Gravitational Wave Generation Using Acoustic Resonators and Detection Using Coupled Resonance Chambers

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Abstract. An experiment is described for the generation and detection of High-Frequency Gravitational Waves (HFGWs) in the laboratory utilizing acoustic piezoelectric resonators for generation, and coupled resonance chambers for detection. Film Bulk Acoustic Resonators or FBARs (similar to those utilized in commercial cellular telephones) that are energized by magnetrons (similar to those utilized in microwave ovens) are distributed in a ring-shaped array several hundred meters in diameter. The magnetrons are phase locked and are sufficient in number to energize millions of FBARs fabricated on thousands of wafers. The FBARs produce jerks (time rate of change of acceleration or a third time derivative motion imparted to their metal electrode masses) at a frequency of 2.45GHz. The resulting 4.9GHz HFGW is focused at the center of the ring and is concentrated there by a high-temperature superconductor (HTSC) to generate a HFGW flux on the order of 34mW m⁻² to 14W m⁻². A miniature version of an existing HFGW detector designed at INFN Genoa, Italy, consisting of a pair of coupled HTSC-surfaced resonance chambers (about one centimeter in diameter) will be situated in the middle, having their axes perpendicular to the plane of the ring of FBARs. Details of the experiment and challenges to be met in its design as well as applications to space technology are discussed.

INTRODUCTION

In order to understand gravitational waves, consider first water waves. These waves are disturbances or undulations on or in a medium: the water. They exhibit an amplitude or height and a frequency that is dependent on the time between the passages of wave crests past some fixed point. They can be sensed, for example, by the motion of a buoy or leaf on the water's surface. Sound waves propagate as compressions and rarefactions in a gas medium – usually air. They have amplitude according to the pressure difference of the compressions and rarefactions and a frequency that is related to the time between adjacent wave pressure crests. They can be sensed by, for example, the motion of a diaphragm in a microphone. Electromagnetic (EM) waves like radio, microwaves, light, or X-rays depend upon the propagation of variations in an EM field that also exhibit a frequency like other waves, but they propagate at a constant speed, $c \approx 3 \times 10^8 \text{m s}^{-1}$ or the "speed of light." Also there is no true medium for EM waves. Such EM waves can be sensed by, for example, the motion of electrons in a photocell.

Einstein (1915) theorized a revolutionary *space-time* fabric or continuum in his general theory of relativity. He called the undulations or waves propagating in this fabric "*gravitational waves*" (Einstein, 1916). He theorized that they propagate at the speed of light and could be generated, for example, by orbiting stars, spinning rods, rims, or spheres. Gravitational waves (GWs) can be sensed by, for example, the change in lengths measured by extremely sensitive interferometers, piezoelectric crystals, superconductors, or resonance chambers. They have never been directly sensed, however, and some scientists believed that these waves were unobservable artefacts of Einstein's theory. The indirect evidence obtained by Taylor (1994) and R.A. Hulse concerning their observations of a contracting binary star pair or pulsar PSR 1913+16, which perfectly matched Einstein's GW theory, garnered them the 1993 Nobel Prize and skepticism concerning GWs evaporated. According to a set of definitions provided by Hawking and Israel (1979), High-Frequency Gravitational Waves (HFGWs) have frequencies in excess of 100 kHz

and have the most promise for terrestrial generation and practical, scientific, and commercial application. Low-Frequency Gravitational Waves (LFGWs) typically generated by most astrophysical sources are expected to be detected by interferometric and resonance devices whose technology is **TOTALLY** different from the technology of high-frequency detector devices – as different as AC-motor technology is from microwave-transmitter technology. Thus LFGW detectors such as LIGO, VIRGO, LISA, DECIGO (Japan), and CEGO (China) are **useless** for HFGW detection.

Since 2001 studies of HFGWs have intensified, including generation, detection, and engineering application. In 2003 the first meeting of the International High-Frequency Gravitational Waves Working Group, sponsored by MITRE Corporation, McLean (VA), was held (Baker, 2003a). This conference attracted 25 scientific papers from nine countries, and discussed many HFGW concepts. It also displayed the strong international activity in this area.

SIGNIFICANCE OF HFGW GENERATION

There has never been any significant effort to generate and detect gravitational waves in the laboratory since only the theory has been considered so far. The experiment proposed here will lead to revolutionary ways in which space propulsion and communication are accomplished 10 to 40 years from now. Remote HFGW generators will modify the gravitational field near an object or spacecraft: "Since it has definite energy, the gravitational wave is itself the source of some additional gravitational field...its field is a second-order effect...But in the case of high-frequency gravitational waves the effect is significantly strengthened..." (Landau and Lifshitz, 1975, page 349). Thus it is possible to change the gravitational field near an object by means of HFGWs and move or perturb it. And, thus there exists a completely new means to propel a spacecraft. The significance of such propellantless propulsion has been considered by many authors. HFGW generators may also be used as a completely new means of space communication. T.A. Prince (2002) recently commented: "Of the applications (of HFGWs), communication would seem to be the most important. Gravitational waves have a very low cross section for absorption by normal matter and therefore high-frequency waves could, in principle, carry significant information content with effectively no absorption unlike any electromagnetic waves." Such an HFGW communication system would represent the ultimate wireless system — point-to-multipoint PHz communication without the need for expensive enabling infrastructure, that is, no need for fiber-optic cable, satellite transponders, or microwave relays.

PROPOSED GENERATOR CONCEPT

The concept proposed here is to generate and detect HFGWs in the laboratory using a novel arrangement of Film Bulk Acoustic Resonators (FBARs). The details of the derivation of the HFGW generation equations, summarized below, are given by Baker (2000 and 2003b), and the concept of generating HFGWs by an array of micro-devices is described by Baker (2004). From Einstein's General Theory of Relativity the power of a GW generator is given by his quadrupole equation. This equation can be written as

$$P(r,\Delta f,\Delta t) = 1.76 \times 10^{-52} (2r\Delta f/\Delta t)^2 \text{ W},$$
(1)

where r is the radius of gyration of a mass (or system of masses) in meters (about 300m to 3km for the experiment proposed here), Δf is an incremental increase in force acting on a mass in newtons during an incremental time period Δt in seconds. This is the **jerk formulation of the quadrupole equation** as developed by Baker (2000 & 2002). For a continuous train of jerks the frequency is $v = 1/\Delta t$, and equation (1) can be phrased as a function of HFGW frequency as

$$P(r, \Delta f, v) = 1.76 \times 10^{-52} (2rv\Delta f)^2 \text{ W}.$$
 (2)

An impractical method of generating HFGWs is to rotate a rod or toroid so that it radiates energy through this quadrupole HFGW generation mechanism. This is impractical because it has long been known that rotation of any material at sufficiently high speed for efficient HFGW generation according to equation (2) is impossible; at much lower rotation speeds than are needed for efficient HFGW generation, centrifugal force causes the rotating mass to disintegrate. This is the origin of the widely-held view that generation of GWs is impossible outside of astronomical bodies.

The basic concept proposed here is to utilize piezoelectric mechanical resonators, specifically Film Bulk Acoustic Resonators (FBARs) similar to those utilized in cellular telephones, energized by magnetrons similar to those utilized in microwave ovens, to produce high frequency jerks in the vibrational elements of the FBARs and thereby generate HFGWs. An FBAR (Fig. 1) is a mechanical (acoustic) resonator consisting of a vibrating membrane (typically around 100×100μm² in plan, and around 1μm thickness), fabricated using well-established IC microfabrication technology. The vibrating membrane is actuated piezoelectrically, typically using aluminum nitride as the piezoelectric excitation material. These devices have recently been highly developed as cost-effective RF filters mainly for use in commercial cellular telephones, in which high-*Q* resonators are needed for the typical carrier frequency band around 1.9GHz. Exploratory FBAR devices have been fabricated operating at frequencies up to 7.5GHz (Ruby and Merchant, 1994; Lakin *et al.*, 2001). The vibrating membrane in an FBAR will also form an ideal vibrating mass for the present application, at slightly higher frequency.



FIGURE 1. Basic FBAR Construction (Side View, Not to Scale).

The FBARS will be arranged in a circle and each vibrating element must vibrate in phase with all the rest. In this way, a ring of masses all oscillating at the same frequency will be produced which reproduces the essentials of a toroidal mass oscillating at the same frequency, except that no centrifugal forces are generated so that the system will not be torn apart by centrifugal forces. The FBAR jerks tangential to the rotating rim are exactly analogous to jerks occurring in a solid toroid. The essential concept here is that a collection of rotating masses, with their attendant centrifugal-force jerks, is emulated by a series of fixed masses that are jerked by piezoelectric forces – energizable elements (FBARs) acted upon by energizing elements (magnetrons). Three important points should now be made:

- The energizable element's action/reaction does not cancel out in GW generation (Einstein and Rosen, 1937).
- The acceleration can be well over 100g and the quadrupole equation will still be valid: e.g., the acceleration of contracting binary star pair or pulsar PSR 1913+16 is 112g at periastron (Baker, 2000 & 2002).
- Δf need *not* be gravitational force (Einstein, 1916; Infeld, quoted by Weber (1964) p. 97). EM forces are ~ 10^{35} times larger than typical gravitational forces and so will give significant advantage in laboratory GW generation.

The generated HFGWs must then be focused to the smallest area possible at the focal point at the center of the ring of FBARs in order to achieve a large HFGW flux (energy/area) for detection. Commercial microwave oven magnetrons are proposed for this experiment because of their low cost in bulk quantities. These typically operate at 2.45GHz. Similarly, suitable FBARs may be fabricated cheaply in large quantities and a small design change will produce resonance at 2.45GHz instead of the cellular telephone band at 1.9GHz. Therefore, the operating frequency of the system will be 2.45GHz, giving a gravitational frequency v = 4.9GHz and a gravitational wavelength of $\lambda = c/v \approx 6.1$ cm. The FBARs need to be accurately oriented around the ring, within a fraction of one HFGW wavelength, and their lines of action (i.e., of the jerks of their vibrational elements) must be coplanar and accurately tangential to the stationary ring to a small fraction of a HFGW wavelength. This means that positional accuracy within at least 1cm must be rigorously maintained. Sketches of the emulation of rotating masses (δm) by a circular array of FBARs are shown in Fig. 2. Fig. 2(a) depicts two masses of a toroid rotating uniformly, and Fig. 2(b) depicts a ring of FBARs (each located on a circle of radius r) that produce the same HFGW effect.

ANALYSIS OF THE FLUX OF A HFGW GENERATOR

The solution to Problem 1, p. 356, of Landau and Lifshitz (1975) shows that the radiation pattern or specific angular intensity, *I*, can be expressed as

$$I = P_0 \times 7.55 \times 10^{-6} (1 + 6\cos^2\theta + \cos^4\theta) \text{ W deg}^{-2},$$
(3)

where P_0 is the power of the GW generator in W, and θ is the polar angle measured from the central axis (perpendicular to the plane of motion through the ring's center). In three dimensions the radiation pattern looks like two candelabra bulbs joined at their bases, or a dumbbell. This is shown in Fig. 2(b). The GW flux has been computed at a "cap" on each side of the focus defined by a cone having an altitude A and a $\pm 10^{\circ}$ vertex angle intersecting the three-dimensional radiation pattern. The GW flux, $F_{\pm 10^{\circ}}$, in either of the caps is given by

$$F_{\pm 10^{\circ}} = P_{\rm o} \times 2.5 (0.282/A)^2 \,\mathrm{W m}^{-2}$$
 (4)

(where A = 0.282m yields a 1m² area sphere). Clearly, the closer the detector device is to the focus along the central axis the better.

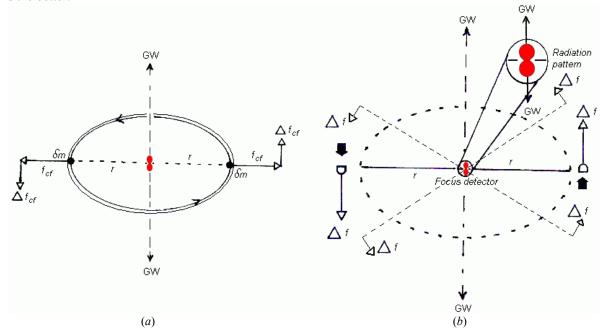


FIGURE 2. Emulation of a Rotating Set of Masses by a Circular Array of FBARs.

The HFGW power from a ring-shaped laboratory HFGW generator is proportional to v^2 according to Einstein's quadrupole relation, equation (2). Li and Torr (1992) showed that a High Temperature Superconductor (HTSC) has strong GW refractive properties that may be used to make a lens for GWs. For a Gaussian beam, perfectly focused (convergence angle 90°), the diffraction-limited spot diameter is $2\lambda/\pi$ (Saleh and Teich, 1991), and so the smallest area into which the GW power may be focused is λ^2/π . (The Gaussian diffraction formula is more nearly applicable to the present case than is the plane-wave formula.) Since $\lambda^2 = c^2/v^2$, the diffraction-limited focus area is inversely proportional to v^2 , so the HFGW flux (power/area) at the focus is proportional to v^4 . Away from the focus the flux will be significantly reduced, so the challenge is to situate the HFGW detector as close as possible to the focus and to make the active detection surface or volume coincident with the diffraction-limited focus. Consider an HFGW generation/detection system in which the generated HFGWs are formed into a collimated beam by an HTSC lens and that at some distance away there is another HTSC lens to concentrate the HFGWs at the detector. The two lenses are diffraction-limited. The beam area of the HFGW beam due to diffraction is inversely proportional to v^2 . The second lens concentrates the intercepted HFGWs at the detector into a diffraction-limited area, also inversely proportional to v^2 . Therefore, the efficiency of this transmitter/receiver system is **proportional to** v^6 (Baker, 2000). If the detector is located at the focus of the HFGW generator the lenses can be essentially coincident. That is, the HTSC surrounding the focus concentrates the HFGWs there by a factor equal to the refractive index = 400 (Li and Torr, 1992, p. 5491) in each axis. The GW wavelength is 400× smaller in the HTSC and consequently the diffraction-limited spot size will be $400^2 \times$ smaller than in free space. In this case the HFGW signal at the detector is **proportional to v^4.** In fact the efficiency is slightly reduced because the sensitivity of many GW detectors decreases with the square root of the GW frequency, but the overall efficiency will still be proportional to $v^{5.5}$ or $v^{3.5}$. Therefore, there is an enormous advantage in working at the highest frequency possible.

ESTIMATE OF PERFORMANCE OF FBARS AS HFGW GENERATORS

In this section a simple model of an HFGW generator assembled from FBARs is presented. Piezoelectric excitation of a small acoustic (mechanical) resonator is far superior to EM excitation at frequencies above 10s of kHz because there are no inductive losses in a typical piezoelectric device. However, the acoustic electrodes must be small so as not to introduce excessive capacitive loss. Existing high-performance FBARs (Ruby and Merchant, 1994; Lakin *et al.*, 2001) demonstrate the feasibility of low-loss operation of this class of device around 2GHz.

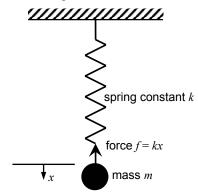


FIGURE 3. Simple Model of Acoustic Resonator.

The basic construction of an FBAR is illustrated in Fig. 1. Accurate modelling of a membrane resonator is a difficult task but here a very simple analog is used to estimate the basic performance of this device as a generator of HFGW. First, the stored energy per FBAR is estimated. The resonance quality factor is given by the well-known result $Q = \omega_0 \times (\text{stored energy})/(\text{power applied})$, where ω_0 is the resonant angular frequency, and so the stored energy is

$$W = O \times (\text{power applied})/\omega_{\text{p}}.$$
 (5)

To calculate the force applied to one FBAR membrane, the simple model of an acoustic resonator shown in Fig. 3 is used here. The vibrational mode of an FBAR membrane is actually a low-order Lamb wave (Auld, 1990) but a simple approximation will be accurate enough for a first estimate. Mass m fixed to the end of a spring of spring (force) constant k oscillates at a natural angular frequency $\omega_0 = (k/m)^{V_2} = 2\pi v_0$ so that

$$k = \omega_0^2 m. (6)$$

Second, the energy stored in the oscillation is given by $W = \frac{1}{2}kx^2 = \frac{1}{2}(kx_0)^2/k = \frac{1}{2}f_0^2/k$ for an oscillation amplitude x_0 and maximum force f_0 , so that from equation (6) the maximum force applied to the mass is

$$f_0 = (2Wk)^{1/2} = (2W\omega_0^2 m)^{1/2}.$$
 (7)

Suppose the total excitation power available is P_{in} divided equally between N FBARs. Then the power supplied to each FBAR is P_{in}/N (assuming, of course, no loss from power distribution or phase adjusting), so that from equation (5) the stored energy per FBAR is $W = QP_{in}/(N\omega_0)$. Combining this with equation (7) gives the force in each FBAR

$$f_0 = (2QP_{in}\omega_0 m/N)^{1/2}.$$
 (8)

Therefore, the total force experienced by N FBARs excited in phase is $(2QP_{in}\omega_0 mN)^{1/2}$. Note that the total force is proportional to $N^{1/2}$ rather than to N, because with more FBARs but fixed total input power P_{in} then the power per FBAR is reduced. To maximize the total force, the largest possible number of FBARs is needed, operating at the highest possible frequency and largest possible input power. A typical FBAR has a resonance curve with a passband resonance width of $2\Delta v = 24$ MHz at a typical passband center frequency $v_0 = 2$ GHz (Lakin *et al.*, 2001). This gives $Q \approx 2000/24 \approx 100$. Mass m is found from density × volume. The values density = 3000kg m⁻³ (typical of materials comprising an FBAR membrane) and volume = $100 \times 100 \times 1\mu m^3$ (typical of an FBAR membrane resonant at $v_0 = 2$ GHz) were estimated here, so that m = 30ng. A typical FBAR takes up rather more area than its nominal

membrane area $100\times100\mu\text{m}^2$ in a fabricated silicon wafer. Current silicon fabrication foundries can process 4" diameter (or larger) silicon wafers as standard. From these figures it is straightforward to calculate that, at a very conservative estimate, 6000 (or more) FBARs may easily be fabricated on one wafer (Lakin *et al.*, 2001). If there are significantly more than 6000 FBARs on each wafer, the total force is proportional to $N^{1/2}$ and so the generated HFGW flux (from equation (2)) is proportional to $(N^{1/2})^2 = N$.

FBARs are quite simple to fabricate using standard microelectronics procedures. A very rough estimate of their cost is \$10 per wafer (in bulk quantities). (In fact it would be economically advantageous to use 6'' or 8'' diameter wafers but here 4'' wafers are assumed.) Suitable magnetrons giving 1kW at 2.45GHz are readily available in bulk *off-the-shelf* at about US\$30 each, intended for OEM microwave oven use. To find the best distribution of magnetrons and FBAR wafers, note that the HFGW flux is proportional to the product ($P_{in}N$); this product must be maximized subject to the constraint of minimum cost. This is an optimization problem and the solution is: *equal sums should be spent on the magnetrons and on the FBAR wafers*. Therefore, the optimum arrangement is to have each 1kW magnetron drive three 4'' FBAR wafers, assuming the rough estimates of costs given above. This excitation corresponds to \sim 56mW per FBAR, well within the power-handling capacity of this type of device (typically \sim 2W per FBAR is reported by Ruby *et al.* (1999)).

Suppose that US\$600k, an arbitrarily chosen sum, is available for the total cost of the magnetrons and FBAR wafers. The optimum design at this price consists of 10,000 magnetrons, costing US\$300k, driving a total of 30,000 FBAR wafers, also costing US\$300k. The magnetrons must be spaced equally around a large circle; its radius is arbitrarily chosen here to be 300m, so that the magnetrons are spaced apart by ~19cm. Laser surveying devices would be necessary to align all the generating elements accurately towards the central focus. Thus there would be ~1.8×10⁸ FBARs. From equation (8), using Q = 100, $P_{in} = 10$ MW, and $\omega_0 = 2\pi \times 2.45$ GHz, the total force each cycle is 4×10^8 N. The magnetron frequency, 2.45GHz, corresponds to a generated HFGW frequency 4.9GHz. From equation (2), the generated HFGW power is P = 250pW. The wavelength of a 4.9GHz HFGW is $\lambda = c/v \approx 6.1$ cm, and dividing by the area of the diffraction-limited spot size, λ^2/π , gives an HFGW flux of 0.2μ W m⁻² in free space over a detector aperture placed at the center of the circular generator (above or below the focus point). Using an HTSC lens at the detection aperture to reduce the HFGW wavelength $400 \times (\text{Li}, 1992)$, making the diffraction-limited spot area $(400)^2 \times \text{smaller}$, gives a flux of 34mW m^{-2} .

AVAILABLE DETECTOR SYSTEMS

INFN Genoa Detector

This detector has been developed by Chincarini and Gemme (2003) of INFN, Genoa, Italy, and is based upon coupling superconducting radio frequency (RF) cavities and exploits the parametric energy conversion between two EM resonant modes. The interaction of the gravitational wave with the superconducting cavity walls induces a motion that is sensed by the EM field stored within the cavity. The energy conversion is maximized when the frequency of the wave is equal to the frequency difference of the cavity resonant modes. Several such devices have been produced by Chincarini, Gemme, and others, proving that this approach is feasible for HFGW detection in the GHz range. Detection at 4.9GHz (the frequency expected in the present proposed configuration) requires a cavity resonance in excess of this frequency, implying overall detector dimensions on the order of ~2cm. In order to build a realistic detector, a suitable cavity shape has to be chosen. From quite general arguments a detector based on two coupled spherical cavities looks very promising (Fig. 4). To match the frequency range relevant here, the mode splitting (i.e., the detection frequency) should cover the range $10\text{kHz} < (\omega_2 - \omega_1)/(2\pi) < 3\text{GHz}$. The internal radius of the spherical cavities would be in the range 20-100mm, corresponding to a TE_{0.11} mode at frequency $\omega/(2\pi) \approx 2$ -10GHz. A tuning cell, or a superconducting bellow, will be inserted in the coupling tube between the two cavities, allowing variation of the coupling strength (i.e., the detection frequency) in a narrow range around the design value. The spherical resonators can be easily deformed in order to favor the field polarization suitable for GW detection: the optimal field spatial distribution is with the field axis of the two cavities orthogonal to each other (Fig. 5).

Chongqing University Detector

This detector (Li *et al.*, 2003) considers the EM response of a Gaussian EM beam passing through a static magnetic field to sense HFGWs in the GHz to PHz regimes. Prof. Fang-Yu Li and his associates at Chongqing University in China found that "...under the synchroresonance condition, the first-order perturbative EM power fluxes will contain a 'left circular wave' and a 'right circular wave' around the symmetrical axis of a Gaussian beam. However, the perturbative effects produced by the states of + polarization and × polarization of the GWs have a different physical behavior." Fang-Yu Li (2003) has confirmed the ability of their detector to measure the HFGW radiation expected to be generated by the configuration proposed here.

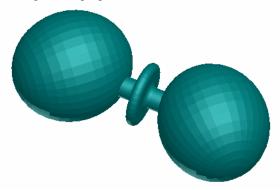


FIGURE 4. Artist's Impression of the Coupled Spherical Cavities with Central Tuning Cell in INFN Genoa Detector. (© A. Chincarini 2003. Courtesy Prof. A. Chincarini.)

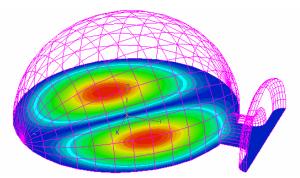


FIGURE 5. Electric Field Magnitude of the TE₀₁₁ Mode in INFN Genoa Detector. Note the Alignment of the Field Axis. (© A. Chincarini 2003. Courtesy Prof. A. Chincarini.)

Feasibility of Detecting Generated HFGWs Using Available Detectors

The Chongqing detector's sensitivity is undergoing improvement, but is predicted by Li *et al.* (2003) to be $1\mu W$ m⁻² at 3GHz. The INFN Genoa detector may be slightly less sensitive. In fact, these detectors may not be quite sensitive enough to detect the flux generated in free space by the generating arrangement described above, but using flux compression by an HTSC lens the flux is well within the detection limit. At very little extra expense, the circle radius could be increased to 3km, giving a HFGW flux $20\mu W$ m⁻² in free space or 3.4W m⁻² using an HTSC focusing lens. With doubled funding the number of magnetrons and FBARs might both be doubled to give $80\mu W$ m⁻² in free space or 14W m⁻² using an HTSC lens. These latter figures would be well within detection capabilities.

CONCLUSIONS

The basic generator proposed here consists of 10,000 phase-locked magnetrons, arranged in a circle of radius 300m, each producing 1kW at 2.45GHz; each supplies three FBAR wafers, each of 6,000 phase-coherent resonant FBARs. This produces an HFGW radiated power of 250pW at 4.9GHz. If focused to a diffraction-limited spot in free space

this corresponds to an HFGW flux of $0.2\mu W$ m⁻². This is not quite within the detection capabilities of current HFGW detectors but increases to well within the sensitivity range if HTSC is used to reduce the HFGW wavelength and therefore reduce the area of the diffraction-limited spot size, and/or if the circle radius is increased to 3km.

NOMENCLATURE

A = cone altitude (m)r = radius of gyration (m) $c = \text{speed of light (m s}^{-1})$ $\Delta t = \text{time increment (s)}$ f = force(N)W = stored energy (J) $F = GW \text{ flux } (W \text{ m}^{-2})$ x = spring extension (m) $k = \text{spring constant (N m}^{-1})$ λ = wavelength (m) Q = resonance quality factor $\Delta v = FBAR$ half-resonance width (Hz) m = mass (kg)v = frequency (Hz)N = number of FBARs v_0 = resonant center frequency (Hz) P = power(W) ω_0 = resonant angular frequency (rad s⁻¹)

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