.02$.

25. a. $V(n) = 100 \cdot 1.06^n$. Solving $V(n) = 200$ for n graphically gives $n \approx 12$ years, which is the approximate doubling time.

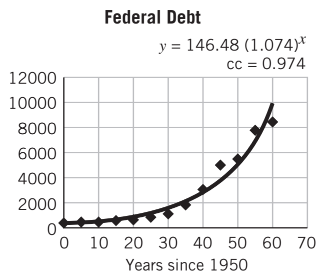
b. $W(n) = 200 \cdot 1.03^n$. Solving $W(n) = 400$ for n graphically gives $n \approx 23$ years, which is the approximate doubling time.

c. The graphs of $V(n)$ and $W(n)$ are in the accompanying diagram. The two graphs intersect at approximately $n = 24$ years, when $V(n) = W(n) \approx \$410$. The values for $V(n)$ are larger than those for $W(n)$ after that.



27. Since 1.00/1.06 = 0.943, then the value of a dollar after t years of such inflation is given by $V(t) = 0.943^t$. Using a calculator or the "rule of 70," when $V(t) = 0.50$, gives $t \approx 12$ years.

29. a. and **b.** Below is a scatter plot of the data and the Excel generated exponential curve that is a best fit.



The model would predict the debt of 2008 to be $146.48 (1.074)^{58} \approx \$9205$ billion.

c. Answers will vary depending on when the student invokes the debt clock.

31. a. In theory, the number recruited is $M_{\text{new}}(n) = 10^n$, where n measures the number of rounds and $M_{\text{new}}(n)$ measures the number of people participating in the nth round of recruiting. Note that this formula assumes that all who are recruited stay and that all recruits are distinct.

b. $M_{\text{Total}}(n) = 1 + 10 + \cdots + 10^n$.

c. $M_{\text{Total}}(5) = 111{,}111$, but only 11,110 of those stem from the originator. After 10 rounds the number recruited (not including the originator) would be 11,111,111,110, which is larger than the 2005 world population.

d. Comments will vary, but all will probably note how fast the number of recruits needed grows and how the amounts expected are not quite what one would have thought from the advertisements. If the chain is initially successful, then you would get a large number of new recruits in a short period. However, as can be seen in part (c), it quickly becomes unrealistic for each new person on the chain to recruit 10 new people.

Section 5.7

Algebra Aerobics 5.7

1. a. Judging from the graph, the number of *E. coli* bacteria grows by a factor of 10 (for example, from 100 to 1000, or 100,000 to 1,000,000) in a little over three time periods.

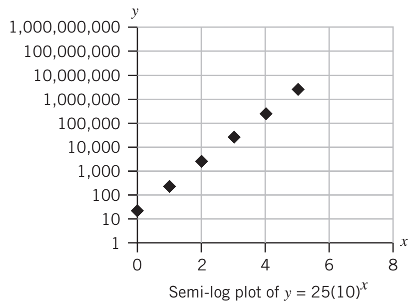
b. From the equation $N = 100 \cdot 2^t$, we know that every three time periods, the quantity is multiplied by 2^3 or 8.

c. Every four time periods, the quantity is multiplied by 2^4 or 16.

d. The answers are consistent, since somewhere between three and four time periods the quantity should be multiplied by 10 (which is between 8 and 16).

2. The graph of $y = 25\,(10)^x$ will be a straight line on a semi-log plot.

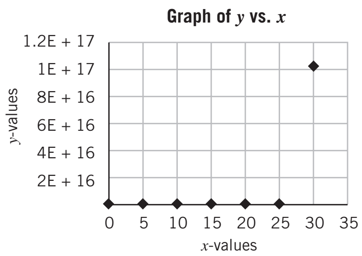
x	$y = 25(10)^x$
0	25
1	250
2	2,500
3	25,000
4	250,000
5	2,500,000



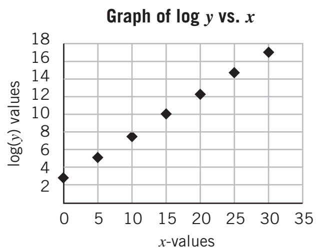
3. a. If $x = 3.5$, y is about 80,000. If $x = 7$, $y = 250{,}000{,}000$.

Exercises for Section 5.7

1. a. Graph of given table of values for $y = 500 \cdot 3^x$



b. Graph of table for $\log(500 \cdot 3^x)$ vs. x



c. $y = 10^{\log(y)}$ and thus each y can be written as 10 to the power listed in the third column.

3. a. A uses the linear scale. B uses a power of 10 scale on the vertical axis and C uses a logarithm scale on the vertical axis.

b. The two graphs look the same because $\log(10^n) = n$ and the powers of 10 are spaced out like n.

c. On Graphs A and B, when $y = 1000$ is multiplied by a factor of 10 to get $y = 10{,}000$, x has increased by about 5.5 units.

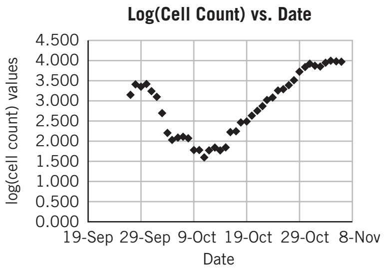
d. On graphs B, y labels go up by factors of 10.

e. On graph A, since the scales on both axes are linear.

5. a. The accompanying graph is of the white blood cell counts on a semi-log plot. The data from October 17 to

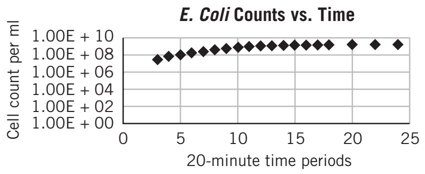
October 30 seem to be exponential, since the data in that range seem to fall along a straight line in the plot. There does not seem to be a discernible exponential decay pattern in this graph of the data.

Semi-log plot for 5(a).

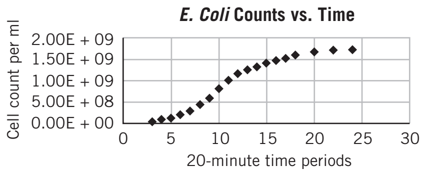


b. Below are two graphs of the *E. coli* counts; one has regular horizontal and vertical axes and the other is a semi-log plot. These data look very exponential from the third to the thirteenth time periods in the regular plot and fairly exponential in the semi-log plot (since that plot looks rather linear). For contrast, the regular linear plot is given as well.

Semi-log plot for 5(b).



Regular plot for 5(b).



7. a. This is a time-series, semi-log plot with time scale linear.

b. It says that the growth was roughly exponential between 1993 and 2000.

c. It says that the decline was roughly exponential between 2000 and 2002.

d. Student answers will vary. Only Fidelity can give the real answer. But one might surmise that Fidelity wanted to show overall that it has done as well as the 500 Index Funds and to visually deemphasize its decline after 2000.

Ch. 5: Check Your Understanding

1. False	**7.** False	**13.** False	**19.** False
2. True	**8.** False	**14.** False	**20.** True
3. False	**9.** True	**15.** True	**21.** True
4. False	**10.** True	**16.** True	**22.** True
5. False	**11.** False	**17.** False	**23.** False
6. True	**12.** False	**18.** True	**24.** True

25. Possible answer: $P = 2.2(1.005)^t$ million people, t = years.

26. Possible answer: $P = 2.2(1.005)^{4t}$ million people, t = years.

27. Possible answer: $M = 1.4(0.977)^{t/10}$ billion dollars, t = years.

28. Possible answer: $y = 3125(0.2)^t$.

29. Possible answer: $y_1 = 300(0.88)^t$ and $y_2 = 300(0.94)^t$.

30. Possible answer: $y = -1.458x + 5$ and $y = 5(0.5)^x$.

31. Possible answer: $V = 1(0.97)^t$, t = years.

32. Possible answer: $y = 5 \cdot (1.051)^t$, t = years.

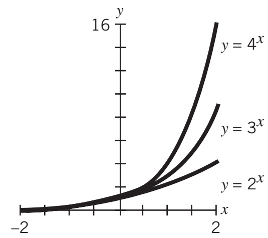
33. Possible answer: $R = 200(0.966)^t$ mg, t = years.

34. True	**37.** True	**40.** False	**43.** False
35. False	**38.** True	**41.** False	
36. True	**39.** False	**42.** True	

Ch. 5 Review: Putting It All Together

1. a. $P = 150 \cdot 2^t$ **c.** $P = 150\,(1.05)^t$
b. $P = 150 - 12t$ **d.** $P = 150 + 12t$

3.



5. a. Males who just stopped smoking are about 22 times more likely (the relative risk) to get lung cancer than a lifelong nonsmoker. Females who stopped smoking 12 years ago are about three times more likely to get lung cancer than a lifelong nonsmoker.

b. One reason could be that the longer it has been since someone quit smoking, the more time the lungs have had to heal. Another reason could be that the death rate of smokers is higher than that of nonsmokers.

c. A relative risk of 1 means each group is equally likely to get lung cancer. It is highly unlikely for the relative risk of smokers vs. nonsmokers to go below 1 since that would mean smokers are less likely to get lung cancer than nonsmokers, which does not make sense.

7. Initial value = 500 and growth rate = $1.5 \Rightarrow G(t) = 500(1.5)^t$, where $G(t)$ = number of bacteria and t = number of days.

9. a. If the world population in 1999 was 6 billion people, and it grew at a rate of 1.3% per year, then it is only in the first year that there is a net addition of 78 million people. The next year the increase would be 1.3% of 6,078,000,000, which is 79,014,000. The population would continue to have an increase that becomes larger and larger than 78 million each year. The increase is a fixed amount only in a linear model.

b. Let P = world population (in millions) and t = number of years since 1999.

Linear model: $P = 6000 + 78t$

Exponential model: $6000(1.013)^t$

c. Linear model prediction for 2006:
$P = 6000 + 78(2006 - 1999) \Rightarrow P = 6546$ million

Exponential model prediction for 2006: $6000(1.013)^7 \Rightarrow$
$P \approx 6568$ million

Answers may vary for best predictor depending on current population.

The U.S Census Bureau website gives the 2006 world population as 6.567 billion, so our exponential model is a more accurate predictor for 2006.

11. If R = the annual growth rate, then using the "rule of 70" we have $70/25 = 2.8 \Rightarrow R = 2.8\%$. So the number of motor vehicles increases by almost 3% per year.

13.

			Projected	
Year	2003	2004	2005	2006
Health care expenditures (billions)	\$1740.6	\$1877.6	\$2016.0	\$2169.5
Annual growth factor	n.a.	$\frac{1877.6}{1740.6}$ ≈ 1.079	$\frac{2016.0}{1877.6}$ ≈ 1.074	$\frac{2169.5}{2016}$ ≈ 1.076
Annual percent growth rate	n.a	7.9%	7.4%	7.6%

15. The ratio of consecutive values over 5-year intervals is approximately constant at 1.09, so an exponential model would be appropriate for the data in the table. If we let $E(t)$ = energy consumption (in quadtrillion Btu), where t = number of 5-year intervals since 2015, then an exponential function to model the data would be: $E(t) = 563\ (1.09)^t$.

17. a. 2002 to 2015:

13 year growth factor = (2 billion/605 million) =

2,000 million/605 million ≈ 3.306

annual growth factor $= 3.306^{1/13} \approx 1.096$

annual growth rate ≈ 0.096 or 9.6%

b. 2015 to 2040:

25 year growth factor = (3 billion/2 billion) = 1.5

annual growth factor $= 1.5^{1/25} \approx 1.016$

annual growth rate ≈ 0.016 or 1.6%

19. Initial value = 20 grams gives $A(t) = 20(1/2)^{t/8}$ grams, where t is in days.

21. The graph shows a semi-log plot. When an exponential function is plotted on a semi-log plot, its graph is a straight line. The data for "Internet hosts" is plotted as a straight line and thus could be modeled with an exponential function. The data for "Pages" and "Websites" is approximately linear on this semi-log plot, and thus an exponential model may be a close approximation.

23. a. Each linear dimension of the model is one-tenth that of the actual village, so the area of the model (which is two-dimensional) would be $(1/10) \cdot (1/10) = 1/100$ or one-hundredth of A_o, the area of the actual village.

b. Here each linear dimension is one-hundredth that of the actual church. The weight depends on volume (which is three-dimensional) and would be $(1/100) \cdot (1/100) \cdot (1/100) = 1/1{,}000{,}000$, or one-millionth of Wo, the weight of the actual church.

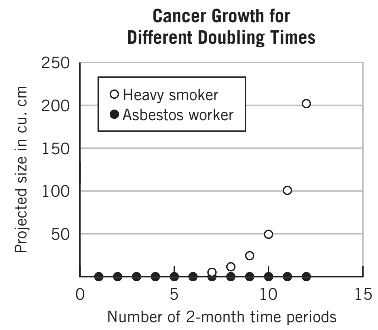
c. $A_n = A_o \cdot \left(\frac{1}{10^2}\right)^n$

$W_n = W_o \cdot \left(\frac{1}{10^3}\right)^n$

25. a. $S(t) = 0.5 \cdot 2^t$, the tumor size in cubic centimeters, where t = number of 2-month time periods

$A(t) = 0.5 \cdot 2^{t/4}$, the tumor size in cubic centimeters, where t = number of 2-month time periods

b.



The faster-growing cancer of the heavy smoker if untreated gets dangerously large very quickly after about 6 time periods (1 year). The slower-growing cancer of the asbestos worker after 12 time periods (2 years) is still relatively small compared to that of the smoker (the graph does not yet show an exponential curve).

c. 1 year = 6 time periods

The tumor size for the smoker after 1 year (or 6 time periods) is

$S(6) = 0.5 \cdot 2^6 = 32$ cubic centimeters $\Rightarrow$

$\text{Volume}_{\text{smoker}} = 32 = \left(\frac{4}{3}\right)\pi r^3 \Rightarrow \frac{32 \cdot 3}{4\pi} = r^3$

$7.64 \approx r^3$

Taking the cube root of both sides, $r \approx 1.97$ cm, or a tumor diameter of about 3.94 cm.

The tumor size for the asbestos worker after 1 year (or 6 time periods) is

$A(6) = 0.5 \cdot 2^{6/4} \approx 1.41$ cubic centimeters $\Rightarrow$

$\text{Volume}_{\text{asbestos}} = 1.41 = \left(\frac{4}{3}\right)\pi r^3 \Rightarrow \frac{1.41 \cdot 3}{4\pi} = r^3$

$0.34 \approx r^3$

Taking the cube root of both sides, $r \approx 0.70$ cm, or a tumor diameter of about 1.4 cm.

27. a. Let $M(t)$ = amount of methane gas in the atmosphere (in parts per billion), where t = number of years since 1850. The initial value in 1850 is 750 parts/billion. In 2005 the methane levels were at 1750 parts/billion, so

$M(t) = 750 \cdot \left(\frac{1750}{750}\right)^{(1/155)t} \approx 750(1.0055)^t$

b. Estimates from graph: Around 1980 methane gas reaches 1500 parts per billion, which is double the 1850 level of

CH. 6 Algebra Aerobics Solutions for Section 6.1

about 750 parts per billion. So the doubling time using this method is about 130 years.

Using the "rule of 70" and a growth rate of 0.55% from part (a), we have $70/0.55 \approx 127$ years. So the estimates are pretty close.

c. Since 1850 the amount of methane gas in the atmosphere has been growing exponentially at a rate of about 0.55% per year. Between 1850 and 1980, the amount of methane gas in the atmosphere doubled.

CHAPTER 6

Section 6.1

Algebra Aerobics 6.1a

1. a. When $t = 0$, $M = 250(3)^0 = 250(1) = 250$.

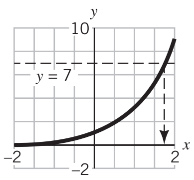
b. When $t = 1$, $M = 250(3)^1 = 250(3) = 750$.

c. When $t = 2$, $M = 250(3)^2 = 250(9) = 2250$.

d. When $t = 3$, $M = 250(3)^3 = 250(27) = 6750$.

2. a. $4 < t < 5$ **b.** $8 < t < 9$

3. To read each value of x from this graph, draw a horizontal line from the y-axis at a given value until it hits the curve. Then draw a vertical line to the x-axis to identify the appropriate value of x.



a. horizontal line $y = 7$, from y-axis to curve, then down to x-axis $\Rightarrow x \approx 1.8$

b. horizontal line $y = 0.5$, from y-axis to curve, then down to x-axis $\Rightarrow x \approx -0.5$.

4. a. $2 < x < 3$ **b.** $-1 < x < 0$

5. a. $2^3 < 13 < 2^4$ **c.** $5^{-1} < 0.24 < 5^0$

b. $3^4 < 99 < 3^5$ **d.** $10^3 < 1500 < 10^4$

Algebra Aerobics 6.1b

1. a. $\log(10^5/10^7) = \log 10^{-2} = -2 \log 10 = -2(1) = -2$ and $\log 10^5 - \log 10^7 = 5 \log 10 - 7 \log 10 = 5 - 7 = -2$. So $\log(10^5/10^7) = \log 10^5 - \log 10^7$.

b. $\log(10^5 \cdot (10^7)^3) = \log(10^5 \cdot 10^{21}) = \log 10^{26} =$
$26 \log 10 = 26$ and $\log 10^5 + 3 \log 10^7 =$
$5 \log 10 + 3(7) \log 10 = 5 + (3 \cdot 7) = 26$.
So $\log[10^5 \cdot (10^7)^3] = \log(10^5) + 3 \log(10^7)$.

2. a. Rule 2: $\log 3 = \log \frac{15}{5} = \log 15 - \log 5$

b. Rule 3: $\log 1024 = \log(2^{10}) = 10 \log 2$

c. Rule 3: $\log \sqrt{31} = \log(31^{1/2}) = \frac{1}{2} \log 31$

d. Rule 1: $\log 30 = \log(2 \cdot 3 \cdot 5) = \log 2 + \log 3 + \log 5$

e. Rule 2: $\log 81 - \log 27 = \log\left(\frac{81}{27}\right) = \log 3$
or Rule 3: $4 \log 3 - 3 \log 3 = \log 3$

3. a. False **d.** True; Rule 3 and that $\log 10 = 1$

b. False **e.** False

c. True; Rule 3 **f.** False

4. a. $\log\sqrt{\frac{2x-1}{x+1}} = \log\left(\frac{2x-1}{x+1}\right)^{1/2} = \frac{1}{2}\log\left(\frac{2x-1}{x+1}\right)$
$= \frac{1}{2}[\log(2x - 1) - \log(x + 1)]$

b. $\log \frac{xy}{z} = \log(xy) - \log z = \log x + \log y - \log z$

c. $\log \frac{x\sqrt{x+1}}{(x-1)^2} = \log \frac{x(x+1)^{1/2}}{(x-1)^2}$
$= \log x(x + 1)^{1/2} - \log(x - 1)^2$
$= \log x + \log(x + 1)^{1/2} - 2 \log(x - 1)$
$= \log x + \frac{1}{2}\log(x + 1) - 2 \log(x - 1)$

d. $\log \frac{x^2(y-1)}{y^3z} = \log x^2(y - 1) - \log y^3z$
$= \log x^2 + \log(y - 1) - [\log y^3 + \log z]$
$= 2 \log x + \log(y - 1) - 3 \log y - \log z$

5. $\frac{1}{3}[\log x - \log(x + 1)] = \frac{1}{3}\log\frac{x}{x+1}$
$= \log\left(\frac{x}{x+1}\right)^{1/3}$
$= \log\sqrt[3]{\frac{x}{x+1}}$

6. a. $\log x = 3 \Rightarrow 10^3 = x, x = 1000$

b. $\log x + \log 5 = 2 \Rightarrow \log 5x = 2 \Rightarrow 10^2 = 5x \Rightarrow x = 20$

c. $\log x + \log 5 = \log 2 \Rightarrow \log 5x = \log 2 \Rightarrow 5x = 2 \Rightarrow x = 2/5$

d. $\log x - \log 2 = 1 \Rightarrow \log \frac{x}{2} = 1 \Rightarrow 10^1 = \frac{x}{2} \Rightarrow x = 20$

e. $\log x - \log(x - 1) = \log 2 \Rightarrow \log \frac{x}{x-1} = \log 2 \Rightarrow$
$\frac{x}{x-1} = 2 \Rightarrow x = 2(x - 1) \Rightarrow x = 2$

f. $\log(2x + 1) - \log(x + 5) = 0 \Rightarrow$
$\log \frac{2x+1}{x+5} = \log 1 \Rightarrow$
$\frac{2x+1}{x+5} = 1 \Rightarrow 2x + 1 = x + 5 \Rightarrow x = 4$

7. $\log 10^3 - \log 10^2 = 3 \log 10 - 2 \log 10 = 3 - 2 = 1$
$\frac{\log 10^3}{\log 10^2} = \frac{3 \log 10}{2 \log 10} = \frac{3}{2} = 1.5$

Since $1 \neq 1.5 \Rightarrow \log 10^3 - \log 10^2 \neq \frac{\log 10^3}{\log 10^2}$

Algebra Aerobics 6.1c

1. a. $60 = 10 \cdot 2^t \Rightarrow 6 = 2^t$ (dividing both sides by 10)
$\log 6 = \log 2^t$ (taking the log of both sides)
$\log 6 = t \log 2$ (using Rule 3 of logs)
$\log 6/\log 2 = t$
$0.7782/0.3010 \approx t$ or $t \approx 2.59$

b. $500(1.06)^t = 2000 \Rightarrow (1.06)^t = \frac{2000}{500} \Rightarrow$
$(1.06)^t = 4 \Rightarrow \log(1.06)^t = \log 4 \Rightarrow$
$t(\log 1.06) = \log 4 \Rightarrow t = \frac{\log 4}{\log 1.06} = \frac{0.6021}{0.0253} \approx 23.8$

c. $80(0.95)^t = 10 \Rightarrow (0.95)^t = 1/8 = 0.125 \Rightarrow$
$\log (0.95)^t = \log 0.125 \Rightarrow t \log 0.95 = \log 0.125 \Rightarrow$
$t = \frac{\log 0.125}{\log 0.95} \approx \frac{-0.9031}{-0.0223} \approx 40.5$

CH. 6 Exercises Solutions for Section 6.1

2. a. $7000 = 100 \cdot 2^t \Rightarrow 70 = 2^t \Rightarrow \log 70 = \log 2^t \Rightarrow$
$\log 70 = t \log 2 \Rightarrow t = \log 70/\log 2 \approx \frac{1.845}{0.301} \approx 6.13$
It will take 6.13 time periods or approximately (6.13)(20) min = 122.6 min (or a little more than 2 hours) for the bacteria count to reach 7000.

b. $12{,}000 = 100 \cdot 2^t \Rightarrow 120 = 2^t \Rightarrow \log 120 = \log 2^t \Rightarrow$ $\log 120 = t \log 2 \Rightarrow t = \log 120/\log 2 \approx 6.907$ time periods or approximately (6.907)(20) min = 138.14 min (or a little more than $2\frac{1}{4}$ hrs.) for the bacteria count to reach 12,000.

3. Using the rule of 70, since $R = 6\%$ per yr, then $70/R = 70/6 \approx 11.7$ yr. More precisely, we have:
$2000 = 1000(1.06)^t \Rightarrow 2 = 1.06^t \Rightarrow$
$\log 2 = \log 1.06^t \Rightarrow \log 2 = t \log 1.06 \Rightarrow$
$t = \frac{\log 2}{\log 1.06} \approx \frac{0.3010}{0.0253} \approx 11.9$ yr

4. $1 = 100(0.976)^t \Rightarrow 0.01 = (0.976)^t \Rightarrow$
$\log 0.01 = \log (0.976)^t \Rightarrow \log 0.01 = t \log 0.976 \Rightarrow$
$t = \frac{\log 0.01}{\log 0.976} = \frac{-2}{-0.0106} \approx 189$ years, or almost 2 centuries!

5. a. $30 = 60(0.95)^t \Rightarrow 0.5 = (0.95)^t \Rightarrow$
$\log(0.5) = \log(0.95^t) \Rightarrow \log 0.5 = t \log 0.95 \Rightarrow$
$t = \frac{\log 0.5}{\log 0.95} \approx \frac{-0.3010}{-0.0223} \approx 13.5$ years for the initial amount to drop in half.

b. $16 = 8(1.85)^t \Rightarrow 2 = (1.85)^t \Rightarrow$
$\log 2 = \log(1.85)^t \Rightarrow \log 2 = t \log 1.85 \Rightarrow t = \frac{\log 2}{\log 1.85} \Rightarrow$
$t \approx \frac{0.3010}{0.2672} = 1.13$ years for the initial amount to double.

c. $500 = 200(1.045)^t \Rightarrow 2.5 = (1.045)^t \Rightarrow$
$\log 2.5 = \log(1.045)^t \Rightarrow \log 2.5 = t \log 1.045 \Rightarrow$
$t = \frac{\log 2.5}{\log 1.045} \Rightarrow t \approx \frac{0.3979}{0.0191} \Rightarrow$
$t \approx 20.8$ years for the initial amount to increase from 200 to 500.

6. a. $60 = 120(0.983)^t \Rightarrow 0.5 = (0.983)^t \Rightarrow$
$\log 0.5 = t \log 0.983 \Rightarrow$
$t = \frac{\log 0.5}{\log 0.983} \Rightarrow t \approx \frac{-0.3010}{-0.0074} \Rightarrow$
$t \approx 40.6$ days

b. $0.25 = 0.5(0.92)^t \Rightarrow 0.5 = (0.92)^t \Rightarrow$
$\log 0.5 = t \log 0.92 \Rightarrow$
$t = \frac{\log 0.5}{\log 0.92} \Rightarrow t \approx \frac{-0.3010}{-0.0362} \Rightarrow$
$t \approx 8.3$ hours

c. $0.5A_0 = A_0(0.89)^t \Rightarrow 0.5 = (0.89)^t \Rightarrow$
$\log 0.5 = t \log 0.89 \Rightarrow t = \frac{\log 0.5}{\log 0.89} \Rightarrow t \approx \frac{-0.3010}{-0.0506} \Rightarrow$
$t \approx 5.9$ years

Exercises for Section 6.1

1. Student estimates will vary for each interest rate.
a. About 38 years. **b.** About 16 years.

3. Eyeball estimates will vary. One set of guesses is:
a. 1.3 hrs. **b.** 2.4 hrs. **c.** 5 hrs.

5. a. 0.001 **b.** 10^6 **c.** 1 **d.** 10 **e.** 0.1

7. a. $2 \cdot \log 3$ **b.** $2 \cdot \log 3 + \log 2$ **c.** $3 \cdot \log 3 + \log 2$

9. a. 12 **b.** 12 **c.** 12 **d.** 2

11. If $w = \log(A)$ and $z = \log(B)$, then $10^w = A$ and $10^z = B$. Thus $A/B = 10^w/10^z = 10^{w-z}$ and therefore $\log(A/B) = \log(10^{w-z}) = w - z = \log(A) - \log(B)$, as desired.

13. a. $\log\left(\frac{K^3}{(K+3)^2}\right)$ **c.** $\log\left(T^4 \cdot \sqrt{T}\right) = \log\left(\sqrt{T^9}\right)$

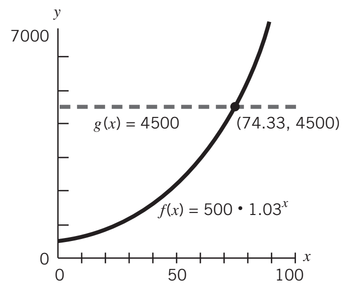
b. $\log\left(\frac{(3+n)^5}{m}\right)$ **d.** $\log\left(\frac{\sqrt[3]{xy^2}}{(xy^2)^3}\right) = \log\left([xy^2]^{-8/3}\right)$

15. Let $w = \log(A)$. Then $10^w = A$ and thus $A^p = (10^w)^p = 10^{wp}$, and then $\log(A^p) = w \cdot p = p \cdot w = p \cdot \log(A)$, as desired.

17. a. 1000
b. 999
c. $10^{5/3} \approx 46.42$
d. 1/9
e. No solution, since x can not be negative.

19. a. Solve $300 = 100 \cdot 1.03^t$ to get $t = \log(3)/\log(1.03) \approx 37.17$ years.
b. Similarly, $\log(3)/\log(1.07) \approx 16.24$ years.

21. a. The graphs of $f(x) = 500 \cdot (1.03)^x$ and $g(x) = 4500$ are in the accompanying diagram.
b. $x = 75$ is a good eyeball estimate.



c. $\log(4500) = \log(500) + x \cdot \log(1.03)$ or
$x = [\log(4500) - \log(500)]/\log(1.03) = \log(9)/\log(1.03) \approx 74.33$
d. The eyeball estimate and the logarithm-computed answer are very close.

23. a. Doubling time: $x = \log(2)/\log(4) = 0.5$ years
b. Half-life $= \log(0.5)/\log(0.25) = 0.5$ years
c. Doubling time $= \log(2)/\log(4) = 0.5$ years
d. Half life $= \log(0.5)/\log(0.25) = 0.5$ years

25. a. $N(t) = 90(1.002)^t$, where $N(t)$ is the number of articles (in thousands) for t days after October 23.
b. The number of articles has doubled when $N(t) = 180$ (thousand articles). So setting $180 = 90(1.002)^t$ and solving for t, we get $2 = (1.002)^t \Rightarrow \log 2 = t \log(1.002) \Rightarrow t = (\log 2)/\log(1.002) \approx 347$ days, or $347/30 \approx 11.6$ months or about one year. So if the rate of growth continues, every 12 months the number of articles will more than double.

27. a. $t = \log(2)/\log(1.5) \approx 1.71$ 20-minute time periods or about 34 min.
b. $t = \log(10)/\log(1.5) \approx 5.68$ 20-minute time periods or about 114 min.

29. a. $B(t) = B_0(0.5)^{0.05t}$, where t is measured in minutes and $B(t)$ and B_0 are measured in some weight unit. None is specified in the problem.

b. In one hour, $t = 60$ and thus $B(60) = B_0(0.5)^{0.05 \cdot 60} = 0.125 \cdot B_0$, or 12.5% is left.

c. If its half-life is 20 minutes, then its quarter-life is 40 minutes.

d. Solving $0.10 = 0.5^{0.05t}$ for t, we get $0.05t \cdot \log(0.5) = \log(0.10)$ or $t = \log(0.1)/[\log(0.5) \cdot 0.05] \approx 66$ minutes.

31. a. $S = 300 \cdot 0.9^W$ if $0 \le W \le 10$, where W is measured in weeks and S is measured in dollars.

b. Solving $S = 150$ for W means solving $0.5 = 0.9^W$ for $W \Rightarrow W = \log(0.5)/\log(0.9) \approx 6.6$ weeks. The selling price when it would be given to charity is at $W = 10$, when $S = 300 \cdot 0.9^{10} \approx \$104.60$

33. a. If we set $4000 = 1355(1.036)^t$ then

$4000/1355 = (1.036)^t \Rightarrow 2.952 \approx 1.036^t \Rightarrow$

$\log(2.952) \approx t\log(1.036) \Rightarrow$

$t \approx \frac{\log(2.952)}{\log(1.036)} \approx 30.6$ years after 1970,

or sometime in late 2000. During that year U.S. fish production actually dropped to a little over 3000 million pounds, almost a million pounds less than the model predicted.

b. Americans are actually increasing their fish and shellfish consumption, probably spurred on by doctors' and nutritionists' advice that "eating fish is good for you." However, the amount of imported fish has grown substantially, probably cutting into U.S. fish production.

Section 6.2

Algebra Aerobics 6.2

1. a. $\$1000(1.085) = \$1085$

b. $\$1000\left(1 + \frac{0.085}{4}\right)^4 = \$1087.75$

c. $\$1000e^{0.085} = \$1088.72$

2. a. $e^{0.04} = 1.0408 \Rightarrow 4.08\%$ is effective rate

b. $e^{0.125} = 1.133 \Rightarrow 13.3\%$ is effective rate

c. $e^{0.18} = 1.197 \Rightarrow 19.7\%$ is effective rate

3. a. Principal = 6000; nominal rate = 5%; effective rate = 5% since $1.05^1 = 1.05$; number of interest periods = 1

b. Principal = 10,000; nominal rate = 8%; effective rate ≈ 8.24% since $1.02^{4(1)} \approx 1.0824$; number of interest periods = 4

c. Principal = 500; nominal rate = 12%; effective rate ≈ 12.68% since $1.01^{12(1)} \approx 1.1268$; number of interest periods = 12

d. Principal = 50,000; nominal rate = 5%; effective rate ≈ 5.06% since $1.025^{2(1)} \approx 1.0506$; number of interest periods = 2

e. Principal = 125; nominal rate = 7.6%; effective rate ≈ 7.90% since $e^{0.076\,(1)} \approx 1.0790$; interest is continuously compounded

4. a. $5e^{0.03t} = 5(e^{0.03})^t \approx 5(1.030)^t$

b. $3500e^{0.25t} = 3500(e^{0.25})^t \approx 3500(1.284)^t$

c. $660e^{1.75t} = 660(e^{1.75})^t \approx 660(5.755)^t$

d. $55{,}000e^{-0.07t} = 55{,}000(e^{-0.07})^t \approx 55{,}000(0.932)^t$

e. $125{,}000e^{-0.28t} = 125{,}000(e^{-0.28})^t \approx 125{,}000(0.756)^t$

5.

n	$1/n$	$1 + 1/n$	$(1 + 1/n)^n$
1	1	2	2
100	0.01	1.01	2.704 813 829
1000	0.001	1.001	2.716 923 932
1,000,000	0.000 001	1.000 001	2.718 280 469
1,000,000,000	0.000 000 001	1.000 000 001	2.718 281 827

The values for $(1 + 1/n)^n$ come closer and closer to the irrational number we define as e and are consistent with the value for e in the text of 2.71828.

6. a. about $12,000

b. about $18,000

c. about $33,000

d. about 6 years

e. 1 year: A = $11,255; 5 years: $18,061; 10 years: $32,620. doubling time:

$1.03^{4t} = 2 \Rightarrow 4t \log 1.03 = \log 2 \Rightarrow$

$t = \frac{\log 2}{4 \log 1.03} \Rightarrow t = \frac{0.3010}{4(0.0128)} \Rightarrow t \approx 5.9$ years

7. a. $8000 \cdot \left(1 + \frac{0.08}{4}\right)^{4 \cdot (18)} = \$33{,}289$

b. $8000 \cdot e^{0.08 \cdot (18)} = \$33{,}766$

c. $8000 \cdot (1.084)^{18} = \$34{,}168$

8. a. $8000 \cdot \left(1 + \frac{0.08}{4}\right)^{4 \cdot (30)} = \$86{,}121$

b. $8000 \cdot e^{0.08 \cdot (30)} = \$88{,}185$

c. $8000 \cdot (1.084)^{30} = \$89{,}943$

9. a. continuous growth rate: 0.6 or 60%; annual effective growth rate ≈ 82.2% since $e^{0.6(1)} \approx 1.822$, which is the growth factor. So $1.822 - 1 = 0.822$ is the growth rate.

b. continuous growth rate: 2.3 or 230%; annual effective growth rate ≈ 897% since $e^{2.3(1)} \approx 9.97$, which is the growth factor. So $9.97 - 1 = 8.97$ is the growth rate.

10. a. continuous decay rate: 0.055 or 5.5%; annual effective decay rate ≈ 5.35% since $e^{-0.055} \approx 0.946$, which is the decay factor. So $1 - 0.946 = 0.054$ is the decay rate.

b. continuous decay rate: 0.15 or 15%; annual effective decay rate ≈ 13.9% since $e^{-0.15} \approx 0.861$, which is the decay factor. So $1 - 0.861 = 0.139$ is the decay rate.

Exercises for Section 6.2

1. a. 1.0353 **b.** 1.0355 **c.** 1.0356 **d.** 1.0356

e. These values represent the growth factors for compounding semi-annually, quarterly, monthly, and continuously. As the number of compoundings increase, the results come closer and closer to continuous compounding.

3. In general, if the nominal rate is 8.5%, then $A_k(n) = 10{,}000 \cdot (1 + 0.085/k)^{kn}$ gives the value of $10,000 after n years if the interest is compounded k times per year and

Ch. 6

$A_c(n) = 10{,}000 \cdot e^{0.085n}$ gives that value if the interest is compounded continuously.

a. annually: $(1 + 0.085/1)^1 = 1.0850$, and thus the effective rate is 8.50%.

b. semi-annually: $(1 + 0.085/2)^2 \approx 1.0868$, and thus the effective rate is 8.68%.

c. quarterly: $(1 + 0.085/4)^4 \approx 1.0877$, and thus the effective rate is 8.77%.

d. continuously: $e^{0.085n} \approx 1.0887$, and thus the effective rate is 8.87%.

5. a. $A(t) = 25{,}000 \cdot (1 + 0.0575/4)^{4t}$, where t = number of years

b. $B(t) = 25{,}000 \cdot e^{0.0575t}$, where t = number of years

c. $A(5) = \$33{,}259.12$ and $B(5) = \$33{,}327.26$

d. The effective rate for compounding quarterly is $\left(1 + \frac{0.0575}{4}\right)^4 - 1 \approx 0.05875$, and the effective rate for compounding continuously is $e^{0.0575} - 1 \approx 0.05919$. Thus continuous compounding has a slightly greater effective rate.

7. a. $e^{0.045} \approx 1.046$

b. $1.0680 < 1.0704 \approx e^{0.068}$

c. $1.2690 > e^{0.238} \approx 1.2687$

d. $e^{-0.10} \approx 0.9048 > 0.9000$

e. $0.8607 \approx e^{-0.15}$

9. a. $U(x) = 10 \cdot (1/2)^{x/5}$, where x is measured in billions of years.

b. $5 = 10 \cdot (1/2)^{x/5}$, and thus $1/2 = (1/2)^{x/5} \Rightarrow 1 = x/5 \Rightarrow x = 5$ billion years.

11. a. $t = \log(2)/\log(1.12) \approx 6.12$ years

b. $t = \log(2)/\log[(1 + 0.12/4)^4] \approx 5.86$ years.

13. a. The 10-year decay factor is 0.85. The yearly decay factor is $0.85^{1/10} \approx 0.9839$. The yearly decay rate is $1 - 0.9839 \approx 0.0161$ or 1.61%.

b. $g(t) = 1.5 \cdot 0.9839^t$, where $g(t)$ is measured in millions and t in years.

c. $h(t) = 1.5 \cdot e^{-0.01625t}$, where $h(t)$ is measured in millions and t in years.

d. $g(20) \approx 1.0842$ million and $h(20) \approx 1.0838$ million. The h function decays a bit faster than the g function, but they are quite close.

15. a. $P(t) = 500 \cdot (1.0202)^t$; the continuous growth rate is 2% and the effective annual growth rate is = 2.02%.

b. $N(t) = 3000 \cdot (4.4817)^t$; the continuous growth rate is 150% and the effective growth rate is 348.17%.

c. $Q(t) = 45 \cdot (1.0618)^t$; the continuous growth rate is 6% and the effective growth rate is 6.18%.

d. $G(t) = 750 \cdot 1.0356^t$; the continuous growth rate is 3.5% and the effective growth rate is 3.56%.

17. a. $0.0343 = e^r - 1$, and thus $r = \log(1.0343)/\log(e) \approx 0.0337$, so the continuous nominal interest rate is 3.37%.

b. $0.046 = e^r - 1$, and thus $r = \log(1.046)/\log(e) \approx 0.04497$, so the continuous nominal interest rate is 4.497% or 4.5% rounded off.

Section 6.3

Algebra Aerobics 6.3

1. a. $\ln e^2 = 2 \ln e = 2(1) = 2$

b. $\ln 1 = 0$

c. $\ln \frac{1}{e} = \ln 1 - \ln e = 0 - 1 = -1$

d. $\ln \frac{1}{e^2} = \ln 1 - \ln e^2 = \ln 1 - 2 \ln e = 0 - 2(1) = 0 - 2 = -2$

e. $\ln \sqrt{e} = \ln e^{1/2} = \frac{1}{2} \ln e = \frac{1}{2}(1) = 1/2$

2. a. $\ln \sqrt{xy} = \ln(xy)^{1/2} = \frac{1}{2} \ln(xy) = \frac{1}{2}[\ln x + \ln y]$

b. $\ln\left(\frac{3x^2}{y^3}\right) = \ln(3x^2) - \ln(y^3) = \ln 3 + 2 \ln x - 3 \ln y$

c. $\ln\big((x + y)^2(x - y)\big) = \ln(x + y)^2 + \ln(x - y) = 2 \ln(x + y) + \ln(x - y)$

d. $\ln \frac{\sqrt{x + 2}}{x(x - 1)} = \ln \frac{(x + 2)^{1/2}}{x(x - 1)} = \ln(x + 2)^{1/2} - \ln x(x - 1)$
$= \frac{1}{2} \ln(x + 2) - \ln x(x - 1)$
$= \frac{1}{2} \ln(x + 2) - [\ln x + \ln(x - 1)]$
$= \frac{1}{2} \ln(x + 2) - \ln x - \ln(x - 1)$

3. a. $\ln x(x - 1)$

b. $\ln \frac{(x + 1)}{x}$

c. $\ln x^2 - \ln y^3 = \ln \frac{x^2}{y^3}$

d. $\ln(x + y)^{1/2} = \ln \sqrt{x + y}$

e. $\ln x - 2 \ln(2x - 1) = \ln x - \ln(2x - 1)^2 = \ln \frac{x}{(2x - 1)^2}$

4. growth factor $= e^r = 1 + 0.064 = 1.064 \Rightarrow \ln e^r = \ln 1.064 \Rightarrow r = \ln 1.064 = 0.062$ or 6.2%

5. $50{,}000 = 10{,}000\, e^{0.078t} \Rightarrow 5 = e^{0.078t} \Rightarrow \ln 5 = 0.078t \Rightarrow t = \ln 5/0.078 \approx 20.6$ yr

6. a. $\ln e^{x+1} = \ln 10 \Rightarrow (x + 1)\ln e = \ln 10 \Rightarrow (x + 1)(1) \approx 2.30 \Rightarrow x \approx -1 + 2.30 \Rightarrow x \approx 1.30$

b. $\ln e^{x-2} = \ln 0.5 \Rightarrow (x - 2) \ln e = \ln 0.5 \Rightarrow x - 2 \approx -0.69 \Rightarrow x \approx 2 - 0.69 \Rightarrow x \approx 1.31$

7. a. true, since $\ln 81 = \ln 3^4 = 4 \ln 3$

b. false; $\ln 7 = \ln \frac{14}{2} = \ln 14 - \ln 2$

c. true, since $\ln 35 = \ln(5 \cdot 7) = \ln 5 + \ln 7$

d. false; $2 \ln 10 = \ln 10^2 = \ln 100$

e. true, since $\ln e^{1/2} = \frac{1}{2} \ln e = \frac{1}{2}$

f. false: $5 \ln 2 = \ln 2^5 = \ln 32$

8. a. $\ln 2 + \ln 6 = x \Rightarrow \ln(2 \cdot 6) = x \Rightarrow \ln 12 = x \Rightarrow x \approx 2.48$

b. $\ln 2 + \ln x = 2.48 \Rightarrow \ln 2x = 2.48 \Rightarrow e^{2.48} = 2x \Rightarrow x = \frac{e^{2.48}}{2} \approx 5.97$

c. $\ln(x + 1) = 0.9 \Rightarrow e^{0.9} = x + 1 \Rightarrow x = e^{0.9} - 1 \approx 1.46$

d. $\ln 5 - \ln x = -0.06 \Rightarrow \ln \frac{5}{x} = -0.06 \Rightarrow e^{-0.06} = \frac{5}{x} \Rightarrow x = \frac{5}{e^{-0.06}} \approx 5.3$

Ch. 6

Exercises for Section 6.3

1. a. $\ln(A \cdot B) = \ln(A) + \ln(B)$ or Rule 1

b. $\ln(A/B) = \ln(A) - \ln(B)$ or Rule 2

c. $\ln(A^p) = p \cdot \ln(A)$ or Rule 3

d. Rules 1 and 3

e. Rules 1 and 3

f. Rules 2 and 1

3. a. $10^n = 35$ **b.** $e^x = 75$ **c.** $e^{3/4} = x$ **d.** $N = N_0 \cdot e^{-kt}$

5. a. $x = 5 \cdot 2 = 10$ **d.** $x = 8 \cdot 36 = 288$

b. $x = 24/2 = 12$ **e.** $x = 64/9 \approx 7.1$

c. $x = 11$ **f.** $x = 16/8 = 2$

7. a. $\frac{1}{2}(\ln 4 + \ln x + \ln y)$ **c.** $\ln(3) + \frac{3}{4}\ln(x)$

b. $\frac{1}{3}(\ln 2 + \ln x) - \ln(4)$

9. a. $\ln\left(\sqrt[4]{(x+1)(x-3)}\right)$ **b.** $\ln\left(\frac{R^3}{\sqrt{P}}\right)$ **c.** $\ln\left(\frac{N}{N_0^2}\right)$

11. a. $r = \ln(1.0253) \approx 0.025$ **c.** $x = \ln(0.5)/3 \approx -0.231$

b. $t = \ln(3)/0.5 \approx 2.197$

13. a. $x = \ln(10) \approx 2.303$

b. $x = \log(3) \approx 0.477$

c. $x = \log(5)/\log(4) \approx 1.161$

d. $x = e^5 \approx 148.413$

e. $x = e^3 - 1 \approx 19.086$

f. no solution [$\ln(-4/3)$ not defined]

15. Let $w = \ln(A)$ and $z = \ln(B)$. Thus $e^w = A$ and $e^z = B$. Therefore $A \cdot B = e^{w+z}$ and therefore $\ln(A \cdot B) = w + z = \ln(A) + \ln(B)$, as desired.

17. $t = [\ln(100{,}000) - \ln(15{,}000)]/0.085 \approx 22.3$ years

19. $1.0338 = e^r$, and thus the nominal continuous interest rate $r = \ln(1.0338) \approx 0.0332 = 3.32\%$.

21. a. $r = \ln(1.025) \approx 0.0247$ **c.** $r = \ln(1.08) \approx 0.0770$

b. $r = \ln(0.5) \approx -0.6931$

23. a. growth; 37% **c.** decay; 19% **e.** growth; 56%

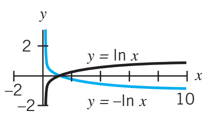
b. growth; 115% **d.** decay; 120% **f.** decay; 29%

Section 6.4

Algebra Aerobics 6.4

1. a. Since $\log x^2 = 2 \log x$ (by Rule 3 of logarithms), the graphs of $y = \log x^2$ and $y = 2 \log x$ will be identical (assuming $x > 0$).

2. The graph of $y = -\ln x$ will be the mirror image of $y = \ln x$ across the x-axis.



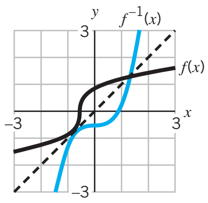
3. a. The graphs are very similar (see Figure 6.5). They intersect at (1, 0), which is the x-intercept of each graph.

b. The graph of $\log x$ reaches a y value of 1 when $x = 10$ while the graph of $\ln x$ reaches a value of 1 when $x = e$ (or approximately 2.7). The graph of $\log x$ reaches 2 at $x = 100$ while the graph of $\ln x$ reaches 2 at $x = e^2 \approx 7.4$.

c. For $0 < x < 1$, both graphs lie below the x-axis and approach the y-axis asymptotically as x gets closer to 0.

d. To the right of $x = 1$, both graphs lie above the x-axis, with the $\ln x$ graph rising slightly faster than the $\log x$ graph, and thus staying above the $\log x$ graph.

4. f and its inverse f^{-1} along with the dotted line for $y = x$:



5. a. $\log 10^3 = 3$

b. $\log 10^{-5} = -5$

c. $3 \log 10^{0.09} = 3(0.09) = 0.27$

d. $10^{\log 3.4} = 3.4$

e. $\ln e^5 = 5$

f. $\ln e^{0.07} = 0.07$

g. $\ln e^{3.02} + \ln e^{-0.27} = 3.02 - 0.27 = 2.75$

h. $e^{\ln 0.9} = 0.9$

6. Acidic. If $4 = -\log[H^+]$, then $-4 = \log[H^+]$. So $[H^+] = 10^{-4}$. Since the pH is 3 less than pure water's, it will have a hydrogen ion concentration 10^3 or 1000 times higher than pure water's.

7. $N = 10 \log\left(\frac{1.5 \cdot 10^{-12}}{10^{-16}}\right) = 10 \log(1.5 \cdot 10^4)$

$= 10(\log 1.5 + \log 10^4) \approx 10(0.176 + 4)$

$= 10(4.176) \approx 42$ dB

8. Multiplying the intensity by $100 = 10^2$ corresponds to adding 20 to the decibel level. Multiplying the intensity by $10{,}000{,}000 = 10^7$ corresponds to adding 70 to the decibel level.

Exercises for Section 6.4

1. Since $\log(5x) = \log(5) + \log(x)$ and $\log(5) > 0$, the graph of $\log(5x)$ (graph A) is above the graph of $\log(x)$ (Graph B).

3. a. $A = \log(x)$, $B = \log(x - 1)$, and $C = \log(x - 2)$; so $f(x)$ matches A, $g(x)$ matches B, and $h(x)$ matches C.

b. $\log(1) = 0$; thus $f(1) = g(2) = h(3) = 0$.

c. f has 1 as its x-intercept; g has 2 and h has 3.

d. The graph moves from crossing the x-axis at $x = 1$ to crossing it at $x = k$. Assuming that $k > 0$, then $f(x - k)$ is the graph of f moved k units to the right.

5. a. The graphs of f and h are mirror images of each other across the y-axis, as are the graphs of g and k.

b. The graphs of f and g are mirror images of each other across the x-axis, as are the graphs of h and k.

Ch. 6

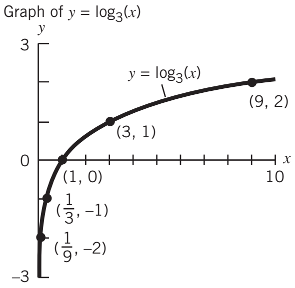
7. The table for $\log_3(x)$ is:

x	1/9	1/3	1	3	9
y	-2	-1	0	1	2

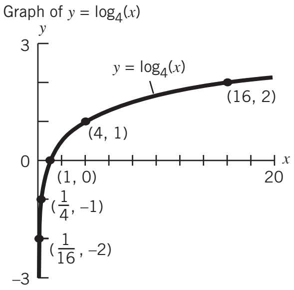
The table for $\log_4(x)$ is:

x	1/16	1/4	1	4	16
y	-2	-1	0	1	2

Graph of $y = \log_3(x)$



Graph of $y = \log_4(x)$



9. dB $= 10 \cdot \log(I/I_0) = 28$ implies that $I/I_0 = 10^{2.8}$ and thus $I = 10^{-13.2}$ watts/cm^2, and dB $= 92$ has $I/I_0 = 10^{9.2}$ and thus $I = 10^{-6.8}$ watts/cm^2. (Note that these answers assume, of course, that $I_0 = 10^{-16}$ watts/cm^2.)

11. If I is the intensity of one crying baby, then $5I$ is the intensity of five crying babies. Thus the perceived noise in decibels is $10 \cdot \log[5(I/I_0)] = 10 \cdot \log(5) + 10 \log(I/I_0) \approx 6.99 + 10 \cdot \log(I/I_0) \approx 7 +$ noise of one baby crying. Thus quintuplets crying are about 7 decibels louder than one baby crying.

13. Total volume $= 2\frac{1}{4} = \frac{9}{4}$ cups or equivalently 9 quarter cups. So lemon juice is $\frac{1}{9}$th of the mixture volume and water is $\frac{8}{9}$ths of the volume. Since the lemon juice has a pH of 2.1, its hydrogen ion concentration is $10^{-2.1}$ (moles per liter). Similarly, since the tap water has a pH of 5.8, its ion concentration is $10^{-5.8}$. So the hydrogen ion concentration of the mixture $= [H^+] = \left(\frac{1}{9}\right) \cdot 10^{-2.1} + \left(\frac{8}{9}\right) \cdot 10^{-5.8} \Rightarrow$ $\text{pH} = -\log[\left(\frac{1}{9}\right) \cdot 10^{-2.1} + \left(\frac{8}{9}\right) \cdot 10^{-5.8}] \approx 3.05$. Thus the mixture is slightly less acidic than orange juice (with a pH of 3).

Section 6.5

Algebra Aerobics 6.5

1. a. decay **b.** decay **c.** growth **d.** growth **e.** decay **f.** decay

2. a. $e^k = 1.062 \Rightarrow k = \ln 1.062 \approx 0.060 \Rightarrow y = 1000e^{0.06t}$

b. $e^k = 0.985 \Rightarrow k = \ln 0.985 \approx -0.015 \Rightarrow y = 50e^{-0.015t}$

3. a. continuous nominal rate $\approx 5.45\%$ since $\ln 1.056 \approx 0.545$; effective rate $= 5.6\%$ since $e^{\ln 1.056} = 1.056$

b. continuous nominal rate $\approx 3.34\%$ since $\ln 1.034 \approx 0.0334$; effective rate $= 3.4\%$ since $e^{\ln 1.034} = 1.034$

c. continuous nominal rate $\approx 7.97\%$ since $\ln 1.083 \approx 0.0797$; effective rate $= 8.3\%$ since $e^{\ln 1.083} = 1.083$

d. continuous nominal rate $\approx 25.85\%$ since $\ln 1.295 \approx 0.2585$; effective rate $= 29.5\%$ since $e^{\ln 1.295} = 1.295$

4. a. growth factor $= e^{0.08} \approx 1.083$

b. decay factor $= e^{-0.125} \approx 0.883$

Exercises for Section 6.5

1. a. $N = 10\, e^{(\ln 1.045)t} = 10 \cdot e^{0.0440t}$

b. $Q = 5 \cdot 10^{-7} \cdot e^{(\ln 0.072)A} = 5 \cdot 10^{-7} \cdot e^{-2.631A}$

c. $P = 500 \cdot e^{(\ln 2.10)x} = 500 \cdot e^{0.742x}$

3. a. decay **b.** decay **c.** growth

5. a. $t = 5$ years; $P = P_0e^{0.1386t}$

b. $t = 25$ years; $P = P_0e^{0.0277t}$

c. $t = 1/2$ year; $P = P_0e^{1.3863t}$

7. If $200 = 760 \cdot e^{-0.128h}$, then $h = \ln(200/760)/(-0.128) \approx 10.43$ km.

9. The half-life is $\ln(1/2)/(-r) = [\ln(1) - \ln(2)]/(-r) = [0 - \ln(2)]/(-r) = \ln(2)/r = 100 \cdot \ln(2)/R \approx 69.3137/R$, which is approximately $70/R$.

11. a. f goes with C **b.** g goes with B **c.** h goes with A

13. a. It is a 2% nominal continuous decay rate.

b. $2500 \cdot e^{-0.02t} = 2500 \cdot 0.5^{t/n} \Rightarrow -0.02t = (t/n) \cdot \ln(0.5) \Rightarrow n = \ln(0.5)/(-0.02) \approx 34.66$.

c. It represents the half-life in whatever time units t is measured in.

15. a. Since $Q(8000) = 0.5Q_0$, the half-life is 8000 years.

b. The annual decay rate is $1 - 0.5^{1/8000} \approx 0.000\,086\,639\,6$.

c. Solving $e^r = 0.5^{1/8000}$ for r gives: $r \approx -0.000\,086\,643\,4$ as the nominal continuous decay rate.

17. Answers from students will vary. Look for an exponential curve that goes roughly through the middle of each cluster. One such curve contains the points (500, 1000) and (2000, 300). Then the best-fit exponential through these two points is $y = 1494(0.9992)^d$, where d measures depth in meters and y measures species density in an unknown unit. (Note that in base e, one gets $y = 1494e^{-0.0008d}$.)

19. Given: $n = n_0 \cdot e^{\ln(2) \cdot t/T}$ and $11 = n_0 \cdot e^{\ln(2) \cdot 2/T}$ and $30 = n_0 \cdot e^{\ln(2)\, \cdot\, 22/T}$.

Therefore: $\ln(11) = \ln(n_0) + 2 \cdot \ln(2)/T$ and $\ln(30) = \ln(n_0) + 22 \cdot \ln(2)/T$

Thus $\ln(30) - \ln(11) = \ln(2)[22 - 2]/T$

$\Rightarrow$ $T = 20 \cdot \ln(2)/[\ln(30) - \ln(11)] \approx 13.863/1.003 \approx 13.8$ seconds, the reactor period

If we substitute 13.8 for T in the equation $11 = n_0 \cdot e^{(2 \cdot \ln 2)/T}$, we get $11 = n_0 \cdot e^{(2 \cdot \ln 2)/13.8} \approx n_0 \cdot e^{0.100456} \approx n_0 \cdot 1.1057$. So $n_0 \approx 9.95$ or 10 neutrons.

21. a. Newton's Law here is of the form $T = 75 + Ce^{-kt}$ where $A = 75°$, the ambient temperature, and T is the temperature of the object at time t. When $t = 0$, $T = 160°$, so

$160 = 75 + Ce^0 \Rightarrow C = 85$. So the equation becomes

$T = 75 + 85e^{-kt}$. When $t = 10$, $T = 100°$ so

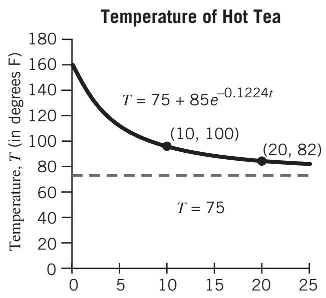
$100 = 75 + 85e^{-10k} \Rightarrow e^{-10k} = \frac{25}{85} \approx 0.2941 \Rightarrow$

$\ln(e^{-10k}) = \ln(0.2941) \Rightarrow$

$-10k \approx -1.224 \Rightarrow k \approx 0.1224$.

So the Law of Cooling in this situation is:
$T = 75 + 85e^{-0.1224t}$

b.



c. When $t = 20$ minutes, then the temperature of the tea $T = 75 + 85e^{-0.1224 \cdot 20} \approx 82°$.

Section 6.6

Algebra Aerobics 6.6

1. a. The graph of $y = 3x + 4$, a linear function, is a straight line on a standard linear plot. The graph of $y = 4 \cdot 3^x$, an exponential function, is a straight line graph on a semi-log plot. The equation $\log y = (\log 3) \cdot x + \log 4$ is equivalent to $y = 4 \cdot 3^x$, whose graph is a straight line on a semi-log plot.

b. The graph of $y = 3x + 4$ has slope $= 3$ and vertical intercept $= 4$. The equations $y = 4 \cdot 3^x$ and $\log y = (\log 3) \cdot x + \log 4$ are equivalent; their graphs have slope $= \log 3$ and vertical intercept $= \log 4$ on a semi-log plot.

2. a. $\log y = \log[5(3)^x] \Rightarrow \log y = \log 5 + x(\log 3)$

b. $\log y = \log[1000(5)^x] \Rightarrow \log y = \log 1000 + x \log 5 = 3 + x \log 5$

c. $\log y = \log[10{,}000(0.9)^x] \Rightarrow \log y = \log 10{,}000 + x \log 0.9 = 4 + x \log 0.9$

d. $\log y = \log[5 \cdot 10^6(1.06)^x] \Rightarrow \log y = \log 5 + \log 10^6 + x \log 1.06 = \log 5 + 6 + x \log 1.06$

3. a. $\log y = \log 7 + (\log 2)x \Rightarrow \log y = \log(7 \cdot 2^x) \Rightarrow 10^{\log y} = 10^{\log 7 \cdot 2^x} \Rightarrow y = 7 \cdot 2^x$

b. $\log y = \log 20 + (\log 0.25)x \Rightarrow \log y = \log(20 \cdot 0.25^x) \Rightarrow 10^{\log y} = 10^{\log(20 \cdot 0.25^x)} \Rightarrow y = 20 \cdot 0.25^x$

c. $\log y = 6 + (\log 3) \cdot x \Rightarrow 10^{\log y} = 10^{6 + \log(3^x)} \Rightarrow y = 10^6 \cdot 10^{\log(3^x)} \Rightarrow y = 10^6 \cdot 3^x$

d. $\log y = 6 + \log 5 + (\log 3) \cdot x \Rightarrow \log y = 6 + \log 5 + \log(3^x) \Rightarrow 10^{\log y} = 10^{6 + \log 5 + \log 3^x} \Rightarrow y = 10^6 \cdot 10^{\log 5} \cdot 10^{\log 3^x} \Rightarrow y = 10^6(5)3^x = 5 \cdot 10^6(3)^x$

4. a. i. slope $= \log 5$; vertical intercept $= \log 2$

ii. slope $= \log 0.75$; vertical intercept $= \log 6$

iii. slope $= \log 4$; vertical intercept $= 0.4$

iv. slope $= \log 1.05$; vertical intercept $= 3 + \log 2$

b. i. $y = 2 \cdot 5^x$

ii. $y = 6 \cdot (0.75)^x$

iii. $y = 10^{0.4}(4)^x$

iv. $y = 10^3 \cdot 2 \cdot 1.05^x = 2000(1.05)^x$

5. a. $a \approx 2.00$ since $10^{0.301} \approx 2$

b. $C \approx 524.81$ since $10^{2.72} \approx 524.81$

c. $a \approx 0.75$ since $10^{-0.125} \approx 0.75$

d. $C \approx 100{,}000$ since $10^5 = 100{,}000$

6. a. $y = 10^{0.301} \cdot 10^{0.477x} \Rightarrow y = 2 \cdot 3^x$

b. $y = 10^3 \cdot 10^{0.602x} \Rightarrow y = 1000(4)^x$

c. $y = 10^{1.398} \cdot 10^{-0.046x} \Rightarrow y = 25 \cdot (0.90^x)$

7. Exponential functions for both since the graphs on semi-log plots are approximately linear.

8. Using the point (0, 4), the vertical intercept, and (2, 5), the slope is $\frac{5-4}{2} = 0.5$. So $\log y = 4 + 0.5x \Rightarrow y = 10^{4+0.5x} \Rightarrow y = 10^4 \cdot 10^{0.5x} \approx 10^4(3.16)^x$.

Exercises for Section 6.6

1. a. goes with **f.**

b. goes with **h.**

c. goes with **e.**

d. goes with **g.**

3. a. $\log(y) = 4.477 + 0.301x$

b. $\log(y) = 3.653 + 0.146x$

c. $\log(y) = 6.653 - 0.155x$

d. $\log(y) = 3.778 - 0.244x$

5. a. goes with Graph **B.**

b. goes with Graph **C.**

c. goes with Graph **A.**

d. goes with Graph **D.**

7. Here are the average rates of change:

a. x	$Y = \log(x)$	Avg. Rt. of Ch.	b. x	$Y = \log(x)$	Avg. Rt. of Ch.
0	2.30103	n.a.	0	4.77815	n.a.
10	4.30103	0.20000	10	3.52876	−0.12494
20	4.90309	0.06021	20	2.27938	−0.12494
30	5.25527	0.03522	30	1.02999	−0.12494
40	5.50515	0.02499	40	−0.21945	−0.12494

a. This is *not* exponential; the average rate of change between consecutive points keeps decreasing.

b. The average rate of change over each decade is −0.12494. Thus the plot of Y vs. x is linear and therefore y is exponential. The exponential function is $y = 60{,}000(0.75)^x$.

9. a. The scatter plot of points $(x, \log(y))$, where $Y = \log(y)$, is nearly linear; thus the growth is very close to being exponential.

b. The equation of the best-fit line is given in the graph and the corresponding exponential equation is approximately $y = 38.788 \cdot 1.3137^x$. The daily growth rate is 31.37%.

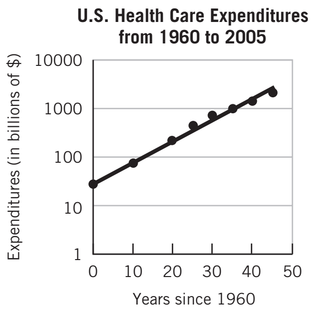
11. a. Approximately 12.2 micrograms/liter.

b. Yes, because its log graph is a straight line with negative slope. The decay factor is $10^{-0.0181} \approx 0.959$, and thus the decay rate is $1 - 0.959 = 0.041$ or 4.1%.

c. The exponential model is approximately $y = 12.2 \cdot 0.959^x$.

13. a. To find out the average health care costs per person, one would divide the total costs by the population estimate for each year. (See Table 2.1 for U.S. population counts.)

b. On a semi-log plot (where years are reinitialized at 1960) the data look rather linear, so it is reasonable to model the growth with an exponential function.



c. The graph shows the best-fit exponential, whose equation is $H(t) = 31(1.102)^t$, where $H(t)$ is the health care costs (in billions) t years after 1960. In 2005, when $t = 45$, $H(45) = 31(1.102)^{45} \approx 2450$ billion or 2.45 trillion dollars. The estimate exceeds the actual cost in 2005 of 2016 billion by 434 billion dollars.

Ch. 6: Check Your Understanding

1. True	**8.** False	**15.** True	**22.** False
2. True	**9.** False	**16.** False	**23.** True
3. False	**10.** False	**17.** False	**24.** False
4. False	**11.** True	**18.** True	**25.** True
5. True	**12.** False	**19.** False	**26.** False
6. False	**13.** True	**20.** False	**27.** True
7. True	**14.** False	**21.** False	**28.** False

29. True

30. True

31. Possible answer: $y = \frac{1}{2}(\log x - \log 3)$

32. Possible answer: $y = 50.3(1.062)^t$

33. Possible answer: $y = 100e^{-0.026t}$

34. Possible answer: $F(t) = 3.3(0.989)^t$, t = number of years since 1966, $F(t)$ in millions

35. Possible answer: $y = 100(1.20)^x$

36. False **37.** False **38.** True **39.** True

40. True

Ch. 6 Review: Putting It All Together

1. a. $100 = 50 \cdot 1.16^t \Rightarrow 2 = 1.16^t \Rightarrow \log 2 = t \log 1.16 \Rightarrow (\log 2)/(\log 1.16) = t \Rightarrow t \approx 0.301/0.064 \approx 4.7$ months (or about 141 days) for the quantity to double.

b. $100 = 200 \cdot 0.92^t \Rightarrow 0.5 = 0.92^t \Rightarrow \log 0.5 = t \log 0.92 \Rightarrow (\log 0.5)/(\log 0.92) = t \Rightarrow t \approx (-0.301)/(-0.036) \approx 8.4$ months (or about 250 days) for the quantity to halve.

3. Expressions (a) and (e) are equivalent and (b), (c), and (f) are equivalent. Expression (d) does not match any other.

5. a. After one time period (20 minutes), $A(1) = 325 \cdot (0.5)^1 = 162.5$ mg (or half the original amount). After two time periods (40 minutes), $A(2) = 325 \cdot (0.5)^2 = 81.25$ mg (or one-quarter of the original amount).

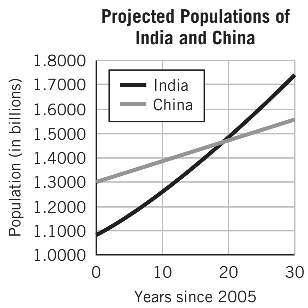
b. Using the graph, it appears that after about 1.7 time periods (34 minutes) $A(t) = 100$. Using the equation, we have $100 = 325 \cdot (0.5)^t \Rightarrow 100/325 = (0.5)^t \Rightarrow 0.3077 \approx (0.5)^t \Rightarrow \log(0.3077) \approx t \log(0.5) \Rightarrow t \approx \log(0.3077)/\log(0.5) \Rightarrow t \approx 1.70$ time periods, or $1.7 \cdot 20 = 34$ minutes. So our estimate was accurate.

c. $B(t) = 81 \cdot (0.5)^t$. So $A(t)$ and $B(t)$ have different initial amounts, but the decay rate is the same.

7. a. Density for India $= (1.08 \cdot 10^9)/(1.2 \cdot 10^6) = 0.9 \cdot 10^3 = 900$ people per square mile. Density for China $= (1.30 \cdot 10^9)/(3.7 \cdot 10^6) \approx 0.35 \cdot 10^3 = 350$ people per square mile. So India's population density is about $900/350 \approx 2.6$ times larger than China's.

b. $C(x) = 1.30(1.006)^x$ and $I(x) = 1.08(1.016)^x$, where $C(x)$ and $I(x)$ are in billions and x = years since 2005.

c.



Note that India's exponential growth rate is so much higher than that for China, the graph for the Chinese population appears almost linear in comparison.

d. The projected populations are the same roughly 18 years after 2005, or in 2023. Using the models, we need the point at which $C(x) = I(x) \Rightarrow 1.30(1.006)^x = 1.08(1.016)^x \Rightarrow 1.204(1.006)^x \approx (1.016)^x \Rightarrow \log(1.204) + x\log(1.006) \approx x\log(1.016) \Rightarrow \log(1.204) \approx x[\log(1.016) - \log(1.006)] \Rightarrow 0.081 \approx x\log[(1.016)/(1.006)] \Rightarrow 0.081 \approx 0.0043x \Rightarrow x \approx 18.8$ years after 2005, that is, in late 2023 our model predicts the two populations will be the same. Evaluating $C(18.8)$, we get $1.30(1.006)^{18.8} \approx 1.455$ billion people. (To double-check you could calculate $I(18.8) \approx 1.456$ billion, with the difference due to rounding.)

9. a. $A(x) = 10{,}000(1.07)^x$, where $A(x)$ is the dollar amount after x years.

b. $A(18) = 10{,}000(1.07)^{18} \approx \$33{,}799$

c. $10{,}000(1.03)^{18} \approx \$17{,}024$, the difference $= \$33{,}799 - \$17{,}024 = \$16{,}775$

d. Investing \$10,000 at 4% for 18 years gives $\$10{,}000(1.04)^{18} \approx \$20{,}258$, which is not equivalent to the result in part (c). However it does represent the amount in the account after 18 years in current dollars (dollars that have been adjusted for inflation).

11. a. $t = \log(2.3) \approx 0.362$

b. $t^2 \cdot 4 = 10^2 \Rightarrow t^2 = 25 \Rightarrow t = 5$ (*Note:* $t = -5$ is not a solution here, since $\log(-5)$ is not defined.)

c. $2 = e^{0.03t} \Rightarrow \ln 2 = 0.03t \Rightarrow t = (\ln 2)/0.03 \approx 0.693/0.03 \approx 23.1$

d. $\ln[(2t-5)/(t-1)] = 0 \Rightarrow (2t-5)/(t-1) = 1 \Rightarrow 2t - 5 = t - 1 \Rightarrow t = 4$

13. a. $1.5 = e^{\ln 1.5} \approx e^{0.405}$

b. $0.7 = e^{\ln 0.7} \approx e^{-0.357}$

c. $1 = e^0$

15. a. Matches Graph *C*

b. Matches Graph *D*

c, d. Both match Graphs *A* and *B* since $y = 100(10)^x \Rightarrow \log y = \log 100 + x \log 10 \Rightarrow \log y = 2 + x$.

17. a. Matches Graph *B* since when $x = 1$, $2 \ln x = 2 \ln 1 = 2 \cdot 0 = 0$. So the horizontal intercept is (1, 0).

b. Matches Graph *A* since when $x = 1$, $2 + \ln x = 2 + \ln 1 = 2 + 0 = 2$. So the graph passes through (1, 2).

c. Matches Graph D since when $x = 0$, $\ln(x + 2) = \ln 2 \approx 0.693 > 0$. So the vertical intercept is above the origin at approximately (0, 0.693).

d. Matches Graph B since the functions $y_1 = 2 \ln x$ and $y_4 = \ln(x^2)$ are the same.

(*Note:* There is no match to Graph *C*.)

19. a. Using the table in the text, five orders of magnitude.

b. Using the function definition, if $30 = 10 \log(I_{30}/I_0)$, where I_{30} = intensity level corresponding to 30 decibels, and $80 = 10 \log(I_{80}/I_0)$, where I_{80} = intensity level corresponding to 80 decibels. Subtracting the two equations gives:

$$80 - 30 = 10 \log(I_{80}/I_0) - 10 \log(I_{30}/I_0) \Rightarrow$$
$$50 = 10 \log[(I_{80}/I_0)/(I_{30}/I_0)] \Rightarrow$$
$$5 = \log[(I_{80}/I_0) \cdot (I_0/I_{30})] \Rightarrow 5 = \log[(I_{80}/I_{30})] \Rightarrow$$
$$10^5 = I_{80}/I_{30},$$

so I_{80} is five orders of magnitude larger than I_{30}.

21. If $R = 6.9$, we have $6.9 = \log(A/A_0) \Rightarrow 10^{6.9} = A/A_0$, so the earthquake's amplitude was 6.9 orders of magnitude (or almost 10 million times) larger than that of the base amplitude, A_0.

23. a. The data appear to be exactly linear, and two estimated points on the best-fit line are (0, 0.3) and (10, 5). The slope $= (5 - 0.3)/(10 - 0) = 0.47$. The equation is then $Y = 0.3 + 0.47x$.

b. Substituting $\log y$ for Y gives $\log y = 0.3 + 0.47x \Rightarrow y = 10^{0.3+0.47x} \Rightarrow y = 10^{0.3}10^{0.47x} \approx 2 \cdot 3^x$.

c. The function is exponential, suggesting that the graph of exponential functions is a straight line on a semi-log plot (where the log scale is on the vertical axis).

CHAPTER 7

Section 7.1

Algebra Aerobics 7.1

1. a. $S(r) = 4\pi r^2$; $S(2r) = 4\pi(2r)^2 = 4 \cdot (4\pi r^2) = 4S(r)$; $S(3r) = 4\pi(3r)^2 = 9 \cdot (4\pi r^2) = 9S(r)$

b. When the radius is doubled, the surface area is multiplied by 4. When the radius is tripled, the surface area is multiplied by 9.

2. a. $V(r) = (4/3)\pi r^3$; $V(2r) = (4/3)\pi(2r)^3 = 8 \cdot ((4/3)\pi r^3) = 8V(r)$; $V(3r) = (4/3)\pi(3r)^3 = 27 \cdot ((4/3)\pi r^3) = 27V(r)$

b. When the radius is doubled, the volume is multiplied by 8. When the radius is tripled, the volume is multiplied by 27.

3. The volume grows faster than the surface area, so as the radius of a sphere increases, the ratio of (surface area)/volume decreases.

4. a. $C(r) = 2\pi r$; $C(2r) = 2\pi(2r) = 2(2\pi r) = 2C(r)$; $C(4r) = 2\pi(4r) = 4(2\pi r) = 4C(r)$. So when the radius is doubled, the circumference is multiplied by 2. When the radius quadruples, the circumference is multiplied by 4.

b. $A(r) = \pi r^2$; $A(2r) = \pi(2r)^2 = 4(\pi r^2) = 4A(r)$; $A(4r) = \pi(4r)^2 = 16(\pi r^2) = 16A(r)$. So when the radius is doubled, the area is multiplied by 4. When the radius is quadrupled, the area is multiplied by 16.

c. The area grows faster than the circumference. So as the radius of the circle increases, the ratio of circumference/area decreases.

5. a. The volume of the sphere is equal to the volume of the cube when $\frac{4}{3}\pi r^3 = r^3$, which is true only if $r = 0$.

b. The volume of the cube is greater than the volume of the sphere if $r^3 > \frac{4}{3}\pi r^3 \Rightarrow 1 > \frac{4}{3}\pi$, which is never true. The volume of the cube is less than the volume of the sphere for all $r > 0$ since $r^3 < \frac{4}{3}\pi r^3 \Rightarrow 1 < \frac{4}{3}\pi$.

6. No, since if the radius is doubled from 5 to 10, the volume is increased by a factor of 4. That is, given $V = \pi r^2 h$, if $r = 5$ then $V = \pi 5^2 \cdot 25 = 625\pi$ cubic feet and if $r = 10$ then $V = \pi 10^2 \cdot 25 = 2500\pi$ cubic feet, which is a factor of 4 larger.

7. Yes; since $V = \pi r^2 h$, if the height is doubled, the volume is doubled. That is, for this example, if $r = 5$, then $V = \pi r^2 h \Rightarrow V = \pi 12^2 \cdot 5 = 720\pi$ cubic feet and if $r = 10$, then $V = \pi 12^2 \cdot 10 = 1440\pi$ cubic feet, which is twice the volume.

8. a. i. The area *B* of the cylinder base $= \pi r^2 \Rightarrow V = (\pi r^2)h = \pi r^2 h$

ii. The area *B* of the triangular prism base $= \frac{1}{2}ab \Rightarrow V = \left(\frac{1}{2}ab\right)h = \frac{1}{2}abh$

b. If the height is doubled, the volume is doubled since $(\pi r^2)2h = 2(\pi r^2)h = 2V$; $\left(\frac{1}{2}ab\right)2h = 2\left(\frac{1}{2}ab\right)h = 2V$

c. If all dimensions are doubled, the volume is increased by a factor of 8 since $(\pi(2r)^2) \cdot 2h = (\pi 4r^2)2h = 8(\pi r^2)h = 8V$; $\left(\frac{1}{2}(2a)(2b)\right)(2h) = 2^3 \cdot \left(\frac{1}{2}abh\right) = 8V$

Exercises for Section 7.1

1. a. $l = \frac{V}{wh}$ **c.** $w = \frac{P - 2l}{2}$

b. $b = \frac{2A}{h}$ **d.** $h = \frac{S - 2x^2}{4x}$

3. a. $r/2$; increasing; as the radius increases, the area increases faster than the circumference.

b. $\frac{4}{3}r$; increasing; as the radius increases, the volume of the sphere increases faster than the area of its radial cross-section.

c. $\frac{2}{3}r^2$; increasing; as the radius increases, the volume of the sphere increases faster than the length of any of its great circles.

5. a. $S \approx 1.26 \cdot 10^{-19}$ m^2 **c.** $S/V \approx 3.0 \cdot 10^{10}$ m^{-1}

b. $V \approx 4.19 \cdot 10^{-30}$ m^3 **d.** ratio $= 3/r$; decreases

7. a. S is multiplied by 16; V is multiplied by 64.

b. S is multiplied by n^2; V is multiplied by n^3.

c. S is divided by 9; V is divided by 27.

d. S is divided by n^2; V is divided by n^3.

9. a. $V = 2632.5$ cm^3

b. quadrupled

c. Quadruple either the length or the width or double both.

11. a. The volume doubles if the height is doubled.

b. The volume is multiplied by 4 if the radius is doubled.

13. a. $V = 3\pi r^3$; $S = 8\pi r^2$

b. Volume eventually grows faster since $V/S = 3r/8$; as $r \to +\infty$, V/S increases without bound.

15. The ratio of area to circumference is $r/2$ and its graph is labeled $h(x)$; the inverse ratio is $2/r$ and its graph is labeled $j(x)$.

Section 7.2

Algebra Aerobics 7.2

1.

	Power Function	Independent Variable	Dependent Variable	Constant of Proportionality	Power
a.	yes	r	A	π	2
b.	yes	z	y	1	5
c.	no	—	—	—	—
d.	no	—	—	—	—
e.	yes	x	y	3	5

2. a. y is directly proportional to x^2.

b. y is not directly proportional to x^2.

c. y is not directly proportional to x.

3. $g(x) = 5x^3$

a. $g(2) = 5(2)^3 = 5(8) = 40$ $g(4) = 5(4)^3 = 5(64) = 320$ So $g(4)$ is eight times larger than $g(2)$

b. $g(5) = 5(5)^3 = 5(125) = 625$ $g(10) = 5(10)^3 = 5(1000) = 5000$ So $g(10)$ is eight times larger than $g(5)$

c. $g(2x) = 5(2x)^3 = 5(8x^3) = 8(5x^3) = 8g(x)$ So $g(2x)$ is eight times larger than $g(x)$

d. $g\left(\frac{1}{2}x\right) = 5\left(\frac{1}{2}x\right)^3 = 5\left(\frac{1}{8}x^3\right) = \frac{1}{8}(5x^3) = \frac{1}{8}g(x)$ So $g\left(\frac{1}{2}x\right)$ is one-eighth the size of $g(x)$

4. a. $h(2) = 0.5(2)^2 = 2$ and $h(6) = 0.5(6)^2 = 18$, an increase by a factor of 9.

b. $h(5) = 0.5(5)^2 = 12.5$ and $h(15) = 0.5(15)^2 = 112.5$, an increase by a factor of 9.

c. $h(x)$ will increase by a factor of 9 since $h(3x) = 0.5(3x)^2 = 9 \cdot [0.5x^2] = 9 \cdot h(x)$.

d. $h(x)$ will decrease by a factor of 9 since $h\left(\frac{1}{3}x\right) = 0.5\left(\frac{1}{3}x\right)^2 = \frac{1}{9} \cdot [0.5x^2] = \frac{1}{9} \cdot h(x)$.

5. a. $V = k \cdot r^3$

b. $V = k \cdot l \cdot w \cdot h$

c. $f = k \cdot c$

6. a. y is equal to 3 times the fifth power of x.

b. y is equal to 2.5 times the cube of x.

c. y is equal to one-fourth of the fifth power of x.

7. a. y is directly proportional to x^5 with a proportionality constant of 3.

b. y is directly proportional to x^3 with a proportionality constant of 2.5.

c. y is directly proportional to x^5 with a proportionality constant of 1/4.

8. a. $P = aR^2 \Rightarrow \frac{P}{a} = \frac{aR^2}{a} \Rightarrow \frac{P}{a} = R^2 \Rightarrow \sqrt{\frac{P}{a}} = \sqrt{R^2} \Rightarrow R = \sqrt{\frac{P}{a}}$

b. $V = \left(\frac{1}{3}\right)\pi r^2 h \Rightarrow 3(V) = 3\left(\frac{1}{3}\right)\pi r^2 h \Rightarrow 3V = \pi r^2 h \Rightarrow \frac{3V}{\pi r^2} = \frac{\pi r^2 h}{\pi r^2} \Rightarrow h = \frac{3V}{\pi r^2}$

9. a. $f(2x) = 0.1(2x)^3 = 0.8x^3$; $2f(x) = 2(0.1x^3) = 0.2x^3$

b. $f(3x) = 0.1(3x)^3 = 2.7x^3$; $3f(x) = 3(0.1x^3) = 0.3x^3$

c. $f(2 \cdot 5) = 0.1(10)^3 = 100$; $2f(5) = 2(0.1 \cdot 5^3) = 25$

Exercises for Section 7.2

1. a. $f(2) = 20$; $f(-2) = 20$ **c.** $h(2) = -20$; $h(-2) = -20$

b. $g(2) = 40$; $g(-2) = -40$ **d.** $k(2) = -40$; $k(-2) = 40$

3. a. $k = 1/2$; $P = 288$ **c.** $k = 4\pi$; $T = 1728\pi$

b. $k = 14$; $M = 84$ **d.** $k = \sqrt[3]{18}$; $N = 6$

Ch. 7

5. a. $y = \frac{1}{5}x$; 4

b. $p = 6\sqrt{s}$; 24

c. $A = \frac{4}{3}\pi r^3$; 64π

d. $P = \frac{1}{2}m^2$; quartered, i.e., P is divided by 4.

7. a. $Y = kX^3$

b. $k = 1.25$

c. Increased by a factor of 125

d. Divided by 8

e. $X = \sqrt[3]{\frac{Y}{k}}$. So X is not directly proportional to Y. It is directly proportional to $\sqrt[3]{Y}$.

9. a. L is multiplied by 32. **b.** M is multiplied by 2^p.

11. In all the formulas below, k is the constant of proportionality.

a. $x = k \cdot y \cdot z^2$ **c.** $w = k \cdot x^2 \cdot y^{1/3}$

b. $V = k \cdot l \cdot w \cdot h$ **d.** $V = k \cdot h \cdot r^2$

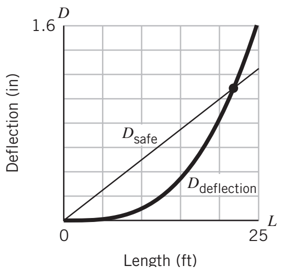
13. a. $C = kAt$

b. $k = 0.06$ dollars per sq. ft, per in.

c. The area of the four walls (without the door) = $2(15 \cdot 8) + 2(20 \cdot 8) = 240 + 320 = 560$ sq. ft. The area of the door is $3 \cdot 7 = 21$ sq. ft. Thus the total wall area = $560 - 21 = 539$ sq. ft.

d. \$129.36; \$194.04

15. a. and **b.**



The graphs of $D_{deflection}$ and D_{safe} (with the deflections measured in inches and the plank length L measured in feet) are given in the accompanying diagram.

c. The safety deflections are well above the actual deflections for all values of L between 0 and 20. It would cease to be safe if the plank were longer than about 22 ft. (This is the L value where D_{safe} and $D_{deflection}$ meet.)

17. In all the formulas below, k is the constant of proportionality.

a. $d = k \cdot t^2$ **d.** $R = k \cdot [O_2] \cdot [NO]^2$

b. $E = k \cdot m \cdot c^2$ **e.** $v = k \cdot r^2$

c. $A = k \cdot b \cdot h$

19. a. V is quadrupled; is multiplied by 9

b. V is doubled; is tripled

c. V is multiplied by n^2

d. V is multiplied by n

21. a. $h(2) = 2$; $h(6) = 18$; the latter is nine times the former.

b. $h(5) = 12.5$; $h(15) = 112.5$; the latter is nine times the former.

c. h's value is multiplied by 9.

d. h's value is divided by 9.

Section 7.3

Algebra Aerobics 7.3

1. a. $f(2) = 4(2)^3 = 4(8) = 32$

b. $f(-2) = 4(-2)^3 = 4(-8) = -32$

c. $f(s) = 4s^3$

d. $f(3s) = 4(3s)^3 = 4(27)s^3 = 108s^3$

2. a. $g(2) = -4(2)^3 = -4(8) = -32$

b. $g(-2) = -4(-2)^3 = -4(-8) = 32$

c. $g\left(\frac{1}{2}t\right) = -4\left(\frac{1}{2}t\right)^3 = -4\left(\frac{1}{8}\right)t^3 = -\frac{1}{2}t^3$

d. $g(5t) = -4(5t)^3 = -4(125)t^3 = -500t^3$

3. a. $f(4) = 3(4)^2 = 48$

b. $f(-4) = 3(-4)^2 = 48$

c. $f(s) = 3s^2$

d. $2f(s) = 2(3s^2) = 6s^2$

e. $f(2s) = 3(2s)^2 = 12s^2$

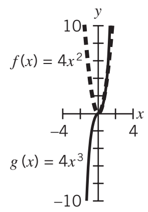
f. $f(3s) = 3(3s)^2 = 27s^2$

g. $f\left(\frac{s}{2}\right) = 3\left(\frac{s}{2}\right)^2 = \frac{3}{4}s^2$

h. $f\left(\frac{s}{4}\right) = 3\left(\frac{s}{4}\right)^2 = \frac{3}{16}s^2$

4. a.

x	$f(x) = 4x^2$	$g(x) = 4x^3$
−4	64	−256
−2	16	−32
0	0	0
2	16	32
4	64	256



b. As $x \to +\infty$, both $f(x)$ and $g(x) \to +\infty$.

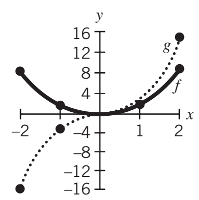
c. As $x \to -\infty$, $f(x) \to +\infty$ but $g(x) \to -\infty$.

d. The domain of both functions is all the real numbers, and the range of g is also all the real numbers; however, the range of f is only all the nonnegative real numbers.

e. They intersect at the origin and at the point (1, 4).

f. All values greater than $x = 1$.

5.

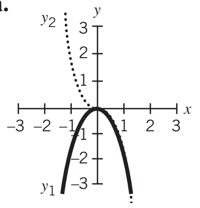


a. $f(x) = g(x)$ for $x = 0$ and $x = 1$.

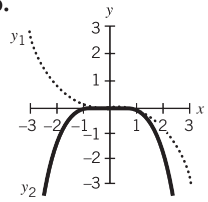
b. $f(x) > g(x)$ for $0 < x < 1$ and for $x < 0$

c. $f(x) < g(x)$ for $x > 1$

6. **a.**



b.



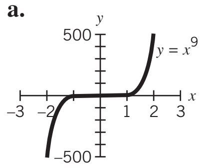
7. a. The graphs are similar; both have similar end behavior; they intersect at the origin; for positive values of x, $y_1 < y_2$; for negative values of x, $y_1 > y_2$.

b. The graphs are similar; both have similar end behavior; they intersect at the origin; for all nonzero values of x, $y_1 > y_2$.

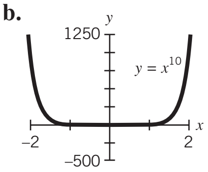
8. a. p is even and a is positive; $a = 3$ since $(1, 3)$ is on the graph of the function.

b. p is odd and a is negative; $a = -2$ since $(1, -2)$ is on the graph of the function.

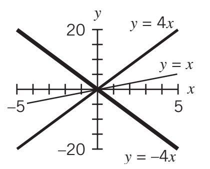
9. **a.**



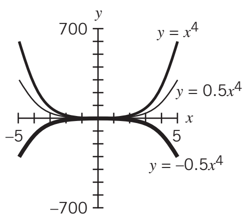
b.



10. a. $y = x$ and $y = 4x$ and $y = -4x$:

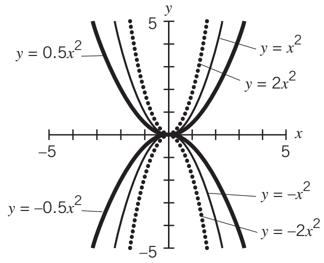


b. $y = x^4$ and $y = 0.5x^4$ and $y = -0.5x^4$



Exercises for Section 7.3

1. The graphs of the six functions are labeled in the accompanying diagram. They are all parabolas since they are graphs of functions of the form $y = ax^2$, for various values of a. The differences are due to the value of a. If $a > 0$, then the graph is concave up. If $a < 0$, then the graph is concave down. The larger the absolute value of a, the narrower the opening of the graph.



3. f goes with Graph C; g goes with Graph A; h goes with Graph D; j goes with Graph B. The graphs in B and D are mirror images of each other across the x-axis; the graph in D is steeper than that in C, the graph in A is flatter than those in C and D for $-1 < x < +1$, but the Graph in A is steeper as $x \to \pm\infty$.

5. a. $g(x) = 6 \cdot x^4$

b. $h(x) = 0.5 \cdot x^4$

c. $j(x) = -2x^4$

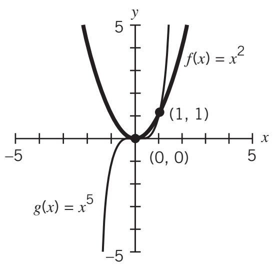
7. a. $f(0) = g(0) = 0$

b. If $0 < x < 1$, then $f(x) > g(x)$

c. $f(1) = g(1) = 1$

d. If $x > 1$, then $f(x) < g(x)$

e. If $x < 0$, then $f(x) > g(x)$



9. a. n is even

b. $k < 0$

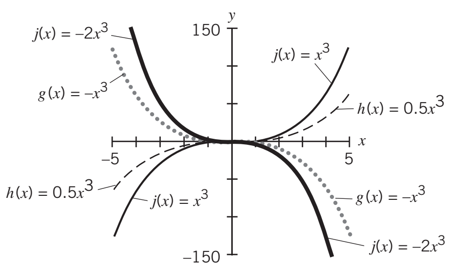
c. Yes, $f(-2) = f(2)$

d. Yes, $f(-x) = f(x)$

e. $-\infty$

f. $-\infty$

11.



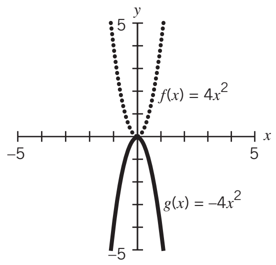
a. The constant of proportionality for f is 1; for g it is –1; for h it is 1/2; for j it is –2.

b. g's graph is the reflection of f's graph across the x-axis.

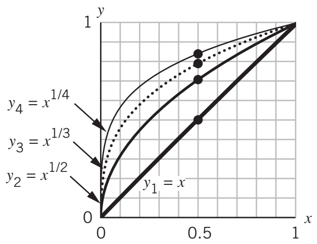
c. j's graph is both a stretch and a reflection of f's graph across the x-axis.

d. h's graph is a compression of f's graph.

13. $g(x) = -4x^2$. Its graph is a reflection of the graph of $f(x)$ across the x-axis.

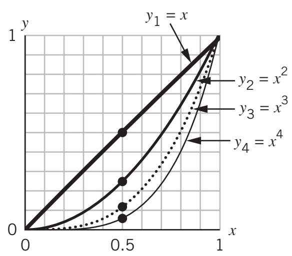


15. a. The graphs for the four functions are given in the diagram. All go through (0, 0) and (1, 1). The smaller the fractional power, the bigger the y value when $0 < x < 1$.



b. The graphs for the four functions are given in the diagram. All graphs go through (0, 0) and (1, 1). The higher the power of x, the smaller the y value when $0 < x < 1$.

Notice that over the interval [0, 1] the powers ≤ 1 all have y values above or on the graph of $y = x$, but the powers ≥ 1 all would have y values below or on the graph of $y = x$.

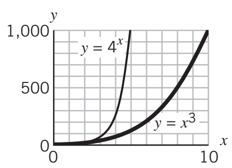


Section 7.4

Algebra Aerobics 7.4

1. a.

x	$y = 4^x$	$y = x^3$
0	1	0
1	4	1
2	16	8
3	64	27
4	256	64
5	1024	125



b. $y = 4^x$ dominates $y = x^3$.

In this case, $4^x > x^3$ for all values of x, but as $x \to +\infty$, the values grow farther and farther apart.

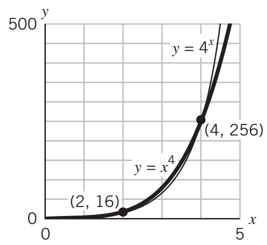
2. Graphs A and D are likely to be power functions; Graphs B and C are likely to be exponential functions.

3. a. $y = 2^x$ eventually dominates.

b. $y = (1.000\,005)^x$ eventually dominates.

Exercises for Section 7.4

1. a. The two graphs intersect at (2, 16) and (4, 256). Thus, the functions are equal for $x = 2$ and $x = 4$. (See the accompanying figure.)



b. As x increases, both functions grow. For $0 \leq x < 2$ we have that $x^4 < 4^x$; at $x = 2$ both have a y value of 16. From $2 < x < 4$, we have $4^x < x^4$. For $x = 4$ they are again equal. For $x > 4$ we have that $x^4 < 4^x$. As x keeps on increasing 4^x will continue to grow faster than x^4.

c. The graph of $y = 4^x$ dominates.

3. a. $f(x) > g(x)$ as $x \to -\infty$

b. $f(x) < g(x)$ as $x \to +\infty$

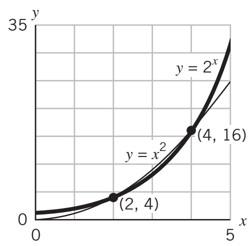
c. $f(x) > g(x)$ for x in $(-\infty, -1)$

d. $f(x) < g(x)$ for x in $(-0.5, 1)$

e. $f(x) > g(x)$ for x in $(1, 6)$

f. $g(x) > f(x)$ for x in $(7, +\infty)$

5. Graphs of $y = 2^x$ and $y = x^2$



Graphs of $y = 3^x$ and $y = x^3$

In the top figure, the graphs of $y = 2^x$ and $y = x^2$ intersect at $x = 2$ and $x = 4$. The graph of $y = 2^x$ lies above the graph of $y = x^2$ for $0 \le x < 2$ and for $x > 4$.

In the bottom figure, the graphs of $y = 3^x$ and $y = x^3$ intersect at $x = 3$ and at $x \approx 2.478$. The latter intersection point can only be seen if we use a graphing device and zoom in. The graph of $y = 3^x$ is above the graph of $y = x^3$ over the intervals $(-\infty, 2.478)$ and $(3, +\infty)$. It is below that graph over the interval $(2.478, 3)$.

7. $3 \cdot 2^x > 3 \cdot x^2$ if $0 < x < 2$ and if $x > 4$. Also, $3 \cdot 2^x < 3 \cdot x^2$ if $2 < x < 4$.

9. **a.** $h(x)$ goes with C

b. $i(x)$ goes with A

c. $j(x)$ goes with B

11. The functions $f(ax)$ and $g(ax)$ are the functions $f(x)$ and $g(x)$, respectively, where the input x has been multiplied by a. Since $f(ax) = (ax)^n = a^n \cdot f(x)$, then $f(ax)$ is $f(x)$ multiplied by a^n. Since $g(ax) = b^{ax} = (b^x)^a = [g(x)]^a$, then $g(ax)$ is $g(x)$ raised to the ath power.

13. Linear function: $m = \frac{32 - 0.5}{4 - 1} = 10.5 \Rightarrow y = 10.5x - 10$.

Exponential function: $y = \frac{1}{8} \cdot 4^x$

Power function: $y = 0.5x^3$

15. **a.** True **c.** False

b. True **d.** False

Section 7.5

Algebra Aerobics 7.5

1. **a.** x is directly proportional to y and z.

b. y is inversely proportional to the square of x

c. D is directly proportional to the square root of y and inversely proportional to the cube of z.

2. (a) and (d) represent direct proportionality; (b) and (c) represent inverse proportionality; (e) does not represent direct or inverse proportionality. It is an exponential function where x, the input, becomes an exponent in the output.

3. The volume will increase by a factor of 4, from 1 ft^3 to 4 ft^3. (See Table 7.6. in Section 7.5.)

4. **a.** **i.** $\frac{15}{x^3}$ **ii.** $\frac{-10}{x^4}$ **iii.** $\frac{3.6}{x}$ **iv.** $\frac{2}{x^2y^3}$

b. **i.** $1.5x^{-2}$ **ii.** $-6x^{-3}$ **iii.** $-\frac{2}{3}x^{-2}$ **iv.** $6x^{-3}y^{-4}z^{-1}$

5. $t = kd^{-2}$ or $t = \frac{k}{d^2}$ for some constant k

6. Intensity at 4 ft is $\frac{k}{4^2} = \frac{1}{16}k$, but at 2 ft intensity is $\frac{k}{(2)^2} = \frac{1}{4}k$. The light is four times as bright at a distance of 2 ft away than it is at 4 ft away.

7. $g(x) = \frac{3}{x^4}$

a. $g(2x) = \frac{3}{(2x)^4} = \frac{3}{16x^4} = \frac{1}{16}\left(\frac{3}{x^4}\right)$, so $g(2x) = \frac{1}{16}g(x)$.

b. $g\left(\frac{x}{2}\right) = \frac{3}{(x/2)^4} = 3 \div \left(\frac{x}{2}\right)^4 = 3 \div \frac{x^4}{16} = 3 \cdot \frac{16}{x^4}$
$= 16\left(\frac{3}{x^4}\right)$, so $g\left(\frac{x}{2}\right) = 16(g(x))$.

8. $h(x) = -\frac{2}{x^3}$

a. $h(3x) = -\frac{2}{(3x)^3} = -\frac{2}{27x^3} = \frac{1}{27} \cdot \left(-\frac{2}{x^3}\right) = \frac{1}{27} \cdot h(x)$, or one-twenty-seventh of the original amount.

b. $h\left(\frac{x}{3}\right) = -\frac{2}{(x/3)^3} = -\frac{2}{x^3/27} = -2 \cdot \frac{27}{x^3} = 27 \cdot \left(-\frac{2}{x^3}\right) =$ $27h(x)$, or 27 times the original amount.

9. **a.** $x = kyz^2$ **b.** $a = \frac{kbc^3}{d}$ **c.** $a = \frac{kb^2\sqrt{c}}{de}$

10. **a.** $x = \frac{k}{\sqrt{y}}$; If $x = 3$ and $y = 4$, then $3 = \frac{k}{\sqrt{4}} \Rightarrow 3 = \frac{k}{2} \Rightarrow$ $k = 6$, so $x = \frac{6}{\sqrt{y}}$.

So if $y = 400$, then $x = \frac{6}{\sqrt{400}} = \frac{6}{20} = 0.3$.

b. $x = k \cdot y \cdot z$; If $x = 3$, $y = 4$ and $z = 0.5$, then $3 = k \cdot 4 \cdot (0.5) \Rightarrow k = 1.5$, so $x = 1.5 \cdot y \cdot z$.
So if $x = 6$ and $z = 2$, then $6 = 1.5y(2)$ or $y = 2$.

c. $x = \frac{ky^2}{\sqrt[3]{z}}$; If $x = 6$, $y = 6$ and $z = 8$, then
$6 = \frac{k(6)^2}{\sqrt[3]{8}} \Rightarrow 6 = \frac{36k}{2} \Rightarrow k = \frac{1}{3}$ so $x = \frac{1}{3}\frac{y^2}{\sqrt[3]{z}}$.

So if $y = 9$ and $z = 0.027$, then
$x = \frac{1}{3}\frac{9^2}{\sqrt[3]{0.027}} = \frac{1}{3} \cdot \frac{81}{0.3} = 90$.

Exercises for Section 7.5

1. **a.** y changes from $\frac{4}{1^3} = 4$ to $\frac{4}{4^3} = \frac{1}{16}$

b. decreases

c. y changes from $\frac{4}{2^3} = \frac{1}{2}$ to $\frac{4}{(1/2)^3} = 32$

d. increases

e. y is inversely proportional to x^3 and 4 is the constant of proportionality.

3. **a.** $h(2x) = \frac{k}{8x^3}$, $h(-3x) = \frac{k}{-27x^3}$ and $h(x/3) = \frac{27k}{x^3}$

b. $j(2x) = \frac{-k}{16x^4}$, $j(-3x) = \frac{-k}{81x^4}$ and $j(x/3) = \frac{-81k}{x^4}$

5. **a.** $P = \frac{0.016}{f}$; 0.0016 **c.** $S = \frac{16}{wp}$; 2

b. $Q = \frac{54}{r^2}$; 2/3 **d.** $W = \frac{2}{3\sqrt{u}}$; 1/6

7. **a.** B's value is divided by 16.

b. Z's value is divided by 2^p.

9. Given: $I(x) = k/x^2$ and $4 = I(6) = k/36$; thus $k = 144 \Rightarrow$ $I(8) = 144/64 = 2.25$ watts per square meter and $I(100) = 144/10000 = 0.0144$ watt per square meter.

11. $I(d) = k/d^2$ and thus $I(4)/I(7) = 49/16 = 3.06$. The light intensity will be more than 3 times as great.

13. **a.** The volume becomes 1/3 of what it was.

b. The volume becomes $1/n$ of what it was.

c. The volume is doubled.

d. The volume becomes n times what it was.

15. a. $x = k \cdot \frac{y}{z}$

b. Solving $4 = k \cdot \frac{16}{32}$ for k gives $k = 8$.

c. $x = 8 \cdot 25/5 = 40$

17. a. Let L = wavelength of a wave and t = time between waves. Using the fact that the speed of the waves is directly proportional to the square root of the wave's length, we then have $t = \frac{L}{k \cdot \sqrt{L}} = \frac{\sqrt{L}}{k}$ and thus time is directly proportional to the square root of the wavelength of the wave.

b. If the frequency of the waves on the second day is twice that of the first, then the waves will be four times as far apart as they were on the previous day.

19. a. $v = \frac{k}{l}$

b. 24 in.

c. decrease

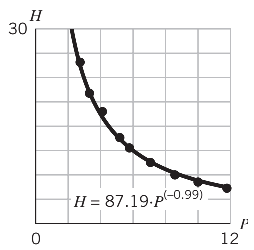
d. decrease

e. It was doubled in length.

21. a. $H = k/P = k \cdot P^{-1}$, where k is a proportionality constant.

b. The software gave $H = 87.19 \cdot P^{-0.99}$, which is very close.

Note that the fit is quite good, as can be seen from the graph of the data and the best-fit graph as given in the accompanying diagram.



Section 7.6

Algebra Aerobics 7.6

1. a. $\frac{1}{x^2}$; $\frac{1}{(-2)^2} = \frac{1}{4}$ or 0.25

b. $\frac{1}{x^3}$; $\frac{1}{(-2)^3} = -\frac{1}{8}$ or -0.125

c. $\frac{4}{x^3}$; $\frac{4}{(-2)^3} = -\frac{1}{2}$ or -0.5

d. $\frac{-4}{x^3}$; $\frac{-4}{(-2)^3} = \frac{-4}{-8} = \frac{1}{2}$ or 0.5

e. $\frac{1}{2x^4}$; $\frac{1}{2(-2)^4} = \frac{1}{32} \approx 0.031$

f. $\frac{-2}{x^3}$; $\frac{-2}{(-2)^3} = \frac{1}{4} = 0.25$

g. $\frac{-2}{x^4}$; $\frac{-2}{(-2)^4} = -\frac{1}{8} = -0.125$

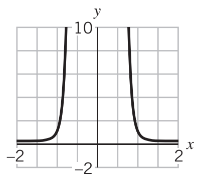
h. $\frac{2}{x^4}$; $\frac{2}{(-2)^4} = \frac{1}{8} = 0.125$

2. a. Graph A: $y = \frac{a}{x^2}$ Graph B: $y = ax^3$

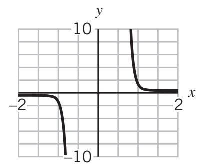
b. Since the point (1, 4) lies on Graph A, $4 = \frac{a}{1^2} \Rightarrow a = 4 \Rightarrow y = \frac{4}{x^2}$.

Since the point (2, 4) lies on Graph B, $4 = a(2)^3 \Rightarrow a = 0.5 \Rightarrow y = 0.5x^3$

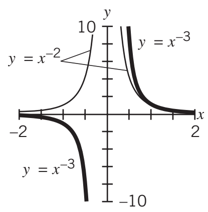
3. a. $y = x^{-10}$



b. $y = x^{-11}$



4. a. $y = x^{-2}$ and $y = x^{-3}$

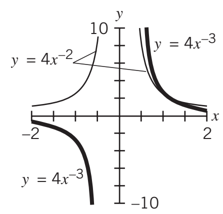


The graphs intersect at (1, 1). As $x \to +\infty$ or $\to -\infty$, both graphs approach the x-axis, but $y = x^{-3}$ is closer to the x-axis at each point after $x = 1$ and before $x = -1$

When $x > 0$, as $x \to 0$, both graphs approach $+\infty$, but $y = x^{-3}$ is steeper.

When $x < 0$, as $x \to 0$, the graph of $y = x^{-2}$ approaches $+\infty$, and the graph of $y = x^{-3}$ approaches $-\infty$.

b. $y = 4x^{-2}$ and $y = 4x^{-3}$



The graphs intersect at (1, 4). As $x \to +\infty$ or $x \to -\infty$, both graphs approach the x-axis, but $y = 4x^{-3}$ is closer to the x-axis after $x = 1$ and before $x = -1$ than $y = 4x^{-2}$.

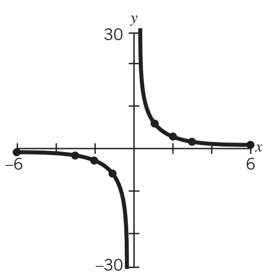
When $x > 0$, as $x \to 0$, both graphs approach $+\infty$.

When $x < 0$, as $x \to 0$, the graph of $y = 4x^{-2}$ approaches $+\infty$, and the graph of $y = 4x^{-3}$ approaches $-\infty$.

5. Graph A matches $f(x)$; Graph B matches $g(x)$; Graph C matches $h(x)$.

Exercises for Section 7.6

1. A table for r is given below.



x	$R(x)$
1	6
2	3
3	2
6	1
−1	−6
−2	−3
−3	−2
−6	−1

Ch. 7

The domain of the abstract function is all real numbers $x \neq 0$. For $x > 0$, as $x \to 0$ we have $R(x) \to +\infty$ and for $x < 0$, as $x \to 0$ we have $R(x) \to -\infty$.

3. a.

x	$g(x) = 5x$
−2	−10
−1	−5
0	0
1	5
2	10

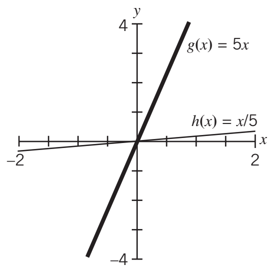
x	$t(x) = 1/x$
−2	−1/2
−1	−1
0.5	2
1	1
2	1/2

x	$h(x) = x/5$
−2	−0.4
−1	−0.2
0	0.0
1	0.2
2	0.4

x	$f(x) = 5/x$
−2	−5/2
−1	−5
0.5	10
1	5
2	5/2

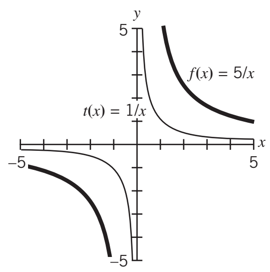
b. The graphs of g and h are both straight lines through the origin with positive slope. The y values of g's graph are 25 times those of h.

Graphs of $g(x) = 5x$ and $h(x) = x/5$



Graphs f and t both have the x- and y-axes as asymptotes, i.e., they approach but never touch these axes, and both are confined to the first and third quadrants. The y values of the graph of f are five times the y values of the graph of t.

Graphs of $t(x) = 1/x$ and $f(x) = 5/x$

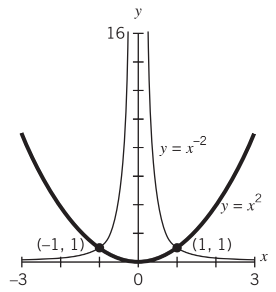


5. a. The graph of $y = x^2$ decreases for $x < 0$ and increases for $x > 0$ (as can be seen in the accompanying diagram).

The graph of $y = x^{-2}$ increases for $x < 0$ and decreases for $x > 0$ (as can also be seen from that diagram).

b. The two graphs intersect at $(-1, 1)$ and $(1, 1)$.

c. As x approaches $\pm\infty$, the graph of $y = x^2$ approaches $+\infty$. As x approaches $\pm\infty$, the graph of $y = x^{-2}$ approaches 0.



7. a. f goes with Graph A. Reason: $f(x)$ values must be positive; rapid decrease in the values of $f(x)$ when $x > 0$.

b. g goes with Graph C. Reason: There must be positive and negative values of $g(x)$.

c. h goes with Graph D. Reason: $h(x)$ values must be positive; when $x > 0$ and $x \to +\infty$ a less rapid decrease in the values in graph D than in Graph A.

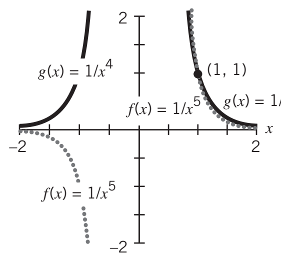
d. j goes with Graph B. Reason: $j(x)$ values must all be negative.

9. a. $g(x) = 4x^{-3}$ **b.** $h(x) = \frac{1}{2}x^{-3}$ **c.** $j(x) = -3x^{-3}$

11. a. If $x > 1$, then $f(x) < g(x)$

b. If $0 < x < 1$, then $f(x) > g(x)$

c. If $x < 0$, then $f(x) < g(x)$



13. a. n is even

b. $k < 0$

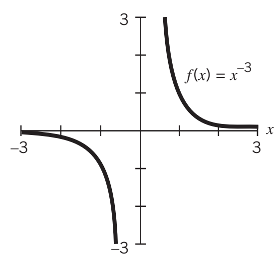
c. $f(-1) < 0$

d. yes

e. 0

f. 0

15. The graph of $f(x) = x^{-3}$



a. g has –1; h has 1/2 and k has –2.

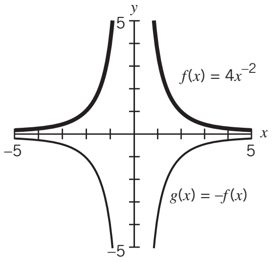
b. g's graph is a reflection of f's across the x-axis.

c. j's graph is a stretch and a reflection of f's across the x-axis.

d. h's graph is a compression of f's.

CH. 7 Algebra Aerobics Solutions for Section 7.7

17. $g(x) = -4x^{-2}$



19. **a.** $f(-x) = \frac{1}{(-x)^4} = \frac{1}{x^4}$ and $-f(x) = -\frac{1}{x^4}$

b. $2f(x) = \frac{2}{x^4}$ and $f(2x) = \frac{1}{(2x)^4} = \frac{1}{16x^4}$

c. $g(-x) = \frac{1}{(-x)^5} = -\frac{1}{x^5}$ and $-g(x) = -\frac{1}{x^5}$

d. $2g(x) = \frac{2}{x^5}$ and $g(2x) = \frac{1}{(2x)^5} = \frac{1}{32x^5}$

e. $h(x) = -f(x) = -\frac{1}{x^4}$

f. $k(x) = -g(x) = -\frac{1}{x^5}$

21. **a.** (0, 0) and (1/2, 1)

b. (0, 0) and (1, 1)

c. $\left(\sqrt{\frac{1}{2}}, 2\right)$ and $\left(-\sqrt{\frac{1}{2}}, 2\right)$

d. (1, 1)

e. (1, 4)

Section 7.7

Algebra Aerobics 7.7

1. Let $Y = \log y$ and let $X = \log x$.

a. $\log y = \log(3 \cdot 2^x) \Rightarrow \log y = \log 3 + \log 2^x \Rightarrow$
$\log y = \log 3 + x \log 2 \Rightarrow$
$\log y = \log 3 + (\log 2)x \Rightarrow Y = 0.477 + 0.301x$

b. $\log y = \log(4x^3) \Rightarrow \log y = \log 4 + \log x^3 \Rightarrow$
$\log y = \log 4 + 3 \log x \Rightarrow Y = 0.602 + 3X$

c. $\log y = \log(12 \cdot 10^x) \Rightarrow \log y = \log 12 + \log 10^x \Rightarrow$
$\log y = \log 12 + x \log 10 \Rightarrow$
$\log y = \log 12 + (\log 10)x \Rightarrow$
$Y = 1.079 + x$ (since $\log 10 = 1$)

d. $\log y = \log(0.15x^{-2}) \Rightarrow \log y = \log 0.15 + \log x^{-2} \Rightarrow$
$\log y = \log 0.15 - 2 \log x \Rightarrow Y = -0.824 - 2X$

2. **a.** $y = 10^{0.067 + 1.63 \log x} \Rightarrow y = 10^{0.067} 10^{1.63 \log x} \Rightarrow$
$y = 10^{0.067} 10^{\log x^{1.63}} \Rightarrow y \approx 1.167x^{1.63}$

b. $y = 10^{2.135 + 1.954x} \Rightarrow y = 10^{2.135} 10^{1.954x} \Rightarrow$
$y = 10^{2.135}(10^{1.954})^x \Rightarrow y \approx 136.458 \cdot 89.95^x$

c. $y = 10^{-1.963 + 0.865x} \Rightarrow y = 10^{-1.963} 10^{0.865x} \Rightarrow$
$y = 10^{-1.963}(10^{0.865})^x \Rightarrow y \approx 0.011 \cdot 7.328^x$

d. $y = 10^{0.247 - 0.871 \log x} \Rightarrow y = 10^{0.247} 10^{-0.871 \log x} \Rightarrow$
$y = 10^{0.247} 10^{\log x^{-0.871}} \Rightarrow y \approx 1.766 \cdot x^{-0.871}$

3. (a), (d), and (f) represent power functions. The graphs of their equations on a log-log plot are linear.

(b), (c), and (e) represent exponential functions. The graphs of their equations on a semi-log plot are linear.

a. $y = 10^{\log 2 + 3 \log x} \Rightarrow y = 10^{\log 2} \cdot 10^{3 \log x} \Rightarrow$
$y = 2 \cdot 10^{\log x^3} \Rightarrow y = 2x^3$

b. $y = 10^{2 + x \log 3} \Rightarrow y = 10^2 \cdot 10^{x \log 3} \Rightarrow$
$y = 100 \cdot 10^{\log 3^x} \Rightarrow y = 100 \cdot 3^x$

c. $y = 10^{0.031 + 1.25x} \Rightarrow y = 10^{0.031} \cdot 10^{1.25x} \Rightarrow$
$y = 1.07 \cdot (10^{1.25})^x \Rightarrow y = 1.07 \cdot (17.78)^x$

d. $y = 10^{2.457 - 0.732 \log x} \Rightarrow y = 10^{2.457} \cdot 10^{-0.732 \log x} \Rightarrow$
$y = 286.4 \cdot 10^{\log x^{-0.732}} \Rightarrow y = 286.4x^{-0.732}$

e. $y = 10^{-0.289 - 0.983x} \Rightarrow y = 10^{-0.289} \cdot 10^{-0.983x} \Rightarrow$
$y = 0.51 \cdot (10^{-0.983})^x \Rightarrow y = 0.51 \cdot (0.104)^x$

f. $y = 10^{-1.47 + 0.654 \log x} \Rightarrow y = 10^{-1.47} \cdot 10^{0.654 \log x} \Rightarrow$
$y = 0.034 \cdot 10^{\log x^{0.654}} \Rightarrow y = 0.034x^{0.654}$

4. **a.** **i.** $y = 4x^3$ is a power function;
ii. $y = 3x + 4$ is a linear function;
iii. $y = 4 \cdot 3^x$ is an exponential function.

Function	**b.** Type of Plot on Which Graph of Function Would Appear as a Straight Line	**c.** Slope of Straight Line
$y = 3x + 4$	Standard linear plot	$m = 3$
$y = 4 \cdot (3^x)$	Semi-log	$m = \log 3$
$y = 4 \cdot x^3$	Log-log	$m = 3$

5. This is a log-log plot, so the slope of a straight line corresponds to the exponent of a power function.

Younger ages: slope $= 1.2 \Rightarrow$ original function is of the form $y = a \cdot x^{1.2}$, where $x =$ body height in cm and $y =$ arm length in cm.

Older ages: slope $= 1.0 \Rightarrow$ original function is of the form $y = a \cdot x^1$, where $x =$ body height in cm and $y =$ arm length in cm.

6. Graph A: Exponential function because the graph is approximately linear on a semi-log plot

Graph B: Exponential function because the graph is approximately linear on a semi-log plot

Graph C: Power function because the graph is approximately linear on a log-log plot

7. **a.** Estimated surface area is $10^4 = 10{,}000$ square cm.

b. Since $S \propto M^{2/3}$, then $S = kM^{2/3}$, where $\log k$ is the vertical intercept of the best-fit line on a log-log plot. In this case the vertical intercept $= \log 10$, so $k = 10$. So the equation is $S = 10M^{2/3}$. When $M = 70{,}000$g, then
$S = 10 \cdot (70{,}000)^{2/3} = 10 \cdot 1700 = 17{,}000$ cm^2.

c. Our estimated surface area was 7000 square cm lower than the calculated area.

d. Since 1 cm = 0.394 in, then 1 cm^2 = (0.394 in)^2 = 0.155 in^2. So 17,000 cm^2 = (17,000)(0.155) = 2635 in^2. Since 1 kg = 2.2 lb, a weight of 70 kg translated to pounds is 70 kg = (70 kg)$\left(2.2 \frac{\text{lb}}{\text{kg}}\right)$ = 154 lb.

CH. 7 Exercises Solutions for Section 7.7

8. **a.** power

b. $y = ax^{-0.23}$

c. When body mass increases by a factor of 10, heat rate is multiplied by $10^{-0.23}$ or about 0.59.

9. Estimated rate of heat production is somewhere between 10^3 and 10^4 kilocalories per day—roughly between 2000–3000 kcal/day.

Exercises for Section 7.7

1. When $x = 4$, then $100(5)^4 = 62{,}500$ and $100(4)^5 = 102{,}400$. So the exponential function $y = 100(5)^x$ in part (a) matches Graph *A*. Since an exponential function is a straight line on a semi-log plot, then the function also matches Graph *D*.

The power function $y = 100x^5$ in part (b) matches Graphs *B* and *C*, since a power function (Graph *C*) is a straight line on a log-log plot (Graph *B*).

3. An exponential function is indicated since the graph is a straight line on a semi-log plot.

5. **a.** $y = 4x^2$

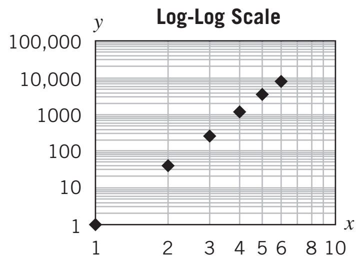
b. $y = 2x^4$

c. $y = 1.25x^4$

d. $y = 0.5x^3$

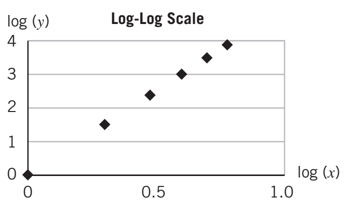
7. **a.**

x	$y = x^5$
1	1
2	32
3	243
4	1024
5	3125
6	7776



b.

x	log (x)	$y = x^5$	log (y)
1	0	1	0
2	0.3010	32	1.5051
3	0.4771	243	2.3856
4	0.6021	1024	3.0103
5	0.6990	3125	3.4949
6	0.7782	7776	3.8908



c. The graphs are exactly the same. The points lie on straight line and are plotted on equivalent scales.

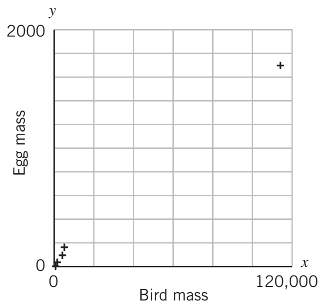
9. **a.**

Species	Adult Mass in Grams	Egg Mass in Grams	Egg/Adult Ratio
Ostrich	113,380.0	1700.0	0.015
Goose	4,536.0	165.4	0.036
Duck	3,629.0	94.5	0.026
Pheasant	1,020.0	34.0	0.033
Pigeon	283.0	14.0	0.049
Hummingbird	3.6	0.6	0.167

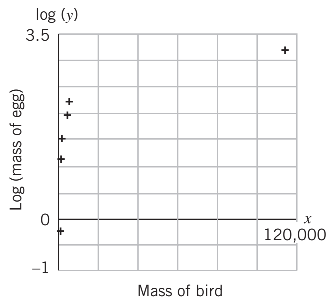
Answers will vary; notable is the fact that the egg/adult mass ratio for the hummingbird is very high and it is very low for the ostrich. The other ratios are not far apart from each other.

b. The plots (where x = adult mass, y = egg mass (both in grams)) are, respectively,

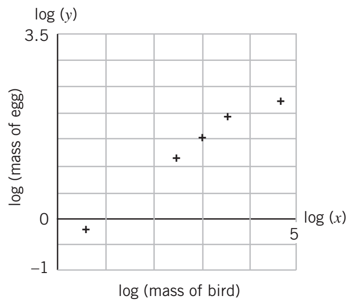
i. y vs. x



ii. log y vs. x



iii. log y vs. log x



c. The log vs. log scatter diagram in part b (iii) is the most linear. Using technology the best-fit straight-line equation is: $\log(y) = \log(0.1918) + 0.7719 \cdot \log(x)$ or, in linear equation form: $Y = -0.717 + 0.7719X$ where $Y = \log(y)$ and $X = \log(x)$.

d. Regrouping, using the laws of logarithms, we get: $\log(y) = \log(0.1918 \cdot x^{0.7719})$. Thus, $y = 0.1918 \cdot x^{0.7719}$, where

x = mass of the adult and y is the corresponding mass of the egg, both measured in grams.

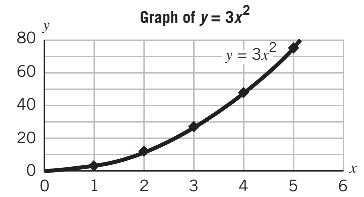
e. If $x = 12.7$ kg (or 12,700 grams) for the weight of an adult turkey, then $y \approx 282$ grams is the predicted weight of its egg.

f. If the egg weighs 2 grams, then the adult bird is predicted to have an adult weight $\approx$ 20.8 grams.

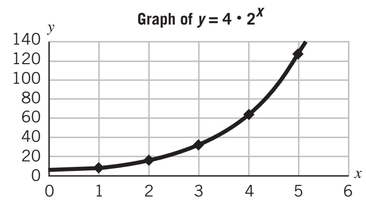
11. a. Since l_1 and l_2 are parallel lines they have the same slope but different vertical intercepts. Thus their equations can be written as $\log(y) = mx + \log(b_1)$ and $\log(y) = mx + \log(b_2)$, with $\log(b_1) \neq \log(b_2)$. Solving each for y in terms of x, we have $y = b_1 \cdot 10^{mx}$ and $y = b_2 \cdot 10^{mx}$ with $b_1 \neq b_2$. Thus they have the same power of 10 but differ in their y-intercepts.

b. The equation of l_3 is $\log(y) = m \cdot \log(x) + \log(b_3)$ and the equation of l_4 is $\log(y) = m \cdot \log(x) + \log(b_4)$, with $\log(b_3) \neq \log(b_4)$. Solving for y in terms of x in each gives: $y = b_3 x^m$ and $y = b_4 x^m$ with $b_3 \neq b_4$. Thus they have the same power of x but differ in their coefficients of x.

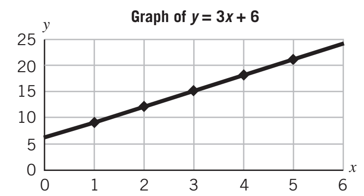
13. a.



b.



c.

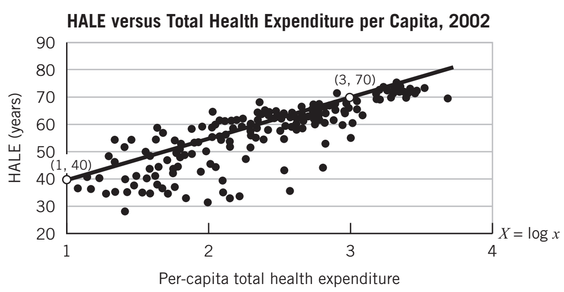


15. a. A power function seems appropriate.

b. The log-log graph is linear and we get $\log(O) = 0.75 \cdot \log(m) + c$, where O stands for oxygen consumption and m stands for body mass. Thus $O = k \cdot m^{0.75}$, a power function, where $k = 10^c$.

c. A slope of 3/4 means that if the body mass, m, is multiplied by 10^4 (=10,000), then the oxygen consumption, O, is multiplied by 10^3 (=1000). If m is multiplied by 10, then O is multiplied by $10^{3/4} \approx 5.6$.

17. a.



b. Two estimated points on the best-fit line are (1, 40) and (3, 70) $\Rightarrow$ slope $= \frac{(70 - 40)}{(3 - 1)} = 15$. So the equation is of the form $y = b + 15X$. Substituting in (1, 40) we get $40 = b + (15 \cdot 1) \Rightarrow 40 = b + 15 \Rightarrow b = 25$. So the equation is $y = 25 + 15X$. Substituting $\log x$ for X, we finally get $y = 25 + 15 \log x$. So HALE is a logarithmic function of per-capita health expenditure.

c. A logarithmic model makes sense, since there will always be a ceiling in life expectancy, so the results of adding funding will increase HALE but at a slower and slower rate.

d. It suggests that the functions are log functions.

Ch. 7: Check Your Understanding

1. True	**6.** False	**11.** False	**16.** False
2. False	**7.** False	**12.** True	**17.** False
3. False	**8.** True	**13.** True	**18.** True
4. False	**9.** True	**14.** True	**19.** True
5. True	**10.** False	**15.** False	**20.** False

21. True

22. Possible answer: $f(x) = 4x^8$ or $f(x) = 10x^6$

23. $g(x) = 3.2x^4$

24. Possible answer: $f(x) = -x^4$

25. Possible answer: $h(m) = m^{12}$ or $h(m) = e^m$

26. Possible answer: $y = 3^x$ or $y = 3 + x$

27. Possible answer: $y = 3x^5$

28. Possible answer: $k(x) = \frac{-1}{x^3}$

29. Possible answer: $T(m) = 3m^4$

30. False	**33.** False	**36.** True	**39.** True
31. True	**34.** False	**37.** False	**40.** False
32. False	**35.** True	**38.** True	

Ch. 7 Review: Putting It All Together

1. a. $V(x) = x(2x)(2x) = 4x^3$. So $V(X) = 4X^3$ and $V(2X) = 4(2X)^3 = 8(4X^3) = 8V(X) \Rightarrow$ the volume is multiplied by 8.

b. The surface area consists of the two top and bottom sides, which are both $(2x)(2x) = 4x^2$ in area, and the other four vertical sides, each $(x)(2x) = 2x^2$ in area. So the surface area $S(x) = 2(4x^2) + 4(2x^2) = 16x^2$. Hence $S(X) = 16X^2$ and $S(2X) = 16(2X)^2 = 4(16X^2) = 4S(X) \Rightarrow$ when the value of x is doubled, the surface area is multiplied by 4. So the volume grows faster than the surface area.

c. The ratio of (surface area)/volume $= R(x) = S(x)/V(x) = (16x^2)/(4x^3) = 4/x$. If you double the value of x, from X to $2X$, since $R(X) = 4/X$, then $R(2X) = 4/(2X) = (1/2)(4/X) = (1/2)R(X)$. So if the value of x doubles, $R(x)$, the ratio of

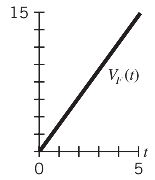
Ch. 7

(surface area)/volume, is multiplied by 1/2—or equivalently, cut in two. This is again confirmation that the volume grows faster than the surface area.

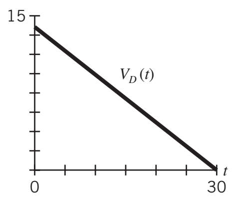
d. In general, as x increases, $R(x)$—the ratio of surface area to volume—will decrease. So as the rectangular solid gets larger, there will be relatively less surface area compared with the volume.

3. Think of the cake as a cylinder, with a volume of $\pi r^2 h$ where r = the radius of the cake and h = the height. The radii of the 10″ and 12″ cakes are 5″ and 6″, respectively. If the corresponding heights are h_1 and h_2, then the respective cake volumes are $25\pi h_1$ and $36\pi h_2$ cubic inches. The site claims that the volume of the 12″ cake is twice that of the 10″ cake (6 quarts vs. 3 quarts). That implies that $36\pi h_2 = 2(25\pi h_1) \Rightarrow h_2/h_1 = 50/36 \approx 1.4 \Rightarrow h_2$ is about 40% larger than h_1. So for the 12″ cake to have twice the volume of the 10″ cake, the height of the 12″ cake would have to be 40% higher than the height of the 10″ cake—which seems rather unlikely.

5. a. $V_F(t) = 3t$; domain is $0 \le t \le 5$; represents direct proportionality.



b. $V_D(t) = 15 - 0.5t$; domain is $0 \le t \le 30$; does not represent inverse proportionality.



7. a. The friend is wrong.

b. If the height is increased by 50% (from 4 feet to 6 feet) or, equivalently, multiplied by 1.5 (since $1.5 \cdot 4' = 6'$), the width (2′) and the length (2′) must both also be multiplied by 1.5, giving 3′ for the new width and length. Hence the new dimensions are $6' \times 3' \times 3'$, which give a volume of 54 cubic feet. The volume of the original block of ice $4' \times 2' \times 2'$ = 16 cubic feet. Since (new volume)/(old volume) = 54/16 = 3.375, the new volume = 3.375 · (old volume) or, equivalently, a 237.5% increase in volume, almost five times what your friend predicted.

c. 10% of 54 cubic feet = 0.10 · 54 = 5.4 cubic feet. So the volume of the melted sculpture would be 54 − 5.4 = 48.6 cubic feet. If h is the height of the melted sculpture, then the width and length are both $0.5h$. So we have $48.6 = (0.5h)^2 h = (0.5)^2 h^3 \Rightarrow h^3 = (48.6)/(0.5)^2 = 194.4 \Rightarrow h \approx 5.8$ feet as the height of the melted sculpture.

9. a. $t = kd/r$ (the k may be needed for unit conversions)

b. $D_1 = kD_2$

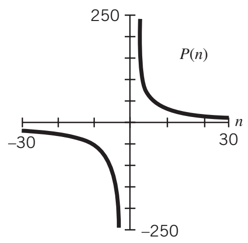
c. $R = kV^2/W$

11. Graph C is symmetric across the y-axis. Graphs A, B, and D are rotationally symmetric about the origin. None of them has an asymptote.

13. a. $P(n) = 450/n$ for integer values of n between 0 and, say, 10.

b. $P(2) = 450/2 = \$225$/person; $P(5) = 450/5 = \$90$/person. The more people, the lower the cost per person.

c. If you take the function $P(n)$ out of context and treat n as any real number ($\neq 0$), then the following graph shows the result.



The abstract function is asymptotic to both the horizontal and vertical axes.

15. a. From the previous problem, we have $I = P/d^2$ where I is measured in foot-candles, P in candlepower, and d in feet. Setting I = 4000 foot-candles and d = 3 feet, we have $4000 = P/(3)^2 \Rightarrow P = 36{,}000$ candlepower.

b. From part (a) we know that the lamp has 36,000 candlepower. Setting $P = 36{,}000$ and $I = 2000$ foot-candles, we have $2000 = 36{,}000/d^2 \Rightarrow d^2 = 18 \Rightarrow d \approx 4.24$ feet or about 51 inches above the operating surface.

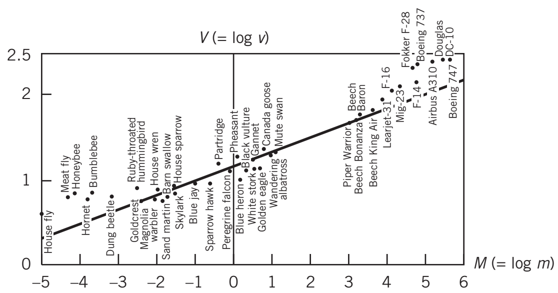
17. Graphs B and C are symmetric about the y-axis. Graph D is symmetric about the origin. Graphs A, B, and D all have the x-axis as a horizontal asymptote. Graphs B and D also have the y-axis as a vertical asymptote.

19. **a**. Y is a linear function of X.

b. Using the estimated points (0, 0.3) and (1, 3.3), we have $Y = 0.3 + 3X$.

c. Substituting $Y = \log y$ and $X = \log x$ in $Y = 0.3 + 3X$, we have $\log y = 0.3 + 3 \log x \Rightarrow \log y = 0.3 + \log(x^3)$. Rewriting using powers of 10, we have $10^{\log y} = 10^{(0.3+\log(x^3))} \Rightarrow y = 10^{0.3} \cdot 10^{\log x^3} \Rightarrow y = 10^{0.3} \cdot x^3 \Rightarrow y \approx 2x^3$. Hence y is a power function and y is directly proportional to x^3.

21. To find the relationship, sketch a best-fit line to the data, where m = mass (in kg) and v = optimal flying speed (in meters per second). Since both axes use a logarithmic scale, we can replace the labels on the axes with $\log(m) = M$ and $\log(v) = V$, respectively. The horizontal labels will now read −5, −4, −3, −2 −1, 0 (= log 1), which is now the beginning of the vertical axis, followed by 1, 2, 3, 5, 5, 6; and on the vertical axis 0, 1, 2, and 2.5 ≈ log 300.



Two points on the line with coordinates of the form (M, V) are $(-1, 1)$ and $(5, 2)$. The slope is then $(2 - 1)/(5 - (-1)) = 1/6$. The vertical intercept is approximately at $(0, 1.2)$. So the equation of the line would be $V = 1.2 + (1/6)M$. Substituting in, we get $\log(v) = 1.2 + (1/6)\log(m) = 1.2 + \log(m^{1/6})$. Rewriting both sides as powers of 10 and simplifying, we get: $v = 10^{(1.2 + \log(m^{1/6}))} = 10^{1.2} \cdot 10^{\log(m^{1/6})} \approx 16m^{1/6}$. So $v \propto m^{1/6}$ $\Rightarrow$ the cruising speed v (in meters per second) is directly proportional to the 1/6 power of body mass m (in kilograms). So Professor Bejan seems to be correct.

CHAPTER 8

Section 8.1

Algebra Aerobics 8.1

1. a. The vertex at (0, 0) is the minimum point. The focal point is $\frac{1}{4a} = \frac{1}{4(3)} = \frac{1}{12}$ units above the vertex at $\left(0, \frac{1}{12}\right)$.

b. The vertex at (0, 0) is the maximum point. The focal point is $\frac{1}{4a} = \frac{1}{4(-6)} = \frac{-1}{24}$ units below the vertex at $\left(0, -\frac{1}{24}\right)$.

c. The vertex at (0, 0) is the minimum point. The focal point is $\frac{1}{4a} = \frac{1}{4(1/24)} = 6$ units above the vertex at (0, 6).

d. The vertex at (0, 0) is the maximum point. The focal point is $\frac{1}{4a} = \frac{1}{4(-1/12)} = -3$ units below the vertex at $(0, -3)$.

2. a. A point on the rim is (15, 10), so if $y = ax^2$, then $10 = a(15)^2$, so $a = 2/45$. The focal point is $\frac{1}{4a} = \frac{1}{4(2/45)} = \frac{45}{8} = 5.625$ ft from the vertex at (0, 0) on the back wall.

b. $y = \frac{2}{45}x^2$

c. You could not hear well if you were more than 15 ft on either side of the stage, because the sound would be traveling in straight lines from the parabolic wall. Also, the sound would not be good if the performer moved around and did not stay at the focal point.

3. a. $g(2) = 2^2 = 4$
$g(-2) = (-2)^2 = 4$
$g(0) = (0)^2 = 0$
$g(z) = (z)^2$

b. $h(2) = -(2)^2 = -4$
$h(-2) = -(-2)^2 = -4$
$h(0) = 0^2 = 0$
$h(z) = -z^2$

c. $Q(2) = -(2)^2 - 3(2) + 1 = -9$
$Q(-2) = -(-2)^2 - 3(-2) + 1 = 3$
$Q(0) = -(0)^2 - 3(0) + 1 = 1$
$Q(z) = -(z)^2 - 3(z) + 1$

d. $m(2) = 5 + 2(2) - 3(2)^2 = -3$
$m(-2) = 5 + 2(-2) - 3(-2)^2 = -11$
$m(0) = 5 + 2(0) - 3(0)^2 = 5$
$m(z) = 5 + 2z - 3z^2$

e. $D(2) = -(2 - 3)^2 + 4 = 3$
$D(-2) = -((-2) - 3)^2 + 4 = -21$
$D(0) = -((0) - 3)^2 + 4 = -5$
$D(z) = -(z - 3)^2 + 4$

f. $k(2) = 5 - 2^2 = 1$
$k(-2) = 5 - (-2)^2 = 1$
$k(0) = 5 - 0^2 = 5$
$k(z) = 5 - z^2$

4. a. $f(x)$
i. concave down
ii. maximum
iii. axis of symmetry: $x = 0$
iv. vertex: (0, 4)
v. approx. horizontal intercepts: $(-2, 0)$, (2, 0)
vertical intercept: (0, 4)

b. $g(x)$
i. concave up
ii. minimum
iii. axis of symmetry: $x = 1$
iv. approx. vertex: $(1, -2.25)$
v. horizontal intercepts: $(-2, 0)$, (4, 0)
vertical intercept: $(0, -2)$

c. $h(x)$
i. concave up
ii. minimum
iii. axis of symmetry: $x = 2$
iv. vertex: $(2, -4)$
v. horizontal intercepts: (0, 0), (4, 0)
vertical intercept: (0, 0)

d. $k(x)$
i. concave down
ii. maximum
iii. axis of symmetry: $x = -4$
iv. vertex: $(-4, -2)$
v. no horizontal intercepts
vertical intercept: $(0, -10)$

5. a. Equation is of the form $y = ax^2$ where $a > 0 \Rightarrow$ focal length $= \frac{1}{4a}$. When $x = 12$, focal length = depth $\Rightarrow$ $\frac{1}{4a} = a(12)^2 \Rightarrow a^2 = \frac{1}{4 \cdot 12^2} \Rightarrow a = \frac{1}{2 \cdot 12} = \frac{1}{24}$.
So focal length $= \frac{1}{4(1/24)} = \frac{1}{(1/6)} = 6$

b. $a = \frac{1}{24} \Rightarrow y = \frac{1}{24}x^2$.

6. $f(x) = x^2 + 2x - 15$ $\quad g(x) = 2x^2 + 7x + 5$
$h(x) = 10x^2 - 80x + 150$ $\quad j(x) = 2x^2 - 18$

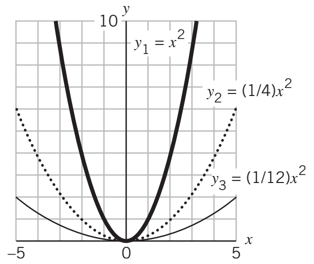
Exercises for Section 8.1

1. a. concave up; minimum; vertex at $(-1, -4)$; the line $x = -1$ is axis of symmetry; (1, 0) and $(-3, 0)$ are horizontal intercepts; $(0, -3)$ is vertical intercept.

b. concave down; maximum; vertex at (2, 9); the line $x = 2$ is axis of symmetry; $(-4, 0)$ and (8, 0) are horizontal intercepts; (0, 8) is vertical intercept.

c. concave down; maximum; vertex at $(-5, 3)$; the line $x = -5$ is axis of symmetry; $(-8, 0)$ and $(-2, 0)$ are horizontal intercepts; $(0, -5)$ is vertical intercept. [Students may have a different y value for vertical intercept.]

3. The graphs of the three functions are in the accompanying diagram.

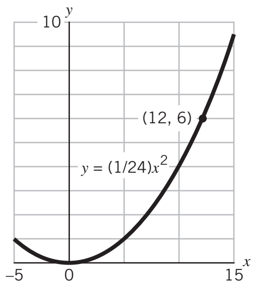


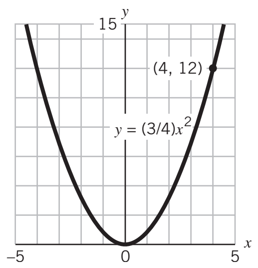
a. For y_1: (0, 1/4); for y_2: (0, 1); for y_3: (0, 3)

b. The graphs of all three functions are concave-up parabolas with vertex at (0, 0), but y_1 rises faster than y_2 and y_3.

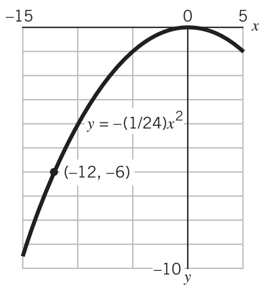
c. Farther from the vertex and the graphs get wider.

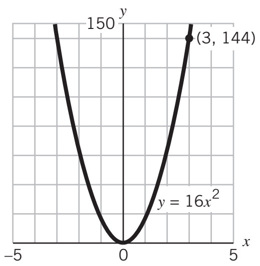
5. **a.** $y = (1/24)x^2$ **c.** $y = (3/4)x^2$





b. $y = -(1/24)x^2$ **d.** $y = 16x^2$





7. **a.** $y = (1/16)x^2$ **c.** $y = 4x^2$
b. $y = -(1/32)x^2$ **d.** $y = -6x^2$

9. **a.** -8 and 12 **b.** 10 and 14 **c.** 2 and -14

11. **a.** $y = (1/6)x^2$

b. $5 = (1/6)x^2$ implies $x = \pm\sqrt{30}$ and thus the reflector is $2\sqrt{30} \approx 10.95$ inches wide.

13. Assuming that we rotate the parabolic model to become concave up:

a. The focus is at (0, 1.25).

b. $y = 0.2x^2$

c. $2.5 = 0.2x^2$ gives $x = \pm\sqrt{12.5}$; thus the reflector should be $2\sqrt{12.5} \approx 7.07$ inches wide.

15. Student answers will vary. The general trend is that the smaller the focal length, the bigger the value of a and thus the narrower the opening of the parabola, since focal length $= \left|\frac{1}{4a}\right|$ when the parabola has the equation $y = ax^2$.

17. **a.** $f(x) = x^2 + 2x - 3$ **c.** $H(z) = -z^2 - z + 2$
b. $P(t) = t^2 - 3t - 10$

19. **a.** $P = 2L + 2W = 200$; Thus $L = 100 - W$ and $A = LW = (100 - W) \cdot W = 100W - W^2$. So A has its maximum when $W = 50$ m, since the vertex is at (50, 50). Thus $L = 50$ m and the maximum area is 2500 m^2.

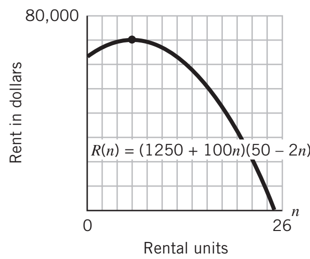
b. The same kind of argument applies. $P = 2L + 2W$; thus $L = (P/2) - W$ and $A = LW = W(P/2 - W) = (P/2) \cdot W - W^2$, and this has its vertex at $(P/4, (P/4)^2)$ and the maximum area is $(P/4)^2$ m^2.

21. **a.** True **c.** False **e.** False
b. False **d.** True

23. **a.** $R(n) = (1250 + 100n) \cdot (50 - 2n)$

b. There will be no apartments rented when $n = 25$. Thus the domain is $0 \le n \le 25$.

c. The graph is given in the accompanying diagram. From inspection, the practical maximum occurs when $n = 6$. At that value $R = \$70{,}300$.



Section 8.2

Algebra Aerobics 8.2a

1. $s(x) = 2x^2 + 2$ is narrower than $r(x) = x^2 + 2$ because, looking at the coefficient of x^2, $2 > 1$. Both have a vertical intercept at (0, 2) since $c = 2$ in both equations.

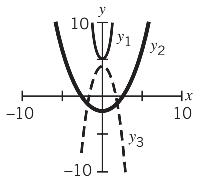
2. $h(t) = t^2 + 5$ is concave up since a (=1) is positive; $k(t) = -t^2 + 5$ is concave down since a (= -1) is negative. They have the same shape because $|a| = 1$ in both. They both cross the vertical axis at 5 since $c = 5$ in both equations.

3. Both are concave down because both have $a < 0$. Both have a vertex at (0, 0) since b and $c = 0$ in both equations. $g(z)$ is flatter than $f(z)$ because $|-0.5| < |-5|$.

4. They have the same shape and are concave up since both have $a = 1$. $g(x)$ is six units higher than $f(x)$ because 8 is 6 more than 2.

5. They have the same shape and are concave down since a (= -3) is negative in both equations. $g(t) = -3t^2 + t - 2$ is three units higher than $f(t) = -3t^2 + t - 5$ since $c = -2$ is three units vertically up from $c = -5$.

6. **a.** $y_1 = 3x^2 + 5$ **b.** $y_2 = \frac{1}{3}x^2 - 2$ **c.** $y_3 = -2x^2 + 4$



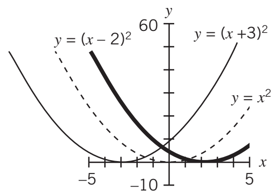
7. **a.** $g(x) = 3x^2$ **c.** $j(x) = 1/2x^2$
   b. $h(x) = -5x^2$ **d.** $k(x) = -x^2$

8. **a.** $y = 5x^2 - 2$ **c.** $y = -0.5x^2 - 4.7$
   b. $y = x^2 + 3$ **d.** $y = x^2 - 71$

Algebra Aerobics 8.2b

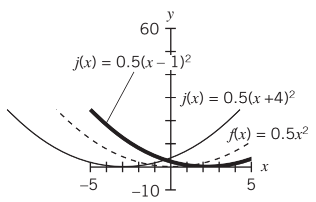
1. **a.** vertex is (0, 0)
   b. vertex is $(-3, 0)$
   c. vertex is (2, 0)

   All vertices lie on the x-axis. Vertex of (b) is three units to the left of the vertex of (a). Vertex of (c) is two units to the right of vertex of (a). All have same shape and are concave up.



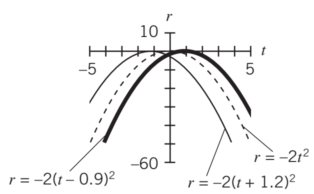
2. **a.** vertex is (0, 0) **c.** vertex is $(-4, 0)$
   b. vertex is (1, 0)

   All vertices lie on the x-axis. The vertex of (b) is one unit to the right of the vertex of (a). The vertex of (c) is four units to the left of vertex of (a). All have the same shape and are concave up.

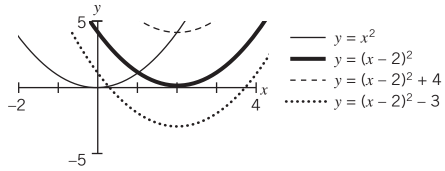


3. **a.** vertex is (0, 0) **c.** vertex is (0.9, 0)
   b. vertex is $(-1.2, 0)$

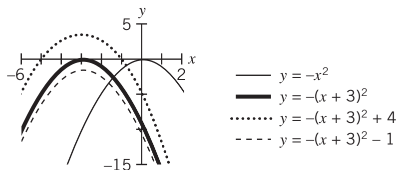
   All of the vertices lie on the horizontal axis. The vertex of (b) is 1.2 units to the left of the vertex of (a). The vertex of (c) is 0.9 units to the right of the vertex of (a). All have the same shape and are concave down.



4. $y = a(x - h)^2 + k$. The value of $h = 2$ in (b), (c), and (d), so the vertices of those parabolas are two units to the right of the vertex of (a), where $h = 0$. All are concave up with the same shape since $a = 1$ in all four equations. (c) is four units above (a) and (b); (d) is three units below (a) and (b) since $k = 0$ in (a) and (b), $k = 4$ in (c), and $k = -3$ in (d).



5. $y = a(x - h) + k$. All are concave down with the same shape since $a = -1$ in all four equations. (b), (c), and (d) have vertices three units to the left of vertex of (a), since $h = -3$ in those equations, and $h = 0$ in (a). $k = 0$ in (a) and (b), but $k = -1$ in (c), so (c) is one unit below (a) and (b). $k = 4$ in (d), so (d) is four units above (a) and (b).



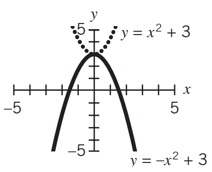
6. **a.** $g(x) = 3(x - 2)^2 - 1$, the vertex is $(2, -1)$
   b. $h(x) = -2(x + 3)^2 + 5$, the vertex is $(-3, 5)$
   c. $j(x) = \frac{1}{5}(x + 4)^2 - 3.5$, the vertex is $(-4, -3.5)$
   d. $k(x) = -(x - 1)^2 + 4$, the vertex is (1, 4)

7. **a.** The vertex at $(3, -4)$ is a minimum.
   b. The vertex at $(-1, 5)$ is a maximum.
   c. The vertex at (4, 0) is a maximum.
   d. The vertex at $(0, -7)$ is a minimum.

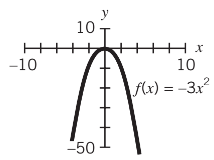
Algebra Aerobics 8.2c

1. **a.** vertex is $(0, -4)$.
   b. vertex is (0, 6).
   c. vertex is (0, 1).

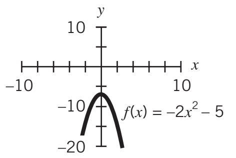
2. **a.** vertex is (0, 3)
   b. vertex is (0, 3)



3. **a.** Vertex is (0, 0). Parabola is concave down since a $(= -3)$ is negative. There is one x-intercept, (0, 0), since vertex is on the x-axis.



b. Vertex is $(0, -5)$, which is below x-axis. Parabola is concave down since $a\ (= -2)$ is negative. So there are no x-intercepts.

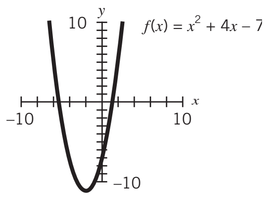


In parts (c) and (d), use the formula $h = \frac{-b}{2a}$ as the horizontal coordinate of the vertex (where $f(x) = ax^2 + bx + c$).

c. $a = 1, b = 4 \Rightarrow h = \frac{-b}{2a} = \frac{-4}{2(1)} = -2,$

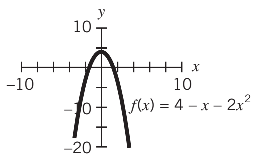
$f(-2) = (-2)^2 + 4(-2) - 7 = 4 - 8 - 7 = -11$

Vertex is $(-2, -11)$. Parabola is concave up since $a\ (= 1)$ is positive. There are two x-intercepts. They are approximately $(-5, 0)$ and $(1, 0)$.



d. $f(x) = 4 - x - 2x^2 = -2x^2 - x + 4$; so $a = -2$, $b = -1$; $h = \frac{-b}{2a} = \frac{-(-1)}{2(-2)} = \frac{1}{-4} = -\frac{1}{4}$

$f\left(-\frac{1}{4}\right) = 4 - \left(-\frac{1}{4}\right) - 2\left(-\frac{1}{4}\right)^2 = 4 + \frac{1}{4} - 2\left(\frac{1}{16}\right) = 4 + \frac{1}{4} - \frac{1}{8} = 4\frac{1}{8}$. So vertex is $\left(-\frac{1}{4}, 4\frac{1}{8}\right)$. Parabola is concave down since $a\ (= -2)$ is negative. There are two x-intercepts. They are approximately $(-2, 0)$ and $(1, 0)$.



4. $y = ax^2 + bx + c$ compared with $y = x^2$ (where $a = 1$)

a. $y = 2x^2 - 5 \Rightarrow a = 2$

i. $a > 0 \Rightarrow$ minimum at the vertex

ii. $|a| > 1$, so parabola is narrower than $y = x^2$.

b. $y = 0.5x^2 + 2x - 10 \Rightarrow a = 0.5$

i. $a > 0 \Rightarrow$ minimum at the vertex

ii. $|a| < 1$, so parabola is flatter than $y = x^2$.

c. $y = 3 + x - 4x^2 \Rightarrow a = -4$

i. $a < 0 \Rightarrow$ maximum at the vertex

ii. $|a| > 1$, so parabola is narrower than $y = x^2$.

d. $y = -0.2x^2 + 11x + 8 \Rightarrow a = -0.2$

i. $a < 0 \Rightarrow$ maximum at the vertex

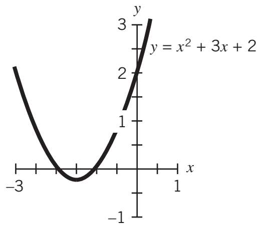
ii. $|a| < 1$, so parabola is flatter than $y = x^2$.

5. a. $a = 1, b = 3$; so horizontal coordinate of vertex is:

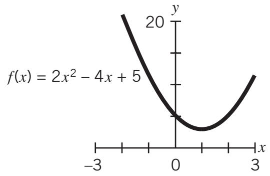
$\frac{-b}{2a} = \frac{-3}{2} = -1\frac{1}{2}$. If $x = -\frac{3}{2}, \Rightarrow$

$y = \left(-\frac{3}{2}\right)^2 + 3\left(-\frac{3}{2}\right) + 2 = \frac{9}{4} - \frac{9}{2} + 2 = 2\frac{1}{4} - 4\frac{2}{4} + 2 = -\frac{1}{4}$, So vertex is $\left(-1\frac{1}{2}, -\frac{1}{4}\right)$.

Vertical intercept is 2.

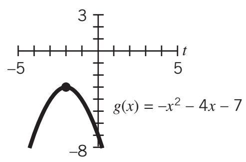


b. $a = 2$, $b = -4$, so horizontal coordinate of vertex is $\frac{-b}{2a} = \frac{-(-4)}{2(2)} = 1$. $f(1) = 2 - 4 + 5 = 3$, so vertex is $(1, 3)$. Since vertex is above x-axis, and $a\ (= 2)$ is positive; the parabola is concave up, it does not cross x-axis, so no horizontal intercepts. Vertical intercept is 5.

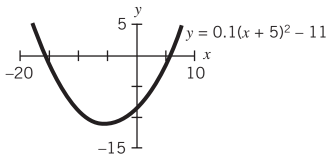


c. $a = -1, b = -4; \Rightarrow \frac{-b}{2a} = \frac{-(-4)}{2(-1)} = \frac{4}{-2} = -2$

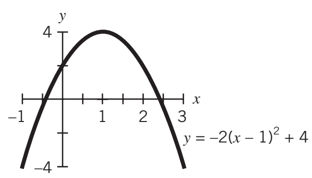
$g(-2) = -(-2)^2 - 4(-2) - 7 = -4 + 8 - 7 = -3$, so vertex is $(-2, -3)$. Since vertex is below t-axis and $a\ (= -1)$ is negative, the parabola is concave down; it does not cross t-axis, so no horizontal intercepts. Vertical intercept $= -7$.



6. a. Vertex is $(-5, -11)$. y-intercept at $x = 0 \Rightarrow$ $y = 0.1(5)^2 - 11 = 0.1(25) - 11 = 2.5 - 11 = -8.5$. So y-intercept is -8.5.



b. Vertex is $(1, 4)$. y-intercept at $x = 0 \Rightarrow y = -2(-1)^2 + 4 = -2 + 4 = 2$. So y-intercept is 2.



7. a. $(x + 3)^2 - 9$

b. $(x - 5)^2 - 25$

c. $(x - 15)^2 - 225$

d. $\left(x + \frac{1}{2}\right)^2 - \frac{1}{4}$

8. a. $f(x) = x^2 + 2x - 1 = x^2 + 2x + (1 - 1) - 1$
$= (x^2 + 2x + 1) + (-1 - 1) \Rightarrow$
$f(x) = (x + 1)^2 - 2$
vertex $(-1, -2)$; stretch factor 1

b. $j(z) = 4z^2 - 8z - 6 = 4(z^2 - 2z) - 6$
$= 4(z^2 - 2z + 1) - 4(1) - 6 \Rightarrow$
$j(z) = 4(z - 1)^2 - 10$
vertex $(1, -10)$; stretch factor 4

c. $h(x) = -3x^2 - 12x = -3(x^2 + 4x)$
$= -3(x^2 + 4x + 4) + 3(4) \Rightarrow$
$h(x) = -3(x + 2)^2 + 12$
vertex $(-2, 12)$; stretch factor -3

d. $h(t) = -16(t^2 - 6t) \Rightarrow h(t) = -16(t^2 - 6t + 9) + 16(9)$
$\Rightarrow h(t) = -16(t - 3)^2 + 144$
vertex $(3, 144)$; stretch factor -16

e. $h(t) = -4.9(t^2 + 20t) + 200$
$\Rightarrow h(t) = -4.9(t^2 + 20t + 10^2) + 200 + 4.9(10^2)$
$\Rightarrow h(t) = -4.9(t + 10)^2 + 690$
vertex $(-10, 690)$; stretch factor -4.9

9. a. $y = 2\left(x - \frac{1}{2}\right)^2 + 5 = 2\left(x - \frac{1}{2}\right)\left(x - \frac{1}{2}\right) + 5$
$= 2\left(x^2 - x + \frac{1}{4}\right) + 5 = 2x^2 - 2x + \frac{1}{2} + 5 \Rightarrow$
$y = 2x^2 - 2x + 5\frac{1}{2}$
vertex $(1/2, 5)$; stretch factor 2

b. $y = -\frac{1}{3}(x + 2)^2 + 4 = -\frac{1}{3}(x + 2)(x + 2) + 4$
$= -\frac{1}{3}(x^2 + 4x + 4) + 4$
$= -\frac{1}{3}x^2 - \frac{4}{3}x - \frac{4}{3} + 4 \Rightarrow$
$y = -\frac{1}{3}x^2 - \frac{4}{3}x + 2\frac{2}{3}$
vertex $(-2, 4)$; stretch factor $-1/3$

c. $y = 10(x^2 - 10x + 25) + 12$
$\Rightarrow y = 10x^2 - 100x + 250 + 12$
$\Rightarrow y = 10x^2 - 100x + 262$
vertex $(5, 12)$; stretch factor 10

d. $y = 0.1(x^2 + 0.4x + 0.04) + 3.8$
$\Rightarrow y = 0.1x^2 + 0.04x + 3.804$
vertex $(-0.2, 3.8)$; stretch factor 0.1

10. a. $y = x^2 + 6x + 7 = x^2 + 6x + 9 - 9 + 7 \Rightarrow$
$y = (x + 3)^2 - 2$

b. $y = 2x^2 + 4x - 11 = 2(x^2 + 2x) - 11$
$= 2(x^2 + 2x + 1) - 2(1) - 11 \Rightarrow$
$y = 2(x + 1)^2 - 13$

11. a. "Completing the square,"
$y = x^2 + 8x + 11 \Rightarrow y = x^2 + 8x + 16 - 16 + 11 \Rightarrow$
$y = (x + 4)^2 - 5$
So vertex is $(-4, -5)$.

b. Using the formula,
$y = 3x^2 + 4x - 2 \Rightarrow a = 3, b = 4, c = -2 \Rightarrow$
$h = \frac{-b}{2a} = \frac{-4}{6} = -\frac{2}{3}$
When $x = \frac{-2}{3}$,
$y = 3\left(-\frac{2}{3}\right)^2 + 4\left(\frac{-2}{3}\right) - 2 \Rightarrow$
$y = 3\left(\frac{4}{9}\right) - \frac{8}{3} - 2 = \frac{4}{3} - \frac{8}{3} - 2 =$
$-\frac{4}{3} - 2 = -1\frac{1}{3} - 2 = -3\frac{1}{3}$, so vertex is at $\left(-\frac{2}{3}, -3\frac{1}{3}\right)$.
In vertex form, $y = 3\left(x + \frac{2}{3}\right)^2 - 3\frac{1}{3}$

12. a. $f(p) = -1875p^2 + 4500p - 2400 \Rightarrow$
$a = -1875, b = 4500, c = -2400 \Rightarrow$
$h = \frac{-b}{2a} = \frac{-4500}{-3750} = 1.2$
$f(1.2) = -1875(1.2)^2 + 4500(1.2) - 2400 \Rightarrow f(1.2) = 300$
So the vertex is at (\$1.20, \$300).

b. The maximum daily profit is \$300 at a price of \$1.20 per pretzel.

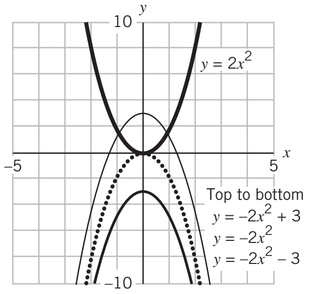
13. a. The vertex is $(-1, 4) \Rightarrow h = -1, k = 4 \Rightarrow$
$y = a(x - (-1))^2 + 4 \Rightarrow y = a(x + 1)^2 + 4$
Passing through the point $(0, 2) \Rightarrow 2 = a(0 + 1)^2 + 4 \Rightarrow$
$a = -2 \Rightarrow$ the equation is $y = -2(x + 1)^2 + 4$.

b. The vertex is $(1, -3) \Rightarrow h = 1, k = -3 \Rightarrow$
$y = a(x - 1)^2 - 3$. Passing through the point $(-2, 0) \Rightarrow$
$0 = a(-2 - 1)^2 - 3 \Rightarrow 3 = 9a \Rightarrow$ the equation is
$y = \frac{1}{3}(x - 1)^2 - 3$.

Exercises for Section 8.2

1. a. downward; the coefficient of x^2 is negative.
b. upward; the coefficient of t^2 is positive.
c. upward; the coefficient of x^2 is positive.
d. downward; the coefficient of x^2 is negative.

3. The graphs of the four functions are given with their labels in the diagram below.



5. a. The compression factor is 0.3, the vertex is at (1, 8).
b. The expansion or stretch factor is 30 and the vertex is at (0, −11).
c. The compression factor is 0.01 and the vertex is at (−20, 0).
d. The expansion or stretch factor is −6 and the vertex is at (1, 6).

7. a. (3, 5); maximum **c.** (−4, −7); maximum
b. (−1, 8); minimum **d.** (2, −6); minimum

9. a. $y = 1(x - 2)^2 - 4 = x^2 - 4x$
b. $y = -(x - 4)^2 + 3 = -x^2 + 8x - 13$
c. $y = -2(x + 3)^2 + 1 = -2x^2 - 12x - 17$
d. $y = 0.5(x + 4)^2 + 6 = 0.5x^2 + 4x + 14$

11. a. $h(x) = -3(x - 4)^2 + 5$ **b.** (4, 5) **c.** (0, −43)

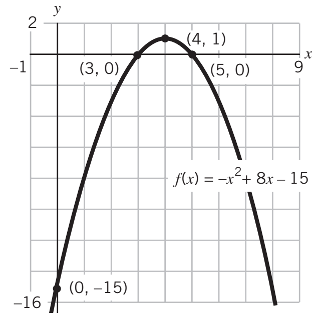
13. a. 9; 3 **c.** 4; 2 **e.** 1; 1
b. 16; 4 **d.** 2.25; 1.5 **f.** 0.25; 0.5

15. a. $y = (x + 3)^2 + 4$
b. $f(x) = (x - 2.5)^2 - 11.25$

c. $g(x) = (x - 1.5)^2 + 3.75$
d. $p(r) = -3(r - 3)^2 + 18$
e. $m(z) = 2(z + 2)^2 - 13$

17. The larger in absolute value the coefficient of the x^2 term, the narrower the opening. Thus the order from narrow to broad is: d, f, a, b, c, and finally e. Technology confirms the principle.

19. a. The diagram exhibits the graph of f, its horizontal and vertical intercepts, and its vertex.



b. $f(x) = -(x - 4)^2 + 1$, and thus its vertex is at (4, 1).

21. a. $y = a(x - 2)^2 + 4$ and $7 = a(1 - 2)^2 + 4$ gives $a = 3$ and thus $y = 3(x - 2)^2 + 4 = 3x^2 - 12x + 16$.

b. If $a > 0$, the graph of $y = a(x - 2)^2 - 3$ is concave up; it is concave down if $a < 0$.

c. The axis of symmetry is the line $y = (-2 + 4)/2 = 1$; since the parabola is concave downward, we have $a < 0$; also we have $y = a(x - 1)^2 + k$ with $5 = a(-2 - 1)^2 + k$ and $5 = a(4 - 1)^2 + k$. Thus we have $9a + k = 5$ or $k = 5 - 9a$. One can choose $a = -1/9$ and then $k = 6$, or one can choose $a = -2/9$ and then $k = 7$. Thus two examples are $y = (-1/9)(x - 1)^2 + 6$ or $y = (-2/9)(x - 1)^2 + 7$.

23. a. $(4, -5)$; no **b.** 0.5 **c.** -2

25. Derivation: from the data we have:

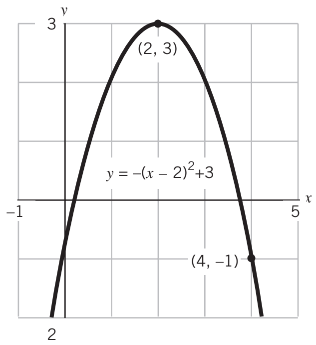
$y = a(x - 2)^2 + 3$; and $-1 = a(4 - 2)^2 + 3$

This implies that $a = -1$.

Thus $y = -(x - 2)^2 + 3$ is its equation.

Check: $-(4 - 2)^2 + 3 = -4 + 3 = -1$

This is confirmed in the accompanying graph.



27. The maximum profit occurs when $x = -20/[2 \cdot (-0.5)] = 20$; the maximum profit is 430 thousand dollars.

29. The maximum occurs when $x = -48/(-6) = 8$ computers and the revenue from selling 8 will be 192 million dollars.

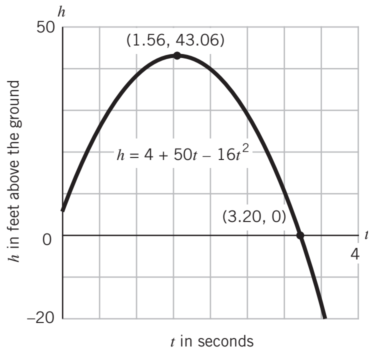
31. a. 4 ft = how high off the ground the baseball was at the instant at which it was hit.

b.

t(sec)	h(feet)
0	4
1	38
2	40
3	10
4	-52

The ball hits the ground between $t = 3$ and $t = 4$ sec, since the height at $t = 3$ is positive and the height at $t = 4$ is negative.

c. The accompanying diagram gives the graph and confirms the estimate given in part (b).

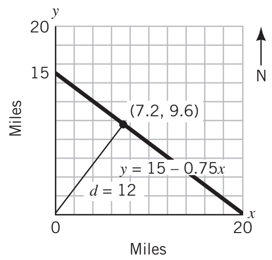


d. Negative values for h make no sense since this would mean that the ball is below ground.

e. Similarly, using technology we can determine that the ball reaches its maximum height of approximately 43.96 ft at $t \approx 1.56$ sec.

33. We are given that $2W + L = 1$ cowhide length. Thus $L = 1 - 2W$ and thus the area formula is $A = W(1 - 2W) = W - 2W^2$. Note this is at its maximum when W is at the vertex, i.e., when $W = -1/[2 \cdot (-2)] = 1/4$. At this value of W we have that $L = 1/2$. Thus, Dido was correct.

35. a. The graph is given in the accompanying diagram, with x and y marked off in miles, and with a sample point (7.2, 9.6) marked on the highway along with the straight line from the origin to that point.



b. The highway goes through the points (0, 15) and (20, 0) and thus has the equation $y = -0.75x + 15$.

c, d. The distance squared:

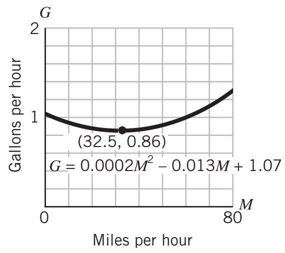
$$d^2 = x^2 + y^2 = x^2 + (15 - 0.75x)^2$$
$$= 1.5625x^2 - 22.5x + 225$$

Ch. 8

e. Letting $D = d^2$, we have $D = 1.5625x^2 - 22.5x + 225$; the minimum occurs at the vertex, which is at $x = 22.5/(2 \cdot 1.5625) = 7.2$. The minimum for D is 144 and thus the minimum distance, d, is $\sqrt{144}$ or 12 miles.

f. The coordinates of the point of this shortest distance from (0, 0) are (7.2, 9.6). [See the graph in part (a).]

37. a, b. The graph is shown below. The minimum gas consumption rate suggested by the graph occurs when M is about 32 mph, and it is approximately 0.85 gph. (Computation on a calculator gives 32.5 for M, and the corresponding gas consumption rate is 0.86 (when rounded off).)



c. In 2 hours 1.72 gallons will be used and you will have traveled 65 miles.

d. If $M = 60$ mph, then $G = 1.01$ gph. It takes 1 hour and 5 minutes to travel 65 miles at 60 mph and one will have used approximately 1.094 gallons.

e. Clearly, traveling at the speed that supposedly minimizes the gas consumption rate does not conserve fuel if the trip lasts only 2 hours.

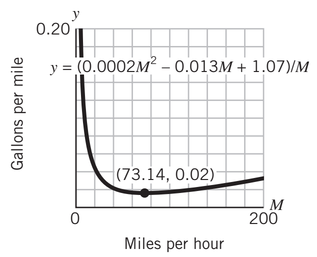
f, g.

M (mph)	G (gph)	G/M (gpm)	M/G (mpg)
0	1.07	—	0.0
10	0.96	0.09600	10.4
20	0.89	0.04450	22.5
30	0.86	0.02867	34.9
40	0.87	0.02175	46.0
50	0.92	0.01840	54.3
60	1.01	0.01683	59.4
70	1.14	0.01629	61.4
80	1.31	0.01638	61.1

For (f) we have: $G/M = (0.0002M^2 - 0.013M + 1.07)/M$.

Its graph is in the accompanying diagram. Eyeballing gives the minimum gpm at $M \approx 73$ mph.

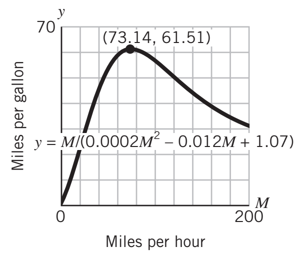
Graph for $y = G/M$



For (g) we have: $M/G = M/(0.0002M^2 - 0.013M + 1.07)$.

Its graph is given in the accompanying diagram. Eyeballing gives the maximum mpg at the same M of approximately 73 mph. This is expected since maximum = 1/minimum.

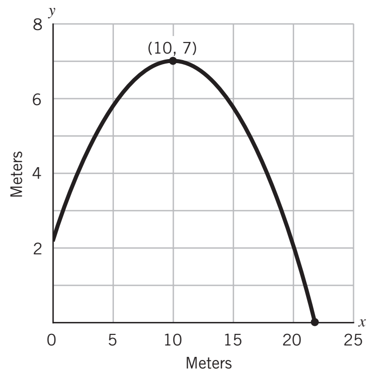
Graph for $y = M/G$



39. a. Time of release is at $x = 0$. The height then is 2 meters.

b. At $x = 4$ m, $y = 5.2$ m; at $x = 16$ m, $y = 5.2$ m.

c, d. The graph in the accompanying diagram shows the highest point, namely when $x = 10$ m and $y = 7$ m, and the point where the shot put hits the ground, namely when x is approximately 22 m.



Section 8.3

Algebra Aerobics 8.3a

1. a. $y = -2t(8t - 25) = 0 \Rightarrow -2t = 0$ or $8t - 25 = 0 \Rightarrow t = 0$ or $t = \frac{25}{8} \Rightarrow$ horizontal intercepts at (0, 0) and $\left(\frac{25}{8}, 0\right)$

b. $y = (t - 5)(t + 5) = 0 \Rightarrow t = 5$ or $t = -5 \Rightarrow$ horizontal intercepts at (5, 0) and (−5, 0)

c. $h(z) = (z - 4)(z + 1) = 0 \Rightarrow z - 4 = 0$ or $z + 1 = 0 \Rightarrow z = 4$ or $z = -1 \Rightarrow$ horizontal intercepts at (4, 0) and (−1, 0)

d. $g(x) = (2x - 3)(2x + 3) = 0 \Rightarrow 2x - 3 = 0$ or $2x + 3 = 0 \Rightarrow x = 3/2$ or $x = -3/2 \Rightarrow$ horizontal intercepts at (3/2, 0) and (−3/2, 0)

e. $y = (5 - x)(3 - x) = 0 \Rightarrow 5 - x = 0$ or $3 - x = 0 \Rightarrow x = 5$ or $x = 3 \Rightarrow$ horizontal intercepts at (5, 0) and (3, 0)

f. $v(x) = (x + 1)^2 = 0 \Rightarrow x + 1 = 0 \Rightarrow x = -1$, or one horizontal intercept at (−1, 0)

g. $p(q) = (q - 3)(q - 3) = 0 \Rightarrow q - 3 = 0 \Rightarrow q = 3 \Rightarrow$ one horizontal intercept at (3, 0)

2. a. $f(x) = (5 + 4x)(1 - x) = 0 \Rightarrow 5 + 4x = 0$ or $1 - x = 0 \Rightarrow x = -5/4$ or $x = 1 \Rightarrow$ horizontal intercepts at (−5/4, 0) and (1, 0)

CH. 8 Algebra Aerobics Solutions for Section 8.3

b. $h(t) = (8 - 3t)(8 + 3t) = 0 \Rightarrow 8 - 3t = 0$ or $8 + 3t = 0 \Rightarrow t = \frac{8}{3}$ or $t = -\frac{8}{3} \Rightarrow$ horizontal intercepts at $\left(\frac{8}{3}, 0\right)$ and $\left(-\frac{8}{3}, 0\right)$

c. $y = (5 + t)(2 - 3t) = 0 \Rightarrow 5 + t = 0,\ 2 - 3t = 0 \Rightarrow t = -5$ or $t = 2/3 \Rightarrow$ horizontal intercepts at $(-5, 0)$ and $(2/3, 0)$

d. $z = (2w - 5)(2w - 5) = 0 \Rightarrow 2w = 5 \Rightarrow w = 5/2 \Rightarrow$ one horizontal intercept $(5/2, 0)$

e. $y = (2x - 5)(x + 1) = 0 \Rightarrow 2x - 5 = 0$ or $x + 1 = 0 \Rightarrow x = \frac{5}{2}$ or $x = -1 \Rightarrow$ horizontal intercepts at $\left(\frac{5}{2}, 0\right)$ and $(-1, 0)$

f. $Q(t) = (3t - 2)(2t + 5) = 0 \Rightarrow 3t - 2 = 0$ or $2t + 5 = 0 \Rightarrow t = \frac{2}{3}$ or $t = -\frac{5}{2} \Rightarrow$ horizontal intercepts at $\left(\frac{2}{3}, 0\right)$ and $\left(-\frac{5}{2}, 0\right)$

3. Product of a sum and difference:

a. $y = x^2 - 9 = (x + 3)(x - 3)$

d. $y = 9x^2 - 25 = (3x + 5)(3x - 5)$

f. $y = 16 - 25x^2 = (4 + 5x)(4 - 5x)$

Square of sum or difference:

b. $y = x^2 + 4x + 4 = (x + 2)^2$

e. $y = x^2 - 8x + 16 = (x - 4)^2$

Neither:

c. $y = x^2 + 5x + 25$

g. $y = 4 + 16x^2$

Ch. 8

4. a. The vertical intercept is $(0, 0)$, which means at time $t = 0$, the object is on the ground. The horizontal intercepts: $h(t) = -16t(t - 4) = 0 \Rightarrow t = 0$ or $t = 4 \Rightarrow (0, 0)$ and $(4, 0)$, which means the object left the ground at time t $= 0$ seconds and returns to the ground at time $t = 4$ seconds.

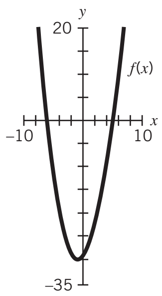
b. The horizontal intercepts: $P(q) = -(q^2 - 60q + 800) \Rightarrow P(q) = -(q - 20)(q - 40) = 0 \Rightarrow q = 20$ or $q = 40$. This means that if either 20 or 40 units are sold, the profit is \$0 (or at breakeven). From the graph, $P(q) > 0$ if $20 < q < 40$. The vertical intercept is $(0, -\$800)$, which means that if no items are sold, there is a loss of \$800.

5. y_1 is Graph C because $(0, 0)$ and $(2, 0)$ are horizontal intercepts, and graph is concave down.
y_2 is Graph A because $(2, 0)$ and $(-1, 0)$ are horizontal intercepts, and graph is concave up.
y_3 is Graph B because $(-4, 0)$ and $(-1, 0)$ are horizontal intercepts, and graph is concave up.

6. a. $f(x) = (x - 5)(x + 6)$

b. $(5, 0)$ and $(-6, 0)$

c.



d. $f(x) = 0 \Rightarrow (x - 5)(x + 6) = 0 \Rightarrow x - 5 = 0$ or $x + 6 = 0 \Rightarrow x = 5$ or $x = -6$. So $f(x)$ has two horizontal intercepts, at $(5, 0)$ and $(-6, 0)$.

Algebra Aerobics 8.3b

1. a. $4x + 7 = 0 \Rightarrow x = -7/4$

b. $4x^2 - 7 = 0 \Rightarrow 4x^2 = 7 \Rightarrow x^2 = \frac{7}{4}$
$\Rightarrow x = \pm\sqrt{\frac{7}{4}} \Rightarrow x = \pm\frac{\sqrt{7}}{2}$

c. $4x^2 - 7x = 0 \Rightarrow x(4x - 7) = 0 \Rightarrow x = 0$ or $x = \frac{7}{4}$

d. $2x + 6 = x^2 \Rightarrow$
$0 = x^2 - 2x - 6 \Rightarrow$
$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(-6)}}{2} \Rightarrow$
$x = \frac{2 \pm \sqrt{28}}{2} \Rightarrow x = \frac{2 \pm 2\sqrt{7}}{2} \Rightarrow$
$x = \frac{2(1 \pm \sqrt{7})}{2} \Rightarrow x = 1 \pm \sqrt{7}$

e. $(2x - 11)^2 = 0 \Rightarrow 2x - 11 = 0 \Rightarrow x = \frac{11}{2}$

f. $(x + 1)^2 = 81 \Rightarrow x + 1 = \pm\sqrt{81} \Rightarrow$
$x = -1 \pm 9 \Rightarrow x = -10$ or $x = 8$

g. $0 = x^2 - x - 5 \Rightarrow$
$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(-5)}}{2} \Rightarrow x = \frac{1 \pm \sqrt{21}}{2}$

2. a. $a = 2, b = 3, c = -1 \Rightarrow D = (3)^2 - 4(2)(-1) = 17 \Rightarrow$ two real, unequal zeros $\Rightarrow$ two horizontal intercepts

b. $a = 1, b = 7, c = 2 \Rightarrow D = (7)^2 - 4(1)(2) = 41 \Rightarrow$ two real, unequal zeros $\Rightarrow$ two horizontal intercepts

c. $a = 4, b = 4, c = 1 \Rightarrow D = (4)^2 - 4(4)(1) = 0 \Rightarrow$ one real zero (also known as a "double zero") $\Rightarrow$ one horizontal intercept

d. $a = 2, b = 1, c = 5 \Rightarrow D = (1)^2 - 4(2)(5) = -39 \Rightarrow$ no real zeros (two imaginary zeros) $\Rightarrow$ no horizontal intercepts

3. a. $h = -4.9t^2 + 50t + 80$. The vertical intercept is the initial height (at 0 seconds), which is 80 m; coordinates are $(0, 80)$. $a = -4.9, b = 50, c = 80$. So when $h = 0$ the horizontal intercepts are:

$$t = \frac{-50 \pm \sqrt{2500 - 4(-4.9)(80)}}{2(-4.9)}$$
$$= \frac{-50 \pm \sqrt{2500 + 1568}}{-9.8} = \frac{-50 \pm \sqrt{4068}}{-9.8} \Rightarrow$$
$$t = \frac{-50 + 63.8}{-9.8} \quad \text{or} \quad t = \frac{-50 - 63.8}{-9.8}$$
$$= \frac{13.8}{-9.8} \qquad = \frac{-113.8}{-9.8}$$
$$= -1.41 \qquad = 11.61 \text{ seconds}$$

Negative values of t have no meaning in a height equation, so the horizontal intercept at $(11.61, 0)$ means that the object hits the ground after 11.61 seconds.

b. $h = 150 - 80t - 490t^2$. The vertical intercept is $(0, 150)$, which means that the initial height is 150 cm. $a = -490$, $b = -80$, $c = 150 \Rightarrow$ when $h = 0$ the horizontal intercepts are at:

$$t = \frac{80 \pm \sqrt{6400 - 4(-490)(150)}}{2(-490)}$$
$$= \frac{80 \pm \sqrt{6400 + 294{,}000}}{-980}$$
$$= \frac{80 \pm \sqrt{300{,}400}}{-980} = \frac{80 \pm 548}{-980} \Rightarrow$$
$$t = \frac{80 + 548}{-980} \quad \text{or} \quad t = \frac{80 - 548}{-980}$$
$$= \frac{628}{-980} \qquad = \frac{-468}{-980}$$
$$= -0.64 \qquad = 0.48 \text{ seconds.}$$

Discard negative solution. The object hits the ground after 0.48 seconds.

c. The vertical intercept is the height in feet at $t = 0$ seconds $\Rightarrow h = 3$ feet.

If $a = -16$, $b = 64$, $c = 3 \Rightarrow$ the horizontal intercepts are:

$$t = \frac{-64 \pm \sqrt{(64)^2 - 4(-16)(3)}}{2(-16)} \Rightarrow t = \frac{-64 \pm \sqrt{4288}}{-32}$$

$\Rightarrow t = \frac{-64 \pm 65.5}{-32} \Rightarrow t \approx 4.05$, which means the object hits the ground after about 4.05 seconds or $t \approx -0.05$ (which is meaningless in this context).

d. The vertical intercept is the height in feet at $t = 0$ seconds $\Rightarrow h = 64(0) - 16(0)^2 = 0$ feet.

$a = -16$, $b = 64$, $c = 0 \Rightarrow$ horizontal intercepts are

$$t = \frac{-64 \pm \sqrt{(64)^2 - 4(-16)(0)}}{2(-16)} \Rightarrow t = \frac{-64 \pm 64}{-32} \Rightarrow$$

horizontal intercepts are $t = 0$ and $t = 4 \Rightarrow$ the object hits the ground after exactly 4 seconds.

4. Discriminant is $b^2 - 4ac$, from $ax^2 + bx + c = y$

a. $a = -5$, $b = -1$, $c = 4 \Rightarrow$ discriminant $= 1 - 4(-5)(4) = 81 > 0 \Rightarrow$ two x-intercepts. $\sqrt{81} = 9$, so roots are rational. The function has two real zeros where:

$$x = \frac{1 \pm 9}{-10} \Rightarrow$$

$$x = \frac{1 + 9}{-10} = \frac{10}{-10} = -1 \quad \text{or} \quad x = \frac{1 - 9}{-10} = \frac{-8}{-10} = \frac{4}{5}$$

So the x-intercepts are $(-1, 0)$ and $(4/5, 0)$.

b. $a = 4$, $b = -28$, $c = 49 \Rightarrow$ discriminant $=$ $784 - 4(4)(49) = 784 - 784 = 0 \Rightarrow$ one x-intercept.

$\sqrt{0} = 0$, so root is rational. The x-intercept is where $x = \frac{28 \pm 0}{8} = \frac{7}{2}$; at $\left(3\frac{1}{2}, 0\right)$.

c. $a = 2$, $b = 5$, $c = 4 \Rightarrow$ discriminant $= 25 - 4(2)(4) = 25 - 32 = -7$, which is negative $\Rightarrow$ no x-intercepts. The function has two imaginary zeros at

$$x = \frac{-5 \pm \sqrt{-7}}{4} = \frac{-5 \pm \sqrt{7}i}{4}$$

d. $a = 2$, $b = -3$, $c = -1 \Rightarrow$ discriminant $=$ $(-3)^2 - 4(2)(-1) = 9 + 8 = 17 \Rightarrow$ two real zeros

$$x = \frac{-(-3) \pm \sqrt{17}}{2(2)} \Rightarrow x = \frac{3 \pm \sqrt{17}}{4} \Rightarrow$$

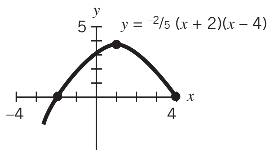
x-intercepts are $\left(\frac{3 + \sqrt{17}}{4}, 0\right)$ and $\left(\frac{3 - \sqrt{17}}{4}, 0\right)$

e. $a = -3$, $b = 0$, $c = 2 \Rightarrow$ discriminant $=$ $(0)^2 - 4(-3)(2) = 24 \Rightarrow$ two real zeros

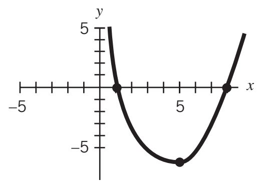
$$x = \frac{-0 \pm \sqrt{24}}{2(-3)} \Rightarrow x = \frac{\pm\sqrt{24}}{6} \Rightarrow x = \frac{\pm 2\sqrt{6}}{6} \Rightarrow$$

$x = \mp \frac{\sqrt{6}}{3} \Rightarrow$ x-intercepts are $\left(-\frac{\sqrt{6}}{3}, 0\right), \left(+\frac{\sqrt{6}}{3}, 0\right)$

5. a. $y = a(x + 2)(x - 4)$. The point $(3, 2)$ is on the parabola $\Rightarrow$ $2 = a(3 + 2)(3 - 4) \Rightarrow a = -\frac{2}{5} \Rightarrow$ $y = -\frac{2}{5}(x + 2)(x - 4)$. The vertex is on the line of symmetry, which lies halfway between the x-intercepts or at $x = 1 \Rightarrow y = \frac{-2}{5}(1 + 2)(1 - 4) \Rightarrow y = \frac{18}{5} \Rightarrow$ the vertex is at $\left(1, \frac{18}{5}\right)$.



b. $y = a(x - 2)(x - 8)$. The vertical intercept is $(0, 10) \Rightarrow$ $10 = a(0 - 2)(0 - 8) \Rightarrow a = \frac{5}{8}$, so $y = \frac{5}{8}(x - 2)(x - 8)$



The vertex is on the line of symmetry, which lies halfway between the x-intercepts or at $x = 5 \Rightarrow y = \frac{5}{8}(5 - 2)(5 - 8) \Rightarrow y = \frac{-45}{8}$ and the vertex is $\left(5, \frac{-45}{8}\right)$.

6. a. Vertex is $(1, 5)$, above x-axis; $a = 3$ is positive, so it opens up; so there are no x-intercepts.

b. Vertex is $(-4, -1)$, below x-axis; $a = -2$ is negative, so it opens down; so there are no x-intercepts.

c. Vertex is $(-3, 0)$, on x-axis; $a = -5$ is negative, so it opens down; so there is one x-intercept.

d. Vertex is $(1, -2)$, below x-axis; $a = 3$ is positive, so it opens up; so there are two x-intercepts.

7. Answers will vary for different values of a and will be of the form:

a. $f(x) = a(x - 2)(x + 3)$
If $a = 1$, then
$f(x) = (x - 2)(x + 3)$

b. $f(x) = ax(x + 5)$
If $a = 2$, then
$f(x) = 2x(x + 5)$

c. $f(x) = a(x - 8)^2$
If $a = -2$, then
$f(x) = -2(x - 8)^2$

8. Graph A: two real, unequal zeros $\Rightarrow$ discriminant is positive
Graph B: one real zero $\Rightarrow$ discriminant is equal to zero
Graph C: no real zeros $\Rightarrow$ discriminant is negative

Exercises for Section 8.3

1. a. $0 = (x - 3)(x + 3)$; thus $x = 3$ or -3

b. $0 = x(4 - x)$; thus $x = 0$ or 4

c. $0 = x(3x - 25)$; thus $x = 0$ or $25/3$

d. $0 = (x + 5)(x - 4)$; thus $x = -5$ or 4

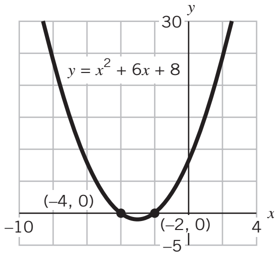
e. $0 = (2x + 3)^2$; thus $x = -3/2$ twice

f. $0 = (3x + 2)(x - 5)$; thus $x = -2/3$ or 5

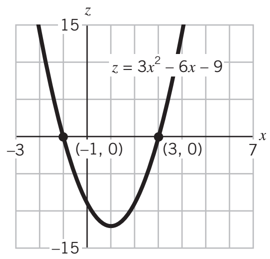
g. $x^2 + 4x + 3 = -1 \Rightarrow x^2 + 4x + 4 = 0 \Rightarrow (x + 2)^2 = 0$; thus $x = -2$ (a double zero)

h. $x^2 + 2x = 3x^2 - 3x - 3 \Rightarrow 2x^2 - 5x - 3 = 0 \Rightarrow$ $(2x + 1)(x - 3) = 0$; thus $x = -1/2$ or 3

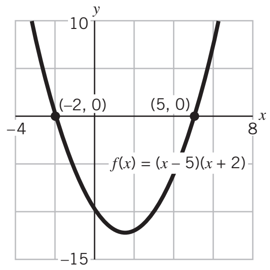
3. a. $y = (x + 4)(x + 2)$



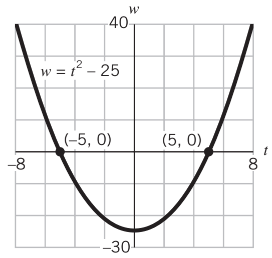
b. $z = 3(x + 1)(x - 3)$



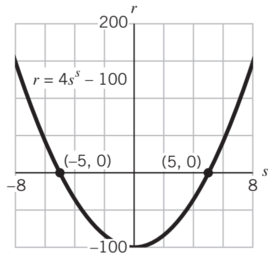
c. $f(x) = (x - 5)(x + 2)$



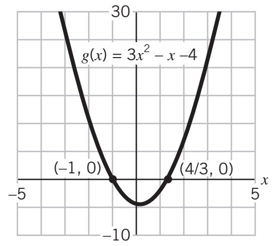
d. $w = (t - 5)(t + 5)$



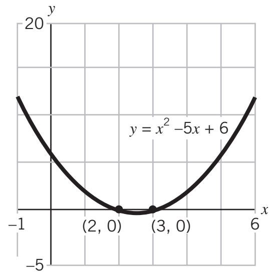
e. $r = 4(s - 5)(s + 5)$



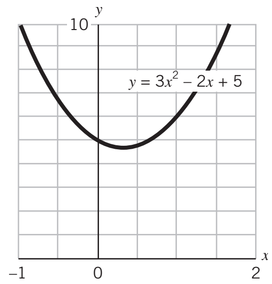
f. $g(x) = (3x - 4)(x + 1)$



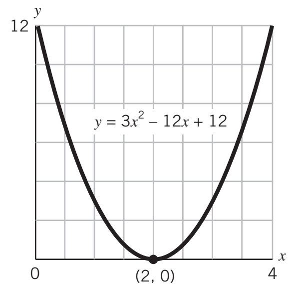
5. a. $y = x^2 - 5x + 6$ has zeros at 2 and 3, as is shown in the graph below.



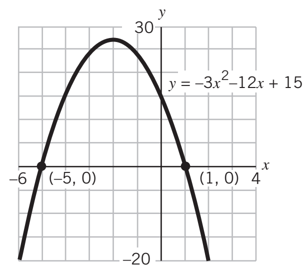
b. $y = 3x^2 - 2x + 5$ has no real zeros, as is shown in the graph below.



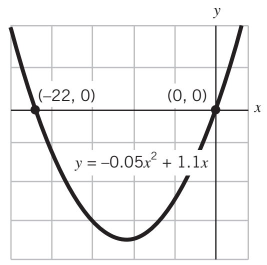
c. $y = 3x^2 - 12x + 12$ has a "double zero" at $x = 2$ since $y = 3(x - 2)^2$. See the graph below.



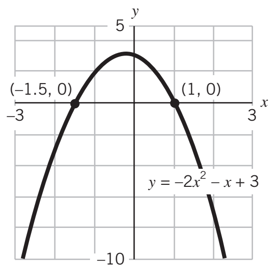
d. $y = -3x^2 - 12x + 15$ has zeros at $x = -5$ and 1; see the graph below.



e. $y = 0.05x^2 + 1.1x$ has zeros at $x = -22$ and 0, as the graph below shows.



f. $y = -2x^2 - x + 3$ has roots at $x = -1.5$ and 1, as the graph below shows.



7. a. $t = \frac{7 \pm \sqrt{49 - 4 \cdot 6 \cdot (-5)}}{2 \cdot 6} = \frac{7 \pm \sqrt{169}}{12}$
$= \frac{7 \pm 13}{12} = \frac{5}{3} \text{ or } -\frac{1}{2}$

b. $x = \frac{12 \pm \sqrt{144 - 4 \cdot 9 \cdot 4}}{2 \cdot 9} = \frac{12 \pm \sqrt{0}}{18} = \frac{2}{3}$

c. $z = \frac{1 \pm \sqrt{1 - 4 \cdot 3 \cdot (-9)}}{2 \cdot 3} = \frac{1 \pm \sqrt{109}}{6} \approx -1.573$ or 1.907

d. $x = \frac{-6 \pm \sqrt{36 - 4 \cdot 1 \cdot 7}}{2 \cdot 1} = \frac{-6 \pm \sqrt{8}}{2} = -3 \pm \sqrt{2}$
$\approx -1.586 \text{ or } -4.414$

e. $s = \frac{-17 \pm \sqrt{17^2 - 4 \cdot 6 \cdot (-10)}}{2 \cdot 6} = \frac{-17 \pm \sqrt{529}}{12}$
$= \frac{-17 \pm 23}{12} = -\frac{10}{3} \text{ or } \frac{1}{2}$

f. $t = \frac{3 \pm \sqrt{9 - 4 \cdot 2 \cdot (-9)}}{2 \cdot 2} = \frac{3 \pm \sqrt{81}}{4} = \frac{3 \pm 9}{4} = 3$ or -1.5

g. $x = \frac{11 \pm \sqrt{121 - 4 \cdot 4 \cdot (-8)}}{2 \cdot 4} = \frac{11 \pm \sqrt{249}}{8} \approx 3.347$ or -0.597

h. $x = \frac{12 \pm \sqrt{144 - 4 \cdot 4 \cdot 2}}{2 \cdot 4} = \frac{12 \pm \sqrt{112}}{8}$
$= \frac{12 \pm 4\sqrt{7}}{8} = \frac{3 \pm \sqrt{7}}{2} \approx 2.823 \text{ or } 0.177$

9. a. y-intercept is -1; $y = (3x - 1)(x + 1)$ and thus x-intercepts are 1/3 and -1.

b. y-intercept is 11; the x-intercepts are $\frac{6 \pm \sqrt{3}}{3} \approx 2.58$ and 1.42.

c. y-intercept is 15; x-intercepts are 5/2 and –3/5.

d. The vertical intercept is $f(0) = -5$ and the x-intercepts are $\pm\sqrt{5}$.

11. Student choices for values of a, b and c will vary. Here are some choices and the accompanying graphs. The equations in the form $y = ax^2 + bx + c$ are in the graph diagrams.

a. $a = 2, b = 3, c = 1; b^2 - 4ac = 9 - 8 = 1$

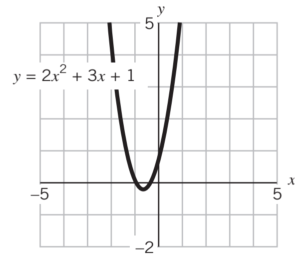
b. $a = 2, b = 3, c = -2; b^2 - 4ac = 9 + 16 = 25$

c. $a = 2, b = 3, c = 4; b^2 - 4ac = 9 - 32 = -23$

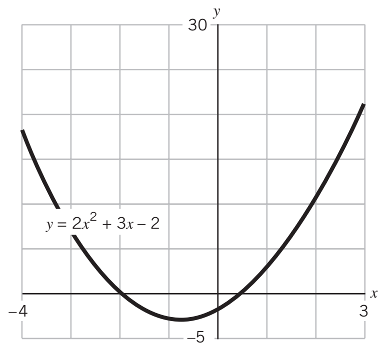
d. $a = -1, b = 2, c = -1; b^2 - 4ac = 4 - 4 = 0$

e. same as in (b) above.

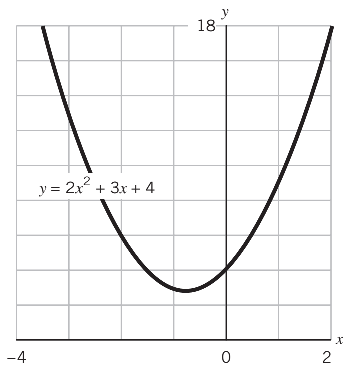
Graph for (a)



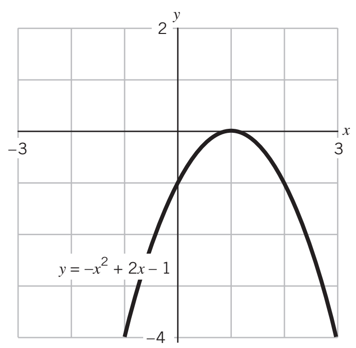
Graph for (b), (e)



Graph for (c)



Graph for (d)



13. a. $Q(t) = -4(t + 1)^2$

b. No, since it must be of the form $Q(t) = a(t + 1)^2$ if 1 is to be a double root and a must be -4 if $Q(0) = -4$.

c. The axis of symmetry is the line whose equation is $t = -1$. The vertex is at $(-1, 0)$.

15. For Graph A: $f(x) = (x + 5)(x - 2) = x^2 + 3x - 10$

For Graph B: $g(x) = -0.5(x + 5)(x - 2) = -0.5x^2 - 1.5x + 5$

17. a. $-1 + 10i$ **b.** 1 **c.** $-1 + 3i$ **d.** $-5 + 11i$

19. $f(x) = (x - (1 + i))(x - (1 - i)) =$
$x^2 - (1 - i)x - (1 + i)x + (1 + i)(1 - i) =$
$x^2 - 2x + (1 - i^2) =$
$x^2 - 2x + 2$

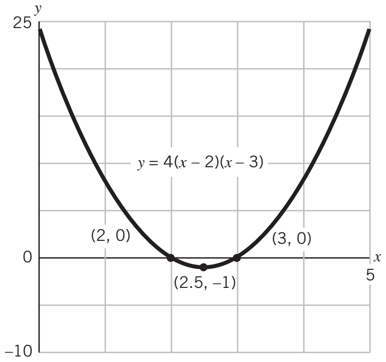
21. $x = \frac{4 \pm \sqrt{16 - 4 \cdot 1 \cdot 13}}{2 \cdot 1} = \frac{4 \pm \sqrt{-36}}{2}$
$= \frac{4 \pm 6i}{2} = 2 \pm 3i.$

Thus $f(x) = (x - 2 - 3i)(x - 2 + 3i)$. Its roots are not real and thus there are no x-intercepts.

23. a. Factoring, $y = (x + 4)(x - 2)$ and thus the roots are -4 and 2 and their average is -1. Thus $h = -1$ and $a = 1$; therefore $y = (x + 1)^2 + k = x^2 + 2x + 1 + k = x^2 + 2x - 8$ and thus $k + 1 = -8$ or $k = -9$. Thus the equation is $y = (x + 1)^2 - 9$.

b. Factoring gives $y = -(x + 4)(x - 1)$ and thus its roots are -4 and 1 and thus $h = -1.5$ and $a = -1$; therefore $y = -(x + 1.5)^2 + k = -x^2 - 3x - 2.25 + k = -x^2 - 3x + 8$ and thus $4 = k - 2.25$ or $k = 6.25$. Thus the equation is $y = -(x + 1.5)^2 + 6.25$.

25. $y = 4(x - 2)(x - 3)$. If the quadratic is to go through (2, 0) and (3, 0), then it must be of the form $y = a(x - 2)(x - 3)$; and if it is to stretch the graph of $y = x^2$ by a factor of 4, then $a = 4$ must hold. But this function's graph does more. It shifts the vertex of the graph of $y = x^2$ to $(2.5, -1)$. Its graph is in the accompanying diagram.

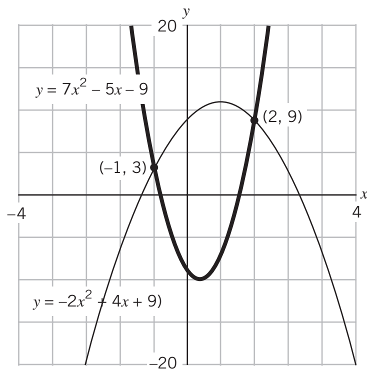


27. a. Setting the two formulas for y equal gives:

$7x^2 - 5x - 9 = -2x^2 + 4x + 9 \Rightarrow$
$0 = 9x^2 - 9x - 18 = 9(x^2 - x - 2)$
$= 9(x - 2)(x + 1)$

and thus the intersection points are where $x = -1$ and $x = 2$. If $x = -1$, then $y = 3$ and if $x = 2$, then $y = 9$ (by substitution into either original equation). The graph confirming this information is given in the accompanying diagram.

b.

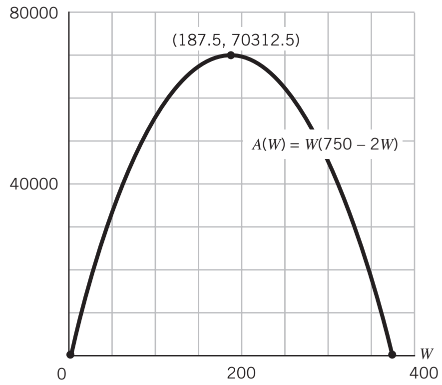


29. a. $1500 = 4W + 2L$ and thus
$L = (1500 - 4W)/2 = 750 - 2W$

b. $A(W) = W(750 - 2W) = 2W(375 - W)$

c. Domain for $A(W)$ is $0 < W < 375$

d. The area is largest when the W value is at the vertex of the parabola graph of $A(W)$, namely, when $W = 187.5$ ft. At that point $L = 375$ ft. and thus the area of the largest rectangle is 70,312.5 sq. ft. See the accompanying graph for verification.



Section 8.4

Algebra Aerobics 8.4

1.

x	y	Average Rate of Change	Average Rate of Change of Average Rate of Change
-1	4	n.a.	n.a.
0	5	$\frac{5 - 4}{0 - (-1)} = 1$	n.a.
1	4	$\frac{4 - 5}{1 - 0} = -1$	$\frac{(-1) - 1}{1 - 0} = -2$
2	1	$\frac{1 - 4}{2 - 1} = -3$	$\frac{(-3) - (-1)}{2 - 1} = -2$
3	-4	$\frac{(-4) - 1}{3 - 2} = -5$	$\frac{(-5) - (-3)}{3 - 2} = -2$
4	-11	$\frac{(-11) - (-4)}{4 - 3} = -7$	$\frac{(-7) - (-5)}{4 - 3} = -2$

b. The third column tells us that the average rate of change of y with respect to x is decreasing at a constant rate, so the relationship is linear. The fourth column tells us that the average rate of change of the average rate of change is constant at -2.

2. a. The function is quadratic since the average rate of change is linear, that is, the average rate of change is increasing at a constant rate.

b. The function is linear since the average rate of change is constant.

c. The function is exponential since both the average rate of change and the average rate of change of the average rate of change are exponential, that is, are multiplied by a factor of 2 or increasing at a constant percentage.

3. a. The slope of the average rate of change $2a = 2$, giving $y = 2t + b$. The vertical intercept $b = 1$, so $y = 2t + 1$ is the equation of the average rate of change.

b. The slope of the average rate of change $2a = 6$, giving $y = 6x + b$. The vertical intercept $b = 5$, so $y = 6x + 5$ is the equation of the average rate of change.

c. The slope of the average rate of change is 10, giving $y = 10x + b$. The vertical intercept $b = 2$, so $y = 10x + 2$ is the equation of the average rate of change.

Exercises for Section 8.4

1. a. linear **b.** positive, negative

3. a. $y = 3$; $y = -2$; $y = a$. It is horizontal line with slope 0 going through the point $(0, a)$.

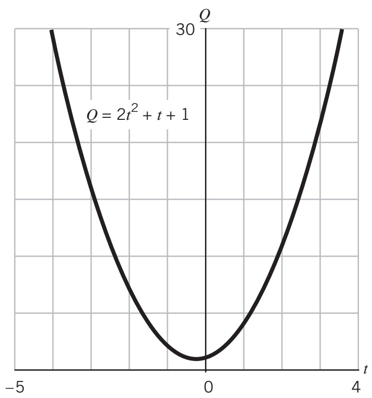
b. It is a straight line with slope $= 2a$ and y-intercept b. No, the slope of linear function is constant. The slope of the quadratic is a linear function.

c. Guesses will vary. You may guess by analogy from the answers to parts (a) and (b) that its function is the quadratic $y = 3ax^2 + 2bx + c$.

5. The table and graph are given below:

t	Q	Average Rate of Change
−3	16	n.a.
−2	7	−9
−1	2	−5
0	1	−1
1	4	3
2	11	7
3	22	11

a. The function Q is quadratic. Its graph is given in the accompanying diagram.



b. The third column indicates that the average rate of change of Q is linear. For each increase of 1 in t it goes up by 4.

7. Graph A goes with Graph F Graph B goes with Graph E Graph C goes with Graph D

9. a. $F(t) = 6t + 1$ **b.** $G(x) = -10x + 0.4$ **c.** $H(z) = 2$

11. a. quadratic **b.** linear **c.** exponential

Section 8.5

Algebra Aerobics 8.5a

1. a. Degree is 5: $f(-1) = 11(-1)^5 + 4(-1)^3 - 11 = 11(-1) + 4(-1) - 11 = -11 - 4 - 11 = -26$

b. Degree is 4: When $x = -1$, $y = 1 + 7(-1)^4 - 5(-1)^3$
$= 1 + 7(1) - 5(-1)$
$= 1 + 7 + 5 = 13$

c. Degree is 4: $g(-1) = -2(-1)^4 - 20 = -2 - 20 = -22$

d. Degree is 2: When $x = -1$, $z = 3(-1) - 4 - 2(-1)^2$
$= -3 - 4 - 2 = -9$

2. a. degree $n = 5 + 3 + 2 = 10$

b. degree $n = 2 + 12 = 14$

3. a. degree 5; a quintic polynomial function

b. The leading term is $-2t^5$.

c. The constant term is 0.5.

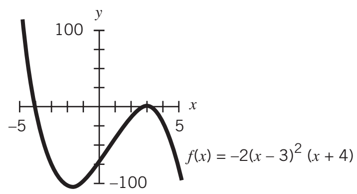
d. $f(0) = 0.5 - 2(0)^5 + 4(0)^3 - 6(0)^2 - (0) = 0.5$

$f(0.5) = 0.5 - 2(0.5)^5 + 4(0.5)^3 - 6(0.5)^2 - (0.5) = -1.0625$

$f(-1) = 0.5 - 2(-1)^5 + 4(-1)^3 - 6(-1)^2 - (-1) = -6.5$

Algebra Aerobics 8.5b

1.



a. degree 3

b. two turning points

c. as $x \to +\infty$, $y \to -\infty$; and as $x \to -\infty$, $y \to +\infty$

d. $y = -2x^3$

e. The horizontal intercepts are (3, 0), where the graph "touches" the x-axis and (−4, 0), where the graph crosses the x-axis.

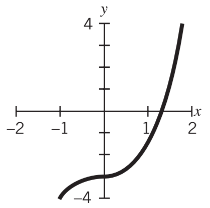
f. $y = -72$; the vertical intercept

2. Graph A: Minimum degree = 3 because two "bumps"; positive leading coefficient

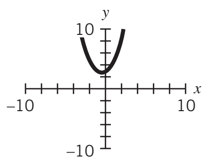
Graph B: Minimum degree = 4 because three "bumps"; negative leading coefficient

Graph C: Minimum degree = 5 because four "bumps"; negative leading coefficient

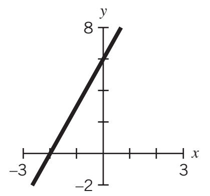
3. a. The y-intercept is at -3. The graph crosses the x-axis only once. It happens at about $x = 1.3$.



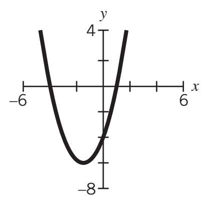
b. The y-intercept is at 3. The graph does not intersect the x-axis.



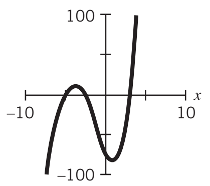
4. a. Degree is 1. x-intercept is -2.



b. Degree is 2. x-intercepts are -4 and 1.

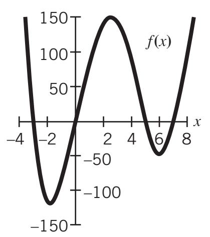


c. Degree is 3. x-intercepts are -5, 3, and $-2\frac{1}{2}$.

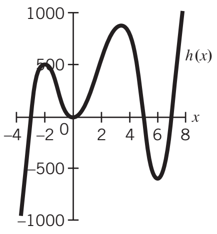


5. There are infinitely many examples of such functions. To have exactly those four x-intercepts, the functions are of the form: $f(x) = ax^n(x + 3)^m(x - 5)^p(x - 7)^r$, where a is a real number, and n, m, p and r are positive integers.

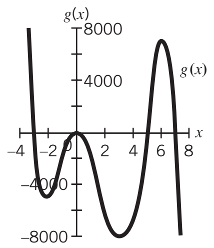
(i) $f(x) = x(x + 3)(x - 5)(x - 7)$



(ii) $h(x) = 2x^2(x + 3)(x - 5)(x - 7)$



(iii) $g(x) = -20x^2(x + 3)(x - 5)(x - 7)$



Exercises for Section 8.5

1. a. polynomial, 1
b. polynomial, 3
c. not polynomial
d. not polynomial
e. polynomial, 5
f. polynomial, 3

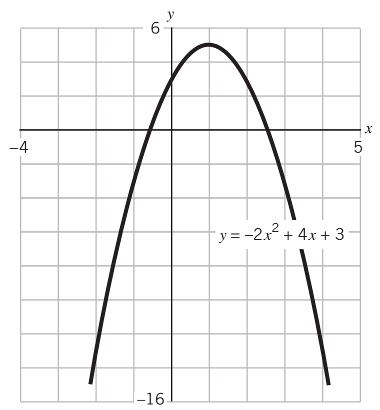
3. a. $\frac{1}{8}, -\frac{1}{8}$
b. $\frac{1}{2}, -\frac{1}{2}$
c. $-\frac{1}{2}, \frac{1}{2}$
d. $-32, 32$

5. a. goes with Graph *A*
b. goes with Graph *C*
c. goes with Graph *B*

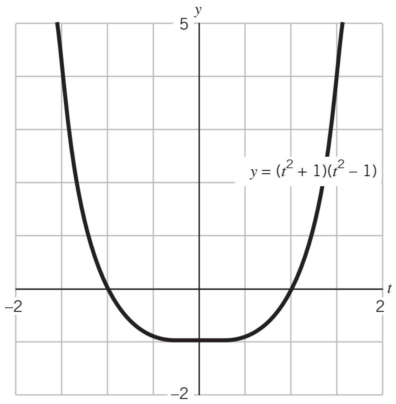
7. For Graph *A*: **i.** 2 **ii.** 2 **iii.** minus **iv.** 3
For Graph *B*: **i.** 3 **ii.** 2 **iii.** minus **iv.** 4
For Graph *C*: **i.** 4 **ii.** 5 **iii.** plus **iv.** 5

9. (a) and (e) are cubics; (b), (d), and (f) are quartics; (c) is by itself, since it is the only quintic.

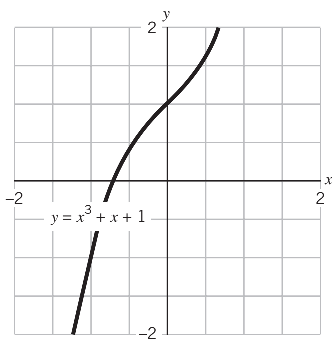
11. a. Always negative; could have up to three turning points; has exactly one turning point (see accompanying graph).



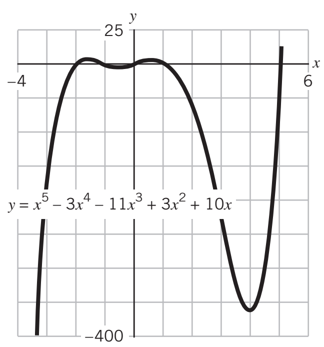
b. Always positive; in general a quartic has at most three turning points; here exactly one turning point (see accompanying graph).



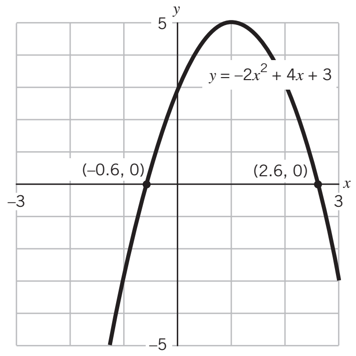
c. Negative if x is negative; positive if x is positive; at most two turning points; here none (see the accompanying graph).



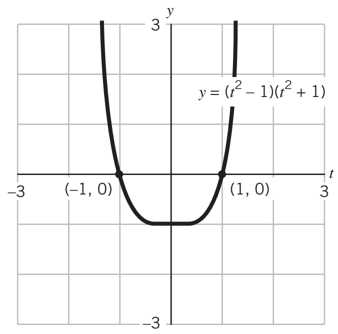
d. Positive if x is positive; negative if x is negative; in general a quintic has at most four turning points; here there are exactly four (see the accompanying graph).



13. a. A quadratic has at most two horizontal intercepts; from the accompanying graph we see that there are two, at $x \approx -0.6$ and 2.6.



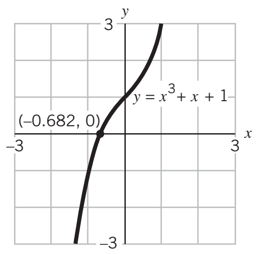
b. A quartic at most four horizontal intercepts; from the accompanying graph we see that it has two: at $t = -1$ and 1.



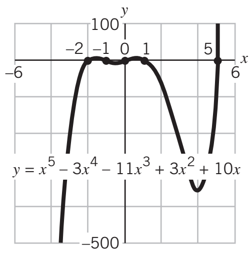
c. A cubic has at most three horizontal intercepts; from the accompanying graph we see that there is only one, at $x \approx 0.682$.

d. A quintic has at most five horizontal intercepts; from the accompanying graph we see that there are five, at $x = -2, -1, 0, 1,$ and 5; in fact we have that $y = (x + 2)(x + 1)x(x - 1)(x - 5)$.

Graph for (c)



Graph for (d)



15. a. $y = 3x^3 - 2x^2 - 3$ has only one real zero at $x \approx 1.28$. (See the accompanying graph.)

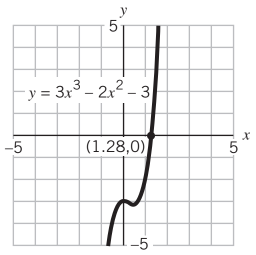
b. $y = x^2 + 5x + 3$ has two real zeros:

$$x = -2.5 \pm 0.5\sqrt{13} \approx -4.302 \text{ and } -0.697.$$

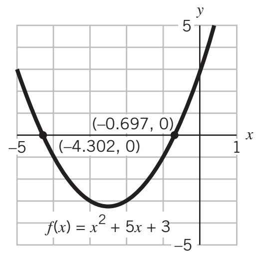
(See the accompanying graph.)

Ch. 8

Graph for (a)

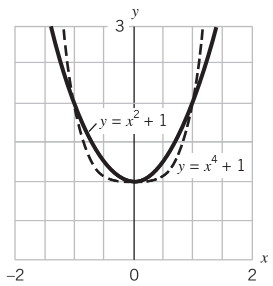


Graph for (b)



17. a. The y-values of the polynomial change sign when going from very large negative values of x to very large positive values of x and thus the graph must cross the x-axis.

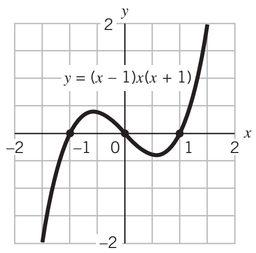
b. $y = x^2 + 1$ and $y = x^4 + 1$ are such polynomials, and their graphs are both in the accompanying graph.

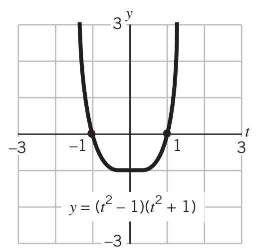


c. i. The polynomial $y = 3x^3 - 2x^2 - 3$ has exactly one real zero and its graph is given in the Solution to Exercise 15(a).

ii. The polynomial $y = (x - 1)x(x + 1)$ has three real zeros at $x = -1$, 0, and 1, as can be seen in the following graph on the left.

d. The polynomial $y = (t^2 - 1)(t^2 + 1)$ has exactly two real zeros and its graph is shown on the right.





19. a. True; it is a function and 0 is in its domain.

b. True; it is a cubic.

c. True.

d. False; it is d and d can be positive, negative, or 0.

e. True, since the cubic term dominates for large positive or negative values of x.

f. False, the origin is a point on the graph if and only if $d = 0$.

21. a. $8701 = 8n^3 + 7n^2 + 0n + 1n^0$

b. $239 = 2n^2 + 3n + 9n^0$

c. The number written in base 2 as 11001 evaluates to 25 when written in base 10 notation, since $(1 \cdot 2^4) + (1 \cdot 2^3) + (0 \cdot 2^2) + (0 \cdot 2^1) + (1 \cdot 2^0) = 25$.

d. Here is one way to find the base two equivalent: find the highest power of 2 in 35. This is $2^5 = 32$. Subtracting that from 35 leaves 3, which is easily written as 2 + 1. Thus 35, in base 10, can be written as

$$(1 \cdot 2^5) + (0 \cdot 2^4) + (0 \cdot 2^3) + (0 \cdot 2^2) + (1 \cdot 2^1) + (1 \cdot 2^0)$$

and this is 100011 in base 2.

Section 8.6

Algebra Aerobics 8.6

1. a. $-f(x)$ matches the graph of $g(x)$ in Graph B because it is a reflection of f across the x-axis.

b. $f(-x)$ matches the graph of $h(x)$ in Graph C because it is a reflection of f across the y-axis.

c. $-f(-x)$ matches the graph of $j(x)$ in Graph A because it is a double reflection of f across both the x- and y-axes.

2. $f(x) = 2x - 3$

a. $f(x + 2) = 2(x + 2) - 3 \Rightarrow f(x + 2) = 2x + 1$

b. $\frac{1}{2}f(x) = \frac{1}{2}(2x - 3) \Rightarrow \frac{1}{2}f(x) = x - \frac{3}{2}$

c. $-f(x) = -(2x - 3) \Rightarrow -f(x) = -2x + 3$

d. $f(-x) = 2(-x) - 3 \Rightarrow f(-x) = -2x - 3$

e. $-f(-x) = -(2(-x) - 3) \Rightarrow$
$-f(-x) = -(-2x - 3) = 2x + 3$

$f(x) = 1.5^x$

a. $f(x + 2) = 1.5^{x+2} \Rightarrow f(x + 2) = 1.5^2 \cdot 1.5^x$ or $2.25(1.5)^x$

b. $\frac{1}{2}f(x) = \frac{1}{2}(1.5^x)$

c. $-f(x) = -(1.5^x)$

d. $f(-x) = 1.5^{-x} \Rightarrow f(-x) = \frac{1}{1.5^x}$

e. $-f(-x) = -(1.5^{-x}) \Rightarrow -f(-x) = \frac{-1}{1.5^x}$

3. Graph B is symmetric across the vertical axis; Graph A is symmetric across the horizontal axis; Graph C is symmetric about the origin.

4. a. $h(t - 2) = e^{t-2}$

b. $-h(t - 2) = -e^{t-2}$

c. $-h(t - 2) - 1 = -e^{t-2} - 1$

5. a. $Q(t + 2) = 2 \cdot 1.06^{t+2}$

b. $Q(t + 2) - 1 = 2 \cdot 1.06^{t+2} - 1$

c. $-(Q(t + 2) - 1) = -(2 \cdot 1.06^{t+2} - 1)$
$\Rightarrow -(Q(t + 2) - 1) = -2 \cdot 1.06^{t+2} + 1$

6. a. $g(x)$ is a reflection of $f(x)$ across the x-axis, since $g(x) = x^2 - 5 = -(5 - x^2) \Rightarrow g(x) = -f(x)$.

b. $g(x)$ is a reflection of $f(x)$ across the y-axis, since $f(x) = 3 \cdot 2^x$ and $f(-x) = 3 \cdot 2^{-x} \Rightarrow f(-x) = g(x)$.

7. a. $g(x)$ is a compression of $f(x)$ by a factor of $\frac{1}{3}$ since $g(x) = \frac{1}{3x - 6} = \frac{1}{3(x - 2)} = \frac{1}{3} \cdot \frac{1}{x - 2} = \frac{1}{3}f(x)$.

b. $g(x)$ is a vertical shift up of $f(x)$ by ln 3 units since $g(x) = \ln 3x = \ln 3 + \ln x \Rightarrow g(x) = \ln 3 + f(x)$.

Exercises for Section 8.6

1. a. g's graph is a reflection of f's graph across the x-axis followed by a stretching of the graph by a factor of 2. Thus $g(x) = -2 \cdot f(x) = -2\sqrt{x}$.

b. g's graph is the graph of $f(x)$ shifted two units to the right. Thus $g(x) = f(x-2) = e^{x-2}$.

c. g's graph is the graph of $f(x)$ shifted three units to the left. Thus $g(x) = f(x+3) = \ln(x+3)$.

3. a. i. $f(-x) = -x^3$; the original graph has been reflected across the y-axis.

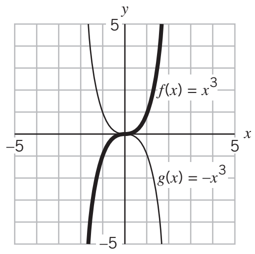
ii. $-2f(x) - 1 = -2x^3 - 1$; the original graph has been first stretched by a factor of 2, then reflected across the x-axis and then lowered one unit in the y direction.

iii. $f(x+2) = (x+2)^3$; the graph has been shifted two units to the left along the x-axis.

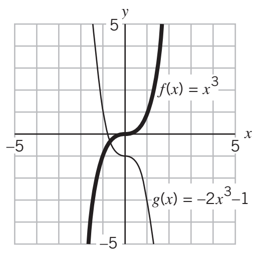
iv. $-f(-x) = x^3$; the graph was reflected across both axes, but the original graph is symmetric with respect to the origin and, effectively, no visual change has occurred.

The graphs are in the diagrams below, each with the graph of the original f.

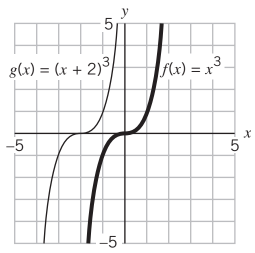
Graph for (i)



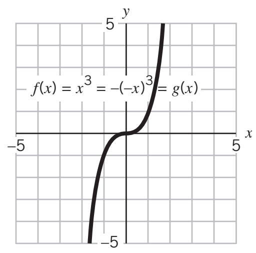
Graph for (ii)



Graph for (iii)



Graph for (iv)



5. a. symmetric about the origin.

b. symmetric across the x-axis

c. symmetric across the y-axis

7. a. If $f(x) = a \cdot x^{2k}$, then $f(-x) = a(-x)^{2k} = ax^{2k} = f(x)$.

b. If $f(x) = a \cdot x^{2k+1}$, then $f(-x) = a(-x)^{2k+1} = -ax^{2k} = -f(x)$.

c. i. $f(-x) = (-x)^4 + (-x)^2 = x^4 + x^2 = f(x)$, so this is an even function.

ii. $u(-x) = (-x)^5 + (-x)^3 = (-x^5) + (-x^3) = -(x^5 + x^3) = -u(x)$, so this is an odd function.

iii. $h(-x) = (-x)^4 + (-x)^3 = x^4 - x^3 \neq h(x)$ and $\neq -h(x)$, and so $h(x)$ is neither even nor odd.

iv. $g(-x) = 10 \cdot 3^{-x} \neq g(x)$ and $\neq -g(x)$, and so $g(x)$ is neither even nor odd.

d. The graphs of even functions are symmetric across the y-axis and the graphs of odd functions are symmetric about the origin, as the graphs of the functions in (c)(i) and show.

9. a. $y = 20(0.5)^{x+2} - 5$

b. $y = 4(x+2)^3 - 5$

c. $y = \log(x+2)^{1/3} - 5$

11. a. $y = \frac{1}{(2-x)^2} - 1$

b. $y = 0.5\frac{1}{(-t)^2} = \frac{0.5}{t^2}$

c. $y = \frac{1}{(\sqrt{s}-3)^2} = \frac{1}{s-3}$

d. $\frac{1}{(x+h)^2} - \frac{1}{x^2} = \frac{x^2}{x^2(x+h)^2} - \frac{(x+h)^2}{x^2(x+h)^2} = \frac{-2hx - h^2}{x^2(x+h)^2}$

13. a. $k(s-2) = \frac{1}{s-2}$

b. $\frac{1}{3}k(s-2) = \frac{1}{3(s-2)}$

c. $-\frac{1}{3}k(s-2) = \frac{-1}{3(s-2)}$

d. $j(s) = -\frac{1}{3}k(s-2) + 4 \Rightarrow j(s) = \frac{-1}{3(s-2)} + 4$

15. $f(x) = \ln x$

a. $f(x+2) = \ln(x+2)$

b. $\frac{1}{2}f(x) = \frac{1}{2}\ln x \Rightarrow \frac{1}{2}f(x) = \ln x^{1/2} \Rightarrow \frac{1}{2}f(x) = \ln\sqrt{x}$

c. $-f(x) = -\ln x \Rightarrow -f(x) = \ln x^{-1} \Rightarrow -f(x) = \ln\frac{1}{x}$

d. $f(-x) = \ln(-x)$

e. $-f(-x) = -\ln(-x) \Rightarrow -f(-x) = \ln(-x^{-1}) \Rightarrow -f(-x) = \ln\left(-\frac{1}{x}\right)$

$f(x) = \frac{1}{x^3}$

a. $f(x+2) = \frac{1}{(x+2)^3}$

b. $\frac{1}{2}f(x) = \frac{1}{2x^3}$

c. $-f(x) = \frac{-1}{x^3}$

d. $f(-x) = \frac{1}{(-x)^3} = -\frac{1}{x^3}$

e. $-f(-x) = -\left(\frac{1}{(-x)^3}\right) = \frac{1}{x^3}$

17. a. The "parent" function is $f(t) = t$. So $g(t)$ is $f(t) = t$ shifted to the right by 1 and compressed by a factor of 1/2 to get $g(t) = \frac{1}{2}f(t-1)$.

b. The "parent" function is $f(t) = (1/2)^t$. So $g(t)$ is $f(t) = \left(\frac{1}{2}\right)^t$ shifted left by 4 and stretched by a factor of 3 to get $g(t) = 3f(t+4)$.

c. The "parent" function is $f(t) = 1/t$. So $g(t)$ is $f(t) = \frac{1}{t}$ shifted to the right by 5, stretched by a factor of 7, reflected across the x-axis, and shifted down by 2 to get $g(t) = -7f(t-5) - 2$.

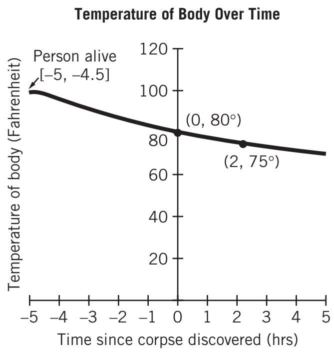
19. a. $g(x) = 3\ln(x+2) - 4$, so the graph of $f(x) = \ln x$ was shifted to the left by 2, with the result stretched by a factor of 3 and then shifted down by 4.

b. To find the vertical intercept, set $x = 0$ to get $g(0) = 3\ln(0 + 2) - 4 \approx -1.92$. So the vertical intercept is at approximately $(0, -1.92)$.

To find any horizontal intercepts, set $g(x) = 0$, to get $0 = 3\ln(x + 2) - 4 \Rightarrow 4/3 = \ln(x + 2) \Rightarrow e^{4/3} = x + 2 \Rightarrow x = e^{4/3} - 2 \approx 1.79$. So there is a single horizontal intercept at approximately $(1.79, 0)$.

21. a. If we let t = time (in hours) since the corpse was discovered and T = temperature of the corpse, then since the ambient temperature is 60° according to Newton's Law of Cooling, $T = 60 + Ce^{-kt}$ for some constants k and C. When $t = 0$, $T = 80$, so we have $80 = 60 + C \Rightarrow C = 20$. So the equation becomes $T = 60 + 20e^{-kt}$. When $t = 2$, $T = 75$, so we have $75 = 60 + 20e^{-2k} \Rightarrow (15/20) = e^{-2k} \Rightarrow (3/4) = e^{-2k} \Rightarrow \ln(3/4) = \ln(e^{-2k}) \Rightarrow -0.288 \approx -2k \Rightarrow k \approx 0.144$. So the full equation is $T = 60 + 20e^{-0.144t}$.

b. If we assume that the normal body temperature is 98.6°, then to find the time of death we need to solve $98.6 = 60 + 20e^{-0.144t}$ for $t \Rightarrow (38.6/20) = e^{-0.144t} \Rightarrow \ln(38.6/20) = \ln(e^{-0.144t}) \Rightarrow 0.658 \approx -0.144t \Rightarrow t \approx -4.6$ hours. So the person died about 4.6 hours before the corpse was discovered.



Section 8.7

Algebra Aerobics 8.7

1. a. $f(2) = 2^3 = 8$

b. $g(2) = 2(2) - 1 = 3$

c. $h(2) = \frac{1}{2}$

d. $(h \cdot g)(2) = h(2) \cdot g(2) = \frac{1}{2} \cdot 3 = \frac{3}{2}$

e. $(f + g)(2) = f(2) + g(2) = 8 + 3 = 11$

f. $\left(\frac{h}{g}\right)(2) = \frac{h(2)}{g(2)} = \frac{1/2}{3} = \frac{1}{6}$

2. a. $Q(1) + P(1) = 7 + 3 = 10$

b. $Q(2) - P(2) = 9 - 24 = -15$

c. $P(-1) \cdot Q(-1) = (-3 \cdot 3) = -9$

d. $\frac{Q(3)}{P(3)} = \frac{11}{81}$

3. a. $f(t) - h(t) = (3 - 2t) - (t^2 - 1) = -t^2 - 2t + 4$

b. $f(t) + h(t) = (3 - 2t) + (t^2 - 1) = t^2 - 2t + 2$

c. $f(t) \cdot h(t) = (3 - 2t)(t^2 - 1) = -2t^3 + 3t^2 + 2t - 3$

d. $\frac{h(t)}{f(t)} = \frac{t^2 - 1}{3 - 2t}$

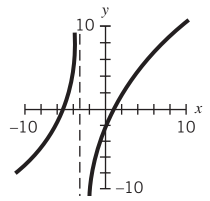
4. a. $f(x + 1) = (x + 1)^2 + 2(x + 1) - 3 = x^2 + 4x$

b. $f(x) + 1 = x^2 + 2x - 3 + 1 = x^2 + 2x - 2$

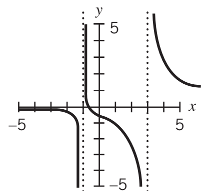
c. $g(x + 1) = \frac{1}{(x + 1) - 1} = \frac{1}{x}$

d. $g(x) + 1 = \frac{1}{x - 1} + 1 = \frac{1}{x - 1} + \frac{x - 1}{x - 1} = \frac{x}{x - 1}$

5. a. horizontal intercepts at $x = 1$ and $x = -5$; vertical asymptote at $x = -3$



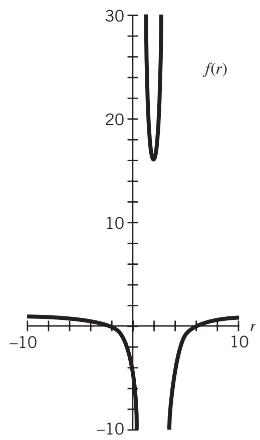
b. horizontal intercept at $x = -2/3$ and vertical asymptotes at $x = -1$ and $x = 3$



6. a. $f(r) = \frac{r^2 - 4r - 12}{r^2 - 4r + 3} = \frac{(r + 2)(r - 6)}{(r - 1)(r - 3)}$

To find any horizontal intercepts, set the numerator = 0 to get $(r + 2)(r - 6) = 0 \Rightarrow r = -2$ or $r = 6$. So the horizontal intercepts are at $(-2, 0)$, $(6, 0)$.

To find any vertical asymptotes, set the denominator = 0 to get $(r - 1)(r - 3) = 0 \Rightarrow r = 1$ or $r = 3$. So $f(r)$ is not defined at $r = 1$ and $r = 3$. So there are two vertical asymptotes at the lines $r = 1$ and $r = 3$. The graph of $f(r)$ follows.

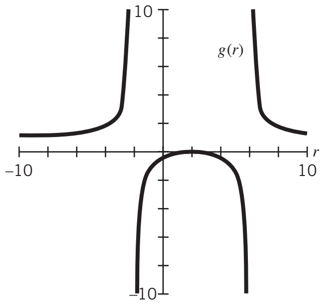


b. $g(r) = \frac{r^2 - 4r + 3}{r^2 - 4r - 12} = \frac{(r - 1)(r - 3)}{(r + 2)(r - 6)}$

To find any horizontal intercepts, set the numerator = 0 to get $(r - 1)(r - 3) = 0 \Rightarrow r = 1$ or $r = 3$. So the horizontal intercepts are at $(1, 0)$, $(3, 0)$.

Ch. 8

To find any vertical asymptotes, set the denominator = 0 to get $(r + 2)(r - 6) = 0 \Rightarrow r = -2$ or $r = 6$. So $g(r)$ is not defined at $r = -2$ and $r = 6$. So there are two vertical asymptotes at the lines $r = -2$ and $r = 6$. The graph of $g(r)$ follows.



7. a. $g(x) = -\frac{1}{(x + 3)^2} + 1$

b. $g(x) = -\frac{1}{(x + 3)^2} + 1 \cdot \frac{(x + 3)^2}{(x + 3)^2}$

$= \frac{-1}{(x + 3)^2} + \frac{x^2 + 6x + 9}{(x + 3)^2}$

$= \frac{-1 + x^2 + 6x + 9}{x^2 + 6x + 9}$

$= \frac{x^2 + 6x + 8}{x^2 + 6x + 9} = \frac{p(x)}{q(x)}$

c. To find any horizontal intercepts, let the numerator $p(x) = 0$ to get $x^2 + 6x + 8 = (x + 4)(x + 2) = 0 \Rightarrow x = -4$ or $x = -2$. So the horizontal intercepts are at $(-4, 0)$ and $(-2, 0)$.

To find any vertical asymptotes, let the denominator $q(x) = 0$ to get $(x + 3)^2 = 0 \Rightarrow x = -3$. So $g(x)$ is not defined when $x = -3$. So there is one vertical asymptote at the line $x = -3$.

8. a. Horizontal intercepts
b. Vertical intercepts
c. Horizontal intercept
d. Vertical asymptote

9. $f(x) = \frac{2x + 6}{x - 3}$

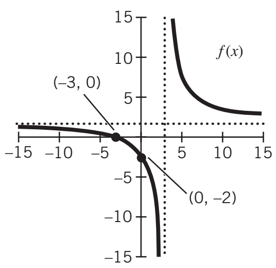
i. To find horizontal intercept(s): set $f(x) = 0$, which is equivalent to setting the numerator $2x + 6 = 0 \Rightarrow x = -3$. So there is one horizontal intercept at $(-3, 0)$.

To find vertical intercept: evaluate $f(0) = 6/-3 = -2$. So the vertical intercept is at $(0, -2)$.

ii. To find vertical asymptote: set $x - 3 = 0 \Rightarrow x = 3$. So the vertical asymptote is the line at $x = 3$.

iii. The end behavior as $x \to \pm\infty$ is $f(x) \to \frac{2x}{x} = 2$. So there is a horizontal asymptote at the line $y = 2$.

iv. Graph of $f(x)$



10. $g(x) = \frac{x^2 + 2x - 3}{3x - 1}$

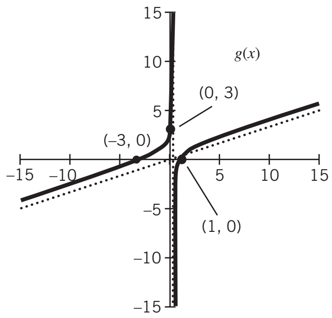
i. To find horizontal intercept(s): set $g(x) = 0$, which is equivalent to setting $x^2 + 2x - 3 = 0 \Rightarrow (x - 1)(x + 3) = 0 \Rightarrow x = 1, x = -3$. So there are horizontal intercepts at $(1, 0)$ and $(-3, 0)$.

To find vertical intercept: evaluate $g(0) = 3$. So the vertical intercept is at $(0, 3)$.

ii. To find vertical asymptote: set $3x - 1 = 0 \Rightarrow x = 1/3$. So the vertical asymptote is the vertical line at $x = 1/3$.

iii. The end behavior as $x \to \pm\infty$ is $g(x) \Rightarrow \frac{x^2}{3x} = \frac{1}{3}x$. So $g(x)$ is asymptotic to the line $y = \frac{1}{3}x$ (called an oblique asymptote).

iv. Graph of $g(x)$



Exercises for Section 8.7

1. a. $3t^2 + 10t - 4$
b. $-3t^2 + 2t + 6$
c. $18t^3 + 27t^2 - 26t - 5$
d. $\frac{3t^2 + 4t - 5}{6t + 1}$

3. a. $j(x) = 3x^5 + x^2 + x - 1$; $k(x) = 3x^5 - x^2 + x + 1$; $l(x) = 3x^7 - 3x^5 + x^3 - x$

b. $j(2) = 101$; $k(3) = 724$; $l(-1) = 0$

5. a. $R(n) = 25n$

b. $R(n) = 25n - 500$

c. $R(n) = 25(n - 30) - 500 = 25n - 1250$

7. If a worker works t hours a week (where $t \geq 40$), then the worker's weekly paycheck, P (in dollars), is the sum of two terms: regular pay + overtime pay. The regular pay is $20 \cdot 40$ = \$800 a week. The overtime pay $= 30 \cdot (t - 40)$, where t = total number of hours worked. So the weekly paycheck is $P = 800 + 30(t - 40)$, where $t \geq 40$.

9.

x	0	1	2	3	4	5
$h(x)$	−6	−6	−6	−6	−6	−6
$j(x)$	0	−4	−16	−36	−64	−100
$k(x)$	9	5	−55	−315	−1015	−2491

11.

x	−3	−2	−1	0	1	2	3
$f(x)$	9	4	1	0	1	4	9
$g(x)$	−4	−3	−2	−1	0	1	2
$f(x) + g(x)$	5	1	−1	−1	1	5	11
$f(x) - g(x)$	13	7	3	1	1	3	7
$f(x) \cdot g(x)$	−36	−12	−2	0	0	4	18
$g(x)/f(x)$	−4/9	−3/4	−2	undefined	0	1/4	2/9

Ch. 8

13. **a.** $C(n) = 500 + 40n$

b. $P(n) = \frac{500 + 40n}{n}$

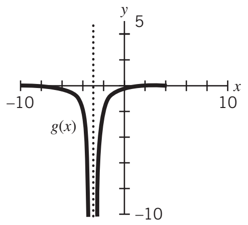
c. $P(25) = 60$; $P(100) = 45$. As the number of people attending increases, the cost per person decreases. If only 25 attend (the minimum size), the cost would be \$60 per person. If 100 attend (the maximum size), the cost would be \$45 per person.

15. Graph A, $f(x)$ asymptotes: horizontal, $y = -3$; vertical, $x = -2$

Graph B, $g(x)$ asymptotes: horizontal, $y = 4$; vertical, $x = 2$

Graph C, $h(x)$ asymptotes: horizontal, $y = -2$; vertical, $x = -2$ and $x = 3$

17. $g(x)$ has no horizontal intercepts and a vertical asymptote at the line $x = -3$. The graph of $g(x)$ verifies this.

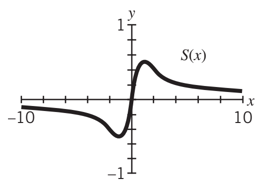


19. **a.** The domain is all real numbers.

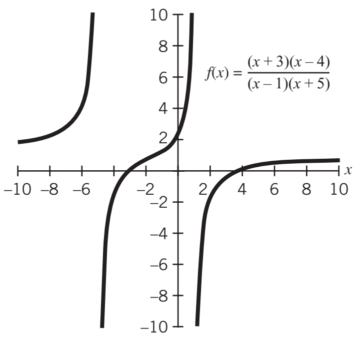
b. Yes, there is one horizontal intercept at $x = 0$, the origin. There are no vertical asymptotes since x is defined for all real values (so there are no singularities).

c. As $x \to \pm\infty$, $S(x) = \frac{x}{x^2 + 1} \approx \frac{x}{x^2} = \frac{1}{x}$. So as $x \to \pm\infty$, $S(x) \to 0$. So $S(x)$ has a horizontal asymptote at the x-axis.

d. To some the graph looks like a slithering serpent.



21. There are infinitely many possibilities. If a and b are nonzero real numbers, then one set of answers would be of the form $f(x) = \frac{a(x + 3)(x - 4)}{b(x - 1)(x + 5)}$. (*Note:* You could create additional rational functions by, say, squaring or cubing one or more terms.) If we let $a = b = 1$, then the graph of $f(x)$ is as follows:



23. $f(x)$ matches Graph C; $g(x)$ matches Graph B; $h(x)$ matches Graph A.

Section 8.8

Algebra Aerobics 8.8a

1. $f(x) = 2x + 3$, $g(x) = x^2 - 4$

a. $f(g(2)) = f(0) = 2(0) + 3 = 3$

b. $g(f(2)) = g(7) = (7)^2 - 4 = 49 - 4 = 45$

c. $f(g(3)) = f(5) = 2(5) + 3 = 10 + 3 = 13$

d. $f(f(3)) = f(9) = 2(9) + 3 = 18 + 3 = 21$

e. $(f \circ g)(x) = f(g(x)) = f(x^2 - 4) = 2(x^2 - 4) + 3$
$= 2x^2 - 8 + 3 = 2x^2 - 5$

f. $(g \circ f)(x) = g(f(x)) = g(2x + 3) = (2x + 3)^2 - 4$
$= 4x^2 + 12x + 9 - 4 = 4x^2 + 12x + 5$

2. **a.** $(P \circ Q)(2) = P(Q(2)) = P(3(2) - 5) = P(1) = \frac{1}{1} = 1$

b. $(Q \circ P)(2) = Q(P(2)) = Q\left(\frac{1}{2}\right) = 3\left(\frac{1}{2}\right) - 5 = \frac{-7}{2}$

c. $(Q \circ Q)(3) = Q(Q(3)) = Q(3(3) - 5) = Q(4)$
$= 3(4) - 5 = 7$

d. $(P \circ Q)(t) = P(Q(t)) = P(3t - 5) = \frac{1}{3t - 5}$

e. $(Q \circ P)(t) = Q(P(t)) = Q\left(\frac{1}{t}\right) = 3\left(\frac{1}{t}\right) - 5 = \frac{3}{t} - 5$

3. $F(x) = \frac{2}{x - 1}$, $G(x) = 3x - 5$

a. $(F \circ G)(x) = F(G(x)) = F(3x - 5) = \frac{2}{(3x - 5) - 1}$
$= \frac{2}{3x - 6}$

b. $(G \circ F)(x) = G(F(x)) = G\left(\frac{2}{x - 1}\right) = 3\left(\frac{2}{x - 1}\right) - 5$
$= \frac{6}{x - 1} - 5$

c. $(F \circ G)(x) = \frac{2}{3x - 6} \neq (G \circ F)(x) = \frac{6}{x - 1} - 5 = \frac{6 - 5(x - 1)}{x - 1} = \frac{11 - 5x}{x - 1}$

4. **a.** $f(-2) = 2$

b. $g(-2) = 0$

c. $f(0) = -2$

d. $g(0) = 1$

e. $(g \circ f)(-2) = g(2) = 2$

f. $(f \circ g)(-2) = f(0) = -2$

g. $(g \circ f)(0) = g(-2) = 0$

h. $(f \circ g)(0) = f(1) = -1$

5. **a.** $f(x) = x^2 - 2$

b. $g(x) = \frac{1}{2}x + 1$

c. $(g \circ f)(x) = g(x^2 - 2) = \frac{1}{2}(x^2 - 2) + 1$
$= \frac{1}{2}x^2 - 1 + 1 = \frac{1}{2}x^2$

d. $(f \circ g)(x) = f\left(\frac{1}{2}x + 1\right) = \left(\frac{1}{2}x + 1\right)^2 - 2$
$= \left(\frac{1}{4}x^2 + x + 1\right) - 2 = \frac{1}{4}x^2 + x - 1$

6. **a.** $(g \circ f)(-2) = \frac{1}{2}(-2)^2 = 2$ and
$(f \circ g)(-2) = \frac{1}{4}(-2)^2 + (-2) - 1 = -2$.
So both answers agree with the answers in Problem 4, parts (e) and (f).

7. **a.** $(h \circ f \circ g)(4) = h(f(g(4))) = h(f(0)) = h(3) = 4$

b. $(f \circ h \circ g)(1) = f(h(g(1))) = f(h(3)) = f(4) = 5$

8. **a.** $(h \circ f \circ g)(3) = h(f(g(3))) = h(f(2)) = h(-1/3) = 3$

b. $(f \circ g \circ h)(100) = f(g(h(100))) = f(g(3)) = f(2) = -1/3$

Algebra Aerobics 8.8b

1. a.

t	$g(t)$
0	5
1	3
2	1
3	−1

t	$h(t)$
−1	3
1	2
3	1
5	0

b. $(g \circ h)(3) = g(h(3)) = g(1) = 3$; $(h \circ g)(3) = h(g(3)) = h(-1) = 3$.

c. Yes, since $(g \circ h)(t) = g(h(t)) = g\left(\frac{5-t}{2}\right) = 5 - 2\left(\frac{5-t}{2}\right) = 5 - (5 - t) = t$ and $(h \circ g)(t) = h(g(t)) = h(5 - 2t) = \frac{5 - (5 - 2t)}{2} = t$ and the domains and ranges of both g and h are all real numbers.

d. They are inverse functions.

2. $f(x) = 2x + 1, \quad g(x) = \frac{x-1}{2}$

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{x-1}{2}\right) = 2\left(\frac{x-1}{2}\right) + 1$$
$$= \frac{2(x-1)}{2} + 1 = x - 1 + 1 = x$$
$$(g \circ f)(x) = g(f(x)) = g(2x+1) = \frac{(2x+1) - 1}{2}$$
$$= \frac{2x}{2} = x$$

3. $f(x) = \sqrt[3]{x+1}, \quad g(x) = x^3 - 1$

$$(f \circ g)(x) = f(g(x)) = f(x^3 - 1) = \sqrt[3]{(x^3-1)+1}$$
$$= \sqrt[3]{x^3} = (x^3)^{1/3} = x^1 = x$$
$$(g \circ f)(x) = g(f(x)) = g(\sqrt[3]{x+1})$$
$$= \left(\sqrt[3]{x+1}\right)^3 - 1 = [(x+1)^{1/3}]^3 - 1$$
$$= (x+1) - 1 = x$$

4. $$f(f^{-1}(x)) = f\left(\frac{1+x}{x}\right) = \frac{1}{\frac{1+x}{x} - 1} = \frac{1}{\frac{1+x}{x} - \frac{x}{x}}$$
$$= \frac{1}{(1/x)} = x$$
$$f^{-1}(f(x)) = f^{-1}\left(\frac{1}{x-1}\right) = \frac{1 + \frac{1}{x-1}}{\frac{1}{x-1}} = \frac{\frac{x-1}{x-1} + \frac{1}{x-1}}{\frac{1}{x-1}}$$
$$= \frac{\frac{x}{x-1}}{\frac{1}{x-1}} = \frac{x}{x-1} \cdot \frac{x-1}{1} = x$$

So $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

5. Letting $f(x) = y$, we have $y = \frac{3}{x} + 5 \Rightarrow y - 5 = \frac{3}{x} \Rightarrow x = \frac{3}{y-5} \Rightarrow f^{-1}(x) = \frac{3}{x-5}$ (using the convention of designating x as the input variable).

6. Letting $g(x) = y$, we have $y = (x-2)^{3/2} \Rightarrow x = y^{2/3} + 2 \Rightarrow g^{-1}(x) = x^{2/3} + 2$ (using the convention of designating x as the input variable).

7. Letting $h(x) = y$, we have $y = 5x^3 - 4 \Rightarrow y + 4 = 5x^3 \Rightarrow x = \left(\frac{y+4}{5}\right)^{1/3} \Rightarrow h^{-1}(x) = \left(\frac{x+4}{5}\right)^{1/3}$ (using the convention of designating x as the input variable).

8. a. Saying "no"

b. Taking the bus from home, then going to class.

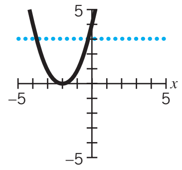
c. Turning off the light, leaving the room, closing the door, and then locking the door.

d. Dividing x by 5 and then adding 3

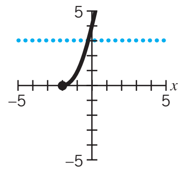
e. Subtracting 2 from z and then dividing the result by −3

9. The functions in Graphs A and D are 1-1 since they pass the horizontal line test. The functions in Graphs B and C are not 1-1, since they fail that test.

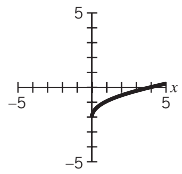
10. a. f is not one-to-one on the domain of all real numbers (since it fails the horizontal line test), so it cannot have an inverse.



b. If we restrict the domain of f to $x \geq -2$, then f has an inverse on this new domain (since it now passes the vertical test).



c. Letting $f(x) = y$, we have $y = (x+2)^2 \Rightarrow x = +\sqrt{y} - 2 \Rightarrow f^{-1}(x) = \sqrt{x} - 2$ (using the convention of designating x as the input variable). The graph of $f^{-1}(x)$ is shown below.



11. a. The radius $R(t)$ as a function of time t is $R(t) = 10t$.

b. $R(2) = 10 \cdot 2 = 20$, so area $A(20) = \pi(20)^2 \approx 1257$ square feet.

c. $(A \circ R)(t) = A(R(t)) \Rightarrow A(10t) = \pi(10t)^2 = 100\pi t^2$

Exercises for Section 8.8

1. a. $f(g(1)) = f(0) = 2$

b. $g(f(1)) = g(1) = 0$

c. $f(g(0)) = f(1) = 1$

d. $g(f(0)) = g(2) = 3$

e. $f(f(2)) = f(3) = 0$

3. a. $g(f(2)) = g(0) = 1$

b. $f(g(-1)) = f(2) = 0$

c. $g(f(0)) = g(4) = -3$

d. $g(f(1)) = g(3) = -2$

5. a. $F(G(1)) = F(0) = 1$

b. $G(F(-2)) = G(-3) = 4$

c. $F(G(2)) = F(0.25) = 1.5$

d. $F(F(0)) = F(1) = 3$

e. $(F \circ G)(x) = 2\left(\frac{x-1}{x+2}\right) + 1$

$= \frac{(2x-2)+(x+2)}{x+2} = \frac{3x}{x+2}$

f. $(G \circ F)(x) = \frac{(2x+1)-1}{(2x+1)+2} = \frac{2x}{2x+3}$

7. a. $A(r) = \pi r^2$, where r is measured in feet and $A(r)$ is measured in square feet.

b. $r = R(t) = 5t$, where t is measured in minutes and $R(t)$ is measured in feet.

c. $A(R(t)) = \pi 25t^2$, where t is measured in minutes and A is measured in square feet.

d. $A(R(10)) = \pi \cdot 25 \cdot 10^2 = 2500\pi \approx 7854$ sq. ft and $A(R(60)) = \pi \cdot 25 \cdot 60^2 \approx 282{,}743$ sq. ft.

9. $r(t) = 13t$ and thus $A(r(t)) = \pi(13t)^2 = 169\pi t^2$

11. a. $T = 32 - 5s$

b. If the road is 40 feet wide, then $k = 20$ and thus $S(x) = \left[1 - \frac{1}{2}\cdot\left(\frac{x}{20}\right)^2\right]S_d = \left[1 - \frac{x^2}{800}\right]S_d$.

c. At the middle of the 40-ft road $x = 0$ and therefore $S(0) = \left[1 - \frac{0}{800}\right]S_d = S_d$. At the edge of the 40-ft road, $x = 20$, so $S(20) = \left[1 - \frac{20^2}{800}\right]S_d = \frac{1}{2}S_d$.

d. $T(S(x)) = T\left(\left[1 - \frac{x^2}{800}\right]S_d\right) = 32 - 5\left[1 - \frac{x^2}{800}\right]S_d$

$= 32 - 5S_d + \frac{x^2}{160}S_d$

e. $T(S(0)) = 32 - 5S_d$ and $T(S(20)) = 32 - 5S_d + \frac{20^2}{160}S_d = 32 - 5S_d + 2.5S_d = 32 - 2.5S_d$

13. a. $M(x) = (L \circ J \circ K)(x) = L(J(K(x))) = L(J(\log x)) = L((\log x)^3) = 1/(\log x)^3$

b. Take the log of x, cube the result, and then place it in the denominator, with 1 as the numerator.

15. If $f(x) = 4x$, $g(x) = e^x$, and $h(x) = x - 1$, then $f(g(h(x))) = f(g(x-1)) = f(e^{x-1}) = 4e^{x-1} = j(x)$

17. $f(g(x)) = f(x^2+1) = \sqrt{(x^2+1)-1} = \sqrt{x^2} = x$ since $x > 0$

$g(f(x)) = g\left(\sqrt{x-1}\right) = \left(\sqrt{x-1}\right)^2 + 1$

$= (x-1) + 1 = x$

19. $f(g(x)) = f\left(\frac{x^3-5}{4}\right) = \sqrt[3]{4\left(\frac{x^3-5}{4}\right)+5} = \sqrt[3]{x^3} = x$

$g(f(x)) = g\left(\sqrt[3]{4x+5}\right) = \frac{\left(\sqrt[3]{(4x+5)}\right)^3 - 5}{4}$

$= \frac{(4x+5)-5}{4} = x$

21. $F(G(t)) = F(\ln(t^{1/3})) = e^{3\ln(t^{1/3})} = e^{\ln(t)} = t$ (where $t > 0$)

$G(F(t)) = G(e^{3t}) = \ln(e^{3t})^{1/3} = \ln(e^t) = t$

23.

x	$f^{-1}(x)$
5	−2
1	−1
2	0
4	1

25. a. Yes, this is a 1-1 function since each letter is associated with a unique number. The inverse function would just consist of matching each number between 1 and 26 wih its letter equivalent. The domain of the inverse function would be the integers 1 through 26.

b. "MATH RULES"

27. a. Yes, $f(x)$ has an inverse since its graph passes the horizontal line test.

b. The domain of $f(x)$ is the interval $[-4, \infty]$. The range of $f(x)$ is the interval $[0, \infty]$.

c. $f(-4) = 0$, $f(0) = 2$, and $f(5) = 3$. This means that the points $(-4, 0)$, $(0, 2)$, and $(5, 3)$ all lie on the graph of $f(x)$.

d. Given the results in part (c), the points $(0, -4)$, $(2, 0)$, and $(3, 5)$ all lie on the graph of $f^{-1}(x)$. So $f^{-1}(0) = -4$, $f^{-1}(2) = 0$, $f^{-1}(3) = 5$.

29. $Q^{-1}(x) = \frac{3x+15}{2}$, $Q(3) = -3$, $Q^{-1}(3) = 12$

31. $Q^{-1}(x) = \frac{3}{x-1}$, $Q(3) = 2$, $Q^{-1}(3) = 3/2$

33. a.

x (cups)	4	8	16	32
$f(x)$ (quarts)	1	2	4	8

x (quarts)	2	4	8	16
$g(x)$ (gallons)	0.5	1	2	4

b. i. $(g \circ f)(8) = g(2) = 0.5$ gal

ii. $g^{-1}(2) = 8$ qt

iii. $(f^{-1} \circ g^{-1})(1) = f^{-1}(4) = 16$ cups

iv. $(f^{-1} \circ g^{-1})(2) = f^{-1}(8) = 32$ cups

c. $(f^{-1} \circ g^{-1})(x)$ is a function that converts gallons to cups.

35. a. $W_{men}(h) = 50 + 2.3(h - 60)$, where a reasonable domain might be $60'' \le h \le 78''$;
$W_{women}(h) = 45.5 + 2.3(h - 60)$, where a reasonable domain might be $60'' \le h \le 74''$.

b. $W_{men}(70) = 50 + 2.3(70 - 60) = 73$ kg, so 73 kg is the "ideal" weight of a 5′10″ man.
$W_{women}(66) = 45.5 + 2.3(66 - 60) = 59.3$ kg, so 59.3 kg is the "ideal" weight of a 5′6″ woman.

c. $W^{-1}{}_{men}(77.6)$ means $77.6 = 50 + 2.3(h - 60) \Rightarrow h = 72$ inches. A man with a IBW of 77.6 kg should be 72 inches or 6 ft tall.

d. $W_{newman}(h) = \frac{50 + 2.3(h-60)}{0.4356}$;
$W_{newwomen}(h) = \frac{45.5 + 2.3(h-60)}{0.4356}$.

e. $W^{-1}{}_{newwomen}(125)$ means $125 = \frac{45.5 + 2.3(h-60)}{0.4356} \Rightarrow h \approx 63.89 \approx 64$ inches. So 125 lb is the IBW for a woman about 5′4″ in height.

37. $F(G(x)) = F(\log_a(x)) = a^{\log_a(x)} = x$

$G(F(x)) = G(a^x) = \log_a(a^x) = x$

Ch. 8: Check Your Understanding

1. False	**6.** True	**11.** False	**16.** False
2. True	**7.** True	**12.** False	**17.** False
3. True	**8.** False	**13.** False	**18.** True
4. False	**9.** True	**14.** True	**19.** True
5. False	**10.** True	**15.** False	**20.** True

 CH. 8 Review: Putting It All Together

21. True

22. Possible answer: $y = x^4 + x^2 + 1$

23. $y = -0.25x^2$

24. Possible answer: $f(x) = (x + 1)(x - 3)(x - 4)$

25. Possible answers: $h(x) = 2(x + 1)(x - 3)(x - 4)$, $g(x) = -3(x + 1)(x - 3)(x - 4)$

26. Possible answer: $y = -(x - 1)^2 + 3$

27. Possible answer: $y = 2(x - 3)^2 - 5$

28. $y = \frac{-1}{2}(x - 2)(x + 2)$

29. $r = s^2 - s + 5$

30. $G(x) = (x + 2)^2 + 2(x + 2)$

31. $-h(t) = -(t - 2)^2$

32. Possible answer: $y = x^2$ and $y = -(x - 1)^2 + 1$

33. Possible answer: $y = (x + 4)^2$

34. $h(t) = \frac{1}{4}(x + 2)(x + 1)(x - 2)(x - 3)$

35. Possible answer: $f(x) = x^3 + 2x^2$, $g(x) = 5x - 2$

36. $H(t) = 3t + 1$, $Q(t) = \sqrt{t}$

37. Possible answer: $f(x) = \frac{x - 2}{x(x + 3)}$

38. False	**44.** False	**50.** True	**56.** True
39. True	**45.** True	**51.** False	**57.** False
40. False	**46.** False	**52.** True	**58.** False
41. True	**47.** False	**53.** False	
42. True	**48.** True	**54.** True	
43. False	**49.** True	**55.** False	

Ch. 8 Review: Putting It All Together

1. In Graph A, the parabola is concave up with an estimated minimum at (2, –4). Hence the axis of symmetry is the line $x = 2$ and there are two horizontal intercepts, at $x = 0$ and $x = 4$.

In Graph B, the parabola is concave down with an estimated maximum at $(0, -3)$. Hence the axis of symmetry is the vertical axis (the line $t = 0$) and there are no horizontal intercepts.

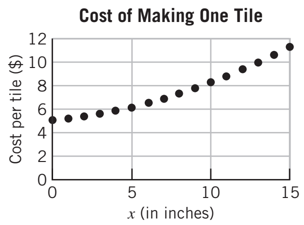
3. a. Area of interior square $= x^2$ square inches; area of each of the maple strips $= (x + 1) \cdot 1 = x + 1$ square inches.

b. Cost of white oak: $(\$2.39/\text{ft}^2) \cdot (1\ \text{ft}^2/144\ \text{in}^2) \approx \$0.02/\text{in}^2$; cost of maple: $(\$4.49/\text{ft}^2) \cdot (1\ \text{ft}^2/144\ \text{in}^2) \approx \$0.03/\text{in}^2$

c. For white oak: $0.02x^2$ (in dollars); for all four maple strips: $4(0.03)(x + 1) = 0.12x + 0.12$ (in dollars)

d. $C(x) = 0.02x^2 + 0.12x + 5.12$, a quadratic function

e.

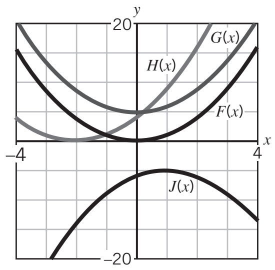


f. Estimating from the graph, when $C(x) = \$7$, then $x \approx 7.5''$, so the width (and length) of the whole tile would be about $9.5''$. To keep the cost/tile at \$7 or below, the dimensions of a tile must be at most $9.5''$ by $9.5''$.

5. $g(x) = 2x^2$ and $h(x) = -0.5x^2$

7. a. Vertex for $F(x)$ is (0, 0), vertex for $G(x) = (0, 5)$, vertex for $H(x)$ is $(-2, 0)$, vertex for $J(x)$ is $(1, -5)$

b.



c. The graph of $G(x)$ is the graph of $F(x)$ shifted up five units. The graph of $H(x)$ is the graph of $F(x)$ shifted left two units. The graph of $J(x)$ is the graph of $F(x)$ shifted right one unit, flipped over the x-axis, and shifted down five units.

9. a. One possibility is $Q(t) = (t - 4)(t + 2) = t^2 - 2t - 8$. The vertex of $Q(t)$ is at $(1, -9)$.

b. One possibility is $M(t) = 3Q(t) = 3(t - 4)(t + 2) = 3t^2 - 6t - 24$. The vertex of $M(t)$ is at $(1, -27)$. So the vertices are not the same, though they share the same t-coordinate.

c. One possibility is $P(t) = Q(t) + 10 = t^2 - 2t + 2$. $P(t)$ has no horizontal intercepts since the discriminant $= (-2)^2 - (4 \cdot 1 \cdot 2) = 4 - 8 = -4$, which is negative.

11. Since we have set the vertex at the origin, then the equation is of the form $y = ax^2$ (where $a < 0$). We know two points on the parabola, $(d, -32)$ and $(-(100 - d), -72) = (d - 100, -72)$. Substituting each set of points into the equation $y = ax^2$, we get the two equations

$$-32 = ad^2 \quad \text{and} \quad -72 = a(d - 100)^2$$

Solving both equations for a, we get

$$a = -32/d^2 \quad \text{and} \quad a = -72/(d - 100)^2$$

Setting both expressions for a equal and solving for d gives us

$$-32/(d^2) = -72/(d - 100)^2$$

cross-multiply $\quad -32(d - 100)^2 = -72d^2$

simplify $\quad -32(d - 100)^2 + 72d^2 = 0$

$$-32(d^2 - 200d + 10{,}000) + 72d^2 = 0$$

$$40d^2 + 6400d - 320{,}000 = 0$$

divide by 40 $\quad d^2 + 160d - 8000 = 0$

Ch. 8

Now we can solve for d either using the quadratic formula or factoring.

Quadratic formula (letting $a = 1, b = 160, c = -8000$) gives:

$$d = \frac{-160 \pm \sqrt{(160)^2 - 4(1)(-8000)}}{2 \cdot 1}$$
$$= \frac{-160 \pm \sqrt{25{,}600 + 32{,}000}}{2}$$
$$= \frac{-160 \pm \sqrt{57{,}600}}{2} = \frac{-160 \pm 240}{2} = -80 \pm 120$$

So $d = 40$ feet or -200 feet. Only $d = 40$ feet makes sense.

Factoring $d^2 + 160d - 8000 = 0$ gives $(d - 40)(d + 200) = 0$, which confirms that either $d = 40$ feet or -200 feet, where $d = 40$ feet is the only valid answer in this context.

Substituting $d = 40$ feet into the equation $-32 = ad^2$, we get $-32 = a(40)^2 \Rightarrow a = -32/1600 = -0.02$. So the equation for the swimming pool parabolic roof is $y = -0.02x^2$, where x and y are both in feet.

13. a. The highest point of her dive will be at the vertex of the height function (which is concave down). Letting $a = -16$, $b = 12$, and $c = 25$, the t-coordinate of the vertex is at $-12/(2 \cdot (-16)) = 0.375$ seconds. Then $H(0.375) = 25 + (12 \cdot 0.375) - 16(0.375)^2 = 27.25$ feet above water will be the highest point of her dive.

b. She will hit the water when $H(t) = 0 \Rightarrow 25 + 12t - 16t^2 = 0$. Using the quadratic formula, letting $a = -16$, $b = 12$, and $c = 25$, we have

$$t = \frac{-12 \pm \sqrt{(12)^2 - 4(-16)(25)}}{2 \cdot (-16)}$$
$$= \frac{-12 \pm \sqrt{(144 + 1600)}}{-32}$$
$$= \frac{-12 \pm \sqrt{1744}}{-32} \approx \frac{-12 \pm 41.8}{-32}$$

≈ -0.93 seconds or 1.68 seconds. Only the positive value makes sense in this context. So about 1.68 seconds (a little under 2 seconds) after she starts her dive, she will hit the water.

15. a.

x	y	Average Rate of Change	Average Rate of Change of Average Rate of Change
−1	5	n.a.	n.a.
0	0	−5	n.a.
1	−3	−3	$\frac{-3 - (-5)}{1 - 0} = 2$
2	−4	−1	2
3	−3	1	2
4	0	3	2
5	5	5	2

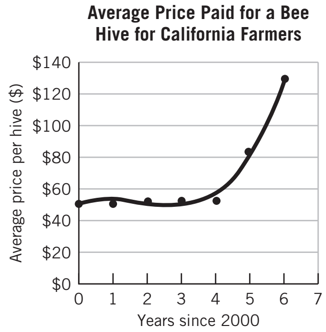
b. A linear function.

c. The fourth column shows that the average rate of change (of the third column with respect to x) is constant, which means that the third column is a linear function of x.

17. a. Behavior looks like a cubic polynomial. (*Note:* The graph doesn't look linear on semi-log plot, so data are not exponential.) The best-fit cubic is

$$P(t) = 1.26x^3 - 6.94x^2 + 9.22x + 50.21$$

[where $P(t)$ = price per hive in t years after 2000]. $P(t)$ is plotted on the accompanying graph along with the data.

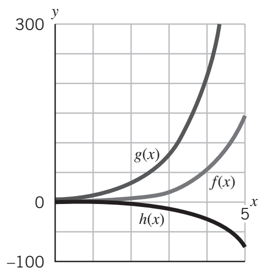


b. $P(10) \approx \$708$/hive

c. (2.5 hives/acre) · (\$708/hive) = \$1770/acre

d. Since healthy beehives would be scarce, the price per beehive would probably go up.

19.

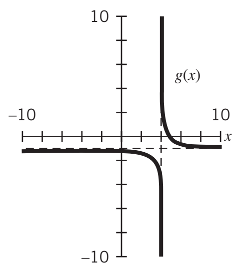


The graph of $g(x)$ is $f(x)$ vertically stretched by a factor of 4; that is, for each value of x, $g(x)$ is four times the value of $f(x)$. The graph of $h(x)$ is the graph of $f(x)$ flipped over the x-axis and then vertically compressed by a factor of 0.5.

21. a. $g(x) = (2/3)f(x - 4) - 1$. So the graph of $f(x)$ was shifted four units to the right, compressed by a factor of 2/3, then shifted down one unit to create the graph of $g(x)$.

b. $g(x) = \frac{2}{3(x-4)} - 1 = \frac{2}{3(x-4)} - \frac{3(x-4)}{3(x-4)} = \frac{-3x + 14}{3x - 12}$

c. The domain of $g(x)$ is all real numbers except 4.



d. If $g(x) = 0$, then $0 = (-3x + 14)/(3x - 12) \Rightarrow 0 = -3x + 14 \Rightarrow x = 14/3 = 4\frac{2}{3}$. So $g(x)$ has a single horizontal intercept at $x = 4\frac{2}{3}$.

Ch. 8

If $x = 0$, then $g(0) = 14/(-12) = -1\frac{1}{6}$. So $g(x)$ has a vertical intercept at $-1\frac{1}{6}$.

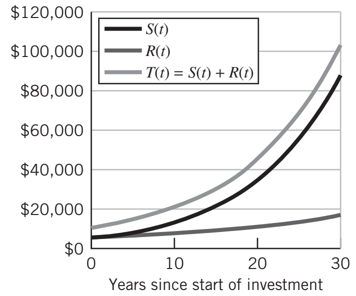
e. If the denominator $3x - 12 = 0 \Rightarrow x = 4$. So $g(x)$ is not defined when $x = 4$, but does have a vertical asymptote at the line $x = 4$.

f. As $x \to \pm\infty$, $g(x) = \frac{-3x + 14}{3x - 12} \approx \frac{-3x}{3x} = -1$. So as, $x \to \pm\infty$, $g(x) \to -1$ (but never reaches -1).

So $g(x)$ has a horizontal asymptote at the line $y = -1$.

23. a. $S(t) = 5000(1.04)^t$; $R(t) = 5000(1.10)^t$; $T(t) = S(t) + R(t) = 5000(1.04)^t + 5000(1.10)^t$ where t = years since the start of the investments.

b. **Individual Return on \$5000 at 4% ($S(t)$) and \$5000 at 10% ($R(t)$), and Total Return $T(t) = R(t) + S(t)$**

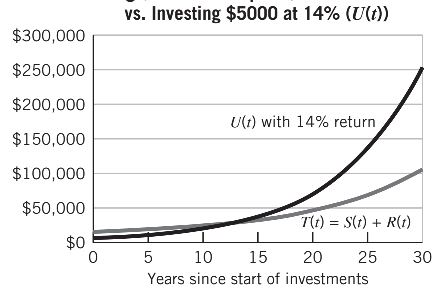


c. She would have only $S(30) = 5000(1.04)^{30} \approx \$16{,}217$.

d. Let $U(t) = 5000(1.14)^t$, where 5000 is the initial value invested at 14% per year. The following table and graph show that after 30 years the $U(t)$ account contains almost 2.5 times as much as the $T(t)$ account (or about 150% more).

Years Since Start of Investment	$T(t) = S(t) + R(t)$	$U(t) = 5000(1.14)^t$
0	\$10,000	\$5,000
5	\$14,136	\$9,627
10	\$20,370	\$18,536
15	\$29,891	\$35,690
20	\$44,593	\$68,717
25	\$67,503	\$132,310
30	\$103,464	\$254,751

Comparing the Return on Investing \$5000 at 4% plus \$5000 at 10% ($T(t)$) vs. Investing \$5000 at 14% ($U(t)$)



25. The function is 1−1 since if $f(x_1) = f(x_2)$, then $(x_1 - 2)^3 + 1 = (x_2 - 2)^3 + 1 \Rightarrow (x_1 - 2)^3 = (x_2 - 2)^3 \Rightarrow x_1 - 2 = x_2 - 2 \Rightarrow x_1 = x_2$. So on an appropriate domain f^{-1} exists. Letting $y = f(x)$, we can solve for x in terms of y.

Given	$y = (x - 2)^3 + 1$
Subtract 1	$y - 1 = (x - 2)^3$
Take the cube root	$(y - 1)^{1/3} = x - 2$
Add 2, switch sides	$x = (y - 1)^{1/3} + 2$

So $f^{-1}(y) = (y - 1)^{1/3} + 2$, or since the function is abstract, we can use any name for the input variable, in particular x, to get the conventional form, $f^{-1}(x) = (x - 1)^{1/3} + 2$.

27. a. i. $\frac{761 \text{ miles}}{1 \text{ hr}} \cdot \frac{5280 \text{ feet}}{1 \text{ mile}} \cdot \frac{1 \text{ hr}}{60 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} \approx 1116$ ft/sec.

ii. $D(t) = 1116t$, where t is in seconds and $D(t)$ is in feet from the lightning strike.

iii. Yes, the rule of thumb is reasonable since for each second after the strike, the sound thunder travels about 1116 feet.

b. i. $A(r) = \pi r^2$, where r is the radius (in feet) of the sound circle.

ii. $A(D(t)) = A(1116t) = \pi(1116t)^2 \approx 1{,}245{,}500\pi t^2 \approx 3{,}913{,}000\, t^2$, where t is in seconds and $A(D(t))$ is in square feet. $A(D(4)) = 3{,}913{,}000(4)^2 \approx 63$ million square feet or $(63 \cdot 10^6 \text{ sq. ft}) \cdot \frac{1 \text{sq. mile}}{5280^2 \text{ sq. ft}} \approx 2.3$ square miles.

iii. When the time doubles, the distance doubles, but the area increases by a factor of 4.

Exercises for EE on Mathematics of Motion

1.

Time (sec)	Distance (cm)	Avg. Vel. over Previous 1/30 sec. (cm/sec)
0.0000	0.00	n.a.
0.0333	3.75	113
0.0667	8.67	147
0.1000	14.71	181
0.1333	21.77	212
0.1667	29.90	243

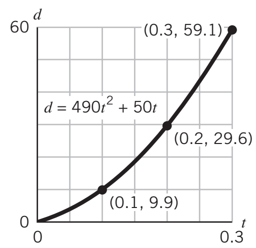
The average velocity (over each 1/60 of a second) increases rapidly as time progresses.

3. For $d = 490t^2 + 50t$:

a. 50 is measured in cm/sec; it is the initial velocity of the object falling; 490 is measured in (cm/sec)/sec and is half the acceleration due to gravity when measured in these units.

b, c. Below is a small table of values and the graph of the equation with the table points marked on it.

t	d
0.0	0.0
0.1	9.9
0.2	29.6
0.3	59.1

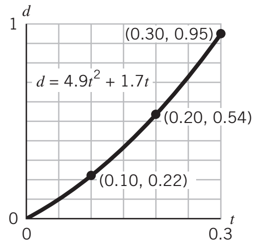


5. For $d = 4.9t^2 + 1.7t$:

a. 1.7 is the initial velocity of the object falling; it is measured in meters per second; 4.9 is half the gravitational constant when it is measured in (meters/sec)/sec.

b, c. Below is a small table of values, and next to it is the graph with the table points marked on it.

t	d
0.0	0.00
0.1	0.22
0.2	0.54
0.3	0.95



d. The results in this question are very similar to those in earlier parts of this chapter. The shape of the graph is that of a quadratic; the coefficient of t^2 is half the gravitational constant, and the coefficient of t is the initial velocity.

7. $m = \frac{m}{sec^2} \cdot \cancel{sec^2} + \frac{m}{\cancel{sec}} \cdot \cancel{sec}$

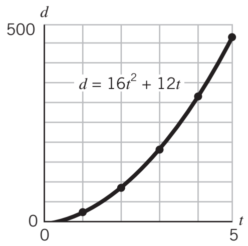
9. a. $d = 490t^2 + 50t$ and $v = 980t + 50$

b. At $t = 1$, $d = 540$ cm and $v = 1030$ cm/sec. At $t = 2.5$, $d = 3187.5$ cm and $v = 2500$ cm/sec.

11. a. $d = 16t^2 + 12t$

b.

t	d
0	0
1	28
2	88
3	180
4	304
5	460



c. The graph and table of the function are given above.

13. The distance is measured in meters if the time is measured in seconds. The use of 4.9 for half of the gravity constant is the indicator of these units.

15. a. Student answers will vary considerably.

b. Since the velocity is changing at a constant rate, a straight line should be a good fit. The graph of this line is a representation of average velocity.

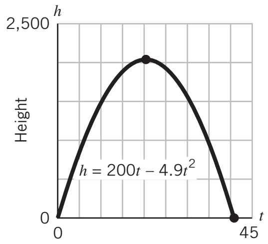
17. a. The coefficient of t^2 is one-half the gravity constant. Since the coefficient of t^2 is approximately 490, distance is measured in centimeters and time in seconds, and 490 is measured in cm/sec^2. The coefficient of t is an initial velocity of 7.6 cm/sec.

b. When $t = 0.05$ sec, $d = 1.59$ cm; when $t = 0.10$ sec, $d = 5.62$ cm, and when $t = 0.30$ sec, $d = 45.99$ cm.

19. It represents an initial velocity of the object measured in meters per second.

21. a. At $t = 0$ sec, $h = 0$ m; at $t = 1$ sec, $h = 195.1$ m; at $t = 2$ sec, $h = 380.4$ m; at $t = 10$ sec, $h = 1510$ m.

b. The graph of h over t is given in the following diagram.



c. The object reaches a maximum height of approximately 2000 meters after 20 seconds. It reaches the ground after approximately 40 seconds of flight.

23. For $h = 85 - 490t^2$:

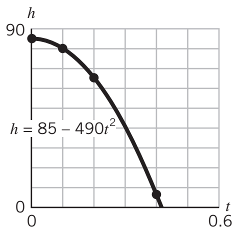
a. 85 is the height in centimeters of the falling object at the start; -490 is half the gravitational constant when measured in (cm/sec)/sec; it is negative in value since h measures height above the ground and the gravitational constant is connected with pulling objects down. This will mean subtraction from the starting height of 85 cm.

b. The initial velocity is 0 cm/sec.

c. Below is a table of values for this function

t	h
0.0	85.0
0.1	80.1
0.2	65.4
0.4	6.6

d. The following diagram is the graph of the function with the table entries marked on it.



25. a. The initial velocity is positive since we are measuring height above ground and the object is going up at the start.

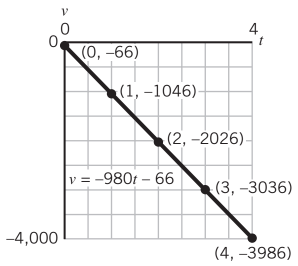
b. The equation of motion is $h = 50 + 10t - 16t^2$, where height is measured in feet and t in seconds.

27. a. 980 cm/sec^2 since we are measuring in cm and in sec.

b.

t	v
0	−66
1	−1046
2	−2026
3	−3006
4	−3986

c. The graph is given below. The object is traveling faster and faster toward the ground. The increase in downward velocity is at a constant rate, as we can see from the constant slope of the graph. This constant acceleration, of course, is due to gravity.



d. Ordinarily, if $t = 0$ corresponds to the actual start of the flight, then the initial condition given would indicate that the object was thrown downward at a speed of 66 meters per second. This interpretation comes from the negative sign given to the initial velocity. But this would contradict the statement in the problem to the effect that it is a "freely falling body." In this context, another interpretation is suggested by the laboratory experiment, namely that the object started being timed at a point along its downward fall.

29. a. Its velocity starts out negative and continues to be so since the object is falling; h is measured in cm above the ground; t is measured in seconds.

b. $h = 150 - 25t - 490t^2$; for $0 \le t \le 0.528$ (the second value being the approximate time in seconds it takes for the object to hit the ground).

c. The average velocity is the slope, i.e., $(15 - 150)/0.5 = -270$ cm/sec; the initial velocity is -25 cm/sec. The average velocity is 10.8 times as great in magnitude as the initial velocity.

31. Forming $\frac{d}{t} = \frac{v_0 + (v_0 + at)}{2}$ and solving for d, we get

$$d = \frac{2v_0 t + at^2}{2} = v_0 t + \tfrac{1}{2}at^2$$

This is very similar in form to the falling-body formula. The acceleration factor increases the velocity in a manner proportional to the square of the time traveled, and the initial velocity increases the distance in a manner proportional to the time.

33. a. After 5 seconds its velocity is 110 cm/sec; after 1 minute (or 60 seconds) its velocity is 660 cm/sec; after t seconds, its velocity is $v(t) = 60 + 10t$ cm/sec.

b. After 5 seconds its average velocity is $(60 + 110)/2 = 85$ cm/sec.

35. a. $v(t) = 200 + 60t$ meters/sec

b. $d(t) = 200t + 30t^2$ meters

37. a. Using units of feet and seconds, the equation governing the water spout is $h = -16t^2 + v_0 t$, where h is measured in feet and t, time, in seconds and where v_0 is the sought-after initial velocity. We are given that the maximum height reached is 120 ft. The maximum height is achieved at the vertex, i.e., when $t = -v_0/(-32) = \frac{v_0}{32}$. Substituting for t and h gives us

$$120 = -16\left(\frac{v_0}{32}\right)^2 + v_0\left(\frac{v_0}{32}\right) = -\frac{v_0^2}{64} + 2\frac{v_0^2}{64} = \frac{v_0^2}{64}$$

Thus $v_0^2 = 7680$ or $v_0 \approx 87.6$ ft per sec

b. $t = v_0/32 = 87.6/32 \approx 2.74$ sec

39. a. $d_c = v_c t + a_c t^2/2$; $d_p = a_p t^2/2$. One wants to solve for the t at which $d_c = d_p$, i.e., when $v_0 t + a_c\frac{t^2}{2} = a_p\frac{t^2}{2} \Rightarrow v_c t + a_c t^2/2 - a_p t^2/2 = 0 \Rightarrow t\,(v_c + [a_c/2 - a_p/2]t) = 0$. This occurs when $t = 0$ (when the criminal passes by the police car) and again when $t = 2v_c/(a_p - a_c)$, (when the police catch up to the criminal).

b. Now $v_c = a_c t + v_c$ and $v_p = a_p t$. One wants to solve for the t at which $v_c = v_p$, i.e., when $a_c t + v_c = a_p t$ or for $t = (a_p - a_c)/v_c$. This does not mean that the police have caught up to the criminal, but rather that the police are at that moment going as fast as the criminal is and that they are starting to go faster than the criminal.