A First Course in Corporate Finance

Preview, Monday 26th September, 2005

"Have I (hic) godda deal for you."

A First Course in Corporate Finance

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Warning: This book is in development.
It is not error-free.

A First Course in Corporate Finance

Preview, Monday 26th September, 2005

Ivo Welch

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"Have I (hic) godda deal for you."

There are a large number of individuals who have helped me with this book. They will eventually be thanked here. Until then, some random collection: Rick Antle. Donna Battista. Randolph Beatty. Wolfgang Bühler. Kangbin Chua. Diego Garcia. Stan Garstka. Roger Ibbotson. Ludovic Phalippou. Matthew Spiegel. John Strong. Julie Yufe. Many anonymous victim students using earlier error-ridden drafts. Most importantly, Mary-Clare McEwing helped me improve the book.

Most of the review comments on early version of this book were very good, and I have tried hard to use them all. Thanks to the reviewers, who really gave a lot of their valuable time and thoughts to help me.

Tony Bernardo
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Warning: This book is in development.
It is not error-free.

Dedicated to my parents, Arthur and Charlotte.

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A Quick Adoption Checklist For Instructors

This checklist will not apply after AFCICF is published (with full supplementary materials) by Addison-Wesley-Pearson. The recommended checklist for this book (AFCICF) while in beta test mode:

- ✓ Read this prologue and one or two sample chapters to determine whether you like the AFCICF approach.

  Although not representative, I recommend that you also read the epilogue.

  • If you do like the AFCICF approach, then please continue. If you do not like AFCICF (or the chapters you read), please email ivo.welch@yale.edu why you did not like it. I promise I will not shoot the messenger: I want to learn how to do it better.

- ✓ You can continue to assign whatever other finance textbook you were planning to use, but now please add AFCICF.

  • AFCICF is a full-service textbook for an introductory finance course. However, this does not mean that it cannot also work as a complement to your previous textbook. The fact that it is so different from the competition means that you and your students can benefit from a test, in which you assign both books for one year. Relative to relying on only your old textbook, AFCICF will *not* increase, but decrease your student confusion and workload. See how well AFCICF works! (Take the Pepsi challenge!) I hope that the majority of your students (and you) will prefer reading AFCICF instead of your old textbook.

  • I believe it should also be a relatively simple matter for you to plug AFCICF into your current class: The chapters are succinct and should map easily into your curriculum. Having the old textbook is also your insurance against using a novel textbook. And it will make students less critical of the remaining shortcomings in AFCICF, such as the limited number of exercises (and their occasionally erroneous solutions). Perhaps most importantly, AFCICF does not *yet* have full supplementary materials. It will, but until then, the auxiliary materials from older textbook may help.

  • For now, the printing cost for AFCICF adds only around $25 to student cost, so affordability should not be a concern.

  You can go wrong if you try out at least a few chapters of AFCICF in this manner.

- ✓ You can receive permission to post AFCICF on your class website. (The website must be secured to allow only university-internal distribution).

- ✓ Ask your copy center to print and bind the version on your website. (If need be, I can send you nicely bound copies at $50/book. Your copy center can probably do it for $25/book.)

  • Although versions on the AFCICF website at http://welch.econ.brown.edu/book will be better than the version you download, it is good for you and your students to have one definitive reference version.

- ✓ If you are using AFCICF and you are looking for lecture notes, feel free to "steal" and adapt my lecture notes (linked at http://welch.econ.brown.edu/book) to your liking. (Please avoid the homeworks for now. Like some of the Q&A in AFCICF, the homeworks are not solid.)

- ✓ At the end of the course, please ask your students which textbook they found more helpful. Please email your conclusions and impressions to ivo.welch@yale.edu. Any suggestions for improvement are of course also very welcome.

To The Instructor: Differences and Innovations

This book is intentionally different.

The main concepts of finance are found in all textbooks, and this textbook is no exception. Thus, most—though not all—of the concepts and subjects in A First Course in Corporate Finance overlap with more traditional finance textbooks. This text's content is evolutionary, not revolutionary. Only its presentation is a revolutionary departure from traditional finance textbooks. Here is my view of how this book differs from what is currently out there:

Conversational Tone.

Conversational Tone The tone is conversational, which (I hope) makes the subject more accessible.

The method of instruction is "step-by-step numerical examples."

Numerical Example Based I learn best by numerical example, and firmly believe that students do, too. Whenever I want to understand an idea, I try to construct numerical examples for myself (the simpler, the better). I do not particularly care for long algebra or complex formulas, precise though they may be. I do not much like many diagrams with long textual descriptions, either—I often find them too vague, and I am never sure whether I have really grasped the whole mechanism by which the concept works. Therefore, I wanted to write a textbook that relies on numerical examples as its primary tutorial method.

This approach necessitates a rearrangement of the tutorial textbook progression. Most conventional finance textbooks start with a bird's eye view and then work their way down. The fundamental difference of this book is that it starts with a worm's eye view and works its way up. In caricature form, the format of other textbooks is "institutional background, hand-waving, formulas, figures, recipe application." The format of this textbook is the posing of a critical question like "what would it be worth?," which is then explained in numerical step-by-step examples from first principles. Right under the numerical computations are the corresponding formulas. In my opinion, this structure clarifies the meaning of these formulas, and is better than either a textual exposition or an algebraic exposition. I believe that the immediate duality of numerics with formulas ultimately helps students understand the material on a higher level and with more ease. (Of course, this book also provides some overviews, and ordinary textbooks also provide some numerical examples.)

This "forward development" approach also goes well with a book that has a conversational, more interactive flavor.

Brevity is important. The book focus is on explanations, not institutions.

Brevity Sometimes, less is more.

This book is intentionally concise, even though it goes into a lot more theoretical detail than other books! Institutional descriptions are short; only the concepts are explained in great detail.

My view is that when students are exposed to too much material, they won't read it, they won't remember it, and they won't know what is really important and what is not. Ten years after our students graduate, they should still solidly remember the fundamental ideas and be able to look up the details when they need them. Aside, many institutional details will have changed—it is the concepts that will last longer.

Self-contained means students can backtrack.

Self-Contained for Clarity Finance is a subject that *every* smart student can comprehend, regardless of background. It is no more difficult than economics or basic physics. The real problem is that many students come into class without much prerequisite knowledge, which we, the instructors, often erroneously believe they have. It is easy to mistake such "lost students" as "dumb students," especially if there is no reference source where lost students can quickly fill in the missing parts.

In this book, I try to make each topic's development as self-contained as possible. I try to explain everything from first principles, but in a way that every student can find interesting. For example, even though the necessary statistical background is integrated in the book for the statistics novice, the statistics-savvy student also should find value in reading it. This is because it is different in our finance context than when it is taught for its own sake in a statistics course.

Because this book tries to be as self-contained as possible, students who have failed to understand a particular lecture or topic (or who simply miss class) can now be referred back to read a self-contained chapter. My experience is that having a textbook that relates closely to the curriculum significantly reduces the need to back up and re-lecture on topics when enough students have become confused. My experience tells me that this reduces the planned time necessary to cover topics by about 10%.

Closer Correspondence with the Curriculum I believe that most finance core courses follow a curriculum that is much closer in spirit to this book—and more logical—than it is to the order in traditional finance textbooks. In the places where this book covers novel material (see below), I hope that you will find that it has merit—and if you agree, that covering the topic is much easier with this book than with another book.

Less Chapter Reordering.

Topical Innovation The book offers a good number of specific topical and expositional innovations. Here is a selection:

Progression to Risk and Uncertainty The book starts with a risk-free world, then adds horizon dependent interest rates, then uncertainty under risk neutrality, then frictions (e.g., taxes), then uncertainty under risk-aversion, and finally uncertainty under risk aversion and with taxes (e.g., WACC and APV). Each step builds on concepts learned earlier. I believe it is an advantage to begin simple and then complicate up, e.g., to first teach uncertainty and default risk in terms of the much simpler concept of expected values (elaborated in my next point). The unique role of the more difficult concepts of risk measurement, risk-aversion, and risk-aversion compensation, then becomes much clearer (there are forward hints to how it will change when we will make the world more complex).

First, no risk; then risk-neutral attitudes to risk; then risk-averse attitudes to risk.

Distinction Between Compensation for Default Risk and Risk Aversion I have always been shocked by how many graduating students think that a *Boston Celtics* bond quotes 400 basis points in interest above a comparable Treasury bond because of the risk-premium, which can be calculated using the CAPM formula. Learning to avoid this fundamental error is more important than fancy theories: the reason why the *Boston Celtics* bond quotes 400 extra basis points is primarily its default risk (compensation for non-payment), not a risk-premium (compensation for risk-aversion). And for bonds, the latter is usually an order of magnitude smaller than the former. Although many instructors mention this difference at some point, 5 minutes of default risk discussion is often lost in 5 hours worth of CAPM discussion. But if students do not understand the basic distinction, the 5 hours of CAPM discussion are not only wasted, they will have made matters worse!

Drive home "default risk."

Traditional textbooks have not helped, because they have not emphasized the distinction. In contrast, in this book, default risk is clearly broken out. The difference between quoted (promised) and expected returns, and default probabilities and default risk are important themes carried through.

Financials from a Finance Perspective Students need to solidly understand the relationship between financial statements and NPV, and they need to understand the basic thought process to construct pro formas. Alas, I could not find good, concise, and self-contained explanations of the important aspects and logic of accounting statements *from a finance perspective*—rather than from the sometimes minutiae-oriented accounting perspective. Consequently, AFCICF offers a good chapter on financials.

Understand accounting without being an accounting textbook.

A fundamental understanding of financials is also necessary to understand comparables: for example, students must know that capital expenditures must be subtracted out if depreciation is not. Therefore, the common use of EBITDA without a capital expenditures adjustment is often wrong.

Pro Forma The final chapter, towards which the book works at, is the creation of a pro forma. It combines all the ingredients from earlier chapters—capital budgeting, taxes, the cost of capital, capital structure, etc.

Explain Pro-Forma, WACC, and APV better.

Better Exposition of Valuation with Corporate Taxes WACC, APV, and direct pro-forma construction all incorporate the tax advantage of debt into valuation. This is bread-and-butter for a CFO. This book offers a clear explanation of how these methods usually come to similar results (and when not!).

Robustness The book covers the robustness of our methods—the relative importance of errors and mistakes—and what first-order problems students should worry about and what second-order problems they can reasonably neglect.

Capital Structure The academic perspective about capital structure has recently changed quite significantly. The empirical capital structure related chapters make it very clear where debt/equity ratios really come from.

...and many more.

Many Other Topical Improvements For example, the yield curve gets its own chapter even before uncertainty is described in much detail, so that students understand that investments over different time horizons can offer different rates of return. There is a self-contained chapter on comparables as a valuation technique—which many of our students will regularly have to do after they graduate. And the book tries to be open and honest about where our knowledge is solid and where it is shaky—and how sensitive our estimates are to the errors we inevitably commit.

Webchapters will allow a-la-carte choice.

Web Chapters and Flexible Topic Choice Although most of our curriculums are the same, covering necessary basics, there are some topics which may or may not appeal to everyone. Your preferences may differ from mine. For example, I find the financials part very important, because this is what most of our graduates will do when they become analysts, brokers, or consultants. However, you may instead prefer to talk more about international finance than I. It is of course impossible to satisfy everyone—and instructors have always chosen their own favorites, adding and deleting topics themselves.

Still, this book tries to help. Some chapters will not be in the printed book, but will be available only on the Web site ("Web chapters"). Chapter style and formatting are unmistakably identical to the book itself. Every instructor can therefore choose his/her own favorite selection and ask students to download it. The existing web chapters are posted. Among them are

Real Options Real options are briefly covered in Chapter 7 now, but not in great detail. This web chapter shows how to use spreadsheets, time-series analysis, Monte Carlo simulation, and optimization to determine the value of a plant that can shut down and reopen (for a cost) as output prices fluctuate.

Option and Derivative Pricing This is a difficult subject to cover in an introductory course, because it really requires a minimum of 4-6 class sessions to do it well. This chapter tries to help you cover the basics in 2-3 class sessions. It explains option contracts, static arbitrage relations (including put-call parity), dynamic arbitrage and the Black-Scholes formula, and binomial trees.

International Finance This chapter explains the role of currency translations and international market segmentation for both investments and corporate budgeting purposes.

Ethics This chapter is experimental—and provocative. There is neither a particular set of must-cover topics, nor is there a template on how to present this material. You may disagree with my choices.

All the material in this book has been covered in one full *semester* course for M.B.A. students at the Yale School of Management. However, it is a tight fit, even for graduate students as talented as those at Yale. It would be impossible to cover all the material in a one quarter course—although I deem all of it essential. However, a two-quarter course sequence (usually one investments and one corporate finance course) should be able to cover the material, even in an undergraduate context. For planning purposes, most chapters should consume either one or two class sessions.

Warning: The title is optimistic.

Is this textbook *too* clear, and does it thereby eliminate the need for an instructor? If you believe this to be true, then you are too familiar with the material and you underestimate how difficult finance is for new students. Neither the book alone nor lectures alone are usually enough. If we get lucky, the two together will work. Redundancy is important to the learning experience. Indeed, in my own classes, I ask the students to read the book *before class*, not after. Having a good idea of what is coming, student ask questions in the classroom that tend to become more informed. Of course, if you find that the book makes it easier to teach finance, you can always speed up and cover more material!

> **SIDE NOTE**: If you use this book, please permit me to use and post *your* homework and exam questions with answers. (Of course, this is not a requirement, only a plea for help.) My intent is for the Website to become collaborative: you will be able to see what other faculty do, and they can see what you do. The copyright will of course remain with you.

To The Student

Prerequisites

What do you need to understand this book? You do not need any specific background in finance. You do need to be thoroughly comfortable with arithmetic and generally comfortable with algebra. You do need mathematical aptitude, but no knowledge of advanced mathematical constructs, such as calculus. (Knowledge of statistics would be helpful, but I will explain the relevant concepts when the need arises.) You should own a $20 scientific calculator. (Financial calculators are not bad but also not necessary.) You should learn how to operate a spreadsheet (such as Excel in Microsoft's office or the OpenCalc spreadsheet in the excellent and free OpenOffice at www.openoffice.org). The financial world is moving rapidly away from financial calculators and toward computer spreadsheets—it is easier to work with information on a large screen with a 2,000 MHz processor than on a small 2-line display with a 2MHz processor. Because I have tried hard to keep the book self-contained and to explain everything from first principles, you should not need to go hunting for details in statistics, economics, or accounting textbooks. This is not to say that you do not need to take courses in these disciplines: they have way more to offer than just background knowledge for a finance textbook.

This book and the subject itself are tough, but they are not forbidding, even to an average student. The main prerequisite is mathematical aptitude, but not mathematical sophistication.

One word of caution: the biggest problem for a novice of *any* field, but especially of finance, is **jargon**: the specialized terminology known only to the initiated. Worse, in finance, much jargon is ambiguous. Different people may use different terms for the same thing, and the same term may mean different things to different people. You have been warned! This book attempts to point out such ambiguous usage. Luckily, the bark of jargon is usually worse than its bite. It is only a temporary barrier to entry into the field of finance.

Jargon can trip up the reader.

How to Read The Book

This book is concise, focusing on the essence of arguments.

This textbook is concise. Its intent is to communicate the essential material in a straightforward (and thus compact), but also conversational (and thus more interactive) and accessible fashion. There are already many finance textbooks with well over a thousand pages. Much of the content of these textbooks is interesting and useful but not essential to an understanding of finance. (I personally find some of this extra content distracting.)

The layout of the book.

The book is organized into four parts: the basics consist of return computations and capital budgeting. Next are corporate financials, investments (asset pricing), and financing (capital structure). Major sections within chapters end with questions that ask you to review the points just made with examples or questions similar to those just covered. **You should not proceed beyond a section *without* completing these questions (and in "closed book" format)!** Many, but not all, questions are easy and/or straightforward replications of problems that you will have just encountered in the chapter. Others are more challenging. Each chapter ends with answers to these review questions. Do not move on until you have mastered these review questions.

This is an annotation.

There are "annotations" on the left side of most paragraphs throughout the text. Suggestion: use the remaining white space in the margin to scribble your own notes, preferably in pencil so that you can revise them.

These are other notices.

Especially important concepts that you should memorize are in red boxes:

> **IMPORTANT**: *This is an important point to remember.*

Interesting, related points that either interrupt the flow of an argument, or that are not absolutely necessary are marked

> **SIDE NOTE**: This is an interesting related note, not crucial for understanding the material. It is usually not excessively technical.

More detailed technical points are "digging-deeper notes," which should be of interest only to the student who is interested in pursuing finance beyond the introductory course:

> **DIGGING DEEPER**: *If you are really interested, here is a curious fact or a derivation that most likely relies on excessive algebra.*

Both can be safely omitted from reading without compromising understanding. Sometimes, an appendix contains further advanced material.

Sense of Humor

A final warning: I have a strange sense of humor. Please do not be easily turned off.

OTHER READINGS

Advice: Follow current coverage of financial topics elsewhere!

This book cannot do it all. It is important for you to keep up with current financial developments. Frequent reading of the financial section of a major newspaper (such as the **New York Times** [**N.Y.T.**]), the **Wall Street Journal** [**W.S.J.**], or the **Financial Times** [**F.T.**] can help, as can regular consumption of good business magazines, such as **The Economist** or **Business Week**. (See the website at http://welch.econ.brown.edu/book for more useful resource links.) Although this is not a book on "how to read and understand the newspaper," you should be able to understand most of the contents of the financial pages after consuming this textbook. You should also know how to cruise the web—sites such as Yahoo!*Finance* contain a wealth of useful financial information, which we shall also use extensively in this book.

Table of Contents

II Corporate Financials 185

IV Financing Choices / Capital Structure 397

VI Appendices 665

Web Chapters

Part I

Investments and Returns

Before we begin tonight's dream,
a word from our sponsor...

(A part of all versions of the book.)

What You Want to Accomplish in this Part

Aside from teaching the necessary background, the two primary goals of this first part of the book is to explain how to work with rates of return, and how to decide whether to take or reject investment projects.

The method of our book is to start with "simple" scenarios and then build on them. For any tool to work in a more complex scenario, it has to also work in the simpler scenario, so what you learn in earlier chapters lays the ground work for later chapters.

- In Chapter 1, you will learn where this book is going. Most of our goal is "relative valuation" (valuing one opportunity relative to others), and everything will come together to be of use only in our final "pro forma" chapter. It will also tell you more about the book's approach—its "method of thinking."

- In Chapter 2, you will start with the simplest possible scenario. There are no taxes, transaction costs, disagreements, or limits as to the number of sellers and buyers in the market. You know everything, and all rates of return in the economy are the same. A one-year investment pays the same and perfectly known rate of return per annum as a ten-year investment. You want to know how one-year returns translate into multi-year returns; and when you should take a project and when you should reject it. The chapter introduces the most important concept of "present value."

  > Typical questions: If you earn 5% per year, how much will you earn over 10 years? If you earn 100% over 10 years, how much will you earn per year? What is the value of a project that will deliver $1,000,000 in 10 years? Should you buy this project if it cost you $650,000?

- In Chapter 3, you will learn how to value particular kinds of projects—annuities and perpetuities—if the economy-wide interest rate remains constant.

  > Typical questions: What is the monthly mortgage payment for a $300,000 mortgage if the interest rate is 4% per annum?

- In Chapter 4, you will abandon the assumption that returns are the same regardless of investment horizon. For example, one-year investments may pay 2% per annum, while ten-year investments may pay 5% per annum. Having time-varying rates of return is a more realistic scenario than the previous chapter's constant interest rate scenario. However, the question that you want to answer are the same questions as those in Chapter 2. (The chapter then also explains some more advanced aspects of bonds.)

  > Typical questions: If you earn 5% in the first year and 10% in the second year, how much will you earn over both years? What is the meaning of a 4% annualized interest rate? What is the meaning of a 4% yield-to-maturity? How can you value projects if appropriate rates of return depend on investment horizon?

- In Chapter 5, you will abandon the assumption that you have perfect omniscience. To be able to study uncertainty, you must begin with statistics. The chapter then explains an important assumption about your risk preferences that makes this easy: risk-neutrality. This lays the ground for discussing the role of uncertainty in finance. (Later, in Part III, you will learn what to do if investors are risk-averse.)

  Uncertainty means that a project may not return the promised amount—the *stated* rate of return would be higher than the *expected* rate of return. Although you are interested in the latter, it is almost always only the former that you are quoted (promised). You must always draw a sharp distinction between promised (stated) rates of return, and expected rates of return. This chapter also explains the difference between debt and equity, which is only meaningful under uncertainty.

Typical questions: If there is a 2% chance that your borrower will not return the money, how much extra in interest should you charge? From an investment perspective, what is the difference between debt and equity? How bad is the role of inevitable mis-estimates in your calculations? If your cost of capital (borrowing from the bank) is 10% and your savings interest rate (saving in the bank) is 5%, should you take a project that will offer a 7% rate of return?

- In Chapter 6, you will abandon the perfect market assumptions and focus on four important frictions: disagreement, transaction costs, taxes, and inflation. This chapter also explains the basics and principles of the tax code. Though not welcome, these frictions matter, so you must consider them!

  Typical questions: What are typical transaction costs, and how do you work with them? Why are capital gains better than ordinary income? If you have to pay 40% income taxes on interest receipts, the inflation rate is 2% per annum, and your investment promises 5% per annum, how much more can you buy in goods tomorrow if you invest? If you can earn 5% in taxable bonds, and 3% in tax-exempt municipal bonds, which is the better investment for you? If the inflation rate is 5% per year, and the interest rate is 10% per year, how much more in goods can you actually buy if you save your money?

- Chapter 7 goes over a number of important issues that you should pay attention to when you have to make investment decisions.

  Typical questions: How should you think of projects that have sideeffects—for example, projects that pollute the air? How should you think of sunk costs? What is a "real option"? How do you value contingencies and your own flexibility to change course in the future? How should your assessment of the value change if someone else makes up the cash flow estimates? How do humans—you—tend to mis-estimate future cash flows.

- Chapter 8 discusses other capital budgeting rules, first and foremost the profitability index and the internal rate of return.

  Typical question: If your project costs \$100, and returns \$50 next year and \$100 in ten years, what is your project's internal rate of return?

CHAPTER 1

Introduction

A First Draft

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Before you set out for your journey into the world of finance, this chapter outlines in very broad strokes what this book is all about.

1·1. The Goal of Finance: Relative Valuation

Finance is such an important part of modern life that almost everyone can benefit from understanding it better. What you may find surprising is that the financial problems facing **PepsiCo** or **Microsoft** are not really different from those facing an average investor, small business owner, entrepreneur, or family. On the most basic level, these problems are about how to allocate money. The choices are many: money can be borrowed or saved; money can be invested into projects, undertaken with partners or with the aid of a lender; projects can be avoided altogether if they do not appear valuable enough. Finance is about how best to decide among these alternatives—and this textbook will explain how.

Theme Number One: Value! Make Decisions Based on Value.

There is one principal theme that carries through all of finance. It is *value*. It is the question "What is a project, a stock, or a house worth?" To make smart decisions, you must be able to assess value—and the better you can assess value, the smarter your decisions will be.

Corporate managers need to know how to value—and so do you.

The goal of a good corporate manager should be to take all projects that add value, and avoid those that would subtract value. Sounds easy? If it only were so. Valuation is often very difficult.

The math is not hard.

It is not the formulas that are difficult—even the most complex formulas in this book contain just a few symbols, and the overwhelming majority of finance formulas only use the four major operations (addition, subtraction, multiplication, and division). Admittedly, even if the formulas are not sophisticated, there are a lot of them, and they have an intuitive economic meaning that requires experience to grasp—which is not a trivial task. But if you managed to pass high-school algebra, if you are motivated, and if you keep an open mind, you positively will be able to handle the math. It is *not* the math that is the real difficulty in valuation.

The tough aspect about valuation is the real world.

Instead, the difficulty is the real world! It is deciding how you should judge the future—whether your Gizmo will be a hit or a bust, whether the economy will enter a recession or not, where you can find alternative markets, and how interest rates or the stock market will move. This book will explain how to use your forecasts in the best way, but it will mostly remain up to you to make smart forecasts. (The book will explain how solid economic intuition can often help, but forecasting remains a difficult and often idiosyncratic task.) But there is also a ray of light here: If valuation were easy, a computer could do your job of being a manager. This will never happen. Valuation will always remain a matter of both art and science, that requires judgment and common sense. The formulas and finance in this book are only the necessary toolbox to convert your estimates of the future into what you need today to make good decisions.

The law of one price.

To whet your appetite, much in this book is based in some form or another on the **law of one price**. This is the fact that two identical items at the same venue should sell for the same price. Otherwise, why would anyone buy the more expensive one? This law of one price is the logic upon which almost all of valuation is based. If you can find other projects that are identical—at least along all dimensions that matter—to the project that you are considering, then your project should be worth the same and sell for the same price. If you put too low a value on your project, you might pass up on a project that is worth more than your best alternative uses of money. If you put too high a value on your project, you might take a project that you could buy cheaper elsewhere.

Value is easier relative.

Note how value is defined in relative terms. This is because it is easier to determine whether your project is better, worse, or similar to its best alternatives than it is to put an absolute value on your project. The closer the alternatives, the easier it is put a value on your project. It is easier to compare and therefore value a new Toyota Camry—because you have good alternatives such as Honda Accords and one-year used Toyota Camry—than it is to compare the Camry against a Plasma TV, a vacation, or pencils. It is against the best and closest alternatives that you want to estimate your own project's value. These alternatives create an "opportunity cost" that you suffer if you take your project instead of the alternatives.

Many corporate projects in the real world have close comparables that make such relative valuation feasible. For example, say you want to put a value on a new factory that you would build in Rhode Island. You have many alternatives: you could determine the value of a similar factory in Massachusetts instead; or you could determine the value of a similar factory in Mexico; or you could determine how much it would cost you to just purchase the net output of the factory from another company; or you could determine how much money you could earn if you invest your money instead into the stock market or deposit it into a savings account. If you understand how to estimate your factory's value relative to your other opportunities, you then know whether you should build it or not. But not all projects are easy to value in relative terms. For example, what would be the value of building a tunnel across the Atlantic, of controlling global warming, or of terraforming Mars? There are no easy alternative projects to compare these to, so any valuation would inevitably be haphazard.

Relative value often works well in the corporate world.

1·2. Learning How to Approach New Problems

This book is not just about teaching finance. It also wants to teach you how to approach novel problems. That is, it would rather not merely fill your memory with a collection of formulas and facts—which you could promptly forget after the final exam. Instead, you should understand *why* it is that you are doing what you are doing, and how you can logically deduce it for yourself when you do not have this book around. The goal is to eliminate the *deus ex machina*—the god that was lowered onto the stage to magically and illogically solve all intractable problems in Greek tragedies. You should understand where the formulas in this book come from, and how you can approach new problems by developing your own formulas. Learning how to logically progress when tackling tough problems is useful, not only in finance, but also in many other disciplines and in your life more generally.

Theme Number Two: Learn how to approach problems.

The method of approaching new problems in this book is to think in terms of the simplest possible example first, even if it may sometimes seem too banal a problem or a step that you would rather brush aside. Some students may even be put off by doing the basics, wanting to move immediately on to the truly interesting, philosophical, or complex problems right away. However, you should try to avoid the temptation of skipping the simpler problems, the foundation. Indeed, arrogance about the basics is often more a sign of insecurity and poor understanding than it is a sign of solid understanding—and even I am always surprised about the many novel insights that I still get from pondering even very basic problems. I have studied finance for almost two decades now, and this is an introductory textbook—and yet I still learned a lot thinking about basic issues while writing this textbook. There was plenty of "simple" material that I had thought I understood, which I then realized I had not.

Always start simple and uncomplicated!

Now, working up from simple examples is done in this book by the method of numerical example. Only after you have understood the simplest numerical form should you translate the nuermics onto algebra and only then make the problem more complex. This will take the sting out of the many formulas that finance will throw at you. Here is an example of how this book will proceed. If you will receive \$150 next year if you give me \$100 today, you probably already know that the rate of return is 50%. How did you get this? You subtracted \$100 from \$150, and divided by your original investment of \$100:

Numerics work well.

$$\frac{\$150 - \$100}{\$100} = 50\% \ . \tag{1.1}$$

The next step is to make an algebraic formula out of this. Name the two inputs, say, CF_1 and CF_0 for cash flow at time 1 and cash flow at time 0. Call your result a rate of return and name it r. To explain the correspondence between formulas and numerics, in this book, the formula is placed under the numerics, so you will read

$$\begin{aligned} 50\% &= \frac{\$150 - \$100}{\$100} \\ r &= \frac{\text{CF}_1 - \text{CF}_0}{\text{CF}_0} \end{aligned} \tag{1.2}$$

Looks silly? Perhaps—but this is how I find it easiest to learn. Now you can ask much more interesting and complex questions, such as what you would end up with if you started with $200 and earned 50% rate of return two years in a row, what the effect of inflation and imperfect competition would be on your rate of return, etc. There will be dozens of other complications to this formula in this book. But, we are getting ahead of ourselves. So trust me. This book will cover a lot of theory—but the theory will not be difficult when properly defanged.

1·3. The Main Parts of This Book

This book has four parts, plus a synthesis pro forma chapter.

This book will now proceed as follows:

1. The first part covers how your firm should make investment decisions, one project at a time. It covers the basics—rates of returns, the time value of money—and capital budgeting. It explains why we often rely on "perfect markets" when we estimate value.
2. The second part explains how corporate financial statements work, and how they relates to firm value.
3. The third part covers "investments." The novel part here is the consideration of how one investment influences the risk of other investments. For example, a coin bet on heads is risky. A coin bet on tails is risky. Half a coin bet on heads and half a coin bet on tails has zero risk. This part explains how ordinary investors should look at your portfolio of bets in overall terms. It then relates this investor problem to what the consequences are in terms of the corporate cost of capital—that is, the opportunity cost of capital that your investors incur if they give their money to your corporation rather to another one.
4. The fourth part covers how your projects should be financed. Should you find partners to join you, or borrow money? The former is called equity financing, the latter is called debt financing. This part also describes how firms have historically financed themselves and how investment banking works. It closes with the subject of corporate governance—how firm owners assure that their firm and other owners will not steal all their money.

The synthesis chapter is not only the standard way of business communication, but it also requires you knowing everything!

The book ends with a keystone chapter—a *pro forma* analysis of a real company, here PepsiCo. A pro forma is a projection of the future for the purpose of valuing the company today. In virtually every corporation, new corporate propositions have to be put into a pro forma. This is how new business ideas are pitched—to the CFO, the board, the venture capitalist, or the investment bank. Pro formas bring together virtually everything that you learn in this book. To do one well, you have to understand how to work with returns and net present values, the subject of the first part of the book. You have to understand how to work with financial statements, the next part of the book. You have to understand how to estimate the firm's cost of capital, the next part of the book. You have to understand how capital structure, taxes and other considerations influence the cost of capital, the final part of the book. You will learn what is easy and what is hard. You will learn what is science and what is art. And you will learn the limits to financial analysis.

Let's set sail.

CHAPTER 2

The Time Value of Money

(Net) Present Values

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In this chapter, we assume that we live in a perfect world of no taxes, no inflation, no transaction costs, no differences of opinion, and infinitely many investors and firms—which is called a "perfect market." Of course, this financial utopia is often unrealistic, but all the tools you will be learning in this chapter will continue to work just as well in later chapters where the world becomes more complex and more "real."

We begin with the concept of a rate of return—the cornerstone of finance. We know that we can always earn interest by depositing our money today into the bank. This means that money today is more valuable than the same amount of money next year. This concept is called the time-value of money—the present value of $1 is above the future value of $1. To begin our study of returns, we will look at the simplest kind of investment—a plain bond.

Now, the other side to our investing money today in order to receive money in the future is a project, company, stock or other investment that requires funding today to pay off money in the future—we want to invest, and companies want to borrow. The process by which firms decide which projects to undertake and which projects to pass up on is called *capital budgeting*. The idea behind this term is that each firm has a "capital budget," and must allocate its capital to the projects within its budgets. Capital budgeting is at the heart of corporate decision-making. You will learn that, to determine the value of projects with given cash flows in the future, the firm should translate all *future* cash flows—both inflows and outflows—into their equivalent *present* values today, and then add them up to find the "net present value" (NPV). The firm should take all projects that have positive net present value and reject all projects that have negative net present values.

This all sounds more complex than it is, so we'd better get started.

2·1. Basic Definitions

Before we can begin, we have to agree on a common language—for example, what we mean by a project, a bond, and a stock.

2·1.A. Investments, Projects, and Firms

To value projects, make sure to use all costs and benefits, including, e.g., opportunity costs and pleasure benefits.

As far as finance is concerned, every **project** is a set of flows of money (**cash flows**). Most projects require an upfront cash outflow (an **investment** or **expense** or **cost**) and are followed by a series of later cash inflows (**payoffs** or **revenues** or **returns**). It does not matter whether the cash flows come from garbage hauling or diamond sales. Cash is cash. However, it is important that all costs and benefits are included as cash values. If you would have to spend more time to haul trash, or merely find it more distasteful than other projects, then you would have to translate these project features into equivalent cash negatives. Similarly, if you want to do a project "for the fun of it," you must translate your "fun" into a cash positive. The discipline of finance takes over after all positives and negatives (inflows and outflows) from the project "black box" have been translated into their appropriate monetary cash values.

The black box is not trivial.

This does not mean that the operations of the firm are unimportant—things like revenues, operations, inventory, marketing, payables, working capital, competition, etc. These business factors are all of the utmost importance in making the cash flows happen, and a good (financial) manager must understand these. After all, even if all you care about is cash flows, it is impossible to understand them well if you have no idea where they come from and how they can change in the future.

These examples show that cash flows must be universal, not just direct cash flows.

Projects need not be physical. For example, a company may have a project called "customer relations," with real cash outflows today and uncertain future inflows. You (a student) are a project: you pay for education and will earn a salary in the future. In addition, some of the payoffs from education are metaphysical rather than physical. If knowledge provides you with pleasure, either today or in the future, education yields a value that should be regarded as a positive cash flow. Of course, for some students, the distaste of learning should be factored in as a cost (equivalent cash outflow)—but I trust that you are not one of them. All such non-financial flows must be appropriately translated into cash equivalents if you want to arrive at a good project valuation!

In finance, firms are basically collections of projects.

A firm can be viewed as just a collection of projects. Similarly, so can a family. Your family may own a house, a car, tuition payments, education investments, etc.,—a collection of projects. This book assumes that the value of a firm is the value of all its projects' net cash flows, and nothing else. It is now our goal to learn how to determine these projects' values, given cash flows.

Stocks and Bonds are just projects with inflows and outflows.

There are two important specific kinds of projects that we may consider investing in—**bonds** and **stocks**, also called **debt** and **equity**. As you will learn later, in a sense, the stock is the equivalent of investing to become a risky owner, while the bond is the equivalent of a lending money. Together, if you own all outstanding bonds (and loans) and stock in a company, you own the firm:

$$\text{Entire Firm} \;=\; \text{All Outstanding Stocks} \;+\; \text{All Outstanding Bonds and Loans}\;. \tag{2.1}$$

Anecdote: The Joy of Cooking: Positive Prestige Flows and Restaurant Failures
In New York City, two out of every five new restaurants close within one year. Nationwide, the best estimates suggest that about 90% of all restaurants close within two years. If successful, the average restaurant earns a return of about 10% per year. One explanation for why so many entrepreneurs are continuing to open up restaurants, despite seemingly low financial rates of return, is that restaurateurs so much enjoy owning a restaurant that they are willing to buy the prestige of owning a restaurant. If this is the case, then to value the restaurant, you must factor in how much the restaurateur is willing to pay for the prestige of owning a restaurant, just as you would factor in the revenues that restaurant patrons generate. (But we also describe an alternative reason why so many restaurants fail on Page 167.)

This sum is sometimes called the **enterprise value**. Our book will spend a lot of time discussing these two forms of financing—but for now, you can consider both of them just investment projects: you put money in, and they pay money out. For many stock and bond investments that you can buy and sell in the financial markets, we believe that most investors enjoy very few, if any, non-cash based benefits.

Solve Now!

Q 2.1 *In computing the cost of your M.B.A., should you take into account the loss of salary while going to school? Cite a few non-monetary benefits, too, and try to attach monetary value to them.*

Q 2.2 *If you purchase a house and live in it, what are your inflows and outflows?*

2·1.B. Loans and Bonds

Why bonds first?

Plain bonds are much simpler than stocks or corporate investment projects in general. You should view bonds as just another type of investment project—money goes in and money comes out—except that bonds are relatively simple because you presumably know what the cash flows will be. For stocks and other projects the complications created by having to guess future cash flows can quickly become daunting. Therefore, it makes sense to first understand the project "plain bond" well before proceeding to other kinds of projects. Aside, much more capital in the economy is tied up in bonds and loans than is tied up in stock, so understanding bonds well is very useful in itself.

Finance Jargon: Loans, Bond, Fixed Income, Maturity.

A **loan** is the commitment of a **borrower** to pay a predetermined amount of cash at one or more predetermined times in the future (the final one being called **maturity**), usually for cash upfront today. A **bond** is a particular kind of loan, named so because it binds the borrower to pay money. Thus, "buying a bond" is the same as "extending a loan." Bond buying is the process of giving cash today and receiving a promise for money in the future. Similarly, instead of "taking a loan," you can just say that you are "giving a bond," "issuing a bond," or "selling a bond." Loans and bonds are also sometimes called **fixed income** instruments, because they "promise" a fixed income to the holder of the bond.

Bond: Defined by payment next year. Savings: Defined by payment this year.

Is there any difference between buying a bond for \$1,000 and putting \$1,000 into a bank savings account? Yes, a small one. The bond is defined by its future promised payoffs—say, \$1,100 next year—and the bond's value and price today are based on these future payoffs. But as the bond owner, you know exactly how much you will receive next year. An investment in a bank savings account is defined by its investment today. The interest rate can and will change every day, and next year you will end up with an amount that depends on future interest rates, e.g., \$1,080 (if interest rates will decrease) or \$1,120 (if interest rates will increase).

A bank savings account is like a sequence of 1-day bonds.

If you want, you can think of a savings account as consecutive 1-day bonds: when you deposit money, you buy a 1-day bond, for which you know the interest rate this one day in advance, and the money automatically gets reinvested tomorrow into another bond with whatever the interest rate will be tomorrow. Incidentally, retirement plans also come in two such forms: **defined benefit** plans are like bonds and defined by how much you will get when you retire; and **defined contribution** plans are like bank deposit accounts and defined by how much money you are putting into your retirement account today—in the real world, you won't know exactly how much money you will have when you will retire.

Interest and Non-Interest. Limited Upside.

You should already know that the net return on a loan is called **interest**, and that the rate of return on a loan is called the **interest rate**—though we will soon firm up your knowledge about interest rates. One difference between interest payments and non-interest payments is that the former usually has a maximum payment, while the latter can have unlimited upside potential. Not every rate of return is an interest rate. For example, the rate of return on an investment in a lottery ticket is not a loan, so it does not offer an interest rate, but just a rate of return. In real life, its payoff is uncertain—it could be anything from zero to an unlimited amount. The same applies to stocks and many corporate projects. Many of our examples use the phrase "interest rate," even though our examples almost always work for any other rates of return, too.

2·1.C. U.S. Treasuries

Start with the simplest and most important bonds: Treasuries.

Bonds may be relatively simple projects, but bonds issued by the U.S. government—called Treasuries—are perhaps the simplest of them all. This is because Treasuries cannot fail to pay. They promise to pay U.S. dollars, and the United States has the right to print more U.S. dollars if it were ever to run out. So, for Treasuries, there is absolutely no uncertainty about repayment. This is convenient because it makes it easier to learn finance—but we study them not just because they are convenient tutorial examples. See, Treasuries are the single most important type of financial security in the world today. As of October 2004, the United States owed about $7.4 trillion, roughly $25,000 per citizen. After Treasuries are sold by the United States government, they are then actively traded in what is one of the most important financial markets in the world today. It would not be uncommon for dedicated bond traders to buy a 5-year Treasury originally issued 10 years ago, and 10 seconds later sell a 3-year Treasury issued 6 years ago—buyers and sellers in Treasuries are easily found, and transaction costs are very low. In 2001, average trading volume in Treasuries was about $300 billion per trading day (about 255 per year). Therefore, the annual trading volume in U.S. Treasuries of about $70 trillion totaled about five to ten times the U.S. economy's gross domestic product (**GDP**) of $10 trillion.

U.S. Treasury Bills, Notes, and Bonds have known and certain payouts.

The name Treasury comes from the fact that the debt itself is issued by the U.S. Treasury Department. They come in three flavors: **Treasury bills** (often abbreviated as **T-bills**) with maturities of less than one year, **Treasury notes** with maturities between one and ten years, and **Treasury bonds** with maturities greater than ten years. The 30-year bond was often called the **long bond**, at least before the Treasury suspended its issuance in October 2001—now, even a ten-year bond is often called the long bond. These three types of obligations are really conceptually the same, so they are usually called **U.S. Treasuries** or just **Treasuries**.

2·2. Returns, Net Returns, and Rates of Return

Defining: Return, Net Return, and Rate of Return.

The most basic financial concept is that of a return. The payoff or (dollar) **return** of an investment is simply the amount of cash it returns. The net payoff or **net return** is the difference between the return and the initial investment, which is positive if the project is profitable and negative if it is unprofitable. The **rate of return** is the net return expressed as a percentage of the initial investment. (**Yield** is a synonym for rate of return.) For example, an investment project that costs $10 today and returns $12 in period 1 has

$$
\begin{aligned}
\text{Return at Time 1} &= \$12 \\
\text{Return}_{t=1} &= \text{CF}_1 \quad , \\
\text{Net Return from Time 0 to Time 1} &= \$12 - \$10 = \$2 \\
\text{Net Return}_{t=0,1} &= \text{CF}_1 - \text{CF}_0 \quad , \\
\text{Rate of Return from Time 0 to Time 1} &= \frac{\$12 - \$10}{\$10} = \frac{\$12}{\$10} - 1 = 20\% \\
r_1 \;=\; r_{t=0,1} &= \frac{\text{CF}_1 - \text{CF}_0}{\text{CF}_0} = \frac{\text{CF}_1}{\text{CF}_0} - 1 \quad .
\end{aligned}
\tag{2.2}
$$

Percent (the symbol %) is a unit of 1/100. So, 20% is the same as 0.20. Also, please note my way to express time. Our most common investment scenario is a project that begins "right here right now this moment" and pays off at some moment(s) in time in the future. We shall use the letter t to stand for an index in time, and zero (0) as the time index for "right now." The length of each time interval may or may not be specified: thus, time $t = 1$ could be tomorrow, next month, or next year. A cash payout may occur at one instant in time, and thus needs only one time index. But investments usually tie up cash over an interval of time, called a **holding period**.

We use a comma-separated pair of time indexes to describe intervals. Whenever possible, we use subscripts to indicate time. When the meaning is clear, we abbreviate phrases such as the interval "$t = 0,1$" to simply $0,1$, or even just as 1. This sounds more complicated than it is. Table 2.1 provides some examples.

Table 2.1. Sample Time Conventions

$\text{Cash}_{t=0}$	Cash Right Now (index time 0). The time index ("t =") is given explicitly.
$\text{Cash}_{\text{Midnight, March 3, 2025}}$	Cash on Midnight of March 3, 2025. We rely on the subscript to tell the reader that the explicit subscript t is omitted.
Cash_1	Cash in the Future (at index time 1).
$\text{Investment}_{0,\text{Midnight March 3 2025}}$	An Investment made right now to pay off on March 3, 2025.
$\text{Investment}_{0,1}$	A One Period Investment, From Right Now To Time 1.
$\text{Return}_{t=1,2}$	A One Period Return, From Time 1 To Time 2.
$\text{Investment}_{0,2}$	A Two Period Investment, From Right Now To Time 2.
Return_2	A Two Period Return, From Right Now To Time 2.

Capital Gains vs. Returns.

Returns can be decomposed into two parts: intermittent payments and final payments. For example, many stocks pay cash dividends, many bonds pay cash coupons, and many real estate investments pay rent. Say, an investment costs \$92, pays a dividend of \$5 (at the end of the period), and then is worth \$110. What would its rate of return be?

$$
\begin{aligned}
r_{0,1} &= \frac{\$110 + \$5 - \$92}{\$92} = \frac{\$110 - \$92}{\$92} + \frac{\$5}{\$92} = 25\% \\
r_{0,1} &= \frac{\text{CF}_1 + \text{Dividend}_{0,1} - \text{CF}_0}{\text{CF}_0} = \underbrace{\frac{\text{CF}_1 - \text{CF}_0}{\text{CF}_0}}_{\text{Percent Price Change}} + \underbrace{\frac{\text{Dividend}_{0,1}}{\text{CF}_0}}_{\text{Dividend Yield}} .
\end{aligned}
\tag{2.3}
$$

The **capital gain** is the difference in the purchase price over the holding period, *not* counting interim payments. Here, the capital gain is the difference between \$110 and \$92, i.e., the \$18 change in the price of the investment. The dividend or coupon divided by the original price is called the **dividend yield** or **coupon yield** when stated in percentage terms. Of course, if the dividend/coupon yield is high, you might earn a positive rate of return but experience a negative capital gain. For example, a bond that costs \$500, pays a coupon of \$50, and then sells for \$490, has a **capital loss** of \$10 (which comes to a −2% capital yield), but a rate of return of $(\$490 + \$50 - \$500)/\$500 = +8\%$. Also, when there are dividends, coupons, or rent, prices follow a predictable pattern—this is because the price has to fall by about the amount of the payment. For instance, if a stock for \$20 were to pay a dividend for \$2 and stay at \$20, you should immediately purchase this stock—you would get \$2 for free. In fact, in a perfect market, anything other than a price drop from \$20 to \$18 at the instant of the dividend payment would not make sense. Such predictable price change patterns do not appear in rates of return. We will almost always work with rates of return, not with capital gains—though sometimes we have to draw the distinction, because the IRS treats capital gains differently from dividends. (We will talk about taxes in Section 6).

(Nominal) interest rates are usually non-negative.

When interest rates are certain, they should logically always be positive. After all, you can always earn 0% if you keep your money under your mattress—you thereby end up with as much money next period as you have this period. So why give your money to someone today who will give you less than 0% (less money in the future)? Consequently, interest rates are indeed almost always positive—the rare exceptions being both bizarre and usually trivial.

People often use incorrect terms, but the meaning is usually clear, so this is harmless.

Most of the time, people (incorrectly but harmlessly) abbreviate a rate of return or net return by calling it just a return. For example, if you say that the return on your $10,000 stock purchase was 10%, you obviously do not mean you received 0.1. You really mean that your rate of return was 10%. This is usually benign, because your listener will know what you mean. Potentially more harmful is the use of the phrase yield, because it is often used as a shortcut for dividend yield or coupon yield (the percent payout that a stock or a bond provide). So, if you say that the yield on a bond is 5%, then some listeners may interpret this to mean that the overall rate of return is 5%, while others may interpret this to mean the coupon yield to be 5%. And there is yet another complication, because coupon yields are often not quoted relative to the current price, but relative to the final payment. If in doubt, ask for a detailed explanation!

Basis Points avoid an ambiguity in the English language: 100 basis points is 1 percent.

Here is a small language problem. What does the statement "the interest rate has just increased by 5%" mean? It could mean either that the previous interest rate, say 10%, has just increased from 10% to $10\% \cdot (1 + 5\%) = 10.5\%$, or that it has increased from 10% to 15%. Because this is unclear, the **basis point** unit was invented. A basis point is simply 1/100 of a percent. So, if you state that your interest rate has increased by 50 basis points, you definitely mean that the interest rate has increased from 10% to 10.5%. If you state that your interest rate has increased by 500 basis points, you mean that the interest rate has increased from 10% to 15%.

IMPORTANT: *100 basis points constitute one percent.*

Solve Now!

Q 2.3 *A project offers a return of $1,050 for an investment of $1,000. What is the rate of return?*

Q 2.4 *A project offers a net return of $25 for an investment of $1,000. What is the rate of return?*

Q 2.5 *If the interest rate of 10% increases to 12%, how many basis points did it increase?*

Q 2.6 *If the interest rate of 10% decreased by 20 basis points, what is the new interest rate?*

ANECDOTE: Interest Rates over the Millenia
Historical interest rates are fascinating, perhaps because they look so similar to today's interest rates. In 2004, typical interest rates may range between 2% and 20% (depending on other factors). Now, for over 2,500 years, from about the thirtieth century B.C.E. to the sixth century B.C.E., normal interest rates in Sumer and Babylonia hovered around 10–25% per annum, though 20% was the legal maximum. In ancient Greece, interest rates in the sixth century were about 16–18%, dropping steadily to about 8% by the turn of the millenium. Interest rates in ancient Egypt tended to be about 10–12%. In ancient Rome, interest rates started at about 8% in the fifth century B.C.E., but began to increase to about 12% by the third century A.C.E. (a time of great upheaval). When lending resumed in the late Middle Ages (12th century), personal loans in England fetched about 50% per annum though they tended to hover between 10–20% in the rest of Europe. By the Renaissance, commercial loan rates had fallen to 5–15% in Italy, the Netherlands, and France. By the 17th century, even English interest rates had dropped to 6–10% in the first half, and even to 3–6% in the second half. Mortgage rates tended to be lower yet. Most of the American Revolution was financed with French and Dutch loans at interest rates of 4–5%.

2·3. The Time Value of Money

Now we turn our rate of return formula 2.2 around to determine how money will grow over time, *given* a rate of return.

2·3.A. The Future Value of Money

Future Payoffs Given a Rate of Return and an Initial Investment.

How much money will you receive in the future if the rate of return is 20% and you invest \$100? The answer is

$$\begin{aligned} 20\% &= \frac{\$120-\$100}{\$100} & \$100\cdot(1+20\%) &= \$100\cdot 1.2 = \$120 \\ r_{0,1} &= \frac{CF_1-CF_0}{CF_0} \quad\Leftrightarrow & CF_0\cdot(1+r_{0,1}) &= CF_1 \quad . \end{aligned} \tag{2.4}$$

Because the interest rate is positive, a given amount of money today is worth more than the same amount of money in the future—after all, you could always deposit your money today into the bank and thereby get back more money in the future. This is an example of the **time value of money**—a dollar today is worth more than a dollar tomorrow. This is one of the most basic and important concepts in finance. Our \$120 next year is therefore called the **future value** (FV) of \$100 today. It is the time-value of our money that causes its future value to be a bigger number than the **present value** (PV) of our money. Using these abbreviations, we could also have written the above as

$$r_{0,1} = \frac{FV-PV}{PV} \quad\Leftrightarrow\quad FV = PV\cdot(1+r_{0,1}) \quad . \tag{2.5}$$

Please note that the time value of money has nothing to do with the fact that the prices of goods may change between today and tomorrow. (In Section 6, we will discuss inflation—the fact that the purchasing power of money can change.) Instead, the time value of money, the present value, and future value are based exclusively on the concept that your money today can earn a positive interest, so the same amount today is better than the same amount tomorrow.

2·3.B. Compounding

Interest on Interest (or rate of return on rate of return) means rates cannot be added.

Now, what if you can earn the same 20% year after year and reinvest all your money? What would your two-year rate of return be? Definitely *not* 20% + 20% = 40%! We know that you will have \$120 in year 1, which you can reinvest at a 20% rate of return from year 1 to year 2. Thus, you will end up with

$$\begin{aligned} \$120\cdot(1+20\%) &= \$144 \\ CF_1\cdot(1+r_{1,2}) &= CF_2 \quad . \end{aligned} \tag{2.6}$$

This \$144—which is, of course, again a future value of \$100 today—represents a total two-year rate of return of

$$\begin{aligned} \frac{\$144-\$100}{\$100} &= \frac{\$144}{\$100}-1 = 44\% \\ \frac{CF_2-CF_0}{CF_0} &= \frac{CF_2}{CF_0}-1 = r_{0,2} \quad . \end{aligned} \tag{2.7}$$

This is more than 40%, because the original net return of \$20 in the first year earned an additional \$4 in interest in the second year. You earn interest on interest! Similarly, what would be your three-year rate of return? You would invest \$144 at 20%, which would provide you with

$$\begin{aligned} \$144\cdot(1+20\%) &= \$172.80 \\ CF_2\cdot(1+r_{2,3}) &= CF_3 \quad , \end{aligned} \tag{2.8}$$

so your three-year rate of return would be

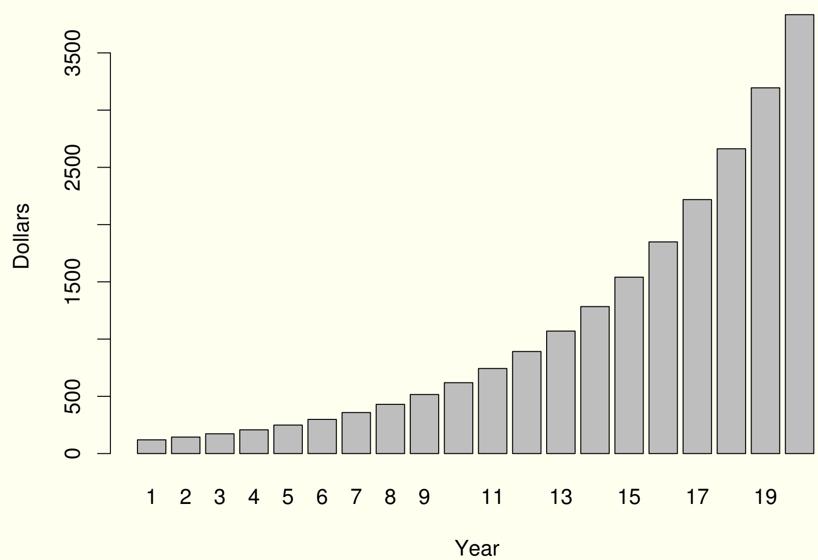
$$\begin{array}{rcccccl} & \dfrac{\$172.80 - \$100}{\$100} & = & \dfrac{\$172.80}{\$100} - 1 & = & 72.8\% & \\ r_{0,3} = & \dfrac{CF_3 - CF_0}{CF_0} & = & \dfrac{CF_3}{CF_0} - 1 & = & r_{0,3} & . \end{array} \quad (2.9)$$

If you do not want to compute interim cash flows, can you directly translate the three sequential one-year rates of return into one three-year **holding rate of return**? Yes! The compounding formula that does this is the "one-plus formula,"

$$\begin{array}{rcl} (1 + 72.8\%) & = & (1 + 20\%) \cdot (1 + 20\%) \cdot (1 + 20\%) \\ (1 + r_{0,3}) & = & (1 + r_{0,1}) \cdot (1 + r_{1,2}) \cdot (1 + r_{2,3}) \quad . \end{array} \quad (2.10)$$

In this case, all three rates of return were the same, so we could also have written this as $r_{0,3} = (1 + 20\%)^3$. Figure 2.2 shows how your \$100 would grow if you continued investing it at a rate of return of 20% per annum. The function is exponential, that is, it grows faster and faster, as interest earns more interest.

Table 2.2. Compounding Over 20 Years at 20% Per Annum



Period	Start Value	One-Year Rate	End Value	Total Factor on \$100	Total Rate of Return
0 to 1	\$100	(1+20%)	\$120.00	1.2	20.0%
1 to 2	\$120	(1+20%)	\$144.00	$1.2 \cdot 1.2 = 1.4$	40.0%
2 to 3	\$144	(1+20%)	\$172.80	$1.2 \cdot 1.2 \cdot 1.2 = 1.728$	72.8%
⋮	⋮	⋮	⋮	⋮	

When money grows at a rate of 20% per annum, each dollar invested right now will be worth \$38.34 in 20 years. The money at first grows about linearly, but as more and more interest accumulates and itself earns more interest, the graph accelerates upward.

IMPORTANT: *The compounding formula translates sequential future rates of return into an overall holding rate of return:*

$$\underbrace{(1+r_{0,T})}_{\text{Holding Rate}} = \underbrace{(1+r_{0,1})}_{\text{Current Spot Rate}} \cdot \underbrace{(1+r_{1,2})}_{\text{Future 1-Period Rate}} \cdot \ldots \cdot \underbrace{(1+r_{T-1,T})}_{\text{Future 1-Period Rate}} \quad . \tag{2.11}$$

The first rate is called the spot rate because it starts now (on the spot). If all spot and future interest rates are the same, the formula simplifies into $(1+r_{0,T}) = (1+r_t)^T$.

Another example of a payoff computation.

We can use the compounding formula to compute all sorts of future payoffs. For example, an investment project costing \$212 today and earning 10% each year for 12 years will yield an overall holding rate of return of

$$\begin{aligned} r_{0,12} &= (1+10\%)^{12} - 1 \approx 213.8\% \\ (1+r)^T - 1 &= r_{0,12} \quad . \end{aligned} \tag{2.12}$$

Our \$212 investment today would therefore turn into a future value of

$$\begin{aligned} \mathrm{CF}_{12} &= \$212 \cdot (1+213.8\%) \approx \$665.35 \\ \mathrm{CF}_0 \cdot (1+r_{0,12}) &= \mathrm{CF}_{12} \quad . \end{aligned} \tag{2.13}$$

Turn around the formula to compute individual holding rates.

Now, what constant two one-year interest rates (r) would give you a two-year rate of return of $r_{0,2} = 50\%$? It is not 25%, because $(1+25\%) \cdot (1+25\%) - 1 = 56.25\%$. Instead, we need to solve

$$(1+r) \cdot (1+r) = (1+r)^2 = 1+50\% \quad . \tag{2.14}$$

The correct answer is

$$\begin{aligned} r &= \sqrt[2]{1+50\%} - 1 \approx 22.47\% \\ &= \sqrt[t]{1+r_{0,t}} - 1 = r \quad . \end{aligned} \tag{2.15}$$

(Appendix 2·3 reviews powers, exponents and logarithms.) Check your answer: $(1+22.47\%) \cdot (1+22.47\%) \approx (1+50\%)$. If the 12 month interest rate is 213.8%, what is the one-month interest rate? By analogy,

$$\begin{aligned} (1+r)^{12} &\approx 1+213.8\% \\ r = \sqrt[12]{1+213.8\%} - 1 &= (1+213.8\%)^{1/12} \approx 10\% \quad , \end{aligned} \tag{2.16}$$

but we already knew this.

You can determine fractional interest rate via compounding, too.

Interestingly, compounding works even over fractional time periods. So, if the overall interest rate is 5% per year, to find out what the rate of return over half-a-year would be that would compound to 5%, compute

$$(1+r_{0,0.5}) = (1+r_{0,1})^{0.5} = (1+5\%)^{0.5} \approx 1+2.4695\% \quad . \tag{2.17}$$

Compounding 2.4695% over two (six-month) periods indeed yields 5%,

$$\begin{aligned} (1+2.4695\%) \cdot (1+2.4695\%) &\approx (1+5\%) \\ (1+r_{0,0.5})^2 &= (1+r_{0,1}) \quad . \end{aligned} \tag{2.18}$$

You need logs to determine time needed to get x times your money.

If you know how to use logarithms, you can also determine with the same formula how long it will take at the current interest to double or triple your money. For example, at an interest rate of 3% per year, how long would it take you to double your money?

$$\begin{aligned} (1+3\%)^x &= (1+100\%) &\Leftrightarrow\quad x &= \frac{\log(1+100\%)}{\log(1+3\%)} \approx 23.5 \\ (1+r_t)^T &= (1+r_{0,t}) &\Leftrightarrow\quad T &= \frac{\log(1+r_{0,t})}{\log(1+r_t)} \; . \end{aligned} \tag{2.19}$$

Solve Now!

Q 2.7 *A project has a rate of return of 30%. What is the payoff if the initial investment is $250?*

Q 2.8 *If 1-year rates of return are 20% and interest rates are constant, what is the 5-year holding rate of return?*

Q 2.9 *If the 5-year holding rate of return is 100% and interest rates are constant, what is the annual interest rate?*

Q 2.10 *If you invest $2,000 today and it earns 25% per year, how much will you have in 15 years?*

Q 2.11 *What is the holding rate of return for a 20 year investment which earns 5%/year each year? What would a $200 investment grow to?*

Q 2.12 *What is the quarterly interest rate if the annual interest rate is 50%?*

Q 2.13 *If the per-year interest rate is 5%, what is the two-year total interest rate?*

Q 2.14 *If the per-year interest rate is 5%, what is the ten-year total interest rate?*

Q 2.15 *If the per-year interest rate is 5%, what is the hundred-year total interest rate? How does this compare to 100 times 5%?*

Q 2.16 *At a constant rate of return of 5% per annum, how many years does it take you to triple your money?*

Q 2.17 *A project lost one-third of its value each year for 5 years. What was its rate of return, and how much is left from a $20,000 investment?*

Q 2.18 *From Fibonacci's Liber Abaci, written in the year 1202: "A certain man gave one denaro at interest so that in five years he must receive double the denari, and in another five, he must have double two of the denari and thus forever. How many denari from this 1 denaro must he have in 100 years?"*

Q 2.19 *(Advanced) In the text, you received the dividend at the end of the period. In the real world, if you received the dividend at the beginning of the period instead of the end of the period, could it change the rate of return? Why?*

2·3.C. Confusion: Interest Rates vs. Interest Quotes

Adding rather than compounding can make forgivably small mistakes in certain situations—but some people are just ignorant.

Unfortunately, when it comes to interest rates, confusion and "sloppy talk" abounds. See, some people mistakenly add interest rates instead of compounding them. When the investments, the interest rates, and the time periods are small, the difference between the correct and incorrect computation can be minor, so this practice can be acceptable, even if it is wrong. For example, when interest rates are 1%, compounding yields

$$\begin{aligned}(1+1\%) \quad \cdot \quad (1+1\%) \quad -1 &= 2.01\% \\ (1+r_{0,1}) \quad \cdot \quad (1+r_{1,2}) \quad -1 &= r_{0,2} \\ 1 + r_{0,1} + r_{1,2} + r_{0,1} \cdot r_{1,2} - 1 &= r_{0,2} \quad ,\end{aligned} \tag{2.20}$$

which is almost the same as the simple sum coming of $r_{0,1}$ and $r_{1,2}$ which comes to 2%. The difference between 2.01% and 2% is the "cross-term" $r_{0,1} \cdot r_{1,2}$. When returns are small—here if both returns are about 0.01—then the cross-product will be even smaller—here, it is 0.0001. This is small enough to be ignored in most situation, and therefore a forgivable approximation. However, when we compound over many periods, we will accumulate more and more cross-terms, and eventually the approximation will not be as good anymore. It is also the same approximation to annualize interest rates by just taking the average interest rate, which again ignores the interest on the interest. This can again be forgivable if the number of time periods and the interest rates are small.

Table 2.3. How Banks Quote Interest Rates

Bank quotes annual rate of (sometimes confusingly called **annual rate, compounded daily**) (should better be called **annual quote**)	10%
Bank pays daily rate of	10%/365 = 0.0274%
Daily rate compounds over 365 days to (sometimes called **effective annual rate**) (EFF or EAR)	$(1 + 0.0274\%)^{365} = 10.5\%$

(Even this is an oversimplication: banks can also compute interest rates based on 360/days per year. Fortunately, this difference between 360 and 365 days compounding is truly trivial.)

Banks add to the confusion, quoting interest rates in a strange but traditional way.

Even banks and many other lenders—who should know how to compound—have adopted a convention of quoting interest rates that may surprise you. What they quote as the annual interest rate, is really lower than the actual annual interest rate your money will earn. The banks will compute daily interest at a rate of their annual interest quote, divided by 365. So, as Table 2.3 shows, in effect, if the bank quotes you an annual interest rate of 10%, it is paying you 10.5% per annum ($10.50 for every $100) if you leave your money in the bank for a year. Similarly, many lenders who receive monthly payments—such as mortgage lenders—use the same method to quote an "annual rate compounded monthly." That is, if they quote 12% per annum, they mean to collect $1.01^{12} \approx 12.68\%$ per year on the money lent to you. Trust me: interest rates are not intrinsically difficult, but they can be tedious and definitional confusions often reign in their world.

IMPORTANT: *My best advice when money is at stake: If in doubt, ask how the interest rate is computed! Even better, ask for a simple illustrative calculation.*

Digging Deeper: *If you want to look up the rate of return on a Treasury bill, you may find that the Wall Street Journal quotes a number like 95. What does this mean?*

For example, in a Treasury auction, in which the government sells 180-day T-bills that will pay $10,000 in 180 days for $9,500, the discount quote would be

$$\begin{aligned}\textit{Quoted TB Price} \;=\; & \$10,000 \cdot [1 - (180/360) \cdot 10] \;=\; \$9,500 \\ & \$10,000 \cdot [1 - (\textit{days to maturity}/360) \cdot \textit{discount rate}] \quad .\end{aligned} \tag{2.21}$$

The Wall Street Journal then simply prints 95, because T-bills are quoted in units of 100. The real interest rate at the 95 quote is $(10,000/9,500) - 1 \approx 5.26\%$. *Therefore, even ignoring the extra 5 days, the 360 day interest rate is* $1.0526^2 - 1 \approx 10.8\%$, *not 10%. Be this as it may, a big advantage is that it is less confusing in that no one will confuse 95 for an interest rate. (Incidentally, I have not memorized the meaning, either. If I need it, I read this box.)*

Solve Now!

Q 2.20 *If you earn an (effective) interest rate of 12% per annum, how many basis points do you earn in interest on a typical day?*

Q 2.21 *If you earn an (effective) interest rate of 12% per annum, and there are 52.15 weeks, how much interest do you earn on a deposit of $100,000 over one week?*

Q 2.22 *If the bank quotes an interest rate of 12% per annum, how many basis points do you earn in interest on a typical day?*

Q 2.23 *If the bank quotes an interest rate of 12% per annum, and there are 52 weeks, how much interest do you earn on a deposit of $100,000 over one week?*

Q 2.24 *How much will your money grow to over the year?*

Q 2.25 *If the bank quotes an interest rate of 6% per year, what does a deposit of $100 in the bank come to after one year?*

Q 2.26 *If the bank quotes a loan rate of 8% per year, what do you have to pay back in one year if you borrow $100 from the bank?*

2·4. Capital Budgeting

Capital Budgeting: should you budget capital for a project?

We now turn to the flip side of our investment problem: if we know how much money we will have next year, what does this correspond to in value today? In a corporate context, our question is, "Given that Project X will return $1 million in 5 years, how much should we be willing to pay to undertake this project today?"

2·4.A. Discount Factor and Present Value (PV)

The "Present Value Formula" is nothing but the rate of return definition—inverted to translate future cash flows into (equivalent) today's dollars.

We start again with our **rate of return** formula 2.2,

$$r_{0,1} = \frac{\text{CF}_1 - \text{CF}_0}{\text{CF}_0} = \frac{\text{CF}_1}{\text{CF}_0} - 1 \quad . \tag{2.22}$$

We only need to turn this formula around to answer the question: if you know the prevailing interest rate in the economy ($r_{0,1}$) and the project's future cash flows (CF_1), what is the project's value to you *today*? For example, if the interest rate is 10%, how much would you have to save (invest) to receive $100 next year? Or, equivalently, if your project will return $100 next year, what is the project worth to you today? The answer lies in the **present value** formula, which translates future money into today's money. We merely need to rearrange the above formula to solve for CF_0,

$$\begin{aligned} & \frac{\$100}{1+10\%} \approx \$90.91 \\ \text{CF}_0 = & \frac{\text{CF}_1}{1+r_{0,1}} = \text{PV}(\,\text{CF}_1\,) \quad . \end{aligned} \tag{2.23}$$

Check this: investing $90.91 at an interest rate of 10% will indeed return $100 next period:

$$\begin{aligned} 10\% &\approx \frac{\$100 - \$90.91}{\$90.91} = \frac{\$100}{\$90.91} - 1 \iff (1+10\%)\cdot\$90.91 \approx \$100 \\ r_{0,1} &= \frac{\text{CF}_1 - \text{CF}_0}{\text{CF}_0} = \frac{\text{CF}_1}{\text{CF}_0} - 1 \iff (1+r_{0,1})\cdot\text{CF}_0 = \text{CF}_1 \quad . \end{aligned} \tag{2.24}$$

Thus, we can also state that the present value (PV) of $100 next year is $90.91—the value today of future cash flows. If you can borrow or lend at the interest rate of 10% elsewhere, you will be indifferent between receiving $100 next year and receiving $90.91 in our project today. In

ANECDOTE: Fibonacci and the Invention of Net Present Value
William Goetzmann argues that Leonardo of Pisa, commonly called Fibonacci, may have invented not only the famous "Fibonacci series," but also the concept of net present value, which is the focus of our chapter.

Fibonacci's family were merchants in the Mediterranean in the 13th century, with trade relations to Arab merchants in Northern Africa. Fibonacci wrote about mathematics primarily as a tool to solve merchants' problems—in effect, to understand the pricing of goods and currencies relative to one another. Think about how rich you could get if you could determine faster than your competition which goods were worth more in relation to others! In fact, you should think of Fibonacci and other Pisan merchants as the "financial engineers" of the 13th century.

Fibonacci wrote his most famous treatise, *Liber Abaci* at age 30, and published it in 1202. We still are solving the same kinds of problems today that Fibonacci explained. One of them—which you will solve at the end of this chapter—is called "On a Soldier Receiving 300 Bezants for his Fief":

> A soldier is granted an annuity by the king of 300 bezants per year, paid in quarterly installments of 75 bezants. The king alters the payment schedule to an annual year-end payment of 300. The soldier is able to earn 2 bezants on 100 per month (over each quarter) on his investment. How much is his effective compensation after the terms of the annuity changed?

To solve this problem, you must know how to value payments at different points in the future—you must understand the time value of money. What is the value of 75 bezants in one quarter, two quarters, etc.? What is the value of 300 bezants in one year, two years, etc.? Yes, money sooner is usually worth more than money later—but you need to determine by exactly how much in order to determine how good or bad the change is for the king and the soldier. To answer, you must use the interest rate Fibonacci gives, and then compare the two different cash flow streams—the original payment schedule and the revised payment schedule—in terms of a common denominator. This common denominator will be the two streams' present values.

contrast, if the standard rate of return in the economy were 12%, our specific project would not be a good deal. The project's present value would be

$$\begin{aligned} & \frac{\$100}{1+12\%} && \approx \quad \$89.29 \\ \text{CF}_0 \quad = \quad & \frac{\text{CF}_1}{1+r_{0,1}} && , \end{aligned} \tag{2.25}$$

which would be less than its cost of $90.91. But if the standard economy-wide rate of return were 8%, the project would be a great deal. Today's present value of the project's future payoffs would be

$$\begin{aligned} & \frac{\$100}{1+8\%} && \approx \quad \$92.59 \\ \text{CF}_0 \quad = \quad & \frac{\text{CF}_1}{1+r_{0,1}} && = \quad \text{PV}(\,\text{CF}_1\,) \quad , \end{aligned} \tag{2.26}$$

which would exceed the project's cost of $90.91. So, it is the present value of the project, weighed against its cost, that should determine whether you should undertake the project today, or whether you should avoid it. The present value is also the answer to the question, "How much would you have to save at current interest rates today if you wanted to have a specific amount of money next year ($\text{CF}_{t=1}$)"?

The PV formula.

Let's extend the example. If the interest rate were 10% per period, what would $100 in two periods be worth today? In two periods, you could earn a rate of return of $r_{0,2} = (1+10\%)\cdot(1+10\%)-1 = 21\%$ elsewhere, so this is your appropriate comparable rate of return. The value of the $100 is then

$$\begin{aligned} & \frac{\$100}{(1+10\%)^2} && = \quad \frac{\$100}{1+21\%} \quad \approx \quad \$82.64 \\ \text{CF}_0 = \quad & \frac{\text{CF}_2}{(1+r_{0,1})\cdot(1+r_{1,2})} && = \quad \frac{\text{CF}_2}{1+r_{0,2}} \quad = \quad \text{PV}(\,\text{CF}_2\,) \quad . \end{aligned} \tag{2.27}$$

The interest rate can now be called the "cost of capital."

In this context, the rate of return, r, with which the project can be financed, is often called the **cost of capital.** It is the rate of return at which you can borrow money elsewhere. This cost of capital is determined by the **opportunity cost** that you bear if you fund your specific project instead of the alternative next-best investment project elsewhere. Remember—you can invest your money at this rate instead of investing it in the project. Now, the better these alternative projects in the economy are, the higher will be your cost of capital, and the lower will be the value of your specific project with its specific cash flows. A project that promises $1,000 next year is worth less today if you can earn 50% elsewhere than when you can earn only 5% elsewhere. (In this first valuation part of our book, I will just inform you of the economy-wide rate of return, here 10%, at which you can borrow or invest. The investments part of the book will explain how we can learn this rate of return.)

Bond present values and prevailing interest rates move in opposite directions.

Remember how bonds are different from savings accounts? The former is pinned down by its promised fixed future payment, while the latter pays whatever the daily interest rate is. This induces an important relationship between the value of bonds and the prevailing interest rates—they move in opposite directions. For example, if we have a bond that promises to pay $1,000 in one year, and the prevailing interest rate is 5%, the bond has a present value of $\$1,000/(1+5\%) \approx \$952.38$. If the prevailing interest rate suddenly increases to 6%, the bond's present value becomes $\$1,000/(1+6\%) \approx \$943.40$. We would have lost $8.98, which is about 0.9% of our original $952.38 investment. The value of our fixed bond payment in the future has gone down, because investors now have relatively better opportunities elsewhere in the economy. They can earn a rate of return of 6%, not just 5%, so if we wanted to sell our bond, we would have to price it to leave the next buyer a rate of return of 6%. If we had invested our money into a savings account, the sudden change to 6% would do nothing to our investment, because we would now earn the 6%. This is a general implication and worthwhile noting:

IMPORTANT: *The price and implied rate of return on a bond with fixed payments move in opposite directions. When the price of the bond goes up, its implied rate of return goes down.*

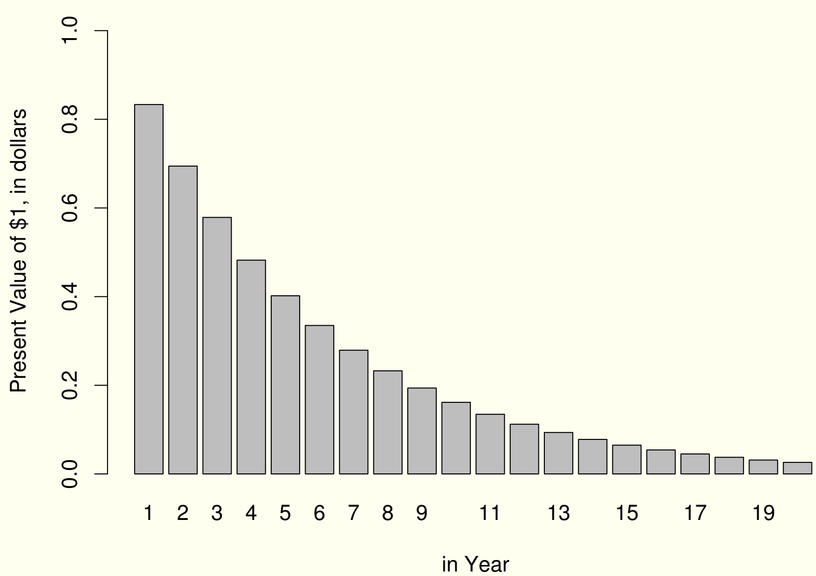
The quantity

The discount factor is closely related to the cost of capital.

$$\text{Discount Factor} = \left(\frac{1}{1+r_{0,t}}\right) \tag{2.28}$$

is called the **discount factor** (or sometimes, less correctly, the **discount rate**, though we use discount rate as a name for $r_{0,t}$ in this context). When you multiply a cash flow by its appropriate discount factor, you end up with its present value. If you wish, you can also think of **discounting**—the conversion of a future cash flow amount into its equivalent present value amount—as the reverse of compounding. In other words, the discount factor translates one dollar in the future into the equivalent amount of dollars today. Because interest rates are usually positive, discount factors are usually less than 1—a dollar in the future is worth less than a dollar today. Figure 2.4 shows how the discount factor declines when the cost of capital is 20% per annum. After about a decade, any dollar the project earns is worth less than 20 cents to you today.

Table 2.4. Discounting Over 20 Years at a Cost of Capital of 20% Per Annum



Each bar is $1/(1+20\%) = 83.3\%$ of the size of the bar to its left. After 20 years, the last bar is 0.026 in height. This means that $1 in 20 years is worth 2.6 cents in money today.

2·4.B. Net Present Value (NPV)

Present Values are alike and thus can be added, subtracted, compared, etc.

An important advantage of present value is that all cash flows are translated into the same unit: cash today. To see this, say that a project generates \$10 in 1 year and \$8 in 5 years. You cannot add up these different future values to come up with \$18—it would be like adding apples and oranges. However, if you translate both future cash flows into their present values, you *can* add them. For example, if the interest rate was 5% per annum (and $(1 + 5\%)^5 = (1 + 27.6\%)$ in over 5 years), the present value of these two cash flows together would be

$$\begin{aligned} \text{PV}(\,\$10 \text{ in one year}\,) &= \frac{\$10}{1+5\%} \approx \$9.52 \;, \\ \text{PV}(\,\$8 \text{ in five years}\,) &= \frac{\$8}{(1+5\%)^5} \approx \$6.27 \;. \\ \text{PV}(\,\text{CF}_t\,) &= \frac{\text{CF}_t}{1+r_{0,t}} \;. \end{aligned} \tag{2.29}$$

Therefore, the project's total value *today* (at time 0) is \$15.79.

The definition and use of NPV.

The **net present value** (NPV) is really the same as present value, except that the word "net" upfront reminds us to add and subtract *all* cash flows, including the *upfront* investment outlay today. The NPV calculation method is always the same:

1. Translate all future cash flows into today's dollars;
2. Add them all up.

So, if obtaining our project costs \$12 today, then this is a positive NPV project, because

$$\begin{aligned} \text{NPV} = -\$12 + \frac{\$10}{1+5\%} + \frac{\$8}{(1+5\%)^5} &\approx \$3.50 \\ \text{CF}_0 + \frac{\text{CF}_1}{1+r_{0,1}} + \frac{\text{CF}_5}{1+r_{0,5}} &= \text{NPV} \;. \end{aligned} \tag{2.30}$$

Thinking what NPV means, and how it can be justified.

There are a number of ways to think about the NPV. One way is is to think of the NPV of \$3.50 as the difference between the market value of the future cash flows (\$15.79) and the project's cost (\$12)—the difference being a sort of value added. You can also think of the equivalent of purchasing bonds that exactly *replicates* the project payoffs. Here, you would want to purchase one bond that promises \$10 next year. If you save \$9.52—at a 5% interest rate—you will receive \$10. Similarly, you would have to save \$6.27 in a bond that promises \$8 in 5 years. Together, these two bonds exactly replicate the project cash flows. The **law of one price** tells us that our project should be worth as much as this bond project—the cash flows are identical. We would have had to put away \$15.79 today to buy these bonds, but our project can deliver these cash flows at a cost of only \$12—much cheaper and thus better than our bond alternative.

Yet another way to justify NPV: opportunity cost.

Still another way is to think of NPV as an indicator of how our project compares to the alternative opportunity of investing at the capital markets, the rates of return being in the denominator (the discount factors). What would we get if we took our \$12 and invested it in the capital markets instead of our project? We could earn a 5% rate of return from now to next year and 27.6% from now to five years. Our \$12 would grow into \$12.60 by next year. Like our project, we could receive \$10, and be left with \$2.60 for reinvestment. Over the next 4 years, at the 5% interest rate, this \$2.60 would grow to \$3.16. But our project would do better for us, giving us \$8. So, our project achieves a higher rate of return than the capital markets alternative would achieve.

Take all positive NPV projects.

The conclusion of this argument is not only the simplest but also the best capital budgeting rule: If the NPV is positive, as it is here, you should take the project. If it is negative, you should reject the project. If it is zero, it does not matter.

IMPORTANT:

- *The **Net Present Value** Formula is*

$$
\begin{aligned}
NPV &= CF_0 + PV(\,CF_1\,) + PV(\,CF_2\,) + PV(\,CF_3\,) + PV(\,CF_4\,) + \ldots \\
&= CF_0 + \frac{CF_1}{1+r_{0,1}} + \frac{CF_2}{1+r_{0,2}} + \frac{CF_3}{1+r_{0,3}} + \frac{CF_4}{1+r_{0,4}} + \ldots \qquad (2.31) \\
&= \sum_{t=0}^{\infty} \left(\frac{CF_t}{1+r_{0,t}} \right) \quad .
\end{aligned}
$$

  The subscripts are time indexes, CF_t is the net cash flow at time t (positive for inflows, negative for outflows), and $r_{0,t}$ is the relevant interest rate for investments from today to time t. (The Greek sigma notation is just a shorter way of writing the longer summation above. You will see it again, so if you are not familiar with it, it is explained in Appendix Section 2·3.)

- *The **Net Present Value Capital Budgeting Rule** states that you should accept projects with a positive NPV and reject projects with a negative NPV.*

- *Taking positive NPV projects increases the value of the firm. Taking negative NPV projects decreases the value of the firm.*

- *NPV is definitively the best method for capital budgeting—the process by which you should accept or reject projects.*

A "free money" interpretation of NPV.

Why is NPV the right rule to use? The reason is that in our perfect world, a positive NPV project is the equivalent of free money. For example, if you can borrow or lend money at 8% anywhere today and you have an investment opportunity that costs $1 and yields $1.09, you can immediately contract to receive $0.01 next year *for free.* Rejecting this project would make no sense. Similarly, if you can sell someone an investment opportunity for $1, which yields only $1.07 next year, you can again earn $0.01 *for free.* Again, rejecting this project would make no sense. (In our perfect world, you can buy or sell projects at will.) Only zero NPV projects ($1 cost for $1.08 payoff) do not allow you to get free money. More interestingly, this allows us to conclude how our perfect world must work: Either positive NPV projects are not easy to come by—they are not available in abundant amounts, but can be available only in limited quantities to a limited number of individuals—or the NPV rule must hold. Positive NPV projects must be scarce, or everyone with access to these positive NPV projects would want to take an infinite amount of these projects, which in turn would continue until the economy-wide appropriate rate of return would go up to equal the project's rate of return. Of course, this argument is not here to show you how to get rich, but to convince you that the NPV rule makes sense and that any rule that comes to other conclusions than NPV would not.

Recap: NPV may be the most important building block in finance.

The translation between future values and present values—and its variant net present value—is the most essential concept in finance. Cash flows at different points in time must first be translated into the same units—dollars today—before they can be compared or added. *You must be comfortable with the mechanics of computing the net present value of projects.* We will solve many more NPV problems throughout the book. As you will find out in later chapters, despite its conceptual simplicity, the application of NPV in the real world is often surprisingly difficult, because you must estimate cash flows and discount factors.

Work a project example.

Let us work another example. A project costs $900 today, yields $200/year for two years, then $400/year for two years, and finally requires a cleanup expense of $100. The prevailing interest rate is 5% per annum. Should you take this project?

First, determine the multi-year costs of capital.

First we need to determine the cost of capital for tieing up money for 1 year, 2 years, 3 years, etc. The formula is

$$(1 + r_{0,t}) \;=\; (1 + 5\%)^t \;=\; 1.05^t \tag{2.32}$$

So, for money right now, the cost of capital $r_{0,0}$ is $1.05^0 - 1 = 0$; for money in one year, it is $r_{0,1}$ is $1.05^1 - 1 = 5\%$; for money in two years, it is $r_{0,2}$ is $1.05^2 - 1 = 10.25\%$; and so on. The discount factors are one divided by one plus our cost of capital. So, a dollar in one year is worth $1/(1 + 5\%) \approx 0.952$ dollars today. A dollar in two years is worth $1/(1 + 5\%)^2 \approx 0.907$. You must now multiply the promised payoffs by the appropriate discount factor to get their present value equivalents. Because present values are additive, you then sum up all the terms to compute the overall net present value. Put this all into one table, as in Table 2.5, and you find that the project NPV is \$68.15. Because this is a positive value, you should take this project.

A negative version of the same project.

However, if the upfront expense was \$1,000 instead of \$900, the NPV would be negative –\$31.85, and we would be better off investing the money into the appropriate sequence of bonds from which the discount factors were computed. In this case, we should have rejected the project.

Table 2.5. Hypothetical Project Cash Flow Table

Time	Project Cash Flow	Interest Rate Annualized	Interest Rate Holding	Discount Factor	Present Value
t	CF_t	r	$r_{0,t}$	$\dfrac{1}{1+r_{0,t}}$	$\text{PV}(\text{CF}_t)$
Today	-\$900	5.00%	0.00%	1.000	-\$900.00
Year +1	+\$200	5.00%	5.00%	0.952	+\$190.48
Year +2	+\$200	5.00%	10.25%	0.907	+\$181.41
Year +3	+\$400	5.00%	15.76%	0.864	+\$345.54
Year +4	+\$400	5.00%	21.55%	0.823	+\$329.08
Year +5	-\$100	5.00%	27.63%	0.784	-\$78.35
			Net Present Value (Sum):		\$68.15

As managers, we must estimate our project cash flows. The appropriate interest rate (also called cost of capital in this context) is provided to us by the opportunity cost of our investors—determined by the supply and demand for capital in the broader economy, where our investors can place their capital instead. These are the two input columns. We compute the remaining columns.

Solve Now!

Q 2.27 *Write down the NPV formula from memory.*

Q 2.28 *What is the NPV capital budgeting rule?*

Q 2.29 *If the cost of capital is 5% per annum, what is the discount factor for a cash flow in two years??*

Q 2.30 *Interpret the meaning of the discount factor.*

Q 2.31 *What are the units on rates of return, discount factors, future values, and present values?*

Q 2.32 *Determine the NPV of the project in Table 2.5, if the per-period interest rate is not 5% but 8% per year. Should you take this project?*

Q 2.33 *A project has a cost of capital of 30%. The final payoff is \$250. What should it cost today? (This will later be called the present value.)*

Q 2.34 *A bond promises to pay \$150 in 12 months. The annual applicable interest rate is 5%. What is the bond's price today?*

Q 2.35 *Work out the present value of your tuition payments for the next two years. Assume that the tuition is \$30,000 per year, payable at the start of the year. Your first tuition payment will occur in six months, your second tuition payment will occur in eighteen months. You can borrow capital at an interest rate of 6% per annum.*

Q 2.36 *Would it be good or bad for you, in terms of the present value of your liability, if your opportunity cost of capital increased?*

Q 2.37 *The price of a bond offering a firm promise of \$100 in 1 year is \$95. What is the implied interest rate? If the bond's interest rate suddenly jumped up by 150 basis points, what would the bond price be? How much would an investor gain/lose if she held the bond while the interest rate jumped up by these 150 basis points?*

2·5. Summary

The chapter covered the following major points:

- Returns must not be averaged, but compounded over time.
- The time-value of money means that one dollar today is worth more than one dollar tomorrow. Put differently, the future value of one dollar is less than the present value of one dollar.
- The discounted present value (PV) translates future cash values into present cash values. The net present value (NPV) is the sum of all present values of a project, including the usually negative upfront cash flow—the investment cost—today.
- The NPV formula can be written as

  $$\text{NPV} = \text{CF}_0 + \frac{\text{CF}_1}{1 + r_{0,1}} + \frac{\text{CF}_2}{1 + r_{0,2}} + \ldots \,. \qquad (2.33)$$

  In this context, r is called the discount rate or cost of capital, and $1/(1+r)$ is called the discount factor.
- The Net Present Value Capital Budgeting Rule states that you should accept projects with a positive NPV and reject projects with a negative NPV.
- 100 basis points are equal to 1%.
- Interest rate quotes are *not* interest rates—confusion often reigns supreme. For example, stated annual rates may not be the effective rates that you will earn if you put money into the bank. Or, a 3% coupon bond does not offer its investors a 3% rate of return.

- Bonds commit to payments in the future. Bank savings deposits are like a sequence of 1-day bonds, where a new interest is set daily.

  A sudden increase in the prevailing economy-wide interest rate decreases the present value of a bonds' future payouts, and therefore decreases today's price of the bond.

Solutions and Exercises

1. Definitely yes. Foregone salary is a cost that you are bearing. This can be reasonably estimated, and many economic consulting firms regularly do so. As to non-monetary benefits, there is the reputation that the degree offers you, the education that betters you, and the consumption pleasure from all the beer that fattens you.
2. Inflows: Value of Implicit Rent. Capital Gain if house appreciates. Outflows: Maintenance Costs. Transaction Costs. Mortgage Costs. Real Estate Tax. Theft. Capital Loss if house depreciates. And so on.
3. $r = \dfrac{\$1,050 - \$1,000}{\$1,000} = 5\%$.
4. $r = \dfrac{\$25}{\$1,000} = 2.5\%$.
5. 200.
6. 9.8%.
7. $r = 30\% = \dfrac{x - \$250}{\$250} \quad \Longrightarrow x = (1 + 30\%) \cdot \$250 = \$325$.
8. $(1 + 20\%)^5 - 1 \approx 149\%$.
9. $(1 + 100\%)^{(}1/5) - 1 \approx 14.9\%$.
10. $\$2,000 \cdot (1 + 25\%)^{15} = \$56,843.42$.
11. $1.05^{20} - 1 \approx 165.3\%$, so you would end up with $\$200 \cdot (1 + 165.3\%) \approx \$530.66$.
12.

$$(1 + r_{0,0.25})^4 \;=\; (1 + r_{0,1}) \qquad r_{0,0.25} \;=\; \sqrt[4]{1 + r_{0,1}} - 1 \;=\; (1 + 50\%)^{1/4} - 1 \;=\; 10.67\% \;. \tag{2.34}$$

13. $r_{0,2} = (1 + r_{0,1}) \cdot (1 + r_{1,2}) - 1 = (1 + 5\%) \cdot (1 + 5\%) - 1 = 10.25\%$.
14. $r_{0,2} = (1 + r_{0,1})^{10} - 1 = 62.89\%$.
15. $r_{0,2} = (1 + r_{0,1})^{100} - 1 == 130.5 = 13,050\%$ In words, this is about 130 times the initial investment, and substantially more than 500% (5 times the initial investment).
16. About 22.5 years.
17. $[1 + (-1/3)]^5 - 1 \approx -86.83\%$. So about \$2,633.74 is left.
18. First, solve for the interest rate: $1d \cdot (1 + r)^5 = 2d \to r = 14.87\%$. Therefore, in 100 years, he will have $(1 + r)^{100} = 1,048,576$ denari. Of course, you can solve this in a simpler way: you have twenty 5-year periods, in each of which the holdings double. So, the answer is 2^{20} denari.
19. Yes, because dividends could then be reinvested, and earn extra return themselves.
20. $(1 + 12\%)^{(1/365)} = 3.105$bp/day.
21. The true daily interest rate, assuming 365 days, is 0.03105%. To get the true rate of return, compound this over 7 days: $(1 + 0.03105\%)^7 \approx 1.0021758$. So, your \$100,000 will grow into \$100,217.58. You can compute this differently: $(1 + 12\%)^{(}1/52.15)$.
22. $12\%/365 = 3.288$bp/day.
23. $(1 + 0.03288\%)^7 \approx 1.003116$. So, your \$100,000 will grow into \$100,230.36.
24. $(1 + 0.12/365)^{365} \approx \$112,747.46$.
25. The bank quote of 6% means that it will pay an interest rate of 6%/365= 0.0164384% per day. This earns an actual interest rate of $(1 + 0.0164384\%)^{365} = 6.18\%$ per year: Each invested \$100 earns \$6.18 over the year.
26. The bank quote of 8% means that you will have to pay an interest rate of 8%/365= 0.021918% per day. This earns an actual interest rate of $(1 + 0.021918\%)^{365} = 8.33\%$ per year: Each borrowed \$100 requires \$108.33 in repayment.
27. If you cannot do this by heart, do not go on until you have it memorized.
28. Accept if NPV is positive. Reject if NPV is negative.
29. $1/[(1 + 5\%) \cdot (1 + 5\%)] = 0.9070$.
30. It is today's value in dollars for one future dollar, i.e., at a specific point in time in the future.

31. The first two are unit-less, the latter two are in dollars (though the former is dollars in the future, while the latter is dollars today).

32. $-\$900 + \$200/(1+8\%)^1 + \$200/(1+8\%)^2 + \$400/(1+8\%)^3 + \$400/(1+8\%)^4 - \$100/(1+8\%)^5 \approx \$0.14$. The NPV is positive, therefore this is a worthwhile project.

33. $r = 30\% = \dfrac{\$250 - x}{x} \quad \Longrightarrow x = \$250/(1+30\%) \approx \$192.31$.

34. $\$150/(1+5\%) \approx \$142.86$.

35. The first tuition payment is worth $\$30,000/(1+6\%)^{1/2} \approx \$29,139$. The second tuition payment is worth $\$30,000/(1+6\%)^{3/2} \approx \$27,489$. Thus, the total present value is \$56,628.

36. Good. Your future payments would be worth less in today's money.

37. The original interest rate is $\$100/\$95 - 1 = 5.26\%$. Increasing the interest rate by 150 basis points is 6.76%. This means that the price should be $\$100/(1+6.76\%) = \$93.67$. A price change from \$95 to \$93.67 is a rate of return of -1.4%.

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

CHAPTER 3

More Time Value of Money

Quick Formulas for Perpetuities and Annuities

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In this chapter, we remain in our world of constant interest rates, perfect foresight, and perfect markets. We have to cover two important remaining questions:

1. What should influence your investment decisions?

   In particular, if you have a need to have cash early on, would this make you value short-term projects more? And should you value companies which grow faster more?

2. Are there any shortcut formulas?

The answer to the first question will be that nothing other than NPV should matter for project valuation, so neither your need to have cash nor the growth pattern of the firm should make any difference

The answer to the second question will be that there are indeed valuation formulas for projects that have peculiar cash flow patterns—perpetuities and annuities. Their values are easy to compute when interest rates are constant. This often makes them useful "quick-and-dirty" tools for approximations. They are not only in wide use, but also are almost necessary to compute cash flows for common bonds (like mortgages) and help understand the economics of corporate growth.

3·1. Separating Investment Decisions and Present Values From Other Considerations

There are two philosophical issues left in our perfect world. First, does it matter *when* we need or generate money (i.e., *who* we are), or does NPV subsume everything important that there is to know about our projects? Second, are fast-growing firms better investments than slow-growing firms?

3·1.A. Does It Matter *When* You Need Cash?

Who owns a project is not important in a perfectly competitive zero-friction capital market.

The answer is no—the value of a project *is* its net present value. It does not depend on who owns it, and/or on when the owner needs cash. Here is why. You already know about the time value of money, the fact that cash today is worth more than cash tomorrow. (A positive time value of money means nothing more than that the interest rate is positive.) If you do not agree—that is, if you value money tomorrow more than you value money today—then just give it to me until you need it back. I can deposit it in my bank account to earn interest in the interim. In our ideal, perfect capital market (without taxes, inflation, transaction costs, or other issues), you can of course do better: you can always shift money between time periods at an "exchange rate" that reflects the time value of money.

An "eager" consumer will take the project.

The shifting-at-will is worth illustrating. Assume that you have $150 cash on hand and that you have exclusive access to a project that costs $100, and returns $200 next year. The appropriate interest rate (cost of capital) is 10%—but you *really*, *really*, *really* want to live it up today. So, how much can you consume? And, would you take the project? Here is the NPV prescription in a perfect market:

An "eager beaver" consumer still takes any positive NPV project.

- Sell the project in the competitive market for its NPV:

$$-\$100 \; + \; \left(\frac{\$200}{1+10\%}\right) \; = \; \$81.82 \quad . \tag{3.1}$$

- Spend the $\$150 + (\$181.82 - \$100) = \$231.82$ today. You will be better off taking the project than consuming just your $150 cash at hand.

A "sleeper" consumer also takes the project.

Now, assume that you are Austin Powers, the frozen detective, who cannot consume this year. So, how much will you be able to consume next year? And, would you take the project? The NPV answer is

- Sell the project in the competitive market for

$$-\$100 \; + \; \frac{\$200}{1+10\%} \; = \; \$81.82 \; . \tag{3.2}$$

- Put the $81.82 into the bank for 10% today. Get $90 next year.
- Also put your $150 into the bank at 10% interest.
- Next year, consume $90 + $165 = $255.20.

Of course, an equally simple solution would be to take the project, and just put your remaining $50 into a bank account.

The point of this argument is simple: regardless of when you need cash, you are better off taking all positive NPV projects, and then using the capital markets to shift consumption into your preferred time for consumption. It makes no sense to let *consumption decisions* influence *investment decisions*. This is called the **separation of decisions**: you can make investment decisions without concern for your consumption preferences. (This separation of investment and consumption decisions does not always hold in imperfect markets, in which you can face different borrowing and lending interest rates. Chapter 6 will discuss this.)

The moral of the story: consumption and investment decisions can be separated in a perfect environment.

SIDE NOTE: Of course, after they have lost their clients' money, many brokers like to confuse this basic truth by claiming that they invested their clients' money for the long-term, and not for the short-term. This presumes that long-term investments do worse in the short-run than short-term investment. This makes little sense, because if this were the case, your broker should purchase the short-term investment, and sell it when it is relatively worth more than the long-term investment in order to purchase relatively more of the (then relatively cheaper) long-term investment. So, the fact is that no matter whether an investor needs money sooner or later, the broker should always purchase the highest NPV investments. This gives clients the most wealth today—if you care about future consumption, you can always save the extra cash from finding the highest NPV investments today.

Solve Now!

Q 3.1 *What is the main assumption that allows you to independently consider investment (project) choices without regard to when you need wealth (or how much money you currently have at hand)?*

Q 3.2 *You have \$500 and a terminal illness. You cannot wait until the following project completes: The project costs \$400 and offers a rate of return of 15%, although equivalent interest rates are only 10%. What should you do?*

3·1.B. Corporate Valuation: Growth as Investment Criteria?

A similar question to that posed in the previous subsection is "Would it make more sense to invest in companies that grow more quickly rather than slowly"? If you wish, you can think of this question as asking whether you should buy stocks in a fast-growing company like Microsoft or in a slow-growing company like Exxon. The answer is that it does not matter in our perfect world. Whether a company is growing fast or growing slow is already incorporated in the firm's price today, which is just the discounted net present value of the firm's cash flows that will accrue to the owners. Therefore, neither is the better deal.

The price should incorporate the attributes of the firms.

For example, consider company "Grow" (G) that will produce

Should we invest in a fast grower or a slow grower?

$$G_1 = \$100 \quad G_2 = \$150 \quad G_3 = \$250 \;, \tag{3.3}$$

and company "Shrink" (S) that will produce

$$S_1 = \$100 \quad S_2 = \$90 \quad S_3 = \$80 \;. \tag{3.4}$$

Is G not a better company to buy than S?

There is no uncertainty involved, and both firms face the same cost of capital of 10% per annum. Then the price of G today is

Let's find out: Compute the values.

$$\text{PV}(\,G\,) = \frac{\$100}{(1+10\%)^1} + \frac{\$150}{(1+10\%)^2} + \frac{\$250}{(1+10\%)^3} \approx \$402.71 \;, \tag{3.5}$$

and the price of S today is

$$\text{PV}(\,S\,) = \frac{\$100}{(1+10\%)^1} + \frac{\$90}{(1+10\%)^2} + \frac{\$80}{(1+10\%)^3} \approx \$225.39 \;. \tag{3.6}$$

Your investment dollar grows at the same rate, disconnected from the cash flow rate.

If you invest in G, then next year you will have \$100 cash, and own a company with \$150 and \$250 cash flows coming up. G's value at time 1 (so PV now has subscript 1) will thus be

$$\mathsf{PV}_1(\,G\,) \;=\; \$100 \;+\; \frac{\$150}{(1+10\%)^1} \;+\; \frac{\$250}{(1+10\%)^2} \;\approx\; \$442.98 \;. \tag{3.7}$$

Your investment will have earned a rate of return of $\$442.98/\$402.71-1 \;=\; 10\%$. If you instead invest in S, then next year you will receive \$100 cash, and own a company with "only" \$90 and \$80 cash flows coming up. S's value will thus be

$$\mathsf{PV}_1(\,S\,) \;=\; \$100 \;+\; \frac{\$90}{(1+10\%)^1} \;+\; \frac{\$80}{(1+10\%)^2} \;\approx\; \$247.93 \;. \tag{3.8}$$

Your investment will have earned a rate of return of $\$247.39/\$225.39-1 \;=\; 10\%$. In either case, you will earn the fair rate of return of 10%. So—whether cash flows are growing at a rate of $+50\%$, -10%, $+237.5\%$, or -92% is irrelevant: *the firms' market prices today already reflect their future growth rates.* There is no necessary connection between the growth rate of the underlying project cash flows or earnings, and the growth rate of your investment money (i.e., your expected rate of return).

3·1.C. The Value today is just "All Inflows" or just "All Outflows"

Dividend Payout Timing can shift around, too.

Now, the same argument applies to dividends: in the end, all earnings must be paid out (i.e., as dividends). This does not need to occur at the same time: your earnings can grow *today*, and your dividends can be zero or be shrinking *today*. In our earlier example, firm G could be a slow dividend payer or a fast dividend payer. It could pay \$100 now, \$150 next year and \$250 in two years. Or, it could reinvest the money, effectively on your behalf, (at the same 10%, of course), and then pay one big lump sum dividend of $\$100\cdot(1+10\%)^2+\$150\cdot(1+10\%)+\$250 = \$536$ at the end of period 2. The dividend payout policy does not affect G's value today. The important point is that the net present value of your total earnings and your total dividends must both be equal to the price of the firm in our idealized world—or you would get something for nothing or lose something for nothing.

<u>Important</u>: *In a perfect market, the price and value of the firm are determined by the net present value of the firm's underlying projects. This net present value of the firm's projects also must be the same as the net present value of the firm's payouts.*

$$\textit{Firm Value} \;=\; \textit{PV}(\underbrace{\text{“All Project Payouts”}}_{\textit{All Future}}) \;=\; \textit{PV}(\underbrace{\text{“All Project Cash Flows”}}_{\textit{All Future}}) \;. \tag{3.9}$$

The same logic applies to stock and debt. Debt receives some cash flows generated by the projects, which are then paid out as principal or interest. Similarly, stock receives some cash flows generated by the projects (sometimes casually called earnings), which are then paid out as dividends.

$$\begin{aligned} \textit{Stock Value} \;&=\; \textit{PV}(\underbrace{\textit{Dividends}}_{\textit{All Future}}) \;=\; \textit{PV}(\underbrace{\textit{Earnings to Stock}}_{\textit{All Future}}) \;. \\ \textit{Debt Value} \;&=\; \textit{PV}(\underbrace{\textit{Principal} + \textit{Interest}}_{\textit{All Future}}) \;=\; \textit{PV}(\underbrace{\textit{Cash Flows to Debt}}_{\textit{All Future}}) \;. \end{aligned} \tag{3.10}$$

The time patterns of inflows or outflows only matters in determining net present values. Beyond this influence, it does not matter whether the firm is a fast-earnings grower, a slow-earnings grower, a fast-dividend payer, or a slow-dividend payer—each firm should be a fair investment. There is no value created by shifting earnings or dividends across periods.

This simple insight is the basis of the "Modigliani-Miller" (M&M) theorems, which won two Nobel prizes in economics. (We will explain them in more detail in Chapter 17.) Remember, though, that the "perfect market" assumption is important—the value of the firm is only the discounted value of all future dividends or all future earnings if markets are not too far from perfect. This is reasonable enough an assumption for large company stocks traded in the United States, but not necessarily the case for small, privately held firms. You should also realize that over any limited time horizon, neither dividends nor earnings may represent value well—dividends can be zero for a while, earnings can be negative, and the firm can still have tremendous and positive value.

This is sometimes called the M&M theorem, but holds in perfect markets only.

There is an important corollary. If General Electric is about to win or has just had some great luck, having won a large defense contract (like the equivalent of a lottery), shouldn't you purchase GE stock to participate in the windfall? Or, if Wal-Mart managers do a great job and have put together a great firm, shouldn't you purchase Wal-Mart stock to participate in this windfall? The answer is that you cannot. The old shareholders of Wal-Mart are no dummies. They know the capabilities of Wal-Mart and how it will translate into cash flows. Why should they give you, a potential new shareholder, a special bargain for something that you contributed nothing to? Just providing more investment funds is not a big contribution—after all, there are millions of other investors equally willing to provide funds at the appropriately higher price. It is competition—among investors for providing funds and among firms for obtaining funds—that determines the expected rate of return that investors receive and the cost of capital that firms pay. There is actually a more general lesson here. Economics tells us that you must have a scarce resource if you want to earn above-normal profits. Whatever is abundant and/or provided by many will not be tremendously profitable.

Any wealth gains accrue to existing shareholders, not to new investors.

Solve Now!

Q 3.3 *Presume that company G pays no interim dividends, so you receive \$536 at the end of the project. What is the G's market value at time 1, 2, and 3? What is your rate of return in each year? Assume that the cost of capital is still 10%.*

Q 3.4 *Presume that company G pays out the full cash flows in earnings each period. What is G's market value at time 1, 2, and 3? What is your rate of return in each year?*

Q 3.5 *Which dividend stream increases the value of the firm? Do you prefer a firm paying a lot of dividends, or a firm paying no dividends until the very end?*

Q 3.6 *The discount rate is 15%/annum over all periods. Firm F's cash flows start with \$500 and grow at 20% per annum for 3 years. Firm S's cash flows also start with \$500 but shrink at 20% per annum for 3 years. What are the prices of these two firms, and what is the expected growth rate of your money that you would invest into these two companies?*

3·2. Perpetuities

"Perpetuities" and "Annuities" are projects with special kinds of cash flows, which permit the use of short-cut formulas.

We now proceed to our second subject of this chapter—the shortcut formulas to compute the present values of certain cash streams. A **perpetuity** is a project with a cash flow that repeats forever. If the cost of capital (the appropriate discount rate) is constant and the amount of money remains the same or grows at a constant rate, perpetuities lend themselves to quick present value solutions—very useful when you need to come up with quick rule-of-thumb estimates. Though the formulas may seem a bit intimidating at first, using them will quickly become second nature to you.

3·2.A. The Simple Perpetuity Formula

Table 3.1. Perpetuity Stream of \$2 With Interest Rate r = 10%

Time	Cash Flow	Discount Factor	Present Value	Cumulative
0	Nothing! You have no cash flow here!			
1	\$2	$^1/(1+10\%)^1$	\$1.82	\$1.82
2	\$2	$^1/(1+10\%)^2$	\$1.65	\$3.47
3	\$2	$^1/(1+10\%)^3$	\$1.50	\$4.97
⋮	⋮	⋮	⋮	⋮
50	\$2	$^1/(1+10\%)^{50}$	\$0.02	\$19.83
⋮	⋮	⋮	⋮	⋮
t	\$2	$^1/(1+10\%)^t$	$^{\$2}/(1+10\%)^t$	
⋮	⋮	⋮	⋮	⋮
			Net Present Value (Sum):	= \$20.00

An Example Perpetuity that pays \$2 forever.

At a constant interest rate of 10%, how much money do you need to invest today to receive the same dollar amount of interest of \$2 each year, starting next year, forever? Such a payment pattern is called a simple perpetuity. It is a stream of cash flows that are the same for each period and continue *forever*. Table 3.1 shows a perpetuity paying \$2 forever if the interest rate is 10% per annum.

The Shortcut Perpetuity Formula.

To confirm the table's last row, which gives the perpetuity's net present value as \$20, you can spend from here to eternity to add up the infinite number of terms. But if you use a spreadsheet to compute and add up the first 50 terms, you will get a PV of \$19.83. If you add up the first 100 terms, you will get a PV of \$19.9986. Trust me that the sum will converge to \$20. This is because there is a nice shortcut to computing the net present value of the perpetuity if the cost of capital is constant.

$$\begin{aligned} \text{Perpetuity PV} &= \frac{\$2}{10\%} = \$20 \\ \text{PV}_t &= \frac{\text{CF}_{t+1}}{r} \ . \end{aligned} \tag{3.11}$$

The "t+1" in the formula is to remind you that the first cash flow begins the *following* period, not this period—the cash flows are the same in 1 period, in 2 periods, etc.

IMPORTANT: *A stream of constant cash flows, CF dollars each period and forever, beginning next period, and is discounted at the same annual cost of capital r forever is worth*

$$PV_t = \frac{CF_{t+1}}{r} \quad . \tag{3.12}$$

The easiest way for you to get comfortable with perpetuities is to solve some problems.

Solve Now!

Q 3.7 *From memory, write down the perpetuity formula. Be explicit on when the first cash flow occurs.*

Q 3.8 *What is the PV of a perpetuity paying \$5 each month, beginning next month, if the monthly interest rate is a constant 0.5%/month (6.2%/year)?*

Q 3.9 *What is the PV of a perpetuity paying \$15 each month, beginning next month, if the annual interest rate is a constant 12.68% per year?*

Q 3.10 *Under what interest rates would you prefer a perpetuity that pays \$2 million a year to a one-time payment of \$40 million?*

ANECDOTE: The Oldest Institutions and Perpetuities
Perpetuities assume that projects last forever. But nothing really lasts forever. The oldest Western institution today may well be the Roman Catholic Church, which is about 2,000 years old. The oldest existing corporation in the United States is The Collegiate Reformed Protestant Dutch Church of the City of New York, formed in 1628 and granted a corporate charter by King William in 1696. The Canadian Hudson's Bay Company was founded in 1670, and claims to be the oldest continuously incorporated company in the world.

Guantanamo Naval Base was leased from Cuba in 1903 as a perpetuity by the United States in exchange for 2,000 pesos per annum in U.S. gold, equivalent to \$4,085. In a speech, Fidel Castro has redefined time as "whatever is indefinite lasts 100 years." In any case, the Cuban government no longer recognizes the agreement, and does not accept the annual payments—but has also wisely not yet tried to expel the Americans.

3·2.B. The Growing Perpetuity Formula

Table 3.2. Perpetuity Stream With CF_{+1} = \$2, Growth Rate g = 5%, and Interest Rate r = 10%

Time	Cash Flow			Discount Rate	Present Value	Cumulative
0	Nothing. You have no cash flows here.					
1	$(1+5\%)^0$	·\$2 =	\$2.000	$(1+10\%)^1$	\$1.818	\$1.82
2	$(1+5\%)^1$	·\$2 =	\$2.100	$(1+10\%)^2$	\$1.736	\$3.56
3	$(1+5\%)^2$	·\$2 =	\$2.205	$(1+10\%)^3$	\$1.657	\$5.22
⋮	⋮	·\$2 =	⋮	⋮	⋮	
30	$(1+5\%)^{29}$	·\$2 =	\$8.232	$(1+10\%)^{30}$	\$0.236	\$30.09
⋮	⋮	·\$2 =	⋮	⋮	⋮	⋮
t	$(1+5\%)^{t-1}$	·\$2 =	⋮	$(1+10\%)^t$	⋮	⋮
⋮	⋮	·\$2 =	⋮	⋮	⋮	⋮
				Net Present Value (Sum):		= \$40.00

A growing perpetuity assumes that cash flows grow by a constant rate forever.

What if the cash flows are larger every period? A generalization of the perpetuity formula is the **growing perpetuity** formula, in which the cash flows grow by a constant rate g each period. The cash flows of a sample growing perpetuity—which pays \$2 next year, grows at a rate of 5%, and faces a cost of capital of 10%—are shown in Table 3.2. The present value of the first 50 terms adds up to \$36.28. The first 100 terms add up to \$39.64. The first 200 terms add up to \$39.98. Eventually, the sum approaches the formula

$$\begin{aligned} \text{PV of Growing Perpetuity}_0 &= \frac{\$2}{10\% - 5\%} = \$40 \\ \text{PV}_t &= \frac{\text{CF}_{t+1}}{r-g} \quad . \end{aligned} \tag{3.13}$$

As before, the "t+1" indicates that cash flows begin next period, not this period, and r is the interest rate minus g, the growth rate of your cash flows. Note that the growth timing occurs one period after the discount factor timing. For example, the time 30 discount factor is power-30, while the growth factor is power-29. We shall see later that the growing perpetuity formula is most commonly used when nominal project cash flows are assumed to grow by the rate of inflation. We will use this formula extensively to obtain "terminal values" in our penultimate chapter on pro formas, where we will use almost all your financial tools to do a strategic financial analysis.

Important: *A stream of cash flows, growing at a rate of g each period and discounted at a constant interest rate r (which must be higher than g) is worth*

$$PV_t = \frac{CF_{t+1}}{r-g} \quad . \tag{3.14}$$

The first cash flow, CF_{t+1} occurs next period, the second cash flow of $CF_{t+2} = CF_{t+1} \cdot (1+g)$ occurs in two periods, and so forth, forever.

What would happen if the cash flows grew faster than the interest rate ($g \geq r$)? Wouldn't the formula indicate a negative PV? Yes, but this is because the entire scenario would be non-sense. The PV in the perpetuities formulas is only less than infinity, because *in today's dollars*, each term in the sum is a little less than the term in the previous period. If g were greater than r, however, the cash flow one period later would be worth more even in today's dollars—and taking a sum over an infinite number of increasing terms would yield infinity as the value. A value of infinity is clearly not sensible, as nothing in this world is worth an infinite amount of money today. And, therefore, the growing perpetuity formula yields a non-sensical negative value if $g \geq r$—as it should!

Non-sensible answers.

Solve Now!

Q 3.11 *From memory, write down the growing perpetuity formula.*

Q 3.12 *What is the PV of a perpetuity paying \$5 each month, beginning this month (in 1 second), if the monthly interest rate is a constant 0.5%/month (6.2%/year), and the cash flows will grow at a rate of 0.1%/month (1.2%/year)?*

3·2.C. A Growing Perpetuity Application: Stock Valuation with Gordon Growth Models

With their fixed interest and growth rates and eternal payment requirements, perpetuities are rarely exactly correct. But they can be very helpful for quick over-the-envelope estimates. For example, consider a stable business with profits of \$1 million next year. Because it is stable, its profits are likely to grow at the inflation rate of, say, 2% per annum. This means it will earn \$1,020,000 in two years, \$1,040,400 in three years, etc. The firm faces a cost of capital of 8%. The growing perpetuity formula indicates that this firm should probably be worth no more than

Perpetuities are imperfect approximations, but often give a useful upper bound.

$$\begin{aligned} \text{Business Value} &= \frac{\$1,000,000}{8\% - 2\%} \approx \$16,666,667 \\ \text{Business Value}_0 &= \frac{\text{CF}_1}{r - g}\quad, \end{aligned} \tag{3.15}$$

because in reality, the firm will almost surely not exist forever. Of course, in real life, there are often even more significant uncertainties: next year's profit may be different, the firm may grow at a different rate (or may grow at a different rate for a while) or face a different cost of capital for one-year loans than it does for thirty-year loans. Thus, \$16.7 million should be considered a quick-and-dirty useful approximation, perhaps for an upper limit, and not an exact number.

The growing perpetuity model is sometimes directly applied to the stock market. For example, if you believe that a stock's dividends will grow by $g = 5\%$ forever, and if you believe that the appropriate rate of return is $r = 10\%$, and you expect the stock to earn and/or pay dividends of $D = \$10$ next year, then you would feel that a stock price of

The "Gordon Growth Model": constant eternal dividend growth.

$$P_0 = \frac{D_1}{r - g} = \frac{\$10}{10\% - 5\%} = \$200 \tag{3.16}$$

would be appropriate. In this context, the growing perpetuity model is often called the **Gordon growth model**, after its inventor Myron Gordon.

Let us explore the Gordon growth model a bit more. In October 2004, Yahoo!*Finance* listed General Electric (GE) with a dividend yield of 2.43%. This is dividends divided by the stock price, D/P, although it may be that dividends are from this year and not forward-looking. (Fixing this would change our numbers only very little, so we shall not bother.) Rearrange our formula 3.16:

Estimating the cost of capital for GE.

$$\frac{D}{P} = r - g = 2.43\% \ . \tag{3.17}$$

Therefore, we know that the market believes that the appropriate cost of capital (r) for General Electric exceeds its growth rate of dividends (g) by about 2.4%. Yahoo!*Finance* further links to a summary of GE's cash flow statement, which indicates that GE paid \$7.643 billion in dividends in 2003, and \$6.358 billion in 2001. Over these two years, the growth rate of dividends was about 9.6% per annum ($\$6.358 \cdot (1+9.6\%)^2 \approx \$7.643$). Therefore, if we believe 9.6%/year is a fair representation of the eternal growth rate of GE's dividends, then the financial markets valued GE as if it had a per-annum cost of capital of about

$$r = \frac{D}{P} + g \approx 2.4\% + 9.6\% \approx 12\% \ . \tag{3.18}$$

You can do the same with earnings.

It is also not uncommon to repeat the same exercise with earnings—that is, presume that stock market values are capitalized as if corporate earnings were eternal cash flows growing at a constant rate g. Again, Yahoo!*Finance* gives us all the information we need. GE's "trailing P/E" ratio—calculated as the current stock price divided by historical earnings—was 21, its "forward P/E" ratio—calculated as the price divided by analysts' expectations of next year's dividends—was 18.5. The latter is P_0/E_1, and thus closer to what we want. Yahoo!*Finance* further tells us that GE's earnings growth was 6.3%—the g in our formula. Therefore,

$$P_0 = \frac{E_1}{r-g} \Rightarrow r = \frac{E_1}{P_0} + g = \frac{1}{P_0/E_1} + g \approx \frac{1}{18.5} + 6.3\% \approx 11.7\% \ . \tag{3.19}$$

Keep perspective!

It is important that you recognize that these are just models—approximations—that you cannot take too seriously (in terms of accuracy). GE will not last forever, earnings are not the cash flows we need (more in Chapter 9), the discount rate is not eternally constant, earnings will not grow forever at 6.3%, etc. However, the numbers are not uninteresting and probably not too far off, either. GE is a very stable company that is likely to be around for a long time, and you could do a lot worse than assuming that the cost of capital (for investing of projects that are similar to GE stock ownership) is somewhere around 12% per annum—say, somewhere between 10% to 14% per annum.

Solve Now!

Q 3.13 *An eternal patent swap contract states that the patentee will pay the patenter \$1.5 million next year. The contract terms state growth with the inflation rate, which runs at 2% per annum. The appropriate cost of capital is 14%. What is the value of this patenting contract?*

Q 3.14 *How would the patent swap contract value change if the first payment did not occur next year, but tonight?*

Q 3.15 *A stock is paying a quarterly dividend of \$5 in one month. The dividend is expected to increase every quarter by the inflation rate of 0.5% per quarter—so it will be \$5.025 in the next quarter (i.e., paid out in four months). The prevailing cost of capital for this kind of stock is 9% per annum. What should this stock be worth?*

Q 3.16 *If a \$100 stock has earnings that are \$5 per year, and the appropriate cost of capital for this stock is 12% per year, what does the market expect the firm's "as-if-eternal dividends" to grow at?*

3·3. The Annuity Formula

The second type of cash flow stream that lends itself to a quick formula is an **annuity**, which is a stream of cash flows for a given number of periods. Unlike a perpetuity, payments stop after T periods. For example, if the interest rate is 10% per period, what is the value of an annuity that pays \$5 per period for 3 periods?

An Annuity pays the same amount for T years.

Let us first do this the slow way. We can hand-compute the net present value to be

$$\begin{aligned} \mathsf{PV}_0 &= \frac{\$5}{1+10\%} + \frac{\$5}{(1+10\%)^2} + \frac{\$5}{(1+10\%)^3} \approx \$12.4343 \\ \mathsf{PV}_0 &= \frac{\mathrm{CF}_1}{(1+r_{0,1})} + \frac{\mathrm{CF}_2}{(1+r_{0,2})} + \frac{\mathrm{CF}_3}{(1+r_{0,3})} \quad . \end{aligned} \tag{3.20}$$

So, what is the shortcut to compute the net present value of an annuity? It is the annuity formula, which is

$$\begin{aligned} \mathsf{PV} &= \$5 \cdot \left\{\frac{1-[1/(1+10\%)]^3}{10\%}\right\} \approx \$12.4343 \quad , \\ \mathsf{PV} &= \mathrm{CF}_{t+1} \cdot \left\{\frac{1-[1/(1+r)]^T}{r}\right\} = \mathsf{PV} \quad . \end{aligned} \tag{3.21}$$

Is this really a short-cut? Maybe not for 3 periods, but try a 360-period annuity, and let me know which method you prefer. Either works.

Important: *A stream of constant cash flows, beginning next period and lasting for T periods, and discounted at a constant interest rate r, is worth*

$$PV_t = \frac{CF_{t+1}}{r} \cdot \left[1 - \frac{1}{(1+r)^T}\right] \quad . \tag{3.22}$$

There is also a **growing annuity** formula, which nobody remembers but which you should know to look up if you need it. It is

$$\mathsf{PV} = \frac{\mathrm{CF}_{t+1}}{r-g} \cdot \left[1 - \frac{(1+g)^T}{(1+r)^T}\right] \quad . \tag{3.23}$$

3·3.A. An Annuity Application: Fixed-Rate Mortgage Payments

Most mortgages are **fixed rate mortgage loans**, and they are basically annuities. They promise a specified stream of equal cash payments each month to a lender. A 30-year mortgage with monthly payments is really a 360-payments annuity. (The "annu-ity" formula should really be called a "month-ity" formula in this case.) So, what would be your monthly payment if you took out a 30-year mortgage loan for \$500,000 at an interest rate of 7.5% per annum?

Mortgages are annuities, so the annuity formula is quite useful.

Before you can proceed further, you need to know one more bit of institutional knowledge here: Mortgage providers—like banks—quote interest by just dividing the mortgage quote by 12, so the true monthly interest rate is $7.5\%/12 = 0.625\%$. (They do not compound; if they did, the monthly interest rate would be $(1+7.5\%)^{1/12} - 1 = 0.605\%$.)

Lenders quote interest rates using the same convention that banks use.

The mortgage payment can be determined by solving the Annuity formula.

So our 30-year mortgage is an annuity with 360 equal payments with a discount rate of 0.625% per month. Its PV of $500,000 is the amount that you are borrowing. We want to determine the fixed monthly cash flow that gives the annuity this value:

$$
\begin{aligned}
\$500,000 &= \frac{\mathrm{CF}_{t+1}}{0.625\%} \cdot \left[1 - \frac{1}{(1+0.625\%)^{360}}\right] \approx CF_{t+1} \cdot 143.02 \\
\mathsf{PV} &= \frac{\mathrm{CF}_{t+1}}{r} \cdot \left[1 - \frac{1}{(1+r)^{T}}\right] \quad .
\end{aligned}
\tag{3.24}
$$

Solving this for the cash flow tells you that the monthly payment on your $500,000 mortgage will be $3,496.07 for 360 months, beginning next month.

SIDE NOTE: Uncle Sam allows mortgage borrowers to deduct the interest, but not the principal, from their tax bills. The IRS imputes interest on the above mortgage as follows: In the first month, Uncle Sam proclaims $0.625\% \cdot \$500,000 = \$3,125$ to be the tax-deductible mortgage interest payment. Therefore, the principal repayment is $\$3,496.07 - \$3,125 = \$371.07$ and remaining principal is $499,628.93. The following month, Uncle Sam proclaims $0.625\% \cdot \$499,628.93 = \$3,122.68$ to be the tax-deductible interest payment, $\$3,496.07 - \$3,122.68 = \$373.39$ as the principal repayment, and $499,255.54 as the remaining principal. And so on.

3·3.B. An Annuity Example: A Level-Coupon Bond

Coupon bonds pay not only at the final time.

Let us exercise our new found knowledge in a more elaborate example—this time with bonds. Bonds come in many different flavors, but one useful classification is into coupon bonds and zero bonds (short for zero coupon bonds). A **coupon bond** pays its holder cash at many different points in time, whereas a **zero bond** pays only a single lump sum at the maturity of the bond. Many coupon bonds promise to pay a regular coupon similar to the interest rate prevailing at the time of the bond's original sale, and then return a "principal amount" plus a final coupon at the end of the bond.

Bonds are specified by their promised payout patterns.

For example, think of a coupon bond that will pay $1,500 each half-year (semi-annual payment is very common) for five years, plus an additional $100,000 in 5 years. This payment pattern is so common that it has specially named features: A bond with coupon payments that remain the same for the life of the bond is called a **level-coupon bond.** The $100,000 here would be called the **principal**, in contrast to the $1,500 semi-annual coupon. Level bonds are commonly named by just adding up all the coupon payments over one year (here, $3,000), and dividing this sum of annual coupon payments by the principal. So this particular bond would be called a "3% semi-annual coupon bond" ($3,000 coupon per year, divided by the principal of $100,000). Now, the "3% coupon bond" is just a naming convention for the bond with these specific cash flow patterns—it is not the interest rate that you would expect if you bought this bond. In Section 2·3.C, we called such name designations interest *quotes*, as distinct from interest *rates*. Of course, even if the bond were to cost $100,000 today (and we shall see below that it usually does not), the interest rate would not be 3% per annum, but $(1 + 1.5\%)^2 - 1 \approx 3.02\%$ per annum.

SIDE NOTE: **Par value** is a vacuous concept, sometimes used to compute coupon payout schedules. Principal and par value, and/or interest and coupon payment need not be identical, not even at the time of issue, much less later. For the most part, par value is best ignored.

We now solve for the value of our coupon bond. Incidentally, you may or may not find the annuity formula helpful—you can use it, but you do not need it. Our task is to find the value of a "3% coupon bond" today. First, we write down the payment structure for our 3% semi-annual coupon bond. This comes from its defined promised patterns,

A Typical Coupon Bond

Year	Due Date	Bond Payment	Year	Due Date	Bond Payment
0.5	Nov 2002	$1,500	3.0	May 2005	$1,500
1.0	May 2003	$1,500	3.5	Nov 2005	$1,500
1.5	Nov 2003	$1,500	4.0	May 2006	$1,500
2.0	May 2004	$1,500	4.5	Nov 2006	$1,500
2.5	Nov 2004	$1,500	5.0	May 2007	$101,500

Second, we need to determine the appropriate expected rates of return to use for discounting. We shall assume that the prevailing interest rate is 5% per annum, which translates into 2.47% for 6 months, 10.25% for two years, etc.

Step 2: find the appropriate costs of capital.

Maturity	Yield	Maturity	Yield
6 Months	2.47%	36 Months	15.76%
12 Months	5.00%	42 Months	18.62%
18 Months	7.59%	48 Months	21.55%
24 Months	10.25%	54 Months	24.55%
30 Months	12.97%	60 Months	27.63%

Our third step is to compute the discount factors, which are just $1/(1 + r_{0,t})$, and to multiply each future payment by its discount factor. This will give us the present value (PV) of each bond payment, and therefore the bond overall value:

Step 3: Compute the discount factor is $1/(1 + r_{0,t})$.

Year	Due Date	Bond Payment	Rate of Return	Discount Factor	Present Value
0.5	Nov 2002	$1,500	2.47%	0.976	$1,463.85
1.0	May 2003	$1,500	5.00%	0.952	$1,428.57
1.5	Nov 2003	$1,500	7.59%	0.929	$1,349.14
2.0	May 2004	$1,500	10.25%	0.907	$1,360.54
2.5	Nov 2004	$1,500	12.97%	0.885	$1,327.76
3.0	May 2005	$1,500	15.76%	0.864	$1,295.76
3.5	Nov 2005	$1,500	18.62%	0.843	$1,264.53
4.0	May 2006	$1,500	21.55%	0.823	$1,234.05
4.5	Nov 2006	$1,500	24.55%	0.803	$1,204.31
5.0	May 2007	$101,500	27.63%	0.784	$79,527.91
				Sum	$91,501.42

We now know that, in our perfect world, we would expect this 3% level-coupon bond to be trading for $91,501.42 today. Because the current price of the bond is below the so-named final principal payment of $100,000, our bond would be said to trade at a **discount**. (The opposite would be a bond trading at a **premium**.)

Common naming conventions for this type of bond: coupon rate is not interest rate!

The above computation is a bit tedious. Can we translate it into an annuity? Yes! We will work in half-year periods. We thus have 10 coupon cash flows, each $1,500, at a per-period interest rate of 2.47%. So, according to our formula, the coupon payments are worth

Using the annuity to make this faster.

$$\begin{aligned}\text{PV} &= \text{CF}_{t+1} \cdot \left\{\frac{1-[1/(1+r)]^T}{r}\right\} \\ &= \$1,500 \cdot \left\{\frac{1-[1/(1+2.47\%)]^{10}}{2.47\%}\right\} \\ &\approx \$13,148.81 \quad .\end{aligned} \tag{3.25}$$

In addition, we have our $100,000 repayment of principal, which is worth

$$\begin{aligned}\text{PV} &= \frac{\$100,000}{1+27.63\%} \approx \$78,352.62 \\ \text{PV} &= \frac{\text{CF}}{(1+r_{0,5})} \approx \$78,352.62 \quad .\end{aligned} \tag{3.26}$$

Together, these present values of our bond's cash flows add up to $91,501.42.

Prevailing Interest Rates and Bond Values: We already know that the value of one fixed future payment and the interest rate move in opposite directions. Given that we now have many payments, what would happen if the economy-wide interest rates were to suddenly move from 5% per annum to 6% per annum? The semi-annual interest rate would now increase from 2.47% to

The effect of a change in interest rates.

$$r = \sqrt[2]{1+6\%} - 1 \approx 2.96\% \quad \Leftarrow \quad (1+2.96\%)\cdot(1+2.96\%) \approx (1+6\%) \quad . \tag{3.27}$$

To get the bond's new present value, reuse our formula

$$\begin{aligned} \text{PV} &= \text{CF}_{t+1} \cdot \left\{ \frac{1 - [1/(1+r)]^T}{r} \right\} + \frac{\text{CF}_T}{1 + r_{0,T}} \\ &= \$1,500 \cdot \left\{ \frac{1 - [1/(1+2.96\%)]^{10}}{2.96\%} \right\} + \frac{\$100,000}{(1+2.96\%)^{10}} \\ &\approx \$12,823.89 + \$74,725.82 \approx \$87,549.70 \ . \end{aligned} \tag{3.28}$$

So, our bond would have lost \$3,951.72, or 4.3% of our original investment—which is the same inverse relation between bond values and prevailing economy-wide interest rates that we first saw on Page 22.

Interest Rates vs. Coupon Rates.

Important Repeat of Quotes vs. Returns: Never confuse a bond designation with the interest it pays. The "3%-coupon bond" is just a designation for the bond's payout pattern. Our bond will not give you coupon payments equal to 1.5% of your \$91,502.42 investment (which would be \$1,372.52). The prevailing interest rate (cost of capital) has nothing to do with the quoted interest rate on the coupon bond. We could just as well determine the value of a 0%-coupon bond, or a 10% coupon bond, given our prevailing 5% economy-wide interest rate. Having said all this, in the real world, many corporations choose coupon rates similar to the prevailing interest rate, so that at the moment of inception, the bond will be trading at neither premium nor discount. So, at least for this one brief at-issue instant, the coupon rate and the economy-wide interest rate may actually be fairly close. However, soon after issuance, market interest rates will move around, while the bond's payments remain fixed, as designated by the bond's coupon name.

Solve Now!

Q 3.17 *If you can, from memory, write down the annuity formula. If not, go back to memorize it.*

Q 3.18 *What is the PV of a 360 month annuity paying \$5 per month, beginning at \$5 next month, if the monthly interest rate is a constant 0.5%/month (6.2%/year)?*

Q 3.19 *Mortgages are not much different from rental agreements. For example, what would your rate of return be if you rented a \$500,000 warehouse for 10 years at a monthly lease payment of \$5,000? If you can earn 5% elsewhere, would you rent out your warehouse?*

Q 3.20 *What is the monthly payment on a 15-year mortgage for every \$1,000 of mortgage at an effective interest rate of 6.168% per year (here, 0.5% per month)?*

Q 3.21 *Solve Fibonacci's annuity problem from Page 21: Compare the PV of a stream of quarterly cash flows of 75 bezants vs. the PV of a stream of annual cash flows of 300 bezants. Payments are always at period-end. The interest rate is 2 bezants per month. What is the relative value of the two streams? Compute the difference for a 1-year investment first.*

Q 3.22 *In L'Arithmetique, written in 1558, Jean Trenchant posed the following question: "In the year 1555, King Henry, to conduct the war, took money from bankers at the rate of 4% per fair [quarter]. That is better terms for them than 16% per year. In this same year before the fair of Toussaints, he received by the hands of certain bankers the sum of 3,945,941 ecus and more, which they called 'Le Grand Party' on the condition that he will pay interest at 5% per fair for 41 fairs after which he will be finished. Which of these conditions is better for the bankers?" Translated, the question is whether a perpetuity at 4% per quarter is better or worse than a 41-month annuity at 5%.*

Q 3.23 *Assume that a 3% level-coupon bond has not just 5 years with 10 payments, but 20 years with 40 payments. Also, assume that the interest rate is not 5% per annum, but 10.25% per annum. What are the bond payment patterns and the bond's value?*

Q 3.24 *Check that the rates of return in the coupon bond valuation example on* *Page 43* *are correct.*

Q 3.25 *In many a defined contribution pension plan, the employer provides a fixed percentage contribution to the employee's retirement. Let us assume that you must contribute $4,000 per annum beginning next year, growing annually with the inflation rate of 2%/year. What is this individual's pension cost to you of hiring a 25-year old, who will stay with the company for 35 years? Assume a discount rate of 8% per year. NOTE: You need the growing annuity formula 3.23, which you should look up.*

3·4. Summary

The chapter covered the following major points:

- In a perfect market, consumption and investment decisions can be made independently. You should always take the highest NPV projects, and use the capital markets to shift cash into periods in which you want it.
- In a perfect market, firms are worth the present value of their assets. Whether they grow fast or slow is irrelevant except to the extent that this determines their PV. Indeed, firms can shift the time patterns of cash flows and dividends without changing the underlying firm value.
- In a perfect market, the gains from sudden surprises accrue to old owners, not new capital provides, because old owners have no reason to want to share the spoils.
- The PV of a growing perpetuity—with constant-growth (g) cash flows CF beginning next year and constant per-period interest rate r—is

$$\mathsf{PV}_t = \frac{\mathrm{CF}_{t+1}}{r-g} \quad . \tag{3.30}$$

- The application of the growing perpetuity formula to stocks is called the Gordon dividend growth model.
- The PV of an annuity—T periods of constant CF cash flows (beginning next year) and constant per-period interest rate r—is

$$\mathsf{PV}_t = \mathrm{CF}_{t+1} \cdot \left\{ \frac{1-[1/(1+r)]^T}{r} \right\} \quad . \tag{3.31}$$

- Fixed-rate mortgages are annuities, and therefore can be valued with the annuity formula.

Solutions and Exercises

1. The fact that you can use capital markets to shift money forth and back without costs.
2. Take the project. If you invest \$400, the project will give $\$400 \cdot (1 + 15\%) = \$460$ next period. The capital markets will value the project at \$418.18. Sell it at this amount. Thereby, you will end up being able to consume $\$500 - \$400 + \$418.18 = \$518.18$.
3. For easier naming, we call year 0 to be 2000. The firm's present value in 2000 is $\$536/1.10^3 \approx \$402.71$—but we already knew this. If you purchase this company, its value in 2001 depends on a cash flow stream that is \$0 in 2001, \$0 in year 2002 and \$536 in year 2003. So, it will be worth $\$536/1.10^2 \approx \$442.98$ in 2001. In 2002, your firm will be worth $\$536/1.10 = \$487.27$. Finally, in 2003, it will be worth \$536. Each year, you expect to earn 10%, which you can compute from the four firm values.
4. Again, we call year 0 2000. The firm's present value in 2000 is based on dividends of \$100, \$150, and \$250 in the next three years. So, the firm value in 2000 is the \$402.71 in Formula 3.5. The firm value in 2001 is in Formula 3.7, but you immediately receive \$100 in cash, so the firm is worth only $\$442.98 - \$100 = \$342.98$. As an investor, you would have earned a rate of return of $\$442.98/\$402.71 - 1 = 10\%$. The firm value in 2002 is

$$\mathsf{PV}_2(\,G\,) \;=\; \frac{\$250}{(1+10\%)} \;\approx\; \$227.27 \;. \tag{3.32}$$

but you will also receive \$150 in cash, for a total firm-related wealth of \$377.27. In addition, you will have the \$100 from 2001, which would have grown to \$110—for a total wealth of \$487.27. Thus, starting with wealth of \$442.98 and ending up with wealth of \$487.27, you would have earned a rate of return of $\$487.27/\$442.98 - 1 = 10\%$. A similar computation shows that you will earn 10% from 2002 (\$487.27) to 2003 (\$536.00).
5. It makes no difference!
6. F's cash flows are \$500, \$600, and \$720. Its value is therefore \$1,361.88. Firm S's cash flows are \$500, \$400, and \$320. Its value is therefore \$947.65. Both firms offer your investment dollar a 15% rate of return.

7. CF_1/r. The first cash flow occurs next period, not this period.
8. $\mathsf{PV} = \dfrac{\text{CF}_1}{r} = \$5/0.005 = \$1,000$.
9. $\mathsf{PV} = \dfrac{\text{CF}_1}{r} = \$15/.01 = \$1,500$.
10. You would prefer the perpetuity if the interest rate/cost of capital was less than 5%.
11. $\text{CF}_1/(r - g)$. The first cash flow occurs next period, not this period.
12. You get \$5 today, and next month you will receive a payment of $(1 + \pi)\cdot\text{CF} = 1.001 \cdot \$5 = \$5.005$. The growing perpetuity is worth $\mathsf{PV} = \dfrac{\text{CF}_1}{r - g} = \$5.005/(0.5\% - 0.1\%) = \$1,251.25$. So, the total value is \$1,256.25.
13. \$12.5 million.
14. The immediate dividend would be worth \$1.5 million. In addition, you now have a growing perpetuity that starts with a payment of \$1.53 million. Therefore, the PV would be $\$1.5 + \$1.53/12\% = \$14.25 million$.
15. First work out what the value would be if you stood at one month. The interest rate is $(1 + 9\%)^{1/12} - 1 = 0.7207\%$ per month, and 2.1778% per quarter. Thus, in one month, you will have \$5.00 plus $\$5.025/(2.1778\% - 0.5\%) \approx \$299.50$. In addition, you get the \$5 for a total of \$304.50. Because this is your value in one month, discount \$304.50 at an 0.7207% interest rate to \$302.32 today.
16. 7% per annum.

17. Remembering this formula is not as important as remembering the other growing perpetuity formula. The annuity formula is $\text{CF}_1 \cdot \left\{\dfrac{1 - [1/(1+r)]^T}{r}\right\}$.
18.

$$\begin{aligned}\text{CF}_1 \cdot \left\{\frac{1 - [1/(1+r)]^T}{r}\right\} &= \$5 \cdot \left\{\frac{1 - [1/(1+0.005)]^{360}}{0.005}\right\} \\ &= \$5 \cdot \left\{\frac{1 - 0.166}{0.005}\right\} \approx \$833.96\end{aligned}$$

19. You need to solve

$$\$500,000 \;=\; \frac{\$5,000}{r}\left(1 - 1/(1+r)^{120}\right) \;. \tag{3.33}$$

The solution is $r \approx 0.3314\%$ per month, or 3.8% per annum. No!

20. For \$1,000 of mortgage, solve for CF_1 in

$$\mathsf{PV} = CF_1 \cdot \left\{ \frac{1 - [1/(1+r)]^T}{r} \right\}$$

$$\$1,000 = CF_1 \cdot \left\{ \frac{1 - [1/(1+0.005)]^{15\cdot 12=180}}{0.005} \right\}$$

$$\$1,000 = CF_1 \cdot 118.504 \iff CF_1 \approx \$8.44$$

In other words, for every \$1,000 of loan, you have to pay \$8.44 per month. For other loan amounts, just rescale the amounts.

21. For 1 year, the 300 bezants are worth $300/1.02^{12} = 236.55$ bezants today. The quarterly interest rate is $1.02^3 - 1 = 6.12\%$. Therefore, the 4-"quartity" is worth $75/.0612 \cdot [1 - 1/1.0612^4] = 300/1.0612^1 + 300/1.0612^2 + 300/1.0612^3 + 300/1.0612^4 = 259.17$ bezants. The soldier would have lost 22.62 bezants, which is 8.7% of what he was promised. (The same 8.7% loss applies to longer periods.)

22. For each ecu (e), the perpetuity is worth $1e/0.04 = 25e$. The annuity is worth $1e/0.05 \cdot (1 - 1/1.05^{41}) = 17.29e$. Therefore, the perpetuity is better.

23. The interest rate is 5% per half-year. Be my guest if you want to add 40 terms. I prefer the annuity method. The coupons are worth

$$\begin{aligned} \mathsf{PV}(\text{Coupons}) &= CF_{t+1} \cdot \left\{ \frac{1 - [1/(1+r)]^T}{r} \right\} \\ &= \$1,500 \cdot \left\{ \frac{1 - [1/(1+0.05)]^{40}}{0.05} \right\} \\ &= \$1,500 \cdot \left\{ \frac{1 - [1/(1+0.05)]^{40}}{0.05} \right\} \\ &\approx \$25,739 \quad . \end{aligned} \tag{3.34}$$

The final payment is worth

$$\mathsf{PV}(\text{Principal Repayment}) = \frac{\$100,000}{(1+0.05)^{40}} \approx \$14,205 \quad . \tag{3.35}$$

Therefore, the bond is worth about \$39,944 today.

24. For six months, $(1 + 2.47\%)^2 - 1 = 5\%$. Now, define six months to be one period. Then, for t 6-month periods, you can simply compute an interest rate of $(1 + 2.47\%)^t - 1$. For example, the 30 months interest rate is $(1 + 2.47\%)^5 - 1 = 12.97\%$.

25.
$$\$4,000/(0.08 - 0.02) \cdot \left[1 - \frac{(1+0.02)^{35}}{(1+0.08)^{35}} \right] \approx \$57,649 \quad . \tag{3.36}$$

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

A. Advanced Appendix: Proofs of Perpetuity and Annuity Formulas

A Perpetuity The formula is

$$\frac{\text{CF}}{1+r} + \frac{\text{CF}}{(1+r)^2} + \cdots + \frac{\text{CF}}{(1+r)^t} + \cdots = \frac{\text{CF}}{r} \; . \tag{3.37}$$

We want to show that this is a true statement. Divide by CF,

$$\frac{1}{1+r} + \frac{1}{(1+r)^2} + \cdots + \frac{1}{(1+r)^t} + \cdots = \frac{1}{r} \; . \tag{3.38}$$

Multiply (3.38) by $(1+r)$

$$1 + \frac{1}{(1+r)} + \cdots + \frac{1}{(1+r)^{t-1}} + \cdots = \frac{(1+r)}{r} \; . \tag{3.39}$$

Subtract (3.39) from (3.38),

$$1 = \frac{(1+r)}{r} - \frac{1}{r} \tag{3.40}$$

which simplifies to be a true statement.

A Growing Perpetuity We know from the simple perpetuity formula that

$$\sum_{t=1}^{\infty} \frac{\text{CF}}{(1+r)^t} = \frac{\text{CF}}{r} \quad \Leftrightarrow \quad \sum_{t=1}^{\infty} \frac{\text{CF}}{f^t} = \frac{\text{CF}}{f-1} \; . \tag{3.41}$$

Return to the definition of a growing perpetuity, and pull out one $(1+g)$ factor from its cash flows,

$$\sum_{t=1}^{\infty} \frac{C \cdot (1+g)^{t-1}}{(1+r)^t} = \left(\frac{1}{1+g}\right) \cdot \sum_{t=1}^{\infty} \frac{C \cdot (1+g)^t}{(1+r)^t} = \left(\frac{1}{1+g}\right) \cdot \sum_{t=1}^{\infty} \frac{\text{CF}}{\left[\frac{1+r}{1+g}\right]^t} \; . \tag{3.42}$$

Let $\left[\dfrac{1+r}{1+g}\right]$ be f, and use the first formula. Then

$$\left(\frac{1}{1+g}\right) \cdot \left\{ \sum_{t=1}^{\infty} \frac{\text{CF}}{\left[\frac{1+r}{1+g}\right]^t} \right\} = \left(\frac{1}{1+g}\right) \cdot \left\{ \frac{\text{CF}}{\left[\frac{1+r}{1+g}\right] - 1} \right\} , \tag{3.43}$$

and simplify this,

$$= \left(\frac{1}{1+g}\right) \cdot \left\{ \frac{\text{CF}}{\left[\frac{(1+r)-(1+g)}{1+g}\right]} \right\} = \left(\frac{1}{1+g}\right) \cdot \left\{ \frac{C \cdot (1+g)}{[r-g]} \right\} = \frac{\text{CF}}{r-g} \; . \tag{3.44}$$

An Annuity Consider one perpetuity that pays \$10 forever, beginning next year. Consider another perpetuity that begins in 5 years and also pays \$10, beginning in year 6, forever. If you purchase the first annuity and sell the second annuity, you will receive \$10 each year for five years, and \$0 in every year thereafter.

	0	1	2	3	4	5	6	7	8	⋯
Perpetuity 1		+\$10	+\$10	+\$10	+\$10	+\$10	+\$10	+\$10	+\$10	⋯
equivalent to	+\$10/r									
Perpetuity 2							−\$10	−\$10	−\$10	⋯
equivalent to						−\$10/r				
Net Pattern		+\$10	+\$10	+\$10	+\$10	+\$10				
equivalent to	+\$10/r					−\$10/r				
discount factor		$\frac{1}{(1+r)^1}$	$\frac{1}{(1+r)^2}$	$\frac{1}{(1+r)^3}$	$\frac{1}{(1+r)^4}$	$\frac{1}{(1+r)^5}$				

This shows that \$10, beginning next year and ending in year 5 should be worth

$$\begin{aligned} \text{PV} &= \frac{\$10}{r} - \frac{1}{(1+r)^5} \cdot \frac{\$10}{r} \\ &= \frac{C}{r} - \frac{1}{(1+r)^5} \cdot \frac{C}{r} \quad = \left(\frac{C}{r}\right) \cdot \left[1 - \frac{1}{(1+r)^T}\right] , \end{aligned} \tag{3.45}$$

which is just our annuity formula.

CHAPTER 4

Investment Horizon, The Yield Curve, and (Treasury) Bonds

Bonds and Fixed Income

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We remain in a world of perfect foresight and perfect markets, but we now delve a little deeper to make our world more realistic. In earlier chapters, the interest rate was the same every period—if a 30-year bond offered an interest rate of 5.6% per annum, so did a 1-year bond. But this is usually not the case in the real world. For example, in May 2002, a 30-year U.S. Treasury bond offered an interest rate of 5.6% per year, while a 1-year U.S. Treasury bond offered an interest rate of only 2.3% per year. The issues that these horizon-dependent interest rates create matter not only for bond traders—who work with time-dependent interest rates every day—but also for companies that are comparing short-term and long-term projects. After all, investors can earn higher rates of return if instead of giving money to your firm's long-term projects, they invest in longer-term Treasury bonds. Thus, if two corporate projects have different horizons, they should not necessarily be discounted at the same cost of capital. In May 2002, building a 30-year power plant probably required a higher cost of capital to entice investors than an otherwise equivalent 1-year factory. Similarly, if your corporation wants to finance projects by borrowing, it must pay a higher rate of return if it borrows long-term.

In this chapter, you will learn how to work with horizon-dependent rates of returns, and you will see *why* rates usually differ. This chapter then takes somewhat of a digression—working a number of issues that are primarily of interest in a bond context. But this digression is germane to the corporate context, because almost all corporations need to borrow money.

4·1. Time-Varying Rates of Return

A second compounding example.

We now switch direction and make the world a bit more realistic—we allow rates of return to differ by horizon. As promised in the previous chapter, all tools you have learned remain applicable. In particular, compounding still works exactly the same way. For example, what is the two-year rate of return if the interest rate is 20% in the first year, and 30% in the second year? (The latter is known as a **reinvestment rate.**) You can determine multiyear rates of return from one-year rates of return using the same compounding formula,

$$\begin{aligned}(1+r_{0,2}) &= (1+r_{0,1}) \cdot (1+r_{1,2}) \\ &= (1+20\%) \cdot (1+30\%) = 1.56 \ .\end{aligned} \tag{4.1}$$

Subtract 1, and the answer is the total two-year rate of return of 56%.

The general formula for compounding over many periods.

So, the compounding formula for obtaining a total rate of return from period i to period j is still the multiplicative "one-plus formula" for each interest rate (subtracting 1 at the end). It now can also help answer questions such as, "If the one-year rate of return is 30% from year 1 to year 2, 40% from year 2 to year 3, and 50% from year 3 to year 4, then what is the rate of return for investing beginning next year for three years?" The answer is

$$\begin{aligned}\text{Given} &: \quad r_{1,2} = 30\% \quad r_{2,3} = 40\% \quad r_{3,4} = 50\% \\ (1+r_{1,4}) &= (1+r_{1,2}) \cdot (1+r_{2,3}) \cdot (1+r_{3,4}) \\ &= (1+30\%) \cdot (1+40\%) \cdot (1+50\%) \\ &= (1+173\%) \ .\end{aligned} \tag{4.2}$$

Subtracting 1, you see that the three-year rate of return for an investment that takes money next year (not today!) and returns money in four years, appropriately called $r_{1,4}$, is 173%. For example, if it were midnight of December 31, 1989 right now, each dollar invested on midnight December 31, 1990, would return \$1.73 on midnight December 31, 1993.

Solve Now!

Q 4.1 *If the first-year interest rate is 2% and the second year interest is 3%, what is the two-year total interest rate?*

Q 4.2 *Although a promising two-year project had returned 22% in its first year, overall it lost half of its value. What was the project's rate of return after the first year?*

Q 4.3 *From 1991 to 2002, the stock market (specifically, the S&P500) had the following annual rates of return:*

Year	$\tilde{r}_{S\&P500}$	Year	$\tilde{r}_{S\&P500}$
1991	+0.2631	1997	+0.3101
1992	+0.0446	1998	+0.2700
1993	+0.0706	1999	+0.1953
1994	−0.0154	2000	−0.1014
1995	+0.3411	2001	−0.1304
1996	+0.2026	2002	−0.2337

What was your rate of return over these 12 years? Over the first 6 years and over the second 6 years?

Q 4.4 *A program lost one third of its value the first year, then gained fifty percent of its value, then lost two thirds of its value, and finally doubled in value. What was the overall rate of return?*

4·2. Annualized Rates of Return

Per-Unit Measures are conceptual aids.

Time-varying rates of return create a new complication, that is best explained by an analogy. Is a car traveling 258,720 yards in 93 minutes fast or slow? It is not easy to say, because you are used to thinking in "miles per sixty minutes," not in "yards per ninety-three minutes." It makes sense to translate speeds into miles per hour for the purpose of comparing speeds. You can even do this for sprinters, who cannot run a whole hour. Speeds are just a standard measure of the rate of accumulation of distance per unit of time.

A Per-Unit Standard for Rates of Returns: Annualization.

The same issue applies to rates of return: a rate of return of 58.6% over 8.32 years is not as easy to compare to other rates of return as a rate of return per year. So, most rates of return are quoted as **average annualized rates**. Of course, when you compute such an average annualized rate of return, you do not mean that the investment earned the same annualized rate of return of, say, 5.7% each year—just as the car need not have traveled at 94.8 mph (258,720 yards in 93 minutes) each instant. The average annualized rate of return is just a convenient unit of measurement for the rate at which money accumulates, a "sort-of-average measure of performance."

An Example of Annualizing a Three-Year Total Holding Return.

So, if you were earning a total three-year holding return of 173% over the three year period, what would your average *annualized* rate of return be? The answer is not $173\%/3 \approx 57.7\%$, because if you earned 57.7% per year, you would have ended up with $(1 + 57.7\%) \cdot (1 + 57.7\%) \cdot (1 + 57.7\%) - 1 = 287\%$, not 173%. This incorrect answer of 57.7% ignores the *compounded interest on the interest* that you would earn after the first year and second year. Instead, you need to find a single hypothetical rate of return which, if you received it each and every year, would give you a three-year rate of return of 173%.

A Problem of finding a three-year annualized interest rate. Solution: Take the N-th Root of the total return (N is number of years).

Call $r_{\overline{3}}$ this hypothetical annual rate which you would have to earn each year for 3 years in order to end up with a total rate of return of 173%. To find $r_{\overline{3}}$, solve the equation

$$\begin{aligned} (1 + r_{\overline{3}}) \cdot (1 + r_{\overline{3}}) \cdot (1 + r_{\overline{3}}) &= (1 + 173\%) \\ (1 + r_{\overline{3}}) \cdot (1 + r_{\overline{3}}) \cdot (1 + r_{\overline{3}}) &= (1 + r_{0,3}) \quad , \end{aligned} \tag{4.3}$$

or, for short

$$\begin{aligned} (1 + r_{\overline{3}})^3 &= (1 + 173\%) \\ (1 + r_{\overline{t}})^t &= (1 + r_{0,t}) \quad . \end{aligned} \tag{4.4}$$

Here $r_{\overline{3}}$ is an unknown. Earning the same rate ($r_{\overline{3}}$) three years in a row should result in a total holding rate of return ($r_{0,3}$) of 173%. The correct solution for $r_{\overline{3}}$ is obtained by computing the third root of the total holding rate of return (Appendix 2·3 reviews powers, exponents and logarithms):

$$\begin{aligned} (1 + r_{\overline{3}}) &= (1 + 173\%)^{(1/3)} = \sqrt[3]{1 + 173\%} \approx 1.3976 \\ (1 + r_{0,t})^{(1/t)} &= \sqrt[t]{1 + r_{0,t}} = (1 + r_{\overline{t}}). \end{aligned} \tag{4.5}$$

Confirm with your calculator that $r_{\overline{3}} \approx 39.76\%$,

$$(1 + 39.76\%) \cdot (1 + 39.76\%) \cdot (1 + 39.76\%) \approx (1 + 173\%) \quad . \tag{4.6}$$

In sum, if you invested money at a rate of 39.76% per annum for three years, you would end up with a total three-year rate of return of 173%. As is the case here, for bonds with maturities away from 1-year, the order of magnitude is often so different that you will intuitively immediately register whether $r_{0,3}$ or $r_{\overline{3}}$ is meant.

IMPORTANT: *The total holding rate of return over N years, called $r_{0,N}$, is translated into an annualized rate of return , called $r_{\overline{N}}$, by taking the N-th root:*

$$(1+r_{\overline{N}}) \;=\; \sqrt[N]{1+r_{0,N}} \;=\; (1+r_{0,N})^{1/N} \quad . \tag{4.7}$$

Compounding the annualized rate of return over N years yields the total holding period rate of return.

Translating long-term net returns into annualized rates of returns.

The need to compute annualized rates of return often arises in the context of investments. For example, what annualized rate of return would you expect from a \$100 investment today that promises a return of \$240 in 30 years? The first step is computing the total holding rate of return. By Formula 2.2, the total 30-year rate of return is

$$\begin{aligned} r_{0,30} &= \frac{\$240 - \$100}{\$100} = 140\% \\ r_{0,30} &= \frac{CF_{30} - CF_0}{CF_0} \quad . \end{aligned} \tag{4.8}$$

The annualized rate of return is the rate $r_{\overline{30}}$, which, if compounded for 30 years, offers a 140% rate of return,

$$\begin{aligned} (1+r_{\overline{30}})^{30} &= (1+140\%) \\ (1+r_{\overline{t}})^{t} &= (1+r_{0,t}) \quad . \end{aligned} \tag{4.9}$$

Solve this equation by taking the 30$^{\text{th}}$ root,

$$\begin{aligned} (1+r_{\overline{30}}) &= (1+140\%)^{1/30} = \sqrt[30]{1+140\%} \approx 1+2.96\% \\ (1+r_{\overline{30}}) &= (1+r_{0,30})^{1/30} = \sqrt[30]{1+r_{0,30}} \quad . \end{aligned} \tag{4.10}$$

Thus, a return of \$240 in 30 years for \$100 investment is equivalent to about a 3% annualized rate of return.

Compounding ≈ Adding. Annualizing ≈ Averaging.

In the context of rates of return, compounding is similar to adding, while annualizing is similar to averaging. If you earn 1% twice, your compounded rate is 2.01%, similar to the rates themselves added (2%). Your annualized rate of return is 1%, similar to the average rate of return of 2.01%/2 = 1.005%. The difference is the interest on the interest.

Compounding vs. Averaging can lead to surprising results.

Now presume that you have an investment that doubled in value in the first year, and then fell back to its original value. What would its average rate of return be? Doubling from, say, \$100 to \$200 is a rate of return of +100%. Falling back to \$100 is a rate of return of (\$100 − \$200)/\$200 = −50%. Therefore, the average rate of return would be [+100% + (−50%)]/2 = +25%. *But you have not made any money!* You started with \$100 and ended up with \$100. If you compound the returns, you get the answer of 0% that you were intuitively expecting:

$$\begin{aligned} (1+100\%)\cdot(1-50\%) &= 1+0\% \\ (1+r_{0,1})\cdot(1+r_{1,2}) &= (1+r_{0,2}) \quad . \end{aligned} \tag{4.11}$$

Therefore, the annualized rate of return is also 0%. Conversely, an investment that produces +20% followed by −20% has an average rate of return of 0%, but leaves you with

$$\begin{aligned} (1+20\%)\cdot(1-20\%) &= (1-4\%) \\ (1+r_{0,1})\cdot(1+r_{1,2}) &= (1+r_{0,2}) \quad . \end{aligned} \tag{4.12}$$

For every \$100 of original investment, you only retain \$96. The average rate of return of 0% does not reflect this. The compounded and therefore annualized rate of return does:

$$1 + r_{\overline{2}} \;=\; \sqrt{(1 + r_{0,2})} \;=\; \sqrt{1 - 4\%} \;=\; 1 - 2.02\% \;. \qquad (4.13)$$

If you were an investment advisor and quoting your historical performance, would you rather quote your average historical rate of return or your annualized rate of return? (Hint: The industry standard is the average rate of return.)

Make sure to solve the following questions to gain more experience with compounding and annualizing over different time horizons.

Solve Now!

Q 4.5 *Assume that the two-year holding rate of return is 40%. The average rate of return is therefore 20% per year. What is the annualized rate of return? Which is higher?*

Q 4.6 *Is the compounded rate of return higher or lower than the sum of the individual rates of return? Is the annualized rate of return higher or lower than the average of the individual rates of return? Why?*

Q 4.7 *Return to Question 4.3. What was the annualized rate of return on the S&P500 over these twelve years?*

Q 4.8 *The following were the daily prices of an investment:*

2-Jan-01	\$1,283.27	4-Jan-01	\$1,333.34	8-Jan-01	\$1,295.86
3-Jan-01	\$1,347.56	5-Jan-01	\$1,298.35	9-Jan-01	\$1,300.80

If returns had accumulated at the same rate over the entire 255 days of 2001, what would a \$100 investment in 2001 have turned into?

Q 4.9 *If the total holding interest rate is 50% for a 5-year investment, what is the annualized rate of return?*

Q 4.10 *If the per-year interest rate is 10% for each of the next 5 years, what is the annualized total 5-year rate of return?*

Q 4.11 *If the annualized 5-year rate of return is 10%, what is the total 5-year holding rate of return?*

Q 4.12 *If the annualized 5-year rate of return is 10%, and if the first year's rate of return is 15%, and if the returns in all other years are equal, what are they?*

Q 4.13 *There is always disagreement about what stocks are good purchases. The typical degree of disagreement is whether a particular stock is likely to offer, say, a 10% (pessimist) or a 20% (optimist) annualized rate of return. For a \$30 stock today, what does the difference in belief between these two opinions mean for the expected stock price from today to tomorrow? (Assume that there are 365 days in the year. Reflect on your answer for a moment, and recognize that a \$30 stock typically moves about ±\$1 on a typical day. This is often called noise.)*

4·3. The Yield Curve

The Yield Curve: annualized interest rate as a function of bond maturity.

Let us now tackle the yield curve, which is also sometimes called the **term structure of interest rates.** The **yield curve** is today's average annualized interest (yield) that investments pay as a function of their time to maturity. If not clarified further, the yield curve usually means investments in **U.S. Treasuries**, although it should more precisely be called the **U.S. Treasuries yield curve.** Bond traders often graph other yield curves, too—such as the yield curve on bonds that were issued by corporations rather than by the government.

4·3.A. An Example: The Yield Curve in May 2002

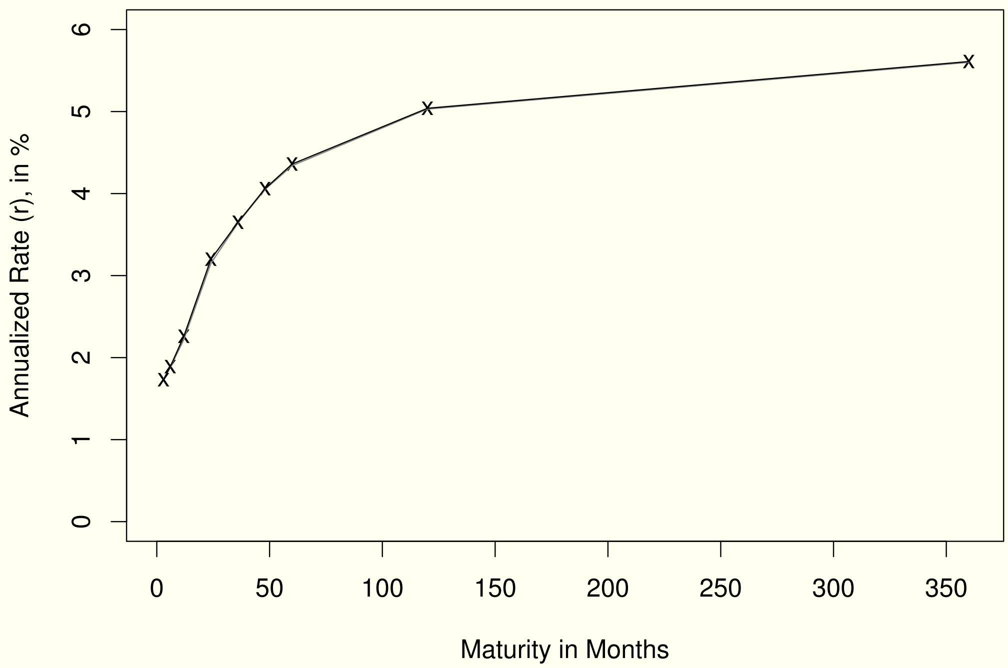
Table 4.1. The Treasury Yield Curves in mid-2002

Maturity	Apr 30	May 30	May 31
1 Month	1.77%	1.72%	1.72%
3 Month	1.77%	1.74%	1.73%
6 Month	1.91%	1.88%	1.89%
1 Year	2.35%	2.22%	2.26%
2 Year	3.24%	3.15%	3.20%
3 Year	3.83%	3.64%	3.65%
4 Year	n/a	4.05%	4.06%
5 Year	4.53%	4.34%	4.36%
10 Year	5.11%	5.03%	5.04%
20 Year	5.74%	n/a	n/a
30 Year	n/a%	5.60%	5.61%

The data for May 30, 2002, and May 31, 2002 were printed in the Wall Street Journal. The data for April 30, 2002, was obtained from the U.S. Treasury website at www.ustreas.gov. As you can see, the yieldcurve changes every day—though day-to-day changes are usually small.

For illustration, I am pretending that the *Wall Street Journal* yield curve is based on zero bonds (which only have one final payment—these would be called Treasury STRIPS). Although this is actually not perfectly correct (the WSJ curve is based on coupon bonds), the differences are usually very small. This is also why the data on the Treasury website is slightly different—in this example, the maximum difference is for the 10-year bond, where it is 4 basis points.

Figure 4.1. The Treasury Yield Curves on May 31, 2002



This is the yieldcurve for May 31, 2002. The data is in the previous table.

The table in Table 4.1 shows the actual Treasury yield table on April 30, May 30, and May 31, 2002. Figure 4.1 graphs the data from May 31, 2002. If you had purchased a 3-month bond at the end of the day on May 30, 2002, your annualized interest rate would have been 1.74%. The following day, a 3-month bond had a yield that was one basis point lower. (In real life, the 90-day bond can also switch identity, because as bonds age, another bond may be closer to 90-days than yesterday's 90-day bond.) If you had purchased a 30-year bond at the end of the day on May 30, 2002, you would have received an annualized interest rate of 5.60% per year, which is one basis point less than a 30-year bond would have been the following day.

We will analyze the actual yield curves at the end of May 2002.

Sometimes, it is necessary to determine an interest rate for a bond that is not listed. This is usually done by interpolation. For example, if the 90-day Treasury note had a known interest rate of 1.73% and the 93-day Treasury note had a known interest rate of 1.76%, a good interest rate for an untraded 91-day Treasury note might be 1.74%.

You can interpolate annualized interest rates on the yield curve.

Table 4.2. Relation Between Holding Returns, Annualized Returns, and Year-by-Year Returns, By Formula and on May 31, 2002

	Rates of Return		
Maturity	Total Holding	Annualized	Individually Compounded
1 Year	$(1+2.26\%)$	$=(1+2.26\%)^1$	$=(1+2.26\%)$
	$(1+r_{0,1})$	$=(1+r_{\overline{1}})^1$	$=(1+r_{0,1})$
2 Year	$(1+6.50\%)$	$=(1+3.20\%)^2$	$=(1+2.26\%)\cdot(1+4.15\%)$
	$(1+r_{0,2})$	$=(1+r_{\overline{2}})^2$	$=(1+r_{0,1})\cdot(1+r_{1,2})$
3 Year	$(1+11.35\%)$	$=(1+3.65\%)^3$	$=(1+2.26\%)\cdot(1+4.15\%)\cdot(1+4.56\%)$
	$(1+r_{0,3})$	$=(1+r_{\overline{3}})^3$	$=(1+r_{0,1})\cdot(1+r_{1,2})\cdot(1+r_{2,3})$

The annualized interest rate was higher for longer time periods.

As notation for the annualized horizon-dependent interest rates, return to our earlier method, calling the two-year annualized interest rate $r_{\overline{2}}$, the three-year annualized interest rate $r_{\overline{3}}$, and so on. When you look at this particular yield curve, it is very clear how important it can be to put a subscript on the annualized yields: the annualized yield varied drastically with maturity. Just to summarize—we now have to be able to recognize a whole set of different interest rates: holding rates of returns, such as $r_{0,3}$; annualized rates of return, such as $r_{\overline{3}}$; and individual 1-year annual interest rates that do not begin today, called forward rates, such as $r_{1,2}$. Table 4.2 relates the different types of returns, so you remember which is which. (Section 4·7.A shows you how you can construct the “individually compounded” column of this table from the “annualized” column that is the yield curve.) Aside, please do not forget that all the interest rates in the yield curve themselves are just computed from the prevailing prices on corresponding Treasury securities. It is much more intuitive to express the yield curve in this annualized implied interest rate fashion than to give you all the Treasury security prices and let you do the calculations—but the two are really one and the same.

4·3.B. Compounding With The Yield Curve

Computing the holding period rate of return for 2-Year bonds.

On May 30, 2002, how much money did an investment of \$500,000 into U.S. 2-Year notes (i.e., a loan to the U.S. government of \$500,000) promise to return in two years? Refer to Table 4.1 on Page 54. Because the yield curve prints annualized rates of return, the total two-year holding rate of return (as in Formula 4.4) is the twice compounded annualized rate of return,

$$\begin{aligned} r_{0,2} &= (1+3.15\%)\cdot(1+3.15\%) - 1 \approx 6.4\% \\ &= (1+r_{\overline{2}})\cdot(1+r_{\overline{2}}) - 1 \ , \end{aligned} \tag{4.14}$$

so the \$500,000 would turn into

$$\begin{aligned} CF_2 &\approx (1+6.4\%)\cdot \$500,000 \approx \$531,996 \\ &= (1+r_{0,2})\cdot CF_0 \ . \end{aligned} \tag{4.15}$$

ANECDOTE: Life Expectancy and Credit
Your life expectancy may be 80 years, but 30-year bonds existed even in an era when life expectancy was only 25 years—at the time of Hammurabi, around 1700 B.C.E. (**Hammurabi** established the Kingdom of Babylon, and is famous for the Hammurabi Code, the first known legal system.) Moreover, four thousand years ago, Mesopotamians already solved interesting financial problems. A cuneiform clay tablet contains the oldest known interest rate problem for prospective students of the financial arts. The student must figure out how long it takes for 1 mina of silver, growing at 20% interest per year, to reach 64 minae. Because the interest compounds in an odd way (20% of the principal is accumulated until the interest is equal to the principal, and then it is added back to the principal), the answer to this problem is 30 years, rather than 22.81 years. This is not an easy problem to solve—and it even requires knowledge of logarithms!

(In the real world, you might have to pay a commission to arrange this transaction, so you would end up with a little less.)

What if you invested \$500,000 into 30-Year Treasuries? The 30-Year total rate of return would be

Computing the holding period rate of return for 30-Year bonds.

$$\begin{aligned} r_{0,30} &= (1+r_{\overline{30}})^{30} - 1 \\ &= (1+5.60\%)^{30} - 1 \approx 5.1276 - 1 \approx 412.76\% \ . \end{aligned} \quad (4.16)$$

Thus, your investment of $CF_0 = \$500,000$ will turn into cash of $CF_{30} \approx \$2,563,820$ in 30 years.

4·3.C. Yield Curve Shapes

What would a flat yield curve mean? It would mean that the interest rate was the same over any time period. This scenario was the subject of the previous chapter. For example, at 5% per annum and borrowing money for two years, the total (non-annualized) interest rate that would have to be paid would be $(1+5\%) \cdot (1+5\%) - 1 = 10.25\%$. More generally, the interest rate over any period can then be quickly computed as

A flat yield curve means that the annualized interest rate is the same regardless of horizon.

$$(1 + r_{0,t}) = (1 + r_{0,1})^t \ . \quad (4.17)$$

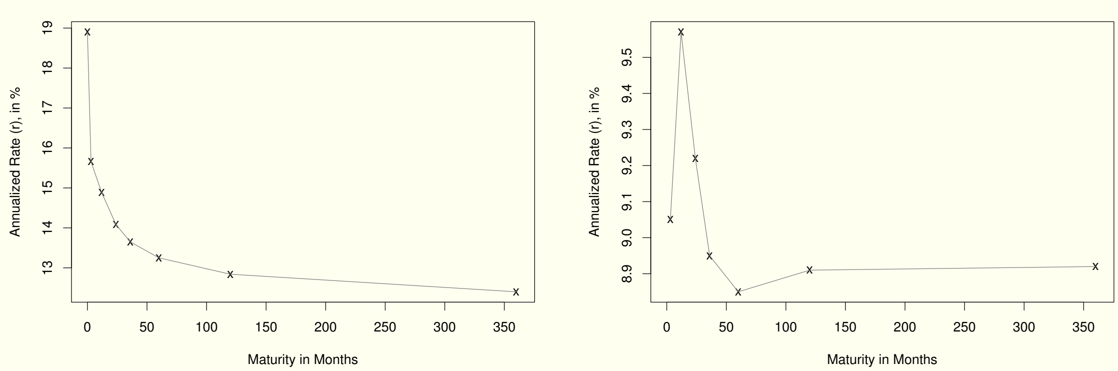
The yield curve is usually upward sloping. This means that longer-term interest rates are higher than shorter-term interest rates. The yield curve at the end of May 2002 was fairly steep—though not the steepest ever. Since 1934, the steepest yield curve (the difference between the long-term and the short-term Treasury rate) occurred in October 1992, when the long-term interest rate was 7.3 percent and the short-term interest rate was 2.9 percent—just as the economy pulled out of the recession of 1991. Another oddity occurred in January 1940, when the long-term interest rate was 2.3 percent—but the short-term interest rate was practically zero.

Yield Curves are often upward sloping.

However, the yield curve is not always upward-sloping. If short-term rates are higher than long-term rates, the yield curve is said to be downward sloping (or **inverted**). Figure 4.2 shows that this was the case in December 1980 during a brief period of rapidly declining inflation rates (and expansion in between two recessions). In fact, it is even possible that medium-term rates are higher than both long-term and short-term rates—the yield curve is then called humped. Inverted or humped yield curves are relatively rare.

They can also be downward-sloping or even be humped!

Figure 4.2. History: One Inverted and One Humped Yield Curve



December 1980 June 1979

SIDE NOTE: Economists have long wondered what they can learn from the shape of the yieldcurve. It appears that it is a useful—though unreliable and noisy—signal of where the economy is heading. Steep yield curves often signal emergence from a recession. Inverted yield curves often signal an impending recession.

Another interesting question is what drives the demand and supply for credit, which is ultimately the determinant of these interest rates. Economic research has shown that the Federal Reserve Bank has good influence on the short end of the Treasury curve—by expanding and contracting the supply of money and short-term loans in the economy—but not much influence on the long end of the Treasury curve. We will revisit this question later in this chapter, and again in Chapter 6 in the context of inflation.

If you want to undertake your own research, you can find historical data at the St. Louis Federal Reserve Bank, which maintains a database of historical interest rates at http://research.stlouisfed.org/fred. There are also the Treasury Management Pages at http://www.tmpages.com/. Or you can look at SmartMoney.com for historical yield curves. PiperJaffray.com has the current yield curve—as do many other financial sites and newspapers. bonds.yahoo.com/rates.html provides not only the Treasury yield curve, but also yield curves for other bonds that will be discussed in the next section.

If you want to learn more about how to work with yield curves, don't forget about the optional "forward interest rates" section below.

Solve Now!

Q 4.14 *Using information from a current newspaper or the WWW, what does an investment of \$1 in 30-year bonds yield in 30 years?*

Q 4.15 *Using information from a current newspaper or the WWW, what does an investment of \$1 in 1-year bonds yield in 1 year?*

4·4. Present Values With Time-Varying Interest Rates

In the previous chapter, you learned that present values allow you to express many future cash flows in the same unit: cash today. With time-varying interest rates, nothing really changes. The only novelty is that you can express the individual holding returns (e.g., $1 + r_{0,2}$) in terms of the individual period interest rates (e.g., $(1 + r_{0,1}) \cdot (1 + r_{1,2})$). So, the **Net Present Value** Formula can be rewritten as

$$\begin{aligned}
\text{NPV} &= \text{PV}(\text{CF}_0) + \text{PV}(\text{CF}_1) + \text{PV}(\text{CF}_2) + \text{PV}(\text{CF}_3) + \cdots \\
&= \text{CF}_0 + \frac{\text{CF}_1}{1+r_{0,1}} + \frac{\text{CF}_2}{1+r_{0,2}} + \frac{\text{CF}_3}{1+r_{0,3}} + \cdots \\
&= \text{CF}_0 + \frac{\text{CF}_1}{1+r_{0,1}} + \frac{\text{CF}_2}{(1+r_{0,1})\cdot(1+r_{1,2})} \\
&\quad + \frac{\text{CF}_3}{(1+r_{0,1})\cdot(1+r_{1,2})\cdot(1+r_{2,3})} + \cdots \quad .
\end{aligned} \tag{4.18}$$

Present Values are alike and thus can be added, subtracted, compared, etc.

Let us return to our earlier example on Page 24, where you had a \$10 payment in year 1 and an \$8 payment in year 2, but assume that the 5-year annualized interest rate is 6% per annum and therefore higher than the 1-year interest rate of 5%. In this case,

$$\begin{aligned}
\text{PV}(\$10 \text{ in one year}) &= \frac{\$10}{1+5\%} \approx \$9.52 \\
\text{PV}(\$8 \text{ in five years}) &= \frac{\$8}{(1+6\%)^5} \approx \$5.98 \quad .
\end{aligned} \tag{4.19}$$

It follows that the project's total value *today* (time 0) would now be \$15.50. If the project still costs \$12, its net present value is

$$\begin{aligned}
\text{NPV} &= -\$12 + \frac{\$10}{1+5\%} + \frac{\$8}{(1+6\%)^5} \approx \$3.50 \\
\text{NPV} &= \text{CF}_0 + \frac{\text{CF}_1}{1+r_{0,1}} + \frac{\text{CF}_5}{1+r_{0,5}} = \text{NPV} \quad .
\end{aligned} \tag{4.20}$$

You can also rework the project from Table 2.5 on Page 26, but you can now use a hypothetical current term structure of interest that is upward sloping. It requires an interest rate of 5% over 1 year, and 0.5% annualized interest more for every year, so it is 7% annualized for the 5-year cash flow. Table 4.3 works out the value of your project. The valuation method works the same way as it did earlier—you only have to use different interest rates now.

Here is a typical NPV Example.

Table 4.3. Hypothetical Project Cash Flow Table

Time	Project Cash Flow	Interest Rate In Year	Interest Rate Compounded	Discount Factor	Present Value
t	CF_t	$r_{t-1,t}$	$r_{0,t}$	$\frac{1}{1+r_{0,t}}$	$PV(CF_t)$
Today	-$900	any	0.0%	1.000	-$900.00
Year +1	+$200	5.0%	5.0%	0.952	$190.48
Year +2	+$200	5.5%	11.3%	0.898	$179.69
Year +3	+$400	6.0%	19.1%	0.840	$335.85
Year +4	+$400	6.5%	28.6%	0.778	$311.04
Year +5	-$100	7.0%	40.2%	0.713	-$71.33
				Net Present Value (Sum):	$45.73

Annualized interest rates apply only within this one year. They are perfectly known today.

4·4.A. Valuing A Coupon Bond With A Particular Yield Curve

Let us now work a more realistic example—determining the price of a coupon bond, for which payments are 100% guaranteed, just like payments on Treasury bonds themselves. We will recycle the 3% coupon bond example from Section 3·3.B. Of course, if you wanted to value a corporate project with risky cash flows instead of a bond, it might be more difficult to determine the appropriate inputs (cash flows and discount rates), but the valuation method itself would proceed in exactly the same way. After all, a bond is just like any other corporate project—an upfront investment followed by subsequent inflows.

First, recall the payment pattern of your bond, which comes from the definition of what a 3%-level semi-annual coupon bond is.

Step 1: Write down the project's payment pattern.

Year	Due Date	Bond Payment	Year	Due Date	Bond Payment
0.5	Nov 2002	$1,500	3.0	May 2005	$1,500
1.0	May 2003	$1,500	3.5	Nov 2005	$1,500
1.5	Nov 2003	$1,500	4.0	May 2006	$1,500
2.0	May 2004	$1,500	4.5	Nov 2006	$1,500
2.5	Nov 2004	$1,500	5.0	May 2007	$101,500

Second, find the appropriate rates of return to use for discounting. Because your bond is assumed default-free, it is just as good as a government bond (in our perfect world). Thus, you can use the government yield curve to extract appropriate discount factors. Assume it is May 30, 2002, so you can use the yield curve from Table 4.1 on Page 54.

Step 2: find the appropriate costs of capital.

Maturity	Yield	Maturity	Yield	Maturity	Yield
3 Month	1.74%	2 Year	3.15%	5 Year	4.34%
6 Month	1.88%	3 Year	3.64%	10 Year	5.03%
1 Year	2.22%	4 Year	4.05%	30 Year	5.60%

To use the PV formula to value your bond, you need to find the appropriate discount factors. Begin by computing the holding rates of return from the yield curve, using the methods from Section 4·1. For example, the 6-month and 2-year holding rates of return are computed as

$$\begin{aligned} 1 + r_{0,0.5} &= (1 + 1.88\%)^{0.5} \approx (1 + 0.94\%) \\ 1 + r_{0,2} &= (1 + 3.15\%)^2 \approx (1 + 6.40\%) \\ 1 + r_{0,t} &= (1 + r_{\bar{t}})^t \quad . \end{aligned} \tag{4.21}$$

The table of holding rates of return that corresponds to the yield curve is

Maturity	Yield	Maturity	Yield	Maturity	Yield
3 Month	not needed	2 Year	6.40%	5 Year	23.67%
6 Month	0.94%	3 Year	11.32%	10 Year	not needed
1 Year	2.22%	4 Year	17.21%	30 Year	not needed

But how do you obtain a holding rate of return for the coupon that will be paid in 18 months? You do not know the annualized 18-month interest rate, but you do know that the 1-year annualized interest rate is 2.22% and the 2-year annualized interest rate is 3.15%. So it is reasonable to guess that the 1.5 year annualized interest is roughly the average interest rate of the 1-year and 2-year annualized interest rates—about 2.7%. Therefore, you would estimate the 1.5 year holding rate of return to be

$$\begin{aligned} 1 + r_{0,1.5} &\approx (1 + 2.7\%)^{1.5} \approx (1 + 4.08\%) \\ 1 + r_{0,t} &= (1 + r_{\bar{t}})^t \quad . \end{aligned} \tag{4.22}$$

You have to do similar interpolations for the coupon payments in 2.5, 3.5 and 4.5 years. Collect this information—our payments, annualized interest rates, and equivalent holding interest rates—into one table:

Year	Due Date	Bond Payment	Annual. Interest	Holding Interest
0.5	Nov 2002	\$1,500	1.88%	0.94%
1.0	May 2003	\$1,500	2.22%	2.22%
1.5	Nov 2003	\$1,500	≈2.7%	4.08%
2.0	May 2004	\$1,500	3.15%	6.40%
2.5	Nov 2004	\$1,500	≈3.4%	8.72%
3.0	May 2005	\$1,500	3.64%	11.32%
3.5	Nov 2005	\$1,500	≈3.8%	13.94%
4.0	May 2006	\$1,500	4.05%	17.21%
4.5	Nov 2006	\$1,500	≈4.2%	20.34%
5.0	May 2007	\$101,500	4.34%	23.67%

Step 3: Compute the discount factor is $1/(1+r_{0,t})$.

Third, compute the discount factors, which are just $1/(1 + r_{0,t})$, and multiply each future payment by its discount factor. This is the present value (PV) of each bond payment, and the overall PV of your bond.

Year	Due Date	Bond Payment	Annual. Interest	Holding Interest	Discount Factor	Present Value
0.5	Nov 2002	\$1,500	1.88%	0.94%	0.991	\$1,486.03
1.0	May 2003	\$1,500	2.22%	2.22%	0.978	\$1,467.42
1.5	Nov 2003	\$1,500	≈2.7%	4.08%	0.961	\$1,441.20
2.0	May 2004	\$1,500	3.15%	6.40%	0.940	\$1,409.77
2.5	Nov 2004	\$1,500	≈3.4%	8.72%	0.920	\$1,379.69
3.0	May 2005	\$1,500	3.64%	11.32%	0.898	\$1,347.47
3.5	Nov 2005	\$1,500	≈3.8%	13.94%	0.878	\$1,316.48
4.0	May 2006	\$1,500	4.05%	17.21%	0.853	\$1,279.75
4.5	Nov 2006	\$1,500	≈4.2%	20.34%	0.831	\$1,246.47
5.0	May 2007	\$101,500	4.34%	23.67%	0.809	\$82,073.26
					Sum	\$94,447.55

Therefore, you would expect this 3% semi-annual level-coupon bond to be trading for $94,447.55 today—because this is lower than the bond's principal repayment of $100,000, this bond is called a discount bond.

Common naming conventions for this type of bond: coupon rate is not interest rate!

4·5. Why is the Yield Curve not Flat?

There is no necessary reason why capital should be equally productive at all times. For example, in agrarian societies, capital could be very productive in summer (and earn a rate of return of 3%), but not in winter (and earn a rate of return of only 1%). This does not mean that investment in summer is a better deal or a worse deal than investment in winter, because cash in winter is *not* the same—not as valuable—as cash in summer, so the two interest rates are not comparable. You could not invest winter money at the 3% interest rate you will be able to invest it with 6 months later.

There is no reason why interest rates have to be the same in all periods.

But although seasonal effects do influence both prices and rates of return on agricultural commodities, and although the season example makes it clear that capital can be differently productive at different times, it is not likely that seasonality is the reason why 30-year Treasury bonds in May 2002 paid 5.6% per annum, and 6-month Treasury notes paid only 1.9% per annum. So why is it that the yield curve was so steep? There are essentially three explanations:

Longer-term Treasury bonds probably have higher yields because they are riskier—though it could also have been investment opportunities that are better in the far-away future than they are today.

1. The 30-year bond is a much better deal than the 1-year bond. This explanation is highly unlikely. The market for Treasury bond investments is close to perfect, in the sense that we have used the definition. It is very competitive and efficient—concepts that we will investigate more in Chapter 6. If there was a great deal to be had, thousands of traders would have already jumped on it. So, more likely, the interest rate differential does not overthrow the old tried-and-true axiom: **you get what you pay for.** It is just a fact of life that investments for which the interest payments are tied down for 30 years must offer higher interest rates now.

   It is important that you recognize that your cash itself is *not* tied down if you invest in a 30-year bond, because you can of course sell your 30-year bond tomorrow to another investor if you so desire.

2. Investors expect to be able to earn much higher interest rates in the future. For example, if the interest rate $r_{0,1}$ is 2% and the interest rate $r_{1,2}$ is 10%, then $r_{0,2} = (1+2\%)\cdot(1+10\%) \approx 1 + 12\%$, or $r_{\overline{2}} = 5.9\%$. If you graph $r_{\overline{T}}$ against T, you will find a steep yield curve, just as you observed. So, higher future interest rates can cause much steeper yield curves.

   However, I am cheating. This explanation is really no different from my "seasons" explanation, because I have given you no good explanation *why* investment opportunities were expected to be much better in May 2032 than they were in May 2002. I would need to give you an underlying reason. One particular such reason may be that investors believe that money will be worth progressively less. That is, even though they can earn higher interest rates over the long run, they also believe that the price inflation rate will increase. Inflation—a subject of Chapter 6—erodes the value of higher interest rates, so interest rates may have to be higher in the future merely to compensate investors for the lesser value of their money in the future.

   However, the empirical evidence suggests that the yield curve is not a good predictor of future interest rates, except on the very shortest horizons (a month or less). So, the expectation of higher interest rates is *not* the most likely cause for the usually upward sloping curve in the real world.

3. Long-term bonds might somehow be riskier than short-term bonds, so investors only want to buy them if they get an extra rate of return. Although we have yet to cover uncertainty more systematically, you can gain some intuition by considering the effects of changes in economy-wide interest rates on short-term bonds vs. long-term bonds. This is the plan of the remainder of this section.

The empirical evidence indeed suggests that it is primarily compensation for taking more risk with long-term bonds than short-term bonds that explains why long-term bonds have higher yields than short-term bonds. That is, investors seem to earn higher expected rates of return on average in long-term bonds, because these bonds are riskier (at least in the interim).

4·5.A. The Effect of Interest Rate Changes on Short-Term and Long-Term Treasury Bond Values

Our agenda is to explore the risk of interim interest rate changes.

Why are 30-year bonds riskier than 1-year bonds? Of course, repayment is no less certain with 30-year Treasury bonds than 1-year Treasury bonds. (This would be an issue of concern if you were to evaluate corporate projects rather than Treasuries: long-term corporate bonds are often riskier than short-term corporate bonds—most firms are unlikely to go bankrupt this week, but fairly likely to go bankrupt over a multi-decade time horizon.) Instead of non-payment risk, the issue here is that economy-wide bond prices (interest rates) can change in the interim, and the effects of interest rate changes can be much more dramatic on 30-year bonds than on 1-year bonds.

First, the effect of a 10bp point change on the 30-year bond.

The 30-Year Bond: Let's compute the value of a $1,000 30-year zero bond today at the prevailing 5.60% interest rate. It is $\$1,000/1.056^{30} \approx \$195.02$. You already know that when prevailing interest rates go up, the prices of outstanding bonds drop and you will have lost money. Now, if interest rates increase by 10 basis points to 5.7%, the bond value decreases to $\$1,000/1.057^{30} \approx \$189.56$. If interest rates decrease by 10 basis points to 5.5%, the bond value increases to $\$1,000/1.055^{30} \approx \$200.64$. Thus, the effect of a 10 basis point increase in the prevailing 30-year yield induces an immediate percent change (a return) in the value of your bond of

$$
\begin{aligned}
r &= \frac{V(r_{\overline{30}} = 5.5\%) - V(r_{\overline{30}} = 5.6\%)}{V(r_{\overline{30}} = 5.6\%)} = \frac{\$200.64 - \$195.02}{\$195.02} \approx +2.88\% \\
r &= \frac{V(r_{\overline{30}} = 5.7\%) - V(r_{\overline{30}} = 5.6\%)}{V(r_{\overline{30}} = 5.6\%)} = \frac{\$189.56 - \$195.02}{\$195.02} \approx -2.80\%
\end{aligned}
\tag{4.23}
$$

For every $1 million you invest in 30-year bonds, you expose yourself to a $29,000 risk for a 10-basis point yield change in the economy.

Second, the effect of a 10bp point change on the 1-year bond.

The 1-Year Bond: To keep the example identical, assume that the 1-year bond also has an interest rate of 5.6%. In this case, the equivalent computations for the value of a 1-year bond are $946.97 at 5.6%, $947.87 at 5.5%, and $946.07 at 5.7%. Therefore, the equivalent change in value is

$$
\begin{aligned}
r &= \frac{V(r_{\overline{1}} = 5.5\%) - V(r_{\overline{1}} = 5.6\%)}{V(r_{\overline{1}} = 5.6\%)} = \frac{\$952.38 - \$946.97}{\$946.97} \approx +0.09\% \\
r &= \frac{V(r_{\overline{1}} = 5.7\%) - V(r_{\overline{1}} = 5.6\%)}{V(r_{\overline{1}} = 5.6\%)} = \frac{\$946.07 - \$946.07}{\$946.07} \approx -0.09\%
\end{aligned}
\tag{4.24}
$$

So for every $1 million you invest in 1-year bonds, you expose yourself to a $900 risk for a 10-basis point yield change in the economy.

Comparison

It follows that the value effect of an equal-sized change in prevailing interest rates is more severe for longer term bonds. It follows, then, that if the bond is due tomorrow, there is very little havoc that an interest rate change can wreak.

In the interim, T-bonds are *not* riskfree!

This brings us to an important insight: Treasury bonds are risk-free in the sense that they cannot default (fail to return the promised payments). But they are risky in the sense that interest changes can change their value. Only the most short-term Treasury bills (say, due overnight) can truly be considered risk-free—virtually everything else is risky.

> **IMPORTANT**: *Though "fixed income," even a Treasury bond does not guarantee a "fixed rate of return" over horizons shorter than the maturity: day to day, long-term bonds are generally riskier investments than short-term bills.*

"Only" a paper loss: A cardinal error!

But, if you really need cash only in 30 years, is this not just a paper loss? This is a cardinal logical error many investors commit. By committing your million dollars one day earlier, you would have lost $29,000 of your net worth in one day! Put differently, waiting one day would have saved you $29,000 or allowed you to buy the same item for $29,000 less. Paper money is actual wealth. Thinking paper losses are any different from actual losses is a common but capital error.

> **IMPORTANT**: *"Paper losses" are actual losses.*

The only exception relates to the fact that realized gains and losses have different tax implications than unrealized gains and losses—a subject which we will discuss in Chapter 6.

> **DIGGING DEEPER**: *I have pulled two tricks on you. First, in the real world, it could be that short-term economy-wide interest rates typically experience yield shifts of plus or minus 100 basis points, while long-term economy-wide interest rates never move. If this were true, long-term bonds could even be safer. But trust me—even though the volatility of prevailing interest rates in 20-year bonds is smaller than that of 1-year bonds, it is not that much smaller. As a consequence, the typical annual variability in the rate of return of an investment in 20-year Treasury bonds is higher (around 10%) than the typical variability in the rate of return of an investment in 1-month Treasury notes (around 3%). Long-term Treasury securities are indeed riskier.*
>
> *Second, when I quoted you value losses of $29,000 and $900, I ignored that between today and tomorrow, you would also earn one day's interest. On a $1,000,000 investment, this would be about $150. If you had invested the money in 1-day Treasury bills at 1.7% instead of 30-year bonds, you would have only received about $30. Strictly speaking, this $120 favors the long-term bond and thus should be added when comparing investment strategies—but it is only about 1 basis point, and so for a quick-and-dirty calculation such as ours, ignoring it was reasonable.*

Solve Now!

Q 4.16 *Using information from a current newspaper or the WWW, what is today's annualized rate of return on a 10-year bond?*

Q 4.17 *Using information from a current newspaper or the WWW, what is today's total rate of return on a 10-year bond over the 10-year holding period?*

Q 4.18 *If you invest $500,000 at today's total rate of return on a 30-day Treasury note, what will you end up with?*

4·6. The Yield To Maturity (YTM)

We want a "sort-of average interest rate" that is implicit in future cash flows.

In Section 4·2, you learned how to annualize rates of return, so that you could better understand the rate at which two different investments accumulate wealth. However, there was only one payment involved. What do you do if each bond has many different payments? For example, what is the interest rate on a bond that costs \$100,000 today, and pays off \$5,000 in 1 year, \$10,000 in 2 years, and \$120,000 in 3 years? This may be an irregular coupon bond, but it is not an illegal one. How should you even name this bond—is there something like an "average" interest rate implicit in these cash flows? Is this bond intrinsically more similar to a bond offering a 4% rate of return or a bond offering a 6% rate of return? Note that this has nothing to do with the prevailing economy-wide yield curve. Our question is purely one of wanting to characterize the cash flows that are implicit to the bond itself. The answer is not obvious at all—until you learn it. The yield-to-maturity gives a sort of "average rate of return" implicit in many bond cash flows.

Important: *The **Yield to Maturity** is the quantity YTM, which, given a complete set of bond cash flows, solves the NPV equation set to zero,*

$$0 = CF_0 + \frac{CF_1}{1 + YTM} + \frac{CF_2}{(1 + YTM)^2} + \frac{CF_3}{(1 + YTM)^3} + \dots \tag{4.25}$$

An example of solving the YTM equation.

So, in this case, you want to solve

$$0 = -\$100,000 + \frac{\$5,000}{1 + \text{YTM}} + \frac{\$10,000}{(1 + \text{YTM})^2} + \frac{\$120,000}{(1 + \text{YTM})^3} \ . \tag{4.26}$$

In general, you solve this equation by trial and error. Start with two values, say 5% and 10%.

$$\begin{aligned} -\$100,000 + \frac{\$5,000}{1+5\%} + \frac{\$10,000}{(1+5\%)^2} + \frac{\$120,000}{(1+5\%)^3} &\approx \$17,493 \ , \\ -\$100,000 + \frac{\$5,000}{1+10\%} + \frac{\$10,000}{(1+10\%)^2} + \frac{\$120,000}{(1+10\%)^3} &\approx \$2,968 \ . \end{aligned} \tag{4.27}$$

To reach zero, you need to slide above 10%. So, try 11% and 12%,

$$\begin{aligned} -\$100,000 + \frac{\$5,000}{1+11\%} + \frac{\$10,000}{(1+11\%)^2} + \frac{\$120,000}{(1+11\%)^3} &\approx \$363 \ , \\ -\$100,000 + \frac{\$5,000}{1+12\%} + \frac{\$10,000}{(1+12\%)^2} + \frac{\$120,000}{(1+12\%)^3} &\approx -\$2,150 \ . \end{aligned} \tag{4.28}$$

Ok, the solution is closer to 11%. Some more trial and error reveals

$$-\$100,000 + \frac{\$5,000}{1+11.14255\%} + \frac{\$10,000}{(1+11.14255\%)^2} + \frac{\$120,000}{(1+11.14255\%)^3} \approx 0 \ . \tag{4.29}$$

So, the cash flows of your bond with payments of \$5,000 in 1 year, \$10,000 in 2 years, and \$120,000 in 3 years have an embedded sort-of-average interest rate—a yield to maturity—that is equal to 11.14%. There are also bonds that the corporation can call back in before maturity. In this case, it is not uncommon to compute a YTM for such a bond assuming the firm will do so, then called a **Yield-to-Call.**

A YTM is (usually) not an interest rate!

You can think of YTM as a generalization of the narrower interest rate concept. If there is only one cash inflow and one cash outflow—as is the case for a zero bond—then the YTM is the same as the annualized interest rate. However, a rate of return is defined by exactly two cash flows, so it is meaningless to talk about it when there are multiple cash flows. In contrast, the YTM can handle multiple cash flows just fine. Although it may help your intuition to think of the YTM as a "sort of" average interest rate that is embedded in a bond's cash flows, you should be clear that the YTM is *not* an interest rate. (An interest rate is a YTM, but not vice-versa.) Instead, a YTM is a characteristic defined by a cash flow pattern.

Should you purchase this bond? The answer is yes if and only if this bond does not have a negative NPV. Fortunately, YTM can often provide the same information. If the yield curve is uniformly below the bond's YTM, then the bond is a positive NPV project. So, if all prevailing economy-wide interest rates were 11%, and your bond's YTM is 11.14%, then this bond would be a positive NPV project and you should buy it. If all prevailing economy-wide interest rates were 12%, and your bond's YTM is 11.14%, then you should not buy this bond. Unfortunately, when the prevailing yield curve is not uniformly above or below the YTM (e.g., if it is 11% on the 1-year horizon climbing to 12% on the 3-year horizon), YTM cannot tell you whether to purchase the bond—though it still gives a nice characterization of bond payments. Instead, you have to go back to NPV.

If the yield curve is flat, YTM can substitute for NPV as a capital budgeting tool.

I must confess some minor sins: First, the Treasury yield curve in Table 4.1 which was used for illustration was not really based on zero-bonds, as I had pretended. Instead, it was based on bonds that had some interim coupon payments—and it was the YTM of these coupon bonds that we graphed, not just the simple zero-bond annualized interest rate. (The zero-bond version of the yield curve would be graphed based on **Treasury STRIPS** [Section **a**]. The STRIP yieldcurve can differ "a little" from the ordinary coupon-bond yieldcurve.) Second, the concept of YTM works with or without the concept of time-varying interest rates, so it may be misplaced in this chapter. It is about bond payments, not about the prevailing economy wide discount rates. I just placed it here, because it allowed us to discuss how you would compare a bond's YTM to the prevailing yield-curve, and how YTM becomes useless if the yield-curve is not uniformly above or below it. Third, there is an easier method than trial-and-error *yourself*—most computer spreadsheets offer a built-in function called "IRR" that solves the YTM equation exactly as you just did, only without trial and error and therefore more conveniently, too. (We will cover IRR in Chapter 8.)

YTM is also called IRR in a more general context, so spreadsheets have it built-in.

Solve Now!

Q 4.19 *What is the YTM of a level-coupon bond whose price is equal to the principal paid at maturity? For example, take a 5-year bond that costs \$1,000, pays 5% coupon (\$50 per year) for 4 years, and finally repays \$1,050 in principal and interest in year 5.*

Q 4.20 *What is the YTM of the following zero-bond? For example, take a 5-year bond that costs \$1,000 and promises to pay \$1,611?*

Q 4.21 *Compute the yield-to-maturity of a two-year bond that costs \$25,000 today, pays \$1,000 at the end of the first year and at the end of the second year. At the end of the second year, it also repays \$25,000. What is the bond's YTM?*

Q 4.22 *Let us learn how to "STRIP" a Treasury coupon bond. (STRIP is a great acronym for Separate Trading of Registered Interest and Principal of Securities.) Presume the 12 month Treasury bond costs \$10,065.22 and pays a coupon of \$150 in 6 months, and interest plus coupon of \$10,150 in 12 months. (Its payment patterns indicate that it was originally issued as a "3-percent semi-annual level-coupon bond.") Presume the 6-month Treasury bond costs \$10,103.96 and has only one remaining interest plus coupon payment of \$10,200. (It was originally issued [and perhaps many years ago] as a "4% semi-annual level-coupon bond.")*

(a) *What is the YTM of these two bonds?*

(b) *Graph a yield curve based on the maturity of these two bonds.*

(c) *What would be the price of a 1-year zero bond?*

(d) *Graph a yield curve based on zero bonds.*

(e) *Do the yield differences between the 1-year zero bond and the 1-year coupon bond seem large to you?*

4·7. Optional Bond Topics

There are many other finer details of bonds that we could dive into, even though they are not absolutely necessary for understanding the basics of capital budgeting. This does not mean that they are unimportant—indeed, any corporate CFO who wants to finance projects by issuing bonds will inevitably run into each of them. So, in this section, we cover a set of related issues that are best dubbed "advanced, but not unimportant."

4·7.A. Extracting Forward Interest Rates

Forward interest rates are implied interest rates in the future, given by today's yield curve.

Can you lock in a 1-year interest rate beginning in 2 years? For example, you may have a project that will generate cash in 2 years and that you need to store for 1 year before the cash can be used in the next project. The answer is yes, and the lock-in rate is right in the yield curve itself. Computing and locking rates may not be important to the ordinary small investor, but it is to bond traders and CFOs. This lock-able interest rate is the **forward interest rate** (or, simply, **forward rate**)—an interest rate for an investment of cash beginning not today, but in the future. You have already used forward rates: we called them, e.g., $r_{2,3}$, the one-year interest rate beginning in 2 years. In contrast to forward rates, interest rates for investments beginning this period are called **spot rates** or **spot interest rates**, because they are the interest that can be obtained on the spot right now.

Working out forward rates step by step from the yield curve.

Begin by working out the future one-year interest rates that were already computed for you in Table 4.2 on Page 56. In Table 4.2, the formulas were

	Rates of Returns		
Maturity	Total Holding	Annualized	Individually Compounded
1 Year	$(1 + r_{0,1})$	$= (1 + r_{\overline{1}})^1$	$= (1 + r_{0,1})$
2 Year	$(1 + r_{0,2})$	$= (1 + r_{\overline{2}})^2$	$= (1 + r_{0,1}) \cdot (1 + r_{1,2})$
3 Year	$(1 + r_{0,3})$	$= (1 + r_{\overline{3}})^3$	$= (1 + r_{0,1}) \cdot (1 + r_{1,2}) \cdot (1 + r_{2,3})$

The *Wall Street Journal* yield curve gives you the annualized interest rates, i.e., the third column. You can read them off and insert them into your table. On May 31, 2002, these interest rates were

	Rates of Returns		
Maturity	Total Holding	Annualized	Individually Compounded
1 Year	$(1 + r_{0,1})$	$= (1 + 2.26\%)^1$	$\approx (1 + r_{0,1})$
2 Year	$(1 + r_{0,2})$	$= (1 + 3.20\%)^2$	$\approx (1 + r_{0,1}) \cdot (1 + r_{1,2})$
3 Year	$(1 + r_{0,3})$	$= (1 + 3.65\%)^3$	$\approx (1 + r_{0,1}) \cdot (1 + r_{1,2}) \cdot (1 + r_{2,3})$

The first step is to compute the holding rates of return in the second column:

	Rates of Returns		
Maturity	Total Holding	Annualized	Individually Compounded
1 Year	$(1 + 2.26\%)$	$\approx (1 + 2.26\%)^1$	$= (1 + r_{0,1})$
2 Year	$(1 + 6.50\%)$	$\approx (1 + 3.20\%)^2$	$= (1 + r_{0,1}) \cdot (1 + r_{1,2})$
3 Year	$(1 + 11.35\%)$	$\approx (1 + 3.65\%)^3$	$= (1 + r_{0,1}) \cdot (1 + r_{1,2}) \cdot (1 + r_{2,3})$

Ultimately, you want to know what the implied future interest rates are. Work your way down. The first row is easy: you know that $r_{0,1}$ is 2.26%. You can also substitute this return into the other rows:

Rates of Returns			
Maturity	Total Holding	Annualized	Individually Compounded
1 Year	(1 + 2.26%)	$\approx (1+2.26\%)^1$	$\approx$ (1+2.26%)
2 Year	(1 + 6.50%)	$\approx (1+3.20\%)^2$	$\approx$ (1+2.26%) $\cdot\ (1+r_{1,2})$
3 Year	(1 + 11.35%)	$\approx (1+3.65\%)^3$	$\approx$ (1+2.26%) $\cdot\ (1+r_{1,2}) \cdot (1+r_{2,3})$

Now you have to work on the two year row to determine $r_{1,2}$: You have one equation and one unknown in the two year row, so you can determine the interest to be

$$(1+\ 6.50\%) \;=\; (1+2.26\%)\cdot(1+r_{1,2}) \;\Rightarrow\; (1+r_{1,2}) \;=\; \left(\frac{1+6.50\%}{1+2.26\%}\right) \;\approx\; 1+4.15\% \;. \tag{4.30}$$

Substitute this solution back into the table,

Rates of Returns			
Maturity	Total Holding	Annualized	Individually Compounded
1 Year	(1 + 2.26%)	$\approx (1+2.26\%)^1$	$\approx$ (1+2.26%)
2 Year	(1 + 6.50%)	$\approx (1+3.20\%)^2$	$\approx$ (1+2.26%) $\cdot$ (1+4.15%)
3 Year	(1 + 11.35%)	$\approx (1+3.65\%)^3$	$\approx$ (1+2.26%) $\cdot$ (1+4.15%) $\cdot\ (1+r_{2,3})$

Now work on row 3. Again, you have one equation and one unknown in the three year row, so you can determine the interest to be

$$(1+11.35\%) \;=\; (1+2.26\%)\cdot(1+4.15\%)\cdot(1+r_{2,3}) \tag{4.31}$$

$$\Rightarrow\; (1+r_{2,3}) \;=\; \frac{1+11.35\%}{(1+2.26\%)\cdot(1+4.15\%)} \;\approx\; 1+4.56\% \;. \tag{4.32}$$

Rates of Returns			
Maturity	Total Holding	Annualized	Individually Compounded
1 Year	(1 + 2.26%)	$\approx (1+2.26\%)^1$	$\approx$ (1+2.26%)
2 Year	(1 + 6.50%)	$\approx (1+3.20\%)^2$	$\approx$ (1+2.26%) $\cdot$ (1+4.15%)
3 Year	(1 + 11.35%)	$\approx (1+3.65\%)^3$	$\approx$ (1+2.26%) $\cdot$ (1+4.15%) $\cdot$ (1+4.56%)

So, given the annualized rates of return in the yield curve, you can determine the whole set of implied forward interest rates. For example, the implied interest rate from year 2 to year 3 is 4.56%.

Think of the annualized interest rate as the average of interest rates.

Behind this arithmetic lies a pretty simple intuition: An annualized two-year interest rate is "really sort of" an "average" interest rate over the interest rates from the first year and the second year. (In fact, the annualized rate is called the **geometric average**.) If you know that the average interest rate is 3.20%, and you know that the first half of this average is 2.26%, it must be that the second half of the average must be a number around 4.2% in order to average out to 3.20%. And, indeed, you worked out that the forward one-year interest rate was 4.15%. It is not exact—due to compounding—but it is fairly close.

Solve Now!

Q 4.23 *Continuing the example, compute the one-year forward interest rate $r_{3,4}$ from year 3 to year 4, if the 4-year annualized interest rate was 4.06%.*

4·7.B. Shorting and Locking in Forward Interest Rates

Table 4.4. The Mechanics of an Apple Short Sale

Three Parties: Apple Lender, You, The Apple Market.

Today:

1. You borrow 1 apple from the lender in exchange for your firm promise to the lender to return this 1 apple next year. (You also pay the lender an extra 1 cent lending fee.)
2. You sell 1 apple into the apple market at the currently prevailing apple price. Say, 1 apple costs \$5 today. You now have \$5 cash, which you can invest. Say, you buy bonds that earn you a 1% interest rate.

Next year:

1. You owe the lender 1 apple. Therefore, you must purchase 1 apple from the apple market.
   - If apples now cost \$6, you must purchase 1 apple from the market at \$6. You return the apple to the lender.
     Your net return on the apple is thus −\$1, plus the \$0.05 interest on \$5, minus the 1 cent fee to the lender. You therefore lost 96 cents.
   - If apples now cost \$4, you must purchase 1 apple from the market at \$4. You return the apple to the lender.
     Your net return on the apple is thus +\$1, plus the \$0.05 interest on \$5, minus the 1 cent fee to the lender. You therefore gained \$1.04.

Net Effects:

- The apple lender has really continued to own the apple throughout, and can sell the apple in Year 1. There is no advantage for the lender to keep the apple in his own apple cellar rather than to lend it to you. In addition, the lender earns 1 cent for free by lending.
- The apple market buyer purchased an apple from you today, and will never know where it came from (i.e., from a short sale).
- The apple market seller next year will never know what you do with the apple (i.e., that you will use it to make good on your previous year's apple loan).
- You speculated that the price of an apple would decline.
- Note that you did earn the interest rate along the way. Except for the fee you paid to the lender, you could sell the apple into the apple market today and use the proceeds to earn interest, just like an apple grower could have.

In the real world, short-selling is arranged so that you cannot sell the apple short, receive the \$5, and then skip town. As a short-seller, you must assure the lender that you will be able to return the apple next year. As the short seller, you must also pay the lender for all interim benefits that the apple would provide—though few apples pay dividends or coupon, the way stocks and bonds often do.

Frictionless borrowing and lending of Treasury bonds allow investors to lock in future interest rates. How shorting works.

Why are forward interest rates so interesting? The reason is that by cleverly buying and selling (shorting) Treasury bonds, you can bet on future interest rates embedded in the yield curve. Working with and speculating on forward rates is the "bread-and-butter" of bond traders. But bond traders are not the only parties here—firms often also want to "lock in" future interest rates today—and they can indeed lock in today's forward interest rates as the future interest rates that they will face. To understand this, assume that you can buy and sell Treasury bonds, even if you do not own them. In effect, you can borrow these securities, sell them, receive the cash, buy back the bonds later, and return them to the lender. This is called a **short sale** (the opposite—buying securities—is said to be a long position). Table 4.4 explains the basic idea behind shorting. In effect, for Treasury bonds, short selling enables you to do what the government does—"issue" a security, take in money, and return it to the lender with interest. For example, you may sell short $89,803.25 of a 3-year Treasury bond today with a 3.65% rate of interest and a maturity of 3 years. This will give you $89,803.25 cash today, but require you to come up with $100,000 for repayment in 3 years. In effect, selling a bond short is a way of borrowing money. In the real world, for professional bond traders, who can prove that they have enough funds to make good *any* possible losses, this is easily possible and with extremely small transaction costs, perhaps 1–2 basis points. Thus, assuming transaction costs away is a reasonable assumption.

Shorting is the opposite of Buying: It speculates that the value will decline.

Holding a security (i.e., being long) speculates that the value will go up, so selling a financial instrument (i.e., being short) speculates that the value will go down. If the price of the bond tomorrow were to go down to $50,000 (an annualized interest rate of 26%), the trader could then purchase the government T-bill for $50,000 to cover the $100,000 commitment he has made for $89,803.25, a profit of $39,803.25. But if the price of the bond tomorrow were to go to $99,000 (an annualized interest rate of 0.33%), the trader would lose $9,196.75.

Future cash flows from the long leg and the short leg.

Now assume that you are able to buy a two-year bond at an annualized interest rate of 3.20%, and able to sell (short) a three-year bond at an annualized interest rate of 3.65%, and do so without transaction costs. For the three-year bond, you would have to promise to pay back $\$100 \cdot (1 + 11.35\%) \approx \$111.35$ in three years (cash outflow to you) for each $100 you are borrowing today (cash inflow to you). For the two-year bond, you would invest these $100 (cash outflow to you) and receive $\$100 \cdot (1 + 6.50\%) \approx \$106.50$ in two years (cash inflow to you).

Table 4.5. Locking in a Future Interest Rate via the Long-Short Forward Interest Rate Spread

Time	Purchased 2-Year Bond Cash Flows		Shorted 3-Year Bond Cash Flows		Net Cash Flow	
Today	–$100.00	(outflow)	+$100.00	(inflow)	$0.00	
Year 1	$0.00		$0.00		$0.00	
Year 2	+$106.50	(inflow)	$0.00		+$106.50	(inflow)
Year 3	$0.00		–$111.35	(outflow)	–$111.35	(outflow)

Looking at your **Payout Table** 4.5, from your perspective, the simultaneous transaction in the two bonds results in an inflow of $106.50 in year two followed by a cash outflow of $111.35. Effectively, you have committed to borrowing $106.50 in year 2 with payback of $111.35 in year 3. The interest rate for this loan is

$$
\begin{aligned}
r_{2,3} &\approx \frac{\$111.35 - \$106.50}{\$106.50} \approx 4.56\% \\
&= \frac{\mathrm{CF}_0 \cdot (1 + r_{0,3}) - \mathrm{CF}_0 \cdot (1 + r_{0,2})}{\mathrm{CF}_0 \cdot (1 + r_{0,2})} \quad ,
\end{aligned}
\tag{4.33}
$$

which is exactly the forward interest rate in the table.

DIGGING DEEPER: *There is an alternative way to work this. Start with the amount that you want to borrow/lend in a future period. For example, say you want to lend \$500 in year 2 and repay however much is necessary in year 3. Lending \$500 in year 2 requires an outflow, which you can only accomplish with an inflow today. (Therefore, the first "leg" of your transaction is that you borrow, i.e., short the 2-year bond!) Specifically, your inflow today is* $\$500/(1+3.20\%)^2 \approx \$469.47$*. Now, invest the entire \$469.47 into the 3-year bond, so that you have zero net cash flow today. (The second "leg" of your transaction is that you lend, i.e., purchase the 3-year bond.) This will earn you an inflow* $\$469.47\cdot(1+3.65\%)^3 \approx \$522.78$ *in 3 years. In total, your financial transactions have committed you to an outflow of \$500 in year 2 in exchange for an inflow of \$522.78 in year 3—otherwise known as 1-year lending in year 2 at a precommitted interest rate of 4.56%.*

Such forward interest rate swaps are so popular that there are markets that make this even simpler.

This particular transaction is called a **forward transaction.** Indeed, this particular type of forward transaction is so popular that an entire financial market on **interest forwards** has developed that allows speculators to easily engage in simultaneously going long or short on bonds.

You get what you pay for: the speculation can end up for better or worse.

Should you engage in this transaction? If the one-year interest rate in 2 years will be higher than 4.56%, you will be able to borrow at a lower interest than what will be prevailing then. Of course, if the interest rate will be lower than 4.56%, you will have committed to borrow at an interest rate that is higher than what you could have gotten.

Solve Now!

Q 4.24 *If you want to commit to saving at an interest rate of* $r_{3,4}$*, what would you have to do? (Assume any amount of investment you wish, and work from there.)*

Q 4.25 *If you want to commit to saving \$500,000 in 3 years (i.e., you will deposit \$500,000) at an interest rate of* $r_{3,4}$ *(i.e., you will receive \$526,498.78), what would you have to do?*

4·7.C. Bond Duration

Maturity ignores interim payment structure.

In Section 4·6, you learned how to summarize or characterize the cash flows promised by a bond with the YTM. But how can you characterize the "term length" of a bond? The final payment, i.e., the maturity, is flawed: zero bonds and coupon bonds may have the same maturity, but a high coupon bond could pay out a good amount of money early on. For example, a coupon bond could pay 99% in coupon in the first month, and leave 1% for a payment in 30 years. It would count as a 30-year bond, the same as a zero-bond that pays 100% in 30 years.

Duration is an "average" payout date.

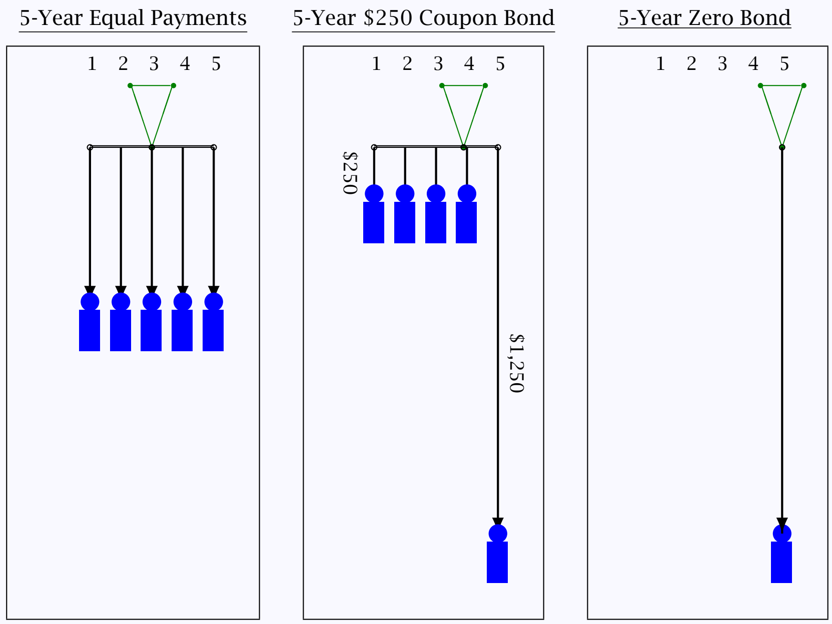
To measure the payout pattern of a bond, investors often rely on both maturity and **duration**—a measure of the *effective* time-length of a project. The simplest duration measure computes the time-weighted average of bond payouts, divided by the sum of all payments. For example, a 5 Year Coupon Bond that pays \$250 for 4 years and \$1,250 in the fifth year, has a duration of 3.89 years, because

$$\text{Plain Duration} = \frac{\$250\cdot 1 + \$250\cdot 2 + \$250\cdot 3 + \$250\cdot 4 + \$1,250\cdot 5}{\$250 + \$250 + \$250 + \$250 + \$1,250} \approx 3.89$$

$$\frac{\text{Payment at Time 1}\cdot 1 + \text{Payment at Time 2}\cdot 2 + \ldots + \text{Payment at Time T}\cdot T}{\text{Payment at Time 1} + \text{Payment at Time 2} + \ldots + \text{Payment at Time T}} . \tag{4.35}$$

(You can think of this as the "payment-weighted" payout year.) The idea is that you now consider this 5-year coupon bond to be shorter-term than a 5-year zero bond (which has a 5-year duration)—and perhaps more similar to a 3.9-year zero bond.

SIDE NOTE: Duration is sometimes explained through a physical analog: If all payments were weights hanging from a (time) line, the duration is the point where the weights balance out, so that the line tilts neither right nor left.



Duration: 3 years — Duration: 3.89 years — Duration: 5 years

Macaulay duration uses PV, and is usually a little bit less than plain duration.

Macaulay Duration alters plain duration by using the present value of payouts, not just nominal payouts. Thus, unlike plain duration which merely characterizes bond cash flows regardless of economy-wide interest rates, Macaulay duration also depends on the prevailing yield curve. If the interest rate on all horizons is 5%, the Macaulay duration for your coupon bond is

$$\text{Macaulay Duration} = \frac{\$238\cdot 1 + \$227\cdot 2 + \$216\cdot 3 + \$206\cdot 4 + \$979\cdot 5}{\$238 + \$227 + \$216 + \$206 + \$979} \approx 3.78$$

$$\frac{\text{PV(Payment at Time 1)}\cdot 1 + \text{PV(Payment at Time 2)}\cdot 2 + \ldots + \text{PV(Payment at Time T)}\cdot T}{\text{PV(Payment at Time 1)} + \text{PV(Payment at Time 2)} + \ldots + \text{PV(Payment at Time T)}} \;. \tag{4.36}$$

Duration Similarity

Duration is used as an interest exposure measure.

Duration can be used as a measure for the "term" of projects other than bonds, too. However, duration only works if all incoming cash flows are positive—otherwise, it may be nonsense. Duration is important, because it helps you judge the exposure (risk) of your projects to changes in interest rates. For example, if you have a project (or bond portfolio) that has an average duration of 6.9 years, then it is probably more exposed to and more similar to the 7-year Treasury bond than the 5-year or 10-year Treasury bonds.

A concrete project example.

Now presume that the yield curve is 5% for 1-year T-bonds, 10% for 2-year T-bonds, and 15% for 3-year T-bonds. You can purchase a project that will deliver $1,000 in 1 year, $1,000 in 2 years, and $1,500 in 3 years, and costs $2,500. This bond would be a good deal, because its present value would be $2,765.10. The project has a YTM of 17.5%, and a Macaulay duration of 2.01 years. (We shall only work with the Macaulay duration.) But, let's presume you are worried about interest rate movements. For example, if interest rates were to quadruple, the project would not be a good one. How does the value of your project change as the yield curve moves around?

The effect of a constant shift of the yield curve.

Let's work out how changes in the yield curve affect your projects and pure zero bonds, each promising $1,000 at maturity. First, your project. Presume that the entire yield curve shifts upward by 1%—the 5% T-bond yield becomes a 6% yield, the 10% becomes 11%, and the 15% becomes 16%. Your project value would now be

$$\text{PV} = \frac{\$1,000}{1+6\%} + \frac{\$1,000}{(1+11\%)^2} + \frac{\$2,500}{(1+16\%)^3} \approx \$2,716.01 \ . \tag{4.37}$$

This is an instant rate of return of $(\$2,716.01 - \$2,765.10)/\$2,765.10 \approx -1.776\%$.

Yield Curve	Project		
	PV(r_L)	PV(r_H)	RoR
Entire yield curve shifts upward by 1%:	\$2,765.10	\$2,716.01	$\boxed{-1.78\%}$

Is this more similar to how the 1-Year zero T-bond changed, how the 2-year zero T-bond changed, or how the 3-year zero T-bond would have changed? Of course, zero bonds are only affected by their specific interest rate, so you can work out the percent change one at a time or all simultaneously, and you would get the same answer.

	Yield Curve			
	$r_L \to r_H$	PV(r_L)	PV(r_H)	RoR
1-Year Bond	1.05%→1.06%	\$952.38	\$943.40	–0.94%
2-Year Bond	1.10%→1.11%	\$826.45	\$811.62	$\boxed{-1.79\%}$
3-Year Bond	1.15%→1.16%	\$640.66	\$657.52	–2.56%

The answer is that your project's value change is most similar to the 2-year zero T-bond value change. This is what your bond's duration of 2.01 year told you—your project behaves most similar to the 2-year bond as far as its interest rate sensitivity is concerned.

Duration Hedging

A hedge matches assets and liabilities to reduce risk.

So, now you know how your project would suffer from a change in the interest rate that you may fear, but what can you do about it? The idea is to **hedge** your risk—you try to own the same assets long and short—you are matching liabilities and assets—so that you are ensured against adverse changes. For example, it would be a perfect hedge if you purchased the project, and also shorted $1,000 in the 1-year bond, $1,000 in the 2-year bond, and $2,500 in the 3-year bond. You would be totally uninterested in where interest rates would be moving—your wealth would not be affected. (This is the "law of one price" in action. In fact, there is absolutely no risk to lose money, so this would be an arbitrage portfolio, explained in Section 15·1.)

Why perfect hedges are rare.

In the real world, perfect hedges, whereby you can match all project cash flows perfectly, are rarely possible. First, it is more common that you know only roughly what cash flows your project will return. Fortunately, it is often easier to guess your project's duration than all its individual cash flows. Second, it may also be difficult for smaller companies to short 137 zero T-bonds to match all project cash flows—the transaction costs would simply be too high. Third, you may not do any active matching, but you would still like to know what kind of exposure you are carrying. After all, you may not only have this project as asset, but you may have liabilities [e.g., debt payments] that have a duration of 2.4 years—and you want to know how matched or mismatched your assets and liabilities are. Or, you may use the newfound duration knowledge to choose among bank or mortgage loans with different durations, so that your assets and liabilities roughly match up in terms of their duration.

Minimizing interest rate risk.

For example, you know your project assets have a duration of 2 years—what kind of loan would you prefer? One that has a 1-year duration, a 2-year duration or a 3-year duration? If you want to minimize your interest rate risk, you would prefer to borrow \$2,716 of a 2-year bond—though the bank loan, too, may not be a zero-bond, but just some sort of loan with a 2-year duration. Would you be comfortable that interest rate would not affect the value of your project very much if you were short the 2-year bond and long the project? Yes and no—you would be comfortable that wholesale shifts of the yield curve would not affect you. You would however be exposed to changes in the shape of the yield curve—if only one of the interest rates were to shift, your project would be impacted differently than your 2-year T-bond. In this case, your project's value would move less than the value of your 2-year bond. In the real world, over short horizons, duration matching often works very well. Over longer horizons, however, you will have to do constant watching and rearranging of assets and liabilities to avoid the gap enlarging too much.

DIGGING DEEPER: *The interest-rate sensitivity of a bond's value is roughly its duration divided by one-plus the bond's yield. Therefore, a bond's price change with respect to a change in interest rate is roughly*

$$\text{Bond Price Return} \approx \left(\frac{\text{Duration}}{1+\text{YTM}}\right) \cdot \text{Interest Rate Change} \quad . \tag{4.38}$$

This ignores complex changes in the term structure, but it is often a useful quick-and-dirty sensitivity measure. For example, take our project, and consider a change in interest rates of 10 basis points. That is, the 1-year interest rate moves to 1.051%, the 2-year to 1.101%, and the 3-year to 1.151%. The value of oyur project would change from \$2,765.10 to \$2,760.12, an immediate percent change of 18 basis points.

$$\begin{array}{rcl} 18bp & \approx & \left(\frac{2.01}{1+17.5\%}\right) \cdot 10bp \quad . \\ \text{Bond Price Return} & \approx & \left(\frac{\text{Duration}}{1+\text{YTM}}\right) \cdot \text{Interest Rate Change} \quad . \end{array} \tag{4.39}$$

Solve Now!

Q 4.26 *Compute the duration of a two-year bond that costs \$25,000 today, pays \$1,000 at the end of the first year and at the end of the second year. At the end of the second year, it also repays \$25,000.*

Q 4.27 *If the yield curve is a flat 3%, compute the Macaulay duration for this two-year bond.*

Q 4.28 *If the yield curve is a flat 10%, compute the Macaulay duration for this two-year bond.*

Q 4.29 *Compute the yield-to-maturity of a 25-year bond that costs \$25,000 today, and pays \$1,000 at year-end for the following 25 years. In the final year ($t = 25$), it also pays \$25,000. What is the YTM?*

Q 4.30 *Compute the plain duration of this 25-year bond.*

Q 4.31 *If the yield curve is a flat 3%, compute the Macaulay duration for this 25-year bond.*

Q 4.32 *If the yield curve is a flat 10%, compute the Macaulay duration for this 25-year bond.*

4·7.D. Continuous Compounding

Continuously compounded interest rates are "as if interest is paid every instant."

A subject of some interest to Wall Street traders, i.e., the people who trade bonds or options for a living, is the concept of a **continuously compounded interest rate.** This is easiest to explain by example.

Progressively more frequently paid interest payments converge to the continuously compounded interest rate.

Assume that you receive \$120 next year for an investment of \$100 today. You already know that this represents a simple rate of return of 20%. What would the interest be if it were paid twice per year, not once per year, the interest rate remained constant, and the \$100 would still come out to be \$120 at the end of the year. You have done this before:

$$(1 + r_{\text{semi-annual}}) \cdot (1 + r_{\text{semi-annual}}) = (1 + 20\%) \qquad \Longrightarrow \qquad r \approx 9.54\% \ . \tag{4.40}$$

If you multiply this semiannual interest rate by two, you get 19.08%. What if you received interest twelve times a year?

$$(1 + r_{\text{monthly}})^{12} = (1 + 20\%) \qquad \Longrightarrow \qquad r \approx 1.53\% \ . \tag{4.41}$$

Multiply this monthly (m) interest rate by 12 and you get 18.36%. What if you received interest 365 times a year?

$$(1 + r_{\text{daily}})^{365} = (1 + 20\%) \qquad \Longrightarrow \qquad r \approx 0.05\% \ . \tag{4.42}$$

The 20% was called an "effective annual rate" in Table 2.3. Multiply this daily (d) interest rate by 365 and you get 18.25% (the annual quote). Now, what would this number be if you were to receive interest every single moment in time—the annual rate, compounded every instant?

The limit: Use logs and exponents to translate simple interest rates to continuously compounded interest rates.

The answer is, you guessed it, the continuously compounded interest rate and it can be computed by taking the **natural logarithm** (abbreviated "ln" on your calculator and below) of one plus the simple interest rate

$$\begin{aligned} r_{\text{continuously compounded}} &= \ln(1 + 20\%) \approx 18.23\% \\ r_{\text{continuously compounded}} &= \ln(1 + r_{\text{simple}}) \ . \end{aligned} \tag{4.43}$$

(Appendix 2·3 reviews powers, exponents and logarithms.)

Warning: Never, ever apply cc rates of return to a cash flow!

You must *never* directly apply a continuously compounded interest rate to a cash flow to compute your return. In this example, investing \$100 would not leave you with \$118.23 after one year. Indeed, if someone quoted you a continuously compounded interest rate, to determine how much money you will end up with, you would first have to convert the continuously compounded return into a simple interest rate

$$r_{\text{simple}} = e^{r_{\text{continuously compounded}}} - 1 \approx e^{18.23\%} - 1 \approx 20\% \ , \tag{4.44}$$

and then apply this interest rate to the cash flow. Alternatively, you can multiply the cash flow not by one plus the simple interest rate, but by $e^{r_{cc}}$.

To obtain multi-period interest returns, continuously compounded interest rates are never compounded, but added instead.

Continuously compounded rates have two nice features: First, if the continuously compounded rate in period 1 is 10% and in period 2 is 20%, the total two-period continuously compounded rate is 30%—yes, continuously compounded interest rates can be added, so no more multiplying one-pluses! This additivity is not a big advantage. Second, they are more "symmetric." See, an ordinary rate of return lies between −100% and +∞, while the continuously compounded rate of return lies between −∞ and +∞. This can be an advantage in statistical work, as can be the fact that the logarithm helps "pull in" large outliers. However, the main need for continuously compounded interest rates arises in other formulas (such as the Black-Scholes option formula, the subject of the Web Chapter on Options and Derivatives).

Solve Now!

Q 4.33 *A bond pays $150 for every $100 invested. What is its continuously compounded interest rate?*

Q 4.34 *Show my claim that you can add continuously compounded interest rates. That is, a bond pays a continuously compounded interest rate of 10%. Upon maturity, the money can be reinvested at a continuously compounded interest rate of 20%. If you invest $100 today, how much money will you end up with? What is the simple and continuously compounded interest rate over the two periods?*

4·8. Summary

The chapter covered the following major points:

- Compounding works just as well for time-varying interest rates.
- A holding rate of return can be annualized for easier interpretation.
- Different interest rates apply to different horizon investments. The graph of interest rates as a function of horizon is called the "term structure of interest" or "yield curve."
- Net present value works just as well for time-varying interest rates. You merely need to use the appropriate opportunity cost of capital as the interest rate in the denominator.
- Different horizon interest rates carry different risks, not because one is a better deal than the other. Instead, it is either that future interest rates are expected to be different, or that longer-term investments carry more interim risk.

  For Treasury bonds, the risk from interim interest rate changes seems to be the primary reason why the yield curve is usually upward sloping.
- More often than not, "paper losses" are no different from real losses.
- The yield curve is usually upward sloping, but can be downward sloping (inverted), humped, or flat.
- The Yield-to-Maturity is a "sort-of-average" interest rate characterizing the payoffs of a bond. It does not depend on economy-wide interest rates (the yield curve).

If you covered the optional bond topics section, you also learned the following.

- The information in the set of annualized rates of return, individual holding rates of return, and total holding rates of return is identical. Therefore, you can translate them into one another. For example, you can extract all forward interest rates from the prevailing yield curve.
- How shorting transactions work.
- If you can both buy and short bonds, then you can lock in forward interest rates today.
- Bond duration is a characterization of *when* bond payments typically come in.
- The continuously compounded interest rate is $\ln(1 + r)$, where r is the simple interest rate.

Solutions and Exercises

1. $r_{0,2} = (1 + r_{0,1}) \cdot (1 + r_{1,2}) - 1 = (1 + 2\%) \cdot (1 + 3\%) - 1 = 5.06\%$.
2. Solve $(1 + 22\%) \cdot (1 + x) = (1 - 50\%)$, so the project had a rate of return of -59%.
3. 166.4%. For checking, 124.3% followed by 18.8%.
4. The returns were $(-33\%, +50\%, -67\%, +100\%)$, so the overall rate of return was -33.3%.

5. The annualized rate of return is 18.3%. It is therefore lower than the average rate of return.
6. The compounded rate of return is always higher, because you earn interest on interest. The annualized rate of return is lower than the average rate of return, again because you earn interest on the interest. For example, an investment of \$100 that turns into an investment of \$200 in two years has a total holding period rate of return of 100%—which is an average rate of return of 50% and an annualized rate of return of $\sqrt{(1 + 100\%)} - 1 = 41\%$. Investing \$100 at 41%/annum would yield \$200, which is higher than 50% per annum.
7. $r_{\overline{12}} = \sqrt[12]{1 + 166.4\%} = 8.5\%$.
8. Your rate of return over the six days here was $r_{0,6} = \$1,300.80/\$1,283.27 - 1 = 1.366\%$. You can compound this over the $255/6 = 42.5$ time periods to obtain 78% per annum. (Alternatively, the 1.366% is a daily rate of 0.2264%, which can compound over 255 days.) So, your \$100 would have turned into \$178. (Actually, these were the index values of the S&P500; the rest of 2001 was a lot bleaker for investors, and they lost 13% in 2001.)
9. $r_{0,5} = 50\% \qquad (1 + r_{\overline{5}})^5 = (1 + 50\%) \qquad \Longrightarrow \qquad r_{\overline{5}} = (1 + 50\%)^{1/5} - 1 = 8.45\%$.
10. The same.
11. $r_{\overline{5}} = 10\% \qquad (1 + r_{0,5}) = (1 + r_{\overline{5}})^5 = (1 + 61.05\%)$.
12. The basic formula is $(1 + r_{\overline{5}})^5 = (1 + r_{0,1}) \cdot (1 + r_{1,2}) \cdot (1 + r_{2,3}) \cdot (1 + r_{3,4}) \cdot (1 + r_{4,5})$. Substituting what you know, $(1 + 10\%)^5 = (1 + 15\%) \cdot (1 + x) \cdot (1 + x) \cdot (1 + x) \cdot (1 + x)$. Calculating, $1.61051 = 1.15 \cdot (1 + x)^4$, $(1 + x)^4 = 1.40$, , and $(1 + x) = 1.40^{1/4} = 1.0878$. So, the return in all other years would have to be 8.78% per year.
13. The daily interest rate is either $(1 + 10\%)^{1/365} - 1 \approx 0.026\%$ or $(1 + 20\%)^{1/365} - 1 \approx 0.05\%$ per day. Thus, the pessimist expects a stock price of \$30.008 tomorrow; the optimist expects a stock price of \$30.015 tomorrow. Note that the 1 cent or so expected increase is dwarfed by the typical day-to-day noise in stock prices.

14. Do it!
15. Do it!

16. Do it!
17. Take the above number, call it $r_{\overline{10}}$, and compute $(1 + r_{\overline{10}})^{10}$.
18. Do it.

19. 5%, because

$$-\$1,000 + \frac{\$50}{1 + 5\%} + \frac{\$50}{(1 + 5\%)^2} + \frac{\$50}{(1 + 5\%)^3} + \frac{\$50}{(1 + 5\%)^4} + \frac{\$1,050}{(1 + 5\%)^5} = 0 \qquad (4.45)$$

The YTM of such a bond is just the coupon itself.

20. The YTM is 10%, because

$$-\$1,000 + \frac{\$1,611}{(1 + 10\%)^5} = 0 \qquad (4.46)$$

21. You are seeking the solution to

$$-\$25,000 + \sum_{t=1}^{2} \frac{\$1,000}{(1 + r)^t} + \frac{\$25,000}{(1 + r)^2} = 0 \ . \qquad (4.47)$$

The correct solution is 4%.

22.

(a)

$$-\$10,065.22 \;+\; \frac{\$150}{\text{YTM}^{0.5}} \;+\; \frac{\$10,150}{\text{YTM}^{1}} \;=\; 0 \quad , \tag{4.48}$$

so YTM = 2.35%. The YTM of the six-month bond is

$$-\$10,103.96 \;+\; \frac{\$10,200}{\text{YTM}^{0.5}} \;=\; 0 \quad , \tag{4.49}$$

so YTM = 1.91%. Okay—I admit I chose the equivalent of the yield curve that we plotted in the text.

(b) Do it.

(c) The \$150 coupon is worth $\$150/1.0191^{0.5} \approx \$1,48.59$. Therefore, the one-year zero bond with one payment of \$10,150 due in one year costs $\$10,065.22 - \$148.59 \approx \$9,916.63$. This means that the 1-year zero bond with payoff of \$10,150 has a YTM of $\$10,150/\$9,916.63 - 1 \approx 2.3533\%$.

(d) Do it.

(e) The difference between the YTM of the coupon and the zero bond is only 0.3 basis points—very small, even though the yield curve here is fairly steep. The reason is that the early 6-month coupon earns a lower interest makes little difference because the coupon payment is only \$150, and most of the YTM comes from the final payment. The coupon effect can become larger on very long horizons when the yield curve is steep, but it is very rarely more than 10-20 basis points.

23. The 4-year holding rate of return is $r_{0,4} \approx (1+4.06\%)^4 \approx 17.26\%$. Therefore, the 1-year forward rate from year 3 to year 4 is $r_{3,4} \approx \frac{(1+r_{0,4})}{(1+r_{0,3})} - 1 \approx \frac{(1+17.27\%)}{(1+11.35\%)} - 1 \approx 5.30\%$.

24. Buy \$1,000 of a 4-year zero bond (4.06%/year) and short \$1,000 of a 3-year zero bond (3.65%/year). Today, you receive and pay \$1,000, so the transaction does not cost you anything. In 3-years, you need to pay the 3-year bond, i.e., you need to pay in \$1,113.55. In 4-years, you receive from the 4-year bond \$1,172.56. This is the equivalent of saving at an interest rate of 5.30%.

25. You can do this from first principles, as before. An alternative is to rely on the previous solution, where you were saving \$1,113.50. So, you now have to do this transaction at a scale that is $\$500,000/\$1113.5 \approx 449.03$ times as much. Therefore, instead of buying \$1,000 of the 4-year bond, you must buy $449.035 \cdot \$1,000 \approx \$449,035$ of the 4-year bond, and short the same amount of the 3-year bond.

26.

$$\begin{aligned} \text{Plain Duration} \;&\approx\; \frac{\sum_{t=1}^{2} \$1,000 \cdot t + \$25,000 \cdot 2}{\sum_{t=1}^{2} \$1,000 + \$25,000} \\ &=\; \frac{53,000}{27,000} \;\approx\; 1.96296 \quad . \end{aligned} \tag{4.50}$$

27.

$$\begin{aligned} \text{Macaulay Duration at 3\%} \;&=\; \frac{\left(\sum_{t=1}^{2} \frac{\$1,000 \cdot t}{(1+3\%)^t}\right) + \frac{\$25,000 \cdot 2}{(1+3\%)^2}}{\left(\sum_{t=1}^{2} \frac{\$1,000}{(1+3\%)^t}\right) + \frac{\$25,000}{(1+3\%)^2}} \\ &=\; \frac{\frac{\$1,000 \cdot 1}{(1+3\%)^1} + \frac{\$1,000 \cdot 2}{(1+3\%)^2} + \frac{\$25,000 \cdot 2}{(1+3\%)^2}}{\frac{\$1,000}{(1+3\%)^1} + \frac{\$1,000}{(1+3\%)^2} + \frac{\$25,000}{(1+3\%)^2}} \\ &=\; \frac{49,986}{25,478} \;\approx\; 1.96189 \quad . \end{aligned} \tag{4.51}$$

The units here are years, because we quoted the multiplication factors “1” and “2” are in years.

28.

$$\begin{aligned} \text{Macaulay Duration at 10\%} \;&=\; \frac{\left(\sum_{t=1}^{2} \frac{\$1,000 \cdot t}{(1+10\%)^t}\right) + \frac{\$25,000 \cdot 2}{(1+10\%)^2}}{\left(\sum_{t=1}^{2} \frac{\$1,000}{(1+10\%)^t}\right) + \frac{\$25,000}{(1+10\%)^2}} \\ &=\; \frac{\frac{\$1,000 \cdot 1}{(1+10\%)^1} + \frac{\$1,000 \cdot 2}{(1+10\%)^2} + \frac{\$25,000 \cdot 2}{(1+10\%)^2}}{\frac{\$1,000}{(1+10\%)^1} + \frac{\$1,000}{(1+10\%)^2} + \frac{\$25,000}{(1+10\%)^2}} \\ &=\; \frac{43,884.3}{22,396.7} \;\approx\; 1.95941 \quad . \end{aligned} \tag{4.52}$$

29. You are seeking the solution to

$$-\$25,000 \;+\; \sum_{t=1}^{25} \frac{\$1,000}{(1+r)^t} \;+\; \frac{\$25,000}{(1+r)^{25}} \;=\; 0 \quad . \tag{4.53}$$

The middle piece is an annuity, so $\sum_{t=1}^{25} \$1,000/(1+r)^t = \$1,000 \cdot \frac{1-[1/(1+r)^{25}]}{r}$. The correct solution is 4%.

30.
$$\text{Plain Duration} = \frac{\sum_{t=1}^{25} \$1,000 \cdot t \;\; + \$25,000 \cdot 25}{\sum_{t=1}^{25} \$1,000 \;\; + \$25,000} \approx 19.62 \tag{4.54}$$

31.
$$\text{Macaulay Duration at 3\%} = \frac{\sum_{t=1}^{25} \frac{\$1,000 \cdot t}{(1+3\%)^t} + \frac{\$25,000 \cdot 25}{(1+3\%)^{25}}}{\sum_{t=1}^{25} \frac{\$1,000}{(1+3\%)^t} + \frac{\$25,000}{(1+3\%)^{25}}} \approx 16.98 \;. \tag{4.55}$$

32.
$$\text{Macaulay Duration at 10\%} = \frac{\sum_{t=1}^{25} \frac{\$1,000 \cdot t}{(1+10\%)^t} + \frac{\$25,000 \cdot 25}{(1+10\%)^{25}}}{\sum_{t=1}^{25} \frac{\$1,000}{(1+10\%)^t} + \frac{\$25,000}{(1+10\%)^{25}}} \approx 11.81 \;. \tag{4.56}$$

33. The simple interest rate is 50%. The cc interest rate is $\log(1+50\%) \approx 40.55\%$.

34. A 10% cc interest rate is a simple interest rate of $r_{0,1} \approx e^{0.10} - 1 \approx 10.52\%$, so you would have \$110.52 after one year. A 20% cc interest rate is a simple interest rate of $r_{1,2} \approx e^{0.20} - 1 \approx 22.14\%$. This means that your \$110.52 investment would turn into $(1 + 22.14\%) \cdot \$110.52 \approx \$134.99$. This means that the simple interest rate is $r_{0,2} \approx 34.99\%$. Thus, the cc interest rate is $\ln(1 + r_{0,2}) \approx \ln(1.3499) \approx 30\%$. Of course, you could have computed this faster: $V_t = e^{0.10} \cdot e^{0.20} \cdot V_0 = e^{0.10+0.20} \cdot V_0 = e^{0.30} \cdot \$100 \approx 1.3499 \cdot \$100 \approx \$134.99$.

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

CHAPTER 5

Uncertainty, Default, and Risk

Promises and Expectations

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We now enter the world of uncertainty—though we shall still pretend that we live in a perfect world of no taxes, no transaction costs, no differences of opinion, and infinitely many investors and firms.

Net present value still rules, but you will now have to face the sad fact that it is not easy to use in the real world. It is not the NPV concept that is difficult—in fact, you already "almost" know it. Instead, it is the present value formula inputs—the expected cash flows and appropriate costs of capital under uncertainty—that can be so very difficult to estimate in the real world.

What does uncertainty really do? There will be scenarios in which you will get more than you expected and scenarios in which you will get less than you expected. This is the case for almost all corporate projects. The single-most important insight under uncertainty is that you must always draw a sharp difference between *promised* (or *quoted*) and *expected* returns. Because firms can default on payments or go bankrupt in the future, promised returns are higher than expected returns.

After setting forth the necessary statistical background, our chapter will cover two important topics: First, we need to determine how lenders should charge borrowers if there is the possibility of default. Second, once we know about how to handle uncertainty, we can really discuss the differences between the two important building blocks of finance—debt and equity.

Anecdote: The Ruin of the First Financial System
The earliest known example of widespread financial default occurred in the year of 1788 B.C.E., when King Rim-Sin of Uruk (Mesopotamia) repealed all loan repayments. The royal edict effectively destroyed a system of flourishing commerce and finance, which was already many thousands of years old! It is not known why he did so.

5·1. An Introduction to Statistics

Statistics is about how to bet in an uncertain world.

Statistics has a reputation of being the most painful of the foundation sciences for finance—but we absolutely need it to describe an uncertain future. Fortunately, although statistics can be a difficult subject, if you have ever placed a bet in the past, chances are that you already have a good intuitive grasp of what you need. In fact, I had already sneaked the term "expected" into previous chapters, even though we only now firm up your knowledge of this important statistical concept.

5·1.A. Random Variables and Expected Values

A "fair" bet means that both sides break even if the bet is repeated infinitely many times.

The most important statistical concept is the **expected value**, which is most often just a fancy phrase for **mean** or **average**. The only necessary clarification is that we use "means" and "averages" for past outcomes and "expected value" for future outcomes.

> **IMPORTANT**: *The expected value is just a mean (or average) that is computed over future outcomes if hypothetical scenarios are repeated (infinitely) often.*

The "Expected Value" is the average outcome. An expected value can be an impossible realization.

For example, say you toss a coin, which can come up with either heads or tails and with equal probability. You receive \$1 if the coin comes up heads and \$2 if the coin comes up tails. Because you know that there is a 50% chance of \$1 and a 50% chance of \$2, the expected value of each coin toss is \$1.50—repeated infinitely often, the mean will be exactly \$1.50. Of course, exactly \$1.50 will never come up—the expected value does not need to be a possible **realization** of a single coin toss.

A random variable is a number whose realization is not yet known.

Statisticians have invented the concept of **random variables** to make it easier to work with uncertainty. A random variable is a variable whose value (i.e., outcome) has not yet been determined. In the coin toss example, we can define a random variable named c (for "coin toss outcome") that takes the value \$1 with 50% probability and the value \$2 with 50% probability. The expected value of c is \$1.50. To distinguish a random variable from an ordinary non-random variable, we use a tilde over the variable. To denote the expected value, we use the notation $\mathcal{E}$. So, in this bet,

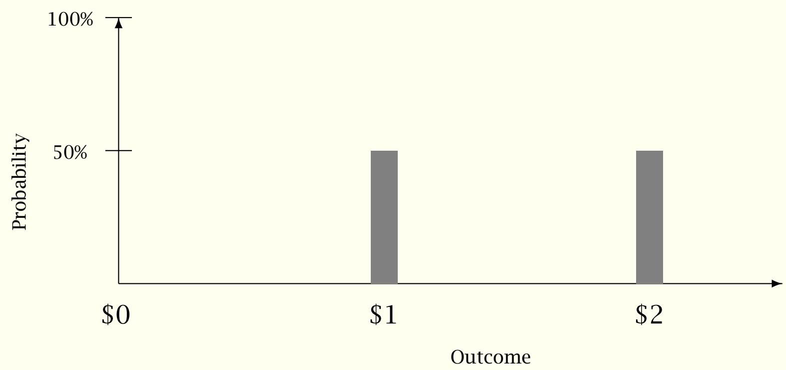
$$\begin{array}{rcccccl} \mathcal{E}(\tilde{c}) & = & 50\% \cdot \$1 & + & 50\% \cdot \$2 & = & \$1.50 \\ \text{Expected Value(of Coin Toss)} & = & \mathcal{P}rob(\text{Heads}) \cdot \$1 & + & \mathcal{P}rob(\text{Tails}) \cdot \$2 & . & \end{array} \tag{5.1}$$

After the coin has been tossed, the actual outcome c could, e.g., be

$$c = \$2 \ , \tag{5.2}$$

and c is no longer a random variable. Also, if you are certain about the outcome, perhaps because there is only one possible outcome, then the actual realization and the expected value are the same. The random variable is then really just a plain simple variable. Is the expected outcome of the coin toss a random variable? No: we know the expected outcome is \$1.50 even before we toss the coin. The expected value is known, the uncertain outcome is not. The expected value is a plain variable; the outcome is a random variable. Is the outcome of the coin throw *after* it has come down heads a random variable? No: we know what it is (heads), so it is not a random variable.

Figure 5.1. A Histogram For a Random Variable With Two Equally Likely Outcomes, \$1 and \$2.



Computing expected values from distributions.

A random variable is defined by the **probability distribution** of its possible outcomes. The coin throw distribution is simple: the value \$1 with 50% probability and the value \$2 with 50% probability. This is sometimes graphed in a histogram, as depicted in Figure 5.1.

An example with 3 possible outcomes.

For practice, let us make this a bit trickier. Assume that there is a 16.7% chance that you will get \$4, a 33.3% chance that you will get \$10, and a 50% chance that you will get \$20. You can simulate this payoff structure by throwing a die and getting \$4 if it comes up ⚀, \$10 if it comes up ⚁ or ⚂, and \$20 if it comes up ⚃, ⚄, or ⚅.

If it costs its expected value to buy a bet, the bet is fair. An example of a fair bet.

Now, a **fair bet** is a bet that costs its expected value. If repeated infinitely often, both the person offering the bet and the person taking the bet would expect to end up even. What is a fair price for our die bet? Call the uncertain payoff $\tilde{D}$. First, you must determine $\mathcal{E}(\tilde{D})$. It is

$$\begin{aligned}
\mathcal{E}(\tilde{D}) \;=\; & \quad 16.7\% \cdot \$4 \\
& + 33.3\% \cdot \$10 \\
& + 50.0\% \cdot \$20 \quad = \$14 \\
\mathcal{E}(\tilde{D}) \;=\; & \quad \mathcal{P}rob(\text{⚀}) \cdot (\text{Payout if ⚀}) \\
& + \mathcal{P}rob(\text{⚁ or ⚂}) \cdot (\text{Payout if ⚁ or ⚂}) \\
& + \mathcal{P}rob(\text{⚃ or ⚄ or ⚅}) \cdot (\text{Payout if ⚃ or ⚄ or ⚅}) \quad .
\end{aligned} \tag{5.3}$$

Therefore, if you repeat this bet a million times, you would expect to earn \$14 million. (On average, each bet would earn \$14, although some sampling variation in actual trials would make this a little more or less.) If it costs \$14 to buy each bet, it would be a fair bet.

The expected value is the probability weighted sum of all possible outcomes.

Generally, you compute expected values by multiplying each outcome by its probability and adding up all these products. If $\tilde{X}$ is a random variable with N possible outcomes, named X_1 through X_N, then you would compute

$$\mathcal{E}(\tilde{X}) = \mathcal{P}rob(\tilde{X} = X_1) \cdot X_1 + \mathcal{P}rob(\tilde{X} = X_2) \cdot X_2 + \cdots + \mathcal{P}rob(\tilde{X} = X_N) \cdot X_N \quad . \tag{5.4}$$

This is the formula that you used above,

$$\begin{aligned}
\mathcal{E}(\tilde{D}) &= 16.7\% \cdot \$4 + 33.3\% \cdot \$10 + 50\% \cdot \$20 = \$14 \quad . \\
&= \mathcal{P}rob(\tilde{D} = D_1) \cdot D_1 + \mathcal{P}rob(\tilde{D} = D_2) \cdot D_2 + \mathcal{P}rob(\tilde{D} = D_3) \cdot D_3 \quad .
\end{aligned} \tag{5.5}$$

Note that N could be a trillion possible different outcomes, and many of them could be impossible, i.e., have probabilities of zero.

> **IMPORTANT**: *You must understand*
>
> 1. *the difference between an ordinary variable and a random variable;*
> 2. *the difference between a realization and an expectation;*
> 3. *how to compute an expected value, given probabilities and outcomes;*
> 4. *what a fair bet is.*

We will consider the expected value the measure of the (average) **reward** that we expect to receive from making an investment.

We will discuss the variance and standard deviation in Part III, which are measures of spread—a coin toss that results in a payoff of either 0 or 4 (with equal probability) has more spread than one that results in a payoff of either 1 or 3 (with equal probability), even though both have an expected outcome of 2.

Solve Now!

Q 5.1 *Is the expected value of a die throw a random variable?*

Q 5.2 *Could it be that the expected value of a bet is a random variable?*

Q 5.3 *An ordinary die came up with a ⚅ yesterday. What was its expected outcome before the throw? What was its realization?*

Q 5.4 *A stock that has the following probability distribution (outcome P_{+1}) costs \$50. Is an investment in this stock a fair bet?*

Prob	P_{+1}	*Prob*	P_{+1}	*Prob*	P_{+1}	*Prob*	P_{+1}
5%	\$41	20%	\$45	20%	\$58	5%	\$75
10%	\$42	30%	\$48	10%	\$70		

5·1.B. Risk Neutrality (and Risk Aversion Preview)

Choosing investments on the basis of expected values is assuming risk-neutrality.

Fortunately, the expected value is all that we need to learn about statistics until we will get to Part III (investments). This is because we are assuming—only for learning purposes—that everyone is **risk-neutral.** Essentially, this means that investors are willing to write or take any fair bet. For example, you would be indifferent between getting \$1 for sure, and getting either \$0 or \$2 with 50% probability. And you would be indifferent between earning 10% from a risk-free bond, and earning either 0% or 20% from a risky bond. You have no preference between investments with equal expected values, no matter how safe or uncertain they may be.

If, instead, you were risk-averse—which you probably are in the real world—you would not want to invest in the more risky alternative if both the risky and safe alternative offered the same expected rate of return. You would prefer the safe \$1 to the unsafe \$0 or \$2 investment. You would prefer the 10% risk-free Treasury bond to the unsafe corporate bond that would pay either 0% or 20%. In this case, if I wanted to sell you a risky project or bond, I would have to offer you a higher rate of return as risk compensation. I might have to pay you, say, 5 cents to get you to be willing to accept the project that pays off \$0 or \$2 if you can instead earn \$1. Or, I would have to lower the price of my corporate bond, so that it offers you a higher expected rate of return, say, 1% and 21% instead of 0% and 20%.

Risk aversion means you would prefer the safe project, so you would demand an extra kicker to take the riskier project.

Would you really worry about a bet for either +\$1 or −\$1? Probably not. For small bets, you are probably close to risk-neutral—I may not have to offer even one cent to take this bet. But what about a bet for plus or minus \$100? Or for plus and minus \$10,000? My guess is that you would be fairly reluctant to accept the latter bet without getting extra compensation. For large bets, you are probably fairly risk-averse—I would have to offer you several hundred dollars to take this bet. However, your own personal risk aversion is not what matters in financial markets. Instead, it is an aggregate risk aversion. For example, if you could share the \$10,000 bet with 100 other students in your class, the bet would be only \$100 for you. And some of your colleagues may be willing to accept even more risk for less extra money—they may have healthier bank accounts or wealthier parents. If you could lay bets across many investors, the effective risk aversion would therefore be less. And this is exactly how financial markets work: the aggregate risk absorption capability is considerably higher than that of any individual. In effect, financial markets are less risk averse than individuals.

Where risk-aversion matters and where it does not matter.

We will study risk aversion in the investments part of the book. There, we will also need to define good measures of **risk**, a subject we can avoid here. (Appendix 2·4 provides more advanced statistical background, if you are interested.) But, as always, all tools we learn under the simpler scenario (risk-neutrality) will remain applicable under the more complex scenario (risk-aversion). And, in any case, in the real world, the difference between promised and expected returns that we shall discuss in this chapter is often more important (in terms of value) than the extra compensation for risk that we shall ignore in this chapter.

The tools we shall learn now will remain applicable under risk-aversion.

5·2. Interest Rates and Credit Risk (Default Risk)

Most loans in the real world are not risk-free, because the borrower may not fully pay back what was promised. So, how do we compute appropriate rates of returns for risky bonds?

5·2.A. Risk-Neutral Investors Demand Higher *Promised* Rates

You can ask the same interest rate from an investor as from the Treasury if payment is certain.

Put yourself into the position of a banker. Assume a one-year Treasury bond offers a safe annual rate of return of 10%. You are about to lend \$1 million to a risky company. The loan maturity is one year. You must decide what interest rate to charge on the loan. If you are 100% certain that the lender will fully pay the agreed-upon amount, you can earn as much charging a 10% interest rate from the borrower as you can from buying the Treasury bond. Both provide \$1,100,000 in repayment.

If you quote a risky borrower a particular interest rate, you must expect to earn a lower interest rate.

However, in the real world, there are few if any borrowers for whom you can be 100% certain that they will fully repay a loan. Let's take an extreme example. Assume you think there is a 50% chance that the company will **default** (fail to pay all that it has promised), and the borrower can only pay back \$750,000. There is also a 50% chance that the company will pay back the principal plus interest (in which case the company is called **solvent**). In this case, if you charge a 10% interest rate, your expected payout would be

$$\underset{\mathcal{Prob}(\text{Default})}{50\%} \cdot \underset{\text{Payment if Default}}{\$750,000} + \underset{\mathcal{Prob}(\text{Solvent})}{50\%} \cdot \underset{\text{Payment if Solvent}}{\$1,100,000} = \$925,000 \;. \tag{5.6}$$

So your *expected* return would not be \$1,100,000, but only \$925,000. Your *expected* rate of return would not be +10%, but only $\$925,000/\$1,000,000 - 1 = -7.5\%$. Extending such a loan would not be—pardon the pun—in your best interest: you can do better by putting your \$1,000,000 into government Treasury bills.

You must ask for a higher promised interest—received only in good times—in order to make up for default risk.

So, as a banker, you must demand a higher interest rate from risky borrowers, even if you just want to "break even" (i.e., earn the same \$1,100,000 that you could earn in Treasury bonds). If you solve

$$\underset{\mathcal{Prob}}{50\%} \cdot \underset{\text{Payment if Default}}{\$750,000} + \underset{\mathcal{Prob}}{50\%} \cdot \underset{\text{Payment if Solvent}}{(\text{promised repayment})} = \underset{\text{Treasury Payment}}{\$1,100,000} \tag{5.7}$$

for the desired promised repayment, you find that you must ask the borrower for \$1,450,000. The promised interest rate is therefore $\$1,450,000/\$1,000,000 - 1 = 45\%$. Of this 45%, 10% is the time premium. Therefore, we can call the remaining 35% the **default premium**—the difference between the promised rate and the expected rate that allows the lender to break even.

Common terminology

We rarely observe expected rates of return directly. Newspaper and financial documents almost always provide only the **promised interest rate**, which is therefore also called the **quoted interest rate** or the **stated interest rate**. (The YTM also is usually merely a promised rate, not an expected rate.) On Wall Street, the **default premium** is often called the **credit premium**, and **default risk** is often called **credit risk**.

Solve Now!

Q 5.5 *For what kind of bonds are expected and promised interest rates the same?*

5·2.B. A More Elaborate Example With Probability Ranges

Borrowers may sometimes not be able to repay.

Now work a similar but more involved example in which we assume that you want to lend money to me. Assume I am often not able to pay you back, despite my best intentions and promises. You believe I will pay you with 98% probability what I promise, with 1% probability half of what I borrowed, and with 1% probability nothing. I want to borrow \$200 from you, and you could alternatively invest the \$200 into a government bond promising a 5% interest rate, so you would receive \$210 for certain. What interest rate would you ask of me?

Table 5.1. Risky Loan Payoff Table: 5% Promised Interest Rate

Receive Cash Flow CF_1	which is a return of	How Likely (Probability)
\$210	5.0%	98% of the time
\$100	−52.4%	1% of the time
\$0	−100.0%	1% of the time

If you ask me to pay the risk-free interest rate, you will on average earn less than the risk-free interest rate.

Table 5.1 shows that if you ask me for a 5% interest rate, your expected cash return will be

$$
\begin{aligned}
\mathcal{E}(\tilde{C}_{t=1}) &= 98\% \cdot \$210 \\
&+ 1\% \cdot \$100 \\
&+ 1\% \cdot \$0 = \$206.80 \\
\mathcal{E}(\tilde{C}_{t=1}) &= \mathcal{P}rob\ \text{CF}_{t=1} \text{ will be case 1} \cdot \text{CF}_{t=1} \text{ cash flow in case 1} \\
&+ \mathcal{P}rob\ \text{CF}_{t=1} \text{ will be case 2} \cdot \text{CF}_{t=1} \text{ cash flow in case 2} \\
&+ \mathcal{P}rob\ \text{CF}_{t=1} \text{ will be case 3} \cdot \text{CF}_{t=1} \text{ cash flow in case 3} \quad .
\end{aligned}
\tag{5.9}
$$

This expected return of \$206.80 is less than the \$210 that the government promises. Put differently, if I *promise* you a rate of return of 5%,

$$
\begin{aligned}
\text{Promised}(\tilde{r}_{0,1}) &= \frac{\$210 - \$200}{\$200} = 5.00\% \\
\text{Promised}(\tilde{r}_{0,1}) &= \frac{\text{Promised}(\tilde{C}_1) - \text{CF}_0}{\text{CF}_0} \quad ,
\end{aligned}
\tag{5.10}
$$

your expected rate of return is only

$$
\begin{aligned}
\mathcal{E}(\tilde{r}_{0,1}) &= \frac{\$206.80 - \$200}{\$200} = 3.40\% \\
\mathcal{E}(\tilde{r}_{0,1}) &= \frac{\mathcal{E}(\tilde{C}_1) - \text{CF}_0}{\text{CF}_0} \quad ,
\end{aligned}
\tag{5.11}
$$

which is less than the 5% interest rate that Uncle Sam promises—and surely delivers.

Determine how much more interest promise you need to break even.

You need to determine how much I have to promise you to "break even," so that you expect to end up with the same \$210 that you could receive from Uncle Sam. In Table 5.1, we computed the expected payoff as the probability-weighted average payoff. You want this payoff to be not

\$206.80, but the \$210 that you can get if you put your money in government bonds. So, you now solve for an amount x that you want to receive if I have money,

$$
\begin{array}{rcccccc}
\mathcal{E}(\tilde{C}_{t=1}) & = & 98\% & \cdot & x & & \\
& + & 1\% & \cdot & \$100 & & \\
& + & 1\% & \cdot & \$0 & = \$210.00 & \\
\mathcal{E}(\tilde{C}_{t=1}) & = & \mathcal{P}rob\ \text{CF}_{t=1} \text{ will be case 1} & \cdot & \text{CF}_{t=1} \text{ cash flow in case 1} & & \\
& + & \mathcal{P}rob\ \text{CF}_{t=1} \text{ will be case 2} & \cdot & \text{CF}_{t=1} \text{ cash flow in case 2} & & \\
& + & \mathcal{P}rob\ \text{CF}_{t=1} \text{ will be case 3} & \cdot & \text{CF}_{t=1} \text{ cash flow in case 3} & . &
\end{array}
\tag{5.12}
$$

The solution is that if I promise you $x = \$213.27$, you will expect to earn the same 5% interest rate that you can earn in Treasury bonds. Table 5.2 confirms that a promise of \$213.27 for a cash investment of \$200, which is a *promised* interest rate of

$$
\begin{array}{rcl}
\text{Promised}(\tilde{r}_{0,1}) & = & \dfrac{\$213.27 - \$200}{\$200} \ = \ 6.63\% \\
\text{Promised}(\tilde{r}_{0,1}) & = & \dfrac{\text{Promised}(\tilde{C}_1) - \text{CF}_0}{\text{CF}_0} \ ,
\end{array}
\tag{5.13}
$$

and provides an *expected* interest rate of

$$
\mathcal{E}(\tilde{r}_{0,1}) = 98\% \cdot (+6.63\%) + 1\% \cdot (-50\%) + 1\% \cdot (-100\%) = 5\% \ . \tag{5.14}
$$

Table 5.2. Risky Payoff Table: 6.63% Promised Interest Rate

Receive Cash Flow CF_1	which is a return of	How Likely (Probability)
\$213.27	+6.63%	98% of the time
\$100.00	-50.00%	1% of the time
\$0.00	-100.00%	1% of the time
\$210.00	+5.00%	in expectation

The difference between the promised and expected interest rate is the default premium.

The difference of 1.63% between the promised (or quoted) interest rate of 6.63% and the expected interest rate of 5% is the default premium—it is the extra interest rate that is caused by the default risk. Of course, you only receive this 6.63% *if* everything goes perfectly. In our perfect world with risk-neutral investors,

$$
\begin{array}{ccccc}
6.63\% & = & 5\% & + & 1.63\% \\
\text{“Promised Interest Rate”} & = & \text{“Time Premium”} & + & \text{“Default Premium”} \ .
\end{array}
\tag{5.15}
$$

Important: *Except for 100% safe bonds (Treasuries), the promised (or quoted) rate of return is higher than the expected rate of return. Never confuse the higher promised rate for the lower expected rate.*

Financial securities and information providers rarely, if ever, provide expected rates of return.

In a risk-neutral world, all securities have the same exp. rate of return.

On average, the expected rate of return is the expected time premium plus the expected default premium. Because the *expected* default premium is zero *on average*,

$$\begin{aligned} \mathcal{E}(\text{Rate of Return}) &= \mathcal{E}(\text{Time Premium}) + \mathcal{E}(\text{Default Premium}) \\ &= \mathcal{E}(\text{Time Premium}) + \quad 0 \quad . \end{aligned} \tag{5.16}$$

If you want to work this out, you can compute the expected default premium as follows: you will receive (6.63% − 5% = 1.63%) in 98% of all cases; −50% − 5% = −55% in 1% of all cases (note that you lose the time-premium); and −100% − 5% = −105% in the remaining 1% of all cases (i.e., you lose not only all your money, but also the time-premium). Therefore,

$$\mathcal{E}(\text{Default Premium}) = 98\% \cdot (+1.63\%) + 1\% \cdot (-55\%) + 1\% \cdot (-105\%) = 0\% \ . \tag{5.17}$$

Solve Now!

Q 5.6 *Recompute the example in Table 5.2 assuming that the probability of receiving full payment of \$210 is only 95%, the probability of receiving \$100 is 1%, and the probability of receiving absolutely no payment is 4%.*

(a) *At the promised interest rate of 5%, what is the expected interest rate?*

(b) *What interest rate is required as a promise to ensure an expected interest rate of 5%?*

5·2.C. Preview: Risk-Averse Investors Have Demanded Higher *Expected* Rates

In addition to the default premium, in real life, investors also demand a risk premium.

We have assumed that investors are risk-neutral—indifferent between two loans that have the same expected rate of return. As we have already mentioned, in the real world, risk-averse investors would demand and expect to receive a little bit more for the risky loan. Would you rather invest into a bond that is known to pay off 5% (for example, a U.S. government bond), or would you rather invest in a bond that is "merely" expected to pay off 5% (such as my 6.63% bond)? Like most lenders, you are likely to be better off if you know exactly how much you will receive, rather than live with the uncertainty of my situation. Thus, as a risk-averse investor, you would probably ask me not only for the higher *promised* interest rate of 6.63%, which only gets you to an expected interest rate of 5%, but an even higher promise in order to get you more than 6.63%. For example, you might demand 6.75%, in which case you would expect to earn not just 5%, but a little more. The extra 12 basis points is called a **risk premium**, and it is an interest component required above and beyond the time premium (i.e., what the U.S. Treasury Department pays for use of money over time) and above and beyond the default premium (i.e., what the promised interest has to be for you to just expect to receive the same rate of return as what the government offers).

A more general decomposition of rates of return.

Recapping, we know that 5% is the time-value of money that you can earn in interest from the Treasury. You also know that 1.63% is the extra default premium that I must promise you, a risk-neutral lender, to allow you to expect to earn 5%, given that repayment is not guaranteed. Finally, if you are not risk-neutral but risk-averse, I may have to pay even more than 6.63%, although we do not know exactly how much.

More intellectually interesting, but otherwise not too useful decompositions.

If you want, you could think of further interest decompositions. It could even be that the time-premium is itself determined by other factors (such as your preference between consuming today and consuming next year, the inflation rate, taxes, or other issues, that we are brushing over). Then there would be a **liquidity premium**, an extra interest rate that a lender would demand if the bond could not easily be sold—resale is much easier with Treasury bonds.

IMPORTANT: *When repayment is not certain, lenders demand a promised interest rate that is higher than the expected interest rate by the default premium.*

$$\begin{aligned} &\textit{Promised Interest Rate} \\ &= \textit{Time Premium} + \textit{Default Premium} + \textit{Risk Premium} . \end{aligned} \tag{5.18}$$

The promised default premium is positive, but it is only paid when everything goes well. The actually earned interest rate consists of the time premium, the realized risk premium, and a (positive or negative) default realization.

$$\begin{aligned} &\textit{Actual Interest Rate Earned} \\ &= \textit{Time Premium} + \textit{Default Realization} + \textit{Risk Premium} . \end{aligned} \tag{5.19}$$

The default realization could be more than negative enough to wipe out both the time premium and the risk premium. But it is zero on average. Therefore,

$$\begin{aligned} &\textit{Expected Interest Rate} \\ &= \textit{Time Premium} + \textit{Expected Risk Premium} . \end{aligned} \tag{5.20}$$

Some real world evidence.

The risk premium itself depends on such strange concepts as the correlation of loan default with the general economy and will be the subject of Part III of the book. However, we can preview the relative importance of these components for you in the context of corporate bonds. (We will look at risk categories of corporate bonds in more detail in the next chapter.) The highest-quality bonds are called investment-grade. A typical such bond may promise about 6% per annum, 150 to 200 basis points above the equivalent Treasury. The probability of default would be small—less than 3% in total over a ten-year horizon (0.3% per annum). When an investment-grade bond does default, it still returns about 75% of what it promised. For such bonds, the risk premium would be small—a reasonable estimate would be that only about 10 to 20 basis points of the 200 basis point spread is the risk premium. The quoted interest rate of 6% per annum therefore would reflect first the time premium, then the default premium, and only then a small risk premium. (In fact, the liquidity premium would probably be more important than the risk premium.) For low-quality corporate bonds, however, the risk premium can be important. Ed Altman has been collecting corporate bond statistics since the 1970s. In an average year, about 3.5% to 5.5% of low-grade corporate bonds defaulted. But in recessions, the default rate shot up to 10% per year, and in booms it dropped to 1.5% per year. The average value of a bond after default was only about 40 cents on the dollar, though it was as low 25 cents in recessions and as high as 50 cents in booms. Altman then computes that the most risky corporate bonds promised a spread of about 5%/year above the 10-Year Treasury bond, but ultimately delivered a spread of only about 2.2%/year. 280 points are therefore the default premium. The remaining 220 basis points contain both the liquidity premium and the risk premium—perhaps in roughly equal parts.

Solve Now!

Q 5.7 *Return to the example in Table 5.2. Assume that the probability of receiving full payment of $210 is only 95%, the probability of receiving $100 is 4%, and the probability of receiving absolutely no payment is 1%. If the bond quotes a rate of return of 12%, what is the time premium, the default premium and the risk premium?*

5·3. Uncertainty in Capital Budgeting, Debt, and Equity

We now turn to the problem of selecting projects under uncertainty. Your task is to compute present values with imperfect knowledge about future outcomes. Your principal tool in this task will be the **payoff table** (or **state table**), which assigns probabilities to the project value in each possible future value-relevant scenario. For example, a floppy disk factory may depend on computer sales (say, low, medium, or high), whether floppy disks have become obsolete (yes or no), whether the economy is in a recession or expansion, and how much the oil price (the major cost factor) will be. Creating the appropriate state table is the manager's task—judging how the business will perform depending on the state of these most relevant variables. Clearly, it is not an easy task even to think of what the key variables are, to determine the probabilities under which these variables will take on one or another value. Assessing how your own project will respond to them is an even harder task—but it is an inevitable one. If you want to understand the value of your project, you must understand what the project's key value drivers are and how the project will respond to these value drivers. Fortunately, for many projects, it is usually not necessary to describe possible outcomes in the most minute detail—just a dozen or so scenarios may be able to cover the most important information. Moreover, these state tables will also allow you to explain what a *loan* (also called *debt* or *leverage*) and *levered ownership* (also called *levered equity*) are, and how they differ.

5·3.A. Present Value With State-Contingent Payoff Tables

Most projects are financed with a mix of debt and equity.

Almost all companies and projects are financed with both debt and levered equity. We already know what debt is. **Levered equity** is simply what accrues to the business owner *after* the debt is paid off. You already have an intuitive sense about this. If you own a house with a mortgage, you really own the house only after you have made all debt payments. If you have student loans, you *yourself* are the levered owner of your future income stream. That is, you get to consume "your" residual income only *after* your loans are paid back. But what will the levered owner and the lender get if the company's projects fail, if the house collapses, or if your career takes a turn towards Rikers Island? What is the appropriate compensation for the lender and the levered owner? The split of net present value streams into loans (debt) and levered equity lies at the heart of finance.

The example of this section: A building in Tornado Alley can have one of two possible future values.

We will illustrate this split through the hypothetical purchase of a building for which the future value is uncertain. This building is peculiar, though: it has a 20% chance that it will be destroyed, say by a tornado, by next year. In this case, its value will only be the land—say, $20,000. Otherwise, with 80% probability, the building will be worth $100,000. Naturally, the $100,000 market value next year would itself be the result of many factors—it could include any products that have been produced inside the building, real-estate value appreciation, as well as a capitalized value that takes into account that a tornado might strike in subsequent years.

Table 5.3. Building Payoff Table

Event	Probability	Value
Tornado	20%	\$20,000
Sunshine	80%	\$100,000
Expected Future Value		\$84,000

The Expected Building Value

To obtain the expected future cash value of the building, multiply each (possible) outcome by its probability.

Table 5.3 shows the payoff table for full building ownership. The expected future building value of \$84,000 was computed as

$$\begin{aligned} \mathcal{E}(\text{Value}_{t=1}) &= 20\% \cdot \$20,000 \\ &+ 80\% \cdot \$100,000 = \$84,000 \\ &= \mathcal{P}rob(\text{Tornado}) \cdot (\text{Value if Tornado}_{t=1}) \\ &+ \mathcal{P}rob(\text{Sunshine}) \cdot (\text{Value if Sunshine}_{t=1}) \quad . \end{aligned} \tag{5.21}$$

Then discount back the expected cash value using the appropriate cost of capital.

Now, assume that the appropriate expected rate of return for a project of type "building" with this type of riskiness and with one-year maturity is 10%. (This 10% discount rate is provided by demand and supply in the financial markets and known.) Your goal is to determine the present value—the appropriate correct price—for the building *today*.

Under uncertainty, use NPV on expected (rather than actual, known) cash flows, and use the appropriate expected (rather than actual, known) rates of return. The NPV principles remain untouched.

There are two methods to arrive at the present value of the building—and they are almost identical to what we have done earlier. We only need to replace the known value with the expected value, and the known future rate of return with an expected rate of return. Now, the first PV method is to compute the expected value of the building next period and to discount it at the cost of capital, here 10 percent,

$$\begin{aligned} \text{PV}_{t=0} &= \frac{\$84,000}{1+10\%} \approx \$76,363.64 \\ &= \frac{\mathcal{E}(\text{Value}_{t=1})}{1+\mathcal{E}(r_{t=0,1})} \quad . \end{aligned} \tag{5.22}$$

Table 5.4. Building Payoff Table, Augmented

Event	Probability	Value	Discount Factor	⇒	PV
Tornado	20%	\$20,000	$1/(1+10\%)$	⇒	\$18,181.82
Sunshine	80%	\$100,000	$1/(1+10\%)$	⇒	\$90,909.09

The second method is to compute the discounted state-contingent value of the building, and then take expected values. To do this, augment Table 5.3. Table 5.4 shows that *if* the tornado strikes, the present value is \$18,181.82. *If* the sun shines, the present value is \$90,909.10. Thus, the expected value of the building can also be computed

Taking expectations and discounting can be done in any order.

$$\begin{aligned} \text{PV}_{t=0} &= 20\% \cdot \$18,181.82 \\ &\quad + 80\% \cdot \$90,909.09 \approx \$76,363.64 \\ &= \mathcal{P}rob(\text{Tornado}) \cdot (\text{PV of Building if Tornado}) \\ &\quad + \mathcal{P}rob(\text{Sunshine}) \cdot (\text{PV of Building if Sunshine}) \ . \end{aligned} \tag{5.23}$$

Both methods lead to the same result: you can either first compute the expected value next year ($20\% \cdot \$20,000 + 80\% \cdot \$100,000 = \$84,000$), and then discount this expected value of \$84,000 to \$76,363.34; or you can first discount all possible future outcomes (\$20,000 to \$18,181.82; and \$100,000 to \$90,909.09), and then compute the expected value of the discounted values ($20\% \cdot \$18,181.82 + 80\% \cdot \$90,909.09 = \$76,363.34$.)

IMPORTANT: *Under uncertainty, in the NPV formula,*

- *known future cash flows are replaced by expected discounted cash flows, and*
- *known appropriate rates of return are replaced by appropriate expected rates of return.*

You can first do the discounting and then take expectations, or vice-versa.

The State-Dependent Rates of Return

What would the rates of return be in both states, and what would the overall expected rate of return be? If you have bought the building for \$76,363.64, and no tornado strikes, your actual rate of return (abbreviated $r_{t=0,1}$) will be

The state-contingent rates of return can also be probability-weighted to arrive at the average (expected) rate of return.

$$\text{if Sunshine:} \qquad r_{t=0,1} = \frac{\$100,000 - \$76,363.64}{\$76,363.64} \approx +30.95\% \ . \tag{5.24}$$

If the tornado does strike, your rate of return will be

$$\text{if Tornado:} \qquad r_{t=0,1} = \frac{\$20,000 - \$76,363.64}{\$76,363.64} \approx -73.81\% \ . \tag{5.25}$$

Therefore, your expected rate of return is

$$\begin{aligned} \mathcal{E}(\tilde{r}_{t=0,1}) &= 20\% \cdot (-73.81\%) \\ &\quad + 80\% \cdot (+30.95\%) = 10.00\% \\ \mathcal{E}(\tilde{r}_{t=0,1}) &= \mathcal{P}rob(\text{Tornado}) \cdot (r_{t=0,1} \text{ if Tornado}) \\ &\quad + \mathcal{P}rob(\text{Sunshine}) \cdot (r_{t=0,1} \text{ if Sunshine}) \ . \end{aligned} \tag{5.26}$$

The probability state-weighted rates of return add up to the expected overall rate of return. This is as it should be: after all, we derived the proper price of the building today using a 10% expected rate of return.

Solve Now!

Q 5.8 *What changes have to be made to the NPV formula to handle an uncertain future?*

Q 5.9 *Under risk-neutrality, a factory can be worth \$500,000 or \$1,000,000 in two years, depending on product demand, each with equal probability. The appropriate cost of capital is 6% per year. What is the present value of the factory?*

Q 5.10 *A new product may be a dud (20% probability), an average seller (70% probability) or dynamite (10% probability). If it is a dud, the payoff will be \$20,000; if it is an average seller, the payoff will be \$40,000; if it is dynamite, the payoff will be \$80,000. What is the expected payoff of the project?*

Q 5.11 *(Continued.) The appropriate expected rate of return for such payoffs is 8%. What is the PV of the payoff?*

Q 5.12 *(Continued.) If the project is purchased for the appropriate present value, what will be the rates of return in each of the three outcomes?*

Q 5.13 *(Continued.) Confirm the expected rate of return when computed from the individual outcome specific rates of return.*

5·3.B. Splitting Project Payoffs into Debt and Equity

State-contingent claims have payoffs that depend on future states of nature.

We now know how to compute the NPV of state-contingent payoffs—our building paid off differently in the two states of nature. Thus, our building was a state-contingent claim—its payoff depended on the outcome. But it is just one of many. Another state-contingent claim might promise to pay \$1 if the sun shines and \$25 if a tornado strikes. Using payoff tables, we can work out the value of *any* state-contingent claims—and in particular the value of the two most important state-contingent claims, debt and equity.

The Loan

Assume the building is funded by a mortgagor and a residual, levered building owner.

We now assume you want to finance the building purchase of \$76,363.64 with a mortgage of \$25,000. In effect, the single project "building" is being turned into two different projects, each of which can be owned by a different party. The first project is the project "Mortgage Lending." The second project is the project "Residual Building Ownership," i.e., ownership of the building but bundled with the obligation to repay the mortgage. This "Residual Building Ownership" investor will not receive a dime until *after* the debt has been satisfied. Such residual ownership is called the **levered equity**, or just the **equity**, or even the **stock** in the building, in order to avoid calling it "what's-left-over-after-the-loans-have-been-paid-off."

The first goal is to determine the appropriate promised interest rate on a "\$25,000 value today" mortgage loan on the building.

What sort of interest rate would the creditor demand? To answer this question, we need to know what will happen if the building were to be condemned, because the mortgage value (\$25,000 today) will be larger than the value of the building if the tornado strikes (\$20,000 next year). We are assuming that the owner could walk away from it and the creditor could repossess the building, but not any of the borrower's other assets. Such a mortgage loan is called a **no-recourse loan.** There is no recourse other than taking possession of the asset itself. This arrangement is called **limited liability**. The building owner cannot lose more than the money that he originally puts in. Limited liability is a mainstay of many financial securities: for example, if you purchase stock in a company in the stock market, you cannot be held liable for more than your investment, regardless of how badly the company performs.

Table 5.5. Payoff to Mortgage Creditor, Providing $25,000 Today

Event	*Prob*	Value	Discount Factor
Tornado	20%	$20,000	$1/(1+10\%)$
Sunshine	80%	Promised	$1/(1+10\%)$

Start with the Payoff Table, and write down payoffs to project "Mortgage Lending."

To compute the PV for the project "Mortgage Lending," we return to the problem of setting an appropriate interest rate, given credit risk (from Section 5·2). Start with the payoff table in Table 5.5. The creditor receives the property worth $20,000 if the tornado strikes, or the full promised amount (to be determined) if the sun shines. To break even, the creditor must solve for the payoff to be received if the sun will shine in exchange for lending $25,000 today. This is the "quoted" or "promised" payoff.

$$\begin{aligned}
\$25,000 &= 20\% \cdot \left(\frac{\$20,000}{1+10\%}\right) + 80\% \cdot \left(\frac{\text{Promise}}{1+10\%}\right) \\
\text{Loan Value}_{t=0} &= \mathcal{P}rob(\text{Tornado}) \cdot (\text{Loan PV if Tornado}) + \mathcal{P}rob(\text{Sunshine}) \cdot (\text{Loan PV if Sunshine})\ .
\end{aligned} \tag{5.27}$$

Solving, the solution is a promise of

$$\begin{aligned}
\text{Promise} &= \frac{(1+10\%)\cdot \$25,000 \;-\; 20\% \cdot \$20,000}{80\%} = \$29,375 \\
&= \frac{[1+\mathcal{E}(r)]\cdot \text{Loan Value} \;-\; \mathcal{P}rob(\text{Tornado}) \cdot \text{Value if Tornado}}{\mathcal{P}rob(\text{Sunshine})}
\end{aligned} \tag{5.28}$$

in repayment, paid by the borrower only if the sun will shine.

ANECDOTE:
The framers of the United States Constitution had the English bankruptcy system in mind when they included the power to enact "uniform laws on the subject of bankruptcies" in the Article I Powers of the legislative branch. The first United States bankruptcy law, passed in 1800, virtually copied the existing English law. United States bankruptcy laws thus have their conceptual origins in English bankruptcy law prior to 1800. On both sides of the Atlantic, however, much has changed since then.

Early English law had a distinctly pro-creditor orientation, and was noteworthy for its harsh treatment of defaulting debtors. Imprisonment for debt was the order of the day, from the time of the Statute of Merchants in 1285, until Dickens' time in the mid-nineteenth century. The common law writs of capias authorized "body execution," i.e., seizure of the body of the debtor, to be held until payment of the debt.

English law was not unique in its lack of solicitude for debtors. History's annals are replete with tales of draconian treatment of debtors. Punishments inflicted upon debtors included forfeiture of all property, relinquishment of the consortium of a spouse, imprisonment, and death. In Rome, creditors were apparently authorized to carve up the body of the debtor, although scholars debate the extent to which the letter of that law was actually enforced.

Direct Source: Charles Jordan Tabb, 1995, "The History of the Bankruptcy laws in the United States." www.bankruptcyfinder.com/historyofbkinusa.html. (The original article contains many more juicy historical tidbits.)

The state-contingent rates of return in the tornado ("default") state and in the sunshine state can be probability weighted to arrive at the expected rate of return.

With this promised payoff of \$29,375 (if the sun will shine), the lender's rate of return will be the **promised rate of return**:

$$\text{if Sunshine:} \qquad r_{t=0,1} = \frac{\$29,375 - \$25,000}{\$25,000} = +17.50\% \quad , \tag{5.29}$$

The lender would not provide the mortgage at any lower promised interest rate. If the tornado strikes, the owner walks away, and the lender's rate of return will be

$$\text{if Tornado:} \qquad r_{t=0,1} = \frac{\$20,000 - \$25,000}{\$25,000} = -20.00\% \quad . \tag{5.30}$$

Therefore, the lender's expected rate of return is

$$\begin{aligned} \mathcal{E}(\tilde{r}_{t=0,1}) &= 20\% \cdot (-20.00\%) \\ &+ 80\% \cdot (+17.50\%) = 10.00\% \\ \mathcal{E}(\tilde{r}_{t=0,1}) &= \mathcal{P}rob(\text{Tornado}) \cdot (r_{t=0,1} \text{ if Tornado}) \\ &+ \mathcal{P}rob(\text{Sunshine}) \cdot (r_{t=0,1} \text{ if Sunshine}) \quad . \end{aligned} \tag{5.31}$$

After all, in a risk-neutral environment, anyone investing for one year expects to earn an expected rate of return of 10%.

The Levered Equity

Now compute the payoffs of the 60% post-mortgage (i.e., levered) ownership of the building. The method is exactly the same.

Our interest now turns towards proper compensation for you—the expected payoffs and expected rate of return for you, the residual building owner. We already know the building is worth \$76,363.64 today. We also already know how the lender must be compensated: to contribute \$25,000 to the building price today, you must promise to pay the lender \$29,375 next year. Thus, as residual building owner, you need to pay \$51,363.64—presumably from personal savings. If the tornado strikes, these savings will be lost and the lender will repossess the building. However, if the sun shines, the building will be worth \$100,000 minus the promised \$29,375, or \$70,625. The owner's payoff table in Table 5.6 allows you to determine that the *expected* future levered building ownership payoff is $20\% \cdot \$0 + 80\% \cdot \$70,625 = \$56,500$. Therefore, the present value of levered building ownership is

$$\begin{aligned} \text{PV}_{t=0} &= 20\% \cdot \left(\frac{\$0}{1+10\%}\right) + 80\% \cdot \left(\frac{\$70,625}{1+10\%}\right) \\ &= \mathcal{P}rob(\text{Tornado}) \cdot (\text{PV if Tornado}) + \mathcal{P}rob(\text{Sunshine}) \cdot (\text{PV if Sunshine}) \\ &= \$51,363.64 \quad . \end{aligned} \tag{5.32}$$

Table 5.6. Payoff To Levered Building (Equity) Owner

Event	$\mathcal{P}rob$	Value	Discount Factor
Tornado	20%	\$0.00	$1/(1+10\%)$
Sunshine	80%	\$70,624.80	$1/(1+10\%)$

Again, knowing the state-contingent cash flows permits computing state-contingent rates of return and the expected rate of return.

If the sun shines, the rate of return will be

$$\text{if Sunshine:} \qquad r_{t=0,1} = \frac{\$70,624.80 - \$51,363.63}{\$51,363.63} = +37.50\% \quad . \tag{5.33}$$

If the tornado strikes, the rate of return will be

$$\text{if Tornado:} \qquad r_{t=0,1} = \frac{\$0 - \$51,363.63}{\$51,363.63} = -100.00\% \quad . \tag{5.34}$$

The expected rate of return of levered equity ownership, i.e., the building with the bundled mortgage obligation, is

$$\begin{aligned} \mathcal{E}(\tilde{r}_{t=0,1}) &= 20\% \cdot (-100.00\%) \\ &+ 80\% \cdot (+37.50\%) = 10.00\% \\ \mathcal{E}(\tilde{r}_{t=0,1}) &= \mathcal{P}rob(\text{Tornado}) \cdot (r_{t=0,1} \text{ if Tornado}) \\ &+ \mathcal{P}rob(\text{Sunshine}) \cdot (r_{t=0,1} \text{ if Sunshine}) \quad . \end{aligned} \tag{5.35}$$

Reflections On The Example: Payoff Tables

Payoff tables are fundamental tools to think about projects and financial claims: you definitely need to start thinking in terms of payoff tables.

IMPORTANT: *Whenever possible, in the presence of uncertainty, write down a payoff table to describe the probabilities of each possible event ("state") with its state-contingent payoffs.*

Admittedly, this can sometimes be tedious, especially if there are many different possible states (or even infinitely many states, as in a bell-shaped normally distributed project outcome—but you can usually approximate even the most continuous and complex outcomes fairly well with no more than ten discrete possible outcomes), but they always work!

Table 5.7. Payoff Table and Overall Values and Returns

Event	$\mathcal{P}rob$	Building Value	Mortgage Value	Levered Ownership
Tornado	20%	$20,000	$20,000	$0
Sunshine	80%	$100,000	$29,375	$70,625
Expected Value$_{t=1}$		$84,000	$27,500	$56,500
	PV$_{t=0}$	$76,364	$25,000	$51,364
	$\mathcal{E}(r_{t=0,1})$	10%	10%	10%

There are three possible investment opportunities here. The bank is just another investor, with particular payoff patterns.

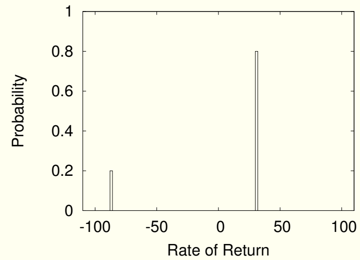
Table 5.7 shows how elegant such a table can be. It can describe everything we need in a very concise manner: the state-contingent payoffs, expected payoffs, net present value, and expected rates of return for your building scenario. Because owning the mortgage and the levered equity is the same as owning the full building, the last two columns must add up to the values in the "building value" column. You could decide to be any kind of investor: a creditor (bank) who is loaning money in exchange for promised payment; a levered building owner who is taking a "piece left over after a loan"; or an unlevered building owner who is investing money into an unlevered project. You might take the whole piece (that is, 100% of the claim—all three investments are just claims) or you might just invest, say $5, at the appropriate fair rates of return that are due to investors in our perfect world, where everything can be purchased at or sold at a fair price. (Further remaining funds can be raised elsewhere.)

Reflections On The Example and Debt and Equity Risk

We have not covered risk yet, because we did not need to. In a risk-neutral world, all that matters is the expected rate of return, not how uncertain you are about what you will receive. Of course, we can assess risk even in a risk-neutral world, even if risk were to earn no extra compensation (a risk premium).

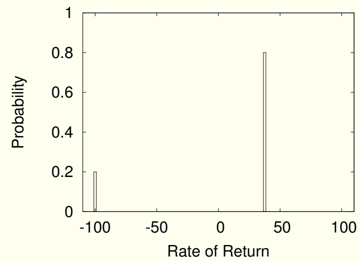
Figure 5.2. Probability Histograms of Project Returns

Histogram of Rate of Return for Project of Type "Full Ownership"



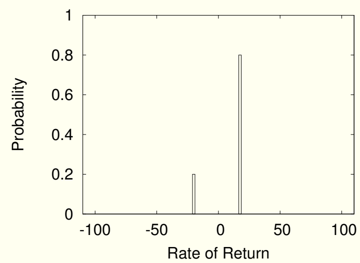
Histogram of Rate of Return for Project of Type "Levered Ownership"

(Most Risky)



Histogram of Rate of Return for Project of Type "Loan Ownership"

(Least Risky)



	20% probability Tornado	80% probability Sunshine	Expected Average	Variability
Loan (Ownership)	$\frac{\$20,000}{\$25,000} - 1 =$	$\frac{\$29,375}{\$25,000} - 1 =$	$\frac{\$27,500}{\$25,000} - 1 =$	
	-20.00%	$+17.50\%$	10.00%	$\approx \pm 20\%$
Levered Ownership	$\frac{\$0}{\$51,364} - 1 =$	$\frac{\$70,625}{\$51,364} - 1 =$	$\frac{\$56,500}{\$51,364} - 1 =$	
	-100.00%	$+37.50\%$	10.00%	$\approx \pm 60\%$
Full Ownership	$\frac{\$20,000}{\$76,364} - 1 =$	$\frac{\$100,000}{\$76,364} - 1 =$	$\frac{\$84,000}{\$76,364} - 1 =$	
	-73.81%	$+30.95\%$	10.00%	$\approx \pm 46\%$

Consider the variability number here to be just an intuitive measure—it will be explained as the "standard deviation" in Part III. For the eager beaver, it is computed as

$$\begin{aligned}
\text{Loan Ownership} \quad Sdv &= \sqrt{20\% \cdot (-20.00\% - 10\%)^2 + 80\% \cdot (17.50\% - 10\%)^2} = 19.94\% \\
\text{Levered Ownership} \quad Sdv &= \sqrt{20\% \cdot (-100.00\% - 10\%)^2 + 80\% \cdot (37.50\% - 10\%)^2} = 59.04\% \\
\text{Full Ownership} \quad Sdv &= \sqrt{20\% \cdot (-73.81\% - 10\%)^2 + 80\% \cdot (30.95\% - 10\%)^2} = 46.07\% \\
Sdv &= \sqrt{\mathcal{P}rob_T \cdot [V_T - \mathcal{E}(V)]^2 + \mathcal{P}rob_S \cdot [V_S - \mathcal{E}(V)]^2} \quad .
\end{aligned} \tag{5.36}$$

Leveraging (mortgaging) a project splits it into a safer loan and a riskier levered ownership, although everyone expects to receive 10% on average.

So, which investment is most risky: full ownership, loan ownership, or levered ownership? Figure 5.2 plots the histograms of the rates of return to each investment type. As the visual shows, the loan is least risky, followed by the full ownership, followed by the levered ownership. Your intuition should tell you that, by taking the mortgage, the medium-risky project "building" has been split into a more risky project "levered building" and a less risky project "mortgage." The combined "full building ownership" project therefore has an average risk.

If everyone is risk-neutral, everyone should expect to earn 10%.

It should not come as a surprise to learn that all investment projects expect to earn a 10% rate of return. After all, 10% is the time-premium for investing money. Recall from Page 88 that the expected rate of return (the cost of capital) consists only of a time-premium and a risk premium. (The **default premium** is a component only of **promised interest rates**, not of **expected interest rates**; see Section 5·2). By assuming that investors are risk-neutral, we have assumed that the risk premium is zero. Investors are willing to take any investment that offers an expected rate of return of 10%, regardless of risk.

Unrealistic, maybe! But ultimately, maybe not.

Although our example has been a little sterile, because we assumed away risk preferences, it is nevertheless very useful. Almost all projects in the real world are financed with loans extended by one party and levered ownership held by another party. Understanding debt and equity is as important to corporations as it is to building owners. After all, stocks in corporations are basically levered ownership claims that provide money only *after* the corporation has paid back its loans. The building example has given you the skills to compute state-contingent, promised, and expected payoffs, and state-contingent, promised, and expected rates of returns—the necessary tools to work with debt, equity, or any other state-contingent claim. And really, all that will happen later when we introduce risk aversion is that we will add a couple of extra basis points of required compensation—more to equity (the riskiest claim) than to the project (the medium-risk claim) than to debt (the safest claim).

Solve Now!

Q 5.14 *In the example, the building was worth $76,364, the mortgage was worth $25,000, and the equity was worth $51,364. The mortgage thus financed 32.7% of the cost of the building, and the equity financed $67.3%. Is the arrangement identical to one in which two partners purchase the building together—one puts in $25,000 and owns 32.7%, and the other puts in $51,364 and owns 67.3%?*

Q 5.15 *Buildings are frequently financed with a mortgage that pays 80% of the price, not just 32.7% ($25,000 of $76,364). Produce a table similar to Table 5.7 in this case.*

Q 5.16 *Repeat the example if the loan does not provide $25,000, but promises to pay off $25,000. How much money do you get for this promise? What is the promised rate of return. How does the riskiness of the project "full building ownership" compare to the riskiness of the project "levered building ownership"?*

Q 5.17 *Repeat the example if the loan promises to pay off $20,000. Such a loan is risk-free. How does the riskiness of the project "full building ownership" compare to the riskiness of the project "levered building ownership"?*

Q 5.18 *Under risk-neutrality, a factory can be worth $500,000 or $1,000,000 in two years, depending on product demand, each with equal probability. The appropriate cost of capital is 6% per year. The factory can be financed with proceeds of $500,000 from loans today. What are the promised and expected cash flows and rates of return for the factory (without loan), for the loan, and for a hypothetical factory owner who has to first repay the loan?*

Q 5.19 *Advanced: For illustration, we assumed that the sample building was not lived in. It consisted purely of capital amounts. But, in the real world, part of the return earned by a* *building owner is rent. Now assume that rent of $11,000 is paid strictly at year-end, and that both the state of nature (tornado or sun) and the mortgage loan payment happens only after the rent has been safely collected. The new building has a resale value of $120,000 if the sun shines, and a resale value of $20,000 if the tornado strikes.*

(a) *What is the value of the building today?*

(b) *What is the promised interest rate for a lender providing $25,000 in capital today?*

(c) *What is the value of residual ownership today?*

(d) *Conceptual Question: What is the value of the building if the owner chooses to live in the building?*

More Than Two Possible Outcomes

Multiple outcomes will cause multiple breakpoints.

How does this example generalize to multiple possible outcomes? For example, assume that the building could be worth $20,000, $40,000, $60,000, $80,000, or $100,000 with equal probability, and the appropriate expected interest rate were 10%—so the building has an PV of $\$60,000/(1+10\%) \approx \$54,545.45$. If a loan promised $20,000, how much would you expect to receive? But, of course, $20,000!

$$\begin{aligned} \mathcal{E}\left(\text{ Payoff(Loan Promise =\$20,000) }\right) &= \$20,000 \quad . \\ \mathcal{E}\left(\begin{array}{c}\text{Payoff of Loan} \\ \text{if } \$0 \le \text{Loan} \le \$20,000\end{array}\right) &= \text{Loan} \quad . \end{aligned} \tag{5.37}$$

If a loan promised $20,001, how much would you expect to receive? $20,000 for sure, plus the extra "marginal" $1 with 80% probability. In fact, you would expect only 80 cents for each dollar promised between $20,000 and $40,000. So, if a loan promised $40,000, you would expect to receive

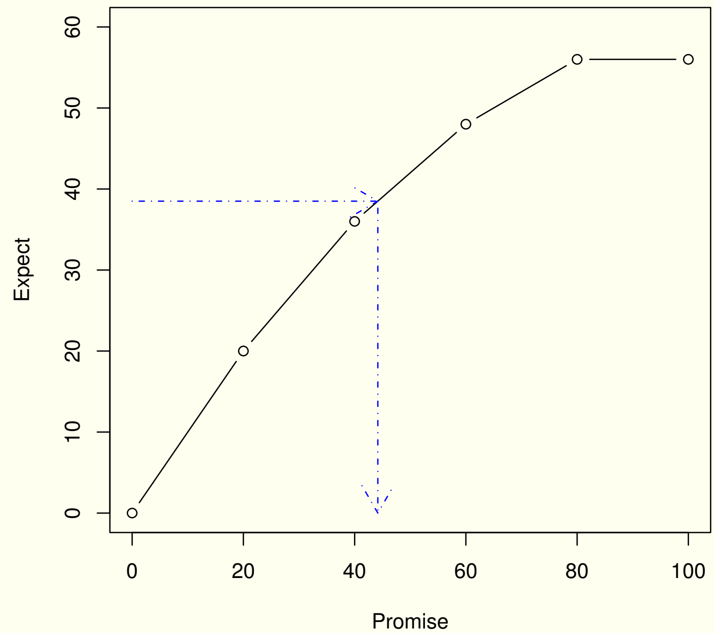
$$\begin{aligned} \mathcal{E}\left(\text{ Payoff(Loan Promise = \$40,000) }\right) &= \$20,000 + 80\% \cdot (\$40,000 - \$20,000) \\ &= \$36,000 \\ \mathcal{E}\left(\begin{array}{c}\text{Payoff of Loan} \\ \text{if } \$20,000 \le \text{Loan} \le \$40,000\end{array}\right) &= \$20,000 + 80\% \cdot (\text{Loan} - \$20,000) \quad . \end{aligned} \tag{5.38}$$

If a loan promised you $40,001, how much would you expect to receive? You would get $20,000 for sure, plus another $20,000 with 80% probability (which is an expected $16,000), plus the marginal $1 with only 60% probability. Thus,

$$\begin{aligned} \mathcal{E}\left(\text{Payoff(Loan Promise = \$40,001)}\right) &= \$20,000 + 80\% \cdot (\$40,000 - \$20,000) \\ &\qquad + 60\% \cdot \$1 \\ &= \$36,000.60 \\ \mathcal{E}\left(\begin{array}{c}\text{Payoff of Loan} \\ \text{if } \$40,000 \le \text{Loan} \le \$60,000\end{array}\right) &= \$20,000 + 80\% \cdot \$20,000 \\ &\qquad + 60\% \cdot (\text{Loan} - \$40,000) \quad . \end{aligned} \tag{5.39}$$

And so on. Figure 5.3 plots these expected payoffs as a function of the promised payoffs. With this figure, mortgage valuation becomes easy. For example, how much would the loan have to promise to provide $35,000 today? The expected payoff would have to be $(1+10\%) \cdot \$35,000 = \$38,500$. Figure 5.3 shows that an expected payoff of $38,500 corresponds to around $44,000 in promise. (The exact number can be worked out to be $44,167.) Of course, we cannot borrow more than $54,545.45, the project's PV. So, we can forget about the idea of obtaining a $55,000 mortgage.

Figure 5.3. Promised vs. Expected Payoffs



Solve Now!

Q 5.20 *What is the expected payoff if the promised payoff is $45,000?*

Q 5.21 *What is the promised payoff if the expected payoff is $45,000?*

Q 5.22 *Assume that the probabilities are not equal: $20,000 with probability 12.5%, $40,000 with probability 37.5%, $60,000 with probability 37.5%, and $80,000 with probability 12.5%.*

(a) Draw a graph equivalent to Figure 5.3.

(b) If the promised payoff of a loan is $50,000, what is the expected payoff?

(c) If the prevailing interest rate is 5% before loan payoff, then how much repayment does a loan providing $25,000 today have to promise? What is the interest rate?

You do not need to calculate these values, if you can read them off your graph.

Q 5.23 *A new product may be a dud (20% probability), an average seller (70% probability) or dynamite (10% probability). If it is a dud, the payoff will be $20,000; if it is an average seller, the payoff will be $40,000; if it is dynamite, the payoff will be $80,000. The appropriate expected rate of return is 6% per year. If a loan promises to pay off $40,000, what are the promised and expected rates of return?*

Q 5.24 *Advanced: What is the formula equivalent to (5.39) for promised payoffs between $60,000 and $80,000?*

Q 5.25 *Advanced: Can you work out the exact $44,167 promise for the $35,000 (today!) loan?*

5·4. Robustness: How Bad are Your Mistakes?

How bad are mistakes?

Although it would be better to get everything perfect, it is often impossible to come up with perfect cash flow forecasts and appropriate interest rate estimates. Everyone makes errors. So, how bad are mistakes? How robust is the NPV formula? Is it worse to commit an error in estimating cash flows or in estimating the cost of capital? To answer these questions, we will do a simple form of **scenario analysis**—we will consider just a very simple project, and see how changes in estimates matter to the ultimate value. Doing good scenario analysis is also good practice for any managers—so that they can see how sensitive their estimated value is to reasonable alternative possible outcomes. Therefore this method is also called a **sensitivity analysis.** Doing such analysis becomes even more important when we consider "real options" in our next chapter.

5·4.A. Short-Term Projects

The benchmark case: A short-term project, correctly valued.

Assume that your project will pay off $200 next year, and the proper interest rate for such projects is 8%. Thus, the correct project present value is

$$\mathsf{PV}_{\text{correct}} = \frac{\$200}{1+8\%} \approx \$185.19 \quad . \tag{5.43}$$

Committing an error in cash flow estimation.

If you make a 10% error in your cash flow, e.g., mistakenly believing it to return $220, you will compute the present value to be

$$\mathsf{PV}_{\text{CF error}} = \frac{\$220}{1+8\%} \approx \$203.70 \quad . \tag{5.44}$$

The difference between $203.70 and $185.19 is a 10% error in your present value.

Committing an error in interest rate estimation.

In contrast, if you make a 10% error in your cost of capital (interest rate), mistakenly believing it to require a cost of capital (expected interest rate) of 8.8% rather than 8%, you will compute the present value to be

$$\mathsf{PV}_{r\text{ error}} = \frac{\$200}{1+8.8\%} \approx \$183.82 \quad . \tag{5.45}$$

The difference between $183.82 and $185.19 is less than a $2 or 1% error.

Important: *For short-term projects, errors in estimating correct interest rates are less problematic in computing NPV than are errors in estimating future cash flows.*

5·4.B. Long-Term Projects

A long-term project, correctly valued and incorrectly valued.

Now take the same example, but assume the cash flow will occur in 30 years. The correct present value is now

$$\text{PV}_{\text{correct}} = \frac{\$200}{(1+8\%)^{30}} \approx \$19.875 \quad . \tag{5.46}$$

The 10% "cash flow error" present value is

$$\text{PV}_{\text{CF error}} = \frac{\$220}{(1+8\%)^{30}} \approx \$21.86 \quad , \tag{5.47}$$

and the 10% "interest rate error" present value is

$$\text{PV}_{r\ \text{error}} = \frac{\$220}{(1+8.8\%)^{30}} \approx \$17.52 \quad . \tag{5.48}$$

Both cash flow and cost of capital errors are now important.

This calculation shows that cash flow estimation errors and interest rate estimation errors are now about equally important. So, for longer-term projects, estimating the correct interest rate becomes relatively more important. However, in fairness, estimating cash flows (and maybe even measuring appropriate interest rates) thirty years into the future is just about as difficult as reading a crystal ball. Of course, as difficult as it may be, we have no alternative. We must simply try to do our best at forecasting.

> **IMPORTANT**: *For long-term projects, errors in estimating correct interest rates and errors in estimating future cash flows are both problematic in computing NPV.*

Solve Now!

Q 5.26 *What is the relative importance of cash flow and interest rate errors for a 10-year project?*

Q 5.27 *What is the relative importance of cash flow and interest rate errors for a 100-year project?*

5·5. Summary

The chapter covered the following major points:

- The possibility of future default causes promised interest rates to be higher than expected interest rates. Default risk is also often called credit risk.
- Quoted interest rates are almost always promised interest rates, and are higher than expected interest rates.
- Most of the difference between promised and expected interest rates is due to default. Extra compensation for bearing more risk—the risk premium—is typically much smaller than the default premium.
- The key tool for thinking about uncertainty is the payoff table. Each row represents one possible state outcome, which contains the probability that the state will come about, the total project value that can be distributed, and the allocation of this total project value to different state-contingent claims. The state-contingent claims "carve up" the possible project payoffs.
- Most real-world projects are financed with the two most common state-contingent claims—debt and equity. The conceptual basis of debt and equity is firmly grounded in payoff tables. Debt financing is the safer investment. Equity financing is the riskier investment.
- If debt promises to pay more than the project can deliver in the worst state of nature, then the debt is risky and requires a promised interest rate in excess of its expected interest rate.
- NPV is robust to uncertainty about the expected interest rate (the discount rate) for short-term projects. However, NPV is not robust with respect to either expected cash flows or discount rates for long-run projects.

Solutions and Exercises

1. No! It is presumed to be known—at least for a die throw. The following is almost philosophy and beyond what you are supposed to know or answer here: It might, however, be that the expected value of an investment is not really known. In this case, it, too, could be a random variable in one sense—although you are presumed to be able to form an expectation (opinion) over anything, so in this sense, it would not be a random variable, either.
2. Yes and no. If you do not know the exact bet, you may not know the expected value.
3. *If* the random variable is the number of dots on the die, then the expected outcome is 3.5. The realization was 6.
4. The expected value of the stock is \$52. Therefore, purchasing the stock at \$50 is not a fair bet, but a good bet.

5. Only for government bonds. Most other bonds have some kind of default risk.
6.
   (a) The expected payoff is now 95%·\$210 + 1%·\$100 + 4%·\$0 = \$200.50. Therefore, the expected rate of return is \$200.50/\$200 = 0.25%.
   (b) You require an expected payoff of \$210. Therefore, you must solve for a promised payment $95\%\cdot P + 1\%\cdot\$100 + 4\%\cdot\$0 = \$210 \rightarrow P = \$209/0.95 = \$220$. On a loan of \$200, this is a 10% promised interest rate.
7. The expected payoff is \$203.50, the promised payoff is \$210, and the stated price is \$210/(1+12%)=\$187.50. The expected rate of return is \$203.50/\$187.50 = 8.5%. Given that the time premium, the Treasury rate is 5%, the risk premium is 3.5%. The remaining 12%-8.5%=3.5% is the default premium.

8. The actual cash flow is replaced by the expected cash flow, and the actual rate of return is replaced by the expected rate of return.
9. $\$750,000/(1+6\%)^2 \approx \$667,497.33$.
10. $\mathcal{E}(P) = 20\% \cdot \$20,000 + 70\% \cdot \$40,000 + 10\% \cdot \$80,000 = \$40,000$.
11. \$37,037.
12. $\$20,000/\$37,037 - 1 = -46\%$, $\$40,000/\$37,037 - 1 = +8\%$, $\$80,0000/\$37037 - 1 = +116\%$.
13. $20\%\cdot(-46\%) + 70\%\cdot(+8\%) + 10\%\cdot(+116\%) = 8\%$.
14. No! Partners would share payoffs proportionally, not according to “debt comes first.” For example, in the tornado state, the 32.7% partner would receive only \$6,547.50, not the entire \$20,000 that the debt owner receives.
15. The mortgage would finance \$61,090.91 today.

Event	*Prob*	Building Value	Mortgage Value	Levered Ownership
Tornado	20%	\$20,000	\$20,000	\$0
Sunshine	80%	\$100,000	\$79,000	\$21,000
Expected Value$_{t=1}$		\$84,000	\$67,200	\$16,800
	PV$_{t=0}$	\$76,364	\$61,091	\$15,273
	$\mathcal{E}(r_{t=0,1})$	10%	10%	10%

16. In the tornado state, the creditor gets all (\$20,000). In the sunshine state, the creditor receives the promise of \$25,000. Therefore, the creditor’s expected payoff is $20\% \cdot \$20,000 + 80\% \cdot \$25,000 = \$24,000$. To offer an expected rate of return of 10%, you can get $\$24,000/1.1 = \$21,818$ from the creditor today. The promised rate of return is therefore $\$25,000/\$21,818 - 1 = 14.58\%$.
17. The loan pays off \$20,000 for certain. The levered ownership pays either \$0 or \$80,000, and costs $\$64,000/(1+10\%) = \$58,182$. Therefore, the rate of return is either −100% or +37.5%. We have already worked out full ownership. It pays either \$20,000 or \$100,000, costs \$76,364, and offers either −73.81% or +30.95%. By inspection, the levered equity project is riskier. In effect, building ownership has become riskier, because the owner has chosen to sell off the risk-free component, and retain only the risky component.
18. **Factory:** The expected factory value is \$750,000. Its price would be $\$750,000/1.06^2 = \$667,497$. The promised rate of return is therefore $\$1,000,000/\$667,497 - 1 \approx 49.8\%$. **Loan:** The discounted (today’s) loan price is $\$750,000/1.06^2 = \$667,497.33$. The promised value is \$1,000,000. The loan must have an expected payoff of $1.06^2 \cdot \$500,000 = \$561,800$ (6% expected rate of return, two years). Because the loan can pay \$500,000 with probability 1/2, it must pay \$623,600 with probability 1/2 to reach \$561,800 as an average. Therefore, the promised loan rate of return is $\$623,600/\$500,000 - 1 = 24.72\%$ over two years (11.68% per annum). **Equity:** The levered equity must therefore pay for/be worth $\$667,497.33 - \$500,000.00 = \$167,497.33$ (alternatively, levered equity will receive $\$1,000,000 - \$623,600 = \$376,400$ with probability 1/2 and \$0 with probability 1/2), for an expected payoff of \$188,200. (The expected two-year holding rate of return is $\$188,200/\$167,497 - 1 = 12.36\%$ [6% per annum, expected].) The promised rate of return is $(\$1,000,000 - \$623,600)/\$167,497.33 - 1 = 124.72\%$ (50% promised per annum).

19.

(a) In the sun state, the value is \$120,000+\$11,000= \$131,000. In the tornado state, the value is \$11,000+\$20,000= \$31,000. Therefore, the expected building value is \$111,000. The discounted building value today is \$100,909.09.

(b) Still the same as in the text: the lender's \$25,000 loan can still only get \$20,000, so it is a promise for \$29,375. So the quoted interest rate is still 17.50%.

(c) $\$100,909.09 - \$25,000 = \$75,909.09$.

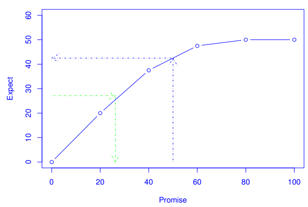
(d) Still \$100,909.09, assuming that the owner values living in the building as much as a tenant would.

Owner-consumed rent is the equivalent of corporate dividends paid out to levered equity. Note: you can repeat this example assuming that the rent is an annuity of \$1,000 each month, and tornadoes strike mid-year.

20. From the graph, it is around \$40,000. The correct value can be obtained by plugging into Formula (5.39): \$39,000.

21. From the graph, it is around \$55,000. The correct value can be obtained by setting Formula (5.39) equal to \$55,000 and solving for "Loan." The answer is indeed \$55,000.

22.



(a)

(b) The exact expected payoff is $1/8 \cdot \$20,000 + 3/8 \cdot \$40,000 + 1/2 \cdot \$50,000 = \$42,500$. The $1/2$ is the probability that you will receive the \$50,000 that you have been promised, which occurs if the project ends up worth at least as much as your promised \$50,000. This means that it is the total probability that it will be worth \$60,000 or \$80,000.

(c) The loan must expect to pay off $(1+5\%) \cdot \$25,000 = \$26,250$. Therefore, solve $1/8 \cdot \$20,000 + 7/8 \cdot x = \$26,250$, so the exact promised payoff must be $x = \$27,142.90$.

23. With 20% probability, the loan will pay off \$20,000; with 80% probability, the loan will pay off the full promised \$40,000. Therefore, the loan's expected payoff is $20\% \cdot \$20,000 + 80\% \cdot \$40,000 = \$36,000$. The loan's price is $\$36,000/(1+6\%) = \$33,962$. Therefore, the promised rate of return is $\$40,000/\$33,962 - 1 \approx 17.8\%$. The expected rate of return was given: 6%.

24.

$$\mathcal{E}\left(\begin{matrix}\text{Payoff of Loan}\\ \text{if } \$60,000 \le \text{Loan} \le \$80,000\end{matrix}\right) = \begin{aligned} & \$20,000 \\ + \; & 80\% \cdot \$20,000 \\ + \; & 60\% \cdot \$20,000 \\ + \; & 40\% \cdot (\text{Loan} - \$60,000) \;\; . \end{aligned} \tag{5.49}$$

25. The loan must yield an expected value of \$38,500. Set formula (5.39) equal to \$38,500 and solve for "Loan." The answer is indeed \$44,166.67.

26. Consider a project that earns \$100 in 10 years, and where the correct interest rate is 10%.

- The correct PV is $\$100/(1+10\%)^{10} = \$38.55$.
- If the cash flow is incorrectly estimated to be 10% higher, the incorrect PV is $\$110/(1+10\%)^{10} = \$42.41$.
- If the interest rate is incorrectly estimate to be 10% lower, the incorrect PV is $\$100/(1+9\%)^{10} = \$42.24$.

So, the misvaluation effects are reasonably similar at 10% interest rates. Naturally, percent valuation mistakes in interest rates are higher if the interest rate is higher; and lower if the interest rate is lower.

27. Although this, too, depends on the interest rate, interest rate errors almost surely matter for any reasonable interest rates now.

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

A. Appendix: A Short Glossary of Some Bonds and Rates

This appendix briefly describes a plethora of different interest rates and bonds that you may encounter.

Other interest rate information sources.

In the real world, there are many different interest rates. Every borrower and every lender may pay a slightly different interest rate, depending on the bond's default risk, risk premium, liquidity, maturity, identity, convenience, etc. It is impossible to describe every common bond or rate. The C-section of the *Wall Street Journal* describes daily interest rates on many common and important interest instruments: the C-1 Page **Markets Diary**; the C-2 Page **Interest Rates and Bonds** section; some boxes on a later page describing the interest rates paid on individual government and government agency bonds (headlines **Treasury Bonds, Notes and Bills** and **Government Agency & Similar Issues**); the **Credit Markets** page thereafter, which includes the important **Yield Comparisons** and **Money Rates** boxes, as well as some Corporate Bonds; and what is often the final page, which contains the **Bond Market Data Bank**. In addition, futures on interest rates (similar to forward rates) are listed in the B-section.

Some real-world interest rates explained.

Here are short descriptions of some of the interest rates printed in the *Wall Street Journal* on a daily basis, as well as some other bond subclasses.

Agency Bonds Issued by quasi-governmental companies, such as FannieMae, FreddieMac, and SallieMae (all described below). These agencies were originally set up by the U.S. government to facilitate loans for a particular purpose, then bundle them, and sell them to the financial markets. These companies are huge. Sometimes they are thought to be implicitly backed by the U.S. government, though no explicit guarantees may exist.

APR (Annual Percentage Rate) A measure of interest due on a mortgage loan that accounts for upfront costs and payments. Unfortunately, there are no clear rules of how to compute APR, so the APR computation can vary across companies.

ARM Rate (Adjustable Rate Mortgage) A mortgage with an interest rate that is usually reset once per year according to a then prevailing interest rate, pre-specified by a formula, but subject to some upper limit (called a cap). Repayable by the borrower.

Bankers Acceptances Loans by banks to importers, used to pay the exporting firm. Backed by the issuing bank if the importer defaults. Usual maturities are 30 to 180 days.

Certificate of Deposit (CD) Rate paid by banks to bank retail customers willing to commit funds for a short-term or medium-term period. Unlike ordinary savings accounts, CDs are not insured by the government if the bank fails.

Callable Bonds Bonds that the issuer can redeem. We will discuss these in Chapter 23.

CMO (Collateralized Mortgage Obligation) A security backed by a pool of real estate mortgages, with specified claims to interest and principal payments. For example, there are **Interest Only** (**IO**) bonds and **Principal Only** (**PO**) bonds, which entitle bond holders to only the interest or principal that the pool of mortgages receives.

Collateralized Trust Bonds Often issued by corporations, these bonds pledge as collateral the securities owned by a subsidiary.

Commercial Paper Short-term bonds issued by corporations to the public markets. Often backed by bank guarantees. Because commercial paper is short-term and often backed by assets, it is usually very low risk.

Consumer Credit Rates The *Wall Street Journal* lists typical **credit-card rates** and **car loan rates**.

Convertible Bonds Bonds that the holder can convert into common equity—we will discuss these in Chapter 23.

Debenture Unsecured general obligation bond.

Discount Rate The interest rate that the Federal Reserve charges banks for short-term loans of reserves.

Equipment Obligations Unlike debentures, these corporate bonds usually pledge specific equipment as collateral.

Eurobond Bonds issued by the U.S. government outside the domain of the Securities Exchange Commission (e.g., in Europe) and purchased by foreign investors. Eurobonds need not be denominated in dollars.

Federal Funds Rate Banks must hold financial reserves at the Federal Reserve Bank. If they have more reserves than they legally need, they can lend them to other banks. The rate at which they lend to one another overnight is the federal funds rate.

FannieMae, originally the *Federal National Mortgage Association* (or **FNMA**), a corporation set up by the government to help facilitate mortgage lending. It holds mortgages as assets. FannieMae and FreddieMac together hold most U.S. mortgages, though they sell off claims against these mortgage bundles into the financial markets. The FNMA bonds are themselves collateralized (backed) by the mortgages, but, despite common perception, *not* by the U.S. government.

FreddieMac, originally the *Federal Home Loan Mortgage Corporation* (**FHLMC**). An agency similar to FNMA.

GICs (Guaranteed Investment Contracts) Usually issued by insurance companies and purchased by retirement plans. The interest rate is guaranteed, but the principal is not.

G.O. Bonds (General Obligation Bonds) Bonds whose repayment is not guaranteed by a specific revenue stream. See also **Revenue Bonds**.

High-Yield Bonds Sometimes also called **Junk Bonds**, high-yield bonds are bonds (usually of corporations) that have credit ratings of BB and lower. This will be discussed in the next chapter.

Home Equity Loan Rate The rate for loans secured by a home. Usually second mortgages, i.e., taken after another mortgage is already in place.

Investment Grade Bonds Bonds that have a rating higher than BBB. This is a common classification for corporate bonds, discussed in the next chapter.

Jumbo Mortgage Rate Like the N-year mortgage rate (see below), but for loans which exceed the FNMA limit on mortgage size.

LIBOR London Interbank Offer Rate Typical rate at which large London banks lend dollars to one another.

Money-Market Rate Rate paid to cash sitting in a brokerage account and not invested in other assets.

Mortgage Bonds Bonds secured by a particular real-estate property. In case of default, the creditor can foreclose the secured property. If still not satisfied, the remainder of the creditor's claim becomes a general obligation.

Municipal Bond Bonds issued by a municipality. Often tax-exempt.

N-year Mortgage Rate A fixed-rate loan, secured by a house, with standard coupon payments. The rate is that paid by the borrower. Usually limited to an amount determined by FNMA.

Prime Rate An interest rate charged by the average bank to their best customers for short-term loans. (This rate is used less and less. It is being replaced by the LIBOR rate, at least in most commercial usage.)

Repo Rate A Repo is a *repurchase* agreement, in which a seller of a bond agrees to repurchase the bond, usually within 30 to 90 days, but also sometimes overnight. (Repos for more than 30 days are called *term repos.*) This allows the bond holder to obtain actual cash to make additional purchases, while still being fully exposed to (speculate on) the bond.

Revenue Bond A bond secured by a specific revenue stream. See also G.O. bond.

SallieMae, originally **Student Loan Marketing Association** (SLMA). Like FannieMae, an agency (corporation) set up by the U.S. government. It facilitates student loans.

Savings Bonds Issued by the U.S. Treasury, Savings Bonds can only be purchased from or sold to agents authorized by the Treasury Department. They must be registered in the name of the holder. **Series E Bonds** are zero bonds; **Series H Bonds** are semi-annual coupon payers and often have a variable interest feature. In contrast to Savings Bonds, other bonds are typically **bearer bonds**, which do not record the name of the owner and are therefore easy to resell (or steal).

Tax-Exempt Bonds Typically bonds issued by municipalities. Their interest is usually exempt from some or all income taxes. The designation **G.O. Bond** means **General Obligation Bond**, i.e., a Bond that was not issued to finance a particular obligation. In contrast, a **Revenue Bond** is a Bond backed by specific municipal revenues—but it may or may not be tax-exempt.

Treasury Security See Section 2·1.C.

Treasury STRIPS , or *Separate Trading of Registered Interest and Principal of Securities.* Financial institutions can convert each coupon payment and principal payment of ordinary Treasury coupon bonds into individual zero bonds. We briefly described these in the previous chapter. See also www.publicdebt.treas.gov/of/ofstrips.htm. for a detailed explanation.

Yankee Bonds U.S. Dollar denominated and SEC-registered bonds by foreign issuers.

Prepayment. Note: mortgage (and many other) bonds can be paid off by the borrower before maturity. Repayment is common, especially if interest rates are dropping.

CHAPTER 6

Dealing With Imperfect Markets

Opinions, Market Depth, Transaction Costs, Taxes, and Inflation

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So far, we have assumed no differences in information (which we might call "opinions"), no transaction costs, no taxes, and a large market with many sellers and buyers. We now leave this frictionless utopian world—the "perfect market."

What these perfect markets assumptions have really done for us is to give us one unique appropriate expected rate of return—whether you want to borrow someone else's money to finance your projects or lend your money to someone else undertaking projects. As convenient as this may be, you know that it is not realistic. Thus, you must now learn how to think about finance in "imperfect markets."

You shall see that if markets are imperfect, it is difficult to attach a unique value to goods. Instead, the value depends on who the seller and who the buyer might be. However, all your tools (and specifically NPV) still work! You only need to apply them with a little more caution.

After we have covered generally what market imperfections are and how they make our lives more difficult, we then discuss each imperfection in greater detail.

6·1. Causes and Consequences of Imperfect Markets

So far, we have not drawn a distinction between the cost of capital at which you can borrow money to finance your projects, and the rate of return at which you can save money. In "perfect markets," these two rates were the same.

6·1.A. Perfect Market Assumptions

A perfect market is defined by the following assumptions:

No Differences in Information Everyone holds the same opinion. How can this assumption be violated? Here is one example. If your bank believes that there is a 50% chance that you will go bankrupt and default, and you believe that there is only a 0.1% chance, then your bank will lend you money only if you pay a much higher expected interest rate than it will pay you if you deposited your money with it.

This is why our perfect markets assumptions includes one that everyone has the same information and agrees on what it means. (It does not mean that there is no uncertainty, however. The important point in a perfect market is that everyone interprets the uncertainty identically.)

A Deep Market You can easily find a buyer or a seller. How could this assumption be violated? Say there is only one bank that you can do business with. This bank will exploit its monopoly power. It will charge you a higher interest rate if you want to borrow money from it than it will pay you if you want to deposit your money with it—and you will have no good alternative. (There is one nitpick qualification: if a project is worth more if it is owned or financed by a particular type—e.g., if a golf range is owned by a golf pro—then there must be a large number of this type of owner.)

This is why our perfect markets assumptions includes one that there are infinitely many buyers and sellers.

No Transaction Costs You can trade without paying any transaction costs. How can this assumption be violated? If it costs $1,000 to process the paperwork involved in a loan, you will incur this cost only if you need to borrow, but not if you want to save. This will make your effective borrowing interest rate higher than your effective savings interest rate.

This is why our perfect markets assumptions includes one that there are no transaction costs.

No Taxes There are no tax advantages or disadvantages to buying or selling securities. Specifically, there are no asymmetric tax treatments to the seller divesting or to the buyer purchasing. How can this be violated? If you have to pay taxes on interest earned, but cannot deduct taxes on interest paid, your de facto savings rate will be lower than your borrowing rate.

This is why our perfect markets assumptions includes one that there are no taxes.

We will soon tackle each of these issues in detail. However, the effect of violating any of these assumptions is really the same. Any violation that breaks the equality between the borrowing and the savings rate also breaks the link between value and one unique price (or cost). In fact, the value of a project may not even have meaning in imperfect markets—a project may not have one unique value, but any from among a range of possible values.

6·1.B. Value in Imperfect Markets

In a perfect market, the value of the project depends only on the project, and not on you personally or on your cash position. This is often called the "separation of investments and financing decisions." You could make investment choices based on the quality of the projects themselves, not based on how you would end up financing them. The NPV formula does not have an input for your identity or wealth—its only inputs are the project's cash flows and the rate of return on alternative investments.

If savings and investment interest rates differ, the project's value (NPV) depends on the owner's position

But if the borrowing and lending rates are not the same, then the value of the project depends on your cash holdings. For example, assume that you can lend money (invest cash) at 3%, and borrow money (receive cash) at 7%. What is the present value of a project that invests \$1,000 today and returns \$1,050 next period?

An example.

- If you have \$1,000 and your alternative is to invest your money in the bank, you will only get \$1,030 from the bank. You should take this project to earn \$20 more than you could earn from the bank.
- If you do not have the \$1,000, you will have to borrow \$1,000 from the bank to receive \$1,050 from the project. But because you will have to pay the bank \$1,070, you will lose \$20 net. You should not take the project.

The proper project decision now depends on how much cash you have. The separation between your project choice and your financial position has broken down. Taking your current cash holdings into account when making investment choices of course makes capital budgeting decisions more difficult. In this example, it is fairly easy—but think about projects that have cash inflows and outflows in the future, and how decisions could interact with your own wealth positions in the future. Equally important, in imperfect markets, the project value is no longer unique, either. In our example, it could be anything between +\$19.42 (\$1,050 discounted at 3%) and −\$18.69 (\$1,050 discounted at 7%).

Solve Now!

Q 6.1 *What are the perfect market assumptions?*

Q 6.2 *What does the assumption of a perfect market buy you that would not be satisfied in an imperfect market?*

Q 6.3 *Evaluate whether supermarkets are competitive markets.*

6·1.C. Perfect, Competitive, and Efficient Markets

Do You Always Get What You Pay For?

Let us expand on this insight that projects may not have unique values. Have you ever heard someone say "it's only worth what people are willing to pay for it" and someone else that "it's worth much more than it's being sold for"? Who is correct? Are there any good deals?

Are there any good deals?

The answer is that both are correct and neither is correct. The first claim is really meaningful only to the extent that markets are perfect: if a market is perfect, items are indeed worth exactly what buyers are willing to pay for them. The second claim is really (sort of) meaningful only to the extent that markets are imperfect: if a market is imperfect, items have no unique value. Different people can place different values on the item, and you may consider an item worth much more than what it was sold for.

Let's waffle a little.

For PepsiCo shares, other perfect market assumptions are not perfectly true, but not too far from the truth.

In finance, we often "conveniently" assume perfect markets. Although not perfectly accurate, this is often reasonably justifiable. For example, take the market for trading shares of stock in PepsiCo. It definitely appears to be a **competitive market**, i.e., a situation in which there are many competing buyers and sellers, so that no single buyer or seller can influence the price. There are lots of potential buyers willing to purchase the shares for the same price (or maybe just a tiny bit less), and lots of potential sellers willing to sell you shares for the same price (or maybe just a tiny bit more). A "perfect market" is a stricter requirement than a "competitive market" assumption. If a market is perfect, then it is also competitive, but not vice-versa. (For example, a competitive market can exist even in the presence of opinions and taxes.) For our discussion, we hope you can further assume that taxes are not distorting rates of return in a way that makes the PepsiCo shares' rates of return to a seller any higher or lower than the equivalent rates of return to a buyer, so that taxes are not distorting holding decisions. (This assumption may be a little, but probably not too far off from reality.) Moreover, few active traders in the market have inside information, so objective information differences should not be too bad either. Everyone should roughly agree to what shares can be sold for tomorrow—which defines value today. Finally, the transaction costs of trading shares on the New York Stock Exchange (NYSE) are very low. There are no costs of having to find out the proper price of PepsiCo shares (it is posted by the NYSE), and there are no costs to searching for a buyer or seller. So, the market for PepsiCo shares may indeed be reasonably close to perfect.

IMPORTANT: *For many financial securities, such as publicly traded stocks, the assumption that the market is competitive and that the security is worth what it is trading for is pretty accurate.*

Buyers get what they pay for in a competitive low-friction market.

Such perfect markets reduce buyers' and sellers' concerns that one deal is better than another—that buying is better than selling, or vice-versa. For a more concrete example, consider gasoline and imagine that you do not yet know when and where on your road trip you will need to pump more gas. Unlike shares of stock, gas is not the same good everywhere: gas in one location can be more valuable than gas in another location (as anyone who has ever run out of gas can confirm). But, in populated areas, the market for gasoline is pretty competitive and close to perfect—there are many buyers (drivers) and sellers (gas stations). This makes it very likely that the first gas station you see will have a reasonable, fair price. If you drive by the first gas station and it advertises a price of \$1.50 per gallon, it is unlikely that you will find another gas station offering the same gas for \$1 per gallon or \$2 per gallon within a couple of miles. Chances are that "the price is fair," or this particular gas station would probably have disappeared by now. (The same applies of course in many financial markets, such as large company stocks, Treasury bonds, or certain types of mortgages.) As long as the market is very competitive, or better yet perfect, most deals are likely to be fair deals. Some shopping around may help a tiny bit, but an extreme amount of shopping would likely cost more in time and effort than what it could save.

Competitive markets leave surplus for average buyers and sellers.

But there is an important conceptual twist here: Paying what something is worth does not necessarily mean that you are paying what you *personally* value the good for. Even in competitive perfect markets, there can be many different types of buyers and sellers. It is only the marginal buyer and the marginal seller who end up trading at their reservation values—but if you are not marginal, having access to the market will allow you to make yourself better off. For example, if you are running out of gas and you are bad at pushing two-ton vehicles, you might very well be willing to pay a lot more for gas than even \$10 per gallon—and fortunately all you need to pay is the market price of \$1.50 per gallon! The difference between what you personally value a good for and what you pay for it is called your "surplus." So, even though everyone may be paying what the good is worth in a perfect market, most buyers and sellers can come away being better off.

Unfortunately, not every good is traded in a perfect market. Let us consider selling a house. What is the value of the house? What if the house is in a very remote part of the country, if potential buyers are sporadic, if alternative houses with the same characteristics are rare, and if the government imposes a 50% transfer fee? Now the value of the house depends on the luck of the draw (how many potential buyers are in the vicinity and see the ad, whether a potential buyer wants to live in exactly this kind of house, and so on), the urgency of the seller (perhaps whether the seller has the luxury to turn down a lowball first offer), and the identity of the seller (the current owner does not need to pay the government transfer fee, so he may value the house more than a potential buyer). So, it is only easy to determine the value of a good if the market is perfect. Because the market for many houses is not even close to perfect, the values of such houses are not unique.

A house's value depends on idiosyncratic factors. There is no single value.

Similarly, not all financial markets are close to perfect. Transaction costs, information differences, special taxes, or the unique power of the seller or market can play a role even in some financial markets. For example, many corporate bonds are traded primarily over-the-counter, meaning that you must call some individual at the brokerage house, who may play the role of the only easy clearinghouse for these particular bonds and who will try to gauge your expertise while negotiating a price with you. You could easily end up paying a lot more for this bond than what you could then sell it for one minute later.

Buyers may not get what they pay for in a non-competitive high-friction market.

Of course, you should not kid yourself: no market, financial or otherwise, is ever "perfectly perfect." The usefulness of the perfect market concept is *not* that you should believe that it actually exists in the real world. Instead, it is to get you to think about how close to perfect a given market actually is. The range in which possible values lie depends on the degree to which you believe the market is not perfect. For example, if you know that taxes or transaction costs can represent at most 2–3% of the value of a project, then you know that even if value is not absolutely unique, it is pretty close to unique—possible values sit in a fairly tight range. On the other hand, if you believe that there are few potential buyers for your house, but that some will purchase the house at much higher prices than others, then it will depend on your financial situation whether you will accept or decline a buyer's low-ball offer.

Learn from the perfect markets concepts, but don't believe it holds.

In sum, when someone claims that a stock or firm is really worth more than he or she is selling it for, there are only a small number of explanations: First, there may be pure kindheartedness toward any buyer or a desire by a seller to lose wealth; this happens so rarely that we just ignore this. Second, the seller may not have access to a perfect market to sell the goods. This may make the seller accept a low amount of money for the good, so depending on how you look at this, the good may be sold for more or less than you think it is worth. Third, the seller may be committing a conceptual mistake. The good is worth neither more nor less than what it is being sold for, but exactly how much it is being sold for. Fourth, the seller may be lying and is using this claim as a sales tactic.

A "salesman" may distort the truth and claim great deals.

Preview: Efficient Markets

There is an important corollary to a perfect market. A market (or a price) is called "efficient" if this market has set the price using *all* available information. If a market is perfect, it will inevitably also be efficient. If it were inefficient, you could become rich too easily. For example, say the market wanted to offer you an expected rate of return of 15% on a particular stock (for whatever reason), and the expected value of the stock is \$115. Then the price of the stock today would have to be \$100 for this market to be efficient. This market would not be efficient if it set the price for this stock at \$99 or \$101, because the stock would then offer other than the 15% expected rate of return. Similarly, you should not be able to locate information that tells you today when/if/that the true expected value tomorrow is really \$120 for the \$100 stock. If you could find this information, you could on average earn more than 15%. If the market has overlooked this information, it is not efficient.

Efficient Market: Use of all information.

What is the model? What is the information set?

The application and use of the "efficient markets" concept faces a number of issues. First, where does the 15% come from? It will have to come from some model that tells you what rate of return a stock should have to offer (given its characteristics, such as risk). Possible models will be discussed at length in Part III of this book. Without such a model, talking about market efficiency is meaningless. Second, what information exactly are we talking about? Insiders often have more information than the public. For example, a drug company executive may know before ordinary investors whether a drug is likely to work. Thus, the market may be efficient with respect to publicly available information, but not efficient with respect to insider information.

Competition and Efficiency

So, to be more accurate, when a market is perfect, we usually believe that it is also efficient with respect to public information. After all, if other buyers and sellers were to ignore a useful piece of information, you could likely earn a lot of money trading on it. For example, what would you do if you learned that the market always goes down on rainy days and up on sunny days? It is unlikely that the average investor requires extra return to hold stocks on sunny days—and, even if the average investor does, you would probably not! You would never buy stocks when the weather forecast predicts that rain is coming, and you would only buy stocks when the weather forecast predicts that the sun will be shining. Investors like yourself—and there are of course many such investors in perfect markets—would rapidly bid up the prices before the sun was shining, so that the prices would no longer systematically go up on sunny days. If markets are efficient, then you should not be able to earn abnormally good sunny-day returns—at least not this easily. To earn higher expected rates of return, you must be willing to take on something that other investors are reluctant to take on—such as higher risk (also the subject of Part III).

Classical vs. Behavioral Finance.

A belief in efficient markets is what defines **classical finance**. In contrast, **behavioral finance** believes that markets sometimes do not use all information. Depending on how strong a believer in classical finance vs. behavioral finance you are, you may believe that there are no such opportunities, that there are few such opportunities, or that there are plenty of such opportunities. Both camps agree, however, that market perfection (and especially competitiveness and transaction costs) play crucial roles in determining whether a market is efficient or not. We will dedicate an entire chapter to market efficiency and its consequences, which will also talk in greater length about classical vs. behavioral finance.

See Chapter 15

Solve Now!

Q 6.4 *Can buyers and sellers be better off if a market is perfect? Or does this mean that everyone just pays what it is worth, and thus is no better off.*

Q 6.5 *What is the use of perfect markets, given that there is obviously not a single market in the world that is perfect?*

Q 6.6 *How does an efficient market differ from a perfect market?*

Q 6.7 *Your borrowing rate is 10%/year. Your lending rate is 4%/year. The project costs $1,000 and returns a rate of return of 8%. If you have $900 to invest, should you take the project?*

Q 6.8 *(Cont'd). You can think of the $900 as the amount of money that you are not consuming. Say, your wealth is $2,000, but in the previous question, you wanted to consume, e.g., $1,100. Could you still consume this much and take the project? How much could you consume and still want to take the project?*

6·2. The Effect of Disagreements

A broad perspective.

As you already learned, no market is truly perfect. Perfect market assumptions are just approximations. They work well in some situations, and poorly in others. They typically work quite well in many (but not all) large financial markets for large and liquidly trading financial securities. You now need to learn how to judge the degree to which markets are imperfect—and how to deal with imperfections as real-world investors and managers.

The assumption "no market power" is straightforward.

Our perfect markets assumptions included one that markets are very deep, consisting of many buyers and sellers. If there is only one investor—say, a capital provider or bank—from which you can obtain funding, this investor will have market power over you. Of course, this investor will exploit it by charging you a higher borrowing rate and offering you a lower deposit interest rate. There is not much more to say about this particular perfect market assumption, so we now turn to the more interesting discussions.

Our agenda.

The remainder of our chapter explores how big typical market imperfections can be, what can mitigate them, and how you should work with them. This section begins by discussing the role of opinions. Then we continue to transaction costs. Next, we will take a look at income taxes. Finally, we will consider hybrid problems—like the role of taxes in the presence of inflation.

6·2.A. Expected Return Differences vs. Promised Return Differences

Under uncertainty, different opinions can lead to disagreements.

The most obvious cause for different borrowing and lending rates are differences in opinion between the lender and borrower. To think about this particular assumption violation, we must work in a world of uncertainty—it would be absurd to believe that such differences in opinion could exist if there is no uncertainty. So, what happens if the lender and borrower have different information or different judgment about the same information? Most prominently, they could disagree about the default risk! For example, if you have no credit history, then a lender who does not know you might be especially afraid of not receiving promised repayments from you—from the perspective of such a lender, you would be extremely high-risk. Your lender might estimate your appropriate default probability to be 20% and thus may demand an appropriate default premium from you of, say, 8% above the risk-free yield. On the other hand, *you* may know that you will indeed return the lender's money, because you know you will work hard and that you will have the money for sure. In your opinion, a fair and appropriate default premium should therefore be only 0-1%.

Expected rates of return for borrowing and lending now become different.

When your potential lenders and you have different opinions, you then face different expected savings interest rates and borrowing costs of capital. That is, if you know that you are a low risk, then your borrowing cost of capital (the *expected* interest rate) would not only be much higher than the lender perceives it to be, but it would also be the case that your borrowing *expected* cost of capital is much higher than your savings *expected* rate of return that you would earn if you deposited your money in the bank. You might be able to borrow at a cost of capital (expected rate of return) of 12%, but save only at an expected rate of return of 5%.

Do not confuse different *promised* borrowing/lending rates (not discussed here) with different *expected* borrowing/lending rates (discussed here).

This issue is very different from the difference between promised and expected returns that was discussed in Chapter 5. It is not just that you must offer a higher *promised* rate of return than what you expect to repay. Instead, you must now offer a higher *expected* rate of return when you want to borrow, compared with the *expected* rate of return that you could earn if you deposited money in the bank. (The bank is unlikely to go bankrupt, and your deposit is probably insured by the government, which means that the bank may pay not more interest than the equivalent short-term Treasury. The bank's promised rate of return is almost the same as the expected rate of return.) The difference in *stated* borrowing interest rates and *stated* saving interest rates at your local bank could just as well be the default premium—the difference in the returns you promise and which you expect to pay, although in a different guise. The bank quoting you a lower savings deposit interest rate than borrowing loan interest rate would just compare promised interest rates, not expected interest rates. Instead, the novel issue discussed in our chapter is that disagreements and information differences are now causing differences in *expected* returns. The borrowing and lending *expected* rates of return are no longer the same.

> **IMPORTANT**: *The fact that credit spreads reflect a default premium—a difference between the promised rate of return and the expected rate of return—is not a market imperfection.*
>
> *The fact that credit spreads reflect differences in opinion between borrower and lender—a difference about the two assessed expected rates of return seen—is a market imperfection.*

6·2.B. Corporate Finance vs. Entrepreneurial or Personal Finance?

For large companies, a perfect market assumption with reasonably equal borrowing and lending rates is reasonable.

Where do such market imperfections apply? In the world of large corporations, the interest rate spread between similarly risky borrowing and lending rates is often mild, so they can pretend they live in a "perfect" market in which they can separate the project choice from their financial situation. Their *promised* borrowing interest rates would still be higher than what they can receive investing their money in Treasury bonds—but, given that these large firms still have some possibility of going bankrupt, their *expected* borrowing cost of capital would probably be fairly similar to the *expected* rate of return that they could earn if they invested money into bonds with characteristics similar to those that they themselves issued.

For entrepreneurs, a perfect market assumption is problematic.

In the world of individuals, entrepreneurs, and small companies, however, expected borrowing interest rates are often higher than expected saving interest rates. In fact, this issue of an extraordinarily high differential between expected borrowing and lending rates—and with it the role of cash-on-hand—is one important difference between "ordinary corporate finance" and "entrepreneurial finance." Entrepreneurs find it very difficult to convey credibly their intent and ability to pay back their loans. As a consequence, many entrepreneurs even resort to financing projects with credit cards, which may charge a thousand basis points above Treasury bonds. These high borrowing costs can thus prevent rational entrepreneurs from taking many projects that they would undertake if they had the money on hand. It also means that more established firms or richer entrepreneurs should optimally take more projects than poorer entrepreneurs.

Don't believe entrepreneurial claims!

But be careful in the real world before you conclude this to be the case: Entrepreneurs tend to have notoriously over-optimistic views of their prospects. (Even venture capitalists, the financing vehicle for many high-tech entrepreneurial ventures, which advertise returns of 30%/year or more seem to have managed to return only a couple of percentage points above the risk-free rate over the last thirty years.) This may actually mean that entrepreneurs face only high *promised* borrowing costs, not high *expected* borrowing costs. Thus, the quoted spread between their borrowing and lending rates, which is really all that you can easily observe, likely has a large component that is due not to information disagreements but simply due to credit risk.

> **SIDE NOTE**: Valuation services are an important revenue business for many finance professors and consulting firms. When asked to value small, non-public businesses—for example, for purposes of assessing the inheritance tax or in disputes among former business partners—it is customary and legally acceptable to first compute the value of an equivalent publicly traded business or company, and then to apply a "private discount" of around 10% to 30% in order to reflect the limited access to capital (because lenders tend not to believe in high default rates for young companies).

Solve Now!

Q 6.9 *Can there be a difference in the borrowing and lending rates quoted by the bank in perfect markets?*

Q 6.10 *What are the possible reasons why entrepreneurs often have to finance their projects with credit cards, which can charge interest rates as high as 1500 basis points above Treasury?*

6·2.C. Covenants, Collateral, and Credit Rating Agencies

You would love to reduce such disagreements—but you may not be able to.

So, if you are an entrepreneur who wants to start a company, what can you do to reduce the expected cost of capital? The answer is that it is in your interest to disclose to the lender all the information you can, provided you are the type of entrepreneur who is likely to pay back the loan. You want to reduce the lender's doubt about future repayment. Unfortunately, this can be very difficult. The lender can neither peer into your brain, nor give you a good lie detector test. Attempts to convey information credibly in the real world are many, but there will always be residual information differences—they are just a fact of life. Still, if you can reduce the information differences, your firm will be able to enjoy lower costs of capital. Also, if you as borrower fail to give your best try to convince the lender of your quality, then the lender should not only assume that you are an average company, but instead assume you are the very worst—or else you would have tried to communicate as much as possible.

Good borrowers want to credibly tell the lender that they are good.

There are at least three important mechanisms that have evolved to alleviate such information differences. The first mechanism is **covenants**, which specify upfront what a debtor must do to maintain credit. This can include such requirements as the maintenance of insurance or a minimum corporate value. The second mechanism is **collateral**—something that the creditor can repossess if payments are not made. But anything that inflicts pain on the debtor will do. For example, if defaulting debtors were thrown into debtor's prison (as they often were until the 19th century), the promise to repay would be more credible and lenders would be more inclined to provide funding at lower rates. Of course, for the unlucky few who just happened to suffer incredibly bad luck ex-post, debtors' prison has some definite drawbacks.

Bond rating agencies indicate probability of default.

The third mechanism to alleviate repayment uncertainty are credit rating agencies, which keep a history of past payments to help assess the probability of future default. This is why you need to give your social security number if you want to take out a substantial personal loan—the lender will check up on you. The same is true for large corporations. It may be easier to judge corporate default risk for large companies than personal default risk, but it is still not easy and it costs both time and money. The two biggest bond credit rating agencies for corporations are **Moody's** and **Standard&Poors**. (The other two are *Duff and Phelps* and *Fitch*.) For a fee that the corporate borrower pays, they will rate the bond's quality, which reflects the issuer's probability that the bonds will default. This fee depends on a number of factors, such as the identity of the issuer, the desired detail in the agencies' investigations and descriptions, and the features of the bond (e.g., a bond that will pay off within one year is usually less likely to default before maturity than a bond that will pay off in thirty years; thus, the former is easier to grade). The credit rating agencies ultimately do not provide a whole set of default probabilities (e.g., 1% chance of 100% loss, 1.2% chance of 99% loss, etc.), but just an overall rating grade. It is up to the ratings' reader to translate the rating into an appropriate compensation for default risk. The top rating grades are called **investment grade**, while the bottom grade are called **speculative grade** (or **junk**).

ANECDOTE: Sumerian Debt Contracts
Among the earliest known collateralized debt contracts is a tablet from Sumeria (Mesopotamia), which promises delivery of silver and gives as security the son of the borrower. (The tablet can be viewed at www.museumofmoney.org/babylon/index.html.) Such contracts are illegal today, but de-facto "debt slavery" for debts not repaid is common in many countries, according to the September 2003 issue of *National Geographic Magazine*.

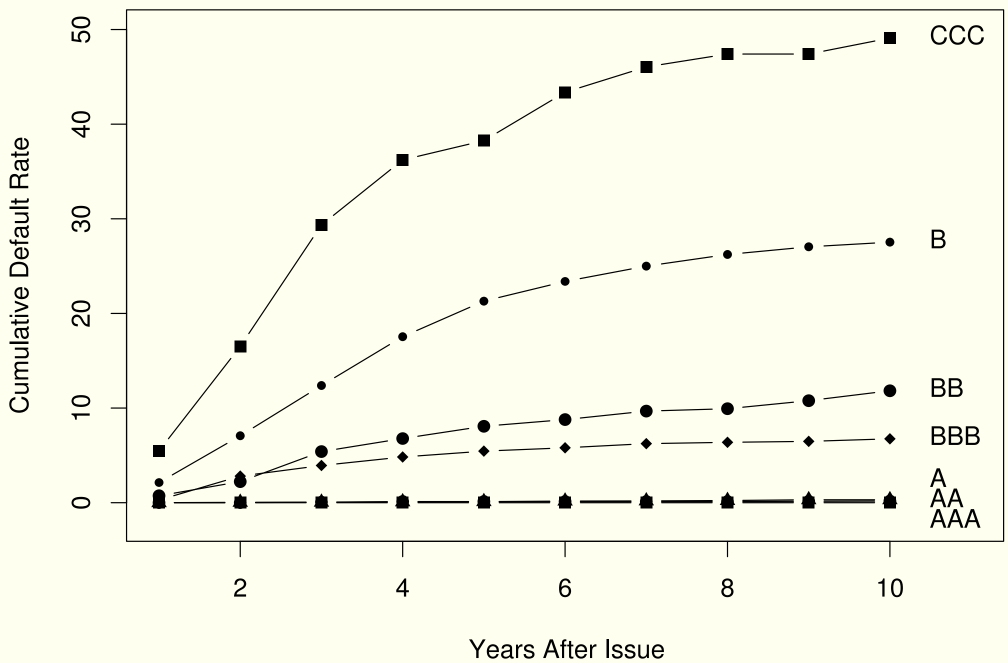
Investment Grade:	Moody's	Standard & Poors
Exceptional	Aaa, Aaa1, Aaa2, Aaa3	AAA, AAA-, AA+
Excellent	Aa, Aa1, Aa2, Aa3	AA, AA-, A+
Good	A, A1, A2, A3	A, A-, BBB+
Adequate	Baa, Baa1, Baa2, Baa3	BBB, BBB-, BB+
Speculative Grade:	**Moody's**	**Standard & Poors**
Questionable	Ba, Ba1, Ba2, Ba3	BB, BB-, B+
Poor	B, B1, B2, B3	B, B-, CCC+
Very Poor	Caa, Caa1, Caa2, Caa3	CCC, CCC-, CC+
Extremely Poor	Ca, Ca1, Ca2, Ca3	CC,CC-, C+
Lowest	C	C

There is often a sharp difference in *quoted* interest rates between the worst investment grade bond and the best speculative grade bond, partly also because many investing institutions are allowed to hold only investment grade bonds.

Here is what ratings mean.

So, is there a difference between bonds of different rating quality? Yes! Altman studied corporate bonds from 1971 to 2003 and reported default and recovery rates. Figure 6.1 gives a sketch of how likely default was for a given credit rating. Very few investment grade bonds default—and especially right after issue when they would have still carried the original credit rating. However, many speculative bonds will eventually miss at least one coupon payment (which is considered default). Upon default, an AAA or AA bond price was worth about 75 cents on the dollar; an A bond price was worth about 50 cents on the dollar; and lower rated bonds were worth about 30 cents on the dollar. (The previous chapter already told you that the net effect is that junk bonds that promised a rate of return of about 5% above the 10-Year Treasury bond delivered only 2.2% above the Treasury on average.)

Figure 6.1. Cumulative Probability of Default by Original Rating



This figure shows the probability of default within x years after issue. For example, at some point during the first seven years of their issue, 25% of all bonds originally issued as B (poor) had not delivered on at least one promised bond payment.

Source: Edward Altman and Gonzalo Fanjul, New York University, February 2004. Moody's also offers similar reports, and publishes an interesting monthly report on credit risk, including corporate bond default forecasts (which change with the business cycle). Older Moody's reports can be found at riskcalc.moodysrms.com/us/research/mdr.asp. For example, in August 2002, the report described that no bonds rated Aaa–A2 bonds had defaulted, 0.3–0.6% of bonds rated A3–Ba3 had defaulted, and 1.8%, 7.6%, 9.9%, and 21.0% of bonds rated B1, B2, B3, and Caa–C had defaulted.

Unfortunately, although bond rating agencies will update their rating if the condition of the firm changes, the empirical evidence suggests that these bond ratings are not very good in helping an investor earn superior rates of returns. In fact, the ratings seem to respond more to drops in the value of the underlying bonds than vice-versa. The bond rating agencies seem to be more reactive than proactive. A longer discussion of bond ratings appears on Page 580f.

Ratings are useless for making money.

How do bond ratings translate into differences in promised (quoted) bond yields? Table 6.1 lists the borrowing rates of various issuers in May 2002. (Many other current interest rates can be found at www.bloomberg.com/markets/rates/index.html and bonds.yahoo.com/rates.html.) Most of the differences between these borrowers' promised interest rates and Treasury interest rates are due to default risk, which compensates lenders for differential default probabilities.

Sample Rates of Returns

Table 6.1. Promised Interest Rates For Some Loans in May 2002.

Security (Bond)	Annual Yield	Similar U.S. Treasury
FNMA May 2003	2.36%	2.22%
FNMA March 2031	6.29%	5.60%
Boston Celtics 6s38	9.40%	5.60%
United Airlines 11.21s14	14.40%	4.82%
Corporate High-Quality 1–10 years	4.89%	≈ 3%–4%
Corporate Medium-Quality 1–10 years	6.24%	≈ 3%–4%
Corporate High-Quality 10+ years	6.76%	≈ 4%–5%
Corporate Medium-Quality 10+ years	7.65%	≈ 4%–5%
High-Yield (Junk Bond) Corporates	11.36%	?

Source: *Wall Street Journal*, Page C13. FNMA is a quasi-governmental agency that underwrites home mortgages. See also Page 107.

Warning, I am cheating a little: not all of the differences in promised rates of return in Table 6.1 are due to default risk. Some of the interest rate differentials can be due to transaction costs, liquidity premia, bond duration mismatches, risk premia (to be discussed in Part III [Investments]), or extra bond features (such as embedded bond options, to be discussed in Part IV [Financing]). For all these reasons, the quoted yields in Table 6.1 are not exactly comparable to treasuries. Nevertheless, the differences are mostly due to default risk.

Solve Now!

Q 6.11 *Using information from a current newspaper or the WWW, what is the annualized yield on corporate bonds (high-quality, medium-quality, high-yield) today?*

6·3. Market Depth and Transaction Costs

The next market imperfections that can drive a wedge between borrowing and lending rates are transaction costs. If it is very difficult and costly to administer loans, an investor must charge you a higher borrowing rate than deposit rate just to break even. This is the subject of this section, in which you will see how corporations and individuals should account for transaction costs.

There can be other causes of different expected rates of return, too.

6·3.A. Typical Costs When Trading Real Goods—Houses

When you engage in transactions, i.e., a purchase or sale, you face costs to facilitate the transactions. Real estate is a perfect example to illustrate transaction costs. The personal residence is the most significant asset that most people own, real estate is a very large part of the total value of the economy, and real estate transaction costs are so high that they will register with anyone who has ever had to sell a house. The real estate example also will allow you to contrast the real-estate transaction costs later with financial securities transaction costs.

Real Estate is an important market in itself, and a great comparison for us.

So, what does selling or buying a house really cost?

Brokerage Commissions, a direct cost: In the United States, if a house is sold, the seller's broker typically receives six percent of the value of the house as commission (and splits this commission with the buyer's agent). Thus, if a real estate agent manages to sell a house for \$300,000, the commission is \$18,000. Put differently, without an agent, the buyer and seller could have split the \$18,000 between them. (Of course, brokers do many useful things, such as matching buyers and sellers, shepherding the selling process, etc., so the \$18,000 may just be the intrinsic transaction cost to selling a house. However, inconsistent with this view, real estate commissions are *much* lower in other countries, and it is difficult to see why the cost of selling houses would be *exactly* 6% in practically *all* markets in the United States.)

Direct transaction costs require a money transfer.

Although only the seller pays the broker's cost, it makes sense to think of transaction costs in terms of **round-trip** costs—how much worse you are off if you buy and then immediately sell an asset. You would mislead yourself if you thought that when you buy a house, you have not incurred any transaction costs because the seller had to pay them—you have incurred an implicit transaction cost in the future when you need to resell your investment. Of course, you usually do not immediately sell assets, so you should not forget about the timing of your future selling transaction costs in your NPV calculations.

Thinking of transactions in "round-trip" form.

Housing transaction costs are so high and so important that they are worth a digression. If you borrow to finance the investment, transaction may be higher than you think. The real estate agent earns 6% of the value of the house, not of the amount of money you put into the house. On a house purchase of \$500,000, the typical loan is 80% of the purchase price, or \$400,000, leaving you to put in \$100,000 in equity. Selling the house the day after the purchase reduces the owner's wealth of \$100,000 by the commission of \$30,000—for an investment rate of return of –30%. This is not a risk component; it is a pure and certain transaction cost.

Transaction costs are on the whole investment, so if you borrow to finance the investment, your part of the transaction costs may be much higher than you think!

Let us briefly consider what happens if the house price decreases or increases by 10%. If house prices decline by 10%, or the buyer overpays by 10%, the house can only be resold for \$450,000, which leaves \$423,000 after agent commissions. The house owner is left with \$23,000 on a \$100,000 investment. A 10% decline in real estate values has reduced the home owner's net worth by 77%! In comparison, a 10% increase in real estate values increases the value of the house to \$550,000, which means that \$517,000 is left after real estate commissions. The house owner's rate of return for the same up movement is thus only 17%.

Let's add some price volatility.

With the tools you already know, you can even estimate how the value of a typical house might change if the Internet could instantly and perfectly replace real estate agents. You cannot be too accurate—you can only obtain a back-of-the-envelope estimate. A typical house in the United States sells every seven years or so. Work with a \$1,000,000 house, and assume that the expected house capital-gain appreciation is 0%—you consume all gains as rental enjoyment. In this case, the house will stay at \$1,000,000 in value, the commission will stay constant at

This is how one estimates the value effects of commissions.

\$60,000 and will be paid every 7 years. If the appropriate 7-year interest rate were 40% (around 5% per annum), then the value of the brokerage fees would be a perpetuity of $\$60,000/40\% = \$150,000$. The capitalized transaction cost would therefore have lowered the value of the \$1,000,000 house by \$150,000. If you could eliminate all commissions, e.g., by selling equally efficiently over the Internet, such a house would increase in value by about 15%. However, if you believed that the brokerage commission were to go up by the inflation rate (2% per annum, or 15% per 7-years), the friction would not be \$150,000 but \$240,000—more like 25% of the value of the house, not just 15%.

Other direct expenses.

Other direct costs: In addition to direct agent commissions, there are also many other direct transaction costs. These can range from advertising, to insurance company payments, to house inspectors, to the local land registry, to postage—all of which cost the parties money.

Indirect transaction costs are the loss of other opportunities.

Indirect and opportunity costs: Then there is the seller's own time required to learn as much about the value of the house as possible, and the effort involved to help the agent sell the house. These may be significant costs, even if they involve no cash outlay. After all, the seller could spend this time working or playing instead. Furthermore, not every house is suitable for every house buyer, and the seller has to find the right buyer. If the house cannot be sold immediately but stays empty for a while, the foregone rent is part of the transaction cost. The implicit cost of not having the house be put to its best alternative use is called an **opportunity cost.** Opportunity costs are just as real as direct cash costs.

Solve Now!

Q 6.12 *If you presumed that the appropriate interest rate was 8%/year rather than 5%/year for the rental flow on a house, what would you presume the value effect of the 6% commission be?*

6·3.B. Typical Costs When Trading Financial Goods—Stocks

Transaction Costs for Stocks are also either direct or indirect.

Similarly, financial markets transactions also incur transaction costs. If an investor wants to buy or sell shares of a stock, the broker charges a fee, as does the stock exchange that facilitates the transaction. In addition, investors have to consider their time to communicate with the broker to initiate the purchase or sale of a stock as an (opportunity) cost.

The typical Transaction Costs for Stocks are relatively low.

Brokerage and Market-Maker Commissions, direct costs: Still, the transaction costs for selling financial instruments are much lower than they are for most other goods. Let's look at a few reasons why. First, even if you want to buy (or sell) \$1 million worth of stock, some Internet brokers now charge as little as \$10 per transaction. Your round-trip transaction, which is a buy and a sale, costs only \$20 in broker's commission. In addition, you have to pay the spread (the difference between the bid and the ask price) to the stock exchange. For example, a large company stock like PepsiCo (ticker symbol PEP) may have a publicly posted price of \$50 per share. But you can neither buy nor sell at \$50. Instead, the \$50 is really just the average of two prices: the **bid price** of \$49.92, at which another investor or the exchange's market-maker is currently willing to buy shares; and the **ask price** of \$50.08, at which another investor or the exchange's market-maker is currently willing to sell shares. Therefore, you can (probably) purchase shares at \$50.08 and sell them at \$49.92, a loss of "only" 16 cents which amounts to round-trip transaction costs of $(\$49.92 - \$50.08)/\$50.08 \approx -0.32\%$. You can compute the total costs of buying and selling 20,000 shares (\$1,000,000 worth) of PepsiCo stock as

Anecdote: Real Estate Agents: Who works for whom?
Real estate agents are conflicted. If they sell sooner, they can spend their time focusing on other properties. Thus, the typical seller's agent will try to get the seller to reduce the price in order to make a quicker sale. Similarly, the buyer's agent will try to get the buyer to increase the offer. In a financial sense, the buyer's agent is working on behalf of the seller, and the seller's agent is working on behalf of the buyer. Interestingly, Steve Levitt found that when agents sell their own houses, their homes tend to stay on the market for about 10 days longer and sell for about 2 percent more. Source: Steve Levitt, University of Chicago.

Financial Round-trip Transaction

Purchase 20,000 Shares	Pay $\$50.08 \cdot 20,000 = \$1,001,600$	
Add Broker Commission	$+\$10$	$= \$1,001,610$
Sell 20,000 Shares	Receive $\$49.92 \cdot 20,000 = \$998,400$	
Subtract Broker Commission	$-\$10$	$= \$998,390$
	Net Round-trip Transaction Costs	–\$3,220

This is not exactly correct, though, because the bid and ask prices that the exchange posts (e.g., on Yahoo!*Finance* or the *Wall Street Journal*) are only valid for 100 shares. Moreover, some transactions can occur inside the bid-ask spread, but for most large round-trip orders, chances are that you may have to pay more than \$50.08 or receive less than \$49.92. So 0.32% is probably a bit too small. (In fact, if your trade is large enough, you may even move the publicly posted exchange price away from \$50!) Your buy order may have to pay \$50.20, and your sell may only get you \$49.85. In real life, the true round-trip transaction cost on a \$1 million position in PEP is on the order of magnitude of 50 basis points.

An even lower cost alternative: limit orders.

The above applies primarily to a **market order**, in which you ask your broker to buy or sell at the prevailing market price. A **limit order** can specify that you only wish to buy or sell at \$50.00, but you are patient and willing to take the chance that your order may not get executed at all. There is a common belief that limit orders are "cheaper" in terms of transaction costs, but also "riskier." For example, if you have a standing limit order to buy at \$50, and the company reveals that it has managed earnings, so its value drops from \$51 to \$20, your limit order could still easily execute at \$50.

Opportunity costs are low, too.

Indirect and Opportunity Costs: Investors do not need to spend a lot of time to find out the latest price of the stock: it is instantly available from many sources (e.g., from the Internet such as Yahoo!*Finance*). So, the information research costs are very low: unlike a house, the value of a stock is immediately known. Finally, upon demand, a buyer can be found practically instantaneously, so search and waiting costs are also very low. Recall the often multi-month waiting periods if you want to sell your house.

Compared to other economic assets, ...

Compare the financial securities transaction costs to the transaction costs in selling a house. Broker fees alone are typically 6%: for the \$100,000 equity investment in the \$500,000 house, this comes to \$30,000 for a round-trip transaction. Add the other fees and waiting time to this cost and you are in for other transaction costs, say, another \$10,000. And houses are just one example: Many transactions of physical goods or labor services (but not all) can incur similarly high transaction costs.

Table 6.2. Comparison of Transaction Costs on Stocks and Real Estate

Cost Type	Explanation	Real Estate (House)	Financial Security (Stock)
Direct	Typical Round-trip Commission etc.	≥6%	0–1%
Search/Research	Time to Determine Fair Price	high	zero
Search/Liquidity	Time Waiting to Find Buyer	variable	zero

ANECDOTE: Payment for Order Flow
The next financial scandal may well be **payment for order flow**. Although well known among finance professionals, and disclosed to customers, most individual investors do not know that especially (but not only) discount brokers, such as Charles Schwab, Ameritrade, and e-Trade, can route their investors' orders routinely to market makers who charge relatively high bid-ask spreads, and then pay the discount broker a rebate kickback (typically 1 to 4 cents per share). Some brokerage houses can even fill customer orders with other customer orders in-house, at execution prices that are not the best prices. In any case, from the point of view of transaction costs, very wealthy investors may not necessarily be better off using discount brokers, but this is difficult for individual investors to determine conclusively. More information can be found at invest-faq.com/articles/trade-order-routing.html. The SEC also publishes "What every Investor should know" at www.sec.gov/investor/pubs/tradexec.htm.

...financial securities have such low transaction costs that they can almost be assumed to be zero.

In contrast, if you want to buy or sell 100 shares in, say, Microsoft stocks, your transaction costs are relatively tiny. Because there are many buyers and many sellers, financial transaction costs are comparably tiny. Even for a $100,000 equity investment in a medium-sized firm's stock, the transaction costs are typically only about $300–$500. To oversimplify, this book will make the incorrect, but convenient assumption that financial transaction costs are zero (unless otherwise described). For individuals buying and selling ordinary stocks only rarely (a **buy-and-hold** investor), a zero transaction cost assumption is often quite reasonable. But if you are a **day trader**—someone who buys and sells stocks daily—you better read another book! (And if you are a company that wants to issue new shares, wait until Section 23.)

Solve Now!

Q 6.13 *What would you guess the order of magnitude to be for a round-trip transaction in $10,000 worth of shares of DELL Computer be? Describe in percent and in absolute terms.*

Q 6.14 *List important transaction cost components, both direct and indirect.*

6·3.C. Transaction Costs in Returns and Net Present Values

Commissions need to be taken out of meaningful rates of return.

As an investor, you usually care about rates of return *after* all transaction costs have been taken into account, not about quoted rates of returns from quoted prices. Let's see how you should take these transaction costs on both sides (buy and sell) into account.

Return to our housing example. If you purchase a house for $1,000,000 and you sell it to the next buyer at $1,100,000 through a broker, your rate of return is not 10%. At selling time, the brokers charge you a 6% commission. There are also some other costs that reduce the amount of money you receive, not to mention the many opportunity costs. Say these costs amount to $70,000 in total. In addition, even when you purchased the house, you most likely had to pay some extra costs (such as an escrow transfer fee) above and beyond the $1,000,000—say, $5,000. Your rate of return would not be $\$1,100,000/\$1,000,000 - 1 = 10\%$, but only

$$r = \frac{(\$1,100,000 - \$70,000) \quad - \quad (\$1,000,000 + \$5,000)}{(\$1,000,000 + \$5,000)} \approx 2.5\% \; . \tag{6.1}$$

$$\text{Rate of Return} = \frac{\begin{array}{c}\text{Dollars Returned,}\\ \text{after Transaction costs}\end{array} - \begin{array}{c}\text{Dollars Invested,}\\ \text{after Transaction costs}\end{array}}{\text{Dollars Invested, after Transaction costs}} \; .$$

Note how the $5,000 must be added to, not subtracted from the price you paid. The price you paid was ultimately higher than $1,000,000. The $5,000 works against you. (Incidentally, in order to make their returns look more appealing, many professional fund managers quote their investors' rates of return before taking their own fees (transaction costs) into account. Usually, a footnote at the bottom satisfies the lawyers that the investors can not sue for being misled—they are supposed to know how to adjust returns for transaction costs themselves, which you now do.)

In NPV, work with after-transaction-cost cash flows—and after-transaction costs of capital.

How do you take care of transaction costs in present value calculations? This is relatively straightforward. In the example, you put in $1,005,000 and receive $1,030,000—say, after one year. So,

$$\text{NPV} = -\$1,005,000 + \frac{\$1,030,000}{1 + \text{Opportunity Cost of Capital}} \; . \tag{6.2}$$

The only thing you must still take care of is to quote your opportunity cost of capital also in after-transaction cost terms. You may not be able to get a 10% rate of return in comparable investments, because you may also be required to pay a transaction cost there, too. In this case, presume that an alternative investment with equal characteristics in the financial markets (not the housing markets) earns an 8%/year return, but has a 50 basis point cost, so this project may have an appropriate NPV of

$$\text{NPV} = -\$1,005,000 + \frac{\$1,030,000}{1 + 7.5\%} \approx -\$141,860 \; . \tag{6.3}$$

Solve Now!

Q 6.15 *Compute your after-transaction costs rate of return on purchasing a house for $1,000,000, if you have to pay 0.5% transaction fees upfront and pay 6% broker's commission (plus 2% in waiting costs). Assume a $4,000/month effective dividend of enjoying living in the house. At what rate of capital appreciation would the NPV be zero if you resold after one year? Assume that your opportunity cost of capital is 7% per year.*

6·3.D. Liquidity

More illiquid investments often have to offer a higher rate of return.

Things get even more interesting when transaction costs influence your upfront willingness to purchase assets. You might not want to purchase a house even if you *expect* to recoup your transaction cost, because you dislike the fact that you do not know whether it will be easy or hard to resell. After all, if you purchase a stock or bond instead, you know you can resell without much transaction cost whenever you want.

Physics in Finance? The Liquidity Analogy.

So, why would you want to take the risk of sitting on a house for months without being able to sell it? To get you to purchase a house would require the seller to compensate you. The seller would have to offer you a **liquidity premium**—an extra expected rate of return—to induce you to purchase the house. (We have already briefly mentioned this premium in the previous chapter.) The liquidity analogy comes from physics. The same way that physical movement is impeded by physical friction, economic transactions are impeded by transaction costs. Financial markets are often considered low-friction, or even close to frictionless. And when the amount of trading activity subsides, pros would even say that "the market has *dried up*."

This is true even for Treasury Bonds: on-the-run vs. off-the-run bonds.

Housing may be an extreme example, but liquidity effects seem to be everywhere and important—and even in financial markets with their low transaction costs. A well-known and startling example is Treasury bonds. One bond is designated to be **on-the-run**, which means that everyone who wants to trade a bond with roughly this maturity (and the financial press) focuses on this particular bond. This makes it easier to buy and sell the on-the-run bond than a similar but not identical off-the-run bond. For example, in November 2000, the 10-year on-the-run Treasury bond traded for a yield-to-maturity of 5.6% per annum, while a bond that was just a couple of days off in terms of its maturity (and thus practically identical) traded at 5.75% per annum. In other words, you would have been able to purchase the off-the-run bond at a much lower price than the on-the-run bond. The reason why you might want to purchase the on-the-run bond, even though it had a higher price, would be that you could resell it much more quickly and easily than the equivalent off-the-run bond. Of course, as the date approaches when this 10-year bond is about to lose its on-the-run designation and another bond is about to become the on-the-run 10-year bond, the old on-the-run bond drops in value and the new on-the-run bond increases in value.

Liquidity provision is a common business.

The provision of liquidity in markets of any kind is a common business. For example, you can think of antique stores or second-hand car dealerships as liquidity providers that try to buy cheap (being a standby buyer), and try to sell expensive (being a standby seller). Being a liquidity provider can require big risks and capital outlays. If it was easy, everyone could do it–and then there would be no more money in liquidity provision!

Solve Now!

Q 6.16 *What is the difference between a liquidity premium and transaction costs?*

6·4. An Introduction to The Tax Code

Our next violation of market perfection is taxes. They are pervasive and usually not small potatoes—they play such an important role that it is worthwhile to make a digression and explain the overall U.S. tax code once, at least in broad strokes. The actual tax code itself is very complex, and its details change every year, but the basics have remained in place for a long time and are similar in most countries. So, let us summarize in this section most of what you shall need in this book. It is an unusual section, in that it covers a subject matter to which we will refer again many times later—and like the tax code itself, it is somewhat tedious. I will try to liven it up with some anecdotes of how crazy the tax code really is, but you will just have to bear with it.

6·4.A. The Basics of (Federal) Income Taxes

The Tax Code basics are simple, the details are complex.

With the exception of **tax-exempt institutions**, such as charitable institutions and pension funds (which suffer no taxes), individuals and corporations in the United States are taxed in a similar fashion, so we can combine our discussion of the two. The name of the **Internal Revenue Service** (**IRS**) **tax form** that individuals have to file is feared by every U.S. tax payer: it is the infamous Form **1040**.

Step 1: Compute your Taxable Income.

Earned income or **ordinary income** is subject to both federal income taxes and state income taxes. There are, however, some deductions that taxpayers can take to result in lower **taxable income**. Most prominently, in the United States, individuals who itemize their deductions can reduce their taxable income through mortgage interest payments. (This does not extend to other kind of interest payments, so mortgage borrowing—rather than, say, car loan borrowing—is often the best choice in terms of after-tax effective interest costs for many individuals.) Further, with some restrictions, individuals may deduct other expenses, such as some educational expenses and certain retirement savings (specifically, through contribution to an **individual retirement account**, such as an ordinary **I.R.A.** or a **401-K**). (These are only tax-advantaged, not tax-exempt. Most contributions are income-tax exempt, but the IRS will collect taxes when the money is withdrawn in the future.) Individuals can also carry forward losses or deductions that they could not legally deduct in the current year into future years.

Corporations are similar.

Corporations are treated similarly, but often more generously by the tax code: they are generally allowed to deduct *all* interest, not just mortgage interest, and many corporations enjoy a plethora of preferential tax exemptions and loopholes, too numerous to list in just one book and ever-changing. Unlike individuals, corporations that have losses or extra deductions can even receive a refund for taxes paid in the most recent three years. This is not necessarily unfair—after all, corporations are just entities owned by individuals. Just as your car is not paying the car tax the DMV imposes—*you, the owner, are paying the car tax*—taxing corporations is just a different mechanism of taxing the individuals who own the corporation.

Anecdote: The Income Tax
The first federal income tax was introduced during the Civil War. It amounted to two percent per year on income above about $80,000 in 2002 dollars. Attorney Joseph H. Choate argued against the federal income tax in the Supreme Court—and won! It took a constitutional amendment to reinstate it. In his argument, Choate warned that the two percent rate might one day rise to twenty percent. (Source: Don Mathews.)

Between 1945 and 1963, the top income tax rate was around 90%; this was also the period in which the United States experienced the greatest economic boom in its history. Presumably, this was just coincidence.

Table 6.3. Sample Taxable Income Computation

	Actual Earned Income	$100,000
–	Allowed Mortgage Interest	$10,000
–	Allowed Retirement Investment Deduction (if investor is a person)	$5,000
=	Taxable Income	$85,000

Corporations are taxed similarly, but may be allowed a plethora of possible deductions. Details vary year to year, state to state, and company to company.

Tax Rates (Brackets) are "progressive."

Table 6.3 shows how your taxable income computation might look like. After you have computed your taxable income, you must apply the appropriate income tax rates. Income tax rates for individuals depend on your marital status and are usually **progressive**—that is, not only do you have to pay higher taxes when making more money, you have to pay *increasingly* higher taxes when making more money. (They are roughly progressive for corporations, but not perfectly so.) For example, Table 6.4 shows the U.S. federal income tax rates in 2004 for single individuals and corporations.

Table 6.4. Federal Income Tax Rate Tables for 2004

Single Individual

Tax Rate	Description	Minimum	Maximum	(Cumulative)
10%	on the first $7,150	$0	$7,150	$715
15%	on the next $21,900	$7,150	$29,050	$4,000
25%	on the next $41,300	$29,050	$70,350	$14,325
28%	on the next $76,400	$70,350	$146,750	$35,717
33%	on the next $172,350	$146,750	$319,100	$92,592
35%	on the remainder	$319,100	unlimited	

Corporations

Tax Rate	Description	Minimum	Maximum	(Cumulative)
15%	on the first $50,000	$0	$50,000	$7,500
25%	on the next $25,000	$50,000	$75,000	$13,750
34%	on the next $25,000	$75,000	$100,000	$22,250
39%	on the next $235,000	$100,000	$335,000	$113,900
34%	on the next $9.6 million	$335,000	$10.0 million	$3,377,900
35%	on the next $5.0 million	$10.0 million	$15.0 million	$5,127,900
38%	on the next $3.3 million	$15.0 million	$18.3 million	$6,381,900
35%	on the remainder	$18.3 million	unlimited	

Source: www.smbiz.com.

Table 6.5. Sample Income Tax Computation

Rate	on Amount		Tax
10%	on $7,150	=	$715
15%	on $21,900	=	$3,285
25%	on $41,300	=	$10,325
28%	on $14,650	=	$4,102
Computed Total Income Tax on $85,000		=	$18,427

Step 2: Look up Your Tax Bracket.

Each of the tax rates in Table 6.4 is also called a **tax bracket**, because each defines a range of income. If you are an individual in the 28% tax bracket, it means that you have taxable earnings between $70,350 and $146,750. Again, as with the computation of the taxable income, be warned that this particular tax rate table also contains many simplifications. (For example, there is also an **Alternative Minimum Tax** (**A.M.T.**) that nowadays applies to many taxpayers.) Table 6.5 shows how you would compute your federal income tax on a taxable income of $85,000, assuming you are single—it would come to $18,427.

6·4.B. Before-Tax vs. After-Tax Expenses

Taxpayers prefer before-tax expenses to (equal) after-tax expenses.

It is important for you to understand the difference between **before-tax expenses** and **after-tax expenses.** Before-tax expenses reduce the income before taxable income is computed. After-tax expenses have no effect on tax computations. Everything else being equal, if the IRS allows you to designate a payment to be a before-tax expense, it is more favorable to you, because it reduces your tax burden. For example, if you earn $100,000 and there were only one 40% bracket, a $50,000 before-tax expense leaves you

$$\begin{aligned} (\$100,000 - \$50,000) \cdot (1 - 40\%) &= \$30,000 \quad , \\ \text{Before-Tax Net Return} \cdot (1 - \text{Tax Rate}) &= \text{After-Tax Net Return} \end{aligned} \tag{6.4}$$

while the same $50,000 expense if post-tax leaves you only with

$$\$100,000 \cdot (1 - 40\%) \;-\; \$50,000 \;=\; \$10,000 \quad . \tag{6.5}$$

We have already discussed the most important tax-shelter: both corporations and individuals can and often reduce their income tax by paying interest expenses, although individuals can do so only for mortgages. (Chapter 19 explores income tax reduction schemes for corporations in great detail.)

ANECDOTE: The Use of Taxes
Where do all the taxes go?

In 2002, there were 128 million households filing federal income taxes. About $1 trillion (or about $7,800 per household) went to entitlement programs (primarily **Social Security** and **Medicare**). About $350 billion (or $2,700 per household) each went to defense spending and to non-defense spending. In inflation-adjusted terms, defense expenditures have remained roughly constant since 1962, non-defense expenditures have doubled, and entitlements have grown eightfold. Entitlements are projected to continue growing rapidly in the future, although this growth is likely not to be economically sustainable by the working population. (Source: The Heritage Foundation.)

State and Local Government expenditures in total are about half as large as federal government expenditures. In 2002, total government expenditures ran at about $2,855 billion per year, for a Gross Domestic Product of $10,150 billion, almost 30% of the total. Put differently, for every two dollars spent in the private economy, one dollar is spent by government somewhere.

However, even the interest tax deduction has an opportunity cost, the oversight of which is a common and costly mistake. Many home owners believe that the deductibility of mortgage interest means that they should keep a mortgage on the house under all circumstances. It is not rare to find a home owner with both a 6% per year mortgage and a savings account (or government bonds) paying 5% per year. Yes, the 6% mortgage payment is tax deductible, and effectively represents an after-tax interest cost of 4% per year for a tax payer in the 33% marginal tax bracket. But, the savings bonds pay 5% per year, which are equally taxed at 33%, leaving only an after-tax interest rate of 3.3% per year. Therefore, for each $100,000 in mortgage and savings bonds, the house owner throws away $667 in before-tax money (equivalent to $444 in after-tax money).

Save yourself some money by not repeating this mistake!

6·4.C. Average and Marginal Tax Rates

It is also important for you to distinguish between the **average tax rate** and the **marginal tax rate.** The average tax rate is the total income tax divided by the income. In our example from Table 6.5, the average tax rate is

The Average Tax Rate is what you pay on your overall income.

$$\begin{aligned} \text{Average Tax Rate} &= \frac{\$18,427}{\$85,000} \approx 21.68\% \\ &= \frac{\text{Total Tax Paid}}{\text{Total Taxable Income}} \ . \end{aligned} \tag{6.6}$$

(Some people prefer computing average tax rates relative to Total Income, rather than relative to Total Taxable Income). In this example, Uncle Sam receives 21.68% of this individual's taxable income.

In contrast, the marginal tax rate is the tax rate that applies to the last dollar earned (i.e., someone's tax bracket, as explained above). In our example with an income of $85,000, Table 6.4 shows that this rate was

The Marginal Tax Rate is what you paid on your last dollar of income, and what you would have to pay on one more dollar of income.

$$\text{Marginal Tax Rate} = 28\% \ . \tag{6.7}$$

The marginal tax rate is important, because it applies to any additional activity you might undertake. For example, if you want to work one extra hour at a $40 an hour pre-tax pay rate, you only receive $(1 - 28\%) \cdot \$40 = \$28.80$ as spendable extra cash, not $(1 - 21.68\%) \cdot \$40 = \$31.32$. Thus, in the decision whether to work (which is itself a *project*) or to play basketball (a sort of consumption "project" that is not taxed!), it is the marginal tax rate that matters, not the average tax rate.

Or, take a company facing the same tax situation (income and tax rate) next year, which now has to decide between investing in a project that costs $100 and will return $110, or investing in a tax-exempt vehicle that costs $100 and will return $107.50. If it takes the project, its earnings will increase from $85,000 to $85,010. At the marginal tax rate of 28%, its taxes will increase to $\$18,427 + 0.28 \cdot 10 \approx 18,429$, slightly raising the average tax rate (still 21.68%, though). So the extra after-tax income will only be $7.20—less than the $7.50 that the firm can get from putting its $100 into the tax-exempt vehicle instead. The firm's average tax rate of 21.89% is irrelevant—whatever the firm was able to avoid in taxes on its first dollars of earnings is the same and thus does not matter to each additional dollar. Economists are almost always more interested in the marginal tax rate than the average tax rate.

In deciding between corporate investments, the marginal tax rate matters!

6·4.D. Dividend and Capital Gains Taxes

Capital gains are taxed less than ordinary income.

While ordinary income applies to products and services sold, **capital gain** applies to income that is earned when an investment asset that was purchased is sold for a higher price. Capital gains are peculiar in three ways:

1. If the asset is held for more than a year, the capital gain is not taxed at the ordinary income tax rate, but at a lower long-term capital gains tax rate. (In 2002, the long-term capital gains tax rate is 15 percent for taxpayers that are in the 25% tax bracket or higher.)

2. Capital losses on the sale of one asset can be used to reduce the taxable capital gain on another sale.

3. The tax obligation occurs only at the time of the realization: if you own a painting that has appreciated by $100,000 each year, you did not have to pay $20\% \cdot \$100,000$ each year in taxes. The painting can increase in value to many times its original value, without you ever having to pay a dime in taxes, just as long as you do not sell it. In contrast, $100,000 in income per year will generate immediate tax obligations—and you even will have to pay taxes again if you invest the labor income for further gains.

Dividends—until 2008 ?!

More on dividends is in Chapter 19.

Dividends, that is, payments made by companies to their stock owners, used to be treated as ordinary income. However, the "Bush 2003 tax cuts" (formally, the *Jobs&Growth Tax Relief Reconciliation Act of 2003*) reduced the tax rate to between 5% and 15%, the same as long-term capital gains taxes—provided that the paying company itself has paid sufficient corporate income tax. However, this will only be in effect until 2008, when dividends may be taxed at the ordinary income tax level again. There is no guarantee that this will not change every couple of years, so you must learn how to think about dividend taxes, not the current details of dividend taxes.

Corporations are "lonelier" in the U.S., because they do not want to be taxed on inter-corporate dividend distributions.

In the United States, corporations holding shares in other companies are also taxed on dividend proceeds. This makes it relatively inefficient for them to hold cross equity stakes in dividend paying companies. However, in Europe, dividends paid from one corporations to another are often tax-exempted or tax-reduced. This has allowed most European corporations to become organized as pyramids or networks, with cross-holdings and cross-payments everywhere. (In effect, such cross-holdings make it very difficult for shareholders to influence management.)

ANECDOTE: Taxing the Rich, or Taxing the Wealthy?

The political rhetoric about who wants to "tax the rich" is just false. Neither Democrats nor Republicans ever debate about taxing the rich. Instead, they debate about taxing high-income individuals. This matters because there is a big difference between wealth increases and income. The lowest tax rates on wealth increases are enjoyed by the richest Americans. Most of these are households that earn most of their wealth increases not in ordinary income, but in capital gains from existing wealth. If not realized, these wealth increases may never have been taxed, at all! For example, Warren Buffett has probably paid about 0.0% in personal income tax on his wealth increase of over $30 billion over the last 30 years.

In contrast, most ordinary households receive their annual wealth increases in ordinary income or interest receipts. These wealth increases are taxed every year, and are taxed at a much higher marginal tax rate than capital gains wealth increases. Roughly, an ordinary worker's wealth increases suffer taxes to the tune of about 25% to 50% per year. (Chapters 18–19 will describe tax sheltering in greater detail.) The primary tax on wealth today seems to come from property taxes—the value of one's house. Again, this does not much to tax the "super-wealthy," because for them, the residence constitutes only a very small part of their wealth.

Although a tax on all wealth increases would be fairer than the current income tax *and* provide better incentives to work, it would also be far more difficult to administer. Nevertheless, most European countries have both annual wealth-based taxes and wealth-based inheritance taxes.

6·4.E. Other Taxes

Other Income Taxes: state, county, international taxes.

In addition to federal income taxes, there are a plethora of other taxes. Most states impose their own income tax. This typically adds another tax rate of between 0% and 10%, depending on state and income. Worse, each state has its own idea not only of what its tax rate and tax brackets should be, but even how taxable income should be computed. Thus, you need to learn not only the federal tax code, but also your state's tax code. For example, California has the highest marginal state income tax bracket that is not federal deductible: 9.3%. Montana has the highest marginal state income tax bracket that is tax deductible on your federal income tax: 11%. Alaska, Florida, Nevada, South Dakota, Texas, Washington, and Wyoming levy no state income tax, and New Hampshire and Tennessee tax only interest and dividend income.

Property Taxes are state/local.

Many counties pay for school education with property tax rates. In the richer counties of southwest Connecticut, the tax is about 1% of the value of the house, but it can reach about 4% in the poorer urban counties. In Maine (the second-highest property tax collector in the nation), residents pay 5.5% of their income in property tax. Many states also levy a sales tax. Tennessee and Louisiana have a sales tax of 8.35%; Alaska, Delaware, Montana, New Hampshire, and Oregon levy no sales tax. (A nice summary can be found at www.retirementliving.com/RLtaxes.html. It also includes a ranking of the tax burdens by state—Alaska [with 6% of total income], New Hampshire, Delaware, and Tennessee have the lowest; New York [with 12.9% of total income], Maine, Ohio, and Hawaii have the highest.)

Complex Complexity.

If you have to file in multiple states or even in multiple countries—although there are rules that try to avoid double taxation—the details can be hair-raisingly complex. If you find yourself in such a situation, may the force be with you!

"Social" Taxes: Social Security and Medicare.

Finally, there are **social security** and **medicare** contributions. Although these are supposedly insurance premia, any money taken in today is immediately spent by the government on the elderly today. Thus, anyone young today is unlikely to receive much in return from the government in 20 to 30 years—when there will be fewer young people around to pay their retirement benefits. Thus, many financial economists consider social taxes to be as much a form of income tax as the statutory income tax.

Non Income Based Taxes: Sales Tax, Real Estate Taxes, Estate Taxes.

This book also ignores many other non-income taxes. For some taxes, such as the **sales tax**, it is not clear how to use expertise in finance to lower them. For other taxes, such as the **estate tax**, you need extremely specialized financial vehicles to avoid or reduce them. These are beyond the scope of this book.

6·4.F. What You Need To Know About Tax Principles In Our Book

IMPORTANT: *You must understand*

1. *how income taxes are computed (the principles, not the details);*
2. *the fact that expenses that can be paid from before-tax income are better than expenses that must be paid from after-tax income;*
3. *how to compute the average tax rate;*
4. *how to obtain the marginal tax rate;*
5. *the fact that capital gains enjoy preferential tax treatment;*
6. *why the average and marginal tax rates differ, and why the marginal tax rate is usually higher than the average tax rate.*

What you will learn later about taxes.

As already noted, you will later have to pay special attention to three facts: that corporations and individuals can deduct certain interest expenses; that capital gains are taxed at lower tax rates; and that some retirement account investment returns are tax-exempt. These features of the tax code offer individuals and corporations opportunities to legally reduce their tax obligations.

Solve Now!

For all questions here, assume that this investor has $1,000 in valid interest deductions.

Q 6.17 *What are the average and marginal federal tax rates for a single individual earning $5,000? Repeat for a corporation.*

Q 6.18 *What are the average and marginal federal tax rates for a single individual earning $50,000? Repeat for a corporation.*

Q 6.19 *What are the average and marginal federal tax rates for a single individual earning $50,000,000? Repeat for a corporation.*

6·5. Working With Taxes

In one sense, taxes are very similar to transaction costs—they take a "cut," making investments less profitable. However, taxes are often orders of magnitude bigger and thus more important than ordinary transaction costs and—except for illustrative examples—you should not simply assume them away, which is quite different from what you can sometimes do with transaction costs. (Ignoring taxes may be a good assumption for the tax-exempt Red Cross, but probably not for you or for the ordinary corporation!) Another difference between taxes and transaction costs is that taxes are higher on profitable transactions, whereas plain transaction costs do not care whether you made money or lost money. In addition, taxes often have many more nuances. We now try to understand better how to work with income taxes.

6·5.A. Taxes in Rates of Returns

Taxable Investors (unlike tax-exempt investors) care about post-tax inflows and outflows.

In the end, all you probably care about are your after-tax returns, not your pre-tax returns. It should not matter whether you receive $100 that has to be taxed at 50% or whether you receive $50 that does not have to be taxed. This leads to an recommendation analogous to that for transaction costs—work only in after-tax money. For example, say you invest $100,000 in after-tax money to earn a return of $160,000. Your marginal tax rate is 25%. Taxes are on the net return of $60,000, so your after-tax net return is

$$\begin{array}{rcl} 75\% \cdot \$60,000 & = & \$45,000 \\ (1-\tau) \cdot \text{Before-Tax Net Return} & = & \text{After-Tax Net Return} \quad . \end{array} \tag{6.8}$$

(The tax rate is often abbreviated with the Greek letter τ, tau.) In addition, you will receive your original investment back, so your after-tax rate of return is

$$r_{\text{After Tax}} = \frac{\$145,000 - \$100,000}{\$100,000} = 45\% \quad . \tag{6.9}$$

6·5.B. Tax-Exempt Bonds and the Marginal Investor

Municipal bonds' interest payments are legally exempt from income taxes.

In the United States, there are bonds that are issued by governmental entities, whose interest payments are legally tax-exempt—the reasoning of the federal government being that it does not want to burden states' or local governments' efforts to raise money. If you own one of these bonds, you do not need to declare the interest on your federal income tax forms, and sometimes not even on your state's income tax form, either. (The arrangement differs from bond to bond.) The most prominent tax-exempt bonds are called **municipal bonds** or **muni bonds** or even **munis** for short. As their name suggests, they are usually issued by municipalities such as the City of Los Angeles (CA) or the City of Canton (OH).

The May 2002 Situation.

On May 31, 2002, the *Wall Street Journal* reported on Page C12 that tax-exempt municipal 7-12 year highly rated bonds (AA) offered an annualized interest rate of 5.24%. Bonds of similar risk issued by corporations offered an interest rate of about 6.76%. Which one would be a better investment *for you*? Well, it depends.

Comparing After-Tax Returns of Tax-Exempt and Taxable Bonds.

If you invested $1,000 into munis at a 5.24% interest rate, you would receive $52.40 at year's end. You would get to keep all of it, because these bonds are tax-exempt. If you invested $1,000 in taxable bonds at a 6.76% interest rate, you would receive $67.60 at year's end. If your income tax rate is 0%, you would clearly prefer the $67.60 to the $52.40. However, if your marginal income tax rate is 30%, Uncle Sam would collect $20.28 and leave you with $47.32. Your after-tax rate of return is

$$\begin{aligned} r_{\text{post-tax}} &= (1-30\%)\cdot 6.76\% = 70\%\cdot 6.76\% \approx 4.73\% \\ r_{\text{post-tax}} &= (1-\tau)\cdot r_{\text{pre-tax}} \quad . \end{aligned} \tag{6.10}$$

With a 30% tax rate, you would prefer the tax-exempt bond that pays $52.40.

High-income tax bracket individuals should prefer tax-exempt bonds; low-income tax bracket individuals should prefer taxable bonds.

Economists sometimes like to talk about a hypothetical marginal investor. This is an investor whose marginal income tax rate is such that she would be exactly indifferent between buying the tax-exempt and the taxable bond. Using Formula 6.10, the marginal investor has a tax rate of

$$\begin{aligned} 5.24\% = (1-\tau_{\text{marginal}})\cdot 6.76\% \quad &\Leftrightarrow \quad \tau_{\text{marginal}} = 1-\frac{5.24\%}{6.76\%} \approx 22.5\% \\ r_{\text{post-tax}} = (1-\tau_{\text{marginal}})\cdot r_{\text{pre-tax}} \quad &\Leftrightarrow \quad \tau_{\text{marginal}} = 1-\frac{r_{\text{post-tax}}}{r_{\text{pre-tax}}} \quad . \end{aligned} \tag{6.11}$$

Any investor with a marginal income tax rate above 22.5% should prefer the tax-exempt bond. Any investor with a marginal income tax rate below this income tax rate should prefer the taxable bond.

Munis do have default (credit) risk. See next chapter.

Unfortunately, unlike the U.S. Treasury, municipalities can and have gone bankrupt, so that they may not fully repay. (The most prominent recent default was the Orange County (CA) default in December 1994.) Municipal bonds are not an entirely risk-free investment.

Solve Now!

Q 6.20 *On May 31, 2002, for short-term bonds, the Bond Market Data Bank Section in the Wall Street Journal (Page C15) indicates the ratio between the equivalent yields of AAA municipal and Treasury securities to be around 74.6%. What is the marginal investor's tax rate?*

Q 6.21 *On May 31, 2002, for long-term bonds, the Bond Market Data Bank Section in the Wall Street Journal (Page C15) computes the ratio between the AAA municipal and Treasury securities to be around 92% (for short-term municipal bonds). What is the marginal investor's tax rate?*

6·5.C. Taxes in NPV

Compute everything in After-Tax Dollars!

Again, as with transaction costs, you should take care to work only with cash in the same units—here, this means cash that you can use for consumption. Again, it should not matter whether you receive \$100 that has to be taxed at 50% or whether you receive \$50 that does not have to be taxed. As far as NPV is concerned, everything should be computed in after-tax dollars. This includes all cash flows, whether today or tomorrow, whether cash inflows or outflows.

> **IMPORTANT**: *Do all NPV calculations in after-tax money.*

You must compute the after-tax opportunity cost of capital.

Unfortunately, you cannot simply discount pre-tax cash flows with the pre-tax cost of capital (wrong!) and expect to come up with the same result as when you discount after-tax cash flows with after-tax costs of capital (right!).

An example—how to pick your opportunity cost of capital.

For example, consider a project that costs \$10,000 and returns \$13,000 next year. Your tax rate is 40%, and 1-year equivalently risky bonds return 25% if their income is taxable, and 10% if their income is not taxable. First, you must decide what your opportunity cost of capital is. Section 6·5.B tells you that if you put \$100 into taxables, you will own \$125, but the IRS will confiscate $(\$125 - \$100) \cdot 40\% = \$10$. You will thus own \$115 in after-tax income. Tax-exempts grow only to \$110, so you prefer the taxable bond—it is the taxable bond that determines your opportunity cost of capital. Your equivalent after-tax rate of return is therefore 15%. This 15% is your after-tax "opportunity" cost of capital—it is your best use of capital elsewhere.

You must apply it to the after-tax expected cash flows.

Return to your \$10,000 project now. You know that your taxable project returns 30% taxable (\$3,000), while taxable bonds return 25% (\$2,500), so NPV should tell you to take this project. Uncle Sam will confiscate $40\% \cdot \$3,000 = \$1,200$, leaving you with \$11,800. Therefore, the NPV of your project is

$$\begin{aligned} \text{NPV} &= -\$10,000 + \frac{\$11,800}{1+15\%} = \$260.87 \\ \text{NPV} &= \text{CF}_0 + \frac{\mathcal{E}(\text{CF}_1)}{1+\mathcal{E}(r_{0,1})} \, . \end{aligned} \tag{6.12}$$

It makes intuitive sense: if you had invested money into the bonds, you would have ended up with \$11,500. Instead, you will end up with \$11,800, the \$300 difference occurring next year. Discounted, the \$261 seems intuitively correct. Of course, there is an infinite number of ways of getting incorrect solutions, but recognize that none of the following calculations that use the pre-tax expected cash flows (and try different discount rates) give the same correct result:

$$\begin{aligned} \text{NPV} &\neq -\$10,000 + \frac{\$13,000}{1+25\%} = \$400 \\ \text{NPV} &\neq -\$10,000 + \frac{\$13,000}{1+15\%} = \$1,304.35 \\ \text{NPV} &\neq -\$10,000 + \frac{\$13,000}{1+10\%} = \$1,818.18 \end{aligned} \tag{6.13}$$

You have no choice: *you cannot work with pre-tax expected cash flows.* Instead, you need to go through the exercise of carefully computing after-tax cash flows and discounting with your after-tax opportunity cost of capital.

Can you compare two projects based on pre-tax NPV?

You know that computing after-tax cash flows is a pain. Can you at least compare two equally taxable projects in terms of their pre-tax NPV? If one project is better than the other in pre-tax terms, is it also better in after-tax terms? If yes, then you could at least do relative capital budgeting with pre-tax project cash flows. This may or may not work, and here is why. Compare project SAFE that costs \$1,000 and will provide \$1,500 this evening; and project UNSAFE that costs \$1,000 and will provide either \$500 or \$2,500 this evening with equal probability. The expected payout is the same, and the cost of capital is practically 0% for 1 day. If you are in the 20% marginal tax bracket, project SAFE will leave the IRS with $20\% \cdot (\$1,500 - \$1,000) = \$100$, and you with +$400 in after-tax net return. Project UNSAFE will either give you $1,500 or –$500 in *taxable* earnings.

- If you can use the losses to offset other gains elsewhere, then you would either send $\$1,500 \cdot 20\% = \$300$ extra to the IRS; or you would send $100 *less* to the IRS (because your taxable profits elsewhere would be reduced). In this case, project SAFE and UNSAFE would have the same expected tax costs and after-tax cash flows.
- If you drop into a different tax bracket beyond an additional net income of $1000, say 25%, then project UNSAFE becomes less desirable than project SAFE. For the $1,500 income, the first $500 would still cost you $100 in tax, but the remaining $1,000 would cost you $250. Thus, your project's marginal tax obligation would be either $350 or −$100, for an expected tax burden of $125. (The same logic applies if your losses would make you fall into a lower tax bracket—the UNSAFE project would become less desirable.)
- If you have no gains elsewhere to use your project tax loss against, then the UNSAFE project would again be worth less. Corporations can ask for a tax refund on old gains, so this factor is less binding than it is for individuals, who may have to carry the capital loss forward until they have sufficient income again to use it—if ever.

Thus, whether you can compare projects on a pre-tax basis depends on whether you have perfect symmetry in the applicable marginal tax rates across projects. If you do, then the project that is more profitable in after-tax terms is also more profitable in pre-tax terms. This would allow you to simply compare projects by their pre-tax NPVs. If gains and losses face different taxation—either because of tax bracket changes or because of your inability to use the tax losses elsewhere—then you cannot simply choose the project with the higher pre-tax NPV. You will have to go through the entire after-tax NPV calculations and compare these.

IMPORTANT: *You can only compare projects on a before-tax NPV basis if the tax treatment is absolutely symmetric. This requires consideration of your overall tax situation.*

WACC and APV unfortunately have to wait.

You now know how to discount projects in the presence of income taxes. However, you do not yet know how to compute the proper discount rate for projects that are financed by debt and equity, because debt and equity face different tax consequences. Unfortunately, you will have to wait until Chapter 18 before we can do a good job discussing the two suitable methods—called APV and WACC—to handle differential taxation by financing. Until we will have covered investments in Part III, you just do not have all the necessary pieces, and your goal must be to understand formulas, rather than just eat them.

Solve Now!

Q 6.22 *You have a project that costs $50,000 and will return $80,000 in three years. Your marginal tax rate is 37.5%. Treasuries pay a rate of return of 8% per year, munis pay a rate of return of 3% per year. What is the NPV of your project?*

6·5.D. Tax Timing

Do not forget that even when inflows require after-tax dollars, sometimes outflows are taxed again.

In many situations, the IRS does not allow reinvestment of funds generated by a project without an interim tax penalty. This can be important when you compare one long-term investment to multiple short-term investments that are otherwise identical. For example, consider a farmer in the 40% tax bracket who purchases grain that costs \$300, and that triples its value every year.

- If the IRS considers this farm to be one long-term two-year project, the farmer can use the first harvest to reseed, so \$300 seed turns into \$900 in one year and then into a \$2,700 harvest in two years. Uncle Sam considers the profit to be \$2,400 and so collects taxes of \$960. The farmer is left with post-tax profits of \$1,440.
- If the IRS considers this production to be two consecutive one-year projects, then the farmer ends up with \$900 at the end of the first year. Uncle Sam collects $40\%\cdot\$600 = \$240$, leaving the farmer with \$660. Replanted, the \$660 grows to \$1,980, of which the IRS collects another $40\%\cdot\$1,980 = \$792$. The farmer is left with post-tax profits of $60\%\cdot\$1,980 = \$1,188$.

The discrepancy between \$1,440 and \$1,188 is due to the fact that the long-term project can avoid the interim taxation. Similar issues arise whenever an expense can be reclassified from “reinvested profits” (taxed, if not with some credit at reinvestment time) into “necessary maintenance.”

Although you should always get taxes right—and really know the details of the tax situation that applies to you—be aware that you must particularly pay attention to getting taxes right if you are planning to undertake real estate transactions. These have special tax exemptions and tax depreciation writeoffs that are essential to getting the project valuation right.

Solve Now!

Q 6.23 *It is not uncommon for individuals to forget about taxes, especially when investments are small and payoffs are large but rare. Presume you are in the 30% tax bracket. Is the NPV of a \$1 lottery ticket that pays off taxable winnings of \$10 million with a chance of 1 in 9 million positive or negative? How would it change if you could purchase the lottery ticket with pre-tax money?*

6·6. Inflation

We have now discussed all violations from the assumptions necessary for our perfect market Utopia. So, what are we doing now? If you return to our perfect markets assumptions, you will see that "no inflation" was not among them. **Inflation** is the process by which goods cost more in the future than they cost today—in which the price level is rising and money is losing its value.

Back to our perfect markets assumptions.

So, inflation is actually not a market imperfection per se. If today we quoted everything in dollars, and tomorrow we quote everything in cents—so that an apple that cost 1 currency unit today will cost 100 currency units tomorrow, an inflation of 10,000%—would it make any difference? Not really. The apple would still cost the same in terms of foregone other opportunities, whether it is 1 dollar or 100 cents.

Known inflation applicable everywhere is irrelevant.

However, we have made a big assumption here—inflation applied equally to everything, and especially applied equally to all contracts across time. See, if you had contracted to deliver apples at 1 currency unit tomorrow, whatever currency units may be, you could be in big trouble—you would have promised to sell your apples at 1 cent (1 currency unit) instead of $1. Most financial contracts are denominated in such "nominal" terms—that is, in plain currency units—so inflation would matter. Of course, inflation would not be much of a concern for a financial contract that would be "inflation-indexed."

...but inflation is often not applicable everywhere.

What effect does inflation have on returns? On (net) present values? This is the subject of this section. As before, we start with interest rates and then proceed to net present values.

Our agenda.

6·6.A. Defining the Inflation Rate

The first important question is how you should define inflation. Is the rate of change of the price of apples the best measure of inflation? What if apples (the fruit) become more expensive, but Apples (the computers) become less expensive? Defining inflation is somewhat tricky. To solve this problem, economists have invented *baskets* or *bundles* of goods that are deemed to be representative, for which they can then measure an average price change. The official source of most inflation measures is the **Bureau of Labor Statistics** (**B.L.S.**), which determines the compositions of a number of prominent bundles (indexes), and publishes the average total price of these bundles on a monthly basis. The most prominent such inflation measure is a hypothetical bundle of average household consumption, called the **Consumer Price Index** (or **CPI**). (The CPI components are roughly: housing 40%, food 20%, transportation 15%, medical care 10%, clothing 5%, entertainment 5%, others 5%.) The *Wall Street Journal* prints the percent change in the CPI at the end of its column *Money Rates.* (On May 31, 2002, the Consumer Price Index was increasing at a rate of 1.6%/year.) A number of other indexes are also in common use as inflation measures, such as the **Producer Price Index** (**PPI**) or the broader **GDP Deflator.** They typically move fairly similarly to the CPI. There are also more specialized bundles, such as computer inflation indexes (the price of equivalent computer power does not inflate, but deflate, so the rate is usually negative), or indexes for prices of goods purchased in a particular region.

The CPI is the most common inflation measure.

ANECDOTE: The German Hyperinflation of 1922
The most famous episode of **hyperinflation** occurred in Germany from August 1922 to November 1923. Prices more than quadrupled every month. The price for goods was higher in the evening than in the morning! Stamps had to be overprinted by the day, and shoppers went out with bags of money—that were worthless at the end of the day. By the time Germany printed 1,000 billion Mark Bank Notes, no one trusted the currency anymore. This hyperinflation was stopped only by a drastic currency and financial system reform. But high inflation is not just a historic artifact. For example, many Latin American countries experienced annual doubling of prices in the early 1980s.

The opposite of inflation is **deflation** (negative inflation)—a process in which the price level falls. Though much rarer, it happens. In fact, in November 2002, Business Week reported that an ongoing recession and low demand continue to force an ongoing decline in Japanese prices.

Many economists now believe that a modest inflation rate between 1% and 3% per year is a healthy number.

The CPI matters—even if it is wrong.

The official inflation rate is not just a number—it is important in itself, because many contracts are **rate-indexed**. For example, even if actual inflation is zero, if the officially reported CPI rate is positive, the government must pay out more to social security recipients. The lower the official inflation rate, the less the government has to pay. You would therefore think that the government has the incentive to understate inflation. But strangely, this has not been the case. On the contrary, there are strong political interest groups that hinder the B.L.S. from even just improving on mistakes in the CPI because it would result in *lower* official inflation numbers. In 1996, the *Boskin Commission*, consisting of a number of eminent economists, found that the CPI overstates inflation by about 74 basis points per annum—a huge difference. The main reasons are that the B.L.S. has been tardy in recognizing the growing importance of such factors as computer and telecommunication effective price declines, and the role of superstores such as Wal-Mart.

One final warning:

> **IMPORTANT**: *The common statement "in today's dollars" is ambiguous. Some people mean "inflation adjusted." Other people mean present values (i.e., "compared to an investment in risk-free bonds"). When in doubt, ask!*

Solve Now!

Q 6.24 *Using information from a current newspaper or the WWW, find out what the current inflation rate is.*

6·6.B. Real and Nominal Interest Rates

Nominal is what is normally quoted. Real is what you want to know.

To work around inflation, you first need to learn the difference between a **nominal return** and a **real return.** The nominal return is what everyone usually quotes—a return that has not been adjusted for inflation. In contrast, the real return somehow "takes out" inflation from the nominal return in order to calculate a return "as if" there had been no price inflation to begin with. It is the real return which reflects the fact that, in the presence of inflation, a dollar in the future will have less purchasing power than a dollar today. It is the real rate of return that measures your tradeoff between present and future consumption, taking into account the change in prices.

An Extreme 100% Inflation Rate Example: Prices Double Every Year.

Consider a simple no-uncertainty scenario: assume that the inflation rate is 100% per year, and you can buy a bond that promises a *nominal* interest rate of 700% (the bond payout is quadruple your pay-in). What is your *real* rate of return? To find out, assume that \$1 buys one apple today. With an inflation rate of 100%, you need \$2 next year to buy the same apple. Your investment return will be $\$1 \cdot (1 + 700\%) = \$8$ for today's \$1 of investment. But this \$8 now applies to apples costing \$2 each. So, your \$8 will buy 4 apples, and not 8 apples. Your real rate of return is

$$r_{\text{real}} = \frac{4 \text{ Apples} - 1 \text{ Apples}}{1 \text{ Apples}} = 300\% \quad . \tag{6.14}$$

For each dollar invested today, you will be able to purchase only 300 percent more apples next year (not 700% more apples) than you could purchase today. This is because the purchasing power of your dollar next year will be reduced by half.

The Conversion Formula from Nominal to Real Rates.

The correct formula to adjust for inflation is again a "one-plus" type formula. In our example, it is

$$\begin{aligned} (1+700\%) &= (1+100\%) \cdot (1+300\%) \\ (1+r_{\text{nominal}}) &= (1+\text{Inflation Rate}) \cdot (1+r_{\text{real}}) \quad . \end{aligned} \tag{6.15}$$

Turning this formula around solves for real rates of return,

$$(1+r_{\text{real}}) = \frac{1+700\%}{1+100\%} = 1+300\%$$
$$(1+r_{\text{real}}) = \frac{(1+r_{\text{nominal}})}{(1+\text{Inflation Rate})} \quad . \tag{6.16}$$

IMPORTANT: *The relation between nominal rates of return ($r_{nominal}$), real rates of returns (r_{real}), and inflation (π) is*

$$(1+r_{nominal}) = (1+r_{real}) \cdot (1+\pi) \quad . \tag{6.17}$$

For small rates, adding/subtracting is ok.

As with compounding, if both inflation and the nominal interest rate are small, the mistake of just subtracting the inflation rate from the nominal interest rate to obtain the real interest rate is not too grave. The difference is a cross-term (see Page 19),

$$r_{\text{real}} = r_{\text{nominal}} - \pi \quad - \underbrace{r_{\text{real}} \cdot \pi}_{\text{cross-term}} \quad . \tag{6.18}$$

For example, as of mid-2004, the official CPI inflation rate fluctuated month-to-month from about 2.5% to 3% per annum. The 10-year Treasury bond paid 4% per annum on October 31, 2004. Therefore, if you believe that the inflation rate will remain at 2.5% per annum, you would presume a real rate of return of about $4\% - 2.5\% \approx 1.5\%$—though this would ignore the cross-term. The more accurate computation would be $(1+4\%)/(1+2.5\%)-1 \approx 1.46\%$. The cross-term difference of 4 basis points is swamped by your uncertainty about the future inflation rate—at least as of 2004. However, when inflation and interest rates are high—as they were, e.g., in the late nineteen-seventies—then the cross-term can make quite a meaningful difference.

Real interest rates can be negative.

A positive time-value of money—the fact that money tomorrow is worth more than money today—is only true for nominal quantities, not for real quantities. Only nominal interest rates are never negative. In the presence of inflation, real interest rates not only *can* be negative, but often *have* been negative. In such situations, by saving money, you would have ended up with more money—but with less purchasing power, not more purchasing power. Of course, if there are goods or project that appreciate with inflation (inflation hedges, such as real estate or gold), and to the extent that these goods are both storable and traded in a perfect market, you would not expect to see negative real rates of return. After all, you could buy these projects today and sell them next year, and thereby earn a real rate of return that is positive.

Solve Now!

Q 6.25 *Using information from a current newspaper or the WWW, find out what the annualized current 30-day nominal interest rate is.*

Q 6.26 *Using the information from the previous two questions, determine the annualized current real interest rate?*

Q 6.27 *From memory, write down the relationship between nominal rates of return ($r_{nominal}$), real rates of return (r_r), and the inflation rate (π).*

Q 6.28 *The nominal interest rate is 20%. Inflation is 5%. What is the real interest rate?*

Q 6.29 *The inflation rate is 1.5% per year. The real rate of return is 2% per year. A perpetuity project that payed $100 this year will provide income that grows by the inflation rate. Show what this project is truly worth. Do this in both nominal and real terms. (Be clear on what never to do.)*

6·6.C. Handling Inflation in Net Present Value

The most fundamental rule is to never mix apples and oranges. Nominal cash flows must be discounted with nominal interest rates.

When it comes to inflation and net present value, there is a simple rule: never mix apples and oranges. The beauty of NPV is that every project, every action is translated into the same units: today's dollars. Keep everything in the same units in the presence of inflation, so that this NPV advantage is not lost. When you use the NPV formula, always discount nominal cash flows with nominal costs of capital, and real (inflation-adjusted) cash flows with real (inflation-adjusted) costs of capital.

A Previous Example Revisited.

Let's show this. Return to our "apple" example. With 700% nominal interest rates and 100% inflation, the real interest rate is $(1 + 700\%)/(1 + 100\%) - 1 = 300\%$. What is the value of a project that gives 12 apples next year, given that apples cost $1 each today and $2 each next year? There are two methods you can use.

1. Discount the nominal value of 12 apples next year ($\$2 \cdot 12 = \$24$) with the nominal interest rate. Thus, the 12 future apples are worth
$$\frac{\text{Nominal Cash Flow}_1}{1 + \text{nominal rate}_{0,1}} = \frac{\$24}{1 + 700\%} = \$3 \quad . \tag{6.19}$$

2. Discount real cash flows (i.e., 12A) with the real interest rate. Thus, the 12 future apples are worth
$$\frac{\text{Real Cash Flow}_1}{1 + \text{real rate}_{0,1}} = \frac{12A}{1 + 300\%} = 3A \quad , \tag{6.20}$$
in today's apples. Because an apple costs $1 today, the eight apples are worth $3.

Both methods arrive at the same result. The opportunity cost of capital is that if you invest one apple today, you can quadruple your apple holdings by next year. Thus, a 12 apple harvest next year is worth 3 apples to you today. The higher nominal interest rates already reflect the fact that nominal cash flows next year are worth less than they are this year.

IMPORTANT:

- *Discount nominal cash flows with nominal interest rates.*
- *Discount real cash flows with real interest rates.*

Either works. Never discount nominal cash flows with real interest rates, or vice-versa.

Usually, use nominal interest rates.

If you want to see this in algebra, the reason that the two methods come to the same result is that the inflation rate cancels out,

$$\begin{aligned} \text{PV} &= \frac{\$24}{1 + 700\%} = \frac{12A}{1 + 300\%} = \frac{12A \cdot (1 + 100\%)}{(1 + 300\%) \cdot (1 + 100\%)} \\ &= \frac{N}{1 + n} = \frac{R}{1 + r} = \frac{R \cdot (1 + \pi)}{(1 + r) \cdot (1 + \pi)} \quad , \end{aligned} \tag{6.21}$$

where N is the nominal cash flow, n the nominal interest rate, R the real cash flow, r the real interest rate, and π the inflation rate. Most of the time, it is easier to work in nominal quantities.

Nominal interest rates are far more common than real interest rates, and you can simply use published inflation rates to adjust the future price of goods to obtain future expected nominal cash flows.

Solve Now!

Q 6.30 *If the real interest is 3% per annum, the inflation rate is 8% per annum, then what is the value of a \$500,000 payment next year?*

Q 6.31 *If the real interest is 3% per annum, the inflation rate is 8% per annum, then what is the value of a \$500,000 payment every year forever?*

Q 6.32 *Inflation is 2% per year, the interest rate is 8% per year. Our perpetuity project has cash flows that grow at 1% faster than inflation forever, starting with \$20 next year.*

(a) *What is the real interest rate?*

(b) *What is the project PV?*

(c) *What would you get if you grew a perpetuity project of \$20 by the real growth rate of 1%, and then discounted at the nominal cost of capital?*

(d) *What would you get if you grew a perpetuity project of \$20 by the nominal growth rate of 3%, and then discounted at the real cost of capital?*

Doing either of the latter two calculation is not an uncommon mistake.

Q 6.33 *You must value a perpetual lease. It will cost \$100,000 each year in real terms—that is, its proceeds will not grow in real terms, but just contractually keep pace with inflation. The prevailing interest rate is 8% per year, the inflation rate is 2% per year forever. The first cash flow of your project next year is \$100,000 quoted in today's real dollars. What is the PV of the project? (Warning: watch the timing and amount of your first payment.)*

6·6.D. Interest Rates and Inflation Expectations

Nominal Interest Rate Levels

Inflation affects the level of the nominal interest rate.

Should you take inflation into account? Absolutely. As an investor, like the market overall, you probably care more about real returns than nominal rates. Therefore, when purchasing financial investments, you must form an expectation of how this investment will affect your purchasing power. For example, if the nominal interest rate is 5%, you may prefer spending more money today if you believe the inflation rate to be 10% than if you believe it to be only 6%. Of course, if you have no better alternatives, you might still want to save money even if your real rate of return is negative. Be this as it may, you would expect nominal interest rates in the economy to be higher when inflation is higher. This also means that you would expect nominal rates to go up when inflation rate expectations are going up. Similarly, you would expect nominal rates to go down when inflation rate expectations are going down. Now, many investors also believe that stocks are good inflation hedges, in that they appreciate automatically in value when the inflation rate increases—after all, they are just claims on real projects, which presumably similarly experience a price increase. In the end, the exact real interest rates in the economy are determined by the demand and supply for capital, which is determined by these kinds of considerations.

TIPS and Short-Term Bonds as "Inflation Hedges"

Inflation is uncertainty

But what if you wanted to purchase a bond that is truly risk-free, i.e., a bond that promises a specified amount of purchasing power (a real amount, not a nominal amount)? The problem is that you do not yet know fully what *future* inflation will be. Inflation is a random variable, because you do not yet know what inflation will be over the bond's holding period. You can estimate it, but you do not really know.

Inflation-Adjusted Treasury Bonds [TIPS].

What you want is a bond that pays out 1% more in interest if inflation were to turn out 1% higher. In 1997, the U.S. Treasury reintroduced such inflation-adjusted bonds. They are called Treasury Inflation Protected Securities (or **TIPS**, or sometimes just **CPI Bonds**).

An example: the October 2004 situation.

In late October 2004, the 10-year T-bond offered 4.02% per annum, while the 10-year TIPS offered 1.6% per annum. If inflation turns out to be above 2.38% per annum over the 10-year interval, then the TIPS will have been the better purchase. If inflation turns out to be lower than 2.56% per annum, then the plain T-bond will have been the better purchase. A volatile oil price in 2004 had caused the inflation rate to fluctuate dramatically—it troughed at 1.69% in March 2004 and peaked at 3.27% in June 2004. Ladies and Gentlemen—place your bet.

Short-term securities also help you "hedge" against inflation.

TIPS are not the only way you can reduce your worry about future inflation. Short-term bonds are another way to reduce the effect of future inflation. Inflation increases are associated with higher interest rates. Thus, an inflation increase would allow a short-term bond investor to earn a higher interest rate upon reinvestment.

Does Future Inflation Drive the Yield Curve Slope?

It is harder to see why the expectation of inflation would affect the slope of the yield curve.

Now let us return to our question about what determines the slope of the ordinary Treasury yield curve. Recall from Page 61 that you might demand a higher long-term interest rate if you believed that future inflation will increase. For example, if you believe that inflation will be much higher from year 5 to year 10, you would be less inclined to accept the same 5% per annum for the 10-year Treasury bond that you might accept for the 5-year Treasury bond. After all, what you will end up getting back from your 10-year bond will be worth much less to you! You could also demand extra compensation if you were less certain about inflation from 5-years out to 10-years out than about inflation from now to 5-years out. Fortunately, you can now put this to the test using TIPS. In October 30, 2004, the yield curves was as follows—with implied inflation rates computed for you:

	Ordinary T-Bonds	TIPS	Implied Inflation
3-month	1.90%	(n/a)	
5-year	3.29%	0.90%	2.4%
10-year	4.02%	1.60%	2.4%
30-year	4.79%	2.06%	2.7%

Remember that the TIPS returns are unaffected by inflation, so neither your expectation nor your uncertainty about future inflation should influence the TIPS yield curve—and yet it is almost as steep as the ordinary yield curve. The 5-year and 10-year T-bond vs. TIPS interest spread even embody the same inflation expectation of 2.4% per annum. The yield difference between the 5-year and the 30-year T-bond is about 1.5%, similar to the 1.2% difference between

ANECDOTE: Inflation-Adjusting Bonds
As it turns out, inflation-adjusted bonds had already been invented once before! The world's first known inflation-indexed bonds were issued by the Commonwealth of Massachusetts in 1780 during the Revolutionary War. These bonds were invented to deal with severe wartime inflation and discontent among soldiers in the U.S. Army with the decline in purchasing power of their pay. Although the bonds were successful, the concept of indexed bonds was abandoned after the immediate extreme inflationary environment passed, and largely forgotten. In 1780, the bonds were viewed as at best only an irregular expedient, since there was no formulated economic theory to justify indexation.

Source: Robert Shiller.

the 5-year and the 30-year TIPS. So, inflation uncertainty can account for only a small fraction of the steepness of this yield curve. There must be something other than inflation that makes investors prefer shorter-term T-bonds to longer-term T-bonds and borrowers prefer longer-term T-bonds to shorter-term T-bonds by so much that they are willing to agree on several hundred basis points less compensation per annum on the short-term rate. Of course, it may be that the horizon-dependent expectations or uncertainties about inflation will play a more important role in the future—but in October 2004, they just did not.

Solve Now!

Q 6.34 *On May 31, 2002, the Wall Street Journal reported on Page C10 that a 30-year CPI bond offered a real yield of about 3.375%/year. The current inflation rate was only 1.6%/year, and a normal 30-year Treasury bond offered a nominal yield of 5.6%/year. Under what scenario would you be better off buying one or the other?*

6·7. Multiple Effects

Of course, in the messy real world, you can suffer inflation, transaction costs, imperfect markets, and taxes all at once, not just in isolation. In fact, there are so many possible real-world problems that no one can possibly give you a formula for each one. Thus, it is more important that you realize you must approach the real world thinking about two issues.

1. To what extent is the assumption of a perfect market appropriate? For example, in the case of large and possibly tax-exempt companies, you may consider it reasonable to get away with assuming a perfect market, thinking about the direction in which market imperfections would push you, and judging the magnitude thereof. This can often give a reasonable answer without enormous complications that a perfect answer would require.

2. How can you handle a new situation in which you face particular sets of market imperfections? To answer such new thorny questions, you should internalize the method of "thinking by numerical example." You really need to become able to work out formulas for yourself when you need them.

6·7.A. How to Work Problems You Have Not Encountered

Taxes and Inflation: Interactions?

For example, let's see how you could approach a situation with both taxes and inflation. Always start by making up some numbers you find easy to work with. Let's say you are considering an investment of \$100. Further, assume you will earn a 10% rate of return on your \$100 investment and Uncle Sam will take $\tau = 40\%$ (or \$4 on your \$10). Therefore, you get \$110 before taxes but end up with only \$106 in nominal terms. What you have just calculated is

$$\$100 \cdot [1 + 10\% \cdot (1 - 40\%)] = \$106 \quad . \tag{6.22}$$

Translate this into an algebraic formula,

$$\begin{aligned} \$100 \cdot [1 + 10\% \cdot (1 - 40\%)] &= \$106 \quad . \\ \text{CF}_0 \cdot [1 + r_{\text{nominal,pre-tax}} \cdot (1 - \tau)] &= \text{CF}_1 \quad . \end{aligned} \tag{6.23}$$

Now you need to determine what your \$106 is really worth, so you must introduce inflation. Pick some round number, say, a rate of $\pi = 5\%$ per annum. Consequently, \$106 is worth in purchasing power

$$\begin{aligned} \frac{\$106}{1 + 5\%} &= \$100.95 \\ \frac{\text{CF}_1}{1 + \pi} &= V_0 \quad . \end{aligned} \tag{6.24}$$

So, your post-tax post-inflation real rate of return is $\$100.95/\$100 - 1 \approx 0.95\%$. Again, knowing the numerical result, you need to translate your numbers into a formula. You computed

$$
\begin{aligned}
r_{\text{post-tax, real}} &= \frac{\$100.95 - \$100}{\$100} &&= \frac{\frac{\$100\cdot[1+10\%\cdot(1-40\%)]}{1+5\%} - \$100}{\$100} \\
&= \frac{10\% \cdot (1-40\%) - 5\%}{1+5\%} &&= 0.95\% \\
r_{\text{post-tax, real}} &= \frac{V_0 - \text{CF}_0}{\text{CF}_0} &&= \frac{\frac{\text{CF}_0\cdot[1+r_{\text{nominal,pre-tax}}\cdot(1-\tau)]}{1+\pi} - \text{CF}_0}{\text{CF}_0} \\
&= \frac{r_{\text{nominal,pre-tax}} \cdot (1-\tau) - \pi}{1+\pi} &&.
\end{aligned}
\tag{6.25}
$$

This is, of course, not a formula that anyone remembers. However, it is both useful and a nice illustration of how you should approach and simplify complex questions—numerical example first, formula second.

6·7.B. Taxes on Nominal Returns?

If the real rate stays constant, does inflation hurt an investor? Yes, if there are taxes!

Here is an interesting question: if the real rate remains constant, does it help or hurt an investor if inflation goes up? Let's assume that the real rate of return is a constant 20%. If inflation is 50%, then the nominal rate of return is 80% (because $(1+50\%)\cdot(1+20\%) = 1+80\%$): you get \$180 for a \$100 investment. Now add income taxes to the tune of 40%. The IRS sees \$80 in interest, taxes \$32, and leaves you with \$48. Your \$148 will thus be worth $\$148/(1+50\%) = \$98.67$ in real value. Instead of a 20% increase in real purchasing power when you save money, you now suffer a $\$98.67/\$100 - 1 \approx 1.3\%$ decrease in real purchasing power. Despite a high real interest rate, Uncle Sam ended up with more, and you ended up with less purchasing power than you started with. The reason is that although Uncle Sam claims to tax only interest gains, because the interest tax is on *nominal* interest payments, you can actually lose in *real* terms. Contrast this with the same scenario without inflation. In this case, if the real rate of return were still 20%, you would have been promised \$20, Uncle Sam would have taxed you \$8, and you could have kept \$112 in real value.

IMPORTANT: *Higher inflation rates hurt taxable investors who earn interest income, even if real interest rates seem to remain constant. This is because the IRS taxes nominal returns, not real returns.*

For much of the post-war U.S. history, real rates of return on short-term government bonds have indeed been *negative* for taxed investors.

When inflation increases, even real interest rates must also increase.

Inflation and taxes have an interesting indirect effect on equilibrium interest rates. You know that holding the agreed-upon interest fixed, inflation benefits borrowers and hurts lenders, because lenders who receive interest must pay taxes on the nominal amount of interest, not the real amount of interest. The reverse holds for borrowers. For example, assume interest rates are 3% and there is no inflation. A savings account holder with \$100 in the 33% tax bracket has to pay 1% to Uncle Sam (\$1), and gets to keep 2% (\$2). Now assume that interest rates are 12% and inflation is 9%. The savings account holder would now have to pay 4% (\$4) in taxes, and own \$108 the coming year. However, because money has lost 9% of its value, the \$108 is worth less than \$100 the following year. In effect, although real rates are identical in the no-inflation and inflation scenarios, a lender who pays taxes on nominal interest receipts gets to keep less in real terms if there is inflation. (It is straightforward to check that the opposite is true for borrowers.) The implication of this argument is simple: to compensate lenders for their additional tax burdens (on nominal interest), *real* interest rates must rise with inflation.

Solve Now!

Q 6.35 *If your tax rate is 20%, what interest rate do you earn in after-tax terms if the pre-tax interest rate is 6%?*

Q 6.36 *If your tax rate is 40%, what interest rate do you earn in after-tax terms if the pre-tax interest rate is 6%?*

Q 6.37 *If the private sector is a net saver, e.g., leaving the public sector as a net borrower, does Uncle Sam have an incentive to reduce or increase inflation?*

Q 6.38 *You are in the 33.33% tax bracket. A project will return $14,000 for a $12,000 investment—a $2,000 net return. The equivalent tax-exempt bond yields 15%, and the equivalent taxable bond yields 20%. What is the NPV of this project?*

Q 6.39 *Compare a 10-year zero bond and a 10% coupon bond, both paying 10%, with an appropriate (economy-wide) interest rate of 10%. If the IRS does not collect interim interest on the zero bond, and the marginal tax rate is 25%, then what is the relative NPV of the two bonds?*

Q 6.40 *Assume you have both taxes and inflation. You are in the 20% tax bracket, and the inflation rate is 5%/year. A 1-year project offers you $3,000 return for a $20,000 investment. Taxable bonds offer a rate of return of 10%/year. What is the NPV of this project? Extra-credit if you can derive the formula yourself!*

Q 6.41 *Advanced question: Return to the apples example from Section 6·6, in which the inflation rate was 100% and the nominal rate of interest was 700%. Now, assume that there is also a 25% default rate. That is, 25% of all apples are returned with worms inside, and will therefore not be sellable (and be worth $0). What is your real rate of return? What is the formula?*

Q 6.42 *Really advanced question: Return to the taxes-and-inflation example from Section 6·7. A 10% nominal rate of return, a tax rate of 40%, and an inflation rate of 5%. (We worked out that the post-inflation, post-tax rate of return was 0.95%.) Now, add a default rate, d, of 2%, where all money is lost (-100% return). What is the real, post-inflation, post-tax, post-default rate of return? (Hint: Losses are tax-deductible, too. Assume that the default rate reduces the nominal rate of return (on which taxes are charged), because you do not just take 1 such loan, but 1 million, which practically assures you of the exact default rate without any sampling variation.)*

6·8. Summary

The chapter covered the following major points:

- If markets are not perfect, even *expected* borrowing and lending rates can be different. This is different from the fact that even in perfect markets, *promised* borrowing and lending rates can be different.
- If markets are not perfect, capital budgeting decisions can then depend on the cash position of the project owner.

  NPV and interest rate computations can still be used, although it then requires special care in working with correct and meaningful inputs (especially for the cost of capital). This is usually best done by thinking in terms of concrete examples first, and translating them into formula later.
- Transaction costs and taxes are market imperfections that reduce earned rates of return.
- Transaction costs can be direct (such as commissions) or indirect (such as search or waiting costs). It is often useful to think of round-trip transaction costs.
- Financial assets' transaction costs tend to be very low, so that it is reasonable in many (but not all) circumstances to just ignore them.
- In the real world, buyers often prefer more liquid investments. To induce them to purchase a less liquid investment may require offering them some additional expected rate of return.
- Many financial markets have such low transaction costs and are often so liquid that they are believed to be fairly efficient—there are so many buyers and so many sellers, that it is unlikely that you would pay too much or too little for an asset. Such assets are likely be worth what you pay for them.
- The tax code is complex. For the most part, individuals and corporations are taxed similarly. You must understand
  1. how income taxes are computed (the principles, not the details);
  2. that expenses that can be paid from before-tax income are better than expenses that must be paid from after-tax income;
  3. how to compute the average tax rate;
  4. how to obtain the marginal tax rate;
  5. that capital gains enjoy preferential tax treatment;
  6. why the average and marginal tax rates differ, and why the marginal tax rate is usually higher than the average tax rate.
- Taxable interest rates can be converted into equivalent tax-exempt interest rates, given the appropriate marginal tax-rate.
- Tax-exempt bonds are usually advantageous for investors in high-income tax brackets. You can compute the critical tax rate investor who is indifferent between the two.
- Long-term projects often suffer less interim taxation than short-term projects.
- You should do all transaction cost and tax net present value calculations with after-transaction cash flows and after-tax costs of capital.
- Like taxes and transaction costs, inflation can also cut into returns. However, in a perfect market, it can be contracted around and therefore neutralized.

- The relationship between nominal interest rates, real interest rates and inflation rates is

$$(1 + r_{\text{nominal}}) \;=\; (1 + r_{\text{real}}) \;\cdot\; (1 + \pi) \;. \tag{6.26}$$

  Unlike nominal interest rates, real interest rates can and have been negative.

- In NPV, you can either discount real cash flows with real interest rates, or discount nominal cash flows with nominal interest rates. The latter is usually more convenient.

- TIPS are bonds whose payments are indexed to the future inflation rate, and which therefore offer protection against future inflation. Short-term bond buyers are also less exposed to inflation rate changes than long-term bond buyers.

- Empirically, inflation seems to be able to explain the level of the yield curve, but not its slope.

- The IRS taxes nominal returns, not real returns. This means that higher inflation rates disadvantage savers and advantage borrowers.

Solutions and Exercises

1. No differences in information, no market power, no transaction costs, no taxes.
2. It means that borrowing and lending rates are identical, and that there is a unique price at which stuff is selling for (i.e., its value).
3. In most neighborhoods, there are plenty of supermarkets, fiercely competing for business. Losing one additional or gaining one additional supermarket probably makes little difference. There are also plenty of buyers. In many ways, supermarkets are a fairly competitive business, and their products are usually priced not far away from their closest competitors. Believe it or not: supermarkets typically earn a gross spread on goods of only about 2%! This has to pay for space and personnel. *However*, in some senses, this "supermarket market" is not perfectly competitive and frictionless: there is sales tax, so third parties cannot easily "arbitrage" product, i.e., sell a product that is less expensive in one supermarket to the other, more expensive supermarket. Plus, once you are at the supermarket, it is often cheaper just to buy the goods there, than it is to drive to another supermarket.
4. Yes! Surplus?
5. It helps you evaluate what violations really mean.
6. An efficient market is one in which the market uses all available information. In a perfect market, market pressures will make this come true, so a perfect market should be efficient. However, an efficient market need not be perfect.
7. You would have to borrow \$100 at an interest rate of 10% in order to take the project. If you take the project, you will therefore have $\$1,000 \cdot 1.08 - \$110 = \$970$ next period. If instead you invest \$900 at the 4% savings rate, you will receive only \$936. So, definitely take the project.
8. Say you invest I. If you put it into the bank, you receive $I \cdot (1 + 4\%)$. If you put I into the project, you receive $\$1,000 \cdot (1 + 8\%)$ from the project, borrow $(\$1,000 - I)$ at an interest rate of $(1 + 10\%)$. Therefore, you must solve

$$I \cdot (1 + 4\%) \;=\; \$1,000 \cdot (1 + 8\%) - (\$1,000 - I) \cdot (1 + 10\%) \tag{6.27}$$

The solution is $I = \$333.33$, which means that if you want to consume more than \$1,666.66, you should not take the project. Check: [1] If you consume \$1,700, you have a remaining \$300 to invest. The bank would pay \$312 next year. The project would pay off \$1,080, but you would have to borrow \$700 and pay back \$770, for a net of \$310. You should not take the project [2] If you consume \$1,600, you have a remaining \$400 to invest. The bank would pay \$416 next year. The project would pay off \$1,080, but you would have to borrow \$600 and pay back \$660, for a net of \$420. You should take the project.

9. Yes! Stated rates include a default premium. A perfect market is about equality of expected rates, not about equality of promised rates?
10. First, default rates are high. (This is not necessarily a difference in expected rates of returns.) Second, information differences about default probabilities are high. Banks cannot easily determine which entrepreneurs are for real, and which ones will die and take the bank's money to their graces. The entrepreneurs may or may not be better at knowing whether their inventions will work. (This can be a market imperfection.)
11. Do it! This information can be found in the **Yield Comparisons** exhibit in the **Credit Markets** section in the WSJ.

12. The appropriate 7-year interest rate would now be about $(1 + 8\%)^7 - 1 \approx 71\%$. Therefore, a \$1 million house that you would resell in 7 years for \$1 million would cost you a direct $\$60,000/(1+8\%)^7 \approx \$35,009$ in present value of commissions. If you were paying all future real estate commissions for this house, the present value of this cost would be \$84,507. Therefore, the capitalized value of all future brokerage commissions would be lower (only about 8.5% of house value) than the 15% that we found in the text for lower interest rates.
13. DELL is an even larger stock than PepsiCo. Therefore, a round-trip transaction would probably cost a bid-ask spread of between 0.1% and 0.3%. On a \$10,000, the bid-ask cost would be around \$20, and broker fees would probably be around \$10 to \$30 with a discount broker. Thus, \$50 is a reasonable estimate.
14. Direct: Broker Costs. Market-Maker or Exchange Costs (Bid-Ask Spread). Indirect: Research Costs; Search (for Buyer/Seller) Costs; Anxiety.
15. You need to assume a proper discount rate for the \$4,000. A reasonable assumption is an annuity. At a 7% interest rate, this value is around \$46,281 today. Therefore

$$-(\$1,000,000 + \$5,000) + \$46,281 + \frac{x \cdot (1 - 8\%)}{1 + 7\%} \;=\; 0 \;. \tag{6.28}$$

Therefore, $x \sim \$1.115,032$ million, so the capital appreciation must be 11.5% per annum. Note how the \$5,000 must be added to the upfront cost, not subtracted!

16. A liquidity premium is an upfront lower price to compensate you for transaction costs later on.

17. Taxable income is \$4,000. **Individual:** Tax Rate of 10%, so taxes are \$400. Average and marginal tax rates are 10%. **Corporation:** Tax Rate of 15%, so taxes are \$600. Average and marginal tax rates are 15%.

18. The taxable income is \$49,000. **Individual:** Taxes are $\$715 + \$3,285 + \$19,950 \cdot 25\% = \$8,987.50$. Average Tax Rate (relative to taxable income) is 18.3%. The marginal tax rate is 25%. **Corporation:** Taxes are \$7,500. The marginal and average tax rate are both 15%.

19. Taxable Income: \$49,999,000. **Individual:** $35\% \cdot (\$49,999,000 - \$319,100) + \$92,592 = \$17,480,557$. The average tax rate is 34.96%. Marginal tax rate is 35%. At very high-income levels, the marginal and average tax rates are close. **Corporation:** $35\% \cdot (\$49,999,000 - \$18,300,000) + \$6,381,900 = \$17,476,550$. The average tax rate is 34.95%. The marginal tax rate is 35%.

20. 25.4%.

21. 8%. The ratio is very high by historical standards, which means that the marginal investor's income tax rate of 8% is quite low.

22. The T-bond will pay \$108 before tax. You will therefore earn \$105 after taxes. The muni will pay only \$103. So, your opportunity cost of capital is 5%. The project itself will have to pay taxes on \$30,000, so you will have \$18,750 net return left after taxes, which comes to an amount of \$68,750. Your project NPV is therefore

$$-\$50,000 + \frac{\$68,750}{(1+5\%)^3} \approx +\$9,389 \quad . \tag{6.29}$$

This is a great project!

23. The \$1 is paid from after-tax income, so leave it as is. The \$10 million is taxed, so you will only receive \$7 million. With a 1 in 9 million chance of winning, the expected payoff is 78 cents. Therefore, the NPV is negative for any cost of capital. If you could pay with pre-tax money, the ticket would cost you only 70 cents in terms of after-tax money, so for interest rates of below 10% or so, the lottery would be a positive NPV investment.

24. Do it! (As of 2002, it should be between 1% and 2% per year.) This rate can be found at the end of the **Money Rates** box in the WSJ.

25. Do it! (This changes too often to give a useful figure here.) It can also found in the Money Rates box.

26. Do it!

27. $(1 + r_{\text{nominal}}) = (1 + r_{\text{real}}) \cdot (1 + \pi)$.

28. $(1 + 20\%)/(1 + 5\%) = (1 + 14.29\%)$. The real interest rate is 14.29%.

29. In nominal terms, the rate of return is $n_{0,1} = (1 + 2\%) \cdot (1 + 1.5\%) - 1 = 3.53\%$, the cash flow will be \$101.50. Therefore, $\text{PV} = \$101.50/(3.53\% - 1.5\%) = \$5,000$. In real, inflation-adjusted terms, the rate of return is 2%, the \$101.50 next year are still worth \$100 in today's dollars, so $\text{PV} = \$100/2\% = \$5,000$. Never discount \$100 by 3.53%, or \$101.50 by 2%.

30. The nominal interest rate is $(1+3\%) \cdot (1+8\%) - 1 = 11.24\%$. Therefore, the cash flow is worth about \$449,478.

31. \$4.448 million.

32.

   (a) 5.88%.

   (b) The *correct* PV is

$$\begin{aligned} \text{PV} &= \frac{\$20}{1+8\%} + \frac{\$20 \cdot (1+3\%)}{(1+8\%)^2} + \frac{\$20 \cdot (1+3\%)^2}{(1+8\%)^3} + \cdots \\ &= \frac{\$20}{8\% - 3\%} = \$400 \quad . \end{aligned} \tag{6.30}$$

   (c) Project value is not $\$20/(8\% - 1\%) \approx \$285.71$.

   (d) Project value is not $\$20/(5.88\% - 3\%) \approx \$694.44$.

33. The first *nominal* cash flow next period is \$102,000. Now, you can switch to nominal quantities throughout (the nominal cash flow next year, the nominal interest rate, and as nominal growth rate the inflation rate). You would therefore use next year's nominal cash flow—a CF of \$102,000—in the formula,

$$\text{PV} = \frac{\$102,000}{8\% - 2\%} = \$1,700,000 \quad . \tag{6.31}$$

It is affirmatively not $\$100,000/6\% \approx \$1,666,666$.

34. If inflation were to remain at 1.6%/year, the plain Treasury bond would offer a higher real rate of return because $(1 + 5.6\%)/(1 + 1.6\%) - 1 \approx 3.9\%$/year. But if inflation were to rise in the future, the TIPS could end up offering the higher rates of return.

35. For every \$100, you receive \$6. Uncle Sam takes 20% of \$6, or \$1.20. So, your after tax rate of return is $\$4.80/\$100 = 4.8\%$. You could have also computed $(1 - 20\%) \cdot 6\% = 4.8\%$ directly.

36. For every \$100, you receive \$6. Uncle Sam takes 40% of \$6, or \$2.40. So, your after tax rate of return is $\$3.60/\$100 = 3.6\%$. You could have also computed $(1 - 40\%) \cdot 6\% = 3.6\%$ directly.

37. Increase. In the real world, interest rates may have to rise to compensate private savers for this extra "tax" on money.

38. Your opportunity cost of capital is determined by the tax-exempt bond, because $66.7\% \cdot 20\% < 15\%$. Your project's \$2,000 will turn into $66.7\% \cdot \$2,000 = \$1,334$ after-tax earnings, or \$13,334 after-tax cash flow. Therefore, your NPV is $-\$12,000 + \$13,334/(1 + 15\%) = -\$405.22$. Check: The after-tax rate of return of the project's cash flow are $\$13,334/\$12,000 - 1 \approx 11\%$. This is less than 15%. You are better off investing in tax-exempt bonds.

39. The coupon bond has an after-tax rate of return of 7.5%. Start with \$1,000 of money. Reinvestment yields an after-tax rate of return of 7.5% (\$75 in the first year on \$1,000). So, after 10 years, you are left with $\$1,000 \cdot 1.075^{10} = \$2,061$. In contrast, the zero bond has a single pre-tax payout of $\$1,000 \cdot (1 + 10\%)^{10} = \$2,593.74$, for which the IRS would collect $\$1,593.74 \cdot 25\% = \$398.43$ in year 10, for a post-tax zero-bond payout of \$2,195. The tax savings on the zero bond are therefore \$134 in 10 years, or \$52 in present value.

40. What is your after-tax rate of return on taxable bonds? \$100 will grow to \$110 ($(1 + 10\%) \cdot \$100 = \$110$) pre-tax, minus the 20% what Uncle Sam collects. Uncle Sam takes $(1 + 10\%) \cdot \$100 = \$110$, subtracts \$100, and then leaves you with only 80% thereof:

$$r_{\text{after-tax}} = \frac{80\% \cdot (\$110 - \$100)}{\$100} = 8\% = \frac{(1 - \tau) \cdot (\text{CF}_1 - \text{CF}_0)}{\text{CF}_0} , \tag{6.32}$$

where τ is your tax-rate of 20%. $(\text{CF}_1 - \text{CF}_0)/\text{CF}_0$ is the pre-tax rate of return, so this is just

$$r_{\text{after-tax}} = 80\% \cdot 10\% = (1 - \tau) \cdot r_{\text{pre-tax}} . \tag{6.33}$$

Now, in pre-tax terms, your project offers a 15% rate of return. In after-tax terms, the project offers $80\% \cdot \$3,000 = \$2,400$ net return, which on your investment of \$20,000 is a 12% after-tax rate of return. (On the same \$20,000, the taxable bond would offer only $80\% \cdot (\$22,000 - \$20,000) = \$1,600$ net return (8%). So, you know that the NPV should be positive.) Therefore, the project NPV is

$$\begin{aligned} \text{NPV} &= -\$20,000 + \frac{\$20,000 + 80\% \cdot (\$22,400 - \$20,000)}{1 + 8\%} \approx \$740.74 \\ &= \text{CF}_0 + \frac{\text{CF}_0 + (1 - \tau) \cdot (\text{CF}_1 - \text{CF}_0)}{1 + r_{\text{after-tax}}} . \end{aligned} \tag{6.34}$$

You can now easily substitute any other cash flows or interest rates into these formulas to obtain the NPV. Note how everything is computed in nominal dollars, so we do not need the information about the inflation rate!

41. Your numeraire is one apple ($1a$) that costs \$1. You will get \$8 in nominal terms, next year ($a \cdot (1 + r_{\text{nominal,pre-tax}}) = a \cdot (1 + 700\%) = 8 \cdot a$). This will purchase apples that cost \$2 each ($(1 + \pi) = (1 + 100\%) = \$2$), i.e., 4 apples ($a \cdot (1 + r_{\text{nominal,pre-tax}})/(1 + \pi) = 1 \cdot (1 + 700\%)/(1 + 100\%) = 4$). However, one of the apples ($d = 25\%$) is bad, so you will get only 3 apples ($a_1 = a_0 \cdot (1 + r_{\text{nominal,pre-tax}})/(1 + \pi) \cdot (1 - d) = 1a_0 \cdot (1 + 700\%)/(1 + 100\%) \cdot 75\% = 3 \cdot a_0$, where d is the 25% default rate) . Therefore, the real rate of return is $(a_1 - a_0)/a_0$ or

$$\begin{aligned} r_{\text{real,post-tax,post-default}} &= \frac{(1a \cdot \frac{1+700\%}{1+100\%} \cdot 75\%) - 1a}{1a} = 300\% - 1 = 200\% \\ r_{\text{real,post-tax,post-default}} &= \frac{[1a \cdot \frac{1+r_{\text{nominal,pre-tax}}}{1+\pi} \cdot (1 - d)] - 1a}{1a} . \end{aligned} \tag{6.35}$$

The "1a" of course cancels, because the formula applies to any number of apples or other goods.

42. WARNING: THIS ANSWER HAS NOT BEEN CHECKED, AND MIGHT BE WRONG: Instead of 10%, you earn only $98\% \cdot 10\% + 2\% \cdot (-100\%) = 7.8\%$. Translated into a formula, this is $(1-d) \cdot r_{\text{nominal,pre-tax}} + d \cdot (-100\%) = r_{\text{nominal,pre-tax}} - d \cdot (1 + r_{\text{nominal,pre-tax}}) = 10\% - 2\% \cdot (1 + 10\%) = 7.8\%$. Now, in Formula 6.25,

$$
\begin{aligned}
r_{\text{post-tax, real, post-default}} &= \frac{V_0 - \mathrm{CF}_0}{\mathrm{CF}_0} = \frac{\frac{\mathrm{CF}_0 \cdot [1 + r_{\text{nominal,pre-tax}} \cdot (1-\tau)]}{1+\pi} - \mathrm{CF}_0}{\mathrm{CF}_0} \\
&= \frac{r_{\text{nominal,pre-tax}} \cdot (1-\tau) - \pi}{1+\pi} \quad ,
\end{aligned} \tag{6.36}
$$

replace the nominal interest rate $r_{\text{nominal,pre-tax}}$ with the default reduced nominal rate $r_{\text{nominal,pre-tax}} - d \cdot (1 + r_{\text{nominal,pre-tax}})$, so the new formula is

$$
\begin{aligned}
r_{\text{post-default, post-tax, real}} &= \frac{V_0 - \mathrm{CF}_0}{\mathrm{CF}_0} \\
&= \frac{\frac{\mathrm{CF}_0 \cdot [1 + (r_{\text{nominal,pre-tax}} - d \cdot (1 + r_{\text{nominal,pre-tax}})) \cdot (1-\tau)]}{1+\pi} - \mathrm{CF}_0}{\mathrm{CF}_0} \\
&= \frac{(r_{\text{nominal,pre-tax}} - d \cdot (1 + r_{\text{nominal,pre-tax}})) \cdot (1-\tau) - \pi}{1+\pi} \\
&= \frac{7.8\% \cdot (1 - 40\%) - 5\%}{1 + 5\%} = -0.3\% \quad .
\end{aligned} \tag{6.37}
$$

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

CHAPTER 7

Capital Budgeting (NPV) Applications and Advice

Tips and Tricks!

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The previous chapters have developed all necessary concepts in capital budgeting. This means that almost everything in the rest of the book will “just” help you estimate and understand the NPV inputs better, help you in particular applications, or elaborate on tradeoffs that you—the decision maker—face. But this does not mean that you are done. Applying the relatively simple concept of NPV in the real world can be very difficult.

In this current chapter, we cover a collection of topics in which the application of NPV is often challenging. You will almost surely encounter these complications in your future corporate practice. In fact, I will try to help you avoid the kind of common mistakes that companies commit almost every day—mistakes that cost them value. In later chapters, we will look at other refinements. In Part II, we will bring in financial accounting information to work with the numbers that corporations use and report. In Part III, we will work on understanding what determines the cost of capital. In Part IV, we will work on how debt and equity financing influences the net present value. And, our crowning achievement will be Chapter 25, in which we will develop a pro forma that will have to bring everything together.

7·1. The Economics of Project Interactions

An example of interacting projects.

So far, we have considered projects in isolation. We computed the costs and benefits necessary to make our decision whether to accept or reject. Unfortunately, in the real world, projects are not always isolated. For example, an aquarium may add a large shark to its exhibition tank at a cost of \$50,000 for projected additional ticket receipts of \$120,000; or it may add a large octopus at a cost of \$75,000 for projected additional ticket receipts of \$200,000.

		Shark		Octopus
Ticket Receipts	+	\$120,000	+	\$200,000
Creature Cost	–	\$50,000	–	\$75,000
Net	=	\$70,000	=	\$125,000

Regrettably, adding both the shark and the octopus would not increase project value by \$195,000, because octopuses are known to have negative effects on similarly sized sharks—they eat them. Thus, the best achievable project value is only \$125,000 (skip the shark!). On the other hand, stocking the aquarium with an octopus plus some lobsters would cost only \$75,000 plus a couple of dollars for the lobsters—which allows the octopus to remain alive. If you do not add the lobsters, you would end up with a starved and expiring octopus and thus not much audience. So, you either want to add the octopus and the lobsters together, or neither. In general, the question we are considering in this section is how you should deal with projects that have mutual interactions. In other words, how should you stock the aquarium?

7·1.A. The Ultimate Project Selection Rule

Important: ***The Ultimate Project Selection Rule:*** *Consider all possible project combinations, and select the combination of projects that gives the highest overall NPV.*

There are too many possible action choices in the real world to evaluate (to compute NPV for). You need rules and heuristics!

Optimal project selection is easier said than done. It is easier for two projects at a time, as it was in our aquarium example, because there are only four options to consider: take neither, take one, take the other, or take both. But the complexity quickly explodes when there are more projects. For three projects, there are eight options. For four projects, there are sixteen options. For ten projects, there are about a thousand options. For twenty projects, there are over a million options. (The formula for the number of choices is 2^N, where N is the number of projects.) Even the simplest corporate projects can easily involve hundreds of decisions that have to be made. For our little aquarium, there are about 54,000 different fish species to consider—and each may interact with many others. These choices do not even consider the fact that some projects may allow other projects to be added in the future, and that many projects are not just “accept” or “reject,” but “how much project to take.”

A Greedy Algorithm?

To help us determine which projects to take, we need to find suitable **heuristics**, i.e., rules that simplify decisions even if they are not always correct. One common heuristic algorithm is to consider project combinations, one at a time. Start with the project combination that, if you were only allowed to take two projects (one pair from a set of many different projects), would give you the highest NPV. Then take this pair as fixed, i.e., treat it as a single project. Now see which project adds the most value to your existing pair. Continue until adding the best remaining project no longer increases value. Computer scientists call this the **greedy algorithm.** It is a good heuristic, because it drastically cuts down the possible project combinations to consider, and usually gives a pretty good set of projects. There are many possible enhancements to this algorithm, such as forward and backward iterations, in which one considers replacing one project at a time with every other option. Full-fledged algorithms and combinatorial enhancements that guarantee optimal choice are really the domain of **computer science** and **operations research**, not of finance. Yet many of these algorithms have been shown to require more time than the duration of the universe, unless you make simplifications that distort the business problem so much that the results seem no longer trustworthy. Fortunately, economics is in our finance domain, and it can also help us simplify our project selection problem.

7·1.B. Project Pairs and Externalities

Project combinations can be classified into positive, zero, and negative interaction combinations.

We just mentioned considering projects in pairs. This is not only common practice, but also clarifies the economic issues. With two projects, we can decompose the total net present value into three terms:

$$\text{Overall NPV} = \text{NPV Project One} + \text{NPV Project Two} + \text{NPV Interactions} \;. \tag{7.1}$$

If you were to stock both the shark and the octopus, you would get ticket receipts of \$200,000 [octopus] but pay \$125,000 [octopus and shark], for a net of \$75,000. Therefore,

$$\begin{array}{ccccccc} \$75,000 & = & \$70,000 & + & \$125,000 & + & (-\$120,000) \\ \text{NPV Aquarium With Both} & = & \text{NPV Shark} & + & \text{NPV Octopus} & + & \begin{array}{c}\text{NPV Octopus eats Shark,}\\ \text{so no more Shark ticket receipts}\end{array} \end{array} \;. \tag{7.2}$$

The final term suggests that we can classify project combinations into one of three different categories:

1. Projects with zero interactions.
2. Projects with positive interactions.
3. Projects with negative interactions.

Interactions are also sometimes called **externalities** in economics, because one project has external influences on other projects—sometimes imposing external costs and sometimes providing external benefits. We now discuss these three cases.

Zero Project Interactions

Project independence is the most common case, and allows simple decision making.

Most projects in this world are **independent**—they have no mutual interactions. For example, a mall in Maine probably has no effect on a mall in Oregon. It neither steals customers from Oregon nor attracts extra customers. Independent project payoffs permit the separate evaluation of each project. This makes decision-making very easy:

- Taking each Positive NPV project increases firm value.
- Taking each Zero NPV project leaves firm value unchanged.
- Taking each Negative NPV project decreases firm value.

(These rules do not hold if projects have non-zero externalities, as we shall explain below.) Project NPVs are additive, because all cash flows have been translated into the same units, today's dollars, and the project interaction term is zero. Project independence makes decisions a lot easier: for twenty projects, only twenty independent decisions (accept or reject) have to be made, not a million.

> **IMPORTANT**: *You can evaluate zero interaction projects independently. In this case, you can simply add project net present values.*

Positive Project Interactions

Positive interactions exist when taking one project increases the value of another project. Projects with positive interactions are often considered as "bundles." Indeed, in many cases, what makes a project a project in the firm's mind is the indivisibility of its components.

Positive interactions mean that the sum of the parts is worth more than the parts individually. If one project has a positive influence on the net present value of another project, you cannot consider it without considering this positive influence. For example, consider creating a product as one project and an advertising campaign as another project. The advertising campaign project is of lesser use without a product, and the product is of lesser use without the advertising campaign. You must consider creating a product and an advertising campaign together. Such positive externalities are even more plentiful in smaller decisions: A secretary with word processing skills is less useful without a word processor, and a word processor is less useful without a secretary who can use it. A computer keyboard is less useful without a computer, and a computer is less useful without a keyboard. In fact, some projects or products only make sense if purchased together. In this case, producers may bundle them together and/or purchasers may only buy them as bundles.

Infrastructure can benefit many different projects.

In the corporate context, investment in *infrastructure* is another classical example of positive project interactions. For example, building a road, hiring a security firm, or laying a fast Internet connection could enhance the value of many divisions simultaneously. The firm should factor in the increase in value to *all* divisions when deciding on how much infrastructure to add.

Positive externalities is why firms exist to begin with.

Don't take positive externalities too lightly: On a philosophical basis, positive project interactions are the reason why firms exist in the first place. If there were no cost savings to having all resources combined in the firm, we might as well not bother and instead all work as individuals.

> **IMPORTANT**: *When deciding whether to take a project, you must credit all positive interactions to the project. The overall NPV is higher than the individual project NPVs alone.*

Agency issues often prevent proper crediting.

Internal conflict and cost allocation procedures issues (further discussed as "agency conflicts" below) often hinder corporations from taking advantage of many positive externalities. For example, in real life, our division managers might argue that they should not be charged for the Internet connection, because they did not request it and therefore do not really need it (even if it were to increase their divisions' values). After all, division managers would prefer getting Internet for free from the company instead of paying for it out of their own division budgets.

Another phrase for positive externalities.

Nowadays, managers who want to acquire other companies usually claim the presence of large positive externalities. **Synergies**, the managerial term for positive externalities between an acquirer and a potential acquisition target, has become an important managerial buzzword. For example, in the 2001 acquisition of Compaq by Hewlett-Packard, HP touted synergies of $2.5 billion dollars—most from cutting employees. Of course, whether synergies will be realized is always another question.

Negative Project Interactions

Negative interactions exist when taking one project decreases the value of another project. Pollution, cannibalization, and limited attention span are examples thereof.

Negative interactions mean that the sum of the parts is worth less than the parts individually. In this case, projects have negative influences on one another, and thereby decrease one another's value. Economists sometimes call negative externalities **diseconomies of scale**. Here are a few examples.

Pollution and Congestion If there is only one major road to two divisions, and the traffic of one division clogs up the traffic to the other division, it can cause a loss of cash flow in the other division. A division that wants to expand and thereby clog up more of the existing infrastructure will not want to pay for the congestion cost that its own expansion will impose on the other divisions. (Of course, it is the overall firm's headquarters that should step in and allow the expansion only if the NPV is positive after taking into account the negative externalities imposed on other divisions.)

Cannibalization If a new Apple computer can produce \$100,000 in NPV compared to the older Windows machine that only produced \$70,000 in NPV, how should we credit the Apple machine? The answer is that the Apple would eliminate the positive cash flows produced by the existing Windows machine, so the cash flow of the project "replace Windows with Apple" is only the \$100,000 minus the \$70,000 that the now unused Windows machine had produced.

Bureaucratization and Internal Conflict If more projects are adopted, project management may find it increasingly difficult to make good decisions in a reasonable time frame. This may require more cumbersome bureaucracy and reduce cash flows for all other divisions.

Resource Exhaustion Perhaps the most common source of negative externalities—and often underestimated—is **limited attention span**. Management can only pay so much attention to so many different issues. An extra project distracts from the attention previously received by existing projects.

Although costs always include opportunity costs, in the case of negative project externalities, they are more obvious. If your project cannibalizes another project or requires more attention, it's clearly an opportunity cost.

> **IMPORTANT**: *When deciding whether to take a project, charge all negative interactions to the project. The overall NPV is lower than the individual project NPVs alone.*

Agency issues often prevent proper costing.

Again, as in the case of positive externalities, agency issues and cost allocation systems often prevent proper accounting for negative externalities in the real world. Whichever division created the negative externality will argue that it is not its problem, and that the complaining division overstates the problem. Clearly, companies that are better at overcoming these issues will end up being more profitable.

7·1.C. One More Project: Marginal Rather Than Average Contribution

The Capital Budgeting Rule for one extra project requires taking all project interactions into account.

Usually, managers do not make the decision for all interacting projects simultaneously. Instead, many projects are already in place. Although existing projects should also constantly be evaluated in an ideal world, the manager often has to make a decision about adding or not adding a single new project (or project complex) only in the real world. For practical purposes, the old projects are present, given, and unalterable. So the new project may have positive or negative externalities on other existing projects, and the question is how best to decide whether to take it or not. This simplifies the decision even further: the question is now only whether the new project adds or subtracts value from the total. In this case, economists use the concept of decision on the **margin**—holding the existing projects as is, what is the *additional* contribution of the new project?

The aquarium haunts us.

Return to our aquarium example.

- If you already have the octopus in the tank [with its NPV of \$125,000], should you add the shark? If you do, you pay an additional ("marginal") \$50,000 and get nothing—because the shark will become octopus food. Thus, the marginal benefit of adding the shark is $-\$50,000$. Therefore you should not add the shark.
- If you already have the shark in the tank with its NPV of \$70,000, should you add the octopus? Your marginal cost to add the octopus is \$75,000 for the beast itself plus the loss of \$70,000 of shark audience ticket sales for a total marginal *cost* of \$145,000. The marginal benefit is \$125,000 in new octopus ticket sales. Therefore, your marginal net gain would be $-\$20,000$. (You can also see this by comparing the previous shark-only NPV of \$70,000 against the octopus NPV of \$125,000, and subtract out the \$75,000 cost.) And therefore, you should not add the octopus.

Of course, if you can sell the shark or put it into its own aquarium, the calculations would change—though you would then also have to consider the marginal cost of selling the shark or getting a new aquarium.

IMPORTANT:

- *The decision on whether to take one additional project should be made based on the rule*

  Take New Project if

  Total Firm NPV with New Project > *Total Firm NPV without New Project*

- *This means that the single new project should be credited with any value increase or value decrease that it confers on other projects.*
- *When considering a project* ***on the margin*** *(i.e., extra), credit/charge to this project all externalities that this project conveys onto the existing firm.*
- *Everything else equal, projects with positive externalities on the rest of the firm have higher marginal benefits than projects with negative externalities.*

We now discuss some more examples of how to think in terms of marginal costs and benefits.

Working with Economies of Scale

An example in which the cost function creates economies of scale.

Consider an example in which there are **economies of scale**—the more product we produce, the lower the average product price. Say, our factory can produce at

$$\text{Average Price Per Good} = \$4 + \frac{\$10}{x+1} \ . \tag{7.3}$$

Thus, producing one good costs $\$4 + \$10/(1+1) = \$9$/good, and one-hundred goods costs $\$4+\$10/(100+1) = \$4.10$/good. The company is currently selling 5 goods domestically, each for a price of \$8.00. It earns a

$$\text{Total Profit @ 5 items} = 5 \cdot \$8 - 5 \cdot \left[\$4 + \frac{\$10}{(5+1)}\right] = \$11.67 \ . \tag{7.4}$$

The company is considering a new foreign sales division that would cost \$16 to open, and that could sell another 5 units at \$8. The average price for the company producing 10 units would be \$4+\$10/11= \$4.91/unit. Therefore, 5 units cost \$24.55 to produce. The total cost of $\$16 + \$24.55 = \$40.55$ exceeds the total profit of $5 \cdot \$8 = \$40$. If considered by itself, opening a foreign sales division would not be a positive NPV project.

The foreign sales division also lowers the cost of domestic production!

Now compute the total firm profit if the firm were to open the foreign sales division. Ten units would sell for a profit of \$80. Subtracting the opening costs of \$16 and production costs of $10 \cdot \$4.91 = \$49.19$ would earn a

$$\text{Total Profit @ 10 items} = 10 \cdot \$8 - 10 \cdot \left[\$4 + \frac{\$10}{(10+1)}\right] - \$16 \approx \$14.91 \ . \tag{7.5}$$

This is more than the \$11.67 that the firm earned without the foreign sales division. The reason is that the foreign office has an additional marginal benefit: it reduces the average production cost experienced by the domestic office. This cost improvement is a positive externality that must be credited to the project—or the firm will make the bad decision of not opening the foreign division.

All this is easier to see when we translate it into terms of marginal costs and benefits. The extra marginal cost of each item changes item by item—it is the difference in total costs of each item:

Think of these economies of scale in terms of marginal costs.

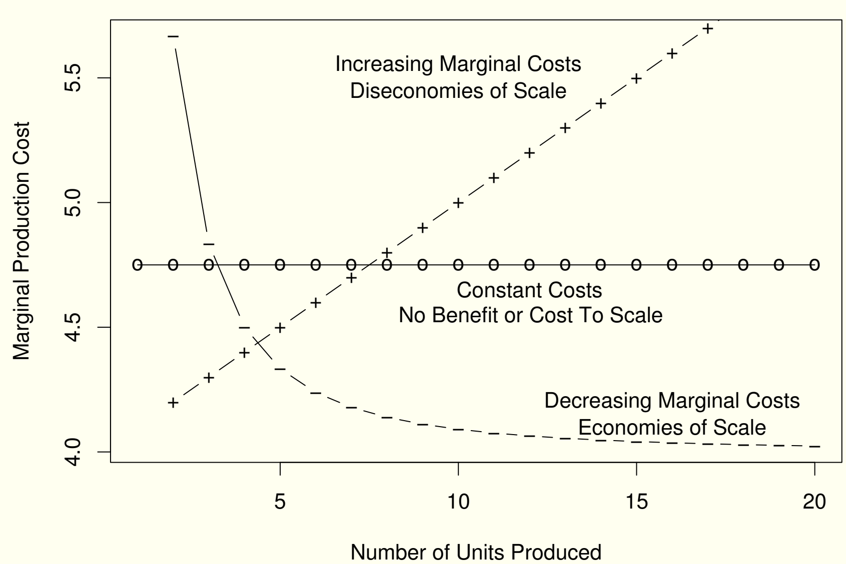
Units	Average	Total	Marginal	Units	Average	Total	Marginal
1	$9.00	$ 9.00	$9.000	6	$5.42	$32.57	$4.238
2	$7.33	$14.67	$5.667	7	$5.25	$36.75	$4.179
3	$6.50	$19.50	$4.833	8	$5.11	$40.89	$4.139
4	$6.00	$24.00	$4.500	9	$5.00	$45.00	$4.111
5	$5.66	$28.33	$4.333	10	$4.90	$49.09	$4.091

Going from 5 items to 10 items, production creates extra costs of $4.333 to $4.091 for a marginal cost of $20.76. There would be an additional marginal cost of $16 to open the foreign office. The total marginal cost would thus be $36.76. The marginal benefit of 5 extra items would be $40. Therefore, the foreign sales division gives us marginal NPV of $40 − $36.76 ≈ $3.242. This is exactly the difference between $11.67 from Formula 7.4 and $14.91 from Formula 7.5. So thinking in terms of marginal costs and benefits is just a more convenient way to compare overall project values.

Graphical Display of Economies of Scale

In Figure 7.1, we show three different cost functions that are based on different externalities. When there are positive externalities, as there are when there are economies of scale, then each item has a positive effect on the next item produced, so the marginal cost is decreasing with the number of units. When there are negative externalities, as there are when there are diseconomies of scale (it may become harder and harder to find the necessary input materials), then each item has a negative effect on the next item produced, so the marginal cost is increasing with the number of units.

Figure 7.1. The Effects of Economies of Scale on Marginal Costs



Here are three examples to show positive externalities, zero externalities, and negative externalities in the production of goods. The decreasing marginal cost function represents economies of scale. In this case, it also looks as if we have diminishing decreasing marginal costs—though the production cost is less for each additional product, it only is a little less good-by-good if we produce many of them. The increasing marginal cost function represents diseconomies of scale—the more units we produce, the more we have to pay for each additional unit.

Economies of scale are often responsible for the big corporate success stories of our time.

In my opinion, decreasing marginal costs are responsible for the biggest corporate success stories. For example, Wal-Mart and Dell have managed not only to use their scale to negotiate considerable supplier discounts, but have also created inventory and distribution systems that allow them to spread their fixed costs very efficiently over the large quantities of goods they sell. They have the lowest costs and highest industry inventory turnover rates—two factors that allow them to benefit tremendously from their economies of scale. Similarly, Microsoft enjoys economies of scale—with a large fixed cost and almost zero variable cost, they can swamp the planet with copies of Windows. No commercial alternative can compete—Microsoft can always drop its price low enough to drive its competitor out of business. The socially optimal number of operating systems software companies is very small and may even be just one—it is what economists call a **natural monopoly**. If we consider our economy as one big firm, we would not want to incur the same huge fixed software development cost twice. The same applies to utilities: we would not want two types of cable strung to our house, two types of telephone lines, and two types of power lines. Of course, companies with monopolies will want to charge higher prices to exploit their monopoly powers. This helps the companies, but hurts the economy overall. Society has therefore often found it advantageous to regulate monopolists. Unfortunately, the regulatory agencies are themselves often "captured" by the companies that they are supposed to regulate, which sometimes can hurt the economy even more than the monopolies themselves. There are no easy and obvious solutions.

Working with Sunk Costs

Sunk costs are ubiquitous, and the opposite of costs that should enter decision-making.

Sunk costs are, in a sense, the opposite of marginal costs. A **sunk cost** is a cost that cannot be altered and that therefore should not enter into your decisions today. It is what it is. Sunk costs are ubiquitous, if only because with the passage of time, everything is past or irrevocably decided and thus becomes a sunk cost.

An example of how capital investments become sunk, and then how production itself becomes sunk.

For example, consider circuit board production—a very competitive industry. If you have just completed a circuit board factory for \$1 billion, it is a sunk cost. What matters now is *not* that you spent \$1 billion, but how much the production of each circuit board costs. Having invested \$1 billion is irrelevant. What remains relevant is that the presence of the factory makes the marginal cost of production of circuit boards very cheap. It is only this marginal cost that matters when you decide whether to produce circuit boards or not. If the marginal board production cost is \$100 each, but you can only sell them for \$90 each, then you should not build boards, regardless of how much you spent on the factory. Though tempting, the logic of "we have spent \$1 billion, so we may as well put it to use" is just plain wrong. Now, presume that the market price for boards is \$180, so you go ahead and manufacture 1 million boards at a cost of \$100 each. Alas, your production run has just finished, and the price of boards—contrary to everyone's best expectations—has dropped from \$180 each to \$10 each. At this point, the board production cost is sunk, too. Whether the boards cost you \$100 to manufacture or \$1 to manufacture is irrelevant. The cost of the production run is sunk. If boards now sell at \$10 each, assuming you cannot store them, you should sell them for \$10 each. Virtually all supply costs eventually become sunk costs, and all that matters when you want to sell a completed product is the demand for the product.

Time is a good proxy, but not the deciding factor.

One more note—time itself often, but not always, decides on what is sunk or not. Contracts may allow you to undo things that happened in the past (thereby converting an ex-post sunk cost into a cost about which you still can make decisions), or bind you irrevocably to things that will happen in the future.

IMPORTANT: *A sunk cost has no cost contribution on the margin. It should therefore be ignored.*

Overhead Allocation and Unused Capacity

Allocating already existing overhead to a project valuation is a common example of bad project decision making.

A closely related mistake is to forget that "overhead" is often a sunk cost. By definition, overhead is not a marginal cost, but something that has been incurred already and is allocated to departments. For example, assume your firm has spent \$500,000 on a computer that is currently idle half the time. It serves only one division. Assume that another division can take an additional project that produces \$60,000 in net present value, but that will consume twenty percent of the computer's time. Should your firm take this project? If twenty percent of the cost of the computer is allocated to this new project (i.e., $20\% \cdot \$500,000 = \$100,000$), the net present value of the new project would appear to be a negative $-\$40,000$. But the correct decision process is not to allocate the existing overhead as a cost to divisions. The \$500,000 on overhead has already been spent. The computer is a sunk cost—assuming that it really would sit idle otherwise and find no better purpose. It may seem unfair to have charged only the original division for the computer and exempt the opportunistic other division. Yet taking this additional project will produce \$60,000 in profits without cost—clearly, a good thing. I personally know of plenty of examples in which overhead allocation has killed very profitable projects.

If capacity is unused, it should have a zero price.

"Capacity" is a subject that is closely related. For example, a garage may be currently only used for half its space. Adding the project "another car" that could also park in the garage would reduce this car's depreciation. The garage would then have a positive externality on project "corporate cars." The marginal cost of storing other cars in the garage should be zero.

Real World Dilemmas

Often we do not have easy, smooth margins.

But should we really charge zippo for parking corporate cars if we suspect that the unused capacity will not be unused forever? What if a new division might come along that wants to rent the five currently unused garage space in the future? Do we then kick out all current parkers? Or, how should we charge this new division if it wanted to rent six spaces? Should we give it the five remaining unused parking spots for free? Presuming that garages can only be built in increments of ten parking spots each, should we build another ten-car garage, and charge it entirely to this new division that needs only one extra parking spot in the new garage? Should this new division get a refund if other divisions were to want to use the parking space—but, as otherwise unused parking space, should we not use the garage appropriately by not charging for the nine extra spaces that will then be a free resource?

Fixed costs are often responsible. The old method works, though—use "on the margin."

When there are high fixed and low variable costs, then capacity is often either incredibly cheap (or even free) or it is incredibly expensive—at least in the short run. Still, the right way to think of capacity is in terms of the relevant marginal costs and marginal benefits. From an overall corporate perspective, it does not matter how or who you charge—just as long as you get the optimal capacity utilization. To the extent that cost allocation distorts optimal marginal decision-making, it should be avoided. In our case, if optimal capacity utilization requires zero parking cost for the old garage, then so be it. Of course, when it comes to the decision to build an entirely new garage, you simply weigh the cost of building the 10-spot garage against the reduced deterioration for 1 car.

It becomes much harder if we do not know the right outcome, so we have to play games with our subordinate managers.

Unfortunately, real life is not always so simple. Return to our example on Page 156 of an Internet connection, that has a positive influence on all our divisions. We know that our division managers will not want to pay for it if they can enjoy it for free—so we cannot rely on them telling us the correct marginal benefit. So, would it solve our problem to charge only divisions that are voluntarily signing up for the Internet connection, and to forcibly exclude those that do not sign up? If we do, then we solve the problem of everyone claiming that they do not need the Internet connection. However, we are then stuck with the problem that we may have a lot of unused network capacity that sits around, has zero marginal cost, and could be handed to the non-requesters at a zero cost. It would not impose a cost on anyone else and create more profit for the firm. Of course, if we do this, or even if we are suspected to do this, then no division would claim that they need the Internet to begin with, so that they will get it for free. In sum, what makes these problems so difficult in the real world is that as the boss, you often do not know the right marginal benefits and marginal costs, and you end up having to "play games" with your division managers to try to make the right decision. Such is real life!

Solve Now!

Q 7.1 *A company must decide if it should move division A to a new location. If division A moves, it will be housed in a new building that reduces its operating costs by \$10,000 per year forever. The new building costs \$120,000. Moving division A allows division B to expand within the old factory. This enables B to increase its profitability by \$3,000 per year forever. If the discount rate is 10%, should division A move?*

Q 7.2 *A firm can purchase a new punch press for \$10,000. The new press will allow the firm to enter the widget industry, and thereby earn \$2,000 in profits per year forever. However, the punch press will displace several screw machines that produce \$1,500 in profits per year. If the interest rate is 10%, should the new punch press be purchased?*

Q 7.3 *A company rents 40,000 square feet of space and is using 30,000 square feet for its present operations. It wishes to add a new division that will use the remaining 10,000 square feet. If it adds the division, equipment will cost \$210,000 once, and the operations will generate \$50,000 in profits every year. Presently, the office staff costs \$160,000 per year. However, the expansion requires a larger staff, bringing costs up to \$180,000 per year. If the cost of capital $r = 10\%$, should the firm expand?*

7·2. Comparing Projects With Different Lives and Rental Equivalents

Comparing contracts with unequal lives.

Let me switch gears, and cover another interesting and not immediately obvious application of NPV. A customer who currently pays \$350 each year (at year end) is offering you a 5-year deal, in which she pays \$1,000 upfront, followed by \$100 each year end, for total payments of \$1,500. The appropriate cost of capital is 10%. Should you accept this?

You get less per year—but more present value if you remember that you currently are paid only at year's end, too..

At first glance, you only get $\$1,500/5 = \$300$ per year, less than the \$350 you are getting now. The present value of this new contract is even less:

$$\begin{aligned} \text{PV} &= +\$1,000 + \frac{\$100}{(1+10\%)} + \frac{\$100}{(1+10\%)^2} + \frac{\$100}{(1+10\%)^3} + \frac{\$100}{(1+10\%)^4} + \frac{\$100}{(1+10\%)^5} \\ &= \$1,379.08 \quad , \end{aligned} \tag{7.6}$$

which comes to only \$275.82 per year. Looks like a bad idea, does it not? Wrong! What you need to realize is that the customer currently pays only \$350 per year *in the future*, too, so the current arrangement of \$350 per year is an annuity that is worth only

$$\begin{aligned} \text{PV} &= \frac{\$350}{(1+10\%)} + \frac{\$350}{(1+10\%)^2} + \frac{\$350}{(1+10\%)^3} + \frac{\$350}{(1+10\%)^4} + \frac{\$350}{(1+10\%)^5} \\ &= \$1,326.78 \quad . \end{aligned} \tag{7.7}$$

The new contract is better than the current annual arrangement.

Rental Value Equivalents.

You can also think of this problem in terms of an equivalent "rental per annum value" of the contract. (It is sometimes abbreviated as **EAC** for equivalent annual cost.) What kind of annuity would be equal to our contract? The answer is

$$\begin{aligned} &\frac{x}{(1+10\%)} + \frac{x}{(1+10\%)^2} + \frac{x}{(1+10\%)^3} + \frac{x}{(1+10\%)^4} + \frac{x}{(1+10\%)^5} \\ &= \text{Annuity}(\,x, 5 \text{ years}, 10\%\,) = \frac{x}{10\%} \cdot \left[1 - \frac{1}{(1+10\%)^5}\right] = \$1,379.08 \quad . \end{aligned} \tag{7.8}$$

The solution is an annual rental value of $x = \$363.80$. So, the new contract would be worth the same as annual contracted payments of \$363.80—more than the \$350 that you are currently receiving.

Comparing multi-period contracts.

Rental values are a good way to think of contracts with different lengths. For example, would you prefer our contract to another one in which the customer pays \$800 upfront and \$200 each year for two years? The NPV of this contract is only

$$\text{NPV} = \$800 + \frac{+\$200}{1+10\%} + \frac{+\$200}{(1+10\%)^2} = \$1,147.11 \;, \tag{7.9}$$

which is lower than the \$1,379.08 that the earlier contract offered—but this is like comparing apples and oranges. Again, if you think about it, with the newer contract, you would still have another three years of product to sell. In this case, the NPV of the new contract of \$1,147.11 for two years worth of production is equivalent to rental payments of

$$\text{NPV} = \frac{x}{10\%} \cdot \left[1 - \frac{1}{(1+10\%)^2}\right] \quad \Leftrightarrow \quad x = \$660.95 \;. \tag{7.10}$$

The annual flow of \$660.95 is much higher than the annual flow of \$363.80 that the older contract offered, so you might prefer the newer contract.

Rental equivalents work only if you believe they are repeatable.

However, as appealing as comparing rental equivalents may be, this technique depends on the very strong assumption that you can repeat contracts when they expire and for quite a number of times (at least until both contracts expire in the same year). To see this, ask yourself which contract you would prefer if you believed that your customer would jump to the competition after the two-year contract is over. If your project would be otherwise useless, you might now prefer the five-year contract. Similarly, you have assumed that you do not value flexibility—for example, the shorter contract may allow you to attract an even better customer in 3 years. Or, it may be that you value the flexibility of not repeating the contract, if you fear that your input costs may be much higher in 3 years (so your net cash flows would not be what you input above). On the other hand, you may not like giving your customer the flexibility of jumping ship. In this case, flexibility would be a cost to you, and you would prefer the 5-year contract. In Section 7·4, we will be talking more about the value of such flexibility.

Cost Bases

Similar rental equivalent value problems also often arise when you compare different technologies—for example, you can purchase a machine that is likely to last for 18 years, and you must compare it against another machine that is likely to last for 22 years. The method for solving these problems is exactly the same, so try it in the next question.

Solve Now!

Q 7.4 *Machine A costs \$10,000 upfront, and lasts for 18 years. It has annual maintenance costs of \$1,000 per year. Machine B costs \$15,000 upfront, lasts for 22 years, and has annual maintenance costs of \$800 per year. Both machines produce the same product. The interest rate is 12% per annum.*

(a) What is the PV of the costs of each machine?

(b) What is the rental equivalent of either machine?

(c) Which machine is the better purchase if you assume neither value to flexibility, nor expect different machine costs or contracting conditions in the future?

7·3. Expected, Typical, and Most Likely Scenarios

The NPV formula requires expected cash flows, not typical cash flows. The difference is low-probability events.

Let us move on to a different, but also common error when managers apply NPV. This error is primarily conceptual. Under uncertainty, the NPV formula requires the **expected cash flows** in the numerator. The mistake is to think of the *typical cash flow* (in statistical terminology, the median) or the *most likely cash flow* (the mode), instead. If you do this, you will fail to consider low-probability events: a plane crash, a legal suit, an especially severe recession, or a terrific new client.

An Example.

For example, your business may have the following payoffs:

Event	Probability	Value
Good Business	50%	\$1,200,000
Normal Business	45%	\$1,000,000
Lawyers Sue For Punitive Damages	5%	$-\$10,000,000$

The most likely payoff is \$1,200,000. The typical payoff is \$1,000,000. The expected payoff, however, is only

$$\begin{aligned} \mathcal{E}(\text{Payoff}) &= 50\% \cdot \$1,200,000 + 45\% \cdot \$1,000,000 + 5\% \cdot (-\$10,000,000) \\ &= \$550,000 \quad . \end{aligned} \tag{7.11}$$

It is the latter that is required in an NPV analysis. If you run this business 100 times, you would receive \$1.2 million 50 times, \$1 million forty-five times, and lose \$10 million 5 times. Fortunately, if the statistical distribution is symmetric, e.g., as it is in the case of the normal distribution, then the center of the distribution is the mean, median, and mode. Unfortunately, few businesses are immune to low-probability shocks, so the distinction between mean, median, and mode can rarely be taken lightly.

7·4. Future Contingencies and Real Options

Now we move on to an important complication in estimating future expected cash flows. When the future is uncertain, then the ability to change course in the future—depending on the economic environment at the time—creates value. A business may expand its size, accelerate its production, and venture out into related or spin-off businesses, if the demand for its products increases, or if the costs of its inputs fall. Similarly, the firm may reduce, delay, or stop production if its economic environment deteriorates. This is called a **real option** (or **strategic option**). Conceptually, these options are just a variant of the problem of assessing the expected cash flows (and their cost of capital) correctly. Practically, the resulting complications can be so difficult that one could write a whole book on the subject–and some have done so.

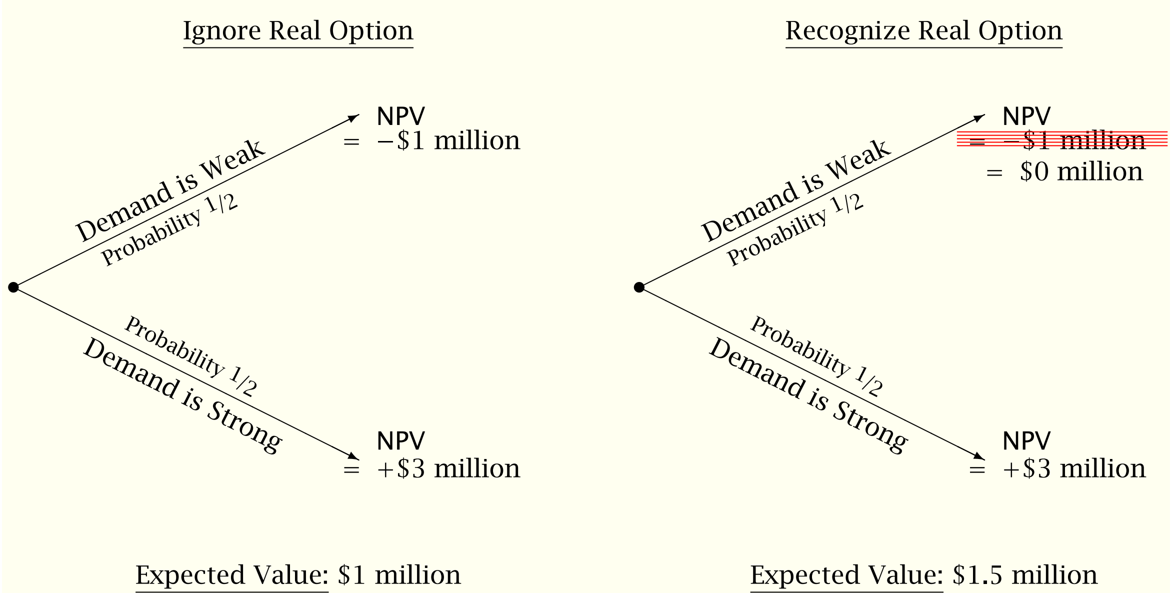
A real option is the value of flexibility to change course in the future.

Let us illustrate real options with a really simple example. A factory may cost $1 million to build in year 1. In year 2, it can produce $2 million worth of inputs into $3 million worth of outputs. The expected profits next year of $1 million are no better than the actual cost today. From an NPV perspective, this factory does not appear worthwhile. Or does it?

An Example.

Table 7.1. State Contingent Factory Payoff Table

		Ignore Real Option	Recognize Real Option
		Always Run Factory	Shut down if Optimal
	Prob	Dumb NPV	Smart NPV
Demand is low	1/2	$1,000,000	$0
		– $2,000,000	
		= –$1,000,000	$0
Demand is high	1/2	$5,000,000	$5,000,000
		– $2,000,000	– $2,000,000
		$3,000,000	$3,000,000
	Expected Value	$1,000,000	$1,500,000



It is not the expected value that matters.

If the product can be either in high demand (yielding \$5 million) or in low demand (yielding \$1 million)—both equally likely—the expected value of output can indeed be \$3 million, as stated. This is what the firm expects to earn if managers always operate the factory, regardless of demand. But if managers can shut down the factory when demand is low, then owning the factory is similar to owning a contingent equity claim with limited liability (Chapter 5): *you can get the upside with less downside.* Take a look at the payoff table in Table 7.1. It shows that if the factory always runs, its expected net cash flows are less than if managers shut down the factory when demand is low. It is the managerial flexibility that increases the expected factory cash flow from \$1 million to \$1.5 million, which means that the factory is well worth building under reasonable cost-of-capital scenarios.

Strategic Options are everywhere.

Strategic options become both more important and more difficult to handle when there are many periods and many possible economic scenarios. Indeed, many projects are nothing but strategic options: For example, the value of unused land around cities is essentially the option that the city might expand enough to make building on the land economically worthwhile. Research and development often has no immediate usefulness, or even usefulness in the most likely scenario—but there is a chance that it might yield a highly profitable discovery. This strategic option value has to be properly considered in the expected cash flow computation, or the project value will be underestimated. Silly as it may sound, the most common mistake that managers commit when it comes to real options is to just not recognize that they are there.

Strategic options are tough to value: use scenario analysis or Monte-Carlo simulations.

This does not mean that the rest is easy. The proper valuation of strategic options is as important as it is difficult. This is especially the case if your decisions in each time period cannot be made independently. For example, if it costs money to close and reopen a plant, then your decision to close the plant must also depend on your assessment of how quickly the product price will recover. If there is a good chance of recovery soon, you may choose to operate the plant even at a small loss right now. The resulting optimal decision rule itself can be incredibly difficult to find. The correct valuation method is **scenario analysis**, conducted in a computer spreadsheet, in which the manager specifies different scenarios, one at a time, each resulting in its own cash flows. The appropriate project NPV is the average over many different scenarios—each with different probabilities and possibly different managerial responses. This is not easy. Think about how difficult it was just to figure out how many different project combinations you could choose *today*. Now, figure out what you could do in the future under an infinite variety of possible *future* scenarios—which you have to do when valuing strategic options. Out of necessity, most businesses just consider the most obvious strategic options.

You can find more sophisticated instructions in advanced books or chapters.

Monte-Carlo simulation is an automated form of scenario analysis, in which managers can specify a whole range of possible future scenarios and let the spreadsheet work out the expected outcome under many different scenarios. The Monte-Carlo simulation computes the probability-weighted average over all scenarios. This method is beyond the scope of this chapter.

There is also a whole web chapter dedicated to the subject of real options, but beware: it is also the most difficult chapter.

Solve Now!

Q 7.5 *List "strategic options" that a firm needs to incorporate in its project valuation.*

Q 7.6 *A business produces 100,000 gadgets, costing \$1 each to produce and sellable for \$1.80 each (last year and just now). To produce another 100,000 gadgets requires running the machine at night, which increases production costs from \$1 to \$2. The business can last for up to 2 years (but think about how you would solve this for 5 years). In every year, with 10% probability, the output price doubles; with 10% probability, the output price halves; with 80% probability, the price stays the same as in the previous year. Shutting down the factory for 1 year costs \$9,000. Reopening it costs \$10,000. The cost of capital is a constant 5% per year. What is the value of this factory? (This is a difficult problem, but unfortunately not an unrealistic one.)*

7·5. Mental Biases

Model inputs are usually not what they should be.

Most cash flow and cost-of-capital estimates rely on judgments. Unfortunately, it is often difficult to obtain accurate judgments. Our brains tend to commit systematic decision errors. Managers who do not recognize these biases will systematically make poor decisions.

Innate Human Decision Biases cause predictable valuation mistakes.

There are literally dozens of well-known behavioral errors, but limited space allows us to highlight just three: **overconfidence**, **relativism**, and **compartmentalization.**

1. **Overconfidence** is the tendency of people to believe that their own assessments are more accurate than they really are. In lab experiments, ordinary people are found to be dramatically overconfident. When asked to provide a 90% confidence interval—which is just a range within which they are confident that their true value will lie in nine out of ten tries—most people end up being correct only five out of ten times.

   It is difficult to empirically document overconfidence—after all, if it were easy, managers would recognize it themselves and avoid it. However, we do have evidence that many managers who are already heavily invested in their own company tend to throw all caution overboard and voluntarily invest the rest of their own money into the corporation—and even in companies going bankrupt lateron. There is also good empirical evidence that those of us who are most optimistic in overestimating our own life-expectancy disproportionately become entrepreneurs. Even if optimism is a disease, it seems to be a necessary one for entrepreneurs.

   To understand this better and to test your own susceptibility to these problems, you can take a self-test at the book website, http://welch.econ.brown.edu/book. Doing so will likely make you remember this problem far more than reading long paragraphs of text in this book. Incidentally, the only population segments who are known not to be systematically overconfident are weather forecasters and clinically depressed patients.

2. **Relativism** is the tendency of people to consider issues of relative scale when they should not. For example, most people are willing to drive 15 minutes to a store farther away to save \$20 on the purchase of \$30 worth of groceries, but they would not be willing to drive the 15 minutes to a car dealer farther away to save \$100 on the purchase of a new \$20,000 car. The savings appears to be less important in the context of the car purchase than in the context of a grocery purchase. This is flawed logic, similar to comparing IRR's while ignoring project scale. The marginal cost is driving 15 minutes extra, and the marginal benefit is a higher \$100 in the context of the car than the \$20 in the context of the groceries. Put differently, the problem is that we tend to think in percentages, and the \$20 is a higher percentage of your grocery bill than it is of your car purchase. The smaller the amount of money at stake, the more severe this problem often becomes. When a gas station advertises a price of \$2 per gallon rather than \$2.10, customers often drive for miles and wait in long lines—all to fill a 20 gallon gas tank at a total savings that amounts to a mere \$2.

3. **Compartmentalization** Compartmentalization is the tendency of people to categorize decisions. Most people are more inclined to spend more when the same category has produced an unexpected windfall earlier. For example, winning a lottery prize while attending a baseball game often makes winners more likely to purchase more baseball tickets, even

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Anecdote: Small Business Failures
In New York City, two of every five new restaurants close within one year. Nationwide, the best estimates suggest that about 90% of all restaurants close within two years. If successful, the average restaurant earns a return of about 10% per year. So, owners seem to lose money on average. So, why open yet another restaurant? We mentioned earlier (Page 10) that restaurateurs may just enjoy owning restaurants. But a more likely explanation is that restauranteurs are over-optimistic, and just do not realize how tough it is to profitably run a restaurant.

More generally, a **Small Business Administration** study of small business failures from 1989 to 1992 found that 33% of businesses failed within 2 years, 50% within 4 years, and 66% within 6 years. Yet in a survey of about 3,000 entrepreneurs, 81% of entrepreneurs believed that their chances of success were at least 70%, and 33% believed that they had zero chance of failure!

though the project "baseball game" has not changed in profitability. Similarly, an unexpected loss may stop people from an otherwise profitable investment that they should make. For example, say an individual likes to attend a particular baseball game. If she loses her baseball game ticket, she is less likely to purchase a replacement, even though the cost and benefit of purchasing the ticket are the same as they were when the original ticket was purchased.

Know thyself to avoid these errors!

Solve Now!

Q 7.7 *Describe common mental decision biases, and how they are likely to bias NPV calculations.*

7·6. Incentive (Agency) Biases

Incentive problems arise when the information provider has incentives that are different from those of the project owner.

Mental biases are not our only bias. Another kind of bias arises when one person is acting on behalf of another. This is called an **agency** problem—a situation in which the owner of a project has to rely on someone else for information, and this someone else has divergent interests. An example may be shareholders who rely on corporate management to undertake projects on their behalves, or a division manager who has to rely on department managers for information about how profitable their proposed projects really are. A cynical synopsis of agency biases would be "all people act and lie in their own self-interest." Now, although everyone does have incentives to lie—or at least color the truth—corporations are especially rife with such agency distortions. Of course, few people sit down and contemplate how to best and intentionally lie. Instead, they convince themselves that what is in their best interest is indeed the best route to take. Thus, mental biases often reinforce incentive problems: "wishful thinking" is a disease from which we all suffer.

These problems are pervasive and important.

You can take the fact that we have already had to mention agency issues repeatedly in this chapter as an indication of how important and pervasive these are. But, again, lack of space forces us to highlight just a few issues with some examples:

1. **Competition for Capital** Managers often compete for scarce resources. For example, division managers may want to obtain capital for their projects. A less optimistic but more accurate estimate of the project cash flows may induce headquarters to allocate capital to another division instead. Thus, division managers often end up in a race to make their potential projects appear in the most favorable and profitable light.

2. **Employment Concerns** Managers and employees do not want to lose their jobs. For example, scientists tend to highlight the potential and downplay the drawbacks of their areas of research. After all, not doing so may cut the project and thereby cost them their jobs.

3. **Perks** Managers do not like to give up perks. For example, division managers may like to have their own secretaries or even request private airplanes. Thus, they are likely to overstate the usefulness of the project "administrative assistance" or "private plane transportation."

4. **Power** Managers typically love to build their own little "empires." For example, they may want to grow and control their department because bigger departments convey more prestige and because they are a stepping stone to further promotion, either internally or externally. For the same reason, managers often prefer not to maximize profits, but sales.

5. **Hidden Slack** Managers like the ability to be able to cover up problems that may arise in the future. For example, division managers may want to hide the profitability of their divisions, fearing that headquarters may siphon off "their" profits into other divisions. They may prefer to hide the generated value, feeling that the cash they produced in good times "belongs" to them—and that they are entitled to use it in bad times.

6. Reluctance to Take Risk Managers may hesitate to take on risk. For example, they may not want to take a profitable NPV project, because they can only get fired if it fails—and may not be rewarded enough if it succeeds. A popular saying used to be "no one was ever fired for buying IBM," although these days Microsoft has taken over IBM's role.

7. Direct Theft Managers and employees have even been known to steal outright from the company. For example, a night club manager may not ring sales into the cash register. Or a sales agent may "forget" to charge her relatives. In some marginal cases, this can be a fine line. For example, is taking a paper clip from the company or answering a personal e-mail from the company account really theft? In other cases, theft is blatantly obvious. In September 2002, Dennis Kozlowski, former CEO, was charged with looting $600 million from Tyco shareholders. His primary defense was that he did so in broad daylight—with approval from the corporate board that he had helped put in place.

We do know where agency problems play a bigger role and where they play a lesser role.

Where agency problems are big, and where they are not.

I. Scale and Owner Engagement In a small company with one owner and one employee, agency conflicts are less important than they are in big corporations with their many layers of management and disengaged owners.

Do you believe that professionally run companies really make the best decisions on behalf of their public shareholders? Remember that agency issues do not just arise between shareholders and management—they start with the lowest level employee and bubble all the way up to the top-level CEO. Decision-making is often based on a chain of deception. It is a testament to the importance of sharing risks among many investors that large, publicly traded companies still manage to net-in-net create shareholder value!

II. Project Duration If the project is short-term and/or comes with good interim progress points, it is easier to reward managers appropriately for success and punish them for failure. For example, think how you would judge and reward a manager who is (supposedly) working on an R&D project that is not likely to have visible results for decades. This is a difficult task. Agency problems for large and very long-term projects may be so intrinsically high that they cannot be undertaken.

III. External Noise If good luck is an integral and important part of the project, it becomes more difficult to judge managerial performance, which in turn aggravates agency issues. For example, we can relatively easily measure the productivity of a line worker in a factory. We know whether she works or slacks off. Therefore, agency problems matter less. In contrast, it is more difficult to determine if our sales agent worked hard but the customer just did not bite, or if our sales agent just failed. Similarly, our nightwatch security guard may or may not be working hard, and it could take years before we could learn (probably the hard way) whether she regularly stayed awake or just dozed off.

IV. Opaqueness If information is very difficult for outsiders to come by, agency problems will be worse. For example, if only your manager sees what projects are available, he can present only those that he would like to undertake and not mention those that have higher NPV, but require different skills that he may not have or more work that he finds unpleasant.

We also know that there are mechanisms that can help alleviate agency problems.

A. Audits If the company runs independent assessments or audits, managers can make decisions based on better information, even if their employees are unwilling to provide it. However, many consultants suffer from the same disease as employees: they know that they are most likely to be rehired if they tell the manager what she wants to hear.

B. Truth-Telling Incentives If managers can be rewarded for telling the truth, agency conflicts will become less important. For example, if your company has a research scientist who has expertise in alpha-proteins and works on an alpha-protein project, your goal as manager should be to allow this scientist to say without suffering any negative consequences: "Do not waste your money putting any more research dollars into alpha-proteins." This means that the scientist's salary and promotion chances must remain the same or even increase—even if this means that she no longer has a good alternative use for her time and effort. You might even offer a reward for any scientists who are voluntarily cancelling their projects.

Would you really be willing to carry through on such a promise? Would your research scientists believe you?

Some companies also undertake **post audits**, which are designed not only to evaluate the quality of the financial numbers (like a usual **audit**), but also the quality of managers' up-front forecasts. Knowing that such post audits will be held will strengthen the incentives of managers to give accurate forecasts to begin with.

C. Contingent Compensation If managers are rewarded more if the project succeeds and punished if the project fails, agency conflicts will become less important. For example, if you pay your managers only when their projects succeed (or throw them into jail when their project fails!), then managers will work harder and choose projects that they believe are more likely to succeed.

Of course, like any other mechanism to control agency problems, this control strategy has its costs, too. Managers have to feed their families and you may not be able to attract the best managers if you force them to take on so much risk. (The capital markets are probably better at taking risk than individual families!) And such managers may also be more reluctant to take good risks on behalf of the company—risks that they *should* take in the interest of shareholders—if they are themselves risk averse and compensated by outcome.

D. Reputation If managers can build a reputation for truth-telling and capable management, they are less likely to undertake bad projects. For example, agency concerns are likely to be a worse problem when it comes to secret one-shot projects, where your managers cannot build a track record that will help them with future projects. On the other hand, sometimes reputational considerations can themselves become the problem. Witness the many dysfunctional but beautifully artistic office buildings that are primarily monuments to some famous architectural firm.

The best "solution" is ample skepticism and common sense.

There is no obvious solution to these decision bias problems. Again, do not believe that just because we have spent only a few pages on agency issues that they are not important—they are both ubiquitous and very important in the real world. The website, http://welch.econ.brown.edu/book, has a full chapter on **corporate governance**, which is all about agency conflicts. As a manager or principal, you must be skeptical of all estimates and judgments and take the biases and incentives of each information provider into account.

Solve Now!

Q 7.8 *The CEO projects earnings of $100 million next year. List three reasons why this is not a good input into an NPV valuation.*

Q 7.9 *Describe common agency biases, and how they are likely to bias NPV calculations.*

ANECDOTE: Fiduciary Responsibility, or the Fox guarding the Hen House.
On Wednesday, December 29, 2004, The Wall Street Journal reported on page 1:

> In the biggest U.S. merger this year, J.P. Morgan Chase&Co. announced last January it would acquire Bank One Corp. To assure investors it was paying fair price, J.P. Morgan told them in a proxy filing it had obtained an opinion from one of "the top five financial advisors in the world."
>
> Itself.
>
> The in-house bankers at J.P. Morgan endorsed the $56.9 billion price—negotiated by their boss—as "fair."

Next to it was a sidebar called Passing Muster, which explained

> A 'fairness' opinion tells a company's board that a deal's terms are fair to shareholders.
>
> **Purpose:** Legal protection from an investor claim that a deal was done without due care.
>
> **Cost:** A few hundred thousand dollars to a few million.
>
> **Potential Conflicts**
>
> - Bankers may have incentives to call a deal fair because most of their advisory fee is paid only if deal closes.
> - Bankers' fee is tied to the deal price.
> - Bankers may support a deal where executives will personally profit, in hopes of securing future work.
> - Bankers use financial data supplied by client that wants deal to go through.
> - When deal maker is a bank, its own bankers often write the fairness opinion.

Remember that everyone—in-house bankers, management, and the corporate boards—are employed by the shareholders, to whom they owe fiduciary responsibility, and for whom they are supposed to look out for. It is a clear agency conflict for an employee to provide a fairness opinion. But it would also be difficult for management to have these in-house bankers fired for doing them a personal favor—another agency conflict.

And there is also the original agency conflict: the incentive of acquiring managers to pay too high a price or of target managers to accept too low a price. Here is how the WSJ story continues:

> But during the negotiations, Bank One Chief Jamie Dimon had suggested selling his bank for billions of dollars less if, among other conditions, he immediately became chief of the merged firm, according to a person familiar with the talks. That suggestion wasn't accepted by J.P. Morgan.

Obviously, Jamie Dimon did not offer to pay his own personal billions for the privilege of becoming CEO, but Bank One's shareholders' billions. Obviously, the J.P. Morgan management did not decline the billions on behalf of their own pockets, but on behalf of their J.P.M. shareholders.

Still, there are of course the corporate boards which could have fired either the in-house bankers or their management teams. Neither happened.

7·7. Summary

The chapter covered the following major points:

- Attribute to each project's NPV its influence on other projects, either positive or negative.
- Think about how you can take advantage of or create positive externalities among projects. If you cannot, there is no reason for these to be organized inside the same firm.
- Think "on the margin." Take all projects that contribute more marginal benefits than they create marginal costs.
- Consider economies of scale.
- Ignore sunk costs.
- Realize that real-world implementation problems—which range from differences in short-term marginal costs and long-term marginal costs, to political reasons and agency considerations inside corporations—often make taking the best set of projects difficult.
- Compare long-term projects in terms of their rental equivalents *if* these projects are easily repeatable.
- Use the *expected* cash flows, not the *most likely* cash flows in the NPV numerator.
- Take "strategic options" (or "real options") into account. This is the value of your ability to change course depending on future conditions. It includes your flexibility to delay or accelerate projects, and to expand or shut down projects.
- Realize that common human and agency biases usually distort expected cash flow estimates.
- Design your operations so as to reduce agency conflicts when it is marginally profitable to do so.

The problem is tough! We can only offer some help.

No doubt about it: good capital budgeting is a difficult problem. Each subsection covered in this chapter can easily be expanded into a full chapter or even a full book—and real options and corporate governance already have their own web chapters. There are pitfalls everywhere. To make your task a little easier, Appendix 2·1 contains an NPV checklist. In the end, though, capital budgeting is an art as much as a science and has to rely as much on common sense and intuition as on rules. The best analysis combines both.

Solutions and Exercises

1. The answer is yes, because it will cost the company $120,000 to move division A. Moving saves $10,000/10%= $100,000 in division A costs and $3,000/10%=$30,000 in division B costs. The total savings are $130,000 which is $10,000 greater than the cost of the building.
2. The answer is no, because the press earns $2,000/0.10=$20,000. But the press costs $10,000 to purchase and eliminates $1,500/0.10=$15,000 of profits from the screw machines. So the total cost of the press, including the $15,000 in opportunity costs, is $25,000. The project's value is $\$20,000 - \$25,000 = -\$5,000$.
3. Yes. The PV of the division's profits will be $50,000/0.10=$500,000. The division costs are $210,000 for new equipment and $20,000 per year in increased overhead. The PV of the increased overhead is $20,000/0.10=$200,000. So the total PV cost of the new division is $\$210,000 + \$200,000 = \$410,000$, and the PV of the benefits equal $500,000.
4. (a)

$$\begin{aligned} \text{PV}(\text{ Cost }) &= \$10,000 + \text{Annuity}(\ \$1,000, 18 \text{ years}, 12\%\ ) \\ &= \quad \$10,000 + \frac{\$1,000}{12\%} \cdot \left[1 - \frac{1}{(1+12\%)^{18}}\right] \quad = \$17,249.67 \ . \\ \text{PV}(\text{ Cost }) &= \quad \$15,000 + \text{Annuity}(\ \$800, 22 \text{ years}, 12\%\ ) \\ &= \quad \$15,000 + \frac{\$1,000}{12\%} \cdot \left[1 - \frac{1}{(1+12\%)^{22}}\right] \quad = \$21,115.72 \ . \end{aligned} \tag{7.12}$$

   (b) The equivalent rental values are

$$\begin{aligned} \text{Annuity}(x, 18 \text{ years}) &= \$17,249.67 \quad \Leftrightarrow \quad x = \$2,379.37 \ . \\ \text{Annuity}(x, 22 \text{ years}) &= \$21,115.72 \quad \Leftrightarrow \quad x = \$2,762.16 \ . \end{aligned} \tag{7.13}$$

   (c) The 18-year machine has the lower rental cost, so it is the better deal.

5. See Page 165.
6. Problems like this need to be solved “backwards.” You can start in period 2 with a prevailing price of $0.45, $0.90, $1.80, $3.60, or $7.20; and your factory can be either open or closed. In this final period,
   - If the price is $0.90 or lower, you definitely want to close the factory, because $9,000 loss is better than $10,000 loss. If the factory is already closed, lucky you.
   - If the price is $1.80 or higher, you definitely want the factory to be open, because $80,000 profit fortunately outweighs all opening and closing costs. If the factory is already open, lucky you.

   Now consider what to do in year 1. If the price drops to $0.90, you have a decision to make: operate the factory for a year, hoping that the future will be better, or close the factory. Operating losses would be $10,000. Closing immediately would cost only $9,000. So, if you operate today, you incur an extra $1,000 loss. In exchange, there is a 10% chance that the price will go back up, in which case you got lucky. In this case, you will have saved $10,000 reopening costs. Thus, you are exactly indifferent between closing and operating if the price has dropped. (Of course, if the price is higher today, operating today is the correct choice.)
   The problem of determining optimal choices as a function of environmental variables can get incredibly complex very easily. Scenario analysis (or just plain real-world experience and intuition) is really the only analysis method. This goes beyond the scope of an introductory textbook.

7. See Page 167.
8. First, it is probably the most likely outcome, not the expected outcome. It is probably more likely that the firm goes bankrupt due to totally unforeseen circumstances than it is likely that the firm will have a windfall. Second, the CEO has an incentive to distort the truth, and report better projections than are most likely. This is an agency problem. And, third, the CEO is probably subject to mental biases, too.

9. See Page 168.

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

CHAPTER 8

OTHER IMPORTANT CAPITAL BUDGETING TOPICS

The Internal Rate of Return (IRR), Duration, and Other Capital Budgeting Rules

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This chapter explains commonly used capital budgeting rules (methods to decide among projects) other than NPV. Two of them often—but not always—yield the same correct answer, because they are really based on the same equation as NPV. They are the profitability index and the internal rate of return, abbreviated IRR. You have already seen the IRR, even though we called it yield-to-maturity (YTM). It is used in other contexts, too, because it can be interpreted as a "sort of average rate of return" when a project has different cash flows at different times. You will encounter IRR many times in your career. Unfortunately, there are also some incorrect capital budgeting rules that you may encounter, and some are in common use. You thus need to know what they are and when and why they are bad.

ANECDOTE: Real-World Use of NPV, IRR, and Payback
Among large Fortune-500 type firms, about 75% of CFOs use NPV and IRR in their capital budgeting decision (and more frequently if they have an MBA). Yet, small firms' CFOs tend to *also* use a flawed payback criterion (and more frequently if they do *not* have an MBA and if they are "mature").
(Source: Graham and Harvey, Duke, 2001.)

8·1. Profitability Index

How it is computed.

The first important alternative capital budgeting measure is the **profitability index.** It divides the present value of future cash flows by the project cost (the negative of the the first cash flow). For example, if you have a project with cash flows

Time	0	1	2	3	PV 1 to 3
Project A Cash Flow	–$100	$70	$60	$50	$128.94

and the interest rate is 20% per annum, you would first compute the present value of future cash flows as

$$\begin{aligned} \text{PV} &= \frac{\$70}{1+20\%} + \frac{\$60}{(1+20\%)^2} + \frac{\$50}{(1+20\%)^3} \approx \$128.94 \\ &= \text{PV}(\text{CF}_1) + \text{PV}(\text{CF}_1) + \text{PV}(\text{CF}_1) \quad . \end{aligned} \tag{8.1}$$

The NPV is $28.94. The profitability index is

$$\begin{aligned} \text{Profitability Index} &= \frac{\$128.94}{-(-\$100)} \approx 1.28 \quad . \\ \text{Profitability Index} &= \frac{\text{PV(Future Cash Flows)}}{-\text{CF}_0} \end{aligned} \tag{8.2}$$

A positive NPV project usually has a profitability index above 1—”usually” because the profitability index is meaningful only if the first cash flow is a cash outflow. When this is the case, you can use either NPV or the profitability index for a simple “accept/reject” decision: the statements “NPV > 0” and “Profitability Index > 1” are the same.

Here it does nicely.

Some managers like the fact that the profitability index gives information about relative performance and use of capital. For example,

Time	0	1	2	3	PV 1 to 3
Project B Cash Flow	–$10.00	$21.14	$18.12	$15.10	$38.94

has the same NPV of $28.94 as our original project, but a higher profitability index than 1.28 because it requires less capital upfront.

$$\begin{aligned} \text{Profitability Index} &= \frac{\$38.94}{-(-\$10)} \approx 3.89 \quad . \\ \text{Profitability Index} &= \frac{\text{PV(Future Cash Flows)}}{-\text{CF}_0} \quad . \end{aligned} \tag{8.3}$$

The reason is that two measures value the scale of the project differently. It is intuitively appealing that we would prefer the second project, even though it has the same NPV, because it requires less capital. It may even be less risky, but this can be deceiving, because we have not specified the risk of our future cash flows.

But here is where it can go wrong.

Unfortunately, the very same feature that we just considered as an advantage can also be a disadvantage. You cannot use the profitability index to choose among different projects. For example, assume that your first project returns twice as much in cash flow in all future periods, so it is clearly the better project now.

Time	0	1	2	3	$\text{PV}(\text{CF}_1, \text{CF}_2, \text{CF}_3)$
Project B Cash Flow	–$10	$21.14	$18.12	$15.10	$38.94
Project C Cash Flow	–$100	$140	$120	$100	$257.87

But the profitability index of project C is only

$$\text{Profitability Index} = \frac{\$257.87}{-(-\$100)} \approx 2.57 \quad . \tag{8.4}$$

which is below the 3.89 profitability index of project B. The reason is that, when compared to NPV, the profitability index *really* "likes" lower upfront investment projects. It can indicate higher index values even when the NPV is lower. So, you should really consider the profitability index to choose among projects only if the NPV of the two projects is equal (or at least very similar).

8·2. The Internal Rate of Return (IRR)

Why study IRR if we already know the correct answer?

The second important alternative to NPV is the **Internal Rate of Return**, often abbreviated as **IRR**. You already know it—it is the same measure as the yield-to-maturity (YTM) that we encountered in Section 4·6. It is important that you understand how to work it—and what its drawbacks are, because it is in wide use.

8·2.A. Definition

Important: *The Internal Rate of Return is the quantity IRR, which, given a complete set of project cash flows, solves the NPV equation set to zero,*

$$0 = CF_0 + \frac{CF_1}{1 + IRR} + \frac{CF_2}{(1 + IRR)^2} + \frac{CF_3}{(1 + IRR)^3} + \dots \tag{8.5}$$

The IRR capital budgeting rule states that if a project's IRR is above its appropriate interest rate (cost of capital), it should be taken.

Computation.

For example, for a project with cash flows −\$100, +\$55, and +\$60.50 in successive years, you find the IRR by solving the equation

$$-\$100 + \frac{\$55}{1 + \text{IRR}} + \frac{\$60.50}{(1 + \text{IRR})^2} = 0 \ . \tag{8.6}$$

For this particular set of cash flows, the solution is an internal rate of return of 10%, because

$$-\$100 + \frac{\$55}{1 + 10\%} + \frac{\$60.50}{(1 + 10\%)^2} = 0 \ . \tag{8.7}$$

If the project's appropriate cost of capital over all years is 9%, then the IRR capital budgeting rule states that the project should be accepted; if the cost of capital is 11%, it should be rejected. As with NPV, it is important with IRR that you know whether you are computing it based on expected or promised cash flows. If you want to use the IRR for capital budgeting, you must use expected cash flows. YTM is most often computed for promised cash flows—thus, it really should be called "quoted YTM" rather than just "YTM."

IRR is a characteristic of a project's cash flows. (It is not an interest rate.)

Even though the internal rate of return is quoted as a percentage and compared against an interest rate (the cost of capital or hurdle rate), the IRR itself is *not* an interest rate. Instead, IRR is a characteristic of a project's cash flow stream. A given cash flow stream directly implies an IRR. In fact, you can compute the IRR for a project, never having looked at financial markets, interest rates, or costs of capital. This is IRRs most important advantage over NPV: *it can be calculated before you know what the appropriate interest rate (cost of capital) is.* It thus can give you useful project information in and of itself. It is also helpful in judging project profitability and thereby allows you to judge the performance of a manager—it may be easier to hold her to her earlier promise of delivering an IRR of 20% than it is to argue with her about what the appropriate cost of capital for her project would be. Armed with the IRR, you can then contact various capital providers to see if the project is worthwhile. This is especially useful if your project can be easily scaled but your cost of capital is increasing with your level of investment (markets are imperfect). In this case, NPV is somewhat cumbersome to use, but IRR makes it easy to help you determine an optimal scale for your investment.

> **Important**: *The IRR is a characteristic of project cash flows. It is not an interest rate, even though the IRR is compared to an interest rate in the IRR capital budgeting rule.*

IRR is safe to use when there is only one positive or only one negative cash flow.

The IRR capital budgeting rule often yields the same (correct) decision as the NPV capital budgeting rule. IRR is guaranteed to work if

1. the first, up-front cash flow is a single negative number (an investment) followed only by positive cash flows (paybacks), or vice-versa.
2. the relevant yield curve for your cost of capital is uniformly above or below the IRR.

This is why IRR has survived as a common method for "capital budgeting": most projects have precisely such cash flow patterns—an upfront investment followed by future profits. Of course, you cannot do any better than doing right, so always using NPV is safer. It is just that if you use IRR *correctly* and in the right circumstances, it can often work equally well and sometimes gives you nice intuition.

Here is how to get the IRR.

You can find IRRs either by trial and error—as we did in Section 4·6—or you can use a common spreadsheet function, which does this in a relatively painless manner. For example, Table 8.1 shows how to obtain an IRR in a simple example—cell **A4** will become 0.13. (It may be painless for you, but finding IRR is actually not a trivial function. The spreadsheet must find the solution to a polynomial equation that is of an order equal to the number of periods.)

Table 8.1. IRR Calculation in Excel

	A	
1	-1000	
2	600	
3	600	
4	=IRR(A1:A3)	← will become 13%

8·2.B. Problems with IRR

Unfortunately, like the profitability index, IRR does not always work. There are a number of problems that you may encounter.

1. Project Comparisons and Scale In fact, the IRR shares the first shortcoming with the profitability index. It can mislead when projects are exclusive alternatives. For example, would you prefer a project with a 100% IRR, or a project with a 10% IRR? Think about it.

What if the first project is an investment opportunity of \$5 (returning \$10), and the second project is an investment opportunity of \$1,000 (returning \$100)? The latter is the better project, even though its IRR is worse.

2. **Direction** If a project starts with inflows and continues with outflows, it may be that a lower IRR is better than a higher IRR. For example, if you have one project that receives \$100 and has to pay \$105 next year, its IRR is 5%. If you have another project that receives \$100 and has to pay \$106, its IRR is 6%. Obviously, you would rather pay less in the future—but the IRR for the second project is higher.

3. **Multiple Solutions** When projects have both future positive and negative cash flows (there are often multiple and sometimes no solutions), all hell can break loose. For example, return to our earlier project from Section 2·4.B on Page 25, where our project cost \$900 today, yielded \$200/year for two years, then \$400/year for two years, and finally required a cleanup expense of \$100. There are at least two internal rates of return: $r = 8\%$ and $r = -79.6\%$ (round to -80%). Confirm this:

$$
\begin{aligned}
-\$900 + \frac{\$200}{1+8\%} + \frac{\$200}{(1+8\%)^2} + \frac{\$400}{(1+8\%)^3} + \frac{\$400}{(1+8\%)^4} + \frac{-\$100}{(1+8\%)^5} &\approx 0 \ , \\
-\$900 + \frac{\$200}{1-80\%} + \frac{\$200}{(1-80\%)^2} + \frac{\$400}{(1-80\%)^3} + \frac{\$400}{(1-80\%)^4} + \frac{-\$100}{(1-80\%)^5} &\approx 0 \ .
\end{aligned}
\tag{8.8}
$$

   So, does this project yield an internal rate of return of 8% or an internal rate of return of -80%? The fact is that both IRRs are valid according to the definition. Should you accept the project if the prevailing interest rate is 5%? The answer is not obvious, and we need to go back to the NPV rule to learn that the correct answer is yes.

   What does Excel do if there are multiple IRRs? You will never know. Excel will just pick one for you.

   SIDE NOTE: There is also "modified IRR" (or MIRR) measure that can sometimes eliminate multiple solutions. It is not worth the bother. If you have alternating-sign cash flows, use NPV instead.

4. **No Solution** What is the proper IRR of a project that yields \$10 today and \$20 tomorrow, and never demands an investment? There is no IRR that makes it zero. Or a project that costs \$10 today and \$20 tomorrow, and never yields a positive cashflow? These projects have no IRR solutions, though they are admittedly far-fetched. But neither does a project with cash flows -10, $+\$28$, and $-\$20$ in consecutive years, where the non-existence is not so obvious.

5. **Cost of Capital Comparison** When the term structure of interest rates is not flat (e.g., when the annualized two-year interest rate is different from the one-year interest rate), and the IRR lies between them, there is no rule as to which one to compare the IRR to. For example, the yield curve may have the 1-year interest rate at 10%, the 2-year interest rate at 9%, the 3-year interest rate at 8%, the 4-year interest rate at 7%, and the 5-year interest rate at 6%. Should you accept a 5-year project with an 8% IRR? After all, its project IRR is above its 5-year cost of capital (interest rate) but below the 1-year cost of capital. There is no clear answer.

These problems may seem obvious when highlighted in isolation. But in the context of complex real-world multiple project analysis, they are surprisingly often overlooked. Don't!

Solve Now!

Q 8.1 *A project has cash flows of -\$100, \$55, and \$70 in consecutive years. What is the IRR?*

Q 8.2 *From memory, write down the equation that defines IRR.*

Q 8.3 *Give an example of a problem that has multiple IRR solutions.*

Q 8.4 *Give an example of a problem that has no IRR solution.*

8·3. So Many Returns: The Internal Rate of Return, the Cost of Capital, the Hurdle Rate, and the Expected Rate of Return

Finance professors like to use terms interchangeably

It is time to recap four rates that are commonly used in finance: the **Internal Rate of Return**, the **Cost of Capital**, the **Expected Rates of Return** and the **Hurdle Rate.** The differences are sometimes subtle, and they are sometimes used interchangeably—which is okay in many, but not all, situations. So, here is a summary.

- **Internal Rate Of Return** The internal rate of return is a feature of cash flows, and has nothing to do with capital markets. It can be calculated before the appropriate cost of capital is known. It is the most different from the three rates below.

- **Cost of Capital** Always think of it as the *opportunity* cost of capital. It is also determined by the prevailing rates for loans of your type in capital markets. It is determined by demand and supply of capital in the economy—the expected rate of return that investors demand in order to give us money willingly. In perfect capital markets, with many lenders and borrowers, loans are usually zero net present value (or the borrower or lender is giving away free money). The cost of capital is sometimes called the "required expected rate of return."

- **Expected Rate of Return** The expected rate of return is a generic term. It could mean your project's expected rate of return, or the cost of capital (the lenders' expected rate of return). If your project's actual expected rate of return is above the required expected rate of return, it is a positive NPV project. If management makes smart decisions, projects' expected rates of return are above the cost of capital. The very last, marginal project often has an expected rate of return just above the cost of capital.

- **Hurdle Rate** The appropriate project hurdle rate is the expected rate of return above which management decides to accept and go forward with the project. It is set neither by the financial markets, nor by the project, but by management. Bad management could choose any arbitrary and even idiotic hurdle rates. Good management should accept all projects that have positive net present value.

  Usually, this means that good managers should set a project's hurdle rate to be equal to the project's cost of capital, and management should then determine whether the project's IRR exceeds this hurdle rate.

  If management makes smart decisions, taking all positive NPV projects, the "hurdle rate," "cost of capital," and "required expected rate of return" are all the same.

We already know that expected project returns are difficult to come by. Managers often incorrectly use promised rates of return. Because corporations are aware that claims based on expected project returns are regularly inflated (agency issues again!), many of them have established hurdle rates high above the firm's cost of capital. It is not uncommon to find project hurdle rates of 15% on claimed project rates of returns in corporations that face costs of capital on the order of 10%. Venture capitalists even regularly employ project hurdle rates as high as 30%!

8·4. Other Capital Budgeting Rules

8·4.A. The Problems of Payback

Alternative Capital Budgeting Rules: Power!

Although most corporations in the real world rely on NPV and IRR, some follow a whole variety of different capital budgeting rules. In some corporations, power rules: the most influential employees get most new funding. In other corporations, even stranger historical methods for deciding whether to take projects have taken hold.

Alternative Capital Budgeting Rules: Payback!

One commonly used alternative rule is the **payback rule.** Projects are assumed to be better if they recover their original investment faster. For the most part, this is a stupid idea. Consider the following three projects:

	Year 1	Year 2	Year 3	Year 4
Project A	−\$5	+\$8		
Project B	−\$5	+\$4	\$1,000	
Project C	−\$5	+\$4	\$0	+\$1 million

Project A has the shortest (best) payback period, but it is the worst of the three projects. Project B has the next shortest payback period, but it is the second-worst of the three projects (assuming reasonable interest rates). Project C has the longest (worst) payback period, but is the best project. (There is also a version of payback in which future paybacks are discounted. This measure asks not how long it takes you to get your money back, but how long it takes you to get the present value of your money back.)

Pluses and Minuses.

To be fair, payback can be an interesting number.

1. There is a beautiful simplicity to payback. It is easier for managers not trained in finance to understand "you will get your money back within 5 years" than it is to understand "the NPV is \$50 million."

2. Payback's emphasis on earlier cash flows helps firms set criteria when there are agency problems inside the firm. For instance, if your department manager claims that you will get your money back within 1 year, and 3 years have already passed without your having seen a penny, then something is probably wrong and you may need a better manager.

3. Payback can also help if you have limited capital—an imperfect market situation—so that your cost of capital is very high and getting your money back in a short amount of time is paramount. In this sense, payback helps you assess your future "liquidity."

4. Finally, in many ordinary situations, in which the choice is a pretty clear-cut yes or no, the results of the payback rule do not lead to severe mistakes, but just to mild mistakes (as would, for example, a rule that ignores all time-value of money). If you have a project in which you get your money back within one year, chances are that it's not a bad one, even from an NPV perspective. If you have a project in which it takes 50 years to get your money back, chances are that it's not a good one.

Having said all this, if you use payback to make decisions, it will lead you to take the wrong projects and ruin your company. So, why take a chance when you know better capital budgeting methods? Feel free to work out the payback period and use it as "interesting side information," but do not base your project choices on it—and certainly don't compare projects merely on the basis of payback.

8·4.B. More Rules

Accounting Based Measures.

There are an infinite number of other possible capital budgeting rules. A set of measures is based on accounting information (which we will cover next, so do not worry if the following makes little sense to you). For example, some firms choose projects based on the book rate of return—net income divided by the book value of equity. Some firms want to choose projects to maximize the book value of equity. Some firms want to choose projects to maximize reported earnings. As you will learn soon, all of these measures are based on complex accounting conventions, and not on economics. Therefore, I can only recommend against using them. I have no idea what kind of projects you will end up with if you use any of these measures—except that in many cases, if the measures are huge (e.g., if your accounting rate of return is 90% per annum), then chances are that the project is also positive NPV.

Sensible Measures.

A more sensible managerial practice is to think about what resources are constrained in the firm, given that we do not live in a perfect market. If the firm is highly levered and has difficulty borrowing more, then the contribution of the new project to the firm's leverage ratio would be an interesting measure. If the firm's management is already stretched thin, then measuring the managerial time required to run the project would be an interesting measure. If the firm is cash-constrained, then the payback time and cash outlay [e.g., the profitability index] would make interesting measures. Of course, all these issues should have already appropriately entered the cash flows that you use in your NPV calculations—but the fact is that this is often difficult to do or was entirely overlooked to begin with. Therefore, even though these measures should not be used as primary capital budgeting tools, you can use them to help you make informed tradeoffs.

IMPORTANT: *Simple Advice: Stick to net present value and, if need be, to the internal rate of return. Do not use alternative capital budgeting rules for investment decisions. Use alternative measures to help you make decisions, but do not be mechanically driven by them.*

8·5. Summary

The chapter covered the following major points:

- The profitability index rearranges the NPV equation. If used by itself, it often provides the same capital budgeting advice as NPV. But relative to NPV, the profitability index favors projects that have a lower upfront scale.
- The IRR (internal rate of return) is computed from a project's cash flows—and without the use of any cost-of-capital information. It solves the NPV formula to equal zero.
- IRR can be interpreted as a "sort of" average rate of return implicit in project cash flows.
- Projects with an IRR above the cost of capital often, but not always, have positive net present value (NPV), and vice-versa.
- IRR can suffer from serious computation problems, having multiple or no solutions. IRR suffers from comparison problems, because it does not adjust for project scale.
- In the context of bonds, IRR is called the "yield to maturity."
- The information that other capital budgeting measures provide can sometimes be "interesting." However, they often provide non-sensical results and therefore should generally be avoided—or at least consumed with great caution.

Solutions and Exercises

1. Using Excel, the answer pops out as 16%. Check: $-\$100 + \$55/1.16 + \$70/1.16^2 = 0$.
2. See equation 8.5 on Page 177.
3. For example, $-\$100$, $+\$120$, $-\$140$, $+\$160$, $-\$1$. (The solutions are -99.3716% and 16.796%. The important aspect is that your example has multiple inflows and multiple outflows.)
4. $-\$100$, $-\$50$. There are no rates of return that can possibly make this a zero net present value.

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

Part II

Corporate Financials

"It's not all bad. We still have enough cash for bribing the prison guards."

(A part of all versions of the book.)

Transition

You now know all the important capital budgeting concepts. The next issue on the agenda is to learn to apply them with the information that companies provide, which is rarely the cash flows that you need as direct inputs into your NPV analyses. So, the goal now is to teach you how to work with the information in corporate financial statements—and how to translate what you can find in financials into what you need for a net present value analysis. Our analysis primarily takes the perspective of the corporate manager, although outside analysts often also need to analyze corporate financials from the same perspective.

We will also try to use some of the information in the financials to perform a different type of valuation estimation, which is based on financials and relies on proper market valuation of comparable firms.

What we want to Accomplish in this Part

The goal of this part of the book is to teach the meaning and intelligent use of corporate financial statements.

- Chapter 9 shows how to extract economic cash flow estimates from corporate financial statements. We will build up some hypothetical firms to show what they would report in their financials. This will help you understand how to translate back from the financials to the firm.

  Typical question: What are the economic cash flows in PepsiCo's financial statements that we would use to estimate the net present value of PepsiCo?

- Chapter 10 shows how to assemble information about your own firm, using publicly available information from comparable firms.

  Typical questions: How do "comparables" differ from NPV? When is the P/E (price earnings) ratio a good number to look at? What should be the P/E ratio of our project? How and when can you average P/E ratios? What can you learn from other financial ratios?

CHAPTER 9

Understanding Financial Statements

Translating Accounting into Finance (Economic Cash Flows)

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Financial Accounting is the "language of business." Although this book is not about financial statements, you must understand both their logic and their fundamentals. They contain information about the cash flows we need for an NPV analysis, as well as a lot of other useful information. Without understanding accounting, you also cannot understand corporate income taxes, a necessary NPV input.

This chapter begins with a simple hypothetical project. Its economics make computing NPV easy. It then explains how accountants would describe the project in a financial statement. We then translate between the finance and the accounting descriptions. Finally, the chapter applies the same analysis to the financial statement of a real corporation, PepsiCo (PEP).

This chapter also gently introduces some more details on corporate income taxes and capital structure, which will be explained in greater detail in Chapter 18.

9·1. Financial Statements

accounting also matters to finance.

We already know that the value of our firm is determined by its underlying projects. We already know that these projects have cash flows that we use in our NPV analyses. So, why should you bother with learning about what companies say in their financials? A rose is a rose is a rose, isn't it? The projects and thus the firm have the same value no matter what we report.

Yes and no. There are many good reasons why you should understand financial statements:

1. If you want to have an intelligent conversation with someone else about corporations, you must understand the language of accounting. In particular, you must understand what earnings are—and what they are not.

2. Companies work internally financial statements, not the pure cash flows that you need for PV discounting. How can you make the decision which projects to take in the first place if you cannot understand the information that your accounting department can provide you with?

3. It may be all the information you will ever get. If you want to get a glimpse of the operation of a publicly traded corporation, or understand its economics better, then you must be able to read what the company is willing to tell you. If you want to acquire it, the corporate financials may be your primary source of information.

4. Unfortunately, the IRS levies corporate income tax. This tax is computed from a tax-specific variant of the corporate income statement, which relies on the same accounting logic as the published financials. (The reported and tax statements are *not* the same!) Because income taxes are definite costs, you must be able to understand and construct financial statements that properly subtract taxes from the cash flows projected from projects when you want to compute NPV. And, if you do become a tax guru, you may even learn how to structure projects to minimize the tax obligations, although most of this is beyond the scope of our textbook.

5. Many contracts are written on the basis of financials. For example, a bond covenant may require the company to maintain a price-earnings ratio above 10. So, even if a change in accounting rules should not matter theoretically, such contracts can create an influence of the reported financials on your projects' cash flows.

6. There is no doubt that managers care about their financial statements. Managerial compensation is often linked to the numbers reported in the financial statements. Moreover, managers can also engage in many maneuvers to legally manipulate their earnings. For example, firms can often increase their reported earnings by changing their depreciation policies (explained below). Companies are also known to actively and expensively lobby the accounting standards boards. For example, in December 2004, the accounting standards board finally adopted a mandatory rule that companies will have to value employee stock options when they grant them. Until 2004, firms' financial statements could treat these option grants as if they were free. This rule was adopted despite extremely vigorous opposition by corporate lobbies, which was aimed at the accounting standards board and Congress. The reason is that although this new rule does not ask firms to change projects, it will drastically reduce the *reported* net income (earnings) especially of technology firms.

   But why would companies care about this? After all, investors can already determine that many high-tech firms (including the likes of Microsoft) may have never had positive earnings if they had had to properly account for the value of all the stock options that they have given. This is a big question. Some behavioral finance researchers believe that the financial markets value companies as if they do not fully understand corporate financials. That is, not only do they share the common belief that firms manage their earnings, but they also believe that the market fails to see through even mechanical accounting computations. Naturally, the presumption that the financial markets cannot understand accounting is a very controversial hypothesis—and, if true, this can lead to all sorts of troublesome consequences.

For example, if the market cannot understand financials, then managers can legally manipulate their share prices. A firm would especially benefit from a higher share price when it wants to sell more of its shares to the public. In this case, managers could and should maneuver their financials to increase their earnings just before the equity issue. There is good evidence that firms do this—and also that the financial markets are regularly disappointed by these firms' performances years after their equity issues.

Even more troublesome, there is also evidence that managers do not take some positive NPV projects, if these projects harm their earnings. Does this sound far-fetched? In fact, in a survey of 401 senior financial executives Graham, Harvey, and Rajgopal found that 55% would delay starting a project and 80% would defer maintenance and research spending in order to meet earnings targets. Starting projects and doing maintenance and R&D are presumably the right kind of (positive NPV) projects, so not taking them decreases the underlying real value of the firm—even though it may increase the financial image the firm projects.

It is of course impossible for this book about finance to explain all the nuances of accounting. Instead, we will focus on only one goal of importance to a financier: how do we obtain the cash flows that we need for an NPV analysis, and why can we not use earnings? Accounting has, of course, more to offer than just this—and fortunately you can learn more about its broader scope in your accounting course.

9·1.A. The Contents of Financials

Companies communicate their internals through standardized financial reports.

Publicly traded companies report their **financial results** (or **financials**) in **financial reports** to their shareholders and to the public. The most important financial report is the **Annual Report**, which is filed with the SEC in Form **10-K**. (There is also a **Quarterly Report**, called **10-Q**.) Almost all Annual Reports begin with a general description of the business and business developments, followed by the more formal presentation of the firm's financials. We are interested primarily in the financials: after all, financiers are primarily interested in *how much* money the firm makes rather than *how* it makes it. However, as much as we would prefer to keep the firm a black box, we usually cannot: knowledge of "how money is made" is usually necessary for good knowledge of the "how much money is made"—and "how can we make more money."

Read some!

If you have not seen a company's Annual Report (which includes the financial statements), please spend some time reading one. Any business library should have a selection. Most large corporations also publish their financials on their websites. If you own shares of stock in a publicly traded company, the Annual Report will automatically be mailed to you once a year. And the SEC runs **EDGAR**, a very comprehensive electronic repository of corporate financials, including annual and quarterly reports.

9·1.B. PepsiCo's 2001 Financials

Table 9.1. Consolidated Balance Sheet PepsiCo, Inc. and Subsidiaries
December 29, 2001 and December 30, 2000

(in millions except per share amounts)	2001	2000
Assets		
Current Assets		
Cash and cash equivalents	$ 683	$ 1,038
Short-term investments, at cost	966	467
	1,649	1,505
Accounts and notes receivable, net	2,142	2,129
Inventories	1,310	1,192
Prepaid expenses and other current assets	752	791
Total Current Assets	5,853	5,617
Property, Plant and Equipment, net	6,876	6,558
Intangible Assets, net	4,841	4,714
Investments in Unconsolidated Affiliates	2,871	2,979
Other Assets	1,254	889
Total Assets	$21,695	$20,757
Liabilities and Shareholders' Equity		
Current Liabilities		
Short-term borrowings	$ 354	$ 202
Accounts payable and other current liabilities	4,461	4,529
Income taxes payable	183	64
Total Current Liabilities	4,644	4,593
Long-Term Debt	2,651	3,009
Other Liabilities	3,876	3,960
Deferred Income Taxes	1,496	1,367
Preferred Stock, no par value	26	49
Deferred Compensation — preferred	-	(27)
Common Shareholders' Equity		
Common stock, par value 1 $^2/_3$ c per share (issued 1,782 and 2,029 shares, respectively)	30	34
Capital in excess of par value	13	375
Deferred compensation	-	(-21)
Retained earnings	11,519	16,510
Accumulated other comprehensive loss	(1,646)	(1,374)
Less: repurchased common stock, at cost (26 and 280 shares, respectively)	(1,268)	(7,920)
Total Common Shareholders' Equity	8,648	7,604
Total Liabilities and Shareholders' Equity	$21,695	$20,757

See accompanying notes to consolidated financial statements.

Table 9.2. Consolidated Statement of Common Shareholders' Equity PepsiCo, Inc. and Subsidiaries
Fiscal years ended December 29, 2001, December 30, 2000 and December 25, 1999.

(in millions)	2001 Shares	2001 Amount	2000 Shares	2000 Amount	1999 Shares	1999 Amount
Common Stock						
Balance, beginning of year	2,029	$ 34	2,030	$ 34	2,037	34
Share repurchases	-	-	(9)	-	(13)	-
Stock option exercises	6	-	-	-	-	-
Quaker stock option exercises	3	-	8	-	6	-
Shares issued to effect merger	(256)	(4)	0	-	-	-
Balance, end of year	1,782	30	2,029	34	2,030	34
Capital in Excess of Par Value						
Balance, beginning of year		375		559		904
Share repurchases		-		(236)		(370)
Stock option exercises[(a)]		82		52		(21)
Reissued shares		150		-		-
Shares issued to effect merger		(595)		-		-
Other		1		-		46
Balance, end of year		13		375		559
Deferred Compensation						
Balance, beginning of year		(21)		(45)		(68)
Net activity		21		24		23
Balance, end of year		-		(21)		(45)
Retained Earnings						
Balance, beginning of year		16,510		14,921		13,356
Net income		2,662		2,543		2,505
Shares issued to effect merger		(6,644)		-		-
Cash dividends declared - common		(1,005)		(950)		(936)
Cash dividends declared - preferred		(4)		(4)		(4)
Balance, end of year		11,519		16,510		14,921
Accumulated Other Comprehensive Loss						
Balance, beginning of year		(1,374)		(1,085)		(1,139)
Currency translation adjustment (CTA)		(218)		(289)		(136)
CTA reclassification adjustment		-		-		175
Cash flow hedges, net of tax:						
Cumulative effect of accounting change		3		-		-
Derivative (losses)/gains, net		(21)		-		-
Minimum pension liability adjustment, net of tax		(38)		(2)		17
Other		2		2		(2)
Balance, end of year		(1,646)		(1,374)		(1,085)
Repurchased Common Stock						
Balance, beginning of year	(280)	(7,920)	(271)	(7,306)	(255)	(6,535)
Shares repurchased	(35)	(1,716)	(38)	(1,430)	(36)	(1,285)
Stock option exercises	20	751	29	816	20	514
Reissued shares	13	374	-	-	-	–
Shares issued to effect merger	256	7,243	-	-	-	–
Balance, end of year	(26)	(1,268)	(280)	(7,920)	(271)	(7,306)
Total Common Shareholders' Equity		$ 8,648		$ 7,604		$ 7,078

(a) Includes total tax benefit of $212 in 2001, $177 in 2000 and $105 in 1999.

See accompanying notes to consolidated financial statements. These include a closing stock price of $49.05/share, which indicates a market capitalization of $87.4 billion.

Table 9.3. Consolidated Statement of Income PepsiCo, Inc. and Subsidiaries
Fiscal years ended December 29, 2001, December 30, 2000 and December 25, 1999.

(in millions except per share amounts)	2001	2000	1999
NET SALES			
New PepsiCo	$ 26,935	$25,479	$22,970
Bottling Operations	–	–	2,123
Total Net Sales	26,935	25,479	25,093
COSTS AND EXPENSES			
Cost of sales	10,754	10,226	10,326
Selling, general and administrative expenses	11,608	11,104	11,018
Amortization of intangible assets	165	147	193
Merger-related costs	356	–	–
Other impairment and restructuring charges	31	184	73
Total Costs and Expenses	22,914	21,661	21,610
OPERATING PROFIT			
New PepsiCo	$ 4,021	$3,818	$3,430
Bottling Operations	–	–	2,123
Total Operating Profit	$ 4,021	$3,818	$3,483
Bottling equity income and transaction gains/(loss), net	160	130	1,083
Interest expense	(219)	(272)	(421)
Interest income	67	85	130
INCOME BEFORE INCOME TAXES	4,029	3,761	4,275
PROVISION FOR INCOME TAXES	1,367	1,218	1,770
NET INCOME	$ 2,662	$ 2,543	$ 2,505
NET INCOME PER COMMON SHARE			
Basic	$ 1.51	$ 1.45	$ 1.41
Diluted	$ 1.47	$ 1.42	$ 1.38

See accompanying notes to consolidated financial statements.

Table 9.4. Consolidated Statement of Cash Flows PepsiCo, Inc. and Subsidiaries
Fiscal years ended December 29, 2001, December 30, 2000 and December 25, 1999.

in millions	52 Weeks Ending 12/29/01	53 Weeks Ending 12/30/00
Cash Flows - Operating Activities		
Net income	$ 2,662	$ 2,543
Adjustments to reconcile net income to net cash provided by operating activities		
Bottling equity income, net	(160)	(130)
Depreciation and amortization	1,082	1,093
Merger-related costs	356	–
Other impairment and restructuring charges	31	184
Cash payments for merger-related costs and restructuring charges	(273)	(38)
Deferred income taxes	162	33
Deferred compensation - ESOP	48	36
Other noncash charges and credits, net	209	303
Changes in operating working capital, excluding effects of acquisitions and dispositions		
Accounts and Notes Receivables	7	(52)
Inventories	(75)	(51)
Prepaid expenses and other current assets	(6)	(35)
Accounts payable and other current liabilities	(236)	219
Income taxes payable	394	335
Net change in operating working capital	84	416
Net Cash Provided by Operating Activities	4,201	4,440
Cash Flows - Investing Activities		
Capital spending	(1,324)	(1,352)
Acquisitions and investments in unconsolidated affiliates	(432)	(98)
Sales of businesses	–	33
Sales of property, plant & equipment	–	57
Short-term investments. by original maturity		
More than three months – purchases	(2,537)	(4,950)
More than three months – payments	2,078	4,585
Three months or less, net	(41)	(9)
Other, net	(381)	(262)
Net Cash Used for Investing Activities	(2,637)	(1,996)

(Continued on the following page.)

(Continued from previous page.)

in millions

	52 Weeks Ending 12/29/01	53 Weeks Ending 12/30/00
Cash Flows - Financing Activities		
Proceeds from issuances of long-term debt	324	130
Payments of long-term debt	(573)	(879)
Short-term borrowings, by original maturity		
More than three months — proceeds	788	198
More than three months — payments	(483)	(155)
Three months or less, net — payments	(397)	1
Cash dividends paid	(994)	(949)
Share repurchases - common	(1,716)	(1,430)
Share repurchases - preferred	(10)	–
Quaker share repurchases	(5)	(254)
Proceeds from issuance of shares in connection with the Quaker merger	524	–
Proceeds from exercises of stock options	623	690
Net Cash Used for Financing Activities	(1,919)	(2,648)
Effect of Exchange Rate Changes on Cash and Cash Equivalents	–	(4)
Net (Decrease)/Increase in Cash and Cash Equivalents	(355)	(208)
Cash and Cash Equivalents - Beginning of year	1,038	1,246
Cash and Cash Equivalents - End of period	$ 683	$ 1,038
Supplemental Cash Flow Information		
Interest Paid	$ 159	$ 226
Income taxes paid	$ 857	$ 876
Acquisitions		
Fair value of assets acquired	$ 604	$ 80
Cash paid and debt issued	(432)	(98)
Liabilities Assumed	$ 172	$ (18)

Tables 9.1–9.4 contain the four main financial statements that PepsiCo reported in its 2001 Annual Report. (The entire annual report is available at http://www.pepsico.com/investors/-annual-reports/2001/pepsico_financials2001.pdf.)

The accounting view: balance sheet, income statement, and cash flow statement.

The balance sheet in Table 9.1 provides a snapshot of the firm's assets and liabilities at a fixed point in time. (It is a measure of "stock," not of "flow" over an interval.)

Some assets (like cash and inventories) are fairly liquid and short-term, and are therefore often called **current assets.** Other assets (like plants and brand reputation [an intangible asset]) are much harder to convert into cold, hard cash if we were to sell the firm, and thus are itemized separately.

As in finance, accounting forces the sum total of all assets to be owned by creditors and shareholders. And, as with assets, some creditors are owed money short-term. These are called **current liabilities.** Other debt is more long-term—and then there are obligations to our "friend," the IRS. The remainder—whatever assets are not accounted for by debt owed to creditors—is called equity. Therefore,

$$\text{Assets} = \text{Liabilities} + \text{Shareholders' Equity} \; . \tag{9.1}$$

If all assets and liabilities were properly valued, this accounting **book value** of shareholders' equity would be the market value, too. However, accounting rules and difficulties in valuing assets and liabilities often render the book value of shareholders' equity into more of a "plug" number that serves to equalize assets and liabilities than into an intrinsically meaningful figure. You have been warned!

The owners' equity statement (or "shareholders' equity statement") in Table 9.2 explains the history of capital originally contributed to the firm, and earnings that were retained (not paid out). We will not use this statement.

The income statement in Table 9.3 reports the revenues and expenses of the company, resulting in earnings (also called net income) over the year. (Thus, it reports a measure of "flow," not of "stock.") Because it is hard to see the trees in so much forest, many financial services produce an abbreviated form thereof, which you can find in Table 9.13 (Page 221).

In the above three statements, accountants seek to "smooth out" temporary hiccups—which you will learn in a moment. It is only in the fourth statement that this is not attempted:

The cash flow statement in Table 9.4 reports the sources and uses of cash over the year. (It is a measure of "flow," not of "stock.") Again, an abbreviated version is in Table 9.14 (Page 222).

You should stare at these four statements for a while. But you can look as hard as you like, and you will not find an item entitled "cash flow for an NPV analysis." And the cash flows on the cash flow statement look nothing like the earnings, which is what the world seems to consider important?! Somehow, we must try to understand what this financials mean so that we can extract what we really need—a "cash flow for our NPV analysis"—from what we have: the four financial statements.

Financial reports follow accounting conventions.

We will be spending a lot of time explaining the income statement and cash flow statement, but the upshot will be that the cash flow statement comes closest to what we want. However, to understand why it is insufficient and where it comes from, we need to take a wider expedition into the logic of accounting (and specifically, of net income), which is different from the logic of finance (and specifically, NPV cash flows). So our next step is to learn how to read, interpret, and transform financial statements into the cash flows that an NPV analysis demands. You also need this expedition to have a better understanding of earnings and financial statements in general.

9·1.C. Why Finance and Accounting Think Differently

Earnings reflect future costs and benefits (in some sense).

The principal difference between accountants and financiers is that the former try to approximate the current value of the firm, while the latter try to understand the current hard cash inflows and outflows. The former want to learn about earnings, the latter want to learn about cash flows.

The difference between income and cash flows are accruals.

The main difference between these two concepts of income and cash flows are **accruals**: economic transactions that have delayed cash implications. For example, if I owe your firm $10,000 and have committed to paying you tomorrow, the accountant would record your current firm value to be $10,000 (perhaps time- and credit-risk adjusted). In contrast, the financier would consider this to be a zero cash-flow—until tomorrow, when the payment actually occurs. The contrast is that the accountant wants the financial statements to be a good representation of the economic value of the firm *today* (i.e., you already own the payment commitment), instead of a representation of the exact timing of inflows and outflows. The financier needs the timing of cash flows for the NPV analysis instead.

Financiers see actual cash flows: an expense spike, followed by years of no expenses.

Accruals can be classified into long-term accruals and short-term accruals. The primary **long-term accrual** is **depreciation**, which is the allocation of the cost of an asset over a period of time. For example, when a financier purchases a maintenance-free machine, he sees a machine that costs a lot of cash today, and produces cash flows in the future. If the machine needs to be replaced every 20 years, then the financier sees a sharp spike in cash outflows every 20 years, followed by no further expenditures (but hopefully many cash inflows).

Accountants smooth asset values over time.

The accountant, however, sees the machine as an asset that uses up a fraction of its value each year. So, an accountant would try to determine an amount by which the machine deteriorates in each year, and would only consider this prorated deterioration to be the annual outflow (called an **expense**). The purchase of a $1 million machine would therefore not be an earnings reduction of $1 million in the first year, followed by $0 in the remaining 19 years. Instead, it would be an expense of $50,000 in each of 20 years. (This is a very common method of depreciation and is called **straight-line depreciation**.)

Accountants use impairment schedules.

To complicate matters further, accountants often use different standardized depreciation schedules over which particular assets are depreciated. These are called impairment rules, and you already know the straight-line rule. Houses, for example, are commonly depreciated straight-line over 30 years—often regardless of whether the house is constructed of wood or brick. The predetermined value schedule is usually not accurate: For example, if investors have recently developed a taste for old buildings, it could be that a building's value has doubled since its construction, even though the financial statements might record this building to be worth nothing. (Even this is oversimplified. On occasion, accountants invoke procedures that allow them them to reduce the value of an asset midway through its accounting life—but more often downward than upward.) Another common impairment rule is accelerated depreciation, which is especially important in a tax context. (But we are straying too far.)

There is some inconsistency when the machine has been fully depreciated.

If the machine happens to continue working after 20 years, the financials which have just treated the machine as a $50,000 expense in Year 20 will now treat it as a $0 expense in Year 21. It remains worth $0 because it cannot depreciate any further—it has already been fully depreciated. The financier sees no difference between Year 20 and Year 21, just as long as the machine continues to work.

For short-term accruals, such as receivables, accounting logic relies on predicted cash flows.

Short-term accruals come in a variety of guises. To a financier, what matters is the timing of cash coming in and cash going out. A sale for credit is not cash *until* the company has collected the cash. To the accountant, if the firm sells $100 worth of goods on credit, the $100 is booked as **revenue** (which flows immediately into **net income**), even though no money has yet arrived. In the accounting view, the sale has been made. To reflect the delay in payment, accountants increase the **accounts receivables** by $100. (Sometimes, firms simultaneously establish an allowance for estimated non-payments [bad debts].)

Another short-term accrual is **corporate income tax**, which a financier considers to be an outflow only when it has to be paid—at least not until (the corporate equivalent of) April 15 of the following year. However, on the income statement, when a firm in the 40% corporate tax bracket makes $100 in profits, the income statement immediately subtracts the corporate income tax of $40 (which will eventually have to be paid on the $100 in profits) and therefore records net income of only $60. To reflect the fact that the full $100 cash is still around, $40 is recorded as **tax payables.**

The logic of finance relies on actual cash flows (or immediate values), only.

In sum, for a financier, the machine costs a lot of cash today (so it is an immediate negative), the accounts receivables are not yet cash inflows (so they are not yet positives), and the corporate income tax is not yet a cash outflow (so it is not yet a negative). For an accountant, the machine costs a prorated amount over a period of years, the accounts receivables are immediate positive earnings, and the corporate income tax is an immediate cost. There is definite sense in the approaches of both accounting and finance: the accounting approach is better in giving a snapshot impression of the firm's value; the finance approach is better in measuring the timing of the cash inflows and cash outflows for valuation purposes. Note that valuation leans much more heavily on the assumption that *all* future cash flows are fully considered. Today's cash flows alone would *not* usually make for a good snapshot of the firm's situation.

Both approaches have relevant advantages and disadvantages.

Solve Now!

Q 9.1 *What are the main differences between how accounting (net income) and finance (NPV cash flows) see projects?*

Q 9.2 *Why is a firm not just a firm and accounting numbers not just "funny money"? That is, what is the most important direct cash flow influence of accounting in most corporations?*

ANECDOTE: Trashy Accounting at Waste Management
On December 14, 1998, Waste Management settled a class action lawsuit by shareholders for $220 million, then the largest such settlement. (This will soon be dwarfed by the Enron, MCI WorldCom, and Arthur Andersen debacles.) The suit alleged that WMX had overstated its income by $1.32 billion over an 8-year period. From 1994 through 1997, about 47% of the company's reported income was fictitious.

One of WMX's dubious practices was that it had changed the accounting life of its waste containers from 12 years to 18 years. Therefore, each year, it subtracted less depreciation, which increased its reported earnings by $1.7 billion. Of course, managers were handsomely rewarded for their superior earnings performance.

9·2. The Bottom-Up Example — Long-Term Accruals (Depreciation)

Rather than starting off trying to understand a creature as complex as the PepsiCo financials, our method is to start with a simple firm and construct hypothetical financials. We will then use our knowledge to reverse the procedure. At the end of this chapter, we will do the same to obtain NPV cash flows from the PepsiCo financials.

9·2.A. Doing Accounting

The hypothetical project to illustrate the accounting perspective and the finance perspective.

We want to understand how depreciation drives a wedge between income and cash flows. Start with a simple project for which we know the cash flows. Our firm is basically just one machine, described in Table 9.5.

Table 9.5. A Hypothetical Project

Project	
Real Physical Life	6 Years
Capital Expenditure	\$75, year 1
	\$75, year 2
Raw Output	\$70/year
− Input Costs (cash)	\$5/year
− Selling Costs (cash)	\$5/year
= Net Output	\$60/year
Overall Cost of Capital	12%/year
Corporate Tax Rate (τ)	40%/year

Available Financing — Executed	
Debt Capacity	\$50
Debt Interest Rate	10%/year
	(=\$5/year)

Accounting Treatment	
Project Life	3 Years

Note: This debt contract provides cash necessary in Year 1, and requires a first interest payment in year 2. Both principal and interest are repaid in Year 6.

Elaborate on the example.

With \$50 of debt at 10% interest, the firm's annual interest payments are \$5. The machine (and its tax depreciation schedule) is rather unusual: it has no maintenance costs; it requires capital expenditures in the second year; it produces fully even in year 1; and divine intelligence tells us that the machine will really last 6 years. But the IRS has decreed that depreciation of this particular machine occurs over 3 years. So, \$75 generates \$25 in depreciation, three years in a row, beginning in the year of the capital expenditure.

By assuming a lower interest rate of 10% on the debt than on the overall firm's cost of capital of 12%, we are in effect assuming that financial markets are risk-averse—as they truly are in the real world. Risk aversion is the subject of Part III.

Table 9.6. Income Statement and Excerpt of Cash Flow Statement of Hypothetical Machine

	Income Statement						
	Year	1	2	3	4	5	6
	Sales (Revenues)	\$70	\$70	\$70	\$70	\$70	\$70
–	Cost of Goods Sold (COGS)	\$5	\$5	\$5	\$5	\$5	\$5
–	Selling, General & Administrative Expenses (SG&A)	\$5	\$5	\$5	\$5	\$5	\$5
=	EBITDA	\$60	\$60	\$60	\$60	\$60	\$60
–	Depreciation	\$25	\$50	\$50	\$25	\$0	\$0
=	EBIT (operating income)	\$35	\$10	\$10	\$35	\$60	\$60
–	Interest Expense	\$0	\$5	\$5	\$5	\$5	\$5
=	EAIBT (or EBT)	\$35	\$5	\$5	\$30	\$55	\$55
–	Corporate Income Tax (at 40%)	\$14	\$2	\$2	\$12	\$22	\$22
=	Net Income	\$21	\$3	\$3	\$18	\$33	\$33

Excerpts From the Cash Flow Statement						
Year	1	2	3	4	5	6
Capital Expenditures	–\$75	–\$75	-	-	-	-
Net Debt Issue	+\$50	-	-	-	-	–\$50

Note: Though broken out in this sample income statement, parts of depreciation are often rolled into COGS or SG&A. It is always fully broken out (and thus can be obtained from) the cash flow statement.

SIDE NOTE: Sign Warning: You might think that a machine purchase of \$75 would be recorded as a Capital Expenditure of +\$75. Alas, the accounting convention is to record this as a negative number, i.e., −\$75. You are assumed to know that a negative expenditure is not a cash inflow, but a cash outflow. (But beware: capital expenditures is recorded as a positive asset on the balance sheet.)

Our project's income statement.

The income statement for our project is shown in Table 9.6. In going down the left-most column of any of these tables, you will notice that—as in finance—accounting has its own jargon. **COGS** abbreviates **cost of goods sold**; **SG&A** abbreviates **selling, general & administrative expenses.** These are expenditures that have to be subtracted from **sales** (or **revenues**) to arrive at **EBITDA**: **earnings before interest and taxes, depreciation, and amortization.** Next we subtract out depreciation, which is a subject that deserves the long sidenote below and that we will return to in a moment. Thus, we arrive at **operating income**, also called **EBIT** (**earnings before interest and taxes**). Finally, we take out interest expense at a rate of 10% per year and corporate income tax (which we can compute from the firm's tax rate of 40%) and arrive at plain **earnings**, also called **Net Income.** Net income is often called the "bottom line," because of where it appears.

Compare our project to PepsiCo.

Note the similarity of our project's income statement to PepsiCo's income statement from Table 9.3. PepsiCo's accountants prefer to put later years in columns to the left. In 2001, PepsiCo had \$26,935 million in sales. COGS and SG&A added up to $\$10,754 + \$11,608 = \$22,362$ million. Therefore, EBITDA was $\$26,935 - \$22,362 = \$4,573$ million. Amortization subtracted \$165 million. Other expenses amounted to \$387 million, leaving us with EBIT of $\$4,573 - \$165 - \$356 - \$31 = \$4,021$ million. In PepsiCo's case, the combination of bottling equity income and transaction gains, interest expenses, and interest income was determined to be its net interest income of $8 million. Uncle Sam demanded $1,367 million, leaving shareholders with net income of $2,662.

SIDE NOTE: The $165 depreciation that we need to add back for our NPV analysis is the $1,082 million on the cash flow statement, not the $165 million amortization that PepsiCo reports on its income statement. The reason is that PepsiCo lumped most but not all depreciation into SG&A. Amortization omits depreciation of non-physical plant assets. For clarity, I have added one line in Table 9.13, which breaks out the depreciation from selling, general & administrative expenses. Edgarscan also reported net interest income/expense of $8, which we therefore added to the table.

Non-Cash Items contain M&A costs of $356 from PepsiCo's acquisition of Quaker Oats in August 2001. Note that financials are often **restated**, i.e., changed ex-post to reflect the acquisition of other businesses. This particular procedure is called **pooling**. The idea is to report financials *as if* the two companies had always been conjoined.

Yes, PepsiCo has a few extra items, and changes some of the names around, but the broad similarity should be apparent.

There is one extra piece of information that we need to record—elsewhere. Depreciation smoothes capital expenditures.

We have used almost all the information of our project. The two exceptions are the capital expenditures and the net debt issue. They do not go onto the income statement. Instead, they are reported on the cash flow statement (also in Table 9.6). In this case, capital expenditures are $75 in Year 1 and $75 in Year 2, followed by $0 in all subsequent years. Net debt issuing is $50 in Year 1, and the debt principal repayment of $50 occurs in Year 6. (For PepsiCo, in Table 9.3, you can find the equivalent items as "capital spending" under Investing Activities and as proceeds from issuances of long-term debt and payments of long-term debt under Financing Activities). This is not to say that capital expenditures and debt play no role in the income statement—they do, but not one-to-one. For example, capital expenditures reduce net income (in the income statement) only slowly through depreciation. In the first year, the first $25 depreciation from the $75 capital expenditures are accounted for; in the second year, the second $25 depreciation from the first $75 capital expenditures are accounted for, plus the first $25 depreciation from the second $75 capital expenditures are accounted for; and so on.

Side Note:

- **Depreciation**, **depletion**, and **amortization** are all "allocated expenses" and not actual cash outflows. The difference in names derives from the asset types to which they apply. Depreciation applies to **Tangible Assets**, such as plants. Depletion applies to **Natural Resources**. Amortization applies to **Intangible Assets**. Because depreciation, depletion, and amortization are really all the same thing, they are often just lumped together under the catch-all phrase "depreciation," a convention that we follow in this chapter.
- For tutorial purposes, our example commits an accounting error. Corporations have no consistent treatment of depreciation on the income statement: some depreciation is typically rolled into cost of goods sold (such as depreciation on manufacturing equipment; thus our machine's depreciation would usually be reported as part of cost of goods sold). Other depreciation may be rolled into selling, general & administrative expenses, and yet other depreciation may be broken out. Therefore, the only complete picture of depreciation can be found on the cash flow statement, where the sum-total of all forms of depreciation is added back to net income.
- The rules for publicly reported financial statements are called **GAAP** (Generally Accepted Accounting Principles) and change rarely. They are set by a number of policy makers, most prominently **FASB** (the Financial Accounting Standards Board). For public reporting purposes, firms are supposed to seek to match true depreciation to reported depreciation.
- Although the IRS financial statements follow the same logic as the public financials, the statements themselves are not the same. The IRS rules change often and are set by Congress. (Even states can have their own rules.) The difference is particularly pronounced when it comes to depreciation schedules. Tax depreciation rules are set by the IRS, and they often apply strict mechanical schedules, regardless of the true asset life. Consequently, there are many companies that report positive earnings to their shareholders, and negative earnings to the IRS.
- In some countries, the financial and tax statements of public companies are identical. In the author's opinion, this is not a bad idea. In the United States, the public debate is how to get companies not to be aggressive in reporting financial earnings (on which executive compensation is often based!). If companies had to release their tax statements to the public, aggressive earnings manipulation would become not only more difficult, but also more costly.
- To compute corporate income tax, our chapter should have used not the depreciation on the firm's reported financial statements, but the depreciation on the firm's unreported *tax* statements. We commit this error to facilitate the exposition.
- There is a small timing difference between interest expense and when interest is actually paid. This is usually about 1 month in timing. The value impact of this difference is small, so we shall ignore it, and just use interest expense as if it were immediately paid.

(These are very important account details—even if they are not particularly exciting.)

9·2.B. Doing Finance

Now, forget accounting for a moment, and instead let us value the machine from an underlying finance perspective. The firm consists of three components: the machine itself, the tax obligation, and the loan.

The finance perspective focuses only on adding actual cash inflows and cash outflows. Note difference between full and levered ownership.

$$\begin{aligned} \text{NPV Project} &= \text{NPV Machine} - \text{NPV Taxes} \;, \\ \text{NPV Levered Ownership} &= \text{NPV Machine} - \text{NPV Taxes} + \text{NPV Loan} \;. \end{aligned} \tag{9.2}$$

Full project ownership is equivalent to supplying both the debt and equity, and earning the cash flows due to both creditors and shareholders. Levered equity ownership adds the project "loan" to the package. As full project owner (debt plus equity), in the first year, you must originally supply \$50 more in capital than if you are just a levered equity owner, but in subsequent years, the interest and principal payments are not negatives for you. (For convenience, we ignore intra-year payment timing complications.)

We first work out the actual cash flows of the first component, the machine itself. Without

Let us look at the first component of the firm—the machine's actual cash flows, without taxes and loan.

taxes and loan, the machine produces

$$
\begin{aligned}
\text{NPV}_{\text{Machine}} &= \frac{\$60 - \$75}{(1+12\%)^1} + \frac{\$60 - \$75}{(1+12\%)^2} + \frac{\$60}{(1+12\%)^3} \\
&+ \frac{\$60}{(1+12\%)^4} + \frac{\$60}{(1+12\%)^5} + \frac{\$60}{(1+12\%)^6} = \$119.93 \\
\text{NPV}_{\text{Machine}} &= \text{CF}_0 + \frac{\text{CF}_1}{1+r_{0,1}} + \frac{\text{CF}_2}{1+r_{0,2}} + \frac{\text{CF}_3}{1+r_{0,3}} \\
&+ \frac{\text{CF}_4}{1+r_{0,4}} + \frac{\text{CF}_5}{1+r_{0,5}} + \frac{\text{CF}_6}{1+r_{0,6}} \; .
\end{aligned}
\tag{9.3}
$$

The tax obligation is a negative NPV project, which must be calculated and then valued.

Unfortunately, corporate income tax—the second component—is an actual cost which cannot be ignored. Looking at Table 9.6, we see that Uncle Sam collects \$14 in the first year, then \$2 twice, then \$12, and finally \$22 twice.

To value the future tax obligations, we need to know the appropriate discount factor. Unfortunately, we need to delay this issue until Section 18·3.B (Page 445). It is both convenient and customary (if not exactly correct) to use the firm's overall cost of capital as the discount rate for its tax obligations.

Value the tax liability, and determine the project NPV.

Assume that the stream of tax obligations has the same discount rate (12%) as that for the overall firm. With this cost-of-capital assumption, the net present cost of the tax liability is

$$
\begin{aligned}
\text{NPV}_{\text{Tax Liability}} &= \frac{\$14}{(1+12\%)^1} + \frac{\$2}{(1+12\%)^2} + \frac{\$2}{(1+12\%)^3} \\
&+ \frac{\$12}{(1+12\%)^4} + \frac{\$22}{(1+12\%)^5} + \frac{\$22}{(1+12\%)^6} = \$46.77 \; .
\end{aligned}
\tag{9.4}
$$

Put together,

$$
\text{NPV Project} = \text{NPV Machine} - \text{NPV Taxes} = \$119.93 - \$46.77 = \$73.16 \; . \tag{9.5}
$$

Table 9.7. Cash Flows and Net Income Summary

	Year 1	2	3	4	5	6	Discount Rate	NPV
CF Machine w/o Tax	-\$15	-\$15	+\$60	+\$60	+\$60	+\$60	12%	\$119.93
CF Uncle Sam	-\$14	-\$2	-\$2	-\$12	-\$22	-\$22	12%	-\$46.77
CF **Project**	-\$29	-\$17	+\$58	+\$48	+\$38	+\$38	12%	\$73.16
CF Loan	+\$50	-\$5	-\$5	-\$5	-\$5	-\$55	10%	\$0.00
Residual CF: **Levered Ownership**	+\$21	-\$22	+\$53	+\$43	+\$33	-\$17	?	\$73.16
For comparison, Net Income	\$21	\$3	\$3	\$18	\$33	\$33	n/a	n/a

Note: The cost of capital (expected rate of return) is higher in this example for the machine than it is for the loan. This will be explained when we discuss the role of risk-aversion, when the safer loan will command a lower cost of capital than the riskier machine.

Now consider the third component—the loan. Assume that we play not the "full project owner," but only the "residual levered equity owner," so we do not extend the loan ourselves. Instead, a perfect capital market extends us a loan. We can assume that our company "got what it paid for," a fair deal: the loan, which provides \$50 and pays interest at a rate of 10%, should be zero NPV. (This saves us the effort to compute the NPV of the loan!)

The loan usually is a "Zero NPV" project, unless you can get an unusually great deal or unusually bad deal on the loan.

$$\text{NPV}_{\text{Loan}} = \$0 \ . \tag{9.6}$$

Be my guest, though, and make the effort:

$$\text{NPV}_{\text{Loan}} = \frac{+\$50}{1.10^1} + \frac{-\$5}{1.10^2} + \frac{-\$5}{1.10^3} + \frac{-\$5}{1.10^4} + \frac{-\$5}{1.10^5} + \frac{(-\$50)+(-\$5)}{1.10^6} = \$0 \ . \tag{9.7}$$

Therefore, the project NPV with the loan, i.e., levered equity ownership, is the same as the project NPV without the loan. This makes sense: we are not generating or destroying any value by walking over to the bank. Therefore

$$\begin{aligned} \text{NPV Levered Ownership} &= \text{NPV Machine} - \text{NPV Taxes} + \text{NPV Loan} \\ &= \$119.93 - \$46.77 + \$0 = \$73.16 \ . \end{aligned} \tag{9.8}$$

Although the NPV remains the same, the cash flows to levered equity ownership are different from the cash flows to the project. The cash flows (and net income) are shown in Table 9.7. Note how different the cash flows and net income are. Net Income is highest in Years 5 and 6, but levered cash flow in Year 6 is negative. In contrast, in Year 3, the year with the highest levered cash flow, net income is lowest.

9·2.C. Translating Accounting into Finance

If you did not know about the details of this machine but saw only the financials, could you compute the correct firm value by discounting the net income? Discounting net income with a cost of capital of 12% would yield

Discounting the Net Income does not give the true project NPV.

$$\begin{aligned} \text{Incorrect NPV}_{\text{via Net Income}} &= \frac{\$21}{(1+12\%)^1} + \frac{\$3}{(1+12\%)^2} + \frac{\$3}{(1+12\%)^3} \\ &+ \frac{\$18}{(1+12\%)^4} + \frac{\$33}{(1+12\%)^5} + \frac{\$33}{(1+12\%)^6} = \$70.16 \ , \end{aligned} \tag{9.9}$$

which is definitely not the correct answer of \$73.16. Neither would it be correct to discount the net income with a cost of capital of 10%,

$$\begin{aligned} \text{Incorrect NPV}_{\text{via Net Income}} &= \frac{\$21}{(1+10\%)^1} + \frac{\$3}{(1+10\%)^2} + \frac{\$3}{(1+10\%)^3} \\ &+ \frac{\$18}{(1+10\%)^4} + \frac{\$33}{(1+10\%)^5} + \frac{\$33}{(1+10\%)^6} = \$75.24 \ . \end{aligned} \tag{9.10}$$

So, how do you reverse-engineer the correct cash flows for the NPV analysis from the financials? You first need to translate the financials back into the cash flows that NPV analysis can use. You just need to retrace our steps. You have both the income statement and cash flow statement at your disposal. First, to obtain the machine cash flow, you can apply the formula

Cash flows can be reverse engineered from the corporate financials.

		Year 1	Year 2
	EBIT	+\$35	+\$10
+	Depreciation	+\$25	+\$50
"+"	(−)Capital Expenditures	+(−\$75)	+(−\$75)
=	CF Project, Pre-Tax	−\$15	−\$15

(9.11)

to the numbers from Table 9.6. You add back the depreciation, because it *was not* an actual cash outflow; and you subtract the capital expenditures, because it *was* an actual cash flow. I find the formula most intuitive if I think of the depreciation + capital expenditures terms as undoing the accountants' smoothing of the cost of machines over multiple periods.

> **IMPORTANT**: *The main operation to take care of long-term accruals in the conversion from net income into cash flows is to undo the smoothing—add back the depreciation and take out the capital expense.*

> **SIDE NOTE**: The formula signs themselves seem ambiguous, because accountants use different sign conventions in different spots. For example, because capital expenditures are usually quoted as negative terms on the cash flow statement, in order to subtract out capital expenditures, you just add the (negative) number. In the formula below, you want to subtract corporate income tax, which appears on the income statement (Table 9.6) as a positive. Therefore, you have to subtract the positive. Sigh... I try to clarify the meaning (and to warn you) with the quotes around the +, and the cumbersome "+″ (−) in the formulas themselves.

Finish the reverse-engineering.

Now you need to subtract corporate income taxes (and, again, look at the numbers themselves to clarify the signs in your mind; income-tax is sometimes quoted as a negative, sometimes as a positive). This gives you the after-tax project cash flow,

		Year 1	Year 2
	EBIT	+$35	+$10
+	Depreciation	+$25	+$50
"+"	(−)Capital Expenditures	+(−$75)	+(−$75)
−	(+)Corporate Income Tax	−(+$14)	−(+$2)
=	CF Project, After-tax	−$29	−$17

(9.12)

Net Income already has corporate income tax subtracted out, but it also has interest expense subtracted out. So, the same cash flow results if you start with net income instead of EBIT, but add back the interest expense,

		Year 1	Year 2
	Net Income	+$21	+$3
+	Depreciation	+$25	+$50
"+"	(−)Capital Expenditures	+(−$75)	+(−$75)
+	Interest Expense (*add* back to Net Income)	$0	+(+$5)
=	CF Project, After-tax	−$29	−$17

(9.13)

Cash flow to levered equity holders.

Investors (equity and debt together) must thus come up with $29 in the first year and $17 in the second year. What part of this is provided by creditors? In the first year, creditors provide $50; in the second year, creditors get back $5. Therefore, levered equity actually receives a positive net cash flow of $21 in the first year, and a negative cash flow of $22 in the second year. Therefore, with the loan financed from the outside, you must add all loan inflows (principal proceeds) and subtract all loan outflows (both principal and interest). Therefore, the cash flow for levered equity shareholders is

		Year 1	Year 2
	EBIT	+$35	+$10
+	Depreciation	+$25	+$50
"+"	(−)Capital Expenditures	+(−$75)	+(−$75)
−	Corporate Income Tax	−(+$14)	−(+$2)
=	CF Project	−$29	−$17
+	Net Debt Issue	+$50	$0
−	Interest Expense	$0	−$5
=	CF Levered Equity Ownership	+$21	−$22

(9.14)

Again, net income already has both corporate income tax and interest expense subtracted out, so again the same result obtains if you instead use the formula

		Year 1	Year 2
	Net Income	+$21	+$3
+	Depreciation	+$25	+$50
"+"	(−)Capital Expenditures	+(−$75)	+(−$75)
+	Net Debt Issue	+$50	$0
=	CF Levered Equity Ownership	+$21	−$22

(9.15)

Although you could now repeat the calculation for PepsiCo, hold your horses—we will do so only after we have covered other issues.

Solve Now!

Q 9.3 *Show that the formulas 9.11–9.15 yield the cash flows in Table 9.7*

Q 9.4 *Using the same cash flows as in the NPV analysis in Table 9.7, how would the project NPV change if you used a 10% cost of capital (instead of 12%) on the tax liability?*

Q 9.5 *Rework the example (income statement, cash flow statement excerpts, cash flows, and NPV) with the following parameters.*

Project	
True Lifespan	*5 Years*
Cost	*$120, year 1*
Raw Output	*$80/year*
− Input Costs	*$6/year*
− Selling Expense	*$8/year*
= Net Output	*$66/year*
Overall Cost of Capital	*8%/year*
Corporate Tax Rate (τ)	*50%*

Available Financing — Executed	
Debt Capacity	*$100*
Debt Interest Rate	*8%/year*

Accounting Treatment	
Accounting Life	*4 Years*

Debt does not require interest payment in Year 1. The world is risk-neutral, because debt and project require the same expected rate of return (cost of capital).

Q 9.6 *For the example in the text, do both the financials and the cash flow analysis using monthly discounting. Assume that the loan is taken at year start, and most expenses and income occur pro-rata. (Warning: Time-Intensive Question. Use Excel. Do not do by hand!)*

ANECDOTE: Solid Financial Analysis

EBITDA was all the rage among consultants and Wall Street for many years, because it seems both closer to cash flows than EBIT and more impervious to managerial earnings manipulation through accruals. Sadly, discounting EBITDA can be worse than discounting EBIT *if* capital expenditures are not netted out—and they usually are not netted out. (Forgetting about capital expenditures when depreciation is not netted out is equivalent to assuming that product falls like manna from heaven. EBIT may spread capital expenditures over time periods in a strange way, but at least it does not totally forget it!) Sometimes, a little bit of knowledge is more dangerous than no knowledge.

In June 2003, a Bear Stearns analyst valued American Italian Pasta, a small N.Y.S.E.-listed pasta maker. Unfortunately, Herb Greenberg from TheStreet.com discovered that he forgot to subtract capital expenditures. This mistake increased the value of American Italian Pasta from $19 to $58.49 (then trading at $43.65). Bear Stearns admitted the mistake, and came up with a new valuation, in which Bear Stern's boosted the estimate of the company's operating cash flows and dropped its estimate of the cost of capital. Presto! The NPV of this company was suddenly $68 per share. How fortunate that Bear Stearns' estimates are so robust to basic errors.

9·3. The Hypothetical Bottom-Up Example — Short-Term Accruals

Here are other items that require cash (inflows or outflows), which have not appeared in this simple example.

You now know how to translate some of the financials into more NPV-relevant cash flows. But you can still improve the accuracy of your cash flow formula by also adjusting for short-term accruals.

9·3.A. Working Capital

The definition of working capital.

To run a business day-to-day requires cash. Firms must put money into cash registers (to make change), into inventories (to have something to sell), and into extending credit to buyers (to get them to bite). This is called **working capital.** Accountants define working capital as current assets minus current liabilities. Current Assets are **cash**, **accounts receivables**, and **inventories.** Current Liabilities are **accounts payables**, **bank overdrafts**, **tax payables**, and other soon-due bills.

$$\begin{aligned} \text{Working Capital} &= (\text{Current Assets}) \quad - \quad (\text{Current Liabilities}) \\ &= (\text{Cash} + \text{Accounts Receivables} + \text{Inventories}) - (\text{Accounts Payables}) \; . \end{aligned} \tag{9.16}$$

Net Income books cash before it comes in, so accounts receivables need to be taken out.

The cash flow effects of working capital changes are best explained with an example. Say that a firm sells \$100 of goods on credit. The firm books \$100 as net income. But because the \$100 is not yet available, the firm also books \$100 into accounts receivables. To compute actual cash flows, recognize that the cash has not yet materialized: we need to subtract out the \$100 accounts receivables from the \$100 net income.

Table 9.8. Multi-Year Working Capital

	Year	0	1	2	3
Finance	1. Sales and Net Income	\$0	\$100	\$300	\$0
	2. Actual Cash Receipts (for NPV CF)	\$0	\$0	\$100	\$300
Accntng	3. Reported Net Income	\$0	\$100	\$300	\$0
	4. Reported accounts receivables	\$0	\$100	\$300	\$0

Table 9.9. Multi-Year Working Capital

	Year	0	1	2	3
Finance	1. Sales and Net Income	\$0	\$100	\$300	\$0
	2. Actual Cash Receipts (for NPV CF)	\$0	\$0	\$100	\$300
Accntng	3. Reported Net Income	\$0	\$100	\$300	\$0
	4. Reported accounts receivables	\$0	\$100	\$300	\$0
	Your Computations				
	5. Change in accounts receivables	*\$0*	*+\$100*	*+\$200*	*–\$300*
	6. Net Income (Line 3) – Change in accounts receivables (Line 5)	*\$0*	*\$0*	*+\$100*	*+\$300*

Line 6 recovers Line 2 from the financials.

The difference between cash flows and Net Income are year-to-year changes in working capital.

This becomes more interesting if you consider multiple years. For example, the firm in Table 9.8 always sells on credit and is always paid by its customers the following year. An NPV analysis requires the firm's actual cash receipts in Line 2, but accountants have provided only the information in lines 3 and 4. How do you get back the information in Line 2? Year 1 has already been discussed: you subtracted accounts receivables from net income to obtain the actual cash inflows of \$0. Year 2 is more interesting: The firm previously had accounts receivables of \$100, but now has accounts receivables of \$300. It is the +\$200 *change* in accounts receivables (= \$300 − \$100) that needs to be subtracted from the \$300 in net income in order to infer the actual cash receipts of \$100. In Year 3, the firm no longer grows and is liquidated, so the remaining receivables turn into cash that can be recaptured from the business. Again, the formula to obtain the NPV cash flow (Line 2) subtracts the change in working capital (accounts receivables) of \$0 − \$300 = −\$300 from the \$0 net income to conclude that we got +\$300 cash inflow. Table 9.9 shows these calculations. (Incidentally, recall how you started this subsection with a Year 1 computation: you subtracted \$100 in accounts receivables from the \$100 net income. This worked only because the accounts receivables were the same as the *change* in accounts receivables, because the original accounts receivables were zero.)

Working Capital already contains other delayed payments, making our life easier.

Other short-term accruals that are components of working capital work similarly. For example, although corporate income tax is deducted on the income statement for the year in which the earnings have occurred, firms do not have to immediately pay these taxes. Instead, they can often defer them—at least until (the corporate equivalent of) April 15 of the following year. To the extent that more taxes can be delayed, more cash is available than is suggested by net income. Therefore, delayed taxes must be added back to net income when computing finance cash flows. Of course, at some point in the future, these tax payables will have to be paid, and they will then have to be counted as a cash outflow of the firm. But, for now, the permitted delay in payment is like a government loan at zero interest—and one that the accounting item net income ignores.

IMPORTANT: *The main operation to take care of short-term accruals in the conversion from net income into cash flows is to undo the smoothing—add changes in working capital.*

SIDE NOTE: Alas, as with capital expenditures (see Page 201), the cash flow statement has its sign conventions. The change in cash, accounts receivables, and inventories is recorded as a negative. But accounts payables do not have the opposite sign from accounts receivables, though they are already an outflow (negative); they are left as is. As a result, to compute the firm's working capital from its line items (accounts receivables, accounts payables, etc.), you do not subtract current liabilities (e.g., accounts payables) from current assets (e.g., accounts receivables), but add them together.

Here is an example of the accounting sign conventions. Table 9.4 on Page 195 listed PepsiCo's changes in working capital as 84, 416, and 79 for the years 2001, 2000, and 1999:

Cash Flow Statement	December		
	2001	2000	1999
Current Assets			
Accounts Receivables	+7	–52	–141
Inventories	–75	–51	–202
Prepaid Expenses, etc.	–6	–35	–209
Current Liabilities			
Accounts Payables, etc.	–236	+219	+357
Corporate Income Tax, Payable	+394	+335	+274
Net Change in Operating working capital	+84	+416	+79

Excludes effects of acquisitions and dispositions.

Because these figures come from the cash flow statement, to obtain the net change in operating working capital, all figures are simply added up, not netted out! (The sign of current liabilities has already been reversed for you.) If you stumble onto the fact that these numbers cannot be inferred from other parts of the financial statements, this is because these numbers exclude the effects of acquisitions and dispositions, as well as non-operating working capital.

Where are changes in cash [in the register] itself? These are not in the changes of working capital, but instead they are what you find at the very bottom of the cash flow statement.

Expand our valuation formula for another source of cash.

We can now expand our formulas to include changes in working capital. For example, Formula 9.12 transmutes into

$$\begin{aligned} \text{NPV Project} = \; & \text{EBIT} + \text{Depreciation} \;\text{“+”}\; \text{Capital Expenditures} \\ & - \text{Corporate Income Tax} - \text{Increase in Working Capital} \; . \end{aligned} \tag{9.17}$$

9·3.B. Earnings Management

There is considerable leeway in financials.

Even though the United States has the tightest accounting regulations of any country, managers still have a lot of discretion when it comes to financials. There is also no clear line where accounting judgments become unethical or even criminal. The border zone between ethical and unethical behavior is a ramp of gray—it may be easy when one is in the clean white zone or in the clean black zone, but in between, it is often a slippery slope.

ANECDOTE: Working Capital Management
Entrepreneurs usually fail for one of two reasons, and both are common: The first is that the business is just not a good idea to begin with. There is not much you can do about this. The second is that the business is too good of an idea, and the entrepreneur is not equipped to handle the success. The growth in sales consumes so much cash for increases in working capital that the firm fails to pay back its own loans: The cash is tied up in production, or in inventory, or in credit extended to customers (payment to be received), when instead it is needed to flow back to the bank. For growing firms, proper working capital management is an issue of first-order importance.

Not only earnings, but also cash flows can be managed.

We already know that managers must make many judgments when it comes to accrual accounting. For example, managers can judge overoptimistically how many products customers will return, how much debt will not be repaid, how much inventory will spoil, how long equipment will last, whether a payment is an expense (fully subtracted from earnings) or an investment (an asset that is depreciated over time), or how much of an expense is "unusual." However, manipulation is possible not only for earnings and accruals but also for cash flows—though doing so may be more difficult and costly. For example, if a firm designates some of its short-term securities as "trading instruments," their sale can then create extra cash—what was not cash before now counts as cash! Similarly, we already know that firms can reduce inventory, delay payments to suppliers, and lean on customers to accelerate payment—all of which will generate immediate cash, but possibly anger suppliers and customers so much that it will hurt the business in the long run. Firms can also sell off their receivables at a discount which may raise the immediate cash at hand but reduce the profit the firm will ultimately receive. A particularly interesting cash flow management play occurs when a firm is lending money aggressively to its customers. The sales generate immediate cash from sales, and the loans can count as investments. Of course, if the customers default, all the company has accomplished is to give away its product for free.

Here is a good warning sign.

One quick measure of comparing how aggressive or conservative financials are is to compare the firm to other similar firms on the basis ratio of its short-term accruals divided by its sales. It is important that "similar" means firms that are not only in the same industry but also growing at roughly the same rate. The reason is that growing firms usually consume a lot of cash—an established firm will show higher cash flow than a growing firm. If the firm is unusual in having much higher accruals—especially short-term accruals—than comparable firms, it is a warning sign that this firm deserves more scrutiny. Managers who decide to manipulate their numbers to jack up their earnings more than likely will try to manage their accruals aggressively in order to create higher earnings, too. Of course, this does not mean that all managers who manage their accruals aggressively do so to deceive the market and will therefore underperform later on. A manager who is very optimistic about the future may treat accruals aggressively—believing in few returns, great sales, and a better future all around. Indeed, as noted earlier, theslope from managerial optimism to illegal earnings manipulation is slippery. Finally, another earnings warning sign for the wary investor is when a firm changes its fiscal year—this is sometimes done in order to make it more difficult to compare financials to the past and to financials of other firms in the same industry.

Solve Now!

Q 9.7 *A firm reports the following financials.*

Year	*0*	*1*	*2*	*3*	*4*	*5*	*6*
Reported Sales= Net Income	*$0*	*$100*	*$100*	*$300*	*$300*	*$100*	*$0*
Reported Accounts Receivables	*$0*	*$100*	*$120*	*$340*	*$320*	*$120*	*$0*

Can you describe the firm's customer payment patterns? Extract the cash flows.

Q 9.8 *Construct the financials for a firm that has quarterly sales and net income of $100, $200, $300, $200, $100, and half of all customers pay immediately, the other half always pay* ***two*** *quarters after purchase.*

Q 9.9 *(Difficult:) Amazonia can pay suppliers after it has sold to customers. Amazonia has 25% margins and is reporting*

Month	*Jan*	*Feb*	*Mar*	*Apr*	*May*
Reported Sales	*$0*	*$100*	*$100*	*$400*	*$0*
Reported Net Income	*$0*	*$25*	*$25*	*$100*	*$0*
Reported Accounts Payables	*$0*	*$75*	*$75*	*$300*	*$0*

What are Amazonia's actual cash flows?

Q 9.10 *Are short-term accruals or long-term accruals easier to manipulate?*

Q 9.11 *Give an example of how a firm can depress the earnings that it reports in order to report higher earnings later.*

Q 9.12 *Give an example of how a firm can depress the cash flows that it reports in order to report higher cash flows later.*

9·4. Completing the Picture: PepsiCo's Financials

Other, so-far neglected sources of cash hidden in the financial statements.

Now, if you take another look at the complete PepsiCo cash flow statement in Table 9.4 or even the abbreviated cash flow statement in Table 9.14 (Page 222), you will see not only the procedures that we have just discussed—starting with net income, adding back depreciation, subtracting off capital spending, and adding changes in working capital—but a whole range of other items that we have not yet even mentioned.

Now "wing it."

There are two pieces of good news here. First, you now understand the main logic of what is going on. Second, you can now rely on the accountants to do most of the hard work for you. The logic of how to handle the remaining items in the cash flow statement is either similar to what we have already discussed and/or obvious from the name. For instance, you hopefully won't need an explanation from me as to why "bottling equity income, net" which appears just below "net income" and which is added to it is probably just another form of net income—even if I knew its meaning better than you, it would not help if I explained it to you, because every company has its own unique collection of named items in their financial statements. Like I, you will have to "wing it"—or, better, seek to understand the specific company you are analyzing.

Here are a few less obvious ones.

There are two other sources and uses of cash flow, which we still want to mention, because it is not (relatively) obvious from the accounting jargon what is really happening.

Changes in Deferred Taxes arise when firms use different depreciation schedules on their tax financials than on their public financials. Many firms are allowed to use accelerated depreciation for tax purposes. The resulting discrepancy in tax timing is then recorded as "changes in deferred taxes."

Note that this item has nothing to do with the fact that income tax is paid after it is incurred (e.g., on April 15). This difference in the timing of taxes incurred and taxes actually sent to the IRS can be computed as changes in Income Tax Payable, which is itself a component of changes in Working Capital.

Investment in Goodwill is an item whose name is even more of a misnomer. It has to do with cash laid out when firms acquire other firms. (It can be a very important item for acquirers.)

A semi-complete cash flow formula:

Putting short-term and long-term accruals and other sources/uses of cash together yields a second and more complete formula for estimating cash flows for an NPV analysis from financial statements. Cash Flows for NPV analysis can be estimated using a formula like the one in Table 9.10.

Table 9.10. A Formula To Compute Cash Flows

		PepsiCo, 2001
	Earnings after Interest before Taxes	$4,029
+	add back Interest Expense	+ ($8)
=	Earnings before Interest and Taxes (EBIT)	$4,021
–	Corporate Income Tax	– $1,367
=	Net Operating Profit	$2,654
+	Changes in Deferred Taxes	+ $162
+	Depreciation	+ $1,082
=	Gross Cash Flow	$3,898
–	Increase in Working Capital (incl. tax payables, etc.)	– ($84)
–	Capital Expenditures	– $1,324
–	Investment in Goodwill	
–	All Sorts of Increases in Net Other Assets	– ($211)
=	Free Cash Flow from Operations	$2,869
+	All Sorts of Non-Operating Cash Flows	+ ($1,313)
=	Total Project Firm Cash Flow (to Debt and Equity)	$1,556
+	Net Issuance of Debt	+ ($341)
–	Interest Expense	– ($8)
=	Total Cash Flow to Levered Equity	$1,223

In this formula, the numbers on the right are from the abbreviated cash flow statement in Table 9.14. You can also see how similar this is to the accounting cash flow statement. It starts with the net income of $4,029 (from the income statement) and adds back depreciation—unlike in the income statement, the depreciation on the cash flow statement includes all depreciation of all sorts of assets. The **deferred taxes** of $162 (i.e., tax depreciation schedule differences) and **non-cash items** of $211 are two more opaque items that provided PepsiCo with extra cash during the year—more than was indicated by its net income of $2,662. The **change in working capital**, explained in Section 9·3.A, is cash that PepsiCo had to add to its inventory, loans made to its retailers, or cash it could recover from its delay of payables and tax payments. In 2001, PepsiCo actually could reduce and thereby generate $84 in cash from its working capital. In total, collecting all positive cash flow terms, PepsiCo's business generated $3,898 + $84 + $211 + $8 = $4,201 million. Of this cash, PepsiCo invested $1,324 + $1,313 = $2,637 million (i.e., negative cash flows) into things like plant and equipment, advertising and sponsorships, etc. Another $8 was spent on interest payments. Again, these were actual cash flows consumed in the course of running PepsiCo.

It's a suggestive formula, not a perfect one.

Unfortunately, the formula cannot cover *all* items in *all* companies—and even for PepsiCo, we had to lump some items and ignore others (such as foreign exchange effects). So please do not consider our cash flow formula to be the perfect, end-all formula to compute NPV cash flows. Again, every business operates and reports differently. Still, the formula is a good start for estimating realized cash flows for an NPV analysis for most firms in the real world, and for understanding the link between earnings and cash flows.

An even easier solution which works better!

Fortunately, most of the time, we do not need to construct the cash flow with this long formula ourselves, because we can instead rely on the corporate **cash flow statement** itself. After all, it tries to construct most of the information for us. Its big items, including even those we forgot in our long formula, are lumped into the sum of cash flows from operating activity and cash flows from investing activity. So we can use this sum instead of fiddling with the components. However, there is one difference between what accountants consider cash flows and what financiers consider cash flows. It is interest payments. Accountants consider interest payments an operating expense. Financiers consider them a distribution to owners. If we take care of this detail, we can then rely on our accounting friends:

IMPORTANT:

Project Cash Flows (CF), *which pay out to owners collectively (both debt holders and equity holders in the corporation) are*

$$\begin{aligned} CF_{Project} = & \quad \textit{Cash Flow from Operating Activity} \\ & + \textit{Cash Flow from Investing Activity} \\ & + \textit{Interest Expense} \quad . \end{aligned} \tag{9.18}$$

Net Income, a component of cash flow from operating activity, has had interest expense subtracted out. But interest expense is cash that is being returned to (debt) investors. Thus, to obtain the total amount generated by the project and available (paid out to) the sum-total of both creditors and shareholders, the interest expense (from the income statement) must be added back.

Equity Cash Flows (CF), *which are available only to levered equity shareholders—that is, they have subtracted out all inflows and outflows to creditors (debtholders)—are*

$$\begin{aligned} CF_{Equity} = & \quad \textit{Cash Flow from Operating Activity} \\ & + \textit{Cash Flow from Investing Activity} \\ & + \textit{Net Issuance of Debt} \\ = & \; CF_{Project} + \textit{Net Issuance of Debt} - \textit{Interest Expense} \quad . \end{aligned} \tag{9.19}$$

DIGGING DEEPER: *Because accountants keep score of both assets and liabilities, the exact same cash flows can also be obtained from the financing side of the business:*

		Change in Marketable Securities
	+	*Decrease in Debt*
	+	*After-Tax Interest Expense*
	+	*Dividends*
	+	*Share Repurchases*
	–	*After-Tax Interest Income*
=		*Total Financial Flow*

(Actually, accountants charge after-tax interest as an operating cost, so care must be taken to recognize it as a distribution.) For complete references, see Corporate Valuation *by Bradford Cornell (Irwin), or* Valuation *by Copeland, Koller, Murrin (McKinsey).*

PepsiCo's cash flow, the easy way.

Will these formulas give us the same result? Let us apply them to PepsiCo. Adding **total operating activity** of $+\$4,201$ and **total investing activity** of $-\$2,637$ gives \$1,564 in **operating activity net of investing activity.** Finally, we need to add back any interest expense that was taken out from net income. (After all, the project generated these funds and they were paid out, just as dividends are paid out.) In PepsiCo's case, it is not an interest expense, but net interest income, so the cash flow that we would use in an NPV analysis of the business of PepsiCo for 2001 is

$$\begin{aligned} \text{CF}_{\text{Project}} &= \text{Cash Flow from Operating Activity} \\ &\quad + \text{Cash Flow from Investing Activity} \\ &\quad + \text{Interest Expense}\ {\scriptstyle\text{(from the income statement)}} \\ &= \$4,201 + (-\$2,637) + (-\$8) = \$1,556 \quad . \end{aligned} \tag{9.20}$$

(PepsiCo is the rare company that did not pay interest income, but earned interest income in 2001!) These are the cash flows accruing to all owners together, debt and equity. We are still interested in the cash flow that is earned by PepsiCo's levered equity (without the creditors). We need to add cash obtained from **net issuance of debt** (the difference of debt principal that we raised and debt principal that we repaid, which we can read from the cash flow statement), and we need to subtract interest that we paid.

$$\begin{aligned} \text{CF}_{\text{Equity}} &= \text{CF}_{\text{Project}} + \text{Net Issuance of Debt} - \text{Interest Expense} \\ &= \$1,556 + (-\$341) - (-\$8) = \$1,223 \ . \end{aligned} \tag{9.21}$$

Both numbers are identical to those on Page 213. It must be noted that you might sometimes need the longer formula with its individual components, because they may need to be discounted by different interest rates. We will see more of this later.

See how much earnings and cash flows can differ.

PepsiCo showed an increase in net income from 1999 to 2001. Did it also have an increase in cash flows? The answer is no. In 1999, PepsiCo had NPV cash flows of $\$3,605-\$1,172-\$792 = \$1,641$; in 2000, it had cash flows of $\$4,440 - \$1,996 + \$57 = \$2,501$; in 2001, it had NPV cash flows of $\$4,201 - \$2,637 - \$8 = \$1,556$. Yet, even in 2000, managers used **changes in working capital** to prevent PepsiCo's cash flows from dropping even further. It may be that PepsiCo did not show stellar three year improvement, after all. On the other hand, the cash was not discarded but used. Naturally, judging whether these were profitable investment uses is a difficult matter.

The cash flow statement in Table 9.14 also continues where we stopped. It proceeds to tell us what PepsiCo did with its projects' (post interest) cash flows: It used \$994 million to pay dividends, \$579 million to repurchase its own equity shares, and \$341 million to repurchase its own debt, for total capital market activities of \$1,919 million. In fact, this means it paid out more than it made in 2001 to the tune of $\$1,919-(\$1,556+\$8) = \$355$ million. (Presumably, PepsiCo still had cash lying around. Of course, this cash, too, was not generated in 1999, as PepsiCo also bled cash in 2000. It was in 1999 that PepsiCo produced the cash it consumed in 2000 and 2001.)

The task is done!

Our task is done—you can now look at a financial statement and obtain an estimate of the information they contain about cash flows that matter to your NPV analysis.

Solve Now!

Q 9.13 *From memory, can you recall the main components of cash flow used in an NPV analysis? Do you understand the logic?*

Q 9.14 *Is the firm's lifetime sum of net income (about) equal to the firm's lifetime sum of cash flows?*

Q 9.15 *A new firm reports the following financials:*

	Income Statement	December		
		2001	2000	1999
=	Revenue	200	162	150
	COGS	60	58	57
	+ SG&A	20	19	18
=	**Operating Income**	120	85	75
-	Net Interest Income (Gains&Losses)	35	35	35
=	**Income Before Tax**	85	50	40
–	Corporate Income Tax at 40%	34	20	16
=	**Income After Tax**	51	39	24
–	Extraordinary Items	0	0	0
=	**Net Income**	51	39	24

The firm also reports

Source	Item	2001	2000	1999
Cash Flow Statement	Capital Expenditures	0	30	200
Cash Flow Statement	Depreciation	25	23	20
Balance Sheet	Deferred Taxes	20	16	0
Balance Sheet	Accumulated Depreciation	68	43	20
Balance Sheet	Working Capital	35	25	20

(You will need to compute changes in deferred taxes, which are $\$20 - \$16 = \$4$ *in 2001, as well as changes in working capital.) Can you compute an estimate of cash flows produced by this firm?*

Q 9.16 *What are the cash flows produced by PepsiCo's projects in 1999, 2000, and 2001? What are the cash flows available to residual equity shareholders in 1999, 2000, and 2001?*

9·5. Summary

The chapter covered the following major points:

- There are four required financial statements: the balance sheet, the income statement, the shareholders' equity statement, and the cash flow statement. Although every company reports its numbers a little different, the major elements of these statements are fairly standard.
- Financial statements also serve more purposes than just NPV calculations, and are well worth studying in more detail—elsewhere.
- Earnings (net income) are *not* the cash flow inputs required in an NPV analysis.
- Accountants use "accruals" in their net income (earnings) computation, which we need to undo in order to extract actual cash flows.
- The primary long-term accrual is "depreciation," an allocation of capital expenditures. The prime operation to undo this is to add back depreciation and subtract out capital expenditures.
- The primary short-term accrual is "changes in working capital," an allocation of soon-expected but not-yet-executed cash inflows and cash outflows. Examples are accounts payables, accounts receivables, and tax payables. The prime operation to undo them is to add back changes in working capital.
- If a cash flow statement is available, it conveniently handles most of the difficulties in undoing accruals for the NPV analysis. However, accountants believe interest expense to be a cost of operations, while financiers believe it to be a payout to owners. Thus, interest expense requires special handling.
- Formula 9.18 shows how to compute cash flows that accrue to project owners (debt plus equity). It is cash flow from operating activity, plus cash flow from investing equity, plus interest expense.
- Formula 9.19 shows how to compute cash flows that accrue to levered equity owners (equity only). It is the cash flow that accrues to project owners, plus net issuance of debt, minus interest expense.

A final observation: the most difficult part to analyzing financial statements for me is getting the signs right.

A. Appendix: Supplementary Financials — Coca Cola

The following tables provide further financial statements for Coca Cola and PepsiCo. They are here to give you a feeling for what real-world financial statements look like—maybe a little more complicated and involved than what we covered, but you should still be able to extract the components that matter.

Solve Now!

Q 9.17 *The 2002 10-K Annual Statement of Coca Cola is available from Edgar. (If need be, you can use the financials provided on Page 219. However, it would be good for you to look this up on the Web, instead.) Approximate the cash flows that you would use in valuing Coca Cola. Then, use the cash flow statement. How different are the numbers that Coca Cola reports from those that you would infer from the income statement combined with capital expenditures and depreciation (i.e., from our first formula, 9.12 on Page 206)? What if you had used our second formula 9.17 on Page 210, which also subtracts out changes in working capital?*

a. Coca Cola's Financials From EdgarScan

Table 9.11. Coca Cola's Financials from EdgarScan, Restated.

	Income Statement	December 2001	December 2000	December 1999
=	Revenues	20,092	19,889	19,284
	COGS	6,044	6,204	6,009
	+ SG&A (incl. Depreciation)	8,696	8,551	8,480
	+ Other Expenses	0	1,443	813
-	= **TOTAL OPERATING EXPENSES**	14,740	16,198	15,302
=	**Operating Income**	5,352	3,691	3,982
	+ Other Net Income	607	155	174
=	**EBIT**	5,959	3,846	4,156
+	Interest Expense	289	447	337
=	**Income Before Tax**	5,670	3,399	3,819
-	Income Tax	1,691	1,222	1,388
=	**Income After Tax**	3,979	2,177	2,431
-	Extraordinary Items	-10,000	0	0
=	**Net Income**	3,969	2,177	2,431

	Cash Flow Statement	December 2001	December 2000	December 1999
	Net Income	3,969	2,177	2,431
+	Depreciation and Depletion	803	773	792
+	Deferred Taxes	56	3	97
+	Non-Cash Items	-256	1,484	1,120
+	Changes In Working Capital	-462	-852	-557
=	**Total Operating Activity**	4,110	3,585	3,883
	Capital Expenditures	-769	-733	-1,069
+	Investments	-1	-218	-342
+	Other Investing	-418	-214	-2,010
=	**Total Investing Activity**	-1,188	-1,165	-3,421
	Dividends	-1,791	-1,685	-1,580
+	Net Issuance of Stock	-113	-198	-153
+	Net Issuance of Debt	-926	-585	+956
=	**Total Financing Activity**	-2,830	-2,072	-471
-	Foreign Exchange Effects	-45	-140	-28
=	**Net Change In Cash**	47	208	-37

b. Coca Cola's Financials From Yahoo!*Finance*

Table 9.12. Coca Cola financial statements from Yahoo!*Finance*, Not Restated

	Income Statement	December 2001	December 2000	December 1999
=	Revenues	20,092	20,458	19,805
	COGS	6,044	6,204	6,009
	+ SG&A	8,696	10,563	9,814
	+ Depreciation and Amortization			
	+ Unusual Expenses			
-	= **Total Operating Expenses**			
=	**Operating Income**	5,352	3,691	3,982
	+ Other Net Income	607	155	174
=	**EBIT**	5,959	3,846	4,156
-	Interest Expense	289	447	337
=	**Income Before Tax**	5,670	3,399	3,819
-	Income Tax	1,691	1,222	1,388
=	**Income After Tax**	3,979	2,177	2,431
-	Extraordinary Items	-10,000	0	0
=	**Net Income**	3,969	2,177	2,431

	Cash Flow Statement	December 2001	December 2000	December 1999
	Net Income	3,969	773	792
+	Depreciation and Depletion	803	773	792
+	Deferred Taxes			
+	Non-Cash Items			
+	Changes In working capital			
=	**Total Operating Activity**	4,110	3,585	3,883
	Capital Expenditures	-769	-733	-1,069
+	Investments	-1	-218	-518
+	Other Investing	-418	-214	-1,834
=	**Total Investing Activity**	-1,188	-1,165	-3,421
	Financing Cash Flow Items			
+	Dividends	-1,791	-1,685	-1,580
+	Net Issuance of Stock	-113	-198	-153
+	Net Issuance of Debt	-926	-585	+956
=	**Total Financing Activity**	-2,830	-2,072	-471
-	Foreign Exchange Effects	-45	-140	-28
=	**Net Change In Cash**	47	208	-37

Q 9.18 *What are the economic project cash flows you would use for Coca Cola from 1999 to 2001?*

B. Appendix: Abbreviated PepsiCo Income Statement and Cash Flow Statement

Abbreviated and summarized statements may appear in a variety of venues, such as www.marketguide.com, the SEC's Edgar, or Price-Waterhouse-Coopers' EdgarScan.

Table 9.13. PepsiCo Income Statement, Restated for Acquisitions.

	Consolidated Income Statement	December		
	(in millions)	2001	2000	1999
=	**Net Sales**			
	New PepsiCo	$26,935	$25,479	$22,970
	Bottling operations	–	–	2,123
=	Total Net Sales	26,935	21,661	25,093
	Costs and Expenses			
	Cost of Goods Sold	10,754	10,226	10,326
	Selling, General & Administrative Expenses	11,608	11,104	11,018
	Note: sg&a incl. depreciation of	*917*	*946*	*963*
	Amortization of Intangible Assets	165	147	193
	Merger-related Costs (Unusual Expenses)	356	–	–
	Other Impairment and Restructuring (Unusual Expenses)	31	184	73
–	= **Total Costs and Expenses**	22,914	21,661	21,610
=	**Operating Income**			
	New PepsiCo	4,021	3,818	3,430
	Bottling operations and equity investments	–	–	53
	Total Operating Project	4,021	3,818	3,483
	a. Bottling equity income and transaction gains/(losses), net	160	130	1,083
	b. Interest Expense	(219)	(272)	(421)
	c. Interest Income	67	85	130
	= Net Interest Income, a-c (Gains&Losses)	*=8*	*=-57*	*=792*
=	**Income Before Income Taxes**	4,029	3,761	4,275
–	**Provision for corporate income tax**	1,367	1,218	1,770
=	**Income After Income Taxes**	2,662	2,543	2,505
–	Extraordinary Items	0	0	0
=	**Net Income**	$ 2,662	$ 2,543	$ 2,505

Table 9.14. PepsiCo Abbreviated Cash Flow Statement, Restated for Acquisitions.

	Cash Flow Statement	December		
	(in millions)	2001	2000	1999
	Net Income	2,662	2,543	2,505
+	Depreciation, Depletion, Amortization	1,082	1,093	1,156
+	Deferred Taxes (changes in)	162	33	573
+	Non-Cash Items	211	355	–708
+	Changes in Working Capital	84	416	79
=	**Total Operating Activity**	4,201	4,440	3,605
	Capital Expenditures	–1,324	–1,352	–1,341
+	Other Investing	–1,313	–644	169
=	**Total Investing Activity**	–2,637	–1,996	–1,172
	Financing Cash Flow Items	–5	–254	–382
+	Dividends	–994	–949	–935
+	Net Issuance of Stock	–579	–740	–902
+	Net Issuance of Debt	–341	–705	391
=	**Total Financing Activity**	–1,919	–2,648	–1,828
–	Foreign Exchange Effects	0	–4	3
=	**Net Change In Cash**	–355	–208	608

Solutions and Exercises

1. Accruals, specifically depreciation and delayed payments/receipts.
2. Uncle Sam uses Accounting methods to compute corporate income taxes. Secondary influences, not discussed in the text, come from the fact that many contracts are contingent on accounting numbers (e.g., debt covenants).
3. Do it!
4. A 12% instead of a 10% interest rate would increase the NPV of the tax obligation from \$46.77 to \$50.16. Therefore, the project value would decrease by \$3.39.
5. The income statementis now

	Year	1	2	3	4	5
	Sales (Revenues)	\$80	\$80	\$80	\$80	\$80
–	Cost of Goods Sold (COGS)	\$6	\$6	\$6	\$6	\$6
–	Selling, General & Administrative Expenses (SG&A)	\$8	\$8	\$8	\$8	\$8
=	EBITDA	\$66	\$66	\$66	\$66	\$66
–	Depreciation	\$30	\$30	\$30	\$30	\$0
=	EBIT (Operating Income)	\$36	\$36	\$36	\$36	\$66
–	Interest Expense	-	\$8	\$8	\$8	\$8
=	EAIBT (or EBT)	\$36	\$28	\$28	\$28	\$58
–	Corporate Income Tax	\$18	\$14	\$14	\$14	\$29
=	Net Income	\$18	\$14	\$14	\$14	\$29

Cash Flow Statement Excerpt

Year	1	2	3	4	5
Capital Expenditures	–\$120	-	-	-	-
Net Debt Issue	+\$100	-	-	-	–\$100

The cash flow formula is EBIT plus depreciation (or use EBITDA instead) minus capital expenditures, minus corporate income tax: $\$36 + \$30 - \$120 - \$18 = -\$72$. The first levered equity cash flows are $-\$72 + \$100 = +\$28$.

Cash Flow	Discount Rate	1	2	3	4	5	NPV
Machine	8%	–\$54	\$66	\$66	\$66	\$66	\$152.41
Uncle Sam	8%	–\$18	–\$14	–\$14	–\$14	–\$29	–\$69.81
Project	8%	–\$72	+\$52	+\$52	+\$52	+\$37	\$82.60
Loan	8%	+\$100	–\$8	–\$8	–\$8	–\$108	\$0
Levered Ownership	8%	+\$28	+\$44	+\$44	+\$44	–\$71	\$82.60

6. The answer will eventually be posted on my website. (It is not there yet.)
7.

Year	0	1	2	3	4	5	6
Reported Net Income	\$0	\$100	\$100	\$300	\$300	\$100	\$0
Reported accounts receivables	\$0	\$100	\$120	\$340	\$320	\$120	\$0
Change in accounts receivables	\$0	\$100	\$20	\$220	–\$20	–\$200	–\$120
Cash Flow	\$0	\$0	\$80	\$80	+\$320	+\$300	+\$120

The firm's customers did not all pay the next period. Therefore, the cash flows were delayed.

8. The cash flows are

	Quarter	0	1	2	3	4	5	6	7
	Reported Net Income	\$0	\$100	\$200	\$300	\$200	\$100	\$0	\$0
	Immediate Cash Flows	\$0	\$50	\$100	\$150+	\$100+	\$50+	\$0	\$0
	+ Delayed Cash Flows				+\$50	+\$100	+\$150	+\$100	+\$50
⇒	= Cash Flows	=\$0	=\$50	=\$100	=\$200	=\$200	=\$200	=\$100	=\$50
⇒	Change in A/R	-	\$50	\$100	\$100	\$0	–\$100	–\$100	–\$50
⇒	Accounts Receivables	\$0	\$50	\$150	\$250	\$250	\$150	\$50	\$0

It is easier to obtain the change in A/R first: we know that Net Income minus the Change in A/R must add up to cash flows. So, Change in A/R = Net Income – cash flows. And, knowing Change in A/R, accounts receivables itself requires simply adding up.

9. In Year 1, Amazonia has cash inflows of \$100 (\$25 net income plus \$75 change in accounts payables). In Year 2, Amazonia has another \$100 in sales, but payables stay the same. (It has to pay its old suppliers \$75, even though it gets to keep \$75 from its new suppliers.) So, Amazonia gets cash inflows of \$25 only. In Year 3, Amazonia gets net income cash inflows of \$100, plus the \$225 change in payables, for cash inflows of \$325. Finally, in Year 4, Amazonia has cash outflows of \$300. The pattern is thus

Month	Jan	Feb	Mar	Apr	May
Cash Flows	\$0	\$100	\$25	\$325	−\$300

Note that Amazonia has total 5-month cash flows of \$150, just as it has total 5-month net income of \$150. The working capital has only influenced the timing attribution.

10. Short term accruals. To manipulate long-term accruals, managers would have to manipulate the depreciation schedule, and though this is possible a few times, if it is done often, it will most surely raise eyebrows.

11. For example, a firm can take out a reserve against a judgment in a pending lawsuit. Or, it could assume that customers will pay their bills less than they actually will.

12. For example, a firm could pay all its payables immediately, instead of delaying them.

13. See Page 213.

14. Yes. Cash flows just have different timing. For example, firm's capital expenditures are not booked immediately, but the sum of all lifetime depreciation adds up to the sum of all lifetime capital expenditures. (This abstracts away from some pathological accounting cases that we have not covered.).

15. Use the Formula on Page 213:

			2001	2000	1999
	Earnings before Interest and Taxes (EBIT)		120	85	75
–	Corporate Income Tax	–	34	20	16
+	Changes in deferred taxes	+	4*	16*	?
=	Net Operating Profit	=	90	81	?
+	Depreciation	+	25	23	20†
=	Gross Cash Flow	=	115	104	?
–	Increase in Working Capital	–	10*	5*	?
–	Capital Expenditures	–	0	30	200
=	Free Cash Flow from Operations	=	105	69	?

* Note that the balance sheet gave the level of deferred taxes and the level of working capital, not the *changes* in these variables. You had to compute the differences yourself. † Depreciation is only available from the cash flow statement, not from the balance sheet.

16. Use Formulas 9.18 and 9.19. PepsiCo's project cash flows, available for satisfaction of both creditors and shareholders, are

	Cash Flow from Operating Activity	4,201	4,440	3,605
+	Cash Flow from Investing Activity	–2,637	–1,996	–1,172
+	Interest Expense	+ (–8)	57	(–792)
=	Cash Flow From Projects	1,556	2,501	1,641

PepsiCo's shareholder cash flows are

	Cash Flow from Operating Activity	4,201	4,440	3,605
+	Cash Flow from Investing Activity	-2,637	-1,996	-1,172
+	Net Issuance of Debt	-341	-705	391
=	Cash Flow To Equity	1,223	1,739	2,824

17. For reference, in 2001, the cash flow statement reports depreciation of +\$803, and capital expenditures of −\$769. Our first formula 9.12 was net income plus depreciation minus corporate income tax, or \$3,969+\$803-\$769=\$4,003. Looking at the actual Coca Cola 2001 cash flow statement on Page 219, our calculation omits deferred taxes (+\$56), non-cash items (-\$256), changes in working capital (-\$462), and "investments" of −\$418 − \$1. In total, our first formula 9.12 therefore would have omitted \$1,081. Our second formula 9.17 would have captured at least changes in working capital, for an error reduction of \$462, and a total error of \$619.

18. Economic project cash flows are operating activity cash flows plus investing activitiy cash flows plus interest:

	1999	2000	2001
Operating Activity	3,883	3,585	4,110
Investing Activity	-3,421	-1,165	-1,188
Interest Paid	337	447	289
Economic Project Cash Flows	799	2,867	3,211

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

However bad my answers to exercises in earlier chapters may have been, the solutions in this chapter are probably worse. I am notoriously bad when it comes to keeping the correct signs. I have no future as an accountant! Before this chapter is formally finished, this section will be quadruply checked.

CHAPTER 10

Valuation From Comparables

A Practical Approach

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You now know how to read financial statements, how to obtain cash flows from financial statements, and how to value them. You also know that forecasting cash flows is a very difficult task. Are there any shortcuts? Are there any good alternatives to NPV? Is there anything else you can do with financial statements?

Surprisingly, the answer is yes. There is one alternative approach often resorted to by practitioners. It is called "valuation by comparables," or "comps" for short. Executed correctly, comps can give answers that are as good as those you can obtain with a thorough NPV analysis—though they are not always the same. In practice, sometimes the NPV study gives a better value estimate, and sometimes the method of comparables does.

The basic idea behind valuation by comparables is simple and best understood by analogy: assume that you want to determine the value of 5 red marbles. If black marbles cost $2 a piece, and if you are willing to make the assumption that red marbles are valued like black marbles, then you can compute that the value of your 5 red marbles should be $10. It is not necessary to forecast what value marbles will have in the future or what discount factor to apply: the market price of black marbles has already taken this information into account.

Of course, the more similar black marbles are to red marbles, the better this method will work. (If black marbles are made from coal and red marbles are made from rubies, you will undervalue your red marbles!) The method of comparables therefore assumes that public markets already value comparable firms appropriately—or at least that they misvalue your firm the same way that they misvalue other firms—so that the value of your firm or your new project can be assessed in terms of its similarity to comparable firms.

10·1. Comparables vs. NPV

NPV is also a comparable in a sense.

The idea of comparables is the same as that of NPV—both are attempts to value your project *relative* to other projects that are available. In NPV analysis, you compare your own project to another-project benchmark through the opportunity cost of capital. NPV also tells you exactly what matters—future cash flows—and how you should weight different cash flows relative to one another. In theory, you cannot do better than NPV. But in practice, it is so very difficult to estimate these future cash flows. There is also no objective standard when you are measuring the future. If you say the expected cash flows in 10 years are \$1 million, and I say that they are \$5 million, which one is right?

Comparables proxy for future cash flows with an available measure today.

Comparables try to measure project similarity not through estimates of future cash flows, but through something that is available *today*. For example, if firms with similar earnings also have similar future cash flows (or at least similar present values of all future cash flows), then earnings are a good proxy for what you really want—plus we can objectively agree on what these earnings are today. In using today's earnings instead of a full estimate of future cash flows, we therefore trade off your judgmental uncertainty about the future cash flow against your judgmental uncertainty about how good your current earnings approximate the future cash flow stream.

When NPV will work better and when comparables will work better.

Both NPV and comparables are based on relative valuation, but comparables lean more heavily on immediately similar projects and the assumption that the market has valued these correctly. NPV is a bit more forgiving, in that the opportunity cost of capital uses a broader swath of alternatives than just a couple of similar-looking firms in an industry. Consequently, NPV makes it easier to compare an investment in PepsiCo to, say, an investment in Treasuries and the stock market. Here, the method of comparables would fail, because these alternatives seem so dissimilar to PepsiCo that you would have no faith in a comparables-derived estimates. In contrast, comparables make it easier to compare an investment in PepsiCo to, say, an investment in Coca Cola. With similar characteristics, you can reasonably assume that you can rely on the financial markets having gotten Coca Cola's valuation based on future cash flows and future discount factors right, so you are in effect free-riding on a wonderfully accurate valuation already provided for you by the financial markets.

Solve Now!

Q 10.1 *When negotiating, would you value your next residence by the method of comparables, or by the method of NPV? If comparables, what kind of ratio might you use?*

10·2. The Price-Earnings (PE) Ratio

Why not Price Cash Flow Ratio?

The kind of ratios that you would be most interested in have value in their numerators—for example, the price-earnings ratio. The reason is that if you obtain a good estimate for what a reasonable price-earnings ratio is, you merely need to multiply the proper ratio by your project's or firm's earnings, and out comes an estimate of price,

$$\underbrace{\left(\frac{\text{Price}}{X}\right)}_{\text{from comparables}} \cdot X_{\text{our project}} \Rightarrow \text{Price Estimate for your Project} \;. \tag{10.1}$$

The most important such ratio is the price-earnings ratio. The reason is that earnings are often seen as best representatives of future cash flows. At first glance, this may seem odd to you. After an entire chapter on how to compute cash flows in order to avoid net income, why compute a price-earnings ratio, rather than a price-cash flow ratio? The reason is that cash flows are usually more "spiky" than earnings. When a firm makes a large capital expenditure or acquisition, it may have a large negative cash flow one year, followed by positive cash flows in the following years. This is not a problem in an NPV analysis, because the higher future cash flows will also enter in the future terms. But, for computing a representative ratio with just one year's information, the current accounting earnings are probably more representative than a current cash flow would be. After all, earnings try to smooth inflows and outflows of large expenditures over many periods. It is a number which accountants have created for the very purpose which we need here: a representative number for a "long-term" picture.

10·2.A. Definition

The price-earnings ratio is price divided by (expected or current) earnings, either per-share or overall.

The **price-earnings ratio** is commonly abbreviated as **P-E ratio**, **P/E ratio**, or **PE Ratio.** The P/E ratio divides the overall firm market value—its market price—by the income flow (earnings) the firm generates. (Later in our chapter, we shall discuss some other ratios—and it will then become clear why the P/E ratio is the most popular comparables measure.) In the real world, price-earnings ratios are often *but not always* quoted as the current market price divided by the *expected* earnings for the *next* year (as determined by a consensus among analysts). The advantage is that expected earnings focus more on the future than on the most recently reported earnings. This suits us well, because valuation is forward-looking, not backward looking. Incidentally, in much of this chapter, we use the growing perpetuity formula 3.13 (on Page 38), which already calls for next year's earnings, anyway. In any case, the intuition would remain the same if you used the most recently reported earnings. Therefore, this chapter keeps the language a bit loose.

Firm-value and price-per-share based ratios are the same.

Let us start by exploring the meaning of the P/E ratio with an example. A firm with a market value of $\mathbf{P}$ = \$200 million and expected earnings of $\mathbf{E}$ = \$10 million next year would have a price-earnings ratio of 20. Both inputs could be expressed in per-share terms, rather than in aggregate value. So, if this firm has 40 million shares outstanding, each share would be worth $\mathbf{P}$ = \$5 and produce earnings of $\mathbf{E}$ = \$0.25. The price-earnings ratio would still be 20.

Our first comparables valuation.

We now want to value a project that is quite similar to this firm and has earnings of $\mathbf{E}$ = \$3 per share. What should your project's per-share value be?

$$\begin{aligned} \text{Price Estimate} &= \text{Comparable Price Earnings Ratio} \cdot \text{Our Earnings} \\ &= 20 \cdot \$3 = \$60 \;. \end{aligned} \tag{10.2}$$

Other interpretations.

One way to look at the price-earnings ratio is that it attaches an implicit overall value to each dollar of earnings. In this case, each extra dollar of earnings translates into an extra \$20 worth of valuation—the shares sell for twenty times earnings. Sometimes you should use the reciprocal of the P/E ratio, the **earnings yield**,

$$\text{Earnings Yield} = \frac{\text{(Expected) Earnings}}{\text{Price}} = \frac{1}{\text{P/E Ratio}} \quad . \tag{10.3}$$

You can view the earnings-yield as telling you the percentage of price that is due to current earnings. In our example, the earnings yield would be 10/200 = 5%. If the earnings are zero or negative, the price-earnings ratio is meaningless, and often indicated as NA or N/A. In contrast, the earnings-price ratio (earnings yield) can reasonably be negative and meaningful. If the earnings are positive, then a higher price-earnings ratio implies a lower earnings-price yield and vice-versa.

10·2.B. Why P/E Ratios differ

Our first goal in constructing a valuation by comparables is to determine where price-earnings ratios come from, and what they say about the firm. The main reason why P/E Ratios differ across firms and industries is that expectations differ as to how today's earnings relate to future cash flows—the expected growth rate. If you believe that today's earnings will be the last, your value estimate per dollar of current earnings should be lower than if you believe that it is a very low proxy of much better times to come.

> **IMPORTANT**: *The price-earnings ratio is higher when the firm has more growth opportunities.*

There is also an influence of leverage here—firms that are more levered have lower price-earnings ratios, a topic that we will cover in Section 10·3.E.

Direct Differences in Earnings Growth

Determining a sensible price-earnings ratio for a hypothetical firm, which is a simple growing perpetuity.

We can illustrate this in a "traditional" growing perpetuity framework. Assume that a firm—call it A—is expected to earn cash of \$100 next year, and its appropriate cost of capital is 15%. This firm is a perpetuity whose income will grow by 5% per annum forever. The growing perpetuity formula 3.13 on Page 38 states that the value of this firm is

$$\begin{aligned} V_A &= \frac{\$100}{15\% - 5\%} = \$1,000 \\ \text{Value}_A \approx \mathbf{P}_A &= \frac{\text{Cash Flow}_A \approx \mathbf{E}_A}{\text{Interest Rate}_A - \text{Growth Rate}_A} \quad . \end{aligned} \tag{10.4}$$

With a price of \$1,000 and expected earnings of \$100, A's price divided by its expected earnings is

$$\begin{aligned} \frac{\mathbf{P}_A}{\mathbf{E}_A} &= \frac{\$1,000}{\$100} = 10 \\ \frac{\mathbf{P}_A}{\mathbf{E}_A} &= \frac{\left[\frac{\mathbf{E}_A}{\mathcal{E}(r_A)-\mathcal{E}(g_A)}\right]}{\mathbf{E}_A} = \frac{1}{\mathcal{E}(r_A) - \mathcal{E}(g_A)} \quad . \end{aligned} \tag{10.5}$$

Faster growing firms have higher price-earnings ratios.

What if the firm grew not by 5% but by 10% per year (forever)? Then the price earnings ratio would be

$$\begin{aligned} \frac{\mathbf{P}_A}{\mathbf{E}_A} &= \frac{1}{15\% - 10\%} = 20 \quad . \\ &= \frac{1}{\mathcal{E}(r_A) - \mathcal{E}(g_A)} \quad . \end{aligned} \tag{10.6}$$

The P/E ratio is higher. So, high price-earnings ratios are a reflection of the market's expectation about how fast the firm will grow relative to its cost of capital.

Slower growing firms have lower price-earnings ratios.

What if the market expected this firm to shrink by 5% each year? Such a firm would have a price-earnings ratio of only

$$\frac{P_A}{E_A} = \frac{1}{15\% - (-5\%)} = 5 \quad . \tag{10.7}$$

Cigarette producers, for example, may suffer from negative annual growth rates and as a result have low price-earnings ratios. In May 2002, RJR Nabisco and Philip Morris (now Altria) had P/E ratios of about 12. Contrast this with high-growth firms, such as AMGEN (a high-tech pharmaceutical), which had a P/E ratio of about 40 and Microsoft, which had a P/E ratio of about 45.

The growth rate of earnings is not the expected rate of return to investors.

Do you find it confusing that the earnings can grow by 5%, but investors expect to receive 15% rate of return? Shouldn't investors' expected rate of return be the growth rate of earnings? No—not at all. (Indeed, the expected rate of return ($\mathcal{E}(r)$) cannot be equal to the growth rate of earnings (g), or the NPV would be infinite.) The reason is that the price today already capitalizes all future earnings. Although we have already discussed why there is no direct link between earnings and rates of return (in Section 3·1.B), this is so important it deserves another example. Say that the appropriate cost of capital for a firm is 10%, and it will produce \$100 next year, \$50 the next year, and \$0 thereafter. There is no uncertainty. Clearly, the cashflows/earnings of the firm are shrinking dramatically. But the value of the firm today is $\$100/1.1 + \$50/1.1^2 \approx \$132.23$. Next year, the investor will receive \$100 and hold a remaining project of $\$50/1.1^1 \approx \$45.45$, for a total wealth of \$145.45. The (expected) rate of r is $\$145.45/\$132.23 - 1 = 10\%$, even though the growth rate of earnings is negative.

The Present Value of Growth Opportunities (PVGO)

PVGO (Present Value of Growth Opportunities).

Another way to express the same information—to give perspective about the meaning of P/E ratios—comes from decomposing a firm into two components: the ratio of one hypothetical firm that has the projected earnings of the company, but has stopped growing; and the ratio of another hypothetical firm that has zero earnings right now and consists just of the projected growth opportunities. The latter part is called the **Present Value of Growth Opportunities** (**PVGO**). You can split the market value of any company—regardless of its actual earnings—into these two components.

An example split of firms' earnings into "steady" and "PVGO."

For example, consider three firms, all priced at \$150 and all with an appropriate cost of capital of 10%. The first firm has expected earnings of \$15, the second firm has expected earnings of \$12, and the third firm has expected earnings of \$20. We will decompose each firm's value into the two components.

Stability: The first firm is worth

$$\begin{aligned} \$150 &= \frac{\$15}{10\%} + \quad ? \quad = \$150 + ? \\ \mathbf{P} &= \frac{\mathbf{E}}{r} + \text{PVGO} \quad . \end{aligned} \tag{10.8}$$

To be an equality, the question mark must stand for \$0. The market has priced this first firm exactly as if it had no expectation of any future growth. Thus, 100% of this firm's value comes from the "steady component," and 0% from the "growth component." Eventually, in the very long-run, you would expect mature and stable companies to settle into this mode.

Growth: In contrast, if the second firm, also trading at \$150, earned only a constant \$12 forever, its constant growth component would only be worth \$120,

$$\begin{aligned} \$150 &= \frac{\$12}{10\%} + \quad ? \quad = \$120 + ? \\ \mathbf{P} &= \frac{\mathbf{E}}{r} + \text{PVGO} \quad . \end{aligned} \tag{10.9}$$

Therefore, this firm's "steady component" is worth \$120, and its growth opportunities must be worth PVGO = \$30. Taking this further, we would say that \$30/\$150 = 20% of the firm's value is due to future growth opportunities above and beyond a steady business.

Decline: Finally, if the third firm were expected to earn a constant \$15 forever, it should have been worth \$150 today. To justify its market value of \$150, you must believe that it will have negative growth in the future,

$$\begin{array}{ccccccc} \$150 & = & \dfrac{\$20}{10\%} & + & ? & = \$200 + ? \\ \mathbf{P} & = & \dfrac{\mathbf{E}}{r} & + & \text{PVGO} & , \end{array} \tag{10.10}$$

specifically, a subtractive part worth PVGO = −\$50. This firm would not maintain a steady business.

Table 10.1. Various E/P Ratios in Early November 2004

	P/E	$\mathcal{E}(r)$	PVGO/P	PEG		P/E	$\mathcal{E}(r)$	PVGO/P	PEG
Google	50	10%	80%	2.2	Coca Cola	20	6%	20%	2
Pixar	45	8%	72%	2	Exxon	15	7%	5%	2
Cisco	20	12%	60%	1.4	Procter&Gamble	19	5%	0%	1.8
PepsiCo	20	10%	50%	2	Altria (P.Morris)	12	6%	−40%	1.3
Microsoft	21	8%	40%	2	GM	8	9%	−40%	1.2
Home Depot	17	9%	35%	1.3	U.S. Steel	6	11%	−50%	0.5
Boeing	20	7%	30%	2	Ford Motor	7	9%	−60%	1.2
Wal-Mart	21	7%	30%	1.5	RJR Nabisco	10	6%	−65%	1.5

All inputs are from Yahoo!*Finance*. No attempt has been made to adjust for debt ratio. The ratio PVGO/**P** = $1 - 1/[\mathcal{E}(r) \cdot \text{P/E ratio}]$. P/E ratios are forward-looking. The cost of capital estimate is rough, and computed as $5\% + 3\% \cdot \beta$ (except Google, which I made up). The cost of capital is the subject of the next part of the book and comes with a good deal of uncertainty. Therefore, PVGO/P is intentionally heavily rounded. PEG ratios are quoted directly from Yahoo!*Finance* as based on 5-yr expected earnings. It divides the P/E ratios by analysts' expected growth rate of earnings.

A sample of firms.

Table 10.1 computes the PVGO as a fraction of firm value from the firm's P/E ratio and an estimate of the cost of capital (the subject of Part III). Apparently, the market believes that the future lies with Google (\$44 billion in market cap) and Pixar (\$5 billion), and not with U.S. Steel (\$5 billion) or Ford Motor (\$25 billion). The table also gives another popular ratio, the PEG ratio, which divides the P/E ratios by analysts' expected growth rate of earnings. It combines information about $\mathcal{E}(g)$ and P/E ratio, thus trying to say something about $\mathcal{E}(r)$. The idea is that stocks with lower P/E ratios and higher growth rates have lower PEG-ratios—and are perhaps better buys. (I do not know how well or how poorly this measure works for this purpose.)

The E/P yield is the interest rate plus the normalized present value of growth opportunities

You can also rearrange Formula 10.8 to get a relationship between a firm's P/E ratio and its cost of capital

$$\mathbf{P} = \frac{\mathbf{E}}{r} + \text{PVGO} \quad \Leftrightarrow \quad \frac{\mathbf{P}}{\mathbf{E}} = \frac{1}{r} + \frac{\text{PVGO}}{\mathbf{E}} \quad \Leftrightarrow \quad \frac{\mathbf{E}}{\mathbf{P}} = r \cdot \left(1 - \frac{\text{PVGO}}{\mathbf{P}}\right) . \tag{10.11}$$

The formula states that a stable company without any growth opportunities ($g = 0 \Rightarrow \text{PVGO} = 0$) has an earnings-price yield equal to its cost of capital, r. A growing firm ($g > 0 \Rightarrow \text{PVGO} > 0$) has an earnings-price yield lower than its cost of capital. And a dying firm (PVGO < 0) has an earnings-price yield higher than its cost of capital.

Empirical Evidence: P/E Ratios and Growth Rates

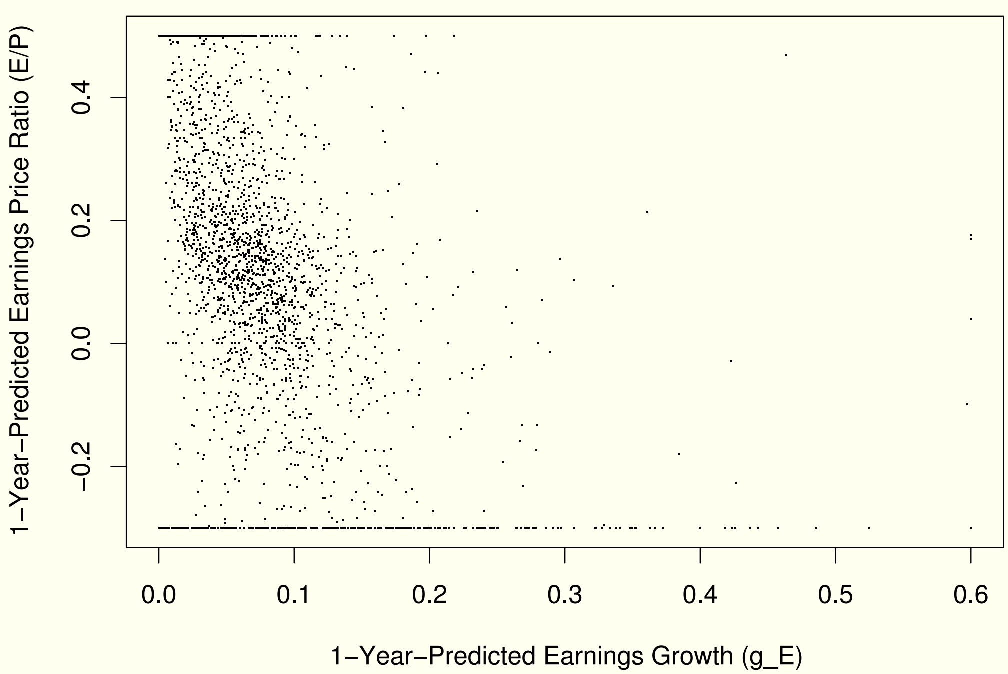
Do high growth firms in the real-world have higher P/E ratios?

The P/E ratio theory works nicely on paper, but does it hold water in the real world? The implication of the theory is that if you plot long-term expected earnings growth (g_E) against E/P, you should get a negative relation.

$$\mathbf{P} = \frac{\mathbf{E}}{r - g_E} \iff \frac{\mathbf{E}}{\mathbf{P}} = r - g_E \ . \tag{10.12}$$

Unfortunately, to make the theory operationally useful, you have to make two more assumptions. First, you do not know the eternal growth rate: you only know the most recent earnings ($\mathbf{E}_0$) and the earnings predicted by analysts for next year (call them $\hat{\mathbf{E}}_{+1}$), so you can only compute the expected growth rate for one year: $\mathcal{E}(g_E) = (\hat{\mathbf{E}}_{+1} - \mathbf{E}_0)/\mathbf{E}_0$. You have to make the leap that firms with higher short-term growth rates also have higher long-term growth rates, so you can use the former as a stand-in for the latter. Second, each firm may have its own costs of capital (unequal r). If firms with high growth rates g_E's also have sufficiently high costs of capital r's, then you might not even be able to see any relationship between earnings yield and earnings growth. Both of these problems could conceivably scramble any negative expected relationship between the earnings yield and the growth rate of earnings. Thus, you need to look at the empirical evidence to determine how practically useful the theory is.

Figure 10.1. Relation between 1-Year Predicted Earnings-Growth Rates and 1-Year Predicted Earnings-Price Yields, as of December 2000.



Analysts' consensus earnings forecasts were obtained from **I/B/E/S**.

DIGGING DEEPER: *To reduce the influence of some extremely unusual firms, a few firms with* E/P yield*s in excess of 100% were truncated ("winsorized") at 100%. More importantly, earnings growth (g_E, the independent variable on the x-axis) was truncated at −30% and +50%, again to reduce the influence of extreme observations. Firms with such negative growth rates and such high positive growth rates are sufficiently unusual that pretending that they had just −30% or +50% growth is reasonable. Indeed, it is particularly unreasonable to pretend that negative earnings growth can last forever, so you would not expect a good relationship between price-earnings ratios and earnings growth for contracting firms. Finally, firms with lagged negative earnings (not earnings growth rates!) had to be ignored, too, because it is impossible to compute a meaningful earnings growth rate when the denominator is negative.*

Some detail information about the figure.

Figure 10.1 plots the predicted next-year earnings-growth rate against the earnings yield (the ratio of predicted earnings over today's stock price), for firms with market capitalization of $50 million or more, as of December 2000. Each dot is one firm.

The evidence supports the theory: high-growth firms have lower E/P (and thus higher P/E) ratios.

The figure shows how firms with higher earnings growth rates had lower earnings-yields (higher price-earnings ratios), just as the theory had predicted. Eyeballing the figure, you can see that firms that are neither growing nor contracting tend to have an earnings price ratio of, say, about 8% ($P/E \approx 12$), firms growing by 20% tend to have a lower earnings price ratio of, say, about 5% ($P/E \approx 20$), and firms growing by about 40% tend to have an even lower earnings price ratio of, say, about 2.5% ($P/E \approx 40$).

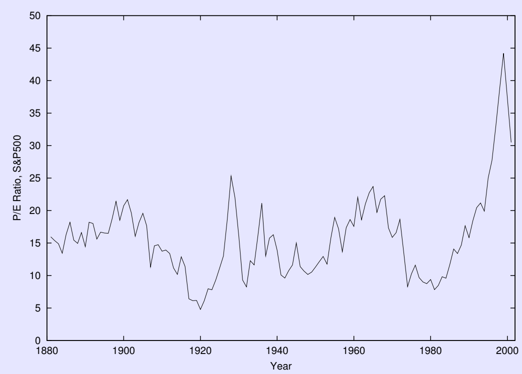
Using the figure to estimate a comparables-based firm value, as of December 2000.

If you had been hired in December 2000 to assess the value of a privately held firm for which you only knew the earnings, Figure 10.1 would have been very useful. For example, if this firm had earnings of $10 million, and was expected to grow them to $12 million by December 2001, the figure would have indicated that this 20% earnings growth rate on average would have translated into likely E/P yields between about 2% and 10%, with 5% being perhaps the best number. Therefore, reasonable value estimates for this company might have been somewhere between $50 \cdot \$12$ million $\approx$ $600 million and $10 \cdot \$12$ million $\approx$ $120 million, with $20 \cdot \$12$ million $\approx$ $240 million being a decent average estimate.

The figure will change, so you must use a current equivalent for valuation.

Unfortunately, you cannot use the December figure to assess appropriate P/E ratios today. The reason is that during economic booms, earnings growth is high, and, although P/E ratios are high, too, they do not seem high enough; after all, such earnings growth is unsustainable. During recessions, earnings growth can be negative. But P/E ratios remain relatively too high, because investors expect that earnings-growth will eventually improve again. For example, in December 2000, corporate earnings growth was running at an average rate of +40%, and therefore unsustainable. Naturally, if such an earnings growth rate could be sustained forever, the price over today's earnings ratio would have to be truly astronomical! By December 2001, i.e., post 9/11, the opposite had happened: the typical current earnings growth rate had dropped to −40%, and yet firms were still worth something! So, it is an important point that the relation between earnings growth and earnings-price yields, using only one-year-ahead earnings forecasts, does not hold over time. Do not use Figure 10.1 to estimate P/E ratios from earnings growth *today*! Instead, if you need to value a firm based on its current growth rates *today*, recreate the graph and implied E/P yield with data as of today.

<u>ANECDOTE</u>: Shiller's Irrational Exuberance



This figure shows the history of the price-earnings ratio for the S&P500. (The S&P500 index contains the 500 largest publicly traded firms in the United States.)

Many experts believe that the expected rate of return on the stock market has not changed dramatically in the last 100 years. In 1999, stock market bulls also believed that the expected rate of return r in the stock market would be <u>at least</u> 10% per year *in real terms (inflation-adjusted)* forever. Therefore, Formula 10.12 tells you that when the stock market's P/E ratio reached 44 at the end of 1999, such optimists must have believed in earnings growth $g_{\mathbf{E}} = r - \mathbf{E}/\mathbf{P} = 10\% - 1/44 \approx 7.5\%$ per year, *in real terms* and *forever*. This is truly an astronomical figure, and should have been tough to swallow. A forecast of GDP growth based on its historical long-run average puts its *real* growth rate at only about 2.5% per year. Put differently, at the historical 4.5% *real* growth rate, the P/E ratio of 44 could have only implied an expected *real* rate of return in the stock market of $2.5\% + 1/44 \approx 5\%$ per year.

This argument—that either the stock market's $\mathbf{P}/\mathbf{E}$ multiple or popular stock return expectations were out of line with reasonable earnings growth estimates (and thus that the stock market was overvalued)—was most forcefully advanced in Robert Shiller's bestseller *Irrational Exuberance*. It was published just before the stock market peaked in 2000. (The plotted data were obtained from Shiller's website.)

10·2.C. P/E Ratio Application Example: Valuing Beverage Companies

Table 10.2. Financial Newspaper Printed Financials, from May 31, 2002

YTD %CHG	52-Week HI	52-Week LO	STOCK (SYM)	DIV	YLD %	P/E	VOL 100s	CLOSE	NET CHG
13.5	31.91	23.55	Cadbury Schweppes (CSG)	.70g	2.4	21	475	29.20	-0.20
15.4	57.91	42.59	Coca Cola (KO)	.80	1.5	35	47,565	54.39	0.24
4.6	53.50	43.08	PepsiCo (PEP)	.60f	1.2	34	26,539	50.93	0.00

The description of the table states that the P/E ratio is based on the closing price and on diluted per-share earnings ignoring extraordinary items, as available, for the most recent four quarters. Fully diluted earnings means that all common stock equivalents (convertible bonds, preferred stock, warrants, and rights) have been included.

You now apply the P/E ratio valuation method to PepsiCo. You can use common newspaper information.

Let us now apply our newly found comparables valuation technique. Table 10.2 reproduces the stock price report from May 31, 2002, in the same format as that of a prominent financial newspaper. (Actually, the most convenient source of financial information on individual stocks may no longer be the newspaper. The World-Wide-Web, such as Yahoo!*Finance* makes it even easier to find more comprehensive financial information.) From Table 10.2, you can see that the price-earnings ratio for Coca Cola was 35, for PepsiCo 34, and for Cadbury Schweppes 21. The (day's closing) price-per-share for Coca Cola was \$54.39, for PepsiCo \$50.93, and for Cadbury Schweppes \$29.20. Using this information, you can back out Coca Cola's earnings-per-share as

$$\frac{\$54.39}{\mathbf{E}_{\text{KO}}} = 35 \quad \Longrightarrow \quad \mathbf{E}_{\text{KO}} = \left(\frac{\$54.39}{35}\right) \approx \$1.55 \ . \tag{10.13}$$

Task: Value PepsiCo now.

We now proceed with the valuation-by-comparables method, once again using PepsiCo as our guinea pig. Pretend that you do not know PepsiCo's value, but you do know PepsiCo's internal financials (earnings). Your task is to value the shares of PepsiCo in light of the value of shares of Coca Cola. If the P/E comparables valuation method works, you can then check whether your estimated value roughly fits the true value of PepsiCo.

Applying Coca Cola's P/E ratio to value PepsiCo, given PepsiCo's earnings.

So, can you really use Coca Cola as a comparable company for PepsiCo? To do so requires making the heroic assumption that Coca Cola is a company quite similar to PepsiCo in terms of ratios and earnings. (The choice of comparable is discussed in general below.) We are indeed heroes (and heroines), so we assume that Coca Cola's P/E ratio of 35 can be applied to PepsiCo earnings of \$50.93/34 ≈ \$1.50 per share,

$$\begin{aligned} \frac{\mathbf{P}_{\text{PEP}}}{\$1.50} &= 35 \quad \Rightarrow \quad \mathbf{P}_{\text{PEP}} = 35 \cdot \$1.50 = \$52.50 \\ \left(\frac{\mathbf{P}_{\text{PEP}}}{\mathbf{E}_{\text{PEP}}}\right) &= \left(\frac{\mathbf{P}_{\text{KO}}}{\mathbf{E}_{\text{KO}}}\right) \ . \end{aligned} \tag{10.14}$$

In PepsiCo's case, valuation-by-comparables against Coca Cola seems to work well.

The valuation-by-comparables method suggests that PepsiCo should be worth \$52.50. This is higher than the \$50.93 that PepsiCo shares are currently trading for, but a difference of \$2 (about 5%) is well within the range of typical valuation uncertainty. So, here the method of comparables works quite well in predicting a correct market value for PepsiCo.

In Cadbury Schweppes's case, valuation-by-comparables against either PepsiCo or Coca Cola does not work well.

Now, let us assume that you instead owned Cadbury Schweppes (CSG), that it was not yet publicly traded, and that it had just earned \$1.39 per share. (This can be inferred from CSG's

P/E ratio of 21 and closing price of \$29.20.) Applying the Coca Cola P/E ratio of 35 to Cadbury Schweppes' earnings, you would expect CSG to trade for

$$\frac{P_{CSG}}{\$1.39} = 35 \quad \Rightarrow \quad P_{CSG} = 35 \cdot \$1.39 = \$48.67 \; . \qquad (10.15)$$
$$\left(\frac{P_{CSG}}{E_{CSG}}\right) = \left(\frac{P_{KO}}{E_{KO}}\right) \; ,$$

You would be far off! The value of Cadbury Schweppes shares in the public markets is \$29.20 per share, not \$48.67 per share. In effect, the method of comparables has not worked well in predicting the correct market value for shares in Cadbury Schweppes. Our next section will be all about why and when P/E Ratios can break down.

Solve Now!

Q 10.2 *Which is likely to have a higher price-earnings ratio: Microsoft or ConAgra?*

Q 10.3 *A firm has earnings of \$230 this year, grows by about 6% each year, and has a price-earnings ratio of 40. What would its price-earnings ratio be if it could grow by 7% each year instead? How much would its value increase?*

Q 10.4 *A firm has earnings of \$200, and a price-earnings ratio of 20. What is its implied growth rate, if its cost of capital is about 10%?*

10·3. Problems With P/E Ratios

If comparables are dissimilar, either the market is wrong or the comparable is wrong.

So, what went wrong in the Cadbury Schweppes valuation? There are basically two possible explanations. The first explanation is that the stock market valuations—either of CSG or KO, or both—are just plain wrong. In this case, it makes little sense to use the methods of comparables. But this scenario is unlikely. If the market values were systematically wrong, you could presumably easily get rich if you purchased undervalued firms. If it is not obvious, Chapter 15 will explain why getting rich is not easy—and which is why only about half of all investors beat the market—so we will assume that misvaluation is not the principal reason. The second explanation is that your assumption that the two firms were basically alike is incorrect. This is the more likely cause. There is a long litany of reasons why comparables are not really comparable, and why the technique failed you in valuing Cadbury Schweppes. Here is an outline of possible problems, on which the remainder of this chapter focuses:

Problems in Selecting Comparable Firms Comparing businesses is almost always problematic. Every firm is a unique combination of many different projects. Cadbury Schweppes owns Dr. Pepper, 7-Up, A&W Root Beer, Canada Dry, Hawaiian Punch, Snapple, Mott's Apple products, Clamato juice, plus some confectionary brands. This may not be comparable to Coca Cola, which owns Coca Cola Bottling, Minute Maid, Odwalla, and some other drink companies. Each of these businesses has its own profitability and each may deserve its own P/E ratio.

Even for the main business, as any soda connoisseur knows, Pepsi Cola and Coca Cola are not perfect substitutes. Different consumer tastes may cause different growth rates, especially in different countries.

Selection of comparable firms will be discussed in Subsection 10·3.A. P/E ratio aggregation issues with multi-division firms will be covered in Subsection 10·3.B. The worst kind of averaging P/E ratios will be examined in Subsection 10·3.C.

Problems in Comparing Accounting Numbers Not all accounting statements are prepared the same way. Here are a few possible discrepancies in regard to the Cadbury Schweppes valuation:

- Maybe as a British firm, Cadbury Schweppes uses altogether different accounting methods.
- Maybe Cadbury Schweppes has just had an unusual year, or a year in which it plowed most of its cash into advertising, thereby causing unusually lower earnings for now and much higher earnings in the future.
- If you did not use earnings from the most recent four quarters, but instead forecast earnings over the next four quarters, maybe the numbers would then be more comparable. How to adjust better for differences in the timing of reports will be the subject of Subsection 10·3.D.
- Maybe Cadbury Schweppes and Coca Cola have different debt ratios. The influence of debt on P/E ratios will be explained in Subsection 10·3.E.
- Maybe extraordinary items (excluded from this measure of earnings) should be included to make these firms more comparable. The use of other financial ratios will be discussed in Section 10·4.

10·3.A. Selection of Comparison Firms

Finding comparables: general criteria for evaluating similarity.

The single biggest problem with comparables may be the selection of appropriately comparable firms. Say, you own a little soda producer, the *Your Beverage Corporation* (YBC), with earnings of $10 million. You want to select public firms to use as your comparables from the universe of firms. Usually, this means publicly traded companies. So, which of the 10,000 or so publicly traded companies are most comparable to your firm (or project)? Are firms more similar if they are similar in assets, similar in their business products and services, similar in their geographical coverage, similar in their age, similar in their size and scale, etc.? Do they have to be similar in all respects? If so, chances are that not a single of the 10,000 firms will qualify!

Which firm is the single best comparable?

Let us assume that after extensive research and much agonizing, you have identified the (same) three companies: KO, PEP, and CSG. Which one is most similar? You know that depending on which firm you select, your valuation could be $250 million (if Cadbury Schweppes, unlevered, was most similar), $410 million (if PepsiCo was most similar), or $500 million (if Coca Cola was most similar). Which shall it be?

Different conclusions about the value of the same firm: Analyst errors and biases can create wide variations in valuations.

Selecting comparables depends both on the judgment and on the motives of the analyst. In the YBC case, one analyst may consider all three firms (KO, PEP, and CSG) to be similar, but CSG to be most similar because it is the smallest comparison firm. She may determine a good P/E ratio would be 30. Another analyst might consider Coca Cola and Pepsi-Co to be better comparables, because they tend to serve the same market as YBC. He may determine a good P/E ratio would be 40. The owner of YBC may want to sell out and try to find a buyer willing to pay as much as possible, so she might claim Coca Cola to be the only true comparable, leading to a P/E ratio of 50. The potential buyer of YBC may instead claim Cadbury Schweppes to be the only comparable, and in fact attribute an extra discount to CSG: after all, YBC is a lot smaller than CSG, and the buyer may feel that YBC deserves only a P/E ratio of 10. There is no definitive right or wrong choice.

Incorrect, but practical averaging. Let's pray!

Another choice may be not to select either the P/E ratio of 10 or the P/E ratio of 45, but to "split the difference." A reasonable P/E ratio that is better than either 10 or 45 may be 30. This might mean valuations of around $300 to $400 million. Unfortunately, though this may be the best solution, it is not a good solution. The next two section shows you why averaging P/E ratios is really not a good procedure.

10·3.B. (Non-) Aggregation of Comparables

Companies are collections of many projects. Is the P/E ratio of the company the same as the weighted average P/E ratio of its subsidiaries, so that you can seamlessly work with either individual subsidiary P/E ratios or with overall company P/E ratios? Unfortunately, the answer is no.

Can you aggregate (take averages of) P/E ratios? No!

Consider two firms. A has a $g_E = 5\%$ growth rate and earnings of \$100 (next year); and B has a $g_E = 14\%$ growth rate and earnings of \$50 (next year). Both have a 15% cost of capital. Their respective values should be

The average of individual P/E ratios is not the overall P/E ratio

$$\begin{aligned} \mathbf{P}_A \equiv V_A &= \frac{\$100}{15\% - 5\%} = \$1,000 \quad \Longrightarrow \quad \mathbf{P}/\mathbf{E} = 10 \quad , \\ \mathbf{P}_B \equiv V_B &= \frac{\$50}{15\% - 14\%} = \$5,000 \quad \Longrightarrow \quad \mathbf{P}/\mathbf{E} = 100 \quad . \end{aligned} \tag{10.16}$$

What would happen if these two firms merged into a single conglomerate, called AB? Assume AB does not operate any differently—the two firms would just report their financials jointly. AB must be worth \$6,000—after all, nothing has changed, and you know that NPVs are additive. It would have earnings of \$150. Thus, its P/E ratio would be $\$6,000/\$150 = 40$.

$$\text{Correct But Unknown AB P/E ratio:} \quad \frac{\mathbf{P}_{AB}}{\mathbf{E}_{AB}} = 40 \quad \Longrightarrow \quad \mathbf{P}_{AB} = 40 \cdot \mathbf{E}_{AB} \quad . \tag{10.17}$$

Your goal is to value AB. Fortunately, you just happen to know a perfectly comparable firm for division A (trading at about $\mathbf{P}/\mathbf{E} = 10$), and a perfectly comparable firm for division B (trading at about $\mathbf{P}/\mathbf{E} = 100$). You even have a good idea of the relative size of the divisions inside AB (5:1). Knowing the combined earnings of AB of \$150, you want to estimate a value for AB, based on your two comparables. Unfortunately, neither the unweighted average P/E ratio nor the weighted average P/E ratio gives you the correct desired P/E ratio of 40:

$$\begin{aligned} &\text{Unweighted P/E ratio Average of A and B} \quad \left(\frac{1}{2}\right) \cdot \left(\frac{\mathbf{P}_A}{\mathbf{E}_A}\right) + \left(\frac{1}{2}\right) \cdot \left(\frac{\mathbf{P}_B}{\mathbf{E}_B}\right) = 55 \quad , \\ &\text{Weighted P/E ratio -Average of A and B} \quad \left(\frac{1}{6}\right) \cdot \left(\frac{\mathbf{P}_A}{\mathbf{E}_A}\right) + \left(\frac{5}{6}\right) \cdot \left(\frac{\mathbf{P}_B}{\mathbf{E}_B}\right) = 85 \quad . \end{aligned} \tag{10.18}$$

Applying either of these two P/E ratios to your \$150 in earnings would result in price assessment for AB that would be too high.

IMPORTANT:

- *Price-earnings ratios cannot be averaged.*
- *Mergers change the* P/E ratio*, even if they do not create value.*

The inability to aggregate divisions is not only an issue for the firm that is to be valued, but it also makes it difficult to extract a single comparable ratio from a division from inside conglomerates. In our case, let's assume that you only wanted to value the U.S. Dr. Pepper division, and that the U.S. Coca-Cola soda division was a perfect comparable for the U.S. Dr. Pepper. But how do you extract a P/E ratio for the Coca-Cola division, if all you know is the P/E ratio of the overall Coca Cola company with all its international subsidiaries, Minute Maid, Odwalla, etc.?

Lack of easy aggregation makes it difficult to value even well-defined firms, if the comparables are divisions inside larger firms.

The consequences of the aggregation failure mean, strictly speaking, that only the most basic single-product firms should be compared.

You have no good methods to aggregate and disaggregate P/E ratios. Therefore, strictly speaking, you can only compare full firms that are similar, which means that P/E ratios are likely to work well only for simple and well-defined companies, and not so well for complex conglomerates. In retrospect, it would have been a coincidence if your naïve attempt to apply the overall P/E ratio of Coca Cola or PepsiCo to Cadbury Schweppes's overall earnings would have worked. Indeed, in retrospect, it was an amazing coincidence that PepsiCo and Coca Cola had such similar P/E ratios. We lived in blissful ignorance.

In Part III, we will introduce "market-beta" as a valuation measure. Unlike P/E ratios, market-beta nicely aggregates and disaggregates. This makes it relatively easy to compute betas for conglomerates from their divisions, and to extract a division beta (given the conglomerate beta and comparable betas for other divisions).

10·3.C. A Major Blunder: Never Average P/E ratios

Using a firm with negative earnings as the comparable makes absolutely no sense.

On top of these unavoidable P/E ratio problems, many an analyst has mistakenly created a much worse and avoidable problem by averaging P/E ratios. Averaging overlooks the fact that earnings can be (temporarily) zero or negative, which can totally mess up any P/E ratio analysis. For example, consider the example where the choice of industry comparables for X consists of A and B.

	Value (**P**)	Earnings (**E**)		P/E ratio	E/P yield
Firm A	\$1,000	+\$10	$\Longrightarrow$	100	1.000%
Firm B	\$20	−\$5	$\Longrightarrow$	−4	−0.250%
		Industry Average:		48	0.375%
Firm X	?	\$2			

If Firm B were the only comparable, it would imply a negative value for Firm X,

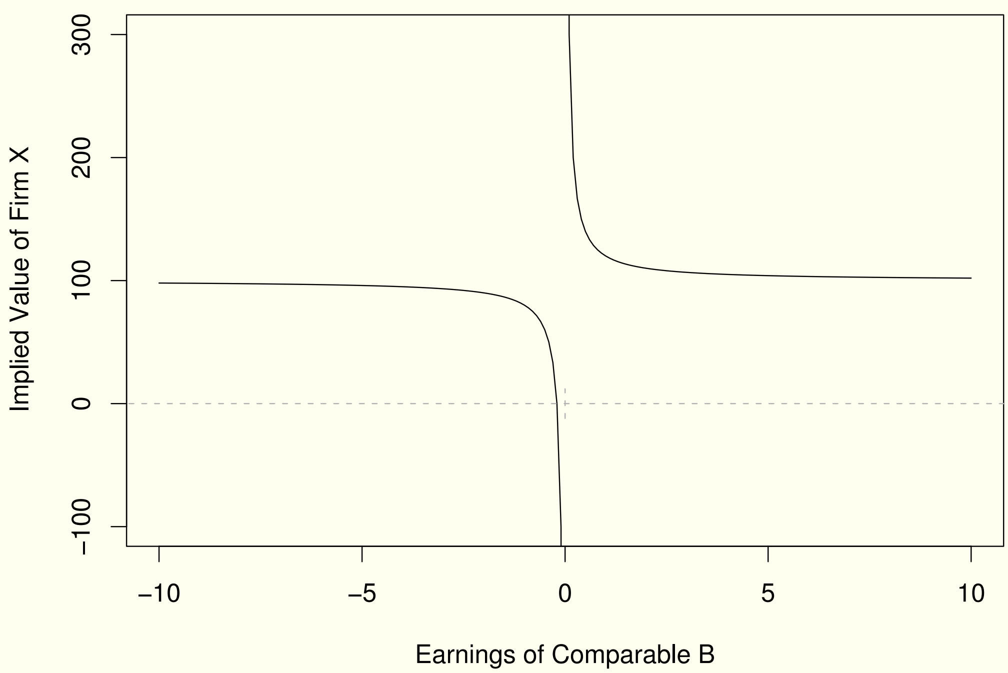
$$V_X = \mathbf{E}_X \cdot (P_B/\mathbf{E}_B) = \$2 \cdot (-4) = -\$8 \ . \tag{10.19}$$

A value of minus eight dollars for a firm with positive earnings and limited liability is not sensible. Luckily, this comparables-derived valuation is so nonsensical that no analyst would not notice it.

Averaging P/E ratios can look reasonable at first glance...

But the problem is often overlooked when an analyst uses a P/E industry average. For example, assume our analyst uses an average of both comparables: Firm A has a P/E ratio of 100, Firm B has a P/E ratio of −4. Thus, the average P/E ratio would be 48 (= [100 + (−4)]/2), which is a reasonable looking average that would not raise a red flag. A thoughtless analyst could conclude that Firm X should be worth $V_X = \mathbf{E}_X \cdot (P_{A,B}/\mathbf{E}_{A,B}) = \$2 \cdot 48 = \$96$.

Figure 10.2. Implied Value vs. Earnings Changes of One Comparable



If the earnings of the comparable B are $1, you get a sensible value for your firm X. If the earnings are a little bit lower, you get a non-sensically high number; if the earnings are a little bit lower, you get a non-sensically low number; and if the earnings are yet a little bit lower, you again get a non-sensical number—but one that can appear at first glance to be of reasonable magnitude.

Figure 10.2 makes the absurdity of this method even clearer. What happens to the implied value of Firm X if Firm B's earnings change? As Firm B improves its performance from about −$5 to about −$1, the average P/E ratio becomes 40, and your implied value remains a seemingly reasonable $80. Beyond −$1, earnings improvements in the comparable B create non-sensically huge negative implied firm values for X. Then, further improvements suddenly create non-sensically huge positive firm values. Finally, once the earnings of B are above $1 or so, you again get seemingly reasonable values (of about $100) for X. So, small changes in earnings can produce either seemingly sensible or non-sensible valuations. In other examples, even one comparable with earnings close to zero among a dozen comparables can totally mess up an average of many comparable P/E ratios.

...but it is not.

In an effort to deal with this problem, a common industry practice is to drop out firms with non-positive earnings from P/E averages. Unfortunately, this is not a good solution, either. First, you want an accurate valuation, and the stock market did value Firm B at $20. You have no good reason to ignore firms with low earnings. Second, dropping out some firms does not solve the problem: the firm would enter the P/E average if its earnings are +5 cents (leading to a very high industry P/E average), but be dropped out if its earnings are −5 cents (potentially leading to a much lower industry average). A small change in the P/E ratio of one comparable among the industry would have a disproportionately large impact on comparables valuation due to arbitrary inclusion/exclusion of comparables, rather than to closeness of earnings to zero. The reason for all these problems with price-earnings ratios is that earnings are in the denominator. The function 1/E is both discontinuous and very steep when earnings are close to zero. In contrast, the price (value) is guaranteed to be positive.

Excluding firms does not help.

The two better alternatives.

Fortunately, there are two easy overall alternatives to obtain good "pseudo averaged P/E ratios," even if some firms' earnings in the industry are low:

1. Work with earnings yields (E/P yields) instead of P/E ratios.

   In the example, the E/P yield of Firm A is $\$10/\$1,000 = 1\%$; the E/P yield of Firm B if it earned -1 cent is $-\$0.01/\$20 = -0.05\%$. The average E/P yield is thus $[1\%+(-0.05\%)]/2 = 0.475\%$. Inverting this back into a P/E ratio provides a halfway sensible value for the P/E ratio ($1/0.475\% \approx 211$).

2. Add up all market capitalizations in the industry and all earnings in the industry, and then divide the two.

   In the example where B earned -1 cent, the total industry earnings would be $\$10.00 - \$0.01 = \$9.99$, the entire industry market value would be $\$1,000 + \$20 = \$1,020$, and the average P/E ratio would be $\$1,020/\$9.99 \approx 102$. Note that in this method, firms are not equally weighted, but weighted by their relative market valuation. This may or may not be desirable: In our example, firm A would become the dominant determinant of your comparable valuation ratio.

Neither of these methods will give a very appealing comparable if the *total* industry average earnings are very small or negative. Our averaging alternatives can only avoid the problem of excessive influence of a small number of negative (or small) earnings firm in the average.

IMPORTANT: *Although neither* P/E ratio*s nor* E/P yield*s can be averaged, strictly speaking, an averaging-like operation can often be performed. We do so only because we lack a better alternative and we do not want to rely on just one single comparable. Never directly average* P/E ratio*s. Instead*

1. *Either average* E/P yield*s and then invert,*
2. *or divide total* **P** *sums by the total* **E** *sums.*

Never take these averages literally. Your goal must be to produce an "intuitively good (industry) average" derived from multiple comparables, not an exact number. You may judge your estimation to be better if you omit outlier firms, for example.

10·3.D. Computing Trailing Twelve Month (TTM) Figures

When comparable firms report annual statements in different months, the time change in economic climate introduces yet another problem.

There is one "small" mechanical detail left: Timing. First, is it meaningful to use annual earnings for a firm if the last annual report was from eleven months ago? Or should you use just the last quarter's numbers? Second, some firms report earnings in June, others in December. Should you compare financials that are timed so differently, especially if the economy has changed during this time lag? For example, consider the following reports:

	2001				2002			
	mar	jun	sep	dec	mar	jun	sep	dec
Firm 1	Q1:$1	Q2:$2	Q3:$3	Ann:$15	Q1:$5	Q2:$6	Q3:$7	Ann:$31

Your own firm closed its financial year in September 2002 with annual earnings of $12 and Q4 earnings of $2. What are the relevant comparable earnings from Firms 1 and 2? Should you work with your annual earnings and choose the comparable but dated annual earnings from December 2001 for firm 1? If you use the dated annual earnings number, to obtain firm 1's P/E ratio, should you divide by the current price or by the December 2001 price? Or should you just work with your quarterly earnings of $2, and compare it to the 4th quarter earnings of Firm 1? Or to the calendar-similar $7 in earnings? Here is some advice.

You could try to work directly with quarterly earnings, but this is usually not a good idea. Most firms do more business in December, and December can be the first month in a quarter or the last month in a quarter. Not only are different quarters difficult to compare across firms, but the fourth quarter may be difficult to compare even to the other three quarters of the same firm. So, generally, the best method to adjust flows (such as earnings) into a "most recent annualized equivalent" is to use a **trailing twelve months** (**TTM**) adjustment. In our example, this means adding the earnings from Q4-2001 through Q3-2002,

This time difference can be reduced, even though quarterly accounting statements themselves should be avoided. Instead, use quarterlies and annuals to compute "trailing twelve month" (TTM) figures.

$$\begin{aligned} \text{As If Annual} &= \$9 + \$5 + \$6 + \$7 = \$27 \\ \text{TTM Earnings} &= \text{Q4-01} + \text{Q1-02} + \text{Q2-02} + \text{Q3-02} \quad . \end{aligned} \tag{10.20}$$

Using the reported earnings, you can also compute this

$$\begin{aligned} \text{As If Annual} &= \$15 + (\$5 - \$1) + (\$6 - \$2) + (\$7 - \$3) = \$27 \\ \text{TTM Earnings} &= \text{Ann-01} + (\text{Q1-02} - \text{Q1-01}) + (\text{Q2-02} - \text{Q2-01}) + (\text{Q3-02} - \text{Q3-01}) \quad . \end{aligned} \tag{10.21}$$

There are two caveats: first, TTM adjusts only "flow" numbers (such as earnings or sales), never "stock" numbers (such as corporate assets). Stock numbers are whatever they have been reported as most recently. Second, firms sometimes change their fiscal year, often to make it intentionally more difficult to compare numbers. In this case, extra care must be exercised.

"Trailing twelve months" only works for "flow" numbers (such as income), not for stock numbers (such as assets).

10·3.E. Leverage Adjustments For P/E Ratios

This section assumes that riskier projects have to offer higher *expected* rates of return. Why this is the case will only fully become clear in Part III. Trust me for now.

Companies can be financed through a mix of debt and equity. You would want to know if the P/E ratio of a firm is changed by its leverage. If the same firm with more debt in its capital structure has a different P/E ratio, then you cannot compare two otherwise identical companies with different debt ratios without adjustment.

Does leverage influence P/E ratios?

Return to a simple example. With \$100 in annual earnings, a growth rate of 0% now, and an appropriate cost of capital of 10%, a hypothetical firm A would be worth \$1,000. A has no debt (leverage). We now work out what happens to the price-earnings ratio of A if it were to recapitalize itself. That is, A takes on a loan of \$500, and pays the cash proceeds from the loan to the equity share owner. This is called a **debt-for-equity** (or **debt-for-stock**) exchange. Such transactions are fairly common.

Recapitalizing a 100-0 firm into a 50-50 firm.

From Chapter 5, you already know that the cost of capital (interest rate) consists of multiple components: the time-premium, the default-premium (on average, zero), and the risk-premium. From Chapter 5·3.B, you know that levered equity is riskier than debt. Also, trust me now when I claim that if investors are not risk-neutral, the cost of capital is generally higher for riskier securities. The *expected* rate of return on risky investments must be higher than the *expected* rate of return on safer investments; it must be higher for equity than for debt; and it must be higher for riskier debt than for safer debt. In the example in this section, we shall assume that the appropriate expected rate of return for a firm financed with 50% debt is 7.5% for the debt, and 12.5% for the remaining 50% "levered equity" (residual ownership).

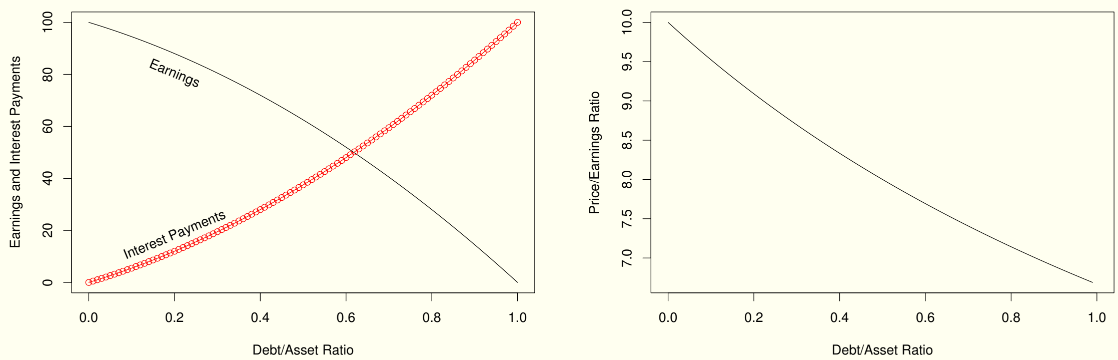
...with different costs of capital for bonds and stocks.

SIDE NOTE: The cost of capital for the firm overall remains at 10% after the firm is financed differently, because

$$\begin{aligned} &50\% \cdot 7.5\% + 50\% \cdot 12.5\% = 10\% \\ &\text{proportion of debt} \cdot \text{cost of capital for debt} + \text{proportion of equity} \cdot \text{cost of capital for equity} = \text{cost of capital for the firm} \quad . \end{aligned} \tag{10.22}$$

The cost of capital for the overall company has not changed. Section 17 will discuss this in more detail.

Figure 10.3. Earnings, Interest Payments, and Price-Earnings ratios as a Function of Leverage



As the firm's debt ratio goes up, its interest payments increase, its earnings decrease—and so does its price-earnings ratio. The same underlying projects can thus have a price-earnings ratio of, say, 7 or 10.

The changed price-earnings ratio under the 50-50 capital structure.

How much total interest payment does the debt have to promise? The loan value is \$500, the interest rate is 7.5%, and payments are forever. Therefore, the perpetuity formula can be solved as follows:

$$\$500 = \frac{\text{Annual Payment}}{7.5\%} \quad \Longrightarrow \quad \text{Annual Payment} = \$37.50/\text{year}$$
$$\text{PV} = \frac{\text{CF}}{\mathcal{E}(r)} \quad . \tag{10.23}$$

Having issued debt, the equity no longer receives \$100 per year, but only what is left after interest payments, i.e., \$100 − \$37.50 = \$62.50 per year. Consequently, the price-earnings ratio of this firm's equity shares will be the value of the equity (now \$500) divided by the earnings available to equity,

$$\text{P/E} = \$500/\$62.50 = 8.0 \text{ (times)} \quad . \tag{10.24}$$

In sum, when the same firm is financed with a 50/50 debt equity ratio, rather than a 0/100 debt equity ratio, its price-earnings ratio is 8 rather than 10. Figure 10.3 shows this graphically for a whole range of different debt-asset ratios: the higher the firm's debt load, the higher the interest payments, the lower the equity earnings, and the lower the price-earnings ratio. (The figure assumes that the expected [not just the promised!] debt interest rate increases in the debt-asset ratio according to the formula $5\% + 5\% \cdot D/A$.)

IMPORTANT: *The price-earnings ratio is higher when the firm has lower leverage (debt-equity ratio).*

Revisiting CocaCola, PepsiCo, and Cadbury: Using leverage information to improve valuation.

To compare firms that you deem to be identical in operations, but different for their capital structures, you need to adjust them to equivalent, unleveraged values. Let us put this insight to practical use. Assume that the companies of Coca Cola, PepsiCo, and Cadbury Schweppes are identical, even if their equities may not be because of their different leverage ratios. Some gathering of information from Yahoo!*Finance* allows you to put together Table 10.3:

Table 10.3. Financial Information from Yahoo

		Leverage		Financials, in billion-$			
STOCK (SYM)	$^P/_E$	D/E	$\frac{D}{D+E} = D/A$	Interest Expense	EBITDA	Earnings	Equity Value
Coca Cola (KO)	35	56%	36%	$0.244	$6.14	$3.91	$136.85
PepsiCo (PEP)	34	33%	25%	$0.207	$4.26	$2.74	$93.16
Cadbury Schweppes (CSG)	21	49%	33%	$0.155	$1.28	$0.72	$15.12

Note: The firm's asset value A is the sum of its equity value E and debt value D. The formula to translate debt-equity (D/E) ratios into debt-asset (D/A) ratios is

$$\frac{1}{^D/_A} = \frac{1}{^D/_{(D+E)}} = \left(\frac{D+E}{D}\right) = 1 + \left(\frac{E}{D}\right) \quad \Longrightarrow \quad \frac{D}{E} = \frac{1}{\left(\frac{1}{D/A} - 1\right)} \,. \tag{10.25}$$

In general, debt-equity ratios could either be based on the market value of equity or the book value of equity. The latter is especially problematic—though commonly used.

Translating P/E ratios of levered firms into P/E ratios as if unlevered.

You are now ready to ask what would happen if the three companies were not leveraged, but were financed only by equity. First, the earnings would not be diminished by the interest expense that the firms are currently paying. You must add back the interest expense to the earnings to get "as if unlevered" earnings. Second, the market capitalization of equity would increase. After all, the firm's projects would no longer be financed by debt and equity, but by equity alone. If a firm has a debt/equity ratio of 33% (1/3), for every dollar worth of debt it currently has 3 dollars worth of equity. Thus, an unlevered equivalent firm would be financed with $4 in equity. To convert a debt/value ratio to an unlevered value, divide the (levered) equity value by one minus the debt/value ratio. In Table 10.4, we can construct "as if unlevered" $^P/_E$ ratios by dividing the unlevered firm capitalization by the unlevered equity earnings (all quoted in billion dollars):

Table 10.4. Computing Unlevered $^P/_E$ Ratios

STOCK (SYM)	$(^P/_E)_{Lv}$	Unlevered Earnings	Unlevered Value	$(^P/_E)_{UnLv}$
Coca Cola (KO)	35	$3.91 + $0.244 = $4.15	$136.85/(1 − 36%) ≈ $214	52
PepsiCo (PEP)	34	$2.74 + $0.207 = $2.95	$93.16/(1 − 25%) ≈ $124	42
Cadbury (CSG)	21	$0.72 + $0.155 = $0.88	$15.12/(1 − 33%) ≈ $23	26

SIDE NOTE: These computations are only approximations and wrong for at least two reasons. First, we assume no taxes. If by unlevering the firm incurred more taxes, these taxes should be subtracted from firm value. Chapter 18 will discuss the role of corporate taxes in more detail. Fortunately, in this particular case (for these three companies), not much would change. Second, the unlevering depends on assuming that the firm continues forever, so that the perpetuity formula is applicable. This could also be wrong. Again, fortunately, in this particular case (for these three companies), not much would change for other growth assumptions.

Unfortunately, in our case, after proper adjustment, the P/E ratios have become more different, not more similar.

Does it appear as if Cadbury Schweppes (the underlying unlevered company) now is a lot more like PepsiCo than levered Cadbury Schweppes shares were to levered PepsiCo shares? Unfortunately, the answer is the opposite of what you should have hoped for. The P/E ratios of Cadbury Schweppes are even more different from those of Coca Cola and PepsiCo than they were before. You also have some more information to evaluate your earlier remarkable finding that PepsiCo could be accurately valued with the comparable of Coca Cola. You chose Coca Cola because you believed that the firm of Coca Cola would be similar to PepsiCo, not that the equity shares of Coca Cola would be similar to those of PepsiCo. But, in this case, the firms of Coca Cola and PepsiCo are less similar than the equity shares of Coca Cola and PepsiCo: their unlevered P/E ratios are farther apart than their levered P/E ratios. So, if you had properly applied the valuation ratio of one firm to the other firm, you would have concluded that PepsiCo and Coca Cola are not so similar, after all, and that you should not have compared the two. You just got lucky. You lived in ignorant bliss.

Solve Now!

Q 10.5 *List the main problems with using comparables.*

Q 10.6 *A firm with a* P/E ratio *of 20 wants to take over a firm half its size with a* P/E ratio *of 50. What will be the* P/E ratio *of the merged firm?*

Q 10.7 *The following are quarterly earnings and assets for Coca Cola and PepsiCo:*

	KO				PEP			
	Earnings (Qua/Ann)		Assets (Qua/Ann)		Earnings (Qua/Ann)		Assets (Qua/Ann)	
6/2002	1,290		25,287		888		24,200	
3/2002	801		23,689		651		22,611	
12/2001	914	3,979	22,417	22,417	667	2,662	21,695	21,695
9/2001	1,074		22,665		627		23,036	
6/2001	1,118		22,387		798		19,503	
3/2001	873		22,248		570		18,660	
12/2000	242	2,177	20,834	20,834	698	2,183	18,339	18,339
9/2000	1,067		23,141		755		17,659	
6/2000	926		23,258		668		17,492	

If it is now in July 2002, what would be good comparable earnings and comparable assets for these two firms?

Q 10.8 *A firm that is 100% equity financed is considering a recapitalization to a 50-50 debt-equity ratio. What will happen to its* P/E ratio*?*

Q 10.9 *A firm has a* P/E ratio *of 12 and a debt-equity ratio of 66%. What would its unlevered* P/E ratio *(i.e., the* P/E ratio *of its underlying business) approximately be?*

Q 10.10 *On October 9, 2002, the seven auto manufacturers publicly traded in the U.S. were*

Manufacturer	Market Cap	Earnings
Volvo (ADR)	\$5.7	–\$0.18
Ford	\$14.1	–\$5.30
GM	\$18.8	\$1.83
Nissan (ADR)	\$27.0	\$2.55
DaimlerChrysler	\$32.3	\$4.63
Honda (ADR)	\$37.7	\$3.09
Toyota (ADR)	\$87.3	\$4.51

(All quoted dollars are in billions. Ignore leverage. ***ADR*** *means* ***American Depositary Receipt****, a method by which foreign companies can list on the New York Stock Exchange.) On the same day, Yahoo! Deutschland reported that Volkswagen AG had earnings of 3.8 billion euro. In terms of sales, Volkswagen was most similar to Volvo and Ford. What would you expect Volkswagen to be worth?*

Q 10.11 *What are the basic assumptions in applying the* P/E ratio *of one company to the earnings of another company?*

10·4. Other Financial Ratios

Financial ratio analysis is extremely widely used in the real world—unfortunately, often without a good understanding of what ratios mean, what they do not mean, how they can be put to good use, and how they can sometimes obscure the right decision. If you do financial ratio analysis, be very, very careful. Know what it is that you are doing and think about meaning.

You should now understand that price-earnings ratios are very useful, but they are no panacea. They have big shortcomings, too. Still, overall, the P/E ratio is the best ratio I am aware of.

However, there are situations in which other value ratios can give you better value estimates. Moreover, there are non-value based ratios, whose purpose is less to give you an estimate of value and more to give you intuition about the economics of the firm.

10·4.A. Value-Based Ratios

Price-earnings ratios are not the only measure. The goal is to find a measure that is proportional to value.

There are a large number of other ratios in use, and this section presents a potpourri of some popular ones. The list is not exhaustive, as indeed only the imagination limits the quantities that can be used in the denominator. Two ratios that are relatively similar to the P/E ratio discussed in the previous section are

Alternative Price-Earnings or Cash Flow Ratios Earnings can be defined in a variety of ways: with or without extraordinary items, diluted, etc. There is no right or wrong way: the goal is to find a ratio that makes the comparables firm appear to be as similar as possible.

For example, one measure of earnings that from Chapter 9 is EBITDA (earnings before interest and taxes, depreciation, and amortization). The rationale is that accounting depreciation is so fictional that it should not be subtracted out. But EBITDA has problems with leverage (interest expense) and capital expenditures—if you use it, please subtract these. Of course, if you do, you will de-facto use a price over cash flow ratio, which can suffer from the shortcomings that capital expenditures are very “lumpy.” This is why we used earnings rather than cash flows in the first place.

Price/Book-Value-of-Equity Ratios This ratio is commonly used, and often abbreviated as the market/book ratio. It should not be recommended for valuation purposes. The reason is that *the book value of equity is often close to meaningless.* Accountants use the book value of equity to balance assets and liabilities, in accordance with all sorts of accounting conventions (such as depreciation). As a result, this measure is especially different across firms with different ages of fixed assets (such as buildings).

Many biotech firms have neither earnings nor sales. So, what to use?

However, sometimes neither earnings nor the book-value of equity are meaningful. For example, biotech firms may need to be valued even before they have meaningful, positive earnings. The idea is to substitute another, more meaningful quantity for earnings. In biotech firms, a better (but still very bad) measure than earnings may be the number of scientists. An analyst might find that biotech firms are worth \$1,000,000 per employed scientist. In principle, the comparables valuation method remains the same as it was when used with price earnings ratios. The alternative ratio typically still has price (either equity value or overall firm value) in the numerator, but a quantity other than earnings (e.g., sales or number of scientists) in the denominator. The analyst chooses comparable firm(s), determines an appropriate typical comparables ratio, and finally multiplies this comparables ratio by the firm’s own quantity to determine its value. This may work well only if firms are comparable enough among the chosen dimensions that application of the ratio of some firms to the ratio of others offers a meaningful price.

Price/Sales Ratios This ratio is especially popular for industries that do not have positive earnings. Therefore, it was commonly used during the Internet bubble, when few Internet firms had positive earnings. Presumably, firms with higher sales should be worth more.

One problem was that firms, such as Amazon during the Internet bubble at the turn of the millenium, were known to sell merchandise at a loss. Naturally, it is relatively easy to sell $100 bills for $99! So, the more Amazon sold, the more money it lost—but the more valuable Amazon appeared to be. After all, with higher sales, the Price/Sales ratio suggested that Amazon would be worth more.

Price/Employees Ratio This ratio is popular when financials are deemed not trustworthy. Of course, it assumes that the employees at the comparable firm are as productive as the employees in the company to be valued.

Price/Scientists Ratio This ratio is popular for upstart technology firms, which have neither earnings nor sales. One problem is that it induces firms to hire incompetent scientists on the cheap in order to increase their valuations. After all, firms with more scientists are presumed to be worth more.

Price/Anything Else Your imagination is the limit.

This latter set of ratios only makes sense if you compute them for the enterprise value of the firm (that is, the value of all equity plus the value of all debt). If you want to obtain the value of the equity, you should compute these ratios using the enterprise value, and then subtract off the current value of all debt.

Solve Now!

Q 10.12 *On July 28, 2003 (all quoted dollars are in billions):*

Firm	*Cash*	*Sales*	*Dividends*	*Value*	*D/E*
CSG	*n/a*	*$9.2*	*$0.4*	*$12.2*	*153%*
KO	*$3.6*	*$20.3*	*$2.2*	*$110.8*	*43%*
PEP	*$1.8*	*$25.9*	*$1.1*	*$81.0*	*22%*

Hansen Natural had $210,000 in cash, $9.22 million in sales, zero dividends, and a debt/equity ratio of 10%. What would a price/cash ratio predict its value to be? A price/sales ratio? A price/dividend ratio? Elaborate on some shortcomings.

10·4.B. Non-Value-Based Ratios Used in Corporate Analyses

Other ratios can be used to judge health and profitability.

Not all ratios are used to estimate firm value. Other ratios can help to assess financial health and profitability—or can be merely interesting. They can help in the "art" of valuation if they can help you learn more about the economics of the firm. For example, a number of ratios are often used to judge financial health (proximity to bankruptcy) and profitability. Most ratios vary systematically by industry and over the business cycle. Thus, they should only be compared to similar firms at the same time. On occasion, however, ratios can be so extreme that they can raise a good warning flag. For example, if you find that the firm has ten times its earnings in interest to pay, you might become somewhat concerned about the possibility of bankruptcy, regardless of what is standard in the industry at the time.

Watch out for the meaning of book value of equity, in particular.

Before discussing the ratios, realize that you should be twice as cautious when ratios involve quantities from the balance sheet and four times as cautious the book value of equity. This quantity is often so far from the true market value that ratios based on the book value can be almost meaningless. (It is not even a good estimate of replacement value, either.) The reason is that after accountants have completed all their bookkeeping, this number will become what is required to equalize the left-hand side and right-hand side of the balance sheet. It is a "placeholder."

Without further ado, here are some of the more interesting and common ratios. For each one, you will find one or more sample computations for PepsiCo in 2001 (which you can compare to the financials from Section 9·1.B), but be aware that many of these ratios exist in various flavors. The ratios are sorted, so that the ones at the top tend to reflect financial health and liquidity, while the ones at the bottom tend to reflect profitability. www.investopedia.com offers a nice reference for many of these ratios.

We now do ratios on PepsiCo.

Let us begin with ratios that consider the firm's debt load. A firm that has high debt ratios (especially compared to its industry) must often be especially careful to manage its cash and inflows well, so as to avoid a credit crunch. Moreover, if you want to borrow more money, then potential new creditors will be very interested in how likely it is that you will not default. They will judge your indebtness relative to your profitability, cash flow, and industry.

Debt-related ratios.

Debt/Equity Ratio The ratio of debt over equity. Many variations exist: debt can be just long-term debt, all obligations, or any other kind of liability. Equity can be measured either in terms of book value or in terms of market value.

$$\text{PepsiCo, 2001:} \quad \frac{\text{Long Term Debt}}{\text{Market Value of Equity}} = \frac{2,651}{87,407} \approx 3\% \ . \tag{10.26}$$

$$\text{PepsiCo, 2001:} \quad \frac{\text{Long Term Debt}}{\text{Book Value of Equity}} = \frac{2,651}{8,648} \approx 31\% \ . \tag{10.27}$$

As stated above, the book value of equity is very problematic. The book value of debt is usually much better—and, as is not the case with equity, you rarely have access to the market value of debt, anyway. (If interest rates have not changed dramatically since issue, this is a good approximation of overall market value.)

$$\text{PepsiCo, 2001:} \quad \frac{\text{All Liabilities}}{\text{Market Value of Equity}} = \frac{13,047}{87,407} \approx 15\% \ . \tag{10.28}$$

Debt Ratio As above, but adds the value of debt to the denominator. Because market value of debt is rarely available, a common variant adds the book value of debt and the market value of equity. For example,

$$\text{PepsiCo, 2001:} \quad \frac{\text{Long Term Debt}}{\text{Market Value of Equity + Debt}} = \frac{2,651}{87,407 + 13,047} \approx 2.6\% \ . \tag{10.29}$$

$$\text{PepsiCo, 2001:} \quad \frac{\text{Long Term Debt}}{\text{Book Value of Assets}} = \frac{2,651}{21,695} \approx 12\% \ . \tag{10.30}$$

$$\text{PepsiCo, 2001:} \quad \frac{\text{All Liabilities}}{\text{Market Value of Equity + Debt}} = \frac{13,047}{87,407 + 13,047} \approx 13\% \ . \tag{10.31}$$

Interest Coverage The ratio of debt payments due as a fraction of cash flows. Many variations exist: debt payments can be only interest due, or include both principal and interest. Cash flows can be any of a number of choices. Popular choices are pure cash flows, operating cash flows, net income plus depreciation minus capital expenditures, and net income plus depreciation. For example,

$$\text{PepsiCo, 2001:} \quad \frac{\text{Short Term Borrowings}}{\text{Cash Flow (Table 9.10)}} = \frac{354}{1,556} \approx 23\% \ . \tag{10.32}$$

Times Interest Earned (TIE) The earnings before interest (usually also before taxes) divided by the firm's interest. In some sense, the inverse of interest coverage.

$$\text{PepsiCo, 2001:} \quad \frac{\text{Times Interest Earned}}{\text{TIE}} = \frac{4,021}{219} \approx 18 \ . \tag{10.33}$$

Current Ratio The ratio of **current assets** (cash, accounts receivables, inventory, marketable securities, etc.) over **current liabilities** (interest soon-due, accounts payable, short-term loans payable, etc.) is a measure of liquidity. Often interpreted to be "healthy" if greater than 1.5.

$$\text{PepsiCo, 2001:} \quad \frac{\text{Current Assets (Page 210)}}{\text{Current Liabilities}} = \frac{74}{158} \approx 0.5 \ . \tag{10.34}$$

Do not read too much into this ratio. PepsiCo is very healthy, even though its current ratio is low.

Quick Ratio or Acid-Test is like the current ratio, but deletes inventories from current assets. It is often considered healthy if it is greater than 1.0. The idea is that you are then likely to be able to cover immediate expenses with immediate income.

$$\text{PepsiCo, 2001:} \quad \frac{\text{Current Assets (Page 210)}}{\text{Current Liabilities}} = \frac{7}{158} \approx 0.0 \ . \tag{10.35}$$

The **cash ratio** further eliminates receivables from current assets.

Duration and Maturity You have already used these concepts in the bond context, but they can also apply to projects and even to firms. They can measure what type of investment the firm is really making—short-term or long-term. This is not an ordinary ratio in that it requires projections of future cash flows.

Turnover The ratio of sales divided by a component of working capital.

- **Inventory Turnover**—how often your inventories translate into sales.

$$\text{PepsiCo, 2001:} \quad \frac{\text{Net Sales}}{\text{Inventories}} = \frac{26,935}{1,310} \approx 21/\text{year} \ . \tag{10.36}$$

  Most financials also provide the components of inventories, so you could further decompose this.

- **Receivables Turnover**—how quickly your customers are paying.

$$\text{PepsiCo, 2001:} \quad \frac{\text{Net Sales}}{\text{Receivables}} = \frac{26,935}{2,142} \approx 13/\text{year} \ . \tag{10.37}$$

- **Payables Turnover**—how quickly you are paying your suppliers.

$$\text{PepsiCo, 2001:} \quad \frac{\text{Net Sales}}{\text{Payables}} = \frac{26,935}{4,461} \approx 6/\text{year} \ . \tag{10.38}$$

These measures are sometimes inverted (one divided by the ratio) and multiplied by 365 to obtain a "number of days" measure. For example,

- **Days of Receivables Outstanding** (DRO), also called **Days of Sales Outstanding** (DSO). Accounts Receivables divided by total sales on credit, times number of days outstanding.

$$\text{PepsiCo, 2001:} \quad \frac{365 \cdot \text{Receivables}}{\text{Net Sales}} = \frac{365 \cdot 2,142}{26,935} \approx 29 \text{ days} \ . \tag{10.39}$$

  PepsiCo collects its bills about every 30 days. A lengthening of this number often indicates that customers are running into financial difficulties, which could impact PepsiCo negatively.

- **Days of Inventories Outstanding.** Inventory divided by total sales on credit, times number of days outstanding:

$$\text{PepsiCo, 2001:} \quad \frac{365 \cdot \text{Inventories}}{\text{Net Sales}} = \frac{365 \cdot 1,310}{26,935} \approx 18 \text{ days} \ . \tag{10.40}$$

PepsiCo turns over its inventory every 18 days.

- **Days of Payables Outstanding** (**DPO**). Accounts Payables divided by total sales on credit, times number of days outstanding.

$$\text{PepsiCo, 2001:} \quad \frac{365 \cdot \text{Payables}}{\text{Net Sales}} = \frac{365 \cdot 4,461}{26,935} \approx 60 \text{ days} \ . \tag{10.41}$$

  A lengthening of this number could mean that PepsiCo is having difficulties coming up with cash to meet its financial obligations—or found a way to pay bills more efficiently (more slowly in this case).

There are also combined versions, such as the **Cash Conversion Cycle**, which is the sum of inventory processing period and the number of days needed to collect receivables. For PepsiCo, this would be 18 + 29, or about one-and-a-half months.

Turnover ratios and their derivatives (below) are especially important for firms in the commodities and retail sector, such as Wal-Mart. Good control often allows firms to leverage economies-of-scale. In this sense, the above measure corporate efficiency, and help managers judge their own efficiency relative to their competition.

Profit Margin (PM) or Return on Sales The net or gross profit divided by sales.

$$\text{PepsiCo, 2001:} \quad \frac{\text{Net Income}}{\text{Sales}} = \frac{2,662}{26,935} \approx 10\% \ . \tag{10.42}$$

Mature cash cow firms should have high ratios; growth firms typically have low or negative ratios.

Return on (Book) Assets (ROA), like return on sales, but divided by book assets.

$$\text{PepsiCo, 2001:} \quad \frac{\text{Net Income}}{\text{Book Value of Assets}} = \frac{2,662}{21,695} \approx 12\% \ . \tag{10.43}$$

You already know that the book value of assets is unreliable, because it calculates firm value based on the book value of equity.

Return on (Book) Equity (ROE) You also know that I *really* do not like this measure.

$$\text{PepsiCo, 2001:} \quad \frac{\text{Net Income}}{\text{Book Value of Equity}} = \frac{2,662}{8,648} \approx 31\% \ . \tag{10.44}$$

Total Asset Turnover (TAT) A measure of how much in assets is required to produce sales. Again, with book value of assets in the denominator, this is not a reliable ratio.

$$\text{PepsiCo, 2001:} \quad \frac{\text{Sales}}{\text{Assets}} = \frac{26,935}{21,695} \approx 1.2 \ . \tag{10.45}$$

The DuPont Model multiplies and divides a few more quantities into the definitions of ROA and ROE:

$$\begin{aligned} \text{ROE} &= \frac{\text{Net Income}}{\text{Book Equity}} = \underbrace{\frac{\text{Net Income}}{\text{Sales}}}_{\text{Profit Margin}} \cdot \underbrace{\frac{\text{Assets}}{\text{Book Equity}}}_{\text{Equity Multiplier}} \cdot \underbrace{\frac{\text{Sales}}{\text{Assets}}}_{\text{Asset Turnover}} \\ &= \frac{\text{Net Income}}{\text{EBIT - Taxes}} \cdot \frac{\text{EBIT - Taxes}}{\text{Sales}} \cdot \frac{\text{Net Income}}{\text{Sales}} \cdot \frac{\text{Assets}}{\text{Book Equity}} \cdot \frac{\text{Sales}}{\text{Assets}} \ . \end{aligned} \tag{10.46}$$

A similar operation can be applied to a variant of ROA:

$$\text{ROA} = \frac{\text{EBIAT}}{\text{Assets}} = \frac{\text{EBIAT}}{\text{Sales}} \cdot \frac{\text{Sales}}{\text{Assets}} \ . \tag{10.47}$$

EBIAT is net income before interest after taxes. With book value of assets and book equity involved, this is often not a particularly trustworthy decomposition, even though it is mathematically a tautology.

The idea of these decompositions is that they are supposed to help you think about what the drivers of ROE and ROA are. Again, I do not like this analysis, because I believe that neither ROA nor ROE are good starting points for an analysis of what drives value. Not everyone agrees with my opinion here—especially not the individuals administering the CFA exam, where the DuPont model remains a staple.

Book-to-Market Ratio The book-value of the equity divided by the market value of the equity. If the book value of equity is representative of how much the assets would cost to replace, then this is a measure of how much the market values the (special) growth opportunities that the company has created from replaceable assets.

$$\text{PepsiCo, 2001:} \quad \frac{\text{Book Equity}}{\text{Market Equity}} = \frac{8,648}{87,407} \approx 10\% \ . \tag{10.48}$$

Dividend Payout Ratio The paid out dividends divided by earnings (any flavor thereof). The idea is that a firms that pays out more earnings today should pay out less in the future, because—in contrast to a firm that retains earnings—it lacks the paid out cash for reinvestment.

$$\text{PepsiCo, 2001:} \quad \frac{\text{Dividends}}{\text{Net Income}} = \frac{994}{2,662} \approx 37\% \ . \tag{10.49}$$

Payout Ratio The dividend payout ratio can be broadened to also include share repurchases, or even net repurchases (i.e., net of equity issuing) to obtain a net payout ratio.

$$\text{PepsiCo, 2001:} \quad \frac{\text{Dividends + Equity Repurchasing}}{\text{Net Income}} = \frac{2,710}{2,662} \approx 100\% \ . \tag{10.50}$$

$$\text{PepsiCo, 2001:} \quad \frac{\text{Dividends + Equity Repurchasing - Issuing}}{\text{Net Income}} = \frac{2,186}{2,662} \approx 82\% \ . \tag{10.51}$$

PepsiCo distributed most of its earnings to shareholders.

Dividend Yield The amount of dividends divided by the share price. Dividends can include share repurchases (in which case, this is called the **payout ratio**). Dividends are a flow measure, while the stock price is a stock measure. Consequently, dividends can be measured at the beginning of the period or at the end of the period (in which case, this is called the **dividend-price ratio**). Incidentally, stock-flow timing was also an issue for some of the measures above, though timing matters more for volatile measures, such as the stock market value. For example,

$$\text{PepsiCo, 2001:} \quad \frac{\text{Dividends + Equity Repurchasing}}{\text{Market Value}} = \frac{2,710}{87,407} \approx 3.1\% \ . \tag{10.52}$$

Firms that have a low payout yield today either have to see smaller stock price increases in the future (which means a lower expected rate of return), or they have to increase their payout yield sometime in the future. Otherwise, no one would want to hold them today.

Retention Ratios The retained earnings, divided either by sales, assets, or income. A firm that decides to start retaining more earnings should pay out more in the future. Because the retained earnings should be reinvested, such firms should have higher expected earnings growth. These measures are usually calculated as one minus the dividend payout ratio or one minus the sum of dividends and equity repurchases divided by net income or one minus the sum of dividends and net equity repurchases divided by net income. For example,

$$\text{PepsiCo, 2001:} \quad \frac{\text{Net Income - Payout}}{\text{Payout}} = \frac{2,662 - 2,710}{2662} \approx -0\% \ . \tag{10.53}$$

PepsiCo also issued \$524 of shares in connection with the Quaker merger, so

$$\text{PepsiCo, 2001:} \quad \frac{\text{Net Income - Net Payout}}{\text{Net Payout}} = \frac{2,662 - 2,710 + 524}{2,662} \approx 19\% \ . \qquad (10.54)$$

so its retention rate could be judged to be around 19%.

How useful are these ratios? It depends on the situation, the industry, and the particular ratio for the particular firm—and what you plan to learn from them. If every firm in the industry has almost the same ratio, e.g., a days of receivables average somewhere between 25 and 32 days, and the firm you are considering investing in reports 7 days, you should wonder about the economics of this shorter number. Is your firm better in obtaining money quickly? Does it do so by giving rebates to faster paying customers? Does it mostly work on a cash basis, while other firms in the industry work on credit? If so, why? Or is your firm simply cooking its books?

10·5. Closing Thoughts: Comparables or NPV?

Use both valuation methods, and use common sense to decide what you believe.

Should you use comparables or net present value? Both methods rely on inputs that you will almost surely not know perfectly. You will never have the perfect comparable, and you will never know the expected future cash flow perfectly. Fortunately, the cause of errors is different for these two methods. Therefore, if you use both, you can get a better idea of where the true value lies. This does not mean that you should average the valuation estimates obtained from NPV and comparables. Instead, you should perform both analyses, and then take a step back and make up your mind as to which combination of methods seems to make most sense in your particular situation.

Yes, valuation is as much an art as it is a science. It consists of the tools that you have now learned *and* your ability to judge. If you can judge better than others, you will end up a rich person.

10·6. Summary

The chapter covered the following major points:

- The P/E ratio (price-earnings) is the price of the firm divided by the firm's earnings, or equivalently the share price divided by the earnings per share.

  Often, these earnings are not the current but the expected earnings.
- The P/E ratio reflects the firm's growth opportunities and cost of capital. Higher growth firms have higher P/E ratios.
- P/E ratios and other methods based on comparables can provide an alternative valuation of firms and projects.
- Comparables suffer from a variety of problems, some of which can be corrected, and some of which are intrinsic and uncorrectable.
- Never average P/E ratios. Either average E/P yields and then invert, or divide total multiple-firm values by total multiple-firm earnings.
- The comparables valuation techniques and NPV have different weaknesses, which therefore often makes it worthwhile to consider and judge both.
- There are many other ratios that can be used to judge the profitability and the financial health of a company. As far as valuation is concerned, their primary purpose is only to provide useful background information.

A. Advanced Appendix: A Formula For Unlevering P/E ratios

Start with a special case in which firms have comparable leverage.

Return to the example in Section 10·2. Your comparison benchmark firm was

Firm B Financing	E	P	P/E
100-0 Equity-Debt Financed	$100.00	$1,000	10
50-50 Equity-Debt Financed	$62.50	$500	8

So, here is your problem: How do you value firm A, if the underlying projects of firm A are similar to those of firm B? Should you use 8 or 10 as your price-earnings ratio? The answer is easy if A has the same debt equity ratio as B. Then you can just use the comparison firm's price equity ratio as a proxy for your own price equity ratio. That is, if Firm A produced $200 in income *for equity holders* each year, after financing with a 50/50 debt equity ratio, then you would expect its equity value to be about

$$\begin{aligned}
\text{Implied A Value} &= \left(\frac{\text{Price}}{\text{Earnings}}\right)_{\text{A}} \cdot \text{Earnings}_{\text{A}} \\
&\approx \left(\frac{\text{Price}}{\text{Earnings}}\right)_{\text{B}} \cdot \text{Earnings}_{\text{A}} \\
&= \quad 8 \cdot \$200 \quad = \$1,600 \ ,
\end{aligned}$$

where we write out Earnings to avoid confusion with equity. As just stated, this equity valuation depends on the assumption that Firm A had borrowed an equal amount in outstanding debt ($1,600), i.e., that it had a 50/50 debt equity ratio to which you could apply your B multiple.

Problem is: leverage may be different. Try to unlever both.

But what do you do if your A, with its reported income of $200 for equity holders, had borrowed $2,500 instead of $1,600? Or, what if A had not borrowed anything? You need to determine an appropriate comparable that you can apply to A and that takes into account the actual leverage of A and of B. The easiest method is to convert all price-earnings ratios to a 0/100 debt equity ratio. In this case, you already know the result that you wish to obtain: a formula should tell you that unlevering B should translate the previous ratio of 8 into a price-earnings ratio of 10. Next, you need to look up the interest rate that A is paying for borrowing. Let us assume that A's records inform you that the interest rate is 8%. This means that A is paying $200 in interest each year, before it paid off the $200 to its equity holders. Consequently, you know that a 0/100 debt equity A would have produced earnings of $200 + $200 = $400. Thus,

$$\begin{aligned}
\text{Implied A Value} &= \left(\frac{\text{Price}}{\text{Earnings}}\right)_{\text{A}} \cdot \text{Earnings}_{\text{A}} \\
&\approx \left(\frac{\text{Price}}{\text{Earnings}}\right)_{\text{B}} \cdot \text{Earnings}_{\text{A}} \\
&= \quad 10 \cdot \$400 \quad = \$4,000
\end{aligned}$$

This would be the value of an all equity firm. Because A has borrowed $2,500, its equity must be worth $1,500.

An attempt at a delevering formula.

So the trick is to convert all debt equity ratios into "all equity" ratios first. Unfortunately, there is no general formula to do so: the translation formula depends on the time when the cash flows are likely to occur. The following formula, based on a plain perpetuity assumption on the firm's earnings, often works reasonably well:

A Debt Adjustment Formula for P/E ratios

$$\left(\frac{\text{Price}}{\text{Earnings}}\right)_{\text{Relevered}} = \frac{1-(r_{\text{Firm}}-r_{\text{Debt}})\cdot\left(\frac{\text{Debt}}{\text{Earnings}}\right)}{r_{\text{Firm}}}$$
$$= \left[1-(r_{\text{Firm}}-r_{\text{Debt}})\cdot\left(\frac{\text{Debt}}{\text{Earnings}}\right)\right]\cdot\left(\frac{\text{Price}_{\text{Firm}}}{\text{Earnings}_{\text{Firm}}}\right)$$

where "firm" means an all-equity no-debt financed firm. Let us apply this formula to firm B. Let us assume that you know that B has a value of \$1,000 and a price-earnings ratio of 10 when it has no debt, but it is now considering a move to a 50/50 debt equity ratio. You want to determine this capital structure change on its new price-earnings ratio. Use the formula:

$$\left(\frac{\text{Price}}{\text{Earnings}}\right)_{\text{Relevered}} = \left[1-(r_{\text{Firm}}-r_{\text{Debt}})\cdot\left(\frac{\text{Debt}}{\text{Earnings}}\right)\right]\cdot\left(\frac{\text{Price}_{\text{Firm}}}{\text{Earnings}_{\text{Firm}}}\right)$$
$$= \left[1-(10\%-7.5\%)\cdot\left(\frac{\$500}{\$62.50}\right)\right]\cdot 10$$
$$= [1-0.025\%\cdot 8]\cdot 10$$
$$= 0.8\cdot 10 = 8 \quad .$$

This formula makes it easy to figure out how an all equity firm's price earnings ratio changes when it takes on debt. If you know the debt equity ratio of a firm and want to convert it into a debt-free equivalent price-earnings ratio, then you can use the following conversion.

To convert the observed leveraged P/E ratio into a hypothetical P/E ratio for an unlevered firm, the following approximation formula can be helpful:

$$\left(\frac{\text{Price}}{\text{Earnings}}\right)_{\text{Firm}} = \frac{1}{\left[1-(r_{\text{Firm}}-r_{\text{Debt}})\cdot\left(\frac{\text{Debt}}{\text{Earnings}}\right)\right]}\cdot\left(\frac{\text{Price}}{\text{Earnings}}\right)_{\text{Equity}} \quad . \tag{10.55}$$

This formula makes very specific assumptions on the type of firm, specifically that earnings are a plain perpetuity. The formula can sometimes be a decent approximation if earnings are not a perpetuity.

Solutions and Exercises

1. You would probably value houses by the method of comps—NPV would be exceedingly difficult to do. However, there are often similar houses that have recently sold. You might use a ratio that has price in the numerator and square-foot in the denominator, and multiply this ratio from comparable houses by the square-foot of your new residence.

2. Microsoft is growing faster, so it would have a higher P/E ratio.

3. $E/P = r - g \Rightarrow r = P/E + g = 1/40 + 6\% = 8.5\%$. Therefore, $E/P = 8.5\% - 7\% = 1.5\%$ and its P/E ratio would shoot from 40 to 66.7. The percent change in value would therefore be $66.6/40 - 1 \approx 66\%$.

4. $E/P = r - g \Rightarrow g = r - E/P = 10\% - 1/20 = 5\%$.

5. See the text.

6. Do an example. The acquirer has value of \$100, so it needs to have earnings of \$5. The target has value of \$50, so it needs to have earnings of \$1. This means that the combined firm will have earnings of \$6 and value of \$150. Its P/E ratio will thus be 25.

7. Earnings: The TTM Earnings for KO is $3,979 + (801 - 873) + (1,290 - 1,118) = 4,079$. The TTM Earnings for PEP is $2,662 + (651 - 570) + (888 - 798) = 2,833$. Assets: You would not compute a TTM, but use the most recent assets: \$25,287 for Coca-Cola and \$24,200 for Pepsico.

8. You know from the text that the P/E ratio would go down.

9. This question cannot be answered if you do not know the different costs of capital. For example, if the firm's cost of capital is equal to the debt cost of capital, the P/E ratio would not change at all!

10. Yahoo reported an actual market value of \$10.52 billion euros, and an earnings yield of 36.9% ($^{P}/_{E}$ of 2.7). The easy part is supplementing the table:

Manufacturer	Market Cap	Earnings	P/E ratio	E/P yield
Volvo (ADR)	\$5.7	-\$0.18	-31.7	-3.2%
Ford	\$14.1	-\$5.30	-2.7	-37.6%
GM	\$18.8	\$1.83	10.3	9.7%
Nissan (ADR)	\$27.0	\$2.55	10.6	9.4%
DaimlerChrysler	\$32.3	\$4.63	7.0	14.3%
Honda (ADR)	\$37.7	\$3.09	12.2	8.2%
Toyota (ADR)	\$87.3	\$4.51	19.4	5.2%
Sum	\$222.9	\$11.13	25.1	6.0%
Average	\$31.8	\$1.59	3.6	0.9%

The hard part is deciding on a suitable $^{P}/_{E}$ comparable. The first method (average E/P yield, then invert) suggests adopting the astronomical ratio of $1/0.9\% = 111$, due to Ford's enormous loss in terms of market capitalization (Ford had \$85 billion in sales, and a positive EBITDA of \$4.8 billion. But Ford also has ongoing depreciation on the order of \$15 billion per year, but capital and other expenditures on the order of \$18 (2001) to \$37 billion (2000 and 1999).) The second method (sum up E's and P's first) suggests $\$222.9/\$11.1 = 20$, but it weighs the larger [and Japanese] firms more highly. Nevertheless, in this case, the second method came closer to the actual Volkswagen $^{P}/_{E}$ multiple of 27.
Incidentally, by mid-2003, VW had introduced a couple of flops, and its earnings had sagged to \$2.5 billion, though its market capitalization had increased to \$15 billion. This meant that Volkswagen's $^{P}/_{E}$ multiple had shrunk from 27 to 6 in just nine months!

11. The firms are alike substitutes in all respects, including product, product lines, and debt ratio.

12. These ratios are usually performed without debt adjustment—the equivalent of surgery without anesthesia. This is a huge problem, but it also makes this exercise relatively easy.

Firm	Value/Cash	Value/Sales	Value/Dividends
CSG	n/a	1.3	30
KO	185	5.5	50
PEP	45	3.1	74

- The cash-based ratio suggests a value between \$10 million and \$39 million. The cash-based ratio values all firms as if only current cash has any meaning, and the ongoing operations are irrelevant (except to the extent that they have influenced current cash).

- The sales-based ratio suggests a value between \$12 million, \$29 million, and \$50.6 million. Because the smaller comparables have lower ratios, one might settle on a lower value. The sales-based ratio ignores that CSG's equity value is relatively low because more of its value is capitalized with debt than with equity.
- The dividend-based ratio suggests a zero value. Obviously, this is not a perfect estimate. Firms can choose different payout policies.

Hansen's actual value on this day was \$51.4 million.

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

Part III

Investor Choices-Short

There is a long version and short version of the Investments part of the book. You are looking at the short version. The long version will appear in a dedicated survey/investments version of the book, and can already be downloaded from the website.

This part appears in the CorpFin text only.

Transition

As a corporate executive, your main concern is to determine which projects to take and which projects to avoid. The net present value method requires knowledge of projects' expected cash flows, $\mathcal{E}(\mathrm{CF})$, and of the cost of capital, $\mathcal{E}(\tilde{r})$.

$$\text{NPV} = \mathrm{CF}_0 + \frac{\mathcal{E}(\widetilde{\mathrm{CF}}_1)}{1+\mathcal{E}(\tilde{r}_{0,1})} + \frac{\mathcal{E}(\widetilde{\mathrm{CF}}_2)}{1+\mathcal{E}(\tilde{r}_{0,2})} + \ldots \qquad (10.56)$$

We now turn our attention towards the root causes which determine the cost of capital, $\mathcal{E}(\tilde{r})$. You should think of it as the **opportunity cost of capital** for your corporation's owners (investors). If they have better alternatives elsewhere, you should return their capital to them and let them invest their money better elsewhere. In valuing your corporation's projects, excellent opportunities elsewhere mean that you should use a high cost of capital. Poor opportunities elsewhere mean that you should use a low cost of capital.

Before you can answer what projects your investors would like you, the corporate manager, to take, you need to first learn about *their* problems. Who are your investors, what do they like and dislike, and how should you evaluate our project relative to what you believe your investors' alternatives are? What *are* your investors' alternatives? And what are the consequences of not using the correct cost of capital? This requires an expedition into the world of uncertainty for measuring risk and reward, and an expedition into the mindset of our investors before you can make your own corporate investment choices. This is the goal of Part III of this book.

This Part III is also where you must finally abandon the assumption that investors are risk-neutral—that they are indifferent between having $1 million for sure, and having $900,000 or $1,100,000 with equal probability. Having risk-averse investors forces you to consider how projects influence one another from a "joint risk" perspective. So, we will assume now that investors are risk-averse—that they would prefer the $1 million for sure—and then determine how this aversion influences their choices from the large universe of available investment projects. We shall still also (cowardly) assume that you are still in a perfect market of no information differences, a deep market, no transaction costs, and no taxes.

> **IMPORTANT**: *Think of the $\mathcal{E}(\tilde{r})$ in the NPV denominator as your investors' "opportunity" cost of capital. If your investors have great opportunities elsewhere, your projects have to be discounted at high discount rates.*

What You Want to Accomplish in this Part

The goal of this part of the book is primarily to investigate where $\mathcal{E}(\tilde{r})$, the cost of capital in the NPV formula, comes from. Think of $\mathcal{E}(\tilde{r})$ as the opportunity cost of capital for your investors. If your investors have great investment opportunities elsewhere, and if your corporation's projects are not as attractive, you should return the money to your investors, so that they themselves can invest it better elsewhere. In the NPV formula, great opportunities elsewhere manifest themselves as a high $\mathcal{E}(\tilde{r})$ that you should apply to your projects.

- Chapter 11 gives you a short tour of historical rates of returns to whet your appetite, and explains some of the setup of equity markets.

  Typical questions: Did stocks, bonds, or cash perform better over the last thirty years? How safe were stocks compared to bonds or cash?

- Chapter 12 explains that investors like more reward and less risk, how reward and risk should be measured, and how diversification reduces risk. It draws a clear distinction between a security's own risk and a security's risk contribution to the investor's overall portfolio.

  Typical questions: What is the standard deviation of the rate of return on my portfolio? What is IBM's beta? Why does it matter to my portfolio? What is the average market-beta of my portfolio? What is the difference between beta and correlation?

- Chapter 13 explains how you should measure your investors' opportunity cost of capital, $\mathcal{E}(\tilde{r})$, given that your own corporate investment projects can help or hurt your investors in their overall risk-reward tradeoff. This is the domain of the "Capital-Asset Pricing Model" (or CAPM).

  Typical questions: What characteristics should influence the appropriate expected rate of return that your investors care about? What should be the appropriate expected rate of return for this particular project? Where do you find all the necessary inputs for use of the CAPM?

- Chapter 14 is an optional chapter, which provides more details on the underpinnings of the CAPM. It explains the mean-variance efficient frontier, and how each security inside a portfolio on the mean-variance efficient frontier must offer a fair rate of return, given its risk contribution. It also talks about where the CAPM is a good model (i.e., in our corporate finance context!), and where it is not (i.e., in a pure investment context).

  Typical questions: How much of security X should you purchase? Where does the CAPM formula come from?

- Chapter 15 explains the concept of efficient markets and arbitrage in more detail than our introductory chapter.

  Typical questions: What is the difference between arbitrage and a great bet? Could it be that market efficiency is not absolute, but comes in different degrees? What processes can stock prices reasonably follow? What processes do stock prices reasonably follow? What should you think of market gurus? What is your expected market value change if you announce that your company has just received FDA drug approval?

CHAPTER 11

A First Look at Investments

Historical Rates of Returns

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The subject of investments is so interesting that we are going to break our rule of starting simple. Instead of laying all the foundations before we look at the evidence, we shall first look at some evidence: the world of returns on stocks, bonds, and "cash." My plan is to show you the annual returns on these investment classes (and on some individual stocks), so that you can visualize the main patterns that matter—patterns of risk, reward, and covariation. Don't worry if you cannot follow everything in this chapter. It will all be explained in due course—and you can always come back later.

11·1. Stocks, Bonds, and Cash, 1970–2004

Common categories.

Financial investment opportunities are often classified into just a few large categories: cash, bonds, and stocks. Cash is actually a misnomer, because it usually designates not physical bills under your mattress, but bonds that are very liquid, very low-risk, and very short-term. Another common designation for cash is **money-market**, a catch-all designation that includes not only very short-term Treasury bills but also a number of other securities (such as CDs, savings deposits, and commercial paper) that were listed in Section 5.**a**. We will just use the term cash, because it is shorter. We have already talked at length about bonds and their many different varieties. Stocks are often further categorized into a few hundred large-firm stocks that are quite visible, liquid and trade very frequently, and mostly make up the popular S&P500 index; and a few thousand small-firm stocks that trade less frequently.

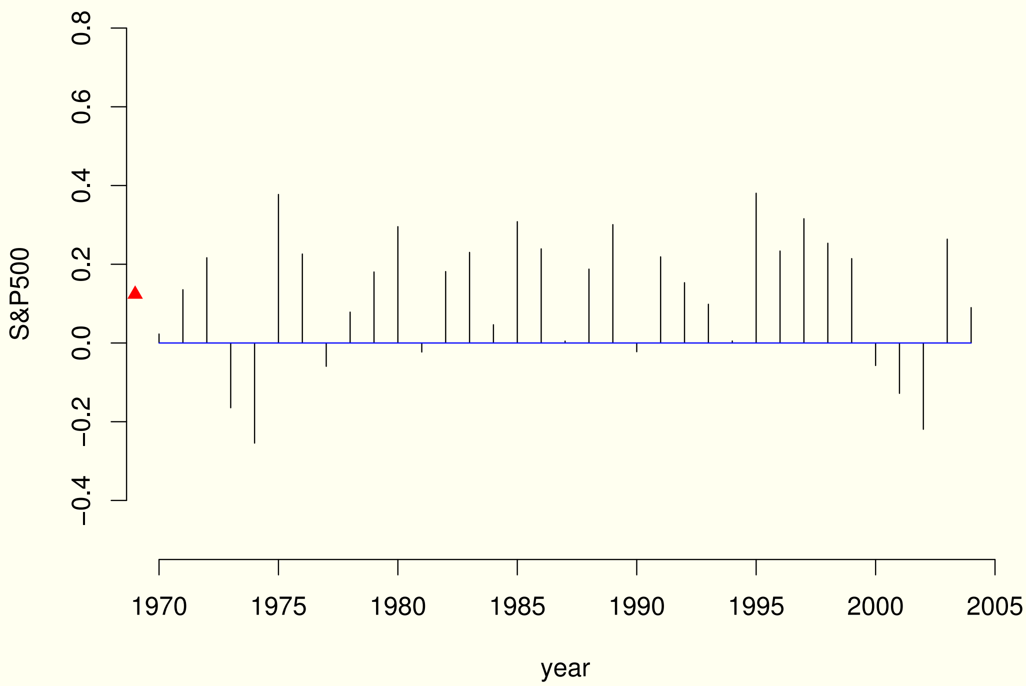
Categories hide a lot of variation. They are only broadly indicative.

You should not take any of these categories too literally, because each of them is quite diverse. For example, we already know that bonds may include anything from Treasury bonds, corporate bonds, municipal bonds, foreign bonds, to even more exotic instruments. Nevertheless, these categories can be useful in giving a broad perspective—because most, though not all, bonds behave more like other bonds than they behave like stocks. The same holds true for stocks and cash—most, but not all stocks behave more like other stocks, and most, but not all money market instruments behave more like other money market instruments. So let us begin our examination of investments by looking at the historical performances of these asset categories.

11·1.A. Graphical Representation of Historical Stock Market Returns

Figure 11.1. The Time Series of Rates of Returns on the S&P500, 1970–2004

1970	+2.3%	1980	+29.6%	1990	−2.2%	2000	−5.7%
1971	+13.5%	1981	−2.3%	1991	+21.9%	2001	−12.8%
1972	+21.7%	1982	+18.2%	1992	+15.3%	2002	−21.9%
1973	−16.5%	1983	+23.0%	1993	+9.8%	2003	+26.4%
1974	−25.4%	1984	+4.6%	1994	+0.5%	2004	+9.0%
1975	+37.7%	1985	+30.8%	1995	+38.0%	Average	12.3%
1976	+22.6%	1986	+23.9%	1996	+23.4%	Std Dev	16.7%
1977	−5.9%	1987	+0.5%	1997	+31.6%		
1978	+7.8%	1988	+18.8%	1998	+25.3%		
1979	+18.0%	1989	+30.1%	1999	+21.4%		

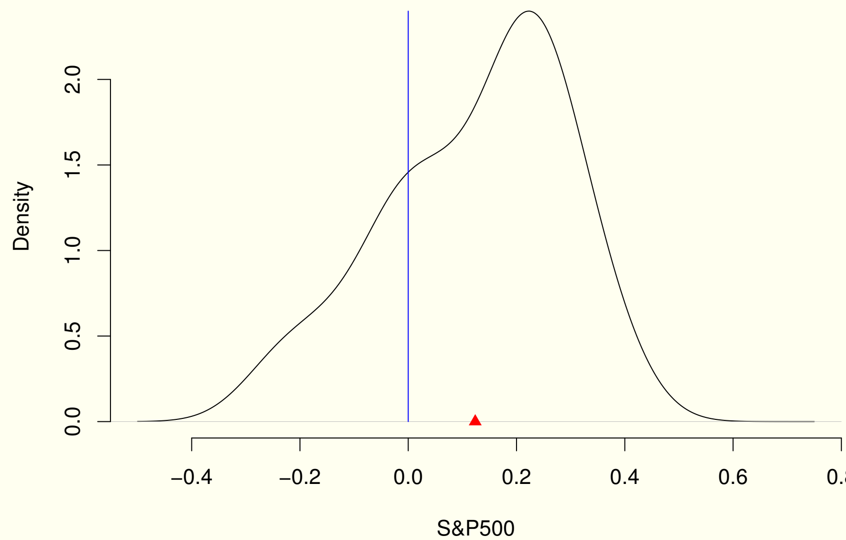


The graph is a representation of the data below.

The time series diagram.

We begin with Figure 11.1, which shows the year-by-year rates of return of the S&P500. It represents the performance of the 500 largest firms in the U.S. stock market. Looking at the data, you would have earned 2.3% in 1970, 13.5% in 1971, and so on. The table in Figure 11.1 allows you to compute the average rate of return over all 35 years as 12.3% per annum—also marked by the red triangle in the graph on the left side, and in the table at the end of the data.

Figure 11.2. Density Function of S&P500 Rates of Return, 1970–2004



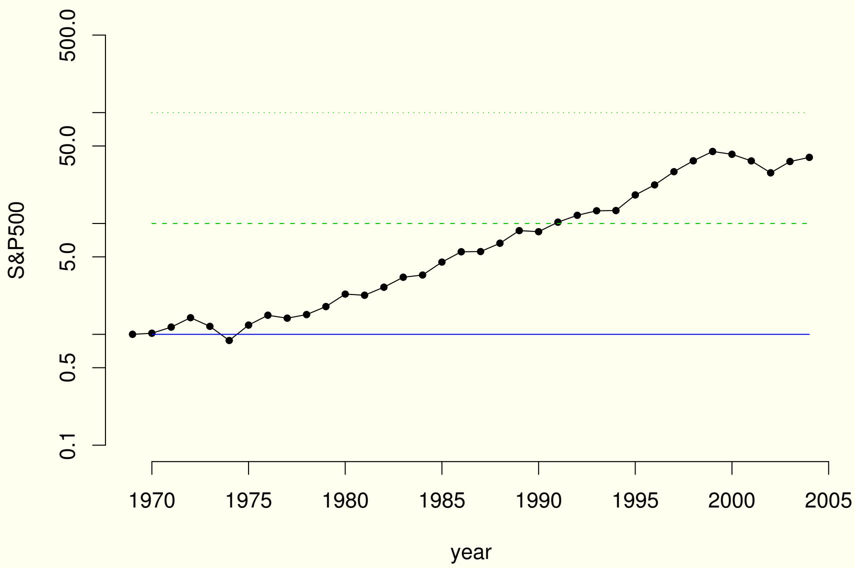
Return Range	$< -20\%$	$-20\%, -10\%$	$-10\%, 0\%$	0%-10%	10%-20%	20%-30%	$30\% - 40\%$	$> 40\%$
Number of Years	2	2	4	7	5	10	5	0

The graph and table are just different representations of the data in Figure 11.1.

The histogram shows how spread out returns are.

Figures 11.2 and 11.3 take the same data as Figure 11.1, but present it differently. The density function—a smooth version of a histogram—in Figure 11.2 is based on the number of returns that fall within a range. For example, the table in Figure 11.1 shows that only 1971, 1979, 1982, 1988, and 1992 had rates of return between 10% and 20%. In this period, the most frequent return range was between 20% and 30%, but there were also a good number of returns below 10%—and even two years in which you would have lost more than 20% of your money (1974 and 2002). The density function makes it easy to see how spread out returns are.

Figure 11.3. Cumulative Rates of Return For the S&P500, 1970–2004



1970	$1.02	1980	$2.31	1990	$8.43	2000	$42.06
1971	$1.16	1981	$2.25	1991	$10.28	2001	$36.68
1972	$1.41	1982	$2.66	1992	$11.85	2002	$28.64
1973	$1.18	1983	$3.27	1993	$13.02	2003	$36.19
1974	$0.88	1984	$3.43	1994	$13.08	2004	$39.45
1975	$1.21	1985	$4.48	1995	$18.06		
1976	$1.49	1986	$5.55	1996	$22.28		
1977	$1.40	1987	$5.58	1997	$29.31		
1978	$1.51	1988	$6.63	1998	$36.74		
1979	$1.78	1989	$8.62	1999	$44.62		

The graph and table are just different representations of the data in Figure 11.1.

The cumulative rate of return graph shows how long-run investments would have fared.

The cumulative return graph in Figure 11.3 offers yet another perspective. It plots the cumulated annual returns (on a logarithmic scale). For example, by the end of 1973, the compound return of $1 invested in 1970 would have been

$$\begin{array}{ccccccccccc} \$1 & \cdot & (1+2.3\%) & \cdot & (1+13.5\%) & \cdot & (1+21.7\%) & \cdot & (1-16.5\%) & \approx & \$1.18 \\ I_0 & \cdot & (1+r_{1970}) & \cdot & (1+r_{1971}) & \cdot & (1+r_{1972}) & \cdot & (1+r_{1973}) & . & \end{array} \tag{11.1}$$

The cumulative return perspective illustrates geometric returns, which adjust for the fact that a return of −50% followed by +100% is a net zero return, even though the average rate of these two returns would be a +25%. The geometric rate of return is always lower than the arithmetic rate of return. For example, our 18% compound rate of return corresponds to a 4.2% annualized rate, which is lower than the 5.25% arithmetic average rate of return from 1970 to 2003. The graph shows that an investment in the stock market of $1 at the start of 1970 would have ended up as $39.70 at the end of 2004—of course, ignoring all taxes.

11·1.B. Comparative Investment Performance

Explore the complex figure first. Stare at it.

What does history tell us about rate-of-return patterns on our three major investment categories—stocks, bonds, and cash? We can find out by plotting exactly the same graphs as those in Figures 11.1–11.3. Figure 11.4 repeats them for a set of historical investment choices *all on the same scale*. It displays a lot of information about the performance of these investments. Do not expect to understand everything at first glance: you need to stare at the elements of Figure 11.4 for a while to comprehend them.

The first row is again "cash."

You have already seen the first row—investments in "cash." Only the scale is different to make direct comparison to the other investments in the graphs below easier. Note how tight the distribution of cash returns is around its 7% mean.

The second row, long-term bonds, offered more reward, but was more risky, too.

The second row describes investments in 20-year Treasury bonds. The graph in column 3 shows that the bars are now sometimes slightly negative (years in which you would have earned a negative rate of return), but there are also years in which you would have done much better. This is why the histogram is much wider for 20-year bonds than it is for cash securities, and this is why the risk was 12% per year—although the average rate of return was a higher 10% per year. By 2004, your \$1 invested in 1970 would have become \$19.29.

The third row, stocks overall, offered even more reward, but was even more risky.

The third row describes an investment in an "index fund" that holds stocks to replicate the rate of return on the S&P500 index. Sometimes, this is colloquially called "stocks" or "the stock market," though it is really only "large stocks." Large stocks would have been even more risky (with a mean rate of return of 12% per year and a risk of 17% per year), but your \$1 invested in 1970 would have been worth \$39.45 in 2004.

Individual stocks can offer more reward, or be even more risky.

Let us see how some individual stock investments would have differed from just the broad category "stocks." The remaining four rows represent the rates of returns for four stalwarts: Coca Cola [KO], PepsiCo [PEP], Sony [SNE], and United Airlines [UAL]. The histograms are all over the place: investing in a single stock would have been a rather risky venture, even for these household names. Indeed, we could not even plot the final year for UAL in the right-most cumulative return graph, because UAL stock investors lost *all* invested money in the 2003 bankruptcy, which on the logarithmic scale would have been minus infinity.

SIDE NOTE: The compound inflation rate from 1970 to 2002 was 5.0. (Put differently, \$1 in 1970 purchased as much as \$5.04 did in 2003.) Therefore, the \$9.69 end result in cash would have been worth \$9.69/\$5.04 ≈ 1.92 in 1970-inflation-adjusted dollars. Over 30 years, you would have only doubled your real purchasing power. You can easily compute equivalent real returns for the other investment opportunities.

Furthermore, the difference between \$39 in stocks and \$9.69 in cash or \$19.29 in bonds is an understatement *for you*. Interest was taxable each year, while the capital gain in stocks was not (the dividend gain would have been taxable). Very roughly, a highly taxed investor would have ended up with about \$5 in "cash," \$13 in bonds, and \$33 in stocks. Therefore, in real *and* after-tax terms, from 1970 to 2004, a highly taxed investor would have ended up just about even if invested in "cash," doubled or tripled if invested in bonds, and quintupled if invested in stocks. This was a great 30 years for stocks!

SIDE NOTE: From 1926 to 2002, the annual risk and reward of some large asset-class investments were approximately as follows:

Asset Class	"Reward" $\mathcal{E}(\tilde{r})$	"Risk" $Sdv(\tilde{r})$
Short-Term U.S. Government Treasury Bills	4%	3%
Long-Term U.S. Government Treasury Bonds	5%+	9%+
Long-Term Corporate Bonds	6%	9%
Large Firm Stocks	10%	20%
Small Firm Stocks	15%	30%

(Inflation was about 3% per year.)

(Source: Ibbotson Associates, and others.)

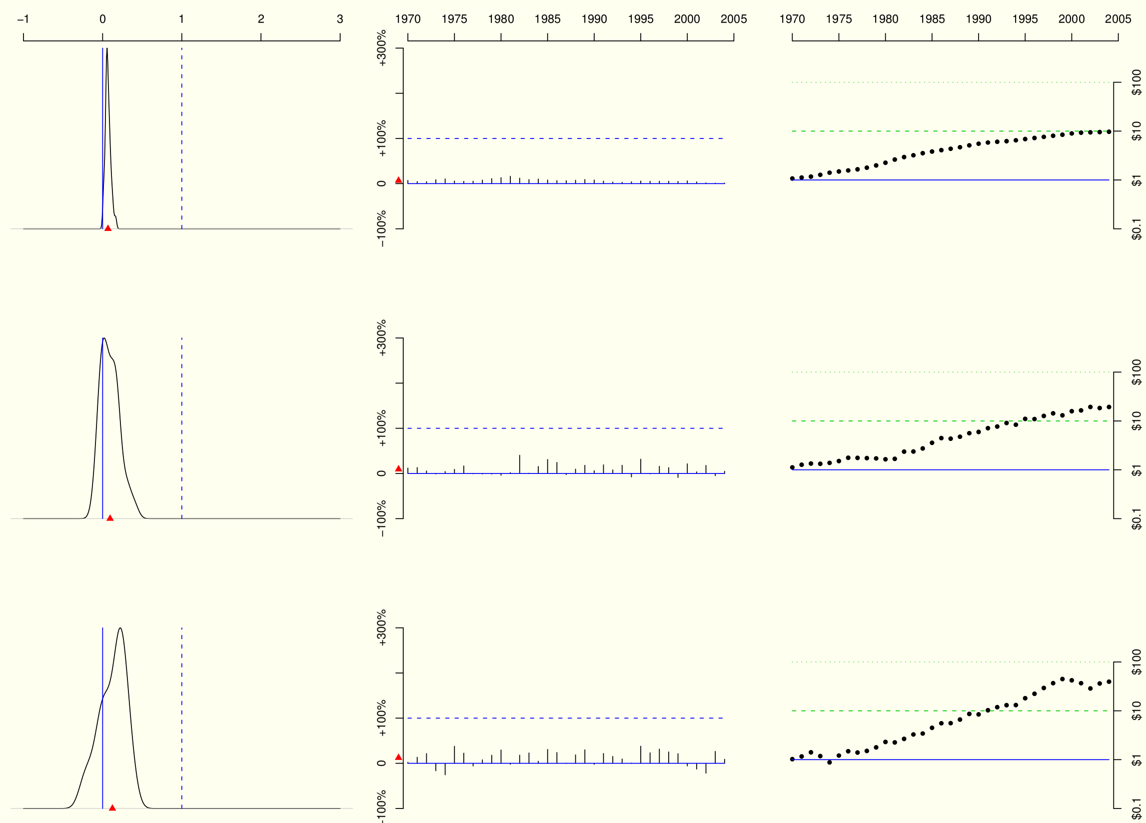
Figure 11.4. Comparative Investment Performance, 1970–2004

Description — Density Function — Rates of Return — Compound Return

"Cash" (Federal Funds Rate)
Mean (Reward): 6.8%/yr
Std.Dev. (Risk): 3.4%/yr
$1 in Jan 1970 would have
become $9.69 in 2004
Correl w/ S&P500: −2.8%
Beta w/ S&P500: −0.0

"Bonds" (20-Year Treasury)
Mean (Reward): 9.4%/yr
Std.Dev. (Risk): 11.8%/yr
$1 in Jan 1970 would have
become $19.29 in 2004
Correl w/ S&P500: +25%
Beta w/ S&P500: +0.2

"Stock Market" (S&P500)
Mean (Reward): 12.4%/yr
Std.Dev. (Risk): 16.7%/yr
$1 in Jan 1970 would have
become $39.45 in 2004
Correl w/ S&P500: +100%
Beta w/ S&P500: +1.0

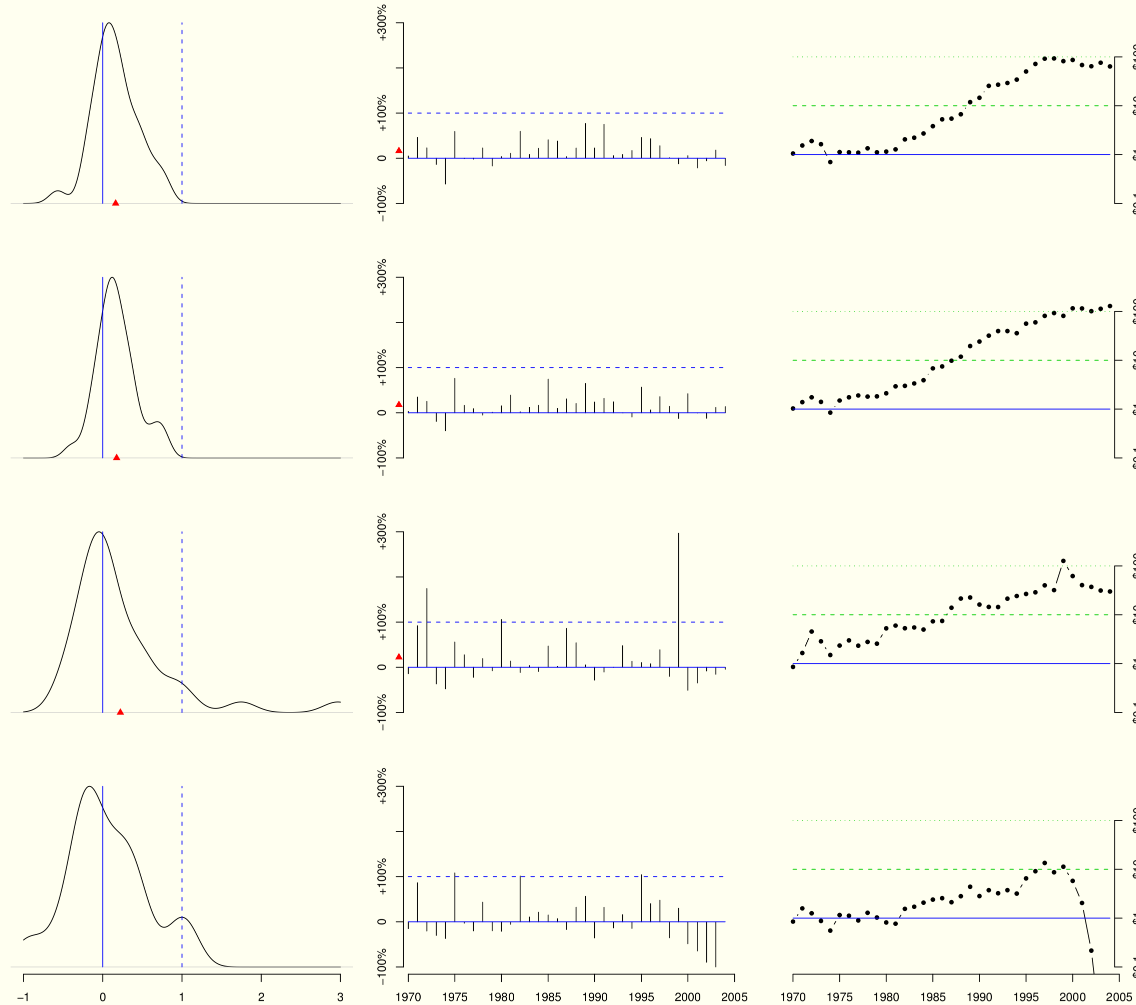


CocaCola (KO)
Mean (Reward): 16.3%/yr
Std.Dev. (Risk): 28.5%/yr
$1 in Jan 1970 would have
become $63.71 in 2004
Correl w/ S&P500: +63%
Beta w/ S&P500: +1.0

PepsiCo (PEP)
Mean (Reward): 17.6%/yr
Std.Dev. (Risk): 25.5%/yr
$1 in Jan 1970 would have
become $128.78 in 2004
Correl w/ S&P500: +59%
Beta w/ S&P500: +0.9

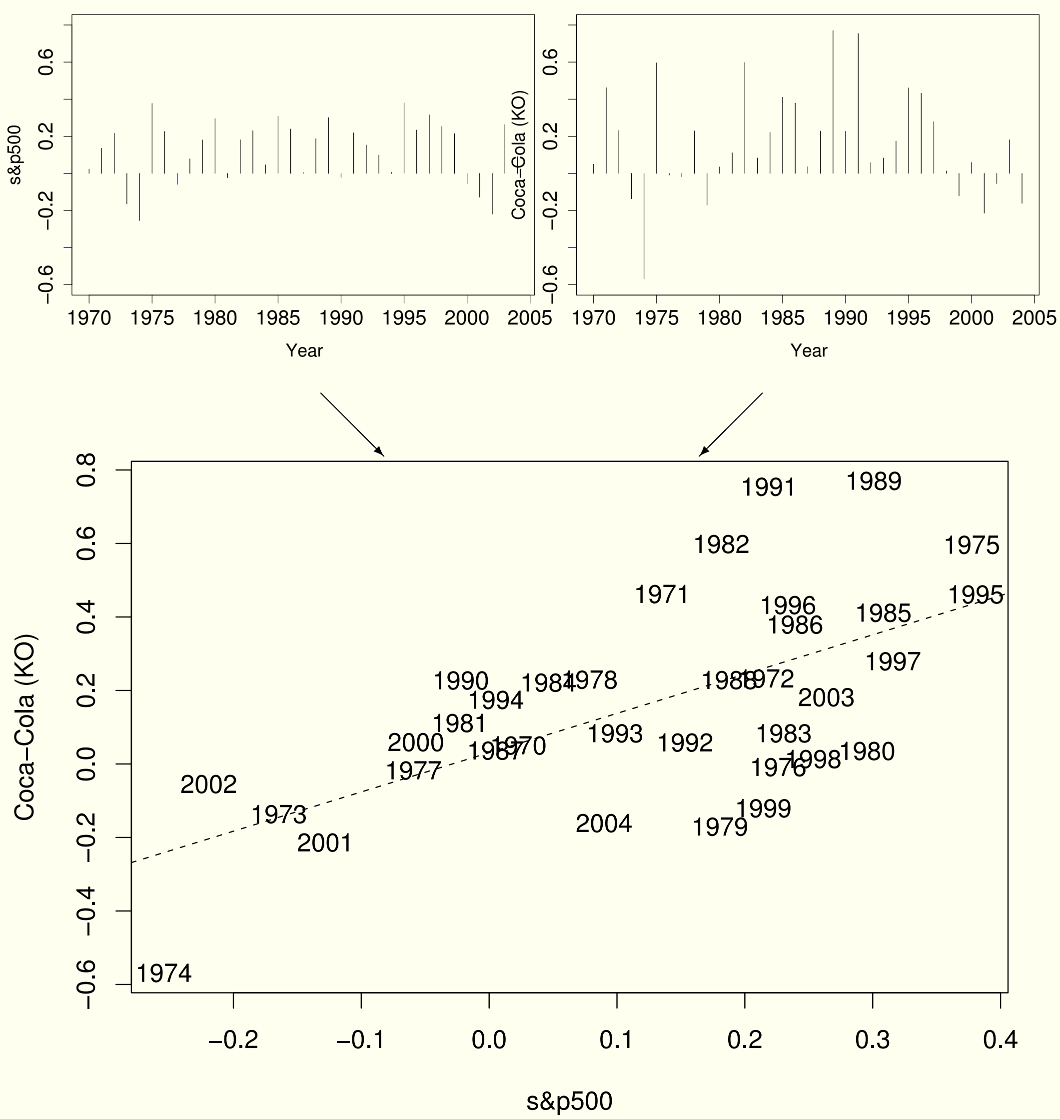
Sony (SNE)
Mean (Reward): 22.3%/yr
Std.Dev. (Risk): 66.9%/yr
$1 in Jan 1970 would have
become $29.94 in 2004
Correl w/ S&P500: +37%
Beta w/ S&P500: +1.5

United (UAL)
Mean (Reward): 4.7%/yr
Std.Dev. (Risk): 51.x%/yr
$1 in Jan 1970 would have
become $0.00 in 2004
Correl w/ S&P500: +57%
Beta w/ S&P500: +1.7



Description | Density Function | Time Series | Compound Return

Figure 11.5. Rate of Returns on The S&P500 and Coca-Cola (KO)



The lower graph combines the information from the two upper graphs. The stock market rate of return is on the X-axis, the Coca-Cola rate of return is on the Y-axis. The figure shows that in years when the stock market did well, Coca-Cola tended to do well, too, and vice-versa. This can be seen in the slope of the best fitting line, which is called the market-beta of Coca-Cola and which will play an important role.

11·1.C. Comovement, Beta, and Correlation

What is the correlation mentioned in the figure?

Figure 11.5 highlights the rates of return on the S&P500 and one specific stock, Coca-Cola (KO). The top row redraws the graphs for these two investments from the third column in Figure 11.4. Do you notice a correlation between these two rates of return? Are the years in which one is positive (or above its mean) more likely to also see the other be positive (or above its mean), and vice-versa? It does seem that way. For example, the worst rates of return for both are 1974. Similarly, 1973, 2000, and 2001 were bad years for investors in either the S&P500 or Coca-Cola. In contrast, 1989 and 1975 were good years for both. The correlation is not perfect: in 1979, the S&P500 had a good year, but Coca-Cola had a bad one. It is very common for all sorts of investments to move together with the stock market: in years of malaise, almost everything tends to be in malaise. In years of exuberance, almost everything tends to be exuberant.

Why do we care about comovement?

This comovement of investments is very important if you do not like risk. An investment that increases in value whenever the rest of your portfolio decreases in value is practically like "insurance" that pays off when you need it most. You might buy into such an investment even if it offers only a very low expected rate of return. In contrast, you might not like an investment that does very badly whenever the rest of the portfolio also does badly. To be included in your portfolio, such an investment would have to offer a very high expected rate of return.

Quantifying comovement—Market-beta, the best-fit line.

How can we measure the extent to which securities covary with others? For example, you might want to know how Coca-Cola performs if your current portfolio is the S&P500 (a common stand-in for the market portfolio). Will Coca-Cola also go down if the market goes down (and make a bad situation worse), or will it go up and thereby serve as useful insurance? How can you quantify such comovement? Graphically, you can plot the two return series against one another, as is done in the lower plot in Figure 11.5. The graph also shows the best line between the two series. This line has a slope of 1.07 and is called the market-beta of Coca-Cola's stock. This market-beta is an important measure of comovement for an investor who owns the market portfolio. Loosely speaking, if a stock has a very steep positive slope, say +3, then if the market and our portfolio drops by 10%, this stock would be expected to drop by 30%—it would make a bad situation worse. In contrast, if a stock has a very negative slope, say −1, then if the market drops by 10%, this investment would "rescue" us, earning a positive 10% rate of return. It would act like insurance.

Market-beta, the best-fit line, is related to correlation, too.

Another common measure of comovement is the correlation. A correlation of 100% indicates that two variables always perfectly move together; a correlation of 0% indicates that two variables move about independently; and a correlation of −100% indicates that two variables always perfectly move in opposite directions. (A correlation can never exceed +100% or −100%.) In this case, the correlation is +63%. Slope and correlation are very similar measures—in fact, a positive correlation implies a positive beta and vica-versa. Of course, beta and correlation are only measures of *average* comovement: even for positive beta investments, there are individual years in which the investment and stock market do not move together. We already mentioned 1979 and 2000 as examples in which Coca-Cola and the S&P500 went their different ways. Negative betas are rare. There are only a very few investment categories that are generally thought to be negatively correlated with the market—principally gold and other precious metals.

Solve Now!

Q 11.1 *What can you see in a time-series graph that is lost in a histogram?*

Q 11.2 *What can you see in a histogram that is more difficult to see in the time-series graph?*

Q 11.3 *What can you see in a cumulative return graph that is difficult to see in the time-series graph?*

Q 11.4 *How do you graph a "market-beta"? What should be on the X-axis, what should be on the Y-axis? What is an individual data point?*

Q 11.5 *What is the market beta of the market?*

11·2. Visible and General Historical Stock Regularities

The main empirical regularities.

What can you learn from these graphs? Actually, almost everything that there is to learn about investments—and I will explain these facts in great detail soon.

- The history indicates that stocks offered higher average rates of return than bonds, which in turn offered higher average rates of return than "cash." However, keep in mind that this was only *on average*. In any given year, the relationship might have been reversed. For example in 2002, stock investors lost 22% of their wealth, while cash investors gained about 1.7%.
- Although stocks did well (on average), you could have lost your shirt investing in them, especially if you had bet on just one individual stock. For example, if you had invested $1 into United Airlines in 1970, you would have had only 22 cents left in 2002—and nothing the following year.
- Cash was the safest investment—its distribution is tightly centered around its mean, so there were no years with negative returns. Bonds were riskier. Stocks were riskier, yet.

  (Sometimes, stocks are called "noisy," because it is really difficult to predict what they will turn out to offer.)
- There was some sort of relationship between risk and reward: the riskiest investments tended to have higher mean rates of return.

  (However, the risk has to be looked at "in context." Thus, please do not overread the simple relationship between the mean and the standard deviation here.)
- Large portfolios consisting of many stocks tended to have less risk than individual stocks. The S&P500 fund had a risk of 17%, much less than the risk of most individual stocks.

  (This is due to diversification.)
- A positive average rate of return usually, but not always, translates into a positive compound holding rate of return. United Airlines had a positive average rate of return, despite having lost all investors' money.

  (You already know why: A stock that doubles and then halves has rates of return of +100% and –50%. It would have earned you a 0% total compound rate of return. But the average rate of return would have been positive, $[100\% + (-50\%)]/2 = +25\%$.)
- Stocks tend to move together. For example, if you look at 2001–2002, not only did the S&P500 go down, but the individual stocks also tended to go down. In 1998, on the other hand, most tended to go up (or at least not down much). The mid-1990s were good to all stocks. And so on. In contrast, money market returns had little to do with the stock market. Long-term bonds were in between.

  On annual frequency, the correlation between cash and the stock market (the S&P500) was about zero; between long-term bond returns and stock market around 30%; and between our individual stocks and the stock market around 40% to 70%. The fact that investment rates of return tend to move together will be important. It will be the foundation for the market-beta, a measure of risk that we shall propose in Chapter 12.

11·3. History or Opportunities?

History is only useful over longer horizons, not over just a few years.

Ultimately, finance is not interested in history for its own sake. We want to know more about the future, and history is useful primarily because it is our best available indicator of the future. But which history? One year? Thirty years? One hundred years? Trust me when I state that if we had drawn the graphs beginning in 1926 instead of 1970, our conclusions would have all remained the same. However, if we had started in 2001, what would we have seen? Two awful years for stock investors. We know intuitively that this would not have been a representative sample period. To make any sensible inferences about what is going on in the financial markets, we need many years of history, not just one, two, or three—and certainly not the 6-week investment performance touted by some funds or friends (who also often display remarkable selective memory!). The flip side of this argument is that we cannot reliably say what the rate of return will be over one year. We will be better in forecasting the average annual rate of return over 5 to 10 years than over 1 year. Any single year will be very noisy.

Still, history can be rather misleading. The Nikkei is a good example.

Instead of relying on just one year, relying on statistics computed over many years is much better. However, although 20 to 30 years of performance is the minimum number necessary to learn something about return patterns, this is still not sufficient. Again, we are really interested in what will happen in the next 5 to 10 years, not what did happen in the last 5 to 10 years. Yes, the historical performance can help us judge, but you should not trust it blindly. For example, an investor in UAL in 2000 might have guessed that the average rate of return for UAL would have been positive—and would have been sorely disappointed. Investors in the Japanese stock market in 1990 had seen the Nikkei-225 stock market index rise from 10,000 to 40,000—a four fold increase in just four years—a 40% rate of return every year. If they had believed history, they would have expected $40,000 \cdot (1 + 40\%)^{13} \approx 3.2$ million by the end of 2002. Instead, the Nikkei had fallen below 8,000 in April 2003, and has only recently recovered to 12,000. History would have been a terrible guide.

But, we do not have much choice other than to use some history.

Nevertheless, despite the intrinsic hazards in using historical information in forecasting stock market returns, having historical data is a great advantage. It is a rich source of forecasting power, so, like everyone else, we will use historical statistics. Yet we must also be careful in not overinterpreting them. For example, if we see crazily high or crazily low past historical rates of return, we should try to exercise additional caution.

Historical risk and betas are better predictors of the future risk and betas.

Fortunately for us, though, it turns out that correlations and risk (explained in the following chapters) tend to be more stable than historical mean rates of return. So, even though we do not believe that a \$1 investment in PepsiCo will return \$100 in 30 years, or that PepsiCo will return 18% next year, or that PepsiCo has an expected rate of return of 18% over the long run, we can believe that PepsiCo has a risk of around 25% to 30% per year, and that its correlation with the S&P500 is around 60%.

Presume we know today the *expected* statistics for the future, although we do not know future realized statistics.

To learn investments, we shall pretend that we know the statistical distributions from which future investment returns will be drawn. That is, we assume that we know the future expected rates of return, their standard deviations, correlations, etc., but we do not know the actual draws. For example, we assume that we may know that the lotto combination "5,10,12,33,34,38" has an expected investment rate of return of –5 cents, but we do not know whether this combination will win the jackpot. Usually, historical statistics will be our guide to future statistics.

Side Note: For the most part, we will work with historical statistics as if they were our expected statistics. So, we will try to slip in "unnoticed" that the historical statistical outcomes tell us something about the future expected statistical outcomes. After the sermon about how historical statistics need not be perfectly indicative of the future statistics, you should realize that equating the two will be a big and not necessarily innocuous jump. Fortunately, as we have already mentioned, historical betas and risks are pretty indicative of future betas and risks. Unfortunately, historical expected rates of return are not too indicative of future expected rates of return. Know what you can and what you cannot trust.

Solve Now!

Q 11.6 *Rank the following asset categories in terms of risk and reward: money-market, long-term bonds, the stock market, and a typical individual stock.*

Q 11.7 *Is the average individual stock safer or riskier than the stock market?*

Q 11.8 *Is it possible for an investment to have a positive average rate of return, but still lose you every penny?*

11·4. Eggs and Baskets

The basic investment choices.

Although the goal of this part of the book is to develop investments in a technical manner, we can explain the intuition with a parable about Easter eggs—a variation of the folk wisdom "Don't put all your eggs in one basket." Assume that you are an Easter-egg-seller and that you have to stock the baskets that you will be carrying to the market tomorrow. Your problem is that you do not know which color eggs will be the most sought after. Say that you can guess that the color that will most likely sell is blue (or colors close to blue)—but you really will not know until you start selling the eggs at the market.

11·4.A. The Overall Basket

Strategies—go for the highest expected rate of return, go for the safest strategy, or choose a little of each.

The first question is, what strategies can you pursue?

- You can paint all your eggs blue. This strategy is the equivalent of purchasing just the investments that have the highest possible expected rate of return. It will work great if you guess right, and this guess is the single most likely outcome—but it also has a very high chance of leaving you entirely destitute. It is a high-risk strategy.

- You can play it safe and not paint your eggs. Uncolored eggs can always be sold for food, so you assume practically no risk. This strategy is the equivalent of purchasing the very lowest-risk investments that also have the very lowest expected rates of return—like Treasury bills. You effectively give up on trying to obtain a high expected rate of return in exchange for more safety. It is a very low-risk strategy.

- You can stock your baskets with eggs of many colors—a strategy called diversification. You will not sell all eggs, but you will likely sell a good number. This strategy is the equivalent of purchasing many different stocks (or a mutual fund that holds many different stocks), where some investments will lose and others will gain. Relative to bringing only blue eggs, you are giving up some expected value today, but you are gaining some extra security, because you will not likely run into a situation where you cannot sell any eggs.

  The more varied the colors that you choose, the safer will be your basket. But you will not want all colors in equal proportions. You will want to tilt the color mix towards blue, because you believe blue offers the highest expected rates of return. Moreover, this basket will probably be similar to the baskets that other smart egg-sellers will choose. In the financial markets, this basket is probably close to something like the market portfolio, which has eggs of all sorts of different colors. Some colors in the market portfolio are relatively more prominent than others. The market portfolio will have relatively more "blue eggs," because they have the highest expected rate of return. Relative to bringing only blue eggs, this strategy is not as high mean, but it is also not as high risk. Relative to bringing only unpainted eggs, it is a higher mean strategy but also with higher risk.

11·4.B. The Marginal Risk Contribution

A very important question is "How much are you willing to pay to have, say, one more yellow egg in your basket?" If you believe blue eggs offer the highest expected rates of return, would you even bring *any* yellow eggs? Yes! Even if you do not believe that yellow is likely to sell tomorrow, a yellow egg will likely sell precisely when most of your blue eggs won't sell. Yellow provides you with the equivalent of "insurance"—it pays off when the rest of your portfolio is losing. Therefore, you may very well be willing to bring some yellow eggs, and even though you expect to make a loss on them—of course, within reasonable bounds. You may be prepared to lose 5 cents on each yellow egg you bring, but you would not be prepared to lose $100.

You might choose some yellow eggs, even though they are not great investments in themselves.

In sum, yellow eggs are valuable to you *because* they are different from the rest of your portfolio. What matters is the insurance that yellow pay off when your blue investments do not.

Yellow is valuable, because it is different.

Perhaps the most important aspect is that you realize that it is not the own risk of each egg color itself that is important, but the overall basket risk and each color's contribution thereto. In fact, you already know that you may even expect to lose money on yellow eggs (just as you may expect to lose money on your homeowner's insurance). This again emphasizes that having yellow eggs as insurance is useful only *because* most of your eggs are not yellow. The risk contribution of yellow thus inevitably must depend on all the other eggs in your portfolios. Of course, it would make no sense to bring *only* yellow eggs—in this case, you would not only expect to lose 5 cents per egg, you would also most likely *always* lose these 5 cents and on all your eggs. In the financial market, the degree to which one stock investment is similar to others in your portfolio will be measured by the aforementioned beta—and if your portfolio is the market portfolio, then it is called the market beta. You will be willing to hold some stocks in the market portfolio that have a low expected rate of return because they are different from the rest of your portfolio—but only some, and only if their expected rate of return is not *too* low.

The risk of yellow eggs is irrelevant. What matters is risk of the overall basket. Yellow eggs help because they are different. If you bought too many yellow eggs, they would no longer be different from the rest of your portfolio!

In sum, when you look at your final basket, you should consider each egg along two dimensions—how does it contribute to your overall expected rate of return (what is its own expected rate of return?), and how does it contribute to your overall portfolio risk (how does its return covary with that of your overall basket?). In a good portfolio, you will try to earn a high expected rate of return with low risk, which you accomplish by having a balanced mix of all kinds of eggs—a balance that evaluates each egg by its expected rate of return versus its uniqueness in your basket.

Assets matter on the margin on two dimensions: expected return and risk contribution.

11·4.C. The Market Equilibrium

Assume now that you own one factory among many that is selling a particular type of colored egg to many smart egg traders. It would make sense for you to assume that your egg traders are smart, that they like to buy eggs in colors that have high expected rates of return, but that they also like to buy some eggs that are different and unique. In other words, you should assume that your traders do the same optimal basket stocking calculations that we have just gone through. You can even work out how much smart egg traders would be willing to pay for eggs of your factory's color. If your egg color is very different from those of the other eggs in traders' baskets, you can charge more for your eggs than if your eggs are very much like the rest of their eggs. In equilibrium, there should be a relationship—the most unusual-colored eggs should command higher prices and thereby earn egg traders lower expected rates of return, but egg traders still like them because of the insurance such eggs offer them, within reasonable bounds, of course.

If everyone is willing to pay more for unique eggs, then unique eggs will sell for higher prices and therefore earn a lower expected rate of return.

For stocks, this model is called the CAPM. It says that stocks that earn high rates of returns when the (market) portfolio of other stocks does poorly are more desirable, therefore priced higher, and therefore offer a lower expected rate of return. And this is what we are ultimately really after. As corporate executives, we want to know how our investors are valuing our projects. If our projects earn our investors money when the rests of their portfolio are doing poorly, then our investors will want us to take these projects on their behalves even if our projects have a (reasonably mildly) low expected rate of return. In finance-speak, we should use a lower

For stocks, this model is called the CAPM.

cost of capital for these projects, because they have lower market-betas—market-beta being a measure of the similarities of our projects' rates of return with those of other investments in the market. The CAPM gives us the precise formula that relates the market beta to the cost of capital, because it presumes that it can work out exactly what smart egg traders (market investors) like and dislike.

11·5. Summary

The chapter covered the following major points:

- Figure 11.4 showed an analysis of historical rate of return patterns of stocks, bonds, and cash investments.
  - Stocks, on average, had higher average rates of return than bonds, which in turn had higher average rates of return than cash investments.
  - Individual stocks were most risky. Large stock market portfolios had lower risk than individual stock holdings. Bonds had lower risk yet, and cash was least risky.
- Stocks (and many other investments) tended to correlate: when the stock market overall had a good year, most stocks also had a good year.
- Historical data can help us in predicting the future. It is especially useful and reliable in predicting future risk and correlation.
- Investments revolves around the following concepts:
  1. Investors can reduce their overall portfolio risk by diversifying—holding many different types of investments.
  2. An individual investment is more desirable if it has a higher expected rate of return, and if it has a lower correlation with the investor's overall portfolio.
  3. The CAPM is a model that corporations can use to assess the value of their projects to their investors. It assumes that all investors follow smart investment rules, which allows the CAPM to relate the expected rate of return of each investment to the correlation of each investment with an investors' portfolios.

A. Appendix: Some Background Information About Equities Market Microstructure

The topic of investments traditionally focuses on equities (stocks) more than on other instruments. The main reasons may be that data on stocks are relatively easy to come by and that stocks are simply more interesting from a corporate perspective than many other financial instruments (e.g., foreign government bonds). So it makes sense to describe a few institutional details as to how investors and stocks "connect"—exchange cash for claims.

a. Brokers

Brokers execute and keep track

Most individuals place their orders to buy or sell stocks with a **retail broker**, such as **Ameritrade** (a "deep-discount broker"), **Charles Schwab** (a discount broker), or **Merrill Lynch** (a full service broker). Investors can place either **market orders**, which ask for execution at the current price, or **limit orders**, which ask for execution if the price is above or below a limit that the investor can specify. (There are also many other types of orders, e.g., stop-loss orders [which instruct a broker to sell a security if it has lost a certain amount of money], GTC [good-to-cancel orders], and fill-or-kill orders.) The first function of retail brokers is to execute these trades. They usually do so by routing investors' orders to a centralized trading location (e.g., a particular stock exchange), the choice of which is typically at the retail broker's discretion, as is the particular agent (e.g., floor broker) engaged to execute the trade. The second function of retail brokers is to keep track of investors' holdings, to facilitate purchasing **on margin** (whereby investors can borrow money to purchase stock, allowing them to purchase more securities than they could afford on a purely cash basis), and to facilitate selling securities "short," which allows investors to speculate that a stock will go down.

Prime Brokers break the two main functions apart. They leave execution to others.

Many larger investors break these two functions apart: the investor can employ its own traders, while the broker takes care only of the bookkeeping of the investor's portfolio, margin provision and the shorting provisions. Such limited brokers are called **prime brokers**.

Side Note: Discount brokers may charge only $10 or so per trade, but they often receive kickback payments from the market-maker [see below] to which they route your order. This is called "payment for order flow." The market-maker in turn recoups this payment to the broker by executing your trade at a price that is less favorable. Although the purpose of such an arrangement seems deceptive, the evidence suggests that discount brokers are still often cheaper in facilitating investor trades—especially small investor trades—even after taking this hidden payment into account. They just are not as (relatively) cheap as they want to make you believe.

b. Exchanges and Non-Exchanges

The standard process.

Exchanges are centralized trading locations where financial securities are traded. The two most important stock exchanges in the United States are the **New York Stock Exchange** (**NYSE**) and **Nasdaq** (originally an acronym for "National Association of Securities Dealers Automated Quotation" System). The NYSE is an **auction market**, in which one designated **specialist** (assigned for each stock) manages the auction process by trading with individual floor brokers. The specialist is often a monopolist. In contrast to this human process in one physical location on Wall Street, Nasdaq is a purely electronic exchange without specialists. (For security reasons, its location—well, the location of its computer systems—is secret!) For each Nasdaq stock, there is at least one **market-maker**, who has agreed to continuously stand by to offer to buy or sell shares, thereby creating a liquid and immediate market for the general public. Most Nasdaq stocks have multiple market-makers, drawn from a pool of about 500 trading firms (such as J.P. Morgan or ETrade), which compete to offer the best price. Market-makers have one advantage over the general public: they can see the **limit order book**, which contains as-yet-unexecuted orders from investors to purchase or sell if the stock price changes—giving them a good idea at which price a lot of buying or selling activity will happen. The NYSE is the older exchange, and for historical reasons, controls considerably more trading than Nasdaq, especially when it comes to "blue chip" stocks. ("Blue chip" now means "well established and serious"; ironically, the term itself came from poker, where the highest-denomination chips were blue.) Nasdaq tends to trade smaller and high-technology firms.

Auctions have lower execution costs, but also lower execution speed.

Continuous trading—trading at any moment an investor wants to execute—relies on the presence of the standby intermediaries (specialists or market-makers), who are willing to absorb shares when no one else is available. This is risky business, and thus any intermediary must earn a good rate of return to be willing to do so. To avoid this cost, some countries have organized their exchanges into non-continuous auction systems, which match buy and sell orders a couple of times each day. The disadvantage is that you cannot execute orders immediately but have to delay until a whole range of buy orders and sell orders have accumulated. The advantage is that this eliminates the risk that an (expensive) intermediary would otherwise have to bear. Thus, auctions generally offer lower trading costs but slower execution.

New Alternative Trading Institutions: ECNs and more.

Even in the United States, innovation and change are everywhere. For example, **electronic communications networks** (**ECNs**) have recently made big inroads into the trading business, replacing exchanges especially for large institutional trades. (They can trade the same stocks that exchanges are trading, and compete with exchanges in terms of cost and speed of execution.) An ECN cuts out the specialist, allowing investors to post price-contingent orders themselves. ECNs may specialize in lower execution costs, more broker kickbacks (see the sidenote below), or faster execution. The biggest ECNs are **Archipelago** and **Instinet**. An even more interesting method to buy and trade stocks are **crossing systems**, such as **ITG POSIT**. ITG focuses primarily on matching large institutional trades with one another in an auction-like manner. If no match on the other side is found, the order may simply not be executed. But if a match is made, by cutting out the specialist or market-maker, the execution is a lot cheaper than it would have been on an exchange. Recently, even more novel trading places have sprung up. For example, **Liquidnet** uses peer-to-peer networking—like the original Napster—to match buyers and sellers in real-time. ECNs or electronic limit order books are now the dominant trading systems for equities worldwide, with only the U.S. exchange floors as holdouts. Such mechanisms are also used to trade futures, derivatives, currencies, and even some bonds.

Other markets, especially OTC.

There are many other financial markets, too. There are financial exchanges handling stock options, commodities, insurance contracts, etc. A fascinating segment are the **over-the-counter** (**OTC**) markets. Over-the-counter means "call around, usually to a set of traders well-known to trade in the asset, until you find someone willing to buy or sell at a price you like." Though undergoing rapid institutional change, most bond transactions are still OTC. Although OTC markets handle significantly more bond trading in terms of transaction dollar amounts than exchanges, their transaction costs are prohibitively high for retail investors—if you call without knowing the market in great detail, the person on the other end of the line will be happy to quote you a shamelessly high price, hoping that you do not know any better alternatives. The **NASD** (National Association of Securities Dealers) also operates a semi-OTC market for the stocks of smaller firms, the **pink sheets**. Foreign securities trade on their local national exchanges, but the costs for U.S. retail investors are again often too high to make direct participation worthwhile.

c. How Securities Appear and Disappear

Firms first sell shares in IPOs.

Most publicly traded equities appear on public exchanges through **initial public offerings** (**IPOs**), whereby a privately traded company first sells shares to ordinary investors. IPOs are usually executed by **underwriters** (investment bankers such as **Goldman Sachs** or **Merrill Lynch**), which are familiar with the complex legal and regulatory process and which have easy access to an investor client base to buy the newly issued shares. Shares in IPOs are typically sold at a fixed price—and for about 10% below the price at which they are likely to trade on the first day of after-market open trading. (Many IPO shares are allocated to the brokerage firm's favorite customers, and can be an important source of profit.)

Money flows into the financial markets through IPOs and SEOs.

Usually, about a third of the company is sold in the IPO, and the typical IPO offers shares worth between \$20 million and \$100 million, although some are much larger (e.g., privatizations, like British Telecom). About two-thirds of all such IPO companies never amount to much or even die within a couple of years, but the remaining third soon thereafter offer more shares in **seasoned equity offerings** (**SEO**). These days, however, much expansion in the number of shares in publicly traded companies, especially large companies, comes not from seasoned equity offerings, but from employee stock option plans, which eventually become unrestricted publicly traded shares.

Publicly traded companies must report financials, and restrict insider trading.

In 1933/1934, Congress established the SEC through the *Securities Exchange Acts*. It further regulated investment advisors through the *Investment Advisors Act of 1940*. (The details of these acts can be obtained at the SEC website.) Today, publicly traded companies must regularly report their financials and other information to the SEC, and their executives have **fiduciary obligations** to their shareholders. Generally, the SEC prohibits insider trading on unreleased specific information, although more general trading by insiders is legal (and seems to be done fairly profitably by these insiders).

Money flows out from the financial markets in dividends and share repurchases.

Capital flows out of the financial markets in a number of ways—through dividends and share repurchases, or more dramatically, through delistings and bankruptcies. Many companies pay some of their earnings in **dividends** to investors. Dividends, of course, do not fall like manna from heaven. For example, a firm worth \$100,000 may pay \$1,000, and would therefore be worth \$99,000 after the dividend distribution. If you own a share of \$100, you would own (roughly) \$99 in stock and \$1 in dividends after the payment—still \$100 in total, no better or worse. (If you have to pay some taxes on dividend receipts, you might come out for the worse.) Alternatively, firms may reduce their outstanding shares by paying out earnings in **share repurchases**. For example, the firm may dedicate the \$1,000 to share repurchases, and you could ask the firm to dedicate \$100 thereof to repurchasing your share. But even if you hold onto your share, you have not lost anything. Previously, you owned $\$100/\$100,000 = 0.1\%$ of a \$100,000 company, for a net of \$100. Now, you will own $\$100/\$99,000 = 1.0101\%$ of a \$99,000 company—multiply this to find that your share is still worth \$100. In either case, the value of outstanding public equity in the firm has shrunk from \$100,000 to \$99,000. We shall discuss dividends and share repurchases in Part IV.

Shares can also shrink out of the financial markets in bankruptcies, liquidations, and delistings.

Firms can shrink more drastically, too: some firms voluntarily liquidate, determining that they can pay their shareholders more if they sell their assets and return the money to them. This is rare, because managers usually like to keep their jobs—even if continuation of the company is not in the interest of shareholders. More commonly, firms make bad investments, and fall in value to the point where they are delisted from the exchange and/or go into bankruptcy. Fortunately, investors enjoy **limited liability**, which means that they can at most lose their investments and do not have to pay for any further sins of management.

ANECDOTE: Trading Volume in the Tech Bubble

During the Tech bubble of 1999 and 2000, IPO underpricing reached one-day returns of 65% *on average*. Getting an IPO share allocation was like getting free money. Of course, ordinary investors rarely received any such share allocations—only the underwriter's favorite clients did. This later sparked a number of lawsuits, one of which revealed that **Credit Suisse First Boston** (**CSFB**) allocated shares of IPOs to more than 100 customers who, in return for IPO allocations, funneled between 33 and 65 percent of their IPO profits back to CSFB in the form of excessive trading of other stocks (like Compaq and Disney) at inflated trading commissions.

How important was this "kickback" activity? In the aggregate, in 1999 and 2000, underwriters left about \$66 billion on the table for their first-day IPO buyers. If investors rebated 20 percent back to underwriters in the form of extra commissions, this would amount to \$13 billion in excessive underwriter profits. At an average commission of 10 cents per share, this would require 130 billion shares traded, or an average of 250 million shares per trading day. This figure suggests that kickback portfolio churning may have accounted for as much as 10 percent of all shares traded.

Source: Ritter-Welch (2002).

Solve Now!

Q 11.9 *What are the two main functions of brokerage firms?*

Q 11.10 *How does a prime broker differ from a retail broker?*

Q 11.11 *What is a specialist? What is a market-maker? When trading, what advantage do the two have over you?*

Q 11.12 *Describe some alternatives to the main exchanges.*

Q 11.13 *Describe some mechanisms by which more shares appear and disappear in the market.*

CHAPTER 12

Investor Choice: Risk and Reward

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Our main purpose in this part of the book is to learn how to estimate the corporate cost of capital ($\mathcal{E}(\tilde{r})$) in the NPV Formula. But before we can understand our own projects' opportunity costs of capital, we have to understand the other opportunities that our investors have. To do this, we must undertake a long venture. We must explore more technically what investors like (reward) and dislike (risk), how we should measure this risk and reward, how diversification works, what overall portfolios smart investors should be holding (something close to the market), and why "market-beta" is a good measure of the contribution of an investment to a portfolio's risk.

12·1. Measuring Risk and Reward

History is practically the same as scenarios.

As always, our method is to cook up simple examples first, and then relate our insights into the broader real-world context. So we will follow five different securities through four different scenarios. You will get to decide which investment strategies you deem better or worse, safer or riskier. The goal is to sharpen your understanding of the tradeoff between risk and reward. In Section 12·5, after the meaning of the formulas has become clear, we will repeat all our calculations in a spreadsheet. Indeed, in a spreadsheet, it won't make any difference whether we work with four scenarios or with hundreds of weeks of historical data. All formulas and conclusions will remain the same. The four scenarios in the text just make it easier to explain the meaning of our computations, so that you fully understand what the spreadsheet functions are really computing for you—and why!

12·1.A. Possible Investment Opportunity Returns

Table 12.1. A Collection of Investment Portfolios

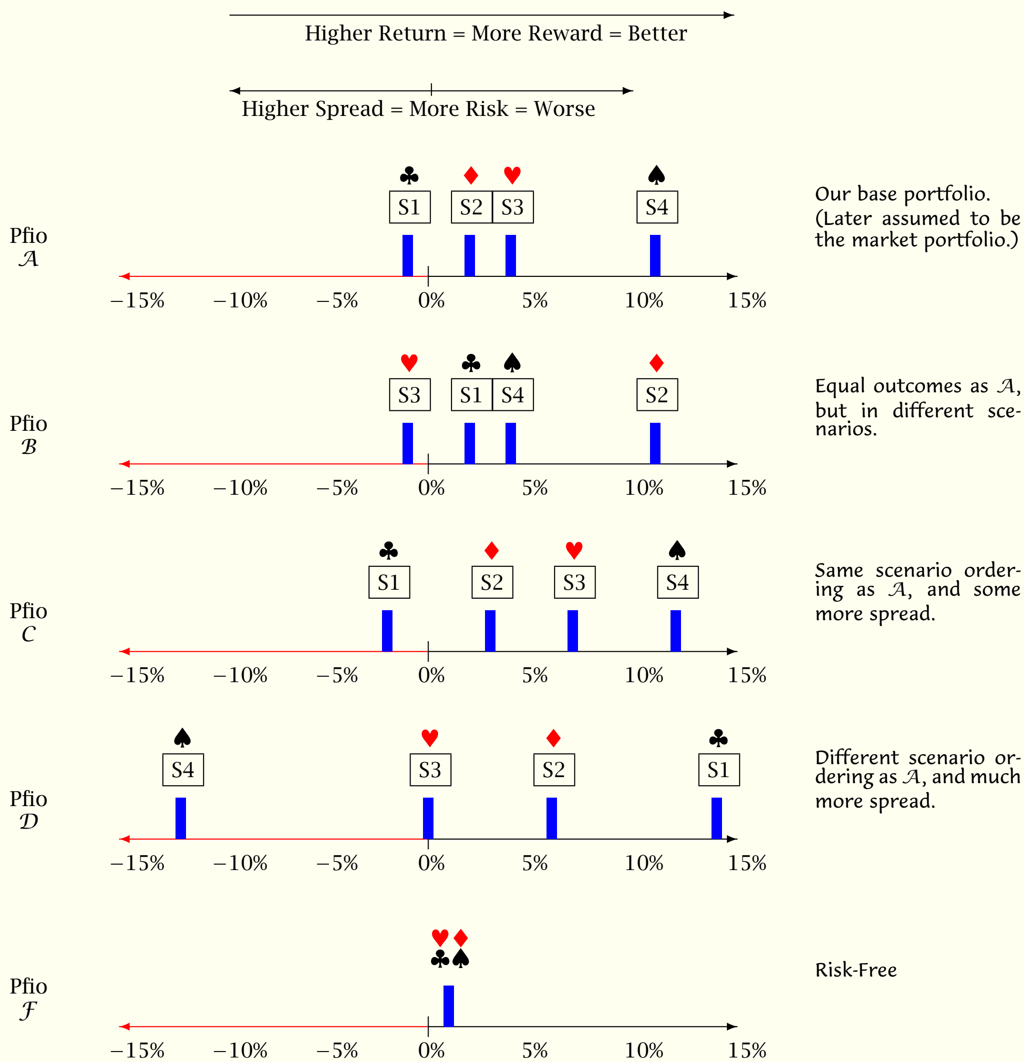
	Portfolio Rates of Return R				
Future	Pfio $\mathcal{A}$ ($\mathcal{M}$)	Pfio $\mathcal{B}$	Pfio $\mathcal{C}$	Pfio $\mathcal{D}$	Pfio $\mathcal{F}$
Scenario S1 ♣	−1.0%	+2.0%	−2.0%	+14.0%	+1.0%
Scenario S2 ♦	+2.0%	+11.0%	+3.0%	+6.0%	+1.0%
Scenario S3 ♥	+4.0%	−1.0%	+7.0%	0.0%	+1.0%
Scenario S4 ♠	+11.0%	+4.0%	+12.0%	−12.0%	+1.0%
"Reward" ($\mathcal{E}(\tilde{r})$)	4.0%	4.0%	5.0%	2.0%	1.0%
$\mathcal{V}ar(\tilde{r})$	19.5%%	19.5%%	26.5%%	90.0%%	0.0%%
"Risk" ($\mathcal{S}dv(\tilde{r})$)	4.42%	4.42%	5.15%	9.49%	0.00%

Variance ($\mathcal{V}ar$) and standard deviation ($\mathcal{S}dv$) are explained in Section 12·1.C.

Plot the outcomes.

Table 12.1 contains rates of return for five hypothetical investments (four risky securities $\mathcal{A}$-$\mathcal{D}$, and one risk-free security $\mathcal{F}$), and we presume we know exactly the probability of each scenario outcome and the return of each portfolio in each scenario. Figure 12.1 graphs these returns. Each outcome is equally likely (the histogram bars are all equally high), so we can just indicate where each scenario lies on the X axis. In this plot, we prefer portfolios that have scenario outcomes further to the right (they have higher rates of return), outcomes that are *on average* further to the right (they have higher *expected* rates of return), and scenario outcomes that are more bunched together (they have less risk; if all outcomes were perfectly bunched at the same spot, they would have zero risk). Visual inspection shows that investment $\mathcal{F}$ is least risky, followed by $\mathcal{A}$ and $\mathcal{B}$, then $\mathcal{C}$, and finally the most risky $\mathcal{D}$.

Figure 12.1. Graphical Perspectives on Performance



Explanation: The graphs are standard histograms. Each outcome is equally likely, so each bar is 25% high—with the exception of the bar in the final graph for the risk-free security, which is 100% high. ♣ is the rate of return outcome in scenario S1, the ♦ in scenario S2, the ♥ in scenario S3, and the ♠ in scenario S4.

12·1.B. Measuring Reward: The Expected Rate of Return

Graphical measures, while helpful, are often less convenient than numerical measures, so we need algebraic risk and reward formulas. The formula for the reward is easy: we can use the **expected rate of return**, the probability-weighted average of all possible returns. For example, the average rate of return for portfolio $\mathcal{A}$ is

We need numerical measures. Reward is easy.

$$\begin{aligned} \mathcal{E}(\tilde{r}_{\mathcal{A}}) &= (1/4)\cdot(-1\%) + (1/4)\cdot(+2\%) + (1/4)\cdot(+4\%) + (1/4)\cdot(+11\%) = +4\% \ . \\ \mathcal{E}(\tilde{r}_{\mathcal{A}}) &= \mathcal{P}rob(S1)\cdot(\tilde{r}_{S1}) + \mathcal{P}rob(S2)\cdot(\tilde{r}_{S2}) + \mathcal{P}rob(S3)\cdot(\tilde{r}_{S3}) + \mathcal{P}rob(S4)\cdot(\tilde{r}_{S4}) \ . \end{aligned} \tag{12.1}$$

Because each outcome is equally likely, this is faster to compute as the average

$$\mathcal{E}(\tilde{r}_{\mathcal{A}}) = \frac{(-1\%) + (+2\%) + (+4\%) + (+11\%)}{4} = 4\% \ . \tag{12.2}$$

So, if we invest in portfolio $\mathcal{A}$, we would expect to earn a rate of return of 4%.

12·1.C. Measuring Risk: The Standard Deviation of the Rate of Return

Risk is more difficult. Here is the variance and the standard deviation.

A good measure of "risk" is less obvious. Figure 12.1 shows that $\mathcal{A}$ is more spread out than $\mathcal{F}$ and less spread out than $\mathcal{D}$. First, we want to rate each data point by how far away it is from the center (average). If our average is +4%, an outcome of 3% is closer than an outcome of 0% would be. (The former is only 1 unit away from the mean; the latter would be 4 units away from the mean.) So, we want to work in deviations,

Outcomes	S1 (♣)	S2 (♦)	S3 (♥)	S4 (♠)
Portfolio $\mathcal{A}$ Rate of Return	−1%	+2%	+4%	+11%
Their Deviations From the 4% Mean	−5%	−2%	+0%	+7%

Unfortunately, the average deviation-from-the-mean would not be a good measure of spread, because it will always be zero—for example, our average here is $(-5-2+0+7)/4 = 0$. Instead, we must compute something like the average *squared*-deviation-from-the-mean, which is called the **variance**.

$$\begin{aligned} \mathcal{V}ar(\tilde{r}_{\mathcal{A}}) &= \frac{(-1\% - 4\%)^2 + (2\% - 4\%)^2 + (4\% - 4\%)^2 + (11\% - 4\%)^2}{N} \\ &= \frac{(-5\%)^2 + (-2\%)^2 + (0\%)^2 + (+7\%)^2}{N} = 19.5\%\% \ , \\ &= \frac{[r_{S1} - \mathcal{E}(\tilde{r})]^2 + [r_{S2} - \mathcal{E}(\tilde{r})]^2 + [r_{S3} - \mathcal{E}(\tilde{r})]^2 + [r_{S4} - \mathcal{E}(\tilde{r})]^2}{N} \\ &= \frac{\sum_{i=1}^{N} [r_{S\,i} - \mathcal{E}(\tilde{r})]^2}{N} \ . \end{aligned} \tag{12.3}$$

The variance has units that are intrinsically difficult to interpret (% *squared* = 0.01·0.01, written as x%%), so the variance carries very little intuition except that a higher variance means more risk. Therefore, a more popular measure of risk is the **standard deviation**. It is just the square root of the variance, so

$$\mathcal{S}dv(\tilde{r}_{\mathcal{A}}) = \sqrt{\mathcal{V}ar(\tilde{r}_{\mathcal{A}})} = \sqrt{19.5\%\%} \approx 4.42\% \ . \tag{12.4}$$

The standard deviation will be our measure of overall portfolio risk. Looking at Figure 12.1, we see that the standard deviation of 4.42% seems like a reasonable measure of how far the typical outcome of $\mathcal{A}$ is away from the mean of $\mathcal{A}$. The last row in Table 12.1 also shows the standard deviations of the other investment portfolios. These standard deviations indeed follow our visual risk perception: $\mathcal{F}$ is risk-free; $\mathcal{A}$ and $\mathcal{B}$ are equally risky at 4.42%; $\mathcal{C}$ is a little more risky at 5.15%; and $\mathcal{D}$ is most risky at 9.49%.

IMPORTANT:

- *We measure our investment "reward" by the expected rate of return on our overall portfolio.*
- *We measure our investment "risk" by the standard deviation of the rate of return on our overall portfolio.*

(Note: we will not measure the investment risk contribution of individual components inside the portfolio via the standard deviation.)

In equilibrium, higher investment risk does not necessarily always lead to higher investment reward—as anyone gambling knows. However, after eliminating all investment mistakes, in order to earn higher rewards, we will have no choice but to take higher risks.

Solve Now!

Q 12.1 *Return to Portfolio $\mathcal{A}$ from Table 12.1. It offers −1%, +2%, +4% and +11%. Now add 5% to each of these returns. This portfolio offers +4%, +7%, +9% and +16%. Compute the expected rate of return, the variance, and the standard deviation of this new portfolio. How does it compare to the original portfolio?*

Q 12.2 *Again, return to Portfolio $\mathcal{A}$ from Table 12.1. Now multiply each return by two. This portfolio offers −2%, +4%, +9% and +22%. Compute the expected rate of return and standard deviation of this new portfolio. How does it compare to the original portfolio?*

12·2. Portfolios, Diversification, and Aggregate Investor Preferences

In the real world, we are usually not constrained to purchase either security $\mathcal{A}$ or security $\mathcal{B}$ in isolation—we can purchase a little of each. This will have the important consequence of reducing our overall portfolio risk. Let's see why.

Start again with investment portfolios $\mathcal{A}$ and $\mathcal{B}$, which offer the same rates of return, but in different future scenarios. If we purchase $100 in either $\mathcal{A}$ or $\mathcal{B}$, we would expect to earn $4 with a risk of $5.10. But what if we purchase $50 in $\mathcal{A}$ and $50 in $\mathcal{B}$? Call this our investment portfolio $\mathcal{P}$. In this case, our $100 investment would turn into

How to compute portfolio combination outcomes.

Scenario Outcome:	S1 (♣)	S2 (♦)	S3 (♥)	S4 (♠)	Average
Return on $50 in $\mathcal{A}$:	$49.50	$51.00	$52.00	$55.50	$52.00
Return on $50 in $\mathcal{B}$:	$51.00	$55.50	$49.50	$52.00	$52.00
⇒ Total Return in $\mathcal{P}$:	$100.50	$106.50	$101.50	$108.00	$104.00
⇒ Rate of Return in $\mathcal{P}$:	0.5%	6.5%	1.5%	7.5%	4.0%

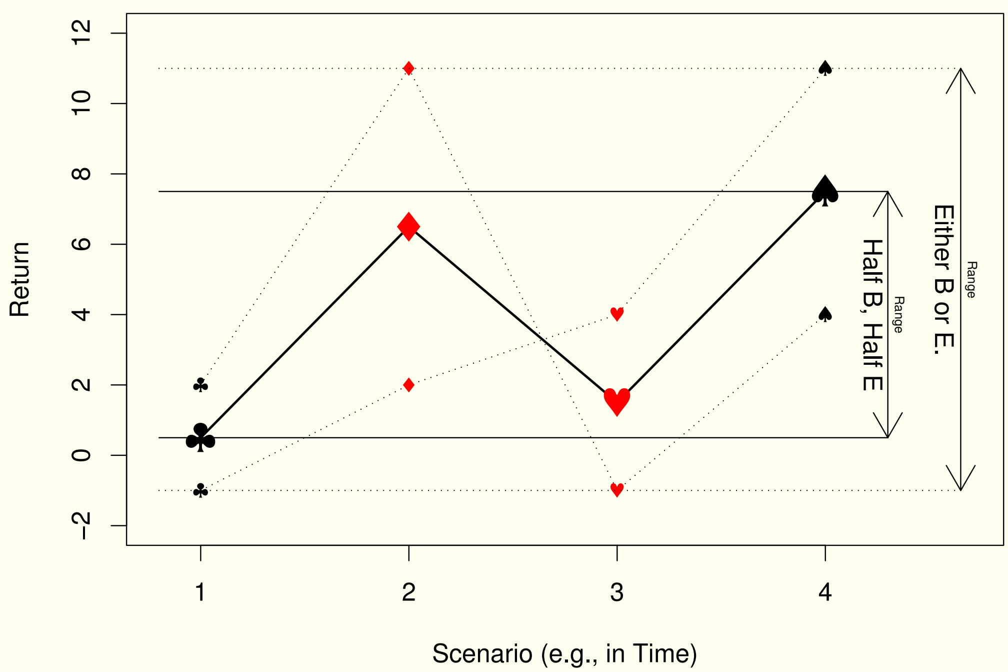
You can do this more quickly by using the returns on $\mathcal{A}$ and $\mathcal{B}$ themselves. In our case, portfolio $\mathcal{P}$ invests $w_{\mathcal{A}} = 50\%$ into $\mathcal{A}$ and $w_{\mathcal{B}} = 50\%$ in $\mathcal{B}$, so its scenario rate of return is always

$$\begin{aligned} \tilde{r}_{\mathcal{P}} &= \tilde{r}_{50\% in \mathcal{A}, 50\% in \mathcal{B}} = 50\% \cdot (\tilde{r}_{\mathcal{A}}) + 50\% \cdot (\tilde{r}_{\mathcal{B}}) \ . \\ \tilde{r}_{\mathcal{P}=(w_1,w_2,\ldots,w_N)} &= w_1 \cdot \tilde{r}_1 + \ldots + w_N \cdot \tilde{r}_N \end{aligned} \tag{12.5}$$

Thus,

$$\begin{array}{lcclclcl}
S1 \clubsuit & : & r_{\mathcal{P}=(50\% \text{ in } \mathcal{A}, 50\% \text{ in } \mathcal{B}) \text{ in S1}} & = & 50\% \cdot (-1\%) & + & 50\% \cdot (+2.0\%) & = 0.5\% \;, \\
S2 \diamondsuit & : & r_{\mathcal{P}=(50\% \text{ in } \mathcal{A}, 50\% \text{ in } \mathcal{B}) \text{ in S2}} & = & 50\% \cdot (+2\%) & + & 50\% \cdot (+11.0\%) & = 6.5\% \;, \\
S3 \heartsuit & : & r_{\mathcal{P}=(50\% \text{ in } \mathcal{A}, 50\% \text{ in } \mathcal{B}) \text{ in S3}} & = & 50\% \cdot (+4\%) & + & 50\% \cdot (-1.0\%) & = 1.5\% \;, \\
S4 \spadesuit & : & r_{\mathcal{P}=(50\% \text{ in } \mathcal{A}, 50\% \text{ in } \mathcal{B}) \text{ in S4}} & = & 50\% \cdot (+11\%) & + & 50\% \cdot (+4.0\%) & = 7.5\% \;, \\
 & & & = & w_{\mathcal{A}} \cdot r_{\mathcal{A} \text{ in S}} & + & w_E \cdot r_{\mathcal{B} \text{ in S}} & .
\end{array} \tag{12.6}$$

Figure 12.2. Return Outcomes for $\mathcal{A}$, $\mathcal{B}$, and the Combination Portfolio



	Portfolio Rates of Return R		
Future	Pfio $\mathcal{A}$	Pfio $\mathcal{B}$	Pfio $\mathcal{P}$
Scenario S1 ♣	−1.0%	+2.0%	+0.5%
Scenario S2 ♦	+2.0%	+11.0%	+6.5%
Scenario S3 ♥	+4.0%	−1.0%	+1.5%
Scenario S4 ♠	+11.0%	+4.0%	+7.5%
"Reward" ($\mathcal{E}(R)$)	4.0%	4.0%	4.0%
"Risk" ($Sdv(\tilde{r})$)	4.42%	4.42%	3.04%

Portfolio $\mathcal{P}$ is half $\mathcal{A}$, half $\mathcal{B}$. Because each half-$\mathcal{A}$/half-$\mathcal{B}$ point is halfway between $\mathcal{A}$ and $\mathcal{B}$, $\mathcal{P}$ has lower spread (risk) than either of its components, $\mathcal{A}$ and $\mathcal{B}$, by itself. (The risk of $\mathcal{A}$ and $\mathcal{B}$ was computed in Formula 12.4.)

We can see the lower risk through dampened oscillations.

Now look at the three investment portfolios next to one another in Figure 12.2. The dotted lines plot the two components, $\mathcal{A}$ and $\mathcal{B}$; the solid line plots the combination portfolio. Even without doing any math, you can see that the $\mathcal{P}$ portfolio is less risky. The range of possible outcomes of portfolio $\mathcal{P}$ is from 0.5% to 7.5%, while the ranges of $\mathcal{A}$ and $\mathcal{B}$ are from −1% to +11%. (If you find it easier to think in terms of history, you can pretend that scenario S1 happened at time 1, S2 at time 2, and so forth.) The range and variability of outcomes are smaller for the combination portfolio $\mathcal{P}$ than they are for each of its components, $\mathcal{A}$ and E.

Compute risk and reward for the combination portfolio: the risk is lower. Diversification can reduce risk!

Can we use our algebraic measures to back up our visual intuition? The average (expected) rate of return of the combination portfolio $\mathcal{P}$ is the same 4% as that of $\mathcal{A}$ and $\mathcal{B}$. The risk of the combination portfolio $\mathcal{P}$ is lower, however, than the risk of either $\mathcal{A}$ or $\mathcal{B}$. In fact, it is

$$\begin{aligned} \mathcal{V}ar_{\substack{50\% \text{ in } \mathcal{A} \\ 50\% \text{ in } \mathcal{B}}} &= \frac{(0.5\% - 4\%)^2 \; + \; (6.5\% - 4\%)^2 \; + \; (1.5\% - 4\%)^2 \; + \; (7.5 - 4\%)^2}{4} \quad = \; 9.25\%\% \\ &= \frac{[r_{S1} - \mathcal{E}(\tilde{r})]^2 \; + \; [r_{S2} - \mathcal{E}(\tilde{r})]^2 \; + \; [r_{S3} - \mathcal{E}(\tilde{r})]^2 \; + \; [r_{S4} - \mathcal{E}(\tilde{r})]^2}{N} \end{aligned} \tag{12.7}$$

$$\Longrightarrow \quad \mathcal{S}dv_{50\% \text{ in } \mathcal{A},\, 50\% \text{ in } \mathcal{B}} \;=\; \sqrt{9.25\%\%} \;=\; 3.04\% \quad . \tag{12.8}$$

An investment in $\mathcal{B}$ also has a risk of 4.42%. But an investment in half of $\mathcal{A}$ and half of $\mathcal{B}$ has a risk of only 3.04%! Why? The reason is **diversification**, the mixing of different investments within a portfolio that reduces the impact of each one on the overall portfolio performance. More simply put, diversification means that not all our eggs are in the same basket. If one investment component goes down, the other investment component at least sometimes happens to go up. The imperfect correlation ("non-synchronicity") has reduced our overall portfolio risk.

12·2.A. Aggregate Investor Preferences

Investors love diversification: almost the more the better.

This intuition suggests that heavily diversified portfolios—portfolios that invest in many different stocks—tend to have lower risk. As corporate managers, we must trust the sanity of our investors. Because diversification helps investors reduce risk, we should reasonably believe that our investors are indeed holding heavily diversified portfolios. The most heavily diversified portfolio contains a little of everything. It is the overall **market portfolio**, consisting of practically all investment opportunities.

Our investors like projects that have high mean and low risk contribution.

As corporate managers, assuming that we can proceed as if investors are holding the market (or something very similar to it) simplifies our task tremendously: instead of asking what each and every investor might possibly like, we can just ask "What makes our investors want us to invest their money for them into our project, given that they currently hold the broad overall stock market portfolio?" The answer will be that our investors like projects that offer more reward (higher expected rates of return) and projects that help them diversify away some of the risk in the market portfolio, so that their overall portfolios end up being less risky. Our corporate management task is to take projects that our investors like to add to their current (market) portfolios. We should therefore search for projects that have high expected rates of return and high diversification benefits with respect to the market. We now turn towards measuring the second characteristic: How can our projects aid our investors' diversification?

IMPORTANT:

- *Diversification among investments reduces the overall portfolio risk.*
- *Diversification could be called imperfect correlation or "non-synchronicity."*
- *Therefore, as corporate managers, we should believe that investors tend to hold diversified portfolios, often heavily diversified portfolios, such as the "entire stock market portfolio."*

Solve Now!

Q 12.3 *What happens if we compute the almost-average deviation from the mean, rather than the almost-average squared deviation from the mean?*

Q 12.4 *What is the risk and reward of a combination portfolio that invests 90% in $\mathcal{A}$ and 10% in $\mathcal{B}$?*

Q 12.5 *Draw the combination portfolio into the figure in Table 12.1. Does it look less spread out?*

12·3. How To Measure Risk Contribution

12·3.A. *Own* Risk is not a Good Measure for Portfolio Risk Contribution

Comovement is the key.

An important insight is that diversification does *not* help if two investment opportunities always move in the same direction. For example, if we try to diversify one \$50 investment in $\mathcal{A}$ with another \$50 investment in $\mathcal{A}$ (which always has the same outcomes), then our risk does not decrease. On the other hand, if two investment opportunities always move in opposite directions, then diversification works extremely well: one is a buffer for the other.

Pretend $\mathcal{A}$ is the market, now called $\mathcal{M}$. Is C better than $\mathcal{D}$, or vice-versa?

Let us formalize this intuition. For explanation's sake, assume that the stock market portfolio held by our investors is $\mathcal{A}$ from Table 12.1, so we rename it $\mathcal{M}$ (for "market"). Return to our investment projects C and $\mathcal{D}$, and assume that you cannot choose both. C offers not only higher expected rate of return than $\mathcal{D}$ (5% vs. 2%) but also lower risk (5.15% vs. 9.49%). As a manager, acting on behalf of your investors, project C is automatically better for them than $\mathcal{D}$. It offers higher reward and less risk. Right?

The combination $C + \mathcal{M}$ has almost the same risk as $\mathcal{M}$

Wrong! (It was a trick question.) Let's assume that our investors start out with the market portfolio. Table 12.3 shows what happens if our investors sell half of their $\mathcal{M}$ portfolio to invest in either C or $\mathcal{D}$. We call these two 50-50 portfolios $\mathcal{M}\&C$ and $\mathcal{M}\&\mathcal{D}$, respectively. Start with $\mathcal{M}\&C$. If our investors reallocate half their money from $\mathcal{M}$ into C, their portfolio $\mathcal{M}\&C$ would have rates of return of

Scenario Outcome:	S1	S2	S3	S4	Reward	Risk
$\mathcal{M}\&C$:	−1.5%	+2.5%	+5.5%	+11.5%	4.5%	4.74%

(The exercises ask you to confirm the mean and standard deviation for yourself.) The left graph in Figure 12.3 plots the $\mathcal{M}\&C$ rates of return, plus the rates of return for both $\mathcal{M}$ and C by themselves. There is not much risk change in moving from a pure $\mathcal{M}$ portfolio to the $\mathcal{M}\&C$ portfolio. The risk actually increases from 4.42% to 4.74%.

The combination $\mathcal{D} + \mathcal{M}$ has much lower risk than the $\mathcal{M}$

Now consider the combination of $\mathcal{M}\&\mathcal{D}$. If our investors instead reallocate half of their wealth from $\mathcal{M}$ into $\mathcal{D}$, their portfolio $\mathcal{M}\&\mathcal{D}$ would have rates of return of

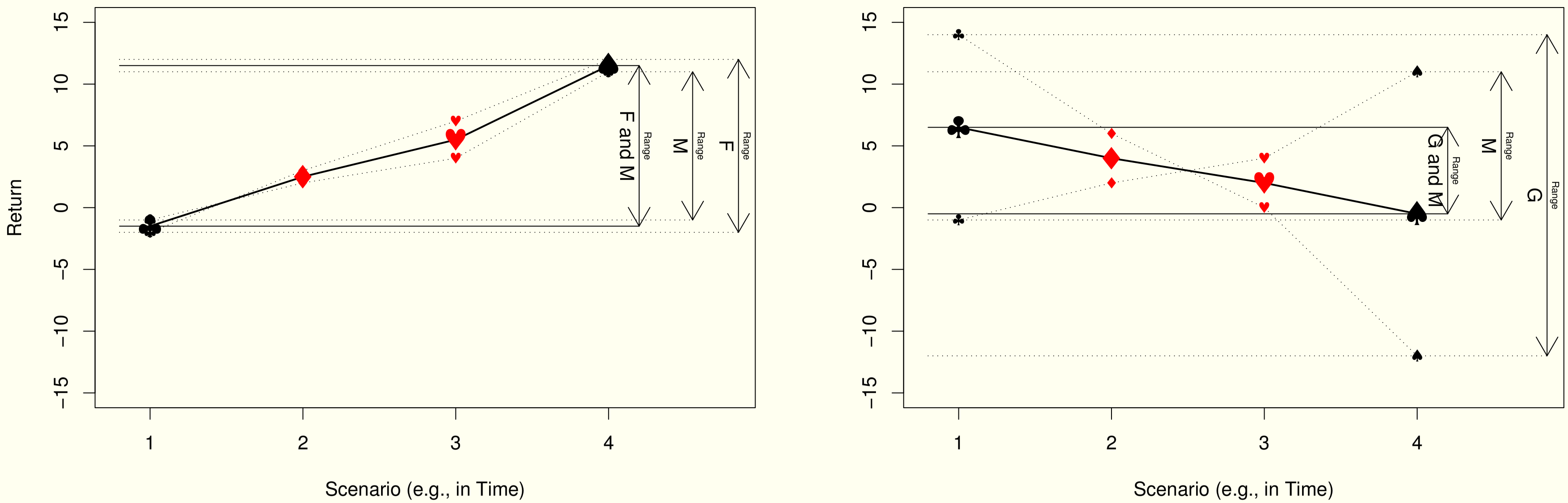
Scenario Outcome:	S1	S2	S3	S4	Reward	Risk
$\mathcal{M}\&\mathcal{D}$:	+6.5%	+4.0%	+2.0%	−0.5%	3.0%	2.57%

The lower graph in Figure 12.3 plots the $\mathcal{M}\&\mathcal{D}$ rates of returns. While the left graph shows that the $\mathcal{M}\&C$ portfolio is about as risky as the $\mathcal{M}$ portfolio, the right graph shows that the $\mathcal{M}\&\mathcal{D}$ portfolio combination portfolio is much safer—even though $\mathcal{D}$ by itself is very risky. Our algebraic risk measure, the standard deviation, confirms this: even though $\mathcal{D}$ by itself is the riskiest choice, adding it to our $\mathcal{M}$ portfolio has reduced our investors' risk from 4.42% to 2.57%, which is an enormous amount. Apparently, $\mathcal{D}$'s own higher standard deviation (9.49%)

Figure 12.3. Combining $\mathcal{M}$ with either C or $\mathcal{D}$

$\mathcal{M}$, C, or $\mathcal{M}\&C$ Combination: Little Risk Reduction

$\mathcal{M}$, $\mathcal{D}$, or $\mathcal{M}\&\mathcal{D}$ Combination: Much Risk Reduction



	Portfolio Rates of Return R				
Future	Pfio $\mathcal{M}$ (=$\mathcal{A}$)	Pfio C	Pfio $\mathcal{D}$	$\mathcal{M}\&C$	$\mathcal{M}\&\mathcal{D}$
Scenario S1 ♣	−1.0%	−2.0%	+14.0%	−1.5%	+6.5%
Scenario S2 ♦	+2.0%	+3.0%	+6.0%	+2.5%	+4.0%
Scenario S3 ♥	+4.0%	+7.0%	0.0%	+5.5%	+2.0%
Scenario S4 ♠	+11.0%	+12.0%	−12.0%	+11.5%	−0.5%
"Reward" ($\mathcal{E}(R)$)	+4.00%	+5.00%	+2.00%	+4.50%	+3.00%
"Risk" ($Sdv(R)$)	4.42%	5.15%	9.49%	4.74%	2.57%

than $\mathcal{C}$'s (5.15%) is not a good indication of whether $\mathcal{D}$ helps investors reduce portfolio risk more or less than $\mathcal{C}$, *if* our investors are holding a lot of $\mathcal{M}$. Thus, as managers, we cannot say that our investors would prefer us to invest in the low-risk project $\mathcal{C}$ over the high-risk project $\mathcal{D}$. What seems higher-risk to us (in itself) allows our owners to manufacture for themselves lower-risk portfolios. Whether we should choose project $\mathcal{C}$ or project $\mathcal{D}$ therefore depends on how our investors would like to trade off the overall reward against the overall risk, and this overall risk depends not only on our project but also on the other investments that our investors are holding. As managers, we cannot yet determine whether $\mathcal{M}$&$\mathcal{C}$ is better or worse for our investors than $\mathcal{M}$&$\mathcal{D}$. We can summarize our findings,

	Reward	Risk	Note
$\mathcal{M}$ ($\mathcal{A}$) alone	4.00%	4.42%	The portfolio that our investors were holding.
$\mathcal{C}$ alone	5.00%	5.15%	$\mathcal{C}$ is less risky than $\mathcal{D}$, if purchased by itself.
$\mathcal{D}$ alone	2.00%	9.49%	
Half $\mathcal{M}$, Half $\mathcal{C}$	4.50%	4.74%	If $\mathcal{C}$ is added to $\mathcal{M}$, portfolio risk barely goes down,
Half $\mathcal{M}$, Half $\mathcal{D}$	3.00%	2.57%	but if $\mathcal{D}$ is added to $\mathcal{M}$, portfolio risk goes down dramatically!

IMPORTANT: *A project's (own) standard deviation is not a good measure of how it helps to reduce the risk in your investors' portfolios. Indeed, it is possible that a project with a very high standard deviation may actually help lower the overall portfolio risk.*

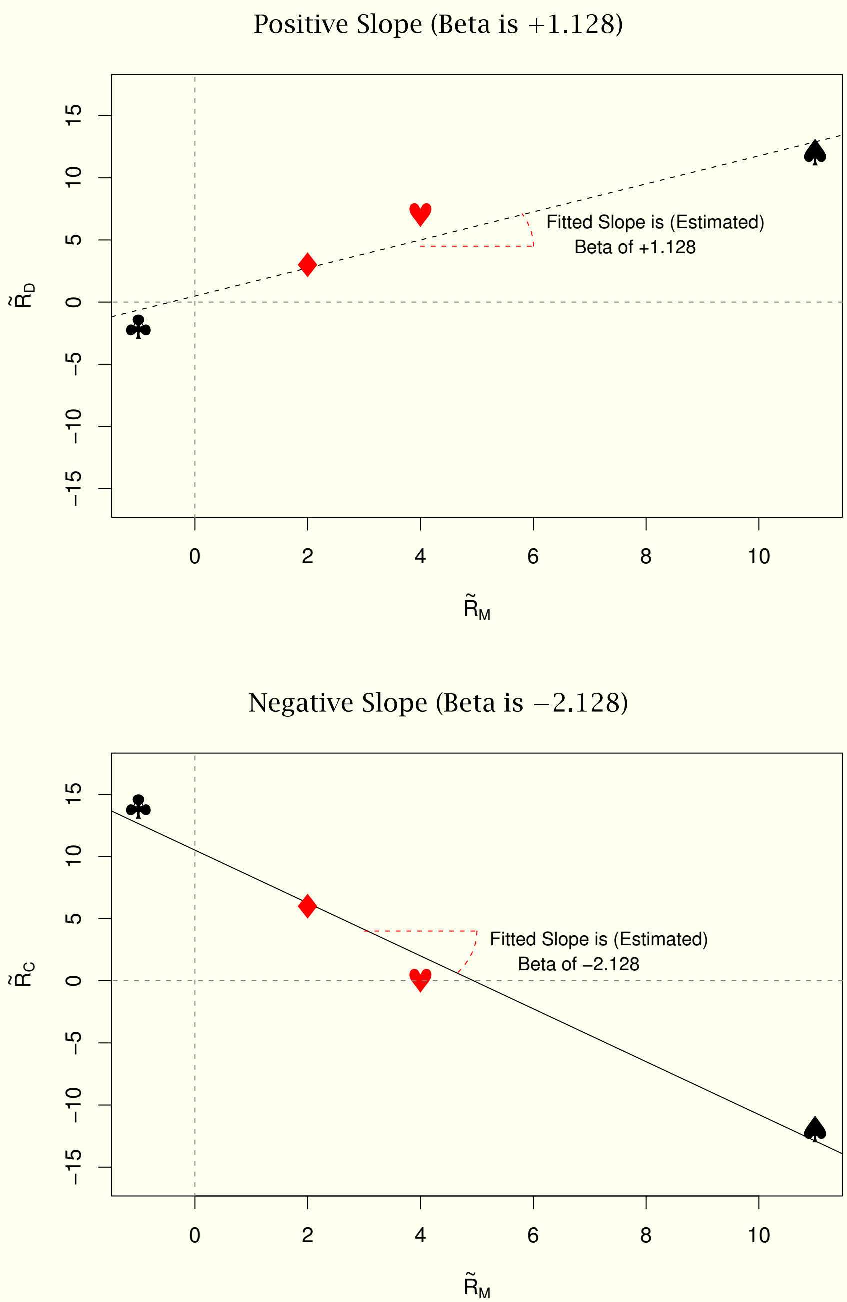
Before we explain it in the next section, can you guess why $\mathcal{D}$ is so much better than $\mathcal{C}$ in reducing the risk in combination with the $\mathcal{M}$ portfolio?

Solve Now!

Q 12.6 *Compute the risk of the $\mathcal{M}$&$\mathcal{C}$ and $\mathcal{M}$&$\mathcal{D}$ combination for yourself.*

12·3.B. Beta is a Good Measure for Portfolio Risk Contribution

Figure 12.4. Possible Outcomes: Rates of Return vs. Market Rate of Return



These are again four data points from Table 12.1: returns on the portfolios $\mathcal{M}$, $\mathcal{C}$, and $\mathcal{D}$. These rates of return are quoted in percent. In our idealized example, we know that these are the four true possible outcomes. In the real world, if the four points were not the true known outcomes, but just the historical outcomes (sample points), then the slope would not be the true unknown beta, but only the "estimated" beta.

$\mathcal{D}$ reduces $\mathcal{M}$'s risk so much because it moves in opposite direction.

We will now answer the question with which we ended the previous section: If an investor already holds $\mathcal{M}$, why does an investment in $\mathcal{D}$ reduce the overall risk so much?

Comovement can be measured by a slope.

The reason is that $\mathcal{D}$ tends to go up if $\mathcal{M}$ tends to go down, and vice-versa. The same cannot be said for C—it tends to move together with $\mathcal{M}$—so C does not help investors very much in their quests to reduce market risk. Figure 12.4 shows this comovement graphically. If you draw the best fitting line between $\mathcal{M}$ and C, the line slopes up. This means that C tends to be higher when $\mathcal{M}$ is higher. But if you draw the best fitting line between $\mathcal{M}$ and $\mathcal{D}$, the line slopes down. This means that $\mathcal{D}$ tends to be higher when $\mathcal{M}$ is lower, and vice-versa. This slope is our measure of expected comovement or countermovement—how much diversification benefit we can obtain from adding a particular new project. A higher slope means more comovement and less diversification; a lower or even negative slope means less comovement and more diversification.

The slope is called beta.

The slope of a line is generally called a beta, because it is common to write the formula for a line as

$$y = \alpha + \beta \cdot x \quad . \tag{12.9}$$

A beta of 1 is a diagonal line, a beta of 0 is a flat line, a positive beta slopes up, a negative beta slopes down, and a beta of infinity is a vertical line. We are interested in a particular beta, the **market beta**, because we can posit that smart investors will hold something very close to it. (The market portfolio is of course highly diversified. To find the market-beta, we draw the rate of return on $\mathcal{M}$ on the X-axis (hence the prefix "market" in market beta) and the rate of return on our project (here, either C or $\mathcal{D}$) on the Y-axis. The market beta of C ($\beta_{C,\mathcal{M}}$) is positive, whereas the market beta of $\mathcal{D}$ ($\beta_{\mathcal{D},\mathcal{M}}$) is negative. In Section 12·3.C, you will get to compute these market betas yourself, but for now trust me that the two lines are

$$\begin{aligned} \tilde{r}_C &\approx 0.49\% + (+1.128) \cdot \tilde{r}_{\mathcal{M}} \\ \tilde{r}_{\mathcal{D}} &\approx 10.51\% + (-2.128) \cdot \tilde{r}_{\mathcal{M}} \\ \tilde{r}_i &= \alpha_{y,\mathcal{M}} + \beta_{y,\mathcal{M}} \cdot \tilde{r}_{\mathcal{M}} \quad . \end{aligned} \tag{12.10}$$

The subscripts on the betas remind you what the variables on the x-axis and the y-axis are. The first subscript is on the y axis, the second is on the x axis. So $\beta_{C,\mathcal{M}} = 1.128$ and $\beta_{\mathcal{D},\mathcal{M}} = -2.128$. In fact, market beta plays such an important role in finance that the term beta itself has become a common abbreviation for market beta, and the second subscript is often omitted.

IMPORTANT:

- *Diversification works better if the new investment project tends to move in the opposite direction of the rest of the portfolio than if it tends to move in the same direction.*
- *It is often reasonable to assume that investors in the market are already holding the market portfolio, and are now considering buying an additional investment—our firm's new project.*
- *If this new investment has a negative beta with respect to the market (its "market-beta"), it means that we expect this investment to go down when the market goes up, and vice-versa.*

  If this new investment has a positive beta with respect to the market, it means that we expect this investment to move together with the market.

  If this new investment has a zero beta with respect to the market, it means that this investment moves for all practical purposes independently of the market.
- *The market-beta of a new investment is a good measure of the investment's risk contribution to an investor who holds the market portfolio. The lower (or negative) the market-beta, the more this investment helps reduce our investor's risk.*
- *The market-beta of an investment can be interpreted as a line slope, where the rate of return on the market is on the X axis and the rate of return on the new investment is on the Y axis. The line states how we expect the new investment to perform as a function of how the market will perform.*
- *Market-beta will become our measure of project risk <u>contribution</u> of an individual project within a larger portfolio. Risk-averse investors that are holding the market portfolio will agree to pay more for investments that have lower market-betas.*

From the corporate perspective of our publicly traded company, it is reasonable to assume that our investors are holding the market portfolio, and our new project is just a tiny new additional component of their overall investments. This "smart investor" assumption is indeed the assumption that we will maintain. However, if our investors were not to hold something close to the market portfolio, then market beta would *not* be a good measure of project risk contribution. Such a situation could be likely if, for example, investors were forced to purchase only our project and no other assets—a situation that often applies to entrepreneurs who have no choice but to put all their money on one egg—and then they would care about the project's standard deviation, not the project's market beta.

Alpha has meaning, too, even though we won't use it.

SIDE NOTE: The intercept (the alpha in Formula 12.9, the 4.87% and 10.51% in Formulas 12.10) also plays an interesting role. Unlike beta, alpha does not measure diversification. However, both alpha and beta figure into how attractive an investment is. For example, if the rate of return on the market is 10%, Formula 12.10 tells us that we would expect the rate of return on $\mathcal{D}$ to be

$$\mathcal{E}(\tilde{r}_{\mathcal{D}})\,|\,\text{if } \tilde{r}_{\mathcal{M}} = 10\% \quad \approx \quad 10.51\% \; + \; (-2.128)\cdot 10\% \quad \approx \quad 10.3\% \; . \tag{12.11}$$

The higher the alpha, the better the investment. Just as investment professionals often call the market-beta just beta, they often call this one intercept just alpha—although there is one small difference: they usually will do this estimation after first subtracting the risk-free interest rate from both $\tilde{r}_{\mathcal{D}}$ and $\tilde{r}_{\mathcal{M}}$.

12·3.C. Computing Betas from Rates of Returns

We need to know how to take historical data and find the beta

We now know what the market beta means, but how can we actually compute it? Let me show you how to compute a market beta for our "four scenarios" example from Table 12.1, specifically the market beta of portfolio C (which you already know is +1.128).

To compute beta, follow a simple four-step procedure.

Fortunately, the procedure to compute a beta is only tedious, but not mysterious.

First, subtract the average from each observation.

1. As with our variance computation, we first translate all returns into deviations from the mean. That is, for each series (from Table 12.1 on Page 284), subtract its mean from every realization.

	Original Rates of Return			Demeaned Rates of Return		
Future	Pfio $\mathcal{M}$	Pfio C	Pfio $\mathcal{D}$	Pfio $\mathcal{M}$	Pfio C	Pfio $\mathcal{D}$
Scenario S1 ♣	−1.0%	−2.0%	+14.0%	−5.0%	−7.0%	+12.0%
Scenario S2 ♦	+2.0%	+3.0%	+6.0%	−2.0%	−2.0%	+4.0%
Scenario S3 ♥	+4.0%	+7.0%	0.0%	0.0%	+2.0%	−2.0%
Scenario S4 ♠	+11.0%	+12.0%	−12.0%	+7.0%	+7.0%	−14.0%
"Reward" ($\mathcal{E}(R)$)	4.00%	5.00%	2.00%	0.00%	0.00%	0.00%
"Risk" ($Sdv(R)$)	4.42%	5.15%	9.49%	4.42%	5.15%	9.49%

The variance is the average product squared.

2. Compute the variance of the series on the x axis, which here is the variance of the rates of return on $\mathcal{M}$. We already have the demeaned $\mathcal{M}$, so this is easy.

$$\begin{aligned} \mathcal{V}ar(\tilde{r}_{\mathcal{M}}) &= \frac{(-5\%)^2 \;+\; (-2\%)^2 \;+\; 0^2 \;+\; (7\%)^2}{4} \;=\; 19.5\%\% \;=\; 0.195\% \;. \\ &= \frac{\sum_{i=1}^{N}[x_i - \mathcal{E}(x_i)]^2}{N} \;. \end{aligned} \tag{12.12}$$

We do not need to compute the variances of C or $\mathcal{D}$.

The covariance is the average cross-product.

3. Compute the average product of the demeaned variables. In our case, we want to compute the market beta for C, so we work with the rates of return on $\mathcal{M}$ and C.

$$\begin{aligned} Cov(\tilde{r}_{\mathcal{M}}, \tilde{r}_C) &= \frac{(-5\%)\cdot(-7\%) \;+\; (-2\%)\cdot(-2\%) \;+\; (0)\cdot(+2\%) \;+\; (+7\%)\cdot(+7\%)}{4} \\ &= 22\%\% \;=\; 0.22\% \\ &= \frac{\sum_{i=1}^{N}[x_i - \mathcal{E}(x_i)]\cdot[y_i - \mathcal{E}(y_i)]}{N} \;. \end{aligned} \tag{12.13}$$

This statistic is called the **covariance** between the rates of return on $\mathcal{M}$ and C. (Incidentally, if you look at the definition, you can see that the covariance of a variable with itself *is* the variance.)

Knowing how to compute variances and covariances is necessary when we will want to compute certainty equivalents (Section A).

Beta is the covariance divided by the variance.

4. The beta of C with respect to the market $\mathcal{M}$, formally $\beta_{C,\mathcal{M}}$ but often abbreviated as β_C, is the ratio of these two quantities,

$$\begin{aligned} \beta_{C,\mathcal{M}} \;\equiv\; \beta_C &= \frac{0.22\%}{0.195\%} \;\approx\; 1.128 \\ &= \frac{Cov(\tilde{r}_{\mathcal{M}}, \tilde{r}_C)}{\mathcal{V}ar(\tilde{r}_{\mathcal{M}})} \;. \end{aligned} \tag{12.14}$$

This slope of 1.128 (a little more than the diagonal) is exactly the market beta we drew in Figure 12.4. Again, it is important that you remember the order of the two variables, because $\beta_{i,\mathcal{M}}$ is not the same as $\beta_{\mathcal{M},i}$: the first correctly divides by the variance of the rate of return on the market, while the second would incorrectly divide by the variance of the rate of return on our new investment project i. Many spreadsheets and statistical programs can compute beta for you: they call the routine that does this a **linear regression**, and the number you want is called the slope. You should always think of the beta of a security i with respect to a portfolio P, written as $\beta_{i,\mathcal{P}}$ as a measure characteristic of security i relative to portfolio $\mathcal{P}$. $\mathcal{P}$ is on the X-axis, i is on the Y-axis. (As we stated earlier, most often—but not always—the portfolio $\mathcal{P}$ will be the market portfolio, $\mathcal{M}$, so $\beta_{i,\mathcal{M}}$ is often just called the market beta, or even just the beta.) This will become clearer soon.

Confirmed!

Now that you know how to compute betas and covariances, you can think of simple scenarios. For example, you might guess that your project will have a rate of return of −5% if the market returns −10%; a rate of return of +5% if the market returns +5%; and a rate of return of 30% if the market returns 10%—and now you know how to compute a market beta or a market covariance for such projects!

Why torture you with computations?

In the real world, we sometimes think in terms of such scenarios, but more often we have historical rates of returns for the overall stock market and for our project (or similar projects). Fortunately, as we noted upfront, the calculations are exactly the same. In effect, when we use historical data, we presume that each period was one representative scenario draw. There are two real-world complications you should worry about: First, how much data should you use? Most researchers tend to use 3–5 years of weekly or monthly historical rate of return data. This is a tradeoff between having enough data and not going too far back into history that may be more and more irrelevant. Also, the frequency with which we sample (annual, monthly, or weekly) data can influence our estimate. Second, you want to know the true future market beta, not the historical market beta—how your project will covary with the market, not how it has covaried with the market. Because history is sometimes deceptive, it is common practice to take the beta we would estimate and "shrink" it towards a beta of 1. For example, in the simplest such shrinker, if you computed a historical market beta of 4, you would predict a future market beta of about $(4 + 1)/2 = 2.5$. In any case, most executives would start with an estimated beta from historical returns data, and then use their intuitive judgment to adjust it.

How to compute market betas in the real world.

DIGGING DEEPER: *The formula for the variance of a portfolio $\mathcal{P}$ based on two components, say $\mathcal{A}$ and $\mathcal{D}$, is*

$$\mathcal{V}ar(\tilde{r}_{\mathcal{P}}) \;=\; w_{\mathcal{A}}^2 \cdot \mathcal{V}ar(\tilde{r}_{\mathcal{A}}) \;+\; w_{\mathcal{D}}^2 \cdot \mathcal{V}ar(\tilde{r}_{\mathcal{D}}) \;+\; 2 \cdot w_{\mathcal{A}} \cdot w_{\mathcal{D}} \cdot Cov(\tilde{r}_{\mathcal{A}}, \tilde{r}_{\mathcal{D}}) \tag{12.15}$$

(To confirm, if you refer to the Excel table at the end of the chapter, the variance of a portfolio that invests half in $\mathcal{A}$ and half in $\mathcal{D}$ is

$$\begin{aligned} \mathcal{V}ar(\tilde{r}_{\mathcal{P}}) &= (1/2)^2 \cdot 19.50\%\% \;+\; (1/2)^2 \cdot (90.00\%\%) \;+\; 2 \cdot (1/2) \cdot (1/2) \cdot (-0.42\%) \;\approx\; 25\% \\ \mathcal{V}ar(\tilde{r}_{\mathcal{P}}) &= w_{\mathcal{A}}^2 \cdot \mathcal{V}ar(\tilde{r}_{\mathcal{A}}) \;+\; w_{\mathcal{D}}^2 \cdot \mathcal{V}ar(\tilde{r}_{\mathcal{D}}) \;+\; 2 \cdot w_{\mathcal{A}} \cdot w_{\mathcal{D}} \cdot Cov(\tilde{r}_{\mathcal{A}}, \tilde{r}_{\mathcal{D}}) \;, \end{aligned} \tag{12.16}$$

which is what we had already computed on Page 292.)

Let's use this formula to show that if you add just a little bit of a new project or stock $\mathcal{D}$ to the market portfolio, then it is only the market beta that matters in changing your risk. So let $\mathcal{A}$ be the market, divide both sides by by $\mathcal{V}ar(\tilde{r}_{\mathcal{M}})$, and use the definition of beta,

$$\begin{aligned} \frac{\mathcal{V}ar(\tilde{r}_{\mathcal{P}})}{\mathcal{V}ar(\tilde{r}_{\mathcal{M}})} &= w_{\mathcal{M}}^2 \;+\; w_{\mathcal{D}}^2 \cdot \frac{\mathcal{V}ar(\tilde{r}_{\mathcal{D}})}{\mathcal{V}ar(\tilde{r}_{\mathcal{M}})} \;+\; 2 \cdot w_{\mathcal{M}} \cdot w_{\mathcal{D}} \cdot \frac{Cov(\tilde{r}_{\mathcal{M}}, \tilde{r}_{\mathcal{D}})}{\mathcal{V}ar(\tilde{r}_{\mathcal{M}})} \\ &= w_{\mathcal{M}}^2 \;+\; w_{\mathcal{D}}^2 \cdot \frac{\mathcal{V}ar(\tilde{r}_{\mathcal{D}})}{\mathcal{V}ar(\tilde{r}_{\mathcal{M}})} \;+\; 2 \cdot w_{\mathcal{M}} \cdot w_{\mathcal{D}} \cdot \beta_{\mathcal{D},\mathcal{M}} \;. \end{aligned} \tag{12.17}$$

This says that the new portfolios $\mathcal{P}$'s relative risk increase depends on the variance of the new investment $\mathcal{D}$ and its market-beta $\beta_{\mathcal{D},\mathcal{M}}$. But if we just add a little bit of $\mathcal{D}$, then $w_{\mathcal{M}}$ would be just a little below 1, $w_{\mathcal{D}}$ would be just a little above 0, and $w_{\mathcal{M}}^2$ would for all practical purposes become zero. So,

$$\begin{aligned} \frac{\mathcal{V}ar(\tilde{r}_{\mathcal{P}})}{\mathcal{V}ar(\tilde{r}_{\mathcal{M}})} &= w_{\mathcal{M}}^2 \;+\; w_{\mathcal{D}}^2 \cdot \frac{\mathcal{V}ar(\tilde{r}_{\mathcal{D}})}{\mathcal{V}ar(\tilde{r}_{\mathcal{M}})} \;+\; 2 \cdot w_{\mathcal{M}} \cdot w_{\mathcal{D}} \cdot \beta_{\mathcal{D},\mathcal{M}} \\ &\approx 1 \;+\; 0 \;+\; (2 \cdot w_{\mathcal{D}}) \cdot \beta_{\mathcal{D},\mathcal{M}} \end{aligned} \tag{12.18}$$

This says that as long as you add just a little of $\mathcal{D}$ to the market portfolio, your new revised portfolio's risk changes (relative to the market portfolio) first and foremost according to the market beta of $\mathcal{D}$.

12·3.D. Beta and Correlation

Covariance and Beta (and Correlation, too!): Always the same sign.

There is a close family relationship between covariance, beta, and correlation. The variance is always positive, because it is the average of squared (i.e., positive) quantities. Thus, if the covariance is positive, so is the beta; if the covariance is negative, so is the beta; and if the covariance is zero, so is the beta. Although we rarely need correlations in finance, the correlation between two variables also always has the same sign as covariance and beta: this is because the correlation is the covariance divided by the square-root of the two multiplied variances:

$$\begin{aligned} Correlation_{i,\mathcal{M}} &= \frac{0.22\%}{\sqrt{0.195\% \cdot 0.265\%}} \;\approx\; 0.97 \\ &= \frac{Cov(\tilde{r}_{\mathcal{M}}, \tilde{r}_{\mathcal{D}})}{\sqrt{\mathcal{V}ar(\tilde{r}_{\mathcal{M}}) \cdot \mathcal{V}ar(\tilde{r}_{\mathcal{D}})}} \;. \end{aligned} \tag{12.19}$$

The nice thing about the correlation is that it has no scale and is always between –100% and +100%. Two variables that have a correlation of 100% always perfectly move in the same direction, two variables that have a correlation of –100% always perfectly move in the opposite direction, and two variables that are independent have a correlation of 0%. This makes the correlation very easy to interpret. The not so nice thing about correlation is that it has no scale and is always between –100% and +100%. This means that an investment that is a million times bigger (multiplies a project return a million times) retains the same correlation with the market. Therefore, this investment would go up or down with any slight tremor in the market a million times more, which would of course mean that this investment would be much riskier. Fortunately, beta takes care of this—indeed, the beta would be a million times larger. This is why we prefer beta over correlation as a measure of risk contribution to a portfolio.

12·3.E. Typical Stock Betas and Interpreting Their Meanings

Beta works so well because investors add only a little of our project.

The market beta is the best measure of "diversification help" for an investor who holds the stock market portfolio and considers adding *just a little* of our firm's project. For the market overall, this is a reasonable assumption. Recall that we assume that our investors are diversified, holding the stock market portfolio. To get our market investors to like a \$10 million project, we just need the average investor to want to buy \$10 million divided by about \$10 trillion [the stock market capitalization], which is $1/1,000,000$ of their portfolios. For our investors, our corporate project is just a tiny addition to their market portfolios.

Table 12.2. Some Betas and Market Capitalizations, from January 2004

Company	Ticker	Beta	Mkt Cap	Company	Ticker	Beta	Mkt Cap
AMD	AMD	2.7	5.4	Liberty Media	L	1.9	38.6
Agilent Tech	A	2.5	16.2	Microsoft	MSFT	1.7	302.9
Barnes Group	B	0.2	0.7	Inco Ltd	N	1.5	7.3
Citigroup	C	1.4	261.4	Realty Income	O	–0.1	1.5
Dominion Resources	D	0.2	20.5	Sears, Roebuck	S	0.5	12.1
ENI SpA	E	0.2	77.2	AT&T	T	0.8	16.1
Entremed	ENMD	2.1	0.2	Vivendi Universal	V	1.9	28.5
Ford Motor	F	1.3	30.1	U.S. Steel	X	1.9	3.8
Gilette	G	0.3	36.7	Alleghany Corp	Y	0.1	1.8
Intel	INTC	2.1	206.6	Starbucks	SBUX	0.6	14.1
Kellogg Co	K	0.0	15.4	Sony	SNE	1.1	38.2

"Mkt cap" is equity stock market value in billion dollars. Betas were reported by Yahoo!*Finance*, and explained as follows:

> The Beta used is Beta of Equity. Beta is the monthly price change of a particular company relative to the monthly price change of the S&P500. The time period for Beta is 5 years when available, and not less than 2.5 years. This value is updated monthly.

Note that Yahoo!*Finance* seems to ignore dividends, but this usually makes little difference.

Financial websites inform of beta.

You can look up the market betas of publicly traded stocks on financial websites. For example, Yahoo!*Finance* quotes the beta in IBM's profile (biz.yahoo.com/p/i/ibm.html) to be 1.5. Table 12.2 lists the betas of some randomly chosen companies. Typical company betas are in the range of around 0 to about 2.5. A beta above 1 is considered risk-increasing for an investor holding the overall stock market (it is riskier than the stock market itself), while a beta below 1 is considered risk-reducing.

An incremental interpretation of beta.

Market beta has yet another nice intuitive interpretation: it is the amount by which the firm value tends to change incrementally if the stock market changes incrementally. So, Vivendi's beta of 1.9 says that if the stock market will return an extra 5% next year (above and beyond expectations), Vivendi will return an extra $1.9 \cdot 5\% = 9.5\%$ (above and beyond expectations). Of course, beta is not a measure of how good an investment Vivendi is: it could be that the expectation for the stock market is 8% and the expectation for Vivendi is 2%. Now, presume that these are the means. Then, if the stock market returns 5% (3% below expectation), then we would expect Vivendi to return –3.7% ($3 \cdot 1.9 = 5.7\%$ below expectations). If the stock market returns 15% (7% above expectation), then we would expect Vivendi to return 15.3% ($7 \cdot 1.9 = 13.3\%$ above expectation). The high beta suggests that if you hold the stock market, adding Vivendi would not help you diversify your market risk very much. Vivendi stock would amplify market swings, not reduce them.

Solve Now!

Q 12.7 *A project returns −5% if the stock market returns −10%, and +5% if the stock market returns +10%. What is the market beta of this project?*

Q 12.8 *A project returns +5% if the stock market returns −10%, and −5% if the stock market returns +10%. What is the market beta of this project?*

Q 12.9 *A project returns −20% if the stock market returns −10%, and +5% if the stock market returns +10%. What is the market beta of this project?*

12·4. Expected Rates of Return and Betas of (Weighted) Portfolios and Firms

How to deal with multiple projects.

Very often, you are considering multiple projects already packaged together as a portfolio. For example, a firm should be seen as a collection of projects that have been packaged together. If division $\mathcal{C}$ is worth \$1 million and division $\mathcal{D}$ is worth \$2 million, then a firm consisting of $\mathcal{C}$ and $\mathcal{D}$ is worth \$3 million, $\mathcal{C}$ constitutes $1/3$ of the portfolio "Firm" and $\mathcal{D}$ constitutes $2/3$ of the portfolio "Firm." This kind of portfolio is called a **value-weighted portfolio**, because the weights correspond to the market values of the components. (An **equal-weighted portfolio** would invest 50% each into $\mathcal{C}$ and $\mathcal{D}$, instead—\$1.5 million each.)

Value-weighting and equal-weighting

Side Note: We usually think of a firm as a value-weighted portfolio of its divisions. This is because value-weighted portfolios are closely related to what we think of as holding portfolios, because maintaining them requires no trading (unless there is a payout, like a dividend). For example, if $\mathcal{C}$ triples and $\mathcal{D}$ halves in value, your original value-weighted portfolio or firm would become \$3 + \$1 = \$4 million—and it would again (or still) be value-weighted. In contrast, in an originally equal-weighted portfolio, your \$1.5 million in $\mathcal{C}$would become \$4.5 million, your \$1.5 million in $\mathcal{D}$ would become \$0.75 million, and your portfolio would be worth \$5.25 million. This means you would want to have \$2.625 million invested in each. To maintain an equal-weighted portfolio, you would have to sell some stock in your past winner to buy some stock in your loser. Only an value-weighted portfolio requires no trading—and if you do not trade in your portfolio, in the very long run, portfolios begin to look more and more like an equal-weighted portfolio.

But how do we handle risk and reward characteristics?

You need to know how you can work with a firm if you have all the information about its underlying projects. Put differently, if I tell you what the expected rate of return on each project is, and what the market beta of each project is, can you tell me what the firm's overall expected rate of return and overall market beta is? Let's try it. Call $\mathcal{CDD}$ a portfolio (or firm) that consists of $1/3$ investment in division $\mathcal{C}$ and $2/3$ investment in division $\mathcal{D}$.

Actual rates of return can be averaged.

You have already worked with portfolios in Section 12·2. You know that actual rates of return can be averaged. For example, Table 12.1 shows that in scenario S4 (♠), investment $\mathcal{C}$ has a rate of return of +12%, and investment $\mathcal{D}$ has a rate of return of −12%. Consequently, your overall investment $\mathcal{CDD}$ has a rate of return of

$$\begin{aligned} r_{\mathcal{CDD},S4} &= 1/3 \cdot (+12\%) + 2/3 \cdot (-12\%) = -4\% \\ &= w_{\mathcal{C}} \cdot r_{\mathcal{C},S4} + w_{\mathcal{D}} \cdot r_{\mathcal{D},S4} . \end{aligned} \tag{12.20}$$

Let us verify this: Put \$100 into $\mathcal{C}$ and \$200 into $\mathcal{D}$. $\mathcal{C}$ turns into $(1 + 12\%) \cdot \$100 = \$112$. $\mathcal{D}$ turns into $(1 - 12\%) \cdot \$200 = \$176$. The total portfolio turns into \$288, which is a rate of return of $\$288/\$300 - 1 = -4\%$ on a \$300 investment.

It is also intuitive that *expected* rates of return can be averaged. In our example, $\mathcal{C}$ has an *expected* rate of return of 5%, $\mathcal{D}$ has an *expected* rate of return of 2%. Consequently, your overall firm $\mathcal{CDD}$ has an expected rate of return of

Expected rates of return can be averaged.

$$\begin{aligned} \mathcal{E}(\tilde{r}_{\mathcal{CDD}}) &= 1/3 \cdot (+5\%) + 2/3 \cdot (+2\%) = 3\% \\ &= w_{\mathcal{C}} \cdot \mathcal{E}(\tilde{r}_{\mathcal{C}}) + w_{\mathcal{D}} \cdot \mathcal{E}(\tilde{r}_{\mathcal{D}}) \quad . \end{aligned} \tag{12.21}$$

Let us verify this. There are four possible outcomes: In S1, your actual rate of return is 8.67%; in S2, it is 5%; in S3, it is 2.33%; and in S4, it is −4%. The average of these four outcomes is indeed 3%.

But here is a remarkable and less intuitive fact: market betas—that is, the projects' risk contributions to our investors' market portfolios—can be averaged, too. That is, we claim that the beta of $\mathcal{CDD}$ is the weighted average of the betas of $\mathcal{C}$ and $\mathcal{D}$. You already computed these in Formula 12.10 on Page 294 as +1.128 and −2.128, respectively. Their value-weighted average is

Market betas can be averaged.

$$\begin{aligned} \beta_{\mathcal{CDD},\mathcal{M}} &= 1/3 \cdot (+1.128) + 2/3 \cdot (-2.128) \approx -1.04 \\ &= w_{\mathcal{C}} \cdot \beta_{\mathcal{C},\mathcal{M}} + w_{\mathcal{D}} \cdot \beta_{\mathcal{D},\mathcal{M}} \quad . \end{aligned} \tag{12.22}$$

To check if this claim is correct, you must compute the market beta for $\mathcal{CDD}$ from the rates of return for the entire firm $\mathcal{CDD}$. Start with the demeaned returns, and compute the cross-product:

	Original Base Rates		Demeaned Rates		
Scenario	$\tilde{r}_B$	$\tilde{r}_{\mathcal{CDD}}$	$\tilde{r}_B$	$\tilde{r}_{\mathcal{CDD}}$	Crossproduct
S1 (♣)	−1%	8.67%	−5%	5.67%	−28.33%%
S2 (♦)	2%	5.00%	−2%	2.00%	−4.00%%
S3 (♥)	4%	2.33%	0%	−0.67%	0.00%%
S4 (♠)	11%	−4.00%	7%	−7.00%	−49.00%%
Mean	5%	3%	0%	0%	−20.33%%

Reusing the beta Formula 12.14 from Page 296, the beta of investment $\mathcal{CDD}$ is

$$\begin{aligned} \beta_{\mathcal{CDD},\mathcal{M}} &= \frac{-0.2033\%}{0.195\%} \approx -1.04 \\ &= \frac{Cov(\tilde{r}_{\mathcal{M}}, \tilde{r}_{\mathcal{C}})}{\mathcal{V}ar(\tilde{r}_{\mathcal{M}})} \quad . \end{aligned} \tag{12.23}$$

So, I did not lie.

IMPORTANT:

- *You can think of the firm as a weighted investment portfolio of its individual divisions and projects. For example, if a firm named ab consists only of two divisions, a and b, then its rate of return is always*

$$\tilde{r}_{ab} \;=\; w_a \cdot \tilde{r}_a + w_b \cdot \tilde{r}_b \quad , \tag{12.24}$$

  where the weights are the relative values of the two divisions. (You can also think of this one firm as a "sub-portfolio" within a larger overall portfolio, such as the market portfolio.)

- *The expected rate of return ("reward") of a portfolio is the weighted average expected rate of return of its components,*

$$\mathcal{E}(\tilde{r}_{ab}) \;=\; w_a \cdot \mathcal{E}(\tilde{r}_a) + w_b \cdot \mathcal{E}(\tilde{r}_b) \quad . \tag{12.25}$$

  Therefore, the expected rate of return of a firm is the weighted average rate of return of its divisions.

- *Like expected rates of return, betas can be weighted and averaged. The beta of a subportfolio (the subportfolio's "risk contribution" to the overall portfolio) is the weighted average of the betas of its components,*

$$\beta_{ab,\mathcal{M}} \;=\; w_a \cdot \beta_{a,\mathcal{M}} + w_b \cdot \beta_{b,\mathcal{M}} \quad . \tag{12.26}$$

  Therefore, the market beta of a firm is the weighted average market-beta of its divisions.

Debt and Equity work, too!

You can think of the firm not only as consisting of divisions, but also as consisting of debt and equity. Therefore, the beta of the overall firm must be the weighted average beta of the debt and equity—if the debt worth \$100 million has a beta of 0.4 and equity worth \$300 million has a beta of 2.0, then the firm is worth \$400 million and the firm's beta is $1/4 \cdot (0.4) + 3/4 \cdot (2.0) = 1.6$.

DIGGING DEEPER: *Do not get the wrong impression: We could average expected rates of returns and market-betas, but averaging does not work for just any statistics. For example, you must not average project variances or standard deviations. You can easily check this. The variance of $\tilde{r}_C$ was 5.15%. The variance of $\tilde{r}_{\mathcal{D}}$ was 9.49%. If we could average project variances, we would expect the variance of the entire firm $C\mathcal{D}\mathcal{D}$ to be $w_C \cdot \tilde{r}_C + w_{\mathcal{D}} \cdot \tilde{r}_{\mathcal{D}} = 1/3 \cdot 5.15\% + 2/3 \cdot 9.49\% \approx 8\%$—but this is the wrong answer. Instead, the variance of $C\mathcal{D}\mathcal{D}$ is $\mathcal{V}ar(\tilde{r}_{C\mathcal{D}\mathcal{D}}) = [(5.67\%)^2 + (2\%)^2 + (-0.67\%)^2 + (-7\%)^2]/4 \approx 85.6\%\%/4 \approx 21.4\%\%$.*

Solve Now!

Q 12.10 *Change the investment proportions of C and $\mathcal{D}$ to $1/3$ $\mathcal{D}$ and $2/3$ C. Call this $CC\mathcal{D}$. Compute the variance, standard deviation, and beta of CCD—and both ways (which means you must compute the covariance).*

Q 12.11 *Say you own a portfolio $\mathcal{P}$ that consists of w_A in $\mathcal{F}$, w_B in $\mathcal{A}$, and w_E in $\mathcal{B}$. What is the weighted average beta of your portfolio with respect to $\mathcal{P}$? What is the weighted average beta of your portfolio with respect to $\mathcal{M}$, more commonly called the weighted average market beta?*

Q 12.12 *Assume that a firm will always have enough money to pay off its bonds, so the beta of its bonds is 0. (The rate of return on the bonds is independent of the rate of return on the stock market.) Assume that the beta of the underlying assets is 2. How does the beta of the equity change if the firm changes its capital structure from all equity to half-debt and half-equity?*

12·5. Practical Application

Doing all these calculations by hand is tedious. We did it within the context of our four scenarios to clarify the meaning of our calculations. We would never do this in the real world with real historical rates of return data.

12·5.A. Spreadsheets

In the real world, we would rely on common spreadsheets, like Excel or OpenOffice, which have the functions we need already built-in. We would put our historical rates of return data into a column (range) in our spreadsheet, and invoke the following functions:

average(range **)** computes the average (rate of return).

varp(range **)** computes the (population) variance. If we use historical data instead of known scenarios, we would instead use the **var(** range **)** formula. (The latter divides by $N - 1$ rather than by N, which is explained in the next subsection.)

stdev(range **)** computes the (population) standard deviation. If we used historical data instead of known scenarios, we would instead use the **stdev(** range **)** formula.

covar(range-1 , range-2 **)** computes the covariance between two series.

correl(range-1 , range-2 **)** computes the correlation between two series.

slope(range-Y , range-X **)** computes a beta. If range-Y are the rates of return of an investment, and range-X are the rates of return on the market, then this formula computes the market-beta.

Table 12.3 shows an Excel spreadsheet that computes everything that we did in this chapter.

Solve Now!

Q 12.13 *The website contains the historical rates of return for Coca-Cola and the S&P500.*

(a) *Compute the average and risk of portfolios that combine the two assets in the following proportions: (0,1), (0.2,0.8), (0.4,0.6), (0.6,0.4), (0.8,0.2), (1.0, 0.0). Then plot them against one another. How does the plot look like?*

(b) *Compute the market-beta of Coca-Cola*

12·5.B. Some Notes on the Statistical Formulas

Sometimes, some formulas divide by $N - 1$, not N as we have.

Statisticians often use a variance formula that divides by $N-1$, not N. Strictly speaking, dividing by $N - 1$ is appropriate if you work with historical data, which is just a sample draw and not the full population, so you do not really know the true forward-looking mean. (Dividing by this smaller number gives unbiased estimates.) Dividing by N is appropriate if you work with "scenarios" and probabilities that you know to be true. This rarely matters in finance, where we usually have a lot of observations—except in our book examples where we have only a few observations. (Dividing by $N = 1,000$ and by $N = 1,001$ gives almost the same number.)

If $N - 1$, we are computing an estimate of the sample, not the population.

The only reason why this may come up is that if you use a program that has a built-in variance or standard-deviation function, you should not be surprised if the numbers you receive from the built-in functions are different from those we have computed in this chapter. Indeed, in Excel, we used the *varp* and *stdevp* population statistical functions, not the *var* and *stdev* sample statistical functions.

Table 12.3. The Excel Spreadsheet

D	E	F	G	H	I	J	K
IF --- Investor Choice --- Excel Spreadsheet							
Portfolios					w_b	w_c	w_d
C	D		F		0.5	0.5	0.5
2.0%	14.0%		1.0%		0.5%	-1.5%	6.5%
3.0%	6.0%		1.0%		6.5%	2.5%	4.0%
7.0%	0.0%		1.0%		1.5%	5.5%	2.0%
2.0%	-12.0%		1.0%		7.5%	11.5%	-0.5%
				Formula: =(I$4*C9+(1-I$4)*$B9)			
5.0%	2.0%		1.0%		4.0%	4.5%	3.0%
50%	0.9000%		0.0000%		0.093%	0.225%	0.066%
15%	9.49%		0.00%		3.04%	4.74%	2.57%
.128	-2.128		0.000		0.474	1.064	-0.564
.49%	10.51%		1.00%		2.10%	0.24%	5.26%
6.8%	-99.1%		#DIV/0!		68.9%	99.1%	-96.8%
.22%	-0.42%		0.00%		0.09%	0.21%	-0.11%

This Excel spreadsheet (also available on the book website) demonstrates the main statistical calculations that are performed in this chapter. Please note that we are using the population variance and population standard deviation formulas, not the sample variance and sample standard deviation formulas.

Beta is not affected by whether you divide the variance/covariance by N or $N-1$, because both numerator (covariance) and denominator (variance) are divided by the same number.

For beta, it does not matter.

Furthermore, statisticians often distinguish between underlying unknown beta and estimated beta from data—the former is occasionally written as β^T, the latter is often written with a little hat ($\hat{\beta}$). We were casual about the difference for lack of space, but whenever we work with historical data, we really are working with $\hat{\beta}$.

We also keep our statistics simple.

12·6. Summary

The chapter covered the following major points:

- The expected rate of return is a measure of expected reward.
- The standard deviation is (roughly) the square root of the average squared deviation from the mean. It is a measure of risk.

$$Sdv(\tilde{r}_P) = \sqrt{\frac{\sum_{i=1}^{N}[r_{S\,i} - \mathcal{E}(\tilde{r})]^2}{N-1}} \;. \tag{12.27}$$

- Diversification reduces the risk of a portfolio.
- We assume that investors are smart enough to hold widely diversified portfolios, which resemble the overall market portfolio.
- Market beta measures how well an individual stock contributes to (increases or reduces) the portfolio risk of an investment in the stock market.
- Market betas for typical stocks are between 0.5 and 2.0.
- It is a straightforward application of formulas to compute beta, correlation, and covariance. They are closely related, and always share the same sign.
- Like expected rates of return, betas can be averaged (using proper weighting).

Solutions and Exercises

1. The mean is 9%, which is 5% higher. The variance and standard deviation remain at the same level, the latter being 4.42%.
2. The mean is twice as high, so it is 8%. The variance is four times as high, the standard deviation is twice as high, the latter being 8.84%.

3. The result is 0.
4. The mean is 4%, the variance is 15.81%%, and the standard deviation is 3.98%.
5. Yes.

6.

$$\begin{aligned}
\mathcal{V}ar_{\mathcal{M}\&C} &= \frac{(-1.5\%-4.5\%)^2+(3.5\%-4.5\%)^2+(5.5\%-4.5\%)^2+(11.5\%-4.5\%)^2}{4} = 22.5\%\% \\
\Longrightarrow \mathcal{S}dv_{\mathcal{M}\&C} &= \sqrt{22.5\%\%} = 4.74\% \quad . \\
\mathcal{V}ar_{\mathcal{M}\&\mathcal{D}} &= \frac{(6.5\%-3\%)^2+(4.0\%-3\%)^2+(2.0\%-3\%)^2+(-0.5\%-3\%)^2}{4} = 6.625\%\% \quad . \\
\Longrightarrow \mathcal{S}dv_{\mathcal{M}\&\mathcal{D}} &= \sqrt{6.625\%\%} = 2.57\% \quad .
\end{aligned} \tag{12.28}$$

7. +0.5
8. -0.5
9. The slope is $[5-(-20)]/[10-(-10)] = 25/20 = 1.25$.

10. The variance of $\mathcal{CCD}$ is 0.47%%. The covariance of $\mathcal{CCD}$ and $\mathcal{M}$ is 0.833%%. The variance of the market is 19.5. Therefore, the beta is 0.04274. Alternatively, compute

$$\begin{aligned}
\beta_{\mathcal{CCD},\mathcal{M}} &= \quad {}^2/_3 \cdot (+1.128) \quad + \; {}^1/_3 \cdot (-2.128) \;\approx\; +.04 \\
&= w_C \cdot \beta_{C,\mathcal{M}} + w_{\mathcal{D}} \cdot \beta_{C,\mathcal{M}} \quad ,
\end{aligned} \tag{12.29}$$

11. The first is $w_{\mathcal{F}} \cdot \beta_{\mathcal{F},\mathcal{P}} + w_{\mathcal{A}} \cdot \beta_{\mathcal{A},\mathcal{P}} + w_{\mathcal{B}} \cdot \beta_{\mathcal{B},\mathcal{P}} = \beta_{w_{\mathcal{F}}\cdot\mathcal{F}+w_{\mathcal{A}}\cdot\mathcal{A}\;+w_{\mathcal{B}}\cdot\mathcal{B},\mathcal{P}} = \beta_{\mathcal{P},\mathcal{P}}$, which applies our law in reverse. Naturally, $\beta_{\mathcal{P},\mathcal{P}}$ is 1, which you can prove if you know how to manipulate covariances; but more intuitively, beta is a measure of how a portfolio varies with another portfolio—and a portfolio varies 1-to-1 with itself. The second is just the formula: $\beta_{\mathcal{P},\mathcal{M}} = w_{\mathcal{F}} \cdot \beta_{\mathcal{F},\mathcal{M}} + w_{\mathcal{A}} \cdot \beta_{\mathcal{A},\mathcal{M}} + w_{\mathcal{B}} \cdot \beta_{\mathcal{B},\mathcal{M}}$.
12. $0.5 \cdot \beta_{\mathsf{E}} + 0.5 \cdot \beta_{\mathsf{D}} = \beta_{\mathsf{W}}$. Thus, $0.5 \cdot \beta_{\mathsf{E}} + 0.5 \cdot 0 = 2$. Thus, $\beta_{\mathsf{E}} = 4$.

13.

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

CHAPTER 13

The Capital Asset Pricing Model

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We are now ready to proceed to our punchline: a formula that relates the appropriate reward (expected rate of return) of our firm's investment project to its risk (market beta). This means that if we can judge the risk of new corporate investment projects, we can determine the appropriate cost of capital in the NPV formula.

Alas, like NPV, the formula may be simple, but the devil is in the details. So, we will first carefully review what we already know, and then we will look at this new model—the CAPM. Finally, you will get to apply it.

13·1. What We Already Know And Where We Want To Go

We are after an opportunity cost of capital, which tells us what our investors want us to do.

First, you already know the philosophy of the question at hand. As corporate managers, our task is to determine whether we should take a project or reject it. We make this decision with the NPV formula. To determine the discount factor in the NPV formula, we need to estimate an appropriate cost of capital, $\mathcal{E}(r)$—or, more precisely, the *opportunity* cost of capital for our investors. We need to judge what a fair expected rate of return for our project is, given our project's risk characteristics. If our project offers less expected return than what our investors can earn elsewhere in similarly risky projects, we should not put their money into our project but instead return their money to them. If our project offers more expected return than what our investors can earn elsewhere in similarly risky projects, we should go ahead and invest their money into our project. Put differently, we want to learn what our investors would voluntarily want us to invest on their behalves.

We pretend we know what the preferences of our investors are.

Second, we are positing what our investors' preferences are—they like overall portfolio reward (expected return) and dislike overall portfolio risk (variance or standard deviation of return). We are also positing that diversification considerations drive our (smart) investors to hold the overall market portfolio (or something close to it).

This leads us to conclude how to measure risk and reward.

Third, we know how to measure risk and reward. The reward of our project is its expected rate of return. The risk of our project is *not* our project's risk itself, but the contribution of our project to our investors' overall portfolio risk. This is best measured by the market beta of our project. A project that increases in value when the market decreases in value and vice-versa has a negative market beta. A project that increases in value when the market increases in value has a positive market beta.

And it will give us a tradeoff between risk and reward.

We can also draw some additional conclusions without any math. In our assumed perfect world, we can guess that investors will immediately have snapped up the best projects—those that have low risk and high expected rates of return. In fact, those selling projects with lower risk will ask for a higher price, which in turn immediately drives down the expected rate of return. Consequently, what is available for purchase in the real world must be subject to some tradeoff: Projects that have more market-risk must offer a higher expected rate of return if they want to be purchased by investors. But what *exactly* does this relation look like? This is the subject of this chapter—it is the domain of the capital asset pricing model, the CAPM.

13·2. The Capital-Asset Pricing Model (CAPM) — A Cookbook Recipe Approach

The **Capital Asset Pricing Model** (**CAPM**) is a model that gives us an appropriate expected rate of return (cost of capital) for each project if we give it the project's risk characteristics. The model states that an investment's cost of capital is lower when it offers better diversification benefits for an investor who holds the overall stock market portfolio. Market beta is its measure of such diversification. Projects contributing more risk require a higher expected rate of return for you to want them; projects contributing less risk require a lower expected rate of return for you to want them. This is the precise relation that the CAPM gives us.

Finally, the model that gives us a cost of capital. It needs three inputs: the risk-free rate, the expected rate of return on the market, and the project's market beta.

Important: *The CAPM formula is*

$$\mathcal{E}(\tilde{r}_i) \;=\; r_{\mathcal{F}} + [\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}] \cdot \beta_{i,\mathcal{M}} \quad , \tag{13.1}$$

where i is the name of our project, $\mathcal{E}(\tilde{r}_i)$ is our project's expected rate of return (the tilde indicates that the return is unknown), $r_{\mathcal{F}}$ is the risk-free rate, $\mathcal{E}(\tilde{r}_{\mathcal{M}})$ is the expected rate of return on the stock market, and $\beta_{i,\mathcal{M}}$ is the market beta of our project ($\beta_{i,\mathcal{M}} \equiv Cov(\tilde{r}_i, \tilde{r}_{\mathcal{M}})/\mathcal{V}ar(\tilde{r}_{\mathcal{M}})$).

The difference between the expected rate of return on the risky stock-market and the risk-free investment, $\left[\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}\right]$, is called the ***equity premium*** *or* ***market risk premium****, discussed in more detail later.*

So, for example, if you believed that the risk-free rate is 3% and the expected rate of return on the stock market is 7%, then the CAPM states that

Plugging into the formula.

$$\begin{aligned} \mathcal{E}(\tilde{r}_i) &= 3\% + (7\% - 3\%) \cdot \beta_{i,\mathcal{M}} = 3\% + 4\% \cdot \beta_{i,\mathcal{M}} \\ \mathcal{E}(\tilde{r}_i) &= r_{\mathcal{F}} + [\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}] \cdot \beta_{i,\mathcal{M}} \quad . \end{aligned} \tag{13.2}$$

Therefore, a project with a beta of 0.5 should have a cost of capital of $3\% + 4\% \cdot 0.5 = 5\%$, and a project with a beta of 2.0 should have a cost of capital of $3\% + 4\% \cdot 2.0 = 11.0\%$. The CAPM gives an opportunity cost for our investors' capital: if the project with the beta of 2.0 cannot earn a rate of return of 11%, we should not take this project and instead return the money to our investors. Our project would contribute too much risk for its reward. Our investors have better opportunities elsewhere.

The CAPM specifically ignores the standard deviation of individual projects' rates of return. The project's standard deviation is only a measure of how risky a stock is *by itself*, and would only be of relevance to an investor who holds just this one stock (and nothing else). It is not of relevance to a "smart" investor who holds the market portfolio. Instead of the project's own variation, the CAPM uses the project's beta—a measure of the project's covariation with the market.

The CAPM Formula is about the suitable risk/reward tradeoff for an investor holding the market portfolio.

So, to estimate an appropriate CAPM expected rate of return for a project or firm, i.e., the cost of capital, you need three inputs:

The CAPM has three inputs.

1. The risk-free rate of return, $r_{\mathcal{F}}$.
2. The expected rate of return on the market, $\mathcal{E}(\tilde{r}_{\mathcal{M}})$.
3. A firm's or project's beta with respect to the market, $\beta_{i,\mathcal{M}}$.

As always, you are really interested in the future expected rate of return on the market, and the future beta of a firm/project with respect to the market, and not in the past average rates of return or beta. And, as always, you usually have no choice but to rely on estimates based on historical data. In Section 13·4, we shall discuss in more detail how to best estimate each CAPM input. But first we will explore the model itself, assuming we already know the inputs.

13·2.A. The Security Markets Line (SML)

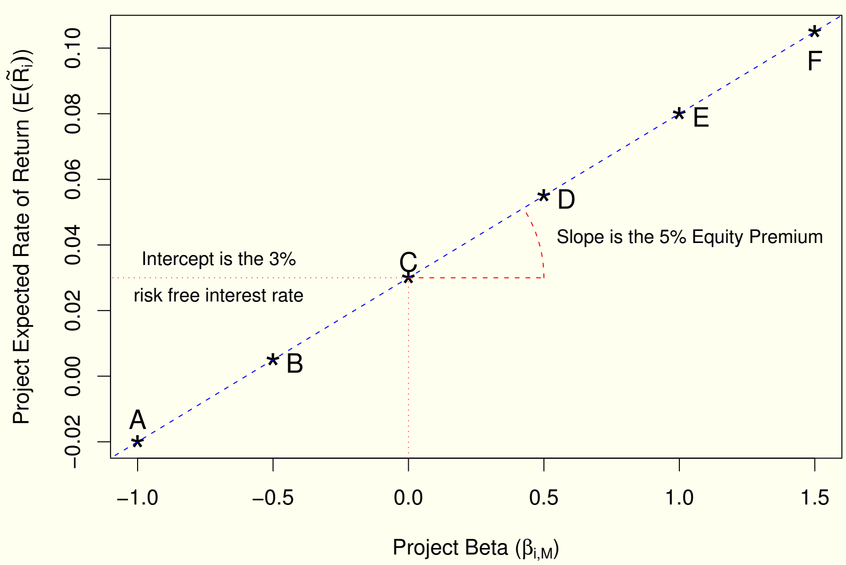
An example of what rate of returns individual securities should offer.

Let us apply the CAPM in a specific example. Assume that the risk-free rate is 3% per year, and that the stock market offers an expected rate of return of 8% per year. The CAPM formula then states that a stock with a beta of 1 should offer an expected rate of return of 3% + (8% − 3%)·1 = 8% per year; that a stock with a beta of 0 should offer an expected rate of return of 3% + (8% − 3%)·0 = 3% per year; that a stock with a beta of 1/2 should offer an expected rate of return of 3% + (8% − 3%)·0.5 = 5.5% per year; that a stock with a beta of 2 should offer an expected rate of return of 3% + (8% − 3%)·2 = 13% per year; and so on.

The Security Markets Line, or SML, is just the CAPM formula.

The CAPM equation is often graphed as the **security markets line**, which shows the relationship between the expected rate of return of a project and its beta. Figure 13.1 draws a first security markets line, using stocks named $\mathcal{A}$ through $\mathcal{F}$. Each stock (or project) is a point in this coordinate system. Because all securities properly follow the CAPM formula in our example, they must lie on a straight line. In other words, the SML line is just a graphical representation of the CAPM Formula 13.1 on Page 309. The slope of this line is the equity premium, $\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}$, the intercept is the risk-free rate, $r_{\mathcal{F}}$.

Figure 13.1. The Security Market Line For Stocks $\mathcal{A}$–$\mathcal{F}$



Stock	$\beta_{i,\mathcal{M}}$	$\mathcal{E}(\tilde{r}_i)$
$\mathcal{A}$	−1.0	−2.0%
$\mathcal{B}$	−0.5	0.5%
$\mathcal{C}$	0.0	3.0%

Stock	$\beta_{i,\mathcal{M}}$	$\mathcal{E}(\tilde{r}_i)$
$\mathcal{D}$	0.5	5.5%
$\mathcal{E}$	1.0	8.0%
$\mathcal{F}$	1.5	10.5%

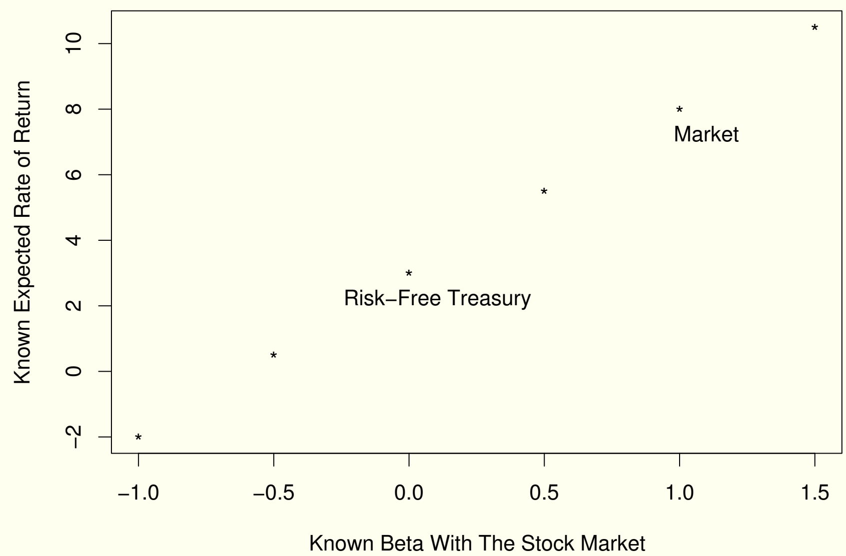
This graph plots the CAPM relation $\mathcal{E}(\tilde{r}_i) = r_{\mathcal{F}} + [\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}] \cdot \beta_{i,\mathcal{M}} = 3\% + (8\% - 3\%) \cdot \beta_{i,\mathcal{M}}$. That is, we assume that the risk-free rate is 3%, and the equity premium is 5%.

The "Security Market Line" in an Ideal CAPM World

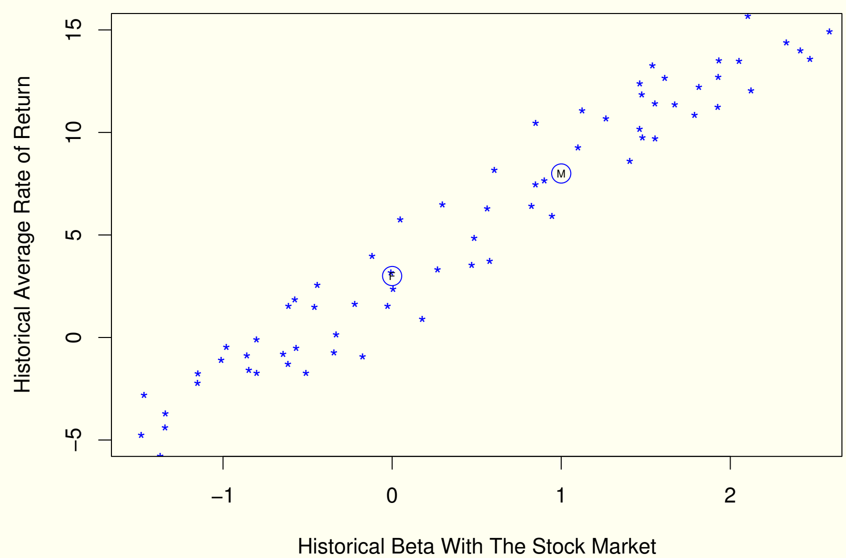
Alas, in the real world, even if the CAPM holds, you would not have the data to draw Figure 13.1. The reason is that you do not know true expected returns and true market-betas. So, Figure 13.2 plots two graphs in a perfect CAPM world. Graph (A) repeats Figure 13.1 and presumes you know CAPM inputs—the true market-betas and true expected rates of return—although in truth you really cannot observe them. This line is perfectly straight. In Graph (B), presume you know only observables—estimates of expected returns and betas, presumably based mostly on historical data averages. Now, you can only plot an "estimated security market line," not the "true security market line." Of course, you hope that our historical averages are good, unbiased estimates of true market-beta and true expected rates of return (and this is a big if), so the line will look at least approximately straight. A workable version of the CAPM thus can only state that there should roughly be a linear relationship between the data-estimated market beta and the data-estimated expected rate of return, just as drawn here.

Figure 13.2. The Security Market Line in an Ideal CAPM World

(A) The Relationship Among Unobservable Variables



(B) The Relationship Among Observable Variables



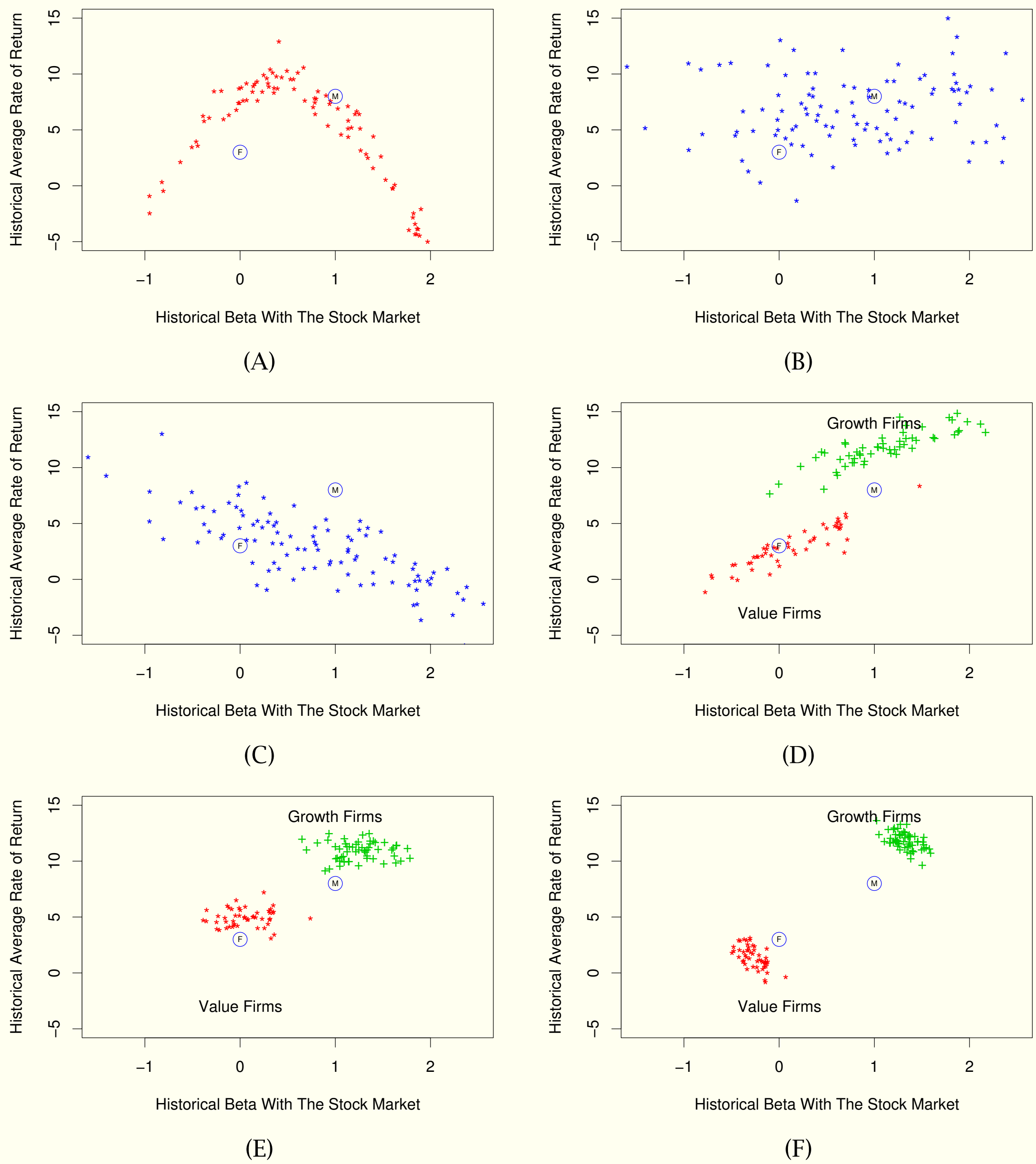
Historical average returns and historical betas are estimated from the data—and hopefully representative of the true underlying mean returns and true betas, which in turn means that they are also indicative of the future mean returns and betas.

13·2.B. Non-CAPM Worlds and Non-Linear SMLs

What happens if a stock offers too much or too little expected rate of return?

What would happen from the CAPM's perspective if a stock offered more than its due expected rate of return? Investors in the economy would want to buy more of the stock than would be available: its price would be too low. It would be too good a deal. Investors would immediately flock to it, and because there would not be enough of this stock, investors would bid up its price and thereby lower its expected rate of return. Eventually, the price of the stock would equilibrate at the correct CAPM expected rate of return. Conversely, what would happen if a stock offered less than its due expected rate of return? Investors would not be willing to hold enough of the stock: the stock's price would be too high, and its price would fall. Neither situation should happen in the real world—investors are just too smart. However, you must realize that if a stock were not to follow the CAPM formula, buying it would still be risky. Yes, such a stock would offer too high or too low an expected rate of return and thus be a good or a bad deal, attracting too many or too few investors chasing a limited amount of project—but it would still remain a risky investment, and no investor could earn risk-free profit by exploiting the pricing inefficiency.

Figure 13.3. The Security Market Line in non-CAPM Worlds



Each point is one stock (or project or fund)—its historical beta and its historical average rate of return. (The market and risk-free rate are noted by a letter inside the circle.) In these figures, the security market line does not appear linear, as the CAPM suggests. Therefore, if these patterns are not just statistical mirages, you should be able to invest better than just in the market: from the CAPM perspective, there are "great deal" stocks that offer too much expected return given their risk contributions to your (market) portfolio, which you would therefore want to overemphasize; and "poor deal" stocks that offer too little expected return, given their risk contribution, which you would therefore want to underemphasize.

The "Security Market Line" if the CAPM is the wrong model (with respect to its own functional form).

Under what circumstances would you lose faith in the CAPM? Figure 13.3 plots what security market relations could look like if the CAPM did not work. In Graph (A), the rate of return does not seem to increase linearly with beta if beta is greater than about 0.5. Because beta is a measure of risk contribution to your market portfolio, as investors, you would not be inclined to add stocks with betas greater than 1 or 2 to our (market) portfolio—these stocks' risk contributions are too high, given their rewards. You would like to deemphasize these firms, tilting your portfolio towards stocks with lower betas. In Graph (B), the rate of return seems unrelated to beta, but the average rate of return on the stock market seems quite a bit higher than the risk-free rate of return. In this case, you again would prefer to tilt your portfolio away from the overall market and towards stocks with lower beta risk. This would allow you to construct a portfolio that has lower overall risk and higher expected rate of return than the market portfolio. (Trust me.) In Graph (C), higher beta securities offer *lower* expected rates of return. Again, you should prefer moving away from your current portfolio (the market) by adding more of stocks with lower market-betas.

The "Security Market Line" if the CAPM is the wrong model (with respect to a specific better alternative).

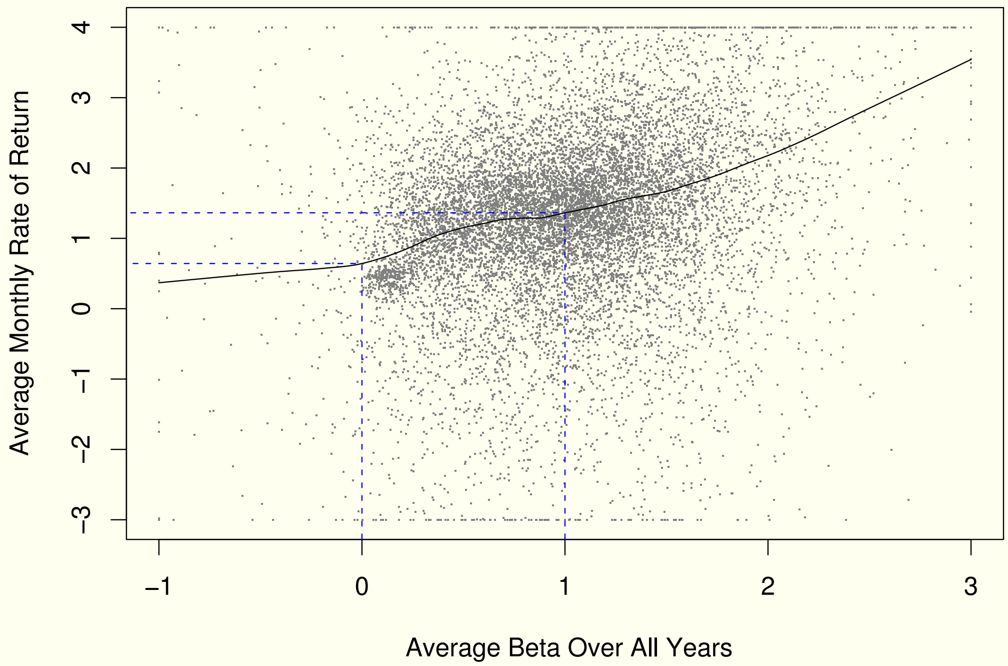
In Graph (D), even though each cluster has a positive relationship between beta and the expected rate of return, growth firms have a different relationship than value firms—and the CAPM says not only that market-beta should matter, but that *market-beta is all that should matter.* If you knew whether a firm was a growth firm or a value firm, you could do better than if you relied on market-beta. Rather than just holding the market portfolio, you would prefer tilting your portfolio towards growth stocks and away from value stocks—for a given beta contribution to your portfolio, you would earn a higher reward in growth firms. Graphs (E) and (F) show the same issue, but more starkly. If you could not identify whether a firm is a growth firm or a value firm, you would conclude that market-beta works—you would still draw a straight positive line between the two clusters of firms, and you would conclude that higher market-beta stocks offer higher rewards. But, truly, it would not be beta that matters, but whether the firm is a growth firm or a value firm. After taking into account what type the firm is, beta would not matter in Graph (E), and even matter *negatively* in Graph (F). In either case, as an investor, you could earn higher expected rates of return buying stocks based on firm type rather than based on beta.

Historical Patterns can be deceptive.

But be warned: these relationships could also appear if your procedures to estimate beta or expected rates of return are poor—after all, when you plot such figures with real-world historical data, you do not have the true beta or true expected rates of return. Even if your statistical procedures are sound, statistical noise makes this a hazardous venture. In particular, in real life, although you can estimate market-betas pretty reliably, you can only roughly estimate expected rates of return from historical rates of return.

13·2.C. Empirical Reality

Figure 13.4. Average Historical Rates of Return Against Historical Market Beta, 1970–2000.



Note: The returns are monthly and *not* annualized. Betas are with respect to the value-weighted stock market. Extreme observations were cut: at −1 and +3 for beta, and at −3% and +4% for monthly returns. The solid black line is "smoothed" to fit points locally, allowing it to show non-linearities. The dashed blue line indicates that this smoothed line suggests that a "beta=0" security had an approximate rate of return of 64 basis points per month, or about 8% per annum. The typical "beta=1" security had an approximate rate of return of 136 basis points per month, or about 18% per annum.

A model is a model.

Before we see where the CAPM succeeds and where it fails, I want to impress on you that a model is just a model—models are never perfect descriptions of reality. They can be useful within a certain domain, even if on closer examination they are rejected. For example, we do not live in a world of Newtonian gravity. Einstein's model of relativity is a better model—though it, too, is not capable of explaining everything. Yet, no one in their right state of mind would use Einstein's model to calculate how quickly objects fall. The Newtonion model is entirely appropriate and much easier to use. Similarly, planetary scientists use Einstein's model, even though we know it, too, fails to account for everything—but it does well enough for the purposes at hand and there are no better alternatives (at least as of yet, even though string theory is trying). This latter situation is pretty much the situation in which corporations find themselves—the CAPM is not really correct, but there are no clear better alternatives. However, ultimately, your concern has to be about the domain within which the CAPM *is* useful, and it usually is very useful for corporate capital budgeting.

The empirical relation looks reasonably linear and upward sloping.

So, what does the security market line (SML) really look like? Figure 13.4 plots the relationships from 1970 to 2000. The typical stock with a beta of 0 earned a rate of return of about 8% per annum, while the typical stock with a beta of 1 (i.e., like the market) earned a rate of return of about 18% per annum. Not drawn in the figure, the average stock with a beta of 2 earned about 217 basis points per month (30% per annum), and the average stock with a beta of 3 earned about 354 basis points per month (50% per annum). You can see that these 30 years were a very good period for financial investments! The figure shows also how there was tremendous variability in the investment performance of stocks. More importantly from the perspective of the CAPM, the relationship between average rate of return and beta was not exactly linear, as the CAPM suggests, but it was not far off. If we stopped now, you would conclude that the CAPM was a pretty good model.

But this is deceptive—the CAPM fails against specific better alternatives.

But look back at Figure 13.3. The empirical evidence is not against the CAPM in the sense of the first three plots (linearity)—it is against the CAPM in the sense of the last three plots (better alternative classifications). So, although you cannot see this in Figure 13.4, the CAPM fails when stocks are split into groups based on different characteristics. The empirical reality is somewhat closer to the latter three figures than it is to the idealized CAPM world. This implies that market beta seems to matter *only if* we do not control for certain other firm characteristics, like value or growth. The "only little problem" (irony warning) is that we finance academics are not exactly sure what these characteristics are, and why they matter.

> **SIDE NOTE**: Some evidence suggests that firms that are classified as "growth firms" by some metrics generally underperform "value firms"—but we do not really know why, nor do we know what we should recommend a corporate manager should do about this fact. Maybe managers should pretend that their firms are growth firms—because investors like this claim so much they are willing to throw money at too cheap a cost of capital at growth firms—but then act like value firms and thereby earn higher returns.

My personal opinion.

Different academics draw different conclusions from this evidence. Some recommend outright against using the CAPM, but most professors recommend "use with caution." Here is my personal opinion: although the CAPM is likely not to be really true, market-beta is still a useful cost-of-capital measure for a corporate finance manager. Why so? Look again at the last three plots in Figure 13.3: If you have a beta of around 1.5, you are more than likely a growth firm with an expected rate of return of 10% to 15%; if you have a beta of around 0, you are more than likely a value firm with an expected rate of return of 3% to 7%. Thus, beta would still provide you with a decent cost of capital estimate, even though it was not market-beta itself that mattered, but whether your firm was a growth or a value firm. (Market beta helped by indicating to you whether the firm was a growth or a value firm.) Admittedly, using an incorrect model is not an ideal situation, but the cost-of-capital errors are often reasonable enough that corporate managers generally can live with them. And the fact is, if they cannot live with them, we really do not know what to recommend as a better alternative to the CAPM! In contrast, my advice to an investor would be *not* to use the CAPM for investing (portfolio choice). Although it is true that wide diversification needs to be an important part of *any* good investment strategy, there *are* better investment strategies than just investing in the market.

Solve Now!

Q 13.1 *The risk-free rate is 4%. The expected rate of return on the stock market is 7%. What is the appropriate cost of capital for a project that has a beta of 3?*

Q 13.2 *The risk-free rate is 4%. The expected rate of return on the stock market is 12%. What is the appropriate cost of capital for a project that has a beta of 3?*

Q 13.3 *The risk-free rate is 4%. The expected rate of return on the stock market is 7%. A corporation intends to issue publicly traded bonds which promise a rate of return of 6%, and offer an expected rate of return of 5%. What is the implicit beta of the bonds?*

Q 13.4 *Draw the security market line if the risk-free rate is 5% and the equity premium is 4%.*

Q 13.5 *What is the equity premium, both mathematically and intuitively?*

13·3. Using the CAPM Cost of Capital in the NPV Context: Revisiting The Default Premium and Risk Premium

We usually use the CAPM expected rate of return in the NPV denominator.

An important reason why you worked through the CAPM in the first place was to obtain the quantities that you need in the denominator of the NPV formula,

$$\text{NPV} \;=\; \text{CF}_0 \;+\; \frac{\mathcal{E}(\tilde{C}_1)}{1+\mathcal{E}(\tilde{r}_{0,1})} \;+\; \frac{\mathcal{E}(\tilde{C}_2)}{1+\mathcal{E}(\tilde{r}_{0,2})} \;+\; \ldots \tag{13.3}$$

The CAPM tells you that cash flows that correlate more with the overall market are of less value to your investors, and therefore require a higher expected rate of return ($\mathcal{E}(\tilde{r})$) in order to pass muster (well, the hurdle rate).

Do not lose the forest: the CAPM has nothing to do with default risk.

Although mentioned before in Chapter 5, it is important to reiterate that the CAPM expected rate of return (based on beta) does not take default risk into account. In the NPV formula, the default risk enters the valuation in the expected cash flow numerator, not in the expected rate of return denominator. So, recall the important box on Page 88, which decomposed rates of return into three parts:

$$\begin{aligned}
\text{Promised Rate of Return} &= \text{Time Premium} + \text{Default Premium} + \text{Risk Premium} \;. \\
\text{Actual Earned Rate} &= \text{Time Premium} + \text{Default Realization} + \text{Risk Premium} \;. \\
\text{Expected Rate of Return} &= \text{Time Premium} + \text{Expected Risk Premium} \;.
\end{aligned} \tag{13.4}$$

The CAPM gives you the expected rate of return, which consists of the time premium and the expected risk premium. It does not give you any default premium. This is important enough to put in a box:

> **Important**: *The CAPM provides an expected rate of return. It does not include a default premium. The probability of default must be handled in the NPV numerator through the expected cash flow, and not in the NPV denominator through the expected rate of return.*

A specific example.

How do you put the default risk and CAPM risk into one valuation? Here is an example. Say you want to determine the PV of a corporate zero bond that has a beta of 0.25, and promises to deliver \$200 next year. This bond pays off 95% of the time, and 5% of the time it totally defaults. Assume that the risk-free rate of return is 6% per annum, and the expected rate of return on the market is 10%. Therefore, the CAPM states that the expected rate of return on your bond must be

$$\mathcal{E}(\tilde{r}_{\text{Bond}}) \;=\; r_{\mathcal{F}} \;+\; \mathcal{E}(\tilde{r}_{\mathcal{M}} - r_{\mathcal{F}}) \cdot \beta_{\text{Bond},\mathcal{M}} \;=\; 6\% \;+\; 4\% \cdot 0.25 \;=\; 7\% \;. \tag{13.5}$$

Of course, this has not yet taken the bond's default risk into account. You must still adjust the numerator (promised payments) for the probability of default—you expect to receive not \$200, but

$$\begin{aligned}
\mathcal{E}(\tilde{C}_{\text{Bond}}) &= 95\% \cdot \$200 \;+\; 5\% \cdot 0 \;=\; \$190. \\
&= \mathcal{P}rob(\text{No Default}) \cdot \text{Promise} \;+\; \mathcal{P}rob(\text{Default}) \cdot \text{Nothing} \;.
\end{aligned} \tag{13.6}$$

Therefore, the CAPM states that the value of the bond is

$$\text{PV}_{\text{Bond}} \;=\; \frac{\mathcal{E}(\tilde{C}_{\text{Bond},t=1})}{1+\mathcal{E}(\tilde{r}_{\text{Bond},0,1})} \;=\; \frac{\$190}{1+7\%} \;\approx\; \$177.57 \;. \tag{13.7}$$

Given this price, you can also compute the promised (or quoted) rate of return on this bond,

$$\text{Promised } r_{0,1} = \frac{\$200 - \$177.57}{\$177.57} \approx 12.6\% \\ = \frac{\text{Promised CF}_1 - \text{CF}_0}{\text{CF}_0} \quad . \tag{13.8}$$

Although you rarely need to decompose quoted interest rates in practice, quantifying the three components in this example helps to better conceptualize the magnitudes of the components of quoted rates. For your bond, the time-premium of money is 6% per annum—it is the rate of return that an equivalent-term Treasury bond offers. The time-premium plus the risk-premium is provided by the CAPM, and it is 7% per annum. Therefore, 1% per annum is your "average" compensation for your willingness to hold this risky bond, rather than the risk-free Treasury bond. The remaining 12.6% – 7% ≈ 5.6% per annum is the default premium: you do not expect to earn money from this part "on average;" you only earn it if the bond does not default.

$$\begin{array}{ccccccccc} 12.6\% & = & 6\% & + & 5.6\% & + & 1\% & . \\ \text{Promised Interest Rate} & = & \text{Time Premium} & + & \text{Default Premium} & + & \text{Risk Premium} & . \end{array} \tag{13.9}$$

As in the example, in the real world, most bonds have fairly small market betas and thus risk premia. Instead, most of the premium that risky bonds quote above equivalent risk-free Treasury rates is due to default risk.

SIDE NOTE: In the real world, corporate bonds also have important liquidity premia built-in, which compensates investors for not being able to easily buy/sell these securities. The broker/market-makers tend to earn this premium. The liquidity premium differs across investors: retail investors are charged higher liquidity premia than bond funds. As a retail investor, it is best not to purchase individual bonds.

Solve Now!

Q 13.6 *A corporate bond with a beta of 0.2 will pay off next year with 99% probability. The risk-free rate is 3% per annum, the risk-premium is 5% per annum. What is the price of this bond, and its promised rate of return?*

Q 13.7 *Continue: Decompose the bond's quoted rate of return into its components.*

Q 13.8 *Going to your school has total additional and opportunity costs of $30,000 this year and up-front. With 90% probability, you are likely to graduate from your school. If you do not graduate, you have lost the entire sum. Graduating from the school will increase your 40-year lifetime annual salary by roughly $5,000 per year, but more so if the stock market rate of return is high than when it is low. For argument's sake, assume that your extra-income beta is 1.5. Assume the risk-free rate is 3%, the equity premium is 5%. What is the value of your education?*

13·4. Estimating CAPM Inputs

Let us now discuss how we can obtain reasonable estimates of the three inputs into the CAPM formula,

$$\mathcal{E}(\tilde{r}_i) \;=\; r_{\mathcal{F}} + [\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}] \cdot \beta_{i,\mathcal{M}} \quad . \tag{13.10}$$

13·4.A. The Equity Premium $\left(\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}\right)$

The equity premium must be provided as a CAPM input. Estimates are all over the map; reasonable ones can range from 2% to 8% per year.

The most-difficult-to-estimate input in the CAPM is the equity premium. It measures the extra expected rate of return that risky projects are offering above and beyond what risk-free projects are offering. The value you choose for the equity premium can have a tremendous influence on your estimated costs of capital. Of course, the CAPM model assumes that you know the *expected* rate of return on the market, not that you have to estimate it. Yet, in real life, the equity premium is not posted anywhere, and *no one really knows the correct number.* There are a number of methods to guesstimate it—but they unfortunately do not tend to agree with one another. This leaves me with two choices: I can either throw you one estimate and pretend it is the only one, or I can tell you the different methods that lead to differing estimates. I prefer the latter, if only because the former would eventually leave you startled to find that someone else has used another number and has come up with another cost of capital estimate. We will discuss the intuition behind each method and the specific estimate the intuition would suggest. In this way, you can make up your own mind as to what you deem to be an appropriate equity premium estimate.

Method 1: Historical Averages.

Historical Averages I: The first estimation method simply relies on historical average equity premia as good indicators of future risk premia. As of 2003, the arithmetic average equity premium since 1926 was about **8.4%** per annum. However, if you start computing the average in 1869, the equity premium estimate drops to around **6.0%.** Maybe you should start in 1771? Or 1971? Which is the best estimation period? No one really knows what the right start date should be. If you choose too few years, your sample average could be unreliable. For example, what happened over the last 20 or 30 years might just have been happen-stance and not representative of the statistical process driving returns. Your estimate of the mean would carry a lot of uncertainty. The more years you use, the lower would be your uncertainty (standard error about the mean). However, if you choose too many years, the data in the earlier part of your sample period may be so different from those today that they are no longer relevant—that is, you may incorrectly believe that the experience of 1880 still has relevance today.

Method 2: Inverse Historical Averages.

Historical Averages II: The second estimation method looks at historical equity premia in the opposite light. You can draw on an analogy about bonds—if stocks become more desirable, perhaps because investors have become less risk-averse, then more investors compete to own them, drive up the price, and thereby lower the future expected rates of return. High historical rates of return are therefore indicative of low future expected rates of returns. An extreme version thereof may even that high past equity premia are not indicative of high future equity premia, but rather indicative of **bubbles** in the stock market. The proponents of the bubble view usually cannot quantify the appropriate equity premium, except to argue that it is lower after recent market runups—exactly the opposite of what proponents of the *Historical Averages I* argue.

Side Note: A bubble is a run-away market, in which rationality has temporarily disappeared. There is a lot of debate as to whether bubbles in the stock market ever occurred. A strong case can be made that technology stocks experienced a bubble from around 1998 to 2000. No one has yet come up with a rational story based on fundamentals that can explain *both* why the Nasdaq Index had climbed from 2,280 in March 1999 to 5,000 on March 27, 2000, *and* why it then dropped back to 1,640 on April 4, 2001.

Method 3: Dividend or Earnings Yields.

Current Predictive Ratios: The third method tries to actively predict the stock market rate of return with historical dividend yields (i.e., the dividend payments received by stockholders). Higher dividend yields should make stocks more attractive and therefore predict higher future equity premia. The equity premium estimation is usually done in two steps: first, you must estimate a statistical regression that predicts next year's equity premium with this year's dividend yield; then, you substitute the currently prevailing dividend yield into your estimated regression to get a prediction. Unfortunately, as of 2003, current dividend yields are so low that the predicted equity premia are negative—which is not a sensible number. Variations of this method have used interest rates of earnings yields, typically with similar results. In any case, the evidence suggests that this method has yielded poor predictions—for example, it had predicted low equity premia in the 1990s, which was a period of superb stock market performance.

Method 4: What is reasonable reward for risk?

Philosophical Prediction: The fourth method wonders how much rate of return is required to entice reasonable investors to switch from bonds into stocks. Even with an equity premium as low as 3%, over 25 years, an equity investor would end up with more than twice the money of a bond investor. Naturally, in an efficient market, nothing comes for free, and the reward for risk-taking should be just about fair. So, equity premia of 8% just seem too high for the amount of risk observed in the stock market. This philosophical method generally suggests equity premia of about **1%** to **3%.**

Method 5: Just ask.

Consensus Survey: The fifth method just asks people or experts what they deem reasonable. The ranges can vary widely, and seem to correlate with very recent stock market returns. For example, in late 2000, right after a huge runup in the stock market, surveys by Fortune or Gallup/Paine-Webber had investors expect equity premia as high as 15%/year. (They were acutely disappointed: the stock market dropped by as much as 30% over the following two years. Maybe they just got the sign wrong?!) The consulting firm, McKinsey, uses a standard of around 5% to 6%, and the social security administration uses a standard of around 4%. In a survey of finance professors in August 2001, the common equity premium estimate ranged between **3.5%** for a 1-year estimate to **5.5%** for a 30-year estimate.

Some recent estimates.

What to choose? Welcome to the club! No one knows the true equity premium. On Monday, February 28, 2005, the C1 page of the WSJ reported the following average annual after-inflation forecasts over the next 44 years:

ANECDOTE: The Power of Compounding
Assume you invested $1 in 1925. How much would you have in December 2001? If you had invested in large-firm stocks, you would have ended up with $2,279 (10.7% compound average return). If you had invested in long-term government bonds, you would have ended up with $51 (5.3%). If you had invested in short-term Treasury bills, you would have ended up with $17 (3.8%). Of course, inflation was 3.1%, so $1 in 2001 was more like $0.10 in real terms in 1926. Source: Ibbotson Associates, Chicago. U

Name	Organization	Stocks	Gov. bonds	Corp. bonds	Equity Premium
William Dudley	Goldman Sachs	5.0%	2.0%	2.5%	3.0%
Jeremy Siegel	Wharton	6.0%	1.8%	2.3%	4.2%
David Rosenberg	Merrill Lynch	4.0%	3.0%	4.0%	1.0%
Ethan Harris	Lehman Brothers	4.0%	3.5%	2.5%	0.5%
Robert Shiller	Yale	4.6%	2.2%	2.7%	2.4%
Robert LaVorgna	Deutsche Bank	6.5%	4.0%	5.0%	2.5%
Parul Jain	Nomura	4.5%	3.5%	4.0%	1.0%
John Lonski	Moody's	4.0%	2.0%	3.0%	2.0%
David Malpass	Bear Stearns	5.5%	3.5%	4.3%	2.0%
Jim Glassman	J.P. Morgan	4.0%	2.5%	3.5%	1.5%
				Average	2.0%

The equity premium is usually quoted with respect to a short-term interest rate, because these are typically safer and therefore closer to the risk-free rate that is in the spirit of the CAPM. This is why you may want to add another 1% to the equity premium estimate in this table—long-term government bonds usually carry higher interest rates than their short-term counterparts. On the other hand, if your project is longer term, you may want to adopt a risk-free rate that is more similar to your project's duration, and thus prefer the equity premium estimates in this table.

Pick a good estimate, and use it for all similar-horizon projects.

You now know that noone can tell you one authoritative number for the equity premium. Everyone is guessing, but there is no way around it—you have to take a stance on the equity premium. I am only able to give you the arguments that you should contemplate when you are picking *your* number. I can however also give you my own take: First, I have my doubts that equity premia will return to the historical levels of 8% anytime soon. (The twentieth century was the American Century for a good reason: there were a lot of positive surprises for American investors.) So, I personally prefer equity premia estimates between 2% and 4%. Interestingly, it appears that I am not the only one, as the above table shows. (It is my impression that there is relatively less dispersion in equity premia forecasts today than there was just five to ten years ago.) But realize that reasonable individuals can choose equity premia estimates as low as 1% or as high as 8%—of course, I personally find such estimates less believable the further they are from my own, personal range. And I find anything outside this 1% to 8% range just too tough to swallow. Second, whatever equity premium you choose, *be consistent.* Do not use 3% for investing in one asset (say, project A), and 8% for investing in another (say, project B). Being consistent will often reduce your relative mistakes in choosing one project over another.

The CAPM is about relative pricing, not absolute pricing.

Unfortunately, you cannot allow our limited knowledge of the equity premium stop you from using the CAPM—in fact, you cannot allow it to stop you from making investment choices. Yes, the equity premium may be difficult to estimate, but there is really no way around taking a stance. Indeed, you can think of the CAPM as telling you the *relative* expected rate of return for projects, not their *absolute* expected rate of return. Given an estimate of how much risky projects should earn relative to non-risky projects, the CAPM can tell you the right costs of capital for projects of riskiness "beta." But the basic judgment of the appropriate spread between risky and non-risky projects is left up to you.

No way around it: the equity premium is the most important number in finance, and we need to pull it out of our hats.

The need to judge the appropriate reward for risky projects relative to risk-free projects is not even just exclusive to the CAPM and corporations. It also matters for your personal investments: if you believe that the equity premium is high, you should allocate a lot of your personal assets to purchasing stocks rather than bonds. So, it is not only because of the CAPM formula that the equity premium may be the single most interesting number in finance.

We shall use the S&P500 as our market.

Finally, I have been deliberately vague about the "market." In CAPM theory, the market should be all investable assets in the economy. In practice, we typically use only a stock market index. And among stock market indexes, it often does not matter too much which index is used—be it the value-weighted stock market index, the S&P 500, or the Dow-Jones 30. The S&P500 is perhaps the most often used standin for the stock market, because its performance is posted everywhere and historical data are readily available. In sum, using the S&P500 as the market is a reasonable simplification from the perspective of a corporate executive.

Solve Now!

Q 13.9 *What are appropriate equity premium estimates? What are not? What kind of reasoning are you relying on?*

13·4.B. The Risk-Free Rate and Multi-Year Considerations ($r_{\mathcal{F}}$)

Which risk-free rate?

The second input of interest is the risk-free rate of return. The risk-free rate is relatively easily obtained from Treasury bonds. There is one small issue, though—which one? What if Treasury bonds yield 2%/year over 1 year, 4%/year over 10 years, and 5%/year over 30 years? How would you use the CAPM? Which interest rate should you pick in a multi-year context?

ANECDOTE: The American Century?
Was this really the "American Century?"

The inflation-adjusted compound rate of return in the United States was about 6% per year from 1920 to 1995. In contrast, an investor who would have invested in Romania in 1937 would have experienced not only the German invasion and Soviet domination, but also a real annual capital appreciation of about −27% per annum over the 4 years of Hungarian stock market existence (1937–1941). Similar fates befell many other East European countries—but even countries not experiencing political disasters often proved to be less stellar investments. For example, Argentina had a stock market from 1947 to 1965, even though its only function seems to have been to wipe out its investors. Peru tried three times: from 1941 to 1953, its stock market investors lost all their money. From 1957 to 1977, its stock market investors again lost all their money. But three times is a charm: From 1988 to 1995, its investors earned a whopping 63% real rate of return. India's stock market started in 1940, and offered its investors a real rate of return of just about −1% per annum. Pakistan started in 1960, and offered about −0.1% per annum.

Even European countries with long stock market histories and no political trouble did not perform as well as the United States. For example, Switzerland and Denmark earned nominal rates of return of about 5% per annum from 1921 to 1995, while the United States earned about 8% per annum.

The United States stock market was indeed an unusual above-average performer in the twentieth century. Will the twenty-first century be the Chinese century?

Source: Goetzmann and Jorion.

Advice: Pick the closest-term interest rate.

Actually, the CAPM offers no guidance, because it has no concept of more than one single time-period. It therefore does not understand why there is a yield curve (different expected rates of return over different horizons). However, from a practical perspective, it makes sense to use the yield on a Treasury bond that is of similar length as a project's approximate lifespan. So, a good heuristic is to pick the risk-free rate closest in some economic sense (maturity or duration) to our project. For example, to value a machine that produces for three years, it makes sense to use an average of the 1-year, 2-year, and 3-year risk-free interest rates, perhaps 2.5% per annum. On the other hand, if you have a 10-year project, you would probably use 4% as your risk-free rate of return. This heuristic has an intuitive justification, too—think about the opportunity cost of capital for a zero-beta investment. If you are willing to commit your money for 10 years, you could earn the 10-year Treasury rate of return. It would be your opportunity cost of capital. If you are willing to commit your money only for 3 months, you could only earn the 3-month Treasury rate—a lower opportunity cost for your capital. One important sidenote, however, is that you should use the same risk-free rate in the calculation of the equity premium—so, if you use a higher risk-free rate because your project is longer-term, you would want to use a lower equity premium where the risk-free rate enters negatively.

Solve Now!

Q 13.10 *What is today's risk-free rate for a 1-year project? For a 10-year project?*

Q 13.11 *Which risk-free rate should you be using for a project that will yield \$5 million each year for 10 years?*

13·4.C. Investment Projects' Market Betas ($\beta_{i,\mathcal{M}}$)

Unlike the risk-free rate and the equity premium, beta is specific to each project.

Finally, you must estimate your project's **market beta**, which measures how your project rates of return fluctuate with the market. Unlike the previous two inputs, which are the same for every project/stock in the economy, the beta input depends on your specific project characteristics: different investments have different betas.

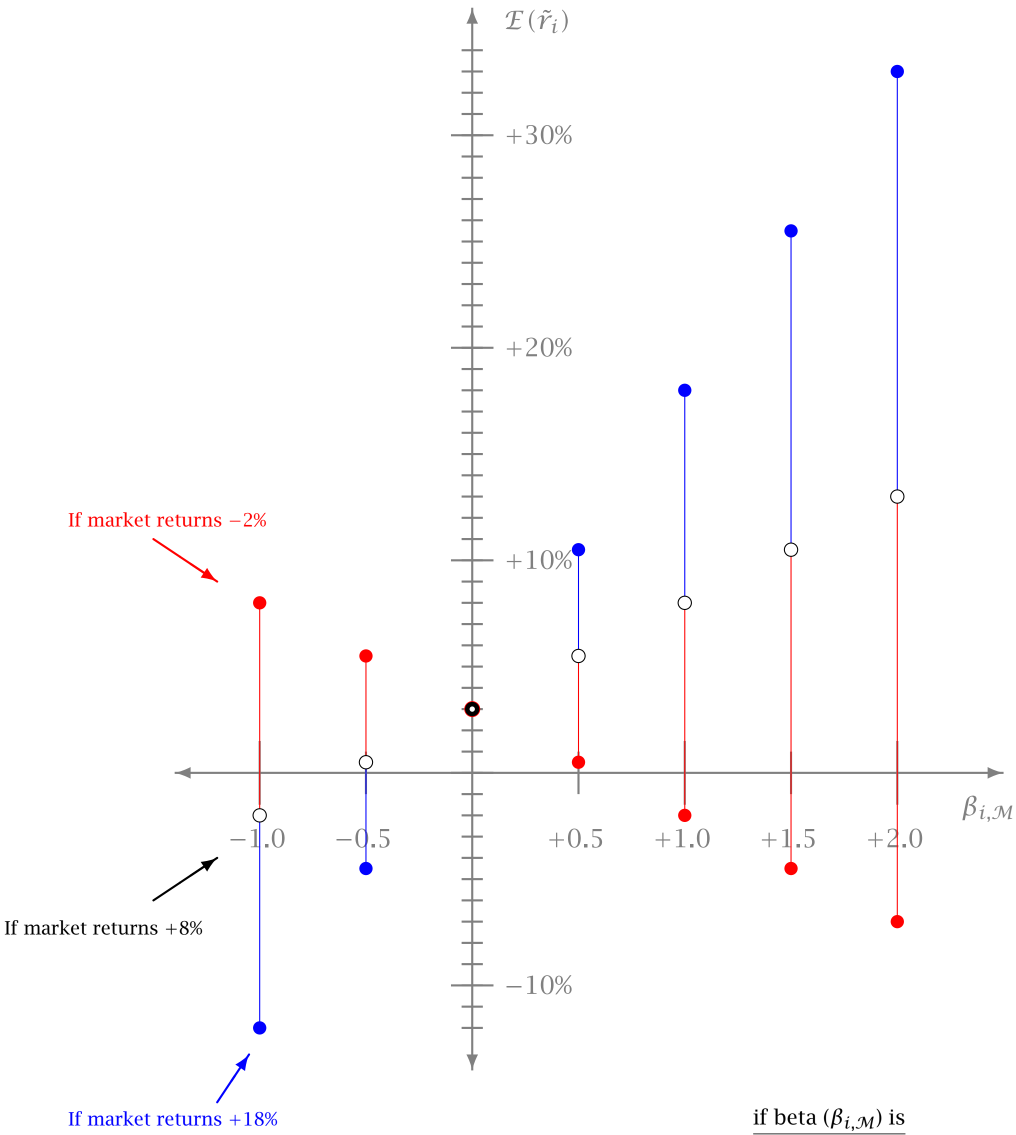
Beta creates both a mean rate of return, and an "amplification factor" of the market rate of return.

Let us gain some intuition on how the market-beta should relate the returns of individual stocks to those of the market. The market-beta has both an influence on the expected return of projects and on the range of observed returns. (This works through the part of a stock's price movement volatility that is caused by the market. There is also very important idiosyncratic price movement, but we shall ignore it in this example.) Say the risk-free rate is 3%, the expected rate of return on the market is 8%, and therefore the equity premium is 5%. A stock with a beta of -1 would therefore have an expected rate of return of -2%, a stock with a beta of $+2$ would have an expected rate of return of $+13\%$. However, more than likely, the stock market rate of return will not be exactly 8%. So, let us entertain one positive and one negative market scenario as a standin for market volatility. If the stock market were to drop by 10% relative to its mean of 8%—i.e., return an absolute -2%—then our first stock would not earn -2%, but $-2\% + \beta_i \cdot (\tilde{r}_{\mathcal{M}} - \mathcal{E}(\tilde{r}_{\mathcal{M}})) = -2\% + (-1) \cdot (-2\% - 8\%) = +8\%$, and our second stock would not earn $+13\%$, but $+13\% + (+2) \cdot (-10\%) = -7\%$. Conversely, if the stock market were to increase by 10% relative to its mean—i.e. return an absolute $+18\%$—we would expect our negative beta stock to do really poorly $(-2\% + \beta_i \cdot (\tilde{r}_{\mathcal{M}} - \mathcal{E}(\tilde{r}_{\mathcal{M}})) = -2\% + (-1) \cdot (+18\% - 8\%) = -12\%$ and our positive beta stock to do really well ($+33\%$).

Beta can be thought of as an amplifier of market movements.

In Figure 13.5, we repeat this computation for stocks with different market-betas. It shows how they are expected to perform, conditional on whether the market beats its mean (of 8% by 10%, i.e., +18%), hits its mean (of 8%), or misses its mean (of 8% by 10%, i.e., −2%). You can see how beta determines both the stock's expected rate of return—the mean given by the CAPM—and how it dampens or amplifies the effect of the stock market performance on our stock. The latter is really just our definition of market-beta—it measures how a project comoves with the stock market. The sign of the market-beta determines whether the investment tends to move with or against the stock market. And it is of course the CAPM that posits how the expected rate of return should be increasing with the market-beta.

Figure 13.5. The Effect of Market Beta on Stock Returns in Good and Bad Markets



	if beta ($\beta_{i,\mathcal{M}}$) is			
	−1.0	0.0	+1.0	+2.0
The stock's expected rate of return	−2%	+3%	+8%	+13%
If market outperforms its mean of 8% by 10% ($\tilde{r}_{\mathcal{M}}$ = +18%), then our stock $\tilde{r}_i$ is expected to return	−2% − 10% = −12%	3% ± 0% = 3%	8% + 10% = 18%	13% + 20% = 33%
If market underperforms its mean of 8% by 10% ($\tilde{r}_{\mathcal{M}}$ = −2%), then our stock $\tilde{r}_i$ is expected to return	−2% + 10% = +8%	3% ± 0% = 3%	8% − 10% = −2%	13% − 20% = −7%

Each line represents the range of return outcomes for one stock (with one particular market beta) if the market rate of return were to be between −10% and +10%. The black circle is the unconditional expected rate of return (or conditional on the market turning in its expected performance of 8%)—i.e., points on the security markets line $\mathcal{E}(\tilde{r}_i) = r_{\mathcal{F}} + [\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}]\cdot\beta_{i,\mathcal{M}} = 3\% + 5\%\cdot\beta_{i,\mathcal{M}}$. The red solid circles show the expected rate of return conditional on a market rate of return of −10%. Stocks with negative beta are expected to perform well in this case. The blue solid circles show the expected rate of return conditional on a market rate of return of +10%. Stocks with a negative beta are expected to perform poorly in this case.

Our plan: discuss various methods to find betas

Depending on the project for which you need a beta, the estimate can be easy or difficult to obtain. We now discuss the three most common sources for beta estimates.

13·4.D. Betas For Publicly Traded Firms

Beta is easy to get for publicly traded stocks.

For publicly trading stocks, finding a market-beta is easy. There are many services (e.g., Yahoo!*Finance*) that publish betas. The average beta in the stock market is 1, and most stocks have betas somewhere between about 0 and 3. Large, low-tech firms tend to have lower betas than small, high-tech firms, but this is not always the case. The published betas themselves are estimated from historical time-series regressions, often monthly data, using our statistical technique that fits the best α and β for the "regression" line $\tilde{r}_i = \alpha_i + \beta_i \cdot \tilde{r}_{\mathcal{M}} + \epsilon$. The regression estimator does exactly what we did in Section 12·3.C: it computes the covariance and divides it by the variance. (Some more sophisticated data providers improve on this simple regression estimate with a little bit of extra statistical wizardry called shrinkage, which we shall ignore.)

Solve Now!

Q 13.12 *Look up the beta for IBM at Yahoo!Finance. How does it compare to the beta of a young upstart growth company? (Pick one!)*

13·4.E. Betas From Comparables and Leverage Adjustments: Equity Beta vs. Asset Beta

Individual Betas can be noisy—we often use similar company betas.

Individual betas are very noisy. For example, a pharmaceutical whose product happened to be rejected by the FDA (usually causing a large negative return) in a month in which the market happened to go up (down) may end up having a negative (positive) market beta estimate—and this would likely be totally unrepresentative of the future market beta. (This month would be a "statistical outlier" or "influential regression observation.") In the long-run, such announcements would appear randomly, so beta would still be the right estimate—but in the long-run, we will all be dead. To reduce such noise in practice, it is common to estimate not just the beta of the firm, but to estimate the beta of a couple of similar firms (comparables similar in size and industry, perhaps), and then to use a beta that reflects some sort of average among them.

Using comparable publicly traded stocks with unlevered Betas.

Indeed, if your project has no historical rate of return experience—perhaps because it is only a division of a publicly traded company or because the company is not publicly traded (though, for the CAPM to be meaningful, the owners need to hold most of their wealth in the market portfolio—you may have little choice other than to consider comparable firms. For example, if you believe your new soda company is similar to PepsiCo, you could adopt the beta of PepsiCo and use it to compute the CAPM expected rate of return. Realizing that smaller firms than PepsiCo tend to have higher betas, you might increase your beta estimate.

Leverage Adjustments: An Intuitive Example.

It is however very important that you draw a clear distinction between equity betas and asset betas. Usually, you have an intuition that your **project beta** (also called **asset beta**) is the same as that of the publicly traded company—but all that you get to see is the comparable's equity beta. You must adjust the **equity beta** for the comparable's leverage, because stocks that are more levered have higher equity market-betas—they are riskier. Recall the example from Chapter 5: when a project was split into debt and equity, the debt became less risky, while the equity became more risky. This turns out to matter for betas, too.

Table 13.1. The Effect of Leverage on Beta

		Choice of Capital Structure		
		(A) Unlevered	(B) Split Project	
	Stock Market	Project	$150 Debt	Equity
Value Today	$10.0 trillion	$200	$150	$50
if Good Times	$13.0 trillion	$230	$156	$74
if Bad Times	$9.0 trillion	$190	$156	$34
Expected Value	$11.0 trillion	$210	$156	$54
if Good Times	+30%	+15%	+4%	+48%
if Bad Times	−10%	−5%	+4%	−32%
Expected Rate of Return	+10%	+5%	+4%	+8%
Dollar Spread	$4 trillion	$40	$0	$40
Relative Spread	40%	20%	0%	80%
Market Beta	1.0	0.5	0.0	2.0

Work one full example.

To determine how leverage changes beta, consider Table 13.1. In this example, the stock market, worth $10 trillion today, is expected to increase by 10% to $11 trillion next year. However, relative to this expected value, the market can either underperform or overperform (by plus or minus 20%). Now, your own unlevered project is worth $200 today and has a beta of $1/2$. Therefore, it is expected to return $210, but either 10% above or 10% below its mean of 5%, and depending on the stock market rate of return. This makes sense: for a 40% difference in the rate of return on the stock market, your project would suffer a 20% difference in its rate of return.

The beta of levered equity scales with leverage.

Now finance your project differently. Use an alternative capital structure that consists of $150 in debt and the rest in equity, i.e.,

$$
\begin{aligned}
\text{Value}_{\text{Project}} &= \text{Value}_{\text{Debt}} + \text{Value}_{\text{Equity}} \ , \\
100\% &= \frac{\text{Value}_{\text{Debt}}}{\text{Value}_{\text{Project}}} + \frac{\text{Value}_{\text{Equity}}}{\text{Value}_{\text{Project}}} \\
100\% &= w_{\text{Debt}} + w_{\text{Equity}} \\
100\% &= 75\% + 25\% \ ,
\end{aligned}
\tag{13.11}
$$

where the weight of each security in the capital structure is called w.

The WACC remains the same regardless of capital structure.

The debt is default-free, so it can command the risk-free rate, which we now assume to be 4% per annum. But being risk-free also means that the debt beta is 0. The value of the levered equity must then be the remaining $50. Working through the remaining cash flows, we find that its expected rate of return is 8%, which is both above the risk-free rate and the unlevered project's expected rate of return. This higher expected rate of return is necessary to compensate investors for risk. More importantly, note how your levered equity has a higher market-beta than the original unlevered project. Instead of translating a market fluctuation of 40% into a project fluctuation of ±20%, the levered equity translates the market fluctuation of 40% into a rate of return fluctuation of ±80%! The beta is now 2, not 0.5.

The weighted average beta is the overall beta.

This example shows that the weighted expected rate of return and the weighted average beta add up to their overall project equivalents:

$$\begin{array}{rcccc} 5\% & = & 75\% \cdot 4\% & + & 25\% \cdot 8\% \\ \mathcal{E}(\tilde{r}_{\text{Project}}) & = & w_{\text{Debt}} \cdot \mathcal{E}(\tilde{r}_{\text{Debt}}) & + & w_{\text{Equity}} \cdot \mathcal{E}(\tilde{r}_{\text{Equity}}) \; . \\ 0.5 & = & 75\% \cdot 0 & + & 25\% \cdot 2 \\ \beta_{\text{Project}} & = & w_{\text{Debt}} \cdot \beta_{\text{Debt}} & + & w_{\text{Equity}} \cdot \beta_{\text{Equity}} \; . \end{array} \tag{13.12}$$

The first equation is called the firm's weighted-average cost of capital, abbreviated WACC, and discussed in detail in Chapter 17. In our perfect world, the cost of capital remains invariant to whatever capital structure you may choose. The latter equation is just a special version of a general linear property of betas: as you learned earlier, you can take weighted averages of betas. Therefore, if you know how the firm is financed, and if you can guess the beta of the debt, it is easy to translate an equity beta into an asset beta.

$$\begin{array}{rcccccc} \beta_{\text{Project},mkt} & = & w_{\text{Debt}} \cdot \beta_{\text{Debt},\mathcal{M}} & + & w_{\text{Equity}} \cdot \beta_{\text{Equity},\mathcal{M}} & & \\ & = & 75\% \cdot 0 & + & 25\% \cdot 2.0 & = & 0.5. \end{array} \tag{13.13}$$

But, in general, where would you get the debt beta from? For large firm stocks that are not in financial distress, it is reasonable to presume that debt betas are reasonably close to zero. This is because the debt is likely to be repaid—and, if not, repayment may not be contingent as much on the stock market overall, as it may depend on the firm's circumstances. For small firm stocks or stocks in financial distress, bond betas can, however, become significantly positive.

IMPORTANT: *If project A consists of part B and part C,*

$$A = B + C \tag{13.14}$$

then the overall market beta of the combined project A is the weighted average market beta of its components,

$$\beta_{A,\mathcal{M}} = w_B \cdot \beta_{B,\mathcal{M}} + w_C \cdot \beta_{C,\mathcal{M}} \; , \tag{13.15}$$

where w are weights according to value today, and add up to 1. The components could be any type of investments, and in particular be the debt and equity of the same firm. Therefore,

$$\begin{array}{rcccc} \textit{Project} & = & \textit{Debt} & + & \textit{Equity} \\ \Longrightarrow \; \beta_{\textit{Project},\mathcal{M}} & = & w_{\textit{Debt}} \cdot \beta_{\textit{Debt},\mathcal{M}} & + & w_{\textit{Equity}} \cdot \beta_{\textit{Equity},\mathcal{M}} \; . \end{array} \tag{13.16}$$

Solve Now!

Q 13.13 *A project i is likely to go up by 20% if the stock market goes up by 10%. It is also likely to go down by 20% if the stock market goes down by 5%. If the risk-free rate of return is 4%, what would you expect the beta to be?*

Q 13.14 *A comparable firm (in a comparable business) has an equity beta of 2.5 and a debt/asset ratio of 2/3. The debt is almost risk-free. Estimate the beta for our firm if projects have alike betas, but our firm will carry a debt/asset ratio of 1/3.*

Q 13.15 *(Continued.) If the risk-free rate is 3% and the equity premium is 2%, what is the expected rate of return on the comparable firm's equity and on our own equity?*

Q 13.16 *A comparable firm (in a comparable business) has an equity beta of 2.5 and a debt/equity ratio of 2. The debt is almost risk-free. Estimate the beta for our firm if projects have alike betas, but our firm will carry a debt/equity ratio of 1/2.*

Q 13.17 *(Continued.) If the risk-free rate is 3% and the equity premium is 2%, what is the expected rate of return on the comparable firm's equity and on our own equity?*

Q 13.18 *You own a stock market portfolio that has a market beta of 2.4, but you are getting married to someone who has a portfolio with 0.4. You are three times as wealthy as your future significant other. What is the beta of your joint portfolio?*

13·4.F. Betas Based on Economic Intuition

Intuitive Betas Guestimating.

Sometimes, there are projects for which there are no good publicly traded firms from which you can extract a beta estimate. In such cases, you need to make a judgment: how will the rate of return of your project covary with the stock market? To find out, rearrange the CAPM Formula:

$$\mathcal{E}(\tilde{r}_i) = r_{\mathcal{F}} + [\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}] \cdot \beta_{i,\mathcal{M}} \quad \Longleftrightarrow \quad \beta_{i,\mathcal{M}} = \frac{\mathcal{E}(\tilde{r}_i) - r_{\mathcal{F}}}{\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}} \quad . \tag{13.17}$$

The right side of this formula helps translate your intuition into a beta estimate. You can ask such questions as "What rate of return (above the risk-free rate) will your project have if the stock market were to have +10% or –10% rate of return (above the risk-free rate)?" Clearly, such guess work is difficult and error-prone—but it can provide a beta estimate when no other is available.

13·4.G. Robustness: How Bad are Mistakes in CAPM Inputs?

Where will we inevitably go wrong?

You know that you do not really know the inputs for the CAPM perfectly. You can only make educated guesses. And even *after* the fact, you will never be sure—you observe only actual rates of returns, never expected rates of return. So, how robust is the CAPM with respect to errors in its inputs?

Errors in the risk-free rate tend to be very small.

The Risk-Free Rate Errors in the risk-free rate ($r_{\mathcal{F}}$) are likely to be modest. The risk-free rate can be considered to be almost known for practical purposes. Just make sure to use a risk-free rate of similar duration and maturity as your project.

This leaves you with having to judge the influence of errors in estimating betas ($\beta_{i,\mathcal{M}}$), errors in estimating the expected market rate of return ($\mathcal{E}(\tilde{r}_{\mathcal{M}})$), and model errors (i.e., that the CAPM itself is false).

Errors in beta estimates tend to be modest.

Market-Beta Reasonable beta estimates typically have some uncertainty, but good comparables can often be found in the public market. If due care is exercised, a typical range of uncertainty about beta might be about plus or minus 0.4. For example, if the equity premium is 3% and if you believe your beta is 2, but it is really 1.6 instead, then you would overestimate the appropriate expected rate of return by $2 \cdot 3\% - 1.6 \cdot 3\% = 1.2\%$. Although this level of uncertainty is not insignificant, it is tolerable in corporate practice.

Disagreement on the equity premium tends to be large, and these differences in equity premium estimates can have a large influence.

Equity Premium Estimates Reasonable equity premium estimates can range from about 2% per year to about 6% per year—a large range. *To date, there is no universally accepted method to estimate the expected rate of return on the market, so this disagreement cannot be easily settled with data and academic studies.* Unfortunately, reasonable differences of opinion in estimating the expected rate of return on the market can have a large influence on expected rate of return estimates. For example, assume the risk-free rate is 3%, and take a project with a beta of 2. The CAPM might advise this corporation that potential investors demand either an expected rate of return of 5% per year (equity premium estimate of 1%) or an expected rate of return of 19% per year (equity premium estimate of 8%), or anything in between. This is—to put it diplomatically—a miserably large range of possible cost of capital estimates. (And this range does not even consider the fact that *actual* future project rates of return will necessarily differ from *expected rates of return*!) Of course, in the real world, managers who want to take a project will argue that the expected rate of return on the market is low. This means that their own project looks relatively more attractive. Potential buyers of projects will argue that the expected rate of return on the market is high. This means that they claim they have great opportunities elsewhere, so that they can justify a lower price offer for this project.

Use the CAPM as guidance, not as gospel!

So, where does this leave the CAPM? First, you need to realize again that there are really no better alternatives in a corporate context. No matter how poor or imprecise the CAPM estimates are, without a better alternative, you have little choice but to use it. Second, as a CAPM user, you need to be aware of its limitations. The CAPM is a model that can often provide a "*reasonable* expected rate of return," but not an "*accurate* expected rate of return." Anyone who believes that CAPM expected rates of return should be calculated with more than one digit after the decimal point is deluded. The CAPM can only offer expected rates of returns that are of the "right order of magnitude." The CAPM also often tends to be better in ranking projects than in providing a good absolute cost of capital. In this case, estimating the equity premium to be too low or too high tends to bias the valuation of all projects—though not necessarily equally so.

Put together NPV and CAPM robustness considerations.

You often use the CAPM expected rate of return as our cost of capital in an NPV calculation. Here, you combine errors and uncertainty about expected cash flows with your errors and uncertainty in CAPM estimates. What should you worry about? Recall that in Chapter 5, you saw the relative importance of getting the inputs into the NPV formula correct. The basic conclusion was that for short-term projects, getting the cash flows right is more important than getting the expected rate of return right; for long-term projects, getting both right is important. We just discussed the relative importance of getting the equity premium and the project beta right. Now recall that your basic conclusion was that the CAPM formula is first and foremost exposed to errors in the market risk premium (equity premium), though it is also somewhat exposed to beta estimates. Putting these two conclusions together suggests that for short-term projects, worrying about exact beta estimates is less important than worrying about estimating cash flows first and the appropriate equity premium second. For long-term projects, the order of importance remains the same, but the difference in the relative importance of good estimates of expected cash flows and good estimates of the equity premium estimates shrinks.

ANECDOTE: "Cost of Capital" Expert Witnessing

When Congress tried to force the "Baby Bells" (the split up parts of the original AT&T) to open up their local telephone lines to competition, it decreed that the Baby Bells were entitled to a fair return on their infrastructure investment—with fair return to be measured by the CAPM. (The CAPM is either the de facto or legislated standard for measuring the cost of capital in many other regulated industries, too.) The estimated value of the telecommunication infrastructure in the United States is about \$10 to \$15 billion. A difference in the estimated equity premium of 1% may sound small, but even in as small an industry as local telecommunications, it meant about \$1,000 to \$1,500 million a year—enough to hire hordes of lawyers and valuation consultants opining in court on the appropriate equity premium. Some of my colleagues bought nice houses with the legal fees.

I did not get the call. I lack the ability to keep a straight face while stating that "the equity premium is exactly x point y percent," which was an important qualification for being such an expert. In an unrelated case in which I testified, the opposing expert witness even explicitly criticized my statement that my cost of capital estimate was an imprecise range—unlike me, he could provide an exact estimate!

In contrast, in most cases, honest mistakes in beta, *given reasonable care*, are relatively less problematic.

Solve Now!

Q 13.19 *To value an ordinarily risky project, that is, a project with a beta in the vicinity of about 1, what is the relative contribution of your personal uncertainty (lack of knowledge) in: the risk-free rate, the equity premium, the beta, and the expected cash flows? Consider both long-term and short-term investments.*

13·5. Value Creation and Destruction

We delay applications until later.

Most of our CAPM applications will be explored in Chapters 17 and 18. Chapters 17 explains how to use the CAPM in a perfect world without taxes. Chapter 18 explains how to use the CAPM in the presence of (corporate) income taxes. Because the primary use of the CAPM is to determine appropriate costs of capital in corporations, it is only in these later chapters that this book offers enough examples to familiarize you with CAPM applications.

Important: How to add value!

However, there are at least two important and basic concepts that were first raised in Chapter 7 that we can finally discuss now, given that the CAPM illuminates the cost of capital. The first concept is almost trivial—it is the question of whether managers should seek to reduce firm risk. The second concept relates to the simplest of insights—that the total net present value of two projects combined without project externalities is the sum of the project's net present value. As always, the concept is straightforward, but the devil is in the details.

13·5.A. Does Risk-Reducing Corporate Diversification (or Hedging) Create Value?

Diversification reduces risk, but does not create value.

In the 1960s through 1970s, many firms became **conglomerates**, that is, companies with widely diversified and often unrelated holdings. Can firms add value through such diversification? The answer is "usually no." Diversification indeed reduces the standard deviation of the rate of return of the company—so diversified companies are less risky—but your investors can just as well diversify risk for themselves. For example, if your \$900 million firm ABC (e.g., with a beta of 2, and a risk of 20%) is planning to take over the \$100 million firm DEF (e.g., with a beta of 1, and also risk of 20%), the resulting firm is worth \$1 billion dollars. ABC+DEF has indeed an idiosyncratic risk lower than 20% if the two firms are not perfectly correlated, but your investors (or a mutual fund) could just purchase 90% of ABC and 10% of DEF and thereby achieve the very same diversification benefits. If anything, you have robbed investors of a degree of freedom here: they no longer have the ability to purchase, say, 50% in ABC and 50% in DEF. (In a CAPM world, this does not matter.) The CAPM makes it explicit that the cost of capital does not change unduly. Say both firms follow the CAPM equation, and say that the risk-free rate is 3% and the equity premium is 5%,

$$
\begin{aligned}
\mathcal{E}(\tilde{r}_{\text{ABC}}) &= 3\% &+& \quad 5\% \cdot 2 &=& \; 13\% \;, \\
\mathcal{E}(\tilde{r}_{\text{ABC}}) &= r_{\mathcal{F}} &+& \left[\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}\right] \cdot \beta_{\text{ABC},\mathcal{M}} & & \;, \\
\mathcal{E}(\tilde{r}_{\text{DEF}}) &= 3\% &+& \quad 5\% \cdot 1 &=& \; 8\% \;, \\
\mathcal{E}(\tilde{r}_{\text{DEF}}) &= r_{\mathcal{F}} &+& \left[\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}\right] \cdot \beta_{\text{DEF},\mathcal{M}} & & \;.
\end{aligned}
\tag{13.18}
$$

The newly formed company will have an expected rate of return—cost of capital—of

$$
\begin{aligned}
\mathcal{E}(\tilde{r}_{\text{ABC+DEF}}) &= 90\% \cdot 13\% &+& \; 10\% \cdot 8\% &=& \; 12.5\% \;, \\
\mathcal{E}(\tilde{r}_{\text{ABC+DEF}}) &= w_{\text{ABC}} \cdot \mathcal{E}(\tilde{r}_{\text{ABC}}) &+& \; w_{\text{DEF}} \cdot \mathcal{E}(\tilde{r}_{\text{DEF}}) & & \;,
\end{aligned}
\tag{13.19}
$$

and a market-beta of

$$
\begin{aligned}
\beta_{\text{ABC+DEF},\mathcal{M}} &= 90\% \cdot 2 + 10\% \cdot 1 = 1.9 \\
\beta_{\text{ABC+DEF},\mathcal{M}} &= w_{\text{ABC}} \cdot \beta_{\text{ABC},\mathcal{M}} + w_{\text{DEF}} \cdot \beta_{\text{DEF},\mathcal{M}} \;.
\end{aligned}
\tag{13.20}
$$

The merged company will still follow the CAPM,

$$
\begin{aligned}
\mathcal{E}(\tilde{r}_{\text{ABC+DEF}}) &= 3\% + 5\% \cdot 1.9 = 12.5\% \\
\mathcal{E}(\tilde{r}_{\text{ABC+DEF}}) &= r_{\mathcal{F}} + [\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}] \cdot \beta_{\text{ABC+DEF},\mathcal{M}} \;.
\end{aligned}
\tag{13.21}
$$

Its cost of capital has not unduly increased or declined. In an ideal CAPM world, no value has been added or destroyed—even though ABC+DEF has a risk lower than the 20% per annum that its two constituents had.

Synergies or Dis-synergies drive M&A value, not diversification. Managers also have agency conflicts in M&A activity.

In the real world, diversified firms often do not operate as efficiently as stand-alone firms, e.g., due to limited attention span of management or due to more bureaucratization. Such mergers destroy firm value. Of course, other mergers can add value due to synergies, as we discussed in Chapter 7. More often, however, the unspoken rationales for mergers are that managers prefer the reduced idiosyncratic uncertainty and higher salaries guaranteed by larger firms to the higher risk and lower salaries in sharply focused, smaller firms. In our context, to justify a merger, managers will want to argue for a lower cost of capital any way they can—including incorrectly using the acquirer's cost of capital. (This is another example of an agency conflict, which we have seen in Chapter 7 and which we will see again in our Chapter 24 on corporate governance.)

IMPORTANT: *If there are no cash flow synergies, combining firms into conglomerates may reduce firm risk, but does not create value for our investors. Investors can diversify risk themselves.*

Managers who want to create value through risk reduction should instead seek to lower their firms' market betas—of course avoiding proportionally similar or higher reductions in their firms' rewards.

Hedging against stock market risk.

Firms can also reduce their overall risk by **hedging**. The simplest example of a hedge would be if the firm itself shorted the stock market. For example, it could sell a contract that promises to deliver the index level of the S&P500 multiplied by 1,000 in one year. Between now and next year, whenever the stock market goes up, the value of this contract goes up. The contract has a negative beta. Because the hedged firm would consist of the unhedged firm plus this contract, the market-beta (or risk) of the hedged firm would be lower than the market-beta of the unhedged firm. In fact, the firm could sell the exact amount of contracts that make the firm's

ANECDOTE: Risk and Conglomeration
In the 1970s, a lot of firms diversified to become conglomerates. Management argued that conglomerates tended to have lower risk, which created value for shareholders. This argument was, of course, total nonsense: Investors could diversify for themselves. It was the managers who valued lower risk, with the lower chance to lose their jobs and the higher compensation due to running a bigger company. Worse, because conglomerates often operate less efficiently than individual stand-alone, focused companies, diversification actually often destroyed firm value. In the 1980s, there were many "bust-up buyouts," which created value by purchasing conglomerates to sell off the pieces.

A good example of such a conglomerate was Gulf and Western. It was simultaneously involved in oil, movies (Paramount), recording (Stax), rocket engines, stereo components, finance, publishing (Simon and Schuster), auto parts, cigars, etc. It promptly crashed and split up in the 1980s. A more current example is Tyco, which has over 260,000 employees in 50 (!) separate business lines, including electronics, undersea fiber optic cables, healthcare, adhesives, plastics, and alarm systems. (Its former executive, Dennis Kozlowski, became famous for his extravagant looting of Tyco's assets. With so many business lines, no wonder no one noticed for years!) The most interesting conglomerate, however, is General Electric. It has hundreds of business lines, but unlike most other conglomerates, GE seems to have been running most of its divisions quite well.

market beta zero or even negative. But, this hedging contract would not create firm value—the firm's expected rate of return would decline proportionally, too. If investors wanted to have less exposure to the overall stock market, they could sell such hedging contracts themselves.

Firms do sometimes hedge against other risks. For example, oil companies often sell contracts on oil that promise delivery in one year. This insulates them from the volatility of the price of oil. In itself, in a perfect market, such fairly priced hedging contracts neither add nor subtract value. But if the market is imperfect, a hedge may allow a firm to operate more efficiently (e.g., generating cash which avoids the need to borrow money), and thereby add value. Corporate hedging is the subject of a web chapter.

Hedging against other risks.

13·5.B. Avoiding Cost-of-Capital Mixup Blunders That Destroy Value

When Acquiring Another Company

Section 7·1.B dropped a cryptic hint that practitioners sometimes forget that NPVs are additive. You may have wondered what was meant. You are now ready to see why this is such a common mistake. The most common abuse of the CAPM arises from the use of the firm's overall cost of capital for individual projects, and here is an example why.

Common misuse of CAPM: a uniform cost of capital.

Assume the risk-free rate of return is 3% and the equity premium is 4%. Your old firm, cleverly named O, is worth \$100 and has a market beta of 0.5. An acquisition target (or just a new project), cleverly named N, costs \$10 and is expected to pay off \$11 next year. (Its rate of return is therefore 10%.) The beta of this new project is 3.

What happens if the firm uses its overall cost of capital for projects, rather than a project specific cost of capital?

For IRR, see Chapter 8.

The simplest method to compute the value of project N relies on the fact that NPVs are additive. You value the new project using its own expected cash flows and own cost of capital. *Who* owns it should matter little: the project is worth what it is worth. So, N should offer an expected rate of return of

The Solution: Compute the NPV of the project.

$$\begin{array}{rcccccccc} \underline{\text{Correct Cost of Capital:}} & \mathcal{E}(\tilde{r}_N) & = & 3\% & + & 4\% & \cdot & 3 & = 15\% \\ & \mathcal{E}(\tilde{r}_N) & = & r_{\mathcal{F}} & + & [\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}] & \cdot & \beta_{N,\mathcal{M}} & , \end{array} \tag{13.22}$$

and the true NPV of the new project is

$$\text{NPV}_N = -\$10 + \frac{\$11}{1+15\%} \approx -\$0.44 \; . \tag{13.23}$$

Therefore, if firm O adopts project N, N's owners would be –\$0.44 poorer than they would be if their managers did not adopt it (i.e., \$100 vs. \$99.56).

Unfortunately, this is not obvious to some practitioners. In many firms, it is standard policy to evaluate *all* projects by the firm's overall cost of capital. Would such a firm take the N project? Evaluated with a market beta of 0.5, the hurdle rate for the project would be

Bad Company Policy.

$$\begin{array}{rcccccccc} \underline{\text{Incorrect Cost of Capital:}} & \mathcal{E}(\tilde{r}) & = & 3\% & + & 4\% & \cdot & 0.5 & = 5\% \; . \\ & & = & r_{\mathcal{F}} & + & [\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}] & \cdot & \beta_{O,\mathcal{M}} & . \end{array} \tag{13.24}$$

With its internal rate of return of \$11/\$10 – 1 = 10%, a (poor) manager would indeed take this project.

If the O firm did take project N, how would its value change? With a beta of 0.5, the old firm had an expected rate of return of 3% + 4%·0.5 = 5%. Its expected value next year would be \$105. Using PV, we see that the present value of the combined firm would be

The loss if the firm makes the mistake.

$$\begin{array}{rcccccc} \text{PV}_{\text{combined}} & = & \dfrac{\$105}{1+5\%} & + & \dfrac{\$11}{1+15\%} & \approx & \$109.56 \; . \\ & = & \text{PV}_O & + & \text{PV}_N & & . \end{array} \tag{13.25}$$

This is \$0.44 less than the original value of \$100 plus the \$10 acquisition cost of the new project. Taking the project has made the N owners 44 cents poorer.

Real World Exceptions. However, contrary to the perfect CAPM world, it is not always true in the real world that mergers *never* add value on the cost-of-capital side. If capital markets are not as efficient for small firms as they are for large firms, it would be possible for a large acquirer to create value. For example, if a target previously had no access to capital markets, as explained in Section 6·1 (Page 110), then the cost of capital to the target can change when it is acquired. The correct cost of capital for valuing the acquisition (the target), however, is *neither* the cost of capital of the acquirer, *nor* the blended post-acquisition cost of capital of the firm. Instead, the correct cost of capital is that appropriate for the target's projects, given the "now ordinary" access to capital markets. For example, if an entrepreneur inventor of holographic displays previously had faced a cost of capital of, say, 303%, primarily due to access only to personal credit card and credit shark financing, and if this inventor's business is purchased by IBM with its cost of capital of 6.5% (market-beta of 1.5), the proper cost of capital is neither IBM's (market-beta based) cost, nor a blended average between 303% and 6.5%. Instead, if part of IBM, the holographic project division should be evaluated at a cost of capital that is appropriate for projects of the market-beta risk class "holographic display projects." This can add value relative to the 303% earlier cost of capital. (Of course, large corporations are often also very adept at destroying all innovation and thereby value in the small companies that they are taking over.)

When Acquiring Another Project

Projects must be discounted by their own beta. It is important to realize that not only firms-to-be-acquired, but also smaller projects themselves consist of components with different market-betas, which therefore have different costs of capital. For example, when firms keep cash on hand in Treasury bonds, such investments have a zero market-beta, which is lower than the beta for the firms' other projects. These bonds should need not earn the same expected rate of return as investments in the firm's risky projects. (The presence of this cash in the firm lowers the average beta of the firm and thus the average cost of capital for the firm by the just-appropriate amounts.)

Another Example Problem. Here is another application example: Assume that you consider purchasing a rocket to launch a Telecomm satellite next year. It would take you 1 year to obtain the rocket, at which point you would have to pay \$100 million. Then you launch it. If the rocket fails (25% chance), then your investment will be lost. If the rocket succeeds, the satellite will produce a revenue stream with an appropriate beta of 2. (Telecomm revenues tend to have a high covariance with the market.) Telecomm's expected cash flows will be \$20 million *forever*. Assume that the risk-free rate is 3% per year and the market equity-premium is 4%.

The Example Solution. The correct solution is to think of the rocket as one project and of the Telecomm revenues as another project. The rocket project has only idiosyncratic risk; therefore, its beta is close to zero, and its discount factor is the same as the risk-free rate of return, 3%. The rocket value (in millions of dollars today is)

$$\mathsf{PV}_{\text{rocket}} \approx \frac{-\$100}{1+3\%} \approx -\$97 \quad . \tag{13.26}$$

You can think of this as the cost of storing the \$100 million in T-bills until we are ready to try our second project. The Telecomm revenues, however, would be a risky perpetuity. With a beta of 2, their cash flows would be discounted at about 11%. However, the cash flows will only occur with a probability of 75%. Therefore,

$$\mathsf{PV}_{\text{Telecomm}} \approx \frac{\mathcal{E}(\text{Telecomm Profits})}{\mathcal{E}(\tilde{r}_{\text{Telecomm Revenues}})} = \frac{75\% \cdot \$20}{3\% + 4\% \cdot 2 = 11\%} = \frac{\$15}{11\%} = \$136 \quad . \tag{13.27}$$

Consequently, this project has a net present value of about \$39 million dollars.

13·5.C. Differential Costs of Capital — Theory and Practice!

There is no doubt that projects must be discounted by their project-specific cost of capital. Yet, Graham and Harvey found that just about half of surveyed CFOs *always*—and *incorrectly*—use the firm's overall cost of capital, rather than the project-specific cost of capital. And even fewer CFOs correctly discount cash flows of different riskiness within projects. The easy conclusion is that CFOs are ignorant—and though some CFOs may indeed use a uniform cost of capital because they are ignorant, some intelligent CFOs are doing so quite deliberately.

In practice, a good number of firms do not use differential costs of capital.

You already know that it is very difficult to correctly estimate the cost of capital. In theory, you just know the market-beta of every project and the other CAPM inputs. In practice, you do not.

Getting project costs of capital is difficult.

1. Even the historical betas of publicly traded corporations are not entirely reliable and indicative of the future. Different estimation methods can come up with different numbers. This is why you may want to use the market betas of similar, publicly traded comparables or the market beta of an entire industry. But many of your projects may be so idiosyncratic, so unusual, or in such far-away locales that no comparable may seem particularly suitable.

2. You could try to estimate your own market beta. To do so, you would need a time-series of historical project values, not just historical project cash flows. This is because you cannot rely on historical cash flow variation as a substitute for historical value variation. You already know that the market values themselves are the present discount value of *all* future cash flows, not just of one period's. Here is an example how this can go awry. Consider a firm whose cash flows are perfectly known. Therefore, its appropriate true discount rate would be close to the risk-free rate. However, if its cash flows occur only every other month (\$200, \$0, \$200, etc.), this firm would have infinite monthly cash flow volatility (-100% followed by $+\infty\%$). Its percent changes in cash flows would not be indicative of its value-based rates of returns. Plus, almost surely, it would have an extreme market-beta estimate, indicating a wrong cost of capital. So in order to estimate your market-beta, you would need to somehow obtain a time series of estimated market values from the known time series of cash flows. Of course, you already know that it is difficult to estimate one market value for our firm—but estimating a time-series of how this market value changes every month is entirely beyond anyone's capability. (When only cash flows but not market-values are known, your estimates must necessarily be less accurate. The best way to estimate an appropriate cost of capital relies on the certainty equivalence formula in Section A.)

3. Many firms may not have *any* historical experience that you can use, not just for market values, but even for cash flows. There would be nothing you could verifiably and credibly use to estimate in the first place.

So, beta estimates are often difficult to estimate, equity premium estimates are very uncertain, and the CAPM is not a perfect model. These uncertainties may not only distort the overall corporate cost of capital, but also the relative costs of capital across different projects. Quite simply, you must be cognizant of the painful reality that your methods for estimating the cost of capital are often just not as robust as you would like them to be.

Consequently, the problem with assigning different costs of capital to different projects may now become one of disagreement. Division managers can argue endlessly why their projects have a lower cost of capital than the company's. Is this how you want your division managers to spend their time? Managers could even shift revenues from weeks in which the stock market performed well into weeks in which the stock market performed poorly in order to produce a lower market-beta. The cost of capital estimate itself becomes a piece in the game of agency conflict and response—every manager would like to convince himself and others that a low cost of capital for her own division is best. What the overall corporation would like to have in order to suppress such "gaming of the system" would be one immutable good estimate of the cost of capital for each division that cannot be argued with. In the reality of corporate politics, however, it may be easier to commit to one and the same immutable cost of capital for

Flexible costs of capital can cause arguments and agency conflicts.

all divisions than it would be to have immutable but different costs of capital for each division. This is not to argue that this one cost of capital is necessarily a good system, but just that there are cases in which having this *one* cost of capital may be a necessary evil.

You will never get this perfectly right. Get it right where it matters!

And finally there is the forest. You know that each component must be discounted at its own discount rate if you want to get the value and incentives right. However, if you want to value each paperclip by its own cost of capital, you will never come up with a reasonable firm value—you will lose the forest among the trees. You need to keep your perspective as to what reasonable errors are and what unreasonable errors are. The question is one of magnitude: if you are acquiring a totally different company or project, with a vastly different cost of capital, and this project will be a significant fraction of the firm, then the choice of cost of capital matters and you should differentiate. However, if you are valuing a project that is uncertain, and the project is relatively small, and its cost of capital is reasonably similar to your overall cost of capital, you can probably live with some error. It all depends—your mileage may vary!

IMPORTANT:

- *Theoretically, all projects must be discounted by their own cost of capital, and not by the firm's overall cost of capital.*
- *Practically, sanity considerations prevent discounting every paper clip by its own cost of capital.*

Therefore, you must judge when it is important to work with different costs of capital and when it is better to use just one cost of capital.

Solve Now!

Q 13.20 *A $300 million firm has a beta of 2. The risk-free rate is 4%, the equity premium is 3%. A supplier has approached the firm for a 1-year loan of $100 million that has a beta of 0. The supplier is willing to pay 6% interest, and there is no default risk. The firm has a policy of only accepting projects with a hurdle rate of 10%.*

(a) *If the firm changes its policy and extends the loan, how would its value change?*

(b) *If the firm changes its policy and extends the loan, approximately how would its beta change?*

(c) *If the firm changes its policy and extends the loan, approximately how would its cost of capital change?*

(d) *If the firm changes its policy and extends the loan, approximately what would its cash flows be expected to be?*

(e) *If the firm changes its policy and extends the loan, can you compute the combined firm's NPV by dividing its expected cash flow by its combined cost of capital?*

(f) *Should the firm change its policy?*

Q 13.21 *Some companies believe they can use the blended cost of capital post-acquisition as the appropriate cost of capital. However, this also leads to incorrect decisions. We explore this now.*

(a) *What is the value of the new project, discounted at its true cost of capital, 15%? (Assume that the combined firm value is around $109.48.)*

(b) *What is the weight of the new project in the firm?*

(c) *What is the beta of the new overall (combined) firm?*

(d) *Use this beta to compute the combined cost of capital.*

(e) *Will the firm take this project?*

(f) *If the firm takes the project, what will the firm's value be?*

13·6. Summary

The chapter covered the following major points:

- The CAPM provides an "opportunity cost of capital" for investors, which corporations can use as the hurdle rate (or cost of capital) in the NPV formula. The CAPM formula is

$$\mathcal{E}(\tilde{r}_i) = r_{\mathcal{F}} + [\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}] \cdot \beta_{i,\mathcal{M}} \quad . \tag{13.28}$$

  Thus, there are three inputs: the risk-free rate of return ($r_{\mathcal{F}}$), the expected rate of return on the stock market ($\mathcal{E}(\tilde{r}_{\mathcal{M}})$), and the project's or firm's market beta ($\beta_{i,\mathcal{M}}$). Only the latter is project specific.

- The line plotting expected rates of return against market beta is called the security markets line (SML).

- The empirical SML from 1970–2000 has a reasonably CAPM-consistent upward slope, even though this is only true if other characteristics (such as growth/value) are not controlled for. Therefore, the CAPM is not a good model for investing purposes, although it remains a reasonable model for capital budgeting purposes.

- The CAPM provides an expected rate of return, consisting of the time-premium and the risk-premium. In the NPV formula, the default-risk and default-premium works through the expected cash flow in the numerator, not through the expected rate of return (cost of capital) in the denominator.

- The expected rate of return on the market is often a critical input, especially if market beta is high—but it is difficult to guess. There are at least five different common guestimation methods, but we really do not know which one is best. Reasonable estimates for the equity premium ($\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}$) range from about 2% to 6% per annum.

- For $r_{\mathcal{F}}$, you should use a risk-free Treasury bond interest rate that is similar to the approximate duration or maturity of the project.

- There are a number of methods to estimate market beta. For publicly traded firms, it can be obtained from commercial data vendors (or self-computed). For private firms or projects, a similar publicly traded firm can often be found. Finally, managerial scenarios can be used to estimate market betas.

- To value a project, corporations should not use the cost of capital (market beta) applicable to the entire firm, but rather the cost of capital (market beta) applicable to the project. However, because the effort involved can be enormous, you should use individual, project-specific costs of capital primarily when it makes a difference.

- Corporations can reduce their risk by diversification—but if investors can do so as easily, diversification per se does not create value.

- In a perfect CAPM market, managers can create value only by increasing cash flows or decreasing market beta (the cost of capital). Diversification for the sake of diversification does not add value.

- Certainty equivalence is discussed in the appendix. You must use the certainty-equivalence form of the CAPM when projects are purchased or sold for a price other than their fair present market-value. It is also often the only method if only underlying cash flows rather than value estimates are available.

A. Appendix: Valuing Goods Not Priced at Fair Value via Certainty Equivalence

How to value a project if the efficient price today is not known?

The CAPM is usually called a pricing model—but then it is presented in terms of rates of return, not prices. This turns out to have one perplexing consequence, which leaves us with one important and difficult conceptual issue best illustrated with a brainteaser: What is today's value of a gift expected to return \$100 next year?

a. Finding The True Value of A Good That is Not Fairly Priced

At a price of zero, is the appropriate cost of capital in the CAPM formula infinite?

How do you even compute the beta of the gift's rate of return with the rate of return on the stock market? The price is \$0 today, which means that your actual rate of return will be infinite! But we clearly should be able to put a value on this gift. Indeed, our intuition tells us that this cash flow is most likely worth a little less than \$100, the specifics depending on how the cash flow covaries with the stock market. But, how do we compute this value? The solution to this puzzle is that the *price* of the gift may be \$0 today, but its *value* today (PV_0) is not—and it is the latter, i.e., the fair value, that is used in the CAPM, not the former.

IMPORTANT:

- *The* CAPM *works only with expected rates of return ($\mathcal{E}(\tilde{r}_i) = [\mathcal{E}(\tilde{P}_1) - P_0]/P_0$) that are computed from the true perfect market asset values today (PV_0) and tomorrow ($\mathcal{E}(\tilde{P}_1)$).*
- *If either the price today or next period is not fair, then you cannot work with the standard* CAPM *formula, $\mathcal{E}(\tilde{r}_i) = r_{\mathcal{F}} + \left[\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}\right] \cdot \beta_{i,\mathcal{M}}$.*

Of course, in a perfect and efficient market, what you get is what you pay for ($P_0 = \text{PV}_0$ and $P_1 = \text{PV}_1$), so this issue would never arise. But, if you buy an asset at a better or worse deal ($P_0 < \text{PV}_0$ or $P_0 > \text{PV}_0$), e.g., from a benevolent or malevolent friend, then you can absolutely not use such a P_0 to compute the expected rate of return in the CAPM formula. The same applies to $\mathcal{E}(\tilde{P}_1)$: the expected value tomorrow must be the true expected value, not a sweetheart deal value at which you may let go of the asset, or an excessive price at which you can find a desperate buyer.

We need to rearrange the CAPM formula into the Certainty Equivalence Formula: We work out an expected value that we can discount with the risk-free rate.

Now, return to our question of how to value a gift. Our specific computational problem is tricky: we could compute a rate of return for the cash flow if we knew PV_0, then from the rate of return we could compute the project beta, which we could use to find the discount rate to translate the expected cash flow back into the price PV_0 today. Alas, we do not know PV_0, so we cannot compute a rate of return. To solve this dilemma, we must use an alternative form of the CAPM formula, called its **certainty equivalence** form. It is

$$\begin{aligned}\text{PV}_0 &= \frac{\mathcal{E}(\tilde{P}_1) - [\mathcal{E}(\tilde{r}_{\mathcal{M}} - r_{\mathcal{F}})/\mathcal{V}ar(\tilde{r}_{\mathcal{M}})] \cdot \mathcal{C}ov(\tilde{P}_1, \tilde{r}_{\mathcal{M}})}{1 + r_{\mathcal{F}}} \\ &= \frac{\mathcal{E}(\tilde{P}_1) - \lambda \cdot \mathcal{C}ov(\tilde{P}_1, \tilde{r}_{\mathcal{M}})}{1 + r_{\mathcal{F}}} \quad ,\end{aligned} \tag{13.29}$$

where λ is $\{[\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}]/\mathcal{V}ar(\tilde{r}_{\mathcal{M}})\}$. So, if we believe that the expected annual equity premium is 5%, and that the variance of the rate of return on the market is around 0.04 (a standard deviation of 20%), we would choose a lambda of around 1.25. (It is the equity premium (8%−3% =

5%) divided by the variance of the rate of return on the stock market, $[(28\% - 8\%)^2 + (-12\% - 8\%)^2]/2 = 0.04$). If the risk-free rate today is 3%, we would value projects as

$$\mathsf{PV}_0 = \frac{\mathcal{E}(\tilde{P}_1) - 1.25 \cdot Cov(\tilde{P}_1, \tilde{r}_{\mathcal{M}})}{1+3\%} = \underbrace{\frac{\mathcal{E}(\tilde{P}_1)}{1+3\%}}_{\text{as-if-riskfree}} - \underbrace{\frac{1.25 \cdot Cov(\tilde{P}_1, \tilde{r}_{\mathcal{M}})}{1+3\%}}_{\text{risk discount}} . \tag{13.30}$$

The name "certainty-equivalence" is apt. The first form shows that, after we have reduced the expected value of the future cash flow ($\mathcal{E}(\tilde{P}_1)$) by some number that relates to the cash flow's covariance with the market, we can then treat this reduced value as if it were a perfectly certain future cash flow and discount it with the risk-free rate. The second form shows that we can decompose the price today into an "as-if-risk-neutral" value discounted only for the time-premium (with the risk-free rate) and an additional discount for covariance risk with the stock market.

Watch out: the covariance is different in this form!

The covariance between the future value $\tilde{P}_1$ and the rate of return on the market is related, but not identical to the project's market-beta. It is *not* the covariance of the project's *rate* of return with the market rate of return, either. It is the covariance of the project's *cash flow* with the market rate of return, instead.

Our problem solved with zero covariance.

With the certainty equivalence formula, we can now value the $100 expected gift. Assuming that the risk-free rate is 3% per annum, and that the lambda is the aforementioned 1.25,

$$\begin{aligned} \mathsf{PV}_0 &= \frac{\$100 - 1.25 \cdot Cov(\tilde{P}_1, \tilde{r}_{\mathcal{M}})}{1+3\%} \\ \mathsf{PV}_0 &= \frac{\mathcal{E}(\tilde{P}_1) - \lambda \cdot Cov(\tilde{P}_1, \tilde{r}_{\mathcal{M}})}{1+r_{\mathcal{F}}} . \end{aligned} \tag{13.31}$$

If we believe that the gift's payout does not covary with the rate of return on the market, so $Cov(\tilde{P}_1, \tilde{r}_{\mathcal{M}}) = 0$, then

$$\begin{aligned} \mathsf{PV}_0 &= \frac{\$100 - 1.25 \cdot 0}{1+3\%} = \frac{\$100}{1+3\%} = \$97.09 \\ \mathsf{PV}_0 &= \frac{\mathcal{E}(\tilde{P}_1) - \lambda \cdot Cov(\tilde{P}_1, \tilde{r}_{\mathcal{M}})}{1+r_{\mathcal{F}}} . \end{aligned} \tag{13.32}$$

Our problem solved with positive covariance.

But what if we believe that our windfall does covary with the market? How can we guesstimate the cash flow's covariance with the rate of return on the stock market? The answer is that we need to write down some scenarios, and then use our covariance computation formula (from Section 12·3.C). This is easiest to understand in an example. Let us assume that we believe that if the market goes up by 28%, our gift will be $200; if the market goes down by 12%, our gift will be $0. We also believe these two outcomes to be equally likely.

Prob:	1/2	1/2			
	Bad	Good	Mean	*Var*	*Sdv*
Stock Market	−12%	+28%	8%	4%	20%
Our Windfall	$0	$200	$100	$^2 10,000	$100

Now use the covariance formula to compute the average product of deviations from the mean. This is

$$\begin{aligned} Cov(\tilde{P}_1, \tilde{r}_{\mathcal{M}}) &= \frac{(\$200 - \$100) \cdot (28\% - 8\%) + (\$0 - \$100) \cdot (-12\% - 8\%)}{2} = \$20 \\ &= \frac{\sum_{j=1}^{N} [P_{1,\text{outcome j}} - \mathcal{E}(\tilde{P}_1)] \cdot [\tilde{r}_{\mathcal{M},\text{outcome j}} - \mathcal{E}(\tilde{r}_{\mathcal{M}})]}{N} \end{aligned} \tag{13.33}$$

Lambda is still 1.25, and we can now use the certainty equivalence formula to value our expected windfall of $100 next year. It is worth

$$\mathsf{PV}_0 = \frac{\$100 - 1.25 \cdot \$20}{1+3\%} = \frac{\$75}{1+3\%} = \$72.82$$
$$\mathsf{PV}_0 = \frac{\mathcal{E}(\tilde{P}_1) - \lambda \cdot Cov(\tilde{P}_1, \tilde{r}_{\mathcal{M}})}{1+r_{\mathcal{F}}} \quad . \tag{13.34}$$

An alternative method to write the CEV formula.

Finally, note that a different way to write the certainty equivalence formula is

$$\mathsf{PV}_0 = \frac{\mathcal{E}(\tilde{P}_1)}{1+r_{\mathcal{F}}} - \left[\frac{\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}}{1+r_{\mathcal{F}}}\right] \cdot b_{\tilde{P}_1, \tilde{r}_{\mathcal{M}}} \quad , \tag{13.35}$$

where $b_{\tilde{P}_1, \tilde{r}_{\mathcal{M}}}$ is the beta of a regression in which the value (not the rate of return) is the dependent variable.

DIGGING DEEPER: *Knowing the fair price of $72.82, we can now easily check that we have really just recomputed the* CAPM *formula. The project will either provide a rate of return of* $\$200/\$72.82 - 1 = 174\%$, *or a rate of return of* -100%, *for an average rate of return of 37%. The beta computed with rates of return is*

$$\beta_{i,\mathcal{M}} = \frac{Cov(\tilde{r}_i, \tilde{r}_{\mathcal{M}})}{\mathcal{V}ar(\tilde{r}_{\mathcal{M}})} = \frac{\frac{(+174\%-37\%)\cdot(+28\%-8\%)+(-100\%-37\%)\cdot(-12\%-8\%)}{2}}{\frac{(+28\%-8\%)^2+(-12\%-8\%)^2}{2}} = \frac{0.274}{0.04} = 6.85 \quad . \tag{13.36}$$

The ordinary CAPM *formula states that the expected rate of return, given this beta of 6.85, should be be*

$$\mathcal{E}(\tilde{r}_i) = r_{\mathcal{F}} + [\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}] \cdot \beta_{i,\mathcal{M}} = 0.03 + (0.08 - 0.03) \cdot 6.85 \approx 0.37 \quad , \tag{13.37}$$

which is indeed what we had computed above. Here is a proof of the certainty equivalence form. Start with the CAPM *formula:*

$$\mathcal{E}(\tilde{r}_i) = r_{\mathcal{F}} + [\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}] \cdot \beta_{i,m} \quad . \tag{13.38}$$

Rewrite beta

$$\mathcal{E}(\tilde{r}_i) = r_{\mathcal{F}} + [\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}] \cdot \frac{Cov(\tilde{r}_i, \tilde{r}_{\mathcal{M}})}{\mathcal{V}ar(\tilde{r}_{\mathcal{M}})} \quad . \tag{13.39}$$

Rewrite the rate of return, $\tilde{r}_i = \tilde{P}_1/\mathsf{PV}_0 - 1$,

$$\mathcal{E}(\tilde{P}_1/\mathsf{PV}_0 - 1) = r_{\mathcal{F}} + [\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}] \cdot \frac{Cov(\tilde{P}_1/\mathsf{PV}_0 - 1, \tilde{r}_{\mathcal{M}})}{\mathcal{V}ar(\tilde{r}_{\mathcal{M}})} \quad . \tag{13.40}$$

We want to simplify $Cov(\tilde{P}_1/\mathsf{PV}_0 - 1, \tilde{r}_{\mathcal{M}})$. *Covariances are easy to manipulate: if* a *and* b *are known constants, then* $Cov(a \cdot \tilde{x} + b, \tilde{y}) = a \cdot Cov(\tilde{x}, \tilde{y})$. *(The constant* b *does not comove with* y, *so it disappears.)* $1/\mathsf{PV}_0$ *plays the role of the constant* a, -1 *plays the role of the constant* b. *So we can write* $Cov(\tilde{P}_1/\mathsf{PV}_0 - 1, \tilde{r}_{\mathcal{M}}) = Cov(\tilde{P}_1, \tilde{r}_{\mathcal{M}})/\mathsf{PV}_0$. *We already know how to manipulate expectations, so* $\mathcal{E}(\tilde{P}_1/\mathsf{PV}_0 - 1) = \mathcal{E}(\tilde{P}_1)/\mathsf{PV}_0 - 1$. *Substitute back in these unrolled expectation and covariances, and you get*

$$\mathcal{E}(\tilde{P}_1)/\mathsf{PV}_0 - 1 = r_{\mathcal{F}} + [\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}] \cdot \frac{Cov(\tilde{P}_1, \tilde{r}_{\mathcal{M}})}{\mathsf{PV}_0 \cdot \mathcal{V}ar(\tilde{r}_{\mathcal{M}})} \quad . \tag{13.41}$$

If you solve this equation for PV_0, *you will arrive at Formula 13.29.*

Solve Now!

Q 13.22 *Although you are a millionaire, keeping all your money in the market, you have managed to secure a great deal: if you give your even richer Uncle Vinny $10,000 today, he will help you buy a Ferrari, expected to be worth $200,000, if his business can afford it. He is an undertaker by profession, so his business will have the money if the stock market drops, but not if it increases. For simplicity, assume that the stock market drops 1 in 4 years and by -10% when it does and increases by 18% per annum if it does not drop. (Write it out as four separate possible state outcomes to make your life simpler.) The risk-free rate is 6%. What is your Uncle's promise worth at market value?*

b. An Application of the Certainty Equivalence Method

The opportunity cost of capital of a privately held corporation.

You are asked to advise a firm on its appropriate cost of capital. The owners of this firm are very wealthy and widely diversified, so that their remaining portfolio is similar to the market portfolio. (Otherwise, our investor's opportunity cost of capital may not be well represented by the CAPM—and therefore, the calculations here are not relevant for the typical cash-strapped entrepreneur.) To make this a more realistic and difficult task, this firm is either privately held or only a division, so you cannot find historical public market values, and there are no obvious publicly traded comparable firms. Instead, the firm hands you its historical annual cash flows,

Year	1999	2000	2001	2002	2003	2004	Average
S&P500	+21.4%	−5.7%	−12.8%	−21.9%	+26, 4%	+9.0%	+2.7%
Cash Flows	\$8,794	\$5,373	\$8,397	\$6,314	\$9,430	\$9,838	\$8,024

In a perfect world, this is an easy problem: you could compute the value of this firm every year, then compute the beta of the firm's rate of return with respect to the market rate of return, and plug this into the CAPM formula. Alas, assessing annual firm value changes from annual cash flows is beyond my capability. You can also not presume that percent changes in the firm's cash flows are percent changes in the firm's value—just consider what would happen to your estimates if the firm had earned zero in one year. So, what cost of capital are you recommending? Having only a time series of historical cash flows (and no rates of return) is not an obscure problem, and you might first want to reflect on how difficult it is to solve this problem without the certainty equivalence formula.

Let us attempt to value this.

First, we have to make our usual assumption that our historical cash flows and market rates of returns are representative of the future. To solve our problem, we begin by computing the beta of the firm's cash flows with respect to the S&P500. This is easier if we work with differences from the mean,

Year	1999	2000	2001	2002	2003	2004	Average
S&P500	+0.187	−0.084	−0.155	−0.246	+0.237	+0.063	0
Cash Flows	+\$770	−\$2, 651	+\$373	−\$1, 710	+\$1, 406	+\$1, 814	\$0

To compute the covariance of the S&P500 returns with our cash flows, we multiply these and take the average (well, we divide by $N-1$, because this is a sample, not the population, but it won't matter in the end),

$$\begin{aligned} Cov_{\text{CF},\tilde{r}_{\mathcal{M}}} &= \frac{(+0.187)\cdot(+\$770) + (-0.084)\cdot(-\$2,651) + \cdots + (+0.063)\cdot(+\$1,814)}{5} \\ &\approx \$235.4 \end{aligned} \quad , \tag{13.42}$$

and compute the variance

$$\mathcal{V}ar(\tilde{r}_{\mathcal{M}}) = \frac{(+0.187)^2 + (-0.084)^2 + \cdots + (0.063)^2}{5} \approx 0.0373 \ . \tag{13.43}$$

The cash flow beta is the ratio of these,

$$\beta_{CF,\mathcal{M}} = \frac{Cov_{\text{CF},\tilde{r}_{\mathcal{M}}}}{\mathcal{V}ar(\tilde{r}_{\mathcal{M}})} = \frac{\$235.4}{0.03734} \approx \$6,304 \ . \tag{13.44}$$

It is easiest now to proceed by considering the historical mean cash flow of \$8,024. We need an assumption of a suitable equity premium and a suitable risk-free rate. Let us adopt 3% and 4%, respectively. In this case, the value of our firm would be

$$\begin{aligned} \text{PV}_0 &= \frac{\$8,024}{1+3\%} - \left[\frac{4\%}{1+3\%}\right]\cdot\$6,304 \\ &\approx \$7,791 - \$245 = \$7,546 \\ &= \frac{\mathcal{E}(\tilde{\text{CF}})}{1+r_{\mathcal{F}}} - \left[\frac{\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}}{1+r_{\mathcal{F}}}\right]\cdot b_{\tilde{\text{CF}},\tilde{r}_{\mathcal{M}}} \end{aligned} \tag{13.45}$$

The certainty equivalence formula tells us that because our firm's cash flows are correlated with the market, we shall impute a risk discount of \$245. We can translate this into a cost of capital estimate—at what discount rate would we arrive at a value of \$7,546?

$$\underset{\text{PV}}{\$7,546} = \underset{\frac{\mathcal{E}(\text{CF})}{1+\mathcal{E}(\tilde{r})}}{\frac{\$8,024}{1+\mathcal{E}(\tilde{r})}} \Rightarrow \mathcal{E}(\tilde{r}) = 6.3\% \ . \qquad (13.46)$$

We now have an estimate of the cost of capital for our cash flow for next year. We can also translate this into an equivalent returns-based market-beta, which is

$$\underset{r_{\mathcal{F}}}{3\%} + \underset{[\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}})] \cdot \beta_{i,\mathcal{M}}}{4\% \cdot \beta_{i,\mathcal{M}}} = 6.3\% \Rightarrow \beta \approx 0.8 \ . \qquad (13.47)$$

Now I can reveal who the firm in this example really was—it was IBM. Because it is publicly traded, we can see how our own estimate of IBM's cost of capital and market beta would have come out if we had computed it from IBM's annual market values. Its rates of return were

Are we close?

Year	1999	2000	2001	2002	2003	2004	Average
IBM's Rate of Return	+17.5%	−20.8%	+43.0%	−35.5%	+20.5%	+7.2%	+5.3%

If you compute the market-beta of these annual returns, you will find an estimate of 0.7—very close to the estimate we obtained from our cash flow series. (For IBM, this is a very low market-beta estimate. If we used monthly cash flows or monthly stock returns, we would obtain a considerably higher market-beta estimate.)

Solve Now!

Q 13.23 *A firm reported the following cash flows:*

Year	*1999*	*2000*	*2001*	*2002*	*2003*	*2004*	*Average*
S&P500	+21.4%	−5.7%	−12.8%	−21.9%	+26,4%	+9.0%	+2.7%
Cash Flows	+\$2,864	+\$1,666	−\$1,040	+\$52	+\$1,478	−\$962	+\$997

(Note that the cash flows are close to nothing in 2002 and even negative in 2004, the latter preventing you from computing percent changes in cash flows.) What cost of capital would you recommend for this firm?

Solutions and Exercises

1. $\mathcal{E}(\tilde{r}) = 4\% + (7\% - 4\%) \cdot 3 = 13\%$.
2. $\mathcal{E}(\tilde{r}) = 4\% + (12\% - 4\%) \cdot 3 = 28\%$.
3. Solve $\mathcal{E}(\tilde{r}) = 4\% + (7\% - 4\%) \cdot \beta_{i,\mathcal{M}} = 5\%$. Therefore, $\beta_{i,\mathcal{M}} = 1/3$.
4. Do it!
5. $\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}$ is the premium that the stock market expects to offer, above and beyond the rate that risk-free investments offer.

6. It does not matter what you choose as the per-unit payoff of the bond. \$100 is expected to return \$99. So, the price of the bond is

$$\text{PV} = \frac{\$99}{1 + (3\% + 5\% \cdot 0.2)} \approx \$95.19 \tag{13.48}$$

Therefore, the promised rate of return on the bond is \$100/\$95.19 − 1 ≈ 5.05%.

7. The risk-free rate is 3%, so this is the time premium. The expected risk premium is 1%. The remaining 1.05% is the default premium.
8. The cost needs to be discounted with the current interest rate. Since payment is upfront, this cost is \$30,000 now! The appropriate expected rate of return for cash flows (of your earnings) is $3\% + 5\% \cdot 1.5 = 10.5\%$. You can now use the annuity formula, to determine the PV if you graduate.

$$\frac{\$5,000}{10.5\%} \cdot \left[1 - \left(\frac{1}{1 + 10.5\%}\right)^{40}\right] = \$47,619 \cdot 98.2\% \approx \$46,741.46 \ . \tag{13.49}$$

With 90% probability, you will do so, which means that the appropriate risk-adjusted and discounted cash flow is about \$42,067.32. The NPV of your education is therefore about \$12,067.

9. An estimate between 2% and 8% per year is reasonable. Anything below 0% and above 10% would be unreasonable. For reasoning, please see the different methods in the chapter.
10. Use the Treasury rate for the 1-year project, e.g., from the *Wall Street Journal*. Because the 10-year project could have a duration of flows anywhere from 5 to 10 years, depending on use, you might choose a risk-free Treasury rate that is between 5 and 10 years.
11. A 5-year interest rate is a reasonably good guess. You should not be using a 30-day Treasury bill, or a 30-year Treasury bond.
12. Beta can be found in Yahoo!*Finance*'s "Profile." In June 2003, IBM's beta was 1.48. Most upstart growth companies have higher betas.
13. Rearrange the CAPM formula to $\beta_{i,\mathcal{M}} = \frac{\mathcal{E}(\tilde{r}_i) - r_{\mathcal{F}}}{\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}}$. So, the first beta estimate would be $\frac{20\% - 4\%}{10\% - 4\%} \approx 2.7$. The second beta estimate would be $\beta_{i,\mathcal{M}} = \frac{-20\% - 4\%}{-5\% - 4\%} = 2.7$. Therefore, a reasonable estimate of beta would be the average, here 2.7.
14. You can compute an unlevered beta.

$$\begin{aligned} \beta_{P,\mathcal{M}} &= \left(\frac{2}{3}\right) \cdot 0 + \left(\frac{1}{3}\right) \cdot 2.5 \\ \beta_{P,\mathcal{M}} &= w_{\text{D}} \cdot \beta_{\text{D},\mathcal{M}} + w_{\text{E}} \cdot \beta_{\text{E},\mathcal{M}} \ . \end{aligned} \tag{13.50}$$

Therefore, $\beta_{P,\mathcal{M}} = 0.83$. We assume our project has the same beta, but a smaller debt ratio:

$$\begin{aligned} 0.833 &= \left(\frac{1}{3}\right) \cdot 0 + \left(\frac{2}{3}\right) \cdot \beta_{\text{E},\mathcal{M}} \Rightarrow \beta_{\text{E},\mathcal{M}} = 1.25. \\ \beta_{P,\mathcal{M}} &= w_{\text{D}} \cdot \beta_{\text{D},\mathcal{M}} + w_{\text{E}} \cdot \beta_{\text{E},\mathcal{M}} \ . \end{aligned} \tag{13.51}$$

15. The comparable project's equity expected rate of return would be $3\% + 2\% \cdot 2.5 = 8\%$. Our own equity's expected rate of return would be $3\% + 2\% \cdot 1.25 = 5.5\%$

16. A debt/equity ratio of 2 is the same as the debt asset ratio of 2/3: two parts debt, one part equity. A debt/equity ratio of 1/2 is the same as the debt asset ratio of 1/3: one part debt, two parts equity. To convert a debt-equity ratio into a debt-asset ratio, recognize that

$$\begin{aligned} \frac{1}{D/A} &= \frac{1}{D/(D+E)} = \left(\frac{D+E}{D}\right) = 1 + \left(\frac{E}{D}\right) \\ \Longrightarrow \frac{D}{E} &= \frac{1}{\left(\frac{1}{D/A} - 1\right)} = \frac{1}{\left(\frac{1}{2/3} - 1\right)} \\ &= \frac{1}{\left(\frac{3}{2} - 1\right)} = \frac{1}{\left(\frac{1}{2}\right)} = 2 \quad . \end{aligned} \tag{13.52}$$

17. This is the same as above, too.
18. $\beta_{\text{combined},\mathcal{M}} = (3/4) \cdot 2.4 + (1/4) \cdot 0.4 = 1.9$.
19. For short investments, the expected cash flows are most critical to estimate well (see Section 5·4). For long-term projects, cost of capital becomes more important to get right. Betas and risk-free rates are usually relatively trouble free, having only modest degrees of uncertainty. The equity premium will be the most important problem factor.

20. The CAPM cost of capital is 10%. Its current projects are expected to provide \$30, the new project would provide \$6, \$2 above the risk-free rate. Therefore, the value of the firm would go up by \$2 next year, which has to be discounted to today. The new project "loan" would be about 1/4 of the new firm. Therefore, the new beta of the firm will be

$$\beta_{\text{W},\mathcal{M}} = {}^{3}/_{4} \cdot 2 + {}^{1}/_{4} \cdot 0 = 1.5 \quad . \tag{13.53}$$

The firm's cost of capital would therefore decline from 10% to 8.5%. At this cost of capital, the extra \$2 would add about \$1.84 to the firm value. The firm's cash flows would change from \$430 to \$330 + \$106 = \$436. Discounted at the 8.5% interest rate, this comes to about \$401.84. Subtracting off the \$100 cost of the loan confirms the NPV. The firm should change its policy.

21.

(a) The value is \$9.57.

(b)

$$w_N = \frac{\text{PV}_N}{\text{PV}_{\text{combined}}} = \frac{\frac{\$11}{1+15\%}}{\$109.48} \approx 8.73\% \quad . \tag{13.54}$$

(c)

$$\beta_{\text{combined}} = w_O \cdot \beta_O + w_N \cdot \beta_N \approx 91.26\% \cdot 0.5 + 8.73\% \cdot 3 = 0.718 \quad . \tag{13.55}$$

(d)

$$\mathcal{E}(\tilde{r}_{\text{combined}}) = 3\% + 4\% \cdot 0.718 = 5.872\% \quad , \tag{13.56}$$

(e) Yes! The IRR of N is 10%. 10% is above the blended cost of capital of 5.872%.

(f) Firm value would be

$$\begin{aligned} \text{PV} &= \frac{\$105 + \$11}{1 + 5.872\%} = \$109.56 \\ &= \frac{\mathcal{E}(\text{CF}_N) + \mathcal{E}(\text{CF}_O)}{1 + \mathcal{E}(\tilde{r}_{\text{combined}})} \quad . \end{aligned} \tag{13.57}$$

So, again, the firm has destroyed \$0.44.

22. This is a certainty equivalence question. Although it is not a gift per se, you cannot assume that \$10,000 is a fair market value, so that you can compute a rate of return of 1,900%—after all, it is your Uncle trying to do something nice for you. There are four outcomes:

Prob:	1/4	1/4	1/4	1/4	
	Crash	No-Crash	No-Crash	No-Crash	Mean
Stock Market	−10%	+18%	+18%	+18%	11%
Ferrari	\$200	\$0	\$0	\$50	

Plug this into the formula

$$Cov(\tilde{P}_1, \tilde{r}_{\mathcal{M}}) = {}^1/_4 \cdot [\$150,000 \cdot (-21\%) + (-\$50,000) \cdot (+7\%) + (-\$50,000) \cdot (+7\%) + (-\$50,000) \cdot (+7\%)] = -\$21,000 \quad . \tag{13.58}$$

We also need to determine the variance of the market. It is

$$Cov(\tilde{r}_{\mathcal{M}}, \tilde{r}_{\mathcal{M}}) = \frac{(-21\%)^2 + (+7\%)^2 + (+7\%)^2 + (+7\%)^4}{4} = 147\%\% \tag{13.59}$$

(which incidentally comes to a standard deviation of 12% per annum, a bit low.) With the risk-free rate of 6%, lambda (λ) is $(11\% - 6\%)/147\%\% \approx 3.4$.

So, we can now use the certainty equivalence formula: the expected value of the Ferrari is \$50,000. If it were a safe payoff, it would be worth \$47,170. Because you get more if the rest of your portfolio goes down, it is actually great insurance for you. So, you value it $3.4 \cdot (-\$21,000)/(1 + 6\%) \approx \$67,358$ higher than \$47,170: the Ferrari is worth \$114,528. You have to pay \$10,000 today, of course, so you have managed to secure a deal for \$104,528.

23. The beta of these cash flows is \$5,104. Therefore, the risk discount on \$997 is about $4\%/1.03 \cdot \$997 \approx \$38.72$, which corresponds to a cost of capital of about 4% (a beta of about 0.25). This firm is Sony. It had returns of −42% (in 2001, computed with the 2000 price), −39%, −10%, +2%, and −8%. (Even these returns depend sensitively on how dividends are reinvested.) The beta computed from market values comes out to just below 0.6. In the real world, the difference of 0.35 would result in about 1% difference in the cost of capital—a reasonable amount of error, especially given that we had to estimate a cost of capital without knowing Sony's historical market values! Yahoo!*Finance* lists a Sony beta of around 1, but also computes this from monthly stock returns.

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

CHAPTER 14

The Optimal Portfolio

An Optional and Advanced Chapter

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This chapter appears in the CorpFin text only.

The material in this chapter is not necessary to your understanding of corporate finance. You may also find it difficult, but if you are more curious about the science of investments, it will explain to you what should be going on in the minds of the investors from whom you want to raise money.

This chapter does two things. First, it explains where the CAPM is coming from:

- It shows that the CAPM formula is a direct result of the fact that investors choose the best overall portfolio. It will show how this best portfolio is constructed. This best overall portfolio lies on the "mean-variance efficient (MVE) frontier."
- If a security offers too little or too much expected rate of return for its beta, all investors would tilt their market portfolio away or towards this stock. This would mean that they would want more or less of this stock than how much is available. If and only if the market portfolio is mean-variance efficient do all stocks follow the CAPM formula.

Second, the chapter gives some additional perspective on the CAPM—what are its assumptions, and should you believe it?

14·1. An Investor's Risk vs Reward Tradeoff

The question at hand: the optimal portfolio.

You know that diversification reduces risk. Therefore, you know that investors like diversification—but this does not tell you how much you should purchase in each security. It may be better to purchase 25% in $\mathcal{A}$ and 75% in $\mathcal{B}$, rather than 50% in each. So how do you determine generally good investment weights—and the best investment weights for *you*? This question—what is the optimal portfolio—is the primary subject of this chapter. You shall see that the optimal portfolio is the force that ultimately shapes the CAPM formula.

Table 14.1. Portfolios

	Base Portfolios		Portfolios				
Future	100% in Pfio $\mathcal{H}$	100% in Pfio $\mathcal{I}$	25% in $\mathcal{H}$ 75% in $\mathcal{I}$	33% in $\mathcal{H}$ 67% in $\mathcal{I}$	50% in $\mathcal{H}$ 50% in $\mathcal{I}$	67% in $\mathcal{H}$ 33% in $\mathcal{I}$	75% in $\mathcal{H}$ 25% in $\mathcal{I}$
Name:	$\mathcal{H}$	$\mathcal{I}$	$\mathcal{J}$	$\mathcal{K}$	$\mathcal{L}$	$\mathcal{M}$	$\mathcal{N}$
Scenario S1 ♣	-6.0%	-12.0%	-10.50%	-10.00%	-9.00%	-8.00%	-7.50%
Scenario S2 ♦	+12.0%	+18.0%	+16.50%	+16.00%	+15.00%	+14.00%	+13.50%
Scenario S3 ♥	0.0%	+24.0%	+18.00%	+16.00%	+12.00%	+8.00%	+6.00%
Scenario S4 ♠	+18.0%	+6.0%	+9.00%	+10.00%	+12.00%	+14.00%	+15.00%
"Reward" ($\mathcal{E}(R)$)	6.00%	9.00%	8.25%	8.00%	7.50%	7.00%	6.75%
Variance ($\mathcal{V}ar(R)$)	90.0%%	189.0%%	128.8%%	114.0%%	92.2%%	81.0%%	79.3%%
"Risk" ($\mathcal{S}dv(R)$)	9.49%	13.70%	11.35%	10.68%	9.60%	9.00%	8.91%

These are the two base portfolios (and their combinations) that you shall use in this chapter to illustrate the mean-variance frontier.

Explaining Table 14.1

Now make up two new portfolios, $\mathcal{H}$ and $\mathcal{I}$. Your task will be to find the best portfolio among them. (The appendix works out the solutions for an arbitrary number of securities, rather than just two.) Table 14.1 shows a set of portfolios constructed from these two base portfolios. Confirm the numbers in the table for portfolio $\mathcal{K}$, which invests $w_{\mathcal{H}} = 0.33$ in $\mathcal{H}$ and $w_{\mathcal{I}} = 0.67$ in $\mathcal{I}$,

$$\begin{array}{lccccccc} \clubsuit & \tilde{r}_{\mathcal{K}} & = & 1/3 \cdot (-6\%) & + & 2/3 \cdot (-12\%) & = & -10\% \\ \diamondsuit & \tilde{r}_{\mathcal{K}} & = & 1/3 \cdot (+12\%) & + & 2/3 \cdot (+18\%) & = & +16\% \\ \heartsuit & \tilde{r}_{\mathcal{K}} & = & 1/3 \cdot (0\%) & + & 2/3 \cdot (+24\%) & = & +16\% \\ \spadesuit & \tilde{r}_{\mathcal{K}} & = & 1/3 \cdot (+18\%) & + & 2/3 \cdot (+6\%) & = & +10\% \\ & \tilde{r}_{\mathcal{K}} & = & w_{\mathcal{H}} \cdot (\tilde{r}_{\mathcal{H}}) & + & w_{\mathcal{I}} \cdot (\tilde{r}_{\mathcal{I}}) & & . \end{array} \tag{14.1}$$

You can compute the expected rate of return of this portfolio as

$$\begin{array}{lcl} \mathcal{E}(\tilde{r}_{\mathcal{K}}) & = & 1/4 \cdot (-10\%) + 1/4 \cdot (+16\%) + 1/4 \cdot (+16\%) + 1/4 \cdot (+10\%) \;=\; 8\% \;. \\ \mathcal{E}(\tilde{r}) & = & \sum_{i=1}^{N} \mathcal{P}rob(\text{scenario i}) \cdot \text{Outcome in scenario i} \quad . \end{array} \tag{14.2}$$

To compute the variance of portfolio $\mathcal{K}$, you follow the procedure laid out in Section 12·1.C. First demean the rates of returns,

$$\begin{array}{lllll} \clubsuit & \tilde{r}_{\mathcal{K}} = & -10\% - 8\% & = & -18\% \\ \diamondsuit & \tilde{r}_{\mathcal{K}} = & +16\% - 8\% & = & +8\% \\ \heartsuit & \tilde{r}_{\mathcal{K}} = & +16\% - 8\% & = & +8\% \\ \spadesuit & \tilde{r}_{\mathcal{K}} = & +10\% - 8\% & = & +2\% \\ & \tilde{r}_{\mathcal{K}} = & \tilde{r}_{\mathcal{K}} - \text{mean}(\tilde{r}_{\mathcal{K}}) & , & \end{array} \tag{14.3}$$

and then compute the average of their squares,

$$\begin{aligned} \mathcal{V}ar(\tilde{r}_{\mathcal{K}}) &= \frac{(-18\%)^2 + (+8\%)^2 + (+8\%)^2 + (+2\%)^2}{4} \\ &= \quad 114\%\% \;=\; 0.0114 \quad . \end{aligned} \tag{14.4}$$

The risk is therefore $\mathcal{S}dv(\tilde{r}_{\mathcal{J}}) = \sqrt{\mathcal{V}ar(\tilde{r}_{\mathcal{K}})} = \sqrt{0.00114} = 10.68\%$. This confirms the number in Table 14.1.

14·1.A. A Short-Cut Formula For the Risk of a Portfolio

The variance formula for a portfolio.

There is a shortcut formula that can make portfolio variance computations faster. This shortcut allows you to compute the variance of a portfolio directly from the variances (and covariances) of its constituent securities, instead of having to first work out the rate of return of the combination portfolio in each and every scenario.

You need to compute the covariance between $\mathcal{H}$ and $\mathcal{I}$.

You need only one extra number to apply the variance formula. You have to compute the covariance between $\mathcal{H}$ and $\mathcal{I}$. You have already worked with the covariance in Section 12·3.C. It is defined as the average product of the two demeaned returns. So, subtract the mean (6% for $\mathcal{H}$ and 9% for $\mathcal{I}$) from each scenario's realization,

$$\begin{array}{lllllll} \clubsuit & \tilde{r}_{\mathcal{H}} - \text{mean}(\tilde{r}_{\mathcal{H}}) & = -12\% & & \tilde{r}_{\mathcal{I}} - \text{mean}(\tilde{r}_{\mathcal{I}}) & = -21\% & \\ \diamondsuit & \tilde{r}_{\mathcal{H}} - \text{mean}(\tilde{r}_{\mathcal{H}}) & = +6\% & & \tilde{r}_{\mathcal{I}} - \text{mean}(\tilde{r}_{\mathcal{I}}) & = +9\% & \\ \heartsuit & \tilde{r}_{\mathcal{H}} - \text{mean}(\tilde{r}_{\mathcal{H}}) & = -6\% & & \tilde{r}_{\mathcal{I}} - \text{mean}(\tilde{r}_{\mathcal{I}}) & = +15\% & \\ \spadesuit & \tilde{r}_{\mathcal{H}} - \text{mean}(\tilde{r}_{\mathcal{H}}) & = +12\% & & \tilde{r}_{\mathcal{I}} - \text{mean}(\tilde{r}_{\mathcal{I}}) & = -3\% & . \end{array} \tag{14.5}$$

Therefore,

$$\begin{aligned} \mathcal{C}ov(\tilde{r}_{\mathcal{H}}, \tilde{r}_{\mathcal{I}}) &= \frac{(-12\%)\cdot(-21\%) + (+6\%)\cdot(+9\%) + (-6\%)\cdot(+15\%) + (+12\%)\cdot(-3\%)}{4} \\ &= +0.0045 \quad . \\ \mathcal{C}ov(\tilde{r}_i, \tilde{r}_j) &= \frac{\sum_{\text{all obs}}[\tilde{r}_i - \mathcal{E}(\tilde{r}_i)]\cdot[\tilde{r}_j - \mathcal{E}(\tilde{r}_j)]}{N} \quad . \end{aligned} \tag{14.6}$$

Therefore, $\mathcal{H}$ and $\mathcal{I}$ are positively correlated—these investments will move together. For example, if the rate of return on portfolio $\mathcal{H}$ exceeds its 6% mean, portfolio $\mathcal{I}$ will also tend to exceed its 9% mean.

Here is the shortcut formula.

Here is the shortcut formula.

IMPORTANT: *The variance of a portfolio $\mathcal{P}$ that consists of $\mathcal{A}$ and $\mathcal{B}$ is*

$$\mathcal{V}ar(\tilde{r}_{\mathcal{P}}) = w_{\mathcal{A}}^2 \cdot \mathcal{V}ar(\tilde{r}_{\mathcal{A}}) + w_{\mathcal{B}}^2 \cdot \mathcal{V}ar(\tilde{r}_{\mathcal{B}}) + 2 \cdot w_{\mathcal{A}} \cdot w_{\mathcal{B}} \cdot \mathcal{C}ov(\tilde{r}_{\mathcal{A}}, \tilde{r}_{\mathcal{B}}) \quad , \tag{14.7}$$

where $w_{\mathcal{A}}$ is the portfolio weight in component $\mathcal{A}$, $w_{\mathcal{B}}$ is the portfolio weight in component $\mathcal{B}$, and therefore equal to $1 - w_{\mathcal{A}}$.

Check whether this is correct. Try it out on portfolio $\mathcal{K}$, which invests $1/3$ in $\mathcal{H}$ and $2/3$ in $\mathcal{I}$,

$$\begin{aligned}\mathcal{V}ar(\tilde{r}_{\mathcal{K}}) &= (1/3)^2 \cdot \mathcal{V}ar(\tilde{r}_{\mathcal{H}}) + (2/3)^2 \cdot \mathcal{V}ar(\tilde{r}_{\mathcal{I}}) + 2 \cdot (1/3) \cdot (2/3) \cdot \mathcal{C}ov(\tilde{r}_{\mathcal{H}}, \tilde{r}_{\mathcal{I}}) \\ &= (1/3)^2 \cdot 0.0090 + (2/3)^2 \cdot 0.01890 + 2 \cdot (1/3) \cdot (2/3) \cdot (+0.0045) \\ &= 0.0114 \quad .\end{aligned} \tag{14.8}$$

This is the same result as in Formula 14.4, so the shortcut indeed gives the same correct answer.

Why can this formula be so useful?

Of course, this formula is not particularly convenient with only one combination portfolio (one weight $w_{\mathcal{A}}$) and four outcome scenarios. However, it is a lot more convenient if you have to compute the portfolio variance of thousands of different combinations of $\mathcal{H}$ and $\mathcal{I}$ and there are hundreds of scenarios. Indeed, recomputing the overall portfolio variance many times is the problem at the heart of determining the best portfolio: you want to know how different portfolio weights influence your consequent portfolio risk. Your alternative to the shortcut would be to recompute the returns for each of hundreds of possible portfolio weight combinations—which would quickly become very painful.

Solve Now!

Q 14.1 *Recompute the portfolio variance if you invest $w_{\mathcal{H}} = 2/3$ in H.*

Q 14.2 *Recompute the portfolio variance if you invest $w_{\mathcal{H}} = 3/4$ in H.*

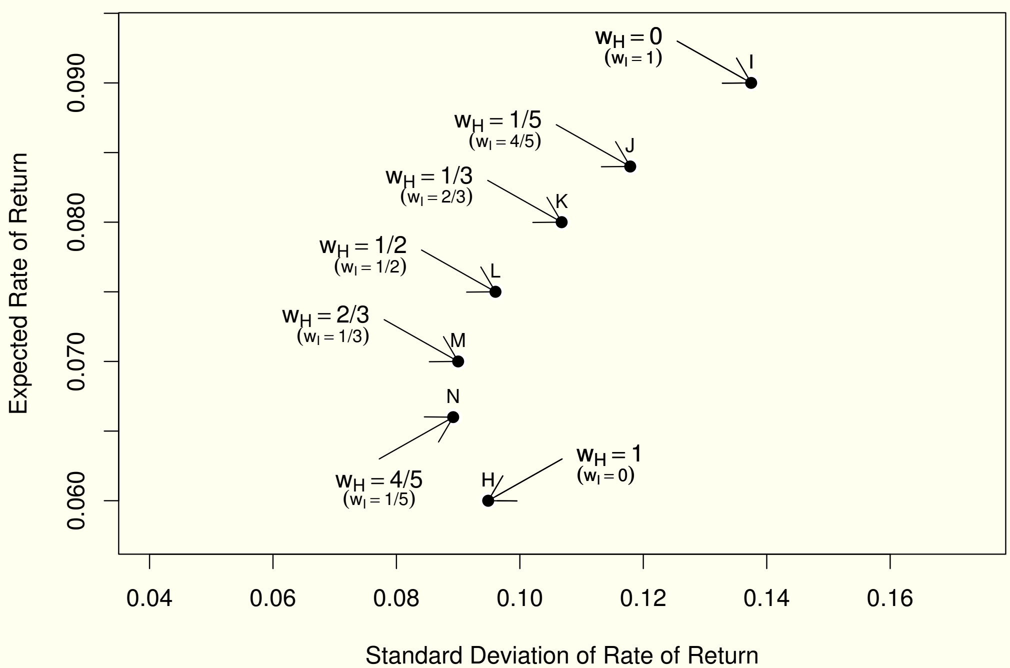
Q 14.3 *Recompute the portfolio variance if you invest $w_{\mathcal{H}} = 1/2$ in H.*

14·1.B. Graphing the Mean-Variance Efficient Frontier

A first plot of risk vs. reward.

For each portfolio from Table 14.1, let us now graph the portfolio risk on the X-axis, and the portfolio reward on the Y-axis. Figure 14.1 does exactly this. Look at it. Do you see a pattern?

Figure 14.1. The Risk-Reward Tradeoff between $\mathcal{H}$ and $\mathcal{I}$

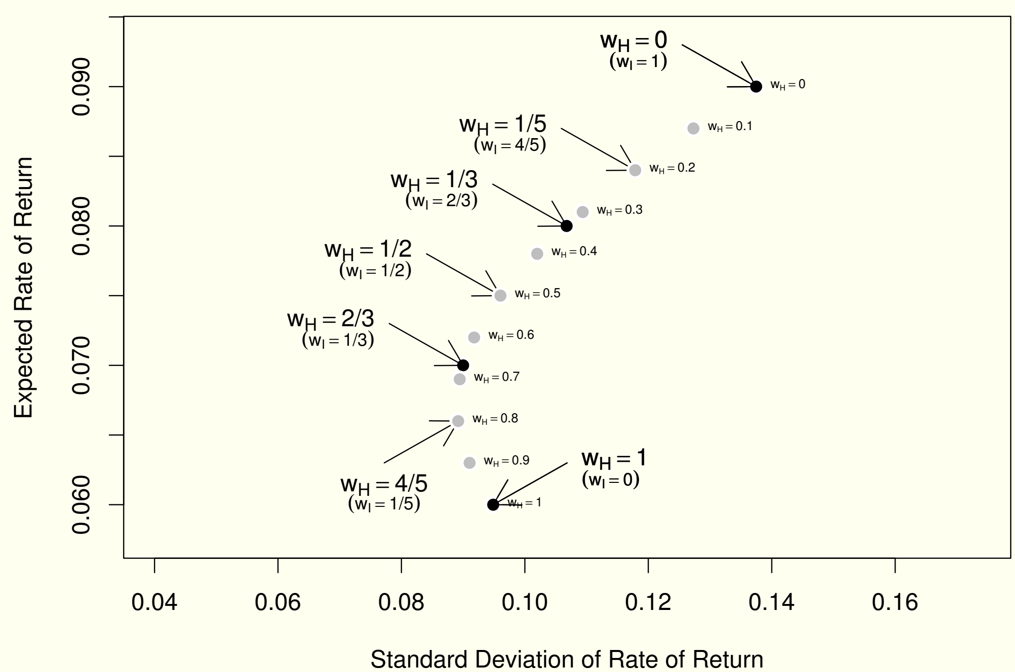


These are the portfolios from Table 14.1.

A few more portfolios make the pattern easier to see.

If you are having trouble seeing a pattern, Figure 14.2 adds a few more portfolios—in fact, these are the portfolios that invest 10%, 20% and so on into $\mathcal{H}$ and the remainder into $\mathcal{I}$. (You can confirm that I have computed the risk and reward of these portfolios correctly in the exercises below.) Do you see a pattern now?

Figure 14.2. The Risk-Reward Tradeoff between $\mathcal{H}$ and I: More Portfolios



This adds some portfolios to Figure 14.1.

The MVE Frontier.

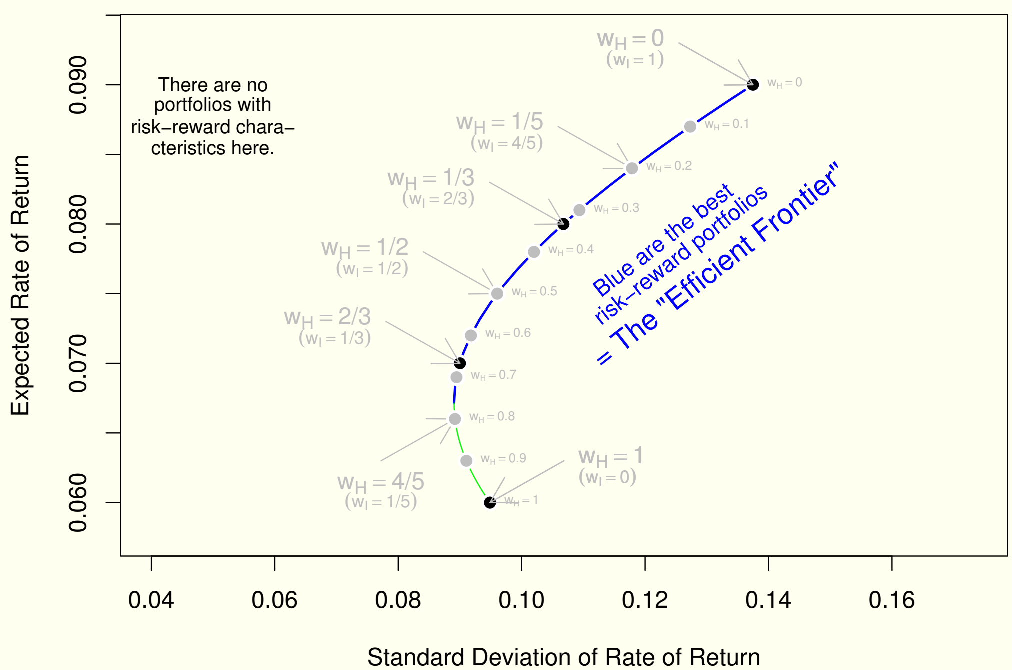
If you picked many more portfolios with weights between 0 and 1, you would eventually end up with Figure 14.3. The curve is called **mean-variance efficient frontier** (MVE Frontier), and it is the locus where the best risk-reward portfolios lie. There are no portfolios that are to the northwest of this frontier—portfolios that would have higher expected rates of return for a given risk, or lower risk for a given expected rate of return. The shape of this frontier is called a hyperbola when graphed in mean vs. standard deviation space.

SIDE NOTE: The very left-most portfolio is called the **minimum variance portfolio**, because you cannot create a portfolio with lower risk. Here, the minimum-variance portfolio has a weight of 76.191% on $\mathcal{H}$ and 23.809% on $\mathcal{I}$, and it achieves as low a risk as 8.9%. Although the graph's scale is too small for you to check this graphically, you can compute the risk of this portfolio, and compare it to the risk of two portfolios that invest either a little more or a little less into $\mathcal{H}$.

$$\begin{aligned}
w_{\mathcal{H}} = 76.0\% &: \quad Sdv(\tilde{r}_P) \approx 8.9042911\% \\
w_{\mathcal{H}} = 76.2\% &: \quad Sdv(\tilde{r}_P) \approx 8.9042526\% \\
w_{\mathcal{H}} = 76.4\% &: \quad Sdv(\tilde{r}_P) \approx 8.9042992\% \\
& \quad Sdv(\tilde{r}_P) = \sqrt{Var(\tilde{r}_P) \equiv Var[w_{\mathcal{H}} \cdot \tilde{r}_{\mathcal{H}} + (1 - w_{\mathcal{H}}) \cdot \tilde{r}_{\mathcal{I}}]} \; .
\end{aligned} \tag{14.9}$$

So, I did not lie to you.

Figure 14.3. The Risk-Reward Tradeoff between $\mathcal{H}$ and $\mathcal{I}$: The Frontier

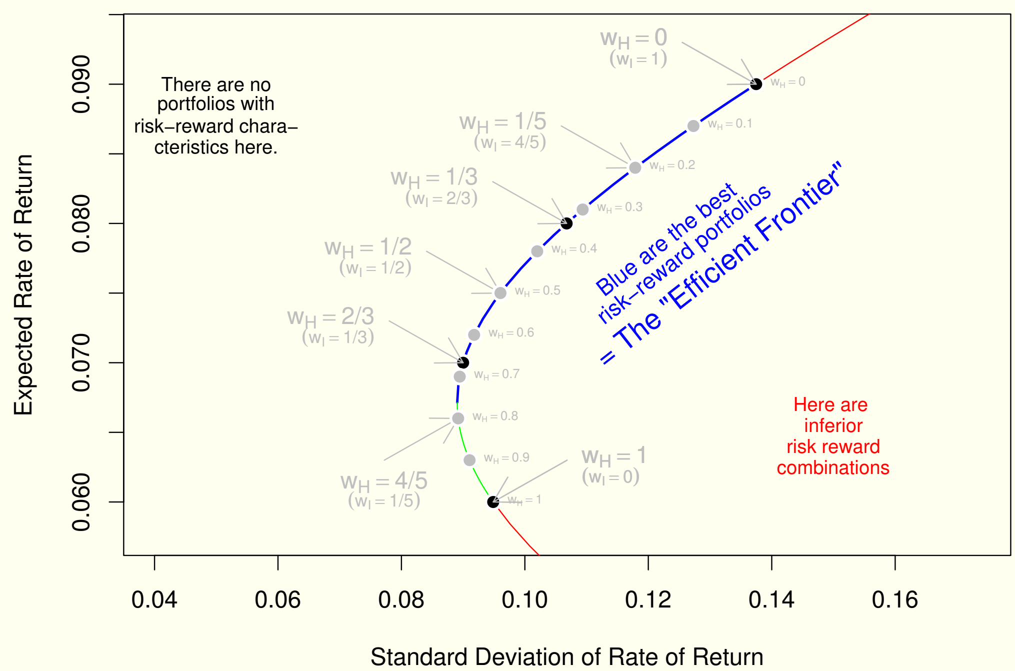


This adds the efficient frontier to Figure 14.2.

A complete figure.

Without working out the details, look at Figure 14.4. It continues the efficient frontier at the top and bottom (in red). These are portfolios that short-sell either $\mathcal{H}$ or $\mathcal{I}$ and purchase more than 100% of the other. (Portfolio weights add up to 1, so if one security has less than 0%, the remaining securities must add to more than 100%.) Second, if you started with more than two basis portfolios $\mathcal{H}$ and $\mathcal{I}$, many consequent combination portfolios would also be outright inferior. They would be a cloud of points inside and south-east of the efficient frontier. However, the efficient frontier would still look similar to what you have drawn—a hyperbola on the upper north-west frontier.

Figure 14.4. The Risk-Reward Tradeoff between $\mathcal{H}$ and $\mathcal{I}$: Sets



This completes the efficient frontier to Figure 14.3, allowing for portfolios that short one or the other portfolio (in red).

Solve Now!

Q 14.4 *Compute the risk and reward of the portfolio $w_{\mathcal{H}} = 0.1, w_{\mathcal{I}} = 0.9$. Confirm that this portfolio is drawn correctly in the figures.*

Q 14.5 *If $\mathcal{H}$ and $\mathcal{I}$ were more correlated, what would the efficient frontier between them look like? If $\mathcal{H}$ and I were less (or more negatively) correlated, what would the efficient frontier between them look like? Hint: Think about the variance of the combination portfolio that invests half in each.*

Q 14.6 *Redraw the efficient frontier for the following two basic securities.*

	$\mathcal{H}$	$\mathcal{A}$
Scenario S1 ♣	-6.0%	-12.0%
Scenario S2 ♦	+12.0%	+18.0%
Scenario S3 ♥	0.0%	+6.0%
Scenario S4 ♠	+18.0%	+24.0%

Also, compute the covariance between $\mathcal{H}$ and $\mathcal{A}$. Is it higher or lower than what you computed in the text for $\mathcal{H}$ and $\mathcal{I}$? How does the efficient frontier compare to what you have drawn in this chapter?

14·1.C. Adding a Risk-Free Rate

Let us now add a risk-free rate ("$\mathcal{F}$") of 4%. Recall that the risk-free rate plays an important role in the CAPM, and you will find that it also plays an important special role for the mean variance frontier. Start with the following three basis portfolios:

Write down the formulas for risk and reward of a combination portfolio that adds the risk-free rate.

Future	$\mathcal{H}$	$\mathcal{I}$	$\mathcal{F}$
Scenario S1 ♣	−6.0%	−12.0%	4.00%
Scenario S2 ♦	+12.0%	+18.0%	4.00%
Scenario S3 ♥	0.0%	+24.0%	4.00%
Scenario S4 ♠	+18.0%	+6.0%	4.00%
"Reward" ($\mathcal{E}(R)$)	6.00%	9.00%	4.00%
"Variance" ($\mathcal{V}ar(R)$)	90.0%%	189.0%%	0.0%%
"Risk" ($\mathcal{S}dv(R)$)	9.49%	13.70%	0.00%

You want to begin by determining the risk and reward of a portfolio S that invests $1/2$ in $\mathcal{H}$ and $1/2$ in $\mathcal{F}$. The reward of this combination is

$$\begin{aligned} \mathcal{E}(\tilde{r}_S) &= 1/2 \cdot 6\% + (1 - 1/2) \cdot 4\% = 5\% \\ \mathcal{E}(\tilde{r}_S) &= w_{\mathcal{H}} \cdot \mathcal{E}(\tilde{r}_{\mathcal{H}}) + (1 - w_{\mathcal{H}}) \cdot r_{\mathcal{F}} \quad . \end{aligned} \tag{14.10}$$

For the risk component, use Formula 14.7. Trust me that a constant number of 4% has neither a variance nor a covariance with anything else. (Makes sense, doesn't it?) Therefore,

$$\begin{aligned} \mathcal{V}ar(\tilde{r}_S) &= (1/2)^2 \cdot 0.009 + (1 - 1/2)^2 \cdot 0\% + 2 \cdot 1/2 \cdot (1 - 1/2) \cdot 0\% \\ &= 1/4 \cdot 0.009 \\ \mathcal{V}ar(\tilde{r}_S) &= w_{\mathcal{H}}^2 \cdot \mathcal{V}ar(\tilde{r}_{\mathcal{H}}) + w_{\mathcal{F}}^2 \cdot \mathcal{V}ar(r_{\mathcal{F}}) + 2 \cdot w_{\mathcal{H}} \cdot w_{\mathcal{F}} \cdot \mathcal{C}ov(\tilde{r}_{\mathcal{H}}, r_{\mathcal{F}}) \\ &= (w_{\mathcal{H}})^2 \cdot \mathcal{V}ar(\tilde{r}_{\mathcal{H}}) \\ & \text{and } (1 - w_{\mathcal{H}}) = w_{\mathcal{F}} \quad . \end{aligned} \tag{14.11}$$

Therefore,

$$\begin{aligned} \mathcal{S}dv(\tilde{r}_S) &= \sqrt{(1/2)^2 \cdot 0.009} = 1/2 \cdot \sqrt{0.009} = 4.75\% \\ \mathcal{S}dv(\tilde{r}_S) &= \sqrt{(w_{\mathcal{H}})^2 \cdot \mathcal{V}ar(\tilde{r}_{\mathcal{H}})} = w_{\mathcal{H}} \cdot \mathcal{S}dv(\tilde{r}_{\mathcal{H}}) \end{aligned} \tag{14.12}$$

Bear with me, and rearrange this into $w_{\mathcal{H}} = \mathcal{S}dv(\tilde{r}_S)/\mathcal{S}dv(\tilde{r}_{\mathcal{H}}) = \mathcal{S}dv(\tilde{r}_S)/9.49\%$, and substitute

Substitute out the w, and you end up with a linear function that relates the risk ($\mathcal{S}dv(\tilde{r})$) to the reward ($\mathcal{E}(\tilde{r})$) of such combination portfolios.

it into Formula 14.10

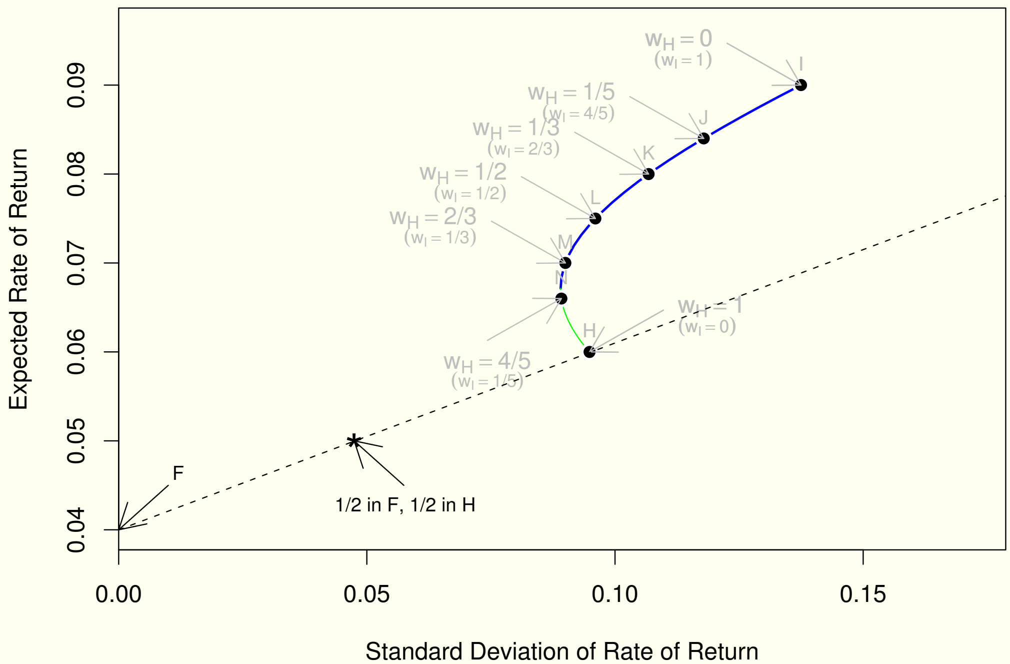
$$\begin{aligned}
\mathcal{E}(\tilde{r}_S) &= w_{\mathcal{H}} \cdot 6\% + (1 - w_{\mathcal{H}}) \cdot 4\% \\
&= w_{\mathcal{H}} \cdot (6\% - 4\%) + 4\% \\
&= \left[\frac{Sdv(\tilde{r}_S)}{9.49\%}\right] \cdot (6\% - 4\%) + 4\% \\
&= 4\% + 0.21 \cdot Sdv(\tilde{r}_S) \\
\mathcal{E}(\tilde{r}_S) &= w_{\mathcal{H}} \cdot \mathcal{E}(\tilde{r}_{\mathcal{H}}) + (1 - w_{\mathcal{H}}) \cdot r_{\mathcal{F}} \\
&= w_{\mathcal{H}} \cdot (\mathcal{E}(\tilde{r}_{\mathcal{H}}) - r_{\mathcal{F}}) + r_{\mathcal{F}} \\
&= \left[\frac{Sdv(\tilde{r}_S)}{Sdv(\tilde{r}_{\mathcal{H}})}\right] \cdot [\mathcal{E}(\tilde{r}_{\mathcal{H}}) - r_{\mathcal{F}}] + r_{\mathcal{F}} \\
&= r_{\mathcal{F}} + \left[\frac{\mathcal{E}(\tilde{r}_{\mathcal{H}}) - r_{\mathcal{F}}}{Sdv(\tilde{r}_{\mathcal{H}})}\right] \cdot Sdv(\tilde{r}_S) \quad .
\end{aligned} \tag{14.13}$$

This is the formula for a line, with $r_{\mathcal{F}}$ being the intercept and $\left[(\mathcal{E}(\tilde{r}_{\mathcal{H}}) - r_{\mathcal{F}})/(Sdv(\tilde{r}_{\mathcal{H}}))\right]$ being the slope. This may become clearer if you graph the portfolio combinations. Figure 14.5 plots the portfolio characteristics of S, which puts half the money into the risk-free rate and the other half into portfolio $\mathcal{H}$. It has a standard deviation of 4.75% on the X-axis, and a mean rate of return of 5% on the Y-axis. The figure then repeats this to plot the line—which gives the risk-reward characteristics of *all* combinations of the risk-free rate $\mathcal{F}$ with portfolio $\mathcal{H}$.

IMPORTANT: *Combination portfolios of a risk-free security and a portfolio P lie on a straight line in mean vs. standard deviation space.*

SIDE NOTE: There is one additional fact of interest here. What are the points to the right of $\mathcal{H}$? These portfolios have a negative weight in $\mathcal{F}$ and a weight above 1 in $\mathcal{H}$. (The portfolio weights must add to 100%!). This means that you would borrow money at a 4% annual interest rate to purchase more of portfolio $\mathcal{H}$ than you otherwise could—purchasing stocks with borrowed money is called **on margin**.

Figure 14.5. The Risk-Reward Tradeoff between $\mathcal{H}$ and $\mathcal{F}$

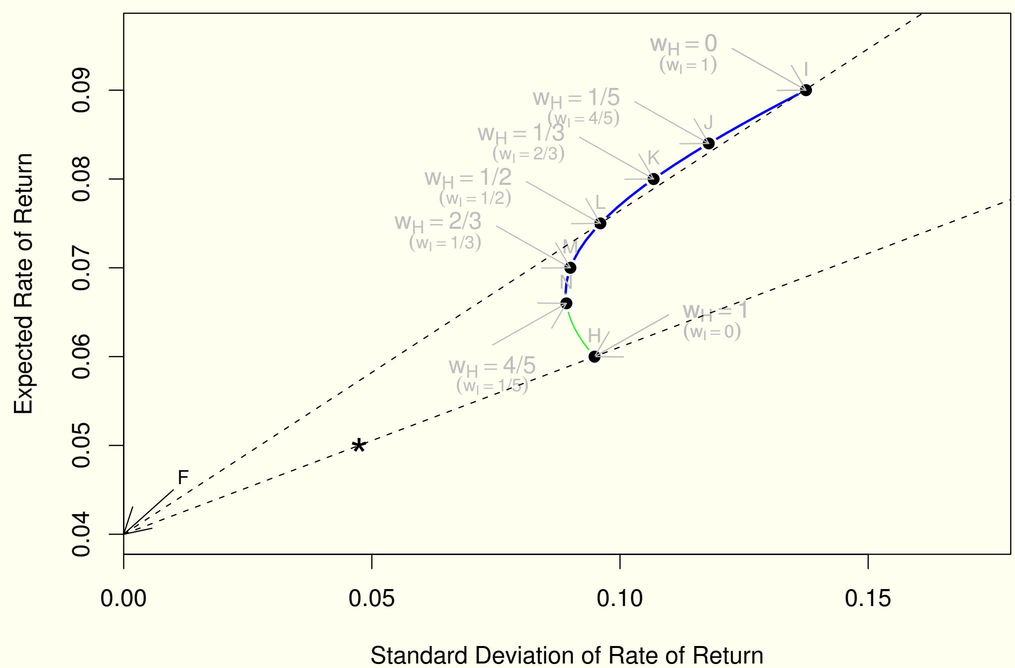


This adds a risk-free rate of 4% to Figure 14.1. The line represents risks and rewards for portfolios that combine portfolio $\mathcal{H}$ and the risk-free rate $\mathcal{F}$.

But would you really want to purchase such a combination of $\mathcal{H}$ and $\mathcal{F}$? Could you purchase a different portfolio in combination with $\mathcal{F}$ that would do better? Would the combination of $\mathcal{L}$ and $\mathcal{F}$ not perform better?

Figure 14.6 draws combinations of the risk-free rate and portfolio $\mathcal{L}$. This combination of $\mathcal{F}$ and $\mathcal{L}$ indeed does a lot better—but you can do better even yet. In fact, what portfolio would you purchase?

Figure 14.6. The Risk-Reward Tradeoff between $\mathcal{L}$ and $\mathcal{F}$

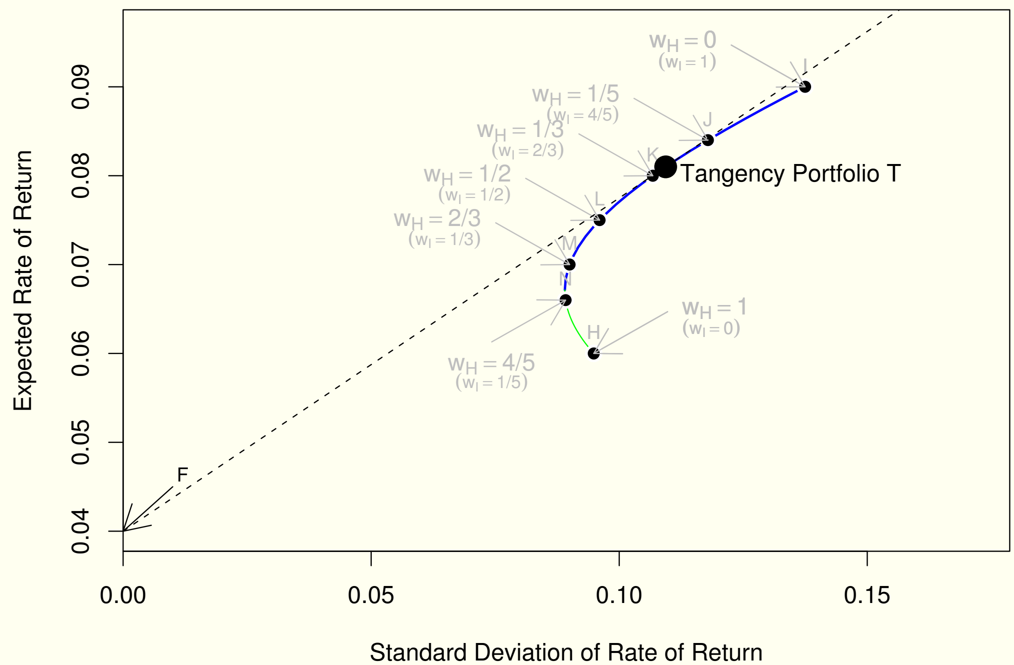


Adding to Figure 14.5, the new line represents risks and rewards for portfolios that combine portfolio $\mathcal{L}$ and the risk-free rate $\mathcal{F}$.

The best portfolio is the line tangent to the efficient frontier of risky assets.

The answer is drawn in Figure 14.7—you would purchase a combination portfolio of the risk-free rate and whatever portfolio on the previous efficient frontier would be tangent—you tilt the line up until it just touches the mean-variance frontier among the risky securities. This line is called the **capital market line.** Here, the exact investment proportions in the risky assets are difficult to see, but if you could blow up the figure, you would see that this is the portfolio that invests about 30% in $\mathcal{H}$, and 70% in $\mathcal{I}$. Let's call it $\mathcal{T}$, for tangency portfolio.

Figure 14.7. The Risk-Reward Tradeoff between $\mathcal{T}$ and $\mathcal{F}$



The capital market line represents risks and rewards for portfolios that combine the tangency portfolio $\mathcal{T}$ and the risk-free rate $\mathcal{F}$. It represents the best opportunities available.

How smart investors make portfolio decisions in the presence of a risk-free security.

Who should purchase a portfolio combination that invests more or less than 30% in $\mathcal{H}$ and 70% in $\mathcal{I}$? Nobody! Each and every smart investor would purchase only a combination of $\mathcal{F}$ and $\mathcal{T}$, regardless of risk-aversion. (This is called the **two-fund separation theorem.**) Investors would only differ in that their personal risk aversion would determine how much they would put into $\mathcal{F}$ versus $\mathcal{T}$.

> **IMPORTANT**: *In the presence of a risk-free security, all investors purchase a combination of the tangency portfolio and the risk-free security.*

What this means for the economy overall

If *all* investors are smart and purchase $\mathcal{T}$, then it must be the market portfolio—of course, this is necessarily true only *if all investors are smart mean-variance optimizers.*

> **IMPORTANT**: *In the* CAPM, *the market portfolio is the tangency portfolio.*

If $\mathcal{T}$ is the market portfolio, your portfolio optimization is even easier—just purchase a combination of the market portfolio and the risk-free rate. (You never even need to compute an efficient frontier.) Of course, in the real world, the market portfolio may not be the tangency portfolio—but then this would mean that the CAPM would not hold. *In fact, the* CAPM *is nothing more and nothing less than the statement that the market portfolio is the tangency portfolio.* This will now be shown to be true.

Solve Now!

Q 14.7 *Compute the covariance of $\mathcal{H}$ and $\mathcal{F}$*

Q 14.8 *The text noted that the minimum-variance portfolio without a risk-free security invests 76.2% on $\mathcal{H}$ and 24.8% on $\mathcal{I}$. With the risk-free security offering 4%, what portfolio would you purchase that has the same risk, and what would its improvement in reward be? First think about how to solve this. However, this is a difficult question, so you will be lead through step by step.*

(a) *What is the risk of this minimum-variance portfolio?*

(b) *What is the reward of the minimum-variance portfolio?*

(c) *The tangency portfolio invests 30% in $\mathcal{H}$ and 70% in $\mathcal{I}$. What are its returns in each of the four scenarios?*

(d) *What is its risk? (Check this visually in the graph!)*

(e) *What is its reward?*

(f) *Using the analog of Formula 14.12, what investment weight $w_{\mathcal{T}}$ in $\mathcal{T}$ would give you the same risk as the minimum-variance portfolio? (If you had \$100, how much would you put into $\mathcal{T}$, and how much would you put into a savings account?)*

(g) *Given this weight $w_{\mathcal{T}}$, what is the reward of this combination portfolio?*

Q 14.9 *Would the tangency portfolio invest in more or less $\mathcal{H}$ if the risk-free rate were 3% instead of 4%?*

14·2. The Efficient Frontier and the CAPM Formula

Every component in the MVE portfolio must follow a fair risk-contribution vs. reward relationship—or the portfolio would not be MVE.

We can now return to the CAPM. It states that the relationship

$$\mathcal{E}(\tilde{r}_i) \;=\; r_{\mathcal{F}} + [\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}] \cdot \beta_{i,\mathcal{M}} \tag{14.14}$$

holds for each security in the market. You can think of the CAPM formula as a relationship that relates the reward and the contribution of risk for *each component* of the market portfolio. But why does the efficient frontier, which graphs only the *overall* portfolio risk and reward, relate to a formula about the portfolio's many components? It turns out that the CAPM formula is really a mathematical property of portfolios on the efficient frontier. The formula says that in the portfolios on the MVE frontier, no component can offer too little or too much reward for its portfolio risk contribution. If it did, you could form a better portfolio by buying more or less of it, and therefore your overall original portfolio would not be on the MVE frontier to begin with!

In portfolio $\mathcal{N}$, you have too much $\mathcal{H}$ and too little $\mathcal{I}$. $\mathcal{H}$ is really too expensive for you given your portfolio. $\mathcal{I}$ is really too cheap.

Let us examine this logic. Take portfolio $\mathcal{N}$, which has 75% investment in $\mathcal{H}$ and 25% investment in $\mathcal{I}$. It is not MVE if you have access to the risk-free security offering a 4% rate of return. Relative to $\mathcal{T}$, $\mathcal{N}$ has too much $\mathcal{H}$ and too little $\mathcal{I}$ in it. (Recall that Portfolio $\mathcal{T}$ invests about 30% in $\mathcal{H}$ and 70% in $\mathcal{I}$.) Put differently, if you owned only $\mathcal{N}$, then security $\mathcal{H}$ would be relatively too expensive and unattractive, and security $\mathcal{I}$ would be relatively too cheap and attractive. You could perform better than $\mathcal{N}$ if you sold some of the expensive $\mathcal{H}$ and bought more of the cheap $\mathcal{I}$. In contrast, this logic should not apply for your tangency portfolio $\mathcal{T}$. If you owned the $\mathcal{T}$, you should not be able to do better. All securities should seem appropriately priced to you. This is the logic underlying the CAPM formula. It gives each security an appropriate reward, given a security's risk contribution (measured by beta with respect to the overall portfolio).

So, let us now confirm that the CAPM formula holds only for the tangency portfolio $\mathcal{T}$, and not for portfolio $\mathcal{N}$.

Table 14.2. Efficient and Inefficient Portfolios

Future	$\mathcal{H}$	$\mathcal{I}$	"$\mathcal{F}$"	$\mathcal{N}$	"$\mathcal{T}$"
Scenario S1 ♣	−6.0%	−12.0%	4.00%	−7.50%	−10.2%
Scenario S2 ♦	+12.0%	+18.0%	4.00%	+13.50%	+16.2%
Scenario S3 ♥	0.0%	+24.0%	4.00%	+6.00%	+16.8%
Scenario S4 ♠	+18.0%	+6.0%	4.00%	+15.00%	+9.6%
"Reward" ($\mathcal{E}(R)$)	6.00%	9.00%	4.00%	6.75%	8.10%
"Variance" ($\mathcal{V}ar(R)$)	90.0%%	189.0%%	0.0%%	79.3%%	119.6%%
"Risk" ($\mathcal{S}dv(R)$)	9.49%	13.70%	0.00%	8.91%	10.94%

These are the two base portfolios and the risk-free rate, plus two combinations of the $\mathcal{H}$ and $\mathcal{I}$ portfolio that we shall use to illustrate the mean-variance frontier with a risk-free security. The portfolio $\mathcal{N}$ appeared in Table 14.1, and invests 75% in $\mathcal{H}$, 25% in $\mathcal{I}$. Portfolio $\mathcal{T}$ invests about 30% in $\mathcal{H}$, 70% in $\mathcal{I}$.

If $\mathcal{N}$ were efficient, it should follow a CAPM formula.

First consider what would happen if $\mathcal{N}$ were a mean-variance efficient portfolio—if it were the market portfolio and not $\mathcal{T}$. Then, as in the CAPM, you must have the mathematical relationship

$$\mathcal{E}(\tilde{r}_i) \;=\; r_{\mathcal{F}} + [\mathcal{E}(\tilde{r}_{\mathcal{N}}) - r_{\mathcal{F}}] \cdot \beta_{i,\mathcal{N}} \quad . \tag{14.15}$$

The beta of security i with respect to portfolio $\mathcal{N}$ ($\beta_{i,\mathcal{N}}$) is your measure of the risk contribution of security i to portfolio $\mathcal{N}$. Therefore, you need to compute the beta of security $\mathcal{H}$ with respect to the overall portfolio $\mathcal{N}$. This is the covariance of $\mathcal{N}$ and $\mathcal{H}$, divided by the variance of $\mathcal{N}$.

Trust me that this is $\beta_{\mathcal{H},\mathcal{N}} = 0.99$. Similarly, $\beta_{\mathcal{I},\mathcal{N}} = 1.02$. Substitute these two betas in, and you find

$$\begin{aligned} \mathcal{E}(\tilde{r}_{\mathcal{H}}) &= 4\% + [8.1\% - 4\%] \cdot 0.99 \approx 8.07\% \\ \mathcal{E}(\tilde{r}_{\mathcal{I}}) &= 4\% + [8.1\% - 4\%] \cdot 1.02 \approx 8.19\% \\ \mathcal{E}(\tilde{r}_i) &= r_{\mathcal{F}} + [\mathcal{E}(\tilde{r}_{\mathcal{N}}) - r_{\mathcal{F}}] \cdot \beta_{i,\mathcal{N}} \quad . \end{aligned} \tag{14.16}$$

But if you look at Table 14.2, you will see that $\mathcal{H}$ offers a reward of only 6% and $\mathcal{I}$ offers a reward of 9%. In this portfolio $\mathcal{N}$, $\mathcal{H}$ is too expensive and I is too cheap. You would do better to get rid of some $\mathcal{H}$ and buy more $\mathcal{I}$. If $\mathcal{N}$ were the market portfolio, the CAPM formula would not hold! $\mathcal{H}$ would be too expensive in the market, and $\mathcal{I}$ would be too cheap in the market. Therefore, $\mathcal{N}$ would not be a mean-variance efficient portfolio.

T is efficient, so it does follow a CAPM formula.

Now consider what would happen if the market portfolio were $\mathcal{T}$. Then, as in the CAPM, you must see the relationship

$$\mathcal{E}(\tilde{r}_i) = r_{\mathcal{F}} + [\mathcal{E}(\tilde{r}_{\mathcal{T}}) - r_{\mathcal{F}}] \cdot \beta_{i,\mathcal{T}} \quad . \tag{14.17}$$

Again, you need to compute the beta of security $\mathcal{H}$ with respect to the overall portfolio $\mathcal{T}$. This is the covariance of $\mathcal{H}$ and $\mathcal{T}$, divided by the variance of $\mathcal{T}$. Trust me again that this is $\beta_{\mathcal{H},\mathcal{T}} = 0.49$. Similarly, $\beta_{\mathcal{I},\mathcal{T}} = 1.22$. Substitute these two betas in, and you find

$$\begin{aligned} \mathcal{E}(\tilde{r}_{\mathcal{H}}) &= 4\% + [8.1\% - 4\%] \cdot 0.49 \approx 6\% \\ \mathcal{E}(\tilde{r}_{\mathcal{I}}) &= 4\% + [8.1\% - 4\%] \cdot 1.22 \approx 9.\% \\ \mathcal{E}(\tilde{r}_i) &= r_{\mathcal{F}} + [\mathcal{E}(\tilde{r}_{\mathcal{T}}) - r_{\mathcal{F}}] \cdot \beta_{i,\mathcal{T}} \end{aligned} \tag{14.18}$$

This is exactly what these two securities are offering, and therefore exactly as CAPM suggests—it is a linear relationship between each security's expected rate of return and beta with respect to the market. You cannot do better by either selling or buying more of $\mathcal{H}$ or $\mathcal{I}$. You are already holding them in the best proportions. And, therefore, $\mathcal{T}$ is indeed mean-variance efficient.

Solve Now!

Q 14.10 *We will now confirm the beta computations. Work with Table 14.2.*

(a) Compute the covariance between $\mathcal{H}$ and $\mathcal{N}$.

(b) Compute the covariance between $\mathcal{I}$ and $\mathcal{N}$.

(c) Compute the variance of $\mathcal{N}$.

(d) Compute the beta of $\mathcal{H}$ with respect to $\mathcal{N}$.

(e) Compute the beta of $\mathcal{I}$ with respect to $\mathcal{N}$.

Repeat this for portfolio $\mathcal{T}$ instead of $\mathcal{N}$.

Q 14.11 *Confirm that the portfolio $\mathcal{H}$ is not mean-variance efficient.*

14·3. Simplifications and Perspective

It is time now for some reflection. What is the logic of the CAPM? Does the CAPM work?

CAPM Logic: The market portfolio must be the tangency portfolio.

In sum, you now know that each investor wants to purchase an MVE portfolio. You know that the MVE is a line from the risk-free asset to the tangency portfolio, so every investor purchases a combination of the risk-free asset and the tangency portfolio only. It follows that the tangency portfolio must be the overall market portfolio. If it were not, it would make no sense: investors would jointly seek to own more or less of this security than there would be available. The CAPM formula is a simple translation of the statement that the market portfolio is on the efficient frontier—that it is the tangency portfolio.

Simplifications: Which do any harm?

The financial markets in this chapter were not only perfect, they were also simplified dramatically. However, everything would still work in a more complex world with more securities, more scenarios, and no risk-free rate:

More Than Two Securities Everything would work the same, except that there would not just be a line, but a whole cloud of points inside the efficient frontier—different portfolios that are inferior because they have not enough expected rate of return for their overall risk, and which therefore no smart investor would choose. The CAPM still says that the tangency portfolio (on the efficient frontier among all risky securities) should be the market portfolio.

More Than Four Scenarios Everything would work the same. It would only take more adding and multiplying. (In the real world, return outcomes are continuous—returns are distributed not too differently from a bell curve. For the finance major, assuming normally distributed returns does make the CAPM a little more robust. If all security returns are jointly normally distributed, then there are good reasons to believe that the CAPM should hold even if investors have non mean-variance preferences.)

The Risk-Free Security Surprisingly, even if there is no risk-free security, the CAPM will still go through. Each and every investor would still want to purchase a portfolio on the MVE frontier, and the combination of such MVE portfolios turns out to still be MVE. Therefore, the market would be MVE, and a CAPM formula will therefore hold—except that the CAPM formula then has some number a instead of the (non-existent) risk-free rate in it.

So, these three simplifications may have appeared quite drastic, but were actually quite harmless.

This will come about, e.g., if everyone holds an MVE efficient portfolio...

Alas, not everything is as pink and rosy as I tried to make you believe in Figure 13.4 and Section 13·2.C. The underlying rationale for why the market portfolio *should* lie on the line is that you know that if each and every investor holds a mean-variance efficient portfolio, the market portfolio will also be mean-variance efficient. (Yes, it could be the case that the market portfolio could be mean-variance efficient just by chance, even if many investors do not hold it. However, this would be highly unlikely.)

...but, of course, no one does. There are good reasons why this is not the case.

So, how many investors do you know who hold mean-variance efficient portfolios? I know of none. Even theoretically, there are good reasons why individuals may not *want* to hold mean-variance efficient portfolios:

- The CAPM relies on perfect markets: no information differences, a deep market in everything, no transaction costs, and no taxes. If these assumptions are not satisfied, different investors may find themselves wanting to hold portfolios different from the market portfolio.

  For example, transaction costs may make it better to forego holding all of the thousands of available securities—a \$10,000 portfolio with 500 stocks would request your broker execute orders for \$20 worth of stock each. (Fortunately, there are funds that do allow even small investors to broadly diversify—but even these funds rarely hold anything close to the value-weighted market portfolio.)

In my opinion, the most important reason why investors do not hold the overall market is because they have different opinions: many individuals seem to believe that they know better than the market that a particular stock will go down or up—an assumption that leads them to buy less or more of this stock. Never mind that such a belief in their superior abilities is often contradicted by all empirical evidence. A closely related issue are ubiquitous agency problems—many brokers seem to be able to convince their clients to let them invest their money on their behalves. But brokers earn money based on trading turnover. Putting the clients' money into a buy-and-hold market portfolio would not be in their self-interest. Again, clients' beliefs in the superior ability of their brokers is often contradicted by the evidence, too.

- The CAPM assumes that everyone can buy the market portfolio of all investment assets. However, different investors may have access to different choices. For example, you may want to invest in and benefit from your education—something I cannot do. If you have access to different investment opportunities than I, the market portfolio could be anywhere. In fact, everyone would face a different mean-variance frontier, so it is not even clear on which MVE frontier the market portfolio should lie.

- The CAPM assumes particular risk-reward preferences. You like reward, but dislike risk. For example, compare the following two portfolios that have about the same mean and standard deviation:

	Skewed	Symmetric
Scenario S1 ♣	−100%	−57%
Scenario S2 ♦	35%	−57%
Scenario S3 ♥	35%	60%
Scenario S4 ♠	35%	60%

The skewed portfolio has a 1 in 4 chance of bankrupting you. You might think of it as crash prone. The symmetric portfolio never bankrupts you, but it leaves you with a significant loss two out of four times. If investors have strong enough preferences, preferring one of these two portfolios enough to be willing to sacrifice expected rate of return, then the CAPM might not hold.

- Academics also worry about time-changing and time-dependent opportunity sets. The most obvious example of this is the risk-free rate. The yield curve typically allows you to earn a higher expected rate of return over longer horizons. If expected rate of returns are time-varying, then investors could possibly tilt their portfolios to "hedge" against adverse future developments. For example, the long-term risk free rate may be higher, because it may be a poorer hedge against future inflation than the short-term risk-free rate. (Personally, I do not believe this is too important—I know of no investors that use their portfolios to hedge against changing opportunity sets.)

All you need for the CAPM is MVE efficiency of the market.

In the end, these are good theoretical reasons why investors need not necessarily choose mean-variance efficient portfolios. If they do not, the market portfolio need not be the tangency portfolio. Of course, like the market portfolio, the tangency portfolio is highly diversified, but this does *not* mean that the tangency portfolio *is* the market portfolio. So, you should now recognize that this one tiny point about market mean-variance efficiency (that the market portfolio lies on the MVE frontier) is not so tiny, after all. Indeed, all the rest is nothing but a little math: if the market is mean-variance efficient, then all securities have to lie on the securities markets line, the CAPM is the right model, and nothing can outperform the CAPM formula's expected rate of return predictions.

The CAPM could still hold—but, unfortunately, it does not.

Theoretically, the market could be mean-variance efficient, but it does not necessarily have to be so. Therefore, it remains an empirical question of how well the CAPM works—is the market portfolio on the MVE-frontier or is it not? This is not easy to measure, because you only have historical estimates of means, variances, and covariances, not the true forward-looking estimates you should have. However, the general consensus of the finance profession is that the market portfolio does *not* lie on the efficient frontier, and therefore that the CAPM is not the final end-all model.

It is still a very useful model in some contexts, though not in others.

So, why torture you with the CAPM? Aside from the Newtonian analogy I mentioned earlier, the fact is that there is no good alternative—it takes a model to beat a model. As I wrote in Section 13·2.C, my own opinion is that the CAPM is a good model if you want to determine the cost of capital in an ordinary corporate finance setting. It has solid intuition, it is reasonably easy to use, and its estimates are usually "close enough." Moreover, everyone is using it, so you must know what everyone around you is using—it is *the* standard. But you should also know that the CAPM is a poor model if you want to invest money into many different securities. There are better models than the CAPM for investment purposes. (Take an investments course!) You should further realize that the CAPM is also a poor model if precision is of the essence. Actually, if accuracy and precision is important, you are thoroughly in trouble. We do not know *any* models that perform reliably better than the CAPM in a corporate context in giving you a reasonable cost of capital estimate. For almost all practical purposes, the CAPM will have to do.

SIDE NOTE: There are some alternatives to the CAPM, first and foremost the **Arbitrage Pricing Theory** (**APT**). Unfortunately, the APT is even harder to use—and it still shares most of the shortcomings of the CAPM. In its favor, it does allow you to specify that factors other than the market and the beta of your project with respect to the market matter. For example, in the APT, you might hypothesize that the unemployment rate is a factor that matters, and so your project's beta with respect to the unemployment rate helps determine your project's expected rate of return. This is both a blessing and a curse: the APT gives you wonderful flexibility, but no guidance on what the factors are!

14·4. Summary

The chapter covered the following major points:

- The efficient frontier plots the achievable combinations of overall portfolio risk and reward.
- With a risk-free security, the real efficient frontier becomes the line connecting the risk-free rate with the tangency portfolio from the efficient frontier, using only the risky securities.
- Portfolios on the mean-variance efficient frontier do not underinvest or overinvest in individual securities. Therefore, for portfolios on the efficient frontier, individual securities must follow the CAPM security markets line (SML).

  If one security were to offer too much or too little reward (measured by expected rate of return) for its risk contribution (measured by portfolio-beta), then this original portfolio could be improved upon by buying more or less of this one security—and therefore it would not have been mean-variance efficient to begin with.
- The CAPM is only one economic statement: the market portfolio lies on the efficient frontier. The rest—the CAPM formula and the securities market line—is just mathematical consequence.
- The CAPM is a reasonable model in a corporate finance context, even though the overall evidence suggests that it flunks in describing the empirical evidence. It is not a good model in an investments context.

14·5. Advanced Appendix: More than Two Securities

How does this work with more than two securities? How would you find the MVE portfolios?

Three Securities If you have three securities to consider, the formula for portfolio variance is

$$\begin{aligned} \mathcal{V}ar(\tilde{r}_P) \;=\;& w_1^2 \cdot \mathcal{V}ar(\tilde{r}_1) + w_2^2 \cdot \mathcal{V}ar(\tilde{r}_2) + w_3^2 \cdot \mathcal{V}ar(\tilde{r}_3) + \\ &+ \; 2 \cdot w_1 \cdot w_2 \cdot Cov(\tilde{r}_1, \tilde{r}_2) + 2 \cdot w_1 \cdot w_3 \cdot Cov(\tilde{r}_1, \tilde{r}_3) + 2 \cdot w_2 \cdot w_3 \cdot Cov(\tilde{r}_2, \tilde{r}_3) \quad . \end{aligned} \tag{14.19}$$

Your objective is to select a best combination of investment weights, w_1 and w_2. The third weight, w_3 is $1 - w_1 - w_2$, because your investment weights add up to 100%. The absolute minimum variance portfolio is relatively easy to find if you know calculus. First, differentiate with respect to the two choice weights, and set them equal to zero

$$\begin{aligned} \frac{\partial \mathcal{V}ar(\tilde{r}_P)}{\partial w_1} &= 2 \cdot w_1 \cdot \mathcal{V}ar(\tilde{r}_1) + w_2 \cdot [Cov(\tilde{r}_1, \tilde{r}_2) - Cov(\tilde{r}_2, \tilde{r}_3)] + w_3 \cdot [\mathcal{V}ar(\tilde{r}_3) - Cov(\tilde{r}_1, \tilde{r}_3)] = 0 \quad , \\ \frac{\partial \mathcal{V}ar(\tilde{r}_P)}{\partial w_2} &= 2 \cdot w_1 \cdot \mathcal{V}ar(\tilde{r}_1) + w_2 \cdot [Cov(\tilde{r}_1, \tilde{r}_2) - Cov(\tilde{r}_2, \tilde{r}_3)] + w_3 \cdot [\mathcal{V}ar(\tilde{r}_3) - Cov(\tilde{r}_1, \tilde{r}_3)] = 0 \quad , \\ w_3 &\equiv 1 - w_1 - w_2 \quad . \end{aligned} \tag{14.20}$$

You can solve this for the three portfolio weights, but the solution is messy and not very insightful—and it gets quickly messier if you have more than three securities.

We now use linear algebra—if you know it, it makes the portfolio choice problem beautifully simple.

N Securities, No Risk-Free Asset To really handle this problem, we have to resort to matrix algebra—so the following will only have meaning to you if you have higher mathematical training. First define the following:

- N The number of securities, indexed i through N.
- $\vec{w}$ The N-vector of investment weights.
- $\vec{E}$ The N-vector of expected rates of return.
- Σ The N by N matrix of covariances. That is, entry i, j in the matrix is the covariance between the rate of return of security i with security j. Therefore, the variances are on the diagonal.
- $\vec{1}$ An N-vector containing only the number 1.

The solutions now are surprisingly quick. The variance of the portfolio, that is the "squared variances and two-times covariances" formula, can now be simply written as

$$\mathcal{V}ar(\tilde{r}_P) \;=\; \vec{w}' \, \Sigma \, \vec{w} \quad . \tag{14.21}$$

The constraint that all investment weights must add to 100% can be written as

$$\vec{1}' \, \vec{w} \;=\; 1 \quad . \tag{14.22}$$

And the constraint that you want to find a portfolio offering an expected rate of return of $\mathcal{E}(\tilde{r}_P)$ (which you can select up-front) can be written as

$$\vec{w}' \, \vec{E} \;=\; \mathcal{E}(\tilde{r}_P) \quad . \tag{14.23}$$

The cleanest way to solve the variance minimization, subject to these two linear constraints, is the Lagrangian method. The solution for the efficient frontier is

$$\mathcal{V}ar(\tilde{r}_P) \;=\; \frac{A \cdot \mathcal{E}(\tilde{r}_P)^2 - 2 \cdot B \cdot \mathcal{E}(\tilde{r}_P) + C}{D} \quad , \tag{14.24}$$

where the four scalar numbers are

$$A \equiv \vec{1}' \, \Sigma^{-1} \, \vec{1} \; , \quad B \equiv \vec{1}' \, \Sigma^{-1} \, \vec{E} \; , \quad C \equiv \vec{E}' \, \Sigma^{-1} \, \vec{E} \; , \quad D \equiv A \cdot C - B^2 \quad . \tag{14.25}$$

To trace the portfolios sitting on the MVE frontier, you need to know two efficient portfolios (the rest are combinations thereof—yes, two fund separation also works if there is no risk-free security). The minimum-variance portfolio is

$$\vec{w} = \frac{\Sigma^{-1}\,\vec{1}}{A} \tag{14.26}$$

A second portfolio would be

$$\vec{w} = \frac{\Sigma^{-1}\,\vec{E}}{B} \tag{14.27}$$

A repeat with a risk-free rate of return.

N Securities, Risk-Free Asset If you have a risk-free rate of return, $r_{\mathcal{F}}$, you must rewrite the desired mean rate of return constraint from Formula 14.23 as

$$\vec{w}'\,(\vec{E} - r_{\mathcal{F}}\,\vec{1}) = \mathcal{E}(\tilde{r}_P) \quad . \tag{14.28}$$

The solution now has the tangency portfolio as

$$\vec{w}_{\mathcal{T}} = \frac{\Sigma^{-1}\,(\vec{E} - r_{\mathcal{F}}\cdot\vec{1})}{B - A\cdot r_{\mathcal{F}}} \quad , \tag{14.29}$$

with an expected rate of return and variance of

$$\begin{aligned} \mathcal{E}(\tilde{r}_{\mathcal{T}}) &= \frac{C - B\cdot r_{\mathcal{F}}}{B - A\cdot r_{\mathcal{F}}} \quad , \\ \mathcal{V}ar(\tilde{r}_{\mathcal{T}}) &= \frac{C - 2\cdot B\cdot r_{\mathcal{F}} + A\cdot r_{\mathcal{F}}^2}{(B - A\cdot r_{\mathcal{F}})^2} \quad , \end{aligned} \tag{14.30}$$

and the tangency line is the

$$Sdv(\tilde{r}_P) = \frac{\mathcal{E}(\tilde{r}_P) - r_{\mathcal{F}}}{\sqrt{C - 2\cdot r_{\mathcal{F}}\cdot B + A\cdot r_{\mathcal{F}}^2}} \quad . \tag{14.31}$$

Show how to use these formulas

Let us breath some life into these formulas. Consider three possible investments: the S&P500, IBM, and Sony. (Their historical returns can be found on the book website, or at Yahoo!*Finance*.) From 1991 to 2002, their annual rates of return had the following characteristics:

		Covariance between $\tilde{r}_i$ and $\tilde{r}_j$		
Security	$\mathcal{E}(\tilde{r}_i)$	1=S&P500	2=IBM	3=Sony
1=S&P500	10.110%	3.6224%	3.2980%	4.7716%
2= IBM	15.379%	3.2980%	15.0345%	2.1842%
3= Sony	24.203%	4.7716%	2.1842%	81.4886%

The most difficult part is to invert the covariance matrix. For this, you need a computer program. In a spreadsheet, the **MINVERSE(**RANGE**)** function will do this for you. The solution is

$$\Sigma^{-1} = \begin{pmatrix} 3.6224\% & 3.2980\% & 4.7716\% \\ 3.2980\% & 15.0345\% & 2.1842\% \\ 4.7716\% & 2.1842\% & 81.4886\% \end{pmatrix}^{-1} = \begin{pmatrix} 37.426 & -7.922 & -1.979 \\ -7.922 & 8.354 & 0.240 \\ -1.979 & 0.240 & 1.336 \end{pmatrix} \quad . \tag{14.32}$$

Therefore,

$$\begin{aligned} A &\equiv \vec{1}'\,\Sigma^{-1}\,\vec{1} = (27.520, 0.672, -0.403)\,\vec{1} = 27.79 \;, \\ B &\equiv \vec{E}'\,\Sigma^{-1}\,\vec{1} = (2.086, 0.542, 0.160)\,\vec{1} = 2.789 \;, \\ C &\equiv \vec{E}'\,\Sigma^{-1}\,\vec{E} = (27.520, 0.672, -0.403)\,\vec{E} = 0.333 \;, \\ D &\equiv A \cdot C - B^2 = 1.481 \;. \end{aligned} \tag{14.33}$$

Therefore, the MVE frontier is

$$\mathcal{V}ar(\tilde{r}_P) = \frac{A \cdot \mathcal{E}(\tilde{r}_P)^2 - 2 \cdot B \cdot \mathcal{E}(\tilde{r}_P) + C}{D} = \frac{27.792 \cdot \mathcal{E}(\tilde{r}_P)^2 - 2 \cdot 2.789 \cdot \mathcal{E}(\tilde{r}_P) + 0.333}{1.481} \;, \tag{14.34}$$

and the global minimum variance portfolio is

$$\vec{w} = \frac{\Sigma^{-1}\,\vec{1}}{A} = (0.9903, 0.0242, -0.0145) \;. \tag{14.35}$$

This makes sense—the minimum variance portfolio has a lot more of the heavily diversified S&P500 in it than it has of the other two securities. You might want a second MVE portfolio to trace out all combination portfolios on the MVE Frontier, so here is one,

$$\vec{w} = \frac{\Sigma^{-1}\,\vec{E}}{B} = (0.7482, 0.1943, 0.0575) \;. \tag{14.36}$$

Now let's presume you have acccess to a risk-free rate of 2%. Then, the tangency portfolio would be

$$\vec{w}_{\mathcal{T}} = \frac{\Sigma^{-1}\,(\vec{E} - r_{\mathcal{F}} \cdot \vec{1})}{B - A \cdot R} = \frac{(1.536, 0.5285, 0.168)}{2.233} = (0.6879, 0.2367, 0.0754) \;. \tag{14.37}$$

which has an expected rate of return of 12.42% and a variance of 0.04667 (a standard deviation of 21.6%). The tangency line is

$$\begin{aligned} \mathcal{S}dv(\tilde{r}_P) &= \frac{\mathcal{E}(\tilde{r}_P) - r_{\mathcal{F}}}{\sqrt{C - 2 \cdot r_{\mathcal{F}} \cdot B + A \cdot r_{\mathcal{F}}^2}} = \frac{\mathcal{E}(\tilde{r}_P) - 0.02}{\sqrt{0.333 - 2 \cdot 0.02 \cdot 2.789 + 27.79 \cdot 0.02^2}} \\ &= 4.146\% + 2.073 \cdot \mathcal{E}(\tilde{r}_P) \;. \\ \mathcal{E}(\tilde{r}_P) &= 2\% + 0.482 \cdot \mathcal{S}dv(\tilde{r}_P) \end{aligned} \tag{14.38}$$

For every extra percent in expected rate of return you choose, your overall portfolio will have to suffer a little more than a two percent increase in standard deviation.

Solutions and Exercises

1. 0.0081
2. 0.00793125
3. 0.009225
4. The mean is 8.7%. The variance is 0.0162. Therefore, the standard deviation (risk) is 12.7%.
5. If the correlation were higher, diversification would help less, so the risk would be higher. Therefore, the efficient frontier would not bend as far towards the west (a risk of 0). An easy way to check this is to rearrange the returns so that they correlate more positively, as you shall do in the next question. If the correlation were lower, diversification would help more, so the risk would be lower. Therefore, the efficient frontier would bend closer towards the west (a risk of 0).
6. The covariance is 0.0126, which is much higher. This means that the correlation between $\mathcal{A}$ and $\mathcal{H}$ shoots from 35% up to 97%. This means that the frontier is more vertical, and the minimum variance portfolio is much more towards the right.
7. Because the demeaned $\mathcal{F}$ is always 0, so is its coproduct with anything else.
8.
   (a) Formula 14.9 noted that this portfolio has a risk of 8.9%.

   (b) The reward is $\mathcal{E}(\tilde{r}) = 76.2\% \cdot 6\% + 24.8\% \cdot 9\% = 6.71\%$.

   (c) See Table 14.2: –10.2%, +16.2%, +16.8%, +9.6%.

   (d) See Table 14.2: 10.94%

   (e) See Table 14.2: 8.1%

   (f) Solve $Sdv(\tilde{r}) = w_{\mathcal{T}} \cdot Sdv(\tilde{r}_{\mathcal{T}}) = 8.9\% = w_{\mathcal{T}} \cdot 10.94\%$. Therefore, $w_{\mathcal{T}} = 81.4\%$. Put in words, a portfolio of the 18.65% in the risk-free security and 81.35% in the tangency portfolio has the same risk of 8.9%.

   (g) Therefore, the expected rate of return of the $(w_{\mathcal{T}}, w_{\mathcal{F}}) = (81.35\%, 18.65\%)$ portfolio is $\mathcal{E}(\tilde{r}) = 81.35\% \cdot \mathcal{E}(\tilde{r}_{\mathcal{T}}) + 18.65\% \cdot r_{\mathcal{F}} = 7.34\%$.

   The capital market line therefore offers $\mathcal{E}(\tilde{r}) - r_{\mathcal{F}} = 7.34\% - 6.71\% \approx 63$ basis points more expected rate of return when compared to the minimum variance portfolio, given the same risk.
9. The line would become steeper. The tangency portfolio would shift from around $\mathcal{K}$ to around $\mathcal{L}$. Therefore, it would involve more $\mathcal{H}$.
10.
    (a) The covariance between $\mathcal{H}$ and $\mathcal{N}$ is 0.00788.

    (b) The covariance between $\mathcal{I}$ and $\mathcal{N}$ is 0.0081.

    (c) The variance of $\mathcal{N}$ is 0.00793. Actually, it was in the table itself.

    (d) This is merely the covariance divided by the variance: $0.00788/0.00793 \approx 0.994$.

    (e) This is $0.0081/0.00793 \approx 1.02$.

    The covariance of $\mathcal{T}$ and $\mathcal{H}$ is 0.0078, between $\mathcal{T}$ and $\mathcal{I}$ is 0.0194.
11. You need to compute the beta of $\mathcal{H}$ and $\mathcal{I}$ with respect to portfolio $\mathcal{H}$. The beta of $\mathcal{H}$ with respect to itself is 1. The beta of $\mathcal{I}$ with respect to $\mathcal{H}$ is 0.5. For a CAPM formula to hold, you need

    $$\mathcal{E}(\tilde{r}_{\mathcal{H}}) = r_{\mathcal{F}} + [\mathcal{E}(\tilde{r}_{\mathcal{H}}) - r_{\mathcal{F}}] \cdot \beta_{\mathcal{H},\mathcal{H}} = 4\% + 2\% \cdot 1 = 6\%. \qquad (14.39)$$

    This is ok.

    $$\mathcal{E}(\tilde{r}_{\mathcal{I}}) = r_{\mathcal{F}} + [\mathcal{E}(\tilde{r}_{\mathcal{H}}) - r_{\mathcal{F}}] \cdot \beta_{\mathcal{I},\mathcal{H}} = 4\% + 2\% \cdot 0.5 = 5\%. \qquad (14.40)$$

    Aha! The CAPM type relationship is violated. This security should offer 5%, but it offers 9% in real life. Therefore, you should purchase more of it.

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

CHAPTER 15

Efficient Markets, Classical Finance, and Behavioral Finance

Or, Do You Get What You Pay For?

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This chapter revisits the concept of competitive, perfect, efficient markets, first mentioned in Section 6·1.C. It develops three basic concepts of finance in more depth: arbitrage, good bets, and efficient markets (**E-M**). No study of finance is complete without an understanding of these concepts.

This chapter also discusses the consequences of the E-M concept: what efficient markets mean for predicting stock performance; how to interpret the success of famous investors; and how to use the efficient markets concept to run an event study to help assess the valuation impact of some corporate events.

15·1. Arbitrage and Great Bets

Although you may have an intuitive notion of what arbitrage is, it is important that you know precisely what it is:

> **IMPORTANT**: *An **arbitrage** is a business transaction*
>
> - *that offers positive net cash inflows in some states of the world,*
> - *and under no circumstance—either today or in the future—a negative net cash outflow. Therefore it is risk-free.*

Arbitrage is the "Perpetuum Mobile" of economics. It is defined in terms of cash outlays and risk.

Let's first be clear about what arbitrage is *not*. It is not the same as "earning money without risk": after all, we know that investments in Treasury securities earn a positive risk-free rate of return. But buying safe bets like T-bills requires you to lay out cash today. Arbitrage is also not the same as "money in today": if you are willing to accept risk, you can often receive cash today. For example, insurance companies take money from you in exchange for the possibility that they may have to pay you in the future.

In a sense, positive NPV projects under certainty are arbitrage.

In theory, what would a hypothetical arbitrage opportunity look like? For example, if you can purchase an item for $1, borrow at an interest rate of 9% (all costs, including your time included), and sell the item tomorrow for $1.10 for sure, you earn 1 cent for certain today without any possible negative outflows in the future. If you ever stumble upon such an opportunity, please execute it—it is a positive NPV project! More than this, it is an arbitrage because you cannot lose money under any scenario (it is without risk!), but it is obviously not a very important arbitrage. Now, in financial markets, many transactions can be scaled up. If you could repeat this transaction one billion times, then you could earn $10 million. Of course, it is even more unlikely that you can find such an arbitrage opportunity that works for one billion items than it is to find an arbitrage that works for one item. After all, you are not the only one searching! True arbitrage opportunities are difficult or outright impossible to find in the real world, especially in very competitive financial markets.

Arbitrage could conceivably occur between different financial markets.

Another hypothetical example of arbitrage involves a violation of the **law of one price**, that is, that the same good should cost the same amount. Assume that PEP shares are quoted for $51 on the Frankfurt Stock Exchange, and for $50 on the **New York Stock Exchange**. The arbitrage could be executed by selling short one share at a price of $51 in Frankfurt, taking the $51, and then investing $50 in one share of PEP on the N.Y.S.E.. You pocket $1 today. What you had to promise to the Frankfurt buyer, which is all PEP payouts (such as dividends), will be covered by your ownership of the N.Y.S.E. PEP share. If you can do this with 20,000 PEP shares worth $1 million, you earn $20,000 without effort or risk.

Consider the hindrances.

But before you conclude that this is an arbitrage, you still have to make sure that you have not forgotten costs or risks. The arbitrage may be a lot more limited than it seems—or not even present. Consider the following issues:

1. There are the direct and indirect transaction costs. How much commission do you have to pay? Do you have to pay extra fees to short a stock? Is $51 the Frankfurt **bid price** at which you can sell, and $50 the NYSE **ask price** at which you can buy? Have you accounted for the value of your own time watching the screen for opportunities?
2. Share prices can move when you want to transact a significant amount of shares. Only the first 100 shares may be available for $50 for a net profit of $100. The next 900 shares may cost $50.50—perhaps still worthwhile, but less profitable. And purchasing the remaining 19,000 shares may cost you $51 or more.
3. By the time you have shorted the shares in Frankfurt at $51, the price in New York may have risen to $52. If such execution timing risk exists, this is not pure arbitrage because

there is a chance of a negative outflow. The real-world evidence suggests that price discrepancies between markets often disappear within a few seconds.

4. You would also have to account for your fixed cost of executing this transaction, such as setting up your own computer operation to do quick arbitrage-like transactions.

My belief is that in the real world, small arbitrage opportunities can occur from time to time, but large financial firms are constantly running automated computer trading programs that search for even tiny arbitrage opportunities in order to exploit them as soon as they appear—and thereby make them disappear.

The difference between arbitrage and a good bet.

The concept of arbitrage is different from the concept of a **good bet**. A good bet would be a chance to win $1,000,000 with 99% probability and to lose $1 with 1% probability. But because there is a chance of losing money in some circumstances, this is "just" a great bet. It is not an arbitrage. The difference is important: everyone would want to take advantage of arbitrage opportunities, but someone sufficiently risk-averse may not like a good bet, even if it is an absolutely wonderful bet. Conversely, a limited arbitrage need not be better than a good bet. For example, a single 1 cent arbitrage that cannot be repeated could be a worse bet than the aforementioned $1 million gain, $1 loss bet.

There are probably neither great bets nor arbitrage in very competitive financial markets.

Unless financial markets are exceedingly strange, we would not expect to find either arbitrage opportunities or great bets. If you agree with this assessment—basically that money does not grow on trees—we can draw some surprisingly strong conclusions about how financial markets work. If you disagree, you should not be sitting in class, but somewhere on a beach, ranking among the richest people in the world. There is little this book can teach you.

Solve Now!

Q 15.1 *Explain when and why you would prefer a good bet to an arbitrage opportunity.*

15·2. Market Efficiency and Behavioral Finance

IMPORTANT: *Warning: Market Efficiency is a different concept from Mean-Variance Efficiency. The reuse of the word "efficiency" is unfortunate.*

15·2.A. Basic Definition and Requirements

Market Efficiency means that markets use all information.

Formally, financial economists call a market efficient when it uses all available information in its price setting. Thus, **market efficiency** is the degree to which prices reflect information. In a fully efficient market, you cannot use available information to predict future returns better than the market can. Unfortunately, this leaves the question vague as to where the market wants to set expected returns. For example, the CAPM might state that the expected rate of return on PepsiCo should be 10% (setting a price of $50 for an expected payoff of $55), but you as an investor could determine when the current price of PepsiCo really offered a rate of

ANECDOTE: Trading Places and Citrus Futures
The 1983 hit comedy **Trading Places**, starring Dan Akroyd and Eddie Murphy, centers around the trading of Orange Juice Frozen Concentrate **Futures Contracts** (securities that promise delivery of oranges) on the **New York Futures Exchange**. If it is going to rain or there is a frost, oranges will be scarcer and the futures price will rise. You can learn more about futures contracts at the website of the **New York Mercantile Exchange** at www.nyce.com.

In a 1984 paper in the *American Economic Review*, Richard Roll found that these citrus futures contracts predict whether the U.S. Weather Service's forecast for central Florida temperatures is too high or too low. It is a great example of how financial markets help aggregate information better than the best non-financial institution. This should not be a surprise. After all, there is a lot of money at stake!

return of 20% (an expected payoff of $60). You could now draw one of two conclusions: first, the CAPM is not the correct model, and the market wanted to set the expected rate of return for PepsiCo at 20% in the first place; second, the stock market is not efficient. In a sense, the problem with market efficiency is that in many circumstances it is almost a matter of faith: if you wish to proclaim a belief in market efficiency, and if you then find that "prices are off," you can still always claim that your model for the appropriate expected returns in financial markets was wrong if you want to deny that the market was inefficient.

Useful?

Even though stock market efficiency is a fairly modest claim—at least as long as we remain vague on what the correct model of appropriate expected rates of return is—it is still a surprisingly useful concept. For example, it is pretty safe to say that no model of financial markets is likely to claim that investors can find great bets "+\$1 million with 99% probability" and "−\$1 with 1% probability." Such an expected return would be way out of line. Even expected rates of return of 100% per year are surely unreasonable for stocks such as PepsiCo. Few people doubt that the stock market is, to such a first approximation, efficient. Still, there is a large gray zone: we do not know the correct model of expected stock returns well enough to know if the stock market set the price of PepsiCo stock so as to offer an expected rate of return on PepsiCo of 10% a year or 12% a year.

Market Perfection and Market Efficiency.

Market efficiency is intimately related to our perfect markets concept from Chapter 6. It leans particularly heavily on the assumptions that there are no transaction costs. That is, even in the presence of some taxes and opinion differences, if it is just cheap enough to arbitrage mispricings, someone will end up doing so. Conversely, it is easier to believe that markets are *not* (or less) efficient if transaction costs are high. If it costs nothing to trade stocks, it would be easy for any investor to trade on any information that the market has not yet incorporated in the stock price—and thereby to earn an unusually good expected rate of return or even an arbitrage. However, the no-free-lunch axiom applies here, too. Low trading costs would make it less likely that you could expect to find violations of efficient markets. But if it is very expensive to trade and therefore if the market is not efficient and does not respond to news immediately, it would also be very difficult for you to take advantage of such inefficiencies. Of course, we also already know from Chapter 6 that if the market is not perfect, it is not even clear what "value" means. There would be a whole range of possible values for financial securities, both now and in the future. As we learned, no market is perfect or perfectly imperfect—market perfection is always a shade of gray. Thus, the range of possible valuations is determined by the extent to which the market is imperfect.

We can assume reasonably efficient markets for large corporate stocks.

In any case, modern financial markets for large corporate stocks and index funds in the United States seem very competitive. There are millions of buyers and sellers, transaction costs are low, and few investors know in advance whether the market will go up or down. It is difficult to believe that you or I could outsmart the prices in such markets. After all, thousands of other traders are likely equally as smart and would flock to good bargains and avoid bad bargains along with us. Of course, the smaller the firm, the less perfect and the less efficient the market in its stock is likely to be. Most stocks on Nasdaq trade only rarely, and can have large transaction costs. (Not only are the bid-ask spread and commissions often very high [which is the instant cost of a roundtrip transaction], but it may be particularly difficult and expensive to short these stocks, i.e., speculate that they will decline.) It is unlikely that these stocks will immediately and fully reflect all information appropriately. So, market efficiency is never white or black, but always a shade of gray—just as it is for perfect markets.

Noise Traders.

One conceptual question that vexed academics for a long time was how markets can be efficient to begin with. After all, if there is no money to be made, why would anyone bother collecting information on firms? And if no one bothers to collect information on firms, how can the market be efficient? Eventually, the resolution to this puzzle was that markets can never be 100% efficient—they can only be, say, "99%" efficient. In equilibrium, good information collectors should earn just about enough trading profits to break even on their costs of information collecting. They earn this money trading against **noise traders**, who do not collect information and who may trade for idiosyncratic reasons (e.g., to pay for a new car).

We will discuss consequences of market efficiency below, but one we can mention right away. The fact that large-firm stock markets are pretty efficient means that, by and large, you can trust these financial markets to get asset values about right—at least within the limits of the arbitrage transaction costs listed in the previous section—and to get it right *immediately*.

In an efficient market, the announcement stock reaction should be a good estimate of the change in NPV, because the market should accurately reflect value at all times.

Would you not rather face an inefficient market? If it were inefficient, you might be able to find some good bets (opportunities that earn unusually high expected rates of returns). But it would not all be gravy. In an inefficient market, you could not rely on market prices being fair—they could be inappropriately too high or too low. You would never really know whether you are overpaying or underpaying. Investing would be a very messy business. The advantage of efficient markets is that if you hold a portfolio of many large and liquid stocks, you do not have to spend a lot of time and money to perform **due diligence** in order to determine whether stocks are fairly priced. All you need to do is to make sure you are appropriately diversified to meet your risk-reward preference. You can probably accomplish this goal by purchasing just a few large index-mimicking portfolios.

The advantage of an efficient market.

15·2.B. Classifications Of Market Efficiency Beliefs

Almost all financial economists believe in basic market efficiency for large markets and liquid securities. No respectable economist believes that it is easy to get very rich trading on easily available information. Instead, the disagreement is, loosely, about whether stock markets are "99% efficient" or "97% efficient." The school of thought that proposes the 99% view is often called **Classical Finance** or **Rational Finance**; the school of thought that proposes the 97% view is often called **Behavioral Finance**. Of course, you can trade millions of dollars in large firm stocks or market indexes relatively easily and at low transaction costs. Thus, it does not require huge efficiency violations for behavioral finance economists to be right and for classical finance economists to be wrong. Exploiting just the tiny—say, 3%—violations from market efficiency could make you a star investor. (This is also not coincidentally why so many fund managers show great interest and publicly proclaim their faith in behavioral finance.) However, don't take me too literally here—the 99% vs. 97% is an analogy, and there is really a spectrum of beliefs in market efficiency among economists and fund managers. Let us now look at some such rough groupings, although you should realize that any classification schemes really identify just segments on a continuous line.

Financial markets are probably close to efficient.

15·2.C. The Fundamentals Based Classification

I like to grade financial economists into camps based on their degrees of belief in market efficiency:

My preferred taxonomy.

True Believer Financial prices always reflect the best estimate of net present value of all future cash flows. This means that stock prices should change only if news about fundamentals appears.

Firm Believer Financial prices may sometimes deviate from the appropriate best estimate of future cash flows. However, transaction costs make it practically impossible to find unusually good bets.

Mild Believer Financial prices occasionally deviate from the appropriate best estimate of future cash flows (and the financial price next period). When they do, the transaction costs are not high enough to prevent investors from obtaining unusually good bets, although the profitabilities of these bets generally remain within economically reasonable magnitudes—maybe a couple of percentage points a year.

Non Believer Financial prices regularly deviate from the appropriate value, and thereby allow investors to obtain arbitrage opportunities or incredibly great bets.

A firm believer need not be a true believer: financial price changes may indeed be unpredictable, but not because of news about fundamentals. (There could be unrelated noise in stock price changes, especially in the short-run.) Occasionally, we are even handed evidence that refutes the true believer—but only in certain specific situations. The most dramatic such example occurred in 2000, when the network company 3COM spun off the PDA company Palm. 3COM retained 95% of Palm's stock—and announced that each shareholder of 3COM would soon receive 1.525 shares of Palm. After the IPO, Palm closed at $95.06 per share. Therefore, 3COM should have been worth at least $145—instead, 3COM shares closed at $81.81. (It was impossible to exploit this discrepancy, because it was impossible to find Palm shares to short. I know—I tried.) A mild believer need not be a firm believer: transaction costs may be low enough to permit great trading strategies based on E-M violation. A non-believer need not be a mild believer: financial markets may just beg to be exploited.

Our best estimate is that we are somewhere between the mild and firm believer camps.

In this classification of market efficiency, virtually no academic is a non-believer, and only a very few remain in the true believer camp. Instead, most finance professors are somewhere between the "mild believer" camp (the center of behavioral finance) and the "firm believer" camp (the center of classical finance). The debates between the two more extreme side of these camps—the "rationalists" and "behavioralists"—is intellectually exciting. After all, bringing new evidence to bear on these disagreements is the process by which we learn more.

My opinion.

I personally am right in the middle between the two schools of thought; that is, I am right in the firm-to-mild camp. In my view, most investors believe that they have more knowledge and control than they actually have. This is why I believe that trading in the stock market seems so (inexplicably) active. In my opinion, investors believe that they can predict when stocks are going to go up or down. Some pundits like to call this **investor psychology**. However, I also believe that an individual investor is unlikely to be able to find rate of return patterns in the stock market to earn high excess returns. A very few sophisticated funds may be able to systematically earn a few extra basis points per year. But these funds are scarce. Even after decades of academic research to identify better performing funds, academics usually find that only about half of all funds outperform the market and half underperform the market even before fund transaction costs.

The low signal-to-noise ratio causes the dispute.

So why is this debate so tough to settle? The reason is that the **signal-to-noise ratio** in financial returns is low. The signal-to-noise description draws on an analogy from physics—the signal (the appropriate average price change that a smart fund manager could predict) is small compared to the noise (the day-to-day price **volatility** that clouds our senses). Here it means that a typical stock may have the signal of an expected rate of return of 0.05% per trading day (14% per year), but the noise of a typical standard deviation of 2-3% per trading day, which is about 50 times as high. This low signal-to-noise means it is difficult for researchers to determine whether a particular trading strategy has earned high returns [a] because it took on risk, and the researcher has just not recognized the risk appropriately; [b] because it had a lucky outcome, which will not repeat; or [c] because the market was inefficient. Although these choices allow us finance professors to continue to write papers to argue one side or the other, most finance professors now agree that when individuals earn an unusual amount of money in a day or a week, it is more likely due to luck than to ability. The burden of proof is on the side claiming superior ability—and a number of former finance professors have taken up the challenge

ANECDOTE: The Limits of Arbitrage in the Internet Bubble
Even in cases where it is probable that the market mispriced stocks, such as technology stocks during the famous "Internet bubble" at the turn of the millenium, it was almost impossible for an individual investor to take advantage of the market inefficiency. Believe me, I know. In 1999, I believed *Yahoo!* (YHOO) was worth less than what it was trading for. So, I speculated that its stock would go down. After I had lost more than three times my original investment, I realized that I had to either close my bet or risk personal bankruptcy. So, I terminated my bet, having lost a lot of money. Yes, I would have been right in the end and made a lot of money if I had held on longer, but I simply could not afford the risk (and mental anguish) any longer. I learned from this episode—after 15 years as a financial economist—that even if the stock market is irrational and even if it overvalues a stock by three times, it can also be irrational enough to overvalue it by yet another three times.

ANECDOTE: A Conversation with Eugene Fama
The book website has an impromptu email conversation between myself and Eugene Fama (perhaps the most famous finance professor alive and a strong defender of market efficiency) at welch.econ.brown.edu/academics/famaconversation.html. This will give you an authentic impression of the ongoing dialogue among finance professors.

and started their own funds.

IMPORTANT: *On a typical day, the typical stock moves up or down by about 10 to 100 times as much as it offers in expected rate of return. Therefore, it is not easy to attribute past observed stock price performance to investor ability or inability.*

DIGGING DEEPER:

Our example was about a model that states that a particular kind of stock should increase by 5 basis points, but has a 200 basis point volatility. If the noise is uncorrelated (which usually means returns on different days), how many trading days would we need to determine whether the true expected rate of return is 5 basis points?

If you have T days, your volatility will decrease with the square-root of T. With 10,000 trading days (about 40 years), the volatility would be roughly $\sqrt{10,000} \cdot 2\% \approx 2$ basis points. This level of volatility would allow you to determine whether the daily expected rate of return on your stocks is 1 basis point, 5 basis points, or 9 basis points, but not whether it is 4 basis points or 6 basis points. Of course, 4 basis points per day is a whopping 3% per year different from 5 basis points per day. To tell apart the difference between 4 basis points and 5 basis points, you would want no larger a volatility than about 0.5 basis points—requiring about 160,000 independent observations. We do not have these 600 years of return history, and even if we did, who would believe that these daily returns were still drawn from the same distribution? So, we usually have to work with "tricks"—primarily forming portfolios that have less than 200 basis points volatility on an average day, which leads to arguments about what proper portfolios for testing market efficiency are. Can you see now why testing for whether stock returns follow one or the other model is such a difficult and contentious task?

15·2.D. The Traditional Classification

The traditional classification of market efficiency.

In contrast to my definition of market efficiency above, which focuses on how rational market prices reflect underlying values, the more standard historical definition of market efficiency focuses on information. This distinction is between weak-form, semi-strong-form, and strong-form market efficiency.

- **Weak Market Efficiency** presumes that markets are efficient enough not to allow the use of historical stock price information to earn inappropriately high rates of return. This means that **technical analysis** (trading based solely on historical price patterns) would not earn excess returns. Put another way, the weak form assumes that all past prices of a stock are reflected in today's price so that technical analysis cannot be used to beat the market.
- **Semi-Strong Market Efficiency** presumes that markets are efficient enough not to allow the use of any publicly available information to earn inappropriately high rates of return. This means that **fundamental trading** (trading based on price and underlying firm information) would not earn excess returns. Put another way, the semi-strong form assumes that all public information is reflected in today's stock price, so that neither fundamental trading nor technical analysis can be used to beat the market.
- **Strong Market Efficiency** presumes that the market incorporates even the most private information held by the deepest insiders in corporations. This means that no trading would earn excess returns. Put another way, the strong form assumes that all information, both public and private, is reflected in today's stock price, so that nothing—not even insider information—can be used to beat the market.

In this classification of market efficiency, all finance professors believe that most large financial markets are not strong-form efficient: insider trading may be illegal, but it works. However, arguments rage on as to whether markets are semi-strong-form or even weak-form efficient, and even for large and liquid financial markets (such as the N.Y.S.E., Nasdaq, or the CBOE). Finance professors regularly publish papers that find new rules that seem to outperform reasonable average rates of return by a large margin. Some strategies seem to work, in particular some forms of **momentum** (buying stocks that have gone up, selling stocks that have gone down) and

value (buying boring old-economy stocks, selling glamour high-growth new-economy stocks). Such strategies can offer seeming "excess returns" as high as 1-2% per month. Unfortunately, many strategies disappear almost as quickly as they are discovered—and may have never been real to begin with. Yet other trading strategies require such high transaction costs that they end up not being profitable in the real world. That is, even though prices may not incorporate all information and the market may not be efficient, the inefficiency may be well within the bounds of transaction costs. Yet some other trading strategies seem to have worked and continue to work—but why and for how long? Personally, I am not claiming that none of these trading strategies works. I am just advising caution when real money is at stake.

Solve Now!

Q 15.2 *What does it mean for a stock market to be efficient?*

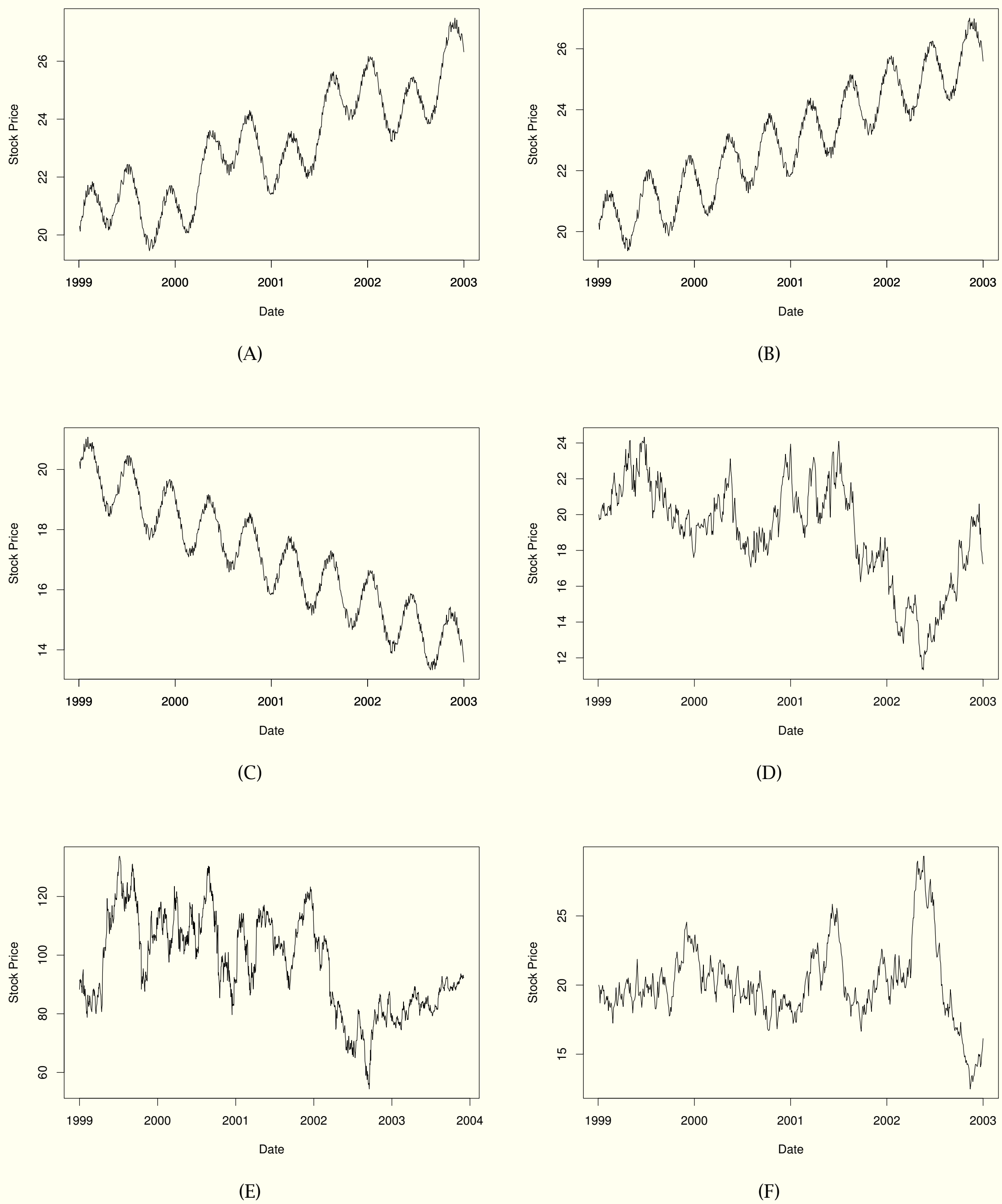
Q 15.3 *Is it more or less likely for a financial market to be efficient when transaction costs are low?*

15·3. Efficient Market Consequences

We already mentioned one consequence of efficient markets: You can trust prices and don't have to waste much time checking that they are appropriate. However, there are a number of other important consequences that deserve further expounding.

15·3.A. Stock Prices and Random Walks

Figure 15.1. Potential Stock Price Patterns



If these patterns were systematic, some of them should make you rich. Which ones?

Cycles in the market?

Here are some trick questions: Look at the various graphs in Figure 15.1. They seem to show what stock market patterns have looked like, do they not? Perhaps. Does it make sense to think that these are representative for the future? Absolutely not! Graphs A, B, and C display a strong regular cycling pattern. If this were representative for the future, you would quickly become a wealthy technical analyst! The patterns would suggest that you should purchase the stock only when it has "bottomed out"—a pattern that you can reasonably detect if you see a multi-month period of losses followed by about a quarter of stable returns. It need not be the kind of regular cycles in the figure: any good predictable patterns (such as "every time the price hits \$22, it drops by \$2") would allow you to get rich. Now, if you look hard enough, can you find some stocks in the real world that have behaved like these graphs? Yes—because with over 10,000 stocks currently trading, by pure chance, maybe one or two could show a pattern that would look remarkably similar to a cycle pattern. But, despite assurances from some stock analysts that you could have made money if you had just trusted their cycle patterns and that you should trust them henceforth, the patterns would *not* be representative of the future—they would just be historical coincidence.

Non cycles are more reasonable—though there are ups and downs, too.

On the other hand, Graphs D, E, and F could actually be representative. On average, each price in the next month is just a tiny bit higher than the previous (i.e., the expected rate of return on stocks is positive), but the important thing is that there is a lot of *noise*, up or down—which is by definition unpredictable. Stock prices must largely be unpredictable, or you could outsmart the stock market. Incidentally, one of these three graphs is a real stock price, while the other two are simulated. Can you see which one? I cannot! The real-world price series fits right in with my simulations of patternless day-to-day changes (called random walks and explained below). In fact, whenever we look at graphical representations of stock prices, they usually look very much like Graphs D-F and very unlike Graphs A-C. (Solution: Graph E is the actual stock price series of IBM.)

Can we predict stock prices? The order of magnitude of typical daily stock price changes is tremendous, but the expected price is not much different from today's price.

Let's look a bit more closely at magnitudes. May 31, 2002 was a decidedly uneventful day for the stock market. The Dow Jones rose 13.56 from 9,911.69 to 9,925.25, a change of 0.14%. On this day, the most actively traded stocks (but not biggest price movers) were MCI WorldCom (rate of return of −1%), Nasdaq100 (−2%), Palm (−30%), Sun (0%), and Oracle (+6%). So, let's put our statistical and financial expertise to good use and ask a fundamental question of Finance: *In a perfect market, if the shares of a company cost \$50 today, what do we expect them to cost tomorrow?* Put another way, could you reasonably expect someone to be able to have predicted this day's stock price movement, e.g., something on the order of ±1% (as was the return of MCI that day)? Think about it: if you could outpredict the stock price by an average of 1% (\$50.00 to \$50.50) on a typical day, you would be the world's most amazing stock picker. Just 1% per day represents an annual return of

$$1 + r_{0,365} = (1 + r_{0,1})^{365} \approx (1 + 1\%)^{365} = (1 + 3,678.34\%) \quad . \tag{15.1}$$

Too bad. Such magical abilities do not exist in the real world, where you can only expect to earn rates between 2% and 50% per year, depending on what risk you are willing to take. Let me put this in perspective: any fund manager who can consistently outperform her peers by 2% per year would be considered a star! Four percent per year makes a super star. In sum, we can conclude that we do not expect individuals to be able to predict returns by an amount of 1%/day, a typical daily stock movement—at least without divine guidance or its equivalent (inside information).

Can the price tomorrow be much lower, on average? No!

Let us return to MCI. It decreased by 1%. Could the expectation be for MCI to decrease by 1%? If the *expectation* were for MCI shares to trade for \$49.50 tomorrow, would you not want to sell the shares for \$50.00 today, instead? After one year, with such daily rates of return, you would be left with only 2.5% of your original investment. Every owner would rush to the market to sell, no one would want to buy, and the price would immediately be lower.

So, what can the price tomorrow be, on average?

These arguments suggest that we would expect very small daily rates of return, not much above 0.1%/day (with 255 trading days, $(1 + 0.1\%)^{255} \approx 29\%/\text{year}$), which is much less than the day-to-day noise in stock prices. Intuitively, this is what an efficient stock market is: we do not believe anyone can get rich easily, so it must be mostly impossible to predict where stocks are going, aside from the very small mean (call it m). The best expectation of the price tomorrow must be roughly the price today. Formally, if time 1 is very close, say just 1 day after time 0,

$$\begin{aligned} &\mathcal{E}(\tilde{P}_1) \approx (1+m)\cdot P_0 \quad \Leftrightarrow \quad \mathcal{E}(\tilde{P}_1) - P_0 \approx m \cdot P_0 \\ \Leftrightarrow\ &\frac{\mathcal{E}(\tilde{P}_1) - P_0}{P_0} \approx m \quad \Leftrightarrow \quad \mathcal{E}(\tilde{r}_{0,1}) \approx m \qquad , \end{aligned} \tag{15.2}$$

where P is the common notation for "price" and m is just a very small number (and determined by a model such as the CAPM). This particular process is just the aforementioned **random walk** (with drift). Thus, we can conclude that in the absence of easy ways to get rich, stock prices approximately follow a random walk, at least in the short run.

IMPORTANT: *In the financial market context, "random walk" refers to a process in which the expected value tomorrow is (almost) the same as the value today. Naturally, actual values tomorrow will most likely be different from the value today.*

The empirical evidence confirms this. Stock prices tend to follow roughly a random walk in the short run. This means that it is not easy to get rich.

Don't wag the tail.

It is important that you realize that a random walk is a necessary consequence of an efficient market, but you cannot conclude that a market is efficient just because prices follow roughly a random walk. In fact, it could be that the true value follows one random walk process for a long time and the market price follows another. Because market prices follow their own random walk, merging with the fundamental value based random walk only in the very long run, transaction costs would prevent you from getting rich, even though we know that market prices would not always be the best estimate of value, given all information.

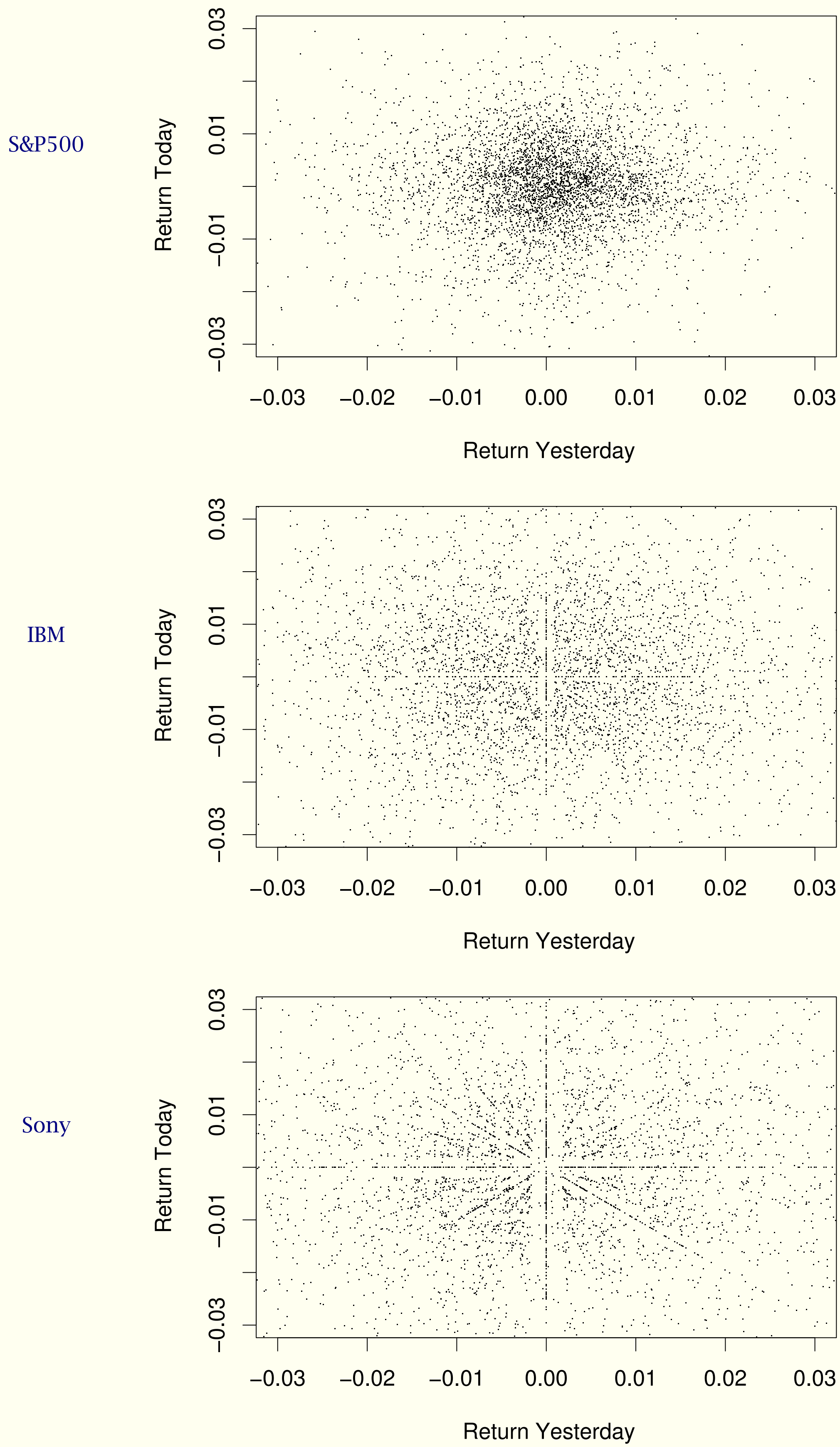
ANECDOTE: Great Mathematicians and Gambling: The Origin of the Random Walk
In the 1700s, it was not beneath mathematicians to study how to gamble in order to gamble better. Jacob Bernoulli (1654–1705) and Abraham DeMoivre (1667–1754) studied the random walk of a gambler's stake in fair games.

Later reinventions and applications of the random walk concept abound: Jan Ingenhausz (1730-1799), a physician and plant physiologist, placed charcoal powder on an alcohol film and observed that the grains moved randomly. The botanist Robert Brown (1773-1858) reported erratic dancing of small particles in fluids at rest. Albert Einstein (1879-1955) considered such fluids to be composed of discrete molecules, whose many collisions with a "Brownian particle" caused the particle to jump in random directions—a random walk. Einstein's analysis not only explained **Brownian motion**, which has itself become a building block of high tech finance nowadays, but also bolstered the case for the existence of atoms, which was not yet universally accepted. The first recorded use of the phrase "random walk" was by Lord Raleigh (1842-1919) in 1899. (Raleigh made a connection between diffusive heat flow and random scattering and showed that a one-dimensional random walk could provide an approximate solution to a parabolic differential equation.) The name is believed to have originated with the description of a drunk who stands on a ladder. The drunk can walk up or down and does so in a random fashion—just like stocks.

Fortunately, in 1900, Louis Bachelier introduced the random walk theory of financial market fluctuations (although Pearson introduced the term "random walk" only later, in 1905), finding that bond prices could diffuse in the same manner as heat. Unfortunately, this has only pointed out the obvious: it is not easy for an investor to outperform the market. The first rigorous and published investigation of the random walk hypothesis was done by Cowles, an eclectic investor and economist at Yale in the 1930s and 1940s.

Source: Mostly Michael F. Schlesinger, Office of Naval Research, Scienceweek.com, 2001.

Figure 15.2. The Relation between Lagged and Current Rates of Return for S&P500, IBM, and Sony.



Note: The figures chop off some outliers, especially the crash of 1987 and mini-crash of 1989, but even if they are

Predicting with past rates of return.

Traders have tried all sorts of strategies in their efforts to become rich. One such strategy is the aforementioned technical analysis, which tries to find patterns in historical stock prices. For example, it is a popular misperception that stocks that rise one day are more likely to fall back the next day. Figure 15.2 shows how the daily rate of return on three stocks (S&P500, IBM, and Sony) related to the previous day's rate of return from 1985 to 2003. The graphs show no pattern that would allow you to get rich quickly. There is definitely not much juice in trying to predict how a stock will perform tomorrow, given how it performed today. (A small reversal that we occasionally observe seems to be caused by the **bid-ask bounce**. This is because if the stock's closing price is a bid price, on average it will fall back the next day (when it will be either a bid or an ask price with roughly equal probability). If the stock's close price is an ask price, on average it will gain the next day.) Similar conclusions apply if we extend our use of historical price information beyond yesterday, although over longer horizons, it appears as if stocks tend to continue their pattern just a little bit. This is the aforementioned "momentum" effect and covered in an investments course.

It is also unlikely that one stock is expected to go up today, and another stock to go up tomorrow, and so on.

Another variation on attempts to find market efficiency violations tries to predict not which stocks systematically go up every day, but just which ones will for the next day: maybe it is possible to predict that one stock should go up today, and another tomorrow. But, if you reflect on this statement, you realize that you could shift your money from one stock to another to take advantage of different stocks on different days. Again, unless the expected daily returns are tiny, it would be too easy to get rich. And, again, with average rates of return being tiny compared to the ups and downs, a good stock market pick is more likely to come from a lucky or unlucky draw than from a systematic ability.

Transaction costs destroy most hope for high turnover strategies.

Of course, in the real world, there are financial transaction costs that would also prevent you from really exploiting misvaluations. You would have to pay money to the broker to buy the shares, and again to sell shares. (This is why financial markets are not exactly perfectly competitive, only approximately perfectly competitive.) Even small transaction costs can render trading strategies with very high turnover unprofitable. After all, even if the bid-ask spread is only 10 basis points, if incurred 255 trading days a year, you would only be left with $(1 - 0.1\%)^{255} = 77.5\%$ of your original investment. So, for a daily trading strategy to earn money, it needs to earn at least an annual rate of return of 25% before it can overcome the trading frictions—which seems almost hopelessly large to me.

Solve Now!

Q 15.4 *From memory, write down the formula for a random walk.*

Q 15.5 *If stocks follow a random walk, can the price tomorrow be different from the price today?*

Q 15.6 *What is the typical movement of a stock on an average day?*

Q 15.7 *What is the typical expected rate of return on a stock on an average day?*

Anecdote: Are women better investors than men?
Analyzing 35,000 households from 1991 to 1997, Terry Odean and Brad Barber found that men trade 45 percent more than women. Apparently men are too overconfident in their trading prowess. (Men also have a higher propensity to suffer from compulsive gambling disorders.) On average, men's investment rates of returns were lower than women's, by a little less than one percent per year. Much, but not all, of the women's better returns could be attributed to the higher transaction costs that the men incurred for transactions that did not gain them higher returns.

Despite strong evidence to the contrary, many investors still believe that stock prices do not follow random walks, as evidenced by the plethora of financial talk shows and investment news letters. It would be better for the general public to watch more sports and cooking shows and fewer investment shows—especially for males like myself!

15·3.B. Are Fund Managers Just Monkeys on Typewriters?

What about celebrity investors?

So, what about all the televised stock analysts who explain which stocks are undervalued and which stocks are overvalued? And what about the aforementioned **technical analysis**, the art of seeing shoulders, price barriers, etc., in historical prices and using them to forecast future prices? (You can try out your own technical analysis at Yahoo!*Finance*—look up any stock, and choose "Charts," then "Technical Analysis"; it is fun, but useless.) And what about famous investors such as Warren Buffett, George Soros, and many others?

Even top investors can have at most mild predictive ability. None can be expected to have even the ability to foresee systematically a 0.1% movement per day.

First, as already stated repeatedly, we are now talking about tiny deviations from the random walk. Even an ability to forecast by 0.01% (yes, 0.0001) better per day yields an annual rate of return that is 3.7% higher than it would otherwise be. So high a superior performance by a fund would be widely considered to be stellar performance. It is unlikely that anyone has good day-to-day predictive ability that are larger than transaction costs.

Pure chance means that some investors succeed many years in a row.

Second, there are about 10,000 **mutual funds** today, which invest money on their investors' behalf. How many of them are likely to outperform the stock market overall next year (at least before they collect fees)? If they have no ability, about 5,000. How many of these will outperform the year thereafter? About 2,500. Even if there is absolutely no ability, pure randomness will mean that about 10 funds will outperform the market every year for ten years in a row. What will happen to the funds that underperform several years in a row? They disappear quietly. What will happen to the funds that outperform several years in a row? They proudly announce their performances, advertise, boast, and collect more investments from outside investors. They are even better dressed, supported by larger "research teams," appear more "professional," and fly in executive jets. They are the ones that will be most visible. Indeed, if you made money ten years in a row in the stock market, would you not believe yourself that you have the ability to pick stocks? From an investor's or analyst's perspective, both the disappearance of funds that have performed poorly and the expansion of funds that have performed well, will make it appear as if managers with a track record who offer to invest your money indeed tend to have an ability to outperform—and even if they have absolutely no ability. This is called **survivorship bias.**

If you look for future performance, past performance may be your best guide, even if it is a very, very poor guide.

In truth, maybe there are some individuals who *can* pick stocks, but the evidence suggests that luck is far, far more important than ability. Whenever academics (or the *Wall Street Journal*) have searched for better forward-looking performance among professional fund managers who outperformed in the past, they have found little or no exceptional performance. For example, if managers were truly capable of systematically earning better rates of returns by picking stocks, you would expect those managers who have picked better in the past to also pick better in the future. The evidence is that about 54% of mutual funds that have outperformed their benchmark over the last 1–3 years tend to outperform their benchmark over the following 1–3 years. (This is better than 50%, but not by much. And if we subtract fund fees, the average performance drops significantly below 50%.) But, as fund prospectuses aptly note, and as the evidence suggests, for the most part, **past performance is no predictor of future performance.**

If there was superior fund performance, an investor could not earn money therefrom. It would be the fund managers who would earn the profits.

Even if the market were inefficient and even if some fund managers could in fact outperform the market, these fund managers would charge appropriately high fees to eliminate investors' advantages. After all, it is the fund manager who would have the scarce skill—picking stocks—and not the typical investor. Investors with money would compete to place money with such managers and accept higher and higher fund fees. In the end, it would be highly unlikely that uninformed investors could earn excess returns by investing in actively trading mutual funds.

> **IMPORTANT**: *In an efficient market, in which no one can pick stocks better than anybody else, a large number of investors will beat the market. A small number of investors will beat the market again and again. In the real world, there is little evidence that investors who did well picking stocks in the past are better picking stocks in the future when compared to investors who did poorly.*

Old-fashioned work and liquidity provision work better than stock picking.

There are, of course, other ways to make money: Warren Buffett's fund, **Berkshire-Hathaway**, for example, runs an insurance and aircraft business. These businesses make money. But it is money earned the old-fashioned way: through hard work and risk taking. Writing insurance is risky business, and deserves extra return.

Funds earn money on the upside—is this a good idea?

Here is my great business idea of the day. I give you stock tips, and I ask for money only if you make money. In fact, I only want 10 percent of your winnings. So "you have nothing to lose." I only get something if I help you make money. Sounds like a deal? Now, if I pick a stock randomly, I have a fifty-fifty chance of making money. If you gain, I get something. If you lose, I pay nothing. I am in effect arbitraging you! Maybe I should give you the advice to buy a stock, and your neighbor the advice to sell it. This way, I will surely make money! My only mistake is that I have told you my plan.

Many funds are compensated on the upside, but the alternative is not palatable, either.

My business model is not as absurd as it sounds. This is exactly how many funds operate: their managers participate in the upside, but not in the downside. (Of course, funds that charge not only when they make you money, but also when they lose you money are not particularly confidence-inspiring, either. What are their incentives?) So, next time someone gives you a great stock tip, regard it with some skepticism: it probably has a fifty-fifty chance of being right.

Solve Now!

Q 15.8 *If a firm employs 10,000 analysts, how many of them are likely to issue forecasts that beat the market ten years in a row if none of them has any ability and there are no transaction costs?*

ANECDOTE: The Three Top Investment Books of 1996

The three best-selling investment books of spring 1996 were David and Tom Gardner's **Motley Fool Investment Guide**, based on a popular investment web site; Matt Seto's **The Whiz Kid of Wall Street's Investment Guide** (Matt Seto was 17 years of age at the time); and the **Beardstown Ladies' Common-Sense Investment Guide**, authored by septuagenarians whose first book mixed cooking recipes with investment advice. All touted "common sense methods" to beat the market, earning 30 percent per year or more. Not a week went by without dozens of prominent radio and TV shows featuring their sound advice. What did I need my Ph.D. in finance for? It is difficult to argue with performance!

Naturally, best-selling books are a great business. However, the stock performance of these three experts was not.

1. From 1996 to 2002, the *Motley Fool* investment recommendations of a number of hypothetical portfolios have been discontinued. In 1997, they launched a real-money portfolio, called DRIP. From 7/28/1997 to 7/31/2002, it lost about 10%, while the S&P500 lost 2.5% and Nasdaq lost 15%. One should not judge a fund by just 5 years of performance (and certainly not without risk adjustment), but it does appear that the Motley Fool has not exactly found the Holy Grail of investment opportunities.
2. Matt Seto has stopped publishing his investment performance and has decided to pursue a career as a student.
3. The Beardstown Ladies, five books richer, were found to have miscalculated their returns: their returns were not 30 percent, but 9 percent—significantly lower than the 15 percent turned in by the S&P500 stock market index during their investment period.

How disappointing: on average, about one of them should have continued beating the market, one should have done about the same as the market, and one should have underperformed it. Now, where are my five minutes of fame?

Source: Time Magazine.

15·3.C. Corporate Consequences

If the Market is Efficient

You can learn from your own market price!

If markets are efficient, then managers can obtain valuable information from their own market prices. The market price is the conglomerate assessment of many investors, who put their money where their mouths are. It aggregates a whole lot of information that managers themselves may not see so easily. If the stock price is very high, it probably means that the market sees great opportunities ahead for the firm. Thus, managers should consider growing the business. Naturally, a high firm value typically will also allow them to raise more funds from the financial markets at favorable rates. On the other hand, if the stock price is very low, it probably means that the financial market anticipates the business to go down and managers to waste the remaining money. In this case, managers should think carefully about whether they should reinvest their money into the business, or into repurchasing their (relatively cheap) stock.

You can learn from other market prices.

In addition to learning from your own company's market price, you can also learn from all sorts of other market prices. You can find out how good your competitor's opportunities are, and whether you should get into the fray. Commodity prices are also often very helpful. If the price of oil in the market is $30/barrel, it probably does not make sense for a firm to plan ahead based on an oil price of $50/barrel. The market price for oil is indeed fairly efficient. A friend of mine sat on the corporate board for a large multinational oil company when the oil price was $13/barrel, and the CEO argued that the firm should plan oil exploration for a target oil price of $20/barrel—the oil price "just had to go up." Not only did this show tremendous hubris, it was also outright stupid. The company could purchase oil in the market at rates of $13/barrel, and thus did not have to do any oil exploration that cost between $13/barrel and $20/barrel. Indeed, if this CEO could predict where the oil price was going, he could make a lot more money as an oil trader than as the CEO of the oil company! Why explore for oil if you can buy oil cheaper in the market?

Adding value cannot be done superficially.

An efficient market also means that it should be impossible to generate value by doing something that investors can do themselves. For example, buying another company to diversify does not add value—investors could buy shares in the target by themselves and thus be themselves diversified. They do not need our firm to recognize that the target is undervalued—in fact, chances are, the target was rightly valued to begin with and it was us who got the target value wrong. In order for us to profitably take over a target, we must have something extra that investors cannot do for themselves—synergies, e.g., in the distribution of product or allocation of overhead.

Fooling investors cannot be done easily.

Market efficiency also means that it should not be easy to fool investors. For example, firms can split their shares—each share trading at $80 would thereby become two shares trading at $40. Nothing fundamental about the underlying project would have changed. If the market is efficient, investors would not believe these new shares to be worth more than $40/share. After all, just renaming shares should add no real value to the projects. A similar argument applies when managers change earnings in ways that investors can see through. For example, if a firm previously reported a foreign division's earnings separately, and now consolidates them into the main earnings, such a change would increase the firm's consolidated earnings on paper, but it would not create anything intrinsically valuable. Such changes should not add or subtract firm value.The same argument applies to dividends. In the absence of taxes or other complications, a $100 firm that pays $10 in dividends should be worth $90 thereafter—no value is magically created or destroyed. However, although these arguments are theoretically appealing, do not believe this too literally. Just because it *should* be this way does not mean that it *is* this way. There is some empirical evidence that paying out money to shareholders and meeting earnings expectations may indeed raise firm value. After all, markets are not perfectly efficient!

If the Market is Not Efficient

What to do if markets are not efficient?

Losely speaking, financial markets tend to be fairly, but not always perfectly efficient. Strong market efficiency is almost surely *not* a good description of reality. Even in a perfectly rational market, executives may know the firm value better than the market—for example, they may know that the company is about to seal a large contract, but has not disclosed it yet. What should an executive do if she knows that the stock price is not equal to the appropriate market value? (Of course, most executives *believe* that the financial markets do not fully reflect the value of their companies even if they have no inside information—as an executive, you should be wary of your own perceptions and biases!) The right way to conceptualize your problem is to consider what you would do if you are the primary owner of most shares, so you really care about firm value. In addition, as an executive, you are supposed to maximize the value of all your shareholders.

If your shares are undervalued, you should recognize that your cost of capital is effectively too high, given the true characteristics of your project. The reason is that you cannot raise risky capital at fair prices—especially equity capital. The CAPM clearly is no longer the right model for the cost of capital.

Assume that you know that your current projects will return \$500 tomorrow. Also assume that you have no cash, and that you can only raise financing through equity. Now assume you come across a new project that costs \$100 and will return a terrific \$200 tomorrow. The problem is that your investors do not believe that the firm will return \$700, but falsely believe that the combined firm will only be worth, say, \$200. Thus, to raise \$100, you would have to sell 50% of your firm, and keep only 50% of the true \$700 return, for a true \$350 share of it. You would therefore be better off passing up this new project and just taking the \$500 from the old project. Put differently, the opportunity cost of new capital to fund this project is way too high for you.

You would definitely not want to raise cash at these high prices. Instead, you would want to do the opposite. The best use of corporate cash may now be to repurchase cheap, underpriced shares, e.g., from other investors. However, there is an intrinsic paradox here: as an executive, you are supposed to act on behalf of your shareholders. Therefore, repurchasing underpriced shares from them at bargain prices would not be what would make the selling shareholders better off.

Shares are overvalued. Now your cost of capital would be very low, so you should be tempted to take more projects. This is easiest to see if you again consider what you would do if you were the primary owner of this overpriced firm. You would want to sell more equity shares at higher prices, and pay the money out in dividends to existing shareholders. (Alternatively, you can just invest in Treasury securities.) Here the paradox is of course that just one instant later, as CEO, you are now the representative of these new shareholders that you have just sold overpriced shares to.

You can see that if the firm is misvalued there are no easy recommendations for a CEO acting on behalf of shareholders. The robust insight, however, is that the CEO of an undervalued firm should assume a relatively low cost of capital, the CEO of an overvalued firm should assume a relatively high cost of capital.

15·3.D. Event Studies Can Measure Instant Value Impacts

Market reactions should be immediate and reflect all value changes.

The immediacy of reaction in an efficient market offers a surprising application: price reactions can allow us to estimate value consequences, using a technique called an **event study**. The idea of an event study is that if the public market is valuing projects appropriately, and if the stock price increases by \$1,000,000 on the minute when the firm first announces the event, then the value of the new project is likely to be worth about \$1,000,000.

Example: The Value Impact of FDA Drug Rejections

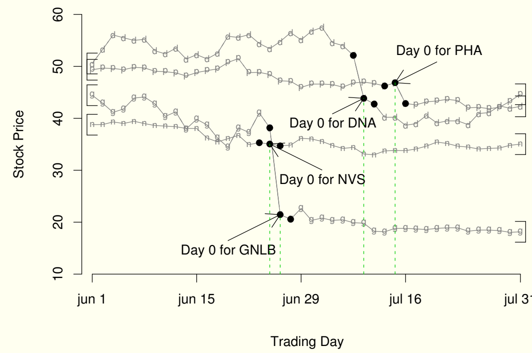
An Example: Estimating the value loss when the FDA rejects a drug application.

For example, say you wanted to know what the value implication of the rejection of a novel drug application by the Food and Drug Administration (F.D.A.) is. You could compute the sudden decline in future expected cash flows, discount it properly, and come up with an appropriate valuation estimate. This is not an easy task. But if stock markets value pharmaceutical stocks appropriately, the stock price reaction to the announcement of the F.D.A. drug application rejection would be a good indicator of the value loss. After all, if the stock market did not react *immediately*, you could on average earn great profits by shorting the relevant stock and waiting for the market to catch up.

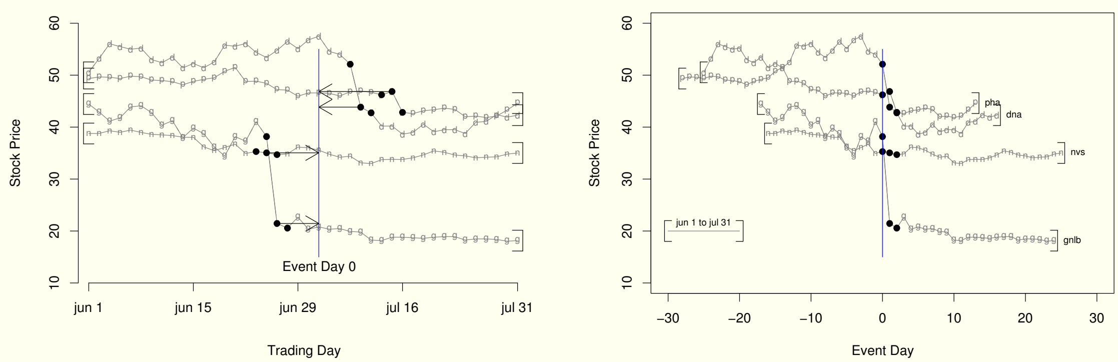
Figure 15.3. Event Study: Lining up Event Date Information

Firm	Date	Event
DNA	7/9/2001	FDA rejects Xolair
GNLB	6/26/2001	FDA rejects Aslera
NVS	6/22/2001	FDA rejects Zelnorm
PHA	7/12/2001	FDA rejects Parecoxib

(A)



(B)



(C) (D)

(GNLB's stock price was scaled by a factor of 11 to fit better into this graph.)

Figure 15.3 starts with four events that we have identified: the FDA rejected Xolair, a Genentech (ticker DNA) drug, on July 9, 2001; Aslera, a Genelabs (ticker GNLB) drug, on June 26; Zelnorm, a Novartis (NVS) drug, on June 22; and Parecoxib, a Pharmacia (PHA) drug on July 12. Graph B plots the price history for these four stocks during June and July 2001. The event day itself is marked in this graph. To find out what happens when the FDA rejects a drug, we need to line up all the returns in event time, as illustrated in graphs C and D. When we compute the rate of return over the three days around our event date (for we are not sure how accurate our identification of the announcement day is, and whether announcements happen before opening or after closing of the stock market), we discover the following:

Firm	Date	Event	"Couple of Days" Return
DNA	7/9/2001	FDA rejects Xolair	dropped about 18%
GNLB	6/26/2001	FDA rejects Aslera	dropped about 46%
NVS	6/22/2001	FDA rejects Zelnorm	stayed put
PHA	7/12/2001	FDA rejects Parecoxib	dropped about 7%
		Our conclusion	FDA rejections are not always, but often, bad news.

Having lined up everything in event time, we can do more analysis. For example, we can subtract the rate of return on the market on each event date to eliminate noise induced by the general movement on the event date. We can compute the average rate of return on the event, which seems to be around –18%—getting rejected by the FDA is not a good thing for a firm's market value. If you know more statistics, you can compute whether this value drop is "statistically significant." (It is.) You can investigate if bigger firms experience a bigger or a smaller drop. For this, you need to locate the market value at the time:

Firm	Event Return	Equity Market Value
GNLB	−46%	\$0.1 billion
NVS	±0%	\$2.7 billion
DNA	−18%	\$26 billion
PHA	−7%	\$58 billion

Therefore, the evidence suggests that smaller firms are harder hit, but that the relationship is not necessarily perfectly monotonic. It would be even more interesting if we knew what fraction of the drug development portfolio the particular rejected drug would constitute—we could then determine whether drug developers whose main portfolio drug was rejected suffer more. We do not have these data, so no such test! We could test whether it has become worse or better over time to have one's drug rejected by the FDA by sorting our events by the event day.

Firm	Event Return	Event Day
NVS	±0%	6/22/2001
GNLB	−46%	6/26/2001
DNA	−18%	7/9/2001
PHA	−7%	7/12/2001

There does not seem to be a clear relationship here—as any sane analyst would have suspected—so the evidence does not suggest that the market looked any more favorably or less favorably upon rejections in June 2001 relative to July 2001. We could investigate returns before or after the event announcement. If information leaks prior to the FDA announcement, we should see a drop even before event day 0. An alternative, though very unlikely, reason for such a pre-announcement price pattern would be if the FDA were more likely to reject a drug if the stock price had recently gone down. We could also investigate whether we can earn profits buying or selling after the event—under market efficiency, this should not be the case. (In a larger sample in Figure 15.4 below, we will examine pre- and post-announcement returns.)

Digging Deeper: *If we had more statistical background, instead of just sorting our events as we did in the in-text tables above, we should run a regression to predict the announcement rate of return with variables we deem to be important determinants of the value change upon FDA rejections.*

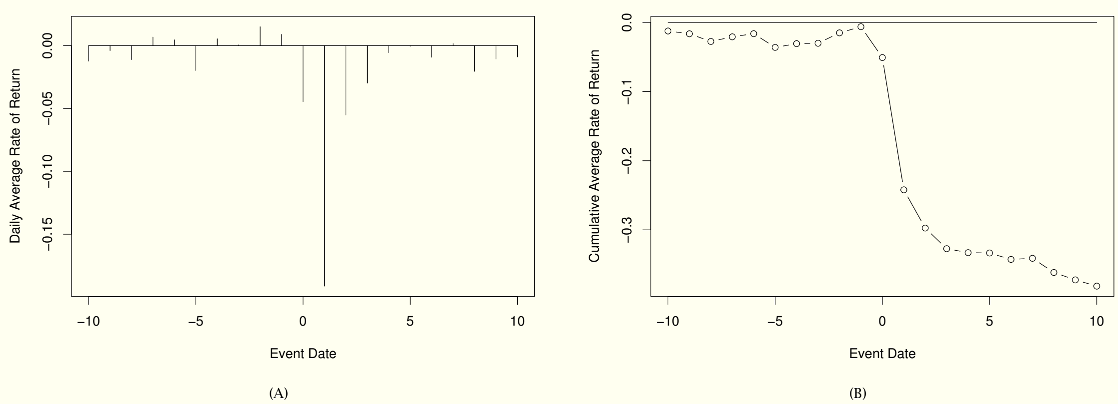
Table 15.1. Sample Event Study of FDA Drug Rejections

	Event		Announcement	Rate of Return		
Symbol	Company	Drug	Day-Day	Stock	Market	Net
AVE	Aventis	Refludan	5/17/00-5/19/00	2.87	−2.82	**+5.69**
AVN	Avanir	Docosanol	12/22/98-12/24/98	−65.56	0.19	**−65.75**
CEPH	Cephalon	Myotrophin	5/7/97-5/9/97	−35.00	1.12	**−36.12**
CRXA	Corixa	Bexxar	3/12/02-3/14/02	−38.32	1.08	**−39.40**
DNA	Genentech	Xolair	7/9/01-7/11/01	−17.96	1.55	**−19.51**
FRX	Forest Labs	Synapton	11/17/98-11/19/98	−3.79	1.17	**−4.96**
GILD	Gilead Sciences	Adefovir	11/01/99-11/3/99	−21.08	0.01	**−21.09**
GNLB	Genelabs	Aslera	6/26/01-6/28/01	−46.11	0.78	**−46.89**
GSK	Glaxo-Smith-Kline	Augmentin	12/14/01-12/18/01	−0.15	1.77	**−1.92**
IMCL	imClone	Erbitux	12/27/01-12/31/01	−20.31	0.08	**−20.39**
JNJ	Johnson& Johnson	Risperdal Consta	6/28/02-7/2/02	−1.24	4.20	**−5.44**
MAXM	Maxim	Maxamine	12/12/00-12/14/00	−44.78	2.20	**−46.98**
NVS	Novartis	Zelnorm	6/22/01-6/26/01	0.35	−0.70	**+1.05**
PFE	Pfizer	Zeldox	6/18/98-6/22/98	−2.51	0.29	**−2.80**
PHA	Pharmacia	Parecoxib	7/12/01-7/16/01	−7.32	0.47	**−7.79**
PRCS	Praecis	Plenaxis	6/11/01-6/13/01	−31.74	1.02	**−32.76**
SCIO	Scios	Natrecor	4/27/99-4/29/99	−57.67	1.47	**−59.14**
SEPR	Sepracor	Soltara	3/6/02-3/8/02	−58.73	0.13	**−58.86**
VPHM	ViroPharma	Picovir	5/9/02-5/13/02	−28.94	0.14	**−29.08**
WPI	Watson	Estradiol/ Progestin	10/30/00-11/1/00	−5.52	1.61	**−7.13**
					Average	−24.96%
					Standard Deviation	±22.31%

To eliminate noise, average many events from many firms.

We already know that individual stocks do move around day by day. So four firms may not be representative enough, given typical stock return noise. Instead, we should average the event returns for many FDA rejections, hoping that the noise averages away and the signal becomes more apparent. (Having more firms is like tuning a radio station with a more powerful antenna.) Table 15.1 presents the same event study—to estimate the value consequences of FDA rejections—for 20 pharmaceutical firms. It is usually a good idea to reduce the influence of overall market returns, so we subtract the overall stock market return on the same day from each firm's stock return. After all, we do not want to attribute the fact that the stock market declined by, e.g., 1% on the announcement day, to the drug announcement. (You also have to hope that there are no other major corporate events that *always* occur on the same day as the FDA drug application rejection.) The larger sample event study in Table 15.1 suggests that when firms announce that their drug applications are rejected by the FDA, they lose on average about 25%, plus or minus 22%.

Figure 15.4. Average Returns and Cumulative Average Returns in Event Time for 20 FDA Rejections



Whereas Table 15.1 gives only the average rate of return on the three days of the announcement at the bottom of the table, Figure 15.4 gives the average rate of return on each event day. Graph A indicates that we most likely misidentified the timing of the announcement day: if the announcement happened after market closing, then we only see a market reaction on day +1. On days +2 and +3, however, there appears to be a slight market imperfection: the stocks seem to go down further, by a total of about 5%—if this pattern is systematic, it is not only a violation of market efficiency, but it would be enough to earn us a lot of money! Graph B plots the cumulative return—it adds the return to a running sum. It shows that not much happens before the announcement, so there is no evidence of widespread news leakage or insider trading prior to our event, but there is some negative trend post announcement. It appears as if the market "underreacted." This would be worth further investigation, e.g., to see if it holds up in bigger samples and if it is more pronounced for certain, identifiable firms—except that this chapter has now told you (and the investing public) about a possible slow reaction here and a possible money-making opportunity now. If the slow market reaction with its profit opportunity was a real effect, it is likely that it will now disappear as traders will try to exploit it.

Event Study Graphs—perhaps we can make money?!

Event studies are not without drawbacks. There are three significant issues to deal with.

Event study are not a panacea.

Event Importance Event studies work well only if the event is significant enough to influence the overall stock market valuation: if a \$1 billion stock fluctuates on average by \$10 million a day, it is practically impossible to use an event study to determine the value of a project worth \$100,000. To use our physics analogy, the noise would drown out the signal. A reasonable rule of thumb is to take the ratio of the typical daily stock market value fluctuation (here, \$10 million) divided by the order of magnitude of the value consequence (here, \$100,000, so the ratio is $\$10,000,000/\$100,000 = 100$), and then require 50 times as many event observations as this ratio. For the example, this would require 5,000 event observations—which is likely too many to make such a study feasible for all but the most frequent events.

Big problem 1 is that we need to have enough event occurences.

Event Anticipation Event studies rely on the fact that stock markets react only to news, i.e., the unanticipated component of an information release. There must be a clear event date. But many events are anticipated, announced over a period of time, or never formally announced. For example, if a company was expected with 80% probability to win a contract worth \$1,000,000, the stock price would have already reflected \$800,000. The news that the company actually won the contract would raise the stock price only by \$200,000, not by \$1,000,000. The news that the company would not have won the contract would drop the stock price by \$800,000, however. Isolating market expectations can be very difficult. More than likely, the analyst would not know after the fact how expected the event was

Problem 2 is that we need to know exactly when news comes out—we want only the unanticipated information.

by the market at the time. (And, worse: insider trading before the event may have already moved the stock price to the $1,000,000 before the public announcement.) Therefore, in many cases, the event study technique is better at helping to determine whether an event is good or bad for a company (e.g., the announcement of a new law), than it is in helping to compute an exact value gain.

Problem 3 is that there are often simultaneous events.

Simultaneous Events (Contamination) The event study technique relies on the fact that the event can be precisely isolated from other events. If other events occur in the same time window, any value consequence may stem from these other events, not from the event that is examined. Unfortunately, many events occur at the same time. For example, at the annual meetings, we may get simultaneous announcements of dividend changes, corporate charter changes, institutional votes, information about successions, tough questions from shareholders, etc. So there is always the danger that what our study may attribute to dividend changes really is due to simultaneous announcements of corporate charter changes, instead. We can only hope that the content in these other simultaneous value events is non-systematic, so that it only adds noise that will average out over many different firms.

Event studies work even if the CAPM does not.

Nevertheless, event studies are a very powerful tool to measure the value effects of many changes. Our usual problem of not trusting the CAPM matters little when it comes to a one- to three-day event, because the average CAPM return is only around five basis points for a stock per day. Whether the true expected rate of return is closer four or six basis points is really irrelevant. Such small differences in mean expected returns are hopefully small compared with the signal that we expect from our event.

Short Preview: Other Event Study Results

Event studies have been used on many different events. We will rely on event studies in later chapters.

There have been event studies on all sorts of events, ranging from new legislation, to corporate name changes, to analysts' opinions, to corporate earnings, to stock splits, to corporate dividends, to corporate debt and equity issuance and retirement, to deaths of the founder, etc. Here are some of the findings. Firm values increase on average

- when firms announce increases in dividends, share repurchases, or stock splits
- when firms are taken over by other firms
- when the founding CEO dies.

Conversely, firm values decrease on average

- when firms announce new stock sales
- when they overpay for other firms in acquisitions
- when they announce bad earnings
- when they fend off an acquirer who has made a bid.

Also, we know that certain legislation can systematically have a positive or negative impact on firms, and this value impact can be measured. For example, it is possible to determine which firms were helped and which firms were hurt when telecommunication and airline markets were deregulated. For another example, we know that when the U.S. Congress imposed banking and tax-related sanctions on firms doing business with South Africa's Apartheid regime, there was again little effect on these firms. Despite the boycott's positive moral effect, it was largely not effective in economic terms.

SIDE NOTE: We may wish sanctions on South Africa's Apartheid regime had been effective, but the evidence is clear that they were not—possibly because there were too many loopholes to evade the boycott. Of course, sanctions may still be appropriate on moral grounds regardless of their economic effectiveness. Whether to boycott socially objectionable behavior is a decision that policy makers should make, not economists. The role of the financial economist is only to inform policy makers of the ultimate effectiveness of their actions.

We shall take a closer look at the capital structure event study evidence (dividends, and debt and equity issuing effects) in Chapter 21.

Solve Now!

Q 15.9 *Which of the following are good candidates for ascertaining the value effects with an event study, and why:*

(a) An acquirer wants to buy the firm.

(b) The CEO dies.

(c) The CEO ages.

(d) Positive earnings surprise at the annual meetings.

(e) Purchase of a new machine.

(f) A law is passed to force the company to reduce its emissions.

(g) An ad campaign.

Q 15.10 *What kind of response ("unusual" stock price change and "unusual" rate of return) would you expect when the company announces that it has struck oil and plans to pay it out next month? What reaction do you expect over this month? What reaction do you expect on the day when it pays the dividends?*

Q 15.11 *What are the factors that make an event study more likely to be more or less informative?*

15·4. Summary

The chapter covered the following major points:

- Arbitrage is a riskless bet with no negative outflows under any circumstances. Anyone would like to take an arbitrage opportunity. When and if they appear, they are likely to be very small.
- A great bet can be risky—although it can be very profitable. If not too risk-averse, an individual may prefer a large, great bet to a tiny arbitrage opportunity. Like arbitrage opportunities, great bets are very rare.
- Market efficiency simply means that the market uses all available information in setting prices to offer "appropriate rates of return."
- In the short run, the appropriate expected rate of return on stocks must be small. Therefore, market efficiency prescribes that stocks roughly follow random walks.
- In the long run, it is rarely clear what this "appropriate rate of return" should be. Because noise makes it difficult to measure the average rate of return, it is very difficult to either test models like the CAPM or to test long-run market efficiency.
- Beliefs in efficient markets come in different flavors.

  A more current E-M classification emphasizes the rationality of the stock market: true believer (stock prices always reflect underlying project NPVs), firm believer (small deviations, but difficult to take advantage of), mild believer (small deviations, and somewhat possible to take advantage of), or non believer (arbitrage opportunities and great bets abound).

  The standard E-M classification emphasizes what information it would take to beat the market: weak form (just past stock price patterns), semi-strong form (other historical information), and strong form (inside information).
- The overall evidence suggests that it is not easy to become rich—a belief shared by most finance professors. The relative strength of their beliefs in market efficiency—the extent to which professors believe that market prices always reflect underlying value—separates finance professors into "rationalists" (or "classical" economists) and "behavioralists."
- Given the millions of investors, many will beat the stock market by chance, and some investors will beat the stock market many years in a row. Market efficiency does not mean that there are not some investors who will beat the stock market ten years in a row *ex-post*, only that any one particular investor is unlikely to beat the stock market *ex-ante* ten years in a row.
- Managers can learn valuable information from market prices, both from their own share price and from other prices. To improve corporate firm value, managers must create fundamental value—they must undertake positive NPV projects. Simple activities such as purchasing a random firm to lower risk or splitting shares will not add value.
- Event studies allow us to ascertain the corporate value impact of sharp events, such as legislative action (FDA rulings) or corporate events (dividend increases).

Solutions and Exercises

1. If the arbitrage opportunity can only be done once and gains \$10, it is probably worse than a good bet that loses 1 cent one percent of the time, and gains \$1,000,000 ninety-nice percent of the time.

2. The market uses all available information in the setting of its price.

3. More likely.

4. See Formula 15.2.

5. The price can definitely and most likely will be different. Only the *expected* price is the same as the price today.

6. The typical movement (variation) is around plus or minus 1% to 3% a day. The average rate of return is much lower.

7. If a stock has an expected rate of return of 20% per year—which is definitely on the high side for most firms—the daily rate of return would be

$$(1 + 20\%)^{1/365} - 1 \approx 0.05\%.$$

8. If each of them has a chance of 50-50 in any given year, then the answer is $10,000/2^{10} \approx 10$.

9.
   (a) An acquirer wants to buy the firm: Super. Usually unannounced and big event.

   (b) The CEO dies: Maybe. Depends on suddenness (anticipation) and replace-ability of CEO.

   (c) The CEO ages: Bad. No sudden information release. Value effect not big enough.

   (d) Positive earnings surprise at the annual meetings: Maybe. Problem is many other things may happen at the same time.

   (e) Purchase of a new machine: Probably Bad. Problem is that one machine is usually too small to make a big value difference.

   (f) A law is passed to force the company to reduce its emissions: Maybe. The value consequences could be large enough, but by the time the law passes, it has long since been anticipated.

   (g) An ad campaign: Bad. First, there is no unique date on which to pin down the information release, and the value effects are often not too overwhelming, either.

10. The share price response would immediately be positive. Over the following month, you would not expect any unusual upward or downward drift: it should be about zero. Finally, when the firm pays out the special dividend, the rate of return should be zero on average, too, although its share price will have to drop by the amount of dividend paid to keep the return around zero.

11. The effect should be big, unanticipated, and there should be many other companies that have already experienced similar events in the past.

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

Part IV

Financing Choices and Capital Structure

"It's not begging if you've been to college.
It's attempting to arrange interim financing
while restructuring your debt load."

Warning: This Part IV of the book has not yet undergone even one edit cycle. It is therefore to be "enjoyed" with caution only.

Transition

We are still plagued by one problem: Although we know how firms should value projects, and although we know the determinants of our cost of capital, we do not yet know how we can best get new investors to part with their cash. We just assumed that if our firm had the right project (with a positive NPV), then the funds would come by themselves.

In the real world, this may not be the case, but our firm does have many venues for raising funds. We now discuss both the types of securities that our firm can sell to potential investors and the selling process itself.

What You Want to Accomplish in this Part

The goal of this part of the book is to learn how firms should finance projects with debt and equity, and how this influences the firm's cost of capital.

- Chapter 23 explains how you should think of securities that firms sell (issue), and how these securities are sold into the financial markets.

  Typical questions: What rights do stock investors have, e.g., if the underlying firm goes bankrupt? What is preferred stock?

- Chapter 17 shows that *if everything is perfect*, then the value of the firm is the value of its underlying assets and not dependent on whether it is financed with debt or equity.

  Typical questions: How does the value and expected rate of return of the firm's equities change if the firm issues a large bond in order to buy back stock?

- Chapter 18 shows how firms should make capital structure and capital budgeting decisions if they have to pay corporate income taxes.

  Typical questions: What is the firm's cost of capital and value if it finances itself with 50% debt and 50% owner's equity, instead of with 100% owner's equity.

- Chapter 19 explains other considerations that can have important influences on capital structure.

  Typical questions: Should a fast growing firm finance itself with more or less debt than a profitable value firm? If investors are concerned that managers like to raise funds primarily when they know that their current projects are not very good, then what can managers do to calm investor fears in order to be able to raise more money?

- Chapter 24 focuses on corporate governance in more detail.

  Typical questions: If investors are concerned that managers like to steal or waste money, what can managers do (and what do they do) to calm investor fears?

CHAPTER 16

Corporate Financial Claims

Who owns What?

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After the firm has determined what projects to take, the question is how they should be financed. This is called the capital structure of the firm. You have already encountered the two basic financing choices that firms have: They can accept new partners, which is called issuing equity (stock). Or they can borrow money, which is called issuing leverage (bonds). There are also many other financial claims, most of which are hybrids of these two basic choices.

This chapter describes some of the choices that corporations have, and how to think of them. The next chapters will explain how to choose among them.

16·1. The Basic Building Blocks

Bonds and Stocks are the basic building blocks. Homes and Corporations are really the same.

The capital structure of a firm consists of the financial claims on its future payoffs. Claims have two aspects to them:

Cash Flow Rights, which describe how future project payoffs will be allocated; and

Control Rights, which allow the claim owners to enforce their cash flow rights.

When claims are issued by a publicly traded company, they are called **securities**, because they are registered with the Securities and Exchange Commission (**SEC**). However, the term is now used much more liberally, e.g., for foreign securities and privately placed securities, which really are not securities in the original sense of the term.

Debt and Equity. Owning the firm really requires owning <u>all</u> claims without further obligations.

The two basic building blocks of capital structure are also the most common financial claims: **bonds** (**leverage**) and **stocks** (**equity**). Just as our buyer from Chapter 5 financed a house purchase with a mortgage and kept the rest as levered equity, so do firms issue bonds and stocks. By definition, you own the firm only if you own *all claims* that the firm has issued. It is not enough for you to own only *all stock.* If you are the levered equity homeowner, you can make a lot of decisions about the house, but you really do not fully own the house, because you cannot dispose of the house and keep the money without repaying the mortgage. It is you plus the mortgage company that together own the house. Similarly, the levered equity holders in a corporation make almost all decisions, but they do not fully own it. The value of the firm is by definition the sum-total value of all its outstanding claims.

16·1.A. Bonds

Bonds are Loans, with specified obligations by the company to its creditors.

In Part I, we have already worked extensively with bonds, though not necessarily corporate bonds. Bonds are just loans that promise specific payoffs at specific times in the future. The borrower (or issuer) receives cash upfront and contractually promises to pay cash in the future. This returned cash can be classified into tax-deductible interest payments and repayment of principal. Most bonds promise payments every 3 or 6 months. At **maturity**, corporate bonds usually pay back the principal. Under the most common arrangement, the **absolute priority rule** (**APR**), bondholders first receive what they have been promised, before more junior claimants (such as equity) can receive anything.

Bondholders have no control rights, unless the firm goes bust.

Control Rights: Unless the firm is near financial distress, bondholders typically do not have the right to participate in the decisions of the firm or the selection of its management. *But* if the firm misses a payment, the bondholders have the right to force the firm into bankruptcy. In the United States, bankruptcy means either corporate reorganization under **Chapter 11** or corporate liquidation under **Chapter 7**, named for their respective chapters in the Federal Bankruptcy Code. In theory, bankruptcy allows bondholders to take over and thereby own the company to recover what they were contractually promised. In practice, this is not as easy as it is in many European countries, but it does happen frequently enough. In any case, few managers survive even Chapter 11 bankruptcy, so they generally try to avoid missing bond payments. In addition to the universal right of repayment through control in default, many lenders contract for additional control rights in the original lending agreement. These are called **bond covenants**. For example, a loan agreement may specify that the firm must maintain a certain liquidity, or its loan can be declared to be in default.

16·1.B. Ordinary Equity (Common Stock)

Stock ownership is ownership post-debt, earning dividends and capital gains, and having control rights.

Stock, a common abbreviation for **ordinary equity** or **common equity**, is like ownership: **stock shareholder** (or just **stockholders**) receive whatever is left over *after* the promises to bondholders have been honored. The bad news is that equity typically has the lowest priority in bankruptcy. The good news is that shareholders enjoy unlimited upside. If they are lucky, they receive dividend payments and capital appreciation. Unlike coupon interest payments, dividend payments not only can be omitted at the corporation's discretion, but also have to be paid from *after-tax* earnings. Then, even though the corporation has already paid corporate income tax on its earnings, individuals have to pay personal income tax on their dividend receipts. This is sometimes called the **double taxation of dividends.** The Bush Tax Cuts of 2003 have greatly reduced the double taxation, which has also been the norm in many other countries (such as in the United Kingdom). In a very real sense, equity holders are like the owners (or partners) of the corporation, with one exception: equity holders are protected by **limited liability** (see also Chapter 5).

Control Rights: In situations outside financial distress, shareholders make all decisions for the company. In publicly traded companies, public shareholders usually delegate this right to a **corporate board**, which they can appoint or dismiss. In turn, the board appoints management, which runs the firm. There is considerable disagreement about how well shareholders' control rights function in large, old, and diffusely held corporations.

Solve Now!

Q 16.1 *What is a control right?*

Q 16.2 *What is limited liability?*

ANECDOTE: Judge Lifland and Eastern Airlines' Creditors
Absolute Priority is the theory. In practice, bankruptcy courts can and sometimes do violate the pre-agreed priority rules in the bankruptcy process. In turn, because corporate managers can choose where to file for bankruptcy, they usually do so in the court where they expect to fare best.

The *Southern District of New York* Bankruptcy Judge Burton Lifland was so notorious for violating creditors' rights, that he attracted not only *Eastern Airlines*' bankruptcy, but also those of *Manville*, *Orion Pictures*, and *LTV*. But it was Eastern Airlines that was Judge Lifland's crowning achievement: When it went bankrupt in March 1989, it was fully solvent. Unsecured creditors would have likely been satisfied in full. Instead, Judge Lifland allowed Eastern to continue operating for two more years, partially on the basis that closing it would have disrupted Christmas travel. Eastern's ongoing operation evaporated about $1.5 billion through operating losses and another $100 million through legal fees. In the end, unsecured creditors received practically nothing of their $2.3 billion claim.

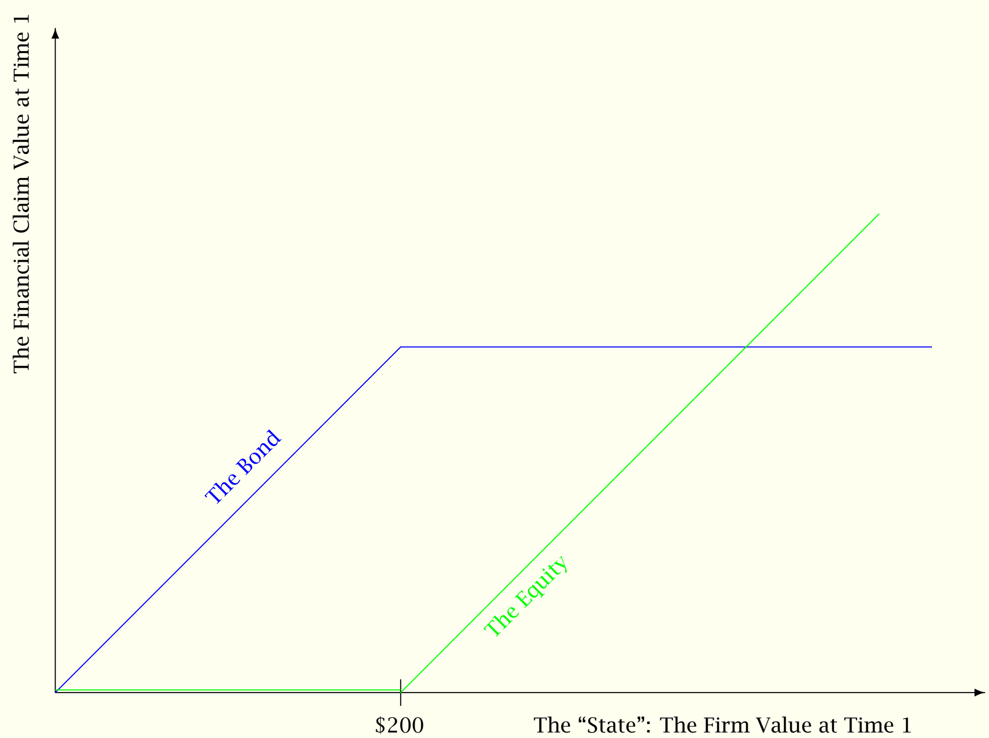
Despite frequent modest APR violations and despite such occasional spectacular examples of drastic APR violations, APR violations are usually mild. (They may even be necessary. After all, society would not want to see our lawyers starve!) These days, creditors are aware of expected violations and legal fees, and therefore take them into account when they purchase bonds and stocks in the first place. Thus, the presence of lawyers increases corporations' costs of financing.

16·1.C. Debt and Equity as State-Contingent Claims

Figure 16.1. Sample Bond and Stock Payoff Table (at Time 1)

Firm Value	Bond Value	Equity Value
\$0	\$0	\$0
\$50	\$50	\$0
\$100	\$100	\$0
\$150	\$150	\$0
\$200	\$200	\$0
\$250	\$200	\$50
\$300	\$200	\$100
\$350	\$200	\$150
...	...	...

Sample Bond and Stock Payoff Diagram (at Time 1)



Payoff diagrams, again.

In Chapter 5, you learned about the main tool for the analysis of promised payments: payoff tables and payoff diagrams. For example, consider a firm with a capital structure that consists of equity and a single bond that promises to pay \$200 next year. The value of the corporation will be the total value promised to bondholders and shareholders. How much each claims holder will receive will depend on the value of the firm. Figure 16.1 shows that if the firm is worth \$100, bondholders will receive \$100 and shareholders will receive nothing. If the firm is worth \$200, bondholders will receive \$200 and shareholders will receive nothing. If the firm is worth \$300, bondholders will receive \$200 and shareholders will receive \$100. If the firm is worth \$400, bondholders will receive \$200 and shareholders will receive \$200. And so on. This is the best way to think of the contractual payment obligations of bonds, stocks, and most other financial claims. (There are other important considerations, though, which are covered in subsequent chapters.) Because you can call the future value of the firm (the base asset) the underlying **state**, debt and equity are often called **state-contingent claims**: their value depends on the future state of the firm.

Timing does not matter to the usefulness of these diagrams.

Note that if the market is perfect, it is not important to our analysis whether the firm continues to exist after the bond comes due. You could imagine that the firm is then sold to new owners for its fair value first. The proceeds are then distributed to stockholders and bondholders according to their claims. Of course, stockholders and bondholders could use these proceeds to repurchase the remaining firm if they so desire. (On Page 89, in our tornado/sunshine example, we explained that it does not matter whether the house is sold at the end of time 1.)

Payoff diagrams cannot tell the part of the story that is time-varying, rather than state-varying.

However, although payoff diagrams are very useful as conceptual aids for a contract that is on one payment at a given point in time, they are not as good at illustrating features that are themselves a function of time or a function of many different points in time. Our example really made it easy to see the value of a zero bond. It would be more difficult to use the payoff diagram to fully describe a coupon bond, because these have multiple payment dates, which can cause quite a set of interesting complications that we would rather avoid. (Similarly, payoff diagrams are less useful to illustrate the value of a claim that consists of randomly timed future payoffs.) Nevertheless, even in such cases, there is usually a link between the value of the firm and the value of the financial claim—so thinking of financial claims as contingent claims often remains a useful conceptual, if not entirely accurate, tool.

16·2. More Financial Claims

Firms can choose any claim features they wish.

Over time, many variations and hybrids of the two basic claims have developed. The features of financial claims are not written in stone. Firms can and do experiment. Naturally, if a claim offers more features or protections that are of value to investors, then these investors will be willing to pay more for the claim upfront. In a perfect market, both companies receive and investors pay the appropriate fair share (price), regardless of the features chosen by corporations offering claims for sale. The features described in this chapter are among those that have survived, evolved, and thrived over the years—those that increase value. Of course, corporations could issue claims that do not maximize value, even if they are fairly priced. For example, a claim might offer its owner the right (or obligation!) to become CEO if it were to rain in Los Angeles next April 21. This claim would fetch an appropriate price, but it would probably significantly lower the value of the firm.

16·2.A. Call Options and Warrants

Firms sometime issue **call options** (or just **calls**), or **warrants.** These are usually more junior even than equity. In publicly traded corporations, they rarely have control rights—except for a right to be transformed into equity itself.

Call Options

Call Options are rights to purchase equity at a predetermined price in the future.

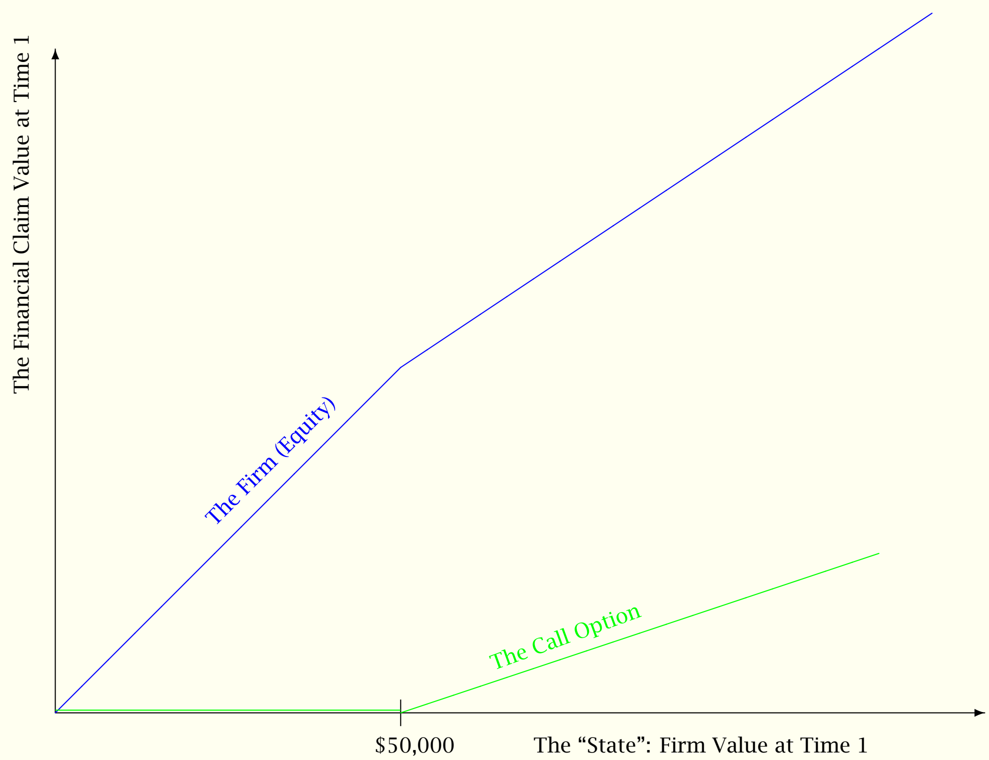
A call option is a right to purchase ("call in") the shares from the call seller for a given price at or by a given point in time. For example, a call might be the right to purchase 300 shares of the firm for $50/share on January 15, 2005. If shares will then be worth more than $50/share, then the call option will have value. Otherwise, the call will be worthless because it will not be not in the interest of the call holder to exercise it. (See also the Web chapter on options and derivatives for a longer explanation of call options.)

Figure 16.2. Sample Call Option Payoff Diagram at Time 1

Firm Value	Option Value	Equity Value
$0	$0	$0
$10,000	$0	$10,000
$20,000	$0	$20,000
$30,000	$0	$30,000
$40,000	$0	$40,000
$50,000	$0	$50,000
$60,000	$3,000	$57,000
$70,000	$6,000	$64,000
...	...	...

The call option strike price is $50 for 300 shares. With 1,000 shares in total, the option is "payment of $15,000 in exchange for 30% of the firm." At a firm value of $60,000, this means $18,000 minus $15,000 payment. At a firm value of $70,000, this means $21,000 minus $15,000 payment.

Sample Call Option Payoff Diagrams



Working the Example. Let us look at the call option in more detail. In our context, the call seller is the corporation having raised money by selling it. Figure 16.2 shows a firm that has 1,000 outstanding equity shares as well as call options for 300 shares at $50/share each. If call option owners were to choose to exercise the call option for 300 shares, they would own the equivalent of 30% of the firm. Now, if the stock price will be $40/share (total firm value $40,000) at option expiration, the call option will be worthless, because owning 30% of $40,000 ($12,000) in exchange for paying 300 shares times the exercise price of $50 ($15,000) would not be in the call owner's interests. However, if the price will be $60/share, call option holders will exercise their option, and demand 300 shares in exchange for payment of $50/share. The firm will have to repurchase

300 of its own shares at \$60/share (for \$18,000), but receive only \$50/share (or \$15,000). This will cost old shareholders a net \$10/share for 300 shares, or \$3,000 in total. This net benefit goes to the new shareholders (the original call option owners). Old equity holders will own 70% of the \$60,000 firm which sums to \$42,000, plus the \$15,000 call payment. In sum, their wealth will be \$57,000. New shareholders will own the remaining \$3,000. (An entirely equivalent transaction—without having to go through the repurchasing and handing over exercise—would be for the old shareholders to just hand over to the option holders $\$3,000/\$57,000 = 5\%$ of the firm, the equivalent of 50 out of 1,000 equity shares.)

Warrants

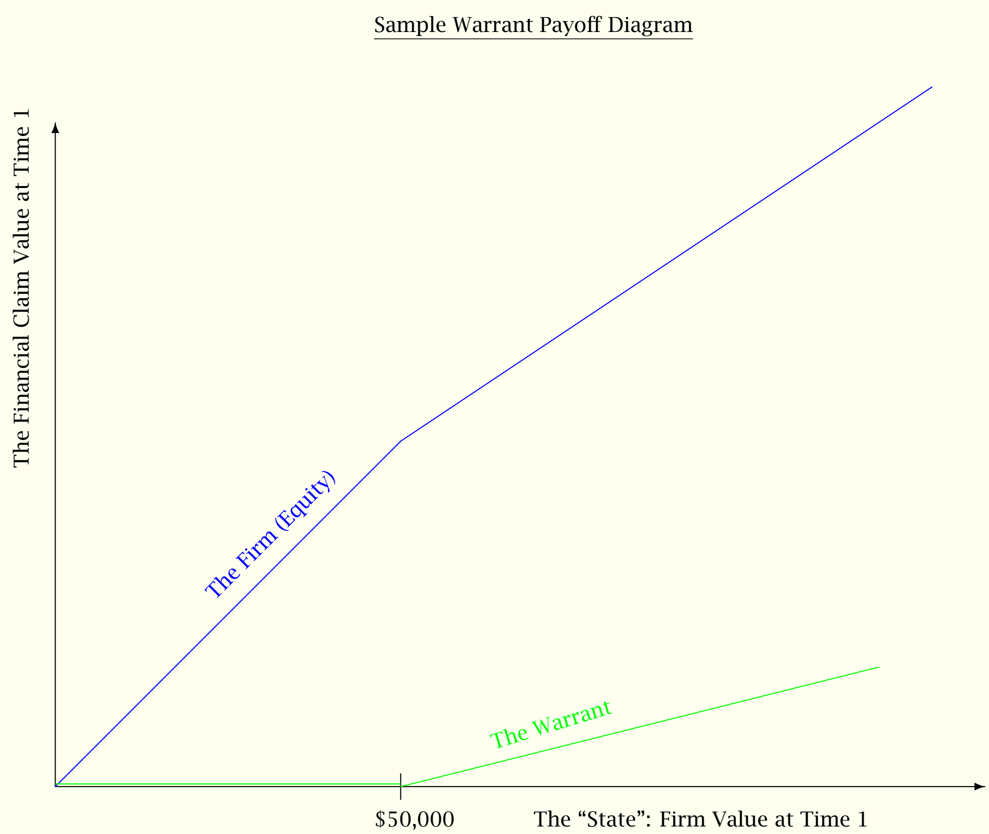
Companies usually issue Warrants, not Call Options.

Companies rarely issue call options, because it is usually easier for them to just issue new shares (to satisfy the option like claim) than it is for them to repurchase existing shares. Such call option-like financial claims are called **warrants.** They give warrant holders exactly the same rights that call option holders would have, except that the money that the warrant holder pays flows into the corporation, and the corporation issues new shares to satisfy the warrants. If the above claim (300 shares at \$50/share) is a warrant rather than an option, and if the firm ends up being worth \$60,000, then the warrant holders would exercise their rights, the firm would be worth \$75,000, and warrant holders would own $300/(1,000 + 300) \approx 23\%$ of this firm—the equivalent of \$17,307.69 in exchange for their \$15,000 payment. Note that this is less than the \$18,000 in stock that a call option holder would have received in exchange for \$15,000 payment. Therefore, a warrant is worth less than an equivalent call option. Exhibits 16.3 shows the warrant payoff diagram.

Figure 16.3. Sample Warrant Payoff Diagram at Time 1

Pre-Warrant Firm Value	Post-Warrant Firm Value	Warrant Value	Old Equity Value
\$0	\$0	\$0	\$0
\$10,000	\$0	\$0	\$10,000
\$20,000	\$0	\$0	\$20,000
\$30,000	\$0	\$0	\$30,000
\$40,000	\$0	\$0	\$40,000
\$50,000	\$0	\$0	\$50,000
\$60,000	\$75,000	\$2,308	\$57,692
\$70,000	\$85,000	\$4,615	\$65,385
...	...	...	

The warrant strike price is \$50 for 300 shares. With 1,000 shares in total, the option is "payment of \$15,000 in exchange for $300/1,300 = 23\%$ of the firm." At a pre-warrant firm value of \$60,000, this means $23\% \cdot \$75,000 \approx \$17,308$ minus \$15,000 payment. At a pre-warrant firm value of \$70,000, this means $23\% \cdot \$85,000 \approx \$19,615$ minus \$15,000 payment. (Old Equity owners now own 77% of a \$85,000 firm, which is \$65,385.)



The Time Dimension

Again, the payoff diagram may fail to capture the fact that time can play an interesting role when the option or warrant contract does not just allow conversion at a particular point in time, but up until a particular point in time. For example, the value of the call or warrant can then depend also on the underlying dividend yield that the stock is paying (and changes therein!)—and indeed the question of when to convert into stock can become quite complex. And, again, although not entirely accurate, the paradigm of thinking of calls and warrants in terms of contingent claims usually remains a very useful one.

16·2.B. Preferred Equity (Stock)

Preferred Equity gets better dividend treatment by the corporation.

Many companies also issue **preferred equity**. Preferred equity is an instrument somewhere between debt and equity. Preferred equity holders usually receive higher dividends than ordinary equity holders—hence the description "preferred." Unlike ordinary equity, where dividends are declared annually at the discretion of management, preferred equity dividends are usually specified at issuance (for example, \$2.25 per calender quarter per share). Unlike bondholders who fail to receive their promised coupon payment, preferred equity holders cannot force the firm into bankruptcy if dividends are not paid—but their contract usually states that they have priority over common equity holders: the preferred equity holders receive dividends before ordinary equity holders.

Preferred Dividends receive better tax treatment when held by other corporations.

But the real advantage of preferred equity over ordinary equity is that, although not tax-deductible to the issuing corporation, the preferred dividend payments are not fully taxable when received by other firms. The IRS considers only 15–30% of preferred dividends taxable for the recipient. Preferred equity is therefore usually not held by individuals, but by corporations investing in other corporations. (Individuals can earn higher post-tax yields in other claims.)

More preferred equity features.

Preferred equity is junior to the company's bonds, which means that in case of bankruptcy, preferred equity holders will be paid only after bondholders are paid. Preferred equity is often retired on a fixed schedule—even though many preferred equities have no formal maturity. Like common stock, some preferred stock is traded on public stock exchanges. Though preferred equity usually enjoys covenant restrictions (specifically promised dividend payments before common equity), it has neither precedence to bonds, nor control rights, nor unlimited upside (rights to the residual value of the firm). Naturally, any such features can be explicitly added by contract. For example, in many small firms, it is not uncommon for preferred equity to be convertible into common equity ("convertible preferred stock") and to explicitly provide for voting rights. The holders of such claims are usually themselves corporations, often venture capitalists, who can write off the claims if the firm fails and convert them into common equity if the firm succeeds.

Solve Now!

Q 16.3 *In what sense is preferred equity like bonds? In what sense is preferred equity like stocks?*

16·2.C. Convertible Bonds

Convertible bonds allow the bondholder to exchange the bond into something else, usually equity. An example.

Convertible bonds can also be considered as claims that are "someway" between bonds and stocks. They permit bondholders to convert their bonds into equity at a predetermined price at predetermined dates. Here is an idealized example, in which the bond can only be converted at one point in time: a firm with 400 outstanding shares of equity also has 200 outstanding convertible bonds that promise \$10 each in January 2005. Each such bond can be converted, at the bondholder's discretion, into 3 shares of stock.

Understanding the specific example: at the time the bond comes due.

If the value of the firm in January 2005 will be $2,000 or less, then bondholders will own the entire firm. If the value of the firm will be $100,000, bondholders will make the following calculation: If they take advantage of the convertibility feature and exchange their 200 bonds for 600 shares, there will be 1,000 shares in total. These 1,000 shares will own 60% of the firm, which will be worth $60,000—a whole lot more than the $2,000 that they will receive if they do not request conversion. Therefore, the bondholders convert.

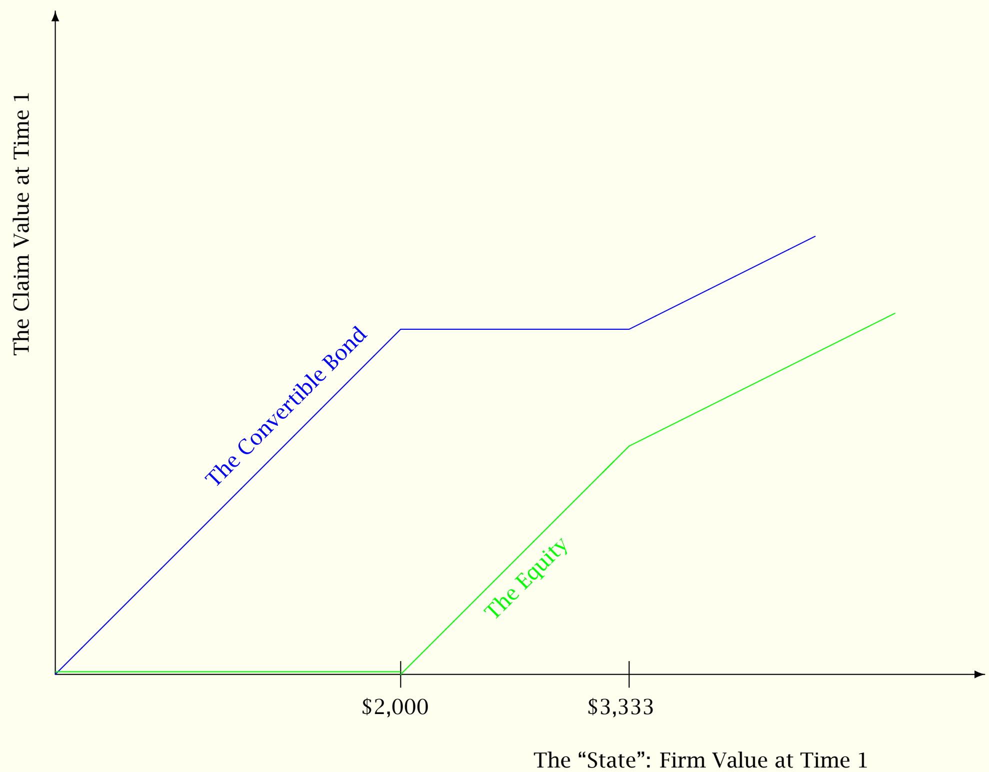
Determining the cutoff at which convertible bondholders will prefer converting.

Can you determine at what firm value the bondholders would be indifferent between converting and not converting? It is where 60% of the firm would be equal to $2,000, so it occurs when the firm value is equal to $3,333. To summarize: below $2,000, the bondholders get everything. Between $2,000 and $3,333, the bondholders get $2,000 and the shareholders get the residual above $2,000. And above $3,333, the shareholders and bondholders both benefit from higher values, with bondholders receiving 60% of the firm's value and shareholders receiving 40% of the firm's value. The payoff diagram helps to understand this claim. Figure 16.4 shows the convertible debt payoff diagram.

Figure 16.4. Sample Convertible Bond and Stock Payoff Diagram at Time 1

Firm Value	Convertible Bond Value	Common Equity Value
$0	$0	$0
$1,000	$1,000	$0
$1,500	$1,500	$0
$2,000	$2,000	$0
$2,500	$2,000	$500
$3,000	$2,000	$1,000
$3,333	$2,000	$1,333
$3,500	$2,100	$1,400
$4,000	$2,400	$1,600
$4,500	$2,700	$1,800
...	...	...

Convertible Bond and Stock Payoff Diagrams



One final question: Why would shareholders be willing to give bondholders this right, which in effect deprives them of much upside? The answer must be that by doing so, bondholders are willing to pay more for the bond upfront. And, indeed, we know that if financial markets are competitive, bondholders will get what they pay for.

Owners are willing to give the convertible right, because it increases the cash they receive upfront.

Solve Now!

Q 16.4 *We want to compare a plain bond plus a warrant to a convertible bond.*

(a) Firm A has a plain bond that promises to pay $200,000. It has 100,000 equity shares outstanding. The bond buyers also receive a warrant to purchase 25,000 equity shares at a price of $8/share. In other words, the bond and warrant form a unit.

(b) Firm B has a convertible bond that promises to pay $200,000 but that can be converted into one-fifth of the firm's equity.

Write down the payoff tables (and draw the payoff diagrams). Reflect on how they differ.

16·2.D. Other Bond Features

Bonds come in a thousand flavors—and then some.

Corporate bonds also have many other flavors—there may be more variety than there is commonality. The issuer can choose what specific rights to offer to buyers and what rights to reserve for the firm. Naturally, the more rights a bond buyer receives, the higher the value of the bond when the issuer sells them, and therefore the lower the interest rate. Some bond flavors are unique to corporations, while others are common. Among the more common bond specifications are the following:

Bond covenants promise that the firm will keep certain promises, or else it will be forced to repurchase (**redeem**) the bond. Among the more common covenants are restrictions on what the firm can do with its assets, how much in dividends it may pay, how many and what kind of other financial claims it may issue, what kind of financial ratios (e.g., debt/equity ratio) it needs to maintain, who the auditor is, what happens if the corporation defaults on any other bond, and how much of its own bonds it will repurchase each year. This last feature is called a **sinking fund** commitment and is very common.

Bond seniority specifies which bond receives first dip in case of bankruptcy and liquidation. A **senior bond** will have to be satisfied in full before a **subordinated bond** (or **junior bond**) may receive any money. In turn, equity receives its funds only after even the most junior bonds have been fully satisfied.

Collateral or **security** are specific corporate assets pledged to a specific bond in case of default. For example, mortgage bonds are collateralized by the value of the underlying real estate. If the issuer fails to pay, the bondholders may repossess the underlying real estate, and use it to satisfy their claim. If the real estate is not enough to satisfy the claim of the **secured bond**, the remaining claim becomes an ordinary bond, waiting in line with other creditors for payment.

Puttability allows bondholders to return the bonds to the issuer, in exchange for pre-agreed-upon payment. This is like convertibility, only that the conversion is into cash, not into equity.

Callability allows the issuer (the firm) to "call in" the outstanding bond at a prespecified price. For example, a callable bond contract may state that the firm can redeem the bond by paying back principal plus 10% rate of interest in May 2020. Usually, callable bonds do not allow a call for the first five years in the life of the bond. The call feature also often comes in connection with convertibility, so the call can be used to force conversion: The corporation calls the bonds, and the holder of the bond finds that it is in her interest to convert the bond into equity, rather than to accept repayment.

While a convertible bond gives bondholders extra rights, callable bonds give the firm extra rights. Therefore, when a bond contains a call feature, it is less valuable than an otherwise identical bond, which means that issuers of bonds with call features receive less money when they include the call feature. Put differently, they must pay a higher interest rate upfront. In effect, every mortgage is a callable bond, because the seller of the bond (the homeowner, that is you) can just pay back the remaining loan balance (the **principal**) and be absolved of all further obligations.

The call feature is a good example of where payoff diagrams do not capture the whole situation. The value of the callable bond is often more a function of the prevailing interest rate than it is a function of the firm value. Corporations tend to call bonds when the economy-wide interest rate has dropped so replacement bonds have become much cheaper. (Similarly, homeowners tend to repay their mortgages and refinance when the mortgage interest rate has dropped.) But, because the interest rate is not a one-to-one function of the firm value in the future, the payoff diagram against the state of the firm at a fixed point in time would not tell the whole story.

Actual calling may depend not just on the value of the firm, but on such things as interest rate.

CFOs must also make decisions on the following corporate bond features. Because these features are shared by non-corporate bonds, we have already described them in Part I.

Bond maturity is the time to final payback. Indeed, borrowing may be very short-term (as little as overnight!), or very long-term (as long as forever). Bonds of different maturities may have different names. For example, **commercial paper** is short-term debt, often guaranteed by a bank's credit line (see below), and therefore is almost risk-free to the lender. On the corporate balance sheet, **funded debt** is the term for debt that has a maturity of less than 1 year. **Unfunded debt** has a maturity of more than 1 year.

Again, our payoff diagrams do not do bond maturity full justice. The reason is that maturity can sometimes be like "super-seniority." That is, a subordinated bond may be repaid before the more senior bond comes due, and, once paid, the money paid to the subordinated bond can often not be reclaimed to satisfy the senior creditor's higher-priority claim.

Bond duration is a measure of how soon payments are made. It was described on Page 70.

Coupon bonds vs. zero bonds: We have already talked extensively about this distinction in Part I of our book. Zero bonds pay a fixed amount of money only at a final date. Coupon bonds make (interest) payments on a regular schedule, typically (but not always) twice a year, and the principal is repaid as a **balloon payment** at the end.

Side Note: In a practice called **bond stripping**, investment banks sometimes separate the coupon obligations from the final balloon payment. That is, they purchase bonds, put them into an escrow account, and offer two types of new financial claims collateralized by the escrowed bonds: one pays all the coupon, the other pays all the principal. The reason for doing this is to exploit tax loopholes in Japan and other countries.

You can think of a coupon bond as a special kind of unit, consisting only of similar payment obligations. More generally, a **unit** is simply a bundle of multiple types of financial claims that are sold together, usually involving a warrant and another claim (like a stock or a bond). The purchaser can keep both types of claims, or similarly unbundle them and sell them separately.

Fixed interest-rate debt vs. floating interest-rate debt: Bonds can promise to pay a predetermined interest rate over the life, or a spread relative to some other interest rate. Floating debt is often issued at a spread relative to the **prime rate** (an average interest rate that banks usually offer their best customers) or relative to **LIBOR** (**London Interbank Offer Rate**). The interest rate on floating rate debt is often **capped** or **collared**; that is, the interest rate will never exceed a predetermined ceiling.

Side Note: Highly reputable companies can borrow at interest rates that are about LIBOR. More risky companies typically pay interest rates that are about 100-300 basis points (or 1-3%) above LIBOR.

There is no limit to the imagination as far as bond features are concerned. For example, the Russian car maker *Avtovaz* issued *Lada bonds* in 1994, which allowed the holders to convert their bonds into Lada cars. Other bonds' payoffs have been linked to commodities, such as the price of oil, to other financial claims, or to exchange rates.

There is an endless array of possible bond features.

Bank Loans may be Credit Lines.

A bank loan is like a bond, but usually with only one bondholder—the bank. The advantage is that the bank may know the firm better, and thereby grant better terms. The disadvantage is that there is less competition among banks for extending loans than there is among public bondholders. Bank loans can also take the form of a **credit line**. Credit lines are like instant debt, permitting borrowers to draw down money (and pay higher interest) only upon need. (Borrowers typically agree to pay a low interest rate even on the unused part of the credit line.) The opposite of a credit line is **negotiated debt**, in which both the bank and the firm commit to a fixed loan.

Solve Now!

Q 16.5 *Write down all bond features (variations) that you remember.*

Q 16.6 *Assume a medical insurance pays 90% of medical expenses, subject to a $500 deductible, and subject to a limit of $10,000 payout by the insurance company per year. Write down an insurance payoff table and graph an insurance payoff diagram, as a function of your medical expenses. What is the slope of the line at each segment?*

Q 16.7 *A firm is financed with a senior bond that promises to pay $100, a junior bond that promises to pay $200 (of lower seniority but of equal maturity to the senior bond), and equity. Write down the payoff table and then draw the payoff diagrams when the two bonds are due.*

16·3. Summary

Large firms usually have multiple claims outstanding.

In the real world, firms are financed by a whole set of different financial claims. The same firm may have senior debt, junior debt (perhaps with a conversion feature), equity, and a warrant. The right way to think about all these claims still often involves the "magic" payoff table (and payoff diagram): if the firm ends up worth very little, only the senior debt is paid. If the firm is worth a little more, both the senior and the junior debt are paid. If the firm is worth even more, the equity will become worth something; and finally, the warrant and/or the conversion feature will be valuable.

The chapter covered the following major points:

- The two most basic building blocks of capital structure are debt and equity. These differ in their cash flow rights and in their control rights:
  - Debt has first rights to the distribution of cash flows. It is "senior." It can force the firm into bankruptcy if payments are not made.
  - Equity gets only what is left over after debt has been satisfied. It is "junior." It is in control of the firm, unless the firm finds itself in financial distress.
- Payoff tables and payoff diagrams are often a good way to describe debt and equity, because they are primarily state-contingent claims, where firm value is the state. The paradigm is not always perfect, because the value of claims can also depend on factors other than firm value.
- Call options and warrants are more junior even than equity. These claims allow their owners to obtain shares at a given price in the future.
- Preferred equity cannot force bankruptcy, but receives its dividends before common equity does. The main advantage of preferred equity is that corporate holders pay lower taxes on preferred equity dividend receipts.

- Convertible bonds allow their owners to convert their bonds into shares. They can therefore be considered as part-debt, part-equity.
- Corporate borrowing comes in thousands of different flavors. It can be plain, convertible, callable, or convertible and callable. It can be fixed-rate or floating-rate. It can be short-term or long-term. It can have detailed covenants of many kinds. And so on.

Solutions and Exercises

1. A control right is the right to influence decisions, specifically by changing management and/or the board.
2. The fact that all that the owner can lose is his investment. He cannot forfeit his house or other possessions because the corporation does bad stuff.

3. Preferred equity is like a bond in that it does not participate in the upside, and in that common shareholders do not get their dividends until preferred shareholders have received their dividends. Preferred equity is like a stock in that its payments are not tax deductible, and in that preferred shareholders have no ability to force the firm into bankruptcy if their dividends are not paid.
4. All numbers are quoted in thousands.

	Firm A				Firm B	
Firm Value	Plain Bond	Equity	Warrant	Plain Bond + Warrant	Convertible Bond	Equity
$0	$0	$0	$0	$0		
$100	$100	$0	$0	$100	$100	$0
$200	$200	$0	$0	$200	$200	$0
$300	$200	$100	$0	$200	$200	$100
$400	$200	$200	$0	$200	$200	$200
$500	$200	$300	$0	$200	$200	$300
$600	$200	$400	$0	$200	$200	$400
$700	$200	$500	$0	$200	$200	$500
$800	$200	$600	$0	$200	$200	$600
$900	$200	$700	$0	$200	$200	$700
$1,000	$200	$800	$0[a]	$200	$200[c]	$800
$1,100	$200	$880	$20[b]	$220	$220[d]	$880
$1,200	$200	$960	$40	$240	$240	$960
$1,300	$200	$1,040	$60	$260	$260	$1,040
$1,400	$200	$1,120	$80	$280	$280	$1,120
$1,500	$200	$1,200	$100	$300	$300	$1,200

Explanations: [a]At $1,000 firm value, Firm A's equity shares are worth $800, which translates into $8/share. At $8 share, warrant holders are indifferent between exercising and not exercising. [b]At $1,100 firm value, Firm A's equity shares are worth $900, which translates into $9/share. At $9/share, warrant holders exercise. They pay in $8 and receive 25,000/(100,000+25,000)=20% of the firm's equity value. [c]Convertible bond holders are indifferent between converting their bond into equity if the firm value is $1,000. After all, they would receive 20% of the firm's total value, which would come to $200. [d]Convertible bond holders prefer to convert their bond into equity, because 20% of $1,100 is more than $200.
In sum, a plain bond plus warrant can be just like a convertible bond.

5. See text.
6.

Medical Costs	Insurance Payout
$0	$0
$250	$0
$500	$0
$750	$225
$1,000	$450
$2,000	$1,350
...	...
$11,500	$9,900
$11,600	$9,990
$11,611	$10,000
$11,700	$10,000
$12,000	$10,000
$13,000	$10,000
...	...

The "slope" is zero until $500 is reached, then 90% until $11,611.11 is reached, then zero again.

7.

Firm	Senior	Junior	Equity
\$0	\$0	\$0	\$0
\$50	\$50	\$0	\$0
\$100	\$100	\$0	\$0
\$150	\$100	\$50	\$0
\$200	\$100	\$100	\$0
\$250	\$100	\$150	\$0
\$300	\$100	\$200	\$0
\$350	\$100	\$200	\$50
\$400	\$100	\$200	\$100
\$450	\$100	\$200	\$150
...	...	...	...

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

CHAPTER 17

Idealized Capital Structure and Capital Budgeting

Should A Company Issue Stocks or Bonds?

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This chapter explains how to look at the basics of financing: in a "perfect" world (no transaction costs, no taxes, etc.) the value of the firm is determined by the value of the projects (the NPV), not by whether the firm is financed with equity or leverage. This chapter also explains the basics of the weighted average cost of capital formula (WACC). Subsequent chapters will explain financing in the real world, not in our perfect world.

17·1. Conceptual Basics

17·1.A. The Firm, The Charter, and The Capital Structure

The firm's charter defines the firm structure. The agreement of who receives what part of future corporate income is the Capital Structure

The best way to think about an optimal firm structure is as follows: the current owners of a firm want to sell it today for the highest possible price. Their goal is to design a **corporate charter** that maximizes the total market value of their firm today—that is, the price that new investors are willing to pay to acquire the firm from them. The corporate charter must not only specify the **voting rules** (the procedures to replace incumbent managers or amend the the charter in the future), but also how future earnings are to be split among possibly different owners and stakeholders (such as customers, workers, and suppliers). This agreement on how to split up future earnings is the firm's financial structure or **capital structure**: it is the rules that specify who receives the proceeds of (usually uncertain) future cash flows. The capital structure is rarely fully explicit or set in stone. Instead, it also encompasses the rules on how the firm may be governed in the future, which in turn may influence the capital structure in the future, which in turn may influence the financing arrangement today.

17·1.B. Maximization of Equity Value or Firm Value?

Should management maximize shareholder value?

Managers are at least in principle appointed by the shareholders. An important question is whether managers should be directed to see themselves as representative of the shareholders or as representatives of the firm. The common view is that it is the task of management to maximize **shareholder wealth**. But equity is only one part of the firm's capital structure. There is also debt—and many financial instruments with both debt and equity features. Does it even make a difference as to whom management is representing?

When is there a potential problem?

The legal situation in the United States is that management has a legal fiduciary duty to shareholders—except if decisions made by the firm can threaten its insolvency. In this case, management's legal fiduciary responsibility extends to creditors, too. In general, managers see themselves more as representatives of shareholders than as representatives of creditors. When both bondholders and shareholders benefit from a manager's actions, there is no problem. But what if there are situations in which optimizing shareholder value (i.e., the value of the equity) is not the same as optimizing the overall firm's value? For example, assume it were possible for managers to increase the value of equity by \$1, but at a cost to the value of debt by \$3. (You will later learn how easy it is to do exactly this.) This "expropriative" transaction would destroy \$2 in the net value of the firm. But even in our idealized world, this is the type of situation that creates a dilemma for management: should management maximize firm value or shareholder value? Recall that it is shareholders who vote managers into office and allow them to stay there. Whether this transaction destroys firm value or not, when the time comes, managers will find it in their interest to execute this transaction because doing so raises equity value and with it management's chances of being reappointed.

Bond buyers understand future conflicts of interest. If managers ignored bondholder concerns, bondholders demand a higher interest rate upfront.

However, there is a fly in the ointment. Put yourself into the shoes of the old shareholders today who own the entire firm and are trying to set up the corporate charter that maximizes the value of their firm, i.e., for sale to new investors at the highest price. You want to find the best capital structure *today*, i.e., before you have found new investors. Clearly, anyone contemplating purchasing your debt will take into consideration what managers may do to them in the future, and they will rationally demand compensation. If you cannot commit the firm today *not* to undertake the \$3-for-\$1 transaction in the future, prospective bond buyers will realize today (before the fact, or **ex-ante**) that you (management) will have the incentive to execute it later (after the fact, or **ex-post**), no matter what you tell them today.

Anecdote: German Stakeholders
In Germany, stakeholders have an explicit role in corporations. In the case of joint stock companies, limited liability companies and cooperatives with more than 500 employees, one third of the members of the **Supervisory Board** must be employees. In the case of companies in the iron, coal and steel industry, provision is made for equal representation on the supervisory board by shareholders and representatives of the work force. A director representing the employees with responsibility for social affairs is also appointed (qualified co-determination).

Therefore, managers need to pay attention to the needs of bondholders—even though they are voted into power only by equity holders and are always tempted to focus only on improving equity value.

If the firm were believed to undertake this transaction in the future, what would it be worth today? It would be worth less than a firm that will not destroy \$2 of value in the future. Therefore, management today has a choice: it can either find a way to commit today not to exploit bondholders in the future; or it can sell the firm today for a lower net present value, which takes into account value destruction tomorrow—because everyone realizes that managers will be trapped tomorrow into destroying \$2 of firm value. To avoid this, managers should want to do everything in their power to constrain themselves from expropriating bondholders in the future. Constraining themselves will maximize the value of the firm in the future, which in turn maximizes the value of the firm today.

IMPORTANT:

- *In deciding on an appropriate price to pay, the buyers of financial securities take into account what the firm is likely to do in the future.*
- *The theoretical notion of the **optimal capital structure** is the structure that maximizes the value of the firm, not the value of the equity.*

Conflicts of interest arise between shareholders and bondholders, but they are dwarfed by the conflict between managers and owners.

In this theoretical world, management should maximize firm owner value, not shareholder value. Practically, the two objectives differ only rarely (and usually only when firms are close to financial distress), so the popular mantra of “shareholder value maximization” is fortunately usually synonymous with “total value maximization.” But strictly speaking, shareholder value maximization is not the goal of an idealized capital structure—firm value maximization is. In any case, in the real world, managers are far more conflicted with respect to their own welfare (the agency conflicts we first discussed in Section 7·6) than they are with respect to favoring shareholders at the expense of bondholders.

Committing to maximizing firm value gives a better interest rate and higher value today.

The advantage of a firm that is committed to maximizing firm value in the future is that it can obtain a better price for its financial securities (a lower interest rate for its bonds) today. Therefore, the firm has the incentive to commit itself today (ex-ante) to treating bondholders well in the future (ex-post). The ex-ante capital structure that results in the highest firm value today is the optimal capital structure.

The basics of capital structure theory is to realize that future events have impact on corporate value today. The goal is to use capital structure to maximize firm value today.

This is the most important insight with respect to capital structure, and worth repeating: the cost of *ex-post* actions against bondholders is not only borne by bondholders tomorrow, but it is also borne by the owners today. Another consequence is that *caveat emptor* (“buyer beware”) applies: bond and stock purchasers can only be hurt to the extent that future opportunistic actions by management are unforeseen surprises. Thus, it is in management’s (owners’) own interest today to commit not to exploit future owners and bondholders tomorrow—especially if everyone knows that when the time comes, management would like to change its mind.

The web chapter on corporate governance will return to this idea.

Competition among management teams pressures firms to improve Capital Structures.

What would happen if the current management team cannot commit to avoid such bad future \$3-for-\$1 exchanges? In this case, in our perfect world, another management team that has the ability to commit to restrain itself would value the firm more highly than the current management team. It could purchase the firm and make an immediate profit. Therefore, competition among management teams can push firms towards the best capital structure. Again, the general thread emphasized throughout this chapter is that firms that can commit to do “the right thing” tomorrow (*ex-post*) are worth more today (*ex-ante*). It is a direct consequence that firms that maximize firm value are worth more than firms that maximize just shareholder value.

Solve Now!

Q 17.1 *Explain the difference between ex-ante and ex-post, especially in the capital structure context. Give an example where the two differ.*

Q 17.2 *Can an ex-post maximizing choice be ex-ante bad?*

17·2. Modigliani and Miller (M&M), The Informal Way

The famous **Modigliani-Miller** (**M&M**) propositions (honored with two Nobel Prizes) are a good start to understanding firms' capital structure decisions. Although the M&M theory involves some complex algebraic calculations, it is actually based on some surprisingly simple ideas—which the following anecdote explains not only in a funnier but also better way than any complex calculations. It is an excerpt from an acceptance speech by Merton Miller for an honorary doctorate at Louvain, Belgium, in 1986. (His coauthor, Franco Modigliani, had just won the first Nobel Prize; Merton Miller would receive his own Nobel Prize a few years later.)

Anecdote: M&M: Milk, Cream, and Pizza

How difficult it is to summarize briefly the contribution of these papers was brought home to me very clearly last October after Franco Modigliani was awarded the Nobel Prize in Economics in part—but, of course, only in part—for the work in finance. The television camera crews from our local stations in Chicago immediately descended upon me. "We understand," they said, "that you worked with Modigliani some years back in developing these M&M theorems and we wonder if you could explain them briefly to our television viewers." "How briefly?", I asked. "Oh, take 10 seconds," was the reply.

Ten seconds to explain the work of a lifetime! Ten seconds to describe two carefully reasoned articles each running to more than 30 printed pages and each with 60 or so long footnotes! When they saw the look of dismay on my face, they said: "You don't have to go into details. Just give us the main points in simple, common sense terms."

The main point of the first or cost-of-capital article was, in principle at least, simple enough to make. It said that in an economist's ideal world of complete and perfect capital markets, and with full and symmetric information among all market participants, the total market value of all the securities issued by a firm would be governed by the earning power and risk of its underlying real assets and would be independent of how the mix of securities issued to finance it was divided between debt instruments and equity capital. Some corporate treasurers might well think that they could enhance total value by increasing the proportion of debt instruments because yields on debt instruments, given their lower risk, are, by and large, substantially below those on equity capital. But, under the ideal conditions assumed, the added risk to the shareholders from issuing more debt will raise required yields on the equity by just enough to offset the seeming gain from use of low cost debt.

Such a summary would not only have been too long, but it relied on shorthand terms and concepts, like perfect capital markets, that are rich in connotations to economists, but hardly so to the general public. I thought, instead, of an analogy that we ourselves had invoked in the original paper. "Think of the firm," I said, "as a gigantic tub of whole milk. The farmer can sell the whole milk as is. Or he can separate out the cream and sell it at a considerably higher price than the whole milk would bring. (Selling cream is the analog of a firm selling low yield and hence high-priced debt securities.) But, of course, what the farmer would have left would be skim milk, with low butter-fat content and that would sell for much less than whole milk. Skim milk corresponds to the levered equity. The M&M proposition says that if there were no costs of separation (and, of course, no government dairy support programs), the cream plus the skim milk would bring the same price as the whole milk."

(continues.)

ANECDOTE: (continued.)

The television people conferred among themselves for a while. They informed me that it was still too long, too complicated and too academic. "Have you anything simpler?", they asked. I thought of another way that the M&M proposition is presented which emphasizes the notion of market completeness and stresses the role of securities as devices for "partitioning" a firm's payoffs in each possible state of the world among the group of its capital suppliers. "Think of the firm," I said, "as a gigantic pizza, divided into quarters. If now, you cut each quarter in half into eights, the M&M proposition says that you will have more pieces, but not more pizza."

Again there was a whispered conference among the camera crew and the director came back and said: "Professor, we understand from the press release that there were two M&M propositions. Maybe we should try the other one."

He was referring, of course, to the dividend invariance proposition and I know from long experience that attempts at brief statements of that one always cause problems. The term "dividend" has acquired too great a halo of pleasant connotations for people to accept the notion that the more dividends the better might not always be true. Dividends, however, as we pointed out in our article, do not fall like manna from heaven. The funds to pay them have to come from somewhere—either from cutting back on real investment or from further sales (or reduced purchases) of financial instruments. The M&M dividend proposition offered no advice as to which source or how much to tap. It claimed, rather, that once the firm had made its real operating and investment decisions, its dividend policy would have no effect on shareholder value. Any seeming gain in wealth from raising the dividend and giving the shareholders more cash would be offset by the subtraction of that part of their interest in the firm sold off to provide the necessary funds. To convey that notion within my allotted 10 seconds I said: "The M&M dividend proposition amounts to saying that if you take money from your left-hand pocket and put it in your right-hand pocket, you are no better off."

Once again whispered conversation. This time, they shut the lights off. They folded up their equipment. They thanked me for my cooperation. They said they would get back to me. But I knew that I had somehow lost my chance to start a new career as a packager of economic wisdom for TV viewers in convenient 10-second sound bites. Some have the talent for it; and some just don't.

These simple, common sense analogies certainly do less than full justice to the M&M propositions; crude caricatures or cartoons they may be but they do have some resemblance. So much, in fact, that looking back now after more than 25 years it is hard to understand why they were so strongly resisted at first. One writer—David Durand, the same critic who had so strongly attacked the Markowitz model—even checked out the prices for whole milk, skim milk and cream in his neighborhood supermarket. He found, of course, that the M&M propositions didn't hold exactly; but, of course, empirical relations never do.

17·3. Modigliani and Miller (M&M), The Formal Way In Perfect Markets

Consider an idealized, perfect world.

To begin with, Modigliani and Miller argued that under perfect conditions, the total value of all financial securities is the same, regardless of whether the firm is financed by equity or debt or anything in between. They proved their argument by showing that there would be arbitrage opportunities if the value of the firm depended on how it is financed. Because there should be no arbitrage in real life, it follows that firms should be able to choose any mix of securities without impact on the firms' values. This perfect world that M&M describe relies on the familiar perfect world assumptions (e.g., in Section 6·1).

- There are no transaction costs. In this context, it also means that there are no such frictions as deadweight losses in bankruptcy. (This includes the assumption that there are no costs to financial distress before bankruptcy occurs, either.)
- Capital markets are perfectly competitive, with a large number of investors competing for many securities.
- There are no taxes.
- There are no differences in opinion and information.

We already know that these assumptions imply that borrowing and lending interest rates are equal. Of course, these assumptions do not hold in reality. However, once you understand how the M&M argument works, it becomes easier to understand what happens when these assumptions are violated, and to understand how important such violations can be. Indeed, the next several chapters will show what happens if the world is not perfect.

The proof is simpler if we assume a fixed investment policy for the moment.

Let's see now how the Modigliani and Miller proof works. For simplicity, take it as given that the firm has already decided on what projects to take. M&M believed that this was a necessary assumption, but it turns out not to matter in their perfect market. (We will discuss this at the end of this section.)

The Modigliani-Miller Proposition state that in this ideal world, capital structure does not matter.

The firm wishes to consider how to finance its projects. Because we all agree on all current and future projects' expected cash flows and proper discount rates, we agree on the present value of these projects today. Call the value of the projects under a hypothetical best capital structure "PV." (This is [almost by definition] the present value that the firm's projects can fetch in our perfect capital market, of course.) The M&M claim is that the present value of the firms' projects must equal the present value of the firms' issued claims today. In other words, if the firm issues 100% equity, the equity must sell for the PV of the projects. If the firm issues 50% debt and 50% equity, the two together must sell for the same PV. If the firm issues x% debt and (1-x%) equity, the two together must sell for PV. According to theory, the value of the firm should be determined by the net present value of its projects, regardless of capital structure. So, why does the capital structure not matter?

Arbitrage!

The Full Repurchase Argument Assume that the managers could find—and actually did choose—a capital structure that makes the firm worth \$1 less than PV. For example, assume that the firm is worth PV=\$100 under the optimal capital structure of 80% equity and 20% debt; and assume further that the firm is worth only \$99 under the capital structure of 50% equity and 50% debt that the firm has actually chosen. Then, all you need to do to get rich is to purchase all old equity and all old debt, i.e., the entire firm, for \$99. Now issue claims duplicating the optimal capital structure (assumed to be 80% equity/20% debt). These claims will sell for \$100, and you pocket an instant arbitrage profit of \$1.

Unfortunately, you would not be the only one to notice this opportunity. After all, information is universally shared. So, the old managers would simply ask for bids from other investors. The only price at which no one will overbid you for the right to purchase the firm's current securities is \$100. But notice that this means that the value of the old securities is equal to the price that the firm is worth under the optimal capital structure. So, regardless of the financial structure that managers choose, they can sell their claims for \$100, i.e., the present value of their projects.

Competition: Others would want to arbitrage, too—until the M&M proposition works (firm value is as if it was financed optimally).

The Partial Repurchase Argument This argument even works if you do not buy 100% of the firm, but only 1% of the firm. That is, if you buy 1% of all the firm's securities, you will receive 1% of the projects' payoffs. You can then sell your securities repackaged to imitate the payoffs under the presumably better capital structure for 1% of the firm's higher value, and receive an arbitrage profit of 1%·\$1.

Actually, the M&M argument should not come as a surprise to you. In Section 5·3.B (Page 92), without calling it the M&M argument, we had already made use of it in the context of financing a house. We found that neither the house value nor the cost of capital were influenced by your debt vs. equity choice: the house was worth what it was worth. This was M&M precisely. It is the same argument but in more detail than the more general but perhaps also more vague argument we just made.

You had already seen this argument!

So, let us put this general proof for our \$100 firm into the framework of a more concrete scenario analysis. To accomplish this as simply as possible, we assume the world is risk-neutral and all securities have to offer an expected rate of return of 10%. (We will work an example in a risk-averse world in Section 17·5.A.)

Linking M&M under uncertainty to state-contingent payoffs (under risk-neutral pricing).

The Full Scenario Analysis Table 17.1 shows the value of a firm if the projects will be worth either \$60 or \$160. The expected future value is \$110, the present value is \$100. Under hypothetical capital structure LD ("little debt"), the firm issues debt with face value \$55. Consequently, bondholders face no uncertainty, and will pay $\$55/(1+10\%) = \$50$. Equity holders will receive either \$5 or \$105, and are thus prepared to pay $\$55/(1+10\%) = \$50$. Simply adding the value of the firm's claims adds up to the same \$100. Under hypothetical capital structure MD ("much debt"), the firm issues debt with face value \$94. Consequently, bondholders will now receive either \$60 or \$94, and are willing to pay \$70 today. Equity holders will receive \$0 or \$66, and are willing to pay \$30 for this privilege. Again, the value of all claims adds to \$100.

Table 17.1. Illustration of the M&M Proposition with Risk-Neutral Investors

		Bad Luck	Good Luck	Future Expected Value	Today's Present Value
Prob:		1/2	1/2		
Project	W	\$60	\$160	\$110	\$100
Capital Structure LD: Bond with Face Value FV=\$55					
Bond(FV=\$55)	D	\$55	\$55	\$55	\$50
Equity	E	\$5	\$105	\$55	\$50
Capital Structure MD: Bond with Face Value FV=\$94					
Bond(FV=\$94)	D	\$60	\$94	\$77	\$70
Equity	E	\$0	\$66	\$33	\$30

The cost of capital in this example is 10% for all securities, which is equivalent to assuming risk-neutrality. Later in this chapter, we will work an example in which the cost of capital is higher for riskier projects.

Arbitraging does not require purchase and sale of the entire firm; it also works with a fraction of the firm.

The Partial Scenario Analysis The more surprising fact is that investors can relever securities themselves—they do not need the firm to do it for them. In our example, assume that the firm has chosen capital structure LD, but you really, really like capital structure MD, because you would really, really like to own a security that pays \$0.60 in the bad state and \$0.94 in the good state. This would cost you 1% of the bond's \$70 price, or \$0.70. How can you purchase the existing LD securities to give you the MD security that you prefer *without* any cooperation by the LD firm?

What happens if you purchase d bonds and e stocks in our LD firm? You will receive payoffs of $d \cdot \$55 + e \cdot \$5$ in the bad scenario, and $d \cdot \$55 + e \cdot \$105$ in the good scenario. You want to end up with \$0.60 in the bad scenario, and \$0.94 in the good scenario—two equations, two unknowns:

$$\begin{array}{lrcrcl} \text{Bad Luck} & d \cdot \$55 & + & e \cdot \$5 & = & \$0.60 \\ \text{Good Luck} & d \cdot \$55 & + & e \cdot \$105 & = & \$0.94 \end{array} \quad \Longrightarrow \quad \begin{array}{rcl} d & \approx & 0.0034 \\ e & \approx & 0.0106 \end{array} \tag{17.1}$$

So, if you purchase 0.0034 bonds and 0.0106 of the equity, you will end up with \$0.60 in the bad state, \$0.94 in the good state—exactly the same that an MD firm would have given you!. How much would you have to pay to get these payoffs? The cost today would be $d \cdot \$50 + e \cdot \$50 = 0.0034 \cdot \$50 + 0.0106 \cdot \$50 = \$0.70$, exactly the same that your desired payoffs would have cost you if the firm itself had chosen an MD capital structure.

In effect, you have manufactured the capital structure payoffs that you like without the cooperation of the firm itself. By repeating this exercise, you can replicate the payoffs of any claims in *any* kind of capital structure.

From here, it is an easy step to the M&M argument. If the value of the firm were higher under the MD capital structure than it is under the LD capital structure, you could manufacture for yourself at lower cost from the current capital structure the securities that would end up with a higher market value, sell them, and earn an arbitrage profit.

IMPORTANT: *In Modigliani and Miller's perfect world:*

- *Arbitrage restrictions force the value of the firm to be the same, regardless of the firm's mix of debt and equity.*
- *Because financing and investing are two entirely separate issues, managers can ignore financing issues when they make investment decisions.*

If the world is not perfect, neither need be the case.

Know what to care about and what not to care about!

This is good news and bad news. It is good news that you now know where to focus your efforts. You should try to increase the value of the underlying projects—by increasing their expected cash flows, or by reducing their cost of capital, or both. It is bad news that you now know that to the extent that the world is perfect, you cannot add value by fiddling around with how you finance your projects.

In a perfect world, firms always undertake the best projects.

The above proof of the Modigliani-Miller proposition followed the M&M 1957 paper in assuming that the firm's real investment decisions had already been decided upon. It turns out that this is not necessary as long as investment decisions are reversible, because the perfect markets assumption also implies that management's project policy should not matter. If the firm were not to undertake all positive NPV projects and reject all negative NPV projects, you could buy all the shares, fire the old management, institute the best underlying firm investment project policy, resell all the shares, and earn the difference in firm value as an immediate arbitrage. Alas, you would again not be the only one: everyone could do this. Therefore, in this perfect world, firms always do the right thing. They maximize the firm's NPV, and they are worth exactly what they should be worth under the optimal operating policy.

Solve Now!

Q 17.3 *Under what assumptions does capital structure not matter?*

Q 17.4 *Explain the M&M argument to your 10-year old brother.*

Q 17.5 *What does risk-neutrality "buy" us in the M&M argument?*

Q 17.6 *In the example from Table 17.1, how would you purchase the equivalent of 5% of the equity of a hypothetical MD firm, if all that was traded were the securities of the LD firm?*

17·4. Dividends

In a perfect world, dividends do not create value.

The second Modigliani-Miller proposition is even simpler than the first: it states that corporate dividend policy should not matter in a perfect market setting, either. From the corporate perspective, if managers pay $1 in dividends, this money has to come from somewhere. As Merton Miller noted, dividends do not fall like manna from heaven, so no value is created or destroyed when firms pay dividends. Money that was previously owned by investors but held inside the corporate shell is just being moved to the same investors, so that it is now outside the corporate shell. The owners do not have any more or any less wealth because of the dividend payment. From the M&M arbitrage perspective, if managers undertook a dividend policy that destroyed value, then any investor could step in to purchase the firm, fire the management, institute the better dividend policy, and resell the firm for the difference. With many investors vying to do this if even just a penny can be earned, the only firm value and dividend policy that do not allow anyone to arbitrage (get rich without effort) is the value of the firm under an optimal dividend policy.

The M&M logic helps us think in our imperfect world.

Like the M&M capital structure proposition, the point of the M&M dividend proposition is not to argue that dividends do not matter. It is to point out what perfect market violations must be the case for dividend policy to matter, and how much these violations can matter. For example, if it costs a roundtrip premium of $10 million to purchase and then resell a firm, then it cannot be that the wrong dividend policy destroys more than $10 million. If it did, you could make money even in our imperfect world.

The situation today, and a preview.

As of 2005, the average dividend yield of large firms is around 1% of firm value per year. This is probably so low that the real-world market frictions are larger than what you could earn by correcting this policy if it were incorrect. That is, if the optimal payout were 0% or 2% instead of 1%, the maximum 1% value increase is too little to overcome the transaction costs that would allow someone to step in and correct it. Interestingly, there is some good evidence that the M&M assumptions are indeed violated: when firms announce dividend increases, their values usually go up, and when they announce dividend decreases, their values usually go down. Can you speculate which M&M assumption is most likely violated? Most finance professors believe that paying dividends sends a credible signal from management about future firm prospects and good managerial behavior (managers will not waste the money on themselves). This violates the M&M assumption that everyone has the same information: in the real world, managers have inside information that investors do *not* have.

17·5. The Weighted Cost of Capital (WACC) in a Perfect M&M World

You now understand why the value of the firm does not depend on the financing in a perfect market. This is equivalent to stating that the overall cost of capital to the firm does not depend on the debt/equity ratio of the firm. To show this, we want to repeat the house example from Section 5·3.B. It is important that you realize that our argument also works in the context of a risk-averse world, just as long as the world is still perfect. Therefore, we shall work our earlier examples again but allow riskier securities to have higher expected rates of returns. Our example will draw on your knowledge of net present value, the capital asset pricing model, and capital structure concepts. The reason why this is important is that it introduces the concept of the "weighted average cost of capital" (or WACC) in the corporate context. (In the next chapter, we will generalize WACC to a world in which corporations pay income tax.)

17·5.A. The Numerical Example

All tools learned in Section 5·3.B still apply under risk aversion.

Our example will again be our house with the mortgage from Section 5·3.A (Page 89), so make sure you remember the concepts from Chapter 5. Briefly, in that Chapter, the *expected* rate of return was the same for projects of all risk classes. In contrast, in this chapter, we take into account the fact that investors are risk-averse, so that riskier securities have to offer higher expected rates of return. Our goal is to compute the **weighted average cost of capital** (or WACC), which is the average cost of capital when a firm is financed with both debt and equity. The basic tools will be exactly the same as those in Section 5·3.A: payoff tables, promised rates of return, and expected rates of return.

This example applies to more than just houses.

From Chapter 23, we know that debt and equity are contingent claims on the underlying project. Although we continue calling this project a house (as we did in Section 5·3.A), you can consider the corporation to be the same as an unlevered house, the mortgage the same as corporate debt, the levered house equity ownership the same as corporate equity, and the possibilities of sunshine and tornadoes the same as future scenarios that the firm might face. There is absolutely no conceptual difference.

Risk Aversion means expected debt interest rates of return are lower than project expected rates of return.

The probability of sunshine is 80%, and the probability of a tornado is 20%. If the sun shines, the project is worth \$100,000; if the tornado strikes, the project is worth only \$20,000. The appropriate cost of capital (at which investors are willing to borrow or save) is 10% for the overall project. We will retain this cost of capital for the project. We had also computed earlier that the house must then be worth \$76,363.64.

Table 17.2. All Provided Inputs For Valuing The House

		Financing Scheme 1	Financing Scheme 2	
		100% Equity	Bond promises \$28,125	Levered Equity after \$28,125 obligation
prob(Sunshine)= 80%	\$100,000.00			
prob(Tornado)= 20%	\$20,000.00			
$\mathcal{E}$ Future Payoff				
$\mathcal{E}$ Rate of Return (r)		10%	6%	
Price P_0 Today				

Here we introduce different costs of capital.

The novelty is that we now assume that Treasury bonds pay a lower *expected* rate of return, which is equivalent to assuming that investors are risk-averse. The debt on the house is not exactly risk-free, though. We assume that a particular risky bond that promises to pay $28,125 does require a 6% *expected* rate of return. (This 6% expected rate of return must be higher than the true risk-free rate of return [e.g., 5%], and lower than the 10% required expected rate of return for projects that are of the riskiness of "unlevered house" ownership.) Table 17.2 summarizes our model inputs. Our goal is to determine now what the appropriate cost of capital for the levered equity is.

Compute the state-contingent payoffs.

Step 1: Determine how much the house owners receive if they own the entire house (Scheme AE for "all equity") vs. if they promise $28,125 to bond holders and retain only the levered equity (Scheme DE for "debt and equity"). Naturally, in each state, the bond and the levered equity together must own the entire house, so:

		Financing Scheme AE	Financing Scheme DE	
		100% Equity	Bond promises $28,125	Levered Equity after $28,125 obligation
prob(Sunshine)= 80%	$100,000.00	$100,000.00	$28,125.00	$71,875.00
prob(Tornado)= 20%	$20,000.00	$20,000.00	$20,000.00	$0.00
$\mathcal{E}$ Future Payoff				
$\mathcal{E}$ Rate of Return (r)		10%	6%	
Price P_0 Today				

Compute the expected payoffs.

Step 2: Compute the expected value of each security, using the probabilities of sunshine vs. tornado. Note that the expected payoffs of the bond and the levered stock together must add up to the expected payoff on the house (i.e., as if the house were 100% equity financed).

		Financing Scheme AE	Financing Scheme DE	
		100% Equity	Bond promises $28,125	Levered Equity after $28,125 obligation
prob(Sunshine)= 80%	$100,000.00	$100,000.00	$28,125.00	$71,875.00
prob(Tornado)= 20%	$20,000.00	$20,000.00	$20,000.00	$0.00
$\mathcal{E}$ Future Payoff		$84,000.00	$26,500.00	$57,500.00
$\mathcal{E}$ Rate of Return (r)		10%	6%	
Price P_0 Today				

Discount the expected payoffs on the overall project and on the debt.

Step 3: Discount the expected cash flows by the appropriate cost of capital demanded by the capital providers:

		Financing Scheme AE	Financing Scheme DE	
		100% Equity	Bond promises $28,125	Levered Equity after $28,125 obligation
prob(Sunshine)=80%	$100,000.00	$100,000.00	$28,125.00	$71,875.00
prob(Tornado)=20%	$20,000.00	$20,000.00	$20,000.00	$0.00
$\mathcal{E}$ Future Payoff		$84,000.00	$26,500.00	$57,500.00
$\mathcal{E}$ Rate of Return (r)		10%	6%	
Price P_0 Today		$76,363.64	$25,000.00	

Determine the value of the levered equity.

Step 4: Invoke our "perfect market" assumptions. Everyone can buy or sell without transaction costs, taxes, or any other impediments. By "absence of arbitrage," the value of the house if financed by a bond plus levered equity must be the same as the value of the house if it is 100% equity financed. Put differently, if you own all of the bond and all of the levered equity ownership, you own the same thing as the house—and vice-versa. Now use the arbitrage condition that the value of the levered equity plus the value of the bond should equal the total house value.

		Financing Scheme AE	Financing Scheme DE	
		100% Equity	Bond promises $28,125	Levered Equity after $28,125 obligation
prob(Sunshine)=80%	$100,000.00	$100,000.00	$28,125.00	$71,875.00
prob(Tornado)=20%	$20,000.00	$20,000.00	$20,000.00	$0.00
$\mathcal{E}$ Future Payoff		$84,000.00	$26,500.00	$57,500.00
$\mathcal{E}$ Rate of Return (r)		10%	6%	
Price P_0 Today		$76,363.64	$25,000.00	$51,363.64

Compute the appropriate expected rate of return on the levered equity.

Step 5: Levered equity ownership, which sells for $51,363.64 and expects to pay off $57,500.00, offers an expected rate of return of $\$57,500.00/\$51,363.64 - 1 = +11.95\%$.

		Scheme AE	Scheme DE	
		100% Equity	Bond promises $28,125	Levered Equity after $28,125 obligation
prob(Sunshine)=80%	$100,000.00	$100,000.00	$28,125.00	$71,875.00
prob(Tornado)=20%	$20,000.00	$20,000.00	$20,000.00	$0.00
$\mathcal{E}$ Future Payoff		$84,000.00	$26,500.00	$57,500.00
$\mathcal{E}$ Rate of Return (r)		10%	6%	11.95%
Price P_0 Today		$76,363.64	$25,000.00	$51,363.64

Small Detour: Compute the riskiness of a dollar investment in each financial instrument.

Now summarize the rates of return in the two possible states on each financial claim:

	Contingent Tornado	Contingent Sunshine	Expected Appropriate
Unlevered Ownership	$\frac{\$20,000}{\$76,364} - 1 =$	$\frac{\$100,000}{\$76,364} - 1 =$	$\frac{\$84,000}{\$76,364} - 1 =$
	−73.81%	+30.95%	+10.00%
Loan Ownership	$\frac{\$20,000}{\$25,000} - 1 =$	$\frac{\$28,125}{\$25,000} - 1 =$	$\frac{\$26,500}{\$25,000} - 1 =$
	−20.00%	+12.50%	+6.00%
Levered (post-Loan) Ownership	$\frac{\$0}{\$51,364} - 1 =$	$\frac{\$71,875}{\$51,364} - 1 =$	$\frac{\$57,500}{\$51,364} - 1 =$
	−100.00%	+39.93%	+11.95%

We started knowing only the cost of capital on our bond (6%) and on our firm (10%), and we were able to determine the cost of capital on our levered equity (11.95%).

Debt is less risky than unlevered ownership which is less risky than levered ownership.

As was also the case in the example with risk-neutral investors (Figure 5.2, Page 97), the rates of return to levered equity are more risky than those to unlevered ownership, which in turn are more risky than those to the corporate loan. But, whereas these risk differences did not affect the expected rates of return in our risk-neutral world, they do in our risk-averse world. The **cost of capital** (the expected rate of return at which you, the owner, can obtain financing) is now higher for levered equity ownership than it is for unlevered ownership, which in turn is higher than it is for loan ownership. We had worked out *exactly* how high this expected rate of return on levered equity ownership must be by resorting to the "absence of arbitrage" argument in the perfect M&M world: Given the expected rate of return on the house and on the bond, we could determine the expected rate of return on levered equity ownership. (Alternatively, if we had known the appropriate expected rate of return on levered equity ownership and the rate of return on the bond, we could have worked out the appropriate expected rate of return on unlevered ownership.)

How the CAPM fits in!

In the real world, house owners would naturally like to pay the lowest rate of return possible on loans. Similarly, anyone selling a firm would like to receive the highest price (lowest cost of capital) possible. What the issuer of the loan can get away with in the end will depend on the demand and supply of capital. Here it was 6% for the debt and 11.95% for the levered equity. However, this does not tell us *why* demand and supply met at 6% for the debt and 11.95% for the levered equity in our example (although one implies the other). So, what model can tell us the appropriate expected rate of return for a risky security? But, *of course*: the CAPM from Chapter 13! To show that everything fits together, we will put the CAPM into the WACC formula in Sections 17·7 and 17·A, below.

17·5.B. The WACC Formula (Without Taxes)

A Line-by-line derivation of the WACC formula.

We can now translate the numerical example into a formula for the "weighted average cost of capital," or WACC. No matter which state will come about, the debt and equity together own the firm, which we shall call W:

$$\begin{array}{lccccc} \text{Sunshine:} & \$28,125 & + & \$71,875 & = & \$100,000 \\ \text{Tornado:} & \$20,000 & + & \$0 & = & \$20,000 \\ \text{Any:} & \mathsf{D}_{t=1} & + & \mathsf{E}_{t=1} & = & \mathsf{W}_{t=1} \quad . \end{array} \tag{17.2}$$

Therefore, the expected value of debt and equity together must be equal to the expected value of the firm:

$$\begin{array}{ccccc} \$57,500 & + & \$26,500 & = & \$84,000 \\ \mathcal{E}(\mathsf{E}_{t=1}) & + & \mathcal{E}(\mathsf{D}_{t=1}) & = & \mathcal{E}(\mathsf{W}_{t=1}) \end{array} \tag{17.3}$$

Each expected value in the future is one plus the expected rate of return multiplied by the value today. (The value today is known; the rate of return is expected.) Thus,

$$\begin{array}{ccccc} \$51,363.64 \cdot (1+11.95\%) & + & \$25,000 \cdot (1+6\%) & = & \$76,363.64 \cdot (1+10\%) \\ \mathcal{E}\left[\mathsf{E}_{t=0} \cdot (1+\tilde{r}_{\mathsf{E}})\right] & + & \mathcal{E}\left[\mathsf{D}_{t=0} \cdot (1+\tilde{r}_{\mathsf{D}})\right] & = & \mathcal{E}\left[\mathsf{W}_{t=0} \cdot (1+\tilde{r}_{\mathsf{W}})\right] \\ \mathsf{E}_{t=0} \cdot [1+\mathcal{E}(\tilde{r}_{\mathsf{E}})] & + & \mathsf{D}_{t=0} \cdot [1+\mathcal{E}(\tilde{r}_{\mathsf{D}})] & = & \mathsf{W}_{t=0} \cdot [1+\mathcal{E}(\tilde{r}_{\mathsf{W}})] \quad . \end{array} \tag{17.4}$$

Divide by $\mathsf{W}_{t=0}$ to express this in terms of percentages of firm value:

$$\begin{array}{ccccc} \left(\frac{\$51,363.64}{\$76,363.64}\right) \cdot (1+11.95\%) & + & \left(\frac{\$25,000.00}{\$76,363.64}\right) \cdot (1+6\%) & = & 1+10\% \\ \left(\frac{\mathsf{E}_{t=0}}{\mathsf{W}_{t=0}}\right) \cdot [1+\mathcal{E}(\tilde{r}_{\mathsf{E}})] & + & \left(\frac{\mathsf{D}_{t=0}}{\mathsf{W}_{t=0}}\right) \cdot [1+\mathcal{E}(\tilde{r}_{\mathsf{D}})] & = & [1+\mathcal{E}(\tilde{r}_{\mathsf{W}})] \quad . \end{array} \tag{17.5}$$

$\mathsf{E}_{t=0}/\mathsf{W}_{t=0}$ is the weight of equity in the firm's value, so we can call it w_{E}. Similarly, $\mathsf{D}_{t=0}/\mathsf{W}_{t=0}$ is better written as w_{D}. (We really should write these as $w_{\mathsf{E},t=0}$ and $w_{\mathsf{D},t=0}$, just as we should write $r_{\mathsf{E},t=0,1}$. But no one is so explicit to put the time subscripts in, so neither will we. When no time subscript is present, we mean "now" or "from now to the future.")

$$\begin{array}{ccccc} 67.26\% \cdot (1+11.95\%) & + & 32.74\% \cdot (1+6\%) & = & 1+10\% \\ w_{\mathsf{E}} \cdot [1+\mathcal{E}(\tilde{r}_{\mathsf{E}})] & + & w_{\mathsf{D}} \cdot [1+\mathcal{E}(\tilde{r}_{\mathsf{D}})] & = & [1+\mathcal{E}(\tilde{r}_{\mathsf{W}})] \quad . \end{array} \tag{17.6}$$

Multiply the weight percentages into the brackets,

$$\begin{array}{ccccccccccc} 67.26\% & + & 67.26\% \cdot 11.95\% & + & 32.74\% & + & 32.74\% \cdot 6\% & = & 1 + 10\% \\ w_E & + & w_E \cdot \mathcal{E}(\tilde{r}_E) & + & w_D & + & w_D \cdot \mathcal{E}(\tilde{r}_D) & = & 1 + \mathcal{E}(\tilde{r}_W) & . \end{array} \tag{17.7}$$

Because debt and equity own the firm, $w_D + w_E = 1$, and the "+1" terms on both sides cancel. We have arrived at the **weighted average cost of capital** (WACC) Formula:

$$\begin{array}{ccccccc} \text{WACC} & = & 67.26\% \cdot 11.95\% & + & 32.74\% \cdot 6\% & = & 10\% \\ \text{WACC} & = & w_E \cdot \mathcal{E}(\tilde{r}_E) & + & w_D \cdot \mathcal{E}(\tilde{r}_D) & = & \mathcal{E}(\tilde{r}_W) & . \end{array} \tag{17.8}$$

Chapter 18 will explain how WACC must be modified in the presence of corporate income taxes.

The promised debt interest rate is not used in the WACC formula; the expected rate is.

One important reminder: the cost of debt in the WACC formula is *not* the interest rate that the bank is charging. The bank's quoted interest rate is the promised rate of return to debt, and therefore higher than the expected interest rate that is in the WACCformula (higher by the default premium). So, how do you find the expected rate of return on debt? Pretty much the same way as you find the expected rate of return on equity or anything else: use the CAPM (if you believe it). Conveniently, the CAPM provides the expected rates of return, which is the sum of the time-premium and the systematic risk premium, and appropriately ignores the debt's idiosyncratic risk and default premium. So you can try to estimate the beta from the debt's historical monthly rates of return, and then substitute it into the CAPM formula. Fortunately, debt betas are often fairly small, especially for short-term and low-risk debt, in which case you will end up with an $\mathcal{E}(\tilde{r}_D)$ reasonably close to the risk-free rate.

17·5.C. The Big Picture: How to Think of Debt and Equity

Important: *Think of debt and equity, contingent claims, and contingent claims valuation, in a perfect M&M world, as follows:*

The Value of Claims

- *An "absence of arbitrage" relationship ensures that the total value of a project equals the sum-total of the values of all its financing instruments.*
- *Debt and equity claims "partition" the firm's payoffs in future states of the world. These payoffs are contractually specified.*

The Risk of Claims

- *Levered equity (i.e., the residual claim after the debt is satisfied) is riskier than full ownership, which in turn is riskier than the debt.*

The Cost of Capital

- *In a world of uncertainty, riskier investments have to offer higher expected rates of return. This implies that levered equity has to offer a higher expected rate of return than outright ownership, which in turn has to offer a higher expected rate of return than the debt.*
- *The absence of arbitrage implies that the capitalization-weighted average expected rate of return is*

$$\text{WACC} = w_{Equity} \cdot \mathcal{E}(\tilde{r}_{Equity}) + w_{Debt} \cdot \mathcal{E}(\tilde{r}_{Debt}) = \mathcal{E}(\tilde{r}_{Firm}) \; , \tag{17.9}$$

where the weights w_{Equity} and w_{Debt} are the values of equity and debt when quoted as a fraction of the overall firm value today.

- *The project's WACC remains the same, no matter how the firm is financed. It is determined by the underlying payoffs of the project.*

17·6. A Major Blunder: If all securities are more risky, is the firm more risky?

Many practitioners commit a serious logical mistake. They argue as follows:

More debt does not increase the firm's cost of capital, because it increases both debt and equity cost of capital.

1. If the firm takes on more debt, the debt becomes riskier and the cost of capital for the debt (r_D) rises.
2. If the firm takes on more debt, the equity becomes riskier and the cost of capital for the equity (r_E) rises.
3. Because the firm consists of only debt and equity, the firm also becomes riskier when the firm takes on more debt.

The fact that both debt and equity become riskier as the firm takes on more leverage does not mean that the firm becomes riskier.

The first two statements are correct. With higher leverage, the cost of capital on debt increases, because the debt becomes riskier: in default, the debt is less likely to receive what it was promised. The equity also becomes riskier: the cost of capital on equity rises, because in financial default, which is now also more likely to occur, more cash goes to the creditors before equity holders receive anything. It is only the final conclusion—"therefore, the firm becomes riskier"—that is wrong. The reason is that when the firm takes on more debt, the weight on the debt (w_D) increases and the weight on the equity ($w_E = 1 - w_D$) decreases. Because the cost of capital for debt (r_D) is lower than the cost of capital for equity (r_E), the weighted sum remains the same. Here is an example, in which I have made up appropriate costs of capital to illustrate the point:

$$\begin{array}{lccccc} \text{Low Debt} & 30\% \cdot 5.0\% & + & 70\% \cdot 12.2\% & = & 10\% \\ \text{High Debt} & 60\% \cdot 6.0\% & + & 40\% \cdot 16.0\% & = & 10\% \\ & w_D \cdot \mathcal{E}(\tilde{r}_D) & + & w_E \cdot \mathcal{E}(\tilde{r}_E) & = & \mathcal{E}(\tilde{r}_W) \;. \end{array} \tag{17.10}$$

This example shows that statements 1 and 2 are correct and statement 3 is incorrect: the costs of capital for both debt and equity are higher when the firm has more leverage, but the overall cost of capital for the firm has not changed. In the perfect M&M world, the overall cost of capital is independent of the mix between debt and equity.

Solve Now!

Q 17.7 *In a perfect market, if the cost of equity for a company is 15% and the cost of debt is 10%, and if the company is financed by 80% debt and by 20% equity, what would be the company's cost of equity if it reduced its debt from 80% to 50%, so that it could qualify as a AAA rated firm with a debt interest rate of 8%?*

Q 17.8 *Compared to Firm B, Firm A has both a higher cost of capital for its debt and a higher cost of capital for its equity. Does this necessarily imply that Firm A has a higher cost of capital for the overall firm than Firm B?*

17·7. Using the CAPM and WACC Cost of Capital in the NPV Formula

NPV, WACC, and CAPM are often all used together.

We have yet to combine NPV, WACC, and the CAPM. This actually turns out to be easy: we use the CAPM to provide appropriate expected rates of return on debt and equity, compute the weighted average to obtain a WACC, and then use this WACC as the denominator in the NPV formula. For example, consider a project that can be financed with low-risk debt with a market-beta of 0.1, worth $400 today; and high-risk equity with a market-beta of 2.5, worth $250 today. The risk-free rate of return is 4%; the equity premium is 3%. What is the cost of capital of this project?

The standard method is to first compute the appropriate expected rates of return for the debt and the equity,

$$\begin{aligned} \mathcal{E}(\tilde{r}_{\text{debt},\mathcal{M}}) &= 4\% + 3\% \cdot 0.1 = 4.3\% \, , \\ \mathcal{E}(\tilde{r}_{\text{equity},\mathcal{M}}) &= 4\% + 3\% \cdot 2.5 = 11.5\% \, , \\ \mathcal{E}(\tilde{r}_{i,\mathcal{M}}) &= r_{\mathcal{F}} + [\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}] \cdot \beta_{i,\mathcal{M}} \, . \end{aligned} \tag{17.11}$$

Second, compute the WACC:

$$\begin{aligned} \text{WACC} &= \left(\frac{\$400}{\$400+\$250}\right) \cdot 4.3\% + \left(\frac{\$250}{\$400+\$250}\right) \cdot 11.5\% \\ &= 61.5\% \cdot 4.3\% + 38.5\% \cdot 11.5\% \\ &= 7.1\% \, . \end{aligned} \tag{17.12}$$

An alternative method relies on the weighted average project beta,

$$\begin{aligned} \beta_{\text{firm},\mathcal{M}} &= \left(\frac{\$400}{\$400+\$250}\right) \cdot 0.1 + \left(\frac{\$250}{\$400+\$250}\right) \cdot 2.5 = 1.025 \, . \\ \beta_{\text{firm},\mathcal{M}} &= w_{\text{debt}} \cdot \beta_{\text{debt},\mathcal{M}} + w_{\text{equity}} \cdot \beta_{\text{equity},\mathcal{M}} \, . \end{aligned} \tag{17.13}$$

This means that the project's cost of capital is

$$\begin{aligned} \mathcal{E}(\tilde{r}_{\text{firm},\mathcal{M}}) &= 4\% + 3\% \cdot 1.025 = 7.1\% \\ \mathcal{E}(\tilde{r}_{i,\mathcal{M}}) &= r_{\mathcal{F}} + [\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}] \cdot \beta_{i,\mathcal{M}} \, , \end{aligned} \tag{17.14}$$

which is the same as the cost of capital estimate in Formula 17.12. We can now use this cost of capital estimate to discount the project's expected cash flows to obtain a present value estimate. For example, if the project earns $800 with probability 48% and $600 with probability 52%, then

$$\text{PV} = \frac{48\% \cdot \$800 + 52\% \cdot \$600}{1 + 7.1\%} \approx \$650. \tag{17.15}$$

Solve Now!

Q 17.9 *Assume the risk-free rate of return is 3% and the equity premium is 4%. A firm worth $100 (all numbers in millions of dollars) has a market beta of 3. A new project costing $10 appears, which is expected to pay off $11 next year. The beta of this new project is 0.5. However, the firm evaluates all projects by its overall cost of capital. Would this firm take the project? How do the beta and the value of the firm change if it takes the project vs. if it does not take it?*

17·8. Summary

The chapter covered the following major points:

- Entrepreneurs have an incentive to set up a capital structure that maximizes firm value, not equity value, even if—or because—managers later would want to behave opportunistically.
- In the perfect market scenario of M&M, the value of all securities is equal to the value of the firm's underlying projects, and thus unrelated both to the financing split between debt and equity and to the firm's dividend policy.
- Arbitrage in this perfect world forces the firm's cost of capital to be invariant to the split between debt and equity. It is equal to the weighted average cost of capital (WACC) of debt and equity.
- Higher leverage does not imply that the overall cost of capital increases, even though both debt and equity become riskier.
- The CAPM is compatible with this perfect-world point of view.

A. Advanced Appendix: Compatibility of Beta, the WACC, and the CAPM Formulas in a Perfect World.

Reconciling Beta, WACC, and the CAPM.

For the nerd, we now show that the "leverage adjustment for beta" Formula 13.16 (Page 328), the WACC Formula 17.9 (Page 434), and the CAPM Formula 13.1 (Page 309) are mutually compatible, at least in the perfect markets scenario.

Recap WACC.

In this chapter, we developed the basic WACC formula (the cost of capital for the overall firm W—not to be confused with $\mathcal{F}$, the subscript for the risk-free security):

$$\mathcal{E}(\tilde{r}_{\text{W}}) \;=\; w_{\text{D}} \cdot \mathcal{E}(\tilde{r}_{\text{D}}) + w_{\text{E}} \cdot \mathcal{E}(\tilde{r}_{\text{E}}) \quad .$$

where W is the firm, D is the total debt, and E is the total equity of the firm.

Recap CAPM.

Substitute the CAPM Formula 13.1 into the three expected rates of return in the WACC Formula:

$$\begin{aligned} r_{\mathcal{F}} + [\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}] \cdot \beta_{\text{W},\mathcal{M}} \;&=\; w_{\text{D}} \cdot \{r_{\mathcal{F}} + [\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}] \cdot \beta_{\text{D},\mathcal{M}}\} \\ &+\; w_{\text{E}} \cdot \{r_{\mathcal{F}} + [\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}] \cdot \beta_{\text{E},\mathcal{M}}\} \quad . \end{aligned}$$

And show the compatibility.

Now pull out the risk-free rates of return,

$$\begin{aligned} & r_{\mathcal{F}} + [\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}] \cdot \beta_{\text{W},\mathcal{M}} \\ &= w_{\text{D}} \cdot r_{\mathcal{F}} + w_{\text{E}} \cdot r_{\mathcal{F}} + w_{\text{D}} \cdot \{[\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}] \cdot \beta_{\text{D},\mathcal{M}}\} + w_{\text{E}} \cdot \{[\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}] \cdot \beta_{\text{E},\mathcal{M}}\} \quad . \end{aligned}$$

Recognize that $(w_{\text{E}} + w_{\text{D}}) = 1$, so $(w_{\text{E}} + w_{\text{D}}) \cdot r_{\mathcal{F}} = r_{\mathcal{F}}$, so

$$[\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}] \cdot \beta_{\text{W},\mathcal{M}} = w_{\text{D}} \cdot [\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}] \cdot \beta_{\text{D},\mathcal{M}} \;+\; w_{\text{E}} \cdot [\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}] \cdot \beta_{\text{E},\mathcal{M}} \quad .$$

Divide by $\left[\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}\right]$ to obtain

$$\beta_{\text{W},\mathcal{M}} \;=\; w_{\text{D}} \cdot \beta_{\text{D},\mathcal{M}} + w_{\text{E}} \cdot \beta_{\text{E},\mathcal{M}} \quad ,$$

which is exactly our relationship in Formula 13.16, which relates betas to one another! Indeed, all three formulas share the same intuition: firms and securities with higher betas are riskier and thus have to offer higher expected rates of return.

Solve Now!

Q 17.10 *A firm consists of 60% equity with a market-beta of 2, and 40% debt with a market-beta of 0.25. The equity premium is 5%, the risk-free rate is 3%. Compute the firm's cost of capital from the overall-firm beta. Then compute the equity's cost of equity capital and debt's cost of debt capital, and compute the firm's cost of capital from these costs of capital.*

Solutions and Exercises

1. Ex-ante means "before the fact"; Ex-post means "after the fact." To the extent that owners can set up a situation (charter) that encourages best (i.e., from a perspective of the firm) ex-post behavior, the ex-ante value (for which the firm can be sold right now) is maximized. However, if the situation (charter) is such that owners will later try to expropriate others or such that managers make bad decisions in the future, the ex-ante value today is less.
2. Yes. See the example of the \$3 for \$1 transaction in the text.
3. A perfect market: no transaction costs, perfect competition, no taxes and bankruptcy costs, and no differences in information.
4. The idea is to explain it really simply. Milk, cream, and pizza are handy metaphors.
5. Nothing really. We do not need it. We only use it because it makes the Tables simpler to compute.
6. $$\begin{array}{rcccccc} \text{Bad Luck} & d\cdot\$55 & + & e\cdot\$5 & = & \$0\cdot 5\% \\ \text{Good Luck} & d\cdot\$55 & + & e\cdot\$105 & = & \$66\cdot 5\% \end{array} \quad\Longrightarrow\quad \begin{array}{rcl} d & \approx & -0.003 \\ e & \approx & +0.033 \end{array} \tag{17.16}$$

   So you would purchase 3.3% of the LD equity and sell (issue) 0.3% of the equivalent of the LD debt. The equity would cost you $e\cdot\$50 = \$1.65$, the debt issue would give you \$0.15 in proceeds. Your net cost would thus be \$1.50—as it should be, because purchasing 5% of the MD equity would have cost you 5% of \$30, which also comes to \$1.50.
7. First, compute the WACC without taxes: $0.8\cdot 10\% + 0.2\cdot 15\% = 11\%$. We now want to compute the cost of debt that satisfies $0.5\cdot 8\% + 0.5\cdot x = 11\%$, so $x = 14\%$.
8. No. The example in the "Important Error" Section illustrates this fallacy.
9. The solution proceeds the same way as in the text on Page 333. The project should have an appropriate rate of return of

   $$\mathcal{E}(\tilde{r}_i) = 3\% + 4\%\cdot 0.5 = 5\% \ . \tag{17.17}$$

   With a 10% expected rate of return, the project would not be taken if the firm used a beta of 3 (implying a cost of capital of 15%, which is higher than the expected rate of return of 10%). Nevertheless, the project has a net present value of

   $$\text{NPV} = -\$10 + \frac{\$10\cdot(1+10\%)}{1+5\%} = \$0.48 \ , \tag{17.18}$$

   which is lost if it is not taken. Please confirm that this is also the outcome if the combined firm value is computed.
10. Using the first method, the overall beta is $60\%\cdot 2 + 40\%\cdot 0.25 = 1.3$. Using the CAPM, the cost of capital is $\mathcal{E}(\tilde{r}_i) = r_{\mathcal{F}} + [\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}]\cdot\beta_{i,\mathcal{M}} = 3\% + 5\%\cdot 1.3 = 9.5\%$. Using the second method, the equity's cost of capital is $\mathcal{E}(\tilde{r}_{\mathsf{E}}) = r_{\mathcal{F}} + [\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}]\cdot\beta_{i,\mathcal{M}} = 3\% + 5\%\cdot 2 = 13\%$, the debt's cost of capital is $\mathcal{E}(\tilde{r}_{\mathsf{E}}) = r_{\mathcal{F}} + [\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}]\cdot\beta_{i,\mathcal{M}} = 3\% + 5\%\cdot 0.25 = 4.25$. Therefore, the firm's cost of capital is $\mathcal{E}(\tilde{r}_i) = 60\%\cdot 13\% + 40\%\cdot 4.25\% = 9.5\%$.

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

CHAPTER 18

Corporate Taxes and A Tax Advantage of Debt

The Tax-Adjusted Weighted Average Cost of Capital (APV, WACC)

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The presence of taxes in the real world may be the most glaring violation of the M&M assumptions. If the firm has to pay corporate income taxes, managers can use capital structure policy to create value. To do so, they rely either on the APV formula or on the tax-adjusted WACC formula. This chapter explains it all.

Note that the relevant tax that this chapter is concerned with is only the *corporate* income tax, not the *personal* income tax. (You can assume our shareholders—or yourself—to be a tax-exempt pension fund, if this helps you.) The next chapter will consider both corporate and personal income taxes.

18·1. Capital Budgeting If Equity and Debt Were Equally Taxed

Table 18.1. Hypothetical Firm

Investment Cost in Year 0	\$200
Before-Tax Return in Year 1	\$280
Before-Tax Net Return From Year 0 to Year 1	\$80
Corporate Income Tax Rate (τ)	30%
Appropriate Cost of Capital From 0 to 1	12%

A Basic Corporate Example

In this chapter, we will follow a simple hypothetical firm that is described in Table 18.1. As in Section 6·5.B, we follow the standard practice to denote the tax rate with the Greek letter τ (Tau). We start with a brief discussion of an entirely unrealistic case, in which the firm faces one tax rate, regardless of how it is financed. This helps to illustrate that investors care about after-tax returns, not before-tax returns.

Taxes mean that the after-tax rate of return is lower than the before-tax rate of return.

Consider financing our firm entirely with equity. With \$280 in before-tax earnings on the \$200 investment, our firm has a "before corporate income tax" internal rate of return of $(\$280 - \$200)/\$200 = 40\%$. But, with taxes to the tune of 30% on its net return of \$80, Uncle Sam collects \$24. The firm's "post corporate income tax" net rate of return is therefore only $(\$256 - \$200)/\$200 = 28\%$.

Investors demand a certain rate of return, regardless of how the firm gets there.

Now assume that financial markets are such that investors are willing to provide capital for projects with this riskiness at a rate of $\mathcal{E}(\tilde{r}_W) = 12\%$. (In this chapter, we will again omit time subscripts if there is little risk of confusion.) Naturally, investor-owners do not care what happens inside the firm, only what the firm will pay them in the end. It is all the same to them if the firm earns 12% before-tax and manages to avoid all corporate income taxes; or if the firm earns 24% and pays half of it in corporate income taxes; or if the firm earns 600%, of which 98% is confiscated by the government $(1 + 600\% \cdot (1 - 98\%) = 1 + 12\%)$. From the perspective of our small firm, we are "price-takers" when it comes to raising capital—we have to accept the rate of return asked for by our investors, which is determined by the competition among capital providers and capital consumers. The investors care only about the cash that will ultimately return to them—and this (after-corporate income tax) rate of return has to be 12%. Now, if our example firm faces a 30% corporate tax rate, it must earn 17.14% in before-tax rate of return in order to be able to offer investors 12% in actual rate of return. Check this: an investment of \$100 that turns into \$117.14 has to pay Uncle Sam 30% in taxes on income of \$17.14 for a total income tax of \$5.14, which leaves the firm \$112 to return to its investors after the corporate income tax is paid. Returning to our "\$280 before-corporate-income-tax" firm, with its 12% required after-corporate-income tax cost of capital, the firm's actual PV is

$$\text{PV} = \frac{\mathcal{E}(\text{CF}_{\text{after-tax}})}{1 + \mathcal{E}(\tilde{r}_{\text{after-tax}})} = \frac{\$256}{1 + 12\%} = \$228.57 \; . \tag{18.1}$$

Again, note that the present value is calculated with both the after-tax expected cash flows and the after-tax cost of the capital.

18·2. Differential Taxation in The U.S. Tax Code

Tax codes worldwide violate the M&M no-tax assumption.

We now move on to a more realistic tax code. In many countries—the United States included—individuals and corporations face similar tax treatments, tax schedules, and tax rates. Although tax code details vary from year to year, country to country, state to state, county to county, and even city to city, most tax codes are pretty similar in spirit. Thus, the tax concepts in this book apply relatively universally.

Tax codes subsidize borrowing: firms can pay interest from before-tax income, but pay dividends from after-tax income.

Recall from Chapter 6·4 that when corporations earn money, the form of payout matters. Firms pay taxes on their earnings *net of interest payments*. That is, unlike dividend distributions or money used to repurchase shares or money reinvested, the IRS considers interest payments to be a cost of production. Therefore, it allows the payment of interest to be treated as a before-tax expense rather than as an after-tax distribution of earnings. The result is that a corporation saves on taxes when it distributes its earnings in the form of interest payments. For example, if PepsiCo's operations really produced \$100, and if \$100 in interest was owed, then PepsiCo could pay the full \$100 to its creditors and Uncle Sam would get nothing. However, if not paid out in interest, Uncle Sam would first collect corporate income taxes, say, 30%. PepsiCo could only keep (or distribute) the \$70 that would be left over. The point of this chapter is to show how to exploit this difference in the relative tax treatment between payments to debt and all other uses of money. It allows the astute CFO to add value by choosing a clever capital structure.

Preview: With too much debt, other forces may increase the cost of capital.

At this point, you are probably wondering why you would not always finance your firm with as much debt as possible. The short answer is that if you were in a world in which corporate incomes taxes are the only distortion, then having as much corporate leverage as possible would indeed be ideal. However, there is more going on. If you take on too much corporate debt, eventually other forces will raise the firm's cost of capital to the point that further increases in debt are no longer value-increasing. These forces will be the subject of the next chapters. But let us first understand how managers should do capital budgeting when there are corporate income taxes.

Solve Now!

Q 18.1 *A debt-equity hybrid security is making a payout of \$500 to its holders. If the firm is in the 33% tax bracket, how much does the firm have to earn if the IRS designates the payment an interest payment? How much does the firm have to earn if the IRS designates the payment a dividend distribution?*

Anecdote: Special Tax Breaks and Corporate Welfare
"Special Income Tax Provisions" are tax breaks enacted by Congress for specific activities, often on behalf of a single corporation. The special income tax provisions amounts are commonly estimated to be about \$1 trillion a year—more than the total amount of federal discretionary spending! These provisions are a main reason why corporations—large corporations, really—have paid less and less in income taxes relative to the rest of the population and relative to other OECD countries. In 1965, corporate income taxes were 4.1% of U.S. GDP; in 2000, about 2.5%; in 2002, about 1.5%. And, for comparison to the 1.5% U.S. tax rate, in 2000, Germany's rate was 1.8%, Canada's rate 4.0%.

It would be wonderful if the low U.S. corporate income tax rate would attract businesses to locate into the United States and to create jobs. Alas, because the low effective corporate income tax rates come about through strange corporate tax shelters (often through relocation of headquarters into foreign countries), the United States often ends up with the worst of both worlds: both incentives for companies to move out of the United States and low corporate income tax receipts. The only president in recent history to buck the trend may have been Ronald Reagan, who slashed both the corporate income tax and the ability of companies to circumvent it.

Source: "Testimony of Robert S. McIntyre" (www.ctj.org), Director of "Citizens for Tax Justice."

18·3. Firm Value Under Different Capital Structures

Introducing an interest tax subsidy leads to a corporate preference for debt.

You know that firms are indifferent between debt and equity in a perfect world. You also know that Uncle Sam subsidizes firms that pay interest, relative to firms that either retain earnings or pay dividends or repurchase shares. Therefore, you know that, *on tax grounds*, firms should have a preference for debt. Your goal now is to determine the exact value of the firm in the presence of this tax subsidy for debt interest payments.

The next chapter describes why too much debt can create other costs, which raise the firm's overall cost of capital. Fortunately, the formulas in this chapter will still hold, because these other costs will be reflected only in the cost of capital ($\mathcal{E}(\tilde{r})$) components.

Table 18.2. Two Financing Scenarios for a 1-Year Firm

Investment Cost in Year 0	$200.00
Before-Tax Return in Year 1	$280.00
Before-Tax Net Return From Year 0 to Year 1	$80.00
Corporate Income Tax Rate (τ)	30%
Appropriate Average Cost of Capital From 0 to 1	12%

Scenario EF: 100% Equity Financing.

Taxable Profits Next Year	$80.00
Corporate Income Taxes Next Year (30% of $80)	$24.00
Owners Will Keep *Next Year*	$56.00

Scenario DF: $200 Debt Financing at 11%. Rest is Levered Equity.

Interest Payments	$22.00
Taxable Profits Next Year	$58.00
Corporate Income Taxes Next Year	$17.40
Equity Owners Will Keep Next Year	$40.60
Equity+Debt Owners Will Keep *Next Year*	$22.00 + $40.60 = $62.60

18·3.A. Future Corporate Income Taxes and Owner Returns

There is more money in distribution for owners if the firm is debt financed.

We begin with Table 18.2, which works out the value of a specific firm under two financing scenarios.

Equity Financing (EF) Scenario In the all-equity scenario, the firm does not exploit the help of the IRS. It earns $280 on an investment of $200. At a 30% corporate income tax rate, it will pay corporate income taxes of 30% · $80 = $24. It can then pay out the remaining $56 in dividends.

Debt Financing (DF) Scenario In the debt financing scenario, the firm borrows $200 today at an interest rate of 11% for interest payments next year of $22. Therefore, its corporate profits will be $80 − $22 = $58, on which it would have to pay Uncle Sam $17.40. This

> permits owners (creditors and shareholders—and you may be both) to receive $62.60, the sum of $22 for its creditors and $40.60 for its equity holders.

Relative to the 100% equity financed case (in which owners keep $56.00), the debt financed case (in which owners keep $62.60) increases the firm's after-tax cash flow by $6.60. A quicker way to compute the tax savings is to multiply the tax rate by the interest payments: If the IRS allows the firm to deduct $22 in interest payments, the firm will save $22·30% = $6.60 in corporate income taxes. These $6.60 in tax savings will occur next year, and will therefore have to be discounted back—but at what cost of capital?

18·3.B. The Discount Factor on Tax Obligations and Tax Shelters

We know the future tax-related cash flows. How do you discount them? Let's work a simple example with risky payoffs.

The question is, What should you use as the appropriate discount rate (cost of capital) for the future tax obligation ($24 in EF, $17.40 in DF) or for the relative tax shelter ($6.60)? To gain intuition about proper discount rates for tax payments and tax shelters, work an example. It will be easiest if the debt is risk-free, so only the equity is risky. Assume that the value of the firm with $280 in expected profits will be either $250 (bad) or $310 (good) with equal probability. Therefore, the $200 debt at 11% interest is risk-free. Because we have constructed it this way, you know that for any cash flow that does not covary with the firm's outcome, you can use the debt's (risk-free) cost of capital of 11%; for any cash flow that covaries with the firm's outcome, you use a different discount rate. Because we now assume that the firm's beta is positive, the firm's equity cost of capital exceeds its debt cost of capital.

Table 18.3. Two Financing Scenarios for a 1-Year Firm

Scenario EF: All Equity Financing.

	$\mathcal{E}$(Value)	Bad	Good
Before-Tax Return Next Year	$280.00	$250.00	$310.00
Taxable Profits Next Year	$80.00	$50.00	$110.00
Corporate Income Taxes Next Year	$24.00	$15.00	$33.00
Owners Will Keep Next Year	$56.00	$35.00	$77.00

Scenario DF: $200 Debt Financing at 11%. Rest is Levered Equity.

	$\mathcal{E}$(Value)	Bad	Good
Before-Tax Return Next Year	$280.00	$250.00	$310.00
Interest Payments	$22.00	$22.00	$22.00
Taxable Profits Next Year	$58.00	$28.00	$88.00
Corporate Income Taxes Next Year	$17.40	$8.40	$26.40
Equity Owners Will Keep Next Year	$40.60	$19.60	$61.60
Equity+Debt Owners Will Keep Next Year	$62.60	$41.60	$83.60

Tax Savings (Scenario EF vs. Scenario DF)

	$\mathcal{E}$(Value)	Bad	Good
Before-Tax Return Next Year	$280.00	$250.00	$310.00
Scenario 1 corporate income taxes	$24.00	$15.00	$33.00
Scenario 2 corporate income taxes	$17.40	$8.40	$26.40
Relative Net Tax Savings Next Year	$6.60	$6.60	$6.60

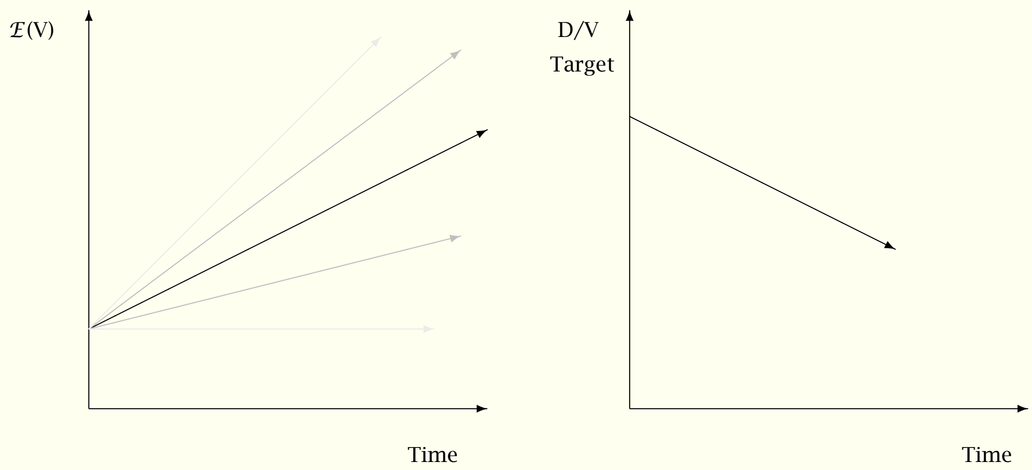
The tax payment is as risky as the firm, and thus warrants a higher cost of capital.

Table 18.3 shows that the income tax obligation is risky and covaries with the firm's return under either financing scenario. Uncle Sam is basically a co-owner, partaking in the good and the bad times. The important point is that you need to use a discount rate on the tax *obligation* that is higher than the risk-free rate. At this point, you could try to work out an exact cost of capital for the tax obligation (using the algebra of betas), but *it is common practice to just apply the firm's cost of capital also to the firm's tax obligation.* Doing so is not exactly correct, but it is a good working assumption. We punt because it would be very difficult to derive the correct discount rate—and you might even end up being inconsistent. If you wanted to get discount rates on individual component cash flows 100% right, why stop with the corporate income tax payments? Why not also determine individual discount rates for every other component of the company (taxes, depreciation, SG&A, marketing, paper clips, etc.)? This is not only impractical, but also beyond anyone's capabilities. More importantly, you have to convince yourselves that that any added valuation precision would be very modest—so we will do so later in this chapter.

The tax shelter is safer than the firm, and thus warrants a lower cost of capital.

The bottom panel in Table 18.3 illustrates a second interesting fact: the tax savings due to debt remain the same \$6.60, regardless of firm performance. Thus, the important point is that you need to use a discount rate on the tax *savings* that is close to the risk-free rate—your firm's debt cost of capital. It turns out that although our example gives good basic intuition that the tax shelter is not risky (the tax saving does not vary with firm value, accruing to owners no matter what the outcome will be), this only works in this simple one-period scenario. The reason is that if your firm value doubles by next year, you can probably borrow twice as much then, and thus enjoy higher tax savings henceforth. If your firm follows such an intelligent dynamic borrowing strategy, the tax shelter obtained by debt financing will not remain constant, but will increase with the firm value, too. To compute the lifetime tax shelter afforded to your firm by its ability to take on more debt, you must therefore realize that intelligent capital structure policies will induce the dollar amount of debt (and thus the tax shelter) to also covary positively with firm value. For this reason, it is often sensible to discount the tax shelter not with the debt's cost of capital, but with the firm's cost of capital (or a discount rate somewhere in between).

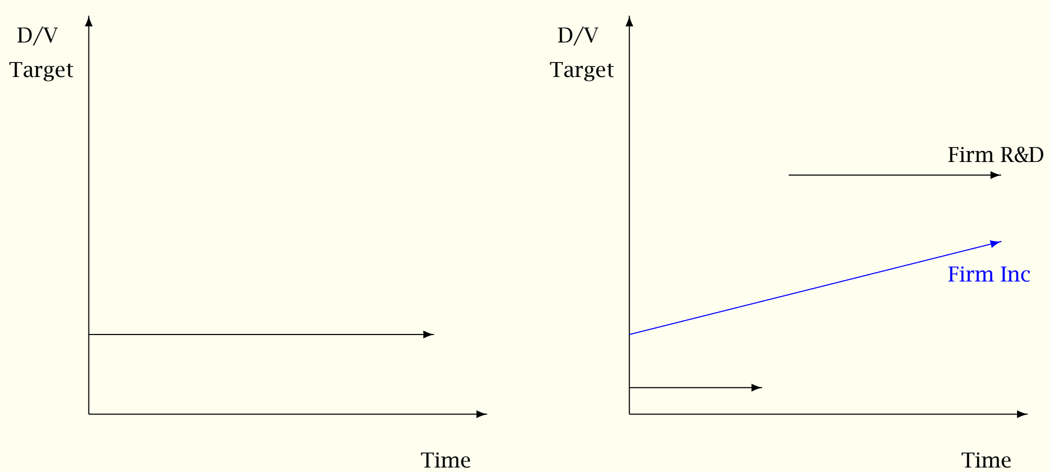
Figure 18.1. Thinking About Proper Discount Rates For The Tax Shelter



The background of the other three graphs: The typical firm value grows over time.

This firm plans to reduce its debt ratio over time, perhaps to keep its dollar debt and its interest payments constant.

Use $\mathcal{E}(\tilde{r}_D)$ to discount the tax shelter.



This firm plans to keep its debt ratio constant.

Use $\mathcal{E}(\tilde{r}_W)$ to discount the tax shelter.

These firms plan to inc A typical R&D Project and a firm that plans to raise its debt ratio over time.

Use $\mathcal{E}(\tilde{r}_E)$ to discount the tax shelter.

Modest errors in the discount rate applied to the tax shelter rarely make a large difference in the valuation.

Reasonable costs of capital for the tax shelter depend on the dynamic debt policy.

Figure 18.1 should help you to think about reasonable choices for the discount rate on the tax shelter. We assume that we are dealing with a typical firm, which tends to grow over time (upper left).

A Decreasing Debt Target The upper right graph shows a firm that plans to reduce its debt ratio over time. (The empirical evidence shows that the debt ratios of most publicly traded firms follow such paths.) This is the case if a growing firm wants to retain the same absolute dollar interest payments. Such a firm would expect to save about the same dollar amount in taxes each year, regardless of firm performance. In this case, you should use some rate close to the debt cost of capital ($\mathcal{E}(\tilde{r}_D)$).

A Constant Debt Target The lower left graph shows a firm that plans to keep a constant debt target. (Many CFOs pay lip service to targeting constant debt ratios.) Firm growth will translate into more and more debt and thus into higher and higher dollar interest payments. Consequently, the tax shelter will grow and shrink with the value of the firm, which means that it will be exposed to about the same risk as the firm overall. In turn, this means that you should use some rate close to the firm's overall cost of capital ($\mathcal{E}(\tilde{r}_W)$) to discount the tax shelter.

An Increasing Debt Target The lower right graph shows two firms with increasing debt targets. (This kind of debt policy is a very rare.) The firm with the discontinuous debt target might be a typical R&D project, which will initially provide no debt capacity and thus no debt tax shelter. Thereafter, if the R&D pays off, the firm has positive cash flows and can take on debt financing. The blue continuous line is a firm that wants to become smoothly more aggressive in its debt policy over time. The values of these tax shelters is even more highly correlated with the value of the firm than if the target is constant. Therefore, the tax shelter should be discounted even more aggressively. You should use some rate above the firm's overall cost of capital, perhaps something close to the equity cost of capital, $\mathcal{E}(\tilde{r}_E)$.

Taxes are important; we are only fudging the divisor, not the numerator!

Of course, this entire discussion is only about the discount factor: it is not about the idiosyncratic risk in the expected tax shelter, a quantity that figures into the present value numerator, not the denominator. For example, the R&D project may not generate any tax shelter half the time—in which case, the expected tax shelter (in the PV numerator) to be discounted would be something like

$$\text{Expected Tax Shelter} \;=\; 50\% \cdot \left(\begin{array}{c}\text{Tax Shelter If R\&D is Successful}\\ \text{riptsize = Tax-Rate Times Interest Paid}\end{array}\right) + 50\% \cdot \$0 \ . \qquad (18.2)$$

Here is why the appropriate discount rate on the tax shelter is not too critical.

At this point, you probably wonder why I am advising you to not worry too much about getting the discount rate on the tax shelter wrong. The reason is that it will make your life a lot easier, without sacrificing meaningful real-world accuracy. Let's put reasonable mistakes into proper perspective: The debt in a typical large firm may be about 25% of the firm value. The interest rate paid in one year is therefore about 5% on this 25% of the firm value. The tax shelter itself is therefore only about 30%–40% (the corporate income tax rate) times 5% times 25% of the firm value. And now you are wondering if you should divide this modest number by, say, 1.10 (a reasonable firm cost of capital) or by 1.05 (a reasonable debt cost of capital)?! (For tax shelters far in the future, the tax shelter will be heavily discounted to be an even smaller number.) For example, in our Scenario DF in Table 18.3, how much does the choice of discount rate on the tax shelter matter? If you use an 11% discount factor on the \$6.60 tax shelter, you would conclude that debt will save you the equivalent of \$5.95. If you use a 12% discount factor, you would conclude that debt will save you \$5.89. The difference of \$0.05 on an expected before-tax return of \$280 is obviously inconsequential. Even if you committed the most "extreme" possible error here, which is using a zero discount rate on the tax shelter, forgetting even about the time value of money, you would misestimate only 60 cents on \$280 in expected return. In almost all real-world corporate decisions, your uncertainty about your \$280 cash flow estimate will dwarf this 60 cents.

Think about the appropriate discount rate on the tax shelter, but don't torture yourself to get it perfect.

Of course, you are already ignoring many other real-world tax issues, which are simply too hard to work out correctly, or where the gained accuracy improvement would again often be trivial. For example, delayed income tax payments, tax-loss carryforwards, recapture of past tax payments, different marginal corporate income tax rates at different income levels, and the possibility to default on income tax payments throw more wrenches into our machine (well, into our tax shelter valuation estimate). I hope you are convinced that your overall project valuation will be robust with respect to moderate variations in the choice of discount rate on the tax shelter. (I typically use whatever is most convenient, although I try to keep track of whether I think my assumptions overestimate or underestimate the true firm value.) Worrying about these issues is not too worthwhile, except to some egghead academics—like myself. So, please, give yourself a break here!

IMPORTANT:

- *The discount rates on the tax obligations and on the tax shelter are usually not exact, but just reasonable and convenient approximations. The value consequences of reasonable errors are minor.*
- *It is common and usually reasonable to value tax liabilities at a discount rate equal to the firm's overall cost of capital ($\mathcal{E}(\tilde{r}_W)$).*
- *For the tax shelter due to interest payments, assuming that the firm will grow over time, it is common and usually reasonable to*
  - *use the debt cost of capital ($\mathcal{E}(\tilde{r}_D)$) if the firm plans on a decreasing debt ratio;*
  - *use the firm cost of capital ($\mathcal{E}(\tilde{r}_W)$) if the firm plans on a constant debt ratio;*
  - *use the equity cost of capital ($\mathcal{E}(\tilde{r}_E)$) if the firm plans on an increasing debt ratio.*

Solve Now!

Q 18.2 *A \$1 million construction project is expected to return \$1.2 million. You are in a 45% combined federal and state marginal income tax bracket. Your annual income is \$200,000 per year. If you finance the project with an \$800,000 mortgage at an interest rate of 5%, how much will Uncle Sam receive? If you finance the project with cash, how much will Uncle Sam receive?*

Q 18.3 *Continue. If the appropriate project interest rate is 8%, what is the PV of the tax savings from financing the project with a mortgage?*

ANECDOTE: The RJR Buyout Tax Loophole
In a **Leveraged Buyout** (**L.B.O.** for short) leverage can increase dramatically—and this can significantly reduce corporate income taxes. In 1988, First Boston's plan to take over R.J.R. Nabisco relied on an esoteric tax loophole just about to be closed. By "monetizing" its food operations, a fancy way to increase leverage, the deferring of taxes would have saved an estimated \$3-\$4 billion dollars of R.J.R.'s corporate income taxes—which would have increased the annual federal U.S. deficit by 2 percent!

18·4. Formulaic Valuation Methods: APV and WACC

You now know how to compute the tax cash flows and the tax shelter. You have also thought about reasonable discount rates. Your next goal is to find a valuation formula that allows you to compute the firm value today not only for the current financing arrangement, but also for other financing arrangements that you might contemplate for your firm. You can essentially use three methods to value the firms under different financing scenarios:

But we need a formula that works for intermediate leverage. The choices are WACC, APV, or financials.

1. You can construct the pro formas for the firm under the new hypothetical capital structure, and then value the cash flows directly. Without describing it as such, you have actually already done this in Chapter 9, but we shall work out another example in the next section.
2. You can compute an **adjusted present value** (or **APV** for short), which adds back the tax subsidy. (This is basically what you have been doing above.)
3. You can generalize the **weighted average cost of capital** (WACC) formula to reflect the preferential treatment of debt by suitably lowering the debt's cost of capital.

This section explains the APV and WACC in some detail. The next section will work out a multi-year example that shows how to use them. Properly applied, all methods should provide very similar answers.

18·4.A. Adjusted Present Value (APV): Theory

APV decomposes the value of the firm into two components: the value of the firm if it were all equity-financed and fully taxed, plus a tax subsidy for each dollar that can be declared as "interest" rather than as "dividend." In our example from Table 18.2, \$256 (\$280 profit minus \$24 in corporate taxes) is the expected cash flow of our firm if it is 100% equity financed. The APV method then adds the tax subsidy. For example,

The main idea of APV: value the firm, then add the tax subsidy.

Zero interest payments If the firm is all equity financed, the tax subsidy is zero.

High interest payments If the firm could have the interest payments of, say a utopian \$80, the IRS would believe that the firm had not earned a penny. Therefore, the owners could keep an extra \$24 above the \$256 all-taxed scenario *next year*.

Normal interest payments If the firm will have interest payments of, say \$19, the IRS would see \$280 – \$19 = \$261 in return minus \$200 investment cost for a net return of \$61. The IRS would therefore collect 30%·\$61 = \$18.30, which is \$5.70 less than the \$24 that the IRS would have collected if the firm had been 100% equity financed. Alternatively, you could have directly calculated the expected tax savings as $\tau \cdot (\mathcal{E}(\tilde{r}_{\mathrm{D}}) \cdot \mathrm{D}) = 30\% \cdot (\$19) = \$5.70$.

Let's make a formula out of this method. Your first step to a more general valuation formula in the presence of corporate income taxes is to relate the amount of debt today to the interest payments next year. Let's return to our example, in which you borrow \$200 at an interest rate of 11%. The expected interest payments are now

Computing tax savings, given the debt level.

$$\begin{aligned} \text{Expected Interest Payment} &= 11\% \cdot \$200 = \$22 \\ \text{Expected Interest Payment} &= \mathcal{E}(\tilde{r}_{\mathrm{D}}) \cdot \mathrm{D} \quad . \end{aligned} \tag{18.3}$$

One important error to avoid is that you must use the *expected* debt interest (of 11%), not the *quoted* bank interest rate, which could be considerably higher than 11%. (This would not matter for large firms that are not too highly levered, but it would matter elsewhere.) Continuing on, the future tax savings *relative to an all-equity financed firm* is the amount of corporate income tax that the firm will *not* have to pay on the interest.

$$\begin{aligned} \text{Expected Tax Saving} &= 30\% \cdot [11\% \cdot \$200] = \$6.60 \\ \text{Expected Tax Saving} &= \tau \cdot [\mathcal{E}(\tilde{r}_D) \cdot D] \quad . \end{aligned} \tag{18.4}$$

In words, Uncle Sam will expect to receive \$6.60 less from the owners of the project, because \$22 in profit repatriation was designated as "interest."

APV discounts tax savings and adds them to an all-equity type hypothetical firm.

The \$6.60 in tax savings have to be discounted, because they will occur next year. So the APV Formula computes the discounted value of an all-equity financed firm (with after-tax cash flows of \$256 next year), and then adds back the *discounted* tax savings:

$$\begin{aligned} &\text{\$200 debt at 11\% interest, i.e.,\$22 interest payment discounted at 11\%:} \quad \text{APV} = \frac{\$256}{1+12\%} + \frac{30\% \cdot \$22}{1+11\%} = \$234.52 \\ &\text{APV} = \frac{\mathcal{E}(\text{CF})}{1+\mathcal{E}(\tilde{r}_W)} + \frac{\tau \cdot [\mathcal{E}(\tilde{r}_D) \cdot D]}{1+\mathcal{E}(\tilde{r}_D)} \\ &\text{APV} = \text{Value as if 100\% equity financed} + \text{Tax Subsidy from Interest Payments} \quad . \end{aligned} \tag{18.5}$$

As described at length in Section 18·3.B, you could also reasonably use the firm's cost of capital to discount the tax savings,

$$\begin{aligned} &\text{\$200 debt at 11\% interest, i.e.,\$22 interest payment discounted at 12\%:} \quad \text{APV} = \frac{\$256}{1+12\%} + \frac{30\% \cdot \$22}{1+12\%} = \$234.46 \\ &\text{APV} = \frac{\mathcal{E}(\text{CF})}{1+\mathcal{E}(\tilde{r}_W)} + \frac{\tau \cdot [\mathcal{E}(\tilde{r}_D) \cdot D]}{1+\mathcal{E}(\tilde{r}_W)} \\ &\text{APV} = \text{Value as if 100\% equity financed} + \text{Tax Subsidy from Interest Payments} \quad . \end{aligned} \tag{18.6}$$

The difference of 6 cents is trivial. APV generalizes easily to multiple years: just compute the tax savings for each year and add them up, the same way that you would add up present values.

IMPORTANT: *The Adjusted Present Value (APV) Formula computes an "as if all equity financed" PV (i.e., after corporate income tax) and then adds back the tax subsidy:*

$$\text{APV}_0 = \textit{Value as if Firm is 100\% equity financed and fully taxed.} + \textit{Tax Subsidies from Interest Payments} \tag{18.7}$$

If the project lasts for only one period, this translates into

$$\text{APV}_0 = \frac{\mathcal{E}(CF_1)}{1+\mathcal{E}(\tilde{r}_W)} + \frac{\mathcal{E}(\tau \cdot \overbrace{\tilde{r}_D \cdot D}^{\text{interest payment}})}{1+\mathcal{E}(\tilde{r}_D)} \quad . \tag{18.8}$$

This generalizes easily to more than one period—you just add up the properly discounted expected cashflows and expected tax savings over all future time periods. We will work such a multi-period example in the next section.

18·4.B. APV: Application to a 60/40 Debt Financing Case

An example of how to value a firm financed with 60% debt.

In our example, the firm with \$200 debt is worth \$234.46 today, which comes to a debt ratio of \$200/\$234.46 ≈ 85%. Now assume that the firm instead considers a new capital structure, in which it would borrow only \$139.16. The firm has determined that this lower-debt capital structure would reduce its debt cost of capital to 9% per annum—after all, at such low levels, it is risk-free, so risk-averse investors would be willing to accept a lower *expected* rate of return. What would the firm's value become?

Problem solved.

According to the APV Formula, you begin with the value of a 100%-equity firm, which is \$256/(1+ 12%), and add back the tax subsidy. Interest payments on \$139.16 of debt will be 9%·\$139.16 = \$12.52 *next year*. Taxes saved will be 30%·\$12.52 = \$3.76 *next year*. This is worth \$3.45 *today*. Therefore

$$
\begin{aligned}
\text{APV} &= \frac{\$256.00}{1+12\%} + \frac{30\%\cdot 9\%\cdot \$139.16}{1+9\%} \\
&= \$228.57 + \$3.45 = \$232.02 \\
\text{APV} &= \frac{\mathcal{E}(\text{CF})}{1+\mathcal{E}(\tilde{r}_W)} + \frac{\tau\cdot\mathcal{E}(\tilde{r}_D)\cdot D}{1+\mathcal{E}(\tilde{r}_D)} \\
&= \text{“as if all equity financed” firm} + \text{tax subsidy} \quad .
\end{aligned}
\tag{18.9}
$$

If you prefer discounting the expected tax shelter with the firm's cost of capital, use

$$
\begin{aligned}
\text{APV} &= \frac{\$256.00}{1+12\%} + \frac{30\%\cdot 9\%\cdot \$139.16}{1+12\%} \\
&= \$228.57 + \$3.36 = \$231.93 \\
\text{APV} &= \frac{\mathcal{E}(\text{CF})}{1+\mathcal{E}(\tilde{r}_W)} + \frac{\tau\cdot\mathcal{E}(\tilde{r}_D)\cdot D}{1+\mathcal{E}(\tilde{r}_W)} \\
&= \text{“as if all equity financed” firm} + \text{tax subsidy} \quad .
\end{aligned}
\tag{18.10}
$$

(Again, the cost of capital on the tax shelter makes little difference, here only \$3.45 − \$3.36 = \$0.09.) This is the APV answer: In the presence of corporate income taxes, a firm financed with \$139.16 in debt would be worth about \$232.

18·4.C. Tax-Adjusted Weighted Average Cost of Capital (WACC) Valuation: Theory

To show that WACC and APV are compatible, we derive the tax-adjusted WACC Formula from the APV Formula.

The second method to compute the value of the firm uses a tax-adjusted weighted average cost of capital formula. If you start with the APV Formula and manipulate it, it will be most apparent that the two methods can yield the same value, at least if you start from Formula 18.10. We will stick with the same parameters: our 60/40 debt/equity financing, a 30% corporate income tax rate, a 9% cost of debt capital, and \$280 in before-tax return (\$256 after-tax return). As before, the firm borrows \$139.16 at a 9% interest rate for net interest payments of \$12.52. The corporate income tax shield is 30% of \$12.52 or \$3.76. The APV Formula 18.10 values the firm at

$$
\begin{aligned}
\text{PV} &= \frac{\$256}{1+12\%} + \frac{\overbrace{30\%\cdot\overbrace{(9\%\cdot\$139.16)}^{\$12.52}}^{=\$3.76}}{1+12\%} = \$231.93 \\
\text{PV} &= \frac{\mathcal{E}(\text{CF})}{1+\mathcal{E}(\tilde{r}_W)} + \frac{\tau\cdot[\mathcal{E}(\tilde{r}_D)\cdot D]}{1+\mathcal{E}(\tilde{r}_W)} \quad .
\end{aligned}
\tag{18.11}
$$

The main difference between APV and WACC is that the WACC method expresses the debt as a ratio of firm value,

$$\begin{array}{rclcrcl} 60\% & = & \$139.16/\$231.93 & & \$139.16 & = & 60\% \cdot \$231.93 \\ w_D & = & \text{D}/\text{PV} & \Longrightarrow & \text{D} & = & w_D \cdot \text{PV} \quad . \end{array} \tag{18.12}$$

Substitute this D into the APV formula,

$$\begin{array}{rcl} \text{PV} & = & \dfrac{\$256}{1+12\%} + \dfrac{30\% \cdot [9\% \cdot (60\% \cdot \$231.93)]}{1+12\%} = \$231.93 \\ \text{PV} & = & \dfrac{\mathcal{E}(\text{CF})}{1+\mathcal{E}(\tilde{r}_{\text{W}})} + \dfrac{\tau \cdot [\mathcal{E}(\tilde{r}_{\text{D}}) \cdot (w_{\text{D}} \cdot \text{PV})]}{1+\mathcal{E}(\tilde{r}_{\text{W}})} \quad . \end{array} \tag{18.13}$$

We now have PV on both sides of the equation, so you want to solve for PV. This requires a couple of algebraic steps.

1. Multiply both sides by $[1 + \mathcal{E}(\tilde{r}_{\text{W}})] = (1 + 12\%)$ to make the denominator disappear,

$$\begin{array}{rcl} (1+12\%) \cdot \$231.93 & = & \$256 + 30\% \cdot [9\% \cdot (60\% \cdot \$231.93)] \\ [1+\mathcal{E}(\tilde{r}_{\text{W}})] \cdot \text{PV} & = & \mathcal{E}(\text{CF}) + \tau \cdot [\mathcal{E}(\tilde{r}_{\text{D}}) \cdot (w_D \cdot \text{PV})] \quad . \end{array} \tag{18.14}$$

2. Move the second term to the left side,

$$\begin{array}{rcl} (1+12\%) \cdot \$231.93 - 30\% \cdot [9\% \cdot (60\% \cdot \$231.93)] & = & \$256 \\ [1+\mathcal{E}(\tilde{r}_{\text{W}})] \cdot \text{PV} - \tau \cdot [\mathcal{E}(\tilde{r}_{\text{D}}) \cdot (w_D \cdot \text{PV})] & = & \mathcal{E}(\text{CF}) \quad . \end{array} \tag{18.15}$$

3. Pull out the PV,

$$\begin{array}{rcl} \$231.93 \cdot [1 + 12\% - 30\% \cdot 9\% \cdot 60\%] & = & \$256 \\ \text{PV} \cdot [1 + \mathcal{E}(\tilde{r}_{\text{W}}) - \tau \cdot \mathcal{E}(\tilde{r}_{\text{D}}) \cdot w_{\text{D}}] & = & \mathcal{E}(\text{CF}) \quad . \end{array} \tag{18.16}$$

4. Divide both sides by the PV multiplier,

$$\begin{array}{rcccl} \$231.93 & = & \dfrac{\$256}{1+12\% - 30\% \cdot 9\% \cdot 60\%} & = & \dfrac{\$256}{1+10.38\%} \\ \text{PV} & = & \dfrac{\mathcal{E}(\text{CF})}{1+\mathcal{E}(\tilde{r}_{\text{W}}) - \tau \cdot [\mathcal{E}(\tilde{r}_{\text{D}}) \cdot w_{\text{D}}]} & = & \dfrac{\mathcal{E}(\text{CF})}{1+\text{WACC}} \quad . \end{array} \tag{18.17}$$

This is the “real” WACC valuation formula, i.e., the WACC formula that also works when firms pay corporate income tax. The big idea of tax-adjusted WACC is to discount the “as-if-100%-equity financed and fully taxed” cash flows (of $\mathcal{E}(\text{CF}) = \$256$), not with firm's cost of capital $\mathcal{E}(\tilde{r}_{\text{W}}) = 12\%$, but with a reduced interest rate that comes from the corporate income tax subsidy on interest payments. The term which does this—relative to our earlier no-tax WACC Formula 17.9—is $\tau \cdot w_{\text{D}} \cdot \mathcal{E}(\tilde{r}_{\text{D}}) = 30\% \cdot 60\% \cdot 9\% \approx 1.62\%$. Therefore, your revised discount rate is $1 + 12\% - 30\% \cdot 9\% \cdot 60\% \approx 1 + 10.38\%$. 10.38% is the (tax-adjusted) WACC.

The more common form of WACC also breaks out equity cost of capital.

The WACC formula is often slightly rearranged. Split $\mathcal{E}(\tilde{r}_W)$ into its cost of equity and cost of debt components, $\mathcal{E}(\tilde{r}_W) = w_D \cdot \mathcal{E}(\tilde{r}_D) + w_D \cdot \mathcal{E}(\tilde{r}_D)$. In our example, to keep the weighted average firm cost of capital at the constant $\mathcal{E}(\tilde{r}_W) = 12\%$, solve $\mathcal{E}(\tilde{r}_W) = w_D \cdot \mathcal{E}(\tilde{r}_D) + w_E \cdot \mathcal{E}(\tilde{r}_E) = 60\% \cdot 9\% + 40\% \cdot \mathcal{E}(\tilde{r}_E) = 12\%$, and find $\mathcal{E}(\tilde{r}_E) = 16.5\%$. Substitute this into Formula 18.17, and you get the more common version of the WACC formula,

$$\begin{aligned}
\text{PV} &= \frac{\$256}{1 + 10.38\%} = \frac{\$256}{1 + 40\% \cdot 16.5\% + (1 - 30\%) \cdot 60\% \cdot 9\%} \\
\text{PV} &= \frac{\mathcal{E}(\text{CF})}{1 + \text{WACC}} = \frac{\mathcal{E}(\text{CF})}{1 + w_E \cdot \mathcal{E}(\tilde{r}_E) + (1 - \tau) \cdot w_D \cdot \mathcal{E}(\tilde{r}_D)} \quad .
\end{aligned} \tag{18.18}$$

The tax-adjusted WACC generalizes ordinary WACC.

Your new WACC formula generalizes your old M&M WACC formula, because when the corporate tax rate τ is zero, the tax subsidy is useless, τ is zero, and the tax-adjusted WACC formula simplifies to your older and simpler WACC formula. But your new WACC formula can also handle firms with positive corporate income tax rates. (Incidentally, about half of all publicly traded firms in the United States have a marginal tax rate of zero, e.g., due to tax-loss carryforwards or due to clever tax shelters. For these companies, the corporate tax rate may indeed be close to zero, which means that they cannot use debt to reduce their cost of capital—and they could therefore use the simplified M&M version of the WACC formula which ignores the tax subsidy of interest.)

IMPORTANT:

- *The (tax-adjusted) weighted average cost of capital (WACC) formula discounts the future cash flows with a lower cost of capital that reflects the corporate income tax shelter,*

$$PV = \frac{\mathcal{E}(CF)}{1 + \text{WACC}} \quad , \tag{18.19}$$

$$\begin{aligned}
\textit{where}\ \text{WACC} &= \mathcal{E}(\tilde{r}_W) - \tau \cdot \mathcal{E}(\tilde{r}_D) \cdot w_D \\
&= w_E \cdot \mathcal{E}(\tilde{r}_E) + w_D \cdot \mathcal{E}(\tilde{r}_D) \cdot (1 - \tau) \quad .
\end{aligned} \tag{18.20}$$

  The expected cash flows must be the cash flows "as if the firm were all equity financed and therefore fully taxed."

- *This formula is a generalization of the* WACC *formula from the perfect M&M world. It is therefore this formula that is usually called the* WACC *formula.*

- *It is not clear how to use the* WACC *formula in a multi-period setting.*

Unfortunately, you can only use the WACC formula in a multi-period setting *if* the cost of capital, the firm's debt ratio, and the firm's tax rate stay constant. In this case, a present value formula would look something like

$$\text{PV}_0 = \sum_{t=1}^{\infty} \frac{\mathcal{E}(\text{CF}_t)}{\{1 + [\ w_E \cdot r_{E,0,t} + w_D \cdot \mathcal{E}(\tilde{r}_{D,0,t}) \cdot (1 - \tau)\ ]\}^t} \quad . \tag{18.21}$$

If these rates are not all constant, no one knows how to compute a proper WACC. Therefore, the WACC formula is often just a rough approximation. Then again, the other errors in our simplified model of taxes that we mentioned on Page 449 would probably dwarf the errors that would come from assuming constant discount rates.

DIGGING DEEPER: *The **Hamada Equation** gives a formula to adjust a firm's beta for both leverage and corporate income taxes:*

$$\beta_{riptsize\ Levered} = \beta_{Unlevered} \cdot [1 + (1 - \tau) \cdot (D/E)] \quad . \tag{18.22}$$

Solve Now!

Q 18.4 *Consider a 25/75 debt/equity financing case for your firm: the before-tax return is \$280, the tax rate is 30%, the overall cost of capital is 12%, and debt when the firm is 25% debt financed must offer an expected rate of return of 8%. First compute the* WACC*, then compute the debt as 25% the* WACC *value, and show how the* APV *yields the same result.*

Q 18.5 *Consider financing your firm with \$100 debt: the before-tax return is \$280, the tax rate is 30%, the overall cost of capital is 12%, and this debt must offer an expected rate of return of 8.7%. First compute the* APV*, then compute the capital structure in ratios, and finally show that the* WACC *yields the same result.*

Q 18.6 *If you are thinking of debt in terms of a (constant) fraction of firm value, would you prefer* WACC *or* APV*? If you are thinking of debt in terms of a (constant) dollar amount, would you prefer* WACC *or* APV*?*

18·4.D. A Major Blunder: Applying APV and WACC to the Current Cash Flows

Another Common Mistake: Make sure you use the correct Cash Flow to Discount.

Unfortunately, both WACC and APV are often used incorrectly. Analysts frequently forget that the correct expected cash flow in the present value numerator is the "as if fully equity financed and fully after-tax" cash flows—\$256 in our example. It is neither the before-tax project cash flows (\$280 in our example), nor the after-tax cash flows under the current financing scheme (e.g., \$231.93 if the firm were currently financed 60/40; or \$234.79 if the firm were currently financed 90/10). If you have worked through the examples in this chapter, you should understand why this would provide the wrong answer.

IMPORTANT: WACC *and* APV *operate with expected "as if 100% equity-financed and after corporate income tax" cash flows, not the firm's current cash flows (which depend on the current debt/equity financing).*

Solve Now!

Q 18.7 *A firm in the 20% marginal tax bracket is currently financed with \$500 debt and \$1,000 equity. The debt carries an interest rate of 6%, the equity's cost of capital is 12%. The risk-free rate is 4%, the equity premium is 3%. What is the firm's beta? The firm is pondering a recapitalization to \$1,000 debt, which would increase the debt's interest rate to 8%. The firm exists only for 1 more year. What would the new equity be worth?*

Solve Now!

Q 18.8 *A firm in the 40% income tax bracket has an investment that costs \$300 in year 0, and offers a pretax return in year 1 of \$500. Assume that the firm's cost of capital, as provided by the external capital markets, is approximately $\mathcal{E}(\tilde{r}_D) = 15\% + w_D \cdot 5\%$. Compute the APV, WACC, and a WACC-based value if the firm borrows \$50 to finance it. Repeat if the firm borrows \$100.*

18·5. A Sample Application of Tax-Adjusting Valuation Techniques

Table 18.4. Income Statement of Hypothetical Machine

	Year	1	2	3	4	5	6
	Gross Sales (Revenues)	$70	$70	$70	$70	$70	$70
–	Cost of Goods Sold (COGS)	$5	$5	$5	$5	$5	$5
–	Selling, General & Administrative Expenses (SG&A)	$5	$5	$5	$5	$5	$5
=	EBITDA (Net Sales)	$60	$60	$60	$60	$60	$60
–	Depreciation	$25	$50	$50	$25	$0	$0
=	EBIT (Operating Income)	$35	$10	$10	$35	$60	$60
–	Interest Expense	$0	$5	$5	$5	$5	$5
=	EAIBT (or EBT)	$35	$5	$5	$30	$55	$55
–	Corporate Income Tax (at 40%)	$14	$2	$2	$12	$22	$22
=	**Net Income**	$21	$3	$3	$18	$33	$33

Excerpts From the Cash Flow Statement

	Year	1	2	3	4	5	6
	Net Income	$21	$3	$3	$18	$33	$33
+	Depreciation	$25	$50	$50	$25	$0	$0
=	**Total Operating Activity**	$46	$53	$53	$43	$33	$33
	Capital Expenditures	–$75	–$75	-	-	-	-
=	**Total Investing Activity**	–$75	–$75	-	-	-	-
	Financing Cash Flow	-	-	-	-	-	-
	Net Equity Issue	+$26	-	-	-	-	-
	Dividends	-	-	–$53	–$43	–$33	–$8
	Net Debt Issue	+$25	-	-	-	-	–$25
=	**Total Financing Activity**	+$51	-	–$53	–$43	–$33	–$33
=	**Net Change In Cash**	+$22	–$22	$0	$0	$0	$0

We now apply our corporate income-tax related valuation techniques on a multi-year, pro-forma firm. (Pro formas will be discussed in detail in Chapter 25. Think of a pro forma as a forward projection of the financial statements.) In Table 18.4, we return to our basic hypothetical machine from Chapter 9, Table 9.6 on Page 201. To make the example more useful, assume an appropriate debt interest rate of 20%, so a loan of $25 must offer an expected $5 in interest per annum. The appropriate overall cost of capital for our corporation is 30%. The corporate income tax rate is 40%. Table 18.4 shows that shareholders pay in $26 and receive a total of $137 in dividends. Debtholders invest $25 and receive $25 in total interest payments. (Your firm follows an odd capital distribution policy, but so be it.) What is your firm worth?

Let's value a pro-forma firm.

18·5.A. Direct Valuations from Pro Forma Financials

Method 1: Direct Flows from the Financials.

Our first method to value the firm uses the pro-forma cash flows directly. First, use the project cash flow formula 9.18 from Page 214, which tells you that the project cash flows that you need for your NPV valuation are

	Computing Project Cash Flows, $25 Debt Financing						
	Year	1	2	3	4	5	6
	Total Operating Activity	$46	$53	$53	$43	$33	$33
+	Total Investing Activity	–$75	–$75	-	-	-	-
+	Interest Expense	-	$5	$5	$5	$5	$5
=	**Project Cash Flows**	–$29	–$17	+$58	+$48	+$38	+$38

Then discount these cash flows, using the assumed 30% firm cost of capital:

$$\text{NPV} = \frac{-\$29}{1+30\%} + \frac{-\$17}{(1+30\%)^2} + \frac{+\$58}{(1+30\%)^3} + \frac{+\$48}{(1+30\%)^4} + \frac{+\$38}{(1+30\%)^5} + \frac{+\$38}{(1+30\%)^6} = \$28.95 \quad . \tag{18.23}$$

So, you would be willing to pay $28.95 *today* for the right to buy (and finance) the firm, which will initiate *next year* with this exact capital structure. But wait: did you not forget about the tax-shelter that came with the debt? No, you did not! The pro forma itself had already incorporated the correct interest expense, which had reduced the corporate income tax, and thus increased our project's cash flows.

18·5.B. APV

Method 2, APV demands a detour: we must construct as-if-100%-equity financials.

Our second method to value our firm is APV. But be careful: the cash flows in Formula (18.23) are *not* the cash flows that you need for the APV analysis, because these are not the cash flows *as if 100% equity financed*. APV states that you want to use the *as if 100% equity financed* cash flows, and then add back the tax shield. If you used the cash flows in Formula 18.23 and then added the tax-shield (due to the interest payment designation), you would mistakenly count the tax-shield twice. We must therefore start over to find the correct expected cash flows. You already know that if the firm were fully equity financed, the tax obligation would go up. By how much? You can intuit this even before you write down the full financials. In years 2–6, the taxable net income would be $5 more, so at your 40% corporate income tax rate, you would have to pay not $2, but $4 in taxes. This means that you would have to pay an extra $2 in taxes each year. To make sure this intuition is right, construct the financials of a 100% equity financed firm:

	Abbreviated Income Statement, As If 100% Equity Financed						
	Year	1	2	3	4	5	6
=	EBIT (Operating Income)	$35	$10	$10	$35	$60	$60
–	Interest Expense	$0	$0	$0	$0	$0	$0
=	EAIBT (or EBT)	$35	$10	$10	$35	$60	$60
–	Corporate Income Tax (at 40%)	$14	$4	$4	$14	$24	$24
=	**Net Income**	$21	$6	$6	$21	$36	$36

	Abbreviated Cash Flow Statement, 100% Equity Financed						
	Year	1	2	3	4	5	6
	Net Income	$21	$6	$6	$21	$36	$36
+	Depreciation	$25	$50	$50	$25	$0	$0
=	**Total Operating Activity**	$46	$56	$56	$46	$36	$36
	Capital Expenditures	–$75	–$75	-	-	-	-
=	**Total Investing Activity**	–$75	–$75	-	-	-	-

We can now reuse our present value cash flow formula on the 100% equity financed version of our firm:

	Computing Project Cash Flows, 100% Equity Financed						
	Year	1	2	3	4	5	6
	Total Operating Activity	$46	$56	$56	$46	$36	$36
+	Total Investing Activity	–$75	–$75	-	-	-	-
+	Interest Expense	$0	$0	$0	$0	$0	$0
	Project Cash Flows	–$29	–$19	+$56	+$46	+$36	+$36

Comparing this to the equivalent table on Page 458, you can see that the project cash flows in your 100% equity financed firm has indeed lost the tax shelter of $2 in each of years 2–6. The intuition was correct!

Now discount these "as if 100% equity financed" cash flows with the firm's appropriate cost of capital, which is assumed to be 30%. Standing at time 0, this gives you

Return to the main task: APV valuation

$$\text{NPV}_{\text{Project, 100\% Equity Financed}} = \frac{-\$29}{(1+30\%)} + \frac{-\$19}{(1+30\%)^2} + \frac{+\$56}{(1+30\%)^3} + \frac{+\$46}{(1+30\%)^4} + \frac{+\$36}{(1+30\%)^5} + \frac{+\$36}{(1+30\%)^6} = \$25.20 \quad . \tag{18.24}$$

The APV formula tells you that you now need to add back the expected tax shield from the debt. The interest tax shields in years 2–6 are the interest payments ($5 per year) multiplied by the corporate tax rate (40%), or $2 per year. What is the value of this tax shelter?

$$\text{NPV}_{\text{Tax Shelter}} = \frac{\$0}{(1+30\%)} + \frac{+\$2}{(1+30\%)^2} + \frac{+\$2}{(1+30\%)^3} + \frac{+\$2}{(1+30\%)^4} + \frac{+\$2}{(1+30\%)^5} + \frac{+\$2}{(1+30\%)^6} = \$3.75 \quad . \tag{18.25}$$

Therefore, the APV method tells you that the firm value is

$$\text{APV}_{t=0} = \$25.20 + \$3.75 = \$28.95 \quad . \tag{18.26}$$

This is the same answer that you found in Formula 18.23.

18·5.C. WACC

Our third method to value the firm is WACC. Start again with the firm's cash flows, as if 100% equity financed.

Method 3: WACC

Computing Project Cash Flows, 100% Equity Financed						
Year	1	2	3	4	5	6
Project Cash Flows	–$29	–$19	+$56	+$46	+$36	+$36

But you shall now use the appropriate tax-adjusted cost of capital for discounting. There is one tricky issue now: what is the firm's debt ratio? That is, WACC requires $w_D = (1 - w_E)$ as an input. In the real world, you could just look up the current firm values, so trust me (and the Digging-Deeper box below) that the debt is about 35 percent of the firm's value today. You know the other two remaining inputs that you need to compute WACC, which are the overall corporate cost of capital at 30%, and the debt cost of capital at 20%.

Return to the main task: WACC valuation

You can now compute the firm's weighted average cost of capital as

$$\begin{aligned} \text{WACC} &\approx 30\% - \overbrace{40\%}^{\text{tax}} \cdot \overbrace{35\%}^{\text{debt}} \cdot \overbrace{20\%}^{\text{c.o.c.}} \approx 27.2\% \\ \text{WACC} &\approx \mathcal{E}(\tilde{r}_{\text{W}}) - \tau \cdot w_{\text{D}} \cdot \mathcal{E}(\tilde{r}_{\text{D}}) \quad . \end{aligned} \tag{18.27}$$

Therefore,

$$\begin{aligned} \text{NPV}_0 = &\frac{-\$29}{(1+27.2\%)} + \frac{-\$19}{(1+27.2\%)^2} + \frac{+\$56}{(1+27.2\%)^3} + \frac{+\$46}{(1+27.2\%)^4} \\ &+ \frac{+\$36}{(1+27.2\%)^5} + \frac{+\$36}{(1+27.2\%)^6} \approx \$29.55 \quad , \end{aligned} \tag{18.28}$$

which is a small 60 cents off the value of the APV formula. Most of the difference comes from the fact that the fraction of debt in the capital structure is 35% in the first year, but a different proportion of the value in subsequent years. Thus, as we noted before, WACC cannot exactly apply in this case, but this error is typically likely to be modest—and dwarfed by errors in what you have assumed about the tax code and by the uncertainty that such projects would carry in the real world. Here you are 60 cents off—only modest harm done.

Digging Deeper: *Your equity will have payments of \$53, \$43, \$33, and \$8 in the final four years. Let's assume for a moment a 35% cost of capital on equity. (We will verify this later.) With a 35% cost-of-equity-capital assumption, the market value of the equity in year 1 (not year 0!) will be*

$$PV_{Equity,t=1} \approx \frac{\$53}{(1+35\%)^2} + \frac{\$43}{(1+35\%)^3} + \frac{\$33}{(1+35\%)^4} + \frac{\$8}{(1+35\%)^5} \approx \$58.28 \quad . \tag{18.29}$$

The market value of the equity in year 1 will not be the \$26 that the equity holders have to put in! The equity can be sold for more and is therefore worth the higher \$58.28. The debt however is priced right: when discounted appropriately, it is worth \$25. So, as a fraction of financing in the capital structure, equity will constitute

$$w_{\text{E}} = \frac{\$58.28}{\$58.28 + \$25.00} \approx 69\% \quad . \tag{18.30}$$

Now, you know that the debt's cost of capital is 20%. To have a consistent example in which the firm's cost of capital is 30% requires an appropriate rate of return on the equity of around 35% without a tax subsidy:

$$\begin{aligned} 69\% \cdot x \;+\; (1-69\%) \cdot 20\% &\approx 30\% \quad \Rightarrow \quad x \approx 35\% \quad . \\ w_{\text{E}} \cdot \mathcal{E}(\tilde{r}_{\text{E}}) \;+\; (1-w_{\text{D}}) \cdot \mathcal{E}(\tilde{r}_{\text{D}}) &= \mathcal{E}(\tilde{r}_{\text{W}}) \quad . \end{aligned} \tag{18.31}$$

so your assumption of a 35% cost-of-equity capital above was correct. Unfortunately, this 35% is correct only in the first year.

Solve Now!

Q 18.9 *Construct a pro forma for the following firm: A 3-year project costs \$150 (year 1), and produces \$70 in year 1, \$60 in year 2, and \$55 in year 3. Depreciation, both real and financial, is 3 years. Projects of this riskiness (and with this term structure of project payoffs) have an 18% cost of capital. The marginal corporate income tax rate is 40%.*

(a) *Assume that the firm is 100% equity financed. Construct the pro forma, and compute expected project cash flows.*

(b) *Compute the Project IRR.*

(c) *Compute the project NPV.*

(d) *Assume that this firm expects to receive an extra bonus of \$2 in years 2 and 3 from a benevolent donor. What would be the project's cash flows and IRR now?*

For the remaining questions, assume that the firm instead has a capital structure financing \$50 in debt raised in year 1 at a 10% (expected) interest rate. There is no interest paid in year 1, just in years 2 and 3. The principal is repaid in year 3.

(e) *Construct the pro forma now. What is the IRR of this project?*

(f) *From the pro forma, what is the NPV of the debt-financed project?*

(g) *Compute the NPV via the APV method.*

(h) *Via the APV method, how much would firm value be if the firm would have taken on \$40 in debt (assuming the same interest rate of 10%)?*

(i) *How much money must the equity provide in year 1? What is the debt ratio of the firm? Does it stay constant over time? Is this a good candidate firm for the WACC method?*

(j) *What is the debt/value ratio in year 1?*

(k) *Assume the equity capital must also be raised in year 0, and the firm can earn a 46% rate of return on this equity capital in year 0. For example, if the firm raises \$10 in equity in year 0, then the firm will have \$14.60 to finance the project in year 1. How much equity will the firm have to raise in year 0 if it wants to cover both interest and project costs?*

(l) *What is the debt/value ratio in year 0?*

(m) *What is the expected debt/expected value ratio in year 1? Why is it different from year 0?*

(n) *Is this a zero NPV project?*

(o) *What is the value of the tax shelter that is part of the project NPV?*

(p) *What is the value of this project if it were purely equity financed and fully taxed?*

(q) *How does firm value change if the firm borrows not \$50, but \$25 at an interest rate of not 10%, but 8%?*

(r) *Tough question: is it legitimate to compute the weighted average cost of capital of this firm in year 0 from debt/equity ratios in year 0, in order to check whether the firm cost of capital is equal to the weighted average cost of capital?*

18·6. The Tax Subsidy on PepsiCo's Financial Statement

Table 18.5. PepsiCo's Income Statement (Revisited).

	Income Statement	December 2000
=	Revenue	25,479
	COGS	10,226
	+ SG&A	11,104
	+ Depreciation and Amortization	147
	+ Unusual Expenses	184
–	= **Total Operating Expenses**	21,661
=	**Operating Income**	3,818
+	Net Interest Income	-57
=	**Income Before Tax**	3,761
–	Income Tax	1,218
=	**Income After Tax**	2,543
–	Extraordinary Items	0
=	**Net Income**	2,543

One can infer the tax subsidy from corporate financial statements.

We now want to apply our theoretical knowledge to a practical example—in fact, the example that you used in Chapter 9 to illustrate how financial statements work. Can you infer the tax subsidy from PepsiCo's Income Statement? The answer is yes. For convenience, Table 18.5 reproduces PepsiCo's Income Statement. In 2000, PepsiCo had \$3.818 billion in operating income, but only had to pay income taxes on \$3.761 billion. With income taxes of \$1.218 billion, PepsiCo's average corporate income tax rate was about 32.4%. If PepsiCo had been purely equity financed, it would have had to pay taxes on its operating income of \$3.818 billion, or about \$1.237 billion. Thus, by having \$57 million in interest, relative to a hypothetical dividend payout of \$57 million, PepsiCo enjoyed a tax shield in 2000 from its interest payments of

$$\begin{aligned} \text{Debt Tax Shield} &= \tau \cdot [\text{Interest Payments}] \\ &= 32.4\% \cdot \$57 \text{ million} = \$19 \text{ million} . \end{aligned} \tag{18.32}$$

Note that you did not need to compute $\mathcal{E}(\tilde{r}) \cdot D$, because you could read the interest payments directly off the financials. Note also that for companies like PepsiCo with high income, the marginal and the average tax rates are practically the same, so you can assume that PepsiCo would have had to pay its average tax rate of 32.4% if it had paid out the \$57 million interest in dividends instead. (Finally note that you are also ignoring more complex tax issues, such as deferred taxes, here.)

Solve Now!

Q 18.10 *Compute the tax shield for a company of your choice (or Coca Cola if you have no favorite!).*

18·7. Odds and Ends

You now understand all there is to know about how managers can adjust to the presence of corporate income taxes. But there are a number of issues that are worth discussing, if only because you may wonder about them in the future.

18·7.A. Which Valuation Method is Best?

None of the method dominates.

First, you might wonder which of the three valuation method is best: the pro forma, the APV, or the WACCmethod. If one were always best, we would not have bothered with the other two, so each must have advantages and disadvantages.

They may provide different answers, but the differences should be modest.

Of course, the three methods should usually come out if not with the same, then at least with very similar results—otherwise, something would be wrong. As the example in Section 18·5 showed, if suitably applied, the differences are usually due to "rounding error." This is especially true if you compare them to the errors that you will inevitably introduce in your simplification of the tax code and your assessment of expected cash flows and appropriate costs of capital.

Here is how I see the three methods:

Pro Forma The advantage of the pro forma is that it is lucid and makes it less likely that you will use an incorrect expected cash flow. The disadvantage of the pro forma is that it requires a lot more effort (you have to construct full financials!), and that it does not break out the tax advantage of debt explicitly. This makes it more difficult to think about the role of changes in contemplated capital structure changes.

APV The APV formula makes it relatively easier to determine how an extra dollar of debt increases firm value. When thinking of a specific addition or project with a specific cost, this may be the easiest formula to use.

WACC The WACC formula makes it relatively easier to determine how an extra percentage in debt increases firm value. When thinking of a target ratio change in capital structure policy, this may be the easiest formula to use.

Personal Author Advice: APV is often simplest.

Still, in many cases, APV is easier to work with than WACC. For example, APV makes it much easier to think about projects that add debt capacity only at some stage in their life. What drives project debt capacity? The simple answer is that more tangible (collateralizable) projects tend to add more debt capacity, because your bank will find it easier to repossess and resell tangible assets. A research and development (R&D) project may require equity investment upfront, followed by the construction of a laboratory that can be debt-financed. The laboratory adds debt capacity, the R&D does not. APV makes it easy to add in the debt capacity only in later stages. APV also makes it easier to assign different discount factors to the firm's projects and to the firm's tax shields (Section 18·3.B).

WACC is often most complex!.

WACC may be the most difficult method. It requires assumptions about the capital structure policy in future years—and not just an assumption of corporate neglect, because if the firm does nothing, the value of the firm and thus the value of equity next year will almost certainly be different. To keep a constant debt ratio, a corporation would have to issue more debt when its equity value increases, and repurchase more debt when its equity value decreases. Even if everything turns out as expected, the equity would likely be expected to increase in value more than the debt each year. This means that without corporate action, the debt ratio would be expected to go down over time. If the debt ratio is not constant over time, you cannot use WACC, because you do not know how to do multi-year compounding with WACC. There is good evidence that publicly traded firms tend not to follow constant debt ratio strategies—but of course you do not care about *other* firms, you care about *your* own firm. If your firm plans to stick to a constant debt ratio, WACC can indeed be the most convenient way to think about the added tax advantages of debt. On a more technical note, WACC also leans more heavily on the assumptions that borrowing rates are competitive and thus zero NPV. Therefore, WACC works only in "normal" situations in which creditors are paid the appropriate cost of capital on the debt. WACC cannot deal with "below-market" or "above-market" unfairly priced loans—much like the CAPM cannot. (We already know that the CAPM rate of return needs to be replaced by the certainty equivalence in this case.)

18·7.B. A Quick-and-Dirty Heuristic Tax-Savings Rule

Why bother with such puny tax savings?

Do not confuse the question of whether tax savings are important with the question of whether the right discount factor for the tax savings is important. The former is much bigger than the latter. But aren't even the tax savings too small to bother with? Before you draw this conclusion, realize that the firm need not invent anything new or work extra hard to obtain the tax savings. And, tax savings materialize year after year after year. In fact, this provides a nice over-the-thumb heuristic of what the firm can gain in value from one dollar extra in debt.

A rule of thumb: Each perpetual dollar of debt increases firm value by the corporate income tax-rate.

Start with the APV formula. If a large firm today takes on and maintains an extra \$1 billion in debt rather than an extra \$1 billion in equity, the interest is on the order of about 6%, or \$60 million per year. The tax rate for many corporation is about 40%, leading to a savings of \$24 million per year—this can pay for a nice executive bonus. But this is only the first year. If the \$24 million per year saving is a perpetuity, at an appropriate cost of capital of 6%, we can compute the total value increase to the firm today to be \$24/6% = \$400 million.

$$\begin{aligned} \text{Value Increase} &= \frac{40\% \cdot 6\% \cdot \$1 \text{ billion}}{6\%} = \$400 \text{ million} \\ \text{Value Increase} &= \frac{\tau \cdot \mathcal{E}(\tilde{r}_D) \cdot D}{\mathcal{E}(\tilde{r}_D)} = \tau \cdot D \quad . \end{aligned} \tag{18.33}$$

This is a nice shortcut: for every dollar extra in eternal debt, the value of the firm increases by the tax rate of the firm. And Formula 18.33 is so easy, you can often compute it in your head. For example, compare financing a \$1 million project with 50% debt (rather than all-equity), in which a firm in the 40% marginal tax bracket plans not to repay any of the debt principal or to take on new debt. The tax savings would be $40\% \cdot \$500,000 = \$200,000$.

18·7.C. Can Investment and Financing Decisions Be Separate?

If the world is not perfect, projects with different financing options can offer different values. Thus, financing and investment decisions must be considered together, not separately.

In the perfect M&M world, investment and financing decisions can be made independently: Managers can focus on production choices and leave the financing to the nerds in the finance department. Unfortunately, if debt is tax advantaged, this will not be the case outside the M&M world.

An example.

For example, consider two projects with equal costs, equal payoffs, and equal costs of capitals. (Alternatively, just consider their NPVs to be the same.) The first project is a research and development project; the second is a building. In the real world, it is difficult to find a bank to lend money for R&D: after all, if the firm fails to pay its interest payments, there is often little that the bank can collect and resell. Buildings, on the other hand, are easy to repossess. Therefore, the building offers more **debt capacity** than the R&D project. This can make it more valuable than the otherwise equally promising R&D project. Managers cannot choose among projects without taking into consideration how each project aids the debt capacity of the firm.

> **IMPORTANT**: *In an imperfect world, unlike the M&M world, managers cannot ignore or delay financing decisions when making real investment decisions. The two decisions are intertwined.*

A second complication derives from the fact that the value of the debt capacity can depend on who the owner is. Although most profitable and older firms are in the same highest tax bracket, some younger, growing, and unprofitable firms are in lower tax brackets. To these younger firms, the debt capacity is worth a lot less than it is to a large firm like PepsiCo (which can immediately use the tax deduction).

18·7.D. Using Our Tax Formulas

With The CAPM

Formally speaking, in the presence of taxes (or other imperfections), the CAPM does not hold and should not be used. But this does not help you in the real world. What choice do you have but the CAPM to provide you with an appropriate discount rate?

Formally, it is wrong to use the CAPM in a world of taxes.

So, it has become common practice to combine the tax-adjusted WACC Formula with a cost of capital derived from a CAPM model. Most often, the CAPM provides the cost of capital for the equity ($\mathcal{E}(\tilde{r}_{\mathsf{E}})$), while the borrowing interest rate provides the cost of capital for the debt ($\mathcal{E}(\tilde{r}_{\mathsf{D}})$).

Informally, no one has a better alternative.

$$\begin{aligned} \mathcal{E}(\tilde{r}_{\mathsf{W}}) &= w_{\mathsf{E}}\cdot\mathcal{E}(\tilde{r}_{\mathsf{E}}) \quad + \quad (1-\tau)\cdot w_{\mathsf{D}}\cdot\mathcal{E}(\tilde{r}_{\mathsf{D}}) \\ &\approx w_{\mathsf{E}}\cdot\{r_{\mathcal{F}} + [\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}]\cdot\beta_{\mathsf{E}}\} \;+\; (1-\tau)\cdot w_{\mathsf{D}}\cdot\{r_{\mathcal{F}} + [\mathcal{E}(\tilde{r}_{\mathcal{M}}) - r_{\mathcal{F}}]\cdot\beta_{\mathsf{D}}\} \;. \end{aligned} \tag{18.34}$$

Of course, corporate debt also has a default premium, so you cannot use the bank's *quoted* interest rate. You must use the *expected* interest rate (cost of capital), which is the estimate that the CAPM provides. Fortunately, for large companies with low default probabilities, the two are often close.

What About Personal Income Taxes or Other Issues?!

From Section 6·5.C, you know that your owners want you to work only with expected after-tax cash flows and expected after-tax costs of capital. As manager, do you need to adjust the corporate expected debt cost of capital and expected equity cost of capital (from the CAPM) for your investors' income taxes? From your perspective, if you use the CAPM or if you ask your investment banker what the appropriate expected cost of capital is at which you can find financing, are these estimates the costs of capital you need in our WACC or APV formulas, or will you need to make further adjustments to these formulas?

Do we need more formulas, one for each M&M distortion?

It turns out that the WACC and APV are the last formulas that you shall need. The reason is that all other effects will be transparent to you. As managers, the costs of capital ($\mathcal{E}(\tilde{r}_{\mathsf{E}})$ and $\mathcal{E}(\tilde{r}_{\mathsf{D}})$) demanded by your investors will take into account any other "issues" they may have with your firm. For example, if your investors suffer higher taxes by investing with you, this will be reflected in your seeing higher costs of capital. If your investors believe you will go bankrupt, it will be reflected in your seeing higher costs of capital. If your investors love us, it will be reflected in your seeing lower costs of capital. And so on. But then why were corporate income taxes so special in needing their own formula? Why did they not flow through the costs of capital, too? The reason is that the corporate income taxes already sat fully budgeted as a line item in your own corporate financials. You had already subtracted them off, and just needed to add them back in when you *as corporate managers* could avoid them. Any other concerns that your investors may have with your firm have not been reflected in your financials, and therefore do not need their own formulas.

For us as corporate managers, the expected costs of capital already reflect investors' other issues.

The fact that you will not need another formula does not excuse you from thinking about what else your investors may care about. After all, you can create value for your investors if you can reduce the necessary cost of capital. For example, if you can minimize your investors' personal tax obligations, you as a firm will face a lower cost of capital, because you are creating more value for our investors.

No formula does not mean no thinking!

The One Mistake To Avoid

Warning

Finally and again, the one big mistake you should never commit is to use the wrong expected cash flows for APV or WACC. Using the wrong discount rate on the tax shelter or tax liability is forgivable (within bounds)—using the wrong expected cash flow is not. So, let's reemphasize what you must do. In the pro forma method, you already have both the projected debt cash flows and the projected equity cash flows, so your life is simple. You can just use these pro forma cash flows which already take the debt tax shield into account. In contrast, in both the APV and WACC methods, you must not use the expected cash flows of the firm under the current capital structure (much less the expected cash flows to the current equity), but the cash flows that would accrue if the firm was fully equity financed.

18·7.E. Other Capital Structure Related Tax Avoidance Schemes

There are too many tax schemes in existence to list in just one book. They are also changing all the time. Here are some examples.

Wall Street and Main Street employ armies of tax experts to help their clients avoid taxes, but this is really an arms race between the IRS (Congress) and investors. Investors keep looking for new tax avoidance schemes and the IRS tries to close these new loopholes. There are a large number of both past (now closed) and current tax avoidance schemes. Some of the more noteworthy remaining tax reduction schemes are as follows:

- Sometimes, high-tax firms may be able to purchase low-tax firms, and thereby immediately use the acquired firm's existing **net operating losses NOLs**.

  For example, the *Financial Times* reported on February 10, 1994, that the £2.5B GKN corporation made a hostile bid for the £300M Westland corporation, solely because GKN needed Westland's NOLs to reduce its own corporate taxes due.

- Compared to purchasing on credit, **leasing** can be a tax advantageous arrangement. If the borrower does not have enough income to efficiently use the interest deduction, someone else should be the official owner of the asset and "lease" it to the borrower, thereby capturing the full benefit of the interest deductability.

- Multinational corporations can shift difficult-to-value assets producing income from a high-tax country into a low-tax country. For example, corporate income taxes in **Switzerland** (federal and canton) can be as low as 7.8% (for holding companies) and as high as 25%. This contrasts with state and federal corporate income tax rates as high as 45% in the United States.

  For example, consider a company that has just developed a patent worth \$10 million per year. If the U.S. branch owned the patent, the firm would retain only $(1 - 45\%) \cdot \$10 = \$4.5$ million per year. If the Swiss branch owned the patent, the firm would retain up to $(1 - 7.8\%) \cdot \$10 = \$9.2$ million per year. Why stop at \$10 million? If the Swiss branch charged the U.S. branch \$20 million per year, the firm's U.S. tax obligations (resulting from profits from other businesses) would decline by \$9 million per year (45% times \$20 million), but Swiss tax obligations would increase by \$1.56 million. Still, this is a healthy \$7.4 million net gain.

  This tax-efficient capital transfer can also be accomplished with capital structure. For example, if the Swiss branch lent funds to the U.S. branch at an interest rate of 36% per year, rather than 6% per year, the effect would be a reduction of the firm's tax liabilities. For every \$1,000 in excess interest paid (at the 36% instead of the 6% rate), the company would retain an extra (45% – 7.8% = 37.2%) \$372 in profits. Companies can play similar, but less drastic, tax games by choosing the U.S. state and municipality in which they are headquartered.

  The IRS is very much aware of these issues. For example, the *Wall Street Journal* reported on June 24, 2002 that the IRS is trying to prevent firms from shifting intellectual property, such as patents, to other countries in which corporations would have fewer taxes to pay.

Should we prevent corporate tax avoidance?

Before such corporate tax avoidance schemes outrage you too much, you should realize that you may even be lucky if tax lawyers and Congress help many U.S. companies succeed in escaping some of their tax burdens. First, corporations are just vehicles owned by investors. Corporate income taxes are really ultimately paid by the investors—often small dispersed investors, like yourself. Second, the United States has no monopoly on corporate locations. If U.S. taxes are too high, some corporations may just leave the United States, others may never come. Many financial services firms have already done so. U.S. disclosure and tax laws and regulations have built strong financial services centers in places like the Bermudas and Cayman Islands, and Switzerland. Greenwich Connecticut is the financial services center that the New York tax code has built. Many European countries have even stronger regulations than the United States and are in fact experiencing dramatic capital flight right now. Of course, this does not mean that the U.S. system cannot be improved. The lawyer-and-accountant-and-legislative-pork method does not seem like the most rational and efficient way to run an economy.

Solve Now!

Q 18.11 *A firm has expected before-tax earnings of \$20 per year forever, starting next year. It is financed with half debt (risk-free, at 5% per year) and half equity (at 10% per year), and this is eternally maintained. If the firm is in the 25% tax bracket, then what is its NPV?*

Q 18.12 *(Continued.) If this firm took on \$50 in debt and maintained the debt load at \$50 forever, rather than maintain a 50/50 debt-equity ratio, then what would this firm's value be?*

ANECDOTE: Stanley Toolworks and Foreign Domiciles
Stanley Toolworks, a hundred-year-old prominent Connecticut-based global manufacturer of tools, was in the process of locating its headquarters to the Bermudas in mid-2002. This would allow Stanley's *foreign* subsidiaries to escape U.S. income taxes. In the end, only unusually strong media attention, public outcries, and the threat of special legislation prevented this departure.

18·8. Summary

The chapter covered the following major points:

- In the imperfect real world, the U.S. tax code favors debt over equity. Managers should take this corporate income tax advantage into account.
- The calculation of the income tax advantage can be done either through a full pro forma (via a financing scenario that subtracts the interest and thereafter the tax burden), through an adjusted WACC method, or through the APV method.
- Both the WACC and the APV method begin with cash flows *as if fully equity-financed and thus fully taxed*, which is why they need to put back the tax advantage derived from the presence of debt.
  - WACC does so by lowering the cost of debt capital:

$$\begin{aligned} \mathsf{PV} &= \frac{\mathcal{E}(\mathrm{CF})}{1+\mathsf{WACC}} \quad , \\ \mathsf{WACC} &= \mathcal{E}(\tilde{r}_{\mathsf{W}}) - \tau \cdot \mathcal{E}(\tilde{r}_{\mathsf{D}}) \cdot \mathsf{D} \\ &= w_{\mathsf{E}} \cdot \mathcal{E}(\tilde{r}_{\mathsf{E}}) + w_{\mathsf{D}} \cdot \mathcal{E}(\tilde{r}_{\mathsf{D}}) \cdot (1-\tau) \end{aligned} \tag{18.35}$$

  - APV does so by adding back the tax benefit:

$$\mathsf{APV} = \frac{\mathcal{E}(\mathrm{CF})}{1+\mathcal{E}(\tilde{r}_{\mathsf{W}})} + \frac{\mathcal{E}(\tau \cdot \overbrace{\mathcal{E}(\tilde{r}_{\mathsf{D}}) \cdot \mathsf{D}}^{\text{interest payment}})}{1+\mathcal{E}(\tilde{r})} \quad . \tag{18.36}$$

    If the firm's debt ratio will shrink over time, use the firm's debt cost of capital to discount the interest payments. If it will remain constant, use the firm's overall cost of capital. If it will increase, use the equity's cost of capital.
- These methods usually arrive at very similar but not exactly identical valuations. We often are not sure about the appropriate discount rate that should be applied to the future tax benefits in the APV formula; and the WACC formula cannot really deal with changing costs of capital or debt ratios over time. However, the errors that an incorrect discount rate on the tax shield would cause are usually dwarfed by other simplifications and uncertainty in expected cash flows and discount rates.

  The one error you should never commit is to use the wrong expected cash flows in one of our three methods.
- A constant extra dollar of debt *forever* increases the value of the firm by the firm's marginal income tax rate. A \$100 eternal debt increase will create \$30 in value for a firm in the 30% marginal income tax bracket.
- In the imperfect real world, financing and investment decisions can no longer be separated: projects that add more debt capacity may add value through the financing channel.
- It is common and reasonable to combine the WACC formula or APV formula with the CAPM formula, even if this is not entirely correct.
- As managers, we will not need a more complex formula than WACC, because other investor considerations will be reflected in the cost of capital that we face. However, not needing a formula does not mean that we do not need to think about how we can address these other investor considerations—quite the opposite.

1. \$500 if designated an interest payment. \$750 if designated a dividend distribution, because only \$500 is left after corporate income taxes have been paid.

2. With an internal rate of return of 20%, Uncle Sam would see \$90,000 if you pay cash. If you finance with 80% debt, you will have \$40,000 in interest to deduct from \$200,000 in return, and thus pay taxes only on \$160,000. This lowers your tax bill to \$72,000. (Side Advice: If you borrow \$800,000, you may have to invest your \$800,000 elsewhere. If you do not choose tax-exempts, Uncle Sam may receive more taxes therefrom.)

3. The net subsidy is $\$90,000 - \$72,000 = \$18,000$ next year. At an appropriate cost of capital of 8%, this is an PV of $\$16,667$.

4. The WACC valuation is

$$\mathsf{PV}_0 = \frac{\$256}{1+12\% - 25\% \cdot 30\% \cdot 8\%} = \$229.80 \ . \tag{18.37}$$

The firm has $\$229.80 \cdot 25\% = \$57.45$ of debt and \$172.35 in equity value today. Its APV is

$$\begin{aligned} \mathsf{APV}_0 &= \frac{\mathcal{E}(\mathrm{CF}_1)}{1+\mathcal{E}(\tilde{r}_\mathsf{W})} + \frac{\tau \cdot \mathcal{E}(\tilde{r}_\mathsf{D}) \cdot \mathsf{D}}{1+\mathcal{E}(\tilde{r}_\mathsf{W})} \\ &= \frac{\$256}{1+12\%} + \frac{30\% \cdot 8\% \cdot \$57.45}{1+12\%} = \$229.80 \ . \end{aligned} \tag{18.38}$$

5. The APV valuation is

$$\begin{aligned} \mathsf{APV}_0 &= \frac{\mathcal{E}(\mathrm{CF}_1)}{1+\mathcal{E}(\tilde{r}_\mathsf{W})} + \frac{\tau \cdot \mathcal{E}(\tilde{r}_\mathsf{D}) \cdot \mathsf{D}}{1+\mathcal{E}(\tilde{r}_\mathsf{W})} \\ &= \frac{\$256}{1+12\%} + \frac{30\% \cdot 8.7\% \cdot \$100}{1+12\%} = \$230.90 \ . \end{aligned} \tag{18.39}$$

Therefore, the \$100 debt is 43.3% of the firm's value today, and

$$\mathsf{PV}_0 = \frac{\$256}{1+12\% - 43.3\% \cdot 30\% \cdot 8.7\%} = \$230.90 \ . \tag{18.40}$$

6. WACC for ratio, APV for dollar amounts. Look at the previous two questions. You cannot figure out the APV in the first question before you determine the WACC, and the opposite in the second question.

7. • The firm's overall cost of capital today is $6\% \cdot 1/3 + 12\% \cdot 2/3 = 10\%$. (Because $4\% + 3\% \cdot 2 = 10\%$, the beta is 2.) • The easy way is to recognize that the firm is sheltering $\$500 \cdot 6\% = \$30$ through interest payments. If it refinanced with \$1,000, it could now shelter $\$1,000 \cdot 8\% = \$80$. Uncle Sam would get to see an additional \$50 less in income, which means that the firm would pay $\$50 \cdot 20\% = \$10$ less in income tax *next year*. • Now you need to determine the appropriate discount rate for \$10 in tax savings. For convenience, use the debt cost of capital: 8%. This means that our recapitalization increases firm value by $\$10/1.08 \approx \$9.26$. (If you prefer to use the overall firm cost of capital, you would obtain \$9.09.) • The question intentionally gave additional irrelevant information.

8. Because you know that the cost of capital if all financed by debt has to be the cost of capital for the firm, you know that the firm's overall cost of capital is $\mathcal{E}(\tilde{r}_\mathsf{D}) = 15\% + 100\% \cdot 5\% = 20\%$. Now, this project will offer \$200 pretax profit in year 1. Discounted back at the firm's cost of capital, the NPV without taxes is $-\$300 + \$500/(1+20\%) = \$116.67$. But, if equity financed, the IRS will declare taxes due on \$200 of profit, or \$80. So, the NPV with taxes and all equity financed is $-\$300 + \$420/(1+20\%) = \$50$.

   the firm has a value of $\$420/1.2 = \$350$. With debt of \$50 (\$100), the firm carries a debt load of around 15% (30%). The cost of debt capital formula given in the question suggests that $\mathcal{E}(\tilde{r}_\mathsf{D}) = 15\% + 15\% \cdot 5\% = 15.75\%$ (16.5%). (Note: the question is a bit ambiguous in that it does not tell you what to use as firm value. The 15% and 30% debt ratios are reasonable values, though.)

   capital of 15.75% (16.5%) are \$7.88 (\$16.50) *next year*. Facing a tax rate of about 40%, Uncle Sam would thereby subsidize the project to the tune of $40\% \cdot \$7.88 = \$3.15$ (\$6.60), which in today's value would be worth around $\$3.15/(1+20\%) \approx \$2.63$ ($\$6.6/1.2 \approx \$5.50$). Therefore, under APV, if financed with \$50 in debt (\$100 in debt), the project is worth $\$50 + \$2.63 = \$52.63$ ($\$50 + \$5.50$).

is financed by debt at an interest rate of 15.75% is the solution to $15\% \cdot 15.75 + 85\% \cdot \mathcal{E}(\tilde{r}_E) = 20\% \rightarrow \mathcal{E}(\tilde{r}_E) = 20.75\%$. So, the WACC is given by the formula, $w_E \cdot \mathcal{E}(\tilde{r}_E) + w_D \cdot \mathcal{E}(\tilde{r}_D) \cdot (1-\tau) = 85\% \cdot 20.75\% + 15\% \cdot 15.75\% \cdot (1-40\%) \approx 19.06\%$. Similarly, if \$100 is borrowed, $\mathcal{E}(\tilde{r}_E) = 21.5\%$, and WACC $= w_E \cdot \mathcal{E}(\tilde{r}_E) + w_D \cdot \mathcal{E}(\tilde{r}_D) \cdot (1-\tau) = 70\% \cdot 21.5\% + 30\% \cdot 16.5\% \cdot (1-40\%) \approx 18.02\%$. The WACC based value is thus $-\$300 + \$420/(1+19.06\%) \approx \$52.76$. Note that you have made enough little assumptions and approximations that it would make little sense to now worry about being off by a little in the APV and WACC computations (\$52.76 and \$52.63).

9.

(a) The pro forma for a 100% equity financed firm is

	Income Statement			
		Year 1	Year 2	Year 3
	EBITDA (=Net Sales)	\$70	\$60	\$55
–	Depreciation	\$50	\$50	\$50
=	EBIT (=Operating Income)	\$20	\$10	\$5
–	Interest Expense	\$0	\$0	\$0
–	Corporate Income Tax	\$8	\$4	\$2
=	**Net income**	\$12	\$6	\$3
	Cash Flow Statement			
	Net income	\$12	\$6	\$3
+	Depreciation	\$50	\$50	\$50
=	**Operating Cash Flow**	**\$62**	**\$56**	**\$53**
	capital expenditures	–\$150	\$0	\$0
=	**Investing Cash Flow**	**–\$150**	**0**	**0**
	Economic Project Cash Flows (Operating CF+ Investing CF+ Interest)			
	Project Cash Flows	**–\$88**	**+\$56**	**+\$53**

(b) The IRR of our project solves

$$\frac{-\$88}{1+\text{IRR}} + \frac{+\$56}{(1+\text{IRR})^2} + \frac{+\$53}{(1+\text{IRR})^2} = 0 \ . \tag{18.41}$$

Thus, the IRR of a purely equity financed project is 15.7%.

(c) The NPV of the purely equity financed project is

$$\text{NPV} = \frac{-\$88}{1+18\%} + \frac{+\$56}{(1+18\%)^2} + \frac{+\$53}{(1+18\%)^3} = -\$2.10 \ . \tag{18.42}$$

This is in line with the fact that the project IRR of 15.7% is less than the 18% cost of capital.

(d) The cash flows would increase to $-\$88$, $+\$58$, and $+\$55$. The IRR would increase to 18.6%.

(e) The debt-financed pro forma would now be

	Income Statement			
		Year 1	Year 2	Year 3
	EBITDA (=Net Sales)	\$70	\$60	\$55
–	Depreciation	\$50	\$50	\$50
=	EBIT(=operating income)	\$20	\$10	\$5
–	Interest Expense	**\$0**	**\$5**	**\$5**
–	Corporate Income Tax	\$8	\$2	\$0
=	**Net income**	\$12	\$3	\$0
	Cash Flow Statement			
	Net income	\$12	\$3	\$0
+	Depreciation	\$50	\$50	\$50
=	Operating Cash Flow	**\$62**	**\$53**	**\$50**
	Capital Expenditures	–\$150	\$0	\$0
=	Investing Cash Flow	**–\$150**	**0**	**0**
	Economic Project Cash Flows (Operating CF + Investing CF + Interest)			
	Project Cash Flows	–\$150+\$62	+\$53+\$5	+\$50+\$5
	=	**–\$88**	**+\$58**	**+\$55**
	The Economics of Financing			
	Debt	+\$50	+\$5	+\$55
	Equity	+\$38	+\$53	+\$0

Not surprisingly, these are the same as our aforementioned cash flows, with a \$2 income-tax subsidy in years 2 and 3. So, the IRR is again 18.6%.

(f) The NPV of the debt-financed firm is

$$\text{NPV} = \frac{-\$88}{1+18\%} + \frac{+\$58}{(1+18\%)^2} + \frac{+\$55}{(1+18\%)^3} = +\$0.55 \quad . \tag{18.43}$$

So, with the tax subsidy, this project becomes worthwhile.

(g) The APV of this project would be the value as-if-100%-equity-financed, which is –\$2.10 (Formula 18.42), plus the discounted tax subsidies in years 2 and 3. These have a value of

$$\text{Tax Subsidy} = \frac{\$2}{(1+18\%)^2} + \frac{\$2}{(1+18\%)^3} = \$1.44 + \$1.22 = \$2.66 \quad . \tag{18.44}$$

Therefore, the APV would be $-\$2.10 + \$2.66 = \$0.56$.

(h) By APV, the expected tax subsidy would shrink from $\tau \cdot \mathcal{E}(IP) = 40\% \cdot \$5 = \$2$ per year to $\tau \cdot \mathcal{E}(IP) = 40\% \cdot \$4 = \$1.60$ per year. The expected value of the tax subsidy would therefore be

$$\text{Tax Subsidy} = \frac{\$1.60}{(1+18\%)^2} + \frac{\$1.60}{(1+18\%)^3} = \$2.12 \quad . \tag{18.45}$$

The net project value would be about \$0.02.

(i) In year 0, the weight of the debt is $w_{D,0} = \$50/\$88 \approx 57\%$. But after year 2 and before year 3, the debt is expected to be 100% of the capital structure, so its weight in the capital structure is drastically changing each year. So, this firm is not at all a good candidate for WACC application.

DIGGING DEEPER: *Do not try to compute a weighted average cost of capital from the debt and equity internal rates of return (10% and 40%, respectively). If the debt would remain at 57% of the firm's capital structure, then the appropriate rate of return of equity would have to be around 30% so that the weighted cost of capital would come out to* $\mathcal{E}(\tilde{r}_W) = w_D \cdot \mathcal{E}(\tilde{r}_D) + w_E \cdot \mathcal{E}(\tilde{r}_E) = 18.6\%$. *This is much lower than the equity IRR of 40% (which is the same as its expected rate of return from year 1 to year 2), because from year 2 to 3, the equity becomes a much smaller part of the firm. What bites you in this case is the fact that you have a strong term structure of investment weights.*

10. Do it!

11. The weighted average cost of capital (WACC) is

$$\begin{aligned} \text{WACC} &= w_D \cdot \mathcal{E}(\tilde{r}_D) \cdot (1 - \tau) + w_E \cdot \mathcal{E}(\tilde{r}_E) \\ &= 50\% \cdot 5\% \cdot (1 - 25\%) + 50\% \cdot 10\% = 6.875\% \ . \end{aligned} \tag{18.46}$$

The numerator has to be post corporate income tax; therefore, it is $(1 - \tau) \cdot \text{CF} = \$15$. This is an annuity, therefore the NPV is

$$\text{PV} = \frac{\$15}{6.875\%} = \$218.18 \ . \tag{18.47}$$

12. The cost of capital for a fully equity financed firm without a tax subsidy would be 7.5%, because it had 50% debt at 5% and 50% debt at 10%. Therefore, the "as if fully equity financed" value is

$$\text{PV} = \frac{\$15}{7.5\%} = \$200.00 \ . \tag{18.48}$$

Now, we need to add back the tax subsidy. With $50 in debt, risk-free and therefore with an interest rate of 5%, the interest payments would be $\mathcal{E}(\tilde{r}_D) \cdot D = \$2.50$ per year. The taxes saved would be $\tau \cdot \$2.50 = \$0.625$, which is an eternal cash flow. At the interest rate of 5%, the value of the tax subsidy today is $12.50. Therefore, the value of this firm is $200+$12.50= $212.50.

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

CHAPTER 19

Other Capital Structure Considerations

Personal Taxes, Bankruptcy Costs, Inside Information, Behavior

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Managers should consider corporate income taxes to be important determinants of capital structure—but not the only one. Personal income taxes, financial distress, agency considerations, and others can all play important roles. They are the subject of this chapter.

Corporate income taxes turn out to be unique in one respect: corporate income taxes distort the debt cost of capital. For example, with a corporate income tax rate of 40%, our corporation would not face the 10% interest rate that it would have to pay creditors, but instead consider the debt cost of capital to be only 6%. In contrast, the capital structure considerations that we shall cover in this chapter work through different conduits—they change the appropriate cost of capital that our corporation has to pay. In other words, they can influence our cost of equity capital $\mathcal{E}(\tilde{r}_E)$, our cost of debt capital $\mathcal{E}(\tilde{r}_D)$, or both. Moreover, their importance is often not easy to quantify—but this does not make them any less important than corporate income taxes.

19·1. The Role of Personal Income Taxes and Clientele Effects

We begin by considering how corporate managers should think about their investors' personal taxes. After all, their own corporate income taxes are just one side of what Uncle Sam receives: he also wants his share from investors' income. (It would be little use to try to economize on corporate income tax through debt financing if Uncle Sam then were to take away 99% of the interest the instant it exits the corporation.) This section will show how investors' personal income taxes can influence the optimal corporate capital structure. The interplay between personal and corporate taxes creates both investor clienteles and firm clienteles: small growth firms should have more equity in their capital structure than large cash-rich firms; and highly-taxed individual investors should invest more in equity-financed firms than tax-exempt investors, while tax-exempt investors should hold more corporate bonds.

19·1.A. Background: The Tax Code For Security Owners

The type of income matters: capital gains income is better for taxable investors than interest income.

When investors receive cash from the company, the form in which this cash arrives matters:

Ordinary Income is taxed at relatively high ordinary income tax rates (up to 35%), and is very difficult to shelter from taxes.

Interest Income is basically taxed like ordinary income.

Dividend Income is taxed at a lower rate. As of 2005, if the corporation has already paid taxes on its earnings, its dividends are considered "qualified," which reduces the personal tax rate required to be paid by the dividend recipients. Individuals in the 10% and 15% ordinary income tax brackets pay a 5% dividend tax, while individuals in higher tax brackets pay a 15% dividend tax. Giving investors credit for dividends paid from already taxed earnings is similar to how the United Kingdom and many other countries have taxed dividends for a long time. However, in the United States, this is not a stable situation. The current dividend taxation scheme was put in place by the Bush tax cuts of 2003—but dividends are scheduled to revert to being taxed at ordinary income tax rates after 2008, just as they were before 2003.

Capital Gains are generally the most tax-advantaged form of income. In 2005, short-term capital gains (on financial assets that you hold for less than 1 year and real assets that you hold for less than 3 years) are federally taxed at 28% (again in 2005), while long-term capital gains are taxed at the same statutory rate as qualifying dividends (i.e., 15% for high-income tax investors). One exception is if the investor is a corporation—it is then taxed at a higher capital gains rate of 25%.

The advantage of capital gains is not only the relatively low statutory tax rate, but also the tax applicability. Unlike interest or dividend income, capital gains can be offset by capital losses. Moreover, capital gains are not incurred on an annual basis, but only when they are realized. Therefore, the best form of income for investors is long-term capital gains.

Details, details, details...

The tax treatment of financial securities—and especially bonds—is an ongoing cat-and-mouse game between the IRS and corporate tax lawyers. For example, there are some very intricate tax rules on how capital gain income and interest income on bonds must be computed. Such regulations prevent firms from paying out interest income in a form that makes them capital gains for their investors. (The specific loophole that these regulations originally closed were **discount bonds**.) In addition, there are hundreds of special clauses—some pure corporate subsidies in the tax codes (and some targeted at one qualifying company only!), and others penalizing particular situations for no obvious reason. We just noted that even the dividend tax treatment is "sunset" in 2008—plus, the tax code may change every other year before and after 2008, too! So, it is more important for you to learn how to think about taxes than it is for you to know the detailed tax clauses and rates. Any details will likely be outdated within 10 years—if not sooner.

19·1.B. The Principle Should Be "Joint Tax Avoidance"

The main point of this section is that managers, who want to best represent corporate owners, should consider not only their own corporate income taxes, but also their investors' personal income taxes. To see this, pretend that you are the owner of a corner shop ("the corporation") and you are also its manager. Do you care care whether the IRS taxes you right at the cash register of your corporate business, or taxes you personally when you move the cash from the corporate register into your personal pocket? Or do you care how much you can ultimately put into your pocket? The finance premise is that you care only about the money in your pocket that you have left over *after* Uncle Sam has had his dip from both. You want to reduce the tax obligation both at the cash register and at your personal pocket. Corporate investors are no different from your corner shop. They really care only about their after-tax personal income in the end, not about whether the corporation pays taxes or they themselves pay taxes.

The owners care not about where taxes are paid (corporation or personal), just that as little as possible is paid in total.

IMPORTANT:

- *Both corporate and personal taxes that can be avoided translates into cash that the owners can keep.*
- *Reducing the total taxes ultimately collected by Uncle Sam (now and in the future) increases the value of the firm.*

How Personal Taxes Enter The Firm's Cost of Capital

How do corporate managers find the personal income tax rate applicable to their investors? Importantly, they do not need to bother finding it. This is because whatever the personal income taxes may be, they manifest themselves in the cost of capital that investors demand from the firm. When a firm offers securities that cause a lot of personal tax liabilities, then investors will demand a higher expected rate of return. Securities that cause no personal tax liabilities will be snapped up by our investors even at lower expected rates of return.

Investors are the corporate owners. Personal taxes matter to the cost of capital demanded by investors.

How do personal taxes enter the corporate cost of capital formula, i.e., the WACC formula in 18.19 and the APV formula in 18.7? The answer is that *the formulas themselves do not change*. You can continue to use them. The personal income taxes only influence the inputs into these two formulas—specifically, the appropriate expected rates of return on debt $\mathcal{E}(\tilde{r}_D)$ and equity $\mathcal{E}(\tilde{r}_D)$ that investors demand. (These are the *before*-personal income tax rates of return, as you will see in the next paragraph.)

Personal taxes do not change the WACC or APV formulas.

An example will make this clearer. Assume that the equilibrium is such that investors demand an after-tax expected rate of return of 6% to hold risky equity capital. If the effective personal income tax rate on equity is 50%, then the market equilibrium would have shareholders demand and receive an expected rate of return of $\mathcal{E}(r_E) = 12\%$ from the corporation. The 12% is the input to the WACC and APV formulas—the rate of return that investors demand before they have to pay their personal taxes. (The 6% rate of return after-personal income tax is what matters to investors, but it does not show up numerically in either formula.)

An example of how to think about personal income taxes.

How should you as a CFO adjust for the effects of changes in the personal taxation of investors (e.g., the Bush dividend tax cuts of 2003)? For argument's sake, presume that the effective personal income tax rate on equity drops from 50% to 25%. At what rate will the corporation now be able to finance projects? Our investors demand an expected after-tax rate of return of 6%, so at an expected rate of return of $\mathcal{E}(r_E) = 9\%$, they will again come out with their required 6% expected rate of return. From the perspective of the corporation, the necessary and appropriate cost of capital $\mathcal{E}(r_E)$ in the formulas would have dropped from 12% to 9%. (Of

An example of how to think about a cut in personal income taxes.

course, this is a simplification. The tax cuts may also change other alternatives available to investors. They could attract more firms and investors into this market, too, which could force the appropriate equilibrium after-tax expected rate of return on equity away from 6%, too.)

IMPORTANT:

- *The* WACC *or* APV *formulas from Chapter 18 remain applicable in the presence of personal income taxes.*
- *Personal income taxes are visible to corporate managers in the cost of capital that they have to pay investors in order to obtain financing. Put differently, they manifest themselves in the expected-rates-of-return inputs to the* WACC *and* APV *formulas.*

In the real world, the financial markets provide managers with costs of capital that reflect not only personal taxes, but also a host of other investor concerns that we will discuss next—but the WACC and APV formulas always remain applicable.

19·1.C. Tax Clienteles

Your Problem — How to Minimize Total IRS Receipts

Investors should care only about value today!

Before we discuss *how* to minimize taxes, should we not weigh doing so (usually by financing with more corporate debt) against such concerns as earnings dilution or the riskiness of the equity? Worrying about these concerns too much is a mistake that the financially naïve often make. The correct view is that reported earnings per sé are a less important concern. Instead, what really matters is the after-tax money that investors get to keep in the end. To the extent that Uncle Sam receives less money, owners ultimately get more money. It is money that ultimately matter to them, not any ephemeral concerns about the debt-equity ratio or the risk split-up or the earnings dilution, or any of a hundred other reasons sometimes given by management. The capital markets are smart enough to know what really matters—money to investors. (There is also good empirical evidence that capital markets indeed appreciate lower income taxes, and reward their reduction with a higher market value.)

Companies can shift tax burdens from themselves to their investors and vice-versa. Distributions in interest save firms money, but not investors. Distributions in capital gains save investors money, but not firms.

You now know the principle of joint tax avoidance, but you do not yet know how to implement it. You know that as manager acting on behalf of your corporate owners, you should try to arrange the capital structure so as to minimize the sum of personal and corporate income taxes, not just your own corporate income tax. You also know that investors cannot shelter interest income, can modestly shelter dividend income, and can easily shelter capital gains income. At this point, you should also realize that you face a dilemma:

- If you plan to pay cash as interest income, you will save on your own corporate income tax—but your investors will face the full brunt of Uncle Sam (and thus demand a relatively higher expected rate of return).
- If you plan to convert cash into capital gains by reinvesting retained earnings, you will pay full corporate income taxes—but your investors can then avoid most personal taxes (and thus demand relatively lower expected rates of return).

Complication: Different investors face different personal tax rates.

To make matters even more interesting, you have to be concerned that, in real life, not every investor faces the same tax rate. There are low-tax investors, like tax-exempt pension funds, who pay low or no (personal) taxes on both dividend and interest distributions. And there are high-tax investors, like most retail investors, who pay especially high taxes on interest income, medium-high taxes on dividends, and lower taxes on capital gains. What should you do?

SimCity Live!

The best way to understand what you should do is to presume for the moment that you are a puppeteer, controlling the private economy. Your opponent is the IRS. Your four pieces are:

1. High-tax corporations—like PepsiCo or R.J.R. Nabisco, bulging with earnings. Often, these are mature "value" firms.

2. Low-tax corporations—like Itar in 1985, a shell company with large tax-loss carryforwards or net operating losses; or Nanotech, a developer of nano-technology with virtual earnings. Growth firms are often low-tax corporations.

3. High-tax investors—like retail investors earning over \$100,000 per year.

4. Low-tax or tax-exempt investors—like a pension fund or the money in a tax-advantaged retirement account, like a 401-K.

These are not perfect classifications, because even low-tax investors must eventually pay some taxes, and even low-tax corporations may run out of tax-shelters (or they can immediately use up all their tax credits and thereby become high-tax companies!). But this classification serves us well in thinking about the problem. How would you arrange your pieces? Would you have the high-tax corporation finance with debt or equity? Would you have the low-tax investor own the high-tax corporation or the low-tax corporation?

ANECDOTE: Tax Reductions for the Needy? For-Profit Corporations with No Tax Obligations
Are all cash-cow corporations in a high marginal tax bracket? The Washington Post reported a study by the Institute on Taxation and Economic Policy which showed that 41 companies not only owed no taxes, but received money back in at least one of the three years studies—1996–1998—although they reported a total of \$25.8 billion in pretax profits. 24 companies—nearly one in 10 studied—received tax rebates in 1998 alone, including such household names as Texaco Inc., Chevron Corp., PepsiCo Inc., MCI WorldCom Inc., Goodyear Tire & Rubber Co. and General Motors Corp. Texaco, for example, received a tax rebate of \$67.7 million, which meant that it paid taxes at a rate of negative 37.2 percent on the \$182 million in profit it reported in 1998. In dollar terms, the study found that tax breaks enabled the companies to reduce their taxes by \$98 billion over the three years, with 25 companies receiving almost half of that amount. General Electric Co. topped the list, with \$6.9 billion in breaks, which cut its tax bill by 77 percent over the three years. A G.E. spokesman also questioned the report's methodology, noting that of the \$6.9 billion in breaks cited, \$2.4 billion was deferred taxes "that we will pay."

The twenty-four companies that payed less than zero in federal income taxes in 1998 were

Company	1998 Profit	1998 Tax	1998 Rate
Lyondell Chemical	\$80.0	–\$44.0	–55.0%
Texaco	\$182.0	–\$67.7	–37.2%
Chevron	\$708.0	–\$186.8	–26.4%
CSX	\$386.6	–\$102.1	–26.4%
Tosco	\$227.4	–\$46.7	–20.6%
PepsiCo	\$1,583.0	–\$302.0	–19.1%
Owens&Minor	\$46.1	–\$7.9	–17.1%
Pfizer	\$1,197.6	–\$197.2	–16.5%
J.P. Morgan	\$481.1	–\$62.3	–12.9%
Saks	\$83.0	–\$7.9	–9.5%
Goodyear	\$400.7	–\$33.2	–8.3%
Ryder	\$227.5	–\$16.4	–7.2%
Enron	\$189.0	–\$12.5	–6.6%
Colgate-Palmolive	\$348.5	–\$19.6	–5.6%
MCI WorldCom	\$2,724.2	–\$112.6	–4.1%
Eaton	\$478.8	–\$18.0	–3.8%
Weyerhaeuser	\$405.0	–\$9.5	–2.3%
General Motors	\$952.0	–\$19.0	–2.0%
El Paso Energy	\$383.7	–\$3.0	–0.8%
WestPoint Stevens	\$142.6	–\$1.2	–0.8%
MedPartners	\$49.6	–\$0.4	–0.7%
Phillips Petroleum	\$145.0	–\$1.1	–0.7%
McKesson	\$234.0	–\$1.0	–0.4%
Northrop Grumman	\$297.7	–\$1.0	–0.3%
Total	\$11,953.0	–\$1,272.9	–10.6%

Your Solution — Rearranging Clienteles

"Clientele effects" mean different firms attract different investors. Carried to extremes, Uncle Sam might not get anything.

Clearly, this is not a difficult problem if 99% of all investors are tax-exempt—you could make all highly taxed corporations issue debt (and thereby avoid corporate income taxes). Neither corporations nor (much of the tax-exempt) investor sector would end up paying any tax. So, we shall assume—realistically—that low-tax investors are in limited supply and therefore in great demand. In this case, as puppeteer with the task of minimizing the IRS' take and maximizing your private sector take, you should sort your pieces into the following clienteles:

High-tax, profitable firms should pay via interest (thus, have debt).

High-Tax Profitable Firms: Make your "cash cow" value firms in the highest tax bracket issue debt, so that their cash flows can be paid out as interest, thereby avoiding the high corporate income tax.

Low-tax investors should hold the debt.

Low-Tax Investors: Make your tax-exempt investors hold this corporate debt, so that the interest receipts remain untaxed at the recipient level. (If you instead made your individual investors hold this debt, Uncle Sam would be better off, and you would be worse off.)

You still have low-tax firms and high-tax investors to allocate. What can you do with them?

High-Tax investors should hold the equity.

High-Tax Investors: Make your high-tax individual investors hold stocks instead of bonds. They will then either receive capital gains (taxed very little) or dividends (taxed just a little more). This way, your high-tax investors will suffer only fairly low tax penalties, too.

Low-tax, unprofitable firms should pay via share repurchases (thus, have equity).

Low-Tax Firms: Make your growth firms and other firms in the lowest corporate tax bracket finance themselves with equity, not with debt. You need this to satisfy the demand for equity by your high-tax investors. You can make your low-tax firms use their cash flows to reinvest in the corporation, repurchase their shares, or pay dividends (but only until 2007!). In any case, it would allow these firms' predominantly high-tax investors not to suffer much in tax. (If you instead made your low-tax firms finance themselves with debt, the firms would have little use for the corporate income tax shelter provided by debt, at least compared to high corporate tax firms—and your high-tax investors would have nothing to buy.)

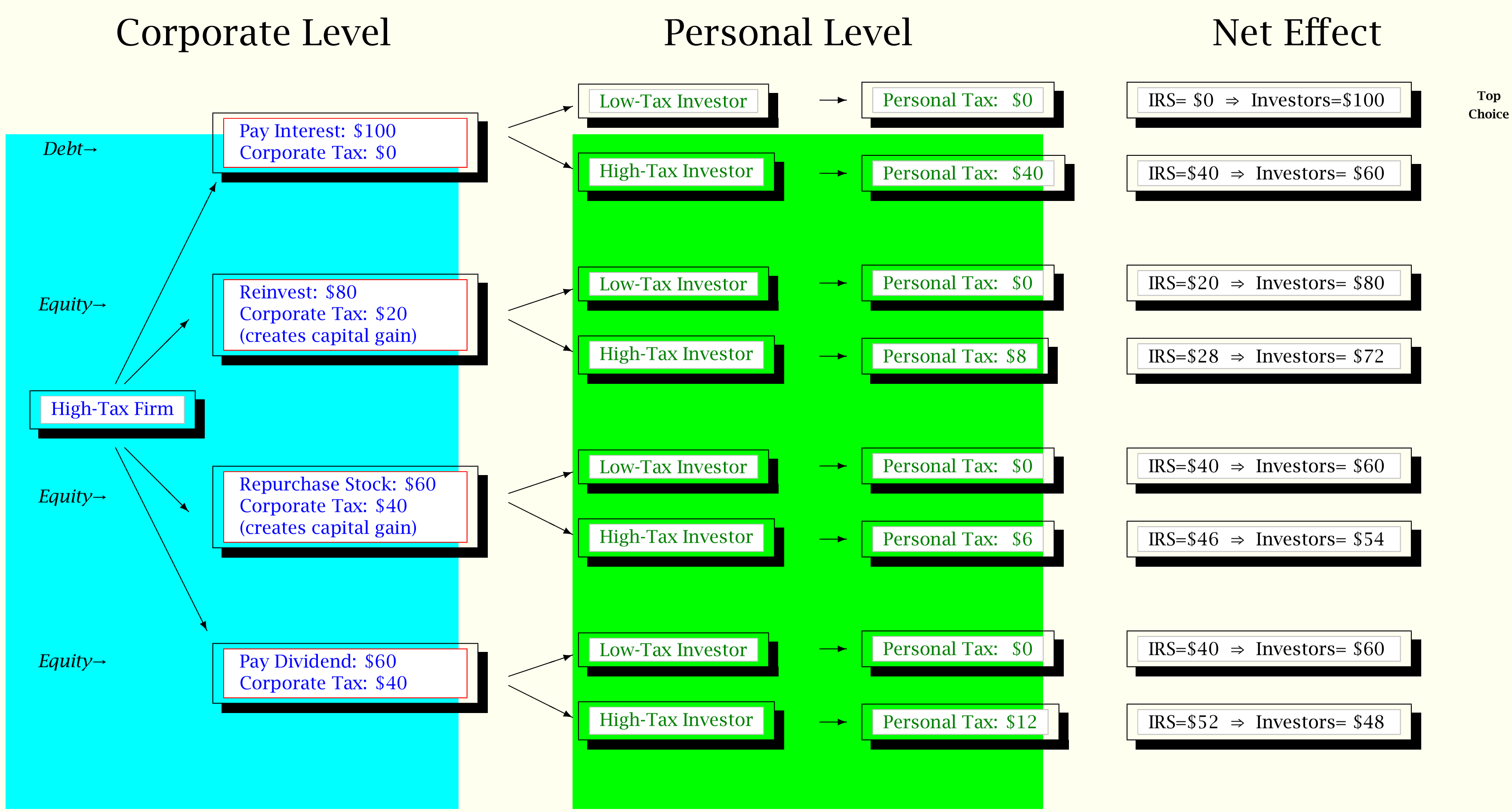
Illustrative Numbers to show our best allocations.

Figures 19.1 and 19.2 try to illustrate the best puppeteering choices for firms and investors—and Uncle Sam's consequent take. All the numbers and tax rates are illustrative only, and not exact. We assign 40% as the ordinary income tax rate for high-tax companies and for high-tax investors' interest receipts. We assign 20% as the high-tax investors' tax on dividend receipts *if* the firm has paid enough in taxes (so investors receive a tax credit). And we assign 10% as the effective capital gains tax rate—partly, because it can be deferred and offset, and partly because the statutory rate is lower. Finally, we fudge some numbers to reflect other freedoms—for example, a firm that reinvests its money may claim that the reinvestment is necessary maintenance and thus not income, or it may receive tax credits for investing (a common situation). Thus, we assign only a 20% effective corporate income tax rate to cash used for reinvestment. Low-tax firm may not mean zero-tax firm—even tax credits have a value, so there is an opportunity cost to using up tax credits today. So, we assign a 10% income tax rate to cash that adds to earnings, and a 5% income tax rate to reinvested cash with its investment/maintenance tax credit.

Do the high-tax firm first.

Figures 19.1 shows that the high-tax firm can pay debt from pre-tax earnings without occurring any tax penalty. If the firm reinvests all cash, we have assigned it our 20% effective corporate income tax rate. Investors get \$80 in the form of capital gains. If these are taxed investors, they suffer only a 10% income tax rate (in present value) on this gain, for a net personal tax obligation of \$80· · 10% = \$8. If the firm instead uses the money to repurchase stock, it can do so only from after-tax money. So, the IRS collects \$40 in ordinary corporate income tax. Investors pay 10% capital gains tax on \$60 in share repurchases (i.e., \$6) and twice this on dividends (\$12). Clearly, the best choice here is the top line—the clientele effect we discussed earlier, in which the IRS receives no money.

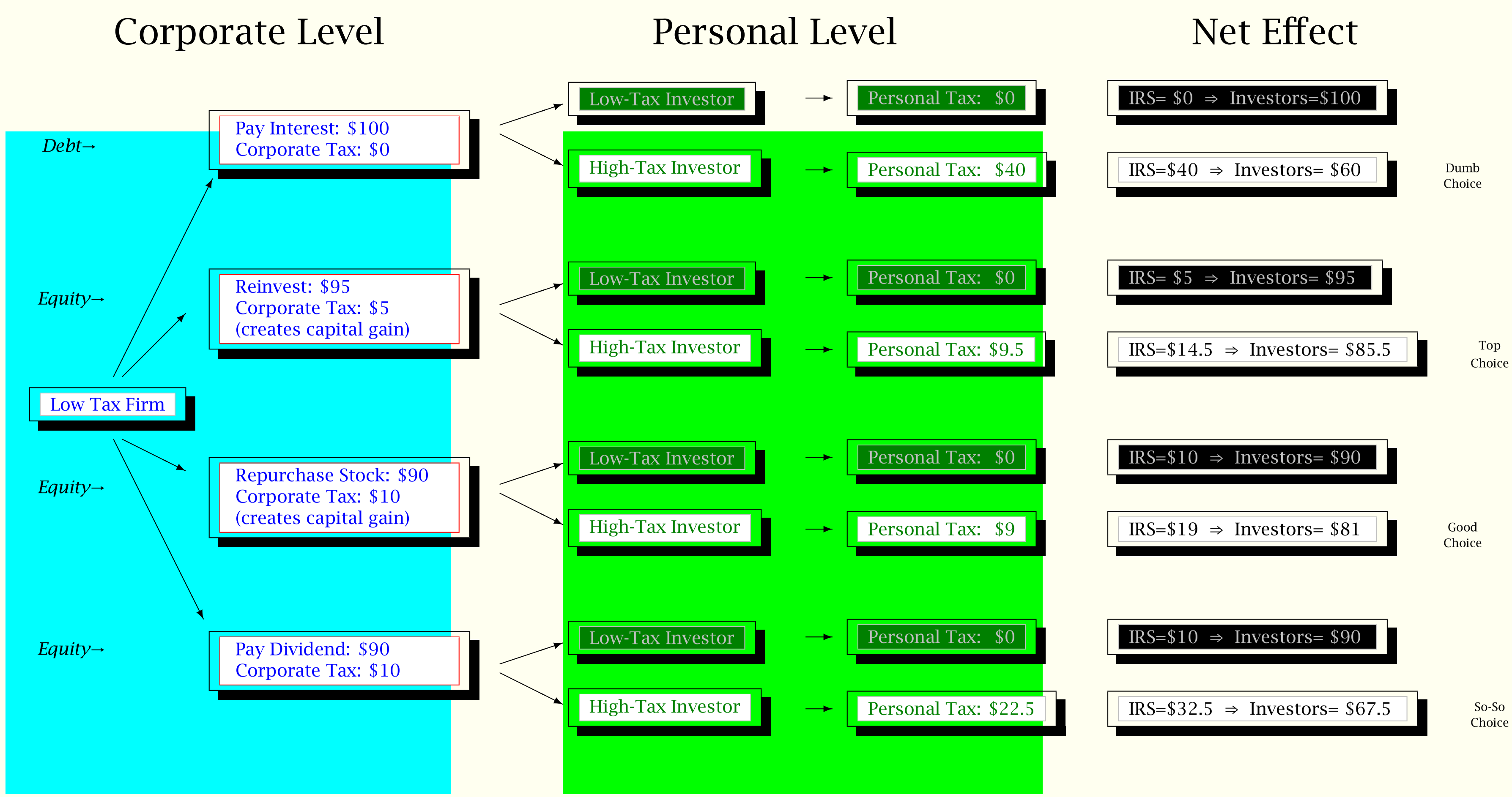
Table 19.1. Taxation Chain of a High Tax Firm



A high-tax firm may be a value firm with lots of earnings and few deductions, such as R.J.R. Nabisco. A low-tax investor may be a tax-exempt pension fund. A high-tax investor may be a typical retail investor.

All numbers are illustrative and not exact! The firm has $100 in cash. Firm Level: If paid as interest, the full $100 can be paid out. If reinvested, some investments and/or maintenance create tax credits. If paid out to shareholders, the firm incurs the full corporate income tax obligation. Personal Level: Low-tax investors never pay. High-tax investors effectively pay 10% on capital gains (in PV terms), 20% on dividend receipts, and 40% on interest receipts.

Table 19.2. Taxation Chain of a Low Tax Firm



A low-tax firm may be a firm with seemingly unlimited tax-loss carryforwards or a growth firm that is unlikely to earn a profit for many years to come. A low-tax investor may be a tax-exempt pension fund. A high-tax investor may be a typical retail investor.

All numbers are illustrative and not exact! The firm has $100 in cash. Dark boxes indicate that we have already used up our low-tax investors on high-tax firms, so that there are few or no low-tax investors left.
Firm Level: Low-tax firms pay no (debt) or almost no corporate income tax (other forms). Personal Level: High-tax investors effectively pay 10% on capital gains (in PV terms), 25% on dividend receipts, and 40% on interest receipts. (The Bush tax cut of 2002 requires the firm having paid dividends, so the dividend tax rate is above the 15% rate after corporate credit.)

The low-tax firm in Figure 19.2 can no longer use our low-tax investors—we have already used them up in Figure 19.1. Focus on our high-tax investors. They pay 10% on capital gains, and 25% rather than 20% because the firm has not paid income taxes—under the Bush dividend tax plan, the investor does not receive much of a tax credit.

Do the low-tax firm next.

Now, put the two figures together: In sum, our puppets are paying \$9.5 in taxes. Can you find a combination that is better? No! This is the best puppeteering that you can do!

Can you do better?

Market Prices as Puppeteers

In the real world, *you are not the puppeteer—market prices (for capital) are*! This is what capitalist markets are really good at—they can allocate resources to their best use, and the best use of capital here is where it avoids paying taxes. By adjusting the required costs of capital on debt and equity, capital markets induce investors and firms to sort themselves to where frictions—such as tax losses—are the lowest. (If the market did not sort, you could make money from better rearranging firms and investors by saving on aggregate taxes.) Here is an example of how prices could adjust to accomplish clientele sorting:

The extreme tax avoidance is illustrative, but not realistic.

Cash Cow Firm The market prices have adjusted so that a firm in the 40% income tax bracket expects a 10% rate of return before corporate income taxes.

Corporate Level

- If it pays out in capital gains, it pays corporate income tax, and can only offer a 6% expected rate of return.
- If it pays out in interest, it can offer the full 10% expected rate of return.

Personal Level

- A tax-exempt investor can receive 10% in income from this company.
- A 30% taxable investor can receive 7% in income from this company.

Growth Firm The market prices have adjusted so that a firm in the 0% income tax bracket expects an 8% rate of return before corporate income taxes.

Corporate Level

- If it pays out in capital gains, it can offer an 8% expected rate of return.
- If it pays out in interest, it can still offer only the 8% expected rate of return—the firm already pays no tax, so deducting interest payments does not help any further.

Personal Level

- A tax-exempt investor can receive 8% in income from this company.
- A 30% taxable investor does not realize capital gain, and thus can keep close to 8% in income from this company.

The result of firm prices, which that create such expected rates of returns, will be that tax-exempt investors will decide to invest with the debt-financed cash cow firms offering a 10% expected rate of return (rather than 8% that they could get from the growth firms); and the taxable investors will prefer to invest with the growth firms, where they can earn a rate of return of 8% (rather than the 7% that they could receive from the debt-financed cash cow firm). The financial market has become the puppeteer!

Of course, in the real world, tax-avoidance is just one force at work—though it is a very important one. In the real world, there are many different types of investors, and they do want to diversify across many different companies. There are also other reasons why tax-exempt investors do not hold only or all of high-corporate-tax firm's debt; reasons why high-tax corporate tax firms have equity in their capital structures; reasons why taxable investors hold debt; and reasons why low-tax bracket firms have debt. But you should now understand the tax rationale for how expected rates of returns will sort firms and investors to minimize taxes. There is good empirical evidence that the tax-clientele ownership effects are important. For example, corporate bonds are indeed overwhelmingly owned by tax-exempt institutions

Other forces are also at work, so this is just a partial model.

DIGGING DEEPER: *In equilibrium, security yields and investor holdings are jointly determined, not sequentially as indicated by this discussion. But, profit seeking means that in equilibrium, proper sorting will take place. If tax-exempt investors were to sort themselves into the wrong stocks, the yields on corporate interest-bearing securities would increase, and tax-exempt investors should find it irresistible to pass up on corporate bonds (which, in order to be purchased by taxable retail investors, would really have to offer high rates of return). Tax-exempts would find it in their interest to sell their equities and move into these bonds. In equilibrium, dividend and interest-bearing securities should have lower pre-tax rates of return and higher post-tax rates of return than zero-dividend stocks.*

But even in the United States, with its armies of academic finance researchers, taxes remain a complex and empirically unresolved issue. Academics are still debating who benefits most from tax arbitrage—dividend-paying issuers or tax-exempt investors. If there are many tax-exempt institutions to compete with one another for the privilege of conducting tax arbitrage, firms could borrow at rates as if all their investors were tax-exempt. Yet, if there are too many firms paying dividends, they will compete to pay a premium to find tax-exempt institutions, up to and including the cost of paying taxes—and it would be the tax-exempt investors who would benefit most from their tax-exempt status. As always, the truth is probably in the middle, where the tax-arbitrage benefits are shared by tax-exempt investors and taxed firms.

Solve Now!

Q 19.1 *Explain the (personal and corporate) tax treatments if a company pays its operating cash flow in interest, repurchase, or dividends.*

Q 19.2 *Would you expect large, stable firms to be predominantly held by pension funds or by individuals? Would you expect young, growing firms to be predominantly held by pension funds or by individuals?*

19·2. Operating Policy Distortions: Behavior in Bad Times

Table 19.3. Illustration of Deadweight Costs in Financial Distress

		Bad Luck	Good Luck	Future Expected Value	Today's Present Value
Prob:		1/2	1/2		
Project	W	\$60	\$160	\$110	\$100
Capital Structure LD: Bond with Face Value FV=\$55					
Bond(FV=\$55)	D	\$55	\$55	\$55	\$50
Equity	E	\$5	\$105	\$55	\$50
Capital Structure MD: Bond with Face Value FV=\$94 and \$10 Deadweight Costs					
		Distress			
Bond(FV=\$94)	D	\$60 −\$10 = \$50	\$94	\$77 −\$5 =\$72	\$70 −\$4.55 =\$65.46
Equity	E	\$0	\$66	\$33	\$30

The cost of capital in this example is 10% for all securities, which is equivalent to assuming risk-neutrality.

A firm that has debt in its capital structure is more likely to experience financial distress or even go bankrupt. Table 19.3 shows how such financial distress can matter. If the firm has little debt, as in capital structure LD with its face value of $55, the firm can always fully meet its debt obligations. Consequently, we assume that it will not experience financial distress, and our LD scenario still matches our perfect world from Table 17.1. However, if the firm has much debt, as in capital structure MD with its face value of $94, the firm may not pay creditors all it has promised. If the world were perfect, as it had been in Table 17.1, this bankruptcy condition would merely change the payoff pattern. Everyone (including bondholders) would have known that the firm would be transferred to bondholders who would liquidate a full $60. The firm value would not be impacted by the financial distress and would therefore still be $100.

Let's rework the perfect markets example from Table 17.1.

However, bankruptcy matters if we introduce deadweight losses—such as legal fees—that are triggered in financial distress. In the lower part of Table 19.3, we assume that these deadweight bankruptcy costs amount to $10. How does this matter?

MD is worse than LD, so a debt increase can lower the firm value today. Distress per se is not the problem — deadweight costs in financial distress are!

- If you choose LD, you would borrow $50 and promise $55. Your cost of capital would be 10%. Your firm value would be $100 today.
- If you choose HD, you would borrow $65.46 and promise $94, for an interest rate of 43.6%. The expected rate of return to creditors would not change—it would still be 10%, because we are in a risk-neutral world. However, the deadweight bankruptcy cost increases *your* cost of capital. You are giving up $60 or $94—for an expected value of $77—in exchange for a payment of $65.46. Your cost of capital would have increased from 10% to to 17.6%! Thus, you could sell your firm only for $95.46, not for $100.

From your perspective, capital structure MD is worse than capital structure LD, in which the firm could never go bankrupt. The important insight with respect to bankruptcy is that it is not bankruptcy *per se* that is the problem, but only the deadweight losses in and around financial distress that matter.

Who ultimately bears the cost of bankruptcy—you or creditors? It would be you, because creditors demand fair compensation upfront. So think about how you want to structure your firm if you face both tax and bankruptcy losses. You should now try to reduce not only the deadweight loss from taxes, but also the deadweight loss from financial distress:

Ceteris paribus, you can increase value today if you reduce future expected financial distress costs—direct and indirect. Capital structures with less debt can do this.

- Too little debt, and you lose too much in taxes.
- Too much debt, and you lose too much in bankruptcy costs.

Therefore, an interior amount of debt will maximize the value of your firm today.

The rest of this section describes the various forms of deadweight losses in financial distress—and it is not just the legal fees if bankruptcy were to occur. It could be that you would spend money prior to formal bankruptcy to avoid worse, or that the fear of bankruptcy will prevent you from taking a positive NPV project if you are sufficiently close to bankruptcy (though not there yet). Indeed, you can think of anything that would distort your choices as a cost. If a particular capital structure induces you to deviate from the optimal set of projects and behavior in the future, the resulting reduction in future cash flows reduces your firm's value. It would be a cost arising from this capital structure. (These are often called **indirect bankruptcy costs**, because they are not direct cash outlays. They can occur even before formal bankruptcy.) But whether they are direct or indirect deadweight costs of financial distress, they all have the same effect—they increase your cost of capital and decrease your firm value today. And of course we already know that the financial distress needs never occur—the probability that it may occur in the future is enough to reduce the value today.

Forms of distress costs to be discussed.

IMPORTANT: *Financial distress costs usually favor equity over debt as a cheaper financing vehicle.*

19·2.A. Direct and Indirect Bankruptcy Costs

The Process

Two forms of bankruptcy.

Although the process and history of bankruptcy, both in the United States and worldwide, are fascinating, the full legal details of bankruptcy are beyond the scope of this book. In the United States, there are two legal forms of financial distress: **Chapter 7 Liquidation** and **Chapter 11 Reorganization**. Larger firms usually petition to enter Chapter 11, which gives them a stay from creditors trying to seize their vital assets. If the court determines that the business is still viable, the firm can reorganize its financial claims and emerge from bankruptcy if its creditors vote to agree to the reorganization. Otherwise, the case is converted into Chapter 7 and the firm is liquidated. Both forms are supervised by a Federal Judge (and/or Federal Bankruptcy Trustee) and last on average about 2-3 years. In real life, creditors in Chapter 11 sometimes agree to modest violations from the Absolute Priority Rule—which we have always used to construct our state-contingent tables—in order to reduce running bankruptcy costs. The firm typically pays for most of the legal fees of all participants—but even if it did not, creditors would ask for compensation for their expected legal fees upfront, so one way or another, the firm has to carry the expected costs of bankruptcy.

Direct vs. Indirect Costs

Direct legal and administrative direct bankruptcy costs are easily visible. But there are also costs that are not cash outlays.

The direct legal fees are just the most obvious costs: the legal fees that the bankruptcy process consumes. There are also hours spent by management, employees, and experts to deal with the running process. But much of the cost of financial distress is indirect and on the real business side. For example, it may become more expensive to produce (e.g., because suppliers may charge more, fearing delayed or no payment), more difficult to focus (as management is distracted with bankruptcy and more talented employees are leaving), more expensive to sell product (e.g., customers fleeing due to loss of confidence), and more expensive to sell assets (e.g., liquidation sales may mean low fire-auction prices). All these costs reduce the value of the firm, and are real welfare losses caused by financial distress. These costs can also arise even before formal bankruptcy. Many of these costs originate from the fact that firms can shed promised claims in bankruptcy, even if they would like to commit themselves today (ex-ante) not to shed them in the future. This inability to commit causes a loss of value when future distress is possible. Consider the following examples:

- When products require customer investments, customers may be reluctant to purchase the products and invest, knowing that their investments could turn out to be wasted if the firm were to disappear.

  For example, the value of a computer is determined not only by its hardware, but also by the manufacturer's continued provision of hardware and software support and development. End-of-life hardware or software, no matter how good, is often close to worthless. Even if the firm promises to continue development of faster hardware to preserve its customers' software investments, if the firm is liquidated, it would not be able to keep such a promise. The inability of the firm to commit to honoring its promises in the future hurts its sales to customers today—and may even cause the bankruptcy itself.

  This channel also works through the product resale market. If you are deciding whether to purchase a Saturn or a Toyota to drive for five years and then resell it, you should worry about whether General Motors will go bankrupt—after all, in 2005, GM's debt just dropped below investment grade. If GM were to go bankrupt, the resale value of used Saturn cars would then drop further. This in turn should make you less inclined to purchase the Saturn in the first place.

- When product sales require promises of future contact, customers may be reluctant to purchase the product, given that the future promised rebate may fail to materialize.

  For example, airlines depend on frequent flier plans to attract business travellers. When the promise of future free flights loses its credibility, an airline becomes severely handicapped. In effect, any firm whose products require warranties should weigh whether issuing debt might not alarm its customers. Such products may require future service, and customers may be reluctant to purchase the product, knowing that the service may become unobtainable in the future.

- When product quality is difficult to judge, customers become afraid that companies may cut corners in order to avoid financial distress.

  Have you ever wondered whether an airline in financial distress cuts corner on airplane maintenance? (You should!) The effect here is not the cutting of corners on the maintenance (which we will discuss below), but the fear of customers that the firm may do so. Consequently, the price at which such an airline can sell tickets may be below that of a financially solid airline. Similarly, wholesalers will not deliver their goods to near-bankrupt retailers unless they are assured of payment. Because bankrupt retailers may no longer be able to purchase credit, the costs of their goods may increase—and their competitive advantage may erode.

Without trustworthy warranty programs, competing in some businesses is very difficult.

- Some business rely on **trade credit**, in which suppliers extend credit to buyers. If suppliers fear that the buyer can go bankrupt, they may not extend trade credit and thereby may even cause the onset of bankruptcy.

Financial Distress Costs As Transaction Cost?

But there is a limit to the importance of bankruptcy costs. We can muster an argument similar in spirit to the M&M proof: if financial distress costs are too high, someone can purchase all the securities, which eliminates the financial distress and with it all bankruptcy costs. In this sense, distress costs can be considered a special form of transaction costs: the transaction costs caused by bankruptcy must be bounded by the costs of purchasing all securities to eliminate deadweight costs. But in a non-perfect world, this is not easy to do, either. One problem is that when all other creditors agree to a bailout, you—as the final bondholder—would not want to agree. You would insist on your full claim, hoping that the other creditors would agree if they want to execute the bailout. Of course, every creditor realizes this, and would prefer to hold out for the other creditors to organize. Given such bargaining considerations, it may be cheaper for a potential saviour to wait until the firm is run down more and then sold in a fire-sale, rather than to try to acquire securities from inflexible creditors.

The limit to the costs' importance is the cost of combining securities.

One attempt to reduce the transaction cost is for firms to bundle their financial claims into **units** (unit securities) of debt and equity. Each creditor would also be a shareholder. If the firm fails to pay interest in the future, creditors will be more inclined to compromise in order to avoid financial distress—after all, there is little reason to force bankruptcy in order to collect assets from oneself.

Disagreement and bargaining among creditors make efficient reorganization difficult.

Anecdote: Fear and Relief: Lotus and Chrysler

On May 1, 1995, the computer trade magazine *PCWeek*'s cover story read "Besieged by ongoing financial turmoil...Latest Lotus woes leave some customers skittish." Customers aware of the possibility of bankruptcy may choose an (inferior) product from a competing vendor with a lower possibility of bankruptcy, which in turn reduces the value of the first firm today.

On July 23, 1981, the *Wall Street Journal* reported on Chrysler's first positive quarterly earnings after a long hiatus:

> Telegraphing even this tiny profit via the nightly news to dealers and potential customers is extremely important for Chrysler, which is trying to shore up its image as a viable automaker. In the past, every time Chrysler's losses mounted, customers fled its showrooms, fearing the company would collapse.

Assessing the Magnitude of Direct Bankruptcy Costs

Orders of magnitude calculations.

Let us compute an estimate of how important potential bankruptcy costs should be for the average Fortune 500 company. Fewer than five Fortune 500 companies enter financial distress (either formal or informal) in a given year. Quadruple this number to get an estimate of 4% probability of bankruptcy at the outset of the year. Further, presume that bankruptcy costs are 5% of the value of such Fortune 500 companies *when they enter bankruptcy.* Although this is a high estimate, again quadruple this number to presume a 20% distress cost. Finally, a Fortune 500 company would drop by about 70% in (market) value before entering financial distress, caused not by deadweight losses but by the fact that the bad state has come about. So, a $1 billion company today would be worth only $200 million in the future, and lawyers' fees of 5% would then come to "only" $10 million—or 1% in expectation of today's value, rather than 5%. Put this all together. At the beginning of the typical year for the typical Fortune 500 company and before there is any indication of financial distress, the expected financial distress costs would be about $4\% \cdot 20\% \cdot 30\% \approx 25$ basis points of value today. This can still be a large amount of money for lawyers to fight over, but it is relatively modest, say, in comparison to the tax savings from another dollar of debt. Bankruptcy costs do not loom too large when a healthy Fortune 500 firm considers taking on another loan in order to save on corporate income taxes.

The fact that some firms go bankrupt "regularly" speaks for their relatively low bankruptcy costs.

This argument does not, of course, apply to each and every firm. Which firms are likely to suffer high deadweight losses in bankruptcy? We know that many U.S. railroads have declared bankruptcy dozens of times, without interruption in service. Even large retailers, like **Federated Department Stores** (Macy's and Bloomingdales), have been in and out of bankruptcy several times. Airlines have some easily transferable and collaterizable assets (airplanes), and thus may have fewer deadweight losses—many airlines have ceased operations with their planes sold, repainted and turned around for another carrier. Airlines' bankruptcy deadweight losses are relatively modest. In contrast, firms with mostly intangible assets (such as reputation or name recognition) need to be more concerned with reducing the probability of future bankruptcy. For example, if Chanel were to go bankrupt, Chanel Number 5 might acquire the odor of death, rather than the odor of high style, and the entire business might disappear. Chanel should therefore choose a lower-leverage capital structure to avoid the loss of prestige that a bankruptcy could bring about.

Expected bankruptcy costs are probably small for healthy, large companies.

The importance of bankruptcy costs as an important determinant of capital structure remains an empirical issue. The current academic consensus is that bankruptcy costs matter for some firms and some industries, particularly during recessions. They can easily be very large, but for most healthy Fortune 500 firms, their expected costs are probably small—Enron notwithstanding.

19·2.B. Operational Distortions of Incentives

A second set of financial distress costs arises from the fact that shareholders' incentives diverge from bondholders' incentives if the firm gets close to financial distress.

Underinvestment

When there is debt, equity holders may not properly take care of the assets.

The **underinvestment** problem is bondholders' concern that managers will not make necessary investments if the promised debt payments end up being too large. That is, owners may prefer to pay out cash to shareholders than spend their money on maintenance and repair (or other projects). This may be in their interest if the project proceeds would more than likely only go to bondholders. Underinvestment in turn reduces the payoffs bondholders will receive, and thus the firm-value that bond purchasers are willing to pay for lending to the firm today.

Grab as much while you can!

For example, in our firm from Table 19.3, assume that the $60 or $160 come about from an investment of $20, which causes eventual payoffs of $80 or $180. Clearly, the firm should undertake this required investment, because it is a positive NPV project. Now assume that the firm has capital structure MD, and learns about the actual state of nature after the firm has been financed, but before it needs to commit the $20. If the firm learns that the state is bad ($80 payoff), it will prefer to pay out the $20 to shareholders instead of investing it to earn the

$80 that will go to creditors.

Again, this "underinvestment problem" is a cost of debt to the firm. If the firm chooses no debt ex ante, such profitable future investments will not be ignored, which in turn will increase the value for which our hypothetical owner can sell the firm.

Fearing future lack of care again makes it more expensive for the firm to raise capital via debt.

> **IMPORTANT**: *Ex post reluctance to do the right thing (such as additional maintenance investment) favors equity over debt as the cheaper financing vehicle.*

Reluctance to Liquidate

A similar problem is the *no-liquidation problem.* Managers acting on behalf of equity holders may not wish to liquidate the firm when it has fallen onto hard times, even if this maximizes firm value. Equity holders always prefer more risky payoffs because equity is essentially like an option. If there is even a small chance of improvement and even if deterioration is more likely, equity holders are better off to take their chances than to give up their options and liquidate. For example, assume that the $60 represents the liquidation value of the factory, and the MD debt is due in two years rather than one year. Further assume that managers can continue running the factory, in which case it will be worth either $100 or $0 with equal probability. The optimal unconflicted behavior would be to liquidate the factory. Unfortunately, shareholders prefer to continue operating—they would get nothing in liquidation, but perhaps $6 if the factory were to be worth $100. In effect, equity holders have an option on the firm. In fact, they might even make running interest and principal payments in order to keep their option alive! This inefficient behavior, caused by the presence of debt in the capital structure, reduces the value of a firm with both debt and equity *today*.

Managers may not want to liquidate the firm, even if they should. This is especially bad for debt holders.

Reversals

In these operational distortions, we have presumed that managers act on behalf of shareholders, which leads them to do the wrong thing from an overall value perspective. They instead exploit debt to help equity. But there is not just an agency conflict between shareholders (and their managers) and creditors, but also between shareholders and their managers. In this case, more debt can actually help to increase firm value. For example, managers may run down the firm's equity substance to enjoy their jobs, rather than make the necessary investment on behalf of equity. Similarly, when the firm finds itself in a contracting industry, both shareholders and creditors may prefer liquidation—but managers may prefer continuing to run the firm instead. In this case, it could be that high enough debt can force bankruptcy and liquidation, and thereby create value for both creditors and shareholders.

OK, the whole story can also work in reverse!

> **IMPORTANT**: *Ex post reluctance by managers not acting on behalf of owners can favor debt over equity as the cheaper financing vehicle.*

We discuss agency problems between managers and owners in the next section, because they tend to be more dramatic in good times. But you should realize that they can occur in financial distress, too—in which case more debt is the same cure as it would be in good times.

19·2.C. Strategic Considerations

Debt can change nature of the competition in the product market.

Finally, there are some theories in which debt is a strategic commitment device. This argument is perhaps easiest to understand by analogy. Consider playing a game of chicken (two cars driving towards one another; the first to "chicken" out and get out of the way loses). How can you make sure you win? If you can tie down your steering controls, remove the steering wheel, and throw it visibly out the window, any smart opponent would surely chicken out! The trick is to visibly commit yourself to not give way. (Some people have suggested that driving an old, large and apparently unstable Buick is the equivalent of throwing out the wheel; other cars will be in a hurry to get out of your way.)

An argument that leverage can make firms more aggressive and better off.

The same argument has been made for debt—that by having debt, firms can commit to squash potential entrant competitors in their product markets. Assume for a moment that a monopolist has levered up. Consider the decision of a potential market entrant who knows this. The market entrant also knows that it is in the interest of the shareholders to increase risk—they will gain more of the upside than the downside. A price war is riskier than accommodation—so the monopolist's managers (acting on behalf of equity holders) may prefer the more risky strategy of a price war over accommodation. Consequently, the potential entrant may chicken out, and the monopolist may never have to start the price war. (Of course, if the market entrant is too stupid to understand the message, both players, the monopolist and the entrant, will be hurt badly—the two cars will crash head-on.)

The argument seems not too important. In fact, the opposite may be true. Leverage may make firms less competitive and worse off.

This argument is clever but perhaps not terribly empirically important. Even if highly levered companies were to have relatively more of a tendency to act in a more risk-seeking fashion in the product market, it is not clear whether they would choose the price war and even less clear whether they really lever up ex ante *in order to* commit themselves to price war. Empirically, researchers have actually found that more debt tends to hurt firms in the product market. Owners tend to take on much debt when they are severely cash-constrained, and this may prevent them from competing effectively. There is some evidence that supermarkets that dramatically increased their leverage were systematically attacked by their competitors with price wars and failed to compete as effectively. To the extent that this weakness reduced their value, it would count as a direct cost of a highly leveraged capital structure.

> **IMPORTANT**: *The competitive product market environment of the firm could favor either equity or debt.*

Solve Now!

Q 19.3 *Describe some non-tax based advantages of equity over debt.*

Q 19.4 *Give examples of bankruptcy costs, both direct and indirect.*

Q 19.5 *Give an example of an underinvestment problem.*

Q 19.6 *Give an example of a no-liquidation problem.*

Q 19.7 *Is leverage a strategic advantage? Describe the arguments on both sides.*

19·3. Operating Policy Distortions: Behavior in Good Times

In most of the previous section, debt was usually worse than equity, because it made it more likely that the firm would enter financial distress. But problems in choosing the wrong projects do not arise only when the firm is in trouble. In this section, we discuss what can happen if the firm operates far from financial distress. Just as too much debt can cause the firm—primarily shareholders—to make poor operating decisions when financial distress looms, too little debt can cause the firm—primarily managers—to make poor operating decision when business is going well.

19·3.A. Agency Issues

We already covered agency conflicts in Chapter 7, and will cover more agency conflicts in Chapter 24 on corporate governance.

Managers like building empires and consumer perks; debt restrains them.

Free Cash Flow Managers usually prefer spending money internally on their pet projects instead of returning money to shareholders. For example, in the 1980s, many large oil companies continued exploring for oil, even though it was well known that oil companies could be bought on the stock exchange for significantly less than the expected cost of finding equivalent oil reserves. How can capital structure counterweigh this? Debt requires coupon payments, which force managers to perform. Managers who fail to generate enough income to pay the coupon are subject to bankruptcy, and (as has been shown empirically) almost always lose their jobs. Therefore, managers who have to service more debt will spend less wastefully, which makes such firms worth more *today*.

Theft and graft.

Theft (and Verification) Another important problem is outright **theft.** If you are a passive partner, you are dependent on true and accurate reporting of what profits really are. The active partners or the managers, however, might try to avoid reporting large profits: they might rather use corporate cash to build more of an empire, to compensate themselves better, or just to outright steal it! Debt has the advantage that the creditor may not even need to know what the profits are: if the agreed-upon payments are not made, the creditor can force bankruptcy.

Holdup costs.

Stakeholder Holdup Higher potential hold-up costs are another important drawback of equity. When a company, especially a public company, rolls in cash, anyone who has the power to hold up business will try to extort more of a share of these profits. For example, a supplier who delivers an important input, a wholesaler who is an important distributor, and any key employees who can bring production to a stop, may want to pressure the firm in order to renegotiate their deals. Yet if the company is financed more via debt than equity, these third parties will recognize that there is less cash to expropriate. After all, if the company does not pay the debt, it can go bankrupt. Thus, in a company with more debt, the equity earnings (which parties can renegotiate) are smaller.

Important: *Free cash flow and agency concerns favor debt over equity as the cheaper financing vehicle.*

Solve Now!

Q 19.8 *Give some examples of perks that management might give up when debt load increases.*

19·4. Bondholder Expropriation

If there is debt, equity shareholders may want management to exploit the debt holders. This has bad ex ante value consequences.

You already know that managers should structure the firm *at the outset* so as to make it in their interest to optimize firm value in the future. But to raise debt at an attractive interest rate, managers must also take into account that bondholders know that managers might later want to transfer value from bondholders to shareholders. After all, creditors realize that it is the shareholders who vote managers into office, not the bondholders. Managers can expropriate bondholders on behalf of shareholders in two ways:

1. They can increase the risk of the firm's projects (a change in operating policies).
2. They can issue further bonds of equal or higher priority. (Bonds that pay cash earlier are de facto higher priority.)

If potential bondholders believe that they can be expropriated, they will demand a higher cost of capital today. Let's understand this better.

Anecdote: Airlines, Unions, and Shareholders
In September 2002, American Airlines (AMR) operated over a thousand airplanes, and owned about half of them. It had assets valued about $30 billion, and debt valued at around $15 billion. Still, its equity market value was only $800 million—about the price of three of its forty top-of-the-line Boeing 777 airplanes. And it is not clear if American was worth even this $800 million: bankruptcy was imminent for all other major U.S. carriers (except Southwest).

In 2002, American lost a significant amount of money operating. But if American is ever to make positive profits again, its unions will surely capture the lion share. After all, it only takes one of its unions (e.g., pilots, flight attendants, mechanics) to ground a fleet worth $30 billion and to wreck customer loyalty. The unions will ultimately make sure that shareholders will receive just enough for them not to kill the golden goose.

How airlines continue to exist as public corporations, instead of as employee-owned organizations, remains a mystery to me. (In my opinion, debt may be the only chance that the major airlines have to restrain union demands.)

19·4.A. Project Risk Changes

Table 19.4. Risk-Shifting

		Bad Luck	Good Luck	Future Expected Value	Today's Present Value
Prob:		1/2	1/2		
Project	W	\$60	\$160	\$110	\$100
Capital Structure LD: Bond with Face Value FV=\$55					
Bond(FV=\$55)	D	\$55	\$55	\$55	\$50
Equity	E	\$5	\$105	\$55	\$50

Adding Risky Project "New"

		Bad Luck		Good Luck		Future Expected Value	Today's Present Value
Prob:		1/4	1/4	1/4	1/4		
Project	W	\$60	\$60	\$160	\$160	\$110	\$100.00
Project	New	\$50	–\$60	\$50	–\$60	–\$5	–\$4.54
Total Projects		\$110	\$0	\$210	\$100	\$105	\$95.45
Capital Structure LD: Bond with Face Value FV=\$55							
Bond(FV=\$55)	D	\$55	\$0	\$55	\$55	\$41.25	\$37.50
Equity	E	\$55	\$0	\$155	\$45	\$63.75	\$57.95

The cost of capital in this example is 10% for all securities, which is equivalent to assuming risk-neutrality.

Adding a risky, but negative NPV project changes the state-contingent payoffs.

The first expropriation risk that creditors face is called "risk-shifting." Table 19.4 returns to our firm with an LD capital structure from Table 19.3, but allows managers to add project "New" after the original debt has been raised. The new project is independent of the old project and pays either +\$50 or −\$60 with equal probability. It is negative NPV, so it would not be too hard for managers to find such projects—any Las Vegas casino provides better investment opportunities. Why would a negative NPV project matter? Would the managers not reject this negative NPV project?

The ex post redistribution.

The lower half of the table shows that if the new negative NPV project is taken, the value of the equity would increase from \$50 to \$57.95. If shareholders are in firm control of their managers and vote them into and out of office, managers would indeed take this project! In essence, the new project would eliminate \$50 – \$37.50 = \$12.50 of bondholder value, waste \$4.54, and hand \$7.95 extra value to shareholders. The intuition is that this risky project gives existing shareholders relatively more of the upside and existing bondholders relatively more of the downside.

Ex ante Effects.

Everyone—managers, shareholders, and bondholders—recognizes that taking the project will be in the interest of the managers if a bond with a face value of $55 was originally sold. Although *ex post* this is good for equity holders, *ex ante* it is bad for them (and the firm). Skeptical creditors would therefore assume that the debt payoff is only $41.25 (not $55), and thus pay no more than $37.50. The firm would have to pay a cost of capital of 46.7%, even if it wanted to finance itself with debt.

This also works with positive NPV Projects.

If you now conclude that it is good for the corporation to commit itself not to take other projects, you would be wrong. This could backfire, too. If a new project were to come along that either pays off –$60 or +$500, it would be highly positive NPV. If creditors had negotiated a commitment at bond issue, they would insist that the project not be taken, because their wealth would still decline. But this would prevent the firm from taking great projects. Therefore, a wholesale *ex ante* commitment not to take any more projects is not necessarily a good thing for the value of the corporation.

19·4.B. Issuance of Bonds of Similar Priority

Table 19.5. Issuance of Equal Seniority or Shorter Term Bonds

		Bad Luck	Good Luck	Future Expected Value	Today's Present Value
Prob:		1/2	1/2		
Project	W	$60	$160	$110	$100
Capital Structure LD: Bond with Face Value FV=$55					
Bond(FV=$55)	D	$55	$55	$55	$50
Equity	E	$5	$105	$55	$50

Adding an Equal Priority Bond

		Bad Luck	Good Luck	Future Expected Value	Today's Present Value
Prob:		1/2	1/2		
Project	W	$60	$160	$110	$100
Capital Structure LD Plus: Two Equal-Priority Bonds					
Bond(FV=$55)	D	$73\% \cdot \$60 = \$44$	$55	$53	$48.18
Bond(FV=$20)	D	$27\% \cdot \$60 = \$16$	$20	$18	$16.36
Equity	E	$0	$85	$42.50	$38.64

The cost of capital in this example is 10% for all securities, which is equivalent to assuming risk-neutrality. 73% is the proportional allocation, $\$55/(\$55 + \$20) \approx 73\%$.

Managers may also exploit bond holders by issuing more debt.

The second expropriation risk that creditors face is the issuance of more bonds of equal or higher priority. (Paying out some cash before the original bond comes due is in effect higher priority.) Table 19.5 shows an example, in which the firm issues another bond with a face value of \$20 that has equal priority. In bankruptcy (the bad state), the old bond would have to share proceeds with the new bond of equal-priority. Being equal, the "spoils" would be often allocated according to face-value within bonds of the same priority. So, because the \$20 bond represents $\$20/(\$20 + \$55) \approx 27\%$ of the debt claim, it would receive $27\% \cdot \$60 = \$16$; and the \$55 bond would receive the remaining $73\% \cdot \$60 = \$44$. This means that when the firm announces the issuance of the new bond, the old bond would immediately drop by $\$50 - \$48.18 = \$1.82$ in value. Would this be in the interest of the equity? It now receives nothing in the bad state and \$85 in the good state—plus the one time dividend of \$16.36. In total, by issuing new debt of equal priority, equity holders would have increased their wealth from \$50 to \$38.64+\$16.36=\$55.

Again, fearing expropriation, the firm has to pay a higher interest upfront to potential bond holders.

This expropriation is not as bad as our risk-shifting example in that no value is destroyed. But it is equally bad insofar as the first creditors will again assume that they will be expropriated, and therefore demand a higher interest rate today. They would demand a quoted interest rate of $\$55/\$48.18 - 1 \approx 14.2\%$. To recoup this higher interest rate, the firm has no choice but to indeed issue more bonds that expropriate the first bond purchasers later. In effect, before deciding on any capital structure, the firm has two choices: either issue no bonds, or be dragged into a capital structure that requires more and more *too much* leverage. (This can in turn increase financial distress costs.) As before, an *ex post* issue has consequences *ex ante*.

SIDE NOTE: Creditors may also face uncertainties if bankruptcy courts do not uphold the agreed-upon **Absolute Priority Rule** (that bond holders are to be paid in full before equity holders receive anything). Known deviations from promised **absolute priority** simply change the contingent payoffs and thus the effective values of the securities. Such known violations do not reduce the total value of the firm. Relative to strict **APR**, the value of the bonds is just lower by the amount that the value of the equity is higher.

19·4.C. Counteracting Forces

Bondholders demand a premium *ex ante* that they would not demand if the firm could commit not to expropriate them *ex post*. The premium may prevent the firm from raising debt at fair interest rates, and thus tilt the optimal capital structure more towards equity. Even managers with the best intentions not to act against bondholders may not be able to shield themselves from the pressures of expropriating creditors later. Who ultimately loses? To the extent that smart bond investors anticipate their fate, they will demand and receive fair compensation. Ultimately, it is the firm that is robbed by itself. Its inability to commit not to expropriate creditors may prevent from issuing debt at fair prices—which may mean it may have to forego debt's other advantages, such as tax savings.

In the real world, there are a number of mechanisms that can help to reduce the fears of bondholders, and thereby allow the firm to issue debt at acceptable interest rates—and thereby lower the firm's cost of capital.

Managerial Risk Aversion

We noted earlier that shareholders like increases in project risk, because they help them at the expense of bondholders. However, it is not clear if managers really act on behalf of shareholders and thus like higher risk, too. After all, if the project fails and the firm enters financial distress, they might get fired themselves. Thus, managerial risk aversion is a natural counterbalance to the shareholders' incentives to increase risk.

Bond Covenants

Bond covenants reduce exploitational opportunities in the future—but at a cost in flexibility.

A variety of bond covenants have developed to mitigate bondholder skepticism.

- Many bonds prohibit excessive dividend payouts.
- Many bonds prohibit large new debt issues, especially of shorter-term and of equal priority debt.
- Many bonds require the maintenance of certain financial ratios. For example, covenants may mandate maximum debt-equity ratios, maximum payout ratios, minimum earnings retention ratios, minimum liquidity ratios, and so on. These ratio restrictions can all help prevent the firm from taking on riskier projects.

If the covenant is broken, creditors can sue or demand their money back. Covenants are never perfect. It is just impossible to enumerate all the things managers can do. In addition, if the firm enters Chapter 11 bankruptcy, the law says that any new debt issued will automatically receive higher priority, no matter what the covenants of the original bond stated.

Bonds with strong covenants often have a "call" feature, that allows the firm to retire the bond before maturity at an agreed-upon price—and thereby free themselves of the covenant requirements.

Corporate Reputation

And, again, covenants reduce the flexibility of the firm to take advantage of other opportunities. Sometimes, reputation can substitute for covenants.

Covenants are inflexible, so they impose costs, too. For example, if the firm happens to come across a project with $1 billion in NPV, the covenants could prevent it from taking it. Again, a firm that fails to take all profitable projects in the future is worth less today. One alternative to formal covenants is for firms to build a less formal "reputation." This is not easy to do, but firms may realize that it is in their interest not to exploit current bondholders, because any future bondholders would henceforth definitely assume the worst behavior. Put differently, if managers were to take advantage of creditors today, then future financing costs would be so much higher that managers would rather not do so. Reputation is not perfect, though, especially if the advantage that can be taken of creditors today becomes very large. The most prominent example of broken reputation was R.J.R. Nabisco. In the 1980s, it was generally believed to be a safe investment for bondholders. However, when it was bought out in 1988 in the largest LBO ever, R.J.R. tripled its debt overnight, its outstanding bonds went from investment grade to junk grade, and bondholders experienced an announcement month loss of 15%.

Convertible Bonds or Strip Financing

Convertible bonds allow bondholders to participate in the upside, and reduce exploitational incentives in the future.

The final way is to try to allow creditors to partake in the upside of equity. The most common such financing vehicles are convertible bonds. Again, they can limit the *ex post* expropriation of bondholders, while still preserving the firm's option to accept new projects. Instead of straight bonds with strong covenants, "convertible bonds" with weak covenants allow creditors to participate if a great new project were to come along. This reduces the risk expropriation problem. One of the following exercises will ask you to show how a convertible bond can reduce the expropriation. Strip financing, in which individuals purchase debt and equity in equal units, is a similar idea—it eliminates the incentives of shareholders to exploit themselves.

Recap: It is in the firm's interest to commit not to take advantage of its creditors in the future. To the extent that the firm fails, debt is too expensive today.

In the real world, firms have to undertake a delicate balancing act. When they issue debt, it can only be issued at favorable terms when the firm can promise not to exploit bondholders after the bonds are issued. Even if such promises can be credibly made, they cause a loss of flexibility, which can be expensive. This can mean that the firm cannot issue debt—and thus that it has to forego some other beneficial effects of debt (such as tax advantages).

IMPORTANT:

- *Creditors can lose value if*
  - *the firm later undertakes riskier projects; or*
  - *the firm adds more debt of equal or higher priority.*
- *Creditors demand higher interest rates if they fear such expropriation. Thus, it is in the interest of the owners to assure creditors that they will not do so. The prime mechanisms to accomplish this are*
  - *Bond Covenants,*
  - *Reputation, and*
  - *Bond Convertibility.*

Solve Now!

Q 19.9 *Why do bond covenants exist?*

Q 19.10 *What is the advantage of adding convertibility features to a bond?*

Q 19.11 *Describe the two basic mechanisms whereby unprotected bondholders can be expropriated by shareholders, preferably with a numerical example.*

Q 19.12 *Return to a project similar to the firm in Table 19.4. The risk-neutral required interest rate is 10%. The firm is worth either \$100 or \$120. The bond promises \$90.*

(a) *Work out the value of the firm. For the bond, create three rows for each state: if bondholders do not convert, if bondholders always convert, if bondholders optimally convert. (Assume project "New" is not available.)*

(b) *Now project "New" becomes available. It will pay off either +\$50 or -\$60. Show that it is in the interest of shareholders for this project to be taken if the bond is not convertible.*

(c) *Now presume that the bond is also convertible into 75% equity. Show that it is no longer necessarily in the interest of shareholders to take the bad project "New."*

19·5. Inside Information

New potential partners/shareholders have less information than current managers and owners.

Our next important determinant of capital structure is inside information. Typically, firm managers (acting on behalf of the old owners) have better information than new investors. New investors should be careful that they are not exploited. As the old adage says, “Never bet with someone better informed than yourself.”

If owners want partners rather than lenders, the project may not be as good.

Consider this scenario: you are a potential investor in an oil well, and you know that the current owner/manager (who has to raise new capital) already knows whether there is oil or not. You do not know. You have to ask yourself:

- What will you believe about the oil well if the present owner offers to make you a full partner sharing in all future profits?
- What will you believe about the oil well if the present owner asks you for a loan to be paid back that she is willing to collateralize with her present assets?

If you are offered partnership, you should be reluctant to believe that there is oil. If, however, the present owners want to keep the profits and simply borrow, she probably knows that the project is profitable. This is sometimes called the **winner’s curse** or **adverse selection.** If you receive the offer to become partner, it does not help you very much. If you do not receive the offer to become partner, you would be better off if you had indeed received it.

Table 19.6. Inside Calculations

		Bad Luck	Good Luck	Future Expected Value	Today’s Present Value
Prob:		1/2	1/2		
Project	W	$60	$160	$110	$100
Capital Structure LD: Bond with Face Value FV=$55					
Bond(FV=$55)	D	$55	$55	$55	$50
Equity	E	$5	$105	$55	$50

To raise $50, if:

Project is	Creditors Believe	Future Payoffs: Percent of equity to raise $50	If equity financed, owners keep	If debt financed, owners keep
Good	Good	$50/$145.45 = 34%	$55 + 65.6% · $160 = $160.00	$160
Bad	Good	$50/$145.45 = 34%	$55 + 65.6% · $60 = $94.36	$60
Good	Unknown	$50/$100 = 50%	$55 + 50% · $160 = $135.00	$160
Bad	Unknown	$50/$100 = 50%	$55 + 50% · $60 = $85.00	$60
Good	Bad	$50/$54.55 = 92%	$55 + 8% · $160 = $68.33	$160
Bad	Bad	$50/$54.55 = 92%	$55 + 8% · $60 = $60.00	$60

The cost of capital in this example is 10% for all securities, which is equivalent to assuming risk-neutrality.

This analogy is directly transferable to capital structure. Sharing in the firm's equity is the equivalent of becoming a partner. Table 19.6 again considers our example, but adds the knowledge of owners and your beliefs as a potential investor.

A numerical example of the inside information problem.

If you also know project quality: Not surprisingly, if the project is good and you believe this, the owners end up with \$160 next year. Similarly, if the project is bad, the owners end up with \$60 next year. Unfortunately, you do not know this.

If you believe either project quality is equally likely: This implies that you are willing to purchase equity based on the expected project payoff of \$110. Thus, you would lend \$50 in exchange for half the firm.

Now consider what current owners would do. If they knew the project was good and financed through debt, they would be better off (\$160) than if they financed through equity (\$135). Conversely, if they knew the project was bad and they financed through equity, they would be better off (\$85) than if they financed through equity (\$60).

It follows that owners that prefer to finance with equity know that their projects are worse than average, and owners that prefer to finance with debt know that their projects are better than average. Therefore, it would not be reasonable for you to believe that the project was equally likely to be good or bad. It would be irrational for you to maintain such beliefs.

If you believe equity-financed projects are bad: If current owners knew the project was good and financed through debt, they would be better off (\$160) than if they financed through equity (\$68.33). Conversely, if current owners knew the project was bad, no matter how they financed the project, they would end up with \$60. For convenience, we can assume that they then prefer to finance with equity. (Think about them getting 1 cent extra.) Thus, your beliefs are confirmed—firms that ask you for equity financing are bad. You would assume that debt-financed firms are better than average, and that equity-financed firms are worse than average.

The "Pecking Order": More equity-like (partner-like) shares are bad news, and can only be sold at a discount.

New equity investors are not inclined to assume that the project is good. They will assume that their new claims are on a project that will eventually develop problems. Thus, when existing owners announce a new equity offering, it releases information that the firm's projects are worse than generally believed, and the new equity can only be sold for a very low price. In real life, we indeed observe that when firms announce that they plan to raise about \$1 by issuing new equity, their old public equity value declines by about 10 cents. But this argument extends not only to equity, but to other claims as well. The more risky the securities are that insiders want to sell rather than keep, the worse are their beliefs. Sharing in more junior (risky) bonds is the equivalent of the present owners making you a "little partner," when they are not willing to collateralize their loan. Consequently, the announcement of a risky junior security releases information that the firm's projects are not too great, but not too bad, either. In contrast, the new issue of a collateralized loan (or a risk-free senior bond) will indicate that the firm's projects are better than expected. The outcome is that the better the firm's projects are, the more senior the security the managers will offer for sale. This is the **pecking order** view of capital structure: the best projects are financed by the most senior debt, worse projects by junior debt, and the worst projects by equity.

Firms may want to avoid issuing more shares of anything, and equity in particular.

What does this imply for the optimal capital structure? Consider a firm that cannot issue debt easily because it has little collateral or because additional debt would unduly increase expected bankruptcy costs. If it cannot issue equity because of these insider concerns, such a firm may have to pass up on some good (but perhaps not stellar) projects, simply because owners do not want to sell their projects at the price of the worst possible scenario. A publicly trading firm thus may take on too much debt (incurring financial distress costs) or ration its projects, failing to take at least some of its positive NPV projects.

IMPORTANT: *Inside information concerns favor debt over equity as the cheaper financing vehicle.*

There is a way out!

When could a firm issue equity without an insider penalty?

- If there is a mechanism—e.g., a detailed audit—by which insiders with good projects can credibly convey the true quality of the project, it would be in their interest to do so. Indeed, if such a mechanism is known to exist, and owners do not undertake it, potential investors should immediately assume that current owners are not doing so because they know that the outcome will be bad.

- If current owners can convince potential investors that they have invested all of their own money, that they have maxed out their personal credit cards, and they just cannot put any more personal capital at risk than they already have, then there is no information in the fact that they are asking to raise equity capital. In this case, external investors can assume that the project is not necessarily bad. Indeed, no venture capitalist will ever invest in a startup in which the current owners do not have most of their personal wealth at stake.

Agency and inside information are closely related.

The pecking order (inside information) and free cash flow (agency) theories have a very close family relationship. The former says that when firms issue equity, managers signal that they believe that the future will be worse. The latter says that when firms issue equity, managers will make the future worse—they will waste the money. Both send information signals to the public about a worse future, although the latter is more causal than the former.

Solve Now!

Q 19.13 *A house up for auction can be worth either $500 or $1,000 with 50-50 probability. The other bidder knows the true value; you do not. If you bid for the house in an auction, what should you bid? If you bid $750, what is your expected rate of return?*

19·6. Transaction Costs and Behavioral Explanations

Transaction costs have played an important role in all capital structure examples above: if transaction costs had been zero, external pressures would force management to choose the best capital structure. But if transaction costs are high, managerial mistakes are difficult or impossible to correct for outsiders. It is not just enough for an outsider to purchase shares and then sell them. The appropriate corrective action requires accumulating enough shares and pressuring management to improve the situation. Without the discipline of external pressure, managers and investors can commit mistakes. They may take too much debt or too much equity, and the market may not be able to correct it.

Transaction costs are everywhere. They definitely can prevent optimal adjustment.

Section 15·2 has already described the link between high transaction costs and **behavioral finance**. In the corporate finance context, the presence of high transaction costs—and, with it, the rarity of correcting mechanisms—means that behavioral finance plays an important role. It can rationalize a lot of managerial behavior, which is otherwise difficult to explain. Unfortunately, on so vague a level, without a further description of what the mistakes are, it is less prescriptive than the above theories. That is, it seems to offer little guidance as to how a smart manager should act differently (i.e., what the best debt/equity ratio is).

Transaction costs “cause” behavioral finance concerns.

Then again, behavioral finance is the most promising new direction in corporate finance. It is still too early to tell where and how it will help us better understand the world. Some early insights suggest that there are certain behavioral mistakes that are more common than others. For example, we now believe that **overconfidence** and **overoptimism** are common traits among both managers and investors. If managers are overoptimistic, it may aggravate agency concerns (they may take some negative NPV projects) and no-liquidation concerns, but alleviate underinvestment problems. If investors are overoptimistic, issuing equity may not be so disadvantageous as the inside information argument suggests. Investors may not necessarily believe the worst—and there is some evidence that such was the case during the Internet bubble at the turn of the millenium. Although it is less likely that markets rather than managers are committing mistakes, there is good evidence that financial markets may be imperfect, too. If markets indeed undervalue securities—either because they are irrational or imperfect—it would be rational for managers to seek to time equity issuing activity.

Specific behavioral errors can have specific consequences.

Another domain where behavioral effects seem particularly important are dividends. There are at least some investors out there who seem to prefer dividends, even if this is not a wise choice from a tax perspective. In response, rational managers may want to pay dividends rather than use share repurchases to send cash to their equity holders, even if this incurs a higher tax obligation. In any case, there is little that financial markets can do about the wrong payout policy. The world is how it is.

Dividend policy is probably a behavioral managerial error.

A more traditional view is that transaction costs also play a direct role. For example, the reporting requirements and liabilities imposed by the *1933 Securities Acts* for publicly traded equity securities can be much larger than those for private borrowing. For many small companies, these costs may be large enough to warrant a capital structure consisting exclusively of private securities and bank debt.

And then there are also direct transaction costs...

IMPORTANT: *Behavioral considerations could favor either debt or equity.*

Solve Now!

Q 19.14 *What are behavioral explanations for dividend payout?*

19·7. Corporate Payout Policy: Dividends and Share Repurchases

You should realize that all these considerations apply to our capital structure, and therefore apply both to the appropriate debt-equity level, as they apply to changes therein—firms may issue debt or equity to send cash into the company, or they may pay out coupon and principal to send cash back to owners. Dividends and equity share repurchases are particularly interesting, because they are at the discretion of management.

Institutional details and "the need for cash"?!

When companies decide to return cash to shareholders, they do so either by dividends or share repurchases. Many public companies pay dividends every quarter, others pay only once a year. Sometimes, firms declare "special dividends" to indicate that they will not repeat. The process to pay dividend is that the board of director votes to pay a dividend (on the declaration date), and then collects the names of all shareholders (on the record date). For shareholders, shares that are owned before the **ex-dividend day** are called **cum dividend** and will receive the dividend payment; shares owned after the ex-dividend date are without the dividend payment. Naturally, shares are worth more with an extra dividend, so the price will fall after the ex-dividend date—usually by just about the value of the dividend itself. Otherwise, a tax-exempt investor can earn close-to-arbitrage profits. For example, if a \$40 stock pays an annual dividend of \$2 (a dividend yield of 5%), this stock will trade at \$38 after the ex-dividend date. Finally, the dividend checks are mailed (on the payment date). Note that it makes no sense to argue that dividends are paid because particular investors "need" money—if you hold 100 shares for \$4,000 and receive \$200 worth of dividends, you would have \$3,800 shares left; but you could just as well have sold 5 shares for \$200 on the stock exchange, which would similarly have left you with \$3,800 and \$200 cash.

Share repurchases and dividends are perfect substitutes in a perfect world.

Share repurchases are direct substitutes for dividend payments. One naïve misconception is that share repurchases are different because they benefit only shareholders tendering their shares into the repurchase—unlike dividends, which benefit all shareholders. This argument is false. A firm with 100 shareholders, each owning \$10 worth of shares, could pay \$50 worth of dividends (50 cents to each shareholder), and the firm would be worth \$950. Each shareholder would have a share worth \$9.50 and \$0.50 in dividends. If the firm repurchased \$50 worth of shares, the firm would now have 95 shareholders, each owning \$10 worth of shares. *In the perfect M&M world, without taxes, all shareholders are equally well off with either a repurchase or a dividend payment.*

In the real world, dividends are tax-disadvantaged on the investor side, relative to share repurchases.

Like the debt/equity choice, dividends and repurchases matter only if we leave the M&M world. For example, investors who need money may now save on transaction costs of having to sell shares, or the dividend payout may reduce agency conflicts and money-wasting by managers, or there may be tax differences. We will focus primarily on the role of personal taxes. If taxes mean that there is less money left over for the dividend payments than there is for the share repurchases, it must follow that share repurchases should be preferred from a tax perspective. And, indeed, we know that dividends are more difficult to shelter than share repurchases, if only because the investors that tender into the share repurchase will be those who find it easiest to shelter the capital gains. The effect is that effective personal income taxes are higher on dividends than on share repurchases. Again, the tax clienteles in the economy will evolve to reduce the effective dividend taxes paid. That is, clientele arguments cannot only explain why some investors prefer holding equity over debt (to receive capital gains instead of taxable interest payments), they can also explain why dividend taxes are not as punishing as they would otherwise seem to be. (Some low-tax pension fund investors can avoid the personal tax on dividend receipts.) But the clienteles can just reduce the penalty of dividends relative to share repurchases—they cannot eliminate it. They can therefore not explain why firms pay dividends—share repurchases remain uniformly better, because they avoid *all* personal income taxes. Share repurchases are both a better and a perfect substitute for dividends.

Share repurchases are becoming relatively more important over time.

Historically, it was an important puzzle why corporations would ever pay dividends. Both the tax advantage of share repurchases over dividends and the average corporate dividend yield in the economy were high. Firms eventually figured this out, too. Bagwell and Shoven (1989) found that big firms paid out about 7 times more through dividends than through share repurchases in 1977—but by 1987, this ratio had dropped to about 1.5. In any case, by today, this payout puzzle has become fairly unimportant. As of 2003, typical **dividend yields** (the amount of dividends divided by the share price) have declined to about two percent per year. Of three tax advantages of capital gains over repurchases—a lower statutory rate, taxation only at realization, and an offset against capital losses—only the latter two remain. The Bush tax cuts (in effect at least until 2008) have much reduced statutory rate differences. Therefore, the effective remaining advantage of capital gains vs. dividends is now fairly small. (A good number of firms responded to the Bush tax cuts in the logical way, most prominently Microsoft—they started paying dividends!) Add to this that some dividend payments are received by tax-exempt institutions (who often purchase shares just before the dividends are paid and sell them just thereafter)—and the question of whether to pay out cash in dividends or share repurchases is no longer a very important for the typical company. Dividends may still not be the smartest method for managers to pay out, but the wasted money is too small to offer large arbitrage opportunities (buying the firm, firing the CFO, changing the payout policy, and selling the firm for a higher price). The presence of dividends is less of a mystery than it once was.

Remaining differences: changes in inside ownership, an informal understanding about persistence of future payments, and "behavioral finance" type investor preferences.

So, with the prime historical difference between dividends and share repurchases (personal income taxes) much reduced, what are the remaining important differences? Not much, but here they are:

1. Executives often receive stock options in the company, whose value depends on the share price. Because share prices drop when companies pay dividends, managers with many options prefer repurchases to dividend payments.

2. Executives and insiders are often not permitted to tender their shares into share repurchase offers, and thus will own relatively more of the company after a share repurchase than after an equivalent dividend payment.

3. Dividends tend to be regular and are expected by shareholders to continue. An ordinary dividend payment therefore informally obliges management to continue, thereby signaling more optimism about the future. In contrast, many share repurchases are done in one-time chunks. The distinction is not perfect, though. Many companies have semi-regular share repurchase programs, which make repurchases almost as regular as dividend payments. And many other companies pay "special dividends," which immediately signal their one-time nature to investors. Such special dividends are as much "one-time" as share repurchases.

4. There is behavioral finance empirical evidence that many retail investors just "like" dividends better than share repurchases.

   Today, this matters less, but it used to paradoxical, when dividend payouts were badly tax-disadvantaged. The argument that investors like dividends "because they need cash" does not hold water. Selling a fraction of the shares in stocks that pay zero dividends provides physical cash, too—except that the investor would not have had to pay as much in personal income tax. Still, it seems that many investors wrongly think only of share sales but not of dividend receipts as reduction in their "investment substance."

   The empirical evidence is still under academic investigation. My guess is that the answer will likely be that these individual investor preference effects are real, but that they are not universal. For example, we also know that many tax-exempt institutions are obliged by their charters to hold *only* dividend paying stocks—just as the clientele theory would have predicted.

Advice?

So what payout policy should a company choose? The most important recommendation is that a company should pay out cash when the alternative uses are not positive NPV projects. Of course, many managers will not like to hear this advice or they will assert that all of their projects are high NPV (even if they are not). Compared to the question of whether the firm should pay out or not pay out, the question of whether the form of payout should be dividends or share repurchases is of secondary importance today. In cases of doubt, share repurchases seem better than corporate dividends. If the Bush dividend tax reductions will disappear in 2009, share repurchases would seem *much* better than dividends.

Stock dividends and splits.

Before we close, you may sometimes hear of **stock dividends** and **stock splits**—these are really not payouts, at all. For example, a \$1 million company whose shares are trading for \$100/share may issue a stock dividend of 1 share for every 10 outstanding shares. Its 10,000 shares would become 11,000 shares, each worth \$90.91. Or it may splits its shares 2-for-1, which means that its 10,000 shares would become 20,000 shares, each worth \$50. Another strange creature is the **dividend reinvestment plan**, in which shareholders can volunteer to automatically reinvest dividend receipts in the company. They therefore never receive any cash. All that they receive is a tax obligation at the end of the year for the dividends that they presumably received. If the company had just kept all the money, it would have saved its investors the tax obligations.

SIDE NOTE: Tax schemes also apply to investors If personal investors want to hold dividend-paying stocks without paying dividend taxes, they can do so for most of the year—as long as they sell them to institutions prior to the **ex-dividend day**, which is the day on which the owner of the shares is determined for the distribution of the dividend.

In real life, this "tax-arbitrage" indeed happens; tax-exempt funds compete to purchase these shares, driving up the share prices before the ex-dividend date. The *Financial Times* reports that such transactions are known as "bed-and-breakfast" deals for equity, and "bond-washing" for bonds—even though both the IRS and the **Bank of England** have specifically prohibited such tax arbitrage. The latter has imposed a 1-week holding period for tax-free institutions purchasing around dividend dates.

Of course, there are limits to such dividend tax arbitrage. Some high-tax investors would have to pay capital gains taxes when they sell their shares, and thus prefer paying taxes on the dividends instead of on their capital gains. In addition, the round-trip transaction costs limit the possible profits from the avoidance of dividend taxes.

Units are combination securities that consist of a debt security and an equity security. If the firm pays interest, it shifts its tax burden to the unit owners. If the firm pays dividends, it shifts its unit owners' tax burdens to itself.

ANECDOTE: Ralph Nader and Microsoft
On January 4, 2002, Ralph Nader wrote an open letter to William H. Gates, III, Chairman of Microsoft, that begins as follows:

> We are writing to ask Microsoft to change its practice of not paying dividends to shareholders. Our reasons are as follows.
>
> 1. The quantitative failure to pay dividends year after year is an inappropriate and we believe unlawful device to shelter Microsoft earnings from federal income taxes.
>
> By not paying dividends, wealthy Microsoft shareholders such as yourself avoid paying the top marginal tax rate of 39.6 percent that would apply to income distributed as dividends. By taking earnings entirely through stock sales, wealthy shareholders lower their tax rate to the maximum 20 percent that applies to capital gains. According to the most recent SEC reports on insider trades, you personally sold more than \$2.9 billion in Microsoft stock last year, benefiting enormously from the lower tax rate that applies to stock sales.

This letter does not even point out that 20% is an overstatement: Bill is taxed only on *realized* capital gains! If Bill does not sell his shares, he suffers zero taxes on increases in his wealth over the year. And, with the **GOP** elimination of the **estate tax**, neither may his heirs. The Bush tax reforms of 2003 have further significantly reduced the taxes on dividend payments. Microsoft promptly started paying dividends in 2003—many billions worth.

Here is an interesting question: Is it the fault of Bill Gates (who is also a prolific political campaign donor) or is it the fault of the U.S. government that Gates has suffered only minimal tax obligations on his wealth gains over the last 20 years?

Solve Now!

Q 19.15 *If a normal investor cannot participate in a share repurchase program, is she better off with a dividend payout than with a share repurchase?*

Q 19.16 *What companies should pay dividends?*

19·8. Synthesis

Of course, we have not covered *everything* possible about capital structure in our chapter. The real world is considerably more complex than the surgical scenarios we have explored. But, you now have a very good grasp of the most important issues. Let me close our chapter with a number of important issues that did not fit earlier.

19·8.A. Cost of Capital Calculations

WACC still works for managers. These factors impact changes in the cost of capital, r, that managers face.

As with personal income taxes, managers need not compute what expected value consequences their decisions will have: if they choose a capital structure leading to inferior future payoffs, they have to give away a larger percentage of the firm today, which manifests itself in the WACC that they experience. (A smart manager could try to compute the value of the firm "as if operated in a purely value-maximizing fashion," and then subtract out the expected costs imposed by the future suboptimal behavior caused by a particular capital structure. However, this would be so difficult that no manager does it.)

IMPORTANT: *Only corporate income taxes require the use of* WACC *or* APV *formulas. All other distortions, ranging from personal investor taxes to bankruptcy distortions, manifest themselves in the cost of capital (the appropriate interest rate) that investors demand. The optimal capital structure allows managers to raise financing at the lowest cost of capital, and thereby maximizes the wealth of current owners.*

19·8.B. Interactions

Executing the value-optimizing strategy may not be possible.

Typically, firms do not face each of the above problems in isolation, but all at the same time. The presence of one problem—or attempts to reduce it—may worsen another. That is, there can be significant costs to move from a suboptimal to an optimal capital structure.

Conflict among different interest groups can prevent optimal solutions.

For example, we already know that if a firm is close to bankruptcy, issuing equity could avoid or reduce bankruptcy costs, which in turn would increase firm value. But the infusion of more equity may mostly benefit bondholders, so equity holders may not agree to put in more equity. So, although a reorganization (i.e., a new start) could install a capital structure to increase firm value, there are problems to be resolved to get there, given the current capital structure.

IMPORTANT: *Interaction effects can make it difficult to optimally adjust capital structure in the future. This can favor a more flexible capital structure (more equity and slack) today.*

Equity Infusions are not always good, either, though.

Of course, equity infusions to stave off bankruptcy are not always good, either. For example, it could be that it would be more efficient to liquidate the firm, and the equity infusion allows the firm to continue to operate inefficiently. Raising more equity might thus facilitate the *wrong* managerial action.

19·8.C. Reputation and Capital Structure Recommendations

Sometimes, owners are best off building a corporate reputation, which can help alleviate investor worries.

We have already mentioned another important factor in the real world—reputation. It can lower financing costs, improve incentives, and increase firm value. Remember an earlier example, in which the presence of the *ex post* ability to expropriate bondholders hurts the firm. If managers had a reputation not to take such bad projects, perhaps overly restrictive covenants could be avoided, in effect lowering financing costs *ex ante*. More importantly, the example assumed that everyone knew exactly what expropriation opportunities existed, and what their probabilities were. But, despite restrictive covenants, bondholders will always have the nagging suspicion that they may be expropriated after all when unforeseen opportunities appear. Only the building of trust and reputation can overcome such suspicions, with their associated increase in financing costs.

Do investors trust managers? Can investors trust managers? Should investors trust managers? When is it worthwhile for a manager/firm to build such a reputation? How can this effectively be accomplished? These are difficult questions to answer empirically, but they are important in the real world.

Choosing the best Capital Structure is a combination of art and science.

Ultimately, the trick in being a good manager is to judge and weigh the plethora of marginal costs and marginal benefits of projects, of debt, and of equity, and to have sound judgment in deciding on a good combination thereof. Choosing a good capital structure remains as much an “art” as it is a “science.” This is good news for today’s business students: capital structure choices are unlikely to be taken over by a computer program anytime soon.

19·9. Summary

The chapter covered the following major points:

- The managerial objective should be to minimize the overall tax burden—the sum of taxes paid by the corporation and its investors.
- Investor clientele effects arise because they reduce overall tax payments. They are

Choice	Low Tax Investors (e.g., Pension Funds)	High Tax Investors (e.g., High-Income Individuals)
Better	Hold bonds (or very high-dividend stocks)	Hold (low-dividend) stocks with high capital gains
Worse	Hold (low-dividend) stocks with high capital gains	Hold bonds (or very high-dividend stocks)

Choice	High Tax Corporations (e.g., "cash cows")	Low Tax Corporations (e.g., "growth firms")
Better	Finance With Bonds	Finance With Stocks (pay out with share repurchases instead of dividends)
Worse	Finance With Stocks	Finance With Bonds

  It is the market prices for the cost of capital that incentivice smart firms and smart investors to arrange themselves in this clientele fashion to reduce taxes.

- Dividends used to be a great mystery, but nowadays they are almost as good as share repurchases. Today, differences between repurchases and dividends are minor.

  One remaining difference is "behavioral": some individual investors just seem to like dividends for their own sake—even when they were severely tax-disadvantaged prior to 2003.
- There are numerous other tax reduction schemes that firms can undertake. Some are mentioned in the final section.

- Capital structure can influence managerial behavior in good times and bad times, and both positively or negatively.
- Equity has the advantage in that it reduces the likelihood of financial distress, and with it deadweight bankruptcy costs in bad times. This includes both direct costs (such as legal fees) and indirect costs (such as underinvestment, reluctance to liquidate, and excessive risk-taking).
- Debt has the advantage in that it imposes discipline on managers and thus reduces money wasting in good times. Managers and employees will work harder if poor performance can lead to bankruptcy.
- Equity has the advantage in that it does not tempt managers to expropriate creditors. If bondholders fear expropriation from subsequent increases in corporate risk or from the issuance of more debt with earlier payments or payments that are equal or higher in priority, they demand a higher cost of capital.

- Debt has the advantage in that it signals confidence. If owners—or managers acting on behalf of owners—prefer to sell partnership shares rather than debt, they probably believe that the project's true quality is worse. Thus, the cost of raising equity is high—new partners will assume the worst.

- Managers can continue to use the WACC or APV formulas from Chapter 18, because the issues described in this chapter flow into the firm's cost of capital through the costs of capital quoted by the financial markets.

A summary of all capital structure effects from Chapter 18 to here is in Table 19.7.

Table 19.7. Summary of Capital Structure Effects

Effect	Favors
Personal Income Taxes	Equity
Debt Expropriation Includes costs arising from the interaction of borrower credibility and borrower flexibility. Includes complete contract specification costs.	Equity
Financial Distress Costs Includes inefficient operations, underinvestment problems, supplier and customer incentives, failure to liquidate or sell at appropriate price, predatory policies by competitors, etc.	Usually Equity
Corporate Income Taxes	Debt
Too Much Cash Flow Sometimes called **Moral Hazard**. Includes overinvestment, free cash flow, excessive managerial perks, verification, etc.	Debt
Inside Information Sometimes called adverse selection. Sometimes called pecking order.	Debt
Behavioral Finance	Situation-Dependent
Transaction Costs	Situation-Dependent

Solutions and Exercises

1. The firm first pays taxes on money used for repurchase and dividends, but can use pre-tax money for interest payments. Investors can easily shelter repurchases (as capital gains), but face the full brunt of Uncle Sam on interest or dividend payments.
2. Stable and old: Pension Funds. Young and growing: Individuals.
3. Lower bankruptcy costs, both direct and indirect. Fewer incentive problems to put up extra "maintenance" money. Fewer incentive problems to avoid liquidation and drag on, instead.
4. For example, legal fees and management attention.
5. Neglected maintenance reduces the value of assets relative to the first-best.
6. Shareholders do not want to sell the firm if they are underwater, even if the offer is more than the firm is worth: all benefits would go to the shareholders.
7. It can get the firm to commit to undertake more risky projects. However, it can also make it more difficult to respond when the company is already levered up.
8. Corporate Planes. Large headquarters. Large staff.
9. They help reduce the incentives of equity shareholders to expropriate bondholders.
10. It reduces the need for some bond covenants and thus gives the firm more flexibility in case a great project were to suddenly appear. Bondholders would be happy, because they, too, would benefit.
11. First, issuance of other securities that have an earlier or equal dip on the firm's cash flows in distress. This could be other bonds of equal or higher priority, or a straight out dividend payment. Second, the adoption of risky projects. Numerical examples are in the text.
12.

(a) The bond is convertible into 75% of the firm's equity.

		Bad Luck	Good Luck	Future Expected Value	Today's Present Value
Prob:		1/2	1/2		
Project	W	\$100	\$120	\$110	\$100
Convertible Bond with Face Value FV=\$90					
Bond(FV=\$90)	D	\$90	\$90		
Converted to 75%	E	\$75	\$90		
Best Choice	D	\$90	\$90	\$90	\$81.82
Equity	E	\$10	\$30	\$20	\$18.18

(b)

		Bad Luck		Good Luck		Future Expected Value	Today's Present Value
Prob:		1/4	1/4	1/4	1/4		
Project	W	\$100	\$100	\$120	\$120	\$110	\$100.00
Project	New	\$50	–\$60	\$50	–\$60	–\$5	–\$4.54
Total Projects		\$150	\$40	\$170	\$60	\$105	\$95.45
Straight Bond with Face Value FV=\$90							
Bond(FV=\$90)	D	\$90	\$40	\$90	\$60	\$70	\$63.64
Equity	E	\$60	\$0	\$80	\$0	\$35	\$31.82

(c) The bond is convertible into 75% of the firm's equity.

		Bad Luck		Good Luck		Future Expected Value	Today's Present Value
Prob:		1/4	1/4	1/4	1/4		
Total Projects		\$150	\$40	\$170	\$60	\$105	\$95.45
Bond(FV=\$90)	D	\$90	\$40	\$90	\$60		
Converted to 75% E	D	\$112.50	\$30	\$127.50	\$45		
Best Choice	D	\$112.50	\$40	\$127.50	\$60	\$85	\$77.27
Equity	E	\$37.50	\$0	\$42.50	\$0	\$20	\$18.18

Note that the shareholders are no longer better off if the project is taken, because they receive \$18.18 either way. If we made the debt convertible into 75.1% of the firm's equity, then the shareholders would be outright worse off.

(m) You should not bid anything (except perhaps \$500). If you bid \$750, then you will get the house only if it is worth \$500, and you would therefore lose 33%.

(n) Managers may behave irrationally, and pay dividends even though this is expensive from a personal income tax perspective. Investors may indeed like dividends irrationally, even if it is not in their self-interest.

(o) No! Even this investor is better off with a share repurchase, but his increase in wealth now comes from in increase in unrealized (and therefore still untaxed) capital gains.

(p) None. If absolutely necessary, it should be firms with many tax-exempt investors.

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

CHAPTER 20

Capital Dynamics

The Evolution of Changes in Debt and Equity

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You now know the considerations that should help management determine the firm's optimal debt-equity ratio. But knowing where it should be does not mean that you already know how to get there. So, how do corporations actually arrive at their current capital structures? And, is it all under the control of managers?

This chapter will show you a framework of how to think about the capitalization process of the firm, how changes in debt-equity ratios come about, and some other institutional details that are relevant to the capital issuing process. The next chapter is closely linked, in that it will lay out what we know empirically about how corporate capital structure has been developing in the United States.

20·1. Tracking IBM's Capital Structure From 2001 to 2003

We will follow IBM from 2001 to 2003. IBM first saw its debt-equity ratio increase from 31% to 55%, and the dropped it to 49%.

Perhaps the best way to understand the evolution of capital structure is to follow a company. So let us track IBM's capital structure from 2001 to 2003. Table 20.1 gives the broad overview. It shows that IBM's debt-equity ratio first increased from

$$D/E_{2001} = \frac{\$64,699}{\$208,437} \approx 31\% \tag{20.1}$$

to

$$D/E_{2002} = \frac{\$73,702}{\$133,484} \approx 55\% \; , \tag{20.2}$$

and then decreased back to

$$D/E_{2003} = \frac{\$76,593}{\$157,047} \approx 49\% \; . \tag{20.3}$$

The table also shows that debt-equity ratio changes are multidimensional—all sorts of debt and equity changes participated. The "delta" lines in the table refer to changes in the rows above, and make it easy to see where big changes were happening. (It makes it immediately obvious that changes in the market-value of equity played the most important role here—an observation to which we will come back later.) Our goal for now is to look over each of the participating components one by one to get a feel for how changes in firms' debt-equity ratios come about. This additional information is usually found in the footnotes accompanying financial statements, so we have to rely on the footnotes from IBM's 2002 and 2003 annual reports. If you wish to read any IBM historical financials, you can find them at www.ibm.com/annualreport/. In any case, it is not important for you to understand every little detail—IBM is just one company, and every company looks somewhat different. Your goal is to follow the basics, and be able to look up what else you might want to know on the Internet.

Use book value of equity?

An alternative method to quote debt-equity ratios uses the book value of equity, rather than the market value of equity.

$$D/BE_{2001} = \frac{\$64,699}{\$23,448} \approx 276\% \tag{20.4}$$

to

$$D/BE_{2002} = \frac{\$73,702}{\$22,782} \approx 324\% \; , \tag{20.5}$$

and then decreased back to

$$D/BE_{2003} = \frac{\$76,593}{\$27,864} \approx 275\% \; . \tag{20.6}$$

As you can see, this can give very different numbers. We also know already that the book value of equity is a "plug" number to equalize both sides of the balance sheet. It is largely a function of past retained earnings and depreciation—and it can even be negative.

Which one should we use?

So, which equity value is correct? Clearly, only the market value of equity has real meaning, so it should be the correct measure. Thinking that IBM carries three times more debt than equity would be misleading. However, our capital structure theories are all about future interest coverage—what part of our future cash flows will go to debt holders and what part will go to equity holders—and whatever debt ratio works better predicting is the one we should use. There are reasons why either may work. The reason why the book value of equity might work better is that it itself is a function of past cash flows. If cash flows remain persistent and steady, it may predict future earnings well. The reason why the market value of equity might work better is that market values are forward looking in contrast to the backward-looking book value. The proof (of which debt equity ratio to use) is in the pudding (which does better in predicting interest rate coverage). Some early empirical evidence on this question is ambiguous—the truth seems to be in the middle. A combination ratio, where we rely on three-quarters of the market-value and one-quarter of the book value does best. However, because this is not conventional wisdom and because it has not yet been suitably scrutinized, we will focus only on market-based debt ratios.

Table 20.1. Major Components of Debt and Equity for IBM, 2001–2003

	2001		2002		2003	
Debt Ratios						
Debt-Equity Ratio	0.31		0.55		0.49	
Delta		*+ 0.24*		*−0.06*		
Debt-Asset Ratio	0.24		0.35		0.33	
Delta		*+0.11*		*−0.02*		
Debt	2001		2002 revised		2003	
Long-Term Debt	$15,963		$19,986		$16,986	see Table 20.2
Delta		*+$4,023*		*−$3,000*		
Short-Term Debt	$35,119		$34,550 $34,220		$37,900	see Table 20.3
Delta		*−$569*		*+$3,350*		
Pension Liabilities	$10,308		$13,215		$14,251	
Delta		*+$2,907*		*+$1,036*		
Other Liabilities	$5,951		$6,281		$7,456	see Table 20.4
Delta		*+$330*		*+$1,175*		
Minority Interest — none						
Negative Goodwill — none						
Total Debt	$64,699		$73,702		$76,593	
Delta		*+$9,003*		*+$2,891*		
Equity	2001		2002 revised		2003	
Total Issued Shares			1920.96		1937.39	
– Treasury Shares =			– 198.59		– 242.88	
Number of Shares	1723.19		1722.38		1694.51	
Delta		*−0.81*		*−27.87*		
Price/Share	$120.96		$77.50		$92.68	
Delta		*−$43.46*		*+$15.18*		
⇒ Market Value	$208,437		$133,484		$157,047	
Delta		*−$74,953*		*+$23,563*		

20·1.A. Debt

Let's look at the four non-zero components of IBM's liabilities.

Liabilities are usually more colorful than equity, and IBM's debt is no exception. Just looking at Table 20.1, we have four sources contributing to IBM's debt: long-term debt, short-term (or current) debt, pension liabilities, and other liabilities. There are two more: minority interest of our business owned by third parties (which is therefore almost like equity, however) and negative goodwill (related to an accounting discount at which IBM might have purchased other companies at), which could (but rarely does) play a role. These two played no role for IBM, either.

20·1.B. Long-Term Debt

Long-Term debt increased and then decreased, mostly drive by IBM's notes.

IBM is a large Fortune-500 company, so it is no surprise that it has a myriad of long-term bonds outstanding. Table 20.2 shows *how* IBM's long-term debt first increased by $4.023 billion and then decreased by $3.0 billion.

Straight Bonds The top part of Table 20.2 are IBM's straight long-term bonds (debentures). (Note how one of IBM's bonds has 90 years remaining to maturity!) These bonds seem to have neither an active call feature (or IBM would surely have retired its 8.375% bond due in 2019), nor a sinking-fund provision (because in most of these, the outstanding principal remained constant from 2001 to 2003). The only bond on which IBM retired any principal was its 6.5% bond, due in 2028. As to new debt, on October 1, 2003, IBM issued a 5.875% bond for $600 million dollars at 97.65 (i.e., below par—given the 5.875% coupon, this bond was a discount bond).

Net in net, IBM did not change its straight bond borrowing from 2001 to 2002, and increased it by only $219 million from 2002 to 2003.

Notes There was more financing action in IBM's notes. Notes are very similar to bonds. The difference is that they are usually not issued in one big underwritten chunk, but instead sold into the market as the firm wants to raise more money—"off the shelf." Notes are also often callable. Together, these two features make it easy to expand or contract long-term debt, as needed.

IBM increased its medium-term notes by $3.5 billion from 2001 to 2002, and then decreased it by $2.4 billion from 2002 to 2003. (Relatively lower interest rates may help explain some of the shift from longer-term notes into medium-term notes in 2002, but not in 2003. In any case, the two do not exactly offset one another.)

Net in net, $3.5 billion of IBM's $4 billion increase in long-term borrowing in 2002 and $2.4 billion of IBM's $3 billion decrease came from its medium-term notes. Other notes were used to offset some of this, but, nevertheless, IBM seems to have mostly used its notes program to expand or contract its long-term borrowing needs.

Hybrid Borrowing: Note also that IBM had one hybrid debt-equity instrument—a convertible 3.43% note. It was issued by IBM to the partners of Price-Waterhouse-Coopers Consulting (PwCC), a firm that IBM acquired in late 2002.

Foreign Borrowing Over this time period, IBM repurchased a good deal of Euro debt. The euro appreciated from about 1.1€/$ in 2001 to about 0.9€/$ by 2002, but the decline in the value of IBM's Euro debt obligations is even steeper. IBM also reduced its Canadian debt, and eliminated its Swiss Franc debt. In contrast, IBM continues to rely heavily on financing in Yen. Nevertheless, you cannot interpret these changes as speculation on exchange rates, because IBM described elsewhere in its financials how it hedges some of its currency risk. Moreover, not only IBM's obligations, but also many of its assets were overseas, so the net exposure of IBM to foreign currency is not easy to determine.

Net in net, IBM decreased its reliance on foreign borrowing by about $600 million in 2002 and $2 billion in 2003.

Fair Value Adjustment Usually, long-term debt is carried at historical value, not market value. However, some of IBM's debt was "hedged"—that is, IBM had financial contracts that would change opposite in value to those of some or all of its bonds. From 2001 to 2003, short-term interests fell, while long-term interest rates remained around 5%. The fair value adjustment reflects the change in value of the hedged bonds. (Somewhere else on IBM's balance sheet will be an opposite item—an asset measuring the the value change experienced by the hedge instruments.)

Current Maturities Some of IBM's long-term debt became current (had less than one year left before coming due), and therefore was reclassified. This could account for about $1.1 billion less in long-term borrowing in 2002, and $543 million in 2003.

In sum, there are many long-term financing instruments that can play a role. In IBM's case, the most important factor influencing changes in borrowing was its expansion and contraction of its medium-term notes program.

Table 20.2. IBM's Long-Term Liabilities

At Dec 31	Maturities	2001		2002		2003
U.S. Dollars:						
Debentures:						
5.875%	2032	–		–		$600
6.22%	2027	$500		$500		$500
6.5%	2028	$700		$700		**$319**
7.0%	2025	$600		$600		$600
7.0%	2045	$150		$150		$150
7.125%	2096	$850		$850		$850
7.5%	2013	$550		$550		$550
8.375%[§]	2019	$750		$750		$750
		$4,100		$4,100		$4,319
	Delta		*±$0*		*+$219*	
3.43% conv.notes[*]	2007	–		$328		$309
Notes, 6%, 5.9%[‡]	2003-32	$2,772		$2,130		$3,034
Med Term Notes, 4%, 3.7%[‡]	2003-18	$3,620		$7,113		$4,690
	Delta		*+$3,493*		*−$2,423*	
Other: 4.9%, 4.0%[‡]	2003-09	$828		$610		$508
		$11,320		$14,281		$12,860
	Delta		*+$2,961*		*−$1,421*	
Other currencies[†]						
Euros (5.4%, 5.3%)[‡]	2003-09	$3,042		$2,111		$1,174
Yen (1.0%, 1.1%)[‡]	2003-15	$4,749		$4,976		$4,363
Canadian (5.8%, 5.8%)[‡]	2003-11	$441		$445		$201
Swiss (4.0%, 4.0%)[‡]	2003	$151		$180		–
Other (6.6%, 6.0%)[‡]	2003-14	$726		$730		$770
		$20,429		$22,723		$19,368
	Delta		*+$2,294*		*−$3,355*	
Unamort. (Prem)/Disc		$47		-$1		$15
SFAS #133 Fair Value Adj.[†]		$396		$978		$806
		$20,778		$23,702		$20,159
	Delta		*+$2,924*		*−$3,543*	
Less current maturities		$4,815		$3,716		$3,173
Total		$15,963		$19,986		$16,986
	Delta		*+$4,023*		*−$3,000*	

[§]: The appendix contains the Standard&Poor's Bond Report on this particular issue. [*]: These convertibles notes were issued in the 2002 acquisition of PwCC to PwCC partners, and some began converting into equity in 2003. [‡] The first interest rate is the average from 2001 to 2002, the second from 2002 to 2003. [†]: This item "marks to market" the value of the debt instruments when interest rates change. The IBM footnotes footnote this further as *In accordance with the requirements of SFAS No. 133, the portion of the company's fixed rate debt obligations that is hedged is reflected in the Consolidated Statement of Financial Position as an amount equal to the sum of the debt's carrying value plus a SFAS No. 133 fair value adjustment representing changes recorded in the fair value of the hedged debt obligations attributable to movements in market interest rates and applicable foreign currency exchange rates.*

20·1.C. Current Liabilities

Table 20.3. IBM's Current Liabilities

		2001		2002 revised		2003
Short-term Debt		$11,188		$6,031		$6,646
	Delta		*−$5,157*		*+$615*	
Commercial Paper		*$4,809*		*$1,302*		*$2,349*
	Delta		*−$3,507*		*+$1047*	
+ Short-Term Loans		*$1,564*		*$1,013*		*$1,124*
	Delta		*−$551*		*+$111*	
+ Long-Term Debt, Current		*$4,815*		*$3,716*		*$3,173*
	Delta		*−$1,099*		*−$543*	
Taxes		$4,644		$5,476		$5,475
	Delta		*+$832*		*−$1*	
Accounts Payable		$7,047		$7,630		$8,460
	Delta		*+$583*		*+$830*	
Comp and Benefits		$3,796		$3,724		$3,671
	Delta		*−$72*		*−$53*	
Deferred Income		$4,223		$5,276 * $4,946		$6,492
	Delta		*+$1,053*		*+$1,546*	
Other Accrued Liabilities		$4,221		$6,413		$7,156
	Delta		*+$2,192*		*+$743*	
Total Current		$35,119		$34,550 $34,220		$37,900
	Delta		*−$569*		*+$3,680*	

* This revision shifted $330 from deferred income into other liabilities, which can be seen in Table 20.4.

Note the many different short-term obligations!

Table 20.3 shows current liabilities, which are due to be paid within one year. The CFO has most influence over short-term debt—at least commercial paper and short-term loan borrowing. We also see the long-term debt that fell into short-term debt. The remaining liabilities are mostly incurred in the course of the firm's operations.

We can see that IBM actively reduced its short-term borrowing from 2001 to 2002, and then expanded it from 2002 to 2003.

20·1.D. Other Liabilities

Table 20.4 shows a set of other obligations that can have an impact on the amount of corporate debt. For IBM, only changes in restructuring actions really mattered in 2002. In 2003, however, both changes in IBM's deferred taxes and deferred income played important roles.

Table 20.4. IBM's Other Liabilities

	2001		2002 revised		2003
Deferred Taxes	$1,485		$1,450		$1,834
Delta		*−$35*		*+$384*	
Deferred Income	$1,145		$1,079* $1,409		$1,842
Delta		*−$66*		*+$433*	
Exec Comp Accrual	$868		$851		$1,036
Delta		*−$17*		*+$185*	
Restructuring Actions	$589		$1,024		$871
Delta		*+$435*		*−$153*	
Postemployment, preretirement	$493		$573 $572		$579
Delta		*+$80*		*+$7*	
Disability Benefits	na		na † $304		$349
Delta		*+$0*		*+$45*	
Environmental Accruals	$215		$208		$214
Delta		*−$7*		*+$6*	
Other	$670		$766† $463		$731
Delta		*+$96*		*+$268*	
Total	$5,465		$5,951 $6,281		$7,456
Delta		*+$486*		*+$1,175*	

* This revision shifted $330 from deferred income into other liabilities, which can be seen in Table 20.3. † IBM broke out $330 million disability benefits in 2003, previously classified as "other."

Other Observations and Discussion

Table 20.1 also shows that just under 20% of IBM's obligations are pension obligations to its more than 300,000 current employees (and also former employees). This is a very important part of IBM's liabilities, but it would be difficult to discuss in less than a chapter in itself—and it would lead us far away from finance.

Pension obligations are very important for firms with many employees—almost as important as long-term debt for IBM!

Interestingly, Table 20.1 can also tell us how IBM's shifted its obligations from short-term debt into medium- and long-term debt in 2002, and then reversed (or no longer continued) this trend in 2003. This can be seen both in IBM's arrangement of long-term vs. current liabilities, and within its long-term liabilities, in its arrangement between long-term notes and medium-term notes. However, the passing of time itself makes outstanding obligations shorter-term, so we might like to know how its financial obligations for each year developed. If we dig deeper into the financial statement footnotes, we can find these, too:

The time dimension of IBM's obligations, and the prevailing yield curve.

	2001	2002	2003	2004	2005	2006	2007	2008	2009
As of 2001	$11,188	$5,186	$3,106	$1,501	$1,904	$2,261	$6,471	←	←
As of 2002		$6,031	$3,949	$3,613	$1,670	$2,705	$846+$9,940		←
As of 2003			$6,646	$4,072	$3,113	$2,760	$1,289+$225+$7,942		

This shows that IBM changed its capital structure dynamically (e.g., it always financed itself with some short-term debt, so the first-year term is large), but for any given year further out, a static shift (i.e., in a given year, like 2006) is not as obvious. When thinking about obligations, IBM's CFO did not operate in a vacuum, but was probably very concerned with the yield-curve.

Relative to 2000, short-term and medium-term interest rates had dropped significantly, but long-term rates were somewhat sluggish. Here is how the economy wide rates changed over this period.

	Maturity	2000	2001	2002	2003
Short-Term Treasury	1 month	>5%	2.47%	1.63%	1.02%
Medium-Term Treasury	3 year	6.22%	4.09%	3.10%	2.10%
Long-Term Treasury	20 year	6.23%	5.63%	5.43%	4.96%
Short-Term Corporate	1 Month	6.3%	3.8%	1.7%	1.1%
Aaa Bonds	Medium Term	7.6%	7.1%	6.5%	5.7%

Some more interesting information.

Finally, the financials also tell us a little bit about IBM's interest payments and unused credit lines.

	2001	2002	2003
Interest Paid and Accrued	$1,235	$815	$663
Unused Credit Lines	$16,121	$16,934	$15,883

To put the interest paid into perspective, in 2001, IBM earned $7.7 billion; in 2002, it earned $5.3 billion; in 2003; it earned $7.6 billion. To put the credit lines into perspective, they are about equal in size to IBM's long-term debt.

20·1.E. Equity

Table 20.5. IBM's Equity and Some Other Information

		2001		2002		2003
Preferred	authorized	150,000,000		–		–
	outstanding	2,546,011		–		–
Common	authorized	4,687,500,000		4,687,500,000		4,687,500,000
	outstanding	1,913,513,218		1,920,957,772		1,937,393,604
	Delta		*7,444,554*		*16,435,832*	
	treasury	190,319,489		198,590,876		242,884,969
	Delta		*8,271,387*		*44,294,093*	
	Net	1,723,193,729		1,722,366,896		1,694,508,635
	Delta		*−826,833*		*−27,858,261*	
	Identifiable Changes					
	PwCC Acquisition Issue, restricted		*−3,677,213*			
	To Pension Fund, from Treasury		*−24,037,354*			
	Repurchase I		*48,481,100*		*49,994,514*	
	Repurchase II ESOP		*189,797*		*291,921*	
	Issue to ESOP, from Treasury		*−979,246*		*−2,120,293*	
	*PwCC Acq Issue**		*−$254*			
	Repurchase I		*$4,212*		*$4,403*	
	Repurchase II ESOP		*$18*		*$24*	
	To Pension Fund		*−$1,871*			
Retained Earnings		$30,142		$31,555		$37,525
Book Equity		$23,448		$22,782		$27,864
	Cash Dividends Paid		*$1,005*		*$1,085*	
	Common Stock Transactions		*$3,087*		*$3,232*	
	For Comparison: Interest Paid		*$831*		*$853*	
	For Perspective: Taxes Paid		*$1,707*		*$1,841*	
Common Price/Share		$120.96		$77.50		$92.68
⇒ Common Market Value		$208,437		$133,484		$157,047

* An additional $30 million is recorded to be issued in future.

Changes in the number of shares ultimately were minor.

Table 20.5 shows some of the evolution of IBM's equity. (There is not enough information in the footnotes to track all changes.) Looking at preferred equity, we see that it must have disappeared by 2002. The background is that in 1995, the IBM board had decided to repurchase all its remaining 7.5% callable preferred stock, and this was ultimately completed on May 18, 2001. Moving on to common equity, note that although 1.9 billion shares of IBM were officially outstanding, IBM itself held about 200 million shares in 2001 and 2002, and 250 million shares in 2003. Thus, if you had owned about 1.7 billion external shares, you would have owned all of IBM's common equity. Interestingly, this number remained fairly constant. Yes, IBM actively repurchased its shares, but although the dollar amount was large, it was only a small fraction of the company's outstanding stock. In addition, IBM then turned around and used these shares in other transactions, e.g., to fund the PwCC acquisition or to fund its employee stock option plans. Consequently, although repurchases and net stock transactions were larger than interest payments and dividend payments combined, the active issuing or repurchasing of shares ultimately did not play much of a role.

Changes in the price per share played a very large role.

Instead, almost all the change in the value of equity came through the one mechanism of changes in the price of each IBM share: from 2001 to 2002, it dropped from \$120.96 to \$77.50, thereby losing about one-third of its market value. From 2002 to 2003, it increased again by 20%.

20·1.F. Observations

Where changes came from.

We can now make some overall observations. IBM's liabilities evolved fairly steadily. About one-quarter of the total debt were pension and other liabilities. The pension obligations, in particular, marched upwards fairly steadily. In terms of IBM's total debt increase, the pension and other obligations accounted for one-third and three-quarters in 2002 and 2003, respectively. About one-quarter of IBM's total liabilities were its long-term debt; one-half was its short-term debt. In 2002, IBM ratcheted up its medium-term notes borrowing, accounting for a debt increase of \$3.5 billion. In 2003, IBM mostly kept its borrowing at the same level, but shifted it relatively from long-term into current debt. These changes in the value of IBM's debt were dwarfed by the changes in the value of IBM's *equity*—and almost all of these came from changes in the per-share price, not from changes in the number of shares outstanding.

Liabilities may be too narrow a view.

As astute financiers, we note that our discussion has focused only on the liabilities, and said little about the assets. But we need to clarify the connection. Creditors and shareholders provide assets (value through cash inflows) in exchange for receiving their claims, which makes them part owners of the firm. The same is true for other obligations, like the pension fund obligation. It is a proper claim on the firm, which therefore makes the pension fund an owner of the firm. It came about because IBM did not fully pay its employees in cash, but deferred some of this employee compensation. This has in effect made the employees creditors. And just like debt and equity, to get this claim on IBM's assets, the pension fund has provided value in terms of having IBM's employees produce goods. In other words, with this pension obligation came a positive impact of employees on the firm's assets. The employees' fund therefore should be considered both in terms of its impact on the scale of the firm (increasing both assets and liabilities) and in terms of its impact on the mix of debt and equity—it is not just an obligation that pops out of a vacuum. Even from the perspective of our theories, it would be artificial to distinguish liabilities such as pensions from debt. They are both subject to the same theoretical factors—for example, they are both paid from pre-tax earnings, and both require servicing (or else, bankruptcy looms).

The immediacy of discretion varies—financial debt can be easily rearranged; pension benefits cannot.

But the two obligations differ in one important respect: Sharp changes often and easily appear in the value of equity affected by stock prices and managerial actions, whereas obligations like pensions tend to march on pretty steadily. You can consider this linked to the speed of control that managers have over these different components of the firm's scale and debt-equity ratio. For example, CFOs probably have relatively little short-term control over the firm's pension obligations, more short-term control over the firm's equity market value and stock price (e.g., by retaining equity), and a lot of short-term control over the firm's debt and equity issuing and repurchasing, and over its dividend policy. Therefore, it still makes sense to think about capital structure in the traditional sense—the mechanisms that the CFO can use to tweak the firm's scale and debt-equity ratio. We therefore proceed now to talk about the financial aspects of capital structure in some more detail.

Solve Now!

Q 20.1 *How would you define the firm's capital structure?*

Q 20.2 *List some of the bigger factors that go into the firm's capital structure.*

Q 20.3 *To purchase all common equity in a firm, do you need to purchase all outstanding shares?*

Q 20.4 *Is debt or equity value usually "spikier"?*

20·2. The Dynamics of Capital Structure and Firm Scale

Most forces are as you would expect—what management chose.

A firm's capital structure is comprised of the claims on its assets. As in IBM's case, this is typically a mix of various forms of debt (long-term, short-term, operating related), equity (common and preferred), and hybrid claims (e.g., convertibles). You also know the forces that managers should be aware of when they think about the value-maximizing mix of these claims—forces like taxes and agency concerns. These theories and considerations are mostly about the capital structure *levels*. As the firm operates, both its needs and its debt and equity levels are changing. In this chapter, we concentrate on these capital structure *changes*. Table 20.6 organizes many of the relevant forces tugging on the firm's debt-equity ratio and the firm's scale (total value). Many of these are the usual suspect transactions stemming from active financial market intervention, as orchestrated by the CFO.

Stock returns are not fully under the control of management.

But not every change is under the control of management. We have already noted these factors in IBM's case. Our Table 20.6 ignores many of the operating influences on the firm's capital structure (and in particular, its liabilities, such as pension liabilities). We have left one factor that is not fully under the financing control of the CFO: firm value changes (aka stock returns) affect both the scale and the debt-equity ratio of the firm. For example, a firm that is financed 50-50 by risk-free debt and equity and that doubles would see its debt-equity ratio decline to 50-150. We have already seen the effects of stock returns in IBM's case—when its stock price tumbled from \$121 to \$78 per share, its equity lost over one-third of its value. This, in turn, dramatically increased IBM's debt ratio. What factors might influence the stock price? Some are out of managers' control. Possibly, investors could have become more risk-averse and therefore were no longer willing to pay \$121 for IBM with its level of risk. Possibly, IBM was hit by additional unexpectedly bad news (an earthquake). Some *are* under managers' control. Perhaps the firm paid out a lot of equity in dividends to shareholders (ok, we know IBM did not!), or managers ran the firm poorly. (Large publicly traded firms cover about 50-90% of their funding needs with retained earnings. The remainder is usually predominantly debt-financed.) As you will see in the next chapter, a considerable proportion of most firms' debt-equity dynamics is determined by such firm value changes, which are reflected most obviously in the firm's stock price.

Side Note: Table 20.6 not only does not mention changes in operating liabilities, but also ignores the effect of bond price changes. When economy-wide interest rates rise or the firm's credit rating deteriorates, the debt usually declines in value—but so does the equity. Conversely, when economy-wide interest rates drop or the firm's credit rating appreciates, the debt usually increases in value—but again, so does the equity. Thus, the effect of changing interest rates on the debt-equity ratio is usually ambiguous. Moreover, there are situations in financial distress in which the debt wrestles power from the equity—there would be no change in overall capitalization, but a good change in the firm's debt-equity ratio.

Table 20.6. Non-Operating Capitalization and Capital Structure Influences

Debt-Equity Ratio	Firm Value: Decreases	Firm Value: Constant	Firm Value: Increases
Decreases	**Debt Repurchase (e.g., sinking fund and interest payment)** **Repayment of Principal or Interest** Debt Call	Equity-for-Debt Exchange	**(Exogenous) Firm Value Rise (possibly through retained earnings)** **Primary Seasoned Equity Issue in M&A context** **Share Creation for Employee Compensation Purposes** Primary Seasoned Equity Issue outside M&A context Warrant Exercise
Constant	Simultaneous Debt-Equity Payout.*		Simultaneous Debt-Equity Issue.* **Hybrid Security Issue.***
Increases	**(Exogenous) Firm Value Drop** **Share Repurchase** **Cash Dividend**	Debt-into-Equity Conversion (e.g., through call) Debt-for-Equity Exchange (e.g., in Chapter 11)	**Debt Issue**
(Depends)	**Sale of Assets (e.g., carveout)**		**Purchase of Assets (e.g., M&A)**

Boldfaced changes are common, though not necessarily of equal quantitative importance. Starred transactions rarely occur in the precise proportionality to maintain a constant debt-equity ratio.

Note that this table ignores the complex interaction with existing capital structure. In particular, if the firm is 100% equity financed, an increase or decrease in firm value, an equity issue or equity repurchase, and a dividend payment have no influence on the firm's debt-equity ratio—it will remain at 0%.

Solve Now!

Q 20.5 *Is all debt at the discretion of management?*

Q 20.6 *Describe some of the financial factors that can change capital structure*

20·3. The Managerial Perspective

An outline.

As CFO, you will arrive at your current capital structure through the forces and mechanisms outlined in Table 20.6, and then some. But you are not helpless. Your current capital structure is at least in part the outcome both of your predecessors' and your own actions. In light of the constant historical change of your capital structure, what questions and issues should be on your mind when you think about your active capital policy for this year?

20·3.A. The Holistic View

Don't think "dividends, yes or no?" Think instead of capital inflows vs. outflows; and of debt vs. equity consequences.

As CFO, should you think narrowly about just one action—say, whether to repurchase shares? Probably not. Table 20.6 should make it clear that you should consider components not in isolation, but within a broader context. It would make more sense for you to think about the overall payin/payout policy of the firm. Dividends, coupon payments, repurchases, and issues are *all* mechanisms for transferring cash from inside the corporation to the outside owners, or vice-versa. To think about the overall debt-equity policy of the firm, you should recognize that equity issues, debt repurchases, and interest payments are *all* mechanisms for lowering the firm's debt-equity ratio.

An example of a firm.

Here is an illustration of the multidimensional nature of your choices. For simplicity's sake, start by assuming we are still in a perfect M&M market, where the mix of financing does not influence total firm value. Your firm is currently worth \$1 billion, of which \$400 million is outstanding debt. You may choose to raise \$100 million in new equity, raise \$200 million in new debt, pay out \$30 million to service old debt (principal and interest), pay out \$20 million in dividends, and repurchase \$50 million of its own equity shares. De facto, your firm has

1. transferred $\$100 + \$200 - \$30 - \$20 - \$50 = \$200$ million of cash from the outside to the inside,
2. and increased its debt-equity ratio from $\$400{:}\$600 \approx 67\%$ to $\$570{:}\$630 \approx 90\%$.

Of course, the real world is not M&M, and this means that you need to amend your choices. You need to consider the reaction of investors. For example, if investors believe that the corporation suffers badly from agency conflict, then they may react negatively (to the \$200 million increase in extra cash available to managers). On the other hand, if investors believe that the higher debt-equity ratio will save the corporation relatively more in taxes, then they may react positively to the increase in the debt-equity ratio. In fact, you should consider each and every value effect that we discussed in Chapters 17–19. Without knowing more about our particular firm, it would be hard to guess whether the financial markets would look fondly or not so fondly on these capital structure changes. Why does this matter? If your capital structure rearrangement created value, for example, it might well be that the outcome is not \$1.2 billion in value and \$570:\$630 in debt-equity ratio, but, say, \$1.3 billion in value and a \$570:\$730 debt-equity ratio. (In Chapter 22, we will look in some depth at how U.S. financial markets have responded to corporate actions.)

An important interaction! Can you think of equity-issuing activity as a decrease in the firm's debt-equity ratio? It is easy to mentally equate the two, but this is not necessarily the case. There are two basic disconnects to consider.

1. The existing capital structure plays an important role in the effect that issuing has on the capital structure. When a firm with a 100% equity structure issues new equity shares, it does not change its debt ratio. But the same equity issue would induce drastic capital structure changes for an equal-sized firm that was previously 90% debt-financed. Thus, even if there were no other influences, studying equity issues is intrinsically not the same as studying debt-equity decreases.

2. An influence may have an effect through multiple channels. For example, contemplated M&A activity may induce equity-issuing activity—but it also induces debt-issuing activity. Thus, M&A activity may not necessarily positively influence debt-equity ratios at all.

In addition, you already know that there are many other factors influencing firms' debt-equity ratios, which have nothing to do with equity issuing. Thus, although equity-issuing and decreases in the firm's debt-equity ratio are linked, studying (equity-) issuing activity is by no means the same as studying capital structure.

Important secondary effects. Before we move on, there is one last interesting capital structure effect worth noting. Dividends tend to be stickier than share repurchases, and thus the fact that our firm pays out more in repurchases than in dividends may send a mixed signal—either managers want to be smart about tax consequences, or they are worried about the firm's ability to pay out cash again next year. Our investors would also draw some inferences about the fact that our sample firm paid out only \$20 million in dividends, but repurchased \$50 in shares. (We will talk about these non-obvious effects later when we zero in on individual actions in more detail.)

20·3.B. Meaningful Questions

The two important questions. So, as CFO, what are the most important questions concerning your target capitalization that you should ask? Is it just the question whether you should pay dividends, repurchase shares, or issue equity? You already know that payin/payout policy is a complex issue, but stating this is not much help to you. Ultimately, capital policy really has to be considered on a firm-by-firm basis. If you want to act on behalf of the firm's owners, the questions that you must ask are:

1. Can you invest the money better than what your investors can find elsewhere?

2. If you are taking an action (e.g., pay out less cash or take in more cash), do your investors share your beliefs that this will increase value—that the additional money will be well spent?

   If your investors agree with your managerial judgment, as they would in a perfect market, then there is no problem. However, if your investors disagree with you, then there is a problem. For example, if you know that investing in a new technology is highly worthwhile but requires cutting dividends, the market may react negatively. This means that all investors would be taking a hit on their market value right now, just as they would if you had thrown away money. If you are correct, investors will eventually realize the value gain, and thus the share price will appreciate again. But this is little consolation to those investors who have to sell their shares this year. Should you represent your current investors or your future investors? There is no easy answer to this difficult question. But note that agency theorists are often skeptical about claims that managers weighed choices and decided to represent the long-run investors—they believe that such claims are often only excuses for managers to represent only themselves. But everyone agrees that good communication from managers to investors can only help.

In any case, these two questions should point out to you that even our holistic view of capital policy is still too narrow. Capital structure has intricate links to the firm's project opportunities, corporate governance, and disclosure policy. If the firm has great opportunities, if managers are well motivated, and if the firm can convince investors of these great opportunities, then the answer to both questions is often yes. Such firms can even create value by reducing dividends and share repurchases and raising more equity. If the answer to both questions is no, then the firm should not issue equity, and instead seek to increase dividends and share repurchases. And if the answers to both questions are fuzzy—*as they often are*—then you have a tough judgment call to make.

Worry about operations and disclosure.

20·3.C. Financial Flexibility and Cash Management

But in many corporations, and especially smaller ones, the CFO faces one issue more important than even long-run capital policy. It is cash management—and fortunately, your goal here is easy and straightforward: Don't let the company run out of cash! We do not mean cash in the cash register, but rather cash necessary to pay creditors. There are many intrinsically profitable companies—especially young, high-growth companies—that have had to fold because of poor liquidity management. The need to pay creditors does not necessarily mean that your company has to have lots of cash on hand. It is enough if you can borrow with ease and rapidity. For example, it is not unusual that when the principal of a bond comes due, it is refinanced by a new bond issue.

Don't run out of money!

But problems can arise when your firm operates too close to the brink of its financial flexibility. This is more often the case for small and private companies. In this instance, it is quite possible that you can get either of two self-fulfilling prophecies ("equilibria") to occur:

Self-fulfilling Prophecy.

1. Lenders are not worried about the company; the company borrows and operates profitably; lenders see their beliefs confirmed and are repaid.
2. Lenders are worried about the company and are unwilling to extend credit; without money, the company goes bankrupt; lenders see their beliefs confirmed that it was wise not to have extended more credit.

What can you do to avoid the disaster equilibrium? You have a number of options, though all of them are costly:

Financial flexibility (credit lines, low debt ratios, matching inflows and outflows) helps, but is expensive.

Match Assets and Liabilities You can try to match expected future income to liabilities. For example, if you are taking out a loan to pay for a new factory, which will come on line and produce income in three years, you could design the loan to require interest and sinking fund payments beginning in three years. Matching future inflows to expected outflows is easier to do if your cash flows are relatively more predictable and if they occur sooner—many lenders would be reluctant to provide long-term credit without any repayment. Moreover, matching inflows and outflows makes more sense on a firm-wide basis, and less sense on a project-by-project basis.

Pay for Flexibility You can pay a bank for an irrevocable credit line. However, although it is often cheap to get a credit line for sunny times, it is often expensive to get one that will hold up (not be revoked) in rainy times. Even IBM's $15 billion credit line, which we mentioned on Page 516, is subject to various bond covenants—and if IBM were to get into trouble and needed it, it might no longer be available.

Hold Liquid Investments You can invest cash in assets that have fairly safe values and allow for relatively quick and cheap liquidation. Unfortunately, unless your company is a Treasury bond fund, your business is not likely to need such assets as much as it needs the kinds of assets that are risky and hard to liquidate—for example, your half-constructed laboratory or half-finished R&D would be very difficult to resell.

Adjust Capital Structure You can keep a low debt ratio, which allows you to maintain high interest rate coverage. (That is, your cash flows easily cover your debt obligations.) Firms that have low debt ratios and high interest rate coverage tend to have an easier time borrowing more cash when they need more. Of course, a low debt ratio may not allow your firm to take advantage of the tax subsidy due to debt.

When CFOs are surveyed, they state that they pay close attention to their “financial flexibility”—they care very much for their interest coverage ratios and bond ratings. This concern is good from a liquidity perspective.

The drawback to too much cash.

From the manager’s perspective, having more cash is always better than having less cash. But there is also a *very* dark side to this flexibility. Access to cash “lying around” tempts managers to waste money or undertake ventures that they otherwise might not undertake. And investors may not be all that thrilled with insulation from financial default—it can lead to satisfaction with the status quo, as well as inefficient investment. After all, management and employees work harder if they know that the company will go bankrupt if they perform poorly. If the company has enough of a financial buffer, the firm may remain stuck with poor management and unmotivated employees.

20·3.D. Market Pressures Towards the Optimal Capital Structure?

The question

A final interesting question is whether you can just avoid thinking about the optimal capital structure altogether and simply copy the existing capital structure of similar firms.

Intriguing evidence—why are firms not more proactive?

The empirical evidence suggests that firms do not counteract what stock market changes do to them—and stock market changes are tremendously large influences on firms’ debt-equity ratios. This finding has led to an academic debate about what this implies:

1. Are the transaction costs too high to make it worthwhile to change the capital structure? (If this is true, all our earlier arguments about what should drive capital structure are fairly unimportant.)
2. Does the optimal capital structure itself change one-to-one with the firm’s market value (so that no changes are necessary)?
3. Are firms making mistakes by failing to optimize their capital structures?

Poor capital structures can persist.

Now, if we believed that an outside investor could make money by fixing a bad capital structure, as in a perfect market, then we would also believe that current capital structures are more than likely fairly close to optimal. Unfortunately, this does not seem to be the case. To “arbitrage” an incorrect financing choice, you would have to mount a corporate takeover. The typical takeover pays a premium of 15 to 30 percent above the current market price, plus a couple of percentage points to the investment banker. A bad capital structure may destroy around 1 to 3 percent per annum in value. This is not an inconsequentially low amount—especially because it is annual and because it requires almost no effort and investment to fix it—but it is usually not enough to make a takeover worthwhile. This also allows for a whole range of capital financing arrangements to persist in the economy—including poor ones.

Empirical evidence is *not* prescriptive, telling you what to do. It is only descriptive.

Unfortunately, this means that there is only so much that a CFO can learn from the capital structures of similar firms. Put differently, knowing your comparables’ capital structure is not very informative about what your firm should do. Given that the high costs of capital structure arbitrage can allow bad capital structures to persist in the economy, it may be that other managers do the wrong thing. So, you cannot have blind faith. Some modest faith may be appropriate, though. Knowing what other managers are doing can still be helpful. Just take this knowledge with a big grain of salt—and realize that they, like you, have incentives that are not about shareholder value maximization.

Solve Now!

Q 20.7 *A \$500 million firm is financed by \$250 million in debt and \$250 million in equity. If the market value does not change, describe some actions that managers can undertake to increase firm size to \$600 million and change its debt-equity ratio to 5:1.*

Q 20.8 *A \$500 million firm is financed by \$250 million in debt and \$250 million in equity. It issues \$150 million in debt, and repurchases \$50 million in equity. The market believes the \$100 million increase in value will result in wasteful spending by managers, which costs \$5 million in NPV. However, the higher \$150 million in new debt will also create \$20 million in additional tax shelter NPV. What is the firm's new value and new debt-equity ratio?*

Q 20.9 *What are the two important questions that a CFO acting on behalf of shareholders should ask?*

Q 20.10 *How can a firm manage cash to avoid running into financial distress? What are the drawbacks?*

Q 20.11 *Are existing capital structures necessarily efficient?*

20·4. Some Process Information

An outline.

Let us now look a little more at the process of capital structure changes, and especially the process of issuing. In the process, we will need to take into account some relevant institutional features of the process, as well as some more available securities.

20·4.A. The Pecking Order (and Financing Pyramid)

Pecking order causes a financing pyramid.

There is good evidence that the more junior the security, the less firms like to obtain funding with it. Firms tend to finance their projects first with retained earnings, then with debt, and only finally with equity. Put differently, many firms perceive the costs of capital lower if the capital comes from internally generated funds than if capital has to be raised by debt and even more so by equity. This characterization goes beyond these three base mechanisms. It extends to grades within the larger categories, too. For example, among debt financings,

- Factored receivables are often safer than debt, so many firms tend to factor their receivables before they issue more debt.
- Collateralized bonds are safer and more senior than plain bonds. Therefore, firms tend to first use collateralized bonds before they issue plain bonds.
- Short-term bonds are safer and more senior than long-term bonds. Therefore, firms tend to first use short-term debt before they issue long-term debt.
- Bonds with stronger covenants are safer than bonds without covenants. Therefore, firms first try to issue bonds with strong bond covenants to assure the lenders.

This preference is often called the **pecking order**—financial markets like and therefore firms tend to issue first securities that are as safe and as senior as possible. Only after the costs to issuing such senior debt become very large (e.g., if the covenants become strangling or if the firm has too much short-term debt) will firms go to the next instruments. As a result, it is often believed that firms may end up financed like a pyramid—a lot of safe (very senior and short-term) debt at the bottom, somewhat less of more risky debt in the middle, and relatively little equity at the top. This belief is, however, incorrect if the firm has experienced much equity appreciation, because it could then end up with a lot more equity than debt in its capital structure, or if the firm's operating debt decreased for exogenous reasons. It could also be that many firms do follow this pyramid financing arrangement, not because they actively issued debt, but because they incurred many operating liabilities along the way.

Explanations.

There are a number of deeper explanations for a pecking order preference, the two most prominent of which are:

1. **Inside Information** We first learned about the pecking order view in Section 19·5 on Page 496, where we discussed inside information. The idea was that when a company wants to raise more financing, it is in its interest to convince investors that managers and owners are confident in the firm's future. Put differently, managers signal their own confidence in the firm by remaining as heavily invested as possible.

2. **Agency Considerations** This explanation is very closely related to inside information. It merely states that managers will put the raised money to good use—that it will not just become free cash flow to be wasted. The more junior the security that the firm issues, the more free cash flow managers could waste. Thus, managers who plan to profitably invest money will not mind as much the more stringent requirements that come with newly issued senior securities.

The inside information and agency theories are the most convincing explanations as to why new debt issues are greeted more warmly than new equity issues, but they are not exclusive. A more functional explanation is that *any* theories in which equity is greeted by the financial market with more skepticism than debt can explain a pecking order, in which firms are more reluctant to issue debt than equity. After all, if the response to the issuance of more junior securities is more negative, firms prefer the consequently cheaper senior securities. To illustrate, here is an alternative theory. It could be that investors do not value firms at their present values, but instead are satiated with shares of a particular firm—the firm issues more and more shares, and it becomes harder and harder to find investors. (In economics lingo, the demand for shares is not perfectly elastic.) This theory could equally well explain a pecking order, in which managers are reluctant to issue equity and more equity-like instruments, because selling additional shares costs more. (Interestingly, this theory implies that investors can find good bets by investing in neglected stocks.)

Not all companies choose this route.

Not all firms opt for pure pecking-order behavior and/or pyramid-like financing arrangements. For example, many leveraged buyout firms such as **Kohlberg, Kravis, Roberts** (**KKR**) purchased many different companies, but kept each of them in its own insulated shell. KKR's financing scheme had different costs and benefits from those of most ordinary companies. If one of KKR's portfolio companies went bankrupt, it would not bring down KKR's other companies. (This arrangement provided good incentives to the management in these companies not to make mistakes! They would not be rescued by their sibling divisions.) Of course, lenders knew that they could not lay claim to other KKR assets if the management were to make mistakes. And they knew that KKR was not confident enough in the quality of a particular acquisition to pledge KKR's remaining assets to the lenders. Therefore, lenders demanded significantly higher interest rates from KKR's individual portfolio companies than they would otherwise have demanded. KKR had to pay the price.

20·4.B. Debt and Debt-Hybrid Offerings

An outline—debt is more important.

We now turn to firms' debt issuing activities. Debt offerings are much more frequent than equity offerings. In fact, except in the context of acquisitions where both equity and debt offerings are common, large, publicly traded firms tend to finance almost all their projects through either retained earnings or debt offerings. Debt offerings are bread-and-butter for both firms and investment banks.

Bond Flavors Revisited

Contract provisions are "priced in."

We have already discussed the many flavors of bonds (such as seniority, security, covenants, collateral, conversion, callability, puttability, maturity, duration, fixed-vs.-floating). IBM's debt structure, described in Table 20.2 and 20.3 is a good example of the variety of debt claims a firm may have outstanding. For most bond features, the basic finance mantra holds: You get what you pay for. For example, if you give bond buyers more rights (e.g., a conversion feature), you get to pay a lower interest rate. If you want to keep more rights (e.g., retain a call feature), you must pay a higher interest rate. Despite some empirical behavioral finance evidence to the contrary, it seems unlikely that managers can guess what features the market generally overvalues or undervalues, and of course whether interest rates will go up or down. But fair pricing does not mean that you cannot add value by choosing debt securities that employ the features that are most appropriate to your own firm. Consider a bond feature that says that all factories will be permanently closed if the NFC team wins the Superbowl. In a competitive market, you will get a fair price for this bond and any other securities that you might issue, but this is not a great security to issue if you want to maximize market value. The point is that you should offer bonds that have features that are well-suited to your company. But if you stay within the limits of ordinary and frequent bond features (say, choosing a convertibility or callability feature), it is often true that it will matter only modestly which exact features your bonds are offering.

Assembling the Building Blocks into a Real Offering

Issues are complex enough to need an investment banker as manager!

So far, we have pretty much taken the buffet approach to bond features—we have discussed each one by itself. Let's now have a full-course dinner. How do large publicly traded corporations really borrow money? The most common way for many mid- to large-cap companies to borrow is to obtain a bank credit facility and issue multiple bonds ("term debt") at the same time. The financing package consists of two parts:

The revolver (i.e., a revolving credit line) is a line of credit on which the company can borrow and repay and borrow again until a termination date/maturity. The bank offering the revolving credit line also receives a fee for the unused/undrawn portion of the revolver.

The term debt is structured in one or more **tranches** (French for "slices"). The principal payment schedule and maturity date are different for each of the tranches. Tranche A would begin to amortize right away and would have the shortest term to maturity. The Tranche B term loan would amortize and mature after the Tranche A term loan but before the Tranche C term loan, and so on.

The revolver and Tranche A loan usually carry the same interest rate spread over LIBOR and are marketed as a package. The Tranche B and C lenders receive wider spreads over LIBOR to compensate creditors for the added credit risk of having a longer term loan to maturity.

Sellers.

Who sells these instruments? If the bond issue is large, a "lead" investment banker ("underwriter") syndicates a large part of the corporate bond to other investment banks to make it easier to place the bond. (Lead underwriters are often the big money banks, such as J.P. Morgan Chase and Citibank. We will discuss investment banks in Chapter 23.) The deal itself is brought to the capital markets with proposed pricing by the syndicate lead and is priced at whatever price (interest rate) clears the market.

Investors. Who are the investors in these multiple loan instruments (all issued simultaneously)? Because institutions and mutual funds are not set up to provide revolving credit, the "pro rata" revolver piece and Tranche A loan is often purchased by commercial banks. The market for subsequent tranches of term debt is more liquid, and these bonds are typically purchased by mutual funds, commercial banks, hedge funds, etc.

Smaller Companies. Smaller companies usually borrow in simpler ways. They often have a relationship with either a smaller syndicate of commercial banks, or perhaps a regional bank in the case of a very small company. The structure would in all likelihood be less complex—a revolver and only one tranche of term debt, or just a revolver. In terms of pricing, their bond must offer premium pricing to compensate the lenders for the added credit risk of lending to a small company and for holding a less liquid financial claim. (The price is negotiated between the borrower and lender.)

Post Placement and Ratings

Bonds usually are illiquid—and, when they do trade, do so "over-the-counter."

As with all securities, issuers can raise financing at lower costs if they can give potential investors more information and the ability to liquidate their investment quickly. You already know that equity securities are usually bought and sold on stock exchanges after the original offering. The two most important exchanges in the United States are Nasdaq and the NYSE. Bonds, on the other hand, often do not trade on any exchange (such as the **New York Bond Exchange**). And when they do trade, the markets tend to be not very liquid. (The bond trading volume on exchanges is very low.) Instead, most bonds are traded **over-the-counter**, that is, by large investors who call up individual investment banks' desks. The transaction price is usually not disclosed in such cases, and trading is fairly rare. Because the vast majority of bond transactions take place between brokers rather than on an exchange, accurate bond prices are difficult to come by. Thus, individual investors are typically better off staying away from purchasing individual bonds, lest they wish to be taken advantage of. A better alternative for individual investors would be to purchase a good mutual fund that just holds bonds.

Coercive Bond Exchange Offers

Most of the time, covenants are firm.

Most bonds include contract provisions by which covenants can be changed. These provisions are usually difficult to change, except in financial distress. So, for the most part, firms must live with whatever covenants they write upfront.

But there is one occasional exception, which appears if creditors are "underwater."

But there is one mechanism that sometimes allows creditors to take advantage of public bondholders and that you should be aware of: the **exchange offer**. Consider a firm that is worth \$500, but which had earlier sold one type of bond with a face value of \$1,000 to 100 creditors. Each bond is a claim on \$10, and it is now really worth only \$5. How can managers reduce the face value of the claim, so that an increase of less than \$500 would allow the equity to be back in the money again?

How to "swindle" the bondholders.

The answer is an exchange offer. For example, if you offer each creditor a higher-seniority (or shorter term) bond for only a claim of \$6 (a total of \$600), it would not be, collectively, a good exchange for them. But consider what *is* in the interest of each creditor.

- If no other creditor accepts the exchange offer, then an unexchanged \$10 bond is worth only \$5. If one creditor accepts the exchange bond, it is paid first, so its value increases from \$5 to \$6.
- If the remaining 99 creditors all accept the exchange offer, then an unexchanged \$10 bond would be worth nothing. The new bonds would collectively claim $99 \cdot \$6 = \$594$ of the firm—and with only \$500 in value, nothing would be left for an original, unexchanged, and thus lower-priority bond.

It is in the interest of every bondholder to participate, but that means they will collectively end up worse off. Thus, the bond exchange offer works only if the firm can play off its creditors against one another—it does not work if one single creditor (a bank) holds the entire bond issue. To eliminate such coercive bond exchange offers, many bond covenants now require firms to first obtain approval by majority vote before a bond can be exchanged or a covenant be waived. In this case, every bondholder would vote against the exchange offer, and thereby come out better off.

20·4.C. Seasoned Equity Offerings

Seasoned equity offerings are rare.

Most publicly traded shares appear on an exchange in the context of a public equity offering. An **initial public offering** is the first sale of shares to the public. A **seasoned equity offering** is the sale of shares in an already publicly traded company. Seasoned equity offerings are rare events for large publicly traded corporations, except in connection with M&A activity. In contrast to bonds, liquidity is not a big problem for after-market stock investors. Over 10,000 large U.S. firms now have their common stock traded on a major public stock exchange, such as Nasdaq or the NYSE. There, any investor can easily purchase and sell shares, and closing prices for the previous day can easily be found in most newspapers. Not all shares are first issued and sold on an exchange. Some shares may simply be granted to employees or managers, and not necessarily from "treasury stock" which is the shares that the company itself is still holding, and into which repurchased share usually go.

How to avoid the SEC process.

The institutional process required to sell new shares in a public offering is lengthy and unwieldy. (For initial public offerings, it is an outright ordeal.) Fortunately, firms with fewer than 100 investors that do not try to sell their claims to the public are not (or are at least less) regulated by the SEC, and thus can avoid the process. (In a famous incident, Google ran into the constraint that it had more than 100 entities owning shares, so it had no choice but to go public, even though it did not need external funds.) Many smaller companies and hedge funds would simply be overwhelmed by the costs of navigating the SEC process.

Choices of Issuing.

Public firms can issue seasoned equity through various mechanisms. The three most important ones are:

Ideally, old shareholders would come out the same.

1. **Standard Issue** For example, a firm with 50 million shares representing \$400 million in outstanding equity (i.e., \$8/share) may announce that its board of directors has approved the issuance and sale of another 10 million shares in 3 months. The shares are to be sold into the market at the then-prevailing stock price three months later. If the stock price will be \$10/share at the time of the offering, the firm value will be \$500 million just before the offering and \$600 million just after the offering. So, both immediately before and after the offering, each old shareholder will still own a claim of \$10/share.

Shelf offerings can be "lazy."

2. **Shelf Offering (Rule 415 Offering)** For new equity shares registered with the SEC under Rule 415, the firm does not set one firm date at which the shares are to be sold into the market. Instead, the firm can sell them over a period of up to two years, at its own discretion and without further announcements. This is similar to the way that companies sell debt notes (Page 512), that is, on demand and off the shelf.

The mechanics.

3. **Rights Offerings** Yet another way to sell new equity shares is a **rights offering**. These are rare in the United States, but popular in some other countries (e.g., the United Kingdom). Instead of issuing new shares to anyone willing to purchase them, the company grants existing shareholders the right to purchase one additional share of equity at \$2 share. If all 50 million shareholders participate, the company will raise \$100 million. Each shareholder will own two shares, so there will now be 100 million shares to represent \$600 million in assets. Each share will be worth \$6/share, and each old investor will have invested \$12 for two shares.

   Rights Offerings force participation.

   So far, there is no difference between the rights offering and the plain offering: both facilitate the raising of \$100 million without loss for existing shareholders. However, what happens to a shareholder who does not participate? This shareholder will then own one share, for which she will have paid \$10, and which will only be worth \$6. This non-participating shareholder will have been expropriated. Therefore, rights offerings allow the firm to effectively force existing shareholders to participate in the offering.

Like bond offerings, equity offerings are usually orchestrated by an underwriter. We shall look at underwriters in more detail in Chapter 23.

SIDE NOTE: There is also a distinction between **primary shares** and **secondary shares**. These are confusing names, because they do not describe the distinction between an initial public offering and a seasoned offering. Instead, primary shares are shares that are newly minted and sold by the firm itself, in which case the proceeds go to the firm itself. (These are really the kinds of offerings that we just discussed.) Secondary shares are shares that are sold by an investor in the firm (e.g., by the founder), in which case the company does not receive the issue proceeds. Secondary offerings are de facto insider sales, so they are also often smaller than primary offerings. But they are usually greeted especially negatively by the market: an owner who wants to abandon ship and sell out is not good news. Because our book focuses on the firm's capital structure, we are concentrating on primary offerings.

20·4.D. Initial Public Offerings

IPOs are special—a lot riskier and with many separate regulations.

In contrast to a seasoned equity offering, an **initial public offering** is the *first* public sale of shares. There are some features that are unique to IPOs: First, there is no established price, so it is considerably more difficult and risky to place IPO shares. Without a public price, we cannot measure how the market responds to the announcement of the IPO. (Also, the diversification benefits for the current owners are usually much larger than they are in seasoned equity offerings.) Second, there are many unusual regulations governing the issuance of IPOs. For example, issuer and underwriter are liable not just for false statements, but even for "material omissions." Moreover, until recently, shares had to be sold at a fixed price that could not be adjusted upward if demand for shares was strong, or downward if demand for shares was weak. Most IPOs are still conducted this way, although it is now possible for strong issuers to auction their shares into the public markets. (The 2004 IPO of Google was the most prominent auction.)

Some of the process.

In a typical IPO, the issuer must provide audited financials for the most recent three years. Thus, unless the firm is so new that it has no recent history, or unless the firm has carefully planned its IPO years ahead, many firms must go back and create audited financials for activities that happened long ago. Similarly, firms often have a lot of other housecleaning to do—folding in or laying out subsidiaries, untangling relationships between the private owners and the firm, and so on. The real IPO process starts when the firms selects an underwriter (usually after presentations by 3-5 investment bankers), who orchestrates the offering. Together with the auditor and legal counsel, underwriter and the firm create a preliminary offering prospectus and file it with the SEC. They then give a set of "roadshow" presentations to solicit interest among potential investors. But neither is allowed to make statements beyond those in the preliminary prospectus. It also does not usually include one fixed price, but only an estimate (a price range). Finally, the underwriter is not allowed to take firm buy orders, but can informally collect a list of interested parties.

IPO Underpricing.

Usually within 48 hours after the SEC approves the prospectus, the offering goes live. The final offer price is set on the morning of the offering. Remarkably, IPOs are usually priced to create excess demand among investors, so shares become rationed. The average IPO experiences a jump of about 10% in one day (not annualized!), called **IPO underpricing**. In the 1999–2000 bubble, however, average underpricing reached as high as 65% *again in just one day*. There are a number of theories that help explain the continued presence of IPO underpricing.

- **Winner's Curse** If uninformed investors ask for allocations, they will disproportionately be stuck with shares in the hard-to-sell offerings. For example, if half the offerings earn +10% and are oversubscribed by a factor of 2, and half the offerings earn –10% and are undersubscribed, it would be 0% on average, but our uninformed investor would receive an allocation of only half as many shares in the +10% offering as in the –10% offering, so the average rate of return would be

$$\underbrace{50\%}_{\substack{\text{\%underpriced}\\ \text{offerings}}} \cdot \overbrace{0.5}^{\substack{\text{expected}\\ \text{share}\\ \text{allocation}}} \cdot \underbrace{(+10\%)}_{\text{underpricing}} \quad + \quad \underbrace{50\%}_{\substack{\text{\%overpriced}\\ \text{offerings}}} \cdot \overbrace{1.0}^{\substack{\text{expected}\\ \text{share}\\ \text{allocation}}} \cdot \underbrace{(+10\%)}_{\text{overpricing}} = -2.5\% \; . \tag{20.7}$$

So, if shares on average earn a 0% rate of return, uninformed investors should not participate, because their return will be less than 0%. To keep them in the market, underwriters underprice IPOs.

- **Information Extraction** Underwriters need to incentivice investors to have them reveal their opinions, and they reward with more underpriced shares those investors who reveal that they believe that the price is reasonably fair and that they want to participate and purchase.
- **Good Taste in Investors' Mouths** Underwriters want to create goodwill among investors to make it easier to place subsequent offerings. A bad or fraudulent issuer would not want to play this game, because the fraud would likely collapse before the goodwill pays off.
- **Cascading Demand** Investors are eying one another, so that offerings end up either a tremendous success or an utter failure. To avoid the latter, underwriters prefer to ensure success by underpricing.
- **Agency Conflict (Underwriter Selling Effort)** It is more efficient for the issuer to make selling easier by underpricing than it is to price the offering correctly and try to ascertain whether underwriters are doing their best to place the offering.
- **Agency Conflict (Additional Underwriter Compensation)** Although not in the interest of the issuer, the underwriter uses IPO underpricing as "currency" to reward their best brokerage customers. This requires that it is the underwriter who is in the driver seat, not the issuer.

Finally, after the firm is publicly trading, the underwriter often tries to "stabilize" the market and promote the firm as well as reasonable trading volume in the after-market. Indeed, for small firms, the underwriter is usually also the Nasdaq market maker, providing investors with the appropriate liquidity.

Other interesting IPO phenomena.

Underpricing is just one among a number of interesting empirical regularities for IPO firms. We do not yet fully understand all of them, but here is an interesting selection of findings about IPOs:

1. On average, IPO firms drastically underperform similar benchmark firms, beginning about 6 months after the IPO and lasting for about three to five years. (A conservative estimate is a risk-adjusted underperformance of about 5% per annum relative to the overall stock market.) But it is not only the IPO firms themselves that seem to perform poorly after the IPO, the same applies to firms that are similarly sized and in the same industry. No one really knows why. We do know that this downward drift is considerably stronger for firms that are very aggressive in the reporting of their financials at the IPO.

   Who would be foolish enough to hold onto IPO shares for more than the first 6 months? Because academic researchers cannot find out where equity shares are located (most stock holdings are confidential), we cannot fully study this phenomenon. The "word on the street" is that many of these shares end up in the accounts of very unsophisticated investors, such as "trust accounts" for widows and orphans.

2. Underwriters' analysts routinely issue "buy" recommendations on their IPOs. This is not surprising. What is surprising is why this still seems to matter. Why would anyone pay attention to these obviously conflicted analysts' opinions?

3. Insiders routinely sell their shares as soon as a pre-agreed lock-up period expires. When the lockup expiration week comes around, the IPO stock price predictably goes down by about 2%. Who would want to hold IPO shares through this lockup expiration?

4. IPOs either happen in droves or do not happen at all. When the overall stock market and the firm's industry have recently performed well, IPOs are pouring in. Professionals call this an "open IPO window." When the opposite occurs, the window is closed and there are zero IPOs. IPOs are not just reduced in price or scale, but they are typically withdrawn wholesale. Why?

5. It is not surprising that the average IPO pays 7% in underwriting commission—the maximum allowed by the National Association of Securities Dealers (NASD)—though many issuers find some backdoor mechanisms to raise the underwriter commissions further. But it is surprising that *virtually every* IPO pays 7% commission. In such a competitive market, why do underwriters not compete more fiercely on the commission front?

20·4.E. Raising Funds Through Other Claims and Means

Think of hybrids as a mix of the debt and equity.

Debt and equity are not the only claims that corporations can issue to raise funds, but they are the broadest categories and best studied. Investment banks regularly help firms to issue all sorts of debt-equity hybrids, and for the most part, you can think of many hybrids as combinations somewhere along a continuum. For example, a bond may be straight, it may have a conversion feature only at a very high firm value (in which case it is almost like a straight bond), or it may have a conversion feature at a very low firm value (in which case it is almost like equity).

There are alternative money providers.

Firms can obtain financing not only from public markets (investment banking), but also from plain old commercial banks—and most large publicly traded corporations do. (Small firms rely on banks almost exclusively as their loan providers.) But insurance companies, pension funds, mutual funds, foundations, venture capital funds, private equity funds, and even a multitude of government support programs have also jumped into the fray, and may help provide certain companies with needed capital.

There are ways to avoid financial markets for raising money altogether.

Another method to obtain financing is not to pay bills on time. A less stark method would be to finance through trade credit from a supplier. Firms can also obtain funds by the issuing of hedging contracts (which may promise future delivery of a good in exchange for cash today), leasing instead of buying, securitization (in which the firm sells off assets such as accounts payable instead of retaining its assets), etc. These are all plausible and common methods to finance operations—whether they are wise or not depends on the situation and the firm.

20·4.F. The Influence of Stock Returns

Capital structure can come about "passively."

We started this chapter with the observation that stock returns have an influence on capital structure, just as active issuing activity does. We could call this the "direct" effect of stock returns. A firm that is financed by $1 billion in debt and $1 billion in equity that loses one quarter of its value ($500 million) will experience a debt-equity ratio increase from 1:1 to 2:1. (If so desired, managers can counteract this effect by issuing more equity and retiring some debt.)

Speculation? How could managers know better?

But stock returns and value changes can have a second conduit by which they can influence capital structure. Indeed, the mechanism is entirely different. Although it is tied directly to past stock returns, it is really about how managers respond through issuing to market returns.

There is good evidence that CFOs *believe* and act as if they can predict ("time") the financial markets. This is not too surprising. Most managers' sense of their firm value is based on the corporate internals, not on how the financial markets have moved recently. If the financial markets have moved up, managers' internal beliefs do not catch up immediately, so they now believe that they can raise equity relatively cheaply at high market valuations. They feel that their stock is relatively more overpriced. Note that this mechanism suggests exactly the opposite behavior to what would be required for the firm to return to its original debt-equity ratio. If the firm wanted to keep a particular debt-equity ratio, it would have to repurchase equity that has gone up and issue more equity that has gone down. If the firm wanted to time the market, it would repurchase equity that has gone down and issue equity that has gone up. (Managers similarly seem to try to time interest rates and the yield curve. If interest rates are higher (lower) than they were in the past, companies tend to avoid (seek out) bonds. If the yield curve is steep by historical standards, corporations tend to borrow more at short-term interest rates than issue long-term bonds.) In an efficient financial market, there is no benefit to attempts at market timing, but also no cost to doing so. You can look at this attempt at market timing as just another investment, which is a fairly harmless attempt by managers to make profitable investments.

Managers seem to believe they know when prices are high or low.

However, what is surprising is not the fact that managers have tried to time financial markets, *but the empirical evidence that this has actually turned out to be profitable*! Even stranger, managers have been good not only in predicting their own stock price level, but in predicting the overall stock market level—an incredibly difficult feat. (In fact, why be a corporate manager if you have this ability?) There is academic controversy as to whether this success has been the result of coincidence or real timing ability. For example, one counter-argument is that seeming timing ability is merely survivorship bias: firms that failed in their timing disproportionally disappeared. It could also just be that when the financial markets go up, more and more firms raise external funds, and this stops when financial markets go down. Thus, even though managers cannot predict the financial markets, when economists look at when firms raised funds, they will find that they did so before the market went down. Either of these two theories could explain seeming market timing ability when there is none.

Weird—it seems to have worked!

Solve Now!

Q 20.12 *What is the financing pecking order?*

Q 20.13 *What is the financing pyramid? Is it a good description of empirical reality?*

Q 20.14 *Does the pecking order imply a financing pyramid?*

Q 20.15 *How does a coercive bond exchange offer work?*

Q 20.16 *How does a coercive seasoned equity rights offering work?*

Q 20.17 *Assume a rights offering for a firm that is worth \$500 million and offers its shareholders to buy one extra share for each share that they already own. The "discount" price for the new shares is 1/5 the price of the current shares. Assume that half the investors do not participate. What is the loss to non-participating investors (shares) and the gain to participating investors (shares)?*

Q 20.18 *What are the some of the main empirical regularities about IPOs?*

Q 20.19 *Why are IPOs underpriced?*

20·5. Summary

This chapter covered the following major points:

1. Both capital scale and capital structure dynamics are influenced by factors under management's immediate control (such as debt issuing or share repurchasing) and factors beyond management's immediate control (such as value changes, aka stock returns).
2. A CFO should consider a holistic view of capital policy. Many activities influence both the firm scale and debt-equity ratio.
3. Appropriate cash management should be a primary concern in many firms.
4. Many firms follow a "pecking order" financing scheme, in which they first use retained earnings, then progressively less senior debt, and finally equity (as a last resort).
5. Debt offerings come in many flavors, and though we surgically dissect their features, the actual debt offerings are often complex packages.
6. Seasoned equity offerings are rare. They can be standard, shelf-registered, or rights offerings. (Secondary shares are more insider sales than corporate capital structure events.)
7. Initial public offerings appear within certain industries and at certain times, i.e., in waves. The average one-day IPO underpricing is about 10%, but IPOs begin to underperform the market beginning about 6 months after the offering for about three to five years.
8. Debt and equity are not the only venues to raise financing. There are other methods, e.g., stretching out the payment of bills.
9. There is empirical evidence that many managers try to time the financial markets. Remarkably, this has often turned out to be profitable, although we do not yet fully understand why this is so.

Solutions and Exercises

1. Usually, this is a debt/equity ratio.
2. Debt consists of long-term debt (bonds and notes), short-term debt (financial, taxes, payables, etc.), pension-debt, and other debt. Equity is a number of shares multiplied by the per-share value.
3. No! Many firms have treasury shares, which they themselves hold.
4. Equity.
5. Yes and no. Firms that operate need to incur debt, so in this sense the answer is no. However, firms could change their operations or refinance their debt by raising equity.
6. See Table 20.6.
7. There are infinitely many different ways, but one that is very easy would be to issue $400 million in debt and repurchase $150 million in equity.
8. Before the market reacts, the firm will have $400 million in debt, and $200 million in equity. The market believes these transactions will create $15 million in equity. If all accrued to shareholders, there would be $215 million in equity for $615 million in value, which would be a 34.96% debt-asset ratio. If none accrued to shareholders, the equivalent ratio would be 32.5%. Most of the value gain is likely to accrue to shareholders, so the real-world answer would probably be above 34%.
9. Can you invest money better than your shareholders can on their own? Do your shareholders understand and agree that you are acting in their interest?
10. The firm can match assets and liabilities, obtain a credit line, invest in more liquid assets, and avoid debt. Doing so is costly—in a public company, too much cash also tempts managers to waste assets and not work as hard.
11. No. The market pressures forcing poorly financed companies back to optimal behavior are weak.
12. Managers prefer issuing safer securities first, before proceeding to less safe alternative.
13. Companies are financed predominantly by safer securities. Equity is a small part of the pyramid at the top. The traditional view of the financing pyramid does not apply to many successful companies, because the equity will have grown in size.
14. No! Equity can change in value (and debt can accumulate during operations). Many firms follow a financing pecking order, but their capital structures do not look like a pyramid.
15. It gives existing bondholders the right to exchange their bond for a more senior bond with lower face value. Creditors who do not participate are effectively expropriated.
16. It gives existing shareholders the right to purchase more shares at a given price. Investors who do not participate are effectively expropriated.
17. Assume that the shares are $10 each. You can then purchase shares for $2 each. Of 50 million shares, 25 million will participate. So, you will raise an extra $50 million. Thus, total corporate assets will be $550 million. There are now 75 million shares in total. Therefore, each share will be worth $7.33. Participating investors will own two shares worth $14.67, for which they will have paid $12. This represents a 22% gain. Non-participating investors will own one share each, for which they will have paid $10. This represents a 26.7% loss.
18. On average, they appreciate by 10% from the offer price to the first after-market price, and then lose about 5-10% per annum over the three years. (Other regularities are described above.)
19. There are a number of explanations—such as the winner's curse, payment to investors for revealing information, the intent to leave goodwill for future offerings, highly elastic cascade-related after-market demand, and agency conflicts between the firm and the underwriter.

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

A. Appendix: Standard&Poor's 04/24/2005 Bond Report on IBM's 2032 5.875% Coupon Bond

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CHAPTER 21

Empirical Evidence on Capital Structure Dynamics

Nothing yet!

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The goal of this chapter is to describe, in broad strokes, how publicly traded corporations in the United States have financed themselves over the last few decades. (There are very little data and research on how private firms are traded.)

You should realize that this chapter is operating at the cutting edge of research. There are different interpretations of the data, so it is unavoidable that what you are reading is an interpretation of the evidence. My goal is give you a taste of what we know—and what we do not know.

21·1. Layers of Causality

What determines the ways by which capital structure can change? Layers of Causality.

We are now interested in exploring empirically how the debt-equity ratios of publicly traded companies have evolved. To do so, we can examine several layers of causality—one can always drill deeper and deeper. (Eventually, if you dig deep enough, you will find yourself in the world of philosophy and theology.) Think about an analogy—say, you want to know why a car is fast. The first layer of causality may be that its speed is due to lots of power, low weight, and low wind resistance. But *why* is there a lot of power? This question brings you to a deeper layer of causality with questions such as how many cylinder and intake valves your engines has. You can then drill down into yet another layer of causality. Why is this particular number of cylinders/valves more powerful? Yet another deeper layer of causality emerges with questions such as why and how gasoline combusts.

Two layers of causality: mechanisms and forces.

We are now going to explore the dynamics of debt-equity ratio changes on two levels. We can call our first, somewhat shallow layer the "mechanistic layer": how important are the various mechanisms through which debt-equity ratios can evolve? These mechanisms are basically the cells you have already seen in Table 20.6, such as debt and equity issuing and repurchasing. The second, deeper layer is more causal and explores the variables, characteristics, and economic forces that induce firms to engage these mechanisms in the first place. There is one factor, which could be classified either in the first or second level—the role of stock value changes: you can think of it either as a mechanism that shifts capital structure around, or as an economic force, partly within and partly outside the domain of the mechanisms that managers can use.

21·2. The Relative Importance of Capital Structure Mechanisms

Stock returns and long-term debt issuing are the most important factors changing debt-equity ratios.

Let us begin with the big picture mechanisms. We have just looked in the last chapter at the various mechanisms that influence capital structure. Now we ask about the relative importance of each of these mechanisms. That is, has the typical company's debt-equity ratio been driven more by the firm's value or by the CFO's net issuing activities (which include issuing, repurchasing and dividends)? This question can be phrased as "If you knew in advance how much every firm would issue over the next x years, what fraction of the change in capital structure could you explain?" Table 21.1 answers this question for five-year horizons. The rest of this section discusses the meaning of this table.

Table 21.1. Relative Importance of Factors Determining Capital Structure Changes Over Five Years

All Net Issuing (and Dividend Activity)			69%
All Net Issuing (without Dividend Activity)			66%
· · · All Net Debt Issuing Activity		40%	
· · · · · · Convertible Debt only	4%		
· · · · · · Short Term Debt only	14%		
· · · · · · Long Term Debt only	32%		
· · · All Net Equity Issuing Activity		16%	
Direct Effect of Stock Returns on Existing Capital Structure			40%

Note: These values measure how much of the change in capital structure from today to five years from now you could explain if you had perfect foreknowledge of each component. Net issuing means issues net of retirements. The samples were all publicly traded U.S. stocks from 1964 to 2003. (The numbers need not add up to 100%, because one component can have information about the other components.) The equity is measured by its market value.
Source: Welch, 2004.

21·2.A. Net Issuing Activity

The first row of Table 21.1 shows that CFOs were by no means inactive in the capital markets. If you had perfectly known how firms had issued and retired debt and equity and paid in and paid out funds, you could have explained 69% of firms' total capital structure changes over a five-year horizon. The remaining 31% are necessarily corporate value changes that have not been directly effected by managerial issuing and repurchasing. Omitting dividends drops the explanatory power from 69% to 66%, so dividends can explain only a meager 3% of capital structure—as far as debt-equity ratio dynamics in publicly traded corporations are concerned, dividends are a sideshow.

Net Debt Issuing

Net Debt Issuing.

Row 2 in Table 21.1 tells us that 40% of all capital structure changes over five years were due to firms' net debt issuing activity. The next three rows can tell us that long-term debt alone can account for 32% of changes in debt-equity ratios, that short-term debt has been somewhat less important, and that convertible debt has been fairly unimportant. It would be interesting to break these debt issuing activities into their components—issuing and repurchasing—and to break the repurchasing in turn into sinking fund payments, interest payments, and principal repayments, so that we could understand better what part of the mechanism really drives capital structure. Remarkably, despite the obvious importance of debt issuing activity, we really do not know this decomposition.

Net and Pure Equity Issuing

Net Equity Issuing.

The next row in Table 21.1 shows that net equity issuing can explain about 16% of changes in firms' debt-equity ratio, and therefore is less important than net debt issuing as a determinant of capital structure. Nevertheless, equity issues are more glamorous, so economists have studied them in more detail.

Table 21.2. Typical Equity Share Activity Among S&P100 Stocks, 1999-2001

Total Seasoned Equity Offering Activity	+	3.77%
· · · M&A Related, +3.68%		
· · · Not M&A Related, +0.09%		
Executive Compensation	+	1.05%
Convertible Debt	+	0.14%
Warrant Exercise	+	0.05%
Share Repurchases	–	1.44%
= Changes in Equity Outstanding	=	+3.57%

Note: Categories describe equity issued in conjunction with an activity. Equity share activity is measured per annum and as a fraction of total assets. For scale, changes in total liabilities were about 10.07% of assets, and changes in retained earnings were 1.37% of assets.
Source: Fama and French, 2004.

Net Equity Issuing.

Table 21.2 decomposes equity issuing into its components, though only for the very largest publicly traded firms. (Unfortunately, we do not have knowledge of a similar decomposition for smaller firms.) The table dispels the popular myth that most shares occur through plain seasoned equity offerings. Instead, from 1999 to 2001, equity shares appeared most commonly through equity offerings in connection with corporate acquisitions. (We also know that firms commonly issue not only equity but also debt to finance acquisitions, so we cannot conclude that firms' debt-equity ratios declines during acquisitions.) Outside an acquisition, seasoned equity offerings are exceedingly rare. We also saw these patterns in IBM's case in Section 20·1—IBM did not issue equity, repurchased some shares into its treasury, and then used equity shares from its treasury in its acquisition of PwCC partners and in its funding of employee stock option plans.

More evidence from elsewhere suggests SEOs are rare in smaller firms, too.

Moreover, other evidence similarly suggests that, even including M&A activity, public equity offerings are rare. The 10,000 or so firms trading on the NYSE and NASDAQ conducted only about 12,000 equity offerings from 1990 to 2000, of which about half were initial public offerings and about half were seasoned equity offerings. With only 300 SEOs in an average year, the typical public firm would issue more equity only about every 20 years.

21·2.B. Firm Value Changes

Value changes are proxied by stock returns.

The final row in Table 21.1 shows the direct effect of stock returns on capital structure. Recall that this is the debt-equity ratio change that a company experiences when it increases or decreases in value— a $200 million firm with $100 million in debt and $100 million in equity, which doubles in value from $200 million to $400 million, will drop its 1:1 debt-equity ratio to a 1:3 debt-equity ratio. As mentioned earlier, corporate stock returns can be viewed both as a mechanism (itself influenced by deeper forces) and as an external force that tugs on firms' debt-equity ratio.

Value changes can account for a little less than half of capital structure changes.

Table 21.1 shows that if you had known perfectly how stock returns would turn out over the next five years, you could have explained 40% of firms' total capital structure changes. (Note how all issuing was able to explain 69%, so a good part of variation must have been explainable by either.) The fact that stock returns are a major factor should not come as a big surprise to you. If your recall our IBM example from Section 20·1, it was changes in the stock price that first reduced IBM's equity value by one-third from 2001 to 2002, and primarily caused its debt-equity ratio to increase from 0.31 to 0.55.

Apparently, managers did not fully rebalance.

Importantly, you can think of these stock returns as the "relevant" changes that were not undone by managers. If firms had undone the value change and rebalanced through issuing and repurchasing, then knowing the stock returns would not have helped in explaining changes in capital structure. Our empirical evidence therefore suggests that even over a five-year horizon, firms do not fully rebalance their capital structure.

Trust me: market timing is only secondary.

You may wonder whether some part of this 40% could also pick up if managers typically tried to time the market, and issued more equity as the stock price goes up. Other empirical evidence suggests that market timing is not a strong force. The reason is that, in response to stock price increases, firms issue not only equity, but also debt, and tend to pay out more in dividends. Therefore, the timing effect on net debt-equity ratios is fairly modest. The 40% that we see is almost entirely the direct value effect of stock returns on debt-equity ratios.

In perspective: observed capital structure today is strongly related to past corporate performance.

Explaining 40% of something that is as variable and firm-specific as corporate debt-equity ratio changes are is quite robust—even though our explanatory variable is conceptually on a fairly shallow level of causality. Consequently, if you want to know why some firms have high debt-equity ratios today and why other firms have low ones, a big part of your explanation has to be not that the former issued a lot of debt and the latter issued a lot of equity, but that the former had experienced negative stock returns and the latter had experienced positive stock returns. This relationship between stock returns and capital structure can also suggest a natural debt-equity lifecycle for firms. Most firms start out being highly levered—the owner must borrow to finance the firm. Eventually, as the firm survives and accumulates equity, its scale increases and its debt-equity ratio declines.

Scale is also not deliberate.

Managers also typically do not pay out large value gains, or raise more funds in response to large value losses. Therefore, like debt-equity ratios, firm scale has a large external component, too—firms that are large today are not large primarily because they raised a lot of funds, but because they appreciated in value. In sum, few firms seem to deliberately choose their target scale and target debt-equity ratio, and then act to retain these targets.

What can a CFO learn that is of immediate guidance?

What can CFOs learn from the fact that stock returns are one major determinant of most firms' capital structures? Let's ask a few pointed questions:

- *Is the empirically observed failure to rebalance size and debt-equity ratio evidence of poor managerial behavior?* Absolutely not. It might well be that the optimal firm size increases and the optimal debt-equity ratio decreases as the firm's underlying business becomes more valuable. In this case, managers should be happy with their capital structures. Or it might be that such rearrangements are fairly expensive, relative to the costs. In this case, managers may be unhappy with their capital structures, but it would not be profitable to do something about it.
- *Could the failure to rebalance be evidence of poor managerial behavior?* Yes, it could be—but it does not need to be! We just don't know yet one way or the other. In some firms, the evidence that managers are mis-capitalized is fairly suggestive. In other firms, we are not so sure. There is lively academic controversy surrounding this question.
- *Does this failure to rebalance mean that if you run a corporation, you should not worry about capital structure or appropriate corporate scale?* Absolutely not. Even if many other managers are passive and/or do not do the right thing, you still can! Your managerial choices should remain intelligent and dynamic.
- *Does this mean that we cannot use the capital structures of other companies to judge what capital structure our own firm has?* Probably yes. Their capital structures are less indicative of deliberate design than they are of their historical performance.

Digging Deeper: *Stock returns are a good proxy for the value changes we discussed in Section 20·4.F. Theoretically, however, stock returns could miss some of the change in the underlying asset values, if these changes benefited or hurt debt holders by making debt repayment more or less likely. However, unless the firm is in—or close to—financial distress, almost all of a firm's own value change goes to equity owners. In the extreme, risk-free debt would not be affected at all by firm value changes, and stock returns would be exactly equivalent to the value change. In any case, we do not mean that debt value changes cannot occur, just that they tend to be so much smaller that our proxy of stock returns will capture most of how firms differ from one another in terms of value changes at any given point in time. Aside, we do not have good market value data for corporate debt, so we could not really measure the whole change in value even if we wanted to.*

Solve Now!

Q 21.1 *What are the most important financial mechanisms influencing capital structure changes over five-year horizons?*

Q 21.2 *How important is non M&A related seasoned equity issuing activity among Fortune-100 firms?*

Q 21.3 *If firms often do not readjust their capital structure, does this mean that capital structure theories are irrelevant?*

21·3. Deeper Causality — Capital Structure Influences

What determines are the forces operating on the channels?

You now know how firms change their capital structure—which mechanisms they are using. You can also think of these mechanisms as "channels" through which other forces can operate—forces that are one layer deeper in terms of causality. There are at least five possible mechanisms (channels), so we can ask:

1. What makes firms issue debt?
2. What makes firms retire debt?
3. What makes firms issue equity?
4. What makes firms retire equity (or pay dividends)?
5. What makes firms experience good/bad corporate value performance? (As noted earlier, you might classify value changes as deeper than a managerial mechanism, though.)

(We are ignoring such channels as pension and operating liabilities.) These questions are getting at the deeper issue of *why* capital structure is what it is. We know how important the mechanisms changing debt-equity ratios are, but we do not yet know *why* firms use them. We also know that if we had a choice, we would determine first what drives debt net issuing (especially long-term debt), then what drives net equity issuing and net short-term debt issuing, and only finally what drives convertible debt issuing and dividends—in that order.

The five channels to work with.

If a variable strongly influences one channel, this influence will likely, but not necessarily, percolate into an influence on the overall capital structure. For example, if solar flares were to make firms issue debt, then we would also expect solar flares to increase firms' debt-equity ratios. However, this is not a necessary outcome. If solar flares had a strong positive influence on debt-equity ratios through one channel and a strong negative influence through another, solar flares could end up having no influence on overall capital structure. Moveover, you learned earlier that it is possible for a variable to explain a lot of equity issuing and yet have no influence on typical debt-equity ratios—if the firms that are subject to this variable are already 100% equity-financed, the firm will still remain all equity. The opposite can also be the case. Some variable could have only a weak influence through every single channel and we would be tempted to discard it as too weak, but if it worked for all five channels, it could end up having a strong influence on the firms' overall debt-equity ratios.

21·3.A. A Large-Scale Empirical Study

The Land of Oz.

A recent large-scale empirical study by Hovakimian, Opler, and Titman (2001) explores how different variables exert influences on the first four channels over one-year horizons. The study shows that our knowledge of the empirical determinants of capital structure is weak. The authors document that there are a multitude of variables that seem to play statistically significant roles—*but all of these variables together can explain only a few percentage points of the total variation in capital structures across firms.* For the most part, there is no smoking gun. The determinants of capital structure remain rather murky. We have only a glimpse of what is going on.

Big Findings.

Here is what the study found:

Asset Performance and Industry Benchmark.

The Debt Issuing Channel Firms issue more *long-term* debt if they have high market/book ratios, good recent stock market performance, and relatively much debt coming due soon. Firms issue more *short-term* debt if they have poor recent asset performance and if they have less short-term debt than their industry peers. In both cases, though, the relationship is very weak—these causes could explain only 2 to 3% of its cross-sectional variation (called R^2)—a minuscule proportion. In sum, we just do not know yet what makes firms issue debt.

The Debt Retirement Channel For the second channel (debt retirement), the authors found that firms reduce their debt if they are above their industry peers in terms of their debt ratios, and if they have had good recent stock market but bad accounting performance. Interestingly, managers' debt retirement actions are thus the opposite of what it would have taken to rebalance to the previous debt-equity ratio. We must again ask how important these causes are, and here we get a much better 12% in explanatory power (R^2).

Industry Benchmark, Recent Performance.

The Equity Net Issuing Channel The third and fourth channels are where most of the academic research has focused—especially insofar as seasoned equity offerings and dividends are concerned. There are three good reasons for this: first, we have robust theories here, specifically the pecking order theory, which seems to be reasonably consistent with some of the evidence; second, the announcement of market-related equity issuing and dividend activity plays a prominent role in the financial press; and third, we have a lot of publicly available data here. Nevertheless, dozens of studies have informed us that equity issuing and retiring activity also remains a mystery.

Recent Performance.

The evidence seems to suggest that firms first and foremost do not like to issue equity, consistent with the pecking order theory and any other theory that imputes high costs to equity issuing activity. When firms do announce that they will issue equity, it is on average greeted with a negative return on its outstanding stock—the subject of our next chapter. On balance, firms tend to issue equity (rather than debt) if they have had worse accounting performance and better stock market performance. (Although firms also tend to issue debt in response to positive stock returns, their tendency to issue equity seems stronger—evidence that managers try to "time" the stock market.) Especially firms with more tax obligations tend to prefer issuing debt over equity.

Altogether, the authors could explain 3% of the variation in firm's equity repurchasing activity, and 15% of firm's equity issuing activity.

Putting this (and other) evidence together, here is my overall impression of what factors play important roles in influencing capital structure outcomes, in rough order of their importance:

The most important factors explaining capital structure choice, IMHO.

Direct Stock Performance Influence If you classify stock returns as a cause rather than a mechanism, then it is by far the most important variable. Because firms do not counteract stock returns, firms with good stock price performance tend to end up with lower debt ratios, while firms with poor stock price performance tend to end up with higher debt ratios. (You may want to dig deeper and ask what causes stock performance, but this would again be a difficult predictive exercise.)

Equity Issuance Avoidance Firms seem to want to avoid issuing equity. A seasoned equity offering is a rarity, and even more so outside an M&A transaction. Given that the costs of an equity issue are high (including the often negative market reaction), this is not surprising behavior.

Peer Similarity Firms seem not only to end up with capital structures similar to those of their industry peers due to their commonality in industry stock returns, but also actively seem to *like* being similar. They often issue or retire debt or equity to come closer to their peers. Some industries (R&D heavy with few tangible assets) have avoided debt financing altogether. (You may want to ask what determines peers' ratios, and why firms want to be similar to their peers, but this is an even deeper level of causality—mostly beyond our current knowledge.)

Corporate Income Taxes Firms with high corporate income tax rates tend to actively issue debt and retire equity, i.e., increase their debt ratios.

Nevertheless, high corporate tax firms usually have *low* debt ratios. How can this be? The reason is that good performance translates not only into high profits and therefore high corporate taxes, but also into positive stock price performance. The latter directly reduces the firm's debt ratio. Although the end effect can be complex, on average, net issuing activity is usually not enough to undo the direct stock return effect.

Accounting Performance Firms prefer net debt issuing over net equity issuing if they have better accounting profitability and more tangible assets (which can be easily collateralized). But as with taxes, good accounting profitability correlates strongly with higher stock prices, which in turn correlates strongly with *lower* debt ratios.

M&A Activity Much debt and much equity are issued in connection with M&A activity, although proportionally more debt is issued than equity. Again, firms usually start acquiring firms after good stock price performance, so the overall capital structure effect can be complex.

Financial Distress Firms that are in dire straits (not the MTV band) have no choice but to retire some debt and issue equity. This seems to be an unusually solid net issuing influence, but only for firms close to the verge of bankruptcy.

Active Market Timing Firms that experience stock price increases tend to issue more securities—through both debt and equity, so the capital structure consequence is not too strong. Moreover, such firms also tend to pay out more in dividends, so even the net equity issuing effect is not yet clear. Nevertheless, when surveyed, CFOs claim that they do watch their stock market value, and respond to it—perhaps even try to time it. In any case, active market timing is the newest and thus the most interesting factor to explore—as more research comes forth, we may see its importance rise.

Uncertainty Firms with more volatile underlying assets tend to have less debt in their capital structures.

21·3.B. Theory vs. Empirics

What we can learn from what we have.

The above variables are interesting, but they are not exactly what the theories were asking for. For example, an interest coverage ratio is often used as a proxy to measure the proximity to financial distress—but it is not exactly financial distress. Some firms have low interest payments relative to earnings, and are in distress; other firms have high interest payments relative to earnings, and are financially sound. Yet ultimately, we study such specific variables only because they are relatively easy to measure empirically. We would have preferred direct measures of our theories of capital structure, but such measures are usually not as easily available. Most of the time, our variables are a compromise between empirical availability and theoretical construct, and we then try to interpret our empirical findings through the lenses of our theories. From our proxies, we can draw two basic conclusions about the theories: First, it appears that agency concerns, pecking order concerns, financial distress (in very few companies) and corporate taxes all matter, at least a little—or they matter in different ways through different channels. Second, there are some other proxies that matter, but we just do not know why they matter. For example, we do not know why firms do not counteract market influences more strongly, and why they seem to “like” capital structures similar to those of their industry peers.

Why so weak?

In sum, we have learned that we do not yet fully understand the factors that are driving firms to actively change their capital structures. It seems to be a complex process, possibly with a lot of idiosyncratic behavior. Our variables are statistically significant, but they leave a lot to be explained. You can read the situation in a number of ways:

1. Our variables may not matter much, because they are poor proxies for our theoretical constructs (e.g., for tax savings or bankruptcy costs). With more research, we may eventually find better proxies that will help us understand capital structure empirically better.

2. There are other theories and factors that we do not yet know which may be more important than those we have now.

3. Our variables may not matter much, because capital structure choice is practically irrelevant. Whatever managers may be acting on—whether based on, say, book market ratios or their horoscopes—may have only minimal value consequences. You could think of this as an empirical validation of Modigliani-Miller.

4. Managers may just act poorly and erratically, and there is nothing outsiders can do to correct it.

It is probably a little of each. Right now, capital structure is an especially fertile area for behavioral finance, because idiosyncratic managerial behavior seems important and because there is no easy way for financial markets to arbitrage misbehavior. So, empirical capital structure remains an exciting field of research. We are definitely making progress in learning *how* managers behave, but we also have a long way to go.

21·3.C. Evidence on Equity Payouts: Dividends and Equity Repurchasing

Dividends and repurchases are the payout mechanisms. Typical dividend yields and changes.

Let us now turn towards dividends and share repurchases, both of which increase the debt-equity ratio. Although they are not too important in terms of changing firms' debt-equity ratios, they are the primary mechanisms by which equity shareholders receive a payback on their investment. As of 2000, the typical firm in the S&P500 paid out about 1% to 2% of its market value and half of its earnings in dividends each year. In the mid-1990s, about one in ten quarterly earnings announcements saw a dividend increase, one in one-hundred quarterly earnings announcements saw a dividend decrease, and all others kept the dividends where they were previously.

Repurchases and dividends are now approximately equally important.

As we discussed in Chapter 19, share repurchases were clearly the tax-smart method to pay out cash until 2003, when the Bush administration reduced the dividend tax from 35% to 15%, and largely eliminated double taxation. The empirical evidence suggests that the historical dividend taxation disadvantage had induced many firms to shift away from dividends and towards share repurchases as their primary means to return money to shareholders. This was clearly reversed. The 2003 tax reform has indeed changed corporate payout policy. Chetty and Saez (2004) found that the elimination of the double taxation has induced nearly 150 firms to initiate dividend payments, adding about $1.5 billion in aggregate, regular dividends. In sum, dividend payments are no longer a mystery.

Remaining Differences: changes in inside ownership, an informal understanding about persistence of future payments, and "behavioral finance" type investor preferences.

So, with the tax disadvantage gone or at least drastically reduced, what are the remaining important institutional differences between dividends and share repurchases? Not many.

1. Executives and insiders are often not permitted to tender their shares into share repurchase offers, and thus will own relatively more of the company after a repurchase than after an equivalent dividend payment. In addition, many executive stock option plans are written on stock prices that are not adjusted for dividends. This means that executives prefer a share repurchase to a dividend payment. For example, if a manager of a $60 company has an option that allows her to purchase shares at $50, then the manager would be reluctant to pay a $20 dividends—after all, the shareprice would drop to about $40, making the right to purchase at $50 a lot less valuable.

2. Dividends tend to be more regular—because shareholders expect them to continue—while share repurchases can be done in occasional chunks. An ordinary dividend payment therefore informally obliges management to continue, signaling more optimism. Of course,

ANECDOTE: The Biggest Dividend Payout Ever?
In the most prominent immediate response to the Bush dividend tax cuts of 2003, Microsoft (MSFT) initiated dividends. Fifteen months later, on July 20, 2004—and 7 minutes after the market had closed—it announced a $32 billion special dividend, plus a $30 billion share repurchase, plus an increase in ordinary dividends from 16 cents to 32 cents per share (a yield increase from 0.56% to 1.12%). With a market capitalization of about $300 billion (a P/E ratio of about 20 [forward-looking earnings] and 37 [current earnings], and a cash hoard of $56 billion), the total payout represented about 20% of Microsoft's market value. A few minutes after market opening on July 22, Microsoft's outstanding shares had jumped in value by a little over 3%, so for every dollar changing hands from investors' company to investors' pocket, shareholders also felt 15 cents happier! Interestingly, two days later, Microsoft announced quarterly earnings 82 percent higher than those from its prior year, but still short of expectations—and shares promptly fell back to where they had been before the payout announcement. It appears as if the payout announcement was a positive signal, and the failure to meet earnings expectations—despite huge earnings growth—just about canceled one another. We researchers are all watching for what will happen in 2008, when the dividend tax cuts are scheduled to expire.

management can also pay a “special dividend,” which signals its one-time nature to investors.

3. There is some evidence that retail investors simply “like” dividends better than share repurchases, and institutions like just the opposite—although no one knows why. (This was a perverse ordering of preferences when dividends were more heavily taxed, and many institutions were tax-exempt. It should have been the case that institutions liked dividends and retail shareholders liked dividends—a mystery.) However, some institutional shareholders are obliged by their charters to hold *only* dividend-paying stocks. This provision excluded them from holding such stocks as Microsoft, at least prior to 2003 when Microsoft initiated its dividends.

Disappearing and reappearing dividends.

There were also other long-term trends at work even before 2003. From 1980 to 2000, share repurchases had increased from 5% to 50% of earnings, and had become just about as important as dividends. (Many shares are repurchased but not retired; instead, they are immediately given out again to compensate employees.) In contrast, dividend initiations had declined—a fact that is sometimes dubbed “disappearing dividends.” Remarkably, this trend reversed in 2000 as more firms began to initiate dividends again—i.e., *prior* to the Bush tax cuts! The reason for this pattern was the age structure of firms in the market. As many of the new tech firms were maturing, they started paying dividends. (Of course, the Bush dividend tax cuts further aided this trend.) In addition, an important factor in the decision of firms to initiate dividends seems to be the relative premium the market places on dividend payers vs. non-dividend payers. In the tech bubble of the late 1990s, firms paying dividends did not trade at a higher P/E ratio than firms not paying dividends. Consequently, firms did not see large valuation incentives for starting dividends.

21·3.D. Forces Acting Through the Equity Payout Channel

Dividends are paid when firms have cash, but smoothed out.

As with our capital structure questions, we would love to know what makes companies pay out cash. Here, our knowledge is relatively good, especially when it comes to dividends. There are two strong empirical regularities:

1. Ordinary (not special!) dividend payouts tend to be persistent: firms are very reluctant to reduce them. As a result, firms also do not raise ordinary dividends too dramatically, because this could force them to cut them again in the future. Lintner (1956) first formally documented that firms only partly adjust dividends towards a target level if they experience abnormally high or low earnings. This phenomenon is called dividend smoothing. Firms still very much smooth dividends, though less so than 50 years ago.

2. Firms pay out money when they have it. That is, they pay out earnings when they have them and when managers are reasonably confident that good earnings will continue. There is some debate, however, whether dividend changes signal primarily that future earnings will be high (so that the dividend yield can continue), or whether they respond more to past earnings.

   The positive association between earnings and dividend payments also means that firms tend to pay out more in dividends when their stock prices have recently risen. (Interestingly, this may not have debt-equity consequences, because the effect through another channel runs opposite: firms also issue more new stock in response to good stock returns.)

Solve Now!

Q 21.4 *What deeper characteristics help explain corporate debt-equity ratios?*

Q 21.5 *How good is our empirical knowledge about the deeper determinants of capital structure?*

Q 21.6 *Firms with larger tax obligations are known to be more inclined to issue debt. Does this mean that firms with high tax obligations usually have high debt-ratios?*

Q 21.7 *If our empirical knowledge about the the deeper determinants of capital structure is modest, does this mean that capital structure theories are irrelevant?*

Q 21.8 *What remaining tax advantage do share repurchases enjoy over dividends?*

Q 21.9 *What are the other differences between a share repurchase and a dividend payment?*

21·4. Survey Evidence From CFOs

Let's ask the CFOs.

There is another way to approach the question of how managers choose capital structures—just ask them. Of course, we should not blindly believe that just because CFOs publicly proclaim a motive, that it *is* their motive. Graham and Harvey (2001) surveyed 392 CFOs to find out what they proclaim makes them issue equity or debt, and they found both interesting and some rather puzzling results that are difficult to interpret.

CFOs recognizes taxes and financial distress costs.

First, the good news: CFOs do care about the tax benefits of corporate debt, at least moderately. But they seem more concerned about their credit ratings. We know that credit ratings are closely related to interest coverage ratios (interest payments divided by earnings) and are a good proxy for possible financial distress costs. So, managers are cognizant of the basic tradeoff between taxes and financial distress.

CFOs do not recognize our other suggestions. They seem to like financial flexibility (more money, more free cash flow!) and less dilution.

Now for the bad news:

1. Many of our other capital structure arguments seem unimportant to managers, from personal income taxes borne by their shareholders, to expropriation concerns by their creditors, to strategic product market factor considerations, to deliberate control of free cash flow incentives, to intentional signaling of good or bad news (inside information), to transaction cost considerations.

   On the one hand, this may not be as bad as it appears. Managers may still care about these considerations, because their cost of capital itself reflects these considerations. (For example, if their investors face higher tax consequences, it increases the firm' cost of capital, and managers do care about their cost of capital.) On the other hand, if a firm does not need to raise money, it is not clear whether managers compute the appropriate cost of capital and hurdle rates for their projects. If they do not take these factors into consideration when estimating the cost of capital that the market would be charging, they could set too high or too low a project hurdle rate.

2. Managers like "financial flexibility," which means that they like having cash around and having untapped debt capacity for possible future activities. Liking this kind of flexibility makes perfect sense from the manager's perspective—but it also hints that free cash flow is a real problem. Managers seem to primarily like this "flexibility" in order to take over other companies—a move that is often not value-enhancing for their shareholders. With almost no chance of bankruptcy in many Fortune 500 companies, it is unlikely that fear of a cash crunch is the driving concern behind the desire for flexibility.

3. Managers worry about lower earnings-per-share (called **earnings dilution**) if they issue more equity. This makes little sense in itself, because the newly raised funds would presumably also produce earnings.

4. Even managers who claim to target a debt ratio tend not to retire equity if their equity has recently increased in value, or to issue more equity if their equity has recently fallen. This makes little sense, because this is exactly what is required to target a debt ratio.

5. Managers believe that they can time the financial markets.

    - About two-thirds of managers feel that the stock market undervalues their firm—a fact that restrains many from issuing equity. When their stock market values have recently increased, then managers feel that they have a "window of opportunity" for an equity issue. In other words, they believe that they can forecast their stock price, and the stock market's usual pessimism is appropriately corrected.
    - Even more remarkably, CFOs believe that they can time overall market interest rates: they issue more debt when interest rates fall or have fallen.

    Amazingly, although it seems almost absurd to believe that they have this ability, there is some new and actively debated empirical evidence that managers have indeed collectively shown some ability to time the market. To explain such corporate issuing activity *and its success*, it appears that we have to look more towards the field of **behavioral finance.**

A survey on payout policy.

In another survey (by Brav et.al. (2004)) prior to the Bush tax cuts, CFOs generally saw the question of dividends vs. repurchases as one of desirable flexibility—dividends being steady, share repurchases being paid "as available." Other answers mirror those in the Graham and Harvey survey. Here, too, managers pretty much considered personal income taxes on dividends to be fairly irrelevant both to them and to the preferences of their shareholders. CFOs also believed that dividends tended to attract more individual retail shareholders than large institutional tax-exempt investors. If the CFOs are correct, it is investors who are acting irrationally. Once again, this seems like a fruitful area of future research for behavioral finance.

Solve Now!

Q 21.10 *What factors do CFOs claim matters to them?*

Q 21.11 *Are answers from managers "prescriptive," i.e., indicative of what corporations should do?*

21·5. Leverage Ratios By Firm Size, Profitability, and Industry

Do these dynamics cause different firms to evolve different leverage patterns over time? For example, do larger firms end up having higher leverage ratios? This is a more nuanced question than it appears to be at first sight, because you can use different definitions of debt and equity. In broad strokes, you have three choices:

This section has been added late, and needs to be edited.

- You can see debt narrowly in terms of the firm's financial indebtedness (long term debt plus debt in current liabilities), or widely in terms of all liabilities (which includes other liabilities, such as pension and other liabilities, for example; refer back to the previous chapter for more detail).

  In 2003, just about about 50%-60% of the typical firm's total liabilities was financial indebtedness. However, this also differed by firm size. Large firms' total liabilities were mostly financial claims, while small firms' liabilities were mostly elsewhere.
- You can see equity in terms of market value or in terms of book value. Although I prefer the former, the latter is also often used in practice.
- You can compute the leverage ratio by dividing only by the value of the financial claims (the sum of financial debt and financial equity claim) or more broadly by the value of all assets.

This gives you six possible measures, but we shall look only at three:

A financial leverage ratio, which we define as long-term debt plus debt in current liabilities, divided by the firm's financial securities' market value.

A broad market-value based leverage ratio, which we define as total liabilities, divided by the sum of total liabilities plus the *market* value of equity.

A broad book-value based leverage ratio, which we define as total liabilities divided by the bookvalue of assets (the sum of total liabilities plus the book value of equity).

Table 21.3 shows the broad patterns of corporate indebtness ratios in 2003. (When repeated for the year 2001, which was a recession year, the table looks very similar.) Publicly traded firms in the United States had average debt ratios of about 25%, although the typical firm had a median debt ratio of a few percent less (16% and 20%, based on the measure). There is quite a bit of variation across firms—one standard deviation is either 26% or 20%, indicating that debt ratios of 0% or 50% are fairly common, too.

Table 21.3 also shows that the answer to the question of which types of firms are more levered depends on the leverage measure you adopt.

Larger firms tend to have more financial leverage but not more indebtness. More profitable firms tend to be less indebtedness.

By Firm Size Larger firms have higher financial leverage ratios. However, if you look at leverage more broadly, you see that larger firms do not tend to be more indebted—and, indeed, by some measures, seem less indebted.

By Profitability More profitable firms are neither more nor less financially leveraged. However, on a broader indebtness measure, more profitable firms tend to be less indebted.

Table 21.4 splits firms into industries.

By Industry Consumer Goods (Drugs, Soap, Perfumes, Tobacco), Machinery And Business Equipment makers, and Mining And Mineral companies tend to have lower debt ratios (market value based). Utilities, steel, and automobile companies tend to have higher financial leverage ratios. Financial services companies are interesting—they tend to have higher financial debt ratios, but relatively low broader indebtedness ratios. (Of course, these industry definitions are still very broad. It is quite possible that many smaller industries have their own, unique debt ratios.)

In sum, it appears that firm size is a good marker for financial leverage, and profitability is a good marker for a broader indebtness ratio.

Table 21.3. Leverage Ratios by Firm Size and Profitability, in 2003

		Financial			Broad (MV)			Broad (BV)			
		Mean	Mdn	S.D.	Mean	Mdn	S.D.	Mean	Mdn	S.D.	N
	All	25%	17%	26%	22%	16%	20%	26%	21%	20%	4,551
Firm Market Value	– $100m	22%	9%	27%	29%	22%	24%	35%	26%	27%	1,419
	$100m – $500m	18%	5%	24%	20%	13%	19%	23%	19%	16%	1,249
	$500m – $2,500m	19%	14%	21%	18%	13%	16%	22%	19%	14%	1,083
	$2,500m – $10,000m	24%	19%	21%	20%	16%	16%	22%	20%	13%	493
	$10,000m –	21%	17%	18%	19%	15%	14%	24%	23%	12%	307
	Interpretation	Increasing			Decreasing			Weak/Decreasing			
Firm Book Value	– $100m	16%	4%	24%	23%	16%	23%	33%	25%	26%	1,706
	$100m – $500m	22%	12%	25%	20%	14%	19%	23%	20%	14%	1,223
	$500m – $2,500m	31%	27%	24%	21%	15%	18%	21%	19%	13%	950
	$2,500m – $10,000m	34%	31%	23%	24%	20%	18%	23%	20%	13%	426
	$10,000 –	37%	36%	24%	26%	22%	17%	22%	21%	11%	292
	Interpretation	Increasing			Weak/Increasing			Decreasing/Weak			
Profitability Income/Sales	Negative	24%	10%	29%	26%	18%	24%	30%	23%	24%	1,639
	0% – 5%	27%	22%	24%	28%	24%	18%	27%	24%	15%	1,153
	5% – 10%	20%	15%	21%	17%	14%	14%	24%	20%	15%	844
	10% – 15%	25%	15%	25%	12%	9%	10%	20%	16%	13%	400
	15%–	28%	24%	25%	13%	7%	18%	22%	15%	23%	562
	Interpretation	None			Decreasing			Decreasing			

The **Financial** Leverage ratio is long-term debt plus debt in current liabilities divided by the firm's financial securities' value (the market value of equity plus the long-term debt plus debt in current liabilities). The **Broad (MV)** Leverage ratio is firms' total liabilities divided by the market value of equity plus the book value of total liabilities. The **Broad (BV)** Leverage ratio is firms' total liabilities divided by the book value of assets (which are the sum of the book value of equity plus the book value of total liabilities. Mdn is the median, S.D. is the standard deviation.) The industry definitions are originally by Fama and French. The original data came from the Compustat financial data base.

The table shows that larger firms seem to have higher financial leverage ratios, but (weakly) lower broad leverage ratios. More profitable firms tend to have lower broad leverage ratios.

Table 21.4. Leverage Ratios by Industry, in 2003

		Financial			Broad (MV)			Broad (BV)			
		Mean	Mdn	S.D.	Mean	Mdn	S.D.	Mean	Mdn	S.D.	N
1	Food	27%	23%	23%	24%	19%	19%	24%	21%	21%	122
2	Mining and Minerals	15%	6%	6%	13%	7%	7%	16%	12%	12%	94
3	Oil and Petroleum Products	26%	23%	23%	19%	13%	13%	19%	15%	15%	196
4	Textiles, Apparel	25%	16%	16%	27%	19%	19%	24%	21%	21%	89
5	Consumer Durables	27%	18%	18%	31%	22%	22%	27%	23%	23%	119
6	Chemicals	29%	24%	24%	26%	21%	21%	21%	19%	19%	93
7	Drugs, Soap, Prfums, Tobacco	11%	5%	5%	11%	8%	8%	22%	18%	18%	221
8	Construction	31%	25%	25%	32%	25%	25%	27%	23%	23%	106
9	Steel Works Etc	38%	34%	34%	34%	26%	26%	22%	20%	20%	62
10	Fabricated Products	33%	25%	25%	32%	25%	25%	26%	22%	22%	33
11	Machinery and Business Equip.	15%	6%	6%	19%	13%	13%	26%	21%	21%	693
12	Automobiles	29%	21%	21%	33%	28%	28%	29%	26%	26%	65
13	Transportation	35%	30%	30%	29%	22%	22%	25%	22%	22%	169
14	Utilities	47%	47%	47%	29%	25%	25%	17%	15%	15%	133
15	Retail Stores	23%	14%	14%	28%	21%	21%	29%	25%	25%	293
16	Banks, Insurance, Financials	39%	40%	40%	18%	11%	11%	29%	19%	19%	151
17	Other Industries	18%	7%	7%	21%	14%	14%	28%	22%	22%	1,901
–	Industry Unknown	29%	19%	19%	33%	26%	26%	40%	29%	29%	61

See Table 21.3 for a table description. The table shows that financial leverage ratios are low for mining, consumer goods, and machinery; and financial leverage ratios are high among utilities, financials, and steel. There are no strong industry patterns on broader indebtedness measures.

21·6. Perspective

What we know.

It is important that you keep the empirical evidence in proper perspective. We *do* know that our theories can explain at least some of the behavior of corporations. So, we should not dismiss them as determinants of observed capital structure. There is a good chance that further refining of our theories and proxies will explain quite a bit more about how firms behave. We also *do* know that we *do not* know why our theories explain relatively little about the differences in behavior across companies. There is a good chance that there are other systematic factors that we do not yet fully understand (probably in the domain of behavioral finance). There is also a good chance that much corporate behavior is just erratic and will never be explained. We should keep an open mind.

It is important that you know what you do not know.

Why torture you in this chapter with something that we do not fully understand? The reason is that capital structure is an important area, and you must know what we do not yet know! As a manager, you will meet many investment bankers mustering arguments about what other firms have been doing, and offering advice as to what you should do. As an investment banker, you should know not only what factors influence firms' capital structures, but also how important or unimportant individual factors are—and how you can measure them to find new potential clients. As a policy maker, you should know how authoritative the capital structure outcomes and choices of firms really are.

The evidence says nothing about the normative implications of the theories—in fact, it may tell you where there is money to be made.

But perhaps most importantly, the empirical evidence here does not suggest that our theories are worthless. For example, does our empirical evidence mean that just because other firms do not exploit the corporate income tax advantage of debt, that you should ignore it, too? Absolutely not! You can still think about how important a corporate income tax advantage is to *your* firm, and what this means for *your* optimal capital structure. Perhaps more important—if many firms are ignoring the factors that they should pay attention to, then over time some will end up with very poor capital structures. In this case, you can think about how you can come in and change these existing firms to increase their value. You can effect change from many different directions. You can work in the firm itself and argue for a capital structure change. You can become an investment banker and advise clients on better capital structures. Or, you can even buy some companies. It has been almost twenty years since there was a wave of "leveraged buyouts," in which many public firms were taken over and restructured to generate value—and much, if not most, of the value was created through better capital structures. Maybe you will start the next wave of takeovers!

21·7. Summary

This chapter covered the following major points:

- We can explore both the mechanisms of capital structure change and the underlying forces (causes). These causes can themselves work through multiple mechanisms.
- Over a five-year horizon, the two most important mechanisms affecting capital structure are stock returns and long-term debt issuing activity. Both can explain about 40% of the changes in debt-equity ratios.
- Long-term debt can explain about 30% of the changes in debt-equity ratios, short-term debt and equity issuing can both explain about 15%, and both convertible debt and payout policy can explain less than 5%.
- Among the Fortune-100 firms, seasoned equity offerings are rare, and appear almost always in the context of acquisitions. (Executive compensation is remarkably high, and about as important as share repurchasing activity.)
- We know a number of statistically significant forces (potential causes), but they can explain only a very small percentage of capital structure dynamics. Among the more important influences are
  - Stock returns.
  - A reluctance to issue equity.
  - A desire to imitate industry peers.
  - Corporate income taxes.
  - Accounting performance, such as profitability.
  - M&A activity.
  - Financial distress.
  - Market timing.
  - Uncertainty.
- Although dividends and repurchases have been fairly modest in acting as capital structure channels, we understand them relatively well. They are both about equally important. Their differences mattered more in the past, before the double taxation of dividends was reduced in 2003. Among the remaining differences are that managers can participate in receiving cash from dividends, but cannot tender into share repurchases, that managers with unadjusted stock option plans prefer repurchases to dividends, and that some investors seem to "like" dividends.
- The most important remaining difference between dividends and share repurchases today is that dividends tend to be more persistent ("stickier") than share repurchases.
- Firms tend to pay out dividends when they have retained earnings. That is, they do not typically finance dividends through other capital markets activity, but through operations.
- In surveys, CFOs claim to be very concerned about their credit ratings and financial flexibility. Together with often largely untapped debt capacity, these findings can be evidence of significant free cash flow problems. CFOs also claim not to care about taxes borne by their investors or many other factors suggested by the theories, but they do believe that they can "time" the market.
- Larger firms tend to have higher financial leverage ratios, but lower total indebtedness ratios. More profitable firms tend to have lower total indebtedness ratios.
- Even if firms do not seem to act according to the theories, the capital structure theories still offer good guidance about how you can add value by doing things differently.

A. Appendix: A List of Some Recent Empirical Capital-Structure Related Publications

Unlike many other subjects of our book—where our knowledge has solidified over several decades—empirical capital structure remains a largely unresolved but actively researched area. Much of what we know appears in recently published or working papers, and remains fluid. My summary in this chapter is my *own* subjective reading thereof.

To allow you to make up your own mind, we will now break our rule that references are not in the book but only on the Web site. Here is a short list of papers published after the turn of the millenium. These papers will in turn reference many related, older, but equally (or possibly more) interesting and relevant papers.

- Franklin Allen and Roni Michaely, 2003. "Payout Policy." North-Holland Handbook of Economics, ed. Constantinides, Harris, and Stulz.
- Malcolm Baker and Jeffrey Wurgler, 2002. "Market Timing and Capital Structure." The Journal of Finance 57–1: p.1–32.
- Alon Brav, and John R. Graham and Campbell R. Harvey and Roni Michaely, 2004, "Payout Policy in the 21st century." Working Paper, June 2004.
- Raj Chetty, and Emmanuel Saez, 2004, "Do dividend payments respond to taxes? Preliminary evidence from the 2003 Dividend Tax Cut." Working Paper, UC/Berkeley and NBER, 2004.
- Eugene F. Fama and Kenneth French, 2004, "Financing Decisions" (check title), Working Paper, University of Chicago and Dartmouth, May 2004.
- John R. Graham, 2003. "Taxes and Corporate Finance: A Review." Review of Financial Studies 16: p.1074–1129.
- John R. Graham and Campbell R. Harvey, 2001. "The Theory and Practice of Corporate Finance: Evidence from the Field." Journal of Financial Economics 60: p.187–243.
- Armen Hovakimian, Timothy C. Opler, and Sheridan Titman, 2001. "The Debt-Equity Choice." Journal of Financial and Quantitative Analysis 36: p.1–24.
- Brandon Julio and David L. Ikenberry, 2004. "Reappearing Dividends." Working Paper, UIUC.
- Mark T. Leary and Michael R. Roberts, 2004. "Do Firms Rebalance Their Capital Structures?" The Journal of Finance, forthcoming.
- Peter MacKay and Gordon M. Philips, 2004. "How Does Industry Affect Firm Financial Structure?" Review of Financial Studies, forthcoming.
- Ivo Welch, 2004. "Capital Structure and Stock Returns." Journal of Political Economy 112-1: p.106–131.

Solutions and Exercises

1. Debt issuing and repurchasing activity, and the direct influence of stock returns can each account for about 40% of the variation in debt-equity ratios. Long-Term debt net issuing can account for about 30%, and short-term debt and equity net issuing can account for about 15%.
2. Non M&A related seasoned equity issuing activity is trivial in magnitude.
3. On the contrary. It may even mean that there is a lot of money left on the table by managers that have not optimized their capital structures.
4. See Page 543.
5. It is pretty modest.
6. Firms with high tax obligations usually have low equity ratios, because they were highly profitable. These firms have low debt ratios.
7. No—the theories tell us what should matter. There are a number of explanations why they may not translate into observed corporate behavior.
8. The capital gains can be realized by investors who have little capital gains tax.
9. Executives and insiders may often not tender into a repurchase, but will enjoy the relatively higher share price from a repurchase through executive compensation that is linked to the shareprice. Dividends tend to be more regular than share repurchases. Some retail investors like dividends.
10. Taxes, credit ratings, financial flexibility, and earnings dilution.
11. No. Managers are conflicted. They do not maximize shareholder wealth, but their own welfare.

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

CHAPTER 22

Financial Market Responses to Capital Structure Changes

What we Know!

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We now turn to the empirical evidence of how the financial markets react to the announcements of managerial capital structure actions. In particular, we want to explore how the stock market responds to new equity issues, new debt issues, and dividend payments. Our goal is to understand better the value consequences of such managerial actions, which in turn will help us to understand better why managers are doing what they are doing.

22·1. Value Changes at Announcements (Event Studies)

Event studies can tell us whether some actions are value increasing or value decreasing...

We will use the event study technique, introduced in Chapter 15, to explore stock market responses to capital structure announcements. As you recall, an event study measures the stock returns of firms from just before an event disclosure to just after it, and attributes the value change to the event. It is a powerful technique, but not a perfect one. To run an event study, you should be aware of the following issues.

- We must to be able to clearly identify when the market learns of the event. Fortunately, we can identify event dates for three of the most important corporate finance events, which are the subject of our chapter: seasoned equity offerings, debt offerings, and dividend declarations. That is, we can usually ascertain quite precisely when firms announce that they will conduct these activities. Offerings must be filed with the SEC, and dividends are usually declared during an identifiable board meeting. To avoid insider trading regulations, firms are also usually careful not to disclose their intent earlier.

  Unfortunately, there are also many events that might have been interesting but for which we cannot find a specific event date—and so we cannot use the event study technique for them. For example, the "non-announcement" of a dividend has no event date. Employee compensation related share issuing and share repurchasing are usually done gradually and so have no sharp event date, either.

- We must isolate the events of interest from other important simultaneous announcements. For example, we know that M&A and capital structure activity can occur together. The simultaneity can make it difficult to disentangle whether such a stock price change is in response to the acquisition or the equity issue. If an acquisition has a positive value impact and an equity offering has a negative value impact, we might even attribute the incorrect sign to an event.

  However, it may not be all that bad, because acquisition announcements often occur *before* the acquirer files with the SEC for the new equity financing. If the acquisition has already been disclosed earlier, it is not news, which thus would not contaminate our event date. But, it could mean that the market had already expected an equity offering, in which case the response would be biased towards zero—the offering would be no news.

- We must have enough observations to overcome the general noise in stock returns. Fortunately, for each of our three events, we have several thousand event observations over several years. This is enough to isolate the signal from the background noise in common stock returns, at least for an overall picture. The drawback is that if we want to find the response for a small biotech firm involved in DNA technology, the subset of similar firms may be much smaller. This makes it harder to judge how our specific company will react, based on the responses of comparable companies.

In sum, even though these three requirements are indeed limiting, they do not prevent us from exploring in broad strokes how financial markets respond to our three events—seasoned equity offerings, debt offerings, and dividend declarations.

22·2. Equity Issuing

Our first topic is seasoned equity offerings. Note that all that happens on our event date is that the firm files its *intent* to issue equity shares *later*. The shares themselves will be sold much later.

22·2.A. The Average Response

Equity offerings are bad news, on average.

How does the stock price react when a firm files with the SEC for a seasoned equity issue? Table 22.1 gives the answer: the average firm underperforms the stock market by a total of about $0.994 + 0.630 \approx 1.6\%$ on the two days around its announcement. (We do not know whether the filing occurred after or before the stock market had closed on day 0, so we must add the two surrounding stock returns.) Thus, our empirical evidence suggests that it is bad news, on average, when firms announce that they are selling more equity.

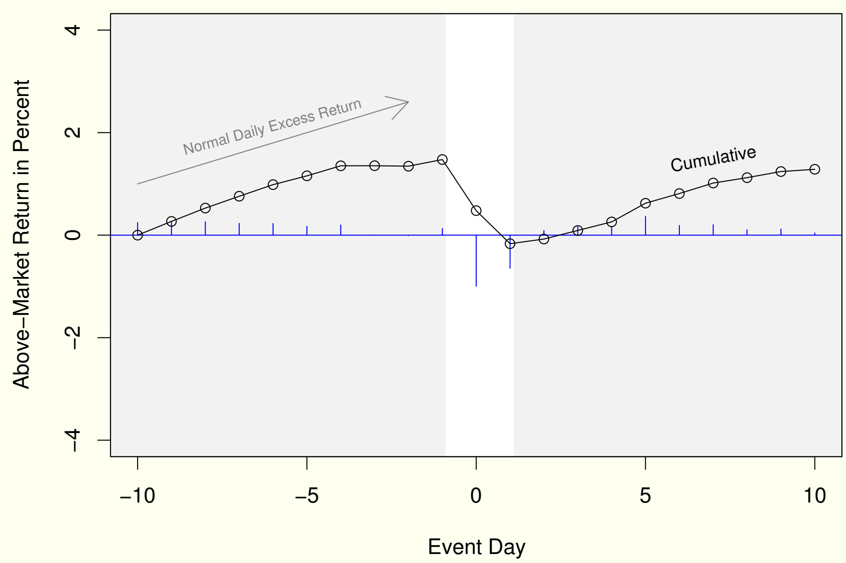
Table 22.1. All Equity Filing Date Reactions, 1990–2000

Event Day	Mean	Std.Dev.	(T-stat)	Observations
–10	0.246	3.81	(5.0)	5,990
–9	0.264	3.67	(5.6)	5,992
–8	0.260	3.78	(5.3)	5,992
–7	0.231	3.59	(5.0)	5,993
–6	0.225	3.81	(4.6)	5,993
–5	0.176	3.66	(3.7)	5,994
–4	0.197	3.89	(3.9)	5,994
–3	–0.001	3.53	(–0.0)	5,994
–2	–0.010	3.55	(–0.2)	5,993
–1	0.126	3.52	(2.8)	5,997
±**0**	–0.994	4.14	(–18.6)	5,997
+1	–0.630	4.52	(–10.8)	6,004
+2	0.093	3.68	(2.0)	6,010
+3	0.163	3.82	(3.3)	6,015
+4	0.166	3.53	(3.6)	6,020
+5	0.365	3.70	(7.6)	6,023
+6	0.185	3.44	(4.2)	6,025
+7	0.200	3.51	(4.4)	6,028
+8	0.106	3.93	(2.1)	6,029
+9	0.123	3.65	(2.6)	6,036
+10	0.045	3.60	(1.0)	6,038

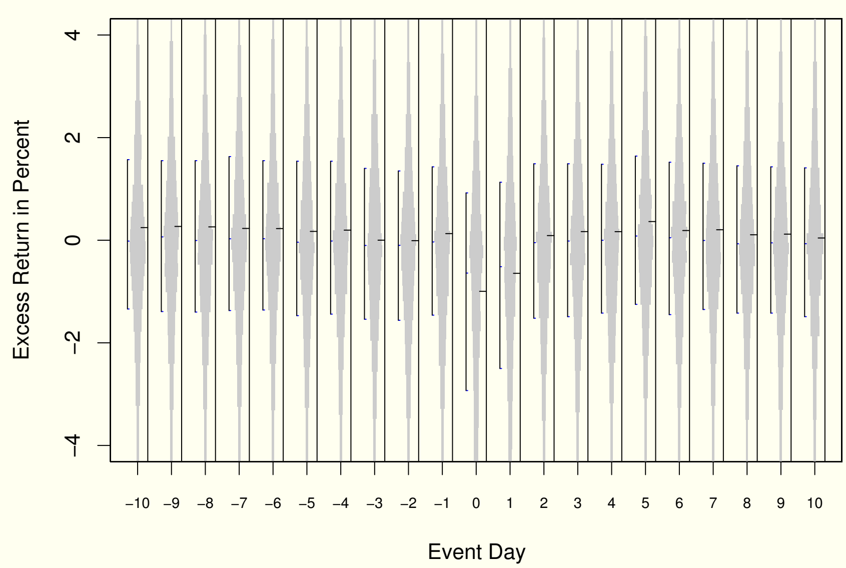
These are statistics of stock returns on an equal-weighted portfolio of event firms, net of the value-weighted stock market rate of return, and quoted in percent. The returns are *not* annualized. In other words, on average, a $100 stock underperformed the market by $0.99 on the announcement day and $0.63 on the following day.

Side Note: The positive average stock return on non-event dates can be explained by the fact that our equity issuers likely have higher risk (market-beta), and thus have to offer expected rates of return that are above the market's. If we adjusted the event date return benchmark for this higher risk—perhaps an extra 20 basis points per day—we would consider the underperformance to be about 1.8%, rather than 1.6%.

Figure 22.1. All Equity Filing Date Reactions, 1990–2000



This graph shows the mean rate of return (above the value-weighted market rate of return), as in Table 22.1. The blue histogram bars are the daily average returns. The connected line adds them up and is called the *cumulative* excess rate of return. This particular sample of firms likely had high betas, meaning that investors expected returns higher than the market. The graph clearly shows the large underperformance at the event days.



This graph shows the same data, but gives more perspective. It plots *all* stock returns on each day, not just the overall daily means that hide its variability. Each bracket on the right represents mean plus or minus one standard deviation; each bracket on the left represents the median and the interquartile range (beginning at the first quartile of the data and ending at the third quartile).

Economic Significance

The 1.6% value loss is highly statistically significant. But how important is it economically?

The dilution is large!

- On the one hand, 1.6% is more important than it seems at first glance. A \$100 firm that announces that it will issue equity drops by about \$1.60. But the typical equity issue is only about 20% as big as the firm's outstanding equity, so the typical issue would be about \$20. This means that about $\$1.60/\$20 \approx 8\%$ of the newly raised money is immediately lost. This effect is often called **offering dilution**—although the term **dilution** is also commonly applied to the fact that the issuance of new shares reduces the fraction of the firm held by previous shareholders.

  Average dilution is very high!

  Because of the statistical properties of ratios, the mean dilution is much higher than 8%. To illustrate why, compare the ratios of means against the means of ratios in a small data set.

Obs	X	Y	X/Y	Y/X
A	10	100	0.10	10
B	10	200	0.05	20
C	20	100	0.20	5
D	20	200	0.10	10
Mean	15	150	0.1125	11.25

  The ratios of the means are 0.1 and 10, while the means of the ratios are 0.1125 and 11.25, respectively, in our small data set. This effect also works in our dilution sample. The average dilution in our sample is about 13.6%, not just 8%. Clearly, an immediate destruction of wealth equal to 13.6% of the equity being raised is *not* small. Other studies have found even larger average dilution effects—as high as 50% in some studies.

  Another win for the pecking order view of financing.

  This empirical evidence can explain why we see a pecking order, as discussed earlier. Given how large the dilution can be, it seems rational for firms to try to avoid issuing equity.

  There is much heterogeneity in response!

- On the other hand, you should also not overestimate the importance and general applicability of the 1.6% average value loss. Figure 22.1 shows how easy it is to get the wrong impression from the average stock return loss. Graph (A) shows the mean rates of return on each day. The graph clearly shows that the announcement of a seasoned equity offering is *not* welcome news for shareholders *on average*. Graph (B) shows practically the same data, except it represents the returns from all equity offerings, not just the mean returns. Although it is still evident that the return on event days 0 and 1 is lower than that on other days, there is still a lot of heterogeneity—many firms experience *positive* announcement reactions on the days that they announce their seasoned equity offerings. The 1.6% loss is highly economically and statistically significant, but for many firms, it is either drowned out by the daily stock market noise or positive because the offering adds value. So the evidence is *not* that *all* firms suffer stock declines when they issue equity, but only that the *average* firm declines in value. It is perfectly consistent with this evidence for you to believe that your own firm has too much debt, and that you *should* issue equity in order to raise the firm's value. Related to this heterogeneity in how firms react is the fact that the median dilution is "only" 2.4% of the new equity issue. That is, even though the mean dilution was 13.6%, half of our firms had dilution of less than 2.4%.

Table 22.2. Two-Day Equity Issue Announcement Reactions

	By "Economic Size"				
	Smallest				Largest
Market Cap Itself (in bill-$)	0.2	0.4	0.6	1.2	4.1
Reaction By Market Cap	–1.29	–1.60	–1.74	–1.93	–1.64
Offer Size/Market Cap Itself	5.4%	11.6%	18.0%	26.5%	47.5% (81%)
Reaction By Offer Size/Market Cap	–1.06	–1.69	–1.89	-2.20	–1.37

By "Issue Year"

Year	≤1989	1990	1991	1992	1993	1994
Reaction	–1.93	–1.89	–1.38	–1.77	–1.93	–1.55
Year		1995	1996	1997	1998	1999
Reaction		–1.17	–1.13	–1.40	–1.84	–1.66
Year		2000	2001	2002	2003	
Reaction		–1.69	–1.91	–2.02	–2.00	

The dependent, classified variable is the sum of the announcement day stock returns on day 0 and on day 1, quoted in percent. The average two day announcement reaction was –1.624%. The "economic size" (market capitalization and relative offer size) classifications are based on ranks *within* the calendar year of the SEO. In my opinion, there is no clear pattern in these data suggesting that larger or smaller firms (or larger or smaller relative issues) experience different stock market responses.

22·2.B. The Cross-sectional Evidence

Smaller firms may or may not be punished more when they issue equity.

Are we able to identify which firms suffer more when they issue equity? For example, do smaller firms experience more of an announcement decline than larger firms? To answer such questions, we can add up the event returns on days 0 and 1 for each firm, and call this the financial market's event reaction. Then we see whether the average announcement drop is different for larger firms than it is for smaller firms. Table 22.2 shows the results.

Firm Size First, we indeed ask whether smaller firms experience different stock market responses than larger firms. We see good dispersion in the types of firms that issue equity, so we are able to address this question. The smallest quintile of our firms are worth around $200 million, on average; our largest quintile are worth around $4.1 billion. But we do not see a clear pattern in the groups' announcement returns. The largest firms experienced a little large drop (−1.64%), but so do the next three quintiles. Only the quintile of smallest firms seemed to experience less of a drop (−1.29%), but this difference is modest. Thus, we can conclude that investors of small and large firms respond in a similar negative manner to SEO announcements.

No obvious systematic return difference based on the relative offering size or year of issue.

Offer Size Next, we ask whether firms that issued more new equity relative to their already outstanding market equity value were punished more by the market. Our quintile of firms with the smallest relative SEO size issued about 5% of the outstanding equity, while the quintile with the largest relative SEO size issued about 50% of the outstanding equity. The middle quintile sees a relative increase in firm size of about 18%. But, more important from our perspective, we do not see a clear relationship between relative offer size and filing date stock price drop—contrary to what one might have expected before looking at these data.

No obvious change in reaction over the years.

Time Changes Finally, we ask whether the market's punishment has changed over the years. Table 22.2 shows that the announcement price drop was a little less in the mid-nineties, but has increased again. There is no clear time trend.

Summary

In sum, although we know that the market responds negatively to the announcement of an equity issue, this response does not seem to vary systematically by firm size, issue size, or year of issue.

22·2.C. Earlier Studies

Older studies were better, but the data are a bit dated. The findings were mostly but not entirely similar.

Of course, the above tables are not the first event study of equity issuing announcements. In the mid 1980s, equity issuing announcement reactions were a prominent area of academic research. Their samples are a little dated by now, which is why we had to conduct our own study above—but the academic studies were done much better and in much more depth. For example, many of these studies carefully excluded offerings that were filed on days of other announcements (like M&A). This research generally found similar announcement reactions as those reported above. The typical equity offering by an industrial company elicited a −3% announcement response; the typical equity offering by a utility company elicited a −0.75% announcement response. (Utility companies are regulated and are forced to issue equity fairly regularly.) Put together, their average abnormal response was about −1.6%, which is almost too eerily similar to what we found for our sample period. But there is an important difference in one finding: these studies suggested much higher offering dilution than our 8%. Their offerings were smaller, but they found that between 30% and 50% of the equity issue value was dissipated on the announcement day. It is not clear where the difference comes from.

Equity cancellation is good. With a previous runup, an equity offering is less bad.

This research also uncovered a host of other interesting relationships. Let's just look at two.

1. Firms that experienced a bigger announcement drop upon announcement were more likely to cancel their equity offering. The cancellation itself was good news—but not good enough. Net in net, such companies still ended up worse than if they had never announced anything in the first place.

2. Firms that experienced large positive runups prior to the offering and then announced an equity offering dropped less. That is, they were punished *less* by the market. This gives more credence to the "market timing" explanation for issuing (from the previous section). It would make sense if managers would be more inclined to issue equity if the penalty is less.

> **DIGGING DEEPER**: *A good starting point to learn more about the existing academic evidence, on which this description is based, is the 1986 symposium issue of the Journal of Financial Economics.*

Equity issuers also underperform in the long run, especially those that were very aggressive prior to the offering.

Some more recent research has examined how stock prices react not just on the *day*, but over the *years* following a seasoned equity offering. The answers here are less robust, because we are not really certain as to what the appropriate multi-year stock return should be—neither the CAPM nor any other model that we know is reliable enough to tell us what annual returns should be in the absence of the issue. Still, the preliminary evidence suggests that the average abnormal multi-year return is negative, perhaps as large as 5% per annum over four years. Even more interesting, firms that are unusually aggressive in managing their earnings prior to the SEO tend to suffer significantly greater post-equity offering value declines in the years thereafter—an Enron effect, perhaps?

22·2.D. Theoretical Perspective

We shall assume that stock price response is equal to overall value response.

Does our empirical evidence tell us that equity issues reduce firm value? Not necessarily. We only know that equity issues are bad news for *stockholders*. If they were great news for bondholders, then it could be that the overall firm value increased rather than decreased. The academic literature provides just a few studies of how debt prices respond to equity issues. This is primarily because most outstanding debt rarely changes hands, and when it is traded, it is usually not publicly traded but "over-the-counter." Thus, researchers cannot learn the trade prices. (The investment banks that are the "counter" consider pricing information on bonds their competitive advantage—and for good reasons.) Fortunately, what little empirical evidence has surfaced in this context and in the context of other events indicates that it is unlikely that bonds would change much in value when the firm issues equity. Especially highly rated debt was probably close to risk-free before the equity issue and would remain so after the equity issue, which would imply that the debt value would not change much upon the announcement. So, we can make a leap to equate that stock value changes represent most of the overall value change, and therefore we believe that when the stock price decreases, so does the overall firm value.

This average reaction is consistent with a number of theories (forces).

With this leap, how does our empirical evidence accord with our theories? (If you do not remember the theories, look back at Table 19.7 on Page 506.) Our evidence of an equity issue announcement drop suggests that any positive effects from reducing personal income taxes, from reducing financial distress costs, and from reducing debt expropriation among issuers were outweighed by equity's negative effects. These negative effects can include the extra additional corporate income taxes (due to tilting away from debt), the inside information problem (managers are not confident about the future, and the market learns this), and the agency problems (managers will have more money to waste). The empirical evidence further suggests that reluctance of managers to issue equity makes sense—it is support for any form of the pecking order hypothesis.

22·3. Debt Issuing

We now turn to the second significant corporate finance event, debt issuing. How does the stock price react when a firm announces an impending debt issue?

22·3.A. The Average Response

The drop upon a debt issue announcement is small or nonexistent.

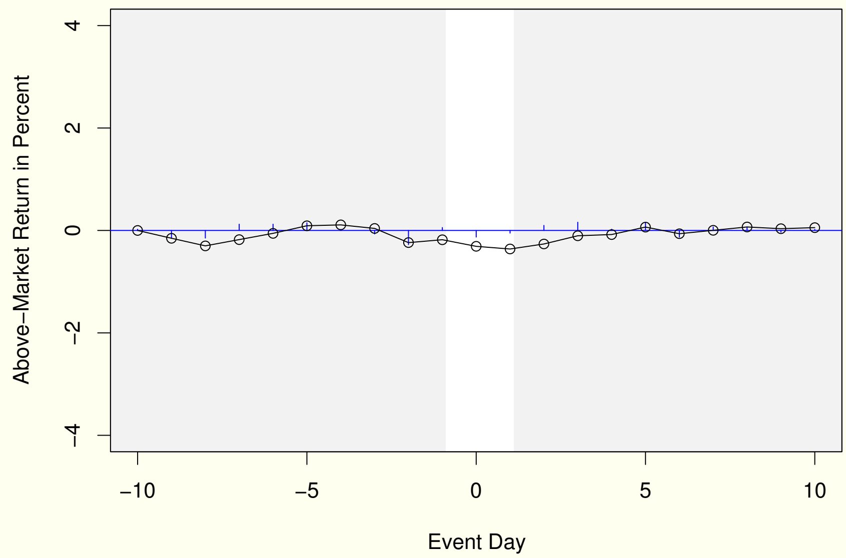
Table 22.3 shows that on the two event days surrounding the announcement of a debt issue, the stock price drop is about 18 basis points—an order of magnitude lower than that of the equivalent equity announcement drop. Further, the table shows that there are many days before and after the debt announcement that have larger stock price responses. Figure 22.2 gives even more perspective by showing how noisy and small the excess returns on the event days are relative to the mean reaction. So I personally judge this evidence to mean that, on average, a debt issue is either a very, very, very mildly negative event, or merely a "no-event" event.

Table 22.3. All Debt Announcement Reactions, 1990–2000

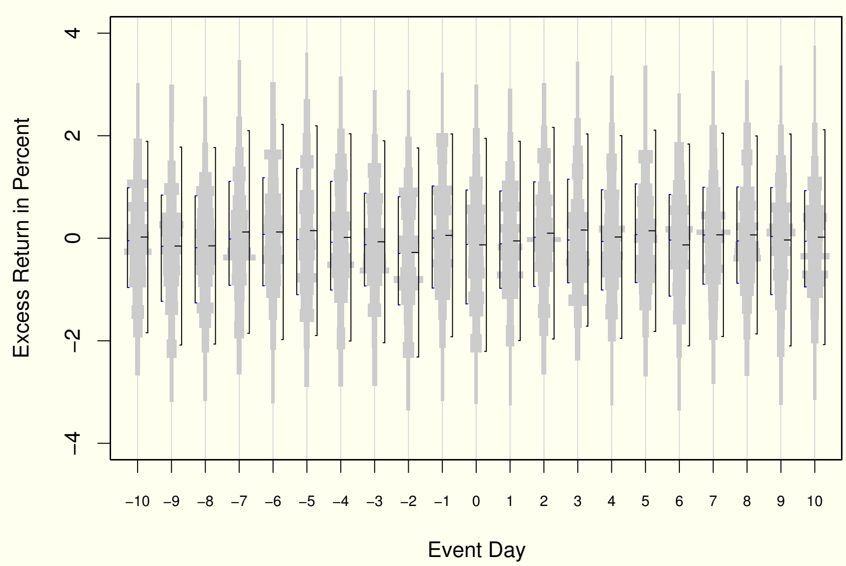
Event Day	Mean	Std.Dev.	(T-stat)	Observations
-10	0.024	1.87	(2.1)	28,422
-9	-0.157	1.93	(-13.3)	28,422
-8	-0.148	1.92	(-13.1)	28,421
-7	0.122	1.98	(10.4)	28,424
-6	0.122	2.10	(9.8)	28,426
-5	0.148	2.04	(12.2)	28,434
-4	0.017	2.02	(1.4)	28,425
-3	-0.069	1.97	(-5.9)	28,413
-2	-0.277	2.04	(-22.9)	28,433
-1	0.056	1.98	(4.8)	28,433
±**0**	-0.129	2.07	(-10.5)	28,440
+1	-0.052	1.94	(-4.5)	28,442
+2	0.098	2.06	(8.0)	28,452
+3	0.160	1.88	(14.4)	28,455
+4	0.024	1.98	(2.1)	28,446
+5	0.146	1.96	(12.5)	28,441
+6	-0.130	1.97	(-11.1)	28,441
+7	0.067	1.99	(5.7)	28,442
+8	0.065	1.93	(5.7)	28,442
+9	-0.034	2.07	(-2.7)	28,441
+10	0.021	2.10	(1.7)	28,440

These are statistics of stock returns on an equal-weighted portfolio of event firms, net of the value-weighted stock market rate of return, and quoted in percent. The returns are *not* annualized. In other words, a $100 stock on average underperformed the market by 18 cents on the two days around the announcement.

Figure 22.2. All Debt Filing Date Reactions, 1990–2000



This graph shows the mean rate of return (above the value-weighted market rate of return), as in Table 22.3. The blue histogram bars are the daily average returns. The connected line adds them up and is called the *cumulative* excess rate of return. Keeping the same scales as those in Figure 22.1 shows how much milder the announcement response to a debt offering is when compared to an equity offering.



This graph shows the same data, but gives more perspective. It plots *all* stock returns on each day, not just the overall daily means that hide its variability. Each bracket on the right represents mean plus or minus one standard deviation; each bracket on the left represents the median and the interquartile range (beginning at the first quartile of the data and ending at the third quartile).

Table 22.4. Two-Day Debt Issue Announcement Reactions

	Mean by Market Value				
	Smallest				Largest
Market Cap Itself (in bill-$)	0.8	2.8	7.5	17.0	47.7
By Market Capitalization	−0.37	−0.09	+0.02	−0.27	−0.19
Offer Size/Market Cap Itself	0.1%	0.5%	1.3%	3.9%	20.6% (71%)
By Offer Size/Market Cap	−0.21	−0.15	−0.12	−0.16	−0.28

Year	≤1989	1990	1991	1992	1993	1994
Reaction	-0.21	−0.73	−0.50	−0.44	−0.19	+0.10
Year		1995	1996	1997	1998	1999
Reaction		−0.36	−0.09	+0.10	−0.70	−0.54
Year		2000	2001	2002	2003	
Reaction		+0.30	+0.15	+0.27	−0.36	

Includes about 1200 Convertible Securities out of 28,000 security offerings. The first two classifications are based on ranks within the year of issue. The dependent, classified variable is the sum of the announcement day stock return on day 0 and on day 1. The average announcement response is -0.18%.

22·3.B. The Cross-sectional Evidence

Smaller firms react no differently from bigger firms.

Does the −0.17% average necessarily mean that there are no firms that have large negative or large positive responses? No! It could be that some firms have a very negative response and other firms have a very positive response. In Table 22.4, we check whether we can identify which firms perform more positively or negatively. We want to see whether larger or smaller firms respond differently when they announce a debt issue. The first row shows that there is no evidence to support such a conjecture. All firm size quintiles have announcement returns that are very small, and there is no consistent relation in how the market responds across quintiles. The next two rows look at whether the relative issue size matters, and again the answer is negative. Even the biggest issuers of debt have an announcement price reaction that is only 7 basis points different from the smallest issuers of debt—which is not an economically meaningful difference.

22·3.C. Earlier Studies

In contrast to equity issuing activity, debt issuing activity has not seen much study. The few academic studies of debt issue announcements have tended to document similar results: the market reaction is either small or nothing—and definitely less significant than the market response to an equity issue. There are also no known strong corporate determinants of when debt issues do seem to solicit a strong value response.

22·3.D. Theoretical Perspective

By itself, the average effect to a debt issue tells us little.

Because the evidence states that it is mostly neither good nor bad news when the firm issues debt, by itself, the debt issue evidence has little to say about the theories from Table 19.7. Net-in-net, the total response from the good aspects of debt seems to weigh about equally with the total response from the bad aspects of debt.

One preliminary conclusion: Its not just that we learn that managers need more money.

But, together with the earlier evidence on the negative response to equity issues, we can draw some interesting conclusions. If the market just disliked learning that managers raise money, we would have expected an equally negative reaction to debt and equity announcements. The fact that the average stock price response is much more negative for equity issues than for debt issues suggests that the cause of the bad response is not just that investors learn about a shortfall in existing projects, which has forced managers to raise more money. If that had been the case, a debt issue should have elicited as negative a response as an equity issue. Instead, the market reactions indicate that either investors learn that managers are less optimistic about the future, or that investors fear that it is the unrestricted nature of equity cash that allows managers to waste money, as opposed to the restricted nature of debt cash with its imposition of future discipline.

Cash to managers is bad news; a higher debt ratio is good news. This may explain why a debt issue is a net zero.

Other studies have explored the market reaction to increases in the firm's debt ratio in a much more clever way. Debt issues and equity issues do two things simultaneously: they can change the existing capital structure towards more debt or more equity, and they bring money into the company. How can we determine whether it is the change in the capital structure that matters, or whether it is the inflow/outflow of cash into the company that matters? The best way is to look for events in which the company has either kept the capital structure constant while issuing (i.e., issuing debt and equity in the same proportion as the existing capital structure), or kept the money in the company constant while changing its capital structure (i.e., issuing debt to retire the same amount of equity, or vice-versa). As we noted in Table 20.6, though rare, such debt-for-equity or equity-for-debt exchanges have occurred. Managers who conduct these exchange offers are definitely not conveying information that they need more or less money, i.e., whether their *existing* projects have fallen short. The empirical evidence that we learn from such exchanges is clear-cut. Firms that announce that they will soon borrow more to retire outstanding common stock experience an immediate 10% to 20% increase in the value of their outstanding stock. Firms that announce that they will soon issue common stock to retire outstanding debt experience an immediate 5% to 10% loss in the value of their outstanding stock. For firms undertaking exchanges, moving towards a higher debt-ratio created value; moving towards a lower debt-ratio destroyed value.

Learning more from the evidence.

Now put our evidence together. We know that there is a positive effect when the firm increases its debt ratio, holding constant cash inflows. We know that there is a negative effect when the firm decreases its debt ratio, holding constant cash inflows. We know that there is a zero overall effect if the firm issues debt. And we know that there is a negative overall effect if the firm issues equity. We can therefore conclude that for debt offerings, the positive signal from the increase in the firm's debt ratio must be roughly equal to the negative signal from more money in the hands of managers; while for equity offerings, both effects are pointing in the same direction.

22·4. Dividend Payment

Finally, we can also look at a payout event—we repeat our analysis with dividend announcements. How is the stock price affected by the declaration of a dividend? Again, we look at the announcement reaction at the declaration date, not the reaction at the actual dividend payment date.

22·4.A. The Average Response

Table 22.3 shows that a dividend declaration is associated with a positive announcement price response of about +0.25%. The magnitude of this response is actually meaningfully large—and surprisingly so. Dividends are usually much smaller than equity or debt issues. (Think of this as the flip side of dilution in the context of an equity offering.) Moreover, because they are so frequent, dividend payments are also more anticipated by investors in the stock market—a fact implying that our measured event study price announcement response of 0.25% is likely below the true total dividend market response. Given these two mitigating factors, we would have expected a smaller announcement reaction.

Dividend announcements are good news—on average, though not for each and every case.

But Figure 22.3 shows again that this applies only for the average dividend, not for each and every one. There is enormous variation in response to dividend announcements. Almost as many firms experience share price declines on the day of the announcements as firms experience increases.

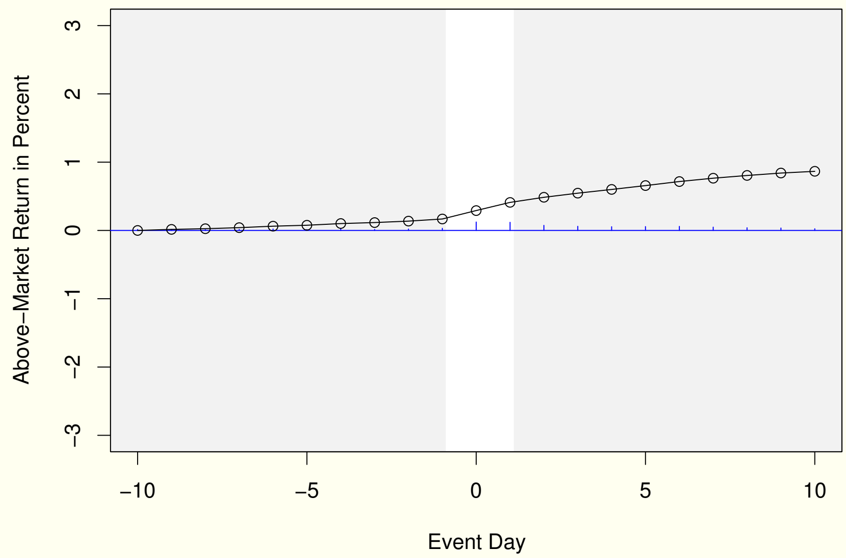
There is again large variation!

Table 22.5. All Ordinary Dividend Announcement Reactions, 1980–2000.

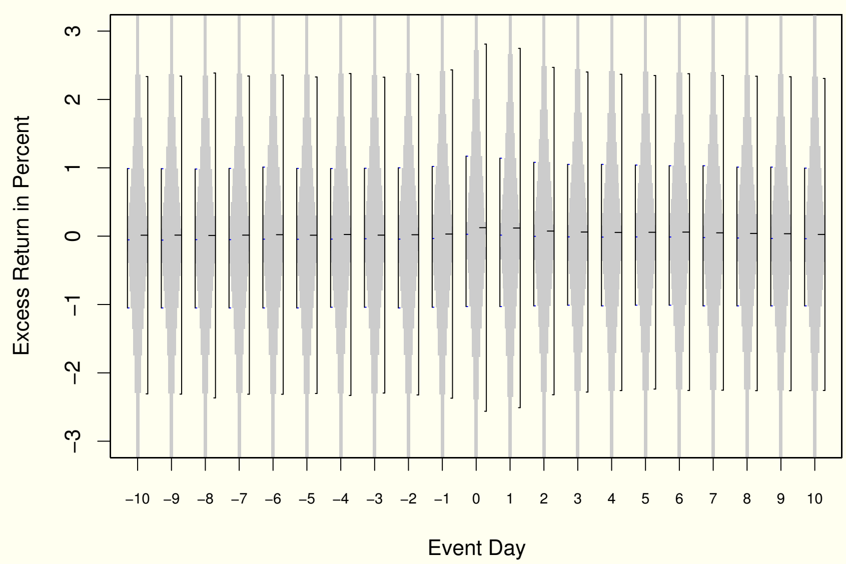
Event Day	Mean	Std.Dev.	(T-stat)	Observations
−10	0.014	2.32	(2.8)	205,693
−9	0.015	2.33	(3.0)	205,754
−8	0.010	2.38	(2.0)	205,805
−7	0.015	2.33	(3.0)	205,829
−6	0.021	2.34	(4.2)	205,870
−5	0.014	2.32	(2.7)	205,906
−4	0.024	2.36	(4.7)	205,954
−3	0.015	2.31	(3.0)	205,969
−2	0.021	2.34	(4.1)	206,012
−1	0.031	2.40	(5.9)	206,036
±**0**	0.124	2.69	(20.9)	206,071
+1	0.119	2.63	(20.6)	206,100
+2	0.075	2.40	(14.1)	206,111
+3	0.060	2.34	(11.7)	206,111
+4	0.054	2.31	(10.7)	206,132
+5	0.057	2.29	(11.2)	206,134
+6	0.059	2.32	(11.7)	206,129
+7	0.049	2.30	(9.6)	206,130
+8	0.040	2.30	(7.8)	206,113
+9	0.036	2.30	(7.1)	206,078
+10	0.025	2.28	(5.0)	206,029

These are statistics of stock returns on an equal-weighted portfolio of event firms, net of the value-weighted stock market rate of return, and quoted in percent. The returns are *not* annualized. In other words, a $100 stock on average outperformed the market by about 24 cents on the two days around the dividend declaration.

Figure 22.3. All Dividend Filing Date Reactions, 1990–2000



This graph shows the mean rate of return (above the value-weighted market rate of return), as in Table 22.5. The blue histogram bars are the daily average returns. The connected line adds them up and is called the *cumulative* excess rate of return. Unlike debt and equity issues, dividends occur fairly regularly, so even a smaller response can be economically more important. Strangely, in our sample, the market seems to have taken a few days to digest the information—the average returns after the announcement were systematically higher than those before the announcement.



This graph shows the same data, but gives more perspective. It plots *all* stock returns on each day, not just the overall daily means that hide its variability. Each bracket on the right represents mean plus or minus one standard deviation; each bracket on the left represents the median and the interquartile range (beginning at the first quartile of the data and ending at the third quartile).

22·4.B. The Cross-sectional Evidence

Finally we find that the market responds differently to announcements based on characteristic!

For both debt and equity offering announcements, we failed to find evidence that the market responded differently to different firms. Fortunately, Table 22.6 shows that we can finally report some real differences across firms! First, small firms (with an average market value of $30 million) experience a stronger dividend announcement reaction than large firms (with an average market capitalization of $8.5 billion). This is probably because dividend announcements are considerably rarer for small firms than for large firms—and any dividend announcements become even better news. Second, the type of dividend declaration matters. Bigger increases in dividends (current payment minus last payment, all divided by share price today) are greeted with a more positive announcement reaction. Interestingly, Table 22.6 shows that the lowest quintile here are firms that decreased their dividends, yet the market did not respond too negatively. Chances are that the market had already digested the reason for the dividend cut prior to the cut's official announcement, and therefore had already lowered the firm's value assessment. (Unfortunately, we can study dividend declines, but it is not as easy to study outright dividend omissions—because we have no clear announcement day when a dividend is *not* announced.) The three middle quintiles are basically firms that did not much change their dividends one way or the other, and their announcement responses are all mildly positive. The largest quintile are firms that significantly changed their dividend yield—and they are the ones that experience the most dramatic stock price increases upon declaring the dividend.

Table 22.6. Two-Day Dividend Declaration Announcement Reactions

	By Market Cap Group:				
	Smallest				Largest
Market Cap Itself (in bill-$)	0.03	0.10	0.26	0.80	8.52
Reaction By Market Cap	0.373	0.247	0.256	0.200	0.137
Change in DivYld Itself	–0.19%	–0.00%	+0.00%	+0.00%	+0.13%
Reaction By Change in DivYld	0.045	0.172	0.214	0.145	0.601

Year	1980	1981	1982	1983	1984
Reaction	0.24	0.32	0.18	0.35	0.15
Year	1985	1986	1987	1988	1989
Reaction	0.16	0.15	0.21	0.28	0.08
Year	1990	1991	1992	1993	1994
Reaction	0.15	0.22	0.41	0.34	0.28
Year	1995	1996	1997	1998	1999
Reaction	0.23	0.19	0.30	0.03	–0.03
Year	2000	2001	2002	2003	
Reaction	0.47	0.43	0.46	0.33	

The dependent, classified variable is the sum of the announcement day stock return on day 0 and on day 1, quoted in percent. The average two day announcement reaction was +0.243%; its standard deviation is 3.62%. The "economic size" (market capitalization and dividend yield change) classifications are based on ranks *within* the calendar year of the SEO. The dividend yield change is dividend delta, normalized by the current price; it is therefore not the change in dividend yield.

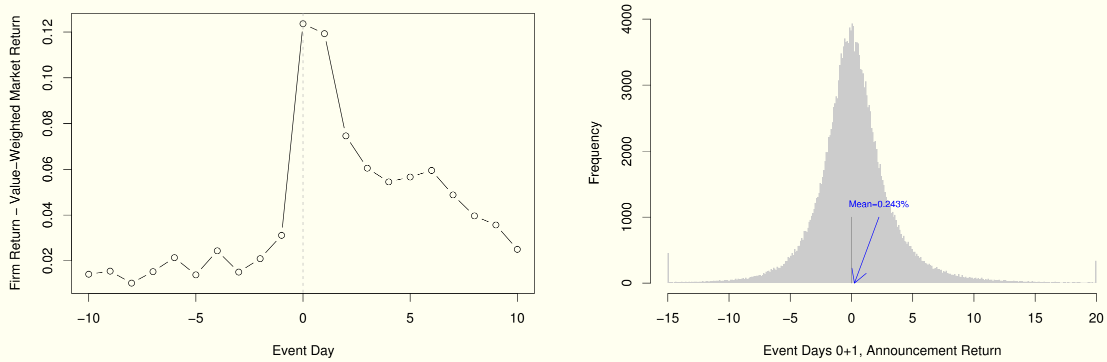
DIGGING DEEPER: *We can also run a regression, rather than report group means.*

$$\begin{array}{lcccccc} \textit{Reaction} & = & 0.70 & + & 35.44 \cdot (\Delta \textit{Dividend Yield}) & - & 0.037 \cdot \log(\textit{Market Cap}) \\ & & (T = 13.14) & & (T = 22.60) & & (T = -8.78) \end{array} \quad (22.1)$$

$R^2 = 0.29\%$

This confirms that changes in dividend yields and firm size matter. Other factors tested (but not reported) played much lesser roles.

Figure 22.4. Perspective Plots



22·4.C. Earlier Studies

Dividend initiations show stronger effects.

The academic literature on dividends is voluminous. Depending on the type of dividend studies, even stronger effects than those we documented above have been found. For example, a firm that pays dividends for the first time is generally greeted with a response of roughly 3% to 4%. Some studies have found bigger effects than our +0.243%: an ordinary dividend increase may generally be greeted by a stock price response as large as 1%; and a dividend decrease may generally be greeted by a stock price response as low as −2% to −3%.

Share repurchases show similar effects as dividends.

As we discussed in Section 19·7, share repurchases are substitutes for dividends. They tend to be greeted with a roughly equally strong positive response as dividends of equal size (although repurchases tend to be bigger). So, over the same time period, the immediate announcement price reaction of an open market share repurchase program was around 2% on a single day. The very largest share repurchase programs, called **intra-firm tender offers**, usually commit to repurchasing a considerable fraction of the firm and are greeted by a much more positive market response (10% to 20%) than the announcement of run-of-the-mill open market repurchase programs.

22·4.D. Theoretical Perspective

The evidence reflects on the theory: dividends are good news, so tax wasting costs must be lower than investor gains.

So why did stock values increase when dividends were declared? If Modigliani-Miller are right, a dividend just shifts money from one pocket of the investor (the firm) into the other pocket (the private account). It must be that increasing dividends sends a signal that helps investors assess the firm's future more favorably. Looking back at Table 19.7, you will see that the theories that immediately come to mind as consistent with our empirical evidence are that either investors learned that the firm was more profitable than they thought (inside information), or that managers have decided not to waste good earnings on pet projects and perks but instead to return money to investors (agency). The evidence also suggests that the personal tax disadvantages of dividends were outweighed by the positive dividend theories—even when they were still theoretically important, as they were in our sample period, which ended before the Bush tax cuts of 2003.

What might the evidence for firms cutting dividends mean?

There is one interesting aspect in interpreting our evidence in light of the theories, though. For the most part, the payment of dividends does not seem to increase bankruptcy costs—or we should have seen a decline in firm value. The wrinkle is that for the firms that are reducing dividends, the stock price did not decline. There are two possible explanations. The first is that despite the cut, dividends remain higher than the market had expected ("good news"). The second is that the cut in dividends reduced bankruptcy costs, a positive event, which thereby mitigated an otherwise negative dividend response.

22·5. Interpreting The Empirical Event Study Evidence

Our findings summarized now.

The event study evidence, on average, can be summarized as follows:

- When firms pay out money, it is good news; when firms raise more money, it is neutral or bad news.
- When firms increase their leverage ratio (holding size constant), firm value increases; when firms decrease their leverage ratio, firm value decreases.
- When firms raise money through debt rather than through equity, these two effects just about cancel one another.
- When firms raise money through equity rather than through debt, both effects work in the same direction, which is particularly bad news.

The last two empirical regularities also go well with the pecking order hypothesis and the empirical evidence that firms raise equity rarely, and debt more frequently.

Our interpretations summarized now.

On average, the empirical evidence is thus favorable to theories that suggest that more money and especially more equity money in the hands of managers is bad. It is not easy to interpret these facts in light of our theories, because the empirical facts are the outcome of many forces. Our facts can primarily tell us only which theories seem to have stronger effects on average, and which theories seem to have weaker effects on average. Moreover, our findings apply primarily to our sample of publicly traded firms—and often very well-capitalized firms:

Agency: yes. More free cash seems to lead managers to waste more.

Corporate Taxes: yes. Lower leverage ratios may reduce the *corporate* income tax shelter, and thus contribute to a decline in value.

Personal Taxes: no. The importance of *personal* income taxes on dividends seems to be not too high, given that the average dividend announcement response was positive.

Bankruptcy Costs: mostly no. On average, financial distress costs seem not to be important. The raising of more money did not increase firm value, but decreased it. (The exception is that we do not see a big announcement price drop when firms reduce dividends. This may indeed be because these are the firms in distress—the market already knows about the trouble and a dividend cut may be a necessity.)

Inside Information: yes. More equity seems to be bad news. It may signal managers' trouble with a current earnings shortfall and/or lack of faith in the future.

Two other factors are difficult to test, because good empirical tests would have to rely more on data that we typically do not have.

Debt Expropriation: unknown. This hypothesis is consistent with the fact that the stock market responds more negatively to an equity announcement than to a debt announcement. To test this hypothesis would require knowledge of how the value of corporate debt changes—which would rely on data that we typically do not have, because corporate debt is often either traded rarely, or primarily over the counter. My own opinion is that debt expropriation is rarely an important factor, except in the most distressed firms.

Transaction Costs: unknown. Transaction costs are not easy to measure. Some direct costs may be known; indirect transactions costs (e.g., time) are much more difficult to describe. However, models of even modest transaction costs suggest that they have the potential to explain a good number of empirical issues we do not yet fully understand.

Again, you should always keep in mind that our evidence is about the *average* importance of these factors. A different set of factors may be important for every firm contemplating a transaction.

Solve Now!

Q 22.1 *What are common limiting factors in applying the event study technique to other managerial actions?*

Q 22.2 *Do financial markets respond more positively to stock issues or debt issues? How do the two events compare?*

Q 22.3 *Is the mean dilution or the median dilution bigger? Why?*

Q 22.4 *Why is the negative 1.6% price drop upon the announcement of an equity offering not small?*

Q 22.5 *Does the –1.6% response suggest that firms should not issue equity?*

Q 22.6 *How do firm size, offer size, and year of issue correlate with the debt and equity announcement effects?*

Q 22.7 *Why would you expect a small response to a dividend declaration that is not an initiation?*

Q 22.8 *What factors seem to matter in determining how firms respond to dividend announcements?*

Q 22.9 *Can you interpret the evidence that debt issues are just about zero-events and equity issues are very negative events from the perspective of the role of changes in payout vs. changes in debt-equity ratio?*

22·6. Summary

The chapter covered the following major points:

- When firms announced new seasoned equity offerings, their stock prices dropped by about 1.5% on average. This corresponded to a mean dilution of about 14% of the money raised by the issue. (Some earlier studies have reported estimates as high as 30-50%, but the median dilution is undoubtedly much smaller than 10%.)
- When firms announced new seasoned debt offerings, their stock prices dropped by a very small amount (perhaps 15 basis points) on average.
- These two reactions can be explained by the fact that raising cash lowers the firm's value, while raising the debt-equity ratio increases the firm's value. For debt issues, the two effects roughly offset one another. For equity issues, they add up.
- When firms declared dividends, their stock prices increased by about 25 basis points—which is large, given the usually small amounts of dividends paid and the fact that many dividend announcements are regularly paid and thus highly anticipated.

  The quintile of firms that raised their dividend yields most (by 13%) saw its stock prices increase by as much as 60 basis points. The quintile of smallest size firms saw its stock prices increase by about 37 basis points.
- These results, together with the evidence from the previous chapter that firms are reluctant to issue equity, are generally consistent with a pecking order view. In turn, a pecking order can be caused by a number of factors, e.g., by inside information, and by agency considerations, although some other theories (such as downward sloping demand curves for stock) are also consistent.
- There is a lot of variation in how the market responded to any of these announcements. Even for the event with the most dramatic average response (equity issuing activity), many firms experienced positive rather than negative stock price reaction on the day of the announcement.

Solutions and Exercises

1. The identification of the event date, the isolation of the event date, and the availability of enough observations to overcome stock return noise.
2. Debt issues are associated with just about a zero response (a tiny value drop, less than 20 basis points, on average). Equity issues are associated with a large price drop, about 160 basis points.
3. The mean dilution. See Page 561.
4. Because of dilution—offerings are usually not to double the firm, so in terms of the offering size, the 1.6% can represent quite a drop.
5. Only on average! For an individual firm, the response might be positive, because an equity issue might create value.
6. They do not, at least not very strongly.
7. Because it is expected, so it is unlikely to be much news. In addition, the amounts paid out are often relatively small.
8. The change in the dividend yield, and the firm size.
9. It appears that both increases in debt-equity ratio and increases in payout improve firm value. These two effects offset one another for debt issues, and go in the same direction for equity issues.

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

CHAPTER 23

Investment Banking

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This chapter looks at the role of the investment bankers. The subject of investment banking is important enough to deserve its own chapter—even if it is only a brief one. After all, investment banks are not only plum employers of finance graduates, but they are also the most important intermediaries through which corporations tap the capital markets. We will focus on two important functions of investment banks: Facilitating the underwriting of securities and advising firms on mergers and acquisitions.

23·1. Investment Bankers

What an investment bank is.

You can think of an investment banker as pretty much the same thing as an ordinary consumer banker. The primary difference is that investment bankers advise corporations rather than individuals, and therefore operate on a larger scale. Like consumer bankers, investment bankers provide the following services:

- lend capital to corporations;
- act as agents on behalf of other capital providers;
- orchestrate the legal and bureaucratic aspects of the capital raising process;
- offer investment advice—solicited and unsolicited.

The last function is especially important when a firm seeks to undertake a large investment, such as a merger or acquisition. We will therefore discuss this at length in the next section.

23·1.A. Underwriting Functions

Underwriters help sell corporate securities.

If you go back to Table 20.6, you will see that with the exception of the direct role of stock returns and the payment of dividends, investment banks facilitate many of the financing activities that corporations undertake. In particular, the underwriting of new debt and equity issues is one of the main businesses of many investment banks. Virtually all offerings of securities for exchange-traded firms are underwritten these days. The term **underwriter** originally came from the guarantee of the issuing proceeds, equivalent to the fact that an insurance company would underwrite a policy. These days, this guarantee is given only on the morning of the offering, when the underwriter practically knows for certain at what price investors are willing to purchase shares. If it looks as if the issue cannot be sold, the underwriter will demand a lower price and/or refuse to bring the issue to market altogether. So, the actual underwriting guarantee itself is no longer as important as it was historically, when it could take weeks to find investors.

Underwriters help sell corporate securities.

Instead, the main function of underwriters today is to provide the legal expertise in carrying through the process (**issue origination**); to make sure that the securities get placed, often with specific investors such as large institutional ones (**issue placement**); and to signal confidence in the firm by putting their own reputations on the line. Underwriters can help in many ways throughout the process. For example, many investment banks have large brokerage arms, and their analysts can spread "positive hype" through optimistic analyst reports for their investment banking corporate clients. This presumably increases the demand for investment in the company, and is thus good for selling more shares and debt in the future.

ANECDOTE: The Analyst Recommends: Buy!
The number of analysts' **buy recommendations** outnumbers the number of **sell recommendations** by a ratio of about 5:1; when limited to **strong buy** and **strong sell** recommendations, this ratio changes to over 10:1.

The primary reason for this imbalance is a conflict of interest. Most brokerage firms and by extension their analysts are owned by investment banks. (They are even called "sell-side" analysts, even though their "advice" goes to investors!) The investment banks are well aware that a sell recommendation is likely to induce the targeted firms not only to exclude the particular analyst from obtaining further information about the firm, but also to induce the targeted firm to select a different underwriter. Therefore, the investment banks discourage their analysts subtly and not so subtly from issuing sell recommendations. Although this analyst bias was always widely recognized by professional investors, it had received scant attention in the press and little recognition by small investors—until 2001, when it suddenly became a public scandal. It is still somewhat of a mystery why then, but not before.

In April 2003, ten of the largest investment banks settled a lawsuit by setting aside funds for making independent research available to brokerage clients and promising a separation of their brokerage analysis from their investment banking functions. It is not yet clear how effective these reforms have been.

Nevertheless, it would be naïve for CFOs to consider underwriters as unconflicted agents on behalf of their clients. Investment banks make their money from financing activity and thus will push for the firm to engage in activity—even if it is value-decreasing. They also sometimes structure transactions in such a way that the valuation is not easy for the CFO to understand. This can allow the investment bank to obtain claims on the company at below fair market pricing. (For example, many bonds include a strippable warrant kicker to lower interest rates, and many CFOs may not understand as well as the bank does just how much value they are giving away.)

Underwriters are also conflicted agents.

23·1.B. The Top Underwriters

Table 23.1. Top Underwriters in the United States in 2004

	Underwriting		M&A Advising		IPOs	
Bookrunner	$	#	$	#	$	#
Bank of America	$204	780	$73	109	$1.5	16
CSFB	$362	1,359	$201	271	$3.6	23
Citigroup (Salomon-Smith-Barney)	$534	1,892	$485	377	$3.6	19
Deutsche Bank	$335	1,299	$247	218	-	-
Goldman Sachs	$286	855	$577	336	$7.1	29
JP Morgan	$386	1,492	$511	396	$4.0	25
Lazard	-	-	$230	207	-	-
Lehman Bros	$370	1,292	$308	175	$2.4	20
Merrill Lynch	$374	1,564	$381	298	$4.5	31
Morgan Stanley	$414	1,334	$381	298	$7.3	21
Rothschild	-	-	$232	269	-	-
UBS	$300	1,175	$219	289	$2.3	16

Firms are in alphabetical order. All dollar figures are in billions. A dash means the information is not available or the firm is not among the top players. Underwriting are all debt and equity offerings, corporate and otherwise, for which the underwriter ran the book (was lead underwriter). IPO underwriting is broken out in the right-side columns. It is small in absolute dollar amounts, but disproportionally rich in commissions and risk. With the exception of *Lazard* and *Rothschild*, who are primarily M&A advisors, the remaining top investment bankers are also top underwriters.

Source: www.thomson.com (Thomson Financial).

Table 23.1 gives you an idea of who the big investment banks are. This market has an interesting history. Until November 1999, the **Glass-Steagall Act of 1933** had prohibited interstate banking (keeping investment banks outside states with large corporate presence, such as California, New York, Illinois, and Massachusetts, relatively small), and the mixing of retail/commercial and investment banking. Glass-Steagall therefore made it impossible for large consumer banks, such as **Citibank** or **Chase Manhattan Bank**, from competing in the investment banking space. Many other countries never had this distinction—they just had "banks" that performed both consumer/commercial and investment banking. Therefore, the United States was unique in fostering a large number of relatively small investment banks. Just around the repeal of Glass-Steagall in 1999, the investment banking sector rapidly began to consolidate. For example, in 1998, Citibank and Travelers Insurance group had merged to become Citigroup. In the same year, Smith Barney purchased Salomon Brothers to become Salomon Smith Barney. One year later, with Glass-Steagull repealed, Citigroup then purchased Salomon Smith Barney, so the five formerly independent financial services providers are now all just parts of one large financial conglomerate. Similarly, Chase Manhattan purchased J.P. Morgan in 2000, and merged with Bank One Corporation (a large credit card issuer) in 2004. CSFB is the combination of Credit Suisse, a very large Swiss bank, and First Boston, an old investment bank. Lehman Brothers and Goldman Sachs still remain independent—and there are even some players that remain

How the top came to be the top.

independent in just one sub-field of investment banking (specifically, Lazard and Rothschild are just M&A advisors).

Like the rest of the world, we now have conglomerates.

Much competition is no longer just in the investment banking field or even just across a whole range of financial services, but even transcends national borders. The same cast of bankers competes not only in the United States, but also in the bigger global market. Even though the United States remains the biggest underwriting market (debt plus equity), it was followed closely by Europe (with the Middle East and Africa), Asia (without Australia and Japan), Japan, Australia, and Latin America. In billions of dollars, the 2004 market was

United States	Europe (incl. Middle East and Africa)	Asia (excl. Japan)	Japan	Australia	Latin America
\$173	166	\$69	\$61	\$17	\$4

Moreover, Europe is now the biggest equity offering market, with the United States being the biggest debt offering market. But, again, these are artificial distinctions: investment banking has truly gone global, with many foreign companies issuing securities in the United States and many domestic companies issuing securities in Europe or Japan. About 70% of securities issues in the United States are sold to the public, and 30% are sold to private parties (without SEC involvement).

UK and US banks are the top competitors.

Table 23.2 shows the rankings of global banks by assets under management as of 2004. (Investment banking is just one of the many businesses these banks engage in.) The table shows some remarkable differences between the more market-oriented banks in the United Kingdom and United States on the one hand, and the more institutionally oriented banks in Europe and Japan on the other hand. For example, Bank of America is at the bottom of this list with “only” \$736 billion under management. Mizuho of Japan is at the top of this list with \$1.285 trillion under management. Yet Bank of America is worth more than three times as much as Mizuho! There is much evidence that foreign banks have been largely unable to translate their financial reserves (a natural scale advantage) into profitability equal to their American and British counterparts. Indeed, many of them are struggling against the more nimble competition.

Table 23.2. The Top 15 Global Banks in 2003, by Assets under Management

Rank	Bank	Country	Assets	Capital	Value
1	Mizuho	Japan	$1,285	$38	$50
2	Citigroup	USA	$1,264	$69	$243
3	UBS	Switzerland	$1,120	$24	$87
4	Credit Agricole	France	$1,105	$55	$36
5	HSBC Holdings	UK	$1,105	$55	$164
6	Deutsche Bank	Germany	$1,014	$27	$43
7	BNP Paribas	France	$989	$32	$55
8	Mitsubishi	Japan	$975	$47	$57
9	Sumitomo	Japan	$950	$34	$41
10	Royal Bank, Scotland	UK	$806	$35	$87
11	Barclays Bank	UK	$791	$27	$57
12	Credit Suisse Group (CSFB)	Switzerland	$778	n/a	$42
13	JP Morgan Chase	USA	$771	$43	$78
14	UFJ Holdings	Japan	$754	$21	n/a
15	Bank of America	USA	$736	$44	$171

All numbers are in billions of U.S. dollars. Assets are assets under management. Capital is "tier one capital" (also called core equity), which is common stock, disclosed reserves, and retained earnings. Although based on book value and therefore unreliable, it is the most common regulatory definition for bank capitalization. Value is market value as of June 17, 2004. The table shows that American and British firms had relatively high capitalizations.

Source: *The Banker* magazine, July 2, 2004.

Solve Now!

Q 23.1 *How important is the guarantee of securities placement success that underwriters provide their clients?*

Q 23.2 *What are the main functions of underwriters today?*

Q 23.3 *How are the interests of investment banks different from those of their clients (investors and firms)?*

Q 23.4 *How good and unbiased are brokerage buy recommendations?*

Q 23.5 *Name some of the top underwriters in the United States today.*

Q 23.6 *In relative terms, how important is the American market compared to the European market?*

Q 23.7 *Name some of the top global banks today. Does it matter whether the criterion is market value or book value of assets under management?*

23·2. The Underwriting Process

We now turn to the question: How much does it cost to issue securities?

23·2.A. Direct Issuing Costs

Table 23.3. Typical U.S. Fees, 1990-1994, in Percent.

Proceeds	Initial Public Offerings			Seasoned Equity Offerings			Convertible Bond Offerings			Plain Bond Offerings		
($ millions)	*N*	UWC	TC	*N*	UWC	TC	*N*	UWC	TC	*N*	UWC	TC
2–10	337	9.1	17.0	167	7.7	13.3	4	6.1	8.8	32	2.1	4.4
10–20	389	7.2	11.6	310	6.2	8.7	14	5.5	8.7	78	1.4	2.8
20–40	533	7.0	9.7	425	5.6	6.9	18	4.2	6.1	89	1.5	2.4
40–60	215	7.0	8.7	261	5.1	5.9	28	3.3	4.3	90	0.7	1.3
60–80	79	6.7	8.2	143	4.6	5.2	47	2.6	3.2	92	1.8	2.3
80–100	51	6.5	7.9	71	4.3	4.7	13	2.4	3.0	112	1.6	2.2
100–200	106	6.0	7.1	152	3.9	4.2	57	2.3	2.8	409	1.8	2.3
200–500	47	5.7	6.5	55	3.3	3.5	27	2.0	2.2	170	1.8	2.2
500–	10	5.2	5.7	9	3.0	3.2	3	2.0	2.1	20	1.4	1.6
All	1,767	7.3	**11.0**	1,593	5.4	**7.1**	211	2.9	**3.8**	1,092	1.6	**2.2**

Source: Lee-Lochhead-Ritter, 1996. *N* is the number of observations. UWC is underwriter compensation (in percent of proceeds). TC is total costs, which includes registration fees, printing fees, and legal and auditing costs (in percent of proceeds).

Typical real-world fees.

Table 23.3 describes cost data from 1990 to 1994. (Although no data are available for issuing costs after 1994, it is unlikely that much has changed in relative terms since then.) The table shows that selling equity shares is more expensive than selling bonds. For example, placing a $100 million bond offering may cost the firm direct expenses of $2.3 million in total, while placing a $100 million equity offering may cost $4.2 million. From the perspective of the firm, there seems to be a price for issuing "capital at risk." The more risk for sale, the higher the underwriting costs. There is usually more value at risk in a $1 million equity offering than in a $10 million bond offering. In this context, you can often think of a typical $10 million corporate bond offering as the combination of, say, a $9.5 million risk-free Treasury bond (with very low commissions, below even what corporate bonds charge) and $0.5 million in highly levered equity (with high commission). Therefore, the cost of issuing corporate debt lies between the cost of issuing Treasury-like securities and the cost of issuing risky equity.

Why underwriters charge for risk.

In turn, underwriters have their reasons for charging more for placing riskier securities. First, investors can be found a lot more easily if the securities are safer. In the extreme, safe bond issues are almost substitutes for Treasury bonds, and so investors are not very concerned about risk analysis and so are easy to engage. Second, the underwriter carries some of the risk of the securities he places with his own reputation capital. For example, when an underwriter takes a firm public in an IPO, he partly carries the risk that the firm will go bankrupt later on, which will not play well with investors that the bank solicited. After a couple of such repeats, the underwriter would probably no longer be able to find IPO investors. Therefore, when companies first sell shares in an initial public offering (IPO), which is the most risky investment banking business around, the costs are usually highest—a fact that Table 23.3 shows quite nicely. As a sidenote, IPO commissions have become considerably more uniform after 1994. Almost all underwriters are now charging *exactly* 7.0% in commission, a fact that has made some observers wonder about how competitive the underwriting market for IPOs truly is. Although there are several dozen underwriters, it could be that the market is segmented enough along the size and industry dimension, so that each IPO really has only one to three natural choices from whom an underwriter can reasonably be selected.

Management Fees!

The costs listed in Table 23.3 are, of course, not even complete as far as direct costs are concerned. Importantly, they do not account for the time and focus that management spends on the issuing process, which could otherwise have been spent more productively (an opportunity cost). The effort is relatively more modest in safer offerings—for IPOs, it is a very lengthy and time-consuming task. Similarly, we have no evidence as to how expensive any time delay in funding would be on the values of projects. These two costs are conceivably just as important, but we cannot assess them because we have no data on the costs of management time and project delay. Finally, there are the indirect costs and benefits that the revised capital structure itself creates—the subject of our earlier chapters and subsection 23·2.C.

Bond Rating Agencies.

There is one additional direct cost to issuing debt that is worthwhile breaking out. We have already mentioned bond rating agencies in Section 6·2.C on Page 117. Issuers can pay **Moody's**, **Standard&Poors**, or **Fitch** to rate their bonds. This typically costs \$5,000 to \$25,000 per bond issue. Having a public bond rating helps potential investors gauge the risk. Indeed, many institutions are prohibited from buying any unrated bonds, making ratings a necessity for many large bond offerings. Only the largest and most stable firms can issue **investment grade bonds**. All other firms can only issue **high-yield bonds**, that is, bonds rated BB or worse (see also Table 6.1 on Page 119). To get a better impression of issuing activity, please browse the issuing calendar in the *Wall Street Journal*, as well as a Moody's or S&P Bond Manual in your local library. (The Moody's descriptions are now published by **Mergent**, a sister company of Moody's.)

23·2.B. Underwriter Selection

Firms are often selected by history. Industry expertise is the next most important factor.

Much of our interest in underwriting is from the perspective of firms wanting to raise capital. So, how do firms select investment bankers? Most of the time, by simple inertia! Firms tend to go with the investment banker that they have always done business with. It is often only when they consider switching—which itself usually occurs after a firm has "outgrown" in capitalization the reputation and size of its original underwriter—that they undertake a comparative analysis. If they do such an analysis, an important factor in the selection process is "industry expertise." Such expertise can help the underwriter navigate the process more smoothly, communicate and understand better the concerns of top management, connect the firm to the right potential investors, and offer the services of specialized analysts who can help hype the offering more to potential retail investors.

ANECDOTE: Legal Monopolies: Bond Ratings

Prior to 2003, federal securities laws had recognized just three "nationally recognized statistical rating agencies" (NRSRO): Moody's, Standard&Poor's, and Fitch. (In 2003, the SEC added Dominion; in 2005, it added A.M. Best.) In the second half of the twentieth century, the SEC began to rely on ratings to determine what sort of securities certain regulated financial institutions could own. The raters had not always enjoyed such privileged status. At the beginning of the twentieth century, they were simply investment service agencies that provided investors with research for a fee. In the 1970s, the revenue model changed, and Moody's and S&P (by far the larger and more important) began to charge issuers instead of investors.

In 1994, the *Jefferson County School District* No R-1 of Colorado decided not to obtain a Moody's ranking. To their surprise, Moody's decided to publish an unsolicited and unusually detailed "Special Comment," anyway. It was a negative rating that downgraded the school district, and interestingly it occurred on the day of the pricing of the bond. Although Jefferson County sued, a judge later ruled that Moody's was protected by the First Amendment's *freedom of speech* clause.

This legal protection also helped the three major credit rating agencies in Enron's case. They received substantial fees from Enron. Interestingly, even when Enron was already trading at \$3 per share and the market was aware of its trouble, all three agencies still gave investment grade ratings to Enron's debt.

Agency issues between managers and the firm may also play a role.

However, there is a puzzle. There is empirical evidence suggesting that it is cheaper for a corporation to ask several investment banks to compete for the underwriting of an issue—but most firms just hire one investment bank (and typically their old investment bank) and stick with it. (The exception are utilities, which are obliged by regulation to bid out their capital raising activities.) Why this inertia? One view is that firms are willing to pay more because hired investment banks provide better service along other dimensions than just underwriting at lowest cost. For example, it may take less management time if the existing underwriter is already well informed about the company through previous interactions. Another view is that firms select their underwriters based on such considerations as personal friendships and convenience. The most cynical view (see Chapter 24 on corporate governance) is that executives like underwriters who "wine and dine" them, give them better and cheaper personal banking services (such as allocations to shares in initial public offerings), and provide a placement network for executives if they want or have to move. After all, an investment banker is not likely to recommend to bigger and better companies those CFO executives who they barely know and who have minimized the investment bank's profits.

23·2.C. Sum-Total Issuing Costs — The Financial Market Reaction

Capital market reactions are a better measure of the overall net cost/gain of an issue.

We have already discussed the financial market reactions to issuing in Chapter 22, but it is important to bring them into our discussion of the costs of issuing here. Recall that the average company dropped by about 1.5% when it announced a new equity issue, which was about 15% in dilution. In a sense, this may be a better measure of the cost of issuing than the direct fees we measured in Table 23.3, because it includes the direct fee. For example, start with a perfect market in which a \$100 million firm raises \$50 million and pays the underwriter \$30 million in commissions. It is the old shareholders who pay the \$30 million. The new shareholders participate only if they can buy at the appropriate price. Because the firm will be worth \$120 million, new shareholders demand 41.7% of the firm. Old shareholders experience an announcement price drop from $100\% \cdot \$100$ million to $58.3\% \cdot \$120$ million = \$70 million. In an efficient market, this value drop must occur in the instant of the announcement, not at the actual issuing. CFOs sometimes make the mistake of adding up the direct cost and the dilution cost to arrive at a total cost of issuing, but this example illustrates that the dilution (announcement drop) is *not* an additional cost, but a total cost.

Adding up costs and benefits.

If we now were to observe that the value of outstanding equity had dropped from \$100 million to \$60 million instead of to \$70 million, then the firm must have lost another \$10 million in value through the issuing of the equity elsewhere. In contrast to the direct fees, we usually do not immediately know the causes for the remaining dilution. It could be that existing owners believe that the firm gave away too much in features, or that it chose the wrong securities features, or that the firm or shareholders will now pay more in taxes, or that shareholders learned the bad news that management was doing so poorly that the firm needed to raise more money. Actually, it is not one or the other, but the sum of all the possible value effects. In the end, the point is that the extra loss of \$10 million is a cost just like the direct cost of \$30 million paid to the underwriter. Of course, if the value of outstanding equity had dropped from \$100 million to \$80 million, the issue must have cost the \$30 million in commissions, but created \$10 million of value elsewhere. In the extreme, if the firm value increased upon the announcement from \$100 million to \$110 million (and we know that some firms increase in value upon the announcement of a new issue), we know that the issue cost \$30 million in underwriting fees, but created \$40 million in value.

Important:

- *A firm that seeks to maximize shareholder value should minimize all costs of issuing—whether underwriter related or deadweight costs (such as taxes)—and maximize all value created by issuing.*
- *In an efficient and perfect market, the instant dilution is a measure of all the costs and benefits of an issue.*

Real Life Difficulties

In real life, it is not so easy to just look at the announcement reaction. First, when managers consider whether to issue, they have not announced it yet, and so they do not know the market's value reaction. Could they perhaps merely announce their intent, and then see what the value consequence is? Unfortunately, if the market can anticipate that managers are just floating a trial balloon, it may not react at all. If the market response is a function of what it believes managers will do, and if what managers will do is a function of what they believe the market will do, then the blind may be leading the blind. The outcome could be anything. If the market believes the managers will carry through an equity offering, responds negatively, and managers then cancel the issue, the net effect is not as bad as carrying through with the bad issue—but firms still end up worse off than if they had never announced an issue to begin with.

Learning from the market response

But can we learn at least something from what other corporations have experienced in the same situation? Yes, we can—and we have. In Chapter 22, we discovered the following outcomes for publicly traded corporations:

- *On average,* when firms raise more external capital, it is bad news and the stock price drops. Conversely, when a firm pays out capital, it is good news and the stock price rises.
- *On average,* when firms replace equity with debt, keeping firm size steady, the stock price generally increases. Conversely, when these firms replace the (remaining) debt with equity, the stock price generally decreases.

We interpreted this evidence that investors seem to infer that management will waste the extra money, or learn that the firm can no longer produce as much money as they thought it could. Our analysis was based on the following "anchor" estimates for the announcement price reactions for three important and frequent capital markets events for publicly traded firms in the United States from 1980 to 2000:

- The two-day announcement price change for an equity issue (increasing firm size and decreasing debt-equity ratio) was a drop of about 1.5-2.0%, with a standard deviation of about 6%.
- The equivalent announcement price change for the typical debt issue was about 0.2%, with a standard deviation of about 3%.
- The equivalent announcement price change for a dividend announcement was a price gain of about 0.25%, with a standard deviation of about 4%. The larger the dividend announcement, the more positive the financial market reaction.

Again, these estimates include *all* costs and benefits of issuing (or paying dividends), including but not limited to underwriting and expert fees.

Where our event study knowledge is limited.

Unfortunately, the historical event studies also have considerable practical limitations when it comes to our assessing the costs and benefits of investment banking activity.

1. **Disentangling Value Effects from Noise** In our example, it was easy to compute that our financial market reaction was the sum of $30 million in underwriting fees, plus other benefits and costs. In the real world, stock prices are very noisy. A $100 billion company

may drop by as much as a $1 billion in value on a day on which it announces nothing. If the firm had happened to announce an issue, we would have erroneously concluded that our issue created total costs of $1 billion. (Some of this noise can be reduced if we knew the exact minute of the day when the issue announcement hit the financial market, but this is usually not the case—if only because many announcements occur after trading hours.)

These problems are borne out by what we see in the data: tremendous heterogeneity in financial market responses. One firm may experience a +5% response, while another firm may experience a −5% response to the same action. It is not clear whether we can disentangle the part of this heterogeneity caused by noise in stock returns from the valuation effect due to the issue.

2. Comparability A big assumption in learning from other companies' responses is that our company is just like these other companies. If you recall our Chapter 10 discussion of comparables, how confident are you in this assessment? Probably not very—every company is different, and every day is different. Are you more like the corporation that increased by 0.4% in value upon the announcement of a new equity issue two years ago, or more like the corporation that decreased by 0.8% in value five years ago?

As indicated, there are important time and industry effects. For example, at certain times and in certain industries, investors may very much like or dislike issues, i.e., react positively or negatively. A good example is high-tech firms around the turn of the millenium. In 1999, many equity issues by almost any Internet firm were greeted very warmly; in 2001, not even the best and most solid Internet businesses could raise funds.

So, would the average 0.2% drop that we see in the data for debt issues apply to you if you issued debt? Given the plus or minus 3% standard heterogeneity, the context of the debt offering is probably more important than the average reaction. Put differently, with this large a standard deviation and this low a mean, about 48% of the firms would have experienced a positive response and 52% a negative response, and it is not clear where *your* company would fall. For debt issues, in particular, this means that, as financial manager, you should probably not care too much that the *average* debt offering dropped by 0.2%. Similarly, how would your stock react if you announced a dividend? The average response is positive, but plenty of firms experience *negative* stock market reactions. (The exception are very, very large dividend initiations, which are met by a clearly positive shareholder response.) On the other hand, the dividend announcements produced large value increases relative to the money being paid out. The one event where the financial markets have been teaching us that the net costs of issuing are very high are equity offerings. Yes, even though this dilution applies to many firms, it does not apply to every firm, and perhaps not to your firm. With empirical estimates of dilution ranging between 10% and 20%, issuing costs can be quite substantial. (If we subtract the 7% total direct costs documented in Table 23.3, we still find an additional 3-13% of value destruction.) Net in net, the benefit of equity issuing for the average firm does not seem to outweigh its costs.

Pinpointing the event reduces noise and event contamination, but leans more heavily on immediate market reaction.

We noted that you can reduce the return noise by shortening the event window. If you do, you are leaning more on the efficient markets assumption—that stock markets immediately react—and on your own ability to pinpoint the time or times when the market fully learned of this information. This brings up yet another important drawback:

Event Anticipation Recall that the markets respond only to the unanticipated part of an event—and even though it is still useful to learn whether the effect was net-positive or net-negative, it would be better to learn exactly what the market assesses the value consequences to be.

Most studies look at the 2-3 day announcement return, because they rarely know exactly when the information of the new issue is released.

Given these shortcomings, you can draw two conclusions:

Use this information, but be careful!

1. The measured market reaction provides very useful information as far as the costs of issuing are concerned.
2. The measured market reaction is so noisy that you cannot get around computing your own independent value estimates of any contemplated investment banking transactions.

Solve Now!

Q 23.8 *Roughly, how expensive is selling $100 million worth of equity? $100 million worth of bonds? $100 million in an IPO?*

Q 23.9 *Why is it more expensive to place equity than debt?*

Q 23.10 *What factors seem important when firms select underwriters?*

Q 23.11 *How would you measure the whole cost of issuing, including deadweight costs that capital structure changes create, direct fees, and everything else? Do you see any implementation problems?*

Q 23.12 *What do you expect the price reaction to be on the day that the new seasoned equity offering shares are sold into the market? (This is not the announcement day.)*

23·3. Mergers and Acquisitions

M&A fits and does not fit.

Though not exclusively a capital raising function, facilitating and advising on mergers and acuisitions (**M&A**) constitutes one of the major functions of the investment banker. It also overlaps with the world of underwriting, because we know that much issuing—and almost all seasoned equity issuing for Fortune-100 corporations—occurs only in the M&A context. Although our focus is the role of the investment banker, we shall start with a brief introduction to M&A.

Various types of M&A.

A merger occurs when two corporations agree to marry on an equal basis. An **acquisition** occurs when one company purchases another. Most mergers and acquisitions are friendly, i.e., the target management agrees to be acquired (often in exchange for extra consideration for managers personally, which is a perfectly legal form of a bribe). Acquisitions are further classified by whether the acquirer pays with cash (a "cash offer") or with the corporation's shares as currency (a "stock offer"). The typical method of execution is the **tender offer**, which simply invites shareholders to present their shares in exchange for cash or stock. Its execution can be contingent on enough shares being tendered. In a **leveraged buyout** (**LBO**), the acquirer is financing the buyout mostly with debt (often high-yield or junk bonds), and the LBO firm usually delists to become a privately owned company. In a **management buyout** (**MBO**), the management is the LBO buyer.

Why Acquire or Merge? Value Gains.

Takeovers naturally increase the scale of the firm (which managers tend to like), but whether they create value (or just more revenue) depends on the situation. Value can come from several sources, the most prominent of which are the following:

Synergies The merging of systems, skills, structures, departments, and staff can improve efficiency. Economies of scale can come from the elimination of duplicate departments or more efficient production and distribution. For example, the merging of ATM networks can attract more bank customers; the elimination of double bureaucracies can reduce overhead; and the reduction in competition can make raising prices easier.

Shutdown Efficiencies Sometimes, it is better to shrink or liquidate a firm, and the current management is unwilling or unable to execute such a drastic measure. A takeover by individuals with less of an institutional history often makes this easier.

Expropriation A transfer of management can allow breaking implicit promises that firms have made but not put into writing. After all, it is difficult to contract out every promise that employers make to employees and vice-versa. Consequently, all companies rely on at least some employee loyalty and all employees rely on at least some company loyalty.

An example of how breaking implicit promises allows a firm to become more profitable is a company that has attracted employees by paying less, but by implicitly promising long-term employment stability and generous pension and health benefits, as long as the firm does well. But as the company and its workers age, these liabilities can become quite significant, and a takeover could allow new management to save money by firing employees, now older and more expensive, or by replacing an overfunded pension fund with a less costly alternative.

Two more value gains come about through the higher leverage often assumed in acquisitions, especially in leveraged or management buyouts:

Tax Benefits Higher debt ratios reduce the amount of profits collected by the IRS.

Better Governance The need to service debt usually makes it easier to convince both managers and employees that they have to work harder and spend less on pet projects—or the firm will go bankrupt. Ironically, management buyouts are often contemplated by the most wasteful managers, who themselves have the incentives to make their own corporations look bad, so that they can buy them on the cheap and improve them.

All of these are important M&A drivers, though not equally in each and every takeover. In some takeovers, it may be primarily synergies; in another, it may be primarily better governance. It is also often difficult to distinguish synergies from expropriation or better governance. If a long-employed but now unproductive worker (or entire department) is eliminated, is this gain expropriative or efficiency-value-enhancing?

Long-Term Success and Failure.

Some takeovers succeed; others fail. The principal negative when a large company takes over a smaller company is often worse focus and poorer management. There is good evidence that takeover activity in the 1960s and 1970s was driven by the desires of managers to increase firm size and form conglomerates, many of which were then run more poorly after the acquisition than before. In the 1980s, the situation reversed: many of these conglomerates were taken over and again dismembered. In addition, many small companies began buying bigger companies. The principal negatives when a small company takes over a larger company, as in an LBO, are the loss of the benefits of access to capital and the lack of diversification. That is, the buyer often has so much of his capital at stake in the firm that he may have to ration capital even for positive NPV projects, and forego the insurance of diversification.

The cost of acquisition.

Of course, in both cases, the acquirer can make the mistake of overpaying for the target. It turns out that the poster child example for the end of the LBO wave of the 1980s was Campeau's 1988 purchase of Federated Department Stores (e.g., Macy's and Bloomingdales). Campeau went bankrupt in 1992. It is true that Campeau had lost a lot of money in buying Federated, but it seems that even this LBO made money—just not for Campeau. Federated had traded for \$4.25 billion in 1988. When it emerged after bankruptcy in 1992, it became clear that Campeau had managed to raise the value to \$5.85 billion (adjusting for market movements over the same period)—a \$1.5 billion value increase. Unfortunately for Campeau, he had paid \$7.67 billion.

Who gets the gains?

This raises the important question of who benefits most from the value gains in M&A's—the acquirer or the target? If the acquirer purchases the target at the prevailing market price before his appearance, all benefits would accrue to the acquirer. If the acquirer purchases the target at a much higher price, many benefits would accrue to the target. Indeed, if the price is high enough, the acquirer may lose money and the target shareholders may gain money.

The empirical evidence suggests that targets make out like bandits. A study by Ernst and Young suggests that the typical announcement price gain is about 25%. From 1996 to 2000, this premium even shot up to between 40% and 50%! It is no surprise, then, that most of the takeover value gains have not accrued to the acquirer. A recent study by Moeller, Schlingemann, and Stulz (2005) looked at publicly trading acquirers. From 1980 to 1998, they lost about 1.6 cents in value for every acquisition dollar. From 1998 to 2001, this shot up to 12 cents per acquisition dollar. (As usual, there was a lot of heterogeneity across M&A, but most of the 12 cent figure was driven by some *really* bad outliers!) For public acquisitions, the total acquisition value gain—the dollar benefit to target shareholders plus the dollar cost to acquiring shareholders (the acquirer is usually larger!)—seems to be just about zero.

It is the target! The acquirer often loses.

Still, there is much heterogeneity in value changes for acquirers, just as there was much heterogeneity in value changes when firms issued securities. Some acquisitions are very good not only for the target, but also for the acquirer. You have to judge acquisitions on a one-by-one basis. Nevertheless, you might ask: Why are bad acquisitions not unusual? The reason is that they are often not only in the interest of the investment banks, who push them because they reap good fees from M&A financing and advice, but also in the interest of the acquiring managers. Running a bigger company usually means more prestige and compensation for target managers. The target management naturally often resists, even though they should be thrilled for their shareholders. The best example of this conflict of interest may have been the merger between Chase and Bank One. The negotiation took place between the Bank One CEO, Dimon, and Chase CEO Harrison, both of whom wanted to become CEO immediately. The original plan was for Dimon to succeed Harrison after two years. Dimon offered to sell Bank One at a zero premium if he just were to become the merged company's CEO immediately. Harrison rejected this offer, and instead paid a $7 billion premium from Chase shareholders to Bank One shareholders in order to retain his post for these two extra years. Chapter 24 will look at M&A activity from a corporate governance perspective.

OK, not all acquirers lose. But why do so many acquirers do the wrong thing for their shareholders?

23·3.A. M&A Participants, Deal Characteristics, and Advisory Fees

In Table 23.4, there are hard statistics for almost all domestic acquisitions that involved a publicly traded corporations from 1980 to 2003, and classified by the quality of advisor (within the industry in which the takeover occurred). Still, the data are not complete. There were many mergers and acquisitions among firms that were not public, and even for the roughly 15,000 acquisitions having involved a public corporation, we have good data on fees for only 6,000.

The typical acquirer was about three to four times as large as the typical target. Also, the mean firm size was much larger than the median firm size, suggesting some disproportionally large firms were in the sample. About one-half to two-thirds of M&A's occurred between firms in the same industry (classified by the "two-digit SIC [standard industry classification] code"). About half to two-thirds of M&A's involved public acquirers or targets.

Participating Firms

The average deal size was about $800 million, but the top tier investment banks advised on disproportionally larger deals. About one in five takeovers occurred through a tender offer (the alternative being a negotiated merger with the target, not involving an offer to shareholders). Only a small fraction of all deals were classified as hostile, where the target management resisted. About one third of all deals were paid for in "all cash," and just a few deals were paid for with "all stock" (in which the acquirer paid target shareholders with its own shares). Somewhere between about 10% and 15% of acquisitions were abandoned. If successful, it took the typical deal about four months to complete. Note that when the deal was hostile, a much larger fraction of targets seem to have engaged top-tier underwriters.

Deal Characteristics

The median advising fees were just about half a percent of the amount of the transaction (usually the target size), on average. The mean fee was much larger, suggesting that there were a few large fee outliers. Remarkably, top-tier investment bankers charged about the same proportional fees as their lower-tier brethren—the reason why they earned more fees is simply that their deals were larger.

Fees

Table 23.4. Descriptive Statistics of U.S. M&A Transactions from 1980 to 2003

	Acquirer Adviser Tier				Target Adviser Tier			
	Top	Mid	Bot	Total	Top	Mid	Bot	Total
Firm Value ($ million)	$7,642	$5,084	$1,020	$4,916	$2,106	$1,237	$265	$1,395
Median ($ million)	$1,765	$711	$213	$736	$440	$251	$65	$241
Acq and Tgt in same industry	63.6%	62.7%	65.9%	64.0%	49.0%	45.5%	60.5%	52.2%
Prop of public acquirers					64.5%	62.0%	72.0%	66.6%
Prop of public targets	58.5%	50.4%	43.3%	51.3%				
Deal (Tgt) value ($ million)	$1,357	$659	$127	$761	$1,821	$663	$126	$840
Median ($ million)	$275	$132	$37	$120	$403	$138	$48	$127
Prop of tender offers	19.7%	17.7%	9.7%	16.1%	24.9%	23.1%	15.3%	20.8%
Prop of hostile deals	3.6%	3.9%	0.8%	2.9%	10.4%	5.3%	2.0%	5.7%
Num of acquirer advisers	1.20	1.11	1.03	1.12	0.84	0.67	0.49	0.66
Num of target advisers	0.90	0.77	0.59	0.76	1.34	1.16	1.06	1.18
Prob of Completion	88.9%	89.2%	91.8%	90.0%	73.6%	79.5%	85.6%	79.8%
Days to Completion	116	100	102	106	141	132	148	141
Prop of all-cash deals	37.6%	38.3%	32.8%	36.3%	42.8%	48.6%	42.8%	44.5%
Prop of all-stock deals	28.8%	27.8%	39.1%	31.6%	23.4%	22.1%	38.9%	28.9%
Pct of cash	47.3%	48.7%	42.2%	46.2%	53.0%	58.2%	48.8%	53.0%
Pct of other	14.5%	14.3%	10.1%	13.1%	16.1%	14.0%	6.4%	11.8%
Pct of stock	38.1%	36.9%	47.7%	40.7%	30.9%	27.8%	44.8%	35.2%
Fees Paid to advisers, in millions of dollars								
Mean	$4.83	$2.65	$0.77	$2.89	$6.47	$2.79	$0.97	$3.06
Median	$2.38	$1.00	$0.25	$1.00	$3.70	$1.40	$0.44	$1.13
Deal Value, in millions of dollars								
Mean	$2,494	$1,092	$208	$1,345	$2,177	$749	$150	$899
Median	$416	$195	$55	$177	$525	$181	$58	$144
Fees paid as percentage of deal value								
Mean	0.91%	0.90%	0.93%	0.91%	0.87%	1.13%	1.15%	1.06%
Median	0.47%	0.58%	0.52%	0.52%	0.67%	0.80%	0.82%	0.76%
N	733	672	591	1,996	1,124	1,113	1,695	3,932

Rows report means (except where noted otherwise), and can have different numbers of observations. In the top rows, there are typically about 15,000 acquisitions; in the middle panel there are typically about 15,000 acquisitions. These are roughly equally split across categories. There is fee information for only about 6,000 acquisitions, and the distribution is somewhat biased, which is why *N* is reported in the last column, and why the deal values here do not match deal values above.
Source: Walter, Yawson, Young, undated (June 2005).

Solve Now!

Q 23.13 *Why do firms like to acquire other firms?*

Q 23.14 *Do acquirer or target shareholders gain more in a takeover?*

Q 23.15 *What are possible sources of value gains in takeovers?*

Q 23.16 *How large is the typical acquirer relative to target?*

Q 23.17 *What is the typical commission for M&A advice that investment bankers earn? How does it differ across the tier of investment bank retained, and across acquirer and target?*

23·4. Summary

The chapter covered the following major points:

- The market for securities underwriting has become part of a global market for investment banking services. Top U.S. investment banks underwrite between \$300 and \$500 billion in securities in a good year.
- The direct costs of underwriting are a function of offering size. An IPO costs around 10% in total costs, a seasoned equity offering about 7%, a convertible bond offering about 4%, and a straight bond offering about 2%.
- There is large heterogeneity in the reaction of the financial markets to the announcement of a new debt or equity issue (or dividend payment).
- The typical firm drops about 2% when it announces a new equity issue. This corresponds to a 10-20% dilution cost for existing shareholders.
- Investment bankers also serve an important advisory function in M&A activity. In an active year, for many investment banks, the M&A they are advising on is often similar in magnitude to the amount they underwrite—\$200 to \$500 billion. Acquirers often overpay for targets, which means that most of any value gains accrue to target shareholders, not acquirer shareholders.
- Based on information from M&A deals among publicly traded corporations between 1980 and 2003, it appears that:
  - Average advisory fees are about 1% of the target (transaction) size.
  - Median advisory fees are about half to two-thirds of the median.
  - The 80-90% of proposed deals that ultimately carry through take about 4 months to complete.
  - Fewer than about 5% of all acquisitions are hostile (and most of these occurred in the 1980s).
  - The typical acquirer is about three or four times larger than the target.
  - Between one-half and two-thirds of acquisitions are within the same industry.
  - About one-third to one-half of acquisitions are paid for with all cash, and about 30% are paid for with all stock.

Solutions and Exercises

1. The guarantee of securities placement sales success is usually fairly unimportant, because it is only given on the day of the offering.
2. Formal process management, selling, certification.
3. Investment bankers like transaction volume and fees, not value creation for their clients. The latter matters primarily to the extent that it helps the former. An investment banker who continually costs its clients money will eventually lose many. For investors, investment bankers are often interested merely in selling securities to, whether they are good or bad investments. For firms, investment bankers are often interested in restructuring, whether it makes sense or not. Furthermore, investment banks sometimes structure issues in a way that makes it hard for firms to value what they are giving away.
4. Most recommendations were not very good, as evidenced by the fact that most recommendations are "buy." This helps the investment banker attract corporate clients.
5. See Table 23.1.
6. They are about equal sized.
7. See Table 23.2. By market value, the U.S. and British banks are far more important.

8. About \$4 million (SEO), \$1.5 million (bond), and \$7 million (IPO) in terms of underwriting costs. Add about \$1 million for other costs in an IPO, and \$300,000 to \$500,000 for bond and seasoned equity offerings.
9. There is more capital at risk, which in turn means that the underwriter has to put more of its reputation on the line and work harder to place the securities.
10. First, inertia. Second, their "outgrowing" their previous underwriter. Third, industry expertise. Fourth, personal relations.
11. Through the degree of dilution at the announcement price reaction! Unfortunately, it is not known by managers before hand, and so depends on comparability assumptions, and stock returns are noisy.
12. It should be about zero, because the share sale is an event that was announced earlier and thus should be almost perfectly anticipated. If the market did not use this information efficiently, and the share price were to go down on the day of the offering, you could short the equity shares the day before the offering, and repurchase them the day after the offering for a profit.

13. Because it can be in the interest of the firm (creating value), or because it can be in the interest of managers (and advising bankers).
14. Target shareholders.
15. Synergies, shutdown efficiencies, and expropriation. In addition, if financed by debt, it could be tax benefits and better governance.
16. About three to four times.
17. The mean is about 1% (0.9% for acquirer, 1.1% for target). The median is about 0.5% (for the acquirer) and 0.8% (for the target). The differences across tier and between target and acquirer seem fairly small.

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

CHAPTER 24

Corporate Governance

More Agency Conflict

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For the most part, we have assumed that managers act on behalf of owners and maximize firm value. We have mentioned agency conflict before, as in Chapter 7. In this chapter, we describe the conflict between corporate owners and top management in more detail. In theory, we know that debt should be paid first; equity should receive the residual; and managers should be compensated according to their marginal value to the company. But we did not ask the simplest of all questions: Why do managers return *any* money to investors? After all, a specific investor contributes little to the corporation *after* the corporation has her money.

This question is the domain of **corporate governance**, which concerns itself with the conflict of interest between the managers of the corporation and its capital providers.

It is also important to understand what corporate governance is not—it is not good management. Instead, governance is the mechanism to control management *if* it is bad. Governance mechanisms may never have to spring into action if management is good.

24·1. Less Fact, More Fiction: In Theory

Firms typically start out tightly controlled but eventually become diffusely held, which brings many problems.

Most companies start out with few **conflict of interest** problems—if only because the entrepreneur owns the entire firm, provides most capital, and makes all decisions. As the company grows, the entrepreneur usually needs to raise more outside capital, either to expand the firm or just to start enjoying the new riches. To be able to get investors to part with their cash, the entrepreneur must create a corporate charter and install safeguards that satisfy potential investors, legal requirements, and common practice. Eventually, the founder's personal role begins to fade, more and more capital is raised from the outside, and management becomes "professional" in the sense that it no longer acts solely based on the whim of the entrepreneur. These professional managers also bring with them unique qualifications and specialization benefits. Legally, these managers now become "agents" acting on behalf of the investors (primarily shareholders), the "principals." Practically, as the distinction between capital providers and decision makers grows over time, so do conflict of interest ("agency") problems—and not just between them, but also among them. Although even two or three co-owners can squabble, when there are thousands of shareholders, as in a publicly traded company, the coordination problems take on an entirely new dimension. Shareholders usually agree that they prefer more money to less money, but often on little else. In any case, they are not capable of constantly voting and communicating their desires to their agent managers, much less checking over what their managers are doing day-to-day—and managers know this, too. Thus, investors in large, publicly traded corporations are typically represented not only by managers, but also by a set of institutions (discussed in this chapter), so that they themselves can fade into a more passive role.

To get money back, investors need control rights. To get money in the first place, firms need to give control rights.

Solemn promises of both corporate value maximization and eventual profit participation are not enough for shareholders and creditors. Investors must be able to coerce their managers to honor their promises. This is primarily (but not exclusively) achieved by **control rights**, which give investors power over managers, and especially when managers do not act appropriately. Giving up such control rights is in the interest of owners, because the terms under which they can obtain capital in the first place are better when the control rights are better. This argument is really the same that we used in Chapters 18-19 to justify why owners want to set up an optimal debt/equity ratio.

The equilibrium outcome is second-best, permitting unavoidable managerial self-enrichment.

In real life, control rights are not perfect—it would be impractical or impossible to protect capital providers *perfectly*. The cost of preventing all managerial opportunism would be prohibitive. It would not maximize firm value if the firm spent \$10 in audits to prevent \$1 in fraud. So corporations and capital providers must live with a **second-best** solution, in which there is a constant tension between investor protection and managerial self-enrichment.

Control rights differ for debt and equity.

Debt and equity are distinct. You already understand the differences in **cash flow rights** between them—debt has first dibs on the promised payments, and the residual cash flows go to equity. But their control rights are distinctly different, too:

Equity Shareholders are legally in charge of decisions, but their primary power is their ability to vote and appoint representatives, usually once a year during the annual meetings. Their elected **corporate board** has the power to hire and fire managers.

Debt The bond contract not only specifies how much the firm obligates itself to repay in the future, but also the immediate legal remedy if the lender fails to do so or fails to meet any number of pre-specified covenants. Usually, this means that the lender receives possession of the firm (collateral)—no ifs, ands, or buts.

When corporate governance usually works and when it does not work.

A firm that has no corporate board may not find shareholders willing to purchase equity shares; a firm that does not give the right to force bankruptcy upon default may not find creditors willing to lend money. But control rights are not all black-and-white. If the firm does not offer perfect protection to its capital providers, it may still be able to obtain capital, although at worse terms that require the surrender of a higher percentage of the firm or the payment of a higher interest rate. This is one important reason why entrepreneurs and firms are interested in good corporate governance in that it enables them to raise capital at better terms. But it also explains when corporate governance is likely to break down—when the entrepreneur (or shareholder-owners) are no longer solidly in charge, and self-interested managers have already taken control. After all, if managers do not care for shareholders, they may not care if they have to give away a larger fraction of the firm to get control of more money from new shareholders. And the problem gets even worse when the firm generates a lot of internal cash and no longer needs to raise much new external capital.

24·2. Managerial Temptations

The human mind for scheming is infinite.

Although the legal fiction is that managers act solely on behalf of the firm and that shareholders own the firm after creditors are paid off, the fact is that all parties act primarily in their own interests. But why is this a problem? How might a manager do harm? Unfortunately, there is a whole battery of tactics managers can employ to enrich themselves at the expense of shareholders. This section outlines a slew of possible managerial self-enrichment schemes. In the next section, we will discuss mechanisms that seek to restrain such managerial behavior.

24·2.A. Illegal Temptations

Theft

Simple Theft is rare, but it does occur.

The most obvious method is simple theft. For example, in March 2003, the *Mercury News* reported that 58-year-old C. Gregory Earls, head of an investment company called USV Partners, simply funneled investor money into a trust fund for his children. What prevents corporate managers from taking corporate diamonds out of the corporate safe? For the most part, it is the law, which criminalizes simple theft, which is therefore fairly rare. (Mr. Earls competes for the prize of “dumbest criminal”—it is hard to leave a clearer paper trail than he did.)

Fraud

Fraud is fairly common...

The next step up is fraud, because it is more complex, and therefore more difficult to detect and prove. For example, in 2003, *Hop-on Wireless* claimed to sell disposable cell phones. It turns out that the prototypes were Nokia phones with plastic cases around them. The COE raised funding, promising not to take a salary—but promptly used the funds to pay off his credit card debts (see theft above) and gave a company he owned a $500,000 contract (see transfer payments below).

...but earnings management is legal.

Usually, fraud involves manipulation of financials. Unlike Hop-on’s extreme case, many accounting choices are not so black-and-white—the line between illegal accounting manipulation and legal earnings management is both wide and gray. There are many judgment calls that corporate executives have to make. There is empirical evidence that *legal* corporate earnings management is particularly aggressive just before the corporation issues more equity, for obvious reasons, and that the most aggressive earnings managers later perform worse. So, appropriate conservatism may not be in the interest of owners, but it could be wise. Nevertheless, too much conservatism is also not in the interest of shareholders. Painting *too* bleak a picture may make the business collapse. What prevents rosy picture painting? GAAP and SEC scrutiny limit the discretion of managers to legally manipulate the financials. And again, there are criminal penalties against fraud.

Insider Trading

Insider trading is very common.

One more step up—and a surprisingly common form of agency conflict—is insider trading. For example, the most well publicized recent insider trading scandal involved Sam Waksal, CEO of ImClone (IMCL). Waksal received advance bad news about clinical tests of an ImClone cancer drug, and proceeded to tip off his family and friends (including Martha Stewart) that they should immediately sell their shares. His next seven years are all booked up now.

Some insider trading is legal and should be allowed—but which?

Like earnings management, insider trading can be either legal or illegal—and again there is a wide gray line. Managers almost always have more information than shareholders. They love to trade on it before the public learns it, and naturally, this does not make other shareholders better off. It would be unwise to prohibit all insider trading, because insiders do need to be able to sell and buy shares just like the rest of us, if only to diversify some of their wealth. But it is illegal for them to trade on information that is not yet public, which is easy to prove if this entails an impending news release. (Because this is so easy to detect and prove, it is curious how someone as smart as Waksal could have made such a big mistake.) More often, the information that executives have is "soft," and the empirical evidence shows that they indeed do well in their private, legal insider trading. They generally tend to buy before the firm gets better and sell before the firm get worse.

Transfers

Complex theft through transfers is more common.

The next step up is yet more difficult to detect and prove—the pilfering of corporate resources through transfers. The CEO of a public company can own private companies that do business with the public company on favorable terms. On occasion, the terms become *so* favorable that they warrant criminal indictment. For example, on May 1, 2003, the U.S. Department of Justice alleged that "in 1997, Fastow [CFO of Enron] conspired with others, including his wife, to create an [entity owned by the Fastows] in order to reap for themselves the profits generated by certain Enron wind farms, while simultaneously enabling Enron to fraudulently receive government financial benefits to which it was not entitled." Naturally, the smarter the manager, the more complex will be the arrangements, so that the true costs and true benefits to the public company are more difficult to assess. Criminal prosecution of such schemes is fairly rare, especially if the corporate executive has followed legal procedures to the letter (e.g., by asking for board approval).

SIDE NOTE: Transfers can also occur to friends of the management or to large shareholders, who then owe more loyalty to the COE. The ambiguous role of large shareholders in corporate governance is described below.

Bribes

Third parties bribe managers all the time.

Yet another way for executives to get rich at the expense of shareholders, and again one step more difficult to detect is bribes. Managers of publicly traded companies need not even create their own temptations: they practically come to them. For example, during the 1998-2000 technology bubble, receiving IPO share allocations was practically like getting free money. (Normal first-day rates of return were around 50%. Ordinary brokerage clients would rarely receive any allocations.) Citigroup was eager to do investment-banking business with WorldCom, a publicly traded company. It therefore allocated $17 million in 21 offerings into CEO Bernie Ebbers' *personal* account. In one IPO (*Rythms Net Connections*) alone, Ebbers was allegedly handed $16 million. De facto, Ebbers was "courted" to direct the business of the publicly traded company, WorldCom, to Citigroup.

Preferential allocations to and treatment of executives' personal accounts were and continue to be common practice. Ebbers was an extreme case, but not the only one. Lesser methods of bribing executives are so commonplace in business that they are considered almost ordinary. For example, there is evidence that competitive bids for high-level professional services (such as the hiring of a search firm or the placement of a bond or equity issue) usually result in better contract terms than negotiated contracts for the firm—and yet most companies negotiate rather than bid out contracts. Although negotiation can be better for other reasons, more commonly the reason lies elsewhere, and here is why. Executives of smaller firms naturally want to be on the candidacy list to become executives of bigger companies. It is therefore in their interests to form good relationships with investment banks and executive search firms. An executive who uses competitive bids, which minimize the profits of the professional service firms, and who constantly switches from one low bidder to the next, is unlikely to build much loyalty and subsequent quid-pro-quo support.

24·2.B. Legal Temptations

Most managers are not criminals, though.

If you now have the impression that fraud, theft, insider trading, and bribes are the most important agency conflicts between shareholders and managers, you would be wrong. The most important conflicts arise in the day-to-day execution of business, and are more judgment calls than outright unethical behavior—few CEOs actively seek out behavior that is obviously unethical.

Empire Building

Most managers want to grow the firm at all cost.

Most managers see it as their task to grow the business. There are a whole slew of reasons why they want to do so. Most managers are less loyal to an abstract, ever-changing shareholder, as they are to their very tangible companies. Running bigger companies is also in the self-interest of managers: Executives of bigger companies are more prominent and receive more compensation. Some decades ago, this was even explicit: managerial compensation schemes were directly tied to sales, not earnings! These days, it is more implicit, and comes about through the choice of "comparable managers" when executive pay schemes are set. Unfortunately, corporate growth is not necessarily shareholder value maximizing. For example, in the 1980s, oil companies were flush with both cash and assets (oil reserves). The industry had overcapacity. Most oil companies did not return excess cash to shareholders, but chose to spend $20/barrel exploring for more oil reserves, even though oil could be purchased in the marketplace for $6/barrel. Managers naturally are paid for operating a company—difficult tasks such as oil exploration, growing the firm, acquiring other companies. It is not in the interests of managers to return cash to shareholders, especially when it entails drastic shrinking (asset sales) or when it means being taken over by another company. The reward for being a good manager who maximizes shareholder wealth would often be unemployment!

This growth is enormously costly to shareholders.

Many academics believe that the highest agency costs in American companies today (in terms of expected costs to shareholders) are not illegal actions, but failure to direct corporate assets towards the activities that maximize shareholder wealth. The agency cost is particularly high for firms that have lots of cash and cash flow (e.g., from prior profitable activities) and few good new opportunities.

Corporate Perks

Perks are goodies that managers have the firm buy for themselves.

A closely related, though smaller, problem is that managers disproportionately enjoy spending money on perks. A public company may buy a corporate airplane that costs $100 million and increases productivity by the equivalent of $10 million—just because it gives managers $1 million worth of pleasure. Plush corporate headquarters and fleets of corporate aircraft are usually sure signs of publicly traded companies, especially in slow-growth industries.

Executive Pay

Executive Pay is often excessive.

Naturally, executives are most conflicted when it comes to higher executive pay. Executive compensation comes in many forms: salary, bonus, stock grants, option grants, retirement benefits, perks, and severance packages. For example, in 2002, *Business Week* reported that Archie Dunham of Conoco (COP) earned $30 million in salary and bonus alone; Alfred Lerner of MBNA (the credit card lender, KRB) earned $200 million in salary, bonus, and long-term compensation grants; Jeff Barbakow of Tenet Healthcare (THC) earned $300 million (Tenet is currently under indictment); and Larry Ellison of Oracle (ORCL) earned $800 million from 2000-2002, mostly from stock and options. Other recent evidence suggests that pension packages that are usually not reported are larger than the reported executive compensation.

Questions about executive pay.

Why do shareholders in the United States pay so much money? How much worse a manager would the next-best executive have been, and would he have done the job for half the compensation? Why do second-in-commands usually earn only a very small fraction of the CEO's salary, even just one year before they succeed the CEO? And why do European CEOs of even the most successful corporations earn only about 10% as much as their U.S. comparables?

- It could be that U.S. managerial talent is scarcer than European managerial talent. Tougher competition for good management in the United States requires paying higher compensation.
- It could be that a large salary is necessary to attract and retain top CEOs in the United States, and to motivate lower-ranked executives to strive for this prize—but less so in Europe.
- It could be that American CEOs are operating in a governance structure that has allowed them to receive higher salaries than their European counterparts. (This could be because they could capture the corporate board better, or because social norms are different.)
- It could be simple error that is not corrected by the market place. It could be that the Europeans have it wrong and are simply paying too little. Or it could be that Americans have it wrong and are simply paying too much.

As to the first argument, it is hard to believe that executive talent is generally scarcer in the United States than it is in Europe. It is also hard to believe that the marginal contribution (the difference between the best executive and the next-best executive) is typically much larger in the United States than in Europe. As to the second argument, it is possible that European social norms are different from American norms, but there is little evidence one way or another. The latter two arguments seem more plausible. The cultural, ethical, and legal constraints in Europe are different from those in the United States. In most European countries, the chief executive is not the chairman of the board, and social norms prevent too high a managerial salary. (Of course, the same social norms and legal regulations make it more difficult for managers to take drastic actions on behalf of shareholders, e.g., when it comes to downsizing and employee layoffs.)

Entrenchment

Take projects where your expertise will be needed!

Managers, like all employees, like to be indispensable. If they decide to take projects for which they are presumably indispensable, their own personal value to the firm and therefore their compensation will likely go up. If they decide to build redundancy—that is, hire someone who can step in for them, thereby making themselves dispensable—their own value to the firm will likely go down. In fact, they might even be replaced by the board. The ability to **hold up** the company, once managers have become indispensable (or at least, very difficult to replace), probably plays an important role in the awarding of high executive compensation contracts. In this case, no managerial board capture is necessary for high executive compensation to occur. The board will not have a choice and will award high compensation "voluntarily."

Bureaucracy often helps promote entrenchment. It can discourage shareholder wealth maximization, but help managers to become indispensable (knowledgeable of the internal process), and even undertake bizarre projects, internally justified by "proper procedure." In contrast, fighting bureaucratization on behalf of shareholders is a painful and prolonged process, with few rewards for the involved executives unless the firm is in dire straits.

Friendship and Loyalty

Nepotism abounds. Few boards have non-friends on them.

Most managers prefer to have loyal friends working around them, instead of gadflies and potential replacements. This natural characteristic naturally promotes nepotism (in the broad sense).

Perverse Incentives

Sometimes, managers even prefer low values.

Though rare, managers can even have the incentive to drive down firm value: they can then negotiate better incentive compensation contracts or even acquire the firm in a **leveraged buyout**, either of which is often followed by seemingly miraculous turnarounds. The most prominent example is that of Ross Johnson, CEO of RJR Nabisco, whose actions are chronicled in the bestselling book *Barbarians at the Gate*.

Ethical Conflicts

A manager may feel special obligations toward many factors that are not in the interest of shareholders: the town in which the factories are located, the workers employed, charitable causes, and so on. Under these circumstances, managers may explicitly or implicitly donate the shareholders' money toward causes that they deem to be more worthwhile than paying dividends.

I am descriptive here and am not stating what is necessarily the most appropriate. The web chapter on ethics deals with this question in more detail.

SIDE NOTE: Some economists' models assume that executives prefer working less (called "shirking"). However, lack of work ethics among executives is rarely a problem in the real world—instead, it is self-enrichment that is the problem.

24·2.C. The Incentive of the Entrepreneur to Control Temptations

We now show that entrepreneurs (i.e., coordinated owners) benefit from good corporate governance.

Are all these problems real? Yes, they are. So, in order to get you, as a private outside investor, to give your money to a firm, it must create mechanisms that will curtail its misbehavior. If these mechanisms are not in place, the firm will be unable to raise more capital at reasonable costs. (Yet, as we shall see later, if manager-agents are already firmly in charge of shareholder-principals, they may not care about "reasonable" costs of capital, either.)

An Example of the Entrepreneur's Incentives

The argument that it is in the interest of the owners to get the governance right is really the same as the argument that it is in the interest of the owners to get the debt-equity ratio right. Here is a simple illustration of how good corporate governance can benefit the original owner:

- Start with a penniless entrepreneur who has an invention that requires \$25 million investment but that will be worth \$100 million in today's dollars. If the entrepreneur cannot raise the money, the project will be captured by the competition. Without capital, the entrepreneur's wealth would be \$0. There are large gains to finding investors. This is why companies go public to begin with: gains from diversification and capital outweigh the costs of agency conflicts.
- If the owner does not need to raise any funds and could avoid all professional management, his net worth is \$75 million. Similarly, if the owner can somehow commit to avoid

all agency problems, investors can give him \$25 million for 25% of the company, leaving him again with \$75 million.

- Investors may rationally believe that agency problems will appear as soon as money is raised. The particular agency problem in our example is that the entrepreneur cannot prevent the grabbing of \$30 million by management-in-charge after the capital has been raised—including by himself. That is, he cannot commit the corporation to control the agency problem. Furthermore, to hide the managerial scheming, this "theft" will require another \$10 million of waste.

  If the entrepreneur must now raise external capital, will investors be satisfied with 25% of the company for a \$25 million investment? Probably not: any future manager—*including the entrepreneur himself*—will want to steal the money. Investors expect this, and therefore value the company only at $\$100 - \$30 - \$10 = \$60$ million. To raise \$25 million requires the entrepreneur to part with $\$25/\$65 = 42\%$ of the company. The entrepreneur's net worth, if he can remain in charge, will be the $58\% \cdot \$60 = \$35$ million that he will own, plus the \$30 million that he can steal. The \$65 million is \$10 million less than what he could have gotten if he could have committed to zero future managerial agency problems—still better than the \$0 if he were not to raise any external funding.

- The entrepreneur's situation becomes outright dire if the project duration exceeds his lifetime, and he must hand it over to professional managers. In this case, he will only earn \$35 million—the \$30 million in theft will go to future management. (The entrepreneur may somehow be able to award the management to the team that agrees to share the \$30 million with him—but this will lead to a whole new set of agency conflicts.)

In sum, our entrepreneur is best off if he can just prevent all future managerial theft (his own and future managers') by instituting good corporate governance. Our entrepreneur is worse off if he cannot prevent future theft by himself. Our entrepreneur is even worse off if he cannot prevent future theft by other managers. And our entrepreneur is worst off if he does not take the project. It is also not difficult to construct examples in which future theft completely prevents any ability to raise funds for otherwise productive projects. The main insight from this example is that owner-entrepreneurs have an incentive to control agency problems in order to be able to raise capital at good terms.

How Strong are the Entrepreneur's Control Incentives?

The control of agency problems is often neither easy nor worthwhile for the original entrepreneurs.

We know that the original owner-entrepreneur—unlike subsequently hired executives—can capture the gains from agency controls. An important question is to what extent an entrepreneur would write contracts upfront (*ex-ante*) that control all these agency issues. There are at least two limiting factors:

1. It is impossible to write contracts for all future contingencies, especially insofar as managerial schemes are concerned. The human mind can be very creative: What a piece of work is man! how noble in reason! how infinite in faculty. Worse, many agency control clauses could even be counterproductive if they rob executives of flexibility that could be used to increase firm value.

2. The entrepreneur's incentives to write the appropriate contracts may be surprisingly modest. Few companies are designed for greatness in the far future. When Thomas A. Edison designed the corporate charter of *General Electric* in 1880, he probably did not do so with an eye towards General Electric managers in the 21st century. Indeed, most companies that go public will never face large agency problems—most will simply end up bankrupt. Only 1 out of 100 may become large enough to indulge significant agency conflict—say, costing 1% of firm value. One percent of a \$100 billion company is \$1 billion (say, \$100 million a year as a 10% perpetuity), but in *ex-ante terms*, it is a cost of $1/100 \cdot 1\% \approx 0.01\%$ of the entrepreneur's value.

   Moreover, it is unlikely that the entrepreneur could even capture this much. The question is to what extent investors would understand better corporate governance controls and

> be willing to pay for them. How many investors would have paid Edison more money for GE in the year 1900 if GE had put better incentives into place for the year 2000? In the next section, we will discuss some mechanisms, such as takeovers, by which shareholders can rein in poor management in already publicly traded companies. Suffice it to say here that these mechanisms are expensive and therefore no panacea, either.

This leads us to a mixed conclusion: The entrepreneur's incentive to control immediate managerial agency conflicts is probably fairly strong. However, the entrepreneur's incentive to set up an effective charter for the long run—if even possible—is modest. Thus, it is not surprising that we see many older Fortune 500 companies in which the entrepreneurs' design no longer plays much of a role in shareholders' control over management. Thus, we need to look towards other mechanisms that can substitute for the failing role of upfront corporate design as the corporation ages.

Solve Now!

Q 24.1 *What are the main control rights of debt and equity*

Q 24.2 *Describe the main illegal and legal temptations that managers face in their duty to maximize shareholder wealth.*

Q 24.3 *When are the incentives to control agency conflict strongest? Can you give a numerical example?*

Q 24.4 *What limits are there to writing a corporate charter that eliminates future agency conflict?*

24·3. Equity Protection

The primary control right of equity is its vote, but there is also the need to raise funds and some external mechanisms.

After the firm has gone public and shareholdings have become diffuse, what can control management? We will discuss a number of possible mechanisms. We begin by reexamining the need to raise financing after the IPO. Recall that the sale of the firm was the primary mechanism to motivate the entrepreneur to control conflict of interest when the firm starts. We then discuss mechanisms that are based on the most important formal control right of public equity, which is its right to vote. Ultimately stemming from the right to vote are three further control mechanisms:

1. Shareholders can vote in the corporate board, which can replace the management or liquidate the firm.
2. Unfortunately, in a widely held company, gathering votes to control management and corporate change is not cheap. The right to vote is therefore of much use only during unusual situations, such as a proxy contest or a hostile takeover. We therefore look at this "external market for corporate control" in more detail.
3. Large-block shareholders can more easily influence management, because managers know that poor performance can lead them to withdraw their support and throw it to a potential buyer. However, large shareholders can also do more harm than good.

Finally, we look at two external control mechanisms:

1. The legal environment regulates what managers and board member can and must do.
2. Ethical considerations and adverse publicity can constrain the norms that govern the behavior of managers.

24·3.A. Subsequent Equity Offerings

Among publicly traded, older, cash cow companies, even the need to raise capital need not help.

We first continue with the example from the previous section. We want to ask whether the need to raise capital provides managers with the incentives to control agency conflicts, just as it provides the incentives for the entrepreneur to control agency conflicts. The answer is often no. Unlike the owners, the managers of an already publicly traded corporation typically own little of the firm. Therefore, their incentives to curtail agency issues are weak or even perverse—they are the immediate beneficiaries of agency problems. Once they are in charge, their desire for more power and control is likely to quickly overcome their desire to control agency issues. This is especially pertinent in widely held, large, old, cash-rich firms, in which the executives/boards have enjoyed long tenure. In this case, the need to raise capital is not necessarily an inducement to institute good corporate governance. In fact, quite the opposite can happen.

- Assume that professional managers are now firmly in charge of our \$60 million firm from above. They now happen to find another project that costs \$50 million, which produces cash flows of \$50 million in today's dollars for an NPV of \$0, but which allows for an additional \$20 million of managerial theft, leaving owners with only \$30 million. (Such projects are easy to find.)

  We know that the existing company is worth \$60 million. If the new project is taken, new shareholders will own a claim on \$30 million in value from the new project and \$60 million in value from the old project. To raise \$50 million in capital for a \$90 million company requires issuing shares worth 55% of the company. Old shareowners will now own only 45% of the company for $45\%\cdot\$90 = \$40$ million in the new firm, down from \$60 million. In effect, the \$20 million agency cost is now paid to buy corporate growth at the expense of old shareholders—growth that the managers will enjoy.

- In fact, fearing similar expropriation in the future, new shareholders may demand even more than 55% of the company—and managers have the incentive to give it to them in order to execute this new project.

Thus, the need to raise capital is not a guarantee that the management of a publicly traded corporation will want to control agency problems. On the contrary, raising capital can become yet another mechanism that helps managers extract shareholder wealth for themselves. Old capital in effect allows new capital to be raised, and thereby allows managers to expand the firm for a long time. Even if managerial looting has reduced the value of $10 million of old equity into just $1 million now, managers might still want to raise another $1 million in capital for their personal consumption by promising 51% of the new firm, leaving old shareholders with only $490,000.

24·3.B. The Corporate Board

The board is supposed to represent shareholders and control managers.

We now move on to the main right of public shareholders: during the annual meetings, shareholders get to vote on the **corporate board**. It is the board's legal responsibility to oversee management and to ensure that managers are acting in the interests of shareholders. The board is the legal principal of the corporation.

The agenda and information available are important sources of real power.

The **Chairman of the Board** not only directs management to produce the necessary information, but also controls the board's agenda. Ultimately, the chairman does have to rely on management, though, in asking for the right information to present for discussion. The power to set the agenda and control the information available should not be underestimated. After all, with only a couple of days per year on the job, and with their own full-time jobs to attend to, board members cannot possibly know the business in great detail. Thousands of pages of readings as preparation for the board meeting—possibly with, possibly without key information—are often just as useful to board members as zero pages. And board members know that if they do not stick to the specific agenda, the risk is that the discussion will degenerate into long-winded, unfocused conversations. Not surprisingly, large boards are usually less effective.

Most corporations have no Chairman to oversee the CEO.

In many U.S. corporations, the power of the chairman relative to that of the CEO does not even matter *because the CEO is also the chairman of the board.* For example, in 2003, out of the thirty Dow-Jones Index companies, only four (General Motors, Intel, Microsoft, and Wal-Mart) had both a CEO and a chairman. This arrangement obviously makes it highly unlikely that the chairman of the board will control and, if necessary, discipline the management.

The board is often controlled by the CEO. There usually is no meaningful shareholder vote for directors.

How independent are other directors? New members of the board are usually either proposed by the chairman of the board or nominated by a committee of other board members. The board is then put forth *as a slate* for an up-or-down vote at the annual shareholder meeting. Shareholders cannot vote for or against particular candidates. (In the extreme, a shareholder with 49% of the shares could find herself with zero board representation.) Most corporate board elections are about as democratic and thrilling as elections in North Korea. The most common outcome is that between one-third and two-thirds of the board are also employees of the company, and thus under the direct day-to-day control of the CEO.

Many firms have a majority of corporate employees on the board. (Such) boards tend not to fire poorly performing managers.

Michael Weisbach studied 495 corporate boards from 1974 to 1983 and classified directors as insiders if they were full employees of the company. This would necessarily put them under the direct control of the CEO. Only about one-half of the 495 NYSE corporate boards even had a majority of outside directors! Only 128 had boards with clear majorities of outside directors, though many of these had their own dealings with the company and were thus also conflicted. Fortunately, the presence of inside rather than outside directors does not seem to matter much, either. In the ten years from 1974 to 1983, the probability that a manager would depart increased only from around 5% to 6% when a company lost 33% (!) of its stock market value (market adjusted)—and this retirement increase may have been because the manager was already close to retirement (and had maximized his own take). Firms with more than 60% outside directors only had an additional 1% resignation frequency; and for firms that lost "only" 10% to 25% of their values, having a majority of outside directors did not even increase the resignation frequency at all.

As corporate control mechanisms, except in extreme situations, corporate boards are ineffective.

In sum, it is usually the case that it is not the board that controls the CEO, but the CEO who controls the board. Almost any CEO, who was originally successful and who is eager to gain full control, can stack the board with dependents and friends within a couple of years. Consequently, in most corporations, the boards are effective control mechanisms only in three cases: first, when there is a large influential shareholder to whom some board members owe loyalty; second, when the CEO/Chairman is fairly new and has not yet taken full control of the board (or has not shown the appropriate interest in doing so, a rare but occasional condition); and third, when the CEO's misbehavior is so egregious that board members begin to fear negative publicity and personal legal liability. We discuss the ability of the media to embarrass managers—and set politicians, lawmakers, and enforcement into motion—below.

Corporate boards can serve other mechanisms.

When it comes to proactive control of managerial misbehavior, most corporate boards in the United States today are more theatrical stages than effective corporate control mechanisms. This is not to say that corporate boards do not serve other useful functions. For example, they can advise executives, they can signal a commitment to diversity, they can help build relationships with suppliers and customers, and they can help to find a new CEO if the current CEO suddenly "evaporates." The discrepancy between the supposed role and the actual role for many boards is so large that many reform ideas focus on improving the independence of corporate boards. If legal reform were to reduce the cozy relationship between board and management, management could indeed be better controlled—but it could come with cost. It might allow large shareholders to extort more value for themselves at the expense of small shareholders, it might reduce other beneficial functions of the board (better relations with suppliers, etc.), and it could even destroy the company, if the relationship between management and board were to degenerate into war.

24·3.C. The Role of Votes

Takeovers, Proxy Contests, and Shareholder Resolutions

Hostile takeovers seek to acquire and then vote shares. Proxy contests just seek votes to change management and the board.

The right to vote becomes relevant during a **hostile takeover** (formally called an **unsolicited bid**), in which an acquirer makes a **tender offer** to purchase shares in order to obtain either the whole firm or a voting majority. In a **proxy contest**, a large shareholder actively solicits other shareholders to vote against management's board and in favor of an alternative board. Often, the two go together, in that a hostile acquirer also launches a proxy contest to eliminate the board and charter provisions that would prevent him from purchasing all shares.

ANECDOTE: Executive Succession in Action

Through board control, CEOs usually can often determine their successors (and more than one successor has found huge skeletons in the closet). Recall that on many boards, the Chairman/CEO has considerable influence over which board members should retire and who the next board's members should be. Of course, these board members in turn nominate the executive compensation committee, who in turn decide on the Chairman/CEO's compensation.

A study of compensation committee membership found that when a director sits on the executive compensation committees (which determine the pay of the managers) of multiple firms, these tend to have similar executive severance pay packages. After all, it is easier to argue for a higher compensation package for oneself, when one can convey authoritatively that the CEO of a similar company required and received a higher pay package. Conversely, would you think it easy to argue to your own board that you should be paid more if you just managed to severely reduce the compensation of another CEO?

For example, the chair of IBM's compensation committee is Charles Knight, whose own exit package from Emersen Electric had a provision similar to one in Jack Welch's package from General Electric. Not surprisingly, IBM CEO Lou Gerstner's separation package was similar to Jack Welch's. (Both Knight's package and Gerstner's package were among the most generous around.) Ivan Seidenberg, CEO of Verizon, was singled out by the report for enjoying one of the most egregious severance packages. Seidenberg sits on Honeywell's compensation committee—*and Honeywell was also singled out.* Source: corporatelibrary.com.

How to get a shareholder proposal up for vote.

A more modest form of the proxy contest is the **shareholder proposal**, which only rarely seeks to eliminate management outright. Shareholder proposals have been particularly successful in removing anti-takeover defenses, including in some cases the staggering of the board (see below). However, they are often not binding, and can therefore be ignored by the board. So they are only useful in setting the stage for later actions against poorly performing management. Any shareholder can put forth a shareholder proposal or proxy contest for vote by all shareholders. The SEC judges whether shareholder proxy suggestions are appropriate for shareholder vote. (The rules by which the SEC accepts or rejects shareholder proposals are explained in www.sec.gov/interps/legal/cfslb14.htm.) In addition, many shareholder proposals are brought by special interest groups, such as churches or labor unions, and are not necessarily in the interest of shareholders, either.

Gains must be very large to make voting action worthwhile. Proxy contests are costly and (only) modestly successful.

Obviously, engineering a hostile takeover is neither cheap nor easy. But even proxy contests and shareholder resolutions are costly and only occasionally successful mechanisms. Clearly, to wage either, the value gains from dislodging management must be large enough. "Modest" governance problems, such as an executive salary of \$100 million in a \$10 billion company (1% of value), are just not enough to make the expense worthwhile. Therefore, proxy contests are rare. For example, *Institutional Shareholder Services* (issproxy.com) reports that there were 17 proxy contests in the first 8 months of 2003, of which only 4 resulted in dissident victories. The average dissident's cost per proxy contest was about \$1 million. (The highest cost was over \$5 million.) Nevertheless, the small success ratio is misleading, because even the threat of a shareholder proxy contest can lead the executives to seek a compromise to rectify some of the problems. And, compared to hostile takeovers, proxy contests are outright cheap.

Defensive Strategies

Management can resist. Staggered boards virtually eliminate all hostile takeovers.

Management can resist hostile takeover and proxy contest attempts through many actions, collectively sometimes called **shark repellants**, such as the following:

Greenmail Management uses shareholders' money to "buy off" the shares of a a potential acquirer at a premium.

Golden Parachutes Management lets itself be bought off by the acquirer.

Acquisitions A bigger company is more difficult to take over—the "blowfish" strategy.

Poison Pills Other shareholders become entitled to purchase more shares at a discount. The potential raider would then have to repurchase those shares at a higher price.

Fair Value Provisions An acquirer is forced to pay every shareholder the same price, i.e., the highest price at which shares are acquired.

Supermajority Rules An acquirer needs to obtain more than just a majority of votes to replace the board.

Litigation Management can delay a potential takeover in the courts, especially if the potential acquirer is in the same industry, in which anti-trust issues can come into play.

Scorched Earth Management can threaten to sell off corporate assets that are of particular interest to the acquirer.

New Share Issuance Management can issue more shares to employees and themselves.

Staggered Boards Each year, only a third of the directors are up for reelection. Therefore, no outsider can take control of a company during one annual meeting—even if the outside purchases 100% of the shares before the annual meetings, he can only replace one-third of the board. The remaining two-thirds will remain in office, which means that the company will remain under the control of the existing board for at least one more year, during which the existing management can do a lot of harm.

Staggered boards are the most effective defense. There have been no successful hostile takeovers of firms with effectively staggered boards. Not all managerial resistance is value-reducing. For example, resistance could force a potential acquirer to pay more for the firm, management could make a competing tender offer for its own shares, or management could pay out free cash to shareholders.

> **SIDE NOTE**: Not all takeovers are driven by poor target management: other reasons are industry consolidation and acquisition of monopoly power, desire by acquirers to increase their own empires, and a desire to take advantage of corporate tax shelters. That is, takeovers may be also occur independent of the target's managerial performance, and they may increase or destroy value.

Some history.

A particular form of a takeover is the **leveraged buyout** (**LBO**). Especially in the 1980s, there was a window when small private holding companies were able to borrow significant amounts to take over much larger publicly-traded companies in **leveraged buyouts** (**LBOs**). The most prominent LBO was the takeover of RJR Nabisco by **Kohlberg, Kravis, Roberts** (**KKR**). Because the majority of financing was debt, KKR owned only a small slice of very high-powered equity and even modest post-LBO underperformance could result in a total investment loss for KKR. This gave them enormous incentives to get everything right. In the typical LBO, they would either fire existing management or completely restructure the existing management compensation contracts in order to dramatically improve managerial incentives. Most LBOs created a lot of value, through better control of agency problems plus tax benefits, and much of it went to the existing shareholders in the price they received for tendering their shares. However, by the 1990s, public market valuations had generally increased, management generally began to pay more attention to shareholders, and it became harder and harder to find companies that could be purchased cheaply and then improved. But perhaps most importantly, companies learned how to institute takeover defenses that would be too expensive for a successful acquirer to overcome. Thus, at least for the time being, corporate governance through external takeovers, and especially through leveraged buyouts, has faded into the background.

Empirical Evidence

Takeover Activity in the United States.

Securities Data Corp (SDC) reports that from 1979 to 2002, there were 40,983 domestic takeovers (incl. LBOs), with $7.6 trillion in total value. Takeovers here include both public and private firms, as well as leveraged buyouts. Most of this acquisition activity was solicited by or occurred with the blessing of target management. Such takeovers are called "friendly." It is only the "hostile," or at least "neutral," takeovers that are likely to be a real threat to poorly performing management.

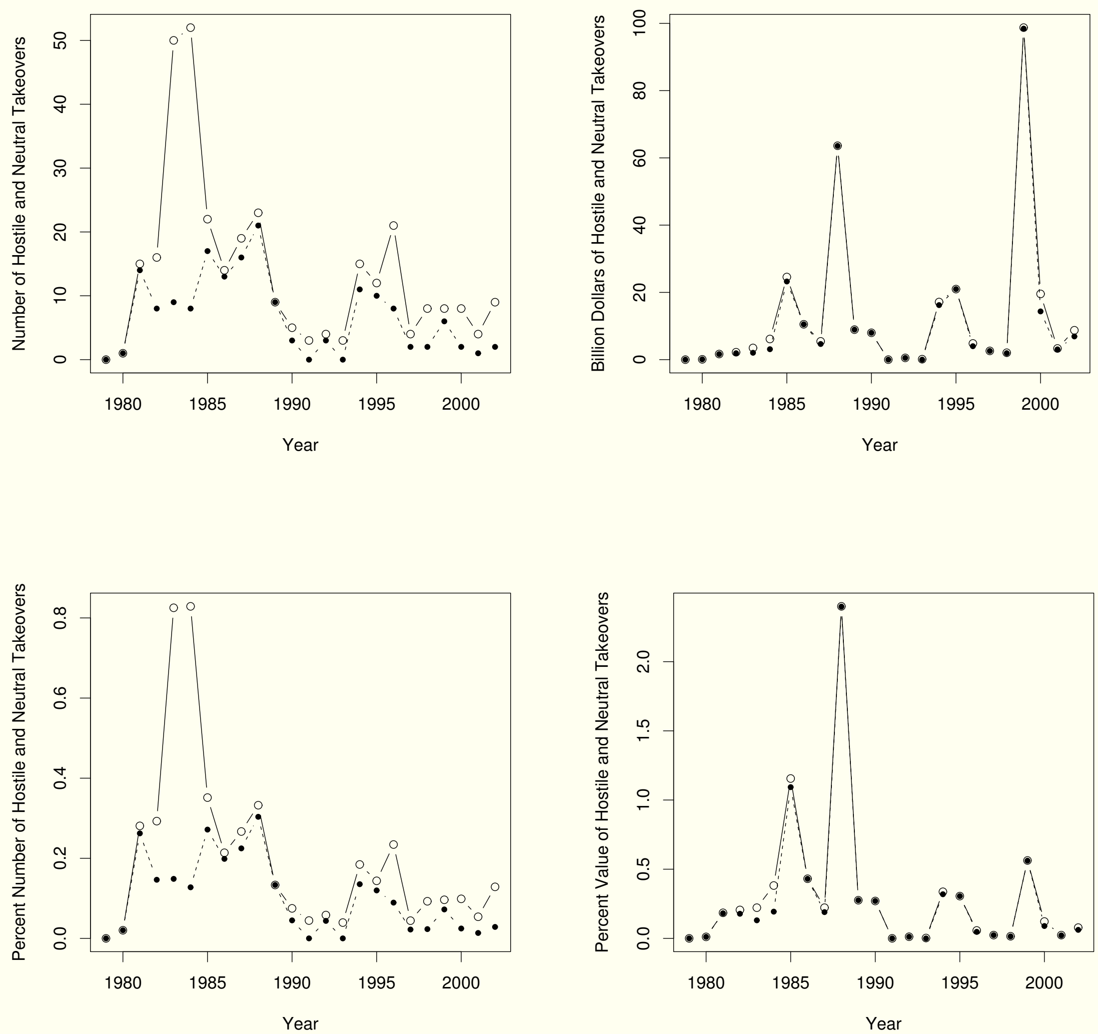
> **ANECDOTE**: RJR, Ego, and Overpayment
> The aforementioned bestseller *Barbarians at the Gate*, also made into a movie, describes the epic takeover battle for *RJR Nabisco* between *Kohlberg Kravis Roberts* (K.K.R.) and RJR management (supported by Shearson Lehman [now Lehman Brothers]). In October 1988, RJR's CEO Ross Johnson and his predecessors had mismanaged the company long enough to allow him to offer RJR shareholders the premium price of $17.6 billion in a leveraged management buyout. This required the resignation of Johnson from the board contemplating the offer, which in turn opened the door to a $20.6 billion counteroffer by KKR. Eventually, KKR purchased RJR for $25 billion, and Johnson got a $53 million golden parachute. This takeover was also probably KKR's biggest miscalculation in that it overpaid for RJR. The prime reasons were personal egos and animosities, which fueled an irrational bidding war—all to the benefit of RJR shareholders.

Table 24.1. Hostile Takeovers with More Than $5 billion in Target Value, United States to 2003

Announced	Target	Acquirer	Value (billion-US$)
1999/11/04	Warner-Lambert	Pfizer	89.2
1988/10/24	RJR Nabisco	Kohlberg Kravis Roberts	30.6
1988/10/17	Kraft	Philip Morris	13.4
1995/10/18	First Interstate	Wells Fargo Capital	10.9
1994/08/02	American Cyanamid	American Home Products	9.6
2000/11/13	Willamette	Weyerhaeuser	7.9
1990/12/02	NCR	AT&T	7.8
2002/02/22	TRW	Northrop Grumman	6.7
2000/02/22	Mirage Resorts	MGM Grand	6.5
1999/08/11	Reynolds Metals	Alcoa	6.1
1985/10/16	Beatrice Co	Kohlberg Kravis Roberts	6.1
1985/09/24	General Foods	Philip Morris	5.7
1988/01/13	Farmer's Group	BAT PLC	5.2

Source: Securities Data Corp. Not inflation adjusted.

Figure 24.1. Takeover activity in the United States



Source: Securities Data Corp. In the upper diagrams, the total number of both types are circles, the hostile type are fat dots. In the lower diagrams, these are reported as a percent of the total number and the total market value of all publicly traded companies on the NYSE, AMEX, and Nasdaq.

Takeover activity in the US in the 1980s.

Figure 24.1 gives an idea of the relative importance of hostile and neutral takeover activity over time. In the 1980s, there were 8,360 takeovers with \$1.2 trillion in total target value. Only about 1.4% of all takeovers were "hostile"; another 1.2% were "neutral." But hostile acquirers managed to take over some pretty big fish relative to non-hostile acquirers: they accounted for 10% of all takeover value in the 1980s (neutral takeovers for less than 0.6%). How does this compare to the number and value of all publicly traded firms in the United States? In 1982, there were about 5,500 firms with just over \$1 trillion in market value. The 50 or so hostile and neutral takeovers in 1982 and 1983 accounted for 0.8% of all publicly traded firms—in one year...and it was entirely a Kohlberg, Kravis, Roberts effect! This was also the peak of hostile takeover activity.

Takeover Activity in the US in the 1990s.

In the 1990s, the stock market boomed. General corporate takeover activity also heated up, with 25,493 takeovers and \$4.4 trillion in total target value. But *hostile* takeover activity declined, accounting for only 0.2% of all takeovers (down from 1.4% in the 1980s), and neutral takeovers for only 0.1% (down from 1.2%). In terms of target value, hostile takeovers accounted only for 3.5% (down from 10%), neutral takeovers only for 1.2%. Again, how does this compare to the number and value of all publicly traded firms in the United States? If we look at the year 2000, for example, there were about 8,100 firms with about $16 trillion in market value. There were 8 hostile takeovers with about $20 billion in market value, which accounted for just about 0.1% of all publicly traded firms.

Does takeover activity matter?

So, does the thread of a hostile takeover discipline managers? It certainly did in the 1980s, and still probably matters a little today. The sheer visibility and novelty of these takeovers were big enough to prevent the managers of many firms from engaging in the worst abuses. However, as Table 24.1 shows, KKR and its colleagues seemed pretty satiated after RJR Nabisco—the hostile takeover threat generally receded after 1990. Any given year now typically sees only about a handful of hostile takeovers. Their dwindling number indicates that they are no longer the sword of Damocles that is hanging over—and thereby controlling—corporate management.

24·3.D. Large Shareholders

The right to vote can matter if it is actively exercised by just a few large shareholders.

The Benevolent Role

Only large shareholders have an incentive to control agency problems.

It is not worth the time of small, diverse shareholders to attempt to vote and/or to influence management. The costs of meaningful action and coordination are too high, and the benefits to each individual shareholder are too low. (This is an example of the *tragedy of the commons*, in which each individual acts in his or her own personal interest, hoping that other individuals will band together to correct the problems that they all jointly face. But it is in the interest of each individual to "free-ride," so this hope is in vain.) Consequently, any active role is most likely to originate from large shareholders with both enough votes to scare management and enough value-at-stake to take an active interest. But to become a large shareholder is in itself costly, because it foregoes the benefit of risk diversification. Typically, the larger the firm, the smaller the stakes of the largest outside shareholders and therefore the smaller the largest shareholder's influence. Indeed, the evidence suggests that the shareholders tend to be more dispersed among firms with more severe agency problems.

Anecdote: Bribing Shareholders in Proxy Fights
Karla Scherer led the *only* successful proxy contest of a major U.S. publicly held company in 1988. As a result, the company founded by her father in 1933 was sold in June 1989 at a price more than double the value of each shareholder's investment the year before, when the proxy contest began.

The most prominent recent proxy contest occurred in 2002, when Walter Hewlett, a Hewlett-Packard (HP) director and son of cofounder William Hewlett (holding 18% of HP), opposed HP's acquisition of Compaq. He lost the proxy vote after Deutsche Bank (DB) switched 17 million of the 25 million shares it controlled—shares of DB's clients held in the DB asset management division—in favor of the $22 billion merger. This happened after DB had become the co-arranger of a new multi-billion-dollar line of credit. In August 2003, the government fined DB $750,000 for failing to disclose another apparent conflict of interest to DB's asset management client. In a memo to her CEO, HP head Fiorina suggested HP do something "extraordinary" for DB and another firm. HP paid DB's investment banking arm $1 million for "market intelligence," with another $1 million contingent upon success. DB's investment banking arm then helped to convince DB's asset management group of DB's interest—and rightly so. During a conference call with DB money managers, Fiorina then reminded DB that their votes would be "of great importance to our ongoing relationship."

Some other institutional shareholders held shares in the target, Compaq, and therefore also voted in favor. (CalPERS, a prominent pension fund and advocate of better corporate governance, voted with Hewlett.) Net, 838 million shares voted in favor, 793 million shares against the deal. Hewlett alleged that HP spent roughly $150 million of shareholders' money on the proxy fight against him (18% of which he had to effectively pay for).

It is little consolation to the HP shareholderst that the acquisition indeed turned out to be a failure, and that Carla Fiorina was fired by the board in 2005.

Large shareholder influence is limited.

But even the power of large shareholders is limited.

1. Even if large shareholders have some incentives to control management, it is usually not profitable. A shareholder who owns 5% of a firm suffers 100% of the cost of any effort to influence the management, yet reaps only 5% of the benefit.

2. Votes are not secret: managers know exactly how their shareholders vote and can seek retribution later on.

3. If the large shareholder is a mutual fund, it cannot actively seek to influence corporate behavior. If it does, it could run into insider trading laws when it wanted to divest itself of its stake upon learning negative information. Therefore, many large institutional shareholders abstain from actively seeking corporate influence.

However, many passive institutional shareholders still can and often do tend to vote their shares *against* management if a third party were to seek an active influence, e.g., in a proxy contest. The presence of large blocks of shares, even passive shares, which could potentially overwhelm the voting power held by management and their allies, is therefore a low-level, but constant restraint on management.

The Malevolent Role

Large shareholders may not seek better governance, but better treatment for themselves!

It is not in the interest of the executives to pick a fight with their largest shareholders. It could be publicly embarrassing, especially in light of management's fiduciary responsibility towards their shareholders (see below). Instead, most corporate executives seek a cordial arrangement with their large shareholders. Special treatment of large shareholders is usually more effective than confrontation. Such "VIP" goodies can include special access to information, the sharing of corporate perks (such as golf outings), special deals (such as sweetheart deals for the firm—or even the manager of the fund controlling the shares), or greenmail targeted share repurchases (in which the company management uses shareholder money to repurchase pesky institutions' shares at a higher price).

Insiders can also be large shareholders. Oi wei!

Company founders in particular often have a special relation with the company. They often consider the company to be their own and hold enough stock to control it. There is strong empirical evidence suggesting that founders are often detrimental to shareholders *on average*: when the founder of a company suddenly dies, the stock price of the company usually goes up, not down! As with founders, managers can often become large shareholders, too, and this is a double-sided sword. On the one hand, they can incentivize managers to be more eager maximizers of share value: they can benefit more. On the other hand, more shares mean more votes, which in turn means that they are more likely to be able to win any votes. In perspective, the best control of agency problems by a founding large shareholder may be managerial retirement and death—the new manager is unlikely to obtain the same high levels of capture immediately after succession.

Anecdote: CalPERS Top-10 List
The most visible corporate governance activist in the United States is the California Public Employee Retirement System. CalPERS publishes an annual list of worst corporate governance companies (in its portfolio). Among its 2003 winners were Gemstar (GMSTE), JDS Uniphase (JDSU), Manugistics (MANU), Midway Games (MWY), Parametric Technology (PMTC), and Xerox (XRX). The detailed corporate governance shortcomings make interesting reading.

But even CalPERS rarely takes on Fortune-100 companies (which are most prone to suffer from agency conflicts). The reason may not only be political, but the fact that CalPERS' ownership share in Fortune-100 companies is too low to make much of a difference.

Anecdote: Graft in Action: Panavision
In August 2002, *Business Week* reported the end of a two-year drama. In 2001, Ronald Perelman had a 53% stake in *M&F Worldwide Group* (MFW), a publicly traded tobacco ingredient company. Perelman initiated an M&F purchase of Perelman's *Panavision* shares at Perelman's cost of $17/share. At the time, *Panavision* (PVIS.OB), a movie camera maker, traded for $4/share. After more than a year in court with a minority shareholder (a hedge fund that had to pay for its court costs), Perelman graciously agreed to reverse the transaction.

More generally, such **tunneling**—transfers from the corporation to a large or controlling stakeholder—are typically not the most important governance issue in the United States—the behavior of managers vis-a-vis all shareholders is. But in many other countries, small shareholders fear not so much that managers expropriate all shareholders, but that large shareholders expropriate small shareholders. For example, in Europe and Asia, a small number of families control large corporate pyramids, in which firms often trade with one another. If a family owns 100% of one company and 10% of another company, it may nevertheless control both managements, and the sale of a $100 million factory from the latter to the former in exchange for a sweetheart price of $20 million can enrich the former by $80 million, and the family by $72 million.

In other countries, large shareholders may be the most important governance issue.

The Evidence

The degree of power of large shareholders to restrain management and the degree to which the presence of a large shareholder aids small shareholders remain a matter of opinion. In some firms, large shareholders serve a useful role in constraining management, and thereby aid small shareholders. In other firms, large shareholders help themselves to corporate assets, and thereby hurt small shareholders. There is some evidence that firms with large public pension fund investors tend to engage in fewer value-reducing takeovers; that firms with external 5% owners tend to perform better than firms without such; and that managers in poorly performing companies are more often replaced when there are large shareholders. But large shareholders are such a diverse group that it is not possible to generalize further.

Some large shareholders help, other large shareholders hurt.

24·3.E. The Legal Environment

The United States' best aspect of corporate governance is probably its legal environment. Investors are protected by a set of laws, regulations, and court rulings, plus appropriate legal enforcement. Much law has come about through court rulings and judicial precedence. This process has an intrinsic flexibility, which continues to fill gaps created by new problems. Such an evolutionary process is more difficult to accomplish by statutory law. In civil law countries, like France or Belgium, where regulations have to be legislated from the top, investor protections tend to be worse.

U.S. law is ever-evolving.

To become a successful publicly traded company, a U.S. company usually has to satisfy laws and regulations imposed by the state, the federal government, the Securities Exchange Commission (SEC), the National Association of Securities Dealers (NASD), and the Financial Accounting Standards Board (FASB). It also has to try to avoid class action lawsuits, which have bankrupted more than one company. These needs together set minimum standards on corporate behavior—especially on appropriate information disclosure and self-dealing—that are not easy to skirt.

There are many regulations that try to ensure minimum decent corporate governance.

Shareholders' single most important and broadest legal protection is management's legal fiduciary responsibility to act on shareholders' behalves. Black's *Law Dictionary* defines a fiduciary relationship as one "in which one person is under a duty to act for the benefit of the others." The seminal opinion on fiduciary duty was written by the New York Court of Appeals in 1984:

Shareholder's best protection is fiduciary responsibility.

> Because the power to manage the affairs of a corporation is vested in the directors and majority shareholders, they are cast in the fiduciary role of "guardians of the corporate welfare." In this position of trust, they have an obligation to all shareholders to adhere to fiduciary standards of conduct and to exercise their responsibilities

ANECDOTE: Board Courage at Citigroup
Although biased, the PBS series Frontline episode *The Wall Street Fix* (www.pbs.org) illuminates many of the conflicts of interest between ordinary shareholders and larger stakeholders. It details how Jack Grubman, star analyst for the investment bank of Salomon Smith Barney, hyped Worldcom in 2000 to its brokerage's small retail investors. At the same time, the CEO of Worldcom, Bernie Ebbers, held a personal $1 billion mortgage from Travelers. Both SSB and Travelers are owned by Citigroup (C). Ebbers' wealth (and therefore his $1 billion mortgage) was closely tied to the Worldcom stock value. In 2005, Ebbers was convicted of corporate fraud.

In a display of less than extraordinary courage, after the indictment of Citigroup for a variety of questionable activities, the Citigroup board voted its full support and confidence in its CEO, Sandy Weill. *Business Week* was not so generous: in January 2003, it ranked Sandy Weil as the worst manager in America.

> in good faith when undertaking any corporate action. Actions that may accord with statutory requirements are still subject to the limitation that such conduct may not be for the aggrandizement or undue advantage of the fiduciary to the exclusion or detriment of the stockholders.
>
> The fiduciary must treat all shareholders, majority and minority, fairly. Moreover, all corporate responsibilities must be discharged in good faith and with "conscientious fairness, morality and honesty in purpose." Also imposed are the obligations of candor and of good and prudent management of the corporation. When a breach of fiduciary duty occurs, that action will be considered unlawful and the aggrieved shareholder may be entitled to equitable relief.

In other words, management's fiduciary responsibility primarily limits excessive self-dealing, especially transactions between the management of a public company and the public company itself. It does not extend to ordinary business decisions. In fact, the **business judgment rule** protects managers against lawsuits if they make poor choices in the execution of most other company affairs. (Otherwise, our litigious climate would paralyze them!)

Actual enforcement is important, too.

The importance of *enforcement* of laws (rather than just what is on the books) is not to be overlooked. The United States has strong civil (financial) and criminal penalties and enforcement for the range of actions detailed in Section 24·2.A. (Although the wheels of American justice are not perfect and only grind slowly, usually taking years to resolve even clearcut cases, they do grind.)

Firms can choose some then-legally-binding mechanisms.

Companies have some discretion to choose the laws and regulations under which they are operating. For example, firms can choose a particular auditor, stock exchange (with exchange rules), a particular investment banker, a particular set of warranties, a particular collateral. Large multinational firms can even choose which country to incorporate in (or reincorporate in). Of course, a firm that reincorporates itself in Russia, hires a no-name auditor, lists on the Moscow Stock Exchange, self-underwrites securities, and gives no promises or collateral is likely not to be able to raise much equity capital.

24·3.F. Ethics, Publicity, and Reputation

Managers are self-interested, but most are not criminals or unethical people.

Ethics is an important factor that constrains many managers (and is often sadly underestimated by economists). Most CEOs want to do well for themselves, but they want to do so only "within the bounds of the normal, accepted, ethical range of actions." Staying within the bounds of the ordinary also reduces the concern for negative publicity and legal liability for violation of their fiduciary duties.

Anecdote: Disclosure Rights Outside the United States
If you believe the U.S. corporate governance situation is bad, wait until you learn the situation in other countries.

In Germany, until recently, insider trading was legal. Disclosure standards are modest. Minority shareholders have few rights against self-dealing by majority shareholders, which are themselves often other corporations. Executives have legal obligations not only to shareholders, but also to employees. But the most amazing fact is that many German firms are owned by complex webs of other firms, which in turn are owned by yet other sets of firms. Ultimately, most large publicly traded firms are owned by the banks. The banks in turn are owned by...themselves! Deutsche Bank holds voting rights for 47.2% of its shares; Dresdner for 59.25%, Commerzbank for 30.29%. (Source: Charkham (1994).) This makes effective control by the ultimate owners very difficult. Many German banks even own themselves!

But Germany looks like investor heaven relative to Russia. In Russia, shares can be declared void by the board at any time; majority share owners cannot force an issue onto the corporate agenda; and even physical threats against pesky shareholders are not unheard of. (And do not look to courts and police for protection: judicial and political corruption in Russia is rampant.)

Ethical standards are relative and changing.

Yet ethical standards are themselves defined by CEOs as a group—and these have slipped over time. In some dimensions, the race seems to have been to the bottom. For example, one hundred years ago, the financier J.P. Morgan argued that no CEO should make more than 20 times what the average company employee earns. The average today is almost 200 times. Consequently, being paid 200 times an average worker's pay does not violate the ethical boundary of a CEO today. Similar arguments apply to almost every other issue in corporate governance: if a practice is commonplace among her peers, it is unlikely to violate an executive's sense of appropriateness.

Lack of transparency hints that pay packages are constrained more by board capture than driven by incentive issues.

The desire to avoid negative publicity is also an important constraint on executive compensation. Negative publicity seems also to be responsible as to why managerial compensation has come to consist of many complex components. The complexity renders pay packages fairly opaque to the press. Researchers are often similarly bewildered when they try to determine whether executive pay is primarily linked to the need to incentivize managers to seek out corporate performance or primarily due to managerial board capture. Both seem to matter, but there is some evidence that obfuscation is particularly important. First, the less-visible retirement packages are often higher even than reported compensation packages. Second, boards often change the terms of executive options that would otherwise expire worthless. Both of these facts indicate that it is not the incentives that are important.

Reputation sometimes constrains managers.

Corporations can also reduce their financing credibility problem through building **reputations**. A manager who has once harmed investors is much less likely to be able to raise capital in the future. Conversely, a company that has a long history of treating investors well (e.g., paying dividends and repurchasing shares) often has an easier time raising capital than a company that has just started up. Reputation may also play a role when a manager is CEO of only a small company, and has his sights set on being selected manager of a larger company in the future. To receive a higher call (with more opportunities to become richer), the manager must constrain his self-interest for a while. One problem with reputation as an agency control mechanism is that managers close to retirement no longer care as much about their reputations as they care about their severance packages. Most CEOs retire, rather than graduate to bigger companies.

ANECDOTE: The fox guarding the hen house: The N.Y.S.E.
It is well-known that the New York Stock Exchange (NYSE) is not a publicly traded company, but is owned by its members, primarily by investment banks like Goldman Sachs. The members appoint the NYSE board. But it is less well-known that the NYSE is an odd creature in another respect. It is both a stock exchange and a regulatory agency. The SEC relies heavily on the NYSE to ensure good corporate governance among its members and its traded firms, which represent almost all large U.S. corporations (with the exception of the technology sector).

As guardian of good corporate governance, arrangements at the NYSE are particularly relevant—but remarkably conflicted. The NYSE board decides on its chairman's compensation package. The chairman regulates its members. The NYSE members appoint the board. The board appoints the chairman and sets the chairman's pay package. The chairman regulates the members who appoint the board. The board pays the chairman. The chain is circular.

In August 2003, the media found out that Richard Grasso, the NYSE Chairman, held a retirement package worth \$140 million—about four times the annual profits of the NYSE. The media later found an additional \$48 million in pay, which Grasso then publicly and graciously declined. (But he never did so in writing.) After more press digging, it was revealed that Grasso also helped pick the executive compensation committee. Many large institutional shareholders then joined the chorus, publicly demanding Grasso's resignation. On September 17, 2003, Grasso finally bowed to the board's discontent—but he did not resign outright. Meeting with his lawyers, he learned that by forcing the board to terminate him (rather than by resigning), he would receive an additional \$57.7 million, in addition to the \$140 million deferred compensation—which he did.

In 2004, Grasso sued the NYSE for \$50 million more, because his contract of 2003 contained a clause that forbade exchange executives from making any statement against Grasso if he left the NYSE. In March 2005, Grasso further sued the former chairman of the exchange's compensation committee for having overseen the approval of Grasso's pay package. As of 2005, Grasso still had one suit against the exchange for \$50 million, but he has received his \$193 million in compensation and pension benefits. (In other litigation, the New York Attorney General seeks to recover \$100 million from Grasso as "excessive compensation.")

24·3.G. Conclusion

Various mechanisms try to constrain agency problems.

Capitalism will not collapse because of managerial theft and misbehavior, even if corporate governance in many public corporations is largely broken—perhaps because theft can only be *so* large. Agency control works in some companies and fails in others. Like the agency problems themselves, the solutions to agency problems are complex. In broad strokes, today's mechanisms involve the combination of corporate obligations (promises by the corporation), legal obligations, and informal and ethical obligations. Ultimately, in today's system, if executives have no scruples, even the best legal and corporate systems are unlikely to succeed in curbing all misbehavior. But even though capitalism as a system will not collapse over poor corporate governance, individual economies may. Arguably, a country that has better corporate governance is likely to outcompete other countries and prosper. It is a matter of great importance to economic competitiveness to seek to improve it.

Solve Now!

Q 24.5 *Does the desire to raise equity capital control managerial agency conflicts?*

Q 24.6 *What are some of the reasons why corporate boards may have limited ability to control the CEO? What other roles may boards serve?*

Q 24.7 *What are some of the reasons why proxy and takeover contests may have limited ability to control the CEO? How is a shareholder proposal different?*

Q 24.8 *What are some of the reasons why large shareholders may have limited ability to control the CEO?*

Q 24.9 *What are some of the reasons why the rule of law may have limited ability to control the CEO?*

Q 24.10 *What are some of the reasons why ethical standards may have limited ability to control the CEO?*

Q 24.11 *What can an executive do to resist a takeover? What has been the most effective anti-takeover device?*

Q 24.12 *What is an LBO? How common are LBOs?*

Q 24.13 *What fraction of takeovers are hostile?*

Q 24.14 *Is the presence of large shareholders always good from an agency perspective?*

24·4. Debt Protection

Equity needs constant, expensive supervision.

Equity payoffs depend very sensitively on good management control and actions and accurate accounting (verification). Even if they are firmly in charge, equityholders have the unenviable task of determining whether poor performance is the fault of management, the market, or both.

Debt has a much easier task: collect promised amounts, or seize assets.

Unlike equity, creditors do not need to play a large role in the day-to-day operations of the company in order to receive most of their due. Ascertaining the value of collateral is cheaper than ascertaining the value of equity (with its future growth options). And if cash is not paid when promised—regardless of whether it is because the market environment is bad, because management has performed poorly, or because management just hides assets—the company falls into automatic default (usually bankruptcy and/or corporate liquidation), and creditors can take control of the company and/or the collateral. Therefore, creditors need not spend much time and money investigating managers.

Some typical covenants.

We have already discussed in Chapter 23 that creditors usually demand and receive covenants, by which the firm must live. Covenants may include collateral, priority, the naming of an auditor, the specification of minimal financial ratios (e.g., dividend payout ratio), and many more terms. Default occurs when covenants are not met. Importantly, coordinated creditor action upon delinquency is not required, because such mechanisms are designed at inception. (If the creditor is a single large bank, this is not necessary.) In the case of a public bond, the covenants designate a trustee to oversee performance of covenants. The trustee has the obligation to declare a bond in default when the covenants are not met. (The process is mechanical.) Therefore, in contrast to equity holders, bond holders do not commonly suffer from free-rider problems.

Bankruptcy is really bad for management.

Management will try to avoid default like the plague. The reason is not just that equity owners, on whose behalves managers supposedly act, lose access to the firm's future projects. The more important reason is that corporate management is replaced in virtually all bankrupt companies. This gives management and shareholder-owners an enormous incentive to avoid default/bankruptcy.

Manipulation of bondholder rights is possible, but it is not easy.

Although there are some escape mechanisms that permit management to manipulate the covenants, these are rare and slow. The first such mechanism is a "forced exchange offer," in which managers set up a prisoner's dilemma that makes it in the interest of every individual bondholder to exchange their current bonds for less worthy bonds but of higher seniority—even though it is not in the bondholders' collective interest. The second mechanism is a covenant amendment, which must be approved by the bond trustee and voted on by bondholders. The third mechanism is asset sales or divisional splits, which require major corporate surgery. For example, when Marriott Corporation announced that it would split into two companies (hotel operator *Marriott International*, MRT, and a real-estate investment trust *Host Mariott*, HMT) in 1992, its share price rose by 10%. Marriott's bondholders sued, because the old Marriott debt now would be owed only by one descendent, *Host Marriott*. Moody's Special Report covering 1970–1992 stated on page 4 that:

> Perhaps the most notorious fallen angel of the year was Marriott Corp., which alone accounted for $2.6 billion of downgraded debt. In October, Marriott announced a controversial spin-off that would relieve the profitable hotel operations business of the heavily indebted real estate and concessions business. Such a move would have the effect of creating one very healthy and essentially debt-free company, Marriott International Inc., and another substantially weaker debt-laded firm, Host Marriott Corp. While issuer-bondholder talks are ongoing in the Marriott case, investors worry that such lopsided spin-offs may become more popular in the future.

Nevertheless, these are the exceptions rather than the rule. It is generally much harder for management to escape bondholder discipline than it is for them to escape stockholder discipline. In turn, this can even help shareholders—even though liquidation almost always hurts shareholders, the threat of future liquidation upon poor managerial performance can motivate managers and thereby help dispersed public shareholders up-front.

The role of large creditors.

We have earlier talked about how large shareholders cannot only discipline managers but also extort special privileges. A similar issue can arise with creditors. That is, although we have discussed primarily the case in which creditors cannot trust corporations, the opposite can also be the case. (And it can just as much prevent the firm from obtaining viable debt financing.) A creditor may be able to pull its line of credit and thereby threaten management or expropriate the firm's equity (receiving control of the firm). Banks attempt to build a reputation for not doing so in order to reduce such borrower concerns.

Solve Now!

Q 24.15 *Why does management often prefer to avoid financial distress?*

24·5. The Effectiveness of Corporate Governance

24·5.A. An Opinion: What Works and What Does not Work

Give up all hope—or don't give up hope.

In the real world, it is impossible to design corporate contracts and arrangements that result in perfect ("first-best") managerial behavior. If we want to maximize wealth, we have to live with imperfection ("second-best"). In equilibrium, we must trade off the advantages of being public (such as access to more capital and better diversification) against the disadvantages (managerial misbehavior). This is not to condone the latter: just because some shoplifting may be unavoidable does not mean it is right.

There are many mechanisms that work in concert. Now comes my opinion.

Our ultimate tradeoff is achieved not through one, but through a variety of mechanisms. Corporate governance consists of many components, of which the corporate and legal structures are perhaps the most important. Although the many mechanisms all need one another, we can wonder what really works. This is a matter of some dispute among economists, so my own view must color my assessment here.

Corporate self-governance in the United States is almost totally broken.

Even if corporate governance in the United States seems to work better than it does elsewhere, it is largely broken. Corporate boards and institutional shareholders have only a modest constraining effect on CEOs in the ordinary course of business. A manager who starts out with a couple of good years and who is bent on taking over control of the company will encounter only mild internal resistance. Once entrenched, it is not corporate self-governance, but only legal and public relations concerns that are likely to constrain the manager. Fortunately, our corporate governance problems are not big enough to destroy most of the wealth created by our multi-billion dollar publicly traded companies, and they won't bring down capitalism, either. But in terms of the wealth siphoned off from the corporate sector into individual pockets and in terms of bad decisions taken, the problem is not modest.

Anecdote: Creditor Protection Outside the United States

In the United States, management can file for Chapter 11 protection, which can delay the turning over of assets to creditors. This option does not exist in many other countries. For example, in Germany, creditors can force practically immediate liquidation of the firm upon non-payment. As a result of poor shareholder protection and strong creditor protection, many German companies are heavily creditor-financed: it is far more difficult for them to find shareholders than it is to find creditors. Many of the largest German companies remain founding-family-financed.

The worst creditor protection usually occurs in the case of sovereign debt (debt issued by countries). There is very little other than a country's desire for a good name and its foreign assets that prevents it from simply repudiating its debt. For example, Argentina owed about $220 billion in 2001, with required repayments of $22 billion a year—during the worst economic crisis the country had ever experienced. Interestingly, in July 2000, an Argentinian Judge named Jorge Ballestero sent down an intriguing ruling on the foreign debt: the ruling attributed responsibility for the debt to the civil servants during the previous dictatorship that contracted it *and co-responsibility* to international organizations like the IMF, who approved the loans, now declared illegal and fraudulent.

Would you lend your money to a country?

Source: odiousdebts.org.

Ethical constraints are decaying or decayed.

Unfortunately, the ethical aspect of corporate governance has also begun to erode, perhaps because the other corporate governance aspects have deteriorated. For example, even as late as 1980, the typical manager earned only 40 times what the average employee earned. Most managers would have felt uncomfortable earning more than 100 times. Nowadays, the average Fortune 500 executive earns over 400 times what the average employee earns, and few executives would deem pay packages of $100 million or more to be obscene. The standards of appropriate managerial behavior today are not the same as they were in the past—and the past itself is nowhere near as rosy as it is often painted.

Legal protection is the only half-way intact mechanism.

Consequently, it seems to be the legal structure in the United States that is our saving grace. The standard of disclosure; the requirement of fiduciary responsibility; the effectively enforced prohibition of theft, fraud, and insider trading; the personalized legal liability; and the strong enforcement of its laws all contribute to a viable governance framework. Oddly, this is enough to rank the United States at the top of locales for equity investors.

Legal protection as a corporate governance mechanism carries a real danger: in the future, it could hurt more than help.

This situation is perplexing to us economists. Our perspective is usually that much of what the government touches comes out for the worse. Private companies usually tend to do better. Yet, it is precisely the legal structure in the United States that has become the most effective corporate governance mechanism. So, should we ask the government to take a more active role in corporate affairs? If so, what is the risk that more government could end up as a cure worse than the disease? The appropriate remedy for managerial abuse is a vexing and thorny problem.

24·5.B. Where are we going?

The corporate scandals of 2001-2003 ironically are not the result of inadequate corporate governance laws, and reform efforts are unlikely to prevent them from repeating.

One might be tempted to just leave a system alone that seems to have worked for centuries. But this system was not static either. There is also a real danger that if no action is taken and corporate governance becomes worse in the United States than in other countries, investors may wander off to other locales. The recent corporate scandals in the United States have helped to highlight the need for corporate governance reform. Ironically, these scandals were the results of already illegal actions, and many perpetrators may end up spending many years in prison. Recent reforms will not eliminate such scandals in the future: Just as bank robberies exist despite laws against bank robbery, so will illegal managerial looting continue despite laws against it. Fortunately, some good may yet come out of the current attempts at corporate reform.

Reforms are proposed by Sarbanes-Oxley, the NYSE, and NASDAQ.

The main legal regulatory functions in terms of corporate governance reside with the SEC and the stock exchanges. The **Sarbanes-Oxley Act of 2002** further reinforces this system. In line with this act, the stock exchanges are trying to tighten their rules for listed companies. (The NYSE enforces a tighter set of corporate governance rules than NASDAQ.)

Listing the current changes to corporate governance.

Most of the post-Enron changes seek to strengthen the independence and function of the corporate board, especially insofar as the audit, executive compensation, and nomination committees are concerned. Here is a selection of the most important reforms of 2003:

- There is now a clear definition of what an independent director is: an individual who has no current or recent material relationship with the company. (Note that independent board members can still have close relationships with the CEO.)
- Independent directors must meet among themselves in regularly scheduled executive sessions without management.
- A large part of the Sarbanes-Oxley Act pertains to the audit committee, as the Act itself was sparked by accounting scandals:
  - The audit committee, which checks over the company's financial reports, must consist entirely of independent directors. There are additional special rules for the audit committee pertaining to large shareholders.

  – The audit committee must have choice of, oversight of, and compensation responsibility for the company's auditors. It can engage additional advisors, and it must institute procedures to handle complaints and whistleblowers.
  – External auditors are also to be limited in the amount of consulting work they can do for companies, which has historically been a great source of conflict for public auditors. In addition, the audit committee must approve any remaining non-audit consulting work by the auditor.
  – The audit committee must identify which of its members is a financial expert, and at least one is required.
  – The audit committee has "code of ethics" responsibility.
  – Auditors must be rotated on a regular basis in order to reduce the tendencies of relationships between firms and auditors to become too cozy. (Of course, this has costs, too: new auditors have to first learn more about the firm, and may be less apt in detecting unusual behavior.)

- The C.E.O. and C.F.O. must certify to the audit committee the accuracy of the company's financial reports/condition. (This is a new feature of Sarbanes-Oxley—or is it? Executives were responsible for the reported financials of their companies even before its enactment. It made for good television, though.)
- Attorneys must alert the SEC if they learn of credible evidence of breaches of fiduciary duty or of United States securities law.
- Companies can select the members of their executive compensation committee and board-nominating committee, but these committees must be majority independent (NASDAQ) or fully independent (NYSE).

In addition to these new legal regulations, there have also been a whole range of institutions that have proposed "best practice" guidelines for corporate governance. The most prominent are the *GM Board Guidelines* (since 1994), the *American Law Institute Principles* (since 1992), the *Business Roundtable Principles* (since 2002), the *National Association of Corporate Directors Report* (since 1996), the *Conference Board Recommendations* (since 2002), the *CalPERS Principles/Guidelines* (at least since 1998), the *Council of Institutional Investors Principles and Positions* (since 1998), the *TIAA-CREF Policy Statement* (since 1997), the *AFL-CIO Voting Guidelines* (since 1997), and the *OECD Principles/Millstein Report* (since 1998).

Here is what I think is missing.

Many of these reforms have positive aspects, but there are also many negative ones. Sarbanes-Oxley was more image then substance, and where it had substance, it focused on process over outcome, and required yet more bureaucracy. Many foreign corporations that had cross-listed on the New York Stock Exchange are currently evaluating whether the added Sarbanes-Oxley cost is so high that they are better off delisting again. There have been a good number of other reform proposals that have been put forward. Here are the four suggestions that I most like:

1. Ira Millstein has proposed that the position of Chairman of the Board should be separate from that of Chief Executive Officer. It should be obvious that if the Chairman is also the CEO, the board at best can only struggle to assert influence over management, rather than direct management to act in the interest of shareholders. Today, in executive circles, a company that has a separate chairman is viewed as not trusting its CEO. It must become an accepted corporate norm for these two positions to be separate.

   The argument against separation, mustered by many CEOs, is that it would cost them time and effort to deal with a separate chairman. It is in effect the argument that a benign dictator is better than checks and balances. This is correct. Good governance—a system of good checks—does not come for free. It can cost money if management is good, but save money if management is bad—which, after all, is the whole point of governance. Good governance is not good management. Good governance is the mechanism to reign in management that is bad.

2. The voting system could be changed to a proportional system, in which minority shareholders are assured some representation. If a shareholder with 10% of the shares can obtain 10% of the seats if so desired, then large institutional shareholders could create mechanisms of "professional trustees" who are not beholden to management.

3. Any insider trading should be disclosed *before* a trade, not after it.

4. Large, publicly traded companies could be forced to disclose their tax financials. This would reduce their incentives to overstate earnings.

The government would not need to legislate governance reform. Instead, it could tighten the legal liability of corporations and individuals that do not follow these recommendations, and offer a "safe harbor" to corporations and individuals that do follow their recommendations. This would put the appropriate pressure on firms to follow them, without absolutely requiring it.

Better than forcible regulation is carrot-and-stick.

Solve Now!

Q 24.16 *What are the main Sarbanes-Oxley reforms?*

ANECDOTE: The Corporate Governance Consulting Industry
A recent phenomenon is the emergence of corporate governance consultants. For example, *Georgeson* publishes an interesting year-end wrap up of shareholder proposals and proxy contests. Unfortunately, some corporate governance consultants not only publish ratings of how well publicly traded companies are governed, but also sell "advice services" to companies. Not surprisingly, following the consultants' advice, the client tends to improve in the consultant's rankings.

24·6. Summary

The chapter covered the following major points:

- Control rights are necessary components of any security in order to defend their cash flow rights.
- Managers have the incentive to act in their own self-interest, not necessarily in the interest of shareholders and creditors.
- Mechanisms have evolved to reduce or rein in managerial theft—such as corporate takeovers, large shareholders, corporate boards, legal environments, ethics, and debt.
- There are a number of possible mechanisms to improve corporate governance in the United States.

Special thanks to Florencio Lopez-De-Silanes, Paul Macavoy, Ira Millstein and Holly Gregory. Holly authored a legal description of Sarbanes Oxley that is synthesized here. Newer versions can be obtained from www.weil.com/weil/corpgov_frames.html.

Solutions and Exercises

1. For debt, it is the right to force bankruptcy if covenants are violated. For equity, it is the right to vote.
2. Illegal: Theft, fraud, insider trading, transfers, bribes. Legal: Empire building, perks, excessive executive pay, entrenchment, friendship and loyalty, and the incentives to drive down the firm value in order to purchase the company on the cheap.
3. Right around the time of the firm going public. The entrepreneur internalizes all future agency conflicts. To the extent that money will be diverted from owners in the future, these owners will be willing to pay less for the firm today. For a numerical example, see the text.
4. First, it is impossible to think of all future contingencies that could happen, and therefore should be considered in the charter. Second, the entrepreneur will care primarily about agency conflicts soon after the IPO, and pretty much ignore what may happen many decades later.
5. No. Quite the opposite can happen—seasoned equity offerings can be a mechanism by which managers enrich themselves at the expense of the company that they are running.
6. The CEO knows the firm better, and through judicious choice of information, control the agenda. The CEO is often the board chair. Elections for the board are usually by slate and uncontested. Outsiders are often CEOs themselves. As to other roles, advice and relationships as well as aid in management succession may play a role.
7. It is very costly to execute a proxy and takeover contest. A typical premium may require a premium as high as 20%—worthwhile only if the current management commits the most egregious breach of appropriate behavior. Shareholder proposals are not binding.
8. In large, widely held publicly traded corporations, Even large shareholders typically hold only small fraction of the shares. Thus, they will not invest too much effort, because they do not receive 100% of the benefits from lobbying. Moreover, management will find out whether a shareholder voted against them.
9. It regulates only the most egregious violations of fiduciary duty. It does not extend to "business judgment" calls.
10. The standards are themselves set by the behavior of CEOs as a group. Moreover, ethical standards tend to be higher when information is publicly available, and not everything is publicly reported.
11. See the list in Section 24·3.C. Staggered boards have virtually eliminated hostile takeovers.
12. An LBO is a leveraged buyout, i.e., one that is financed with a significant amount of debt. They were very common in the 1980's, but have largely faded.
13. Around 1 percent.
14. The presence of large shareholders can be very bad from an agency perspective if these shareholders use their voting power to arrange special deals for themselves.
15. Even if the company continues to exist, management is usually replaced!
16. Independent directors are now clearly defined. They must meet by themselves regularly without management. The audit committee and the independence of auditors was beefed up. The CEO and CFO must certify the accuracy of the company's financial reports. Attorneys must report certain breaches of fiduciary duty or securities laws. And the executive compensation and board-nominating committees must be majority independent.

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

Part V

Putting It All Together – Pro Formas

Sorry, no cartoon yet.

(A part of all versions of the book.)

CHAPTER 25

Pro Forma Financial Statements

Projecting Financial Statements.

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In a sense, pro formas are what corporate finance is all about—the standard way in business to think about and propose financing or investing. For example, when you propose a new project to your boss, to the board of directors, or to an external venture capitalist, you will almost surely be asked to produce a business plan. The most critical part of this business plan will have to be your "pro forma" financials. These financials will be used as the baseline for discussion and valuation of your proposed project.

Managers and entrepreneurs are not the only producers of pro formas. Analysts for major investment banks or for firms seeking acquisitions or mergers also have to produce pro formas to back up their analyses of corporate value. Their task is both easier and harder than that of the entrepreneur: analysts can rely on historical financial statistics and sometimes a stable history upon which to base their pro formas, but they also often lack the detailed knowledge of the business internals and of the corporate intentions that the internal managers and entrepreneurs would have.

Every business is different, and thus every pro forma is different. Still, this chapter tries to give some guidance to the process of creating pro formas. In this chapter, we will produce pro forma analyses for PepsiCo, which means that we will make extensive use of its financial statements on Pages 192–196. We will alternate between a number of perspectives—that of an analyst valuing a privately traded company that has no market value yet, that of an analyst proposing a capital structure change for a publicly traded corporation, and that of an economist who has the advantage of hindsight.

Anecdote: Pro Forma
According to Merriam-Webster, *pro forma* is a Latin term meaning "for form" and dated ca. 1580. Pro forma has two definitions: "provided in advance to prescribe form or describe items"; and "made or carried out in a perfunctory manner or as a formality." In many (rejected) business plans, the latter may be a better description than the former!

25·1. The Goal and Logic

Detailed pro formas help us think about the business.

A **pro forma** is a model of a hypothetical future scenario. In our context, a pro forma usually means a model of the financial performance in this hypothetical scenario. Creating a pro forma is a similar challenge to what you encountered in earlier chapters, where you had to compute a project's present value. You needed to understand how everything fits together—the expected cash flows, the appropriate cost of capital, the role of the corporate and capital structure, the agency conflicts. The main novelty of a pro forma is that you now need to forecast the future in the context of the financial statements, rather than just in the context of the NPV formula. Creating such a full pro forma is not work for nothing: it will help impose some discipline and structure on your thinking about the design and value of the proposed project. It forces you to think about "details" such as what you believe sales and costs will be, how you will manage working capital, how quickly your contribution to corporate earnings can turn positive, whether taxes will be an important factor, and so on.

But forecasting for pro formas is hard and different from business to business.

Finance professors would not dispute the importance of pro formas, but constructing a pro forma is difficult, and we finance professors naturally prefer the "easy" tasks! There are at least two reasons for our reluctance:

1. **Idiosyncracy:** In contrast to the many beautifully simple theoretical concepts in finance, financials and pro formas are messy and unique for each business. Forecasting the financials for a new cancer drug is different from forecasting the financials for a new toy fad, which is different from forecasting the financials for a retail store, which is different from forecasting the financials for aluminum mining, and so on. So, many of the guidelines for creating good pro formas are necessarily less universal and more ad hoc than other finance concepts.

2. **Relativity:** The difficulties of making good financial projections for a specific project are often tremendous. It is important that you realize the limits of what you can and cannot do. You should be able to do it better than your peers—a *relative* rather than an *absolute* concept. Looking in retrospect at what later actually happened in relation to what you predicted in your pro forma is often a great lesson in humility. At the very least, you will learn in this chapter that you are not alone in this dilemma.

Still, this chapter seeks to give you some general guidance, because in the end, there is no way around pro formas: to be an effective entrepreneur, manager, or analyst, you must learn both how to produce and how to critically consume financial pro formas. After reading this chapter, your next step in learning pro formas should be to work through and critique many case studies—necessarily a trial-and-error-and-experience process.

25·1.A. The Template

Decide on a detailed projection period and a terminal value period.

The standard method for creating a pro forma separates the future into a "detailed projection" time period, for which you forecast the financials in great detail, and a **terminal value**, which you can think of as the "then market value" of the business—a going-concern value of the business if you were to sell it at this point in the future. You have to decide for how many years you want to project in detail before capping your value analysis with your terminal value.

Here is the template of what we need to do.

As our guinea pig, let's use PepsiCo, because you have already studied its historical financials in Chapter 9. Your goal now is to construct a good pro forma as of December 2001 to estimate PepsiCo's market value, presuming you already know the 2001 financials. The construction template is in Table 25.1. It shows the three big areas you must work on:

1. A choice of horizon T, up to which you estimate in great detail
2. The detailed financials during the initial projection phase, from time $+1$ to $T-1$
3. A terminal market value at time T

Table 25.1. The Pro Forma Problem for PepsiCo

To be determined

Pro Forma Net Income Statement

Year	−2	−1	0	+1	+2	+3	...	+T	Terminal Value
	1999	2000	2001	2002	2003	2004	...		
Net Sales	$22,970	$25,479	$26,935						
– COGS	$10,326	$10,226	$10,754	to be projected in detail					“wholesale” projection needed
...									
= Net Income	$2,505	$2,543	$2,662						

Pro Forma Cash Flow Statement

Net Income	$2,505	$2,543	$2,662		
+ Depreciation	$1,156	$1,093	$1,082	to be projected in detail	“wholesale” projection needed
...	...	...	...		
= Operating Cash Flow	$3,605	$4,440	$4,201		

The numbers for PepsiCo’s income statement were taken from Table 9.13 (Page 221). The numbers for PepsiCo’s cash flow statement were taken from Table 9.14 (Page 222).

Your goal is to project future cash flows—T periods worth of detailed financials—followed by a wholesale market value estimate of the remaining cash flows until eternity.

Your first perspective is that of an external analyst who has to construct a pro forma to value a firm for which you do not know the real market value. That is, pretend you do not know PepsiCo’s market value. As an outsider, you also do not have detailed knowledge of PepsiCo’s operation—certainly not enough to pretend you are an insider or manager. (PepsiCo is an established company, so the pro forma is not exactly following the mindset of an entrepreneur who knows his firm inside out and who is now proposing a new project that has no prior history.) You do know that PepsiCo is an established firm and so its cash flows will *not* start with a sharp initial business growth curve and a high cost of capital, followed only later by a more stable period with a lower cost of capital. (The end of the upstart growth phase is often a natural break and thus a natural choice for T.) Instead, PepsiCo is already in its mature, stable state. You really have no detailed knowledge of how the next year will be different from what will happen in ten years. You could even just rely on a terminal value right now and dispense with the initial detailed projection phase altogether. Nevertheless, we will work out the detailed projections to illustrate the process. Another issue that is less important for PepsiCo than for upstart companies is good working capital projection—PepsiCo is so big and stable that it can easily borrow more capital if it needs more. An entrepreneur, on the other hand, would have to pay close attention to avoid running out of cash—which could lead to loss of the entire business even if its underlying economics is sound.

PepsiCo is an imperfect example, because it could be done a lot more simply.

There is another unusual feature here—our pro forma is constructed in 2001, which is now a number of yours ago. This allows us to use hindsight knowledge to see how good or bad our forecasts turned out.

We can peek at “illegal” information.

25·2. The Detailed vs. Terminal Time Break

How many years of detail?

Your first goal is to understand how to choose a suitable value for the horizon choice T in Table 25.1. Remember that the horizon is the span of time up to which you project detailed financials and beyond which you substitute your "wholesale" terminal value estimate.

In relative terms, the very-long run may not be more daunting than the intermediate run. Future cash flows may be equally uncertain, and present values would be less uncertain.

As an initial step, let us take a brief detour into forecasting. There is one surprising and key insight: you may be able to project future cashflows as well in the very long term as in the intermediate term—and this means that you may be able to estimate long-term present values *better* than intermediate-term present values. This would be the case if the business is in a stable phase and is best explained by analogy.

Uncertainty may not grow dramatically with horizon. In NPV terms, long-term uncertainty can often be less problematic.

If the business and environment is stable, then your uncertainty is not growing with your horizon after some point. If you have to forecast the temperature in two hours, your (short-term) forecast will be pretty good. What if you have to forecast the temperature on August 1, 2055? You will not be as precise, but most likely still be reasonably accurate. (Here in New York, the temperature will probably be 85 degrees, plus or minus 15 degrees.) But if you have to forecast the temperature at an intermediate horizon—say, August 1 of next year—your one-year forecast will likely be neither different nor better than your fifty-year forecast. How will this affect your estimated present value components? Say you want to value an ice cream store. The effect of the temperature uncertainty in August of next year is less discounted and thus more important than the effect of the temperature uncertainty in August of 2055. For example, say your store expects to earn \$100,000, and a 15 degree temperature difference can cause you to earn anything between \$75,000 and \$125,000. At a 15% discount rate (cost of capital), the temperature uncertainty for August of next year can cause a value difference of about $\$50,000/(1+15\%)^1 \approx \$43,478$ in present value today. But the same temperature uncertainty in 50 years causes only a value difference of about $\$50,000/(1+15\%)^{50} \approx \$46$ in present value. Consequently, if you want to estimate the present value correctly today, then the intermediate-term uncertainty is of more concern to you than the long-term uncertainty.

Economics and Strategy: Scarce resources make rents!

The role of intermediate term vs. long term uncertainty generalizes beyond ice cream stores, because knowledge of economics and strategy allows you to put reasonable bounds on long-term future profitability (in 10, 20, or 30 years). At such far-out horizons, you should not expect businesses to still have unusually large growth rates and to earn **economic rents**—where economic rents are defined as investment rates of return that are much higher than the cost of capital. Economic rents can only be achieved when a firm has assets and capabilities that are scarce, valuable, and difficult to imitate. Examples of such scarce resources are the presence of a unique and excellent manager (e.g., a Jack Welch), economies of scale (e.g., Microsoft's computer software or Walmart's mass logistics), unduplicable corporate reputation (e.g., Sony's brand name), legally protected intellectual property (e.g., Glaxo's retroviral drug patents or Disney's Mickey Mouse), or consumer switching costs (e.g., Comcast's cable television). In the very long run, over decades, scarce resources tend to become less scarce, as new technologies and consumers make old advantages obsolete. *Wal-Mart Stores* (WMT) may seem like a juggernaut today, but in 50 years, it will almost surely not have the scarce resources that will allow its owners to continue earning rates of return much above their investment cost of capital. (If Wal-Mart did maintain its historical growth rate, it would have to colonize other planets!)

The force (of economics) has worked on products historically, too.

To determine how long it might take before a product becomes a commodity and thus produce only normal profits, you need to apply economic thinking to your specific business knowledge. If there are few scarce resources and entry barriers, then it may only take a couple of years before unusually high corporate growth rates slow down and there are no more economic rents. For example, there are few entry barriers to flat-screen television technology today. Consequently, you can count on the industry that produces flat-screen televisions to earn few excess rents within 10 years. (If you do not believe this, think back to the days of \$500 DVD players; today, all entry barriers have disappeared, and you can purchase a DVD player for \$20.) Other products, however, can enjoy more scarcity and entry barriers for longer periods of time. For example, if you can get a patent on an effective cancer drug, you will be able to earn economic rents for 15 to 25 years—although better competitors' drugs will eventually come onto the scene and your patent will eventually run out. If you still do not believe me, think back. Can you name the companies from the 1980s that still earn large economic rents? If you had picked two companies that looked similar in 1985, are both companies still around? For example, Dell may still be doing well, but Gateway looked just as good in 1985—and there are literally dozens of now bankrupt companies that looked no different then, either. Standing in 1985, you should not have expected to earn large economic rents if you had bet on any one computer hardware vendor then.

The strategy model: what delays erosion of economic rents?

The economics that help you decide on when a firm is likely to settle into a lower economic growth rate are taught in great detail in business strategy courses and carry different labels (e.g., Porter's five forces). To determine when your business' economic rents are likely to erode, strategy suggests you ask such questions as:

- How long before your entry barriers will erode?
- How long before your success will be mimicked by the competition?
- How long before you can be squeezed by suppliers or customers?

The two considerations for setting T: business economics and discount factors.

So, to choose your horizon T, you should consider the "underlying firm economics." If you set T at a point where long-run economic forces will have eroded most of the economic edge of the company—where growth will return from the initial but unsustainably high short-term rates to sustainable ordinary long-term rates—then you can assume that the company will henceforth earn only "ordinary profits." This suggests that your goal should be to capture the initial rapid and possibly unstable growth phase with detailed financial forecasts, and the stable period with the terminal value. Another way to say this is that a good T is the point in time when you expect the present value of growth opportunities (PVGO) to be zero. But there is also a second consideration to your choice of T. You want to pick a horizon such that the discount factor is high enough so that the precise choice of T would not matter *too* much. For example, at a 10% discount rate, each dollar in 20 years is worth only about 15 cents in present value today. The high discount factor can help plaster over the errors that your crude terminal value estimate will inevitably commit. Whether you choose 20 years or 25 years as the detail horizon, then, does not matter too much in terms of present value. When it comes to exit values on horizons that are so far away, the best you can hope for is a *halfway* reasonable estimate of market value, anyway.

Typical values for T: 5-10 years.

For most businesses, you would pick a terminal value somewhere between 3 and 20 years, with 5 to 10 years being most common. Let's apply economic intuition to choose a T for PepsiCo. PepsiCo is a very stable company, so it is not necessary to project 20 years of financials in great detail. You can instead "lump" the value created in all future years into one terminal market (sale) value fairly soon. This is a relief—it saves you from guessing too many years of numbers about which you—as an outside analyst—really have no clue. Thus, for convenience, let us choose a horizon $T = 5$ years.

25·3. The Detailed Projection Phase

In real life, you must use all your economic knowledge to do a good projection.

Your second goal is to determine the project value during the beginning growth period, from next year up to some year $T-1$. The good news is that, in real life, analysts usually know your business quite well and thus are able to reasonably predict the immediate future. They can use historical cash flows for some guidance about future cash flows. Of course, to do this well, you have to understand a lot about the underlying economics of the business, and you have to make many assumptions. In this process, much additional information that you have so far mostly ignored—such as the specific industry economics or the corporate balance sheet—would come in very handy.

Initial growth projections are highly product specific. External analysts can use the historical financials as one of their inputs.

The bad news is that illustrating this process is difficult. You probably do not know much about PepsiCo's business—and even if this chapter fully explained PepsiCo's business, it would not help you elsewhere. Pharmaceutical drug research, aluminum mining, fad toys, and a new stamping machine each have their unique business, financial, and accounting patterns. There is little generality here. In contrast to the terminal value, long-run economic forces are unlikely to apply in the projection phase period.

The detailed projections will also influence your terminal values.

Unfortunately, despite the need for specific information that we lack, we cannot simply brush over the need to obtain accurate forecasts of the initial growth phase. These projections will have a significant impact on project wealth, and not just because they have a direct contribution to the present value over the next five years. The terminal value itself is usually relative to a baseline expected cash flow in year $T-1$ or T, which must be established from your initial detailed projections. Consequently, although this chapter can give you only modest guidance for the general case of coming up with initial projections, we must make up some numbers in order to illustrate the process. Be warned: our financial projections for PepsiCo will necessarily remain naïve. Again, because we know very little about PepsiCo's business or the plans of its managers, accuracy is not the goal—illustration is.

Projecting economic cash flows directly or indirectly (via detailed financials).

The two primary methods of projecting financials are explained in the next two subsections:

1. Direct extrapolation of the accounting component that you are interested in (i.e., the economic NPV cash flows for the project, though sometimes also the earnings).
2. Detailed financial modeling of all or most items in the financial statements.

The first is a drastic shortcut, used by analysts only when time and knowledge are limited, whereas the second is more common. Incidentally, computer spreadsheets were originally invented primarily to facilitate the projections in pro formas, and are therefore the preferred tool for designing them.

25·3.A. Method 1: Direct Extrapolation of Historical Cash Flows

Directly project the final cash flows themselves forward. Here, it gives bad results. Recognize such problems!

The first method is really a "cheat," which avoids having to do the full-blown financial pro forma analysis. It directly projects the historical cash flows forward, e.g., by assuming a constant growth rate forever. For example, using Formula 9.18, PepsiCo's cash flows for 1999 to 2001, you can see that

From Formula 9.18:

$$\begin{array}{lcccccccc}
\text{Cash Flow}_{1999} & = & \$3,605 & - & \$1,172 & + & (-\$792) & = & \$1,641 \\
\text{Cash Flow}_{2000} & = & \$4,440 & - & \$1,996 & + & (+\$57) & = & \$2,501 \\
\text{Cash Flow}_{2001} & = & \$4,201 & - & \$2,637 & + & (-\$8) & = & \$1,556 \\
\text{Economic Project Cash Flow} & = & \text{Operating Cash Flow} & + & \text{Investing Cash Flow} & - & \text{Interest Income} & . &
\end{array} \tag{25.1}$$

Over the three years, PepsiCo showed a cash flow decline of about $\$1,556/\$1,641 - 1 \approx 5\%$. This comes to an annual decline of about $(\$1,556/\$1,641)^{1/2} - 1 \approx 2.6\%$. Over the most recent

12 months, cash flows even dropped by one-third! You could assume that PepsiCo's cash flows will continue to decline at this rate forever. But do you really believe that PepsiCo's cash flow decline will continue, or do you believe that it will reverse? If you investigate PepsiCo's cash flow statement in Table 9.14 further, you can find that much of PepsiCo's decline was due to a heavy increase in (other) investing activity, not to a decline in its business (sales). Some of it was due to the acquisition of Quaker, which PepsiCo hopes will eventually pay off in *more* cash, not less cash. This demonstrates how hazardous simplistic extrapolation of cash flows can be: You really need to know more about the business itself and the reasons behind the financial trends. Purely mechanical rather than economic models of the business usually just don't work well. Again, always remember that valuation requires much economic and common sense and that it is as much an art as it is a science.

Table 25.2. Pro Forma: Direct Cash Flow Projections

	Known		"Detailed" Model Growth at 10%			Terminal Value See Next Section	
	2000	2001	2002	2003	2004	2005	2006-
	Year −1	Year 0	Year +1	Year +2	Year +3	Year +4	to ∞
Projected CF[1]	\$1,556	\$1,712	\$1,883	\$2,071	\$2,278	\$2,506	?

Explanations (Notes):
[1]: Projecting 10% due to investments, until (incl.) 2005.

You could project earnings instead of cash flows—which has advantages and disadvantages.

In Chapter 10, we considered forecasting earnings rather than cash flows. In the very long-run, earnings and cash flows should be roughly equal—after all, earnings "just" shift the time-series accruals. Historical net income may be a better representation not only of future earnings, but even of future cash flows compared to historical cash flows. The advantage is that earnings are less lumpy; the disadvantage is that accruals are not really cash. PepsiCo had earnings of \$2,662 in 2001, having grown at rates of 1.5% and 4.7% over the two prior years. If PepsiCo were to grow its earnings by 3% per year, the following earnings trend would emerge:

	Known		"Detailed" Model Growth at 3%			Terminal Value See Next Section	
	2000	2001	2002	2003	2004	2005	2006-
	Year −1	Year 0	Year +1	Year +2	Year +3	Year +4	to ∞
Projected Earnings	\$2,543	\$2,662	\$2,742	\$2,824	\$2,909	\$2,996	?

In this future, earnings would reach \$3 billion in earnings by 2005—about 20% higher than the equivalent cash flow projection. Because earnings are more stable (have lower variance), their forecasts are also often more reliable than cash flow forecasts.

25·3.B. Method 2: Pro Forma Projections With Detailed Modeling of Financials

A more sophisticated method attempts to model the entire financials, not just the "end product," the economic cash flows.

The second and more common method of projecting economic cash flows during the initial period is to project entire financial statements, which provide the individual components for the economic cash flows you seek. Doing so is often (but not always) better than projecting economic cash flows directly for three reasons:

1. As we just noted, on the one hand, cash flows are difficult to directly project, because they tend to be volatile. Capital expenditures occur in lumps, and thus do not follow steady, constant growth paths. On the other hand, the smoother net income contains many fictional accounting accruals that are not really cash. It's the "rock and the hard place."

2. The full projection method can make it easier to intelligently incorporate any knowledge of the underlying business into the economic cash flow estimates. For example, you may happen to know that unusual expenses will be zero next year, or that a new payment system may speed the collection of receivables. By forecasting the individual items, such economic knowledge automatically flows into your cash flow estimates.

3. The full projection method can help you judge other important information—such as working capital availability, suitable debt-equity ratios, and interest rate coverage. Especially for entrepreneurs who are often in danger of a liquidity crisis, such information can be just as important as the economic cash flows themselves. In fact, *all* the ratio analysis , such as the financial health and profitability ratios, are often more useful when applied to pro forma financials than when applied to current financials. Such analysis can help you judge whether the firm is on a sound path or on a critical path.

See Section 10·4.B

The Income Statement: Sales

Table 25.3. A Possible PepsiCo Pro Forma Income Statement Model for 2002

	Income Statement	December 1999	2000	2001	Estimated 2002	2003	···
=	Sales[1]	$25,093	$25,479	$26,935	$27,906	···	···
	COGS[2]	$10,326	$10,226	$10,754	$10,760	···	···
	+ SG&A[3]	$11,018	$11,104	$11,608	$12,279	···	···
	+ Deprec/Amort[4]	$193	$147	$165	$168	···	···
	+ Unusual Expenses[5]	$73	$184	$387	$279	···	···
–	= **Operating Expenses**[6]	$21,610	$21,661	$22,914	$23,486	···	···
=	**Operating Income**[7]	$3,483	$3,818	$4,021	$4,420	···	···
+	Net Interest Income[8]	$792	–$57	$8	$0	···	···
=	**Income Before Tax**[9]	$4,275	$3,761	$4,029	$4,420	···	···
–	Corporate Income Tax[10]	$1,770	$1,218	$1,367	$1,591	···	···
=	**Income After Tax**[11]	$2,505	$2,543	$2,662	$2,828	···	···
–	Extraordinary Items[12]	$0	$0	$0	$0	···	···
=	**Net Income** [13]	$2,505	$2,543	$2,662	$2,828	···	···

Explanations (Notes):

[1]: grows by historical 3.6%.
[2]: $3,506+26% of sales.
[3]: 44% of sales.
[4]: 3-year historical average.
[5]: 1% of sales.
[6]: sum the above.
[7]: subtract the above.
[8]: too ignorant and lazy.
[9]: subtract the above.
[10]: 36% of IBT.
[11]: subtract the above.
[12]: too ignorant and lazy.
[13]: subtract the above.

The base for detailed pro formas is sales prediction.

The detailed projection method usually starts by forecasting future sales in the income statement. This sales forecast is the single most critical aspect of any pro forma, because it becomes the baseline number from which many other financial item estimates will follow. In PepsiCo's case:

- You could use a mechanistic model that extrapolates sales growth from historical financials. For example, in Table 25.3, you can compute that PepsiCo sales grew at an annualized rate of $(\$26,935/\$25,093)^{1/2} - 1 \approx 3.6\%$ from 1999 to 2001. Let us assume that PepsiCo sales will continue in 2002 at the same growth rate. Therefore, projected PepsiCo sales in 2002 would be $\$26,935 \cdot (1 + 3.6\%) \approx \$27,906$.

Like other pro forma line items, sales has a footnote that explains our assumption. Indeed, every good pro forma must have detailed footnotes explaining the assumptions behind each and every line item projection. Admittedly, our notes in Table 25.3 are too perfunctory and do not even explain where the 3.6% came from. But, in the real world, you must carefully explain the background assumptions behind each and every critical component of your pro forma!

- You could and should use an economic model that uses detailed business knowledge. For example, as a real-world analyst, you might know whether PepsiCo was about to launch many exciting new products or whether it had few new projects in the pipeline. You might use knowledge of how much PepsiCo did not pay out in dividends but kept in retained earnings for reinvestment into its operations—which eventually would turn into more sales or profitability. You might look at the forecast of the macroeconomic climate, which might tell you something about how PepsiCo sales would perform next year, e.g. during the then-predicted recession of 2002. And so on. Any such information would help you to adjust the sales estimates for more accurate projections.

In a real pro forma where your money is on the line, it would be reckless to forecast sales through a mechanistic model without an economic model!

The Income Statement: Other Components

You would then go down item by item on the income statement, the next being COGS. You have a whole range of options:

Direct extrapolation of COGS is possible. But it can now also be projected in relation to (as a fraction of) sales.

- You could repeat the sales exercise with COGS: a pure growth model would project that COGS' historical growth rate of $(\$10,754/\$10,326)^{1/2} - 1 \approx 2.05\%$ will continue in 2002. If applied to the year 2001 COGS of \$10,754, your 2002 COGS forecast would thus be $\$10,754 \cdot (1 + 2.05\%) \approx \$10,975$.

But, armed with the sales scenario of \$27,906 in 2002, you can now consider a much wider set of models.

- You could forecast COGS not only relative to its own history, but also relative to projected sales for 2002, which you have already estimated. You also know the historical relationship between COGS and sales, which you can use to predict a relationship between 2002 sales and 2002 COGS. For example, PepsiCo's COGS was $\$10,326/\$25,093 \approx 41.15\%$ of sales in 1999, 40.14% of sales in 2000, and 39.93% of sales in 2001. The simplest sales-based model might just project that COGS would be a slowly declining fraction of sales in 2002. In this case, your COGS forecast might be

$$\begin{aligned} \mathcal{E}(\text{COGS}_{2002}) &\approx 0 + 39.5\% \cdot \mathcal{E}(\text{sales}_{2002}) \\ &= 39.5\% \cdot 27,906 \approx \$11,023 \,. \end{aligned} \tag{25.2}$$

- A more sophisticated model might pose that there are **economies of scale.** In this case, COGS would not go up one-to-one with sales. Instead, COGS would have both a "fixed component," whose cost would not change with sales (e.g., the factories), and a "variable component," whose costs would increase with sales (e.g., the cola syrup). You might try to plot COGS against sales for 1999–2001, and determine visually that a good line fit would be

$$\begin{aligned} \mathcal{E}(\text{COGS}_{2002}) &= \$3,500 + 25\% \cdot \mathcal{E}(\text{sales}_{2002}) \\ &= a + b \cdot \mathcal{E}(\text{sales}_{2002}) \,. \end{aligned} \tag{25.3}$$

This says that \$3.5 billion is unalterable factory costs, but for each extra dollar of sales, you have to purchase only 25 cents of syrup. Substituting in our estimated 2002 sales of \$27,906 million, you would project COGS for 2002 to be

$$\mathcal{E}\,(\,\text{COGS}_{2002}\,) \;\approx\; \$3,500 \;+\; 25\% \cdot (\$27,906) \;\approx\; \$10,500 \;. \tag{25.4}$$

Or, you could use heavier statistical artillery and run a regression relating PepsiCo's COGS to sales over its most recent three years. (Don't worry if you do not know what this is.) Such a regression suggests that a better line fit would be

$$\mathcal{E}\,(\,\text{COGS}_{2002}\,) \;\approx\; \$3,506 \;+\; 26\% \cdot \mathcal{E}\,(\,\text{sales}_{2002}\,) \;, \tag{25.5}$$

so your prediction would change to

$$\mathcal{E}\,(\,\text{COGS}_{2002}\,) \;\approx\; \$3,506 \;+\; 26\% \cdot \$27,906 \;\approx\; \$10,760 \;. \tag{25.6}$$

- You could draw on information from other firms, such as Coca Cola. In 2001, Coca Cola had COGS of \$6,044 on sales of \$20,092, a ratio of 30%, which is much lower than PepsiCo's. This may not only suggest that Coca Cola's business is different, but also that PepsiCo may be able to lower its COGS in the future to meet "better practice" standards. Thus, you might want to lower PepsiCo's COGS estimate from \$10,760.
- If you were even more sophisticated, you could recognize that COGS contains some depreciation. Thus, the history of PepsiCo's past capital expenditures could also influence your COGS estimate. You could throw past capital expenditures into your statistical regression, too, to come up with a better prediction equation.

The sky—your economic and econometric background knowledge—is your limit. For illustration's sake, let's adopt \$10,760 as our reference predicted COGS in Table 25.3.

Other items in the table may follow other models.

You can repeat these forecasting processes to predict other income statement items. Like COGS, SG&A contains both fixed and variable expenses, as well as depreciation that relates to past investments. SG&A might thus best be modeled as a combination of a fixed component, plus a sales-variable component, plus a past capital-expenditure-based component. But given that no money (only scarce book space) is at stake, for the rest of the income statement, let's play it simple. The footnotes describe the method of projection for each item. Clearly, if your money was at stake, you would want to know as much about the business as possible and use this knowledge to come up with better models for PepsiCo's business relations. Again, the limit is your business knowledge, your imagination, and your economic and accounting knowledge. Different items could receive totally different treatments, too. For example, you could relate net interest income to how much debt PepsiCo currently has, and what you know current and what you believe future interest rates to be.

SIDE NOTE: In the appendix to this chapter, there are similar formulas for many pro forma components estimated with data from the universe of publicly traded companies. These can be used "in-a-pinch"—or even help you gain some intuition about how important the fixed and variable components are in a particular data item. However, the formulas there are mechanistic and therefore definitely not particularly reliable in any individual case—so be careful.

The Cash Flow Statement

The cash flow statement model would rely on the income statement model.

Next, you might model the cash flow statement. Table 25.4 is an attempt for PepsiCo. It starts by transferring the projected net income from the pro forma income statement model into the cash flow statement model. For the remaining cash flow items, we can only remain perfunctory—after all, this is only an illustration without real economic knowledge. Although we really have no idea what causes depreciation and depletion, a number on the order of $1,100 looks "reasonably reasonable," given that you know nothing about PepsiCo's physical plant, and given the stabilities of PepsiCo's prior history of depreciation and capital expenditures. (We also ignore the fact that some parts of depreciation have already been modelled in components of the income statement, although we really should check consistency.)

Table 25.4. A Possible PepsiCo Pro Forma Cash Flow Statement Model

	Cash Flow Statement	December 1999	December 2000	December 2001	Estimated 2002	
	Net Income [1]	$2,505	$2,543	$2,662	$2,828	···
+	Depreciation and Depletion[2]	$1,156	$1,093	$1,082	$1,100	···
+	Deferred Taxes[3]	$573	$33	$162	$300	···
+	Non-Cash Items[4]	–$708	$355	$211	$0	···
+	Changes in Working Capital [5]	$79	$416	$84	–$200	···
=	**Total Operating Activity**[6]	$3,605	$4,440	$4,201	$3,700	···
	Capital Expenditures[7]	–$1,341	–$1,352	–$1,324	–$1,300	···
+	Other Investing[8]	$169	–$644	–$1,313	$0	···
=	**Total Investing Activity**[9]	–$1,172	–$1,996	–$2,637	–$1,300	···
	Operating Plus Investing				$2,400	···

Explanations (Notes):

[1]: transfer $2,828 from IS.
[2]: cat in the hat.
[3]: 15%-20% of Income Tax.
[4]: too ignorant and lazy.
[5]: 27% of revenue *Increase*.
[6]: sum of above, rounded.
[7]: $-\$1,200 + 4\% \cdot$ Earnings.
[8]: too ignorant and lazy.
[9]: sum the above, rounded.

Other cash flow statement components.

Working down the cash flow statement, you must adopt a ratio for your model for deferred taxes, which fits the history reasonably well—let's go with around 18% of PepsiCo's income taxes. You know nothing about non-cash items, and PepsiCo's history does not suggest a clear pattern, so choose zero. Changes in working capital are more interesting, because their relation to sales contain interesting economics. We know that it is not the absolute level of sales, but sales growth that determines the working capital that the business consumes—but not one-to-one. For example, you may have to carry more inventory to satisfy sales growth, although economies of scale may allow you to grow inventory less than one-to-one. Your receivables collection policies and technologies (and your willingness to sell to dubious customers) may influence how much your receivables should grow with sales. Your willingness to pay your suppliers may influence your payables, and so on. With a projected sales increase for 2002 of just under $1 billion, it would suggest that PepsiCo will need more working capital. Yet, PepsiCo also grew in prior years, and still managed to pull working capital out of the business, rather than put it in! This is rather unusual, and may contain some interesting choices PepsiCo has made. We could dig further to find out, but without further knowledge, and after much (pretend) analysis of the underlying business, just presume that PepsiCo will need to put $200 million into the business to finance sales growth. The result of all these forecasts is a projected operating cash flow of $3.7 billion. Finally, after equally long consideration of PepsiCo's business, and equally long interviews with PepsiCo management, you determine that PepsiCo is planning to invest $1.3 billion into capital expenditures, and nothing into other activities.

Financing Policy, the Balance Sheet, and Linkages

More Linkages arise.

Your next step would be to think more about your financing policy. This will influence not only the remainder of your cash flow statement (the financing cash flows), but also your balance sheet (debt and equity positions) and even your income statement (interest payments). In fact, depending on what you assume, you may have to go back to the income statement and go through your forecasts again. Other linkages will arise, too. For example, what you assume about financing cash flows will force your end-of-period cash position on your balance sheet, because the cash position next year is the cash position this year plus the net of all cash flows. For another example, you must also think hard about what you believe current assets and current liabilities will be—e.g., how your technology may change your inventory or your collection abilities. This assumption has direct implications not only for your balance sheet, but also for your changes in working capital on your cash flow statement. Of course, you would also need to provide detailed projections for the remaining detailed projection period, 2003–2005. The principles are the same as they were for your projection of 2002. We will skip all these for lack of space.

25·3.C. Policy and Calculations off the Pro Forma Components

After you have projected your balance sheet and the statement of owners' equity, i.e., a full set of detailed financial forecasts up to the terminal value, T, what can you do with these numbers?

Economic Project Cash Flows

The projected cash flow is now much higher, due to our Other Investing assumptions.

The first important use of the pro forma is project value analysis. Having guestimated the components of the cash flow statement for 2002, you can now compute the economic cash flow for your NPV analysis, using the basic cash flow formula 9.18 from Page 214: economic project cash flow for PepsiCo is the sum of operating cash flows and investing cash flows plus interest expense, which comes to around $2.4 billion—about 50% higher than your alternative $1.7 billion direct projection in Table 25.2. This is not because the forecasting technique is different, but primarily because here you projected other investing activity to be zero. (It accounted for around $1 billion of consumed cash in 2001.)

Ratio and Soundness Analysis

Ratio or Financial Health Analysis.

A second common use for detailed financial projections is forward-looking ratio and soundness analysis. Such an analysis can serve to check the reasonableness of your forecasts—and the viability of the firm under your presumed scenario. For example, if an upstart firm were to end up with a very high debt/equity ratio and very little cash, the implied future interest coverage ratio should ring an alarm. Or, a growth path may have an interim negative cash position—which would doom the firm. Either the firm is on a collision course with reality, or (if you are the manager) you should change course to preserve cash before the entire firm evaporates. However, because most ratio analysis requires aspects of the financials that we do not have space to model—specifically, the financing policy on the cash flow statement, and the full balance sheet—we will not discuss this any further. Once you have the full pro forma model, the ratio analysis principles and soundness principles remain exactly the same as they were in Chapter 10.

Policy Influences

Historical projections work only if the world is stable.

Pro forma projections depend not only on external factors—e.g., whether the economy is going into a recession—but also on many choices that managers make—how quickly to pay for or collect outstanding bills, how much to invest into new projects vs. how much to pay out in dividends, how much to finance with debt vs. how much to finance with equity, and so on. You have to be careful to realize that historical extrapolations may no longer work if either the external environment or the corporate policy is changing.

If policy is changing, the world may no longer be stable.

This is even more important to recognize when you are not an external analyst, but a manager constructing a pro forma in order to contemplate a corporate policy change. For example, if you invest more in new factories, all sorts of relationships—some of them nonobvious—may change. For instance, the relation between COGS and sales may change if the consumers of your product ask for more or less complementary products from other producers, which in turn may change the cost of raw materials that you require for production. Just be careful.

25·4. Pro Forma Terminal Values

We have decided on T and the cash flows up to T—let's work on the terminal value.

Your third goal is to determine the firm's terminal market value. The growing perpetuity formula 3.13 from Page 38 is the most common way to estimate it. That is, you would take your detailed estimated value of cash flows for time T, presume that it will grow forever at some sustainable, long-term growth rate g, and discount it back:

$$\mathcal{E}(\text{Terminal Value}_{T-1}) = \frac{\mathcal{E}(\text{Cash Flow}_T)}{\mathcal{E}(r) - \mathcal{E}(g)} \quad . \tag{25.7}$$

For illustration's sake, the remainder of the chapter will rely only on the direct cash flow forecasts from Table 25.2. This means that Cash $\text{Flow}_{T=2005}$ is \$2,506. We need a combined estimate for the eventual, stable, and eternal growth rate g and cost of capital r.

25·4.A. The Cost of Capital

The first goal—determine the appropriate expected rate of return for PepsiCo—or, if you do not have historical data, a company like PepsiCo that is in its stable phase.

To determine the cost of capital for PepsiCo as of late 2001, you would probably rely on the CAPM. If PepsiCo is publicly traded, you can use its own information. You can also use information from one or multiple comparables, such as Coca Cola—and this would be your only good option if PepsiCo were a privately held company. Table 25.6 gathers a couple of years of (dividend-adjusted) stock prices from Yahoo!*Finance* for the S&P500, PepsiCo, and Coca Cola.

Table 25.6. Four Years of Historical Stock Prices

Date	S&P500	PEP	KO	Date	S&P500	PEP	KO
2-Jan-98	980.28	$32.86	$58.87	3-Jan-00	1,394.46	$31.94	$53.21
2-Feb-98	1,049.34	$33.20	$62.39	1-Feb-00	1,366.42	$30.07	$45.05
2-Mar-98	1,101.75	$38.95	$70.56	1-Mar-00	1,498.58	$32.79	$43.65
1-Apr-98	1,111.75	$36.22	$69.12	3-Apr-00	1,452.43	$34.49	$43.94
1-May-98	1,090.82	$37.24	$71.40	1-May-00	1,420.60	$38.25	$49.64
1-Jun-98	1,133.84	$37.70	$78.04	1-Jun-00	1,454.60	$41.92	$53.58
1-Jul-98	1,120.67	$35.64	$73.48	3-Jul-00	1,430.83	$43.22	$57.19
3-Aug-98	957.28	$25.52	$59.44	1-Aug-00	1,517.68	$40.23	$49.11
1-Sep-98	1,017.01	$27.06	$52.73	1-Sep-00	1,436.51	$43.54	$51.59
1-Oct-98	1,098.67	$31.02	$61.82	2-Oct-00	1,429.40	$45.85	$56.51
2-Nov-98	1,163.63	$35.56	$64.24	1-Nov-00	1,314.95	$42.95	$58.78
1-Dec-98	1,229.23	$37.70	$61.43	1-Dec-00	1,320.28	$47.06	$57.19
4-Jan-99	1,279.64	$35.97	$59.88	2-Jan-01	1,366.01	$41.84	$54.43
1-Feb-99	1,238.33	$34.64	$58.57	1-Feb-01	1,239.94	$43.75	$49.77
1-Mar-99	1,286.37	$36.26	$56.42	1-Mar-01	1,160.33	$41.86	$42.53
1-Apr-99	1,335.18	$34.18	$62.56	2-Apr-01	1,249.46	$41.59	$43.51
3-May-99	1,301.84	$32.85	$62.97	1-May-01	1,255.82	$42.63	$44.64
1-Jun-99	1,372.71	$35.94	$57.13	1-Jun-01	1,224.38	$42.23	$42.55
1-Jul-99	1,328.72	$36.17	$55.80	2-Jul-01	1,211.23	$44.55	$42.17
2-Aug-99	1,320.41	$31.70	$55.11	1-Aug-01	1,133.58	$44.91	$46.02
1-Sep-99	1,282.71	$28.44	$44.59	4-Sep-01	1,040.94	$46.48	$44.30
1-Oct-99	1,362.93	$32.35	$54.53	1-Oct-01	1,059.78	$46.68	$45.27
1-Nov-99	1,388.91	$32.23	$62.36	1-Nov-01	1,139.45	$46.61	$44.57
1-Dec-99	1,469.25	$32.99	$53.96	3-Dec-01	1,148.08	$46.80	$44.75

Prices on December 1, 1997, were 970.43, $60.64, and $32.98. All prices were obtained from Yahoo!*Finance*.

Compute the Historical Beta.

You can compute historical rates of returns from historical prices. For example, using also the price from December 1997,

Date	S&P500	KO	PEP	$\tilde{r}_{S\&P500}$	$\tilde{r}_{KO}$	$\tilde{r}_{PEP}$
2-Jan-98	980.28	$58.87	$32.86	1.015%	−2.919%	−0.3639%
2-Feb-98	1,049.34	$62.39	$33.20	7.045%	5.979%	1.0347%
2-Mar-98	1,101.75	$70.56	$38.95	4.995%	13.095%	17.3193%
...	...	...	...	...	...	...

With these rates of return, you can compute the relevant historical statistics:

Statistic	$\tilde{r}_{S\&P500}$	$\tilde{r}_{KO}$	$\tilde{r}_{PEP}$
Mean	0.49%	−0.21%	1.08%
Variance	27.77%%	84.46%%	67.03%%
Standard Deviation	5.27%	9.19%	8.19%
Cov with $\tilde{r}_{S\&P500}$	27.77%%	12.76%%	19.30%%
Corr with $\tilde{r}_{S\&P500}$	100%	26%	45%

These statistics make it easy to calculate the historical beta of PepsiCo and Coca Cola:

$$\beta_{\text{KO,S\&P500}} = \frac{0.001276}{0.00157} = 0.46 \; , \qquad \beta_{\text{PEP,S\&P500}} = \frac{0.001930}{0.00157} = 0.70$$
$$\beta_{i,\text{S\&P500}} = \frac{Cov(\tilde{r}_i, \tilde{r}_{\text{S\&P500}})}{Var(\tilde{r}_i, \tilde{r}_{\text{S\&P500}})} \tag{25.8}$$

You have a number of choices to estimate PepsiCo's future beta. You could just use its historical beta of 0.70, perhaps adjusting this a little down because PepsiCo had about 10% of its market value in various liabilities, so its asset beta would be somewhat lower than the equity beta. Or, you could compute an industry beta, which may be more reliable than a one-firm beta. Or, you could assume that Coca Cola is similar to PepsiCo, so it gives us information about PEP's future beta, too. Stare at these two historical betas for a while, and then use your judgment. Often, one would "shrink" historical beta estimates towards 1. So, you might decide on a beta of 0.9.

CAPM inputs: risk-free rate choice.

To use the CAPM, you also need a risk-free rate and an equity premium. Start with the risk-free rate. At the end of 2001, the 5-year Treasury Yield was about 4.4%, and the 20-year Treasury Yield was about 5.7%, holding pretty steady throughout 2001. Given that PepsiCo is likely to be around for a while, the 20-year interest rate is the better choice. Out of curiosity, you can see how this compares to PepsiCo's historical average interest rate. The income statement suggests an interest expense of \$219 in 2001 on balance sheet short-term borrowings of \$354 and long-term debt of \$2,651. This interest debt ratio suggests a nominal interest rate of about 7.3%—although we do not know whether some of the interest expense went to pay for other liabilities, when PepsiCo contracted to its debt, what the interest rate would be if it could refinance in 2001, or what PepsiCo bonds' relative liquidity premium would be. A quick look would show that PepsiCo's bond rating was A+, which at the time carried nominal interest rates of just about 7.5%. In any case, for CAPM purposes, you want a purely risk-free interest rate (not PepsiCo's interest rate, which contains a default and small risk premium). You could choose a rate of around 6% per annum, perhaps plus or minus 1-2%.

Computing the Levered Equity Cost of Capital.

You also need to determine an expected equity premium. Pretend that your manager and the board of PepsiCo have unanimously declared that 4% per annum is the standardized estimate. This gives an appropriate CAPM cost of capital of

$$\mathcal{E}(\tilde{r}_{\text{PEP}}) \approx 6\% + 4\% \cdot 0.9 \approx 9.6\% \; . \tag{25.9}$$

Let's just round this up to 10% to make computations easier—the CAPM is not a model with accuracy after the decimal point, anyway. As noted, this 10% is the equity premium for the levered equity, because it was computed from the rates of return of PepsiCo's levered equity beta. However, PepsiCo had so little debt that this happens to be almost the same as PepsiCo's underlying enterprise beta. Reasonable variations on our estimate for the risk-free rate and the equity premium could justify costs of capital between about 8% and 12%.

25·4.B. The Cost of Capital Minus the Growth Rate of Cash Flows

For choice of g, a wide range is often easy to come up with.

It is easy to come up with *high* upper limits for sustainable growth rates. For example, g cannot be above the firm's cost of capital, or the value would be infinite. You would also not expect g to be much above growth rates of such quantities as world GDP—you would not expect our world to consist of nothing but PepsiCo. You can also think of *low* lower bounds. Although it is not impossible to imagine PepsiCo fading away in terms of its importance, this probably will not happen too quickly, so we might want to choose a growth rate no less than, say, −1% per annum. Sometimes, it is more intuitive to think of such changes not in terms of nominal growth rates, but in terms of real growth rates. With an assumption of an inflation rate of 2% per annum, the −1% growth rate would correspond to a real rate of decline of about 3% per annum.

For choice of g, a narrow range is more difficult—and subjective.

But you need to do better than these wide limits, or your valuation will have too large a range of possible values to be useful. You can draw on information from two sources:

1. **Within Company Information:** For example, you can assume that managers will not be drastically overinvesting or underinvesting forever. This means you should be consistent in your choice of expected cash flows and the expected growth rate of your cash flows. Would you really want to assume that a firm invests 20% of its value each year forever, but will grow its cash flows by only 1% forever? Probably not.

   In PepsiCo's case, cash flow from investing activity was \$2,637 in 2001. You can also easily look up its market value to be \$90 billion. (For a privately traded company, you may have to use your final estimated market value, instead.) This is a reinvestment rate of about 3% per annum. A number in this vicinity for $\mathcal{E}(g)$ would thus make sense.

   Having estimated the cost of capital at 10%, this would imply an $\mathcal{E}(r-g)$ of 7%.

2. **Industry Information:** For example, you may analyze Coca Cola to better understand PepsiCo. Its economic cash flows were computed on Page 221. You can learn that Coca Cola had earnings of \$2,431 in 1999, \$2,177 in 2000, and \$3,969 in 2001. Its economic cash flows were \$799, \$2,867, and \$3,211, respectively—driving home yet again how lumpy cash flows are compared to earnings! Moreover, throughout 2001, Coca Cola was valued at just about \$100 billion.

   For Coca Cola's financials, see Tables 9.11 and 9.12.

   If you think of Coca Cola in 2000 or 2001 as a comparable for a then-stable PepsiCo as of 2005, you can back out an estimate of $\mathcal{E}(r-g)$ from Coca Cola's value:

$$\$100,000 = \frac{\$3,211}{\mathcal{E}(r-g)} \Rightarrow \mathcal{E}(r-g) = 3.2\%$$
$$\text{Terminal Value}_{2000} \approx \frac{\text{CF}_{2001}}{\mathcal{E}(r)-\mathcal{E}(g)} . \tag{25.10}$$

   Although you should formally have used the cash flow forecasts for 2002 rather than those for 2001, the real problem is elsewhere: just two years earlier, the same calculation would have yielded 0.8%! Clearly, the lumpiness of cash flows makes backing out eternal growth rates hazardous. Maybe you would be better off with our "earnings stand in for cash flows" assumption, which would yield an estimate for $\mathcal{E}(r-g)$ of about 4%. This is about 2% above the inflation rate, and roughly in line with generally predicted long-run real growth rates of GDP.

Differences in estimates of $\mathcal{E}(r-g)$ matter even for a company as large as PepsiCo, which does not have almost all its earnings power far in the future.

How big is the value difference that would come from eternal growth rates between 4% and 7%? Unfortunately—and as is often the case in the real world, too—your estimated range for PepsiCo's eternal cost of capital—or more accurately, your uncertainty about $\mathcal{E}(r)-\mathcal{E}(g)$—is not only wide, but also has a significant influence on our valuation. Apply the formula

$$\text{Terminal Value}_{2004} \approx \frac{\$2,506}{\mathcal{E}(r)-3\%}$$
$$\text{Terminal Value}_{T-1} \approx \frac{\text{CF}_T}{\mathcal{E}(r)-\mathcal{E}(g)} . \tag{25.11}$$

Again, this terminal value TV represents the value of all future cash flows that PepsiCo will create from Year 2005 to eternity—the presumed market value if you had to sell PepsiCo at the end of 2004.

$\mathcal{E}(r)$	$\mathcal{E}(r)-\mathcal{E}(g)$	$\text{TV}_{T=2004}$	$\mathcal{E}(r)$	$\mathcal{E}(r)-\mathcal{E}(g)$	$\text{TV}_{T=2004}$
6%	3%	\$84 billion	9%	6%	\$42 billion
7%	4%	\$63 billion	10%	7%	\$36 billion
8%	5%	\$50 billion	11%	8%	\$31 billion

Of course, because both estimates are for 2004, you have to discount them back to 2001. In upstart firms, the early discount rate would often be higher than the long-run discount rate, which would be used in the growing perpetuity formula, because there is more uncertainty and market dependence before the firm reaches its more stable phase. For PepsiCo, however, risk is probably the same in 2001 as it is after 2004, so you can use the same discount rate. This gives valuations of:

$\mathcal{E}(r)$	$\mathcal{E}(r-g)$	$\text{PV}_{T=2001}(\text{TV}_{T=2004})$	$\mathcal{E}(r)$	$\mathcal{E}(r-g)$	$\text{PV}_{T=2001}(\text{TV}_{T=2004})$
6%	3%	$66 billion	9%	6%	$30 billion
7%	4%	$48 billion	10%	7%	$24 billion
8%	5%	$37 billion	11%	8%	$21 billion

This is a big problem—the spread between $24 billion and $48 billion in value is very large. Clearly, your uncertainty about the appropriate eternal growth rate and cost of capital makes a real difference in your valuation. What to do now? In real life, you would probably entertain a range of possible values, do more research, and pick estimates based on the purpose for which you wanted to use the pro forma. If you wanted to sell the company, you would pick a low discount and high growth rate. If you wanted to buy the company, you would want to claim a high discount and low growth rate. Yes, it is not all science!

25·5. Complete Pro Formas

You now have the components necessary to produce a pro forma: economic cash flow forecasts, a terminal value based on the cost of capital and eternal growth, and your discount factor. Let's put it all together.

25·5.A. An Unbiased Pro Forma

Reasons why our pro forma value estimate for PepsiCo is too low.

Table 25.7 uses one set of assumptions that we deemed to be reasonable. If you assume that $\mathcal{E}(r) - \mathcal{E}(g) = 10\% - 3\% = 7\%$, the total market value in 2001 that our pro forma indicates for PepsiCo is only about

$$\begin{aligned} \text{PV} &= \$1{,}556 + \$1{,}556 + \$1{,}556 + \$1{,}556 + \$24{,}452 \approx \$31{,}000 \\ &= \text{PV}(\text{CF}_{2001}) + \text{PV}(\text{CF}_{2002}) + \text{PV}(\text{CF}_{2003}) + \text{PV}(\text{CF}_{2004}) + \text{PV}(\text{TV}_{2004}) \;. \end{aligned} \tag{25.12}$$

If you had instead relied more on Coca Cola information and its growth capitalization, you would have inferred the much higher

$$\begin{aligned} \text{PV} &= \$1{,}556 + \$1{,}556 + \$1{,}556 + \$1{,}556 + \$48{,}000 \approx \$54{,}000 \\ &= \text{PV}(\text{CF}_{2001}) + \text{PV}(\text{CF}_{2002}) + \text{PV}(\text{CF}_{2003}) + \text{PV}(\text{CF}_{2004}) + \text{PV}(\text{TV}_{2004}) \;. \end{aligned} \tag{25.13}$$

If PepsiCo were a privately traded company, and you had no market value, you might have to settle on one of these two values as your best estimate of PepsiCo's market value as of 2001—or do more research to improve your forecasts.

Table 25.7. Direct Economic Cash Flow Projections

Pro Forma Cash Flow Statement

	Known		Cash Flow Model Growth at 10%			Terminal Value Growth at 3%	
	2000	2001	2002	2003	2004	2005	2005
	Year −1	Year 0	Year +1	Year +2	Year +3	Year +4	to ∞
Projected Annual CF[1]	\$1,556	\$1,712	\$1,883	\$2,071	\$2,278	\$2,506	next row
Terminal Value Equivalent for 2005 to eternity[2]					$\frac{\$2,506}{10\% - 3\%}$	← \$35,800	
Total Cash Flows	\$1,556	\$1,712	\$1,883	\$2,071	\$2,506+\$35,800≈\$38,306		
Discount Factor[3]		1	0.91	0.83	0.75	0.68	
Present Value of Cash Flows			\$1,712	\$1,712	\$28,780		
Total Present Value ≈ \$32,000							

Explanations (Notes):
[1]: Projecting 10% due to investments, until (incl.) 2005.
[2]: Projecting 3% eternal growth, beginning 2005 to 2006.
[3]: Discount factor based on 10% cost of capital.
It is coincidence that the present value of the cash flows is \$1,556 in years 2001-2004.

25·5.B. A Calibrated Pro Forma

We have more information about PepsiCo! We need to "calibrate" our model to the current market value.

Now switch perspective to that of an analyst who is analyzing not the hypothetical privately held company, but the actual publicly traded PepsiCo. This allows you to check whether your pro forma value is in line with the actual market value. It turns out that PepsiCo's stock market-value was actually around $87.4 billion in 2001 (plus about $3 billion of debt and another $10 billion in other liabilities). This suggests that your pro forma value estimate was *way low*. We will later investigate using as-yet-unavailable *ex-post information* why this is so, but for now let's just presume the following scenario: You are an analyst, and you are evaluating PepsiCo's capital structure—for example, to suggest changes that not only increase PepsiCo's value, but also generate fees for your employer, an investment bank. Naturally, you will have to present your pro forma to PepsiCo's management. But before you can do so, you would want to have a value estimate that fits the current market value of PepsiCo—otherwise, PepsiCo would likely be so displeased with your original pro forma value estimate that they would not listen to any proposals. It would also be silly for you to pretend that you believe that PepsiCo is worth only $40 billion when it is trading for $100 billion. You must somehow coerce your model to fit reality better—this is not called model fudging, but model **calibration**. In your case, you would look for reasons why PepsiCo would be worth more than what your original pro forma suggested.

We can tinker with the numbers.

You have basically three tools at your disposal that can do the job:

1. You can depart from your current projected cash flow path. The original pro forma relied on the direct-projection cash flows assuming a growth rate of 10%. Altering the cash flow path changes both the initial period cash flow projections and the 2005 cash flows of $2,506, upon which your terminal value was based. There are a number of alternative choices you could entertain.

   One way to justify higher cash flows is to argue for higher sales, lower expenses, higher future cash flows, etc. This could flow into a faster growth path for directly projected cash flows. For example, your calibrated model could assume that PepsiCo would be valued off cash flows that grow faster than 10%—say 15%.

	2001	2002	2003	2004	2005
	Year 0	Year +1	Year +2	Year +3	Year +4
Projected Economic CF	$1,789	$2,058	$2,366	$2,721	$3,130

   Another way to increase value is to work off the detailed financials rather than the direct projections, because they were higher, reaching $2,400 even for 2002.

   Yet another way is to (surreptitiously) shift your focus to earnings, either from the detailed financials or from the direct projection. You know that in the very long-run, discounted earnings and discounted cash flows should be roughly equal—after all, earnings "just" shift the time-series accruals; and that earnings may be more suitable to a growing-perpetuity valuation, because they are less affected by temporary and possibly lumpy investment patterns. Perhaps PepsiCo accelerated its investments in 2001, sacrificing immediate cash flows for higher future cash flows. So, relying on earnings growing at 3% per annum, you have the following revised

	2001	2002	2003	2004	2005
	Year 0	Year +1	Year +2	Year +3	Year +4
Projected Earnings	$2,662	$2,742	$2,824	$2,909	$2,996

   Or, you could rely on the detailed projections, which again were higher, reaching $2,828 already for 2002.

2. You can reduce your estimate of PepsiCo's cost of capital from 10% to a lower number. You can check if comparable firms like Coca Cola had lower market betas. (It did, so you may want to return to the original PepsiCo beta estimate of 0.7, and not shrink the beta.)

You can assume a lower equity premium than 4% per annum (say, 2%). You can assume the 5-year lower risk-free rate of 4.4%. This would give you a cost of capital of

$$\mathcal{E}(\tilde{r}_{\text{PEP}}) \approx 4.4\% + 2\% \cdot 0.7 \approx 5.8\% . \tag{25.14}$$

Rounded, the 6% cost of capital is significantly lower than the 10% that you assumed earlier.

3. You can increase PepsiCo's eternal earnings growth rate estimate g, thereby changing its growth profile—thereby assuming it has more of the characteristics of a growth firm. The original pro forma estimate had a $\mathcal{E}(r - g) = 7\%$ spread to the cost of capital. Now move to a Coca Cola-like 3% spread. With the reduced cost of capital estimate of 6% (rather than 10%), the divisor on the cash flow yielding the terminal value would be much lower, which means that the terminal value would be much higher.

Voi-la!

Table 25.8 contains one calibrated version of the PepsiCo pro forma—you could make up others. PepsiCo's management would be pleased with your calibrated pro forma—of course, assuming you do not show them your original version. It would indicate to them that their market value is justified.

Know what you are doing here when you are doing this!

In any case, you must be conceptually clear about what you are doing if you calibrate your pro forma: you are fudging numbers to make the outcome fit a market value that you believe is correct in a reasonably efficient market. You can do so appropriately and responsibly or inappropriately and irresponsibly. For example, by adopting Coca Cola's 3% growth rate as appropriate, you are accepting the market's assessment of Coca Cola at face value—even though this seems economically like an optimistically low cost of capital and optimistically high growth rate. In one sense, you are adopting a "deus ex machina"—a number that is dropped on you from another part of the stage (the financial markets) and that you therefore do not fully understand. In another sense, you are just doing what you have always done: you are doing *relative* valuation, accepting the known market-value of a comparable as a good baseline in your quest to compute another project's valuation.

Table 25.8. Calibrated Economic Cash Flow Projections

	Known		Pro Forma Cash Flow Statement: Cash Flow Model Growth at 15%			Terminal Value Growth at 3%	
	2000	2001	2002	2003	2004	2005	2005
	Year −1	Year 0	Year +1	Year +2	Year +3	Year +4	to ∞
Projected Annual CF[1]	$1,556	$1,789	$2,058	$2,366	$2,721	$3,130	next row
Terminal Value Equivalent for 2005 to eternity[2]					$\frac{\$3,130}{6\% - 3\%}$	← $104,333	
Total Cash Flows	$1,556	$1,789	$2,058	$2,366	$2,721 + $104,333 ≈ $107,054		
Discount Factor[3]		1	0.94	0.89	0.84	0.79	
Present Value of Cash Flows			$1,942	$2,106	$89,885		
Total Present Value ≈ $95,000							

Explanations (Notes):
[1]: Projecting 15% due to investments, until (incl.) 2005.
[2]: Projecting 3% eternal growth, beginning 2005 to 2006.
[3]: Discount factor based on a 6% cost of capital.

25·6. Alternative Assumptions and Sensitivity Analysis

You need sensitivity analysis

What should you learn from this chapter? Perhaps most importantly: do not trust any single pro forma estimate. But you can do more analysis to help you understand how robust your estimates actually are. Most such techniques are easiest to perform in spreadsheets, which allow you to try out different assumptions and scenarios.

25·6.A. Fiddle With Individual Items

You want to find a best estimate of value—not the simplest or most complex, easiest or hardest pro forma.

Always keep your ultimate goal in mind—you want to obtain the best (value) estimates for your business. Your goal is not an exercise in NPV analysis. It is not beauty or simplicity, either. Though both are nice to have, if elegance requires sacrificing important value drivers, you cannot do it. Use your head and your imagination!

You can use ad-hoc assumptions if you believe they offer better estimates.

You are allowed to use all sorts of other information—and even your opinion. For example, in our PepsiCo valuation, \$2,506 was the estimated expected cash flow in year 5. If you have good reason to believe that this is a low estimate, you can "fudge it." For example, if you believed that a new drink were to come on-line and give cash flows a one-time upward value transition of \$500 million, then you can use \$3,000 instead of \$2,506. It does not have to be formal, scientific forecasting. Of course, whoever is the consumer of your pro forma may not agree with your estimate, so you'd better be ready to mount a good and credible defense of your number.

You can use alternative terminal value estimates, too

Similarly, there are no laws that say that you have to use the growing perpetuity formula on cash flows to obtain your terminal market value. Instead of using the assumption that growth will remain eternally the same (say, 3%/year), you could develop another formula that assumes high growth rates for a few years (say, 5% next year), followed by growth rate declines until the growth rate reaches the inflation rate (say, 2% per year). Or, you might deem it best if you assumed that you could find a buyer for PepsiCo who will be paying \$200 billion in 5 years—ultimately, it is this quantity that you modeled with the terminal value. Again, you'd better be ready to argue why this is the best estimate.

More analysis can help to determine expected (rather than just most likely) cash flows.

Modeling the pro forma as a spreadsheet will also allow you to consider specific future scenarios. For example, what would happen if the new product were to be wildly successful, or if it were to fall on hard times (though few pro formas in the real world consider complete failure–a mistaken omission)? What would happen in a recession? What has happened in past recessions? What would happen if sales were to decline by 5% next year, rather than grow by 3.6% per year? What would happen if sales were to decline for a number of years, not just for one year? How bad would one or many inputs have to be for you to regret having bought into the project in the first place? And, of course, you can ask the venerable payback question: how long will it take before you get your money back? Admittedly, with more time, technology, and printing space, you really should look at many different modified scenario analyses to understand our PepsiCo pro forma better. Computer spreadsheets were invented precisely to make such analyses relatively easy.

25·6.B. Do Not Forget Failure

The biggest problem. This is a scenario, not an expected value! Overall failure is often not considered.

The biggest problem in most pro formas, however, is not even in the details. It is the fact that a pro forma is just one particular scenario, and usually a reasonably optimistic one. Many pro formas are modeling just a "typical" or median outcome (recall Section 7·3). This would not be dissimilar to an average outcome *but conditional on the project not aborting altogether.*

Entrepreneurial ventures—especially tech ventures—often have almost all value in the terminal value estimate.

Obviously, this is more important for entrepreneurial ventures or startups than it is for PepsiCo. For example, if someone pitches you a new magazine, most of the time, the pro forma will project a mildly optimistic scenario—*on condition that the magazine succeeds.* It probably won't take into account the fact that 50% of all magazines fold within a year. It is your task as the consumer of the pro forma to determine for yourself the probability of overall magazine failure, or you will end up misled. (Immediate death does not matter for our PepsiCo pro forma. PepsiCo is likely to stay around for a few more years.)

25·6.C. Assessing the Fudge Factor

What you should ask and what I can tell you.

By now, you should have realized that the question, “Which PepsiCo pro forma is correct?” is not a good one . *No* pro forma is correct! A better question is, “What kind of PepsiCo pro forma is better?” But perhaps the best question is, “How can I judge how good a pro forma is?” There are no easy answers.

An interesting diagnostic: what fraction of the value comes from the final value estimate?

Most importantly, consider your level of uncertainty about each item. In most valuations, the most important source of uncertainty is the terminal value. An interesting statistic is therefore what fraction of the value comes from the terminal value. In PepsiCo’s case, the value estimate was \$32 billion, of which roughly \$29 billion was the terminal value and \$5 billion was the initial period forecast. After calibration, the value estimate was about \$95 billion, of which \$90 billion came from the terminal value. So most of PepsiCo’s value is buried in the terminal value estimate. To the extent that you do not trust this terminal value, you should be particularly careful. Of course, if you had stretched T, more value would have been part of the detailed period rather than the terminal value—but this would not mean that your forecast would automatically have had more reliability. Consequently, the fraction of terminal value in the overall value should not be seen as more than an interesting diagnostic. Startup companies typically have more of the future cash flows far in the future, but they also have higher costs of capital. As a result, many entrepreneurial venture business plans have 80% to 95% of their value in this “gray box” called terminal value. Watch it!

Monte-Carlo Estimation.

There is an altogether different issue, too. A plain spreadsheet is really the wrong tool for a pro forma. Your input into each cell of your pro forma spreadsheet should really contain not just one number for your best estimate, but also a second number that tells you how reliable you deem your best estimate to be. It turns out that there is an even more sophisticated method of analysis—**Monte-Carlo Simulation**—which is like automated sensitivity analysis for a whole range of different scenarios. It takes this second number for each cell (your uncertainty!) into account, and produces a whole range of possible outcomes (NPV values). But this is beyond the scope of a first textbook in finance.

Monte-Carlo Analysis is explained in the web chapter on real options.

25·7. Proposing Capital Structure Change

We want to propose capital structure changes.

Return to the scenario in which you are an investment banker seeking to propose a capital structure change. Equipped with your calibrated pro forma, you can now go in front of PepsiCo's management and present two capital structure scenarios—the current structure and the proposed change. Your exposition of any proposed capital structure change will again be through a hypothetical pro forma, including the full balance sheet and financing section on the cash flow statement, but space limits our discussion to be in-text only.

The current situation.

Let's begin evaluating PepsiCo's current capital structure. In 2001, it had short-term debt of \$354 and long-term debt of \$2,651. Other liabilities and deferred income taxes added another \$5,372. The income statement tells us that this caused PepsiCo to pay \$219 in interest and provide \$1,367 for corporate income taxes. With \$4,029 in pre-tax earnings, this is a 34% average tax rate.

Judge the reasons pro and con capital structure.

On its market value of over \$90 billion, PepsiCo had very little debt, so the principal question is whether it would make sense for PepsiCo to take on more. To answer this question, you must weigh the various capital structure rationales from Part IV—questions like: How likely is PepsiCo to go into financial distress if it increases its leverage? How much could it save in corporate income taxes if it takes on more debt? How important are free cash flow concerns? And so on. In PepsiCo's case, the answers are relatively easy—the probability that PepsiCo will experience financial distress is very low. Moody's rated PepsiCo's current debt an A1, just below Aa3; Standard and Poor's rated it an A. For each dollar extra in debt rather than equity financing *forever*, you know that the corporate income tax savings would have a present value of $\tau \cdot D$.

See Section 18·7.B.

To sell to PepsiCo, you must estimate the cost of debt, and the change in value that management can expect.

To pitch a new debt issue, you would have to inform PepsiCo what you believe its cost of debt would be if it took on more debt. You would probably begin by looking at the credit ratings of other companies. For example, Table 25.9 gives some relevant statistics for firms with different credit ratings, debt ratios and interest coverages. In 2001, PepsiCo had a book-value based debt/assets ratio of 14%, and its EBIT/interest ratio was about 25. In fact, PepsiCo seemed like an outlier—its S&P rating should have been AA, not just A+.

Let's speculate on alternative capital structure interest rates.

Table 25.9 suggests that firms with long-term debt of about 30% and an EBIT/Interest ratio to 7 still tend to rank as "investment grade," a category that many investment professionals consider an important break. How much debt could PepsiCo take on to reach this high a level? The answer is around \$4 billion. With about \$4 billion additional debt, even at an 8% interest rate PepsiCo would still likely remain in the category of BBB bonds. A quick look at prevailing interest rates on financial websites further reveals that AAA bonds paid about 7%, BB bonds about 7.95% on average. Consequently, a PepsiCo with \$6.5 billion in debt would likely have to promise an interest rate of about 7.7%. Of course, to really convince PepsiCo, you should spend many more hours researching a good interest rate estimate for PepsiCo's new debt.

Table 25.9. Characteristics of Firms by S&P Bond Ratings, December 2001

		Investment Grade			Speculative Grade		
		AA	A	BBB	BB	B	C
Long-Term Debt / Book-Assets	Mean	23%	26%	34%	43%	54%	62%
	Std.Dev.	15%	16%	16%	20%	26%	56%
	Quart 1	11%	15%	23%	30%	36%	22%
	Median	20%	26%	33%	42%	52%	56%
	Quart 3	32%	37%	44%	53%	67%	86%
EBIT / Interest	Mean	17	11	7	5	4	1
	Std.Dev	15	15	11	14	25	4
	Quart 1	6	4	3	2	0	-1
	Median	14	7	5	3	1	0
	Quart 3	24	12	8	5	3	1

PepsiCo had an equivalent total debt over assets ratio of $(\$2,651 + \$354)/\$21,695 \approx 14\%$, and an equivalent operating income over interest ratio of $\$5,490/\$219 \approx 25$. Assets are book value based, and, for an old firm such as PepsiCo, severely understate assets.

What would the effect of such leveraging be on PepsiCo's value? With an interest rate of 7.7% on $4 billion of new debt, PepsiCo's interest proceeds would increase by $300 million. In turn, at its 33% tax rate, this would create a net present value of tax savings of about $100 million in the first year alone, and over $1.5 billion in tax savings in perpetuity—not bad for a day's work.

To create the tax savings, the money would need to be returned to shareholders—or else it would earn more taxable net income. This can be done either through dividend payments or through a share repurchase. Both have the disadvantage that if PepsiCo were overvalued in the market, as the original pro forma suggested, we should raise more money in the equity markets, too, and not repurchase our shares. As you learned, overvalued shares allow you to raise capital at very low expected inerest rates.

You can return the cash to shareholders either as dividends or in a repurchase. This makes sense primarily if you do not believe that shares are already overvalued.

See Section 15·3.C.

In real life, your problem would not be maximizing firm value; your problem would be convincing management—an example of an agency issue.

But your most important problem in proposing more leverage would almost surely be something else—it would be convincing PepsiCo's management. You could tell them that if they raised $4 billion in debt to repurchase $4 billion in equity, they would probably create an instant value increase of at least $1 billion—more than just one-year's $100 million savings, though less than the $1.5 billion perpetuity income tax savings. Unfortunately, this is unlikely to sway them. Clearly, with more debt and less equity, they would have less ability to take over other companies, start new projects, purchase corporate airplanes, or build empires. As an investment banker, in thinking about how to pitch to PepsiCo's management, you would have to ask yourself—what's in it for PepsiCo's management? The answer would most likely have to lie in the compensation package of management. But on an equity value of $90 billion, even $1 billion in more value is only about 1% of PepsiCo's stock market value—clearly, you would have an uphill struggle on your hands, even though a debt-for-equity issue would just as clearly create shareholder value. (And hindsight knowledge tells us not only that PepsiCo maintained its capital structure, but also that it continues to pay around $300 in interest expense, and continues to incur tax obligations of around $1,400 every year.) Your best shot may be to convince PepsiCo to take over another company and lever up in the process.

25·8. Hindsight

Let's now switch perspective again. This time, you will look at the preceding analysis as an economist with hindsight. Remember that in our previous perspective, PepsiCo was a publicly traded firm. Consequently, you had a real market value upon which you could calibrate your pro forma estimate. But why was this real value so much higher than our original unbiased pro forma estimate? Were the financial markets too optimistic, or were you too pessimistic?

This is unfair—you would not have this information.

In fact, we chose PepsiCo as of 2001 not only because you could then compare the pro forma estimate to the market value, but also because you can now see how your financial forecasts turned out in hindsight. Before you do this, you should realize, however, that actual ex-post performance is not necessarily the best estimate, because it contains subsequent and possibly unexpected developments. For example, if you bet on an Indian summer for next October, you can still lose if it were to snow—unlikely, but possible. This does not mean that your sunshine forecast was bad, or that you should have bet on snow—even though this may be of little consolation to you if it were snowing and you would have lost your money. Nevertheless, more often than not, ex-post analysis will come up with sunshine, helping us to judge what we predicted. Analogously, in our case, PepsiCo's later performance can help tell you why the markets were more optimistic than your pro forma was, though not perfectly so. The autopsy can give you some hints where we were wrong.

Table 25.10. Actual Vs. Forecast Cash Flows and Earnings for PepsiCo

Year	1999	2000	2001	2002	2003	2004
Actual Econ. Cash Flow	\$1,641	\$2,501	\$1,556	\$4,242	\$2,169	\$2,817
Projected, Direct, Table 25.2				\$1,883	\$2,071	\$2,278
Projected, Detailed, Table 25.4				\$2,400^{+}		
Actual Net Income	\$2,505	\$2,543	\$2,662	\$3,000	\$3,568	\$4,212
Projected, Direct, Page 633				\$2,742	\$2,824	\$2,909
Projected, Detailed. Table 25.3				\$2,828		

$^{+}$: The detailed projected cash flow omits interest paid, and is therefore a little too low.

If your cash flow forecasts were too low, your pro forma forecast would have been too pessimistic. Indeed, Table 25.10 shows this was indeed the case. In 2002, PepsiCo sold off some subsidiaries and therefore produced cash of over \$4.2 billion. In 2003, PepsiCo invested more than usual, and its cash flows dropped back to just above \$2 billion. Again, this confirms what you already knew—cash flows are too lumpy to be well suited to direct projections. But what about your earnings forecasts? They grew more smoothly than cash flows—but also much faster than what you had projected. By 2004, actual earnings were almost 50% higher than your detailed forecast. No wonder that the pro forma was too pessimistic!

Your cash flow and especially earnings forecasts were too low!

A closer reading of the 2002 annual report reveals what happened. After adjusting for changes in the reporting of sales and COGS, PepsiCo's 2002 sales actually increased by about \$1.6 billion, much more than the \$971 million sales growth forecast in the pro forma, and almost all of the increased sales ended up as profit. Higher sales in later years, too, can explain why most of the pro forma forecasts were so mistakenly low. Our method—mechanistic projection models from past financial data—is rarely very good, and PepsiCo was no exception. Unless you had known the business and market well enough to forecast sales this high, you would have stood no chance!

Further information.

You can also autopsy the pro forma estimate of $\mathcal{E}(r-g)$. As of mid 2005, PepsiCo had a market cap of \$90 billion on earnings of \$4.3 billion. Consequently, it is now capitalized at about $\mathcal{E}(r-g) \approx 4.8\%$, in between the two pro forma estimates you entertained, the pessimistic one at 7% and the optimistic one at 4%. Next, autopsy the forecast for $\mathcal{E}(r)$, again as of 2005. PepsiCo had a lower beta of only about 0.35—closer to the optimistic historical 0.7 beta than the pessimistic, shrunk beta of 0.9. Interest rates also turned out to remain low, so the 2005 cost of capital estimate might be

How to reach the \$100 billion!

$$\mathcal{E}(r) \;=\; 5\% + 3\% \cdot 0.35 \;\approx\; 6\% \;, \tag{25.15}$$

which was at the lower range of the cost of capital estimate. Together with the $\mathcal{E}(r-g) \approx 4.8\%$, this implies that PepsiCo is capitalized as if its earnings were to grow only by about 1.2% per year—not a very optimistic valuation, and indeed even lower both than the 2005 rate of inflation and than your pro forma growth estimate.

In sum, in hindsight, the primary driver of PepsiCo's higher value was its higher sales. The secondary driver was its lower cost of capital.

This was an "easy" pro forma—and we were still off by a factor of two.

Let this be a lesson in humility: even for a large and established company with a solid history, valuation is difficult and suffers from plenty of uncertainties—though economic knowledge could have done much to improve our estimates. But how much more uncertain are pro formas of upstart projects, in which most the value may lie far in the future?! This should not discourage you, however. Just as the CAPM is the premier model for the cost of capital, the pro forma is the premier model to write business plans—*simply, there is no better alternative.* Forecasting the future is the tough job that economic value is all about. Fortunately, you don't even need to be able to forecast *well.* All that matters is that you can forecast *better* than the rest of us. If you can, you will become rich.

A reasonable way to approach public market values.

Finally, how would I, as an investor in 2001, have looked at your pro forma? Most of my faith would have been in the market value of PepsiCo, not in your pro forma value analysis. I would not have trusted your ability to forecast the economics. However, if you had had more knowledge of the underlying sales dynamics, your value analysis would have raised enough doubts in me to believe that PepsiCo might be a little overvalued. After all, any public market value is the clearing price where the bears and bulls on PepsiCo are in equilibrium—and your analysis would have led me to join the bears. But I would have kept it all in proper perspective—it would have been irrational to believe that the appropriate market price of PepsiCo would be the pro forma \$35 billion when I could have seen the market value of \$100 billion—a reasonable synthesis of the PepsiCo value estimates would instead have concluded a value closer to the market value than to the pro forma value—say, a synthesis of \$95 billion.

25·9. Caution — The Emperor's New Clothes

Do not automatically trust pro formas! They often look very professional even if they are not credible.

Did our projections seem arbitrary to you? They should, *because they were arbitrary*—and our chapter made a point of telling you so throughout. But look back at the financials in Tables 25.3 and 25.4. If you did not round but quoted a few more digits (for pseudo-accuracy), if you expanded the footnotes with some more mumbo-jumbo, and if you added a few more columns for 2003 to 2005, a naïve reader might be fooled into thinking that we were sophisticated analysts who knew what we were doing! It is important that you not end up being such a naïve consumer of pro formas. A well-written pro forma can easily convey an image of professional knowledge where there is none. (Form over content may work here!) But in the case of pro formas, even the best emperor wears only a bathing suit.

Do not lose the forest and discuss mini-details.

Another danger for the unwary pro forma reader is falling into the trap of looking at the trees, rather than the forest. You can easily get involved in endless discussions of a particular projected item in someone else's pro forma. In real life, most pro formas rely on plenty of heroic assumptions—in some cases, there are just one or two critical assumptions, in other cases, there may be many. You must look at the big picture as well as at the minor assumptions. There is devil in both detail and in the sum-total.

What a good pro forma is and is not.

I hope I am not sounding dismissive of pro formas. On the contrary—again, you really have *no* alternative, and forecasting the future is inherently a difficult task. The universal use of heroic assumptions does not mean that there is no difference between a good and a bad pro forma. A less naïve reader can certainly distinguish a good one from a bad one. A good pro forma pitched to a sophisticated audience must use solid economics and have detailed footnotes explaining and justifying just about every important line item. It is a starting point for a good discussion, not an end in itself.

Closing the circle.

Ultimately, finance is about value, so it must revolve around projections, and pro formas are a good tool to organize projections. Projecting is very hard. Remember how the book started? I told you that valuation is both an art and a science. The formulas are easy, the application is hard. I trust that you believe me now. Welcome to the club of financiers!

25·10. Summary

The chapter covered the following major points:

- The purpose of pro formas is to project financials, which are then often used to compute a project's NPV today. You can also use pro formas to perform a ratio analysis to test the financial soundness of a business plan.
- Pro formas are often not only idiosyncratic, but also not very reliable—but there is no alternative.
- Pro formas are usually split into a detailed forecast period and a terminal value.
- A good horizon choice for the detailed forecast period depends on the economics of the business and the prevailing discount rate.
- A quick-and-dirty pro forma analysis may just project the line items of direct use. A more complete pro forma analysis can try to project many intermediate components.
- A useful distinction is to think of fixed vs. variable (sales-contingent) forecasts for individual components.
- Scenario analysis helps to better understand pro forma uncertainty.
- Use caution in constructing and interpreting pro formas.

A. Appendix: In-a-Pinch Advice: Fixed vs. Variable Components

What is fixed, what is variable? Some advice.

Is it possible to predict *in general* how firms' income statements and cash flow statements are likely to develop in the future? Is depreciation better modeled as consisting of fixed+variable components, or is it better modeled as a fixed component only, or as a variable component only? Is COGS more sales-variable or more stable, or are dividends? Of course, every business is different, so there are no uniform answers here. Some firms rely more on fixed-cost technologies, others on variable cost technologies. However, rather than not provide any guidance, I will now describe how corporate financials have evolved *on average* in publicly traded companies. Our specific interest is whether particular accounting items have been better explained by their own history or by sales growth. Although such knowledge of how the average publicly traded firm has evolved can sometimes help you in a pinch (when you need something quickly and without much thought), it is better if you regard this section as a "jumpstart" to get you to do more economic thinking, exploration, and business modeling of your particular company.

> **IMPORTANT**: *If you can, ignore the crutches provided for you in this section. Instead, execute your modeling based on specific and sound intelligence about your business.*

Our projections consist only of a fixed component and a variable (sales-related) component.

Our basic public company financial item prediction model will be

$$\mathcal{E}\left(X_{t+1}\right) \approx \gamma_{\text{fixed}} \cdot X_t + \gamma_{\text{variable}} \cdot \left\{X_t \cdot \left[\frac{\mathcal{E}\left(\text{sales}_{t+1}\right)}{\text{sales}_t}\right]\right\} . \tag{25.16}$$

where X is a financial statement number, such as COGS or SG&A, and t is a year index. For example, statistical history suggests that

$$\begin{aligned}\mathcal{E}\left(\text{SG\&A}_{t+1}\right) &\approx 36\% \cdot \text{SG\&A}_t + 68\% \cdot \left\{\text{SG\&A}_t \cdot \left[\frac{\mathcal{E}\left(\text{sales}_{t+1}\right)}{\text{sales}_t}\right]\right\} \\ &= \gamma_{\text{fixed}} \cdot \text{SG\&A}_t + \gamma_{\text{variable}} \cdot \left\{\text{SG\&A}_t \cdot \left[\frac{\mathcal{E}\left(\text{sales}_{t+1}\right)}{\text{sales}_t}\right]\right\} .\end{aligned} \tag{25.17}$$

This says that the typical firm's SG&A was about one-third related to its own past SG&A value, and two-thirds related to SG&A adjusted for sales growth. How would you use this prediction in our PepsiCo pro forma? In 2001, PepsiCo had SG&A of \$11,608, and sales of \$26,935. Projected 2002 sales were \$27,906 for a 3.6% increase. Thus, Formula 25.17 suggests

$$\begin{aligned}\mathcal{E}\left(\text{SG\&A}_{2002}\right) &\approx 36\% \cdot \$11,608 + 68\% \cdot \left[\$11,608 \cdot \left(\frac{\$27,906}{\$26,935}\right)\right] . \\ &\approx 36\% \cdot \$11,608 + 68\% \cdot [\$11,608 \cdot (1+3.6\%)] \\ &\approx 36\% \cdot \$11,608 + 68\% \cdot \$12,025 \approx \$12,356 .\end{aligned} \tag{25.18}$$

The left part in the formula measures the "fixed effect," i.e., the degree to which SG&A remains the same as last year's SG&A, independent of PepsiCo's 2002 sales growth. The right part in the formula measures the "variable effect," i.e., how SG&A has to increase with sales growth in 2002.

SIDE NOTE: The reason why the coefficients in formula 25.18 do not add up to 1 is that SG&A increased on average in the sample—perhaps due to inflation. If γ_{fixed} is 1 and γ_{variable} is 0, then the best prediction of X next year is the same as X this year. If γ_{fixed} is 0 and γ_{variable} is 1, then the best prediction of X next year is obtained by multiplying last year's X by the observed or predicted sales increase from this year to next year.

It is important that you do not believe that the precise coefficient estimates of 36% and 68% are applicable to *your* company. They are based on mechanical statistical models, which rely only on historical information for publicly trading companies that may be totally unrelated to your own, and on a time period that is ancient history. The coefficient estimates can serve only as "quick-and-dirty" stand-ins until you use your skills and smarts to produce something better. They are here only to help give you some initial guidance in your own economic exploration of whether a particular financial item in your firm tends to be more fixed or more variable.

Again, use the estimates for guidance, and—if need be—as stand-ins, but do not believe they fit your project well.

Moreover, keep in mind that most of the time, you will be asked to create a pro forma when the company contemplates a change in policy, or when you want to propose a new project. The historical behavior of large publicly traded companies is unlikely to be a good representation of what will happen in such circumstances. Instead, your pro forma forecasts must be specific in addressing contemplated policy changes. So, please do better than the formulas below.

Projection formulas can definitely be hazardous to your wealth. Watch it.

Enough words of caution. Here are some nuggets of forecasting advice:

Sales This is the most important variable. You must forecast this number as diligently as you possibly can. Other variables below can depend on this critical estimate. For illustration, we shall forecast PepsiCo's 2002 sales to be \$27,906, which means that PepsiCo's 2002 sales growth is $\$27,906/\$26,935 - 1 \approx 3.6\%$.

COGS In our average publicly traded companies,

$$\mathcal{E}(\text{COGS}_{t+1}) \approx 6\% \cdot \text{COGS}_t + 95\% \cdot \left\{\text{COGS}_t \cdot \left[\frac{\mathcal{E}(\text{sales}_{t+1})}{\text{sales}_t}\right]\right\} . \tag{25.19}$$

Coefficients so close to 0 and 1, respectively, suggest that cost of goods sold is best explained as a constant ratio of sales (unless the firm deliberately shifts production into different [fixed cost] production). Like all other formulas below, this formula is based on the history of reasonably large publicly traded U.S. firms (and thus is neither necessarily applicable to smaller firms nor to the future).

To use this formula to forecast PepsiCo's COGS for 2002, you would compute

$$\begin{aligned} \mathcal{E}(\text{COGS}_{2002}) &\approx 6\% \cdot \text{COGS}_{2001} + 95\% \cdot \left\{\text{COGS}_{2001} \cdot \left[\frac{\mathcal{E}(\text{sales}_{2002})}{\text{sales}_{2001}}\right]\right\} . \\ &\approx 6\% \cdot \$10{,}754 + 95\% \cdot \{\$10{,}754 \cdot [1.036]\} \\ &\approx \$11{,}229 . \end{aligned} \tag{25.20}$$

SG&A Selling, general & administrative expenses tend to have both a fixed and a variable component. A typical firm may be modeled by assuming that two-thirds is related to the sales increase, and one-third is related to historical SG&A. A formula estimated on reasonably large publicly traded U.S. firms suggests that

$$\mathcal{E}(\text{SG\&A}_{t+1}) \approx 36\% \cdot \text{SG\&A}_t + 68\% \cdot \left\{\text{SG\&A}_t \cdot \left[\frac{\mathcal{E}(\text{sales}_{t+1})}{\text{sales}_t}\right]\right\} . \tag{25.21}$$

For PepsiCo,

$$\begin{aligned} \mathcal{E}(\text{SG\&A}_{2002}) &\approx 36\% \cdot \text{SG\&A}_{2001} + 68\% \cdot \left\{\text{SG\&A}_{2001} \cdot \left[\frac{\mathcal{E}(\text{sales}_{2002})}{\text{sales}_{2001}}\right]\right\} . \\ &\approx 36\% \cdot \$11{,}608 + 68\% \cdot \{\$11{,}608 \cdot [1.036]\} \\ &\approx \$12{,}356 . \end{aligned} \tag{25.22}$$

Unusual Expenses No particular advice.

Operating Income Either construct from the items above (i.e., use the accounting identities), or forecast as

$$\mathcal{E}\,(\,\text{Oper.Inc.}_{t+1}\,) \;\approx\; -41\% \cdot \text{Oper.Inc.}_t \;+\; 120\% \cdot \left\{ \text{Oper.Inc.}_t \cdot \left[\frac{\mathcal{E}\,(\,\text{sales}_{t+1}\,)}{\text{sales}_t} \right] \right\} \;. \tag{25.23}$$

Note that operating income is extremely sensitive to sales growth: any extra sales on the margin has more than a one-to-one effect on operating income. This is why the first coefficient is negative and the second is above 1. It makes economic sense: operating income goes positive only above some break-even sales point. (A strong sensitivity to sales growth also appears in some other variables below.) However, there is one unusual feature of this formula that you should understand: the two coefficients sum up to considerably less than 100%. This means that the formula indicates a strong "drift" of operating income towards zero. For example, for PepsiCo,

$$\begin{aligned} \mathcal{E}\,(\,\text{Oper.Inc.}_{2002}\,) &\approx -41\% \cdot \text{Oper.Inc.}_{2001} \;+\; 120\% \cdot \left\{ \text{Oper.Inc.}_{2001} \cdot \left[\frac{\mathcal{E}\,(\,\text{sales}_{2002}\,)}{\text{sales}_{2001}} \right] \right\} \;. \\ &\approx -41\% \cdot \$4{,}021 \;+\; 120\% \cdot \{\$4{,}021 \cdot [1.036]\} \\ &\approx \$3,350 \;. \end{aligned} \tag{25.24}$$

You would estimate declining operating income even in the face of increasing sales! This also occurs in a number of formulas below. You must watch out for this—and think about whether such a drift towards zero would make sense for your particular company and pro forma!

Interest Income/Payments Either construct from debt and/or previous year's interest payments, or forecast as

$$\mathcal{E}\,(\,\text{Intst Inc.}_{t+1}\,) \;\approx\; 22\% \cdot \text{Intst Inc.}_t \;+\; 67\% \cdot \left\{ \text{Intst Inc.}_t \cdot \left[\frac{\mathcal{E}\,(\,\text{sales}_{t+1}\,)}{\text{sales}_t} \right] \right\} \;. \tag{25.25}$$

Remember: If a change in capital structure policy is contemplated, this item needs to reflect it.

$$\begin{aligned} \mathcal{E}\,(\,\text{Intst Inc.}_{2002}\,) &\approx 22\% \cdot \text{Intst Inc.}_{2001} \;+\; 67\% \cdot \left\{ \text{Intst Inc.}_{2001} \cdot \left[\frac{\mathcal{E}\,(\,\text{sales}_{2002}\,)}{\text{sales}_{2001}} \right] \right\} \;. \\ &\approx 22\% \cdot \$8 \;+\; 67\% \cdot \{\$8 \cdot [1.036]\} \\ &\approx \$7 \;. \end{aligned} \tag{25.26}$$

Income Before Tax Either construct from items above, or forecast as

$$\mathcal{E}\,(\,\text{Inc.bef.Tax}_{t+1}\,) \;\approx\; -32\% \cdot \text{Inc.bef.Tax}_t \;+\; 116\% \cdot \left\{ \text{Inc.bef.Tax}_t \cdot \left[\frac{\mathcal{E}\,(\,\text{sales}_{t+1}\,)}{\text{sales}_t} \right] \right\} \;. \tag{25.27}$$

For PepsiCo,

$$\begin{aligned} \mathcal{E}\,(\,\text{Inc.bef.Tax}_{2002}\,) &\approx -32\% \cdot \text{Inc.bef.Tax}_{2001} \;+\; 116\% \cdot \left\{ \text{Inc.bef.Tax}_{2001} \cdot \left[\frac{\mathcal{E}\,(\,\text{sales}_{2002}\,)}{\text{sales}_{2001}} \right] \right\} \;. \\ &\approx -32\% \cdot \$4{,}029 \;+\; 116\% \cdot \{\$4{,}029 \cdot [1.036]\} \\ &\approx \$3,553 \;. \end{aligned} \tag{25.28}$$

Income Tax Either construct from items above, or forecast as

$$\mathcal{E}\,(\,\text{Income Tax}_{t+1}\,) \;\approx\; -55\% \cdot \text{Income Tax}_t \;+\; 123\% \cdot \left\{ \text{Income Tax}_t \cdot \left[\frac{\mathcal{E}\,(\,\text{sales}_{t+1}\,)}{\text{sales}_t} \right] \right\} \;. \tag{25.29}$$

For PepsiCo,

$$\begin{aligned}\mathcal{E}\,(\,\text{Income Tax}_{2002}\,) &\approx -55\%\cdot\text{Income Tax}_{2001} + 123\%\cdot\left\{\text{Income Tax}_{2001}\cdot\left[\frac{\mathcal{E}\,(\,\text{sales}_{2002}\,)}{\text{sales}_{2001}}\right]\right\} \;.\\ &\approx -55\%\cdot\$1{,}367 + 123\%\cdot\{\$1{,}367\cdot[1.036]\}\\ &\approx \$990 \;.\end{aligned} \tag{25.30}$$

Income After Tax Either construct from items above, or forecast as

$$\mathcal{E}\,(\,\text{Inc.aft.Tax}_{t+1}\,) \approx -30\%\cdot\text{Inc.aft.Tax}_{t} + 113\%\cdot\left\{\text{Inc.aft.Tax}_{t}\cdot\left[\frac{\mathcal{E}\,(\,\text{sales}_{t+1}\,)}{\text{sales}_{t}}\right]\right\} \;. \tag{25.31}$$

For PepsiCo,

$$\begin{aligned}\mathcal{E}\,(\,\text{Inc.aft.Tax}_{2002}\,) &\approx -30\%\cdot\text{Inc.aft.Tax}_{2001} + 113\%\cdot\left\{\text{Inc.aft.Tax}_{2001}\cdot\left[\frac{\mathcal{E}\,(\,\text{sales}_{2002}\,)}{\text{sales}_{2001}}\right]\right\} \;.\\ &\approx -30\%\cdot\$2{,}662 + 113\%\cdot\{\$2{,}662\cdot[1.036]\}\\ &\approx \$2,318 \;.\end{aligned} \tag{25.32}$$

Extraordinary Items No specific advice.

Net Income Either construct from items above, or forecast as

$$\mathcal{E}\,(\,\text{Net Inc.}_{t+1}\,) \approx -42\%\cdot\text{Net Inc.}_{t} + 114\%\cdot\left\{\text{Net Inc.}_{t}\cdot\left[\frac{\mathcal{E}\,(\,\text{sales}_{t+1}\,)}{\text{sales}_{t}}\right]\right\} \;. \tag{25.33}$$

For PepsiCo,

$$\begin{aligned}\mathcal{E}\,(\,\text{Net Inc.}_{2002}\,) &\approx -42\%\cdot\text{Net Inc.}_{2001} + 114\%\cdot\left\{\text{Net Inc.}_{2001}\cdot\left[\frac{\mathcal{E}\,(\,\text{sales}_{2002}\,)}{\text{sales}_{2001}}\right]\right\} \;.\\ &\approx -42\%\cdot\$2{,}662 + 114\%\cdot\{\$2{,}662\cdot[1.036]\}\\ &\approx \$2,026 \;.\end{aligned} \tag{25.34}$$

Depreciation, Depletion, Amortization Either construct from items above, or forecast as

$$\mathcal{E}\,(\,\text{DDA}_{t+1}\,) \approx 42\%\cdot\text{DDA}_{t} + 62\%\cdot\left\{\text{DDA}_{t}\cdot\left[\frac{\mathcal{E}\,(\,\text{sales}_{t+1}\,)}{\text{sales}_{t}}\right]\right\} \;. \tag{25.35}$$

For PepsiCo,

$$\begin{aligned}\mathcal{E}\,(\,\text{DDA}_{2002}\,) &\approx 42\%\cdot\text{DDA}_{2001} + 62\%\cdot\left\{\text{DDA}_{2001}\cdot\left[\frac{\mathcal{E}\,(\,\text{sales}_{2002}\,)}{\text{sales}_{2001}}\right]\right\} \;.\\ &\approx 42\%\cdot\$1{,}082 + 62\%\cdot\{\$1{,}082\cdot[1.036]\}\\ &\approx \$1,149 \;.\end{aligned} \tag{25.36}$$

Deferred Taxes Very strongly related to sales growth and/or capital investment.

Non-Cash Items Very sticky, but negatively related to sales growth.

Changes in Working Capital Changes in working capital are cash consumers, especially when the firm is growing fast! Consequently, this is one of the cases where a negative coefficient on the sales-growth-adjusted term makes sense! And, indeed, we find that a decent model for large firms is

$$\mathcal{E}\,(\,\Delta WC_{t+1}\,) \;\approx\; 46\% \cdot \Delta WC_t \;+\; (-43\%) \cdot \left\{\Delta WC_t \cdot \left[\frac{\mathcal{E}\,(\,\text{sales}_{t+1}\,)}{\text{sales}_t}\right]\right\} \;. \tag{25.37}$$

For PepsiCo,

$$\begin{aligned} \mathcal{E}\,(\,\Delta WC_{2002}\,) &\approx 46\% \cdot \Delta WC_{2001} + (-43\%) \cdot \left\{\Delta WC_{2001} \cdot \left[\frac{\mathcal{E}\,(\,\text{sales}_{2002}\,)}{\text{sales}_{2001}}\right]\right\} \;. \\ &\approx 46\% \cdot \$84 + (-43\%) \cdot \{\$84 \cdot [1.036]\} \\ &\approx 1 \;. \end{aligned} \tag{25.38}$$

Capital Expenditures Capital expenditures seem to be strongly related to sales growth.

$$\mathcal{E}\,(\,\text{CapExp}_{t+1}\,) \;\approx\; 0\% \cdot \text{CapExp}_t \;+\; 100\% \cdot \left\{\text{CapExp}_t \cdot \left[\frac{\mathcal{E}\,(\,\text{sales}_{t+1}\,)}{\text{sales}_t}\right]\right\} \;. \tag{25.39}$$

For PepsiCo,

$$\begin{aligned} \mathcal{E}\,(\,\text{CapExp}_{2002}\,) &\approx 0\% \cdot \text{CapExp}_{2001} + 100\% \cdot \left\{\text{CapExp}_{2001} \cdot \left[\frac{\mathcal{E}\,(\,\text{sales}_{2002}\,)}{\text{sales}_{2001}}\right]\right\} \;. \\ &\approx 0\% \cdot \$1{,}324 + 100\% \cdot \{\$1{,}324 \cdot [1.036]\} \\ &\approx \$1,324 \;. \end{aligned} \tag{25.40}$$

Note: If a change in capital expenditures policy is contemplated, this item needs to reflect it.

Other Investing Very sticky, but negatively related to sales growth.

Total Cash Flows From Investing Activity

$$\mathcal{E}\,(\,\text{CF-Inv}_{t+1}\,) \;\approx\; (-320\%) \cdot \text{CF-Inv}_t \;+\; 340\% \cdot \left\{\text{CF-Inv}_t \cdot \left[\frac{\mathcal{E}\,(\,\text{sales}_{t+1}\,)}{\text{sales}_t}\right]\right\} \;. \tag{25.41}$$

For PepsiCo,

$$\begin{aligned} \mathcal{E}\,(\,\text{CF-Inv}_{2002}\,) &\approx (-320\%) \cdot \text{CF-Inv}_{2001} + 340\% \cdot \left\{\text{CF-Inv}_{2001} \cdot \left[\frac{\mathcal{E}\,(\,\text{sales}_{2002}\,)}{\text{sales}_{2001}}\right]\right\} \;. \\ &\approx (-320\%) \cdot \$2{,}637 + 340\% \cdot \{\$2{,}637 \cdot [1.036]\} \\ &\approx \$850 \;. \end{aligned} \tag{25.42}$$

Very strongly related to sales growth.

Financing Cash Flow Items No useful relationship.

Dividends Very sticky, but negatively related to sales growth.

$$\mathcal{E}\,(\,\text{Dividends}_{t+1}\,) \;\approx\; 159\% \cdot \text{Dividends}_t \;+\; (-82\%) \cdot \left\{\text{Dividends}_t \cdot \left[\frac{\mathcal{E}\,(\,\text{sales}_{t+1}\,)}{\text{sales}_t}\right]\right\} \;. \tag{25.43}$$

This estimated formula often does not make much economic sense: Why would dividends go down if sales go up? It is not altogether impossible, of course. For example, if the firm experiences great sales surprises, it may decide that it needs the money to cover working capital or that it wants to reinvest the money rather than pay it out as dividends. However, you should consider this on a case-by-case basis. You might be better off just assuming last year's dividends.

Net Stock Issuing No useful relationship. Strongly related to sales growth.

Net Debt Issuing Strongly related to sales growth.

$$\mathcal{E}\,(\,\text{Debt-Issue}_{t+1}\,) \;\approx\; (-192\%)\cdot\text{Debt-Issue}_t \;+\; 195\%\cdot\left\{\text{Debt-Issue}_t\cdot\left[\frac{\mathcal{E}\,(\,\text{sales}_{t+1}\,)}{\text{sales}_t}\right]\right\}\;. \tag{25.44}$$

Total Cash Flows From Financing Activity Mildly related to sales growth.

$$\mathcal{E}\,(\,\text{CF-Fin}_{t+1}\,) \;\approx\; (-0.07)\cdot\text{CF-Fin}_t \;+\; 0.25\cdot\left\{\text{CF-Fin}_t\cdot\left[\frac{\mathcal{E}\,(\,\text{sales}_{t+1}\,)}{\text{sales}_t}\right]\right\}\;. \tag{25.45}$$

For PepsiCo,

$$\begin{aligned}\mathcal{E}\,(\,\text{CF-Fin}_{2002}\,) &\approx (-0.07)\cdot\text{CF-Fin}_{2001} \;+\; 0.25\cdot\left\{\text{CF-Fin}_{2001}\cdot\left[\frac{\mathcal{E}\,(\,\text{sales}_{2002}\,)}{\text{sales}_{2001}}\right]\right\}\;.\\ &\approx (-0.07)\cdot\$1{,}919 \;+\; 0.25\cdot\{\$1{,}919\cdot[1.036]\}\\ &\approx \$363\;.\end{aligned} \tag{25.46}$$

Foreign Exchange Effects Sticky.

$$\mathcal{E}\,(\,\text{FX}_{t+1}\,) \;\approx\; 0.75\cdot\text{FX}_t \;+\; (-0.52)\cdot\left\{\text{FX}_t\cdot\left[\frac{\mathcal{E}\,(\,\text{sales}_{t+1}\,)}{\text{sales}_t}\right]\right\}\;. \tag{25.47}$$

For PepsiCo,

$$\begin{aligned}\mathcal{E}\,(\,\text{FX}_{2002}\,) &\approx 0.75\cdot\text{FX}_{2001} \;+\; (-0.52)\cdot\left\{\text{FX}_{2001}\cdot\left[\frac{\mathcal{E}\,(\,\text{sales}_{2002}\,)}{\text{sales}_{2001}}\right]\right\}\;.\\ &\approx 0.75\cdot\$4 \;+\; (-0.52)\cdot\{\$4\cdot[1.036]\}\\ &\approx \$1\;.\end{aligned} \tag{25.48}$$

Total Net Cash Flows

$$\mathcal{E}\,(\,\text{Net CF}_{t+1}\,) \;\approx\; 272\%\cdot\text{Net CF}_t \;+\; (-267\%)\cdot\left\{\text{Net CF}_t\cdot\left[\frac{\mathcal{E}\,(\,\text{sales}_{t+1}\,)}{\text{sales}_t}\right]\right\}\;. \tag{25.49}$$

Here is an example of an estimated formula that serves as a warning: a negative coefficient on the sales-growth adjusted number probably makes little sense for most large companies. Yes, it could be that the company does consume more working capital as it grows, but it just does not seem to be applicable in many cases—such as PepsiCo. You might just want to avoid this formula.

> **Digging Deeper**: *The formulas are estimated using a statistical technique called "regression analysis." For super-nerds, to normalize firms, all variables were normalized by sales, regressions were run firm-by-firm, and the coefficients were then averaged over firms. Even more sophisticated modeling assumptions and techniques did no better than the simple regression approach adopted here.*

In conclusion, do not trust these formulas. They are merely tools you can use for constructing a first draft of your pro forma—they are not good blueprints. Forecasting the performance of any business, but especially a new business, remains an art that relies on the underlying sciences of economics, statistics, accounting and finance. Don't just rely on statistics alone. Use common sense. Use good knowledge of the economics of the business and the industry. Document your reasoning in informed and detailed footnotes. And then—pray!

Solve Now!

Q 25.1 *Complete the 2002 forecast in the cash flow statement model in Table 25.4. Create a forecast for 2003. (Iterate on depreciation and investing to determine sensible inputs into both.)*

Solutions and Exercises

1. There is no clear and unique answer to this question. Here is a reasonable attempt.

	Income Statement	December			
		2001	2002	2003	Model Used
=	Sales	\$26,935	\$27,906	\$28,911	grows by 3.6%
	COGS	\$10,754	\$10,761	\$11,023	\$3,506+26% of revenue
	+ SG&A	\$11,608	\$12,279	\$12,721	44% of revenue
	+ Deprec/Amort	\$165	\$168	\$168	3-year average
	+ Unusual Expenses	\$387	\$279	\$289	1% of revenue
–	= **Operating Expenses**	\$22,914	\$23,486	\$24,201	Sum The Above
=	**Operating Income**	\$4,021	\$4,420	\$4,710	Subtract The Above
+	Net Interest Income	\$8	\$0	\$0	Too Ignorant and Lazy
=	**Income before Tax**	\$4,029	\$4,420	\$4,710	Subtract The Above
–	Corporate Income Tax	\$1,367	\$1,591	\$1,696	36% of IBT
=	**Income After Tax**	\$2,662	\$2,828	\$3,014	Subtract The Above
–	Extraordinary Items	\$0	\$0	\$0	Too Ignorant and Lazy
=	**Net Income**	\$2,662	\$2,828	\$3,014	

	Cash Flow Statement	December		
		2002	2003	Model Used
=	Net Income	\$2,828	\$3,014	transferred
+	Depreciation and Depletion	\$1,149	\$1,216	formula
+	Deferred Taxes	\$286	\$305	18% of Income Tax
+	Non-Cash Items	–\$46	–\$46	Average Historical
+	Changes in Working Capital	\$43	\$71	27% of Revenue *Increase*
=	**Total Operating Activity**	\$4,260	\$4,560	Sum Above
=	Capital Expenditures	–\$1,313	–\$1,321	$-\$1,200 - 4\% \cdot$ Earnings
+	Other Investing	–\$1,000	–\$1,000	Arbitrary. Sticky.
=	**Total Investing Activity**	–\$2,313	–\$2,321	Sum The Above

Depreciation: 42% times prior year depreciation plus 62% times sales-grossed-up prior depreciation.

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

Part VI

Appendices

APPENDIX A

Epilogue

Afterthoughts and Opinions. **Preliminary**

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You have traveled a long distance with me throughout this book. We have now reached the Epilogue, where by tradition, I am allowed to voice my own personal and perhaps unscientific opinions. I want to leave you with some of my thoughts on business and finance education, finance as a discipline, and financial research.

Anecdote: Yogi Berra's Theory and Practice

In theory, there is no difference between theory and practice. In practice, there is.

— Yogi Berra

A·1. Thoughts on Business and Finance Education

By nature, most disciplines in business schools, but especially finance, are closely related to practice. It is not an overstatement to claim that the majority of ideas in finance were either invented or developed in Academia, before they crossed over into practice. Unfortunately, over the years, fundamental misunderstandings have developed, which have become the source of much frustration among both faculty and students. Let me try to correct some of them.

A·1.A. Common Student Misconceptions

Some students seem to believe that business schools exist primarily to increase salaries and to enhance job opportunities. As a result, they expect a "vocational education." It is no wonder that they are especially fond of some practitioner-teachers, who can share plenty of war stories, vouch for the importance of their own teaching in their business environment, and may even help some students to get a job at their own or their friends' businesses.

This is a sad and limited view of what business schools have to offer. It will necessarily cause their finance education to be a rather unrewarding experience. Vocational training is not what top business schools are good at. The top business schools are without exception *not* vocational training centers, but research centers. Community colleges teach job-specific skills; universities do not!

Business schools provide—or at least should provide—a profoundly intellectual experience. Such an experience allows students to take a fresh look at the world, to explore other business areas for the first time, to learn how to think in economic and business terms, to consider the intellectual foundations of business, and to learn about the most novel ideas—those that have not yet permeated practice. Chances are that the practice in any given company is based on knowledge that the previous generation of managers learned in business schools *ten to twenty years ago*.

So, the value of an M.B.A. graduate—even to the first employer—is not his/her immediate business knowledge. It makes no sense for M.B.A. students to learn how the fixed-income department at Goldman Sachs works *this year*, which is well known by anyone working there (including the secretaries), and which will surely be best explained by the Goldman Sachs traders to any new hire upon arrival. Instead, the value of M.B.A. students to an employer is the intellectual ability; knowledge of the fundamentals, basic theories and their application; cutting edge ideas; human skills; team skills; sales skills, etc. Some of these skills are native, but most can be taught or at least improved upon by studying. In the end, it is an individual's versatility and curiosity, an ability to generalize and synthesize, and a talent for bringing an aerial perspective to specific problems that will allow the newly minted M.B.A. to be of value for many years to come.

Naturally, many students feel a great deal of anxiety about job prospects, and therefore they tend to prefer skills that they believe will facilitate immediate placement upon graduation. Trust me: pretending to have been taught business practice in business schools is not what employers want. Employers first and foremost want to hire smart, curious, and enthusiastic individuals, who are solid on the basic concepts and who can apply them to new situations. They can teach their own practices better than business schools can.

Anecdote: The Time Warp
Do you really want to just learn what the CFO knows *today*? In October 2003, City&Guilds (U.K.) released their study of 405 random financial directors. One in seven needs help even switching his or her computer on and off. One in five struggle to save a document. More than one in five need assistance in printing. And a quarter cannot understand spreadsheets. (Source: The Register.)

Student Heterogeneity

Realize that there are distinct student populations.

There is another factor at play which may make you initially unhappy in your introductory finance class—but it is important that you realize why this is so. Chances are that you will find yourself in a classroom with considerable heterogeneity in student preparation. Some students will be more comfortable with math then other students. If you are taking this course in business school, half the students may have come from a background in which their prime function was finance-related. Usually, such finance work experience will not have left them with solid enough knowledge to skip the finance core course, but it will have left them with the knowledge to help them better integrate the new information. Large and distinct student populations are a fact of life in many introductory finance courses. It is thus inevitable that you find yourself in a classroom in which many students find the tempo of the first course finance too fast and many other students find it too slow. On the plus side, I have found that it can work very well if students with worse backgrounds are tutored by students with better backgrounds. On the minus side, the temptation is high to just let the the "finance jocks" take care of the group assignments. Do *not* let this happen, or the preparation problem will accumulate and become unsurmountable.

It is impossible to time a finance course in a business school core, so that both the well-prepared finance nerds and novices will be happy all the way.

Now put yourself into the shoes of your finance instructor. There is plenty of material that can absolutely not be skipped. Interviewers expect students to have a solid grasp of the finance basics (but fortunately not of practical esoterics). It is not uncommon for an interviewer to ask questions that could go right onto the midterm or final. To appreciate the difficult task of the instructor, now add the heterogeneity in student background. The need to grade does not improve student happiness much, either. The well-prepared students start out with a considerable headstart when it comes to test performance relative to students who come from non-quantitative and non-financial backgrounds. The world is not fair—and neither is the grade competition in such a course.

Advice to the "non-quants": As a less prepared student, *you* must struggle.

In the end, there is no way around it: it will be a challenge for previously unprepared and non-technically inclined students to keep up. It is the task of the instructor to make this a surmountable challenge. This is the most important goal of a finance course—*all* motivated students must be able to acquire a solid finance background. But if you are one of those students without quantitative and financial preparation, you will inevitably feel overwhelmed by your class experience. Let me advise patience, practice, and reflection: it will all eventually fall into place, kemosabe, and you can do well *in the end*. Some of my best and brightest students felt frustrated during the course, but they kept at it, studied and learned twice as hard, and ended up at the top of their class. Struggling and anxiety along the way are necessary, maybe even desirable, and in the end unavoidable.

A·1.B. Common Faculty Misconceptions

Some faculty are as mistaken as students. They seem to believe that ideas in Academia are too difficult to communicate to M.B.A. students in an exciting and interesting fashion. They deemphasize current academic research in their classes. They rarely talk about what it is that drew themselves to business schools rather than to practice: the excitement of new knowledge and research, and the opportunity to convey ideas to students and the world at large. If academic research is not universally incorporated into the curriculum and identified as such, then it is not surprising that students find little value in it. In fact, if the research ideas are so obscure that they cannot be explained to and appeal to M.B.A. students, they probably are of little interest to begin with.

So, here is my personal appeal to faculty in core courses: in addition to integrating current research throughout the curriculum, please reserve your final teaching session of class to talk about academic research in finance in general terms—and the academic research in *your* own department, specifically. My own experience tells me that students will find this to be the single most popular session of the entire course.

A·1.C. Business School vs. Practice

Table A.1. Advantages and Disadvantages of Business Schools over Business Practice

Some Examples of	
What Business School Teaches Better Than Practice	What Practice Teaches Better Than Business School
General, universal knowledge	Job specific knowledge
Concepts of business	The specific business
General tools (statistics, data, economics, etc.)	Specific tools (e.g., a particular accounting system)
Marketing methods	Our product or service marketing
Method of thinking	Method of company's practice
Concepts of ideas for the next 20 years	Implementation of ideas from the last 10 years
Knowledge for a lifetime	Knowledge for this year
Leadership principles and theories	Learning how to lead a particular Set of people
Source of conflict	Conflict resolution with a specific person
Learning by study	Learning by doing
Reflection	Action
Selling principles	Selling our product or service
Negotiation principles	Negotiating with specific customers
Forests	trees

Business schools can teach some subjects better than practice, but not all. This is *not* to say that practice is any less interesting than Academia. It *is* to say that practice is best taught by practice (the employer) than by business schools. As an M.B.A. student, be patient: the fixed income department at Goldman Sachs will explain in its own training program the specialized fixed income and institutional knowledge that it will require. The fixed income department does not seek individuals who already know what Goldman Sachs will teach in its first week. Instead, the fixed income department seeks smart, flexible, and open-minded individuals, with a solid understanding of fundamentals—of forests, not of trees. Table A.1 is my perspective on who does what better.

Business schools should focus on subjects that they can teach both well and better than practice. One or the other is not enough. For example, there is ample research that has shown that taller people are more successful. But height is not something that business schools can contribute much to, so we should not teach it. Take the second: I wish I knew how to teach you how to "sell" anything—products, services, ideas. In my opinion, the ability to sell to other people—to get them excited—may be the single most important skill and key for success in life. Now, some people are naturally adept at selling, others can learn it, and still others will never be good at it. Unfortunately, although selling ability is undoubtedly enormously important, this does not mean that business schools can and should teach it. It may be better learned by following the company's best salesperson. (I will let you know when I figure this one out!) In sum, do not expect to learn *everything* you need for success either only in practice or only in school! If you do, you will be disappointed.

Anecdote: Success in Business: Grow up!
Timothy Judge, a University of Florida management professor, finds that controlling for gender, weight, and age, each inch in height seems to add about $789 a year in salary. In his study, greater height boosted subjective ratings of work performance, including supervisors' evaluations of how effective someone was on the job. It also raised objective measures of performance, such as sales volume. The relationship between height and earnings was particularly strong in sales and management, but was also present in less social occupations such as engineering, accounting and computer programming.
Source: Yahoo.

A·1.D. The Rankings

In 1988, *Business Week* (BW) began to publish a bi-annual ranking of business schools. This rankings issue has become one of BW's top sellers. Unfortunately, the quality of the rankings is only mediocre. Worse, the influence of the rankings on business education has been both enormous and negative.

The BW rankings are based primarily on "customer satisfaction" surveys of students and recruiters. Consequently, the BW ratings end up mostly as a popularity contest, and are not based on criteria that measure the quality of education. For example, consider another prominent survey: students at California State University at Chico were #1 in *Playboy*'s Party School Rankings. They would probably rate their satisfaction very highly—but this does not make CalState Chico a good school. The same issue applies to recruiters sampled by BW. Most recruiters are themselves alums of *one* of the schools they are asked to rank. Most business school alums have never studied at any school beyond their own—a fact that naturally makes them relatively ill-equipped to make comparisons. (They also see themselves reflected in the students from their alma mater.) Because larger schools have more alums that are sampled, the size of the pool of alums ends up being the primary predictor of "recruiter opinion" in the BW survey. The result is inevitable: the average recruiter ranks his or her own alma mater highest (or at least very highly). Finally, all schools, students, and alums are now catering to and manipulating the BW rankings. Students and alums know that if they do not rank their own school highly, the values of their degrees will go down. And in almost every school, some faculty member will explain this to those students who have not yet understood this basic fact. In sum, popularity ratings are not a great measure of educational quality.

But the most important error of the BW survey is that it treats education as if it were a consumption good sold by vendors. Instead, education is something that is coproduced by the school *and the student.* Almost anyone with an above-average IQ can get a degree in a business school today, but its usefulness is largely determined by the depth of engagement of the student. A student who coasts will gain little, no matter how good the school is.

This is not to say that there are no quality differences between schools. There are quality differences, but the BW rankings do not fairly reflect them. My advice to any student is to consider many rankings only as useful supplementary indicators. For example, Harvard Business School (HBS) should probably be ranked as the #1 business school for a general M.B.A. education today, although a ranking somewhere between #1 and #5 would be more appropriate. But HBS is not #1 in every field. Its finance education, though superb, is not the world's #1. There are other schools that are at least as good. In contrast, HBS' strategy education—where its world-renowned case method works well—is undoubtedly #1. Yale, my prior school, may not boast a top 3 M.B.A. program, but it offers the #1 ranked education for not-for-profit management today. And so on. Finally, quality differences among similarly ranked schools are often modest: most schools teach similar curricula. The material in this book should appeal to students of any school. My personal guess is that the educational quality difference between the #1 school and the #10 school is very small (as it would be between #10 and #30, or between #30 and #100). *The variation in what an individual gets out of an M.B.A. program within one individual school just swamps the average quality variations across schools.* It is up to you to make your education top-ranked.

Fortunately, although deciding on the right school is a tough problem, there are many good choices to pick from. It is especially encouraging to me that many schools that never show up in any of the rankings are offering excellent business educations. Again, by selection of classes and instructors, a student can easily get a worse business education at, say, Harvard Business School, than at, say, Notre Dame, even though Harvard clearly outranks Notre Dame in any ratings.

SIDE NOTE: In my opinion, there are no good distance-learning universities in existence today. (This may change in the future.) The most prominent, the so-called University of Phoenix, is a great business for its owners, but not for its students. Its degrees are not recognized by others and it is not accredited by the AACSB. (This is not an absolute necessity for an established top-10 school, but it is necessary for an upstart school.)

A·2. Finance: As A Discipline

A·2.A. Art or Science?

I have stated several times throughout the book that finance is as much an art as it is a science. All three parts of finance—valuation, investments, and financing—have simple conceptual underpinnings, but their applications in real life are difficult. And for all three of them, there is no alternative: finding the proper value, the proper portfolio, the proper capital structure may be tough, but this is what it is all about. The difficulty of these questions is good news for practitioners and academics alike: it means that computers will not replace them for a long time to come.

What to do for now? Given that all methods have their errors, the best advice is to use common sense, to employ a number of different techniques to come up with a whole range of possible answers, and then to make a judgment at the end of the day as to what appears most reasonable in light of different models and estimates.

A·2.B. Will We Ever Fully Understand Finance?

No! It is the nature of the beast. Most of finance is a social science. When there are no arbitrage conditions to constrain permissible behavior and prices, behavior and prices can and will deviate from the theory. On occasion, this leads some to conclude that finance is less worthy of study or even a lesser science than, say, mathematics or physics. This is a mistake. The questions are different. Finance is not interested in the big bang, and physics is not interested in the behavior of C.F.O.'s. The study of one is not more or less worthy than the study of the other.

Finance and physics even share many similar philosophical issues: Some questions permit more precise answers than others. Some systems (like the weather and stock prices) are chaotic and difficult to predict, while others (like Newtonian mechanics and option prices) are more exact. It may even surprise you that I am comfortable stating that economics and finance ask many questions to which the answers are more difficult and complex than those often pondered in mathematics and physics. For example, economic agents can react to economic forecasts, which makes predicting the stock market even harder than predicting the weather. Imagine how much more difficult it would be for atmospheric physicists to predict the weather if the weather read the weather forecast, and changed its behavior after reading the weather forecast!

Unfortunately, we are now encountering a new hindrance to progress in finance. Financial institutions have come to consider their data to be their proprietary competitive advantage. Fear of legal liability is further limiting the data that becomes available for public study—and given the litigiousness of U.S. society, justly so. Sadly, many of the most interesting questions in finance therefore may no longer be researchable or answerable.

The fact that we do not have all the answers is good news and bad news. The bad news is that we will never fully understand financial markets and individuals. The good news is that our knowledge will continue to improve, and that there is plenty of space for new and exciting research in finance. For me, this means finance is still intellectually challenging enough to remain "fun."

A·3. Finance Research

Finance research is not just for aspiring academics: consulting firms are basically research firms. Academics and consultants may have different audiences, production speeds, team systems, and evaluation processes, but they both research issues of interest to business and do so using similar methodologies. There is also much cross-fertilization: many professors work regularly with major consulting firms—and some have even quit Academia altogether and departed for higher paying jobs in consulting.

A·3.A. Accomplishments of Finance

Rather than taking up space here, let me just refer you to my paper called The Top Achievements, Challenges, and Failures of Finance, available for free download at the book's website or the Social Science Research Network (www.ssrn.com).

A·3.B. Interesting Current Academic Research

Fortunately, finance is by nature a very applied discipline. If you have read this book, you already understand the main questions and problems in finance and financial research today. You do not need a higher finance degree. Unfortunately, academic finance journals (and many academics) love obscure jargon and algebra. It may or may not require some extra training in "language" for you to follow the writeups of academic papers in academic journals. But, in the end, with just a little bit of extra jargon, you should be able to pick up the important journals and understand the most cutting-edge and interesting research ideas in finance today.

A·3.C. Getting Involved in Academic Research

My own recommendation to an aspiring student of finance is first to learn what the top professors (and especially the younger professors) in your own school are working on. Then, browse SSRN for current working papers. Finally, you should work for a professor in your finance or economics department, even if it is unpaid—though you should pick a professor who does not have too many assistants already. You will learn more in this one-on-one contact than you will learn from taking many classes.

A·3.D. Finance Degrees

The most common finance degree in many of the top schools is the M.B.A. with a specialization in finance. But increasingly, many universities, such as UC/Berkeley, Princeton, and Wharton, are offering undergraduate degrees in finance. The Harvard economics department may well feature the best finance department in the world right now, and it teaches only undergraduates and Ph.D. students. Similarly, universities like Brown and the University of Chicago are just beginning to expand financial economics curricula into undergraduate education. Finance definitely qualifies as a subject with no less intellectual rigor than economics, and no more of a specialization/vocational education component than, say, pre-med or biochemistry.

There are also some other programs that offer masters programs in finance, e.g., the N.Y.U. program in mathematical finance, offered by the Courant Institute. Typically, these programs have a bent towards financial engineering. Their graduates tend to come from specific backgrounds (usually some other engineering discipline), and their graduates tend to work in specific types of jobs (typically in derivatives and fixed income modeling). Finally, there is the Ph.D. track, discussed next.

A·3.E. Academic Careers in Finance and Economics: A Ph.D.?

Finance is a subfield of economics. About one-third of its professors have an economics Ph.D. instead of a finance Ph.D. Either degree is sufficient—although it is imperative for the future academic to have solid grounding in both disciplines.

The typical Ph.D. program in finance takes between 4 and 8 years. Unlike most degree programs, success is not guaranteed. About one-third of accepted students drop out, typically after 2 to 4 years—not a cheap outcome. Although qualifying exams, usually taken in the first two years of the program, are very challenging, the biggest hurdle for almost every Ph.D. student to overcome is the transition from classroom work to academic research. This is a Gordian knot, and success is difficult to predict. Although intelligence and smarts are necessary, it is not mathematical sophistication that determines success. Very little of finance uses more than plain algebra—although it does use lots of it. Instead, the successful Ph.D. student must develop a problem-relevant intuition and creativity. If I only knew how to translate this skill into a recipe!

Although the first 4 years in Ph.D. programs are usually paid for by full stipends by the university, the opportunity costs and the uncertainty of ultimate success mean that only the most intellectually interested students will find a Ph.D. program to be a rewarding endeavor. For the successful graduate, job opportunities tend to be plenty and lucrative. Even academic careers are not exactly a vow of poverty. In 2003, the typical first year Assistant Professor in a top business school earned somewhere between \$130,000 and \$180,000 per year. Industry jobs in financial or consulting institutions sometimes pay more even in the first year, but their big advantage is that salaries tend to escalate far more rapidly than those in Academia in subsequent years. Finally, many economics and finance Ph.D.s pursue governmental careers, e.g., at the *International Monetary Fund* or the *World Bank*.

It is very encouraging that many universities and institutions today conduct terrific academic research in finance and economics. Thirty years ago, only a handful of schools were able to produce great papers, but this time of exclusivity has passed. This does not mean that there are no differences in average academic quality. I will volunteer here my personal impression of the rank order of academic finance departments today, which is based on the tendency of departments to successfully attract faculty from other departments. In my opinion, the top academic department today is the University of Chicago. It is followed closely by "Cambridge," which is really the combination of Harvard (economics and finance) and M.I.T. (economics and finance). A large number of schools vie for the ranking spots right after. Among them, but not exclusively, are (in alphabetical order) Columbia, Duke, N.Y.U., Northwestern, Stanford, U.C./Berkeley, U.C.L.A., Wharton, and Yale. These schools each have their unique advantages and disadvantages, and regularly succeed in stealing faculty from one another.[1] There are also a large number of excellent schools, many of which have individual faculty who are every bit as good as some faculty at, say, Chicago, but which typically do not have the same overall average academic quality or resources.

The average quality of a finance or economics department can make an important difference for Ph.D. students, however. They benefit greatly from the variety of interaction. Therefore, a Ph.D. from any top academic institution would make an excellent springboard into a top-notch academic economics or finance department, or into a very high-quality investment or consulting career.

[1] I almost surely have omitted some schools by mistake.

A·3.F. Being a Professor — A Dream Job for the Lazy?

So, what does a professor do? Multiply the number of classes per year by the hours per class, and you arrive at a number of 120–180 hours per year. Is being a finance professor the ultimate dream job for the lazy?

Sorry to disappoint you—the opposite is the case. The classroom hours during which you see your instructor are just a small part of the job—most comparable perhaps to the small number of hours in which a litigation lawyer is in the courtroom. The rule of thumb is that every hour of teaching of a new course requires about ten hours of preparation. This includes topic selection, comparative evaluations of various textbooks, reading of the relevant literature, preparation of slides and homeworks, and so on. Many finance professors do not teach exactly what is in any one textbook, but inform *themselves* about what *they* should teach, how *their* material fits together in one coherent set, what relevant papers have recently appeared in the literature, what relevance their courses and subjects have to current events and their own locale and audience, where they think the textbooks are wrong, how their finance courses relate to other academic areas, and so on. (Fortunately, once prepared, a course would take only about two hours of preparation for each hour of teaching.) Add this all up, and the 150 hours have already increased to about 600–800 hours. In addition to course preparation and lecturing, there are class handling tasks, office hours, teaching assistance coordination, and grading. This easily adds another 100 hours per year. Finally, many finance professors get roped into holding speeches at school events, and giving lectures not within the context of their regular classes. So, a typical finance professor may spend about 800 to 1,000 hours per year on teaching related issues.

Is this it? Of course not! Tenure-track finance professors are promoted based on their research. Where do you believe the insights in this book have originally come from? Yes, most financial concepts are now heavily used in practice, so even practitioners know them—especially if they were taught concepts in their own academic training decades ago—but it is the academic published research that is responsible for 99% of what you have read in this book. After all, if smart practitioners invent something useful, they do not teach it—they keep it secret and try to sell it. So how much time do professors spend on creating knowledge? Writing an academic paper can take anywhere from 100 hours to 1,000 hours. I know this from painful experience, having written papers that fall into both extremes of this spectrum. Moreover, a good amount of research flops and thus never ends up in a published paper. After all, this is why it is called research and not development! In total, a research-active professor will publish one or two papers per year spending about 500 to 1,500 hours per year on research. Attending conferences and seminars that are necessary to keep up with the profession and publicize one's work may require another 100 hours.

Is this it? Sorry, still no. There is service. Students need advising—undergraduate students, masters students, and Ph.D. students. Universities are governed by the faculty and run by committees that need to be staffed. Alumni and potential donors need to be charmed. Depending on the particular university and one's particular role, this can be anything from 2 hours per week to 10 hours per week. In-school service therefore sums to another 100-500 hours per year.

For all of the aforementioned tasks, you may be able to catch your professor in the act—that is, you are the direct beneficiary or may be present when he or she is spending time working thereon. However, an important part of a professor's job is service to the profession overall. Academia lives by peer evaluation. This applies both to papers and careers. Journals need referees to judge papers. Schools need outsiders to write academic letters for promotion. Refereeing a paper or writing a reference for another professor at another university (should) take at least a day (10 hours). Different professors get different number of external evaluation requests—my own number sits at about 30 per year, consuming about 300 hours per year of my time. There is very little direct reward for doing a good, conscientious job on refereeing and referencing, but it is necessary to make Academia work. As an external referee or evaluator, you are also literally making or breaking someone else's career. It is every professor's duty to take these tasks very seriously.

Putting this all together, my typical year has about 2,500-3,000 hours of work per year. On an hourly basis, my compensation would probably be five times higher if I worked for a top consulting firm or investment bank. So, why do I work for a university? Simple—I love my work. I love teaching, I love doing research, and I love the relative independence to do what I want to do that only an academic job can provide. Yes, not every single task is enjoyable, but overall, it is the best job for me.

Now I must admit that telling you all about what I do in a typical year had a second hidden agenda. I want you to understand the difference between a full-time professor and a part-time professor. Understanding the full scope of professorial obligations will hopefully make you appreciate why you need "the real deal." Yes, both faculty and students can benefit from some lecturers who know practice well, who are only teaching what they themselves learned in their programs (often decades ago, though supplemented with their practical experience), and who do not participate in academic research and in the running of the university and of the academic profession. In fact, many lecturers are very valuable, both to the research faculty and to the students. They can complement our academic knowledge with some practical experience. And a small number start out as lecturers and over time turn into full faculty and excellent researchers. But it is the regular faculty that remains the backbone of financial economics—who provide you with new knowledge to navigate the broad continent of finance over the next few years.

A·3.G. Top Finance Journals

The top academic journals in finance today are *The Journal of Finance*, the *Journal of Financial Economics*, the *Journal of Financial and Quantitative Analysis*, the *Review of Financial Studies*, and the *Journal of Business*. However, there are also many other good outlets for academic research. For example, economics journals have published some of the most influential work in finance. Other journals are written with more of a practitioner audience in mind, such as *Financial Analysts Journal*.

Although numbers do not tell the whole story (it is impact that counts!), the tenure standards for professors range from about 8–10 papers in the top journals for a Chicago professor, to 5–7 papers for a school ranking at around #10, to 3–5 for a school ranking at around #30. The top journals have rejection rates of about 90%. A successful academic will write about 2–3 papers per year, but publish only one of them in a top journal.

A·4. Bon Voyage

Our book has covered the principles of finance in some depth and breadth. You can trust me when I say that if you have read and understood these chapters, you are very well prepared for the next steps in your finance/business education. (You can choose your next courses á la carte: investments, derivatives, corporate finance, fixed income, financial institutions, international finance, or something else. If you are still curious to learn more, visit the book's web site at http://welch.econ.brown.edu/book.)

But even more important to me than teaching you finance has been teaching you how to approach problems: when you need to solve a new problem, think in terms of the easiest numerical example that you can come up with, and only then translate whatever you have learned from your simple example into something more complex—be it a formula or a more complex scenario. So, if you are facing a new problem, even if you do not know or remember any of our formulas, given time, you should now be able to "reinvent" them. When you encounter a complex new problem in your company, do not despair, but gradually work your way up from the simplest versions.

I have enjoyed writing this book in the same way that I enjoy writing my academic research papers, and pretty much for the same reason: it has been like solving an intriguing puzzle that no one else has figured out in quite the same way—a particular way to see and explain finance. Of course, writing it has taken me far longer than I had anticipated—four years and still counting just for the first edition.

But it will all have been worth it if you have learned from my presentation. If you have studied the book, you should now know about 90% of what I know about finance. Interestingly, there were a number of topics that I thought I had understood, but had not—and it was only my having to explain it that made this clear to myself. And this brings me to a key point that I want to leave you with—never be afraid to ask questions, even about first principles. To do so is not a sign of stupidity—on the contrary, it is often a sign of deepening awareness and understanding.

I have no illusions that you will remember all the fine details in this book as time passes—nor will I. But more than the details, I hope that I will have left you with an appreciation for the big ideas, an arsenal of tools, a method to approach novel problems, and a new perspective. You can now think like a financier.

Ivo Welch

APPENDIX B

MORE RESOURCES

NPV Checklist, Some Used Data Links, Algebra, Statistics, Portfolios, !

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2·1. An NPV Checklist

Here is an abbreviated list of issues to worry about when using NPV.

The NPV formula is easy. For most projects, its application is hard. It is usually very difficult to estimate future cash flows (and even their appropriate interest rates), especially for far-in-the-future returns. It is usually more important and more difficult to avoid errors for the expected cash flow (the NPV numerator) than it is for the cost of capital (the NPV denominator). The NPV formula is less robust to cash flow errors than it is to cost of capital (r) errors, and it is "easier" to commit dramatic errors in the cash flow estimation than in the cost of capital estimation.

Here is an abbreviated checklist of items to consider when working out NPV estimates.

✓ **Real After-Tax Dollars** (Page 137, Page 140, Page 140):

- ✓ Have all relevant inputs and outputs been quoted in what-is-relevant-to-you after-tax dollars? This applies to both expected cash flows and to appropriate discount rates.
- ✓ Has inflation been properly included? Preferably, have all computations used nominal expected future cash flows and nominal costs of capital, with inflation used only to gross up nominal cash flows appropriately?

✓ **Interactions** (Page 154, Page 333):

- ✓ Have all projects been properly credited with their contributions, positive or negative, to the values of other projects (externalities)?
- ✓ Have all projects been judged "on the margin," i.e., without charging them for unalterable or previously made choices, such as sunk costs, overhead, etc.?
- ✓ Has the cost of capital applicable to each project component, respectively, been used, and not the (incorrect) overall average cost of capital? (Note: some errors and simplifications here are unavoidable in the real world, because it is impossible to put a different cost of capital on each paper clip.)

✓ **Conditionals (Strategic Options)** (Page 165, and in the Web Chapter on Options and Derivatives):

- ✓ Have all possible future options been considered (using scenario analyses) in order to find the correct *expected* cash flows, e.g.:
  - ✓ The ability to leverage a product into future markets?
  - ✓ The ability to find product spinoffs?
  - ✓ The ability to learn about (how to do) future products?
  - ✓ The ability to stop the project if conditions are bad.
  - ✓ The ability to delay the project if conditions are bad.
  - ✓ The ability to mothball the project if conditions are bad and restart the project if conditions improve.
  - ✓ The ability to accelerate the project if conditions are good.
  - ✓ The ability to expand the project if conditions are good.

✓ **Accuracy** (Page 101, Page 167, Page 168, Page 204, Page 329):

- ✓ How accurate are the estimated project cash flows?
- ✓ If project success and project cash flows were estimated by someone else, what are the motives of the estimator? Does the estimator want the project taken or rejected?
- ✓ Can the cash flow estimates be improved by doing more research?
- ✓ Is it possible to get another independent evaluation/audit of the project estimates?
- ✓ Given unavoidable simplifications, assumptions, and errors, how sensitive/robust is the NPV computation to changes?

✓ **Correct Inputs** (Page 163):

- ✓ Are the cash flows *expected*, rather than just *promised*? Are the interest rates *expected*, rather than *promised*? (Recall: expected interest rates are below promised interest rates due to default premia, not just due to risk premia.)
- ✓ Are the expected cash flows the "average outcome" (correct!), and not the "most likely outcome"?
- ✓ Do the expected cash flow estimates include the correct weighted probabilities of low-probability events, especially for negative outcomes?
- ✓ If money needs to be borrowed to execute the project, is the used cost of capital r the borrowing rate? If capital is already available, is the used cost of capital r the lending (investments) rate?

✓ **Corporate Income Taxes** (Page 444):

- ✓ For use of WACC and APV, is the numerator in the NPV calculation the expected cash flows "as if all equity financed"? (This means that the company bears the full brunt of its corporate income tax load.)
- – In the weighted cost of capital, is the debt cost of capital the *expected* (not the promised!) interest rate on debt? Is the numerator the *expected* cash flow, not the *promised* cash flow?

A final warning: although many of these issues seem obvious in isolation, they are much harder to spot and take care of in complex real-world situations than in our highlighted expositions. Watch out! The most common error is worth its own box:

> **IMPORTANT**: *The most common NPV method is to estimate cash flows for the numerator, and to use an expected rate of return (cost of capital) from the CAPM formula (see Chapter 13).*
>
> - ✓ *The default risk is handled only in the numerator, i.e., in the computation of expected cash flows.*
> - ✓ *The time-premium and risk-premium are handled only in the denominator. The CAPM formula provides an expected rate of return, which contains only these two components.*
> - ✓ *Do not try to adjust the numerator for the time premium or the risk premium. Do not try to add a default-premium to the rate of return in the denominator. (This would yield a promised, not an expected rate of return on capital.) Do not believe that by using the CAPM expected rate of return, you have taken care of the default risk.*

Q B.1 *Recall as many items from the NPV checklist as you can remember. Which are you most likely to forget?*

2·2. Prominently Used Data Websites

The following data and information websites have been prominently used in this book. (If you are reading this on the Acrobat reader , you can click on the links!) Please note that the list is not complete, and that the links may have changed by the time you read this.

	Overall Market Information	
Marketgauge.com	*Various market gauges (incl. S&P500 dividend and earnings yields).*	http://tal.marketgauge.com/dvmgpro/gauges/dvplast.htm
Yahoo!*Finance*	*Stock and Index Quotes, Current and Historical*	http://quote.yahoo.com
Yahoo!*Finance*	*Current Interest Rates*	http://bonds.yahoo.com/rates.html
CNN Money	*General Information and Quotes*	http://money.cnn.com
Federal Reserve Data	*Historical Interest Rates*	http://www.federalreserve.gov/releases/h15/data.htm
Fred (Federal Reserve)	*U.S. economic time series, macroeconomic and financial.*	http://research.stlouisfed.org/fred
SmartMoney	*Animated Yield Curve*	http://www.smartmoney.com/onebond/index.cfm?story=yieldcurve
Treasury Direct	*Inflation Protected Interest Rates*	http://www.publicdebt.treas.gov/gsr/gsrlist.htm
Treasury – Debt	*Office of Public Debt*	http://www.publicdebt.treas.gov/
Bloomberg	*Index rates (incl. muni bonds)*	http://www.bloomberg.com/markets/rates/index.html
R. Shiller's Website	*Very long-run indexes*	http://aida.econ.yale.edu/~shiller/
Yahoo!*Finance*	*Foreign financial market websites*	e.g., Germany: http://de.finance.yahoo.com/

	Individual Stock and Fund Information	
Yahoo!*Biz*	*Firm-specific corporate profiles (earnings, sales, etc.). Here, IBM.*	http://biz.yahoo.com/p/i/ibm.html
Edgar	*All public corporate SEC filings*	http://www.edgar.sec.gov
PWC	*Price-Waterhouse-Coopers' Edgarscan*	http://edgarscan.pwcglobal.com/EdgarScan/
Vanguard	*Funds Information*	http://www.vanguard.com
PepsiCo	*Investor Information*	http://www.pepsico.com/investors/
PepsiCo	*Annual Reports*	http://www.pepsico.com/investors/annual-reports/
PepsiCo	*2000 10-K Filing*	http://www.pepsico.com/filings/200010k.shtml

	Other Information	
Ivo Welch	*General Website*	http://welch.econ.brown.edu/
SSRN	*Finance Working Papers*	http://www.ssrn.com
AFA	*The American Finance Association*	http://www.afajof.org
AEA	*The American Economics Association*	http://www.aeaweb.org
Moody's	*Monthly bond default reports.*	http://riskcalc.moodysrms.com/us/research/mdr.asp
Moody's	*Extended report on default rates, 1992.*	http://riskcalc.moodysrms.com/us/research/defrate/0085.pdf
BankruptcyFinger	*Bankruptcy Related Information.*	http://bankruptcyfinder.com/
BLS	*Bureau of Labor Statistics (Inflation).*	http://www.bls.gov/
SEC	*Securities Exchange Commission*	http://www.sec.gov

	Governance Web Chapter	
CalPers	*Corporate Governance Focus*	http://www.calpers-governance.org/alert/focus/

	International Web Chapter	
Barchart	*Currencies*	http://www2.barchart.com/mktcom.asp?section=currencies
PACIFIC	*Exchange Rate Related Information and Data*	http://pacific.commerce.ubc.ca/
Bloomberg	*Market Indices*	http://www.bloomberg.com/markets/rates/index.html
The Economist	*The "Big-Mac" Price Index*	http://www.economist.com/markets/Bigmac/Index.cfm

2·3. Necessary Algebraic Background

- Finding a base:

$$\begin{aligned} 3^2 &= 9 &\Leftrightarrow\quad 3 &= 9^{1/2} \\ x^a &= b &\Leftrightarrow\quad x &= b^{1/a} \quad . \end{aligned} \tag{B.1}$$

- Finding an exponent:

$$\begin{aligned} 3^2 &= 9 &\Leftrightarrow\quad 2 &= \frac{\ln(9)}{\ln(3)} \\ a^x &= b &\Leftrightarrow\quad x &= \frac{\ln(b)}{\ln(a)} \quad . \end{aligned} \tag{B.2}$$

- Summation Notation:

$$\sum_{i=1}^{N} f(i) \;=\; f(1) + f(2) + \cdots + f(N) \quad . \tag{B.3}$$

This should be read as the "sum over all i from 1 to N." There are N terms in this sum. i is not a real variable: it is simply a dummy counter to abbreviate the notation. When 1 and N are omitted, it usually means "over all possible i."

- Summation Rules:

$$\begin{aligned} \sum_{i=1}^{N} [a \cdot f(i) + b] &= [a \cdot f(1) + b] + [a \cdot f(2) + b] + \cdots + [a \cdot f(N) + b] \\ &= a \cdot \left[\sum_{i=1}^{N} f(i)\right] + N \cdot b \quad . \end{aligned} \tag{B.4}$$

Here is an illustration:

$$\sum_{i=1}^{3} \left[5 \cdot i^i + 2\right] \;=\; [5 \cdot 1^1 + 2] + [5 \cdot 2^2 + 2] + [5 \cdot 3^3 + 2] \;=\; 7 + 22 + 137 \;=\; 166 \quad . \tag{B.5}$$

- The following is not necessary but interesting. A function $\mathcal{L}(\cdot)$ is called a linear function, if and only if $\mathcal{L}(a + b \cdot x) = a + \mathcal{L}(b \cdot x) = a + b \cdot \mathcal{L}(x)$, where a and b are constants.

  Here is an illustration. (Weighted) averaging is a linear function. For example, start with (5,10,15) as a data series. The average is 10. Pick an $a = 2$ and a $b = 3$. For averaging to be a linear function, it must be that

$$\text{Average}(2 + 3 \cdot \text{Data}) = 2 + 3 \cdot \text{Average}(\text{Data}) \tag{B.6}$$

  Let's try this—the LHS would become the average of 17, 32, 47, which is 32. The RHS would become $2+3\cdot10 = 32$. So, in our example, averaging indeed behaves like a linear function. In contrast, the square-root is not a linear function, because $\sqrt{-2 + 3\cdot9} \neq -2 + 3\cdot\sqrt{9}$. The LHS is 5, the RHS is 7.

  – Similar to averaging, expected values are linear functions. This is what has permitted us to interchange expectations and linear functions:

$$\mathcal{E}(a + b \cdot \tilde{X}) \;=\; a + b \cdot \mathcal{E}(\tilde{X}) \quad . \tag{B.7}$$

  This will be explained in the next section.

  – The rate of return on a portfolio is also a linear function of the investment weights. For example, a portfolio rate of return may be $r(x) = 20\% \cdot r_x + 80\% \cdot r_y$, where r_x

a is the rate of return on the component into which you invested \$20. For $r(x)$ be a linear function, we need

$$\begin{aligned} 2+3\cdot r(x) &= r(2+3\cdot x) \\ a+b\cdot r(x) &= r(a+b\cdot x) \end{aligned} \tag{B.8}$$

Substitute in

$$2+3\cdot[20\%\cdot r_x+80\%\cdot r_y] = 20\%\cdot(2+3\cdot r_x)+80\%\cdot(2+3\cdot r_y) \tag{B.9}$$

Both sides simplify to $2+60\%\cdot r_x+240\%\cdot r_y$, so our statement is true and a portfolio return is indeed a linear function.

However, not all functions are linear. The variance is not a linear function, because

$$\mathcal{V}ar(a+b\cdot\tilde{X}) \neq a+b\cdot\mathcal{V}ar(\tilde{X}) \ . \tag{B.10}$$

This will also be explained in the next section.

Solve Now!

Q B.2 *If* $(1+x)^{10} = (1+50\%)$, *what is* x*?*

Q B.3 *If* $(1+10\%)^x = (1+50\%)$, *what is* x*?*

Q B.4 *Write out and compute* $\sum_{x=1}^{3}(3+5\cdot x)$. *Is* x *a variable or just a placeholder to write the expression more conveniently?*

Q B.5 *Write out and compute* $\left(\sum_{y=1}^{3} 3\right)+5\cdot\left(\sum_{x=1}^{3} y\right)$. *Compare the result to the previous expression.*

Q B.6 *Is* $\sum_{i=1}^{3}(i\cdot i)$ *the same as* $\left(\sum_{i=1}^{3}\right)\cdot\left(\sum_{i=1}^{3} i\right)$*?*

2·4. Laws of Probability, Portfolios, and Expectations

This section describes some of the algebra that we are using in our investments chapters. The material is exposited in a more mathematical fashion that in the chapters, which you may find easier or harder depending on your background.

2·4.A. Single Random Variables

The **Laws of Expectations** for single random variables (illustration will follow):

- Definition of Expectation

$$\mathcal{E}(\tilde{X}) \;=\; \sum_{i=1}^{N} \mathcal{P}rob(i) \cdot [\tilde{X} = X(i)] \tag{B.11}$$

- The expected value of a linear transformation (a and b are known constants):

$$\mathcal{E}(a \cdot \tilde{X} + b) \;=\; a \cdot \mathcal{E}(\tilde{X}) + b \;\;. \tag{B.12}$$

  This works because expectation is a linear operator. Similarly, you could rename $\tilde{X}$ as $f(\tilde{X})$, so

$$\mathcal{E}[a \cdot f(\tilde{X}) + b] \;=\; a \cdot \mathcal{E}[f(\tilde{X})] + b \;\;. \tag{B.13}$$

  However, you cannot always "pull" expectations in, so $\mathcal{E}(f(\tilde{X}))$ is not always $f(\mathcal{E}(\tilde{X}))$ For example, if $f(x) = x^2$, it is the case that

$$\mathcal{E}(\tilde{X} \cdot \tilde{X}) \;\neq\; \cdot\mathcal{E}(\tilde{X}) \cdot \mathcal{E}(\tilde{X}) \;\;. \tag{B.14}$$

  To see this, consider a fair coin that can be either 0 or 1. $\mathcal{E}(\tilde{X}^2) = 0.5 \cdot 0^2 + 0.5 \cdot 1^2 = 0.5$, but $\mathcal{E}(\tilde{X})^2 = (0.5 \cdot 0 + 0.5 \cdot 1)^2 = 0.25$.

- Definition of Variance:

$$\mathcal{V}ar(\tilde{X}) \;=\; \mathcal{E}\left([\tilde{X} - \mathcal{E}(\tilde{X})]^2\right) \;\;. \tag{B.15}$$

  It is sometimes easier to manipulate this formula $\mathcal{V}ar(\tilde{X}) = \mathcal{E}\,(\tilde{X}) - [\mathcal{E}(\tilde{X})]^2$.

- Definition of a Standard Deviation:

$$\text{Standard Deviation}(\tilde{X}) = \sqrt{\mathcal{V}ar(\tilde{X})} \;\;. \tag{B.16}$$

- The variance of a linear transformation (a and b are known constants):

$$\mathcal{V}ar(a \cdot \tilde{X} + b) \;=\; a^2 \cdot \mathcal{V}ar(\tilde{X}) \;\;. \tag{B.17}$$

Here is an extended illustration. A coin, outcome called $\tilde{X}$, with 4 and 8 written on the two sides. These two outcomes can be written as $4 \cdot i$ where i is either 1 or 2. So, the expected value of $\tilde{X}$ is

$$\begin{aligned} \mathcal{E}(\tilde{X}) &= \sum_{i=1}^{2} \mathcal{P}rob[\tilde{X} = (4 \cdot i)] \cdot (4 \cdot i) \\ &= \mathcal{P}rob(\tilde{X} = 4) \cdot (4) \;+\; \mathcal{P}rob(\tilde{X} = 8) \cdot (8) \\ &= 50\% \cdot 4 \;+\; 50\% \cdot 8 \qquad = \;6 \;\;. \end{aligned} \tag{B.18}$$

$$\begin{aligned} \mathcal{V}ar(\tilde{X}) &= \sum_{i=1}^{2} \mathcal{P}rob[\tilde{X} = (4 \cdot i)] \cdot [(4 \cdot i) - 6]^2 \\ &= \mathcal{P}rob(\tilde{X} = 4) \cdot (4-6)^2 + \mathcal{P}rob(\tilde{X} = 8) \cdot (8-6)^2 \\ &= 50\% \cdot 4 + 50\% \cdot 4 = 4 \ . \end{aligned} \tag{B.19}$$

The standard deviation is the square root of the variance, here 2.

As we noted earlier, $\mathcal{E}(\tilde{X}^2)$ is of course not the same as $[\mathcal{E}(\tilde{X})]^2 = [3]^2 = 9$, because

$$\begin{aligned} \mathcal{E}(\tilde{X}^2) &= \sum_{i=1}^{2} \mathcal{P}rob[\tilde{X} = (2 \cdot i)] \cdot (2 \cdot i)^2 \\ &= \mathcal{P}rob(\tilde{X} = 2) \cdot (2^2) + \mathcal{P}rob(\tilde{X} = 4) \cdot (4^2) \\ &= 50\% \cdot 4 + 50\% \cdot 16 = 10 \ . \end{aligned} \tag{B.20}$$

Now we work with a linear transformation of the X, say $\tilde{Z} = \$2.5 \cdot \tilde{X} + \$10$. (In finance, the rate of return on portfolios are such linear transformation; for example, if you own 25% in A and 75% in B, you will earn $0.25 \cdot \tilde{r}_A + 0.75 \cdot \tilde{r}_B + 0$.) Thus,

$\mathcal{P}rob$	Coin	$\tilde{X}$	$\tilde{Z}$
$1/2$	Heads	4	\$20
$1/2$	Tail	8	\$30

We want to convince ourselves that the expected value of $\tilde{Z}$, defined as is $\$2.5 \cdot \tilde{X} + \$10$, is $\$2.5 \cdot \mathcal{E}(\tilde{X}) + \$10 = \$25$. So, we hand-compute the expected value the long way from $\tilde{Z}$,

$$\begin{aligned} \mathcal{E}(\tilde{Z}) &= \sum_{i=1}^{2} \mathcal{P}rob[\tilde{X} = (4 \cdot i) \text{ same as } \tilde{Z} = \$2.5 \cdot X + \$10] \cdot (Z_i) \\ &= \mathcal{P}rob(\tilde{X} = 4 \text{ same as } \tilde{Z} = \$20) \cdot (\$20) \\ &+ \mathcal{P}rob(\tilde{X} = 8 \text{ same as } \tilde{Z} = \$30) \cdot (\$30) \\ &= 50\% \cdot \$20 + 50\% \cdot \$30 = \$25 \ . \end{aligned} \tag{B.21}$$

Unlike the mean (the expected value), the variance is *not* a linear function, so the variance of $\tilde{Z} = \$2.5 \cdot \tilde{X} + \$10$ is *not* $\$2.5 \cdot \mathcal{V}ar(\tilde{X}) + \$10 = \$2.5 \cdot 4 + \$10 = \$20$. Instead, $\mathcal{V}ar(\tilde{Z}) = \mathcal{V}ar(a \cdot \tilde{X} + c) = a^2 \cdot \mathcal{V}ar(\tilde{X}) = (\$2.5)^2 \cdot \mathcal{V}ar(\tilde{X}) = \$^2 \cdot 2.5^2 \cdot 4 = \$^2 25$. We can confirm this working with $\tilde{Z}$ directly:

$$\begin{aligned} \mathcal{V}ar(\tilde{Z}) &= \sum_{i=1}^{2} \mathcal{P}rob[\tilde{X} = (4 \cdot i)] \cdot [(\tilde{Z}_i) - \mathcal{E}(\tilde{Z})]^2 \\ &= \mathcal{P}rob(\tilde{X} = 4 \text{ same as } \tilde{Z} = \$20) \cdot (\$20 - \$25)^2 \\ &+ \mathcal{P}rob(\tilde{X} = 8 \text{ same as } \tilde{Z} = \$30) \cdot (\$30 - \$25)^2 \\ &= 50\% \cdot (\$5)^2 + 50\% \cdot (\$5)^2 = \$^2 25 \ . \end{aligned} \tag{B.22}$$

The standard deviation of $\tilde{Z}$ is therefore $\sqrt{\$^2 \cdot 25} = \$5$.

You should confirm Formula B.12: the expected value of $\tilde{Z}$ should be (\$5) times the expected value of $\tilde{X}$ plus \$10. Confirm Formula B.17: the variance of $\tilde{Z}$ should be the variance of $\tilde{X}$ multiplied by (\$5) squared.

Solve Now!

Q B.7 *What is the expected value and standard deviation of a bet B that pays off the number of points on a fair die, squared? For example, if the die comes down 3, you receive \$9.*

Q B.8 *Assume that you have to pay \$30, but you receive twice the outcome of the previous bet $\tilde{B}$. This is a new bet, called $\tilde{C}$. That is, your payoff is $\tilde{C} = -\$30 + 2 \cdot \tilde{B}$. What is the expected payoff and risk of your position? Make your life easy!*

2·4.B. Portfolios

Portfolios are defined as follows (illustration will follow):

$$\tilde{r}_p \equiv \sum_i w_i \cdot \tilde{r}_i \quad , \tag{B.23}$$

where w_i is the known investment weights in security i and $\tilde{r}_i$ is the security return on security i. Unlike the above, simpler definitions, portfolios are the weighted sum of multiple random variables.

- Portfolio Expectations
$$\mathcal{E}\left(\sum_i w_i \cdot \tilde{r}_i\right) = \sum_i w_i \cdot \mathcal{E}(\tilde{r}_i) \quad . \tag{B.24}$$
Although the weights are fixed and known constants, they cannot be pulled out of the summation, because they are indexed by i (each could be different from the others).
-
$$\begin{aligned} \mathcal{V}ar\left(\sum_i w_i \cdot \tilde{r}_i\right) &= \sum_{i=1}^{N}\left\{\sum_{j=1}^{N}\left[w_i \cdot w_j \cdot Cov(\tilde{r}_i, \tilde{r}_j)\right]\right\} \\ &= \sum_{i=1}^{N}\sum_{j=1}^{N}\left[w_i \cdot w_j \cdot Cov(\tilde{r}_i, \tilde{r}_j)\right] \end{aligned} \tag{B.25}$$

Here is an illustration. A coin toss outcome is a random variable, $\tilde{T}_i$, and it will return either \$2 (head) or \$4 (tail). You have to pay \$2 to receive this outcome. This looks like a great bet: The mean rate of return on each coin toss is 50%, the variance on *each* coin toss is

$$\mathcal{V}ar(\tilde{r}_1) = {}^1/_2 \cdot (0\% - 50\%)^2 + {}^1/_2 \cdot (100\% - 50\%)^2 = 0.50 \quad . \tag{B.26}$$

Therefore, the standard deviation of each coin toss is \$0.707.

Now, bet on two independent such coin toss outcomes. You have \$10 invested on the first bet and \$20 on the second bet. In other words, your overall actual and unknown rates of return R are

$$\begin{aligned} r &= \sum_{i=1}^{2} w_i \cdot r_i \quad , \\ \tilde{r} &= \sum_{i=1}^{2} w_i \cdot \tilde{r}_i \quad . \end{aligned} \tag{B.27}$$

(The second equation is in random variable terms.) Now, your investment portfolio consists of the following investments

$$\begin{aligned} w_1 &= \frac{\$10}{\$30} = 0.33 \quad , \\ w_2 &= \frac{\$20}{\$30} = 0.67 \quad . \end{aligned} \tag{B.28}$$

We can now use the formulas to compute your expected rate of return ($\mathcal{E}(\tilde{r})$) and risk ($Sdv(\tilde{r})$). To compute your expected rate of return, use

$$\begin{aligned}\mathcal{E}(\tilde{r}) = \sum_{i=1}^{2} w_i \cdot \mathcal{E}(\tilde{r}_i) &= w_1 \cdot \mathcal{E}(\tilde{r}_1) + w_2 \cdot \mathcal{E}(\tilde{r}_2) \\ &= {}^1/_3 \cdot (50\%) + {}^2/_3 \cdot (50\%) = 50\% \;.\end{aligned} \tag{B.29}$$

To compute your variance, use

$$\begin{aligned}\mathcal{V}ar(\tilde{r}) &= \sum_{i=1}^{2}\sum_{j=1}^{2} w_i \cdot w_j \cdot Cov(\tilde{r}_i, \tilde{r}_j) \\ &= w_1 \cdot w_1 \cdot Cov(\tilde{r}_1, \tilde{r}_1) + w_1 \cdot w_2 \cdot Cov(\tilde{r}_1, \tilde{r}_2) \\ &\quad + w_2 \cdot w_1 \cdot Cov(\tilde{r}_2, \tilde{r}_1) + w_2 \cdot w_2 \cdot Cov(\tilde{r}_2, \tilde{r}_2) \\ &= w_1^2 \cdot Cov(\tilde{r}_1, \tilde{r}_1) + 2 \cdot w_1 \cdot w_2 \cdot Cov(\tilde{r}_1, \tilde{r}_2) \\ &\quad + w_2^2 \cdot Cov(\tilde{r}_2, \tilde{r}_2) \\ &= w_1^2 \cdot \mathcal{V}ar(\tilde{r}_1) + 2 \cdot w_1 \cdot w_2 \cdot Cov(\tilde{r}_1, \tilde{r}_2) \\ &\quad + w_2^2 \cdot \mathcal{V}ar(\tilde{r}_2) \\ &= ({}^1/_3)^2 \cdot \mathcal{V}ar(\tilde{r}_1) + 2 \cdot w_1 \cdot w_2 \cdot 0 + ({}^2/_3)^2 \cdot \mathcal{V}ar(\tilde{r}_2) \\ &= ({}^1/_9) \cdot \mathcal{V}ar(\tilde{r}_1) + ({}^4/_9) \cdot \mathcal{V}ar(\tilde{r}_2) \\ &= ({}^1/_9) \cdot 0.5 + ({}^4/_9) \cdot 0.5 = 0.278 \;.\end{aligned} \tag{B.30}$$

The standard deviation is therefore $\sqrt{0.278} = 52.7\%$. This is lower than the 70.7% that a single coin toss would provide you with.

Solve Now!

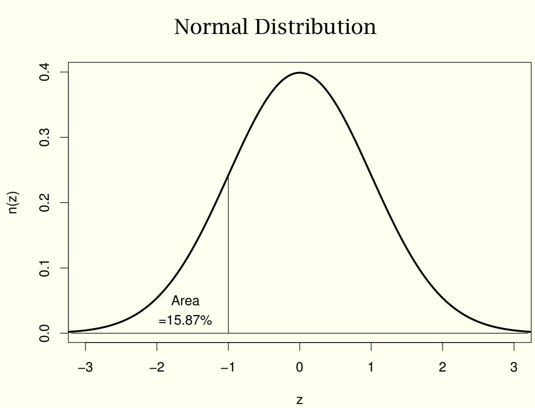
Q B.9 *Repeat the example, but assume that you invest \$15 into each coin toss, rather than \$10 and \$20 respectively. Would you expect the risk to be higher or lower? (Hint: What happens if you choose a portfolio that invests more and more into just one of the two bets.)*

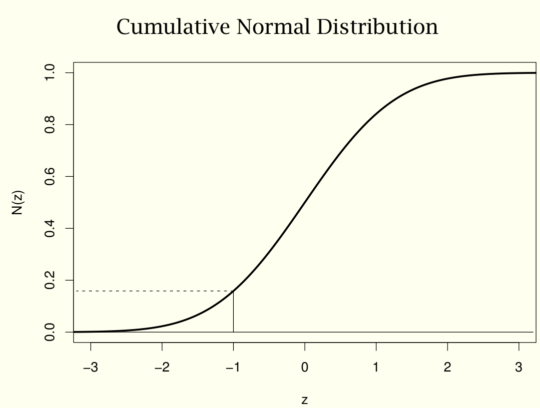
2·5. Cumulative Normal Distribution Table

Table B.1. Cumulative Normal Distribution Table

z	$\mathcal{N}(z)$	z	$\mathcal{N}(z)$	z	$\mathcal{N}(z)$	z	$\mathcal{N}(z)$	z	$\mathcal{N}(z)$	z	$\mathcal{N}(z)$
-4.0	0.00003										
-3.5	0.00023										
-3.0	0.0013	-2.0	0.0228	-1.0	0.1587	0.0	0.5000	1.0	0.8413	2.0	0.9772
-2.9	0.0019	-1.9	0.0287	-0.9	0.1841	0.1	0.5398	1.1	0.8643	2.1	0.9821
-2.8	0.0026	-1.8	0.0359	-0.8	0.2119	0.2	0.5793	1.2	0.8849	2.2	0.9861
-2.7	0.0035	-1.7	0.0446	-0.7	0.2420	0.3	0.6179	1.3	0.9032	2.3	0.9893
-2.6	0.0047	-1.6	0.0548	-0.6	0.2743	0.4	0.6554	1.4	0.9192	2.4	0.9918
-2.5	0.0062	-1.5	0.0668	-0.5	0.3085	0.5	0.6915	1.5	0.9332	2.5	0.9938
-2.4	0.0082	-1.4	0.0808	-0.4	0.3446	0.6	0.7257	1.6	0.9452	2.6	0.9953
-2.3	0.0107	-1.3	0.0968	-0.3	0.3821	0.7	0.7580	1.7	0.9554	2.7	0.9965
-2.2	0.0139	-1.2	0.1151	-0.2	0.4207	0.8	0.7881	1.8	0.9641	2.8	0.9974
-2.1	0.0179	-1.1	0.1357	-0.1	0.4602	0.9	0.8159	1.9	0.9713	2.9	0.9981
										3.5	0.99977
										4.0	0.99997

Normal Score (z) vs. standardized Normal Cumulative Distribution Probability $\mathcal{N}(z)$ Table: This table allows determining the probability that an outcome X will be less than a prespecified value x, when standardized into the score z. For example, if the mean is 15 and the standard deviation is 5, an outcome of $X = 10$ is one standard deviation below the mean. This standardized score can be obtained by computing $z(x) = [x - \mathcal{E}(x)]/\mathcal{S}dv(x) = (x-15)/5 = (10-15)/5 = (-1)$. This table then indicates that the probability that the outcome of $\tilde{X}$ (drawn from this distribution with mean 15 and standard deviation 5) will be less than 10 (or less than its score of $z = -1$) is 15.87%.





These two figures show what the table represents. The left-side is the classical bell curve. Recall that at $z = -1$, the table gave $N(z = -1) = 15.87\%$. This 15.87% is the area under the left curve up to an including $z = -1$. The right figure just plots the values in the table itself, i.e., the area under the graph to the left of each value from the left-side figure.

If you need to approximate the cumulative normal distribution, you can use the formula

$$\begin{aligned} \mathcal{N}(z) &\approx 1 - \frac{e^{-z^2/2}}{\sqrt{2\pi}} \cdot \left(b_1 \cdot k_z + b_2 \cdot k_z^2 + b_3 \cdot k_z^3 + b_4 \cdot k_z^4 + b_5 \cdot k_z^5\right) \\ k_z &\equiv \frac{1}{1 + a \cdot |z|} \ . \end{aligned} \tag{B.31}$$

where $a = 0.2316419$, $b_1 = 0.319381530$, $b_2 = (-0.356563782)$, $b_3 = 1.781477937$, $b_4 = (-1.821255978)$, $b_5 = 1.330274429$, and $\pi = 3.141592654$.

Solutions and Exercises

1. See text for list. Your personal propensity to forget is probably unique to yourself.

2. $x \approx 4.138\%$. Check: $(1 + 4.138\%)^{10} \approx 1.5$.
3. $x \approx 4.254$. Check: $1.1^{4.254} \approx 1.5$.
4. The expression is

$$\sum_{x=1}^{3}(3 + 5 \cdot x) \;=\; (3 + 5 \cdot 1) + (3 + 5 \cdot 2) + (3 + 5 \cdot 3) \;=\; 8 + 13 + 18 \;=\; 39 \;. \tag{B.32}$$

x is not an unknown. It is simply a counter dummy used for writing convenience. It is not a part of the expression itself.

5. The expression is

$$\left(\sum_{y=1}^{3} 3\right) + 5 \cdot \left(\sum_{y=1}^{3} y\right) \;=\; (3 + 3 + 3) + 5 \cdot (1 + 2 + 3) \;=\; 39 \;. \tag{B.33}$$

The result is the same. This is an example why $\sum_i a + b \cdot x = \left(\sum_i a\right) + b \cdot \sum_i x$.

6.

$$\begin{aligned} \sum_{i=1}^{3}(i \cdot i) \;&=\; 1 + 4 + 9 \;=\; 14 \;. \\ \left(\sum_{i=1}^{3} i\right) \cdot \left(\sum_{i=1}^{3} i\right) \;&=\; (1 + 2 + 3) \cdot (1 + 2 + 3) \;=\; 36 \;. \end{aligned} \tag{B.34}$$

The two are not the same. So, be careful not to try to pull out multiplying i's! You can only pull out constants, not counters. Incidentally, is also why $\mathcal{E}(\tilde{X}^2) \neq \mathcal{E}(\tilde{X})^2$, as stated in the next section.

7. The expected value is

$$\begin{aligned} \mathcal{E}(\tilde{B}) \;&=\; (^1/_6) \cdot \$1 + (^1/_6) \cdot \$4 + (^1/_6) \cdot \$9 + (^1/_6) \cdot \$16 + (^1/_6) \cdot \$25 + (^1/_6) \cdot \$36 \\ &=\; \$15.17 \end{aligned} \tag{B.35}$$

The variance is

$$\begin{aligned} \mathcal{V}ar(\tilde{B}) \;&=\; (^1/_6) \cdot (\$1 - \$15.17)^2 + (^1/_6)(\$4 - \$15.17)^2 + (^1/_6) \cdot (\$9 - \$15.17)^2 \\ &+\; (^1/_6) \cdot (\$16 - \$15.17)^2 + (^1/_6) \cdot (\$25 - \$15.17)^2 + (^1/_6) \cdot (\$36 - \$15.17)^2 \\ &=\; \$^2 149.14 \end{aligned} \tag{B.36}$$

The standard deviation is therefore

$$\mathcal{S}dv(\tilde{B}) \;=\; \sqrt{\mathcal{V}ar(\tilde{B})} \;=\; \sqrt{149.14} \;=\; 12.21 \tag{B.37}$$

8. You expect to receive

$$\begin{aligned} \mathcal{E}(\tilde{C}) \;&=\; -\$30 + 2 \cdot \mathcal{E}(\tilde{B}) = -\$30 + 2 \cdot \$15.17 \;=\; \$0.34 \;, \\ \mathcal{V}ar(\tilde{C}) \;&=\; 2^2 \cdot \mathcal{E}(\tilde{B}) = 4 \cdot \$149.14 \;=\; \$595.56 \;, \\ \mathcal{S}dv(\tilde{C}) \;&=\; \sqrt{\mathcal{V}ar(\tilde{B})} \;=\; \$24.42 \;. \end{aligned} \tag{B.38}$$

9. Your investment weights are now $w_1 = w_2 = 0.5$. The mean rate of return remains the same 50%. The variance of the rate of return is computed similarly to the example in the text,

$$\mathcal{V}ar(\tilde{r}) = ({}^1/_4) \cdot 0.5 + ({}^1/_4) \cdot 0.5 = 0.25 \quad . \tag{B.39}$$

Therefore, the risk (standard deviation) is 50%. This is lower than where you put more weight on one of the coin tosses. This makes sense: as you put more and more into one of the two coin tosses, you lose the benefit of diversification.!

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

APPENDIX C

Sample Exams

Applied Torture!

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The following are the midterm and final exams that I gave in my introductory finance course in Spring 2005. The exams did not cover all subjects that were covered in the course, but students did not know which subjects would be on the exam and which subjects would be omitted. The exam answers follow.

The student instructions common to both exams were

- This is a closed-book, closed-notes exam. You are allowed to use your prepared 3*5 index card, and a calculator. No Internet connections are allowed, either.
- The *final answer* must be in the right units, so make sure to distinguish between raw numbers and percent, between dollars and dollars-squared, etc.
- We will try to give partial credit, so show your work.
- You have enough time to write clearly: we will mercilessly penalize hard-to-read and hard-to-comprehend answers. It is your task to make it clear to us that you know the answer, not our task to decipher what you mean. Be concise.
- If you believe a question is ambiguous, please make reasonable assumptions, and spell them out in your answer.
- We will liberally subtract points for wrong answers—in particular, we do not like the idea of 3 different answers, one of which is correct, two of which are incorrect. So, if you show us two different solutions, you can at best only get half credit, unless you clearly outline assumptions that you have to make because my question is ambiguous.
- Assume a perfect market, unless otherwise indicated.

3·1. A Sample Midterm

Students were told that the midterm was 80 minutes for 12 questions, and that each question was worth 10 points, regardless of difficulty or time required to solve.

Q C.1 *Market Perfection Questions:*

(a) *What are the four conditions that make a market "perfect"?*

(b) *What kind of ambiguity happens if the market is not perfect? (You do not need to spell it out for each reason why the market can be imperfect. You need to tell us what breaks generally.)*

Q C.2 *The interest rate (at a zero-tax rate) is 12 basis points per week, A year is always 52 weeks.*

(a) *What is the payoff on a $200 investment in 5 years?*

(b) *If the inflation rate is 5 basis points per week, what is the PV of your answer?*

(c) *Now introduce an imperfect market. Your tax rate is now 20%, and due immediately each Jan 1. What will your cash flow in 5 years be? What is this worth in real terms (in 2005 dollars), i.e., adjusted for purchasing power using the inflation rate?*

Q C.3 *If it takes 9 years for you to triple your investment, what is your annualized rate of return?*

Q C.4 *Risk-free Treasury bonds earn holding rates of return (not annualized) of 10% over 1 year, 25% over 2 years, and 40% over 3 years.*

(a) *Draw the yield curve and provide the appropriate table that you use to draw your yield curve. (Use the same abbreviations that we have been using in class.)*

(b) *What are the two forward rates?*

Use at least 4 significant digits in your calculation, so we know you are computing the right thing.

Q C.5 *What is the IRR of a project that costs $100 today, earns $100 next year, and costs $50 the year after?*

Q C.6 *What is the monthly payment on a fixed 30-year 8% home mortgage for $500,000? (Interpret the 8% quote the same way a normal mortgage company or bank interprets it.)*

Q C.7 *Tomorrow, a project will be worth either $200 million (60% probability), or $10 million liquidation value 940% probability). Today, the project is worth and can be bought for $100 million. You only have $80 million, so you borrow $20 million today from a bank.*

(a) *If the world is risk-neutral, what interest rate do you have to promise the bank?*

(b) *If the world is not risk-neutral, but you know that in equilibrium the bank asks for a 50% promised rate of return, what would you as residual equity holder demand as your expected rate of return?*

Q C.8 *A project reports the following:*

	Year 1	Year 2	Year 3	Beyond
Sales = Income	$200	$300	$500	$0
A/R	$100	$100	$50	$0

What are the cash flows?

Q C.9 *Some accounting questions:*

(a) *What is the main difference between how an accountant thinks of cash flows (not earnings!) and how a financier thinks of the same?*

(b) *Why can EBITDA be a very "incomplete" and therefore often worse number than EBIT for valuation purposes? Can you modify EBITDA to be better?*

Q C.10 *Compare two equal underlying firms (projects). One, however, is levered. Which one has the higher P/E ratio? Do you have to assume a risk-averse world, or will your analysis also hold just the same in a risk-neutral world?*

Q C.11 *If you believe that the underlying growth rate of GDP of 5% nominal (2.0% real) is also applicable to the earnings of firms in the stock market forever, and if the P/E ratio of the stock market is 20 (as it is in December 2004), then what do you expect to be an appropriate expected rate of return on the stock market?*

Q C.12 *If the interest rate is 12% per annum, what is the rental equivalent of a machine that costs $50,000 upfront, $2,000 per year in maintenance, and lasts for 10 years?*

3·2. A Sample Final

Students were told that the final was 160 minutes for 24 questions. The number of points was provided for each questions, and is noted at the beginning of each question.

Q C.13 ***(4p)*** *Market Perfection Questions:*

(a) *What are the four conditions that make a market "perfect"?*

(b) *What kind of ambiguity happens if the market is not perfect? (You do not need to spell it out for each reason why the market can be imperfect. You need to tell us what breaks generally.)*

Q C.14 ***(12p)*** *The following are all possible future outcomes, all equally likely:*

	T-bond	Market	Project A	Project B
Bad	5%	–5%	–10%	$800
Medium	5%	+10%	–5%	$1,000
Good	5%	+25%	+90%	$15,000

(a) *What are the risks and rewards of the projects in the first three data columns?*

(b) *What is the risk and reward of an investment of 20% in A and 80% in the market?*

(c) *What is correlation between project A and the market?*

(d) *What is the market beta of project A?*

(e) *If the CAPM (almost) held, is project A overpriced or underpriced?*

(f) *What is a fair price of project B if the CAPM holds?*

(g) *What are the rates of return for project B?*

Q C.15 ***(3p)*** *If the average rate of return in the market had a standard deviation of about 20% per year, then what was its monthly standard deviation?*

Q C.16 ***(4p)*** *What is the definition of an arbitrage opportunity? How does it differ from a great bet?*

Q C.17 ***(2p)*** *If the stock market is efficient, what kind of advantages does this carry for corporations?*

Q C.18 ***(3p)*** *Evaluate: If the market is efficient, all goods are fairly priced. Therefore, there are no gains to trade.*

Q C.19 ***(4p)*** *What are the two main kinds of owner rights for debt and equity?*

Q C.20 ***(3p)*** *Evaluate each in the context of an example that you make up.*

(a) *If a firm increases its leverage, its cost of debt will generally increase (or at least not decrease).*
(b) *If a firm increases its leverage, its cost of equity will generally increase (or at least not decrease).*
(c) *If a firm increases its leverage, its cost of capital will generally increase (or at least not decrease).*

Q C.21 ***(4p)*** *What do the two M&M propositions say?*

Q C.22 ***(6p)*** *Name three deeper reasons that favor debt over equity as a value-maximizing claim. (In other words, saying debt is cheaper than equity is not deep enough a reason.)*

Q C.23 ***(6p)*** *Name three deeper reasons that favor equity over debt as a value-maximizing claim.*

Q C.24 ***(10p)*** *A firm consists of the following:*

This Year, Value			Next Year, expected		
			Revenues	=	\$230
			Cost*	=	\$200
Debt Today	=	\$100	Interest	=	\$10
Equity Today	=	\$100	Taxes	=	\$5
			Net Income	=	\$15

* *The cost is covered by the financing that debt and equity are providing.*

If everything is fairly priced:

(a) *What is the expected rate of return on equity?*
(b) *What is the expected rate of return on debt?*
(c) *What is the tax rate?*
(d) *What is the WACC?*
(e) *What is the total net payout to debt and equity investors?*
(f) *Using the WACC method, what is the project value?*

Q C.25 ***(3p)*** *From a pure tax perspective, what sort of clientele would you expect would be attracted by cash cow firms, and how would they do this?*

Q C.26 ***(5p)*** *Capital Structure Dynamics:*

(a) *What seems to be the main determinant in firms' debt-equity ratios?*
(b) *When do firms typically issue public seasoned equity?*
(c) *As far as capital structure is concerned, is long-term debt net issuing or equity net issuing more important?*
(d) *In company's debt balance, is there debt that is not created in the financial markets?*
(e) *How strong are the forces pulling towards an optimal capital structure?*

Q C.27 ***(4p)*** *What can firms do to avoid liquidity problems?*

Q C.28 ***(3p)*** *What is the typical fee charged in M&A transactions?*

Q C.29 ***(4p)*** *What is the pecking order? What is the financing pyramid? Does the pecking order hypothesis imply a financing pyramid?*

Q C.30 ***(4p)*** *What is the typical announcement response in a debt offering? in an equity offering? what does this suggest about the market's beliefs about capital inflows vs. outflows, and debt-equity ratio changes?*

Q C.31 ***(5p)*** *What legal temptations that do not maximize shareholder wealth do managers face?*

Q C.32 ***(3p)*** *When do companies have the strongest incentives to control agency problems? Why?*

Q C.33 ***(3p)*** *What considerations and caveats should flow into the "Terminal Value" in a pro forma analysis?*

Q C.34 ***(2p)*** *What is the golden rule of ethics?*

Q C.35 ***(2p)*** *In the most common economic point of view, is it a seller's fault if he misrepresents the good that is for sale?*

Q C.36 ***(4p)*** *Is there a problem with averaging P/E ratios? If so, how can you avoid it?*

A. Q&A: Answers

1.
   (a) No Taxes. No opinion/information differences. No Transaction costs. No big buyers/sellers.
   (b) Project value is ambiguous because it depends on owner's wealth.
2.
   (a) The annual interest rate is $1.0012^{52} - 1 = 6.43\%$. This comes to a 5-year rate of return of 36.5%. The \$200 would grow to \$273.18.
   (b) Still \$200.
   (c) Your interest rate is now 5.144%, because you have to pay taxes every year. Over 5 years, you will have $1.05144^5 - 1 = 28.5\%$ rate of return, or \$257.01. The inflation rate is 2.6% inflation per year. In real terms, you will end up with \$226.06 in 5 years. [rounding error may lead you to a number off by a dollar or so.]
3. $(1 + r)^9 = (1 + 100\%) \Rightarrow r = 3^{1/9} - 1 = 12.98\%$. Check: $1.1298^9 \approx 3$.
4.
   (a) $$r_{\overline{2}} = \sqrt{1.25} - 1 = 11.80\%, r_{\overline{3}} = \sqrt[3]{1.40} - 1 = 11.87\%$$
   (b) $$f_{1,2} = 1.25/1.1 - 1 = 13.64\%, f_{2,3} = 1.4/1.25 - 1 = 12.00\%$$
5. This project has no IRR!
6. Annuity: $\$500,000 = \dfrac{C}{8\%/12}\left(1 - \dfrac{1}{(1 + 8\%/12)^{30\cdot 12}}\right) \Rightarrow C = \$3,668.82$.
7.
   (a) The expected project payoff is \$124. At a price of \$100, this is an expected 24% rate of return. In a risk-neutral world, this will be the expected rate of return for all projects. The bank must expect to receive $\$20 \cdot 1.24 = \$24.8$. With 40% probability, the bank will get \$10 million, with 60% probability it will get what it was promised. The promise must therefore be \$34.667 million. This represents a 73.33% promised rate of return.
   (b) The project payoff is still the same. The bank will receive \$30 million [50% rate of return] with 60% probability, \$10 million with 40% probability. This gives the bank an expected payoff of \$22 million—or an expected rate of return of 10%. The project is financed by 20% debt, so to keep the project expected rate of return at 24%, it must be that
   $$20\% \cdot 10\% + 80\% \cdot x = 24\% \qquad x = 27.5\%$$
   So, you demand an unconditional expected rate of return of 27.5%.
8. Cash Flows: \$100. \$300-\$0=\$300. \$500-(-\$50)=\$550. \$0-(-\$50)=\$50.
9.
   (a) The financier considers interest to be a distribution to investors, while the accountant considers it a cost.
   (b) Because it has absolutely no adjustment for capital expenditures. EBIT has at least an amortized version therein. The alternative is to work with EBITDA - capexp.
10. Unlevered has higher P/E. See classnotes. You need risk-aversion, or else the P/E ratio will be the same.
11. $$P/E = 1/(r - g) \qquad \to \qquad 20 = 1/(r - 5\%) \qquad r = 10\%$$
12. The 10 years of maintenance are the equivalent of
   $$\frac{\$2,000}{0.12} \cdot \left(1 - \frac{1}{1.12^{10}}\right) = \$11,300.4$$
   Add to this the cost of \$50,000, and you have a total cost of \$61,300. This comes to a rental equivalent of
   $$\frac{x}{0.12} \cdot \left(1 - \frac{1}{1.12^{10}}\right) = \$61,300 \qquad \Rightarrow \qquad x = \$10,849$$

13.

(a) No Taxes. No opinion/information differences. No Transaction costs. No big buyers/sellers.

(b) Project value is ambiguous because it depends on owner's wealth.

14.

(a) T-bond: mean 5%, sd 0%. Market: 10%, 12.25%. A: 25%. 46.01%

(b) Returns are (-6,7,38). So, mean is 13%, sd is 18.5%.

(c) Cov=500%. Cor=88.8%

(d) Cov/Var= $500/12.25^2 = 3.33$.

(e) The CAPM prescribes $5\% + 5\% \cdot 3.33 = 21.65\%$. The actual rate of return is 25%. Therefore, the price of A is too low—it is underpriced.

(f) You need the certainty equivalence formula here. Mean(B) = \$5,600. cov(B,M)=\$710 . Var(M)=150%%=0.015. E(M)-RF= 5%. lambda= 3.33 .
So, the value is

$$P = \frac{\$5,600}{1 + 5\%} - 3.3 \cdot \frac{\$710}{1 + 5\%} = \$5,333 - \$2,253 = \$3,080$$

(g) This translates into rates of return of -74%, -68%, and +287%.

15. $20/\sqrt{12} \approx 5.8\%$

16. A zero upfront investment, with no possibility of a cash outflow, and occasionally positive cash flows. A great bet may have a negative in the future, e.g., $-\$5$ in one state of the world, $+\$100,000$ in another state of the world.

17. Rely on and learn from your own market prices. Rely on and learn from your competitors' market prices. Rely on and learn from input's and output's market prices. Cannot add value by doing things that investors can do for themselves.

18. False. There is consumer and producer surplus. e.g., a gas station may have fair prices, but if you are out of gas, having the ability to buy gas there is very useful to you.

19. Control rights (debt=force bankruptcy, equity=vote management), and cash flow rights (debt=first dib, equity=remainder incl unlimited upside).

20. True. True. False.

21. [1] In a perfect market, the financing mix (debt/equity) of the company makes no difference to its value. [2] In a perfect market, the payout policy of the company makes no difference to its value.

22. See Table 19.7

23. See Table 19.7

24.

(a) $\$100 \to \$115$, so 15%.

(b) $\$100 \to \$110$, so 10%.

(c) \$20 in earnings. \$5 in taxes. Thus, tax rate is 25%.

(d) $0.5 \cdot 10\% \cdot (1 - 25\%) + 0.5 \cdot 15\% = 22.5\%/2 = 11.25\%$ \$25

(e) It is not $-\$200 + \$225/1.1125 = \$2.25$! The reason is that the payouts have already taken care of the taxes. so, you need to use the non-tax adjusted WACC, which is $-\$200 + \$225/1.125 = \$0$. Actually, another way to see this is to just rely on the statement that everything is fairly priced—this means that there is positive NPV value to be gained here.

25. Cash cows would be debt-financed, have high interest payments, and be held by non-profits.

26.

(a) How their value has changed around, i.e., their recent stock prices.

(b) In M&A transactions.

(c) Long-Term debt.

(d) Yes, e.g., pension fund obligations.

(e) Not very strong. They are unlikely to be big enough to stop managers from maintaining lousy capital structures if they so wish.

27. Match assets and liabilities. Obtain a credit line. Have more liquid investments. Have more equity and less debt financing.

28. About 1% of deal value.

29. Pecking Order: firms prefer issuing debt to equity. Financing pyramid: a lot of debt at the bottom, less equity at the top. The pecking order does not imply a financing pyramid, perhaps most of all, because equity gains in value over time.

30. Debt carries about a zero (or tiny negative) announcement event. Equity is a large negative announcement event, –1.5% (dilution of >10%). Together, this implies that markets like capital payouts and higher leverage.
31. [Legal Bribes]. Empire building. Corporate perks. High executive pay. Entrenching. Friendship+Loyalty. Employees. Perverse incentives (MBO). Any 5 are enough.
32. Before the company is originally sold, because by reducing future agency problems, the entrepreneur is increasing the value of the firm that she benefits from immediately. Thus, it should be a great corporate charter!
33. You are trying to forecast the firm value at some point in the future. Pick a horizon where the discount rate is high, and the growth rate has settled down towards its longer-run equilibrium (i.e., where t

(All answers should be treated as suspect. They have only been sketched, and not been checked.)

WEB CHAPTER A

Index

Index

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n/a

1·1. Main Index

- Please note that page numbers here can be off by a couple of pages (most hopefully no more than 1 page). This has to do both with infrequent updating of the index by myself, and with LaTeX's way of processing lines and pages. Underline means frequent mention on the same page.
- **Boldface** of a page number (or range) means an important occurrence (or specific definition) of the phrase on the particular page.
- underline of a page number (or range) means multiple occurrences.

INDEX