CHAPTER FOUR

Understanding Money and Its Management

Hybrid Mortgages Can Cause Pain Should Rates Start to Rise[1] Are you shopping for a mortgage to finance a home that you expect to own for no more than a few years? If so, you should know about a hybrid mortgage. Hybrid loans give prospective home buyers the ability to buy a lot more home than they can afford, thanks to the initially lower interest rate. But with such flexibility comes greater risk. Since lenders are free to design loans to fit borrowers' needs, the terms and fees vary widely and homeowners can get burned if rates turn higher.

Hybrid mortgages allow homeowners to benefit from the best aspects of both fixed-rate and adjustable-rate mortgages (ARMs). With hybrids, borrowers choose to accept a fixed interest rate over a number of years—usually, 3, 5, 7, or 10 years—and afterward the loan

Rates Rising

Mortgages are more costly than they were a year ago.

	2006	2005
30-year fixed-rate mortgage	6.48%	5.85%
Hybrid ARM with fixed rate for first 10 years	6.32	5.58
Hybrid ARM with fixed rate for first 5 years	6.02	5.01
ARM with rate that adjusts annually	5.29	4.21

[1] "Hybrid mortgages can cause pain should rates start to rise," by Terri Cullen, *Wall Street Journal Online*, December 5, 2002. @2002 Dow Jones & Company, Inc.

converts to an ARM. But therein lies the danger: While you're getting an extraordinarily low rate up front for a few years, when the fixed-rate period expires you could very well end up paying more than double your current rate of interest.

At today's rate of 6.16% for a 30-year mortgage, a person borrowing $200,000 would pay $1,220 a month. With a 7-year hybrid, more commonly called a 7/1 loan, at the going rate of 5.61% that monthly payment drops to $1,150. By the end of the seventh year, the homeowner would save $7,700 in interest charges by going with a 7-year hybrid.

To say that there are drawbacks is an understatement. Despite the surge in popularity, a hybrid loan can be a ticking time bomb for borrowers who plan on holding the loan for the long term.

Let's say a borrower takes out a 30-year $200,000 hybrid loan that will remain at a fixed rate of 5.19% for 5 years and then will switch to an adjustable-rate mortgage for the remaining period. The lender agrees to set a cap on the adjustable-rate portion of the loan, so that the rate will climb no more than 5 percentage points. Conceivably, then, if rates are sharply higher after that initial 7-year period, a borrower could be looking at a mortgage rate of more than 10%. Under this scenario, the homeowner's monthly payment on a $200,000 mortgage would jump to $1,698 from $1,097 after the 5-year term expires—a 55% increase! But if there's a very real chance you'll be looking to sell your home over the next 10 years, hybrids can make a lot of sense, since shorter term loans usually carry the lowest rates.

In this chapter, we will consider several concepts crucial to managing money. In Chapter 3, we examined how time affects the value of money, and we developed various interest formulas for that purpose. Using these basic formulas, we will now extend the concept of equivalence to determine interest rates that are implicit in many financial contracts. To this end, we will introduce several examples in the area of loan transactions. For example, many commercial loans require that interest compound more frequently than once a year—for instance, monthly or quarterly. To consider the effect of more frequent compounding, we must begin with an understanding of the concepts of nominal and effective interest.

CHAPTER LEARNING OBJECTIVES

After completing this chapter, you should understand the following concepts:

- The difference between the nominal interest rate and the effective interest rate.
- The procedure for computing the effective interest rate, based on a payment period.
- How commercial loans and mortgages are structured in terms of interest and principal payments.
- The basics of investing in financial assets.

4.1 Nominal and Effective Interest Rates

In all the examples in Chapter 3, the implicit assumption was that payments are received *once a year*, or *annually*. However, some of the most familiar financial transactions, both personal and in engineering economic analysis, involve payments that are not based on one annual payment—for example, monthly mortgage payments and quarterly earnings on savings accounts. Thus, if we are to compare different cash flows with different compounding periods, we need to evaluate them on a common basis. This need has led to the development of the concepts of the **nominal interest rate** and the **effective interest rate**.

4.1.1 Nominal Interest Rates

Take a closer look at the billing statement from any of your credit cards. Or if you financed a new car recently, examine the loan contract. You will typically find the interest that the bank charges on your unpaid balance. Even if a financial institution uses a unit of time other than a year—say, a month or a quarter (e.g., when calculating interest payments)—the institution usually quotes the interest rate on an *annual basis*. Many banks, for example, state the interest arrangement for credit cards in this way:

18% compounded monthly.

Annual percentage rate (APR) is the yearly cost of a loan, including interest, insurance, and the origination fee, expressed as a percentage.

This statement simply means that each month the bank will charge 1.5% interest on an unpaid balance. We say that 18% is the **nominal interest rate** or **annual percentage rate** (APR), and the compounding frequency is monthly (12). As shown in Figure 4.1, to obtain the interest rate per compounding period, we divide, for example, 18% by 12, to get 1.5% per month.

Although the annual percentage rate, or APR, is commonly used by financial institutions and is familiar to many customers, the APR does not explain precisely the amount of interest that will accumulate in a year. To explain the true effect of more frequent compounding on annual interest amounts, we will introduce the term *effective interest rate*, commonly known as *annual effective yield*, or *annual percentage yield* (APY).

Month	1	2	3	4	5	6	7	8	9	10	11	12
Interest rate (%)	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5

Nominal interest rate.
1.5% × 12 = 18%

Figure 4.1 The nominal interest rate is determined by summing the individual interest rates per period.

4.1.2 Effective Annual Interest Rates

The **effective annual interest rate** is the one rate that truly represents the interest earned in a year. For instance, in our credit card example, the bank will charge 1.5% interest on any unpaid balance at the end of each month. Therefore, the 1.5% rate represents the effective interest rate per month. On a yearly basis, you are looking for a cumulative rate—1.5% each month for 12 months. This cumulative rate predicts the actual interest payment on your outstanding credit card balance.

Suppose you deposit $10,000 in a savings account that pays you at an interest rate of 9% compounded quarterly. Here, 9% represents the nominal interest rate, and the interest rate per quarter is 2.25% (9%/4). The following is an example of how interest is compounded when it is paid quarterly:

End of Period	Base amount	Interest Earned 2.25% × (Base amount)	New Base
First quarter	$10,000.00	2.25% × $10,000.00 = $225.00	$10,225.00
Second quarter	$10,225.00	2.25% × $10,225.00 = $230.06	$10,455.06
Third quarter	$10,455.06	2.25% × $10,455.06 = $225.24	$10,690.30
Fourth quarter	$10,690.30	2.25% × $10,690.30 = $240.53	$10,930.83

Clearly, you are earning more than 9% on your original deposit. In fact, you are earning 9.3083% ($930.83/$10,000). We could calculate the total annual interest payment for a principal amount of $10,000 with the formula given in Eq. (3.3). If $P = \$10,000$, $i = 2.25\%$, and $N = 4$, we obtain

$$\begin{aligned} F &= P(1 + i)^N \\ &= \$10,000(1 + 0.0225)^4 \\ &= \$10,930.83. \end{aligned}$$

The implication is that, for each dollar deposited, you are earning an equivalent annual interest of 9.38 cents. In terms of an effective annual interest rate (i_a), the interest payment can be rewritten as a percentage of the principal amount:

$$i_a = \$930.83/\$10{,}000 = 0.093083, \text{ or } 9.3083\%.$$

In other words, earning 2.25% interest per quarter for four quarters is equivalent to earning 9.3083% interest just one time each year.

Table 4.1 shows effective interest rates at various compounding intervals for 4%–12% APR. As you can see, depending on the frequency of compounding, the effective interest earned or paid by the borrower can differ significantly from the APR. Therefore, truth-in-lending laws require that financial institutions quote both the nominal interest rate and the compounding frequency (i.e., the effective interest) when you deposit or borrow money.

Certainly, more frequent compounding increases the amount of interest paid over a year at the same nominal interest rate. Assuming that the nominal interest rate is r, and M compounding periods occur during the year, we can calculate the effective annual interest rate

$$i_a = \left(1 + \frac{r}{M}\right)^M - 1. \tag{4.1}$$

When $M = 1$, we have the special case of annual compounding. Substituting $M = 1$ in Eq. (4.1) reduces it to $i_a = r$. That is, when compounding takes place once annually, the effective interest is equal to the nominal interest. Thus, in the examples in Chapter 3, in which only annual interest was considered, we were, by definition, using effective interest rates.

TABLE 4.1 Nominal and Effective Interest Rates with Different Compounding Periods

	Effective Rates				
Nominal Rate	Compounding Annually	Compounding Semiannually	Compounding Quarterly	Compounding Monthly	Compounding Daily
4%	4.00%	4.04%	4.06%	4.07%	4.08%
5	5.00	5.06	5.09	5.12	5.13
6	6.00	6.09	6.14	6.17	6.18
7	7.00	7.12	7.19	7.23	7.25
8	8.00	8.16	8.24	8.30	8.33
9	9.00	9.20	9.31	9.38	9.42
10	10.00	10.25	10.38	10.47	10.52
11	11.00	11.30	11.46	11.57	11.62
12	12.00	12.36	12.55	12.68	12.74

EXAMPLE 4.1 Determining a Compounding Period

The following table summarizes interest rates on certificates of deposit (CDs) offered by various lending institutions during November 2005:

Product	Bank	Minimum	Rate	APY*
3-Month CD	Imperial Capital Bank	$2,000	4.03%	4.10%
6-Month Jumbo CD	IndyMac Bank	$5,000	4.21%	4.30%
1-Year Jumbo CD	VirtualBank	$10,000	4.50%	4.60%
1.5-Year CD	AmTrust Bank	$1,000	4.50%	4.60%
2-Year CD	Ohio Savings Bank	$1,000	4.59%	4.70%
2.5-Year Jumbo CD	Countrywide Bank	$98,000	4.66%	4.77%
3-Year CD	ING Direct	$0	4.70%	4.70%
5-Year CD	Citizens & Northern Bank	$500	4.70%	4.78%

*Annual percentage yield = effective annual interest rate (i_a)

Annual percentage yield (APY) is the rate actually earned or paid in one year, taking into account the affect of compounding.

In the table, no mention is made of specific interest compounding frequencies. (a) Find the interest periods assumed for the 2.5-year Jumbo CD offered by Countrywide Bank. (b) Find the total balance for a deposit amount of $100,000 at the end of 2.5 years.

SOLUTION

Given: $r = 4.66\%$ per year, i_a (APY) $= 4.77\%$, $P = \$100{,}000$, and $N = 2.5$ years.
Find: M and the balance at the end of 2.5 years.

(a) The nominal interest rate is 4.66% per year, and the effective annual interest rate (yield) is 4.77%. Using Eq. (4.1), we obtain the expression

$$0.0477 = \left(1 + \frac{0.0466}{M}\right)^M - 1,$$

or

$$1.0477 = \left(1 + \frac{0.0466}{M}\right)^M.$$

By trial and error, we find that $M = 365$, which indicates daily compounding. Thus, the 2.5-year Jumbo CD earns 4.66% interest compounded daily.

Normally, if the CD is not cashed at maturity, it will be renewed automatically at the original interest rate. Similarly, we can find the interest periods for the other CDs.

(b) If you purchase the 2.5-year Jumbo CD, it will earn 4.66% interest compounded daily. This means that your CD earns an effective annual interest of 4.77%:

$$\begin{aligned} F &= P(1 + i_a)^N \\ &= \$100{,}000(1 + 0.0477)^{2.5} \\ &= \$100{,}000\left(1 + \frac{0.0466}{365}\right)^{365\times 2.5} \\ &= \$112{,}355. \end{aligned}$$

4.1.3 Effective Interest Rates per Payment Period

We can generalize the result of Eq. (4.1) to compute the effective interest rate for periods of *any duration*. As you will see later, the effective interest rate is usually computed on the basis of the payment (transaction) period. For example, if cash flow transactions occur quarterly, but interest is compounded monthly, we may wish to calculate the effective interest rate on a quarterly basis. To do this, we may redefine Eq. (4.1) as

$$\begin{aligned} i &= \left(1 + \frac{r}{M}\right)^C - 1 \\ &= \left(1 + \frac{r}{CK}\right)^C - 1, \end{aligned} \tag{4.2}$$

where

M = the number of interest periods per year,

C = the number of interest periods per payment period, and

K = the number of payment periods per year.

Note that $M = CK$ in Eq. (4.2). For the special case of annual payments with annual compounding, we obtain $i = i_a$ with $C = M$ and $K = 1$.

EXAMPLE 4.2 Effective Rate per Payment Period

Suppose that you make quarterly deposits in a savings account that earns 9% interest compounded monthly. Compute the effective interest rate per quarter.

SOLUTION

Given: $r = 9\%$, C = three interest periods per quarter, K = four quarterly payments per year, and M = 12 interest periods per year.

Find: i.

Using Eq. (4.2), we compute the effective interest rate per quarter as

$$i = \left(1 + \frac{0.09}{12}\right)^3 - 1$$
$$= 2.27\%.$$

COMMENTS: Figure 4.2 illustrates the relationship between the nominal and effective interest rates per payment period.

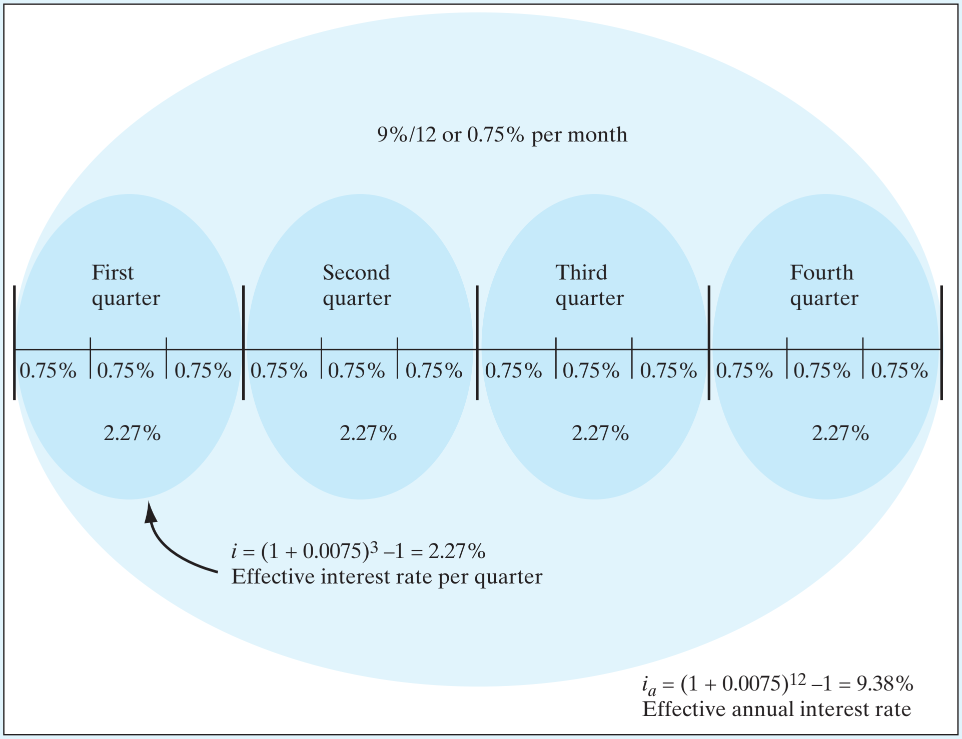


Figure 4.2 Functional relationships among r, i, and i_a, where interest is calculated based on 9% compounded monthly and payments occur quarterly (Example 4.2).

4.1.4 Continuous Compounding

To be competitive on the financial market or to entice potential depositors, some financial institutions offer frequent compounding. As the number of compounding periods (M) becomes very large, the interest rate per compounding period (r/M) becomes very small. As M approaches infinity and r/M approaches zero, we approximate the situation of **continuous compounding**.

By taking limits on both sides of Eq. (4.2), we obtain the effective interest rate per payment period as

$$i = \lim_{CK \to \infty}\left[\left(1 + \frac{r}{CK}\right)^{C} - 1\right]$$
$$= \lim_{CK \to \infty}\left(1 + \frac{r}{CK}\right)^{C} - 1$$
$$= (e^{r})^{1/K} - 1.$$

Continuous compounding: The process of calculating interest and adding it to existing principal and interest at infinitely short time intervals.

Therefore, the effective interest rate per payment period is

$$i = e^{r/K} - 1. \tag{4.3}$$

To calculate the effective annual interest rate for continuous compounding, we set K equal to unity, resulting in

$$i_a = e^{r} - 1. \tag{4.4}$$

As an example, the effective annual interest rate for a nominal interest rate of 12% compounded continuously is $i_a = e^{0.12} - 1 = 12.7497\%$.

EXAMPLE 4.3 Calculating an Effective Interest Rate with Quarterly Payment

Find the effective interest rate per *quarter* at a nominal rate of 8% compounded (a) quarterly, (b) monthly, (c) weekly, (d) daily, and (e) continuously.

SOLUTION

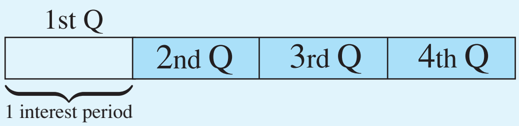
Given: $r = 8\%$, M, C, and $K = 4$ quarterly payments per year.

Find: i.

(a) Quarterly compounding:

$r = 8\%$, $M = 4$, $C = 1$ interest period per quarter, and $K = 4$ payments per year. Then

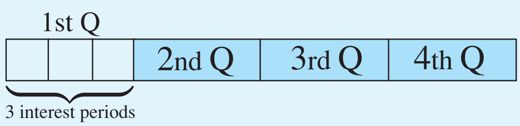
$$i = \left(1 + \frac{0.08}{4}\right)^{1} - 1 = 2.00\%.$$



(b) Monthly compounding:

$r = 8\%$, $M = 12$, $C = 3$ interest periods per quarter, and $K = 4$ payments per year. Then

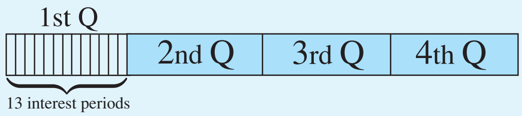
$$i = \left(1 + \frac{0.08}{12}\right)^{3} - 1 = 2.013\%.$$



(c) Weekly compounding:

$r = 8\%$, $M = 52$, $C = 13$ interest periods per quarter, and $K = 4$ payments per year. Then

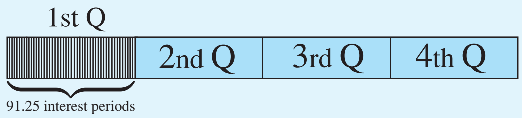
$$i = \left(1 + \frac{0.08}{52}\right)^{13} - 1 = 2.0186\%.$$



(d) Daily compounding:

$r = 8\%$, $M = 365$, $C = 91.25$ days per quarter, and $K = 4$. Then

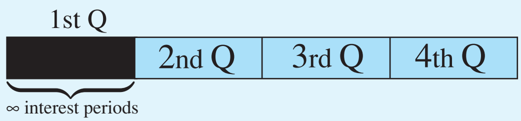
$$i = \left(1 + \frac{0.08}{365}\right)^{91.25} - 1 = 2.0199\%.$$



(e) Continuous compounding:

$r = 8\%$, $M \rightarrow \infty$, $C \rightarrow \infty$, and $K = 4$. Then, from Eq. (4.3),

$$i = e^{0.08/4} - 1 = 2.0201\%.$$



COMMENTS: Note that the difference between daily compounding and continuous compounding is often negligible. Many banks offer continuous compounding to entice deposit customers, but the extra benefits are small.

4.2 Equivalence Calculations with Effective Interest Rates

All the examples in Chapter 3 assumed annual payments and annual compounding. However, a number of situations involve cash flows that occur at intervals that are not the same as the compounding intervals often used in practice. Whenever payment and compounding periods differ from each other, *one or the other must be transformed so that both conform to the same unit of time*. For example, if payments occur quarterly and compounding occurs monthly, the most logical procedure is to calculate the effective interest rate per quarter. By contrast, if payments occur monthly and compounding occurs quarterly, we may be able to find the equivalent monthly interest rate. The bottom line is that, to proceed with equivalency analysis, the compounding and payment periods must be in the same order.

4.2.1 When Payment Period Is Equal to Compounding Period

Whenever the compounding and payment periods are equal $(M = K)$, whether the interest is compounded annually or at some other interval, the following solution method can be used:

1. Identify the number of compounding periods (M) per year.
2. Compute the effective interest rate per payment period—that is, using Eq. (4.2). Then, with $C = 1$ and $K = M$, we have

$$i = \frac{r}{M}.$$

3. Determine the number of compounding periods:

$$N = M \times (\text{number of years}).$$

EXAMPLE 4.4 Calculating Auto Loan Payments

Suppose you want to buy a car. You have surveyed the dealers' newspaper advertisements, and the following one has caught your attention:

College Graduate Special: New 2006 Nissan Altima 2.55 with automatic transmission, A/C, power package, and cruise control

MSRP:	\$20,870
Dealer's discount:	\$1,143
Manufacturer rebate	\$800
College graduate cash:	\$500
Sale price:	\$18,427

Price and payment is plus tax, title, customer service fee, with approved credit for 72 months at 6.25% APR.

You can afford to make a down payment of \$3,427 (and taxes and insurance as well), so the net amount to be financed is \$15,000. What would the monthly payment be (Figure 4.3)?

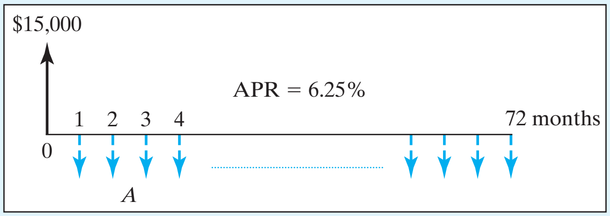


Figure 4.3 A car loan cash transaction (Example 4.4).

DISCUSSION: The advertisement does not specify a compounding period, but in automobile financing, the interest and the payment periods are almost always monthly. Thus, the 6.25% APR means 6.25% compounded monthly.

SOLUTION

Given: $P = \$25{,}000$, $r = 6.25\%$ per year, K $= 12$ payments per year, $N =$ 72 months, and $M = 12$ interest periods per year.
Find: A.

In this situation, we can easily compute the monthly payment with Eq. (3.12):

$$i = 6.25\%/12 = 0.5208\% \text{ per month,}$$

$$N = (12)(6) = 72 \text{ months,}$$

$$A = \$15{,}000(A/P, 0.5208\%, 72) = \$250.37.$$

4.2.2 Compounding Occurs at a Different Rate than That at Which Payments Are Made

We will consider two situations: (1) compounding is more frequent than payments and (2) compounding is less frequent than payments.

Compounding Is More Frequent than Payments

The computational procedure for compounding periods and payment periods that cannot be compared is as follows:

1. Identify the number of compounding periods per year (M), the number of payment periods per year (K), and the number of interest periods per payment period (C).
2. Compute the effective interest rate per payment period:
   - For discrete compounding, compute
   $$i = \left(1 + \frac{r}{M}\right) - 1.$$
   - For continuous compounding, compute
   $$i = e^{r/K} - 1.$$
3. Find the total number of payment periods:
   $$N = K \times (\text{number of years}).$$
4. Use i and N in the appropriate formulas in Table 3.4.

EXAMPLE 4.5 Compounding Occurs More Frequently than Payments Are Made (Discrete-Compounding Case)

Suppose you make equal quarterly deposits of $1,500 into a fund that pays interest at a rate of 6% compounded monthly, as shown in Figure 4.4. Find the balance at the end of year 2.

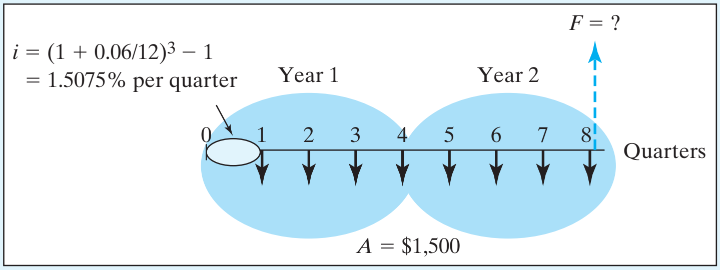


Figure 4.4 Quarterly deposits with monthly compounding (Example 4.5).

SOLUTION

Given: $A = \$1{,}500$ per quarter, $r = 6\%$ per year, $M = 12$ compounding periods per year, and $N = 8$ quarters.
Find: F.

We follow the aforementioned procedure for noncomparable compounding and payment periods:

1. Identify the parameter values for M, K, and C, where

$$M = 12 \text{ compounding periods per year,}$$
$$K = 4 \text{ payment periods per year,}$$
$$C = 3 \text{ interest periods per payment period.}$$

2. Use Eq. (4.2) to compute the effective interest:

$$i = \left(1 + \frac{0.06}{12}\right)^3 - 1$$
$$= 1.5075\% \text{ per quarter.}$$

3. Find the total number of payment periods, N:

$$N = K(\text{number of years}) = 4(2) = 8 \text{ quarters.}$$

4. Use i and N in the appropriate equivalence formulas:

$$F = \$1{,}500(F/A, 1.5075\%, 8) = \$12{,}652.60.$$

COMMENT: No 1.5075% interest table appears in Appendix A, but the interest factor can still be evaluated by $F = \$1{,}500(A/F, 0.5\%, 3)(F/A, 0.5\%, 24)$, where the first interest factor finds its equivalent monthly payment and the second interest factor converts the monthly payment series to an equivalent lump-sum future payment.

EXAMPLE 4.6 Compounding Occurs More Frequently than Payments Are Made (Continuous-Compounding Case)

A series of equal quarterly receipts of \$500 extends over a period of five years as shown in Figure 4.5. What is the present worth of this quarterly payment series at 8% interest compounded continuously?

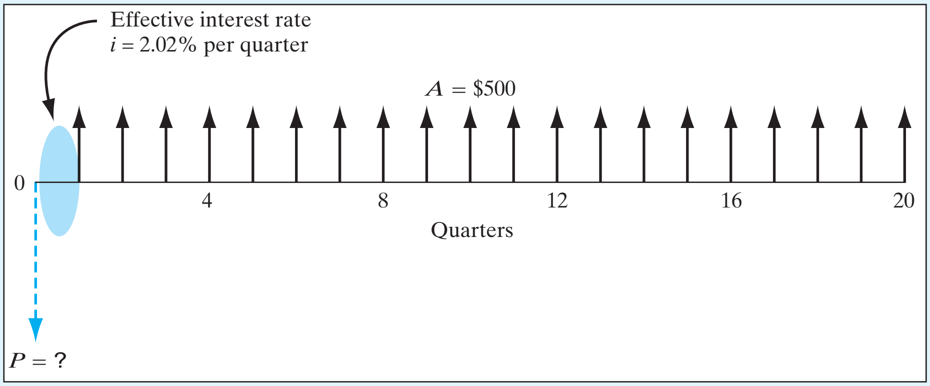


Figure 4.5 A present-worth calculation for an equal payment series with an interest rate of 8% compounded continuously (Example 4.6).

Discussion: A question that is equivalent to the preceding one is "How much do you need to deposit now in a savings account that earns 8% interest compounded continuously so that you can withdraw \$500 at the end of each quarter for five years?" Since the payments are quarterly, we need to compute i per quarter for the equivalence calculations:

$$\begin{aligned} i &= e^{r/K} - 1 = e^{0.08/4} - 1 \\ &= 2.02\% \text{ per quarter} \end{aligned}$$

$$\begin{aligned} N &= (4 \text{ payment periods per year})(5 \text{ years}) \\ &= 20 \text{ quarterly periods.} \end{aligned}$$

SOLUTION

Given: $i = 2.02\%$ per quarter, $N = 20$ quarters, and $A = \$500$ per quarter.

Find: P.

Using the $(P/A, i, N)$ factor with $i = 2.02\%$ and $N = 20$, we find that

$$\begin{aligned} P &= \$500(P/A, 2.02\%, 20) \\ &= \$500(16.3199) \\ &= \$8{,}159.96. \end{aligned}$$

Compounding Is Less Frequent than Payments

The next two examples contain identical parameters for savings situations in which compounding occurs less frequently than payments. However, two different underlying assumptions govern how interest is calculated. In Example 4.7, the assumption is that, whenever a deposit is made, it starts to earn interest. In Example 4.8, the assumption is that the deposits made within a quarter do not earn interest until the end of that quarter. As a result, in Example 4.7 we transform the compounding period to conform to the payment period, and in Example 4.8 we lump several payments together to match the compounding period. In the real world, which assumption is applicable depends on the transactions and the financial institutions involved. The accounting methods used by many firms record cash transactions that occur within a compounding period as if they had occurred at the end of that period. For example, when cash flows occur daily, but the compounding period is monthly, the cash flows within each month are summed (ignoring interest) and treated as a single payment on which interest is calculated.

Note: *In this textbook, we assume that whenever the time point of a cash flow is specified, one cannot move it to another time point without considering the time value of money (i.e., the practice demonstrated in Example 4.7 should be followed).*

EXAMPLE 4.7 Compounding Is Less Frequent than Payments: Effective Interest Rate per Payment Period

Suppose you make \$500 monthly deposits to a tax-deferred retirement plan that pays interest at a rate of 10% compounded quarterly. Compute the balance at the end of 10 years.

SOLUTION

Given: $r = 10\%$ per year, $M = 4$ quarterly compounding periods per year, $K = 12$ payment periods per year, $A = \$500$ per month, $N = 120$ months, and interest is accrued on deposits made during the compounding period.
Find: i, F.

As in the case of Example 4.5, the procedure for noncomparable compounding and payment periods is followed:

1. The parameter values for M, K, and C are

$$M = 4 \text{ compounding periods per year,}$$
$$K = 12 \text{ payment periods per year,}$$
$$C = \tfrac{1}{3} \text{ interest period per payment period.}$$

2. As shown in Figure 4.6, the effective interest rate per payment period is calculated with Eq. (4.2):

$$i = 0.826\% \text{ per month.}$$

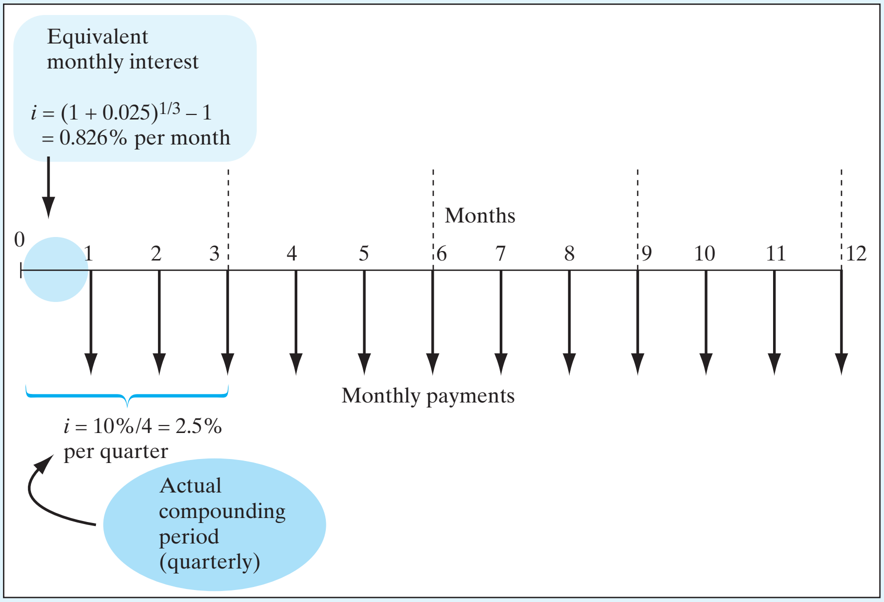


Figure 4.6 Calculation of equivalent monthly interest when the quarterly interest rate is specified (Example 4.7).

3. Find N:

$$N = (12)(10) = 120 \text{ payment periods.}$$

4. Use i and N in the appropriate equivalence formulas (Figure 4.7):

$$\begin{aligned} F &= \$500(F/A, 0.826\%, 120) \\ &= \$101{,}907.89. \end{aligned}$$

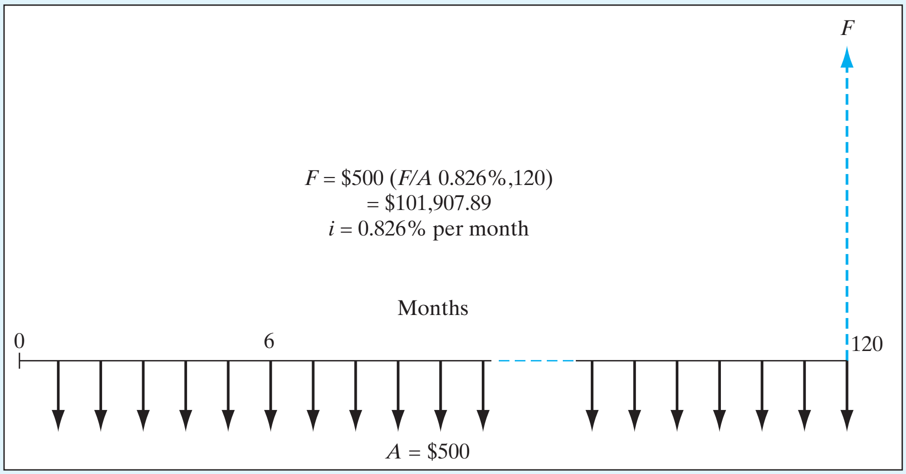


Figure 4.7 Cash flow diagram (Example 4.7).

EXAMPLE 4.8 Compounding Is Less Frequent than Payment: Summing Cash Flows to the End of the Compounding Period

Some financial institutions will not pay interest on funds deposited after the start of the compounding period. To illustrate, consider Example 4.7 again. Suppose that money deposited during a quarter (the compounding period) will not earn any interest (Figure 4.8). Compute what F will be at the end of 10 years.

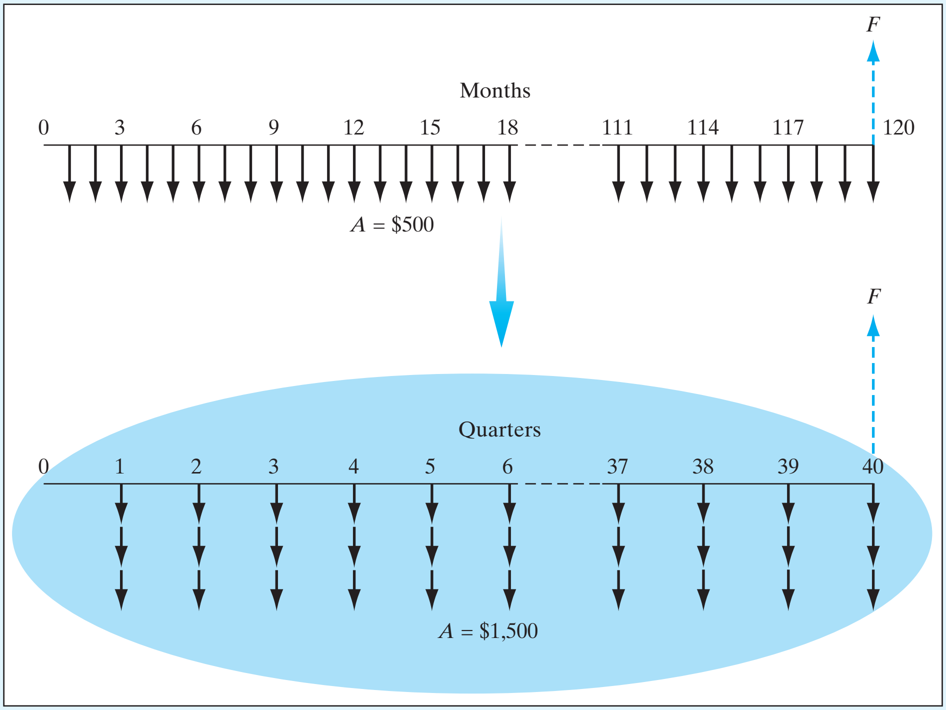


Figure 4.8 Transformed cash flow diagram created by summing monthly cash flows to the end of the quarterly compounding period (Example 4.8).

SOLUTION

Given: Same as for Example 4.7; however, no interest on flow during the compounding period.

Find: F.

In this case, the three monthly deposits during each quarterly period will be placed at the end of each quarter. Then the payment period coincides with the interest period, and we have

$$i = \frac{10\%}{4} = 2.5\% \text{ per quarter},$$

$$A = 3(\$500) = \$1{,}500 \text{ per quarter},$$

$$N = 4(10) = 40 \text{ payment periods},$$

$$F = \$1{,}500(F/A, 2.5\%, 40) = \$101{,}103.83.$$

COMMENTS: In Example 4.8, the balance will be \$804.06 less than in Example 4.7, a fact that is consistent with our understanding that increasing the frequency of compounding increases the future value of money. Some financial institutions follow the practice illustrated in Example 4.7. As an investor, you should reasonably ask yourself whether it makes sense to make deposits in an interest-bearing account more frequently than interest is paid. In the interim between interests compounding, you may be tying up your funds prematurely and forgoing other opportunities to earn interest.

Figure 4.9 is a decision chart that allows you to sum up how you can proceed to find the effective interest rate per payment period, given the various possible compounding and interest arrangements.

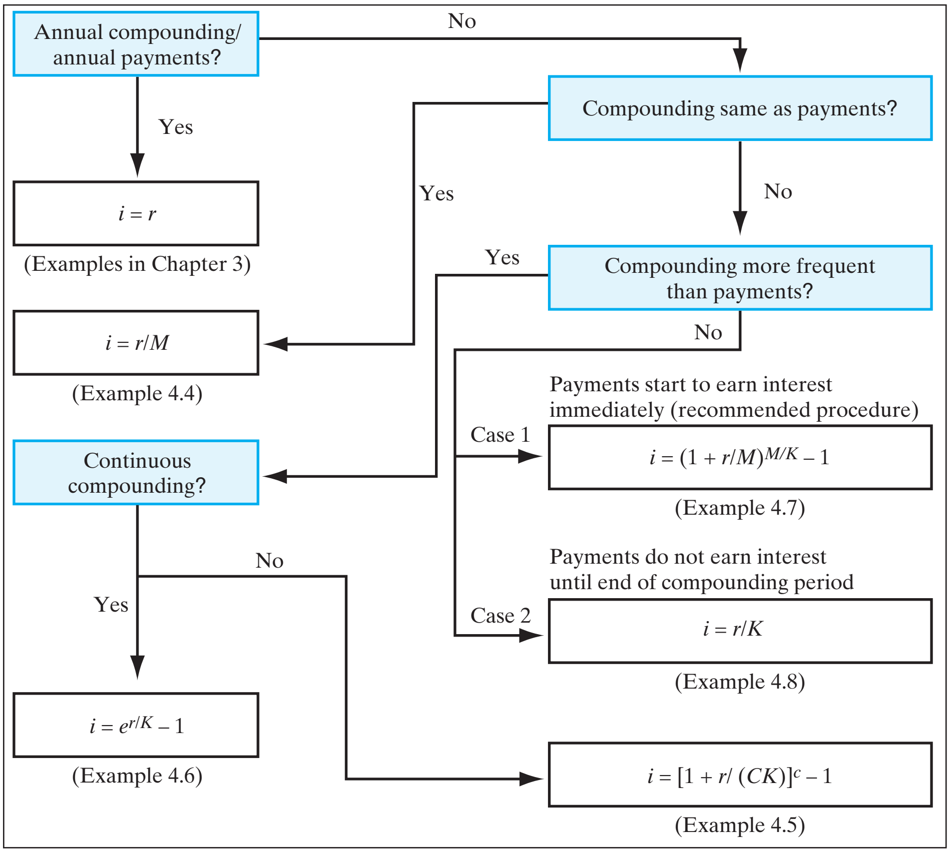


Figure 4.9 A decision flowchart demonstrating how to compute the effective interest rate i per payment period.

4.3 Equivalence Calculations with Continuous Payments

As we have seen so far, interest can be compounded annually, semiannually, monthly, or even continuously. Discrete compounding is appropriate for many financial transactions; mortgages, bonds, and installment loans, which require payments or receipts at discrete times, are good examples. In most businesses, however, transactions occur continuously throughout the year. In these circumstances, we may describe the financial transactions as having a continuous flow of money, for which continuous compounding and discounting are more realistic. This section illustrates how one establishes economic equivalence between cash flows under continuous compounding.

Continuous cash flows represent situations in which money flows continuously and at a known rate throughout a given period. In business, many daily cash flow transactions can be viewed as continuous. An advantage of the continuous-flow approach is that it more closely models the realities of business transactions. Costs for labor, for carrying inventory, and for operating and maintaining equipment are typical examples. Others include capital improvement projects that conserve energy or water or that process steam. Savings on these projects can occur continuously.

4.3.1 Single-Payment Transactions

First we will illustrate how single-payment formulas for continuous compounding and discounting are derived. Suppose that you invested P dollars at a nominal rate of r % interest for N years. If interest is compounded continuously, the effective annual interest is $i = e^r - 1$. The future value of the investment at the end of N years is obtained with the F/P factor by substituting $e^r - 1$ for i:

$$\begin{aligned} F &= P(1 + i)^N \\ &= P(1 + e^r - 1)^N \\ &= Pe^{rN}. \end{aligned}$$

This implies that \$1 invested now at an interest rate of r% compounded continuously accumulates to e^{rN} dollars at the end of N years. Correspondingly, the present value of F due N years from now and discounted continuously at an interest rate of r% is equal to

$$P = Fe^{-rN}.$$

We can say that the present value of \$1 due N years from now and discounted continuously at an annual interest rate of r% is equal to e^{-rN} dollars.

4.3.2 Continuous-Funds Flow

Suppose that an investment's future cash flow per unit of time (e.g., per year) can be expressed by a continuous function ($f(t)$) that can take any shape. Suppose also that the

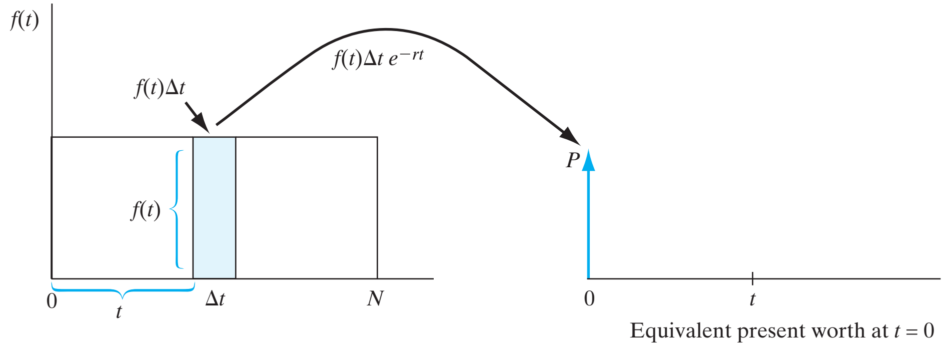


Figure 4.10 Finding an equivalent present worth of a continuous-flow payment function $f(t)$ at a nominal rate of $r\%$.

investment promises to generate cash of $f(t)\Delta t$ dollars between t and $t + \Delta t$, where t is a point in the time interval $0 \leq t \leq N$ (Figure 4.10). If the nominal interest rate is a constant r during this interval, the present value of the cash stream is given approximately by the expression

$$\Sigma(f(t)\Delta t)e^{-rt},$$

where e^{-rt} is the discounting factor that converts future dollars into present dollars. With the project's life extending from 0 to N, we take the summation over all subperiods (compounding periods) in the interval from 0 to N. As the interval is divided into smaller and smaller segments (i.e., as Δt approaches zero), we obtain the expression for the present value by the integral

$$P = \int_0^N f(t)e^{-rt}\,dt. \tag{4.5}$$

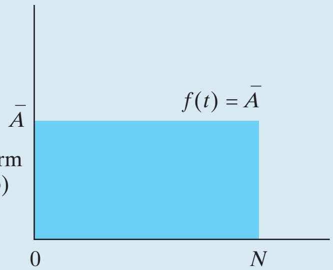
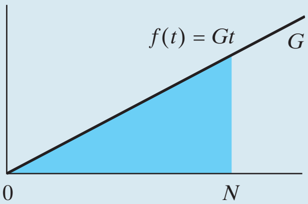
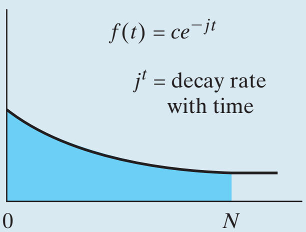
Similarly, the expression for the future value of the cash flow stream is given by the equation

$$F = Pe^{rN} = \int_0^N f(t)e^{r(N-t)}\,dt, \tag{4.6}$$

where $e^{r(N-t)}$ is the compounding factor that converts present dollars into future dollars. It is important to observe that the time unit is the *year*, because the effective interest rate is expressed in terms of a year. Therefore, all time units in equivalence calculations must be converted into years. Table 4.2 summarizes some typical continuous cash functions that can facilitate equivalence calculations.[2]

[2] Chan S. Park and Gunter P. Sharp-Bette, *Advanced Engineering Economics*. New York: John Wiley & Sons, 1990. (Reprinted by permission of John Wiley & Sons, Inc.)

TABLE 4.2 **Summary of Interest Factors for Typical Continuous Cash Flows with Continuous Compounding**

Type of Cash Flow	Cash Flow Function	Parameters Find	Parameters Given	Algebraic Notation	Factor Notation
Uniform (step)		P	$\overline{A}$	$\overline{A}\left[\frac{e^{rN}-1}{re^{rN}}\right]$	$(P/\overline{A}, r, N)$
		$\overline{A}$	P	$P\left[\frac{re^{rN}}{e^{rN}-1}\right]$	$(\overline{A}/P, r, N)$
		F	$\overline{A}$	$\overline{A}\left[\frac{e^{rN}-1}{r}\right]$	$(F/\overline{A}, r, N)$
		$\overline{A}$	F	$F\left[\frac{r}{e^{rN}-1}\right]$	$(\overline{A}/P, r, N)$
Gradient (ramp)		P	G	$\frac{G}{r^2}(1 - e^{-rN}) - \frac{G}{r}(Ne^{-rN})$	
Decay		P	c, j	$\frac{c}{r+j}(1 - e^{-(r+j)N})$	

EXAMPLE 4.9 Comparison of Daily Flows and Daily Compounding with Continuous Flows and Continuous Compounding

Consider a situation in which money flows daily. Suppose you own a retail shop and generate \$200 cash each day. You establish a special business account and deposit your daily cash flows in an account for 15 months. The account earns an interest rate of 6%. Compare the accumulated cash values at the end of 15 months, assuming

(a) Daily compounding and
(b) Continuous compounding.

SOLUTION

(a) With daily compounding:

Given: $A = \$200$ per day $r = 6\%$ per year, $M = 365$ compounding periods per year, and $N = 455$ days.
Find: F.

Assuming that there are 455 days in the 15-month period, we find that

$$\begin{aligned} i &= 6\%/365 \\ &= 0.01644\% \text{ per day}, \\ N &= 455 \text{ days}. \end{aligned}$$

The balance at the end of 15 months will be

$$\begin{aligned} F &= \$200(F/A, 0.01644\%, 455) \\ &= \$200(472.4095) \\ &= \$94{,}482. \end{aligned}$$

(b) With continuous compounding:

Now we approximate this discrete cash flow series by a uniform continuous cash flow function as shown in Figure 4.11. In this situation, an amount flows at the rate of $\overline{A}$ per year for N years.

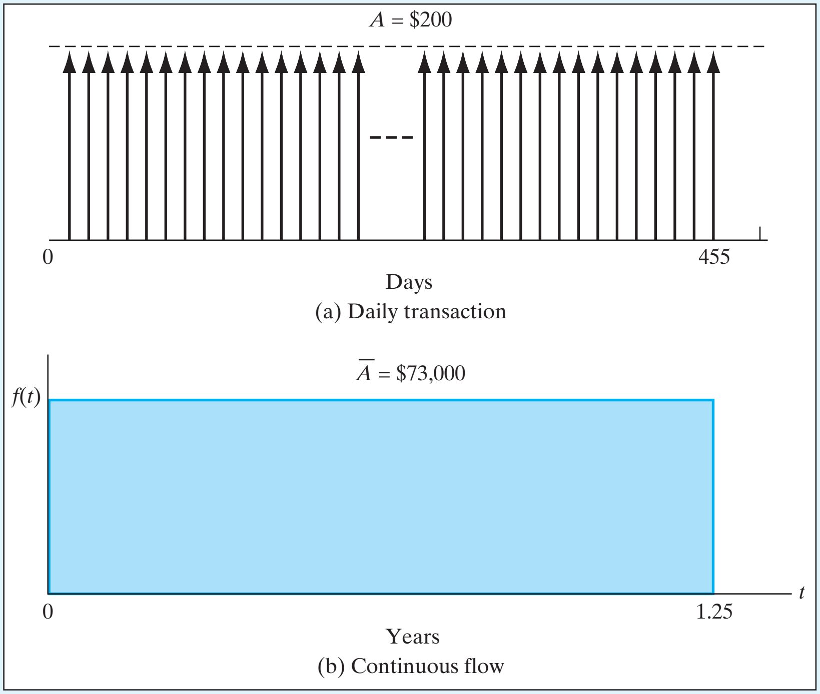


Figure 4.11 Comparison between daily transaction and continuous-funds flow transaction (Example 4.9).

Note: *Our time unit is a year*. Thus, a 15-month period is 1.25 years. Then the cash flow function is expressed as

$$\begin{aligned} f(t) &= \overline{A}, 0 \leq t \leq 1.25 \\ &= \$200(365) \\ &= \$73{,}000 \text{ per year.} \end{aligned}$$

Given: $\overline{A}$ = \$73,000 per year, r = 6% per year, compounded continuously, and N = 1.25 years.
Find: F.
Substituting these values back into Eq. (4.6) yields

$$\begin{aligned} F &= \int_0^{1.25} 73{,}000e^{0.06(1.25-t)}\,dt \\ &= \$73{,}000\left[\frac{e^{0.075}-1}{0.06}\right] \\ &= \$94{,}759. \end{aligned}$$

The factor in the bracket is known as the **funds flow compound amount factor** and is designated $(F/\overline{A}, r, N)$ as shown in Table 4.2. Notice that the difference between the two methods is only \$277 (less than 0.3%).

COMMENTS: As shown in this example, the differences between discrete daily compounding and continuous compounding have no practical significance in most cases. Consequently, as a mathematical convenience, instead of assuming that money flows in discrete increments at the end of each day, we could assume that money flows continuously at a uniform rate during the period in question. This type of cash flow assumption is common practice in the chemical industry.

4.4 Changing Interest Rates

Up to this point, we have assumed a constant interest rate in our equivalence calculations. When an equivalence calculation extends over several years, more than one interest rate may be applicable to properly account for the time value of money. That is to say, over time, interest rates available in the financial marketplace fluctuate, and a financial institution that is committed to a long-term loan may find itself in the position of losing the opportunity to earn higher interest because some of its holdings are tied up in a lower interest loan. The financial institution may attempt to protect itself from such lost earning opportunities by building gradually increasing interest rates into a long-term loan at the outset. Adjustable-rate mortgage (ARM) loans are perhaps the most common examples of variable interest rates. In this section, we will consider variable interest rates in both single payments and a series of cash flows.

4.4.1 Single Sums of Money

To illustrate the mathematical operations involved in computing equivalence under changing interest rates, first consider the investment of a single sum of money, P, in a savings account for N interest periods. If i_n denotes the interest rate appropriate during period n, then the future worth equivalent for a single sum of money can be expressed as

$$F = P(1 + i_1)(1 + i_2)\cdots(1 + i_{N-1})(1 + i_N), \quad (4.7)$$

and solving for P yields the inverse relation

$$P = F[(1 + i_1)(1 + i_2)\cdots(1 + i_{N-1})(1 + i_N)]^{-1}. \quad (4.8)$$

EXAMPLE 4.10 Changing Interest Rates with a Lump-Sum Amount

Suppose you deposit $2,000 in an individual retirement account (IRA) that pays interest at 6% compounded monthly for the first two years and 9% compounded monthly for the next three years. Determine the balance at the end of five years (Figure 4.12).

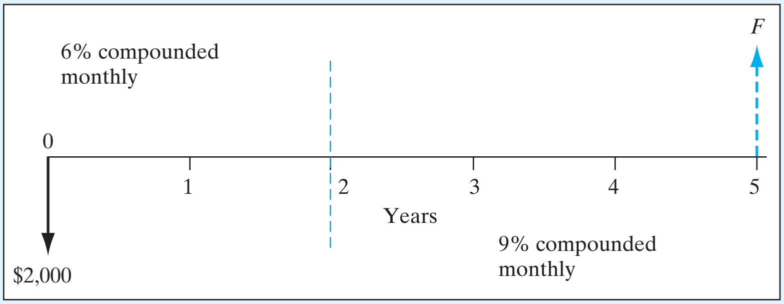


Figure 4.12 Changing interest rates (Example 4.10).

SOLUTION

Given: $P = \$2{,}000$, $r = 6\%$ per year for first two years, 9% per year for last three years, $M = 4$ compounding periods per year, $N = 20$ quarters.
Find: F.

We will compute the value of F in two steps. First we will compute the balance B_2 in the account at the end of two years. With 6% compounded quarterly, we have

$$i = 6\%/12 = 0.5\%$$

$$N = 12(2) = 24 \text{ months}$$

$$B_2 = \$2{,}000(F/P, 0.5\%, 24)$$

$$= \$2{,}000(1.12716)$$

$$= \$2{,}254.$$

Since the fund is not withdrawn, but reinvested at 9% compounded monthly, as a second step we compute the final balance as follows:

$$i = 9\%/12 = 0.75\%$$
$$N = 12(3) = 36 \text{ months}$$
$$F = B_2(F/P, 0.75\%, 36)$$
$$= \$2{,}254(1.3086)$$
$$= \$2{,}950.$$

4.4.2 Series of Cash Flows

The phenomenon of changing interest rates can easily be extended to a series of cash flows. In this case, the present worth of a series of cash flows can be represented as

$$P = A_1(1 + i_1)^{-1} + A_2[(1 + i_1)^{-1}(1 + i_2)^{-1}] + \dots + A_N[(1 + i_1)^{-1}(1 + i_2)^{-1} \dots (1 + i_N)^{-1}]. \quad (4.9)$$

The future worth of a series of cash flows is given by the inverse of Eq. (4.9):

$$F = A_1[(1 + i_2)(1 + i_3) \dots (1 + i_N)] + A_2[(1 + i_3)(1 + i_4) \dots (1 + i_N)] + \dots + A_N. \quad (4.10)$$

The uniform series equivalent is obtained in two steps. First, the present-worth equivalent of the series is found from Eq. (4.9). Then A is obtained after establishing the following equivalence equation:

$$P = A(1 + i_1)^{-1} + A[(1 + i_1)^{-1}(1 + i_2)^{-1}] + \dots + A[(1 + i_1)^{-1}(1 + i_2)^{-1} \dots (1 + i_N)^{-1}]. \quad (4.11)$$

EXAMPLE 4.11 Changing Interest Rates with Uneven Cash Flow Series

Consider the cash flow in Figure 4.13 with the interest rates indicated, and determine the uniform series equivalent of the cash flow series.

DISCUSSION: In this problem and many others, the easiest approach involves collapsing the original flow into a single equivalent amount, for example, at time zero, and then converting the single amount into the final desired form.

SOLUTION

Given: Cash flows and interest rates as shown in Figure 4.13; $N = 3$.
Find: A.

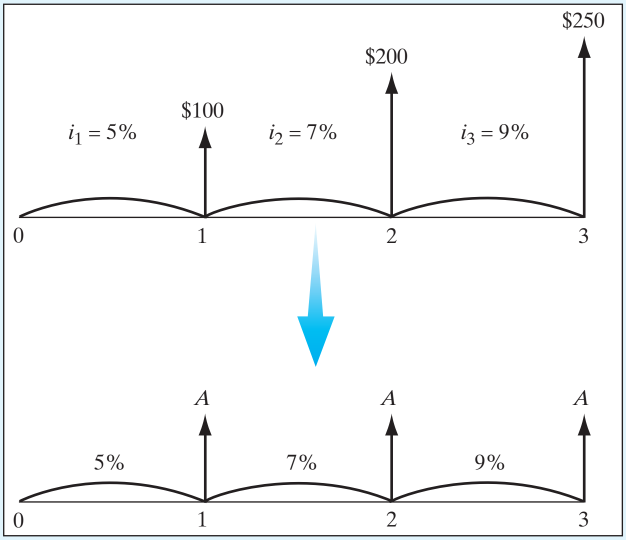


Figure 4.13 Equivalence calculation with changing interest rates (Example 4.11).

Using Eq. (4.9), we find the present worth:

$$\begin{aligned} P &= \$100(P/F, 5\%, 1) + \$200(P/F, 5\%, 1)(P/F, 7\%, 1) \\ &\quad + \$250(P/F, 5\%, 1)(P/F, 7\%, 1)(P/F, 9\%, 1) \\ &= \$477.41. \end{aligned}$$

Then we obtain the uniform series equivalent as follows:

$$\begin{aligned} \$477.41 &= A(P/F, 5\%, 1) + A(P/F, 5\%, 1)(P/F, 7\%, 1) \\ &\quad + A(P/F, 5\%, 1)(P/F, 7\%, 1)(P/F, 9\%, 1) \\ &= 2.6591A \\ A &= \$179.54. \end{aligned}$$

4.5 Debt Management

Credit card debt and commercial loans are among the most significant financial transactions involving interest. Many types of loans are available, but here we will focus on those most frequently used by individuals and in business.

4.5.1 Commercial Loans

One of the most important applications of compound interest involves loans that are paid off in **installments** over time. If the loan is to be repaid in equal periodic amounts (weekly, monthly, quarterly, or annually), it is said to be an **amortized loan**.

Examples of installment loans include automobile loans, loans for appliances, home mortgage loans, and the majority of business debts other than very short-term loans. Most commercial loans have interest that is compounded monthly. With an auto loan, a local bank or a dealer advances you the money to pay for the car, and you repay the principal plus interest in monthly installments, usually over a period of three to five years. The car is your collateral. If you don't keep up with your payments, the lender can repossess the car and keep all the payments you have made.

Two things determine what borrowing will cost you: the finance charge and the length of the loan. The cheapest loan is not the one with the lowest payments or even the one with the lowest interest rate. Instead, you have to look at (1) the total cost of borrowing, which depends on the interest rate plus fees, and (2) the term, or length of time it takes you to repay the loan. While you probably cannot influence the rate or the fees, you may be able to arrange for a shorter term.

- **The annual percentage rate** (APR) is set by lenders, who are required to tell you what a loan will actually cost per year, expressed as an APR. Some lenders charge lower interest, but add high fees; others do the reverse. Combining the fees with a year of interest charges to give you the true annual interest rate, the APR allows you to compare these two kinds of loans on equal terms.
- **Fees** are the expenses the lender will charge to lend the money. The application fee covers processing expenses. Attorney fees pay the lender's attorney. Credit search fees cover researching your credit history. Origination fees cover administrative costs and sometimes appraisal fees. All these fees add up very quickly and can substantially increase the cost of your loan.
- **Finance charges** are the cost of borrowing. For most loans, they include all the interest, fees, service charges, points, credit-related insurance premiums, and any other charges.
- **The periodic interest rate** is the interest the lender will charge on the amount you borrow. If lender also charges fees, the periodic interest rate will not be the true interest rate.
- **The term of your loan** is crucial in determining its cost. Shorter terms mean squeezing larger amounts into fewer payments. However, they also mean paying interest for fewer years, saving a lot of money.

Amortized Installment Loans

So far, we have considered many instances of amortized loans in which we calculated present or future values of the loans or the amounts of the installment payments. An additional aspect of amortized loans that will be of great interest to us is calculating the amount of interest versus the portion of the principal that is paid off in each installment. As we shall explore more fully in Chapter 10, the interest paid on a loan is an important element in calculating taxable income and has repercussions for both personal and business loan transactions. For now, we will focus on several methods of calculating interest and principal paid at any point in the life of the loan.

In calculating the size of a monthly installment, lending institutions may use two types of schemes. The first is the conventional amortized loan, based on the compound-interest method, and the other is the add-on loan, based on the simple-interest concept. We explain each method in what follows, but it should be understood that the amortized loan is the most common in various commercial lending. The add-on loan is common in financing appliances as well as furniture.

In a typical amortized loan, the amount of interest owed for a specified period is calculated on the basis of the remaining balance on the loan at the beginning of the period. A set of formulas has been developed to compute the remaining loan balance, interest payment, and principal payment for a specified period. Suppose we borrow an amount P at an interest rate i and agree to repay this principal sum P, including interest, in equal payments A over N periods. Then the size of the payment is $A = P(A/P, i, N)$, and each payment is divided into an amount that is interest and a remaining amount that goes toward paying the principal.

Let

$$B_n = \text{Remaining balance at the end of period } n, \text{ with } B_0 = P,$$
$$I_n = \text{Interest payment in period } n, \text{ where } I_n = B_{n-1}i,$$
$$P_n = \text{Principal payment in period } n.$$

Then each payment can be defined as

$$A = P_n + I_n. \tag{4.12}$$

The interest and principal payments for an amortized loan can be determined in several ways; two are presented here. No clear-cut reason is available to prefer one method over the other. Method 1, however, may be easier to adopt when the computational process is automated through a spreadsheet application, whereas Method 2 may be more suitable for obtaining a quick solution when a period is specified. You should become comfortable with at least one of these methods; pick the one that comes most naturally to you.

Method 1: Tabular Method. The first method is tabular. The interest charge for a given period is computed progressively on the basis of the remaining balance at the beginning of that period. Example 4.12 illustrates the process of creating a loan repayment schedule based on an iterative approach.

EXAMPLE 4.12 Loan Balance, Principal, and Interest: Tabular Method

Suppose you secure a home improvement loan in the amount of \$5,000 from a local bank. The loan officer computes your monthly payment as follows:

$$\text{Contract amount} = \$5{,}000,$$
$$\text{Contract period} = 24 \text{ months},$$
$$\text{Annual percentage rate} = 12\%,$$
$$\text{Monthly installments} = \$235.37.$$

Figure 4.14 is the cash flow diagram. Construct the loan payment schedule by showing the remaining balance, interest payment, and principal payment at the end of each period over the life of the loan.

SOLUTION

Given: $P = \$5{,}000$, $A = \$235.37$ per month, $r = 12\%$ per year, $M = 12$ compounding periods per year, and $N = 24$ months.

Find: B_n and I_n for $n = 1$ to 24.

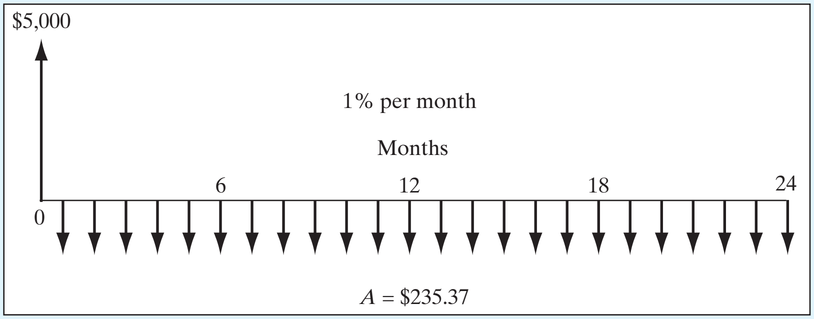


Figure 4.14 Cash flow diagram of the home improvement loan with an APR of 12% (Example 4.12).

TABLE 4.3 Creating a Loan Repayment Schedule with Excel (Example 4.12)

Payment No.	Size of Payment	Principal Payment	Interest Payment	Loan Balance
1	$235.37	$185.37	$50.00	$4,814.63
2	235.37	187.22	48.15	4,627.41
3	235.37	189.09	46.27	4,438.32
4	235.37	190.98	44.38	4,247.33
5	235.37	192.89	42.47	4,054.44
6	235.37	194.83	40.54	3,859.62
7	235.37	196.77	38.60	3,662.85
8	235.37	198.74	36.63	3,464.11
9	235.37	200.73	34.64	3,263.38
10	235.37	202.73	32.63	3,060.65
11	235.37	204.76	30.61	2,855.89
12	235.37	206.81	28.56	2,649.08
13	235.37	208.88	26.49	2,440.20
14	235.37	210.97	24.40	2,229.24
15	235.37	213.08	22.29	2,016.16
16	235.37	215.21	20.16	1,800.96
17	235.37	217.36	18.01	1,583.60
18	235.37	219.53	15.84	1,364.07
19	235.37	221.73	13.64	1,142.34
20	235.37	223.94	11.42	918.40
21	235.37	226.18	9.18	692.21
22	235.37	228.45	6.92	463.77
23	235.37	230.73	4.64	233.04
24	235.37	233.04	2.33	0.00

We can easily see how the bank calculated the monthly payment of \$235.37. Since the effective interest rate per payment period on this loan is 1% per month, we establish the following equivalence relationship:

$$\$235.37(P/A, 1\%, 24) = \$235.37(21.2431) = \$5{,}000.$$

The loan payment schedule can be constructed as in Table 4.3. The interest due at $n = 1$ is \$50.00, 1% of the \$5,000 outstanding during the first month. The \$185.37 left over is applied to the principal, reducing the amount outstanding in the second month to \$4,814.63. The interest due in the second month is 1% of \$4,814.63, or \$48.15, leaving \$187.22 for repayment of the principal. At $n = 24$, the last \$235.37 payment is just sufficient to pay the interest on the unpaid principal of the loan and to repay the remaining principal. Figure 4.15 illustrates the ratios between the interest and principal payments over the life of the loan.

COMMENTS: Certainly, constructing a loan repayment schedule such as that in Table 4.3 can be tedious and time consuming, unless a computer is used. As you can see in the website for this book, you can download an Excel file that creates the loan repayment schedule, on which you can make any adjustment to solve a typical loan problem of your choice.

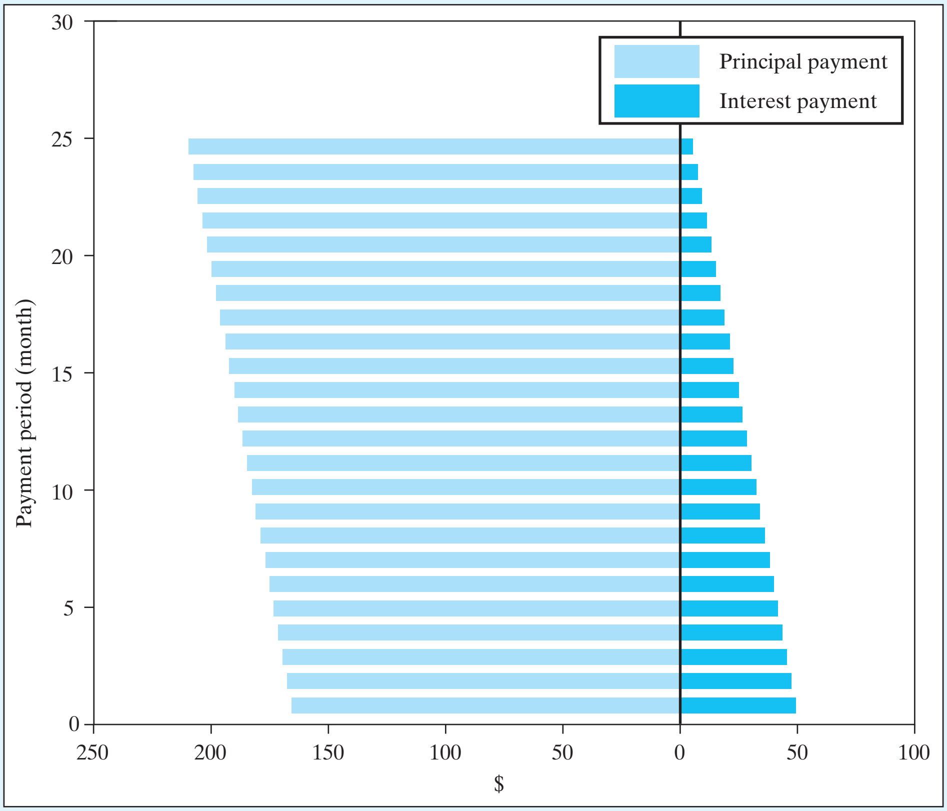


Figure 4.15 The proportions of principal and interest payments over the life of the loan (monthly payment = \$235.37) (Example 4.12).

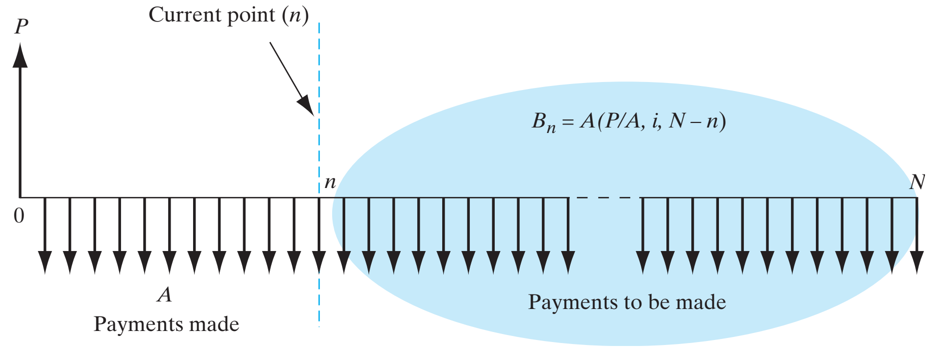


Figure 4.16 Calculating the remaining loan balance on the basis of Method 2.

Method 2: Remaining-Balance Method. Alternatively, we can derive B_n by computing the equivalent payments remaining after the nth payment. Thus, the balance with $N - n$ payments remaining is

$$B_n = A(P/A, i, N - n), \tag{4.13}$$

and the interest payment during period n is

$$I_n = (B_{n-1})i = A(P/A, i, N - n + 1)i, \tag{4.14}$$

where $A(P/A, i, N - n + 1)$ is the balance remaining at the end of period $n - 1$ and

$$P_n = A - I_n = A - A(P/A, i, N - n + 1)i$$
$$= A[1 - (P/A, i, N - n + 1)i].$$

Knowing the interest factor relationship $(P/F, i, n) = 1 - (P/A, i, n)i$ from Table 3.4, we obtain

$$P_n = A(P/F, i, N - n + 1). \tag{4.15}$$

As we can see in Figure 4.16, this method provides more concise expressions for computing the balance of the loan.

EXAMPLE 4.13 Loan Balances, Principal, and Interest: Remaining-Balance Method

Consider the home improvement loan in Example 4.12, and

(a) For the sixth payment, compute both the interest and principal payments.
(b) Immediately after making the sixth monthly payment, you would like to pay off the remainder of the loan in a lump sum. What is the required amount?

SOLUTION

(a) Interest and principal payments for the sixth payment.
Given: (as for Example 4.12)

Find: I_6 and P_6.

Using Eqs. (4.14) and (4.15), we compute

$$I_6 = \$235.37(P/A, 1\%, 19)(0.01)$$
$$= (\$4{,}054.44)(0.01)$$
$$= \$40.54.$$
$$P_6 = \$235.37(P/F, 1\%, 19) = \$194.83,$$

or we simply subtract the interest payment from the monthly payment:

$$P_6 = \$235.37 - \$40.54 = \$194.83.$$

(b) Remaining balance after the sixth payment.

The lower half of Figure 4.17 shows the cash flow diagram that applies to this part of the problem. We can compute the amount you owe after you make the sixth payment by calculating the equivalent worth of the remaining 18 payments at the end of the sixth month, with the time scale shifted by 6:

Given: $A = \$235.37$, $i = 1\%$ per month, and $N = 18$ months.

Find: Balance remaining after six months (B_6).

$$B_6 = \$235.37(P/A, 1\%, 18) = \$3{,}859.62.$$

If you desire to pay off the remainder of the loan at the end of the sixth payment, you must come up with \$3,859.62. To verify our results, compare this answer with the value given in Table 4.3.

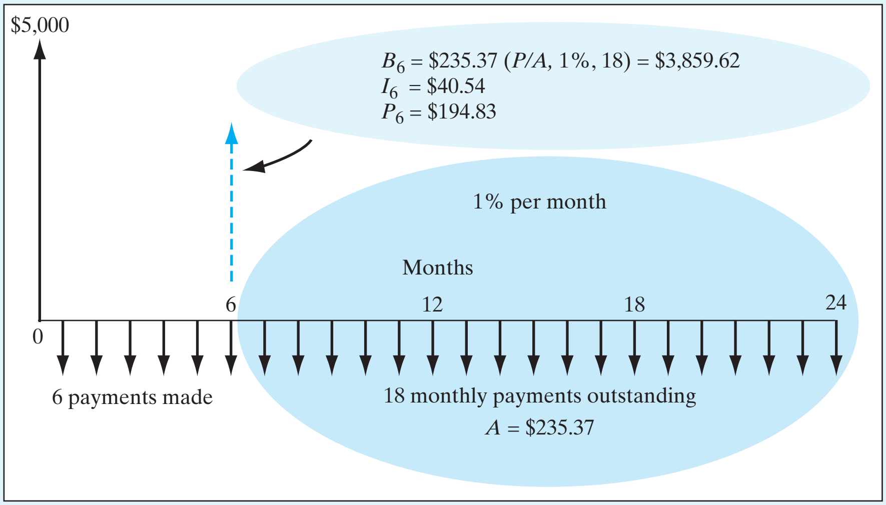


Figure 4.17 Computing the outstanding loan balance after making the sixth payment on the home improvement loan (Example 4.13).

Add-On Interest Loans

Add-on Interest: A method of computing interest whereby interest charges are made for the entire principal amount for the entire term, regardless of any repayments of principal made.

The add-on loan is totally different from the popular amortized loan. In the add-on loan, the total interest to be paid is precalculated and added to the principal. The principal and this precalculated interest amount are then paid together in equal installments. In such a case, the interest rate quoted is not the effective interest rate, but what is known as **add-on interest**. If you borrow P for N years at an add-on rate of i, with equal payments due at the end of each month, a typical financial institution might compute the monthly installment payments as follows:

$$\text{Total add-on interest} = P(i)(N),$$

$$\text{Principal plus add-on interest} = P + P(i)(N) = P(1 + iN)$$

$$\text{Monthly installments} = \frac{P(1 + iN)}{(12 \times N)}. \tag{4.16}$$

Notice that the add-on interest is *simple interest*. Once the monthly payment is determined, the financial institution computes the APR on the basis of this payment, and you will be told what the value will be. Even though the add-on interest is specified along with the APR, many ill-informed borrowers think that they are actually paying the add-on rate quoted for this installment loan. To see how much interest you actually pay under a typical add-on loan arrangement, consider Example 4.14.

EXAMPLE 4.14 Effective Interest Rate for an Add-On Interest Loan

Consider again the home improvement loan problem in Example 4.12. Suppose that you borrow \$5,000 with an add-on rate of 12% for two years. You will make 24 equal monthly payments.

(a) Determine the amount of the monthly installment.
(b) Compute the nominal and the effective annual interest rate on the loan.

SOLUTION

Given: Add-on rate = 12% per year, loan amount (P) = \$5,000, and $N = 2$ years.
Find: (a) A and (b) i_a and i.

(a) First we determine the amount of add-on interest:

$$iPN = (0.12)(\$5{,}000)(2) = \$1{,}200.$$

Then we add this simple-interest amount to the principal and divide the total amount by 24 months to obtain A:

$$A = \frac{(\$5{,}000 + \$1{,}200)}{24} = \$258.33.$$

(b) Putting yourself in the lender's position, compute the APR value of the loan just described. Since you are making monthly payments with monthly compounding, you need to find the effective interest rate that makes the present \$5,000 sum equivalent to 24 future monthly payments of \$258.33. In this situation, we are solving for i in the equation

$$\$258.33 = \$5{,}000(A/P, i, 24),$$

or

$$(A/P, i, 24) = 0.0517.$$

You know the value of the A/P factor, but you do not know the interest rate i. As a result, you need to look through several interest tables and determine i by interpolation. A more effective approach is to use Excel's RATE function with the following parameters:

=RATE(N,A,P,F,type,guess)
=RATE(24,258.33,−5000,0,0,1%)→1.7975%

The nominal interest rate for this add-on loan is $1.7975 \times 12 = 21.57\%$, and the effective annual interest rate is $(1 + 0.01975)^{12} - 1 = 26.45\%$, rather than the 12% quoted add-on interest. When you take out a loan, you should not confuse the add-on interest rate stated by the lender with the actual interest cost of the loan.

COMMENTS: In the real world, truth-in-lending laws require that APR information always be provided in mortgage and other loan situations, so you would not have to calculate nominal interest as a prospective borrower (although you might be interested in calculating the actual or effective interest). However, in later engineering economic analyses, you will discover that solving for implicit interest rates, or rates of return on investment, is performed regularly. Our purpose in this text is to periodically give you some practice with this type of problem, even though the scenario described does not exactly model the real-world information you would be given.

4.5.2 Loan versus Lease Financing

When, for example, you choose a car, you also choose how to pay for it. If you do not have the cash on hand to buy a new car outright—and most of us don't—you can consider taking out a loan or leasing the car to spread the payments over time. Deciding whether to pay cash, take a loan, or sign a lease depends on a number of personal as well as economic factors. Leasing is an option that lets you pay for the portion of a vehicle you expect to use over a specified term, plus a charge for rent, taxes, and fees. For instance, you might want a \$20,000 vehicle. Suppose that vehicle might be worth about \$9,000 at the end of your three-year lease. (This is called the residual value.)

- If you have enough money to buy the car, you could purchase it in cash. If you pay cash, however, you will lose the opportunity to earn interest on the amount you spend. That could be substantial if you know of an investment that is paying a good return.
- If you purchase the vehicle using debt financing, your monthly payments will be based on the entire \$20,000 value of the vehicle. You will continue to own the vehicle

at the end of your financing term, but the interest you will pay on the loan will drive up the real cost of the car.

- If you lease the same vehicle, your monthly payments will be based on the amount of the vehicle you expect to "use up" over the term of the lease. This value ($11,000 in our example) is the difference between the original cost ($20,000) and the estimated value at the end of the lease ($9,000). With leasing, the length of your lease agreement, the monthly payments, and the yearly mileage allowance can be tailored to your driving needs. The greatest financial appeal for leasing is low initial outlay costs: Usually you pay a leasing administrative fee, one month's lease payment, and a refundable security deposit. The terms of your lease will include a specific mileage allowance; if you put additional miles on your car, you will have to pay more for each extra mile.

Discount Rate to Use in Comparing Different Financing Options

In calculating the net cost of financing a car, we need to decide which interest rate to use in discounting the loan repayment series. The dealer's (bank's) interest rate is supposed to reflect the time value of money of the dealer (or the bank) and is factored into the required payments. However, the correct interest rate to use in comparing financing options is the interest rate that reflects *your* time value of money. For most individuals, this interest rate might be equivalent to the savings rate from their deposits. To illustrate, consider Example 4.15, in which we compare three auto financing options.

EXAMPLE 4.15 Financing your Vehicle: Paying Cash, Taking a Loan, or Leasing

Suppose you intend to own or lease a vehicle for 42 months. Consider the following three ways of financing the vehicle—say, a 2006 BMW 325 Ci 2-D coupe:

- **Option A:** Purchase the vehicle at the normal price of $32,508, and pay for the vehicle over 42 months with equal monthly payments at 5.65% APR financing.
- **Option B:** Purchase the vehicle at a discount price of $31,020 to be paid immediately.
- **Option C:** Lease the vehicle for 42 months.

The accompanying chart lists the items of interest under each option. For each option, license, title, and registration fees, as well as taxes and insurance, are extra.

For the lease option, the lessee must come up with $1,507.76 at signing. This cash due at signing includes the first month's lease payment of $513.76 and a $994 administrative fee. The lease rate is based on 60,000 miles over the life of the contract. There will be a surcharge at the rate of 18 cents per mile for any additional miles driven over 60,000. No security deposit is required; however, a $395 disposition fee is due at the end of the lease, at which time the lessee has the option to purchase the car for $17,817. The lessee is also responsible for excessive wear and

	Option A	Option B	Option C
	Debt Paying	Lease	
Item	Financing	Cash Financing	
Price	$32,508	$32,508	$32,508
Down payment	$4,500	$0	$0
APR(%)	5.65%		
Monthly payment	$736.53		$513.76
Length	42 months		42 months
Fees			$994
Cash due at end of lease			$395
Purchase option at end of lease			$17,817
Cash due at signing	$4,500	$31,020	$1,507.76

use. If the funds that would be used to purchase the vehicle are presently earning 4.5% annual interest compounded monthly, which financing option is a better choice?

DISCUSSION: With a lease payment, you pay for the portion of the vehicle you expect to use. At the end of the lease, you simply return the vehicle to the dealer and pay the agreed-upon disposal fee. With traditional financing, your monthly payment is based on the entire $32,508 value of the vehicle, and you will own the vehicle at the end of your financing terms. Since you are comparing the options over 42 months, you must explicitly consider the unused portion (resale value) of the vehicle at the end of the term. In other words, you must consider the resale value of the vehicle in order to figure out the net cost of owning it. *As the resale value, you could use the $17,817 quoted by the dealer in the lease option*. Then you have to ask yourself if you can get that kind of resale value after 42 months' ownership.

Note that the 5.65% APR represents the dealer's interest rate used in calculating the loan payments. With 5.65% interest, your monthly payments will be $A = (\$32{,}508 - \$4{,}500)(A/P, 5.65\%/12, 42) = \$736.53$. Note also, however, that the 4.5% APR represents your earning opportunity rate. In other words, if you do not buy the car, your money continues to earn 4.5% APR. Therefore, 4.5% represents your opportunity cost of purchasing the car. So which interest rate do you use in your analysis? Clearly, the 4.5% rate is the appropriate one to use.

SOLUTION

Given: Financial facts shown in Figure 4.18, $r = 4.5\%$, payment period = monthly, and compounding period = monthly.

Find: The most economical financing option, under the assumption that you will be able to sell the vehicle for $17,817 at the end of 42 months.

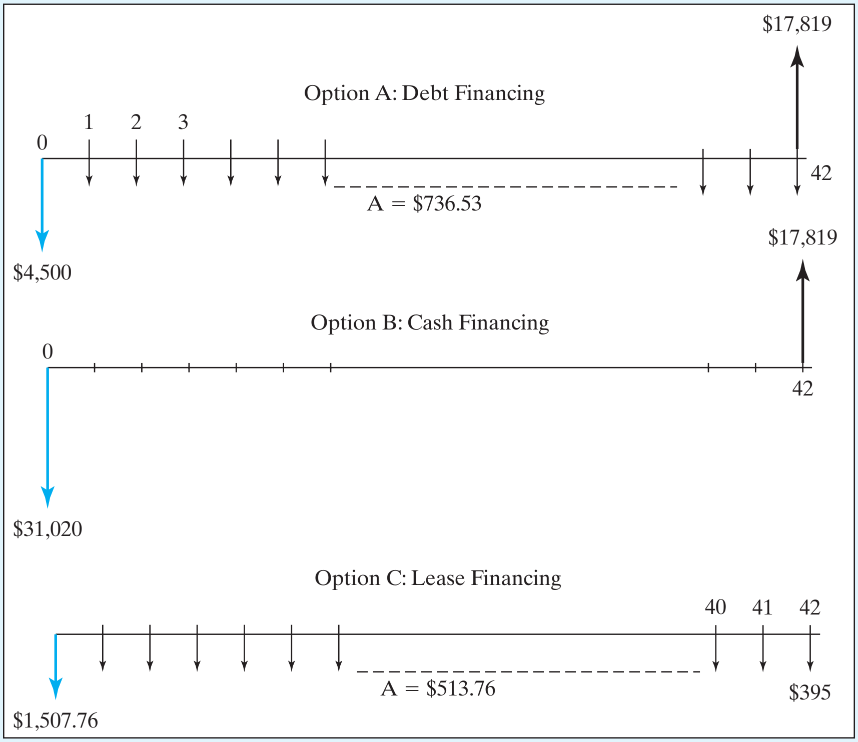


Figure 4.18 Comparing different financing options.

For each option, we will calculate the net equivalent total cost at $n = 0$. Since the loan payments occur monthly, we need to determine the effective interest rate per month, which is 4.5%/12.

- **Option A: Conventional debt financing**

  The equivalent present cost of the total loan payments is

$$P_1 = \$4{,}500 + \$736.53(P/A, 4.5\%/12, 42)$$
$$= \$33{,}071.77.$$

  The equivalent present worth of the resale value of the car is

$$P_2 = \$17{,}817(P/F, 4.5\%/12, 42) = \$15{,}225.13.$$

  The equivalent net financing cost is

$$P_{\text{Option A}} = \$33{,}071.77 - \$15{,}225.13$$
$$= \$17{,}846.64.$$

- **Option B: Cash financing**

$$P_{\text{Option B}} = \$31{,}020 - \$17{,}817(P/F, 4.5\%/12, 42)$$
$$= \$31{,}020 - \$15{,}225.13$$
$$= \$15{,}844.87.$$

- **Option C: Lease financing**

  The equivalent present cost of the total lease payments is

$$\begin{aligned} P_{\text{Option C}} &= \$1{,}507.76 + \$513.76(P/A, 4.5\%/12, 41) \\ &\quad + \$395(P/F, 4.5\%/12, 42) \\ &= \$1{,}507.76 + \$19{,}490.96 + \$337.54 \\ &= \$21{,}336.26 \end{aligned}$$

  It appears that, at 4.5% interest compounded monthly, the cash financing option is the most economical one.

4.5.3 Home Mortgage

The term **mortgage** refers to a special type of loan for buying a piece of property, such as a house. The most common mortgages are fixed-rate amortized loans, adjustable-rate loans, and graduated-payment loans. As with many of the amortized loans we have considered so far, most home mortgages involve monthly payments and monthly compounding. However, as the names of these types of mortgages suggest, a number of ways determine how monthly payments and interest rates can be structured over the life of the mortgage.

Mortgage: A loan to finance the purchase of real estate, usually with specified payment periods and interest rates.

The Cost of a Mortgage

The cost of a mortgage depends on the amount you borrow, the interest you pay, and how long you take to repay the loan. Since monthly payments spread the cost of a mortgage over a long period, it is easy to forget the total expense. For example, if you borrow $100,000 for 30 years at 8.5% interest, your total repayment will be around $277,000, more than two-and-a-half times the original loan! Minor differences in the interest rate—8.5% versus 8%—can add up to a lot of money over 30 years. At 8%, the total repaid would be $264,240, almost $13,000 less than at the 8.5% rate. Other than the interest rate, any of the following factors will increase the overall cost, but a higher interest rate and longer term will have the greatest impact:

ARM: A mortgage with an interest rate that may change, usually in response to changes in the Treasury Bill rate or the prime rate.

- **Loan amount.** This is the amount you actually borrow after fees and points are deducted. It is the basis for figuring the real interest, or APR, on the money you are borrowing.
- **Loan term.** With a shorter term, you will pay less interest overall, and your monthly payments will be somewhat larger. A 15-year mortgage, as opposed to a 30-year mortgage for the same amount, can cut your costs by more than 55%.
- **Payment frequency.** You can pay your mortgage biweekly instead of monthly, or you can make an additional payment each month. With biweekly payments, you make 26 regular payments instead of 12 every year. The mortgage is paid off in a little more than half the time, and you pay a little more than half the interest. With an additional payment each month, you can reduce your principal. With a fixed-rate mortgage, you pay off the loan more quickly, but regular monthly payments remain the same.
- **Points (prepaid interest).** Points are interest that you prepay at the closing on your home. Each point is 1% of the loan amount. For example, on a $100,000

Origination fee: A fee charged by a lender for processing a loan application, expressed as a percentage of the mortgage amount.

loan, three points represents a prepayment of \$3,000. This is equivalent to financing \$97,000, but your payments are based on a \$100,000 loan.

- **Fees.** Fees include application fees, loan origination fees, and other initial costs imposed by the lender.

Lenders might be willing to raise the rate by a fraction (say, $\frac{1}{8}\%$ or $\frac{1}{4}\%$) and lower the points—or the reverse—as long as they make the same profit. The advantages of fewer points are lower closing costs and laying out less money when you are apt to need it most. However, if you plan to keep the house longer than five to seven years, paying more points to get a lower interest rate will reduce your long-term cost. Example 4.16 examines the effect of points on the cost of borrowing.

EXAMPLE 4.16 Points or No Points?

When you are shopping for a home mortgage loan, you frequently encounter various types of borrowing options, including paying points up front and paying no points but accepting a slightly higher interest rate. Suppose you want to finance a home mortgage of \$100,000 at a 15-year fixed interest rate. Countrywide, a leading independent home lender offers the following two options with no origination fees:

- **Option 1:** Pay one point with 6.375% interest.
- **Option 2:** Pay no points with 6.75% interest.

A point is equivalent to 1% of the face value of the mortgage. In other words, with Option 1, you are borrowing only \$99,000, but your lender will calculate your monthly payments on the basis of \$100,000. Compute the APR for Option 1.

DISCUSSION: Discount fees or points are a fact of life with mortgages. A point is a fee charged by a lender to increase the lender's effective yield on the money borrowed. Points are charged in lieu of interest; the more points paid, the lower is the rate of interest required on the loan to provide the same yield or return to the lender. One point equals 1% of the loan amount. Origination fees are the fees charged by a lender to prepare a loan document, make credit checks, and inspect and sometimes appraise a property. These fees are usually computed as a percentage of the face value of the mortgage.

SOLUTION

Given: $P = \$100{,}000$, $r = 6.375\%$, $N = 180$ months, and discount point = 1 point.
Find: APR.

We first need to find out how much the lender will calculate your monthly payments to be. Since the mortgage payments will be based on the face value of the loan, your monthly payment will be

$$A = \$100{,}000(A/P, 6.375\%/12, 180) = \$863.98.$$

Because you have to pay \$1,000 to borrow \$100,000, you are actually borrowing \$99,000. Therefore, you need to find out what kind of interest you are actually paying. In other words, you borrow \$99,000 and you make \$863.98 monthly payments over 15 years. To find the interest rate, you can set up the following equivalence equation and solve for the unknown interest rate.

$$\$99{,}000 = \$863.98(P/A, i, 180),$$

$$i = 0.545\% \text{ per month},$$

$$\text{APR} = 0.545\% \times 12 = 6.54\%.$$

Note that with a one-point fee, the lender was able to raise its effective yield from 6.375% to 6.54%. However, the lender is still earning less than Option 2 affords.

Variable-Rate Mortgages

Mortgages can have either fixed or adjustable rates (or both, in which case they are known as hybrid mortgages). As we mentioned earlier, interest rates in the financial marketplace rise and fall over time. If there is a possibility that market rates will rise above the fixed rate, some lenders may be reluctant to lock a loan into a fixed interest rate over a long term. If they did, they would have to forgo the opportunity to earn better rates because their assets are tied up in a loan that earns a lower rate of interest. The variable-rate mortgage is meant to encourage lending institutions to commit funds over long periods. Typically, the rate rises gradually each year for the first several years of the mortgage and then settles into a single rate for the remainder of the loan. By contrast, a hybrid loan offers a plan with a fixed interest rate for the first few years and then converts to a variable-rate schedule for the remaining periods. Example 4.17 illustrates how a lender would calculate the monthly payments with varying interest rates.

EXAMPLE 4.17 A 5/1 Hybrid Mortgage Plan

Consider again the hybrid mortgage issue discussed in the chapter opening story. Suppose that you finance a home on the basis of a 5/1 hybrid mortgage (five-year fixed/adjustable) plan over 30 years. The $100,000 hybrid loan plan offers an initial rate of 6.02% fixed for 60 months. The loan rate would be adjusted thereafter every 12 months to the lowest of three options: the then-current rate on one-year Treasury bills plus 2.75 percent, the previous rate plus a maximum annual cap of 2.0 percent, or a lifetime cap of 11.02 percent. There is no prepayment penalty for this type of loan. The projected interest rates by the lender after 5 years are as follows:

Period	Projected APR
Years 1–5	6.02%
Year 6	6.45%
Year 7	6.60%
Year 8	6.80%
Year 9	7.15%
Year 10	7.30%

(a) Develop the payment schedule for the first 10 years.
(b) Determine the total interest paid over a 10-year ownership.

SOLUTION

Given: Varying annual mortgage rates and $N = 30$ years.

Find: (a) the monthly payment; (b) the total interest payment over the 10-year ownership of the home.

(a) *Monthly payment calculation*. During the first 5 years, you borrow \$100,000 for 30 years at 6.02%. To compute the monthly payment, use $i = 6.02\%/12 = 0.5017\%$ per month and $N = 360$ months:

$$A_{1-60} = \$100{,}000(A/P, 0.5017\%, 360) = \$600.84.$$

The balance remaining on the loan after you make the 60th payment will be

$$B_{60} = \$600.84(P/A, 0.5017\%, 300) = \$93{,}074.$$

During the 6th year, the interest rate changes to 6.15%, or 0.5125% per month, but the remaining term is only 300 months. Therefore, the new monthly payment would be

$$A_{60-72} = \$93{,}074(A/P, 0.5375\%, 300) = \$625.54.$$

After you make the 72nd payment, the balance remaining on the mortgage is

$$B_{72} = \$625.54(P/A, 0.5375\%, 288) = \$91{,}526.$$

During the 7th year, the interest rate changes to 6.60%, or 0.5500% per month. The new monthly payment and the remaining balance after you make the 84th payment are then

$$A_{73-84} = \$91{,}526(A/P, 0.5500\%, 288) = \$634.03$$

and

$$B_{84} = \$634.03(P/A, 0.5500\%, 276) = \$89{,}908.$$

You can compute the monthly payments in the same fashion for the remaining years. The accompanying table gives the details over the life of the loan.

(b) To determine the total interest paid over 10 years, we first determine the total monthly mortgage payments over 10 years. Since we know the ending balance at the end of 10 years, we can easily calculate the interest payments during this home ownership period:

$$\begin{aligned}\text{Total mortgage payment} &= 60 \times \$600.84 \\ &\quad + 12\begin{pmatrix}\$625.54 + \$634.03 + \$645.09 \\ + \$664.09 + \$672.05\end{pmatrix} \\ &= \$74{,}940.\end{aligned}$$

	Year	Month	Forecast Rate	Monthly Payment	Ending Loan Balance
	1	1–12	6.02%	$600.84	$98,777
	2	13–24	6.02%	$600.84	$97,477
Fixed rate	3	25–36	6.02%	$600.84	$96,098
	4	37–48	6.02%	$600.84	$94,633
	5	49–60	6.02%	$600.84	$93,704
	6	61–72	6.45%	$625.54	$91,526
	7	73–84	6.60%	$634.03	$89,908
Variable rate	8	85–96	6.80%	$645.09	$88,229
	9	97–108	7.15%	$664.09	$86,513
	10	109–120	7.30%	$672.05	$84,704

$$\begin{aligned}\text{Interest payment} &= \text{Ending balance} + \text{Total mortgage payment} \\ &\quad - \$100{,}000 \\ &= \$84{,}704 + \$74{,}940 - \$100{,}000 \\ &= \$59{,}644.\end{aligned}$$

4.6 Investing in Financial Assets

Most individual investors have three basic investment opportunities in financial assets: stocks, bonds, and cash. Cash investments include money in bank accounts, certificates of deposit (CDs), and U.S. Treasury bills. You can invest directly in any or all of the three, or indirectly, by buying mutual funds that pool your money with money from other people and then invest it. If you want to invest in financial assets, you have plenty of opportunities. In the United States alone, there are more than 9,000 stocks, 7,500 mutual funds, and thousands of corporate and government bonds to choose from. Even though we will discuss investment basics in the framework of financial assets in this chapter, the same economic concepts are applicable to any business assets examined in later chapters.

4.6.1 Investment Basics

Selecting the best investment for you depends on your personal circumstances as well as general market conditions. For example, a good investment strategy for a long-term retirement plan may not be a good strategy for a short-term college savings plan. In each case, the right investment is a balance of three things: liquidity, safety, and return.

- **Liquidity: How accessible is your money?** If your investment money must be available to cover financial emergencies, you will be concerned about liquidity: how easily you can convert it to cash. Money-market funds and savings accounts are very liquid; so are investments with short maturity dates, such as CDs. However, if you are investing for longer term goals, liquidity is not a critical issue. What you are after in that case is growth, or building your assets. We normally consider certain stocks and stock mutual funds as growth investments.

- **Risk: How safe is your money?** Risk is the chance you take of making or losing money on your investment. For most investors, the biggest risk is losing money, so they look for investments they consider safe. Usually, that means putting money into bank accounts and U.S. Treasury bills, as these investments are either insured or default free. The opposite, but equally important, risk is that your investments will not provide enough growth or income to offset the impact of inflation, the gradual increase in the cost of living. There are additional risks as well, including how the economy is doing. However, *the biggest risk is not investing at all*.
- **Return: How much profit will you be able to expect from your investment?** Safe investments often promise a specific, though limited, return. Those which involve more risk offer the opportunity to make—or lose—a lot of money. Both risk and reward are time dependent. On the one hand, as time progresses, low-yielding investments become more risky because of inflation. On the other hand, the returns associated with higher risk investments could become more stable and predictable over time, thereby reducing the perceived level of risk.

4.6.2 How to Determine Your Expected Return

Return is what you get back in relation to the amount you invested. Return is one way to evaluate how your investments in financial assets are doing in relation to each other and to the performance of investments in general. Let us look first at how we may derive rates of return.

Risk-free return: A theoretical interest rate that would be returned on an investment which was completely free of risk. The 3-month Treasury Bill is a close approximation, since it is virtually risk-free.

Basic Concepts

Conceptually, the rate of return that we realistically expect to earn on any investment is a function of three components: (1) risk-free real return, (2) an inflation factor, and (3) a risk premium.

Suppose you want to invest in stock. First, you should expect to be rewarded in some way for not being able to use your money while you are holding the stock. Then, you would be compensated for decreases in purchasing power between the time you invest the money and the time it is returned to you. Finally, you would demand additional rewards for any chance that you would not get your money back or that it will have declined in value while invested.

Risk premium: The reward for holding a risky investment rather that a risk-free one.

For example, if you were to invest \$1,000 in risk-free U.S. Treasury bills for a year, you would expect a real rate of return of about 2%. Your risk premium would be also zero. You probably think that the 2% does not sound like much. However, to that you have to add an allowance for inflation. If you expect inflation to be about 4% during the investment period, you should realistically expect to earn 6% during that interval (2% real return + 4% inflation factor + 0% for risk premium). Here is what the situation looks like in tabular form:

Real return	2%
Inflation (loss of purchasing power)	4%
Risk premium (U.S. Treasury Bills)	0%
Total expected return	6%

How would it work out for a riskier investment, say, in an Internet stock such as Google.com? Because you consider this stock to be a very volatile one, you would increase the risk premium to something like the following:

Real return	2%
Inflation (loss of purchasing power)	4%
Risk premium (Google.com)	20%
Total expected return	26%

So you will not invest your money in Google.com unless you are reasonably confident of having it grow at an annual rate of 26%. Again, the risk premium of 20% is a perceived value that can vary from one investor to another.

Return on Investment over Time

If you start out with \$1,000 and end up with \$2,000, your return is \$1,000 on that investment, or 100%. If a similar investment grows to \$1,500, your return is \$500, or 50%. However, unless you held those investments for the same period, you cannot determine which has a better performance. What you need in order to compare your return on one investment with the return on another investment is the **compound annual return**, the average percentage that you have gained on each investment over a series of one-year periods. For example, if you buy a share for \$15 and sell it for \$20, your profit is \$5. If that happens within a year, your rate of return is an impressive 33.33%. If it takes five years, your return (compounded) will be closer to 5.92%, since the profit is spread over a five-year period. Mathematically, you are solving the following equivalence equation for i:

Compound annual return: The year-over-year growth rate of an investment over a specified period of time.

$$\$20 = \$15(1 + i)^5,$$
$$i = 5.92\%.$$

Figuring out the actual return on your portfolio investment is not always that simple. There are several reasons:

1. The amount of your investment changes. Most investment portfolios are active, with money moving in and out.
2. The method of computing the return can vary. For example, the performance of a stock can be averaged or compounded, which changes the rate of return significantly, as we will demonstrate in Example 4.18.
3. The time you hold specific investments varies. When you buy or sell can have a dramatic effect on the overall return.

EXAMPLE 4.18 Figuring Average versus Compound Return

Consider the following six different cases of the performance of a $1,000 investment over a three-year holding period:

	Annual Investment Yield					
Investment	**Case 1**	**Case 2**	**Case 3**	**Case 4**	**Case 5**	**Case 6**
Year 1	9%	5%	0%	0%	−1%	−5%
Year 2	9%	10%	7%	0%	−1%	−8%
Year 3	9%	12%	20%	27%	29%	40%

Compute the average versus compound return for each case.

SOLUTION

Given: Three years' worth of annual investment yield data.

Find: Compound versus average rate of return.

Average annual return: A figure used when reporting the historical return of a mutual fund.

As an illustration, consider Case 6 for an investment of $1,000. At the end of the first year, the value of the investment decreases to $950; at the end of second year, it decreases again, to $\$950(1 - 0.08) = \$874$; at the end of third year, it increases to $\$874(1 + 0.40) = \$1{,}223.60$. Therefore, one way you can look at the investment is to ask, "At what annual interest rate would the initial $1,000 investment grow to $1,223.60 over three years?" This is equivalent to solving the following equivalence problem:

$$\$1{,}223.60 = \$1{,}000(1 + i)^3,$$

$$i = 6.96\%.$$

If someone evaluates the investment on the basis of the average annual rate of return, he or she might proceed as follows:

$$i = \frac{(-5\% - 8\% + 40\%)}{3} = 9\%.$$

If you calculate the remaining cases, you will observe that all six cases have the same average annual rate of return, although their compound rates of return vary from 6.96% to 9%:

	Compound versus Average Rate of Return					
	Case 1	**Case 2**	**Case 3**	**Case 4**	**Case 5**	**Case 6**
Average return	9%	9%	9%	9%	9%	9%
Balance at end of three years	$1,295	$1,294	$1,284	$1,270	$1,264	$1,224
Compound rate of return	9.00%	8.96%	8.69%	8.29%	8.13%	6.96%

Your immediate question is "Are they the same indeed?" Certainly not: You will have the most money with Case 1, which also has the highest compound rate of return. The average rate of return is easy to calculate, but it ignores the basic principle of the time value of money. In other words, according to the average-rate-of-return concept, we may view all six cases as indifferent. However, the amount of money available at the end of year 3 would be different in each case. Although the average rate of return is popular for comparing investments in terms of their yearly performance, it is not a correct measure in comparing the performance for investments over a multiyear period.

COMMENTS: You can evaluate the performance of your portfolio by comparing it against standard indexes and averages that are widely reported in the financial press. If you own stocks, you can compare their performance with the performance of the Dow Jones Industrial Average (DJIA), perhaps one of the best-known measures of stock market performance in the world. If you own bonds, you can identify an index that tracks the type you own: corporate, government, or agency bonds. If your investments are in cash, you can follow the movement of interest rates on Treasury bills, CDs, and similar investments. In addition, total-return figures for the performance of mutual funds are reported regularly. You can compare how well your investments are doing against those numbers. Another factor to take into account in evaluating your return is the current inflation rate. Certainly, your return needs to be higher than the inflation rate if your investments are going to have real growth.

4.6.3 Investing in Bonds

Bonds are loans that investors make to corporations and governments. As shown in Figure 4.19, the borrowers get the cash they need while the lenders earn interest. Americans have more money invested in bonds than in stocks, mutual funds, or any other types of securities. One of the major appeals is that bonds pay a set amount of interest on a regular basis. That is why they are called *fixed-income securities*. Another attraction is that the issuer promises to repay the loan in full and on time.

Bond versus Loan

A bond is similar to a loan. For example, say you lend out $1,000 for 10 years in return for a yearly payment of 7% interest. Here is how that arrangement translates into bond terminology. You did not make a loan; you bought a bond. The $1,000 of principal is the **face value** of the bond, the yearly interest payment is its **coupon**, and the length of the loan, 10 years, is the bond's **maturity**. If you buy a bond at face value, or **par**, and hold it until it matures, you will earn interest at the stated, or coupon, rate. For example, if you buy a 20-year $1,000 bond paying 8%, you will earn $80 a year for 20 years. The yield, or return on your investment, will also be 8%, and you get your $1,000 back. You can also buy and sell bonds through a broker after their date of issue. This is known as the **secondary market**. There the price fluctuates, with a bond sometimes selling at more than par value, at a premium price (premium bonds), and sometimes below, at a discount. Changes in price are tied directly to the interest rate of the bond. If its rate is higher than the rate being paid on similar bonds, buyers are willing to pay more to get the higher interest. If its rate is lower, the bond will sell for less in order to attract buyers. However, as the price goes up, the yield goes down, and when the price goes down, the yield goes up.

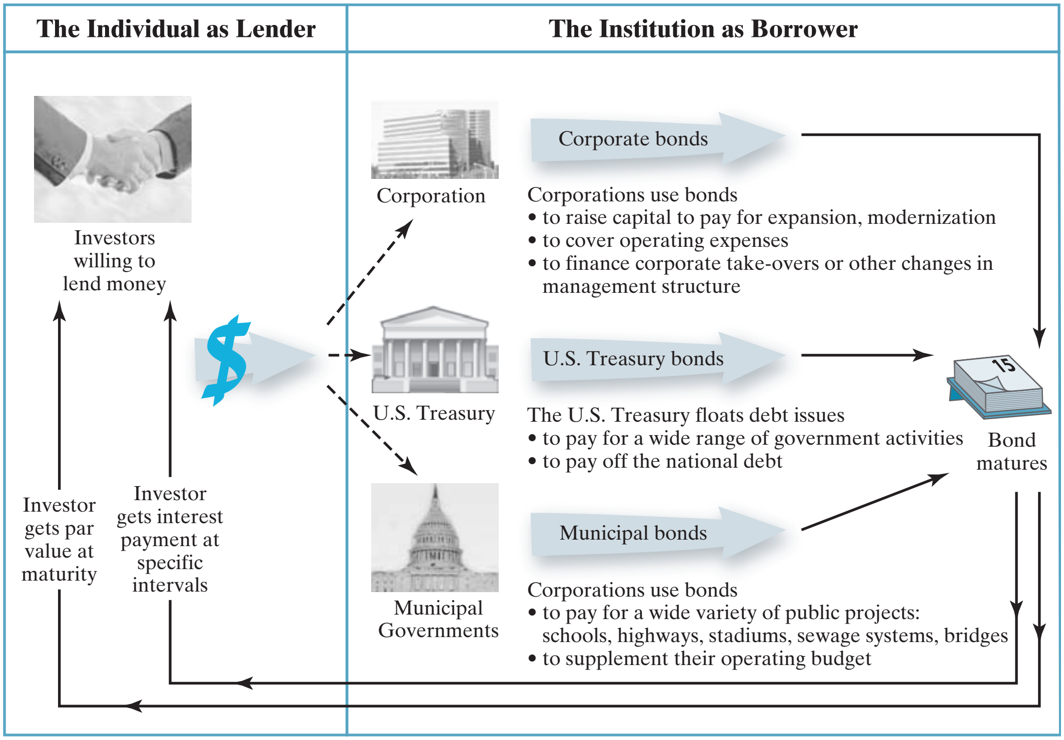


Figure 4.19 Types of bonds and how they are issued in the financial market.
(*Source*: Kenneth M. Morris and Alan M. Stegel, *The Wall Street Guide to Understanding Money and Investing*, © 1993 by Lightbulb Press, Inc.)

Types of Bonds

You can choose different types of bonds to fit your financial needs—be they investing for college, finding tax-free income, or operating over a range of other possibilities. That is why it is important to have a sense of how the various types work. Several types of bonds are available in the financial market:

- **Corporate bonds.** Bonds are the major source of corporate borrowing. *Debentures* are backed by the general credit of the corporation. *Specific corporate assets, such as property or equipment, may back bonds*.
- **Municipal bonds.** State and local governments issue bonds to finance many projects that could not be funded through tax revenues. *General-obligation* bonds are backed by the full faith and credit of the issuer, *revenue bonds* by the earnings of the particular project being financed.
- **Treasury notes and bonds.** The U.S. Treasury floats debt issues to pay for a wide range of government activities, including financing the national debt. Intermediate (2 to 10 years) and long-term (10 to 30 years) government bonds are a major source of government funding.
- **Treasury bills.** These are the largest components of the money market, where short-term (13 weeks to 52 weeks) securities are bought and sold. Investors use T-bills for part of their cash reserve or as an interim holding place. Interest is the difference between the buying price and the amount paid at maturity.

- **Zero-coupon bonds.** Corporations and governments sell zero-coupon bonds at a deep discount. Investors do not collect interest; instead, the value of the bond increases to its full value when it matures. In this way, zero-coupon bonds are similar to old Series savings bonds that you bought for $37.50 and could cash in for $50 after seven years. Organizations like to issue zeros because they can continue to use the loan money without paying periodic interest. Investors like zeros because they can buy more bonds for their money and then time the maturities to coincide with anticipated expenses.
- **Callable bonds.** Callable bonds do not run their full term. In a process called redemption, the issuer may call the bond—pay off the debt—before the maturity date. Issuers will sometimes call bonds when interest rates drop, so that they can reduce their debt. If they pay off their outstanding bonds, they can float another bond at the lower rate. It is the same idea as that of refinancing a mortgage to get a lower interest rate and make lower monthly payments. Callable bonds are more risky for investors than noncallable bonds, because the investors are often faced with reinvesting the money at a lower, less attractive rate. To protect bondholders who expect long-term steady income, call provisions often specify that a bond cannot be called before a certain number of years, usually 5 or 10 years.
- **Floating-rate bonds.** These bonds promise periodic adjustments of the interest rate to persuade investors that they are not locked into what seems like an unattractively low rate.

Under the U.S. Constitution, any type of government bond must include a tax break. Because federal and local governments cannot interfere with each other's affairs, income from local government bonds (municipals, or simply **munis**) is immune from federal taxes, and income from U.S. Treasury bonds is free from local taxes.

Understanding Bond Prices

Corporate bond prices are quoted either by a percentage of the bond's face value or in increments of points and eight fractions of a point, with a par of $1,000 as the base. The value of each point is $10 and of each fraction $1.25. For example, a bond quoted at $86\frac{1}{2}$ would be selling for $865, and one quoted at $\$100\frac{3}{4}$ would be selling for $1,007.50. Treasury bonds are measured in thirty-seconds, rather than hundredths, of a point. Each $\frac{1}{32}$ equals 31.25 cents, and we normally drop the fractional part of the cent when stating a price. For example, if a bond is quoted at 100.2, or $100 + \frac{2}{32}$, the price translates to $1,000.62.

Are Bonds Safe?

Just because bonds have a reputation as a conservative investment does not mean that they are always safe. There are sources of risk in holding or trading bonds:

- To begin with, not all loans are paid back. Companies, cities, and counties occasionally do go bankrupt. Only U.S. Treasury bonds are considered rock solid.
- Another source of risk for certain bonds is that your loan may be called, or paid back early. While that is certainly better than its not being paid back at all, it forces you to find another, possibly less lucrative, place to put your money.
- The main danger for buy-and-hold investors, however, is a rising inflation rate. Since the dollar amount they earn on a bond investment does not change, the value of that money can be eroded by inflation. Worse yet, with your money locked away in the bond, you will not be able to take advantage of the higher interest rates that are usually available in an inflationary economy. Now you know why

bond investors cringe at cheerful headlines about full employment and strong economic growth: These traditional signs of inflation hint that bondholders may soon lose their shirts.

How Do Prices and Yields Work?

You can trade bonds on the market just as you do stocks. Once a bond is purchased, you may keep it until it matures or for a variable number of interest periods before selling it. You can purchase or sell bonds at prices other than face value, depending on the economic environment. Furthermore, bond prices change over time because of the risk of nonpayment of interest or par value, supply and demand, and the economic outlook. These factors affect the **yield to maturity** (or **return on investment**) of the bond:

Yield to maturity (YTM): Yield that would be realized on a bond or other fixed income security if the bond was held until the maturity date.

- The **yield to maturity** represents the actual interest earned from a bond over the holding period. In other words, the yield to maturity on a bond is the interest rate that establishes the equivalence between all future interest and face-value receipts and the market price of the bond.
- The **current yield** of a bond is the annual interest earned, as a percentage of the current market price. The current yield provides an indication of the annual return realized from investment in the bond. To illustrate, we will explain these values with numerical examples shortly.

Current yield: Annual income (interest) divided by the current market price of the bond.

Bond quotes follow a few unique conventions. As an example, let's take a look at a corporate bond with a face value of $1,000 (issued by Ford Motor Company) and traded on November 18, 2005:

Price:	94.50
Coupon (%):	6.625
Maturity Date:	16-Jun-2008
Yield to Maturity (%):	9.079
Current Yield (%):	7.011
Debt Rating:	BBB
Coupon Payment Frequency:	Semiannual
First Coupon Date:	16-Dec-2005
Type:	Corporate
Industry:	Industrial

- Prices are given as percentages of face value, with the last digits not decimals, but in eighths. The bond on the list, for instance, has just fallen to sell for 94.50, or 94.5% of its $1,000 face value. In other words, an investor who bought the bond when it was issued (at 100) could now sell it for a 5.5% discount.
- The discount over face value is explained by examining the bond's coupon rate, 6.625%, and its current yield, 7.011%. The current yield will be higher than the coupon rate whenever the bond is selling for less than its par value.
- The coupon rate is 6.625% paid semiannually, meaning that, for every six-month period, you will receive $\$66.25/2 = \$33.13$.

- The bond rating system helps investors determine a company's credit risk. Think of a bond rating as the report card on a company's credit rating. Blue-chip firms, which are safer investments, have a high rating, while risky companies have a low rating. The following chart illustrates the different bond rating scales from Moody's, Standard and Poor's (S&P), and Fitch Ratings—the major rating agencies in the United States.

Bond Rating			
Moody's	**S&P/Fitch**	**Grade**	**Risk**
Aaa	AAA	Investment	Highest quality
Aa	AA	Investment	High quality
A	A	Investment	Strong
Baa	BBB	Investment	Medium grade
Ba, B	BB, B	Junk	Speculative
Caa/Ca/C	CCC/CC/C	Junk	Highly speculative
C	D	Junk	In default

Notice that if the company falls below a certain credit rating, its grade changes from investment quality to junk status. Junk bonds are aptly named: They are the debt of companies that are in some sort of financial difficulty. Because they are so risky, they have to offer much higher yields than any other debt. This brings up an important point: Not all bonds are inherently safer than stocks. Certain types of bonds can be just as risky, if not riskier, than stocks.

- If you buy a bond at face value, its rate of return, or yield, is just the coupon rate. However, a glance at a table of bond quotes (like the preceding one) will tell you that after they are first issued, bonds rarely sell for exactly face value. So how much is the yield then? In our example, if you can purchase the bond for $945, you are getting two bonuses. First, you have effectively bought a bond with a 6.625% coupon, since the $66.25 coupon is 7.011% of your $945 purchase price. (Recall that the coupon rate, adjusted for the current price, is the current yield of the bond.) However, there's more: Although you paid $945, in 2008 you will receive the full $1,000 face value.

Example 4.19 illustrates how you calculate the yield to maturity and the current yield, considering both the purchase price and the capital gain for a new issue.

EXAMPLE 4.19 Yield to Maturity and Current Yield

Consider buying a $1,000 corporate (Delta Corporation) bond at the market price of $996.25. The interest will be paid semiannually, the interest rate per payment period will be simply 4.8125%, and 20 interest payments over 10 years are required. We show the resulting cash flow to the investor in Figure 4.20. Find (a) the yield to maturity and (b) the current yield.

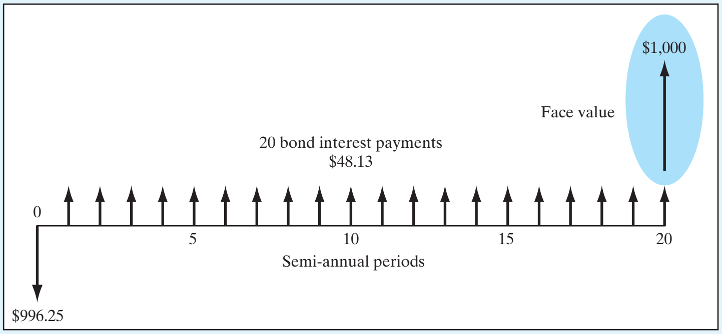


Figure 4.20 A typical cash flow transaction associated with an investment in Delta's corporate bond.

DISCUSSION

- **Debenture bond:** Delta wants to issue bonds totaling \$100 million, and the company will back the bonds with specific pieces of property, such as buildings.
- **Par value:** Delta's bond has a par value of \$1,000.
- **Maturity date:** Delta's bonds, which were issued on January 30, 2006, will mature on January 31, 2016; thus, they have a 10-year maturity at the time of their issue.
- **Coupon rate:** Delta's coupon rate is 9.625%, and interest is payable semiannually. For example, Delta's bonds have a \$1,000 par value, and they pay \$96.25 in simple interest $\left(9\frac{5}{8}\%\right)$ each year (\$48.13 every six months).
- **Discount bond:** At 99.625%, or a 0.375% discount, Delta's bonds are offered at less than their par value.

SOLUTION

Given: Initial purchase price = \$996.25, coupon rate = 9.625% per year paid semiannually, and 10-year maturity with a par value of \$1,000.

Find: (a) Yield to maturity and (b) current yield.

(a) *Yield to maturity*. We find the yield to maturity by determining the interest rate that makes the present worth of the receipts equal to the market price of the bond:

$$\$996.25 = \$48.13(\boldsymbol{P/A}, i, 20) + \$1{,}000(\boldsymbol{P/F}, i, 20).$$

The value of i that makes the present worth of the receipts equal to \$996.25 lies between 4.5% and 5%. We could use a linear interpolation to find the yield to maturity, but it is much easier to obtain with Excel's **Goal Seek** function. As shown in Figure 4.21, solving for i yields $i = 4.8422\%$.

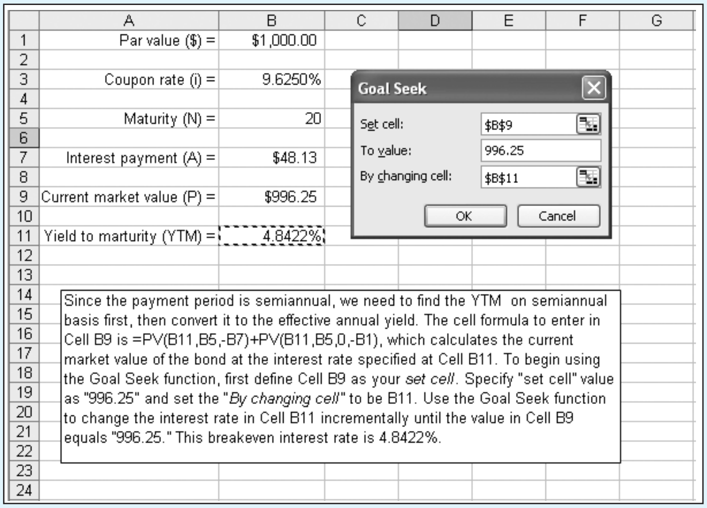


Figure 4.21 Finding the yield to maturity of the Delta bond with Excel's Goal Seek function.

Note that this 4.8422% is the yield to maturity *per semiannual period*. The nominal (annual) yield is $2(4.8422) = 9.6844\%$, compounded semiannually. Compared with the coupon rate of $\left(9\frac{5}{8}\%\right)$ (or 9.625%), purchasing the bond with the price discounted at 0.375% brings about an additional 0.0594% yield. The effective annual interest rate is then

$$i_a = (1 + 0.048422)^2 - 1 = 9.92\%.$$

The 9.92% represents the **effective annual yield** to maturity on the bond. Notice that when you purchase a bond at par value and sell at par value, the yield to maturity will be the *same* as the coupon rate of the bond.

Until now, we have observed differences in nominal and effective interest rates because of the frequency of compounding. In the case of bonds, the reason is different: The stated (par) value of the bond and the actual price for which it is sold are not the same. We normally state the nominal interest as a percentage of par value. However, when the bond is sold at a discount, the same nominal interest on a smaller initial investment is earned; hence, your effective interest earnings are greater than the stated nominal rate.

(b) *Current yield*. For our example of Delta, we compute the current yield as follows:

$$\frac{\$48.13}{996.25} = 4.83\% \text{ per semiannual period,}$$

$$4.83\% \times 2 = 9.66\% \text{ per year (nominal current yield),}$$

$$i_a = (1 + 0.0483)^2 - 1 = 9.90\%.$$

This effective current yield is 0.02% lower than the 9.92% yield to maturity we just computed. If the bond is selling at a discount, the current yield is smaller than the yield to maturity. If the bond is selling at a premium, the current yield is larger than the yield to maturity. A significant difference between the yield to maturity and the current yield of a bond can exist because the market price of a bond may be more or less than its face value. Moreover, both the current yield and the yield to maturity may differ considerably from the stated coupon value of the bond.

EXAMPLE 4.20 Bond Value over Time

Consider again the Delta bond investment introduced in Example 4.19. If the yield to maturity remains constant at 9.68%, (a) what will be the value of the bond one year after it was purchased? (b) If the market interest rate drops to 9% a year later, what would be the market price of the bond?

SOLUTION

Given: The same data as in Example 4.19.

Find: (a) The value of the bond one year later and (b) the market price of the bond a year later at the going rate of 9% interest.

(a) We can find the value of the bond one year later by using the same valuation procedure as in Example 4.19, but now the term to maturity is only nine years:

$$\$48.13(P/A, 4.84\%, 18) + \$1{,}000(P/F, 4.84\%, 18) = \$996.80.$$

The value of the bond will remain at \$996.80 as long as the yield to maturity remains constant at 9.68% over nine years.

(b) Now suppose interest rates in the economy have fallen since the Delta bonds were issued, and consequently, the going rate of interest is 9%. Then both the coupon interest payments and the maturity value remain constant, but now 9% values have to be used to calculate the value of the bond. The value of the bond at the end of the first year would be

$$\$48.13(P/A, 4.5\%, 18) + \$1{,}000(P/F, 4.5\%, 18) = \$1{,}038.06.$$

Thus, the bond would sell at a premium over its par value.

COMMENTS: The arithmetic of the bond price increase should be clear, but what is the logic behind it? We can explain the reason for the increase as follows: Because the going market interest rate for the bond has fallen to 9%, if we had \$1,000 to invest, we could buy new bonds with a coupon rate of 9%. These would pay \$90 interest each year, rather than \$96.80. We would prefer \$96.80 to \$90; therefore, we would be willing to pay more than \$1,000 for Delta's bonds to obtain higher coupons. All investors would recognize these facts, and hence the Delta's bonds would be bid up in price to \$1,038.06. At that point, they would provide the same yield to maturity (rate of return) to a potential investor as would the new bonds, namely, 9%.

SUMMARY

- Interest is most frequently quoted by financial institutions as an **annual percentage rate**, or **APR**. However, compounding frequently occurs more often than once annually, and the APR does not account for the effect of this more frequent compounding. The situation leads to the distinction between nominal and effective interest:
- **Nominal interest** is a stated rate of interest for a given period (usually a year).
- **Effective interest** is the actual rate of interest, which accounts for the interest amount accumulated over a given period. The **effective rate** is related to the APR by the equation

$$i = \left(1 + \frac{r}{M}\right)^{M} - 1,$$

where r is the APR, M is the number of compounding periods, and i is the effective interest rate.

In any equivalence problem, the interest rate to use is the effective interest rate per payment period, or

$$i = \left[1 + \frac{r}{CK}\right]^{C} - 1,$$

where C is the number of interest periods per payment period, K is the number of payment periods per year, and r/K is the nominal interest rate per payment period. Figure 4.9 outlines the possible relationships between compounding and payment periods and indicates which version of the effective-interest formula to use.

- The equation for determining the effective interest of continuous compounding is

$$i = e^{r/K} - 1.$$

The difference in accumulated interest between continuous compounding and very frequent compounding $(M > 50)$ is minimal.

- Cash flows, as well as compounding, can be continuous. Table 4.2 shows the interest factors to use for continuous cash flows with continuous compounding.
- Nominal (and hence effective) interest rates may fluctuate over the life of a cash flow series. Some forms of home mortgages and bond yields are typical examples.
- **Amortized loans** are paid off in equal installments over time, and most of these loans have interest that is compounded monthly.
- Under a typical **add-on loan**, the lender precalculates the total simple interest amount and adds it to the principal. The principal and this precalculated interest amount are then paid together in equal installments.
- The term **mortgage** refers to a special type of loan for buying a piece of property, such as a house or a commercial building. The cost of the mortgage will depend on many factors, including the amount and term of the loan and the frequency of payments, as well as points and fees.
- Two types of mortgages are common: fixed-rate mortgages and variable-rate mortgages. Fixed-rate mortgages offer loans whose interest rates are fixed over the period

of the contract, whereas variable-rate mortgages offer interest rates that fluctuate with market conditions. In general, the initial interest rate is lower for variable-rate mortgages, as the lenders have the flexibility to adjust the cost of the loans over the period of the contract.

- Allocating one's assets is simply a matter of answering the following question: "Given my personal tolerance for risk and my investment objectives, what percentage of my assets should be allocated for **growth**, what percentage for **income**, and what percentage for **liquidity**?"
- You can determine the **expected rate of return** on a portfolio by computing the weighted average of the returns on each investment.
- You can determine the **expected risk** of a portfolio by computing the weighted average of the volatility of each investment.
- All other things being equal, if the expected returns are approximately the same, choose the portfolio with the lowest expected risk.
- All other things being equal, if the expected risk is about the same, choose the portfolio with the highest expected return.
- **Asset-backed bonds:** If a company backs its bonds with specific pieces of property, such as buildings, we call these types of bonds **mortgage bonds**, which indicate the terms of repayment and the particular assets pledged to the bondholders in case of default. It is much more common, however, for a corporation simply to pledge its overall assets. A **debenture bond** represents such a promise.
- **Par value:** Individual bonds are normally issued in even denominations of $1,000 or multiples of $1,000. The stated face value of an individual bond is termed the **par value**.
- **Maturity date:** Bonds generally have a specified **maturity** date on which the par value is to be repaid.
- **Coupon rate:** We call the interest paid on the par value of a bond the **annual coupon rate**. The time interval between interest payments could be of any duration, but a semiannual period is the most common.
- **Discount or premium bond:** A bond that sells below its par value is called a **discount bond**. When a bond sells above its par value, it is called a **premium bond**.

PROBLEMS

Nominal and Effective Interest Rates

4.1 If your credit card calculates interest based on 12.5% APR,
(a) What are your monthly interest rate and annual effective interest rate?
(b) If you current outstanding balance is $2,000 and you skip payments for two months, what would be the total balance two months from now?

4.2 A department store has offered you a credit card that charges interest at 1.05% per month, compounded monthly. What is the nominal interest (annual percentage) rate for this credit card? What is the effective annual interest rate?

4.3 A local bank advertised the following information: Interest 6.89%—effective annual yield 7.128%. No mention was made of the interest period in the advertisement. Can you figure out the compounding scheme used by the bank?

4.4 College Financial Sources, which makes small loans to college students, offers to lend \$500. The borrower is required to pay \$400 at the end of each week for 16 weeks. Find the interest rate per week. What is the nominal interest rate per year? What is the effective interest rate per year?

4.5 A financial institution is willing to lend you \$40. However, \$450 is repaid at the end of one week.

(a) What is the nominal interest rate?

(b) What is the effective annual interest rate?

4.6 The Cadillac Motor Car Company is advertising a 24-month lease of a Cadillac Deville for \$520, payable at the beginning of each month. The lease requires a \$2,500 down payment, plus a \$500 refundable security deposit. As an alternative, the company offers a 24-month lease with a single up-front payment of \$12,780, plus a \$500 refundable security deposit. The security deposit will be refunded at the end of the 24-month lease. Assuming an interest rate of 6%, compounded monthly, which lease is the preferred one?

4.7 As a typical middle-class consumer, you are making monthly payments on your home mortgage (9% annual interest rate), car loan (12%), home improvement loan (14%), and past-due charge accounts (18%). Immediately after getting a \$100 monthly raise, your friendly mutual fund broker tries to sell you some investment funds with a guaranteed return of 10% per year. Assuming that your only other investment alternative is a savings account, should you buy?

Compounding More Frequent than Annually

4.8 A loan company offers money at 1.8% per month, compounded monthly.

(a) What is the nominal interest rate?

(b) What is the effective annual interest rate?

(c) How many years will it take an investment to triple if interest is compounded monthly?

(d) How many years will it take an investment to triple if the nominal rate is compounded continuously?

4.9 Suppose your savings account pays 9% interest compounded quarterly. If you deposit \$10,000 for one year, how much would you have?

4.10 What will be the amount accumulated by each of these present investments?

(a) \$5,635 in 10 years at 5% compounded semiannually.

(b) \$7,500 in 15 years at 6% compounded quarterly.

(c) \$38,300 in 7 years at 9% compounded monthly.

4.11 How many years will it take an investment to triple if the interest rate is 9% compounded

(a) Quarterly? (b) Monthly? (c) Continuously?

4.12 A series of equal quarterly payments of \$5,000 for 12 years is equivalent to what present amount at an interest rate of 9% compounded

(a) Quarterly? (b) Monthly? (c) Continuously?

4.13 What is the future worth of an equal payment series of \$3,000 each quarter for five years if the interest rate is 8% compounded continuously?

4.14 Suppose that \$2,000 is placed in a bank account at the end of each quarter over the next 15 years. What is the future worth at the end of 15 years when the interest rate is 6% compounded

(a) Quarterly? (b) Monthly? (c) Continuously?

4.15 A series of equal quarterly deposits of \$1,000 extends over a period of three years. It is desired to compute the future worth of this quarterly deposit series at 12% compounded monthly. Which of the following equations is correct?

(a) $F = 4(\$1{,}000)(F/A, 12\%, 3)$. (b) $F = \$1{,}000(F/A, 3\%, 12)$.

(c) $F = \$1{,}000(F/A, 1\%, 12)$. (d) $F = \$1{,}000(F/A, 3.03\%, 12)$.

4.16 If the interest rate is 8.5% compounded continuously, what is the required quarterly payment to repay a loan of \$12,000 in five years?

4.17 What is the future worth of a series of equal monthly payments of \$2,500 if the series extends over a period of eight years at 12% interest compounded

(a) Quarterly? (b) Monthly? (c) Continuously?

4.18 Suppose you deposit \$500 at the end of each quarter for five years at an interest rate of 8% compounded monthly. What equal end-of-year deposit over the five years would accumulate the same amount at the end of the five years under the same interest compounding? To answer the question, which of the following is correct?

(a) $A = [\$500(F/A, 2\%, 20)] \times (A/F, 8\%, 5)$.

(b) $A = \$500(F/A, 2.013\%, 4)$.

(c) $A = \$500\left(F/A, \frac{8\%}{12}, 20\right) \times (A/F, 8\%, 5)$.

(d) None of the above.

4.19 A series of equal quarterly payments of \$2,000 for 15 years is equivalent to what future lump-sum amount at the end of 10 years at an interest rate of 8% compounded continuously?

4.20 What will be the required quarterly payment to repay a loan of \$32,000 in five years, if the interest rate is 7.8% compounded continuously?

4.21 A series of equal quarterly payments of \$4,000 extends over a period of three years. What is the present worth of this quarterly payment series at 8.75% interest compounded continuously?

4.22 What is the future worth of the following series of payments?

(a) \$6,000 at the end of each six-month period for 6 years at 6% compounded semiannually.

(b) \$42,000 at the end of each quarter for 12 years at 8% compounded quarterly.

(c) \$75,000 at the end of each month for 8 years at 9% compounded monthly.

4.23 What equal series of payments must be paid into a sinking fund to accumulate the following amount?

(a) \$21,000 in 10 years at 6.45% compounded semiannually when payments are semiannual.

(b) \$9,000 in 15 years at 9.35% compounded quarterly when payments are quarterly.

(c) \$24,000 in 5 years at 6.55% compounded monthly when payments are monthly.

4.24 You have a habit of drinking a cup of Starbucks coffee (\$2.50 a cup) on the way to work every morning. If, instead, you put the money in the bank for 30 years, how much would you have at the end of that time, assuming that your account earns

5% interest compounded *daily*? Assume also that you drink a cup of coffee every day, including weekends.

4.25 John Jay is purchasing a $24,000 automobile, which is to be paid for in 48 monthly installments of $543.35. What effective annual interest is he paying for this financing arrangement?

4.26 A loan of $12,000 is to be financed to assist in buying an automobile. On the basis of monthly compounding for 42 months, the end-of-the-month equal payment is quoted as $445. What nominal interest rate in percentage is being charged?

4.27 Suppose a young newlywed couple is planning to buy a home two years from now. To save the down payment required at the time of purchasing a home worth $220,000 (let's assume that the down payment is 10% of the sales price, or $22,000), the couple decides to set aside some money from each of their salaries at the end of every month. If each of them can earn 6% interest (compounded monthly) on his or her savings, determine the equal amount this couple must deposit each month until the point is reached where the couple can buy the home.

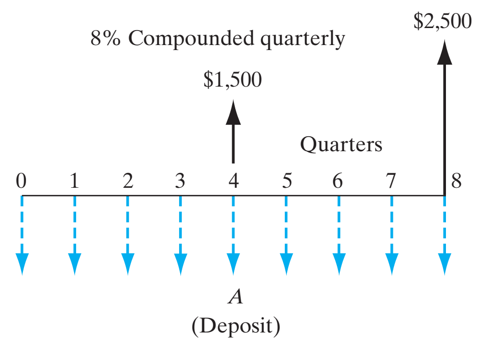
4.28 What is the present worth of the following series of payments?

(a) $1,500 at the end of each six-month period for 12 years at 8% compounded semiannually.

(b) $2,500 at the end of each quarter for 8 years at 8% compounded quarterly.

(c) $3,800 at the end of each month for 5 years at 9% compounded monthly.

4.29 What is the amount of the quarterly deposits A such that you will be able to withdraw the amounts shown in the cash flow diagram if the interest rate is 8% compounded quarterly?



4.30 Georgi Rostov deposits $15,000 in a savings account that pays 6% interest compounded monthly. Three years later, he deposits $14,000. Two years after the $14,000 deposit, he makes another deposit in the amount of $12,500. Four years after the $12,500 deposit, half of the accumulated funds is transferred to a fund that pays 8% interest compounded quarterly. How much money will be in each account six years after the transfer?

4.31 A man is planning to retire in 25 years. He wishes to deposit a regular amount every three months until he retires, so that, beginning one year following his retirement, he will receive annual payments of $60,000 for the next 10 years. How much must he deposit if the interest rate is 6% compounded quarterly?

4.32 You borrowed $15,000 for buying a new car from a bank at an interest rate of 12% compounded monthly. This loan will be repaid in 48 equal monthly installments over four years. Immediately after the 20th payment, you desire to pay the remainder of the loan in a single payment. Compute this lump-sum amount of that time.

4.33 A building is priced at $125,000. If a down payment of $25,000 is made and a payment of $1,000 every month thereafter is required, how many months will it take to pay for the building? Interest is charged at a rate of 9% compounded monthly.

4.34 You obtained a loan of $20,000 to finance an automobile. Based on monthly compounding over 24 months, the end-of-the-month equal payment was figured to be $922.90. What APR was used for this loan?

4.35 *The Engineering Economist* (a professional journal) offers three types of subscriptions, payable in advance: one year at $66, two years at $120, and three years at $160. If money can earn 6% interest compounded monthly, which subscription should you take? (Assume that you plan to subscribe to the journal over the next three years.)

4.36 A couple is planning to finance its three-year-old son's college education. Money can be deposited at 6% compounded quarterly. What quarterly deposit must be made from the son's 3rd birthday to his 18th birthday to provide $50,000 on each birthday from the 18th to the 21st? (Note that the last deposit is made on the date of the first withdrawal.)

4.37 Sam Salvetti is planning to retire in 15 years. Money can be deposited at 8% compounded quarterly. What quarterly deposit must be made at the end of each quarter until Sam retires so that he can make a withdrawal of $25,000 semiannually over the first five years of his retirement? Assume that his first withdrawal occurs at the end of six months after his retirement.

4.38 Michelle Hunter received $250,000 from an insurance company after her husband's death. Michelle wants to deposit this amount in a savings account that earns interest at a rate of 6% compounded monthly. Then she would like to make 120 equal monthly withdrawals over the 10-year period such that, when she makes the last withdrawal, the savings account will have a balance of zero. How much can she withdraw each month?

4.39 Anita Tahani, who owns a travel agency, bought an old house to use as her business office. She found that the ceiling was poorly insulated and that the heat loss could be cut significantly if 6 inches of foam insulation were installed. She estimated that with the insulation, she could cut the heating bill by $40 per month and the air-conditioning cost by $25 per month. Assuming that the summer season is three months (June, July, and August) of the year and that the winter season is another three months (December, January, and February) of the year, how much can Anita spend on insulation if she expects to keep the property for five years? Assume that neither heating nor air-conditioning would be required during the fall and spring seasons. If she decides to install the insulation, it will be done at the beginning of May. Anita's interest rate is 9% compounded monthly.

Continuous Payments with Continuous Compounding

4.40 A new chemical production facility that is under construction is expected to be in full commercial operation 1 year from now. Once in full operation, the facility will generate $63,000 cash profit daily over the plant's service life of 12 years.

Determine the equivalent present worth of the future cash flows generated by the facility at the beginning of commercial operation, assuming

(a) 12% interest compounded daily, with the daily flows.

(b) 12% interest compounded continuously, with the daily flow series approximated by a uniform continuous cash flow function.

Also, compare the difference between (a) discrete (daily) and (b) continuous compounding.

4.41 Income from a project is expected to decline at a constant rate from an initial value of \$500,000 at time 0 to a final value of \$40,000 at the end of year 3. If interest is compounded continuously at a nominal annual rate of 11%, determine the present value of this continuous cash flow.

4.42 A sum of \$80,000 will be received uniformly over a five-year period beginning two years from today. What is the present value of this deferred-funds flow if interest is compounded continuously at a nominal rate of 9%?

4.43 A small chemical company that produces an epoxy resin expects its production volume to decay exponentially according to the relationship

$$y_t = 5e^{-0.25t},$$

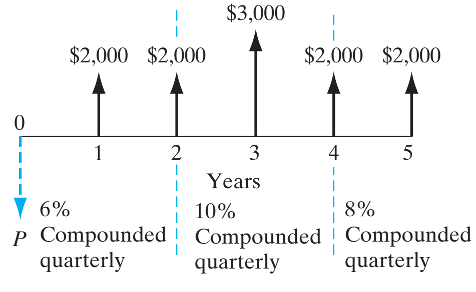
where y_t is the production rate at time t. Simultaneously, the unit price is expected to increase linearly over time at the rate

$$u_t = \$55(1 + 0.09t).$$

What is the expression for the present worth of sales revenues from $t = 0$ to $t = 20$ at 12% interest compounded continuously?

Changing Interest Rates

4.44 Consider the accompanying cash flow diagram, which represents three different interest rates applicable over the five-year time span shown.



(a) Calculate the equivalent amount P at the present time.

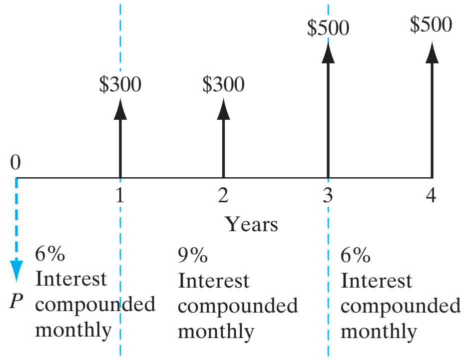
(b) Calculate the single-payment equivalent to F at $n = 5$.

(c) Calculate the equal-payment-series cash flow A that runs from $n = 1$ to $n = 5$.

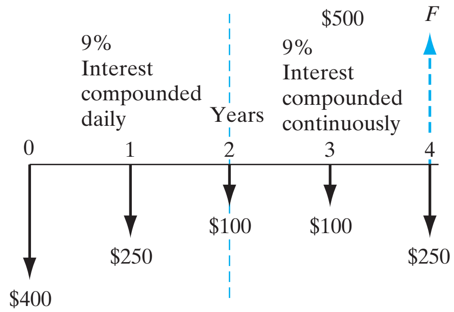
4.45 Consider the cash flow transactions depicted in the accompanying cash flow diagram, with the changing interest rates specified.

(a) What is the equivalent present worth? (In other words, how much do you have to deposit now so that you can withdraw \$300 at the end of year 1, \$300 at the end of year 2, \$500 at the end of year 3, and \$500 at the end of year 4?)

(b) What is the single effective annual interest rate over four years?



4.46 Compute the future worth of the cash flows with the different interest rates specified. The cash flows occur at the end of each year over four years.



Amortized Loans

4.47 An automobile loan of $20,000 at a nominal rate of 9% compounded monthly for 48 months requires equal end-of-month payments of $497.70. Complete the following table for the first six payments, as you would expect a bank to calculate the values:

End of Month (*n*)	Interest Payment	Repayment of Principal	Remaining Loan Balance
1			$19,652.30
2			
3		$352.94	
4	$142.12		
5	$139.45		
6			$17,874.28

4.48 Mr. Smith wants to buy a new car that will cost $18,000. He will make a down payment in the amount of $8,000. He would like to borrow the remainder from a bank at an interest rate of 9% compounded monthly. He agrees to pay off the loan monthly for a period of two years. Select the correct answer for the following questions:

(a) What is the amount of the monthly payment A?

(i) $A = \$10{,}000(A/P, 0.75\%, 24)$.

(ii) $A = \$10{,}000(A/P, 9\%, 2)/12$.

(iii) $A = \$10{,}000(A/F, 0.75\%, 24)$.

(iv) $A = \$12{,}500(A/F, 9\%, 2)/12$.

(b) Mr. Smith has made 12 payments and wants to figure out the balance remaining immediately after 12th payment. What is that balance?

(i) $B_{12} = 12A$.

(ii) $B_{12} = A(P/A, 9\%, 1)/12$.

(iii) $B_{12} = A(P/A, 0.75\%, 12)$.

(iv) $B_{12} = 10{,}000 - 12A$.

4.49 Tony Wu is considering purchasing a used automobile. The price, including the title and taxes, is \$12,345. Tony is able to make a \$2,345 down payment. The balance, \$10,000, will be borrowed from his credit union at an interest rate of 8.48% compounded daily. The loan should be paid in 36 equal monthly payments. Compute the monthly payment. What is the total amount of interest Tony has to pay over the life of the loan?

4.50 Suppose you are in the market for a new car worth \$18,000. You are offered a deal to make a \$1,800 down payment now and to pay the balance in equal end-of-month payments of \$421.85 over a 48-month period. Consider the following situations:

(a) Instead of going through the dealer's financing, you want to make a down payment of \$1,800 and take out an auto loan from a bank at 11.75% compounded monthly. What would be your monthly payment to pay off the loan in four years?

(b) If you were to accept the dealer's offer, what would be the effective rate of interest per month the dealer charges on your financing?

4.51 Bob Pearson borrowed \$25,000 from a bank at an interest rate of 10% compounded monthly. The loan will be repaid in 36 equal monthly installments over three years. Immediately after his 20th payment, Bob desires to pay the remainder of the loan in a single payment. Compute the total amount he must pay.

4.52 You plan to buy a \$200,000 home with a 10% down payment. The bank you want to finance the loan suggests two options: a 20-year mortgage at 9% APR and a 30-year mortgage at 10% APR. What is the difference in monthly payments (for the first 20 years) between these two options?

4.53 David Kapamagian borrowed money from a bank to finance a small fishing boat. The bank's terms allowed him to defer payments (including interest) on the loan for six months and to make 36 equal end-of-month payments thereafter. The original bank note was for \$4,800, with an interest rate of 12% compounded monthly. After 16 monthly payments, David found himself in a financial bind and went to a loan company for assistance in lowering his monthly payments. Fortunately, the loan company offered to pay his debts in one lump sum if he would pay the company \$104 per month for the next 36 months. What monthly rate of interest is the loan company charging on this transaction?

4.54 You are buying a home for $250,000.

(a) If you make a down payment of $50,000 and take out a mortgage on the rest of the money at 8.5% compounded monthly, what will be your monthly payment to retire the mortgage in 15 years?

(b) Consider the seventh payment. How much will the interest and principal payments be?

4.55 With a $350,000 home mortgage loan with a 20-year term at 9% APR compounded monthly, compute the total payments on principal and interest over the first 5 years of ownership.

4.56 A lender requires that monthly mortgage payments be no more than 25% of gross monthly income with a maximum term of 30 years. If you can make only a 15% down payment, what is the minimum monthly income needed to purchase a $400,000 house when the interest rate is 9% compounded monthly?

4.57 To buy a $150,000 house, you take out a 9% (APR) mortgage for $120,000. Five years later, you sell the house for $185,000 (after all other selling expenses). What equity (the amount that you can keep before tax) would you realize with a 30-year repayment term?

4.58 Just before their 15th payment,

- Family *A* had a balance of $80,000 on a 9%, 30-year mortgage;
- Family *B* had a balance of $80,000 on a 9%, 15-year mortgage; and
- Family *C* had a balance of $80,000 on a 9%, 20-year mortgage.

How much interest did each family pay on the 15th payment?

4.59 Home mortgage lenders usually charge points on a loan to avoid exceeding a legal limit on interest rates or to be competitive with other lenders. As an example, for a two-point loan, the lender would lend only $98 for each $100 borrowed. The borrower would receive only $98, but would have to make payments just as if he or she had received $100. Suppose that you receive a loan of $130,000, payable at the end of each month for 30 years with an interest rate of 9% compounded monthly, but you have been charged three points. What is the effective interest rate on this home mortgage loan?

4.60 A restaurant is considering purchasing a lot adjacent to its business to provide adequate parking space for its customers. The restaurant needs to borrow $35,000 to secure the lot. A deal has been made between a local bank and the restaurant so that the restaurant would pay the loan back over a five-year period with the following payment terms: 15%, 20%, 25%, 30%, and 35% of the initial loan at the end of first, second, third, fourth, and fifth years, respectively.

(a) What rate of interest is the bank earning from this loan?

(b) What would be the total interest paid by the restaurant over the five-year period?

4.61 Don Harrison's current salary is $60,000 per year, and he is planning to retire 25 years from now. He anticipates that his annual salary will increase by $3,000 each year (to $60,000 the first year, $63,000 the second year, $66,000 the third year, and so forth), and he plans to deposit 5% of his yearly salary into a retirement fund that earns 7% interest compounded daily. What will be the amount accumulated at the time of Don's retirement?

4.62 Consider the following two options for financing a car:

- **Option A.** Purchase the vehicle at the normal price of $26,200 and pay for the vehicle over three years with equal monthly payments at 1.9% APR financing.

- **Option B.** Purchase the vehicle for a discount price of $24,048, to be paid immediately. The funds that would be used to purchase the vehicle are presently earning 5% annual interest compounded monthly.

(a) What is the meaning of the APR of 1.9% quoted by the dealer?
(b) Under what circumstances would you prefer to go with the dealer's financing?
(c) Which interest rate (the dealer's interest rate or the savings rate) would you use in comparing the two options?

Add-On Loans

4.63 Katerina Unger wants to purchase a set of furniture worth $3,000. She plans to finance the furniture for two years. The furniture store tells Katerina that the interest rate is only 1% per month, and her monthly payment is computed as follows:

- Installment period = 24 months.
- Interest = 24(0.01)($3,000) = $720.
- Loan processing fee = $25.
- Total amount owed = $3,000 + $720 + $25 = $3,745.
- Monthly payment = $3,745/24 = $156.04 per month.

(a) What is the annual effective interest rate that Katerina is paying for her loan transaction? What is the nominal interest (annual percentage rate) for the loan?
(b) Katerina bought the furniture and made 12 monthly payments. Now she wants to pay off the remaining installments in one lump sum (at the end of 12 months). How much does she owe the furniture store?

4.64 You purchase a piece of furniture worth $5,000 on credit through a local furniture store. You are told that your monthly payment will be $146.35, including an acquisition fee of $25, at a 10% add-on interest rate over 48 months. After making 15 payments, you decide to pay off the balance. Compute the remaining balance, based on the conventional amortized loan.

Loans with Variable Payments

4.65 Kathy Stonewall bought a new car for $15,458. A dealer's financing was available through a local bank at an interest rate of 11.5% compounded monthly. Dealer financing required a 10% down payment and 60 equal monthly payments. Because the interest rate was rather high, Kathy checked her credit union for possible financing. The loan officer at the credit union quoted a 9.8% interest rate for a new-car loan and 10.5% for a used car. But to be eligible for the loan, Kathy has to be a member of the union for at least six months. Since she joined the union two months ago, she has to wait four more months to apply for the loan. Consequently, she decided to go ahead with the dealer's financing, and four months later she refinanced the balance through the credit union at an interest rate of 10.5%.

(a) Compute the monthly payment to the dealer.
(b) Compute the monthly payment to the union.
(c) What is the total interest payment on each loan?

4.66 A house can be purchased for $155,000, and you have $25,000 cash for a down payment. You are considering the following two financing options:

- **Option 1.** Getting a new standard mortgage with a 7.5% (APR) interest and a 30-year term.

- **Option 2.** Assuming the seller's old mortgage, which has an interest rate of 5.5% (APR), a remaining term of 25 years (the original term was 30 years), a remaining balance of $97,218, and payments of $597 per month. You can obtain a second mortgage for the remaining balance ($32,782) from your credit union at 9% (APR) with a 10-year repayment period.

(a) What is the effective interest rate of the combined mortgage?
(b) Compute the monthly payments for each option over the life of the mortgage.
(c) Compute the total interest payment for each option.
(d) What homeowner's interest rate makes the two financing options equivalent?

Loans with Variable Interest Rates

4.67 A loan of $10,000 is to be financed over a period of 24 months. The agency quotes a nominal rate of 8% for the first 12 months and a nominal rate of 9% for any remaining unpaid balance after 12 months, compounded monthly. Based on these rates, what equal end-of-the-month payment for 24 months would be required to repay the loan with interest?

4.68 Emily Wang financed her office furniture from a furniture dealer. The dealer's terms allowed her to defer payments (including interest) for six months and to make 36 equal end-of-month payments thereafter. The original note was for $15,000, with interest at 9% compounded monthly. After 26 monthly payments, Emily found herself in a financial bind and went to a loan company for assistance. The loan company offered to pay her debts in one lump sum if she would pay the company $186 per month for the next 30 months.

(a) Determine the original monthly payment made to the furniture store.
(b) Determine the lump-sum payoff amount the loan company will make.
(c) What monthly rate of interest is the loan company charging on this loan?

4.69 If you borrow $120,000 with a 30-year term at a 9% (APR) variable rate and the interest rate can be changed every five years,

(a) What is the initial monthly payment?
(b) If the lender's interest rate is 9.75% (APR) at the end of five years, what will the new monthly payments be?

Investment in Bonds

4.70 The Jimmy Corporation issued a new series of bonds on January 1, 1996. The bonds were sold at par ($1,000), have a 12% coupon rate, and mature in 30 years, on December 31, 2025. Coupon interest payments are made semiannually (on June 30 and December 31).

(a) What was the yield to maturity (YTM) of the bond on January 1, 1996?
(b) Assuming that the level of interest rates had fallen to 9%, what was the price of the bond on January 1, 2001, five years later?
(c) On July 1, 2001, the bonds sold for $922.38. What was the YTM at that date? What was the current yield at that date?

4.71 A $1,000, 9.50% semiannual bond is purchased for $1,010. If the bond is sold at the end of three years and six interest payments, what should the selling price be to yield a 10% return on the investment?

4.72 Mr. Gonzalez wishes to sell a bond that has a face value of $1,000. The bond bears an interest rate of 8%, with bond interests payable semiannually. Four years ago, $920 was paid for the bond. At least a 9% return (yield) on the investment is desired. What must be the minimum selling price?

4.73 Suppose you have the choice of investing in (1) a zero-coupon bond, which costs $513.60 today, pays nothing during its life, and then pays $1,000 after five years, or (2) a bond that costs $1,000 today, pays $113 in interest semiannually, and matures at the end of five years. Which bond would provide the higher yield?

4.74 Suppose you were offered a 12-year, 15% coupon, $1,000 par value bond at a price of $1,298.68. What rate of interest (yield to maturity) would you earn if you bought the bond and held it to maturity (at semiannual interest)?

4.75 The Diversified Products Company has two bond issues outstanding. Both bonds pay $100 semiannual interest, plus $1,000 at maturity. Bond A has a remaining maturity of 15 years, bond B a maturity of 1 year. What is the value of each of these bonds now, when the going rate of interest is 9%?

4.76 The AirJet Service Company's bonds have four years remaining to maturity. Interest is paid annually, the bonds have a $1,000 par value, and the coupon interest rate is 8.75%.

(a) What is the yield to maturity at a current market price of $1,108?

(b) Would you pay $935 for one of these bonds if you thought that the market rate of interest was 9.5%?

4.77 Suppose Ford sold an issue of bonds with a 15-year maturity, a $1,000 par value, a 12% coupon rate, and semiannual interest payments.

(a) Two years after the bonds were issued, the going rate of interest on bonds such as these fell to 9%. At what price would the bonds sell?

(b) Suppose that, two years after the bonds' issue, the going interest rate had risen to 13%. At what price would the bonds sell?

(c) Today, the closing price of the bond is $783.58. What is the current yield?

4.78 Suppose you purchased a corporate bond with a 10-year maturity, a $1,000 par value, a 10% coupon rate, and semiannual interest payments. All this means that you receive a $50 interest payment at the end of each six-month period for 10 years (20 times). Then, when the bond matures, you will receive the principal amount (the face value) in a lump sum. Three years after the bonds were purchased, the going rate of interest on new bonds fell to 6% (or 6% compounded semiannually). What is the current market value (P) of the bond (three years after its purchase)?

Short Case Studies

ST4.1 Jim Norton, an engineering junior, was mailed two guaranteed line-of-credit applications from two different banks. Each bank offered a different annual fee and finance charge.

Jim expects his average monthly balance after payment to the bank to be $300 and plans to keep the credit card he chooses for only 24 months. (After graduation, he will apply for a new card.) Jim's interest rate on his savings account is 6% compounded daily. The following table lists the terms of each bank:

Terms	Bank A	Bank B
Annual fee	$20	$30
Finance charge	1.55%	16.5%
	monthly interest rate	annual percentage rate

(a) Compute the effective annual interest rate for each card.

(b) Which bank's credit card should Jim choose?

(c) Suppose Jim decided to go with Bank B and used the card for one year. The balance after one year is $1,500. If he makes just a minimum payment each month (say, 5% of the unpaid balance), how long will it take to pay off the card debt? Assume that he will not make any new purchases on the card until he pays off the debt.

ST4.2 The following is an actual promotional pamphlet prepared by Trust Company Bank in Atlanta, Georgia:

> "Lower your monthly car payments as much as 48%." Now you can buy the car you want and keep the monthly payments as much as 48% lower than they would be if you financed with a conventional auto loan. Trust Company's *Alternative Auto Loan* (AAL)^{SM} makes the difference. It combines the lower monthly payment advantages of leasing with tax and ownership of a conventional loan. And if you have your monthly payment deducted automatically from your Trust Company checking account, you will save $\frac{1}{2}$% on your loan interest rate. Your monthly payments can be spread over 24, 36 or 48 months.

Amount Financed	Financing Period (months)	Monthly Alternative Auto Loan	Payment Conventional Auto Loan
	24	$249	$477
$10,000	36	211	339
	48	191	270
	24	498	955
$20,000	36	422	678
	48	382	541

> The amount of the final payment will be based on the residual value of the car at the end of the loan. Your monthly payments are kept low because you make principal payments on only a portion of the loan and not on the residual value of the car. Interest is computed on the full amount of the loan. At the end of the loan period you may:
>
> 1. Make the final payment and keep the car.
> 2. Sell the car yourself, repay the note (remaining balance), and keep any profit you make.

3. Refinance the car.
4. Return the car to Trust Company in good working condition and pay only a return fee.

So, if you've been wanting a special car, but not the high monthly payments that could go with it, consider the *Alternative Auto Loan*. For details, ask at any Trust Company branch.

Note 1: The chart above is based on the following assumptions. Conventional auto loan 13.4% annual percentage rate. *Alternative Auto Loan* 13.4% annual percentage rate.
Note 2: The residual value is assumed to be 50% of sticker price for 24 months; 45% for 36 months. The amount financed is 80% of sticker price.
Note 3: Monthly payments are based on principal payments equal to the depreciation amount on the car and interest in the amount of the loan.
Note 4: The residual value of the automobile is determined by a published residual value guide in effect at the time your Trust Company's *Alternative Auto Loan* is originated.
Note 5: The minimum loan amount is $10,000 (Trust Company will lend up to 80% of the sticker price). Annual household income requirement is $50,000.
Note 6: Trust Company reserves the right of final approval based on customer's credit history. Offer may vary at other Trust Company banks in Georgia.

(a) Show how the monthly payments were computed for the *Alternative Auto Loan* by the bank.

(b) Suppose that you have decided to finance a new car for 36 months from Trust Company. Suppose also that you are interested in owning the car (not leasing it). If you decided to go with the *Alternative Auto Loan*, you would make the final payment and keep the car at the end of 36 months. Assume that your opportunity cost rate (personal interest rate) is an interest rate of 8% compounded monthly. (You may view this opportunity cost rate as an interest rate at which you can invest your money in some financial instrument, such as a savings account.) Compare Trust Company's alternative option with the conventional option and make a choice between them.

ST4.3 In 1988, the Michigan legislature enacted the nation's first state-run program, the *Pay-Now, Learn-Later Plan*, to guarantee college tuition for students whose families invested in a special tax-free trust fund. The minimum deposit now is $1,689 for each year of tuition that sponsors of a newborn want to prepay. The yearly amount to buy into the plan increases with the age of the child: Parents of infants pay the least, and parents of high school seniors pay the most—$8,800 this year. This is because high school seniors will go to college sooner. Michigan State Treasurer Robert A. Bowman contends that the educational trust is a better deal than putting money into a certificate of deposit (CD) or a tuition prepayment plan at a bank, because the state promises to stand behind the investment. "Regardless of how high tuition goes, you know it's paid for," he said. "The disadvantage of a CD or a savings account is you have to hope and cross your fingers that tuition won't outpace the amount you save." At the newborns' rate, $6,756 will prepay four years of college, which is 25% less than the statewide average public-college cost of $9,000 for four years in 1988. In 2006, when a child born in 1988 will be old enough for college, four years of college could cost $94,360 at a private institution and $36,560 at a state school if costs continue to rise the expected average of at least 7% a year. The Internal Revenue Service issued its opinion, ruling that the person who sets aside the money would not be taxed on the amount paid into the fund. The agency said that the student would be subject to federal tax on the difference between the amount paid in and the amount paid out. Assuming that you are interested in the program for a newborn, would you join it?

ST4.4 Suppose you are going to buy a home worth $110,000 and you make a down payment in the amount of $50,000. The balance will be borrowed from the Capital Savings and Loan Bank. The loan officer offers the following two financing plans for the property:

- **Option 1.** A conventional fixed loan at an interest rate of 13% over 30 years, with 360 equal monthly payments.
- **Option 2.** A graduated payment schedule (FHA 235 plan) at 11.5% interest, with the following monthly payment schedule:

Year (n)	**Monthly Payment**	**Monthly Mortgage Insurance**
1	$497.76	$25.19
2	522.65	25.56
3	548.78	25.84
4	576.22	26.01
5	605.03	26.06
6–30	635.28	25.96

For the FHA 235 plan, mortgage insurance is a must.

(a) Compute the monthly payment under Option 1.
(b) What is the effective annual interest rate you are paying under Option 2?
(c) Compute the outstanding balance at the end of five years under each option.
(d) Compute the total interest payment under each option.
(e) Assuming that your only investment alternative is a savings account that earns an interest rate of 6% compounded monthly, which option is a better deal?

ST4.5 Consider the following advertisement seeking to sell a beachfront condominium at SunDestin, Florida:

95% Financing $8\frac{1}{8}$% interest!!

5% Down Payment. Own a Gulf-Front Condominium for only $100,000 with a 30-year variable-rate mortgage. We're providing incredible terms: $95,000 mortgage (30 years), year 1 at 8.125%, year 2 at 10.125%, year 3 at 12.125%, and years 4 through 30 at 13.125%.

(a) Compute the monthly payments for each year.
(b) Calculate the total interest paid over the life of the loan.
(c) Determine the equivalent single-effect annual interest rate for the loan.