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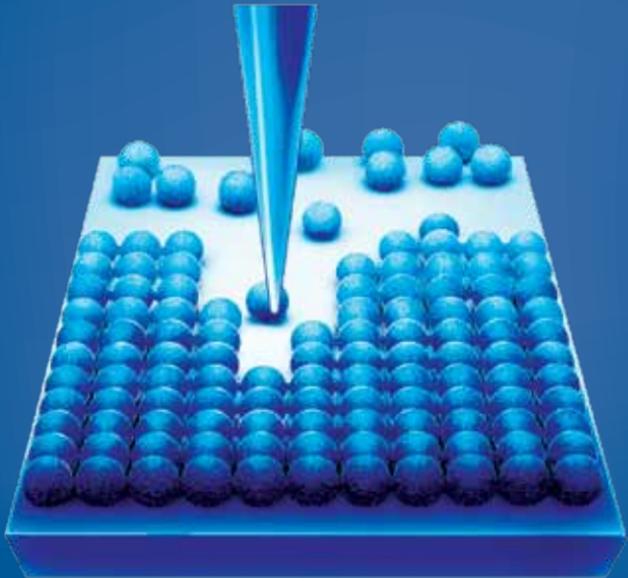
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Quantum Mechanics: The Physics of the Microscopic World

Course Guidebook

Professor Benjamin Schumacher
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Professor Schumacher is the author of numerous scientific papers and two books, including *Physics in Spacetime: An Introduction to Special Relativity* (Rinton Press, 2005). As one of the founders of quantum information theory, Professor Schumacher introduced the term “qubit,” invented quantum data compression (also known as “Schumacher compression”), and established several fundamental results about the information capacity of quantum systems. For his contributions, he won the 2002 Quantum Communication Award, the premier international prize in the field, and was named a Fellow of the American Physical Society. Besides quantum information theory, Professor Schumacher has done physics research on black holes, thermodynamics, and statistical mechanics.

Professor Schumacher has spent sabbaticals working at Los Alamos National Laboratory and as a Moore Distinguished Scholar at the Institute for Quantum Information at California Institute of Technology. He has also done research at the Isaac Newton Institute of Cambridge University, the Santa Fe Institute, the Perimeter Institute, the University of New Mexico, the University of Montreal, the University of Innsbruck, and the University of Queensland. At Kenyon College, Professor Schumacher mostly teaches physics, but he also regularly ventures into astronomy, mathematics, scientific computing, and the humanities. ■

Table of Contents

INTRODUCTION

Professor Biography	i
Course Scope	1

LECTURE GUIDES

LECTURE 1

The Quantum Enigma	4
--------------------------	---

LECTURE 2

The View from 1900	7
--------------------------	---

LECTURE 3

Two Revolutionaries—Planck and Einstein	10
---	----

LECTURE 4

Particles of Light, Waves of Matter	13
---	----

LECTURE 5

Standing Waves and Stable Atoms	16
---------------------------------------	----

LECTURE 6

Uncertainty	19
-------------------	----

LECTURE 7

Complementarity and the Great Debate	22
--	----

LECTURE 8

Paradoxes of Interference	26
---------------------------------	----

LECTURE 9

States, Amplitudes, and Probabilities	29
---	----

LECTURE 10

Particles That Spin	33
---------------------------	----

Table of Contents

LECTURE 11	
Quantum Twins.....	37
LECTURE 12	
The Gregarious Particles.....	40
LECTURE 13	
Antisymmetric and Antisocial.....	43
LECTURE 14	
The Most Important Minus Sign in the World	45
LECTURE 15	
Entanglement	49
LECTURE 16	
Bell and Beyond	53
LECTURE 17	
All the Myriad Ways.....	56
LECTURE 18	
Much Ado about Nothing	59
LECTURE 19	
Quantum Cloning.....	62
LECTURE 20	
Quantum Cryptography	66
LECTURE 21	
Bits, Qubits, and Ebits	68
LECTURE 22	
Quantum Computers	71
LECTURE 23	
Many Worlds or One?.....	74

Table of Contents

LECTURE 24

The Great Smoky Dragon.....	78
-----------------------------	----

SUPPLEMENTAL MATERIAL

Timeline	81
Glossary	87
Biographical Notes	101
Bibliography.....	117

Quantum Mechanics: The Physics of the Microscopic World

Scope:

Quantum mechanics is the fundamental physics of the microscopic world, the domain of atoms and photons and elementary particles. The theory was developed in the early 20th century by Planck, Einstein, Bohr, Heisenberg, and others. Though physics has advanced quite far in the decades since quantum mechanics was born, it remains the basic framework for our deepest insights into nature.

Yet, although it is a cornerstone of modern physics, quantum mechanics remains a profoundly strange picture of reality. The quantum world confronts us with mind-boggling questions. How can light be both wave and particle? Are all electrons truly identical, and what difference can that possibly make? What does it mean when 2 quantum particles are “entangled”—a relationship so weird that Einstein called it “spooky”? Is there really a vast amount of energy in empty space? Can the laws of quantum physics someday make our computers faster and our messages more private?

This course is an introduction to the fundamentals of quantum mechanics, accessible to students without any previous preparation in math and physics. In 24 lectures, using a small toolkit of simple concepts and examples, we will trace the origins of the theory of quantum mechanics, describe its basic principles, and explore some of the most remarkable features of the quantum world.

After surveying the way ahead, the course begins by describing the theories of physics that prevailed before the quantum revolution. We will see how Max Planck and Albert Einstein introduced quantum ideas to explain certain mysterious properties of light. These ideas soon spread to all of physics, affecting our understanding of all types of matter and energy. A central idea is the notion of “wave-particle duality,” in which the entities of nature (electrons and so on) can exhibit the characteristics of both waves and particles. This will lead us to Werner Heisenberg’s famous uncertainty

relation. The new physics posed many puzzles for its founders. This was nowhere better exemplified than in the great debate between Einstein and Niels Bohr over the validity and meaning of quantum mechanics, which we will also explore.

Next we will turn to the task of presenting quantum theory in its clearest and simplest form. We will do this through a careful analysis—a thought experiment involving a single photon traveling through an apparatus called an “interferometer.” As we will discover, even so simple a system has a few surprises in store.

The description of particle spin provides us with another useful illustration of quantum principles. Armed with these principles, we will discuss in detail the concept of identical quantum particles. A simple distinction—the difference, in effect, between $+1$ and -1 in our quantum rules—leads to particles with very different properties. We will see how this distinction plays a role in phenomena including lasers, superfluids, the structure of atoms, and the properties of solids.

This will lead us to our next topic, the riddle of quantum entanglement. As we will show, the behavior of entangled particles challenges some of our most deeply held intuitions about the physical world. Almost as bizarre is Richard Feynman’s startling idea that a quantum particle moves from point A to point B by following *every possible path* from A to B, each path making its own contribution. By applying Feynman’s principle and the uncertainty principle to “empty space,” we find that even the vacuum is a realm of ceaseless quantum activity.

Quantum information theory is a relatively new branch of quantum physics. In our lectures, we will describe some of its remarkable concepts. Unlike ordinary information, quantum information cannot be perfectly copied. It can, on the other hand, be used to send perfectly secret messages and to perform “quantum teleportation.” It may even be possible to use quantum physics to construct a quantum computer, a novel and extremely powerful kind of machine for solving mathematical problems.

Our course concludes with a discussion of some philosophical questions. What is the real meaning of quantum mechanics? What does it tell us about the nature of our world? Do our choices and observations help to bring reality into being? Does the randomness of the quantum realm disguise a deeper, universe-spanning order? Or are the myriad possibilities of quantum physics all part of a complex “multiverse” beyond our imagining? What deep principle links together the many mysteries of the quantum world?

A note about mathematics: Quantum mechanics is often framed in highly abstract mathematical terms. (Open up any advanced textbook on the subject and see for yourself!) Yet the central ideas of quantum mechanics are not at all complicated and can be understood by almost anyone. With a few careful simplifications and a very little math, it is possible to embark on a serious exploration of the quantum world. That journey is ours. ■

The Quantum Enigma

Lecture 1

In this course, we are embarking on a journey to a distant world, a world that is governed by strange and unfamiliar laws. By distant, I mean a world far from our everyday experience. I mean a world that is distant not in space, but in size. It's the world of the microscopic, the microscopic world.

What is quantum mechanics? “Mechanics” is the branch of physics that studies force and motion—the way things in the universe evolve over time. “Classical mechanics” is based on Newton’s laws of motion. This was the prevailing view of the world before about 1900. It is a branch of classical physics, which also includes thermodynamics and electromagnetism. “Quantum mechanics” is a new theory developed between 1900 and 1930 to replace Newton’s laws, especially to account for the behavior of microscopic pieces of matter. “Quantum theory” is a more encompassing term, including a wider application of quantum ideas. “Quantum physics” is the most general term for the physics of the microscopic realm. In everyday usage, however, these terms are practically synonymous.

Quantum mechanics is the most successful physical theory ever devised. It explains the structure of atoms, their combination into molecules, the interaction of light with matter, the behavior of solids and liquids near absolute zero, and many other phenomena. Quantum theory remains the general framework within which modern theories of physics are formed. For example, superstring theory (an exciting but speculative theory of elementary particles and forces) is a quantum theory.

Quantum physics challenges our imaginations in new and unexpected ways. First, quantum theory has a number of surprising implications for probability, the motion of particles, the properties of energy, the strange connectedness of separated systems, and the behavior of information at the smallest physical scales. The “weirdness” of quantum theory is not an incidental feature. It is at the center of the theory, required to make a consistent, accurate physical theory of the microscopic world.

Second, quantum physics has inspired profound philosophical discussions about the basic nature of the physical world. These will also be part of our story. Albert Einstein and Niels Bohr carried on a famous debate on the new physics in the early years of quantum mechanics. Also, the phenomenon of quantum entanglement has led us further and further away from a “common-sense” view of the microscopic realm. Even today, there are several competing ideas about how to interpret the mathematical theory of quantum mechanics.

Before we start off, we need to set some ground rules for our course. First, we will simplify our discussion to highlight the fundamental principles, and we will try to note when this happens. Don’t be too worried about this. The course explains the real theory, in simplified form. Also, we will often consider “thought experiments”—highly idealized experiments that might be possible in principle, although they may be impractical. In most cases, a more complicated and realistic version can actually be done in a lab.

The “weirdness” of quantum theory is not an incidental feature. It is at the center of the theory, required to make a consistent, accurate physical theory of the microscopic world.

How will we use mathematics? We will sometimes express the ideas of quantum mechanics symbolically, and we will learn a few simple rules for manipulating and interpreting the symbols. (These rules are no harder than very elementary algebra, though the details differ.) Venturing a short way into the abstract mathematics of quantum theory will allow us to explore the quantum world in a much deeper way. ■

Questions to Consider

1. In the remarkable short film *Powers of Ten*, Charles and Ray Eames “zoom in” on a scene by a magnification of $\times 10$ every few seconds. Imagine creating such a film of your own. You’ll need 8 of these $\times 10$ stages to go from a 1-inch aluminum cube down to a single aluminum atom. What sorts of common things (bugs, dust specks, cells, molecules) would you show at each stage from cube to atom?

2. In the lecture, we described several conceptual puzzles posed by quantum theory. Which of these seems most intriguing to you?

The View from 1900

Lecture 2

Throughout the history of human thought, there have been essentially 2 ideas about the fundamental nature of the physical world. ... In a nutshell, those 2 ideas are either the world is made out of things [discrete, indivisible units] or the world is made out of stuff [smooth continuous substances].

In modern terms, you can think of things as digital and stuff as analog. But if we travel back to a much earlier historical period, we can find a version of this debate among the Greek philosophers. The atomists, led by Democritus, considered the world to be composed of discrete, indivisible units called “atoms.” Everything is made of atoms, with empty space between them. All phenomena are due to the motions and combinations of atoms. Other philosophers, including Aristotle, believed on the contrary that the basic substances of the world are continuous and infinitely divisible.

The debate resurfaced in the 17th century as early physicists tried to understand the nature of light. Isaac Newton believed that light is a stream of discrete corpuscles that move in straight lines unless their paths are deflected. Different colors of light correspond to different types of corpuscles. Christiaan Huygens believed that light is a continuous wave phenomenon analogous to sound.

These waves propagate through space, and different colors correspond to different frequencies of the waves. Waves are characterized by their speed v , their wavelength λ , and their frequency f . These are related by the equation $v = \lambda f$.

In the 19th century, classical physicists arrived at a very successful synthesis of these ideas to explain the physical world. Matter is discrete, they said, while light is composed of continuous waves.



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Is nature essentially discrete, made of things, or is nature continuous, made of stuff? Sand is composed of individual units, grains of sand, but if you stand back from sand dunes they look smooth and continuous.

In the 19th century, classical physicists arrived at a very successful synthesis of these ideas to explain the physical world. Matter is discrete, they said, while light is composed of continuous waves. It was in the 1800s that John Dalton realized that chemical compounds could be explained by assuming that elements are composed of atoms of differing weights, which can combine into molecules. This became the fundamental idea of chemistry. Also in the 1800s, James Clerk Maxwell and Ludwig Boltzmann showed how the properties of a gas (pressure, temperature, and viscosity) can be explained by viewing the gas as a swarm of huge numbers of tiny molecules, moving according to Newton's laws. Heat energy is just the random motion of these molecules. The theory of heat became unified with the theory of mechanics.

Also in the 19th century, scientists conducted experiments that indicated that light is made of waves. Thomas Young devised his famous 2-slit experiment, in which light shows constructive and destructive interference. This

demonstrated that light travels in waves. Young measured the wavelength of visible light, which is less than 1 millionth of a meter. Maxwell showed that light is a traveling disturbance in electric and magnetic fields—in short, an electromagnetic wave. The theory of optics became unified with the theory of electromagnetism.

In 1900, Lord Kelvin gave a lecture at the Royal Institution in which he pointed out “two dark clouds” on the horizon of classical physics. Each dark cloud would turn out to be a hurricane. The first dark cloud was the curious result of an experiment by Michelson and Morley, who tried to detect the presence of the ether (the medium of light waves). This experiment later led to the development of Einstein’s theory of relativity, revolutionizing our ideas of space and time. The second dark cloud was the thermal radiation (“blackbody radiation”) given off by a warm object. If we try to explain this using classical physics, we get a very wrong result. This problem became the origin of quantum physics. ■

Questions to Consider

1. In 1900, no one had ever “seen” an atom or even knew exactly how large they were. Why, then, was it reasonable for physicists and chemists to believe in the existence of atoms?
2. The speed of sound is about 343 m/s. The human ear can detect sounds with a frequency range of 20 to 20,000 cycles/s. What range of wavelengths can the ear detect?
3. In the 2-slit experiment, imagine that 1 slit is somewhat larger than the other, so that the light waves coming from the 2 slits are not equal in intensity. What would the interference pattern look like?

Two Revolutionaries—Planck and Einstein

Lecture 3

At the beginning of the 20th century, 2 revolutionary thinkers, Max Planck and Albert Einstein, began to question the 19th-century synthesis, and to introduce quantum ideas into physics. ... There were just a few leftover experimental puzzles about light and matter, and to solve them, they needed to change the entire structure of physics.

The first puzzle was the problem of thermal radiation. When a solid object is heated, like the filament of an incandescent light bulb, it gives off radiation. The details are hard to reconcile with classical physics. This is sometimes called “blackbody radiation,” since the simplest case occurs when the object is black in color. At a given temperature, all black bodies radiate in the same way. When the classical theory of heat is applied to the radiation, it predicts the lower-frequency radiation (infrared) pretty well. But it predicts a lot more high-frequency radiation (ultraviolet) than is actually observed, or even possible. This is called the “ultraviolet catastrophe.”

In 1900, Planck made a strange hypothesis. He supposed that light energy can only be emitted or absorbed by a black body in discrete amounts, called “light quanta.” The energy of a light quantum is related to the light frequency by $E = hf$, where h is called “Planck’s constant.” Because the value of h is so tiny (6.6×10^{-34} J·s), the individual quanta are extremely small. An ordinary light bulb emits around a billion trillion (10^{20}) quanta each second. Since higher frequencies mean higher-energy quanta, groups of atoms cannot emit high-frequency light as readily. Therefore the ultraviolet catastrophe is avoided. A wave can have any intensity and therefore can carry any amount of energy. Planck’s quantum hypothesis is a radical change in the way we look at light.

Einstein examined the problem of the “photoelectric effect.” This problem arises from the fact that, when light falls on a polished metal surface in a vacuum, electrons can be emitted from the metal, and this process has several features that are hard to explain if light is a wave. The energy of the electrons does not depend on the intensity of the light. If we use a brighter light, we

get more electrons, but each one has the same energy as before. Instead, the electrons' energy depends on the frequency of the light. If the frequency is too low, no electrons are produced. The higher the frequency, the higher the electron energy.

In 1905, Einstein realized that Planck's quantum hypothesis amounts to assuming that light comes in the form of discrete particles, later called "photons." This is the key to understanding the photoelectric effect. Each photoelectron gets its energy from a single photon. Some of this energy goes into "prying it loose" from the metal; the electron flies away with the rest. Photons in bright light or dim light have the same energy, so the electron energies are the same in each case. Since $E = hf$, high-frequency light has photons of higher energy, and therefore electrons of higher energy are produced.

The third puzzle is the problem of heat capacities. There was a long-standing puzzle about the heat capacities of pure solids—that is, solids made from 1 type of atom. We will consider the examples of platinum and diamond (carbon). The "heat capacity" is the heat energy needed to raise the temperature of the solid by 1°C. Classical heat theory predicts that all pure solids should have the same heat capacity for the same number of atoms. This is because at a given temperature T all vibrating atoms should have the same energy on average. Carbon atoms, being less massive, would vibrate more times per second than platinum atoms, but they would have the same average

energy. Experimental results, however, are quite different. At 1000°C, both platinum and diamond have about the expected heat capacity. At room temperature, around 20°C, platinum behaves as expected but diamond's heat capacity is too small. At -200°C, both platinum and diamond have unexpectedly low heat capacities.

In 1905, Einstein realized that Planck's quantum hypothesis amounts to assuming that light comes in the form of discrete particles, later called "photons." This is the key to understanding the photoelectric effect.

In 1908, Einstein applied quantum ideas to the vibration of atoms. He proposed that atomic vibration energy only comes in discrete quanta of size $E = hf$. This is the first application of quantum physics to matter rather than light. It directly challenges Newton's mechanics in which a vibrating atom can have any amount of energy. For any pure solid, at high T there is enough heat energy for all the atoms to vibrate as expected. But at low T there is not enough heat energy for all of the atoms to vibrate, so the heat capacity is lower than expected. For diamond (carbon atoms), higher vibration f means that the energy quanta are larger. Both -200°C and 20°C count as "low" T . For platinum, only -200°C is a "low" T . Einstein's idea, with a few refinements of detail, precisely explains the heat capacities of pure solids at all temperatures. ■

Questions to Consider

1. In the phenomenon of photoluminescence, atoms absorb light of one frequency, then reemit light of a different frequency. According to Stokes's rule, the emitted light has a lower frequency than the absorbed light. Explain why this fact makes sense given the photon theory. (Einstein discussed Stokes's rule in his photoelectric effect paper.)
2. "The quantum discoveries of Planck and Einstein tell us that what we once supposed to be continuous is actually discrete." Discuss how this statement applies (if it does) to each of the 3 problems we have described.

Particles of Light, Waves of Matter

Lecture 4

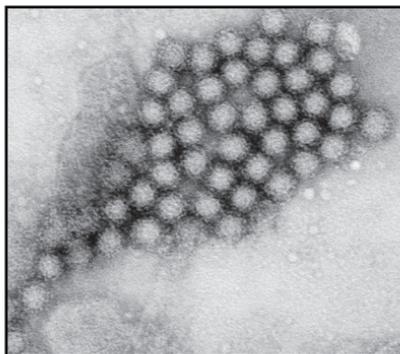
Young's 2-slit experiment demonstrates that light is a wave. ... On the other hand, Einstein's analysis of the photoelectric effect demonstrates that light is composed of discrete particles. ... Our understanding of light must somehow encompass both the wave and the particle ideas.

The quantum view can be summed up as wave-particle duality. The true nature of light cannot be described in simple terms. Both particle and wave pictures are required to explain the behavior of light. The rule of thumb is this: Light travels in the form of waves, with frequency and wavelength, interference effects, etc. But light interacts, is emitted or absorbed, in the form of particles, with discrete energies, etc.

In 1924, Louis de Broglie, following up the suggestion of Einstein, proposed that quantum wave-particle duality must also apply to matter. Particles such as electrons must also have wave properties of frequency, wavelength, interference effects, etc. De Broglie's idea was rapidly confirmed for electrons, which exhibit interference effects when they pass through the regularly spaced atoms in a crystal. Electron waves constructively interfere in some directions and destructively interfere in others. Modern experiments have demonstrated the wave properties of even larger pieces of matter, including neutrons and entire atoms. In one recent experiment, a 2-slit experiment was done with C_{60} molecules, which are more than a million times more massive than electrons.

A particle has a definite position, but a wave is spread out. How can we reconcile this? The Born rule states that the intensity of a wave ... tells us the probability of finding the particle at any given location.

There is a connection between wave and particle properties. The Planck-de Broglie relations connect the mechanical properties of waves and particles. A particle of mass m moving with a speed v has a momentum $p = mv$ and an energy $E = \frac{1}{2}mv^2$. Waves on the other hand are characterized by their wavelength λ and frequency f . Particle properties are connected to wave properties by Planck's constant (h): $E = hf$ and $p = \frac{h}{\lambda}$. The typical wavelength of electrons in atoms is extremely small, $<1 \text{ nm}$ (10^{-9} m).



© CDC/Charles D. Humphrey

This transmission electron micrograph (TEM) revealed some of the ultrastructural morphology displayed by norovirus virions, or virus particles.

The Born rule, named after physicist Max Born, provides another connection between wave and particle properties. A particle has a definite position, but a wave is spread out all over the place. How can we reconcile this? The Born rule states that the intensity of a wave, given by the square of its amplitude, tells us the probability of finding the particle at any given location. We illustrate the Born rule by examining an electron 2-slit experiment, 1 particle at a time. Each particle lands randomly, but after billions of particles arrive a statistical pattern emerges. Constructive interference enhances the probability of a particle being found in a given location, while destructive interference suppresses it. ■

Questions to Consider

- Let's put some numbers to wave-particle duality. The value of Planck's constant is about $6.6 \times 10^{-34} \text{ kg}\cdot\text{m}^2/\text{s}$. An electron has a mass of $9.1 \times 10^{-31} \text{ kg}$. Suppose an electron is moving at a speed of $2.0 \times 10^6 \text{ m/s}$. (This might seem fast, but it is actually a typical speed for an electron in an atom.) First find the electron's energy and momentum; then calculate its quantum frequency and wavelength.

2. In a 2-slit experiment, if we open only 1 slit or the other, suppose the probability that a photon reaches a given point is P (the same in either case). Now we open both slits and repeat the experiment. Explain why the probability that a photon reaches the given point might be anything between 0 and $4P$.
3. In “classical” wave physics, the intensity of a light wave gives the amount of energy it carries. The Born rule tells us that in quantum physics, the intensity gives the probability of finding a photon in that region. How are these 2 ideas related?

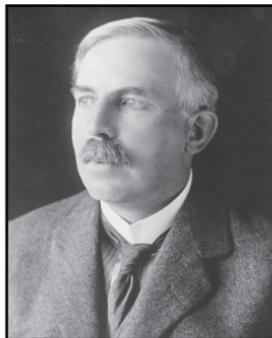
Standing Waves and Stable Atoms

Lecture 5

In the last lecture, we saw and explored the strange quantum idea of wave-particle duality, an idea that applies both to light and to matter. Everything has both wave properties and particle properties. ... This time we're going to see how the wave characteristics of matter explain the structure of atoms.

In 1909, Ernest Rutherford supervised experiments to scatter fast-moving particles from gold foil. These experiments led him to propose a “solar system” model of atomic structure. In this model, most of the atom’s mass lies in the heavy, positively charged nucleus at its center. Electrons, with a negative charge and relatively low mass, orbit the nucleus, held in place by the attractive electric force between positive and negative charges. This leads to a puzzle: In classical mechanics, an orbiting electron should emit electromagnetic radiation. It should therefore lose energy and spiral inward toward the nucleus. Rutherford’s atom should implode in less than 1 microsecond!

In 1913, Niels Bohr, a postdoctoral student in Rutherford’s lab, used the new quantum ideas to explain atomic structure. Bohr proposed that only certain discrete orbits are possible for the electron in the atom. If the electron is in the innermost possible orbit, it can no longer spiral inward. Thus atoms can be stable. When an electron “jumps” from one orbit to another, it absorbs or emits a photon. We can also imagine more abstractly: Different Bohr orbits are “rungs” on an “energy ladder” for the electron. To climb up to a higher rung, the electron must absorb a photon; to descend, it must emit a photon. The photon energies are determined by the spacing of these energy levels. Bohr was able to predict the pattern of energy levels in



Nobel Prize winner Ernest Rutherford (1871–1937) was the first researcher to produce an artificial transmutation of elements, using alpha particles to transform nitrogen into oxygen.

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hydrogen atoms, which only have a single electron. This pattern accounts for the discrete colors of light (photon energies) produced by hot hydrogen gas. This is called the “emission spectrum” of the element.

Bohr’s orbits correspond to “standing wave patterns” of electrons moving around the nucleus. They can be nicely explained by de Broglie’s electron waves, although this was not Bohr’s original explanation. In de Broglie’s version of Bohr’s model, in any wave system enclosed in space, only certain wave patterns are possible. An easy example of this is a stretched piano wire. The wave must “fit” between the fixed ends of the wire. Only certain wavelengths and frequencies (or combinations of these) can occur, which is why the piano wire vibrates with a definite note when struck. As an electron orbits an atom, only certain wave patterns “fit” around the atom. Thus only certain wavelengths and frequencies are possible. These standing wave patterns determine the possible Bohr orbits.

In de Broglie’s version of Bohr’s model, in any wave system enclosed in space, only certain wave patterns are possible.

In 1926, Erwin Schrödinger provided a detailed mathematical description of de Broglie’s waves. His description is embodied in the famous Schrödinger equation, which is one of the fundamental equations in all of physics. Here is one form of the equation:

$$-\frac{\hbar^2}{2m}\nabla^2\Psi + U(x,y,z)\Psi = i\hbar\frac{\partial\Psi}{\partial t}.$$

The Ψ in this equation is the “wave function” of the electron. The wave intensity $|\Psi|^2$ gives the probability of finding the electron at any given point in space. Solving the Schrödinger equation gives 3-dimensional standing wave patterns for an electron in an atom. Each wave pattern corresponds to a different energy level. The wave patterns are changed by the emission or absorption of photons. In an ordinary advanced quantum mechanics course, students spend at least 90% of their time learning methods for solving the Schrödinger equation. This can be a very hard task, especially when the situation is complicated.

The Schrödinger equation and quantum mechanics do a good job of explaining energy levels for atoms and molecules, the emission and absorption of light by atoms, and the way atoms are affected by outside forces (e.g., stretched by electric fields or twisted by magnetic fields). All of these can be calculated from the standing wave patterns of de Broglie waves, determined by the Schrödinger equation. ■

Questions to Consider

1. Before Rutherford's scattering experiment, a leading idea of atomic structure was J. J. Thomson's "plum-pudding" theory. In this model, negatively charged electrons were embedded in a diffuse, positively charged "pudding." If Thomson's model had been correct, how would the scattering experiment have turned out differently?
2. A piano wire can vibrate at a certain fundamental frequency f and also at higher "overtone" frequencies $2f$, $3f$, and so on. If you have access to a piano, try the following experiment. Hold down the key for middle C (262 cycles/s) without playing the note. Now briefly play each of the following notes and listen to how the open C string responds: C (1 octave up, or 524 cycles/s), G (1.5 octaves up, or 784 cycles/s), and C (2 octaves up, or 1048 cycles/s). Also try this with other notes. What do you observe?
3. Excited hydrogen atoms emit violet, blue-green, and red light. These correspond to electrons dropping to the second energy level from the ones above it: $3 \Rightarrow 2$, $4 \Rightarrow 2$ and $5 \Rightarrow 2$. Which jumps correspond to which emitted colors and why? (Recall that violet light has a higher frequency than red light.)

Uncertainty

Lecture 6

Our business today is to explore the implications of [the quantum idea of wave-particle duality], to say what it means for the wave to spread out, and to say what it means for the quantum wave describing a quantum particle to spread out in space. Today we're going to talk about the uncertainty principle.

Particles and waves have contrasting properties. In classical physics, a particle like an electron has both an exact location in space and a definite velocity or momentum at every moment. In other words, the particle has an exact “trajectory” through space. On the other hand, we have the basic wave phenomenon of diffraction of waves through a single slit. After passing through the slit, the waves spread out into the space beyond. The diffraction effect depends on the ratio λ/w , the wavelength divided by the width of the slit. A narrow slit (large ratio) produces a wide pattern of waves, while a wide slit (small ratio) produces a narrow pattern. This allows waves to “go around corners.” A thought experiment illustrates this: A friend behind a wall speaks to us through an open door. We can hear the friend because the wavelength of the sound waves is large, and the sound waves passing through the door spread out. But we do not see the friend because light waves have very short wavelengths, and diffraction through the door is negligible.

Diffraction and wave-particle duality set a basic limit on how well a particle's properties are defined. We consider an electron, described by de Broglie waves, passing through a barrier with a single slit. If the electron passes through the slit, this means that we know the particle's lateral position (x), though not exactly. Our uncertainty in the particle position is just the slit width: $\Delta x \approx \Delta w$. Because of diffraction, the de Broglie wave spreads out past the slit. The lateral velocity of the particle is not exactly known, which means we cannot tell exactly where the particle will be found. It turns out to be easier to consider the particle's lateral momentum p . The spreading of the wave pattern means there is an uncertainty Δp in this momentum. A wide pattern means a larger Δp . The relation between slit width and diffraction spreading

means there is a trade-off between Δx and Δp . The smaller one is, the larger the other must be.

Werner Heisenberg realized that this represents a basic trade-off in nature, which is his famous “uncertainty principle.” Suppose Δx and Δp are our uncertainties in a particle’s position and momentum. Then it must be true that $\Delta x \Delta p \geq h$ (where h is Planck’s constant).

There are some important things to note. First, this is an inequality. We can always be less certain about x and p than this, but never more certain. Second, our definitions of uncertainty here are informal or “fuzzy.” With more careful technical definitions, there may be a factor of 2 or 4 (or $4\pi!$) in the right-hand side. This does not change the basic point. Additionally, Planck’s constant is extremely small, so a large-scale object can have a pretty well-defined location and momentum. This is part of the reason why large-scale objects can behave like classical particles. Lastly, for microscopic particles like electrons, the uncertainty principle can be very important. An electron confined to an atom has Δx no larger than the diameter of the atom. The resulting momentum uncertainty Δp is large enough that we do not even know which direction the electron is moving in the atom!

The point is not that we do not know the exact values of x and p for an electron; the electron in fact does not have exact values of x and p .

Heisenberg argued that the uncertainty principle is actually an “indeterminacy principle.” The point is not that we do not know the exact values of x and p for an electron; the electron in fact does not have exact values of x and p .

Another uncertainty principle relates time and energy. If a process happens over a period of time Δt , and the energy involved in the process is uncertain by an amount ΔE , then $\Delta E \Delta t \geq h$. ■

Questions to Consider

1. Heisenberg used several different terms to describe his basic idea. He said that a particle's position might have "uncertainty" or "indeterminacy" or "imprecision" or "latitude" or "statistical spread." Remark on the different shades of meaning that these various terms suggest.
2. There is a classical "uncertainty principle" for any sort of wave, including sound. A musical note is a mixture of a range of frequencies Δf . If the note lasts for a time period Δt , it turns out that the spread of frequencies must satisfy $\Delta f \geq \frac{1}{\Delta t}$. (This means that very short notes do not have very definite pitch.) What version of the quantum uncertainty principle is most closely related to this fact?

Complementarity and the Great Debate

Lecture 7

This lecture is about an argument. The protagonists are 2 giants of 20th-century science, Albert Einstein and Niels Bohr. They're 2 of the founders of quantum theory and the subject of their argument is the meaning of quantum mechanics. At stake, are our most fundamental ideas about the nature of nature.

Albert Einstein was the father of the idea of wave-particle duality, but he found much to criticize in quantum mechanics. In his view, one key flaw was that quantum mechanics failed to answer the question of why a particle ended up in one place rather than another. The theory only predicts probabilities. Einstein believed this to be a flaw—he thought a theory should explain individual events, not just tendencies. Also, he was predisposed to “determinism,” the idea that the future of the universe is completely determined by the present. He said, “God does not play dice with the universe.” Additionally, he at first thought that quantum mechanics was not logically consistent.

Niels Bohr was a deeply philosophical thinker and a powerful personality. Much of quantum mechanics was developed by his followers and worked out at his theoretical physics institute in Copenhagen. Bohr believed that the new quantum theory required physicists to abandon old concepts, including determinism. He said, “Einstein, stop telling God what to do.” Bohr worked out a sophisticated framework of concepts for using quantum ideas without contradictions. This framework came to be called the “Copenhagen interpretation” of quantum mechanics and has been the principal way that physicists have made sense of quantum theory. (We will see other approaches later.)

The Copenhagen interpretation rests on Bohr’s “principle of complementarity.” This is a subtle idea that requires a careful explanation. Bohr says that we must consider 2 physical realms. There is a microscopic realm of electrons, photons, etc., that cannot be described in “ordinary language.” There is also a macroscopic realm of large objects, people, etc., that can be described in

“ordinary language” and in which classical physics is approximately valid. To make a measurement of the quantum realm, we must always “amplify” the result to the macroscopic realm, so that we can record it and communicate it in ordinary language. This act of amplification is crucial!

Every experiment on a quantum particle is an “interaction” between the experimental apparatus and the particle, not just a passive observation. Interaction goes both ways, and the particle is always affected in some way. How the quantum particle responds depends on what interaction occurs.

Different types of experiments are logically exclusive—we can do one or the other, but not both at the same time. The uncertainty principle tells us that we cannot exactly measure the position and momentum of a particle at the same time. Why not? The interaction needed for a position measurement is not the same as that needed for a momentum measurement. They are complementary quantities.

Measuring one logically excludes measuring the other at the same time. Consequently, when we try to use ordinary language to describe the quantum world, we must use different complementary descriptions in different situations—but the mathematics of quantum mechanics guarantees that we can do this without contradiction.

Einstein proposed several puzzles and paradoxes designed to show some loophole in quantum mechanics. Bohr responded to them one by one, in each case trying to expose the flaw in Einstein’s thinking and defend quantum mechanics.

Bohr and Einstein engaged in a long-running debate about the validity and meaning of quantum mechanics. Much of the debate centered on a series of thought experiments. Einstein proposed several puzzles and paradoxes designed to show some loophole in quantum mechanics. Bohr responded to them one by one, in each case trying to expose the flaw in Einstein’s thinking and defend quantum mechanics. This might appear to be a debate about details and examples, but it was really a profound argument about basic principles.

The debate reached its crescendo at the Solvay Conferences of 1927 and 1930. Einstein proposed several clever thought experiments to try to prove that the uncertainty principle could be beaten. We will examine one of these.

Einstein asked us to suppose a particle passes through a barrier with 1 slit. This gives us a lateral position uncertainty Δx , which implies a minimum lateral momentum uncertainty Δp . But if the barrier is moveable, then the deflection of the particle will cause a sideways recoil of the barrier. By measuring this recoil, we should be able to determine the particle's new momentum. We can violate the uncertainty principle!

Not so fast, replied Bohr. We must also consider how the uncertainty principle applies to the moveable barrier, which has position X and momentum P . To measure the recoil precisely, then our uncertainty ΔP in the barrier's momentum must be extremely small. But then the barrier's position is uncertain by a large ΔX . The uncertainty Δx in the particle's position cannot be smaller than our (large) uncertainty ΔX in the location of the slit. The uncertainty principle is not violated.

After 1930, Einstein was forced to accept that quantum mechanics was a consistent theory of nature. Nevertheless, he was still dissatisfied and kept looking for clues to a deeper view. The Bohr-Einstein debate shifted but did not end. ■

Questions to Consider

1. De Broglie's original view was that both waves and particles existed at the same time and that the wave exerted quantum forces that guided the particle through space. The particle thus had a definite trajectory, though the trajectory would be complicated and difficult to predict. Would this idea have appealed more to Einstein or to Bohr?
2. In a 1928 letter to Schrödinger, Einstein referred to complementarity as a "tranquilizing philosophy" that merely allowed quantum physicists to avoid uncomfortable questions. Is this fair? How would you respond to—or defend—Einstein's remark?

3. Should we think of Einstein's sharp critique of quantum theory as an obstacle or a spur to its development? What is the role of criticism in the creation of new ideas?

Paradoxes of Interference

Lecture 8

In the first section of the course, we've been tracing how a generation of the most brilliant scientists in human history created the theory of quantum mechanics and wrestled with its perplexities. ... In the second section, we embark on the task of introducing the theory itself, quantum mechanics in a simplified form.

To begin to look at quantum mechanics, we will use an interferometer as a simple conceptual “laboratory.” An interferometer is an optical apparatus made up of several components. A light source generates a beam of light with a definite wavelength as input to the apparatus. The intensity of the light can be reduced so that only 1 photon is traveling through the apparatus at a time. Photon detectors can be placed to register any photons that strike them. (Our detectors are somewhat idealized.) Mirrors are used to guide the light beams in various directions.

The crucial components of an interferometer are “half-silvered mirrors.” A half-silvered mirror splits a beam of light into 2 beams of equal intensity. These mirrors are also sometimes called “beam splitters.” It's important to note that light waves that reflect from the silvered side of the mirror are inverted—that is, the electromagnetic fields are reversed in the reflected waves. If we send a single photon through a beam splitter, photon detectors do not both register “half a photon.” Instead, the entire photon is registered in one place or the other, each with probability $\frac{1}{2}$.

Our apparatus is a Mach-Zehnder interferometer. An incoming light beam is split at a half-silvered mirror. The 2 beams are then recombined at a second half-silvered mirror, and the output beams are observed. If everything is set up properly, all of the light emerges in 1 beam. This is because the light constructively interferes in that direction but destructively interferes in the other. If we send 1 photon at a time through the interferometer, the photon is always registered in 1 beam rather than the other. The probabilities exhibit constructive and destructive interference!

We can explore quantum ideas by thought experiments using the single-photon version of the Mach-Zehnder interferometer. Interference can only happen if the photon travels “both ways” through the interferometer. Suppose we block 1 beam with our hand; 50% of the time, the photon hits our hand. Otherwise it travels along the other beam to the second half-silvered mirror and is registered by each detector 25% of the time. Suppose we introduce a nonabsorbing detector into 1 beam. This tells us which beam the photon traveled, but in so doing we completely lose the interference effect. Each detector registers the photon 50% of the time. For interference to occur, the photon must leave no “footprints” behind that would tell which way it went.

“Both ways” and “which way” experiments illustrate the principle of complementarity. With the second half-silvered mirror present, we find interference effects. The photon must have traveled “both ways” through the interferometer. If we remove the half-silvered mirror, the photon

detectors tell us “which way” the photon traveled through the interferometer.

Wheeler’s quantum motto is “No phenomenon is a phenomenon until it is an *observed* phenomenon.”

We must choose which experiment to do, and we cannot later say what would have happened if we had done the other one. Keep in mind Asher Peres’s quantum motto: “Unperformed experiments have *no* results.” In 1978, John Wheeler proposed the “delayed-choice experiment”:

We decide whether to leave the mirror in or take it out when the light has already traveled 99% of the way through the apparatus. We decide whether the photon went “both ways” or “one way” after it has almost completed its journey! Wheeler’s quantum motto is “No phenomenon is a phenomenon until it is an *observed* phenomenon.”

The Elitzur-Vaidman bomb problem leads us to even stranger conclusions. In this scenario, a factory produces bombs with extremely sensitive light triggers. Some bombs are “good” and some are defective. We want to test them. A good bomb will explode if even 1 photon hits the trigger. But a defective bomb lacks the trigger mechanism and photons pass through. Suppose we send just 1 photon into a bomb to test it. A good bomb will explode and a defective one will not. This tests the bomb, but only by

blowing it up. Can we ever find out that a bomb is good without exploding it? This seems impossible!

A quantum trick solves the puzzle. Elitzur and Vaidman suggested that we put the bomb in 1 beam of a Mach-Zehnder interferometer and send 1 photon through. If the bomb is defective, then the light shows interference. The photon always winds up in one detector and never in the other. If the bomb is good, then 50% of the time it will explode. But the other 50% of the time, the photon travels the other beam to the second beam splitter. Thus 25% of the time it will strike a detector that would be impossible if the bomb were defective. We can certify some good bombs without exploding them! ■

Questions to Consider

1. In our interferometer experiment, suppose we flip the second beam splitter so that its metal coating is on the other side. How would this affect the constructive and destructive interference? What if we flip both beam splitters?
2. In the bomb-testing experiment, one possible outcome is inconclusive, since both working and defective bombs can produce it. Suppose we repeat the test once more if this happens. What percentage of working bombs are (a) blown up, (b) certified as working, and (c) still undetermined in this double test? Suppose we repeat the test as many times as necessary to achieve a conclusive result. What percentage of the working bombs are blown up?

States, Amplitudes, and Probabilities

Lecture 9

Now we want to formulate symbolic ways of working with quantum ideas. We want to introduce a kind of mathematical language. I mean, after all, if we want to explore Mongolia, it's a good idea to learn some Mongolian, especially if the best maps are all written in Mongolian. Our destination is a place that's even more exotic than Mongolia. Our destination is the microscopic world.

Our aim here is to introduce a formal language to describe quantum ideas. First we introduce a few terms and abstract symbols. A “system” is any piece of the quantum world that we wish to consider. For example, we might consider a single photon in an interferometer. A “state,” on the other hand, is a physical situation of some system. We represent a state by a “ket,” like so: $|state\rangle$. What we put inside the ket $|\dots\rangle$ is just a convenient label for the state. A “basis” is a set of distinct states that cover all of the outcomes of some measurement. For example, the photon in the interferometer would be found in one beam or the other, so the 2 states $|upper\rangle$ and $|lower\rangle$ make up a basis. There can be different possible measurements, so there can be different possible basis sets for a quantum system.

Besides basis states, there are also “superposition” states. The term superposition is meant to suggest a composite, like 2 pictures “superimposed” on one another in a double exposure. We represent a superposition as an abstract sum:

$$|state\rangle = a|upper\rangle + b|lower\rangle.$$

The numerical factors a and b are called “amplitudes.” In full quantum mechanics, these amplitudes might include imaginary numbers (like $\sqrt{-1}$). We can omit this complication, but we will use both positive and negative amplitudes. We define the number $s = 0.7071 \dots$, for which $s^2 = \frac{1}{2}$.

(We give this number a special name for convenience because we will use it a lot in our examples.)

Next, we need rules for working and interpreting the abstract quantum symbols. The “rule of superposition” says that a superposition of 2 or more basis states is also a quantum state. This means that a quantum system has more possibilities than we might expect. For the photon in the interferometer, besides $|\text{upper}\rangle$ and $|\text{lower}\rangle$ states, we also have lots of superposition states $a|\text{upper}\rangle + b|\text{lower}\rangle$ for many different choices of amplitudes a and b . A superposition state represents the photon divided among the beams in some way, as happens in an interferometer. The amplitudes determine the details.

Quantum mechanics only predicts probabilities, not definite results. What is probability? For any event, its probability P is a number between 0 and 1.

The “rule of probability” (also called the Born rule) says that if we make a measurement, the probability of any result is determined by the amplitude for that result:

$$\text{probability} = |\text{amplitude}|^2 .$$

Quantum mechanics only predicts probabilities, not definite results. What is probability? For any event, its probability P is a number between 0 and 1. The value $P = 0$ means the event is impossible, and $P = 1$ means that it is certain. An intermediate value like $P = 0.37$ means that, if we tried the same experiment many times, the event would happen about 37% of the time. Probabilities predict statistics. Both positive and negative amplitudes give positive probabilities.

Suppose our photon is in the state $a|\text{upper}\rangle + b|\text{lower}\rangle$. If we make a measurement to find which beam the photon is in, we will get results with probabilities:

$$P(\text{upper}) = |a|^2 \text{ and } P(\text{lower}) = |b|^2 .$$

This means we must have $|a|^2 + |b|^2 = 1$, since probabilities must always add up to 1.

In the state $s|\text{upper}\rangle + s|\text{lower}\rangle$, each beam has probability $|s|^2 = \frac{1}{2}$. The same thing is also true for the different quantum state $s|\text{upper}\rangle - s|\text{lower}\rangle$, because:

$$|s|^2 = |-s|^2 = \frac{1}{2}.$$

There are 2 “update rules” that tell how the state changes when something happens to the system. Update rule I says that when there is no measurement, the state changes in a definite way that maintains any superposition. If we know how to update the basis states, we can determine how to update superposition states. Update rule II says that when there is a measurement, we use the results to find the new state. In this case, the state is updated randomly.

We will now look at an example for update rule II. In our example, the photon is in the state $a|\text{upper}\rangle + b|\text{lower}\rangle$, and we use photon detectors to determine which beam it is in. Then:

$$a|\text{upper}\rangle + b|\text{lower}\rangle \begin{array}{l} \vdots \\ \vdots \\ \vdots \end{array} \begin{array}{l} |\text{upper}\rangle \quad \text{with } P = |a|^2 \\ \\ |\text{lower}\rangle \quad \text{with } P = |b|^2 \end{array}.$$

These are (almost) the only rules of quantum mechanics!

To understand the meaning of the quantum rules, we apply them to the photon in an interferometer. At a beam splitter, the basis states change in this way:

$$\begin{aligned} |\text{upper}\rangle &\Rightarrow s|\text{upper}\rangle + s|\text{lower}\rangle. \\ |\text{lower}\rangle &\Rightarrow s|\text{upper}\rangle - s|\text{lower}\rangle. \end{aligned}$$

This is an example of update rule I, since no measurement is made. The minus sign indicates reflection from the silvered side of the mirror.

In the interferometer, we keep track of the quantum state at each stage to figure out what happens to the photon. The photon starts out in the upper beam, so its state is $|\text{upper}\rangle$. At the first beam splitter, the state changes: $|\text{upper}\rangle \Rightarrow s|\text{upper}\rangle + s|\text{lower}\rangle$. The beams recombine at the second beam splitter. We apply the beam splitter state change to each part of the superposition, according to update rule I:

$$\begin{aligned} & s|\text{upper}\rangle + s|\text{lower}\rangle \\ & \Rightarrow s(s|\text{upper}\rangle + s|\text{lower}\rangle) + s(s|\text{upper}\rangle - s|\text{lower}\rangle). \end{aligned}$$

We now multiply amplitudes and combine terms as we would in an ordinary algebraic expression. This gives us the final state:

$$(s^2 + s^2)|\text{upper}\rangle + (s^2 - s^2)|\text{lower}\rangle = |\text{upper}\rangle.$$

At the end, the photon is certain to be in the upper beam. Constructive and destructive interference take place in the amplitudes. The quantum amplitude keeps track of the wave properties of the photon. ■

Questions to Consider

1. One of the questions for the last lecture asked what happens when the second beam splitter is flipped so that its metal coating is on the other side. Write down how a flipped beam splitter affects the $|\text{upper}\rangle$ and $|\text{lower}\rangle$ basis states, and work out the final quantum state for the photon. Does this agree with your previous answer? (It should.)
2. If we simply allow the 2 beams to cross without a beam splitter, this simply exchanges the basis states: $|\text{upper}\rangle \Rightarrow |\text{lower}\rangle$ and $|\text{lower}\rangle \Rightarrow |\text{upper}\rangle$. Use this fact to find the final quantum state if the second beam splitter is removed (as in Wheeler's delayed-choice experiment).
3. Suppose we place a nonabsorbing detector in one of the beams of the interferometer. Using both update rules, explain what happens to the quantum state at various stages of the apparatus.

Particles That Spin

Lecture 10

This time, we're going to take up a new example. We're going to use lots of 3-dimensional geometry, lots of angles and directions. We'll get to practice our spatial skills. This time we're going to talk about the physics of spin.

The physics of “spin” offers us another example of the quantum rules. An electron in orbit is analogous to a planet moving around the Sun. Each planet moves through space and also rotates on its axis. Something similar is true for quantum particles like electrons. They not only move through space—they also have internal spin. Spin is a kind of angular momentum, a physical measure of the amount of rotation in a particle.

Classical spins can have any value for any component. A “spin component” is the degree of spin a particle has along a particular axis. Classically, this depends on (1) the total amount of spin, and (2) the angle between the rotation axis and the axis we are interested in. For a classical spinning object, a given spin component can have any value, and all spin components have definite values at the same time.

A “spin component” is the degree of spin a particle has along a particular axis. Classically, this depends on (1) the total amount of spin, and (2) the angle between the rotation axis and the axis we are interested in.

Quantum spins have quite different characteristics. We can measure any component of a particle's spin by a “Stern-Gerlach apparatus,” which measures the deflection of the particle in a nonuniform magnetic field. The orientation of the apparatus determines which spin component we measure. For electrons, the measurement of any spin component only can give 2 possible results, the values $\pm\frac{1}{2}$ (in units of $h/2\pi$). Electrons are said to be spin- $\frac{1}{2}$ particles, as are protons and neutrons. Other quantum particles can be spin 0 (no spin at all), spin 1, spin $\frac{3}{2}$, etc.

Let's look at measurements and states for a spin- $\frac{1}{2}$ particle. We can measure an electron's spin along any axis in space. Two axes that we are especially interested in are the perpendicular axes z and x . A z measurement gives us 2 basis states $|\uparrow\rangle$ and $|\downarrow\rangle$, corresponding to the results $z = +\frac{1}{2}$ and $z = -\frac{1}{2}$, respectively. We call these "spin up" and "spin down." An x measurement gives us different basis states $|\rightarrow\rangle$ and $|\leftarrow\rangle$, corresponding to the results $x = +\frac{1}{2}$ and $x = -\frac{1}{2}$, respectively. We call these "spin right" and "spin left." We can write the x basis states as superpositions of z basis states and vice versa:

$$\begin{aligned} |\rightarrow\rangle &= s|\uparrow\rangle + s|\downarrow\rangle & |\uparrow\rangle &= s|\rightarrow\rangle + s|\leftarrow\rangle \\ |\leftarrow\rangle &= s|\uparrow\rangle - s|\downarrow\rangle & \text{and} & \\ |\downarrow\rangle &= s|\rightarrow\rangle - s|\leftarrow\rangle \end{aligned}$$

We call x and z "complementary quantities" for the electron. The x and z measurements are mutually exclusive—the Stern-Gerlach apparatus must be aligned one way or the other. There is an uncertainty principle for spin. A particle cannot have definite values for both x and z at the same time. For z basis states, z is definite but x is indeterminate; for x basis states, x is definite but z is indeterminate. The complementarity of different spin components for large-scale spinning objects (baseballs, planets) is negligible because h is so small.

From here we can extend the theory of spin $\frac{1}{2}$. There are other spin components besides z and x . Consider the spin component at an angle α from the z -axis (in the x - z plane). The basis vectors for this spin component can be called $|\alpha+\rangle$ and $|\alpha-\rangle$. For instance, $|\rightarrow\rangle = |90^\circ+\rangle$. Suppose we prepare $|\alpha+\rangle$ and measure spin component z . What is the probability P that we obtain the result $z = +\frac{1}{2}$? Here is a table:

α	0°	45°	90°	135°	180°
P	1.00	0.85	0.50	0.15	0.00

(The values for 45° and 135° are rounded off here.) This table helps us calculate the probabilities if we have a spin with a definite component along any axis and then measure it along another axis at an angle α .

What happens to the spin of a particle if we rotate it in space? We can rotate an electron spin (or the spin of a proton or neutron) by using magnetic fields. Since no measurement is involved, the spin state should change according to update rule I. Suppose we rotate around the y -axis by 90° . How do basis states change?

$$|\uparrow\rangle \Rightarrow s|\uparrow\rangle + s|\downarrow\rangle = |\rightarrow\rangle.$$

$$|\downarrow\rangle \Rightarrow -s|\uparrow\rangle + s|\downarrow\rangle = -|\leftarrow\rangle.$$

From this, we can also find the following:

$$|\rightarrow\rangle \Rightarrow |\downarrow\rangle \text{ and } |\leftarrow\rangle \Rightarrow |\uparrow\rangle.$$

Why the minus sign in the rotation of $|\downarrow\rangle$? We cannot work things out consistently without it. But it should not matter. Because of the rule of probability, the states $|\leftarrow\rangle$ and $-|\leftarrow\rangle$ will yield exactly the same probabilities in any measurement. The 2 kets $|state\rangle$ and $-|state\rangle$ describe equivalent physical situations.

To rotate the spin by 360° , we can do it 90° at a time:

$$|\uparrow\rangle \Rightarrow |\rightarrow\rangle \Rightarrow |\downarrow\rangle \Rightarrow -|\leftarrow\rangle \Rightarrow -|\uparrow\rangle.$$

This curious minus sign does not worry us—but we will remember it. It turns out to be very interesting and significant later on! ■

Questions to Consider

1. In a Stern-Gerlach experiment, a beam of particles with spin is deflected by a magnetic field. The amount of deflection depends on the z component of the spin. In the real experiment, quantum particles emerge in just 2 different directions, corresponding to spin components $+\frac{1}{2}$ or $-\frac{1}{2}$. But imagine a world in which these particles had “classical” spin, like the spin of a top. How would the experiment turn out?
2. Here is an algebraic exercise suggested in the lecture. We gave formulas for the x basis states written in terms of the z basis states:

$|\rightarrow\rangle = s|\uparrow\rangle + s|\downarrow\rangle$ and $|\leftarrow\rangle = s|\uparrow\rangle - s|\downarrow\rangle$. Starting only with these, figure out the formulas that give the z basis states in terms of the x basis states. (You will need to remember that $s^2 = \frac{1}{2}$.)

3. Show that we cannot “do without” the funny minus signs in the rotation rule for spins. It would be nicer if a 90° rotation worked something like this: $|\uparrow\rangle \Rightarrow |\rightarrow\rangle \Rightarrow |\downarrow\rangle \Rightarrow |\leftarrow\rangle \Rightarrow |\uparrow\rangle$, with no minus signs at all. Show that this “nice rule” is inconsistent with update rule I. (Hint: Write $|\rightarrow\rangle$ and $|\leftarrow\rangle$ as superpositions of the basis states $|\uparrow\rangle$ and $|\downarrow\rangle$.)

Quantum Twins

Lecture 11

Now we'll be moving into ... several different particular topics in quantum theory. We'll begin ... with ... the theory of identical particles. All electrons are identical. All photons are identical. What does this mean? How can quantum theory describe that? What are the implications? This is an amazing story that we'll be telling. ... Here's our essential point: Macroscopic classic objects and microscopic quantum particles have a different sense of identity.

Macroscopic objects obey the “snowflake principle”: No 2 are exactly alike. Every object can be uniquely identified, at least in principle. No 2 snowflakes are alike (though some appear quite similar). Even identical twins have slightly different fingerprints. If we put 2 pennies in a box and then draw 1 out, it makes sense to ask which penny we have. There are always microscopic differences that can be used as identifying marks, like the serial numbers on currency.

In contrast, quantum particles do not obey this snowflake principle. All electrons are exactly identical to each other. They may differ in location and spin, but they are otherwise exactly the same. If we put 2 electrons in a box and then draw 1 out, it does not make sense to ask which electron we have. There are no microscopic differences to be used as serial numbers. The same is true for 2 photons, or 2 protons, or even 2 atoms of the same type.

The point here is not simply a philosophical one; it changes how we apply the quantum rules. We already know the quantum rules for a single-particle system. We imagine 2 “boxes,” A and B. A quantum particle can be in either of the 2 boxes. Thus 1 particle has basis states $|A\rangle$ and $|B\rangle$ (and could be in any superposition of these).

“Distinguishable” quantum particles have simple rules. These particles can be discriminated in some way. For example, in a 2particle system, our first particle might be a proton and the second one an electron. The 2-particle states $|AB\rangle$ and $|BA\rangle$ are distinct physical situations. In $|AB\rangle$, the first

particle is in box A and the second in box B; in $|BA\rangle$, they are reversed. We can tell these situations apart. Distinguishable particles might also be in the same box, as in the states $|AA\rangle$ and $|BB\rangle$.

“Identical” particles force us to reexamine our assumptions. For 2 electrons, the states $|AB\rangle$ and $|BA\rangle$ do not represent distinct physical situations. We can express this by using the “SWAP” operation, which exchanges the 2 particles. For instance, $\text{SWAP}|AB\rangle = |BA\rangle$. (If we had more particles, we would have a SWAP operation for each possible pair.) For any state of 2 identical particles, $|state\rangle$ and $\text{SWAP}|state\rangle$ must be physically equivalent. If we swap twice, we must return to the original situation: $\text{SWAP}^2|state\rangle = |state\rangle$.

Quantum particles come in 2 possible types, depending on how the SWAP operation works. First we will consider Bose-Einstein particles, or “bosons,” named for Satyendra Bose and Albert Einstein, who did groundbreaking work related to them. The boson rule says that for a pair of identical bosons, $\text{SWAP}|state\rangle = |state\rangle$. The quantum state is completely unchanged when we swap the particles. Examples of bosons include photons and helium atoms.

Next we will consider Fermi-Dirac particles, or “fermions,” about which Enrico Fermi and Paul Dirac did important work. The fermion rule says that for a pair of identical fermions, $\text{SWAP}|state\rangle = -|state\rangle$. The quantum state acquires a negative sign when we swap the particles. Two swaps still cancel out:

$$\text{SWAP}^2|state\rangle = -(-|state\rangle) = +|state\rangle.$$

Examples of fermions include electrons, protons, and neutrons. Notice that the basic constituents of ordinary matter are all fermions.

Quantum particles come in 2 possible types, depending on how the SWAP operation works: [Bose-Einstein particles, or “bosons,” and Fermi-Dirac particles, or “fermions”].

Consider the 2 boxes again, with 1 particle in each. For distinguishable particles, we have distinct states $|AB\rangle$ and $|BA\rangle$. For bosons, there is only 1 distinct state, which is $s|AB\rangle + s|BA\rangle$. This is called a “symmetric” state because it is unchanged if we swap the particles. For fermions there is also only 1 distinct state, which is $s|AB\rangle - s|BA\rangle$. This is called an “antisymmetric” state because it acquires a minus sign if we swap the 2 particles:

$$\begin{aligned} \text{SWAP}(s|AB\rangle - s|BA\rangle) &= s|BA\rangle - s|AB\rangle \\ &= -(s|AB\rangle - s|BA\rangle) . \end{aligned}$$

If 2 (or an even number) of identical fermions combine to make a “composite” particle, then the result is a boson, because swapping 2 fermions yields 2 minus signs. This is why ordinary helium atoms (with 2 electrons + 2 protons + 2 neutrons) are bosons. ■

Questions to Consider

1. Think of the 2 most nearly identical macroscopic objects in your house. How could they in fact be distinguished?
2. If we have 3 identical particles (labeled 1, 2, and 3), then there are at least 3 different SWAP operations: SWAP(12), SWAP(13), and SWAP(23). Show how SWAP(23) can be created out of a combination of SWAP(12) and SWAP(13). Also show how to use these pairwise SWAPs to create the “cyclic” swap that takes $1 \Rightarrow 2 \Rightarrow 3 \Rightarrow 1$.
3. A composite particle of several fermions can act like a boson. Can we have the opposite—a composite of bosons that acts as a fermion? If so, how? And if not, why not?

The Gregarious Particles

Lecture 12

We're going to be considering bosons, the symmetric ones, and examples of bosons include photons, the particles of light, helium atoms. ... Where did all this boson stuff come from originally? It's actually rooted in the same thing that quantum theory itself is rooted in; remember the original impetus for quantum theory was to explain black body radiation.

Because of the boson rule, 2 identical bosons can exist in the same state. In fact, they prefer it that way. We consider a pair of particles in 3 boxes: A, B, and C. A single particle has basis states $|A\rangle$, $|B\rangle$, and $|C\rangle$. A pair of distinguishable particles has 9 basis states:

$$\begin{aligned} &|AA\rangle, |AB\rangle, |AC\rangle, |BA\rangle, |BB\rangle, |BC\rangle, \\ &|CA\rangle, |CB\rangle, |CC\rangle. \end{aligned}$$

In $\frac{1}{3}$ of these states ($|AA\rangle, |BB\rangle, |CC\rangle$), the particles will be found in the same box. Just by chance, we would expect to find the particles together $\frac{1}{3}$ of the time.

If the 2 particles are bosons, there are fewer basis states, since the states must be symmetric under a particle swap. A pair of identical bosons has 6 symmetric basis states:

$$\begin{aligned} &|AA\rangle, s|AB\rangle + s|BA\rangle, s|AC\rangle + s|CA\rangle, |BB\rangle, \\ &s|BC\rangle + s|CB\rangle, |CC\rangle. \end{aligned}$$

In $\frac{1}{2}$ of these states, the particles are found in the same box. Just by chance, we would expect to find the particles together $\frac{1}{2}$ of the time, more often than we would a pair of distinguishable particles. Bosons have a “gregarious” streak, not because of some special force but simply because they are bosons. This effect gets stronger when more bosons are together.

The boson rule explains how a laser works. Einstein identified 3 ways that an atom can interact with a photon. An atom can absorb a photon, if one is present with the right energy. The atom jumps to an excited state. Alternatively, an atom already in an excited state can emit a photon spontaneously, which then emerges in some random direction. Another possibility is stimulated emission: Suppose we have an excited atom, and there are already some photons present that are moving in a particular direction. Because photons are bosons, the atom has a greater probability of adding its own photon to this group.

**Stimulated emission
is what enables us
to build lasers.**

Stimulated emission is what enables us to build lasers. Here is the simplified version of how it works: First, get a lot of atoms together. Add some energy so that most of the atoms are excited. This is called “optical pumping.” We need to have more excited atoms than unexcited ones—called a “population inversion”—since otherwise absorption will defeat us. Next, make sure that we have some photons around that are moving in a particular direction. This is usually done by bouncing the light we want back and forth with mirrors. Because photons are bosons, lots and lots of photons will be emitted in that same direction. (“laser” stands for “light amplification by stimulated emission of radiation.”) The result will be a highly directional beam of light having just 1 wavelength. This is called “coherent light.”

The boson rule also explains some amazing low-temperature phenomena. Our first example is superfluid helium. Helium atoms are bosons. Helium gas liquefies at about 4° above absolute zero, and the resulting liquid is called Helium I. At about 2° above absolute zero, helium forms a superfluid (Helium II). A superfluid can flow without any friction, leak through tiny pores less than 1 millionth of a meter across, and literally “creep” out of an open container. The superfluid state represents trillions of helium atoms in a single quantum state—a macroscopic example of the quantum gregariousness of bosons.

Our second example of amazing low-temperature phenomena is superconductivity. In metals, electric current is carried by the flow of electrons. But there is some friction in the form of electrical resistance, which

is why a current-carrying wire can heat up. Under some circumstances, the electrons can combine into “Cooper pairs.” Cooper pairs can carry electric current, but they are bosons. Near absolute zero, Cooper pairs flow as a superfluid in the metal. Electric current can be carried with zero resistance! This is called “superconductivity.” If we set up an electric current in a superconducting circuit, it will continue to flow for millions of years without any addition of energy. This has many technological applications, especially to make powerful electromagnets. A lot of research involves looking for superconductors that work at higher temperatures.

Our third example is a Bose-Einstein condensate. In this example, a supercold cloud of atoms can be created in which thousands or millions of atoms are in exactly the same quantum state. These atoms act like a single quantum system. This state of matter was first predicted in 1925 but was not created in the lab until 1995. ■

Questions to Consider

1. Three particles each have 4 basis states $|A\rangle$, $|B\rangle$, $|C\rangle$, and $|D\rangle$. If the particles are distinguishable, how many 3-particle basis states are there? If they are identical bosons, how many basis states are there?
2. Helium III is a rare isotope of helium that only has 1 neutron in its nucleus, so that helium III atoms are fermions. Nevertheless, at extremely low temperatures (only $\frac{1}{400}$ of a degree above absolute zero) it is observed that helium III can become a superfluid. How is this possible?

Antisymmetric and Antisocial

Lecture 13

Last time we talked about bosons and their curiously gregarious behavior. It was a lecture full of laser physics and exotic states and super cold matter. Lots of particles were always doing the same thing. ... This time we're going to discuss the other kind of quantum particle, the fermions. The fermions include electrons, protons, and neutrons. They're anything but rare. ... They are very different from bosons.

Because of the fermion rule, 2 identical fermions can never exist in the same state. Consider again 2 particles in 3 boxes: A, B, and C. For a pair of identical fermions, there are only 3 antisymmetric basis states:

$$s|AB\rangle - s|BA\rangle, s|AC\rangle - s|CA\rangle, s|BC\rangle - s|CB\rangle.$$

We cannot have states with the fermions in the same box, because those states cannot be antisymmetric: $s|AA\rangle - s|AA\rangle = 0$, which is no state at all.

This is the basis of the “Pauli exclusion principle” discovered by Wolfgang Pauli in 1925 as he investigated atomic structure. Pauli said that no 2 electrons can be in exactly the same quantum state. The same principle holds for any sort of fermion (e.g., protons and neutrons). Fermions are antisocial simply because they are fermions—no actual “repelling forces” are involved.

The exclusion principle for electrons explains many of the properties of ordinary matter. The structure of atoms with many electrons depends on Pauli's principle. An atom has various energy levels corresponding to standing wave patterns. If electrons were bosons, they could all just collect in the bottom level. The Pauli exclusion principle means that the electrons can “fill up” the lower rungs on the ladder. Note that, since electrons also have spin, there can be 2 electrons for each standing wave pattern. The chemical properties of the various elements depend on how the electrons have filled the energy levels. Generally, only the outermost electrons (on the top rungs) are involved in chemical reactions.

The structure of atomic nuclei works in a similar way. There are 2 kinds of fermions involved: protons and neutrons. Both are called nucleons. The way that nucleons fill their nuclear shells determines nuclear properties. For instance, certain numbers of nucleons make unusually stable nuclei, while others make unstable nuclei.

The Pauli exclusion principle explains why matter occupies space.

The Pauli exclusion principle explains why matter occupies space. A gas is easily compressible. It is not very hard to push twice as much gas into the same volume. A liquid or a solid is much, much less compressible. It is almost impossible to push twice as much material into the same volume. Why is solid matter solid? Electric repulsion between electrons cannot be the whole story, since ordinary matter contains both positive and negative charges and thus attracts and repels the same amount. To push 2 solid objects into the same volume, we would have to add more electrons into the same region of space. To do this, we must give the electrons a very high energy, since all of the low-energy states in that volume are already occupied. Thus, it takes a lot of energy to get twice as many electrons into the same space. The Pauli exclusion principle affects almost everything we see around us. ■

Questions to Consider

1. Three particles each have 4 basis states $|A\rangle$, $|B\rangle$, $|C\rangle$, and $|D\rangle$. If the particles are distinguishable, how many 3-particle basis states are there? If they are identical fermions, how many basis states are there?
2. Look around the room and begin to make a list of the phenomena you can see that are directly affected by the Pauli exclusion principle. (You may stop your list after you reach a dozen items. That should not take long!)

The Most Important Minus Sign in the World

Lecture 14

I want to tell you the story of a mathematical idea and what [it] means for the quantum world. What mathematical idea? It's a minus sign. A minus sign seems like a pretty minor piece of mathematical paraphernalia. ... In quantum mechanics, a minus sign can make the difference between constructive and destructive quantum interference and that's not a trivial matter.

What's the difference between bosons and fermions? At the fundamental level, bosons and fermions differ only in a single minus sign. For a system of identical bosons, $\text{SWAP} |state\rangle = |state\rangle$. For a system of identical fermions, $\text{SWAP} |state\rangle = -|state\rangle$. Yet the difference between bosons and fermions is extremely important. Bosons are more likely to be found together, fermions less likely. Physicists sometimes say that bosons and fermions have different “statistics”—Bose-Einstein statistics versus Fermi-Dirac statistics. Boson properties are especially important for light and for matter at low temperatures, while fermion properties, especially the Pauli exclusion principle, determine atomic structure, chemical properties, nuclear structure, the solidity of matter, etc. This is undoubtedly the most important minus sign in the universe!

But where does it come from? Nature provides a clue: There is a link between a particle's spin and the swapping rule it obeys. Physicists call this the “spin-statistics connection.” Bosons always have spin 0, spin 1, spin 2, etc. Fermions on the other hand always have spin $\frac{1}{2}$, spin $\frac{3}{2}$, etc.

Let's revisit spin and rotation. Richard Feynman created a useful “magic trick” based on an idea of Dirac's. In the trick, two pencils are connected by a flexible ribbon. Start with the ribbon untwisted. Rotate 1 pencil by 360° , which is 1 full turn. Now the ribbon is twisted, and it stays twisted even if we shift it around in space. Now start again with the ribbon untwisted. Rotate 1 pencil by 720° , 2 full turns. The ribbon appears twice as twisted—but this twist is not real, since we can remove it simply by shifting the ribbon around.

The moral of this story is that a 360° rotation is not the same as no rotation—but a 720° rotation is!

What does this have to do with quantum mechanics? Recall the quantum physics of a spin- $\frac{1}{2}$ particle. When we rotated a spin- $\frac{1}{2}$ particle by 360° ($4 \times 90^\circ$), we wound up with an unexpected minus sign in the quantum state:

$$|\uparrow\rangle \Rightarrow |\rightarrow\rangle \Rightarrow |\downarrow\rangle \Rightarrow -|\leftarrow\rangle \Rightarrow -|\uparrow\rangle.$$

This is part of a general rule about 360° rotation. For spin 0, spin 1, spin 2, etc., $\text{ROTATE}|state\rangle = |state\rangle z$. For spin $\frac{1}{2}$, spin $\frac{3}{2}$, etc., $\text{ROTATE}|state\rangle = -|state\rangle$.

The effects of this minus sign can be observed in a clever experiment. We can make a Mach-Zehnder interferometer that works with neutrons instead of photons. The neutrons enter in the upper beam and undergo state changes at the beam splitters.

$$|upper\rangle \Rightarrow s|upper\rangle + s|lower\rangle \Rightarrow |upper\rangle.$$

The neutrons are always detected by the upper neutron detector.

Neutrons have spin $\frac{1}{2}$. We can rotate the spin of the neutron by using a magnetic field. Suppose we rotate the spins by 360° , but only on the lower beam. This introduces a sign change for the $|lower\rangle$ state but not the $|upper\rangle$ state. Now,

$$\begin{aligned} |upper\rangle &\Rightarrow s|upper\rangle + s|lower\rangle \\ &\Rightarrow s|upper\rangle - s|lower\rangle \Rightarrow |lower\rangle. \end{aligned}$$

In this case, the neutrons are always detected by the lower neutron detector.

We can make a table relating the amount of rotation and the fraction of neutrons that are detected by the upper detector.

Rotation	0°	180°	360°	540°	720°
Upper beam	100%	50%	0%	50%	100%

To restore the original situation, we must rotate the neutrons by 720°. Electrons, protons, and neutrons see a “720° world.” This is very difficult to imagine!

The spin-statistics connection is as follows: Spin- $\frac{1}{2}$ fermions have 2 mysterious minus signs, with 1 for particle swapping and 1 for 360° rotation. In fact, these are the same minus sign!

Electrons, protons, and neutrons see a “720° world.” This is very difficult to imagine!

We return to the Feynman magic trick with 2 pencils connected by a flexible ribbon. If we start with the ribbon untwisted and swap the positions of the pencils, the ribbon becomes twisted. To restore an untwisted ribbon, we have to rotate 1 pencil by 360°. The pencils represent 2 identical particles with spin.

Swapping the particles involves an easy-to-miss relative rotation by 360°, which is revealed by the twist in the ribbon. This leads to the minus sign in the fermion rule!

We have not exactly “explained” the most important minus sign in the universe. However, we do understand much better what it means and why there is a connection between spin and statistics. ■

Questions to Consider

- Use a ribbon or a belt to create your own version of the Feynman ribbon trick and try the following experiments. In each case, you should find out whether the belt ends up twisted or not. (Remember that a ribbon might appear to be twisted when in fact it can be straightened out by simply shifting it around.)
 - Each end of the ribbon is individually rotated by 180° in the same direction.

- (b) The ends are exchanged by rotating the whole setup by 180° around a central point, then unrotating each end individually to restore their original orientation. (In the lecture, we did not rotate the ends as we did the exchange.)
- (c) Each end of the ribbon is individually rotated by 360° in the same direction.
2. In the neutron interferometer, suppose the neutrons enter with spin $|\uparrow\rangle$ and then the lower beam spin is rotated by 180° . The upper and lower beams now have distinct spins $|\uparrow\rangle$ and $|\downarrow\rangle$. The spin state thus amounts to a “measurement” of which beam the neutron is in, and thus there should be no interference effects. How does this analysis compare with the results we described?

Entanglement

Lecture 15

In this lecture, we're going to talk about the quantum mechanics of composite systems, systems that are composed of 2 or more individual particles. ... We skimmed this topic for our discussion of identical particles. ... [Now] we're going to follow this road to a different destination. It's going to lead us to the idea of quantum entanglement ... a key feature of the quantum world.

As mentioned above, a composite system composed of 2 or more particles can have “quantum entanglement.” What states are possible for a pair of particles? Assume that they are distinguishable in some way, so that we can designate them #1 and #2. “Simple states” arise when each particle has a state of its own. If the state of #1 is $|U\rangle$ and the state of #2 is $|V\rangle$, then the state of the composite system is just $|UV\rangle$. (Note that $|U\rangle$ and $|V\rangle$ do not have to be basis states.) Simple states work like multiplication and thus are sometimes called “product states.” If particle #1 is in the state $|U\rangle$ and particle #2 is in the state $a|V\rangle + b|W\rangle$, then the composite state is:

$$|U\rangle \text{ "times" } a|V\rangle + b|W\rangle = a|UV\rangle + b|UW\rangle.$$

This fact is called the “composition rule” and is the last of our basic rules of quantum mechanics.

Not every state of the 2 particles is a simple state. The ones that are not are “entangled states,” or states with entanglement. (We may also say that the particles themselves are entangled or have entanglement.)

Entangled particles display some interesting features. First, if 2 particles are in an entangled state, neither particle has a definite quantum state of its own, but the pair does. This is a strange situation. In classical physics, every particle has its own state—its own position and momentum—no matter what. Also, if we measure 1 particle, update rule II applies at once to both particles, even if they are far apart.

A very useful example of entanglement is a pair of spin- $\frac{1}{2}$ particles in a total spin 0 state. The total spin 0 state looks like this: $s|\uparrow\downarrow\rangle - s|\downarrow\uparrow\rangle$. (The minus sign is important.) We can arrange for 2 spins to be in such a state. For example, the spins of the electrons in a helium atom in its ground state are in a total spin 0 state.

The total spin 0 state has 2 key properties. First, for any spin-axis measurement on 1 spin, the probability of either result is always $\frac{1}{2}$. Second, if we measure both spins along the same spin axis, we must always get opposite results, since the total spin is 0.

For example: If we measure the z -axis on spin #1 and get the result $|\uparrow\rangle$, we must immediately assign the state $|\downarrow\rangle$ to spin #2; if we measure the x -axis on #1 and obtain $|\rightarrow\rangle$, we must immediately assign the state $|\leftarrow\rangle$ to #2; and so on.

Quantum entanglement became the focus of the last stage of the Bohr-Einstein debate. After 1930, Einstein accepted that quantum mechanics is consistent. However, he still did not regard it as a complete description of nature.

Einstein thought that there must be things in nature that are real but are not described by quantum mechanics. In 1935, Einstein, Boris Podolsky, and Nathan Rosen (EPR for short) wrote one of the most consequential papers in history: “Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?”

In this paper, EPR called attention to the strange nature of quantum entanglement. Before a measurement on spin #1, spin #2 does not have a definite state. Particle #2 gains a definite state instantly—even if it is far away—when the measurement is made on #1. (Einstein called this “spooky action at a distance.”) How do we know when something is real? EPR gave their answer: If we can predict something about a system without interacting with the system in any way, then that something must be real. Quantum mechanics says that no spin can have definite values of x and z at the same

How do we know when something is real? EPR gave their answer: If we can predict something about a system without interacting with the system in any way, then that something must be real.

time. In quantum physics, the values of x and z cannot simultaneously be real.

The EPR experiment worked like this: 2 spins are created in a total spin 0 state. If we measure x on #1, then we know the value of x on #2. (It must be opposite.) If we measure z on #1, then we know the value of z on #2. (It must be opposite as well.) Without “touching” spin #2 in any way, we can determine either its x value or its z value. By the EPR criterion, both x and z must be real. Therefore, quantum mechanics is not a complete description of nature!

Bohr’s reply was rather tricky and hard to understand. He argued that we must regard the 2 entangled particles as a single system, not 2 systems. The x and z measurements on #1 are still complementary. We cannot make both measurements at once, and so we cannot actually know both x and z for particle #2 at the same time. If we measure 1 of the 2, we cannot say what would have happened if we had measured the other. (Recall Peres’s motto: “Unperformed experiments have no results!”) There is no “action at a distance,” as Einstein called it. But there is a sort of “complementarity at a distance” in the entangled system, and this knocks down the EPR argument. Einstein, however, was not convinced.

The final round of the Bohr-Einstein debate seems inconclusive. Among the remaining questions are these: Is the EPR argument correct? That is, does quantum mechanics demonstrate its own incompleteness? Do quantum variables like x and z really have definite (though hidden) values? Can a particle affect another instantaneously at a distance? Or is Bohr’s subtle rejoinder correct? ■

Questions to Consider

1. A coin is sliced into 2 thinner pieces, each piece bearing 1 of the coin’s faces. The 2 half coins are randomly put in 2 envelopes and mailed to 2 separate locations. Before an envelope is opened, we do not know whether it contains the head or the tail, but afterward we know the contents of both envelopes. What does the EPR argument say about this

situation? How are the properties of the total spin 0 state different from the half coin experiment?

2. Write the total spin 0 state $s|\uparrow\downarrow\rangle - s|\downarrow\uparrow\rangle$ in terms of the basis states $|\rightarrow\rangle$ and $|\leftarrow\rangle$. You will need to write $|\uparrow\rangle$ and $|\downarrow\rangle$ in terms of $|\rightarrow\rangle$ and $|\leftarrow\rangle$, then use the composition rule. This calculation involves a bit of work, but the final result is satisfying!

Bell and Beyond

Lecture 16

Bohr responded to the EPR argument. ... Based on his critique, we concluded that the EPR argument was not airtight. So the matter stood for almost 30 years. Then, in 1964, John Stewart Bell, an Irish particle physicist, reconsidered the EPR argument, and thereby changed the world. ... He showed that the physics of quantum entanglement actually leads us to a very different conclusion about the nature of reality.

In his paper, John Bell carefully analyzed the EPR argument. He noted that it includes 3 identifiable propositions about the world. One proposition is that quantum mechanics correctly predicts the behavior of entangled states—specifically, the total spin 0 state of 2 particles, which we can check by experiment. Another proposition pertains to hidden variables; it says that the results of measurements are actually predetermined. We use probabilities only because we lack detailed information about the hidden variables that determine the results. The third proposition relates to locality. Specifically, the behavior of any particle is locally determined—that is, it is governed only by the particle’s own variables and the immediate circumstances, including any measurement apparatus. According to Bell, the EPR argument can be summarized this way:

entanglement + locality \Rightarrow hidden variables.

Bell decided to try something different: Assume both locality and hidden variables, then study the consequences for entanglement.

Bell derived an inequality that any “local hidden variable theory” must satisfy. We imagine 2 spins: #1 and #2. On #1 we measure spin components A or B, and on #2 we measure spin components C or D. This gives us 4 possible joint measurements: (A,C), (A,D), (B,C), and (B,D). Let $P(A = C)$ be the probability that A and C measurements give the same results (either both $+\frac{1}{2}$ or both $-\frac{1}{2}$). In a similar way, define $P(B = D)$, etc.

Assume that there are hidden variables and that locality holds. A particular example illustrates Bell's argument. A, B, C, and D all have values every time we do the experiment, even though we only find out some of the values. Furthermore, by locality, the value of A on #1 does not depend on whether we are measuring C or D on #2. Assume the following: $P(A = C) = 0.85$, $P(C = B) = 1.00$ and $P(B = D) = 0.85$. What can we say about $P(A = D)$? Our assumptions mean that B and C are always the same, so A agrees with B 85% of the time. (We can conclude this even though we never measure both A and B together.) If B agrees with D 85% of the time, then A must agree with D at least 70% of the time. Therefore, $P(A = D) \geq 0.70$. This is a special case of Bell's inequality.

Quantum systems can violate Bell's inequality. We create our 2 spins in a total spin 0 state. The probability of agreement between 2 spin measurements A and C depends on the angle α between the axes. By applying the quantum rules and what we have already learned about spin, we arrive at the following table:

α	0°	45°	90°	135°	180°
$P(A = C)$	0.00	0.15	0.50	0.85	1.00

We can choose the 4 spin axes so that the AC angle is 135° , the BC angle is 180° , the BD angle is 135° and the AD angle is 45° . This will satisfy our assumptions about $P(A = C)$, $P(C = B)$, and $P(B = D)$. However, quantum mechanics predicts that $P(A = D) = 0.50$, which is less than 0.70. Quantum entanglement violates Bell's inequality!

Bell finds the fatal flaw in EPR. The 3 propositions—entanglement, hidden variables, and locality—cannot all be true at the same time!

Therefore, the latter 2 cannot imply the first. Experiments confirm quantum mechanics, even when the 2 spins are very far apart. Therefore, we must either give up determinism (hidden variables), or we must imagine that entangled particles can influence each

Bell finds the fatal flaw in EPR. The 3 propositions—entanglement, hidden variables, and locality—cannot all be true at the same time!

other instantaneously over great distances (faster than the speed of light), or both! Bohr would have said that the hidden-variables assumption is flawed because of complementarity. Bell himself preferred to say that quantum mechanics was “nonlocal.”

A postscript: Einstein died in 1955, Bohr in 1962. Neither of them got to see the surprise twist in the debate about EPR. ■

Questions to Consider

1. In a classroom simulation of the Bell experiment, 2 students are given separate instructions and sent to opposite ends of campus. The students then answer yes or no to questions posed to them. Student 1 is asked either question A or question B, and student 2 is asked either question C or question D. After doing the experiment many times, we find that the answers to A and C agree 85% of the time, as do the answers to B and D, while B and C answers always agree. What can we say about how often A and D agree?
2. Now suppose the 2 students are provided with radios so that they can coordinate their answers “instantaneously” at a distance. Can we draw the same conclusion? Is this a fair representation of the Bell experiment?

All the Myriad Ways

Lecture 17

Now [we're] going to turn our attention to another important part of quantum theory. We're going to talk about a new way of looking at the theory. It's a way that has proved to be terribly important in making detailed theories about elementary particles in the present day.

In the 1940s, Richard Feynman devised a startling new way to look at quantum mechanics. The new perspective that he provided stemmed from his answer to the question of how an electron travels from point A to point B. Specifically, he looked at the question of what determined the probability $P(A \rightarrow B)$ that an electron makes the trip. According to Schrödinger, who also had looked at this question, the electron's quantum wave travels through space, and the intensity of the quantum wave determines the probability of finding the particle there. Must we imagine that the electron somehow "solves" Schrödinger's wave equation? That would be a pretty smart electron!

Feynman says this is how it works:

- Write down all of the possible ways (paths) to get from A to B.
- Assign an amplitude to each path according to a simple rule. (We will skip the details.)
- Add up all of the amplitudes for all paths to get a total amplitude $A(A \rightarrow B)$. This adding of amplitudes may involve a lot of constructive or destructive interference.
- The total probability is just $P(A \rightarrow B) = |A(A \rightarrow B)|^2$.

In this scenario, the electron does not have to be smart; it simply tries everything, and the amplitudes add up. This is called the "sum-over-histories"

approach to quantum mechanics. The general idea of the sum-over-histories approach can be illustrated by our favorite example, the photon in a Mach-Zehnder interferometer.

Feynman's idea turned out to be extremely useful for working out the quantum interactions between electrons and light—the field of quantum electrodynamics (QED). He drew little cartoons to represent possible histories of electrons and photons. These are called Feynman diagrams. In these cartoons, time points upward—the future is at the top, the past at the bottom. Solid lines pointing up represent electrons. Solid lines going down represent “positrons,” which are antiparticles to electrons. Positrons and electrons have the same mass and spin but opposite electric charge. In QED, a positron is an electron going “backward in time.” Wavy lines represent photons. These are either “real photons” (wavy lines that come out of the diagram), which can be detected, or “virtual photons” (wavy lines that begin and end within the diagram), which are not directly detectable. A “vertex” represents an event where a photon is created or destroyed by an electron or a positron. Feynman gave a mathematical rule for assigning amplitudes to each diagram. More complex diagrams (with more vertices) make smaller contributions, so we can often just consider the simplest ones.

He drew little cartoons to represent possible histories of electrons and photons. These are called Feynman diagrams.

QED gives a quantum description of the electrical repulsion between electrons. The simplest diagram involves an exchange of a virtual photon. Where does the energy for this photon come from? A usable though imperfect answer is that we can “borrow” energy ΔE for a time Δt without violating any laws, provided we stay below the “uncertainty limit.” Thus $\Delta E \Delta t < \hbar$. Virtual photons can be exchanged even over long distances because photon energy can be as small as we like. However, the resulting force will be weaker at large distances.

QED also describes the collision of a photon and electron, called “Compton scattering.” There are several possible diagrams for this process, and all contribute to the quantum amplitude for it. The most important ones have 2 vertices. In these diagrams, the electron may absorb the incoming photon,

then emit the outgoing photon. It may also emit the outgoing photon first, then absorb the incoming one. Alternatively, the incoming photon may create an electron-positron pair, and then the positron annihilates the incoming electron.

To get more precise results in QED, we must simply include more and more complicated diagrams in the calculation. There might be a lot of these. Electrons continually emit and absorb virtual photons. This changes their observed properties. Mathematically, the process can lead to an apparently infinite result. However, in the not-quite-magic procedure called “renormalization,” the infinities can be persuaded to cancel out, leaving only the finite answer.

QED is the most accurate physical theory ever developed. It predicts certain phenomena, like the magnetic properties of the electron, to about 1 part in 1 trillion (1 in 10^{12}). QED is also the prototype for modern theories of fundamental forces. All forces are carried by the exchange of virtual bosons of one sort or another. For nuclear forces, the exchanged particles have mass, which means there is a lower limit to the energy ΔE that must be “borrowed” to make them. These forces act only over very short ranges. ■

Questions to Consider

1. Feynman regarded his sum-over-histories way of thinking as simply an extension of the quantum 2-slit experiment. Give an explanation of that experiment in Feynman’s terms.
2. Consider the Compton scattering process, in which a photon “bounces off” an electron. Draw several Feynman diagrams for this process. How many different diagrams can you find with exactly 3 vertices? Four vertices?
3. Japanese physicist Hideki Yukawa proposed in the 1930s that the nuclear force between protons and neutrons is carried by particles. From the observed short range of the nuclear force, he deduced that these particles had to have about 200 times the mass of an electron. Explain how such a deduction was possible. (Yukawa was proved correct a decade later with the discovery of the pi meson, or “pion.”)

Much Ado about Nothing

Lecture 18

Swarms of virtual particles are present in these [Feynman] diagrams. They come and go unobservably, underneath the limit set by the time-energy uncertainty principle. ... This time we're going to analyze what is going on in so-called "empty space," and we'll find that the quantum mechanical answer is quite a lot is going on; and that this fact, that there's a lot going on where nothing appears to exist, has enormous consequences.

At its absolute minimum energy, a quantum system still has some energy in it. This is called "zero-point energy." One way to think about this idea is to consider both classical and quantum pendulums. A classical pendulum has energy both in its motion (kinetic energy) and by virtue of its displacement from the bottom (potential energy). If it is exactly at rest at the bottom point, its energy is zero. This cannot be true for a quantum pendulum. To be exactly at rest at the bottom point, we would need both Δx and Δp to be 0. The uncertainty principle forbids this. Even in its ground state, the lowest energy level, the quantum pendulum has a zero-point energy of $hf/2$, where f is the pendulum's frequency. This is half of a "quantum of energy" for the oscillating system.

Zero-point energy can make a real difference. One example is the strange difficulty of freezing helium. Almost any substance will freeze if it is made cold enough. Molecules are slightly "sticky" due to the van der Waals force, so if they are moving slowly enough they will stick together and be "frozen" in place. Helium condenses into a liquid near absolute zero. But helium atoms have a very low mass and the van der Waals force between them is extremely weak. This means that, even at absolute zero, helium atoms have enough zero-point energy to prevent freezing. It is possible to freeze helium, but only by imposing very high pressures to make up for the lack of "stickiness" between the atoms.

Even "empty space"—the vacuum—has quantum zero-point energy. In the electromagnetic field, energy comes in the form of photons. Even with zero

photons—the vacuum state—the electromagnetic field has zero-point energy. The vacuum is filled with electromagnetic fluctuations at all frequencies. Spontaneous emission of a photon from an atom can be viewed as “stimulated emission” by the quantum fluctuations of the vacuum.

In 1948, Hendrik Casimir discovered a way to observe vacuum energy directly. The presence of metal objects slightly reduces the number of ways that the vacuum can fluctuate. The vacuum state is distorted to “fit around” the objects. Between 2 parallel metal plates, the vacuum fluctuations are reduced. Therefore, there is less vacuum energy (less “nothing”) between the 2 plates than there is outside of them. This leads to a tiny attractive force between the plates called “the Casimir effect.” This effect was soon detected experimentally, but the first really accurate measurements had to wait for the 1990s.

Vacuum energy may have cosmic implications. In 1998, cosmologists learned that the expansion of the universe is actually getting faster over time. Not only are galaxies getting further apart—an aftereffect of the Big Bang—they are doing so at an increasing rate. This was a surprise—simple gravity would suggest that the expansion should be slowing down, not accelerating. The physical cause of the acceleration is called “dark energy”—“dark” because we do not see it directly; “energy” to distinguish it from “dark matter,” which is matter of an unknown type that is also present but has a different effect.

One leading hypothesis is that the dark energy is quantum vacuum energy, the energy of empty space. As space expands, more dark energy appears, driving the expansion faster. One major difficulty with this idea is that, if we plug in some obvious numbers, there should be a lot of vacuum energy—an amount that is much, much, *much* too large to account for the dark energy. We have to assume that the vacuum energy is almost, but not exactly, irrelevant to the cosmic expansion. On the other hand, cosmologists believe that, immediately

One leading hypothesis is that the dark energy is quantum vacuum energy, the energy of empty space. As space expands, more dark energy appears, driving the expansion faster.

after the Big Bang, the universe experienced a short period of superfast expansion called “cosmic inflation.” Vacuum energy could well account for this. ■

Questions to Consider

1. As we saw in Lecture 5, a stretched wire can vibrate in standing wave patterns at many different frequencies. Explain why such a wire can never be absolutely still, even at its minimum possible energy.
2. In some highly speculative cosmological theories, the entire visible universe had its origin as a quantum “fluctuation” in a primordial quantum vacuum. Does this really count as “making the universe out of nothing?” (Does a quantum vacuum really count as “nothing?”)

Quantum Cloning

Lecture 19

With this lecture we're starting Section 4 of our course, in which we will explore a contemporary topic in quantum mechanics research: quantum information and quantum computing—my own field of research specialty. ...Our question is how can we use quantum systems to store, retrieve, transmit, and process data?

We can use single photons, atoms, and electrons to perform our tasks. In this part of the course, we will think about and understand the limitations imposed by quantum physics as well as the opportunities it affords. This is not really a question about futuristic technology. It is mostly a deep question about nature. Rolf Landauer said, "Information is physical." All information is related to physical states and physical processes of physical systems. We will consider what quantum physics can tell us about the basic concept of information.

Classical and quantum information are alike in many ways. Classical information is the type of information that can be stored in classical (macroscopic) systems. This is the sort of information that we are familiar with in everyday life. It can be changed from one physical form to another. If we consider just the classical information generated by and corresponding to this lecture, we see the diverse range of forms it can take. For this lecture, classical information includes light and sound in the studio, electrical signals in the camera, magnetic patterns on a videotape, tiny dimples on a DVD, reflected laser light in a DVD player, more electrical signals, and finally light and sound again. Yet the information remains the same throughout.

The basic unit of classical information is the "bit." A bit is a binary digit, which can be either 0 or 1. This can stand for "yes" or "no," "on" or "off," etc. We can use different physical systems to represent bits, and any sort of information can be encoded into a series of bits. We can use bits to measure "how much information" something contains. How many bits do we need to store a novel? A nice photograph? A minute of music on my digital

player? All of these have an information content of about 1 megabyte (8 million bits).

On the other hand, quantum information is the type of information that can be stored in quantum systems. Like classical information, we can transform the physical form of quantum information. The basic unit of quantum information is the “qubit.” A qubit is a quantum system with just 2 basis states. (We have seen a couple of examples already: a single photon in an

The fundamental difference between classical and quantum information is that, while quantum information cannot be exactly copied, we can always in principle copy classical information.

interferometer and a spin- $\frac{1}{2}$ particle.) We can call the basis states $|0\rangle$ and $|1\rangle$. In addition to the basis states, a qubit may be in any superposition state $a|0\rangle + b|1\rangle$. If we have more than 1 qubit, they can be entangled with each other. Qubits have lots of possibilities!

Qubits can be used to send classical information, if we wish. For example, Alice wishes to send Bob a 1bit message (0 or 1). She prepares a spin-

$\frac{1}{2}$ particle in the state $|\uparrow\rangle$ for 0, $|\downarrow\rangle$ for 1. The spin is sent to Bob, who makes a z measurement and reads the message. But there are more spin states available. Can Alice send more than 1 bit in a single qubit? Suppose she wants to send a 2bit message. She encodes 00 by $|\uparrow\rangle$, 01 by $|\downarrow\rangle$, 10 by $|\rightarrow\rangle$ and 11 by $|\leftarrow\rangle$. All of these available states of a single spin. This message will not get through, because Bob cannot read it. He can correctly tell $|\uparrow\rangle$ from $|\downarrow\rangle$ using z , or $|\rightarrow\rangle$ from $|\leftarrow\rangle$ using x , but no measurement will let him distinguish all 4 spin states. The capacity of a qubit for sending classical information is 1 bit.

It is not so straightforward to send qubits via bits. This depends on exact definitions. At worst, it is impossible. At best, it will take very many bits to describe the exact superposition state $a|0\rangle + b|1\rangle$ of a single qubit.

The fundamental difference between classical and quantum information is that, while quantum information cannot be exactly copied, we can always in principle copy classical information. In classical physics, observing a system

need have no effect on it. By carefully measuring our bits, we can duplicate them exactly. The ability to copy classical information is a huge problem of copyright law, intellectual property, and privacy!

To consider how we would copy, or try to copy, quantum information, we imagine a “quantum cloning machine” that would take as input a single qubit and produce as output 2 qubits with exactly the original state: $|state\rangle \Rightarrow |state\ state\rangle$. Imagine that the cloning machine works for the states $|\uparrow\rangle$ and $|\downarrow\rangle$ of a spin- $\frac{1}{2}$ particle. That is, $|\uparrow\rangle \Rightarrow |\uparrow\uparrow\rangle$ and $|\downarrow\rangle \Rightarrow |\downarrow\downarrow\rangle$. For simplicity, assume there are no measurements, so that only update rule 1 applies. How does the cloning machine work for $|\rightarrow\rangle = s|\uparrow\rangle + s|\downarrow\rangle$?

$$|\rightarrow\rangle = s|\uparrow\rangle + s|\downarrow\rangle \Rightarrow s|\uparrow\uparrow\rangle + s|\downarrow\downarrow\rangle.$$

But this result is an entangled state of 2 spins, not the product state $|\rightarrow\rightarrow\rangle$ we wished for. The quantum cloning machine therefore has to fail for some input states!

In 1982, the “quantum no-cloning theorem” was proved in this way by William Wootters and Wojciech Zurek, and in a different way by Dennis Dieks. A perfect quantum cloning machine is impossible. Quantum information cannot be exactly copied.

We now move on to a science fiction story to illustrate the fact that if we *did* have a perfect cloning machine, then we could send 2 bits in 1 qubit. In our story, Alice sends Bob the 4 states $|\uparrow\rangle$, $|\downarrow\rangle$, $|\rightarrow\rangle$, $|\leftarrow\rangle$ as before. Bob uses a cloning machine to make 2000 copies. He now has: $|\uparrow\uparrow\uparrow\dots\rangle$, $|\downarrow\downarrow\downarrow\dots\rangle$, $|\rightarrow\rightarrow\rightarrow\dots\rangle$, $|\leftarrow\leftarrow\leftarrow\dots\rangle$.

Bob measures z on the first 1000 spins.

If he has $|\uparrow\uparrow\uparrow\dots\rangle$, he will obtain $+\frac{1}{2}$ all 1000 times.

If he has $|\downarrow\downarrow\downarrow\dots\rangle$, he will obtain $-\frac{1}{2}$ all 1000 times.

If he has $|\rightarrow\rightarrow\rightarrow\cdots\rangle$ or $|\leftarrow\leftarrow\leftarrow\cdots\rangle$, he will obtain $+\frac{1}{2}$ and $-\frac{1}{2}$ about 500 times each. Bob also measures x on the next 1000 spins. Combining his results, Bob can determine which of the 4 original states Alice sent and read her 2bit message.

Why can a qubit only convey 1 bit of classical information? Part of the answer lies in the quantum no-cloning theorem. ■

Questions to Consider

1. Write a paragraph that clearly explains to your Aunt Mary the essential difference between bits and qubits. (If your own Aunt Mary happens to be a quantum physicist, pick someone else's Aunt Mary.)
2. Think of some technical and legal methods by which we try to make it hard to copy certain kinds of classical information. (This is done for privacy, copyright, and other reasons.) Are any of them foolproof?

Quantum Cryptography

Lecture 20

Now, this time, we're going to see how quantum information and the no-cloning theorem play out in action. We're going to see how the laws of quantum physics will help us to keep secrets. We're going to talk about the subject of quantum cryptography. So, let's begin.

The science of cryptography is about keeping certain information private. To think about cryptography, we begin with an example involving Alice and Bob, two characters we met in our last lecture (and whose names also appear frequently in examples in journals of mathematical cryptography). In our example here, Alice wishes to send a message to Bob that cannot be read by any eavesdropper, whom we'll call Eve. They do this by agreeing on a secret code for their messages. Many codes can be "broken" by cryptanalysis. However, there is a type of secret code that cannot be broken, called a "1time pad." A 1time pad uses secret "key" information to encode the message. If Eve lacks the key, she cannot read the message. We can describe the 1time pad using strings of bits. There is a "plaintext string," a "key string," and a "ciphertext string." If Eve intercepts the ciphertext but lacks the key, she cannot read it. Bob, with the key, can decrypt the message and read the plaintext.

The big problem with this involves key distribution. If Alice and Bob use the same key over and over, it becomes insecure, and a clever Eve can begin to read their messages. They must only use the key once! (This is why it is called a 1time pad.) How can Alice send Bob a new key without Eve reading it? Alice might send the key in a tamper-proof box. Bob could check it for Eve's fingerprints, etc. But Eve might be able to make a copy of the key without leaving any traces, so that Alice and Bob

In 1984, Charles Bennett and Gilles Brassard showed how to use quantum mechanics to solve the problem of key distribution. Their idea, known as "BB84," marks the birth of "quantum cryptography."

would be fooled and think their key is still secret. No classical method of key distribution can be 100% safe from Eve.

In 1984, Charles Bennett and Gilles Brassard showed how to use quantum mechanics to solve the problem of key distribution. Their idea, known as “BB84,” marks the birth of “quantum cryptography.” We will use another example, with our stock characters, to describe the BB84 method.

In our example, Alice sends to Bob a series of spins, their states chosen randomly from the set $|\uparrow\rangle, |\downarrow\rangle, |\rightarrow\rangle, |\leftarrow\rangle$. Bob measures each spin, randomly choosing z or x for each. Then Alice and Bob talk on the phone. (Eve may be listening.) They do not say which states were sent, but they do discuss the measurements Bob made. Alice tells him which spins were measured using the “right” axis. They use the good ones for their secret key and throw out the others.

In our example, why can’t Eve intervene and learn the secret key? She cannot do so because she cannot simply make exact copies of the spins as they go from Alice to Bob. The no-cloning theorem prevents this. If she makes measurements on the spins, she is bound to choose the “wrong” axis a lot of the time. This will necessarily introduce errors at Bob’s end. If Alice and Bob compare a few hundred of their key bits over the phone, they can detect this. Eve must leave “quantum fingerprints.” BB84 works because of complementarity of x and z , plus the no-cloning theorem. ■

Questions to Consider

1. To use the BB84 scheme, Alice and Bob must individually generate some random sequences of zeros and 1s. Otherwise, if Eve can guess what sequences they are using, she can also guess their key. Make some suggestions for generating these random bits. (Extra points for using quantum physics to do it!)
2. Imagine that Eve possesses a quantum cloning machine that can perfectly duplicate qubit states. How can she use this magical device to “break” the BB84 quantum key distribution?

Bits, Qubits, and Ebits

Lecture 21

Every age in history has a basic metaphor ... a way of organizing our thinking about the world around us. ... And today ... we sort of inevitably think of the world as a huge network, a vast system of information exchange. ... What we want to do is we want to discover the fundamental rules of that information network. ... All of this is based on something called “information theory.”

“Information theory” is the mathematical theory of communication and related subjects, invented by Claude Shannon in 1948. The concepts of information theory include bits, codes, errors, and so on. The study has been vital to the development of telecommunications, computing, and many other fields. Information theory is all about information “resources” and information “tasks.” It focuses on which resources are required to perform a given task. These resources may include time, storage space, power, etc. Tasks may include storage of data, overcoming noise, and keeping a message private.

Shannon’s information theory does not take quantum mechanics into account. A “quantum information theory” would include quantum resources and tasks. We can identify 3 types of quantum resources: bits (Alice sends 1 bit of classical data to Bob), qubits (Alice sends 1 qubit of quantum data to Bob), and ebits (Alice and Bob share an entangled pair of qubits—like 2 spins in a total spin 0 state—which amounts to “1 bit of entanglement”). Bits and qubits are “directed resources” (Alice to Bob or Bob to Alice), but ebits are “undirected.” Together we will work to answer the question of how these different resources are related to each other.

Charles Bennett put together some simple principles about quantum information resources. We’ll call these “Bennett’s laws.” Each law is of the form $X \succeq Y$, which is read, “ X can do the job of Y .” This means that the resources labeled X can perform the same task as the resources labeled Y .

Bennett's first law says that 1 qubit \succeq 1 bit. We have already seen this in our example in which Alice can use a qubit to send a 1bit message to Bob. However, notice that 1 qubit $\not\preceq$ 2 bits because Alice cannot transmit 2 bits in 1 qubit.

Bennett's second law is that 1 qubit \succeq 1 ebit. This is also easy to understand. Alice can make a pair of spins in a total spin 0 state, then send 1 of the 2 entangled qubits to Bob. Qubits are the most capable of the 3 resources. We can use them for anything. We cannot send messages using only an ebit. This is because entanglement by itself cannot be used to send either classical or quantum messages, although it can assist in sending messages.

Qubits are the most capable of the 3 resources. We can use them for anything.

Bennett's third law says that 1 ebit + 1 qubit \succeq 2bits. This was discovered by Bennett and Stephen Wiesner in 1992 and is sometimes called "dense coding." We return to Alice and Bob to consider this law. Alice and Bob initially share an ebit (say, 2 spins in a total spin 0 state). Alice makes 1 of 4 possible rotations on her spin. These are either no rotation or a 180° rotation about the x , y , or z axes. Her choice of rotation represents a 2bit message: 00, 01, 10, or 11. Alice sends her qubit to Bob. Bob now makes a special measurement called the "Bell measurement" on the pair of qubits. From this, he is able to deduce which rotation Alice made—and thus he can read the 2bit message.

Dense coding appears to be very strange, because it seems that the 2 bits are carried by 1 qubit. However, there are 2 qubits involved, though 1 of them stays in Bob's possession the whole time. If we make a diagram of the process, it appears that some information has traveled "backward in time"!

Bennett's fourth law says that 1 ebit + 2 bits \succeq 1 qubit. Here is how this law plays out in another Alice-and-Bob example. The two initially share an ebit. In addition, Alice has a qubit that she'd like to transfer to Bob. Alice makes a Bell measurement on the 2 qubits she has. She sends the result to Bob as a 2bit classical message. Bob can use this information to choose a rotation for his qubit (either no rotation or a 180° rotation about the x , y , or z axis).

Afterward, his qubit is in exactly the same state as Alice's original was! This process, discovered in 1993 by Bennett and several co-workers, is called quantum teleportation.

It's important to note here that teleportation is about information transfer, not transportation. It is barely possible to do teleportation of 1 qubit in the lab. Teleporting the quantum information in a human being is at least 10^{27} times harder—and we would need a *lot* of entangled matter. Because of the no-cloning theorem, the original qubit is necessarily wiped out. Suppose Alice can send qubits to Bob only occasionally, but she can send classical bits at any time. They may store up ebits when quantum communication is possible, then send their qubits whenever they like using teleportation.

How much are different resources worth? If classical bits cost nothing, then qubits and ebits are worth an equal amount. We turn one resource into the other for free. If ebits cost nothing (an odd assumption), then the value of a qubit is exactly 2 bits. ■

Questions to Consider

1. It is impossible to send more than 1 bit of classical information using just 1 qubit. Why doesn't Bennett and Wiesner's dense coding disprove this rule?
2. Suppose we consider a situation with 3 protagonists: Alice, Bob, and Charles. At the outset, Alice and Bob share 1 ebit, as do Bob and Charles, but Alice and Charles do not share any entanglement. If the 3 can send only classical bits to each other, how can Alice and Charles end up with a shared ebit? Can this be done even if Charles is unable to communicate at all? (It is interesting to try to work out some basic rules of 3-party quantum information theory.)

Quantum Computers

Lecture 22

Our everyday language struggles to cope with the nature of quantum information. [In] this lecture we're going to explore the full power of quantum information. ... We will imagine a quantum computer, and we'll see what we can do with it.

Is quantum computing the future of computers? According to Moore's law, computer power is increasing exponentially over time. Roughly speaking, computer capabilities double every 2 years. Basic units of computers are growing smaller at about the same rate. They operate faster, using less energy. If Moore's law continues to hold, in a couple of decades we will be trying to use individual quantum particles for basic computer components. We will need to design quantum computers.

In a quantum computer, the memory elements are qubits. These can be in superposition states (not just $|0\rangle$ and $|1\rangle$), and huge numbers of qubits may be entangled together. While performing computations, a quantum computer operates without any measurements of any kind, even inadvertent ones. Its state therefore changes according to update rule I. (At the end of the computation, of course, we must make a measurement to read the output.) A quantum computer cannot merely do ordinary computations faster or with smaller components. It can do computations in fundamentally new ways, completely unlike any classical computer.

A quantum computer could solve some mathematical problems much more efficiently than a classical computer. In 1992, Richard Jozsa and David Deutsch proposed the Deutsch-Jozsa problem, which first showed that quantum computers could be more powerful than classical ones. The computer can evaluate a function $f(n)$, where n ranges from 1 to N . The value of the function is always either 0 or 1. We happen to know that the function is either constant (always 0 or always 1) or balanced (0 or 1 equally often). How many times must we evaluate f to determine which one it is? On a classical computer, we might have to evaluate f more than $N/2$ times to be certain. If the function is really hard to compute, this might take a while.

However, on a quantum computer, we can answer the question by evaluating f only once, on a superposition of all possible inputs.

In 1996, Lov Grover showed that a quantum computer could help solve the “inverse phonebook problem.” A phonebook is an alphabetical list of names, together with phone numbers. But suppose we only have the phone number and want to find the name. How many names do we have to look up to find it? Suppose there are 1 million entries. A classical computer would have to look up about 500,000 names on average, and 999,999 names in the worst-case scenario. A quantum computer could do the same job by consulting the phone book only 1000 times, each time looking up a superposition of all the names.

The most exciting application is “quantum factoring,” discovered by Peter Shor in 1994. In this application, we are given a very large number, perhaps with hundreds of digits. This number is the product of 2 smaller numbers. Can we find the factors? On a classical computer, this is a very hard problem. A 200-digit number was recently factored by hundreds of computers working together for over a year. A 500-digit number is so much harder that no imaginable computer could ever do the job. Shor proved that a quantum computer could factor integers very efficiently. A 500-digit number is only about 16 times harder than a 200-digit one. Because much modern cryptography is based on factoring, if someone invents a quantum computer, a lot of secret data will no longer be secret!

Can a quantum computer actually be built? Many scientists are working very hard to build one. Design ideas include atoms suspended in laser beams, nuclear spins in magnetic fields, superconducting loops near absolute zero, and single electrons in semiconductors. Even the best efforts so far involve just a few qubits working for a few seconds.

Many scientists are working very hard to build [a quantum computer]. Design ideas include atoms suspended in laser beams, nuclear spins in magnetic fields, superconducting loops near absolute zero, and single electrons in semiconductors.

The would-be builder of a quantum computer faces a fundamental dilemma. On the one hand, the computer must be extremely well isolated from the outside. Otherwise, stray molecules and photons would make inadvertent measurements of the computer's state, interrupting the magic of update rule I. On the other hand, the different parts of the computer must interact extremely rapidly with each other, so that the computation can be done. The good news is that we need not be perfect. By using "quantum error correction," the computer can tolerate a little outside interference. However, the bad news is that nobody knows how to resolve the fundamental dilemma.

In the mid-19th century, Charles Babbage designed mechanical equivalents of modern computers. His computing ideas were never put into real practice until the development of electronics. Our present ideas about how to do quantum computing may be the modern equivalent of Babbage's gears and wheels. ■

Questions to Consider

1. Your lecturer has a bet with a colleague about whether or not quantum computers will become practical within 20 years. Which way would you bet?
2. Suppose a working quantum computer became available tomorrow. What would be its main practical impact?
3. We said that the builder of a quantum computer faces a fundamental dilemma. Why does this same dilemma not apply to an ordinary "classical" computer?

Many Worlds or One?

Lecture 23

In the final 2 lectures of this course we will probe some philosophical issues about quantum mechanics. We'll ask: What does quantum mechanics mean? What does it tell us about the nature of reality? ... In this lecture we're going to examine three different ways that physicists have come to interpret the meaning of quantum mechanics.

Even though quantum mechanics is more than 80 years old (and some parts are more than 100), there is still a lot of debate about its interpretation. Physicists agree on how to use quantum mechanics. The question is what the theory is telling us about the nature of reality. Some issues are philosophical: Is the world really nondeterministic? Is a quantum state objective or subjective—something “out there” or “all in our heads”?

One key issue is the question of measurement. Measurement seems special. It forces us to use the probabilistic and instantaneous update rule II rather than the smooth and predictable update rule I. Yet any measurement apparatus is made of atoms. Why can we not treat it as just another quantum system?

There have been 3 main schools of interpretation: the Copenhagen interpretation, the hidden-variables interpretation, and the many-worlds interpretation. The Copenhagen interpretation is the standard approach. Championed by Bohr, this interpretation rests on the principle of complementarity. This interpretation says that the microscopic world does not really “exist” on its own, independent of an observer. You can sum up the idea here in this way: “No phenomenon is a phenomenon until it is an observed phenomenon.” In this interpretation, measurement is special because it is the process by which quantum things are amplified into macroscopic reality.

The line between “microscopic” and “macroscopic” may be drawn in various places. We can analyze at least some of the workings of a measurement

apparatus in a quantum-mechanical way. In the thought experiment of Schrödinger's cat, the cat is seemingly brought into a superposition state $a|\text{alive}\rangle + b|\text{dead}\rangle$. Eugene Wigner imagined his friend examining Schrödinger's cat. Does Wigner's friend now exist in a superposition state?

The Copenhagen interpretation has its drawbacks. It is not clear, for example, that orthodox approaches to quantum theory will be good enough for challenges like quantum gravity or quantum cosmology.

The hidden-variables interpretation is somewhat less popular. It was discussed most deeply by David Bohm, beginning in 1952. The work was based on an earlier idea of de Broglie, who thought that both the quantum wave and the quantum particle exist together. The wave acts to "pilot" the particle through space. Bohm was able to create a theory that would appear exactly like quantum mechanics in any experiment, but the particles always had definite positions and velocities at every given moment, and they moved in a complicated but deterministic way.

No phenomenon is a phenomenon until it is an observed phenomenon.

What about Bell and entanglement? Bell's argument means that Bohm's hidden-variable theory must work in a nonlocal way. That is, distant parts of the universe

can instantaneously affect each other. Bohm did not regard this as a flaw. He saw this as an expression of a large-scale cosmological order, something quite different from the "reductionist" ideas common to modern science. Relatively few quantum physicists subscribe to Bohm's ideas, though they continue to be discussed and developed.

The many-worlds interpretation was proposed by Hugh Everett III in 1957. The basic idea is that the measurement apparatus and observers are all quantum systems and that the whole universe always evolved according to update rule I. The quantum state thus evolves deterministically. The apparent randomness in quantum mechanics arises because we can only see part of the whole.

The many-worlds interpretation gives a strange account of measurement. Consider a simple universe containing an observer Joe and a spin- $\frac{1}{2}$ particle. The spin starts out in the state $a|\uparrow\rangle + b|\downarrow\rangle$, and Joe starts out in the state $|\text{Joe}_0\rangle$.

The initial state of the universe is

$$a|\text{Joe}_0, \uparrow\rangle + b|\text{Joe}_0, \downarrow\rangle.$$

Joe now measures z on the spin. This is an interaction (update rule I) that works like this on basis states:

$$|\text{Joe}_0, \uparrow\rangle \Rightarrow |\text{Joe sees "up," } \uparrow\rangle \text{ and } |\text{Joe}_0, \downarrow\rangle \Rightarrow |\text{Joe sees "down," } \downarrow\rangle.$$

The new state of the universe is

$$a|\text{Joe sees "up," } \uparrow\rangle + b|\text{Joe sees "down," } \downarrow\rangle.$$

In each branch of this superposition, Joe sees only 1 thing, and what Joe sees agrees with the state of the spin. But both branches are still present in the overall state of the universe. It is as if the world has split in 2, each branch invisible to the other. In the process of measurement, the observer becomes entangled with the observed.

The many-worlds interpretation is controversial but increasingly popular. One positive aspect of this interpretation is that it gets rid of any special measurement process and lets us apply quantum theory to the entire universe. This makes it attractive for physicists trying to develop a “theory of everything.” However, it does have problems. One key one is that it asserts the existence of vast numbers of unobservable branches other than what we see, which seems to violate the logical principle of Occam’s Razor. It involves another difficult puzzle as well: In our example, why does Joe see “up” with probability $|a|^2$ and “down” with probability $|b|^2$? Both branches are present, but why does he seem to experience them with this likelihood? The universe of the many-worlds interpretation contains all quantum possibilities in a vast, ever-more-complicated, stupendously entangled quantum superposition. ■

Questions to Consider

1. What do you find least satisfactory about each of the 3 main interpretations of quantum theory described in this lecture?
2. The principle of Occam's Razor has been invoked both to criticize the many-worlds interpretation (why imagine so many other worlds?) and to defend it (why imagine that the principle of superposition has any limits?). Which argument seems more sensible to you, and why?
3. Try to imagine how Bohr and Einstein might have responded to the many-worlds interpretation of quantum mechanics. Write a short fictional dialogue between them, discussing the idea.

The Great Smoky Dragon

Lecture 24

The quantum realm is a wonderfully strange place. Indeed, we are not entirely sure just what kind of a place it is. There is significant disagreement about the meaning of quantum mechanics. ... In this final lecture, I would like to reflect on what it is that makes quantum mechanics so strange and so mysterious.

We return to our example of the photon in the Mach-Zehnder interferometer. In this example, interference challenges our intuition. Block either beam, and either detector might register the photon. Leave both beams open, and only 1 detector can possibly register it. Interference can occur when no measurement is made of which path the photon travels. This leads us to an astonishing point: When interference occurs, no physical record is made *anywhere in the universe* of the path of the photon. The photon on its journey is “informationally isolated” from the rest of the universe. Remember, “Quantum mechanics is what happens when no one is looking.”

The “magic” of quantum mechanics is like a stage magician’s trick box. With the box, we believe that we could peer inside and find out how the trick works. With the quantum mechanics box, the trick only works when the box is absolutely closed. We cannot find out how it works, even in principle.

Our description of the magic in the box is quantum mechanics, which is full of strange mathematical abstractions: states, amplitudes, etc. We can use quantum mechanics to perform amazing tricks, but the magic box remains no less mysterious.

Why is it hard to observe quantum interference of a baseball? Large objects are extremely difficult to isolate from the outside world. To observe baseball interference, we would have to remove all photons and gas molecules, then cool the baseball fantastically close to absolute zero. We would even have to worry about how the baseball’s gravity is affecting nearby atoms! The point here is that macroscopic atoms are very strongly connected to the rest of

the world. If we are very careful, we can observe interference for photons, electrons, atoms, etc. But we cannot cut a baseball away from the rest of the world and close the lid of the magic box.

Quantum mechanics is sometimes called the “Great Smoky Dragon.” John Wheeler introduced a cartoon to illustrate the nature of quantum mechanics. The dragon’s tail appears at the start of the experiment. The dragon’s head bites one of the particle detectors at the end. In between, the dragon is shrouded in smoke, and we can never say exactly what its shape is.

The Great Smoky Dragon is a metaphor for the elusiveness of the quantum realm. It is found in every part of quantum mechanics, shrouded by the uncertainty principle and shielded by the principle of complementarity. It has a delicate touch; it can tickle a hair-trigger bomb without setting it off. A pair of identical particles is less like 2 dragons than a single dragon with 2 tails and 2 heads. Feynman’s ribbon trick gives us a hint of how the dragon twists among particles with spin. In Feynman’s view, the dragon gets from here to there by wriggling through everywhere in between. Virtual dragons stretch invisibly from particle to particle, carrying forces between them. Even when space appears empty, it fluctuates with the stirrings of the dragon. Quantum information reminds us of the Great Smoky Dragon, for it cannot be pinned down and copied. We can use the hiddenness of the dragon for our own purposes, sending secret messages that no eavesdropper can penetrate. Dragons carry signals in strange ways, even snaking backward in time; and with a quantum computer, we can quickly solve hard mathematical problems entirely inside the cloud of smoke.

The Great Smoky Dragon is a metaphor for the elusiveness of the quantum realm. It is found in every part of quantum mechanics, shrouded by the uncertainty principle and shielded by the principle of complementarity.

Entanglement is the most dragonish aspect of quantum mechanics. If 2 particles are in an entangled state together, then neither of them can be entangled with any other particles in the universe. The relationship of

entanglement is entirely “private.” This fact is called the monogamy of quantum entanglement. (Abner Shimony called entanglement “passion at a distance.”)

How should we regard the Great Smoky Dragon? It is a Copenhagen picture. The tail and head are where the dragon emerges into the macroscopic world; the smoky in-between is the indescribable quantum realm. (John Wheeler was a student of Niels Bohr.)

Other interpretations deal with the dragon in different ways. The hidden-variables interpretation asserts that the dragon has a definite shape. This shape is strange because parts of the world that seem far apart are actually close together on the dragon.

According to the many-worlds interpretation, the dragon has no tail and no head. Everything is inside the smoke, including us! When we think we see a tail or a head, we are only seeing a tiny part of the whole dragon, which encompasses every possible world.

None of this makes the Great Smoky Dragon less mysterious. Though its actions shape everything we see in the world, elusiveness is the quantum dragon’s most essential feature. ■

Questions to Consider

1. In his writings on complementarity, Bohr laid great stress on “amplification”—the process by which a quantum event is magnified into a macroscopic measurement result. Based on the ideas in this lecture, explain how this takes the effect across the boundary between the quantum and classical realms.
2. Think about what you have learned about quantum mechanics from the previous lectures and pick out the phenomenon that you find most strange or striking. How does Wheeler’s metaphor of the Great Smoky Dragon illuminate quantum physics in this example?

Timeline

- 5th century B.C. Democritus proposes that all matter is composed of tiny, indivisible atoms.
- 4th century B.C. Aristotle develops a sophisticated theory of physics in which matter is continuous and infinitely divisible.
- 1678..... Christiaan Huygens writes his *Treatise on Light*, exploring the wave theory. (The book is eventually published in 1690.)
- 1687..... Isaac Newton publishes his *Principia Mathematica Naturalis Philosophiae* (*Mathematical Principles of Natural Philosophy*), establishing the basic laws of classical mechanics.
- 1704..... Isaac Newton publishes his *Opticks*, exploring the corpuscular theory of light.
- 1803..... John Dalton proposes the laws of chemical combination can be explained by assuming each element is made of its own type of atom; Thomas Young publishes the results of his 2-slit experiment, establishing the wave character of light and measuring its wavelength.
- 1862..... James Clerk Maxwell shows that light is an electromagnetic wave, a traveling disturbance in electric and magnetic fields.

- 1866..... Maxwell develops the “kinetic theory” of gases, based on the idea that gases are composed of huge numbers of tiny molecules; a decade later, Ludwig Boltzmann independently duplicates Maxwell’s work and considerably extends the theory.
- 1887..... The photoelectric effect is discovered by Heinrich Hertz.
- 1900..... William Thomson, Lord Kelvin, delivers a lecture at the Royal Institution noting “two dark clouds” over the classical theory of heat and radiation: the Michelson-Morley experiment and blackbody radiation; Max Planck introduces the quantum hypothesis to explain the properties of blackbody radiation.
- 1905..... Albert Einstein elaborates the quantum hypothesis and explains the photoelectric effect.
- 1907..... Einstein applies the quantum hypothesis to the vibration of atoms in a solid, explaining the anomalously low heat capacity of some materials.
- 1911..... Ernest Rutherford shows that the atom consists of a massive central nucleus surrounded by orbiting electrons; discovery of superconductivity.

- 1913..... Niels Bohr publishes his quantum theory of atomic structure.
- 1922..... Otto Stern and Walter Gerlach do the first experiment showing that atomic spins can only have discrete values.
- 1924..... Louis de Broglie proposes, in his doctoral thesis, that wave-particle duality applies to matter as well as to light (a few years later, this was confirmed in diffraction experiments with electrons); Satyendra Bose develops the quantum statistical theory of photons, which Einstein later extends to other particles (“bosons”); Wolfgang Pauli proposes the exclusion principle for electrons in an atom.
- 1925..... Werner Heisenberg develops his version of quantum mechanics, sometimes called “matrix mechanics.”
- 1926..... Erwin Schrödinger develops his version of quantum mechanics, called “wave mechanics,” based on de Broglie’s matter waves (this is later shown to be exactly equivalent to Heisenberg’s matrix mechanics); Max Born proposes his rule for interpreting Schrödinger’s waves as probability amplitudes; Enrico Fermi and Paul Dirac develop the quantum statistical theory of particles that obey the exclusion principle (“fermions”).

- 1927..... Heisenberg proposes the uncertainty principle; Bohr proposes the principle of complementarity, the basis for the Copenhagen interpretation; the Bohr-Einstein debate begins with vigorous discussions at the Fifth Solvay Conference on Physics in Belgium.
- 1930..... The Bohr-Einstein debate ends its first phase during further vigorous discussions at the Sixth Solvay Conference on Physics; after this date, Einstein no longer argues that quantum mechanics is inconsistent, but he still believes it to be incomplete.
- 1935..... Einstein, Boris Podolsky, and Nathan Rosen draw attention to quantum entanglement (a term coined by Schrödinger in the same year) and argue that quantum mechanics must be incomplete; Bohr responds, but the question remains unresolved.
- 1937..... Discovery of superfluidity in He II.
- 1942..... Richard Feynman, in his doctoral thesis, proposes the “sum-over-histories” approach to quantum mechanics.

- 1948..... Feynman applies the “sum-over-histories idea to quantum electrodynamics, introducing Feynman diagrams; Hendrik Casimir shows that 2 metal plates must attract one another due to their effect on the quantum vacuum; Claude Shannon develops (classical) information theory.
- 1952..... David Bohm proposes the hidden-variables interpretation of quantum mechanics.
- 1957..... Hugh Everett III proposes the many-worlds interpretation of quantum mechanics.
- 1960..... Invention of the laser.
- 1964..... John Bell proves that no local hidden-variable theory can account for quantum entanglement.
- 1978..... John Wheeler proposes his delayed-choice experiment.
- 1982..... The quantum no-cloning theorem is proved by William Wootters and Wojciech Zurek and independently by Dennis Dieks.
- 1984..... Charles Bennett and Gilles Brassard propose quantum key distribution, the beginning of quantum cryptography.

- 1992..... Bennett and Stephen Wiesner invent dense coding, in which 1 ebit and 1 qubit can be used to transmit 2 classical bits of information; David Deutsch and Richard Jozsa show that a quantum computer could solve a particular mathematical problem much faster than any classical computer.
- 1993..... A collaboration of quantum physicists (including Bennett, Brassard, Jozsa, and Wootters) invents quantum teleportation, in which 1 ebit and 2 classical bits can be used to transmit a qubit; Avshalom Elitzur and Lev Vaidman devise their bomb-testing thought experiment.
- 1994..... Peter Shor shows that a quantum computer could factor a large integer much faster than any classical computer.
- 1995..... First Bose-Einstein condensate is created in the laboratory.
- 1996..... Lov Grover shows that a quantum computer could solve the inverse phonebook problem faster than any classical computer.
- 1998..... The expansion of the cosmos is discovered to be accelerating due to an unknown “dark energy,” possibly related to quantum vacuum energy.

Glossary

absorption: A process in which a photon deposits its energy in matter and is destroyed.

amplitudes: The numerical coefficients in a superposition. If the amplitude is a , then the probability of finding that result in a measurement is $|a|^2$.

angular momentum: A measure of how much rotational motion is present in a system, analogous to momentum.

antiparticles: Particles that have the same mass but otherwise opposite properties to “ordinary” particles. Every type of particle has an antiparticle (although photons are their own antiparticles).

antisymmetric: The mathematical property of the quantum state of fermions, which pick up a negative sign when 2 identical particles are swapped.

atom: To Greek philosophers, a tiny and indivisible particle out of which all matter is made. In modern usage, atoms are the basic constituents of chemical elements, but they are in turn made up of smaller particles, including protons, neutrons, and electrons.

basis: A set of quantum states corresponding to the various possible outcomes of a measurement on a quantum system. Since there are many possible complementary measurements, there are many possible basis sets for that system.

beam splitter: *See* half-silvered mirror.

Bell’s inequality: A mathematical relation that holds true in any local hidden variable theory but may be violated in quantum mechanics.

bit: The basic unit of classical information, defined as the information carried by a single binary digit (0 or 1).

blackbody radiation: The electromagnetic radiation emitted by a hot, absorbing object called a “blackbody.” All blackbodies at a given temperature emit radiation with the same characteristics.

Bohr model: The atomic model proposed by Niels Bohr in 1913 in which electrons can only move in discrete orbits around the nucleus. When light is absorbed or emitted, the electron “jumps” from one orbit to another.

Born rule: The rule introduced by Max Born to interpret quantum waves. The intensity of the wave, which is the square of the absolute value of the amplitude, gives the probability of finding the particle.

Bose-Einstein condensate: A low-density cloud of atoms extremely close to absolute zero, so that all of the atoms are found in the same quantum state. Though this was predicted by Einstein in the 1920s, it was not created in the lab until 1995.

bosons: Identical quantum particles such as photons, helium atoms, etc., whose states do not change when the particles are swapped. Bosons have a tendency to be in the same state.

branch: In the many-worlds interpretation, one part of the superposition state of the whole universe—in effect, one “world.”

Casimir effect: The weak attraction between metal plates, predicted by Hendrik Casimir in 1948 and later observed in the lab. The force is due to the plates’ effect on the quantum vacuum.

ciphertext: In cryptography, the representation of the message that is actually transmitted. Generally, an eavesdropper only has access to the ciphertext and wishes to determine the plaintext. *See also* **key**.

classical information: The familiar type of information contained in text, audio, video, or data messages, measured in bits and described by Shannon’s information theory.

classical mechanics: The theory of mechanics based on Newton's laws of motion.

classical physics: A general term that includes classical mechanics, thermodynamics, and electromagnetism. Classical physics prevailed before 1900.

code: Any way of representing information. Specifically, a code is an association of a particular message with a particular representation—representing “no” with “0,” for example. In cryptography, a code may be used to conceal the meaning of the message.

coherent light: Light of a single wavelength and direction.

complementarity: The principle that different observations are incompatible. Thus we cannot design an experiment that measures both a particle's position and its momentum. Complementary quantities cannot both have exact values at the same time.

continuous: Having a whole connected range of values. The real numbers are continuous; between any 2 different real numbers there is an infinite range of intermediate values.

Cooper pairs: Bound pairs of electrons in a low-temperature metal. Although electrons themselves are fermions, Cooper pairs are bosons.

Copenhagen interpretation: The standard interpretation of quantum mechanics developed by Bohr and others, based on the principle of complementarity. In this interpretation, we cannot ascribe a definite meaning to quantum events until a measurement is made and the result is amplified to the macroscopic realm.

cosmic inflation: A brief period of extremely rapid expansion early in the history of the universe, likely driven by quantum vacuum energy.

cryptanalysis: The effort to “break” a secret code by mathematical analysis.

cryptography: The science of maintaining the privacy and integrity of information.

dark energy: A kind of unseen energy, nature unknown, that drives the accelerating expansion of the universe. One theory is that dark energy is the energy of the quantum vacuum.

de Broglie wave: A wave associated with a particle such as an electron, in accordance with the proposal of Louis de Broglie.

delayed-choice experiment: A thought experiment proposed by John Wheeler in which the decision between complementary measurements is made after the experiment is almost completed.

determinism: The belief that future events are completely determined by the present state of the universe—for example, by the exact positions and momenta of all the particles in the world. In this view, “randomness” is simply due to the practical inability to know the present and calculate the future in sufficient detail; in fact, nothing can be truly “random.”

Deutsch-Jozsa problem: The problem of determining whether a binary function is “balanced” or “constant.” Deutsch and Jozsa determined that a quantum computer can answer this question much faster than any ordinary computer.

diffraction: The spreading of waves that pass through a single opening in a barrier.

discrete: Having only disconnected values. The whole numbers are discrete because they are separated from each other; for example, there is no whole number between 2 and 3. (The opposite of continuous.)

distinguishable: Possible to tell apart, at least in principle, by some measurement. Quantum particles of different types (a proton and a neutron, say) are distinguishable. (The opposite of identical.)

eavesdropper: A person who tries to intercept private information without authorization.

ebit: The basic unit of quantum entanglement, defined as a pair of entangled qubits. As an example, 2 spins in a total spin 0 state form an ebit.

electromagnetic wave: A traveling disturbance in the electromagnetic field. Light is an electromagnetic wave; other examples with other wavelengths include radio waves, infrared radiation, ultraviolet radiation, X-rays, and gamma rays.

electromagnetism: The branch of physics that deals with the behavior of electric and magnetic fields.

electron: A low-mass, negatively charged particle that orbits the nucleus of an atom.

Elitzur-Vaidman bomb-testing problem: A thought experiment proposed by Avshalom Elitzur and Lev Vaidman in 1993, showing the surprising features of quantum interference.

entanglement: The correlation of 2 distinct quantum systems. Einstein drew attention to the strange features of entanglement, and Bell used those properties to prove that quantum mechanics is inconsistent with local hidden variable theories.

EPR argument: The argument made by Einstein, Podolsky, and Rosen in 1935 that the properties of quantum entanglement imply quantum mechanics must be an incomplete description of nature. The EPR argument is based on a “criterion of reality” that was later criticized by Bohr.

excited state: A state of a quantum system, such as an atom, that has a greater energy than the ground state.

exclusion principle: The physical principle, first discovered by Pauli for electrons, that no 2 identical fermions can be in exactly the same quantum state.

fermions: Identical quantum particles such as electrons, protons, etc., whose state acquires a negative sign when the particles are swapped. Fermions obey the exclusion principle, so that no 2 identical fermions can be in the same state.

Feynman diagram: A cartoonlike representation of a process in QED involving electrons and photons. (More general diagrams arise in more general particle theories.)

frequency: The number of wave cycles per second that pass a fixed point in space.

ground state: The state of lowest energy of a quantum system such as an atom. (The opposite of excited state.)

half-silvered mirror: A partially reflecting mirror, also known as a beam splitter. A light beam shining on a half-silvered mirror is divided into a reflected and transmitted beam, each of which has half of the original intensity.

heat capacity: The amount of heat energy necessary to increase the temperature of a material by 1°C. The anomalously low heat capacity of some solids was first explained by Einstein.

hidden variables: The conjectured unknown factors that might underlie quantum mechanics and predetermine the outcomes of measurements. Also, the assumption that such variables exist.

hidden-variables interpretation: The alternate interpretation of quantum mechanics proposed by David Bohm. Quantum mechanics is thought to be an incomplete description of nature. There are additional, hidden variables that make nature deterministic and that function in a highly nonlocal way.

identical: Impossible to tell apart by any conceivable measurement. Quantum particles of the same type (2 electrons, say) are identical. (The opposite of distinguishable.)

information theory: The mathematical science of communication developed by Claude Shannon in 1948. This theory, however, did not take quantum mechanics into account.

informationally isolated: Leaving no “footprints” behind to record what happened. A photon in an interferometer is informationally isolated, so that it is impossible to say which beam it followed. Quantum interference effects only appear in systems that are informationally isolated.

interference: The phenomenon in which 2 or more waves can reinforce each other (constructive interference) or cancel each other out (destructive interference).

interferometer: An optical apparatus in which 2 or more light beams are split, redirected, and combined by beam splitters, demonstrating interference effects.

inverse phonebook problem: Given only an alphabetical phonebook, the problem of finding a name associated with a given phone number. Lov Grover showed that this could be done more efficiently on a quantum computer.

ket: A mathematical object describing a quantum state. Symbolically, the ket is written this way: $|state\rangle$, where “state” is just a label designating the state.

key: In cryptography, the mathematical recipe for transforming plaintext into ciphertext and vice versa.

key distribution: In cryptography, the problem of distributing secret keys to users while keeping them secret from any eavesdropper. There is no perfect solution to this in classical cryptography.

kinetic energy: For a particle of mass m moving with velocity v , the kinetic energy is $K = \frac{1}{2}mv^2$.

laser: A device that uses stimulated emission to produce coherent light.

local hidden variable theory: A hypothetical type of theory studied by Bell. In this sort of theory, the quantum realm is assumed to be governed by hidden variables that act in a local way. Bell showed that such theories are incompatible with quantum entanglement.

locality: The assumption that what happens to a particle depends only on its own variables and its immediate circumstances, not what is happening to other particles far away.

Mach-Zehnder interferometer: A particular type of interferometer including just 2 beams. We use this as our basic thought experiment for understanding quantum mechanics.

macroscopic: A generic term for phenomena and objects at the large scale. Everything that we can directly perceive may be regarded as macroscopic.

many-worlds interpretation: The alternate interpretation of quantum mechanics proposed by Hugh Everett III. Macroscopic systems, including observers themselves, are considered to be part of the quantum system. Measurement creates entanglement between system and observer, and all measurement outcomes (all “worlds”) are present in various branches of the state of the universe.

mechanics: The branch of physics that deals with force and motion.

microscopic: A generic term for phenomena and objects at the small scale. When we use this term in connection with quantum physics, we mean atomic-scale phenomena and objects (which are in fact too small to see under an ordinary microscope).

momentum: For a particle of mass m moving with velocity v , the momentum (usually denoted p) is $p = mv$.

Moore’s law: An observation by Gordon Moore that computer power doubles about every 2 years. This has held true for 4 decades and counting.

neutron: A massive, uncharged particle found in the atomic nucleus.

one-time pad: A type of unbreakable secret code that only uses its key once. If an eavesdropper does not have the key, the message is perfectly secure. If the key is used more than once, however, an eavesdropper may be able to break the code.

optical pumping: In a laser, adding energy to a collection of atoms to produce a population inversion.

Pauli exclusion principle: *See* exclusion principle.

photoelectric effect: The emission of electrons from a polished metal surface that is exposed to light of a sufficiently high frequency. Einstein explained this effect using quantum ideas in 1905.

photon: A light quantum; the basic particle of light.

plaintext: In cryptography, the original message to be protected by a secret code. *See also* key and ciphertext.

Planck-de Broglie relations: Mathematical relations (involving Planck's constant h) between wave and particle properties. The particle energy E is connected to the wave frequency f by $E = hf$. The particle momentum p is connected to the wavelength λ by $p = h/\lambda$.

Planck's constant: A fundamental constant of nature, usually denoted h , with a value of 6.63×10^{-34} J-sec. The tiny value of h tells us that quantum effects are most important only at the microscopic scale and that macroscopic physics appears classical.

population inversion: A situation in a laser in which there are more excited atoms than atoms in the ground state.

positrons: The antiparticles of electrons, having the same mass but opposite electric charge. Positrons and electrons can be created or annihilated in pairs.

potential energy: A particle subject to a force has energy due to its position in space. For a simple pendulum, for instance, the potential energy is lowest at the low point of the pendulum and higher at either end of its swing. Kinetic plus potential energy will remain constant as the pendulum swings.

product state: A quantum state of a pair of particles in which each particle has its own definite quantum state. Such particles are completely independent. Not all states are product states, however. If the pair is not in a product state, it is said to have quantum entanglement.

proton: A massive, positively charged particle found in the atomic nucleus.

QED: Quantum electrodynamics, the highly precise theory of electron-photon interactions developed in the 1940s by Richard Feynman and others.

quantum cloning: A hypothetical process, impossible in the real world, by which an exact duplicate is made of the quantum state of a particle.

quantum computing: The use of quantum particles to process information.

quantum electrodynamics: *See* QED.

quantum factoring: A superefficient method of finding the factors of large numbers by using a quantum computer. Discovered by Peter Shor in 1994.

quantum hypothesis: Max Planck's radical idea, proposed in 1900, that a hot object only emits or absorbs light energy in discrete units, or quanta. The energy of 1 quantum of light is $E = hf$, where h is Planck's constant and f is the light frequency.

quantum information: The distinctive kind of information that is carried by quantum particles. Quantum information is measured in qubits.

quantum mechanics: The theory of mechanics developed between 1900 and 1930 that replaced classical mechanics based on Newton's laws.

quantum no-cloning theorem: The mathematical proof by Wootters, Zurek, and Dieks that it is impossible to perfectly duplicate the state of a quantum particle.

quantum physics: A general term for the physics of the microscopic world.

quantum theory: A more general term for quantum mechanics and related theories.

qubit: The basic unit of quantum information, defined as the information carried by a binary quantum system such as a spin- $\frac{1}{2}$ particle.

real photon: In quantum electrodynamics, a photon in a Feynman diagram that connects to the “outside world” and thus is subject to measurement. The opposite of virtual photon.

Schrödinger equation: The equation discovered by Erwin Schrödinger that controls how the quantum wave function behaves over time.

simple state: *See product state.*

snowflake principle: Heuristic principle that no 2 macroscopic objects are ever exactly the same in every detail.

spin: The internal angular momentum of a quantum particle, such as an electron. The spin of a particle can only have values of $0, \frac{1}{2}, 1, \frac{3}{2}, 2$, etc. (in units of $\hbar/2\pi$).

spin component: The total amount of spin angular momentum parallel to a particular axis in space. For a quantum spin- $\frac{1}{2}$ particle, any component of spin can only have the values $+\frac{1}{2}$ or $-\frac{1}{2}$ (in units of $\hbar/2\pi$).

spin-statistics connection: The physical principle that particles with spin $0, 1, 2$, and so on must be bosons, while those with spin $\frac{1}{2}, \frac{3}{2}$, and so on must be fermions.

spontaneous emission: A process in which matter emits a photon, even without the presence of other photons.

state: A physical situation for a quantum system, described by a ket.

Stern-Gerlach apparatus: A laboratory device in which a particle with spin is passed through an inhomogeneous magnetic field. This permits us to measure the particle's spin along any 1 axis we choose (but not along all axes at the same time).

stimulated emission: A process in which matter emits a photon with the same wavelength and direction as some already existing photons. The more photons are present, the more likely this process becomes.

sum-over-histories: An approach to quantum mechanics developed by Richard Feynman. An electron going from here to there takes all possible paths, each one contributing its own amplitude to the process. The total amplitude gives the total probability for the trip.

superconductivity: The phenomenon of zero electrical resistance in some materials at very low temperatures. Such materials are called superconductors. Superconductivity is due to the superfluid-like properties of Cooper pairs of electrons in the material.

superfluid: A liquid at extremely low temperatures that has many surprising properties, including zero viscosity.

superposition: A combination of basis states, written: $a|state\ 1\rangle + b|state\ 2\rangle + \dots$.

superstring theory: A contemporary speculative theory of elementary particles and their interactions, developed within the general framework of quantum theory.

symmetric: The mathematical property of the quantum state of bosons, which is unchanged when 2 identical particles are swapped.

system: Any part of the quantum world that we wish to consider. A system may include 1 or more particles.

thermodynamics: The branch of physics that deals with heat and energy transformations.

thought experiment: A highly idealized experiment that is used to illustrate physical principles.

ultraviolet catastrophe: A prediction of classical physics that a blackbody should emit more and more intensely at higher and higher frequencies. This prediction is not correct.

uncertainty principle: The principle discovered by Heisenberg in 1927 that sets a fundamental trade-off between how precisely a particle's position and momentum may be defined. This is sometimes expressed by the relation $\Delta x \Delta p \geq h$. A variation of the principle gives a trade-off between uncertainties in energy and time.

vacuum: The physical situation in which no particles are present. In quantum theory, the vacuum actually contains considerable energy.

Van der Waals force: The “stickiness” between atoms and molecules that causes them to condense into liquids and solids at low temperatures.

vertex: A point in a Feynman diagram representing a photon interacting with an electron or a positron.

virtual photons: In QED, an internal photon in a Feynman diagram. Such photons can never be directly observed. The energy in a virtual photon is “borrowed,” subject to the terms of the uncertainty principle.

wave: A periodic disturbance, such as sound. Waves may either be traveling (like a moving sound wave) or standing (like the vibrations of a wire with fixed ends).

wave function: The mathematical function, usually denoted Ψ , that describes how a quantum wave depends on space and time.

wavelength: The distance between adjacent crests in a wave.

wave-particle duality: The idea that light can show wave and particle characteristics in different experiments. Later, this idea was extended to matter as well.

zero-point energy: The energy present in any quantum system, even in its ground state, due to the uncertainty principle.

zero total spin state: A special state of a pair of spin- $\frac{1}{2}$ particles. If the same spin component is measured on the 2 particles, opposite results are always obtained. This state is useful for studying the properties of quantum entanglement.

Biographical Notes

Aristotle (384–322 B.C.): Greek philosopher and polymath; the most notable pupil of Plato. Aristotle had one of the widest-ranging intellects in human history. His works on logic, metaphysics, science, medicine, ethics, and law established systems of thought that remain influential to this day. Aristotle believed that matter comprises 5 basic elements (earth, air, fire, water, and a fifth element found in the heavens). However, he viewed these as continuous substances, not discrete atoms.

Babbage, Charles (1791–1871): English mathematician and engineer. Babbage, the son of a banker, studied mathematics at Cambridge. He spent his subsequent career trying to create mechanical calculating “engines” of increasing complexity. His designs followed principles closely resembling those of modern electronic computers, but the mechanical technology of 19th-century England was not advanced enough to realize his most ambitious designs. His Difference Engine, abandoned, was designed to compute the values of complex mathematical functions. His more complex Analytical Engine would have been a computer of a much more general and powerful sort. With different “programs” (encoded on punch cards), the Analytical Engine would have been capable of any sort of calculation at all. None of Babbage’s engines were completed during his lifetime, but a working model of Difference Engine No. 2 (designed in 1849) was finally constructed in 2002.

Bardeen, John (1908–1991): American physicist and one of the few individuals in history to win two Nobel Prizes, one in 1956 and the other in 1972. The first was with W. Shockley and W. Brattain for the discovery of the transistor, which revolutionized electronics. The second was with Leon Cooper and John Robert Schrieffer for their “BCS” theory of superconductivity, a phenomenon that had been first observed as long ago as 1911. Bardeen spent the early part of his career at Bell Labs, then moved to the University of Illinois.

Bell, John (1928–1990): British physicist. Although he was trained and worked as a particle physicist, spending most of his career at the European particle physics lab CERN in Geneva, Bell found time to think deeply about the foundations of quantum theory. Inspired by the work of David Bohm on hidden variables, he did a careful reanalysis of the argument of Einstein, Podolsky, and Rosen. In 1964 he proved his remarkable theorem, stating that no mechanism of local hidden variables could ever reproduce the statistical correlations between entangled quantum systems. The exact conclusion to be drawn from this has been a subject of debate ever since; Bell’s own view seems to have been that the concept of locality could not be maintained in quantum theory.

Bennett, Charles (b. 1943): American physicist and computer scientist. Bennett has been among the most profound thinkers about the physical nature of information and computation. In the 1970s, he showed that any computation can be done by a computer that operates in a thermodynamically reversible way—that is, with arbitrarily little “waste heat.” With Gilles Brassard, he developed the BB84 scheme for quantum key distribution, essentially founding quantum cryptography. Later, in his office at IBM, he built the first working demonstration of the BB84 method. Bennett helped to discover quantum teleportation, dense coding, entanglement “distillation,” and a host of other basic ideas in quantum information theory. Bennett is known for his creativity, his collegiality, his ability to communicate (one colleague admiringly called him a “troubadour”) and his unfailing sense of humor. He has spent his career at IBM Research.

Bohm, David (1917–1992): American-born physicist who later became a British subject. After service on the Manhattan Project during World War II, Bohm was called upon to testify before the House Un-American Activities Committee. He declined, invoking the Fifth Amendment, leading to his suspension from the faculty of Princeton University. Bohm left the United States and eventually settled in England. Meanwhile, Bohm did important research on the basic concepts of quantum theory. He proved that a hidden-variables theory could in principle reproduce the observed phenomena of quantum mechanics. With Yakir Aharonov, Bohm demonstrated that

a quantum particle can respond to a magnetic field even if the particle has zero probability of being found in the region of the field. This Aharonov-Bohm effect is one of the great insights of modern mathematical physics and has led to a deeper understanding of so-called gauge fields. In his writings, Bohm was unafraid to engage deep philosophical questions about the nature of the world and the human condition. Bohm's work on hidden variables in quantum theory, together with his classic discussion of the EPR argument, later inspired John Bell.

Bohr, Niels (1885–1962): Danish physicist and one of the fathers of quantum mechanics. After receiving his doctorate in Denmark, Bohr spent several years in England, where he worked for Ernest Rutherford. Bohr applied quantum ideas to atomic structure, explaining atomic spectra by the discrete orbits allowed for the electron in the atom. After returning to Denmark, he established the Institute for Theoretical Physics in Copenhagen. This became the center of work on the new quantum physics, and young physicists from all over Europe and America studied and worked there. While others such as Heisenberg and Schrödinger created the mathematical theory of quantum mechanics, Bohr carefully laid its conceptual foundations. His principle of complementarity, the foundation of the so-called Copenhagen interpretation of quantum mechanics, allowed physicists to use the strange new concepts without contradictions. His fierce but friendly debate with Einstein about the nature and meaning of quantum physics explored many of the puzzles of the quantum realm. He was awarded a Nobel Prize in 1922. In 1939, on the eve of World War II, Bohr and John Wheeler developed the liquid-drop model of the atomic nucleus, the basis for the theory of nuclear fission. Bohr spent the first part of the war in occupied Copenhagen, but then, forced to make a daring escape because of his Jewish ancestry, he participated in the U.S. Manhattan Project to develop the nuclear bomb. After the war, he returned to Denmark. Bohr's ideas and personality were tremendously influential among theoretical physicists. He was always ready to consider radical new thinking; to one colleague, he said, "Your theory is crazy, but it's not crazy enough to be true."

Boltzmann, Ludwig (1844–1906): Brilliant but troubled Austrian physicist, most notable for his work in connecting atomic theory to macroscopic physics. Boltzmann showed how very simple assumptions about the chaotic world of atoms and molecules lead to detailed predictions about the laws of gas behavior, together with many other phenomena. He was often involved in controversy and left the University of Vienna for some years due to a dispute with his fellow professor, Ernst Mach. Boltzmann suffered from bouts of severe depression, however, and a few years after his return to Vienna he committed suicide. On his tombstone in Vienna is inscribed his greatest discovery, a mathematical relation between the thermodynamic concept of entropy and the statistics of the microscopic world.

Born, Max (1882–1970): German physicist who later became a British subject and who contributed decisively to the development of quantum theory. Born assisted Heisenberg in developing the mathematics of his version of quantum mechanics. He also provided a key insight for the interpretation of the waves in de Broglie and Schrödinger's version: the Born rule, which states that the intensity of the wave at a point determines the probability of finding a particle there. Born taught for many years at the University of Göttingen, and among his students and postdoctoral assistants are numbered many of the most famous names in 20th-century physics. He received a Nobel Prize in 1954.

Bose, Satyendra (1894–1974): Indian physicist most notable for the discovery, in 1922, of the statistical laws governing one type of identical particle. Bose made his discovery in the middle of a lecture at the University of Dakha, in which he was attempting to demonstrate that classical statistical physics could not explain Planck's blackbody radiation law. During the lecture he made a "mistake" that unexpectedly led to the correct answer. Bose soon realized that he had stumbled on a new insight into the quantum world. Bose sent his paper to Einstein, who recognized it as an important contribution, saw to its publication, and worked to develop its ideas further. Bose became an important figure in the growth of science in India.

Brassard, Gilles (b. 1955): Canadian computer scientist at the University of Montreal. Brassard started out studying the mathematics of cryptography, but his collaboration with Charles Bennett on the BB84 protocol in quantum cryptography soon made him into a quantum physicist. He helped to discover quantum teleportation—indeed, it was invented at a workshop that he hosted at the University of Montreal. Brassard has also made fundamental contributions to entanglement “distillation” and the theory of quantum computing.

Casimir, Hendrik (1909–2000): Dutch physicist who contributed to both low-temperature physics and quantum electrodynamics. Casimir studied with the great Paul Ehrenfest, then worked with Bohr in Copenhagen and Pauli in Zurich. Although he was an industrial scientist, directing the Philips Research Laboratories in the Netherlands, he made numerous contributions to pure research. In 1948 he predicted the phenomenon that later bore his name (the Casimir effect), in which 2 metal plates are attracted to each other due to their modification of the quantum vacuum.

Cooper, Leon (b. 1930): American physicist who helped discover the mechanism of superconductivity and received a Nobel Prize in 1972. Cooper proposed that electrons in a superconductor join up in pairs, later called “Cooper pairs,” that behave as bosons in a superfluid. This allows the material to conduct electricity without resistance. Cooper is a faculty member at Brown University, where he has most recently done research in theoretical neuroscience.

Dalton, John (1766–1844): English chemist and the father of modern atomic theory. After studying the known facts of chemical composition, Dalton proposed in 1803 that elements are made up of atoms of a uniform mass, that the atoms of different elements have different masses, and that these atoms combine in definite ways to create chemical compounds. The atoms themselves are neither created nor destroyed in a chemical process but simply change their combinations. This idea revolutionized chemistry and shed new light on the behavior of gases.

de Broglie, Louis (1892–1987): French physicist who, in one of the most influential doctoral dissertations in history, proposed that electrons and other quantum particles must have wave characteristics. De Broglie’s work “closed the circle” of quantum ideas and in short order became the basis for the wave mechanics of Schrödinger. De Broglie, who was a member of the French nobility, received a Nobel Prize in 1929 and became one of the most eminent men in European science after World War II.

Democritus (c. 460–370 B.C.): Greek natural philosopher and one of the originators of “atomism,” the idea that everything in the world is made of tiny, indivisible units. Democritus’s theory is summarized in a famous quotation: “By convention there is sweet, by convention there is bitter, by convention hot and cold, by convention color; but in reality there are only atoms and the void.”

Deutsch, David (b. 1953): Israeli-English physicist and one of the most creative and eccentric thinkers in contemporary quantum theory. Long a proponent of Everett’s many-worlds interpretation of quantum mechanics, Deutsch became interested in the idea of a quantum computer. An intelligent quantum computer, he reasoned, could be a type of observer that was “aware” of the branching of the universe’s quantum state. His development of the theory of quantum computing led to the discovery of the Deutsch-Jozsa problem, which in turn sparked widespread interest in the powers of quantum computers. Deutsch has also applied his combination of rigorous mathematics and powerful imagination to other topics, such as the quantum physics of time machines. Deutsch is affiliated with, but not a faculty member at, Oxford University. He is seldom seen outside of Oxford, but his ideas are closely followed by quantum physicists worldwide.

Dieks, Dennis (b. 20th century): Dutch philosopher of physics. Trained as a theoretical physicist, Dieks has spent his career studying the philosophical aspects of relativity and quantum physics. In 1982 he proved the quantum no-cloning theorem independently of William Wootters and Wojciech Zurek, using a different mathematical method. He is a member of the philosophy faculty at the University of Utrecht.

Dirac, Paul (1902–1984): English physicist who contributed deeply to the mathematical tools of quantum theory and received a Nobel Prize in 1933. As a graduate student at Cambridge University in the 1920s, Dirac seized upon the new theories of Heisenberg and Schrödinger, demonstrating their mathematical equivalence. The “ket” notation for quantum states used in our lectures was introduced by Dirac. In 1928 he proposed a new form of quantum theory compatible with Einstein’s special theory of relativity, including a relativistic version of the Schrödinger equation later known as the Dirac equation. Consideration of this equation led Dirac to predict the existence of antiparticles. These were discovered only a few years later in studies of cosmic rays. Dirac laid the groundwork for the quantum theory of fields (including quantum electrodynamics) and was one of the first to analyze the statistical properties of identical particles—to mention only 2 of his remarkable contributions. For over 30 years he held Newton’s old post as Lucasian Professor of Mathematics at Cambridge. Dirac’s scientific work was guided by a passionate belief in the mathematical elegance of nature. He is buried in Florida, where he spent the last decade of his life, but his monument in Westminster Abbey is just a few steps from Newton’s tomb.

Einstein, Albert (1879–1955): German physicist, later an American citizen, whose epoch-making contributions to physics during the early 20th century turned him into a public icon of a scientific genius. His fame was entirely deserved. In a series of brilliant papers in 1905, the young Einstein (then working as a patent clerk in Switzerland) made fundamental discoveries in statistical mechanics, established the special theory of relativity, and used Planck’s quantum hypothesis to explain the photoelectric effect. More contributions followed, including his quantum explanation of the heat capacities of solids, many papers on the interaction of light with matter, and the statistical behavior of identical particles. Einstein’s 1915 discovery of the general theory of relativity, which explains gravitation as the curvature of space and time, was as astonishing as it was profound. The confirmation of this theory came a few years later, just after World War I, when the deflection of starlight by the Sun’s gravity was precisely measured. This was the event that catapulted Einstein to international celebrity. He received a Nobel Prize in 1921. Although Einstein was one of the pioneers of quantum theory, he later became its sharpest critic. His debates with Bohr at the 1927 and 1930 Solvay conferences were decisive turning points in

the history of the subject. Einstein, a Jew, left Europe for America in 1932 and never returned. In 1935, Einstein, Podolsky, and Rosen argued that the phenomenon of quantum entanglement proved that quantum theory was an incomplete description of reality. (This argument, and Bohr's subtle reply, led John Bell to his remarkable work 3 decades later.) In later years, Einstein worked unsuccessfully to combine the known laws of physics into a "unified field theory." Einstein was never fully reconciled with quantum physics, never quite accepting that God "played dice with the Universe." In all of his scientific work, he was guided by the maxim, "The Lord God is subtle, but He is not malicious."

Everett, Hugh, III (1930–1982): American physicist. As a student of John Wheeler at Princeton in 1957, Everett developed the many-worlds interpretation of quantum mechanics. He saw this interpretation as a way to avoid the problems of the Copenhagen interpretation and give a solid framework for applying quantum theory to Einstein's general relativity (a problem still unsolved today). In the same Ph.D. thesis, Everett also pioneered the use of concepts from information theory in the analysis of quantum systems. Possibly discouraged by the cool reception his ideas received from most physicists, Everett switched fields and spent the rest of his career doing operations research for the U.S. defense establishment. His departure from physics research and his early death at age 51 deprived the world—this one, anyway—of a radical and creative thinker about the meaning of quantum theory.

Fermi, Enrico (1901–1954): Italian physicist, later an American citizen, who made brilliant contributions to both theoretical and experimental physics. In 1926, while still in Rome, Fermi helped to develop the statistical theory of identical particles such as electrons that obey the Pauli exclusion principle. Later he became even more famous for his remarkable experiments on neutron-induced nuclear transformation, for which he won the Nobel Prize in 1938 and in which he narrowly missed discovering nuclear fission. His groundbreaking theory of beta decay included Pauli's undiscovered "ghost" particle, which Fermi christened the "neutrino." After leaving fascist Italy and emigrating to the United States, Fermi worked on the Manhattan Project. His experimental reactor achieved the first sustained nuclear chain reaction in 1942.

Feynman, Richard (1918–1988): American physicist whose astounding scientific insight and quirky personality left an indelible stamp on 20th-century physics. As a graduate student of John Wheeler in 1942, Feynman developed his “sum-over-histories” approach to quantum mechanics. Like so many physicists, he worked on the Manhattan Project to develop the atomic bomb during World War II. Returning to theoretical pursuits after the war, he made decisive contributions to the development of quantum electrodynamics, introducing the remarkable Feynman diagrams to assist in calculations. He also made advances in the theory of superfluids and superconductors, in the theory of weak nuclear interactions, and in the quark model of nucleons, receiving a Nobel Prize in 1965. Feynman spent most of his career as a faculty member at Caltech, where he became a legend as a brilliant teacher. His 3-volume *Lectures on Physics* is standard equipment on any physicist’s bookshelf. Feynman had the knack of seeing new possibilities in nature; both nanotechnology and quantum computing trace their origins in part to lectures given by Feynman. Many people first heard of Feynman during his work on the commission investigating the loss of the space shuttle *Challenger* in 1986; Feynman performed a dramatic demonstration using a clamp, a sample of material from the shuttle, and a glass of ice water that identified the root cause of the disaster. He was a remarkable raconteur, and his books of personal reminiscences gained a wide audience. It was said that Feynman’s graduate students at Caltech learned 3 things from him: theoretical physics, safe-cracking (a talent Feynman had developed playing pranks on the security officers at Los Alamos during the war), and bongo drumming.

Grover, Lov (b. 1961): Indian-American computer scientist. Like Peter Shor, a fellow computer scientist at Bell Labs, Grover began moonlighting as a quantum physicist, studying the emerging field of quantum computing. In 1996 he discovered his quantum search algorithm, which would allow a quantum computer to “find a needle in a haystack” far more rapidly than any classical computer.

Heisenberg, Werner (1901–1976): German physicist and one of the creators of quantum mechanics. In 1924–1925, Heisenberg came to Copenhagen to work with Bohr on the new physics. There he discovered his own highly abstract version of quantum mechanics, which came to be called “matrix mechanics.” Although the mathematics of the theory was very strange, it soon became clear that it gave a precise account of the strange behavior of the microscopic realm. The theory was at first seen as a competitor to Schrödinger’s wave mechanics, until Paul Dirac showed that they were mathematically equivalent. Heisenberg also formulated the famous “uncertainty principle,” which establishes limits on our ability to know about the microscopic world. Heisenberg later made fundamental contributions to quantum field theory, nuclear physics, and elementary particle physics. He received a Nobel Prize in 1932. During World War II, Heisenberg remained in Nazi Germany and directed part of the German nuclear program. This later led to considerable strain on his relationships with physicists from other countries, and his long friendship with Niels Bohr came to an end. After the war, Heisenberg wrote extensively about the philosophical ideas embedded in quantum theory.

Huygens, Christiaan (1629–1695): Dutch physicist and astronomer. As an astronomer, Huygens discovered Saturn’s rings and its largest moon, Titan. As a mathematician, he contributed to the foundations of probability theory. As an inventor, he was responsible for several advances in the construction of accurate clocks. But it was as a physicist that he made his most notable contributions. Huygens was particularly interested in the nature of light, which he regarded as a wave phenomenon like sound. He introduced what is now called the “Huygens principle,” which states that each point on a traveling wave front acts as a source for further waves. This principle allowed him to analyze the reflection and refraction of light based on his wave theory.

Jozsa, Richard (b. 20th century): British mathematician and physicist. After studying mathematical physics with the great Roger Penrose, Jozsa worked with David Deutsch on what came to be called the “Deutsch-Jozsa problem,” the first-proposed mathematical problem that could be solved more efficiently by a quantum computer than by any classical one. He also helped

to invent quantum teleportation. Jozsa is now a professor in the Department of Computer Science at the University of Bristol.

Maxwell, James Clerk (1831–1879): Scottish mathematician and physicist who made fundamental contributions to mechanics and electromagnetism. Maxwell applied Newtonian mechanics to the behavior of huge numbers of colliding molecules, deriving the statistical distribution of molecular speeds in a gas. He also derived many useful mathematical relations in the science of thermodynamics. By collecting together and analyzing the known laws of electromagnetism, Maxwell realized that the system was mathematically incomplete. When he supplied the missing pieces, he discovered that electromagnetic disturbances would travel through space in the form of polarized waves with a speed equal to that of light. He concluded that light is an electromagnetic wave, an idea that unified optics and electromagnetism, and his work indicated the possible existence of other related waves. The later discovery by H. Hertz of radio waves vindicated Maxwell's theory. Maxwell himself was a religious man, a guitar player, and the author of several amusing songs about physics and its study.

Newton, Isaac (1642–1727): English physicist and mathematician and without doubt the greatest scientific mind of his age. In his book *Mathematical Principles of Natural Philosophy* (1687), Newton established the science of mechanics based on universal laws of motion and gravitation. This work explained motions ranging from projectiles on Earth to the orbits of the planets, together with a host of other phenomena. Newton invented calculus, which he called “the method of fluxions,” to deal with his new system of mechanics. Newtonian mechanics was the basis for physics for more than 2 centuries. Newton also made tremendous contributions to optics, including the invention of the reflecting telescope and the discovery that white light is a mixture of all colors. Newton's view, expounded in his book *Opticks* (1704), was that light was a stream of discrete corpuscles. In this he disagreed with the wave view of Huygens and others, but the matter was not settled experimentally for another century. In addition to his scientific pursuits, Newton commented on scripture, wrote about theology, and studied alchemy. Newton was a powerful and influential figure in the English science of his day and served as president of the Royal Society of London from 1701 until his death.

Pauli, Wolfgang (1900–1958): Austrian physicist, later an American citizen and a resident of Switzerland, and winner of a Nobel Prize in 1945; famous for his brilliant discoveries in theoretical physics and his sharp critique of shaky reasoning. Pauli developed his “exclusion principle” in 1924 to explain the structure of many-electron atoms. He was the first to use Heisenberg’s quantum mechanics to explain atomic spectra, and he contributed a great deal to the theory of particle spin. He also proved the “spin-statistics” theorem, the connection between a particle’s spin and its character as a boson or fermion. In 1929 he proposed that the mysteries of beta decay (one of the main types of radioactivity) could be explained by the existence of an almost-invisible “ghost particle,” later called the neutrino by Fermi. (When the neutrino was finally discovered almost 30 years later, the discoverers sent a telegram congratulating Pauli. His reply: “Thanks for the message. Everything comes to him who knows how to wait.”) Pauli was well known for his ready and caustic wit, and anecdotes about his various remarks are favorites among physicists. (Of one paper he said, “This isn’t right. This isn’t even wrong.”)

Peres, Asher (1934–2005): Israeli physicist. After a perilous childhood during World War II hiding out in occupied France, Peres emigrated to Israel, where he studied theoretical physics at Technion under Nathan Rosen, one of the authors of the EPR paper. Peres went on to be a faculty member at Technion and to make many contributions to physics, especially to the foundations of quantum theory. He drew attention to the fundamental role that the concept of information plays in the theory and later was one of the inventors of quantum teleportation. He was once asked by a reporter, “Can you teleport only the body, or also the spirit?” He replied, “Only the spirit.”

Planck, Max (1858–1947): German physicist and the originator of the quantum hypothesis; winner of a Nobel Prize in 1918. For most of his career, Planck was a professor at the University of Berlin. In the last years of the 19th century, he turned his attention to the problem of understanding the electromagnetic radiation emitted by hot bodies of all sorts. Since all black bodies, regardless of composition, emit radiation with the same characteristics, Planck recognized this as a problem of fundamental

importance. His early work met with only partial success. Finally, in 1900 he adopted the quantum hypothesis as, in his words, “an act of despair.” Though it involved a radical departure from previous ideas about energy, Planck’s new theory accounted for blackbody radiation with great exactness. Planck observed the subsequent development of quantum theory with great interest. With a sad wisdom, he wrote, “A new scientific truth does not triumph by convincing its opponents and making them see the light, but rather because its opponents eventually die, and a new generation grows up that is familiar with it.”

Rutherford, Ernest (1871–1937): New Zealand physicist and one of the great experimentalists in the history of science, he received a Nobel Prize in 1908. Though born in New Zealand, Rutherford spent most of his career in England. He identified the main kinds of radioactivity and discovered the law governing the rate of radioisotope decay. He supervised the scattering experiment of Hans Geiger and Ernest Marsden and correctly interpreted its results to construct the “solar system” model of the atom. Rutherford was the first researcher to produce an artificial transmutation of elements, using alpha particles to transform nitrogen into oxygen. Rutherford’s students and assistants included many who won Nobel Prizes in their own right (including Niels Bohr). Rutherford had no false modesty about his remarkable accomplishments. When someone suggested that he had been lucky to ride “the crest of the wave” in discovering new physics, he answered, “Well, I made the wave, didn’t I?” His untimely death in 1937 came when he was still at the height of his powers; just a few years earlier, his suggestion that the nucleus must contain a neutral particle had been confirmed by James Chadwick’s discovery of the neutron.

Schrieffer, John Robert (b. 1931): American physicist who, as a graduate student at the University of Illinois, helped to formulate the theory of superconductivity; he received a Nobel Prize in 1972. Schrieffer figured out how to describe the flow of Cooper’s electron pairs through a material. He holds posts as professor of physics at universities in both California and Florida.

Schrödinger, Erwin (1887–1961): Austrian physicist and one of the developers of quantum mechanics. Schrödinger’s version, called “wave mechanics,” was at first seen as a competitor to Heisenberg’s wave mechanics, before Paul Dirac showed them to be mathematically equivalent. His basic equation, the Schrödinger equation, is one of the most fundamental relations of mathematical physics. Like so many of the physicists of Germany, Italy, and Austria, Schrödinger was obliged to leave in the early 1930s as the Nazis took power. He settled in Dublin, founding the Institute for Advanced Study at the university there and writing an influential book, *What is Life?*, about the physical nature of biological systems. This book inspired physicist Francis Crick to switch fields and become one of the discoverers of the structure of DNA. Schrödinger returned to Vienna for the last few years of his life. Schrödinger received a Nobel Prize in 1933, but in the popular mind he is most strongly linked to his 1935 thought experiment in which a cat enters a quantum superposition of being alive and dead. He wrote, “The [quantum state] of the entire system would express this by having in it the living and the dead cat (pardon the expression) mixed or smeared out in equal parts.” (In his defense, one should note that this idea was introduced with the words, “One can even set up quite ridiculous cases.”)

Shannon, Claude (1916–2001): American mathematician and engineer, founder of information theory. Shannon’s many discoveries have been of incalculable importance in creating the “information age.” His MIT master’s thesis in 1937 laid the abstract groundwork for the digital computer. After World War II, he developed the mathematical theory of communication and soon applied it to everything from signal processing to human language to cryptography. He was a prolific inventor and game player, applying his genius to gambling, the stock market, and computer chess. Shannon did much of his work at Bell Labs, later joining the faculty at MIT.

Shor, Peter (b. 1959): American computer scientist who has made key discoveries in quantum computing and quantum information. As a computer scientist at Bell Labs, Shor became fascinated by the new idea of a quantum computer. In 1994 he discovered that a quantum computer algorithm could factor a large integer exponentially faster than any known procedure on a classical computer. Given the huge importance of the factoring problem in cryptography and number theory, this has provided much of the impetus for

experimental work on quantum computers. Such computers are difficult to build, since their operation is very sensitive to environmental noise. Shor helped to find a possible answer, however: In 1995 he discovered the first method of “quantum error correction.” Shor is now a professor of applied mathematics at MIT.

Wheeler, John (1911–2008): American physicist who made fundamental contributions to several areas of physics, from elementary particles to cosmology. Wheeler was deeply influenced by Niels Bohr, with whom he developed the theory of nuclear fission in 1939. An intensely patriotic man, he helped to develop both nuclear and thermonuclear weapons in the 1940s and 1950s. In the 1950s, Wheeler became interested in the implications of Einstein’s general relativity. His work helped to revive the field of gravitational physics, and in 1967 he coined the term “black hole” to describe a completely collapsed star. Though he mentored Hugh Everett III in the creation of the many-worlds interpretation, Wheeler eventually rejected it and came to espouse a version of Bohr’s Copenhagen interpretation. For Wheeler, the world itself comes into being through innumerable “elementary quantum phenomena.” These elementary quantum phenomena are themselves not localized in space and time—as illustrated by his “delayed-choice experiment”—but form the real underlying structure of space, time, matter, and energy. The world is therefore essentially made of information—an idea Wheeler christened “it from bit.” Wheeler was famous for his penetrating (if slightly oddball) questions and his striking way of expressing ideas in phrases and images. He spent most of his career at Princeton University, with a 10-year sojourn at the University of Texas. Wheeler was teacher and mentor to many physicists mentioned in this course, including Richard Feynman, Hugh Everett III, Wojciech Zurek, William Wootters, and your lecturer.

Wootters, William (b. 20th century): American physicist. While a graduate student at the University of Texas, Wootters, together with Wojciech Zurek, proved the quantum no-cloning theorem. Under the influence of John Wheeler, he became fascinated by the relation between quantum physics and information. He helped to discover quantum teleportation and protocols by which noisy entanglement may be “distilled,” among many other contributions to quantum information theory. Wootters is a professor of physics at Williams College.

Young, Thomas (1773–1829): English physicist and polymath. Young was a physician who contributed to many areas of science, including the theory of elasticity. He is most famous for his decisive 2-slit experiment (performed in 1801) that demonstrated the wave nature of light and measured its wavelength. This experiment settled for a century the long-standing debate about whether light was made of continuous waves or discrete corpuscles. Young was also a linguist who made fundamental contributions to reading the Rosetta Stone, laying the groundwork for Champollion’s later decipherment of Egyptian hieroglyphs.

Zurek, Wojciech (b. 1951): Polish physicist, now an American citizen, who has made contributions to statistical physics, quantum mechanics, black holes, and cosmology. Long interested in the relation between information and quantum physics, he proved (with William Wootters) the quantum no-cloning theorem and has long studied quantum decoherence. Decoherence is the process by which the environment of a system, by constantly “monitoring” it, destroys the coherence of quantum superpositions. Zurek is presently a researcher at Los Alamos National Laboratory.

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Rhodes, R. *The Making of the Atomic Bomb*. New York: Simon and Schuster, 1986. This massive, Pulitzer Prize–winning history does ultimately focus on the Bomb, of course, but the first 300 pages or so are among the best accounts of the development of 20th-century physics that I have ever read.

Styer, D. F. *The Strange World of Quantum Mechanics*. Cambridge: Cambridge University Press, 2000. A slender but rewarding book based on

Prof. Styer's fine course at Oberlin College. Spins, amplitudes, probability, entanglement, and more are given a very lucid and accessible treatment.

Tipler, P. *Elementary Modern Physics*. New York: Wirth, 1992. This is a physics textbook intended for first-year or second-year undergraduates and is mostly about quantum theory and its applications. I recommend this as a good textbook for the graduate of our course who is confident about math and ready for more of the details. The chapters are equipped with plenty of homework problems at various levels of difficulty.

Wheeler, J. A., and W. H. Zurek, eds. *Quantum Theory and Measurement*. Princeton: Princeton University Press, 1983. The best 1-volume collection of important papers on the basic concepts of quantum mechanics. Bohr's own description of the Bohr-Einstein debate is here, with the original EPR paper, Bohr's response to EPR, John Bell's deeper analysis, Everett's many-worlds interpretation, original accounts of basic entanglement and delayed-choice experiments, and many other treasures. My copy is about to fall apart from use. (The level of some of the papers is pretty advanced and probably not for the general reader.)

Whitaker, A. *Einstein, Bohr and the Quantum Dilemma*. Cambridge: Cambridge University Press, 1996. This is a marvelous treatment of the development of quantum theory, the Bohr-Einstein debate, and beyond. Whitaker has a good sense for the conceptual issues at stake.

Wick. D. *The Infamous Boundary*. New York: Copernicus, 1995. An excellent book about the history, experiment, and theory involved in the great controversies of quantum mechanics. The book itself is descriptive and qualitative, but there is a mathematical appendix about quantum probability (by William Faris) that should please even the most technical reader.