Peter J. Riggs

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Quantum Causality

Conceptual Issues in the Causal Theory of Quantum Mechanics

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QUANTUM CAUSALITY

CONCEPTUAL ISSUES IN THE CAUSAL THEORY OF QUANTUM MECHANICS

By

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Springer

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Preface

There is no sharp dividing line between the foundations of physics and philosophy of physics. This is especially true for quantum mechanics. The debate on the interpretation of quantum mechanics has raged in both the scientific and philosophical communities since the 1920s and continues to this day. (We shall understand the unqualified term 'quantum mechanics' to mean the mathematical formalism, i.e. laws and rules by which empirical predictions and theoretical advances are made.) There is a popular rendering of quantum mechanics which has been publicly endorsed by some well known physicists which says that quantum mechanics is not only more weird than we imagine but is weirder than we can imagine.[1] Although it is readily granted that quantum mechanics has produced some strange and counter-intuitive results, the case will be presented in this book that quantum mechanics *is not as weird* as we might have been led to believe!

The prevailing theory of quantum mechanics is called Orthodox Quantum Theory (also known as the Copenhagen Interpretation). Orthodox Quantum Theory endows a special status on measurement processes by requiring an intervention of an observer or an observer's proxy (e.g. a measuring apparatus). The placement of the observer (or proxy) is somewhat arbitrary which introduces a degree of subjectivity. Orthodox Quantum Theory only predicts probabilities for measured values of physical quantities. It is essentially an instrumental theory, i.e. a theoretical 'instrument' or 'tool' for making predictions for the possible results of experiments on quantum systems. However, instrumental theories do not explain the results that they predict. If we are to explain the physical universe then a better understanding of the foundations of quantum mechanics is needed than can be provided by Orthodox Quantum Theory. A realistic and viable alternative to this quantum orthodoxy is the Causal Theory of Quantum Mechanics. The Causal Theory is not well known within the general physics community and many

[1] The original phrase is due to J.B.S. Haldane (Haldane 1927, 286).

physicists who do know of the Causal Theory are generally dismissive in their attitudes towards it.

This book is a contribution to the debate over the fundamentals of quantum mechanics and causality in the quantum realm. Much of what is argued for here will be controversial but, at the very least, these arguments may engender some lively discussions on the various issues raised. One of the most influential professional figures in physics in the second half of the twentieth century was the late Professor John Wheeler of Princeton University. He once remarked that doing scientific research means that you "stick up for something". Wheeler's advice is taken in this book and an unashamed stand is made on a number of contentious issues. Readers are encouraged to approach the issues presented with an open mind and weigh up the arguments dispassionately and on their merits. Hopefully, these arguments will be found convincing to those unfamiliar with the Causal Theory and assist in changing the opinions of those who reject the Causal Theory out of hand.

This book is not an introductory text on quantum mechanics and although most of the technical terms and various concepts are defined, a familiarity with the basic mathematical methods of physics in general and of quantum mechanics in particular is assumed. The mathematical expositions can be by-passed on a first reading (or taken on faith) without affecting the arguments presented.

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Chapter 1

General Introduction

I am now convinced that theoretical physics is actual philosophy.
— Max Born

Science in effect creates philosophy.
— Gaston Bachelard

Abstract This chapter begins with a commitment to scientific realism and reasons for accepting the existence of an objective quantum realm. The notion of causality in the context of physical theories is discussed together with a (defined) Principle of Causality. The Causal Theory of Quantum Mechanics is then introduced together with a list of its advantages and another of its alleged flaws which will be addressed in later chapters. Criteria for the appraisal of rival scientific theories are presented which will be used in assessing the merits of both the Causal Theory of Quantum Mechanics and Orthodox Quantum Theory. The distinction between conceptual and theoretical problems as found in scientific theories is made explicit and examples are provided from the history of physical science.

1.1 Quantum Reality

This book is a treatise devoted to the foundations of quantum physics and the role that causality plays in the microscopic world governed by the laws of quantum mechanics. It is most unfortunate that there has been a great deal of ill-informed commentary about quantum mechanics since its beginnings. Much of this commentary has been speculation and/or confusion based upon the following:

- identifying probabilistic outcomes with an absence of causality;
- an ontological interpretation of the uncertainty relations;

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- unconditionally accepting the quantum 'no-go' theorems as final proof that a more complete description of quantum phenomena cannot be given.

Together with the conclusion of the most influential of the founders of quantum mechanics about the impossibility of depicting a quantum ontology, these points led to the abandonment of a set of concepts and principles that were strongly held prior to the advent of quantum mechanics (Cushing 1993, 816; Woit 2006, 147). The dominant paradigm of quantum physics, Orthodox Quantum Theory (also known, for historical reasons, as the Copenhagen Interpretation of Quantum Mechanics) is a *broad interpretive framework* adhered to by a majority of physicists (Jammer 1966, 361; Baggott 1992, 82; Stapp 1993, 49 and 234; Cushing 1994b, 289; Beller 1999, 2). Orthodox Quantum Theory came about, in part, from the abandonment of a number of previously held physical concepts and principles. Examples of such abandoned principles include: event-by-event causality; deterministic evolution of physical systems; continuity of processes; and (occasionally) energy conservation (Cushing 1998, 284; Kragh 1999, 209).

However, well established physical concepts and principles should not be given away until such time as they are unambiguously shown to be inappropriate, not applicable, or simply false. There are certainly many, new features to be learnt about the microworld and which quantum mechanics does inform about. This does not necessarily require abandoning physical principles and ontological concepts that have served physics well. In the case of Orthodox Quantum Theory, abandoning prior held physical concepts and principles was done for a number of reasons, including sociological ones (Baggott 1995, 41; Norris 2000, 52). In hindsight and in light of the explanatory achievements of the Causal Theory of Quantum Mechanics (see Section 1.3 and details in later chapters), it can be reasonably argued that this abandonment occurred too hastily in the quantum revolution of the early twentieth century.

Realism as applied to the microworld is a case in point. Scientific realism asserts that there is an objective physical realm that exists independent of any knowledge obtained about it and indeed, independent of any sentient beings to observe it. Several of the principal founders of quantum mechanics denied the existence of

a quantum realm that is independent of human 'observations'. Two of the most prominent founders, Niels Bohr and Werner Heisenberg, were especially forthright in expressing this opinion. Although there are several differing accounts of Bohr's views, there is sufficient evidence to indicate that Bohr did hold an attitude towards quantum phenomena that was anti-realist in the sense of denying an independent quantum realm (much of the relevant evidence is cited in Beller 1999, Chapter 8 and Murdoch 1987, Chapter 10). Bohr was quoted, for example, as stating:

> … *There is no quantum world*. There is only an abstract quantum physical description. It is wrong to think that the task of physics is to find out how nature is … (Petersen 1963, 8, italics added).

Heisenberg wrote in even stronger terms:

> … the idea of an objective real world whose smallest parts exist objectively in the same sense as stones or trees exist, independently of whether or not we observe them … is impossible (Heisenberg 1989, 117).

This anti-realist attitude to the microworld is ingrained in many physicists and philosophers of physics (although thankfully not all) as a result of the legacy of the early debates on the foundations of quantum mechanics and subsequent dominance of the field by Orthodox Quantum Theory. The anti-realist sentiment has permeated into the wider academic community as can be appreciated from the following passage in a textbook on the topic of thinking about 'weird things':

> Something strange is going on in physics. … This weirdness is taking place in the branch of physics known as quantum mechanics … The notorious weirdness is this: In the quantum realm, particles don't acquire some of their characteristics *until they're observed by someone*. They seem not to exist in a definite form until scientists measure them. … It has caused some people to speculate that reality is subjective … that the universe is a product of our imagination (Schick and Vaughn 1995, 8–9, italics in original).

Rather than renouncing physical principles and ontological concepts, the modification of existing principles and concepts as required in order to suit new knowledge should be the case. This is a better way to proceed if we are to gain a fuller understanding of the foundations of quantum physics. In regard to quantum theory and its postulated entities, the respected American philosopher of science Ernan McMullin cogently expressed:

> … problems do arise when we consider such microentities as electrons … The moral is not that … [quantum] physics makes no sort of realist claim, but that the claim it makes must be construed with caution (McMullin 1984, 14).

Developments in experimental atomic and quantum physics in the last two decades of the twentieth century and since the advent of this century have borne out reality claims about quantum entities. Experimental techniques have advanced to the stage that individual atoms can be 'mapped' in detail and imaged (Humphreys 1999, 21–22; Zuo et al. 1999, 49–52; Herz et al. 2003, 45301–5308; Gericke et al. 2008). Single atoms and even single electrons can now be isolated, moved, manipulated (Hey and Walters 2003, 79–87), and trapped in containment vessels for periods of time. This has allowed researchers to 'examine' them over extended time scales (Hey and Walters 2003, 70–71; Haroche and Raimond 2006, 18). In particular, the entrapment of atoms and elementary particles allows their specific properties to be measured to great accuracy (Cohen-Tannoudji and Dalibard 2005, 151). In their book, *Invitation to Contemporary Physics*, Ho-Kim, Kumar and Lam acknowledge the existence of the quantum realm that has become evident through new experimental techniques:

> Perhaps the most convincing proof of the reality of the quantum world would be to capture some of its creatures and hold them in place for all to see. This has become feasible … Observations can be made on single particle systems (Ho-Kim et al. 2004, 81).

Evidence for the existence of an objective physical realm in the form of trapped quantum particles, manipulation of single quantum systems, and the detailed imagery of atoms is extensive and continues to mount. This evidence has brought to light a number of important facts. We can now explore the same individual quantum system over and over again and get the same data each time. Not only can we trap a quantum particle, we find that it is still in its trap after intervals of time where there have been no interactions. Single atoms can be imaged and re-imaged with the same results. Individual atoms can be 'pushed around', arranged into patterns (which can also be imaged) and otherwise manipulated. These experiments, probings, and other interactions all yield *consistent* results and information about quantum entities using a variety of techniques and under different conditions. This is only possible because quantum systems and elementary particles exist whether we 'observe' them, conduct experiments with them, or not. In other

words, the entities of the quantum world are physically present independently of any actions by human beings. The quantum realm has shown itself to exist in verifiable ways!

The amount and diversity of the amassed evidence confirms the assumption of realism, i.e. that the quantum realm does exist independently of the observations of any sentient beings. Further, the means by which this has been established is an extension of scientific methods used in previous decades to show the existence of physical entities that were too small to be directly 'observed' in the early stages of their scientific investigation. The discovery of the structure of DNA in the mid-twentieth century based on X-ray diffraction data is one example of such an earlier demonstration (Gribbin 2002, 567–568). The extension of these earlier methodologies to the scale of quantum phenomena provides additional confidence that the experimental results obtained are due to an objective quantum reality.

The methods of physics have yielded compelling evidence for accepting the existence of the quantum realm. We might take a leaf out of the pages of the history of science and say that it now remains a matter of how much evidence is required to convince the sceptics. Whether it be the acceptance of a heliostatic over an Earth-centred solar system or the Einsteinian over the Newtonian worldview, it was the weight of evidence which finally decided the question. In respect to the attitudes of older physicists, perhaps many of them will never overcome the dominant thought patterns of the prevailing paradigm of Orthodox Quantum Theory (such as the denial of an independently existing quantum realm). If it proves to be the case that most older physicists cannot ever overcome these thought patterns, then a complete generational change in the discipline of quantum physics will be required to remove all of the sceptics (Kuhn 1996, 151–152; Riggs 1992, 47–48).

1.2 Causation, Causality, and Determinism

The word 'causation' has different meanings in different contexts. Some of these meanings are relevant to scientific theories and some are not. The extensive philosophical analyses of causation

undertaken over the last few centuries (and particularly in the second half of the twentieth century) have generated a substantial amount of 'ink on paper' but, as one commentator has described, all this is a jumbled mess of disparate accounts and approaches (Hall 2006, 9). 'Causation' as applied in science concerns processes involving connections between physical events such that one or more events bring about other events. Given that there are causal connections, the 'bringing about' aspect of causation may be expressed in terms of a causality principle. Although there are several versions, for the purposes of this book, the Principle of Causality is defined as follows:

- *Principle of Causality*

  The same cause or set of causes always produces the same effect or effects (other things being equal) and the cause(s) temporally precedes, or is simultaneous with, its effect(s).

The above statement of the Principle of Causality is consistent with the notion of cause and effect generally accepted in science (Campbell 1952, 49–50; Hempel 1966, 52–53; Bunge 1979, 4, 50–51; Skyrms 1986, 84; Newton 2000, 124–128), even if this is only implicit in some physical descriptions (Wallace 1974, 278). Of course, the *ceteris paribus* clause ('other things being equal') allows for a variety of circumstances such as whether there is multiple causation, whether the cause(s) is sufficient or necessary, etc. These issues will not be addressed as this book is not a detailed philosophical analysis of the nature of causation or of causal relations. Such an analysis would require a book of its own.

The Principle of Causality is an underlying aspect of modern physics and has been central to debates on the foundations of relativity and quantum mechanics for over a century. Despite this, there is a philosophical train of thought that the concept of causation should be purged from physics.[1] This train of thought follows Bertrand Russell's 1912 expressed view:

> All philosophers, of every school, imagine that causation is one of the fundamental axioms or postulates of science, yet, oddly enough, in advanced sciences … the word "cause" never appears. … The law of causality, I believe, like much that passes muster

[1] Papers on this and related themes are to be found in Suárez 2000, and in Price & Corry 2007.

> among philosophers, is a relic of a bygone age surviving, like the monarchy, only because it is erroneously supposed to do no harm (Russell 1913, 1).

Causal reasoning has always been a basic methodology in physics and in science generally, together with the search for causes. Whether philosophers eventually conclude that causation should or should not be part of the world's fundamental ontology, the Principle of Causality is going to remain indispensable to physics. Ewan Fales draws a similar conclusion about the future of the notion of causality in physics in his book devoted to causation:

> Russell … went so far as to foresee a time when physics would entirely abandon causal concepts and causal laws. Physics has not, however, abandoned these concepts, and it is hard to see how it could (Fales 1990, 78).

In modern daily life the causal aspects of scientific research is nowhere as obvious than in the advance of medical science where finding the causes of diseases and ailments is a principal objective. In physics too, we look for the causes of phenomena (Bohm 1957b, Chapter 1; Cartwright 1983, 11; Cartwright 2007, 52; Frisch 2008, 1). This may be plainly seen with reference to the history of physics[2] and to some of its contemporary technical literature. Relativity and electrodynamic texts and articles, for example, regularly invoke causal notions and postulates such as causal connectiveness, causality conditions, causal spacetime structures, causal boundaries, causality-violating circumstances, etc.[3] Relativistic quantum theory also appeals to the Principle of Causality (Sakurai 1982, 59).

It is, of course, well known that the laws of physics are not of the form: 'A causes B', but instead are expressed by functional relationships (e.g. equations that relate how a physical quantity changes with respect to other quantities). Expression of the laws of physics as functional relationships does not entail that the concept of causality is absent from physics, as Mathias Frisch (amongst others) has pointed out:

> … of course it does not follow from the fact that physical theories present us with functional dependencies that these dependencies themselves cannot be understood causally (Frisch 2008, 5).

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[2] For an account of the notions of cause and effect in the history of physics, see: Oldroyd 1986. Also see: Wallace 1974.

[3] Examples include: Hawking and Ellis 1973, Chapter 6; Jackson 1975, 306–309; Wald 1984, Chapter 8. An internet search of the physics literature will display many more examples.

Much of the formal language of physics has developed in such a way that it rarely employs the terms ‘cause’ and ‘effect’ explicitly (when used as nouns). Occasionally, one finds ‘cause’ used as a verb, as in the following passage from a nuclear physics textbook:

> … one disadvantage of the cloud chamber is that the density of the gas is not high enough to cause an appreciable amount of interaction … (Enge 1978, 219).

The word ‘cause’ (when used as a verb) is more commonly found in engineering rather than in physics textbooks. Engineering texts explicitly describe physical processes in causal terms such as: force causes acceleration; friction causes heating; increasing speed will cause a body to move outwards; the force transverse to the beam will cause it to shear; etc. (see, for example, Hannah and Hillier 1981). These kind of explicit causal descriptions are mostly absent in the physics literature even though the subject matter is the same in both engineering and physics articles and textbooks. This is because the language of physics tends to obscure the notion of causality (with the exception of relativity and electrodynamics) by employing synonyms of ‘cause’ (when used as a verb) such as ‘produce’ or ‘generates’. An example is the following exposition of aspects of supernovae dynamics:

> Not only does the shock wave carry with it energy and matter at high speeds and violently heat up the outer layers [of the star], it also produces in its wake more heavy elements from the pre-existing materials (Ho-Kim et al. 2004, 293).

An alternative rendering of this passage is that the shock wave *causes* more heavy elements to be formed in its wake. This re-statement is just as valid a description of the process as the above but also makes explicit the role of causality. Mario Bunge provided a suitable summary of these issues when he wrote:

> Russell … prophesised that … the word ‘cause’ will not occur in any statement of invariable laws … But it does not follow … that the concept of cause will finally be extruded … The *word* ‘cause’, which denotes a *generic* concept, need not occur explicitly in any particular scientific statement … The fact that science employs less and less of the word ‘cause’, which belongs to the philosophical vocabulary, cannot be regarded as a sign of decrepitude of the causal principle (Bunge 1979, 345, italics in original).

In later chapters, the transfer of energy (or energy-momentum) and the reality of processes involving energy will be discussed at length. This might tend to imply that a physicalist version of causation should be advocated. Indeed, sophisticated versions of physicalist theories, such as developed by the Australian

philosopher of science Phil Dowe (Dowe 2000), are very encouraging. However, these theories have yet to provide a complete explanatory account of causation and any further judgement on the viability of physicalist theories of causation would go beyond the scope of this book.

In regard to the notions of causality and determinism, it is a mistake regularly made to conflate them. We need to be careful not to identify the concept of 'causality' with the doctrine of 'determinism'. The doctrine of determinism has many different definitions (and variations thereof). Roughly speaking, determinism is the thesis that past events uniquely entail future events (where the entailment is not logical necessity). The philosophical issues of determinism will not be explicitly addressed here.

Rather, the term 'deterministic' is used in the following sense (as applied in physics). A physical theory is deterministic if it predicts the future state of a system uniquely from the specification of the system's present state. In other words, if the equation governing a particular phenomenon together with the relevant boundary conditions provide a unique solution, then the description of the phenomenon is deterministic. Conversely, if the governing equation does not provide a unique solution then the description of the phenomenon is non-deterministic. It does not logically follow that if a phenomenon is not describable by a completely deterministic formalism then the phenomenon is itself uncaused. Indeed, it turns out that even Newtonian mechanics (which is the paradigmatic case of a deterministic physical theory) is deterministic only in simple situations (Cushing 1998, 173). Yet, causal interactions in Newtonian mechanics would not be denied purely on the basis of whether its equations provide unique solutions in all circumstances or not.

The notions of determinism and of causality, although related, are distinct. The usage in the literature does not assist in making the distinction clear. A few words about the terminology found in the literature, therefore, are in order as these can be quite confusing. The term 'causality' is routinely identified with 'causal determinism' by which is meant the thesis that the laws of nature are deterministic in their form. In addition to 'causality' and 'determinism', it is not uncommon to find other terms which only

add to possible confusion, such as ‘deterministic causality’, ‘strict causality’, ‘statistical causality’, and ‘probabilistic determinism’. The first two of these terms are used somewhat synonymously to assert that the laws of nature are deterministic. The latter two are especially common in discussions of quantum theory and are used to assert that the laws of nature are non-deterministic. The word ‘indeterminate’ (as in ‘the theory is indeterminate’) is improperly used as a synonym for ‘non-deterministic’.

1.3 Introducing the Causal Theory of Quantum Mechanics

One focus of this book is the major conceptual issues that confront the Causal Theory of Quantum Mechanics. This theory is, as will be seen in later chapters, a micro-realistic theory of quantum physics that is consistent with the Principle of Causality. The names under which the Causal Theory of Quantum Mechanics has developed have varied somewhat in the literature and include: Causal Interpretation of Quantum Mechanics; Bohm Interpretation; Pilot Wave Theory; Bohm’s Theory; deBroglie-Bohm Interpretation; the Bohm Formulation; Ontological Interpretation of Quantum Mechanics; deBroglie-Bohm Theory; Quantum Theory of Motion; and Bohmian Mechanics. We shall use the designation – The Causal Theory of Quantum Mechanics (or Causal Theory, for short). The principal rationale for preferring this title is that the Causal Theory of Quantum Mechanics is *a theory in its own right* rather than merely being just one of the many existing interpretations of the formalism of quantum mechanics. Here we understand ‘formalism’ and ‘interpretation’ in the following senses. The formalism of a physical theory comprises the laws and rules which allow the theory to make theoretical advances and empirical predictions. An interpretation of a physical theory is an account of what the various terms postulated in the theory represent (Cushing 1992, 37).

The claim that the Causal Theory is a theory in its own right is justified for the following reasons:

(i) The Causal Theory is a mature scientific endeavour with more than fifty years of progress.

(ii) The axioms of Orthodox Quantum Theory are not identical to the axioms of the Causal Theory (see Chapter 2 for the former and Chapter 3 for the latter).

(iii) The conceptual structures postulated in the two theories (Causal and Orthodox) are radically distinct. This is an important point that was emphasised in 1957 by Hans Freistadt. He wrote:

> … a physical theory is not only a computational tool; it is also a conceptual model … the underlying models [Causal and Othodox] are so diametrically opposed that one is faced, indeed, with two different *physical* theories (Freistadt 1957, 3, italics in original).

(iv) The Causal Theory offers the *possibility* of making predictions that differ from those of the Orthodox Quantum Theory in areas that have not been subject to experimental investigation (see Section 5.9). The possibility of making different predictions is usually taken as *the* criterion that distinguishes between being a theory or being an interpretation of a theory.

Why use the name 'Causal'? Orthodox Quantum Theory is known for doing away with causality in the sense of event-by-event causality. Event-by-event causality refers to the existence of the causes of contiguous events in space and time (Bunge 1979, 14; Cushing 1998, 290 and 298). Orthodox Quantum Theory denies that this is the case. A prime example of the absence of event-by-event causality in Orthodox Quantum Theory is its claim that a quantum particle (such as an electron) *does not have* a trajectory unless it is 'observed' (or on some accounts, *does not exist* until a 'measurement' is made). The Causal Theory, on the other hand, embraces causality by explaining micro-phenomena in terms of entities with definite properties and physical processes that occur in space over time. In the Causal Theory, the future state of a quantum system is determined by the dynamics of the system and its interactions with the surrounding environment, i.e. the Causal Theory is deterministic.

There are many advantages to be found in accepting the Causal Theory over Orthodox Quantum Theory, not the least of which is the ability of the Causal Theory to *make sense* of the

quantum realm, as was argued by Michael Dickson in his detailed book on interpretations of quantum mechanics:

> [The Causal Theory provides] … a tolerably clear picture of the quantum world (Dickson 1998, 125)

and by Detlef Dürr and his colleagues who cogently argue that:

> When all is said and done, Bohmian mechanics [i.e. the Causal Theory] emerges as a precise and coherent "quantum theory" providing a microscopic foundation for the quantum formalism (Berndl et al. 1995, 748).

In particular, we shall see in later chapters that the Causal Theory has the following benefits:

- a ontology in terms of entities and processes in space and time;
- no arbitrary division between classical and quantum realms;
- a single, continuous dynamics;
- no measurement problem;
- no need to postulate hypothetical mechanisms designed to overcome conceptual difficulties inherent in Orthodox Quantum Theory;
- the paradoxes found in Orthodox Quantum Theory are solvable or do not occur; and
- Heisenberg's Uncertainty Principle does not have ontological implications.

Despite these desirable features, the Causal Theory's reception from its beginning to the present day has been most unfavourable. The general assessment by the physics community of the Causal Theory is plagued with a variety of 'myths' and widely-held misconceptions. The most common of these are as follows:

① The Causal Theory is a return to classical physics.

② The Causal Theory contains 'hidden variables'.

③ The Causal Theory is disproved by the various impossibility theorems.

④ The Causal Theory has been refuted by experiments on Bell-type inequalities.

⑤ The Causal Theory is pure metaphysics.

⑥ The Causal Theory is inconsistent.

⑦ The Causal Theory cannot be made relativistic.

⑧ The Causal Theory cannot incorporate intrinsic angular momentum (spin).

These eight alleged flaws/misconceptions are quite commonly held by physicists who are aware of the existence of the Causal Theory (although many are not). However, these (and other) criticisms have either not been properly substantiated or have been shown to be false, as commented on by the late American physicist and philosopher of science James T. Cushing:

> Most physicists do not really know much about Bohm's [Causal] theory and those who are even vaguely aware of its existence usually "know" that it is wrong, although they are not exactly certain just why. … the folklore-wisdom charges against Bohm's program … can be seen to be either specious or inconclusive … (Cushing 1996, 5–6).

Since these alleged flaws and misconceptions regarding the Causal Theory continue to have strong currency in the physics community, it will be shown (or otherwise indicated) during the course of this book why they are ill-founded and/or unwarranted. Further motivations for embracing the Causal Theory will be discussed in Chapter 3.

This book deals only to non-relativistic quantum mechanics. Relativistic quantum mechanics and relativistic quantum field theory are concerned with quantum particles with velocities near the speed of light, their creation and annihilation, and associated quantum fields. However, there is still much to be understood in the arena of non-relativistic quantum phenomena which does not require a relativistic description. In particular, we shall not be concerned with the relativity of time. Since the Principle of Causality (as defined above) makes explicit reference to simultaneous events, this implies that a preferred frame of reference will be required for some interactions. In practice, there are preferred coordinate reference frames for non-relativistic quantum mechanics. It is common to take the frame in which experiments on the validity of quantum mechanics are conducted, i.e. the (approximately inertial) frame attached to the Earth's surface. When a many-particle quantum system is studied, it is also common to use the centre-of-mass reference frame. We shall see in Chapter 3 that the description of a

many-particle quantum system in the Causal Theory needs a preferred frame of reference since the motion of one particle at any given time depends on the positions of all the other particles in the system. This need for a preferred reference frame cannot be avoided in a non-relativistic theory. Where appropriate, the limitations of the non-relativistic Causal Theory of Quantum Mechanics will be explicitly mentioned.

1.4 Assessing Rival Scientific Theories

We shall also have to take on board a few points about scientific theories in general and assess (to some degree) the various merits of the Causal Theory versus Orthodox Quantum Theory. It is important when comparing rival scientific theories to acknowledge that they are postulated in order to meet a number of needs. A scientific theory must be empirically successful, i.e. the predictions of the theory must be borne out by experiment to within the range of the accuracy available. However, empirical success is *not all* that is required from scientific theories. One might think that a theory's ability to describe why events occur and why instruments record the numerical results that they do, for example, would perhaps have equal standing with empirical success.

In addition to empirical adequacy, there are other criteria for the assessment of the virtues of rival scientific theories. The following list has a large degree of consensus in both the scientific and philosophical communities (Riggs 1992, 51):

- explanatory success;
- predictive power;
- consistency; and
- conceptual coherence.

The sometimes used criteria of simplicity and aesthetic appeal will not be entertained as explicit criteria for theory choice as both of these are very subjective (Giere 1988, 224–225). Nature need not conform to a specific account of simplicity that one finds

attractive or to particular notions of what constitutes ‘beauty’ in a physical theory (Riggs 1992, 13).

As with all physical theories, the Causal Theory embodies a mathematical model. It is an uncontroversial point that no model captures all aspects of the phenomena under study. A scientific theory then, should not be taken *literally in all respects*. Nor is it the case that any one theory (or version thereof) is the final word regarding the phenomena described. However, it shall be the contention here that some suitably developed and tested scientific theories, when interpreted realistically, have features that ‘mirror’ aspects of an observer independent reality (Bunge 1973, 7–8; Penrose 2004, 508). It will be argued that the Causal Theory is one such theory and that this is supported by both the evidence for the objective existence of quantum entities (as discussed in Section 1.1 above) and the evidence in favour of matter waves (discussed in Chapter 4).

1.5 Conceptual Issues, Theoretical and Conceptual Problems

Conceptual issues may be broadly defined as those issues that are relevant to the concepts embodied in a theory rather than its empirical content. Physics has always been a scientific discipline where conceptual issues play important (and sometimes crucial) roles in the development and success of its theories. When conceptual issues in physics are scrutinised they usually bring into focus two broad categories of problems – theoretical and conceptual. Physics is a discipline where both theoretical and conceptual problems abound. Depending on the specific context, the definitions of these two types of problems have not always been made clear and, in some instances, there is overlap between them. Additionally, the solution of a conceptual problem may lead to theoretical advances, i.e. the resolution of conceptual difficulties in physical theory may open up new avenues for solution of previously unsolved (or perhaps unknown) theoretical problems. In order to avoid any confusion, theoretical and conceptual problems will now be defined for the purposes of this book.

We shall follow standard usage in relation to defining theoretical problems. A theoretical problem is one relating to some unresolved technical aspect of the theory under examination. Further theoretical development is required for its solution. One example was the 1927 theoretical solution of the empirical difficulty regarding losses of energy and angular momentum in nuclear beta-decay processes. The problem was solved by the theoretical development of postulating the existence of two new subatomic particles, the neutrino and the anti-neutrino. This allowed each of the equations for beta-decays to be rewritten to include an additional particle. The addition of another particle then accounted for the otherwise missing energy and angular momentum. This was purely a theoretical solution at that time (1927) for the neutrino was not experimentally detected until 1954 (Gasiorowicz 1974, 450).

In respect to conceptual problems, we shall use the definition formulated by the American philosopher of science Larry Laudan. A conceptual problem is generated either when a theory contains internal inconsistencies – an internal conceptual problem, or where a theory is in conflict with another well-established theory or widely held belief or doctrine – an external conceptual problem (Laudan 1977, 49–51).

The criterion for internal consistency is fairly obvious since any theory that is not internally consistent will contain within itself, logical contradictions. Conflict with another well-established theory might be due to one theory being logically inconsistent with another theory. External conceptual problems are also frequently the result of a different form of conflict, i.e. conflict between the claims of a theory and a widely held belief. This notion of conflict needs some elaboration and a well known example should suffice to illustrate the point.

In 1917, the initial relativistic model of the cosmos invented by Albert Einstein was not static. This conflicted with the prevailing view of the era that the universe was neither expanding nor contracting (Cushing 1998, 262). The conflict with this view generated an external conceptual problem for Einstein's theory. Consequently, Einstein introduced another term into the field equations of General Relativity (called the Cosmological Constant) which resulted in a static universe model and thereby resolved the

external conceptual problem faced by the theory at that time. (This turned out to be an unnecessary step, as Edwin Hubble announced in 1929 that observations of distant galaxies showed that the universe was, in fact, expanding.)

Finding solutions to external conceptual problems is not generally as straight forward as solving theoretical ones. This is due to a number of factors. The solution to an external conceptual problem may involve the generation of new ideas and/or new interpretations of existing concepts. Indeed, the process of finding a solution to a given (external) conceptual problem may not require any formal theoretical development of a theory at all!

Another well known example from the history of science that illustrates this point is Tycho Brahe's model of the universe. This model avoided the conceptual problem inherent in the Copernican model, i.e. that of not having the Earth at the centre of the universe. It did so by postulating that although the Sun revolved around the Earth, all the other planets revolved around the Sun. Brahe's model was mathematically equivalent to the Copernican model (and so had all of its technical advantages) but without the unacceptable conflict of the time of having the centre of the universe at a place other than the centre of the Earth (Dreyer 1953, 363–364).

The solution of conceptual problems, as noted above, may open up new avenues for the solution of theoretical problems. Conceptual problems may also arise from new theoretical advances and it is not uncommon for the two to go hand in hand. In cosmology, Stephen Hawking's set of theorems about the singular nature of the origin of the universe is a first-class example. Hawking showed that, based on some very reasonable physical assumptions, the General Theory of Relativity implied that there must have been a beginning to the whole universe at a finite time in the past and that this was a singularity (i.e. a point of infinite density and zero volume) commonly called the 'Big Bang' origin (Hawking 2003, 111). This created a host of both theoretical and conceptual problems for relativistic cosmology. The presence of one or more singularities in a physical theory is an example of a theoretical problem for many theoreticians consider this to be a serious blemish on the theory that needs to be formally eliminated. The 'Big Bang' origin of the universe itself constitutes a conceptual problem in the

view of many cosmologists for the ‘Big Bang’ is where all the laws of physics apparently break down.

The above examples indicate the significance of conceptual problems in physical theories and also the importance of resolving them. Similar to other physical theories, the Causal Theory of Quantum Mechanics also has its own set of conceptual problems, some of which have been used to criticise the theory. Several of the more important conceptual problems will be addressed in later chapters.

Chapter 2

Preliminaries

I think I can safely say that nobody understands quantum mechanics.
— Richard Feynman

Abstract This chapter displays the core concepts and principles of Orthodox Quantum Theory together with its mathematical formalism as a set of axioms. The general form of Heisenberg's uncertainty relation is derived. Discussions of quantum uncertainty and the 'Measurement Problem' are presented. Two of the best known quantum paradoxes, Einstein-Podosky-Rosen and Schrödinger's Cat, are outlined. 'Hidden Variable' theories are defined. Formulations of the Kochen-Specker Theorem and Bell's Theorem are provided with a brief description of GHZ states.

2.1 Orthodox Quantum Theory and its Mathematical Formalism

The early days of quantum theory were a period of great puzzlement and disillusion for those involved in trying to formulate a consistent theoretical scheme of atomic phenomena. This scheme had to provide an empirically satisfactory account of diverse sets of experimental data, such as the Photoelectric and Compton Effects, and atomic spectral lines. Werner Heisenberg expressed the frustration experienced when he wrote:

> … an intensive study of all questions concerning the interpretation of quantum theory in Copenhagen finally led to a complete … clarification of the situation. But it was not a solution which one could easily accept. I remember discussions with Bohr which went through many hours till very late at night and ended almost in despair; and when at the end of the discussion I went alone for a walk in the neighboring park I repeated to myself again and again the question: Can nature possibly be as absurd as it seems to us in these atomic experiments? (Heisenberg 1989, 30).

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Heisenberg was right to be worried about the apparent absurdity of quantum physics. The concerns that he enunciated in the above quotation are just as relevant to the contemporary debate on the foundations of quantum mechanics, as they were when he first conceived them. Yet, as absurd as it may sometimes appear, quantum mechanics is one of the two best experimentally confirmed theories in the whole history of physics (the other being, of course, Relativity). There exists little disagreement about the mathematical apparatus of quantum mechanics, but what does this formalism tell us about the nature of the quantum realm?

We noted in Chapter 1 that the (Copenhagen) version of quantum theory originally due to Bohr and Heisenberg, and progressed by Max Born, Wolfgang Pauli, John von Neumann, Paul Dirac, and others (i.e. Orthodox Quantum Theory) achieved dominance in the physics community. Further, it is generally accepted by a majority of physicists that Orthodox Quantum Theory has *essentially dealt with* the interpretative problems that plagued the beginnings of quantum mechanics. Fortunately, there have been substantial views to the contrary occasionally expressed by leading physicists. Murray Gell-Mann for example, the recipient of the 1969 Nöbel Prize for Physics, stated in his Nöbel acceptance speech that:

> Bohr brainwashed a whole generation of physicists into believing that the problem [of the interpretation of quantum mechanics] had been solved fifty years ago (Gell-Mann 1972).

The alternative designations to 'Copenhagen Interpretation' found in the literature include: 'Standard Quantum Theory'; 'Copenhagen Orthodoxy'; 'Received Quantum View'; 'Conventional Quantum Mechanics'; 'Usual Interpretation'; 'Orthodox Interpretation'; and 'Quantum Orthodoxy'. These shall be taken as synonymous with 'Orthodox Quantum Theory'. The formal aspects of Orthodox Quantum Theory are defined by the set of axioms that appear below. However, before presenting these axioms, it will be of assistance to state the core concepts and principles of Orthodox Quantum Theory.

Let's begin with the concept of wave-particle duality. This is the expression of the apparent dual nature of quantum phenomena as manifest in either wave-like *or* particle-like behaviour but *not both* in a single experimental arrangement, as Heisenberg once explained:

> … Bohr advocated the use of both [wave and particle] pictures, which he called "complementary" to each other. The two pictures are of course mutually exclusive, because a certain thing cannot be at the same time a particle … and a wave (Heisenberg 1989, 37).

The notion of wave-particle duality is an essential component of Orthodox Quantum Theory as is evidenced by the in-depth discussions of the topic by the founders of quantum mechanics and by the number of textbooks that present it as such (examples include: Enge et al. 1972, 142–143; Eisberg and Resnick 1985, 62–63; Kragh 1999, 210; Rae 2002, 7–10.). There are, however, some physicists who are clearly not keen to acknowledge this any more (e.g. Griffiths 2005, 420 note 1). Bohr embodied this notion into his Principle of Complementarity which he considered to be at the heart of quantum theory. Although Bohr's statements on this matter suffered from a lack of clarity, the Principle may be broadly stated as follows (Jammer 1974, 95):

- *Principle of Complementarity*

  Any application of a classical concept precludes the simultaneous use of other classical concepts which in a different connection are equally necessary for the description of phenomena.

Bohr also proposed a principle designed to 'bridge' the gap between classical and quantum realms. This is called the Correspondence Principle of Orthodox Quantum Theory (Gasiorowicz 1974, 19):

- *Correspondence Principle*

  Quantum states and measurements will tend to the corresponding classical case in the limit of large quantum numbers.

Two other fundamental features of Orthodox Quantum Theory are its postulation of 'indeterminacy' and the role of measurement. These features have turned out to be major departures from classical physics.

Quantum 'indeterminacy' is understood as meaning that the quantum level of description is one where physical quantities do not have definite values (or, on some accounts, do not exist) unless they are measured. This is inferred on the basis of Heisenberg's famous Uncertainty Principle. An accepted statement of the Uncertainty Principle is as follows (Schiff 1968, 7; Ho-Kim et al. 2004, 467):

- *Uncertainty Principle*

  It is impossible to specify precisely and simultaneously the values of a pair of canonically conjugate variables that describe a quantum state.

The Uncertainty Principle is given an exact mathematical form as the uncertainty relations (see Section 2.2 below). In Orthodox Quantum Theory, these relations are taken as providing an *in-principle* limitation on the precision of the simultaneous measurement of some quantities.

The process of making measurements on a quantum system (sometimes referred to as 'observing' a quantum system) is specified in the axioms of Orthodox Quantum Theory (see below). The inclusion of an explicit measurement axiom endows a special status on measurement within Orthodox Quantum Theory (e.g. Sudbery 1986, 185; see also Section 2.3). Further, the actual placement of a measuring device is somewhat arbitrary which introduces a degree of subjectivity.

The mathematical formalism of Orthodox Quantum Theory can be represented in several different ways. The term 'quantum mechanics' is usually taken to encompass Erwin Schrödinger's wave mechanics (Schrödinger 1926, 361–376, 489–527), Heisenberg's matrix mechanics (Heisenberg 1926, 411–426), and abstract generalisations of them. It is a standard practice to present quantum mechanics as a set of axioms and it will be useful to follow this practice. More detailed remarks about the axiomization of a physical theory will be made in Chapter 3. The axioms appearing below are to be taken as defining the formal aspects of Orthodox Quantum Theory. These axioms refer to a quantum system in a pure state. Such states are uniquely definable by state vectors evolving in time. States other than pure states, i.e. mixed states, also appear in quantum mechanics. However, mixed states cannot be represented by state vectors but instead are represented by density operators which are used to describe ensembles of systems of which there is incomplete information (Saxon 1968, 387–388). (Mixed states will not be considered as these will not be of assistance in resolving the stated conceptual issues.) Axiomizations similar to those presented below are found in the literature (e.g. Sudbery 1986, Chapter 2) but these tend to combine some of the twelve axioms presented here.

2.1.1 Axioms of Orthodox Quantum Theory

State Vectors Axiom

Any (pure) quantum state is described by a state vector which is an element of a Hilbert space. Multiplication of a state vector by a complex number results in a description of the same physical state.

Completeness Axiom

A state vector contains all possible information about the quantum state.

Linear Superposition Axiom

The addition of two or more state vectors results in a state vector which describes another quantum state.

Inner Product Axiom

The inner product of two state vectors ψ_1 and ψ_2, denoted by (ψ_1, ψ_2) on a Hilbert space is defined as a mapping of an ordered pair of state vectors into the complex numbers with the following properties:

$$(\psi_1, \psi_2) = (\psi_2, \psi_1)^*$$

$$(\psi_1, c\psi_2) = c\,(\psi_1, \psi_2)$$

$$(c\psi_1, \psi_2) = c^*\,(\psi_1, \psi_2) \text{ and } (\psi_i, \psi_i) \geq 0$$

where c is a complex number and * denotes the complex conjugate.

Hermitian Operators Axiom

Corresponding to every physical observable is a linear, Hermitian operator on the Hilbert space. These operate on state vectors to give other state vectors.

Eigenstate Axiom

A state vector ϕ is in an eigenstate of an operator **A** if the equation: $\mathbf{A}\phi = \alpha\phi$ holds, where α is a real number called the eigenvalue.

Expansion Postulate

An arbitrary state vector ψ can be expanded into a complete set of eigenstate vectors (eigenvectors) ϕ_i, where $\psi = \sum_i c_i \phi_i$ is a linear superposition and the coefficients are given by $c_i = (\psi, \phi_i)$.

Canonical Commutation Relations

The canonical commutation relations are defined by the following equations:

$$[\hat{\mathbf{q}}_i, \hat{\mathbf{q}}_i] = 0 = [\hat{\mathbf{p}}_i, \hat{\mathbf{p}}_i] \text{ and } [\hat{\mathbf{p}}_i, \hat{\mathbf{q}}_i] = i\hbar\ \delta_{ij}$$

where the commutator [A, B] = AB – BA, $\hat{\mathbf{p}}_i, \hat{\mathbf{q}}_j$ are operators corresponding to canonical conjugate variables, δ_{ij} is the Kronecker Delta, $\hbar$ is Planck's Constant divided by 2π, and $i = \sqrt{-1}$.

Time Development of States Axiom

A state vector ψ develops in time according to the equation: $i\hbar\,(\partial\psi/\partial t) = \mathbf{H}\psi$, where **H** is the Hamiltonian operator.

Projection Postulate

A measurement of an observable on a system in a (superposition) state given by $\psi = \sum_i c_i\phi_i$ will project (or reduce) ψ into one of the eigenvectors ϕ_i of the superposition.

Quantisation Algorithm

A measurement of an observable only can yield an eigenvalue of the operator corresponding to that observable.

Born Statistical Postulate

The probability that a measurement on an observable A of a system in a state described by a state vector $\psi = \sum_i c_i\phi_i$ will yield an eigenvalue α_n is $|c_n|^2$, where $\mathbf{A}\phi_n = \alpha_n\,\phi_n$, with **A** is the operator corresponding to observable A, with ψ and ϕ_i normalised: $(\psi,\psi) = 1 = (\phi_i, \phi_i)$.

A few points about the axioms of Orthodox Quantum Theory are in order. These axioms specify that the (pure) state of a quantum system is represented by a vector in a Hilbert space, a formalism that has proved high successful at the empirical level. Basically, Hilbert space is a complex vector space on which an inner product is defined **(**Wallace-Garden 1984**,** 7). The vectors in this mathematical space are 'normalised' to unity, i.e. have unit length. An observable is always represented by a Hermitian operator which, by definition,

is an operator that is equal to its adjoint (Szekeres 2004, 348). This ensures that the operator's eigenvalues are real-valued, as must be the case for the measured value of any observable.

However, the Hilbert space representation is very different from the way that the state of a physical system is characterised in classical mechanics where the system's state may be represented by a point in a phase space. Why have a physical theory that uses abstract (Hilbert) vector spaces? Reasons for adopting the Hilbert space formalism for quantum mechanics include:

- the failure of classical mechanics and the old quantum theory of Bohr and Sommerfeld to successfully account for empirical results at the atomic level;
- the need for a mathematical space in which the represented form of quantum states have certain desirable properties, e.g. being normalisable (Penrose 2004, 534);
- representing the uncertainty in measurement in terms of a non-commutative algebra (Bub 1996, 211).

However, the representation of physical systems by means of a Hilbert space has always been somewhat odd and counter-intuitive, as observed by Robert Carroll:

> Perhaps no subject has been the focus of as much mystery as "classical" quantum mechanics (QM) even though the standard Hilbert space framework provides an eminently satisfactory vehicle … So why all the fuss? The erection of the Hilbert space edifice … has an air of magic. It works but exactly why it works and what it really represents remain shrouded in ambiguity (Carroll 2004, 1).

In Orthodox Quantum Theory, the state vector is taken as containing all possible information about a quantum system (Completeness Axiom). No physical reality is ascribed to a state vector (Rae 2002, 284; Penrose 2004, 805). Consequently, any questions relating to a quantum system that go beyond what can be found from the state vector are considered *meaningless* in Orthodox Quantum Theory (Jammer 1966 330; Bohm 1957b, 92). It is in this sense that Orthodox Quantum Theory is claimed to be 'complete'. Examples of such questions include: What is the exact nature of quantum entities? What paths do quantum particles trace out in space when they are not being observed?

It is usual for a given state of a quantum system to be in a superposition of other states, consistent with the Principle of Linear Superposition which applies to classical as well as quantum waves and fields. The Principle may be stated as follows:

- *Principle of Linear Superposition*

  When several individual states are superimposed, the resultant state is the addition of the individuals.

When a quantum system is in a superposition, its state is described by a sum of state vectors. This appears formally in Orthodox Quantum Theory as the Linear Superposition Axiom. Moreover, if not specifically prepared and left 'unobserved', a quantum system will be in a *superposition of eigenstates*. An eigenstate may be defined as a state pertaining to a particular Hermitian operator (call it **A**) where the eigenstate is described by a state vector (call it ϕ) such that the equation: $\mathbf{A}\phi = \alpha\phi$ holds, with α being a real number called the eigenvalue (Eigenstate Axiom). The time evolution of a state vector when 'unobserved' is governed by the time-dependent Schrödinger equation (Time Development Axiom):

$$i\hbar\ (\partial\psi/\partial t) = \mathbf{H}\psi$$

where the state vector is $\psi = \sum_i c_i\phi_i$ with the ϕ_i being eigenstate vectors.

Orthodox Quantum Theory is principally about predicting the results that would be obtained if the value of some physical parameter (or quantity) of a quantum system, i.e. an observable, were to be measured. The result of measuring an observable is always an eigenvalue of the Hermitian operator corresponding to the observable (Quantisation Algorithm). However, unless a quantum system is already in an eigenstate of a Hermitian operator rather than a superposition of states, a measurement (or more generally, an 'observation') of some physical quantity will instantaneously 'reduce' the superposition to a single eigenstate (Projection Postulate). This is a process that is inherently *non-deterministic* and for which the theory only can predict the *probability* that a particular value for the relevant physical quantity will be found on measurement.

Probability in Orthodox Quantum Theory is supposed to convey a notion of objective chance. The interpretation of probability in quantum mechanics has been the subject of much debate but at a basic level, probability may be understood in the sense of a relative frequency. The probability for a particular value of an observable to be found on a single measurement is taken as the fraction of the total number of results that yield this value in a large number of such measurements which are repeated under identical conditions (Gibbins 1987, 8; Ballentine 1998, 35). There are, therefore, two types of evolution of quantum states, i.e. evolution in conformity with the Schrödinger equation and evolution on measurement (Ballentine 1970, 369; Penrose 2004, 528–532).

It should be clear from the above discussion that Orthodox Quantum Theory is both an algorithm for obtaining statistical predictions for the results of experiments and a prescription for avoiding fundamental questions. In other words, Orthodox Quantum Theory is essentially an *instrumental* theory (Smart 1968, 159). This was well expressed by Christopher Norris in his book on realism and quantum theory:

> [Orthodox Quantum Theory] … effectively debars all attempts to interpret the quantum formalism aside from their purely instrumental yield … What is thereby excluded is any prospect of advancing beyond that stage to the point where it becomes possible to achieve a more adequate (realist or causal-explanatory) account of quantum phenomena (Norris 2000, 34).

2.2 Uncertainty at the Quantum Level

The common account of the uncertainty relations is that they are an *in-principle* limitation on the precision of simultaneous measurements of some quantities of a quantum system, such as position and momentum. Is this claim necessitated (in some sense) by the mathematics? In order to address this question, the general uncertainty relation will be derived. In mathematical statistics, the mean value of a quantity A is the average value obtained from a large set of individual values of A. This mean is defined by:

$$\langle A \rangle = \sum_{i=1}^{N} A_i \Big/ N \qquad (2.1)$$

where the A_i are individual values of A of which the total number of values is N. In quantum mechanics, the expression for the expectation value (or mean) of a quantity A in a given state may be derived from its axioms. Measurements of the value of an observable A of a system in a state with state vector $\psi = \sum_i c_i\phi_i$ will yield eigenvalues α_i with frequency $|c_i|^2$. The expectation value will then be given by summing all the products of individual eigenvalues with their associated frequency (Boas 1966, 697), i.e. $\sum_i \alpha_i |c_i|^2$. This provides agreement with the statistical definition (Equation 2.1). Since $\mathbf{A}\phi_n = \alpha_n\phi_n$ where $\mathbf{A}$ is the Hermitian operator corresponding to an observable A, it follows that: $\mathbf{A}\psi = \sum_i c_i\mathbf{A}\phi_i = \sum_i c_i\alpha_i\phi_i$ and the inner product $(\psi, \mathbf{A}\psi) = \sum_i (c_i\phi_i, c_i\alpha_i\phi_i) = \sum_i \alpha_i|c_i|^2 (\phi_i, \phi_i) = \sum_i \alpha_i |c_i|^2$ if $(\phi_i, \phi_i) = 1$, i.e. if the ϕ_i are normalised. More generally, the expectation value of an observable A in a state described by a state vector ψ is given by:

$$\langle A \rangle_\psi = (\psi, \mathbf{A}\psi) \,/\, (\psi, \psi) \qquad (2.2)$$

if ψ is not normalised. The variance (or dispersion) of a quantity A, denoted $(\Delta A)^2$, is defined by (Ballentine 1970, 364):

$$(\Delta A)^2 = \langle \, (A - \langle A \rangle)^2 \, \rangle \qquad (2.3)$$

Variance is a measure of the spread or scatter of values about the mean, i.e. the width of the statistical distribution of the value of A (Boas 1966, 697).

The derivation of the general uncertainty relation uses a form of the Schwarz Inequality (familiar from pure mathematics) as applied to inner products. This derivation appears in the literature in varying forms (examples include: Redhead 1987, 59–61; Sakurai 1985, 34–36; and Griffiths 2005, 110–111). Here is a short version of the 'proof'. Consider the inner product: (uv + w, uv + w) where v and w are arbitrary state vectors and u is a number. Using the rules set out in the Inner Product axiom, we have:

$$(uv + w, uv + w) = (uv + w, uv) + (uv + w, w)$$

$$= u^2 (v, v) + u \{(w, v) + (v, w)\} + (w, w)$$

$$\Rightarrow (uv + w, uv + w) = au^2 + bu + c$$

where the coefficients $a = (v, v)$, $b = \{(w, v) + (v, w)\}$, and $c = (w, w)$ must be non-negative numbers. The condition for this quadratic expression to be non-negative (which it must be as the left-hand side is an inner product) is ($b^2 \leq 4ac$) (Sudbery 1986, 59). Therefore

$$(v, v)(w, w) \geq ¼ \{(w, v) + (v, w)\}^2 \qquad (2.4)$$

Now let $v = (\mathbf{A} - a)\chi$ and $w = i(\mathbf{B} - b)\chi$, where **A** and **B** are non-commuting Hermitian operators, χ is a normalised state vector, $a = \langle \mathbf{A} \rangle$, $b = \langle \mathbf{B} \rangle$, and $i = \sqrt{-1}$. Then substituting for v gives:

$$(v, v) = \big((\mathbf{A} - a)\chi, (\mathbf{A} - a)\chi \big) = \big(\chi, (\mathbf{A} - a)^2 \chi\big)$$

$$= \big\langle (\mathbf{A} - a)^2 \big\rangle = \big\langle \big(\mathbf{A} - \langle \mathbf{A} \rangle\big)^2 \big\rangle$$

Similarly, we have:

$$(w, w) = \big\langle (\mathbf{B} - b)^2 \big\rangle = \big\langle \big(\mathbf{B} - \langle \mathbf{B} \rangle\big)^2 \big\rangle$$

and

$$(w, v) + (v, w) = i \big(\langle \mathbf{AB} \rangle - \langle \mathbf{BA} \rangle\big) = \big\langle i [\mathbf{A}, \mathbf{B}] \big\rangle$$

where $[\mathbf{A}, \mathbf{B}] = \mathbf{AB} - \mathbf{BA}$. The product of the variances of the two operators **A** and **B** may now be found from the above results using Equations (2.3) and (2.4):

$$(\Delta \mathbf{A})^2 (\Delta \mathbf{B})^2 \geq \big\langle i [\mathbf{A}, \mathbf{B}] \big\rangle^2 = ¼ \langle \mathbf{C} \rangle^2 \qquad (2.5)$$

where $[\mathbf{A}, \mathbf{B}] = i\mathbf{C}$. This inequality shows that the product of the statistical variances of **A** and of **B** has a lower bound.

The uncertainty of a quantity is taken to be the root mean square (RMS) deviation (also known as the standard deviation). The RMS deviation is defined as the positive square root of its statistical variance (Boas 1966, 697). Dispersion-free states are those for which the RMS deviation is zero. The general uncertainty relation

follows from the definition of the RMS deviation and the inequality (2.5):

$$(\Delta\mathbf{A})\,(\Delta\mathbf{B}) \geq \tfrac{1}{2}\,|\langle\mathbf{C}\rangle| \quad (2.6)$$

The interpretation of this general uncertainty relation is if a system is prepared in a pure state, then repeated measurements of A in an ensemble of identically prepared systems will yield a standard deviation $\Delta\mathbf{A}$ around the mean value $\langle\mathbf{A}\rangle$. Likewise, measurements of **B** will yield a standard deviation $\Delta\mathbf{B}$ (Redhead 1987, 61). If **A** and **B** are canonically conjugate operators, then $[\mathbf{A}, \mathbf{B}] = i\hbar$ (Canonical Commutation axiom) and the expression for the lower bound (2.6) reduces to the recognisable form of Heisenberg's uncertainty inequality: $(\Delta\mathbf{A})\,(\Delta\mathbf{B}) \geq \hbar/2$.

Many critiques emphasise that the common account of the uncertainty relations is often misunderstood and misrepresented. The world renowned philosopher of science, the late Sir Karl Popper, for example, argued continually against this common account:

> … Heisenberg's famous formulae … are, beyond all doubt, validly derivable *statistical formulae* of the quantum theory. But they have been *habitually misinterpreted* by those quantum theorists who said that these formulae can be interpreted as determining some upper limits to the *precision of our measurements* … (Popper 1982, 53–54, italics in original).

Other analyses of the uncertainty relations have made similar conclusions (Margenau 1950, 375–377; Ballentine 1970, 365). What the general uncertainty relation asserts is that there are limitations on the preparation of dispersion-free states for all observables (Redhead 1987, 62). Based on its formal statistical origin alone, the uncertainty relations would not constitute an in-principle limitation on the precision of *simultaneous* measurements on 'conjugate' observables of a quantum system (Jammer 1966, 330). Such a limitation would need *at the very least* one further assumption (Popper 1975, 216). In any case, the simultaneous measurement of different observables in the same quantum system is not possible in practice. Nor do the uncertainty relations indicate the absence of possessed values for some physical quantities. The mathematics does not necessitate these claims. (It can be argued that these are demanded by other considerations but this is, of course, a different

issue.) Further discussion of the nature of the uncertainty relations will appear in Chapter 3, in the context of the Causal Theory.

2.3 The Measurement Problem and Quantum Paradoxes

The concept and meaning of 'measurement' has always held central stage in discussions of the foundations of quantum mechanics. This is certainly the case in Orthodox Quantum Theory which provides little more than predictions of the results that can be obtained if one *were to measure* some physical parameter of a quantum system. Measurement holds a privileged place in Orthodox Quantum Theory as it appears in the theory's axioms. This special status is afforded to measurement on the basis that its effects cannot be made arbitrarily small (Redhead 1987, 52) and that a system's evolution on measurement is different from its non-measurement evolution (cf. Section 2.1). Yet, the term 'measurement' is used carelessly in the literature and it is not hard to find objections to this poor usage (Bell 1990, 20).

Much has been written about the Measurement Problem in quantum mechanics and the various quantum paradoxes. Indeed, whole books and dissertations have been devoted to these topics. It is not the intention here to try to do justice to these extensive discussions. Since there are many detailed descriptions of these in the literature, a level of familiarity will be assumed and only a brief outline is provided.

2.3.1 The Measurement Problem

What exactly is the 'Measurement Problem'? The problem, according to some accounts (e.g. Stone 1994, 250), arises by simultaneously holding firm to the following propositions: (i) the state vector ψ is a complete description of any system's physical state; (ii) the state vector evolves in time according to: $\psi = \mathbf{U}\psi_o$, where $\mathbf{U}$ is a linear unitary operator; (iii) to each observable quantity

there corresponds a linear operator **A** with at least one nonzero eigenvector; (iv) the quantity measured will always be an eigenvalue of **A**; (v) the probability that a measurement will yield a specific eigenvalue α is: $|(\psi, \psi_j)/(\psi, \psi)|^2$, where ψ_j are all eigenvectors of **A** with eigenvalue α. These propositions are merely restatements of the relevant axioms of Orthodox Quantum Theory. Although the assertion that the Measurement Problem is due to holding the above propositions simultaneously is correct, this presentation of the problem does not illuminate its essential features.

In order to bring out these features, we shall need to consider a particular case. The intrinsic angular momentum (spin) state of a quantum system is an example typically used for this purpose (Rae 2002, 274–276; Cushing 1998, 309–311). The spin of a quantum system along a particular reference direction can be 'up' or 'down' and is described by state vectors. Let the 'spin-up' state vector be ψ_1 and the 'spin-down' state vector be ψ_2. The apparatus with which a measurement is to be made has an initial state described by the state vector ϕ_o. If the system's spin is either 'up' or 'down', then the combination of quantum system and apparatus would have an initial state described by either the state vector $(\psi_1\phi_o)$ or the state vector $(\psi_2\phi_o)$. The coupling of the quantum system with the measuring apparatus allows for a correlation between the state of the system and the state of the apparatus, i.e. this coupling allows the apparatus to give a readout (i.e. a result of measurement).

Let ϕ_1, ϕ_2 be the state vectors of the apparatus that correspond to a readout of 'spin-up' or 'spin-down' respectively. The initial state of the coupled quantum system plus apparatus will evolve linearly into a state described by the state vector $(a\psi_1\phi_1)$ or by the state vector $(b\psi_2\phi_2)$, where a and b are numbers. However, if the quantum system is not initially prepared in a particular spin state, i.e. if not in an eigenstate of either 'spin-up' or 'spin-down', then the quantum system will be in the superposition given by: $\psi = a\psi_1 + b\psi_2$, where $|a|^2 + |b|^2 = 1$. In this case, the initial state of the quantum system plus apparatus will evolve linearly from a state described by the state vector: $(a\psi_1 + b\psi_2)\phi_o$ into a state described by the state vector: $(a\psi_1\phi_1 + b\psi_2\phi_2)$, which is also a superposition. According to Orthodox Quantum Theory, the apparatus (as a part of

the coupled combination) will be in a superposition after measurement, i.e. the result of the measurement should be something that is 'smeared out'. The Measurement Problem resides in accepting two propositions: (i) the measurement apparatus is governed by the laws of quantum mechanics; and (ii) superpositions of macroscopic objects do not occur. In other words, after an actual measurement our apparatus is only in a state that corresponds to a readout of a single result, e.g. 'spin-up' or 'spin-down'.

The Measurement Problem constitutes an (external) conceptual problem for Orthodox Quantum Theory since a 'smeared out' measurement result conflicts with the prevailing belief that superpositions of (large) macroscopic objects, such as instrument pointers, do not occur. Orthodox Quantum Theory does not solve the Measurement Problem but merely circumvents it by applying the Projection Postulate, i.e. a measurement instantaneously 'projects' the superposition ψ into eigenstate ψ_1 with probability $|(\psi, \psi_1)|^2 = |a|^2$ or eigenstate ψ_2 with probability $|(\psi, \psi_2)|^2 = |b|^2$, since $(\psi, \psi) = 1$. (In the chapters that follow, wavefunctions will necessarily be used in preference to state vectors. The Projection Postulate then refers to the collapse of a wavefunction on measurement.)

Those who advocate Orthodox Quantum Theory do not tend to see this as anything much to worry about:

> For most physicists, the measurement problem of quantum mechanics would hardly rate as even a 'small cloud' on the horizon. The standard view is that Bohr had it more or less right, and that anyone willing to waste a little time on the subject could easily straighten out the sort of muddle philosophers might get themselves into (Bub 1997, 212).

In Chapter 3, it will be shown that there is no Measurement Problem within the context of the Causal Theory.

The paradoxes of Orthodox Quantum Theory are indicative of problems in the foundations of the theory. The two best known quantum paradoxes, Schrödinger's Cat and the EPR Paradox, are introduced below.

2.3.2 Einstein-Podosky-Rosen (EPR) Paradox

EPR is not a paradox in the formal sense, i.e. it does not contain an explicit logical contradiction. Instead, it is a thought experiment originally designed to show that Orthodox Quantum Theory is incomplete (Pais 1982, 456). EPR's necessary condition for completeness is that every element of physical reality must have a counterpart in physical theory (Einstein et al. 1935, 777).

According to EPR, elements of physical reality are discovered by experiment. In quantum mechanics, if the state vector ψ of a system is an eigenstate of an operator **A** (which corresponds to a physically measurable quantity A) then $\mathbf{A}\psi = \alpha\psi$, i.e. the physical quantity A will have value α with certainty. Then for the state described by the state vector ψ, EPR claim that there is an element of reality which corresponds to the physical quantity A. They wrote:

> *If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity* (Einstein et al. 1935, 777, italics in original).

The EPR argument is based on an assumption of realism (i.e. the existence of objective 'elements of reality') and a Principle of Locality. The Principle of Locality used in the EPR paper is sometimes referred to as "Einstein Locality" (Redhead 1987, 61), and may be stated as follows:

- *Principle of Locality*

  Elements of reality pertaining to one system cannot be affected by measurements performed at a space-like distance on another system, even if the systems previously interacted.

The Principle of Locality asserts that there is no action-at-a-distance for spatially separated systems. The argument proceeds by considering two quantum systems (denoted as System I and System II) each consisting of one particle whose state vector is known. The systems interact for a short period, then separate. Once interaction has occurred, these systems are said to be *entangled*. If two or more quantum systems have interacted, quantum mechanics shows that the state of one system after the interaction will depend on the state

of the others even if they become well separated in space (Rae 2002, 245). This is what is meant by the term 'entanglement'(see also Vedral 2008). In the EPR example, the state of the combined system (i.e. I + II) may be calculated using quantum mechanics at any subsequent time after interaction. When the systems have separated, a measurement on one system is assumed not to affect the other (by the Principle of Locality). If we measure a physical quantity of System I, say the particle's momentum, quantum mechanics allows an inference to be made on the value of the momentum of the other particle (System II) which, by EPR's assumption of realism, constitutes an 'element of reality'. However, the measurement of System I could just as easily been made on its particle's position from which the position of the other particle would likewise constitute an 'element of reality'.

This leads to a conflict with the Completeness Axiom of Orthodox Quantum Theory as the position and momentum of a particle are quantities represented by non-commuting operators and therefore (as conventionally interpreted) cannot simultaneously both have values predicted with certainty. On this basis, position and momentum cannot simultaneously be 'elements of reality' (Einstein et al. 1935, 780). Given the assumption of realism and the Principle of Locality, the conclusion of the EPR argument is that the completeness assumption of Orthodox Quantum Theory is false.

2.3.3 Schrödinger's Cat

"Schrödinger's Cat" is the most famous of the quantum paradoxes (Schrödinger 1935, 807–812, 824–828, 844–849) and is a graphic example of the Measurement Problem in Orthodox Quantum Theory. Schrödinger apparently proposed the Cat Paradox after corresponding with Einstein over the EPR paper (Cushing 1998, 311). Schrödinger's aim was also to show that Orthodox Quantum Theory is incomplete. Here the cat is in a sealed box and its life or death depends on a (random) quantum event happening, such as a radioactive decay. If the event occurs, the cat dies. If it does not occur, the cat lives. We do not know the result until we make a measurement (e.g. look into the box). If we assume that the cat's

state is described by a state vector, then prior to the observation Orthodox Quantum Theory dictates that the cat's state vector is in a superposition corresponding to the cat being both alive and dead! Although a microscopic quantum entity being described by a superposition may be palatable in the view of most physicists, such a result is unacceptable for macroscopic objects, such as cats (Penrose 2004, 804–805).

Further, *if* the state vector gives a complete description of the state of the cat, then the observation (i.e. measurement) will project the cat's (superposition) state vector into either the eigenstate where the cat is alive or the eigenstate where the cat is dead. Something appears wrong here for a cat will either be alive or dead and it seems preposterous that a simple act of observation could so drastically alter the state of the cat.

Outlines of the solutions of these two paradoxes within the context of the Causal Theory will be presented in Chapter 3.

2.4 'Hidden Variable' Theories and Impossibility Proofs

Quantum mechanical 'hidden variables' were originally proposed to be those variables that determine the values of measurable quantities but which are not themselves accessible to empirical investigation (Hughes 1989, 172). A (so-called) 'hidden-variable' quantum theory (also known as a 'hidden-variable' extension to quantum theory) is a recasting of quantum theory into some classical (or classical-like) form which contain these 'hidden variables'. There have been various proofs of the impossibility of such 'hidden-variable' theories (the 'no-go' theorems) advanced in the literature since the 1930s. The first of these theorems was derived by John von Neumann (von Neumann 1932). The conclusion drawn by von Neumann from his impossibility proof is as follows:

> It is therefore not, as is often assumed, a question of a re-interpretation of quantum mechanics — the present system of quantum mechanics would have to be objectively false, in order that another description of the elementary processes than the statistical one be possible (von Neumann, Beyer's English translation 1955, 325).

This theorem was later strongly endorsed by Niels Bohr himself (Selleri 1990, 35) and by Max Born (Cushing 1993, 829). It took several decades before von Neumann's Theorem was shown to have premises that are too wide (Hughes 1989, 173). In any case, the existence of a consistent counter-example to von Neumann's Theorem (i.e. Bohm's Causal Theory) indicated that its conclusion cannot hold for all 'hidden-variable' theories. However, this consistent counter-example seemed to make little difference to most researchers who knew about it, as they assumed that there must be something wrong with Bohm's theoretical arguments (Cushing 1996, 5).

2.4.1 Kochen and Specker Theorem

Of even more significance than von Neumann's Theorem is the theorem of Simon Kochen and Ernst Specker. This theorem purports to show that extending quantum mechanics by the addition of 'hidden-variables' is not possible because the algebraic structure of self-adjoint operators on a Hilbert space cannot be 'embedded' (in a sense defined below) into the commutative algebra of real-valued functions on a phase space (Kochen and Specker 1967, 59–67. Reprinted in Hooker 1975). Kochen and Specker stated the problem of making a 'classical reinterpretation' of quantum theory (i.e. what they referred to as a 'hidden-variable' extension of quantum mechanics) as follows:

> The proposals … for a classical reinterpretation usually introduce a phase space of hidden pure states in a manner reminiscent of statistical mechanics. The attempt is then shown to succeed in the sense that the quantum mechanical average of an observable is equal to the phase space average. However, this statistical condition does not take into account the algebraic structure of the quantum mechanical observables. A minimum such structure is given by the fact that some observables are functions of others. This structure … should be preserved in a classical reinterpretation … this is not provided for by the above statistical condition … (Kochen and Specker in Hooker 1975, 293).

Before proceeding any further, let's define a 'hidden-variable' extension of quantum mechanics (as accepted by Kochen and Specker). This will also provide a more precise meaning for their term 'embedding' (Jammer 1974, 262):

(1) Each individual quantum system is specified by a (statistical) state function ψ and additional 'hidden' states denoted by the parameter λ. The totality of hidden states is the phase space Γ. The result of measuring any observable of the quantum system (i.e. the value of the observable) is determined by both ψ and λ.

(2) Each state function ψ is associated with a probability measure $\rho_\psi(\Lambda)$ on Γ. The measure $\rho_\psi(\Lambda)$ is the probability that the state (i.e. the phase point of the system) lies in Λ, where Λ is a measurable subset of Γ.

(3) An observable A (represented by a Hermitian operator **A** on a Hilbert space) is interpreted as denoting an attribute of a physical object (Bub 1969, 102). Each observable A is associated with a single-valued, real-valued function $f_{\mathbf{A}} : \Gamma \rightarrow \mathfrak{R}$, i.e. $f_{\mathbf{A}}$ maps Γ into the set of real numbers $\mathfrak{R}$.

(4) Let $\mathcal{M}$ be a measurable subset of $\mathfrak{R}$ and let $\mu_\psi^{\mathbf{A}}$ be the quantum mechanical probability measure such that $\mu_\psi^{\mathbf{A}}(\mathcal{M})$ is the probability that the value of A lies in $\mathcal{M}$. Then, the measure of the set of phase space points in Γ that are mapped by $f_{\mathbf{A}}$ onto the set $\mathcal{M}$ is equal to the measure of the set $\mathcal{M}$ specified by quantum theory, i.e.

$$\mu_\psi^{\mathbf{A}}(\mathcal{M}) = \rho_\psi [f_{\mathbf{A}}^{-1}(\mathcal{M})].$$

This sense of 'embedding' means that the statistical (quantum) theory is expressible in terms of a more fundamental one whose states are *not statistically* related to the physical parameters (Bub 1969, 102–103). This is, of course, not the only possible general definition of a hidden-variable extension of quantum theory. Jammer labels this as *Definition II* of a hidden-variable theory (Jammer 1974, 262).

A 'hidden-variable' theory is constituted by the parameters (or hidden variables) λ, the space Γ, the set of measures $\{\rho\}$ and the set of functions $\{f_i\}$which satisfy the above four constraints. If a system is in a state given by ψ and λ, precise predictions could be made about the result of any measurement if the values of λ were known. Alternatively, if the probability distribution of the parameters λ is known, then the obtainable statistical results will be

in accord with those of quantum mechanics (Jammer 1974, 262). Also, such a theory is *non-contextual* in that the result is not dependent on the context of the performed measurement, i.e. the result $f_{\mathbf{A}}$ does not depend on whether any other observables of the system are measured simultaneously (Jammer 1974, 263).

Whether such an 'embedding' is possible depends on the algebraic structure of the statistical theory involved (in this case, quantum mechanics). The minimum algebraic structure of the quantum mechanical observables is taken into account by 'embedding' the partial algebra of *comeasurable* observables into the commutative algebra of real-valued functions on a phase space. This is known as the Kochen and Specker Condition (Jammer 1974, 323). Kochen and Specker defined the term 'comeasurable' to mean that for a set of observables A_i (represented by operators $\mathbf{A_i}$), $i \in I$ (where I is the set of integers), there exists another observable B (represented by the operator **B**) and functions f_i such that $\mathbf{A_i} = f_i(\mathbf{B})$. Redhead calls this the Functional Composition Principle (Redhead 1987, 121). The value of any of the observables A_i can be ascertained simply by measuring the value of B and applying the function f_i. These form a partial algebra if the following conditions apply (Kochen and Specker in Hooker 1975, 299–300):

If $\mathbf{A_1} = f_1(\mathbf{B})$, $\mathbf{A_2} = f_2(\mathbf{B})$ and r_1, r_2 are real numbers, then

$$\mathbf{A_1A_2} = f_1f_2(\mathbf{B}) \text{ and } r_1\mathbf{A_1} + r_2\mathbf{A_2} = (r_1f_1 + r_2f_2)(\mathbf{B})$$

The Kochen and Specker Condition imposes some restrictions on the functions f_i. Suppose that $f_{\mathbf{A}}$ equals some eigenvalue of the operator **A** (corresponding to an observable A), e.g. $f_{\mathbf{A}} = \alpha_n$ where $\mathbf{A}\psi = \alpha_n\psi$, with ψ being an eigenvector of a system in an eigenstate of **A**. Further suppose that the operator **B** (corresponding to observable B) which commutes with **A** is given by: $\mathbf{B} = g(\mathbf{A})$, where g is a real-valued function. Since physical parameters (as represented by Hermitian operators) are supposed to denote the physical attributes of objects, it is assumed that the value of $g(\mathbf{A})$ is equal to $g(\alpha_n)$. Then, in the state ψ, the measured value of observable B is $g(\alpha_n)$ where $g(\alpha_n) = g(f_{\mathbf{A}}) = f_{\mathbf{B}}(\lambda)$. Alternatively, we could write that if $f_{\mathbf{A}}(\lambda) = \alpha_n$ then $f_{g(\mathbf{A})}(\lambda) = g(\alpha_n)$.

The functions f: $\Gamma \rightarrow \Re$ preserve the partial algebra P of 'comeasurable' operators (Bub 1969, 104). If Σ is the set of all real-valued functions on Γ which constitute a commutative algebra, then a 'hidden-variable' extension exists only if P can be 'embedded' into Σ. In other words, the Kochen and Specker condition requires the existence of a homomorphism (i.e. a structure preserving mapping) h_λ for each $\lambda \in \Gamma$ defined as: $h_\lambda (\mathbf{A}) = f_{\mathbf{A}}(\lambda)$, which maps P into $\Re$ (Jammer 1974, 323). Such a homomorphism associates a value with every physical parameter simultaneously (Bub 1969, 104). Another way of putting this is that the algebraic structure of 'comeasurable' self-adjoint operators should be reflected in the possessed values of the observables (Redhead 1987, 121). The Kochen-Specker Theorem proves that no homomorphism h_λ exists if the Hilbert space has more than two dimensions (Redhead 1987, 121). The proof by Kochen and Specker was unnecessarily complicated involving a set of *117 observables* which were associated with the components of the square of an angular momentum operator (Redhead 1987, 121–130; Ballentine 1998, 607).

Kochen and Specker did provide, however, an example to illustrate the correctness of their theorem. The example concerned exciting Orthohelium (i.e. causing an energy perturbation by applying an electric field) and then measuring the emitted photon energy which corresponds to the change in energy levels. They suggested that all the components of the square of the spin angular momentum of Orthohelium could be inferred from such measurements. Each of the components commutes with the others and so meets the criterion of being comeasurable. The energy perturbation can be achieved by applying an electric field of rhombic symmetry to the atom (Kochen and Specker in Hooker 1975, 308–312). Given that the components are simultaneously measurable, it should be possible in a 'hidden-variable' extension to define a function which assigns values which would be obtained on measurement of the energy perturbation. In terms of their assumptions, Kochen and Specker showed by means of a geometrical argument that this assignment function cannot be defined (Jammer 1974, 324).

What is of particular importance in the assumptions of Kochen and Specker (and in keeping with the definition of a non-contextual 'hidden-variable' extension given above) is that the manner in which the measurement is done has no effect on the result of the measurement (Belinfante 1973, 43). Therefore, the Kochen-Specker Theorem does show that *non-contextual* 'hidden-variable' extensions to quantum theory (with a Hilbert space of more than two dimensions) cannot exist. It does not prove the impossibility of all 'hidden-variable' extensions since the theorem is not applicable to contextual 'hidden-variable' theories.

2.4.2 Bell's Theorem

The EPR Paradox was aimed at showing that Orthodox Quantum Theory is incomplete given that the Principle of Locality holds. It was, however, nearly another thirty years before the late John S. Bell showed that predictions based on the assumption of the Principle of Locality are inconsistent with some predictions of quantum mechanics (Bell 1964, 195–200). He did this by deriving an inequality which quantum mechanics violates. There are several versions of this inequality which are collectively referred to as Bell's Inequalities (Clauser and Shimony 1978, 1889). The version reproduced below follows Bell's 1971 argument (Bell 1971).

Suppose we have two particles and two measuring apparatuses that each can register the value of a particular variable associated with each of the particles. Further suppose that this particular variable has only two possible values which can be chosen to be ± 1 in some appropriate units. The measuring apparatuses, however, have a range of possible settings, i.e. possible ways in which an individual apparatus may be configured. Let 'a' denote the settings for the first apparatus and 'b' for the second. The outcome of any measurements may depend on these settings together with unknown (and uncontrollable) variables which are associated with the particles and/or the apparatuses. These will be collectively denoted by λ. Then suppose there is a continuous function which determines the outcome of measurements on the first particle. This function (which shall be denoted A) depends on the variables λ and

a, i.e. $A = A(a, \lambda)$. Likewise, we can suppose that there is a continuous function (denoted B) which determines the outcome of measurements on the second particle and depends on the variables λ and b, i.e. $B = B(b, \lambda)$.

We shall assume that A and B provide average values so that $|A(a,\lambda)| \leq 1$ and $|B(a,\lambda)| \leq 1$. If the Principle of Locality holds and the two measuring apparatuses are spatially well separated, then A cannot depend on the variable b and B cannot depend on the variable a. However, both A and B will depend on the unknown variables λ. We also shall assume that these variables have a distribution described by a probability function $\rho(\lambda)$, where

$$\rho(\lambda) \geq 0 \text{ and } \int \rho \, d\lambda = 1 \qquad (2.7)$$

This distribution does not depend on the type of measurements made on the particles. Now let E(a,b) be the expectation value of the quantity AB which is defined by:

$$E(a,b) = \int A(a,\lambda)\, B(a,\lambda)\, \rho(\lambda)\, d\lambda \qquad (2.8)$$

where an integral rather than a simple summation is used as A and B are continuous functions. E(a,b) is a function that gives a measure of the correlation between the variables that are measured. Let a, a′ be two different settings for the first apparatus. Likewise, let b, b′ be two different settings for the second apparatus. Then we can form the difference between two correlation functions as follows:

$$E(a,b) - E(a,b') = \int [A(a,\lambda)\, B(b,\lambda) - A(a,\lambda)\, B(b',\lambda)]\, \rho(\lambda)\, d\lambda$$

This may be more usefully expressed by a little reorganisation to yield:

$$E(a,b) - E(a,b') = \int \{A(a,\lambda)\, B(b,\lambda)\, [1 \pm A(a',\lambda)\, B(b',\lambda)]\}\, \rho(\lambda)\, d\lambda$$

$$- \int \{A(a,\lambda)\, B(b',\lambda)\, [1 \pm A(a',\lambda)\, B(b,\lambda)]\}\, \rho(\lambda)\, d\lambda$$

$$\Longrightarrow |E(a,b) - E(a,b')| \leq$$

$$\int \{[1 \pm A(a',\lambda)\, B(b',\lambda)] + [1 \pm A(a',\lambda)\, B(b,\lambda)]\}\, \rho(\lambda)\, d\lambda$$

Now

$$[1 \pm A(a',\lambda)\, B(b',\lambda)] + [1 \pm A(a',\lambda)\, B(b,\lambda)]$$
$$= [2 \pm \{A(a',\lambda)\, B(b',\lambda) + A(a',\lambda)\, B(b,\lambda)]$$

Using the properties of the probability function ρ (relations (2.7)) and the definition of E (Equation (2.8)), we find:

$$\int \{[1 \pm A(a',\lambda)\, B(b',\lambda)] + [1 \pm A(a',\lambda)\, B(b,\lambda)]\}\rho(\lambda)\, d\lambda$$
$$= 2 \pm \{E(a',b') + E(a',b)\}$$

Thus,

$$\left|E(a,b) - E(a,b')\right| + \left|E(a',b') + E(a',b)\right| \leq 2 \qquad (2.9)$$

This is Bell's Inequality (Bell 1987b, 36–37). No use has been made of the formalism of quantum mechanics in deriving this inequality. Quantum mechanics, however, indicates that there is greater correlation between the particles than would be expected on the basis of assuming that the Principle of Locality holds (Sudbery 1986, 200).

Some results of quantum mechanics readily show that the quantity $\left|E(a,b) - E(a,b')\right| + \left|E(a',b') + E(a',b)\right| > 2$ (Clauser and Shimony 1978, 1893–1894). The theorem that inequality (2.9) conflicts with the predictions of quantum mechanics is called Bell's Theorem (Ballentine 1998, 590). Experiments that test Bell inequalities have confirmed that they are indeed violated (e.g. Grangier et al. 1986; Weihs et al. 1998; Rowe et al. 2001), as is demanded by quantum mechanics.

Since Bell's original derivation (published in 1964), there have been assumptions other than the Principle of Locality identified in his theorem. This has been a significant advance as it has allowed further proofs of Bell's Theorem to be produced which do not depend on these assumptions. Of particular importance in this regard has been to eliminate dependence on the assumptions of: determinism; probability factorisation; counterfactual definiteness; and the presence of 'hidden variables' (Ballentine 1998, 608). These issues are well covered in the literature and so will not be discussed here. The over-riding result of these later proofs has been to show that the Bell's Inequalities crucially depend on assuming the Principle of Locality.

A further important advance was made in 1989 when Daniel Greenberger, Michael Horne, and Anton Zeilinger considered correlated states with three or more entangled particles (Greenberger et al. 1989; Greenberger et al. 1990). They showed that, after two of the three particles were measuring, a measurement of the third particle becomes a test between local realism and quantum mechanics as each predicts a different value for the measurement result. This eliminated the statistical dependence found in Bell's Theorem. The required three particle entanglement states have become known as 'Greenberger-Horne-Zeilinger' (or GHZ) states.

Experiments have been performed on GHZ states and confirm the predictions of quantum mechanics (Pan et al. 2000; Zhao et al. 2003). One experimental report concludes:

> Bell's theorem states that certain statistical correlations predicted by quantum physics for measurements on two-particle systems cannot be understood within a realistic picture based on local properties … A more striking conflict between quantum mechanical and local realistic predictions (for perfect correlations) has been discovered; but experimental verification has been difficult, as it requires entanglement between at least three particles. Here we report experimental confirmation of this conflict … The results of three specific experiments, involving measurements of polarization correlations between three photons, lead to predictions for a fourth experiment; quantum physical predictions are mutually contradictory with expectations based on local realism. We find the results of the fourth experiment to be in agreement with the quantum prediction and in striking conflict with local realism (Pan et al. 2000, 515–516).

We shall see in Chapter 3 that the Causal Theory is not refuted by experiments that show Bell's Inequalities are violated.

Chapter 3

The Causal Theory of Quantum Mechanics

De Broglie showed in detail how the motion of a particle … could be influenced by waves … This idea seems to me so natural and simple … that it is great mystery to me that it was so generally ignored.
— J.S. Bell

Abstract This chapter is an exposition and discussion of the Causal Theory of Quantum Mechanics. A set of axioms is presented for single particle quantum systems which is later generalised to the many-particle case. The equations of the Causal Theory are derived from its axioms and realistically interpreted. The ontology of the Causal Theory, i.e. quantum particles and the quantum field (wave field), is described. An important issue addressed in this chapter is the contention that the wave field is a physical field that propagates through three-dimensional space. This view is considered problematic in the literature since the wavefunction of an N-particle quantum system (N > 1) is defined on a 3N-dimensional configuration space. It is also shown that there is no 'Measurement Problem' in the Causal Theory and that the two quantum paradoxes discussed in Chapter 2 are readily solvable. Non-locality is introduced together with discussions of the transition to the classical level of description and the status of the Quantum Equilibrium Condition within the Causal Theory.

3.1 Motivations for the Causal Theory

In Chapter 1, it was stated that speculations and ill-informed commentary regarding what quantum mechanics asserts about nature have led to the abandonment of some important physical concepts

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and principles. This is evident in the formulation of Orthodox Quantum Theory whose axioms are essentially 'geared' to provide predictions for results that could be obtained if a measurement were performed on a quantum system. The British philosopher of science, Nicholas Maxwell, once asked the following question in relation to what Orthodox Quantum Theory asserts about the physical world:

> What sort of physical objects are … --the entities of the quantum world-- in view of the contradictory wave and particle properties that these objects appear to possess? … Orthodox quantum theory (OQT) *evades* and does not *solve* this key problem. The creators of OQT … decided, in effect, that no consistent, fully micro realistic theory of quantum objects evolving and interacting in space and time could be developed … (Maxwell 1988, 1, italics in original).

Orthodox Quantum Theory does not concern itself with questions about what the physical world consists of.

In order to account for quantum phenomena and avoid the problems in Orthodox Quantum Theory, alternative interpretations of the quantum formalism have either postulated bizarre entities (including infinitely many parallel universes), or resorted to rather odd and problematic mechanisms (such as spontaneous state vector reduction, i.e. wavefunction collapse), or have invoked non-standard logic. Such moves are not particularly justifiable based on the formalism and experiment. Would it not be preferable to have a realist theory of quantum phenomena that not only gives correct predictions but also solves the outstanding problems and paradoxes without the undesirable aspects present in alternative interpretations? It is accepted by most scientists and philosophers of science that *one* aim of science is to provide explanations of physical phenomena. The way to achieve this in the context of the quantum realm is to specify both the ontology and the laws that govern the realm in addition to those rules by which we predict the outcome of experiments.

Louis de Broglie is generally credited with formulating the first causal interpretation of quantum mechanics in the 1920s (de Broglie, 1923). A consistent causal theory was postulated independently of de Broglie's ideas by David Bohm in the 1950s which also answered the principal objections that had been levelled at de Broglie's original interpretation (Bohm 1952a,b). The hypothesis that quantum particles are directed by some sort of guiding field (as postulated in the Causal Theory) offers perhaps the

best current possibility of providing an empirically adequate, realist theory of quantum mechanics. The axioms of the Causal Theory (see Section 3.2. below) provide, *inter alia*, a basis for realistic explanations of quantum phenomena. A main attraction of the Causal Theory is its ontology – quantum particles and guiding fields are prescribed to have an objective existence in space and time. Quantum particles also have well-defined trajectories in the Causal Theory just as the tracks in particle detection chambers have always indicated. The Causal Theory's capacity to make realist claims about quantum entities was suitably described by Christopher Norris:

> Bohm's [Causal] theory is thus premised on the realist assumption that any adequate account of QM [quantum mechanical] phenomena will indeed 'do more' than establish a high degree of predictive correlation or empirical warrant. … Where Bohm's theory is at its strongest … is in putting up a realist interpretation of the evidence … [and] takes scientific theories to be warranted by their jointly observational, predictive, *and* causal-explanatory power (Norris 2000, 27, italics in original).

The Causal Theory has attracted more attention in the 1990s and the first decade of the twenty-first century than it did in all previous decades since its inception. Surprisingly, much of this attention has come, not from the physics community (although there is a small minority following there), but from physical chemists who do not carry a legacy of philosophical prejudice against quantum realism and especially, the existence of quantum trajectories (such as appear in the Causal Theory). One physical chemist who has been a leader in the use of quantum trajectory methods is Robert Wyatt who explicitly addressed this issue:

> … [one] compelling reason for running [i.e. calculating] quantum trajectories is that we may *gain new insights* into the dynamics. Unlike conventional computational methods, quantum trajectories provide detailed information about *how* the process takes place. These insights may lead to improved algorithms for treating [quantum] systems … This has already started to happen … It is hoped that the prejudice displayed by some against the use of trajectories will not carry over … (Wyatt 2005, 4, italics in original).

A further motivation for attempting to comprehend the fundamentals of non-relativistic quantum mechanics in realist terms is that without such comprehension it is unlikely that we will ever arrive at a coherent understanding of (relativistic) quantum field theory, of quantum gravity, or indeed of the universe at large.

This chapter is devoted to providing a formal exposition of the Causal Theory of Quantum Mechanics. However, before embarking on the technical details, there is an important point to be

acknowledged. The historical selection of Orthodox Quantum Theory over the Causal Theory was *not* dictated by empirical results. John Bell sought, in several published papers, to dispel the idea that Orthodox Quantum Theory had been chosen because it was in better agreement with experiment. He advocated the pilot wave picture (i.e. the Causal Theory) as a legitimate alternative to Orthodox Quantum Theory:

> Why is the pilot wave picture [Causal Theory] ignored in the text books? Should it not be taught, not as the only way, but as an antidote to prevailing complacency? To show that vagueness, subjectivity, and indeterminism, are not forced on us by experimental facts, but by theoretical choice? (Bell 1987b, 160).

3.2 An Axiomatic Foundation

In this section, a set of axioms will be presented which will serve as a foundation for the Causal Theory of Quantum Mechanics. We begin with some initial remarks about the axiomization of physical theories. Any axiomatic treatment is necessarily limited and cannot present all the concepts and technicalities required for a mathematical theory. Max Jammer presented a rather barren view of axiomizations in his well-known exposition on the different interpretations of quantum mechanics. He writes:

> … an axiomization cannot dispense with undefined primitive concepts and relations whose concrete meaning can be conveyed only in terms of the language of ordinary experience. Since an axiomization of quantum mechanics is intended to clarify the latter it is not only sterile, as axiomizations usually are, but also necessarily circular; it can, at best, serve as a test for the consistency of reasoning … (Jammer 1974, 472).

Although Jammer's point about undefined concepts and relations (be they primitives or not) cannot be disregarded, it is also the case that axiomizations that attempt to cover all relevant concepts become impossibly long and/or hopelessly complicated. There always remains much that is assumed in any axiomization of physical theories (Bunge 1973, 9), such as the rules of the propositional logic, the basis of geometry, the operations of vector algebra, the foundations of the differential calculus, and so forth. Despite obvious practical limitations, an axiomization of a physical

theory has distinct advantages. In particular, the following possible benefits may be gained:

- assistance in verifying the theory's consistency;
- minimisation of the semantic content needed to present the theory;
- revealing gaps in any previous renderings of the theory (van Fraassen 1991, 5); and
- assistance in showing any mutual dependence of various parts of a physical system (Popper 1975, 72).

It also will be beneficial to note relevant comments about axiomizations due the mathematical physicist J.L. Synge:

> Physical concepts, being by their nature vague, cannot be treated with logical rigour. … it would seem right that any systematic treatment … should start with axioms, carefully laid down, on which the whole structure would rest as a house rests on its foundations.
>
> The analogy to a house is, however, a false one. Theories are created in mid-air, so to speak, and develop upward and downward. Neither process is ever completed. …
>
> To a physicist … there is an element of artificiality in the creation of a complete axiomatic base, for he knows that the axioms will be chosen to fit the theory … (Synge 1960, 5).

We will have cause to refer again to these comments by Synge later in this chapter.

Throughout the rest of this book, it will be appropriate to employ the configuration space representation in the Schrödinger picture of quantum mechanics, where the wavefunction Ψ of a quantum system is used instead of the state vector ψ. The wavefunction Ψ is given by the inner product: (x,ψ) where x is the position observable. The axioms presented below refer to a single particle quantum system. These are framed in terms of the wavefunction of the system, the quantum particle's position and inertial mass, Planck's Constant, and time. Similar axiomizations appear in the literature (examples include: Bohm 1952a, 169–171; Freistadt 1957, 9–13; Holland 1993, 66–68) although they are not as explicit in detail.

3.2.1 Axioms of the Causal Theory (Single Particle Case)

I. *Particle*

A particle is a point-like object localised in (three-dimensional) Galilean space with an inertial mass.

II. *Wave Field*

A wave field is a physical process that propagates in (three-dimensional) Galilean space over time. A wave field is described by its wavefunction Ψ which is a continuous, bounded function of the space and time coordinates.

III. *Quantum System*

A single particle quantum system consists of a quantum particle and an accompanying wave field, i.e. the set $\{\Psi, x\}$ where x is the particle's position.

IV. *Lagrangian Density*

A single particle quantum system has a Lagrangian density $\mathcal{L}$ which is expressed in terms of its wavefunction Ψ:

$$\mathcal{L} = \tfrac{1}{2} i\hbar(\Psi^*\dot{\Psi} - \dot{\Psi}^*\Psi) - (\hbar^2/2m)\,(\nabla\Psi^*)\cdot(\nabla\Psi) - V\,\Psi^*\Psi$$

where $i = \sqrt{-1}$, Ψ^* is the complex conjugate of Ψ, $\dot{\Psi}$ is the partial derivative of Ψ with respect to time, V is an external (classical) potential, $\hbar$ is Planck's Constant divided by 2π, ∇ is the standard three-dimensional differential operator, and m is the particle's inertial mass.

V. *Guidance Condition*

A quantum particle is guided by its wave field in accordance with the condition:

$$\frac{d\boldsymbol{x}}{dt} = \left(\frac{\hbar}{2im}\right) \nabla \left[\log\left(\frac{\Psi}{\Psi^*}\right)\right]$$

where 'log' denotes the natural (Naperian) logarithm, and other terms are as defined above.

VI. *Quantum Equilibrium Condition*

The probability density ρ (x) of possible values of the initial particle position in an ensemble of similarly prepared quantum systems satisfies the condition:

$$\rho = |\Psi|^2.$$

3.3 One Particle States

Axiom I states that a particle is 'point-like'. This is, of course, an abstraction for it is assumed that the particle's size is negligible and that it can be treated as if all its mass were concentrated at a single point. However, 'point-like' particles are a feature of many mathematical models. The term 'quantum particle' will refer to those particles found in quantum systems, as distinct from classical particles.

Axiom II states that there exists a physically real quantum field. This field is commonly called the 'wave field' for historical reasons. Other names include: pilot wave, deBroglie wave, Schrödinger wave, and matter wave. We shall follow the usage of 'wave field'. The wave field is described mathematically by its wavefunction which is defined on a configuration space and is usually a single-valued, square-integrable, complex function. In the case of a single particle, the quantum system's configuration space coincides with ordinary three-dimensional space. The wave field is not a theoretical fiction, it is postulated to have an *objective* existence in space and time. The essential distinction to be understood here is that the wavefunction is a mathematical entity whereas the wave field is a physical entity whose behaviour is described by its wavefunction. Thus the wavefunction is not merely a device for calculating the results of experiments, as in Orthodox Quantum Theory. Bohm describes this aspect of the wavefunction with reference to an electron as follows:

> … we have effectively been led to regard the wave function of an individual electron as a mathematical representation of an objectively real field (Bohm 1952a, 170).

Since the wave field is a physically real field that propagates as a wave through space, it shares the same characteristics as other types of physical waves. In particular we note that wave fields will, under appropriate physical conditions, be subject to the following wave processes:

- diffraction;
- reflection;
- refraction;
- interference; and
- superposition.

This is an important point to establish about wave fields, as stated by Ian Main in his monograph on wave phenomena:

> … the behaviour of the [deBroglie] waves themselves is not different from that of any other waves (Main 1978, 270).

Evidence for the physical existence of wave fields is presented in Section 4.5.

The manifestation of both wave-like and particle-like behaviours in experiments at the atomic scale offers a partial justification for accepting Axioms I and II since such results taken literally indicate the co-existence of both wave and particle. Axiom III states that the particle and wave field together constitute a quantum system. Indeed, they are physically inseparable aspects of the system in the sense that the particle is always accompanied by its wave field. However, different traits of these two aspects can be given a limited, individual description. A quantum particle possesses some characteristics which are familiar from macroscopic objects, i.e. mass, energy, and (in principle) localisability. Given the co-existence of particle and wave field, it is conceivable from the perspective of the Causal Theory, that there may be experimental circumstances in which the Principle of Complementarity fails.

The simple polar form of the wavefunction Ψ of a quantum system, i.e. $\Psi = \mathrm{R}e^{i\mathrm{S}/\hbar}$, provides a natural decomposition into two functions representing the amplitude and phase of the wave. R and S

are real-valued functions of the space and time coordinates with $R \geq 0$ and $i = \sqrt{-1}$. There is no loss of generality by expressing the wavefunction in polar form. If the wavefunction Ψ is single-valued (as usually assumed) so must be its amplitude R. However, the value of the wave field's phase ($S/\hbar$) may change by an integral multiple of 2π. Also, as Ψ is bounded (from Axiom II), it must tend to zero with increasing distance from the quantum particle.

Axiom IV gives the system's Lagrangian density in terms of its wavefunction:

$$\mathcal{L} = \tfrac{1}{2} i\hbar(\Psi^* \dot{\Psi} - \dot{\Psi}^* \Psi) - (\hbar^2/2m)(\nabla\Psi^*) \cdot (\nabla\Psi) - V \Psi^* \Psi$$

Substitution of $Re^{iS/\hbar}$ for Ψ into this Lagrangian density gives the following expression:

$$\mathcal{L} = -R^2 \left(\frac{\partial S}{\partial t}\right) - \left(\frac{R^2}{2m}\right) |\nabla S|^2 - \left(\frac{\hbar^2}{2m}\right) |\nabla R|^2 - R^2 V$$

A fundamental principle of theoretical physics is the Principle of Stationary Action. This Principle has the advantage in that it allows a range of different physical systems to be treated in a uniform manner (Doughty 1990, 5). The Principle may be stated as follows (Weyl 1952, 210–211):

- *Principle of Stationary Action*

  The change in the total Action for each infinitesimal variation of the state of a physical system is zero.

In the current context, the Principle of Stationary Action requires that the variation of the integral of the Lagrangian density is zero. In mathematical notation this is written: $\delta \int \mathcal{L}\, d^4\mathbf{x} = 0$. The δ symbol is shorthand for a differential and will not concern us except for noting that the requirement that this variation is zero is equivalent to applying the Euler-Lagrange equation (Sakurai 1982, 5). If we first vary the parameter R and let $x_1 = x$, $x_2 = y$, $x_3 = z$, $x_4 = t$, then we obtain the following Euler-Lagrange equation:

$$\sum_{\mu=1}^{4} \frac{\partial}{\partial x_\mu} \left[\frac{\partial \mathcal{L}}{\partial(\partial R / \partial x_\mu)}\right] - \left(\frac{\partial \mathcal{L}}{\partial R}\right) = 0$$

$$= -\left(\frac{\hbar^2}{m}\right)\nabla^2 R - \left\{-2R\left(\frac{\partial S}{\partial t}\right) - \left(\frac{R}{m}\right)|\nabla S|^2 - 2RV\right\}$$

Dividing by 2R and rearranging terms gives:

$$-\frac{\partial S}{\partial t} = \frac{(\nabla S)^2}{2m} + V(\mathbf{x},t) - \frac{\hbar^2}{2m}\left(\frac{\nabla^2 R}{R}\right) \qquad (3.1)$$

where $(\nabla S)^2 = (\nabla S)\cdot(\nabla S) = |\nabla S|^2$ is introduced to conform with the notation in the literature. Equation (3.1) is sometimes referred to as a modified Hamilton-Jacobi equation as it differs from its classical counterpart by the presence of the last term. A more appropriate name is the Quantum Hamilton-Jacobi Equation (Bohm and Hiley 1993, 29; Wyatt 2005, 48 and 56ff).

Axiom V tells us that a quantum particle is guided by its wave field in accordance with the condition:

$$\frac{d\mathbf{x}}{dt} = \left(\frac{\hbar}{2im}\right)\nabla\left[\log\left(\frac{\Psi}{\Psi^*}\right)\right]$$

Since $\Psi = R\, e^{iS/\hbar}$, $\log(\Psi/\Psi^*) = \log[(R\, e^{iS/\hbar})/(R\, e^{-iS/\hbar})] = 2iS/\hbar$. Therefore

$$\frac{d\mathbf{x}}{dt} = \left(\frac{\hbar}{2im}\right)\nabla\left[\log\left(\frac{\Psi}{\Psi^*}\right)\right] = \left(\frac{\nabla S}{m}\right) \qquad (3.2)$$

In other words, the Guidance Condition requires that the momentum $\mathbf{p}$ ($= d\mathbf{x}/dt$) of a single quantum particle be equal to (∇S). These are continuous (possessed) values of momentum. The justification for the Guidance Condition is as follows. First, in classical Hamilton-Jacobi theory, the momentum of a classical particle is given by the same equation, i.e. $\mathbf{p} = \nabla S$, with S being Hamilton's Principal Function (Goldstein 1980, 439–440). Thus, by direct analogy, it would be reasonable to expect that (∇S) is the particle's momentum in the case of the Quantum Hamilton-Jacobi Equation (Maudlin 2002, 120). Second, by taking $\mathbf{p} = \nabla S$, the Causal Theory provides correct empirical predictions (Bohm 1952a, 170). This is, of course, an essential ingredient if the theory is to be accepted and

may be considered sufficient justification in itself. Recall the apt remark of J.L. Synge (quoted in Section 3.2) that the choice of axioms is made to fit the theory! In Orthodox Quantum Theory, it is denied that the quantity (∇S) is the particle's momentum on the basis that this would violate the Uncertainty Principle (Sakurai 1985, 102–103). This is not a concern within the Causal Theory for Heisenberg's uncertainty relations merely express the statistical scatter of measured values of complementary variables in an ensemble of systems. Therefore, the existence of definite, sharp values for observables (such as particle position) is not inconsistent with the Uncertainty Principle (see the discussion in Section 3.5). This was concluded in the last chapter and has also been explicitly stated by several commentators, including Dewdney and Malik who wrote:

> There is no contradiction with Heisenberg's uncertainty relations in the assumption of definite values for both the position and momentum of the particle (or other sets of noncommuting observables), since the uncertainty relations simply refer to the inevitable statistical scatter in the values obtained for complementary variables in an ensemble of measurements (Dewdney and Malik, 1993, 3513).

The research group led by Detlef Dürr has argued that considerations of symmetry and simplicity suggest the form of the Guidance Condition. Given Schrodinger's equation, they contend that there is a simplest choice for an evolution equation for the system's configuration which is compatible with overall Galilean and time-reversal invariance. The velocity vector field on configuration space associated with this 'simplest choice' is, they argue, given by the Guidance Condition (Dürr et al. 1992a, 851–853). Dickson and also Brown, Elby and Weingard have rightly criticised Durr et al. for this argument on several grounds. Dickson points out that they do not have a well defined notion of simplicity (Dickson 1998, 223, note 4). Brown, Elby and Weingard raise concerns about whether the spacetime symmetries invoked have any bearing at all on the form of the Guidance Condition (Brown et al. 1996, 310–311). Despite the appeal of the argument of Dürr's group, its validity has not been established. It is also worth noting that even if their argument is sound, it still would not constitute a rigorous mathematical derivation.

Taking $\mathbf{p} = \nabla S$ indicates that the term $[(\nabla S)^2/2m]$ in Equation (3.1) is the particle's kinetic energy and this is an energy

equation for the quantum particle, where [– (∂S/∂t)] is the total energy available to the particle. The last term in Equation (3.1) is called the Quantum Mechanical Potential Energy Q (or just quantum potential, for short):

$$Q(\mathbf{x},t) = -\frac{\hbar^2}{2m}\left(\frac{\nabla^2 R}{R}\right) \qquad (3.3)$$

It turns out that the presence of the quantum potential accounts for most of the differences between classical and quantum physics. The mathematical form of the quantum potential is independent of the amplitude of the wave field. This can readily be seen by multiplying the amplitude R by some (real) constant b, say:

$$Q = -\frac{\hbar^2}{2m}\left(\frac{\nabla^2 (bR)}{bR}\right) = -\frac{\hbar^2}{2m}\left(\frac{\nabla^2 R}{R}\right)$$

i.e. the value of Q remains unchanged. This amplitude independence produces highly *non-classical* behaviour in quantum systems. The form of the quantum potential will be further considered in Chapter 5.

The Hamilton-Jacobi formalism is the most appropriate representation for the Causal Theory. This idea can be traced back to de Broglie's original conception, as Cushing has remarked:

> DeBroglie did believe that *one* theory should best conform to nature. He felt that the classical Hamilton-Jacobi formalism provided an embryonic theory of the union of waves and particles, all in a manner consistent with a realist conception of matter (Cushing 1994a, 224, italics in original).

Also, the Hamilton-Jacobi formalism allows both quantum and classical mechanics to be assessed using the same terminology (Holland 1993, 78). Of course, other representations are possible (minimalist positions) but these exclude the quantum potential (e.g. Dürr et al. 1992). There are, however, important advantages in retaining the quantum potential. These advantages will be addressed in Chapter 4.

The state of a quantum system at a given time is specified by both the wavefunction and the particle's position (Holland 1993, 75). Since the form of the wavefunction is influenced by the system's surroundings (i.e. how the wave field is altered as it propagates), the quantum state has a holistic dependence on its

environment (Holland 1993, 79). This is called state dependence and is a feature not found in the paradigm of classical physics. This is contrary to the contention of alleged flaw ① in Section 1.3.

In Section 2.4, it was stated that quantum mechanical 'hidden variables' were originally proposed to be variables that determine the values of measurable quantities but are not themselves measurable. A 'hidden-variable' quantum theory contains such variables. The Causal Theory is historically a member of the set of 'hidden-variable' theories and is still labelled as such in much of the literature. In the Causal Theory, the position of a particle is the (so-called) 'hidden variable'. Although there are restrictions on the measurement of particle positions, it is clearly possible to measure these positions. Roderick Tumulka has provided an appropriate description of the contradictory attitude held by advocates of Orthodox Quantum Theory in regard to 'hidden variables':

> … the orthodox view has a contradictory attitude towards the idea of 'true values', often (and misleadingly) called 'hidden variables'. The typical orthodox physicist openly condemns hidden variables as impossible, but in his heart cannot abandon them, and continues to talk as if particles had energies and angular momentum vectors. Ironically, Bohmian mechanics is often called a 'hidden-variables theory' … because it can be regarded, … as postulating actual values for the position observable … (Tumulka, 2007, 3249).

The use of the term 'hidden variables' carries with it the implication that any theory containing such variables is physically meaningless and therefore not worthy of any attention. John Bell made strong protests against the Causal Theory being labelled in this way. He wrote:

> Absurdly, such theories are known as 'hidden variable' theories. Absurdly, for there it is not in the wavefunction that one finds an image of the visible world, and the results of experiments, but in the complementary 'hidden'(!) variables (Bell 1987b, 201).

Since it is possible to measure particle positions, alleged flaw ② in Section 1.3 is not applicable.

Quantum particles have well-defined trajectories in the Causal Theory. The position of a particle, in conjunction with Equation (3.2), allows the trajectory for a quantum particle $\mathbf{x} = \mathbf{x}(t)$, to be determined. Trajectories are calculated by specifying the wavefunction (as ∇S is found from the wavefunction) and the particle's initial position. A unique trajectory can then be found by integrating Equation (3.2), i.e.

$$\mathbf{x}(t) = \int \left(\frac{\nabla S}{m}\right) dt + \mathbf{x}_o$$

where $\mathbf{x}_o$ is the initial particle position. In the Causal Theory, therefore, a quantum particle has a distinct trajectory in Galilean space with an attached inertial mass. The existence of particle trajectories is specifically ruled out in Orthodox Quantum Theory *by fiat*. Although $S(\mathbf{x}, t)$ is a multi-valued function, (∇S) is single-valued and is not defined at nodal points or surfaces, i.e. where $R = 0$. This requires that trajectories do not pass through the nodes of the wavefunction (Holland 1993, 85). This is reflected in the dynamics of the Causal Theory which ensure that quantum particles cannot pass through nodes.

An expression for the total time rate of change of momentum of a quantum particle can also be derived. The total derivative of the momentum $\mathbf{p}$ with respect to time is:

$$\frac{d\mathbf{p}}{dt} = \sum_{i=1}^{3} \frac{\partial \mathbf{p}}{\partial x_i} \frac{dx_i}{dt} + \frac{\partial \mathbf{p}}{\partial t}$$

Since $\mathbf{p} = \nabla S$, we have

$$\frac{d\mathbf{p}}{dt} = \sum_{i=1}^{3} \frac{p^i}{m} \frac{\partial \nabla S}{\partial x_i} + \frac{\partial \nabla S}{\partial t}$$

$$= \frac{(\nabla S)}{m} \cdot \nabla (\nabla S) + \frac{\partial \nabla S}{\partial t}$$

$$= \nabla \left[\frac{(\nabla S)^2}{2m} + \frac{\partial S}{\partial t}\right]$$

Using Equations (3.1) and (3.3), we can make the following substitution:

$$\frac{(\nabla S)^2}{2m} = \left[\frac{\partial S}{\partial t} + V(\mathbf{x}, t) + Q(\mathbf{x}, t)\right]$$

then

$$\left(\frac{d\mathbf{p}}{dt}\right) = \nabla\left(-\left[\frac{\partial S}{\partial t} + V(\mathbf{x},t) + Q(\mathbf{x},t)\right] + \frac{\partial S}{\partial t}\right)$$

It can be seen now that the total time rate of change of momentum of a quantum particle (i.e. the net force acting) is given by:

$$(d\mathbf{p}/dt) = -\nabla(V + Q) = -\nabla V - \nabla Q \qquad (3.4)$$

The term $(-\nabla Q)$ is called the quantum mechanical force (or just, quantum force). Since $R = |\Psi|$ the quantum force acting on the particle depends (in part) on a function of the absolute value of the wavefunction, evaluated at the particle's position (Bohm 1952a, 170). Bohm's description is as follows:

> … This [wave] field exerts a force on the particle in a way that is analogous to, but not identical with, the way in which an electromagnetic field exerts a force on a charge, and a meson field exerts a force on a nucleon (Bohm 1952a, 170).

It is clear from Equation (3.4) that the motion of a quantum particle cannot be, in general, derived entirely from the classical potential V. We shall have more to say about the quantum mechanical force in Chapters 4 and 5.

The existence of well-defined trajectories for quantum particles together with the explicit recognition of the role of a *causal agent* (i.e. the quantum mechanical force) in the Causal Theory is consistent with event-by-event causality in space and time, subject to the Principle of Causality (as defined in Section 1.2).

3.4 Statistical Predictions

If we again substitute $Re^{iS/\hbar}$ for Ψ into the equation for the Lagrangian density (in Axiom IV) and apply the Principle of Stationary Action in the form: $\delta \int \mathcal{L}\ d^4\mathbf{x} = 0$, but this time vary S, then we obtain another Euler-Lagrange equation:

$$\sum_{\mu=1}^{4} \frac{\partial}{\partial x_\mu}\left[\frac{\partial \mathcal{L}}{\partial(\partial S/\partial x_\mu)}\right] - \left(\frac{\partial \mathcal{L}}{\partial S}\right) = 0$$

which gives the equation:

$$\nabla \cdot \left\{ R^2 \frac{(\nabla S)}{m} \right\} + \left(\frac{\partial R^2}{\partial t} \right) = 0 \qquad (3.5)$$

The Quantum Equilibrium Condition (Axiom VI) states that when a system has a wavefunction Ψ, the probability density $\rho(\mathbf{x})$ of possible values of the initial particle position in an ensemble of similarly prepared quantum systems is equal to $|\Psi|^2$. The essence of this condition is also assumed in Orthodox Quantum Theory as the Born Statistical Postulate. The probability density $\rho(\mathbf{x})$ is also called the quantum equilibrium distribution. The status of Axiom VI will be reviewed later in this chapter. Since $R^2 = |\Psi|^2 = \rho$, we can rewrite Equation (3.5) in the alternative form:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho \frac{\nabla S}{m} \right) = 0 \qquad (3.6)$$

This is an equation of continuity for ρ which ensures that if the initial value is $|\Psi|^2$ then it will remain so at all subsequent times (Holland 1993, 99). In Orthodox Quantum Theory, the quantity known as the probability current density $\mathbf{j}$ is equal to $(\rho \nabla S/m)$. In the Causal Theory, $\mathbf{j}(\mathbf{x}, t) = \rho \mathbf{v}$ (where $\mathbf{v} = \nabla S/m$), i.e. this 'current' lies on the tangent to each point on a trajectory $\mathbf{x}(t)$ (Holland 1993, 75).

It should be clear now that the function R plays a *dual* role in the Causal Theory: (i) R represents the amplitude of the wave field and therefore (in part) determines the value of the quantum potential. Hence R helps to determine individual particle motion; and (ii) $R^2 = |\Psi|^2$ describes the ensemble quantum state (Holland 1993, 100). Although the particle trajectories are causally determined, they depend on initial conditions. Initial particle positions may be unknown or fluctuate in a random manner. The Quantum Equilibrium Condition (Axiom VI) requires that we cannot know the distribution of a particle's position better than that given by $|\Psi|^2$. We may then interpret R^2 as a probability density such that the probability that a particle's position lies in an interval between $\mathbf{x}$ and $(\mathbf{x} + d\mathbf{x})$ at a given time t is equal to $R^2(\mathbf{x},t)\, d^3\mathbf{x}$ (Holland 1993, 67). Probability, therefore, is not inherent to the Causal Theory but

merely expresses a lack of knowledge of initial conditions. This is a situation similar to the use of probability in classical statistical mechanics. In the absence of exact knowledge of a system's initial conditions, we can make statistical predictions if we have the system's wavefunction and the initial value of the amplitude R.

Incidentally, if Equation (3.5) is combined with the polar form of the wavefunction, then this allows the equation describing the propagation of the wave field to be obtained. Since we now have $(\partial S/\partial t)$ and $(\partial R^2/\partial t)$, we might expect to find $(\partial \Psi/\partial t)$ as a first step.

$$\frac{\partial \Psi}{\partial t} = R \frac{\partial}{\partial t}(e^{iS/\hbar}) + (e^{iS/\hbar}) \frac{\partial R}{\partial t}$$

$$= \left[\left(\frac{i}{\hbar}\right) R \frac{\partial S}{\partial t} + \frac{\partial R}{\partial t}\right](e^{iS/\hbar}) =$$

$$\left[\left(\frac{i}{\hbar}\right) R \left\{-\frac{(\nabla S)^2}{2m} + \left(\frac{\hbar^2}{2m}\right)\left(\frac{\nabla^2 R}{R}\right) - V(\mathbf{x},t)\right\} + \frac{\partial R}{\partial t}\right](e^{iS/\hbar})$$

where we have substituted from Equation (3.1) for $(\partial S/\partial t)$. It can be shown readily that Equation (3.5) is equivalent to the following equation:

$$\frac{\partial R}{\partial t} = \left(\frac{-1}{2m}\right)\{R\nabla^2 S + 2(\nabla R)\cdot(\nabla S)\} \qquad (3.7)$$

If we substitute for $(\partial R/\partial t)$ in the expression for $(\partial \Psi/\partial t)$ and rearrange some terms, then we get: $(i\hbar)(\partial \Psi/\partial t) =$

$$\left[\left(\frac{R}{2m}\right)(\nabla S)^2 - \left(\frac{i\hbar R}{2m}\right)(\nabla^2 S) - \left(\frac{\hbar^2}{2m}\right)\nabla^2 R - i\hbar\frac{(\nabla R)\cdot(\nabla S)}{m}\right]e^{iS/\hbar}$$

$$+ RV(e^{iS/\hbar}) = -(\hbar^2/2m)\nabla^2(R\, e^{iS/\hbar}) + (R\, e^{iS/\hbar})\, V$$

This is just the time-dependent Schrödinger equation:

$$i\hbar\frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi \qquad (3.8)$$

which describes the propagation of the wave field.

3.5 Dynamic Theory of Measurement

We saw in Section 2.3 that a special status is afforded to 'measurement' in Orthodox Quantum Theory. On the question of this status, John Bell wrote:

> It would seem that the [Orthodox Quantum] theory is exclusively concerned with 'results of measurement' and has nothing to say about anything else. … And does not any *analysis* of measurement require concepts more *fundamental* than measurement? And should not the fundamental theory be about these more fundamental concepts? (Bell 1987b, 117–118, italics in original).

Bell is certainly correct about physical theories not just being about predicting the outcomes of experiments. What does the 'measurement process' entail within the Causal Theory? Measurement is not fundamental in the Causal Theory as can be seen with reference to its axioms (as presented in Section 3.2). Measurement is merely a special type of interaction between two systems, the quantum system under investigation and a measurement apparatus.

One principle that is frequently discussed in quantum measurement theory is the Principle of Faithful Measurement:

- *Principle of Faithful Measurement*

  The result of measurement is numerically equal to the value possessed by an observable immediately prior to measurement.

In detailing what the 'measurement process' entails, we shall answer the question of whether the Principle of Faithful Measurement generally holds in the Causal Theory.

An exposition of the measurement process in the Causal Theory will now be presented. Let $\psi(\mathbf{x}, t)$ be the single-particle wavefunction of a quantum system. A hermitian operator **A** corresponds to an observable A of the system. In an ensemble of similar systems with the wavefunction ψ, the initial value A_o is given by:

$$A_o = (\mathrm{Re}\ \psi_o^*)\, \mathbf{A} \psi_o \,/\, |\psi_o|^2$$

where only the real part contributes due to the operator **A** being hermitian (Holland 1993, 92). The quantum system interacts with a measurement apparatus which has an initial wavefunction $\phi_o(y)$.

This wavefunction is conveniently described as a wave packet, i.e. an envelope of waves. The packet's coordinate y denotes the 'readout' (or pointer display) of the measurement apparatus. The measurement of A is an interaction which is assumed to be impulsive such that any independent evolution of apparatus or quantum system is negligible. The interaction Hamiltonian is given by (Holland 1993, 339):

$$\mathbf{H_I} = g\,\mathbf{A}\,\mathbf{p_y} \qquad (3.9)$$

where g is a coupling constant and $\mathbf{p_y}$ is the momentum operator conjugate to y.

The total initial wavefunction for the combination of quantum system and apparatus is given by the product of their initial individual wavefunctions:

$$\Psi_o(\mathbf{x},y) = \psi_o(\mathbf{x})\,\phi_o(y)$$

During the time of the interaction, Schrödinger evolution with the above Hamiltonian (Equation (3.9)) requires:

$$i\hbar\frac{\partial\Psi}{\partial t} = \mathbf{H_I}\Psi = g\mathbf{A}\mathbf{p_y}\Psi = -i\hbar\, g\mathbf{A}\frac{\partial\Psi}{\partial y} \qquad (3.10)$$

where Ψ is the total combined wavefunction for time $t > 0$ and the operator $\mathbf{p_y} = (-i\hbar)(\partial/\partial y)$. The wavefunction Ψ can be expanded into a complete set of (orthonormal) eigenfunctions $\psi_\alpha(\mathbf{x})$ of the operator **A**. ('Orthonormal' describes normalised wavefunctions which have zero inner product.) This expansion has coefficients $f_\alpha(y,t)$ where $\mathbf{A}\psi_\alpha = \alpha\psi_\alpha$ and α is an eigenvalue, i.e.

$$\Psi(\mathbf{x},y,t) = \sum_\alpha f_\alpha(y,t)\,\psi_\alpha(\mathbf{x}) \qquad (3.11)$$

Substitution of Equation (3.11) into Equation (3.10) and applying the orthonormal conditions of the eigenfunctions ψ_α we find that the partial derivatives of the coefficients f_α are related by (Holland 1993, 340):

$$\frac{\partial f_\alpha(y,t)}{\partial t} = -\,g\alpha\frac{\partial f_\alpha(y,t)}{\partial y} \qquad (3.12)$$

Using a standard method (such as separation of variables), the partial differential Equation (3.12) can be shown to have the solution:

$$f_\alpha (y,T) = f_{\alpha o} (y - gAT) \qquad (3.13)$$

where the $f_{\alpha o}$ are initial values and T is the period of the impulsive interaction. Now if we let

$$\psi_o(\mathbf{x}) = \sum_\alpha c_\alpha (y,t)\, \psi_\alpha(\mathbf{x})$$

then

$$\Psi(\mathbf{x},y,0) = \sum_\alpha f_\alpha(y,0)\, \psi_\alpha(\mathbf{x}) = \Psi_o(\mathbf{x},y) = \sum_\alpha c_\alpha\, \psi_\alpha(\mathbf{x})\, \phi_o(y)$$
$$\Rightarrow f_{\alpha o} = c_\alpha\, \phi_o(y) \qquad (3.14)$$

If we substitute Equation (3.14) into (3.13) and then the result into Equation (3.11), this provides the wavefunction at the end of the interaction (Holland 1993, 341):

$$\Psi(\mathbf{x},y,t) = \sum_\alpha c_\alpha\, \phi_o(y - g\alpha T)\ \psi_\alpha(\mathbf{x}) \qquad (3.15)$$

The wavefunction is split into non-overlapping packets which are represented by the summands in Equation (3.15). Only one of these represents the part of the wave field in which the particle is present. Since there is no overlap, the other packets will have no further effect on the particle and consequently are not relevant to subsequent system evolution (Holland 1993, 341). The wavefunction then effectively becomes:

$$\Psi = c_\alpha\, \psi_\alpha(\mathbf{x})\, \phi_o(y - g\alpha T) \qquad (3.16)$$

In the Causal Theory, this is understood in terms of the wave field dividing into separate parts which continue to exist albeit as empty quantum waves. (The term ‘empty wave’ means that the wave field no longer contains a quantum particle but still possesses some energy and momentum (Holland 1993, 86)). There is *no* ‘collapse’ of the wavefunction and the particle has a definite position at all times.

The measurement apparatus will give a ‘readout’ which will be a single value of y as the combined wavefunction is effectively given by Equation (3.16). Thus, there is no Measurement Problem. The initial value A_o of the observable A will have evolved into a value which would be identified in Orthodox Quantum Theory as an

eigenvalue of the operator **A** (Holland 1993, 342–343). Since the measurement apparatus has an enormous number of degrees of freedom, once the measurement interaction is over, the process is essentially irreversible (Holland 1993, 348; Cushing 1994b, 51).

Occasionally one finds in the literature, claims that the Causal Theory fails to solve the 'Measurement Problem' (e.g. Stone 1994, 250–266; Zeh 1999, 197–200; Brown and Wallace 2005, 517–540). These claims have been successfully answered by Tim Maudlin (Maudlin 1995, 479–483) and by Peter J. Lewis (Lewis 2007b, 787–803) and will not be discussed here.

In general, the measurement process introduces an uncontrollable (and unpredictable) disturbance to the wave field of the quantum system. The interaction with a measurement device transforms the wavefunction of the system into an eigenfunction of the observable being measured (Bohm 1952b, 182–183; Holland 1993, 343). Since the wave field is a real, physical entity there is no sudden collapse on measurement. Instead we have seen that there is a change in the wave field and this may alter the particle's momentum, position, energy, etc. The statistical results of measurement coincide with the probabilistic predictions for the measured values of physical quantities, i.e. observables, and not necessarily the statistical distribution of possessed values (Holland 1993, 360–365). Therefore the measurement of physical observables, according to the Causal Theory, does not provide necessarily the (pre-existing) possessed value of the observable prior to measurement. The exception is measurements of a particle's position which does yield pre-measurement values (Holland 1993, 351; Home 1997, 45). Since measurement is a dynamic process (in the sense that the measured value of a physical quantity need not be identical with its possessed value prior to the measurement process), the Principle of Faithful Measurement cannot be generally upheld in the Causal Theory (Dewdney and Malik 1993, 3522–3533).

It was argued in Chapter 1 that the Heisenberg uncertainty relations specify a lower bound of the variance of two kinds of ('incompatible') measurements made on an ensemble of similarly prepared quantum systems, as stated by Holland:

> … [Heisenberg's uncertainty relations] asserts a limitation on the scatter in the results of a statistical ensemble of identical experiments. …[this] follows from the usual axioms of quantum mechanics regarding measurements (Holland 1993, 360).

This statistical spread is explained by the Causal Theory as due to the change in the wave field (and therefore to the quantum potential) caused by the measurement process (Cushing 1994b, 53). Further consideration of physical measurement within the Causal Theory will appear in Chapter 5.

In Chapter 1, we saw that the Kochen and Specker Theorem does not apply to contextual 'hidden variable' theories. The context dependence aspect of measurement in the Causal Theory requires that the value of an observable obtained on measurement depends on the evolution of the quantum state and this, in turn, depends on the system's Hamiltonian. Measurements of 'incompatible' variables will alter the wave field resulting in a different outcome from what would be otherwise, i.e. a measured value depends on what other observables are measured. This is summarised in the following quotation from Dewdney and Malik:

> … in Bohm's [i.e. Causal] theory, the value that would be obtained for the measurement of any observable is predetermined in a particular instance, with a given set of hidden variables, but this value does not depend solely on the initial value of the hidden parameter … it also depends on the evolution of the quantum state in the system's configuration space … Just how the state evolves depends on the system's Hamiltonian and hence the value that will be obtained for a particular measurement with a given initial position can depend on which, if any, other observables are measured … it is this feature of Bohm's theory that precludes the argument of Kochen and Specker (Dewdney and Malik 1993, 3523).

Clearly then, this context dependence is what makes the Causal Theory immune from the conclusion of Kochen and Specker and similar arguments. Alleged flaw ③ in Section 1.3, viz., that the Causal Theory has been disproved by impossibility theorems, is thereby invalidated.

It was noted in Section 2.4 that experiments that test Bell inequalities have confirmed that they are violated and, *a fortiori*, with experiments on GHZ states. There remains, however, a widespread myth that this confirmation of the violation of Bell inequalities has decided the question about (so-called) 'hidden-variable' theories in the negative (i.e. alleged flaw ④ in Section 1.3). Unfortunately, this is a 'myth' that continues to be promulgated in the literature (e.g. Demtröder 2006, 497). Given the assumptions

behind Bell's Theorem (as set out in Section 2.4), these experiments only show that the class of *local*, realistic 'hidden-variable' theories are ruled out (Hardy 1996, 68–69), as the findings of Rowe and his colleagues have reported:

> Local realism is the idea that objects have definite properties whether or not they are measured, and that measurements of these properties are not affected by events taking place sufficiently far away. … Bell and others constructed mathematical inequalities whereby experimental tests could distinguish between quantum mechanics and local realistic theories. … experiments are still being refined to overcome 'loopholes' that might allow a local realistic interpretation. Here we have measured correlations … these correlations violate a form of Bell's inequality. … this violation of Bell's inequality was obtained by use of a complete set of measurements. Moreover, the high detection efficiency of our apparatus eliminates the so-called 'detection' loophole (Rowe et al. 2001, 791)

These considerations dispose of alleged flaw ④, i.e. that the Causal Theory has been refuted by experiments on Bell-type inequalities, since the Causal Theory is non-local (see Section 3.6).

3.6 Many Particle States and Non-Locality

3.6.1 Many Particle Systems in the Causal Theory

In a system consisting of N 'point-like' particles (where N is an integer > 1), if each particle is unconstrained then its position can be given by assigning it three coordinates (not necessarily Cartesian ones). The minimum number of coordinates (variables) required to specify the positions of all the particles in an unconstrained system at a given time, i.e. the configuration of the system, must be 3N. An N-particle quantum system may be considered as a generalisation from the single particle case in which the wavefunction is a field on a 3N-dimensional configuration space. All the single particle quantities and equations have many-particle analogues. However, there is only one guiding wave field described by a wavefunction $\Psi = \Psi(\mathbf{x}_1, \ldots, \mathbf{x}_N, t)$ where each of the $\mathbf{x}_i$ is a set of Cartesian coordinates (Holland 1993, 277). This wavefunction evolves according to the many-particle Schrödinger equation:

$$i\hbar \frac{\partial \Psi}{\partial t} = \sum_{i=1}^{N} \left(\frac{-\hbar^2}{2m_i}\right) \nabla_i^2 \Psi + V\Psi$$

where ∇_i^2 is the Laplacian evaluated at the position of the i^{th} particle.

Let's consider, for simplicity, a two-particle system where the particles have equal mass m. We denote the particles by numerical subscripts 1 and 2. At a given time t, let particle 1 have coordinates $\mathbf{x}_1$ and particle 2 have coordinates $\mathbf{x}_2$. As before, the wavefunction of the system may be written as: $\Psi = Re^{iS/\hbar}$, then the two-particle equivalent of Equation (3.1) is:

$$-\frac{\partial S}{\partial t} = \frac{(\nabla_1 S)^2}{2m} + \frac{(\nabla_2 S)^2}{2m} - \left(\frac{\hbar^2}{2m}\right)\left[\frac{(\nabla_1^2 + \nabla_2^2)R}{R}\right] + V(\mathbf{x},t)$$

This is the quantum Hamilton-Jacobi equation in six-dimensional configuration space. The subscripts on the Laplacian operators refer to explicit dependence on the coordinates of the individual particle. The two-particle equivalent of Equation (3.5) is:

$$\frac{\partial R^2}{\partial t} + \nabla_1 \cdot \left(R^2 \frac{\nabla_1 S}{m}\right) + \nabla_2 \cdot \left(R^2 \frac{\nabla_2 S}{m}\right) = 0$$

The two-particle quantum potential, Q = Q ($\mathbf{x_1}$, $\mathbf{x_2}$, t), is given by:

$$Q = -\left(\frac{\hbar^2}{2mR}\right)\left(\nabla_1^2 R + \nabla_2^2 R\right)$$

The respective momenta of the two particles is:

$$\mathbf{p}_1 = \nabla_1 S \text{ and } \mathbf{p}_2 = \nabla_2 S$$

The momentum of each particle will, in general, depend on the position of the other. This is a manifestation of the state dependence which, in a many-particle system, finds expression as a holistic, non-local connection between the particles of the system. The quantum mechanical force **F** for a two-particle system is given by:

$$\mathbf{F} = -(\nabla_1 Q + \nabla_2 Q)$$

The above two-particle system equations are easily generalised to their many-particle equivalents (Holland 1993, 279–280), which will be used in later chapters.

3.6.2 The Reality of the Many-Particle Wave Field

The reality of the wave field is not placed in jeopardy because its wavefunction is defined on a multi-dimensional configuration space. Bohm correctly described the wavefunction as a mathematical representation of an objectively real field (Bohm 1952a, 170). Nor does the use of such a description imply that a multi-dimensional space has an existence in the same sense that physical three-dimensional space may be said to exist. There is an acceptance by some researchers that the multi-dimensional configuration space of the many-particle Causal Theory is a real aspect of nature, as Holland claims:

> … an *individual physical system resides in a multidimensional (configuration) space*. While the particles each move in 3-space, the guiding wave is, in general, irreducibly defined in 3*n*-space [n > 1]. Since we conceive of the wave as a physical influence on the particles, *we ascribe to configuration space as much physical reality as we do to three-dimensional Euclidean space in the one-body theory* (Holland 1993, 277–278, italics in original).

This position is rejected unequivocally as it confuses the formal machinery of the model with the reality that the model represents. This is an important distinction. The mathematical technique of using a multi-dimensional (mathematical) space to model physical phenomena is well established. In the configuration space description of a many-particle system in classical mechanics, the system is represented by a *single* point in the space. The empirical predictions of classical mechanics are correct within its domain even though this space is not physical space. The use of a multi-dimensional mathematical space to model phenomena does not necessarily require an ontological commitment to the physical existence of such a space.

Although the configuration space representation in classical mechanics is a convenient summary of the positions of all the particles in a system, the situation is different in quantum mechanics for there is information in the configuration space wavefunction that

is not present in the individual wavefunctions for the particles (due to their entanglement). The problem is then that we cannot represent the physics in terms of a wavefunction (or even wavefunctions) in three-dimensional space. This is seen by some commentators as a conceptual problem for the Causal Theory. These commentators hold the view that wavefunctions that describe real, physical fields need to be defined in three-dimensional space. So the argument goes that either we accept the multi-dimensional space as a real physical space or we don't accept the wavefunction as representing a real field. Yet, as argued above, the *use* of a multi-dimensional mathematical space to model phenomena does not necessarily require an ontological commitment to the physical existence of a multi-dimensional space. The position adopted here (in common with Bohm's 1952 account) is that the many-particle wavefunction defined on 3N-dimensional space is a *mathematical representation* of an objectively real field in physical three-dimensional space. Is this a coherent position to take? In defence of this position, it was stated that a scientific model should not be taken literally in all respects but this does not directly address the issue that we cannot describe a system by wavefunctions in three-dimensional space without a loss of information. Yet, there are good arguments and even better evidence for the physical existence of wave fields (see Section 4.5). Given their physical existence, the justification for claiming that the wavefunction in 3N-dimensional configuration space is a representation of a real field in physical three-dimensional space is as follows.

First, a simply connected three-dimensional space alone cannot describe the holistic quantum connectiveness and non-locality features of multi-particle quantum systems (see below). Instead, this is done *formally* by employment of a multi-dimensional configuration space. Ontological commitment is not made to a multi-dimensional space since the problems associated with such a commitment are considerable and include:

- needing at least three separate dimensions for every particle in the universe;

- the total number of dimensions in the universe varying from moment to moment along with the creation and annihilation of particles;
- the extra dimensions always being completely unnoticeable at macroscopic scales; and
- a complete lack of any experimental evidence for the existence of multi-dimensional physical spaces (Randall 2002, 1422; Tuttle 2006, 19; Smolin 2006, xvi).

These are strong reasons for not having an ontological commitment to a physical multi-dimensional space in the same sense as we do for physical three-dimensional space.

Second, we do not know the 'means' by which quantum non-local connections are actualised. This is not because of the non-relativistic context for non-locality is also present in relativistic versions of quantum theory (see Section 4.6).

Given the strong reasons against taking multi-dimensional space as real, the evidence in favour of physically real wave fields (see Section 4.5), and the absence of information about the 'means' of non-local connections, it is a coherent position to take the wavefunction to be a mathematical representation of a real field in physical space.[1] This disarms the conflicting view that the wavefunction *must* be defined in three-dimensional space and thereby dissolves the potential conceptual problem for the Causal Theory. The empirical predictions of quantum mechanics are some of the best confirmed in the whole history of physics even though its multi-dimensional configuration space is not physical space. Hopefully, when we have discovered (or have developed a model of) the 'means' by which quantum non-local connections are actualised then we will be able to describe the wave field in physical three-dimensional space (see also Section 4.6).

The motion of an N particle system is described in the Causal Theory by its trajectory which is traced out in 3N-dimensional configuration space. Even though this description is given using a multi-dimensional space, the motion of individual particles can be calculated for there exists a natural mapping from the system's

[1] For a different approach to the wave field in three dimensions, see: Lewis 2004.

trajectory in 3N-dimensional configuration space to trajectories in three-dimensional space. Such calculations are relatively straightforward (Dewdney 1988b, c, d).

3.6.3 The Non-Locality Aspect

Consider now the issue of locality in quantum theory. In the Causal Theory, a many-particle quantum system exhibits non-local effects as its quantum potential allows for a strong and direct interconnection between the particles. In particular, the non-local influence on a particle depends on the positions of all other particles in the system at a given time (Bohm and Hiley 1975, 99). Consequently, the description of a many-particle quantum system is made by reference to a simultaneity frame (i.e. a preferred frame of reference). This cannot be rectified in a non-relativistic theory. The non-local aspect provides a physical explanation for the motions of quantum particles in a many-particle system. Such motion is consistent with the Principle of Causality (Aharonov and Rohrlich 2005, 87) but violates the Principle of Locality.

Non-locality is another of the various criticisms that has been laid at the feet of the Causal Theory (Rae 2002, 261). The experimental tests of the various Bell Inequalities have come down 'fair and square' on the side of non-locality, i.e. experiments continue to confirm that the Bell Inequalities are indeed violated, as predicted by the formalism of quantum mechanics (as already discussed in Section 3.5). Such criticism of the Causal Theory is completely misdirected, as Maudlin explains:

> Violations of Bell's Inequality show that the *world* is non-local. It can be no criticism of a theory that it displays this feature of the world in an obvious way (Maudlin 2002, 121, italics in original).

The continued criticism of the Causal Theory for its explicit non-locality is a little surprising for this reason but also because it has become clear that Orthodox Quantum Theory also requires some kind of non-locality (i.e. action-at-a-distance), as Redhead concludes in his detailed analysis of quantum theory:

> … some sort of action-at-a-distance … seems to be built into a reasonable attempt to understand the quantum view of reality (Redhead 1987, 169).

Fortunately, parts of the universe are sufficiently separable (i.e. do not constantly exhibit non-local behaviours) that we can still use established methods of scientific investigation and analysis to obtain knowledge of the physical world (Bohm and Hiley 1993, 59; Cushing 1994b, 185).

The emergence of non-locality (by the formal means of a multi-dimensional configuration space) is the expression in the model of a holistic quantum connectiveness. In spite of the presence of non-locality, it is a curious result that neither the Causal Theory nor Orthodox Quantum Theory violates the Special Theory of Relativity, as Hooker notes:

> … one arrives at the rather bizarre conclusion that that a theory (QM) which is itself not relativistically formulated, nonetheless seems uncannily prevented from coming into conflict with relativity theory by a diverse, if related, set of principles (Hooker 1989, 244).

There is no violation of Special Relativity for the connections between quantum systems cannot be used for the purposes of signalling (i.e. the transmission of information) or the transfer of energy faster than the speed of light in vacuum (Ballentine and Jarrett 1987, 696; Maudlin 2002, 125). We shall further consider issues concerning quantum non-locality in Section 4.6 and the non-violation of Special Relativity by the Causal Theory in Section 5.5.

3.7 Resolution of the Quantum Paradoxes

The two ‘paradoxes’ of Orthodox Quantum Theory that were summarised in Chapter 1 may now be shown to be readily solvable using the Causal Theory.

3.7.1 EPR (Einstein-Podosky-Rosen)

The EPR Paradox is resolved within the Causal Theory by the existence of a non-local connection between the particles and by rejecting the Completeness Axiom of Orthodox Quantum Theory. A change in the wave field (and therefore the quantum potential) results from a measurement on one of the particles. The quantum

potential allows for a direct connection between the particles which depends on the state of both. The connection between the particles via the quantum potential is instantaneous, but as noted in the previous section, Special Relativity is not violated (Bohm and Hiley 1975, 107).

3.7.2 Schrödinger's Cat

The root of the problem here is the Completeness Axiom of Orthodox Quantum Theory. If this axiom is rejected (as in the Causal Theory) then the solution is straight-forward. Whether the radioactive decay occurs (which leads directly to the cat dying) will depend on the position of the relevant particle in the radioactive source. The initial position of the particle together with the many-particle wavefunction of the source determines its future behaviour (i.e. decay or not). The usual expected events will then follow. The cat is not, of course, ever in a superposition of live and dead states. It is either alive or dead, but this will not be known until an observation is made.

3.8 Transition to the Classical Realm

It is commonly asserted in textbooks on Orthodox Quantum Theory that the transition to classical mechanics arises either:

(i) in the limit of large quantum numbers, i.e. when the principal quantum number (n) tends to infinity ($n \to \infty$); or

(ii) when allowing $\hbar \to 0$.

The former criterion is an application of the Correspondence Principle where quantum states tend to classical ones in the limit of large quantum numbers. What exactly constitutes 'large quantum numbers' is, however, not rigorously specified. Indeed, there are examples where the principal quantum number (which specifies the energy of a state) can be made arbitrarily large with the system still

being governed by quantum mechanics (Holland 1993, 221). The failure of this criterion in some situations indicates its unsuitability.

The latter criterion is used to cover situations where a system grows to macroscopic proportions and thereafter develops according to the laws of classical mechanics. This criterion (strictly speaking) is *nonsense*, for $\hbar$ is a physical constant (with a non-zero magnitude in units of energy-time) and not a parameter that can be adjusted or taken to be zero.

The formal transition from quantum to classical realms has nothing to do with the above two criteria. Classical mechanics emerges naturally when the value of the quantum potential becomes negligible with respect to the other terms in the Quantum Hamilton-Jacobi equation. When this occurs, the Quantum Hamilton-Jacobi equation tends to the classical version and the wave field no longer affects particle motions. Therefore, the problem of having an arbitrary boundary between classical and quantum realms (as found in Orthodox Quantum Theory) does not arise in the Causal Theory. The role of the quantum potential here also shows that there is value in holding to a non-minimalist version of the Causal Theory.

3.9 Quantum Equilibrium, Metaphysics, and Consistency

3.9.1 The Quantum Equilibrium Condition

Axiom VI of the Causal Theory is called the Quantum Equilibrium Condition. This axiom is necessary in order for the statistical predictions of the Causal Theory to agree with experiment. One might again cite Synge's comment (quoted in Section 3.2) that the choice of axioms is made to fit the theory, as a sufficient reason to accept the Quantum Equilibrium Condition as an axiom.

Whether the Quantum Equilibrium Condition should be of axiomatic status has been the subject of dispute. In 1952, J.D. Keller suggested that if the Causal Theory is to bear a similar relationship to Orthodox Quantum Theory that classical mechanics bears to

classical statistical mechanics, then the probability density $P(\mathbf{x},t) = |\Psi(\mathbf{x},t)|^2$ would have to be derivable in the Causal Theory from the other assumptions (Keller 1953, 1040). In response to this and other criticisms, Bohm attempted to show that the initial probability density $\rho(\mathbf{x}) = P(\mathbf{x},0) = |\Psi|^2$ was a theorem, for then the equation of continuity (Equation 3.6) would ensure that $P(\mathbf{x},t) = |\Psi(\mathbf{x},t)|^2$ holds at all subsequent times (Bohm 1953a). Bohm's 1953 proof was not successful. Indeed, Hans Freistadt was later to point out that the mathematics in Bohm's argument was somewhat suspect (Freistadt 1957, 29). Bohm also tried an alternative approach with J.-P. Vigier in 1954 where a fluctuating 'sub-quantum realm' was assumed (Bohm and Vigier 1954). There was no general acceptance of this approach either.

There have been more recent attempts too. Antony Valentini claimed in 1991 to have done what Bohm failed to do in 1953 (Valentini 1991a; 1991b). Dürr, Goldstein and Zanghì also claimed to have shown that $P(\mathbf{x},t) = |\Psi(\mathbf{x},t)|^2$ using different assumptions (Dürr et al. 1992a, 856–858). What's more, they assert that Valentini's derivation is not only unnecessary, it is mathematically incorrect (Dürr et al. 1992c, 11). Who one believes depends on what premises are found acceptable and whether the derivations are judged to be mathematically rigorous. It is not the intention here to attempt to decide this question. The status of the Quantum Equilibrium Condition and its possible derivation is a continuing area of research within the Causal Theory. Dickson has summarised this situation:

> … we do not seem to have a justification for the [Quantum Equilibrium] hypothesis. The arguments of Dürr et al. and Valentini are suggestive, but far from providing a satisfactory justification … (Dickson 1998, 124).

A generally accepted proof of $P(\mathbf{x},t) = |\Psi(\mathbf{x},t)|^2$ would be a boost to the fortunes of the Causal Theory and one can only hope that a proof which gains general acceptance will be forthcoming in the future.[2]

[2] For a recent survey of this issue, see: Callender 2007.

3.9.2 Metaphysical Labelling

We also note that the metaphysical labelling of the Causal Theory is invalid. In Section 1.3, alleged flaw ⑤ is the statement that the Causal Theory is pure metaphysics. The term 'pure metaphysics' applied in this context implies that the Causal Theory is devoid of physical content. This is because it has been claimed that the entities postulated in the Causal Theory are unobservable and that the particle paths described by the Causal Theory also cannot be observed. The so-called 'pure metaphysics' criticism is used as an *emotive condemnation* of the Causal Theory. John Bell explicitly commented on this:

> [the opponents of the Causal Theory] … could produce no more devastating criticism of Bohm's [Causal] version [of Quantum Theory] than to brand it 'metaphysical' and 'ideological' (Bell 1987b, 160).

This criticism can be immediately dismissed as the Causal Theory is no more pure metaphysics than is any other physical theory that postulates the existence of entities and/or processes that, at present, cannot be *directly* observed, e.g. quarks, black holes, dark matter, event horizons, etc. If these were considered only to be metaphysical then we ought not to take seriously the Standard Model of Elementary Particles and the General Theory of Relativity.

The history of science reveals other instances where the label of 'metaphysical' has been used to belittle ideas in science with great explanatory promise but where there was no *direct* evidence to support the ideas at the time when they were articulated. A paradigm example must surely be the denial of the existence of atoms and molecules in the late nineteenth century. At that time the notions of atoms and molecules were described as being metaphysical by some of the leading chemists of the era (Kragh 1999, 8). Unfortunately, the 'pure metaphysics' criticism is still employed to this day as a passionate indictment of the Causal Theory despite being an illegitimate criterion for theory rejection.

It is interesting that Orthodox Quantum Theory itself includes a postulate that *cannot ever be* experimentally tested – the Completeness Axiom, i.e. that the state vector (or wavefunction) contains all information about the quantum state (Holland 1993, 25). If the 'pure metaphysics' criterion is accepted then it is also the case

that Orthodox Quantum Theory would qualify as a piece of metaphysical speculation!

3.9.3 Consistency of the Causal Theory

We shall now deal with the claim that the Causal Theory is inconsistent (alleged flaw ⑥ in Section 1.3). In the period immediately after publication of Bohm's 1952 papers, many in the quantum physics community assumed that either Bohm had made some errors in the formalism or that there were inconsistencies present. This was assumed because to admit otherwise would have gone against the supreme authority of Bohr and Heisenberg (both of whom were still alive at that time) and the unquestioned belief in the correctness of von Neumann's proof. Basil Hiley and David Peat were later to describe these circumstances:

> Because of the high (and justified) mathematical reputation of von Neumann … together with the various writings of Bohr, Heisenberg and Pauli, etc., gave rise to the dogma that there is no alternative. … Unfortunately the physics community did not take very kindly to the appearance of this alternative [i.e. Bohm's] view. … [it was] felt there was some fundamental flaw in Bohm's argument. … *But exactly where the logical contradictions lie is never made clear* (Hiley and Peat 1987, 6–7, italics added).

The formalism of the Causal Theory, as presented in this chapter, can be seen on close inspection to be a fully consistent mathematical scheme. Further, after more than fifty years since the appearance of Bohm's original papers, if there were any logical contradictions in the Causal Theory then these would have been exposed with much enthusiasm by opponents of Bohm's ideas. However the occasional claims of inconsistency have never been established – indeed the opposite has been the case (Home 1997, 55). This suffices to dismiss alleged flaw ⑥.

Chapter 4

Energy and the Wave Field

… the principle of energy in its generality … is nowadays no longer disputed.
— Max Planck

Abstract This chapter begins with a discussion of the concept of energy and how energy is defined in physical theories. The concept of energy proves especially useful in dealing with a number of issues that arise in the Causal Theory. Two alternative renderings of the wave field, the Active Information Hypothesis and a Non-Interactive approach to the wave field, are surveyed and their shortcomings highlighted. The physical nature of potential energy is discussed at some length for this topic will be important to the arguments presented in Chapter 5. It is concluded that potential energy is an attribute of physical fields. The characteristics of wave fields are summarised together with a discussion of the reasons for accepting their existence. The examples of the Double Slit Experiment, the Aharonov-Bohm Effect, and the laboratory manipulation of matter waves are cited as providing evidence in favour of the existence of wave fields. Lastly, non-locality and its relation to the wave field are discussed. Some possibilities for accounting for non-locality are offered.

4.1 The Wave Field and the Concept of Energy

Since the initial development of quantum mechanics the wave field has been the subject of different views as to its role and ontological status. The name 'wave field', although of historical origin, is appropriate since it obeys the Principle of Linear Superposition (Holland 1993, 69). In the early days of quantum mechanics, the

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mathematical formalism came first then its interpretation (or rather interpretations) ultimately resulting in acceptance by the majority of the physics community of Orthodox Quantum Theory. It is interesting to note the early ideas of one of the pioneers of quantum mechanics, Max Born, in regard to the wave field. Born initially ascribed some kind of reality to both particles and waves but thought that the waves did not carry energy or momentum (Jammer 1974, 495). Born later changed his conception of wave fields to that of 'waves of probability' and postulated that the square of the wavefunction provides a probability density for finding a particle (Pais 1982, 442). There have been, of course, several other accounts postulated since Born's era. These range from the subjective view where the wavefunction merely represents an observer's knowledge of a quantum system, through to Everett's universal wavefunction which gave rise to the Many-Universes interpretation of quantum mechanics. (A suitable summary of these views may be found in Sudbery 1986, 212–224.)

Even amongst adherents to the Causal Theory, there is not, unfortunately, unanimous agreement on the nature of the wave field. Franco Selleri considers the wave field to be real but, like Born's original view, he argues that it has zero energy content (Selleri 1982, 1087–1112). Another view appears in the later writings of David Bohm wherein the role of the wave field is presented in terms of his notion of 'active information'. Here the quantum potential is seen only as an 'information potential' (Bohm and Hiley 1993, 32). The Active Information Hypothesis will be examined later in this chapter. Peter R. Holland furnishes yet another description of the wave field in his comprehensive text on the Causal Theory in which he not only argues that wave field carries energy, momentum and angular momentum through space but can do so far from the particle's location (Holland 1993, p. 84.). Holland's position is very close to that taken in this book.

The physics literature pays little or no attention to elucidating a general definition of energy. Further, it is occasionally flagged that the relevant quantity that is the same for all inertial frames is not energy but relativistic energy-momentum (Rindler 1982, 78–81). This implies that it is energy-momentum that is objectively real. However, in a non-relativistic context, we have to make do with

energy and momentum separately. Given this, it is consistent to take energy as a relevant (real) aspect of system's energy-momentum in the non-relativistic domain.

In classical mechanics, energy is defined as the capacity of a physical system to perform work. The definition of energy then becomes dependent on the definition of work. This is usually given by an integral of the scalar product of the force acting on an object and the object's displacement, i.e. $\int \mathbf{F} \cdot d\mathbf{x}$ which, in turn, depends on an intuitive notion of force. If one accepts this definition of work as a starting point, it suffices (in most part) for purely mechanical systems but has severe limitations in other contexts.

Consider now a system that is mechanically and thermodynamic isolated from events external to itself. One could imagine for example, a system consisting of a non-conducting, sealed cylinder containing a piston. This piston is made of a material that conducts heat. The piston is locked in place half-way along the length of the cylinder with one half of the cylinder containing gas at high temperature and the other half containing gas at low temperature. We further suppose that this system is situated in intergalactic space (to avoid problems with gravitation). If the piston became unlocked, the system would perform work, i.e. the piston would move towards the low temperature end of the cylinder. If the piston remained locked, the system would still have the capacity to perform work. However, in either case, this system eventually will come to a state of thermodynamic equilibrium when the temperature throughout the inside of the cylinder equalises. Then the system will not be able to perform any work (nor have the capacity to do so) despite the fact that it has a constant energy content as evidenced by the resulting uniform temperature. The standard definition of work (as given above) is useless for defining energy in such a case.

Typically, different forms of energy (e.g. kinetic, gravitational, heat, etc.) are defined in each specific domain of physics. However, no general, quantitative definition of energy which covers all its aspects is currently known. The lack of such a general definition was explicitly acknowledged as far back as 1902 in an insightful statement by the French philosopher of science Henri Poincaré in his treatise, *Science and Hypothesis*:

> … In every particular case we clearly see what energy is, and we can give it at least a provisory definition: but it is impossible to find a general definition of it (Poincaré 1905, 132).

It may or may not be impossible to find a general definition of energy but, as we do not currently have one, we shall draw on the well-known and accepted concept of energy that uses particular examples in order to illustrate itself. The concept of energy is sufficiently well understood that it will be referred to without necessarily requiring elaboration. However, the relevant characteristics of energy that will be drawn on in arguments presented in this book are as follows. Energy:

- is conserved;
- exists in different forms;
- can be stored;
- can be transferred through space or from one material body to another;
- can be transformed into other forms of energy.

In regard to the conservation of energy, this is either postulated as a law or derived as a theorem from a set of axioms depending on the area of physics involved. An example of the former is the First Law of Phenomenological Thermodynamics (Zemansky 1968, 78–79). An example of the latter is Noether's Theorem where invariance under continuous time translation requires a system's energy to be a conserved quantity (Szekeres 2004, 473–474). Given the importance in physics of both the concept of energy and its conservation, it is a little surprising that chemistry texts tend to discuss these issues in more detail than do most physics texts. One undergraduate chemistry textbook, for example, makes the following statement:

> No single theory of physics is more widely accepted or more generally useful [than conservation of energy], yet the statement [energy is conserved] refers to an abstract concept about a quantity never directly measured. We measure velocity and mass to calculate energy of motion. We measure an altitude … to determine energy of position. We measure moles of a substance to infer its chemical energy. We measure the change in the density of mercury to infer transfer of heat. Frequently, the main evidence for the existence of a quantity or type of energy is that apparently energy is not conserved unless some unseen energy is assumed (Pimentel and Spratley 1971, 248).

The concept of energy is abstract *only* in the sense that energy is not measured directly. However, there are a number of extremely compelling reasons for accepting that energy is a real attribute of all physical systems. Our best confirmed physical theories, Quantum Mechanics and General Relativity, both posit energy to be an essential attribute of physical systems. Physical events such as elementary particle pair creation and annihilation, and the expansion of the universe could not be explained without energy being real. If energy is not a real attribute of physical systems, then we might ask questions such as: What is released on the detonation of a nuclear weapon that causes so much devastation? Why do machines (including biological machines) 'run down' if not provided with fuel (i.e. sources of energy)? What causes stars (including our Sun) to continue to shine over the course of billions of years? Why does the Earth's surface regularly erupt with hot volcanic flows? At the level of human affairs, we build enormous (and costly) electrical generating stations and pump natural gas through pipelines stretching thousands of kilometres – if energy is not real, what do these stations produce and what does the natural gas provide that is so useful and precious? If energy is not real, what then makes our vehicles run, heats our homes, lights the darkness, and generally drives the universe? None of this makes any sense unless energy is a real attribute of physical systems.

In practice, energy differences provide a way of keeping track of changes in physical systems and assist in gaining an understanding of physical processes that would otherwise be unintelligible. We shall have more to say about the nature of energy in Section 4.4. However, it will be useful to end this section by explicitly stating what is meant by 'conservation of energy' as a principle within the non-relativistic context (Halliday et al. 1993, 201):

- *Principle of the Conservation of Total Energy*

  The energy of a physical system is neither created nor destroyed, but may be transformed from one kind of energy into another, such that it is always theoretically possible to account for the total energy of a system.

The Principle of the Conservation of Total Energy is widely accepted and is one of the most empirically confirmed principles of physics, i.e. it is based on an enormous and extensive experimental basis (*McGraw-Hill Concise Encyclopedia of Physics* 2005, 116). In addition, it is not only at the 'observational level' that we have reason to believe in energy conservation. There are also theoretical reasons for accepting energy conservation such as Noether's Theorem (Ho-Kim et al. 2004, 428).

Accounts of the wave field that mention 'energy' take the meaning of the term to be fully understood or at least understood from the context. However, throughout this book, the concept of energy as outlined above will be used extensively. If the Conservation of Total Energy is construed as a law of nature applicable to individual processes (as is usually the case), then we shall find that there is a conceptual problem for the Causal Theory in accounting for the energy of isolated quantum systems. This will be explicitly addressed in Chapter 5.

4.2 The Active Information Hypothesis

One influential rendering of the wave field's nature is due to David Bohm and Basil Hiley. This account incorporates their idea of 'active information'. Bohm's original description of causal quantum phenomena included a contribution to the total force exerted on a quantum particle given by the gradient of the quantum potential (Bohm 1952a, 170). Later in his work with Hiley, Bohm abandoned the view that the wave field exerts a force on quantum particles in favour of one in which the quantum potential becomes only an *information potential*. Bohm and Hiley postulated the existence of what they called 'active information' where the quantum potential is interpreted as representing information that encodes details relevant to the whole of a given experimental arrangement or environment. The information becomes 'active' upon entering an entity that can process the information (such as a quantum particle). Their basic hypothesis is that information carried by something with only a small amount of energy can direct something else with much greater energy (Bohm and Hiley 1993, 35).

Why did Bohm abandon his earlier view of the quantum potential in favour of the Active Information Hypothesis? Consider the following analogy offered by Bohm and Hiley about a cork bobbing up and down as water waves pass by. The energy of the cork depends on the intensity of the water wave (where intensity is proportional to the square of the wave's amplitude). The greater the distance the cork is away from the cause of the water oscillations, the smaller will be the effect on the cork. The quantum potential effects do not, however, depend on the wave field's intensity since multiplication of the amplitude R by a constant cancels out in the expression for Q (as noted in Section 3.3). Bohm and Hiley described this as follows:

> … the effect of the quantum potential is independent of the strength (i.e., the intensity) of the quantum field but depends only on its *form*. By contrast, classical waves, which act mechanically (i.e., to transfer energy and momentum, for example, to push a floating object) always produce effects that are more or less proportional to the strength of the wave (Bohm & Hiley 1987, 326, italics in original).

Bohm and Hiley seemed to be overly conscious that the effect of the quantum potential is independent of the intensity of the wave field, whereas the effects of a classical wave are due to the strength of the wave. In light of this, they appear to have inferred that the wave field must act in a totally non-mechanical way, which relies only on the form of the wave field. (For a discussion of why Bohm might have altered his opinion, see Guarini 2003).

In the context of quantum physics, 'active information' (which is postulated to be carried by the wave field and represented by the quantum potential) determines a quantum particle's path and its velocity by using the particle's own energy. Bohm and Hiley illustrated this idea with an analogy concerning a ship being automatically guided by a radio signal. The effect of the signal on the ship does not depend on its intensity, for a weak signal will do just as well as a strong one (provided the radio signal is received properly). What is important is the form of the signal for this carries information which, when processed by the ship's autopilot, determines how the ship's own energy will be utilised. The information is described as 'active' when it has entered something which exploits its form (i.e. when the information is processed):

> … the effect of the radio waves is independent of their intensity and depends only on their form. The essential point is that the ship is moving with its own energy, and that the *form*

> of the radio waves is taken up to direct the much greater energy of the ship (Bohm & Hiley 1993, 32, italics in original).

Bohm and Hiley argued that the quantum potential works in a similar manner – by 'informing' a quantum particle about how it will move under its own energy. The other illustrations provided by Bohm and Hiley are not especially helpful to their case and will not be discussed here.

The Active Information Hypothesis opens up a whole host of questions and issues that are extremely problematic. Consider first the difficulties encountered with particle structure. Quantum particles would require complex internal structures with which the 'active information' is processed in order that the particle be directed through space. Bohm and Hiley readily acknowledge this:

> The fact that the particle is moving under its own energy, but being guided by the information in the quantum field suggests that an electron or other elementary particle has a complex and subtle inner structure (e.g. perhaps even comparable to that of a radio) (Bohm & Hiley 1993, 37).

It has not been specified what these complex structures consist of or how they might be arranged within elementary particles. Nor has it been suggested how the actual processing of the 'active information' could occur. Bohm and Hiley's account is presented solely by way of alluding to a number of indirect analogies (e.g. portable radios, computers, DNA) and not by detailed and specific arguments. What's more, it seems likely that at least some fundamental particles do not have the kind of structure necessary. Electrons, which are a prime example for Bohm and Hiley, do not seem to have any constituent parts (Veltman 2003, 54–55; Close 2004, 40) and therefore cannot have a complex internal structure.

Second, consider the difficulties with satisfying physical laws. If information carried by something with only a small amount of energy is to direct something else with much greater energy, where does this greater energy come from in the case of quantum particles? Marcello Guarini has also expressed this question, stating:

> Radios have batteries or some other power source to draw on. Metaphorically speaking, where are the electron's batteries? (Guarini 2003, 82).

Energy conservation necessitates that either the quantum particle would have to have an internal energy content to draw on or that energy be transferred to the particle from a source external to

itself. Further, in the case of a particle that is increasing its speed, it would need a continuous supply of energy during periods of (positive) acceleration. After a large number of such speed increases (which might be interspersed with periods of deceleration), any internal energy content would become depleted. The particle would not be able to 'speed up' thereafter. If the particle's energy comes from an external source, what is it? Other than a mere conjecture about vacuum fluctuations as a possible reservoir of energy (Bohm & Hiley 1993, 48), there is no explanation provided of where the required energy might originate from or how such energy might be 'tapped into'.

The Law of Inertia (better known as Newton's First Law: 'Every body remains at rest or in uniform motion unless acted upon by an unbalanced force') requires that there be some change made to a body's momentum for its path to be altered. If the Active Information Hypothesis is correct, then a quantum particle would have to be deviated from its initial trajectory (i.e. its momentum changed) as a result of the internal processing of the 'active information'. If we relate this to the ship analogy, a ship can have the highest quality radio receiver, a state-of-the-art autopilot, a large reserve of fuel (i.e. energy content), but if it has no engines then none of these other components will affect any change in the ship's momentum. We might ask, metaphorically, what constitutes the electron's engines? Bohm and Hiley give no indication as to how the interior make-up of a quantum particle can possibly affect its momentum.

Serious and substantial flaws have been highlighted in the Active Information Hypothesis. In summary, it is clear that the Active Information Hypothesis:

- leaves too many questions unanswered about its operation;
- cannot be applied to some elementary particles;
- would seem to require violations of the Law of Inertia; and
- does not provide a proper account of energy conservation.

These problems of the Active Information Hypothesis are sufficiently severe that they warrant its abandonment.

4.3 A Non-Interactive Approach to the Wave Field

An approach to the wave field that might be labelled 'non-interactive' has been proposed by Parmenter and DiRienzo. In their assessment, the Causal Theory has several attractive features which include the possibility of addressing fundamental questions of quantum phenomena. Most of the familiar features of the Causal Theory are present in their account but the wave field does *not* exert any direct influence on quantum particles. Parmenter and DiRienzo pose the following questions:

> There are, however, weaknesses in the original [deBroglie-Bohm Causal] theory. One of the most obvious of these relates to the quantum potential Q: What is its source? Typically in physics a force, and its associated potential, have a source. However, nowhere in the literature is this fundamental question addressed in a physically reasonable way (Parmenter and DiRienzo 2004, 2).

Parmenter and DiRienzo provide their own answer to the origin of the quantum force.

They begin with an isolated, many-particle quantum system which has a quantum potential $Q = Q(x_1, x_2, x_3, ..., x_N, t)$ given by:

$$Q = -\sum_{i=1}^{N} \left(\frac{\hbar^2}{2m_i R}\right) \nabla_i^2 R$$

such that for the *i*-th particle:

$$(d\mathbf{p}_i / dt) = -\nabla_i V - \nabla_i Q$$

where $\mathbf{p}_i$ is the momentum of the *i*-th particle, $-\nabla_i V$ is the sum of all the classical forces on the *i*-th particle, and $-\nabla_i Q$ is interpreted as the quantum force on the *i*-th particle. The total momentum $\mathbf{p}$ of an N-particle system is:

$$\mathbf{p} = \sum_{i=1}^{N} \mathbf{p}_i$$

For a classically free system (i.e. where $V = 0$) we have:

$$\frac{d\mathbf{p}}{dt} = -\sum_{i=1}^{N} \nabla_i Q$$

Parmenter and DiRienzo assume that for an isolated quantum system, ($d\mathbf{p}$ /dt) = 0. This condition requires that:

$$- \sum_{i=1}^{N} \nabla_i Q = \sum_{i=1}^{N} \mathbf{F}_i = 0 \qquad (4.1)$$

where $\mathbf{F}_i$ is the quantum force on the i-th particle. If we let N = 2, then $\mathbf{F}_1 = - \mathbf{F}_2$. This suggests to Parmenter and DiRienzo that the source of the quantum force on one particle is just the other particle. More generally, they conclude that the 'quantum force' on a given particle is a force of constraint which originates from all the other particles in the N-particle system (Parmenter and DiRienzo 2004, 7). The exact nature of the 'quantum force' is *unspecified* with the quantum potential Q acting as an intermediary. The quantum potential is also unspecified by them, except to hypothesise that Q results from a non-holonomic constraint on the system (Parmenter & DiRienzo 2004, 4 and 7), i.e. a constraint that cannot be expressed as an equation in the form $f(x_1, x_2, x_3, ..., x_N, t) = 0$, which relates the coordinates of the particles and time (Goldstein 1980, 12).

Parmenter and DiRienzo use the rather curious argument that *if* Q is removed from the Quantum Hamilton-Jacobi equation, *then* the time-dependent Schrödinger equation would be modified by the addition of the term:

$$\frac{\hbar^2}{2m}\left(\frac{\nabla^2|\Psi|}{|\Psi|}\right)\Psi = - Q\Psi$$

which would make the Schrödinger equation non-linear. There would, of course, be many consequences (both mathematical and empirical) that would follow from such a non-linear equation. One consequence would be a different Hamiltonian. This Hamiltonian suggests to Parmenter and DiRienzo that there is a possibility of deterministic chaos in the time development of different wavefunctions (Parmenter and DiRienzo 2004, 9). They then hypothesise that the quantum potential Q results from a constraint which prevents such deterministic chaos for wavefunctions, presumably because when Q is present there is no possibility of the kind of deterministic chaos envisaged. The argument for the existence of this constraint is invalid for it is a not legitimate

approach to simply 'pluck-out' a term in an equation of physics without detailed and careful justification. This cannot be the case with the quantum potential for Q is not just another potential function that can be added to the classical Hamilton-Jacobi equation (Kyprianidis 1988a, 412). Nor can Q be adjusted to zero as can some types of externally imposed, classical potentials. Further, the notion of a mathematical constraint is that it restricts the possible solutions of the equation governing the phenomenon under study, not that it can add or subtract terms from the governing equation. In any case, the removal of the quantum potential from the Quantum Hamilton-Jacobi equation formally changes this to its classical counterpart, i.e. the subject matter is no longer quantum mechanics!

The overall solution of Parmenter and DiRienzo must also be seen to be unsuccessful for it cannot explain the motion of a single quantum particle such as occurs in the Double Slit experiment when only one particle is present between the slits and the screen at any particular time. The defect in their argument occurs in the assumption of Equation (4.1). In the one-particle Double Slit arrangement (N = 1), $\mathbf{F} = -\nabla Q = (d\mathbf{p}/dt) = 0$, from which it follows that the momentum $\mathbf{p}$ = constant. In other words, the sole quantum particle will execute rectilinear motion and consequently, the familiar two slit diffraction pattern cannot be formed over time. This example clearly shows that quantum force cannot be due to the particles of a quantum system (Riggs 2008, 30).

The 'non-interactive' approach to the wave field of Parmenter and DiRienzo, like the Active Information Hypothesis, needs to be abandoned in favour of a more promising line of development. This will be presented in Chapter 5 in terms of the physical characteristics of the wave field.

4.4 The Physical Nature of Potential Energy

Before proceeding to an alternative account of the motion of quantum particles, it will be necessary to deal with a 'thorny' issue of the physical underpinnings of potential energy, as this will be pivotal to subsequent discussion. In undergraduate studies of

(classical and quantum) mechanical systems, the energy of a system is divided into kinetic and potential quantities. Potential energy is introduced to account for the ability of a physical entity to perform work on its surroundings (where work has its usual definition of the product of force and displacement) and for the purposes of energy conservation. The formal potential energy term is (explicitly or implicitly) defined as the potential energy of a particle or object. In electrostatics for example, a (point) particle with an electric charge q_1 at a distance r from another particle with charge q_2 is defined to have a potential energy V given by:

$$V = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r} \qquad (4.2)$$

where ε is the electric permittivity constant. Explicit statements that particles possess potential energy may be found in many introductory texts.[1] Such definitions of potential energy are drummed into students to the extent that it is dogmatically accepted to a large degree that potential energy is a particle characteristic. This is despite the existence of strong counter-examples (such as those provided below). Attributing potential energy to a particle is only a convenient description which is a statement of convention and not a matter of physical reality.

Defining potential energy as a particle characteristic is, in a strict sense, fundamentally incorrect. The familiar potential energy term is a potential energy function that represents an amount of field energy that is available to a particle situated within the field. In other words, potential energy is energy properly associated with fields, not particles. Hans Freistadt stressed this very point in an article written in the 1950s:

> V [potential energy] is merely a shorthand way of writing … an energy which really resides in the field (Freistadt 1957, 17).

A field, for example, may be present in a spatial region which is totally devoid of any particles because fields can propagate enormous distances into otherwise empty regions of space, regions which might be many cubic light-years in size. Yet, despite the

[1] Textbooks with explicit examples include: Burghes & Downs 1975, 130; Hannah and Hillier 1981, 179; Nolan 1993, 189; Cutnell and Johnson 1998, 165 and 558.

absence of particles, such a spatial region possesses a (potential) energy density due to the presence of a field (Jackson 1975, 46). The fact that it is physical fields that are repositories of potential energy (and not particles) was emphasised by D.W. Theobald in his classic treatise, *The Concept of Energy*:

> … the field is characterised by the presence of energy … A field is nothing more than a spatial distribution of energy which varies with time. [The concept of] Energy has thus been freed from its dependence upon physical vehicles such as particles … (Theobald 1966, 98).

Potential energy being a field attribute is rarely stated in the physics literature and indeed, assigning potential energy to particles is a standard and almost totally unquestioned practice. This is because, in the majority of physical contexts (such as particle mechanics), it makes no difference to the final result by assigning potential energy to a particle. Further, regarding potential energy as a particle property is easier to use and simpler for students to assimilate for this treatment acts as a kind of 'shortcut' to the actual location of potential energy in a field. This 'shortcut', however, *is not possible in all physical situations,* in particular those involving non-linear interactions (Freistadt 1957, 17).

A notable exception to labelling potential energy as a particle attribute appears in the works of the respected physicist Wolfgang Rindler. He writes:

> In classical mechanics, a particle moving in an electromagnetic (or gravitational) field is often said to possess potential energy, so that the sum of its kinetic and potential energies remains constant. This is a useful 'book-keeping' device, but energy conservation can also be satisfied by debiting the *field* with an energy loss equal to the kinetic energy gained by the particle (Rindler 1977, 83, italics in original).

The 'book-keeping device' referred to by Rindler is sometimes called the 'accountant's model' and is the view that energy is a mathematical attribute that always tallies if calculated correctly (Sefton 2002, 2). The message that energy conservation is satisfied by a change in field energy is repeated throughout Rindler's writings and he has emphasised this in a number of texts (although this message seems to have 'fallen on deaf ears'). In another textbook, Rindler states:

> … [particle] potential energy, which is really nothing but a useful 'book-keeping' device. But physically it is more satisfactory to credit the field *itself* with whatever momentum or energy is required to 'balance the books' (Rindler 1982, 132, italics in original).

Surely then, this is the critical point – a physically satisfactory account of the nature of potential energy in both linear and non-linear interactions requires that fields, not particles, possess potential energy. We can again quote Rindler:

> … the 'real' location of any part of the [field] energy is *no longer a mere convention* … (Rindler 2006, 113, italics added).

In order to illustrate this, consider the following two examples. The first example concerns the everyday supply of household electricity. In most industrialised countries, electricity is supplied by power generating stations through heavy duty metallic cables using alternating current, i.e. current that changes direction over a short time interval (typically with a frequency of 50–60 Hertz). The regular change in the polarity of the electricity requires the electrons in the cables to oscillate back and forth about equilibrium positions. Consequently, there is no net electron flow along the cables from an electricity power station to the end-users of the supplied electricity. The electrons cannot, therefore, transport the electrical energy since they do not travel from source to user. Instead, the energy is transferred as potential energy in the generated electric field. It is the field and not the particles that possess potential energy. Ian Sefton is physics educator who has also strongly argued against the mistaken view that electrons possess potential energy. He writes:

> … [There is a] basic misconception that is often implied and sometimes explicitly stated in texts. The mistake is to speak of the electric potential energy (PE) of an electron as though the electron owns all the PE – it doesn't. This error seems to be a reflection of a similar sloppy way of talking about gravitational PE. When you lift a brick do you increase its PE? No, … PE is not stored in either Earth or the brick … electrons don't have PE of their own … and in a circuit they generally don't go anywhere much but energy is transferred very rapidly … (Sefton 2002, 2).

A second example may bring this into sharper focus. Consider an electrically charged particle placed in an external electric field. Such an external field may be produced by applying an electrical potential difference to two (usually parallel) metal plates. Also assume that this is done in inter-galactic space so that the effects of gravity and air resistance will be totally negligible. If the charged particle is released at rest between the plates before they become charged, the particle remains at rest. However, if the particle is released at rest between the plates when they are charged, the

particle will immediately accelerate. (We can also keep the acceleration's magnitude low by ensuring that the charge is small, thereby making negligible any loss of energy from the particle by electromagnetic radiation.) The electric field between the charged plates imparts energy to the particle as it had no kinetic energy initially. This energy is gained at the expense of some (but not all) of the potential energy stored in the field between the charged plates, i.e. by a small fraction of the potential energy contained within the external electric field.

Moreover, if we were to 'shoot' the charged particle in a direction towards the plate of similar charge to itself, the particle would decelerate and then come to a (momentary) stop. The particle's kinetic energy would then be instantaneously zero. If at the instant when the particle stops, we arrange for the electric field between the plates to be zero, then the value of the potential energy would also be zero. If potential energy is taken to be a particle property, then *all* the particle's energy (i.e. kinetic and potential) would have just disappeared from existence! This situation is physically inexplicable. The loss of potential energy when the external field is turned off can only be accounted for in a manner that is physically reasonable if potential energy is contained in the field.

It is also important to distinguish between the potential energy available to a particle situated in a field and the total energy stored in the field. In the current example, if the plates are the same size and shape, are parallel, and the particle is a perpendicular distance y from the plate of opposite charge, then the former energy is given by (qEy), where q is the particle's electric charge and E is the strength of the electric field. The total energy stored in the field is given by ($\frac{1}{2}\varepsilon AdE^2$) where A is the surface area of one plate, and d is the separation of the plates (Johnk 1975, 210). The amount of potential energy available to the particle depends on a number of factors such as the particle's location in the field and how the field's amplitude varies. In this respect, the physicist and mathematician Hermann Weyl wrote:

> Not only the field as a whole, but every portion of the field has a definite amount of potential energy … (Weyl 1952, 70).

The proper characterisation of potential energy as field energy will permit the solution of a significant conceptual problem of the Causal Theory, viz. energy conservation in quantum systems. This will be done in Chapter 5.

4.5 The Existence and Characteristics of the Wave Field

If quantum entities consist of both particles and waves, then it should not only be not surprising that atomic and elementary particle experiments show particle and wave aspects, it should be expected. The objective existence of the wave field is an essential characteristic of the Causal Theory, as John Bell has commented:

> *No one can understand this* [Causal] *theory until he is willing to think of* **...** [the wave field] *as a real objective field rather than just a 'probability amplitude'* (Bell 1987b, 128, italics in original).

In this section will be discussed some of the reasons for accepting the existence of the wave field and for holding to the quantum potential approach. In doing so, we shall review three empirically significant phenomena.

There are 'minimalist' accounts of the Causal Theory that postulate only the Schrödinger and guidance equations, the Quantum Equilibrium Condition and the existence of quantum particles with definite positions (e.g. Dürr et al. 1996). Mathematically speaking, it is the case that these accounts will produce all the predictions made by Orthodox Quantum Theory in addition to the trajectories of quantum particles. However, they will fall short of a full causal explanation of quantum behaviour because such accounts are primarily *kinematic* descriptions. Just as in classical mechanics a complete explanation of physical phenomena requires the explication of the dynamics of the system under study, so too in the microscopic realm, an account of the dynamics of a quantum system is required. We shall see that such an explanation is provided by the existence and role of the quantum potential. In particular, the quantum potential is essential to account for the conservation of energy (Holland 1993, 78). The requirement to conserve energy is, of course, one reason why the concept of potential energy was

originally introduced into physics. Indeed, solutions to problems in theoretical chemistry and solid state physics within the context of the Causal Theory *require* application of the quantum potential approach (e.g. Grubin et al. 1994; Shifren et al. 2001; Garashchuk and Rassolov 2003a; 2004b; 2007; Rassolov and Garashchuk 2005). Use of the quantum potential is essential to these solutions.

In regard to the question of the energy content of the wave field, it should be recognised that to claim something to be real and yet possess no energy at all would go against long established physical results. This notion is embodied in the basic concepts of physics (for example by the laws of Thermodynamics) and will be stated as the following general principle:

- *Principle of Energy Content*

  Every physically real entity contains some finite quantity of energy.

In accordance with this principle, the wave field will always possess some amount of energy, although it may be exceedingly small at times in comparison to the kinetic energy of the accompanying quantum particle.

Consider now the three examples of empirically significant phenomena. Two of these (the Double Slit Experiment and the Aharonov-Bohm Effect) are exemplars wherein the assumption of an objectively existing wave field provides coherent, realistic and causal explanations of the phenomena. The third, the manipulation of matter waves, is an example of physical processes made possible only by recent developments in laser technology, the *causal* explanation of which requires accepting wave fields (matter waves) as objectively real.

4.5.1 The Double Slit Experiment

The classic double slit experiment with light is credited to Thomas Young in 1803. The experiment was first performed with electrons by Claus Jönsson in 1959 (Jönsson 1961). The behaviour of a single quantum particle passing through a double slit arrangement can be found using the quantum potential approach. At a large distance from the slits, the value of the wave field's amplitude R is taken to

be constant. The quantum potential is assumed to have a zero value since $\nabla^2 R = 0$ for constant R. The particle's trajectory at some distance from the slits will then be as predicted by classical mechanics.

It is well known that the diffraction pattern obtained in the double slit experiment is different when two slits are open from when only one is open. The Causal Theory gives a explanation for this behaviour. When both slits are open, a quantum particle will pass through one, and only one, of the slits (or will impact on the barrier in which the slits are cut). The wave field, however, passes through *both* slits and the emergent waves interfere with each other. Once a quantum particle passes through a slit, it experiences rapidly varying values of the quantum potential, as the value of R changes with position due to self-interference of the wave field (Bohm 1952a, 174). Bohm and Hiley provided a succinct description:

> … A particle is incident on this system, along with its quantum wave. While the particle can only go through one slit or the other, the wave goes through both. On the outgoing side of the slit system, the waves interfere to produce a complex quantum potential which does not in general fall off with distance from the slits (Bohm & Hiley 1987, 326).

Particles that pass through the slits will have their trajectories altered from a straight line path by the action of the quantum mechanical force in a manner such that the familiar two slit interference pattern emerges if sufficient numbers of particles are allowed to pass through the slits, as shown in Fig. 4.1 (below).

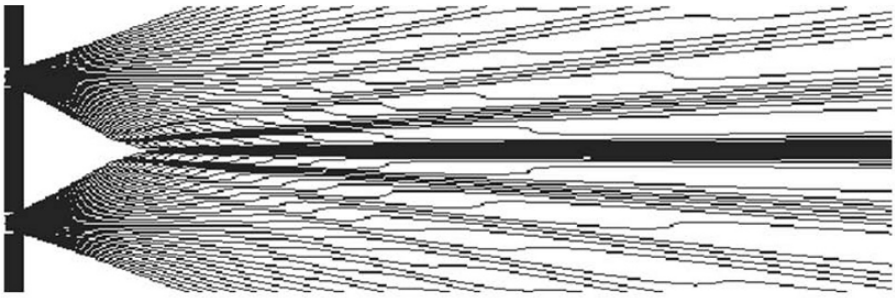

Fig. 4.1 An ensemble of trajectories for the double-slit experiment. (Used with permission of Società Italiana di Fisica and the author. Adapted from Philippidis et al. 1979, 23)

Since the probability density is $P(\mathbf{x}) = |\Psi|^2$, it may be concluded that the particle cannot be found at any point where the wavefunction vanishes. What's more, the particle trajectories do not cross the line of symmetry between the slits, so that particles that are incident on the left (right) side of the screen passed through the left (right) slit. Further theoretical calculations into the Double Slit experiment for electrons have also been made by Holland and Philippidis whose research has made corrections to previous plots by taking into account the effects of an electron's intrinsic angular momentum (Holland and Philippidis 2003).

The calculated particle trajectories in the double slit experiment constitute an example of what Bohr, Heisenberg, and Richard Feynman (to name a few) explicitly declared to be impossible. Their attitude is summarised in Feynman's well known textbook, *The Feynman Lectures on Physics*:

> We choose to examine a phenomenon [the double-slit experiment] which is impossible, *absolutely* impossible, to explain in any classical way, and which has in it the heart of quantum mechanics (Feynman et al. 1963, Vol. 3, Chapter 37, italics in original).

It is clear from the context of Feynman's book that by 'classical way', he meant an explanation in terms of the particles having well-defined trajectories through space from the slits to the screen. Another well known physics textbook by leading Russian physicists, Lev Landau and Evgeny Lifshitz, resonates the finality of Feynman's sentiment:

> It is clear that this result [electron diffraction] can in no way be reconciled with the idea that electrons move in paths. … In quantum mechanics there is no such concept as the path of a particle (Landau and Lifshitz 1958, 2).

The notion that the trajectories of quantum particles is a meaningless concept, as demanded by Orthodox Quantum Theory, is clearly flawed. This is evident from the many and various illustrations in the literature and the computational techniques developed principally by physical chemists (e.g. Bittner 2000; Garashchuk and Rassolov 2002, 2003a, b, 2004c; Gindensperger et al. 2000; Lopreore & Wyatt 1999; Wyatt 1999; Nerukh and Frederick 2000; Wyatt 2005; Rassolov et al. 2006). The usefulness and empirical insight gained from quantum trajectories is summarised in the conclusion of Wyatt's book devoted to this very topic. He writes:

> It has only been since 1999 that quantum trajectories have been used as a computational technique for solving the time-dependent Schrödinger equation. Since then a number of promising methods have been introduced … quantum trajectories have been used to formulate new approaches to old problems … From analyses of quantum dynamics, new insights have been revealed about fundamental dynamical processes … (Wyatt 2005, 387–388).

One practical example of quantum trajectories is the modelling of electron transport in nano-scale devices. This method employs quantum trajectories for calculating the behaviour of many-particle systems which is useful for designing electronic devices, as described by Oriols:

> Electron transport in mesoscopic systems is analyzed in terms of quantum (Bohm) trajectories associated to wave-function solutions of a many-particle (effective-mass) Schrödinger equation. … The approach opens a new path for studying electron transport and quantum noise in nanoscale systems. … (Oriols 2007b, 239).

In respect to trajectories of quantum particles as displayed by the Causal Theory, the research group led by B.-G. Englert has alleged that the double slit trajectories are 'surrealist', i.e. not observable and therefore physically meaningless (Englert et al. 1992). This objection to the Causal Theory is made by ignoring the fact that *the theory itself explains why* quantum trajectories are, in practice, unobservable (Nikolić 2006, 6). The 'surrealist' claim has been explicitly addressed by Dürr's group (Dürr, Fusseder and Goldstein 1993) and independently by Hiley and his associates (Hiley et al. 2000). The latter response is very detailed and concludes that the trajectories provide a deep insight into quantum processes – a conclusion similar to that made independently by physical chemists! The 'surrealist' attitude is just a more recent expression of the Orthodox Quantum Theory requirement that the wavefunction (or state vector) gives a complete description of a quantum system.

4.5.2 The Aharonov-Bohm Effect

In 1959, Yakir Aharonov and David Bohm calculated that there would be a shift in the fringes of a double slit arrangement with electrons when an energised cylindrical solenoid is placed in the geometric shadow of the two electron beams emanating from the slits (Aharonov and Bohm 1959). This is known as the Aharonov-Bohm

Effect and the calculated trajectories for the electrons are shown in Fig. 4.2.

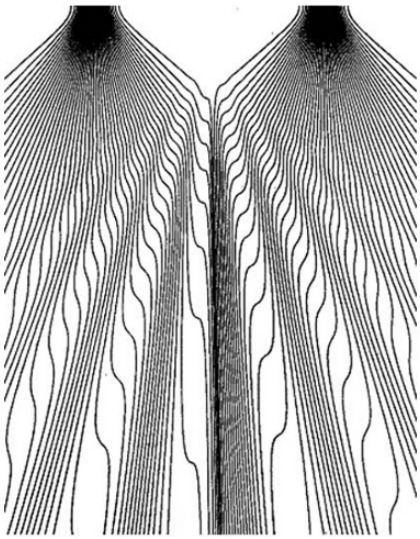

Fig. 4.2 An ensemble of two slit trajectories showing the Aharonov-Bohm Effect. (Used with permission of Società Italiana di Fisica and the author. Adapted from Philippidis et al. 1982, 84)

The Aharonov-Bohm Effect has been experimentally confirmed (Tonomura et al. 1986, 792). The electromagnetic vector potential **A** related to a magnetic field may be defined (up to a gauge transformation) by: $\mathbf{B} = (\nabla \times \mathbf{A})$, where **B** is the (classical) magnetic induction. The Schrödinger equation including the electromagnetic vector potential takes the form:

$$i\hbar \frac{\partial \Psi}{\partial t} = \frac{1}{2m} \left(-i\hbar \nabla - \frac{e}{c}\mathbf{A}\right)^2 \Psi \qquad (4.3)$$

where *e* is the electronic charge and c is the speed of light in vacuum (Bohm & Hiley 1993, 51). The interesting feature of the Aharonov-Bohm Effect is that the electrons only pass through space where there is no magnetic field strength (i.e. regions where $\mathbf{B} = 0$). The solenoid produces a closed loop of magnetic flux which is zero outside the solenoid. The vector potential, however, cannot be zero in these regions or the enclosed magnetic flux loop would also be zero. The presence of the vector potential **A** in Equation (4.3) induces a phase shift in the wavefunctions of the electrons emerging

from both slits from what they would be with **A** = 0. This phase shift alters the interference pattern that results when the two electron beams combine. The shift of the interference fringes is evident if one compares the pattern of trajectories as shown in Fig. 4.1 (no phase shift) with the pattern shown in Fig. 4.2.

The Causal Theory provides an understanding of why the electron trajectories change (and therefore the interference pattern) in terms of the quantum potential Q. The quantum mechanical force, i.e. $(d\mathbf{p}/dt) = -\nabla Q$, is present even if the magnetic field and the (classical) Lorentz Force (i.e. $\mathbf{F} = e\mathbf{v} \times \mathbf{B}/c$, where **v** is the electron's velocity) are zero in the regions through which the electrons pass. The usual double slit pattern is explained by the variation of the quantum potential due to self-interference of the wave field (as outlined above). The additional phase shift changes the values of the quantum potential from those that occur when the vector potential is absent. The quantum mechanical force is also changed resulting in different electron trajectories and an altered interference pattern. Bohm and Hiley described these circumstances as follows:

> We see then that in general … there is a quantum force which is present even when the magnetic field is zero. … It is clear that Q [the quantum potential] will be large only where … the two beams overlap … The phase shift will then alter the quantum potential in a significant way and this will explain the origin in the shift of the interference pattern (Bohm and Hiley 1993, 52).

The Aharonov-Bohm Effect is therefore explained if the reality of the electromagnetic vector potential **A** is accepted and its effect on electron trajectories through the modification of the quantum potential (Holland 1993, 195).

4.5.3 Matter Wave Manipulation

There is mounting evidence for the existence of wave fields from the new experimental area of Atom Optics (also called Matter Wave Optics), where the term 'matter waves' is used in preference to 'wave fields'. Applications of laser technology have now made possible the control of atoms consistent with the manipulation of their matter waves. This is done using 'laser cooling' and other cooling methods which reduce the temperature of a gas to a fraction

of a degree above Absolute Zero. In such a low temperature state, the speed of the atoms is so slow that the deBroglie wavelength of an atom is approximately equal to the spacing between individual atoms. The atoms then have a dominant wave behaviour that allows manipulation by laboratory atom-optical devices (Helmerson 1999, 587; Balykin et al. 2005, 45). Although the matter wave (i.e. wave field) is not directly observable, the fact that significant quantities of matter can be diffracted, focussed, reflected, etc., using essentially optical devices is clear evidence that wave fields are physically real.

Experiments utilising 'matter wave amplification' offer further evidence for the existence of wave fields (Kozuma et al. 1999; Inouye et al. 1999; Schneble et al. 2004). The term 'matter wave amplification' refers to the production of an output of atoms with particular properties from a holding reservoir of atoms (an atom trap) using a process similar to the stimulated emission of light in a laser. The atoms of some gases, for example, can be put into the same, low energy quantum state by the process known as Bose-Einstein Condensation (Ho-Kim et al. 2004, 123). In this state, the atoms act together in a holistic fashion. An initial matter wave can be 'amplified' by using the Bose-Einstein condensate (BEC) atoms in the reservoir as a gain medium. This produces atoms with the desired properties in large numbers, as shown in Fig. 4.3.

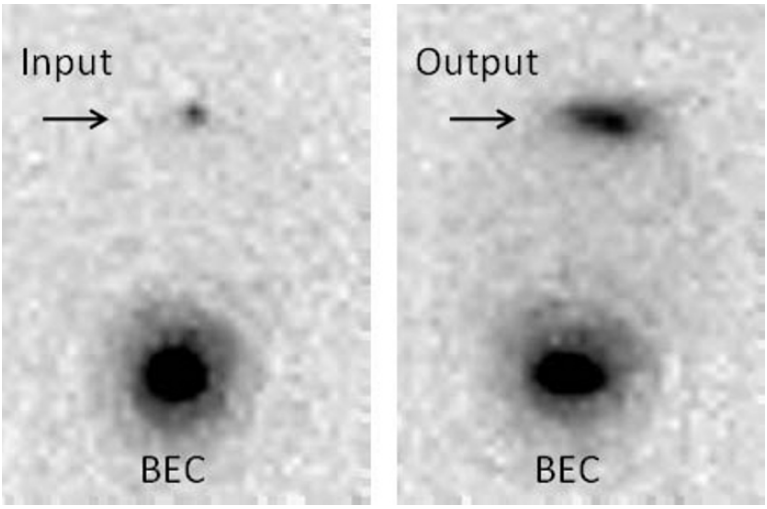

Fig. 4.3 Matter Wave Amplification. Left image – input without amplification; Right image (20 milliseconds later) – output contains far more atoms than input. (Used with permission of Professor W. Ketterle. Adapted from Ketterle 2001, 46)

The particular properties that the output atoms acquire is that they have the same momentum and phase relations as the atoms used as input. This process has been described by one of the experimental groups conducting research into 'matter wave amplification' as follows:

> … we report the observation of phase-coherent amplification of atomic matter waves. The active medium is a Bose-Einstein condensate … An atomic wave packet is split off the condensate by diffraction from an optical standing wave, and then amplified. We verified the phase coherence of the amplifier by observing interference of the output wave with a reference wave packet (Inouye et al. 1999, 641).

In these experiments, the coherence of the matter wave produced in the output was established using interferometers. If a matter wave can be subject to and utilised in such amplification processes, then it logically follows that the matter wave must exist in order to act and be acted upon. This requires accepting the matter wave (i.e. wave field) as physically real.

The above three examples (Double Slit, Aharonov-Bohm Effect and Matter Wave Manipulation) lend strong support to the proposition that the wave field has an objective existence and, most importantly, is *causally efficacious* in bringing about observed quantum phenomena. This is especially the case with the mounting evidence from Atom Optics as indicated by the number of articles on matter waves appearing in both general physics journals and specialist optics journals. Even mainstream physics journals are taking the subject of matter waves very seriously, so much so that some explicitly dedicate whole sections of their journals to matter waves.[2] One article on matter waves described the situation as follows:

> Due to the rapid developments and the explosion in activities and publications following the first realization of Bose–Einstein condensation in dilute atomic gases a comprehensive coverage of the entire field is beyond the scope of a single review article. … the ability to produce macroscopically occupied matter wavefunctions via Bose–Einstein condensation is the basis for many new insights into the physics of coherent matter waves (Bongs and Sengstock 2004, 907).

The objectivity of wave fields leads to the realisation that they will have characteristics in common with classical fields and waves. The wave field will, of course, also have non-classical

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[2] See: *Physical Review A* homepage <http://pra.aps.org/>.

features. The wave field and its quantum particle are *physically* inseparable aspects of a single quantum entity. Bohm himself, stressed that quantum theory needed *some* non-mechanistic descriptions and emphasised the importance of a holistic view (Bohm 1951, 166–167). These stipulations, however, do not prevent an *in-principle* analysis of the characteristics and causal role exhibited by wave fields.

The quantum potential performs similar roles to those of classical potentials. This is evident in situations where a quantum particle is subject to both classical and quantum potentials. Given the explanation of potential energy in Section 4.4, a (partial) answer to the question of what constitutes the quantum potential may be 'fleshed out' in terms of Q being the potential energy function of the wave field (see Section 5.5). The quantum potential has some features in common with classical potentials for this reason, such as the relationship expressed by Equation (3.4): $(d\mathbf{p}/dt) = -\nabla (V + Q)$, which shows that classical and quantum potentials are on an 'equal footing' in regard to affecting the particle's motion (Holland 1993, 74). However, Q is not completely equivalent to an external classical potential and could not be so for the following reasons. Classical potentials are due to fields which do not, in general, travel along with the particle, i.e. a quantum particle is embedded in the wave and together they constitute a single quantum entity. Nor is a classical field that is externally imposed on a particle intrinsic to the physical system so created in the way that the wave field is intrinsic to a quantum system. The quantum potential is also not a pre-assigned function of coordinates as are classical potentials (Holland 1993, 74). Holland uses the term 'internal potential' to distinguish Q from potentials that are externally imposed (Holland 1993, 63).

4.6 Non-Locality and the Wave Field

The results of experiments on Bell-type arrangements have forced the conceptual issue of quantum non-locality into focus. It now seems clear that any physically adequate quantum theory must violate the Principle of Locality (Maudlin 1996, 286; Lange 2002, 279). Yet the idea of non-locality still sits uncomfortably with

most of us. Bohm and Hiley described the sense of non-locality as follows:

> For several centuries, there has been a strong feeling that non-local theories are not acceptable in physics. … One can understand this feeling, but if one reflects deeply and seriously on this subject one can see nothing basically irrational about such an idea. Rather it seems to be most reasonable to keep an open mind on the subject and therefore allow oneself to explore this possibility. If the price of avoiding non-locality is to make an intuitive explanation impossible, one has to ask whether the cost is not too great (Bohm and Hiley 1993, 57).

It is the case that we really know very little about non-locality and only the advent of quantum mechanics has demanded its explicit recognition in modern physics. Indeed, the Causal Theory was initially criticised for being non-local until John Bell showed that this was a characteristic of any empirically satisfactory quantum theory. The kind of non-locality found in quantum mechanics has been described as 'benign' since it cannot be used for any kind of signalling (Cushing 1994a, 230; Bohm and Hiley 1993, 158) i.e. does not allow the transfer of information between locations at a spacelike separation and therefore does not explicitly violate Special Relativity (see Section 5.2). It is fortunate (as noted in Section 3.6) that parts of the universe do not constantly exhibit non-local behaviours so that we can continue to use established scientific methods to investigate the physical world.

However (and perhaps somewhat surprisingly), non-locality does not necessitate acausality, as noted by Aharonov and Rohrlich:

> … we find that *quantum nonlocality obeys causality* (Aharonov & Rohrlich 2005, 87, italics in original).

In the Causal Theory, non-local connections are manifest in that the coordinates of all particles in a quantum system are needed to specify the state of the system. Measurement on a particular particle affects all others in the system through the many-particle quantum potential, which depends on the form of the wave field. In non-relativistic theory, the wave field can be altered instantaneously, e.g. by an act of measurement.

Many questions come to mind in relation to the existence of non-locality in the Causal Theory. A few of the more obvious questions are:

- What are the features of non-local connections?
- Does the wave field really change instantaneously?
- Will there be an effect on the wave field's energy content?
- What is the 'means' by which the non-local connection is actualised?

Unfortunately, these questions are not properly answerable within a non-relativistic context for relativity imposes limitations not present in non-relativistic theory. It should be emphasised that non-locality does not just appear in non-relativistic quantum mechanics but is also present in relativistic versions. However, it is not possible to send a *signal* between events with a spacelike separation in relativistic quantum mechanics (Bohm and Hiley 1993, 285–286; Holland 1993, 523–524; Cushing 1994b, 191).

However, we can offer some possible responses to the last of the above questions:

(1) The connection is done through ordinary (simply connected) three-dimensional space and is mediated by particles or fields that propagate at superluminal speeds.
(2) The connection propagates backwards in time.
(3) Physical space has more than three spatial dimensions.
(4) The causal connection travels at sub-light speed but physical space is not simply connected (Shimony in Boyd et al. 1991, 525).

These responses are not meant to be exhaustive. Since Option (2) violates the Principle of Causality, it will not be entertained here. (The reader interested in a 'backwards' causation explanation of quantum phenomena will find the subject discussed in Price 1996.) Option (3) would entail a theory that postulates the existence of multi-dimensional physical space. If such dimensions exist they are completely unnoticeable at macroscopic scales and experimental evidence for them is totally lacking (as already noted in Section 3.6).

Providing a limited answer to the first of the above questions might assist in deciding which of the remaining Options (1) or (4) is better. Based on the results of tests of Bell-type inequalities, it would seem that non-local connections have at least the features of (Maudlin 2002, 22–23; Lange 2002, 281; Ghirardi 2005, 263):

(a) not decreasing with distance;

(b) cannot be shielded against; and

(c) are highly selective in what they affect.

The features (a–c) are consistent with Option (4) for the following reasons:

— The relativistic ban on the motion of real rest mass particles and the transmission of information at speeds greater than the speed of light in vacuum is upheld;

— The distance between different spatial locations in a multi-connected physical space will depend on the path taken;

— A wormhole (i.e. a passage of short distance in a physically multi-connected space) can directly link the events in question which would account for only the relevant events being affected and why there is no shielding of the effects.

Option (4) also provides an answer to the fourth question above, i.e. the 'means' by which non-locality is actualised. This is not to suggest that Option (4) is the only answer or indeed, the most plausible solution. Rather, Option (4) neatly explains how non-local connections might be actualised and why they have the features listed. The discussion presented here only 'scratches the surface' of the topic (see also Holland 1993, Chapter 11).

4.7 Can the Causal Theory be made Relativistic?

We might also put to rest in this chapter, the claim that the Causal Theory cannot be made relativistic (alleged flaw ⑦ in Section 1.3). This claim is the final appeal for those opposed to the Causal Theory. It is an objection of the 'if all else fails' type, as Bell has commented:

> When the cogency of Bohm's reasoning is admitted, a final protest is often this: it is all nonrelativistic. This is to ignore that Bohm himself, … applied his scheme to the electromagnetic field (Bell 1987b, 173).

This issue falls outside the scope of the book *per se*. However, the objection is plainly false, not only because of Bohm's preliminary 1952 application to the electromagnetic field (as Bell mentioned) but

also because a proper relativistic version of the Causal Theory was later developed by Bohm and his co-workers (Bohm, Hiley and Kaloyerou 1987; Bohm and Hiley 1993, Chapter 12). This relativistic version of the Causal Theory has since been extended by several other theoreticians (Holland 1993, Chapter 12; Dewdney 1994; Holland and Philippidis 2003; Dürr et al. 2004; Nikolić 2005; Carroll 2006). What's more, the statistical predictions of the Causal version of the quantised electromagnetic field are the same as found from orthodox quantum electrodynamics (Cushing 1994b, 191).

It has already been acknowledged that the non-relativistic Causal Theory has its limitations. However, some new avenues for further research that require relativistic Causal Theory are identified in Chapter 5.

Chapter 5

Energy-Momentum Transfer and the Quantum Potential

Quantum mechanics is very impressive. But an inner voice tells me that it is not yet the real thing. The theory produces a good deal but hardly brings us closer to the secret of the Old One. I am at all events convinced that *He* does not play dice. Waves in 3*n*-dimensional space whose velocity is regulated by potential energy …

— Albert Einstein

Abstract This chapter deals with a number of important conceptual issues and some theoretical problems of the Causal Theory. Contrary to claims in the literature, it is shown that energy conservation does hold in the Causal Theory. The function and role of the quantum potential is described in some detail with particular reference to the case of a Gaussian wave field. A novel explanation is developed of how quantum particles can 'tunnel' out of a finite potential well. Arguments are made in favour of the absence of a classical reaction for quantum entities. An account is outlined of what occurs physically when a measurement is made on a quantum system. Reasons are advanced for granting the same ontological status upon the quantum mechanical force as is given to the accepted fundamental forces of nature. A possible experimental test of the Causal Theory is suggested and some empirical consequences considered.

5.1 Energy Conservation in the Causal Theory?

It has been stated in the literature on the Causal Theory that energy is not conserved for a quantum system as a whole (i.e. wave field and particle together). The non-conservation of energy is claimed because, although the wave acts on the particle, the particle does not appear to react back on the wave (Holland 1993, 120). Both the

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apparent energy non-conservation and the absence of a classical reaction constitute conceptual problems for the Causal Theory. In the former case, this is because of the conflict with the Principle of the Conservation of Total Energy. In the latter case, this is because of the conflict with Newton's Third Law (also known as the Principle of Reaction). The role played by the quantum potential in the conservation of energy will be examined by building on the account of the nature of potential energy given in Chapter 4. This will allow the solution of some conceptual and theoretical problems in connection with energy conservation, energy transfer, and action-reaction in quantum systems.

5.2 Energy-Momentum Exchange in Single Particle States

The claim that the Causal Theory fails to conserve energy is a serious indictment of the theory as well as generating a conceptual problem. The key to understanding energy processes in the Causal Theory is accepting that the quantum potential is the potential energy function of the wave field. The wave field acts on its particle(s) via the quantum potential and, as such, it is the wave field that is the origin of the quantum mechanical force (i.e. the particle's rate of change of momentum with respect to time). Where then does the energy that is necessary for the wave field to act upon the particle come from? An isolated, one-particle quantum system provides the answer for, in such a system, the only possible repositories of energy are the wave field and its accompanying particle. (Isolated one-particle quantum systems are essentially achievable in practice not just in theory, see: Haroche and Raimond 2006, 17 and 20). In this case, the wave field may gain energy at the expense of the particle's kinetic energy or may lose energy to the particle, as may be seen from the following example of an isolated, classically-free (i.e. $V = 0$), one-particle system.

Consider a free quantum particle of mass *m* that is initially not subject to any force fields or barriers.[1] Its corresponding wave field is represented by a plane wave with a constant amplitude. Since $\nabla^2 R = 0$ for a wave of constant amplitude, the value of its quantum potential Q is zero. The particle moves with a constant velocity as both V and Q are zero, i.e. all the particle's energy is kinetic. If we were to trap the particle in a sealed enclosure that had no classical force fields within, we find that the wave field takes up a stationary wave pattern due to its new boundary conditions. This is the case with the simple example of an 'infinite' well (also known as a particle in a box). In three dimensions, imagine a cubical well of side length L with zero classical potential inside and 'infinite' potential outside. If we take one corner of the well as the origin of a rectangular Cartesian coordinate system then the stationary state wavefunction Ψ for a particle of mass *m* inside the well is given by:

$$\Psi = (2/L)^{3/2} \left| \sin(n_1 \pi x/L) \sin(n_2 \pi y/L) \sin(n_3 \pi z/L) \right| e^{-iE_n t/\hbar}$$
$$= R\, e^{iS/\hbar} \qquad (5.1)$$

The total energy is: $E = (n_1^2 + n_2^2 + n_3^2)(\pi^2 \hbar^2/2mL^2)$, where the n_i are positive integers. In Orthodox Quantum Theory, the particle must be in motion – it cannot be at rest as this would violate the Uncertainty Principle (Saxon 1968, 77) and since V = 0, the particle has kinetic energy only. In the Causal Theory the situation is different. Since $S = -Et$, $\nabla S = 0$, i.e. the particle's momentum is zero and therefore has zero kinetic energy! In keeping with the Principle of the Conservation of Total Energy, we should be asking: where has the particle's kinetic energy gone? The only possibility in this case for the location of the energy is in the wave field. If we calculate the quantum potential corresponding to Equation (5.1), we find:

$$Q = -(\hbar^2/2m)(\nabla^2 R)/R = (n_1^2 + n_2^2 + n_3^2)(\pi^2 \hbar^2/2mL^2) \qquad (5.2)$$

---

[1] The quantum system in this example would also need to have zero intrinsic angular momentum (i.e. would be spinless). See Chapter 6 for a discussion of the nature of spin.

This is the *same* magnitude as the particle's kinetic energy as given by Orthodox Quantum Theory.

Clearly then, the particle has come to rest (as $\nabla S = 0$) and all its energy is taken up by the quantum potential, as shown by Equation (5.2). Since the quantum potential is the potential energy function of the wave field, it is the wave field in which the energy is stored (Riggs 1999, 3072). Surprisingly, this explanation appears in Bohm's original papers. He wrote:

> … the ψ-field is able to bring the particle to rest and to transform the entire kinetic energy into potential energy of interaction with the ψ-field (Bohm 1952b, 184).

What's more, this energy will be returned to the particle if the wave field's stationary state is disturbed, e.g. if any side of the box is removed. This idea too, was suggested by Bohm when he wrote:

> … the kinetic energy of the particle will come from the ψ field, which is able to store up even macroscopic orders of energy when its wave-length is small (Bohm 1953c, 14).

Unfortunately, Bohm did not develop these ideas opting in later years for his Active Information Hypothesis.

It might be objected, however, that the above explanation cannot be so in the case of the 'infinite' well because the value of the quantum potential inside the well is independent of position, whereas the wave field varies from a maximum at its anti-nodes to zero at its nodes. This objection is ill-founded. In response, we repeat again that it is physical fields that are the repositories of potential energy. Second, we must ask what is it that the quantum potential represents in such a stationary state. In a stationary state with one particle, the quantum potential does not give the value of the potential energy at a particular spatial location but instead gives the value of the *total* field energy of the system (Riggs 1999, 3072).

Although the above example shows that the wave field may gain or lose energy to the quantum particle, it does not provide the exact mechanism for these energy transfers. However, this is also the case in classical physics, e.g. Newtonian Gravitation (which is also a non-local theory) does not give a mechanism for energy transfers between a massive particle and a (classical) gravitational field (Doughty 1990, 123). The explication of a mechanism for energy transfer in quantum systems will require a relativistic quantum field approach.

5.2.1 Aspects of Energy Transfer and Storage in a Quantum System

We shall now examine, in more detail, aspects of energy transfer and storage for a one-particle quantum system, as this will bring out the essential features under examination. The energy *available* to the particle (denoted E) is:

$$E = T + Q = -\frac{\partial S}{\partial t} = \frac{(\nabla S)^2}{2m} + Q$$

where T is the particle's kinetic energy. The time rate of change of E is given by:

$$\frac{dE}{dt} = \left(\frac{1}{2m}\right)\frac{d}{dt}(\nabla S)^2 + \frac{dQ}{dt} = (\nabla S)\cdot\left(\frac{-\nabla Q}{m}\right) + \frac{dQ}{dt} \quad (5.3)$$

Now

$$\frac{dQ}{dt} = \sum_{i=1}^{3}\frac{\partial Q}{\partial x^i}\frac{dx^i}{dt} + \frac{\partial Q}{\partial t} = (\nabla Q)\cdot\left(\frac{\nabla S}{m}\right) + \frac{\partial Q}{\partial t} \quad (5.4)$$

where $(\nabla S/m) = (d\mathbf{x}/dt)$. The term $[(\nabla Q)\cdot(\nabla S/m)]$ is equal to minus the rate of change of the particle's kinetic energy with respect to time, i.e. $(-dT/dt)$, as can be seen with reference to Equation (5.3). Substitution of Equation (5.4) into Equation (5.3) yields:

$$\frac{dE}{dt} = (\nabla S)\cdot\left(\frac{-\nabla Q}{m}\right) + (\nabla Q)\cdot\left(\frac{\nabla S}{m}\right) + \frac{\partial Q}{\partial t} = \frac{\partial Q}{\partial t} \quad (5.5)$$

When $(\partial Q/\partial t) = 0$, the energy available to the particle E is constant and changes in kinetic energy are exactly balanced by changes in the quantum potential. If $[(\nabla Q)\cdot(\nabla S/m)] > 0$, energy passes from particle to field. If $[(\nabla Q)\cdot(\nabla S/m)] < 0$, energy passes from field to particle. Thus any change in the particle's kinetic energy is a straight-forward energy conversion process, familiar from classical mechanics (Riggs 1999, 3070).

What if $(\partial Q/\partial t) \neq 0$? What does this term represent? Holland calls it 'the quantum power' (Holland 1993, 119). This label is misleading, for if any single term should be called the quantum power, it should be $[-(\nabla Q)\cdot(\nabla S/m)]$ since this term is the scalar product of the quantum mechanical force and the particle's instantaneous velocity (i.e. the formal definition of instantaneous power). Holland also does not suitably characterise the role of $(\partial Q/\partial t)$. The quantum potential itself gives the potential energy available to the quantum particle at its specific position in the field but Q does not, in general, coincide with the total field energy. Acceptance of this point requires a separate mathematical expression for total field energy (to be provided below).

In an isolated one-particle state, the wave field is the only repository of energy other than the particle itself. This being so, it follows that $(\partial Q/\partial t)$ gives the time rate of change of the quantum potential due to energy stored in the wave field other than at the particle's location. This indicates that the particle's energy will increase (decrease) with decreases (increases) in the amount of energy stored in the wave field as a whole. Therefore, we can account for the total energy of a classically-free single particle quantum system without recourse to external sources and without the need to conjecture about the existence of vacuum fluctuations and the like, as Bohm and Hiley did (Bohm and Hiley 1993, 38).

The above account resolves the external conceptual problem of conflict with the Principle of the Conservation of Total Energy. It also provides a direction for further development of the Causal Theory, i.e. it allows for the solution of related theoretical problems (see below).

5.2.2 Non-Violation of Special Relativity

It was noted in both Sections 3.6 and 4.6 that there is no violation of the Special Theory of Relativity by the Causal Theory as connections between quantum systems cannot be used for the purposes of signalling (i.e. the transmission of information) or the transfer of energy faster than the speed of light in vacuum. The

restriction on our knowledge of initial particle positions to $|\Psi|^2$ guarantees that the non-locality aspect of many-particle quantum systems cannot be used for signalling (Cushing 1994b, 58). Clearly though, satisfying Special Relativity is not merely a matter of ruling out superluminal transmission of information (signalling) or superluminal energy transfer. There is a larger question here, i.e. that *satisfying* Special Relativity requires there not be a preferred frame of reference and that of superluminal causal influences (as distinct from superluminal signalling).

In respect to the issue of a preferred frame, the effect on an individual quantum particle in a many-particle system depends on all particles in the system at a given instant (and therefore needs a preferred frame). We have already recognised that this cannot be rectified in a non-relativistic theory and so falls outside the scope of this book. In regard to the issue of superluminal causal influence, if it is meant that a measurement or other disturbance on one of a pair of spatially distant, entangled particles produces an effect on the other, then Special Relativity is not satisfied in this respect. However, this is not disputed in the literature (Cushing 1994b, 337). The issue of energy transfer is considered in Section 5.5. The further implications for Special Relativity of any kind of superluminal causal influence are serious but again, these are topics outside the current work.

5.3 Wave Field Energy and its Transfer

Now that it has been established that isolated quantum systems (as described by the Causal Theory) do conserve energy, we shall focus our attention in this section on theoretical problems relating to the energy content of the wave field and changes to it. The Hamiltonian density $\mathcal{H}$ of a physical system is the total density of mass-energy in an observer's frame of reference (Misner, et al. 1973, 137). In the non-relativistic context, the Hamiltonian density is the total energy density of the system. In the case of a classically-free quantum system, the Hamiltonian density is given by (Holland 1993, 115):

$$\mathcal{H} = \mathrm{R}^2(\nabla \mathrm{S})^2/2m + (\hbar^2/2m)(\nabla \mathrm{R})^2 \qquad (5.6)$$

Let

$$\mathrm{H} = \iiint_{-\infty}^{\infty} \mathcal{H}\, d^3\mathbf{x} \qquad (5.7)$$

Integration of the classically-free Hamiltonian density as given by Equation (5.6) shows that H is constant (Holland 1993, 116). The quantity H may now be interpreted as the total energy of the isolated, classically-free system (i.e. wave field and particle) and not just the energy of the wave field alone for the following reasons:

- particle and wave field are intrinsic parts of a single quantum system (the particle is not an 'add-on' to, or an enlargement of, the system);
- the quantum particle receives energy from the wave field (Holland 1993, 120);
- the quantum potential represents part of the wave field's energy;
- there are isolated, classically-free quantum systems where the field energy decreases (such as a Gaussian wave field described below);
- in any isolated system, total energy is a conserved quantity.

It then follows that the energy of the wave field is (H − T). Now let the quantity U be defined as the energy content of the wave field (in a non-stationary state) minus that given by the quantum potential, i.e.

$$\mathrm{U} = \mathrm{H} - (\mathrm{T} + \mathrm{Q}) \qquad (5.8)$$

Consequently,

$$\frac{d\mathrm{U}}{d\mathrm{t}} = \frac{d\mathrm{H}}{d\mathrm{t}} - \left(\frac{d\mathrm{T}}{d\mathrm{t}} + \frac{d\mathrm{Q}}{d\mathrm{t}}\right) = -\frac{d\mathrm{E}}{d\mathrm{t}} = -\frac{\partial \mathrm{Q}}{\partial \mathrm{t}} \qquad (5.9)$$

using Equation (5.5) and the fact that H is constant in the classically-free case (Riggs 1999, 3071). It can be seen therefore, that a change of the energy content of the wave field appears as a change in the quantum potential (as surmised in the previous section).

Since Q represents the potential energy available to the particle at a specific position in the wave field, both deviations from inertial motion and conservation of energy can be accounted for in individual quantum processes provided the single particle state is isolated. Depending on the prevailing circumstances, some (or all) of a particle's energy-momentum can be transferred and temporarily stored in its wave field. Once stored in the field, energy-momentum can be returned to the particle if circumstances change. This transfer back and forth of energy-momentum affects the particle's motion for a change in the momentum of the particle over time is, by definition, the force acting. Therefore the motion of a quantum particle need not be in a straight line even if there is no external field present. Since the particle is inseparable from its 'guiding' wave field, exchanges of energy-momentum occur between wave field and particle as they travel along together.

Equation (5.9) shows that $(\partial Q/\partial t)$ gives the change of the quantum potential due to changes in U, i.e. the time rate of change of Q due to changes in the amount of energy stored in the wave field other than at the particle's position. Then, by Equation (5.5), the particle's kinetic energy can be shown to increase (decrease) with decreases (increases) in the amount of energy stored in the wave field. Any change in the particle's kinetic energy is then explained by an energy conversion process, the concept of which is common to all branches of physics. Energy transfers, therefore, occur through a process whereby $T \leftrightarrows Q \leftrightarrows U$, with the direction of the arrows depending on whether the particle is losing or gaining energy. The quantum potential is the physical interface between particle and wave field and its role is to channel energy (or more generally, energy-momentum) from wave field to particle and back again (Riggs 2008, 33). These conversions need to be registered when accounting for the total energy of an isolated, classically-free quantum system. A suitable summary of such energy exchange was provided by Hermann Weyl (albeit from another field context):

> The total energy … remains unchanged: they merely stream from one part of the field to another, and become transformed from field energy … into kinetic-energy … and *vice versa* (Weyl 1952, 168, italics in original).

Unlike a classical field, the wave field's form has greater physical significance than its amplitude. The form of the wave field

may be described with reference to its wavefronts. A wavefront is defined as a surface over which the phase of the wave ($S/\hbar$) is constant. Common examples of wavefronts include spherical waves which have expanding spheres as their wavefronts and plane waves whose wavefronts are flat planes perpendicular to the direction of wave propagation. The shape of a wavefront depends (in part) on what the wave field encounters, i.e. whether its initial shape has been altered by passing over or through an obstruction. It is generally the case that when a wave changes its shape there will be a change in its amplitude. The total rate of change of the amplitude R with respect to time is:

$$\frac{dR}{dt} = \sum_{i=1}^{3} \frac{\partial R}{\partial x_i} \frac{dx_i}{dt} + \frac{\partial R}{\partial t} = (\nabla R) \cdot \left(\frac{\nabla S}{m}\right) + \frac{\partial R}{\partial t}$$

What do the terms $[(\nabla R) \cdot (\nabla S/m)]$ and $(\partial R/\partial t)$ represent? The simple example of a uniformly expanding spherical wave field will be useful in illustrating this. The wavefunction for a spherical wave field is:

$$\Psi = (A/r) \exp [i(kr - \omega t)]$$

where A is a constant, k is the wave number, $S = \hbar(kr - \omega t)$, amplitude $R = (A/r)$, where $r = (x^2 + y^2 + z^2)^{1/2}$, and the other symbols have their usual meanings (Holland 1993, 141). The term $[(\nabla R) \cdot (\nabla S/m)]$ gives the change in the value of R due to any changes in the size of a radius vector. However, as the wavefront expands, the value of R will decrease with time as r increases, thus $(\nabla R) \neq 0$. The term $(\partial R/\partial t)$ gives the rate of change of R explicitly due to changes over time in the shape of its wavefronts since changes in wavefront shape are generally accompanied by changes in amplitude. In the case of an undisturbed spherical wave, its wavefronts retain their shape as they expand so the amplitude of a spherical wave does not alter due changes in shape over time, i.e. $(\partial R/\partial t) = 0$ since the spherical wave retains its shape.

However, it can be seen that (dR/dt), in general, will depend on changes in the wave field's shape. The explicit dependence of $(\partial Q/\partial t)$ on $(\partial R/\partial t)$ is given by (Riggs 2008, 35):

$$\frac{\partial Q}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial}{\partial t}\left(\frac{\nabla^2 R}{R}\right) = -\frac{\hbar^2}{2mR}\nabla^2\left(\frac{\partial R}{\partial t}\right) - \frac{Q}{R}\left(\frac{\partial R}{\partial t}\right) \quad (5.10)$$

which clearly shows that $(\partial Q/\partial t) \neq 0$ over the time interval of a change in the shape of the wave field. The more pronounced the change in shape is, the greater will be the amount of energy exchanged between particle and wave field. Since, from Equation (5.9), $(dU/dt) = -(\partial Q/\partial t)$, it can be seen from Equation (5.10) that the condition for energy exchange between wave field and particle (and vice-versa) is $(\partial R/\partial t) \neq 0$.

The shape of the wave field depends, in large part, on whether it has encountered any obstructions which have distorted it. (In this limited sense, the form of the wave field indirectly carries information about the surrounding environment.) The surrounding environment modifies the form of the wave field which, in turn, acts by altering the motion of the particle. The environment changes the shape of its wave field and it is *changes in shape* of the wave field that are a major determining factor of the extent of any energy-momentum exchanges. Thus both the form of the wave field and the energy stored within it depends on whether the wave has encountered any obstructions which have distorted its shape.

A free particle not being subject to any external barriers or force fields has a wave field *represented* by a plane wave with constant amplitude. However, it can be seen that such a plane wave is an idealisation for the following reasons. First, a plane wave is usually given as infinite in spatial extent as pointed out in de Broglie's original reasoning:

> The plane monochromatic wave must in a certain sense be considered as an abstraction, for it would fill the whole of space and last throughout all time. In practice a wave always occupies a limited region of space at a particular instant, and at any particular point it has a beginning and an end (de Broglie 1930, 51).

Second, this would violate the Principle of Energy Content for something real not to possess any energy at all. This suggests that the wave field never divests itself completely of energy gained from its accompanying quantum particle or external interactions. The wave field retains a very small amount of energy at all times

and the shape of the wave field approaches a constant amplitude plane wave only as a limiting process.

In any real situation, the wave field will be of finite extent in all directions and initially localised about the particle. In his original pilot wave theory, de Broglie postulated that a quantum particle is always situated inside an envelope of waves that guides the particle (de Broglie 1924, 450). Such a wave packet can be described mathematically by the superposition of an infinite number of monochromatic plane waves differing only slightly in wavelength. A wave packet description is consistent with Axiom II of the Causal Theory for if the wavefunction is bounded, then its amplitude tends to zero with increasing distance from the quantum particle, i.e. $\Psi \rightarrow 0$ as $r \rightarrow \infty$. However, it should be kept in mind that the superposition that forms the wave envelope is part of a model and although we take the wave field to be a real entity, the infinite number of plane waves used in the superposition description of the packet is a mathematical convenience only. This is an example of not taking a theory *literally in all respects* (as discussed in Chapter 1).

5.3.1 Application to the Gaussian Wave Field

Although there are several types of (mathematical) wave packets, one description of an envelope of waves which has particularly useful properties (such as being able to be solved exactly) is the Gaussian wave packet. Gaussian distribution functions are standardly employed in statistics and probability theory (Boas 1966, 708). Gaussian curves have the same shape as normal distribution curves with a standard deviation σ. A Gaussian wave packet with the quantum particle located somewhere within the packet can be used to model a variety of physical phenomena such as diffraction by a slit with imperfect edges (Holland 1993, 163). The wavefunction for a Gaussian wave packet at any time t (> 0) is given by:

$$\Psi(\mathbf{x}, t) = (2\pi\sigma_t^2)^{-3/4} \exp\{i\mathbf{k} \cdot (\mathbf{x} - \tfrac{1}{2}\mathbf{u}t) - (\mathbf{x} - \mathbf{u}t)^2/4\sigma_o s_t\}$$

where σ_o is the initial root-mean-square (RMS) width of the packet in each coordinate direction, with $s_t = \sigma_o\,(1 + i\hbar t/2m\sigma_o^2)$, and **u** is the initial group velocity (Holland 1993, 163). The Hamiltonian H for a classically-free quantum system may be found by integrating the Hamiltonian density $\mathcal{H}$. From Equations (5.6) and (5.7), we have:

$$H = \iiint_{-\infty}^{\infty} \mathcal{H}\, d^3\mathbf{x} = \iiint_{-\infty}^{\infty} \left[R^2(\nabla S)^2/2m + (\hbar^2/2m)(\nabla R)^2\right] d^3\mathbf{x}$$

The relevant functions for a classically-free Gaussian wave packet are derived in Appendix A. Using these Gaussian functions, and the integrals shown in Appendix B, the above triple integration of $\mathcal{H}$ in the Gaussian case is evaluated in Appendix C. The result is the following Hamiltonian for a classically-free Gaussian quantum system:

$$H = {}^1/_2 m|\mathbf{u}|^2 + (3\hbar^2/8m\sigma_o^2) \quad (5.11)$$

which is the total energy of this isolated, classically-free quantum system. The first term on the right-hand side of Equation (5.11) is obviously the particle's initial kinetic energy. The second term can be seen to be the initial field energy as this term depends on the initial RMS width of the wave packet and does not involve any velocities or time-dependent quantities. The arguments presented in this section show that claims in the literature that the wave field contains no energy, such as Alastair Rae's (Rae 2004, 33), are mistaken (see also expression below for the quantity (H – T)).

The time dependence of the RMS width σ of the wave packet, i.e. $[\sigma_o^2 + (\hbar\, t/2m\sigma_o)^2]^{1/2}$, shows that the packet will expand with increasing time t and be accompanied by a change in shape. Therefore, both the energy of the particle and the amount of energy stored in the wave field will be time-dependent. The total energy of the system, however, will remain constant whilst the system is isolated. Transfer of energy-momentum will occur from the wave field to the quantum particle. The particle, in turn, will accelerate until such time as the value of the quantum potential drops effectively to zero. This can be shown quantitatively as follows. Consider a quantum particle positioned in the front of a wave

packet, so that $(\mathbf{x} - \mathbf{u}t) > 0$. The quantum potential derived from the Gaussian wavefunction is:

$$Q = (\hbar^2/4m\sigma^2)\ \{3 - (\mathbf{x} - \mathbf{u}t)^2/2\sigma^2\} \quad (5.12)$$

from which we find by partial differentiation:

$$\frac{\partial Q}{\partial t} = \frac{\hbar^4\, t}{8m^3\,\sigma_o^2\,\sigma^6}\ (\mathbf{x} - \mathbf{u}t)^2 + \frac{\hbar^2}{4m\sigma^2}\,[\mathbf{u}\cdot(\mathbf{x} - \mathbf{u}t)] - \frac{3\,\hbar^4 t}{8m^3\,\sigma_o^2\,\sigma^4} \quad (5.13)$$

(Riggs 1999, 3073) and the total rate of change of the particle's momentum with respect to time:

$$\frac{d\mathbf{p}}{dt} = -(\nabla Q) = \frac{\hbar^2}{4m\sigma^4}\ (\mathbf{x} - \mathbf{u}t) \qquad (5.14)$$

Since $(\mathbf{x} - \mathbf{u}t) > 0$, $(d\mathbf{p}/dt) > 0$ which shows that the particle's momentum is increasing, implying that its kinetic energy is also increasing. This is confirmed by checking the rate of change of the particle's kinetic energy with respect to time (Riggs 1999, 3073):

$$(dT/dt) = -(\nabla Q)\cdot(\nabla S)/m$$

$$= (\hbar^2/4m\sigma^4)\ [\mathbf{u}\cdot(\mathbf{x} - \mathbf{u}t)] + (\hbar^4 t/16m^3\sigma_o^2\,\sigma^6)(\mathbf{x} - \mathbf{u}t)^2 > 0 \quad (5.15)$$

i.e. the total rate of change of the particle's kinetic energy with respect to time is positive.

The dominant terms for large values of time t will be those containing powers of σ. Since the σ's are all denominator terms, Q, $(\partial Q/\partial t)$ and (dT/dt) will all tend to zero as time $t \to \infty$. Provided no further obstacles or disturbances are encountered, the wave field will expand extensively for large values of t, and correspondingly, the quantum potential and the energy contained in the wave field as a whole will rapidly approach zero, resulting in the energy of the quantum system becoming overwhelmingly kinetic (Riggs 1999, 3073). It is interesting to note that although Bohm and Hiley denied any explanation in which the wave field itself transferred energy, a reading of the their account of the spread of a wave packet might lead one to *question their consistency* on this issue:

> It is clear then that the particles are accelerated … This acceleration is evidently a result of the quantum potential … the quantum potential decreases as the wave packet spreads, falling eventually to zero.
>
> The picture is then that as the wave packet spreads, the particle gains kinetic energy, the amount depending upon where it was initially in the packet. … *the energy represented by the quantum potential was turned into kinetic energy* (Bohm and Hiley 1993, 47, italics added).

The time taken for the transfer of energy in the case of a free Gaussian wave packet is a theoretical problem which has been worked out in detail (Riggs 1999, 3073–3074). This calculation was made possible by resolving the conceptual problem involving energy conservation and by deducing the equations that govern energy transfer. If the particle is in a forward and central region of the wave packet, then the time taken for a complete transfer of energy is (Riggs 1999, 3074):

$$t = \frac{2m\sigma_o^2(\sqrt{T_f} - \sqrt{T_i})}{\sqrt{4m^2\sigma_o^4 - \hbar^2(T_f + T_i - 2\sqrt{T_f T_i})}}$$

where T_i and T_f are the initial and final kinetic energies of the particle respectively.

We have seen that the energy of the wave field is (H − T). There are very few references to the energy content of the wave field to be found in the literature. Given that the role of energy is essential to the complete description of any physical system, the potential energy stored within the wave field is an important quantity. In the case of a classically-free Gaussian wave field, an expression for the energy stored may be found in terms of the functions R, S and their derivatives (although it does not have a simple form). The derivation may be found in Appendix D. The result is: (H − T) =

$$\left(\frac{3}{4}\right)\frac{R^2\nabla^2(dS/dt)}{(\nabla R)^2 - R(\nabla^2 R)} + \frac{R^2(\nabla^2 S)}{(\nabla R)^2 - R(\nabla^2 R)}\left(\frac{\nabla S}{m}\right)\cdot\left(\frac{\nabla R}{R}\right)$$

$$+ \left(\frac{1}{2m}\right)\left[\frac{R^2(\nabla^2 S)}{(\nabla R)^2 - R(\nabla^2 R)}\right]^2 \qquad (5.16)$$

We shall have reason to consider Equation (5.16) in Section 5.5.

5.4 Quantum Reaction?

The wave field acts on the quantum particle but the particle does not react back on the wave in the sense that the shape or size of the wave field is not directly affected by the particle. This lack of a classical reaction is viewed as a flaw in the Causal Theory by some commentators (Anandan and Brown 1995, 359). The absence of a classical reaction constitutes a conceptual problem for the Causal Theory for it conflicts with the Principle of Reaction (i.e. Newton's Third Law) which may be stated as follows (Doughty 1990, 116–117):

- *Principle of Reaction*

  Any interaction between two physical entities has a mutual effect on both entities. The forces of interaction are equal and opposite, and act along straight lines joining the locations of the entities.

(The Principle of Reaction is commonly paraphrased as 'for every action there is an equal and opposite reaction'.) In the example of a charged particle accelerated by an external electric field between charged plates used in Section 4.4, there is an obvious action of the external field on the particle but what is the reaction and how is it mediated? Before explicitly answering this question, consider the following description of fields by Noel Doughty in his text *Lagrangian Interaction*:

> Fields are thus of two forms, those like gravity or electromagnetism which are generated by a source (for example mass or electric charge), and those which are not and represent the sources themselves, such as the non-relativistic Schrödinger wave function … The field equations of a sourced, or mediated field, can be recognised by the presence in them of a term, *the source term*, which does not contain the field itself (Doughty 1990, 139, italics in original).

A charged particle is surrounded by its own very small electric field which is independent from any external field. Both the particle's field and an external field (each with its own source) are distorted in shape when they interact. The particle reacts back on the external field via its own electric field. The standard answer to the above question is, of course, that the particle exerts a force (albeit almost totally negligible) on the plates equal and opposite to that which it experiences. This ensures agreement with the Principle of Reaction.

However, the issue of action and reaction is really a more general question of the status of the Principle of Reaction and whether it has universal validity. Indeed, it does not appear that the Principle of Reaction can be universally valid as there are counter-examples in electrodynamics where an action is not accompanied by a corresponding *equal and opposite* reaction (Goldstein 1980, 7–8; Fowles 1977, 44). This situation is not the same as for energy conservation since action-reaction is not found to apply in all circumstances. It seems to be the case that classical action-reaction applies in cases of contact phenomena but (as specific counter-examples indicate) not necessarily to all interactions (Lange 2002, 163 n.2 and 234). We are, therefore, justified in holding to the Principle of the Conservation of Total Energy and to denying the applicability of the Principle of Reaction to individual quantum systems.

One suggested approach that has appeared in the literature to 'rectify' the action-reaction problem within the Causal Theory is to add a source term to the Schrödinger equation (Squires 1994a, 131; Abolhasani and Golshani 1999, 304). This, however, would lead to a non-linear wave equation which would produce predictions in conflict with well-established empirical results. Instead of viewing the absence of a classical reaction as a defect in the Causal Theory, this should be seen as a *new insight* into the quantum domain. Doughty rightly points out that the Schrödinger wave field is not a mediated field. Therefore there is no familiar means to carry a reaction from the quantum particle to the wave field. Indeed, Cushing has suggested that our intuitions about classical action-reaction might not be reliable in the quantum realm (Cushing 1994b, 46). This is a very plausible suggestion for we have seen that the total energy of a quantum system is conserved because energy transformations between particle and wave field are facilitated through the quantum potential, despite the absence of a classical reaction on the wave field.

Part of the difficulty of applying the Principle of Reaction to a quantum system is thinking of the quantum particle and its wave field as if they were on par with say, an external electric field and an introduced charged particle. The electric field example, useful as it is in demonstrating field characteristics, cannot be taken as

accurately representing all aspects of quantum entities because the wave field is not a mediated field. The assumption that wave field and particle can be treated as separate but interacting entities that the Principle of Reaction applies to equally is the cause of the problem. The wave field and its quantum particle(s) constitute a single entity, as has been emphasised throughout this book. The Principle of Reaction, however, was formulated to apply to the interaction of separate entities.

We should also ask whether further consideration of energy transfer processes might enlighten this issue. Equations (5.9) and (5.10) together indicate that the energy transfer from particle to wave field is due to a change in the form of the wave field. This transfer of energy is an event that results from changes in the wave field, i.e. one part of a quantum system influencing another part of the system. This implies that the classical ideal of an action accompanied by an equal and opposite reaction need not be realised in the quantum domain. This is also echoed in Holland's response to the action-reaction issue. He writes:

> … But while it may be reasonable to require reciprocity of actions in classical theory, this cannot be regarded as a logical requirement of all theories that employ the particle and field concepts, especially one involving a nonclassical field (Holland 1993, 26).

The Principle of Reaction, as classically formulated, cannot account for all types of field interactions and quantum processes. This conclusion avoids conflict with the Causal Theory. The Principle of Reaction needs revision if it is to be applicable to quantum entities.

5.5 The Wave Field and the Quantum Potential

The mathematical expression of the quantum potential is very different from potential functions for classical fields. Why is the quantum potential so dissimilar? In Bohm's original account, the form of the quantum potential was merely accepted as given by the mathematics and not requiring further explanation. Others have seen a need to specify an origin for the quantum potential (Hiley and Peat 1987, 12). A summary of efforts to derive the quantum potential may be found in Carroll 2005b. However, as Dickson has rightly

pointed out, it is not exactly reasonable to expect a theory to itself justify the existence of its fundamental entities (Dickson 1998, 109).

We are now in a position to summarise the relevant features of the wave field and the quantum potential:

(i) The wave field exhibits the usual wave properties (e.g. reflection, transmission, diffraction, interference, etc.) and obeys the Principle of Linear Superposition.

(ii) Since the Schrödinger equation is homogeneous, the wave field is not a radiated field and there is no source term for the field.

(iii) The environment surrounding a quantum particle (in part) determines the shape of its wave field.

(iv) The wave field is the repository of potential energy in a quantum system.

(v) The wave field acts on the quantum particle similar to an external field and receives or imparts energy and momentum to the particle.

(vi) The quantum potential represents a portion of the energy contained in the wave field and is the amount of potential energy available to the particle at its specific position in the wave field.

(vii) The magnitude of the quantum potential is independent of the intensity of the wave field.

(viii) Non-local connections between particles in a many-particle quantum system are facilitated through the operation of the quantum potential.

Given these features, what might be inferred about the 'origin' of the quantum potential Q? At a coarse level of description, Q is the potential energy function of the wave field and its 'origin' may be understood as deriving from the potential energy of the wave field. In classical mechanics, the form of a potential energy function is found by integrating the expression for the force between classical particles where the force expression contains a source term for the classical field. This is not the case in the Causal Theory for there is no source term and the force expression does not have a general form (such as the inverse square law). Therefore, the form of the

quantum potential cannot be derived, in general, by this means. This is sometimes expressed by stating that the quantum potential is not a pre-assigned function of the coordinates.

This identification of Q as the potential energy function is not an entirely satisfactory response to the question of the quantum potential's origin and is, by no means, a complete explanation of its nature. We have not, for example, explained why the quantum potential takes the form $(-\hbar^2/2m)(\nabla^2 R/R)$ rather than one more akin to the familiar classical potential functions which essentially have a (1/r) dependence, where r is the distance from the relevant particle. Nor have we explained why the effect of the wave field is independent of its intensity. Clearly, these two issues are not mutually exclusive and deserve some further comment.

Consider first the latter issue. The effect of the wave field on its quantum particles depends on how much of its energy is available to the particles via the quantum potential. This, in turn, is related to the total energy stored in the wave field which cannot depend on field intensity (i.e. amplitude squared) for if it did, the wave field could not have an amplitude large enough to store energy up to macroscopic orders of magnitude (cf. Bohm's comment quoted in Section 5.2). Equation (5.16) shows that the energy content of the wave field is independent of the field's intensity. This can be seen in the same way as we did for the quantum potential, i.e. multiplication of the amplitude R in Equation (5.16) by a constant does not change the value of (H − T). The mechanism of energy storage and transfer cannot be the same as for classical fields.

In relation to the former issue, the quantum potential is structured in such a manner so as to facilitate energy exchange between wave field and quantum particles without the presence of a mediated field. Clearly, if there is no wave field source term then the quantum potential cannot include such a term. This is an obvious restriction on the form of Q. In a many-particle quantum system, non-local effects are orchestrated by the operation of the quantum potential and these do not necessarily fall off with increasing distance. A potential energy function that can perform these roles could not be of the same form as classical potential functions, i.e. since the effect may not fall off with distance, the quantum potential cannot be proportional to (1/r). A potential energy function that can

perform the roles discussed above would need a very specific form. In particular, the form of potential functions found in classical electrodynamics and Newtonian gravitational theory is ruled out since these have a (1/r) dependence (just as classical amplitudes do). Here again this implies that the mechanism must be different to the classical case.

We may now deal with a related conceptual problem involving the quantum potential itself. There is an objection to the Causal Theory occasionally made in the literature which claims that the quantum potential is not a physical potential (Rae 2002, 260–261), where 'physical potential' is presumably used in the same sense as in electrostatics. A conceptual problem arises here because the nature of the quantum potential conflicts with the belief that physical potentials must be due to a source, such as an electric charge (Parmenter and DiRienzo 2004, 1). However, as detailed above, the quantum potential should be understood as deriving from the potential energy of the wave field. The 'recycling' of energy by the wave field in an isolated quantum system indicates how the quantum potential operates. Although the operation of the quantum potential would be unexpected on the basis of our understanding of classical potentials, this has not prevented a consistent account of the dynamics of quantum systems and energy conservation being developed. The quantum potential is an integral part of these physical processes which do not require classical sources. The stipulation that all physical potentials must have (classical) sources does not apply to quantum fields. Bohm and Hiley have also been careful to outline this:

> … it should be pointed out that … the Schrödinger equation for the quantum field does not have sources … This of course constitutes an important difference between quantum fields and other fields … the quantum theory can be understood completely in terms of the assumption that the quantum field has no sources …(Bohm and Hiley 1993, 30).

On the question of the possible violation of Special Relativity with respect to the potential energy of the wave field, what about the instantaneous changes that occur when a quantum system is 'manipulated'? When such manipulation occurs, the wave field will be altered. This appears in the formalism as an instantaneous change to the wavefunction which, in turn, means that the value of the quantum potential for a particle in a spatially distant part of the

quantum system may also change. The instantaneous change to the form of the wavefunction is a feature of the model being non-relativistic but does not produce superluminal transfer of energy. The transfer of energy to/from the i^{th} particle in a many-particle quantum system from its wave field occurs through the quantum potential. The value of the quantum potential at the i^{th} particle's position is $Q_i = -[\hbar^2/2m_i R]\,\nabla_i^2 R$. Since Q_i depends on the location within the wave field, the energy transfer to a quantum particle is from a portion of the wave field at that location, i.e. this process occurs locally. (Recall the apt comment of Hermann Weyl in Section 4.4 that every portion of a field has a definite amount of potential energy.) Therefore, there is no violation of Special Relativity when energy is transferred and/or transformed within a quantum system (see also Section 5.7).

The above considerations imply that the level of analysis which is appropriate to the issue of origin of the quantum potential is one where the nature of the wave field itself is the subject. An in-depth ontological account of the wave field would provide a more substantial explanation of the origin of the quantum potential (as the potential energy function of the wave field) but such an account would need a relativistic approach. In relation to the wave field, however, a relevant question is why does the time-dependent Schrödinger equation describe the propagation of the wave field (in the non-relativistic domain)? Even before quantum mechanics was given any interpretation, no rigorous derivation of the Schrödinger equation had even been attempted from basic physical assumptions. Instead, the Schrödinger equation is justified by appeal to its predictions and the results of experiments, as the following extract from a leading quantum mechanics text indicates:

> Various assumptions have to be made as regards the structure of the [Schrödinger] wave equation … These assumptions are given a high degree of plausibility … by relating them to experimental results … However, no attempt is made to derive the formalism uniquely from a consideration of the experiments (Schiff 1968, 19).

Many 'derivations' of the Schrödinger equation have been made, each of which is based on different premises (Carroll 2004, 1–2). There is no general agreement on its derivation.

The mathematical form of the (classically-free) Schrödinger equation is similar to the Heat equation of classical physics rather

than the classical wave equation (a similarity that has been noticed since the advent of quantum mechanics). The principal difference between the classically-free Schrödinger equation and the Heat equation is the appearance of the imaginary number $i = \sqrt{-1}$ in the Schrödinger equation. However, this is a mathematical convenience as the Schrödinger equation can be rewritten as two coupled, differential equations involving two real functions. The presence of i in the Schrödinger equation is ultimately traceable to wavefunctions being defined as complex functions. In the Causal Theory, the wavefunction represents the wave field within a mathematical model (which must be the case as any physically acceptable but otherwise arbitrary wavefunction can be normalised). The definition of the wavefunction as a complex function should not, therefore, be considered problematic (cf. the discussion in Section 2.6 of the distinction between the formal machinery of a model representing an aspect of reality and reality itself).

The Heat equation of classical physics (without any heat sources) is:

$$\nabla^2 u = \frac{1}{\beta}\left(\frac{\partial u}{\partial t}\right) \qquad (5.17)$$

where $u = u(\mathbf{x},t)$ is temperature and β is a constant (Boas 1966, 630). Does the similarity of the (classically-free) Schrödinger equation to the Heat equation hold any significance? We can answer this in the affirmative for the similarity arises through a common functional role of both equations. If one looks at a standard derivation of the Heat equation without the presence of any sources of heat, the derivation proceeds by specifying the energy 'flux' from one region to another due to temperature variation. The resulting equation then describes the transfer of energy (in this case, heat). The Schrödinger equation describes the propagation through space of a physical field with no sources (in the classical sense) but has a finite energy content. The wave field's time development conserves energy and thereby describes the energy transfer from one spatial region to another. Doughty provides an apt description:

> The Schrödinger equation can be considered an example of a classical field equation of Galilean relativity which is local in space and time with a simple well-defined additive and conserved local energy (Doughty 1990, 122).

The spatial energy 'flux' due to the propagation of the wave field together with the lack of sources places restrictions on the form of the equation describing the wave field's time development (i.e. the Schrödinger equation). This, in part, explains the similarity of the classically-free time-dependent Schrödinger equation to Equation (5.17).

5.6 The Wave Field and Physical Measurement

It was noted in Chapter 2 that what is commonly called 'measurement' in Orthodox Quantum Theory is only one type of interaction between different physical systems (albeit a most important interaction from the perspective of experimental physics). In the Causal Theory's account of the measurement process, the wavefunction is split into non-overlapping packets which move off independently. The spatially separated, empty wave packets do not affect the particle and the wavefunction Ψ effective evolves:

$$\Psi \rightarrow c_a \psi_a \phi_o$$

There is no 'collapse' of the wavefunction. The various separated parts of the wave field continue to objectively exist albeit as empty quantum waves. The interaction with a device designed to measure an 'observable' of a quantum system transforms the wavefunction Ψ into what would be called an eigenstate of the observable in Orthodox Quantum Theory. This measurement account is described by means of a multi-dimensional configuration space as the interaction involves a many-particle system (viz. the measurement apparatus). Although the mathematical description is in terms of such a configuration space, the actual measurement interaction occurs in physical three-dimensional space, not in a multi-dimensional configuration space, and not in a mathematical Hilbert space. Recall that in Chapter 3, the view that a multi-dimensional configuration space is a real aspect of nature was rejected. Therefore, we ought to be able to give a *physical* account of what happens to particle and field in three-dimensional space during a measurement process.

We saw in Chapter 3 that measurement processes generally introduce uncontrollable (and unpredictable) disturbances to a quantum system. Instead of a collapse on measurement, there is a physical change caused to the wave field with the following results: (i) the wavefunction is altered (a description in terms of a split into non-overlapping packets); and (ii) the particle's position, momentum, etc. (which depend on the wave field) will generally be changed as a consequence. Since an 'act of measurement' generally changes the form of the wave field, such acts would be better labelled as 'disturbance measurements', for it is conceivable to have 'non-disturbance measurements', i.e. those that do not alter the form of the wave field. This kind of measurement is also known as 'protective' (Aharonov and Rohrlich 2005, 214–215).

In order to visualise what occurs during one kind of 'disturbance measurement', consider a quantum particle trapped inside a cubical box of side length L. If one of the walls is removed suddenly then the particle and its wave field would no longer be contained. The wave field will evolve from a stationary state and will change in form as it propagates out of the box from a standing wave to a travelling wave. (There will also be a corresponding change to the wavefunction.) During this transition process, energy will transfer from the wave field to the particle (as seen in Section 5.3). This change would be interpreted in Orthodox Quantum Theory as an *instantaneous collapse of the wavefunction* since the removal of the wall would be part of a 'measurement' process on the particle. The collapse of the wavefunction on measurement (a requirement of the Projection Postulate) is necessary in Orthodox Quantum Theory to avoid the 'Measurement Problem' but can be seen here as a surrogate for the physical situation where a quantum particle gains energy from its wave field. A measurement in these circumstances will only reveal a quantum particle with definite properties. Therefore, it is no wonder that Orthodox Quantum Theory has to postulate wavefunction collapse to achieve the result found on measurement.

However, the restoration of the particle's kinetic energy on measurement (or indeed on any other physical process) is *not* and *cannot be* instantaneous, otherwise the Special Theory of Relativity would be violated. We have already discussed that even though the

non-relativistic Causal Theory is non-local, this does not lead to violations of Special Relativity. It was found for the case of a classically-free quantum particle, that the rate of change of kinetic energy with respect to time is equal to:

$$(d\,\mathrm{T}/d\mathrm{t}) = -(\nabla \mathrm{Q}) \cdot (\nabla \mathrm{S}/m)$$

from which the time of energy transfer would be given by:

$$\mathrm{t} = -m \int_{\mathrm{T_i}}^{\mathrm{T_f}} \frac{d\mathrm{T}}{(\nabla \mathrm{Q}) \cdot (\nabla \mathrm{S})}$$

where $\mathrm{T_i}$ and $\mathrm{T_f}$ are the initial and final kinetic energies of the particle respectively. This integral can be evaluated in specific instances but not in the case a 'particle in a box' since the integral is not defined as $\nabla \mathrm{S} = 0$.

There is a change in amplitude that accompanies the change in the shape of the wave field over a short time interval immediately after the wall of the box is removed. From Equation (5.9) and Equation (5.10), we have:

$$\frac{d\mathrm{U}}{d\mathrm{t}} = \frac{\hbar^2}{2m\mathrm{R}} \nabla^2 \left(\frac{\partial \mathrm{R}}{\partial \mathrm{t}}\right) + \frac{\mathrm{Q}}{\mathrm{R}} \left(\frac{\partial \mathrm{R}}{\partial \mathrm{t}}\right)$$

The more pronounced the change in shape is, as indicated by $(\partial \mathrm{R}/\partial \mathrm{t})$, the greater will be the amount of energy exchanged between particle and wave field.

What form does the wave field take upon leaving the box? It would seem that the *exact* mathematical form of such a wave is not possible to find analytically (Main 1978, 309–310). Given this, one has to assume a form for the wave field as it emerges from the open part of the box. A physically reasonable approximation for the form of the wave field is represented by the initial, normalised Gaussian wave packet:

$$\Psi_\mathrm{o} = (2\pi\sigma_\mathrm{o}^2)^{-3/4} \exp\{i\mathbf{k} \cdot \mathbf{x} - (|\mathbf{x}|^2/4\sigma_\mathrm{o}^2)\}$$

Then the corresponding initial quantum potential is:

$$\mathrm{Q_o} = (\hbar^2/4m\sigma_\mathrm{o}^2)\ \{3 - (|\mathbf{x}_\mathrm{o}|^2/2\sigma_\mathrm{o}^2)\}$$

where $\sigma_o = (L/2)$ and the particle's position being at the end of the box, i.e. $|\mathbf{x}_o| = L$. This gives a value of: $Q_o = (\hbar^2/4mL^2)$. In the stationary state inside the closed box, the quantum potential had a minimum value of $(3\pi^2\hbar^2/2mL^2)$. The difference between these two values of the quantum potential shows a decrease of slightly less than $(15\hbar^2/mL^2)$. This is a loss of energy from the wave field which appears as the kinetic energy of the moving quantum particle. Once clear of the obstruction (i.e. the open end of the box) the wave field will tend to the form of a travelling plane wave of constant amplitude. This latter evolution of the wave field is essentially the same as given in the account provided in Section 5.3.

F.J. Belinfante once stated that it is a contradiction-in-terms to claim that *free* quantum particles would accelerate. He further asserted that there is no sense in maintaining such a claim without experimental support (Belinfante 1973, 121). Similar sentiments were more recently expressed by Parmenter and DiRienzo (Parmenter and DiRienzo 2004, 5) which were motivations for their 'non-interactive' approach to the wave field (as presented in Section 4.3). Belinfante's conclusion is, of course, not warranted in the context of the Causal Theory, for classically-free particles are not necessarily quantum mechanically free. The above case of a particle moving out of an opened box is one such example.

5.7 Tunnelling from a Quantum Well

Tunnelling is a quantum phenomenon with no classical analogue. In the case of a potential well (such as a Coulomb force field) quantum mechanics predicts that there is a small but finite probability that a particle can be found in a classically forbidden region, i.e. outside the well. Tunnelling arises formally as a consequence of the mathematics (by the constraints of continuity for the wavefunction and its first derivative at boundaries) but has no other explanation in Orthodox Quantum Theory. Since tunnelling has been experimentally confirmed, it is clearly a real, physical process rather than an artefact of the mathematics. A number of papers have been devoted to aspects of tunnelling within the Causal Theory (e.g.

Dewdney and Hiley 1982; Cushing 1995; Leavens 1996; Oriols et al. 1996; Bittner 2000).

How can a particle free itself when bound in a potential well, i.e. where the magnitude of the particle's kinetic energy is less than the potential energy inside the well? Consider such a situation where quantum particles are trapped in a (finite) well, such as may be produced by an electric field, with insufficient kinetic energy to escape. Despite this, quantum mechanics predicts that there is a small probability that some particles can be found outside the well. If we have an N-particle system (N > 1), classically one would expect the particles to be held in a well with a (finite) potential V if

$$[(\nabla_i \mathrm{S})^2/2m_i] < |\mathrm{V}|$$

for all i, $1 \leq i \leq \mathrm{N}$. (It is usual for a well exerting an attractive force to have its potential energy defined to be zero at the 'top' of the well which then requires V to be negative inside the well.) The solution to the question of how quantum particles can become free if bound by an attractive force field becomes evident when the role of the quantum potential is recognised as facilitating the exchange of energy between the wave field and particles. In the many-particle case, as previously indicated, individual particles can gain energy from, or lose energy to, the wave field through the associated value of the quantum potential Q_i depending on their positions in the wave field, where $\mathrm{Q}_i = -[\hbar^2/2m_i \mathrm{R}](\nabla_i^2 \mathrm{R})$. These transfers of energy occur locally so that there is no violation of Special Relativity.

The condition for the i-th particle to escape from the well is:

$$\left(\frac{1}{2m_i}\right)(\nabla_i \mathrm{S})^2 > \left|\mathrm{V} + \left(\frac{-\hbar^2}{2m_i \mathrm{R}}\right)\nabla_i^2 \mathrm{R}\right| \qquad (5.18)$$

This condition can be satisfied in two ways depending on the nature of the potential well, the form that the wave field takes within the well, and the positions of the particles in the well. First, an individual particle might gain sufficient energy from the wave field that its kinetic energy becomes large enough to satisfy the inequality (5.18). Second, a small part of the wave field might increase its energy content (and thereby increase the magnitude of the quantum potential Q_i associated with individual particles situated

in that part of the wave field) so that the absolute value of the net potential energy in this region of the wave field (i.e. $|V + Q_i|$) is less than an individual particle's kinetic energy. The additional energy in both cases is gained at the expense of a portion of the kinetic energies of other (non-tunnelling) particles in the system. Either way, this would allow a small fraction of the total number of particles to break free of the binding force of the well (Riggs 2008, 34).

5.8 The Quantum Mechanical Force

Are the forces postulated in physical theories real? Our human senses would seem to indicate that they are since we can 'feel' contact forces (i.e. pushes and pulls), we can levitate magnetised objects, and we can deflect charged particles from straight paths by generating electromagnetic fields, etc. It would seem to be an obvious fact that forces are real. Also, our most successful theories posit the existence of forces in order to explain and predict all kinds of phenomena in the physical world and at all scales from rotating galaxies to vehicle crashes to nuclear decays. All standard physics textbooks tell us that there are four 'fundamental forces of nature', i.e. gravitation, electromagnetism, the weak nuclear force, and the strong nuclear force.[2] There are also other forces discussed in the physics and chemistry literatures including inter-atomic and inter-molecular forces, although these are not considered to be fundamental.

The case for the reality of physical forces is a strong one, as has been has argued in many accounts, such as that by Australian philosopher of physics Adrian Heathcote, who writes:

> … We do not *just* have access to the nature of forces through what they do, we also have some information about where they come from, how one force differs from another; in other words their embedding in a total physical theory … Physics is a theory that is (in large part) a theory of the nature of forces and how systems behave under the influence of forces. Forces are real (Heathcote 1989, 79, italics in original).

[2] A typical example of the descriptions of 'forces' given by standard physics textbooks may be found in Nolan 1993, 1002–1003.

The concept of force plays a unifying role in physics as forces are the *causes* of the accelerations of objects. Accelerations are experienced by all matter in the universe (to some degree) and are explained in terms of the action of different types of forces, in accord with the Principle of Causality (Bigelow et al. 1988, 619–620). It is not the intention here, however, to provide detailed arguments for the reality of forces. Instead, it will be granted that there are sufficient grounds to accept the existence of physical forces and the reader is referred to philosophical arguments in their favour (such as in the works by Bigelow et al. 1988; Bigelow and Pargetter 1990; and Wilson 2007).

An important conceptual issue raised by the Causal Theory is whether there exists a purely quantum mechanical force in addition to the four accepted fundamental forces of nature. We have seen that the Causal Theory indicates the existence of such a force **F** given by:

$$\mathbf{F} = -\nabla Q$$

What can be reasonably said about the existence and nature of the quantum mechanical force? It was found in Section 4.3 that the origin of the quantum mechanical force on a given particle cannot be the other particles in the relevant quantum system. Instead, the wave field was identified as exerting the force on quantum particles. If the wave field is an objectively existing, physical field that is distinct from other physical fields, then there should be a force that the wave field exerts on quantum particles, just as other real fields exert forces on particles. In the Causal Theory, this is particularly evident in the deviation from inertial motion of quantum particles as shown by their calculated trajectories in different situations (e.g. Dewdney 1988a; b; c, 869–871; Bohm and Hiley 1993, 33 and 53; Wyatt 1999; Colijn and Vrscay 2002, 338). The presence of energy (or more generally, energy-momentum) within the wave field allows it to exert a force on quantum particles in the field. This is in keeping with our understanding of how *all* physical fields interact with matter for empirical knowledge of the existence of all forces in nature comes by their effects on objects or particles. The absence of any ‘quantum charge’ should not viewed as counting against the existence of the quantum force which has its origin in the wave field and only affects other parts of a quantum

system, i.e. the particles that are embedded in, and intrinsic to, the system.

The non-classical nature of the quantum mechanical force is especially striking – its strength need not decrease with distance (Holland 1993, 282). This is directly related to the effect of the quantum potential being independent of the intensity of the wave field. Although unexpected from a classical perspective, it is not the only example in the quantum realm. The inter-quark force, for example, increases with increasing distance between two quarks (Penrose 2004, 679).

If we accept the reality of physical forces and that current physical theory should be our guide to what exists then we should also accept the reality of the quantum mechanical force in much the same way as we accept say, the reality of the electromagnetic force. The quantum mechanical force should be considered as another fundamental force of nature on ontological parity with the four accepted fundamental forces. Yet, the existence of a quantum mechanical force is not generally recognised in the physics community. Indeed, it should come as no surprise that its possibility is *not even known* to most physicists! This lack of acknowledgement of the existence of the quantum mechanical force as a different force arising from a different kind of (non-mediated) field constitutes a serious explanatory gap in quantum physics.

Circumstances in which the quantum mechanical force is present but not acknowledged fall into three categories:

(a) The quantum mechanical force acts but the resulting phenomenon is merely described as a quantum effect having no classical analogue;

(b) Another force is postulated which is the quantum mechanical force under a different name;

(c) The quantum mechanical force acts together with one (or more) of the four accepted fundamental forces of nature but is not recognised.

A prime example of category (c) is the covalent bond which binds atoms together. In basic chemistry texts the covalent bond is described as being due to the equal sharing between two atoms of a pair of electrons, one from each atom (Pimentel and Spratley 1971,

612). This force of attraction is electrostatic, i.e. it is the Coulomb force between the atomic nuclei and each of the electrons. The covalent bond is one of the strongest found in chemistry, yet classical electrostatic attraction would yield only a fairly weak bond (Enge et al. 1972, 316). The covalent bond needs a quantum approach as it cannot be explained by classical physics. The strength of the force between the two atoms may be calculated in Orthodox Quantum Theory but this does not provide an understanding of the underlying processes involved as indicated by Bohm and Hiley:

> Classically it is incomprehensible why chemical bonds can form, … [Orthodox] quantum theory has explained this … But this has been achieved at the expense of a loss of intuitive comprehension of what is involved physically in such a bond (Bohm and Hiley 1993, 63).

The Orthodox Quantum Theory calculation may be found in many standard quantum mechanics texts (e.g. Gasiorowicz 1974, Chapter 21) and will not be reproduced here.

In the Causal Theory, covalent bonding is readily explained. Consider two initially separated hydrogen atoms which move together to form a hydrogen molecule by means of a covalent bond. Using the formalism of the Causal Theory, the total (effective) potential energy available to the electrons is found to be the sum of a quantum potential term (Q) and a Coulomb term. This total potential energy is a function only of the inter-nuclear distance (r). The effective inter-nuclear force $\mathbf{F}_{\text{eff}}$ is also just a function of the distance r and its magnitude is given by:

$$|\mathbf{F}_{\text{eff}}| = -(\partial Q/\partial r) + k(e^2/r^2)$$

where e is the electronic charge and k is a constant (Holland 1993, 319). The presence of the quantum force term, $-(\partial Q/\partial r)$, shows why covalent bonding cannot be explained classically.

An example of category (b), i.e. the quantum mechanical force under a different name, is the (so-called) 'Pauli Force'. This is postulated by some researchers because of effects observed when the Exclusion Principle applies to physical systems such as 'cavities' in liquid helium (Günther et al. 1995, 395). The 'Pauli Force' is discussed in Chapter 6.

The lack of any acknowledgement of the quantum mechanical force in Orthodox Quantum Theory has also raised the

question of whether quantum mechanics is compatible with the Law of Inertia because there are situations where quantum particles do not move in straight lines (Rabinowitz 2008). This is particularly evident in the Causal Theory where non-inertial motion is obvious from the calculated trajectories of quantum particles in different situations. Such situations include the Aharonov-Bohm Effect where particles only pass through spatial regions where there is no electric or magnetic field present (and therefore no classical forces) but there is still a shift of visible interference fringes (as shown in Section 4.5). In Orthodox Quantum Theory, this falls under the above category (a), i.e. the phenomenon is characterised as being due to a quantum effect which has no classical analogue (Sakurai 1985, 139). The Causal Theory explains why the particle trajectories change (and therefore the interference pattern) because the quantum mechanical force on the particles is non-zero even when other forces are absent.

In much the same way as we account for the behaviour of macroscopic physical systems by the action of the four accepted fundamental forces of nature, we may also account for the behaviour of quantum systems by the quantum mechanical force acting by itself or in concert with other forces. This leads to the conclusion that the reality of the quantum mechanical force should be recognised in the same manner as the accepted fundamental forces are recognised. Further, the disregarding of the quantum mechanical force has been a barrier to gaining a more comprehensive understanding of natural processes in which quantum effects are manifest (e.g. quantum gravity, superconductivity, quantum cosmology, etc.) and of the Standard Model of elementary particle interactions.

In Section 5.5, it was argued that an in-depth ontological account of the wave field would provide a better explanation of the quantum potential. A similar line of reasoning leads to the conclusion that such an in-depth ontological explication of the wave field would also provide a better explanatory account of the quantum mechanical force.

5.9 Empirical Consequences

There have been a variety of experiments proposed in recent years to test quantum mechanics. (A detailed coverage of many of these experiments may be found in Ghose 1999.) In respect to the statistical predictions of the Causal Theory, Antony Valentini has suggested that astronomical observations might reveal events where the Quantum Equilibrium Condition does not hold. Valentini proposes searching for non-equilibrium violations in primordial inflation fluctuations imprinted on the cosmic microwave background, relic cosmological particles, Hawking radiation, photons whose entangled partners are inside black holes, neutrino oscillations, and particles from very distant sources. Such observations might be used to discriminate between the Causal Theory and Orthodox Quantum Theory (Valentini 2007; 2008). However, it is always taken that the Quantum Equilibrium Condition does hold in everyday circumstances and this guarantees empirical equivalence between the two theories. Since both the Causal Theory and Orthodox Quantum Theory share the formalism of the Schrödinger equation and the Born Statistical Postulate, both give the same *probabilities* for the results of terrestrial experiments!

Despite this, a number of articles have claimed that either the Causal Theory makes predictions that are not realised or makes predictions contrary to the predictions of Orthodox Quantum Theory. Indeed, nothing would 'kill off' the Causal Theory more quickly than a situation where its predictions are consistently and repeatedly not confirmed by experiment and where those of Orthodox Quantum Theory are reliably corroborated. Of particular interest are the criticisms of the Causal Theory by Ghose and by Golshani and Akhavan, especially their double slit predictions (Golshani and Akhavan 2001; Ghose 2000a; b). The intention here is to acknowledge this work rather than analysing it as this has already been performed by Marchildon and Guay, Struyve and De Baere, and by Nikolić, who have adequately dealt with these particular criticisms (Marchildon 2000; Struyve and De Baere 2001; Marchildon 2001; Nikolić 2003; Guay and Marchildon 2003). Other possibilities for testing the Causal Theory have also been mooted

from time to time, such as measuring transit times for some quantum tunnelling situations (Cushing 1995; Lan and Liang 2008).

5.9.1 A Crucial Experiment?

There are instances in the history of science where competition between rival theories has occurred over long periods of time (even centuries). This is because there is not usually what might be called a 'crucial experiment', i.e. a repeatable experiment (or set of experiments) for which one theory gives the correct predictions and its competitors do not (Riggs 1992, 87). This section is primarily devoted to looking at the possibility of a new experiment which might yield results in accordance with the Causal Theory but different from that predicted by Orthodox Quantum Theory. Is such a crucial experiment a realistic possibility? There are very few situations in which the Causal Theory and Orthodox Quantum Theory make different *theoretical* predictions and it is to these few that we must look. The experimental circumstances for testing such situations require technologies that have only become available in relatively recent times or perhaps will have to await more advanced technological developments.

One example to which we have already referred in detail is that of the infinite potential well. This constitutes a situation that would offer different predictions for the same phenomenon by the two theories. Recall from Section 5.2 that Orthodox Quantum Theory does not give the same answer to the question of what a quantum particle with zero net intrinsic angular momentum (i.e. spinless[3]) is doing within the well from that predicted by the Causal Theory. According to Orthodox Quantum Theory, the particle must be in motion (or the Uncertainty Principle would be violated). Whereas, according to the Causal Theory, the particle has zero velocity. Is it possible to conduct experiments to test this? The Uncertainty Principle limits the information obtainable about particle positions and momenta. However, in the Causal Theory, the Uncertainty Principle is a practical not an in-principle limitation.

[3] See Section 6.2 for why this must be the case.

The question to be answered is whether experiments utilising state-of-the-art technology would allow some measurements that 'get around' the practical limitations inherent in older types of experimental arrangements.

The rapidly developing field of Atom Optics was introduced in Section 4.5. It has become, for example, a common technique in Atom Optics to 'cool' and then trap a single atom by the optical dipole force induced by laser irradiation. The dipole force is due to a distortion of an atom's electron distribution caused by a focussed laser beam (Milburn 1996, 67). In the case of an atom trapped in a cavity, it is also possible to ascertain the atom's trajectory by looking at how laser light leaving a cavity is modulated (Walls and Milburn 2008, Section 18.4). Atom Optics provides a new and exciting experimental venue for testing quantum mechanics that might be able to discriminate empirically between the rival quantum theories. Suitable approximations to an infinite well, for example, are now feasible (e.g. Crommie et al. 1993; Dowling and Gea-Banacloche 1995; Meyrath et al. 2005) and open up possibilities for experimental tests on trapped particles or atoms using the techniques of Atom Optics.

If we only want to ascertain whether an atom is in motion or not (as in the case of an infinite well) then measurements of the atom's momentum would suffice. Such tests could be attempted with a (spinless) 'ultra-cooled' atom. We shall now consider one possible experimental setup to test whether an atom trapped in a well is in motion. Imagine a horizontal containment vessel (so that gravity will not affect the atom's motion along the length of the vessel) with evanescent light wave reflectors at each end (Riggs 1999, 3072). The process of total internal reflection (i.e. the reflection of light without transmission from an interface between two media) is used to generate an evanescent light wave. Evanescent light remains close to the interface surface and its intensity decays exponentially (of the order of one wavelength) with the distance from the interface surface. A laser beam that is totally internally reflected away from the interface produces the required evanescent light wave (Milburn 1996, 69; Meystre 2001, 52–53).

The atom would first need to be 'laser cooled' in order to have a sufficiently slow speed to be reflected by an evanescent light

wave and then placed in the containment vessel. Once the atom is in place, any fields or optical forces used initially to hold the atom would be turned off in order not to affect the atom's motion. The lasers used to produce the evanescent waves at each end of the containment vessel could then be turned on. A suitable choice of laser frequency (called 'blue' detuning) will result in evanescent waves that will reflect the atom if it is incident at the ends of the containment vessel without the atom touching the containment vessel's wall (Henkel et al. 1994, 1047). If the atom's speed is low enough, it will be reflected elastically and its momentum will become oppositely directed to what it was before reflection (Cohen-Tannoudji and Dalibard 2005, 156). The resultant *change in momentum* of the atom will then be equal to twice its momentum just prior to reflection by the evanescent light wave. The atom will not absorb any light as it is being reflected by an evanescent wave but will cause a small phase shift in the total internally reflected laser light to occur due to a change in the refractive index of the region just above the interface (Aspect et al. 1995, 4705). The phase shift of the laser light will be proportional to the change in momentum of the atom (Milburn, private communication). A measurement of any resulting phase shift of the reflected laser beam, e.g. by interfering the reflected beam with another (reference) laser (Aspect et al. 1995, 4705) could then be used to calculate the value of this change in momentum and thereby determine if any motion has occurred.

Ideally, if the Causal Theory prediction is correct, one would expect to detect no phase shift in the reflected laser light (i.e. no motion of the atom). However, this would depend on the wave field of the atom not being disturbed once the atom has been placed in the containment vessel. This is essential for, as we have seen in Section 5.6, a disturbance (in the relevant sense) to the wave field will cause a quantum particle to accelerate. If it proved to be the case that such disturbance was unavoidable, then measurements of the phase shift which consistently indicate values of momenta smaller than the minimum value predicted by Orthodox Quantum Theory would also be acceptable as confirming the prediction of the Causal Theory. Note that measurements of these smaller values of momentum are not inconsistent with the limits set by the uncertainty relations.

The experiment described above would be very difficult to conduct in practice. However, it may not be the only method available in Atom Optics to test this prediction of the Causal Theory. The techniques of Atom Optics are also available to test other aspects of quantum mechanics that were previously only considered to be (and to remain) 'thought experiments' as commented on in an experimemental report by S. Maniscalco:

> Quantum mechanics is a theory peppered with counterintuitive and bizarre aspects. For this reason, since its very early days, it has given rise to a heated debate—still far from being concluded—on its interpretation and consequences. … during the last two decades, extraordinary experimental advances in the control and manipulation of single or small numbers of atoms and ions have made it possible to realize experiments which have been considered for a long time as 'gedanken experiments' (Maniscalco 2005, R15).

The history of science shows that measurements that were thought to be impossible (or practically so) in one era, have later become routine. This has proven to be the case with quantum mechanics in a fundamental way. We should leave the possible conduct of tests of quantum mechanics to experimental physicists and take the advice of David Wick when he wrote:

> The moral is clear: be modest about the implications of your theories — and never underestimate the cleverness of the experimentalists (Wick 1995, 130).

Chapter 6

The Exclusion Principle

THE EXCLUSION PRINCIPLE plays an important role in quantum physics and has effects that are almost as profound and as far-reaching as those of the principle of relativity … [the Exclusion Principle] enacts vetoes on a very basic level of physical description.

— Henry Margenau

The reason why the Pauli Exclusion Principle is true and the physical limits of the principle are still unknown.

— National Aeronautics and Space Administration (US) website

Abstract In this chapter, a principle that is absolutely pivotal to the structure of matter but has remained beyond our understanding is examined – the Exclusion Principle. This Principle is crucial to the operation of many physical processes, including all of chemistry. The issues of permutation invariance, the indistinguishability of quantum particles, and the antisymmetrization of the wavefunction are discussed in reference to prior justifications of the Exclusion Principle. The nature of quantum mechanical spin is examined and its connection to the Exclusion Principle is described. This allows a basis for the Exclusion Principle to be presented within the context of the Causal Theory which is particularly well suited to providing this basis as the motion of individual quantum particles in the Causal Theory depends on the quantum state as a whole.

6.1 What is the Exclusion Principle?

Why is it that electrons within an atom do not all collect in the lowest orbital? This is a long-standing, unsolved theoretical problem of atomic physics. In 1925, Wolfgang Pauli published a limited version of the Exclusion Principle from his studies of the fine

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structure of atomic energy levels and the earlier suggestions of E.C. Stoner (Jammer 1966, 143; Duck and Sudarshan 1997, 21–22). This limited version is known as Pauli's Principle and may be stated as follows (Pauli 1925, 776):

- *Pauli's Principle*

  In an atom there cannot be two or more electrons with the same quantum numbers.

Since a set of quantum numbers specifies a unique state of a particular physical system, Pauli's Principle provides a *simple rationale* for the existence of the observed atomic electron 'shells'. A complete and consistent explanation of why Pauli's Principle holds has never been advanced. Indeed, since its inception, the status of Pauli's Principle has been axiomatic. This is well summarised in the following statement by Lindsay and Margenau:

> There is *no way of deducing* Pauli's principle; its validity has to be inferred from its results … (Lindsay and Margenau 1957, 491, italics added).

Pauli's Principle was generalised when it was realised that the Principle applies not just to electrons but to all fermions of the same type. Quantum particles are identical if they have the same mass, electric charge, etc. Fermions are sometimes defined to be those identical quantum particles that, when part of a quantum system consisting of two or more of the same particles (e.g. all electrons), the system has a wavefunction that is antisymmetrical in its form (see below). The generalisation of Pauli's Principle is called the Exclusion Principle (Sudbery 1986, 72; Ballentine 1998, 476):

- *Exclusion Principle*

  In a quantum system, two or more fermions of the same kind cannot be in the same (pure) state.

The antisymmetrical form of the wavefunction is generally taken as a 'brute fact', i.e. as a defining characteristic of fermions or as a feature of nature that cannot be otherwise explained. Both of these characterisations of 'brute fact' are problematic. The history of science indicates that what is taken as a 'brute fact' at one time may be explained after further advances in scientific knowledge occur. Archimedes' Principle (i.e. the upthrust on a body immersed in a fluid is equal to the weight of the fluid displaced by the body), for

example, was originally accepted as a 'brute fact' about the natural world without further explanation. Only after many centuries was it shown that Archimedes' Principle follows from the physical attributes of a fluid, e.g. force, density, and pressure.

Where fermions are defined by their multi-particle system wavefunctions being antisymmetrical, this definition cannot be used for a single particle system since a quantum system with an antisymmetrical wavefunction must contain at least two particles. A better definition is that a fermion is a quantum particle whose system exhibits a half-odd integer value for its intrinsic angular momentum, or spin (see Section 6.2). The latter definition is more basic than the former as it can be given for just one fermion.

The Exclusion Principle acts primarily as a selection rule for non-allowed quantum states and cannot be deduced as a theorem from the axioms of Orthodox Quantum Theory. Pauli himself, admitted (with some frustration) that the Exclusion Principle could not be deduced. He wrote:

> … I was unable to give a logical reason for the exclusion principle or to deduce it from more general assumptions. … in the beginning I hoped that the new quantum mechanics … [would] also rigorously deduce the exclusion principle (Pauli 1947, 136).

This is a serious admission of incompleteness given that the Exclusion Principle is acknowledged as acting at a very basic physical level and having ramifications for most physical phenomena. This lack of a theoretical basis has led to descriptions of the Exclusion Principle such as:

> … one of the *oddest* of the instruments of microphysics (Heilbron 1983, 261, italics added).

6.1.1 The Appeal to the 'Indistinguishability' of Identical Particles

The standard approach in introductory textbooks on quantum mechanics is to justify the Exclusion Principle by appealing to the 'indistinguishability' of identical particles. In Orthodox Quantum Theory, it is assumed that identical particles cannot be distinguished from each other. Suppose we have two non-interacting, identical

quantum particles. Let the first particle have coordinates $\mathbf{x}_1$ and its wavefunction be denoted $\psi_A(\mathbf{x}_1)$. Similarly, let the second particle's coordinates be $\mathbf{x}_2$ with wavefunction denoted $\psi_B(\mathbf{x}_2)$. The composite system consisting of these particles is represented by a single wavefunction, denoted $\psi(\mathbf{x}_1, \mathbf{x}_2)$. This wavefunction is a solution of the two-particle Schrödinger equation and is equal to the product of the individual wavefunctions, i.e.

$$\psi(\mathbf{x}_1, \mathbf{x}_2) = \psi_A(\mathbf{x}_1)\psi_B(\mathbf{x}_2) \qquad (6.1)$$

The 'indistinguishability' argument proceeds by claiming that since the identical particles are indistinguishable, their coordinates merely serve to label the particles and an exchange of such 'labels' cannot be empirically meaningful. This would require the two-particle wavefunction to yield the same probability density regardless of whether the particle 'labels' are exchanged or not. If an exchange of particle 'labels' has no empirical import then the following equalities should hold:

$$|\psi(\mathbf{x}_1, \mathbf{x}_2)|^2 = |\psi_A(\mathbf{x}_1)\ \psi_B(\mathbf{x}_2)|^2 = |\psi_A(\mathbf{x}_2)\ \psi_B(\mathbf{x}_1)|^2 = |\psi(\mathbf{x}_2, \mathbf{x}_1)|^2$$

Using Equation (6.1), it is easily shown that the probability densities found before and after exchange of the particle 'labels' are *not* equal (e.g. French and Taylor 1978, 561). However, this may be corrected by the technique of linearly combining wavefunctions. Since $\psi_A(\mathbf{x}_1)\psi_B(\mathbf{x}_2)$ and $\psi_A(\mathbf{x}_2)\psi_B(\mathbf{x}_1)$ are both solutions of the (two-particle) Schrödinger equation, so is any linear combination of them (as the Schrödinger equation is itself linear). Using this method, the composite system's wavefunction may be expressed as the following two kinds of linear combinations:

$$\Psi(\mathbf{x}_1, \mathbf{x}_2) = (1/\sqrt{2})\ [\psi_A(\mathbf{x}_1)\psi_B(\mathbf{x}_2) \pm \psi_A(\mathbf{x}_2)\psi_B(\mathbf{x}_1)] \quad (6.2)$$

where the factor $(1/\sqrt{2})$ is required for normalisation.

If the sign between the two terms in Equation (6.2) is positive then Ψ is said to be symmetric with respect to the exchange of coordinates as $\Psi(\mathbf{x}_1, \mathbf{x}_2) = \Psi(\mathbf{x}_2, \mathbf{x}_1)$. If the sign is negative then Ψ is said to be antisymmetric with respect to the exchange as $\Psi(\mathbf{x}_1, \mathbf{x}_2) = -\Psi(\mathbf{x}_2, \mathbf{x}_1)$. It is the case that only symmetrical and antisymmetrical

wavefunctions are 'found' in nature (Greenhow 1990, 213; Zettili 2001, 444; Omar 2005, 443). Both these types of wavefunction satisfy the required probability density equality, but only antisymmetrical wavefunctions entail the Exclusion Principle. This is seen if, in Equation (6.2), we take the negative sign and make $\mathbf{x}_1 = \mathbf{x}_2$ then $\psi = 0$, i.e. there is no corresponding quantum state. Note that the presence of the two terms inside the brackets of Equation (6.2) indicates that the particles do not act independently unlike those represented by Equation (6.1). Equation (6.2) shows that the particles are entangled.

A full treatment of the antisymmetry will also take into account a system's spin. The Exclusion Principle arises from the wavefunction of a system of fermions being antisymmetric in its form, as was grasped initially (and independently) by both Heisenberg and Dirac in 1926 (Dirac 1926; Heisenberg 1926). However, the Exclusion Principle is not equivalent to the condition that fermionic systems have antisymmetrical wavefunctions (as asserted in many quantum mechanics texts) but follows from this condition.

The standard textbook argument is not valid for it has been shown elsewhere that the conclusion that the wavefunction of a fermionic system is antisymmetric in form does not follow from the indistinguishability criterion alone. Assumptions in addition to the indistinguishability of identical particles are needed to arrive at the result of antisymmetrical wavefunctions (Harris and Loeb 1963, 244; Messiah and Greenberg 1964, B248–B249; De Muynck and van Liempd 1986, 478; Kaplan 2002, 268), as will be discussed below. Nor does the antisymmetric form of fermionic wavefunctions arise from the requirements of relativistic invariance. This is a justification for the antisymmetric form that is erroneously claimed in the literature as having been conclusively established by Pauli (Reif 1981, 332; Itzykson and Zuber 1987, 149–150). It is only the case that relativistic invariance is merely *consistent* with antisymmetric wavefunctions (Dieks 1990, 134–135; van Fraassen 1991, 384).

The fact that the total wavefunction for a system of fermionic particles takes an antisymmetric form rather than a symmetric one, or a form exhibiting another symmetry, or a form that exhibits no symmetry, has not been satisfactorily explained. The assumption

that fermionic wavefunctions are antisymmetric is added to Orthodox Quantum Theory as an additional postulate (Harris and Loeb 1963, 244; Sudbery 1986, 72). Further, this antisymmetry cannot be given a *physical* explanation within the confines of Orthodox Quantum Theory because the wavefunction is only considered to be an abstract entity that does not represent anything physically real.

6.1.2 'Indistinguishability' and Wave Field Overlap

Discussion on the indistinguishability and identity of quantum particles has a long history in the literature.[1] It remains an area of intense philosophical debate and also of disagreement between physicists and philosophers of physics.[2] Some of the philosophers concerned tend to believe that physicists who comment on these issues are philosophically *naïve* and consequently no attention need be paid to the physicists' views! Although many accounts of identical quantum particles assume that these particles are *always* indistinguishable, it can be forcibly argued that this is not the case (see below). In this book, the criterion for identical particles to be indistinguishable is that the particles' individual wave fields spatially overlap or have spatially overlapped at some particular time in the past. This means that identical quantum particles can be distinguished if they are sufficiently separated (such as when each is in different and well separated atoms) and have remained so for then the overlap of their individual wave fields is zero. The physics literature expresses this condition by referring to the 'overlap of wavefunctions' (Schiff 1968, 364; Eisberg and Resnick 1985, 303; Sakurai 1985, 365–366; Townsend 2000, 341; Omar 2005, 439; Haroche and Raimond 2006, 42–43).

If it is assumed that a composite quantum system's state is described by the simple product of the individual wavefunctions, i.e.

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[1] Readers interested in the background should consult the references in French and Rickles 2003.

[2] More recent additions to the debate include: Pniower 2005; Saunders 2006; French and Krause 2006.

as given by Equation (6.1), then this is also assuming that there is no overlap of their individual wave fields. This can be seen by evaluating the expectation value of the square of the distance between two quantum particles (Griffiths 2005, 207–208). Consider two particles in a combined state with the normalised wavefunction $\Psi(x_1, x_2)$ with coordinates x_1 and x_2 respectively. The distance between the particles is $(x_1 - x_2)$ and the expectation value of the square of the distance is:

$$\langle (x_1 - x_2)^2 \rangle = \int\int \Psi^*(x_1, x_2)\, [(x_1 - x_2)^2]\, \Psi(x_1, x_2)\, dx_1\, dx_2$$

$$= \int\int \Psi^*(x_1, x_2)\, [x_1{}^2 + x_2{}^2 - 2\, x_1\, x_2]\, \Psi(x_1, x_2)\, dx_1\, dx_2$$

$$= \langle x_1{}^2 \rangle + \langle x_2{}^2 \rangle - 2\, \langle x_1\, x_2 \rangle$$

Now if we let $\Psi(x_1, x_2) = \psi(x_1, x_2) = \psi_A(x_1)\psi_B(x_2)$, then we find that:

$$\langle x_1{}^2 \rangle = \int\int \psi^*(x_1, x_2)\, [x_1{}^2]\, \psi(x_1, x_2)\, dx_1\, dx_2$$

$$= \int \psi^*{}_A(x_1)\, [x_1{}^2]\, \psi_A(x_1)\, dx_1 \int \psi^*{}_B(x_2)\, \psi_B(x_2)\, dx_2 = \langle x^2 \rangle_A$$

where $\langle x^2 \rangle_A$ is the expectation value of x^2 in the (single-particle) state denoted A.

The wavefunctions ψ_A and ψ_B are, for convenience, taken to be orthonormal (i.e. wavefunctions that are normalised and have a zero inner product). Likewise we find: $\langle x_2{}^2 \rangle = \langle x^2 \rangle_B$ and $\langle x_1\, x_2 \rangle = \langle x \rangle_A \langle x \rangle_B$, where $\langle x \rangle_A$ is the expectation value of x in the (single-particle) state denoted A, $\langle x \rangle_B$ is the expectation value of x in the (single-particle) state denoted B, etc. Therefore, for $\psi(x_1, x_2) = \psi_A(x_1)\psi_B(x_2)$, we get:

$$\langle (x_1 - x_2)^2 \rangle = \langle x^2 \rangle_A + \langle x^2 \rangle_B - 2\, \langle x \rangle_A \langle x \rangle_B.$$

This is the result for particles that exhibit no symmetry (or antisymmetry) in the form of their wavefunctions i.e. are distinguishable (Griffiths 2005, 208). The wave fields represented by these wavefunctions have *no overlap*.

Now let's find the expectation value of the square of the distance between two identical fermions by using an antisymmetrical wavefunction, i.e. let $\Psi(x_1, x_2) = \Psi(x_1, x_2) = (1/\sqrt{2})\,[\psi_A(x_1)\psi_B(x_2) - \psi_A(x_2)\psi_B(x_1)]$. Then, as above, we have:

$$\langle (x_1 - x_2)^2 \rangle = \langle x_1^2 \rangle + \langle x_2^2 \rangle - 2\,\langle x_1\, x_2 \rangle$$

Using $\Psi(x_1, x_2)$, we find that:

$$\langle x_1^2 \rangle = \int\int \Psi^*(x_1, x_2)\,[x_1^2]\,\Psi(x_1, x_2)\,dx_1\,dx_2$$

$$= \tfrac{1}{2}\iint [\psi_A^*(x_1)\psi_B^*(x_2) - \psi_A^*(x_2)\psi_B^*(x_1)]\,x_1^2$$

$$\times\,[\psi_A(x_1)\psi_B(x_2) - \psi_A(x_2)\psi_B(x_1)]\,dx_1\,dx_2$$

$$= \tfrac{1}{2}\int x_1^2\,|\psi_A(x_1)|^2\,dx_1 \int |\psi_B(x_2)|^2\,dx_2$$

$$+ \tfrac{1}{2}\int x_1^2\,|\psi_B(x_1)|^2\,dx_1 \int |\psi_A(x_2)|^2\,dx_2$$

$$- \tfrac{1}{2}\int x_1^2\,[\psi_A^*(x_1)\,\psi_B(x_1)]\,dx_1 \int [\psi_B^*(x_2)\psi_A(x_2)]\,dx_2$$

$$- \tfrac{1}{2}\int x_1^2\,[\psi_B^*(x_1)\,\psi_A(x_1)]\,dx_1 \int [\psi_A^*(x_2)\psi_B(x_2)]\,dx_2$$

The last two terms are zero due to ψ_A and ψ_B being orthonormal wavefunctions. This gives the result:

$$\langle x_1^2 \rangle = (1/2)\,\left[\,\langle x^2 \rangle_A + \langle x^2 \rangle_B\,\right]$$

Likewise we find that: $\langle x_2^2 \rangle = (1/2)\,\left[\langle x^2 \rangle_B + \langle x^2 \rangle_A\right]$ and $\langle x_1\ x_2 \rangle = \langle x \rangle_A\,\langle x \rangle_B - |\langle x \rangle_{AB}|^2$ where the quantity $\langle x \rangle_{AB} = \int x\,\psi_A^*(x)\,\psi_B(x)\,dx$, is a measure of the overlap between individual wave fields represented by ψ_A and ψ_B. Therefore, we find by using $\Psi(x_1, x_2) = (1/\sqrt{2})\,[\psi_A(x_1)\psi_B(x_2) - \psi_A(x_2)\psi_B(x_1)]$, we get:

$$\langle (x_1 - x_2)^2 \rangle = \langle x^2 \rangle_A + \langle x^2 \rangle_B - 2\,\langle x \rangle_A \langle x \rangle_B + 2\,|\langle x \rangle_{AB}|^2$$

If there is no overlap then $\langle x \rangle_{AB} = 0$ (Griffiths 2005, 209) and the above expression for $\Psi(x_1, x_2)$ would reduce to: $\psi_A(x_1)\psi_B(x_2)$.

The fermions would then be distinguishable. In order for this to be the case, the particles must be widely separated and have remained so.

In addition, those that dispute the argument that identical quantum particles can be distinguished by means of their spatial relations do so despite the fact that distinguishability can be demonstrated in experiments and can be shown to be lost when an overlap of wave fields occurs (Bongs and Sengstock 2004, 917). This has been described by Cornish and Cassettari in relation to the 'ultra-cooling' of a quantum gas:

> The de Broglie wavelength [of a gas, λ_{dB}] … characterizes the degree of position uncertainty (or fuzziness) associated with the thermal momentum distribution. The colder the gas, the longer λ_{dB}. If the gas is cooled to the point where λ_{dB} is comparable with the interatomic spacing, the atomic wave packets 'overlap' and the atoms become indistinguishable … (Cornish and Cassettari 2003, 2700).

Since much has been made of the indistinguishability of identical particles and a great deal of attention has been given to the notion, we shall briefly review the main argument. This argument claims that indistinguishability extends to encompass what is called the Permutation Invariance Postulate (van Fraassen 1991, 381):

- *Postulate of Permutation Invariance*

  If φ is the state of a composite system whose components are identical particles, then the expectation value of any observable A is the same for all permutations of φ.

Permutation Invariance allows for quantum states that are symmetric, antisymmetric, and of higher symmetry. This is an important point – Permutation Invariance does not restrict states to just symmetric and antisymmetric ones, nor does it assign a type of particle to any particular symmetry class (Massimi 2005, 154). The restriction to just symmetric and antisymmetric states requires accepting another assumption called the Symmetrization Postulate (Omar 2005, 439):

- *Symmetrization Postulate*

  The only possible states of a system of identical particles are described by state vectors (or wavefunctions) that are either completely symmetrical or completely antisymmetrical.

The Symmetrization Postulate is *not* implied by the indistinguishability of identical particles (Massimi 2005, 155; Ballentine 1998, 475) but by the results of experiments, i.e. the Symmetrization Postulate is an empirical rule (Omar 2005, 444; Ballentine 1998, 475). Some advance in placing the Symmetrization Postulate on a theoretical footing in the context of the Causal Theory has been made by Brown and his colleagues (Brown et al. 1999), although purely on a mathematical basis. However, this is not the only way to explain the restriction to particular symmetry classes (Huggett 1999, 326). Indeed, inferences based only on permutation invariance carry little weight in realist versions of quantum mechanics, as has been acknowledged by Huggett:

> … scientific realists cannot accept, as a legitimate argument form, inferences from the unobservability of a distinction to the irreality of the distinction … (Huggett 1999, 335).

Symmetrization is not mandatory and the problem can be dealt with by other approaches. In this book, a realist approach will be pursued which attempts to explain the antisymmetrical form of the wavefunction of a fermionic system using physical arguments. This will, therefore, offer a very different account to those that stress the notions of indistinguishability and/or permutation invariance.

The antisymmetry of the wavefunction of a fermionic system constitutes a conceptual problem for the Causal Theory since, if the wave field is a physical field that propagates through space, it should be able to be represented by wavefunctions that do not have any particular symmetry (or antisymmetry). The arguments presented below do not depend on identical particles of the same kind being indistinguishable. In any case, the criterion of 'indistinguishability' itself fails within the context of the Causal Theory. Although the particles are identical in that they have the same mass, charge, etc., they can be distinguished in the Causal Theory by their individual trajectories (Holland 1993, 284). What will be required is that individual wave fields physically overlap for particles that form a single quantum system. This is consistent with the arguments presented above.

6.2 Quantum Mechanical Spin

Intrinsic angular momentum (otherwise known as 'spin') is a characteristic of quantum systems that is very relevant to the Exclusion Principle. We have already seen that the Exclusion Principle prescribes that if the fermions of a particular physical system share the same set of quantum numbers (and this includes the spin quantum number) then they cannot be at the same location. A coherent account of why the Exclusion Principle holds will require a realistic explanation of spin.

The initial concept of spin, as formulated by Uhlenbeck and Goudsmit in 1925, has its origin in the experiments of Stern and Gerlach in which a beam of silver atoms was split in two by passage through a non-uniform magnetic field (Stern and Gerlach 1922a; 1922b). Uhlenbeck and Goudsmit proposed that an electron had a magnetic dipole moment which they explained using the classical idea of an extended particle (in this case, an electron) spinning about an axis through its centre (Uhlenbeck and Goudsmit 1926). They used this idea to explain the results of the Stern-Gerlach experiments, although the concept of a spinning particle was suggested earlier by R. Kronig (Jammer 1966, 146–147). However, it has become clear that what is called the 'spin of a quantum particle' is *not* the rotational angular momentum of a spinning particle. In other words, spin cannot be due to an extended body rotating about an axis through its centre of mass. The reasons against the axial rotation explanation are readily provided:

- the rotation of an extended particle would not require an additional variable for its specification;
- the spin's vector does not depend on the particle's position and momentum;
- angular momentum due to rotation about the centre of mass cannot take half-odd-integer values (Sudbery 1986, 138); and
- the rate of rotation required to give results in agreement with experiment would need tangential velocities exceeding the speed of light in vacuum (Jammer 1966, 149–150).

The rotational characterisation has merely assisted in 'picturing' the extra degree of freedom (i.e. spin) required for an accurate description of quantum states. What has also become clear about quantum mechanical spin is that total spin is a conserved quantity and that the square of the spin operator commutes with all other dynamical operators. These two points together with the other characteristics of spin listed above imply that spin must be (in some sense) internal to a quantum system. This has led some theorists to speculate that spin results from an internal structure of the particle (Saxon 1968, 317; Penrose 1989, 341). Yet, it does not follow that because spin is internal to a quantum system that it must be due to the particle's structure. We shall return to this issue below.

Pauli claimed that the quantum mechanical spin has a discreteness that is not describable in classical terms, e.g. spin for electrons has two discrete values. In 1927, he introduced the equation which carries his name, in order to accommodate the spin variable of the electron in non-relativistic quantum mechanics. Erwin Schrödinger had postulated his scalar wave equation in 1926. The Pauli (or Pauli-Schrödinger) equation for a single spin-½ particle (i.e. with the third component of the spin along an arbitrary axis of value $\hbar/2$), of mass m, electric charge e, and magnetic moment μ, has a two-component wavefunction:

$$\Psi = (\psi_a) = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

and is given by:

$$i\hbar(\partial\Psi/\partial t) = \mathbf{H}\Psi$$

where the Hamiltonian operator is:

$$\mathbf{H} = \frac{-\hbar^2}{2m}\left[\nabla - \frac{ie}{\hbar c}\mathbf{A}\right]^2 + \mu\,\mathbf{B}\cdot\boldsymbol{\sigma} + eA_o + V$$

with $\mathbf{A}$ and A_o being the electromagnetic potentials, $\mathbf{B} = \nabla \times \mathbf{A}$ is an external magnetic field, c is the speed of light in vacuum, $i = \sqrt{-1}$, and V is a (classical) scalar potential (Davydov 1976, 258). The vector quantity $\boldsymbol{\sigma}$ has Pauli's 'spin matrices' as its components:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},\ \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

where $\sigma^2 = \sigma_x^2 + \sigma_y^2 + \sigma_z^2$. These spin matrices are operators that represent the spin observables, e.g. the z-component of spin would be given by: $s_z = \frac{1}{2}\hbar\sigma_z = \frac{1}{2}\hbar \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. The eigenfunctions of spin represent the states of 'spin up' and 'spin down' are given respectively by the following two-component wavefunctions (called spinors): $\chi_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\chi_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. The general expression for a system that is not in an eigenstate of 'spin up' or 'spin down' is the superposition: $\chi = a\chi_1 + b\chi_2$ where a, b are complex numbers. These spinor wavefunctions give the required measured values of spin, i.e. $\pm (\hbar/2)$ with certainty when the system is in an eigenstate, or when in a superposition, with probability $|a|^2$ for 'spin up' and $|b|^2$ for 'spin down'.

In order to meet the need for incorporating spin into Orthodox Quantum Theory, much attention has been given to developing spinor representation and spin algebra as a way of dealing with an aspect of quantum systems (i.e. spin) that was not properly understood. The late Sir Karl Popper, for example, acknowledged this lack of comprehension of the nature of spin when he wrote:

> Spin … differs from what we usually … mean by spin. Spin is really something very queer … (Popper 1982, 24).

Although it is the case that spinor methods have been *formally* successful, they are really a technical means of not addressing the underlying nature of the spin phenomenon. Indeed, the Pauli equation does not provide any insight into the origin or characteristics of spin, as noted by Lindsay and Margenau:

> Pauli's theory does not explain the origin of the spin, nor does it give any reason for its magnitude. It merely provides a method for incorporating it into quantum mechanics (Lindsay and Margenau 1957, 487).

We shall address the underlying nature of quantum mechanical spin at the end of this section.

Pauli's approach does have its uses though, for even inadequate formulations can lead to important insights. In the Causal Theory, a version of the Pauli Equation has also been developed (Bohm et al. 1955; Bohm and Schiller 1955; Dewdney et al. 1988).

The approach is to let the wavefunction be represented by a spinor of the form:

$$\psi = \mathrm{R}e^{i\chi/2}\begin{pmatrix}\cos(\theta/2)\,e^{i\phi/2}\\ i\sin(\theta/2)\,e^{-i\phi/2}\end{pmatrix}$$

where (ϕ, θ, χ) are the Euler Angles for a rigid body undergoing rotation. This leads to a quantum potential given by (Holland 1993, 391):

$$Q = -(\hbar^2/2m)(\nabla^2 R)/R + (\hbar^2/8m)\,[(\nabla\theta)^2 + (\sin^2\theta)\,(\nabla\phi)^2] \quad (6.3)$$

where the first term is the usual quantum potential and the second term is spin-dependent. This approach has proved quite useful and provides a better account of atomic processes than is possible with Orthodox Quantum Theory. The lowest energy level of hydrogen, for example, in the context of the Causal Theory was originally dealt with by Bohm in the first of his 1952 papers. He argued that an electron in this state is at rest since the Coulomb force was exactly balanced by the quantum mechanical force but with a statistical distribution of possible positions that would be found on measurement (Bohm 1952a, 173). Indeed, the statement that an electron would be at rest has been used to criticise the Causal Theory (Humphreys 1968, 229–230; Audi 1973, 74).

In 1955, Bohm and his co-workers published two articles which incorporated spin into the Causal Theory. They should have explicitly documented that the presence of a spin-dependent term in the quantum potential would produce a different answer. The spin-dependent term gives rise to a situation where the Coulomb and quantum forces do not balance each other. The electron would, therefore, not be at rest. One might have expected that P.R. Holland would have dealt with this problem in his detailed text on the Causal Theory by taking account of the spin-dependent term as given in Equation (6.3). Alas, Holland's comments were not helpful either and might sit better with an advocate of Orthodox Quantum Theory. Holland writes:

> Readers who … exclaim 'I don't believe it' when confronted with a stationary electron in m = 0-states should … be prepared to put aside expectations based on acquaintance with classical physics … (Holland 1993, 155).

The solution of the motion of the electron in a hydrogen atom was not worked out in sufficient detail until 2002/2003 when spin-dependent trajectories for several hydrogen eigenstates and transitions were calculated by Caroline Colijn and E.R. Vrscay (Colijn and Vrscay 2002; 2003). Here there is a spin-dependent term in the expression for the momentum of a particle with spin **s**. The total momentum is given by: $\mathbf{p} = \nabla S + \nabla(\log \rho) \times \mathbf{s}$ (where $\rho = R^2$) and yields non-stationary trajectories. Figure 6.1 (below) is an example of such trajectories (Colijn and Vrscay 2002, 338).

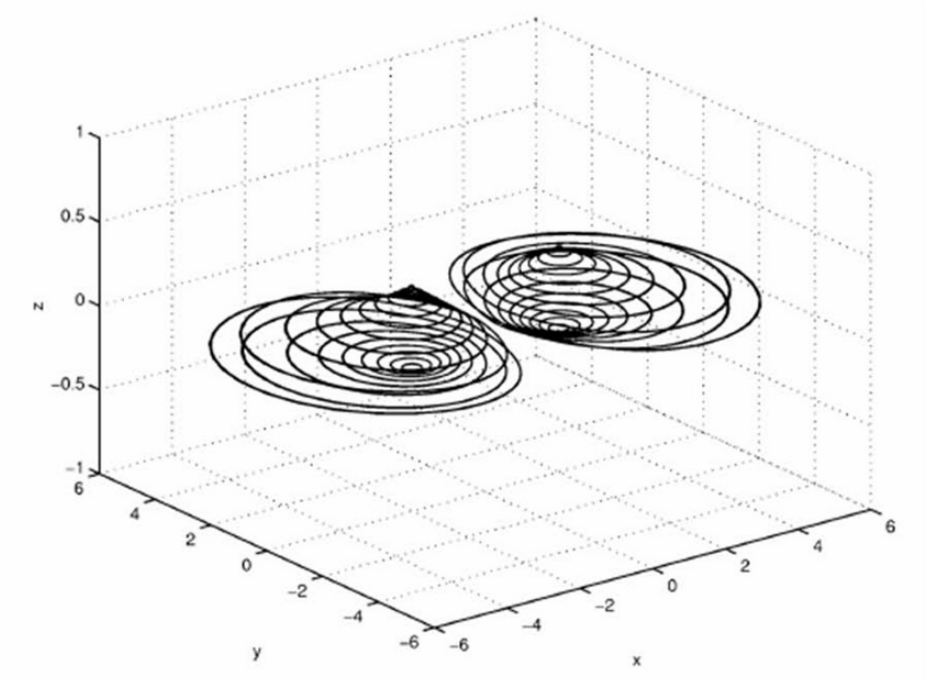

Fig. 6.1 Spin-dependent trajectory for the hydrogen $2p_X$ state. (Used with permission of Elsevier B.V.)

The trajectories in Fig. 6.1 compare well with the calculated probability densities for the $2p_X$ orbital and imaging of such orbitals, as shown in Fig. 6.2 (Herz et al. 2003, 45301–45304).

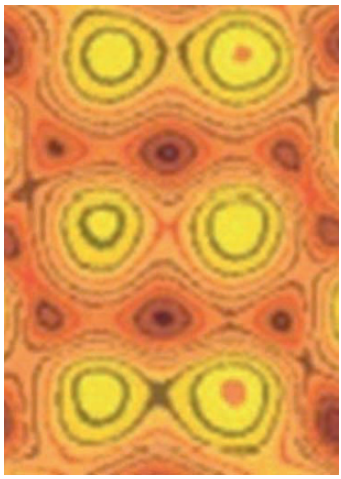

Fig. 6.2 Imaged 2p orbitals. (Used with permission of the American Physical Society)

This research has been extended by Stenson and Van Huele, as may be seen in Fig. 6.3 and 6.4 below for the hydrogen 1s and $2p_Z$ states (Stenson and Van Huele 2004).

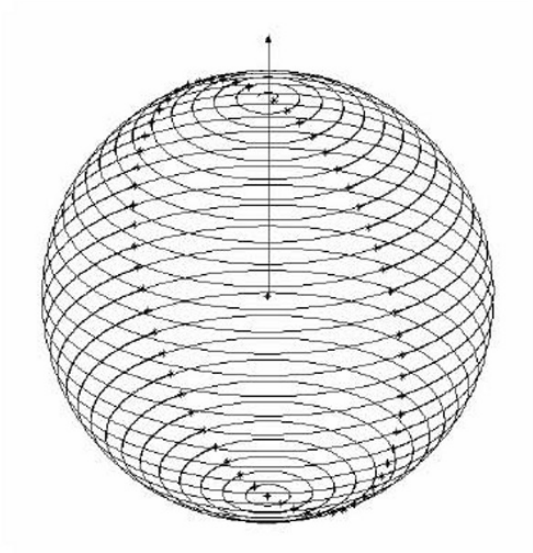

Fig. 6.3 Several spin-dependent trajectories for the hydrogen *1s* state each corresponding to a different initial position (spin direction indicated arrow). (Used with permission of the *Journal of the Idaho Academy of Science*)

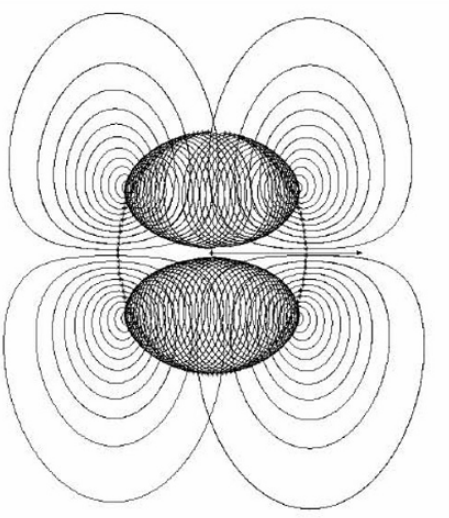

Fig. 6.4 Several trajectories in the hydrogen $2p_z$ state showing the nodal plane (spin direction indicated by arrow). (Used with permission of the *Journal of the Idaho Academy of Science*)

The above visualisations are completely at odds with statements from some of the founders of Orthodox Quantum Theory, such as Heisenberg who wrote:

> In classical physics the aim of research was to investigate objective processes occurring in space and time … In the quantum theory, however, the situation is completely different. The very fact that the formalism of quantum mechanics cannot be interpreted as visual description of a phenomenon occurring in space and time shows that quantum mechanics is in no way concerned with the objective determination of space-time phenomena (Heisenberg 1965, 296).

Figures 6.1, 6.3, and 6.4 and the associated calculations used to generate them indicate just how wrong the above comments by Heisenberg have proved to be. The discussion of spin-dependent trajectories also disposes of alleged flaw ⑧ in Section 1.3 (i.e. that the Causal Theory cannot incorporate spin) which may now be clearly seen as false.

Although the causal version of the Pauli equation is very useful for calculation purposes, it will not be pursued here as its ontology (i.e. a rotating particle) cannot be physically realisable for the reasons cited above. So what is quantum mechanical spin? It was previously mentioned that some theorists speculated that quantum

mechanical spin arises from the (presumed) internal structures of quantum particles. However, it was pointed out in Section 4.2 that some quantum particles, e.g. electrons, do not appear to have such internal structure. Most quantum systems (including electrons) have non-zero spins and since their spin cannot originate from internal particle structure (if they do not have any such structure), we have to look elsewhere for the origin of quantum mechanical spin.

We are now in a position to make an important inference about the nature of spin, based on prior findings in this book and the existence of a spin-dependent term in the quantum potential. In the Causal Theory, the wavefunction represents an objectively existing field that propagates through space as a wave and shares characteristics found with other types of waves. Spin can also be seen to be a property of the wave field because the quantum potential (which represents a portion of the wave field's energy) has a spin dependence, as shown by equation (6.3). The conclusion that spin is a property of the wave field furnishes a realistic description of spin and will assist in providing a basis for the Exclusion Principle.

It turns out that the notion that spin is not a property of particles but of waves is not new. In respect to electromagnetic waves, the conclusion that spin is a wave property has been around for years – spin is part of an electromagnetic wave's angular momentum, the part which is dependent on the wave's polarisation (Belinfante 1939; Wallace 1972, 288–291; Jackson 1975, 333; Ohanian 1986). This reveals the connection between spin and polarisation. Consider, for example, a circularly polarised plane electromagnetic wave with a vector potential **A** given by:

$$\mathbf{A} = (\hat{\mathbf{x}} \pm i\,\hat{\mathbf{y}})\,(iE_o/\omega)\exp[i\,\omega(t - x/c)]$$

where E_o is the electric field strength, ω is the angular frequency, c is the speed of light in vacuum, $i = \sqrt{-1}$, and $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$ are the usual Cartesian unit vectors. The polarisation dependent part of the wave's angular momentum (i.e. its spin s) is:

$$s = \pm \frac{1}{\mu_o c^2} \int \frac{E_o^{\,2}}{\omega}\,\hat{\mathbf{z}}\,d^3\mathbf{x}$$

where μ_o is the (magnetic) permeability of free space constant, $\hat{\mathbf{z}}$ is a unit vector in the z direction, and the ± sign indicates the dependence on polarisation (Ohanian 1986, 502). Further, the explanation of spin as part of a wave's angular momentum has been extended to electrons and other fermions (Gsponer 2003 and references therein) as Ohanian has commented:

> The lack of a concrete picture of the spin leaves a *grievous gap* in our understanding of quantum mechanics … spin could be regarded as due to a circulating flow of energy, or a momentum density in the electron wave field … this picture of the spin is valid not only for electrons … (Ohanian 1986, 501, italics added).

Similar to the case of an electromagnetic wave, wave fields will also have states of polarisation. It is obvious that wavefunctions in non-relativistic quantum mechanics represent scalar waves when describing spinless quantum systems. It might be objected, therefore, that if wave fields have states of polarisation, then quantum mechanical wavefunctions would have to represent vector waves and this might conflict with the representation of quantum systems with spin by spinors. The crucial word here is 'represent' for there is more than one formal way to achieve this. In particular, either vector waves or scalar waves plus spinors can be used. Indeed, spinors are used in this way in classical wave theory (Rogalski and Palmer 2006, 401–403). Therefore, as previously noted, the representation of spin by the use of spinors is only a method of dealing with the spin phenomenon without needing an understanding of its fundamental nature. (A vector wave approach is also possible, however this will not be developed here.)

The connection between spin and wave field polarisation accounts for the empirical fact that the spin related to protons, electrons and neutrons, i.e. spin ½ fermions, has a two-valued discreteness (commonly called 'spin-up' and 'spin-down'). The observed two-valued discreteness related to spin ½ fermions is determined by the polarisation state of their wave fields. The explanation of spin as the polarisation dependent part of the wave field's angular momentum has not only *not been accepted* by most physicists who are aware of this explanation, it is almost universally ignored. One principal reason for the non-acceptance is that, in Orthodox Quantum Theory, the wave field is not considered to be a real field.

6.3 The Exclusion Principle in the Causal Theory

We shall delay discussion of why fermionic wavefunctions are antisymmetric in their form until the next section. If we tentatively accept their antisymmetric form (in the context of the Causal Theory) then it is relatively straight forward to provide a causal mechanism to explain the Exclusion Principle.

Consider first the response within Orthodox Quantum Theory to the consequences of assuming an antisymmetrical wavefunction for a system of fermions. This is sometimes expressed in textbooks by statements such as electrons in an atom 'avoid one another' (or words to that effect) with no further qualification (e.g. French and Taylor 1978, 569; Sakurai 1985, 365; Penrose 2004, 596). Very occasionally one finds a statement in the literature that acknowledges that there is no satisfactory explanation in Orthodox Quantum Theory:

> … it is still quite mysterious why or how fermions with common values in their internal degrees of freedom [i.e spin] will resist being brought close together, as in the dramatic example of the formation of neutron stars, this resistance resulting in an effective force, completely different from the other interactions we know … (Omar 2005, 445).

It is more common for texts to gloss over the lack of a proper explanation in Orthodox Quantum Theory for fermions 'avoiding each other' by making some kind of covering statement. Consider, for example, the following assertion:

> A system in an antisymmetric state … exhibits what is called a *statistical repulsion* … (Park 1974, 409, italics in original).

Strange notions such as 'statistical repulsion' come from dismissing any possibility of a realistic, causal description of quantum phenomena and leaves this kind of correlated particle motion completely unexplained.

However, the causal description of a many-particle quantum system provides an explanation, as Holland has previously stated:

> The symmetrization or antisymmetrization of the wavefunction has nothing to do with the 'indistinguishability', but in fact, implies the introduction of forces between the particles making up the system, which bring about correlations in their motion (Holland 1993, 284).

In Chapter 3, we saw that in the Causal Theory, the trajectories of quantum particles do not pass through the nodes of the wave field. An antisymmetrical wavefunction that represents a wave field obviously will have nodal points. This led Holland to make the following conclusion:

> … the exclusion principle is incorporated into the [Causal] quantum theory of motion in that particles cannot pass through nodes (Holland 1993, 310).

Holland's conclusion may be better appreciated from a dynamical perspective which provides a causal description of particle motion in terms of the effects of the quantum mechanical force. Indeed, Bohm himself was initially very much in favour of such an explanation. He wrote:

> … the [quantum mechanical] force between any two particles may depend significantly on the location of every other particle in the system. An example of such a force is given by the exclusion principle (Bohm 1952a, 175).

Let's see how this is achieved in the Causal Theory. The study of atomic electrons shows that a total antisymmetrical wavefunction can occur in a number of ways (ignoring the interaction between the electrons). In the case of two electrons, we can label them by the numerals 1, 2 and suppose that particle 1 is in state A at position $\mathbf{x}_1$ and particle 2 is in state B at position $\mathbf{x}_2$. The states of interest are those for which the electrons 'avoid each other'. There are three of these which have the collective name of the 'triplet state'. The total 'z-component of spin' for these three states has values of $\hbar$, $-\hbar$, and 0 respectively. Since the Hamiltonian for a system of identical quantum particles does not involve spin operators (in the absence of a magnetic field), their wavefunctions may be given as a product of a spatial component and a spin component (Davydov 1976, 297–298; Eisberg. and Resnick 1985, 303; Greenhow 1990, 212). These are, respectively, as follows:

$$\Psi = \{\psi_A(\mathbf{x}_1)\psi_B(\mathbf{x}_2) - \psi_A(\mathbf{x}_2)\psi_B(\mathbf{x}_1)\}\ \alpha(1)\,\alpha(2) \qquad (6.4)$$

$$\Psi = \{\psi_A(\mathbf{x}_1)\psi_B(\mathbf{x}_2) - \psi_A(\mathbf{x}_2)\psi_B(\mathbf{x}_1)\}\ \beta(1)\,\beta(2) \qquad (6.5)$$

$$\Psi = \{\psi_A(\mathbf{x}_1)\psi_B(\mathbf{x}_2) - \psi_A(\mathbf{x}_2)\psi_B(\mathbf{x}_1)\}\ \{\alpha(1)\beta(2) + \alpha(2)\beta(1)\} \qquad (6.6)$$

where ψ_A, ψ_B are the spatial components of the wavefunctions of particles 1 and 2; α, β denote 'spin up' and 'spin down' respectively; and the normalisation factors have been ignored (Enge et al. 1972, 252; Greenhow 1990, 213). The values of the spin components α and β are discrete, independent of position, and each is an eigenfunction of the z-component of spin. Then, in the above notation, $\psi_A(\mathbf{x}_2)$ is the value of ψ_A at position $\mathbf{x}_2$, $\psi_B(\mathbf{x}_1)$ is the value of ψ_B at position $\mathbf{x}_1$, $\alpha(1)$ is 'spin up' applied at particle 1's position, $\beta(2)$ is 'spin down' applied at particle 2's position, etc. Equations (6.4) and (6.5) describe situations where the spins are the same for both electrons (both 'spin up' or both 'spin down'). Note that the above equations all have antisymmetrical spatial components, so that $\Psi = 0$ if $\mathbf{x}_1 = \mathbf{x}_2$.

We can rewrite Equation (6.6) to incorporate generalised spins χ_A, χ_B for particles 1, 2 respectively so that these may include the possibility of spin states which consist of superpositions of 'spin-up' and 'spin-down', e.g. $\chi = a\alpha + b\beta$, where a, b are numbers with $|a|^2 + |b|^2 = 1$. Then the general two-particle, antisymmetrical wavefunction is:

$$\Psi = \{\psi_A(\mathbf{x}_1)\psi_B(\mathbf{x}_2) - \psi_A(\mathbf{x}_2)\psi_B(\mathbf{x}_1)\}\{\chi_A(1)\chi_B(2) + \chi_A(2)\chi_B(1)\} \quad (6.7)$$

(again suppressing the normalisation constant). Now let the spatial part of Ψ be given by:

$$\psi = \psi_A(\mathbf{x}_1)\psi_B(\mathbf{x}_2) - \psi_A(\mathbf{x}_2)\psi_B(\mathbf{x}_1) = \mathrm{Re}\, e^{iS/\hbar} \quad (6.8)$$

When $\psi = 0$, the amplitude R in equation (6.8) must be zero (since $e^{iS/\hbar}$ cannot be zero, by definition). Thus, as a nodal region of the wave field is approached, the value of R will tend to zero. The (repulsive) quantum mechanical force on each particle is:

$$\mathbf{F}_j = (d\mathbf{p}_j/dt) = -\nabla_j Q \; (j = 1, 2) \quad (6.9)$$

where

$$Q = -\left(\frac{\hbar^2}{2mR}\right)\left(\nabla_1^2 R + \nabla_2^2 R\right) + \text{spin dependent terms} \quad (6.10)$$

Substitution of Equation (6.10) into equation (6.9) and ignoring the contribution of the spin dependent terms (since the spatial terms will dominate as R tends to zero) gives:

$$\mathbf{F}_j = \left(\frac{\hbar^2}{2m\,R^2}\right) \sum_{k=1}^{2} \left[R\,\nabla_j(\nabla_k^2 R) - (\nabla_j^2 R)(\nabla_k R)\right] \quad (6.11)$$

It can be seen from Equation (6.11) that as $R \to 0$, $\mathbf{F}_j \to \infty$. The force $\mathbf{F}_j$ exerted by the wave field on the two fermions prevents them coming into close proximity of each other when their spins are the same (i.e. in cases where the spatial part of the wavefunction is antisymmetric). However, we would not expect literal infinities to occur, only that $\mathbf{F}_j$ can become quite large. The numerator terms in Equation (6.11), for example, may serve in some instances to cancel out the 'blowing up' of (1/R) (Holland 1993, 227). Also, in some cases of motion in external potentials there are instances where there is compensation due to the external potential resulting in a finite value for $\mathbf{F}_j$ (Belinfante 1973, 187). More generally, the dynamics as shown by the Causal Theory prevent fermions occupying the same quantum state. This arises as a natural consequence in the Causal Theory.

In Section 5.8, the interesting fact was presented that, although the quantum mechanical force is not generally accepted by the physics community, a (so-called) 'Pauli Force' has been postulated/acknowledged by some researchers because of observed physical effects (Simons and Bloch 1973, 2755; Apkarian and Schwentner 1999, 1484; Kanorsky et al. 1995, 3645). These effects are not just restricted to electrons 'avoiding each other' but include the creation of 'cavities' in liquid helium which are occupied by electrons or by neutral atoms (Günther et al. 1995, 395). In solids, similar 'cavities' are called Fermi Holes:

> … in terms of the short-range Pauli repulsive force between parallel-spin electrons … each electron creates a Fermi hole around itself which is due to the repulsion of other electrons with the same spin polarization (Payami 2001, 4133–4134).

This 'Pauli Force' is just the quantum mechanical force under a different name!

If we consider two electrons moving in an atom's electric field in the context of Orthodox Quantum Theory we find that, in addition to the Coulomb potential, another potential has to be postulated as a pragmatic means of dealing with the circumstances of the electrons 'avoiding each other'. In the literature, it is called the 'exchange potential' and, in the usual quantum mechanical notation, is given by the following integral (Gasiorowicz 1974, 289):

$$\pm \frac{e^2}{4\pi\varepsilon} \int\int \psi^*_{\mathrm{A}}(\mathbf{x}_1)\psi^*_{\mathrm{B}}(\mathbf{x}_2)\left[\frac{1}{|\mathbf{x}_1 - \mathbf{x}_2|}\right]\psi_{\mathrm{A}}(\mathbf{x}_1)\psi_{\mathrm{B}}(\mathbf{x}_2)\, d^3\mathbf{x}_1\, d^3\mathbf{x}_2$$

where e is the electronic charge, ε is the electric permittivity constant, $\mathbf{x}_1$ and $\mathbf{x}_2$ are the particle coordinates, and the sign depends on the 'relative orientation' of the spins. Although this exchange term appears in the expression for total energy, it is not considered in Orthodox Quantum Theory to originate from a real (i.e. physical) potential and is justified as merely a formal expression of the effect of the Exclusion Principle:

> … a term having no classical analogue, called the *exchange potential*. This exchange term has its origin in the Pauli Principle and may be regarded as an expression of an effective repulsion of electrons with the same spin.
> … There is no real 'potential' in the *N*-electron problem corresponding to this exchange repulsion … The essentially nonclassical nature of the exchange potential is clear, since it takes a 'nonlocal' form … (Rybicki and Lightman 1979, 245, italics in original).

The 'exchange potential' is necessary for the accurate description of an N-electron system in Orthodox Quantum Theory but, like other aspects of the theory, has no physical explanation. The 'effective repulsion' and the 'exchange potential' are explained in the Causal Theory by the existence of a quantum mechanical force on each particle and the extra energy due to the effects of the quantum potential. Holland has explained such effects of the quantum potential as follows:

> … we may say that *classical potentials have nonclassical effects in quantum mechanics because their influence is made manifest in the motion of particles via the mediating role of the quantum potential* (Holland 1993, 81, italics in original).

This extra energy is especially large in cases of white dwarf stars and neutron stars. These stellar objects are held from complete gravitational collapse only by the action of the Exclusion Principle (Doughty 1990, 132; Paty 2003, 462). Since the electrons and

neutrons in these stars need enormous amounts of energy to resist gravity, the operation of the Exclusion Principle in such stars has been described as 'a huge energy storage mechanism' (Clark 2005, 30), but without any further explanation. The existence of the wave field and the quantum potential provide such an explanation which accounts for stability of these stars and the otherwise enigmatic existence of the energy necessary for them to avoid total gravitational collapse.

6.4 A Basis for the Exclusion Principle

All quantum mechanical textbooks describe the effects of the Exclusion Principle but its explanation is either avoided or put down to symmetry considerations. The importance of the Exclusion Principle as a foundational pillar of modern physics cannot be overstated for atomic structure, the rigidity of matter, stellar evolution, and the whole of chemistry depends on the operation of the Exclusion Principle. Given its absolutely crucial nature to understanding physical processes, an explanation of why the Exclusion Principle holds is long overdue. In order to provide a basis for the Exclusion Principle, it needs to be explained why the wavefunction of a fermionic system takes an antisymmetric form. We have already noted that this antisymmetry does not follow from the indistinguishability of identical particles nor from satisfying relativistic invariance. If a plausible account of the antisymmetry of wavefunctions of fermionic systems was provided, this would lay a much needed basis for the Exclusion Principle as well as resolving the conceptual problem for the Causal Theory of the existence of the antisymmetric form.

In 1946, Pauli expressed the following thoughts about the history of the Exclusion Principle to that time and its status:

> … [the exclusion principle] remains an independent principle which excludes a class of mathematically possible solutions of the wave equation. … The history of the exclusion principle is thus already an old one, but its conclusion has not yet been written. … it is not possible to say beforehand where and when one can expect the further development … (Pauli 1946, 215).

The considerable magnitude of the task of 'deducing' the Exclusion Principle should not be underestimated. Even a brief survey of the history of the subject matter (e.g. van der Waerden 1960; Jammer 1966, Chapter 3; Duck and Sudarshan 1997, Chapters 1, 2 and 4; and Massimi 2005) indicates that this problem is not going to be *readily* solvable in a mathematically rigorous way. Further, as Pauli said, it is not possible to predict where further development will occur. Granting this, we shall only attempt in what follows to shed some light on why wavefunctions of simple fermionic systems are antisymmetric and thereby suggest a basis for understanding the Exclusion Principle.

In the context of the Causal Theory, the antisymmetrical form should be explicable in terms of the well established behaviour of physical waves as the wave field is a physically real wave. We begin with a couple of important observations about the Exclusion Principle. First, if fermions are localised particles (as postulated in the Causal Theory) then the Exclusion Principle cannot operate as a local causal effect. It must be physically manifested as non-local and holistic as explained by Gibbons:

> The Pauli Exclusion Principle provides a näive example of holistic nonlocality. If electrons are localized particles there must be some nonlocal mechanism which stops the two electrons in (for example) a helium atom occupying the same state. On a particle view of electrons, chemistry is possible only because of the nonlocal structuring of electrons in complex atoms (Gibbins 1987, 117).

Similar comments have also been made by Hooker (Hooker 1989, 246) and by Bohm and Hiley (Bohm and Hiley 1993, 156–157). This being the case, the Causal Theory is particularly well suited to providing an explanation as it is a non-local theory in which the motion of an individual particle depends on the quantum state as a whole.

Second, the Exclusion Principle is assumed to apply to all physical situations involving fermions of the same kind. However, it has been argued elsewhere that the applicability of the Exclusion Principle should be restricted. Such an argument was presented by Toyoki Koga in his *Foundations of Quantum Physics*, where he claimed that applying the Exclusion Principle in situations other than stationary states can lead to absurdities (where the term 'stationary state' is understood in its standard meaning). He wrote:

> If we treat such a [non-stationary] state carelessly … we may get solutions which imply unreal states. In order to avoid such mistakes, it is necessary to set forth a criterion by which those solutions that appear to be possible … but [are physically] impossible … are eliminated. Pauli's [Exclusion] principle serves this purpose of elimination. For the same reason, the [Exclusion] principle should not be applied, for instance, to the treatment of nonstationary states.
> Pauli's [Exclusion] principle as such cannot be applied thoughtlessly without causing paradoxical results … (Koga 1980, 66)

This was also implied in an earlier article by Margenau (Margenau 1966, 89). Physically, a stationary state results when two travelling waves that are propagating in opposite directions, superimpose on each other (Hirose and Lonngren 1985, 97; Rogalski and Palmer 2006, 311). This can be achieved with quantum systems by containing the system in an enclosure (e.g. a rigid container) or restricting it to a finite spatial region, e.g. an atomic orbital, the lattice structure within a metal, etc. (Main 1978, 273 and 276; Ingard 1988, 422). Clearly, the Exclusion Principle does not apply to widely separated fermions of the same kind, e.g. an atom of helium on the Moon can be in the same state as one on Mars. In order for the Exclusion Principle to apply, two or more fermions must interact, i.e. there must be substantial overlap of their individual wave fields (Herbut and Vujicic 1987, 5562). However, Koga's criterion of applicability to just stationary states is too restrictive, as the Exclusion Principle may be applied to some non-stationary situations (Mott 1929, 222–230). What Koga should have inferred is that the Exclusion Principle will apply to a system of identical fermions that has constraints imposed upon it which are necessary but not always sufficient for the establishment of a stationary state.

The Exclusion Principle is most readily manifested in the case of identical spin-½ fermions in a stationary state and this situation is easiest to describe. We shall proceed by applying the characteristics of the wave field of a system of two identical spin-½ fermions in order to see what progress this approach can yield. The magnitude of the current task is such that this approach to finding a complete explanation of the Exclusion Principle is, at best, only a promising line of investigation. In what follows, the terms 'wave field' and 'wavefunction' will be used in close association and the reader is reminded of the distinction that 'wave field' refers to the physical quantum field whereas 'wavefunction' refers to its mathematical representation.

Consider a system of fermions, say two neutrons (as this avoids the problem of electrical interaction between the particles) which are in motion but are also well separated. The individual wave field of each neutron is initially a travelling wave as both are in motion. We can formally treat the neutrons and their wave fields as a single quantum system. Since the neutrons are well separated, there is no overlap of the individual wave fields and a system consisting of both neutrons may be described by a wavefunction which is the product of the individual wavefunctions associated with each neutron (i.e. as a solution of the Schrödinger equation for two non-interacting particles).

Let the neutrons move sufficiently close so that significant overlap of their individual wave fields occurs. Each neutron will then be subject to the other's wave field. In the case of non-overlapping wave fields it is clear that the wavefunction of the two-neutron system is just the simple product of the individual wavefunctions. However, without invoking the antisymmetry assumption, there is *no obvious expression* for the form of the two-neutron wavefunction when the individual wave fields first overlap.

Although we do not have an obvious expression for the wavefunction when there is initial overlap of individual wave fields, we can theorise that this situation may be described by a single wavefunction (denoted Ψ_I). This wavefunction represents a travelling wave in which both neutrons move under the influence of the wave field described by Ψ_I. We noted above that a stationary state is achieved by containing a system in a fixed enclosure or a finite spatial region. The case of atomic electrons is an example of a system contained in a finite region (described by Equations (6.4), (6.5) and (6.6)). We shall suppose, for current purposes, that the neutrons move only within a box with rigid walls. The example of two identical particles in a box (i.e. an infinite potential well) has occasionally appeared in the textbooks but such examples merely assume that the wavefunction for the two particle system is antisymmetric (Griffiths 2005, 205–206). Once inside the box, the wave field of the two-neutron system (initially represented by the wavefunction Ψ_I) will be successively reflected from one end of the box and then from the other. In the case of a fermionic wave field,

reflection at a rigid wall causes a change of the wave field's phase of π radians. This is a well-known effect when a physical wave (such as an electromagnetic wave) is reflected from a fixed boundary. However, it is the polarisation of the incident wave field (and not the total spin) that determines whether there is a change of phase on reflection at a fixed boundary.[3]

The wave field that is reflected back from the end of the box (and described by a wavefunction denoted Ψ_R) travels in the opposite direction to the wave field that is first incident at the box's wall. (Here we shall define the direction of motion of the wave field described by Ψ_I to be the positive x-axis.) The wavefunction Ψ_R will therefore differ from Ψ_I in two respects. First, the phase difference between Ψ_R and Ψ_I requires that each term in Ψ_R carry a negative sign. Second, the wave speed which appears in each term of Ψ_R will be of the opposite sign to the corresponding term in Ψ_I (as incident and reflected wave fields are moving in opposite directions). The spin part of the wavefunction remains the same on reflection of the wave field (Greenhow 1990, 215). The interference between the incident and reflected wave fields will produce a resultant wave field that is in a stationary state within the box (Main 1978, 286). This may be described by a total wavefunction (denoted Ψ_T) which has a standing wave pattern given by the sum of the incident wavefunction Ψ_I and the reflected wavefunction Ψ_R. The total wavefunction Ψ_T will have an antisymmetric form due to the negative signs in the terms of Ψ_R. This is the origin of the minus sign in the antisymmetrical wavefunction for this kind of stationary state.

This account of forming a total wavefunction that is antisymmetric can also be related to atomic systems and not just 'waves in boxes'. It may be employed, for example, to explain the case of two electrons in the same atomic orbital (such as found in neutral helium). Here the account is analogous to the waves generated in a free floating wire (or similar) loop when the wire is twisted and then released. Two waves are produced which are half a

[3] Cf. the treatment of electromagnetic waves in Jackson 1975, 280–282.

wavelength out of phase and propagate in opposite directions when the wire is released. In an atomic orbital, we would have two wave fields superimposing with the necessary phase difference of π radians, one propagating 'clockwise' and the other 'anticlockwise' around the nucleus. The antisymmetrical form of the total wavefunction for this system would then result from describing the behaviour of these two wave fields when they superimpose.

6.5 Modelling of Fermionic Wave Fields

How might the presented explanation for the behaviour of wave fields be analytically modelled? The main difficulty with modelling this phenomenon is to provide a valid mathematical description of the *initial overlap* of individual wave fields. What appears in the literature when two (or more) fermions of the same kind are involved is simply to assume that the overall wavefunction for a combined system is antisymmetric without showing how this is achieved. Even quantitative calculations in quantum chemistry assume an antisymmetrical wavefunction for multi-fermion systems or directly import empirical values into their calculations (Atkins and Friedman 1997, 276). There are no rigorous mathematical methods to be found in the literature to analytically determine the resultant form of wavefunctions which describe significant overlap of individual wave fields.

A tentative approach to finding a mathematical description of significantly overlapping wave fields is to model the overlap using superpositions of their individual wavefunctions. Consider again the example of the two neutrons and label them with numerals 1, 2. We can specify, as before, that neutron 1 is in state A at position $\mathbf{x}_1$ and neutron 2 is in state B at position $\mathbf{x}_2$. We shall denote the spatial components of their individual wavefunctions by ψ_A, ψ_B and spin components by χ_A, χ_B. Now when the individual wave fields overlap, they superimpose so that there will be new values for the wave field at each neutron's position. The net field at the position of neutron 1 will be a resultant of neutron 1's own wave field superimposed with the value of neutron 2's wave field at neutron 1's

position. Likewise, the net field at the position of neutron 2 will be a resultant of neutron 2's own wave field superimposed with the value of neutron 1's wave field at neutron 2's position. This will be manifest in the values of the wavefunctions at coordinates $\mathbf{x}_1$ and $\mathbf{x}_2$. Let these values be Ψ_1 and Ψ_2 respectively. Expressions for Ψ_1 and Ψ_2 can then be formed by superimposing the individual wavefunctions.

The superposition expression with the smallest number of terms leads to the following expressions for wavefunctions Ψ_1 and Ψ_2:

$$\Psi_1 = [\psi_A(\mathbf{x}_1)\chi_A(1) + \psi_B(\mathbf{x}_1)\chi_B(1)] \quad (6.12)$$

$$\Psi_2 = [\psi_A(\mathbf{x}_2)\chi_A(2) + \psi_B(\mathbf{x}_2)\chi_B(2)] \quad (6.13)$$

where 1, 2 refer to the values at positions $\mathbf{x}_1$ and $\mathbf{x}_2$. Since the overlap of individual wave fields is explicitly taken into account by Equations (6.12) and (6.13), we might try defining Ψ_I by forming the product $\Psi_1\Psi_2$, by analogy with the case of non-overlapping wave fields. This gives:

$$\begin{aligned}\Psi_I &= [\psi_A(\mathbf{x}_1)\chi_A(1) + \psi_B(\mathbf{x}_1)\chi_B(1)]\,[\psi_A(\mathbf{x}_2)\chi_A(2) + \psi_B(\mathbf{x}_2)\chi_B(2)] \\ &= \psi_A(\mathbf{x}_1)\psi_A(\mathbf{x}_2)\chi_A(1)\chi_A(2) + \psi_A(\mathbf{x}_1)\psi_B(\mathbf{x}_2)\chi_A(1)\chi_B(2) \\ &+ \psi_A(\mathbf{x}_2)\psi_B(\mathbf{x}_1)\chi_A(2)\chi_B(1) + \psi_B(\mathbf{x}_1)\psi_B(\mathbf{x}_2)\chi_B(1)\chi_B(2) \quad (6.14)\end{aligned}$$

Using Equation (6.14) and taking account of the differences between Ψ_R and Ψ_I (as stated above) the wavefunction representing the reflected wave field is:

$$\begin{aligned}\Psi_R = -[&\psi_A^{\ddagger}(\mathbf{x}_1)\,\psi_A^{\ddagger}(\mathbf{x}_2)\,\chi_A(1)\,\chi_A(2) + \psi_A^{\ddagger}(\mathbf{x}_1)\,\psi_B^{\ddagger}(\mathbf{x}_2)\,\chi_A(1)\,\chi_B(2) \\ &+ \psi_A^{\ddagger}(\mathbf{x}_2)\,\psi_B^{\ddagger}(\mathbf{x}_1)\,\chi_A(2)\,\chi_B(1) + \psi_B^{\ddagger}(\mathbf{x}_1)\,\psi_B^{\ddagger}(\mathbf{x}_2)\,\chi_B(1)\,\chi_B(2)] \quad (6.15)\end{aligned}$$

where $\psi_A^{\ddagger}$ represents the same function as ψ_A but with the opposite sign of the wave speed appearing in its argument and likewise for $\psi_B^{\ddagger}$. Then, using Equations (6.14) and (6.15), an expression for the total wavefunction would be:

$$\Psi_T = \Psi_I + \Psi_R$$
$$= [\psi_A(\mathbf{x}_1)\psi_A(\mathbf{x}_2) - \psi_A^{\ddagger}(\mathbf{x}_1)\psi_A^{\ddagger}(\mathbf{x}_2)]\,\chi_A(1)\,\chi_A(2)$$
$$+ [\psi_A(\mathbf{x}_1)\psi_B(\mathbf{x}_2) - \psi_A^{\ddagger}(\mathbf{x}_1)\psi_B^{\ddagger}(\mathbf{x}_2)]\,\chi_A(1)\,\chi_B(2)$$
$$+ [\psi_A(\mathbf{x}_2)\psi_B(\mathbf{x}_1) - \psi_A^{\ddagger}(\mathbf{x}_2)\psi_B^{\ddagger}(\mathbf{x}_1)]\,\chi_A(2)\,\chi_B(1)$$
$$+ [\psi_B(\mathbf{x}_1)\psi_B(\mathbf{x}_2) - \psi_B^{\ddagger}(\mathbf{x}_1)\psi_B^{\ddagger}(\mathbf{x}_2)]\,\chi_B(1)\,\chi_B(2)$$
(6.16)

Although expected terms (such as $\psi_A(\mathbf{x}_1)\psi_B(\mathbf{x}_2)$) do appear by this process, the required antisymmetrical wavefunction, i.e.

$$\Psi_T = \{\psi_A(\mathbf{x}_1)\psi_B(\mathbf{x}_2) - \psi_A(\mathbf{x}_2)\psi_B(\mathbf{x}_1)\}\,\{\chi_A(1)\,\chi_B(2) + \chi_A(2)\,\chi_B(1)\}$$
(6.17)

does not result from Equation (6.16). This implies that the wavefunction Ψ_I as formed from the product of Equations (6.12) and (6.13) cannot be a faithful representation of a quantum system where individual wave fields first overlap.

Given that the form of Ψ_R is derived from the form of Ψ_I and that Ψ_T is found from their summation, the above example indicates that the correct expression for Ψ_I is not likely to be of a simple form. This was not completely unforeseen, as mentioned above in relation to the magnitude of the task of 'deducing' the Exclusion Principle. In order to find the correct expression for Ψ_I a better understanding of what occurs when individual wave fields overlap will be required. It should then become apparent how to provide a correct mathematical description of the process. What constitutes such a description of overlapping individual wave fields and a method of deriving the relevant wavefunction remain open questions.

However, a plausible case has been presented which indicates that the antisymmetric form of fermionic wavefunctions in a stationary state arises from the *description* of the interference between physical wave fields within a bounded region. This explains why the Exclusion Principle is best known in regard to stationary

quantum states. It also resolves the conceptual problem for the Causal Theory that arises due to the wavefunctions of fermionic systems being restricted to the antisymmetric form. Since the Exclusion Principle is a consequence of the antisymmetric form of fermionic wavefunctions and wavefunctions describe physical wave fields, this provides a basis for the Exclusion Principle which cannot be duplicated in Orthodox Quantum Theory. Recall that wavefunctions in Orthodox Quantum Theory do not represent physical fields.

There is no empirical evidence to doubt the soundness of the Exclusion Principle (e.g. Reines and Sobel 1974; Logan and Ljubicic 1979; Nolte et al. 1991; Kishimoto et al. 1992; Baron et al. 1999; Javorsek et al. 2000). Despite this, debate about the possibility of violations of the Exclusion Principle continues with speculations that it might only be an approximation to something more fundamental.[4] These discussions and speculations all occur without any theoretical underpinning for the Exclusion Principle. The group led by Bartalucci, for example, see violations of the Exclusion Principle as a real possibility:

> The Pauli exclusion principle is one of the basic principles of modern physics and, even if there are no compelling reasons to doubt its validity, it is still debated today because an intuitive, elementary explanation is still missing … (Bartalucci et al. 2006, 18).

This statement conveys the motive behind the search for violations of the Exclusion Principle, i.e. the lack of a relatively simple explanation for why the Exclusion Principle holds. This has never been forthcoming from the physics community. Despite the absence of such an explanation, it is possible to conduct meaningful tests of the Principle's limits, as reported by Javorsek:

> The Pauli exclusion principle (PEP) has played a central role in quantum mechanics since its formulation in 1925. Notwithstanding its many successful predictions, the PEP remains somewhat enigmatic, particularly with respect to the question of whether small deviations from it are possible. … Nonetheless, since the PEP makes clear predictions which have direct experimental implications, tests of the PEP are possible even without a fully consistent theoretical framework (Javorsek et al. 2000, 2701).

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[4] Such debates have occurred at: International Conference on Spin-Statistics Connection and Commutation Relations: Experimental Tests and Theoretical Implications, Anacapri, 31 May – 3 June 2000; International Conference of Fundamental Symmetries and Fundamental Constants, 15–18 September 2004, Trieste, Italy.

It has been argued in this chapter that an explanation of the Exclusion Principle is not possible within Orthodox Quantum Theory. In the absence of any theoretical basis for the Exclusion Principle, a series of further experiments are being planned and conducted by the Violation of the Pauli Exclusion Principle (VIP) Experimental Group:

> The Pauli Exclusion Principle is one of the basic principles of modern physics and is at the very basis of our understanding of matter: thus it is fundamental importance to test the limits of its validity … the VIP (Violation of the Pauli Exclusion Principle) experiment, where we search for anomalous X-rays emitted by copper atoms in a conductor: any detection of these anomalous X-rays would mark a Pauli-forbidden transition. … VIP is currently taking data at the Gran Sasso underground laboratories, and its scientific goal is to improve by at least four orders of magnitude the previous limit on the probability of Pauli violating transitions … (Curceanu et al. 2008, 1).

A basis for the Exclusion Principle has been set out above which provides the intuitive and relatively simple explanation that has been missing since Pauli first postulated the Principle. Further, this basis allows the possibility that the Exclusion Principle might be violated in some extreme circumstances.

Concluding Remarks

We need a notion of physical reality … for without it our objective universe, and hence the whole of science, simply evaporates before our contemplative gaze!

— Sir Roger Penrose

After nearly a century of debate over the fundamentals of quantum mechanics, we are in a much stronger position to decide whether the Causal Theory of Quantum Mechanics or Orthodox Quantum Theory is the better theory. Many physicists have no interest in this question since it will not change the manner in which they do physics or the results they obtain. However, the foundational problems that have plagued Orthodox Quantum Theory since its origin are still with it and the debate will continue until these problems are satisfactorily resolved one way or another.

Both the Causal Theory and Orthodox Quantum Theory are now theoretically well developed. In addition, there has been an accumulation of information about the quantum realm gained from advanced technologies and techniques that were not even imaginable at the time when Orthodox Quantum Theory achieved dominance in the physics community. These theoretical and empirical developments have changed significantly the situation in which an informed, rational decision can be made between the two theories.

The criteria for the assessment of rival scientific theories was listed in Section 1.3, viz.: empirical adequacy; explanatory success; predictive power; consistency; and conceptual coherence. Both the Causal Theory and Orthodox Quantum Theory are empirically adequate. Neither theory has yet been shown to be empirically better than the other. Both theories are mathematically consistent. However, new quantum mechanical methods involving particle trajectories and quantum potential energies, as detailed in this book, have now been developed. These new methods are not available to Orthodox Quantum Theory and therefore count in favour of the Causal Theory. This does not invalidate the technical methods of Orthodox Quantum Theory which will continue to be used for many types of calculations.

In this book, arguments have been presented (and supported by cited evidence) that the quantum realm exists independently of

human 'observation'. It has been further argued that quantum phenomena can be described in terms of causal processes and that the explanatory success of the Causal Theory is due to the theory's capacity to 'mirror' aspects of this objective quantum reality.

It has been shown that an absence of causality is not necessitated by the formalism of quantum mechanics. The denial of causality was a deliberate choice made in the formulation of Orthodox Quantum Theory. Quantum causality was not 'killed-off' by two of the principal founders of quantum mechanics, Bohr and Heisenberg, although they did a good job of convincing most physicists that they had done just this. The Heisenberg uncertainty relations were central to the arguments of Bohr and Heisenberg. They presented these relations as *in-principle* limitations on the precision of the simultaneous measurement of some quantities. We have seen that the uncertainty relations specify a lower bound of the variance of two kinds of (incompatible) measurements, i.e. a statistical spread when made on an ensemble of similarly prepared quantum systems. The Causal Theory explains this spread in terms of changes to the quantum potential caused by measurement processes. Therefore, the Heisenberg Uncertainty Principle only has epistemic, not ontological, implications.

The Causal Theory is a viable physical theory which provides a realistic account of observable quantum phenomena, such as diffraction patterns and quantum mechanical tunnelling. Arguments and evidence for the existence of wave fields (matter waves), as postulated in the Causal Theory, was presented. This strongly supported the proposition that wave fields are causally efficacious in bringing about the observed wave aspects of quantum phenomena.

Arguments were presented for the existence of the quantum mechanical force. The case was made that the quantum mechanical force should be recognised in the same manner as the accepted fundamental forces of nature.

The concept of energy and the processes of energy transfer proved to be important ingredients in explaining quantum phenomena within the Causal Theory. The prior absence of detailed analyses of the role of energy in quantum systems has been detrimental to gaining a better understanding of quantum

phenomena. In the conduct of a (disturbance) measurement and in quantum tunnelling, for instance, energy transfers occur within a quantum system from a physically real wave field to quantum particles. Such transfers provide a causal underpinning for quantum processes that would otherwise not be explicable in an intelligible way.

Without the acknowledgement of an objective quantum reality, as postulated in the Causal Theory, quantum mechanics is doomed to be little more than an instrument for the generation of numbers. The Causal Theory incorporates a much needed notion of physical reality at the quantum level, a notion strongly advocated by a number of commentators on the foundations of quantum mechanics (e.g. Norris 2000, 4; Penrose 2004, 508; Smolin 2006, 9–10). In providing a realist framework, the Causal Theory offers the potential to contribute to the solution of many of the contemporary mysteries of physics. This has been stated with even greater force by R.W. Carroll who wrote:

> The conclusion seems inevitable that dBB [deBroglie-Bohm] Theory is essentially all pervasive and represents perhaps the most powerful tool available for understanding not only QM [Quantum Mechanics] but the universe itself (Carroll 2006, 275).

A rational decision cannot yet be made between the Causal Theory and Orthodox Quantum Theory based purely on empirical results. However, the case has been made in this book that the Causal Theory is not only a legitimate alternative to Orthodox Quantum Theory but is superior, for we have seen that when it comes to explanatory success, predictive power, and conceptual coherence, the Causal Theory wins ‘hands-down’!

A number of conceptual issues confronting the Causal Theory have been discussed. Solutions were offered to some important conceptual and theoretical problems that arose in the course of illuminating these issues. The conceptual problems to which solutions were provided were:

- the wavefunction on 3N-dimensional configuration space legitimately represents a physical field in three-dimensional space (pp. 70–71);
- energy conservation (pp. 113–114);
- the absence of a classical reaction (pp. 124–126);

- the quantum potential being a physical potential without conventional sources (p. 129); and
- the antisymmetric form of fermionic wavefunctions (pp. 174–175).

Some related theoretical problems were also solved:

- the transfer time for energy from wave field to particle (p. 123);
- the energy content of the wave field (pp. 123, 197–198);

and (in outline),

- a basis for the Exclusion Principle (pp. 176–178).

In the process of offering accounts of quantum phenomena from the perspective of the Causal Theory, it has also been shown that causal explanations can be given for circumstances which Orthodox Quantum Theory provides no explanation, such as:

- the physical role of the Schrödinger equation (p. 131);
- the (alleged) 'collapse of the wavefunction' (p. 133);
- quantum tunnelling (p. 136); and
- quantum mechanical spin (Section 6.2).

The rapidly developing field of Atom Optics was suggested as the most appropriate experimental arena to test the Causal Theory. In Section 5.9, an experiment was proposed utilising an 'ultra-cooled', trapped atom. This experiment offers the possibility of empirically discriminating between the Causal Theory and Orthodox Quantum Theory. It is left to the experimentalists to devise a practical version of this test.

In addition, it has been shown why eight commonly held 'myths' and misconceptions about the Causal Theory are ill-founded and/or unwarranted. In summary, the reasons for dismissing these 'myths' and misconceptions are as follows:

① A return to classical physics – the Causal Theory has a number of non-classical features.

② Contains ‘hidden variables’ – the (so-called) hidden variables in the Causal Theory are the particle positions and not some mysterious quantity that can never be measured.

③ Disproved by the various impossibility theorems – the Causal Theory is contextual and the impossibility theorems are not applicable to contextual theories.

④ Refuted by experiments on Bell-type inequalities – these are applicable only to local, realistic theories. The Causal Theory is non-local.

⑤ Is pure metaphysics – the Causal Theory is no more metaphysics than any other contemporary physical theory that proposes the existence of entities that, at present, cannot be directly observed.

⑥ Is inconsistent – the formalism of the Causal Theory can be seen on close inspection to be a fully consistent mathematical scheme and claims of alleged inconsistency over more than fifty years have never been substantiated.

⑦ Cannot be made relativistic – a full relativistic version of the Causal Theory was developed in the 1980s and has been extended since that time.

⑧ Cannot incorporate spin – this was first done in 1955 and subsequently extended.

In Chapter 1, it was observed that the resolution of conceptual difficulties in physical theory may open up new avenues for solution of previously unsolved (or perhaps unknown) theoretical problems. Several topics treated have shown new avenues for further research. The following questions are particularly noteworthy:

- What further fundamental properties of the wave field might be identified?
- To what degree will an in-depth account of the wave field require relativistic Causal Theory?
- What is the exact mechanism by which energy transfers are achieved between a quantum particle and its wave field?

- What further insights into the wave field would studies of many-particle quantum systems disclose?
- What spatial structure and/or 'means' of obtaining non-local connections would allow the wave field to be described entirely as a wave in three-dimensional space?

and

- What constitutes a valid mathematical description of the overlap of individual wave fields and how is this to be found?

These questions are indicative of those issues that should be the subject of subsequent studies.

Appendix A

The Gaussian Wave Packet

The form of the Gaussian wave packet for a classically-free quantum system and the associated Gaussian expressions are derived in this Appendix. We shall start with a physically acceptable (i.e. normalisable) configuration space wavefunction $\psi(x, t)$ which may be expressed by the following integral in one-dimension (Saxon 1968, 31 & 60):

$$\psi(x, t) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} \phi(p) \exp\left[\frac{ipx}{\hbar} - \frac{ip^2 t}{2m\hbar}\right] dp$$

where $\phi(p)$ are the corresponding momentum space wavefunctions, and $i = \sqrt{-1}$. Then at time t = 0, we have:

$$\psi(x, 0) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} \phi(p) \exp\left[ipx/\hbar\right] dp$$

A Fourier transform will give $\phi(p)$ if $\psi(x, 0)$ is specified, viz.:

$$\phi(p) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} \psi(x', 0) \exp\left[-ipx'/\hbar\right] dx'$$

where position coordinate x' relates to the particle at time t = 0. Then it follows that:

$$\psi(x, t) = \frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \psi(x', 0) \exp\left[\frac{ip(x - x')}{\hbar} - \frac{ip^2 t}{2m\hbar}\right] dx' dp$$

$$= \frac{m}{\sqrt{2\pi i\hbar t}} \int_{-\infty}^{\infty} \psi(x', 0) \exp\left[\frac{-im(x - x')^2}{2\hbar t}\right] dx' \quad \text{(A.1)}$$

where the standard definite integrals (as listed in Appendix B) have been employed.

It is stated in the relevant literature that the initial form of a normalised Gaussian wave packet is given by (Belinfante 1973, 194):

$$\Psi_o(\mathbf{x}) = \Psi(\mathbf{x}, 0) = (2\pi\sigma_o^2)^{-3/4} \exp\{i\mathbf{k}\cdot\mathbf{x} - (\mathbf{x}^2/4\sigma_o^2)\}$$

where σ_o is the initial root-mean-square (RMS) width of the packet in each coordinate direction with $\sigma_o^2 = \langle x^2\rangle = \langle y^2\rangle = \langle z^2\rangle$. The expression for $\Psi_o(x)$ is also called a minimum uncertainty wave packet because it yields the equality (Schiff 1968, 62; Sakurai 1985, 58): $(\Delta p)\,(\Delta x) = (\hbar/2)$. At times $t > 0$, the Gaussian wave packet will evolve according to the classically-free (i.e. $V = 0$) Schrödinger equation. In one dimension, the initial normalised Gaussian packet is:

$$\psi(x, 0) = (2\pi\sigma_o^2)^{-1/4} \exp\{(ipx/\hbar) - (x^2/4\sigma_o^2)\} \quad \text{(A.2)}$$

with $k = p/\hbar$ (Saxon 1968, 64; Ashby and Miller 1970, 181–182). If we replace x by x' in Equation (A.2) and then substitute into Equation (A.1), we get:

$$\psi(x, t) =$$

$$\frac{m}{(2\pi)^{1/4}\sqrt{i\hbar t\sigma_o}} \int_{-\infty}^{\infty} \exp\left[\frac{ip\,x'}{\hbar} - \frac{x'^2}{4\sigma_o^2} - \frac{im}{2\hbar t}(x^2 - 2x\,x' + x'^2)\right] dx'$$

$$= \frac{m}{(2\pi)^{1/4}\sqrt{i\hbar t\sigma_o}} \exp\left[\frac{imx^2}{2\hbar t}\right]$$

$$\int_{-\infty}^{\infty} \exp\left[-\left(\frac{1}{4\sigma_o^2} - \frac{im}{2\hbar t}\right)x'^2 - \left(\frac{imx}{\hbar t} - \frac{ip}{\hbar}\right)x'\right] dx'$$

$$= \frac{m}{(2\pi)^{1/4}\sqrt{i\hbar t\sigma_o}}$$

$$\exp\left[\frac{imx^2}{2\hbar t}\right] \exp\left[\frac{im\hbar t\,x^2 + 4m\sigma_o^2\,pt\,x - 2\sigma_o^2 p^2 t^2}{2\hbar t\,(\hbar t - 2im\sigma_o^2)}\right]$$

Using u_1 to denote the initial velocity, with $u_1 = p/m = \hbar k_1/m$, the above expression then becomes:

$$\psi(x, t) = (2\pi s_t^2)^{-1/4} \exp[ik_1(x - \tfrac{1}{2}u_1 t) - (x - u_1 t)^2/4\sigma_o s_t]$$

where $s_t = \sigma_o (1 + i\hbar\, t/2m\sigma_o^2)$. In three dimensions, the normalised Gaussian packet is the product of wave packets in each of the coordinate directions (Holland 1993, 158):

$$\Psi(\mathbf{x}, t) = \psi(x, t)\ \psi(y, t)\ \psi(z, t)$$

Using the one-dimensional expression for $\psi(x, t)$ and similar expressions for $\psi(y, t)$ and $\psi(z, t)$, we find the form of the three dimensional wavefunction:

$$\Psi(\mathbf{x}, t) = (2\pi s_t^2)^{-3/4} \exp\{i\,\mathbf{k}\cdot(\mathbf{x} - \tfrac{1}{2}\,\mathbf{u}t) - (\mathbf{x} - \mathbf{u}t)^2/4\sigma_o\, s_t\} \quad \text{(A.3)}$$

which describes a Gaussian wave packet at times $t > 0$, with $\mathbf{k}\cdot\mathbf{x} = (k_1x + k_2y + k_3z)$, $\mathbf{k}\cdot\mathbf{u} = k_1u_1 + k_2u_2 + k_3u_3$, and $\Psi(\mathbf{x}, t)$ is a solution of the classically-free Schrödinger equation.

Since $\Psi = Re^{iS/\hbar}$, the functions $R(\mathbf{x}, t)$ and $S(\mathbf{x}, t)$ for the normalised Gaussian wave packet may now be derived from Equation (A.3). Looking at the first term of Ψ, i.e. $(2\pi s_t^2)^{-3/4}$, we can express the complex number s_t as follows:

$s_t = \sigma_o (1 + i\hbar\, t/2m\sigma_o^2) = (a + ib)$, where $a = \sigma_o$ and $b = (\hbar\, t/2m\sigma_o)$

or in polar form:

$$s_t = |s_t| \exp(i\varphi), \text{ where } |s_t| = (a^2 + b^2)^{1/2} \text{ and } \varphi = \arctan(b/a).$$

So

$$|s_t| = [\sigma_o^2 + (\hbar\, t/2m\sigma_o)^2]^{1/2} = \sigma_o [1 + (\hbar\, t/2m\sigma_o^2)^2]^{1/2}$$

and

$$\varphi = \arctan[(\hbar\, t/2m\sigma_o)/\sigma_o] = \arctan(\hbar\, t/2m\sigma_o^2).$$

In order to conform with notation in the recent literature, let $\sigma = |s_t|$ then

$$\sigma^2 = \sigma_o^2 [1 + (\hbar^2 t^2/4m^2\sigma_o^4)]$$

where σ is the RMS width of the packet in each coordinate direction at time $t > 0$ (Holland 1993, 159) with

$$\sigma^2 = \langle x^2 - \langle x\rangle^2\rangle = \langle y^2 - \langle y\rangle^2\rangle = \langle z^2 - \langle z\rangle^2\rangle.$$

Then

$$4m^2\sigma_o^2\sigma^2 = 4m^2\sigma_o^4 + \hbar^2 t^2$$

and

$$(2\pi s_t^2)^{-3/4} = [(2\pi\sigma^2)\exp(2i\varphi)]^{-3/4} = (2\pi\sigma^2)^{-3/4}\exp(-3i\varphi/2).$$

The second term of Ψ is: $\exp\{i\mathbf{k}\cdot(\mathbf{x} - \tfrac{1}{2}\mathbf{u}t) - (\mathbf{x} - \mathbf{u}t)^2/4\sigma_o s_t\}$ which can also be separated into real and imaginary terms. The factor:

$$(1/\sigma_o s_t) = [\sigma_o^2(1 + i\hbar t/2m\sigma_o^2)]^{-1} = 2m/(2m\sigma_o^2 + i\hbar t)$$

$$= \frac{2m(2m\sigma_o^2 - i\hbar t)}{4m^2\sigma_o^4 + \hbar^2 t^2} = \frac{(4m^2\sigma_o^2 - 2im\hbar t)}{4m^2\sigma_o^2\sigma^2}$$

$$= (1/\sigma^2) - i(\hbar t/2m\sigma_o^2\sigma^2)$$

Thus

$$\exp\{-(\mathbf{x} - \mathbf{u}t)^2/4\sigma_o s_t\} =$$

$$\exp\{-(\mathbf{x} - \mathbf{u}t)^2/4\sigma^2\}\exp\{(i\hbar t)(\mathbf{x} - \mathbf{u}t)^2/8m\sigma_o^2\sigma^2\}$$

The wavefunction Ψ is then expressed as:

$$(2\pi\sigma^2)^{-3/4}\exp\{-(\mathbf{x} - \mathbf{u}t)^2/4\sigma_o^2\}\exp\{i[\mathbf{k}\cdot(\mathbf{x} - \tfrac{1}{2}\mathbf{u}t) + (\hbar t/8m\sigma_o^2\sigma^2)(\mathbf{x} - \mathbf{u}t)^2 - 3\varphi/2]\}$$

from which can be identified the following expressions for R and S:

$$R(\mathbf{x}, t) = (2\pi\sigma^2)^{-3/4}\exp\{-(\mathbf{x} - \mathbf{u}t)^2/4\sigma^2\} \qquad (A.4)$$

and

$$S(\mathbf{x},t) = -(3\hbar/2)\arctan(\hbar t/2m\sigma_o^2) + m\mathbf{u}\cdot(\mathbf{x} - \tfrac{1}{2}\mathbf{u}t) + \hbar^2 t(\mathbf{x} - \mathbf{u}t)^2/8m\sigma_o^2\sigma^2 \qquad (A.5)$$

where $\hbar\,\mathbf{k} = m\mathbf{u}$.

Using Gaussian derivatives (as given in Appendix B), the gradient of $S(\mathbf{x}, t)$ gives the particle's (possessed) momentum:

$$\nabla S = m\,\nabla[\mathbf{u}\cdot(\mathbf{x} - \tfrac{1}{2}\mathbf{u}t)] + (\hbar^2 t/8m\sigma_o^2\,\sigma^2)\,\nabla\,[(\mathbf{x} - \mathbf{u}t)^2]$$

$$= m\mathbf{u} + (\hbar^2 t/4m\sigma_o^2\,\sigma^2)\,(\mathbf{x} - \mathbf{u}t) \qquad \text{(A.6)}$$

and

$$(\nabla^2 S) = (3\hbar^2 t/4m\sigma_o^2\sigma^2)$$

The particle's velocity is then:

$$\mathbf{v} = (\nabla S/m) = \mathbf{u} + (\hbar^2 t/4m^2\sigma_o^2\,\sigma^2)\,(\mathbf{x} - \mathbf{u}t) \qquad \text{(A.7)}$$

This expression can be used to find the particle's trajectory $\mathbf{x}(t)$ as a function of time. One way of achieving this is to solve a first-order differential equation using the integrating factor method (see: Boas 1966, 328–329). Equation (A.7) may be rearranged as:

$$(d\mathbf{x}/dt) - (\hbar^2 t/4m^2\sigma_o^2\,\sigma^2)\mathbf{x} = [1 - (\hbar^2 t^2/4m^2\sigma_o^2\,\sigma^2)]\mathbf{u} = (\sigma_o^2/\sigma^2)\mathbf{u}$$

In this case, the integrating factor I is given by:

$$I = -(\hbar^2/4m^2\sigma_o^2)\int (t/\sigma^2)\,dt = -\log\sigma$$

Then

$$\mathbf{x}(t) = e^{-I}\int [(\sigma_o^2/\sigma^2)\,\mathbf{u}\,e^{I}]\,dt + \mathbf{c}\,e^{-I}$$

$$= (\mathbf{u}\,\sigma_o^2\,\sigma)\int (1/\sigma^3)\,dt + \mathbf{c}\,\sigma = \mathbf{u}\,t + \mathbf{c}\,\sigma$$

where $\mathbf{c}$ is a constant (vector) of integration. At $t = 0$, $\sigma = \sigma_o$, $\mathbf{x} = \mathbf{x}_o$ (initial position) and $\mathbf{c} = \mathbf{x}_o/\sigma_o$.

$$\Rightarrow \mathbf{x}(t) = \mathbf{u}t + (\sigma/\sigma_o)\,\mathbf{x}_o$$

where the integrals and identities used in this derivation are presented in Appendix B.

The quantum potential for a Gaussian wave system is easily calculated as follows:

$$Q = -(\hbar^2/2m)\,(\nabla^2 R/R)$$

$$= -(\hbar^2/2m)\,\nabla^2\,[\exp\{-(\mathbf{x} - \mathbf{u}t)^2/4\sigma^2\}]/\exp\{-(\mathbf{x} - \mathbf{u}t)^2/4\sigma^2\}$$

Now

$$\nabla\{-(\mathbf{x}-\mathbf{u}t)^2/4\sigma^2\} = -(\mathbf{x}-\mathbf{u}t)/2\sigma^2$$

$$\text{and } \nabla^2\{-(\mathbf{x}-\mathbf{u}t)^2/4\sigma^2\} = -3/2\sigma^2$$

as $(\nabla\cdot\mathbf{x}) = 3$. We then find:

$$\nabla R = -(2\pi\sigma^2)^{-3/4}[(\mathbf{x}-\mathbf{u}t)/2\sigma^2]\exp\{-(\mathbf{x}-\mathbf{u}t)^2/4\sigma^2\}$$

and

$$\nabla^2 R = (2\pi\sigma^2)^{-3/4}\{\nabla^2[-(\mathbf{x}-\mathbf{u}t)^2/4\sigma^2]$$

$$+(\nabla[-(\mathbf{x}-\mathbf{u}t)^2/4\sigma^2])^2\}\exp\{-(\mathbf{x}-\mathbf{u}t)^2/4\sigma^2\}$$

$$= (2\pi\sigma^2)^{-3/4}\{-(3/2\sigma^2)+[(-1/2\sigma^2)(\mathbf{x}-\mathbf{u}t)]^2\}\exp\{-(\mathbf{x}-\mathbf{u}t)^2/4\sigma^2\}$$

which yields:

$$Q = (\hbar^2/4m\sigma^2)\{3-(\mathbf{x}-\mathbf{u}t)^2/2\sigma^2\} \qquad (A.8)$$

Using $(d\mathbf{p}/dt) = -\nabla(V+Q)$ with V set to zero, the time rate of change of the particle's momentum in the classically-free case is then:

$$\frac{d\mathbf{p}}{dt} = -\nabla Q = -\frac{\hbar^2}{4m\sigma^2}\nabla\left[3-\frac{(\mathbf{x}-\mathbf{u}t)^2}{2\sigma^2}\right] = \frac{\hbar^2}{4m\sigma^4}(\mathbf{x}-\mathbf{u}t) \qquad (A.9)$$

The particle's kinetic energy is: $T = (\nabla S)^2/2m =$

$$\tfrac{1}{2}m|\mathbf{u}|^2 + (\hbar^2 t/4m\sigma_o^2\sigma^2)[\mathbf{u}\cdot(\mathbf{x}-\mathbf{u}t)]$$

$$+(\hbar^4 t^2/32m^3\sigma_o^4\sigma^4)(\mathbf{x}-\mathbf{u}t)^2 \qquad (A.10)$$

so that in an isolated, classically-free Gaussian quantum system, $(dS/dt) = (T-Q)$

$$= m|\mathbf{u}|^2 - (3\hbar^2 t/4m\sigma^2) + (\hbar^2 t/4m\sigma_o^2\sigma^2)[\mathbf{u}\cdot(\mathbf{x}-\mathbf{u}t)]$$

$$+(\hbar^2/8m\sigma^4)[1+(\hbar^2 t^2/4m^2\sigma_o^4)](\mathbf{x}-\mathbf{u}t)^2$$

$$= \tfrac{1}{2}m|\mathbf{u}|^2 - (3\hbar^2 t/4m\sigma^2) + (\hbar^2 t/4m\sigma_o^2\sigma^2)[\mathbf{u}\cdot(\mathbf{x}-\mathbf{u}t)]$$

$$+(\hbar^2/8m\sigma_o^2\sigma^2)(\mathbf{x}-\mathbf{u}t)^2 \qquad (A.11)$$

Then

$$\nabla (d\mathrm{S}/d\mathrm{t}) = (\hbar^2 \mathrm{t}/4m\sigma_o^2\sigma^2)\,\mathbf{u} + (\hbar^2/4m\sigma_o^2\sigma^2)\,(\mathbf{x} - \mathbf{u}\mathrm{t})$$

and

$$\nabla^2(d\mathrm{S}/d\mathrm{t}) = (3\hbar^2/4m\sigma_o^2\sigma^2) \qquad \text{(A.12)}$$

Appendix B

Derivatives and Integrals

Let $\sigma^2 = \sigma_o^2 [1 + (\hbar^2 t^2 / 4m^2 \sigma_o^4)]$, then

$$\frac{d\sigma}{dt} = \frac{\hbar^2 t}{4m^2 \sigma_o^2 \sigma} = \frac{\partial \sigma}{\partial t}$$

$$\frac{\partial^2 \sigma}{\partial t^2} = \frac{\hbar^2}{4m^2 \sigma^3}$$

$$\frac{\partial}{\partial t} \left(\frac{1}{\sigma}\right) = \frac{-\hbar^2 t}{4m^2 \sigma_o^2 \sigma^3}$$

$$\frac{\partial}{\partial t} \left(\frac{1}{\sigma^2}\right) = \frac{-\hbar^2 t}{2m^2 \sigma_o^2 \sigma^4}$$

$$\frac{\partial}{\partial t} \arctan\left[\frac{\hbar t}{2m\sigma_o^2}\right] = \frac{\hbar}{2m\sigma^2}$$

$$\int \frac{dt}{\sigma^2} = \left(\frac{2m}{\hbar}\right) \arctan\left[\frac{\hbar t}{2m\sigma_o^2}\right]$$

$$\int \frac{t}{\sigma^2}\, dt = \left(\frac{4m^2 \sigma_o^2}{\hbar^2}\right) \log \sigma$$

Standard Integrals (Dwight 1961):

$$\int_{-\infty}^{\infty} \exp\left[-\alpha x^2 - \beta x\right] dx = (\pi/\alpha)^{1/2} \exp\left[\beta^2/4\alpha\right]$$

$$\int_{-\infty}^{\infty} x \exp\left[-\alpha x^2 - \beta x\right] dx = (-\beta/2\alpha)(\pi/\alpha)^{1/2} \exp\left[\beta^2/4\alpha\right]$$

$$\int_{-\infty}^{\infty} x^2 \exp\left[-\alpha x^2 - \beta x\right] dx = \left[(1/2\alpha) + (\beta/2\alpha)^2\right](\pi/\alpha)^{1/2} \exp\left[\beta^2/4\alpha\right]$$

Appendix C

Hamiltonian of a Classically-free Gaussian Quantum System

The Hamiltonian H for a classically-free quantum system may be found by integrating the Hamiltonian density $\mathcal{H}$. From Equations (5.6) and (5.7), we have:

$$H = \iiint_{-\infty}^{\infty} \mathcal{H}\, d^3\mathbf{x} = \iiint_{-\infty}^{\infty} [R^2(\nabla S)^2/2m + (\hbar^2/2m)(\nabla R)^2\,]\, d^3\mathbf{x}$$

The relevant functions for a classically-free Gaussian wave packet derived in Appendix A are:

$$R^2 = (2\pi\sigma^2)^{-3/2} \exp[-(\mathbf{x}-\mathbf{u}t)^2/2\sigma^2]$$

$$(\nabla R)^2 = (2\pi\sigma^2)^{-3/2}\,[(\mathbf{x}-\mathbf{u}t)^2/4\sigma^4] \exp[-(\mathbf{x}-\mathbf{u}t)^2/2\sigma^2]$$

$$= R^2\,[(\mathbf{x}-\mathbf{u}t)^2/4\sigma^4]$$

$$(\nabla S) = m\mathbf{u} + (\hbar^2 t/4m\sigma_o^2\sigma^2)\,(\mathbf{x}-\mathbf{u}t)$$

From equation (A.10), the particle's kinetic energy is:

$$T = {}^1/_2 m|\mathbf{u}|^2 + (\hbar^2 t/4m\sigma_o^2\sigma^2)\,[\mathbf{u}\cdot(\mathbf{x}-\mathbf{u}t)]$$

$$+ (\hbar^4 t^2/32m^3\sigma_o^4\sigma^4)\,(\mathbf{x}-\mathbf{u}t)^2 \qquad \text{(C.1)}$$

where $\sigma = |s_t| = \sigma_o\,[1+(\hbar t/2m\sigma_o^2)^2]^{1/2}$ is the RMS width of the packet at time t.
Since

$$H = \iiint_{-\infty}^{\infty} [(R^2(\nabla S)^2/2m + (\hbar^2/2m)(\nabla R)^2\,]\, d^3\mathbf{x}$$

and using the above Gaussian functions and the integrals shown in Appendix B, this triple integration yields:

$$H = (3\hbar^2/8m\sigma^2) + (3\hbar^4 t^2/32m^3\sigma_o^4\sigma^2) + {}^1/_2 m|\mathbf{u}|^2$$

$$= (3\hbar^2/8m\sigma^2)\,[1 + (\hbar^2 t^2/4\,m^2\sigma_o^4)] + {}^1/_2 m|\mathbf{u}|^2 \;>\; 0$$

With the aid of the identity $\sigma^2 = \sigma_o^2 [1+(\hbar t/2m\sigma_o^2)^2]$, we arrive at:

$$H = {}^1/_2 m|\mathbf{u}|^2 + (3\hbar^2/8m\sigma_o^2) \qquad \text{(C.2)}$$

The first term on the right-hand side of Equation (C.2) is the particle's initial kinetic energy and the second term is the initial field energy. H is the total energy of this isolated, classically-free Gaussian quantum system.

Appendix D

Energy Content of a Gaussian Wave Field

The energy content of the wave field is given by (H – T). In the case of a classically-free Gaussian wave packet, an expression for the energy stored may be found in terms of the functions R, S and their derivatives. Using Equations (C.1) and (C.2) we find:

$$(H - T) = (3\hbar^2/8m\sigma_o^2)$$
$$- (\hbar^2 t/4m\sigma_o^2\sigma^2)\,[\mathbf{u}\cdot(\mathbf{x} - \mathbf{u}t)] - (\hbar^4 t^2/32m^3\sigma_o^4\sigma^4)\,(\mathbf{x} - \mathbf{u}t)^2$$
$$= (3\hbar^2/8m\sigma_o^2)$$
$$- (\hbar^2 t/4m^2\sigma_o^2\sigma^2)\,[m\mathbf{u} + (\hbar^2 t/8m\sigma_o^2\sigma^2)(\mathbf{x} - \mathbf{u}t)]\cdot(\mathbf{x} - \mathbf{u}t)$$
$$= (3\hbar^2/8m\sigma_o^2)$$
$$- (\hbar^2 t/4m^2\sigma_o^2\sigma^2)\,[m\mathbf{u} + (\hbar^2 t/4m\sigma_o^2\sigma^2)(\mathbf{x} - \mathbf{u}t)]\cdot(\mathbf{x} - \mathbf{u}t)$$
$$+ (\hbar^4 t^2/32m^3\sigma_o^4\sigma^4)(\mathbf{x} - \mathbf{u}t)^2$$
$$= (3\hbar^2/8m\sigma_o^2) + (\hbar^2 t/2m\sigma_o^2)[(\nabla S)/m]\cdot[(\nabla R)/R]$$
$$+ (1/2m)\,(\hbar^2 t/2m\sigma_o^2)^2\,[(\nabla R)/R]^2 \qquad (D.1)$$

where $(\nabla R)/R = [-(\mathbf{x} - \mathbf{u}t)/2\sigma^2]$ and

$(\nabla S) = m\mathbf{u} + (\hbar^2 t/4m\sigma_o^2\sigma^2)\,(\mathbf{x} - \mathbf{u}t)$.

Now we shall make use of the identity:

$$\nabla\cdot\left(\frac{\nabla R}{R}\right) = \frac{R(\nabla^2 R) - (\nabla R)^2}{R^2} = -\frac{3}{2\sigma^2}$$

so that

$$\frac{\hbar^2 t}{2m\sigma_o^2} = -(\nabla^2 S)\Big/\left[\nabla\cdot\left(\frac{\nabla R}{R}\right)\right] = \frac{R^2(\nabla^2 S)}{(\nabla R)^2 - R(\nabla^2 R)} \qquad (D.2)$$

where $(\nabla^2 S) = (3\hbar^2 t/4m\sigma_o^2\sigma^2)$.

Now, from equation (A.12), $\nabla^2(dS/dt) = (3\hbar^2/4m\sigma_o^2\sigma^2)$ and with Equation (D.2) we find that:

$$\frac{3\hbar^2}{8m\sigma_o^2} = \left(\frac{-3}{4}\right)\nabla^2\left(\frac{dS}{dt}\right)/\left[\nabla\cdot\left(\frac{\nabla R}{R}\right)\right] = \left(\frac{3}{4}\right)\frac{R^2\nabla^2(dS/dt)}{(\nabla R)^2 - R(\nabla^2 R)}$$

Then, using Equation (D.1), we have (H – T) =

$$\left(\frac{3}{4}\right)\frac{R^2\nabla^2(dS/dt)}{(\nabla R)^2 - R(\nabla^2 R)} + \frac{R^2(\nabla^2 S)}{(\nabla R)^2 - R(\nabla^2 R)}\left(\frac{\nabla S}{m}\right)\cdot\left(\frac{\nabla R}{R}\right)$$

$$+ \left(\frac{1}{2m}\right)\left[\frac{R^2(\nabla^2 S)}{(\nabla R)^2 - R(\nabla^2 R)}\right]^2\left(\frac{\nabla R}{R}\right)^2$$

Appendix E

Summary of Defined Principles

Principle of Causality

The same cause always produces the same effect or effects (other things being equal) and the cause temporally precedes, or is simultaneous with, its effects.

Principle of Complementarity

Any application of a classical concept precludes the simultaneous use of other classical concepts which in a different connection are equally necessary for the description of phenomena.

Correspondence Principle

Quantum states and measurements will tend to the corresponding classical case in the limit of large quantum numbers.

Uncertainty Principle

It is impossible to specify precisely and simultaneously the values of a pair of canonically conjugate variables that describe a quantum state.

Principle of Linear Superposition

When several individual states are superimposed, the resultant state is the addition of the individuals.

Principle of Locality

Elements of reality pertaining to one system cannot be affected by measurements performed at a space-like distance on another system, even if the systems previously interacted.

Functional Composition Principle

Given a set of observables A_i (represented by operators $\mathbf{A_i}$), $i \in I$, where I is the set of integers, there exists another observable B (represented by the operator **B**) and functions f_i such that $\mathbf{A_i} = f_i(\mathbf{B})$.

Principle of Stationary Action

The change in the total Action for each infinitesimal variation of the state of a physical system is zero.

Principle of Faithful Measurement

The result of measurement is numerically equal to the value possessed by an observable immediately prior to measurement.

Principle of the Conservation of Total Energy

The energy of a physical system is neither created nor destroyed, but may be transformed from one kind of energy into another, such that it is always theoretically possible to account for the total energy of a system.

Principle of Energy Content

Every physically real entity contains some finite quantity of energy.

Principle of Reaction

Any interaction between two physical entities has a mutual effect on both entities. The forces of interaction are equal and opposite, and act along straight lines joining the locations of the entities.

Pauli's Principle

In an atom there cannot be two or more electrons with the same quantum numbers.

Exclusion Principle

In a quantum system, two or more fermions of the same kind cannot be in the same (pure) state.

Postulate of Permutation Invariance

If φ is the state of a composite system whose components are identical particles, then the expectation value of any observable A is the same for all permutations of φ.

Symmetrization Postulate

The only possible states of a system of identical particles are described by state vectors (or wavefunctions) that are either completely symmetrical or completely antisymmetrical.

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Causal (de Broglie-Bohm) Theory of Quantum Mechanics / Bohmian Mechanics

University of Portsmouth – de Broglie-Bohm website:
<http://www.phys.port.ac.uk/fpweb%20dell/>
University of Texas at Austin – Wyatt Research Group: <http://research.cm.utexas.edu/rwyatt/>
University of South Carolina – Physical Division, Department of Chemistry and Biochemistry:
<http://www.chem.sc.edu/people/facultyStaffDetails.asp?SID=83>
Collaboration Bohmian Mechanics website: <http://www.bohmian-mechanics.net>
Bohmian Mechanics Workgroup Munich:
<http://www.mathematik.uni-muenchen.de/~bohmmech>
University of Innsbruck – Bohmian Mechanics Group: <http://bohm-c705.uibk.ac.at/index.htm>
Rice University – Java applets wavefunction and Bohmian particle visualisations:
<http://yepes.rice.edu/PhysicsApplets>
Stanford Encyclopedia of Philosophy – Bohmian Mechanics webpage:
<http://plato.stanford.edu/entries/qm-bohm>

Atom Optics

Many research institutions worldwide have established Atom Optics centres. Most of these may be accessed via the Quantum Optics and Atom Optics links webpage:
<http://www.quantumoptics.net/>
The Atom Optics research centres that are of greatest relevance to discussions presented in this book are:
MIT-Harvard University – Centre for Ultracold Atoms:
<http://www.rle.mit.edu/cua/new/research/research.asp>
U.S. National Institute of Standards and Technology – Laser Cooling and Trapping Group:
<http://physics.nist.gov/Divisions/Div842/Gp4/AtomOptics/index.html>
Australian National University – Atom Laser Group:
<http://atomlaser.anu.edu.au/research/>
University of Queensland, Australia – Centre for Quantum-Atom Optics:
<http://www.physics.uq.edu.au/BEC/research.html>
Laboratoire Charles Fabry de l'Institut d'Optique: <http://www.atomoptic.fr>

University of Amsterdam – Van der Waals-Zeeman Institute for Experimental Physics:
<http://www.science.uva.nl/research/aplp/index.php>
University of Vienna – Quantum Optics, Quantum Nanophysics and Quantum Information
<http://www.quantum.at/research.html>

Exclusion Principle

Violation of the Pauli Exclusion Principle (VIP) experiment site:
<http://www.lnf.infn.it/esperimenti/vip/>
National Aeronautics and Space Administration (NASA) website:
<http://apod.nasa.gov/apod/ap030219.html>

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