SUMMARY AND CONCEPT REVIEW

SECTION 5.1 Angle Measure, Special Triangles, and Special Angles

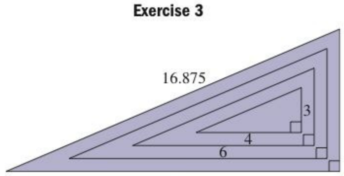
KEY CONCEPTS

- An angle is defined as the joining of two rays at a common endpoint called the vertex.
- An angle in standard position has its vertex at the origin and its initial side on the positive x-axis.
- Two angles in standard position are coterminal if they have the same terminal side.
- A counterclockwise rotation gives a positive angle, a clockwise rotation gives a negative angle.
- One degree ($1°$) is defined to be $\frac{1}{360}$ of a full revolution. One (1) radian is the measure of a central angle subtended by an arc equal in length to the radius.
- Degrees can be divided into a smaller unit called minutes: $1° = 60'$; minutes can be divided into a smaller unit called seconds: $1' = 60''$. This implies $1° = 3600''$.
- Two angles are complementary if they sum to $90°$ and supplementary if they sum to $180°$.
- Properties of triangles: (I) the sum of the angles is $180°$; (II) the combined length of any two sides must exceed that of the third side and; (III) larger angles are opposite larger sides.
- Given two triangles, if all three corresponding angles are equal, the triangles are said to be similar. If two triangles are similar, then corresponding sides are in proportion.
- In a 45-45-90 triangle, the sides are in the proportion $1x$: $1x$: $\sqrt{2}x$.
- In a 30-60-90 triangle, the sides are in the proportion $1x$: $\sqrt{3}x$: $2x$.
- The formula for arc length: $s = r\theta$; area of a circular sector: $A = \frac{1}{2}r^2\theta$, θ in radians.
- To convert degree measure to radians, multiply by $\frac{\pi}{180°}$; for radians to degrees, multiply by $\frac{180°}{\pi}$.
- Special angle conversions: $30° = \frac{\pi}{6}$, $45° = \frac{\pi}{4}$, $60° = \frac{\pi}{3}$, $90° = \frac{\pi}{2}$.
- Angular velocity is a rate of rotation per unit time: $\omega = \frac{\theta}{t}$.
- Linear velocity is a change in position per unit time: $V = \frac{\theta r}{t}$ or $V = r\omega$.

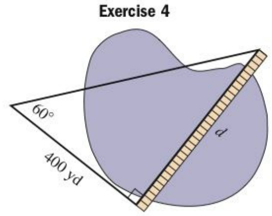
EXERCISES

1. Convert $147°36'48''$ to decimal degrees.
2. Convert $32.87°$ to degrees, minutes, and seconds.
3. All of the right triangles given are similar. Find the dimensions of the largest triangle.

Exercise 3



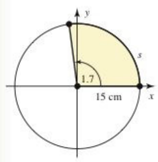
Exercise 4



4. Use special angles/special triangles to find the length of the bridge needed to cross the lake shown in the figure.
5. Convert to degrees: $\frac{2\pi}{3}$.
6. Convert to radians: $210°$.
7. Find the arc length if $r = 5$ and $\theta = 57°$.
8. Evaluate without using a calculator: $\sin\left(\frac{7\pi}{6}\right)$.

Find the angle, radius, arc length, and/or area as needed, until all values are known.

9.



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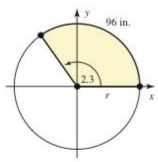

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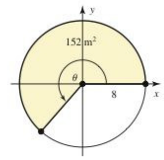


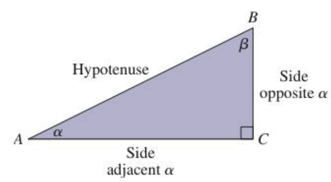
12. With great effort, 5-year-old Mackenzie has just rolled her bowling ball down the lane, and it is traveling painfully slow. So slow, in fact, that you can count the number of revolutions the ball makes using the finger holes as a reference. (a) If the ball is rolling at 1.5 revolutions per second, what is the angular velocity? (b) If the ball's radius is 5 in., what is its linear velocity in feet per second? (c) If the distance to the first pin is 60 feet and the ball is true, how many seconds until it hits?

SECTION 5.2 The Trigonometry of Right Triangles

KEY CONCEPTS

- The sides of a right triangle are named using their location with respect to a given angle.
- The ratios of two sides are named as follows:

$$\sin\alpha = \frac{\text{opp}}{\text{hyp}} \qquad \cos\alpha = \frac{\text{adj}}{\text{hyp}} \qquad \tan\alpha = \frac{\text{opp}}{\text{adj}}$$



- The reciprocal ratios are likewise given special names:

$$\csc\alpha = \frac{\text{hyp}}{\text{opp}} \qquad \sec\alpha = \frac{\text{hyp}}{\text{adj}} \qquad \cot\alpha = \frac{\text{adj}}{\text{opp}}$$

$$\csc\alpha = \frac{1}{\sin\alpha} \qquad \sec\alpha = \frac{1}{\cos\alpha} \qquad \cot\alpha = \frac{1}{\tan\alpha}$$

- Each function of α is equal to the cofunction of its complement. For instance, the complement of sine is *co*sine and $\sin\alpha = \cos(90° - \alpha)$.
- To solve a right triangle means to apply any combination of the trig functions, along with the triangle properties, until all sides and all angles are known.
- An angle of elevation is the angle formed by a horizontal line of sight (parallel to level ground) and the line of sight. An angle of depression is likewise formed, but with the line of sight below the line of orientation.

EXERCISES

13. Use a calculator to solve for A:

a. $\cos 37° = A$

b. $\cos A = 0.4340$

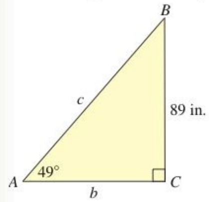
14. Rewrite each expression in terms of a cofunction.

a. $\tan 57.4°$

b. $\sin(19°30'15'')$

Solve each triangle. Round angles to the nearest tenth and sides to the nearest hundredth.

15.



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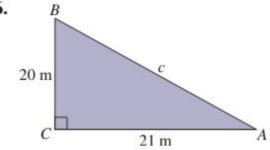
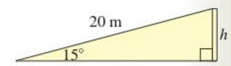
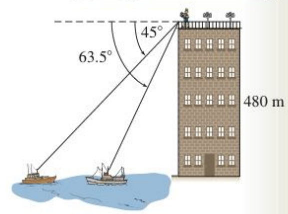


17. Josephine is to weld a vertical support to a 20 m ramp so that the incline is exactly 15°. What is the height h of the support that must be used?



18. From the observation deck of a seaside building 480 m high, Armando sees two fishing boats in the distance. The angle of depression to the nearer boat is 63.5°, while for the boat farther away the angle is 45°. (a) How far out to sea is the nearer boat? (b) How far apart are the two boats?



19. A slice of bread is roughly 14 cm by 10 cm. If the slice is cut diagonally in half, what acute angles are formed?

SECTION 5.3 Trigonometry and the Coordinate Plane

KEY CONCEPTS

- In standard position, the terminal sides of 0°, 90°, 180°, 270°, and 360° angles coincide with one of the axes and are called quadrantal angles.
- Given $P(x, y)$ is any point on the terminal side of an angle θ in standard position, with $r = \sqrt{x^2 + y^2}$ the distance from the origin to this point. The six trigonometric functions of θ are then defined as follows:

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x} \qquad \csc\theta = \frac{r}{y} \qquad \sec\theta = \frac{r}{x} \qquad \cot\theta = \frac{x}{y}$$
$$x \neq 0 \qquad y \neq 0 \qquad x \neq 0 \qquad y \neq 0$$

- A reference angle θ_r is defined to be the acute angle formed by the terminal side of a given angle θ and the x-axis.
- Reference angles can be used to evaluate the trig functions of any nonquadrantal angle, since the values are fixed by the ratio of sides and the signs are dictated by the quadrant of the terminal side.
- If the value of a trig function and the quadrant of the terminal side are known, the related angle θ can be found using a reference arc/angle, or the $\sin^{-1}$, $\cos^{-1}$, or $\tan^{-1}$ features of a calculator.
- If θ is a solution to $\sin\theta = k$, then $\theta + 360°k$ is also a solution for any integer k.

EXERCISES

20. Find two positive angles and two negative angles that are coterminal with $\theta = 207°$.

21. Name the reference angle for the angles given: $\theta = -152°$ $\qquad \theta = 521°$ $\qquad \theta = 210°$

22. Find the value of the six trigonometric functions, given $P(x, y)$ is on the terminal side of angle θ in standard position.

a. $P(-12, 35)$ **b.** $(12, -18)$

23. Find the values of x, y, and r using the information given, and state the quadrant of the terminal side of θ. Then state the values of the six trig functions of θ.

a. $\cos\theta = \frac{4}{5}$; $\sin\theta < 0$ **b.** $\tan\theta = -\frac{12}{5}$; $\cos\theta > 0$

24. Find all angles satisfying the stated relationship. For special angles, express your answer in exact form. For other angles, use a calculator and round to the nearest tenth.

a. $\tan\theta = -1$ **b.** $\cos\theta = \frac{\sqrt{3}}{2}$ **c.** $\tan\theta = 4.0108$ **d.** $\sin\theta = -0.4540$

SECTION 5.4 Unit Circles and the Trigonometry of Real Numbers

KEY CONCEPTS

- A central unit circle is a circle with radius 1 unit having its center at the origin.
- A central circle is symmetric to both axes and the origin. This means that if (a, b) is a point on the circle, then $(-a, b)$, $(-a, -b)$, and $(a, -b)$ are also on the circle and satisfy the equation of the circle.

- On a unit circle, the length of a subtended arc is numerically the same as the subtended angle θ (θ in radians), making the arc a "circular number line" and associating any given rotation with a unique real number.
- For functions of a real number we refer to a reference arc rather than a reference angle.
- For any real number t and a point on the unit circle associated with t, we have:

$$\cos t = x \qquad \sin t = y \qquad \tan t = \frac{y}{x},\ x \neq 0 \qquad \sec t = \frac{1}{x},\ x \neq 0 \qquad \csc t = \frac{1}{y},\ y \neq 0 \qquad \cot t = \frac{x}{y},\ y \neq 0$$

EXERCISES

25. Given $\left(\frac{\sqrt{13}}{7}, y\right)$ is on a unit circle, find y if the point is in QIV, then use the symmetry of the circle to locate three other points.

26. Given $\left(\frac{3}{4}, -\frac{\sqrt{7}}{4}\right)$ is on the unit circle, find the value of all six trig functions of t without the use of a calculator.

27. Without using a calculator, find two values in $[0, 2\pi)$ that make the equation true: $\csc t = \frac{2}{\sqrt{3}}$.

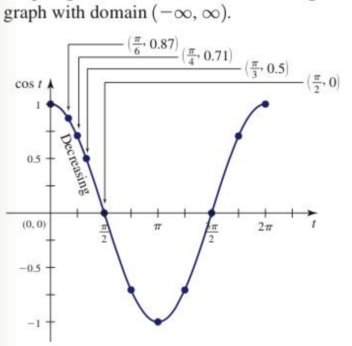
28. Use a calculator to find the value of t that corresponds to the situation described: $\cos t = -0.7641$ with t in QII.

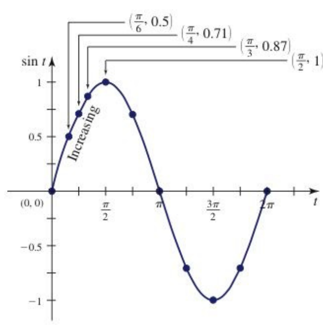
29. A crane used for lifting heavy equipment has a winch-drum with a 1-yd radius. (a) If 59 ft of cable has been wound in while lifting some equipment to the roof-top of a building, what radian angle has the drum turned through? (b) What angle must the drum turn through to wind in 75 ft of cable?

SECTION 5.5 Graphs of the Sine and Cosine Functions; Cosecant and Secant Functions

KEY CONCEPTS

- Graphing sine and cosine functions using the special values from a unit circle results in a periodic, wavelike graph with domain $(-\infty, \infty)$.





- The characteristics of each graph play a vital role in their contextual application, and these are summarized on pages 559 and 562.
- The amplitude of a sine or cosine graph is the maximum displacement from the average value. For $y = A\sin(Bt)$ and $y = A\cos(Bt)$, the amplitude is $|A|$.
- The period of a periodic function is the smallest interval required to complete one cycle. For $y = A\sin(Bt)$ and $y = A\cos(Bt)$, the period is $P = \frac{2\pi}{B}$.
- If $|A| > 1$, the graph is vertically stretched. If $0 < |A| < 1$ the graph is vertically compressed. If $A < 0$ the graph is reflected across the x-axis.

- If $B > 1$, the graph is horizontally compressed (the period is smaller/shorter). If $B < 1$ the graph is horizontally stretched (the period is larger/longer).
- To graph $y = A\sin(Bt)$ or $A\cos(Bt)$, draw a reference rectangle $2A$ units high and $P = \frac{2\pi}{B}$ units wide, centered on the x-axis. Then use the *rule of fourths* to locate the zeroes and max/min values (see page 560), and connect these points with a smooth curve.
- The graph of $y = \sec t = \frac{1}{\cos t}$ will be asymptotic everywhere $\cos t = 0$, increasing where $\cos t$ is decreasing, and decreasing where $\cos t$ is increasing.
- The graph of $y = \csc t = \frac{1}{\sin t}$ will be asymptotic everywhere $\sin t = 0$, increasing where $\sin t$ is decreasing, and decreasing where $\sin t$ is increasing.

EXERCISES

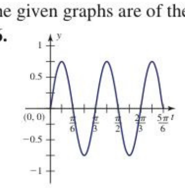
Use a reference rectangle and the *rule of fourths* to draw an accurate sketch of the following functions through at least one full period. Clearly state the amplitude (as applicable) and period as you begin.

30. $y = 3\sin t$ **31.** $y = 3\sec t$ **32.** $y = -\cos(2t)$

33. $y = 1.7\sin(4t)$ **34.** $f(t) = 2\cos(4\pi t)$ **35.** $g(t) = 3\sin(398\pi t)$

The given graphs are of the form $y = A\sin(Bt)$ and $y = A\csc(Bt)$. Determine the equation of each graph.

36.



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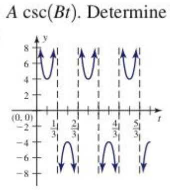
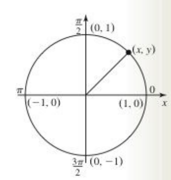


38. Referring to the chart of colors visible in the electromagnetic spectrum (page 572), what color is represented by the equation $y = \sin\left(\frac{\pi}{270}t\right)$? By $y = \sin\left(\frac{\pi}{320}t\right)$?

SECTION 5.6 Graphs of Tangent and Cotangent Functions

KEY CONCEPTS



- Since $\tan t$ is defined in terms of the ratio $\frac{y}{x}$, the graph will be asymptotic everywhere $x = 0$ on the unit circle, meaning all odd multiples of $\frac{\pi}{2}$.
- Since $\cot t$ is defined in terms of the ratio $\frac{x}{y}$, the graph will be asymptotic everywhere $y = 0$ on the unit circle, meaning all integer multiples of π.
- The graph of $y = \tan t$ is increasing everywhere it is defined; the graph of $y = \cot t$ is decreasing everywhere it is defined.
- The characteristics of each graph play a vital role in their contextual application, and these are summarized on page 577.
- To graph $y = A\tan(Bt)$, note $A\tan(Bt)$ is zero at $t = 0$. Compute $P = \frac{\pi}{B}$ and draw asymptotes a distance of $\frac{P}{2}$ on either side of the y-axis. Plot zeroes halfway between asymptotes and use symmetry to complete the graph.
- To graph $y = A\cot(Bt)$, note it is asymptotic at $t = 0$. Compute $P = \frac{\pi}{B}$ and draw asymptotes a distance P on either side of the y-axis. Plot zeroes halfway between the asymptotes and use symmetry to complete the graph.

- If $|A| > 1$, the graph is vertically stretched.
  If $0 < |A| < 1$ the graph is vertically compressed.
  If $A < 0$ the graph is reflected across the x-axis.
- If $B > 1$, the graph is horizontally compressed (the period is smaller/shorter).
  If $B < 1$ the graph is horizontally stretched (the period is larger/longer).

EXERCISES

39. State the value of each expression without the aid of a calculator:

$\tan\left(\frac{7\pi}{4}\right)$ $\cot\left(\frac{\pi}{3}\right)$

40. State the value of each expression without the aid of a calculator, given that t terminates in QII.

$\tan^{-1}(-\sqrt{3})$ $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

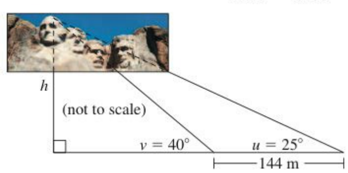
41. Graph $y = 6\tan\left(\frac{1}{2}t\right)$ in the interval $[-2\pi, 2\pi]$.

42. Graph $y = \frac{1}{2}\cot(2\pi t)$ in the interval $[-1, 1]$.

43. Use the period of $y = \cot t$ to name three additional solutions to $\cot t = 0.0208$, given $t = 1.55$ is a solution. Many solutions are possible.

44. Given $t = 0.4444$ is a solution to $\cot^{-1}(t) = 2.1$, use an analysis of signs and quadrants to name an additional solution in $[0, 2\pi)$.

45. Find the approximate height of Mount Rushmore, using $h = \frac{d}{\cot u - \cot v}$ and the values shown.



46. Model the data in the table using a tangent function. Clearly state the period, the value of A, and the location of the asymptotes.

Input	Output	Input	Output
−6	$-\infty$	1	1.4
−5	−19.4	2	3
−4	−9	3	5.2
−3	−5.2	4	9
−2	−3	5	19.4
−1	−1.4	6	∞
0	0		

SECTION 5.7 Transformations and Applications of Trigonometric Graphs

KEY CONCEPTS

- Many everyday phenomena follow a sinusoidal pattern, or a pattern that can be modeled by a sine or cosine function (e.g., daily temperatures, hours of daylight, others).
- To obtain accurate equation models of sinusoidal phenomena, vertical and horizontal shifts of a basic function are used.

- The equation $y = A\sin(Bt \pm C) + D$ is the *standard form* of a general sinusoid. The equation $y = A\sin\left[B\left(t \pm \frac{C}{B}\right)\right] + D$ is the *shifted form* of a general sinusoid. $\frac{C}{B}$ gives the horizontal shift.
- In either form, D represents the average value of the function and a vertical shift D units upward if $D > 0$, D units downward if $D < 0$. For a maximum value M and minimum value m, $\frac{M+m}{2} = D$, $\frac{M-m}{2} = A$.
- To graph a shifted sinusoid, locate the primary interval by solving $0 \le Bt + C < 2\pi$, then use a reference rectangle along with the *rule of fourths* to sketch the graph in this interval. The graph can then be extended as needed, then shifted vertically D units.
- One application of sinusoidal graphs involves phenomena in harmonic motion, or motion that can be modeled by functions of the form $y = A\sin(Bt)$ or $y = A\cos(Bt)$ (with no horizontal or vertical shift).
- If the period P and critical points (X, M) and (x, m) of a sinusoidal function are known, a model of the form $y = A\sin(Bx + C) + D$ can be obtained using:

$$B = \frac{2\pi}{P} \qquad A = \frac{M-m}{2} \qquad D = \frac{M+m}{2} \qquad C = \frac{3\pi}{2} - Bx$$

EXERCISES

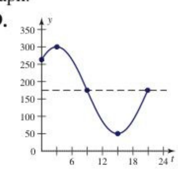
For each equation given, (a) identify/clearly state the amplitude, period, horizontal shift, and vertical shift; then (b) graph the equation using the primary interval, a reference rectangle, and *rule of fourths*.

47. $y = 240\sin\left[\frac{\pi}{6}(t-3)\right] + 520$

48. $y = 3.2\cos\left(\frac{\pi}{4}t + \frac{3\pi}{2}\right) + 6.4$

For each graph given, identify the amplitude, period, horizontal shift, and vertical shift, and give the equation of the graph.

49.



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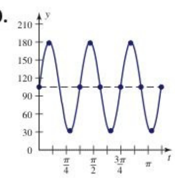


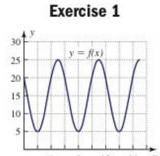
51. Monthly precipitation in Cheyenne, Wyoming, can be modeled by a sine function, by using the average precipitation for July (2.26 in.) as a maximum (actually slightly higher in May), and the average precipitation for February (0.44 in.) as a minimum. Assume $t = 0$ corresponds to March. (a) Use the information to construct a sinusoidal model, and (b) use the model to estimate the inches of precipitation Cheyenne receives in August ($t = 5$) and December ($t = 9$).

Source: 2004 Statistical Abstract of the United States, Table 380.

MIXED REVIEW

1. For the graph of periodic function f given, state the (a) amplitude, (b) average value, (c) period, and (d) value of $f(4)$.

Exercise 1



2. Name two values in $[0, 2\pi)$ where $\tan t = 1$.

3. Name two values in $[0, 2\pi)$ where $\cos t = -\frac{1}{2}$.

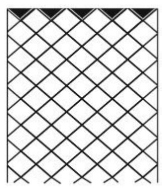
4. Given $\sin \theta = \frac{8}{\sqrt{185}}$ with θ in QII, state the value of the other five trig functions.

5. Convert to DMS form: $220.81\overline{38}°$.

6. Find two negative angles and two positive angles that are coterminal with (a) 57° and (b) 135°.

7. To finish the top row of the tile pattern on our bathroom wall, 12″ by 12″ tiles must be cut diagonally. Use a standard triangle to find the length of each cut and the width of the wall covered by tiles.

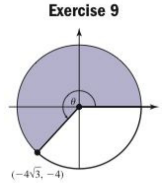
Exercise 7



8. The service door into the foyer of a large office building is 36″ wide by 78″ tall. The building manager has ordered a large wall painting 85″ by 85″ to add some atmosphere to the foyer area. (a) Can the painting be brought in the service door? (b) If so, at what two integer-valued angles (with respect to level ground) could the painting be tilted?

9. Find the arc length and area of the shaded sector.

Exercise 9



10. Monthly precipitation in Minneapolis, Minnesota, can be modeled by a sine function, by using the average precipitation for August (4.05 in.) as a maximum (actually slightly higher in June), and the average precipitation for February (0.79 in.) as a minimum. Assume $t = 0$ corresponds to April. (a) Use the information to construct a sinusoidal model, and (b) Use the model to approximate the inches of precipitation Minneapolis receives in July ($t = 3$) and December ($t = 8$).

Source: 2004 Statistical Abstract of the United States, Table 380

11. Convert from DMS to decimal degrees: 86° 54′ 54″.

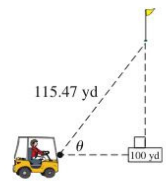
12. Name the reference angle θ_r for the angle θ given.

a. 735° **b.** −135° **c.** $\frac{5\pi}{6}$ **d.** $-\frac{5\pi}{3}$

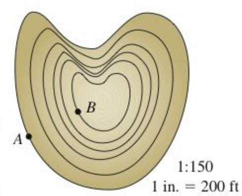
13. Find the value of all six trig functions of θ, given the point $(15, -8)$ is on the terminal side.

14. Verify that $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ is a point on the unit circle and find the value of all six trig functions at this point.

15. On your approach shot to the ninth green, the Global Positioning System (GPS) your cart is equipped with tells you the pin is 115.47 yd away. The distance plate states the straight line distance to the hole is 100 yd (see the diagram). Relative to a straight line between the plate and the hole, at what acute angle θ should you hit the shot?



16. The electricity supply lines to the top of Lone Eagle Plateau must be replaced, and the new lines will be run in conduit buried slightly beneath the surface. The scale of elevation is 1:150 (each closed figure indicates an increase in 150 ft of elevation), and the scale of horizontal distance is 1 in. = 200 ft. (a) Find the increase in elevation from point A to point B, (b) use a proportion to find the horizontal distance from A to B if the measured distance on the map is $2\frac{1}{4}$ in., (c) draw the corresponding right triangle and use it to estimate the length of conduit needed from A to B and the angle of incline the installers will experience while installing the conduit.



17. State the amplitude, period, horizontal shift, vertical shift, and endpoints of the primary interval (as applicable), then sketch the graph using a reference rectangle and the rule of fourths.

a. $y = 5\cos(2t) - 8$ **b.** $y = \frac{7}{2}\sin\left[\frac{\pi}{2}(x - 1)\right]$

c. $y = 2\tan\left(\frac{1}{4}t\right)$ **d.** $y = 3\sec\left(x - \frac{\pi}{2}\right)$

18. Solve each equation in $[0, 2\pi)$ without the use of a calculator. If the expression is undefined, so state.

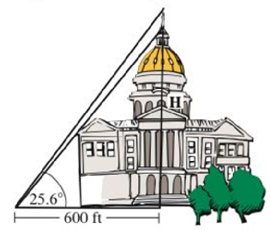
a. $x = \sin\left(-\frac{\pi}{4}\right)$ **b.** $\sec x = \sqrt{2}$

c. $\cot\left(\frac{\pi}{2}\right) = x$ **d.** $\cos \pi = x$

e. $\csc x = \frac{2\sqrt{3}}{3}$ **f.** $\tan\left(\frac{\pi}{2}\right) = x$

19. A salad spinner consists of a colander basket inside a large bowl, and is used to wash and dry lettuce and other salad ingredients. The spinner is turned at about 3 revolutions per second. (a) Find the angular velocity and (b) find the linear velocity of a point on the circumference if the basket has a 20 cm radius.

20. Virtually everyone is familiar with the Statue of Liberty in New York Bay, but fewer know that America is home to a second "Statue of Liberty" standing proudly atop the iron dome of the Capitol Building. From a distance of 600 ft, the angle of elevation from ground level to the top of the statue (from the east side) is 25.60°. The angle of elevation to the base of the statue is 24.07°. How tall is the statue *Freedom* (the name sculptor Thomas Crawford gave this statue)?



PRACTICE TEST

1. State the complement and supplement of 35°.

2. Name the reference angle of each angle given.

a. 225° **b.** −510°

c. $\frac{7\pi}{6}$ **d.** $\frac{25\pi}{3}$

3. Find two negative angles and two positive angles that are coterminal with $\theta = 30°$. Many solutions are possible.

4. Convert from DMS to decimal degrees or decimal degrees to DMS as indicated.

a. 100°45′18″ to decimal degrees

b. 48.2125° to DMS

5. Four Corners USA is the point at which Utah, Colorado, Arizona, and New Mexico meet. The southern border of Colorado, the western border of Kansas, and the point P where Colorado, Nebraska, and Kansas meet, very nearly approximates a 30-60-90 triangle. If the western border of Kansas is 215 mi long, (a) what is the distance from Four Corners USA to point P? (b) How long is Colorado's southern border?

Exercise 5

6. Complete the table from memory using exact values. If a function is undefined, so state.

t	$\sin t$	$\cos t$	$\tan t$	$\csc t$	$\sec t$	$\cot t$
0						
$\frac{2\pi}{3}$						
$\frac{7\pi}{6}$						
$\frac{5\pi}{4}$						
$\frac{5\pi}{3}$						
$\frac{13\pi}{6}$						

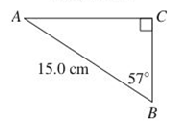
7. Given $\cos\theta = \frac{2}{5}$ and $\tan\theta < 0$, find the value of the other five trig functions of θ.

8. Verify that $\left(\frac{1}{3}, -\frac{2\sqrt{2}}{3}\right)$ is a point on the unit circle, then find the value of all six trig functions associated with this point.

9. In order to take pictures of a dance troupe as it performs, a camera crew rides in a cart on tracks that trace a circular arc. The radius of the arc is 75 ft, and from end to end the cart sweeps out an angle of 172.5° in 20 seconds. Use this information to find (a) the length of the track in feet and inches, (b) the angular velocity of the cart, and (c) the linear velocity of the cart in both ft/sec and mph.

10. Solve the triangle shown. Answer in table form.

Exercise 10

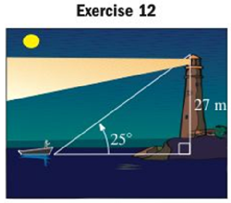


11. The "plow" is a yoga position in which a person lying on their back brings their feet up, over, and behind their head and touches them to the floor. If distance from hip to shoulder (at the right angle) is 57 cm and from hip to toes is 88 cm, find the distance from shoulders to toes and the angle formed at the hips.

Exercise 11

12. While doing some night fishing, you round a peninsula and a tall light house comes into view. Taking a sighting, you find the angle of elevation to the top of the lighthouse is 25°. If the lighthouse is known to be 27 m tall, how far from the lighthouse are you?

Exercise 12



13. Find the value of $t \in [0, 2\pi]$ satisfying the conditions given.

a. $\sin t = -\frac{1}{2}$, t in QIII

b. $\sec t = \frac{2\sqrt{3}}{3}$, t in QIV

c. $\tan t = -1$, t in QII

14. In arid communities, daily water usage can often be approximated using a sinusoidal model. Suppose water consumption in the city of Caliente del Sol reaches a maximum of 525,000 gallons in the heat of the day, with a minimum usage of 157,000 gallons in the cool of the night. Assume $t = 0$ corresponds to 6:00 A.M. (a) Use the information to construct a sinusoidal model, and (b) Use the model to approximate water usage at 4:00 P.M. and 4:00 A.M.

15. State the domain, range, period, and amplitude (if it exists), then graph the function over 1 period.

a. $y = 2\sin\left(\frac{\pi}{5}t\right)$ **b.** $y = \sec t$

c. $y = 2\tan(3t)$

16. State the amplitude, period, horizontal shift, vertical shift, and endpoints of the primary interval. Then sketch the graph using a reference rectangle and the rule of fourths:

$$y = 12\sin\left(3t - \frac{\pi}{4}\right) + 19.$$

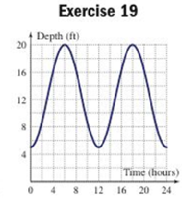
17. An athlete throwing the shot-put begins his first attempt facing due east, completes three and one-half turns and launches the shot facing due west. What angle did his body turn through?

18. State the domain, range, and period, then sketch the graph in $[0, 2\pi)$.

a. $y = \tan(2t)$ **b.** $y = \cot\left(\frac{1}{2}t\right)$

19. Due to tidal motions, the depth of water in Brentwood Bay varies sinusoidally as shown in the diagram, where time is in hours and depth is in feet. Find an equation that models the depth of water at time t.

Exercise 19



20. Find the value of t satisfying the given conditions.

a. $\sin t = -0.7568$; t in QIII

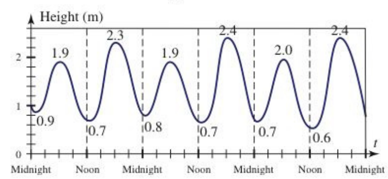
b. $\sec t = -1.5$; t in QII

CALCULATOR EXPLORATION AND DISCOVERY

Variable Amplitudes and Modeling the Tides

Tidal motion is often too complex to be modeled by a single sine function. In this *Exploration and Discovery*, we'll look at a method that combines two sine functions to help model a tidal motion with variable amplitude. In the process, we'll use much of what we know about the amplitude, horizontal shifts and vertical shifts of a sine function, helping to reinforce these important concepts and broaden our understanding about how they can be applied. The graph in Figure 5.93 shows three days of tidal motion for Davis Inlet, Canada.

Figure 5.93



As you can see, the amplitude of the graph varies, and there is no *single* sine function that can serve as a model. However, notice that the amplitude *varies predictably*, and that the high tides and low tides can independently be modeled by a sine function. To simplify our exploration, we will use the assumption that tides have an exact 24-hr period (close, but no), that variations between high and low tides takes place every 12 hr (again close but not exactly true), and the variation between the "low-high" (1.9 m) and the "high-high" (2.4 m) is uniform. A similar assumption is made for the low tides. The result is the graph in Figure 5.94.

Figure 5.94

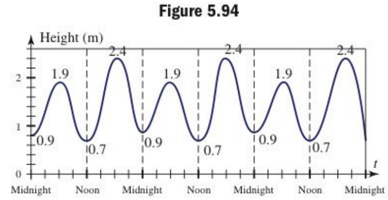


Figure 5.95

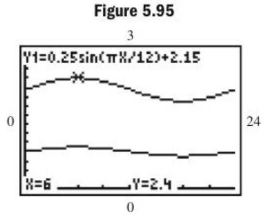
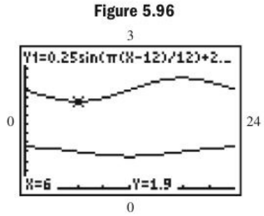
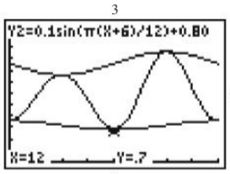


Figure 5.96



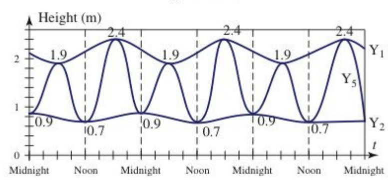
First consider the high tides, which vary from a maximum of 2.4 to a minimum of 1.9. Using the ideas from Section 5.7 to construct an equation model gives $A = \frac{2.4 - 1.9}{2} = 0.25$ and $D = \frac{2.4 + 1.9}{2} = 2.15$. With a period of $P = 24$ hr we obtain the equation $Y_1 = 0.25 \sin\left(\frac{\pi}{12}x\right) + 2.15$. Using 0.9 and 0.7 as the maximum and minimum low tides, similar calculations yield the equation $Y_2 = 0.1 \sin\left(\frac{\pi}{12}x\right) + 0.8$ (verify this). Graphing these two functions over a 24-hr period yields the graph in Figure 5.95, where we note the high and low values are correct, but the two functions are in phase with each other. As can be determined from Figure 5.94, we want the high tide model to start at the average value and decrease, and the low tide equation model to start at high-low and decrease. Replacing x with $x - 12$ in Y_1 and x with $x + 6$ in Y_2 accomplishes this result (see Figure 5.96). Now comes the fun part! Since Y_1

Figure 5.97



represents the low/high maximum values for high tide, and Y_2 represents the low/high minimum values for low tide, *the amplitude and average value for the tidal motion at Davis Inlet are* $A = \frac{Y_1 - Y_2}{2}$ and $D = \frac{Y_1 + Y_2}{2}$! By entering $Y_3 = \frac{Y_1 - Y_2}{2}$ and $Y_4 = \frac{Y_1 + Y_2}{2}$, the equation for the tidal motion (with its variable amplitude) will have the form $Y_5 = Y_3 \sin(Bx \pm C) + Y_4$, where the value of B and C must be determined. The key here is to note there is only a 12-hr difference between the changes in amplitude, so $P = 12$ (instead of 24) and $B = \frac{\pi}{6}$ for this function. Also, from the graph (Figure 5.94) we note the tidal motion begins at a minimum and increases, indicating a shift of 3 units to the right is required. Replacing x with $x - 3$ gives the equation modeling these tides, and the final equation is $Y_5 = Y_3 \sin\left[\frac{\pi}{6}(x - 3)\right] + Y_4$. Figure 5.97 gives a screen shot of Y_1, Y_2, and Y_5 in the interval [0, 24]. The tidal graph from Figure 5.94 is shown in Figure 5.98 with Y_3 and Y_4 superimposed on it.

Figure 5.98



Exercise 1: The website www.tides.com/tcpred.htm offers both *t*ide and *c*urrent *predictions* for various locations around the world, in both numeric and graphical form. In addition, data for the "two" high tides and "two" low tides are clearly highlighted. Select a coastal area where tidal motion is similar to that of Davis Inlet, and repeat this exercise. Compare your model to the actual data given on the website. How good was the fit?

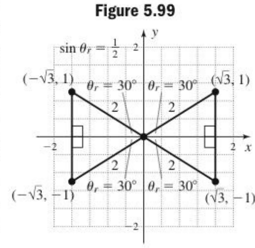
STRENGTHENING CORE SKILLS

Standard Angles, Reference Angles, and the Trig Functions

A review of the main ideas discussed in this chapter indicates there are four of what might be called "core skills." These are skills that (a) play a fundamental part in the acquisition of concepts, (b) hold the overall structure together as we move from concept to concept, and (c) are ones we return to again and again throughout our study. The first of these is *(1) knowing the standard angles and standard values.* These values are "standard" because no estimation, interpolation, or special methods are required to name their value, and each can be expressed as a single factor. This gives them a great advantage in that further conceptual development can take place without the main points being obscured by large expressions or decimal approximations. Knowing the value of the trig functions for each standard angle will serve you very well throughout this study. *Know* the chart on page 550 and the ideas that led to it.

The standard angles/values brought us to the trigonometry of any angle, forming a strong bridge to the second core skill: *(2) using reference angles to determine the value of the trig functions in each quadrant.* For review, a 30-60-90 triangle will always have sides that are in the proportion $1x: \sqrt{3}x: 2x$, regardless of its size.

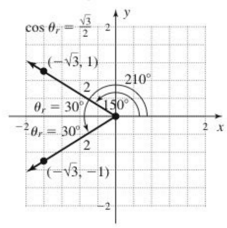
Figure 5.99



This means for any angle θ, where $\theta_r = 30°$, $\sin\theta = \frac{1}{2}$ or $\sin\theta = -\frac{1}{2}$ since the *ratio is fixed* but the sign *depends on the quadrant of* θ: $\sin 30° = \frac{1}{2}$ [QI], $\sin 150° = \frac{1}{2}$ [QII], $\sin 210° = -\frac{1}{2}$ [QIII], $\sin 330° = -\frac{1}{2}$ [QIV], and so on (see Figure 5.99).

In turn, the reference angles led us to a third core skill, helping us realize that if θ was not a quadrantal angle, *(3) equations like* $\cos(\theta) = -\frac{\sqrt{3}}{2}$ *must have two solutions in* $[0, 360°)$. From the standard angles and standard values we learn to recognize that for $\cos\theta = -\frac{\sqrt{3}}{2}$, $\theta_r = 30°$, which will occur as a reference angle in the two quadrants where cosine is negative, QII and QIII. The solutions in $[0, 360°)$ are $\theta = 150°$ and $\theta = 210°$ (see Figure 5.100).

Figure 5.100



Of necessity, this brings us to the fourth core skill, *(4) effective use of a calculator.* The standard angles are a wonderful vehicle for introducing the basic ideas of trigonometry, and actually occur quite frequently in real-world applications. But by far, most of the values we encounter will be nonstandard values where θ_r must be found using a calculator. However, once θ_r is found, the reason and reckoning inherent in these ideas can be directly applied.

The *Summary and Concept Review Exercises*, as well as the *Practice Test* offer ample opportunities to refine these skills, so that they will serve you well in future chapters as we continue our attempts to explain and understand the world around us in mathematical terms.

Exercise 1: Fill in the table from memory.

t	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin t = y$					
$\cos t = x$					
$\tan t = \frac{y}{x}$					
$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$

Exercise 2: Solve each equation in $[0, 2\pi)$ without the use of a calculator.

a. $2\sin t + \sqrt{3} = 0$

b. $-3\sqrt{2}\cos t + 4 = 1$

c. $-\sqrt{3}\tan t + 2 = 1$

d. $\sqrt{2}\sec t + 1 = 3$

Exercise 3: Solve each equation in $[0, 2\pi)$ using a calculator and rounding answers to four decimal places.

a. $\sqrt{6}\sin t - 2 = 1$

b. $-3\sqrt{2}\cos t + \sqrt{2} = 0$

c. $3\tan t + \frac{1}{2} = -\frac{1}{4}$

d. $2\sec t = -5$

CUMULATIVE REVIEW CHAPTERS 1–5

1. Solve the inequality given: $2|x + 1| - 3 < 5$

2. Find the domain of the function: $y = \sqrt{x^2 - 2x - 15}$

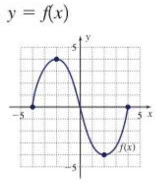
3. Given that $\tan\theta = \frac{80}{39}$, draw a right triangle that corresponds to this ratio, then use the Pythagorean theorem to find the length of the missing side. Finally, find the two acute angles.

4. Without a calculator, what values in $[0, 2\pi)$ make the equation true: $\sin t = -\frac{\sqrt{3}}{2}$?

5. Given $\left(\frac{3}{4}, -\frac{\sqrt{7}}{4}\right)$ is a point on the unit circle corresponding to t, find all six trig functions of t.

State the domain and range of each function shown:

6. $y = f(x)$



7. **a.** $f(x) = \sqrt{2x - 3}$

b. $g(x) = \frac{2x}{x^2 - 49}$

8. $y = T(x)$

x	$T(x)$
0	−7
1	−5
2	−3
3	−1
4	1
5	3
6	5

9. Analyze the graph of the function in Exercise 6, including: (a) maximum and minimum values; (b) intervals where $f(x) \geq 0$ and $f(x) < 0$; (c) intervals where f is increasing or decreasing; and (d) any symmetry noted. Assume the features you are to describe have integer values.

10. The attractive force that exists between two magnets varies inversely as the square of the distance between them. If the attractive force is 1.5 newtons (N) at a distance of 10 cm, how close are the magnets when the attractive force reaches 5 N?

11. The world's tallest indoor waterfall is in Detroit, Michigan, in the lobby of the International Center Building. Standing 66 ft from the base of the falls, the angle of elevation is 60°. How tall is the waterfall?

12. It's a warm, lazy Saturday and Hank is watching a county maintenance crew mow the park across the street. He notices the mower takes 29 sec to pass through 77° of rotation from one end of the park to the other. If the corner of the park is 60 ft directly across the street from his house, (a) how wide is the park? (b) How fast (in mph) does the mower travel as it cuts the grass?

13. Graph using transformations of a parent function: $f(x) = \dfrac{1}{x + 1} - 2.$

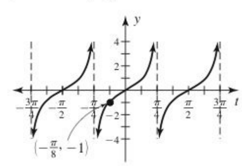
14. Graph using transformations of a parent function: $g(x) = e^{x-1} - 2.$

15. Find $f(\theta)$ for all six trig functions, given the point $P(-9, 40)$ is a point on the terminal side of the angle. Then find the angle θ in degrees, rounded to tenths.

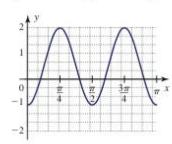
16. Given $t = 5.37$, (a) in what quadrant does the arc terminate? (b) What is the reference arc? (c) Find the value of $\sin t$ rounded to four decimal places.

17. A jet-stream water sprinkler shoots water a distance of 15 m and turns back-and-forth through an angle of $t = 1.2$ rad. (a) What is the length of the arc that the sprinkler reaches? (b) What is the area in m^2 of the yard that is watered?

18. Determine the equation of graph shown given it is of the form $y = A\tan(Bt)$.

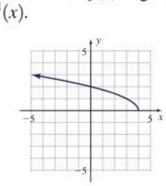


19. Determine the equation of the graph shown given it is of the form $y = A\sin(Bt \pm C) + D$.

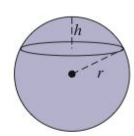


20. In London, the average temperatures on a summer day range from a high of 72°F to a low of 56°F (Source: 2004 Statistical Abstract of the United States, Table 1331). Use this information to write a sinusoidal equation model, assuming the low temperature occurs at 6:00 A.M. Clearly state the amplitude, average value, period, and horizontal shift.

21. The graph of a function $f(x)$ is given. Sketch the graph of $f^{-1}(x)$.



22. The volume of a spherical cap is given by $V = \dfrac{\pi h^2}{3}(3r - h)$. Solve for r in terms of V and h.



23. Find the slope and y-intercept: $3x - 4y = 8$.

24. Solve by factoring: $4x^3 - 8x^2 - 9x + 18 = 0$.

25. At what interest rate will \$1000 grow to \$2275 in 12 yr if compounded continuously?