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The Switching Function

Analysis of power electronic circuits

C.C. Marouchos

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Preface

This book is about the switching function, a technique employed to analyse power electronic circuits. The mathematical models of generic circuits of power electronics are derived by applying a procedure suggested in this book. The analysis that follows gives expressions for the voltage and current at any point of the circuit; usually the output voltage, output current and input current are derived. In ac to dc converters the input current is distorted. This method of analysis gives frequency spectra, power factor and distortion figures, usually displayed for a range of operating conditions such as the firing angle. In dc to dc converters the output ripple voltage might be of some interest and the expression derived in this book is used to calculate its rms value. Frequency changers are also analysed and the current in each semiconductor switch is derived together with the ripple current at the input. The latter is useful in order to choose the appropriate capacitor which might be taking this current as is the case of a dc link inverter. The complexity of the matrix converter makes the application of this technique for their analysis very attractive. The current at each branch is clearly defined as a function of time by introducing appropriate switching functions for each switch. The ability of this technique to give exact expressions of the current in each semiconductor device in the circuit enables the circuit designer to collect all the relevant data to set the ratings of the device such as rms, average and peak values of voltage and current. The order of the voltage and current harmonics at any point in the circuit is derived with simple arithmetic.

The validity of this method is also tested when very well-known expressions of the output voltage of ac to dc and dc to dc converters are derived rather elegantly.

The application of the switching function technique is based on the simple realisation that amplitude modulation takes place in most power electronic circuits. The switching function is modulating the power frequency. According to amplitude modulation theory the modulated signal, the output, is equal to the input times the modulating signal.

The application of the technique is perhaps obvious and relatively simple when the switched network is directly connected to a voltage source with negligible source impedance feeding an RL load. In this case the output voltage is equal to the input times the switching function. The input current is a reflection of the output current to the input. Again we have amplitude modulation with the same or similar switching function acting on the output current.

The presence of capacitors in the load and other parts of the circuit and non-zero source impedance make the application of the switching function technique less obvious and more interesting. Capacitors store energy between successive switching cycles, as are the cases of the ac to dc converter with capacitive load, the dc to dc converter with smoothing capacitors and the active filters. The action of the switches reflects this voltage to the input and other points in the circuit. In order to apply standard circuit analysis techniques, as is the application of the Kirchoff's laws, the voltage across the switched part of the network has to be expressed by a continuous time expression valid at all times and not only within a mode. This demands the application of the superposition theorem for the switched part of the circuit in a way suggested in this book. A complete procedure is suggested for the application of the switching function technique in power electronic circuits in Chapter 1.

This book is arranged into five parts.

Part 1 deals with the switching function itself in three chapters. The switching function is defined and its properties presented in the first chapter together with a procedure for the application of the switching function technique in power electronic circuits. In the second chapter the voltage and current relationships of basic switched circuits are presented with reference to the Kirchoff's laws and the superposition theorem. In the third chapter the switching function technique is applied to construct PWM signals representing sine waves or composite modulating signals.

Part 2 consists of five chapters on ac to dc converters. Two single phase and two three-phase circuits are analysed together with overlap. In Part 3, of five chapters, the switching function technique is applied on the standard types of dc to dc conversion. Part 4 deals with frequency changers. There are four chapters on the matrix converter, the dc to ac inverter and the envelope cyclo-converter. Active filters are presented in Part 5 in four chapters: the reactor static VAR controller, the switched capacitor, the inverter filter and the active line current shaping circuits.

All circuits are analysed using the switching function technique by applying the procedure outlined in Chapter 1. For every circuit, voltage and current expressions are derived together with frequency spectrums. Mathcad (R) is successfully employed to represent all the expressions developed in the book and a web link is included **http://www.iee.org/Publish/Books/Cds/index.cfm?book=CS%20017**

The sample mathematics in this book generated from Mathcad (R) software are courtesy of Mathsoft Engineering & Education, Inc., http://www.mathcad.com. Mathcad is a registered trademark of Mathsoft Engineering & Education, Inc.

The switching function technique is rather new and the presentation of its application for the analysis of a wide range of power electronic circuits in this book in a systematic way will help to appreciate its potential and limitations. One of the limitations, at the moment, is that it is only applied for the steady state.

Alternatively you can find the Mathcad examples by searching the IEE website using 'The Switching Function' as the keyword.

Acknowledgements

This book is the result of a number of iterations and revisions based upon research work and applications. Many people have contributed either directly or indirectly and I am thankful to all of them.

The material was also partly tested in the classroom with encouraging results. I would like, therefore, to express my sincere thanks to my many HTI students who help me in various ways with this project and in particular Miss Eleni Tsirtou and Andreas Poullaides.

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To my wife Despo I am most grateful for putting up with me during the writing of this book and indeed for the many years together.

Many thanks also, to my sons Andreas and Philippos who have been a source of support and understanding.

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My appreciation is also expressed to the authors recorded in my references and whose work has been a source of inspiration.

Part 1

The switching function

This book is about the systematic application of the switching function technique for the analysis of power electronic circuits. The switching function method of analysis is based on the derivation of the voltage–current expressions of a switched circuit covering all modes into a single expression, a ‘unified expression’. A ‘unified expression’ is the result of applying the superposition theorem in order to combine the expressions of all modes of the circuit into one expression with time varying parameters.

Once this is done the Kirchoff’s laws can be applied. In applying this technique a procedure has evolved together with a number of properties of the switching function itself. These are reported in Chapter 1. The application of Kirchoff’s laws and the Superposition Theorem in these circuits is discussed in Chapter 2 together with voltage–current relationships of switched-electric elements. Chapter 3 is a presentation of the application of the switching function technique to produce pulse width modulation (PWM) signals. A sine-wave, a rectified sine and a composite waveform are presented.

The switching function is bidirectional. State 1 (or -1) implies that the input is connected to the output and current can flow in either direction. Hence care must be taken in cases where the actual circuit cannot allow this bidirectional flow of current as is the case of single thyristors and diodes. The switching function mathematical model must be a true picture of the operation of the circuit. Hence the switching function must always be derived from the actual input and output voltage and currents and not from the driving signals.

Here we have the question of the Switching Function Algebra. A procedure is suggested and applied in this book for the application of the switching function and a number of rules are identified. Nevertheless, the author of this book believes that not enough work is done yet for the Switching Function Algebra to be presented as a single entity.

The switching function method of analysis gives the steady state response of a switched circuit. When the parameters of the switching function are changed, the new steady state can be defined but it does not tell us how it goes there and how long it takes. Hence at the moment the switching function technique is limited to the steady state.

The switching function has been referred to also as 'existence' and 'modulation' function [1,2,11]. The term 'switching' function [3,4] is perhaps more appropriate and is adopted here; it describes the switching action of the ideal semiconductor switch, hence the word switching seems more appropriate.

Chapter 1

The switching function: Application and properties

1.1 Introduction

This type of analysis is applicable for the steady state of a circuit. Transient response is not investigated yet. A procedure is set up for the application of the technique for the steady state and a list of rules is suggested to follow. The derived list of rules is based on the study of the behaviour of many very well-known circuit configurations and some ‘newer’ circuits, the active filters.

The standard approach to derive the mathematical model of a power electronic converter is usually done in terms of its modes or states. Because of its switching nature the circuit changes states-configurations. These are the modes of the circuit and a set of differential equations describing the circuit in each mode is derived. The linking parameter between modes is usually a variable, that is, a voltage or a current or both. These equations are solved for each state and then linked together: the final value of a variable within a mode is the initial value in the next mode. The solution is repeated until the initial and final value is the same within a mode. This is the steady state of the circuit and it might take many cycles until steady state is reached. One might speed up things by setting appropriate non-zero values for certain key variables. The switching function technique is applied in a different way. It attempts to derive analytical expressions that represent the voltages and currents at all times and for the circuit as a whole. At the moment the effort is concentrated for the steady state but the transient response cannot be excluded in the future.

1.2 Application of the switching function technique

For the application of the switching function technique, a switching function is defined. It is a signal which takes the value of zero or one thus representing the on and off state of a semiconductor switch. A semiconductor switch operating in a regular manner, is acting as an amplitude modulator. Its switching action is defined by a mathematical expression $F(t)$ which is a series of pulses representing the periods that the switch is on and off. This is the 'unipolar' switching function and it can be used to derive other switching functions for more complex switching patterns such as the one operating on the bridge configuration, the 'bipolar' switching function or PWM signals.

The switching function is a statement of the time instances that both the input and output of a switch or a switch configuration are the same; the input is reflected to the output. The switching function relates the input to the output in a similar way that the transfer function relates input to output in control systems. Hence:

$$\mathrm{OUTPUT}(t) = \mathrm{INPUT}(t)F(t) \tag{1.1}$$

The bridge configuration is very popular in power electronic circuits. The input voltage of a bridge is reflected to the output by the bridge switching function, the 'bipolar' switching function. The bipolar switching function is derived from the unipolar switching function and it is a quasi-square wave signal, which takes the values of 1, 0 and -1. When it has no dead periods, that is, when it oscillates directly from 1 to -1, it takes the form of a square wave signal and the output is given by Expression (1.1). Also the following applies:

$$\mathrm{INPUT}(t) = \mathrm{OUTPUT}(t)F(t)$$

We refer to it as a 'transparent' switching function. A transparent switching function has no dead periods. Reference to the transparent switching function is made in Chapters 5 and 20.

1.2.1 Procedure for the application of the switching function technique

The suggested procedure for the application of the switching function technique is in two major parts: mathematical modelling and analysis. In the first part the mathematical model is derived and in the second part the model is used to perform the analysis of the circuit. Alternatively the model can be simulated by matlab.

In developing the mathematical model, the modes of the circuit are first derived and for each mode the expressions for voltage and current at key points are stated. Each mode exists for a specific period of time, the 'existence' period. Hence the associated expressions of voltages and currents are valid only for that slot of time, the existence period. The existence period is the on period of a switching function for which these expressions are valid. It is also the existence period of the mode.

In applying this technique, the switching function for each mode must be defined; its on-period is the existence period of the mode. In other words a switching function is attached to each mode. Therefore more than one switching function might exist for the same circuit. A single mode might be repeated more than once within a circuit period. Hence the sequence of the modes must be established. The 'period' of the circuit is easily identified as that period of time over which the sequence of modes is completed; the addition of all the 'existence' periods makes up the 'period' of the circuit. It usually corresponds to the input power frequency or the switching frequency, depending on the circuit. In the same way that the 'existence' period of a mode is the ON period (magnitude 1) of the switching function, the OFF period of the same switching function is the rest of the time interval of the 'period' of the circuit. Once the sequence is established, the unified voltage–current expressions for the circuit as a whole can be derived. These unified expressions of voltage and current at a point are made up from the contribution of voltage or current from all modes at that point. This is an application of the superposition theorem. These unified expressions are functions of the appropriate switching functions.

Usually power electronic circuits have a rather simple topology with a few nodes. It is easy to see the common points of the modes: there are points in the circuit where more than one mode contributes voltage and/or current. These are the key parameters in the circuit, which need to be defined. It is advised that the unified expressions include these 'key' parameters.

Once the unified expressions of voltage and current are derived, the mathematical model can be built using the elements of Table 1.1. Matlab–simulink could be employed at this stage to simulate the circuit and investigate its behaviour. Alternatively, the expressions that make up the mathematical model are combined and expanded to derive compact expressions for the output voltage, output current, input current and the current and voltage of the semiconductor switches. Consequently expressions for power factor, distortion factor and frequency spectrums are derived as necessary.

The suggested procedure for the application of the switching function might be as follows:

A. Mathematical modelling

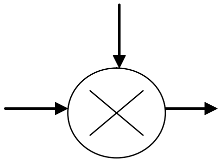
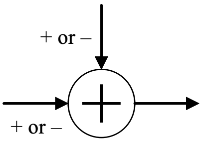
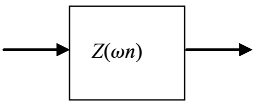
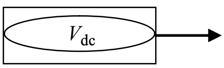
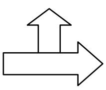
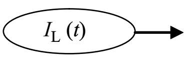
1. Derive the modes of the circuit and write the voltage–current expressions for each mode.
2. Identify the appropriate switching function for each mode. During state 1 of the switching function the expressions of the mode are valid. Take note of the state of the switching function, 1, 0 or -1.
3. Establish the mode sequence and hence the period of the circuit: make sure that all instances of the period of the circuit are represented by a switching function.
4. Derive unified expressions by employing the switching functions and applying the superposition theorem.
5. Build a simple mathematical model to represent the operation of the circuit based on the unified expressions of step 4.

B. Analysis

6. Combine and expand the unified expressions derived in step 4 and use the mathematical model (step 5) as a guide in order to derive voltage and currents at each point of the circuit.

The recommended symbols for the mathematical model are given in Table 1.1. The mathematical model is a block functional diagram that represents the operation of the circuit. Simple symbols are employed. The mathematical model could be used to simulate the circuit with matlab or other similar software.

Table 1.1 The recommended symbols for the mathematical model

	*Amplitude modulator*: Two inputs and an output. The output is the product of the two inputs (amplitude modulation).
	*Summer*: Two inputs and an output. The output is the algebraic sum of the two inputs.
	*Operator*: One input and one output, the impedance is acting on the input to give the output, Ohm's Law.
	*Calculation box*: Only one output. The quantity shown in the box (and its output) is calculated in the box and supplied to the model.
	*Splitter*: One input, two outputs. The input is split into two components usually the dc and ac component. It applies to both voltage and current.
	The output quantity shown in the oval circle – and its output – is calculated already elsewhere in the model.

1.3 Properties of the switching function

1.3.1 Definition of the unipolar switching function

The switching function is that signal that will give the output when it is multiplied by the input voltage (or current). Consider the simple circuit below where a voltage source, $V(t)$ is connected on the LHS of the semiconductor switch (Fig. 1.1). The output voltage, the voltage on the right of the switch will take the value of the input voltage when the switch is closed for the period t_1–t_2. When the switch is open for the period t_2–t_3, the output voltage is zero. If the state of the closed switch is attributed the logic value of 1 and the state of the open switch the logic value of 0 then a function of time, $F(t)$, can be defined as:

$$F(t) = 1 \text{ switch closed for } \quad t_1 < t < t_2 \;\text{ and }\; V_{AB}(t) = V(t)$$

$$F(t) = 0 \text{ switch open for } \quad t_2 < t < t_3 \;\text{ and }\; V_{AB}(t) = 0$$

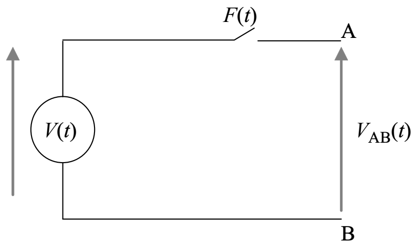


Figure 1.1 Definition of the switching function

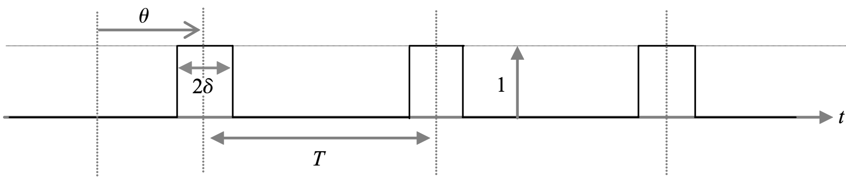


Figure 1.2 The basic switching function

Then the period of the switching function is $t_3 - t_1$ and the switching frequency, f_s, is

$$f_s = \frac{1}{t_3 - t_1}$$

$F(t)$ is a pulse-function as shown in Fig. 1.2 and it can be expressed by a sum of sinusoids according to the Fourier series as

$$F(t) = K_o + 2\sum_{n=1}^{\infty} K_n \cos(n\omega t - \theta * n) \tag{1.2}$$

where

n is an integer number
K_o is the duty cycle of the switch
$K_n = (\sin(n\delta))/\pi n$
δ is half the on period of the switch
w_s = switching frequency, $2\pi f_s$
θ is the phase angle of the switching function relative to a reference.

This switching function is termed 'unipolar' because it takes only positive values. It is a series of pulses of unit magnitude and pulse width 2δ, at a switching frequency w_s phase displaced by θ relative to a reference, Fig. 1.2. It describes the basic repetitive action of the semiconductor switch. As it will be shown later, a more complex switching function can be derived from the unipolar switching function. Its frequency spectrum is displayed in Fig. 1.3.

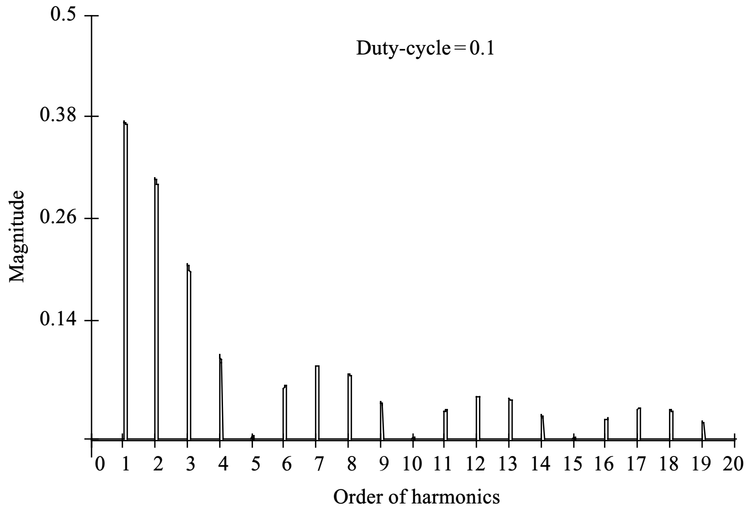


Figure 1.3 Frequency spectrum of the unipolar switching function

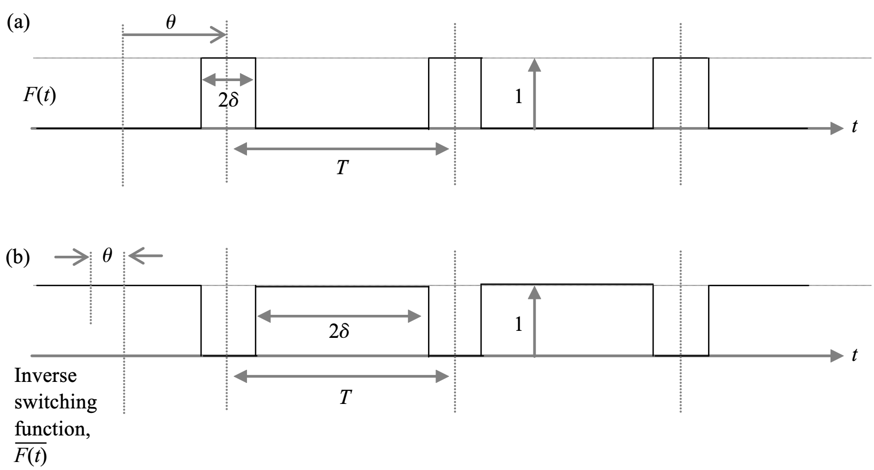


Figure 1.4 (a) The unipolar switching function and (b) its inverse

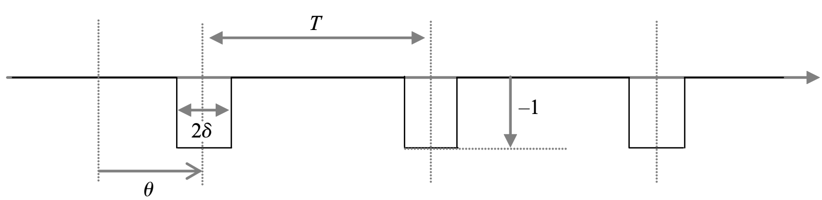


Figure 1.5 The negative pulse unipolar switching function

1.3.2 The inverse of the unipolar switching function, $\overline{F(t)}$

By definition the inverse of the unipolar switching function, $\overline{F(t)}$ is a new function that represents its off periods. In other words, the function $\overline{F(t)}$ takes the magnitude one when $F(t)$ is zero. With reference to Fig. 1.4 this function is $1 - F(t)$

$$\overline{F(t)} = 1 - F(t) \tag{1.3}$$

1.3.3 Negative pulse unipolar switching function

$$F_N(t) = -K_o - 2\sum_{n=1}^{\infty} K_n \cos(n\omega t - \theta * n)$$

The negative pulse unipolar switching function is shown in Fig. 1.5.

1.3.4 Phase displacement

The phase displacement of the switching function is set by θ; it is changed by either adding or subtracting to θ. Figure 1.6 is an example of a switching function which is phase delayed by π radians.

1.3.5 The bipolar switching function – bridge configuration

In a bridge configuration, Fig. 1.7, S_1 is operated with S_4 and S_2 with S_3. Each group is operated in an anti-parallel fashion. Definitely there are no overlaps but there might be dead periods. Each group is operated by a unipolar switching function: S_1 and S_4 are operated by $F_{14}(t)$, Fig. 1.8(a), and S_2 and S_3 are operated by $F_{23}(t)$, Fig. 1.8(b).

The two unipolar switching functions are of the same pulse width but phase displaced by 180°, Fig. 1.8. In order to derive the input–output voltage relationship, the modes of the circuit are derived.

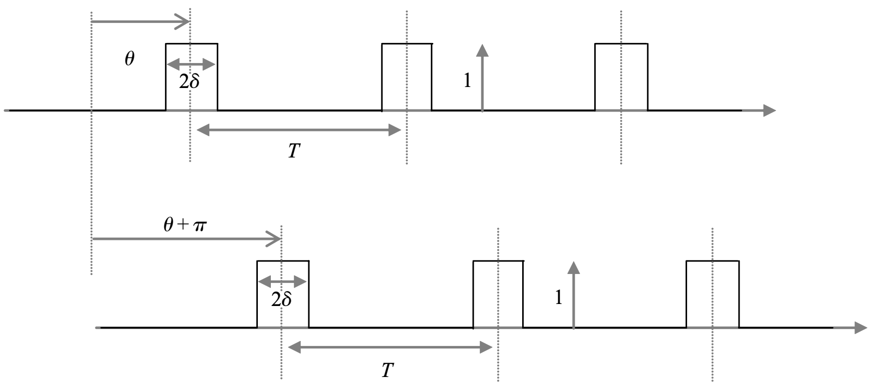


Figure 1.6 Phase displacement of the unipolar switching function

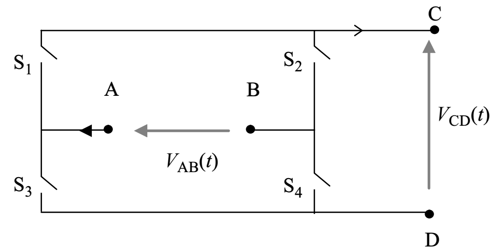


Figure 1.7 The circuit diagram of the bridge configuration

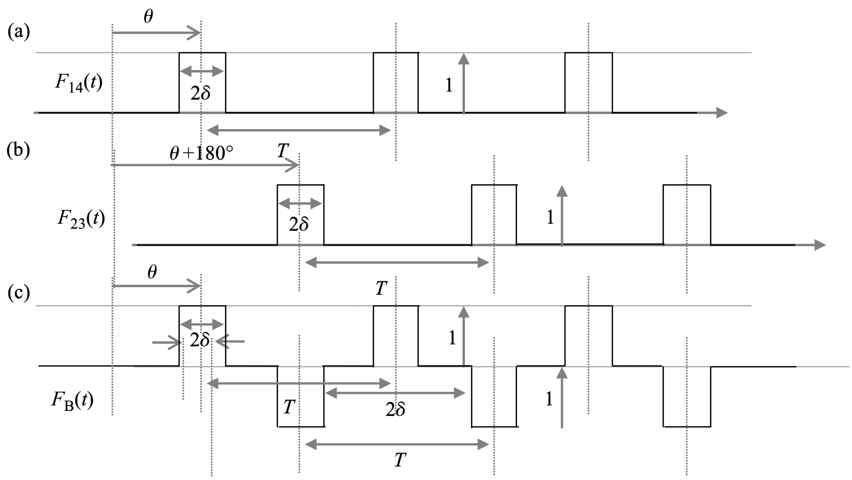


Figure 1.8 The switching functions of the bridge

Switches S_1 and S_4 are on when switching function $F_{14}(t)$ is at state 1. The output voltage is given by $V_{CD}(t) = F_{14}(t)V_{AB}(t)$. This is the contribution of Mode I to the output voltage (Fig. 1.9(a)).

Switches S_2 and S_3 are on when switching function $F_{23}(t)$ is at state 1. The output voltage is given by

$$V_{CD}(t) = -F_{23}(t)V_{AB}(t).$$

The output is reversed. This is the contribution of Mode II to the output voltage (Fig. 1.9(b)).

There is a third mode where none of the four switches is conducting, Fig. 1.9(c). None of the switches is on and both switching functions are at the zero state,

$$F_{23}(t) = F_{14}(t) = 0$$

The output voltage is zero. There is no contribution of Mode III to the output voltage.

$$V_{CD}(t) = 0$$

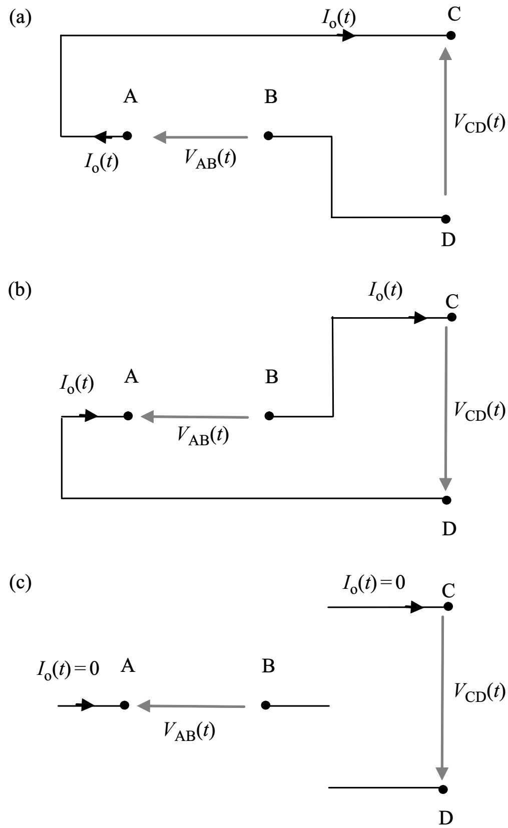


Figure 1.9 Bridge configuration: (a) Mode I, (b) Mode II and (c) Mode III

Superposition theorem applies and $V_{CD}(t)$ is given from the contribution of the two modes I and II; the third does not contribute anything. Hence the reflection of $V_{AB}(t)$ on the DC terminals, $V_{CD}(t)$ is given by

$$V_{CD}(t) = F_1(t)V_{AB}(t) - F_2(t)V_{AB}(t)$$

$$V_{CD}(t) = [F_1(t) - F_2(t)]V_{AB}(t)$$

and

$$V_{CD}(t) = F_B(t)V_{AB}(t)$$

where

$$F_B(t) = [F_1(t) - F_2(t)]$$

$V_{AB}(t)$ is the input voltage, $V_{in}(t)$; $V_{CD}(t)$ is the output voltage, $V_o(t)$. Hence for the bridge configuration

$$V_o(t) = F_B(t)V_{in}(t) \tag{1.4}$$

Expression (1.4) is in accordance with Expression (1.1) where the switching function $F_B(t)$ is relating the input voltage $V_{AB}(t)$ to the output voltage $V_{CD}(t)$ in a switched circuit. Both expressions represent an amplitude modulation where the switching function is acting on the input voltage to give the output voltage.

The bridge switching function $F_B(t)$ is simplified to Expression (1.5):

$$F_B(t) = [F_1(t) - F_2(t)]$$

$$F_B(t) = K_o + 2\sum_{n=1}^{\infty} K_n \cos(n\omega t - \theta * n) - \left[K_o + 2\sum_{n=1}^{\infty} K_n \cos(n\omega t - \theta * n - n\pi)\right]$$

After expansion and simplification, it reduces to the compact form:

$$F_B(t) = 4\sum_{n=1}^{\infty} K_n \cos(n\omega t - \theta * n) \qquad n = 1, 3, 5 \ldots$$

The compact form of $F_B(t)$ is valid only for odd values of n. This is achieved by including the absolute value of the term $\sin(n\pi/2)$

$$F_B(t) = 4\sum_{n=1}^{\infty} K_n \cos(n\omega t - \theta * n) \left|\sin\left(\frac{n\pi}{2}\right)\right| \tag{1.5}$$

As it appears from Fig. 1.8, $F_B(t)$ – the bridge switching function – is a quasi-square signal of the same frequency and pulse width as the other two unipolar switching functions. The output voltage $V_{CD}(t)$ is pushing an output current $I_o(t)$ through the load impedance according to Ohm's Law. This current is reflected to the input.

During Mode I $\quad I_{in}(t) = I_o(t) \qquad F_{14}(t) = 1$

During Mode II $\quad I_{in}(t) = -I_o(t) \qquad F_{23}(t) = 1$

Hence

$$I_{in}(t) = I_o(t)F_{14}(t) - I_o(t)F_{23}(t)$$

This is simplified to

$$I_{in}(t) = F_1(t)I_o(t) - F_2(t)I_o(t)$$

$$I_{in}(t) = [F_1(t) - F_2(t)]I_o(t)$$

$$I_{in}(t) = F_B(t)I_o(t) \tag{1.6}$$

Therefore the input current is a reflection of the output current to the input. The input and output voltage with the switching function are shown in Fig. 1.10. The frequency spectrum of the bipolar switching function is shown in Fig. 1.11.

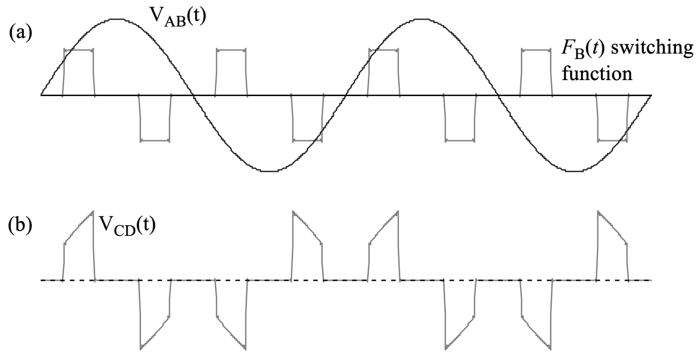


Figure 1.10 *Time waveforms of the reflected voltage or current in a bridge configuration: (a) $V_{AB}(t)$ input voltage and (b) $V_{CD}(t)$ output voltage*

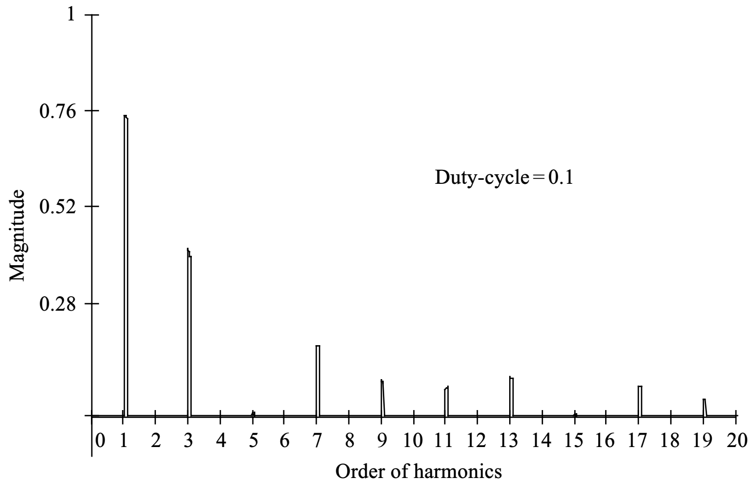


Figure 1.11 *The frequency spectrum of the bipolar switching function*

1.3.6 The square of the 'unipolar' switching function

The square of the 'unipolar' switching function is given by raising every point of the waveform of Fig. 1.2 to the square. It is obvious from the same graph that if every point of $F(t)$ is raised to the square, it will give exactly the same shape, hence:

$$F(t)^2 = F(t) \tag{1.7}$$

1.3.7 The square of the 'bipolar' switching function

A bipolar switching function is displayed in Fig. 1.12(a) with period T, phase delay θ and half-pulse width δ. The square of this switching function is derived graphically by raising every point of this signal, Fig. 1.12(a), to the square. The resultant is a unipolar

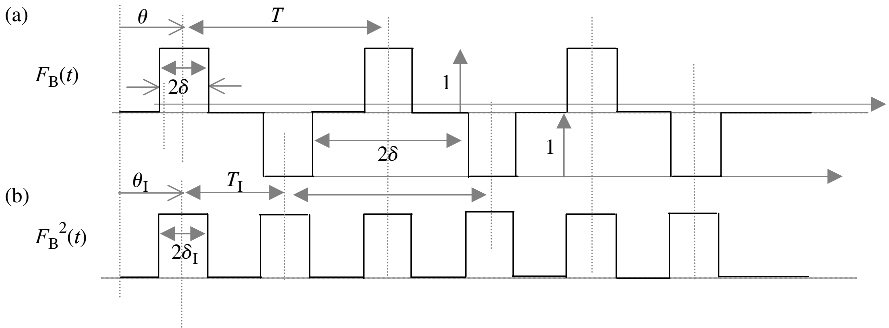


Figure 1.12 The square of the 'bipolar' switching function

switching function, Fig. 1.12(b) with twice the frequency of the original signal with period T_I, phase delay θ_I and half-pulse width δ_I.

$$F_B(t)^2 = F_I(t)$$

$$K_o = \frac{4\delta}{2\pi}$$

The parameters of the two switching functions are related by $T_I = T/2$, $\theta_I = 2\theta$ when measured in radians ($\theta_I = \theta$ when measured in seconds) and $\delta_I = 2\delta$ when measured in radians ($\delta_1 = \delta$ when measured in seconds).

Therefore the new switching function is written as

$$F_I(t) = K_o + 2\sum_{n=1}^{\infty} K_n \cos(2n\omega t - \theta 2{*}n) \tag{1.8}$$

1.3.8 *Transparent switching function, $F_T(t)$*

A transparent switching function is a special case of switching function that has no dead periods; a dead period is allowed if during the dead period both input and output are zero. A square wave bipolar switching function is a typical case of a transparent switching function as it has no dead periods. In this case both Expressions (1.1) and (1.9) apply.

$$\mathrm{OUTPUT}(t) = \mathrm{INPUT}(t)F(t) \tag{1.1}$$

$$\mathrm{INPUT}(t) = \mathrm{OUTPUT}(t)F_T(t) \tag{1.9}$$

1.3.9 *Harmonic impedance*

The harmonic impedance refers to the impedance as a function of the order of the harmonics of the applied voltage. The output voltage of a switched network is usually forcing current through an impedance. The output voltage contains a frequency spectrum determined by the frequency of the input voltage and the switching function itself. Such complex voltage waveforms are applied to complex impedances such as resistance–inductance or resistance–capacitance. The frequency content of the resultant current is the same as the frequency content of the voltage that is creating it. Hence each frequency component of the applied voltage is forcing current through an impedance of the same frequency order. Therefore a simple inductance which normally has an impedance of ωL at a single frequency ω, has a harmonic impedance of $n\omega L$ when it is supplied from a complex voltage waveform of $n\omega$ components.

A more detailed account of harmonic impedances is given below.

(a) Single component such as an inductance L

$$X_{\mathrm{L}}(\omega n) = n\omega L \qquad \varphi = -\frac{n\pi}{2}$$

(b) A series R–L impedance

$$Z(\omega n) = \sqrt{(n\omega)^2 + R^2} \qquad \varphi = -\tan^{-1}\left(\frac{n\omega L}{R}\right)$$

(c) A parallel R–C impedance is presented after simplification

$$Z(\omega n) = \frac{R}{\sqrt{(\omega nCR)^2 + 1}} \qquad \varphi = -\tan^{-1}(n\omega C)$$

The variable n accounts for the impedance at the nth harmonic component of the applied voltage.

Chapter 2

Voltage–current relations in switched circuits

In applying of the Kirchoff's laws of current and voltage in switched circuits, the modulating action of the switches must be taken into account.

It is common in power electronics circuits for a number of switched paths to feed a junction as is the case of a three phase half-wave controlled rectifier or a single inductive path feeding two switched paths as is the case of a dc to dc boost converter. In this case the current is 'diverted' from branch to branch. Hence the outgoing branches carry the current entering the junction for part of the time. The switching function will allow a unified expression of time for the current to be derived in each branch according to superposition theorem and then the Kirchoff's Law of current can be applied. In order to achieve this, the modes must be derived and an appropriate switching function for each mode must be defined. It is absolutely necessary that all instances of the 'period' of the circuit are represented by a switching function. The period of the circuit was defined in the previous chapter as the time it takes for a single sequence of the modes to be completed.

It is also common in power electronic circuits for the voltage across two points to be a composite waveform where more than one circuit loop is contributing to it 'at different times' due to the action of the switches. The voltage is then described by different expressions for different periods of time and an appropriate switching function is defined. The switching function will allow a unified expression of time for the voltage across these points to be derived according to the superposition theorem and then the Kirchoff's Law of voltage can be applied. Again it is absolutely necessary that all instances of the 'period' of the circuit are represented by a switching function.

In this chapter voltage and current relations across a switch, combination of switches and switched electric elements are derived. The Kirchoff's laws and Superposition are also examined in the light of the switching nature of circuits.

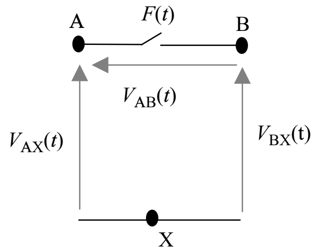


Figure 2.1 *Single switch*

2.1 Single switch

The voltage source, $V_{AX}(t)$ is connected on the LHS of the switch and the output is collected on the RHS, Fig. 2.1. The voltage on the RHS of the switch will take the value of the input voltage when the switch is closed and it will be zero when the switch is open.

$$V_{BX}(t) = V_{AX}(t) \quad \text{for } F(t) = 1 \;\text{ Mode I}$$

$$V_{BX}(t) = 0 \quad \text{for } F(t) = 0 \;\text{ Mode II}$$

The output voltage, $V_{BX}(t)$ is given by the contribution of both modes:

$$V_{BX}(t) = V_{AX}(t)F(t)$$

$V_{BX}(t)$ is the output voltage and $V_{AX}(t)$ is the input

$$V_o(t) = V_{in}(t)F(t) \tag{2.1}$$

Expression (2.1) is in accordance to Expression (1.1) of Chapter 1.

Perhaps it is important to note that $V_{AX}(t) \neq V_{BX}(t)F(t)$ and that $V_{AX}(t) = V_{BX}(t)/F(t)$ is not permitted because it encounters division by zero.

The voltage across the switch, $V_{AB}(t)$, is derived by applying Kirchoff's Second Law around the loop

$$V_{AX}(t) = V_{AB}(t) + V_{BX}(t)$$

Re-arranging

$$V_{AB}(t) = V_{AX}(t) - V_{BX}(t)$$

Substituting $V_{BX}(t)$ from Expression (2.1)

$$V_{AB}(t) = V_{AX}(t) - V_{AX}(t)F(t)$$

and

$$V_{AB}(t) = V_{AX}(t)[1 - F(t)] \tag{2.2}$$

Expression (2.2) gives the voltage across the switch.

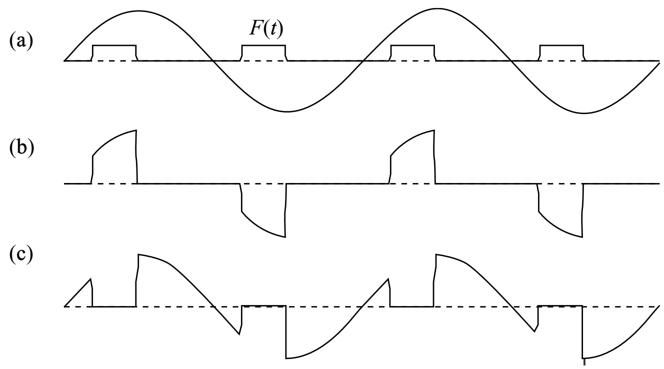


Figure 2.2 *Time waveforms of the basic switching function acting on a single switch connecting points A and B: (a) $V_{AX}(t)$, input voltage, (b) $V_{BX}(t)$, output voltage and (c) $V_{AB}(t)$, voltage across the switch*

The current, which will flow in the presence of a load:

$$I_B(t) = \frac{V_{BX}(t)}{Z_{BX}(n\omega)}$$

The current entering a switch is the same as the current leaving it.

$$I_A(t) = I_B(t)$$

Figure 2.2 displays the three voltages associated with the single switch and the switching function.

2.2 Parallel switches

The voltage source, $V_{AX}(t)$ is connected on the LHS of the parallel switches and the output is collected on the RHS, Fig. 2.3. The voltage on the right of the switch will take the value of the input voltage when any one of the switches is closed and it will be zero when none of the switches is closed.

With no overlap of the switching functions,

$$\sum_{n=1}^{N} F_n(t) \leq 1$$

The loop equation is given by

$$V_{AX}(t) = V_{AB}(t) + V_{BX}(t)$$

The output voltage is made up from the contributions of all switched branches (modes).

$$V_{BX}(t) = V_{AX}(t) \sum_{n=1}^{N} F_n(t) \tag{2.3}$$

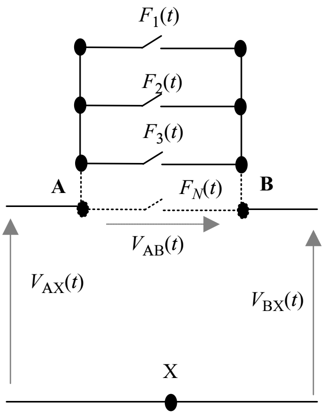


Figure 2.3 Combination of parallel switches

And the voltage across the switch combination

$$V_{AB}(t) = V_{AX}(t) - V_{BX}(t)$$

$$V_{AB}(t) = \left\{1 - \sum_{n=1}^{N} F_n(t)\right\} V_{AX}(t)$$

2.3 Parallel switched-resistors

The source $V_{AX}(t)$ is feeding a number of parallel resistor – switch branches giving a current $I(t)$ (Fig 2.4). No overlap between the switching functions,

$$\sum_{n=1}^{N} F_n(t) \leq 1$$

Applied voltages across the nth resistor

$$V_{Rn}(t) = F_n(t) V_{AX}(t)$$

Current through the nth resistor

$$I_{Rn}(t) = \frac{F_n(t) V_{AX}(t)}{R_n}$$

The total current is the sum of the currents in all branches

$$I(t) = \sum_{n=1}^{N} I_{Rn}(t) \tag{2.4}$$

In the event of an overlap of the switching functions, the switching function technique is applied by deriving the modes of the circuit and applying the procedure outlined in Chapter 1.

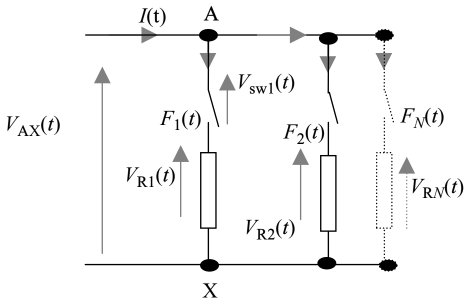


Figure 2.4 Combination of parallel switched-resistors

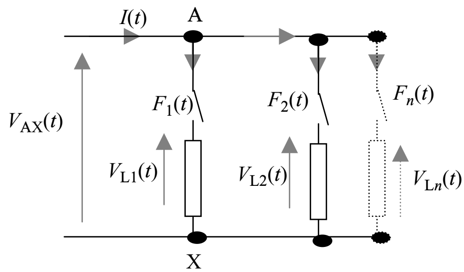


Figure 2.5 Parallel-switched inductors

2.4 Switched-inductors

2.4.1 Parallel switched-inductors

The source $V_{AX}(t)$ is feeding a number of parallel inductor – switch branches giving a total current $I(t)$ (Fig. 2.5). A charged inductor must always remain in a closed circuit. This implies that the series switch will not open unless the current through the inductor is zero. Switched-inductors are examined in Chapter 18 where single thyristors are used in each branch. The charged inductor will keep a thyristor conducting as long as there is current flowing. Overlap is also taking place in order to improve the current waveform.

Applied voltages across the nth inductor

$$V_{Ln}(t) = F_n(t)V_{AX}(t)$$

Current through the nth inductor

$$I_{Ln}(t) = \frac{F_n(t)V_{AX}(t)}{X(\omega n)}$$

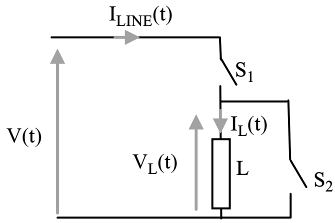


Figure 2.5(a) Single-switched inductors

The total current is the sum of all branches

$$I(t) = \sum_{n=1}^{N} I_n(t)$$

In the event of an overlap of the switching functions, the switching function technique is applied by deriving the modes of the circuit and applying the procedure outlined in Chapter 1. This is done in Chapter 18.

2.4.2 Single-switched inductor

In order to investigate the properties of the single switched inductor the circuit of Fig. 2.5(a) is suggested, where switch S_1 is modulating the applied voltage to the inductor and S_2 secures a closed path for the inductor current when S_1 is open. The two switches are working in anti-parallel because the current through a charged inductor can not be interrupted. The switching function for S_1 is $F_1(t)$ and the switching function for S_2 is $F_2(t)$.

Therefore

$$F_1(t) = K_o + 2\sum_{n=1}^{\infty} K_n \cos(nm\omega t - \theta_* n)$$

$$F_2(t) = 1 - F_1(t)$$

$$V_L(t) = F_1(t)V(t)$$

Let $V(t) = V_p \sin \omega t$

$$V_L(t) = K_o V_p \sin(\omega t) + \sum_{n=1}^{\infty} K_n\{[\sin[(nm+1)\omega t - n\theta] \\ - \sin[(nm-1)\omega t - \theta n]\}$$

The current in an inductor in the steady state is given by

$$I_L(t) = \frac{V_L(t)}{\omega n L}$$

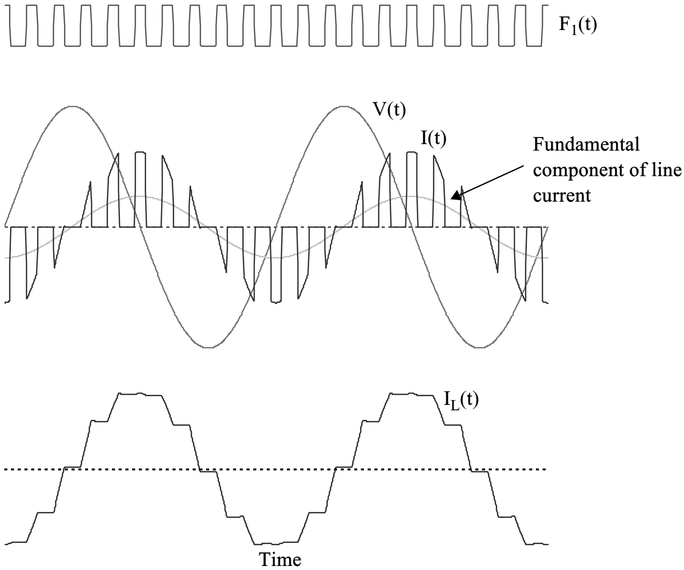


Figure 2.5(b) Voltage and current waveforms for the single switched inductor

$$I_L(t) = \frac{K_o V_p}{\omega L}\sin(\omega t - 90^\circ) + \sum_{n=1}^{\infty}\frac{V_p K_n}{\omega L(n+1)}\sin[(nm+1)\omega t - (\theta + 90^\circ)n]$$

$$-\sum_{n=1}^{\infty}\frac{V_p K_n}{\omega L(n-1)}\sin[(nm-1)\omega t - (\theta + 90^\circ)n]$$

or

$$I_L(t) = \frac{K_o V_p}{\omega L}\sin(\omega t - 90^\circ) + \text{Higher Harmonics}$$

The current supplied by the source,

$$I(t) = I_L(t)F(t) = \frac{K_o^2 V_p}{\omega L}\sin(\omega t - 90^\circ) + \text{Higher Harmonics}$$

The equivalent inductor impedance at fundamental frequency, $X(\omega)$

$$X(\omega) = \frac{wL}{K_o^2}$$

where ωL is the "static" impedance of the inductor.

Figure 2.5(b) is a display of the voltage and currents in the inductor circuit. Figure 2.5(c) is a display of the impedance of the switched inductor against the duty-cycle of the switch.

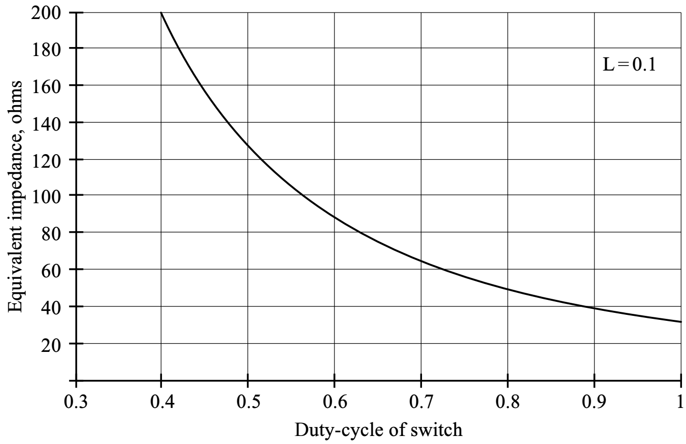


Figure 2.5(c) *The equivalent reactance of the single switched inductor at the fundamental frequency*

2.5 Parallel switched-capacitors

A current source on the left, $I(t)$ is feeding a number of parallel capacitor-switch branches, Fig. 2.6. In practice the current source is a voltage source with a large inductance in series.

First we consider a single branch, Fig. 2.6(a). The current through the nth capacitor

$$I_{Cn}(t) = F_n(t)I(t)$$

The voltage across the nth capacitor

$$V_{Cn}(t) = \frac{1}{C}\int F_n(t)I(t)\,\mathrm{d}t$$

The reflection of the nth capacitor voltage across AB

$$V_{ABn}(t) = F_n(t)*V_{Cn}(t)$$

$$V_{ABn}(t) = F_n(t)\frac{1}{C}\int F_n(t)I(t)\,\mathrm{d}t$$

The associated waveforms related to a single branch are shown in Fig. 2.7.

Now we consider the N branches, Fig. 2.6(b). The switching functions have no overlap and no dead periods

$$\sum_{n=1}^{N} F_n(t) = 1$$

Contribution of the N capacitors to the voltage across AB, $V_{AB}(t)$

$$V_{AB}(t) = \sum_{n=1}^{N} F_n(t)\frac{1}{C}\int F_n(t)I(t)\,\mathrm{d}t \tag{2.5}$$

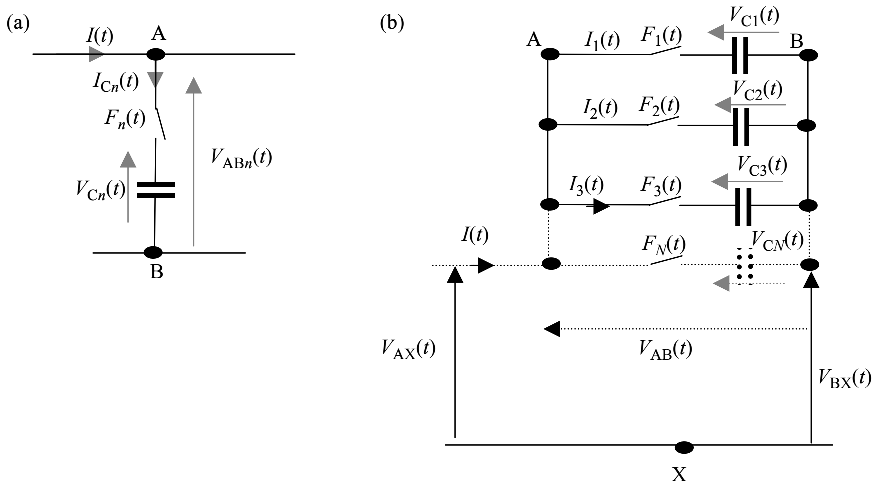


Figure 2.6 Parallel switched-capacitors: (a) single branch and (b) N-parallel switched-capacitors

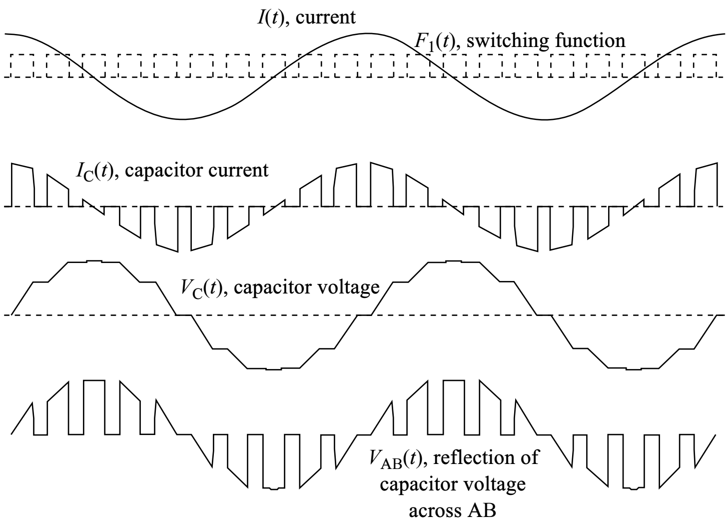


Figure 2.7 The associated waveforms related to a single branch switched capacitor circuit

Expression (2.5) gives the voltage which is reflected back to the input from a current fed network of switched capacitors. Usually an inductor is connected between the voltage source and the network [3]. Figure 2.8 displays the associated voltage, current and switching function waveforms for three parallel switched capacitors.

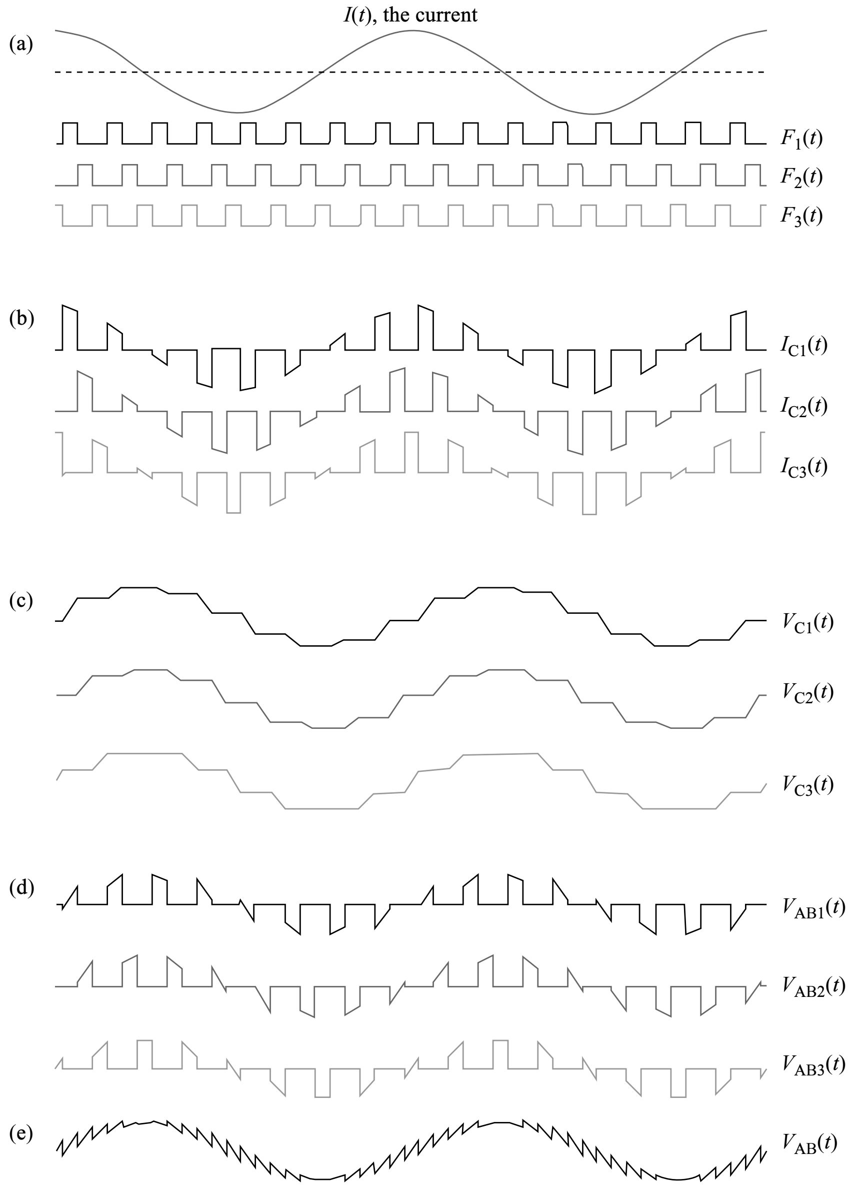


Figure 2.8 Parallel switched capacitors: time waveforms (a) the switching functions, (b) currents through the switched capacitors, (c) voltages across the switched capacitors, (d) reflected voltage from the switched capacitors to the terminals AB and (e) total reflected voltage across AB

2.6 Kirchoff's First Law (current law)

The textbook definition of the Kirchoff's First Law or the junction theorem, states that 'the sum of the currents into a specific junction in the circuit equals the sum of the currents out of the same junction. Electric charge is conserved: it does not suddenly appear or disappear'.

For a switched circuit Fig. 2.9(a) with a current source on the LHS, the junction theorem is applied in the following way. The modes of the switched circuit give the current with the associated switching functions. The mode sequence also must be established making sure that all instances of the 'period' of the circuit are represented by a switching function. 'Period' of the circuit is the time taken for one mode sequence to be completed.

The modes of this simple circuit are:

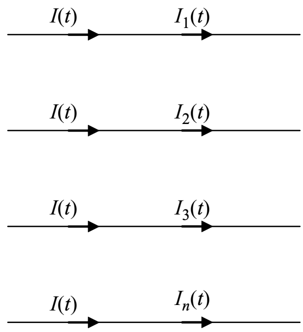
$I_1(t) = I(t)$ for $F_1(\mathrm{t}) = 1$ Mode I

$I_2(t) = I(t)$ for $F_2(\mathrm{t}) = 1$ Mode II

$I_3(t) = I(t)$ for $F_3(\mathrm{t}) = 1$ Mode III

$\vdots$

$I_n(t) = I(t)$ for $F_n(t) = 1$ Mode n



Therefore the current flowing away from the junction in the nth path is given by

$$I_n(t) = I(t)F_n(t) \tag{2.6}$$

The total current flowing into the junction for a simple sequence where one switch is closed once per cycle with no overlaps

$$I(t) = \sum_{n=1}^{N} I(t)F_n(t) \tag{2.7}$$

Condition $\sum_{n=1}^{N} F_n(t) = 1$, that is, no overlap and no dead periods.

Expressions (2.6) and (2.7) give the current relationships in a junction of a switched network. In the event of an overlap of the switching functions, the switching function technique is applied by deriving the modes of the circuit and applying the procedure outlined in Chapter 1. Figure 2.9(b) is a display of the associated switching functions and the currents at the various points for a 3-switch system.

2.7 Kirchoff's Second Law (voltage)

The second rule, the loop equation, states that 'around each loop in an electric circuit the sum of the emf's (electromotive forces, or voltages, of energy sources such as

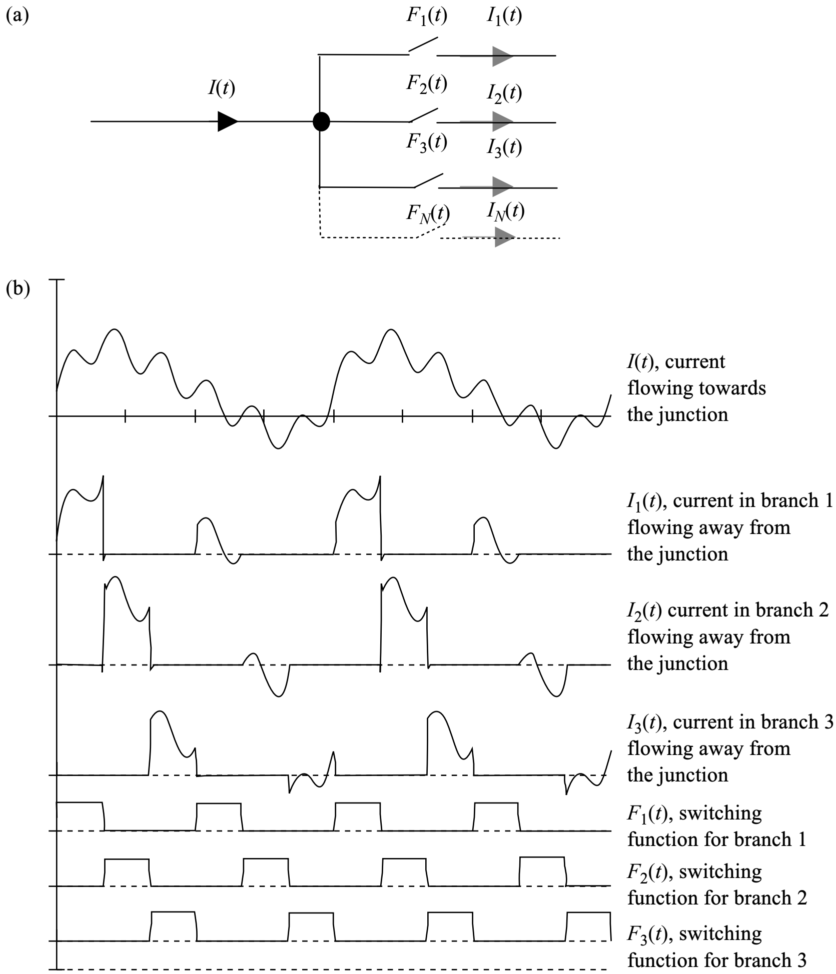


Figure 2.9 Kirchoff's Law: (a) circuit diagram and (b) time waveforms

batteries and generators) is equal to the sum of the potential drops, or voltages across each of the impedances, in the same loop'.

Kirchoff's Second Law is applied in the first instance in switched circuits in the same way that it is applied in non-switched circuits.

The loop equation of Fig. 2.10 gives

$$V_{\text{in}}(t) = I(t)Z(\omega n) + V_{\text{SW}}(t)$$

The challenge is to express the voltage across the switched circuit, $V_{\text{SW}}(t)$ in a single expression which is valid at all times. This voltage is shaped by the action of the switches in that network; the procedure outlined in the first section of Chapter 1 is to be applied in order to derive that single expression.

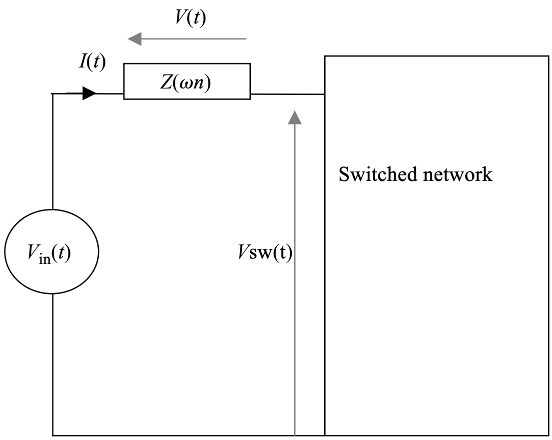


Figure 2.10 Kirchoff's Law of voltage

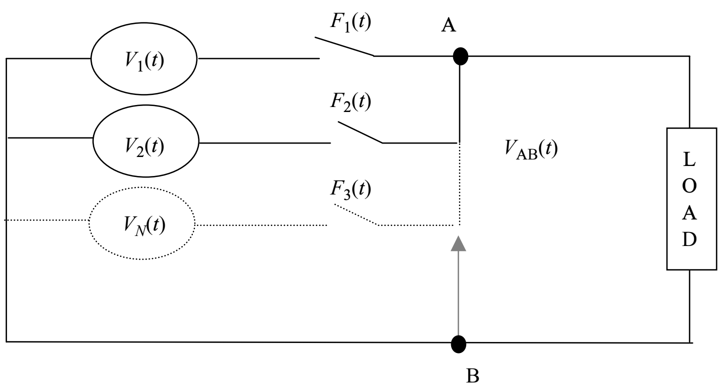


Figure 2.11 Superposition theorem

For the application of the Kirchoff's Second Law (voltage) around a loop with switched parts, the voltage across the switched part must be expressed by an expression that gives that voltage at all times.

2.8 Superposition theorem in switched circuits

The superposition theorem is already applied many times in this chapter; here it is introduced to the switched circuits in a more formal way. Consider the circuit in Fig. 2.11 where N voltage sources are supplying a single load connected across AB. The switching functions do not overlap, thus implying that no two switches are closed

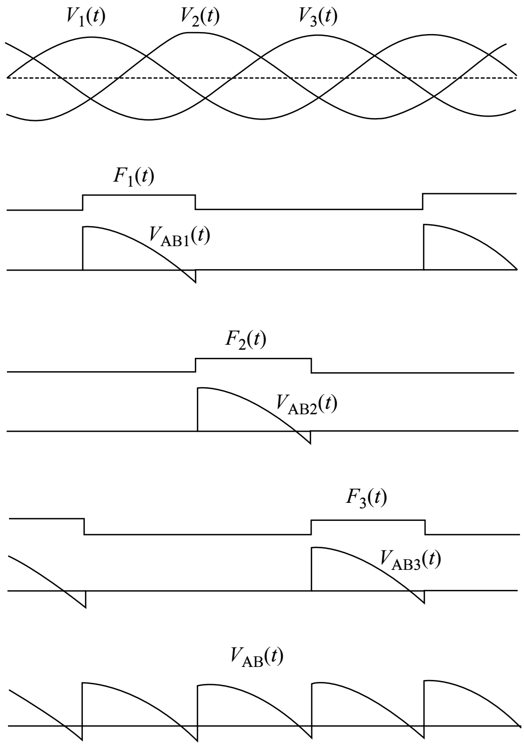


Figure 2.12 Superposition theorem as it applies in switched circuits.

at the same time, hence

$$\sum_{n=1}^{N} F_n(t) \leq 1$$

Every voltage source contributes to the output voltage, $V_{AB}(t)$, during the period that the series switch is closed according to Expression (1.2) of Chapter 1.

$$V_{ABn}(t) = F_n(t)V_n(t)$$

Hence the contributions of all the voltage sources make up the output voltage, $V_{AB}(t)$. This is also demonstrated graphically in Fig. 2.12.

$$V_{AB}(t) = F_1(t)V_1(t) + F_2(t)V_2(t) + \cdots + F_N(t)V_N(t)$$

$$V_{AB}(t) = \sum_{n=1}^{N} F_n(t)V_n(t) \tag{2.8}$$

The condition $\sum_{n=1}^{N} F_n(t) \leq 1$ implies no overlap. Overlap means violation of Kirchoff's laws in the absence of source impedance. In the presence of source impedance overlap is allowed. More than one switch can be closed during the overlap period. In that event, the mode of the circuit under overlap is derived and the appropriate overlap switching function is set, Chapter 8.

2.9 Current sharing in a parallel RC switched network

In many power electronic circuits, the output stage is represented by Fig. 2.13. The current $I(t)$ is diverted from the switch to the diode branch as the switch closes and opens at a high frequency. The procedure recommended in Chapter 1 for the application of the switching function is used here to show that the current through the capacitor is given by:

$$I_C(t) = I_D(t) - I_{dco}$$

I_{dco} is the dc component of the current through the diode when it is conducting. The expression is valid on the assumption that the output current through the load resistor can be approximated to its average value, I_{dco}.

Three modes are identified:

Mode I with switch on $I(t) > 0$, diode is not conducting, Fig. 2.14(a)
Mode II with switch off $I(t) > 0$, diode is conducting, Fig. 2.14(b)
Mode III with switch off $I(t) = 0$, diode is not conducting, Fig. 2.14(c)

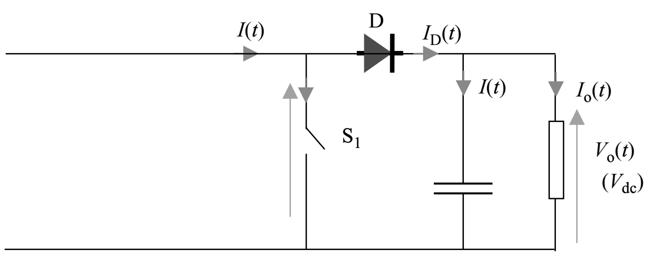


Figure 2.13 Parallel RC switched network

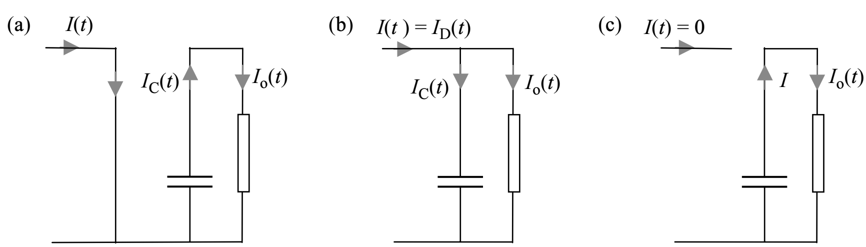


Figure 2.14 (a) Mode I, (b) Mode II and (c) Mode III

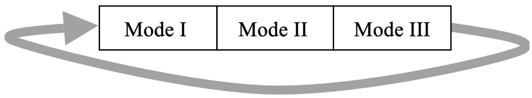


Figure 2.15 Mode sequence

The mode sequence is shown in Fig. 2.15. The switching function $F_1(t)$ is attached to Mode I, $F_2(t)$ is attached to Mode II and $F_3(t)$ is attached to Mode III. The three switching functions are not overlapping and there are no dead periods

$$F_1(t) + F_2(t) + F_3(t) = 1$$

In Modes III and I, the capacitor is discharging and this action must be considered in the mathematical model. The sequence of the modes, Fig. 2.15 is such that Mode III is succeeded by Mode I, therefore their effect can be considered together. During Modes I and III the capacitor is discharging with current, $I_{\text{disch}}(t)$. The discharging current, $I_{\text{disch}}(t)$, during these modes is approximated to the output current, $I_{\text{dco}} = V_{\text{dco}}/R$. Hence the contribution of Modes I and III to the capacitor current is given by:

$$I_{\text{C}}(t) = -I_{\text{dco}}[1 - F_2(t)].$$

During Mode II it is charging with a current $I_{\text{ch}}(t)$. This current is the diode current $I_{\text{D}}(t)$, less the current which flows in the load. The load current is rightly approximated to the dc component I_{dco} in the presence of a large smoothing capacitor. Hence the contribution of Mode II to the capacitor current is given by:

$$I_{\text{Ccharging}}(t) = [I_{\text{D}}(t) - I_{\text{dco}}]F_2(t)$$

Hence, by considering the sequence of modes and their contribution to the capacitor current, the capacitor current is given by

$$I_{\text{C}}(t) = -I_{\text{dco}}[1 - F_2(t)] + [I_{\text{D}}(t) - I_{\text{dco}}]F_2(t)$$

The diode current, according to Expression (2.6) is given by

$$I_{\text{D}}(t) = I(t)F_2(t)$$

Substituting expression for the diode current

$$I_{\text{C}}(t) = -I_{\text{dco}}[1 - F_2(t)] + [I(t)F_2(t) - I_{\text{dco}}]F_2(t)$$

Simplifying

$$I_{\text{C}}(t) = -I_{\text{dco}}[1 - F_2(t)] + I(t)F_2(t)^2 - I_{\text{dco}}F_2(t)$$

$F_2(t)$ is a unipolar switching function, and $F_2(t)^2 = F_2(t)$, Chapter 1.

$$I_{\text{C}}(t) = -I_{\text{dco}}[1 - F_2(t)] + I(t)F_2(t) - I_{\text{dco}}F_2(t)$$

Simplifying

$$I_{\text{C}}(t) = I(t)F_2(t) - I_{\text{dco}}$$

The product $I(t)F_2(t)$ is the diode current, hence

$$I_C(t) = I_D(t) - I_{dco} \tag{2.9}$$

In a more rigorous approach, the accuracy of Expression (2.9) can be improved by considering the ripple current through the load resistance R as well. In other words, the diode current $I(t)F_2(t)$ is shared by the load resistance R and the smoothing capacitor C as indicated by Expression (2.10). The dc component, I_{dco}, is taken by the load because the capacitor is not taking any dc current under steady-state conditions.

$$I_C(t) = F_2(t)I(t)\frac{\overline{X(\omega n)}}{\overline{X(\omega n)} + R} - I_{dco} \tag{2.10}$$

Expressions (2.9) and (2.10) could also be derived in a simpler way from the distribution of currents at the diode–capacitor–resistor junction. The current going into the junction is the diode current; the resistance takes its dc component as the capacitor is taking no dc under steady-state conditions. The capacitor and the resistance share the remainder, the ac components.

Chapter 3

Pulse width modulation

Pulse width modulation is a form of pulse coding where the modulated signal contains a chosen frequency or band of frequencies as the main components plus harmonics of very high order that can be easily filtered. Even though PWM originated in telecommunication systems, it is now very common in power electronic circuits: dc to dc converters, dc to ac and wave-shaping circuits. In its simplest form we have a single frequency coded on the PWM signal known as sinusoidal Pulse Width Modulation. In the standard way, the reference signal is compared with a triangular signal in order to generate the PWM signal. The reference or modulating signal bears the characteristics – amplitude, frequency and phase – of the desired voltage or current to be produced. The triangular signal – called the carrier – has a much higher frequency f_c, compared with the frequency, f_m of the modulating signal. The 'frequency modulation ratio' m is the ratio of the two frequencies, $m = f_c/f_m$ [5]. The spectral content of the PWM signal is very important to be known. The order of the produced high frequency unwanted harmonics is standard for a single frequency modulating frequency. The magnitude of the produced frequency components is derived by computer simulation and/or the Bessel functions or other methods [6].

The switching function is employed in this work to construct pulse width modulated unipolar and bipolar signals. The modulating signal is used as a reference to produce a series of pulses; the width of each pulse is set according to the amplitude of the modulating signal at that instant. This is achieved by splitting the cycle of the modulating signal into m number of sectors, Fig. 3.1. At the centre of each sector a pulse of unit amplitude is produced. The width of the pulse is set in such a way that its area is proportional to the area under the signal for that sector. More accurately stated, 'the area of the kth pulse $P_A(k)$ is proportional to the area under the reference signal of the kth sector $S_A(k)$'.

$$P_A(k) = S_A(k) \tag{3.1}$$

Since there are m sectors, the switching frequency is m times the modulating frequency. This is more widely known as the frequency modulation ratio. It will be

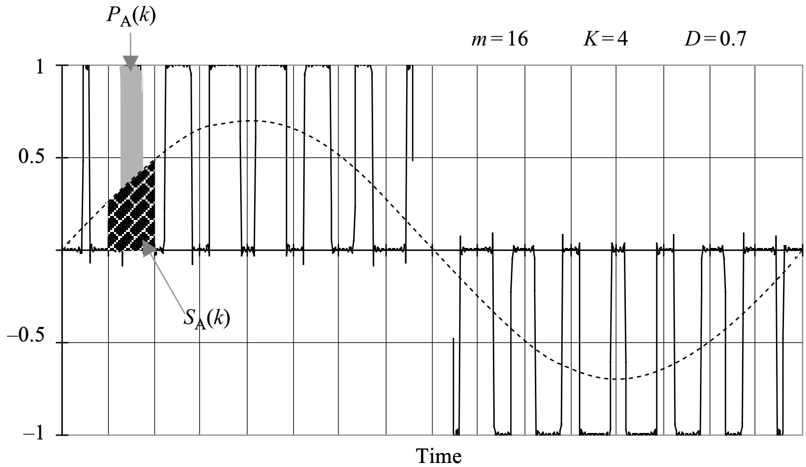


Figure 3.1 Unipolar PWM waveform with 16 pulses ($m = 16$)

shown later that for certain applications with periodic symmetry and symmetry about the 90° axis, m is a number divisible by two or four.

In this chapter a number of signals are pulse width modulated: a unipolar signal for a sine wave a unipolar signal for a rectified sine wave, a composite signal consisting of a rectified sine wave and its derivative, and a bipolar signal for a sine wave. For each signal a 'component switching function' is derived which gives a single pulse; the width of the pulse is set according to its position. The PWM signal is the sum of m such component switching functions. Some of the derived signals are used in the next chapters.

3.1 Sinusoidally modulated PWM signal – unipolar

Figure 3.1 shows a Unipolar Sinusoidal PWM signal with 16 pulses, $m = 16$. The switching function technique is employed here to produce such a signal. The switching frequency is $m\omega$ where m is an integer number and ω is the frequency of the signal to be produced. As demonstrated in Fig 3.1, a unipolar PWM modulated signal exhibits periodic symmetry and symmetry about the 90° axis. The symmetry about the 90° axis implies that the k^{th} pulse is of the same width as the $[m/2 - k]^{\text{th}}$ pulse; the periodic symmetry implies that these pulses appear inverted with the same spacing in the negative half-cycle, 180° later. A 'component switching function', $F_{\text{comp}}(t)$, is introduced to contain this information, Expression (3.2) and displayed in Fig. 3.2.

$$F_{\text{comp}}(t) = 4\sum_{n=1}^{\infty} K_n[\cos(n\omega t - n\theta_1) - \cos(n\omega t - n\theta_2)] \tag{3.2}$$

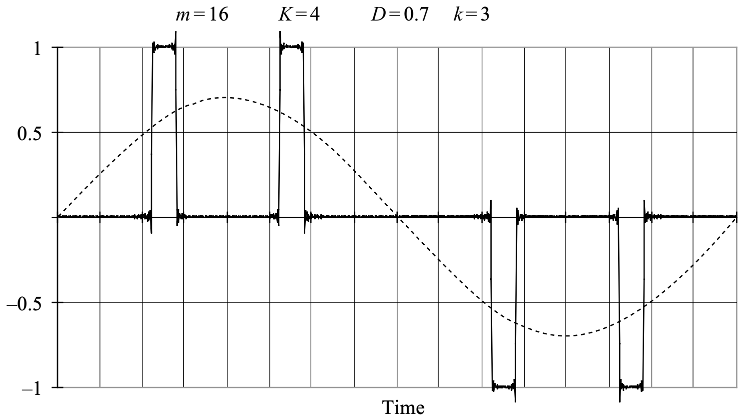


Figure 3.2 Component switching function, second

where

n = odd integer
$\theta_1 = Tk - (T/2)$, the phase displacement of the first pulse
$\theta_2 = Tk - (T/2) + \pi$, the phase displacement of the second pulse
$K_n = (\sin(n\delta))/n\pi$
δ = half the width of the kth pulse (Expression (3.5))
$m = f_c/f_m$, frequency modulation ratio, an integer number; f_c is the carrier frequency and f_m is the modulating frequency
$T = 2\pi/m$, period of the PWM switching function
$K = m/4$, number of component switching functions necessary to construct the PWM signal
k = integer number indicating the kth component switching function $1, 2, 3, \ldots K$.

The Expression (3.2) is simplified to

$$F_{comp}(t) = 8\sum_{n=1}^{\infty} K_n \sin(n\theta_1)\sin(n\omega t) \tag{3.3}$$

Expression (3.3) is valid for odd values of n. The 'component switching function', $F_{comp}(t)$, is a quasi-square signal, Fig. 3.2, and it has to be repeated $m/4$ times to make up the PWM signal. Therefore the complete Unipolar PWM switching function, $F_{PWM}U(t)$ is the summation of $m/4$ such component switching functions, Expression (3.3).

$$F_{PWM}U(t) = 8\sum_{k=1}^{K}\sum_{n=1}^{\infty} K_n \sin(n\theta_1)\sin(n\omega t) \tag{3.4}$$

$$K = \frac{m}{4}$$

The half-pulse-width δ is derived from Expression (3.1). As stated earlier, the area of the kth pulse, $P_A(k)$ is equal to the area under the sine wave for the kth sector, $S_A(k)$.

The modulating signal is

$$V_{\text{mod}}(t) = D \sin \omega t$$

where D is the amplitude and it can vary from 0 to 1; this is usually known as the modulation ratio.

$$\begin{aligned}\text{Area of the } k\text{th sector, } S_A(k) &= \int_{(k-1)T}^{T} V_{\text{mod}}(t)\, \mathrm{d}\omega t \\ &= \int_{(k-1)T}^{T} D \sin(\omega t)\, \mathrm{d}\omega t \\ &= [\cos(k-1)T - \cos(kT)]D\end{aligned}$$

The area of the kth pulse, $P_A(k)$, is the product of its width and its height. By definition the width of the pulse is 2δ, where δ is half-pulse-width. The height of the switching function pulse is by definition 1.

$$P_A(k) = 2\delta$$

Hence the δ parameter of the switching function is given by substituting the expressions of both $S_A(k)$ and $P_A(k)$ into (3.1):

$$\delta := \tfrac{1}{2}[\cos[(k-1)T] - \cos(kT)]0.5D \tag{3.5}$$

The order and magnitude of each harmonic is derived directly from Expression (3.4) by setting the appropriate value of n. It appears that all the harmonics from the fundamental to infinity are present but this is not true. The magnitude of each frequency component is made up from the contributions of all pulses. As it happens, all the harmonics except the fundamental are practically of zero magnitude at the lower end of the frequency spectrum, even for low switching frequencies ($m > 12$). Figure 3.3 is a display of the frequency spectrums for the unipolar PWM signal at a switching frequency of 52 times the fundamental (2.6 kHz for a 50 Hz fundamental). The first harmonics to appear of significant magnitude are near the switching frequency, $m \pm 1$. The components at $m \pm 2$ are at much lower magnitude; the switching frequency is absent. This is repeated for $2m \pm 1$ and $2m \pm 2$.

Low order harmonics such as the third harmonic are insignificant even at low switching frequencies. For a switching frequency of 600 Hz ($m = 12$) the third harmonic is noticeable but down to 0.25% of the fundamental.

In order to preserve both the periodic symmetry and the symmetry about the 90° axis, the switching frequency must be a number, which can be divided by four. If this symmetry is not respected, low-order harmonics manifest themselves in the PWM signal. This is more acute for low switching frequencies and it is demonstrated in Fig. 3.4 where the magnitude of the harmonics is displaced against the switching frequency ratio, m. As shown at $m = 4, 8, 12, 16, \ldots$, the harmonic level is the lowest if not zero.

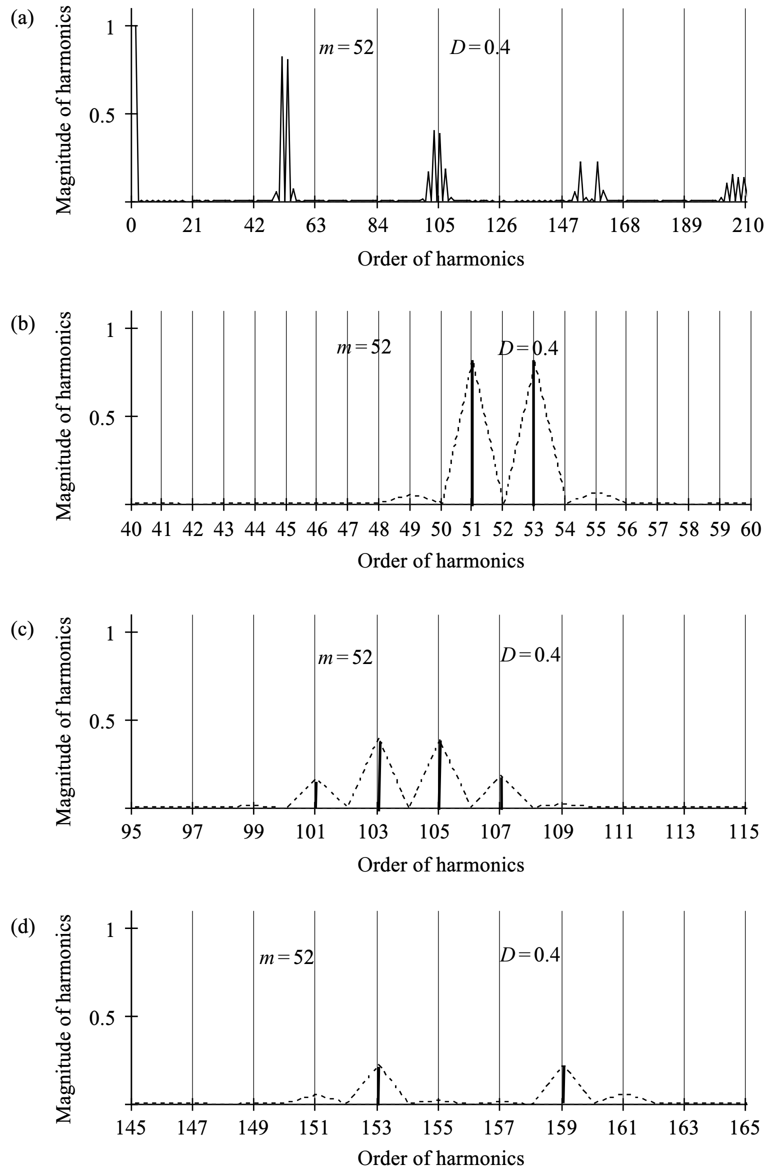


Figure 3.3 The frequency spectrum of a sinusoidal PWM signal (unipolar)

3.2 The rectified sine-wave PWM signal

A rectified sine-wave is pulse width modulated by the same method as we did for the sine-wave. Naturally the pulse width, δ is derived in the same way, Expression (3.5).

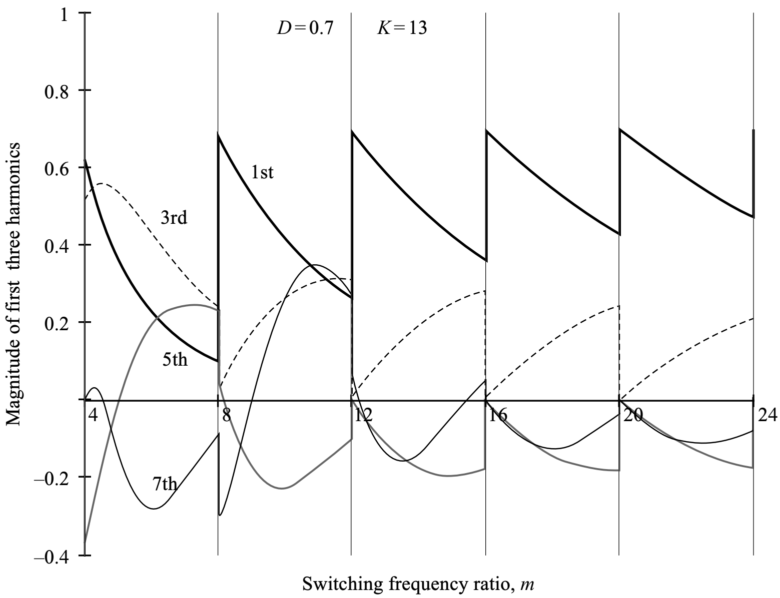


Figure 3.4 Magnitude of low-order harmonics against the switching frequency ratio, m (unipolar)

Hence,

$$\delta := \tfrac{1}{2}[\cos[(k-1)T] - \cos(kT)]D$$

T is the switching period and k is an integer number giving the order of the component switching function. The PWM signal for the rectified sine though, does not display the same symmetry as the sine signal. It is symmetrical about the 90° axis and the 180° axis, hence the four pulses of the component switching function, $F_{\text{comp}}R(t)$, are arranged as follows: the first two are symmetrical about the 90° and the other two are symmetrical about the 270° vertical axis, Fig. 3.5. The component switching function is easily derived below where each term represents one pulse.

$$F_{\text{comp}}R(t) = K_o + 2\sum_{n=1}^{\infty} K_n\{\cos(n\omega t - n\beta_1) + \cos[n\omega t - n(\pi - \beta_1)] + \cos[n\omega t - n(\pi + \beta_1)] + \cos[n\omega t - n(2\pi - \beta_1)]\}$$

$$\beta_1 = Tk - \frac{T}{2}$$

where K_o is the average value of the switching function.

$$K_o = \frac{4(2\delta)}{2\pi}$$

K_n, and m are defined in the usual way, Expression (3.2).

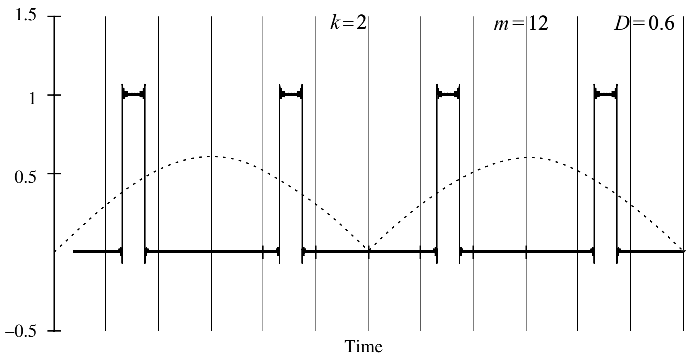


Figure 3.5 The component switching function for the rectified sine-wave

The component switching function is simplified to

$$F_{\mathrm{comp}}R(t) = K_{\mathrm{o}} + 8\sum_{n=1}^{\infty} K_n \cos(n\omega t)\cos(n\beta_1)\left|\cos\frac{n\pi}{2}\right| \tag{3.6}$$

where n is an even integer. The term $\{\cos(n\pi/2)\}$ is included to take care of that.

The PWM switching function for the rectified sine-wave, $F_{\mathrm{PWM}}R(t)$, is given by considering all the component switching functions, K.

$$F_{\mathrm{PWM}}R(t) = K_{\mathrm{PWM}} + 8\sum_{k=1}^{K}\sum_{n=1}^{\infty} K_n \cos(n\omega t)\cos(n\beta_1)\left|\cos\frac{n\pi}{2}\right| \tag{3.7}$$

where

$$K_{\mathrm{PWM}} = \frac{2D}{\pi}$$

$K = m/4$ and m is the number of pulses in one cycle of the PWM signal.

Figure 3.6 displays the PWM switching function for the rectified sine-wave, $F_{\mathrm{PWM}}R(t)$. Its frequency spectrum is displayed in Fig. 3.7(a). The modulating signal, which is the rectified sine is displayed in Fig. 3.7(b). As shown, the PWM signal contains all the frequency components that make up the rectified sine plus higher order harmonics. There is no evident mathematical justification for the order of the harmonics of the PWM signal but they seem to obey a simple rule: they are of the order $xm \pm 2y$ plus the switching frequency m, where x and y are positive integers. Large values of y might suggest low order harmonics near the useful bandwidth of the rectified sine. As y and x are increasing though, the magnitude of these harmonics diminishes, Fig. 3.7(a). For $m = 40$ there is a 'safe' distance between the useful bandwidth of the rectified sine and the high-frequency components centred around m. This 'safe' distance, increases with m.

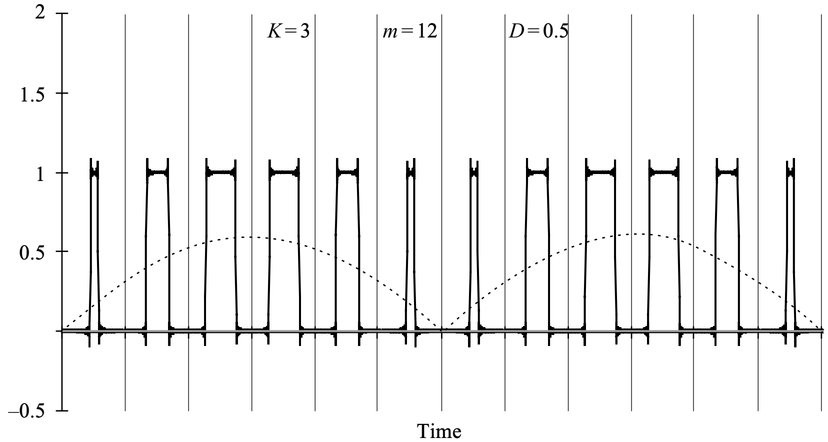


Figure 3.6 The PWM signal for the rectified sine

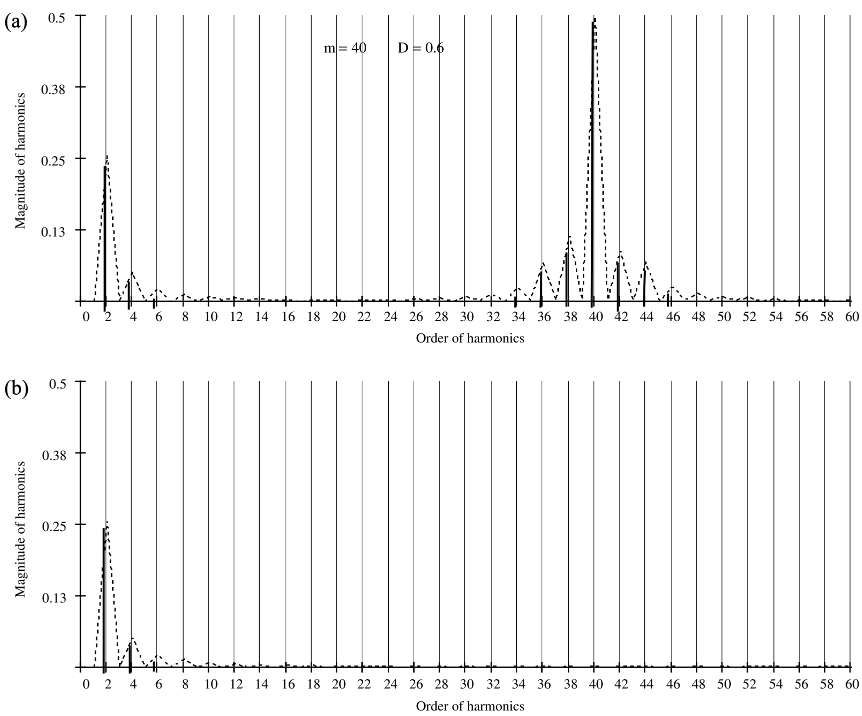


Figure 3.7 Frequency spectrum: (a) of the pulse width modulated rectified sine $F_{\mathrm{PWM}}R(t)$ and (b) of the rectified sine

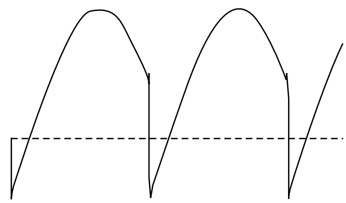


Figure 3.8 The modulating composite signal

3.3 The PWM signal of a composite function

The signal to be coded in this section, $M(t)$, is a composite signal consisting partly of a sine and partly a cosine wave. It is described by Expression (3.8) in the specified period π and it is repeated for 2π, 3π, etc.

$$M(t) = \sin\omega t - D_2 \cos\omega t \quad \text{for } 0 < \omega t < \pi \tag{3.8}$$

It is chosen to present the modulating signal over a period of ω, Fig. 3.8. D_1 and D_2 are the amplitudes of the two ac components. The dc component, K_o, is the average value of the two ac components and in this case it is reduced to the average of the first term since the second term, being a cosine from 0 to π, has a zero average value.

$$K_o = \frac{2D_1}{\pi}$$

Area under the curve for the period $k-1$ to k, $S_A(k)$

$$\begin{aligned} S_A(k) &= \int_{(k-1)T}^{kT} [D_1 \sin\omega t - D_2 \cos\omega t]\, \mathrm{d}\omega t \\ &= [\cos(k-1)T - \cos(kT)]D_1 - D_2[\sin(kT) - \sin\{(k-1)T\}] \end{aligned}$$

Area of the kth pulse of the switching function, $P_A(k) = 2\delta$.

The area under the curve is equal to the area of the pulse, Expression (3.1). The half-pulse-width, δ, is

$$\delta = \{[\cos(k-1)T - \cos(kT)]D_1 + D_2[\sin(k-1)T - \sin(kT)]\}/2$$

The switching function is constructed on the basis of the period of one complete cycle of ω, Expression (3.8). It exhibits periodic symmetry: for every pulse in the first half-cycle there is a similar one (same width) in the second half-cycle at π later. Hence the component switching function is a switching function with two similar pulses

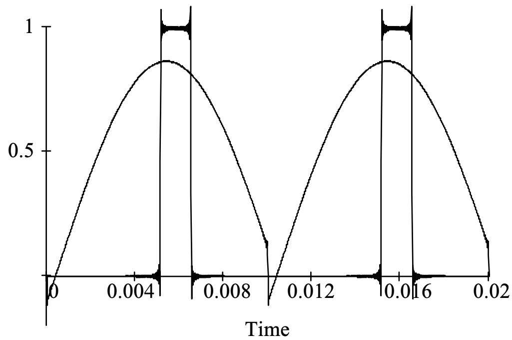


Figure 3.9 The component switching function of the composite signal

separated by 180° with a switching frequency ω, the power frequency, Fig. 3.9.

$$F_{\mathrm{compC}}(t) = K_{\mathrm{o}1} + 2\sum_{n=1}^{\infty} K_n\{\cos(n\omega t - n\beta_1) + \cos[n\omega t - n(\pi - \beta_1)]\}$$

$$K_{\mathrm{o}_1} = \frac{2\delta}{\pi}$$

$$\beta_1 = Tk - \frac{T}{2}$$

For the construction of the switching function with m pulses per mains cycle, $m/2$ component switching functions are required, hence $K = m/2$. Hence the PWM switching function of the composite signal, $F_{\mathrm{PWMC}}(t)$ is given by Expression (3.9)

$$F_{\mathrm{PWMC}}(t) = K_{\mathrm{o}} + 2\sum_{k=1}^{K}\sum_{n=1}^{\infty} K_n\{\cos(n\omega t - n\beta_1) + \cos[n\omega t - n(\pi - \beta_1)]\}$$

$$K_{\mathrm{o}} = \frac{1}{\pi}\sum_{k=1}^{k}\delta \tag{3.9}$$

The display of the PWM composite waveform in Fig. 3.10 seems to be a unipolar signal. This is misleading; if the switching frequency is increased to the point that the switching period is small enough to accommodate the period for which the signal is negative, the PWM composite waveform will display negative pulses. This implies that the actual switching must be done by bridge configuration; it is the only one which produces inversion of the input quantity, voltage or current. Nevertheless it will be shown in Chapter 21 that a single switch is used to implement it. A single switch cannot perform the inversion implied by the negative pulses of the PWM switching function. In the particular case $D_1 \gg D_2$ and the error introduced is apparently small.

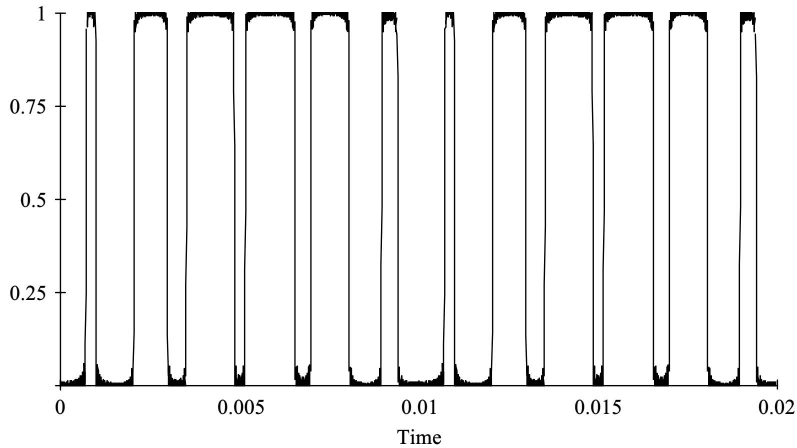


Figure 3.10 The composite rectified PWM signal

3.4 PWM sine-wave – bipolar square wave modulation

A bipolar switching function oscillates between two values: 1 and -1. The negative state implies that the output is reversed with respect to the input and the circuit associated with this inversion is the bridge. The principle adopted to produce the bipolar PWM signal is that of a high frequency carrier in the form of a bipolar square wave whose width is modulated according to the area of the modulating signal within the half-period period of the bipolar square wave.

The half-pulse width of a unipolar switching function modulated by a sine-wave modulating signal with amplitude D is found earlier in Expression (3.5) as:

$$\delta := \tfrac{1}{2}[\cos[(k-1)T] - \cos(kT)]D$$

It can be used for this application as well with a small difference. T is the period of the switching frequency and k is an integer number indicating the kth pulse. In this application the modulating signal needs to take only half of its maximum amplitude that is, $D = 0.5$. This is because the un-modulated pulse width of the square wave is already at its half value before any modulation is applied. D will be allowed to take the full range from zero to one in this application by introducing a factor of 0.5 in Expression (3.10).

The half-width of the pulse of the carrier is $T/4$ and it will be modulated by δ. The half-width of the modulated pulse of the carrier will be

$$\delta = \tfrac{1}{2}\{\cos[(k-1)T] - \cos(kT)\}0.5D + \frac{T}{4} \tag{3.10}$$

where $T/4$ is the un-modulated half-width of the carrier.

The component switching function will be a signal with a single pulse per modulating signal cycle, hence K, the number of component switching functions is m.

$$F_{\text{comp}}(t) = K_o + 2\sum_{n=1}^{\infty} K_n \cos[n\omega t - n\theta]$$

$\theta = Tk - (T/2)$ is the position of the pulse relative to reference signal.

$$K_o = \frac{2\delta}{\pi}$$

The unipolar switching function is given by all K component switching functions

$$F_{\text{UNIP}}(t) = \sum_{k=1}^{K} K_o + 2\sum_{n=1}^{\infty} K_n \cos[n\omega t - n\theta]$$

In order to convert the unipolar switching function into a bipolar one, the following operation is performed:

$$F_{\text{PWM}}B(t) = 2F_{\text{UNIP}}(t) - 1$$

It is easily shown that $\sum_{k=1}^{K} K_o = 0.5$ and the bipolar switching function is:

$$F_{\text{PWM}}B(t) = 4\sum_{k=1}^{m}\sum_{n=1}^{\infty} Kn \cos[n\omega t - n\theta] \tag{3.11}$$

The bipolar sinusoidaly modulated PWM signal, $F_{\text{PWM}}B(t)$ described by Expression (3.11), is displayed in Fig. 3.11 for a switching frequency $m\omega$ 50 times higher than the fundamental, $m = 50$ and an amplitude modulation ratio, $D = 0.7$. Expression (3.11) can be employed to give a single component, $V_n(t)$ by setting the appropriate value of n. Hence for individual values of n, Expression (3.11) is reduced to

$$V_n(t) = 4\sum_{k=1}^{m} K_n \cos\left[n\omega t - n\left(\theta - \frac{T}{2}\right)\right] \tag{3.12}$$

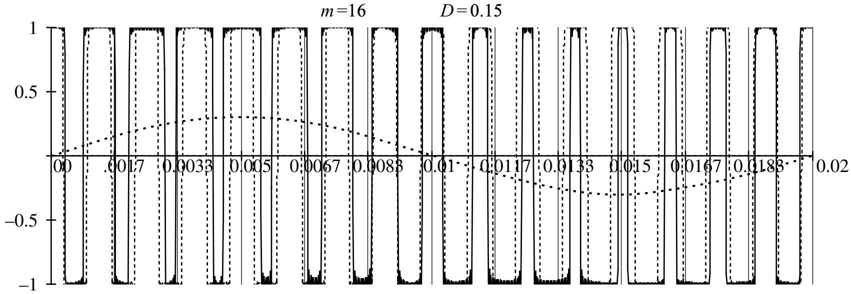


Figure 3.11 The bipolar PWM switching function of a sine wave

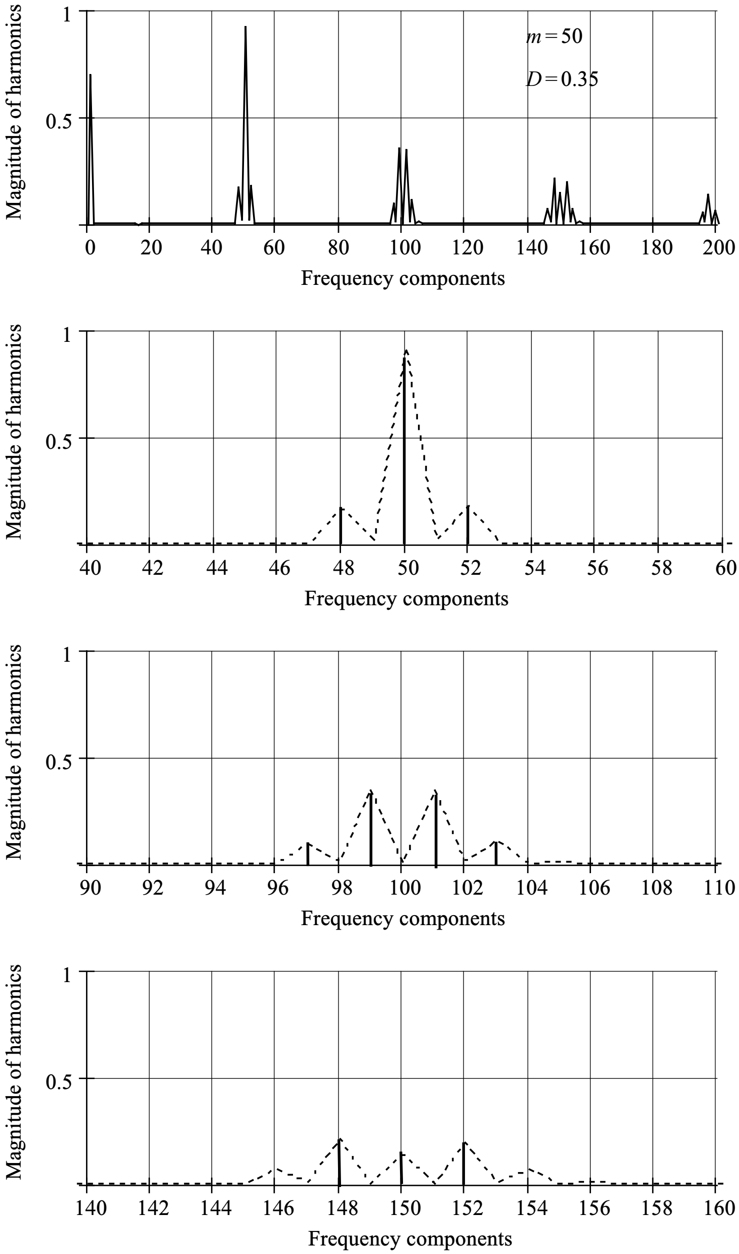


Figure 3.12 The frequency spectrum of the unipolar PWM sine wave

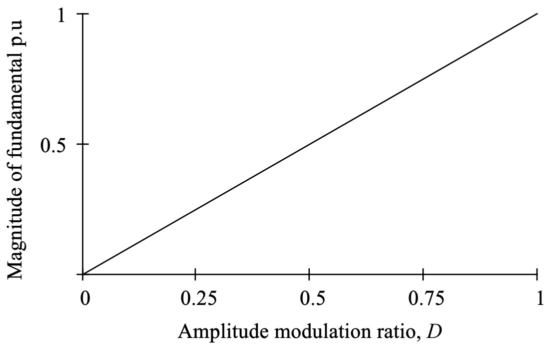


Figure 3.13 The fundamental of the bipolar PWM signal against the depth of modulation D

Both the magnitude and the phase of the single component are made up from the contributions of the m pulses of the signal. In order to extract the magnitude, Expression (3.12) is expanded into an in-phase and a quadrature component and then Pythagoras' theorem is used to give the actual magnitude. This result is used to display the frequency spectrum of the sinusoidally modulated pulse width signal as shown in Fig. 3.12. As shown the harmonic components are concentrated around the switching frequency and its multiples: $m\omega$, $2m\omega$, $3m\omega$ etc. The more important component is the fundamental. It is the useful part of the signal. A simple relationship seems to exist between the amplitude modulation ratio D and the magnitude of the fundamental of the PWM signal as shown in Fig. 3.13. Simply the amplitude modulation ratio is equal to the p.u. value of the fundamental component of the bipolar PWM sine wave signal.

Part 2

AC to DC conversion

Chapter 4

Analysis of the single phase ac to dc phase controlled converter with R–L load

4.1 Introduction

In the normal operation of a full-wave single phase converter the four thyristors conduct in two pairs: one from the upper group and one from the lower group. Thyristors TH1 and TH4 form the first group and TH3 and TH2 the other group (Fig. 4.1).

The analysis of this circuit is straightforward and the application of the switching function presents no special problems. Expressions for the output voltage, output current and input current are derived for continuous conduction and displayed. The analysis is extended to discontinuous conduction as well. The line current expression is further expanded by introducing a new variable, p, which represents the order of the harmonic. In this way the Total Harmonic Distortion factor, the Distortion factor, the frequency spectrum and the power factor are derived and displayed. The average and RMS values of the current through the semiconductor switches are readily available.

4.2 Mathematical modelling

In this section the mathematical model of the single phase ac to dc phase controlled converter with inductive load is derived. A procedure is suggested in Chapter 1 for the application of the switching function technique for the analysis of power electronic circuits. This demands the derivation of the modes of the circuit and many other things. Such a procedure is more applicable to more complicated circuits such as the dc to dc converters and the ac to dc converters with RC load. The analysis of this simple circuit does not demand the application of this procedure.

The input–output voltage relationship together for a bridge configuration with the input–output current relationship are derived in Chapter 1 and are given by

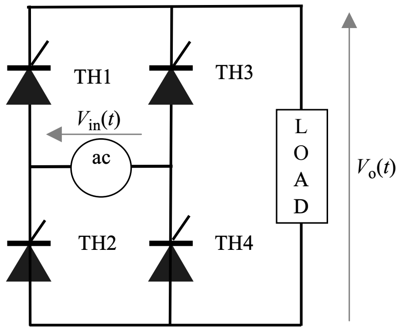


Figure 4.1 Single phase, phase controlled ac to dc converter with R–L load

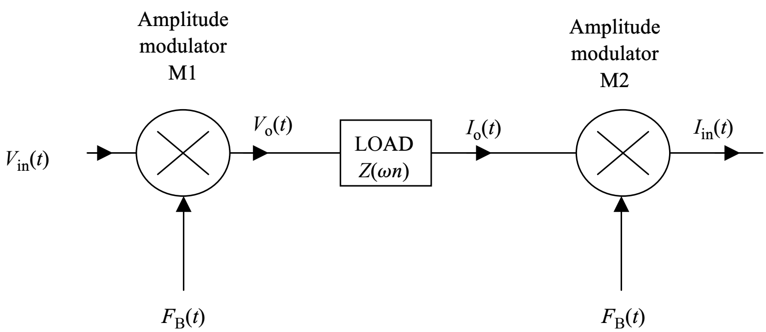


Figure 4.2 Simple mathematical model of the single phase, phase-controlled ac to dc converter

Expressions (1.4) and (1.6). These are re-represented here

$$V_o(t) = F_B(t)V_{in}(t) \tag{4.1}$$

$$I_{in}(t) = F_B(t)I_o(t) \tag{4.2}$$

4.2.1 The mathematical model

The output voltage, $V_o(t)$ is the result of an amplitude modulation of the input supply voltage and the appropriate switching function, $F_B(t)$. The output voltage forces the output current, $I_o(t)$, through the 'harmonic impedance', $Z(\omega n)$. The input current is a reflection of the output current to the input. It is the result of an amplitude modulation of the output current, $I_o(t)$, with the same switching function, $F_B(t)$. The mathematical model is shown in Fig. 4.2.

In the simple model of Fig. 4.2 two modulation processes are shown. M1 modulates the input voltage to give the output voltage and M2 modulates the output current to give the input current. In both cases the carrier is the switching function $F_B(t)$. The load and the firing delay angle of the thyristors dictate the output voltage and current.

4.2.2 The switching function for continuous and discontinuous conduction

The modulating signal, $F_B(t)$ is the bipolar switching function presented in Chapter 1 in Part 1 and reproduced in Expression (4.3). This expression applies for both continuous and discontinuous conduction. For continuous conduction the on period of the conducting thyristors and consequently the pulse width of the pulse of the switching function is 180°; for discontinuous conduction it is less and it is determined by the extinction angle, β. Hence the switching function is a square wave for continuous conduction and a quasi-square for discontinuous conduction. The pulse width of the quasi-square signal must be pre-calculated before the application of the technique by considering the firing angle α and the load.

For a circuit with very well-known behaviour deriving the switching function parameters is an easy task. Reference is made to Fig. 4.3 where the output voltage

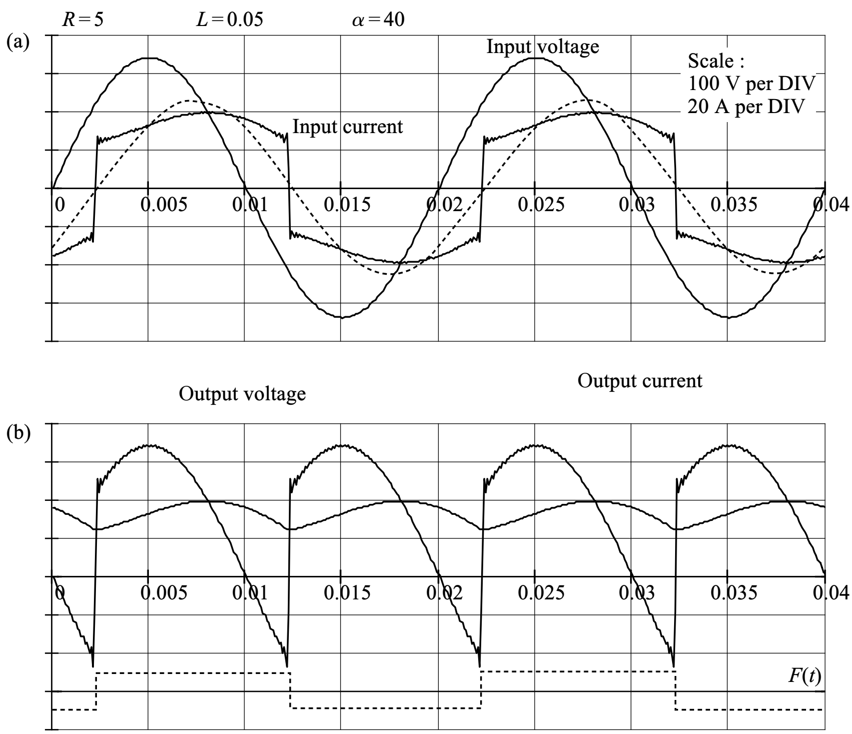


Figure 4.3 Voltage and current waveforms for the single phase ac to dc converter with inductive load (a) input voltage and current and (b) output voltage and current. The dotted line in (a) is the fundamental component of the line current

and current are displayed together with the line current for continuous conduction case. The conduction of the thyristors persists into the next half-cycle because of the inductive load. The switching function is that signal which when multiplied with the input voltage will give us the output voltage. A close examination of Fig. 4.3 reveals that the required switching function is a square wave of unit magnitude with its positive pulse starting at the firing instant α and finishing π radians later; the negative pulse of the switching function starts at the firing instant of the mains negative half-cycle $\pi + \alpha$ and finishes π radians later, Fig. 4.3(b). So the conduction period is π and the switching function is expressed in (4.3).

The ac to dc converter of Fig. 4.2 can enter discontinuous current conduction, depending on the relative amount of inductance in the circuit. Then we observe dead periods between the two half half-cycles. The positive pulse of current ends at some point just after π and before thyristors TH2 and TH3 connect the negative half-cycle to the load. This is the extinction angle, β, and must be calculated beforehand. Hence there is a dead period, which must be accounted for by the switching function. The positive pulse of the switching function starts at the firing instant α and finishes at $\pi + \beta$. The negative pulse starts at the firing instant $\pi + \alpha$ and finishes at $2\pi + \beta$. This is a quasi-square switching function as described in Chapter 1.

Modelling for discontinuous conduction does not differ from the one presented already. The switching function is a bipolar quasi-square signal defined above, Expression (4.3); the on-period differs and consequently the phase θ of the switching function. For the calculation of the K_n coefficients, the half-width, δ, of the on period of the switching function is required; this is

$$\delta = \frac{\beta - \alpha}{2}$$

where β is the extinction angle and α the firing angle. The delay angle θ is given by

$$\theta = \frac{\beta - \alpha}{2} + \alpha$$

The extinction angle must be calculated using standard methods such as the Laplace transforms.

$$F_{\mathrm{B}}(t) = 4\sum_{n=1}^{\infty} K_n \cos(n\omega t - \theta * n) \tag{4.3}$$

where

n is an odd number
$K_n = \sin(n\delta)/\pi n$
$\delta = \pi/2$, for continuous conduction
$\delta = (\beta - \alpha)/2$, for discontinuous conduction
$\theta = \alpha + \delta$, the phase of the switching function relative to supply voltage for continuous conduction
$\theta = (\beta - \alpha)/2 + \alpha$, for discontinuous conduction
$\alpha =$ delay firing angle.

4.3 Analysis

The model of Fig. 4.2 is employed for the analysis of this circuit.

4.3.1 Output voltage

In accordance with the model of Fig. 4.2 the output voltage is given as:

$$V_o(t) = V_p \sin(\omega t) F(t)$$

$$V_o(t) = \sum_{n=1}^{\infty} 2V_p K_n \{\sin[(n+1)\omega t - n\theta] - \sin[(n-1)\omega t - n\theta]\} \tag{4.4}$$

The output voltage, displayed in Fig. 4.3, is free from odd harmonics because n is an odd integer.

4.3.2 Output dc voltage

The output dc voltage (zero frequency) is created when n takes the value of one in Expression (4.4)

$$V_o(t)_{n=1} = 2V_p K_1 \{\sin[2\omega t - \theta] + \sin(\theta)\}$$

The dc component is located in the second term, $\sin(\theta)$; the first term $\sin[2\omega t - \theta]$ is dropped as it represents an ac component at twice the mains frequency. Hence the dc component, V_{dc} is given by:

$$V_{dc} = 2V_p K_1 \sin\theta \tag{4.5}$$

Expression (4.5) gives the output dc voltage for both continuous and discontinuous conduction; in each case K_1 and θ are derived. For continuous conduction it is further simplified here by substituting, $\theta = (\alpha + \pi/2)$.

$$\sin\theta = \sin(\alpha + \pi/2) = \cos\alpha \quad \text{and} \quad K_1 = \frac{\sin(\pi/2)}{\pi} = \frac{1}{\pi}$$

Therefore for continuous conduction

$$V_{dc} = 2V_p \frac{1}{\pi} \cos\alpha \tag{4.5a}$$

Expression (4.5a) is recognised as the same found by conventional methods. For discontinuous conduction K_1 and θ of Expression (4.5) must be derived from Expression (4.3).

4.3.3 Output current

The output voltage pushes through the ‘harmonic impedance’ of the load the output current. The harmonic impedance is the impedance presented to the output voltage as explained in Chapter 1. The dc component of the output voltage will give rise to a dc current, which is only limited by the resistance in the load. Therefore the output current consists of this dc component and the harmonics, which are limited by the complex 'harmonic impedance' of inductance and resistance, $Z(\omega n)$. The complex harmonic impedance is given by two expressions, one for each component of the output voltage, Expression (4.4), Z_{n+1} and Z_{n-1}.

$$Z_{n+1} = \sqrt{R^2 + (\omega(n+1)L)^2} \qquad \varphi_{n+1} = \tan^{-1}\left(\frac{\omega(n+1)L}{R}\right) \tag{4.6a}$$

and

$$Z_{n-1} = \sqrt{R^2 + (\omega(n-1)L)^2} \qquad \varphi_{n-1} = \tan^{-1}\left(\frac{\omega(n-1)L}{R}\right) \tag{4.6b}$$

The output current is given by simple application of Ohm's Law, Expression (4.7).

$$I_o(t) = \frac{V_o(t)}{Z(\omega n)}$$

$$\begin{aligned} I_o(t) = \sum_{n=1}^{\infty} 2\frac{V_p K_n}{Z_{n+1}} \sin[(n+1)\omega t - n\theta - \varphi_{n+1}] \\ - 2\frac{V_p K_n}{Z_{n-1}} \sin[(n-1)\omega t - n\theta - \varphi_{n-1}] \end{aligned} \tag{4.7}$$

The output current and voltage are displayed in Fig. 4.3.

Selecting the appropriate value of n in Expression (4.7) derives the dc component of the output current. This is the value of n that will give zero value to the coefficient of ω. This happens in the second term for $n = 1$. The impedance Z_{n-1}, Expression (4.6b) is reduced to R.

$$I_{dco} = 2\frac{V_p K_1}{R} \sin(\theta) \tag{4.8}$$

In the same way the various harmonic components of the output current are derived from Expression (4.7). For example, the second harmonic is derived for $n = 1$ in the first term of Expression (4.7) and for $n = 3$ in the second term of Expression (4.7)

$$I_{o2}(t) = 2\frac{V_p K_1}{Z_2}\left\{\sin[2\omega t - \theta - \varphi_2] - 2\frac{V_p K_3}{Z_2} - \sin[2\omega t - 3\theta - \varphi_2]\right\}$$

4.3.4 Line current

The line current is a 'reflection' of the output current to the input and it is given by Expression (4.2) in accordance with the mathematical model of Fig. 4.2. Expression (4.2) is reproduced here:

$$I_{in}(t) = I_o(t) F_B(t)$$

The output current $I_o(t)$ is given by Expression (4.7) and the switching function $F_B(t)$ by Expression (4.3). Substituting in (4.2)

$$I_{in}(t) = \left\{ \sum_{n=1}^{\infty} 2\frac{V_p K_n}{Z_{n+1}} \{\sin[(n+1)\omega t - n\theta - n\varphi_{n+1}] - \sin[(n-1)\omega t - n\theta - n\varphi_{n-1}]\} \right\} 4 \sum_{n=1}^{\infty} K_n \cos(n\omega t - \theta * n)$$

After expansion it yields

$$\begin{aligned}
I_{in}(t) = {} & \sum_{n=1}^{N}\sum_{m=1}^{N} 4\frac{V_p(\sin(n\delta)/n\pi)(\sin(m\delta)/m\pi)}{\sqrt{R^2+[\omega(n+1)L]^2}} \\
& \times \sin\left[(n+m+1)\omega t - (n+m)\gamma - \operatorname{atan}\left[\omega\frac{(n+1)L}{R}\right]\right] \\
& + \sum_{n=1}^{N}\sum_{m=1}^{N} 4\frac{V_p(\sin(n\delta)/n\pi)(\sin(m\delta)/m\pi)}{\sqrt{R^2+[\omega(n+1)L]^2}} \\
& \times \sin\left[(n-m+1)\omega t - (n-m)\gamma - \operatorname{atan}\left[\omega\frac{(n+1)L}{R}\right]\right] \\
& - \sum_{n=1}^{N}\sum_{m=1}^{N} 4\frac{V_p(\sin(n\delta)/n\pi)(\sin(m\delta)/m\pi)}{\sqrt{R^2+[\omega(n-1)L]^2}} \\
& \times \sin\left[(n+m-1)\omega t - (n+m)\gamma - \operatorname{atan}\left[\omega\frac{(n-1)L}{R}\right]\right] \\
& - \sum_{n=1}^{N}\sum_{m=1}^{N} 4\frac{V_p(\sin(n\delta)/n\pi)(\sin(m\delta)/m\pi)}{\sqrt{R^2+[\omega(n-1)L]^2}} \\
& \times \sin\left[(n-m-1)\omega t - (n-m)\gamma - \operatorname{atan}\left[\omega\frac{(n-1)L}{R}\right]\right]
\end{aligned} \tag{4.9}$$

Expression (4.9) gives the input line current and is given in the form as used in a Mathcad program. Note that $\gamma = \theta$ where θ is defined in Expression (4.3). The counter variable n appears twice so it is replaced once by m. The line current is given by Expression (4.9) and is displayed in Fig. 4.3.

4.3.4.1 Fundamental line current

Expression (4.9) gives the frequency of each component of the input current in terms of the two counter variables m and n as $n \pm m \pm 1$. The line fundamental current is given for those combinations of m and n that give 1 in the expression below

$$n \pm m \pm 1 = \pm 1$$

In this way five terms are derived to form Expression (4.10), the expression of the fundamental line current, $I_1(t)$.

$$
\begin{aligned}
I_1(t) = &\sum_{n=1}^{N} 4\left[\frac{V_\mathrm{p}(\sin(n\delta)/n\pi)(\sin(n\delta)/n\pi)}{\sqrt{R^2+[\omega(n+1)L]^2}}\right.\\
&\left.\times \sin\left[\omega t - \operatorname{atan}\left[\omega\frac{(n+1)L}{R}\right]\right]\right]\\
&+\sum_{n=1}^{N} 4\left[\frac{V_\mathrm{p}(\sin(n\delta)/n\pi)(\sin[(n+2)\delta]/(n+2)\pi)}{\sqrt{R^2+[\omega(n+1)L]^2}}\right.\\
&\left.\times \sin\left[-\omega t-(-2\gamma)-\operatorname{atan}\left[\omega\frac{(n+1)L}{R}\right]\right]\right]\\
&-4\frac{V_\mathrm{p}(\sin(\delta)/\pi)(\sin(\delta)/\pi)}{R}\sin(\omega t-2\gamma)\\
&-\sum_{n=1}^{N} 4\frac{V_\mathrm{p}(\sin(n\delta)/n\pi)(\sin(n\delta)/n\pi)}{\sqrt{R^2+[\omega(n-1)L]^2}}\\
&\times \sin\left[-\omega t-\operatorname{atan}\left[\omega\frac{(n-1)L}{R}\right]\right]\\
&-\sum_{n=3}^{N} 4\frac{V_\mathrm{p}(\sin(n\delta)/n\pi)(\sin[(n-2)\delta]/(n-2)\pi)}{\sqrt{R^2+[\omega(n-1)L]^2}}\\
&\times \sin\left[\omega t-2\gamma-\operatorname{atan}\left[\omega\frac{(n-1)L}{R}\right]\right]
\end{aligned}
\tag{4.10}
$$

Note that $\gamma = \theta$ where θ is defined in Expression (4.3).

4.3.5 *Identification of frequency components of the input current*

This is necessary in order to display the frequency spectrum of the input current. The frequency components of the line current can be extracted from Expression (4.9). In this Expression there are four terms and the frequency is given by the two counter variables m and n.

$$n \pm m \pm 1$$

A new counter variable is introduced; P represents the frequency of a single component

$$P = n \pm m \pm 1$$

There are four combinations for P

$P = n + m + 1$	for the first term of Expression (4.9)
$P = n - m + 1$	for the second term of Expression (4.9) for $n > m$
$P = n + m - 1$	for the third term of Expression (4.9)
$P = n - m - 1$	for the fourth term of Expression (4.9) for $n > m$

But P can be either positive or negative depending on the relative magnitudes of m and n; it is positive for $n > m$ and it is negative for $m > n$. In this way there are two more terms to accommodate the negative values of P; an extra term is created for the second term of Expression (4.9) and another one for the fourth term of the same expression.

The six combinations for m including the cases where $m > n$ are listed:

$-P = n - m + 1$	for the second term of Expression (4.9) for $m > n$
$-P = n - m - 1$	for the fourth term of Expression (4.9) for $m > n$

From these values of P, the variable m is expressed in terms of n and P as

$$\begin{aligned} m &= P - n - 1 \\ m &= P - n + 1 \\ m &= -P + n + 1 \\ m &= P + n + 1 \\ m &= -P + n - 1 \\ m &= P + n - 1 \end{aligned} \tag{4.11}$$

Expression (4.11) for m can be substituted in Expression (4.9) in its four terms accordingly. Two more are created for $n - m + 1$ and $n - m - 1$ to account for $m > n$. In this way the order of a single frequency component is easily identified. The counter variable P represents the frequency of a single component and n accounts for the contributions to that component from all the inter-modulations.

Expression (4.12), taken from a Mathcad program, is an example of the first term of the line current where m is replaced by $m = P - n - 1$. In the same way the rest of the terms of (4.11) are replaced in (4.9). Note that $\gamma = \theta$ where θ is defined in Expression (4.3).

$$\begin{aligned} I_{\text{in}_1} P(t) := \sum_{n=1}^{N} \sum_{P=n+2}^{N} 4 \frac{V_{\text{p}}(\sin(n\delta)/n\pi)(\sin[(P-n-1)\delta]/(P-n-1)\pi)}{\sqrt{R^2 + [\omega(n+1)L]^2}} \\ \times \sin\left[P\omega t - [n + (P - n - 1)]\gamma - \text{atan}\left[\omega\frac{(n+1)L}{R}\right]\right] \end{aligned} \tag{4.12}$$

The line current is then expressed as a function of P and n. In this way individual harmonics can be plotted against time, Fig. 4.4.

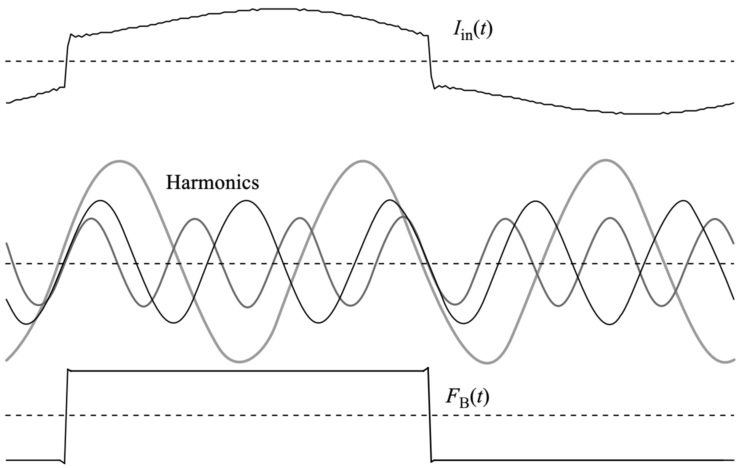


Figure 4.4 Individual line current harmonics

For large values of L where the output current is smoothed to an almost dc, I_{dc}, the input current is given by Expression (4.13). In this case the phase of the fundamental and the harmonics have a fixed relationship with the firing angle α.

$$I_o(t) = I_{dc}$$

From (4.2)

$$I_{in}(t) = F_B(t) I_{dc}$$

$$I_{in}(t) = 4I_{dc} \sum_{n=1}^{\infty} K_n \cos(n\omega t - \theta * n) \tag{4.13}$$

Hence the magnitude of any frequency component of $I_{in}(t)$ for large values of inductance is simply given by

$$4I_{dc}K_n \tag{4.14}$$

And the phase angle of the nth frequency component of $I_{in}(t)$ for large values of inductance is simply given by $\theta * n$. The phase angle between the fundamental component of current and the mains voltage is therefore θ. The fundamental is expressed as a cosine, Expression (4.13).

$$I_1(t) = 4I_{dc}K_1 \cos(\omega t - \theta)$$

The phase difference between $I_1(t)$ and the supply voltage can be shown from this expression to be α. Hence the Displacement Power Factor is given by:

$$\text{Displacement Power Factor} = \cos(\alpha) \tag{4.15}$$

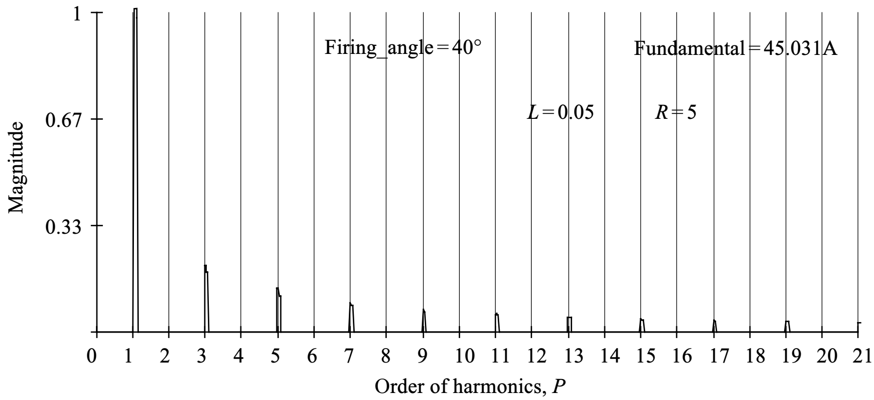


Figure 4.5 The frequency spectrum of the line current

The firing angle α alone sets the magnitude of the harmonic since θ is a function of α. The frequency spectrum, the displacement power factor, the distortion factor and the total harmonic distortion are displayed in Figs 4.5–4.7 for large value of L.

4.3.6 Frequency spectrum of line current harmonics

The frequency spectrum of the line current is displayed in Fig. 4.5 for a load of $L = 0.05$ H, $R = 5\ \Omega$ and $\alpha = 40°$ securing a good quality dc current at the output. The magnitude of each frequency component is divided by the magnitude of the fundamental component of current.

4.3.7 Distortion

This is quantified by two factors: the Total Harmonic Distortion, %THD and the Distortion Factor. The %THD is defined as the ratio of the rms value of the distortion current divided by the rms value of the fundamental current. The Distortion Factor is the ratio of the rms value of the fundamental current divided by the rms value of the total current. The Total Harmonic Distortion is given by Expression (4.16) where K sets the highest harmonic to be considered.

$$\%\mathrm{THD} = \frac{\sqrt{\sum_{P=3}^{K} I(P)}}{I_1} 100 \tag{4.16}$$

$I(P)$ is the rms magnitude of the Pth harmonic component; its order is set by P. I_1 is the rms value of the fundamental component. %THD for the ac to dc phase controlled converter is displayed in Fig. 4.6.

The Distortion Factor is given by Expression (4.17).

$$\text{Distortion factor} = \frac{I_1}{\sqrt{\sum_{P=1}^{K} I(P)}} \tag{4.17}$$

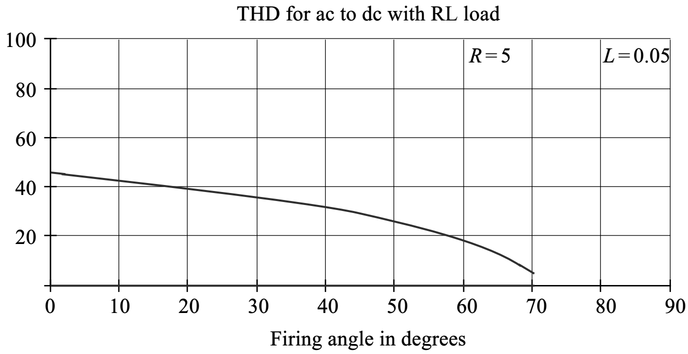


Figure 4.6 The total harmonic distortion

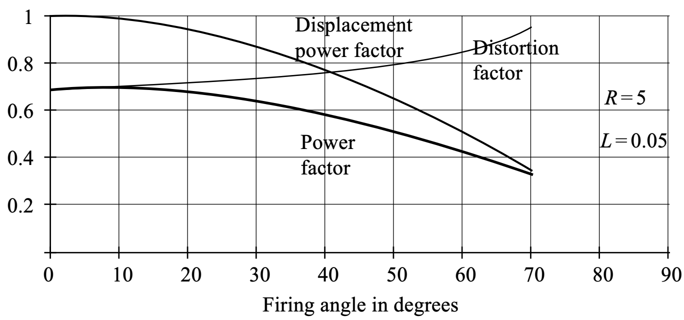


Figure 4.7 Displacement, power and distortion factors

$I(P)$ is the rms magnitude of the Pth harmonic component; its order is set by P. I_1 is the rms value of the fundamental component.

4.3.8 Power factor

The Power factor (PF) in power electronic systems is given by the product of the Distortion Factor and the Displacement Power Factor. The Displacement Power Factor is the cosine of the phase angle between the fundamental component of current and the mains voltage, Expression (4.15).

$$\mathrm{PF} = \cos(\theta)\frac{I_1}{\sqrt{\sum_{P=1}^{K} I(P)}} \tag{4.18}$$

The Distortion factor, Displacement factor and the Power factor for the ac to dc phase controlled converter are all displayed in Fig. 4.7.

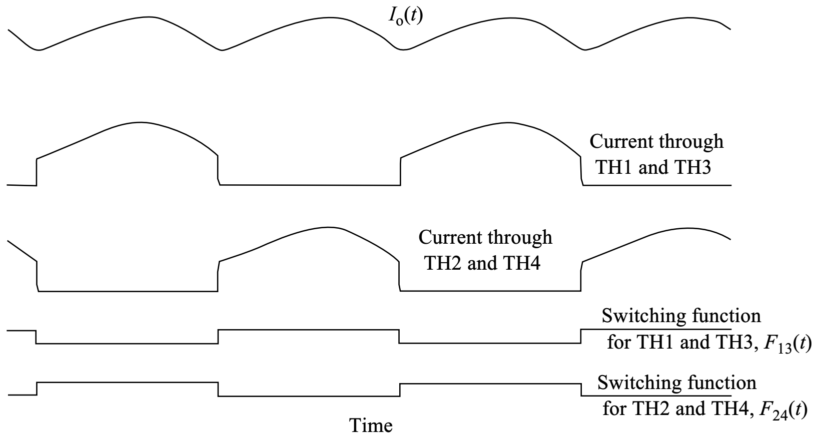


Figure 4.8 The current through the thyristors

4.3.9 Semiconductor current ratings

The required data for the current rating of the semiconductor devices of the circuit is readily available. The current through thyristors (Fig. 4.8) is given by

$$I_{\text{TH13}}(t) = F_{13}(t)I_o(t) \quad \text{and} \quad I_{\text{TH24}}(t) = F_{24}(t)I_o(t) \tag{4.19}$$

The switching functions $F_{13}(t)$ and $F_{24}(t)$ are unipolar functions with on periods equal to π for continuous conduction and switching frequency equal to the power frequency with $\pi/2$ phase delay. From Expressions (4.19) the rms, average, peak values and duty cycles for each group of thyristors are found.

$$F_{13}(t) := \frac{1}{2} + 2\sum_{n=1}^{200}\left(\frac{\sin(n\delta)}{n\pi}\right)(\cos(n\omega t - n\gamma))$$

$$F_{24}(t) := \frac{1}{2} + 2\sum_{n=1}^{200}\left(\frac{\sin(n\delta)}{n\pi}\right)(\cos(n\omega t - n\gamma - n\pi))$$

$$I_{13}(t) := F_{13}(t)I_o(t) \quad I_{24}(t) := F_{24}(t)I_o(t)$$

$$\text{AVE\_I}_{13} := \frac{1}{T}\int_0^T I_{13}(t)\mathrm{d}t \quad \text{AVE\_I}_{13} = 9.373$$

$$\text{RMS\_I}_{13} := \sqrt{\frac{1}{T}\int_0^T I_{13}(t)^2\mathrm{d}t} \quad \text{RMS\_I}_{13} = 13.577.$$

The average and rms values are derived for $R = 10\Omega$, $l = 0.05$, $\alpha = 30°$.

Chapter 5

The single phase full-wave diode rectifier – RC load

5.1 Introduction

The analysis of the rectifier with capacitive load is a classical case of a 'circuit determined' switching function. The switching instances are circuit determined and they must be known before the application of the technique.

The analysis is based on the input loop equation. The voltage at the input of the bridge is made up from two components each at a different time slot: the input supply voltage and the 'reflection' of the output dc voltage. For this purpose two switching functions are introduced, both circuit determined (Fig. 5.1).

The output voltage ripple is ignored when the line current is calculated; it makes the derivation simpler. This is justified for a practical system where large capacitors are used. The output ripple voltage itself is later calculated in terms of the output ripple current. The line current is derived as a function of the parameters of the two switching functions, the output dc voltage and the other circuit parameters.

The output dc voltage is calculated from the expression of the line current by considering the two points in time where it is zero.

5.2 Mathematical modelling

The mathematical modelling of this circuit is a good candidate for the procedure outlined in Chapter 1 for the application of the switching function technique. The analysis of this circuit is based on the input loop voltage expression. The input loop comprises the input voltage $V_{\text{in}}(t)$, the voltage across the source impedance, $V_{\text{L}}(t)$ and the voltage across the input of the bridge, $V_{\text{AB}}(t)$. These voltages must be mathematically described for one period of the driving frequency (mains frequency) in order to apply the Kirchoff's Law of voltages. The expression for the voltage at the input of the diode bridge, $V_{\text{AB}}(t)$, though is fragmented. It is made up from the

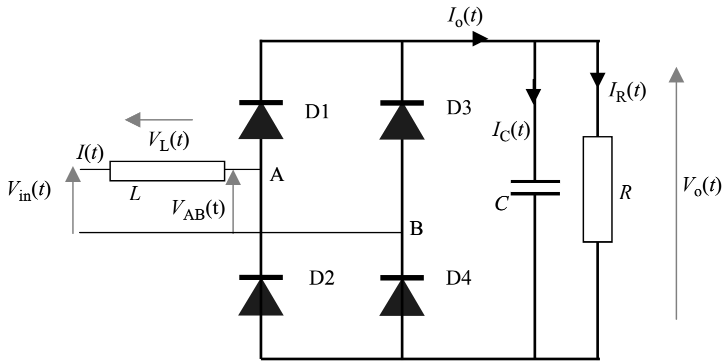


Figure 5.1 AC to DC diode converter: capacitive load

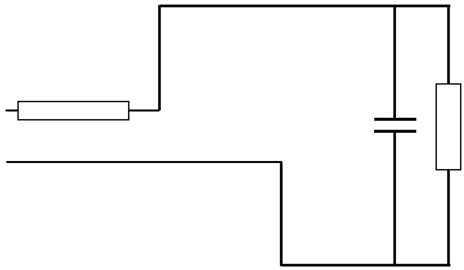
contributions of both the input voltage, $V_{in}(t)$, when no diode conducts and the output voltage, $V_o(t)$ when the diodes conduct. This demands the appropriate definitions of two switching functions, $F_B(t)$ and $F(t)$. The input loop voltage equation is finally solved for $V_L(t)$ and the line current $I(t)$ is derived by integrating $V_L(t)$. Solution of the input loop voltage equation demands knowledge of the output dc voltage. The dc value of the output voltage is calculated from the input loop equation by setting the input current to zero at t_1 or t_2, the instants at which this current is zero. The switching instances, t_1 and t_2 must be known for the analysis; they are provided from a real circuit, a simulation or by iteration.

Even though the contribution of the output voltage harmonics to the input current is neglected, considering the capacitor current derives the output voltage harmonics.

5.2.1 Operation and modes

The circuit undergoes three modes, Fig. 5.2. Two switching functions are required to describe the switching action of the circuit and the transfer from one mode to the other. A bipolar switching function, $F_B(t)$, describes the action of the bridge. A second unipolar switching function $F(t)$ represents the off periods of the bipolar switching function; it takes the value of one only, when $F_B(t) = 0$. This is in order to satisfy the basic requirement of the switching function method of analysis that all instances of the period of the circuit are represented by a switching function. The period of the circuit is discussed in Chapter 1 and in this circuit it is the period of the mains frequency.

During Mode I, Fig. 5.2(a) diodes D1 and D4 are conducting and the input current is flowing to the output. Hence, $F_B(t) = 1$ and $F(t) = 0$. The voltage appearing across the bridge input terminals $V_{AB}(t)$ is the output voltage, $V_o(t)$. The current at the output of the bridge, $I_o(t)$ is the input current, $I_{in}(t)$. The output current is shared between the load resistance R and the smoothing capacitor C. Actually it is the ac component of the output current which is shared between R and C. The dc component



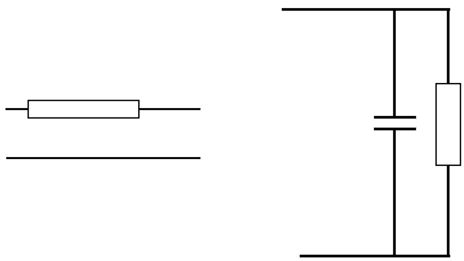
(a) Mode I: Diodes 1 & 4 are conducting.

$F_B(t) = 1$ and $F(t) = 0$

$V_{AB}(t) = V_o(t)$

$I_{in}(t) = I_o(t)$

$$I_C(t) = I_{oAC}(t) \frac{R}{\sqrt{R^2 + X_C(\omega n)^2}}$$



(b) Mode II: No diodes are conducting.

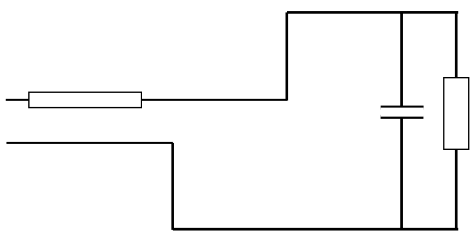
$F_B(t) = 0$ and $F(t) = 1$

$V_{AB}(t) = V_{in}(t)$

$I_{in}(t) = 0$

$I_o(t) = 0$

$I_C(t) =$ discharging current through R



(c) Mode III:
Diodes 2 and 3 are conducting.

$F_B(t) = -1$ and $F(t) = 0$

$V_{AB}(t) = -V_o(t)$

$I_{in}(t) = -I_o(t)$

$$I_C(t) = I_{oAC}(t) \frac{\mathrm{R}}{\sqrt{\mathrm{R}^2 + X_C(\omega n)^2}}$$

Figure 5.2 The modes of the rectifier with RC load

is flowing through R.

$$F_B(t) = 1 \quad \text{and} \quad F(t) = 0$$

$$V_{AB}(t) = V_o(t)$$

$$I_o(t) = I_{in}(t)$$

$$I_C(t) = I_{charging}(t)$$

During Mode II, Fig. 5.2(b) no diode is conducting so $F_B(t) = 0$ therefore $F(t) = 1$. The current at the output of the bridge, $I_o(t)$ is zero together with the input current, $I_{in}(t)$. The capacitor current is the discharging current through R. The voltage appearing across the bridge input terminals $V_{AB}(t)$ is the supply input voltage, $V_{in}(t)$.

$$F_B(t) = 0 \quad \text{and} \quad F(t) = 1$$

$$V_{AB}(t) = V_{in}(t)$$

$$I_o(t) = 0$$

$$I_{in}(t) = 0$$

$$I_C(t) = I_{discharging}(t)$$

During Mode III, Fig. 5.2(c) diodes D2 and D3 are conducting and the input current is flowing to the output but inverted. Hence $F_B(t) = -1$ and $F(t) = V$. The ac component of the output current is shared between the load resistance R and the smoothing capacitor C as in Mode I. The voltage appearing across the bridge input terminals $V_{AB}(t)$ is the output voltage, $V_o(t)$ inverted.

$$F_B(t) = -1 \quad \text{and} \quad F(t) = 0$$

$$V_{AB}(t) = -V_o(t)$$

$$I_o(t) = -I_{in}(t)$$

$$I_C(t) = I_{charging}(t)$$

There are two contributions of voltage at the input of the bridge.

(a) The output voltage $V_o(t)$ appears at the input of the bridge when the diodes are conducting, that is, during the on periods of $F_B(t)$, Modes I and III.
(b) The input voltage $V_{in}(t)$ appears at the input of the bridge when the diodes are not conducting, that is, during $F(t)$, Mode II.

Therefore

$$V_{AB}(t) = V_o(t)F_B(t) + V_{in}(t)F(t) \tag{5.1}$$

In the same way, there are two contributions to the capacitor current

$$I_C(t) = I_{charging}(t)F_B(t) + I_{discharging}(t)F(t) \tag{5.2}$$

The output current is a reflection of the input current via the transparent switching function $F_B(t)$

$$I_o(t) = I_{in}(t)F_B(t) \tag{5.3}$$

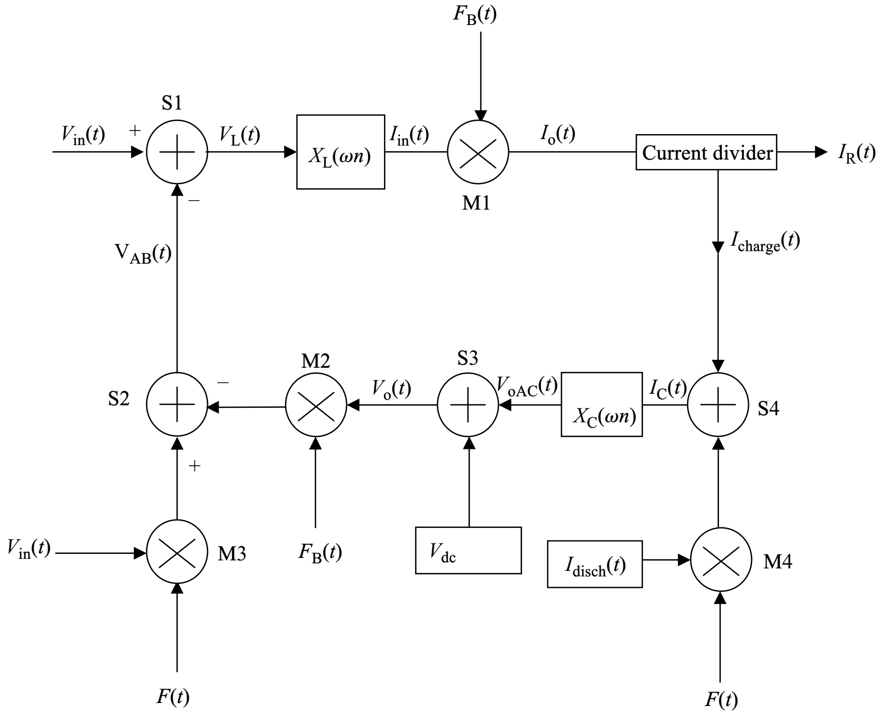


Figure 5.3 The mathematical model

This current is the current at the output of the bridge, $I_o(t)$ and it has a dc component, I_{dc} and an ac component $I_{o_{ripple}}(t)$

$$I_o(t) = I_{dc} + I_{oAC}(t) \tag{5.3a}$$

5.2.2 *The mathematical model*

The input loop voltage equation can now be written as

$$V_{in}(t) = V_L(t) + V_{AB}(t) \tag{5.4}$$

In the mathematical model (Fig. 5.3) the adder S1 represents Expression (5.4).

The inductor voltage $V_L(t)$ acts on the inductor reactance, $X(\omega n)$ to give the line current, $I_{in}(t)$.

$$I_{in}(t) = \frac{V_L(t)}{X_C(\omega n)} \tag{5.5}$$

The input current is reflected to the output via $F_B(t)$ (Modulator M1) according to Expression (5.3). The load represented by R and the smoothing capacitor, C, shares the ripple output current; the dc component, I_{dc}, flows through the resistance R.

$$I_{charging}(t) = I_{oAC}(t)\frac{R}{\sqrt{R^2 + X_C(\omega n)^2}}$$

The discharging current can be approximated to the dc value of the output current.

$$I_C(t) = I_{oAC}(t)\frac{R}{\sqrt{R^2 + X_C(\omega n)^2}} F_B(t) + I_{dc} F(t)$$

This Expression is shown to be simplified in Chapter 2 to

$$I_C(t) = I_{oAC}(t) - I_{dc} \tag{5.6}$$

The dc component is given by

$$I_{dc} = \frac{V_{dc}}{R} \tag{5.7}$$

V_{dc} is the output dc voltage.

5.2.3 *Definition of the switching functions*

The action of the semiconductor switches (diodes) of the bridge rectifier of Fig. 5.1 is described by the bipolar switching function $F_B(t)$ as shown in Fig. 5.4(a). The circuit itself dictates the switching instances, t_1 and t_2. One pair of diodes switch-on at t_1 when the input voltage is higher than the output dc voltage; this is the instant that they are forward biased. The diodes will switch-off at t_2 when the input voltage is lower than the output voltage. The other pair of diodes operates half a period later.

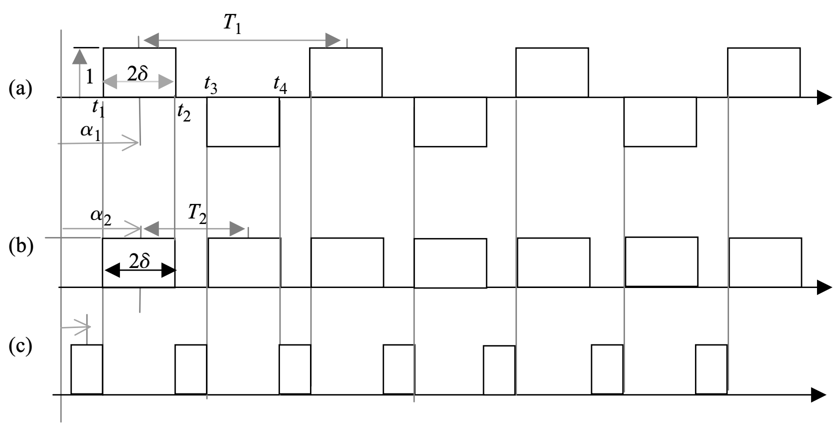


Figure 5.4 The switching function: (a) $F_B(t)$, (b) $F_I(t)$ and (c) $F(t)$

The switching function $F_B(t)$ describes the action of the diode bridge and it reflects input and output quantities to the other side. In this analysis it is used to reflect the output voltage to the input of the bridge and $I_{in}(t)$ to its output; it is a transparent switching function as far as the current on both sides of the converter is concerned because its dead periods coincide with the dead periods of the current, Fig. 5.5. The bipolar switching function $F_B(t)$ is given by Expression (5.8)

$$F_B(t) = 4\sum_{n=1}^{\infty} K_n \cos(n\omega t - n\alpha_1) \tag{5.8}$$

where

$n =$ odd integer
$K_n = \sin(n\delta)/n\pi$
$\alpha =$ phase displacement of $F_1(t)$ relative to the supply voltage
$\delta =$ half the on period of the diodes.

The on and off instances are dictated by the instantaneous values of input and output voltages. The switch on instant takes place when $V_{in}(t) > V_o(t)$ and the switch off instant takes place when $V_{in}(t) < V_o(t)$. Hence it is a hidden switching function instead of a forced one.

When the diodes are not conducting, the input voltage appears at the input of the bridge since no current flows through the source impedance. This component is derived by considering a second switching function, $F(t)$. This switching function is unipolar, it has only positive values, zero or one. It takes the value of one when $F_B(t)$ is zero. In order to derive $F(t)$ an intermediate switching function is defined, $F_I(t)$ that takes the value of one when $F_B(t)$ is 'not' zero, Fig. 5.4(b). $F_I(t)$ is therefore the inverse of $F(t)$ and the two switching functions are related by Expression (1.3) in Chapter 1.

$$F(t) = 1 - F_I(t)$$

The intermediate switching function $F_I(t)$ is derived graphically, Fig. 5.4 by raising every point of $F_B(t)$ to the square.

$$F_I(t) = F_B^2(t)$$

It is shown in Chapter 1 that the square of a bipolar switching function is given by Expression (1.8) which is repeated here.

$$F_I(t) = M_o + 2\sum_{m=1}^{\infty} M_n \cos(2m\omega t - m\alpha_2) \tag{1.8}$$

where

$m =$ an integer $1, 2, 3, \ldots$
$M_n = \sin(n\delta_I)/n\pi$

$\alpha_2 = 2\alpha_1$ the phase displacement of the switching function relative to the supply voltage (in radians)
$\delta_I = 2\delta$ half the on period of the diodes (in radians)
$M_o = 2\delta_1/2\pi$.

The wanted switching function, $F(t)$ is the inverse of the intermediate switching function $F_I(t)$.

$$F(t) = 1 - F_I(t) \tag{5.9}$$

5.3 Analysis

Expression (5.9) is substituted in Expression (5.1)

$$V_{AB}(t) = V_o(t)F_B(t) + V_{in}(t)[1 - F_I(t)]$$

Substituting in (5.4)

$$V_{in}(t) = V_L(t) + V_o(t)F_B(t) + V_{in}(t)[1 - F_I(t)]$$

The output voltage which appears across the capacitor has a dc component V_{dc} and an ac component $V_{oAC}(t)$

$$V_o(t) = V_{dc} + V_{oAC}(t)$$

Substituting for $V_o(t)$

$$V_{in}(t) = V_L(t) + [V_{dc} + V_{oAC}(t)]F_B(t) + V_{in}(t)[1 - F_I(t)]$$

Re-arranging and simplifying

$$V_{in}(t)F_I(t) = V_L(t) + V_{dc}F_B(t) + V_{oAC}(t)F_B(t)$$

The term $V_{oAC}(t)$ can be ignored for the derivation of the line current. It is too small compared with the dc value, V_{dc}. In this way the effect of the output voltage harmonics on the input current is ignored. Though the output ripple voltage is not ignored, it is derived later. Ignoring at the moment $V_{oAC}(t)$ leads to a simpler and quicker solution.

$$V_{in}(t)F(t) = V_L(t) + V_{dc}F_B(t)$$

Solve for the source inductance voltage, $V_L(t)$

$$V_L(t) = V_{in}(t)F(t) - V_{dc}F_B(t)$$

And replacing with the expressions for $F_{\mathrm{I}}(t)$ and $F_{\mathrm{B}}(t)$, (5.8), (1.8)

$$V_{\mathrm{L}}(t) = V_{\mathrm{p}}\sin(\omega t)\left[M_{\mathrm{o}} + 2\sum_{m=1}^{\infty} M_n \cos(2m\omega t - m\alpha_2)\right] - V_{\mathrm{dc}}4\sum_{n=1}^{\infty} K_n \cos(n\omega t - n\alpha_1)$$

Expanding

$$V_{\mathrm{L}}(t) = V_{\mathrm{p}}\sin\omega t M_{\mathrm{o}} + V_{\mathrm{p}}\sum_{m=1}^{\infty}\{M_m \sin((2m+1)\omega t - m\alpha_2) - M_m \sin((2m-1)\omega t - m\alpha_2)\} - V_{\mathrm{dc}}\,4\sum_{n=1}^{\infty} K_n \cos(n\omega t - n\alpha_1) \tag{5.10}$$

5.3.1 *Input line current*

The input current, $I_{\mathrm{in}}(t)$, is also the current through the series (source) inductance. This current is derived from $V_{\mathrm{L}}(t)$ according to Expression (5.5)

$$I_{\mathrm{in}}(t) = -\frac{M_{\mathrm{o}}}{\omega L}V_{\mathrm{p}}\cos\omega t - V_{\mathrm{p}}\sum_{m=1}^{\infty}\frac{1}{\omega L(2m+1)}M_m\cos((2m+1)\omega t - m\alpha_2) + V_{\mathrm{p}}\sum_{m=1}^{\infty}\frac{1}{\omega L(2m-1)}M_m\cos[(2m-1)\omega t - m\alpha_2] - V_{\mathrm{dc}}4\sum_{n=1}^{\infty}\frac{1}{\omega L n}K_n\sin(n\omega t - n\alpha_1) \tag{5.11}$$

5.3.2 *Output dc voltage V_{dc}*

The current through the inductance, $I_{\mathrm{in}}(t)$, Expression (5.11) is zero at four points in time during one mains cycle, at t_1, t_2, t_3 and t_4, Fig. 5.5. Hence by setting Expression (5.11) to zero for $t = t_1$, V_{dc} is calculated. This is done by using Mathcad. Of course, t_1 or t_2 or t_3 or t_4 must be known.

5.3.3 *Fundamental line current*

The line current contains the fundamental as well. The first element is clearly fundamental, the second has no fundamental, the third and fourth give a fundamental component for $m = 1$ and $n = 1$, correspondingly.

$$I_1(t) = -\frac{M_{\mathrm{o}}}{\omega L}V_{\mathrm{p}}\cos\omega t + V_{\mathrm{p}}\frac{1}{\omega L}M_1\cos[\omega t - \alpha_2] - V_{\mathrm{dc}}4\frac{1}{\omega L}K_1\sin(\omega t - \alpha_1) \tag{5.12}$$

5.3.4 Output current

The output current is given by Expression (5.3). Substituting for $F_B(t)$ and $I_{in}(t)$ gives:

$$
\begin{aligned}
I_o(t) = &-\frac{M_o}{\omega L}V_p 2\sum_{n=1}^{\infty} K_n[\cos((n+1)\omega t - n\alpha_1) + \cos((n-1)\omega t - n\alpha_1)] \\
&- 2V_p\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} M_m K_n \frac{1}{\omega L(2m+1)}[\cos((2m+n+1)\omega t - m\alpha_2 - n\alpha_1) \\
&+ \cos((2m-n+1)\omega t - m\alpha_2 + n\alpha_1)] \\
&+ 2V_p\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} K_n M_m \frac{1}{\omega L(2m-1)}[\cos((2m+n-1)\omega t - m\alpha_2 - n\alpha_1) \\
&+ \cos((2m-n-1)\omega t - m\alpha_2 + n\alpha_1)] \\
&- V_{dc} 8\sum_{x=1}^{\infty}\sum_{n=1}^{\infty} \frac{1}{\omega L n}[K_x K_n \sin((x+n)\omega t - x\alpha_1 - n\alpha_1) \\
&- \sin((x-n)\omega t - x\alpha_1 + n\alpha_1)] \qquad (5.13)
\end{aligned}
$$

$$K_x = \frac{\sin(x\delta)}{x\pi}$$

where x is an odd integer.

5.3.5 DC component at the output, I_{dc}

A dc component manifests itself in the expression for the output current for those terms for which the frequency is zero. In each term we look for this condition. In all terms this condition is satisfied in their second part. In the first term we have a dc component when $n = 1$, in the second term when $2m - n + 1 = 0$, in the third term when $2m - n - 1 = 0$. The fourth term gives no dc component because when $m - n = 0$, the magnitude of the term is zero as well.

$$
\begin{aligned}
I_{dc} = &-\frac{M_o}{\omega L}V_p 2K_1[\cos(-\alpha_1)] && \text{for } n = 1 \\
&- 2V_p\sum_{n=1}^{\infty} M_m K_{2m+1}\frac{1}{\omega L(2m+1)} \\
&\times [\cos(-m\alpha_2 + (2m+1)\alpha_1)] && \text{for } n = 2m+1 \\
&+ 2V_p\sum_{m=1}^{\infty} K_{2m-1} M_m \frac{1}{\omega L(2m-1)} \\
&\times [\cos(-m\alpha_2 + (2m-1)\alpha_1)] && \text{for } n = 2m-1 \qquad (5.14)
\end{aligned}
$$

The dc output current is also found by dividing the dc output voltage with R. The result for the dc component given by (5.14) can be compared with this value.

5.3.6 Output ripple voltage

The capacitor current is given by Expression (5.6) for a large value of C. Hence the output ripple voltage, $V_{\text{oAC}}(t)$ is given by

$$V_{\text{oAC}}(t) = I_{\text{C}}(t)\frac{1}{\omega pC\angle{-90^\circ}} \tag{5.15}$$

where p is the order of the harmonic current.

5.3.7 Identification of line current harmonics

For the display of the input current frequency spectrum and the derivation of %THD and distortion factor the magnitude of each individual harmonic is evaluated. This is done by introducing a new counter variable, p as we have done in Chapter 4.

$$P = 2m + 1 \quad \text{giving } m = (P - 1)/2$$

and

$$P = 2m - 1 \quad \text{giving } m = (P + 1)/2$$

This new variable, P represents the order of each component and replaces the coefficients of ω in the expression for the line current (5.11). The first term of Expression (5.11) is a fundamental component and no replacement is done. The second term contains $2m + 1$ and it is replaced by P; m is replaced by $(P - 1)/2$. The third term contains $2m - 1$ and it is replaced by P; m is replaced by $(P + 1)/2$. The fourth term contains n and it is simply replaced by P. The resulting expression is (5.16). The first and third terms produce fundamental and all the odd harmonics, the second term does not produce a fundamental component but all the odd harmonics starting from the third. The fourth term produces only a fundamental component. Each component is expanded to sine and cosine components, all sine and cosine components of all terms are collected together separately and the magnitude is found by Pythagoras for each harmonic. Based on that, the frequency spectrum and the distortion factors are derived. The input current expressed as a function of P is given as $I_{\text{LINE}}(t)$, Expression (5.16); it is an extract from a Mathcad program.

$$
\begin{aligned}
I_{\mathrm{LINE}}(t) = & -\frac{M_0}{wL} V_{\mathrm{p}} \cos(\omega t) - V_{\mathrm{p}} \sum_{P=3}^{N} \frac{\sin(((P-1)/2)\delta_1)}{((P-1)/2)\pi} \frac{1}{wPL} \\
& \times \cos\left(wPt - \frac{P-1}{2}\alpha_2\right) \\
& + V_{\mathrm{p}} \sum_{P=1}^{N} \frac{\sin(((P+1)/2)\delta_1)}{((P+1)/2)\pi} \\
& \times \left(\frac{1}{wPL} \cos\left(wPt - \frac{P+1}{2}\alpha_2\right)\right) \\
& - 4V_{\mathrm{dc}} \sum_{P=1}^{N} \frac{\sin(P\delta)}{P\pi} \\
& \times \left(\frac{1}{wPL} \sin(wPt - P\alpha_1)\right)
\end{aligned} \tag{5.16}
$$

where P is an odd integer and N is the number of harmonics, theoretically ∞.

5.3.8 *Displacement power factor*

This is the phase delay, φ of the fundamental component. Expression (5.12) of the fundamental line current is expanded and from the in-phase and quadrature components the delay phase angle is found.

$$
\begin{aligned}
\varphi := & \operatorname{atan}[((4V_{\mathrm{dc}}/wL)(\sin(\delta)/\pi)\sin(\alpha_1) \\
& + (\sin(\delta_1)/wL\pi)V_{\mathrm{p}}\cos(\alpha_2) - (M_0/wL)V_{\mathrm{p}})] \\
& \times [((\sin(\delta_1)/wL\pi)V_{\mathrm{p}}\sin(\alpha_2) \\
& - (4V_{\mathrm{dc}}/wL)(\sin(\delta)/\pi)\cos(\alpha_1))]^{-1}
\end{aligned} \tag{5.17}
$$

5.3.9 *Input current distortion*

The total harmonic distortion and the distortion factor are both derived in the same way as in Chapter 4 and for source inductance 4 mH, $C = 10{,}000\ \mu\mathrm{F}$ and $R = 40\ \Omega$:

%THD $= 0.609$

Distortion factor $= 0.621$

5.3.10 *Display of waveforms*

The input current and voltage of a single phase rectifier with capacitive load are displayed in Fig. 5.5 together with the bipolar switching function – $F_{\mathrm{B}}(t)$ – associated with this circuit. The current is discontinuous and it contains all the odd harmonics as

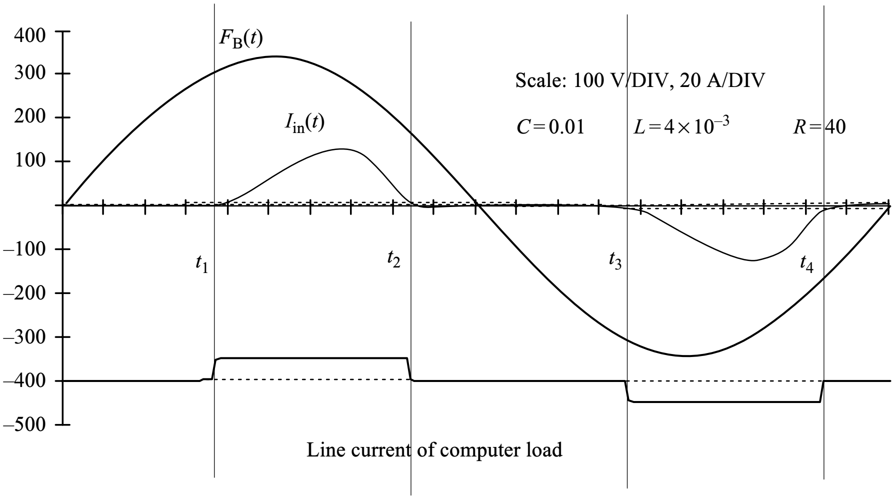


Figure 5.5 Input voltage and current and the bipolar switching function (lower trace)

indicated by the frequency spectrum, Fig. 5.6. Therefore the current can be described by the general Expression (5.18). The parameters of this expression – θ_k, φ, I_k, I_1 – can be derived tediously from Expressions (5.11).

$$I(t) = I_1 \sin(\omega t - \varphi) + \sum_{3}^{\infty} I_k \cos(k\omega t - \theta_k) \tag{5.18}$$

where I_1 is the peak value of the fundamental, φ is the phase difference between the mains voltage and the fundamental component of the current, I_k is the peak value of the kth component of harmonic current, θ_k is the phase delay of the kth component of harmonic current and k is an odd integer.

As indicated in Fig. 5.6, the most prominent harmonic is the third which is about 60% of the fundamental. This can lead to serious problems in three phase systems as explained in the next section. Figure 5.7 is a display of the current at the output of the bridge, the output voltage and the input line current.

5.4 Neutral current in three phase systems

5.4.1 Introduction

The current in the neutral wire in an electrical installation, where there are no power electronic converters, can be made to be zero by balancing the loads per phase. This is not always possible when a power electronics converter processes the power before it is supplied to the load. So we can expect high neutral conductor currents. As it happens, the geometry of the distorted phase currents is such as to cancel out the

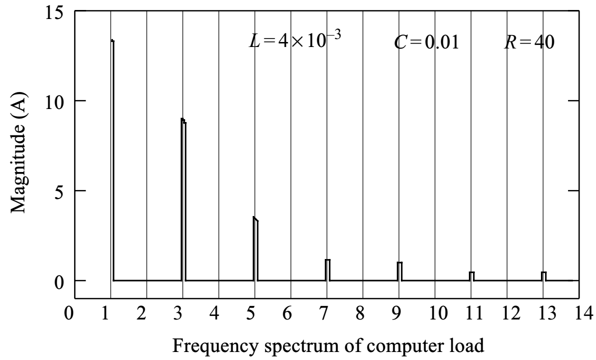


Figure 5.6 The frequency spectrum of the input current

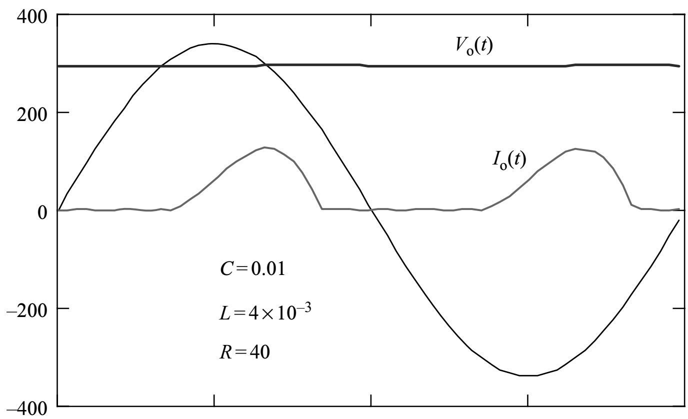


Figure 5.7 Output voltage (top trace), output current and input voltage

fundamental and the non-triplen harmonics if the loads are 'balanced'. The triplen harmonics – third, ninth etc. – are not cancelled out, they add up. The strongest harmonic of the line current is the third and we can expect the current of this harmonic to be three times in the neutral wire. This is a fact that cannot be ignored. Fires are reported due to overloading of the neutral conductor in computer loads. Typical loads of this type are computer loads where the power supply consists of a simple rectifier and a large smoothing capacitor; a circuit which is adequately represented by Fig. 5.1.

5.4.2 Analysis

The line current of a computer load, is shown in the previous section to be in the form

$$I(t) = I_1 \sin(\omega t - \varphi) + \sum_{3}^{\infty} I_k \cos(k\omega t - \theta_k) \tag{5.18}$$

It is possible to have a lot of computer loads in large buildings distributed among the three phases in a star configuration. In this case the three line currents are given by:

$$I_R(t) = I_{1R}\sin(\omega t - \varphi_R) + \sum_3^{\infty} I_k \cos(k\omega t - \theta_{Rk})$$

$$I_Y(t) = I_{1Y}\sin(\omega t - \varphi_Y - 120^\circ) + \sum_3^{\infty} I_k \cos(k\omega t - \theta_{Yk} - 120^\circ k)$$

$$I_B(t) = I_{1B}\sin(\omega t - \varphi_B - 240^\circ) + \sum_3^{\infty} I_k \cos(k\omega t - \theta_{Bk} - 240^\circ k) \qquad (5.19)$$

5.4.3 Neutral current

The neutral current is the vectorial addition of the three phase currents.

$$I_N(t) = I_R(t) + I_Y(t) + I_B(t)$$

In order to minimise the current through the neutral line the phase loads are usually balanced. For a balanced load we have

$$|I_R| = |I_Y| = |I_B| \quad \text{and} \quad \varphi_R = \varphi_Y = \varphi_B \qquad (5.20)$$

It will be shown now that it is possible to eliminate the fundamental and non-triplen harmonics from the neutral; the third, ninth and the other triplen harmonics unfortunately add up.

5.4.4 Fundamental current in the neutral wire

The fundamental current in the neutral is given by

$$\begin{aligned} I_{1N}(t) = {} & I_{1R}\sin(\omega t - \varphi_R) + I_{1Y}\sin(\omega t - \varphi_Y - 120^\circ) \\ & + I_{1B}\sin(\omega t - \varphi_B - 240^\circ) \end{aligned}$$

The three vectors of neutral current are shown in Fig. 5.8. For a balanced load Expression (5.20) applies. The three vectors are equal in magnitude and phase displaced by 120°. The resultant is zero. Hence no fundamental (50 Hz) current flows in the neutral.

$$I_{1N}(t) = 0$$

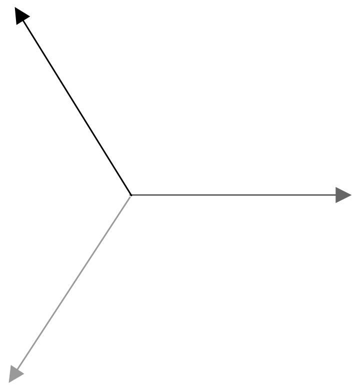


Figure 5.8 Fundamental current in the neutral wire: resultant is zero

5.4.5 Harmonic current in the neutral line

The harmonic current in the neutral line is derived from Expression (5.19)

$$I_{\mathrm{H}}(t) = \sum_{3}^{\infty} I_k \cos(k\omega t - \theta_{\mathrm{R}k}) + \sum_{3}^{\infty} I_k \cos(k\omega t - \theta_{\mathrm{Y}k} - 120^\circ k) + \sum_{3}^{\infty} I_k \cos(k\omega t - \theta_{\mathrm{B}k} - 240^\circ k)$$

A careful study of the above expression reveals that for $k = 3, 9, 15, 21, \ldots$ all the three components are in phase and they add up to give a neutral harmonic current at three times the phase current. The most serious problem is the third harmonic because as it is shown in the previous section, its value is more than half the fundamental. This means that for a 100 A load, the neutral current can be more than 150 A.

5.4.5.1 Third harmonic neutral current, 150 Hz

$$I_{3\mathrm{N}}(t) = I_3 \cos(3\omega t - \theta_{\mathrm{R}3}) + I_3 \cos(3\omega t - \theta_{\mathrm{Y}3} - 120^\circ \times 3) + I_3 \cos(3\omega t - \theta_{\mathrm{B}3} - 240^\circ \times 3)$$

$$I_{3\mathrm{N}}(t) = I_3 \cos(3\omega t - \theta_{\mathrm{R}3}) + I_3 \cos(3\omega t - \theta_{\mathrm{Y}3} - 360^\circ) + I_3 \cos(3\omega t - \theta_{\mathrm{B}3} - 720^\circ)$$

$$I_{3\mathrm{N}}(t) = I_3 \cos(3\omega t) + I_3 \cos(3\omega t - \theta_{\mathrm{Y}3}) + I_3 \cos(3\omega t - \theta_{\mathrm{B}3})$$

For a balanced load

$$\Theta_{y3} = \Theta_{R3} - 120^\circ \times 3 \quad \text{and} \quad \Theta_{B3} = \Theta_{R3} - 240^\circ \times 3$$

Figure 5.9 Third harmonic current in the neutral conductor

The three add up to give three times the line third harmonic current in the neutral (Fig. 5.9).

$$I_{3\mathrm{N}}(t) = 3I_3 \cos(3\omega t)$$

5.4.6 Discussion

The switching function technique was applied to analyse the diode rectifier with RC load. The switching instances of the diodes were known from other sources and the technique was applied in a straightforward manner. When the switching instances are not known, something to be expected, a simulation or a real circuit might be used. Alternatively the analysis presented here can be extended in the following way. Certain parameters of the circuit such as the dc output voltage and the dc output current are derived from different starting points and different data but they are related by Ohm's Law. Hence it might be possible to start from a set of switching instances and change them until the calculated values of the dc voltage and current converge.

The various expressions for voltage and currents presented in this chapter are tested and used to derive the various waveforms, (see CD with this book). The reader must be careful to use 'n' as an odd integer and 'm' as integer.

Chapter 6

The three-phase half-wave phase controlled converter

6.1 Introduction

Two cases are examined here: resistive load and inductive load. In the latter case continuous and discontinuous conduction, with and without a free-wheeling diode are considered. Triggering of the thyristor is effected by supplying a positive train of pulses to their gates. Only one thyristor is conducting at a time and it remains conducting until the voltage across it is reversed or the current through it becomes zero. The effect of overlap for the three-phase half-wave phase controlled converter is not examined. Overlap is examined in the next chapter for the full wave system. Its findings can easily be adopted for the half-wave converter.

The delay-firing angle α for the red phase is measured in the usual way from the point of the first crossing of the blue and red phases; hence a fixed delay of 30° always exists for α with reference to the red line. The delay angle for the other two phases is measured in a similar way. The switching function of these circuits is a single pulse, which is repeated for every mains cycle. There are three switching functions, one for each phase, displaced by 120°.

6.2 Mathematical modelling of the three-phase half-wave phase controlled converter

The circuit diagram of a half-wave three-phase controlled rectifier with resistive or inductive load is shown in Fig. 6.1. There is one mathematical model valid for continuous and discontinuous conduction including the presence of a free wheeling diode. In each case though the parameters of the switching function, half-pulse width δ and phase angle θ, are different. It will be shown in the next chapter that the same mathematical model applies for the full-wave thyristor controlled rectifier; again it is the parameters of the switching function that change.

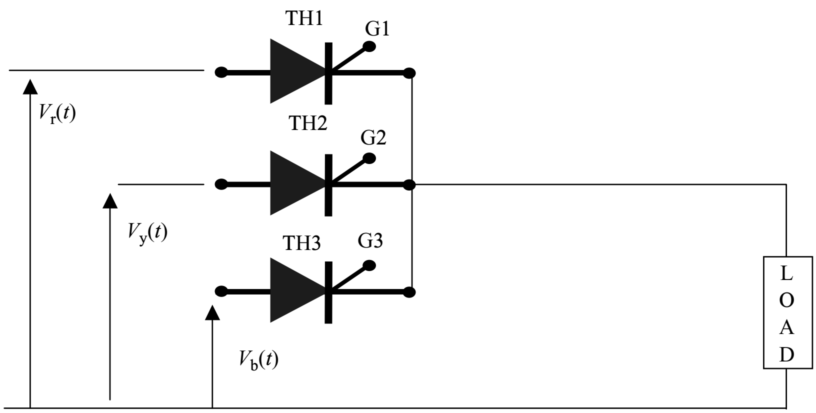


Figure 6.1 The three-phase half-wave phase controlled converter

The modes of the circuit are three for continuous conduction and four for discontinuous conduction. Continuous conduction takes place if the firing angle α is $<30°$ irrespective of the load. Discontinuous conduction might take place for $\alpha > 30°$ if the inductance is not adequate to sustain the current until the next thyristor is fired.

The presence of the free wheeling diode limits the conduction period of the line current from the firing point to the end of the half-cycle. In this case the modes are four.

The derivation of the modes leads to the definition of the time intervals over which each mode exists. This is useful data in order to set the parameters of the switching function, half pulse width δ and phase angle θ, in each case. Furthermore, the mathematical model is derived from the study of the modes.

The mathematical model dictates the analysis of the circuit. Expressions of the output voltage and current are derived. In the mathematical model it is shown that the line current, is a reflection of the output current to the input and implies simple multiplication with the appropriate switching function. The presence of a large load inductance almost eliminates the output current harmonics and the output current smooths almost to its dc value. This makes the calculation for the line current, the power factor and the total harmonic distortion simpler.

In each case though the parameters of the switching function (half-pulse width δ and phase angle θ) are different. It will be shown in the next chapter that the same mathematical model applies for the full-wave thyristor controlled rectifier; again it is the parameters of the switching function that change.

6.2.1 The modes and operation of the circuit

The three-phase half-wave phase controlled converter is shown in Fig. 6.1. Thyristor TH1 is forward biased at the instant that the blue and red phases first cross, 30° from the reference point. It remains forward biased for the next 150° unless TH2

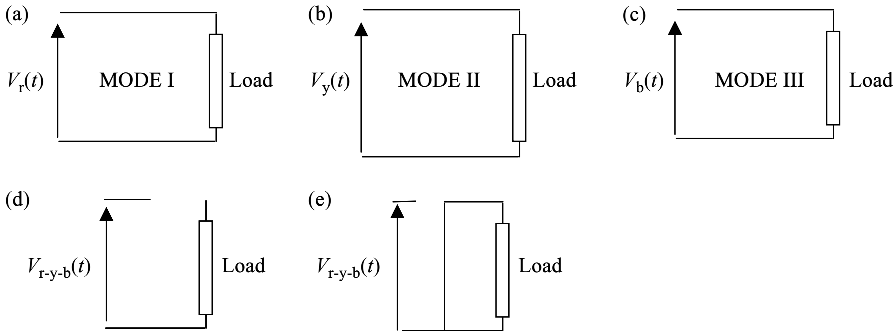


Figure 6.2 The modes of the three-phase half-wave controlled rectifier: (a) Mode I: thyristor TH1 is connecting the red line to the load, (b) Mode II: thyristor TH2 is connecting the yellow line to the load, (c) Mode III: thyristor TH3 is connecting the blue line to the load, (d) Mode I/II: discontinuous conduction with, R–L load with inadequate inductance or purely resistive load, no free-wheeling diode and (e) Mode I/IID: R–L load with free-wheeling diode

is fired at 120° or later. If it is fired within that period it will conduct. With TH1 conducting, the circuit enters Mode I, as shown in Fig. 6.2(a). During Mode I the input voltage $V_{\mathrm{r}}(t)$ appears at the output, hence

$$V_{\mathrm{o}}(t) = V_{\mathrm{r}}(t)$$

The relation of the output and input voltage of a single switch operated by a switching function is given by Expressions (2.1), Chapter 2. A switching function $F_{\mathrm{r}}(t)$ is introduced which describes the operation of thyristor TH1.

$$V_{\mathrm{o}}(t) = F_{\mathrm{r}}(t)V_{\mathrm{r}}(t)$$

Thyristor TH2 is forward biased at 150° and it will conduct if it is fired. It will be fired though only with the same delay as TH1 was fired at $\omega t = \alpha$, measured from the first crossing of red and yellow phase voltages. The delay-firing angle α for the red phase is measured from the point of the first crossing of the blue and red phases, hence a fixed delay of 30° always exists with reference to the red line. In the same way for the yellow phase the delay-firing angle α is measured from the first crossing of the red and yellow. Once TH2 is fired, Mode II is entered, Fig. 6.2(b). During Mode II the input voltage $V_{\mathrm{y}}(t)$ appears at the output, hence

$$V_{\mathrm{o}}(t) = V_{\mathrm{y}}(t)$$

The relation of the output and input voltage of a single switch operated by a switching function is given by Expressions (2.1), Chapter 2. A switching function $F_{\mathrm{y}}(t)$ is introduced.

$$V_{\mathrm{o}}(t) = F_{\mathrm{y}}(t)V_{\mathrm{y}}(t)$$

Before TH2 is fired though, the current might become zero and the circuit enters Mode I/II, Fig. 6.2(d), which is the discontinuous mode. Mode I/II takes place for a resistive load if $\alpha > 30°$. In this case the conducting thyristor (TH1) will be reverse biased by its own voltage $V_r(t)$ at 180° and it will commutate. Hence a dead period exists between 180° and $150° + \alpha$. Discontinuous conduction is also possible if the inductance is not adequate to maintain the current for the above period. In this case the extinction angle β has to be calculated in order to define the switching function. During Mode I/II the no input voltage appears at the output, hence

$$V_o(t) = 0$$

During Mode II the voltage of the yellow phase, $V_y(t)$ is transferred to the cathodes of both TH1 and TH3 thus reverse biasing them. TH3 will be forward biased at 270° and it will be fired at $270° + \alpha$. This is Mode III, Fig. 6.2(c), and $V_b(t)$ is transferred at the cathodes of TH1 and TH2, thus keeping them reverse biased.

During Mode III the input voltage $V_b(t)$ appears at the output, hence

$$V_o(t) = V_b(t)$$

The relation of the output and input voltage of a single switch operated by a switching function is given by Expressions (2.1), Chapter 2. A switching function $F_b(t)$ is introduced

$$V_o(t) = F_b(t)V_b(t)$$

Before TH3 is fired, the circuit might enter the discontinuous mode, Mode I/II again for a resistive load or not adequate inductance. For a resistive load, this mode will last from 300° to $270° + \alpha$. In this case the conducting thyristor (TH2) will be reverse biased by its own voltage $V_y(t)$ at 300° and it will commutate (switch-off). Hence a dead period exists between 300° and $270° + \alpha$. TH3 is conducting from $270° + \alpha$ until 420° for a resistive load and α larger than 30°. For an inductive load with adequate inductance for continuous conduction it will carry on until TH1 is fired. If the inductance is not adequate to sustain the current until the next thyristor is fired, the current will be extinct before the next thyristor is fired. This is the discontinuous operation of the circuit. The extinction angle, β has to be calculated in order to define the switching function.

Table 6.1 contains the relevant timing information for continuous conduction; that includes both resistive load at $\alpha < 30°$ and resistive-inductive load with adequate inductance. Table 6.2 contains the timing information for a purely resistive load and Table 6.3 contains the relevant timing information for discontinuous conduction due to inadequate inductance in the load. The circuit will operate with discontinuous current if the conduction period is less than 120°.

A free wheeling diode is sometimes included across the load. This will limit the conduction of the thyristors up to the end of each positive phase half-cycle. A new Mode is now entered, Mode I/II/D to replace Mode I/II, Fig. 6.2(e) and Table 6.4 contains the relevant timing information which is the same as for the resistive load. The output voltage, $V_o(t)$ during Mode I/II/D is zero since the output is short-circuit

Table 6.1 Continuous conduction

Conducting thyristor	Mode	Conduction
TH1 red phase	I	$30^\circ + \alpha$ to $150^\circ + \alpha$
TH2 yellow phase	II	$150^\circ + \alpha$ to $270^\circ + \alpha$
TH3 blue phase	III	$270^\circ + \alpha$ to $390^\circ + \alpha$

Table 6.2 Discontinuous conduction resistive load with firing angle α higher than 30°

Conducting thyristor	Mode	Conduction
TH1 red phase	I	$30^\circ + \alpha$ to 180°
None	I/II	180° to $150^\circ + \alpha$
TH2 yellow phase	II	$150^\circ + \alpha$ to 300°
None	I/II	300° to $270^\circ + \alpha$
TH3 blue phase	III	$270^\circ + \alpha$ to 420°
None	I/II	420° to $390^\circ + \alpha$

Table 6.3 Discontinuous conduction, RL load with inadequate inductance ($\alpha > 30^\circ$)

Conducting thyristor	Mode	Conduction
TH1 red phase	I	$30^\circ + \alpha$ to $180^\circ + \beta$
None	I/II	$180^\circ + \beta$ to $150^\circ + \alpha$
TH2 yellow phase	II	$150^\circ + \alpha$ to $300^\circ + \beta$
None	I/II	$300^\circ + \beta$ to $270^\circ + \alpha$
TH3 blue phase	III	$270^\circ + \alpha$ to $420^\circ + \beta$
None	I/II	$420^\circ + \beta$ to $390^\circ + \alpha$

β is the current extinction angle.

by the free-wheeling diode.

$$V_o(t) = 0$$

The parameters of the switching functions $F_r(t)$, $F_y(t)$ and $F_b(t)$ are modified for free-wheeling operation as shown in Table 6.4.

Table 6.4 RL load with free-wheeling diode

Conducting thyristor	Mode	Conduction
TH1 red phase	I	$30° + \alpha$ to $180°$
Diode	I/II	$180°$ to $150° + \alpha$
TH2 yellow phase	II	$150° + \alpha$ to $300°$
Diode	I/II	$300°$ to $270° + \alpha$
TH3 blue phase	III	$270° + \alpha$ to $420°$
Diode	I/II	$420°$ to $390° + \alpha$

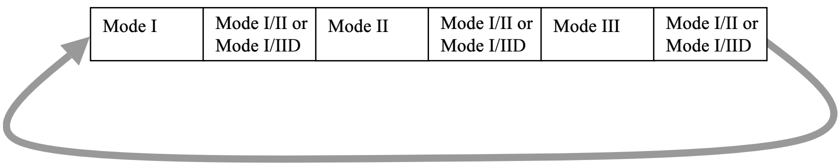


Figure 6.3 Mode sequence for the three-phase half-wave phase controlled converter

The information of these four tables is directly related to the appropriate switching functions.

6.2.2 Mode sequence

Figure 6.3 shows the mode sequence for the three-phase half-wave phase controlled converter. Mode I/II or Mode I/IID are omitted for continuous conduction, Mode I/II replaces Mode I/IID for discontinuous conduction and Mode I/IID replaces Mode I/II for free-wheeling action.

6.2.3 The mathematical model

It follows from the four modes of the circuit that:

(i) The output voltage is made up from the contributions of the three phases. Each contribution is time displaced as described by the Tables 6.1–6.4. Appropriate switching functions must be defined.
(ii) The output current is the result of the output voltage acting on the load.
(iii) The output current is diverted from line to line. Appropriate switching functions must be defined.

These facts are represented in the model of Fig. 6.4. The circuit topology suggests that the output voltage is made up from the contributions of all three phases. This

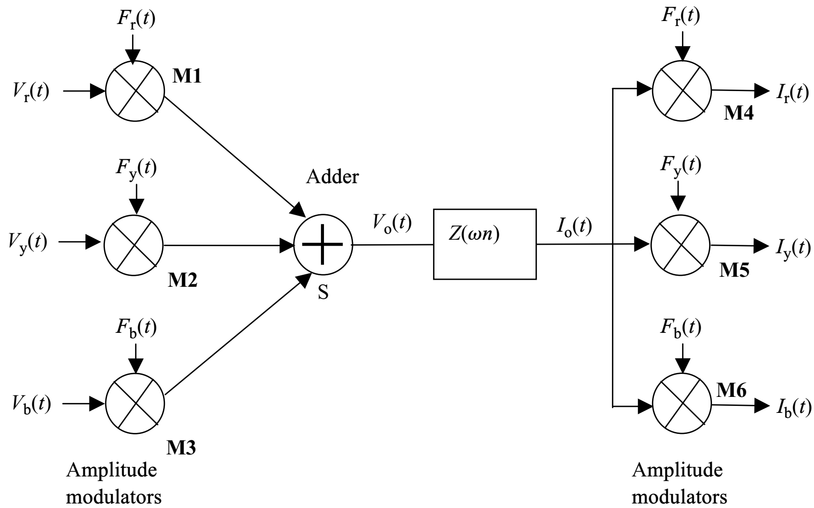


Figure 6.4 The steady-state mathematical model

is an application of the superposition theorem in switched circuits as discussed in Chapter 2, Expression (2.8). The voltage at the output is given by

$$V_{AB}(t) = \sum_{n=1}^{N} F_n(t) V_n(t) \tag{6.1}$$

There are three branches, hence $N = 3$. $V_n(t)$ represents the three phase voltages, $V_r(t)$, $V_y(t)$ and $V_b(t)$. We need to define the three switching functions $F_r(t)$, $F_y(t)$ and $F_b(t)$. The three modulation processes and the summation of their output are represented by the modulators M1, M2 and M3 and the adder S, Fig. 6.4.

Hence the output voltage is derived from Expression (6.1) as

$$V_o(t) = V_r(t)F_r(t) + V_y(t)F_y(t) + V_b(t)F_b(t) \tag{6.2}$$

The output voltage $V_o(t)$, is pushing a current $I_o(t)$, into the load.

$$I_o(t) = \frac{V_o(t)}{Z(\omega n)} \tag{6.3}$$

The output current is reflected back to the input and the current in each of the three input lines is the result of amplitude modulation (modulators M4, M5 and M6) with the appropriate switching functions. This is in accordance to the Kirchoff's law of current as applied to switched circuits, Chapter 2, Expression (2.6).

$$I_r(t) = F_r(t)I_o(t) \quad I_y(t) = F_y(t)I_o(t) \quad I_b(t) = F_b(t)I_o(t) \tag{6.4}$$

6.2.4 The switching functions

In order to describe mathematically any switching function, we need four pieces of information.

(i) The type of the switching function, *unipolar* type or *bipolar* type.
(ii) The switching frequency.
(iii) The pulse duration, δ.
(iv) The phase angle, θ.

All the information needed to express the switching functions mathematically is found in Tables 6.1–6.4. The switching functions are of the 'unipolar' type as no inversion of the input voltage takes place at the output. The switching frequency is the same as the mains frequency and its timing details are derived from the tables. Hence the general form of the switching functions is:

$$F(t) = K_o + 2\sum_{n=1}^{\infty} K_n \cos(n\omega t - \theta * n) \tag{6.5}$$

where

n is an integer number
K_o = duty cycle of the switch
$K_n = \sin(n\delta)/\pi n$
δ = half the on period of the switch
θ = the phase angle of the switching function relative to the red phase.

The phase angle θ is the sum of the inherent 30° delay of the three-phase system, the firing angle α and δ. This is true for all four cases: for both resistive and inductive loads, continuous and discontinuous conduction, with and without a free-wheeling diode

$$\theta = 30^\circ + \alpha + \delta \tag{6.6}$$

The half-width δ has to be defined for each of the four cases of continuous and discontinuous conduction. For continuous conduction, the duration of the on period of the switching function is 120° hence

$$\delta = 60^\circ$$

For discontinuous conduction due to resistive load and firing angle α higher than 30°, the duration of the on period of the switching function is $120^\circ - \alpha$ hence

$$\delta = (120^\circ - \alpha)/2$$

For discontinuous conduction with extinction angle β due to inadequate inductance in the load and firing angle α higher than 30°, the duration of the on period of the

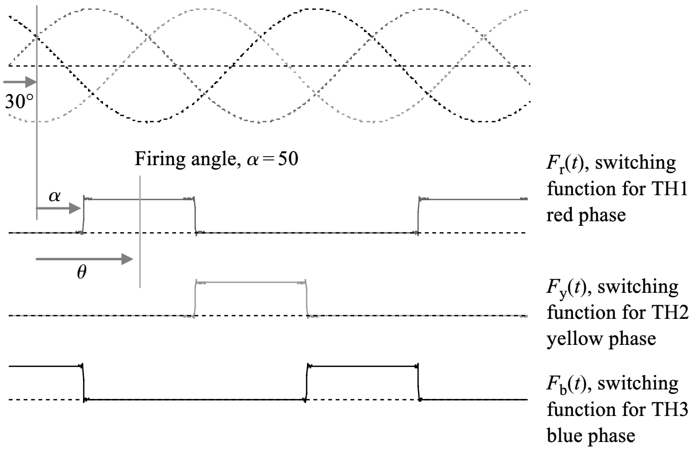


Figure 6.5 *The switching functions*

switching function is $120° - \alpha + \beta$ hence

$$\delta = (120° - \alpha + \beta)/2$$

For the circuit with the free-wheeling diode, the conduction period is the same as for the resistive load. Furthermore there is 120° phase difference between the three phases (Fig. 6.5) in all cases.

$$F_r(t) = K_o + 2\sum_{n=1}^{\infty} K_n \cos(n\omega t - n\theta) \tag{6.7a}$$

$$F_y(t) = K_o + 2\sum_{n=1}^{\infty} K_n \cos(n\omega t - n\theta - n120°) \tag{6.7b}$$

$$F_b(t) = K_o + 2\sum_{n=1}^{\infty} K_n \cos(n\omega t - n\theta - n240°) \tag{6.7c}$$

6.3 Analysis

Two types of loads for the half-wave three-phase-controlled rectifiers are presented here: the resistive load and the inductive load. The model shown in Fig. 6.5 represents all cases. The difference lies on the pulse width and the phase displacement of the switching functions.

The modes for the four cases are the same as well. Mode I/II is valid for discontinuous conduction that is, resistive load with α larger than 30°, and RL load where the inductance is not adequate to maintain continuous conduction.

6.3.1 The output voltage

From the mathematical model, the output voltage is given by

$$V_o(t) = F_r(t)V_r(t) + F_y(t)V_y(t) + F_b(t)V_b(t)$$

where

$$\begin{aligned} V_r(t) &= V_p \sin(\omega t) \\ V_y(t) &= V_p \sin(\omega t - 120^\circ) \\ V_b(t) &= V_p \sin(\omega t - 240^\circ) \end{aligned}$$

$$\begin{aligned} V_o(t) = {} & K_o V_p \sin(\omega t) + K_o V_p \sin(\omega t - 120^\circ) + K_o V_p \sin(\omega t - 240^\circ) \\ & + V_p \sin(\omega t) 2 \sum_{n=1}^{\infty} K_n \cos(n\omega t - n\theta) \\ & + V_p \sin(\omega t - 120^\circ) 2 \sum_{n=1}^{\infty} K_n \cos(n\omega t - n\theta - n120^\circ) \\ & + V_p \sin(\omega t - 240^\circ) 2 \sum_{n=1}^{\infty} K_n \cos(n\omega t - n\theta - n240^\circ) \end{aligned}$$

This is expanded according to the identity,

$$\sin B \cos A = \tfrac{1}{2}[\sin(A + B) - \sin(A - B)],$$

$$\begin{aligned} V_o(t) = {} & K_o V_p \sin(\omega t) + K_o V_p \sin(\omega t - 120^\circ) + K_o V_p \sin(\omega t - 240^\circ) \\ & + V_p \sum_{n=1}^{\infty} K_n \sin[(n+1)\omega t - n\theta] - V_p \sum_{n=1}^{\infty} K_n \sin[(n-1)\omega t - n\theta] \\ & + V_p \sum_{n=1}^{\infty} K_n \sin[(n+1)\omega t - n\theta - (n+1)120^\circ] \\ & - V_p \sum_{n=1}^{\infty} K_n \sin[(n-1)\omega t - n\theta - (n-1)120^\circ] \\ & + V_p \sum_{n=1}^{\infty} K_n \sin[(n+1)\omega t - n\theta - (n+1)240^\circ] \\ & - V_p \sum_{n=1}^{\infty} K_n \sin[(n-1)\omega t - n\theta - (n-1)240^\circ] \end{aligned}$$

The first three terms are equal in magnitude and phase displaced by 120°, hence their resultant is zero.

$$
\begin{aligned}
V_o(t) = {} & V_p \sum_{n=1}^{\infty} K_n \sin[(n+1)\omega t - n\theta] - V_p \sum_{n=1}^{\infty} K_n \sin[(n-1)\omega t - n\theta] \\
& + V_p \sum_{n=1}^{\infty} K_n \sin[(n+1)\omega t - n\theta - (n+1)120°] \\
& - V_p \sum_{n=1}^{\infty} K_n \sin[(n-1)\omega t - n\theta - (n-1)120°] \\
& + V_p \sum_{n=1}^{\infty} K_n \sin[(n+1)\omega t - n\theta - (n+1)240°] \\
& - V_p \sum_{n=1}^{\infty} K_n \sin[(n-1)\omega t - n\theta - (n-1)240°]
\end{aligned}
\tag{6.8}
$$

6.3.2 *DC component of output voltage*

This is derived from Expression (6.8) by selecting the components with zero frequency. This is happening by substituting $n = 1$ in the second terms containing $n - 1$.

$$V_{o_{dc}} = V_p K_1 \sin(\theta) + V_p K_1 \sin[\theta] + V_p K_1 \sin(\theta)$$

$$V_{o_{dc}} = 3V_p K_1 \sin(\theta)$$

$$K_1 = \frac{\sin\delta}{\pi}$$

$$V_{o_{dc}} = 3V_p \frac{\sin\delta}{\pi} \sin(\theta) \tag{6.9}$$

Depending on the type of load, resistive inductive with and without free-wheeling diode, θ and δ take different values as described in Section 6.1.

6.3.3 *Output current*

The output voltage, $V_o(t)$, is forcing a current, $I_o(t)$, through the load harmonic impedance, $Z(n\omega)$. There are two harmonic impedances for $n + 1$ and $n - 1$,

$$Z_{n+1} = \sqrt{R^2 + (\omega(n+1)L)^2} \qquad \varphi_{n+1} = \tan^{-1}\left(\frac{\omega(n+1)L}{R}\right)$$

and

$$Z_{n-1} = \sqrt{R^2 + (\omega(n-1)L)^2} \qquad \varphi_{n-1} = \tan^{-1}\left(\frac{\omega(n-1)L}{R}\right)$$

$$\begin{aligned}
I_o(t) = {} & V_p \sum_{n=1}^{\infty} \frac{K_n}{Z_{n+1}} \sin[(n+1)\omega t - n\theta - \varphi_{n+1}] \\
& - V_p \sum_{n=1}^{\infty} \frac{K_n}{Z_{n-1}} \sin[(n-1)\omega t - n\theta - \varphi_{n-1}] \\
& + V_p \sum_{n=1}^{\infty} \frac{K_n}{Z_{n+1}} \sin[(n+1)\omega t - n\theta - (n+1)120^\circ - \varphi_{n+1}] \\
& - V_p \sum_{n=1}^{\infty} \frac{K_n}{Z_{n-1}} \sin[(n-1)\omega t - n\theta - (n-1)120^\circ - \varphi_{n-1}] \\
& + V_p \sum_{n=1}^{\infty} \frac{K_n}{Z_{n+1}} \sin[(n+1)\omega t - n\theta - (n+1)240^\circ - \varphi_{n+1}] \\
& - V_p \sum_{n=1}^{\infty} \frac{K_n}{Z_{n-1}} \sin[(n-1)\omega t - n\theta - (n-1)240^\circ - \varphi_{n-1}] \qquad (6.10)
\end{aligned}$$

6.3.4 Input current

The input current is a reflection of the output current to the input as demonstrated in the mathematical model of Fig. 6.4

$$I_r(t) = F_r(t) I_o(t) \quad I_y(t) = F_y(t) I_o(t) \quad I_b(t) = F_b(t) I_o(t) \qquad (6.11)$$

For a highly inductive load the output current is approximated to its dc value, I_{dc}.

$$I_r(t) = F_r(t) I_{dc} \quad I_y(t) = F_y(t) I_{dc} \quad I_b(t) = F_b(t) I_{dc} \qquad (6.12)$$

Considering the red line current

$$I_r(t) = I_{dc} K_o + 2 I_{dc} \sum_{n=1}^{\infty} K_n \cos(n\omega t - n\theta)$$

Extracting and re-arranging the fundamental component in the current expression

$$= I_{dc}K_o + 2I_{dc}K_1\cos(\omega t - \theta) + 2I_{dc}\sum_{n=2}^{\infty} K_n\cos(n\omega t - n\theta)$$

Reverting to a sine term for the fundamental current

$$= I_{dc}K_o + 2I_{dc}K_1\sin(\omega t - \theta + 90^\circ) + 2I_{dc}\sum_{n=2}^{\infty} K_n\cos(n\omega t - n\theta)$$

For $\theta = \alpha + \pi/2$

$$I_r(t) = I_{dc}K_o + 2I_{dc}K_1\sin(\omega t - \alpha) + 2I_{dc}\sum_{n=2}^{\infty} K_n\cos(n\omega t - n\theta) \tag{6.13}$$

Hence, for a highly inductive load.

(i) There is a dc component circulating in the mains, $I_{dc}K_o$
(ii) There is a fundamental component $2I_{dc}K_1$ lagging the voltage by α degrees. The displacement power factor (DPF) is $\cos(\alpha)$

$$\text{DPF} = \cos(\alpha) \tag{6.14}$$

(iii) The distortion current is

$$2I_{dc}\sum_{n=2}^{\infty} K_n\cos(n\omega t - n\theta)$$

6.3.5 Harmonics distortion

Both the total harmonic distortion and the distortion factor are derived for the line current, assuming a smoothed output dc current

Total harmonic distortion factor, %THD from a Mathcad program

$$\%\text{THD} := 100\frac{\sqrt{\sum_{n=3}^{X} I(n)^2}}{I_1} \tag{6.15}$$

where $I(n)$ is the magnitude of each harmonic and I_1 is the rms magnitude of the fundamental.

$$I(n) = 2I_{dc}K_n$$

$$I_1 = 2I_{dc}K_1$$

x – it is the number of harmonics to be considered, theoretically ∞.

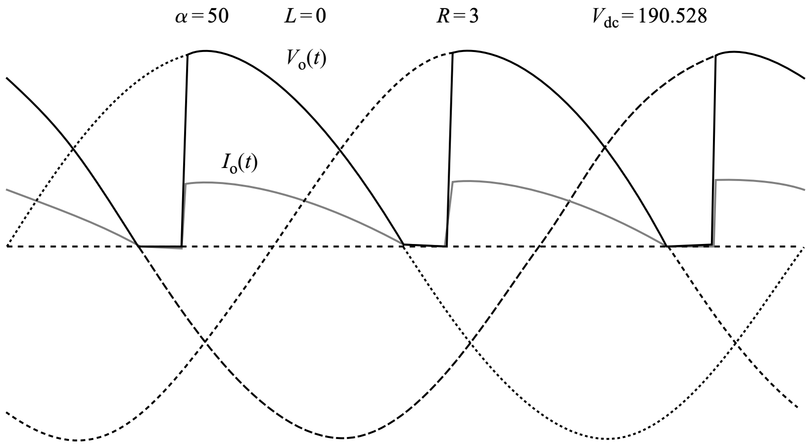


Figure 6.6 Output voltage and current, resistive load

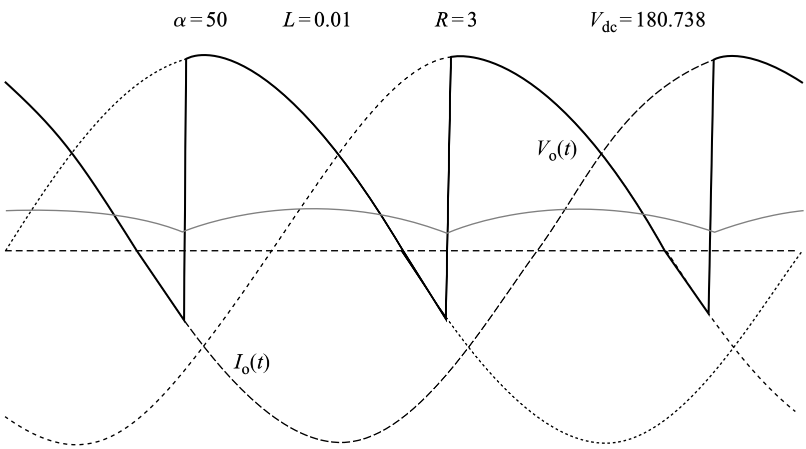


Figure 6.7 Output voltage and current, RL load

Distortion factor, DistF

$$\mathrm{DistF} := \frac{I_1}{\sqrt{\sum_{n=1}^{X} I(n)^2}} \tag{6.16}$$

Power factor, PF

$$\mathrm{PF} = \mathrm{DistF} \times \mathrm{DPF} \tag{6.17}$$

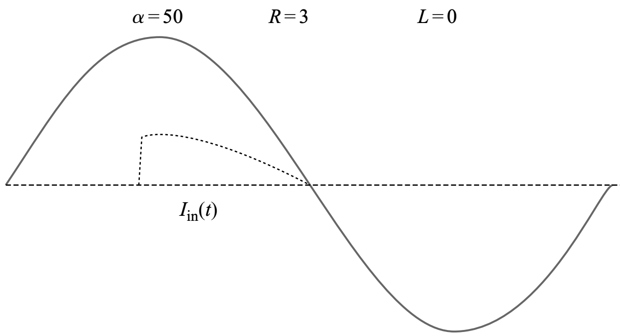


Figure 6.8 Input voltage and current, resistive load

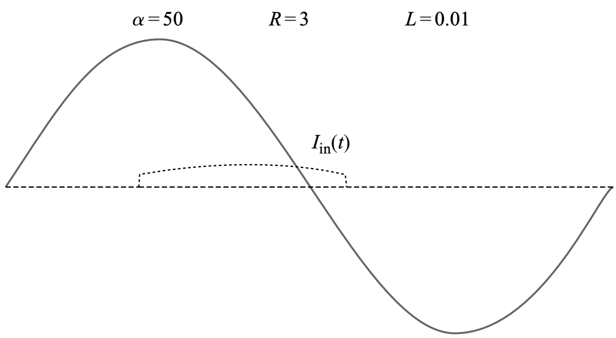


Figure 6.9 Input voltage and current, inductive load, continuous conduction

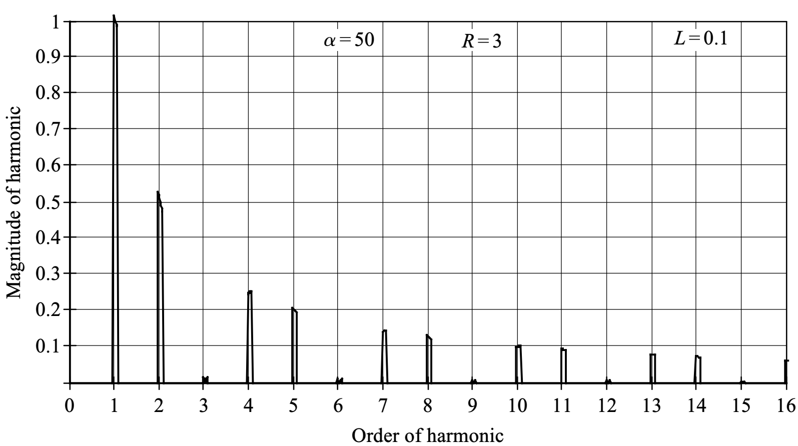


Figure 6.10 Frequency spectrum of line current (fully smoothed output current)

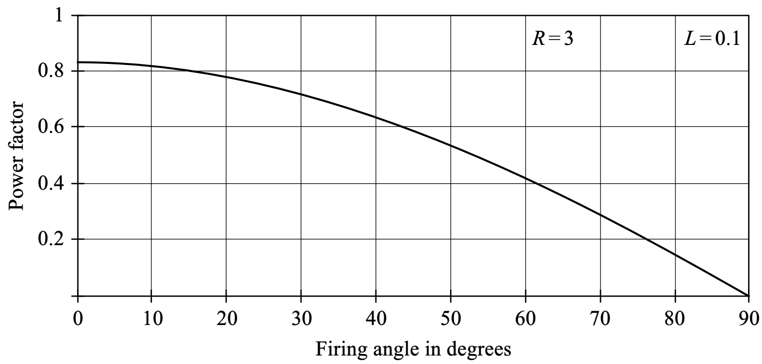


Figure 6.11 Power factor fully smoothed output current

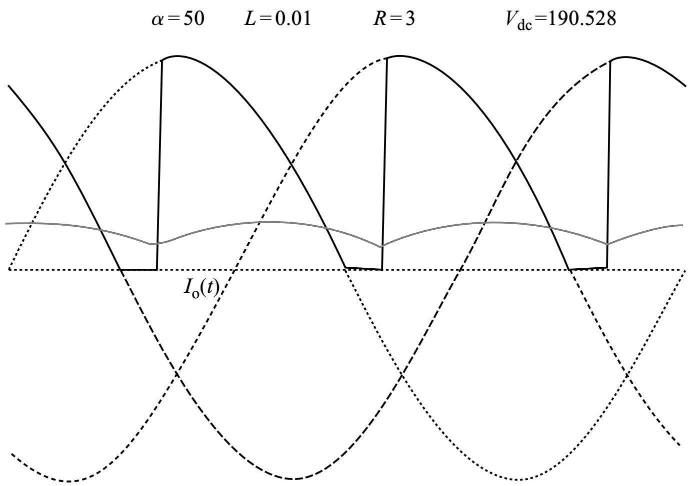


Figure 6.12 Output voltage and current, inductive load, with free-wheeling diode

6.4 Results

Expressions for the output voltage, output and input current, %THD, Distortion factor and the PF are derived in the previous section. Furthermore the necessary information for the frequency spectrum for these quantities is also available. MATHCAD is employed to produce the waveforms.

The output current of the half-wave three-phase controlled rectifier with resistive load might be continuous or discontinuous depending on the firing angle. It is continuous as long as the firing angle α is less than 30° and discontinuous for α larger than 30°. The output voltage and current waveforms are shown in Fig. 6.6 for

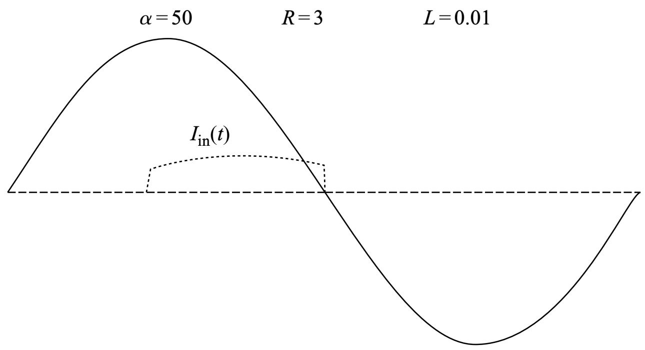


Figure 6.13 Input voltage and current, inductive load, with free-wheeling diode

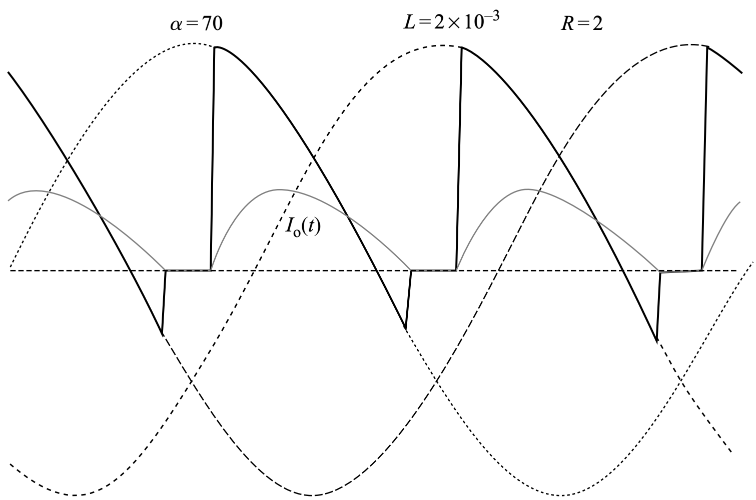


Figure 6.14 Output voltage and current, inductive load, discontinuous conduction

discontinuous conduction. In this case the parameter δ which is half the on period of the switching function is set to $\delta = (120^\circ - \alpha)/2$.

The current might become continuous in the presence of inductance in the load, even for a firing angle α higher than 30°. Due to the presence of inductance, the conducting thyristor will not commutate until either the current is zero – discontinuous conduction – or the next thyristor is fired, continuous conduction. The thyristors will persist to conduct well into the negative half-cycle as long as the load inductance keeps the current flowing. The associated time waveforms are shown in Fig. 6.7 for $\alpha = 50$, $R = 3\ \Omega$ and $L = 0.01$ H. In this case the parameter δ which is half the on period of the switching function is 60°.

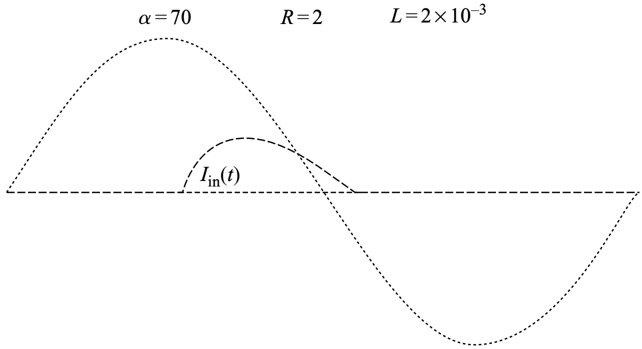


Figure 6.15 Input voltage and current, inductive load, discontinuous conduction

The input current is a part-sinusoid for a resistive load and it is limited to the positive half-cycle only, Fig. 6.8. For an inductive load it is a positive pulse, Fig. 6.9. Conduction in one half-cycle only gives rise to odd and even harmonics, Fig. 6.10, which shows the frequency spectrum of the input current for a perfectly smoothed output current. As expected, due to the geometry of the current pulse, the triplen harmonics are absent. The magnitude of each harmonic is divided by the magnitude of the fundamental in Fig. 6.10.

The displacement of the fundamental component of the line current of a fully smoothed output current is shown to be the firing angle α. Hence the displacement power factor is the cosine of that angle. The power factor is the product of the distortion factor and the displacement power factor and it is displayed in Fig. 6.11.

The presence of the free-wheeling diode limits the conduction of the thyristors up to the end of the positive half-cycle. Figure 6.12 displays the output voltage and current. Figure 6.13 displays the input voltage and current.

Discontinuous conduction is displayed in Fig. 6.14 for the output voltage and current and in Fig. 6.15 for the input voltage and current.

Chapter 7

The three-phase full-wave phase controlled rectifier

7.1 Introduction

The three-phase full-wave phase controlled rectifier is analysed in this chapter by considering an R–L load and continuous conduction. Voltage and current expressions, frequency spectrum, power and distortion factors are derived using the switching function. The overlap is dealt with in the next chapter in order to derive expressions for the output voltage 'notches'. In both cases the relevant switching functions are identified.

The various modes of the circuit are set by the switching action of the six thyristors; the voltage and current at any point is set by the mode of the circuit. In deriving the switching functions a careful study of the action of the semiconductor switches is required. This is done by developing the modes of the circuit, Fig. 7.1. We identify three switching functions that contribute to the output voltage, one for each line voltage: $F_{\mathrm{ry}}(t)$, $F_{\mathrm{yb}}(t)$ and $F_{\mathrm{br}}(t)$. The switching functions are of the quasi-square shape expressed as a sum of cosines and they are phase displaced between them by 60°. The delay firing angle α is measured in the normal way from the positive going crossing of the red phase voltage.

7.2 The mathematical modelling of the three-phase full-wave controlled rectifier circuit

7.2.1 Modes and operation

Six Modes are identified as shown in Fig. 7.2; the overlap is neglected. During any mode two thyristors are conducting, one from the upper group and one from the lower group are connecting two phases to the load. Table 7.1 contains all the information relevant to the timing of the modes. In the first column the mode and the conducting thyristors are listed; in the second column, the phases connected to the load by the conducting thyristors are listed and the third column contains the corresponding

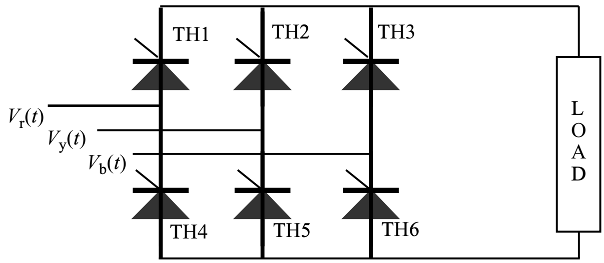


Figure 7.1 The three-phase full-wave controlled rectifier circuit

segments of the mains cycle: the starting and finishing instances for the conducting thyristors. For each mode a switching function is defined. Three switching functions are introduced $F_{ry}(t)$, $F_{yb}(t)$ and $F_{br}(t)$ for the output voltage and three for the input current, $F_r(t)$, $F_y(t)$ and $F_b(t)$. The state of these switching functions (1, 0 or -1) is dictated by the relationship of the 'input' and 'output'. In the current case the 'input' is the output current $I_o(t)$ and the 'output' is the line current $I_r(t)$, $I_y(t)$ or $I_b(t)$. For the voltage the six modes of the three-phase full-wave controlled rectifier circuit are presented here.

Mode I: Thyristor TH1 connects red line and thyristor TH5 connects yellow line to the load from $30° + \alpha$ to $90° + \alpha$. During that period the line voltage $V_{ry}(t)$ appears at the output. Hence:

$$V_o(t) = V_{ry}(t)$$

A switching function, $F_{ry}(t)$ is introduced associated with this mode. Mode I exists when

$$F_{ry}(t) = 1$$

During this mode the current is entering the load from the red line and returning through the yellow line. Hence:

$$I_r(t) = I_o(t) \qquad I_y(t) = -I_o(t) \qquad I_b(t) = 0$$

Obviously the state of the switching functions associated with the current during Mode I is:

$$F_r(t) = 1 \qquad F_y(t) = -1 \qquad F_b(t) = 0$$

Mode II: Mode II is taking place 60° after Mode I. Thyristor TH1 connects red line and thyristor TH6 connects blue line to the load at $90° + \alpha$ to $150° + \alpha$. During that period the line voltage $V_{br}(t)$ appears at the output inverted. Hence:

$$V_o(t) = -V_{br}(t)$$

A switching function $F_{br}(t)$ is introduced associated with Mode II. The output is inverted during Mode II and $F_{br}(t) = -1$; this implies that its first pulse is negative. Hence the phase displacement of $F_{br}(t)$ with reference to $F_{ry}(t)$ is 60° to account for the delay of Mode II plus 180° to account for the inversion.

During this mode the current is entering the load from the red line and returning through the blue line. Hence:

$$I_r(t) = I_o(t) \qquad I_y(t) = 0 \qquad I_b(t) = -I_o(t)$$

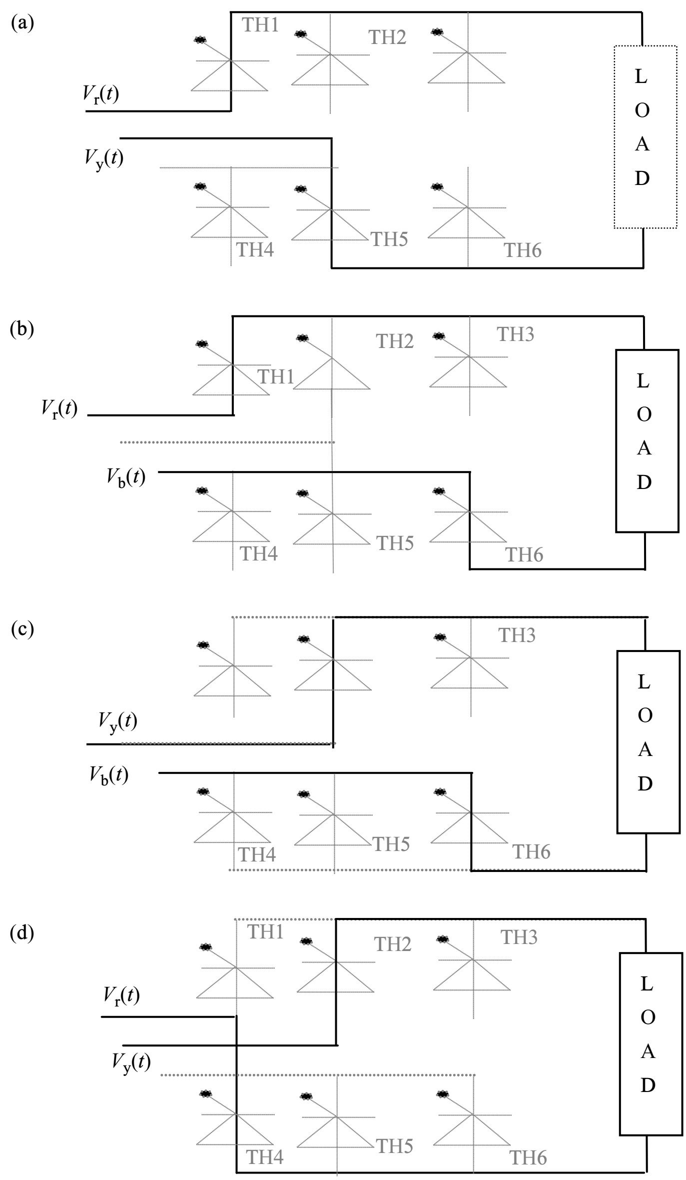


Figure 7.2 Continued

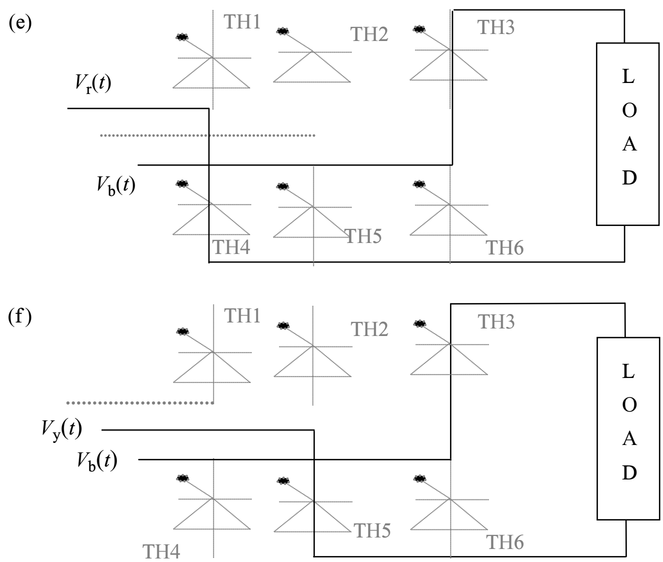


Figure 7.2 Three-phase full-wave controlled rectifier: (a) Mode I; (b) Mode II; (c) Mode III; (d) Mode IV; (e) Mode V and (f) Mode VI

Table 7.1 Conducting thyristor pairs (three-phase inductive load, continuous conduction)

Modes and conducting thyristor	Connected phases to the load	Conduction period with reference to red phase voltage
Mode I TH1–TH5	Red–Yellow	$\alpha + 30°$ to $\alpha + 90°$
Mode II TH1–TH6	Red–Blue	$\alpha + 90°$ to $\alpha + 150°$
Mode III TH2–TH6	Yellow–Blue	$\alpha + 150°$ to $\alpha + 210°$
Mode IV TH2–TH4	Yellow–Red	$\alpha + 210°$ to $\alpha + 270°$
Mode V TH3–TH4	Blue–Red	$\alpha + 270°$ to $\alpha + 330°$
Mode VI TH3–TH5	Blue–Yellow	$\alpha + 330°$ to $\alpha + 390°$

Obviously the state of the switching functions for the current during Mode II is:

$$F_r(t) = 1 \qquad F_y(t) = 0 \qquad F_b(t) = -1$$

Mode III: Thyristor TH2 connects yellow line and thyristor TH6 connects blue line to the load at $150° + \alpha$ to $210° + \alpha$. During that period we have mode III and the line voltage $V_{yb}(t)$ appears at the output. Hence:

$$V_o(t) = V_{yb}(t)$$

A switching function $F_{yb}(t)$ is introduced associated with this mode; Mode III exists when $F_{yb}(t) = 1$. $F_{yb}(t)$ is delayed by 120° relative to $F_{ry}(t)$ since Mode III is taking place 120° after Mode I, Table 7.1. During this mode the current is entering the load from the yellow line and returning through the blue line. Hence:

$$I_r(t) = 0 \qquad I_y(t) = I_o(t) \qquad I_b(t) = -I_o(t)$$

Obviously the state of the switching functions for the current during Mode III is:

$$F_r(t) = 0 \qquad F_y(t) = 1 \qquad F_b(t) = -1$$

Mode IV: Thyristor TH2 connects yellow line and thyristor TH4 connects red line to the load at $150° + \alpha$ to $210° + \alpha$. During that period the line voltage $V_{ry}(t)$ appears at the output, inverted. Hence:

$$V_o(t) = -V_{ry}(t)$$

The switching function $F_{ry}(t)$ already introduced for Mode I is also appropriate for this mode. It takes the value of -1 to account for the inversion of the output voltage.

$$F_{ry}(t) = -1$$

During this mode the current is entering the load from the yellow line and returning through the red line, the opposite to Mode I. Hence:

$$I_r(t) = -I_o(t) \qquad I_y(t) = I_o(t) \qquad I_b(t) = 0$$

Obviously the state of the switching functions for the current during Mode IV is:

$$F_r(t) = -1 \qquad F_y(t) = 1 \qquad F_b(t) = 0$$

Mode V: Thyristor TH3 connects blue line and thyristor TH4 connects red line to the load at $270° + \alpha$ to $330° + \alpha$. During that period the line voltage $V_{br}(t)$ appears at the output, inverted. Hence:

$$V_o(t) = -V_{br}(t)$$

The switching function $F_{rb}(t)$ already introduced for Mode II is also appropriate for this mode. It takes the value of -1 to account for the inversion of the output voltage.

$$F_{br}(t) = -1$$

During this mode the current is entering the load from the blue line and returning through the red line, the opposite to Mode II. Hence:

$$I_r(t) = -I_o(t) \qquad I_y(t) = 0 \qquad I_b(t) = I_o(t)$$

Obviously the state of the switching functions for the current during Mode V is:

$$F_r(t) = -1 \qquad F_y(t) = 0 \qquad F_b(t) = 1$$

Mode VI: Thyristor TH3 connects the blue line and thyristor TH5 connects the yellow line to the load at $330° + \alpha$ to $390° + \alpha$. During that period the line voltage $V_{yb}(t)$ appears at the output, inverted. Hence:

$$V_o(t) = -V_{yb}(t)$$

The switching function $F_{yb}(t)$ already introduced for Mode III is also appropriate for this mode. It takes the value of -1 to account for the inversion of the output voltage.

$$F_{yb}(t) = -1$$

During this mode the current is entering the load from the blue line and returning through the yellow line, the opposite to Mode III. Hence:

$$I_r(t) = 0 \qquad I_y(t) = -I_o(t) \qquad I_b(t) = I_o(t)$$

Obviously the state of the switching functions for the current during Mode II is:

$$F_r(t) = 0 \qquad F_y(t) = -1 \qquad F_b(t) = 1$$

7.2.2 Mode sequence

All the information concerning the six modes is collected in Table 7.1. The first three modes are related to the last three in the following way. In both Modes I and IV phases red and yellow lines are connected to the loads but in Mode IV the connection is inverted. This suggests a single switching function, $F_{ry}(t)$ of the bipolar type for both modes. This applies for Modes II–V and Modes III–VI. The mode sequence is

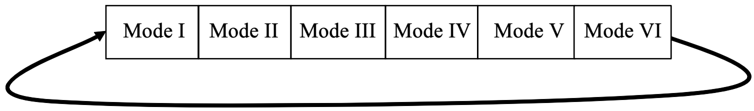


Figure 7.3 Mode sequence for the three-phase full-wave phase controlled rectifier

shown in Fig. 7.3. Each mode contributes to the output voltage according to the state of the associated switching function. The contribution of Mode I and Mode IV to the output voltage is given by

$$V_o(t) = V_{ry}(t)F_{ry}(t)$$

In the same way, the contribution of Mode II and Mode V to the output voltage is given by

$$V_o(t) = V_{yb}(t)F_{yb}(t)$$

And, the contribution of Mode III and Mode VI to the output voltage is given by

$$V_o(t) = V_{br}(t)F_{br}(t)$$

According to the superposition theorem as applied to switched circuits, Chapter 1, the output voltage is made up from the contributions of the three modulations

$$V_o(t) = V_{ry}(t)F_{ry}(t) + V_{yb}(t)F_{yb}(t) + V_{br}(t)F_{br}(t) \tag{7.1}$$

The relationship between the input and output current is directly derived from the modes of the circuit as well. The red line current is carrying the output current in the positive direction during Modes I and II and in the negative direction during Modes IV and V. Hence the switching function $F_r(t)$ relates the output current and red line input current.

$$I_r(t) = I_o(t)F_r(t) \tag{7.2a}$$

This switching function is of the bipolar type since current inversion is taking place. It takes the value of 1 during Modes I and II and the value of -1 during Modes IV and V. In the same way,

$$I_y(t) = I_o(t)F_y(t) \tag{7.2b}$$

$$I_b(t) = I_o(t)F_b(t) \tag{7.2c}$$

7.2.3 The mathematical model

The mathematical model for the three-phase full-wave controlled rectifier circuit is shown in Fig. 7.4. The output voltage appearing across the load is the result of an amplitude modulation between the input line-to-line mains voltage and appropriate switching functions. The output voltage is made up from the contributions of the three modulations (Modulators M1, M2, M3) according to Expression (7.1).

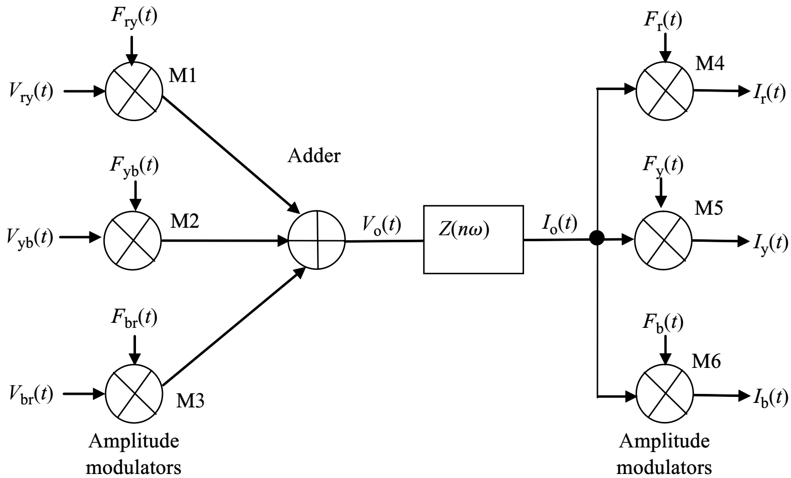


Figure 7.4 The three-phase full-wave controlled rectifier: block diagram mathematical model

The output voltage is forcing a current, $I_o(t)$, through the 'harmonic impedance' of the load. The harmonic impedance refers to the impedance as a function of the order of the harmonics of the output voltage, as explained in Chapter 1. The line current is a reflection of the output current to the input, according to Expressions (7.2). It is the result of amplitude modulation (Modulators M4, M5 and M6) of the output current and appropriate switching functions: $F_r(t)$, $F_y(t)$ and $F_b(t)$.

7.2.4 The switching functions

In order to derive the switching functions we need to study the mechanism of operation of the circuit. This is done by deriving the modes of the circuit the way presented above. Reference is made to Table 7.1 as well which contains the information concerning the time intervals for each mode. A switching function is associated with an input quantity and an output quantity. It takes the value of one when the output quantity takes the value of the input and it takes the value of zero when the output is zero; if there is an inversion of the output, the switching function takes the value of -1. This is in accordance with Chapter 1, Expression (1.1). Within a mode we might have more than one switching function. There are two types of switching functions for the circuit under investigation: the switching functions related to the voltage $F_{ry}(t)$, $F_{yb}(t)$ and $F_{br}(t)$ and the switching functions related to the current, $F_r(t)$, $F_y(t)$ and $F_b(t)$.

7.2.5 Voltage switching functions

Certain parts of the line voltage will appear at the output when certain pairs of thyristors are conducting. The thyristor pairs listed in Table 7.1 are forward biased

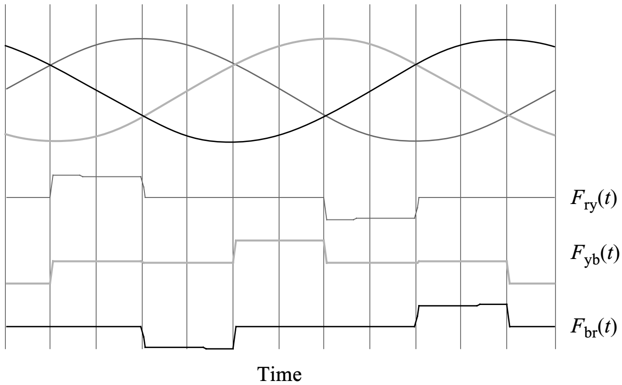


Figure 7.5 The switching functions for the output voltage

in sequence for 60° for each half-cycle. The derivation of the switching function connecting the red and yellow lines to the load, $F_{ry}(t)$, is presented in detail now.

It is stated in Table 7.1 and in Mode I that the thyristor pair TH1–TH5 is forward biased in the period starting at the positive half-cycle at 30° from the positive going crossing point of the reference red phase voltage, Mode I. The starting of conduction is further delayed by the delay firing angle α and conduction will be maintained for the next 60° for an inductive load. The thyristor pair TH1–TH5 connects the red and blue to the load. Hence the switching functions $F_{ry}(t)$ must have a positive pulse starting at $\alpha + 30°$ and ending 60° later, Fig. 7.5. During the negative half-cycle of the red line we have a similar situation. Now the red and yellow lines are connected to the load by the thyristors TH2 and TH4, Mode IV, and the input voltage appears inverted at the output. These thyristors are forward biased for 60° of the negative half-cycle of the r–y line voltage starting 30° from the negative going crossing point of the reference red phase voltage. The starting of conduction is further delayed by the delay firing angle α and conduction will be maintained for the next 60° for an inductive load. That part of the line voltage appears at the output inverted. Hence the switching function $F_{ry}(t)$ must have a negative pulse starting at $\alpha + 30° + 180°$ and ending 60° later for continuous conduction, Fig. 7.5. Therefore the switching function that modulates the red–yellow line voltage consists of two 'unit' switching functions separated by 180° having pulse width 60°. The first unit switching function is phase delayed by $\theta°$ relative to reference red phase voltage and the second is further displaced by 180° with a negative pulse. Hence a single bipolar switching function can be formed, $F_{ry}(t)$, as discussed in Chapter 1.

$$F_{ry}(t) = 4\sum_{n=1}^{\infty} K_n[\cos(n\omega t - n\theta)] \tag{7.3a}$$

In the same way the other two switching functions for the yellow–blue and blue–red lines are derived, $F_{yb}(t)$ and $F_{br}(t)$. $F_{br}(t)$ is delayed by 60° relative to $F_{ry}(t)$ but the first pulse is negative as mentioned above. Hence the phase displacement of $F_{br}(t)$

with reference to $F_{ry}(t)$ is 60° to account for the delay of Mode II plus 180° for the inversion. The minus sign in (7.3b) accounts for the 180°.

$$F_{br}(t) = -4\sum_{n=1}^{\infty} K_n[\cos(n\omega t - n\theta - n60°)] \tag{7.3b}$$

The switching function associated with the yellow and blue lines $F_{yb}(t)$ is taking place 120° after $F_{ry}(t)$ since Mode III is taking place 120° after Mode I.

$$F_{yb}(t) = 4\sum_{n=1}^{\infty} K_n[\cos(n\omega t - n\theta - n120°)] \tag{7.3c}$$

where

$K_n = \sin(n\delta)/n\pi$
$\delta =$ Half the on period of the switch, 30°
$\theta = \alpha + \delta + 30°$
n is an odd integer number.

The three switching functions modulating the input voltage to give the output voltage are shown in Fig. 7.5.

7.2.6 Current switching functions

The line current is a reflection of the output current to the input and the appropriate switching functions must be derived. The switching function relating the output current to the red line current, $F_r(t)$, is derived in the next paragraph.

The red line is conducting for a period of 120° during each half-cycle. It is shown in Table 7.1 that during the positive half-cycle it conducts for 60° in conjunction with the yellow line and 60° with the blue line. And this is repeated during the negative half-cycle. Therefore the switching function for the red line current, $F_r(t)$, is a quasi-square signal, the on-period is 120° and it is phase displaced by $90° + \alpha$ with reference to the red phase voltage.

$$F_r(t) = 4\sum_{n=1}^{\infty} K_n[\cos(n\omega t - n(\alpha + 90°))] \tag{7.4a}$$

In the same way, the other two switching functions are derived.

$$F_y(t) = 4\sum_{n=1}^{\infty} K_n[\cos(n\omega t - n(\alpha + 90° + 120°))] \tag{7.4b}$$

$$F_b(t) = 4\sum_{n=1}^{\infty} K_n[\cos(n\omega t - n(\alpha + 90° - 120°))] \tag{7.4c}$$

The switching functions for the input current are shown in Fig. 7.6.

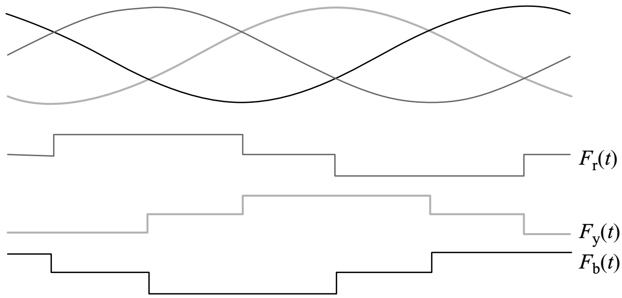


Figure 7.6 The switching functions for the input current

7.3 Analysis of three-phase full-wave phase controlled rectifier

In this section, the expressions for the output voltage, the output current and the input current are derived from the mathematical model of Fig. 7.4. The frequency spectrum, the power factor, distortion factor and Total Harmonic Distortion are derived from the line current expression.

7.3.1 The output voltage

The expressions for the line voltages are given below, with reference to the red phase.

$$V_{\mathrm{ry}}(t) = \sqrt{3}V_{\mathrm{p}} \sin\left(\omega t - \frac{\pi}{6}\right)$$

$$V_{\mathrm{yb}}(t) = \sqrt{3}V_{\mathrm{p}} \sin\left(\omega t - 5\frac{\pi}{6}\right)$$

$$V_{\mathrm{yb}}(t) = \sqrt{3}V_{\mathrm{p}} \sin\left(\omega t - 4\frac{\pi}{6}\right)$$

Expressions (7.3) for the switching functions and the expressions of the line voltages above are substituted in Expression (7.1) for the output voltage, $V_{\mathrm{o}}(t)$.

$$\begin{aligned} V_{\mathrm{o}}(t) = {} & \sqrt{3}V_{\mathrm{p}} \sin\left(\omega t - \frac{\pi}{6}\right) 4 \sum_{n=1}^{\infty} K_n[\cos(n\omega t - n\theta)] \\ & + \sqrt{3}V_{\mathrm{p}} \sin\left(\omega t - 5\frac{\pi}{6}\right) 4 \sum_{n=1}^{\infty} K_n[\cos(n\omega t - n\theta - 120^\circ)] \\ & - \sqrt{3}V_{\mathrm{p}} \sin\left(\omega t - 4\frac{\pi}{6}\right) 4 \sum_{n=1}^{\infty} K_n[\cos(n\omega t - n\theta - 60^\circ)] \end{aligned}$$

The above is expanded into Expression (7.5) by simple trigonometry.

$$
\begin{aligned}
V_o(t) = {} & \sum_{n=1}^{N}\left[\frac{\sin(n\delta)}{\pi n}\left[\sin\left[(n+1)\omega t - n\theta_{\text{ryr}} + \frac{\pi}{6}\right]\right.\right. \\
& \left.\left. - \sin\left[(n-1)\omega t - n\theta_{\text{ryr}} - \frac{\pi}{6}\right]\right]\right] \\
& - \sum_{n=1}^{N}\left[\frac{\sin(n\delta)}{\pi n}\left[\sin\left[(n+1)\omega t - n(\theta_{\text{ryr}} + \pi) + \frac{\pi}{6}\right]\right.\right. \\
& \left.\left. - \sin\left[(n-1)\omega t - n(\theta_{\text{ryr}} + \pi) - \frac{\pi}{6}\right]\right]\right] \\
& + \sum_{n=1}^{N}\left[\frac{\sin(n\delta)}{\pi n}\left[\sin\left[(n+1)\omega t - n\theta_{\text{ybr}} - \frac{\pi}{6}\right]\right.\right. \\
& \left.\left. - \sin\left[(n-1)\omega t - n\theta_{\text{ybr}} + \frac{\pi}{6}\right]\right]\right] \\
& - \sum_{n=1}^{N}\left[\frac{\sin(n\delta)}{\pi n}\left[\sin\left[(n+1)\omega t - n(\theta_{\text{ybr}} + \pi) - \frac{\pi}{6}\right]\right.\right. \\
& \left.\left. - \sin\left[(n-1)\omega t - n(\theta_{\text{ybr}} + \pi) + \frac{\pi}{6}\right]\right]\right] \\
& + \sum_{n=1}^{N}\left[\frac{\sin(n\delta)}{\pi n}\left[\sin\left[(n+1)\omega t - n\theta_{\text{brr}} - \frac{\pi}{2}\right]\right.\right. \\
& \left.\left. - \sin\left[(n-1)\omega t - n\theta_{\text{brr}} - \frac{\pi}{2}\right]\right]\right] \\
& - \sum_{n=1}^{N}\left[\frac{\sin(n\delta)}{\pi n}\left[\sin\left[(n+1)\omega t - n(\theta_{\text{brr}} + \pi) - \frac{\pi}{2}\right]\right.\right. \\
& \left.\left. - \sin\left[(n-1)\omega t - n(\theta_{\text{brr}} + \pi) + \frac{\pi}{2}\right]\right]\right] \qquad (7.5)
\end{aligned}
$$

where: $\theta_{\text{ryr}} = \alpha + 60°$, $\theta_{\text{ybr}} = \theta_{\text{ryr}} + 60°$, $\theta_{\text{brr}} = \theta_{\text{ybr}} + 60°$. Expression (7.5) is an extract from a Mathcad program. The output voltage is displayed in Fig. 7.7.

7.3.2 DC output voltage, V_{dco}

A dc component manifests itself in the expression for the output voltage for those terms where the frequency is zero. In each term of Expression (7.5), we look for this condition. In all terms this condition is satisfied in their second part for $n = 1$.

$$\begin{aligned} V_{odc} = -V_p\sqrt{3}\, &\frac{\sin(\delta)}{\pi}\left(\sin\left(-\theta_{ryr} - \frac{\pi}{6}\right)\right) - \frac{\sin(\delta)}{\pi}\left(\sin\left(-\theta_{ryr} - \frac{\pi}{6} - \pi\right)\right) \\ &+ \frac{\sin(\delta)}{\pi}\left(\sin\left(-\theta_{ybr} + \frac{\pi}{6}\right)\right) - \frac{\sin(\delta)}{\pi}\left(\sin\left(-\theta_{ybr} + \frac{\pi}{6} - \pi\right)\right) \\ &+ \frac{\sin(\delta)}{\pi}\left(\sin\left(-\theta_{brr} + \frac{\pi}{2}\right)\right) - \frac{\sin(\delta)}{\pi}\left(\sin\left(-\theta_{brr} + \frac{\pi}{2} - \pi\right)\right) \end{aligned}$$

Replacing for $\delta = 30°$, $\theta_{ryr} = \alpha + 60°$, $\theta_{ybr} = \theta_{ryr} + \theta_{brr} + 60° = \theta_{ybr} + 60°$ and simplifying

$$V_{odc} = \frac{3\sqrt{3}}{\pi} V_p \cos\alpha \tag{7.6}$$

This is also the standard textbook expression for the output dc voltage of a three-phase ac to dc converter with continuous conduction.

7.3.3 The output current

The output current is dictated by the output voltage and the load harmonic impedance, $Z(pw)$, where p is the order of the output harmonic voltage.

$$I_o(t) = \frac{V_o(t)}{Z(p\omega)} \tag{7.7}$$

There are two terms in the output voltage $(n+1)w$ and $w(n-1)$. Hence the harmonic impedance is given by two expressions for each component of the expression of the output voltage, Z_{n+1} and Z_{n-1}.

$$Z_{n+1} = \sqrt{R^2 + (\omega(n+1)L)^2} \qquad \varphi_{n+1} = \tan^{-1}\left(\frac{\omega(n+1)L}{R}\right)$$

and

$$Z_{n-1} = \sqrt{R^2 + (\omega(n-1)L)^2} \qquad \varphi_{n-1} = \tan^{-1}\left(\frac{\omega(n-1)L}{R}\right)$$

The current is found by dividing each term of (7.6) by Z_{n+1} or Z_{n-1} accordingly. There are six terms as it is suggested by Expression (7.5) and they are found in

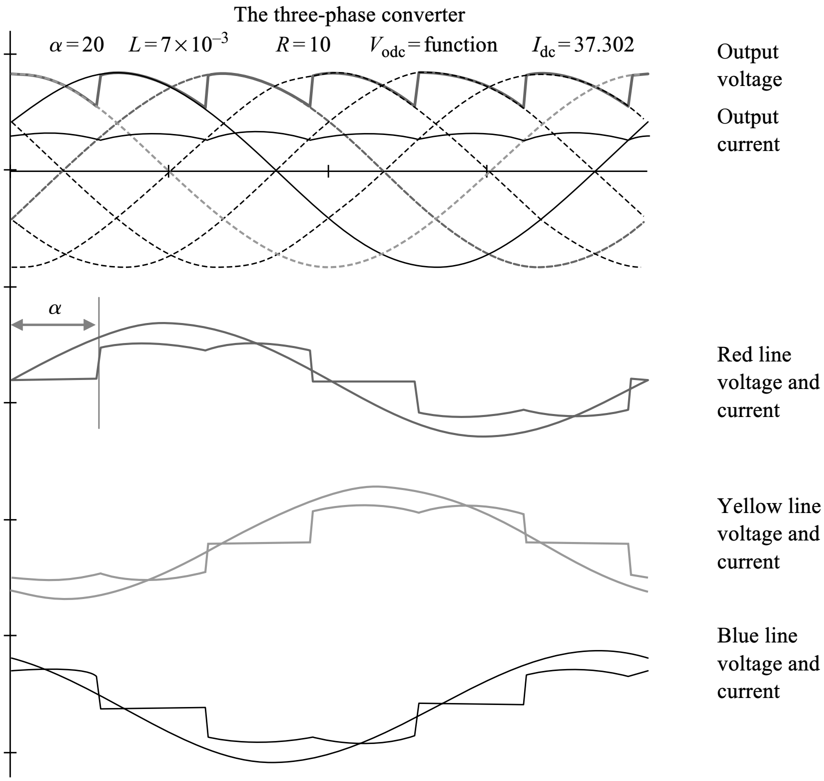


Figure 7.7 Phase voltages and line currents for the three-phase converter with inductive load with the output voltage and current

Appendix A7.1. They are in the form of a short MATHCAD program. The output current is displayed in Fig. 7.7.

7.3.4 Input line current

Expressions (7.2) give the line currents $I_r(t)$, $I_y(t)$ and $I_b(t)$. These expressions for the line currents are not expanded here. They are easily displayed though by MATHCAD, Fig. 7.7.

7.3.5 Displacement power factor

The displacement power factor is the cosine of the phase delay angle of the fundamental component of the line current. In order to extract the fundamental current, Expressions (7.2) must be expanded. This is a tedious task. In a three-phase full-wave rectifier the output current can be approximated to its dc component if the inductance is adequate, Fig. 7.7. For the sake of simplicity this route is followed here.

$$I_o(t) = I_{dc}$$

Hence the line current in the red line is approximated to:

$$I_r(t) = I_{dc} F_r(t)$$

Substituting for $F_r(t)$, the red line current is given by the Mathcad Expression (see website link in page x)

$$I_r(t) := 4I_{dc} \sum_{n=1}^{\infty} K_n[\cos(n\omega t - n(\alpha + 90°))]$$

The magnitude of the frequency components is derived from above $I(n) = 4I_{dc}K_n$ Replacing the K_n coefficient

$$I(n) = 4I_{dc}\frac{\sin(2n\delta)}{n\pi} \tag{7.8}$$

The fundamental component is derived from the Expression of $I_r(t)$ above by setting $n = 1$

$$I_r(t) = 4I_{dc}\frac{\sin(2\delta)}{\pi}\cos[\omega t - (\alpha + 90°)]$$

Simple trigonometry can show that this can be written also as

$$I_r(t) = 4I_{dc}\frac{\sin(2\delta)}{\pi}\sin[\omega t - \alpha]$$

Hence the magnitude of the fundamental is given by

$$I_r = 4I_{dc}\frac{\sin(2\delta)}{\pi} \tag{7.9}$$

And the phase displacement is simply α giving the displacement power factor as:

$$\text{Displacement Power Factor} = \cos(\alpha) \tag{7.10}$$

7.3.6 Distortion factor, DistF

The distortion factor is defined as the ratio of the rms value of the fundamental over the rms of the total current.

$$\text{Distortion Factor, DistF} = \frac{I_1}{\sqrt{\sum_{K=1}^{\infty} I_k^2}} \tag{7.11}$$

where I_1 is the rms value of the fundamental and I_k the kth component of current. Substituting (7.8) and (7.9) in (7.11) gives after simplification.

$$\text{DF} := \frac{(\sin(\delta 2))/\pi}{\sqrt{\sum_{P=1}^{N} ((\sin(P\delta 2)/\pi P)\sin(P\pi/2))^2}}$$

N, number of harmonics to be used, ideally ∞. The distortion factor is found to be 0.955 for a perfectly smoothed output current.

7.3.7 Power factor

The power factor is the product of the displacement power factor and the distortion factor. Both factors are displayed in Fig. 7.8.

7.3.8 Total harmonic distortion

The distortion of current is also measured by the total harmonic distortion factor,

$$\text{THD} = \frac{\sqrt{\sum_{k \neq 1}^{\infty} I_k^2}}{I_1} \tag{7.12}$$

Expressions (7.8) and (7.9) are substituted in (7.12) to give

$$\text{THD} := \frac{\sqrt{\sum_{P=3}^{N} ((\sin(P\delta 2)/\pi P)\sin(P\pi/2))^2}}{\sin(\delta 2)/\pi}$$

N, number of harmonics, ideally ∞.

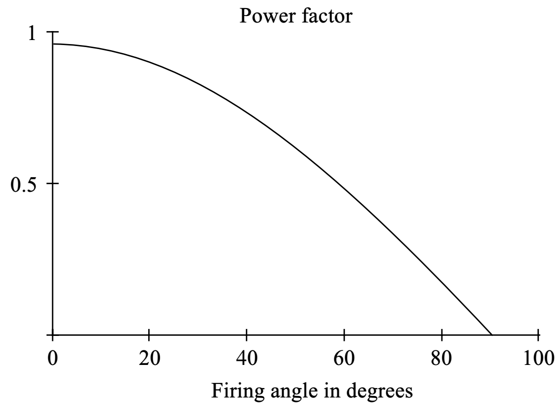


Figure 7.8 *Power factor*

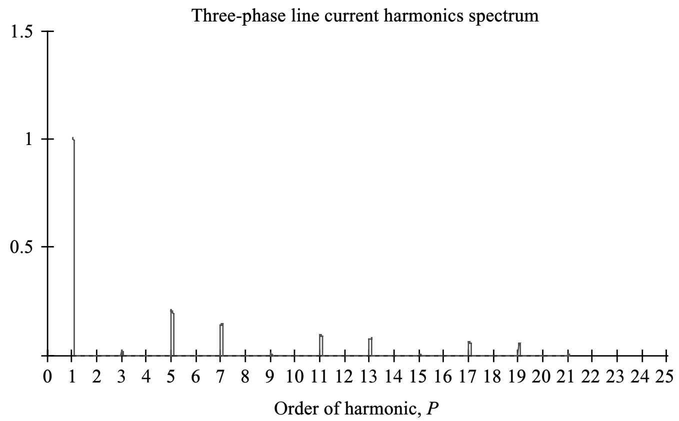


Figure 7.9 *The frequency spectrum of the line current (ratio to the fundamental)*

For the case of simplifying the output current to its dc component, this is found to be 0.311 irrespective of the firing angle.

7.3.9 Frequency spectrum of input current

The magnitude of each harmonic including the fundamental is extracted from Expression (7.8) and displayed in Fig. 7.9. The magnitude of the harmonics is shown as a ratio to the fundamental. The triplen harmonics are missing due to the geometry of the fully smoothed current at the output.

Appendix A7.1 The output current

$$
I_{\text{ory1}}(t) := \left|
\begin{array}{l}
I \leftarrow 0 \\
\gamma \leftarrow \theta_{\text{ryr}} \\
\text{for } n \in 1 \ldots N \\
\quad \left|
\begin{array}{l}
K_n \leftarrow \dfrac{\sin(n\delta)}{n\pi} \\
Z_1 \leftarrow \sqrt{R^2 + [\omega(n+1)L]^2} \\
\phi_1 \leftarrow \text{atan}\left[\omega \dfrac{(n+1)L}{R}\right] \\
\phi_2 \leftarrow \text{atan}\left[\omega \dfrac{(n-1)L}{R}\right] \\
Z_2 \leftarrow \sqrt{R^2 + [\omega(n-1)L]^2} \\
I_1 \leftarrow \sqrt{3}\dfrac{V_{\text{p}}K_n}{Z_1}\sin\left[\left[(n+1)\omega t - n\gamma - \phi_1 + \dfrac{\pi}{6}\right]\right] \\
I_2 \leftarrow \sqrt{3}\dfrac{V_{\text{p}}K_n}{Z_2}\sin\left[(n-1)\omega t - n\gamma - \phi_2 - \dfrac{\pi}{6}\right] \\
I \leftarrow I + I_1 - I_2
\end{array}
\right. \\
II \leftarrow 1
\end{array}
\right.
$$

$$
I_{\text{ory2}}(t) := \left|
\begin{array}{l}
I \leftarrow 0 \\
\gamma \leftarrow \theta_{\text{ryr}} + \pi \\
\text{for } n \in 1 \ldots N \\
\quad \left|
\begin{array}{l}
K_n \leftarrow \dfrac{\sin(n\delta)}{n\pi} \\
Z_1 \leftarrow \sqrt{R^2 + [\omega(n+1)L]^2} \\
\phi_1 \leftarrow \text{atan}\left[\omega \dfrac{(n+1)L}{R}\right] \\
\phi_2 \leftarrow \text{atan}\left[\omega \dfrac{(n-1)L}{R}\right] \\
Z_2 \leftarrow \sqrt{R^2 + [\omega(n-1)L]^2} \\
I_1 \leftarrow \sqrt{3}\dfrac{V_{\text{p}}K_n}{Z_1}\sin\left[\left[(n+1)\omega t - n\gamma - \phi_1 + \dfrac{\pi}{6}\right]\right] \\
I_2 \leftarrow \sqrt{3}\dfrac{V_{\text{p}}K_n}{Z_2}\sin\left[(n-1)\omega t - ny - \phi_2 - \dfrac{\pi}{6}\right] \\
I \leftarrow I + I_1 - I_2
\end{array}
\right. \\
II \leftarrow 1
\end{array}
\right.
$$

$$I_{\text{oyb2}}(t) := \left| \begin{array}{l} I \leftarrow 0 \\ \gamma \leftarrow \theta_{\text{ybr}} + \pi \\ \text{for } n \in 1 \ldots N \\ \quad \left| \begin{array}{l} K_n \leftarrow \dfrac{\sin(n\delta)}{n\pi} \\ Z_1 \leftarrow \sqrt{R^2 + [\omega(n+1)L]^2} \\ \phi_1 \leftarrow \text{atan}\left[\omega\dfrac{(n+1)L}{R}\right] \\ \phi_2 \leftarrow \text{atan}\left[\omega\dfrac{(n-1)L}{R}\right] \\ Z_2 \leftarrow \sqrt{R^2 + [\omega(n-1)L]^2} \\ I_1 \leftarrow \sqrt{3}\dfrac{V_{\text{p}}K_n}{Z_1}\sin\left[\left[(n+1)\omega t - n\gamma - \phi_1 - \dfrac{\pi}{6}\right]\right] \\ I_2 \leftarrow \sqrt{3}\dfrac{V_{\text{p}}K_n}{Z_2}\sin\left[(n-1)\omega t - n\gamma - \phi_2 + \dfrac{\pi}{6}\right] \\ I \leftarrow I + I_1 - I_2 \end{array} \right. \\ II \leftarrow 1 \end{array} \right.$$

$$I_{\text{oyb1}}(t) := \left| \begin{array}{l} 1 \leftarrow 0 \\ \gamma \leftarrow \theta_{\text{ybr}} \\ \text{for } n \in 1 \ldots N \\ \quad \left| \begin{array}{l} \text{K}_{\text{n}} \leftarrow \dfrac{\sin(n\delta)}{n\pi} \\ Z_1 \leftarrow \sqrt{R^2 + [\omega(n+1)L]^2} \\ \phi_1 \leftarrow \text{atan}\left[\omega\dfrac{(n+1)L}{R}\right] \\ \phi_2 \leftarrow \text{atan}\left[\omega\dfrac{(n-1)L}{R}\right] \\ Z_2 \leftarrow \sqrt{R^2 + [\omega(n-1)L]^2} \\ I_1 \leftarrow \sqrt{3}\dfrac{V_{\text{p}}K_n}{Z_1}\sin\left[\left[(n+1)\omega t - n\gamma - \phi_1 - \dfrac{\pi}{6}\right]\right] \\ I_2 \leftarrow \sqrt{3}\dfrac{V_{\text{p}}K_n}{Z_2}\sin\left[(n-1)\omega t - n\gamma - \phi_2 + \dfrac{\pi}{6}\right] \\ I \leftarrow I + I_1 - I_2 \end{array} \right. \\ II \leftarrow 1 \end{array} \right.$$

$$I_{\mathrm{obr2}}(t) := \left| \begin{array}{l} I \leftarrow 0 \\ \gamma \leftarrow \theta_{\mathrm{brr}} + \pi \\ \text{for } n \in 1 \ldots N \\ \quad \left| \begin{array}{l} K_n \leftarrow \dfrac{\sin(n\delta)}{n\pi} \\ Z_1 \leftarrow \sqrt{R^2 + [\omega(n+1)L]^2} \\ \phi_1 \leftarrow \operatorname{atan}\left[\omega\dfrac{(n+1)L}{R}\right] \\ \phi_2 \leftarrow \operatorname{atan}\left[\omega\dfrac{(n-1)L}{R}\right] \\ Z_2 \leftarrow \sqrt{R^2 + [\omega(n-1)L]^2} \\ I_1 \leftarrow \sqrt{3}\dfrac{V_{\mathrm{p}}K_n}{Z_1}\sin\left[\left[(n+1)\omega t - n\gamma - \phi_1 - \dfrac{\pi}{2}\right]\right] \\ I_2 \leftarrow \sqrt{3}\dfrac{V_{\mathrm{p}}K_n}{Z_2}\sin\left[(n-1)\omega t - n\gamma - \phi_2 + \dfrac{\pi}{2}\right] \\ I \leftarrow I + I_1 - I_2 \end{array} \right. \\ II \leftarrow 1 \end{array} \right.$$

$$I_{\mathrm{obr1}}(t) := \left| \begin{array}{l} I \leftarrow 0 \\ \gamma \leftarrow \theta_{\mathrm{brr}} \\ \text{for } n \in 1 \ldots N \\ \quad \left| \begin{array}{l} K_{\mathrm{n}} \leftarrow \dfrac{\sin(n\delta)}{n\pi} \\ Z_1 \leftarrow \sqrt{R^2 + [\omega(n+1)L]^2} \\ \phi_1 \leftarrow \mathrm{a}\tan\left[\omega\dfrac{(n+1)L}{R}\right] \\ \phi_2 \leftarrow \mathrm{a}\tan\left[\omega\dfrac{(n-1)L}{R}\right] \\ Z_2 \leftarrow \sqrt{R^2 + [\omega(n-1)L]^2} \\ I_1 \leftarrow \sqrt{3}\dfrac{V_{\mathrm{p}}K_n}{Z_1}\sin\left[\left[(n+1)\omega t - n\gamma - \phi 1 - \dfrac{\pi}{2}\right]\right] \\ I_2 \leftarrow \sqrt{3}\dfrac{V_{\mathrm{p}}K_n}{Z_2}\sin\left[(n-1)\omega t - n\gamma - \phi_2 + \dfrac{\pi}{2}\right] \\ I \leftarrow I + I_1 - I_2 \end{array} \right. \\ II \leftarrow 1 \end{array} \right.$$

$$I_{\mathrm{o}}(t) := I_{\mathrm{ory1}}(t) - I_{\mathrm{ory2}}(t) + I_{\mathrm{oyb1}}(t) - I_{\mathrm{oyb2}}(t) + I_{\mathrm{obr1}}(t) - I_{\mathrm{obr2}}(t)$$

Chapter 8

Overlap in ac to dc three-phase converters

8.1 Introduction

In the normal operation of a full-wave three-phase converter with inductive load the output dc current is diverted from thyristor to thyristor and from phase to phase in a regular manner. Two thyristors are normally conducting at a time: one from the upper group and one from the lower group, Fig.7.1. There are six thyristor commutations per mains cycle where the current is diverted through a thyristor to another phase. The output current is usually smoothed to its dc value by the load inductance. At the point of commutation the current does not become zero immediately in the conducting thyristor and it does not rise immediately from zero to the dc value in the thyristor which is taking the current because of the source inductance; it will take a finite time dictated by the value of the dc current, the magnitude of the line voltage and the value of the source inductance. During this finite time two thyristors are conducting from the same group and one from the other group. The two thyristors from the same group in effect constitute a short circuit of two phases via the source impedance; this is called overlap. No surge of current is observed but the fact that two phases are shorted reduces the voltage available to the load. More importantly 'notches' distort the line voltage.

The modes are increased from 6 to 12 to account for the commutation instances. Three new groups of switching functions are introduced in this chapter for the three-phase full-wave controlled rectifier. They will enable the derivation of an expression of the output voltage and the derivation of an expression for the line voltage. For the output voltage, one group of switching functions deals with the contribution of the modes with no overlap and the other deals with the contribution of the modes during overlap. For the line voltage the switching function accounts for the reduction of line voltage during overlap.

8.2 Operation and modes

There are six commutations per mains cycle. During each commutation there is a brief short-circuit of two phases, the outgoing and the incoming phase and two thyristors are conducting from the same group; this is known as overlap. The overlap angle γ is given as [5]

$$\gamma = \operatorname{acos}\left(1 - \frac{2I_{\mathrm{dc}}2\pi f L_{\mathrm{s}}}{V_{\mathrm{p}}\sqrt{3}}\right)$$

where

I_{dc} = dc value of output current
L_{s} = source inductance
f = mains frequency
V_{p} = peak value of mains voltage.

The calculated value of γ is in radians at 50 Hz.

8.2.1 Calculation of line voltage during overlap

Consider the change from Mode I to Mode II. Red (TH1) and yellow (TH5) phases are conducting (Mode I) and the blue phase (TH6) is coming in to replace the yellow phase (Mode II), Fig. 8.1. A brief short circuit between the yellow and the blue phase (TH5 and TH6) takes place (Mode I/II). The phase voltage under overlap conditions is half the normal phase voltage and phase advanced by 60° relative to the blue line, the incoming phase voltage. This is derived below.

Taking three loops of the circuit in Fig. 8.1

$$V_{\mathrm{ry}}(t) = V_{\mathrm{r}}(t) - V_{\mathrm{sYB}}(t)$$

$$V_{\mathrm{sYB}}(t) = V_{\mathrm{y}}(t) + V_{\mathrm{L}}(t)$$

$$V_{\mathrm{sYB}}(t) = V_{\mathrm{b}}(t) - V_{\mathrm{L}}(t)$$

Hence the phase voltage under overlap conditions (yellow–blue shorted) is given by

$$V_{\mathrm{sYB}}(t) = \tfrac{1}{2}[V_{\mathrm{y}}(t) + V_{\mathrm{b}}(t)]$$

By expanding and simplifying

$$= \tfrac{1}{2}[V_{\mathrm{p}}\sin(\omega t - 120^\circ) + V_{\mathrm{p}}\sin(\omega t + 120^\circ)]$$

$$V_{\mathrm{sYB}}(t) = -\frac{V_{\mathrm{p}}}{2}\sin(\omega t)$$

Expression above applies in Mode I/II and Mode IV/V. In the same way the other two short-circuit phase voltages are

$$V_{\mathrm{sBR}}(t) = \frac{V_{\mathrm{p}}}{2}\sin(\omega t - 60^\circ)$$

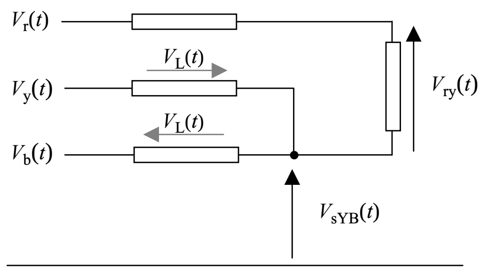


Figure 8.1 Mode I/II: overlap

Expression above applies in Mode II/III and Mode V/VI. In the same way the red and yellow lines are shortened in the next change over of phases.

The red and blue lines are shorted in the next change over Mode III/IV. And for mode III/IV

$$V_{sBR}(t) = \frac{V_p}{2}\sin(\omega t + 60^\circ)$$

It also applies for Mode VI/I.

8.2.2 Modes

Because of the overlap, the modes of the circuit are increased by six to become twelve; there are six new modes describing the overlap during the changeover of phases. Every mode under no overlap condition is followed by a new mode describing the overlap conditions. These are the Modes I/II, II/III, III/IV, IV/V, V/VI and VI/I. Their time duration is γ, the duration of the overlap. They are preceding the next mode: Mode II is preceded by Mode I/II, Mode III is preceded by Mode II/III, etc.

The six modes under no overlap condition are described already in Chapter 7: Modes I, II, III, IV, V and VI. The important difference is that their timing characteristics are slightly modified due to overlap. Their duration, the existence period as described in Chapter 1, is shorter by γ radians and they are also delayed by the same amount. Hence the contribution of these modes to the output voltage is still given by Expression (7.1) but the switching functions have to be modified to account for the overlap on the half-pulse width δ and the phase delay θ.

8.2.3 Mode I/II

8.2.3.1 Output voltage

This mode, Fig. 8.2, takes place when the red and yellow lines (Mode I) are supplying the load (TH1 and TH5, Fig. 7.1) and TH6 is fired so that the blue line takes over from the yellow; all three thyristors conduct for a period γ until the load current is transferred from the yellow line to the blue line.

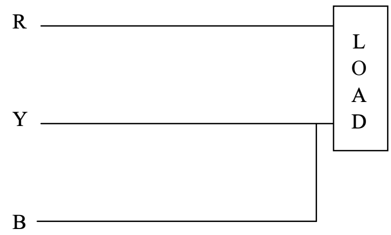


Figure 8.2 *Mode I/II of the three-phase converter with overlap. Lines Y–B are shorted*

The time slot for the transfer from the yellow to blue line is from $\alpha + 90^\circ$ to $\alpha + 90^\circ + \gamma$. During that period the phase voltage of the shorted lines is reduced to $V_{\mathrm{sYB}}(t)$. A switching function $F_{\mathrm{rYBO}}(t)$ is introduced associated with the output voltage for the period $\alpha + 90^\circ$ to $\alpha + 90^\circ + \gamma$.

The output voltage during Mode I/II

$$V_{\mathrm{o}}(t) = V_{\mathrm{r}}(t) - V_{\mathrm{sYB}}(t) \quad \text{for } F_{\mathrm{rYBO}}(t) = 1$$

8.2.3.2 Reductions on line voltages

The line voltages for this period are derived from Fig. 8.2

$$V_{\mathrm{RY}}(t) = V_{\mathrm{r}}(t) - V_{\mathrm{sYB}}(t)$$

$$V_{\mathrm{YB}}(t) = 0$$

$$V_{\mathrm{BR}}(t) = V_{\mathrm{sYB}}(t) - V_{\mathrm{r}}(t)$$

Hence the reductions of the line voltages are:

Reduction on red–yellow line voltage,

$$V_{\mathrm{ry}}(t)_{\mathrm{Reduction}} = V_{\mathrm{ry}}(t) - [V_{\mathrm{r}}(t) - V_{\mathrm{sYB}}(t)]$$

Reduction on yellow–blue line voltage, $V_{\mathrm{yb}}(t)_{\mathrm{Reduction}} = V_{\mathrm{yb}}(t)$

Reduction on blue–red line voltage, $V_{\mathrm{br}}(t)_{\mathrm{Reduction}} = [V_{\mathrm{r}}(t) - V_{\mathrm{sYB}}(t)] - V_{\mathrm{br}}(t)$

The associated switching function for the line voltages for this mode is $F_{\mathrm{YBS}}(t)$ indicating that yellow–blue lines are shorted, $F_{\mathrm{YBS}}(t) = 1$. This switching function is going to be used to calculate the reductions on the input line voltages.

8.2.4 Mode II/III

8.2.4.1 Output voltage

This mode, Fig. 8.3, takes place when the red and blue lines are supplying the load (TH1 and TH6, Fig. 7.1) and TH2 is fired so that the yellow line takes over from the red; all three thyristors are conducting for a period γ until the load current is transferred from the red line to the yellow line

The time slot for the transfer from the red to yellow line is from $\alpha + 150^\circ$ to $\alpha + 150^\circ + \gamma$. During that period the phase voltage of the shorted lines is reduced

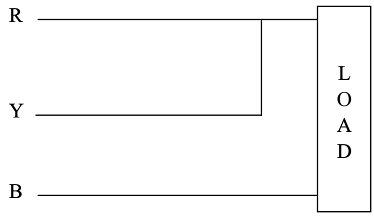


Figure 8.3 Mode II/III of the three-phase converter with overlap. Lines R–Y are shorted

to $V_{\mathrm{sRY}}(t)$. A switching function $F_{\mathrm{RYbO}}(t)$ is introduced associated with the output voltage for the period $\alpha + 150^\circ$ to $\alpha + 150^\circ + \gamma$.

The output voltage during Mode II/III

$$V_{\mathrm{o}}(t) = V_{\mathrm{b}}(t) - V_{\mathrm{sRY}}(t) \quad \text{and} \quad F_{\mathrm{RYbO}}(t) = 1$$

8.2.4.2 Reduction on input line voltage

Therefore the line voltages for this period are:

$$V_{\mathrm{RY}}(t) = 0 \quad V_{\mathrm{YB}}(t) = V_{\mathrm{sRY}}(t) - V_{\mathrm{b}}(t) \quad V_{\mathrm{BR}}(t) = V_{\mathrm{b}}(t) - V_{\mathrm{sRY}}(t)$$
$$\text{for } F_{\mathrm{RYbO}}(t) = 1$$

Hence the reductions of the line voltages are:

Reduction on red–yellow line voltage, $V_{\mathrm{ry}}(t)_{\mathrm{Reduction}} = V_{\mathrm{ry}}(t)$

Reduction on yellow–blue line voltage,

$$V_{\mathrm{yb}}(t)_{\mathrm{Reduction}} = V_{\mathrm{yb}}(t) - [V_{\mathrm{sRY}}(t) - V_{\mathrm{b}}(t)]$$

Reduction on blue–red line voltage,

$$V_{\mathrm{br}}(t)_{\mathrm{Reduction}} = V_{\mathrm{br}}(t) - [V_{\mathrm{b}}(t) - V_{\mathrm{sRY}}(t)]$$

The associated switching function for the line voltages for this mode is $F_{\mathrm{RYS}}(t)$ indicating that red and yellow lines are shorted, $F_{\mathrm{RYS}}(t) = 1$. This switching function is going to be used to calculate the reductions on the input line voltages.

8.2.5 MODE III/IV

8.2.5.1 Output voltage

This mode, Fig. 8.4, takes place when yellow and blue lines are supplying the load (TH2 and TH6, Fig. 7.1) and TH4 is fired so that the red line takes over from the blue; all three thyristors are conducting for a period γ until the load current is transferred from the blue line to the red line

The time slot for the transfer from the blue to red line is from $\alpha + 210^\circ$ to $\alpha + 210^\circ + \gamma$. During this period the phase voltage of the shorted lines is reduced to $V_{\mathrm{sBR}}(t)$. A switching function $F_{\mathrm{RyBO}}(t)$ introduced is associated with the output

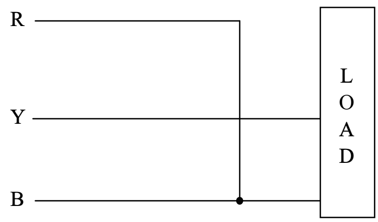


Figure 8.4 Mode III/IV of the three-phase converter with overlap. Lines R–B are shorted

voltage for the period $\alpha+210^\circ$ to $\alpha+210^\circ+\gamma$. The output voltage during Mode II/III is

$$V_o(t) = V_y(t) - V_{sBR}(t) \quad \text{for } F_{RyBO}(t) = 1$$

8.2.5.2 Reduction on line voltages

Therefore the line voltages for this period are:

$$V_{RY}(t) = V_{sBR}(t) - V_y(t) \quad V_{YB}(t) = V_y(t) - V_{sBR}(t) \quad V_{BR}(t) = 0$$
$$\text{for } F_{RyBO}(t) = 1$$

Hence the reductions of the line voltages are:

Reduction on red–yellow line voltage,

$$V_{ry}(t)_{\text{Reduction}} = V_{ry}(t) - [V_{sBR}(t) - V_y(t)]$$

Reduction on yellow–blue line voltage,

$$V_{yb}(t)_{\text{Reduction}} = V_{yb}(t) - [V_y(t) - V_{sBR}(t)]$$

Reduction on blue–red line voltage, $V_{br}(t)_{\text{Reduction}} = V_{br}(t)$

The associated switching function for the line voltages for this mode is $F_{RBS}(t)$ indicating that red and blue lines are shorted, $F_{RBS}(t) = 1$. This switching function is going to be used to calculate the reductions on the input line voltages.

8.2.6 Mode IV/V

8.2.6.1 Output voltage

This mode, Fig. 8.5, takes place when yellow and red lines are supplying the load (TH2 and TH4, Fig. 7.1) and TH3 is fired so that the blue line takes over from the yellow; all three thyristors conduct for a period γ until the load current is transferred from the yellow line to the blue line.

The time slot for the transfer from the yellow to blue line is from $\alpha + 270^\circ$ to $\alpha + 270^\circ + \gamma$. During that period the phase voltage of the shorted lines is reduced to $V_{sYB}(t)$. The switching function $F_{rYBO}(t)$ introduced in Mode I/II is associated with the output voltage for the period $\alpha + 270^\circ$ to $\alpha + 270^\circ + \gamma$. The output voltage during

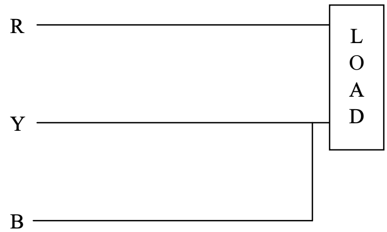


Figure 8.5 Mode IV/V of the three-phase converter with overlap. Lines Y–B are shorted

Mode IV/V

$$V_{\mathrm{o}}(t) = V_{\mathrm{r}}(t) - V_{\mathrm{sYB}}(t)$$

$F_{\mathrm{rYBO}}(t) = -1$ because the phases are connected to the load in reverse compared with Mode I/II. Hence the switching function $F_{\mathrm{rYBO}}(t)$ is clearly defined from the two modes I/II and IV/V as a bipolar function delayed by $\alpha + 90° + \gamma/2$ and of duration γ Fig. 8.8. Hence $\delta = \gamma/2$.

8.2.6.2 Reduction on line voltages

Therefore the line voltages for this period are:

$$V_{\mathrm{RY}}(t) = V_{\mathrm{r}}(t) - V_{\mathrm{sYB}}(t)$$

$$V_{\mathrm{YB}}(t) = 0$$

$$V_{\mathrm{BR}}(t) = V_{\mathrm{sYB}}(t) - V_{\mathrm{r}}(t)$$

Hence the reductions of the line voltages are:

Reduction on red–yellow line voltage,

$$V_{\mathrm{ry}}(t)_{\mathrm{Reduction}} = V_{\mathrm{ry}}(t) - [V_{\mathrm{r}}(t) - V_{\mathrm{sYB}}(t)]$$

Reduction on yellow–blue line voltage, $V_{\mathrm{yb}}(t)_{\mathrm{Reduction}} = V_{\mathrm{yb}}(t)$

Reduction on blue–red line voltage, $V_{\mathrm{br}}(t)_{\mathrm{Reduction}} = V_{\mathrm{br}}(t) - [V_{\mathrm{sYB}}(t) - V_{\mathrm{r}}(t)]$

The associated switching function for the line voltages for this mode is $F_{\mathrm{YBS}}(t)$ already introduced for Mode I/II . Since it is associted with a reduction and no inversion takes place, $F_{\mathrm{YBS}}(t) = 1$ for this mode. Hence the switching function $F_{\mathrm{YBS}}(t)$ Fig. 8.8 is clearly defined from the two modes I/II and IV/V as a unipolar function delayed by $\alpha + 90° + \gamma/2$ and of duration γ, that is, $\delta = \gamma/2$.

8.2.7 Mode V/VI

8.2.7.1 Output voltage

This mode, Fig. 8.6, takes place when red and blue lines are supplying the load (TH3 and TH4, Fig. 7.1) and TH5 is fired so that the yellow line takes over from the red;

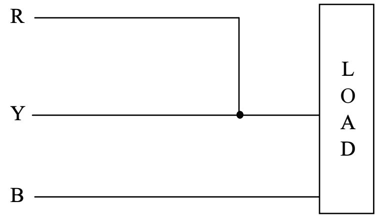


Figure 8.6 Mode V/VI of the three-phase converter with overlap

all three thyristors are conducting for a period γ until the load current is transferred from the red line to the yellow line.

The time slot for the transfer from the red to yellow line is from $\alpha + 330^\circ$ to $\alpha + 330^\circ + \gamma$. During that period the phase voltage of the shorted lines is reduced to $V_{\mathrm{sRY}}(t)$. The switching function $F_{\mathrm{RYbO}}(t)$ already introduced in Mode II/III is associated with the output voltage for the period $\alpha + 330^\circ$ to $\alpha + 330^\circ + \gamma$. The output voltage during Mode V/VI

$$V_{\mathrm{o}}(t) = V_{\mathrm{b}}(t) - V_{\mathrm{sRY}}(t) \quad \text{for } F_{\mathrm{RYbO}}(t) = -1$$

8.2.7.2 Reduction on line voltages

Therefore the line voltages for this period are:

$$V_{\mathrm{RY}}(t) = 0$$

$$V_{\mathrm{YB}}(t) = V_{\mathrm{sRY}}(t) - V_{\mathrm{b}}(t)$$

$$V_{\mathrm{BR}}(t) = V_{\mathrm{b}}(t) - V_{\mathrm{sRY}}(t)$$

Hence the reductions of the line voltages are:

Reduction on red–yellow line voltage, $V_{\mathrm{ry}}(t)_{\mathrm{Reduction}} = V_{\mathrm{ry}}(t)$

Reduction on yellow–blue line voltage,

$$V_{\mathrm{yb}}(t)_{\mathrm{Reduction}} = V_{\mathrm{yb}}(t) - [V_{\mathrm{sRY}}(t) - V_{\mathrm{b}}(t)]$$

Reduction on blue–red line voltage,

$$V_{\mathrm{br}}(t)_{\mathrm{Reduction}} = V_{\mathrm{br}}(t) - [V_{\mathrm{b}}(t) - V_{\mathrm{sRY}}(t)]$$

The associated switching function for the line voltages for this mode is $F_{\mathrm{RY}}(t)$ indicating that red and blue lines are shorted, $F_{\mathrm{RY}}(t) = 1$.

8.2.8 Mode VI/I

8.2.8.1 Output voltage

This mode, Fig. 8.7, takes place when yellow and blue lines are supplying the load (TH3 and TH5, Fig. 7.1) and TH1 is fired so that the red line takes over from the blue; all these thyristors are conducting for a period γ until the load current is transferred from the blue line to the red line.

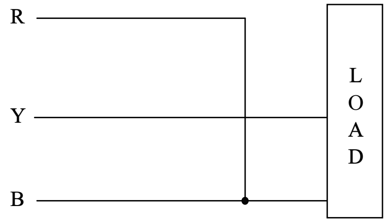


Figure 8.7 Mode VI/I of the three-phase converter with overlap

The time slot for the transfer from the blue to red line is from $\alpha + 390°$ to $\alpha + 390° + \gamma$. During that period the phase voltage of the shorted lines is reduced to $V_{\mathrm{sBR}}(t)$. The switching function $F_{\mathrm{RyBO}}(t)$ introduced in Mode III/IV is associated with that voltage for the period $\alpha + 390°$ to $\alpha + 390° + \gamma$. The output voltage during Mode II/III

$$V_{\mathrm{o}}(t) = V_{\mathrm{y}}(t) - V_{\mathrm{sBR}}(t) \quad F_{\mathrm{RyBO}}(t) = -1$$

8.2.8.2 Reduction on line voltages

Therefore the line voltages for this period are:

$$V_{\mathrm{RY}}(t) = V_{\mathrm{sBR}}(t) - V_{\mathrm{y}}(t)$$
$$V_{\mathrm{YB}}(t) = V_{\mathrm{y}}(t) - V_{\mathrm{sBR}}(t)$$
$$V_{\mathrm{BR}}(t) = 0$$

Hence the reductions of the line voltages are:

Reduction on red–yellow line voltage,

$$V_{\mathrm{ry}}(t)_{\mathrm{Reduction}} = V_{\mathrm{ry}}(t) - [V_{\mathrm{sBR}}(t) - V_{\mathrm{y}}(t)]$$

Reduction on yellow–blue line voltage,

$$V_{\mathrm{yb}}(t)_{\mathrm{Reduction}} = V_{\mathrm{yb}}(t) - [V_{\mathrm{y}}(t) - V_{\mathrm{sBR}}(t)]$$

Reduction on blue–red line voltage, $V_{\mathrm{br}}(t)_{\mathrm{Reduction}} = V_{\mathrm{br}}(t)$

The associated switching function for the line voltages for this mode is $F_{\mathrm{BRS}}(t)$, which indicates that red and blue lines are shorted, $F_{\mathrm{BRS}}(t) = 1$.

All information related to the modes and the switching functions is summarised in Table 8.1.

8.2.9 Switching functions

There are three types of switching functions for the circuit under overlap. Two are for the output voltage and one is for the line voltage.

Table 8.1 Three-phase full-wave controlled rectifier. Timing information

Conducting thyristor	Connected phases to the load	Conduction period with reference to red phase voltage
TH1–TH5	Red–Yellow	$\alpha + 30° + \gamma$ to $\alpha + 90°$ (Normal) Mode I
TH1–TH5 + TH6	Red–Yellow + Blue	$\alpha + 90°$ to $\alpha + 90° + \gamma$ (Overlap) Mode I/II
TH1–TH6	Red–Blue	$\alpha + 90°$ to $\alpha + 150°$ (Normal) Mode II
TH1 + TH2–TH6	Red + Yellow–Blue	$\alpha + 150°$ to $\alpha + 150° + \gamma$ (Overlap) Mode II/III
TH2–TH6	Yellow–Blue	$\alpha + 150°$ to $\alpha + 210°$ (Normal) Mode III
TH2–TH6 + TH4	Yellow–Blue$^-$ + Red	$\alpha + 210°$ to $\alpha + 210° + \gamma$ (Overlap) Mode III/IV
TH2–TH4	Yellow–Red	$\alpha + 210°$ to $\alpha + 270°$ (Normal) Mode IV
TH2 + TH3–TH4	Yellow$^+$ +Blue$^+$ – Red	$\alpha + 270°$ to $\alpha + 270° + \gamma$ (Overlap) Mode IV/V
TH3–TH4	Blue–Red	$\alpha + 270°$ to $\alpha + 330°$ (Normal) Mode V
TH3–TH4 + TH5	Blue–Red$^-$ + Yellow$^-$	$\alpha + 330°$ to $\alpha + 330° + \gamma$ (Overlap) Mode V/VI
TH3–TH5	Blue–Yellow	$\alpha + 330°$ to $\alpha + 390°$ (Normal) Mode VI
TH3 + TH1–TH5	Blue + Red–Yellow	$\alpha + 390°$ to $\alpha + 390° + \gamma$ (Overlap) Mode VI/I

8.2.9.1 Switching functions for the periods free from overlap for the output voltage

These switching functions are exactly the same as those derived in Chapter 7 – $F_{\mathrm{ry}}(t)$, $F_{\mathrm{yb}}(t)$ and $F_{\mathrm{br}}(t)$ – but they are delayed by γ radians due to overlap hence θ and δ are modified as shown below

$$\delta := \frac{(\pi/3) - \gamma}{2} \qquad \theta := \delta + \frac{\pi}{6} + \gamma + \alpha$$

α is the delay firing angle in radians.

And the switching functions are shown below (8.1) and displayed in Fig. 8.1.

$$F_{\mathrm{ryO}}(t) := 4\sum_{n=1}^{N}\left|\sin\left(\frac{n\pi}{2}\right)\right|\frac{\sin(n\delta)}{\pi n}\cos(n\omega t - n\theta)$$

$$F_{\mathrm{ybO}}(t) := 4\sum_{n=1}^{N}\left|\sin\left(\frac{n\pi}{2}\right)\right|\frac{\sin(n\delta)}{\pi n}\cos\left(n\omega t - n\theta + \frac{\pi}{3}n - n\pi\right) \tag{8.1}$$

$$F_{\mathrm{brO}}(t) := 4\sum_{n=1}^{N}\left|\sin\left(\frac{n\pi}{2}\right)\right|\frac{\sin(n\delta)}{\pi n}\cos\left(n\omega t - n\theta - \frac{\pi}{3}n - n\pi\right)$$

These switching functions are displayed in Fig. 8.8.

8.2.9.2 Switching functions for the periods during overlap for the output voltage

These are the switching functions $F_{\mathrm{rYBO}}(t)$, $F_{\mathrm{RyBO}}(t)$ and $F_{\mathrm{RYbO}}(t)$ with a short on-period of γ radians. $F_{\mathrm{rYBO}}(t)$, applies for Mode I/II and Mode IV/V with an inversion

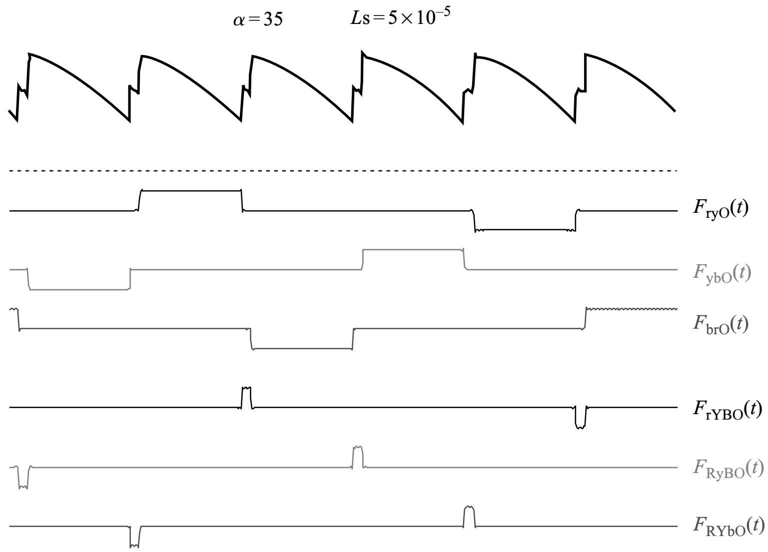


Figure 8.8 The output voltage with overlap and the associated switching functions

suggesting that they are bipolar switching functions. In the same way $F_{RyBO}(t)$ applies for Mode II/III and Mode V/V I and $F_{RYbO}(t)$ applies for Mode III/IV and Mode VI/I. The delay angle for the switching functions is θ_0.

$$\theta_0 := \frac{\pi}{3} + \frac{\gamma}{2} + \frac{\pi}{6} + \alpha$$

where α is the delay firing angle in radians.

Each switching function is phase delayed by 60° from each other as shown in Expression (8.1). These switching functions are displayed in Fig. 8.8.

$$F_{rYBO}(t) = 4\sum_{n=1}^{\mathrm{N}} \left|\sin\left(\frac{n\pi}{2}\right)\right| \frac{\sin(n(\gamma/2))}{\pi n} \cos(n\omega t - n\theta_0)$$

$$F_{RYbO}(t) = 4\sum_{n=1}^{\mathrm{N}} \left|\sin\left(\frac{n\pi}{2}\right)\right| \frac{\sin(n(\gamma/2))}{\pi n} \cos\left(n\omega t - n\theta_0 - \left(\frac{\pi}{3}\right) n\right)$$

$$F_{RyBO}(t) = 4\sum_{n=1}^{N} \left|\sin\left(\frac{n\pi}{2}\right)\right| \frac{\sin(n(\gamma/2))}{\pi n}$$

$$\times \cos\left(n\omega t - n\theta_0 + \left(\frac{\pi}{3}\right) n - n\pi\right) \tag{8.2}$$

8.2.9.3 Switching functions for the line voltage

There is only one type of switching function for the line voltage, one for each phase. As is suggested from the mode-pairs I/II–IV/V, II/III–V/VI and III/IV–VI/I these switching functions are of the unipolar type and they have two pulses per mains cycle. Their duration is γ delayed by $\alpha + 90° + \gamma/2$ and of duration $\delta_o = \gamma/2$. As usual there is a further delay of 60° between each switching function due to the three-phase environment.

$$
\begin{aligned}
F_{\mathrm{BRS}}(t) &= \mathrm{Ko}1 + 2\sum_{\mathrm{n}=1}^{\mathrm{N}} \frac{\sin(n\delta_o)}{\pi n}\left[\cos\left[n\omega t2 - n2\left(\alpha + \frac{\gamma}{2} + \frac{\pi}{6}\right)\right]\right] \\
F_{\mathrm{YBS}}(t) &= \mathrm{Ko}1 + 2\sum_{n=1}^{N} \frac{\sin(n\delta_o)}{\pi n}\left[\cos\left[n\omega t2 - n2\left(\alpha + \frac{\gamma}{2} + \frac{\pi}{2}\right)\right]\right] \qquad (8.3)\\
F_{\mathrm{RYS}}(t) &= \mathrm{Ko}1 + 2\sum_{n=1}^{N} \frac{\sin(n\delta_o)}{\pi n}\left[\cos\left[n\omega t2 - n2\left(\alpha + \frac{\gamma}{2} + \frac{\pi}{6}5\right)\right]\right]
\end{aligned}
$$

8.3 Analysis

8.3.1 Output voltage

The output voltage is made from the contribution of each mode. This is already done in Chapter 7 for the circuit without overlap in Expression (7.1). This expression is expanded here to include the M I/II, II/III, III/IV, IV/V, V/VI and VI/I. The first three terms of Expression (8.4) account for the contributions of the modes under no overlap and the last three for the modes with overlap.

$$
\begin{aligned}
V_o(t) = {} & V_{\mathrm{ry}}(t)F_{\mathrm{ryO}}(t) + F_{\mathrm{brO}}(t)V_{\mathrm{br}}(t) + F_{\mathrm{ybO}}(t)V_{\mathrm{yb}}(t) \\
& + [V_r(t) - V_{\mathrm{sYB}}(t)]F_{\mathrm{rYBO}}(t) + [V_b(t) - V_{\mathrm{sRY}}(t)]F_{\mathrm{RYbO}}(t) \\
& + [V_y(t) - V_{\mathrm{sBR}}(t)]F_{\mathrm{ByRO}}(t) \qquad (8.4)
\end{aligned}
$$

The output voltage is displayed in Fig. 8.8.

8.3.2 Line voltage under overlap

The line voltages under overlap are derived in the following way. A single switching function is introduced for each line voltage describing the circuit under overlap conditions. This switching function will give the reduction of the line voltage due to overlap. We have reductions only during overlap, that is, during mode I/II, II/III, III/IV and the corresponding modes IV/V, V/VI and VI/I, 180° later. Let us consider the derivation of the red–yellow line voltage with overlap in some detail. During mode I/II the actual line red–yellow line voltage is:

$$V_r(t) - V_{\mathrm{sYB}}(t)$$

Hence the reduction for that mode is:

$$V_{ry}(t) - [V_r(t) - V_{sYB}(t)]$$

The associated switching function is $F_{YBS}(t)$. In the same way by considering the reduction in the first three overlap modes, the overall reduction in this line voltage is

$$V_{ryReduction}(t) = [V_{ry}(t) - V_r(t) + V_{sYB}(t)]F_{YBS}(t) + V_{ry}(t)F_{RYS}(t) + [V_{ry}(t) + V_y(t) - V_{sBR}(t)]F_{BRS}(t)$$

And the red–yellow line voltage under overlap is

$$V_{ryO}(t) = V_{ry}(t) - V_{ryReduction}(t) \tag{8.5a}$$

In the same way the other two line voltages are derived.

$$V_{ybReduction}(t) = V_{yb}(t)F_{YBS}(t) + [V_{yb}(t) - V_{sRY}(t) + V_b(t)]F_{RYS}(t) + [V_{yb}(t) + V_{sBR}(t) - V_y(t)]F_{BRS}(t)$$

And the yellow–blue line voltage under overlap is

$$V_{ybO}(t) = V_{yb}(t) - V_{ybReduction}(t) \tag{8.5b}$$

$$V_{brReduction}(t) = [V_{br}(t) - V_{sYB}(t) + V_r(t)]F_{YBS}(t) + [V_{br}(t) + V_{sRY}(t) - V_b(t)]F_{RYS}(t) + V_{br}(t)F_{BRS}(t)$$

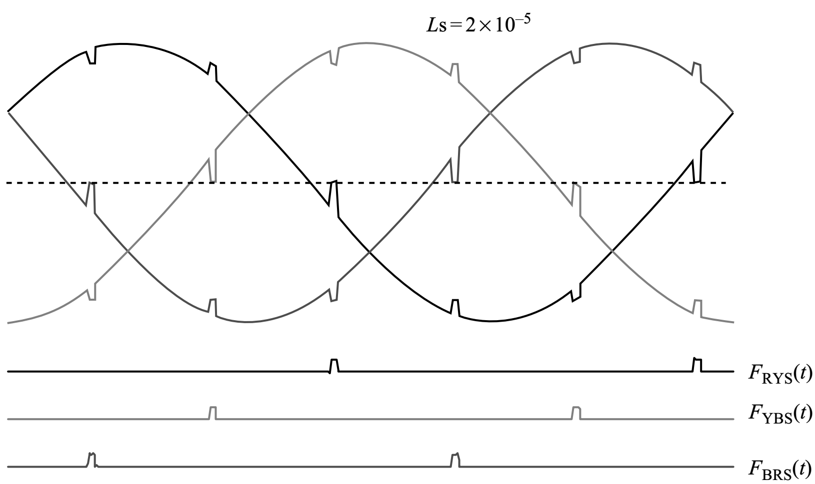


Figure 8.9 Line voltage notches and the associated switching functions

And the blue–red line voltage under overlap is

$$V_{\text{brO}}(t) = V_{\text{br}}(t) - V_{\text{ybReduction}}(t) \tag{8.5c}$$

The line voltages with overlap are displayed in Fig. 8.9.

It is possible, by further expansion, to collect all the reductions of the output voltage in a single expression. By expanding this expression, both the dc value of the loss of voltage and the related harmonics can be derived.

Part 3

DC to DC converters

Fine common circuits are presented here for both continuous and discontinuous conduction. The dc to dc step down is a rather simple application of the technique. The rest of the circuits present a challenge because an impedance, an inductor, is inserted between the voltage source and the semiconductor switches. In this case the voltage equations for the loops are derived by considering the various modes of the circuit in order to derive the switching functions and the derived expressions.

Chapter 9
The step down converter

9.1 Introduction

Figure 9.1 shows a simple circuit of the step down converter. The chopped voltage of a dc source is applied to a series inductor L and a parallel combination of a smoothing capacitor C and a resistor R. The resistor represents the power-consuming element of the load. The inductor has inductance L and ohmic resistance r. The switch is a MOSFET or an IGBT transistor. A control circuit, not shown in the diagram, generates the gate pulses to switch on and off the transistor. The switch opens and closes at a fixed frequency. The duty-cycle of the switch determines the level of the output voltage.

It is possible for this circuit to enter discontinuous conduction and the relevant mode is derived in this chapter. The analysis though will be restricted to continuous conduction only. The interested reader may apply the procedure introduced in Chapter 1 in order to extend the analysis to discontinuous conduction as well.

9.2 Mathematical modelling of the step down converter

9.2.1 Operation and modes of operation

When the switch is open, the current circulates through the diode. Thus the diode conducts in anti-parallel to the switch. The switching action of the semiconductor switch is described by the unipolar switching function, $F(t)$ and the switching action of the diode is described by the inverse of $F(t)$, for continuous conduction. For discontinuous conduction the diode is conducting for only part of the off period of $F(t)$ and a new switching function must be defined.

The circuit exhibits three modes: Mode I, the switch is closed and the input voltage is applied to the RLC combination. Mode II, the switch is open and the diode carries the inductor current. In Mode III, neither the switch nor the diode is conducting (Figure 9.2).

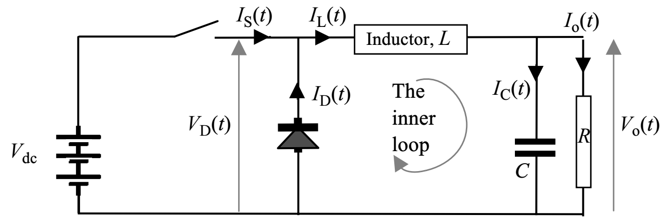


Figure 9.1 The step down converter

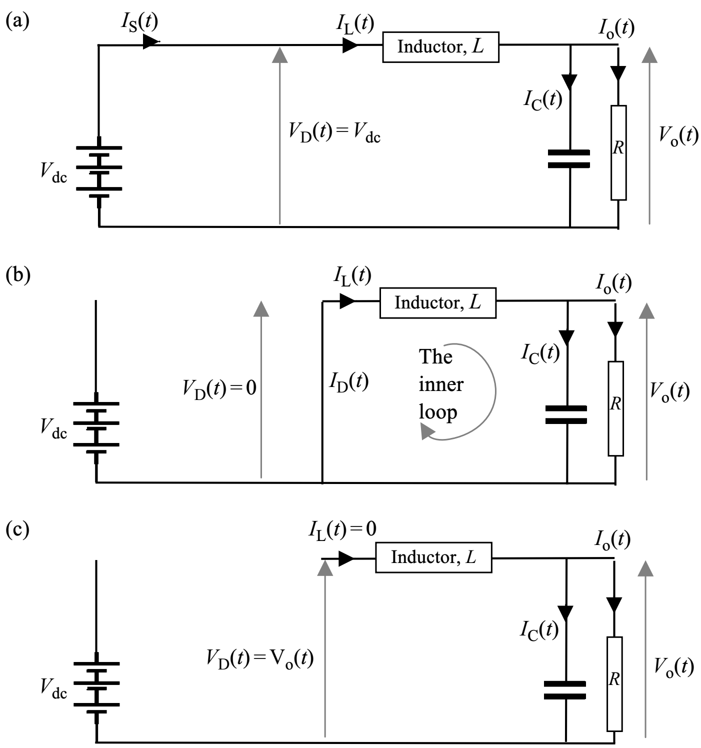


Figure 9.2 The modes of the circuit: (a) Mode I, (b) Mode II and (c) Mode III

The various voltages and currents are marked on Fig. 9.1. The voltage across the diode, $V_D(t)$ takes the value of V_{dc} in Mode I and zero in Mode II. In Mode III it takes value of the output voltage $V_o(t)$. It is the case of a switch connecting a voltage source to a load, Chapter 2, Expression (2.1). Hence, for continuous conduction where Mode III does not exist

$$V_D(t) = F(t)V_{dc} \tag{9.1}$$

The switching function is of the unipolar type

$$F(t) = K_o + 2\sum_{n=1}^{\infty} K_n \cos(n\omega t) \tag{9.2}$$

The current, $I_L(t)$, through the inductor is diverted from the switch, Mode I to the diode, Mode II.

With the switch closed, $F(t) = 1, \quad I_L(t) = I_S(t)$

With the switch open, $F(t) = 0, \quad I_L(t) = I_D(t)$

Therefore the current through the switch, $I_S(t)$ takes the value of $I_L(t)$ during the on periods of the switching function.

$$I_S(t) = I_L(t)F(t) \tag{9.3}$$

The diode current, $I_D(t)$ takes the values of $I_L(t)$ during the off periods of the switching function

$$I_D(t) = I_L(t)[1 - F(t)] \tag{9.4}$$

The current at the diode-switch-inductor junction is given by

$$I_L(t) = I_S(t) + I_D(t) \tag{9.5}$$

The ac component of the voltage across the LRC load, $V_{Dac}(t)$, is pushing an ac current $I_{Lac}(t)$ into the 'harmonic impedance', $Z_{RLC}(\omega n)$, consisting from the series combination of L–r and the parallel combination of R–C.

$$I_{Lac}(t) = \frac{V_{Dac}(t)}{Z_{RLC}(t)} \tag{9.6}$$

The harmonic impedance of the network $Z_{RLC}(wn)$ is given by

$$Z_{RLC}(\omega n) = \frac{\sqrt{(L\omega n + \omega n r RC)^2 + [R + r - (\omega n)^2 CLR]^2}}{\sqrt{1 + (\omega CRn)^2}}$$

$$\Phi(n) := \operatorname{atan}\left[\frac{\omega Ln}{R + r - (\omega n)^2 CLB}\right] - \operatorname{atan}(\omega CRn) \tag{9.7}$$

The voltage equation of the inner loop, Fig. 9.1 is given by

$$V_D(t) = V_L(t) + V_o(t) \tag{9.8}$$

The output voltage consists of a dc component, V_{dc} and the ripple component, $V_{oac}(t)$

$$V_o(t) = V_{dco} + V_{oac}(t) \tag{9.9}$$

The voltage across the diode, $V_D(t)$, is the applied voltage to the load. It has a dc and an ac component

$$V_D(t) = V_{Ddc} + V_{Dac}(t) \tag{9.10}$$

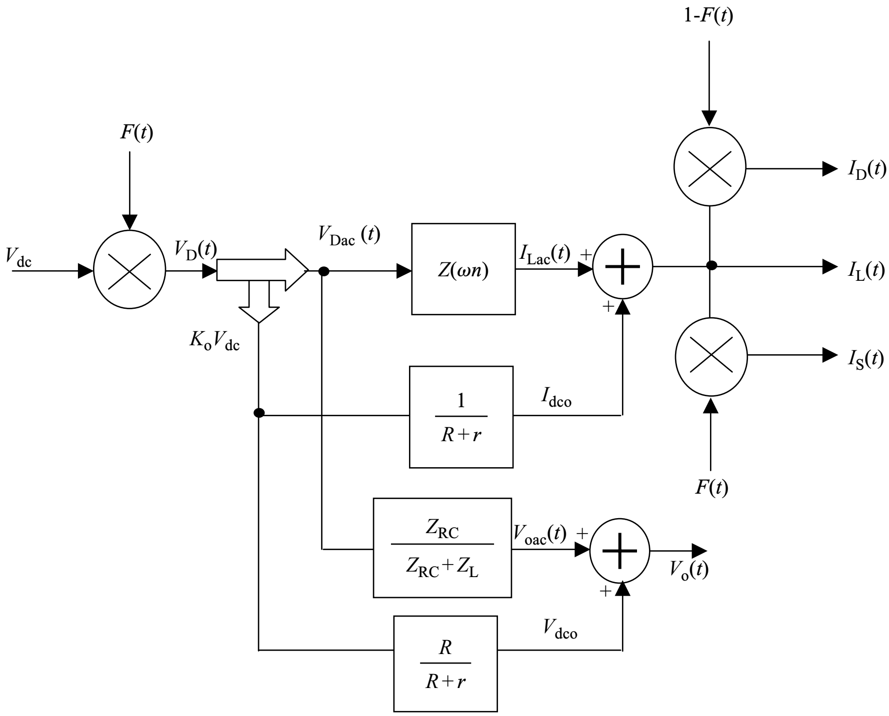


Figure 9.3 The mathematical model of the step down converter

9.2.2 The mathematical model of the step down converter

Expressions (9.1)–(9.10) are employed to build the mathematical model of the step down converter (Fig. 9.3).

9.2.3 Analysis

The voltage across the diode is given by Expression (9.1). Replacing $F(t)$ and expanding

$$V_D(t) = V_{dc}K_o + 2V_{dc}\sum_{n=1}^{\infty} K_n \cos(n\omega t)$$

The dc component and the ac components of $V_D(t)$ are now derived from this expression as

$$V_{Ddc} = V_{dc}K_o \tag{9.11}$$

$$V_{Dac}(t) = 2V_{dc}\sum_{n=1}^{\infty} K_n \cos(n\omega t) \tag{9.12}$$

The dc component V_{Ddc} is shared between the internal resistance of the inductor r and the load resistor R. Hence the output dc voltage V_{dco} is given by

$$V_{\text{dco}} = \frac{R}{R+r} V_{\text{Ddc}} K_{\text{o}} \tag{9.13}$$

The ac component of $V_{\text{D}}(t)$ is pushing an ac current through the 'harmonic impedance', $Z_{\text{RLC}}(\omega n)$. This is the inductor ac component of current, $I_{\text{Lac}}(t)$. From Expressions (9.6) and (9.7) we have

$$I_{\text{Lac}}(t) = 2V_{\text{dc}} \sum_{n=1}^{\infty} \frac{K_n}{Z_{\text{RLC}}(\omega n)} K_n \cos[n\omega t - \Phi(n)] \tag{9.14}$$

Practically all of this ac current will flow through the capacitor. Its rms value, RMS_IL is important in selecting the smoothing capacitor.

$$\text{RMS\_IL} := \sqrt{\frac{1}{T} \int_0^T I_{\text{Lac}}^2(t)\, \mathrm{d}t}$$

The value of RMS_IL shown below is for the circuit and load parameters

$$f = 1 \times 10^5 \quad V_{\text{dc}} = 110 \quad K_{\text{o}} = 0.3$$

$$L = 5 \times 10^{-6} \quad R = 1.25 \quad C = 6 \times 10^{-5}$$

$$\text{RMS\_IL} = 13.429\ \text{A}$$

The dc component of the inductor current, I_{LDC} is also the output dc current.

$$I_{\text{Ldc}} = \frac{K_{\text{o}} V_{\text{dc}}}{R+r} \tag{9.15}$$

Hence

$$I_{\text{L}}(t) = I_{\text{Ldc}} + I_{\text{Lac}}(t) \tag{9.16}$$

The harmonic output voltage, $V_{\text{oac}}(t)$, is of some interest to the designer and it is easily found by considering the voltage divider rule. The ac component of the voltage $V_{\text{D}}(t)$ is divided between the inductor impedance Z_{L} and the RC network impedance Z_{RC}.

$$V_{\text{oac}}(t) = V_{\text{Dac}}(t) \frac{Z_{\text{RC}}}{Z_{\text{RC}} + Z_{\text{L}}} \tag{9.17}$$

The rms value of this ripple voltage, RMS_Vor is derived from

$$\text{RMS\_Vor} := \sqrt{\frac{1}{T} \int_0^{\text{T}} \text{V}_{\text{oac}}(t)^2\, \mathrm{d}t} \qquad \text{RMS\_Vor} = 0.345\ \text{A}$$

The value of RMS_Vor shown is for the circuit and load parameters

$$f = 1 \times 10^5 \quad V_{\text{dc}} = 110 \quad K_{\text{o}} = 0.3$$

$$L = 5 \times 10^{-6} \quad R = 1.25 \quad C = 6 \times 10^{-5}$$

The voltage and current waveforms are shown in Fig. 9.4.

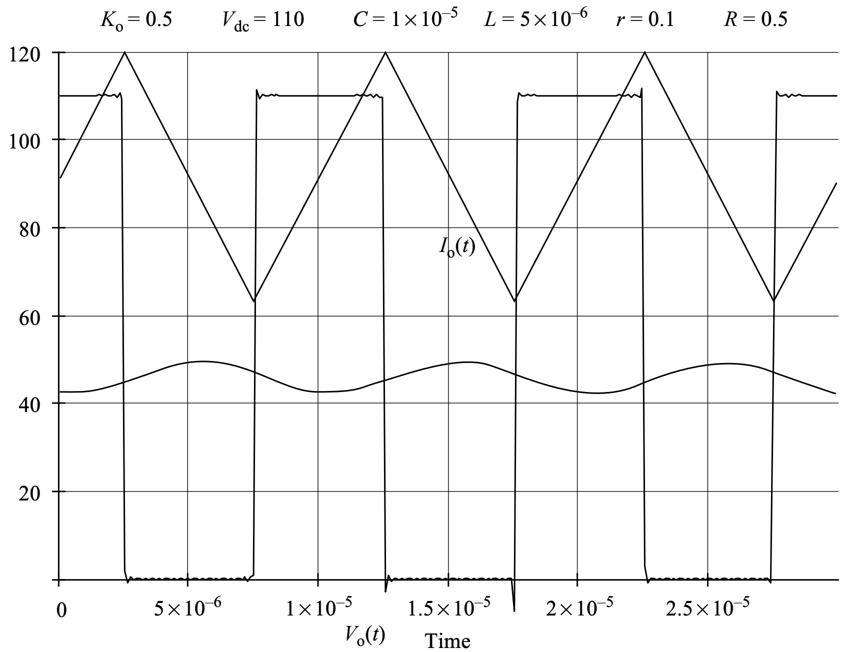


Figure 9.4 The voltage and current waveforms for the step down dc to dc converter

9.2.4 Semiconductor current ratings

The average and rms values of the currents through the transistor and the diode are derived from Expressions (9.3) and (9.4). They are of interest to the designer for the ratings of the semiconductor devices. The values shown are for the following circuit and load parameters:

$$f = 1 \times 10^5 \quad V_{dc} = 110 \quad K_o = 0.3$$
$$L = 5 \times 10^{-6} \quad R = 1.25 \quad C = 6 \times 10^{-5}$$

The average value of the current through the diode, ID_AVE

$$\mathrm{ID\_AVE} := \frac{1}{T}\int_0^{\mathrm{T}} \mathrm{ID}(t)\,\mathrm{d}t \quad \mathrm{ID\_AVE} = 16.68\,\mathrm{A}$$

The average value of the current through the transistor switch, IS_AVE

$$\mathrm{IS\_AVE} := \frac{1}{T}\int_0^{\mathrm{T}} \mathrm{IS}(t)\,\mathrm{d}t \quad \mathrm{IS\_AVE} = 7.765\,\mathrm{A}$$

The rms value of the current through the transistor switch, RMS_IS

$$\mathrm{RMS\_IS} := \sqrt{\frac{1}{T}\int_0^{\mathrm{T}} \mathrm{IS}(t)^2 \, \mathrm{d}t} \quad \mathrm{RMS\_IS} = 16.68\,\mathrm{A}$$

The rms value of the current through the diode, RMS_ID

$$\mathrm{RMS\_ID} := \sqrt{\frac{1}{T}\int_0^{\mathrm{T}} \mathrm{ID}(t)^2 \, \mathrm{d}t} \quad \mathrm{RMS\_ID} = 7.765\,\mathrm{A}$$

Chapter 10

The step up or boost converter

10.1 Introduction

The analysis of the circuit shown in Fig. 10.1 is based on the voltage equation of the input loop: dc source V_{dc}, voltage across the inductance $V_L(t)$ and the voltage across the switched network, $V_A(t)$. Kirchhoff's Voltage Law cannot be applied unless the voltage across the switched network is expressed as a single expression valid for all modes of the circuit, a 'unified expression'. This is done by applying superposition theorem on the switched network in the way explained in Chapter 2.

Three switching functions associated with the three modes of the circuit are identified; the forced switching function of the semiconductor switch, the switching function of the diode and the switching function for the circuit at the state where the source current is zero. The last two switching functions are 'hidden' in the sense that they are circuit determined. Continuous and discontinuous conduction are both treated in the same way; simply the parameters of the 'hidden' switching functions change.

10.2 Mathematical modelling of the dc to dc step up (boost) converter

10.2.1 *Operation and modes of the circuit*

The switch is operated by a 'unipolar' switching function $F_1(t)$ and the circuit undergoes three modes (Fig. 10.2). With the switch closed, $F_1(t) = 1$, the diode is reversed biased, it is not conducting and the circuit is in Mode I. The voltage across the switch, $V_A(t)$ is zero. A charging current $I_L(t)$ flows through the inductor.

$$V_A(t) = 0 \quad \text{for } F_1(t) = 1$$

With the switch open, $F_1(t) = 0$, the diode is forward biased, it is conducting and the circuit is in Mode II. The voltage across the switch, $V_A(t)$, is the output voltage less

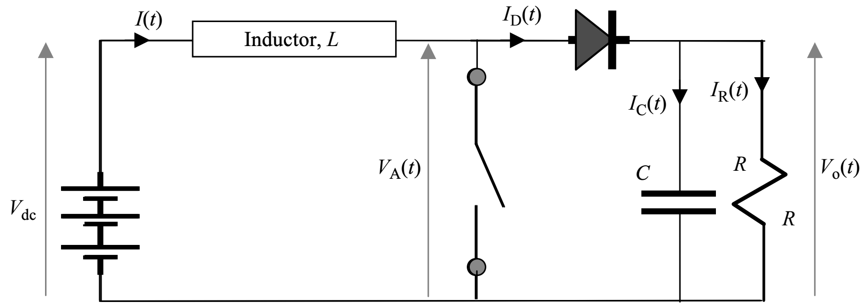


Figure 10.1 DC to DC step up (boost) converter

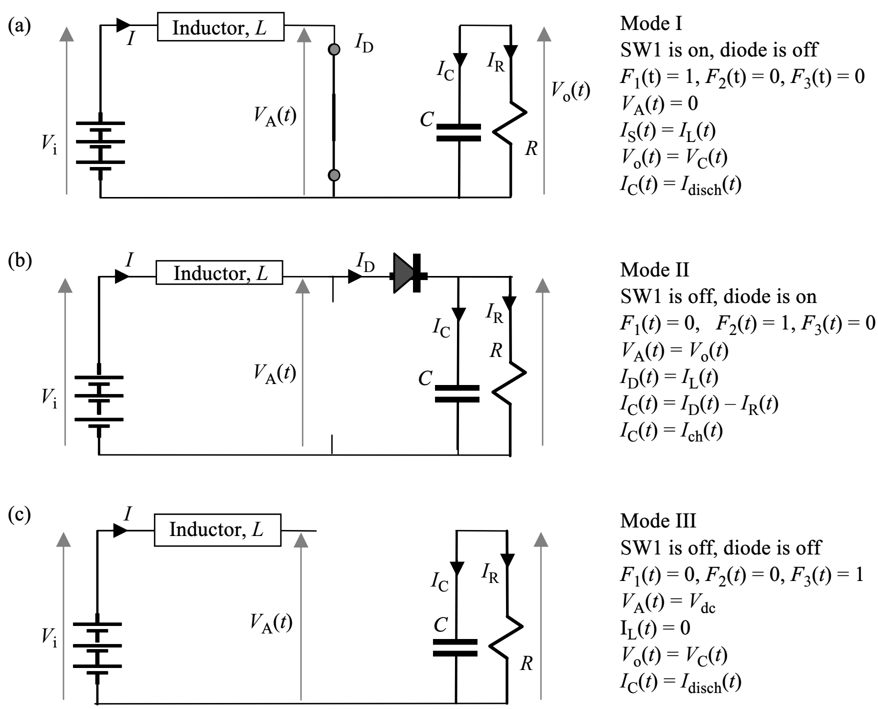


Figure 10.2 The modes of the boost dc to dc converter

the diode voltage drop; the diode voltage drop is ignored at the moment. A discharging current $I_L(t)$ flows through the inductor. Another unipolar switching function $F_2(t)$ is introduced to describe this mode: it takes the value of one as long as the diode is conducting.

$$V_A(t) = V_o(t) \quad \text{for } F_2(t) = 1$$

In the event of discontinuous conduction, a third mode is entered. In this mode the discharging inductor current becomes zero before the switch is closed again and the diode is not conducting. With the inductor current zero, the voltage across the switch takes the value of the input dc voltage, V_{dc}. A third unipolar switching function is introduced, $F_3(t)$ for this state.

$$V_A(t) = V_{dc} \quad \text{for } F_3(t) = 1$$

$F_3(t)$ is a unipolar switching function which takes the value of one as long as no current is flowing in the inductor. The switch is open $F_2(t) = 0$, $F_1(t) = 0$.

The sequence of modes is Mode I $\rightarrow$ Mode II $\rightarrow$ Mode III (Fig 10.3) and the mathematical model is shown in Fig. 10.4.

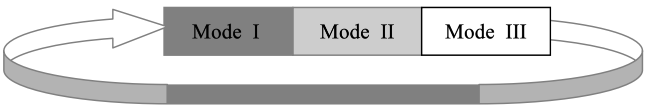


Figure 10.3 Mode sequence for the dc to dc step up converter

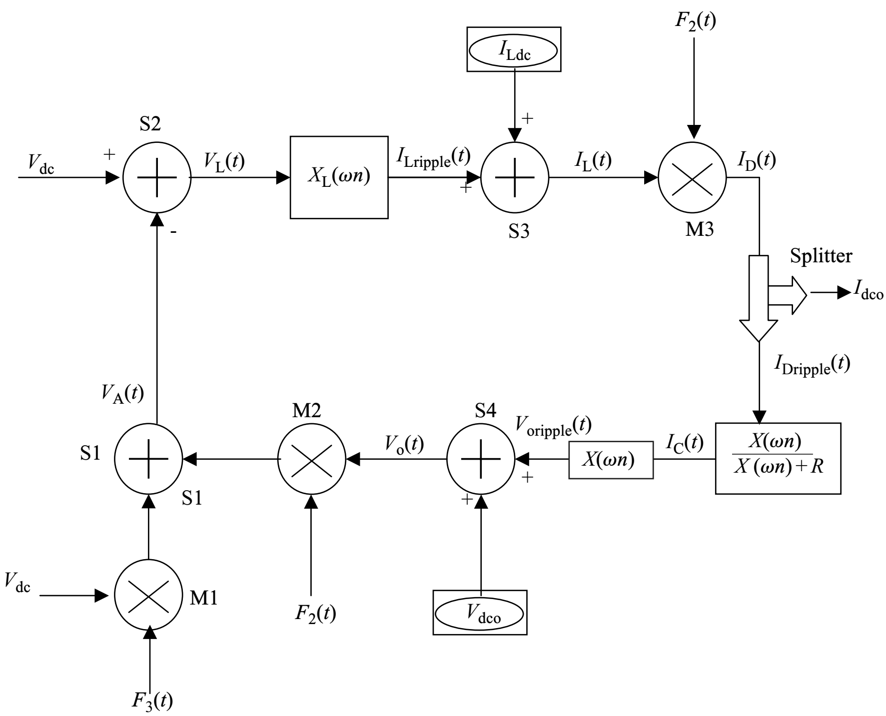


Figure 10.4 The mathematical model for the step up dc to dc converter

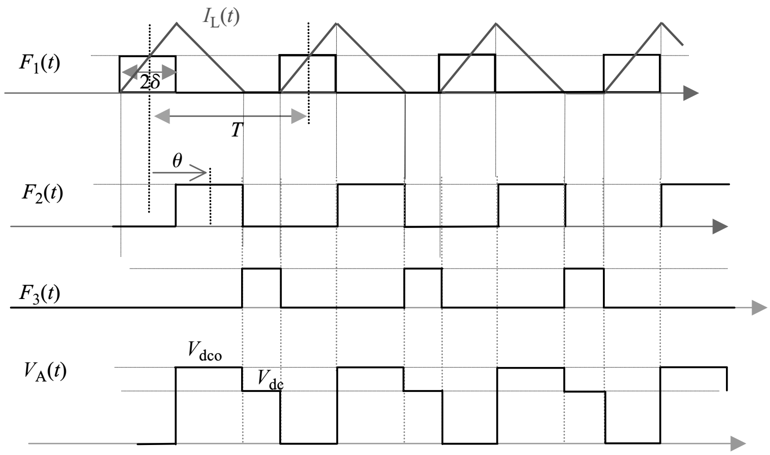


Figure 10.5 Switching functions for the step up dc to dc converter with idealised inductor current and switch voltage $V_A(t)$ waveforms – with discontinuous conduction. Straight line approximation

10.2.2 Voltage across the transistor switch

The voltage across the switch, V_A(t), can now be derived by considering the contributions of all the modes. Only Modes II and III contribute to it and according to Expression (2.8), Chapter 2:

$$V_A(t) = V_o(t)F_2(t) + V_{dc}F_3(t) \tag{10.1}$$

Expression (10.1) is valid for both continuous and discontinuous conduction. For continuous conduction $F_3(t)$ takes the value of zero for all times and (10.1) is reduced to

$$V_A(t) = V_o(t)F_2(t)$$

10.2.3 Current through the transistor switch and the diode

The three switching functions and the voltage across the switch, $V_A(t)$ are displayed in Fig. 10.5 as a theoretical straight line approximation. The inductor current is diverted from the switch to the diode in a manner dictated by the switching functions.

$$I_D(t) = I_L(t)F_2(t) \tag{10.2}$$

The current through the switch is the difference between the inductor current and the diode current

$$I_{SWITCH}(t) = I_L(t) - I_D(t) \tag{10.3}$$

10.2.4 Capacitor ripple current

In Modes III and I the capacitor discharges and this action must be considered in the mathematical model. The sequence of the modes (Fig. 10.3) is such that Mode III is succeeded by Mode I, therefore their effect can be considered together. During Modes I and III the capacitor is discharging with current, $I_{\text{disch}}(t)$. The discharging current, $I_{\text{disch}}(t)$, during these modes is approximated to the output current, $I_{\text{dco}} = V_{\text{dco}}/R$. Hence the contribution of Modes I and III to the capacitor current is given by

$$I_{\text{C}}(t) = -I_{\text{dco}}[1 - F_2(t)]$$

During Mode II it is charging with a current $I_{\text{ch}}(t)$. This current is the diode current, $I_{\text{D}}(t)$, less the current which flows in the load. The load current is rightly approximated to the dc component I_{dco} in the presence of a large smoothing capacitor. Hence the contribution of Modes II to the capacitor current is given by

$$I_{\text{Ccharging}}(t) = [I_{\text{D}}(t) - I_{\text{dco}}]F_2(t)$$

Hence, by considering the sequence of modes and their contribution to the capacitor current, the capacitor current is given by

$$I_{\text{C}}(t) = -I_{\text{dco}}[1 - F_2(t)] + [I_{\text{D}}(t) - I_{\text{dco}}]F_2(t)$$

Substituting Expression (10.3) for the diode current, $I_{\text{D}}(t) = I_{\text{L}}(t)F_2(t)$

$$I_{\text{C}}(t) = -I_{\text{dco}}[1 - F_2(t)] + [I_{\text{L}}(t)F_2(t) - I_{\text{dco}}]F_2(t)$$

$$I_{\text{C}}(t) = -I_{\text{dco}}[1 - F_2(t)] + I_{\text{L}}(t)F_2(t)^2 - I_{\text{dco}}F_2(t)$$

$F_2(t)$ is a unipolar switching function, and $F_2(t)^2 = F_2(t)$ (Chapter 1).

$$I_{\text{C}}(t) = -I_{\text{dco}}[1 - F_2(t)] + I_{\text{L}}(t)F_2(t) - I_{\text{dco}}F_2(t)$$

Simplifying

$$I_{\text{C}}(t) = I_{\text{L}}(t)F_2(t) - I_{\text{dco}}$$

The product $I_{\text{L}}(t)\ F_2(t)$ is the diode current, Expression (10.3), hence

$$I_{\text{C}}(t) = I_{\text{D}}(t) - I_{\text{dco}} \tag{10.4}$$

In a more rigorous approach, the accuracy of Expression (10.4) can be improved by considering the ripple current through the load resistance R as well. In other words the diode current $I_{\text{L}}(t)\ F_2(t)$ is shared by the load resistance R and the smoothing capacitor C as indicated by Expression (10.5). The dc component, I_{dco}, is taken by the load because the capacitor is not taking any dc current under steady-state conditions.

$$I_{\text{C}}(t) = I_{\text{L}}(t)F_2(t)\frac{\overline{X(\omega n)}}{\overline{X(\omega n)} + R} - I_{\text{dco}} \tag{10.5}$$

Expressions (10.4) and (10.5) are also directly derived in a simpler way from the distribution of currents at the diode–capacitor–resistor junction. The current going into the junction is the diode current; its dc component is taken by the resistance as the capacitor takes no dc under steady-state conditions. The capacitor and the resistance share the remainder, the ac components.

10.2.5 The mathematical model of the dc to dc step up (boost) converter

From the discussion regarding the modes of this circuit it follows that:

(i) Both the input voltage, V_{dc} and the output voltage, $V_{\text{o}}(t)$, contribute to the voltage across the switch, $V_{\text{A}}(t)$ Modes II and III.
(ii) For the three modes, three switching functions are defined. One for the state of the switch, $F_1(t)$ associated to Mode I, one for the state of the diode, $F_2(t)$ associated to Mode II and one when both the switch and the diode are not conducting, $F_3(t)$ associated to Mode III.
(iii) All three modes take place within one period of the switching frequency.

The contribution of the input and the output voltage to $V_{\text{A}}(t)$ is quantified by Expression (10.1). Modulators M1 and M2 together with Adder S1 represent this expression in the mathematical model, Fig. 10.4. The input loop equation for the converter, Fig. 10.1 is given by Expression (10.6)

$$V_{\text{dc}} = V_{\text{L}}(t) + V_{\text{A}}(t) \tag{10.6}$$

Adder S2 makes the necessary arithmetic manipulation to give at its output $V_{\text{L}}(t)$. Dividing $V_{\text{L}}(t)$ by the 'harmonic impedance' of the inductor, $X_{\text{L}}(\omega n)$, the ac component, $I_{\text{Lac}}(t)$ of the inductor current is given as

$$I_{\text{Lac}}(t) = \frac{V_{\text{L}}(t)}{X_{\text{L}}(\omega n)} \tag{10.7}$$

The dc component through the inductor, I_{Ldc}, is also the current supplied by the dc source and it has to be calculated by equating the input to the output power; a loss-free circuit is assumed.

$$P_{\text{in}} = V_{\text{dc}} I_{\text{Ldc}}$$

$$P_{\text{out}} = V_{\text{dco}} I_{\text{dco}}$$

$$I_{\text{Ldc}} = I_{\text{dco}} \frac{V_{\text{dco}}}{V_{\text{dc}}} \tag{10.8}$$

Adder S3 gives the inductor current $I_L(t)$. The current flowing in the inductor, in the steady state, is in the general form of

$$I_L(t) = I_{Ldc} + I_{Lac}(t) \tag{10.9}$$

The output voltage $V_o(t)$ can be expressed as

$$V_o(t) = V_{dco} + V_{oac}(t) \tag{10.10}$$

The dc component of the output voltage is calculated by expanding (10.6) which will be shown in Section 10.3. The ripple component of the output voltage is calculated by considering the ripple current, Expression (10.4) or (10.5), flowing into the capacitor reactance, $X(\omega n)$.

$$V_{oac}(t) = X(\omega n) I_C(t) \tag{10.11}$$

Adder S4 gives the output voltage $V_o(t)$ by adding both dc and ac components.

10.2.6 The switching functions

We need to define $F_1(t)$, $F_2(t)$ and $F_3(t)$; all are of the unipolar type. $F_1(t)$ describes the operation of the switch and it is externally determined.

$$F_1(t) = K_o + 2\sum_{n=1}^{\infty} K_n \cos(n\omega t) \tag{10.12}$$

where

n is an integer
$K_n = \sin(n\delta)/n\pi$
$\delta =$ half the on-period for which the switch is conducting.
$K_o =$ duty-cycle of the switching function.

The switching function $F_2(t)$ describes the action of the diode as a switch. It is not a forced switching function; for discontinuous conduction the duration of the on-period is determined by the circuit parameters. The instant at which the current becomes zero has to be calculated. Using a textbook expression [7] a constant K is defined as

$$K = \frac{2L}{RT}$$

If the duty-cycle of the switch, K_o is less than K the circuit is in continuous conduction.

Otherwise the circuit is in discontinuous conduction and the duty-cycle of the mode is given by

$$M_{\text{o}} = \frac{2K_{\text{o}}}{-1 + \sqrt{1 + 4(K_{\text{o}}^2/K)}}$$

Otherwise,

$$M_{\text{o}} = 1 - K_{\text{o}}$$

For continuous conduction the diode conducts for the period the switch is off and the timing details are easily determined.

$$F_2(t) = M_{\text{o}} + 2\sum_{x=1}^{\infty} M_x \cos(x\omega t - x\theta) \tag{10.13}$$

where

x is an integer
$M_x = \sin(r\gamma)/r\pi$
$\theta =$ phase displacement of the switching function relative to $F_1(t)$. $\theta = \delta + \gamma$
$\gamma =$ half the on-period for which the diode is conducting. It has to be calculated beforehand. $\gamma = M_{\text{o}} \cdot \pi$
$M_{\text{o}} =$ duty-cycle of the switching function.

The third switching function $F_3(t)$ refers to the period where the switch and the diode are both not conducting. For continuous conduction, $F_3(t) = 0$. The sum of the three switching functions is one, Expression (10.14). A straight line approximation of the three switching functions, the inductor current and the voltage across the switch is shown in Fig. 10.5. Figure 10.6 displays the same quantities derived by the switching function method of analysis.

$$F_3(t) = 1 - F_1(t) - F_2(t) \tag{10.14}$$

10.3 Analysis

The step up converter may have continuous and discontinuous conduction. The following analysis applies for both continuous and discontinuous inductor current (Figs. 10.7 and 10.8). During discontinuous conduction, the diode current conducts as long as the switch is open AND current still flows in the inductor, Fig. 10.5. Therefore the switching action of the diode is described by its own switching function defined as $F_2(t)$. The Kirchoff's Law of voltages for the input loop – dc source, inductor, transistor switch – is given by Expression (10.6)

$$V_{\text{dc}} = V_{\text{L}}(t) + V_{\text{A}}(t)$$

Replacing $V_{\text{A}}(t)$ from (10.1)

$$V_{\text{dc}} = V_{\text{L}}(t) + V_{\text{o}}(t)F_2(t) + V_{\text{dc}}F_3(t)$$

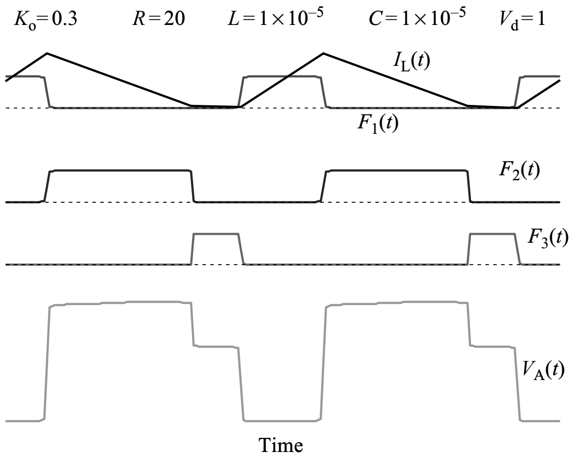


Figure 10.6 *The three switching functions with the inductor current $I_L(t)$ and $V_A(t)$*

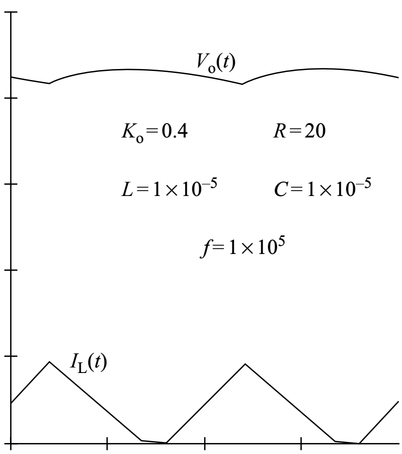


Figure 10.7 *Discontinuous conduction. The inductor current $I_L(t)$ and output voltage $V_o(t)$*

And solving for the inductor voltage, $V_L(t)$

$$V_L(t) = V_{dc} - V_o(t)F_2(t) - V_{in}F_3(t)$$
$$V_L(t) = V_{dc} - V_o(t)F_2(t) - V_{dc}[1 - F_1(t) - F_2(t)]$$
$$V_L(t) = V_{dc} - V_o(t)F_2(t) - V_{dc} + F_1(t)V_{dc} + F_2(t)V_{dc}$$
$$V_L(t) = -V_o(t)F_2(t) + F_1(t)V_{dc} + F_2(t)V_{dc}$$

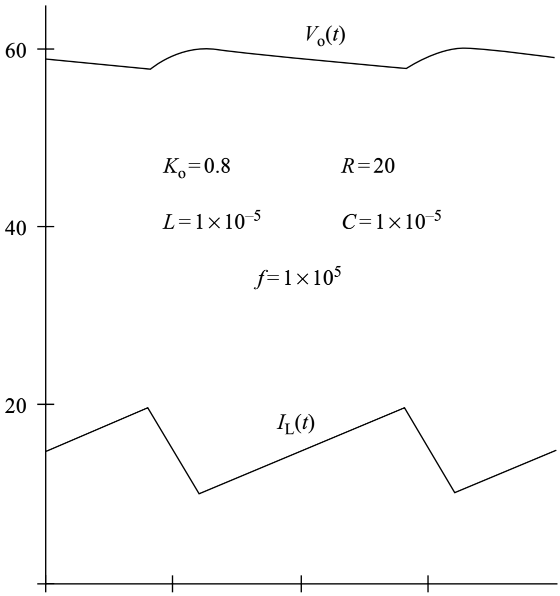


Figure 10.8 *Continuous conduction, inductor current $I_L(t)$ and output voltage $V_o(t)$*

Replacing $V_o(t)$, $F_1(t)$ and $F_2(t)$ from Expressions (10.10), (10.12) and (10.13)

$$V_L(t) = -[V_{dco} + V_{oac}(t)]\left[M_o + 2\sum_{x=1}^{\infty} M_x \cos(x\omega t - x\theta)\right]$$
$$+ \left[K_o + 2\sum_{n=1}^{\infty} K_n \cos(n\omega t)\right]V_{dc} + \left[M_o + 2\sum_{x=1}^{\infty} M_x \cos(x\omega t - x\theta)\right]V_{dc}$$

Expanding

$$V_L(t) = -V_{dco}M_o - M_o V_{oac}(t) - V_{dco}2\sum_{x=1}^{\infty} M_x \cos(x\omega t - x\theta)$$
$$+ 2V_{oac}(t)\sum_{x=1}^{\infty} M_x \cos(x\omega t - x\theta) + \mathrm{K}_o V_{dc} + 2V_{dc}\sum_{n=1}^{\infty} K_n \cos(n\omega t)$$
$$+ M_o V_{dc} + 2V_{dc}\sum_{x=1}^{\infty} M_x \cos(x\omega t - x\theta) \tag{10.15}$$

The output ripple voltage, $V_{\mathrm{oac}}(t)$ is too small in the presence of a large smoothing capacitor to have any significant effect on the inductor voltage and current; it is therefore neglected in Expression (10.15). Furthermore the dc terms are omitted as their algebraic sum is zero; the average voltage across an inductance is zero.

$$V_{\mathrm{L}}(t) = 2[V_{\mathrm{dc}} - V_{\mathrm{dco}}]\sum_{x=1}^{\infty} M_x \cos(x\omega t - x\theta) + 2V_{\mathrm{dc}}\sum_{n=1}^{\infty} K_n \cos(n\omega t) \tag{10.16}$$

The output ripple voltage, $V_{\mathrm{oac}}(t)$ is later derived from Expression (10.11).

10.3.1 DC output voltage

The dc components of Expression (10.15) add up to zero since the average voltage across an inductor is zero. It is assumed that the inductor is loss free.

$$0 = -V_{\mathrm{dco}}M_{\mathrm{o}} + K_{\mathrm{o}}V_{\mathrm{dc}} + M_{\mathrm{o}}V_{\mathrm{dc}}$$

Hence the average dc voltage at the output is given by

$$V_{\mathrm{dco}} = \frac{V_{\mathrm{dc}}(K_{\mathrm{o}} + M_{\mathrm{o}})}{M_{\mathrm{o}}} \tag{10.17}$$

And the output dc current

$$I_{\mathrm{dco}} = \frac{V_{\mathrm{dco}}}{R} \tag{10.18}$$

Expression (10.17) compares well with the text-book expression [5].

10.3.2 Calculation of input dc current, I_{Ldc}

The input dc current is also the inductor average current supplied by the source, I_{Ldc}. For a loss-free circuit as the one under investigation the input power is equal to the output power. Hence from Expressions (10.8), (10.17) and (10.18)

$$I_{\mathrm{Ldc}} = \frac{V_{\mathrm{dc}}}{R}\left(\frac{K_{\mathrm{o}} + M_{\mathrm{o}}}{M_{\mathrm{o}}}\right)^2 \tag{10.19}$$

10.3.3 Inductor current

The inductor current consists of a dc component, Expression (10.9) and the ac component

$$I_{\mathrm{L}}(t) = I_{\mathrm{Lac}}(t) + I_{\mathrm{Ldc}}$$

The ac component, $I_{\mathrm{Lac}}(t)$ is found from Expression (10.7) by dividing the inductor voltage, Expression (10.16) by the harmonic impedance of the inductor.

$$I_{\mathrm{Lripple}}(t) = \sum_{x=1}^{\infty} \frac{2(V_{\mathrm{dc}} - V_{\mathrm{dco}})}{n\omega L} M_x \sin(x\omega t - x\theta) + \sum_{n=1}^{\infty} \frac{2V_{\mathrm{dc}}}{n\omega L} K_n \sin(n\omega t)$$

And the inductor current is given by

$$I_{\mathrm{L}}(t) = I_{\mathrm{Ldc}} + \sum_{x=1}^{\infty} \frac{2(V_{\mathrm{dc}} - V_{\mathrm{dco}})}{n\omega L} M_x \sin(x\omega t - x\theta) + \sum_{n=1}^{\infty} \frac{2V_{\mathrm{dc}}}{n\omega L} K_n \sin(n\omega t) \tag{10.20}$$

I_{Ldc} is given by Expression (10.19).

10.3.4 Diode current

This is given by (10.2) as $I_{\mathrm{D}}(t) = F_2(t) I_{\mathrm{L}}(t)$. Expressions (10.13) and (10.20) are substituted into (10.2) and expanded in Appendix A10.1. It is shown that this current is given by

$$\begin{aligned} I_{\mathrm{D}}(t) = {} & I_{\mathrm{Ldc}} M_{\mathrm{o}} + M_{\mathrm{o}} \sum_{x=1}^{\infty} \frac{2(V_{\mathrm{dc}} - V_{\mathrm{o}})}{n\omega L} M_x \sin(x\omega t - x\theta) \\ & + M_{\mathrm{o}} \sum_{n=1}^{\infty} \frac{2V_{\mathrm{dc}}}{n\omega L} K_n \sin(n\omega t) \\ & + 2 \sum_{y=1}^{\infty} \sum_{x=1}^{\infty} \frac{(V_{\mathrm{dc}} - V_{\mathrm{o}})}{n\omega L} M_y M_x \{ \sin[(x+y)\omega t - (x+y)\theta] \\ & + \sin[(x-y)\omega t - (x-y)\theta] \} \\ & + 2 \sum_{n=1}^{\infty} \sum_{x=1}^{\infty} \frac{V_{\mathrm{dc}}}{n\omega L} K_n M_x \{ \sin[(x+n)\omega t - x\theta] \\ & + \sin[(-x+n)\omega t + x\theta] \} + 2 I_{\mathrm{indc}} \sum_{x=1}^{\infty} M_x \cos(x\omega t - x\theta) \end{aligned} \tag{10.21}$$

The dc component of the diode current is important because this is also the dc component of the load current already calculated above, Expression (10.18).

The dc component of the diode current is not limited to the obvious component $I_{\text{Ldc}}M_{\text{o}}$. There is also a hidden component as a result of the inter-modulation process taking place in the fourth term of Expression (10.21) when the two counter variables x and n are equal. The fourth term of the diode current is reproduced here from Expression (10.18).

$$2\sum_{n=1}^{\infty}\sum_{x=1}^{\infty}\frac{V_{\text{dc}}}{n\omega L}K_n M_x\{\sin[(x+n)\omega t - x\theta] + \sin[(-x+n)\omega t + x\theta]\}$$

For $x = n$ a dc component is produced in the second term

$$I_{\text{dcmod}} = 2\sum_{z=1}^{\infty}\frac{V_{\text{dc}}}{n\omega L}K_z M_z \sin(z\theta)$$

z is an integer.

The dc component of the diode current

$$I_{\text{Ddc}} = I_{\text{Ldc}}M_{\text{o}} + 2\sum_{z=1}^{\infty}\frac{V_{\text{dc}}}{n\omega L}K_z M_z \sin(z\theta) \tag{10.22}$$

This is also the dc current flowing into the load resistor R, I_{dco}.

10.3.5 Capacitor current and output ripple voltage

The capacitor current is given by Expressions (10.4) and (10.5) as a function of the diode current and the dc current in the load, I_{dco}. The diode current is given by Expression (10.21) and I_{dco} by Expression (10.18).

The capacitor current is causing a ripple voltage across the capacitor, Expression (10.11). With the capacitor current known the output ripple voltage $V_{\text{oac}}(t)$ is derived. In Appendix A10.2 a Mathcad Expression gives the output ripple voltage.

10.3.6 Peak inductor current

The peak value of the inductor current takes place at the end of the on period of the switch, at the end of the on period of $F_1(t)$, at $\omega t = \delta$. Replacing this value of time in Expression (10.20) gives the peak inductor current. For $R = 10\,\Omega$, $L = 10\,\mu\text{H}$, and a large smoothing capacitor the peak values of the inductor current are given for a range of values of the duty-cycle of the switch in Table 10.1.Consequently,

Table 10.1 Peak value of current through the inductor, the switch, diode and dc source $R = 10\,\Omega$, $L = 10\,\mu H$

Duty cycle of the switch, K_o	0.2	0.4	0.6	0.8
Peak value of current (A) through the inductor, the switch, diode and dc source	2.396	4.794	7.342	19.785

these are the peak values of current through the diode, the switch and the battery.

10.3.7 The current through the semiconductor switch

Expression (10.3) is expanded to give the current through the switch, Expression (10.23).

$$
\begin{aligned}
I_{\text{switch}}(t) = I_{\text{indc}} K_o + K_o \sum_{x=1}^{N} \frac{V_{\text{in}} - V_{\text{dco}}}{x\omega L} \frac{\sin(x\gamma)}{x\pi} \sin(x\omega t - x\beta) \\
+ K_o \sum_{n=1}^{N} \frac{V_{\text{in}}}{n\omega L} \frac{\sin n\delta}{n\pi} \sin(n\omega t) \\
+ \sum_{y=1}^{N} \sum_{x=1}^{N} \frac{V_{\text{in}} - V_{\text{dco}}}{y\omega L} \frac{\sin(y\gamma)}{y\pi} \frac{\sin(x\delta)}{x\pi} \\
\times [\sin[(y+x)\omega t - y\beta] + \sin[(y-x)\omega t - y\beta]] \\
+ \sum_{n=1}^{N} \sum_{x=1}^{N} \frac{V_{\text{in}}}{n\omega L} \frac{\sin(n\delta)}{n\pi} \frac{\sin(x\delta)}{x\pi} \\
\times [\sin[(n+x)\omega t] + \sin[(n-x)\omega t]] \\
+ I_{\text{indc}} \sum_{x=1}^{N} \frac{\sin(x\delta)}{x\pi} \cos(x\omega t)
\end{aligned}
\tag{10.23}
$$

Alternatively, the current through the switch is found from Expression (10.3) as the difference between $I_L(t)$ and $I_D(t)$.

10.3.8 Current ratings of the semiconductor devices

The rms values of switching power converters can be derived in various ways [6]. By employing the switching function, expressions for the currents and voltages are available and used to derive their rms values.

Expression (10.21) gives the current through the diode and Expression (10.23) the current through the switch. The average, peak and rms values of these currents are useful data for the circuit designer together with their duty-cycle. The average value, the peak value and duty-cycle are readily available from Expressions (10.21) and (10.23).

The peak value of current through the switch and the diode is the same; it is the peak value of the inductor current discussed above; some values are given in Table 10.1. The duty-cycles are also readily available; they are K_{o} for the switch and M_{o} for the diode.

The rms value of the current through the diode is given by

$$I_{\mathrm{Drms}} = \sqrt{\frac{1}{T}\int_0^T I_{\mathrm{D}}(t)^2\,\mathrm{d}t}$$

The rms value of the current through the semiconductor switch is given by

$$I_{\mathrm{Srms}} = \sqrt{\frac{1}{T}\int_0^T I_{\mathrm{S}}(t)^2\,\mathrm{d}t}$$

In a similar way the average values are derived by application of the textbook expression:

$$\text{Average} = \frac{1}{T}\int_0^T f(t)\,\mathrm{d}t$$

10.3.9 RMS of the ripple current through the smoothing capacitor

Expression (10.5) gives the current through the capacitor. The rms, average, peak to peak values and duty-cycle are easily extracted and used to set its current rating.

The peak to peak value of current is the peak value of the current through the inductor, the diode and the semiconductor switch. Table 10.1 contains these values for a range of values of the duty-cycle.

The rms value of the current through the capacitor switch is given by

$$I_{\mathrm{Crms}} = \sqrt{\frac{1}{T}\int_0^T I_{\mathrm{C}}(t)^2\,\mathrm{d}t} \tag{10.24}$$

Appendix A10.1: Current through the diode

$$
\begin{aligned}
I_{\mathrm{D}}(t) &= I_{\mathrm{L}}(t) * F_2(t) \\
&= \left\{ \sum_{x=1}^{\infty} \frac{2(V_{\mathrm{dc}} - V_{\mathrm{o}})}{n\omega L} M_x \sin(x\omega t - x\theta) + \sum_{n=1}^{\infty} \frac{2V_{\mathrm{dc}}}{n\omega L} K_n \sin(n\omega t) + I_{\mathrm{Ldc}} \right\} \\
&\quad \times \left[M_{\mathrm{o}} + 2 \sum_{x=1}^{\infty} M_x \cos(x\omega t - x\theta) \right] \\
&= M_{\mathrm{o}} \sum_{x=1}^{\infty} \frac{2(V_{\mathrm{dc}} - V_{\mathrm{o}})}{n\omega L} M_x \sin(x\omega t - x\theta) \\
&\quad + M_{\mathrm{o}} \sum_{n=1}^{\infty} \frac{2V_{\mathrm{dc}}}{n\omega L} K_n \sin(n\omega t) + I_{\mathrm{Ldc}} M_{\mathrm{o}} \\
&\quad + \left\{ \sum_{x=1}^{\infty} \frac{2(V_{\mathrm{dc}} - V_{\mathrm{o}})}{n\omega L} M_x \sin(x\omega t - x\theta) \right\} \left\{ 2 \sum_{x=1}^{\infty} M_x \cos(x\omega t - x\theta) \right\} \\
&\quad + \left\{ \sum_{x=1}^{\infty} \frac{2(V_{\mathrm{dc}} - V_{\mathrm{o}})}{n\omega L} M_x \sin(x\omega t - x\theta) \right\} \left\{ 2 \sum_{y=1}^{\infty} M_y \cos(y\omega t - y\theta) \right\} \\
&\quad + \left\{ \sum_{n=1}^{\infty} \frac{2V_{\mathrm{dc}}}{n\omega L} K_n \sin(n\omega t) \right\} \left\{ 2 \sum_{x=1}^{\infty} M_x \cos(x\omega t - x\theta) \right\} \\
&\quad + 2I_{\mathrm{Ldc}} \sum_{x=1}^{\infty} M_x \cos(x\omega t - x\theta)
\end{aligned}
$$

$$
\begin{aligned}
I_{\mathrm{D}}(t) &= I_{\mathrm{Ldc}} M_{\mathrm{o}} + M_{\mathrm{o}} \sum_{x=1}^{\infty} \frac{2(V_{\mathrm{dc}} - V_{\mathrm{o}})}{n\omega L} M_x \sin(x\omega t - x\theta) \\
&\quad + M_{\mathrm{o}} \sum_{n=1}^{\infty} \frac{2V_{\mathrm{dc}}}{n\omega L} K_n \sin(n\omega t) + 2 \sum_{y=1}^{\infty} \sum_{x=1}^{\infty} \frac{(V_{\mathrm{dc}} - V_{\mathrm{o}})}{n\omega L} M_y M_x \\
&\quad \times \{\sin[(x+y)\omega t - (x+y)\theta] + \sin[(x-y)\omega t - (x-y)\theta]\} \\
&\quad + 2 \sum_{x=1}^{\infty} \sum_{x=1}^{\infty} \frac{V_{\mathrm{dc}}}{n\omega L} K_n M_x \{\sin[(x+n)\omega t - x\theta] \\
&\quad + \sin[(-x+n)\omega t + x\theta]\} + 2I_{\mathrm{indc}} \sum_{x=1}^{\infty} M_x \cos(x\omega t - x\theta)
\end{aligned}
$$

There is a dc component for $n = k$ given by the second part of the expression provided that $\varphi_k \neq \alpha_n$.

$$I_{\mathrm{dc_o}} = -\sum_{n=1}^{\infty} K_n I_k \sin[-\varphi_k + \alpha_n] \quad \text{for all values of } n = k \ \text{ for } \varphi_k \neq \alpha_n$$

The phase angle α_n can be set to zero since it is externally controlled and if $\varphi_k \neq 0$, it means that the inductor is consuming power. This is only true if its ohmic resistance R_{IND} is not negligible. Hence for small, this dc component can be ignored.

Appendix A10.2: Output (capacitor) ripple voltage

This is to calculate the ripple due to the charging current.

Capacitor ripple voltage

$$VC1(t) := M_o \sum_{x=1}^{N} \frac{V_{\mathrm{dc}} - V_{\mathrm{dco}}}{x\omega L} \frac{\sin(x\gamma)}{x\pi} \frac{1}{x\omega C} \sin\left(x\omega t - x\beta - \frac{\pi}{2}x\right)$$

$$VC2(t) := M_o \sum_{n=1}^{N} \frac{V_{\mathrm{dc}}}{n\omega L} \frac{\sin(n\delta)}{n\pi} \frac{1}{n\omega C} \sin\left(n\omega t - \frac{\pi}{2}n\right)$$

$$VC31(t) := \sum_{y=1}^{N}\sum_{x=1}^{N} \frac{V_{\mathrm{dc}} - V_{\mathrm{dco}}}{y\omega L} \frac{\sin(y\gamma)}{y\pi} \frac{\sin(x\gamma)}{x\pi} \frac{1}{(y+x)\omega C}$$
$$\times \sin\left[(y+x)\omega t - (x+y)\beta - (y+x)\frac{\pi}{2}\right]$$

$$VC321(t) := \sum_{y=1}^{N}\sum_{x=y+1}^{N+1} \frac{V_{\mathrm{dc}} - V_{\mathrm{dco}}}{y\omega L} \frac{\sin(y\gamma)}{y\pi} \frac{\sin(x\gamma)}{x\pi} \frac{1}{(y-x)\omega C}$$
$$\times \sin\left[(y-x)\omega t - (y-x)\beta - (y-x)\frac{\pi}{2}\right]$$

$$VC322(t) := \sum_{x=1}^{N}\sum_{y=x+1}^{N+1} \frac{V_{\mathrm{dc}} - V_{\mathrm{dco}}}{y\omega L} \frac{\sin(y\gamma)}{y\pi} \frac{\sin(x\gamma)}{x\pi} \frac{1}{(y-x)\omega C}$$
$$\times \sin\left[(y-x)\omega t - (y-x)\beta - (y-x)\frac{\pi}{2}\right]$$

$$VC41(t) := \sum_{n=1}^{N}\sum_{x=1}^{N} \frac{V_{\mathrm{dc}}}{n\omega L} \frac{\sin(n\delta)}{n\pi} \frac{\sin(x\gamma)}{x\pi} \frac{1}{(n+x)\omega C}$$
$$\times \sin\left[(n+x)\omega t - x\beta - (n+x)\frac{\pi}{2}\right]$$

$$VC421(t) := \sum_{n=1}^{N} \sum_{x=n+1}^{N+1} \frac{V_{\text{dc}}}{n\omega L} \frac{\sin(n\delta)}{n\pi} \frac{\sin(x\gamma)}{x\pi} \frac{1}{(n-x)\omega C} \times \sin\left[(n-x)\omega t + x\beta - (n-x)\frac{\pi}{2}\right]$$

$$VC422(t) := \sum_{x=1}^{N} \sum_{n=x+1}^{N+1} \frac{V_{\text{dc}}}{n\omega L} \frac{\sin(n\delta)}{n\pi} \frac{\sin(x\gamma)}{x\pi} \frac{1}{(n-x)\omega C} \times \sin\left[(n-x)\omega t + x\beta - (n-x)\frac{\pi}{2}\right]$$

$$VC5(t) := I_{\text{indc}} \sum_{x=1}^{N} \frac{\sin(x\gamma)}{x\pi} \frac{1}{x\omega C} \cos\left(x\omega t - x\beta - \frac{\pi}{2}x\right)$$

$$V\text{CrippleCharg}(t) := 2(\text{VC1}(t) + \text{VC2}(t) + \text{VC31}(t) + \text{VC321}(t) + \text{VC322}(t) + \text{VC41}(t) + \text{VC421}(t) + \text{VC422}(t) + \text{VC5}(t))$$

Chapter 11

The buck boost dc to dc converter

11.1 Introduction

The switch, Fig. 11.1, operates at a rather high frequency (in the region of 20 kHz) and its duty-cycle sets the magnitude of the output voltage. The inductor is alternately connected to the dc supply and then across the output via the diode. The circuit configuration allows the inductor to charge and store energy. This happens when the switch is closed. When the switch opens, it discharges into the resistor – capacitor combination in such a way that the output voltage is reversed. The voltage is reversed because the direction of the current in the inductor cannot change; it carries on in the downward direction, Fig. 11.1. The current is decreasing because the battery is not connected to it any more. With the current having a negative rate of change, the voltage across the inductor is reversed according to

$$V_{\mathrm{L}}(t) = L\frac{\mathrm{d}I(t)}{\mathrm{d}t}$$

Hence the voltage across the inductor is reversed; it is opposite to the battery polarity voltage. The presence of the diode ensures that the capacitor will only charge with

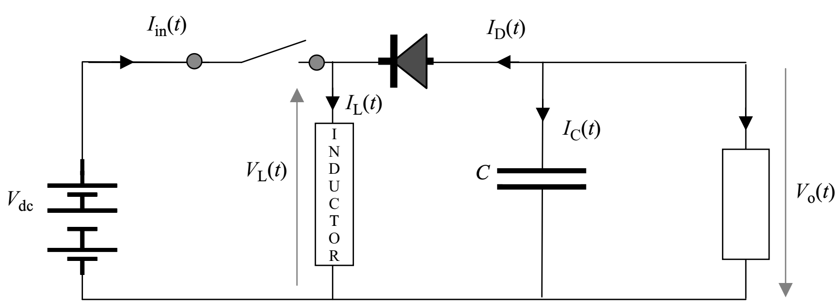


Figure 11.1 The buck boost dc-to-dc converter

the polarity shown, Fig. 11.1. In the steady state the capacitor is charged to a reverse voltage determined by the on-period of the switching function, as we will show later. A small ripple voltage exists on the capacitor voltage and hence the output. The choice of the capacitor is such as to keep this ripple as low as possible.

11.2 Mathematical modelling of the buck boost converter

11.2.1 Operation and modes

The switch is operated by a unipolar switching function, $F_1(t)$ and its action gives two modes, I and II. In the case of discontinuous conduction there is a third mode, III. All modes are demonstrated in Fig. 11.2 and the theoretical idealised waveforms are shown in Fig. 11.3. When the switch is closed, Mode I, the battery voltage V_{dc} is applied to the inductor.

$$V_L(t) = V_{dc} \quad F_1(t) = 1$$

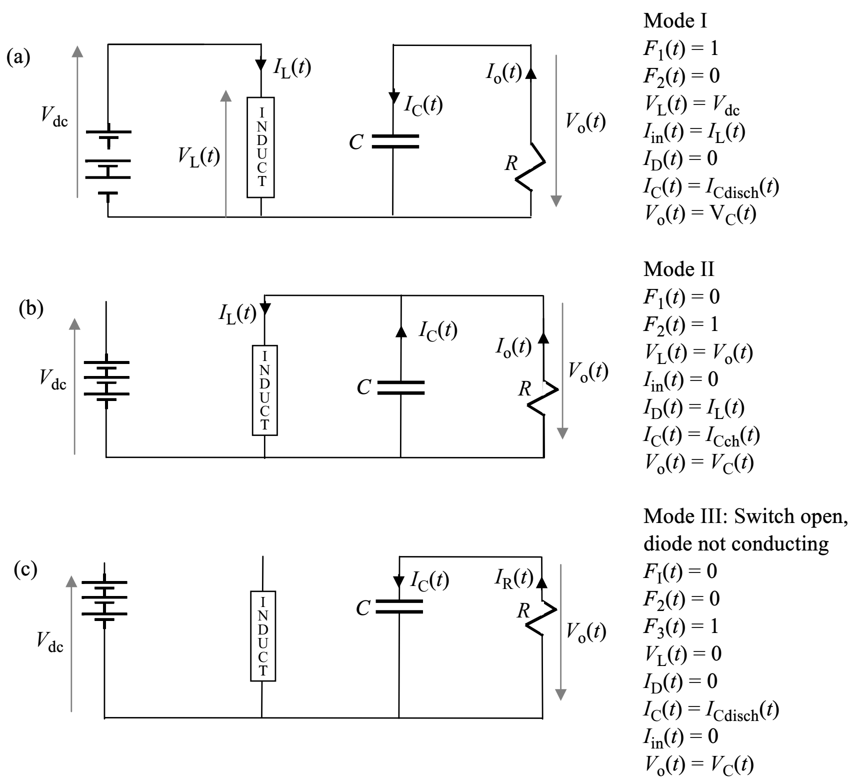


Figure 11.2 The modes of the buck boost converter

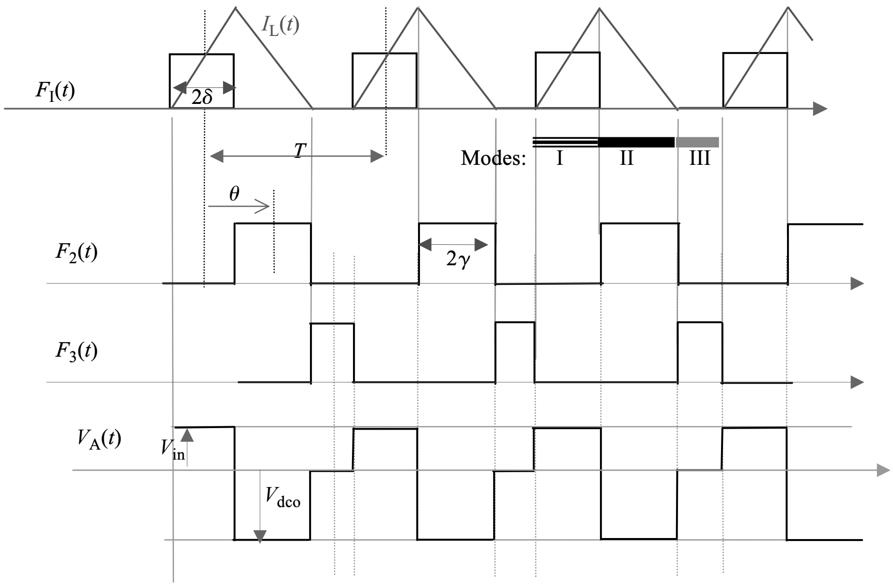


Figure 11.3 Idealised waveforms for the boost converter

$F_1(t)$ is a unipolar switching function and it takes the value of one as long as the switch is closed. The inductor current is rising and energy is stored in it. The diode is reverse biased by the capacitor voltage. The capacitor discharges into the load R; the voltage across it falls and the current is practically constant for a large capacitor but negative.

When the switch is open, the circuit is at Mode II and the inductor transfers its stored energy to the load. Its current falls thus reversing the polarity of the inductor voltage. The diode is forward biased securing a closed loop for the inductor current. The inductor voltage appears across the output via the diode and this gives the opposite polarity to the output voltage relative to the input V_{dc}.

$$V_L(t) = V_o(t) \quad F_2(t) = 1$$

$F_2(t)$ is a unipolar switching function associated to this mode and it takes the value of one as long as the diode conducts. In this mode the capacitor voltage is on the increase and its current is almost constant because of its large value.

It is possible for the inductor discharging current, $I_L(t)$ to become zero before the switch is closed again. This is discontinuous conduction, Mode III. In Mode III the inductor current is zero. With the inductor current zero, the diode is not conducting and the inductor voltage becomes zero as well.

The sequence of the modes is shown in Fig. 11.4. The voltage across the inductor, $V_L(t)$, is made up from the contributions of Modes I and II.

$$V_L(t) = V_{dc}F_1(t) + V_o(t)F_2(t) \tag{11.1}$$

Figure 11.4 Mode sequence for the buck boost dc-to-dc converter

11.2.2 Diode current

Current flows through the diode during Mode II. It is the inductor current $I_L(t)$ diverted through the diode when the semiconductor switch is closed. The switching function $F_2(t)$ is associated with this mode of the circuit. According to Expression (2.6) of Chapter 2 the diode current is given by

$$I_D(t) = F_2(t)I_L(t) \tag{11.2}$$

11.2.3 Semiconductor switch current

Current flows through the semiconductor switch during Mode I. It is the inductor current $I_L(t)$ diverted through the semiconductor switch. The switching function $F_1(t)$ is associated with this mode of the circuit. According to Expression (2.6) of Chapter 2 the switch current is given by

$$I_{SWITCH}(t) = F_1(t)I_L(t) \tag{11.3}$$

This is also the input current, $I_{in}(t)$, Fig. 11.1.

11.2.4 Capacitor current

The capacitor current is directly derived from the distribution of currents at the diode–capacitor–resistor junction. The current which flows into the junction is the diode current; its dc component I_{dco} is taken by the resistance as the capacitor takes no dc under steady-state conditions. The remainder, the ac component, is shared by the capacitor and the resistance. Therefore

$$I_C(t) = I_L(t)F_2(t)\frac{\overline{X(\omega n)}}{\overline{X(\omega n)} + R} - I_{dco} \tag{11.4}$$

Assuming a large capacitor taking practically all the ac component of the diode current Expression (11.4) is reduced to

$$I_C(t) = I_D(t) - I_{dco} \tag{11.5}$$

Expression (11.5) is further discussed in Section 2.9

11.2.5 Output voltage

The output voltage, $V_o(t)$, has a dc component V_{dco} and an ac component $V_{oac}(t)$ voltage, the sum of the harmonics.

$$V_o(t) = V_{dco} + V_{oac}(t) \tag{11.6}$$

V_{dco} is the output dc voltage and it will be derived by expanding Expression (11.1). The ac component, $V_{oac}(t)$, is the result of $I_C(t)$ flowing through the harmonic impedance of the capacitor.

$$V_{oac}(t) = I_C(t)X_C(\omega n) \tag{11.7}$$

11.2.6 Inductor current

The current flowing in the inductor, in the steady state, has dc component I_{Ldc} and the ac components, $I_{Lac}(t)$ as the sum of the harmonics.

$$I_L(t) = I_{Ldc} + I_{Lac}(t) \tag{11.8}$$

11.2.7 The mathematical model of the buck boost converter

Please refer to Fig. 11.5.

11.2.8 Switching functions

Three switching functions are defined for this circuit, one for each mode of the circuit; they are all of the unipolar type, a series of pulses. In Mode I the switching function

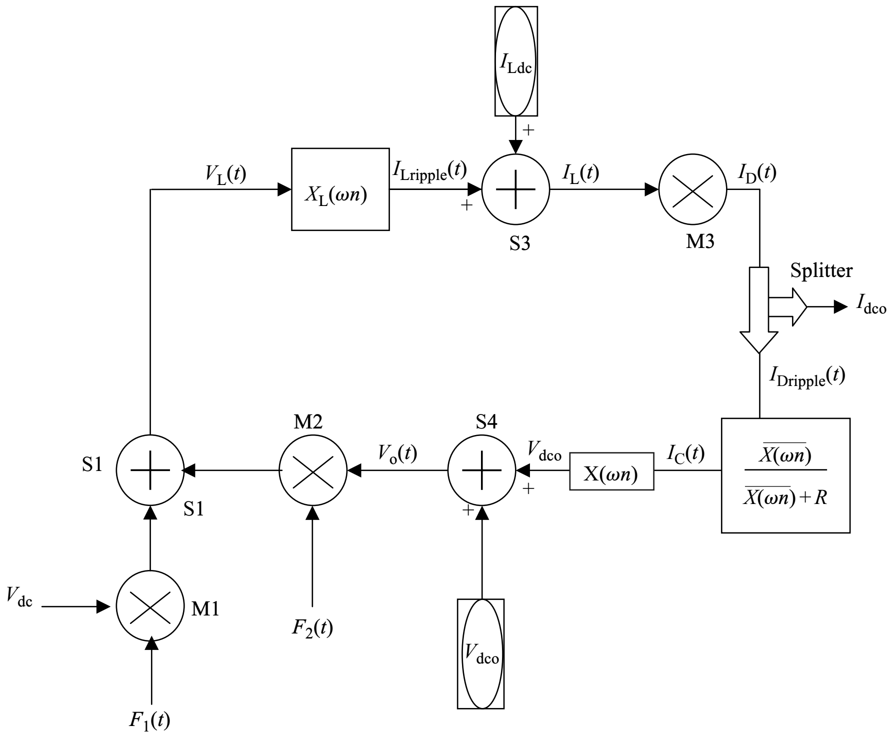


Figure 11.5 The mathematical model of the buck boost dc-to-dc converter

of the semiconductor switch is $F_1(t)$; it is a forced function and its duty cycle is externally determined. In Mode II the action of the diode as a switch is described by $F_2(t)$. It is not a forced switching function; the duration of the on-period is determined by the circuit parameters. The starting of its on-period is the instant that the switch opens but the end of the pulse is either the instant that the semiconductor switch closes – continuous conduction – or the instant when the current in the inductor – and the diode – becomes zero – for discontinuous conduction. In Mode III, we have the third switching function, $F_3(t)$ that takes the value of one during the off periods of the inductor current. It does not need to be defined separately because it is derived from the previous two.

$$F_1(t) + F_2(t) + F_3(t) = 1 \tag{11.9}$$

The three switching functions are displayed in the idealised line approximations of Fig. 11.3.

$$F_1(t) = K_{\mathrm{o}} + 2\sum_{n=1}^{\infty} K_n \cos(n\omega t) \tag{11.10}$$

where

n is an integer
$K_n = \sin(n\delta)/n\pi$
$\delta =$ half the on-period of the switch
$K_{\mathrm{o}} =$ duty cycle of the switching function.

$$F_2(t) = M_{\mathrm{o}} + 2\sum_{x=1}^{\infty} M_x \cos(x\omega t - x\theta) \tag{11.11}$$

where

x is an integer
$M_x = \sin(x\gamma)/x\pi$
$\theta =$ phase displacement of the switching function relative to $F_1(t)$
$\gamma =$ half the on-period for which the diode is conducting
$M_{\mathrm{o}} =$ duty-cycle of the switching function.

11.3 Analysis of the buck boost converter

11.3.1 Inductor voltage

The inductor voltage is given by Expression (11.1)

$$V_{\mathrm{L}}(t) = V_{\mathrm{dc}} F_1(t) + V_{\mathrm{o}}(t) F_2(t)$$

Substituting (11.6), (11.10) and (11.11) into (11.1)

$$V_{\mathrm{L}}(t) = V_{\mathrm{dc}}\left[K_{\mathrm{o}} + 2\sum_{n=1}^{\infty} K_n \cos(n\omega t)\right] + [V_{\mathrm{dco}} + V_{\mathrm{oac}}(t)]$$
$$\times \left[M_{\mathrm{o}} + 2\sum_{x=1}^{\infty} M_x \cos(x\omega t - \theta)\right]$$

$$V_{\mathrm{L}}(t) = V_{\mathrm{dc}}K_{\mathrm{o}} + 2V_{\mathrm{dc}}\sum_{n=1}^{\infty} K_n \cos(n\omega t) + V_{\mathrm{oac}}(t)M_{\mathrm{o}}$$
$$+ 2V_{\mathrm{o}}(t)\sum_{n=1}^{\infty} M_n \cos(n\omega t - x\theta)$$
$$+ V_{\mathrm{dco}}M_{\mathrm{o}} + 2V_{\mathrm{dco}}\sum_{n=1}^{\infty} M_n \cos(x\omega t - x\theta)$$

By neglecting the effect of the output voltage harmonics $V_{\mathrm{oac}}(t)$ on $V_{\mathrm{L}}(t)$

$$V_{\mathrm{L}}(t) = V_{\mathrm{dc}}K_{\mathrm{o}} + V_{\mathrm{dco}}M_{\mathrm{o}} + 2V_{\mathrm{dc}}\sum_{n=1}^{\infty} K_n \cos(n\omega t)$$
$$+ 2V_{\mathrm{dco}}\sum_{n=1}^{\infty} M_n \cos(x\omega t - x\theta)$$

The dc component is zero since a lossless inductor carries no dc voltage

$$V_{\mathrm{dc}}K_{\mathrm{o}} + V_{\mathrm{dco}}M_{\mathrm{o}} = 0$$

$$V_{\mathrm{dco}} = -\frac{V_{\mathrm{dc}}K_{\mathrm{o}}}{M_{\mathrm{o}}} \tag{11.12}$$

And the inductor voltage is now given by,

$$V_{\mathrm{L}}(t) = 2V_{\mathrm{dc}}\sum_{n=1}^{\infty} K_n \cos(n\omega t) + 2V_{\mathrm{dco}}\sum_{n=1}^{\infty} M_n \cos(n\omega t - x\theta) \tag{11.13}$$

In developing the expression of the inductor voltage, the effect of the output voltage harmonics, $V_{\mathrm{oac}}(t)$ is neglected. This is because their small magnitude in real circuits has little effect on the inductor voltage; the smoothing capacitor is chosen to do exactly that. The output ripple voltage is derived though later.

11.3.2 Inductor current ac component

The inductor current consists of a dc component and the ac component as described by Expression (11.8). The ac component, $I_{\mathrm{Lac}}(t)$ is found by dividing the inductor voltage, Expression (11.13) by the harmonic impedance of the inductor, wLn.

$$I_{\mathrm{L}}(t) = I_{\mathrm{Ldc}} + 2V_{\mathrm{dc}} \sum_{n=1}^{\infty} \frac{K_n}{\omega Ln} \sin(n\omega t) + 2V_{\mathrm{dco}} \sum_{x=1}^{\infty} \frac{M_x}{\omega Lx} \sin(n\omega t - x\theta) \tag{11.14}$$

The dc component, I_{Ldc} is derived below.

11.3.3 Inductor current dc component

The Kirchoff's First Law is applied at the switch–diode–inductor junction.

$$I_{\mathrm{L}}(t) = I_{\mathrm{SWITCH}}(t) + I_{\mathrm{D}}(t)$$

Consider only dc components.

$$I_{\mathrm{Ldc}} = I_{\mathrm{dc}} + I_{\mathrm{Ddc}}$$

where I_{dc}, is the dc current from the dc source.

The diode dc component of current, I_{Ddc} is also the dc component of the output current through the resistance R as the capacitor does not take any dc current in the steady state under investigation.

$$I_{\mathrm{odc}} = -I_{\mathrm{Ddc}}$$

$$I_{\mathrm{odc}} = -\frac{V_{\mathrm{dc}} K_{\mathrm{o}}}{M_{\mathrm{o}} R} \tag{11.15}$$

The input dc component is calculated by equating the input to the output power, thus ignoring the losses within the converter. The input power is equal to the output power.

$$V_{\mathrm{dc}} I_{\mathrm{dc}} = I_{\mathrm{dco}} V_{\mathrm{dco}}$$

$$V_{\mathrm{dc}} I_{\mathrm{dc}} = \left\{ -\frac{V_{\mathrm{dc}} K_{\mathrm{o}}}{M_{\mathrm{o}}} \right\} \frac{1}{R} \left\{ -\frac{V_{\mathrm{dc}} K_{\mathrm{o}}}{M_{\mathrm{o}}} \right\}$$

$$V_{\mathrm{dc}} I_{\mathrm{dc}} = \left\{ \frac{V_{\mathrm{dc}} K_{\mathrm{o}}}{M_{\mathrm{o}}} \right\}^2 \frac{1}{R}$$

$$I_{\mathrm{dc}} = \left\{ \frac{V_{\mathrm{dc}} K_{\mathrm{o}}}{M_{\mathrm{o}}} \right\}^2 \frac{1}{R V_{\mathrm{dc}}} \tag{11.16}$$

DC component of inductor current, I_{Ldc}

$$I_{\mathrm{Ldc}} = \frac{V_{\mathrm{dc}} K_{\mathrm{o}}}{M_{\mathrm{o}} R} + \left(\frac{V_{\mathrm{dc}} K_{\mathrm{o}}}{M_{\mathrm{o}}} \right)^2 \frac{1}{R V_{\mathrm{dc}}} \tag{11.17}$$

11.3.4 Diode current

Expression (11.2) gives this current as a product of the inductor current and $F_2(t)$; this expression is expanded in Appendix 11A.1 to give:

$$
\begin{aligned}
I_D(t) = {} & I_{Ldc}M_o + 2M_oV_{dc}\sum_{n=1}^{\infty}\frac{K_n}{\omega Ln}\sin(n\omega t) \\
& + 2M_oV_{dco}\sum_{x=1}^{\infty}\frac{M_x}{\omega Lx}\sin(n\omega t - x\theta) + 2I_{Ldc}\sum_{x=1}^{\infty}M_x\cos(x\omega t - x\theta) \\
& + 2V_{dc}\sum_{x=1}^{\infty}\sum_{n=1}^{\infty}\frac{K_n}{\omega Ln}M_x\{\sin[(n+x)\omega t - x\theta] \\
& + \sin[(n-x)\omega t + x\theta]\} \\
& + 2V_{dco}\sum_{y=1}^{\infty}\sum_{x=1}^{\infty}\frac{M_x}{\omega Lx}M_y\{\sin[(x+y)\omega t - (x+y)\theta] \\
& + \sin[(x-y)\omega t + (y-x)\theta]\}
\end{aligned} \tag{11.18}
$$

11.3.5 DC component of diode current

The dc component of the diode current is not limited to the rather obvious component $I_{Ldc}M_o$. It is possible that there is a hidden component as a result of the inter-modulation process taking place in the fourth term of Expression (11.18) when the two counter variables x and n are equal. The hidden dc component, I_{DdcH}, is derived from the fourth term of the diode current when $x = n$.

$$
I_{DdcH} = 2V_{dc}\sum_{x=1}^{\infty}\frac{K_x}{\omega Lx}M_x\sin[x\theta] \tag{11.19}
$$

11.3.6 Capacitor current

The capacitor current is given by Expressions (11.4) and (11.5). Assume that a large capacitor takes practically all the ac component of the diode current, Expression (11.5) is employed to derive the capacitor current. The diode current $I_D(t)$ is replaced from Expression (11.18) and I_{dco} from Expression (11.15).

11.3.7 Output ripple voltage

This is the ac component of the capacitor voltage, $V_{oac}(t)$ given by Expression (11.7). Assume a large capacitor where the ac components of the diode current are taken by the capacitor and replace $I_{\mathrm{C}}(t)$ from (11.18); the output ripple voltage is given by

$$
\begin{aligned}
V_{\mathrm{oac}}(t) = &-2M_{\mathrm{o}}V_{\mathrm{dc}}\sum_{n=1}^{\infty}\frac{K_n}{\omega Ln\omega Cn}\cos(n\omega t)\\
&-2M_{\mathrm{o}}V_{\mathrm{dco}}\sum_{x=1}^{\infty}\frac{M_x}{\omega Lx\omega Cx}\cos(x\omega t - x\theta)\\
&-2I_{\mathrm{Ldc}}\sum_{x=1}^{\infty}\frac{M_x}{\omega Cx}\sin(x\omega t - x\theta)\\
&-2V_{\mathrm{dc}}\sum_{x=1}^{\infty}\sum_{n=1}^{\infty}\frac{K_n}{\omega Ln}\frac{M_x}{(n+x)\omega C}\sin[(n+x)\omega t - x\theta]\\
&-2V_{\mathrm{dc}}\sum_{x=1}^{\infty}\sum_{n=1}^{\infty}\frac{K_n}{\omega Ln}\frac{M_x}{(n-x)\omega C}\sin[(n-x)\omega t + x\theta]\\
&-2V_{\mathrm{dco}}\sum_{x=1}^{\infty}\sum_{y=1}^{\infty}\frac{M_x}{\omega Lx}\frac{M_y}{(x+y)\omega C}[\sin(x+y)\omega t + (x-y)\theta]\\
&-2V_{\mathrm{dco}}\sum_{x=1}^{\infty}\sum_{y=1}^{\infty}\frac{M_x}{\omega Lx}\frac{M_y}{(x-y)\omega C}[\sin(x-y)\omega t + (y-x)\theta]
\end{aligned}
\tag{11.20}
$$

In deriving the harmonic impedance of the capacitor care must be taken for the terms $x-y$. They appear in the denominator and when $y = x$ we divide by zero! There is another good reason to exclude the cases when $y = x$ because a dc component of the diode current is materialising. The dc component is not flowing through the capacitor but through the resistance as explained above. The diode current, $I_{\mathrm{D}}(t)$, has five terms and three different frequency combinations n, $x \pm y$, $n \pm x$. The harmonic impedance of the capacitor must be calculated for each combination. The terms $x - y$ which appear in the denominator lead to division by zero, when $x = y$, something that is prohibited. When $x = y$, a dc component of diode current is materialising and its exclusion is allowed since only ac terms make up the ripple voltage. The counters in Expression (11.20) are set in a manner shown in the CD accompanying the book to exclude these terms.

11.3.8 Peak inductor current

The peak of the inductor current takes place at the end of the on-period of the switch, at the end of the on-period of $F_1(t)$ (Fig. 11.6). This happens at $\omega t = \delta$. Replacing this value of time in Expression (11.14) gives the peak inductor current (Table 11.1).

For $R = 20\,\Omega \quad L = 30 \times 10^{-6} \quad V_{\mathrm{dc}} = 24\,\mathrm{V}$

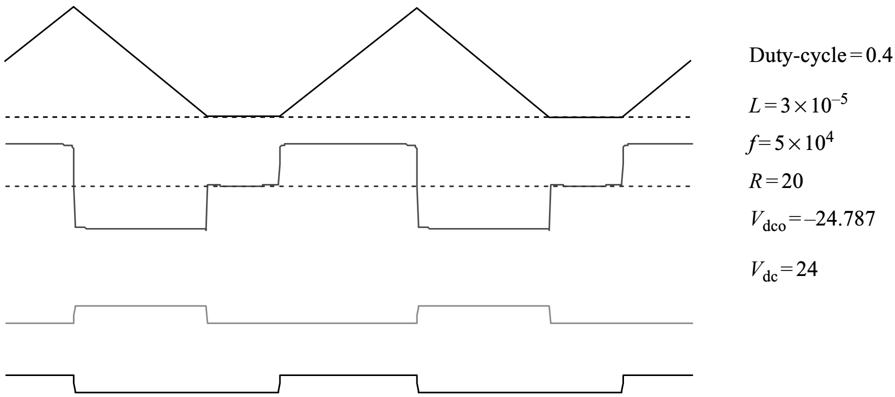


Figure 11.6 *The inductor current, the associated switching functions and the inductor voltage for discontinuous conduction*

Table 11.1 *Peak value of current through the inductor, the switch, diode and dc source*

Duty-cycle of the switch, K_o	0.2	0.4	0.6	0.8
Peak value of current (A) through the inductor, the switch, diode and dc source	1.8	3.28	6.9	27.2

11.3.9 Current rating of components

The rms, average and peak values of the current through the semiconductor devices can easily be derived by employing the expression for the current through the diode and the transistor switch, Expressions (11.3) and (11.18).

The average value is given by the expression

$$\text{Average} = \frac{1}{T}\int_0^T I(t)\,\mathrm{d}t$$

And the rms value

$$\text{rms} = \sqrt{\frac{1}{T}\int_0^T I(t)^2\,\mathrm{d}t}$$

Mathcad is employed with success to derive average and rms values for the diode and the transistor switch currents.

Appendix A11.1: Diode current $I_D(t)$

$$
\begin{aligned}
I_D(t) &= F_2(t) I_L(t) \\
&= \left\{ I_{Ldc} + 2V_{dc} \sum_{n=1}^{\infty} \frac{K_n}{\omega L n} \sin(n\omega t) + 2V_{dco} \sum_{x=1}^{\infty} \frac{M_x}{\omega L x} \sin(x\omega t - x\theta) \right\} \\
&\qquad \times \left\{ M_o + 2 \sum_{x=1}^{\infty} M_x \cos(x\omega t - x\theta) \right\} \\
&= I_{Ldc} M_o + M_o 2V_{dc} \sum_{n=1}^{\infty} \frac{K_n}{\omega L n} \sin(n\omega t) \\
&\quad + 2M_o V_{dco} \sum_{x=1}^{\infty} \frac{M_x}{\omega L x} \sin(x\omega t - x\theta) + 2I_{Ldc} \sum_{x=1}^{\infty} M_x \cos(x\omega t - x\theta) \\
&\quad + 2V_{dc} \sum_{n=1}^{\infty} \frac{K_n}{\omega L n} \sin(n\omega t) 2 \sum_{x=1}^{\infty} M_x \cos(x\omega t - x\theta) \\
&\quad + 2V_{dco} \sum_{x=1}^{\infty} \frac{M_x}{\omega L x} \sin(x\omega t - x\theta) 2 \sum_{x=1}^{\infty} M_x \cos(x\omega t - x\theta)
\end{aligned}
$$

$$
\begin{aligned}
I_D(t) &= I_{Ldc} M_o + 2M_o V_{dc} \sum_{n=1}^{\infty} \frac{K_n}{\omega L n} \sin(n\omega t) \\
&\quad + 2M_o V_{dco} \sum_{x=1}^{\infty} \frac{M_x}{\omega L x} \sin(x\omega t - x\theta) + 2I_{Ldc} \sum_{x=1}^{\infty} M_x \cos(x\omega t - x\theta) \\
&\quad + 2V_{dc} \sum_{x=1}^{\infty} \sum_{n=1}^{\infty} \frac{K_n}{\omega L n} M_x \{\sin[(n+x)\omega t - x\theta] + \sin[(n-x)\omega t + x\theta]\} \\
&\quad + 2V_{dco} \sum_{y=1}^{\infty} \sum_{x=1}^{\infty} \frac{M_x}{\omega L x} M_y \{[\sin(x+y)\omega t - (x+y)\theta] \\
&\quad + [\sin(x-y)\omega t + (y-x)\theta]\}
\end{aligned}
$$

Chapter 12
The CUK dc to dc converter

12.1 Introduction

This circuit is modelled and analysed in order to derive unified expressions for the currents and voltages in the circuit. The investigation is limited to continuous conduction but the interested reader has all that is needed to extend it to discontinuous conduction according to the procedure of Chapter 1.

12.2 Mathematical modelling of the CUK dc to dc converter

The conventional directions of voltage and current are shown in Fig. 12.1. The Kirchoff's laws cannot be applied directly. In order to do that the modes of the circuit have to be derived; there are two modes, I and II. One switching function is introduced for Mode I and its inverse for Mode II. Discontinuous operation is not investigated.

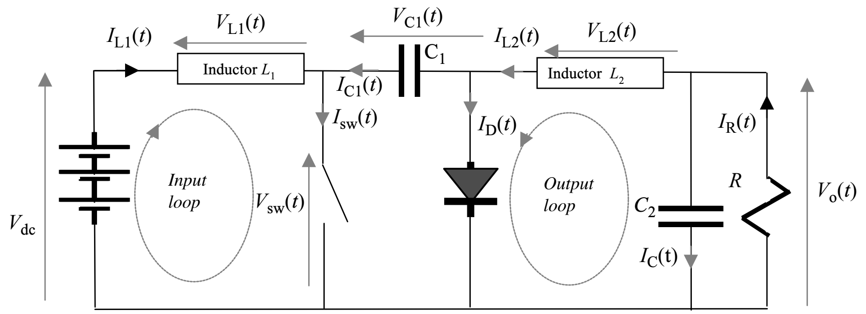


Figure 12.1 The circuit diagram of the CUK converter

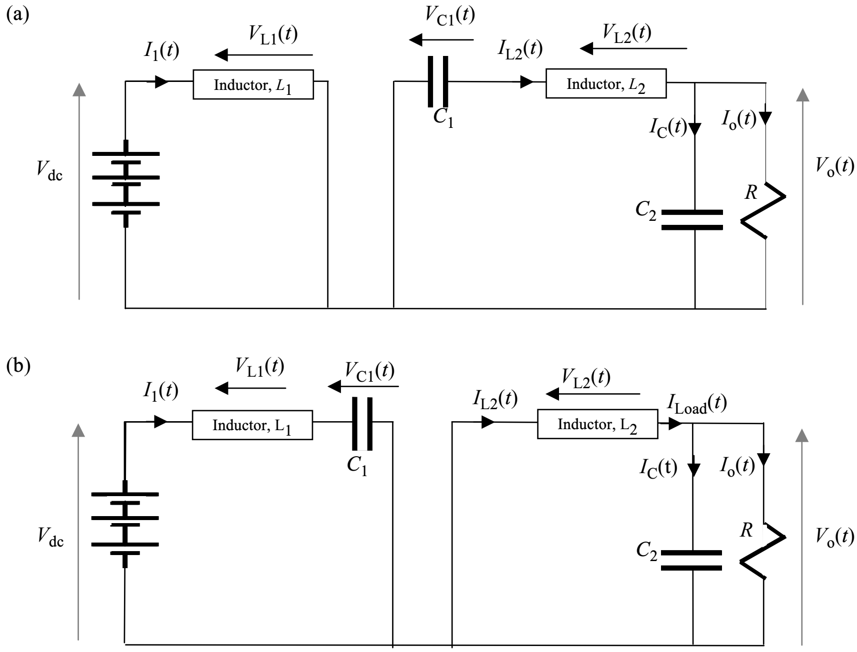


Figure 12.2 The modes of the CUK dc to dc converter: (a) Mode I: the switch is closed, $F(t) = 1$ and (b) Mode II: the switch is open, $\overline{F(t)} = 1$

12.2.1 Operation and modes of the CUK dc to dc converter

The inductor L_1 is charged from the dc source when the switch is closed. When the switch is open, the energy is transferred to the capacitor C_1. C_1 is chosen to be large so that the voltage across it remains practically constant throughout the operation of the converter. With the switch closed C_1 is charged from L_1 and it supplies the load via inductor L_2. With the switch open, the diode is forward biased and C_1 is charging. This circuit is not examined for discontinuous conduction; hence there are only two modes. The semiconductor switch is operated by the switching function $F(t)$ and the diode by the switching function $1 - F(t)$.

Mode I, Fig. 12.2(a), represents the circuit with the switch closed and the unipolar switching function, $F(t)$ takes the value of 1, $F(t) = 1$. During this mode the diode is reverse biased and not conducting. The RHS of inductor L_1 and the LHS of capacitor C_1 are both connected to the ground. L_1 is charging directly from the source and C_1 is discharging into the load via L_2. Hence the input dc voltage is applied to the inductor L_1 and the output voltage is the difference of V_{C1} and voltage across L_2.

$$V_{dc} = V_{L1}(t) \quad \text{for } F(t) = 1$$

$$V_o(t) = -V_{C1}(t) - V_{L2}(t) \quad \text{for } F(t) = 1$$

With the switch open we have Mode II, Fig. 12.2(b). The diode is forward biased thus connecting the RHS of C_1 to the ground; C_1 is now being charged by both L_1 and the source. The diode carries now both the charging current of C_1 and the current of L_2. The switching function takes the value of zero, $F(t) = 0$. At the same time its inverse takes the value of 1, Chapter 1, $\overline{F(t)} = 1$

$$V_{\mathrm{dc}} = V_{\mathrm{C1}}(t) + V_{\mathrm{L1}}(t) \quad \text{for } \overline{F(t)} = 1$$

$$V_{\mathrm{o}}(t) = -V_{\mathrm{L2}}(t) \quad \text{for } \overline{F(t)} = 1$$

The complete expression for the input, V_{dc} is derived by considering the contributions of both modes. This is in accordance to the application of the superposition theorem as applied to the switched circuits, Chapter 2.

$$V_{\mathrm{dc}} = V_{\mathrm{L1}}(t)F(t) + \{V_{\mathrm{C1}}(t) + V_{\mathrm{L1}}(t)\}\overline{F(t)}$$

This expression is further simplified by considering that the inverse of a function, as given in Chapter 1 is $\overline{F(t)} = [1 - F(t)]$.

$$V_{\mathrm{dc}} = V_{\mathrm{L1}}(t) + V_{\mathrm{C1}}(t)[1 - F(t)] \tag{12.1}$$

In the same way the unified expression of the output voltage, $V_{\mathrm{o}}(t)$ is given by

$$V_{\mathrm{o}}(t) = -V_{\mathrm{L2}}(t)\overline{F(t)} + \{-V_{\mathrm{C1}}(t) - V_{\mathrm{L2}}(t)\}F(t)$$

simplified to

$$V_{\mathrm{o}}(t) = -V_{\mathrm{L2}}(t) - V_{\mathrm{C1}}(t)F(t) \tag{12.2}$$

Capacitor C_1 is taking the current of L_1 during Mode II, $\overline{F(t)} = 1$, and the current of L_2 during Mode I, $F(t) = 1$,

$$I_{\mathrm{C1}}(t) = I_{\mathrm{L1}}(t) \quad \text{for } \overline{F(t)} = 1 \ \text{ Mode II}$$

$$I_{\mathrm{C1}}(t) = I_{\mathrm{L2}}(t) \quad \text{for } F(t) = 1 \ \text{ Mode I}$$

Therefore the unified expression for the current through C_1 is

$$I_{\mathrm{C1}}(t) = I_{\mathrm{L2}}(t)F(t) + I_{\mathrm{L1}}(t)[1 - F(t)] \tag{12.3}$$

Capacitor C_2 shares the current of L_2 with R during both modes. Under steady-state conditions that are investigated here, the capacitor takes no dc component. Hence it shares the ac component of $I_{\mathrm{L2}}(t)$ with R. For large value of capacitance where the output current through R can be approximated to a dc value, $I_{\mathrm{C2}}(t)$ is given by

$$I_{\mathrm{C2}}(t) = I_{\mathrm{Lac2}}(t)\frac{\overline{X(\omega n)}}{\overline{X(\omega n)} + R}$$

In a practical circuit the values of C_2 and R are such as to assume that all of it is passing through C_2

$$I_{\mathrm{C2}}(t) = I_{\mathrm{Lac2}}(t) \tag{12.4}$$

The voltage across C_1 and the voltage across C_2, in the steady state, can be expressed in the general form of Expression (12.5). V_{Cdc} is the dc component and $V_{\mathrm{Cac}}(t)$ is the ac component.

$$V_{\mathrm{C1}}(t) = V_{\mathrm{Cdc1}} + V_{\mathrm{Cac1}}(t) \tag{12.5a}$$

$$V_{\mathrm{C2}}(t) = V_{\mathrm{Cdc2}} + V_{\mathrm{Cac2}}(t) \tag{12.5b}$$

In the same way the current flowing in the inductor, in the steady state, has a dc component I_{Ldc} and the ripple current $I_{\mathrm{Lac}}(t)$.

$$I_{\mathrm{L1}}(t) = I_{\mathrm{Ldc1}} + I_{\mathrm{Lac1}}(t) \tag{12.6a}$$

$$I_{\mathrm{L2}}(t) = I_{\mathrm{Ldc2}} + I_{\mathrm{Lac2}}(t) \tag{12.6b}$$

12.2.2 The mathematical model of the CUK dc to dc converter

Expression (12.1) is re-arranged to give the voltage across the inductor L_1.

$$V_{\mathrm{L1}}(t) = V_{\mathrm{dc}} - V_{\mathrm{C1}}(t)[1 - F(t)]$$

Modulator M1 and adder S1 in Fig. 12.3 are employed to represent this part of the function of the circuit. Expression (12.2) is re-arranged to give the voltage across the inductor L_2.

$$V_{\mathrm{L2}}(t) = -V_{\mathrm{o}}(t) - V_{\mathrm{C1}}(t)F(t)$$

Modulator M2 add adder S2 in Fig. 12.3 are employed to represent this part of the function of the circuit.

The ac components of the inductor currents, $I_{\mathrm{L1ac}}(t)$ and $I_{\mathrm{L2ac}}(t)$ are derived from the inductor voltages by dividing with the harmonic reactance $X_{\mathrm{L1}}(\omega n)$ and $X_{\mathrm{L2}}(\omega n)$, respectively. The dc components of the two inductor currents are added to their ac components to give the inductor currents in adders S3 and S4.

The capacitor C_1 takes $I_{\mathrm{L1}}(t)$ when the switch is open and $I_{\mathrm{L2}}(t)$ when the switch is closed. Modulator M3 and M4 together with adder S5 represents this action in the mathematical model to give $I_{\mathrm{C1}}(t)$. The ripple voltage across C_1 is created by considering the harmonic impedance of this capacitor, $X_{\mathrm{C1}}(\omega n)$. The voltage across C_1 is the summation of its ripple voltage and its dc voltage, V_{C1}. This is done in adder S6.

The output voltage, $V_{\mathrm{o}}(t)$, is also the voltage across the second capacitor C_2, $V_{\mathrm{C2}}(t)$. This voltage is described by Expression (12.5b). The ac component of $V_{\mathrm{C2}}(t)$ is derived by multiplying the current through it, $I_{\mathrm{C2ac}}(t)$, with the harmonic impedance of this capacitor $X_{\mathrm{C2}}(\omega n)$ and adding its dc component, adder S7.

12.2.3 The switching function

The switching Function of the transistor switch:

$$F(t) = K_{\mathrm{o}} + 2\sum_{n=1}^{\infty} K_n \cos(n\omega t) \quad \text{or} \quad F(t) = K_{\mathrm{o}} + 2\Phi(t) \tag{12.7}$$

K_n and K_{o} are defined in the usual way.

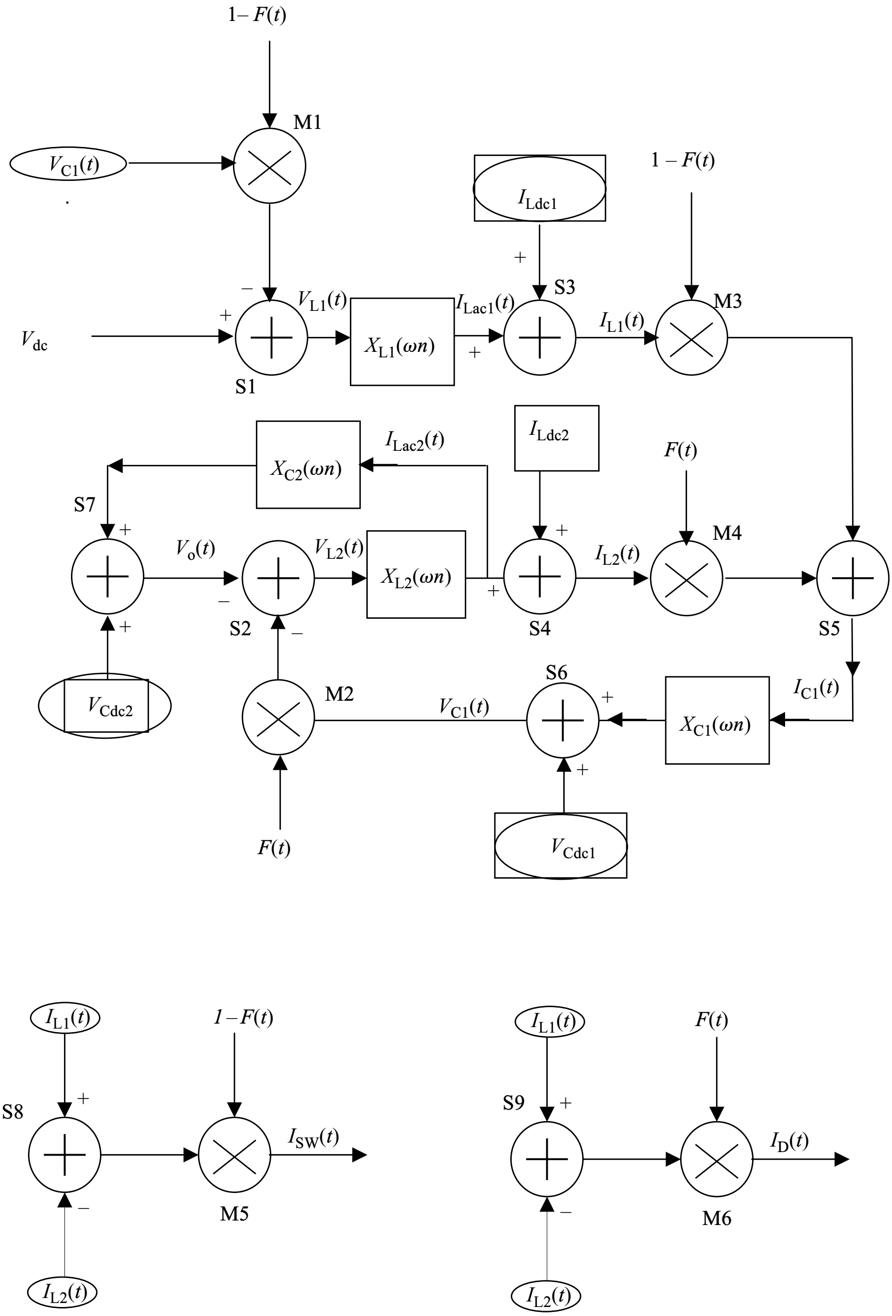


Figure 12.3 The mathematical model of the CUK dc to dc converter

12.3 Analysis of the CUK dc to dc converter

First, we consider Expression (12.1). The switching function, $F(t)$ and the voltage across C_1, $V_{C1}(t)$ are substituted from Expressions (12.5a) and (12.7), respectively.

$$V_{dc} = V_{L1}(t) + \{V_{C1dc} + V_{C1ac}(t)\}[1 - K_o - 2\Phi(t)]$$

The above expression is used to derive the current through the inductor L_1. The valid assumption that the ripple voltage across the capacitor C_1 is too small to have a significant effect on the inductor current is used to reduce this voltage to its dc value, V_{C1dc}; C_1 chosen to be large. This ripple voltage though can be estimated after the expression of the current through it is derived.

Therefore after simplification

$$V_{dc} = V_{L1}(t) + V_{C1dc}(1 - K_o) - V_{C1dc}2\Phi(t)$$

Equating dc components on both sides

$$V_{dc} = V_{C1dc}(1 - K_o)$$

$$V_{C1dc} = +\frac{V_{dc}}{1 - K_o} \tag{12.8}$$

Equating ac components on both sides

$$0 = V_{L1}(t) - V_{Cdc1}2\Phi(t)$$

Substituting $\Phi(t)$ from (12.7)

$$V_{L1}(t) = 2V_{C1dc}\sum_{n=1}^{\infty} K_n \cos(n\omega t) \tag{12.9}$$

The current through the first inductor L_1, is found by dividing the voltage across it, $V_{L1}(t)$ of Expression (12.11) with the harmonic impedance of L_1, ωLn and adding its dc component, Expression (12.6a).

$$I_{L1}(t) = I_{L1} + 2V_{C1}\sum_{n=1}^{\infty} \frac{K_n}{wLn}\sin(n\omega t) \tag{12.10}$$

The dc component, I_{L1}, will be derived later.

$V_{C1}(t)$ and $V_o(t)$ are simplified to their dc components, V_{C1dc} and V_{C1dc} and substituted with $F(t)$ from Expression (12.7) into Expression (12.2). Note that $V_o(t) = V_{C2}(t)$.

$$V_{C2dc} = -V_{L2}(t) - V_{C1dc}K_o - V_{C1dc}2\sum_{n=1}^{\infty} K_n \cos(n\omega t) \tag{12.11}$$

12.3.1 Output dc voltage

Equating dc components on both sides of Expression (12.11)

$$V_{\mathrm{Cdc2}} = -V_{\mathrm{Cdc1}} K_{\mathrm{o}}$$

Substituting V_{C1dc} from (12.8) and considering that the output voltage is the voltage across C_2, then the output dc voltage is given as a function of the input voltage and the duty-cycle of the switch as

$$V_{\mathrm{odc}} = -K_{\mathrm{o}} \frac{V_{\mathrm{dc}}}{1 - K_{\mathrm{o}}} \tag{12.12}$$

The output is inverted. This is also the dc component of the voltage across C_2, V_{C2dc}.

12.3.2 Voltage and current for L_2

Equating ac components on both sides of Expression (12.11)

$$\begin{aligned} 0 &= -V_{\mathrm{L2}}(t) - V_{\mathrm{Cdc1}} 2 \sum_{n=1}^{\infty} K_n \cos(n\omega t) \\ V_{\mathrm{L2}}(t) &= -2 V_{\mathrm{Cdc1}} \sum_{n=1}^{\infty} K_n \cos(n\omega t) \end{aligned} \tag{12.13}$$

And the current through the second inductor L_2, is found by dividing this voltage with the 'harmonic impedance' of L_2 and adding to its dc component, Expression (12.6b).

$$I_{\mathrm{Ldc2}}(t) = I_{\mathrm{Ldc2}} - 2 V_{\mathrm{Cdc1}} \sum_{n=1}^{\infty} \frac{K_n}{\omega L_2 n} \sin(n\omega t) \tag{12.14}$$

The dc component, I_{Ldc2}, will be derived later.

12.3.3 Current through C_2

The current through C_2, is equal to the ac component of the current through L_2, according to Expression (12.4)

$$I_{\mathrm{C2}}(t) = -2 V_{\mathrm{Cdc1}} \sum_{n=1}^{\infty} \frac{K_n}{\omega L_2 n} \sin(n\omega t) \tag{12.15}$$

12.3.4 Current through C_1

By comparing Expressions (12.10) and (12.14) we observe that the ac components of the current through L_2 and L_1 are related by,

$$I_{\mathrm{Lac2}}(t) = -I_{\mathrm{Lac1}}(t)$$

This result is used in Expression (12.3) which gives the current through C_1 and after simplification it takes the form of

$$I_{C1}(t) = I_{Ldc2}K_o + I_{Ldc1}[1 - K_o] + (I_{Ldc2} - I_{Ldc1})2\Phi(t) + [1 - 2K_o - 4\Phi(t)]I_{Lac1}(t)$$

Equating ac components

$$I_{C1}(t) = (I_{Ldc2} - I_{Ldc1})2\Phi(t) + [1 - 2K_o - 4\Phi(t)]I_{Lac1}(t) \tag{12.16}$$

Expression (12.16) gives the current through the capacitor C_1. It is not simplified any further; the rms value is derived from this expression as it is.

Equating dc components

$$0 = I_{Ldc2}K_o + I_{Ldc1}[1 - K_o]$$

The dc current through L_1, I_{Ldc1} is given by

$$I_{Ldc1} = -I_{Ldc2}\frac{K_o}{1 - K_o} \tag{12.17}$$

The dc component of L_2 passes through R as the capacitor C_1 does not take dc current. Hence I_{Ldc2} is easily derived from

$$I_{Ldc2} = \frac{V_{dco}}{R} \tag{12.18}$$

12.3.5 Voltage and currents of the semiconductor switch and the diode

Current through the transistor switch, $I_{SW}(t)$, is the algebraic addition of $I_{L1}(t)$ and $I_{L2}(t)$ and it exists in the time slot when $F(t) = 1$

$$I_{SW}(t) = [I_{L1}(t) - I_{L1}(t)]F(t) \tag{12.19}$$

The diode takes $I_{L1}(t)$ when the switch is open

$$I_D(t) = [I_{L1}(t) - I_{L1}(t)][1 - F(t)] \tag{12.20}$$

According to the Kirchoff's Second Law (voltage), when the switch is open, the voltage across it is the difference between the input voltage and the voltage across the inductor, $V_{dc} - V_{L1}(t)$; it is zero when the switch is closed. Therefore,

$$V_{SW}(t) = [V_{dc} - V_{L1}(t)][1 - F(t)] \tag{12.21}$$

In the same way, the voltage across the diode is the voltage across C_1, $V_{C1}(t)$ during Mode I. During Mode II it is zero as the diode is conducting

$$V_D(t) = V_{C1}(t)F(t) \tag{12.22}$$

Table 12.1 contains the RMS values of the currents through the transistor switch, the diode and the two capacitors. Table 12.2 contains the average values of the currents through the transistor switch and the diode for $L_1 = 10\,\mu H$, $L_2 = 10\,\mu H$, $C_2 = 5\,\mu F$, $R = 5\,\Omega$, $K_o = 0.65$ and $V_{dc} = 10\,V$.

Table 12.1 RMS values

$I_{\text{Drms}} := \sqrt{(1/T)\int_0^T I_{\text{D}}(t)^2\,\text{d}t}$
$I_{\text{Drms}} := 4$
$I_{\text{Srms}} := \sqrt{(1/T)\int_0^T I_{\text{SW}}(t)^2\,\text{d}t}$
$I_{\text{Srms}} := 9.325\ \text{A}$
$I_{\text{C1rms}} := \sqrt{(1/T)\int_0^T I_{\text{C1}}(t)^2\,\text{d}t}$
$I_{\text{C1rms}} := 5.278$
$I_{\text{C2rms}} := \sqrt{(1/T)\int_0^T I_{\text{C2}}(t)^2\,\text{d}t}$
$I_{\text{C2rms}} := 1.885\ \text{A}$

Table 12.2 Average values

$I_{\text{Dave}} := (1/T)\int_0^T I_{\text{D}}(t)\,\text{d}t$
$I_{\text{Dave}} := 3.384\ \text{A}$
$I_{\text{Save}} := (1/T)\int_0^T I_{\text{SW}}(t)\,,\text{d}t$
$I_{\text{Save}} := 7.228\ \text{A}$

12.3.6 *Output voltage*

The output voltage is the voltage across C_2. The voltage, in the steady state, across the capacitor is given by the general Expression (12.5b). The dc component is derived above in the dc analysis and it is the dc component of the capacitor voltage V_{Cdc2} given by Expression (12.12). The harmonic ac component is derived here; it is set up by the harmonic current through the capacitor C_2. The ripple current of $I_{\text{L2}}(t)$ is shared between C_2 and R.

$$I_{\text{C2}}(t) = I_{\text{L2ac}}(t)\frac{R}{\overline{X(\omega n)} + R}$$

For large values of C_2 as is the real case, practically all the ac current is taken by the capacitor, Expression (12.4).

$$I_{\text{C2}}(t) = I_{\text{L2ac}}(t)$$

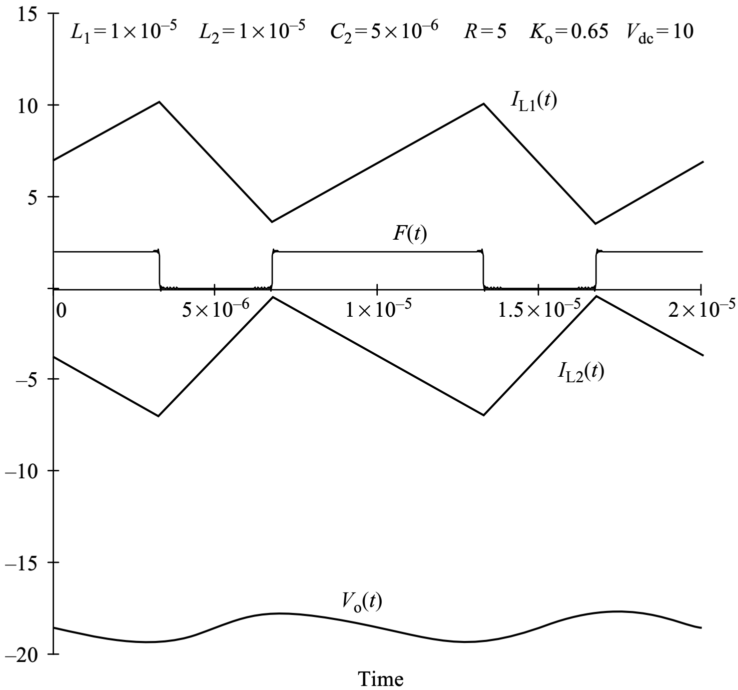


Figure 12.4 *The inductor currents of L_1 and L_2, the output voltage and the switching function of the CUK dc to dc converter*

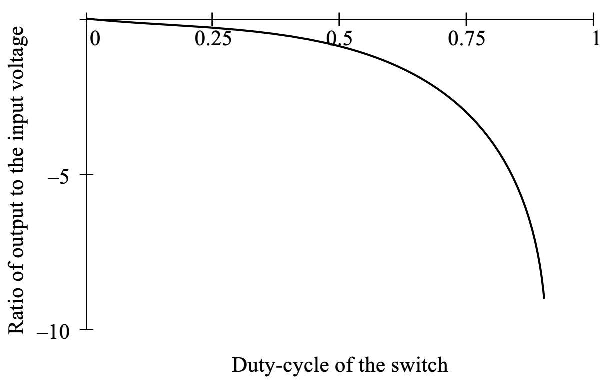


Figure 12.5 *Ratio of output voltage to input voltage against the duty-cycle of the switch, K_o*

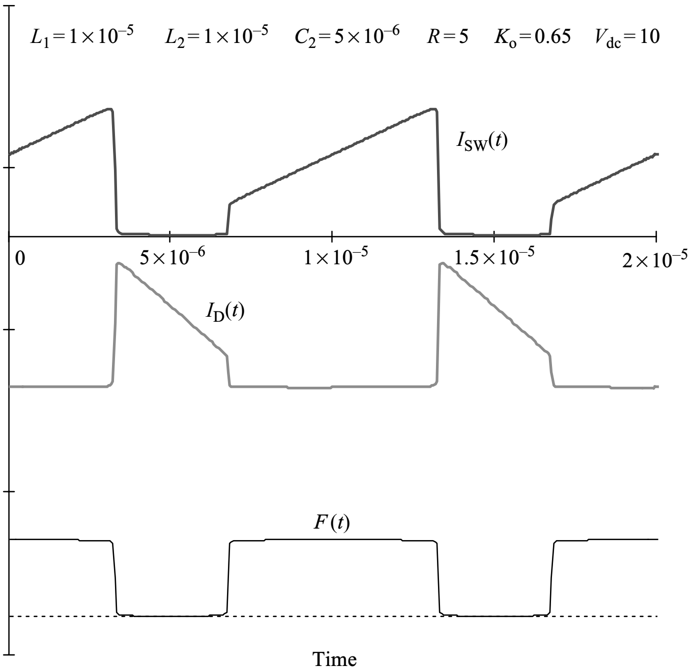


Figure 12.6 The current through the switch, $I_{sw}(t)$ and the diode, $I_D(t)$

The current, $I_{C2}(t)$ produces the output ripple voltage

$$V_{oac}(t) = I_{C2}(t)X(\omega n)$$

$$V_o(t) = -K_o\frac{V_{dc}}{1-K_o} - 2V_{Cdc1}\sum_{n=1}^{\infty}\frac{R}{\sqrt{1+(n\omega C_2 R)^2}}\frac{K_n}{\omega L_2 n} \times \sin(n\omega t - \tan^{-1}(\omega n C_2 R) + 90^o) \qquad (12.23)$$

Figure 12.4 displays the currents through inductors L_1 and L_2, the output voltage and the switching function of the CUK dc to dc converter. Figure 12.5 represents the ratio of the output voltage to the input voltage against the duty-cycle of the switch, K_o. Figure 12.6 represents the currents through the switch and the diode.

Chapter 13

The PWM full bridge dc to dc converter

13.1 Introduction

The full bridge rectifier converter, Fig. 13.1 is very likely to be used to drive a dc motor as it provides four-quadrant operation. Hence the load is composed from L_a the armature inductance, R_a the armature resistance and E_a the back emf induced in the armature of the dc motor.

Two schemes of control for the output voltage are examined: bipolar and unipolar. The switches in each leg S1 to S4 are combinations of a transistor and a diode in anti-parallel thus allowing flow of current in both directions. There is no loss of generality to consider the single switches S1 to S4 in Fig. 13.1 as bi-directional switches.

Expressions for the output voltage and current are derived and time waveforms are displayed. Expressions for the distortion of the output voltage and current are also developed. A mathematical model is not used because of the simplicity of the circuit. A mathematical model is included in Appendix A13.1. It can be used to extend the analysis in investigating the input current. This is useful in deciding the size and rating of the filter capacitors that usually accompany the output stage of a dc supply.

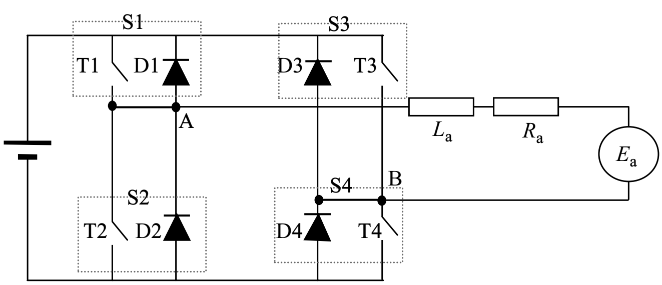


Figure 13.1 The PWM full bridge dc to dc converter

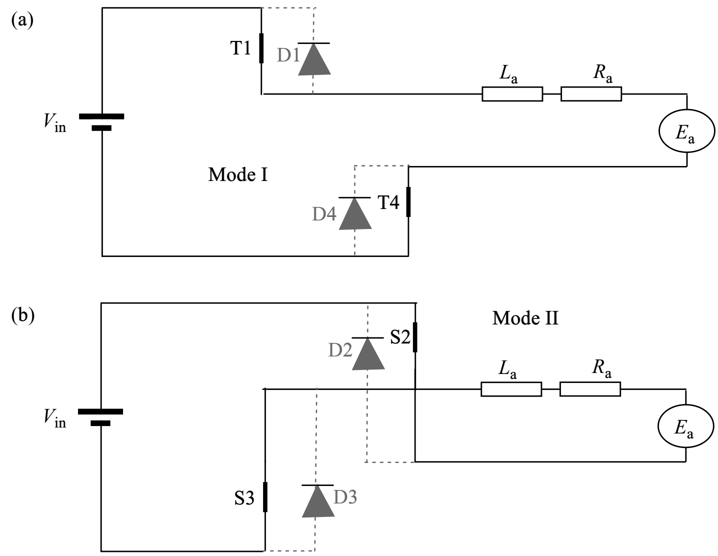


Figure 13.2 PWM full bridge dc to dc converter: bipolar operation. (a) Mode I and (b) Mode II

13.2 Operation and modes of the PWM full bridge dc to dc converter: bipolar operation

In bipolar operation the output voltage swings between V_{in}^{+} and V_{in}^{-}. The pair T1 and T4 conducts simultaneously giving positive voltage at point A, Mode I Fig. 13.2(a). If the current becomes negative, diodes D1 and D4 carry the current. With transistors T1–T4 or diodes D1–D4, the dc source is connected to the load with positive polarity. The switching function $F_1(t)$ is attached to this mode.

The pair T2 and T3 also conducts simultaneously giving negative voltage at point A, Mode II Fig. 13.2(b). If the current becomes positive, diodes D2 and D3 carry the current. With transistors T2–T3 or diodes D2–D3, the dc source is connected to the load with negative polarity. The switching function $F_2(t)$ is attached to this mode.

The two pairs of switches operate in anti-parallel. Figure 13.3 displays the two switching functions together with the output voltage. Both switching functions are of the 'unipolar' type.

The switching function for the switches S1–S4

$$F_1(t) = K_o + 2\sum_{n=1}^{\infty} K_n \cos(n\omega t) \tag{13.1}$$

where

n is an odd integer
$K_o = T_{on}/T$

$K_n = \sin(n\delta)/n\pi$
$\delta = \pi K_o$, half the on-period of the diodes.

And the switching function for the switches S2–S3

$$F_2(t) = 1 - F_1(t) \quad \text{(anti-parallel operation)}$$

$$F_2(t) = 1 - K_o - 2\sum_{n=1}^{\infty} K_n \cos(n\omega t)$$

where n is an odd integer.

During Mode I, the output voltage is given by:

$$V_{Ao}(t) = V_{in} F_1(t) \tag{13.2a}$$

During Mode II, the output voltage is given by:

$$V_{Bo}(t) = V_{in} F_2(t) \tag{13.2b}$$

13.3 Analysis of the PWM full bridge dc to dc converter: bipolar operation

The output voltage, $V_o(t)$, is the potential difference across A and B. The potential at each point is given by Expressions (13.2a) and (13.2b).

$$\begin{aligned} V_o(t) &= V_{in} F_1(t) - V_{in} F_2(t) \\ &= V_{in}[F_1(t) - F_2(t)] \\ &= V_{in}[F_1(t) - [1 - F_1(t)]] \\ &= V_{in}[2F_1(t) - 1] \\ &= V_{in} 2K_o + 4V_{in} \sum_{n=1}^{\infty} K_n \cos(n\omega t) - V_{in} \end{aligned}$$

$$V_o(t) = V_{in}(2K_o - 1) + 4V_{in} \sum_{n=1}^{\infty} K_n \cos(n\omega t) \tag{13.3}$$

Figure 13.3 is a display of the two switching functions, $F_1(t)$ and $F_2(t)$ and the output voltage (negative) and current.

13.3.1 Output dc voltage

From Expression (13.2), we identify the output dc component as

$$V_{odc} = V_{in}(2K_o - 1) \tag{13.4}$$

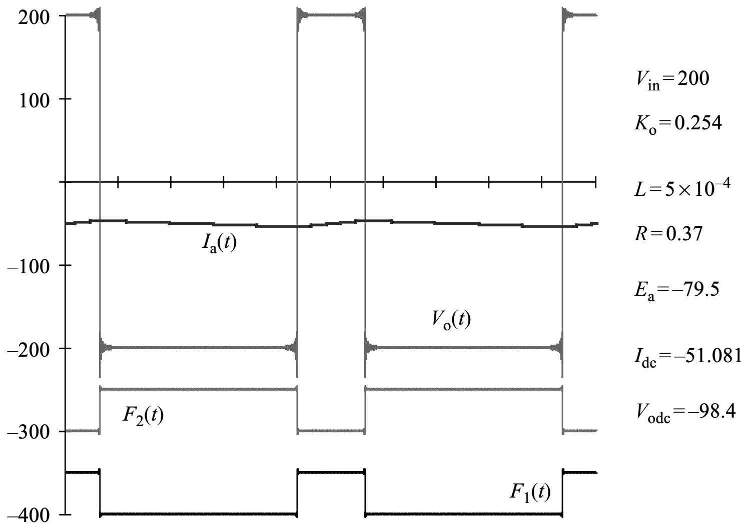


Figure 13.3 Output voltage and current with the associated switching functions. Negative voltage output, bipolar voltage

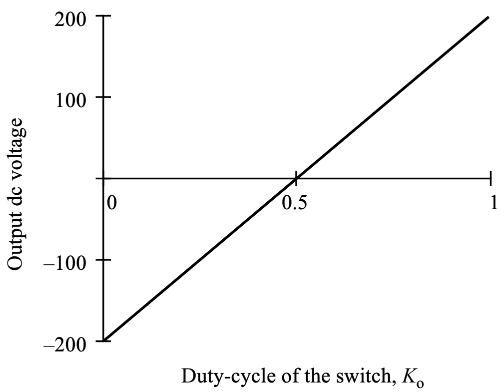


Figure 13.4 Output voltage variation with duty-cycle

For $K_o > 0.5$ the output is positive and for $K_o < 0.5$ the output is negative. The maximum value in both cases is V_{in}. Figure 13.4 displays the output dc voltage against the duty-cycle of the switch, for $V_{in} = 200$ V.

13.3.2 Output current

The output voltage, Expression (13.3), is pushing a current through the armature of the dc motor. The dc component of the output voltage creates a dc component and the ac components of the output voltage create ac components of the output current. The

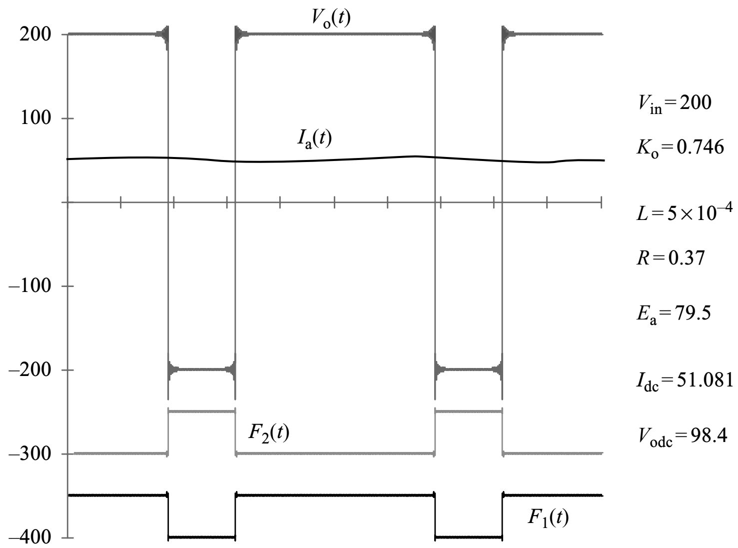


Figure 13.5 Output voltage and current with the associated switching functions. Positive voltage output, bipolar voltage

dc component of current is opposed by the resistance of the load R_a and the back emf E_a. The ac components of current are opposed by the 'harmonic impedance' of the armature, $Z(nw)$.

The harmonic impedance $Z(nw)$ consists of the armature impedance R_a and the armature inductance L_a.

$$Z(n\omega) = \sqrt{R_a^2 + (\omega n L_a)^2} \qquad Q = \tan^{-1}\frac{\omega n L_a}{R_a} \tag{13.5}$$

The output current is given by considering (13.3) and (13.5)

$$I_o(t) = \frac{V_{in}(2K_o - 1) - E_a}{R_a} + 4V_{in}\sum_{n=1}^{\infty}\frac{K_n}{\sqrt{R_a^2 + (n\omega L_a)^2}}\cos(n\omega t - Q) \tag{13.6}$$

Figure 13.5 is a display of the output voltage (positive) together with the output current and the associated switching functions.

13.3.3 Distortion

The level of distortion [5] of the output voltage, $V_{dist}(K_o)$, is expressed as a ratio of the rms value of the output ripple voltage to the input dc voltage, V_{in}. The magnitude

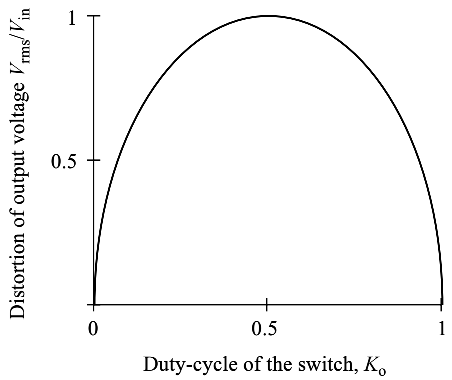


Figure 13.6 Display of the voltage distortion against the duty-cycle of the switch (bipolar operation)

of the harmonics is deduced from Expression (13.3).

$$4V_{in}\sum_{n=1}^{\infty} K_n$$

Hence the output voltage distortion is given in Expression (13.7) as a function of the duty-cycle of the switch, K_o; K_n is substituted from (13.1). Figure 13.6 is a display of the voltage distortion against the duty-cycle of the switch, K_o. Expression (13.7) is an extract from a Mathcad program.

$$V_{dist}(K_o) := \sqrt{\sum_{n=1}^{N}\left(4\frac{\sin(nK_o\pi)}{n\pi\sqrt{2}}\right)^2} \tag{13.7}$$

The ac components of the output current produce no useful power at the dc motor; it is a form of distortion. The level of distortion of the output current, I_{dist}, is expressed as a ratio of the rms value of output ripple current to the maximum value of the output dc current. The magnitude of the output current harmonics is deduced from Expression (13.3). It is given by

$$4V_{in}\sum_{n=1}^{\infty}\frac{K_n}{\sqrt{R_a^2+(n\omega L_a)^2}}$$

And the dc output current,

$$\frac{V_{in}(2K_o-1)-E_a}{R_a}$$

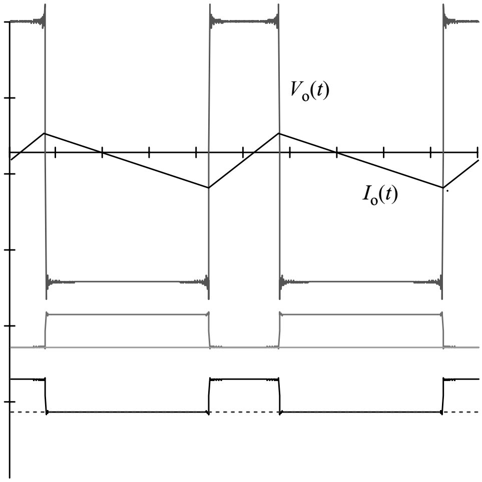


Figure 13.7 Output current at high distortion: bipolar operation

Hence the output current distortion is given in Expression (13.8) as a function of the duty-cycle of the switch, K_o.

$$I_{\mathrm{DIST}} = \frac{\sqrt{\sum_{n=1}^{100}[(V_{\mathrm{in}}4\sin[n(K_o\pi)])/(\sqrt{2}(n\pi)\sqrt{(\omega L_a n)^2 + R_a})^2]^2}}{(V_{\mathrm{in}}(2K_o - 1) - E_a)/R_a} \tag{13.8}$$

K_n is substituted from Expression (13.1). Expression (13.8) must be used with care. It is not valid for all values of K_o because as K_o varies, so does the speed of the motor and E_a. Hence it can only be used for a set of values of K_o and E_a for which the system is working. Figure 13.7 displays high distortion current.

13.4 Operation and modes of the PWM full bridge dc to dc converter: unipolar operation

The same circuit, Fig. 13.1, under a different scheme of control produces an output voltage that swings between zero and V_{in} thus producing a unipolar output voltage. The output can be either positive or negative.

13.4.1 The modes

There are four modes, shown in Fig. 13.8. For positive voltage output the circuit takes the Modes I, II and III in the sequence shown in Fig. 13.9. Effectively the dc source is connected to the load without inversion via switches S1 and S4 (Mode I) for a predetermined period of time and then the current is allowed to circulate in the load outside the dc source first through S3–S4 (Mode II) and then through S1, S2 (Mode III). Each of the four switches S1, S2, S3 and S4 is operated by its own switching function, $F_1(t)$, $F_2(t)$, $F_3(t)$ and $F_4(t)$, Fig. 13.8.

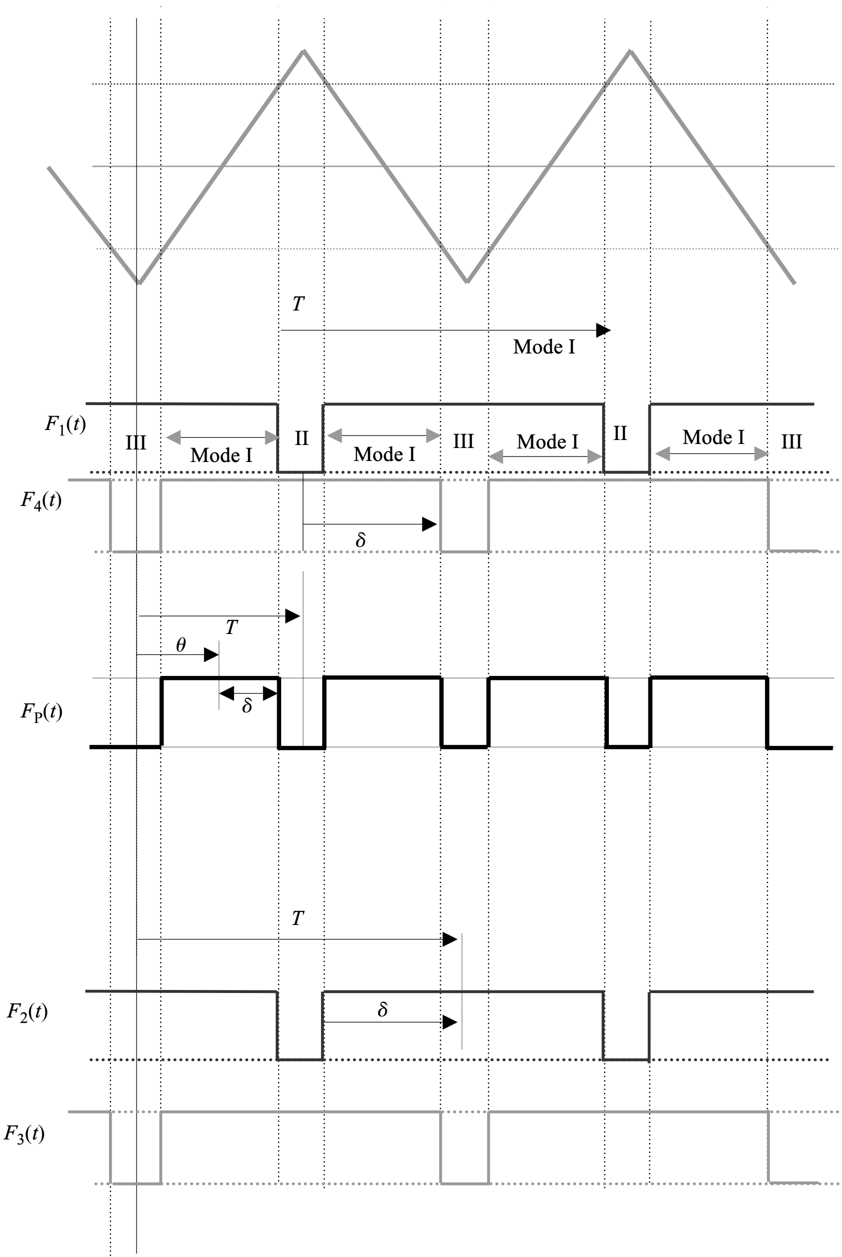


Figure 13.8 Control signals for the PWM full bridge dc to dc converter in unipolar operation

Figure 13.9 Mode sequence for the PWM full bridge dc to dc converter: unipolar operation. Positive voltage

The four modes are shown in Fig. 13.10. During Mode I switches S1 and S4 are closed and the input voltage V_{in} appears at the output without inversion; positive at A, Fig. 13.1. The switching function attached to this mode is $F_{\text{P}}(t)$.

$$V_{\text{o}}(t) = V_{\text{in}} \quad \text{for } F_{\text{P}}(t) = 1$$

$F_{\text{P}}(t)$ is a new switching function describing the state of S_1 and S_4 together. It is related to $F_1(t)$ and $F_4(t)$ by the logic 'and' operator or the arithmetic 'times' operator. It is the switching function for Mode I. During this mode the current might take a negative value and the current is carried by the diodes D1 and D4.

Mode I is followed by Mode II where the dc source is disconnected from the load and the load current is circulating via S3 and S4; in effect it is circulating through D3 and T4. The output voltage is zero and this is taking place when $F_{\text{P}}(t)$ is zero, Fig. 13.8.

$$V_{\text{o}}(t) = 0 \quad F_{\text{P}}(t) = 0$$

Mode I is repeated just after Mode II and then Mode III follows. In Mode III, the dc source is disconnected from the load and the load current is circulating via S1 and S2; in effect it circulates through T1 and D2 in a similar manner as in Mode II. The output voltage is zero.

$$V_{\text{o}}(t) = 0 \quad F_{\text{P}}(t) = 0$$

13.4.2 The switching functions

The control scheme of the PWM full bridge dc to dc converter for unipolar operation is such that each semiconductor switch is supplied with its own drive, Fig. 13.8. Figure 13.8 also displays the control scheme of the PWM full bridge dc to dc converter for unipolar operation for positive output voltage. The action of each semiconductor switch is described by the appropriate switching function. $F_1(t)$ is acting on S1, $F_2(t)$ is acting on S2, $F_3(t)$ is acting on S3 and $F_4(t)$ is acting on S4. All switching functions are of the 'unipolar' type.

$$F_1(t) = K_{\text{o}} + 2\sum_{n=1}^{\infty} K_n \cos(n\omega t)$$

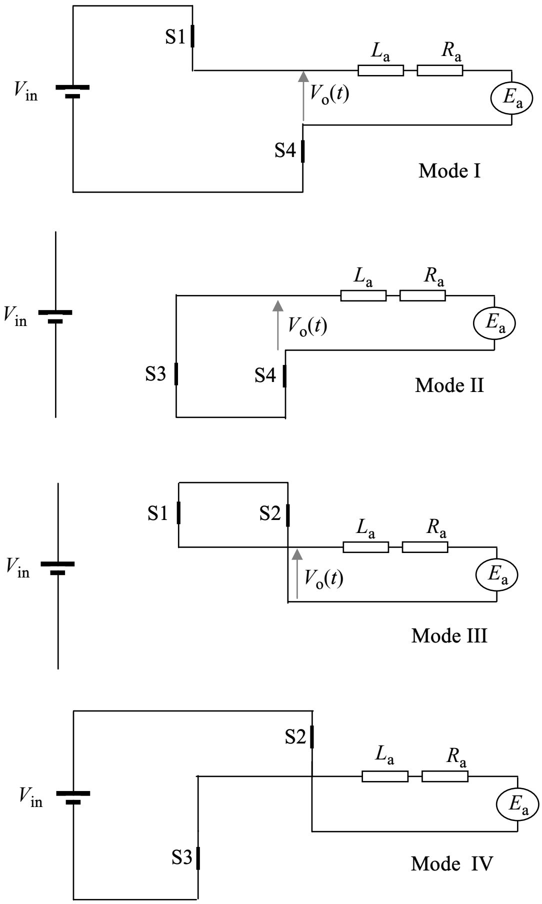


Figure 13.10 The four modes of the full bridge dc to dc converter, unipolar operation

The switching function of S4, $F_4(t)$, is phase delayed by 180° relative to $F_1(t)$.

$$F_4(t) = K_o + 2\sum_{n=1}^{\infty} K_n \cos(n\omega t - n\pi)$$

Mode I is taking place when both switching functions $F_1(t)$ and $F_4(t)$ are one. It is the only mode that contributes to the output. This is when the input dc voltage appears at the output across the load. The switching function, $F_P(t)$, describing the operation of the switches S1 and S4 together, has to be defined. It takes the magnitude of one if and only if both switching functions take the magnitude one, that is, both switches are on. $F_P(t)$ is related to $F_1(t)$ and $F_4(t)$ by the logic 'and' operator or the arithmetic 'times' operator. It is the switching function for Mode I.

$$F_P(t) = F_1(t) \times F_4(t)$$

The above manipulation is tedious and it can be avoided by deriving $F_P(t)$ directly from the diagram of Fig. 13.8. We observe that for the above condition, $F_P(t)$ is derived graphically as a unipolar switching function at twice the switching frequency of $F_1(t)$ and $F_4(t)$ and twice the frequency of the control triangular signal, Appendix A13.2. It is delayed by 180° relative to $F_1(t)$, its half-pulse width δ_1 is $\delta_1 = 2\delta - \pi$ and its average value is $2K_o - 1$.

It is further simplified to give

$$F_P(t) = 2K_o - 1 + 2\sum_{n=1}^{\infty} K_n \cos(2n\omega t) \tag{13.9}$$

where

n is an odd integer
$K_o = T_{on}/T$, duty-cycle of switches S1, S2, S3 and S4
$K_n = \sin(2n\delta)/n\pi$ operation.

This expression of $F_P(t)$ is valid for $1 > K_o > 0.5$.

13.5 Analysis of the PWM full bridge dc to dc converter: unipolar operation

The sequence of the modes shown in Fig. 13.9 for the unipolar operation of the circuit gives positive voltage at the output. Mode I is repeated at a rate twice the frequency of the control triangular signal, Fig. 13.8.

The contribution of Mode I to the output voltage is the only contribution from the three modes. The output voltage is therefore given by

$$V_o(t) = V_{in}F(t)$$

$$V_o(t) = V_{in}[2K_o - 1] + 2V_{in}\sum_{n=1}^{\infty} K_n \cos(n2\omega t) \tag{13.10}$$

The dc component,

$$V_{dco} = [2K_o - 1]V_{in} \tag{13.11}$$

Figure 13.11 is a display of the output voltage V_{dco}, against the duty-cycle of the switches, K_o.

$$V_{oripple}(t) = 2V_{in}\sum_{n=1}^{\infty} K_n \cos(n2\omega t) \tag{13.12}$$

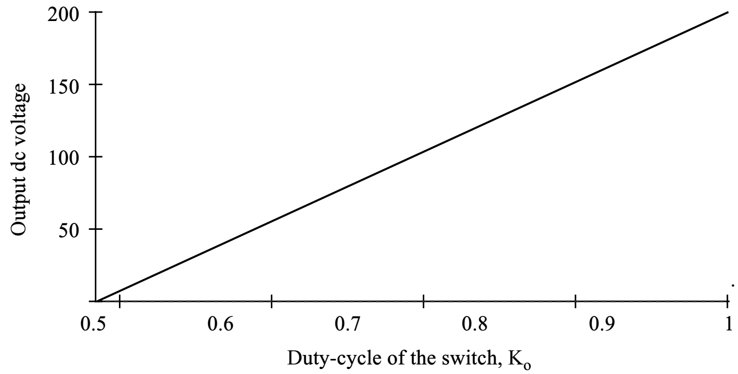


Figure 13.11 Display of the output voltage V_{dco}, against the duty-cycle of the switches, K_o

The output current is given by simple application of Ohm's Law

$$I_o(t) = \frac{V_{in}(2K_o - 1) - E_a}{R_a} + 2V_{in}\sum_{n=1}^{\infty}\frac{K_n}{\sqrt{R_a^2 + (2n\omega L_a)^2}}\cos(2n\omega t - Q) \tag{13.13}$$

$$Q = \tan^{-1}\frac{2n\omega L_a}{R_a}$$

$$I_{dco} = \frac{V_{dco} - E_a}{R_a} \tag{13.14}$$

Figure 13.12 is a display of the output voltage, output current and the associated switching functions. The same quantities are displayed in Fig. 13.13 for a lower current.

13.5.1 Distortion

The level of distortion of the output voltage, V_{DIST}, is expressed as a ratio of the rms value of the output ripple voltage to the input dc voltage, V_{in}. The magnitude of the harmonics of the output voltage is deduced from Expression (13.12) as:

$$2V_{in}\sum_{n=1}^{\infty}K_n$$

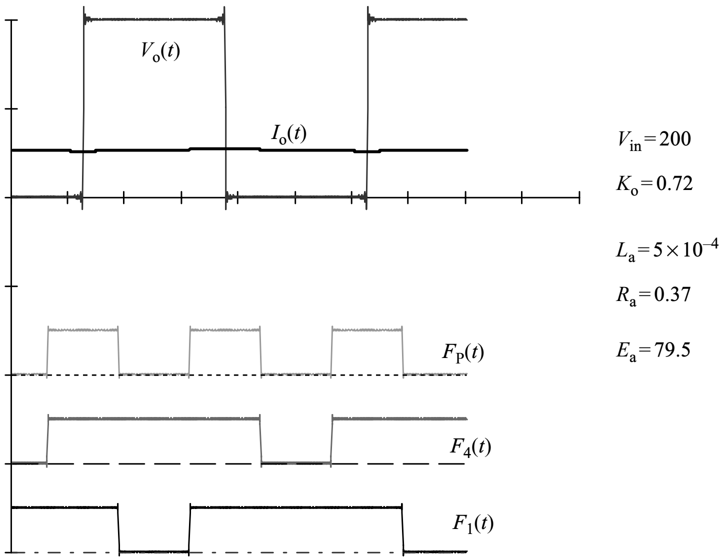


Figure 13.12 *Unipolar operation of the PWM full bridge dc to dc converter: positive voltage output, output current and associated switching functions*

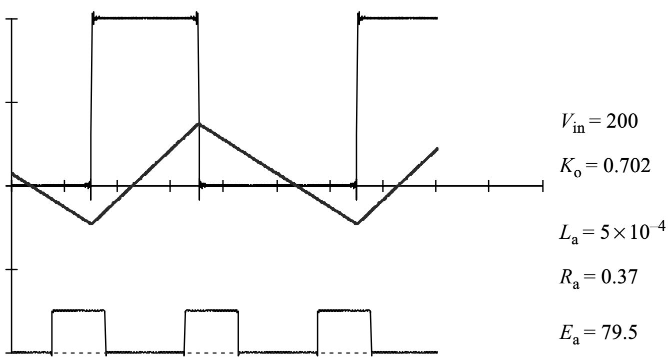


Figure 13.13 *Unipolar operation of the PWM full bridge dc to dc converter: positive voltage output, low output current and associated switching functions*

Hence the output voltage distortion is given in Expression (13.15) as a function of the duty-cycle of the switch, K_o.

$$V_{\text{DIST-U}} = \sqrt{\sum_{n=1}^{N}\left[2\frac{\sin[n2(K_o\pi)]}{(n\pi)\sqrt{2}}\right]^2} \tag{13.15}$$

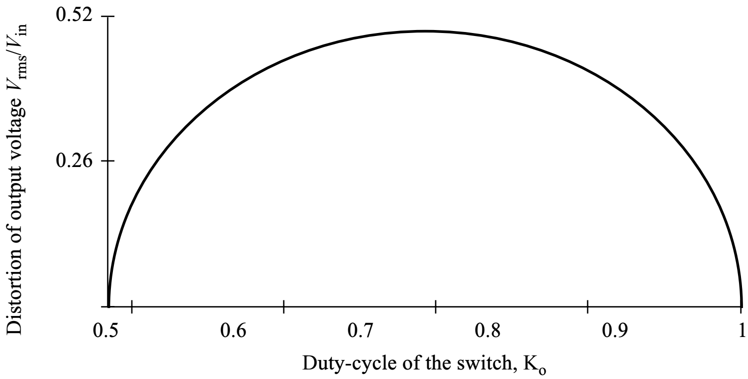


Figure 13.14 Display of the voltage distortion against the duty-cycle of the switch (unipolar operation)

K_n is replaced from (13.9) and N is the number of the harmonics considered in the calculation of the output voltage distortion; typical number, 200. Figure 13.14 is a display of the voltage distortion against the duty-cycle of the switch.

The level of distortion of the output current, I_{DIST}, is expressed as a ratio of the rms value of output ripple current to the output dc current. The output ripple and the magnitude of the harmonics is deduced from Expression (13.13). Hence the output current distortion is given in Expression (13.16) as a function of the duty-cycle of the switch, K_o.

$$I_{\mathrm{DIST}} = \frac{\sqrt{\sum_{n=1}^{N}[V_{\mathrm{in}}2\sin\lfloor 2n(K_o\pi)\rfloor/\sqrt{2}(n\pi)\sqrt{(2\omega L_a n)^2 + R_a^2}]^2}}{(V_{\mathrm{in}} - E_a)/R_a} \tag{13.16}$$

K_n is substituted from Expression (13.9). Expression (13.16) must be used with care. It is not valid for all values of K_o because as K_o varies, so does the speed of the motor and E_a. Hence it can only be used for a set of values of K_o and E_a for which the system is working. Figure 13.13 displays a high distortion current.

The dc component for the unipolar operation of the converter is the same as for the bipolar operation, Expressions (13.7) and (13.15). The harmonic component though is improved. The magnitude of the harmonics is reduced by a factor of two and the lowest harmonic present at the output is the second; the first is cancelled. These result in lower voltage distortion at the output, Fig. 13.14.

13.5.2 Negative voltage generation

For the converter to produce negative voltage, Modes IV, II and III are employed in the sequence shown in Fig. 13.15. Switches S2 and S3 are switched-on (Mode IV) to connect the dc source to the load with polarity inverted. Modes II and III simply provide the freewheeling path for the load current. The switching functions describing the action of the switches S1–S4 are displayed in Fig. 13.8.

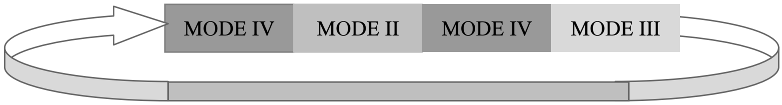
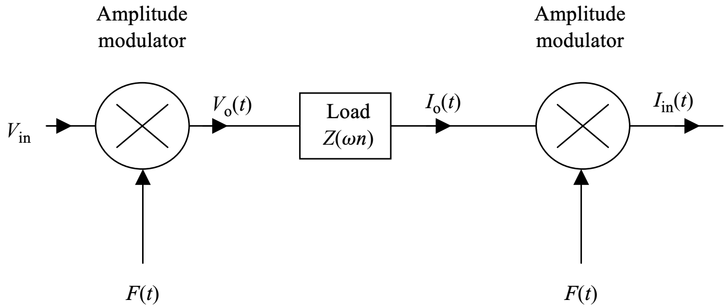


Figure 13.15 *Mode sequence for the PWM full bridge dc to dc converter: unipolar operation negative voltage*

Appendix A13.1: A mathematical model for bipolar operation



It is shown that the output voltage is related to the input by:

$$V_o(t) = V_{in}[2F_1(t) - 1]$$

Therefore the input current is related to the output current

$$I_{in}(t) = [2F_1(t) - 1]I_o(t)$$

For the bipolar

$$I_o(t) = \frac{V_{in}(2K_o - 1) - E_a}{\sqrt{R_a^2 + (n\omega L_a)^2}} 4V_{in} \sum_{n=1}^{\infty} K_n \sin(n\omega t - Q)$$

The switching function

$$F(t) = [2K_o - 1] + 4\sum_{n=1}^{\infty} K_n \cos(n\omega t)$$

The input current

$$\begin{aligned} I_{in}(t) = [2K_o - 1]&\frac{V_{in}(2K_o - 1) - E_a}{\sqrt{R_a^2 + (n\omega L_a)^2}} 4V_{in} \sum_{n=1}^{\infty} K_n \sin(n\omega t - Q) \\ &+ 2\frac{V_{in}(2K_o - 1) - E_a}{\sqrt{R_a^2 + (n\omega L_a)^2}} 4V_{in} \sum_{n=1}^{\infty}\sum_{n=1}^{\infty} \sin[(m+n)\omega t - Q] \\ &+ \sin[(m-n)\omega t - Q] \end{aligned}$$

Appendix A13.2: The switching function for the PWM full bridge dc to dc converter

A13.2.1 Unipolar operation

The switching function, $F_P(t)$, is shown in Fig. 13.9 and it can be described from there. It has a frequency twice the $F_1(t)$, phase delayed by π and it is of the unipolar type.

$$F(t) = K_{o1} + 2\sum_{n=1}^{\infty} K_n \cos(n2\omega t - n\pi)$$

The off-period T_{OFF} is the same as for $F_1(t)$ but the on-period, $T_{ON} = (T_{ON1} - T_{OFF})/2$

$$T_{ON} = (2T_{ON1} - T_1)/2$$

Also the period T is $T_1/2$.

$$T_{OFF} = T_1 - T_{ON1}$$

$$K_o = T_{ON}/T = \{(2T_{ON1} - T_1)/2\}/[T_1/2]$$

$$K_o = 2K_{o1} - 1$$

And $\delta t = (2T_{ON1} - T_1)/4 = T_{ON1}/2 - T_1/4$

$F(t)$ is at twice the frequency of $F_1(t)$. Therefore $T_{ON1}/2$ gives 2δ in 2ω for $F(t)$ in radians and $T_1/4$ is half the cycle of $F(t)$ that is, π.
Now in radians, $\delta = \delta_1 - \pi$

$$K_n = \sin n[2\delta_1 - \pi]/n\pi = \sin[n2\delta_1 - n\pi]/n\pi$$

$$K_n = \frac{\sin(2n\delta)}{n\pi}$$

$$F_p(t) = 2K_{o1} - 1 + 2\sum_{n=1}^{\infty} K_n \cos(n2\omega t - n\pi)$$

This is the same as $F_p(t) = 2K_o - 1 + 2\sum_{n=1}^{\infty} K_n \cos(n2\omega t)$

$$K_n = \frac{\sin(2n\delta)}{n\pi} \quad \text{and} \quad K_o = 2K_{o1} - 1$$

Part 4

Frequency changers

Chapter 14
Three by three matrix converter

14.1 Introduction

Matrix converters were suggested many years ago [8] and have been attracting the interest of many researchers [9,10] recently. In this chapter the switching function method of analysis is employed first to derive the appropriate modulating frequency and then to produce input and output waveforms. Perhaps the switching function is best suited for the matrix converter because of the complexity of the switching patterns and the strict requirements imposed for the safety of the semiconductor switches.

The analysis is first performed with a two pulse switching function and the switching frequency is m times the mains frequency, ω. The converter is analysed for input and output phase voltage, input and output current for a star connected load; m can be chosen to be smaller or larger than one. The two pulse switching function gives amplitude control of the output voltage and at the same time it satisfies the two safety criteria applied in matrix converters: no two input lines must always be connected at the same time to a single output line and the load must always be connected to the input.

A two pulse switching function subjects the circuit to a wide spectrum of modulating frequencies, $\sum_{n=1}^{\infty} nm\omega$. The switching matrix is derived on the basis that each of the three input lines is connected to a single output line with the normal three phase sequence separated by 120°.

It will be shown that the output voltage has a strong $(m-1)\omega$ frequency component or a strong $(m+1)\omega$ frequency component depending on the switching matrix. In both cases there is a wide range of harmonics in the output voltage. It is also observed from the expression of the output voltage that when the two pulse switching functions are replaced by their average value and their fundamental ac component, the harmonics of the output voltage are eliminated. The output becomes a pure sinewave at $(m-1)\omega$ or at $(m+1)\omega$ depending on the modulating matrix. This implies that the output frequency will be set by m and can be lower or higher than the mains frequency. A switching function is approximated to its average value and its fundamental ac component by PWM coding.

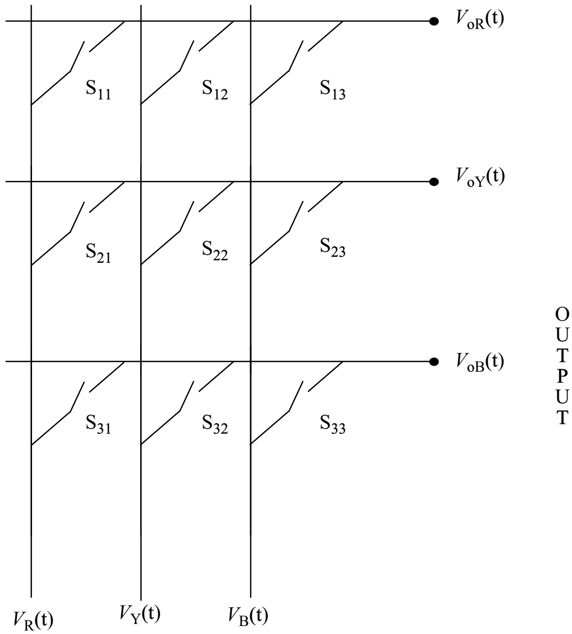


Figure 14.1 The matrix converter

The output current for an RL passive load is calculated for both the two pulse switching function and the single frequency (PWM) modulating signal. As the input current is a reflection of the output current, the appropriate switching matrix is derived from the modes of the circuit.

14.2 Operation and mathematical model

The three input lines are connected to the three output lines via nine bidirectional switches S_{rc}, Fig. 14.1, to form the matrix converter. Each switch is operated by a switching function, $F_{rc}(t)$, where r denotes the row and c the column.

The connecting matrix for the output voltage is $M_V(t)$,

$$M_V(t) = \begin{bmatrix} F_{11}(t) & F_{12}(t) & F_{13}(t) \\ F_{21}(t) & F_{22}(t) & F_{23}(t) \\ F_{31}(t) & F_{32}(t) & F_{33}(t) \end{bmatrix} \tag{14.1}$$

The choice of the switching matrix determines the frequency, the amplitude and the phase of the output. The switches must be operated in such a way that the voltage and current Kirchoff's laws are not violated. This means

(i) No two input lines must be connected at the same time to the same output line, otherwise a short circuit will take place.

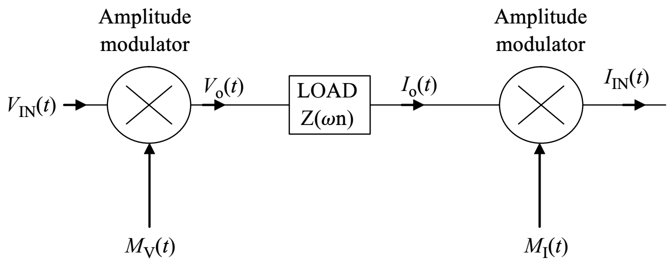


Figure 14.2 The mathematical model of the matrix converter

(ii) At all times, each of the three output lines is connected to an input line otherwise an inductive load will destroy the semiconductor switches.

Both restrictions are met if the summation of the three switching functions operating on a single output line is one.

$$\begin{aligned} &\sum_{c=1}^{3} F_{1c}(t) = 1 \quad \text{For the red output line} \\ &\sum_{r=1}^{3} F_{2c}(t) = 1 \quad \text{For the yellow output line} \\ &\sum_{r=1}^{3} F_{3c}(t) = 1 \quad \text{For the blue output line} \end{aligned} \tag{14.2}$$

The output current, $I_o(t)$, is forced to flow through the load by the output voltage. The input current is a reflection of the output current to the input as it was discussed in Chapter 1. The input current, $I_{IN}(t)$, in each input line is made up from the contributions of each output line. Hence a connecting switching function matrix exists, $M_I(t)$ and it will be derived in the next section. The mathematical model for the Matrix converter representing its basic operation is very simple, Fig. 14.2

The output voltage is given by

$$V_{OUT}(t) = M_V(t)V_{IN}(t) \tag{14.3}$$

The output current by

$$I_o(t) = \frac{V_o(t)}{Z(\omega n)} \tag{14.4}$$

And the input current by

$$I_{IN}(t) = M_I(t)I_o(t) \tag{14.5}$$

14.3 The modes of operation and the switching functions

14.3.1 The switching functions

A two pulse switching function is employed; the switches connecting input and output lines are switched-on $2m$ times per mains cycle; m is the ratio of the switching frequency to the mains frequency. If a single pulse switching function is used, the restrictions of Expression (14.2) are only satisfied when the on-period of each pulse is fixed to 120°. This implies a fixed on-period which cannot give amplitude control. Hence in order to satisfy the two requirements of Expression (14.2) and at the same time provide amplitude control, a switching function with at least two pulses is required; each pulse is separated by 180°. Perhaps this is not the only choice; it is adopted though because it is the simplest switching function that can satisfy the two safety requirements above. More importantly, such a switching function contains a wide frequency spectrum starting from the fundamental $m\omega$ and extending to infinity. In this way the response of the system to this wide modulating frequency spectrum is studied. It will be shown that this approach will help us to choose the appropriate modulating frequency for the required output frequency.

The three switches in each row of Fig. 14.1 are switched on connecting the three input lines to a single output line at a phase delay of 120° to each other in order to match the phase-displacement of the regular three-phase supply; reference is the red phase input voltage. There is a switching function for each switch and there are three groups of switching functions, one for each output line. The first group contains $F_{11}(t)$, $F_{12}(t)$ and $F_{13}(t)$ connecting the three input lines to the output red line, the second group contains $F_{21}(t)$, $F_{22}(t)$ and $F_{23}(t)$ connecting the three input lines to the output yellow line and the third group contains $F_{31}(t)$, $F_{32}(t)$ and $F_{33}(t)$ connecting the three input lines to the output blue line. Each group is phase delayed by 120° to each other in order to create the output three-phase voltage. These are displayed in Fig. 14.3 and the general expression of the switching function is given in Expression (14.7).

The total on-period for each switching function is $\pi/3$ in order to satisfy the two restrictions set above, hence if 2δ is the on-period of the first pulse, $\pi/3 - 2\delta$ is the on-period of the second pulse. Hence $F_{\mathrm{rc}}(t)$, the general expression for the switching functions is given by,

$$\begin{aligned}
F_{\mathrm{rc}}(t) &= K_{\mathrm{oF}} + 2\sum_{n=1}^{\infty} K_{\mathrm{nF}} \cos\{nm\omega t - [\alpha + (r+c-2)120^\circ]n\} \\
&\quad + K_{\mathrm{oS}} + 2\sum_{n=1}^{\infty} K_{\mathrm{nS}} \cos\{nm\omega t - [\alpha + 180^\circ + (r+c-2)120^\circ]n\} \\
K_{\mathrm{oF}} &= \frac{\delta}{\pi} \quad K_{\mathrm{nF}} = \frac{\sin(n\delta)}{n\pi} \quad K_{\mathrm{nS}} = \frac{\sin(n\delta_{\mathrm{o}})}{n\pi} \quad K_{\mathrm{oS}} = \frac{\delta_{\mathrm{o}}}{\pi} \\
\delta &= \frac{\pi}{3}D \quad \delta_{\mathrm{o}} = \frac{\pi}{3} - \delta
\end{aligned} \tag{14.7}$$

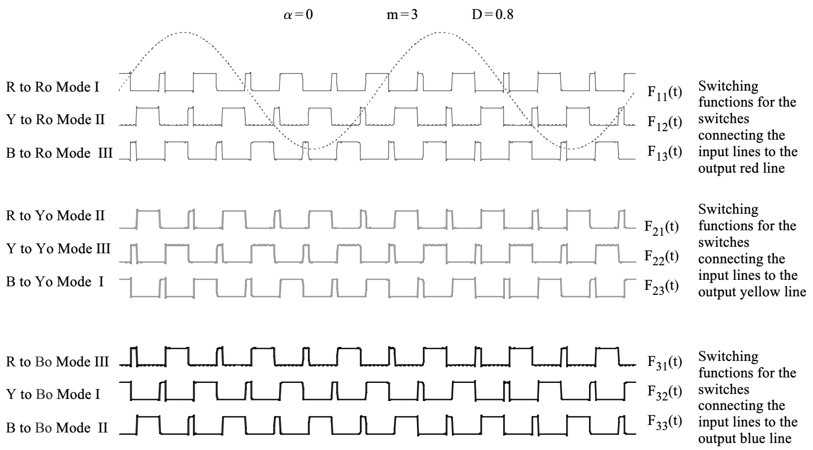


Figure 14.3 *The switching functions for the matrix converter: three phase to three phase*

where

$\alpha =$ The value of α is set externally and it is the phase of the switching function connecting the input red line to the output red phase.
$m =$ Ratio of the switching frequency to the input frequency.
$D =$ Duty cycle of the switches.
$r =$ row and $c =$ column.

The ratio of the switching frequency w_s to the mains frequency w is m. Hence the adopted switching function contains all the frequency harmonics of the switching frequency mw.

14.3.2 The modes

The circuit undergoes three modes, Fig. 14.4 under the modulating matrix of Expression (14.1). Each mode is repeated twice per mode sequence, Fig. 14.5; once with the first pulse of the switching functions and once with the second pulse of the switching function.

14.3.3 The switching matrix for the input current

Each input line is connected to the output lines in the sequence R_o, Y_o and B_o. Therefore the input line current is made up from the contributions of each output line in a way described by the mode sequence Expression (14.5) and the switching functions Fig. 14.3. Therefore, three expressions can be written for the input currents as a function of the switching functions and the output currents.

$$I_R(t) = I_{Ro}(t)F_{11}(t) + I_{Yo}(t)F_{21}(t) + I_{Bo}(t)F_{31}(t)$$

$$I_Y(t) = I_{Ro}(t)F_{12}(t) + I_{Yo}(t)F_{22}(t) + I_{Bo}(t)F_{32}(t)$$

$$I_B(t) = I_{Ro}(t)F_{13}(t) + I_{Yo}(t)F_{23}(t) + I_{Bo}(t)F_{33}(t)$$

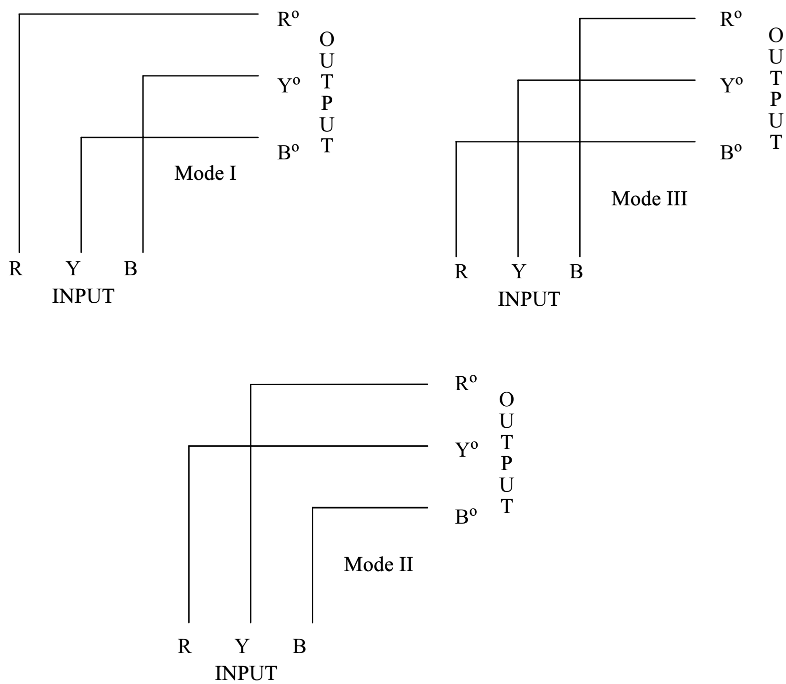


Figure 14.4 The modes of the three phase to three phase matrix converter

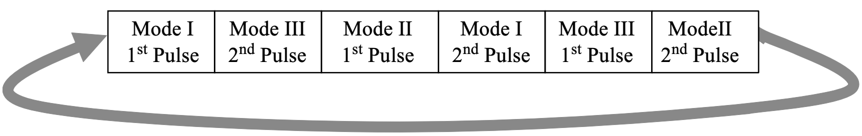


Figure 14.5 Mode sequence for the matrix converter

And in matrix form

$$\begin{bmatrix} I_{\mathrm{R}}(t) \\ I_{\mathrm{Y}}(t) \\ I_{\mathrm{B}}(t) \end{bmatrix} = \begin{bmatrix} F_{11}(t) & F_{21}(t) & F_{31}(t) \\ F_{12}(t) & F_{22}(t) & F_{32}(t) \\ F_{13}(t) & F_{23}(t) & F_{33}(t) \end{bmatrix} \begin{bmatrix} I_{\mathrm{Ro}}(t) \\ I_{\mathrm{Yo}}(t) \\ I_{\mathrm{Bo}}(t) \end{bmatrix}$$

Therefore the switching matrix for the input current is $M_{\mathrm{V}}(t)^{\mathrm{T}}$

$$M_{\mathrm{I}}(t) = \begin{bmatrix} F_{11}(t) & F_{21}(t) & F_{31}(t) \\ F_{12}(t) & F_{22}(t) & F_{32}(t) \\ F_{13}(t) & F_{23}(t) & F_{33}(t) \end{bmatrix} \tag{14.8}$$

14.4 Analysis of the matrix converter as a three-phase to three-phase system

14.4.1 Output phase voltage with strong $(m-1)\omega$ *component*

The output voltage is given by Expression (14.3). The input voltages, $V_R(t) = V_p \sin \omega t$, $V_Y(t) = V_p \sin(\omega t - 120°)$ and $V_B(t) = V_p \sin(\omega t - 240°)$ form the input voltage vector, $V_{IN}(t)$,

$$V_{IN}(t) = \begin{matrix} V_R(t) \\ V_Y(t) \\ V_B(t) \end{matrix} \tag{14.9}$$

The output voltage is given by Expression (14.3) where $M_V(t)$ is given in Expression (14.1). The output red phase voltage is derived by multiplying the first row of the matrix $M_V(t)$ with the input voltage vector of Expression (14.9), $(r = 1)$. In the same way the output yellow phase voltage is derived by multiplying the second row of $M_V(t)$ with the input voltage vector of Expression (14.9), $(r = 2)$. Finally, the output blue phase voltage is derived by multiplying the third row of $M_V(t)$ with the input voltage vector of Expression (14.9), $(r = 3)$. The input phase voltage is expressed by the general Expression (14.9a)

$$V_{in}(t) = V_p \sin[\omega t - 120°(c-1)] \tag{14.9a}$$

The output phase voltage is derived in the general form

$$\begin{aligned} V_{or}(t) = {} & V_p \sum_{c=1}^{3} [K_{oF} + K_{oS}] V_p \sin[\omega t - 120°(c-1)] \\ & + V_p \sum_{c=1}^{3} \sum_{n=1}^{\infty} K_{nF} \sin\{(nm+1)\omega t - [\alpha + (r+c-2)120°]n - 120°(c-1)\} \\ & - V_p \sum_{c=1}^{3} \sum_{n=1}^{\infty} K_{nF} \sin\{(nm-1)\omega t - [\alpha + (r+c-2)120°]n + 120°(c-1)\} \\ & + V_p \sum_{c=1}^{3} \sum_{n=1}^{\infty} K_{ns} \sin\{(nm+1)\omega t - [\alpha + 180° + (r+c-2)120°]n \\ & - 120°(c-1)\} \\ & - V_p \sum_{c=1}^{3} \sum_{n=1}^{\infty} K_{ns} \sin\{(nm-1)\omega t - [\alpha + 180° + (r+c-2)120°]n \\ & + 120°(c-1)\} \end{aligned} \tag{14.10}$$

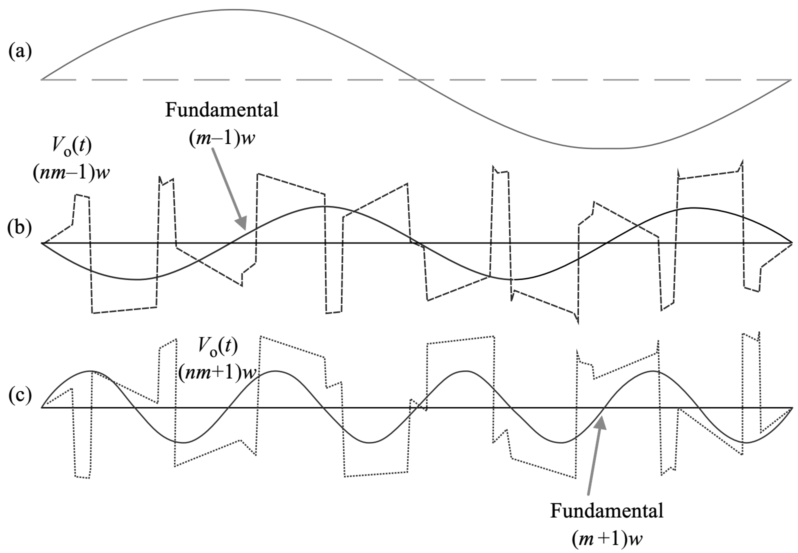


Figure 14.6 Output voltage

The output voltage for each phase is derived from the Expression (14.10) by setting the appropriate value of r (row) and c (column). The value of r gives the output phase voltage as follows:

For the output red phase output, $r = 1$

For the output yellow phase output, $r = 2$

For the output blue phase output, $r = 3$

Expression (14.10) gives the output phase voltage, Fig. 14.6(b). The output phase voltage for the red line is also given in a Mathcad format in Appendix A14.1. A careful study of Expression (14.10) reveals that

1. The output voltage consists of two bands of frequencies: $nm + 1$ and $nm - 1$.
2. In the first band of $nm + 1$, the vectorial addition of the three terms (one for each value of c), is zero for odd values of n. Hence the lowest frequency term is for $n = 2$, that is, $2m + 1$.
3. In the second band of $nm - 1$, for $n = 1$ all three terms add up giving a strong $m - 1$ term. For $n = 2$ and $n = 3$, the sum is zero. For $n = 4$ all three terms add up, hence the lowest term is $4m - 1$.
4. The observations of points 2 and 3 above lead to the conclusion that the strongest term is the $(m - 1)\omega$.
5. The output does not contain the power input frequency, at 50 Hz.

6. For $n = 1$ the output is a single frequency component at a frequency $(m-1)\omega$. In this case the output is given by

$$\begin{aligned} V_{m-1}(t) = &-V_{\mathrm{p}} \sum_{c=1}^{3} K_{\mathrm{nF}} \sin\{(m-1)\omega t \\ &- [\alpha + (r+c-2)120^\circ] + 120^\circ(c-1)\} \\ &- V_{\mathrm{p}} \sum_{c=1}^{3} K_{\mathrm{ns}} \sin\{(m-1)\omega t \\ &- [\alpha + 180^\circ + (r+c-2)120^\circ] + 120^\circ(c-1)\} \end{aligned} \tag{14.11}$$

Fig. 14.6(b) gives the output voltage of the output red phase of Expression (14.11). The result of Expression (14.11) is very useful; it suggests direct ac to ac conversion without the need for passive reactive components [8]. For $n = 1$ the switching functions, Expression (14.7), are reduced to their average value and the fundamental component $m\omega$ only, Expression (14.11a). In practice this is achieved by employing PWM techniques to construct single ac component ($n = 1$) switching functions. The general Expression (14.7) of the switching function in this case is reduced to $\mathrm{MS}_{rc}(t)$ a modulating signal of Expression (14.11a).

$$\begin{aligned} \mathrm{MS}_{rc}(t) = &K_{\mathrm{oF}} + 2K_{1\mathrm{F}} \cos\{m\omega t - [\alpha - (r+c-2)120^\circ]\} \\ &+ K_{\mathrm{oS}} + 2K_{1\mathrm{S}} \cos\{m\omega t - [\alpha + 180^\circ - (r+c-2)120^\circ]\} \end{aligned} \tag{14.11a}$$

The output voltage under this condition is displayed in Fig. 14.7 for $m = 3$.

14.4.2 Generation of output phase voltage with strong $(m+1)\omega$ component

The term $(m-1)\omega$ in Expression (14.10) can be removed and the strongest term becomes $(m+1)\omega$ if the phase delays of the second and third term of

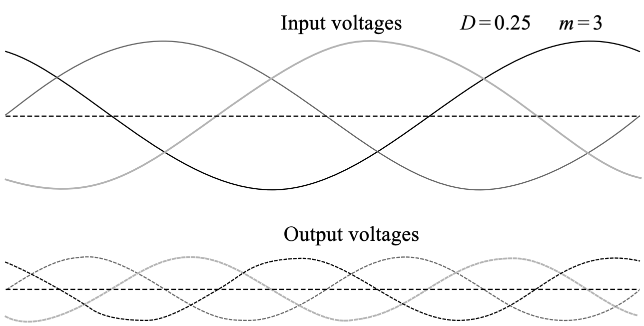


Figure 14.7 Output phase voltages with the single component modulating signal

Expression (14.10) are interchanged. This is achieved by modulating $V_y(t)$ with $F_{13}(t)$ instead of $F_{12}(t)$ and $V_b(t)$ with $F_{12}(t)$ instead of $F_{13}(t)$. This implies interchanging yellow and blue lines at the input of the converter or a new switching matrix is employed, Expression (14.12). This is derived from Expression (14.1) by interchanging the positions of the switching functions as per discussion above.

$$M_V(t) = \begin{matrix} F_{11}(t) & F_{13}(t) & F_{12}(t) \\ F_{21}(t) & F_{23}(t) & F_{22}(t) \\ F_{31}(t) & F_{33}(t) & F_{32}(t) \end{matrix} \tag{14.12}$$

Expression (14.3) is used again but now the switching matrix of Expression (14.12) is employed in order to eliminate the $(m-1)\omega$. Expression (14.13) is derived in the same way as (14.10) from Expressions (14.3), (14.12) and (14.9a) and it gives a strong $(m+1)w$ component.

$$\begin{aligned} V_o(t) = {} & \sum_{c=1}^{3}[K_{oF} + K_{oS}]V_p \sin[\omega t + 120^\circ(c-1)] \\ & + \sum_{c=1}^{3}\sum_{n=1}^{\infty} K_{nF} \sin\{(nm+1)\omega t \\ & - [\alpha + (r+c-2)120^\circ]n + 120^\circ(c-1)\} \\ & - \sum_{c=1}^{3}\sum_{n=1}^{\infty} K_{nF} \sin\{(nm-1)\omega t \\ & - [\alpha + (r+c-2)120^\circ]n - 120^\circ(c-1)\} \\ & + \sum_{c=1}^{3}\sum_{n=1}^{\infty} K_{ns} \sin\{(nm+1)\omega t \\ & - [\alpha + 180^\circ + (r+c-2)120^\circ]n + 120^\circ(c-1)\} \\ & - \sum_{c=1}^{3}\sum_{n=1}^{\infty} K_{ns} \sin\{(nm-1)\omega t \\ & - [\alpha + 180^\circ + (r+c-2)120^\circ]n - 120^\circ(c-1)\} \end{aligned} \tag{14.13}$$

Figure 14.6(c) gives the output voltage of the output red phase as derived from Expression (14.13). If the switching functions are reduced to their average value and the fundamental component $m\omega$ only, the counter n in Expression (14.13) takes the value of 1 and the output voltage of Expression (14.13) is now given by Expression (14.14). A single ac component switching function is achieved

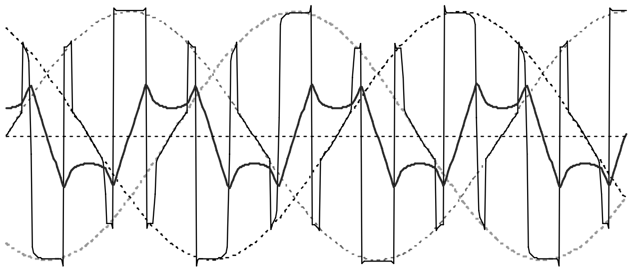


Figure 14.8 Output phase voltage and current

by PWM.

$$V_o(t) = +\sum_{c=1}^{3} K_{nF} \sin\{(m+1)\omega t - [\alpha + (r+c-2)120°] + 120°(c-1)\}$$
$$+\sum_{c=1}^{3} K_{ns} \sin\{(m+1)\omega t - [\alpha + 180° + (r+c-2)120°] + 120°(c-1)\} \tag{14.14}$$

Expression (14.14) gives the output voltage when the switching function is reduced to its average value and its fundamental ac component.

14.4.3 Output current

The output voltage and the load 'harmonic impedance' dictate the output current, $I_O(t)$ according to Expression (14.4). The output current for the red phase is derived for a star load and for the output voltage given by Expression (14.10) where the two pulse switching function is used; it is displayed in Fig. 14.8. The expression for the output current is shown in Appendix A14.2 for the red phase in a Mathcad format.

The output current for the red phase is also derived from Expression (14.11), for the sinusoidal waveform given by Expression (14.15); it is displayed in Fig. 14.9.

$$I_{oR}F(t) := \frac{-3V_p}{\sqrt{[(m-1)wL]^2 + R^2}} \frac{\sin(\delta)}{\pi} \times \sin\left[(m-1)\omega t - \alpha r - \operatorname{atan}\left[\frac{[(m-1)wL]}{R}\right]\right]$$
$$- \frac{3V_p}{\sqrt{[(m-1)wL]^2 + R^2}} \frac{\sin(\delta_o)}{\pi} \times \sin\left[(m-1)\omega t - \alpha - \pi - \operatorname{atan}\left[\frac{[(m-1)wL]}{R}\right]\right] \tag{14.15}$$

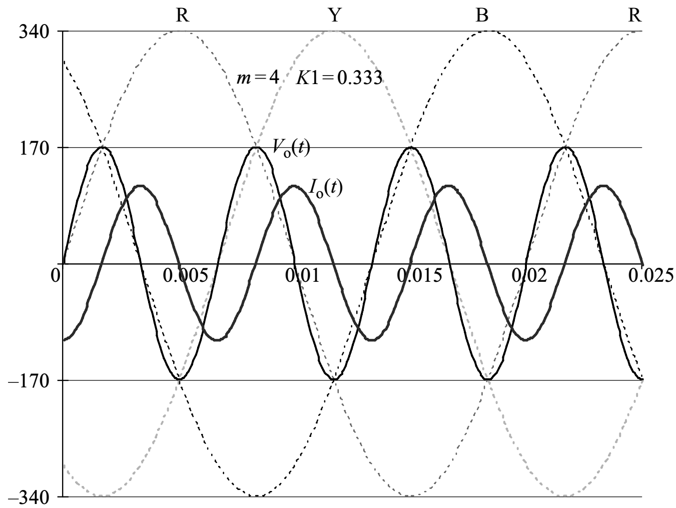


Figure 14.9 Output voltage and current when the switches are operated by the modulating signal $M_{\mathrm{rc}}(t)$

A more general way to present Expression (14.15) is Expression (14.15a)

$$I_{\mathrm{oC}}(t) = I_{\mathrm{p}} \sin[(m-1)\omega t - \varphi - (c-1)120°] \tag{14.15a}$$

where c is the counter indicating the phase: $c = 1$ for red, $c = 2$ for yellow and $c = 3$ for blue. The load angle is φ.

14.4.4 Input current

A matrix converter is more likely to be used to produce a sinusoidal output voltage and consequently a sinusoidal output current is expected. This implies modulation of the converter by the modulating signal $\mathrm{MS}_{rc}(t)$, Expression (14.11a). The input current is given by Expression (14.5) and from it the phase current for modulating signals $\mathrm{MS}_{rc}(t)$ instead of two pulse switching functions is given by

$$I_{\mathrm{C}}(t) = [\mathrm{MS}_{11}(t) \quad \mathrm{MS}_{13}(t) \quad \mathrm{MS}_{12}(t)] \begin{bmatrix} I_{\mathrm{OR}}(t) \\ I_{\mathrm{OB}}(t) \\ I_{\mathrm{OY}}(t) \end{bmatrix} \tag{14.16}$$

The switching functions are replaced by the modulating signal $\mathrm{MS}_{rc}(t)$ of Expression (14.11a) which is further simplified

$$\mathrm{MS}_{rc}(t) = K_{\mathrm{oF}} + K_{\mathrm{oS}} + 2(K_{1\mathrm{F}} - K_{1\mathrm{S}}) \cos\{m\omega t - [\alpha + (r + c - 2)120°]\}$$

$$r = 1 \text{ for the row and } c = 1, 2, 3 \text{ depending on the column}$$

The dc component is $K_{\mathrm{oF}} + K_{\mathrm{oS}}$ and is $\frac{1}{3}$ for all values of D. The average value of the modulating signal has to be limited to $\frac{1}{3}$ in order to allow all three switches in a single row of Fig. 14.1 to connect the three input lines in succession to a single output line without overlap, Expression (14.2), within the period of the switching frequency.

The coefficient of the ac component is $K_1 = 2(K_{1F} - K_{1S})$. It is a function of the duty-cycle of the switch, D. But D has no meaning now that the switching function is replaced by a PWM signal. Therefore the modulating signal takes the form of:

$$\mathrm{MS}_{rc}(t) = \tfrac{1}{3} + K_1 \cos\{m\omega t - [\alpha + (r + c - 2)120°]\} \tag{14.16a}$$

It is important though to stress that in coding the modulating signal $M_{\mathrm{rc}}(t)$ by PWM, it is Expression (14.11a) that is used in order to satisfy the restrictions of Expression (14.2)

A single switch is only operated by a unipolar switching signal, that is, the switching signal cannot take negative values. Therefore the peak value of $M_{\mathrm{rc}}(t)$ cannot exceed the dc component, that is, $K_1 \leq \frac{1}{3}$.

$$I_{\mathrm{inC}}(t) = \sum_{c=1}^{3} \left\{ \frac{1}{3} + K_1 \cos[m\omega t - [\alpha + (r + c - 2)120°]] \right\}$$
$$\times \{I_{\mathrm{p}} \sin[(m - 1)\omega t - [\varphi + (c - 1)120°]]\}$$

Expanded and simplified

$$\begin{aligned} I_{\mathrm{inC}}(t) = \sum_{c=1}^{3} \Bigg\{ & \frac{1}{3} I_{\mathrm{p}} \sin[(m - 1)\omega t - \varphi - (c - 1)120°] \\ & + 0.5 K_1 I_{\mathrm{p}} \sin[(2m-1)\omega t - [\alpha + (r + c - 2)120°] - [\varphi + (c - 1)120°]] \\ & + 0.5 K_1 I_{\mathrm{p}} \sin[(\omega t) - [\alpha + (r + c - 2)120°] + [\varphi + (c - 1)120°]] \Bigg\} \end{aligned}$$

The first two terms represent three vectors each that cancel out for all values of r. Hence, the input current is given by:

$$I_{\mathrm{inC}}(t) = \sum_{c=1}^{3} 0.5 K_1 I_{\mathrm{p}} \sin[(\omega t) - [\alpha + (r + c - 2)120°] + [\varphi + (c - 1)120°]]$$

And finally

$$I_{\mathrm{inC}}(t) = 0.5 K_1 I_{\mathrm{p}} \sin[(\omega t) + \varphi - \alpha - (r - 1)120°] \tag{14.17}$$

Therefore the input current under PWM switching is a single component at the mains frequency. Figure 14.10 displays input and output voltages and input and output currents with the same modulation process.

14.4.5 Discussion

In order to respect the restrictions of Expressions 14.2, the duty-cycle of the switches must be limited to $\frac{1}{3}$; this implies that its dc component is $\frac{1}{3}$. Therefore when the

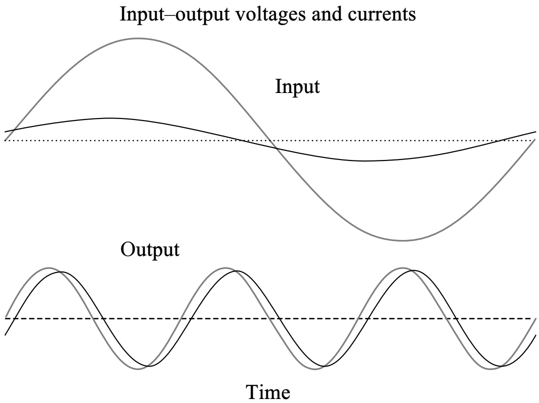


Figure 14.10 Input and output voltages and input and output currents (PWM modulation)

double pulse switching functions $F_{\mathrm{rc}}(t)$ are replaced by the PWM signal $\mathrm{MS}_{rc}(t)$ this restriction still applies. Hence the peak value of $M_{\mathrm{rc}}(t)$ cannot exceed its dc component, that is, $K_1 \le \frac{1}{3}$ because a single switch is only operated by a unipolar switching signal, that is, the switching signal cannot take negative values. Under these conditions the output voltage cannot exceed 50% of the input.

The investigation of the matrix converter is not exhausted in this chapter. It has shown how such a converter can produce directly an ac voltage from an ac source. More importantly it has shown the way that very serious matters of this circuit can be investigated.

The matrix converter consists of nine switches and the timing of their switching action is very important in order to avoid short-circuiting of the input or an open circuit of the output. The complexity of the many switching signals of the semiconductor devices can be mathematically modelled with the appropriate switching functions. This will help the designer to adopt the best control strategies for the converter.

14.5 The matrix converter as an ac to dc voltage converter

It is evident from the above discussions and Expression (14.10) that for $m = 1$, the output has zero frequency, hence dc is produced at the output, Fig. 14.11.

The matrix converter can be operated in such a way in order to produce dc output voltage. The output voltage, Expression (14.10), has a strong component at $m - 1$. Therefore, for $m = 1$ the strongest output component will be at zero frequency, that is, dc. Even better, for single frequency modulation, $n = 1$, the output will be a perfect dc. For dc generation, $m = 1$ with $n = 1$. The level of the voltage output is set by both the delay angle of the switching function α_{R} and the duty-cycle of the switches.

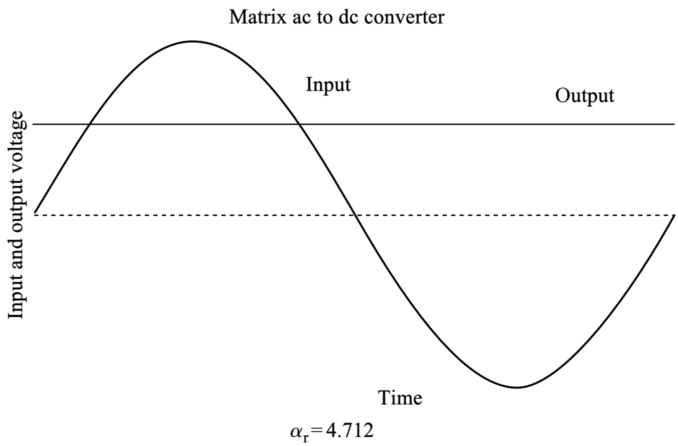


Figure 14.11 *Rectifier operation. Output voltage at $m = 1$*

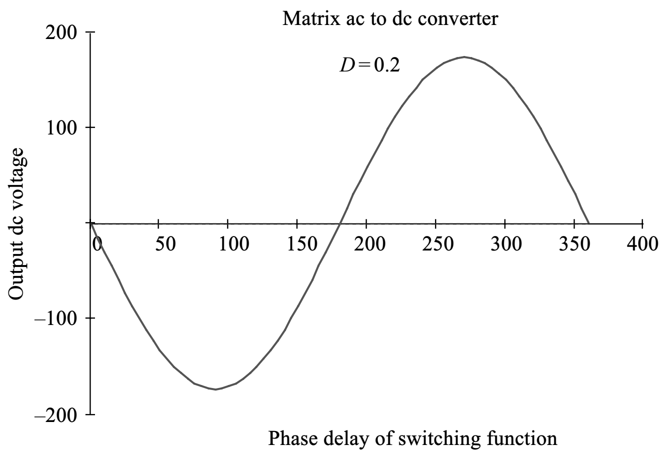


Figure 14.12 *Rectifier operation. Output voltage against the delay angle*

From Expression (14.10) and for $m = 1$ and $n = 1$

$$V_{\text{dc}} = 3V_{\text{p}}K_{1\text{F}}\sin(\alpha_{\text{R}}) + 3V_{\text{p}}K_{1\text{S}}\sin(\alpha_{\text{R}} + 180^\circ)$$

$$V_{\text{dc}} = 3V_{\text{p}}K_{1\text{F}}\sin(\alpha_{\text{R}}) - 3V_{\text{p}}K_{1\text{S}}\sin(\alpha_{\text{R}})$$

$$V_{\text{dc}} = 3V_{\text{p}}\sin(\alpha_{\text{R}})\{K_{1\text{F}} - K_{1\text{S}}\}$$

Hence the output voltage is set by both the phase of the modulating signal, α_{R}, Fig. 14.12 and the magnitude of the modulating signal, $\{K_{1\text{F}} - K_{1S}\}$, Fig. 14.13.

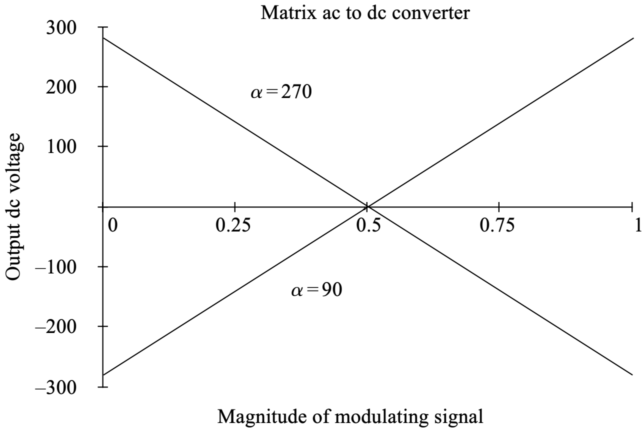


Figure 14.13 Rectifier operation. Level of output dc voltage

Appendix A14.1: The output voltage

$$
\begin{aligned}
V_o(t) := \;& V \leftarrow 0 \\
& \alpha r \leftarrow \frac{\alpha \pi}{180} \\
& \text{for } n \in 1 \ldots \mathrm{N} \\
& \quad Kn1 \leftarrow \frac{\sin(n\delta)}{n\pi} \\
& \quad Kn2 \leftarrow \frac{\sin(n\delta_o)}{n\pi} \\
& \quad \Omega 1 \leftarrow \omega t(nm + 1) \\
& \quad \Omega 2 \leftarrow \omega t(nm - 1) \\
& \quad \Theta 1 \leftarrow \alpha rn + (n + 1)\frac{2\pi}{3} \\
& \quad \Theta 2 \leftarrow \alpha rn + (n - 1)\frac{2\pi}{3} \\
& \quad \Theta 3 \leftarrow \alpha rn + (2n - 1)\frac{2\pi}{3} \\
& \quad \Theta 4 \leftarrow \alpha rn + (2n + 1)\frac{2\pi}{3} \\
& \quad \Theta 11 \leftarrow (\alpha r + \pi)n + (n + 1)\frac{2\pi}{3}
\end{aligned}
$$

$$
\begin{array}{l}
\Theta 21 \leftarrow (\alpha r + \pi)n + (n-1)\dfrac{2\pi}{3} \\
\Theta 31 \leftarrow (\alpha r + \pi)n + (2n-1)\dfrac{2\pi}{3} \\
\Theta 41 \leftarrow (\alpha r + \pi)n + (2n+1)\dfrac{2\pi}{3} \\
\mathrm{V1} \leftarrow \mathrm{Kn1}(\sin(\Omega 1 - \alpha r n) - \sin(\Omega 2 - \alpha r n)) \\
\mathrm{V2} \leftarrow \mathrm{Kn1}(\sin(\Omega 1 - \Theta 1) - \sin(\Omega 2 - \Theta 2)) \\
\mathrm{V3} \leftarrow \mathrm{Kn1}(\sin(\Omega 1 - \Theta 3) - \sin(\Omega 2 - \Theta 4)) \\
\mathrm{V4} \leftarrow \mathrm{Kn2}(\sin(\Omega 1 - (\alpha r + \pi)n) - \sin(\Omega 2 - (\alpha r + \pi)n)) \\
\mathrm{V5} \leftarrow \mathrm{Kn2}(\sin(\Omega 1 + \Theta 11) - \sin(\Omega 2 - \Theta 21)) \\
\mathrm{V6} \leftarrow \mathrm{Kn2}(\sin(\Omega 1 + \Theta 31) - \sin(\Omega 2 - \Theta 41)) \\
\mathrm{VT} \leftarrow \mathrm{Vp}(\mathrm{V1} + \mathrm{V2} + \mathrm{V3} + \mathrm{V4} + \mathrm{V5} + \mathrm{V6}) \\
\mathrm{V} \leftarrow \mathrm{V} + \mathrm{VT}
\end{array}
$$

Appendix A14.2: The output current

$$
\mathrm{I_o}(t) := \left|
\begin{array}{l}
I \leftarrow 0 \\
\alpha r \leftarrow \dfrac{\alpha \pi}{180} \\
\text{for } n \in 1 \ldots \mathrm{N} \\
\quad \left|
\begin{array}{l}
\mathrm{Kn1} \leftarrow \dfrac{\sin(n\delta)}{n\pi} \\
\mathrm{Kn2} \leftarrow \dfrac{\sin(n\delta_o)}{n\pi} \\
\Omega 1 \leftarrow \omega t(nm + 1) \\
\Omega 2 \leftarrow \omega t(nm - 1) \\
\Theta 1 \leftarrow \alpha r n + (n+1)\dfrac{2\pi}{3} \\
\Theta 2 \leftarrow \alpha r n + (n-1)\dfrac{2\pi}{3} \\
\Theta 3 \leftarrow \alpha r n + (2n-1)\dfrac{2\pi}{3} \\
\Theta 4 \leftarrow \alpha r n + (2n+1)\dfrac{2\pi}{3}
\end{array}
\right.
\end{array}
\right.
$$

$$\Theta 11 \leftarrow (\alpha r + \pi)n + (n+1)\frac{2\pi}{3}$$

$$\Theta 21 \leftarrow (\alpha r + \pi)n + (n-1)\frac{2\pi}{3}$$

$$\Theta 31 \leftarrow (\alpha r + \pi)n + (2n-1)\frac{2\pi}{3}$$

$$\Theta 41 \leftarrow (\alpha r + \pi)n + (2n+1)\frac{2\pi}{3}$$

$$\Phi 1 \leftarrow \mathrm{atan}\left[\frac{[\omega L(nm+1)]}{R}\right]$$

$$\Phi 2 \leftarrow \mathrm{atan}\left[\frac{[\omega L(nm-1)]}{R}\right]$$

$$\mathrm{Y1} \leftarrow \frac{1}{\sqrt{[\omega L(nm+1)]^2 + R^2}}$$

$$\mathrm{Y2} \leftarrow \frac{1}{\sqrt{[\omega L(nm-1)]^2 + R^2}}$$

$$\mathrm{I1} \leftarrow \mathrm{Kn1}(\mathrm{Y1}\sin(\Omega 1 - \alpha r n - \Phi 1) - \mathrm{Y2}\sin(\Omega 2 - \alpha r n - \Phi 2))$$

$$\mathrm{I2} \leftarrow \mathrm{Kn1}(\mathrm{Y1}\sin(\Omega 1 - \Theta 1 - \Phi 1) - \mathrm{Y2}\sin(\Omega 2 - \Theta 2 - \Phi 2))$$

$$\mathrm{I3} \leftarrow \mathrm{Kn1}(\mathrm{Y1}\sin(\Omega 1 - \Theta 3 - \Phi 1) - \mathrm{Y2}\sin(\Omega 2 - \Theta 4 - \Phi 2))$$

$$\mathrm{I4} \leftarrow \mathrm{Kn2}[\mathrm{Y1}\sin[\Omega 1 - (\alpha r + \pi)n - \Phi 1] - Y2\sin[\Omega 2 - (\alpha r + \pi)n - \Phi 2]]$$

$$\mathrm{I5} \leftarrow \mathrm{Kn2}(\mathrm{Y1}\sin(\Omega 1 - \Theta 11 - \Phi 1) - \mathrm{Y2}\sin(\Omega 2 - \Theta 21 - \Phi 2))$$

$$\mathrm{I6} \leftarrow \mathrm{Kn2}(\mathrm{Y1}\sin(\Omega 1 - \Theta 31 - \Phi 2) - \mathrm{Y2}\sin(\Omega 2 - \Theta 41 - \Phi 2))$$

$$\mathrm{IT} \leftarrow V_{\mathrm{p}}(\mathrm{I1} + \mathrm{I2} + \mathrm{I3} + \mathrm{I4} + \mathrm{I5} + \mathrm{I6})$$

$$\mathrm{I} \leftarrow \mathrm{I} + \mathrm{IT}$$

Chapter 15

The single pulse PWM inverter

15.1 Introduction

The output of this converter is sometimes described as 'modified sine-wave'; in effect it is a quasi-square signal. The level of the output voltage is controlled by the width of the single pulse, which makes up each half-cycle of the output voltage. It is analysed here not only because it presents some academic interest but more importantly, its findings are to be applied to the more practical case of the sinusoidal PWM inverter. The switches and the source are considered ideal. Continuous and discontinuous conduction are both considered.

15.2 Operation and modes of the circuit

There is a transistor and a diode connected in anti-parallel in each branch of the bridge configuration of Fig. 15.1. The current is conducted either by the transistor or the diode in each of the four branches; hence bi-directionality is secured. A transistor conducts if it is correctly biased and the appropriate drive is applied to its gate. Conduction through the transistors is controlled by the gate signals and a suitable

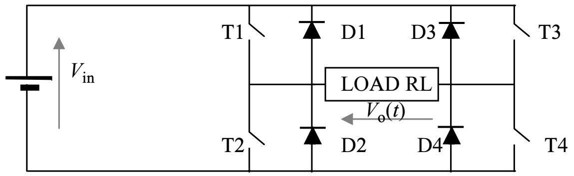


Figure 15.1 The single pulse PWM inverter

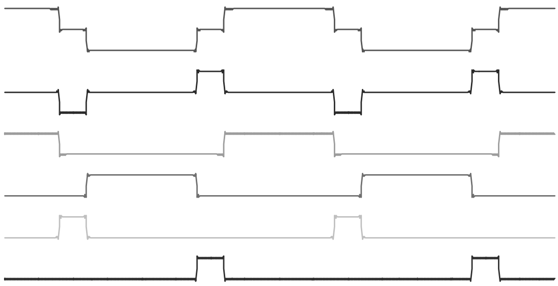


Figure 15.2 The switching functions

switching function, $F_{B2}(t)$ shown in Fig. 15.2 describes their action. A diode will conduct when it is forward biased and there is no control over it. In Fig. 15.1, the diodes are reverse biased by the dc voltage supply and they will only conduct if the load provides the required forward biasing. The presence of an active load such as an ac motor or a simple RL load can provide these conditions. In this case the load parameters, the input dc voltage and the output current determine the timing and duration of the conduction of the diodes. Hence a second switching function, $F_{B2}(t)$ shown in Fig. 15.2 is defined, to describe the action of the switches. It will be shown that both functions are of bridge or bipolar type and $F_{B1}(t)$ refers to the transistors and $F_{B2}(t)$ refers to the diodes. The first is a forced function since the control circuit produces it and the second is called 'hidden or inherent' because the circuit itself sets its parameters. Nevertheless there is a restriction on $F_{B1}(t)$ as well; the transistors can conduct current in one direction only and the gate signals can only switch them on when the current in the inductive load is extinguished. Hence the positive going pulse of $F_{B1}(t)$ starts when the current in the diodes is zero. For this reason the operation of this inverter is limited to discontinuous conduction; in a more sophisticated application the direction of current can be sensed so the appropriate drive signals are applied. For the multi-pulse configuration of the next chapter this restriction is only true at the transition instance when the current changes polarity.

The modes of the circuit, Fig. 15.3 are seven. Two modes, I and IV present the transistors in the on state and they exist during the on state of the switching function $F_{B1}(t)$. Another two modes, II and V present the circuit with the diodes in the on state and the current freewheeling through the load and the dc supply. These modes exist during the on state of the switching function $F_{B2}(t)$. Mode III presents the circuit when all the semiconductor switches are off; no new switching function needs to be defined as this is taking place during the off states of $F_{B1}(t)$ and $F_{B2}(t)$. Modes VI and VII present the circuit under freewheeling mode through the load. They take place during the off states of $F_{B1}(t)$ and $F_{B2}(t)$ and the output voltage is zero. A new switching function though needs to be defined for the output current during that period.

The circuit can be operated in two ways:

(i) During the off-periods of the transistors, the load current is allowed to flow through the diodes and return energy to the dc source, Fig. 15.3, giving rise to Modes II and V. They exist during the on state of the second switching function, $F_{B2}(t)$. The sequences A and C in Figs. 15.4 and 15.6, respectively refer to this case.
(ii) The current can be made to circulate through a diode and a transistor (freewheeling) thus disconnecting completely the supply from the load, Fig. 15.3, giving rise to Modes VI and VII. The sequence in Fig. 15.5 refers to this case.

In both cases it is possible for the current to extinguish and the transistors to be gated to re-connect the dc source to the load after a delay giving rise to Mode III, Fig. 15.3. The current direction with the connected transistors is opposite to the one during the preceding freewheeling mode so the transistors can only be fired at the instant the current is zero or any time after. The output voltage is zero and the load is open circuited during Mode III. Fig. 15.6 shows the mode sequence for this case.

15.2.1 Mode sequences

15.2.1.1 Mode sequence A

Sequence A suggests continuous conduction. Without sensing of the load current direction, this sequence applies only on the onset of continuous and discontinuous conduction. With reference to Figs. 15.2 and 15.3 Modes I and IV exist with switching function $F_{B1}(t)$

During Mode I $\quad V_o(t) = V_{dc} \quad F_{B1}(t) = 1$

During Mode IV $\quad V_o(t) = -V_{dc} \quad F_{B1}(t) = -1$

The change of polarity of the output voltage suggests that $F_{B1}(t)$ is of the bipolar type. The contribution of the Modes I and IV to the output is

$$V_o(t) = F_{B1}(t)V_{dc}$$

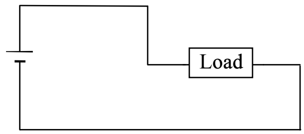
Modes II and V exist with switching function $F_{B2}(t)$

During Mode II $\quad V_o(t) = -V_{dc} \quad F_{B2}(t) = -1$

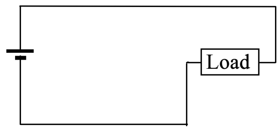
During Mode V $\quad V_o(t) = V_{dc} \quad F_{B2}(t) = 1$

The change of polarity of the output voltage suggests that $F_{B2}(t)$ is also of the bipolar type. The contribution of the Modes II and V to the output is

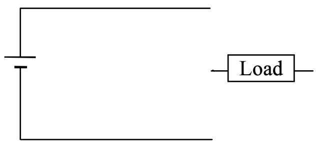
$$V_o(t) = F_{B2}(t)V_{dc}$$



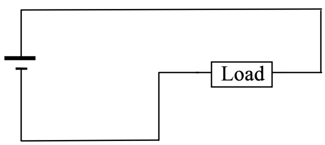
Mode I: Semiconductor switches T1 and T4 are ON, $I_{T14}(t) = \mathrm{I_o}(t)$. Load voltage is positive on LHS of load. The current is increasing.
$V_o(t) = V_{in}$ $F_{B1}(t) = 1$ $F_{B2}(t) = 0$



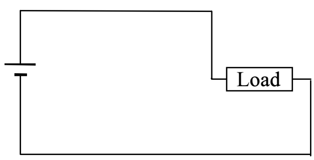
Mode II: Semiconductor switches T1,T2,T3,T4, D1 and D4 are OFF. Diodes D3 and D2 are forward biased and conducting, $I_{D23}(\mathrm{t}) = I_o(t)$. Current is decreasing and the voltage across the inductor is reversed. Load voltage is negative on LHS of load.
$V_o(t) = -V_{in}$ $F_{B1}(t) = 0$ $F_{B2}(t) = -1$



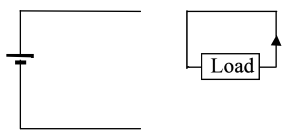
Mode III: The current is extinguished and all diodes and transistors are OFF. The beginning of this mode marks the end of the on-state of $F_{B2}(t)$.
$F_{B1}(t) = 0$ $F_{B2}(t) = 0$ $V_o(t) = 0$



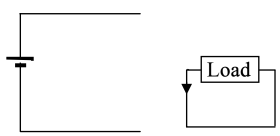
Mode IV: Semiconductor switches T2 and T3 are ON. Load voltage is negative on LHS of load. The current is increasing.
$V_o(t) = -\mathrm{V_{in}}$ $F_{B1}(t) = -1$ $F_{B2}(t) = 0$
Current through T2 and T3, $I_{T23}(t) = I_o(t)$. Current through the rest of the semiconductor switches is zero.



Mode V: Semiconductor switches T1,T2,T3,T4, D2 and D3 are OFF. Diodes D1 and D4 are forward biased and conducting, $I_{D14}\,(t) = I_o(t)$. Current decreases and the voltage across the inductor is reversed. Load voltage is positive on LHS of load.
$V_o(t) = V_{in}$ $F_{B1}(t) = 0$ $F_{B2}(t) = 1$



Mode VI: The load current decreases and it circulates through D3 and T1 from left to right in the load. Current and the voltage across inductor is reversed. Diodes D2 and D3 are forward biased but T1 is gated and conducts. Load voltage is 0. $I_{in}(t) = 0$



Mode VII: Semiconductor switches T2 and T3 are just OFF. Current decreases and the voltage across inductor is reversed. Diodes D4 and D1 are forward biased but T2 is gated and conducts. The load current circulates through D4 and T2 from right to left in the load.
$V_o(t) = 0$ $F_{B1}(t) = 0$ $F_{B2}(t) = 0$

Figure 15.3 Single pulse dc to ac converter: the modes of operation

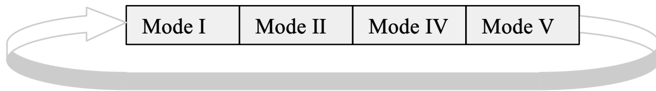


Figure 15.4 *Mode sequence A: continuous conduction and energy return to the source via the diodes*

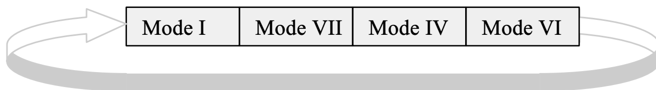


Figure 15.5 *Mode sequence B: continuous conduction with free-wheeling action through the load*

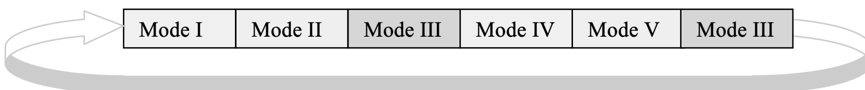


Figure 15.6 *Mode sequence C: discontinuous conduction and energy return to the source via the diodes through the load*

Hence the output voltage of the inverter with continuous conduction under sequence A is

$$V_{\mathrm{o}}(t) = F_{\mathrm{B1}}(t)V_{\mathrm{dc}} + F_{\mathrm{B2}}(t)V_{\mathrm{dc}}$$

Expression (15.1) refers to continuous conduction and the parameters of $F_{\mathrm{B2}}(t)$ are derived from $F_{\mathrm{B1}}(t)$. Specifically,

$$F_{\mathrm{B2}}(t) = 1 - F_{\mathrm{B1}}(t)$$

15.2.1.2 Mode sequence B

Modes II and V in sequence A are replaced by VI and VII in sequence B. For these modes the output voltage is zero, Fig. 15.3 and the only contribution to the output is from Modes I and IV. Expression (15.1) is modified to:

$$V_{\mathrm{o}}(t) = F_{\mathrm{B1}}(t)V_{\mathrm{dc}}$$

Expression (15.3) gives the output voltage for free-wheeling through the load.

A third switching function must be defined to give the current through the load during Modes VII and VI.

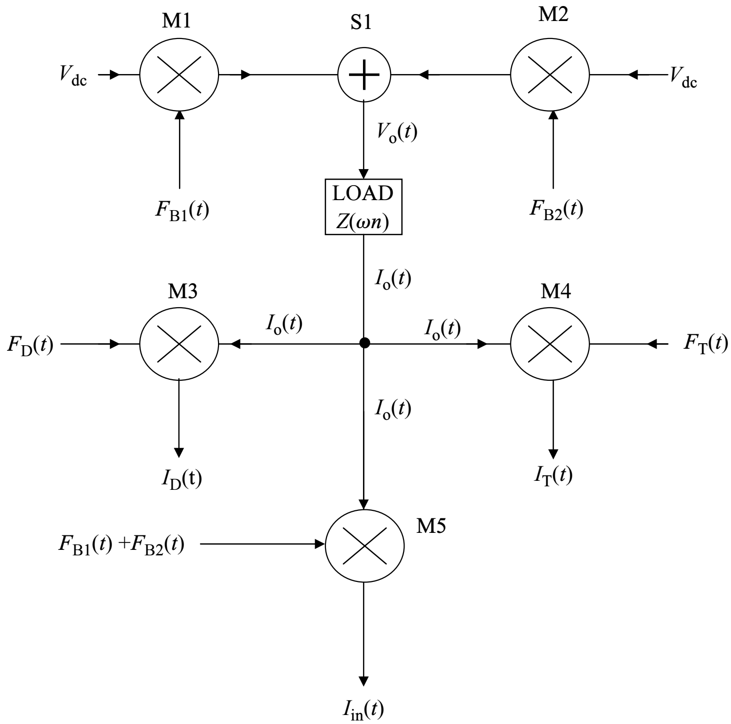


Figure 15.7 The mathematical model of the single pulse inverter (Mode sequence A and C)

15.2.1.3 Mode sequence C

Mode III is added to sequence A to get sequence C. Mode III, refers to discontinuous conduction where the current and the output voltage are zero. The timing parameters of $F_{B2}(t)$ have to be calculated as they are load determined. The output voltage is made up from the contributions of Modes I and IV, and II and V. Expression (15.1) is still applicable

$$V_o(t) = F_{B1}(t)V_{dc} + F_{B2}(t)V_{dc} \tag{15.1}$$

In this case the parameters of $F_{B2}(t)$ are calculated from the parameters of the circuit. The current I_o at the instant the transistors are switched-off (Mode I) is calculated

$$I_o := \left[\frac{V_{dc}}{R}\left(1 - e^{(-R/L)(2\delta/2\pi f)}\right)\right]$$

Then the time it takes to decay to zero (Mode II) is calculated in a simple mathcad program

$\mathtt{t} := 0.006$ ...an estimate

Given

$$\frac{V_{dc}}{R} 1 - e^{(-R/L)t} = I_o e^{(-t/L)}$$

$\mathtt{To} := \mathtt{Find(t)}$

$\mathtt{To} = 2.083 \times 10^{-3}$

$\gamma = 2\pi \times T_o$

The calculated value of time is for specific values of circuit parameters. In this case, the on period of the diode is 2.083 msec; therefore $\gamma = 0.653$ radians.

15.2.2 The switching functions

The bipolar switching function for the transistors associated for the Modes I and IV is given as:

$$F_{B1}(t) = -4 \sum_{n=1}^{\infty} K_n \cos(n\omega t) \qquad (15.4)$$

For odd values of n

$$K_n = \frac{\sin(n\delta)}{\pi n}$$

$\delta =$ half the on-period of the switch

$\omega =$ switching frequency, $2\pi f_s$

The bipolar switching function for the diodes associated for the Modes II and V is given as:

$$F_{B2}(t) = 4 \sum_{m=1}^{\infty} K_m[\cos(m\omega t - m\delta - m\beta) \qquad (15.5)$$

For odd values of n

$$K_m = \frac{\sin(m\beta)}{\pi m}$$

$\beta =$ half the on-period of the diodes

$\beta = \gamma/2$ for discontinuous conduction

$\beta = (\pi - 2\delta)/2$ for continuous conduction

15.3 The mathematical model and analysis

The mathematical model of this circuit is derived in Fig. 15.7 for the mode sequence A and C. Expressions (15.1) is used to present the generation of the output voltage, $V_o(t)$, and modulators M1 and M2 and adder S implement this operation. The output current is dictated by Ohm's Law: the output voltage is acting upon the harmonic impedance of the load, $Z(\omega n)$, to give the output current, $I_o(t)$. In a bridge configuration the input current is a reflection of the output current to the input and the input current is given by Expression (1.6) of Chapter 1. In this case the two switching functions are added to give the full switching action of the bridge.

$$I_{in}(t) = I_o(t)[F_{B1}(t) + F_{B2}(t)] \tag{15.6}$$

In the mathematical model this action is presented by modulator M5.

It is always useful to be able to express the current through the semiconductor devices. The rms, average and peak values of current are necessary for choosing the right device. In the mathematical model of Fig. 15.7 the current through the diodes, $I_D(t)$ and through the transistors, $I_T(t)$ are both derived via modulators M3 and M4, respectively.

$$I_D(t) = I_o(t)F_{B2}(t) \tag{15.7a}$$

$$I_T(t) = I_o(t)F_{B1}(t) \tag{15.7b}$$

15.3.1 Output voltage

The output voltage is given by Expression (15.1)

$$V_o(t) = F_{B1}(t)V_{dc} + F_{B2}(t)V_{dc}$$

$$V_o(t) = V_{dc}4\sum_{n=1}^{\infty} K_n \cos(n\omega t) - V_{dc}4\sum_{m=1}^{\infty} K_m \cos(m\omega t - m\delta - m\beta) \tag{15.8}$$

15.3.2 Output current

The output voltage is forcing a current $I_o(t)$ through the load. The load harmonic impedance is

$$Z(\omega n) = \sqrt{(\omega Ln)^2 + R^2} \quad \text{and} \quad \Phi = \tan^{-1}\frac{\omega Ln}{R}$$

And the output current is

$$I_o(t) = \frac{V_o(t)}{Z(\omega n)}$$

$$I_o(t) = 4V_{dc}\sum_{n=1}^{\infty}\frac{K_n}{\sqrt{(\omega Ln)^2 + R^2}}\cos(n\omega t - \Phi) \tag{15.9}$$

$$- V_{dc}4\sum_{m=1}^{\infty}\frac{K_m}{\sqrt{(\omega Lm)^2 + R^2}}\cos(m\omega t - m\delta - m\beta - \Phi)$$

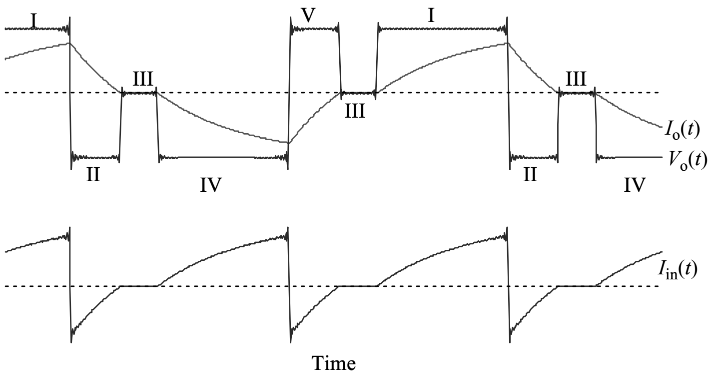


Figure 15.8 *Output voltage, output current and input current: discontinuous conduction. The modes are marked on the output voltage. Duty-cycle = 0.25, R = 1 Ω, L = 4 mH and dc source voltage 100 V*

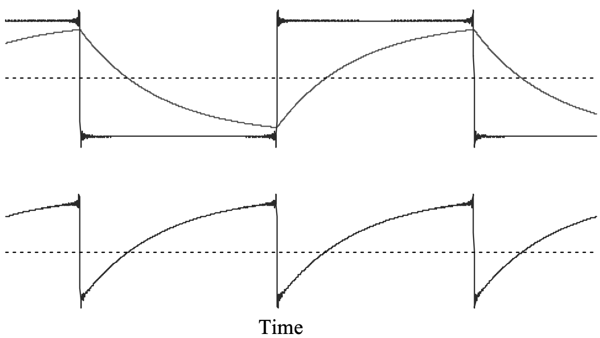


Figure 15.9 *Output voltage, output current and input current: continuous conduction. Duty-cycle = 0.38, R = 1 Ω, L = 4 mH and dc source voltage 100 V*

The output voltage, output current and input current are displayed in Fig. 15.8 for discontinuous conduction. Figure 15.9 displays the same quantities but for a larger duty-cycle (0.38) giving just continuous conduction. The duty-cycle cannot be increased any further unless the direction of current is sensed; this is not done in this modelling.

15.3.3 Current through the semiconductor devices

The switching function method of analysis can provide data related to the ratings of the semiconductor switches and other circuit elements. Transistors T1 and T3 carry the same current hence a unipolar switching function $F_{T13}(t)$ is appropriate. In the same way $F_{T24}(t)$ applies for transistors T2 and T4, $F_{D13}(t)$ applies for

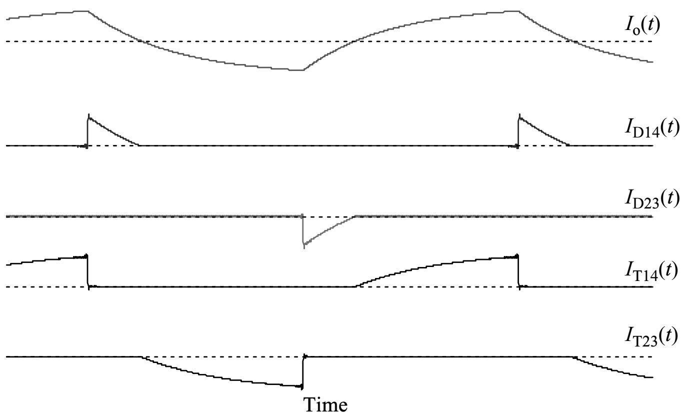


Figure 15.10 Display of output current and currents through the various semiconductor devices. Duty-cycle = 0.25, R = 1 Ω, L = 4 mH and dc source voltage 100 V

diodes D1 and D3 and $F_{D24}(t)$ applies for diodes D2 and D4.

$$
\begin{aligned}
F_{T14}(t) &:= K_o + \sum_{n=1}^{\infty} \frac{\sin(n\delta)}{n\pi} 2\cos(n\omega t) \\
F_{T23}(t) &:= -\left(K_o + \sum_{n=1}^{\infty} \frac{\sin(n\delta)}{n\pi} 2\cos(n\omega t - n\pi)\right) \\
F_{D14}(t) &:= -\left[K_{o2} + \sum_{n=1}^{\infty} \frac{\sin(n\beta)}{n\pi} 2(\cos(n\omega t - \beta n - \delta n))\right] \\
F_{D23}(t) &:= K_{o2} + \sum_{n=1}^{\infty} \frac{\sin(n\beta)}{n\pi} 2\cos(n\omega t - n\pi - \beta n - \delta n)
\end{aligned}
\tag{15.10}
$$

Transistor current through T1 and T4

$$I_{T14}(t) = F_{T14}(t) I_o(t)$$

Transistor current through T2 and T3

$$I_{T23}(t) = F_{T23}(t) I_o(t)$$

Diode current through D1 and D4 (15.11)

$$I_{D14}(t) = F_{D14}(t) I_o(t)$$

Diode current through D2 and D3

$$I_{D23}(t) = F_{D23}(t) I_o(t)$$

Figure 15.10 is a display of the currents through the various semiconductor devices. The peak, RMS and average values can be derived from Expressions (15.10) for the current rating of the various semiconductor devices. The RMS values of the current through the semiconductor switches and the rms value of the output current are derived from the text-book expression for rms values for a duty-cycle of the switches 0.38, $R = 1\,\Omega$, $L = 4\,\text{mH}$ and dc source voltage 100 V as:

$$
\begin{aligned}
&\mathrm{RMS\_I_{D14}} := \sqrt{\frac{1}{T}\int_0^T I_{\mathrm{D14}}(t)^2\,\mathrm{d}t} \quad \mathrm{RMS\_I_{D14}} = 15.801\ \mathrm{A}\\
&\mathrm{RMS\_I_{D23}} := \sqrt{\frac{1}{T}\int_0^T I_{\mathrm{D23}}(t)^2\,\mathrm{d}t} \quad \mathrm{RMS\_I_{D23}} = 15.801\ \mathrm{A}\\
&\mathrm{RMS\_I_{T14}} := \sqrt{\frac{1}{T}\int_0^T I_{\mathrm{T14}}(t)^2\,\mathrm{d}t} \quad \mathrm{RMS\_I_{T14}} = 36.79\ \mathrm{A}\\
&\mathrm{RMS\_I_{T23}} := \sqrt{\frac{1}{T}\int_0^T I_{\mathrm{T23}}(t)^2\,\mathrm{d}t} \quad \mathrm{RMS\_I_{T23}} = 36.79\ \mathrm{A}\\
&\mathrm{RMS\_I_o} := \sqrt{\frac{1}{T}\int_0^T I_{\mathrm{o}}(t)^2\,\mathrm{d}t} \quad \mathrm{RMS\_I_o} = 56.69\ \mathrm{A}
\end{aligned}
\tag{15.12}
$$

In the same way the average values are derived from the text book expression for the same circuit parameters:

$$
\begin{aligned}
&\mathrm{AVE\_I_{D14}} := \frac{1}{T5}\int_0^T I_{\mathrm{D14}}(t)\mathrm{d}t \quad \mathrm{AVE\_I_{D14}} = 4.679\ \mathrm{A}\\
&\mathrm{AVE\_I_{D23}} := \frac{1}{T}\int_0^T I_{\mathrm{D23}}(t)\,\mathrm{d}t \quad \mathrm{AVE\_I_{D23}} = -4.679\ \mathrm{A}\\
&\mathrm{AVE\_I_{T14}} := \frac{1}{T}\int_0^T I_{\mathrm{T14}}(t)\,\mathrm{d}t \quad \mathrm{AVE\_I_{T14}} = 20.743\ \mathrm{A}\\
&\mathrm{AVE\_I_{T23}} := \frac{1}{T}\int_0^T I_{\mathrm{T23}}(t)\,\mathrm{d}t \quad \mathrm{AVE\_I_{T23}} = -20.743\ \mathrm{A}\\
&\mathrm{AVE\_I_{in}} := \frac{1}{T}\int_0^T I_{\mathrm{in}}(t)\,\mathrm{d}t \quad \mathrm{AVE\_I_{in}} = 32.138\ \mathrm{A}
\end{aligned}
\tag{15.13}
$$

The input ripple current is found by deducting from $I_{\mathrm{in}}(t)$ its average value, $\mathrm{AVE\_}I_{\mathrm{in}}$. Figure 15.11 displays the input ripple current at the dc side of the inverter.

$$I_{\mathrm{in}}\mathrm{Ripple}(t) = I_{\mathrm{in}}(t) - \mathrm{AVE\_}I_{\mathrm{in}} \tag{15.14}$$

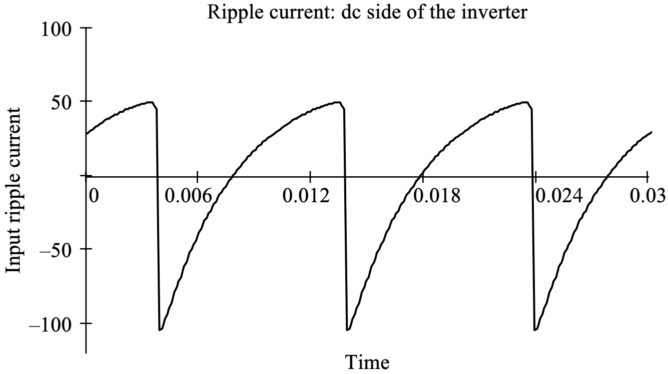


Figure 15.11 Ripple current at the dc side of the inverter. Duty-cycle =0.38, $R = 1\ \Omega$, $L = 4$ mH and dc source voltage 100 V

The ripple current at the input is very likely to flow through the smoothing capacitor in the case of a UPS system. Its peak and rms values are needed for the rating of the capacitor. The peak value is read from Fig. 15.11 and its RMS value is calculated below for a duty-cycle = 0.38, $R = 1\ \Omega$, $L = 4$ mH and dc source voltage 100 V as:

$$\mathrm{RMS\_Iripple} := \sqrt{\frac{1}{T}\int_0^T I_{\mathrm{in}}\mathrm{Ripple}(t)^2\,\mathrm{d}t} \quad \mathrm{RMS\_Iripple} = 46.542\ \mathrm{A} \qquad (15.15)$$

Chapter 16

The sinusoidally PWM inverter

16.1 Introduction

The inverter circuit of Fig. 15.1 is operated by a sinusoidally pulse width modulated signal to produce an output with minimum distortion. The same modes apply as in Fig. 15.3 of the single pulse system of the previous chapter. The most likely mode sequence though is A, continuous conduction of current. This is because of the rather high switching frequencies used. It will be shown in this chapter that a modified sequence A is used to account for the multiple pulses per half-cycle.

16.2 Mathematical modelling

16.2.1 Modes of operation and mode sequence

Transistors T1 and T4 of the circuit of Fig. 15.1 are switched on many times together to construct the positive half-cycle of the output; during their off-periods within the positive half-cycle diodes D2 and D3 provide the freewheeling path of the current to the inductive load via the dc source. Hence Modes I and II are alternating during the positive half-cycle of the current, Fig. 16.1. Modes I and II are shown in Fig. 15.3 of the previous chapter.

During the negative half-cycle of the current, transistors T2 and T3 are switched together to construct the negative half-cycle of the output; during their off-periods within the negative half-cycle diodes D1 and D4 provide the freewheeling path of the current to the load via the dc source. Hence Modes IV and V are alternating during the negative half-cycle of the current. Modes IV and V of the circuit are shown in Fig. 15.3 of the previous chapter.

Sequence A presented in the previous chapter is slightly modified in its application in this converter: for m pulses per cycle of the output voltage, Modes I and II are repeated $m/2$ times during the positive half-cycle of the output current. In the same way Modes IV and V are repeated $m/2$ times during the negative values of current. In an inductive circuit the current is lagging the voltage by an amount determined by the inductance and resistance in the circuit. This delay accounts for m_1 pulses and it marks the beginning and end of the two groups of modes as indicated in Fig. 16.1.

The semiconductor switches – transistors and diodes – are unidirectional; the current in the inverter must be controlled in such a way that this property is respected. The direction of current is dictated by the inductance of the load. During the positive and negative half-cycles the appropriate transistors are switched on as shown by the modes of Fig. 15.3 and the modes sequence of Fig. 16.1. At the pulse of the switching function just before the instant the current is changing direction, a gated transistor will not carry the current if the current is in the wrong direction; the diodes will carry the current until it becomes zero. Then the transistors will carry the current. This implies that during a single pulse of the switching function the current might change direction. This detail is not included in the mode sequence of Fig. 16.1. This is not a very serious omission since for large values of m the error becomes insignificant. For the safe operation of the inverter though this is something not to be ignored.

16.2.2 The switching functions

16.2.2.1 The switching function of the bridge, $F_{\mathrm{PWM}}(t)$

The inverter, Fig. 15.1 is subjected to a three level switching pattern, Fig. 16.2, by switching on and off T1 and T4 for the positive half-cycle of the current (Mode I) and

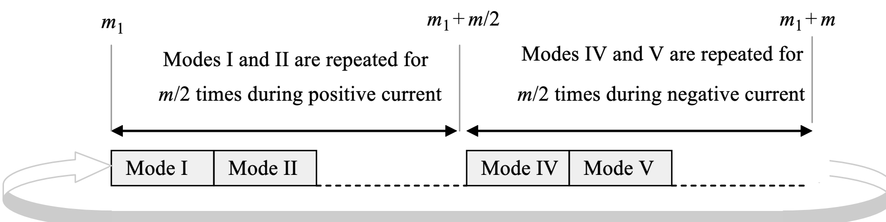


Figure 16.1 Mode sequence for the sinusoidal PWM inverter

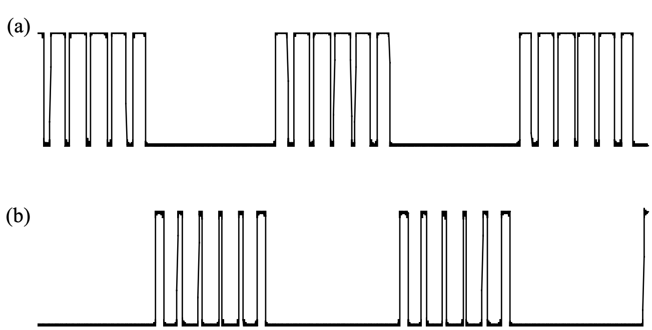


Figure 16.2 Gate signals to the transistors: (a) T1 and T4, (b) T2 and T3

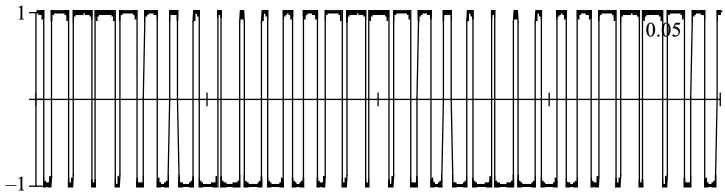


Figure 16.3 Switching function for the inverter

then switching on T2 and T3 for the negative half-cycle of the current (Mode IV). During the off-periods of the switching signal the diodes carry the current in a free-wheeling manner giving rise to Modes II and V. Considering continuous conduction, the voltage appearing at the output adopts the two level shape, changing from $+V_{dc}$ to $-V_{dc}$. Hence the switching function connecting the input and the output is a bipolar signal, sinusoidally pulse modulated as shown in Fig. 16.3. This switching function is derived in Chapter 3, Expression (3.10) and it is repeated below.

$$F_{PWM}(t) = \sum_{k=1}^{m} \sum_{n=1}^{\infty} K_n \cos[n\omega t - n\theta] \tag{3.11}$$

$n =$ integer number

$$K_n = \frac{\sin(n\delta)}{\pi n}$$

$$\delta = \frac{1}{2}\{\cos[(k-1)T] - \cos(kT)\}0.5D + \frac{T}{4}$$

$$T = \frac{2\pi}{m}, \quad \text{the period of the switching frequency}$$

$\frac{T}{4} =$ the un-modulated half-width of the carrier

$m =$ number of pulses per reference signal cycle or frequency modulation ratio

$\theta = Tk - \frac{T}{2}$ the position of the pulse relative to reference signal

$D =$ amplitude modulation ratio

16.2.2.2 The switching functions of the current through the semiconductor devices

The average, rms and peak values of the current through the semiconductor devices are useful for the circuit designer. In order to derive the current through the transistors and diodes, separate switching functions are defined. Furthermore T1 and T4 conduct together, T2 and T3 conduct together, D1 and D4 conduct together and D2 and D3 conduct together; hence we need to introduce two switching functions

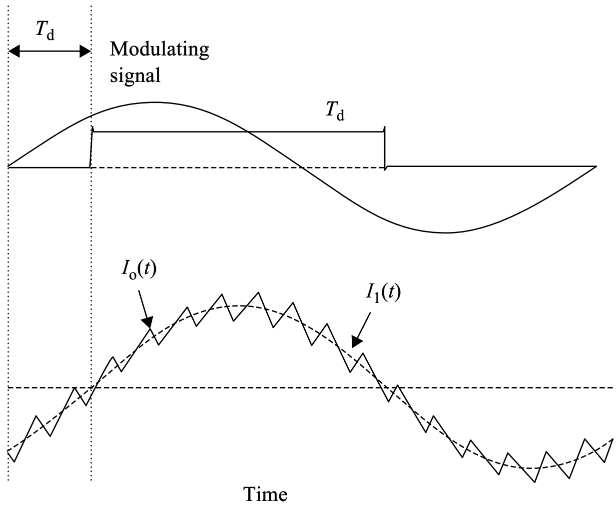


Figure 16.4 Display of the $F_{\text{current}}(t)$ switching function with the output current

for the transistors, $F_{T14}(t)$ and $F_{T23}(t)$ and two for the diodes $F_{D14}(t)$ and $F_{D23}(t)$. First, we need to define the periods for which each group is conducting.

In an inductive circuit the current is lagging the voltage by an amount, T_d, determined by the inductance and resistance in the circuit.

$$T_d = \frac{1}{2\pi f} \tan^{-1}\left[\frac{\omega L}{R}\right] \tag{16.1}$$

Expression (16.1) gives the delay for the fundamental component of current and it only approximates the instant that the current is changing direction as shown in Fig. 16.4. This information is to be used for the derivation of average, rms and peak values of the current through the semiconductor switches and this loss of accuracy is acceptable. The loss of accuracy decreases as the switching frequency is increased. Hence a switching function is introduced, $F_{\text{current}}(t)$ which takes the value of one when the current is positive and it is zero when the current is negative.

$$F_{\text{current}}(t) = 0.5 + 2\sum_{n=1}^{\infty} \frac{\sin(n\pi/2)}{n\pi} \cos\left(n\omega t - n\,\text{atan}\left(\frac{\omega L}{R}\right) - \frac{n\pi}{2} + n\frac{T}{2}\right) \tag{16.2}$$

Transistors T1 and T4 are conducting when both $F_{\text{current}}(t)$ and $F_{\text{PWM}}(t)$ are at state one; hence the switching function for these two transistors, $F_{T14}(t)$ is given by

$$F_{T14}(t) = F_{\text{current}}(t) \times F_{\text{PWM}}(t) \tag{16.3}$$

Transistors T2 and T3 are conducting in the negative current half cycle when $F_{\text{current}}(t)$ is zero, that is, for $\overline{F_{\text{current}}(t)}$ the inverse of $F_{\text{current}}(t)$. The inverse of a switching

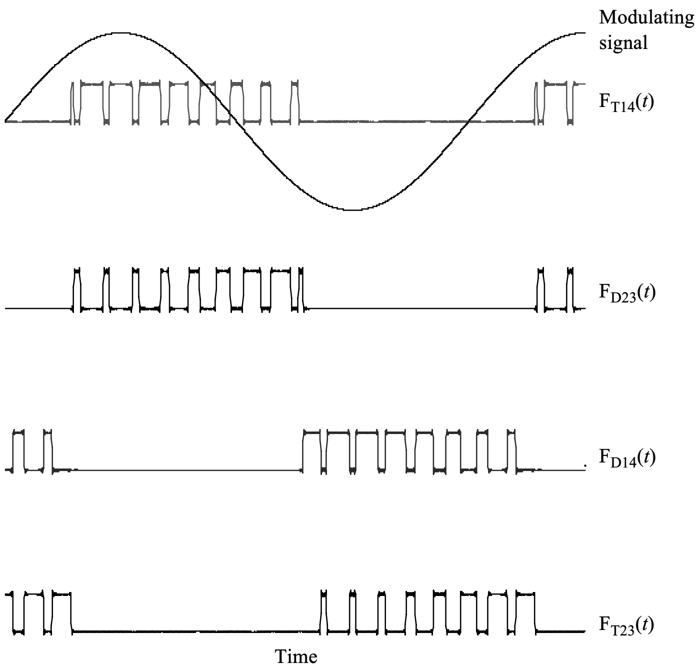


Figure 16.5 The switching functions for the semiconductor switches

function is defined in Chapter 1 and for this function it is

$$1 - F_{\text{current}}(t)$$

Therefore the switching functions for the current through T2 and T3 is given by

$$F_{\text{T23}}(t) = [1 - F_{\text{current}}(t)]F_{\text{PWM}}(t) \tag{16.4}$$

In the same way the switching functions for the diodes are given by:

$$F_{\text{D14}}(t) = F_{\text{current}}(t)[1 - F_{\text{PWM}}(t)] \tag{16.5}$$

$$F_{\text{D23}}(t) = [1 - F_{\text{current}}(t)][1 - F_{\text{PWM}}(t)] \tag{16.6}$$

The switching functions for the semiconductor devices are displayed in Fig. 16.5

The input dc voltage is reflected to the output through the bridge according to Expression (1.4) Chapter 1. Hence

$$V_{\text{o}}(t) = V_{\text{dc}}F_{\text{PMW}}(t) \tag{16.7}$$

The output current is forced through the harmonic impedance $Z(\omega n)$ by the output voltage according to Ohm's Law

$$I_o(t) = \frac{V_o(t)}{Z(\omega n)} \tag{16.8}$$

The harmonic impedance is explained in Chapter 1.

The input current is a reflection of the output current to the input according to Expression (1.6), Chapter 1. Modulator M4 presents this in the model of Fig. 16.6.

$$I_{\text{in}}(t) = I_o(t)\ [2F_{\text{current}}(t) - 1] \tag{16.9}$$

The harmonic content of the input current will flow through the dc source and more likely through a smoothing electrolytic capacitor. Hence this information might be useful for the choice of the capacitor ripple current rating. The harmonic content is readily derived if the average value of $I_{\text{in}}(t)$ is known.

$$I_{\text{in\_ripple}}(t) = I_{\text{in}}(t) - I_{\text{AVE\_}I_{\text{in}}} \tag{16.10}$$

The average value of the input current, $I_{\text{AVE\_}I_{\text{in}}}$, is derived later.

The current through the semiconductor switches is derived from the output current. The output current is diverted from the transistors (Modes I and IV) to the diodes (Modes II and V); the modes are shown in Fig. 15.3 of Chapter 15. Hence use is made of the four switching functions defined in the previous section, one for each group of semiconductors.

Current through transistors T1 and T4, $I_{\text{T14}}(t)$ is given from modulator M2

$$I_{\text{T14}}(t) = F_{\text{T14}}(t) I_o(t) \tag{16.11}$$

Current through transistors T2 and T3, $I_{\text{T23}}(t)$ is given from modulator M3

$$I_{\text{T23}}(t) = F_{\text{T23}}(t) I_o(t) \tag{16.12}$$

Current through diodes D1 and D4, $I_{\text{D14}}(t)$ is given from modulator M5

$$I_{\text{D14}}(t) = F_{\text{D14}}(t) I_o(t) \tag{16.13}$$

Current through diodes D2 and D3, $I_{\text{D23}}(t)$ is given from modulator M6

$$I_{\text{D23}}(t) = F_{\text{D23}}(t) I_o(t) \tag{16.14}$$

16.2.3 The mathematical model

Expressions (16.1)–(16.14) are employed to build the mathematical model of the inverter shown in Fig. 16.6. The input dc voltage V_{dc} is modulated (M1) by the switching function $F_{\text{PWM}}(t)$ to give the output voltage. The output voltage is pushing a current $I_o(t)$ through the harmonic impedance of the load, $Z(\omega n)$. The output current and the currents through the various semiconductor switches are the result of amplitude modulation of the input current and the relevant switching functions. Modulators M2, M3, M4, M5 and M6 give the currents $I_{\text{T14}}(t)$, $I_{\text{T23}}(t)$, $I_{\text{in}}(t)$, $I_{\text{D14}}(t)$ and $I_{\text{D23}}(t)$, respectively.

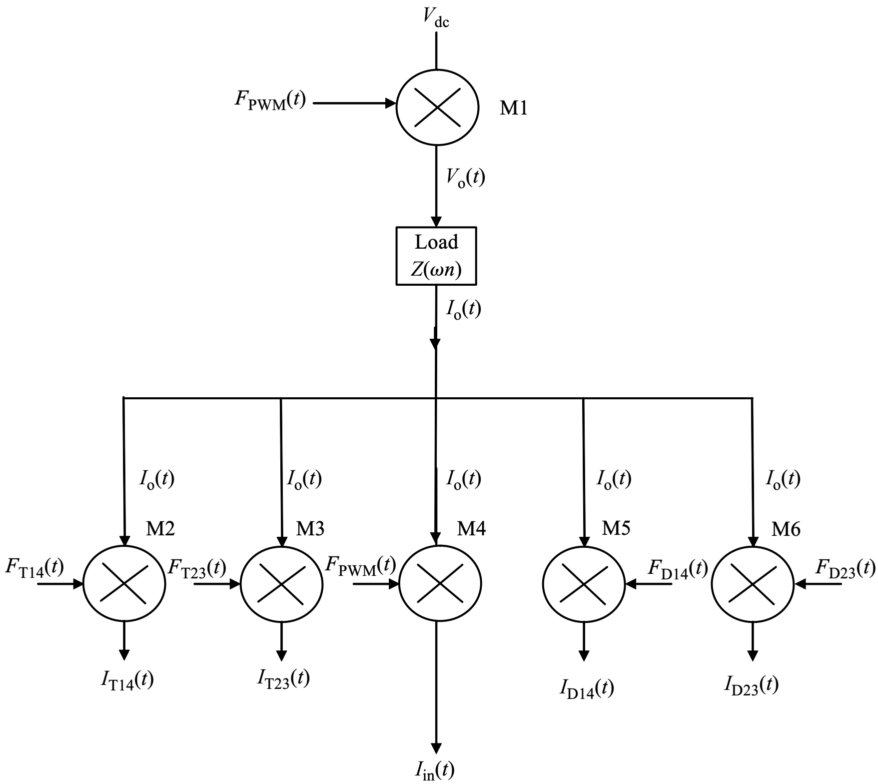


Figure 16.6 The mathematical model of the sinusoidally modulated inverter

16.3 Analysis

The output voltage is derived from Expression (16.7) as

$$V_o(t) := V_{dc} 4 \sum_{k=1}^{m} \sum_{n=1}^{\infty} \left[\frac{\sin[n[(1/2)[\cos[(k-1)T] - \cos(kT)]D0.5 + (T/4)]]}{n\pi} \right] \times [\cos[n\omega t - n(Tk - T)]] \tag{16.15}$$

The various terms are defined in Section 3.5 where the switching function is introduced. The output current is derived from Expression (16.8) as

$$I_o(t) = 8 \sum_{k=1}^{K} \sum_{n=1}^{\infty} \frac{K_n}{\sqrt{R^2 + (\omega L n)^2}} \cos(n\omega t - Q) \qquad Q = \tan^{-1}\left[\frac{n\omega L}{R}\right] + n\theta \tag{16.16}$$

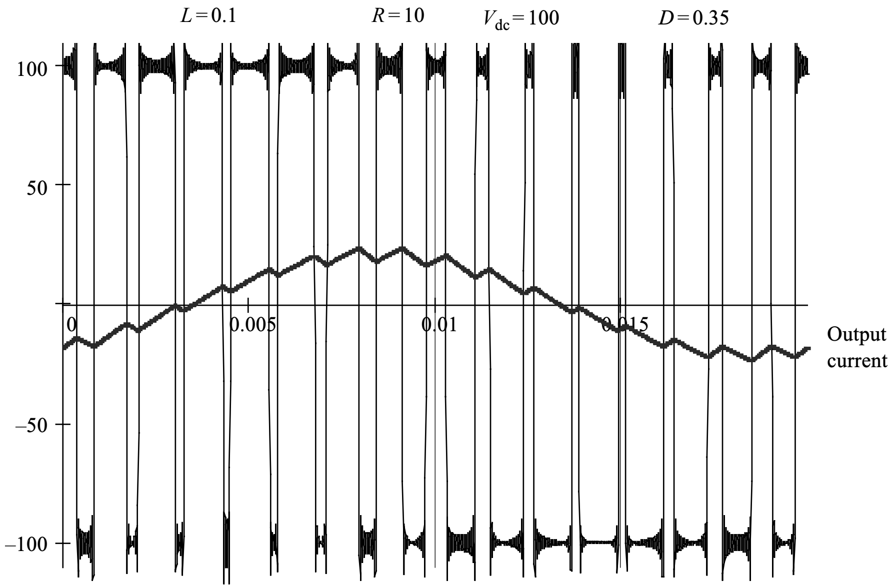


Figure 16.7 Output voltage and current for a low switching frequency, 1040 Hz

Figure 16.7 is a display of the output current and voltage. The input current is derived from Expression (16.9). The current through the transistors and diodes is derived from Expressions (16.11) to (16.14) and displayed in Fig. 16.8.

16.3.1 Frequency content of the output voltage

The switching patent applied to the semiconductor switches, Fig. 16.2, dictates the output voltage waveform. The spectral content of the output voltage is the same as the spectral content of the bipolar switching function applied to the system, Fig. 3.12 of Chapter 3, Section 3.4. Low-order harmonics are absent. The lowest unwanted harmonics to be observed are m, $m \pm 2$ where m is the ratio of the switching frequency to the frequency of the signal to be created or frequency modulation ratio.

16.3.2 Ratings of the semiconductor devices

The expressions for the currents through the diodes and transistors are employed to derive the peak, RMS and average values. The peak value is the peak value of the output current and it is derived from the current Expression (16.16). The average and RMS values are derived for a set of circuit and load parameters. The parameters used for the results below are:

$$L = 0.02 \quad R = 5 \quad V_{dc} = 300 \quad D = 0.3 \quad m = 16$$

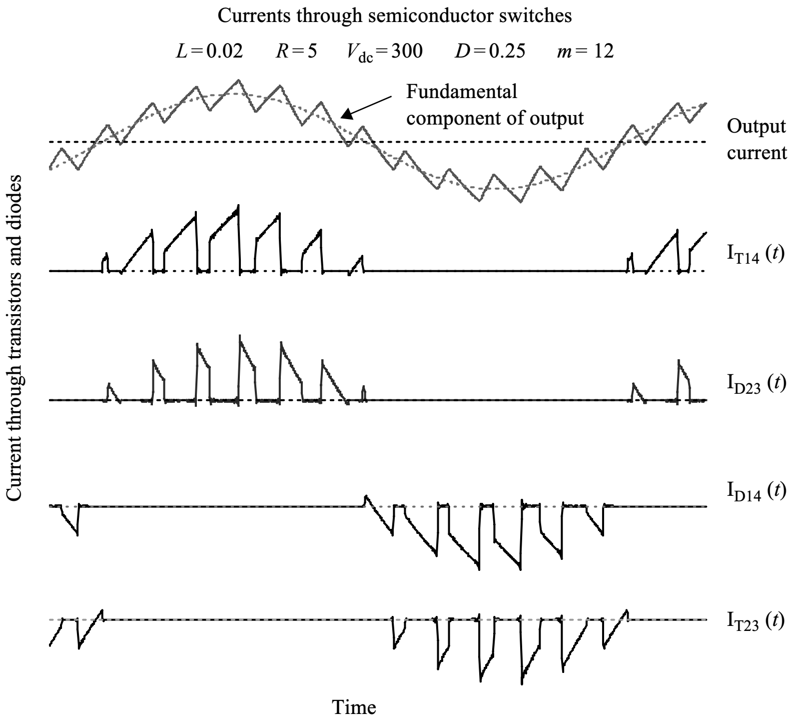


Figure 16.8 Current through transistors T1–T4 and diodes D1–D4

Average of current supplied by dc source

$$\text{AVE\_Iin} := \frac{1}{T_o}\int_0^{T_o} (\text{Iin}(t))\,\mathrm{d}t \quad \text{AVE\_Iin} = 2.791\ \text{A}$$

Now the ripple current through the dc source can be displayed, Fig. 16.9 by subtracting the average value of the input current (AVE_Iin) from the input current, Expression (16.9).

The rms value of the ripple current is given by:

$$\text{RMS\_IINripple} := \sqrt{\frac{1}{T_o}\int_0^{T_o} (\text{IINripple}(t))^2\,\mathrm{d}t} \quad \text{RMS\_IINripple} = 3.33\ \text{A}$$

The rms value of the current through transistors T1 and T4 is derived as :

$$\text{RMS\_ITI4} := \sqrt{\frac{1}{T_o}\int_0^{T_o} \text{ITI4}(t)^2\,\mathrm{d}t} \quad \text{RMS\_ITI4} = 7.489\ \text{A}$$

The current through transistors T2 and T3 has the same RMS value.

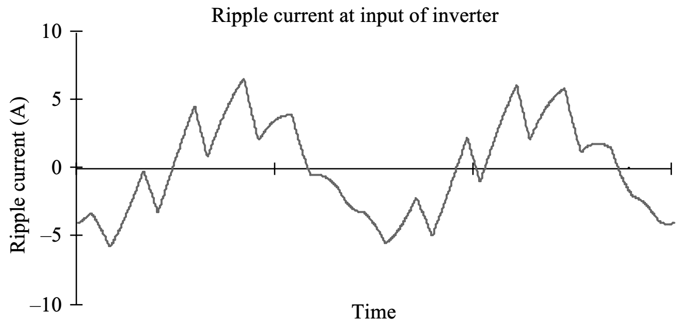


Figure 16.9 Ripple current through the dc source, m = 12

The rms values of the current through diodes D2 and D3 is derived as

$$\mathrm{RMS\_ID23} := \sqrt{\frac{1}{T_o}\int_0^{T_o} \mathrm{ID23}(t)^2\,\mathrm{d}t} \quad \mathrm{RMS\_ID23} = 5.612$$

The rms value of the current through diodes D1 and D4 has the same value.

The average value of the current through transistors T1 and T4 is derived as :

$$\mathrm{AVE\_IT14} := \frac{1}{T_o}\int_0^{T_o} \mathrm{IT14}(t)\,\mathrm{d}t \quad \mathrm{AVE\_IT14} = 3.659$$

The average value of the current through transistors T2 and T3 has the same value.

The average value of the current through diodes D2 and D3 is derived as:

$$\mathrm{AVE\_ID23} := \frac{1}{T_o}\int_0^{T_o} \mathrm{ID23}(t)\,\mathrm{d}t \quad \mathrm{AVE\_ID23} = 2.176$$

The average value of the current through diodes D1 and D4 has the same value.

The rms value of the output current is derived as

$$\mathrm{RMS\_Io} := \sqrt{\frac{1}{T_o}\int_0^{T_o} (I_o(t))^2\,\mathrm{d}t} \quad \mathrm{RMS\_Io} = 13.326$$

T_o is the period of the power frequency.

Chapter 17
The envelope cyclo-converter

17.1 Introduction

The cyclo-converter is employing thyristors which are naturally commutated. The circuit of Fig. 17.1 is a combination of a positive and a negative phase-controlled rectifier. There are two types: the envelope cyclo-converter and the sinusoidally pulse width modulated. Here we will examine the envelope cyclo-converter.

Cyclo-converters are becoming obsolete and their use seems to be limited to very high power at very low-frequency applications. The application of the switching function technique is applied in this short chapter to derive the output voltage of the envelope cyclo-converter for a resistive load. Further development of the technique for the operation of the circuit with inductive load and its application to the phase-controlled cyclo-converter is left to the interested reader.

17.2 The mathematical model

The mathematical model of the envelope cyclo-converter is shown in Fig. 17.2. Half cycles of the mains are arranged in order to construct the output waveform. For the

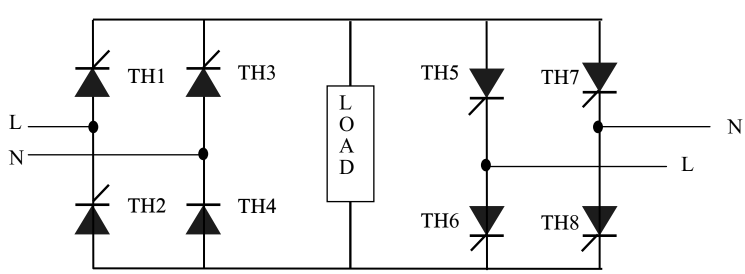


Figure 17.1 The envelope cyclo-converter

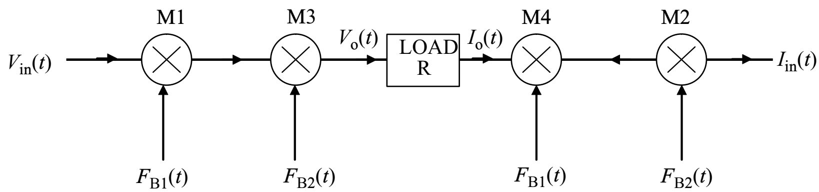


Figure 17.2 The mathematical model of the envelope converter

positive part of the output voltage the thyristors of the rectifier on the left are fired accordingly: TH1 with TH4 for positive input half-cycles and TH2 with TH3 for negative input half-cycles. For the negative part of the output voltage the thyristors of the rectifier on the right are fired accordingly: TH6 with TH7 for positive input half-cycles and TH5 with TH8 for negative input half-cycles, Fig. 17.1. This action of the rectifiers is described by a modulation process (modulator M1, Fig. 17.2) where the input voltage, $V_{in}(t)$ is modulated by a bipolar switching function $F_{B1}(t)$. The switching function of the rectifier is introduced in Chapter 1 as a square wave at the mains frequency. Each rectifier is connected to the load for a number of half-cycles that corresponds to the half cycle of the output signal. This action is a second modulation process (M2) described by a bipolar switching function, $F_{B2}(t)$, at the frequency of the output voltage. The combined action of the two modulation processes is the product of two switching functions $F_{B1}(t)F_{B2}(t)$ since the first modulation process is succeeded by the second modulation.

$$V_o(t) = V_{in}(t)F_{B1}(t)F_{B2}(t) \tag{17.1}$$

The output voltage is forcing a current through the load impedance (R)

$$I_o(t) = \frac{V_o(t)}{R} \tag{17.2}$$

The output current is reflected back to the input by the same modulation process.

$$I_{in}(t) = I_o(t)F_{B1}(t)F_{B2}(t) \tag{17.3}$$

17.3 The switching functions

$F_{B1}(t)$ is square-wave at the power frequency, phase delayed by 90°. It is defined as a bipolar switching function in Chapter 1. Its pulse width is π and it is in phase with the power frequency. $F_{B1}(t)$ is the sum of cosines, hence its phase delay, σ, relative to the reference is $\pi/2$.

$$F_{B1}(t) = 4\sum_{n=1}^{\infty} K_n \cos(n\omega t - n\sigma) \tag{17.4}$$

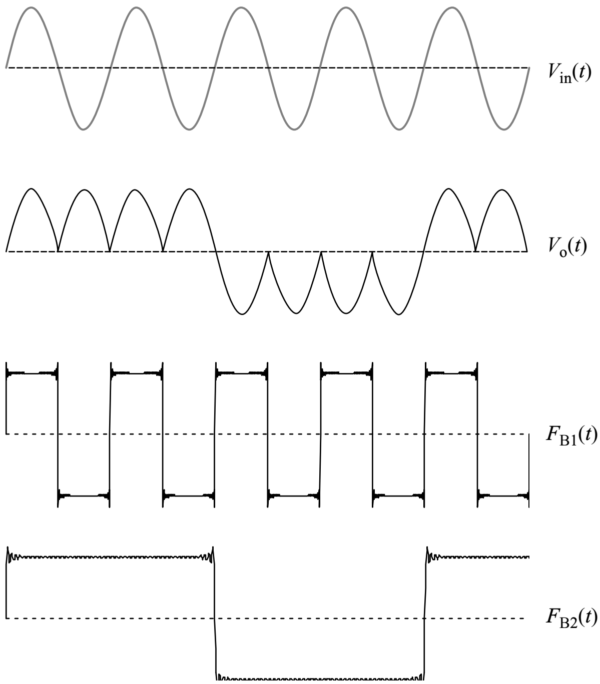


Figure 17.3 Input–output voltage and switching function waveforms

The second switching function, $F_{B2}(t)$ is of the bipolar type as well with a switching frequency $K\omega$, where K is the ratio of the wanted output frequency to the power frequency; it can take the values of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$ etc. Again, its phase delay is $\pi/2$.

$$F_{B2}(t) = 4 \sum_{m=1}^{\infty} K_m \cos(mK\omega t - m\sigma) \qquad (17.5)$$

The output voltage waveform is displayed in Fig. 17.3 together with the switching functions.

Part 5

Active filters

Chapter 18
The thyristor-controlled reactor

18.1 Introduction

There are many configurations for active power filters [12]. The thyristor-controlled reactor is perhaps one of the first configurations [13].

The current is controlled in an inductor by delaying the triggering of the thyristors. There are two versions; the single reactor and the two-reactor circuits. For the single reactor the firing angle is limited to the range 90–180° and for the two-reactor circuit the firing angle is extended from 0–180°. In the former case, the current is discontinuous giving rise to a high %THD figure. In the latter case, the current is continuous up to a firing angle of 90° giving a low-distortion figure. The three phase configuration – even though not presented here – with a delta or star connection can easily be derived from the presented material.

A parallel capacitor is included to provide leading reactive power.

18.2 The single reactor arrangement

18.2.1 Operation and the switching function

Triggering for each thyristor is done at the correct half-cycle: hence thyristor TH1 is fired during the positive half-cycle and TH2 during the negative half-cycle, Fig. 18.1. The delay angle, α, is set to allow a smaller or bigger part of the half-cycle to pass to the output. It can be varied from the load phase angle to 180° allowing smooth control of the level of the produced reactive power. The load angle, $\tan^{-1}(\omega L/R)$ is close to 90° for a good quality reactor with minimum resistance. R represents the losses of the reactor.

Conduction is maintained for as long as the current flows. The extinction angle, β must be calculated; it is the end of the on-period of the switching function, $F(t)$. Otherwise β can be approximated to $\beta = (180 - \alpha)2 + \alpha$ for a low loss system [16]. The beginning of the on-period of the switching function is the firing angle α, Fig. 18.2. The switching frequency is twice the mains frequency and the switching

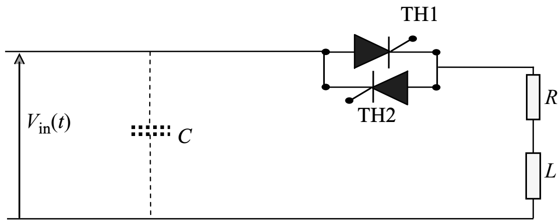


Figure 18.1 The thyristor-controlled reactor

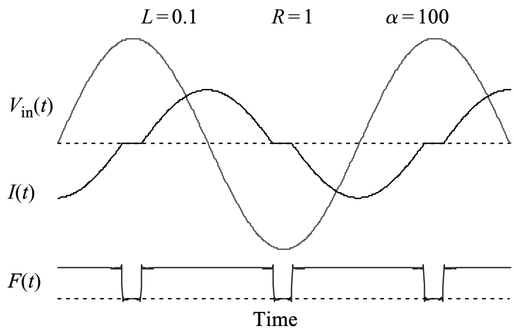


Figure 18.2 Voltage current and the switching function

function is of the unipolar type since no inversion takes place between the input and the output. The on-period is therefore $\beta - \alpha$ at 50 Hz and $2(\beta - \alpha)$ at 100 Hz, the switching frequency of the switching function. It is multiplied by two because the switching frequency is twice the power frequency at which both α and β are measured. The delay angle of the switching function, θ, is derived in the usual way as

$$\theta = \left[\frac{\beta - \alpha}{2} + \alpha\right] 2$$

$$\delta = (\beta - \alpha)$$

The average value of the switching function or duty-cycle of the switch, K_o, is

$$K_o = \frac{\delta}{\pi}$$

Therefore the switching function of the circuit is given by,

$$F(t) = K_o + 2\sum_{n=1}^{\infty} K_n \cos(n2\omega t - n\theta) \tag{18.1}$$

where n is an integer.

18.2.2 *Output voltage*

The output voltage is derived in a simple fashion; it is a straightforward case of a single electric element controlled by a switch, Chapter 1. The switch in this case is the combination of the two thyristors.

$$\begin{aligned} V_o(t) &= V_{in}(t)F(t) \\ &= V_p \sin \omega t \left\{ K_o + 2\sum_{n=1}^{\infty} K_n \cos(n2\omega t - n\theta) \right\} \end{aligned}$$

$$\begin{aligned} V_o(t) = K_o V_p \sin \omega t &+ V_p \sum_{n=1}^{\infty} K_n \sin((2n+1)\omega t - n\theta) \\ &- V_p \sum_{n=1}^{\infty} K_n \sin((2n-1)\omega t - n\theta) \end{aligned}$$

We can extract the fundamental and the distortion part. For the fundamental we have the obvious component, $K_o V_p \sin \omega t$ and another component when $n = 1$ in the third term.

Hence the output is expressed as

$$\begin{aligned} V_o(t) = K_o V_p \sin \omega t - V_p K_1 \sin(\omega t - \theta) &+ V_p \sum_{n=1}^{\infty} K_n \sin((2n+1)\omega t - n\theta) \\ &- V_p \sum_{n=2}^{\infty} K_n \sin((2n-1)\omega t - n\theta) \end{aligned} \tag{18.2}$$

18.2.3 *Conversion into single harmonics*

Expression (18.2) gives the fundamental and the harmonic content of the output voltage but components of the same frequency exist in both terms depending on the value of n. For example, the fifth harmonic, 5ω, is given for $n = 2$ in the first term and for $n = 3$ in the second term. Expression (18.2) can be arranged to give the order of the harmonic straight away by changing the counter variable from n to p, an odd integer.

For the first term

$$p = 2n + 1 \qquad n = \frac{p-1}{2} \text{ and starting from } n = 1 \text{ hence } p = 3$$

For the second term

$$p = 2n - 1 \qquad n = \frac{p+1}{2} \text{ and starting from } n = 2 \text{ hence } p = 3$$

Expression (18.2) is modified to

$$V_o(t) = K_o V_p \sin\omega t - K_1 \sin(\omega t - \theta) + V_p \sum_{p=3}^{\infty} K_{p1} \sin\left(p\omega t - \frac{p-1}{2}\theta\right)$$
$$- V_p \sum_{p=3}^{\infty} K_{p2} \sin\left(p\omega t - \frac{p+1}{2}\theta\right) \quad (18.3)$$

And it is separated into the fundamental and the distortion parts.

Fundamental component

$$V_{o1}(t) = K_o V_p \sin\omega t - V_p K_1 \sin(\omega t - \theta) \quad (18.4)$$

Distortion components

$$V_{oH}(t) = V_p \sum_{p=3}^{\infty} K_{p1} \sin\left(p\omega t - \frac{p-1}{2}\theta\right)$$
$$- V_p \sum_{p=3}^{\infty} K_{p2} \sin\left(p\omega t - \frac{p+1}{2}\theta\right) \quad (18.5)$$

In calculating the coefficient K_{p1}, n is replaced by $n = (p-1)/2$ and for the coefficient K_{p2}, n is replaced by $n = (p+1)/2$.

18.2.4 Harmonic impedance

Dividing the output voltage with the ‘harmonic impedance’ derives the line current. The harmonic impedance is given below as a function of p.

$$Z(p\omega) = \sqrt{R^2 + (p\omega L)^2} \qquad Q_p = \tan^{-1}\frac{p\omega L}{R}$$

18.2.5 Line current

The fundamental current is derived by dividing the fundamental component of output voltage, Expression (18.4), with the harmonic impedance.

$$I_1(t) = \frac{K_o V_p}{Z(\omega)}\sin(\omega t - Q_1) - \frac{K_1 V_p}{Z(\omega)}\sin(\omega t - Q_1)\cos\theta$$
$$+ \frac{K_1 V_p}{Z(\omega)}\cos(\omega t - Q_1)\sin\theta \quad (18.6)$$

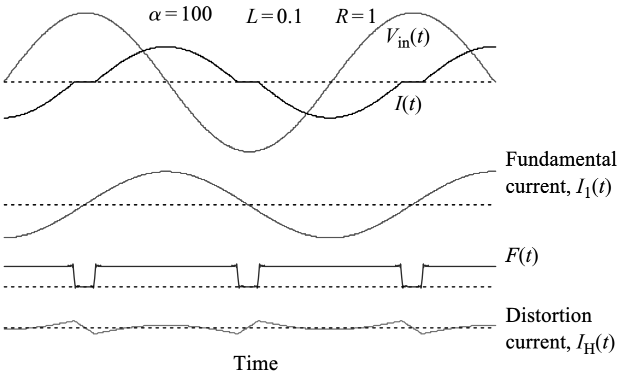


Figure 18.3 Line current, fundamental and distortion components

The harmonic current is derived from (18.5)

$$I_H(t) = V_p \sum_{p=3}^{\infty} \frac{K_{p1}}{Z(p\omega)} \sin\left(p\omega t - \frac{p-1}{2}\theta - Q_p\right) - V_p \sum_{p=3}^{\infty} \frac{K_{p2}}{Z(p\omega)} \sin\left(p\omega t - \frac{p+1}{2}\theta - Q_p\right) \tag{18.7}$$

The line current is the sum of the fundamental and the distortion components (Fig. 18.3).

$$I(t) = I_1(t) + I_H(t)$$

18.2.6 Displacement power factor

This is derived from the expression of the fundamental current (18.6) by extracting the phase angle of the current.

$$\begin{aligned}
\varphi &= \tan^{-1} \frac{K_1 \sin(\theta)}{K_o - K_1 \cos(\theta)} - Q_1 \\
I_{peak} &= \frac{V_p}{\sqrt{R^2 + (\omega L)^2}} \\
Q_1 &= \tan^{-1} \frac{\omega L}{R} \\
\mathrm{DPF} &= \cos(\varphi)
\end{aligned} \tag{18.8}$$

The DPF is close to zero for a good quality reactor. The losses of the thyristors and the reactor itself (R) will give a slightly higher value.

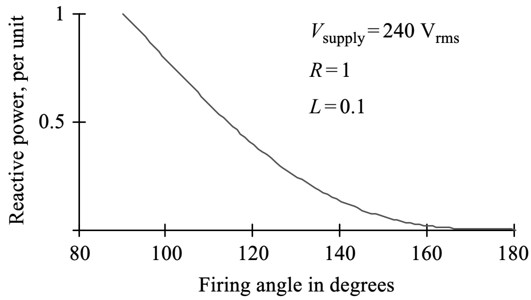


Figure 18.4 Produced inductive reactive power

18.2.7 Amount of reactive power

This is the product of the rms value of the reactive component of the fundamental current and the rms value of the supply voltage.

$$P_r = [V_p I_{1R}]\frac{1}{2} \tag{18.9}$$

The reactive component of the fundamental current, $I_{reactive}(t)$ is extracted from Expression (18.6).

$$I_{reactive}(t) = I_{peak}\{\sin(Q_1)[K_1\cos\theta - K_o] + K_1\sin\theta\cos Q_1\}\cos\omega t$$

And the peak value of the reactive current, I_{1P} is

$$I_{1R} = I_{peak}\{\sin(Q_1)[K_1\cos\theta - K_o] + K_1\sin\theta\cos Q_1\}$$

where

$$K_1 = \frac{\sin(\beta-\alpha)}{\pi} \quad \text{from } K_n = \frac{\sin(n\delta)}{n\pi} \text{ for } n = 1$$

Figure 18.4 is a display of the produced inductive reactive power as a function of the firing angle of the thyristors.

18.2.8 Distortion

This is quantified by two factors: the Total Harmonic Distortion, %THD and the Distortion Factor (DF) given by Expressions (18.10) and (18.11), respectively. X sets the highest harmonic to be considered and P sets the lowest which is the third for %THD and one for DF for this circuit.

$$\%\text{THD} = \frac{\sqrt{\sum_{P=3}^{X} I(P)^2}}{I_1} \tag{18.10}$$

$$\text{DF} = \frac{I_1}{\sqrt{\sum_{P=1}^{X} I(P)^2}} \tag{18.11}$$

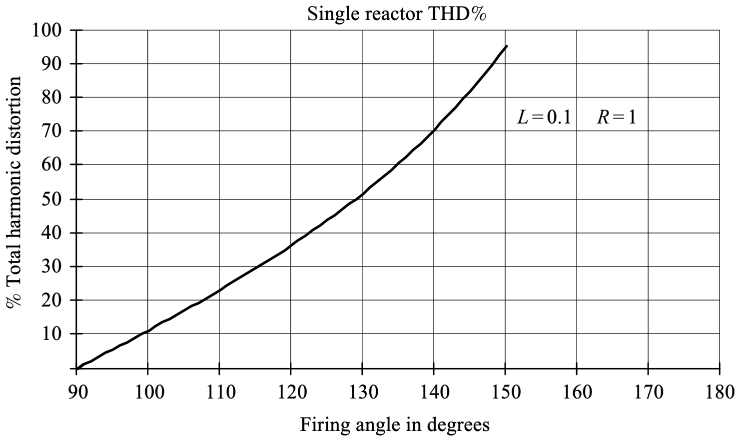


Figure 18.5 Total harmonic distortion for the single reactor circuit

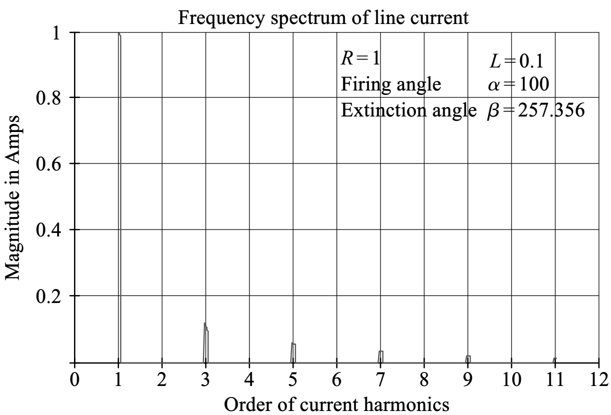


Figure 18.6 Frequency spectrum of the single reactor circuit

Where I_1 is the rms of the fundamental current and $I(P)^2$ is the rms of the harmonic component. Figure 18.5 is a display of the %THD for the single reactor circuit. As the firing angle progresses from 90° upwards so does the distortion. Another measure of the harmonic content of the line current is the display of the magnitude of the harmonics, the frequency spectrum of the line current, Fig. 18.6.

18.3 The two reactor arrangement

The two branches, Fig. 18.7, are fired separately allowing a firing angle from 0–180°; this implies overlap from 0 to 90°. It will be shown that it is this overlap that gives the reduced distortion for that range of operation.

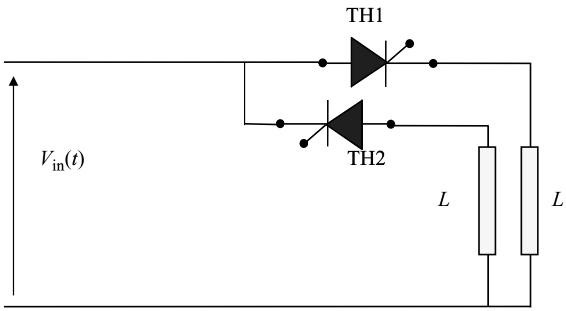


Figure 18.7 The double reactor circuit

18.3.1 The switching functions

There is a separate switching function for each branch. $F_1(t)$ operates in the branch with thyristor TH1 and $F_2(t)$ operates in the branch with thyristor TH2.

$$F_1(t) = K_o + 2\sum_{n=1}^{\infty} K_n \cos(n\omega t - n\theta) \tag{18.12}$$

And for the second branch,

$$F_2(t) = K_o + 2\sum_{n=1}^{\infty} K_n \cos(n\omega t - n\theta - n\pi) \tag{18.13}$$

The parameters of the switching functions (θ and δ) are defined in the previous section, Expression (18.1). Figure 18.8 displays the two switching functions and the input voltage.

18.3.2 The output voltage

The input voltage is applied to each reactor separately giving two output voltages

$$V_1(t) = F_1(t)V_{in}(t)$$

$$V_2(t) = F_2(t)V_{in}(t)$$

18.3.3 The input current

The current in each branch is given by Expressions (18.14) and (18.15). The harmonic impedance is defined in the previous section.

$$I_1(t) = \frac{V_1(t)}{Z(\omega n)} \tag{18.14}$$

$$I_2(t) = \frac{V_2(t)}{Z(\omega n)} \tag{18.15}$$

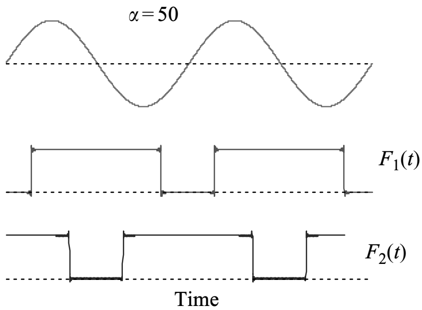


Figure 18.8 The switching functions for the double reactor circuit

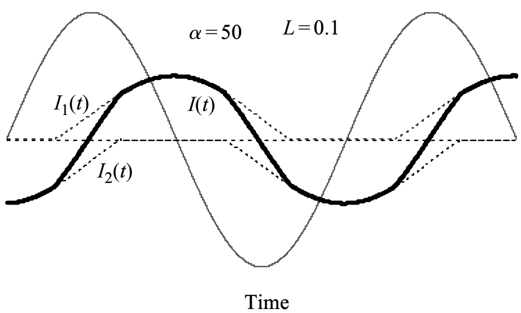


Figure 18.9 Currents in the double inductor circuit

The input current is the sum of the two branch currents

$$I(t) = I_1(t) + I_2(t)$$

$$I(t) = \frac{V_1(t)}{Z(\omega n)} + \frac{V_2(t)}{Z(\omega n)}$$

$$I(t) = 2\frac{V_\mathrm{p}}{Z_\omega} K_\mathrm{o} \sin(\omega t - Q) + \sum_{n=2}^{\infty} \frac{2V_\mathrm{p}}{Z_{(n+1)\omega}} K_n \sin((n+1)\omega t - n\theta - Q_{n+1})$$
$$+ \sum_{n=2}^{\infty} \frac{2V_\mathrm{p}}{Z_{(n-1)\omega}} K_n \sin((n-1)\omega t - n\theta - Q_{n-1}) \tag{18.16}$$

V_p is the peak value of the mains voltage and n is an even integer.

The two branch currents and the line current are displayed in Fig. 18.9 with the input voltage. The input current is further expanded to the fundamental and distortion components.

Let

$$I(t) = I_1(t) + I_{\mathrm{H}}(t)$$

From Expression (18.16), the fundamental component is derived as

$$I_1(t) = 2\frac{V_{\mathrm{p}}}{Z_{\omega n}}[K_o \sin(\omega t - Q_1) - K_2 \sin(\omega t - \theta - Q_1)] \tag{18.17}$$

$$\begin{aligned} I_{\mathrm{H}}(t) = \sum_{n=2}^{\infty} & \frac{2V_{\mathrm{p}}}{Z_{(n+1)\omega}} K_n \sin((n+1)\omega t - n\theta - Q_{n+1}) \\ & - \sum_{n=4}^{\infty} \frac{2V_{\mathrm{peak}}}{Z_{(n-1)\omega}} K_n \sin((n-1)\omega t - n\theta - Q_{n-1}) \end{aligned} \tag{18.18}$$

where n is an even integer.

18.3.4 Conversion into single harmonic

Expression (18.18) gives the harmonic content of the output current. Components of the same frequency exist in both terms depending on the value of n. For example, the fifth harmonic 5ω is given for $n = 4$ in the first term and for $n = 6$ in the second term. This expression can be arranged to give the order of the harmonic straight away by changing the counter variable from n to P.

For the first term

$P = n + 1 \quad n = (p - 1)$ and starting from $n = 2$ hence $p = 3$

For the second term

$P = n - 1 \quad n = (p + 1)$ and starting from $n = 4$ hence $p = 3$

$$\begin{aligned} I_{\mathrm{H}}(t) = \sum_{p=3}^{\infty} & \frac{V_{\mathrm{p}}}{Z_{p\omega}} K_{p-1} \sin(p\omega t - (P-1)\theta - Q_{\mathrm{p}}) \\ & - \sum_{p=3}^{\infty} \frac{V_{\mathrm{p}}}{Z_{p\omega}} K_{\mathrm{p}+1} \sin(p\omega t - (P+1)\theta - Q_{\mathrm{p}}) \end{aligned} \tag{18.19}$$

18.3.5 Displacement power factor

This is derived from Expression (18.17) of the fundamental current by extracting the phase angle of the fundamental component of the line current,

$$\Phi = \operatorname{atan}\left(\frac{2(\sin(2\delta)/2\pi)\sin(2\theta)}{2K_{\mathrm{o}}(\sin(2\delta)/2\pi)2\cos(2\theta)}\right) - Q$$

$$\mathrm{DPF} = \cos(\varphi) \tag{18.20}$$

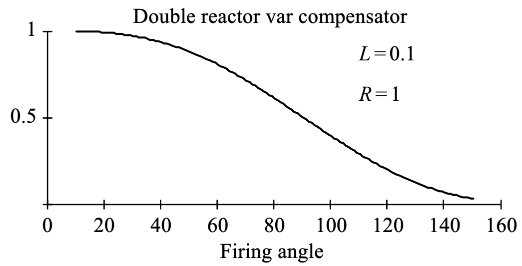


Figure 18.10 Produced inductive reactive power

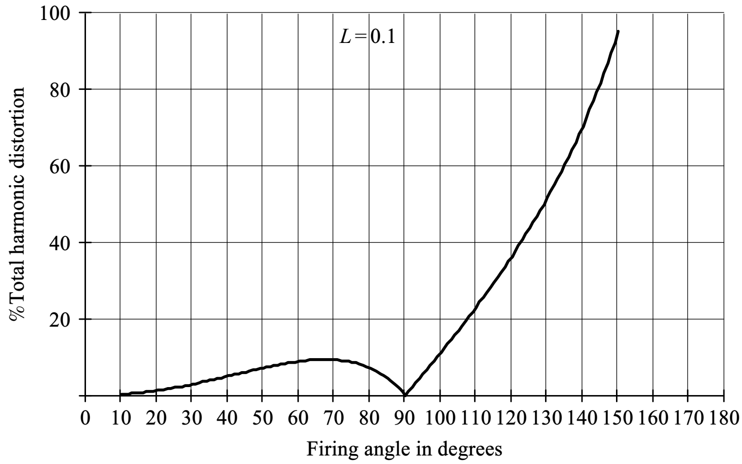


Figure 18.11 Total harmonic distortion for the double inductor circuit

The DPF is close to zero for a good quality reactor. The losses of the thyristors and the reactor itself will give a power factor slightly higher than zero. For $R = 1\,\Omega$ and $L = 100\,\text{mH}$ it is found to be 0.032.

18.3.6 The amount of reactive power

The reactive power is derived in this case in a slightly different way (Fig. 18.10).

$$P_{\text{reactive}} = I_{1\text{rms}} V_{\text{rms}} \sin(\Phi)$$

Hence we need to derive the rms value of the fundamental line current. This is derived from Expression (18.17) as

$$I_{1\text{rms}} = \frac{V_{\text{p}}}{\sqrt{2}} \frac{\sqrt{(2K_{\text{o}} - (\sin(2\delta)/2\pi)2\cos(2\theta))^2 + (2(\sin(2\delta)/2\pi)\sin(2\theta))^2}}{\sqrt{R^2 + (\omega L)^2}}$$

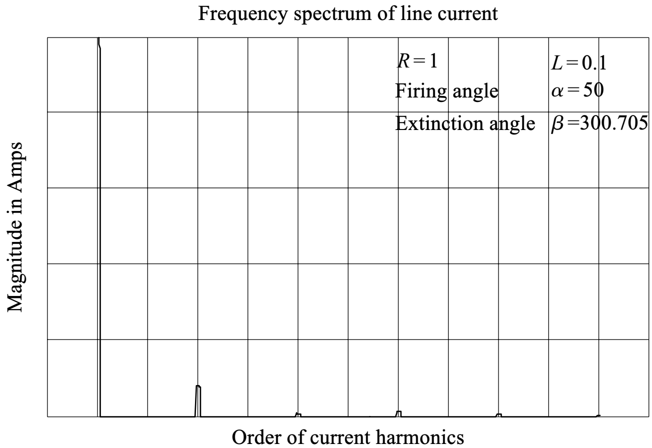


Figure 18.12 Frequency spectrum of the double reactor circuit

18.3.7 Distortion

This is quantified by two factors: the Total Harmonic Distortion, %THD and the Distortion Factor (Fig. 18.11). They are derived using Expressions (18.17) and (18.18) as applied to this circuit.

The frequency content of the line current is displayed in Fig. 18.12.

Chapter 19
The switched capacitor active filters

19.1 Introduction

This circuit can produce both reactive power and harmonic current [3]. From the general circuit a number of practical circuits can be derived. The double-switch double-capacitor circuit is employed in this chapter for reactive power generation. The reactive power is smoothly varied over a range from 60 to 100 per cent of its value for the chosen values. For the analysis use is made of a general mathematical model.

19.2 The general model for the switched-capacitor active filters

A capacitor with a series semiconductor switch is the basic building block used to build these circuits. In the general circuit Fig. 19.1 a number of switched capacitors are placed in parallel and connected to the mains via an inductor. The general model of the switched capacitor circuits is shown in Fig. 19.2. The switching functions are not overlapping and they have no dead periods ensuring always a continuous and single path for the inductor current, Fig. 19.3.

The input loop equation is given by Expression (19.1)

$$V_{\mathrm{in}}(t) = V_{\mathrm{L}}(t) + V_{\mathrm{SW}}(t) \tag{19.1}$$

The voltage, $V_{\mathrm{SW}}(t)$, across the switched combination is the summation of all the contributions from each switched-capacitor as shown in Chapter 2.

$$V_{\mathrm{SW}}(t) = \sum_{n=1}^{N} \frac{F_n(t)}{C_n} \int I(t) F_n(t)\,\mathrm{d}t \tag{19.2}$$

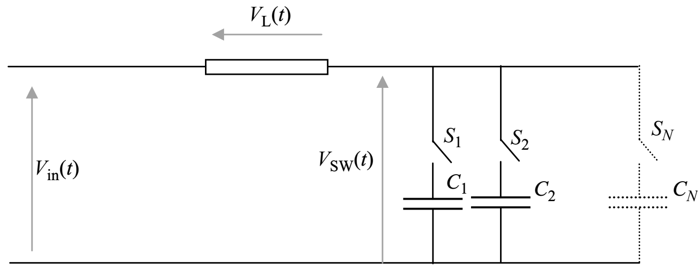


Figure 19.1 The general circuit of switched capacitor active filters

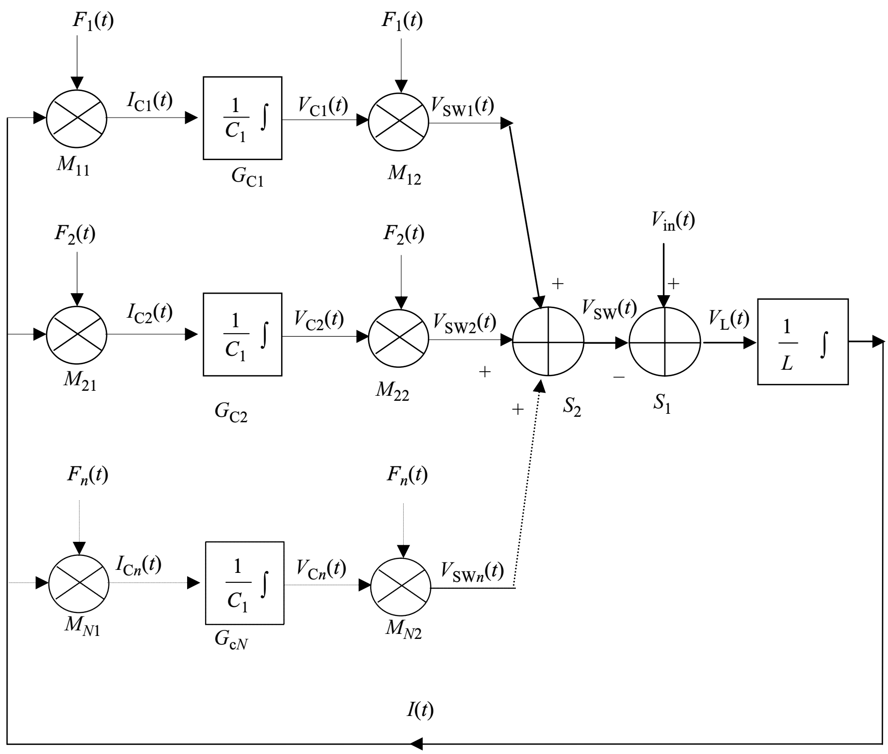


Figure 19.2 The mathematical model of the switched capacitor circuit

And the input loop equation can be written as

$$V_{in}(t) = L\frac{d(I(t))}{dt} + \sum_{n=1}^{N} \frac{F_n(t)}{C_n} \int I(t)F_n(t)dt \tag{19.3}$$

where N is the number of parallel paths.

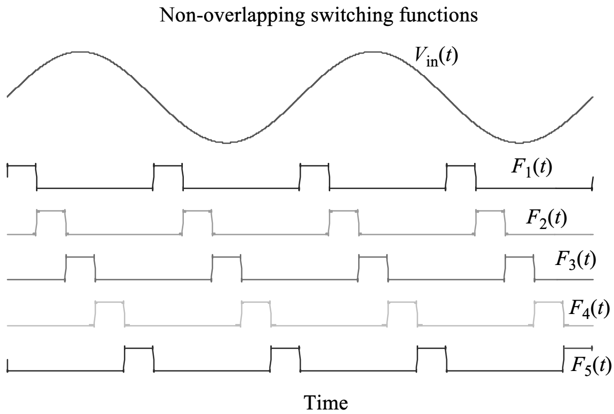


Figure 19.3 Switching functions for the switched capacitor active filters

Expressions (19.2) and (19.3) are used to present a mathematical 'block-diagram' model for the operation of the general circuit in Fig. 19.1. This mathematical 'block-diagram' is illustrated in Fig. 19.2 and is modelled in the following way:

- The line current $I(t)$ is diverted from one branch to another and it undergoes amplitude modulation by the action of the switches S_1, S_2 to S_N and the corresponding switching functions $F_1(t)$, $F_2(t), \ldots, F_N(t)$. This operation is represented by modulators M_{11}, $M_{21}, \ldots, M_{N1}$. The result is the capacitor currents $Ic_1(t)$, $Ic_2(t), \ldots, Ic_N(t)$.
- The capacitor currents are converted to the capacitor voltages $V_{c1}(t)$, $V_{c2}(t), \ldots, V_{cN}(t)$ by the integrators $G_{c1}, G_{c2}, \ldots, G_{cN}$.
- Each capacitor voltage is reflected to the input of the switched network after undergoing amplitude modulation by the action of the switches S_1, $S_2, \ldots, S_N$ and the corresponding switching functions $F_1(t)$, $F_2(t), \ldots, F_N(t)$. This is carried out through any of the modulators M_{12}, $M_{22}, \ldots, M_{N2}$ and then through the adder 'S_1'.
- The voltage across the inductor $V_L(t)$ is then evaluated by subtracting $V_{sw}(t)$ from $V_{in}(t)$, through the adder 'S_2'.
- Finally, the integrator G_L converts $V_L(t)$ back to $I(t)$ and the whole sequence is repeated again.

19.3 The switching functions

In order to secure a path for the current at all times and at the same time avoid connecting two capacitors together of different voltage, Fig. 18.1:

$$\sum_{n=1}^{N} F_n(t) = 1 \tag{19.4}$$

where

$$F_n(t) = K_o + \sum_{x=1}^{\infty} K_n \cos(x\omega t - x\theta) \tag{19.5}$$

The switching function $F_n(t)$ is of the unipolar type as no inversion of voltage takes place. These circuits can be made to generate reactive power at both leading and lagging power factor. The level of the produced reactive power is smoothly controlled by the on period of the switches. Harmonic current can also be produced by the appropriate choice of the switching functions operating the switches and the relative values of L and C.

A number of circuits can be derived from the general model of Fig. 19.1, which are reported elsewhere [3]. Here the double switch double capacitor is presented.

19.4 The line current

The parameters of this expression are time varying and the current flowing can be expressed by

$$I(t) = \sum_{k=1}^{\infty} I_k \cos(k\omega t - \varphi_k) \tag{19.6}$$

$k = mn \pm 1$, where m is the ratio of the switching frequency to the mains frequency and n is a positive integer.

19.5 The double-switch double-capacitor

The double-switch double-capacitor circuit has two branches, Fig. 19.4 and it is derived from the general circuit of Fig. 19.1

$$N = 2$$

$$V_{\text{in}}(t) = L\frac{\mathrm{d}(I(t))}{\mathrm{d}t} + \frac{F_1(t)}{C}\int I(t)F_1(t)\mathrm{d}t + \frac{F_2(t)}{C}\int I(t)F_2(t)\mathrm{d}t \tag{19.7}$$

$F_1(t) = 1 - F_2(t)$ according to Expression (19.4). The two capacitors are set to be of the same value just for simplicity of the calculations.

19.6 Reactive power generation

For reactive power generation the switched part of the circuit must create a voltage component $V_{\text{SW1}}(t)$ at the power frequency and its magnitude must be bigger than

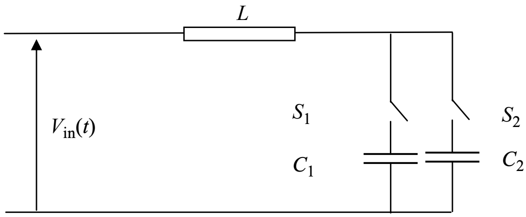


Figure 19.4 The circuit of the double switch double capacitor active filter

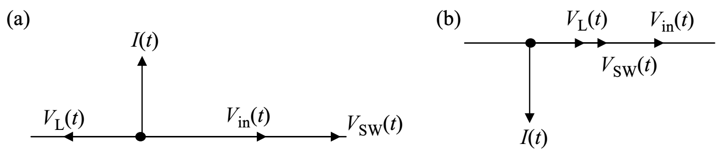


Figure 19.5 Generation of compensating fundamental current: (a) leading; (b) lagging

$V_{\text{in}}(t)$ for leading current, Fig. 19.5(a) or smaller than $V_{\text{in}}(t)$ for lagging current through the inductor, Fig. 19.5(b).

The choice of the switching function must be such as to generate the lowest voltage harmonics across the switched network, $V_{\text{SW}}(t)$. The switched circuit will generate a strong power frequency if the switching function is a series of pulses at high frequency.

For a switching frequency of m times the power frequency, the lowest harmonics in the line current above the fundamental are $m-1$ and $m+1$. Hence the current in the capacitor after the first modulation, Fig. 19.2 will contain also the same harmonics. These current harmonics will produce a harmonic voltage across each capacitor. This harmonic voltage is heavily attenuated if the switching frequency is many times the power frequency, that is, $m \gg 1$. Hence we can safely assume a good quality sine-wave voltage across the capacitor. During the second modulation, the voltage of each capacitor is modulated by the corresponding switching function in order to produce $V_{\text{SW}n}(t)$. This voltage contains $m \pm 1$ harmonics. These harmonics in the two branches are in anti-phase (as their switching functions) and their effect on $V_{\text{SW}}(t)$ is greatly reduced. Hence $V_{\text{SW}}(t)$ can be considered as a good quality sine-wave at 50 Hz with the lowest harmonic components limited to $m \pm 1$ and their magnitude greatly reduced. The line current consists of the fundamental and the same order harmonics, Expression (19.6). It is therefore concluded that the order and magnitude of the line current harmonics are directly controlled by the switching frequency $(m\omega)$ and the value of the circuit parameters L and C. The first two harmonics with the fundamental are considered in Reference 3 and it is shown that they are significantly reduced for a high switching frequency, $m > 50$, Fig. 19.6. Here, in order to demonstrate the simplicity of applying the switching function, we assume a harmonic free current.

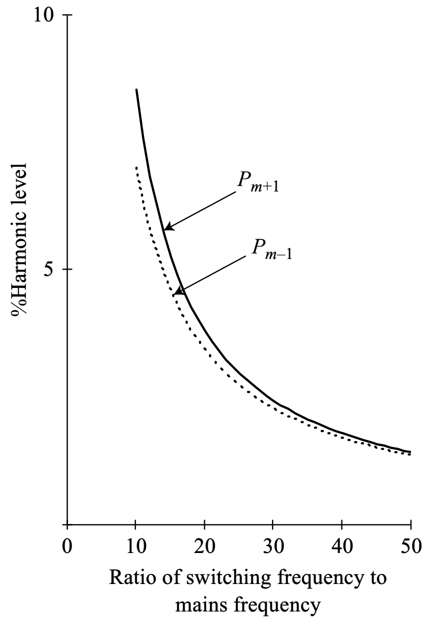


Figure 19.6 Harmonic level of the Class A compensator as function of 'm'

Hence Expression (19.6) is reduced to:

$$I(t) = I_1 \cos(\omega t)$$

For a harmonics free line current the switching functions can also be reduced to their average values, K_o since their higher order harmonics contribute nothing to the fundamental. Expression (19.7) is expanded for these two conditions.

$$V_p \sin(\omega t) = -X_L I_1 \sin(\omega t) + I_1 X_C K_o^2 \sin(\omega t) + I_1 X_C [1 - K_o]^2 \sin(\omega t)$$

Simplifying

$$V_p \sin(\omega t) = I_1 \sin(\omega t)\{-X_L + X_C K_o^2 + X_C[1 - K_o]^2\}$$

The impedance of the circuit at fundamental frequency

$$Z = \frac{V_p}{I_1}$$

Re-arranging

$$Z = X_C - X_L + 2X_C K_o^2 - 2K_o X_C \tag{19.8}$$

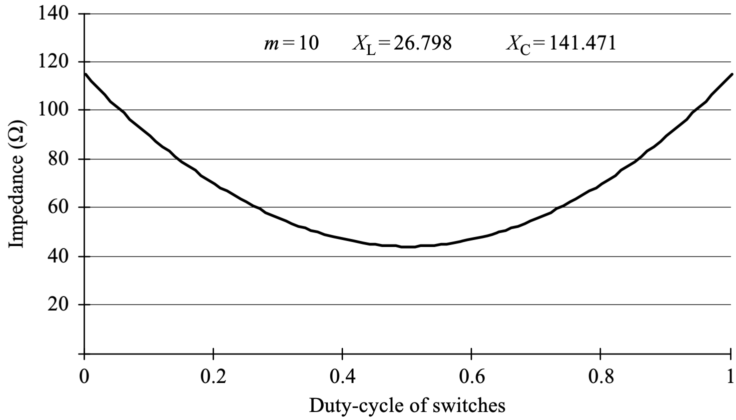


Figure 19.7 The impedance of the circuit against the duty-cycle of the switch, K_o

K_o is the duty-cycle of the switch and it can be set to any value from 0 to 1.

For $K_o = 0$ $\quad Z = X_C - X_L$

For $K_o = 1$ $\quad Z_o = X_C - X_L$

For $K_o = 0.5$ we have $Z_{min} = 0.5X_C - X_L$

Expression (19.8) is used to display the impedance of the circuit against the duty-cycle of the switch, Fig. 19.7.

Chapter 20
The inverter configuration active filter

20.1 Introduction

This is a very well-known configuration; usually in three-phase and there are many references in the literature [12]. It can produce both fundamental reactive current and harmonic current. In this chapter the single phase of the inverter of Chapter 15, Fig. 15.1 is employed to produce reactive power.

20.2 Operation and analysis

The switching pattern of the inverter, Fig. 20.1 is set to generate the required compensating current. The voltage across the capacitor is kept constant to a dc value higher than the peak input voltage; this may be achieved with an extra three-phase rectifier. The relationships of voltage and currents at the terminals of a bridge configuration were discussed in Chapter 1. The voltage on the dc side, V_{dc} is reflected to the ac side, $V_{\mathrm{INV}}(t)$ by the switching function operating the bridge according to Expression (1.4) of Chapter 1; we have

$$V_{\mathrm{INV}}(t) = F_{\mathrm{pwmB}}(t)\,V_{\mathrm{dc}} \tag{20.1}$$

The switching function, $F_{\mathrm{pwmB}}(t)$ takes the form of a sinusoidally pulse width modulated signal given by Expression (3.11) of Chapter 3.

According to Expression (20.1), $V_{\mathrm{INV}}(t)$ is controlled by the switching function, $F_{\mathrm{pwmB}}(t)$ and it can take a magnitude and frequency content as required. By controlling $V_{\mathrm{INV}}(t)$, the current of the required shape can be forced through the inductor into the mains. This current is the compensating current drawn from the supply. Irrespective of the harmonic content of the compensating current to be produced, $V_{\mathrm{INV}}(t)$ must contain a term at fundamental frequency. This component controls the amount of lagging or leading reactive power; the produced reactive power can be set to zero if the fundamental component of $V_{\mathrm{INV}}(t)$ is equal in both phase and magnitude to the input main voltage.

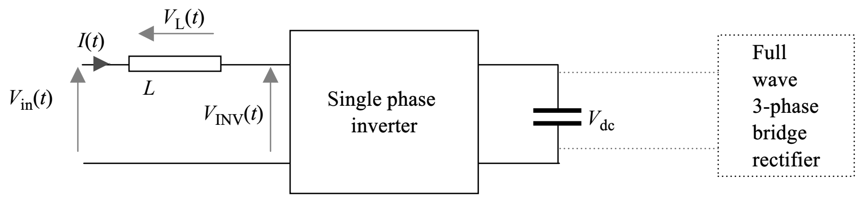


Figure 20.1 The single phase inverter configuration active filter

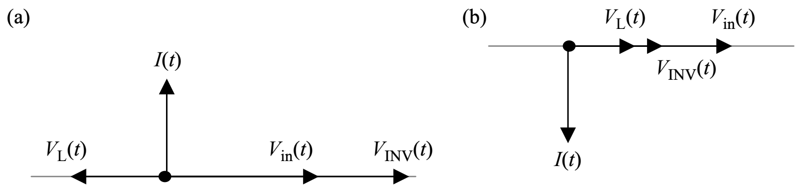


Figure 20.2 Generation of compensating line current: (a) leading and (b) lagging

The input loop voltage equation of this circuit is given by

$$V_{in}(t) = V_L(t) + V_{INV}(t) \tag{20.2}$$

Replacing Expression (20.1) into (20.2)

$$V_{in}(t) = V_L(t) + F_{pwm}(t)V_{dc} \tag{20.3}$$

For reactive power generation the switched part of the circuit must create a voltage $V_{INV}(t)$ which is a pure sinusoid at the power frequency and its magnitude is bigger than $V_{in}(t)$ for leading current, Fig. 20.2(a). $V_{INV}(t)$ must be smaller than $V_{in}(t)$ for lagging current through the inductor, Fig. 20.2(b). At the same time the voltage at the ac side of the inverter, $V_{INV}(t)$ must be a 'clean' sinusoidal voltage. Therefore the switching function must be a sinusoidally PWM signal.

For harmonic current generation (and no reactive power generation) the fundamental component voltage is equal to the mains voltage. Furthermore the modulating function must contain the frequency components for the required harmonic current compensation.

The fundamental voltage at the ac side of the inverter, V_1, is derived from Expression (20.1) by considering the fundamental component of switching function $F_{pwmB}(t)$. It was shown in Chapter 3 that the peak value of the fundamental component of switching function is D, the amplitude modulation ratio.

$$V_1 = V_{dc}D$$

Hence for known values of ac supply voltage and dc voltage at the dc side of the inverter, the fundamental current can be calculated for a given value of L. Also it can be clearly stated if it is leading or lagging. This is demonstrated in Fig. 20.3 and

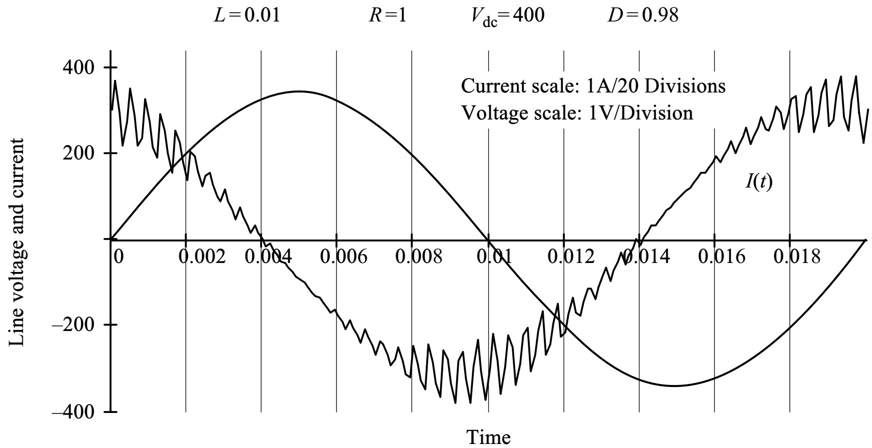


Figure 20.3 Voltage and current, leading

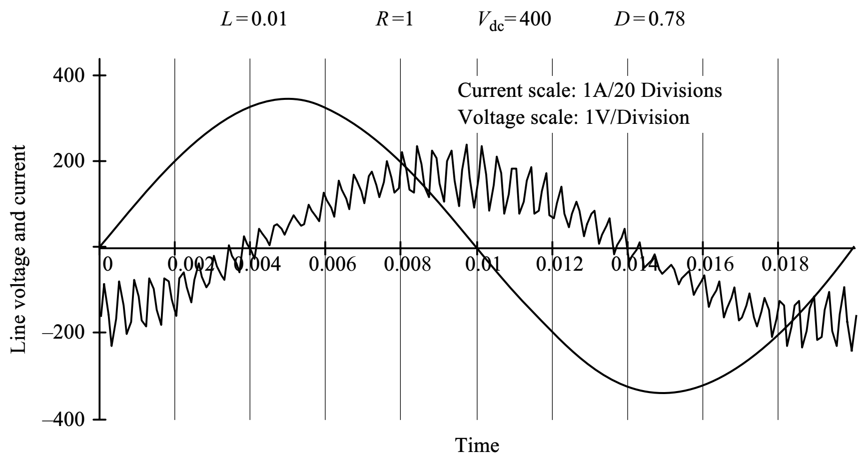


Figure 20.4 Voltage and current, lagging

Fig. 20.4. The switching frequency is chosen to be low in order to show the switching action of the compensator.

For harmonic generation the switching function can be extended to include the necessary harmonic components with independent control of the reactive power.

Chapter 21

Single phase rectification with active line shaping

The current drawn by a rectifier with a smoothing capacitor consists of sharp pulses, Chapter 2. The level of line current distortion is unacceptable. In an attempt to alleviate this problem, a boost dc-to-dc converter is established at the dc side of the rectifier in order to shape the line current into a sinusoid in phase with the supply voltage, Fig. 21.1. The analysis, operation, application and modifications of this circuit are the subject of many published papers using various techniques [15]. In this chapter the circuit is analysed using the switching function technique. The object of this exercise is to derive the modulating signal of the semiconductor switch in order to establish unity power factor with no harmonics. From the modulating signal, the switching function can be derived; this is already done in Part 1, Chapter 3. The derived switching function is then applied in order to calculate the inductor current for a set of circuit parameters (L, R and output dc voltage). The input current, current through the switch and capacitor current are also derived.

The action of the switch is to create a current through the inductor in the shape of a rectified sine wave. This current is reflected to the input via the bridge and its rectifying switching function $F_B(t)$. The reflected current will be a pure sinusoid clean from harmonics and at unity power factor; this is the goal of introducing the boost converter at the dc side of the rectifier.

The gain of this exercise is to show that a switching function exists to make this circuit produce the necessary shape of the line current. Since the switching function is directly related to the action of the semiconductor switch, proof of the existence of the switching function is a good indication that the circuit can be implemented. Of course, this circuit is already implemented and is the subject of many research papers. The method though, can be proved useful in new circuits under investigation.

21.1 Mathematical modelling of the active shaping circuit

The mathematical model includes all the modulation processes within the single phase rectification with active line shaping. It will be used to derive the switching function

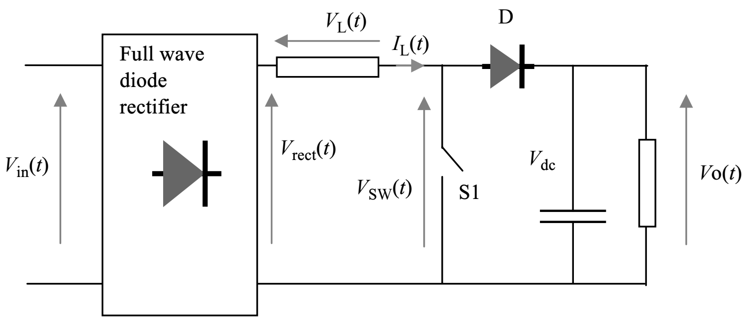


Figure 21.1 The active line current shaping circuit

of the transistor switch, the currents through it and the current through the diode on the dc side.

21.1.1 The modes of operation

The circuit can be operated with discontinuous or continuous inductor current. This analysis is limited to continuous operation but in principle it can be expanded to non-continuous operation. There are two modes, Fig. 21.2. During Mode I the switch is open and during Mode II the switch is closed. A general switching function is introduced, $F_M(t)$, operating on the switch S1.

During Mode I the inductor current flows through the diode and then is shared by the parallel combination of R and C. The output voltage, $V_o(t)$ appears across the switch, $V_{SW}(t)$. By neglecting the diode voltage drop

$$V_{SW}(t) = V_o(t) \quad \text{for } F_M(t) = 0$$

The diode current

$$I_D(t) = I_L(t) \quad \text{for } F_M(t) = 0$$

And the switch current

$$I_{switch}(t) = 0 \quad \text{for } F_M(t) = 0$$

With the switch closed, Mode II, the voltage across the switch is zero, the diode is reverse biased by the capacitor voltage. The capacitor is discharging into the resistance.

$$\begin{aligned} V_{SW}(t) &= 0 & F_M(t) &= 1 \\ I_D(t) &= 0 & F_M(t) &= 1 \\ I_{switch}(t) &= I_L(t) & F_M(t) &= 1 \end{aligned}$$

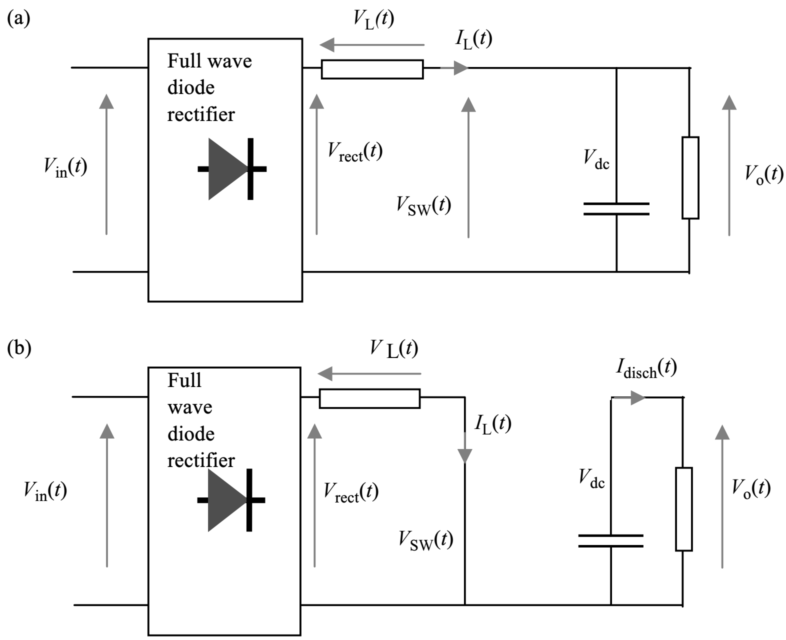


Figure 21.2 Active current shaping circuit: (a) Mode I and (b) Mode II

The unified expressions that are valid at all times and for both modes are derived from the set of expressions associated to each mode. The reader is reminded that the inverse of a switching function is given by Expression (1.3) in Chapter 1 as

$$\overline{F(t)} = 1 - F(t)$$

It follows from the two modes that $V_{SW}(t)$ takes the value of the output voltage, $V_o(t)$ when the switching function $F_M(t)$, is zero or when the inverse of $F_M(t)$ is 1, Mode I and the value of zero during Mode II.

$$V_{SW}(t) = [1 - F_M(t)]V_o(t) \tag{21.1}$$

The switch current $I_{switch}(t)$ takes the value of the inductor current, $I_L(t)$ when the switching function $F_M(t)$, is one.

$$I_{switch}(t) = I_L(t)F_M(t) \tag{21.2}$$

The diode current $I_D(t)$ takes the value of the inductor current, $I_L(t)$ when the switching function $F_M(t)$, is zero or when the inverse of $F_M(t)$ is 1.

$$I_D(t) = [1 - F_M(t)]I_L(t) \tag{21.3}$$

The diode current is shared between the load resistor R and the smoothing capacitor C the way shown in Chapter 2. Expression (2.10)

$$I_C(t) = F_2(t)I_L(t)\frac{\overline{X(\omega n)}}{\overline{X(\omega n)} + R} - I_{dco} \tag{2.10}$$

For large values of C the capacitor current is simplified to

$$I_C(t) = I_D(t) - I_{dco} \tag{21.4}$$

This is based on the assumption that the capacitor is large enough to take practically all the ac components of the current entering the RC combination and the discharging current of the capacitor is approximated to the dc current through the load resistor R.

The inner loop voltage equation for Fig. 21.1

$$V_{rect}(t) = V_L(t) + V_{SW}(t) \tag{21.5}$$

The voltage at the output of the rectifier, $V_{rect}(t)$ is the result of an amplitude modulation of the input voltage, $V_{in}(t)$ and the switching function of the bridge, $F_B(t)$ as

$$V_{rect}(t) = F_B(t)V_{in}(t) \tag{21.6}$$

$V_{rect}(t)$ has an ac and a dc component, $V_{rectAC}(t)$ and V_{rectDC}

The wanted current at the input is a single frequency component at the frequency of the supply voltage and in phase with the input voltage

$$I(t) = I_p \sin \omega t$$

The bridge switching function, $F_B(t)$, is of the transparent type hence Expression (1.9) of Chapter 1 applies. $I(t)$ is reflected to the output via the bridge and according to Expression (1.9)

$$I_L(t) = I(t)F_B(t) \tag{21.7}$$

The output voltage, $V_o(t)$, has a dc component, V_{dco}, and an ac component, $V_{oAC}(t)$

$$V_o(t) = V_{dco} + V_{oAC}(t) \tag{21.8}$$

It is desired that the ac component is as small as possible and the smoothing capacitor, C, is chosen to be of such large value to keep it within acceptable limits. For many practical purposes the ac components may be ignored.

21.1.2 Mathematical model

Expressions (21.1)–(21.8) are employed to build the mathematical model of the active current shaping circuit (Fig. 21.3). Expression (21.1) is implemented by Modulator M2 to give the voltage across the switch, $V_{SW}(t)$. Modulator M1 implements Expression (21.6) to give the output of the diode rectifier, $V_{rect}(t)$. The output of M1 and M2 are fed to adder S1 according to Expression (21.5); the output of S1 is the voltage across the inductor, $V_L(t)$. The inductor current $I_L(t)$ is a reflection of the input current $I(t)$ – Modulator M3. The current through the diode $I_D(t)$ and the current through the switch $I_{switch}(t)$ are given by Modulators M4 and M5 according to Expressions (21.2) and (21.3). The diode current is shared between the smoothing capacitor and the load resistance R according to (2.10). The capacitor current $I_C(t)$ gives rise to a ripple output voltage $V_{oAC}(t)$ through the harmonic impedance $X_C(\omega n)$. Adder S3 adds the dc component across the capacitor V_{dco} to $V_{oAC}(t)$ to give $V_o(t)$.

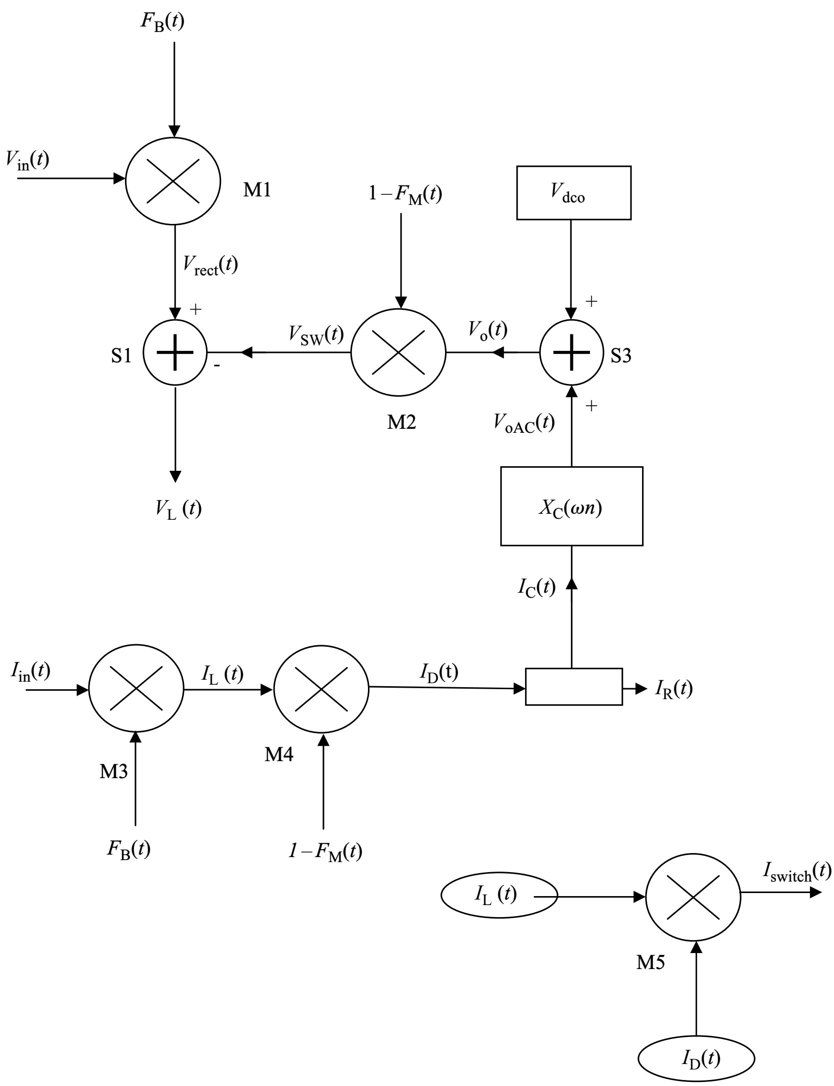


Figure 21.3 The mathematical model of the current shaping circuit

21.1.3 The switching functions

There are two switching functions associated with this circuit. $F_B(t)$ is a bipolar transparent switching function and it represents the action of the bridge. It is a square wave at the same frequency as the input voltage

$$F_B(t) = 4\sum_{n=1}^{\infty} \frac{\sin(n\pi/2)}{n\pi} \cos\left(n\omega t - \frac{n\pi}{2}\right) \qquad \text{for odd values of } n \tag{21.9}$$

The second switching function is associated with the switch at the dc side of the rectifier, $F_M(t)$ and it is of the unipolar type. (Single switches can only be operated by a unipolar type switching function.) It is the goal of this exercise to determine its parameters. $F_M(t)$ has a dc component Ψ_o and an ac component $\Psi(t)$ and it is implemented by PWM.

$$F_M(t) = \Psi_o + \Psi(t) \tag{21.10}$$

21.1.4 Analysis

The goal of the analysis is to derive the switching function of the switch, S1, at the dc side of the rectifier. The wanted switching function $F_M(t)$ is introduced into Expression (21.5) from Expression (21.1). $F_M(t)$ is replaced by Expression (21.10) into its dc and ac components and the ac component of $V_o(t)$ is dropped in view of the fact that it is too small to have any effect on $V_{SW}(t)$.

$$V_{rect}(t) = V_L(t) + [1 - \Psi_o - \Psi(t)]V_{dco} \tag{21.11}$$

The terms of the above expression contain both ac and dc components; hence two new expressions are derived

$$V_{rectAC}(t) = V_L(t) - \Psi(t)V_{dco} \tag{21.11a}$$

$$V_{rectDC} = [1 - \Psi_o]V_{dco} \tag{21.11b}$$

The ac component of the switching function, $\Psi(t)$ is derived from Expression (21.11a)

$$\Psi(t) = \frac{1}{V_{dco}}[V_L(t) - V_{rectAC}(t)] \tag{21.12}$$

The dc component of the switching function, Ψ_o is derived from Expression (21.11b)

$$\Psi_o = 1 - \frac{V_{rectDC}}{V_{dco}} \tag{21.13}$$

The wanted switching function is derived from Expressions (21.10), (21.12) and (21.13) as

$$F_M(t) = \Psi_o + \frac{1}{V_{dco}}[V_L(t) - V_{rectAC}(t)] \tag{21.14}$$

For these Expressions to be of any use, $V_{rectAC}(t)$, V_{rectDC} and $V_L(t)$ have to be derived. As $F_M(t)$ is a pulse width modulated signal, both $V_L(t)$ and $V_{rectAC}(t)$ must be also be converted into a pulse width modulated in Expressions (21.12) and (21.14). This is done in the next section.

21.1.5 Parameters of the switching function $F_M(t)$

In order to derive the parameters of the wanted switching function the modulating signal must be defined. This is derived from Expression 21.14 by considering $V_L(t)$ and $V_{rectAC}(t)$. $V_L(t)$ is the voltage across the inductor supporting a current in the shape of a rectified sine wave. Hence the inductor current is defined as

$$I_L(t) = I_p \sin \omega t \quad \text{for } 0 < \omega t < \pi$$

This is repeated for the period π–2π, 2π–3π ....

The voltage across an inductor is given in general by

$$V_L(t) = L\frac{d[I_L(t)]}{dt}$$

Therefore

$$V_L(t) = I_p \omega L \cos \omega t \quad \text{for } 0 < \omega t < \pi \tag{21.15}$$

This is repeated for the period π–2π, 2π–3π ....

The rectified voltage at the output of the rectifier, $V_{rect}(t)$ is defined as

$$V_p \sin \omega t \quad \text{for } 0 < \omega t < \pi \tag{21.16}$$

Based on Expressions (21.15) and (21.16) the wanted reference signal $R(t)$ can be defined in the range $0 < \omega t < \pi$ as

$$R(t) = -\mathrm{D1} \sin \omega t + \mathrm{D2} \cos \omega t \tag{21.17}$$

where

$$\mathrm{D1} = \frac{V_p}{V_{dco}} \quad \text{and} \quad \mathrm{D2} = \frac{I_p \omega L}{V_{dco}} \tag{21.17a}$$

Expression 21.17 gives the reference signal to be pulse width modulated. The pulse width code of a similar signal is already developed in chapter 3 section 3.3. This is $F_{pwm}C(t)$ (Expression 3.9) and it is based on the reference signal of Expression 3.8. Comparison of Expression 21.17 and Expression 3.8 suggests that the wanted switching function $F_M(t)$ is given by

$$F_M(t) = 1 - F_{pwm}C(t),$$

$\Psi_o = 1 - K_o$ where K_o is given by Expression 3.9.

Where

$$\Psi(t) = -2\sum_{k=1}^{K}\sum_{n=1}^{\infty} K_n\{\cos(n\omega t - n\beta_1) + \cos[n\omega t - n(\pi - \beta_1)]\} \tag{3.9}$$

The inductor current, $I_L(t)$ is a rectified sine wave with a peak value of I_p and it is composed from a dc component, I_{LDC} and an ac component $I_{LAC}(t)$

$$I_L(t) = I_{LDC} + I_{LAC}(t) \tag{21.18}$$

Its average value is given by

$$I_{LDC} = \frac{2I_p}{\pi} \tag{21.18a}$$

21.1.6 Derivation of the switching function

The switching function is a PWM signal in the form of the difference of two sinusoids, Expression (21.14) coded in PWM in the form of Expression (3.9). In order to derive the parameters of the switching function, (D1, D2 and K_o) knowledge of the values of the required output voltage V_{dco}, the input peak voltage V_p, the load resistor and value of L are required. The average value of the switching function K_o is calculated for the required output dc voltage, V_{dco} from Expression (21.13). For a specific ac supply, V_p is known and I_p is derived by equating input and output power.

In order to demonstrate how this analysis can work we derive the parameters of a switching function for a specific set of circuit and load parameters.

Example

The input is 240 V, 50 Hz and the output is set to 400 V for an 80 Ω load. The inductor on the dc side is set to 6 mH. (i) Derive the parameters of the appropriate PWM switching function and display it, (ii) derive and display the input and output currents, (iii) the current through the diode and switch, (iv) the voltages across the diode and switch.

Solution

$$\text{Output dc current, } I_{dco} = 400/80 = 5\,\text{A}$$

$$\text{Output power, } P_o = 400 \times 5 = 2\,\text{kW}$$

Input and output power are equal if losses are neglected; the peak input current is derived as

$$I_p = \frac{2P_o}{V_p}$$

$$I_p = 11.765\,\text{A}$$

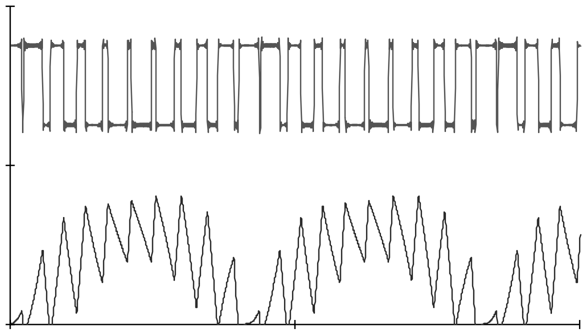


Figure 21.4 *The switching function and the inductor current at a low switching frequency, 1200 Hz*

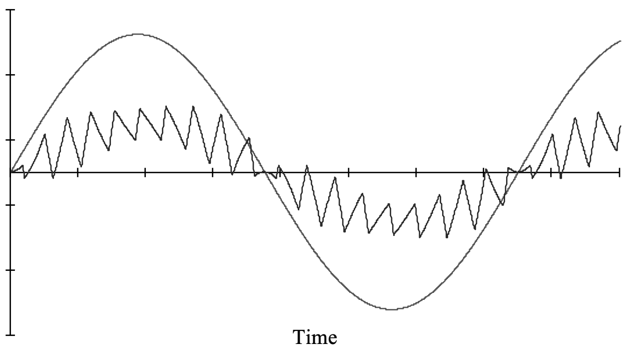


Figure 21.5 *Input voltage and current at low switching frequency, 1200 Hz*

The average value of the inductor current is found from (21.18a)

$$I_{\mathrm{LDC}} = \frac{2I_{\mathrm{p}}}{\pi}$$

$$I_{\mathrm{LDC}} = 7.496\,\mathrm{A}$$

From (21.17a) the coefficients of the modulating function are:

$$\mathrm{D1} = 0.847 \quad \mathrm{D2} = 0.067$$

The switching function for these parameters is given in Fig. 21.4 together with the inductor current. Input voltage and input current are displayed for the same low switching frequency of 1200 Hz. Input voltage and current are displayed in Fig. 21.5 at low switching frequency, 1200 Hz.

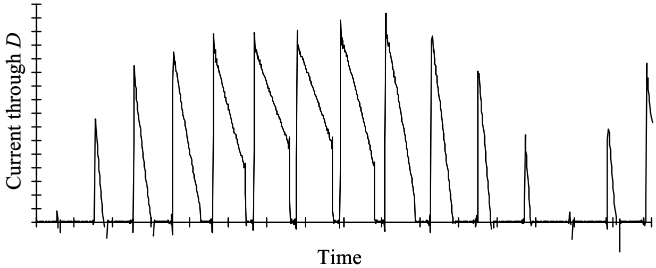


Figure 21.6 Current through the diode

Current through the diode

The inductor current, $I_L(t)$ is a rectified sine wave with a peak value of I_p and it is composed from a dc component, I_{LDC} and an ac component $I_{LAC}(t)$ as indicated in Expression (21.18).

$$I_L(t) = I_{LDC} + I_{LAC}(t)$$

Its average value is given by

$$I_{LDC} = \frac{2I_p}{\pi}$$

For the diode current, $I_D(t)$, Expression (21.3) is expanded by substituting $I_L(t)$ from (21.18).

$$I_D(t) = [1 - K_o]I_{LDC} - \Psi(t)I_{LAC}(t) - [1 - K_o]I_{LAC}(t) - \Psi(t)I_{LDC} \quad (21.19)$$

The first term, $[1 - K_o]I_{LDC}$, is purely dc. The second term, even though it is the product of two ac components, has also a dc component produced when same frequency components are multiplied (intermodulation). Hence the dc component through the diode cannot be approximated to the first term, $[1 - K_o]I_{LDC}$. The dc component through the diode is also the output current through the load resistance; it is the ratio of the output dc voltage V_{dco}, and the load resistance.

$$I_{Ddc} = \frac{V_{dco}}{R} \quad (21.20)$$

The current through the diode is displayed in Fig. 21.6.

The voltage across the diode is

$$V_D(t) = F_M(t)V_{dco}$$

It is displayed in Fig. 21.7.

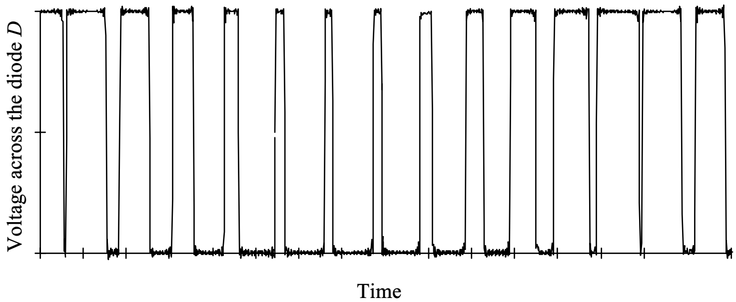


Figure 21.7 *The voltage across the diode*

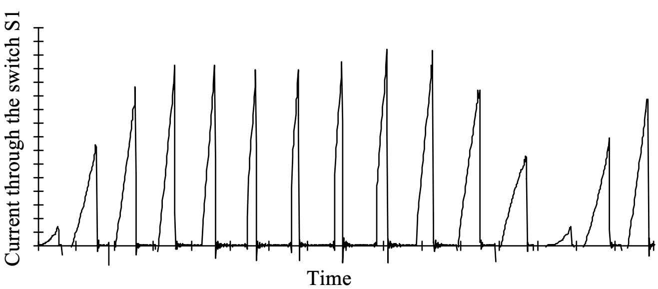


Figure 21.8 *Current through the transistor switch*

Current through the transistor switch

Expanding Expression (21.3)

$$I_{\text{switch}}(t) = K_o I_{\text{LDC}} + \Psi(t) I_{\text{LAC}}(t) + \Psi(t) I_{\text{LDC}} + K_o I_{\text{LAC}}(t) \tag{21.21}$$

For the same reason as that for the diode current the dc component of the current through the transistor switch cannot be the first term of (21.21), $K_o I_{\text{LDC}}$. But the switch dc current $I_{\text{switch DC}}$ is related to the inductor dc current I_{LDC} and the diode dc current, I_{Ddc}, by

$$I_{\text{switchDC}} = I_{\text{LDC}} - I_{\text{Ddc}}$$

The current through the semiconductor switch is displayed in Fig. 21.8. The voltage across the transistor switch is simply $V_{\text{SW}}(t)$ and it is displayed in Fig. 21.9.

RMS and average values of currents

The rms, average and peak values of the currents through the semiconductor switch and the diode are derived from Expressions (21.2), (21.3) and (21.7). The peak value

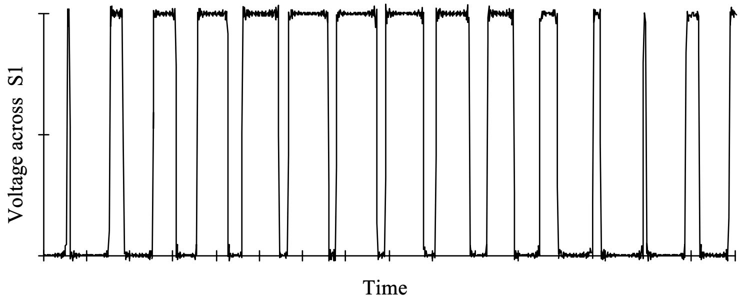


Figure 21.9 Voltage across the transistor switch

of current for both semiconductor devices is the peak value of the input current I_p. The RMS and average values are derived below for the following set of parameters:

$$L = 6 \times 10^{-3} \quad V_{dco} = 400 \quad R = 80 \quad K_o = 0.539$$

RMS values

The RMS of the current through the inductor for the above set of parameters

$$\text{RMS\_}I_L := \sqrt{\frac{1}{T_o}\int_0^{T_o} I_L(t)^2\,dt} \quad \text{RMS\_}I_L = 8.547\text{ A}$$

The RMS of the current through the diode for the above set of parameters

$$\text{RMS\_DIODE} := \sqrt{\frac{1}{T_o}\int_0^{T_o} I_D(t)^2\,dt} \quad \text{RMS\_DIODE} = 7.108\text{ A}$$

The RMS of the current through the transistor switch for the above set of parameters

$$\text{RMS\_SWITCH} := \sqrt{\frac{1}{T_o}\int_0^{T_o} I_{switch}(t)^2\,dt} \quad \text{RMS\_SWITCH} = 4.462\text{ A}$$

Average values

The average value of the current through the inductor for the above set of parameters

$$\text{AVE\_I}_L := \frac{1}{T_o}\int_0^{T_o} I_L(t)\,dt \quad \text{AVE\_I}_L = 7.493\text{ A}$$

The average value of the current through the diode for the above set of parameters

$$\text{AVE\_DIODE} := \frac{1}{T_o}\int_0^{T_o} I_D(t)dt \quad \text{AVE\_DIODE} = 4.988\text{ A}$$

The average value of the current through the transistor switch for the above set of parameters

$$\text{AVE\_SWITCH} := \frac{1}{T_o}\int_0^{T_o} I_{\text{switch}}(t)\,\mathrm{d}t \quad \text{AVE\_SWITCH} = 2.508$$

Output voltage ripple

The effect of the output ripple voltage, $V_{oAC}(t)$, on the voltage across the switch, $V_{SW}(t)$, was ignored on the assumption of a large smoothing capacitor. Nevertheless, the output ripple voltage is there and however small it can be calculated. It is the capacitor current $I_C(t)$ through its harmonic impedance $X_C(\omega n)$ that gives rise to this ripple voltage. The capacitor voltage is given by Expression (21.22).

$$V_{oAC}(t) = X_C(\omega n) I_C(t)$$

Effect of parasitic elements

The effect of the voltage drops across the rectifier diodes, diode of boost converter, semiconductor switch, resistance of the inductor, ESR of capacitor, source impedance are neglected. The mathematical model of Fig. 21.3 can be extended to include these voltage drops at the expense of more tedious calculations.

Discussion

The switching function is applied systematically for the analysis of power electronic circuits. The Kirchoff's Laws and the Superposition theorem are applied by introducing the appropriate switching functions. A procedure is suggested for the effective application of this method for the analysis of all types of power electronic circuits.

The application of the switching function to build PWM signals to represent sine waves and other composite signals resulted in compact expressions that give the order, magnitude and phase of the frequency components in a simple fashion.

The method is limited at the moment for the steady-state response of switched circuits; it is not developed yet to give the response of a power electronic circuit that is just switch-on. But for a system that is already in the steady state and the duty-cycle of the switching function is changed, the present method of analysis will give the new steady-state response. It will not tell us how it went to the new state and how long it took for the change.

Even though most generic power electronic circuits are analysed in this book, the application of this technique is not exhausted yet. The mathematical models can be expanded to include parasitic components such as the ESR of electrolytic capacitors, voltage drops across diodes and semiconductor switches, losses in inductors, source impedances, etc.

In applying this technique some rather new and certainly very interesting features came to light.

'Unified expressions' of voltage and current in switched circuits valid at all times, for all modes are derived.

It is giving the response of power electronic circuits in the steady state of all known circuits including circuits with 'charged capacitors'.

It has 'proved that' in a matrix converter the output frequency is $w(m-1)$ if the modulating frequency is $m\omega$. It can be made to be $w(m+1)$.

For the current shaping circuit of a rectifier with capacitive load, it has shown that a switching function 'exists' to operate the circuit in order to correct the input current to a nice sine-wave, in phase with the supply voltage.

'Compact expressions' are derived for Sinusoidal PWM signals based on the switching function. The order, magnitude and phase of each component are derived directly from the expressions with simple arithmetic.

This book will not attempt to replace or compete with existing books dealing with the operation, analysis and design of Power Electronics; it will 'complement' them with a proposal for a new way of examining circuits and analysing them. There are many choices for the analysis of power electronic circuits. No detailed comparison is done between this method and the existing ones but certainly the major advantage of this method is that it allows the modelling of the converter in a simple fashion, which is actually very close to its true switching nature. This might prove useful to both the researcher of new circuit configurations and the educator.

The researcher can have a simple model of his new circuit, which represents its mechanism of operation and it will allow him to make valid approximations in order to see very quickly if the circuit is behaving the general way he expects it to be.

The educator has a simple way to present to his students the mechanism of operation of complex switched circuits where all the statements regarding their operation are presented in the switching function model of the circuit.

The author of this book believes that this area is open for more research and this book is hoped to be of help to the future researcher.

Many waveforms are produced in this book and many results such as distortion factors are displayed in the form of graphs. They are generated from Mathcad(R) software, courtesy of Mathsoft Engineering & Education, Inc., http://www.mathcad.com. Mathcad is a registered trademark of Mathsoft Engineering & Education, Inc. In producing these waveforms and graphs two parameters determine their quality: the number of harmonics and the time step at which Mathcad calculates. Theoretically the number of harmonics is infinite. For the display of the waveforms in this book 300 harmonics are chosen even though 50 harmonics give a recognisable picture. The x-axis of the displays (usually time) is chosen to give about 200 points in order to enhance the quality. Large numbers of harmonics and small time intervals increase the time taken by Mathcad to produce these waveforms. Mathcad is also employed to derive rms and average values.

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