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Spring Design and Calculation

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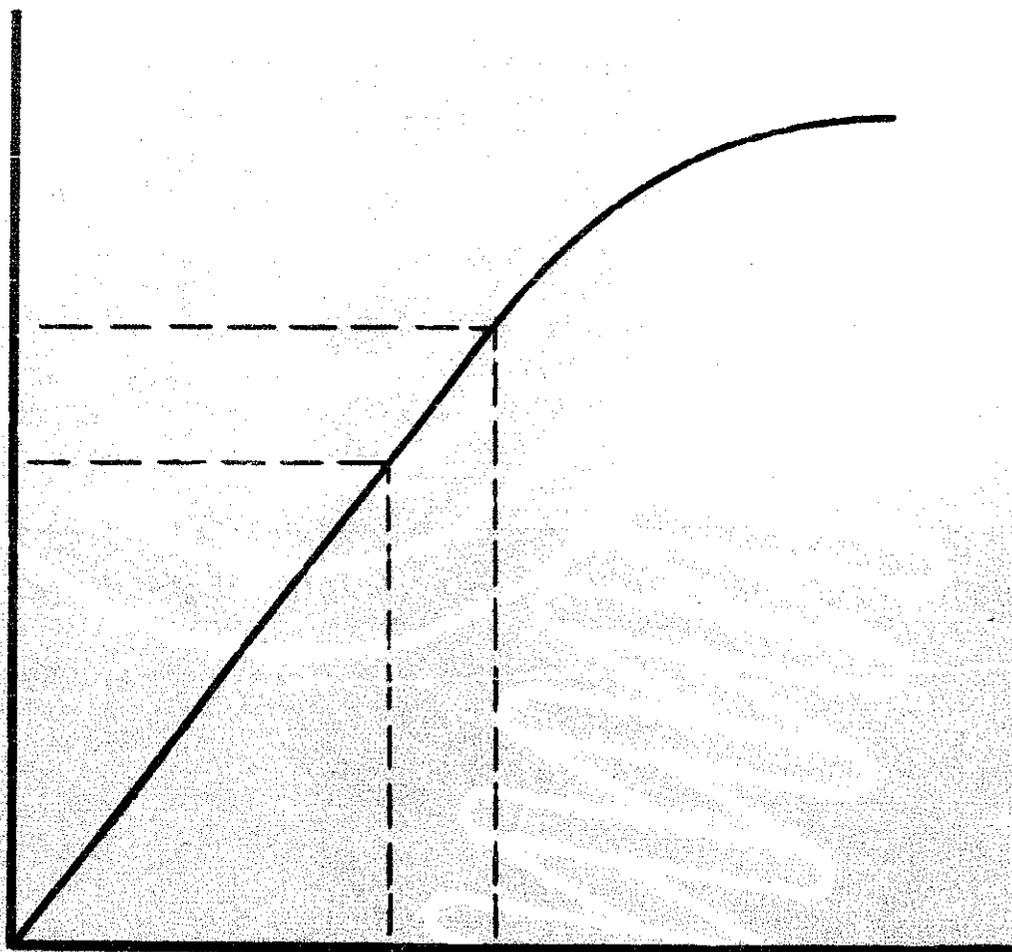
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spring design AND CALCULATION



MAP

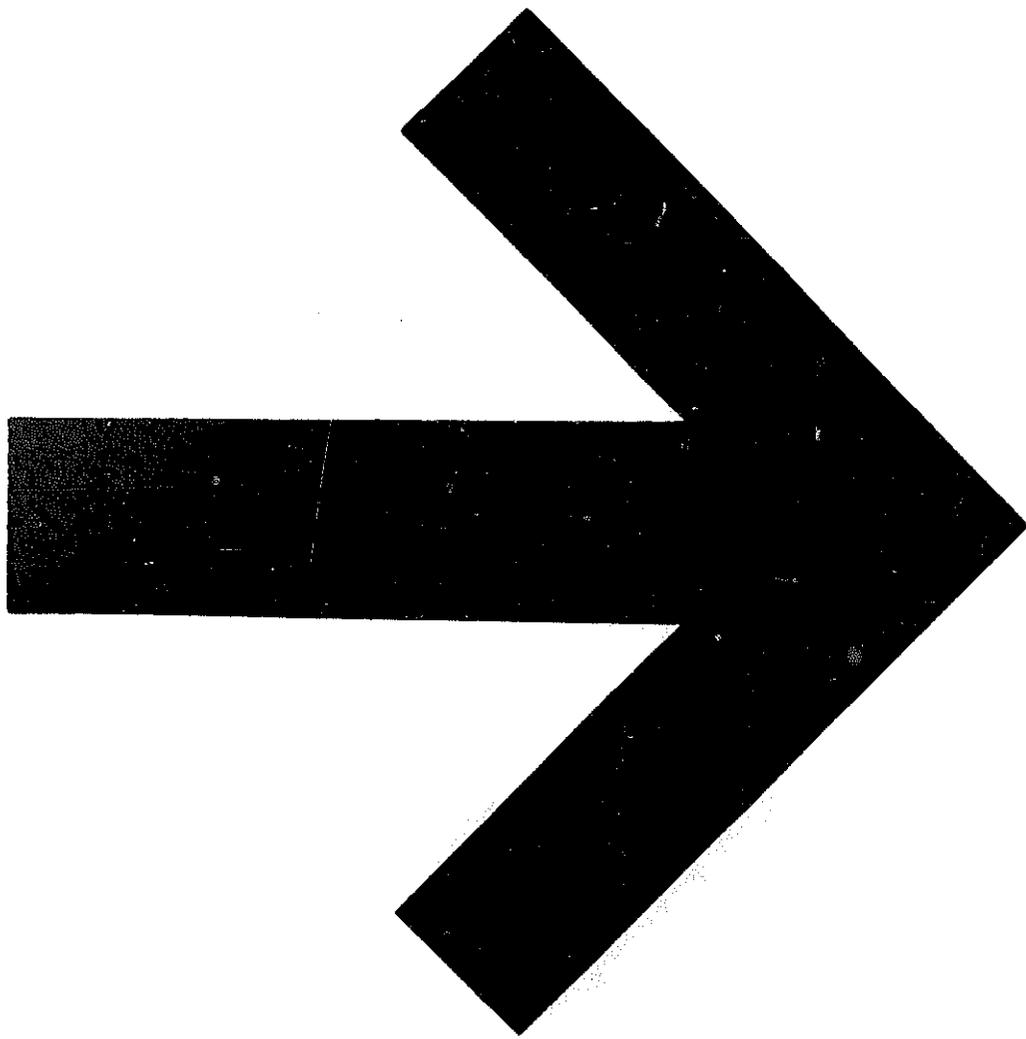
TECHNICAL PUBLICATION

Spring Design and Calculation

R. H. WARRING

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The springiness of metals is related in a general way to their hardness. Lead, for example, is a soft metal, with virtually no 'spring' properties. The same with aluminium. Extreme hardness, on the other hand, again results in lack of 'spring' properties because the material is brittle rather than 'elastic'. The range of suitable spring materials are thus those which combine suitable hardness with 'elasticity'.

It is also important, if spring performance is to be consistent, that the material retains its original properties. Many metals are subject to 'work-hardening' or a change of hardness when stressed – and all working springs are subject to cycles of stress. Brass, for example, is a metal which is relatively soft, but repeated stressing or 'working' causes its hardness to increase, with the metal becoming more springy as a consequence. Thus whilst soft brass is quite useless as a spring material, fully hardened brass possesses reasonably good spring properties.

The hardness of many metals can also be improved by heat treatment, and as a result their spring properties enhanced. This is quite common practice in the preparation of basic spring 'stock'. Heat, however, can also produce the opposite result. Thus a hard, springy metal can often be permanently softened by heating and slow cooling (or annealing). On the other hand, heating and rapid cooling a spring material can increase its hardness to the point of brittleness. Without considerable experience in the techniques of heat treatment, therefore, spring materials should always be used 'as is'.

With suitable knowledge, however, processed springs may often be heat treated to advantage – e.g. to remove internal stresses remaining in the material after cold working to shape or form. The temperature and method of heat treatment employed is dependent on the composition of the spring material and the method of spring application.

Another form of treatment which can produce embrittlement in a spring is electroplating. This applies particularly in the case of carbon steel springs, where plating may sometimes be thought desirable to provide resistance to corrosion. If such springs are plated, regardless of the method used they require to be baked immediately after plating to drive out hydrogen absorbed by the material during plating. Any hydrogen remaining in the pores of the spring material will cause embrittlement. Similar comment applies to plated steel wire, used as a spring material.

The range of true spring materials is fairly limited. Ordinary carbon steel rendered in 'spring temper' form is the most common choice for general purpose springs of all types. In the case of wire, the necessary temper may be produced by the method of fabrication – e.g. cold

SPRING MATERIALS

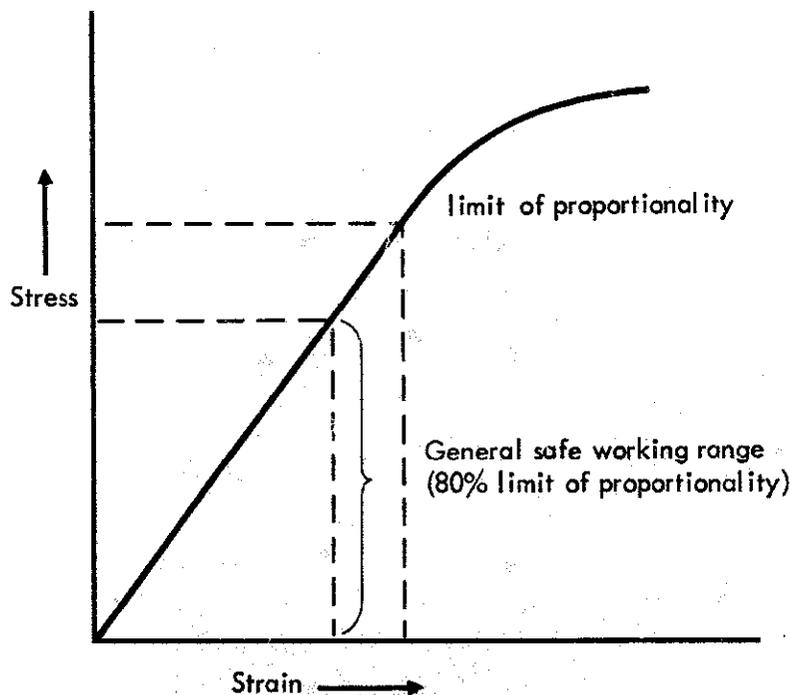
drawing. The spring temper may, however, be further improved by heat treatment or oil tempering. Such spring materials are suitable for use under ordinary temperatures, in normal stress ranges – i.e. without the limit of proportionality of the material (see later). For use under higher stresses, or higher temperatures, special alloy steels may be needed.

Where corrosion may be a problem, the choice of stainless steel or non-ferrous spring materials may be necessary – the former where high stresses have to be carried by the spring, and the latter for lower cost, easier working, where stresses are not so high. Beryllium copper is an attractive choice where high resistance to stress and corrosion are necessary, and good electrical conductivity is also required. If electrical conductivity is the main requirement, phosphor-bronze provides a cheaper alternative; and brass even lower cost (although brass is a 'marginal' spring material, even at full 'spring temper'). A nickel alloy (e.g. monel) may be specified where high temperatures have to be accommodated.

Mechanical Considerations

However good the spring material, there are limits over which it can be expected to work consistently and show a long 'spring' life. The critical factor involved is the actual stress born by the material when

Fig. 1



SPRING MATERIALS

the spring is working. Up to a certain point, with increasing stress the corresponding strain in the material follows a linear relationship – Fig. 1. Beyond this *limit of proportionality* this linear relationship no longer applies and subjecting the material to these higher stress values may permanently change the mechanical properties of the material.

The limit of proportionality thus represents the upper stress limit for the material. In practice, a lower limit is normally employed – 80 per cent of the limit of proportionality – to allow a safety factor in spring design.

Working within this limit will then ensure a consistent performance from a spring material.

This, however, only presents part of the picture. The strength of any material is different for different ways in which it is stressed. Maximum strength is usually available when stressed in pure *compression*, with an almost similar value when stressed in pure *tension*. If subject to twisting or *torsion*, the material strength available is considerably reduced.

Basically, in fact, the life of a spring depends on four main factors:

- (i) The manner in which the spring material is stressed.
- (ii) The maximum working stress.
- (iii) The range of stress over which the spring material is worked.
- (iv) the number of cycles of stress or the effects of fatigue on material properties.

Items (i) and (ii) are directly related. Once the manner in which the material is stressed is established, a safe maximum working stress can be established for a particular material – see Table I.

The stress range is more difficult to establish. In general, the higher the range of stress over which the spring is worked, the lower should be the maximum permissible stress to ensure long spring life. However, this will vary with both differences in material properties and heat treatment, and with frequency of working. For simplicity of design it is best to adopt 'safe' figures which err on the side of underestimating material performance, such as given in Table I.

Whilst material strength and stress determine the load which can be carried by a spring of given geometry, and the life of the spring, deflection characteristics are determined by the moduli of the material. Again this depends on the manner in which the spring material is deflected or stretched. If the spring material is under tension, then it is the *modulus of elasticity* of Young's modulus which is the parameter involved. For a spring material subject to torsion it is the *modulus of rigidity* which is involved in calculating deflection.

Values of modulus of elasticity (E) and modulus of rigidity (G) have, therefore, to be known for the spring materials used before the full

performance of a spring can be evaluated. These are also given in Table I. The modulus of elasticity largely governs the material performance of flat springs and torsion springs. The modulus of rigidity governs the material performance in helical springs. The actual stress produced in a spring, on the other hand, is dependent only on the load carried by the spring and the spring geometry. All these individual parameters appear in the spring design formulas in subsequent chapters.

Formulas and Units

Spring design proportions are not something that can be 'guesstimated' with any degree of accuracy – and trial-and-error design can produce a succession of failures. Thus this book on spring design is full of formulas, as the only accurate method of predicting spring performance. However, all are essentially *practical working formulas*; and all are quite straightforward to use. Each calculation is nothing more elaborate than an arithmetical calculation – aided by a slide rule or log tables.

No units are given with the formulas, since these follow quite logically depending on whether you are working to English or metric standards. Most quantities are linear dimensions, and it is only necessary to remember that stress values, etc., should be rendered in the same units. Thus for working with all dimensions in inches, stresses, etc., must be in pounds per square inch. Answers will then work out logically in the right units.

For example, the deflection per coil of a helical compression spring is given by

$$\text{deflection} = \frac{8PD^3}{Gd^4}$$

where P is the load

D is the mean coil diameter

d is the wire diameter

G is the modulus of rigidity of the spring material.

In English units, P would be in pounds. Dimensions D and d would be in inches. To be consistent, G must then be in pounds per square inch. The deflection, calculated from the formula, is another linear dimension and so would be given directly in inches.

Using metric units the point to watch is that the modulus or stress values used (or calculated) are in the same units as the linear dimensions. The latter, for example, will usually be in millimetres. Moduli and stress figures may, however, be quoted in kilograms per *square centimetre* and would need adjusting for consistency when used with millimetre linear units.

The basic form of a simple flat spring is shown in Fig. 2. The following are the two design formulas concerned:

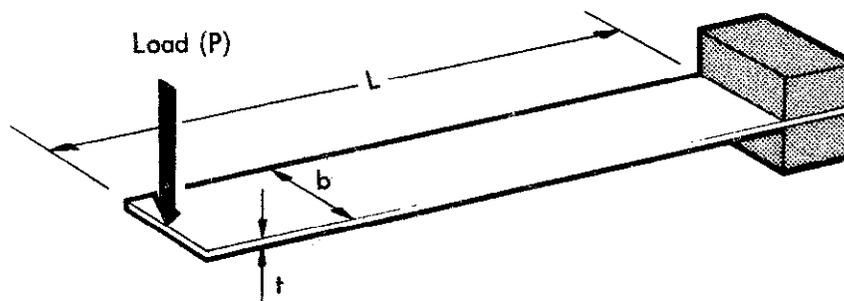
$$\text{Stress (tension)} = \frac{6PL}{bt^2}$$

$$\text{Deflection} = \frac{4PL^3}{Ebt^3}$$

in consistent

units – i.e. Stress is given in lb/sq.in when P is in pounds and L, b and t are in inches. The modulus of elasticity (E) is in lb/sq.in.

Fig. 2



As a general guide it follows that: **Stress** in the spring material increases in direct proportion to spring length, and in inverse proportion to width and (thickness)². Thus, for example, increasing the thickness of the spring will decrease the stress more effectively than increasing the width. **Deflection** increases with the cube of the length (thus small changes in length will have a marked effect on deflection); but can be decreased by increasing the width or thickness (the latter being much more effective in stiffening the spring).

Design Procedure

The design of a simple flat spring commonly calls for a certain deflection not to be exceeded under a given load. The length (L) of the spring may also be predetermined, or can be given a suitable value. The formula for Deflection can then be rearranged as a solution for bt^3 , viz:

$$bt^3 = \frac{4PL^3}{E \times \text{deflection}}$$

All values on the right hand side of this equation are known (the value of E following from the spring material selected), and can thus be calculated as a single quantity – say X

SIMPLE FLAT SPRINGS

We then have

$$bt^3 = X$$

From this point, either 'guesstimate' a value of b and from this calculate the corresponding value of t to satisfy the equation; or 'guesstimate' t and from this calculate b . The latter is the usual method since thickness is governed by the standard sizes of materials available, and thus there is a choice of *specific* values of t (e.g. 20 swg, 18 swg, etc.). Note: See Appendix B for tabular values of t^3 .

Any solutions derived by the above method will give spring proportions satisfying the deflection under load requirements. It is now necessary to enter these values in the Stress formula, together with load (P) and calculate the stress resulting. Providing this is lower than the maximum permissible stress for the material used, then the spring geometry is satisfactory. If the calculated stress is *higher* than the maximum permissible stress, then the spring geometry must be recalculated from the deflection formula, using different values. This is simplest if the spring length is left unaltered. It is then only necessary to return to the formula

$$bt^3 = X$$

and use a greater thickness to calculate a new value for b . Check if this reduced the stress to below the maximum permissible value. If not, try again.

Square Wire Springs

Use the same formulas, and procedure, substituting a^3 for bt^2 in the stress formula; and a^4 for bt^3 in the Deflection formula; where a = dimension of square.

Round Wire Flat Springs

Exactly the same formulas (and design procedure) follow in the case of a flat spring made from round wire – Fig. 3 – except that b and t are replaced by the wire diameter (d).

$$\text{Stress} = \frac{6PL}{d^3} \qquad \text{Deflection} = \frac{4PL^3}{Ed^4}$$

Flat Spring Supported at Each End

The stress and deflection formulas are modified when a flat spring is supported at both ends – Fig. 4 – and become

$$\text{Stress} = \frac{3PL}{2bt^2} \qquad \text{Deflection} = \frac{PL^3}{4Ebt^3}$$

Design procedure is the same again.

SIMPLE FLAT SPRINGS

Fig. 3

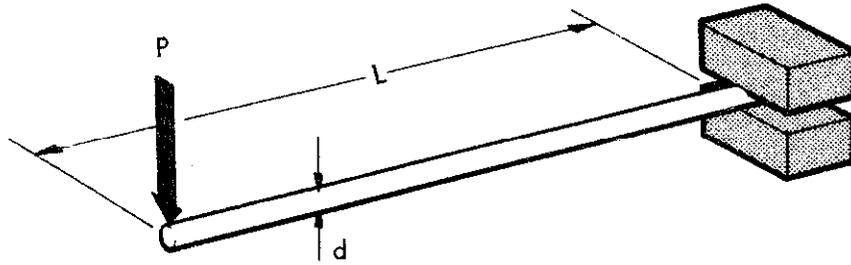
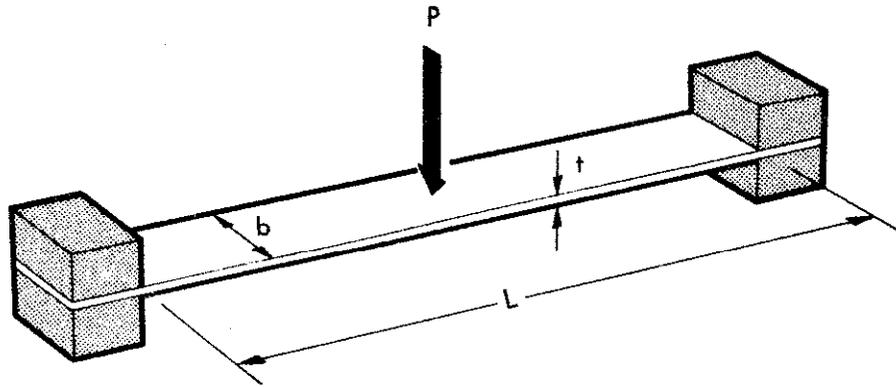


Fig. 4



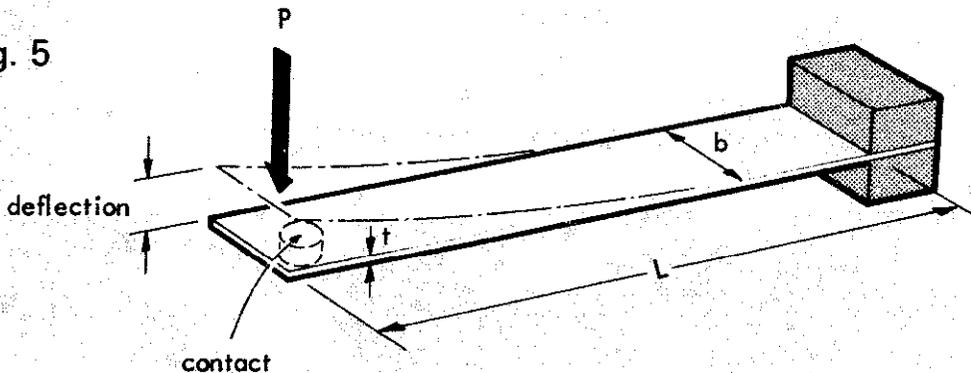
In the case of a flat spring made from round wire, supported at both ends, bt^2 in the 'Stress' formula is replaced by d^3 ; and bt^3 in the 'Deflection' formula is replaced by d^4

Design of Contact Springs

A contact spring is simply a flat spring designed to apply a certain pressure at a particular point (contact point) along its length – Fig. 5. It can be derived from the standard 'Stress' formula, rewritten as a solution for load (P) or actual contact pressure produced when deflected, viz

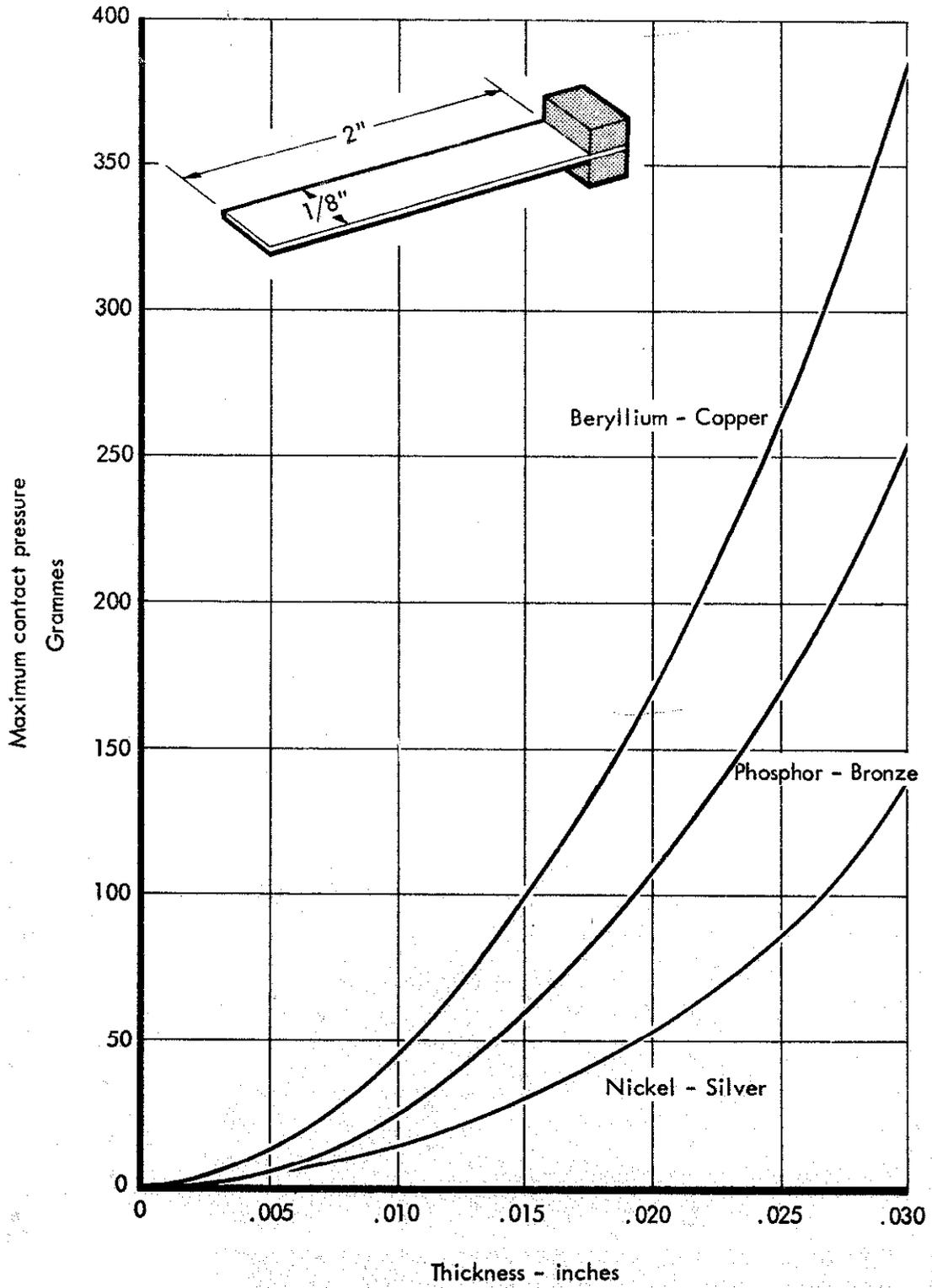
$$P = \frac{bt^2S}{6L}$$

Fig. 5



SIMPLE FLAT SPRINGS

Fig. 6



HELICAL SPRINGS

where S is the design maximum working stress for the material to be used (i.e. use 80 per cent of the limit of proportionality of the spring material from Table I).

The simplest way of tackling design is to fix suitable values of spring length L and width (b), from which the required material thickness can be calculated. If this yields a not standard thickness, then the nearest standard thickness (up or down) can be adopted, and the corresponding width re-calculated to provide the required contact pressure.

It can also be instructive, having decided on a suitable spring length and width, to calculate the maximum contact pressure available over a range of thicknesses for different spring materials. This is done in Fig. 6 for a spring length of 2" and width $\frac{1}{8}$ ", and clearly indicates the superiority of beryllium copper as a contact spring material.

Limitations to Flat Spring Calculations

Whilst the design formulas provide accurate theoretical solutions, actual performance may be modified somewhat by the manner in which the end (or ends) of a flat spring is (are) clamped.

In the case of contact springs, performance may be further modified by the fact that such springs are not necessarily simple beam shapes, but may be irregular in width. Calculation applied to such shapes is tedious. It is best to design the spring on the basis of a 'mean' or 'typical' width and check the performance by practical experiment.

HELICAL SPRINGS

3

In the case of helical springs which are either compressed or extended under load, the spring material is stressed in torsion and so the following basic formulas apply for round wire springs:

$$\text{Torsional stress} = \frac{8PD}{\pi d^3}$$

$$\text{Deflection} = \frac{8PD^3}{Gd^4} \times N$$

where P = load

D = mean diameter of spring

d = wire diameter

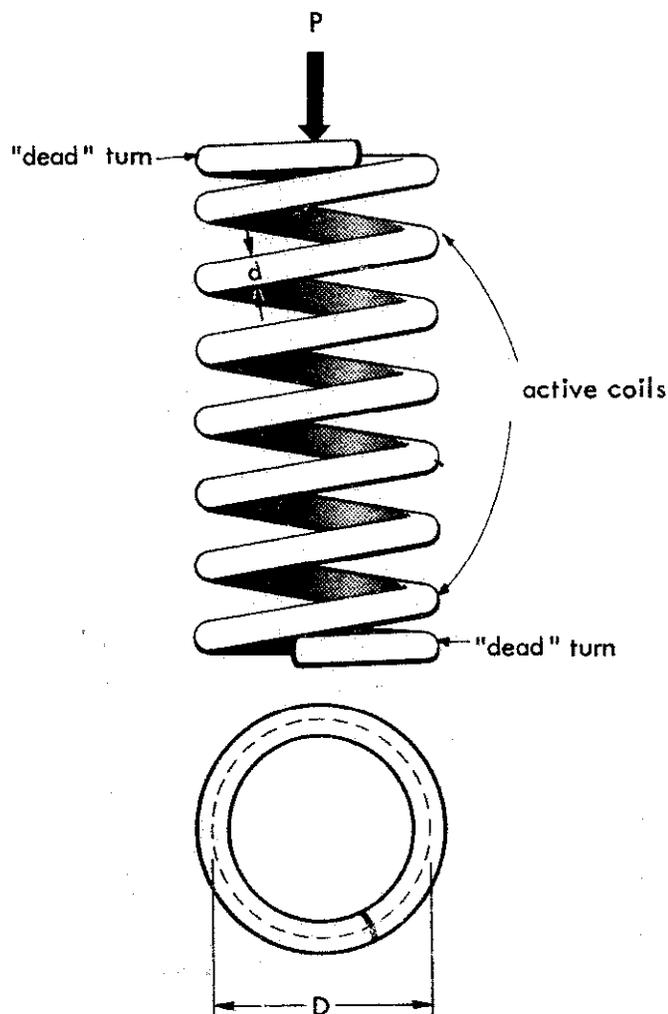
G = modulus of rigidity of spring material.

N = number of active coils in the spring

(see also Fig. 7)

HELICAL SPRINGS

Fig. 7



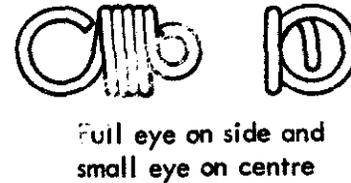
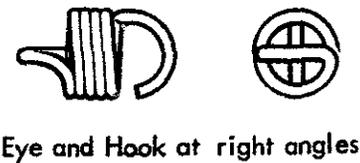
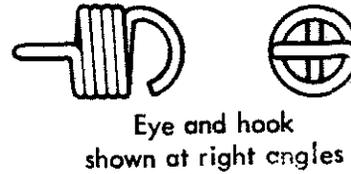
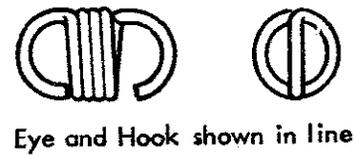
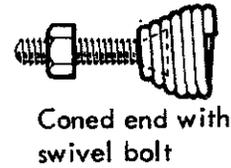
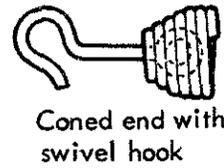
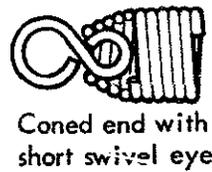
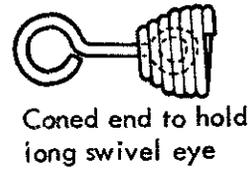
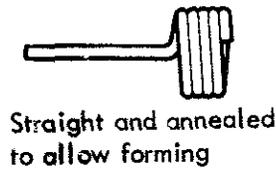
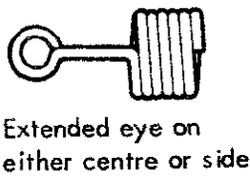
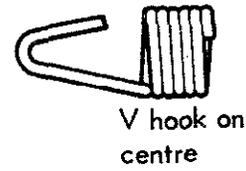
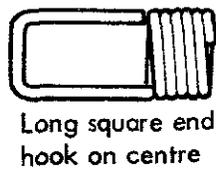
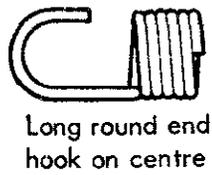
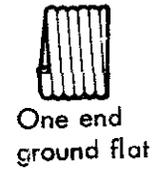
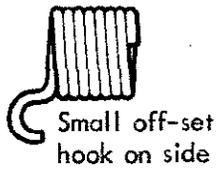
The stiffness of a helical spring, therefore, is proportional to the *fourth* power of the wire diameter, and varies inversely as the *cube* of the mean diameter. Both d and D thus have a marked effect on spring performance. Using a wire size only one gauge up can appreciably reduce the deflection, and vice versa. Similarly, only a small increase in spring diameter D can considerably increase the deflection; or a small decrease in D can make the spring much stiffer.

The latter effect, particularly, should be borne in mind when making a helical spring by wrapping around a mandrel. There will be an inevitable 'spring back' resulting in a spring inner diameter size greater than that of the mandrel. An undersize mandrel is thus required to form a spring of required diameter. The degree of undersize can only be estimated from experience since it will vary with the quality of the spring material used, and also the coiling technique.

The number of active coils is those actually 'working' as a spring. Usual practice is to allow $\frac{3}{4}$ of a turn (or 1 complete turn) at each end

HELICAL SPRINGS

Fig. 8 TENSION SPRING ENDS



HELICAL SPRINGS

in the case of a plain compression spring to produce parallel ends. Thus geometrically the spring has a total number of coils equal to $N + 1\frac{1}{2}$ (or $N + 2$), the number of active coils being calculated for the required deflection performance. Extension springs, on the other hand, commonly have all the coils 'active', the ends being made off at right angles to the main coil, e.g. see Fig. 8.

Another important parameter is the *spring rate* (or load rate), which is simply the load divided by the deflection.

$$\begin{aligned}\text{Spring Rate} &= \frac{P}{\text{deflection}} \\ &= \frac{Gd^4}{8ND^3}\end{aligned}$$

Where the spring is of constant diameter and the coils are evenly pitched, the spring rate is constant. A spring can be given a variable rate by tapering the coil, or using a variable pitch. Constant rate springs are the more usual, and much easier to work out.

Basically, spring design involves calculating the spring diameter and wire size required to give a safe material stress for the load to be carried. It is then simply a matter of deciding how many coils are required (i.e. how many active turns) to give the necessary spring rate or 'stiffness' in pounds per inch of movement. This may also be affected by the amount of free movement available for the spring.

The same considerations apply to both compression and extension springs, with one difference. Extension springs may be wound with initial tension, which in some cases can be as high as 25 per cent of the safe load. To open the coils of the spring this load must be applied, and only the *remainder* of the load is then available for deflection. This does not modify the spring design formula – merely the value of the applied load effective in producing deflection.

Whilst the working formulas are straightforward, spring design is complicated by the fact that three variables are involved in the spring geometry – diameter (D), wire diameter (d) and number of active coils (N). However, only D and d appear in the Stress formula, which is the one to start with. So here it is a case of 'guesstimating' one figure and calculating the other on that basis.

Design Procedure

- (i) Either
(a) fix a value for D and calculate d for the safe value of working stress from

$$d^3 = \frac{8PD}{\pi S}$$

HELICAL SPRINGS

(Note: From the value of d^3 so found the corresponding wire diameter can be found from the tables of Appendix B – there is no need to work out the cube root of the answer to the formula)

or

(b) fix the value of d (from an estimated suitable or readily available wire size), and from this calculate the required value or D from

$$D = \frac{\pi S d^3}{8P}$$

(Note: Again you can look up d^3 directly in the tables of Appendix B.)

(ii) Check that the sizes are practical. For example, if the value of D is fixed the calculated value of d may be a non-standard wire size. In this case, recalculate for the nearest standard size to yield an acceptable value of S . This can be avoided by fixing the value of d to start with, but could yield an impractical value for D .

(iii) Having arrived at suitable values for D and d , calculate the number of active turns required for the deflection to be accommodated.

$$N = \frac{Gd^4 \times \text{deflection}}{8PD^3}$$

(Note: you can look up values of d^4 directly in the Appendix tables).

That, in fact, is all there really is to designing helical compression or extension springs, provided extreme accuracy is not required. Remember to add on $\frac{3}{4}$ or 1 turn to each end for closed end compression springs.

More Accurate Working

Stress calculation by the above method assumes that the spring material is stressed in pure torsion. In fact, further stress is added because of the curvature in the wire. Thus the true stress in the material is higher than predicted from simple calculation, viz

$$\text{True stress} = K \times S$$

where K is a correction factor for wire curvature (normally known as the Wahl correction factor).

Unfortunately, the value of K depends on the spring geometry and thus the spring diameter (D) and wire diameter (d) have to be determined before the correction factor can be found.

$$K = \frac{4c - 1}{4c + 4} + \frac{0.615}{c}$$

where $c = D/d$ (which ratio is also known as the spring index).

Having determined a suitable size of spring, therefore, the true stress should be calculated, using the Wahl correction factor calculated as above. If this true stress works out higher than the maximum permissible material stress, then the whole spring geometry must be recalculated through.

To save a lot of working, values of K are shown graphically against spring index in Fig. 9, and also in Table II.

Solid Height of Spring

The solid or 'closed' length of a helical spring follows by multiplying wire diameter (d) by the total number of coils (N + 'dead' turn at each end, where applicable). This length may be reduced somewhat by grinding the 'dead' turns flat on a closed end spring see Fig. 10.

Helical Springs in Rectangular Wire Section

Similar formulas apply, with wire width (b) and thickness (a) replacing d – Fig. 11. Also additional stress factors are introduced to take into account the additional stresses imparted by bending rectangular section wire into a helical coil.

$$\text{Stress} = \frac{PDKK_2}{2a^2b} \qquad \text{Deflection} = \frac{PD^3NK_3}{Ga^3b}$$

Values of K_2 and K_3 are given in Table III. The spring index, for determining the value of K the Wahl correction factor K, is found as follows.

For rectangular wire coiled on edge, $c = D/a$

For rectangular wire coiled on flat, $c = D/b$

For non-critical applications the design of helical coil springs wound from rectangular section wire can ignore the corrections to stress, by adopting an appreciably lower value of maximum permissible material stress. This will not utilise the full spring potential of the material, but considerably simplifies calculation.

$$\text{Stress} = \frac{PD}{2a^2b} \qquad \text{Deflection} = \frac{PD^3N}{Ga^3b}$$

HELICAL SPRINGS

Fig. 9

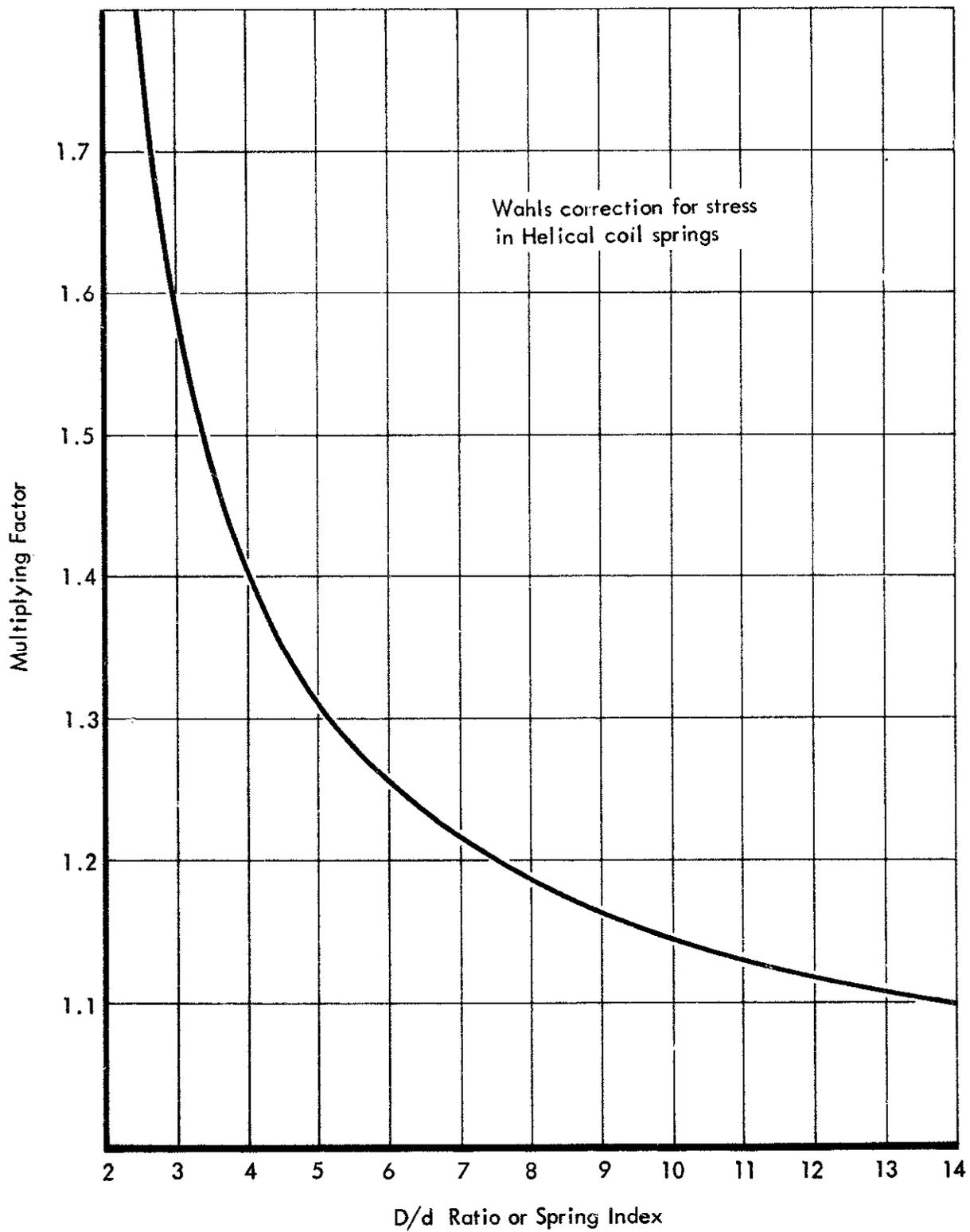
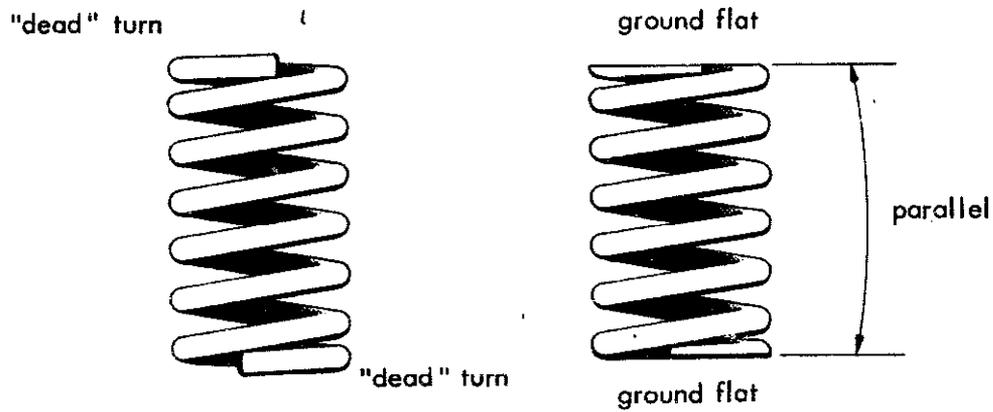


Fig. 10

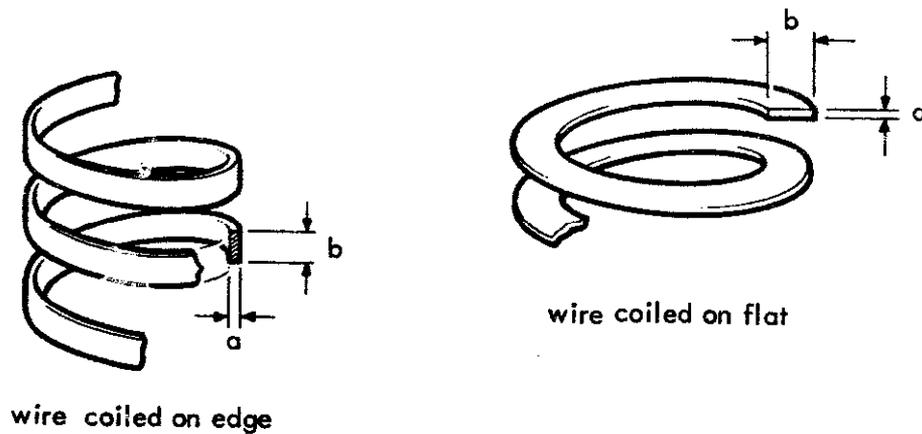


Energy Stored in Helical Springs

The energy stored in a compression or extension spring can easily be calculated from

$$\text{Energy} = \frac{P \times \text{deflection}}{2}$$

Fig. 11



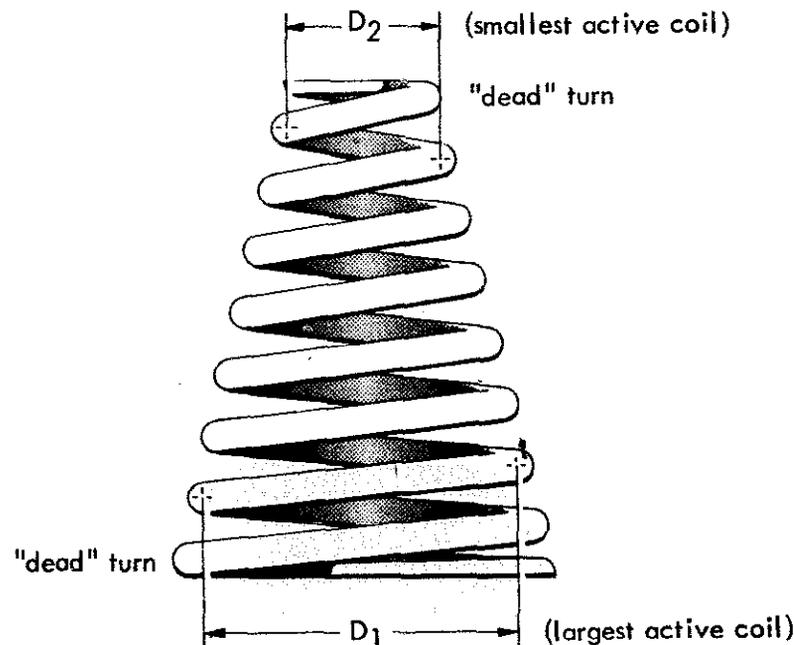
With the tapered or conical spring, each coil is of different diameter. This gives the spring a variable rate. The stress imposed by any load causing deflection is also variable from coil to coil. For design purposes it is the *maximum* stress, which is most important. This will occur in the largest active coil – Fig. 12 – and the stress is largely tension.

$$\text{Max. stress} = \frac{8PD_1}{\pi d^3} \times K$$

where K is the Wahl correction factor

(Note: since stress is proportional to spring diameter D, it follows that the stress in any coil can be calculated by using the appropriate coil diameter; also that the maximum stress will occur when D is a maximum, i.e. equal to that of the largest coil).

Fig. 12



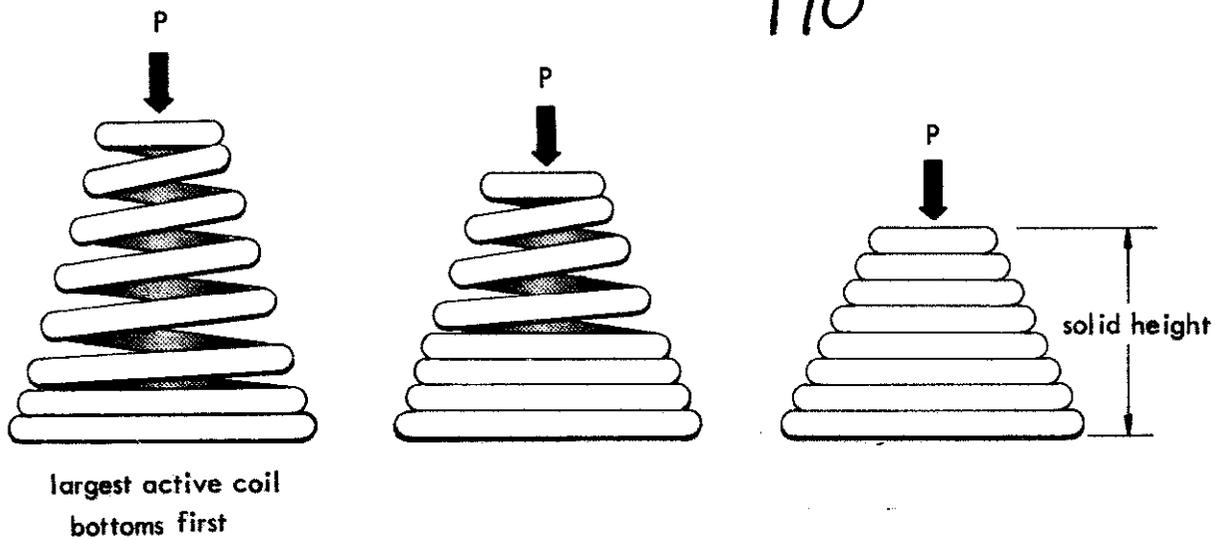
Deflection under a constant load will vary. In the case of a compression spring, first the largest coil will bottom, then the next largest, and so on – Fig. 13. An extension spring will 'open' in a similar progressive manner.

$$\text{Total deflection} = \frac{8PD_2^3N}{Gd^4}$$

where D_2 is the mean diameter of the smallest active coil

N is the number of active turns

Fig. 13



This formula can also be rewritten in terms of the maximum load to close the spring solid

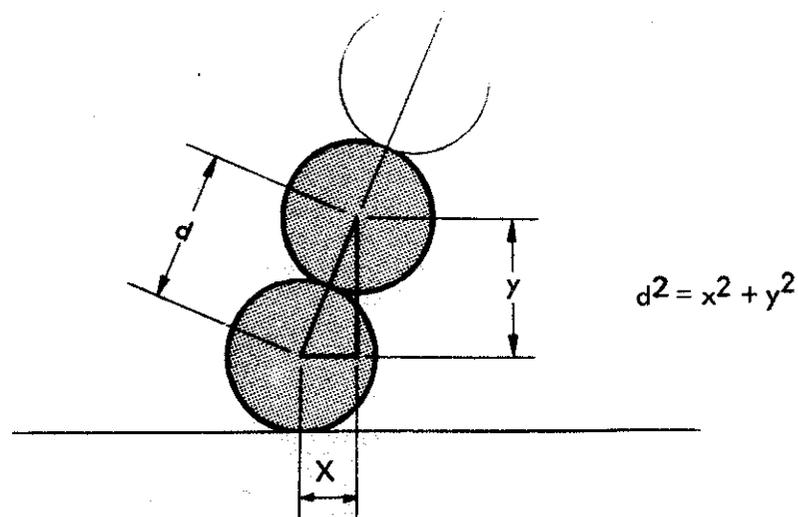
$$P_{\max} = \frac{Gd^4 \times \text{deflection}}{8D_2^3N}$$

The design of tapered springs, therefore, follows the same lines as for helical coil springs (Section 3), using these modified formulas.

Solid Height

The solid height of a tapered spring is less than that of a helical spring since the individual turns 'stack' to a certain extent – Fig. 14. The

Fig. 14



TORSION SPRINGS

effective height (y) per coil can be determined from the right-angled triangle shown, where

$$d^2 = x^2 + y^2$$

or $y = \sqrt{d^2 - x^2}$

The solid height of the spring then follows as

Solid height = Ny

where N = number of active turns.

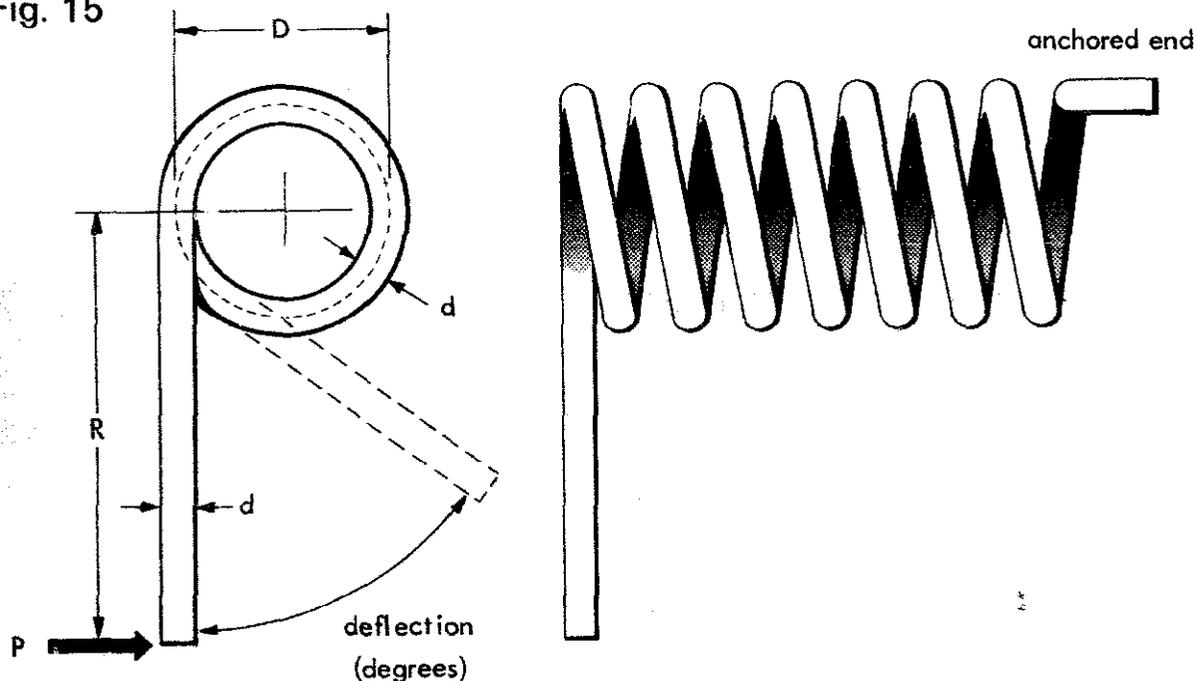
Remember to add $2d$ to this to account for one 'dead' turn at each end in the case of springs with closed ends.

TORSION SPRINGS

5

A helical torsion spring is designed to provide an angular deflection of an arm at one end of the spring – see Fig. 15 – the other end of the spring being anchored. The stiffness of such a spring (or its resistance to deflection) is directly proportional to the *fourth* power of the wire

Fig. 15



diameter; and inversely proportional to its diameter. The coil diameter is commonly fixed (e.g. the spring has to fit over a shaft or spindle); and thus choice of different wire sizes will have a considerable effect on spring performance.

TORSION SPRINGS

The following basic formulas apply:

$$\text{Stress} = \frac{32PR}{\pi d^3} \times K_4$$

where K_4 is the stress correction factor for round wire springs in torsion (see Table IV)

$$\text{Angular deflection (degrees)} = \frac{3665PRDN}{Ed^4}$$

where E = Young's modulus of spring material

Design calculations are again based on working the spring material within acceptable limits of stress. The force (P) acting on the spring is applied over a radius (R), equal to the effective length of the free arm of the spring. Design calculations can proceed as follows:

(i) Knowing the force to be accommodated and the spring arm leverage required (R), use the stress formula (without correction factor K_4) to calculate a suitable Wire size:

$$d^3 = \frac{32PR}{\pi S} = \frac{10.18PR}{S}$$

where S is the maximum permissible material stress.

(ii) Adjust to a standard wire size, if necessary.

(iii) Calculate the angular deflection of such a spring, using a specified value of diameter D , from the deflection formula, and ignoring the factor N . This will give the deflection per coil. Then simply find out how many coils are needed to produce the required deflection.

This stage may, of course, be varied. The load moment PR may be the critical factor – i.e. the spring is required to exert (or resist) a certain force (P) at a radius R with a specific deflection. In this case, having adopted a specific value for D , the deflection formula can be used to find a solution for the number of turns required.

(iv) Having arrived at a possible spring geometry, recalculate the true stress as a check, using the correction factor K_4 .

If necessary, readjust the spring geometry to reduce the stress and recalculate the spring.

If the spring is to be fitted over a shaft or spindle a check should also be made that in its tightened position it does not bind on the shaft.

TORSION SPRINGS

$$\text{Final mean diameter} = D \times \frac{N}{N_i}$$

where N_i is the final number of turns when tightened.

This is simple enough to work out. A deflection of x degrees is equivalent to $x/360$ turns.

$$\text{Thus } N_i = N + \frac{x}{360}$$

Remember that the final inner diameter of the coil will be equal to the final mean diameter *minus* d .

Torsion Springs in Rectangular Section Wire

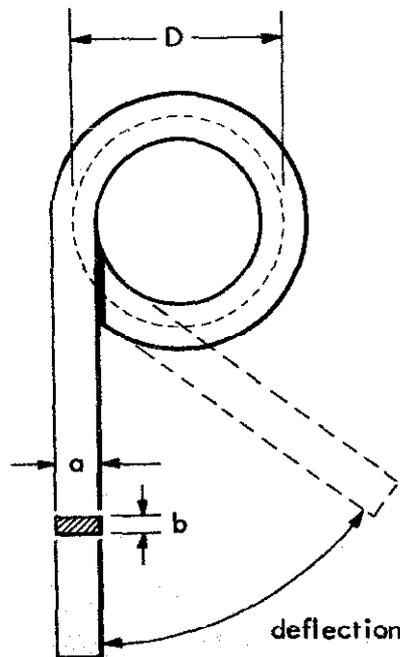
Exactly the same procedure is involved, except that the basic formulas are modified slightly (see also Fig. 16).

$$\text{Stress} = \frac{6PR}{a^2b} \times K_s$$

$$\text{Angular deflection (degrees)} = \frac{2160PRDN}{Ea^3b}$$

Square wire section is simply a special case of rectangular wire section where $a = b$.

Fig. 16



Energy Stored in Torsion Springs

This is easily calculated from the deflection and moment.

$$\text{Stored energy} = \frac{PR \times \text{deflection (degrees)}}{115}$$

The same formula applies to both round and rectangular wire sections.

CLOCK SPRINGS

6

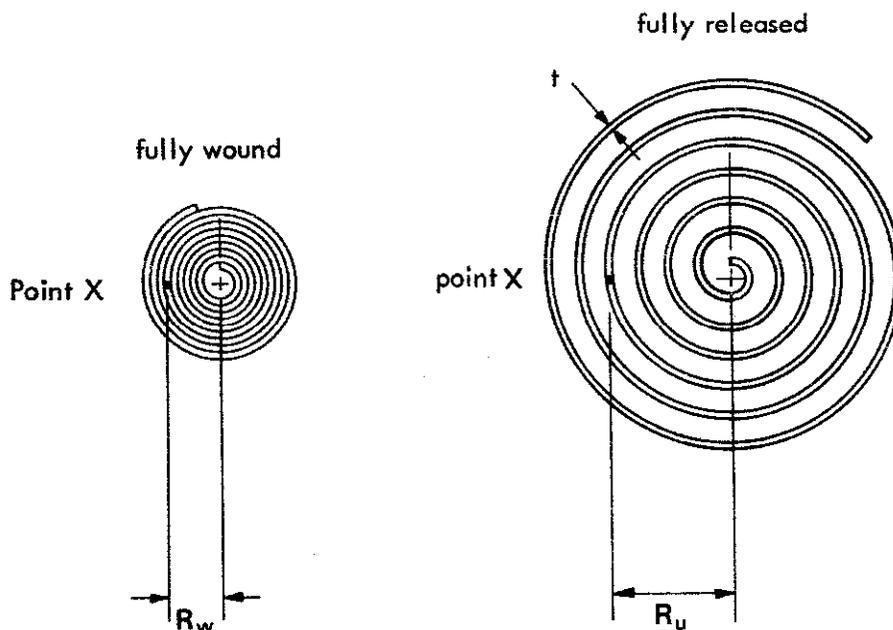
A clock spring is a special type of spiral or torsion spring, wound from flat strip. Main interest is in the turning moment or *torque*, and the *power* such a spring can develop.

The *stress* developed in the spring material can be calculated from the spring dimensions in a close wound and fully released conditions. If R_w is the radius of a particular point in the spring in a fully wound condition and R_u the radius of the same point in an unwound condition, a close approximation to the stress is given by:

$$\text{Bending stress } (S_b) = E \frac{t}{2} \left(\frac{1}{R_u} - \frac{1}{R_w} \right)$$

(see Fig. 17 for notations)

Fig. 17



Note that it is the material stress in bending (or torsion) which applies in this case, not the tensile stress (which is lower).

CLOCK SPRINGS

The 'Deflection' formula can be rendered in terms of the number of turns (T) the spring can be wound up.

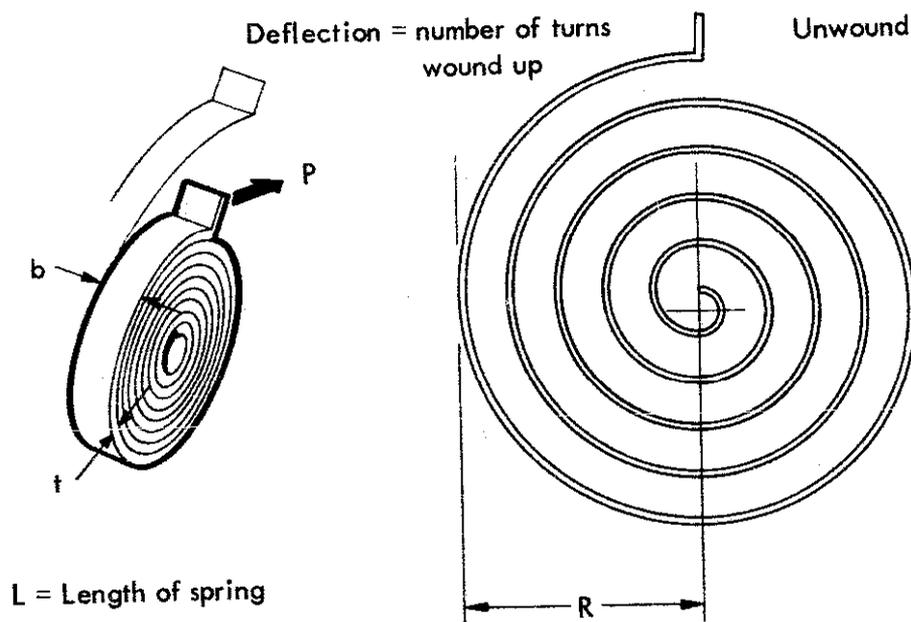
$$T = \frac{6PRL}{\pi Et^3 b}$$

this can also be rewritten in terms of the stress (S_b)

$$T = \frac{LS_b}{\pi Et}$$

(see Fig. 18 for notation)

Fig. 18



This is by far the more convenient form, but is not strictly correct since it does not allow for the effect of curvature on stress (see Torsion Springs). The complete formula for number of turns is thus

$$T = \frac{LS_b}{\pi Et K_s}$$

where K_s is the curvature stress factor (see Table V)

The *length* of spring (L) can be derived from basic geometry.

$$L = \pi DN$$

where N = number of active coils

CLOCK SPRINGS

In the fully wound condition

$$D = 2 \left(R_w + \frac{R_u - R_w}{2} \right)$$

but $R_u - R_w = Nt$

or $R_u = Nt + R_w$

Thus
$$D = 2 \left(R_u + \frac{Nt + R_u - R_u}{2} \right)$$
$$= 2R_u + Nt$$

Substituting in the first formula

$$L = \pi N(2R_w + Nt)$$

These formulas can be used to determine the required spring geometry, with the mechanical output given by

$$\text{Turning moment or torque } Q = PR_u$$

If the applied torque is known, then the number of turns to wind up the spring also follows directly as

$$T = \frac{6QL}{\pi Et^3 \gamma}$$

Horsepower Calculation

The stored energy in a clock spring can be released at various rates, according to the manner in which the movement is governed or restrained.

Note the relationship between number of turns (T) and stress.

No. of turns (T) to produce

stress S in spring material = $\frac{LS}{\pi Et}$

Thus stress (S) = $\frac{\pi EtT}{L}$

also:

$$\text{Energy per revolution} = \frac{\pi Sbt^2}{6}$$

To determine the energy produced by a clock spring, proceed as follows:

(i) Calculate length L from the geometry

CONSTANT FORCE SPRINGS

(ii) Calculate stress produced from the number of turns available to wind up (this must not exceed the maximum permissible bending stress of the material).

(iii) From the stress, calculate the energy per revolution (E_r)

If the energy per revolution (E_r) is determined in units of inch-pounds (which will follow using inch units for the spring geometry and stress in lb/sq.in).

$$\text{Horsepower} = \frac{E_r \times \text{rpm}}{396,000}$$

or say $\frac{E_r \times \text{rpm}}{400,000}$ as a suitable approximation.

The *time* for which the spring will develop power also follows as T/rpm , in minutes – i.e. the number of turns which can be wound on, divided by the rate of unwinding in revolutions per minute.

The whole series of calculations can, of course, be worked in reverse. That is, starting with a horsepower output requirement, and a known value of maximum permissible stress, suitable geometric proportions for the spring can be determined, together with the number of 'winding' turns available for the required rate of revolution and number of complete revolutions.

Note: as a practical design feature the diameter of the inner coil of the spring, in the fully wound condition, should not be less than 12 times the spring strip thickness. That is, the spring should be wound on an arbor of this minimum size. If wound up to a smaller diameter the spring is likely to suffer from fatigue effects.

CONSTANT FORCE SPRINGS

7

Constant force springs are a special type of flat strip spring, prestressed to have a uniform tendency to curl along its whole length.* They can be used in two ways (see also Fig. 19).

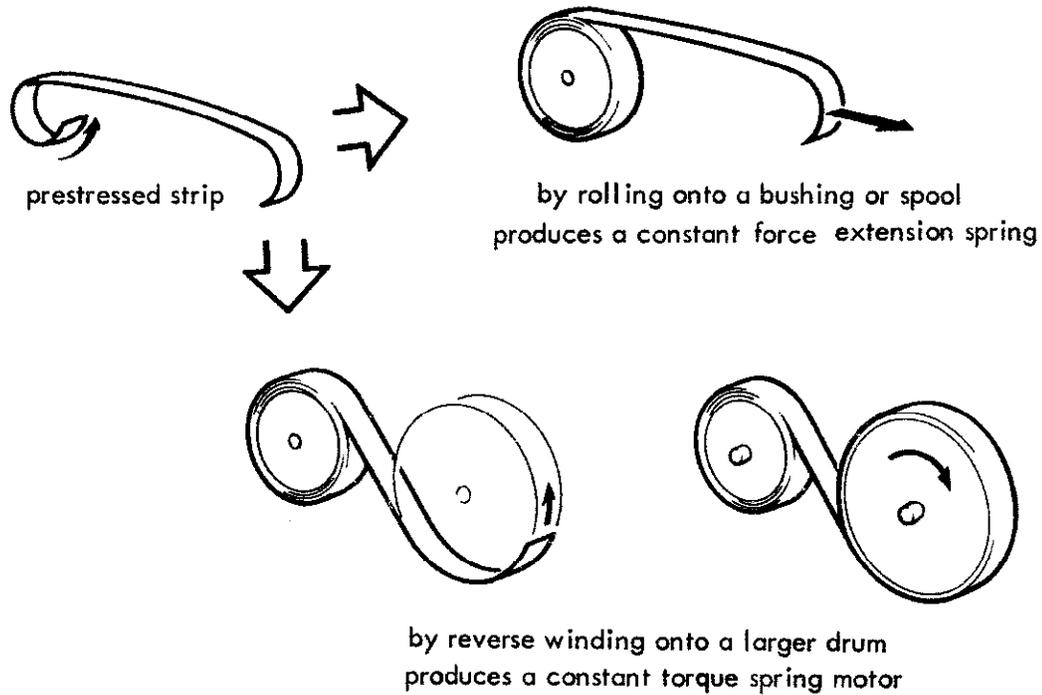
(i) Rolled onto a bushing to form a *constant force* extension spring, because the resistance to unrolling is the same at any extension.

(ii) Reverse-wound around a second drum to provide a *constant torque* spring, or constant torque spring motor.

* Springs of this type are made by Tensator Limited, Acton Lane, Harlesden, London, NW10. They are known as 'Tensator' springs in this country; and 'Neg'ater' springs in America.

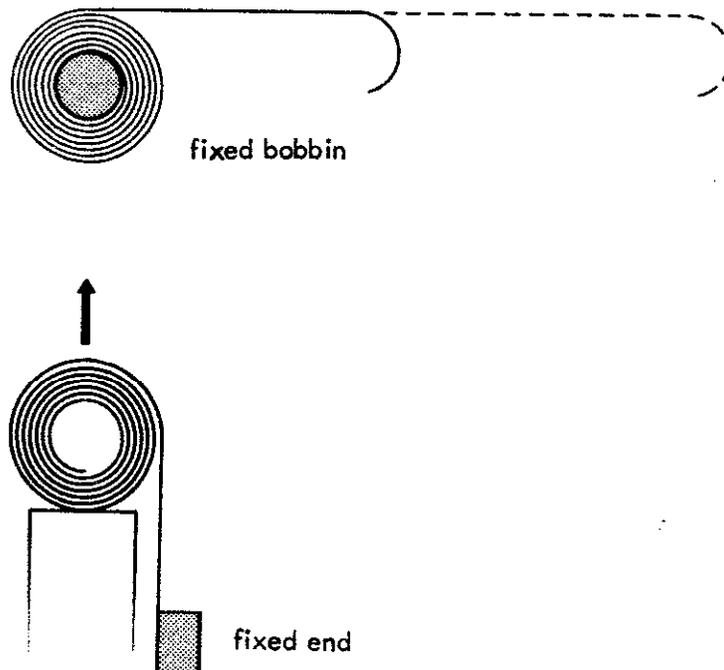
CONSTANT FORCE SPRINGS

Fig. 19



Constant force springs of this type have the advantage of being more compact when relaxed, compared with helical springs, plus the fact that very long extensions are possible. Either end can be fixed to produce an extension spring as shown diagrammatically in Fig. 20. The fixed free-end configuration, for example, has proved particularly effective for brush springs on electric motors.

Fig. 20



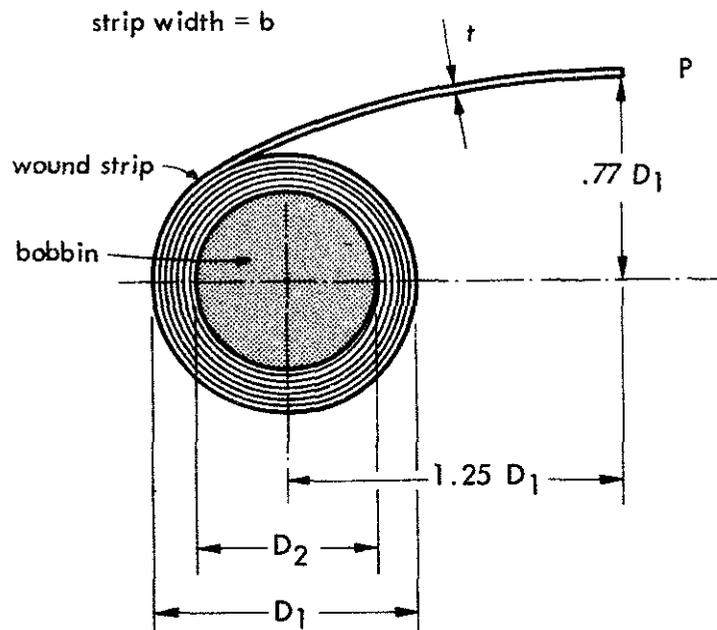
CONSTANT FORCE SPRINGS

The *constant torque* or spring motor form is particularly interesting since it offers a performance far superior to an ordinary clockwork motor, particularly in the length of run possible and the greater mechanical efficiency because of the absence of intercoil friction. Its performance can also be predicted quite accurately.

Extension Spring Design

Fig. 21 shows the static parameters of a 'Tensator' extension spring.

Fig. 21



The load to extend can be calculated directly from the load factor for the material (see Table V) and the spring width and thickness.

$$P = Qbt$$

The working extension of the spring (X) will be specified, but can also be determined from the actual length of spring strip.

$$X = L - 6D_2$$

where L is the total length of spring.

(Note: this formula allows for $1\frac{1}{2}$ dead turns on the coil).

The following formulas can also be used to determine D_1 and D_2 .

$$D_1 = \sqrt{1.275(X + 4.75D_2)t + D_2^2}$$

In design this should be increased by at least 10 per cent to be on the safe side, to allow for air space between the coils.

$$D_2 = 1.2 \times \text{natural free diameter of spring, as made.}$$

CONSTANT FORCE SPRINGS

Torque Motor Design (see Fig. 22)

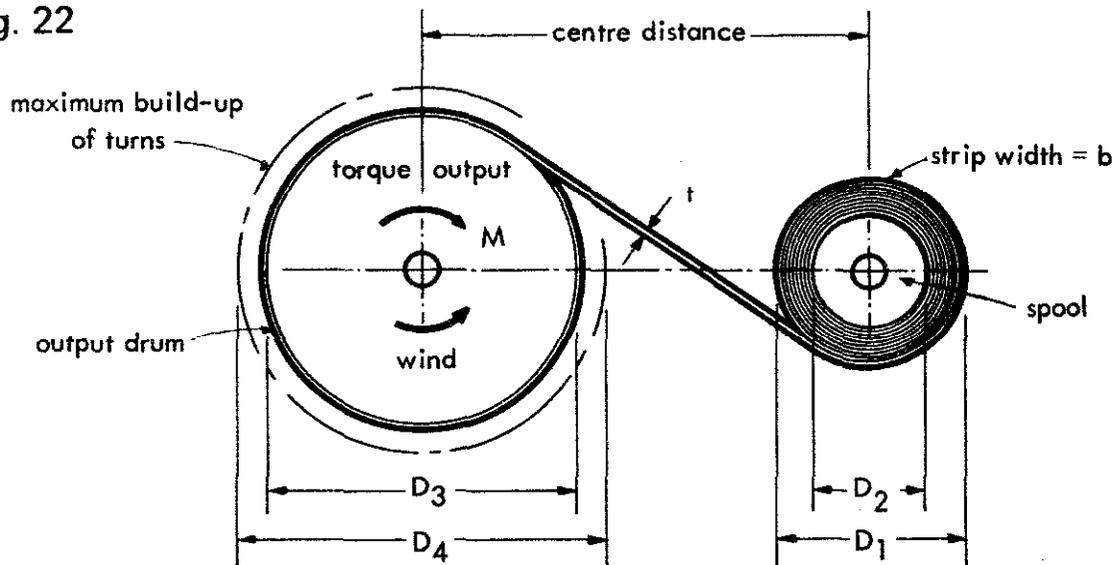
The torque output (M) available from a constant torque 'Tensator' spring motor is given by the formula

$$M = \frac{Qbt D_3}{2}$$

where Q is the load factor (see Table V)

The horsepower output can be derived from the rate of unwinding, as with clockwork motors (see Section 6).

Fig. 22



The following formulas will also be useful in design.

$$\text{Optimum value of } D_3 = \frac{6t}{Sf}$$

where Sf = bending factor (see Table V)

$$\text{Optimum value of } t = \sqrt{\frac{M \cdot Sf}{3bQ}}$$

$$\text{Optimum centre distance} = \frac{D_1 + D_3}{2} + 30t$$

$D_2 = 1.2$ times natural free diameter of spring, as made

$$D_1 = \sqrt{1.275Lt + D_2^2}$$

where L = total length of spring

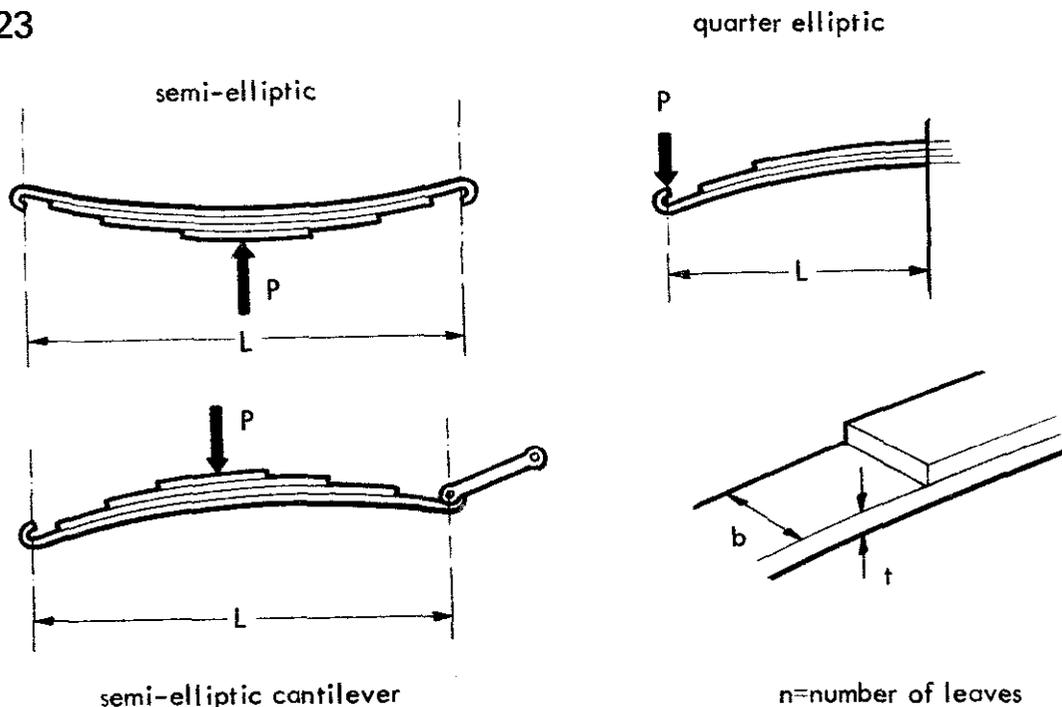
$$D_4 = \sqrt{1.275Lt + D_3^2}$$

$$L = 11 (D_3 N + t N^2) \text{ approx}$$

where N = number of working revolutions of D_3

Basically a laminated spring consisting of a number of individual leaves is no different to a single leaf spring, except that the additional leaves increase the effective thickness and thus reduce both deflection and stress in the individual leaves for a given load. Stress calculations are usually based on the assumption of a proportionate load on each leaf (i.e. proportionate to the number of leaves).

Fig. 23



Three common configurations for multiple leaf springs are shown in Fig. 23. The following deflection formulas apply:

Half elliptic:

$$\text{Deflection} = \frac{PL^3}{4Ebt^3n}$$

Quarter elliptic:

$$\text{Deflection} = \frac{4PL^3}{Ebt^3n}$$

Half elliptic Cantilever:

$$\text{Deflection} = \frac{PL^3}{2Ebt^3n}$$

where E = modulus of elasticity of spring material
 n = number of leaves

MULTIPLE LEAF SPRINGS

The corresponding stress formulas are (the material being stressed in bending as with simple flat springs).

Half elliptic:

$$\text{Stress} = \frac{1.5 PL}{bt^2n}$$

Quarter elliptic:

$$\text{Stress} = \frac{6PL}{bt^2n}$$

Half elliptic Cantilever:

$$\text{Stress} = \frac{3PL}{bt^2n}$$

There are several possible design approaches. If the thickness of each leaf (t) is decided, the spring width (b) necessary to produce the required deflection with 2, 3, 4, etc., leaves can be calculated, using the appropriate deflection formula. For example, in the case of a quarter elliptic spring

$$b = \frac{4PL^3}{Et^3n \times \text{deflection}}$$

This will give suitable spring geometry with 2, 3, 4 leaves, etc., from which the most attractive can be selected. This value of b can then be used in the stress formula to check that the maximum permissible material stress is not exceeded. If so, then an alternative solution must be adopted (e.g. more leaves and smaller width); or the calculations re-done starting with a different (higher) value of thickness (t).

Sometimes it is simpler to work directly from the load the spring will carry, which can be arrived at by rewriting the stress formulas:

Load capability:

$$\text{Half elliptic} = \frac{bt^2nS_p}{1.5L}$$

$$\text{Quarter elliptic} = \frac{bt^2nS_p}{6L}$$

$$\text{Half elliptic Cantilever} = \frac{bt^2nS_p}{3L}$$

where S_p is the maximum permissible material stress in bending.

A series of alternative spring designs can then be worked out in

APPENDIX A

terms of different values of width (b), thickness (t) and number of leaves, all of which would be capable of carrying the required load. It is then a matter of calculating the deflection of each of these springs and deciding on the most suitable one. If none give a suitable value for deflection, then further alternatives must be worked out, bearing in mind:

- spring stiffness* increases in direct proportion to spring width (b) and number of leaves (n);
- ... increases in direct proportion to the *cube* of the leaf thickness.

APPENDIX A SPRING TERMINOLOGY (and standard units)

Load (P) is the force in pounds (or kilograms) exerted on or by a spring producing or modifying motion, or maintaining a force system in equilibrium. Load is directly proportional to deflection and is limited by the elastic limit of the spring material.

Deflection is the maximum movement of a spring from its free length or free position to a specified operating position. In the case of helical coil springs, deflection per coil is equal to the total deflection divided by the number of active coils.

Rate or load rate is equal to load divided by deflection, and is thus inversely proportional to the number of active coils in a coil spring.

Free length is the true dimensional length of a spring in its unloaded position.

Solid height is the geometric height (or length) of a coil spring when it is fully compressed.

Active coils – the number of coils in a coil spring which deflect under load. End turns or part-turns on a compression spring which do not take part in deflection are referred to as 'dead' coils.

Pitch is the spacing or pitch dimension between adjacent active coils in a coil spring. Pitch determines the number of coils per unit length. Spring rate is also dependent on pitch, being substantially constant if the pitch is constant.

APPENDIX A

Stress is the operating stress on the spring material under working conditions. It is important both to use the right stress value for the material (e.g. depending on whether the spring material is being subject to tension or compression, bending or torsional loading); and also ensure that a maximum permissible stress figure is not exceeded. The latter depends on both load and frequency of deflection.

Mean diameter (D) The mean diameter of a helical coil spring is specified as the diameter to the centreline of the coil. The *overall* diameter of a coil spring is thus equal to $D + d$; and the inner diameter of a coil spring to $D - d$. Note that diameters can vary with working in the case of a torsion spring.

Wire diameter (d) the actual diameter or wire size used in a spring made from round wire.

Spring index. This is the ratio D/d and is used to determine stress correction factors where the stress loading on a spring is not simple (e.g. helical compression and extension springs, and torsion springs).

APPENDIX B: WIRE SIZES AND VALUES OF d^3 AND d^4

d			
swg	in.	d^3	d^4
33	0.010	0.000001000	0.000000010000
32	0.011	0.000001331	0.000000014641
30	0.012	0.000001728	0.000000020736
	0.013	0.000002197	0.000000028561
28	0.014	0.000002744	0.000000038416
	0.015	0.000003375	0.000000050625
27	0.016	0.000004096	0.000000065536
	0.017	0.000004913	0.000000083521
26	0.018	0.000005832	0.000000104976
	0.019	0.000006859	0.000000130321
	0.020	0.000008000	0.000000160000
	0.021	0.000009261	0.000000194481
24	0.022	0.000010648	0.000000234256
	0.023	0.000012167	0.000000279840
23	0.024	0.000013824	0.000000331780
	0.025	0.000015625	0.000000390620
	0.026	0.000017576	0.000000456980
	0.027	0.000019683	0.000000531440
22	0.028	0.000021952	0.000000614660
	0.029	0.000024389	0.000000707280
	0.030	0.000027000	0.000000810000
	0.031	0.000029791	0.000000923520
21	0.032	0.000032768	0.000001048580
	0.033	0.000035937	0.000001185920
	0.034	0.000039304	0.000001336340
	0.035	0.000042875	0.000001500620
20	0.036	0.000046656	0.000001679620
	0.037	0.000050653	0.000001874160
	0.038	0.000054872	0.000002085140
	0.039	0.000059319	0.000002313440
19	0.040	0.000064000	0.000002560000
	0.041	0.000068921	0.000002825760
	0.042	0.000074088	0.000003111700
	0.043	0.000079507	0.000003418800
	0.044	0.000085184	0.000003748100
	0.045	0.000091125	0.000004100620
	0.046	0.000097336	0.000004477460
	0.047	0.000103823	0.000004879680
18	0.048	0.000110592	0.000005308420
	0.049	0.000117649	0.000005764800
	0.050	0.000125000	0.000006250000
	0.051	0.000132651	0.000006765200
	0.052	0.000140610	0.000007311600
	0.053	0.000148880	0.000007890500
	0.054	0.000157460	0.000008503100
	0.055	0.000166380	0.000009150600
17	0.056	0.000175650	0.000009834500
	0.057	0.000185190	0.000010556000
	0.058	0.000195110	0.000011316500
	0.059	0.000205380	0.000012117400

d			
swg	in.	d^3	d^4
	0.060	0.00021600	0.0000129600
	0.061	0.00022698	0.0000138458
	0.062	0.00023833	0.0000147763
	0.063	0.00025005	0.0000157530
16	0.064	0.00026214	0.0000167772
	0.065	0.00027463	0.0000178506
	0.066	0.00028750	0.0000189747
	0.067	0.00030076	0.0000201511
	0.068	0.00031443	0.0000213814
	0.069	0.00032851	0.0000226670
	0.070	0.00034300	0.0000240100
	0.071	0.00035791	0.0000254117
15	0.072	0.00037325	0.0000268739
	0.073	0.00038902	0.0000283982
	0.074	0.00040522	0.0000299866
	0.075	0.00042188	0.0000316406
	0.076	0.00043898	0.0000333622
	0.077	0.00045653	0.0000351530
	0.078	0.00047455	0.0000370151
	0.079	0.00049304	0.0000389501
14	0.080	0.00051200	0.0000409600
	0.081	0.00053144	0.0000430467
	0.082	0.00055137	0.0000452122
	0.083	0.00057179	0.0000474583
	0.084	0.00059270	0.0000497871
	0.085	0.00061412	0.0000522006
	0.086	0.00063606	0.0000547008
	0.087	0.00065850	0.0000572898
	0.088	0.00068147	0.0000599695
	0.089	0.00070497	0.0000627422
	0.090	0.00072900	0.0000656100
	0.091	0.00075357	0.0000685750
13	0.092	0.00077869	0.0000716393
	0.093	0.00080436	0.0000748052
	0.094	0.00083058	0.0000780749
	0.095	0.00085738	0.0000814506
	0.096	0.00088474	0.0000849347
	0.097	0.00091267	0.0000885293
	0.098	0.00094119	0.0000922368
	0.099	0.00097030	0.0000960596
	0.100	0.00100000	0.0001000000
	0.101	0.00103030	0.0001040600
	0.102	0.00106121	0.0001082400
	0.103	0.00109273	0.0001125500
12	0.104	0.00112486	0.0001169900
	0.105	0.00115762	0.0001215500
	0.106	0.00119102	0.0001262500
	0.107	0.00122504	0.0001310800
	0.108	0.00124971	0.0001360500
	0.109	0.00129503	0.0001411600

APPENDIX B: WIRE SIZES AND VALUES OF d^3 AND d^4

d		d^3	d^4	
swg	in.			
11	0-110	0-0013310	0-00014641	
	0-111	0-0013676	0-00015181	
	0-112	0-0014049	0-00015735	
	0-113	0-0014429	0-00016305	
	0-114	0-0014815	0-00016890	
	0-115	0-0015209	0-00017490	
	0-116	0-0015609	0-00018106	
	0-117	0-0016016	0-00018739	
	0-118	0-0016430	0-00019388	
	0-119	0-0016852	0-00020053	
	10	0-120	0-0017280	0-00020736
		0-121	0-0017716	0-00021436
		0-122	0-0018158	0-00022153
		0-123	0-0018609	0-00022889
		0-124	0-0019066	0-00023642
		0-125	0-0019531	0-00024414
		0-126	0-0020004	0-00025205
		0-127	0-0020484	0-00026014
		0-128	0-0020972	0-00026844
0-129		0-0021467	0-00027692	
9		0-130	0-0021970	0-00028561
		0-131	0-0022481	0-00029450
		0-132	0-0023000	0-00030360
		0-133	0-0023526	0-00031290
		0-134	0-0024061	0-00032242
		0-135	0-0024604	0-00033215
		0-136	0-0025155	0-00034210
		0-137	0-0025714	0-00035228
		0-138	0-0026281	0-00036267
	0-139	0-0026856	0-00037330	
	8	0-140	0-0027440	0-00038416
		0-141	0-0028032	0-00039525
		0-142	0-0028633	0-00040659
		0-143	0-0029242	0-00041816
		0-144	0-0029860	0-00042998
		0-145	0-0030486	0-00044205
		0-146	0-0031121	0-00045437
		0-147	0-0031765	0-00046695
		0-148	0-0032418	0-00047979
0-149		0-0033079	0-00049288	
7		0-150	0-00337500	0-00050625
		0-151	0-00344295	0-00051989
		0-152	0-00351181	0-00053379
		0-153	0-00358160	0-00054798
		0-154	0-00365230	0-00056245
		0-155	0-00372390	0-00057720
		0-156	0-00379640	0-00059224
		0-157	0-00386990	0-00060757
		0-158	0-00394430	0-00062320
	0-159	0-00401970	0-00063913	

d		d^3	d^4	
swg	in.			
8	0-160	0-0040960	0-00065536	
	0-161	0-0041733	0-00067190	
	0-162	0-0042515	0-00068875	
	0-163	0-0043307	0-00070591	
	0-164	0-0044109	0-00072339	
	0-165	0-0044921	0-00074120	
	0-166	0-0045743	0-00075933	
	0-167	0-0046575	0-00077780	
	0-168	0-0047416	0-00079659	
	0-169	0-0048268	0-00081573	
	7	0-170	0-0049130	0-00083521
		0-171	0-0050002	0-00085504
		0-172	0-0050884	0-00087521
		0-173	0-0051777	0-00089575
		0-174	0-0052680	0-00091664
		0-175	0-0053594	0-00093789
		0-176	0-0054518	0-00095951
		0-177	0-0055452	0-00098051
		0-178	0-0056398	0-00100388
0-179		0-0057353	0-00102663	
6		0-180	0-0058320	0-00104976
		0-181	0-0059297	0-00107328
		0-182	0-0060286	0-00109720
		0-183	0-0061285	0-00112151
		0-184	0-0062295	0-00114623
		0-185	0-0063316	0-00117135
		0-186	0-0064349	0-00119688
		0-187	0-0065392	0-00122283
		0-188	0-0066447	0-00124920
	0-189	0-0067513	0-00127599	
	5	0-190	0-0068590	0-0013032
		0-191	0-0069679	0-0013309
		0-192	0-0070779	0-0013590
		0-193	0-0071891	0-0013875
		0-194	0-0073014	0-0014165
		0-195	0-0074149	0-0014459
		0-196	0-0075295	0-0014758
		0-197	0-0076454	0-0015061
		0-198	0-0077624	0-0015370
0-199		0-0078806	0-0015682	
4		0-200	0-008000	0-001600
		0-212	0-009528	0-002020
		0-232	0-012490	0-002897
		0-252	0-016000	0-004003
		0-276	0-021020	0-005803
		0-300	0-027000	0-008100
		0-324	0-034000	0-011000

TABLE I SPRING MATERIALS AND THEIR MECHANICAL PROPERTIES

Material	Limit of proportionality lb/sq. in †	Maximum safe working stress in torsion	Maximum safe working stress in tension	Modulus of elasticity E (lb/sq. in)	Modulus of rigidity G (lb/sq. in)
Piano wire up to 0.1" dia.	—	180,000	120,000	30,000,000	12,000,000
Oil tempered steel wire	—	150,000	100,000	30,000,000	11,500,000
Hard drawn steel wire	—	150,000	100,000	30,000,000	11,500,000
Stainless steel 18-8 wire	—	90-120,000	60-80,000	28,000,000	9,700,000
Stainless steel Ni wire	—	120,000	80,000	29,000,000	11,500,000
Chrome vanadium	—	—	—	30,000,000	11,500,000
Phosphor bronze*	70,000	90,000	60,000	15,000,000	6,300,000
Brass	—	52,500	35,000	9,000,000	5,500,000
Monel	—	90,000	60,000	26,000,000	9,000,000
Beryllium copper*	100-110,000	—	—	16-18,500,000	6-7,000,000
Nickel silver*	45-50,000	—	—	16,000,000	5,500,000

* Contact spring materials.

† Use 80 per cent of this value for design of flat contact springs.

TABLE II WAHL'S CORRECTION FACTOR K FOR ROUND WIRE HELICAL COIL SPRINGS

Spring index D/d	K
2	2.06
3	1.58
4	1.40
5	1.31
6	1.25
8	1.18
10	1.14
12	1.12
15	1.09
20	1.07
25	1.06

TABLE III CORRECTION FACTORS FOR RECTANGULAR WIRE HELICAL COIL SPRINGS

Number of active coils	Stress factor K ₂	Deflection factor K ₃
1.0	4.80	5.55
1.9	4.28	4.01
2.0	3.90	3.32
2.5	3.72	3.07
3.0	3.60	2.92
4.0	3.45	2.76
6.0	3.30	2.61
10.0	3.15	2.50
20	3.09	2.40
50	3.04	2.38
100	3.02	2.37

TABLE IV STRESS CORRECTION FACTORS FOR TORSION SPRINGS

Number of active coils	Stress factor for round wire K ₄	Stress factor for rectangular wire K ₅
2	1.61	1.54
3	1.33	1.29
4	1.23	1.20
5	1.18	1.15
6	1.14	1.12
8	1.10	1.09
10	1.08	1.07
12	1.06	1.06
15	1.05	1.04
20	1.09	1.03
25	1.04	1.03
30	1.03	1.02
above 30	1.0	1.0

TABLE V DESIGN VALUES FOR 'TENSATOR' SPRINGS

Design life no. of cycles	Carbon steel		Stainless steel	
	Q	S _f	Q	S _f
5000	521	0.023	660	0.027
10,000	418	0.020	502	0.023
20,000	271	0.015	350	0.019
40,000	169	0.010	233	0.012
70,000	123	0.009	151	0.009
200,000	101	0.008	87	0.008
2,000,000	81		70	